Macroeconomic Effects of Fiscal Policy

Pedro Batista Maia Gomes

Declaration

I certify that the thesis I have presented for examination for the MPhil/PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

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Abstract

Government spending has several components. The government buys intermediate goods and services from the private sector, it invests in infrastructure, it hires workers and pays them a wage and it also makes transfer and interest payments. While most of the theoretical papers studying the effects of government spending focus on purchases of intermediate goods and services, the main objective of this thesis is to examine two other types of expenditure.

The first part studies the effects of public sector employment and wages through the labour market and their role over the business cycles in a model with search and matching frictions. The first conclusion is that different components of spending can potentially have distinct macroeconomic effects. The second conclusion is that government wages are an important element to achieve efficiency in the labour market. High wages induce too many unemployed to queue for public sector jobs and raise private sector wages, which lowers job creation in the private sector and raises unemployment. Throughout the business cycle it is optimal to have procyclical public sector wages.

The second part is devoted to the study of the role of public sector capital and its interaction with the determination of labour and profit taxation. Over the past 30 years in developed countries we have observed a decline in the corporate tax rate and public investment offset by an increase in the labour income tax and government consumption. I study these trends in an optimal dynamic taxation model where the government also chooses how to allocate spending between government consumption and investment in public capital. I find that the government’s decision of how to allocate spending is not independent of the decision of how to raise taxes. I then discuss several hypotheses that are consistent with the observed trends.

The last part of the thesis gathers two empirical essays on labour market flows and on the determinants of sovereign debt ratings.
Para a minha Vovocas.
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Macroeconomics has traditionally been a divided field. Yet after the *Rational Expectations* revolution led by Lucas, Sargent, Kydland and Prescott and the posterior developments of New-Keynesian models, a new consensus has emerged. As pointed out in recent papers by prominent macroeconomists – for example Mankiw (2006), Blanchard (2008) or Woodford (2009) – this consensus is settled on two pillars: on the one hand the adoption of the methodological tools of Real Business Cycle models, namely the explicit microfoundations and the role of rational expectations; on the other hand the acknowledgment of the role of monetary policy in managing fluctuations in demand and controlling inflation.

In this synthesis there is hardly any room for fiscal policy. This lack of interest in fiscal policy was driven by the belief that the role of demand management should be assigned to the Central Bank and that the governments should do as little as possible. This point was well emphasised by the Nobel Prize winner Paul Krugman in a recent lecture at the London School of Economics.

> Even among the Saltwater economists there was certainly a loss of focus on things that have turned out to be terribly important right now. And just to say: fiscal policy. (...) How much work have we done on fiscal policy? (...) Between 1985 and 2000 there are about 7000 NBER working papers released, of which 5 said anything in their title or abstract about fiscal policy. (...) The whole discussion of fiscal policy essentially disappeared from Macroeconomics.

(Paul Krugman, 10th of June 2009, Lionel Robbins Memorial Lectures)

The objective of this thesis is to study the effects of government spending. However, it does not focus on the demand channels. Instead, its contribution is to analyse the effects of the different components of spending. Most of the theoretical papers studying government spending focus mainly on one of them: purchases of intermediate goods and services. For example, Barro (1990) studies the effects of spending in an endogenous growth model. Baxter and King (1993) examine their effects in a Neo-Classical setting,
Linnemann and Schabert (2003) extend it to the New Keynesian model and Gali, López-Salido, and Vallés (2007) introduce rule-of-thumb agents. All these papers share the approach of considering government spending as goods bought from the private sector. However, this simplification is too restrictive. Government spending has several other components. The government hires workers and pays them a wage, it invests in roads, bridges, infrastructure and it also does transfer and interest payments. Figure 1.1 shows the evolution of each component of government spending in the United States. As we can see, purchases of intermediate goods and services correspond only to 20 percent of the total spending.

The main hypothesis of this thesis is that the different components of spending have distinct impacts on the economy, even in the absence of nominal rigidities. So it is important to understand what distinguishes each type of expenditure and examine their role in achieving efficient outcomes. The first part of the thesis deals with the effects of public sector employment and wages through the labour market and their role over the business cycles. The second part is devoted to the study of the role of public sector capital and its interaction with the determination of labour and profit taxation.

Part I studies the effects of public sector employment and wages. The public sector wage bill represents 30 percent of total government expenditure in the United States. Furthermore, around 16 percent of all employees are working in the public sector. Given its size, it seems natural that part of the transmission mechanism of fiscal policy occurs through the labour market. Chapter 2 sets up a comprehensive yet simple framework to study the macroeconomic effects of public sector employment and wages, and the optimal policy over the business cycle. As the focus of the chapter is to study public sector employment and wages, it seems important to leave the assumption of frictionless labour markets and attempt a more realistic setting. I build a dynamic stochastic general

Figure 1.1: Government expenditure and its components (United States)
equilibrium model with search and matching frictions along the lines of Pissarides (2000) with both public and private sectors.

The main positive conclusion is that the response of the labour market variables to fiscal shocks depends on the type of shock considered. A reduction of separations lowers unemployment, an increase in wages raises it, while hiring more people can increase or decrease unemployment. All shocks raise the wages and crowd out employment in the private sector contrary to shocks in purchases of private goods and services.

The main normative conclusion is that the government's wage policy plays a key role in attaining the efficient allocation. In steady-state, high wages induce too many unemployed to queue for public sector jobs, which lowers job creation in the private sector and raises unemployment. If wages are too low, the government faces recruitment problems. Over the business cycle public sector wages should follow the wages in the private sector. Otherwise, in recessions too many people queue for public sector jobs and in expansions few people apply for them. An acyclical public sector wage policy increases the volatility of business cycles.

I test the empirical validity of the model in two ways. First, still in Chapter 2, using quarterly data I employ Bayesian methods to estimate the model for the United States between 1948 and 2007. I find evidence that the share of unemployed searching for public sector jobs fluctuates over the business cycle and that the government follows a slightly procyclical wage policy.

Chapter 3 tests the model in an alternative way. First, it presents an empirical study on the interaction between public and private sector wage growth for the OECD countries. I have found that in the OECD countries public sector wage growth is driven by expected private sector wage growth, as well as by fiscal conditions. Regarding the private sector wage growth, I find that it is influenced by market variables but also by expected public sector wage growth. I then extend the baseline model to include exogenous growth in the private sector technology and distortionary taxation. I check the validity of the model by running the same regressions on simulated data, where I find very similar coefficients.

The second part of the thesis focuses on the interactions between the revenue and the expenditure side of fiscal policy. Over the past 30 years, most developed countries have experienced opposite trends in several government instruments. On the taxation side, while statutory corporate tax rate has declined, the marginal labour-income tax has steadily risen. On the expenditure side, public investment has declined sharply at the same time as government consumption increased. I argue that these trends are related.

Chapter 4 studies this issue in the context of an optimal dynamic taxation model where the government chooses labour income and profit taxes, as well as decides how
to allocate spending between government consumption and investment in public capital. Public capital is an input of the production function, so it generates economic rents for the firm, rents that can be captured by profit taxation. This creates an interdependency between the government's decisions of how to raise revenue and how to allocate spending. When public capital stock is higher, there are more rents for the firms so the optimal profit tax is higher. Also, if the taxes are higher, particularly the profit tax, the return of public investment in the form of future revenues is higher, so it is optimal for the government to increase investment relative to consumption. One important implication of the interdependence of the two sides of the government problem is that any exogenous factor affecting one of the instruments will have repercussions on the optimal choice of the other three instruments.

In light of the model, I discuss three possible explanations for the trends in the fiscal instruments. One hypothesis is that public capital has become a less important input in the production function. The second explanation is that we are simply observing the transition to the steady state, starting from a low level of public capital. The third explanation is that globalization has simultaneously put a downward pressure on profit taxation (through international tax competition) and increasing pressure in government consumption (through the demand for public transfers).

Chapter 5 explores in detail the last of these hypotheses. It argues that international tax competition, an exogenous event that drives the profit tax rate down, has contributed to the decline of public investment and the consequent reduction of the supply of public capital. To illustrate this, I set up a simpler model with the same properties as the one in Chapter 4 adding an element of tax competition. I then estimate two policy functions for 18 OECD countries. I find that corporate tax rate and public investment are endogenous and that 20 percentage points decline in the corporate tax rate, driven by competition, reduces public investment by 0.5 to 0.9 percent of GDP.

Finally, Part III gathers two empirical essays. In Chapter 6, I study the main facts of the UK labour market worker gross flows using survey data. I document the properties of the flows and transition probabilities between employment, unemployment and inactivity from several angles. I examine conditional transition probabilities, job-to-job flows, employment separations by reason, flows between inactivity and the labour force, flows by education and the differences between the public and the private sector. Although this chapter is not directly related to the main topic of this thesis, I use the information summarised in it in Chapter 3.

Chapter 7 analyses the determinants of sovereign debt ratings. Given the recent worries about sovereign default in several developed countries (The Economist 2010a, 2010b), it seems important to understand which variables the rating agencies look at
when deciding on a country's rating. The main interest in the context of this thesis is to find how government variables, such as fiscal balance or government debt, can affect a country's rating. The main contribution of this chapter is technical. Using linear and ordered response models, I employ a specification that allows the distinction between short and long-run effects of macroeconomic and fiscal variables on a country's rating. The results show that four core variables have a consistent short-run impact on sovereign ratings: the level of GDP per capita, real GDP growth, the public debt level and government balance. Government effectiveness is found to be an important long-run determinant.

This thesis uses a wide range of research approaches. Chapters 2 and 4 use recursive macroeconomic methods and focus mainly on how the government can achieve optimality, which allows us to draw normative conclusions. Chapters 3 and 5 set variants of the baseline models, in order to draw some positive conclusions and guide the empirical study. Most of the empirical work in the thesis uses a panel of OECD countries for a period of 40 years. Panel data methods are also used in Chapter 7. On the other hand, Chapter 6 applies a more descriptive approach to micro data. Each of these chapters is self-contained. Each one has a detailed introduction that includes a discussion of the relevant literature and of its main contributions. In the conclusion of every chapter there is a summary of the main findings and its implications both for research and for economic policy. Chapter 8 suggests some of the potential areas for future research.
Part I

Public sector employment and wages
Chapter 2

Fiscal policy and the labour market: the effects of public sector employment and wages

2.1 Introduction

If you seek advice from a macroeconomist on how to model government consumption, you are likely to hear: government consumption should be modelled as goods bought from the private sector. However, the main component of government consumption is compensation to employees. As shown in Table 2.1, in most OECD countries the public sector wage bill represents between 50 to 60 percent of government consumption expenditures. Government employment is an important aspect of fiscal policy, but it is also a sizable element of the labour market. In OECD economies, between 10 to 30 percent of all employees are working in the public sector. Given its relevance, it seems plausible that part of the transmission mechanism of fiscal policy occurs through the labour market.

The level of employment and wages in the public sector are relevant, not just because of their weight in the economy or in the government budget, but also because they play an important role over the business cycle. Since 2004, the Internet search engine Google releases a weekly index of keyword searches. Figure 2.1 shows the growth rate of keyword searches of “Jobs” and “Government jobs” for the United States, relative to the previous year. From August 2008, as the recession worsened, the number of searches for jobs has increased dramatically, but it is clear that since February 2009,

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1 At least this is the approach taken by most articles that study the aggregate effects of government spending. Barro (1990) studies the effects of productive and unproductive spending in an endogenous growth model. Baxter and King (1993) examine their effects in a Neo-Classical setting, Linnemann and Schabert (2003) extends it to the New Keynesian model and Galí, López-Salido, and Vallés (2007) introduces rule of thumb agents. All these papers share the feature of considering government spending as goods bought from the private sector.
people are turning more towards government jobs. The difference between the growth rates is around 20 percentage points. Repeating the exercise for the United Kingdom gives a similar picture. Indeed, the change in the searching patterns of the unemployed has gained such proportions that it has been noticed by the press. The following quote is particularly insightful regarding its causes:

Wall Street may be losing its luster for new U.S. college graduates who are increasingly looking to the government for jobs that enrich their social conscience, if not their wallet. In the boom years, New York’s financial center lured many of the brightest young stars with the promise of high salaries and bonuses. But the financial crisis has tainted the image of big banks, and with fewer financial jobs available, Uncle Sam may be reaping the benefit. (Reuters, 11th of June 2009)

The quote hints that in the current recession more people are searching for public sector jobs for two reasons. First, as the wages in the private sector have fallen, more people are turning to the public sector where the wages are insulated from the market forces. Second, there are less jobs available in the private sector relative to the public sector. Indeed, as shown in the fourth column of Table 2.1, in all but one country, public sector employment goes up during recessions. These two facts suggest that government employment and wages are important elements in explaining the business cycle fluctuations of unemployment.

Compared to the theoretical research that focuses on government spending as buying part of the production of the economy, the literature that studies the effects of public sector employment and wages is scarce. Finn (1998) finds that in an RBC model with a perfectly competitive labour market, contrary to government purchases of goods and services, the purchase of hours reduces output, employment and investment in the private sector. Cavallo (2005) extends the model to include capital adjustment cost and exogenous growth in technology and Pappa (2009) to allow for nominal rigidities. Both conclude that private sector hours and output go down and real wages go up after an increase in government hours. Ardagna (2007) study the issue in a dynamic general equilibrium model with a unionised labour market. In her setting, an increase in public sector employment, wages or unemployment benefits, raises the wage in the private sector and thus unemployment. Algan, Cahuc, and Zylberberg (2002) in a partial equilibrium version find that, if public sector wages are low, an increase in public sector employment can reduce unemployment.

Looking at this issue in a frictionless labour market framework might be a useful starting point, but as Figure 2.1 shows clearly, to fully understand the transmission
## CHAPTER 2. FISCAL POLICY AND THE LABOUR MARKET

### Table 2.1: Public sector and the labour market

<table>
<thead>
<tr>
<th>Country</th>
<th>Public wage bill (% gov. consumption)</th>
<th>Public Employment (% total employment)</th>
<th>Unemployment rate</th>
<th>Correlation ($u_t, P_t$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>52.2%</td>
<td>14.1%</td>
<td>6.3%</td>
<td>0.51</td>
</tr>
<tr>
<td>Austria</td>
<td>53.4%</td>
<td>13.1%</td>
<td>4.7%</td>
<td>0.34</td>
</tr>
<tr>
<td>Belgium</td>
<td>53.8%</td>
<td>17.9%</td>
<td>6.9%</td>
<td>0.91</td>
</tr>
<tr>
<td>Canada</td>
<td>59.8%</td>
<td>20.5%</td>
<td>6.8%</td>
<td>0.55</td>
</tr>
<tr>
<td>Denmark</td>
<td>67.8%</td>
<td>30.5%</td>
<td>4.4%</td>
<td>0.78</td>
</tr>
<tr>
<td>Finland</td>
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Note: Public wage bill, public employment and unemployment rate refer to the year 2000. The correlation between public sector employment and the unemployment rate is computed from quarterly data (1970 to 2007). Source: OECD.

### Figure 2.1: Growth rate of Google keyword searches in the United States

![Graph](image)

Note: The growth rate of the four-weeks average index of keyword searches, relative the same four weeks in the previous year.
mechanisms of fiscal policy through the labour market it is crucial to model the existing search and matching frictions. There have been some attempts to do it. According to Holmlund and Linden (1993), an increase in public employment has a direct negative effect in unemployment but crowds out private employment due to an increase in wages. But, for all realistic calibrations, the direct effect of reducing unemployment is stronger than the indirect effect through wages. Quadrini and Trigari (2007) examine the impact of public sector employment on business cycle volatility and find that the presence of the public sector increases the volatility of both private and total employment. Hörner, Ngai, and Olivetti (2007) study the effect of turbulence on unemployment when the wages in the public sector are insulated. They conclude that an increase in turbulence induces more unemployed, who are risk averse, to search for jobs in public companies, resulting in higher aggregate unemployment than if the companies were privately managed.

The aim of this work is to provide a comprehensive, yet simple, framework to study the macroeconomic effects of public sector employment and wages, and their role over the business cycle. I build a dynamic stochastic general equilibrium model with search and matching frictions along the lines of Pissarides (2000) with both public and private sectors. The model shares several features with Quadrini and Trigari (2007). One of its main difficulties is the calibration of the friction parameters in the public sector. In order to do it accurately, I explore information from several sources from the United States and the United Kingdom.

As the first stage, I solve the social planner’s problem to find the constrained efficient allocation. I then solve the decentralized equilibrium and determine the public sector wage consistent with the optimal steady-state allocation. The optimal wage premium depends mainly on the differences of the labour market frictions parameters of the public sector relative to the private sector. For the chosen calibration, the optimal wage is 3 percent lower than in the private sector. If the government sets a higher wage, it induces too many unemployed to queue for public sector jobs and raises private sector wages, thus reducing private sector job creation and increasing unemployment. Conversely, if it sets a lower wage, few unemployed want a public sector job and the government faces recruitment problems.

I also examine the properties of the model when subject to technology shocks. The optimal government policy consists of a countercyclical vacancy posting and a procyclical wage. If the public sector wages are acyclical, in recessions they become more attractive relative to the wages in the private sector, inducing more unemployed to queue for public sector jobs. This further dampens job creation in the private sector and amplifies the business cycle. Deviations from the optimal policy can entail significant welfare losses. If, for instance, the public sector wage does not respond to the cycle, unemployment
volatility doubles relative to the scenario under optimal policy.

The model allows us to disaggregate fiscal shocks into wage and employment shocks and the latter into separation and hiring shocks. The response to the three shocks varies. Paying more to public sector workers raises unemployment through two channels. On the one hand, more unemployed direct their search towards the public sector. On the other hand, as it increases the value of unemployment, it spills over to private sector wages. These two channels are also in place under a separation or hiring shock, but they are offset by the direct effect of increasing public sector employment. In general, reducing separations always lowers unemployment, but increasing hiring can have opposite effects on unemployment, depending on the steady-state level of public sector wages. If the wages are high, when the government opens new vacancies it induces many more unemployed to search for these new jobs, enhancing the crowding out effect in the private sector and raising unemployment.

The opposite effects of the different components of fiscal policy is one of the key results of the paper. The extensive empirical literature that evaluates the macroeconomic effects of government spending tends to find mixed effects on private consumption, real wage or private employment. As a consequence, the center of the debate has been on the technical methodology, particularly on the identification of fiscal shocks. I argue that the mixed evidence might be more related to the data, rather than the methodological strategy used. Fiscal shocks can have distinct effects depending on the type of expenditure we are considering: employment, wages, purchases of privately produced goods or government investment. By including all components together, some in particular or using different samples in which the composition of spending has changed, we cannot expect to identify properly one type of fiscal shock. This hypothesis is consistent with evidence from Caldara and Kamps (2008) who, using the same variables and sample, conclude that alternative identification strategies yield similar results.

The argument that the various types of spending can have mixed macroeconomic effects is not new. In their seminal paper, Baxter and King (1993) find that government investment has different quantitative and qualitative effects than government consumption because it affects the marginal productivity of factors. As mentioned above, Finn (1998) find that, contrary to government purchases of goods and services, an increase in government hours reduces output, employment and investment in the private sector. I show that if we disaggregate employment compensation into employment and into per-employee wage, they can have opposite effects on unemployment. To strengthen my argument, I do a simple extension to the model, replacing public sector employment with services bought directly from the private sector. In such an economy, increases in

\[^2\text{See Caldara and Kamps (2008) for an overview.}\]
the government purchase of goods lowers the wage and raises employment in the private sector, contrary to shocks in employment and wages.

Some of the model's results are driven by the assumption that the unemployed direct their search towards the private or the public sector. The purpose of the rest of the paper is to argue that this is a relevant mechanism. First, I review the evidence from microeconometric studies on public sector wages that suggest that individuals self-select into the private or public sector based on the expected wage differential. Then, I employ Bayesian methods to estimate the parameters of the model for the United States, between 1948 and 2007, using quarterly data on: government employment and wages, private sector wages, unemployment rate, job-separation and job-finding rates. I find evidence that the share of unemployed searching for public sector jobs fluctuates over the business cycle. Additionally, the government follows a countercyclical vacancy and a slightly procyclical wage policy.

2.2 Model

2.2.1 General setting

The model is a dynamic stochastic general equilibrium model with public and private sectors. The only rigidities present in the model are due to search and matching frictions. Public sector variables are denoted by the superscript \( g \) while private sector variables are denoted by \( p \). Time is denoted by \( t = 0, 1, 2, \ldots \)

The labour force consists of many individuals \( j \in [0, 1] \). Part of them are unemployed \( u_t \), while the remaining are working either in the public \( l^p_t \) or in the private \( l^g_t \) sectors.

\[ 1 = l^p_t + l^g_t + u_t. \tag{2.1} \]

Total employment is denoted by \( l_t \). The presence of search and matching frictions in the labour market prevents some unemployed from finding jobs. The evolution of employment in both sectors depends on the number of new matches \( m^p_t \) and \( m^g_t \) and on the separations. In each period, jobs are destroyed at constant fraction \( \lambda^i \), potentially different across sectors.

\[ l_{t+1}^i = (1 - \lambda^i)l_t^i + m_t^i, \quad i = p, g. \tag{2.2} \]

The new matches are determined by two Cobb-Douglas matching functions:

\[ m_t^i = \mu^i(u_t^i)\eta^i(v_t^i)^{1-\eta^i}, \quad i = p, g. \tag{2.3} \]

I assume the unemployed choose which sector they want to search in, so \( u_t^i \) represents
the number of unemployed searching in sector \(i\). The vacancies in each sector are denoted by \(v^i_t\). The parameter \(\eta^i\) is the matching elasticity with respect to unemployment and \(\mu^i\) the matching efficiency. An important part of the analysis focuses on the behaviour of the share of unemployed searching for a public sector job, defined as: \(s_t = \frac{u^2_t}{u^i_t}\).

From the matching functions we can define the probabilities of vacancies being filled \(q^i_t\), the job-finding rates conditional on searching in a particular sector \(p^i_t\), and the unconditional job-finding rates \(f^i_t\):

\[
q^i_t = \frac{m^i_t}{v^i_t}, \quad p^i_t = \frac{m^i_t}{u^i_t}, \quad f^i_t = \frac{m^i_t}{u^i_t}, \quad i = p, g.
\]

The assumption of directed search implies that the number of vacancies posted in one sector only affects contemporarily the probability of filling a vacancy in the other sector through the endogenous reaction of \(s_t\).

### 2.2.2 Households

In the presence of unemployment risk we would observe consumption differences across different individuals. Following Merz (1995), I assume all the income of the members is pooled so the private consumption is equalised across members. The household is infinitely-lived and has preferences over private consumption goods, \(c_t\), and public goods \(g_t\). It also has utility from unemployment \(\nu(u_t)\), which captures leisure and home production.

\[
E_t \sum_{t=0}^{\infty} \beta^t[u(c_t, g_t) + \nu(u_t)], \quad (2.4)
\]

where \(\beta \in (0, 1)\) is the discount factor. The budget constraint in period \(t\) is given by:

\[
c_t + B_t = (1 + r_{t-1})B_{t-1} + w^p_t l^p_t + w^g_t l^g_t + \Pi_t, \quad (2.5)
\]

where \(r_{t-1}\) is the real interest rate from period \(t - 1\) to \(t\) and \(B_{t-1}\) are the holdings of one period bonds. \(w^p_t l^p_t\) is the total wage income from the members working in sector \(i\). Finally, \(\Pi_t\) encompasses the lump sum taxes that finance the government’s wage bill and possible transfers from the private sector firms. I assume there are no unemployment benefits.

The household chooses \(c_t\) to maximize the expected utility subject to the sequence of budget constraints, taking the public goods as given. The solution is the Euler equation:

\[
\nu_c(c_t, g_t) = \beta(1 + r_t)E_t[\nu_c(c_{t+1}, g_{t+1})]. \quad (2.6)
\]
2.2.3 Workers

The value of each member to the household depends on their current state. The value of being employed in sector $i$ is given by:

$$W^i_t = w^i_t + E_t \beta_{t,t+1} [(1 - \lambda^i)W^i_{t+1} + \lambda^i U_{t+1}], \ i = p, g,$$

where $\beta_{t,t+k} = \beta^k \frac{\mu c_{t+k} \cdot g_{t+k}}{u_c(c_t, g_t)}$ is the stochastic discount factor. The value of being employed in a sector depends on the current wage, as well as, the continuation value of the job that depends on the separation probability. Under the assumption of directed search, the unemployed are searching for a job either in the private or in the public sector, with value functions given by:

$$U^i_t = \frac{\nu_u(u_t)}{u_c(c_t, g_t)} + E_t \beta_{t,t+1} \left[ p^i_t W^i_{t+1} + (1 - p^i_t) U_{t+1} \right], \ i = p, g. \quad (2.8)$$

Beside the marginal utility from unemployment, the value of being unemployed and searching in a particular sector, depends on the probabilities of finding a job and the value of working in that sector. Optimality implies that there are movements between the two segments that guarantee that there is no additional gain of searching in one sector vis-à-vis the other:

$$U^p_t = U^g_t = U_t. \quad (2.9)$$

This equality determines the share of unemployed searching in each sector. We can re-write it as:

$$\frac{m^p_t E_t[W^p_{t+1} - U_{t+1}]}{(1 - s_t)} = \frac{m^g_t E_t[W^g_{t+1} - U_{t+1}]}{s_t}, \quad (2.10)$$

which implicitly defines $s_t$. An increase in the value of being employed in the public sector, driven either by an increase in the wage or by a decrease in the separation rate, raises $s_t$, until there is no extra gain from searching in that sector. Under the directed search assumption the public sector wage plays a key role in determining $s_t$. If the search was random between sectors, the public sector wage would not affect any variable of the model.

2.2.4 Private sector firms

The representative firm hires labour to produce the private consumption goods. The production function is linear in labour, but part of the resources produced have to be used to pay the cost of posting vacancies $\zeta^p v^p_t$.

$$y_t = a^p_t \rho^p_t - \zeta^p v^p_t. \quad (2.11)$$
CHAPTER 2. FISCAL POLICY AND THE LABOUR MARKET

At time $t$, the level of employment is predetermined and the firm can only control the number of vacancies it posts. The value of opening a vacancy is given by:

$$V_t = E_t \beta_{t+1} [a_t^p J_{t+1} + (1 - q_t^p) V_{t+1}] - \zeta^p,$$

(2.12)

where $J_t$ is the value of a job for the firm, given by:

$$J_t = a_t^p - w_t^p + E_t \beta_{t+1} [(1 - \lambda^p) J_{t+1}].$$

(2.13)

Free entry guarantees that the value of posting a vacancy is zero ($V_t = 0$), so we can combine the two equations into:

$$s^p_t = E_t \beta_{t+1} [a_t^{p+1} - w_t^{p+1} + (1 - \lambda^p) \frac{s^p_t}{q_{t+1}^p}].$$

(2.14)

The condition states that the expected cost of hiring a worker must equal its expected return. The benefit of hiring an extra worker is the discounted value of the expected difference between its marginal productivity and its wage, plus the continuation value, knowing that with a probability $\lambda^p$ the match is destroyed.

Finally, I consider the private sector wage is the outcome of a Nash bargaining between workers and firms. The sharing rule is given by:

$$(1 - b)(W_t^p - U_t) = bJ_t.$$ 

(2.15)

2.2.5 Government

The government produces its goods using a linear technology on labour. As in the private sector, the costs of posting vacancies are deducted from production.

$$g_t = a_t^{p \gamma} - \zeta^p v_t^p.$$ 

(2.16)

The government collects lump sum taxes to finance the wage bill:

$$\tau_t = w_t^{p \gamma} r_t.$$ 

(2.17)

The numeraire of this economy is the private consumption good. As the public good is not sold, it has no actual price. However, there is an implicit relative price given by the marginal rate of substitution. The formulation of the production function (2.16) implies that the cost of recruiting is given in units of the public good. Alternatively, if the cost was included in the budget constraint it would be expressed in units of private consumption.
Finally, the government sets a policy for the sequence of vacancies and wage \( \{v_t^p, w_{t+1}^g\}_{t=0}^{\infty} \). I assume it sets the wage one period in advance, at the time it posts the vacancies. As \( s_t \) is determined based on the expected future wages in the two sectors, the current public sector wage does not affect any variable in the model. There is no time inconsistency problem because, as taxes are lump sum, the government does not gain from setting a current wage different than promised. Throughout the paper I contrast two types of policies: exogenous policies to help us understand the functioning of the model and the transmission mechanisms of fiscal policy and the optimal policy - the one arising from the social planner’s problem.

### 2.2.6 Decentralised equilibrium

**Definition 1** A decentralised equilibrium is a sequence of prices \( \{r_t, w_t^f\}_{t=0}^{\infty} \) such that, given a sequence of government vacancies and wages \( \{v_t^p, w_{t+1}^g\}_{t=0}^{\infty} \), the household chooses a sequence of consumption \( \{c_t\}_{t=0}^{\infty} \), and the fraction of unemployed members searching in the public sector \( s_t \) and firms choose private sector vacancies \( v_t^p \), such that: (i) the household maximises its lifetime utility; (ii) the share of unemployed searching in the public sector is such that the values of searching in the two sectors equalise (equation 2.10); (iii) private sector vacancies satisfy the free entry condition (2.14); (iv) the private wage \( w_t^p \) solves the bargaining condition (2.15); (v) the private goods market clears: \( c_t = y_t \); and (vi) the lump sum taxes \( r_t \) are chosen to balance the government budget (equation 2.17).

### 2.2.7 Social planner’s solution

As a benchmark for analysis, I consider the constrained efficient solution. The social planner’s problem is to maximize the consumers lifetime utility (2.4) subject to the labour market and technology constraints (2.1-2.3, 2.11 and 2.16). The first-order conditions are given by:

\[
\begin{align*}
\frac{\zeta^p}{q_t^f} &= \beta E_t \left\{ \frac{u_c(c_{t+1}, g_{t+1})}{u_c(c_t, g_t)} \left[ (1 - \eta^p)(a^p_{t+1} - \frac{\nu_u(u_{t+1})}{u_c(c_{t+1}, g_{t+1})}) + (1 - \lambda^p) \frac{\zeta^p}{q_{t+1}^f} - \frac{\eta^p \zeta^p v_{t+1}^p}{(1 - s_{t+1})u_{t+1}} \right] \right\}, \\
\frac{\zeta^g}{q_t^g} &= \beta E_t \left\{ \frac{u_g(c_{t+1}, g_{t+1})}{u_g(c_t, g_t)} \left[ (1 - \eta^g)(a^g_{t+1} - \frac{\nu_u(u_{t+1})}{u_g(c_{t+1}, g_{t+1})}) + (1 - \lambda^g) \frac{\zeta^g}{q_{t+1}^g} - \frac{\eta^g \zeta^g v_{t+1}^p}{s_{t+1}u_{t+1}} \right] \right\}, \\
\frac{u_g(c_t, g_t) \zeta^g v_{t+1}^g \eta^g}{(1 - \eta^g) s_t} &= \frac{u_c(c_t, g_t) \zeta^p v_{t+1}^p \eta^p}{(1 - \eta^p)(1 - s_t)}. \tag{2.20}
\end{align*}
\]

Conditions 2.18 and 2.19 describe the optimal private and public sector vacancies. On the left hand side we have the expected cost of hiring an extra worker. The right hand side gives us the marginal social benefit of hiring an additional worker. It consists of its expected marginal productivity minus the utility cost of working, weighted by
CHAPTER 2. FISCAL POLICY AND THE LABOUR MARKET

the matching elasticity with respect to vacancies, plus the continuation value. The last element that enters with a negative sign reflects the fact that hiring an additional worker makes it harder for both sectors to recruit a worker in the future.

The optimal split of the unemployed between sectors, pinned down in (2.20), depends on the marginal utility of consumption of both goods, on the number of vacancies and their cost, and on the matching elasticity with respect to unemployment in both sectors.

2.3 Calibration

To solve the model, I assume a CES utility function in logs, which allows us to address different elasticities of substitution between the two consumption goods. The utility of unemployment is linear.

\[ u(c_t, g_t) = \frac{1}{\gamma} \ln[c_t^\gamma + \zeta g_t^\gamma], \quad \nu(u_t) = \chi u_t. \]

The model is calibrated to match the US economy at a quarterly frequency. The first graph in Figure 2.2 shows the government employment in the United States since 1947. Under the baseline calibration, the steady-state vacancies in the public sector are such that public sector employment corresponds to the sample average i.e. 16 percent of total employment.

The second graph shows the monthly separation rate for the two sectors, taken from the Job Opening and Labour Turnover Survey (JOLTS). The separation rate in the private sector is almost 3 times higher than in the government: 4.3 against 1.5 percent. The last graph plots the new hires of each sector as a share on the total unemployed, a proxy for the job-finding rate. The probability of finding a job in the government sector is only 4.5 percent compared with 62.5 percent in the private sector. To retrieve the quarterly separation rate, I first calculate the aggregate monthly separation rate (0.038) and job-finding rate (0.67). I then compute the quarterly transition probabilities, allowing for multiple transitions within the quarter. I find that an employed person has a 5.3 percent probability of being unemployed in the following quarter. I fix the separation rate in the private and public sectors at 0.06 and 0.03. These values imply an aggregate separation rate close to 0.053 while preserving the difference between the two sectors.

To estimate the matching elasticity with respect to vacancies, I regress for each sector the log of the job-finding rate (the ratio between hires in that sector and unemployment) on the log of tightness (the ratio between job openings in that sector and unemployment).

---

3 I compute these probabilities using the following formulas:

\[ \lambda^m = (\lambda^m)(f^m)(\lambda^m) + (\lambda^m)(1 - f^m)(1 - f^m) + (1 - \lambda^m)(1 - \lambda^m)(\lambda^m) + (1 - \lambda^m)(\lambda^m)(1 - f^m), \]

\[ f^m = (f^m)(\lambda^m)(f^m) + (f^m)(1 - \lambda^m)(1 - \lambda^m) + (1 - f^m)(1 - f^m)(f^m) + (1 - f^m)(f^m)(1 - \lambda^m). \]
PART I. PUBLIC SECTOR EMPLOYMENT AND WAGES

Figure 2.2: Evidence for the United States

Note: The government employment series is taken from the Current Employment Statistics survey (Bureau of Labor Statistics). The grey bars indicate the NBER recession dates. The job-separation and job-finding rates are calculated from the Job Opening and Labour Turnover Survey.

The estimated coefficients are 0.63 for the private sector and 0.79 for the public sector which suggest that vacancies are more important determinants of matches in the public sector.\(^4\) I set the public sector matching elasticity with respect to unemployment, \(\eta^p\), at 0.2 and \(\eta^g\) at 0.5, slightly higher than the estimated value but in line with estimates from the literature (Petrongolo and Pissarides (2001)).

A recent paper by Davis, Faberman, and Haltiwanger (2009) provides some insights into the duration of vacancies by sector. They use JOLTS data to study the behaviour of vacancies and hiring. After adjusting the data, they estimate that the duration of a vacancy is 30 days for the government and 20 days for the private sector. I calibrate the matching efficiency \(\mu^i\) to reproduce these numbers (\(\overline{v}^p = 3.9\) and \(\overline{v}^g = 2.5\)).

The United Kingdom has a unique source of data on recruitment costs. Every year, the Chartered Institute of Personal Development carries out a survey of recruitment practices of around 800 organizations from different sectors: manufacturing and production, private sector services, public sector services and voluntary, community and not-for-profit (CIPD (2009)). The costs of recruiting a worker, which encompass advertising and agency costs, are for the median firm around £4000, corresponding to roughly 8 weeks of the median income in the United Kingdom. On average, these costs are 40 percent lower in the public sector.\(^5\) I take these values as indicative that the cost per hire is lower in the public sector. I consider the cost of posting a vacancy \(c^i\) to be 2 in the private sector and 1.1 in the public sector. Given that the duration of a vacancy is higher in the public sector, these values imply that the average cost of recruiting expressed in the same units is 15 percent lower than in the private sector. Under this calibration, the sum of recruitment costs is

\(^4\)Strictly speaking, these regressions are only correct if the share of unemployed searching in the public sector is constant. However, in Section 2.8 I estimate the structural model and find similar values.

\(^5\)Also, the median firm takes 12 weeks to recruit a new worker while in the public sector it takes 30 percent longer. See appendix for the disaggregated values. Another study by the National Audit Office (2009) that analyses the recruitment practices in the central government finds that it takes 16 weeks to recruit a new worker, costing between £1600 and £2200, which is consistent with the CIPD study.
close to 3 percent of the total labour costs, value found in Russo, Hassink, and Gorter (2005). It also implies that the cost of recruiting per hire equals to around 5 to 7 weeks of wages, which is consistent with the evidence for the United Kingdom and with the study by Boca and Rota (1998).

Estimates of public sector wage premium have proved quite sensitive to the country choice, education and sex of a worker or even the sub-sector of the government. The survey by Gregory and Borland (1999) places the premium between 0 and 10 percent. I set it close to the lower bound, at 2 percent \( (\pi \equiv \frac{w^g}{w^p} = 1.02) \).

The empirical evidence relative to the substitution elasticity between private and government consumption is not conclusive. Evans and Karras (1998) find that private consumption is complement to military expenditure and substitute to non-military expenditure. Fiorito and Kollintzas (2004) disaggregate expenditure into “public goods” (defence, public order, and justice) and “merit goods” (health, education, and other services). They find that “public goods” are substitutes and “merit goods” are complements to private consumption. As it is hard to select one value for \( \gamma \), I consider an elasticity of substitution of 1 \( (\gamma = 0.0) \). In Section 2.7 I discuss the cases where the goods are substitutes \( (\gamma = 0.5) \) and complements \( (\gamma = -0.5) \). The parameter \( \zeta \), that reflects the preference for government services, is chosen such that the optimal level of public sector employment is 0.15.

For the model to satisfy the Hosios condition in the private sector, the worker’s share in the Nash bargaining is set at 0.5. The value of leisure in the utility function is calibrated, such that the unemployment rate in steady-state is 0.06 and implies an outside option equivalent to 42 percent of the average wage. Technology in both sectors is normalised to 1 and the discount factor is set at 0.99. Table 2.2 summarises the baseline calibration and the implied steady-state values for some of the variables.

| \( a^p \) | 1 | \( \eta^p \) | 0.5 | \( \zeta^p \) | 2.0 | \( \mu^p \) | 1.71 | \( \lambda^p \) | 0.06 | \( \bar{y} \) | 0.15 |
| \( a^g \) | 1 | \( \eta^g \) | 0.2 | \( \zeta^g \) | 1.1 | \( \mu^g \) | 1.97 | \( \lambda^g \) | 0.03 | \( \pi \) | 1.02 |
| \( \gamma \) | 0 | \( \zeta \) | 0.18 | \( \chi \) | 0.46 | \( \beta \) | 0.99 | \( b \) | 0.5 |

<table>
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<th>Steady-state variables</th>
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</tr>
<tr>
<td>( \bar{p} )</td>
</tr>
</tbody>
</table>
2.4 Attaining the steady-state constrained efficient allocation

The constrained efficient steady-state allocation consists of a triplet of \( \{v^p, v^g, s\} \). In order to achieve it, the government can post the optimal number of vacancies directly, but it still has to induce an optimal share of the unemployed searching for public sector jobs. The government can do so by choosing an appropriate level of the public sector wage.

**Proposition 1** If the government sets the optimal level of public sector vacancies and sets a wage such that the share of unemployed searching for public sector jobs is optimal then, if the bargaining power of the workers is equal to the matching elasticity with respect to unemployment in the private sector \((b = \eta^p)\), the steady-state level of vacancies in the private sector is optimal.

The proof is in the Appendix. In a one-sector model, a firm's vacancy posting behaviour entails a positive and a negative externality: it increases the probability of an unemployed finding a job but reduces the other firms' probability of filling a vacancy. The decentralised equilibrium is efficient if the share of the surplus of a match that goes to the firm \((1 - b)\) is equal to the importance of vacancies in the matching process \((1 - \eta^p)\), in what is usually called the Hosios condition. When we include the public sector, besides the externalities of public sector vacancies there are also the ones arising from the directed search. If more unemployed search in the public sector, the probability of filling a vacancy is higher in the public sector but lower for private sector firms. What this proposition states is that if the government is able to internalise the externalities in \(v^g, w^g\), the vacancies in the private sector will also be efficient, provided that the Hosios condition is satisfied.

Let us assume the government sets its wage as a premium over the private sector wage: \(\tilde{w}^g = \pi \tilde{w}^p\). Even though we cannot get an analytical solution for the optimal wage ratio, we can find it numerically. Under the baseline calibration the optimal public sector wage is 3% lower than in the private sector. This value depends mainly on the difference between the friction parameters in the public and private sectors. Figure 2.3 shows how the optimal wage ratio varies with the parameters of the public sector.6

When the cost of posting vacancies is lower or when the matching depends more on vacancies (lower \(\eta^p\)), it is more efficient to have more vacancies and fewer unemployed searching in the public sector. In order to induce it, the government should pay less to its workers. When the separation rate decreases or the matching becomes more efficient,

---

6The Appendix shows how the optimal share of unemployed searching in the two sectors, unemployment rate and wages in the two sectors vary with the parameters.
more unemployed turn into the public sector, but it is optimal to have fewer. The private incentive is not efficient and, thus, the government should offer lower wages to correct it.

The optimal wage ratio does not depend on the coefficients of the utility function: $\gamma$ and $\zeta$, but it depends on the disutility of working ($\chi$) and on the productivity of the public sector. Higher $\chi$, raises the value of employment in the private sector relative to the public sector, because people are more likely to have another spell of unemployment there. As it induces more unemployed to search in the private sector, the government needs to offer higher wages to offset it. If government jobs are less productive, the relative cost of posting vacancies is higher because the marginal utility of public sector goods goes up. Although the social planner wants fewer government jobs, it prefers the new matches to be driven by the unemployment side, which requires higher public sector wages.

To investigate the consequences of paying more to public sector employees, I compare the unemployment rate and households’ welfare when the public sector wage is optimal (a gap of 3 percent) with the baseline case (a premium of 2 percent). The unemployment rate which was calibrated to 6 percent in the baseline steady-state, falls to 5 percent when the government sets the optimal wage. This happens because many unemployed that were queuing for public sector jobs, now find it more attractive to search in the private sector (from 20 to 3 percent), boosting job creation. The public sector wage is an important determinant of equilibrium unemployment. In terms of welfare, moving to the optimal wage generates a gain of 0.6 percent of steady-state consumption.
2.5 The effects of fiscal shocks

In this framework there are several fiscal shocks. We can distinguish shocks to wages from shocks to employment. Furthermore, an employment shock can be driven by hirings or by separations. We can represent the fiscal shocks as:

\[
\begin{align*}
\ln(A_t^f) &= \ln(A^f) + \epsilon_t^f, \quad w_t^f = \bar{w}^f, \quad v_t^f = \bar{v}^f; \\
\ln(u_t^f) &= \ln(u^f) + \epsilon_t^u, \quad w_t^f = \bar{w}^f; \\
\ln(w_t^g) &= \ln(w^g) + \epsilon_t^w, \quad l_t^g = \bar{l}^g.
\end{align*}
\]

The shocks \( \epsilon_t^f \) follow and AR(1) process with autocorrelation coefficient of 0.8. We start from the baseline steady-state. I assume that, under a hiring shock, the government holds the public sector wage constant, while under a wage shock it maintains the level of employment constant.\(^7\) Finally, under the separation shock, I consider that both the wage and vacancies are kept at their steady-state level.

Figure 2.4 shows the impulse responses of the variables to a separation rate and a vacancies shock that generate an increase of 6.6 percent of public sector employment, equivalent to 1 percentage point of the labour force. The peak in government employment takes place 10 quarters after the shock. For comparison, I consider a shock to wages of 6.6 percent. In terms of magnitude, they are equivalent to a fiscal stimulus of 1 percent of aggregate income.

Both employment shocks crowd out private sector employment through three channels. First, as there are fewer unemployed available, the cost of hiring an extra worker increases. Second, either because the probability of getting a job is higher or the separation rate is lower, more unemployed search in the public sector, which further reduces the firms’ vacancy-filling probability. Finally, as the overall job-finding probability increases so does the value of being unemployed, which raises the private sector wage through the wage bargaining.

Now, the question is whether the crowding out of private sector employment is partial, or whether it outweighs the increase in public sector employment and raises unemployment. Following the separation rate shock, the unemployment rate declines by 0.2 percentage points, but a vacancies shock raises the unemployment rate by 0.4 percentage points. There are two explanations for these reversed effects. First, an increase in employment through hiring induces many more unemployed to look for public sector jobs,\(^7\)

---

\(^7\)I could alternatively assume that under the wage shock the vacancies are constant. If the government sets the number of vacancies, as more unemployed search for government jobs, public sector employment increases after a wage shock. Under this policy, the shock to wages also incorporates a shock to employment. This does not change qualitatively the results.
rather than if it is done through retention of workers. Under the hiring shock, the share of unemployed searching for public sector jobs goes up by 12 percentage points, but only by 2 percentage points following a separation shock. Additionally, the effect of a vacancy shock on the private sector wage is four times stronger than the shock to separations.

An increase in the public sector wage reduces private sector employment via two channels. On the one hand, the increase of the public sector wage spills over to the private sector, with an elasticity of around 0.05. On the other hand, it induces more unemployed to search for a job in the public sector, which reduces the probability of filling a vacancy for the firms. As a consequence, they posts fewer vacancies and unemployment rises.

All the fiscal shocks raise the private sector wage, even in the presence of a negative wealth effect. As they crowd out private production, they raise the marginal utility of private consumption lowering the relative value of leisure. The increase in the probability of finding a job in the public sector or its value is large enough to offset this effect.
Figure 2.5 compares the response of the unemployment rate to fiscal shocks when we start from the efficient steady-state. With lower steady-state public sector wages, a hiring shock reduces unemployment, as opposed to when we start from the baseline steady-state. When the government opens new vacancies, if the wage rate is high, many more unemployed queue for these positions, thus enhancing the crowding out effect on private sector job creation.

The opposite effect of the different types of fiscal shocks on unemployment is an important result. The vast literature that tries to understand the effects of government spending tends to be inconclusive. Rotemberg and Woodford (1992) find that after a military expenditure shock (both military purchases and employment) real wages go up, but Edelberg, Eichenbaum, and Fisher (1999) and Ramey and Shapiro (1998) find that after a government military purchases shock real wages go down. Blanchard and Perotti (2002), as well as Fatás and Mihov (2001) find that private consumption increases after a government consumption shock but Mountford and Uhlig (2008) and Ramey (2009) report a negative or zero response. Most of the discussion has focused on the technical methodology, particularly on the identification of fiscal shocks. In light of my results, I think the contradictory evidence might not due to methodological issues. Fiscal policy shocks can have different effects depending on the type of expenditure considered. Increasing the wage of all employees by 1 percent is different from increasing employment by 1 percent. The model even suggests that the effects of government employment can be different, depending if the adjustment takes place through hiring or separations.

2.6 Public sector policies and the business cycle

One of the main conclusions of the Real Business Cycle literature is that governments should not pursue active business cycle policies. Although the model is, in essence, a real business cycle model with only real frictions, the policy prescription is quite differ-
ent. Let us examine the effects of a 1 percent negative private technology shock on the economy, under alternative government policies. I again consider an AR(1) shock with autoregressive coefficient of 0.9.

\[ \ln(a_t^p) = \ln(\bar{a}^p) + \epsilon_t^p. \]

Figure 2.6 shows the impulse responses, starting from the efficient steady-state, when the government follows the optimal rule. I contrast the optimal policy with simple rules for vacancies and wage as follows:

\begin{align*}
\log(v_t^p) &= \log(\bar{v}^p) + \psi^v[\log(v_t^p) - \log(\bar{v}^p)], \\
\log(w_{t+1}^p) &= \log(\bar{w}^p) + \psi^w[\log(w_t^p) - \log(\bar{w}^p)].
\end{align*} \tag{2.21}

Existing evidence by Lane (2003) and Lamo, Pérez, and Schuknecht (2008) suggest that public sector wages are less procyclical than private sector wages, particularly in the United States.\footnote{Additionally, a study by Devereux and Hart (2006) using micro data for the United Kingdom finds that for job movers in the private sector the wages are procyclical but for the public sector they are acyclical.} For the sake of simplicity, I consider two cases where the public sector wage is acyclical ($\psi^w = 0$). In the first one, the public sector vacancies decline proportionally to increases in private sector vacancies ($\psi^v = -1$). In the second, they are acyclical ($\psi^v = 0$).

After the negative productivity shock, private sector firms post fewer vacancies, the probability of finding a job there falls and the unemployed increase their search for public sector jobs. The unemployment rate increases at most by 0.05 percentage points, much less than after fiscal shocks. As pointed out by Shimer (2005), search and matching models are not able to generate enough fluctuations on unemployment in response to technology shocks.

The optimal government policy is to have countercyclical vacancies and procyclical wages. The argument for hiring more people in recessions is one of sector reallocation, different from the traditional demand argument (bringing to mind the famous metaphor of digging holes and covering them). If the private sector has lower productivity, it is better for the economy to absorb part of the unused labour force into the public sector. If the government jobs were not productive, it would not be optimal to hire anyone in the first place.

On the other hand, the public sector wage should follow the decline of the private sector wage. In recessions, if the government keeps its wage constant, it becomes more attractive relative to the private sector, thus increasing the share of unemployed searching
Figure 2.6: Response to a private sector technology shock under different policies

![Graphs showing response to private sector technology shock under different policies](image)

Note: Solid line (optimal policy); dash line (countercyclical vacancies and acyclical wages) and dotted line (acyclical vacancies and wages). The response of the variables is in percentage of their steady-state value, except for the unemployment rate and the share of unemployed searching for public sector jobs, which is in percentage points difference from the steady-state.

for public sector jobs. This reduces the vacancy-filling probability in the private sector, which further dampens job creation and amplifies the business cycle. We can see that under the two exogenous rules, the response of unemployment is much stronger. There is an increase of 1.6 percentage points of the share of unemployed searching of public sector jobs, much higher than under the optimal policy (0.02 percentage points).

Table 2.3 compares the standard deviation of the key variables under the alternative policies, as well as when there is no public sector. If the government follows the optimal rule, the presence of public sector employment stabilises unemployment. However, if public sector wages are acyclical the volatility of unemployment increases twofold. The effects of the presence of public sector employment on the volatility of unemployment depends crucially on the government’s business cycle policy. The last column presents the welfare cost of business cycles under the different scenarios. See the Appendix for details.
CHAPTER 2. FISCAL POLICY AND THE LABOUR MARKET

Table 2.3: Business cycle properties under the different policies

<table>
<thead>
<tr>
<th>Policy</th>
<th>Standard deviations</th>
<th>Correl</th>
<th>Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$l_t^p$</td>
<td>$l_t^f$</td>
<td>$u_t$</td>
</tr>
<tr>
<td>No government</td>
<td>0.0007</td>
<td>-</td>
<td>0.0007</td>
</tr>
<tr>
<td>Optimal policy</td>
<td>0.0008</td>
<td>0.0007</td>
<td>0.0006</td>
</tr>
<tr>
<td>Rule ($\psi^u = 0$, $\psi^v = -1$)</td>
<td>0.0159</td>
<td>0.0830</td>
<td>0.0014</td>
</tr>
<tr>
<td>Rule ($\psi^u = 0$, $\psi^v = 0$)</td>
<td>0.0132</td>
<td>0.0673</td>
<td>0.0014</td>
</tr>
</tbody>
</table>

is absent, the welfare cost of fluctuations is very small - around 0.028 percent of steady-state consumption. This is a well known result. When the public sector is present, under the optimal policy, the welfare cost of fluctuations is lower, but if the government wages do not respond to the cycle, it can be up to four times higher.

In their paper, Quadrini and Trigari (2007) have two conclusions contrary to mine. First, that the best policy to stabilize total employment is to have procyclical public sector employment. Second, that the presence of the public sector increases the volatility of unemployment. In their model, the government does not choose their wage optimally. Instead, it sets a wage premium exogenously, which explains the disparity of the conclusions. As we have seen in the previous section, under a high public sector wage premium, after a hiring shock, the crowding out of the private sector employment can be more than complete, resulting in higher unemployment. This switch alters the policy recommendations for government employment.

2.7 Extensions$^{10}$

2.7.1 Government services as goods bought from the private sector

To compare the results with the ones from a typical model of government consumption, I construct an extension where there is no public sector employment ($l_t^f = 0$), but where the government buys its goods from the private sector ($c_t + g_t = y_t$). I am interested in the response to a government consumption shock of 6.6 percent (Figure 2.7) and the optimal response of government consumption to a negative technology shock.

There are three main differences relative to the benchmark model. First, the effects of a fiscal shock on private sector employment and wages are the opposite from the model with public sector employment. The wages go down because the reduction of private consumption raises its marginal utility, lowering the value of unemployment. Because of the direct stimulus, private employment goes up and unemployment goes down.

$^{10}$All figures can be found in the Appendix.
PART I. PUBLIC SECTOR EMPLOYMENT AND WAGES

Figure 2.7: Response to a government consumption shock

Note: The response of the variables is in percentage of their steady-state value, except for the unemployment rate which is shown as percentage points difference from the steady-state.

The second difference is the magnitude of the response of unemployment. A shock of 6.6 percent in government spending only reduces unemployment by 0.008 percentage points. Both technology and government consumption shocks have a small quantitative effect on unemployment. However, as public sector employment or wage shocks strike directly in the labour market, they have a much stronger effect. Finally, the differences are also visible in the optimal business cycle policy. In recessions, the government should buy fewer goods from the private sector, in order to equate the marginal utility of the two goods.

2.7.2 Productive sector public employment

A recent paper by Linnemann (2009) finds, in the context of a VAR, that a government employment shock generates a positive response in the private sector employment. I want to see if this can be generated within the model, if we consider that public sector employment affects the productivity of the private sector. I consider that private sector technology follows:

\[ \ln(\alpha_f^p) = \ln(\bar{\alpha}^p) + \alpha[\ln(\bar{l}_f^p) - \ln(\bar{l}^p)]. \]  

I look at the responses of unemployment and private sector employment to a separation and vacancies shocks for different values of \( \alpha \). For higher levels of \( \alpha \), the crowding out effect on private employment is lower and, therefore, it has a larger negative impact on unemployment. However, even with a value as high as 0.4, the crowding out is still substantial.

2.7.3 Different elasticities of substitution between goods

I have also examined the cases where the goods are substitutes or complements. Regarding employment shocks, the qualitative results do not depend on \( \gamma \), and even quantitatively the differences are small. If the goods are complements, the increase in government services raises the marginal utility of the private good, so the negative effect on private
sector employment is smaller. If they are substitutes, the household reduces more private consumption, leading to a bigger crowding out of private sector employment.

With respect to the optimal business cycle policy, the result of counter-cyclical vacancies is only overturned if the goods are strong complements. If that is the case, during a recession, as the marginal utility of the government services falls with the consumption of the private good, the government should also decrease its vacancies. However, in all scenarios the public sector wage should follow the decline of the private sector wage.

### 2.7.4 Optimal policy under alternative sources of fluctuations

When discussing the optimal policy along the business cycle, I assumed it was generated by technology shocks from which the public sector was isolated. Now, I consider two alternative sources of fluctuations: an aggregate technology shock and a shock to the discount factor.

The result of procyclical wages holds for the two shocks, but the result of counter-cyclical vacancies is reversed. Following an economy-wide technological shock, as the public sector is also less productive, the argument for sector reallocation does not hold and the government should also decrease its vacancies. If people become more impatient, the present discounted value of a vacancy goes down. As it affects both sectors symmetrically, both the private sector and the government should decrease their vacancies and wages.

### 2.8 How important is the directed search between the public and the private sector?

The theoretical model has one important policy prescription: government wages should keep track of the private sector wages over the business cycle. If not, the volatility of unemployment is higher because of the fluctuation of the share of unemployed searching for public sector jobs. It is clear that this result is entirely driven by the directed search assumption. The aim of this section is to show that the assumption is realistic. I begin by presenting some evidence from micro-econometric studies.

As mentioned previously, public sector wage premium varies substantially within groups. As reported in Gregory and Borland (1999), the premium is much higher for females, veterans and minorities, and it is higher for federal government employees compared to state or local government employees. There are also differences across education levels. Katz and Krueger (1991) find that in the two previous decades, more educated individuals tend to be paid less in the public sector, while individuals with less education...
tend to receive a higher premium. If people can direct their search, these differences should have repercussions.

Gregory and Borland (1999) report a number of studies that have found the existence of queues for federal public jobs. For example, Venti (1985) finds that for each federal government job opening, there are 2.8 men and 6.1 times as many women that want the job. Katz and Krueger (1991) find that blue collar workers are willing to queue to obtain public sector jobs, whereas highly-skilled workers are difficult to recruit and retain in the public sector. A recent study for the United Kingdom by Postel-Vinay and Turon (2007) also finds evidence of job queuing for public sector jobs among low-employability individuals, who face larger potential premia from working there.

Most studies that estimate the public sector wage premium use switching regression models. The idea is that the unemployed can self-select to work in the sectors in which they have more advantages. Blank (1985) finds that, among other factors, sectoral choice is influenced by wage comparison. Heitmueller (2006) manages to quantify this effect and finds that an increase of 1 percent in the expected wage in the public sector increases the probability of being employed in that sector by 1.3 for men and 2.9 percent for women.

The micro evidence supports the directed search assumption, but it does not imply that, from a macroeconomic perspective, the mechanism plays a role over the business cycle. In this section, I estimate a log-linearized version of the model using Bayesian methods as in Smets and Wouters (2007) and Sala, Söderström, and Trigari (2008). The main purpose is to evaluate the mechanism of directed search between the two sectors. Additionally, I can also assess the cyclicality of the public sector wages and vacancies, as well as get estimates for some of the key friction parameters.

2.8.1 Estimation preliminaries

In order to test if the share of unemployed searching for public sector jobs fluctuates over the business cycle, I modify the equation determining it (2.10). The log-linearised expression is:

$$\dot{s}_t = \kappa (1 - \bar{s}) E_t (\tilde{z}_t - \tilde{z}_t^{p} - \tilde{m}_t^{p} + \tilde{m}_t^{q}),$$

where \(\tilde{z}_t^{i}\) is the log-deviations from steady-state of \(W_t^{i} - U_t^{i}\). From the original expression, I have added the parameter \(\kappa\) that measures the significance of the mechanism. If it is close to 0, the data does not support the assumption. As in the theoretical section, I assume two rules for public sector wages and vacancies. However, I consider that each
variable responds to a moving average of the private counterpart:

\[
\ln(v^p_t) = \ln(\bar{v}^p) + \psi_v^p \left[ \frac{1}{4} \sum_{i=0}^{3} \ln(v^p_{t-i}) - \ln(\bar{v}^p) \right] + \ln(\omega_v^p),
\]

\[
\ln(w^p_{t+1}) = \ln(\bar{w}^p) + \psi_w^p \left[ \frac{1}{4} \sum_{i=0}^{3} \ln(w^p_{t-i}) - \ln(\bar{w}^p) \right] + \ln(\omega_w^p).
\]

Following one of the extensions, I allow the private sector technology to depend partially on the level of public sector employment, though a coefficient \(\alpha\), to be estimated.

\[
\ln(a^p_t) = \ln(\bar{a}^p) + \alpha (\ln(I^p_t) - \ln(\bar{I}^p)) + \ln(\omega_a^p).
\]

I use US quarterly data from 1948:1 to 2007:1 for 6 variables: unemployment rate, government employment (% of labour force), government per employee real wage, private sector per hour real wage, aggregate job-separation rate and aggregate job-finding rate. The series of government per employee real wage is calculated by dividing the compensation of government workers from the NIPA tables by the government employment. The monthly job-finding and job-separation rates are taken from Shimer (2007) and are transformed into quarterly, by allowing for multiple transitions between the two states within the quarter. All other variables are taken from the Bureau of Labor Statistics.

I include 6 different shocks: government vacancies, government wages, separation rates in both sectors, bargaining power and private sector technology. The variables enter the estimation in demeaned log-differences.\(^\text{11}\)

I calibrate the utility function parameter \(\zeta\) to be equal to 0.18, \(\beta\) to 0.99 and I normalise the technology in both sectors to 1. In each iteration, the steady-state public sector vacancies are set such that, in equilibrium, the employment in the sector is 0.15 while the steady-state public sector wage is set as a premium over the private sector. Instead of establishing the prior over the matching efficiencies, I opt for doing it on the steady-state probability of filling a vacancy \(\bar{q}\).

I assume that the matching elasticities with respect to unemployment, the steady-state bargaining power of the unemployed and the autoregressive coefficients of the shock process have a Beta distribution. I assume that the standard deviations of the shocks have an inverse gamma distribution. All other parameters are assumed to be normally distributed. Given the strong evidence presented in Section 2.3, the prior mean for the separation rates is 0.06 for the private and 0.03 for the public sector. Also, the prior mean

\(^{11}\text{With the exception of the wages, all other variables are stationary. As a robustness check, I have also estimated the model with the stationary variables entering in levels and the wages in demeaned log-differences. The posterior distributions are quite close between the two versions. The results, as well as all the equations of the model in its log-linearized form and the relation of the observable variables to the model's variables can be found in the Appendix.}\)
of $q$ is 3.9 for the private (duration of a vacancy of 20 days) and 2.5 for the public sector (30 days). However, as the matching elasticity in the public sector came from a back-of-the-envelope calculation, I start with the prior that the mean and standard deviation are the same across sectors. The prior distribution of $\kappa$ and of the business cycle policy parameters is centered around 0 with a standard deviation of 0.3.

2.8.2 Results

I estimate the model with Bayesian methods (see An and Schorfheide (2007) for a review). The likelihood function of the model is combined with the prior distribution of the parameters to obtain the posterior distribution. Subsequently, 2,000,000 draws of the posterior are generated with the Metropolis Hastings algorithm, where the step size is chosen such that the acceptance rate is around 1/3. The draws are divided into two chains with different starting values. The first 2,500 draws of each chain are dropped. Given the recent studies alerting for identification problems in DSGE models (Canova and Sala (2009)), I have done estimations with simulated data. Although there are several parameters that are not identified using the data, the main parameters of interest are, namely: $\kappa$, $\psi^w$, $\psi^v$, $\tau^p$, $\tau^q$ and all the parameters of the shock processes.

Table 2.4 reports the prior distribution and the mean, the 5th and the 95th percentile of the posterior distribution of the parameters. The mean of the posterior distribution of $\kappa$ is close 0.5 with a 90 percent interval between 0.4 and 0.6. This suggests that, although $s_t$ does not fluctuate as much as the model predicts, the mechanism still has explanatory power. With respect to the policy, there is a strong countercyclical policy in vacancies with an estimated mean close to $-0.9$. Public sector wage policy seems slightly procyclical, with the posterior mean of $\psi^w$ of around 0.4.

The elasticity of the matching function with respect to unemployment is much lower in the public sector. The posterior mean for the private sector is around 0.65, but only 0.16 in the public sector. The steady-state vacancy filling probability in both sectors do not seem to be identified, as well as the cost of posting vacancies in the public sector and the elasticity of substitution between the two consumption goods. The posterior mean of the cost of posting vacancies in the private sector is around 1.6. The posterior distributions of both separation rates are very similar, both centered at 0.015.

The posterior mean of the flow value of unemployment is around 0.35, while of the bargaining power is around 0.6. The posterior distribution of $\alpha$ is centered around 0.15. This value suggests that public employment might increase the productivity of the private sector or, alternatively, it might be capturing demand effects that are absent from the model.\footnote{I have also estimated the model for three subsamples of roughly 20 years: 1948:1 to 1967:3, 1967:4 to 1983:4, and 1984:1 to 2005:4.}

\footnote{I have also estimated the model for three subsamples of roughly 20 years: 1948:1 to 1967:3, 1967:4 to 1983:4, and 1984:1 to 2005:4.}
### Table 2.4: Prior and posterior distribution of structural parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
<th>Mean</th>
<th>5th-95th Percentil</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Elasticity of substitution</strong></td>
<td>$\gamma$ Normal (0, 0.1)</td>
<td>0.014 (-0.129, 0.146)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Utility of unemployment</strong></td>
<td>$\chi$ Normal (0.5, 0.1)</td>
<td>0.352 (0.245, 0.470)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Separation rate (private sector)</strong></td>
<td>$\lambda^p$ Normal (0.06, 0.01)</td>
<td>0.016 (0.010, 0.023)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Separation rate (public sector)</strong></td>
<td>$\lambda^g$ Normal (0.03, 0.01)</td>
<td>0.015 (0.011, 0.019)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Cost of posting vacancy (private sector)</strong></td>
<td>$\zeta^p$ Normal (2, 0.3)</td>
<td>1.628 (1.136, 2.121)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Cost of posting vacancy (public sector)</strong></td>
<td>$\zeta^g$ Normal (1.1, 0.2)</td>
<td>1.200 (0.886, 1.512)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Vacancy filling probability (private sector)</strong></td>
<td>$\varphi^p$ Normal (3.9, 0.2)</td>
<td>3.989 (3.700, 4.300)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Vacancy filling probability (public sector)</strong></td>
<td>$\varphi^g$ Normal (2.5, 0.2)</td>
<td>2.486 (2.152, 2.796)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Matching elasticity w.r.t unemp. (private)</strong></td>
<td>$\eta^p$ Beta (0.5, 0.15)</td>
<td>0.647 (0.560, 0.753)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Matching elasticity w.r.t unemp. (public)</strong></td>
<td>$\eta^g$ Beta (0.5, 0.15)</td>
<td>0.159 (0.060, 0.258)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Bargaining power</strong></td>
<td>$b$ Beta (0.5, 0.10)</td>
<td>0.638 (0.537, 0.735)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Public sector wage premium</strong></td>
<td>$\pi$ Normal (1.02, 0.01)</td>
<td>1.031 (1.017, 1.043)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Productivity of public employment</strong></td>
<td>$\alpha$ Normal (0, 0.1)</td>
<td>0.167 (0.082, 0.249)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Cyclicity of public sector wages</strong></td>
<td>$\psi^w$ Normal (0, 0.3)</td>
<td>0.428 (0.165, 0.685)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Cyclicity of public sector vacancies</strong></td>
<td>$\psi^v$ Normal (0, 0.3)</td>
<td>-0.937 (-1.225, -0.656)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Importance of directed search</strong></td>
<td>$\kappa$ Normal (0, 0.3)</td>
<td>0.479 (0.381, 0.579)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Autoregressive parameters</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Productivity</strong></td>
<td>$\rho^a$ Beta (0.5, 0.15)</td>
<td>0.987 (0.980, 0.994)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Public sector wage</strong></td>
<td>$\rho^w$ Beta (0.5, 0.15)</td>
<td>0.973 (0.956, 0.990)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Public sector vacancies</strong></td>
<td>$\rho^v$ Beta (0.5, 0.15)</td>
<td>0.281 (0.182, 0.383)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Private sector separation rate</strong></td>
<td>$\rho^p$ Beta (0.5, 0.15)</td>
<td>0.952 (0.917, 0.988)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Public sector separation rate</strong></td>
<td>$\rho^g$ Beta (0.5, 0.15)</td>
<td>0.500 (0.265, 0.768)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Bargaining power</strong></td>
<td>$\rho^b$ Beta (0.5, 0.15)</td>
<td>0.936 (0.892, 0.977)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Standard deviations</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Productivity</strong></td>
<td>$\sigma^a$ IGamma (0.01, 0.15)</td>
<td>0.008 (0.007, 0.008)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Public sector wage</strong></td>
<td>$\sigma^w$ IGamma (0.01, 0.15)</td>
<td>0.011 (0.010, 0.012)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Public sector vacancies</strong></td>
<td>$\sigma^v$ IGamma (0.01, 0.15)</td>
<td>0.439 (0.305, 0.554)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Private sector separation rate</strong></td>
<td>$\sigma^p$ IGamma (0.01, 0.15)</td>
<td>0.071 (0.064, 0.077)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Public sector separation rate</strong></td>
<td>$\sigma^g$ IGamma (0.01, 0.15)</td>
<td>0.011 (0.002, 0.021)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Bargaining power</strong></td>
<td>$\sigma^b$ IGamma (0.01, 0.15)</td>
<td>0.033 (0.017, 0.056)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### 2.8.3 Model comparison

To show how the model with directed search performs, I compare it with two alternative models: one where there are no fluctuations in the share of unemployed searching in the public sector ($\kappa = 0$) and the other with random search, where the new matches depend only on the relative number of vacancies. I compare them from two angles. First, to see how well they explain the variables used in the estimation, I compare the logarithms of the marginal data density, computed using the Mean-Harmonic Estimator. An alternative way to compare the models is to look at the predictions for an unobserved

---

13Details in the Appendix.
PART I. PUBLIC SECTOR EMPLOYMENT AND WAGES

Table 2.5: Model Comparison

<table>
<thead>
<tr>
<th>Model</th>
<th>Marginal Density</th>
<th>Prediction of vacancies</th>
<th>Prediction of tightness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\text{std}(v^e)$</td>
<td>$\text{std}(v^d)$</td>
<td>Correlation $v^e, v^d$</td>
</tr>
<tr>
<td>Directed search</td>
<td>3185.0</td>
<td>0.90</td>
<td>0.52</td>
</tr>
<tr>
<td>Directed search ($\kappa = 0$)</td>
<td>3154.0</td>
<td>0.90</td>
<td>0.49</td>
</tr>
<tr>
<td>Random search</td>
<td>3151.0</td>
<td>0.79</td>
<td>0.68</td>
</tr>
</tbody>
</table>

Note: $v$ represents private sector vacancies. The superscript $e$ refers to the estimated series and $d$ refers to the data (Help-wanted index).

variable. I use the Help-Wanted index as a proxy for private sector vacancies and look at its correlation with the predicted series, as well as compare the volatility of the two series. I also compare the prediction for labour market tightness. The results are shown in Table 2.5.

The marginal data density is higher for the directed search model. The numbers imply that we would need a prior probability over the directed search model parameters $5.8 \times 10^{14}(= \exp(3185 - 3151))$ times larger than our prior over the random search model in order to reject the fact that the share of unemployed searching for public sector jobs fluctuates in response to shocks. The Random search model does slightly better than the Directed search model in terms of the correlation of predicted vacancies with the actual values, but it does worse in predicting the volatility of vacancies, as well as the moments of labour market tightness.

2.9 Conclusion

This chapter examined the links between the public and the private sector through the labour market. I have built a dynamic stochastic general equilibrium model with search and matching frictions to analyse the effects of fiscal shocks, as well as to understand the optimal employment and wage policy.

The main normative conclusion is that the government's wage policy plays a key role in attaining the efficient allocation. In steady-state, the optimal public sector wage premium should reflect differences in the labour market friction parameters. Under the baseline calibration, the optimal wage is 3 percent below the private sector. In reality, all studies point to the existence of a wage premium in the public sector. Although there are other reasons for governments to set higher wages, namely to induce effort or to avoid corruption, they should weight its costs in terms of inefficiency in the labour market.

Over the business cycle, public sector wages should follow the wages in the private sector. Otherwise, in recessions too many people queue for public sector jobs and in expansions few people apply for them. Although I have abstracted from financing issues,
a procyclical public sector wage policy has the advantage of requiring a lower tax burden in recessions. However, it also has problems. First, lowering public sector wages in recessions might be politically difficult to implement. Yet, to achieve efficiency in the labour market the only relevant wages are those of the new hires which are potentially easier to reduce in recessions. The second problem related to implementation is that wages in the public sector are usually decided annually. One possible solution is to index the wage growth in the public sector to private sector wage growth. Finally, I have ignored the insurance role of the government. If agents are risk averse, they would prefer to have a constant income profile throughout the business cycle, which is an argument for acyclical wages. While this is a valid line of reasoning, one has to realise that the intertemporal insurance is achieved at the cost of stronger fluctuations in unemployment.

Additionally, the baseline model suggests that it is optimal to have a \textit{leaning-against-the-wind} vacancies policy. While the result of procyclical wages is very robust, the result of counter-cyclical vacancies should be interpreted with caution because it does not hold in a number of settings: if the goods are complements, if the shocks affect both sectors symmetrically or if the steady-state public sector wage is higher than optimum.

The main positive conclusion is that the response of the labour market variables to fiscal shocks depends on the type of shock considered. A reduction of separations lowers unemployment, an increase in wages raises it, while hiring more people can increase or decrease unemployment. All shocks raise the wage and crowd out employment in the private sector. Conversely, when the government buys goods from the private sector, a fiscal shock lowers the wage and increases employment in the private sector. The mixed effects of the different components of government consumption on the labour market might be one reason why many empirical studies on the effects of government spending find ambiguous results.

Many of the model’s results rely on the assumption that the unemployed can direct their search between the private and the public sector. I believe that this mechanism is playing a significant role during the current recession. A casual look through the newspapers gives the impression that the unemployed are turning to the public sector for jobs, but also that the wages there have not suffered as much as in the private sector. Although governments were praised for their reactions against the economic crisis, they can still do better.
PART I. PUBLIC SECTOR EMPLOYMENT AND WAGES

2.10 Appendix

2.10.1 Data

<table>
<thead>
<tr>
<th>Table A2.1: Data - CIPD</th>
<th>All sectors</th>
<th>Manufacturing and production</th>
<th>Private sector services</th>
<th>Public services</th>
<th>Voluntary and not-for-profit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cost of recruiting (£)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Senior managers</td>
<td>15123</td>
<td>13396</td>
<td>18964</td>
<td>10452</td>
<td>8534</td>
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<tr>
<td>Managers and professionals</td>
<td>9738</td>
<td>8050</td>
<td>12393</td>
<td>6067</td>
<td>6471</td>
</tr>
<tr>
<td>Administrative, secretarial and technical</td>
<td>4519</td>
<td>3680</td>
<td>5628</td>
<td>1935</td>
<td>4976</td>
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<tr>
<td>Services (costumer, personal and sales)</td>
<td>8996</td>
<td>4565</td>
<td>13980</td>
<td>2327</td>
<td>1399</td>
</tr>
<tr>
<td>Manual, craft workers</td>
<td>2381</td>
<td>2498</td>
<td>2978</td>
<td>1898</td>
<td>1379</td>
</tr>
<tr>
<td><strong>Time to fill a vacancy (weeks)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Senior managers</td>
<td>17.1</td>
<td>16.8</td>
<td>16.5</td>
<td>18</td>
<td>16.6</td>
</tr>
<tr>
<td>Managers and professionals</td>
<td>12.5</td>
<td>12.1</td>
<td>11.8</td>
<td>14.3</td>
<td>11.8</td>
</tr>
<tr>
<td>Administrative, secretarial and technical</td>
<td>6.5</td>
<td>6.0</td>
<td>7.1</td>
<td>9.1</td>
<td>7.1</td>
</tr>
<tr>
<td>Services (costumer, personal and sales)</td>
<td>7.0</td>
<td>6.7</td>
<td>5.6</td>
<td>9.9</td>
<td>7.4</td>
</tr>
<tr>
<td>Manual, craft workers</td>
<td>5.9</td>
<td>5.2</td>
<td>4.5</td>
<td>8.3</td>
<td>6.3</td>
</tr>
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<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition and source</th>
<th>Availability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l^p$</td>
<td>Government employment</td>
<td>1939q1-2008q3</td>
</tr>
<tr>
<td>$w^p$</td>
<td>Government per employee real wage</td>
<td>Compensation per hour (BLS)</td>
</tr>
<tr>
<td>$u^p$</td>
<td>Business sector hourly real wage</td>
<td>Business Sector: Compensation Per Hour (BLS)</td>
</tr>
<tr>
<td>$v^p$</td>
<td>Unemployment rate</td>
<td>Civilian Unemployment Rate (BLS)</td>
</tr>
<tr>
<td>$v^p$</td>
<td>Vacancies</td>
<td>Index of Help Wanted Advertising in Newspapers (The Conference Board)</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>Separation rate</td>
<td>Job-separation rate (Shimer, own calculation for quarterly aggregation)</td>
</tr>
<tr>
<td>$f$</td>
<td>Job-finding rate</td>
<td>Job-finding rate (Shimer, own calculation for quarterly aggregation)</td>
</tr>
</tbody>
</table>
Figure A2.1: Looking at the data

- Government Employment (% of labour force)
- Unemployment Rate
- Government per employee nominal wage
- Business sector hourly nominal wage
- Business sector hourly real wage
- Business sector hourly real wage
- Vacancies (Help wanted index)
- Quarterly job-separation rate
- Quarterly job-finding rate

Figure A2.2: Growth rate of Google keyword searches in the United Kingdom

Note: The growth rate of the four-weeks average index of keyword searches, relative the same four weeks in the previous year.
2.10.2 Steady-state optimal wages, search and unemployment

Figures A2.3, A2.4 and A2.5 show the optimal steady-state wages in both sectors, the share of unemployed searching in the public sector and the unemployment rate as a function of the labour market friction parameters in the public sector, as well as the technology and utility function parameters.

Note: The solid line is the optimal public sector wage and the dash line is the optimal private sector wages.
Figure A2.4: Optimal steady-state share of unemployed searching in the public sector
Figure A2.5: Optimal steady-state unemployment rate
2.10.3 Extensions

**Gov. services as goods bought from the private sector**

Figure A2.6: Optimal policy with government consumption

Note: The response of the variables is in percentage of their steady-state value, except for unemployment rate which is in percentage points difference from steady-state.

**Productive sector public employment**

Figure A2.7: Response to a public employment shock

Note: Solid line ($\alpha = 0.0$); dash line ($\alpha = 0.2$) and dotted line ($\alpha = 0.4$). The response of the private employment is in percentage of its steady-state value and unemployment rate in percentage points difference from steady-state.
PART I. PUBLIC SECTOR EMPLOYMENT AND WAGES

DIFFERENT ELASTICITIES OF SUBSTITUTION BETWEEN GOODS

As it is hard to select one value for $\gamma$, I distinguish three cases: if the goods are substitutes ($\gamma = 0.5$), complements ($\gamma = -0.5$) and one where the elasticity of substitution of 1 ($\gamma = 0.0$). Figures A2.8, A2.9 and A2.10 show the impulse responses to the different fiscal shocks, for different levels of $\gamma$.

Figure A2.8: Response to a public sector wage shock (Baseline steady-state)

Note: Solid line ($\gamma = 0.0$); dash line ($\gamma = 0.5$) and dotted line ($\gamma = -0.5$). The response of the variables is in percentage of their steady-state value, except for the unemployment rate and the share of unemployed searching for public sector jobs, which is in percentage points difference from the steady-state.
Figure A2.9: Response to a public sector vacancies shock (Baseline steady-state)

Note: Solid line ($\gamma = 0.0$); dash line ($\gamma = 0.5$) and dotted line ($\gamma = -0.5$). The response of the variables is in percentage of their steady-state value, except for the unemployment rate and the share of unemployed searching for public sector jobs, which is in percentage points difference from the steady-state.
Figure A2.10: Response to a public sector separation shock (Baseline steady-state)

Note: Solid line ($\gamma = 0.0$); dash line ($\gamma = 0.5$) and dotted line ($\gamma = -0.5$). The response of the variables is in percentage of their steady-state value, except for the unemployment rate and the share of unemployed searching for public sector jobs, which is in percentage points difference from the steady-state.
Finally, Figure A2.11 shows the optimal business cycle policy for different elasticities. As we see the result of countercyclical vacancies does not hold if the private and public consumption good are complements.

Figure A2.11: Optimal business cycle policy under different elasticities

Note: Solid line ($\gamma = 0.0$); dash line ($\gamma = 0.5$) and dotted line ($\gamma = -0.5$). The response of the variables is in percentage of their steady-state value, except for the unemployment rate and the share of unemployed searching for public sector jobs, which is in percentage points difference from the steady-state.
Optimal policy under alternative sources of fluctuations

Figure A2.12: Optimal policy under an economy-wide technology shock

Note: The response of the variables is in percentage of their steady-state value, except for unemployment rate which is in percentage points difference from steady-state.

Figure A2.13: Optimal policy under a discount factor shock

Note: The response of the variables is in percentage of their steady-state value, except for unemployment rate which is in percentage points difference from steady-state.
2.10.4 Derivations

SOCIAL PLANNER'S PROBLEM

The social planner maximises the consumer’s utility (2.4) subject to the technology constraints (2.16) and (2.11) and the labour market frictions (2.1)-(2.3). Setting up the Lagrangian:

\[
\sum_{k=0}^{\infty} \beta^{t+k} \left\{ u(c_{t+k}^{P}, p_{t+k}^{P}, \alpha_{t+k}^{P}, p_{t+k}^{Q}, \alpha_{t+k}^{Q}, \lambda_{t+k}^{P}, \lambda_{t+k}^{Q}, \eta_{t+k}^{P}, \eta_{t+k}^{Q}) + \nu(1 - l_{t+k}^{P} - l_{t+k}^{Q}) 
- \Omega_{t+k}^{P} l_{t+k+1}^{P} - (1 - \lambda_{t+k}^{P}) l_{t+k}^{Q} - \mu_{t+k}^{P}((1 - s_{t+k})(1 - l_{t+k}^{P} - l_{t+k}^{Q}))^{p} (v_{t+k}^{P})^{1-\eta_{t+k}^{P}} 
- \Omega_{t+k}^{Q} l_{t+k+1}^{Q} - (1 - \lambda_{t+k}^{Q}) l_{t+k}^{P} - \mu_{t+k}^{Q}((1 - s_{t+k})(1 - l_{t+k}^{P} - l_{t+k}^{Q}))^{q} (v_{t+k}^{Q})^{1-\eta_{t+k}^{Q}} \right\}.
\]

The first order conditions are given by:

\[
v_{c}^{t} = u_{c}(c_{t}, g_{t}) \sigma_{t}^{P} = \Omega_{t}^{P} (1 - \eta_{t}^{P}) q_{t}^{P},
\]

\[
v_{g}^{t} = u_{g}(c_{t}, g_{t}) \sigma_{t}^{Q} = \Omega_{t}^{Q} (1 - \eta_{t}^{Q}) q_{t}^{Q},
\]

\[
s_{t} = \frac{\Omega_{t}^{Q} \eta_{t}^{Q} m_{t}^{Q}}{s_{t}} = \frac{\Omega_{t}^{P} \eta_{t}^{P} m_{t}^{P}}{1 - s_{t}}.
\]

\[l_{t+1}^{P} = \beta \{ a_{t+1}^{P} u_{c}(c_{t+1}, g_{t+1}) - \nu_{u}(u_{t+1}) + \Omega_{t+1}^{P} (1 - \lambda_{t+1}) - \Omega_{t+1}^{Q} \eta_{t}^{P} m_{t+1}^{P} / u_{t+1}^{P} - \Omega_{t+1}^{Q} \eta_{t}^{Q} m_{t+1}^{Q} / u_{t+1}^{Q}\},
\]

\[l_{t+1}^{Q} = \beta \{ a_{t+1}^{Q} u_{g}(c_{t+1}, g_{t+1}) - \nu_{u}(u_{t+1}) + \Omega_{t+1}^{Q} (1 - \lambda_{t+1}) - \Omega_{t+1}^{P} \eta_{t}^{Q} m_{t+1}^{Q} / u_{t+1}^{Q} - \Omega_{t+1}^{P} \eta_{t}^{P} m_{t+1}^{P} / u_{t+1}^{P}\}.
\]

Plugging the first two equations in the third one gives the implicit expression for optimal level of search in each sector:

\[
\frac{u_{g}(c_{t}, g_{t}) \sigma_{t}^{Q} \eta_{t}^{P} q_{t}^{Q}}{(1 - \eta_{t}^{Q}) s_{t}} = \frac{u_{c}(c_{t}, g_{t}) \sigma_{t}^{P} \eta_{t}^{P} q_{t}^{P}}{(1 - \eta_{t}^{P})(1 - s_{t})}.
\]

If we rewrite the third condition as \(\Omega_{t}^{P} \eta_{t}^{P} m_{t}^{P} + \Omega_{t}^{Q} \eta_{t}^{Q} m_{t}^{Q} = \frac{\Omega_{t}^{Q} \eta_{t}^{Q} m_{t}^{Q}}{s_{t}} = \frac{\Omega_{t}^{P} \eta_{t}^{P} m_{t}^{P}}{1 - s_{t}}\), we can use it to simplify the last two conditions and get:

\[
\frac{\sigma_{t}^{P}}{q_{t}^{P}} = \frac{\beta u_{c}(c_{t+1}, g_{t+1})}{u_{c}(c_{t}, g_{t})} \{ (1 - \eta_{t}^{P}) (\alpha_{t+1}^{P} - \nu_{u}(u_{t+1}) / u_{c}(c_{t+1}, g_{t+1}) ) + (1 - \lambda_{t}^{P}) \frac{\sigma_{t}^{P}}{q_{t}^{P}} - \frac{\eta_{t}^{P} \sigma_{t+1}^{P} v_{t+1}^{P}}{(1 - s_{t+1}) u_{t+1}^{P}} \},
\]

\[
\frac{\sigma_{t}^{Q}}{q_{t}^{Q}} = \frac{\beta u_{g}(c_{t+1}, g_{t+1})}{u_{g}(c_{t}, g_{t})} \{ (1 - \eta_{t}^{Q}) (\alpha_{t+1}^{Q} - \nu_{u}(u_{t+1}) / u_{g}(c_{t+1}, g_{t+1}) ) + (1 - \lambda_{t}^{Q}) \frac{\sigma_{t}^{Q}}{q_{t}^{Q}} - \frac{\eta_{t}^{Q} \sigma_{t+1}^{Q} v_{t+1}^{Q}}{s_{t+1} u_{t+1}^{Q}} \}.
\]

\[\text{Proof of Proposition 1}\]

Plugging the steady-state expressions for the value of job, unemployment and em-
employment in the Nash sharing rule gives us:

\[(1 - b)\frac{\bar{w}^p - \nu_u}{1 - \beta(1 - \lambda^p - \frac{m^p}{(1 - \bar{g}^p)})} = b\frac{\bar{a}^p - \bar{w}^p}{1 - \beta(1 - \lambda^p)}.
\]

The decision rule for private sector vacancies is given by the free-entry condition of firms:

\[\frac{c^p}{\beta g^p}(1 - \beta(1 - \lambda^p)) = [\bar{a}^p - \bar{w}^p].\]

Combining the two equations using \((\bar{a}^p - \bar{w}^p)\) we get the following expression:

\[\bar{w}^p - \frac{\nu_u}{u_c} = (1 - \beta(1 - \lambda^p - \frac{\bar{m}^p}{(1 - \bar{g}^p)})\frac{bc^p}{(1 - b)\bar{q}^p\beta}.
\]

Adding it to the free-entry condition:

\[[\bar{a}^p - \frac{\nu_u}{u_c}] = (1 - \beta(1 - \lambda^p - \frac{\bar{m}^p}{(1 - \bar{g}^p)})\frac{bc^p}{(1 - b)\bar{q}^p\beta} + \frac{c^p}{\beta \bar{q}^p}(1 - \beta(1 - \lambda^p)).\]

We can simplify it into

\[[\bar{a}^p - \frac{\nu_u}{u_c}] = (1 - \beta(1 - \lambda^p))\frac{c^p}{(1 - b)\bar{q}^p\beta} + \frac{bc^p\bar{m}^p}{(1 - \bar{g}^p)(1 - b)\bar{q}^p\beta},\]

which can be re-written as:

\[(1 - \beta(1 - \lambda^p))\frac{c^p}{\bar{q}^p} = \beta[(1 - b)(\bar{a}^p - \frac{\nu_u}{u_c}) - \frac{bc^p\bar{m}^p}{(1 - \bar{g}^p)\bar{u}}].\]

This is equivalent to the social planner’s first order condition for private vacancies if \(b = \eta^p\).

**Welfare Costs of High Public Sector Wages**

Let \(\{c_{opt}, g_{opt}, u_{opt}\}\) be the steady-state private and government consumption, and unemployment under the optimal public sector wage. The \(\{c, g, u\}\) is the allocation under an exogenous public sector wage. We want to find what is the welfare gain as a percentage of steady-state private consumption of having the optimal steady-state public sector wage (Section 2.4). This is given by \(x\) that solves the following equation:

\[u(c_{opt}, g_{opt}) + \nu(u_{opt}) = u((1 + x)\bar{c}, \bar{g}) + \nu(\bar{u}).\]

Using the utility function:

\[x = \frac{[\exp[\ln(c_{opt}^\gamma + \zeta g_{opt}^\gamma) + \gamma(\chi(u_{opt} - \bar{u}) - \zeta \bar{g}^\gamma)]^{\frac{1}{\gamma}} - 1}{\bar{c}} - 1, \gamma \neq 0.\]

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If \( \gamma = 0 \), the utility function is not defined, so I use the equivalent \( u(c_t, g_t) = \ln(c_t) + \zeta \ln(g_t) \). The welfare cost in terms of steady state consumption is then given by:

\[
x = \frac{\exp[\ln(c_{opt}) + \zeta(\ln g_{opt} - \ln \bar{g}) + x(u_{opt} - \bar{u})]}{\bar{c}} - 1, \gamma = 0.
\]

**WELFARE COSTS OF BUSINESS CYCLES**

In Section 2.6 I show the welfare costs of business cycles under different policies for \( \{v_t^f, w_t^f\} \). Let us start by defining the variables in log-deviations from the steady-state:

\[
\begin{align*}
\tilde{c}_t &= \log(\frac{c_t}{c}) \quad c_t = \tilde{c} \exp(\tilde{c}_t) \\
\tilde{g}_t &= \log(\frac{g_t}{\bar{g}}) \quad g_t = \tilde{g} \exp(\tilde{g}_t) \\
\tilde{u}_t &= \log(\frac{u_t}{\bar{u}}) \quad u_t = \tilde{u} \exp(\tilde{u}_t).
\end{align*}
\]

If we do a second-order approximation to the variables around the steady state \( \{\tilde{c}, \tilde{g}, \tilde{u}\} \):

\[
\begin{align*}
c_t &= \tilde{c}(1 + \tilde{c}_t + \frac{1}{2}\tilde{c}_t^2) + o(3), \\
g_t &= \tilde{g}(1 + \tilde{g}_t + \frac{1}{2}\tilde{g}_t^2) + o(3), \\
u_t &= \tilde{u}(1 + \tilde{u}_t + \frac{1}{2}\tilde{u}_t^2) + o(3).
\end{align*}
\]

The second-order approximation of the utility function gives:

\[
U(c_t, g_t, u_t) = U(c, g, u) + U_c(c, g, u)c_t + U_g(c, g, u)g_t + U_u(c, g, u)u_t + \frac{1}{2}U_{cc}(c, g, u)c_t^2 + \frac{1}{2}U_{gg}(c, g, u)g_t^2 + \frac{1}{2}U_{uu}(c, g, u)u_t^2 + U_{cg}(c, g, u)c_t g_t + o(3).
\]

But for it to be correct, we have to plug in the second-order approximation of the variables. Given the additive separability of the utility functions, we can drop the cross-terms between the consumption goods and unemployment.

\[
U(c_t, g_t, u_t) = U(c, g, u) + U_c(c, g, u)c_t + U_g(c, g, u)g_t + U_u(c, g, u)u_t + \frac{1}{2}U_{cc}(c, g, u)c_t^2 + \frac{1}{2}U_{gg}(c, g, u)g_t^2 + \frac{1}{2}U_{uu}(c, g, u)u_t^2 + U_{cg}(c, g, u)c_t g_t + o(3).
\]

Collecting terms and substituting the derivatives,

\[
U(c_t, g_t, u_t) = U(c, g, u) + u_c \tilde{c} + u_g \tilde{g} + u_u \tilde{u} + \frac{\tilde{c}}{2}(c_u c + u_c)\tilde{c}_t^2 + \tilde{g}(g_u g + u_g)\tilde{g}_t^2 - \frac{\tilde{u}}{2}(u_u u + \nu_u)\tilde{u}_t^2 + u_{cg}(c, \tilde{g})c \tilde{g}_t \tilde{g}_t + o(3).
\]
PART I. PUBLIC SECTOR EMPLOYMENT AND WAGES

Taking the unconditional expectation, we can write the welfare cost in terms of the moments of the variables:

\[ E[u(c_t, g_t) + \nu(u_t) - u(\bar{c}, \bar{g}) - \nu(\bar{u})] \approx u_c \bar{c} E[\bar{c}_t] + u_g \bar{g} E[\bar{g}_t] + \nu_u \bar{u} E[\bar{u}_t] + \frac{\bar{c}}{2} (\bar{c} u_{cc} + u_c) E[\bar{c}^2_t] + \frac{\bar{g}}{2} (\bar{g} u_{gg} + u_g) E[\bar{g}^2_t] + \frac{\bar{u}}{2} (\bar{u} u_{uu} + u_u) E[\bar{u}^2_t] + u_{cg}(\bar{c}, \bar{g}) \bar{c} \bar{g} E[\bar{c}_t \bar{g}_t] \equiv \Xi. \]

I solve the model up to a second-order using perturbation methods and compute the moments of the variables to find the value of \( \Xi \). To express the welfare costs as a percentage of steady-state consumption, we solve the following equation:

\[ u((1 - x)\bar{c}, \bar{g}) - u(\bar{c}, \bar{g}) = \Xi. \]

For the CES function, the derivatives are given by:

\[ u_c(\bar{c}, \bar{g}) = \frac{\bar{c}^{\gamma-1}}{\bar{c}^{\gamma} + \zeta \bar{g}^{\gamma}}, \]
\[ u_g(\bar{c}, \bar{g}) = \frac{\zeta \bar{g}^{\gamma-1}}{\bar{c}^{\gamma} + \zeta \bar{g}^{\gamma}}, \]
\[ u_{cc}(\bar{c}, \bar{g}) = \frac{(\gamma - 1) \bar{c}^{\gamma-2}}{\bar{c}^{\gamma} + \zeta \bar{g}^{\gamma}} - \frac{\gamma \bar{c}^{2\gamma-2}}{(\bar{c}^{\gamma} + \zeta \bar{g}^{\gamma})^2}, \]
\[ u_{gg}(\bar{c}, \bar{g}) = \frac{(\gamma - 1) \zeta \bar{g}^{\gamma-2}}{\bar{c}^{\gamma} + \zeta \bar{g}^{\gamma}} - \frac{\zeta^2 \bar{g}^{2\gamma-2}}{(\bar{c}^{\gamma} + \zeta \bar{g}^{\gamma})^2}, \]
\[ u_{cg}(\bar{c}, \bar{g}) = -\frac{\gamma \zeta \bar{g}^{\gamma-1} \bar{c}^{\gamma-1}}{(\bar{c}^{\gamma} + \zeta \bar{g}^{\gamma})^2}, \]
\[ \nu_u(\bar{u}) = \chi, \]
\[ \nu_{uu}(\bar{u}) = 0. \]

And the expression for the welfare cost is:

\[ x = 1 - \frac{\exp[\gamma \Xi + \ln(\bar{c}^{\gamma} + \zeta \bar{g}^{\gamma})] - \zeta \bar{g}^{\gamma}}{\bar{c}}, \gamma \neq 0. \]

If \( \gamma = 0 \) the solution is given by:

\[ x = 1 - \frac{\exp[\Xi + \ln \bar{c}]}{\bar{c}}. \]
CHAPTER 2. FISCAL POLICY AND THE LABOUR MARKET

EXTENSION: GOVERNMENT CONSUMPTION

The Lagrangian of the social planner’s problem is:

\[
\sum_{k=0}^{\infty} \beta^{t+k} \{ u(a_{t+k}^{p} p_{t+k}^{p} - \zeta^{p} p_{t+k}^{p} - g_{t+k}, g_{t+k}) + \nu(1 - p_{t+k}^{p}) - \\
\Omega_{t+k}^{1} [p_{t+k}^{p} (1 - \lambda^{p}) p_{t+k}^{p} - m(1 - p_{t+k}^{p}, v_{t+k}^{p})] \}.
\]

The first order conditions are given by:

\[
\frac{\zeta^{p}}{u_{t}} = \beta E_{t} \left\{ \frac{u_{t}(c_{t+1}, g_{t+1})}{u_{t}(c_{t}, g_{t})} \right\} (1 - \eta^{p}) a_{t+1}^{p} + (1 - \eta^{p}) \frac{\nu_{u}(u_{t+1})}{u_{c}(c_{t+1}, g_{t+1})} + (1 - \lambda^{p}) \frac{\zeta^{p}}{g_{t+1}} - \eta^{p} \zeta^{p} v_{t+1}^{p},
\]

\[
u_{c}(c_{t}, g_{t}) = u_{p}(c_{t}, g_{t}).
\]

2.10.5 Bayesian estimation

MODEL - ALL EQUATIONS

The labour market is described by the following equations:

\[
1 = l_{t}^{p} + l_{t}^{q} + u_{t},
\]

\[
l_{t+1}^{p} = (1 - \lambda^{p}) l_{t}^{p} + m_{t}^{p},
\]

\[
l_{t+1}^{q} = (1 - \lambda^{q}) l_{t}^{q} + m_{t}^{q},
\]

\[
m_{t}^{p} = \mu^{p}((1 - s_{t}) u_{t})^{\eta^{p}} (u_{t}^{p})^{1-\eta^{p}},
\]

\[
m_{t}^{q} = \mu^{q}(s_{t} u_{t})^{\eta^{q}} (v_{t}^{q})^{1-\eta^{q}},
\]

\[
q_{t}^{p} = \frac{m_{t}^{p}}{v_{t}^{p}},
\]

\[
p_{t}^{p} = \frac{m_{t}^{p}}{(1 - s_{t}) u_{t}},
\]

\[
p_{t}^{q} = \frac{m_{t}^{q}}{s_{t} u_{t}}.
\]
PART I. PUBLIC SECTOR EMPLOYMENT AND WAGES

The marginal utility of consumption and the stochastic discount factor are:

\[ u_c(c_t, g_t) = \frac{c_t^{\gamma-1}}{c_t^{\gamma} + \zeta g_t^{\gamma}}, \quad (A2.9) \]

\[ \nu_t(u_t) = \chi, \quad (A2.10) \]

\[ \beta_{t,t+1} = \beta \frac{u_c(c_{t+1}, g_{t+1})}{u_c(c_t, g_t)}, \quad (A2.11) \]

I define a variable \( x_t \) as the difference between the value of working and being unemployed. I use it to re-write the equation pinning down \( s_t \) and the Nash bargaining:

\[ x_t^e = W_t^e - U_t^e = w_t^e - \frac{\nu_t(u_t)}{u_c(c_t, g_t)} + E_t \beta_{t,t+1}(1 - \lambda_t^e - p_t^e)x_{t+1}^e, \quad (A2.12) \]

\[ x_t^g = W_t^g - U_t^g = w_t^g - \frac{\nu_t(u_t)}{u_c(c_t, g_t)} + E_t \beta_{t,t+1}(1 - \lambda_t^g - p_t^g)x_{t+1}^g, \quad (A2.13) \]

\[ J_t = a_t^p - w_t^p + E_t \beta_{t,t+1}[(1 - \lambda_t^p)J_{t+1}], \quad (A2.14) \]

\[ \frac{m_t^p E_t[x_{t+1}^e]}{(1 - s_t)} = \frac{m_t^p E_t[x_{t+1}^g]}{s_t}, \quad (A2.15) \]

\[ (1 - b_t)x_t^e = b_t J_t. \quad (A2.16) \]

The production functions, the vacancy posting condition and the policy rules are given by:

\[ c_t = a_t^p q_t^e - \varsigma^p v_t^e, \quad (A2.17) \]

\[ g_t = a_t^p q_t^g - \varsigma^g v_t^g, \quad (A2.18) \]

\[ \varsigma^p = E_t \beta_{t,t+1}[a_{t+1}^p - w_{t+1}^p + (1 - \lambda_{t+1}^p)\frac{\varsigma^p}{q_{t+1}}], \quad (A2.19) \]

\[ \ln(v_t^e) = \ln(\bar{v}^e) + \psi^e \left[ \frac{\sum_{i=0}^{3} \ln(v_{t-i}^e)}{4} - \ln(\bar{v}^e) \right] + \ln(\omega_t^e), \quad (A2.20) \]

\[ \ln(w_t^e) = \ln(\bar{w}^e) + \psi^e \left[ \frac{\sum_{i=0}^{3} \ln(w_{t-i}^e)}{4} - \ln(\bar{w}^e) \right] + \ln(\omega_t^e), \quad (A2.21) \]

I include 6 different shocks: a shock to government vacancies, to government wages, to private and public separation rates, private sector bargaining power and to technology.
These shocks are described by the following equations:

\[ \ln(a_f^p) = \ln(\bar{a}^p) + \alpha(\ln(t_f) - \ln(\bar{p})) + \ln(\omega_f^o). \]  
(A2.22)

\[ \ln(\omega_f^p) = \rho^p \ln(\omega_{f-1}^p) + \epsilon_f^p, \]  
(A2.23)

\[ \ln(\omega_f^w) = \rho^w \ln(\omega_{f-1}^w) + \epsilon_f^w, \]  
(A2.24)

\[ \ln(\omega_f^o) = \rho^o \ln(\omega_{f-1}^o) + \epsilon_f^o, \]  
(A2.25)

\[ \ln(\lambda_f^g) = (1 - \rho^g) \ln(\bar{\lambda}_f^g) + \rho^g \ln(\lambda_{f-1}^g) + \epsilon_f^g, \]  
(A2.26)

\[ \ln(\lambda_f^p) = (1 - \rho^p) \ln(\bar{\lambda}_f^p) + \rho^p \ln(\lambda_{f-1}^p) + \epsilon_f^p, \]  
(A2.27)

\[ \ln(b_t) = (1 - \rho^b) \ln(\bar{b}) + \rho^b \ln(b_{t-1}) + \epsilon_t^b. \]  
(A2.28)

Finally, I define that overall job-separation and job-finding rates:

\[ f_t = \frac{m_t^p + m_t^g}{u_t}, \]  
(A2.29)

\[ \Lambda_t = \frac{\lambda_t^p + \lambda_t^g}{\bar{p}_t + \bar{l}_t}. \]  
(A2.30)

**MODEL - STEADY STATE**

I set the steady-state government employment at \( \bar{b} \). As there is no recursive way to write the steady-state variables, they solve the following non-linear system of equations:

\[ \bar{p} = 1 - \bar{b} - \bar{u}, \]

\[ m_t^p = \lambda_t^p \bar{p}, \]

\[ m_t^g = \lambda_t^g \bar{p}, \]

\[ m_t^p = \mu_t^p((1 - \bar{\bar{s}})\bar{u})^{\eta_p}(\bar{p})^{1-\eta_p}, \]

\[ m_t^g = \mu_t^g(\bar{\bar{s}}\bar{u})^{\eta_g}(\bar{p})^{1-\eta_g}, \]

\[ p_t^f = \frac{m_t^p}{(1 - \bar{\bar{s}})\bar{u}}. \]

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The variables with tilde are expressed in deviations from steady-state.

\[
\begin{align*}
q^p &= \frac{\bar{n}^g}{(\bar{s}) \bar{u}}, \\
\bar{q}^p &= \frac{\bar{n}^p}{\bar{v}^p}, \\
\bar{x}^g &= \frac{\bar{w}^g - \bar{v}_u}{1 - \beta(1 - \lambda^g - \bar{p}^g)} \\
\bar{x}^p &= \frac{\bar{w}^p - \bar{v}_u}{1 - \beta(1 - \lambda^p - \bar{p}^p)} \\
\bar{m}^p \bar{\bar{s}} = \bar{m}^g \bar{\bar{s}} (1 - \bar{s}), \\
(1 - b)(\bar{x}^p) &= b \bar{J}, \\
\bar{J} &= \frac{\bar{a}^p - \bar{w}^p}{1 - \beta(1 - \lambda^p)}, \\
\frac{\bar{c}^p}{\bar{q}^p} (1 - \beta(1 - \lambda^p)) &= \beta (\bar{a}^p - \bar{w}^p), \\
\bar{w}^g &= \pi \bar{w}^p, \\
\bar{c} &= \bar{a}^p \bar{p} - \bar{c}^p \bar{w}^p, \\
\bar{g} &= \bar{a}^g \bar{p} - \bar{g}^p \bar{w}^g, \\
u_u (\bar{c}, \bar{g}) &= \frac{\bar{c}^{-\gamma} - 1}{\bar{c}^{-\gamma} + \zeta \bar{g}^{-\gamma}}, \\
\nu_u &= \chi, \\
\bar{f} &= \frac{\bar{m}^p + \bar{m}^g}{\bar{u}}, \\
\bar{\Lambda} &= \frac{\lambda^p \bar{p}^p + \lambda^g \bar{p}^g}{\bar{p}^p + \bar{p}^g}.
\end{align*}
\]

**Estimated log-linearized model**

The variables with tilde are expressed in deviations from steady-state.

\[
0 = \bar{p}^p \bar{\bar{t}} + \bar{p}^g \bar{\bar{t}} + \bar{\bar{u}} \bar{u}_t, \\
(\text{L1})
\]

\[
\bar{p}^p_{t+1} = (1 - \lambda^p) \bar{p}^p_t - \bar{\lambda}^p \bar{\lambda}^p + \bar{\lambda}^p \bar{\bar{m}}^p_t, \\
(\text{L2})
\]

\[
\bar{p}^g_{t+1} = (1 - \lambda^g) \bar{p}^g_t - \bar{\lambda}^g \bar{\lambda}^g + \bar{\lambda}^g \bar{\bar{m}}^g_t, \\
(\text{L3})
\]

\[
\bar{m}^p_t = \eta^p (\bar{u}_t - \frac{\bar{s} - \bar{\bar{s}}_{t}}{1 - \bar{\bar{s}}_{t}}) + (1 - \eta^p) \bar{\bar{v}}^p_t, \\
(\text{L4})
\]

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\[ \tilde{m}_t = \eta^o (\tilde{u}_t + \tilde{s}_t) + (1 - \eta^o) \tilde{v}_t^o, \]  

\[ q_t = \tilde{m}_t^p - \tilde{v}_t^p, \]  

\[ \tilde{p}_t = \tilde{m}_t^p + \frac{\tilde{s}}{1 - \tilde{s}} \tilde{s}_t - \tilde{u}_t, \]  

\[ \tilde{p}_t^p = \tilde{m}_t^p - \tilde{s}_t - \tilde{u}_t, \]  

\[ \tilde{u}_c(\tilde{t}, \tilde{g}_t) = \tilde{c}_t(\gamma - 1 - \frac{\gamma \tilde{c}^o}{v_t} + \tilde{g}_t \frac{v_t}{v_t}) - \tilde{g}_t \left( \frac{v_t ^v}{v_t ^c} + \tilde{g}_t \right), \]  

\[ \tilde{v}_u(\tilde{u}_t) = 0, \]  

\[ \tilde{b}_{t+1} = E_t[\tilde{u}_c(\tilde{c}_{t+1}, \tilde{g}_{t+1}) - \tilde{u}_c(\tilde{c}_t, \tilde{g}_t)], \]  

\[ \tilde{x}_t = \tilde{w}^p \tilde{w}_t^p - \frac{\tilde{v}_u}{\tilde{x}_t \tilde{u}_c}(\tilde{u}_u - \tilde{u}_c) - \beta(\tilde{x}_t \tilde{p}_t + \tilde{p}_t \tilde{P}_t) + \beta(1 - \tilde{x}_t - \tilde{p}_t)E_t(\tilde{x}^p_{t+1} + \tilde{b}_{t+1}), \]  

\[ \tilde{x}^p_t = \tilde{w}^p \tilde{w}_t^p - \frac{\tilde{v}_u}{\tilde{x}_t \tilde{u}_c}(\tilde{u}_u - \tilde{u}_c) - \beta(\tilde{x}_t \tilde{p}_t + \tilde{p}_t \tilde{P}_t) + \beta(1 - \tilde{x}_t - \tilde{p}_t)E_t(\tilde{x}^p_{t+1} + \tilde{b}_{t+1}), \]  

\[ \tilde{J}_t = \frac{\tilde{a}^p}{\tilde{a}_t} \tilde{w}_t^p - \frac{\tilde{v}_u}{\tilde{x}_t \tilde{u}_c}(\tilde{u}_u - \tilde{u}_c) - \beta(\tilde{x}_t \tilde{p}_t + \tilde{p}_t \tilde{P}_t) + \beta(1 - \tilde{x}_t - \tilde{p}_t)E_t(\tilde{x}^p_{t+1} + \tilde{b}_{t+1}), \]  

To test the relevance of the directed search assumption, I have added the parameter \( \kappa \) to the log-linearized equation that determines \( \tilde{s}_t \)

\[ \tilde{s}_t = \kappa(1 - \tilde{s})E_t(\tilde{x}_t \tilde{a}^p_{t+1} - \tilde{w}^p \tilde{w}_t^p + \tilde{m}_t^p + \tilde{m}_t^o), \]  

\[ \tilde{J}_t + \frac{1}{1 - \tilde{b}} \tilde{b}_t = \tilde{x}_t^p, \]  

\[ \tilde{c}_t = \frac{\tilde{a}^p \tilde{p}_t}{\tilde{c}}(\tilde{a}_t + \tilde{p}_t) - \frac{\tilde{c}^o \tilde{v}_t^p}{\tilde{c}} \tilde{v}_t^p, \]  

\[ \tilde{g}_t = \frac{\tilde{a}^p \tilde{g}_t}{\tilde{g}}(\tilde{a}_t + \tilde{p}_t) - \frac{\tilde{g} \tilde{v}_t^o}{\tilde{g}} \tilde{v}_t^o, \]  

\[ -\frac{\tilde{c}^o}{\tilde{q}^p} \tilde{a}^p = \beta[\tilde{a}^p \tilde{a}^p_{t+1} - \tilde{w}^p \tilde{w}_t^p + (1 - \tilde{x}_t)^p \tilde{q}^p \tilde{q}^p + (\tilde{a}^p - \tilde{w}^p + (1 - \tilde{x}_t)^p \tilde{q}^p) \tilde{b}_t], \]
\[ \ddot{v}_t = \psi' \ddot{v}_t + \ddot{v}_{t-1} + \ddot{v}_{t-2} + \ddot{v}_{t-3} + \ddot{\omega}_t, \tag{L20} \]
\[ \ddot{w}_{t+1} = \psi' \ddot{w}_t + \ddot{w}_{t-1} + \ddot{w}_{t-2} + \ddot{w}_{t-3} + \ddot{\omega}_t, \tag{L21} \]
\[ \ddot{\alpha}_t = \alpha t_0 + \omega_t, \tag{L22} \]
\[ \ddot{\omega}_t = \rho \ddot{\omega}_{t-1} + e_t, \tag{L23} \]
\[ \ddot{\omega}_t = \rho \ddot{\omega}_{t-1} + e_t, \tag{L24} \]
\[ \ddot{\omega}_t = \rho \ddot{\omega}_{t-1} + e_t, \tag{L25} \]
\[ \ddot{\lambda}_t = \rho \ddot{\lambda}_{t-1} + e_t, \tag{L26} \]
\[ \ddot{\lambda}_t = \rho \ddot{\lambda}_{t-1} + e_t, \tag{L27} \]
\[ \ddot{d}_t = \rho \ddot{d}_{t-1} + e_t, \tag{L28} \]
\[ f_t = \frac{\bar{m}_t}{\bar{m}_p + \bar{m}_g} + \frac{\bar{m}_g}{\bar{m}_p + \bar{m}_g} - \ddot{u}_t, \tag{L29} \]
\[ \ddot{\lambda}_t = \left( \ddot{\lambda}_t + \ddot{p}_t \right) \frac{\ddot{\lambda}_t \ddot{p}_t}{\ddot{\lambda}_t \ddot{p}_t + \ddot{\lambda}_t \ddot{g}_t} + \left( \ddot{\lambda}_t + \ddot{p}_t \right) \frac{\ddot{\lambda}_t \ddot{g}_t}{\ddot{\lambda}_t \ddot{p}_t + \ddot{\lambda}_t \ddot{g}_t} + \ddot{u}_t \frac{\ddot{u}_t}{1 - \ddot{u}}. \tag{L30} \]

**Definition of Observable Variables**

\[
\begin{bmatrix}
\ell_{t}^{Ob} \\
\ell_{t}^{Ob} \\
w_{t}^{Ob} \\
w_{t}^{Ob} \\
w_{t}^{Ob} \\
w_{t}^{Ob} \\
A_{t}^{Ob} \\
f_{t}^{Ob}
\end{bmatrix}
\begin{bmatrix}
\bar{\ell}_{t}^{Ob} - \bar{\ell}_{t-1}^{Ob} \\
\bar{u}_{t} - \bar{u}_{t-1} \\
\bar{w}_{t}^{Ob} - \bar{w}_{t-1}^{Ob} \\
\bar{w}_{t}^{Ob} - \bar{w}_{t-1}^{Ob} \\
A_{t}^{Ob} - \bar{A}_{t-1} \\
f_{t}^{Ob} - f_{t-1}^{Ob}
\end{bmatrix}
\begin{bmatrix}
\ell_{t}^{Ob} \\
\ell_{t}^{Ob} \\
w_{t}^{Ob} \\
w_{t}^{Ob} \\
w_{t}^{Ob} \\
w_{t}^{Ob} \\
A_{t}^{Ob} \\
f_{t}^{Ob}
\end{bmatrix}
\begin{bmatrix}
\bar{\ell}_{t}^{Ob} - \bar{\ell}_{t-1}^{Ob} \\
\bar{u}_{t} - \bar{u}_{t-1} \\
\bar{w}_{t}^{Ob} - \bar{w}_{t-1}^{Ob} \\
\bar{w}_{t}^{Ob} - \bar{w}_{t-1}^{Ob} \\
A_{t}^{Ob} - \bar{A}_{t-1} \\
f_{t}^{Ob} - f_{t-1}^{Ob}
\end{bmatrix}
\]

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CHAPTER 2. FISCAL POLICY AND THE LABOUR MARKET

MODEL WITH RANDOM SEARCH

Equation A1-A3, A6 and A9-A11 are the same. As there is no directed search, we drop equation A15 and the matches in each sector are given by the relative vacancies:

\[ m_t^p + m_t^q = p_t^p (u_t)^{p^p} (v_t^p + v_t^q)^{1-p^p}, \quad (R4) \]

\[ v_t^q m_t^p = v_t^p m_t^q, \quad (R5) \]

\[ p_t^p = \frac{m_t^p}{u_t}, \quad (R7) \]

\[ p_t^q = \frac{m_t^q}{u_t}, \quad (R8) \]

\[ x_t^p = u_t^p - \frac{\nu_t(u_t)}{u_t(c_t, g_t)} + E_t \beta_{t+1}((1 - \lambda_t^p - p_t^p)x_{t+1}^p - p_t^q x_{t+1}^q), \quad (R12) \]

\[ x_t^q = u_t^q - \frac{\nu_t(u_t)}{u_t(c_t, g_t)} + E_t \beta_{t+1}((1 - \lambda_t^q - p_t^q)x_{t+1}^q - p_t^p x_{t+1}^p). \quad (R13) \]

For the log-linearized model the expressions are:

\[ \tilde{m}_t^p - \tilde{m}_t^q = \tilde{v}_t^p - \tilde{v}_t^q, \quad (RL4) \]

\[ \tilde{m}_t^p - \tilde{v}_t^p = \eta^p \tilde{u}_t + \eta^p \left( \frac{\tilde{m}_t^p}{\tilde{m}_t^p + \tilde{m}_t^q} \tilde{v}_t^p + \frac{\tilde{m}_t^q}{\tilde{m}_t^p + \tilde{m}_t^q} \tilde{v}_t^q \right), \quad (RL5) \]

\[ \tilde{x}_t^p = \frac{\tilde{v}_t^p}{\tilde{\varepsilon}} \tilde{u}_t^p - \frac{\tilde{p}_u}{\tilde{\varepsilon} \tilde{u}_c} (\tilde{v}_u - \tilde{u}_c) - \beta (\tilde{\lambda}_t^p \tilde{p}_t^p + \tilde{\lambda}_t^q \tilde{p}_t^q) + \beta (1 - \tilde{\lambda}_t^p - \tilde{\lambda}_t^q) E_t(\tilde{x}_{t+1}^p + \tilde{\beta}_{t+1}). \quad (RL12) \]
### Estimation results (levels)

#### Table A2.3: Prior and posterior distribution of structural parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Structural parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elasticity of substitution</td>
<td>( \gamma )</td>
<td>Normal (0, 0.1)</td>
</tr>
<tr>
<td>Utility of unemployment</td>
<td>( \chi )</td>
<td>Normal (0.5, 0.1)</td>
</tr>
<tr>
<td>Separation rate (private sector)</td>
<td>( \lambda_p )</td>
<td>Normal (0.06, 0.01)</td>
</tr>
<tr>
<td>Separation rate (public sector)</td>
<td>( \lambda_g )</td>
<td>Normal (0.03, 0.01)</td>
</tr>
<tr>
<td>Cost of posting vacancy (private sector)</td>
<td>( \phi_p )</td>
<td>Normal (2, 0.3)</td>
</tr>
<tr>
<td>Cost of posting vacancy (public sector)</td>
<td>( \phi_g )</td>
<td>Normal (1.1, 0.2)</td>
</tr>
<tr>
<td>Vacancy filling probability (private sector)</td>
<td>( \theta_p )</td>
<td>Normal (3.9, 0.2)</td>
</tr>
<tr>
<td>Vacancy filling probability (public sector)</td>
<td>( \theta_g )</td>
<td>Normal (2.5, 0.2)</td>
</tr>
<tr>
<td>Matching elasticity w.r.t unemp. (private)</td>
<td>( \tau_p )</td>
<td>Beta (0.5, 0.15)</td>
</tr>
<tr>
<td>Matching elasticity w.r.t unemp. (public)</td>
<td>( \tau_g )</td>
<td>Beta (0.5, 0.15)</td>
</tr>
<tr>
<td>Bargaining power</td>
<td>( \beta )</td>
<td>Beta (0.5, 0.10)</td>
</tr>
<tr>
<td>Productivity of public employment</td>
<td>( \alpha )</td>
<td>Normal (0, 0.1)</td>
</tr>
<tr>
<td>Cyclicality of public sector wages</td>
<td>( \psi_w )</td>
<td>Normal (0, 0.3)</td>
</tr>
<tr>
<td>Importance of directed search</td>
<td>( \kappa )</td>
<td>Normal (0, 0.3)</td>
</tr>
<tr>
<td><strong>Autoregressive parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Productivity</td>
<td>( \rho_p )</td>
<td>Beta (0.5, 0.15)</td>
</tr>
<tr>
<td>Public sector wage</td>
<td>( \rho_w )</td>
<td>Beta (0.5, 0.15)</td>
</tr>
<tr>
<td>Public sector vacancies</td>
<td>( \rho_v )</td>
<td>Beta (0.5, 0.15)</td>
</tr>
<tr>
<td>Private sector separation rate</td>
<td>( \rho_{plp} )</td>
<td>Beta (0.5, 0.15)</td>
</tr>
<tr>
<td>Public sector separation rate</td>
<td>( \rho_{plg} )</td>
<td>Beta (0.5, 0.15)</td>
</tr>
<tr>
<td>Bargaining power</td>
<td>( \rho_b )</td>
<td>Beta (0.5, 0.15)</td>
</tr>
<tr>
<td><strong>Standard deviations</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Productivity</td>
<td>( \sigma^\alpha )</td>
<td>IGamma (0.01, 0.15)</td>
</tr>
<tr>
<td>Public sector wage</td>
<td>( \sigma^\alpha )</td>
<td>IGamma (0.01, 0.15)</td>
</tr>
<tr>
<td>Public sector vacancies</td>
<td>( \sigma^\alpha )</td>
<td>IGamma (0.01, 0.15)</td>
</tr>
<tr>
<td>Private sector separation rate</td>
<td>( \sigma^\alpha )</td>
<td>IGamma (0.01, 0.15)</td>
</tr>
<tr>
<td>Public sector separation rate</td>
<td>( \sigma^\alpha )</td>
<td>IGamma (0.01, 0.15)</td>
</tr>
<tr>
<td>Bargaining power</td>
<td>( \sigma^\beta )</td>
<td>IGamma (0.01, 0.15)</td>
</tr>
</tbody>
</table>

#### Table A2.4: Model Comparison (Levels)

<table>
<thead>
<tr>
<th></th>
<th>Marginal Data Density</th>
<th>Prediction of vacancies Correlation ( (v^e,v^d) )</th>
<th>Prediction of tightness Correlation ( ((\bar{Z})^e, (\bar{Z})^d) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Directed search</td>
<td>4381.7</td>
<td>2.39 0.40 0.94 0.58</td>
<td></td>
</tr>
<tr>
<td>Directed search (( \kappa = 0 ))</td>
<td>4357.2</td>
<td>0.97 0.59 1.15 0.73</td>
<td></td>
</tr>
<tr>
<td>Random search</td>
<td>4345.0</td>
<td>0.71 0.50 0.76 0.71</td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER 2. FISCAL POLICY AND THE LABOUR MARKET

SUBSAMPLE ESTIMATION RESULTS

Figure A2.14: Subsample stability of parameters

Note: Solid line is the estimation in differences and the dash line the estimation in levels.
Chapter 3

Interactions between public and private wages

3.1 Introduction

The relevance of public wages for total government spending has increased gradually over the past decades in several European countries. Apart from the importance that such a budgetary item has for the development of public finances and for attaining budgetary objectives, public sector employment and wages play a key role in the labour market. In this context, the main objective of this paper is to study one aspect of the relation between fiscal policy and the labour market, namely the interaction between private and public sector wages.

First, we analyse the interactions between the wages in the two sectors empirically. We examine the determinants of private sector wage growth, paying attention to the role of public sector wage and employment growth, as well as other market related variables. Additionally, we look at the determinants of public sector wage growth. Although there is evidence of some pro-cyclicality of public wages (Lamo, Pérez, and Schuknecht (2008)), their developments may be less aligned with those of the private sector. For instance, public wages can also depend on the fiscal position. In fact, Poterba and Rueben (1995) and Gyourko and Tracy (1989) find that fiscal conditions affect wages of public employees at a local level. Moreover, they might be used as an instrument for income policies, thus they can depend on political factors such as the political alignment of the ruling party or election cycles. For instance, Matschke (2003) finds evidence of systematic public wage increases prior to a federal election in Germany.

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1 This chapter was written in co-authorship with António Afonso from the European Central Bank.
2 According to the European Commission, the average share of public wages (compensation of employees) in general government total spending was around 23 per cent in 2007 for the European Union, that is, around 11 percent of GDP. Interestingly, the public wages-to-total government spending ratio was 28 per cent in 2006 in the US.
We develop our analysis for OECD countries for the period between 1973 and 2000. We carefully discuss the econometric issues involved, particularly the problem of endogeneity, and how we subsequently address them. In a nutshell, we find that a number of variables affect private sector wage growth, for instance: changes in the unemployment rate (negative relationship), total factor productivity growth and changes in the urbanisation rate. Moreover, public sector wages and employment growth also affect private sector wage growth. A 1% increase in public sector wages raises the wages in the private sector by 0.3 percent. Public sector wage growth seems to respond mainly to private sector wage growth, but also to the budget balance, the tax wedge and the position of the countries in the political spectrum.

Second, we set up a dynamic two-sector labour market equilibrium model in order to understand the interaction mechanisms. The model, that features search and matching frictions in the labour market and exogenous growth, captures the essence of the interaction between the wages in the two sectors. The government determines wage increases depending on the expected growth rate of private sector wages and an error correction that depends on the public-private wage differential. The long-run growth rate is determined in the private sector and then spreads out to the public sector, but similar to other models that address this issue, we also find that public sector wages and employment affect private sector wages.3

Public sector wages and employment impinge on private sector wages via three channels. First, they affect the outside option of the unemployed, either by increasing the probability of being hired (public sector employment) or by increasing the value of being employed in the public sector (public sector wages). Therefore, they put pressure on wage bargaining. Second, they both crowd out private sector employment which, due to the presence of diminishing marginal productivity of labour, raises the average productivity. Finally, both public wages and employment have to be financed by an increase in taxes, which will also affect the wages paid by the firm. In addition, the model also features the effects from private sector wages to public sector wages in response to technology shocks. Re-doing the empirical exercise with simulated data, yields very similar coefficients to the ones estimated for the OECD countries.

The chapter is organised as follows. In Section 3.2 we present the empirical setting and in Section 3.3 we report and discuss the results. In Section 3.4 we present the theoretical model. Section 3.5 summarises the main findings of the chapter.

3See, for instance, Holmlund and Linden (1993), Algan, Cahuc, and Zylberberg (2002), and Ardagna (2007).
3.2 Empirical framework

In this section, we estimate the determinants of both private sector and public sector wages. Our underlying idea is to estimate two different wage functions that link private and public wages, while carefully addressing the problem of endogeneity between the two.

Most papers provide an aggregate perspective of the relation between the two wages focussing on wage levels per employee (see, for instance, Nunziata (2005), Jacobson and Ohlsson (1994) and Friberg (2007)). However, we prefer to model the growth rates of real compensation per employee to assess the behaviour of the two variables in the short run. Since we have annual data, the use of growth rates eliminates the low frequency movements, but preserves the movements at business cycle frequency, which we are more interested in uncovering (Abraham and Haltiwanger (1995)).

In the long-run it is natural that the two variables are cointegrated with a slope coefficient of one, if not one would observe a constant divergence of the wages in the two sectors. This does not exclude differences in the levels of the wages, but simply that these differences do not show a trend. In fact, we observe a public sector wage premium or a gap, either due to different skills composition of employment or because of barriers between the two sectors.

3.2.1 Empirical specification for private sector wages

Our baseline wage function for the developments in private sector salaries is given by

\[ \omega_{it}^p = \alpha_i + \theta^p \omega_{it-1}^p + \theta^p X_{it}^p + \pi^p Z_{it}^p + \kappa^p E_{it-1} + \mu_{it}. \]  

(3.1)

In (3.1) the index \( i (i = 1, ..., N) \) denotes the country, the index \( t (t = 1, ..., T) \) indicates the period, \( \alpha_i \) stands for the individual effects to be estimated for each country \( i \), and it is assumed that the disturbances \( \mu_{it} \) are independent across countries. \( \omega_{it}^p \) is the growth rate of the real compensation per employee in the private sector.

\( X_{it}^p \) is a vector of macroeconomic variables that might be endogenous to private sector wage growth. This vector includes the growth rate of real compensation per employee in the public sector, \( \omega_{it}^p \), the growth rate of the consumer price index, growth rate of total factor productivity, change in the unemployment rate, change in urbanization rate, growth rate of the per worker average hours worked, growth rate of the countries' terms of trade, change in the tax wedge and the growth rate of public employment. The latter can also positively impinge on the growth rate of private sector wages if higher labour demand in the public sector pushes private sector wages upwards.
PART I. PUBLIC SECTOR EMPLOYMENT AND WAGES

On the other hand, $Z_{it}^m$ is a vector of institutional exogenous variables. It includes the change in union density, an index of bargaining coordination, the change in benefit duration and the change in the benefit replacement ratio. Previous work by Nunziata (2005) concluded that these institutional variables are important determinants of the level of wages. While union density should contribute to increase wages, the benefit replacement rate and duration affect the outside option of workers and may also influence their wages. Additionally, if the bargaining process is centrally coordinated it is likely to restrain private sector wage growth. Finally, we include an index of central bank independence to capture potential credibility effects on inflation expectations as well as a variable that measures the change in education attainment of the working age population to control for composition effects.

Finally, $E_{it-1}$ in (3.1) is defined as the percentage difference between public and private sector wages - the public wage premium or gap:

$$E_{it-1} = \ln \left( \frac{w_{it-1}^p}{w_{it-1}^g} \right) \times 100. \tag{3.2}$$

where $w_g$ and $w_p$ are respectively the nominal public and private per employee wage in levels. This term can be interpreted as an error correction mechanism. There are two ways through which public sector wages can affect private sector wages. There is the direct effect, captured in $\theta^p$, in equation (3.1), and there is the indirect effect through the error correction mechanism of magnitude $\kappa^p$. If the ratio of public-to-private wages increases, private sector wages may rise in order to correct the wage differential downwards. This can be seen both as a demonstration effect stemming from the public sector or a catching up effect in salaries implemented in the private sector. Therefore, $\kappa^p$ is expected to be positive.

In addition, one can assess the cyclicality of private wages. If the coefficient on the change in the unemployment rate is negative this implies a pro-cyclical behaviour of private wages. While the idea of wage counter-cyclicality was put forward by Keynes (1939), empirical results actually produce evidence of both pro-cyclical and counter-cyclical private sector behaviour. Abraham and Haltiwanger (1995) offer several arguments for the possibility of both outcomes.

3.2.2 Empirical specification for public sector wages

We also estimate an equation for public sector real wage growth. The baseline wage function for the developments of public sector salaries can be assessed with the following
We consider that government wages can respond to the same variables as private wages, except for the average hour worked per worker, central bank independence and the growth rate of public employment growth. Indeed, the hours worked in the public sector are more standardized than in the private sector, and the central bank independence is more relevant for the private sector. \( X_{it}^p \) also includes the growth rate of private sector wages. Additionally, \( F_{it} \) includes fiscal variables, such as the general government budget balance as a percentage of GDP and the general government debt-to-GDP ratio. \( P_{it} \) contains the political variables, which consist of the percentage of votes for left wing parties and a dummy variable for parliamentary election years. While the variables in \( F_{it} \) are endogenous, we consider the variables in \( P_{it} \) as exogenous. \( \beta_i \) stands for the individual effects to be estimated for each country \( i \).

Similar to the specification for the private sector wages, \( \kappa_g \) now measures to what extent public wages correct the imbalances of the long-term relation between the two. In this case, increases in the public-to-private wages ratio can produce a future reduction in public sector wages, implying an expected negative value for \( \kappa_g \).

While one would expect that recent fiscal developments may impinge on the public sector wages per employee, this hypothesis seems less relevant for the development of private sector wages. On the other hand, if one expects the unemployment rate to impinge negatively on the development of private sector wages, this effect may be mitigated in the case of public sector wages, given the higher rigidity of the labour force in the government sector and a possible higher degree of unionisation.

### 3.2.3 Econometric issues

There are two main econometric issues when estimating the wage functions (3.1) and (3.3). The first issue is the presence of endogenous variables, particularly the simultaneous determination of public and private sector wage growth. To deal with this, we estimate each equation separately and instrument all the endogenous variables by the remaining pre-determined variables and two lags of all variables. We compute the Sargan over-identifying test to access the validity of the instruments. As we are using the lagged variables as instruments, what we are essentially doing is predicting the value of the regressors based on past information. Thus, the interpretation of the coefficients should be, for instance, the effect of expected public sector wage growth on the growth rate of private sector wages.
Although our distinction between endogenous and exogenous variables is arbitrary, we run a Hausman test to examine the exogeneity of each block of variables.

The second econometric issue is that the regressors and the error term are correlated, because we allow for a country specific error and include a lagged dependent variable. Although we also tried the Arellano and Bond GMM estimator, our preferred methodology is a simple panel 2SLS estimation. First, the Arellano and Bond methodology implies estimating the equation in first differences (of growth rates) which adds a lot of noise to the estimates. Furthermore, as Nickell, Nunziata, and Ochel (2005) point out, the bias created by the presence of a lagged dependent variable in panel data tends to zero if we have a long time series component. As we have close to 30 time observations for most countries in the sample, we proceed with the estimation with a panel 2SLS. We also include country fixed effects.

3.3 Estimation results and discussion

3.3.1 Data

We study this issue in a panel framework for eighteen OECD countries, covering essentially the period between 1973 and 2000.\(^4\)

For the employment and wage data our main data source is the OECD Economic Outlook database, the European Commission database AMECO and the Labour Market Institutions Database used in Nickell, Nunziata, and Ochel (2005) and expanded by Nickell (2006). Private sector wages are defined as total compensation of employees minus compensation of government employees. Private sector wages per employee are defined as private compensation of employees divided by private sector employees (total employment minus government employees minus self-employed persons).\(^5\) We compute the real wages per employee using the consumption price deflator.

Using aggregate data has its limitations. On the one hand, it ignores the composition of public and private employment, in particular with respect to the skills level of employment. On the other hand, it is difficult to get a completely clean identification strategy. Despite these problems we still think using aggregate data is an advantage. First, no other type of data would allow for such a long time span for so many countries. Second, the identification using lags as instruments has been used quite successful in several studies, for instance by Nickell, Nunziata, and Ochel (2005) and Nunziata (2005).

---

\(^4\)Given data availability, the countries used in the empirical analysis are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, United Kingdom and United States. See the Appendix for details and sources.

\(^5\)This approach is also used by Lamo, Pérez, and Schuknecht (2008)
The share of government employment in total employment increased for most countries in the 1980s, while there was an even more generalised decline after the beginning of the 1990s. Regarding real wages per employee an upward trend occurred for most countries, both for private and public wages. In addition, although the ratio of public-to-private wages per employee is relatively constant, it has followed an upward path for the majority of European countries since the beginning of the 1990s.\(^6\)

### 3.3.2 Private wage determinants

The first two columns of Table 3.1 report the results for the growth rate of the real private sector wages per employee. One can observe that the growth rate in public sector wages affects their private counterpart both directly and through the error correction model. Both coefficients are positive and statistically significant. A 1 percent increase in real public sector wage growth increases private sector real wage growth by 0.3 percent. The growth rate of public employment also has a positive and significant effect on the growth rate of nominal private sector wages. A 1 percent increase in public sector employment increases private sector wage growth by close to 0.2 percent.

The change in the unemployment rate exerts a negative effect on private sector wages growth. In other words, private sector wages have a pro-cyclical behaviour: a 1 percentage point increase in the unemployment rate reduces the growth rate of private sector wages by around 0.5 percentage points. On the other hand, such a growth rate increases with total factor productivity growth.

The inflation rate does not affect the growth rate of real private sector wages per employee, which supports the idea that agents have rational expectations. Some wage stickiness is captured by the statistically significant lagged dependent variable, while there are no statistically significant effects reported for the terms of trade or for the tax wedge.

Regarding the set of pre-determined explanatory variables (in vector Z), it is interesting to note that the growth rate of real private sector wages is negatively affected by the index of central bank independence. Changes in union density, bargaining coordination, the benefit replacement rate and education do not statistically affect the growth rate of real private sector wages. Moreover, the change in benefit duration has a negative significant coefficient.

In the estimations, the Hausman test clearly supports that the institutional variables block is exogenous and that the variables in the macroeconomic block are endogenous. The Sargan test points to the validity of the instruments.

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\(^6\)See Afonso and Gomes (2008) for a more detailed assessment of the data trends.
### Table 3.1: Determinants of the real growth rate of wages per employee (1973-2000)

<table>
<thead>
<tr>
<th>Private sector</th>
<th>Public sector</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>Lagged dependent variable</td>
<td>0.120** (2.46)</td>
</tr>
<tr>
<td>Error correction component</td>
<td>0.034** (2.47)</td>
</tr>
<tr>
<td>Growth rate of public sector wages</td>
<td>0.304*** (4.72)</td>
</tr>
<tr>
<td>Δ Unemployment rate</td>
<td>-0.501*** (-2.97)</td>
</tr>
<tr>
<td>Inflation rate</td>
<td>-0.005 (-0.10)</td>
</tr>
<tr>
<td>Growth rate of TFP</td>
<td>0.349*** (2.78)</td>
</tr>
<tr>
<td>Δ Urbanization rate</td>
<td>0.873 (1.58)</td>
</tr>
<tr>
<td>Growth rate of terms of trade</td>
<td>0.031 (0.64)</td>
</tr>
<tr>
<td>Growth rate of hours per worker</td>
<td>-0.604 (-0.96)</td>
</tr>
<tr>
<td>Growth rate of public employment</td>
<td>0.229** (2.09)</td>
</tr>
<tr>
<td>Tax wedge</td>
<td>0.187 (1.42)</td>
</tr>
<tr>
<td>Budget balance</td>
<td>0.102 (1.61)</td>
</tr>
<tr>
<td>Government debt</td>
<td>0.003 (0.24)</td>
</tr>
<tr>
<td>Δ Union density</td>
<td>0.001 (1.03)</td>
</tr>
<tr>
<td>Δ Bargaining coordination</td>
<td>1.975 (1.60)</td>
</tr>
<tr>
<td>Δ Benefit duration</td>
<td>-4.567* (-1.84)</td>
</tr>
<tr>
<td>Δ Benefit replacement rate</td>
<td>-0.018 (-0.55)</td>
</tr>
<tr>
<td>Δ Education</td>
<td>0.392 (0.35)</td>
</tr>
<tr>
<td>Central bank independence</td>
<td>-2.416*** (-3.33)</td>
</tr>
<tr>
<td>Election year</td>
<td>-0.171 (-0.59)</td>
</tr>
<tr>
<td>% Left wing votes</td>
<td>0.069* (1.81)</td>
</tr>
</tbody>
</table>

**Notes:** The coefficients are estimated using 2SLS. Endogenous variables: the change in unemployment rate, the change in the urbanisation rate, the growth rate of total factor productivity, inflation rate, growth rate of real per worker private sector wages, growth rate of hours per worker, growth rate of public employment, growth rate of terms of trade, budget balance, government debt and tax wedge. They are instrumented by the remaining pre-determined variables and two lags of all explanatory variables. The t statistics are in parentheses. *, **, *** - statistically significant at the 10, 5, and 1 percent. 

* The null hypothesis of the Sargan overidentification test is that the instruments are uncorrelated with the error term and that the excluded instruments are correctly excluded from the estimated equation. Under the null, the test statistic is distributed as chi-squared in the number of overidentifying restrictions. The p-value is in brackets.

$ The null hypothesis is that the block of institutional variables is exogenous. Under the null, the estimator used is efficient but it is inconsistent under the alternative hypothesis. The consistent estimator would be to consider all variables as endogenous and instrument them with lags. The p-value is in brackets.

§ The null hypothesis is that the block of macroeconomic variables is exogenous. Under the null, the most efficient estimator is fixed effects estimation taking all variables as exogenous. Under the alternative hypothesis the estimates are consistent. The p-value is in brackets.
3.3.3 Public wage determinants

We now turn to the analysis of the determinants of the growth rate of real public sector wages per employee, which are presented in columns (3) and (4) of Table 3.1. The estimations pass the Sargan test. When we include all regressors, the p-value of the test is low, albeit above 0.05. In the reduced specification the p-value is around 0.24.

The growth rate of real public sector wages per employee reacts positively to real private sector wages, with a coefficient between 0.6 and 0.7. On the other hand, it responds negatively to an increase in the ratio between public and private sector wages in line with our previous conjecture. Therefore, this correction mechanism adjusts public wages downward when the differential vis-à-vis private wages rises. Note that the absolute value of the coefficient is roughly three times higher than the one from the similar error correction component of the coefficient estimated in the private sector model. This means that three quarters of the adjustment is done via public sector wages.

The lagged dependent variable is statistically significant with a magnitude of around 0.24, denoting a higher degree of wage stickiness than in the private sector. The growth of public sector wages is not affected by any of the market variables. Regarding the explanatory fiscal variables, improvements in the budget balance increase the growth rate of nominal public sector wages. An increase in the budget balance ratio of 1 percentage point translates into an increase of the growth rate of public sector wages of around 0.1 percentage points. Increases in the tax wedge are associated with lower public sector wage growth. In terms of the pre-determined exogenous variables, there is a statistically significant positive effect of the percentage votes for left wing parties.

3.3.4 Robustness, system estimation

Given that the two wage variables are jointly determined, a more efficient approach is to perform a 3SLS estimation of the system of two equations. The drawback is that, if there is a problem with one of the equations all the parameter estimates are inconsistent. We start the estimation with all variables and then exclude the variables that are not statistically significant. According to the results shown in Table 3.2, we do not reject the validity of all the instruments at a 5 percent significance level.

We note that the results are close to the estimations of the single equations. The lagged dependent variable is significant in both equations with a higher value in the public sector. In addition, the error correction mechanism is also significant, but its magnitude in the private sector equation is somewhat higher than in the baseline estimation. Interestingly, the magnitude of the direct effects of wage growth in the other sector are
Table 3.2: System estimation (1973-2000)

<table>
<thead>
<tr>
<th></th>
<th>Private sector</th>
<th>Public sector</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Lagged dependent variable</td>
<td>0.082** (2.15)</td>
<td>0.166*** (4.27)</td>
</tr>
<tr>
<td>Error correction component</td>
<td>0.056*** (4.66)</td>
<td>-0.109*** (-7.55)</td>
</tr>
<tr>
<td>Growth rate of public sector wages</td>
<td>0.473*** (8.59)</td>
<td></td>
</tr>
<tr>
<td>Growth rate of private sector wages</td>
<td></td>
<td>0.881*** (8.11)</td>
</tr>
<tr>
<td>Δ Unemployment rate</td>
<td>-0.311*** (-2.80)</td>
<td></td>
</tr>
<tr>
<td>Growth rate of total factor productivity</td>
<td>0.276*** (2.61)</td>
<td></td>
</tr>
<tr>
<td>Δ Urbanization rate</td>
<td>1.148** (2.34)</td>
<td>-1.572** (-2.26)</td>
</tr>
<tr>
<td>Growth rate of public employment</td>
<td>0.172*** (2.59)</td>
<td></td>
</tr>
<tr>
<td>Tax wedge</td>
<td>0.208* (1.79)</td>
<td>-0.404** (-2.30)</td>
</tr>
<tr>
<td>Budget balance</td>
<td>0.103** (2.34)</td>
<td></td>
</tr>
<tr>
<td>Δ Benefit duration</td>
<td>-3.422 (-1.63)</td>
<td></td>
</tr>
<tr>
<td>Central bank independence</td>
<td>-2.071*** (-3.57)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>437 (18)</td>
<td>437 (18)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.341</td>
<td>0.310</td>
</tr>
<tr>
<td>Hansen-Sargan test #</td>
<td>103.1 (0.057)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: the coefficients are estimated using 3SLS. Endogenous variables: the change in unemployment rate, the change in the urbanisation rate, growth rate of total factor productivity, inflation rate, growth rate of real per worker public and private sector wages, growth rate of terms of trade, growth rate of hours per worker, change in tax wedge, the growth rate of public employment, budget balance and government debt. These endogenous variables are instrumented by the remaining pre-determined variables and two lags of all explanatory variables. # The null hypothesis of the Hansen-Sargan overidentification test is that the instruments are uncorrelated with the error term and that the excluded instruments are correctly excluded from the estimated equation. Under the null, the test statistic is distributed as chi-squared in the number of overidentifying restrictions (82). The p-values are in brackets.

The change in the unemployment rate, growth rate of total factor productivity, change in the urbanisation rate, growth rate of public sector wages, employment, and Central Bank independence are all significant determinants of private sector wages, with the coefficients having the same magnitude as previous estimates. The estimate of the effect of the budget balance on public sector wage growth is still 0.1, but the percentage of left wing votes is not significant. The tax wedge is now statistically significant for both private and public sector.

3.3.5 Robustness, including further dynamics

In our baseline estimations, the only dynamic element is the inclusion of the lagged dependent variable. Even if that coefficient is small one could argue that there might be direct effects of past explanatory variables on the regressors. Therefore, we included one lag of all explanatory variables in the regressions.
### CHAPTER 3. PUBLIC AND PRIVATE SECTOR WAGE INTERACTIONS

#### Table 3.3: Estimation with lags (1973-2000)

<table>
<thead>
<tr>
<th></th>
<th>Private sector</th>
<th>Public sector</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Lagged dependent variable</td>
<td>0.125** (0.033)</td>
<td>0.259*** (0.000)</td>
</tr>
<tr>
<td>Error correction component</td>
<td>0.038** (0.044)</td>
<td>-0.098*** (0.000)</td>
</tr>
<tr>
<td>Growth rate of public sector wages</td>
<td>0.262*** (0.000)</td>
<td>0.682** (0.011)</td>
</tr>
<tr>
<td>Growth rate of private sector wages</td>
<td>0.262*** (0.000)</td>
<td>0.682** (0.011)</td>
</tr>
<tr>
<td>Δ Unemployment rate</td>
<td>-0.121 (0.569)</td>
<td>-0.158 (0.811)</td>
</tr>
<tr>
<td>Inflation rate</td>
<td>-0.040 (0.595)</td>
<td>0.076 (0.517)</td>
</tr>
<tr>
<td>Growth rate of total factor productivity</td>
<td>0.338 (0.220)</td>
<td>-0.011 (0.982)</td>
</tr>
<tr>
<td>Δ Urbanization rate</td>
<td>-1.000 (0.622)</td>
<td>-1.370 (0.415)</td>
</tr>
<tr>
<td>Growth rate of terms of trade</td>
<td>0.068 (0.473)</td>
<td>0.009 (0.948)</td>
</tr>
<tr>
<td>Growth rate of hours per worker</td>
<td>-1.026 (0.285)</td>
<td>0.009 (0.948)</td>
</tr>
<tr>
<td>Growth rate of public employment</td>
<td>0.230 (0.168)</td>
<td>0.009 (0.948)</td>
</tr>
<tr>
<td>Tax wedge</td>
<td>0.155 (0.433)</td>
<td>-0.715 (0.140)</td>
</tr>
<tr>
<td>Budget balance</td>
<td>0.105 (0.585)</td>
<td>0.009 (0.948)</td>
</tr>
<tr>
<td>Government debt</td>
<td>-0.022 (0.349)</td>
<td>0.009 (0.948)</td>
</tr>
<tr>
<td>Δ Union density</td>
<td>0.001 (0.188)</td>
<td>-0.003 (0.186)</td>
</tr>
<tr>
<td>Δ Bargaining coordination</td>
<td>2.160 (0.128)</td>
<td>-3.199 (0.340)</td>
</tr>
<tr>
<td>Δ Benefit duration</td>
<td>-4.664 (0.118)</td>
<td>3.583 (0.529)</td>
</tr>
<tr>
<td>Δ Benefit replacement rate</td>
<td>-0.019 (0.633)</td>
<td>0.059 (0.396)</td>
</tr>
<tr>
<td>Δ Education</td>
<td>0.491 (0.702)</td>
<td>-0.272 (0.906)</td>
</tr>
<tr>
<td>Central bank independence</td>
<td>-2.340** (0.025)</td>
<td>0.009 (0.948)</td>
</tr>
<tr>
<td>Election year</td>
<td>-0.235 (0.712)</td>
<td>0.009 (0.948)</td>
</tr>
<tr>
<td>% Left wing votes</td>
<td>0.071 (0.199)</td>
<td>0.009 (0.948)</td>
</tr>
</tbody>
</table>

| Observations            | 437 (18)       | 437 (18)       |
| $R^2$                   | 0.398          | 0.126          |
| Zero effect of lagged variables% | 24.18 (0.062) | 9.76 (0.879) |
| Sargan test#            | 15.05 (0.719)  | 26.64 (0.032)  |
| Overidentifying restrictions | 19         | 15            |

Notes: the coefficients are estimated using 2SLS. Estimation included a lag of both endogenous and exogenous variables. The coefficient refers to the sum of the coefficients of the contemporaneous and lagged variable. In parenthesis is reported the p-value of the test that the sum of the coefficients is zero. For the lagged dependent variable and the error correction mechanism we present the p-value of the usual significance test. *, **, *** - statistically significant at the 10, 5, and 1 percent. Endogenous variables: the change in unemployment rate, the change in urbanisation rate, growth rate of total factor productivity, inflation rate, growth rate of real per worker public and private sector wages, growth rate of terms of trade, growth rate of hours per worker, change in tax wedge, the growth rate of public employment, budget balance and government debt. These endogenous variables are instrumented by the remaining pre-determined variables and two lags of all explanatory variables. # The null hypothesis of the Sargan overidentification test is that the instruments are uncorrelated with the error term and that the excluded instruments are correctly excluded from the estimated equation. Under the null, the test statistic is distributed as chi-squared in the number of overidentifying restrictions. % The null hypothesis is that the coefficients of all lagged explanatory variables are jointly equal to zero. The p-values are in brackets.

In general, the inclusion of lags does not carry much explanatory power in this case. Indeed, the R-square changes very little from the baseline estimation, and the test that all coefficients of the lagged explanatory variables are jointly equal to zero is not rejected at the 5 percent significance level for both equations. Consequently, not much is gained from
the inclusion of the above mentioned lags, as they just increase the standard deviations and reduce the significance of some variables.

As we are interested in the overall effect of a variable on the wage growth, in Table 3.3 we only report the sum of the two coefficients (contemporaneous and lagged) and the p-value of the test that the sum of the two coefficients is different from zero. The main results from the baseline specification remain; notably, there is a spillover between private and public wages, through the direct effect and via the error correction mechanism, with similar magnitude to the baseline results.\textsuperscript{7}

3.4 Analytical framework

3.4.1 Model

The empirical section offered several important conclusions, useful for the setting up of the model. First, the relation between public and private wages is bi-directional, with market forces and productivity having an effect on private sector wage growth, which is then followed by the public sector. Second, developments in the public sector wages caused by, for instance, political issues or by the need for fiscal tightening also affect the private sector wages. Moreover, and in addition to the contemporaneous relation, there is also an error correction mechanism that corrects the gap between the wages in the two sectors. Most of such correction occurs in the public sector.

In this section we set up a dynamic labour market equilibrium model that captures the qualitative essence of the interaction between private and public sector wage growth. The purpose is twofold. The first objective is to uncover the transmission mechanisms of fiscal policy through the labour market. The second is to find out if the model with only frictions in the labour market is able to replicate the findings of the empirical section.

The model is an extension of the model in Chapter 2. The economy has a public and a private sector and search and matching frictions, along the lines of Pissarides (2000). The unemployed can only search for a job in one sector. There is some micro-econometric evidence on the assumption of directed search between the private and public sector. Blank (1985) finds that sectoral choice is influenced by wage comparison. Heitmueller (2006) is able to quantify this effect and finds that an increase in 1 percent in the wages in the public sector relative to the private sector increases the probability of choosing public sector employment by 1.3 and 2.9 percent respectively for men and for women. The model has several differences from the one in Chapter 2, that make it more realistic: it

\textsuperscript{7}We have also performed alternative robustness checks: estimations with only the subset of European Union and euro area countries, estimations with only macroeconomic variables and a longer sample, GMM Arellano and Bond estimation and also for the nominal wages. See Afonso and Gomes (2008)
features exogenous growth in the private sector technology; the public sector wage bill is financed through a distortionary labour income taxation; the production function in the private sector has a more general form with diminishing marginal returns and we analyse different fiscal rules for the setting of public sector vacancies and wages, as opposed to discussing the optimal policy.

General setting

Public sector variables are denoted with superscript $g$ while private sector variables are denoted by superscript $p$. Time is denoted by $t$. The labour force consists of many individuals $j \in [0, 1]$. A proportion $u_t$ are unemployed, while the remainder are working either in the public ($l^p_t$) or in the private ($l^g_t$) sector

$$1 = l^p_t + l^g_t + u_t. \quad (3.4)$$

The presence of search and matching frictions in the labour market prevents some unemployed individuals from finding work. The evolution of public and private sector employment depends on the number of new matches $m^p_t$ and $m^g_t$ and on separations in each sector. We consider that, in each period, a constant fraction of jobs is destroyed, and this fraction ($\lambda$) might be different between the two sectors

$$l^i_{t+1} = (1 - \lambda^i)l^i_t + m^i_t, \quad i = p, g. \quad (3.5)$$

We assume that the unemployed choose in which sector they want to conduct their search, so $u_t^i$ represents the number of unemployed searching in sector $i$. The number of matches formed in each period is determined by two Cobb-Douglas matching functions:

$$m^i_t = \bar{m}u^i_t \eta^i v^i_t (1 - \eta^i), \quad i = p, g. \quad (3.6)$$

We define the share of unemployed searching for a public sector job as $s_t = \frac{u^p_t}{u_t}$. From the matching functions we can define the probabilities of vacancies being filled $q^i_t$, the job-finding rates conditional on searching in a particular sector, $p^i_t$, and the unconditional job-finding rates, $f^i_t$:

$$q^i_t = \frac{m^i_t}{u^i_t}, \quad p^i_t = \frac{m^i_t}{u_t}, \quad f^i_t = \frac{m^i_t}{u_t}, \quad i = p, g.$$ 

Households

In the presence of unemployment risk we could observe consumption differences across different individuals. As in Merz (1995), we assume all the income of the household members is pooled so that private consumption is equalised within the household.
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The household is infinitely-lived and has preferences over the private consumption good, $c_t$, and a public consumption good $g_t$

$$E_0 \sum_{t=0}^{\infty} \beta^t (\ln c_t + \zeta \ln g_t), \quad (3.7)$$

where $\beta \in (0, 1)$ is the discount factor. The budget constraint in period $t$ is given by

$$c_t + B_t = (1 + r_{t-1})B_{t-1} + (1 - \tau_t)w_t^p l_t^p + (1 - \tau_t)w_t^g l_t^g + z_tu_t + \Pi_t. \quad (3.8)$$

$r_{t-1}$ is the real interest rate from period $t-1$ to $t$, and $B_{t-1}$ are the holdings of one period bonds. $(1 - \tau_t)w_t^p l_t^p$ is the wage income from the members working in sector $i$, net of taxes, being $\tau$ the distortory tax rate. The unemployed members receive unemployment benefits $z_t$. Finally, $\Pi_t$ encompasses all lump sum transfers from the firm.

The household chooses consumption and bond holdings to maximize the expected lifetime utility subject to the sequence of budget constraints, taking the public consumption good as given. The solution is the consumption Euler equation:

$$1 = \beta(1 + r_t)E_t[\frac{c_t}{c_{t+1}}]. \quad (3.9)$$

Workers

The value to the household of each member depends on their current state. The value of being employed is given by:

$$W_t^i = (1 - \tau_t)w_t^i + E_t[\beta_{t+1}[(1 - \lambda^i)W_{t+1}^i + \lambda^i U_{t+1}], \ i = p, g \quad (3.10)$$

where $\beta_{t+1} = \beta E_t[\frac{\sigma_t}{\sigma_{t+1}}]$ is the stochastic discount factor. The value of being employed depends on the current wage as well as on the continuation value of the job, which depends on the separation probability in each sector. Under the assumption of directed search the agents are either searching in the private or in the public sector. The value functions are given by

$$U_t^i = z_t + E_t[\beta_{t+1}[p_t^i W_{t+1}^i + (1 - p_t^i)U_{t+1}], \ i = p, g. \quad (3.11)$$

The value of unemployment depends on the level of unemployment benefits and on the probabilities of finding a job in the two sectors. Optimality implies the existence of movements between the two sectors to guarantee that there is no additional gain of searching in one sector vis-à-vis the other

$$U_t^p = U_t^g = U_t. \quad (3.12)$$
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This equality determines the share of unemployed searching in each sector, and the respective expression is implicitly given by

$$m_t^p E_t[\frac{W_t^p - U_{t+1}}{(1 - s_t)}] = m_t^q E_t[\frac{W_t^q - U_{t+1}}{s_t}].$$ (3.13)

The optimal search of public sector jobs increases with the number of vacancies in the public sector and the value of a such a job, which depends positively on the public sector wages and negatively on the separation probability.

Firms

The private sector representative firm hires labour to produce the private consumption good. The production function depends on labour, but part of the resources produced have to be used to pay for the cost of posting vacancies $c_{t+1}^p$,

$$c_t = a_t^p (P_t^{(1 - \alpha)} - c_{t+1}^p).$$ (3.14)

The technology, $a_t^p$, has a unit root and grows at an average rate on $\gamma$. Its law of motion is given by

$$\ln a_t^p = \ln a_{t-1}^p + \gamma + \epsilon_t.$$ (3.15)

The firm's objective is to maximize the present discounted value of profits given by

$$E_t \sum_{k=0}^{\infty} \beta_k a_{t+k}^p (P_{t+k}^{(1 - \alpha)} - c_{t+k}^p v_{t+k}^p) - w_{t+k}^p L_{t+k}^p.$$ (3.16)

and faces the law of motion for private sector employment given by

$$L_{t+1}^p = (1 - \lambda^p) L_t^p + q_t^p v_t^p.$$ (3.17)

The firm takes the probability of filling a vacancy, $q_t^p$, as given. At any given point the level of employment is predetermined and the firm can only control the number of vacancies it posts. The solution to the problem is given by equation (3.18)

$$\frac{c_t^p}{q_t^p} = E_t \beta_{t+1} a_{t+1} \frac{a_{t+1}}{a_t} [(1 - \alpha) L_{t+1}^{P-\alpha} - w_{t+1}^p L_{t+1}^{P-\alpha} + (1 - \lambda^p) \frac{c_t^p}{q_{t+1}}].$$ (3.18)

The optimality condition of the firm states that the expected cost of hiring a worker must equal its expected return. The benefits of hiring an extra worker is the discounted value of the expected difference between its marginal productivity and its wage and the continuation value, knowing that with probability $\lambda^p$ the match can be destroyed.

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Private sector wage bargaining

We consider that the private sector wage is the outcome of a Nash bargaining between workers and firms,

$$w^p_t = \arg \max_{w^p_t} (W^p_t - U_t)^b(J_t)^{1-b},$$  \hspace{1cm} (3.19)

where $b$ is the bargaining power of the unemployed and $J_t$ is the value of the marginal job for the firm, given by the following expression

$$J_t = (1 - \alpha)\ell_t^{\alpha-\alpha} - w^p_t + E_t \beta_{t,t+1}[(1 - \lambda^p)J_{t+1}].$$  \hspace{1cm} (3.20)

The Nash bargaining solution is given by:

$$(W^p_t - U_t) = \frac{b(1 - \tau_t)}{1 - b\tau_t}(W^p_t - U_t + J_t).$$  \hspace{1cm} (3.21)

In the presence of distortionary taxes the share of the surplus going to the worker is lower than its bargaining power. The reason is that for every unit that the firm gives up in favour of the worker, the pair lose a fraction $\tau$ to the government. So they economise on their tax payments by agreeing a lower wage.

Government

The government produces its consumption good using a linear technology on labour. As in the private sector, the costs of posting vacancies are deduced from production

$$g_t = \ell_t - \zeta^g v^g_t.$$  \hspace{1cm} (3.22)

It sets a labour income tax to finance the wage bill and the unemployment benefits

$$\tau_t(w^p_t | \ell_t^p) = (1 - \tau_t)(w^p_t | \ell_t^p) + z_t u_t,$$  \hspace{1cm} (3.23)

and the unemployment benefits are given by

$$z_t = za^p_t.$$  \hspace{1cm} (3.24)

Finally, the government follows a policy for public sector vacancies and public sector wages $\{v^g_t, w^g_{t+1}\}_{t=0}^{\infty}$. We assume the government sets the wage one period in advance, at the time it posts the vacancies. As $s_t$ is determined based on the expected value of both public and private sector wage in $t+1$, the current period public sector wage only affects
the current level of taxes. We assume the following rule for public sector wages:

\[
\frac{w^g_{t+1}}{w^g_t} = E_t[\frac{w^p_{t+1}}{w^p_t}] + \kappa_1[L^g_{t-1} - 1 - \psi_t] + \epsilon_t^w. \tag{3.25}
\]

In every period, the government sets its wage for the next period based on the expected growth of private sector wages and on an error correction mechanism mimicking the public wage premium in (3.3), that adjusts the differences from the actual to the target public sector wage premium ($\psi$). Public sector vacancies are set at their steady state level, designed to target a steady state level of public sector employment

\[
v^p_t = \bar{v}^p + \epsilon_t^v. \tag{3.26}
\]

Both public sector vacancies and wages are subject to shocks. We can interpret a shock to wages ($\epsilon_t^w$) as a short-run phenomenon coming from the need for fiscal tightness, because of pressure from the trade union or arising from a change in government.8

### 3.4.2 Calibration

We calibrate the model at a quarterly frequency to be close to the UK economy. Figure 3.1 shows the level of government employment and the job-separation and job-finding rates in the two sectors. The data are taken from the UK Labour Force Survey.9

We calibrate the steady-state public sector vacancies such that the steady-state employment in that sector is 20% of the total labour force. The separation rate in the public sector is set to 1%, half the one in the private sector (2%). The public sector wage is set such that in steady-state, the public sector wage premium is equal to 4%. This value is in line with several empirical estimates (see Gregory and Borland (1999) for an overview of the literature).

We also calibrate the two matching functions differently. We set $\eta^p$ equal to 0.5 as it is common in the literature. In contrast, $\eta^g$ is set equal to 0.2, which implies that vacancies are relatively more important than the pool of unemployed in the public sector matching. This was the value found in an estimation for the United States in Chapter 2. The parameters $\bar{m}^i$ are set in such a way that the duration of a vacancy is 12 weeks for the private sector and 16 weeks for the public sector. These values are taken from two studies for the United Kingdom by the National Audit Office (2009) and the CIPD (2009). The latter study also estimates that the average cost of recruiting a worker is between £4600 and £5800. This corresponds to between 10 and 12 weeks of the median income in United Kingdom (£479 according to the ONS). In the public sector, the costs

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8The model written in efficiency units can be found in Appendix.

9See Chapter 6 for a detailed study on UK labour market flows.
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Figure 3.1: Evidence for the United Kingdom

Note: The data is taken from the Labour Force Survey. Government employment includes employment in local and central government, health authorities, universities and armed forces.

of recruiting are between 20% to 80% lower than for the total economy, depending on occupation category. We therefore set $\psi^p$ to be such that the recruitment cost per employee in the private sector is equivalent to wages paid over three-month period and $\psi^g$ such that the cost of recruitment per hire in the public sector is 20 percent lower than the private sector.

The unemployment benefit is set around 0.40, which implies a net replacement rate around 0.7 while $\alpha$ is set to 0.3. The discount factor is set to 0.985 and the quarterly growth rate of technology to 0.005, implying a steady-state interest rate of 4 per cent a year. As there is not much empirical evidence on the coefficient of the private sector wage bargaining we set it such that the equilibrium unemployment rate is around 7%, close to the average unemployment rate of the last 20 years in the United Kingdom. The baseline value of $\kappa$ is set at $-0.025$, which implies an annual correction of around $-0.10$.

<table>
<thead>
<tr>
<th>Table 3.4: Baseline Calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Structural Parameters</strong></td>
</tr>
<tr>
<td>$\eta^p$ 0.50  $\zeta^p$ 0.75  $m^p$ 0.59  $\lambda^p$ 0.02  $\gamma$ 0.005</td>
</tr>
<tr>
<td>$\eta^g$ 0.20  $\zeta^g$ 0.50  $m^g$ 0.49  $\lambda^g$ 0.01  $\beta$ 0.985</td>
</tr>
<tr>
<td>$\alpha$ 0.30  $\bar{b}$ 0.43  $\psi$ 0.04  $z$ 0.40  $\kappa$ $-0.05$</td>
</tr>
<tr>
<td><strong>Steady-state values</strong></td>
</tr>
<tr>
<td>$\bar{f}^p$ 0.20  $\bar{q}^p$ 1.01  $\tilde{a}$ 0.07  $\tilde{s}$ 0.42  $\bar{r}$ 0.26</td>
</tr>
<tr>
<td>$\bar{f}^g$ 0.03  $\bar{q}^g$ 0.81  $\bar{b}$ 0.20  $\bar{w}$ 1.04  $(1-\bar{r})\bar{w}$ 0.72</td>
</tr>
<tr>
<td><strong>Shocks</strong></td>
</tr>
<tr>
<td>$\rho^w$ 0.65  $\rho^p$ 0.80  $\rho^g$ 0.70</td>
</tr>
<tr>
<td>$\sigma_w$ 0.0054  $\sigma_v$ 0.0009  $\sigma_a$ 0.0061</td>
</tr>
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<td><strong>Moments at annual frequency</strong></td>
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</table>

<table>
<thead>
<tr>
<th>Standard deviation</th>
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<td>Private wage growth</td>
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<tr>
<td>Public wage growth</td>
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<td>Public employment growth</td>
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<table>
<thead>
<tr>
<th>Autocorrelation</th>
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</thead>
<tbody>
<tr>
<td>Private wage growth</td>
</tr>
<tr>
<td>Public wage growth</td>
</tr>
<tr>
<td>Public employment growth</td>
</tr>
</tbody>
</table>
the value found in the empirical section. Much of the analysis compares the responses of the model with alternative values for the error correction mechanism. Overall, the calibration implies a steady-state overall job-finding rate of 0.23: 0.20 in the private and 0.03 in the public sector.

Finally the standard deviation and the autocorrelation of the shocks are calibrated such that the standard deviations and the autocorrelation of the annual growth rates of public and private wages and public employment are close to the ones from the United Kingdom and from the average of the OECD countries used in the estimation. Table 3.4 summarises the baseline calibration, the implied steady-state values of the relevant variables and the moments of the growth rates of public sector employment and wages and private sector wages.

3.4.3 Simulation

Figures 3.2, 3.3 and 3.4 show the impulse responses of the private and public wage growth, the public-to-private wage ratio, the tax rate and of the unemployment rate to the three shocks: public sector wages ($\epsilon^{pw}_t$), public sector vacancies ($\epsilon^{v}_t$), and technology ($\epsilon^{t}_t$).

A positive shock to public sector wage growth has a direct spillover effect to the private sector wage growth, mainly through three channels. First, it increases the value of being unemployed because if those individuals are hired by the public sector they get paid more. This effect is amplified because of the endogenous increase of the share of unemployed searching for public sector jobs. Second, as the shock crowds out private sector employment, it raises the marginal productivity and consequently private sector wages. Finally, the increase in the tax rate necessary to finance the wage bill, has two opposite effects. On the one hand, it reduces the match surplus and raises the private sector wage. On the other hand, it reduces the share of the surplus going to the worker, which puts a downward pressure on wages. The contemporaneous and annual elasticity of private wages with respect to public sector wages is around 0.10.

The subsequent dynamics are driven by the error correction mechanism. As the premium of working in the public sector increases relative to the target, there is a correction of public sector wages that, after 6 quarters have a growth rate below the long-run value. This adjustment is quicker the higher the magnitude of the error correction coefficient ($\kappa$). In addition, an increase in the unemployment rate occurs after the shock.

A positive shock to public sector vacancies initially raises private sector wage growth (see Figure 3.3). The annual elasticity is around 0.3. Both the tax and the bargaining channel drive the wages up but, additionally, public employment crowds out the employment in the private sector raising the average productivity of workers which serves as a
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Figure 3.2: Response to a public sector wage shock

Note: the solid, dashed and dotted lines corresponds to the case where $\kappa$ is -0.025, -0.05 and -0.10 respectively. The growth rates of wages are expressed in annual terms.

Figure 3.3: Response to a public sector employment shock

Note: the solid, dashed and dotted lines corresponds to the case where $\kappa$ is -0.025, -0.05 and -0.10 respectively. The growth rates of wages are expressed in annual terms.
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Figure 3.4: Response to a technology shock

Note: the solid, dashed and dotted lines correspond to the case where $\kappa$ is $-0.025$, $-0.05$ and $-0.10$ respectively. The growth rates of wages and technology are expressed in annual terms.

reference for the bargaining process. The increase in the private sector wages reduces the premium paid in the public sector relative to the target, therefore, after the initial period the public sector wages grow above the average to catch up with the private sector.

Regarding a technology shock, depicted in Figure 3.4, the private sector wage growth increases substantially and contemporaneously, and stays above average for several periods. As the public-private wage premium is reduced, the wages in the public sector grow at a faster pace in the subsequent periods.

3.4.4 Reconciling the model with the data

The model we set up does not have a straightforward connection with the estimated equation, nor are the elasticities necessarily comparable. The empirical exercise uses annual data and the IV estimation retrieves the effect on the wage growth in one sector of the expected wage growth in the other. In the model, set at a quarterly frequency, we show how the variables respond to unexpected shocks in technology, public sector wages and employment.

In order to reconcile the two, we simulate the model to generate quarterly observations, aggregate data into annual frequency and then perform IV regressions of the type used in section 3.2. We estimate 5000 regressions with 100 observations each, as well as 1000 regressions with 500 observations. Table 3.5 shows the results.
PART I. PUBLIC SECTOR EMPLOYMENT AND WAGES

Table 3.5: IV estimation with simulated data

<table>
<thead>
<tr>
<th></th>
<th>Private sector</th>
<th></th>
<th>Public sector</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Nobs=100</td>
<td>Nobs=500</td>
<td>Nobs=100</td>
<td>Nobs=500</td>
</tr>
<tr>
<td></td>
<td>Nreg=5000</td>
<td>Nreg=1000</td>
<td>Nreg=5000</td>
<td>Nreg=1000</td>
</tr>
<tr>
<td>Lagged dependent variable</td>
<td>-0.139 (0.083)</td>
<td>-0.058 (0.059)</td>
<td>0.243 (0.090)</td>
<td>0.230 (0.045)</td>
</tr>
<tr>
<td>Error correction component</td>
<td>0.018 (0.021)</td>
<td>0.016 (0.010)</td>
<td>-0.103 (0.043)</td>
<td>-0.075 (0.016)</td>
</tr>
<tr>
<td>Growth rate of public sector wages</td>
<td>0.192 (0.125)</td>
<td>0.210 (0.073)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growth rate of private sector wages</td>
<td></td>
<td></td>
<td>0.697 (0.267)</td>
<td>0.857 (0.159)</td>
</tr>
<tr>
<td>Growth rate of TFP</td>
<td>0.807 (0.164)</td>
<td>0.626 (0.111)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growth rate of public employment</td>
<td>-0.027 (0.083)</td>
<td>-0.051 (0.042)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.867</td>
<td>0.801</td>
<td>0.294</td>
<td>0.199</td>
</tr>
</tbody>
</table>

Notes: the coefficients are estimated using 2SLS. Endogenous variables: total factor productivity, growth rate of real per worker private and public sector sector wages, growth rate of public employment. They are instrumented by two lags of all explanatory variables. The standard deviation of the estimated coefficients are in parentheses.

In the equation determining the public sector wage growth the three coefficients are very close to the ones estimated for the OECD countries (see Table 3.1). The error correction mechanism is between -0.08 and -0.10 and the response to expected wages is also around 0.7. Another similar feature is that the R-square of the estimation for the public sector wage growth tends to be quite low.

In the equation determining the private sector wage growth, the coefficient of public sector wages (0.2) and the error correction mechanism (0.02), all have magnitudes similar to the ones we estimated for the OECD countries. The coefficient of total factor productivity growth is slightly bigger, which is expected since it is the only source of fluctuations directly affecting private sector wages in the model. This also translates into a high R-square. The autocorrelation coefficient is slightly negative, which means that for OECD countries there must be some other sources of autocorrelation, perhaps wage stickiness. Still, the difference is relatively small. On the other hand, the estimated coefficient of the growth rate of public employment tends to be close to zero.

3.5 Conclusion

The purpose of this chapter was to analyse the interactions between public and private sector wages per employee in OECD countries, and to uncover the determinants of public and private sector wage growth. We find that the public sector wage growth is mainly driven by private sector wages and the government budget balance.

Regarding the private sector wage growth, we find that it is influenced by the unemployment rate, total factor productivity and urbanization rate. More important, public sector wages and employment also affect private sector wage growth. The empirical estimates show that a 1 percent increase in public sector wages raises the wages in the private sector by 0.2 percent. The error correction mechanism is between -0.08 and -0.10 and the response to expected wages is also around 0.7. Another similar feature is that the R-square of the estimation for the public sector wage growth tends to be quite low.

In the equation determining the private sector wage growth, the coefficient of public sector wages (0.2) and the error correction mechanism (0.02), all have magnitudes similar to the ones we estimated for the OECD countries. The coefficient of total factor productivity growth is slightly bigger, which is expected since it is the only source of fluctuations directly affecting private sector wages in the model. This also translates into a high R-square. The autocorrelation coefficient is slightly negative, which means that for OECD countries there must be some other sources of autocorrelation, perhaps wage stickiness. Still, the difference is relatively small. On the other hand, the estimated coefficient of the growth rate of public employment tends to be close to zero.
sector by 0.3 percent, while the regressions with simulated data point to an elasticity of
around 0.2 percent.

The dynamic labour market equilibrium model that we set up captures the main essence of the interaction between public and private wages, and is quantitatively consistent with the main estimation findings. This is true even if it abstracts from other channels that may be relevant. For instance, higher public sector wages might translate into higher demand, increasing the pressure on the private sector labour market. Alternatively, public sector wage growth may also carry a signal to the private sector about the government's expectations for inflation. In addition, in the presence of on-the-job search, the transmission mechanism of public sector wages can be amplified.

In light of our results, and as discussed in Pedersen et al. (1990), governments could use their role as an employer to reduce public sector wages. This policy, in addition to reducing the tax burden necessary to finance government spending, would have a downward impact on private sector wages, unemployment and, possibly, on inflation. Nevertheless, one has to bear in mind the issue of the composition of public sector employment. It is a known fact that high-skilled workers have a negative premium from working in the public sector (Postel-Vinay and Turon (2007)), which makes it harder for the government to recruit them (Nickell and Quintini (2002)). Therefore, wage moderation for this group could worsen the problem and make retention of high-skilled workers even harder in the public sector.
3.6 Appendix

3.6.1 Model in efficiency units

As the technology has a unit root with drift, the model does not have a steady state. There is a balance growth path in which employment, unemployment, vacancies and labour market flows are constant; and in which wages, consumption, the value of employment and unemployment and the value of a job for the firm are growing at rate \( \gamma \) values. We define the variables in efficiency units with tilde (as a ratio of technology),

\[
\tilde{w}_t = \frac{w_t}{a_t}, \quad \tilde{w}_{t+1} = \frac{w_{t+1}}{a_t}, \quad \tilde{c}_t = \frac{c_t}{a_t}, \quad \tilde{W}_t = \frac{W_t}{a_t}, \quad \tilde{U}_t = \frac{U_t}{a_t}, \quad \tilde{J}_t = \frac{J_t}{a_t}, \quad \tilde{z}_t = z, \quad \tilde{e}_t = e_t.
\]

We can re-write the non-stationary equations in efficiency units. The Euler equation becomes:

\[
1 = \beta(1 + r_t)E_t[\frac{\tilde{c}_{t+1}}{a_{t+1}}]. \tag{A3.1}
\]

The value functions become:

\[
W_t^p = (1 - r_t)\tilde{w}_t^p \frac{a_{t+1}}{a_t} + E_t\beta_{t,t+1} \frac{a_{t+1}}{a_t}[(1 - \lambda^p)\tilde{W}^q_{t+1} + \lambda^p\tilde{U}_{t+1}], i = p, g, \tag{A3.2}
\]

\[
\tilde{W}_t^p = (1 - r_t)\tilde{w}_t^p + E_t\beta_{t,t+1} \frac{a_{t+1}}{a_t}[(1 - \lambda^p)\tilde{W}^q_{t+1} + \lambda^p\tilde{U}_{t+1}], i = p, g, \tag{A3.3}
\]

\[
\tilde{U}_t = z + E_t\beta_{t,t+1} \frac{a_{t+1}}{a_t}[(1 - \lambda^p)\tilde{W}^q_{t+1} + \lambda^p\tilde{U}_{t+1}], i = p, g, \tag{A3.4}
\]

\[
\tilde{J}_t = (1 - \alpha)\tilde{I}_t^p - \tilde{w}_t^p + E_t\beta_{t,t+1} \frac{a_{t+1}}{a_t}[(1 - \lambda^p)\tilde{J}_{t+1}]. \tag{A3.5}
\]

The first order condition from the firm and the Nash bargaining becomes

\[
\frac{\zeta^p}{\tilde{q}_t} = E_t\beta_{t,t+1} \frac{a_{t+1}}{a_t}[(1 - \alpha)\tilde{I}_{t+1}^p - \tilde{w}_{t+1}^p + (1 - \lambda^p)\tilde{J}_{t+1}], \tag{A3.6}
\]

\[
(\tilde{W}_t^p - \tilde{U}_t) = \frac{b(1 - \tau_t)}{1 - b\tau_t}(\tilde{W}_t^p - \tilde{U}_t + \tilde{J}_t). \tag{A3.7}
\]

Finally, the equations for the public sector become:

\[
\tau_t(\tilde{w}_t^p) = (1 - \tau_t)\left(\tilde{w}_t^p + zu_t\right), \tag{A3.8}
\]

\[
\frac{\tilde{w}_{t+1} - \tilde{w}_t}{\tilde{a}_{t-1}} = E_t[\frac{\tilde{w}_{t+1} - \tilde{w}_t}{\tilde{a}_t} + \kappa(\tilde{w}_{t-1} - 1) + e_t^{w}], \tag{A3.9}
\]
where

\[ \frac{a_t^p}{a_{t-1}^p} = \exp(\gamma + \epsilon_t^p). \]  \hspace{1cm} (A3.10)

### 3.6.2 Data

<table>
<thead>
<tr>
<th>Table A3.1: Summary statistics and sources</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
</tr>
<tr>
<td>-------------------------------------------</td>
</tr>
<tr>
<td>Growth rate of real private sector wages</td>
</tr>
<tr>
<td>Growth rate of real public sector wages</td>
</tr>
<tr>
<td>Error correction mechanism</td>
</tr>
<tr>
<td>Δ Unemployment rate</td>
</tr>
<tr>
<td>Total factor productivity growth rate</td>
</tr>
<tr>
<td>Δ Urbanisation rate</td>
</tr>
<tr>
<td>Inflation rate</td>
</tr>
<tr>
<td>Terms of Trade growth rate</td>
</tr>
<tr>
<td>Hours per worker growth rate</td>
</tr>
<tr>
<td>Growth rate of public employment</td>
</tr>
<tr>
<td>Budget Balance</td>
</tr>
<tr>
<td>Government Debt</td>
</tr>
<tr>
<td>Δ Tax wedge</td>
</tr>
<tr>
<td>Δ Union density</td>
</tr>
<tr>
<td>Δ Bargaining Coordination</td>
</tr>
<tr>
<td>Δ Benefit duration</td>
</tr>
<tr>
<td>Δ Replacement rate</td>
</tr>
<tr>
<td>Δ Education attainment</td>
</tr>
<tr>
<td>Central bank independence</td>
</tr>
<tr>
<td>Election</td>
</tr>
<tr>
<td>% Left wing votes</td>
</tr>
</tbody>
</table>

**Note:** I use two datasets that expand the Labour Market Institutions Database created by Nickell, Nunnziata, and Ochel (2005): the BHHS expanded by Baker, Glyn, Howell, and Schmidt (2003) and the CEP - OECD Institutions Data Set by Nickell (2006). The comparatives party dataset was created by Duane Swank and it is available on [http://www.msu.edu/polisci/Swank.htm](http://www.msu.edu/polisci/Swank.htm).

### Employment and wage variables

The data on public employment and wages is taken from the OECD (Economic Outlook database). For most countries there is information on Government employment (EG). To calculate the per employee wage we divide Government final wage consumption expenditure (CGW) by Government employment. To get the wage in real terms we deflate it using the Private final consumption expenditure deflator (PCP).

We also have the value for the Compensation of employees (WSSS) and Total employment (ET), which refers to the total economy. We define Private sector compensation as the total Compensation of employees minus the Government final wage consumption (WSSS-CGW). We define the private employment (EP) as Total employment minus Government employment minus Self Employed (ES): EP = ET - EG - ES. The private sector nominal wage per employee is Private sector compensation divided by private sector employees.
For the case of Australia, there is no information on government employment but there is on *Private sector employment* and *Compensation of private sector employees*. In this case, *Government employment* is defined as *Total employment* minus *Private sector employment* and Compensation of public sector employees defined as the value of *Compensation of employees* minus *Compensation of private sector employees*.

**Other variables**

- Benefit replacement rate - Benefit entitlement before tax as a percentage of previous earnings before tax. Source: CEP.
- Benefit duration index. Source: CEP.
- Coordination index - Captures the degree of consensus between actors in collective bargaining (1 low, 3 high). Source: CEP.
- Trade union density - Ratio of total reported union members (minus retired and unemployed) to all salaried employees. Source: CEP.
- Educational attainment - Average years of schooling from total population aged 15 and over (taken from Barro and Lee dataset and intrapolated). Source: CEP.
- Tax wedge - Payroll tax plus income tax plus the consumption tax rate. Source: BHHS.
- Productivity growth - Growth rate of productivity per worker. Source: OECD.
- Terms of trade - Growth rate of terms of trade. Source: BHHS.
- Urbanisation rate - Percentage of the population living in urban areas (taken from the World Bank World Development Indicators). Source: CEP.
- Inflation - Source: OECD.
- Unemployment rate - Source: CEP.
- Budget Balance - Government balance as percentage of GDP. Source: AMECO European Commission database, complemented with IMF data for early years.
- Government debt - Government debt as percentage of GDP. Source: AMECO European Commission database, complemented with IMF data for early years.
- Election year - Dummy if there was a parliamentary of presidential election. Source: Comparative parties dataset.
- Left wing - Percentage of left with votes of last parliamentary elections. Source: Comparative parties dataset.
- Central Bank Independence Index. Source: BHHS
Figure A3.1: Public and private sector wages in OECD countries
CHAPTER 3. PUBLIC AND PRIVATE SECTOR WAGE INTERACTIONS
Part II

Taxation and public capital
Chapter 4

Optimal labour and profit taxation and the supply of public capital

4.1 Introduction

Over the past four decades most developed countries have experienced similar trends in several government instruments, both on the expenditure and on the taxation side. In a nutshell, compared to the 1960s many governments are now allocating relatively more resources to government consumption than to public investment and shifted part of the taxation burden from corporate profits to labour income.

As shown in Figure 4.1, in the G7 countries government consumption has increased on average from 14 to 20 percent of GDP, while public investment has declined from 4.5 to below 3 percent of GDP. Such a decrease in public investment has implied a decline in the stock of public capital from 60 to 50 percent of GDP. At the same time the statutory corporate tax rate has declined on average by 20 percentage points, while the marginal labour income tax has increased by 15 percentage points.

Among the possible explanations we investigate whether such trends can be generated as the outcome of an optimal policy plan chosen by a benevolent government. The literature on optimal dynamic taxation, following Judd (1985) or Chamley (1986), has typically analyzed the choice of taxing labour or capital income where the provision of public expenditure is treated as exogenous. This is no coincidence. Under standard assumptions the choice of how much public good to provide does not influence the choice of how to spread the tax burden between different sources of income. However, since the taxation and the expenditure sides are analyzed independently, those models are limited

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1This chapter was written in co-authorship with Davide Debortoli from the University of California, San Diego.
2In appendix we show the variables disaggregated by country as well as for an average of 18 OECD countries. Most of them share the same trends.
PART II. TAXATION AND PUBLIC CAPITAL

in their ability to explain the co-movements between the composition of expenditure and the sources of taxation. Additionally, with a few exceptions, they focus on capital taxation while in reality perhaps their major component is the taxation of corporate profits.3

We build on the dynamic optimal taxation literature in considering a model where the government can choose how to allocate the public expenditure between two types of goods, namely public consumption and investment in productive public capital. Public expenditure is financed by levying taxes on labour income and corporate profits. The corporate profit tax plays a different role than a simple tax on capital income. Since public capital increases corporate profits, taxing corporate profits becomes a way to extract the private rents generated by public capital.

The presence of productive public capital and taxes on corporate profits generates a link between the choice of how to allocate expenditure and how to finance it. The provision of public capital affects the rents of firms, and alters the trade-off between taxing profits or wages. Similarly, the level and the composition of the tax rates affect the trade-off between government consumption and investment. In other words, the composition of expenditures and the structure of taxation are two interrelated choices.

As a result, in our model it is optimal for the government to have a positive profit tax also in steady-state. As a tax on capital income, the corporate profit tax distorts the accumulation of private capital. However, it also allows to extract the firms' rents that exist due to public capital. In the same spirit as in Correia (1996), in our model a factor of production, public capital, cannot be taxed directly. Then, taxing corporate profits becomes an indirect way of taxing public capital. As a result, the optimality of zero capital taxation established by Judd (1985) or Chamley (1986) is overturned. Under the baseline calibration, the steady-state corporate tax rate is 32 percent and implies an effective marginal tax rate on private capital of around 20 percent.

In light of the model we find three possible explanations for the trends in the fiscal instruments. One hypothesis is that public capital has become a less important input in the production function. This is a plausible scenario given the change in the economic structure towards a "weightless economy", more centered in services and knowledge-based (Quah (1998)). As the importance of public infrastructure declines, the government reduces public investment and raises government consumption. Additionally, as the firms enjoy fewer rents, the government also reduces the profit tax and compensates it with an increase in the taxation of labour income.

CHAPTER 4. OPTIMAL TAXATION AND THE SUPPLY OF PUBLIC CAPITAL

Figure 4.1: Taxes and allocation of public spending in the G7 countries

Note: The sources for the statutory and effective marginal tax rate are Michigan World tax database and Devereux, Lockwood, and Redoano (2008). The marginal labour income is from Mendoza, Razin, and Tesar (1994). The public investment and the government consumption series are from OECD. The estimates of public capital are from Kamps (2006). The countries included are: Canada, France, Germany, Italy, Japan, United Kingdom and United States.

The second explanation is that we are simply observing the transition to the steady state, starting from a low level of public capital. At the beginning, the Ramsey planner sets a labour income tax at low values to induce labour supply, and a high corporate tax for several periods. This tax structure together with a reduction in government consumption allows for high rates of public investment and rapid accumulation of public capital. Along the transition, public investment and the corporate tax go down, while government consumption and labour income tax experience the opposite trend.

Under the optimal plan, the four instruments are determined endogenously and they respond to changes in the parameters. However, the interdependence of the instruments has an important implication. Any exogenous factor affecting one of them will also affect the optimal choice of all three other instruments. We find that exogenous movements in any one instrument alters both the optimal level and the composition of spending and taxes. Although exogenous changes in only one instrument, cannot explain in isolation the three other trends, if one considers that globalization has simultaneously put a downward pressure of profit taxation (through international tax competition) and increasing pressure in government consumption (through the demand for public transfers), it would also lead to an increase in labour income tax and a decline in public investment.

This paper relates to other papers in the fiscal policy literature. Baxter and King (1993) analyzed the business cycle implications of exogenous changes in taxes and in
the supply of an unproductive public good and of productive public capital. We build on their framework in two dimensions. First, we consider that the government has two instruments available to raise revenue: labour income and profit taxation. Second, we consider the government chooses optimally the tax rates, as well as the allocation of spending.

Other authors have also explored the relation between profit and capital taxation. Abel (2007) finds that if firms can have immediate expensing, the effective tax rate on capital is always zero, independently of the corporate tax rate. The corporate tax does not, therefore, affect the investment decision of the firms, so the government can raise a substantial amount of revenue in a non-distortionary way by setting the maximum possible profit tax. Conesa and Domínguez (2009) also distinguish between corporate taxation and dividend taxation in a setting where the firms’ rents are generated by the presence of intangible capital. In their setting, corporate tax should be zero in every period. On the other hand, as the government has a non-distortionary dividend tax available, it should set it at the highest possible level, in order to capture the economic rents. In our framework this is not possible because the government has only one instrument available, and the extent to which profit tax is tied to the effective marginal tax rate on capital is exogenous. One interesting aspect of our framework is that the size of the rents is influenced by the government’s choices.

The chapter continues as follows. In Sections 4.2 and 4.3 we describe the model and characterize the solution of the optimal policy problem. In Sections 4.4 and 4.5 we calibrate the model and we analyze the different possible sources of the observed trends in fiscal policy instruments. Section 4.6 concludes the chapter.

### 4.2 Model

The model is a standard deterministic neoclassical growth model, augmented with productive public capital. The economy is populated by a representative infinitely lived household, whose standard preferences are given by

\[
\sum_{t=0}^{\infty} \beta^t u(c_t, g_t, n_t, p_t),
\]  

where \(c_t\) and \(n_t\) denote private consumption and hours worked. The utility function is also assumed to be increasing and concave in the amount of public consumption \((g_t)\) and public capital \((p_t)\) provided by the government.

The household rents capital \((k_t)\) and supplies labour to a representative firm, facing distortionary taxes on labour income \((\tau^L_t)\) and corporate profits \((\tau^P_t)\). Its period-by-period
budget constraint is then
\[ c_t + k_{t+1} = w_t n_t (1 - \tau^*_t) + (1 + r_t) k_t + \Upsilon_t \quad \forall t = 0, 1, 2, \ldots \tag{4.2} \]

where \( w_t \) denotes the hourly wage, \( r_t \) denotes the return on capital net of taxes and depreciation, and \( \Upsilon_t \) represents all the lump-sum transfers to the households in terms of profits or government subsidies.

Output is produced by a representative firm using labour, private and public capital. Following Arrow and Kurz (1970) we consider a constant return to scale production function \( y_t = f(k_t, p_t, n_t) \). Taking as given prices \((r_t, w_t)\) and policies \((\tau^*_t, p_t)\), firms choose the production factors to maximize their after-tax profits, given by
\[ (1 - \zeta) \left[ f(k_t, p_t, n_t) - w_t n_t - \delta k_t - \zeta r_t k_t \right] - (1 - \zeta) r_t k_t. \tag{4.3} \]

In writing equation (4.3) we assume that only a fraction \( \zeta \) of the financial cost of capital can be deduced from the tax base.\(^4\) We motivate this assumption on two grounds. First, the parameter \( \zeta \) introduces a wedge between the statutory tax and the effective marginal tax rate on capital, a feature consistent with what observed in the data. In the limiting case of \( \zeta = 0 \), the effective marginal tax rate and the statutory tax rate coincide. Second, the parameter \( \zeta \) may reflect the firm’s financing structure, divided between bonds and equity, which is assumed to be exogenous in our model. Typically, bond interest payments can be deduced from the tax base, while dividends to shareholders cannot.

In order for the firm’s problem to be well-defined, we impose a limit on corporate tax rate \( \tau^* < 1 \). Otherwise firm’s profits would always be negative, as can be seen in equation (4.3). Once this limit is imposed, given the positive externality produced by public capital, firm’s profits are strictly positive in equilibrium, despite the assumption of a perfectly competitive market.\(^5\)

The government provides the public good \( g_t \) and the public capital \( p_t \), raising taxes on labour income and corporate profits, subject to the balanced budget condition\(^6\)
\[ g_t + p_{t+1} - (1 - \delta^p) p_t = \tau^*_t (w_t n_t) + \tau^*_t (y_t - w_t n_t - \delta k_t - \zeta r_t k_t). \tag{4.4} \]

\(^4\)If all the financial costs of capital could be deduced from the tax base (i.e. when \( \zeta = 1 \)), the profit taxation would be non-distortionary and corporate taxes would always be used to its maximum extent.

\(^5\)Profits remain positive as long as the production function is not homogenous of degree one in private capital and labour. Even in that case, positive profits could arise in the presence of other frictions like monopolistic competitions and limited entry, not explicitly modeled here for simplicity.

\(^6\)Since the government can accumulate public capital, the balanced budget condition only limits the possibility of the government to borrow from the private sector. This assumption is made for simplicity and is largely irrelevant for our considerations.
Finally, the aggregate feasibility constraint is given by
\[ c_t + g_t + p_{t+1} - (1 - \delta^r)p_t + k_{t+1} - (1 - \delta^k)k_t = f(k_t, p_t, n_t). \] (4.5)

We can now define the competitive equilibrium as follows.

**Definition 2** Given an initial stock of private capital \((k_0)\) and public capital \((p_0)\), a competitive equilibrium is a feasible allocation \(\{c_t, k_{t+1}, n_t\}_{t=0}^{\infty}\), a policy \(\{p_{t+1}, g_t, \tau_t^p, \tau_t^n\}_{t=0}^{\infty}\) and a price system \(\{r_t, w_t\}_{t=0}^{\infty}\) such that (i) the allocation maximizes (4.1) subject to the sequence of constraints (4.2), a no-Ponzi scheme constraint, for given prices, policies and initial capital \(k_0\); (ii) In any period \(t\), firms maximize (4.3), given prices and public capital \(p_t\); (iii) the government satisfies its period by period budget constraint (4.4).

It then follows that, in addition to the feasibility constraint (4.5), the competitive equilibrium is characterized by the following relations:
\[ u_{c,t} = \beta u_{c,t+1} \left[ 1 + \frac{1 - \tau_{t+1}^p}{1 - \zeta \tau_{t+1}^n} (f_{k,t+1} - \delta^k) \right], \] (4.6)
\[ -\frac{u_{n,t}}{u_{c,t}} = f_{n,t}(1 - \tau_t^n), \] (4.7)
\[ g_t + p_{t+1} - (1 - \delta^p)p_t = \tau_t^n (w_t n_t) + \tau_t^p \left[ \frac{1 - \zeta}{1 - \zeta \tau_t^n} (f_{k,t} - \delta^k)k_t + f_{p,t}p_t \right]. \] (4.8)

where we used the firm’s optimality conditions to substitute for the equilibrium values of \(w_t\) and \(r_t\). Moreover, to obtain (4.8) we made use of the homogeneity of degree one of the production function.\(^7\)

Some considerations are in order. First, as indicated by the Euler Equation (4.6), the ratio \(\frac{1 - \tau_t^n}{1 - \zeta \tau_t^n}\) constitutes a wedge between the rate of intertemporal substitution and the marginal returns on capital. That ratio can, therefore, be considered as the effective marginal tax rate on private capital. Second, the government budget constraint (4.8) indicates that the tax base for corporate taxes is composed by two elements: the returns on private capital \((f_{k,t} - \delta^k)k_t\) and the returns on public capital \((f_{p,t}p_t)\). The presence of the latter term shows why taxing capital income is different than taxing corporate profits. Taxing corporate profits allows the government to appropriate part of the rents associated with the provision of public capital. This indicates that the supply of public capital and the corporate profit tax rate are two interrelated choices. In what follows, we characterize the choices of the composition of taxes and expenditures when policies are chosen optimally by a benevolent policymaker.

\(^7\)As usual, the household’s budget constraint is also satisfied, as implied by Walras’ Law.
CHAPTER 4. OPTIMAL TAXATION AND THE SUPPLY OF PUBLIC CAPITAL

4.3 Optimal fiscal policy problem

The goal of a Ramsey planner is to choose the best competitive equilibrium implied by a sequence of policies \( \{p_{t+1}, g_t, \tau^n_t, f^k_t\}_{t=0}^{\infty} \). The planner’s problem is therefore

\[
\max_{\{c_t, n_t, k_{t+1}, p_{t+1}, g_t, \tau^n_t, f^k_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, g_t, n_t, p_t)
\]

s.t. \( (4.5) - (4.8) \)

\( \tau^n < 1 \quad \forall t = 0, 1, ... 
\)

\( p_0, k_0 \) given

To better understand the interactions between the tax rates and the composition of public expenditure, we can look at the first-order optimality condition with respect to the corporate tax rate, given by

\[
\mu_{1,t} \left[ f_{p,t} P_t + \frac{1 - \zeta}{(1 - \zeta \tau^n_t)^2} \left( f_{k,t} - \delta^k \right) k_t \right] = \lambda_{t-1} u_{c,t} \left( f_{k,t} - \delta^k \right) \frac{1 - \zeta}{(1 - \zeta \tau^n_t)^2}, \tag{4.9}
\]

where \( \lambda_{t-1} \) and \( \mu_{1,t} \) represent the shadow values of relaxing constraints \((4.6)\) and \((4.8)\), respectively. The left-hand side of \((4.9)\) represents the marginal benefits of increasing profit taxes due to the higher tax revenues. An increase in \( \tau^n \) increases the revenues from public capital income (first term in the square brackets) and increases the effective marginal tax rate (as given by the ratio \( \frac{1 - \zeta}{(1 - \zeta \tau^n_t)^2} \)) applied to private capital income (last term in the brackets). An increase in \( \tau^n \) also generates some welfare costs due to the interest rate distortions, as indicated in the right-hand side of \((4.9)\). At an optimum, the planner equalizes these marginal costs and benefits. We can rewrite equation \((4.9)\) as

\[
\zeta \tau^n_t = 1 - \sqrt{\frac{(f_{k,t} - \delta^k) k_t (\lambda_{t-1} u_{c,t} - \mu_{1,t}) (1 - \zeta)}{\mu_{1,t} f_{p,t} P_t}}, \tag{4.10}
\]

There are three elements that affect the choice of the tax rate. The first one is the extent to which the profit tax is tied to the effective marginal tax rate. If \( \zeta = 1 \), the firm can deduce all costs of capital from the tax base, corporate taxation becomes non-distortionary and the optimal tax rate is 100 percent. If \( \zeta \neq 1 \), the tax rate is distortionary and the government chooses it by balancing two opposite effects. On the one hand, the more distortionary the tax rate is, captured by the multiplier of the Euler equation, the lower the tax rate. On the other hand, it is increasing on the size of the rents \( f_{p,t} P_t \) and on the shadow value of government revenue \( \mu_{1,t} \). From these considerations it follows that

\( ^8 \) For illustrative purposes only, we are assuming that the constraint \( \tau^n < 1 \) is not binding. Notice that the latter constraint will always be binding in \( t = 0 \). In that period, being \( \lambda_{-1} = 0 \), equation \( 4.9 \) implies that by raising corporate profit taxes the government can achieve the Pareto-optimal solution where the constraint \( 4.8 \) is never binding.
the optimal level of corporate taxes depends on the amount of public capital supplied. For a given level of public expenditure, the higher the share of public investment, the higher the benefits from increasing \( \tau_t^n \).

Also, the composition of public spending depends on the tax rates. This is clear when combining the first order conditions of government consumption and public capital, given by

\[
U_{g,t} = \beta u_{p,t+1} + \beta \mu_{2,t+1} [(1 - \delta^p) + f_{p,t+1}] + \beta \mu_{3,t}(1 - \tau_t^n) f_{m,t+1} + \beta \lambda_t u_{c,t+1} \left[ \frac{1 - \tau_t^n}{1 - \zeta_t^n} f_{p,k,t+1} + \frac{1 - \tau_t^n}{1 - \zeta_t^n} f_{k,t+1} \right] + \beta \mu_{1,t+1} [(1 - \delta^p) + R_{F,t+1}],
\]

where \( R_{F,t+1} \equiv \frac{\tau_t^n}{1 - \zeta_t^n} f_{p,t+1} f_{p,t+1} + \frac{\tau_t^n}{1 - \zeta_t^n} f_{k,t+1} f_{k,t+1} + \tau_t^n f_{n,t+1} n_{t+1} \) is the derivative of future government revenues with respect to public capital, while \( \mu_{2,t} \) and \( \mu_{3,t} \) are the shadow values of relaxing constraints (4.5) and (4.7), respectively.\(^9\)

When choosing the allocation of spending between public investment and government consumption, the Ramsey planner equates the marginal benefit of the two types of public goods. If the government had lump sum taxes available, the marginal benefit of public investment would be the present value of the marginal utility of public capital and the effect on future aggregate resources (first line). However, the presence of distortionary taxation gives more incentive for the government to invest. First, by increasing the productivity of private factors, it counteracts the effects of distortionary taxes (second line). Second, public capital also increases future tax revenues (third line). In other words, public capital raises the marginal productivity of factors and increases the firm’s rents that are taxed. Thus, higher tax rates, particularly the corporate tax rate, increase the return to public investment in terms of future tax revenues and raise the incentive for the government to invest instead of consume.

4.4 Calibration

Before proceeding to our numerical analysis, we have to make some specific assumptions regarding the functional forms of the utility and the production function and calibrate the relevant parameters. We consider a separable log utility function and a Cobb-Douglas production function:

\[
 u (c_t, g_t, n_t, P_t) = \ln(c_t) + \psi^n \ln(1 - n_t) + \psi^g \ln(g_t) + \psi^p \ln(p_t)
\]

\(^9\)The other first-order conditions are reported in the appendix.
The discount factor is such that the steady-state annual real interest rate is 4 percent. The elasticity of output with respect to private capital is 0.3. The parameter $\theta$ is more controversial. Estimates of the elasticity of output with respect to public capital in the literature are quite dispersive (see Bom and Ligthart (2008) for a meta-analysis). We set it at 0.05, the benchmark value used by Baxter and King (1993) at the lower end of the empirical literature.

The three parameters of the utility function $\psi^n$, $\psi^g$, $\psi^p$ are such that, in steady state, the household works one third of the available time, the ratio of government consumption to output is around 0.2 and the stock of public capital amounts to 56 percent of output. The parameter $\zeta$ is set such that the effective marginal tax rate on capital is close to 10 percentage points below the statutory tax rate. The depreciation rate of private and public capital are set at 8 and 6 percent. These numbers imply that private investment ($i^k$) corresponds to 18 percent of GDP and public investment ($i^p$) to 3.4 percent of GDP.

Table 4.1 shows the baseline calibration and the implied steady-state values of the main variables. Under this calibration, in steady state, the corporate tax rate is 32 percent and the labour income tax is 30 percent.

### Table 4.1: Baseline calibration

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\theta$</th>
<th>$\alpha$</th>
<th>$\zeta$</th>
<th>$\delta^k$</th>
<th>$\delta^p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi^n$</td>
<td>0.05</td>
<td>0.3</td>
<td>0.35</td>
<td>0.08</td>
<td>0.06</td>
</tr>
<tr>
<td>$\psi^g$</td>
<td>1.6</td>
<td>0.35</td>
<td>0.01</td>
<td>0.96</td>
<td></td>
</tr>
<tr>
<td>$\psi^p$</td>
<td>0.06</td>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Steady-state value of variables</th>
<th>$\tau^n$</th>
<th>$1-\tau^n$</th>
<th>$\tau^g$</th>
<th>$\tau^p$</th>
<th>$\frac{1}{\gamma}$</th>
<th>$\frac{1}{\bar{y}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i^k$</td>
<td>0.32</td>
<td>0.23</td>
<td>0.30</td>
<td>0.21</td>
<td>0.034</td>
<td></td>
</tr>
<tr>
<td>$\bar{y}$</td>
<td>0.56</td>
<td>2.24</td>
<td>2.33</td>
<td>0.18</td>
<td>0.58</td>
<td></td>
</tr>
</tbody>
</table>

### 4.5 Explaining the co-movement of fiscal variables

The purpose of this section is to understand the interaction mechanisms that characterize the model and to discuss some hypotheses that are consistent with the evolution of fiscal instruments described above. First, we examine how the steady state varies with the different parameters. Then we look at the transition dynamics. Finally, we analyse how the optimal choice of the instruments is made, when one of them is set exogenously.
4.5.1 Decreasing need of public infrastructure

Figures 4.2 show how the several fiscal variables change in steady state with the elasticity of output with respect to public capital ($\theta$). As the importance of public capital in the production function decreases, the Ramsey government reduces the corporate tax rate to extract a smaller fraction of the rents. Labour income tax increases but the magnitude is small. In the expenditure side, as $\theta$ decreases, there is a transfer of resources from public investment to government consumption; on the one hand, because public capital becomes a less important input of production; on the other hand, because it has a dimmer impact on the stream of future tax revenues. At the limit, when the weight of public capital in the production function is zero, the model is equivalent to the standard model of optimal dynamic taxation. There are no rents in production, so the optimal steady-state profit tax rate is zero.

In order to better understand the role of the deduction allowance, we show how it affects the steady state (Figure 4.3). If the firms can deduct all financial costs from their reported profit, the corporate tax rate is not distortionary. The optimal policy in that case is to have a 100 percent corporate tax rate, which allows the government to capture all the rents from public capital. The most interesting element is that the effective marginal tax rate on capital is not very responsive to $\zeta$. Even with a $\zeta$ close to 0.9, the effective marginal tax rate on capital is around 20 percent. As the effective tax rate on capital becomes detached from the profit tax rate, the government prefers to capture a larger fraction of the rents, instead of reducing the intertemporal distortion.

Figure 4.2: Steady-state effects of the elasticity of output w.r.t. public capital
In the Appendix we illustrate how the steady-state of the model changes with the other parameters of the model, namely the relative weights of government consumption, leisure and public capital in the utility function and the elasticity of output with respect to private capital ($\psi^g, \psi^n, \psi^p, \alpha$). All of these parameters alter the composition of taxation and spending, but none of them is consistent with the observed comovements.

### 4.5.2 Transition to steady state

When examining the transition dynamics of the model, our aim is to understand the role of the accumulation of both private and public capital along the path to steady-state. We consider two starting points: one with low public capital, where public and private capital are 60 and 20 percent below steady state (solid line), and one with low private capital with the inverse proportions (dashed line). The results are shown in Figure 4.4.

The transition dynamics do not change much from the traditional model. When the government re-optimizes, as the previous plan is made obsolete, it sets the profit tax rate at the maximum possible. At the same time, it reduces labour income tax to induce labour supply. When we start with a low private capital stock, the decline of labour income tax is so strong that it turns into a subsidy. This is achieved with the increase of profit tax, a sharp reduction of public consumption and a disaccumulation of public capital.

When we start with a lower public capital stock, corporate tax stays at the maximum value for several periods and the labour income does not decrease as much. Together with
the reduction of government consumption, it allows for a rapid accumulation of public capital. Along the transition path that takes roughly 20 years, public investment goes down, government consumption increases, corporate tax decreases and labour income tax goes up.

### 4.5.3 Effects of globalisation

As we have seen, under the optimal plan, the four instruments are determined jointly and changes in the parameters of the model alter the optimal allocation of spending and of the tax burden. An important corollary of this interdependence is that exogenous changes in one of the instruments can affect the optimal choices of all other instruments. In this sub-section we try to illustrate why this is relevant. We focus our discussion on how the model responds to exogenous changes in corporate tax rate (Figure 4.5) and in government consumption (Figure 4.6). The responses to the other two instruments are in appendix.
As corporate tax goes down, the government tries to get additional revenue by raising the labour income tax. However, as total revenue falls, all types of expenditures go down, particularly the government consumption. As government consumption increases, both taxes go up in order to raise revenue, particularly the labour income tax. On the other hand, although there are more incentives for the government to invest because of higher taxes, government consumption drains so much revenue that it is optimal to reduce public investment.

When one instrument changes for exogenous reasons, there is a revenue and a substitution effect. The substitution effect comes directly from the first-order conditions of the Ramsey problem. However when the government exogenously increases one type of expenditure, it will require higher revenue which will push both tax rates up. Similarly, when it decreases one tax rate, it will generate lower revenue, so it forces both types of spending to go down. From these exercises it seems that the revenue effect always overcomes the substitution effect, so we can never get the appropriate co-movement between the instruments.

Exogenous changes in one instrument are not sufficient to explain the trends in the four instruments. However, they can be attributed to the combination of a decline in profit tax and an increase in government consumption, which in turn can be seen as the effects of globalisation. Epifani and Gancia (2009) argue that the increase in the size of government is a consequence of the increase in openness, partially because it raised the demand for insurance and public transfers. The decline in the corporate tax rate is

Figure 4.5: Steady-state effects of exogenous changes in profit tax
usually attributed to international tax competition and a higher degree of capital and profit mobility (see Devereux, Lockwood, and Redoano (2008)). Our model shows why in turns those movements may result in a decline in public investment and an increase in labour income taxes. Quantitatively, if we combine a decline in the corporate tax rate of 20 percentage points and an increase in government consumption by 6 percent of GDP, labour income tax would increase by 12 percentage points and public investment would decline by one quarter.

While the first two explanations for the evolution of the fiscal variables consist of an optimal response through time or to changes in technology, this last hypothesis is very different in nature. It implies that exogenous events are constraining the choice of one or more instruments. Even if the government is responding optimally to an exogenous event, there are some welfare costs associated. To illustrate this we do a simple exercise. If the government is limited by a corporate tax rate of 12 percent, the welfare costs is 1.4 percent of consumption compensating variation relative to the Ramsey solution. On the other hand, if government consumption is constrained to be 6 percent of GDP above the optimal steady-state level, the associated cost is 1.2 percent of steady-state consumption.

4.6 Conclusion

In this chapter we have argued that the decisions on both the level and composition of expenditure and revenue of the government are intrinsically related and should not be analysed in isolation. The optimal choice of government consumption weights the
marginal benefit to the household’s utility against the costs of the distortions needed to raise revenue and the aggregate resources used. As for public capital, apart from entering the utility of the households, it raises output, generating more resources. Additionally, an increase in public capital raises the firm’s profits and the household’s wages, thus enlarging the tax base. The tax rates, particularly the profit tax, can, therefore, be interpreted as one of the returns of public capital, in terms of government revenue. This element is absent in the case of lump-sum taxation, but becomes relevant when taxes are distortionary. The level and the composition of the tax rates affect the trade-off between government’s consumption and investment. On the one hand, the government’s decision on how to allocate spending affects the size of the rents of the firm, which alters the trade-off between taxing profits or wages.

Although we cannot give a definitive answer on the origin of the trends we observe in the data, the model allows us to identify three scenarios that are consistent with them. First, if the production function is becoming less dependent on public capital, we would observe a decline in government investment and corporate tax rate, but an increase in government consumption and labour income tax. These trends are also consistent with the transition path to steady state, when we start from a low public sector capital. Finally, if we characterize globalization as both reducing the corporate tax rate and increasing government consumption, it can also explain the trends in both public investment and labour income tax rate.

Trying to identify the true source of the evolution of the fiscal instruments is an important task with defining welfare implications. If the data is driven by changes in technology or it is the optimal path towards the steady state, there is no particular reason for concern. However, if these trends are driven by exogenous factors such as globalisation, they generate welfare losses that governments should try to avoid.
4.7 Appendix

4.7.1 Model

The consumer's first-order conditions:

\[ u_{c,t} = \beta u_{c,t+1} (1 + r_{t+1}), \]  
(A4.1)

\[ -\frac{u_{n,t}}{u_{c,t}} = w_t (1 - \tau^n_t). \]  
(A4.2)

The firm's first-order conditions:

\[ r_t = \frac{(1 - \tau^n_t)}{(1 - \zeta^n_t)} (f_{kt} - \delta^k), \]  
(A4.3)

\[ (1 - \tau^n_t)(f_{n,t} - w_t) = 0. \]  
(A4.4)

Combining these equations and imposing \( \tau^n_t < 1 \), we get (4.6) and (4.7). If we want to determine the first-best allocation, achievable if the government has a lump sum tax we just have to solve the following problem

\[
\max_{\{c_t, k_{t+1}, g_t, n_t, P_{t+1}\}} \sum_{t=0}^{\infty} \beta^t u (c_t, g_t, n_t, P_t) - \beta^t \mu_{2,t} [c_t + g_t + P_{t+1} - (1 - \delta^p)P_t + k_{t+1} - (1 - \delta^k)k_t - f(k_t, P_t, n_t)]
\]

\[ P_0, k_0 \text{ given} \]

The first-order conditions of the Pareto problem are:

\[ c_t : \quad u_{c,t} - \mu_{2,t} = 0, \]
\[ g_t : \quad u_{g,t} - \mu_{2,t} = 0, \]
\[ n_t : \quad u_{n,t} + \mu_{2,t}f_{n,t} = 0, \]
\[ k_{t+1} : \quad -\mu_{2,t} + \beta\mu_{2,t+1}[1 - \delta^k] + f_{k,t+1} = 0, \]
\[ P_{t+1} : \quad -\mu_{2,t} + \beta u_{p,t+1} + \beta\mu_{2,t+1}[1 - \delta^p] + f_{p,t+1} = 0. \]
The Lagrangian formulation of the Ramsey problem is

$$\max_{\{c_t, k_{t+1}, g_t, n_t, P_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t, g_t, n_t, P_t)$$

$$- \beta^t \mu_{1,t} \left[ g_t + P_{t+1} - (1 - \delta^P) P_t - \tau_t^n (f_{n,t} n_t) - \tau_t^v \left( \frac{1 - \zeta}{1 - \zeta} (f_{k,t} - \delta^k) k_t + f_{p_t} P_t \right) \right]$$

$$- \beta^t \mu_{2,t} \left[ c_t + g_t + P_{t+1} - (1 - \delta^P) P_t + k_{t+1} - (1 - \delta^k) k_t - f(k_t, P_t, n_t) \right]$$

$$- \beta^t \mu_{3,t} \left[ - \frac{u_{n,t}}{u_{c,t}} - f_{n,t}(1 - \tau_t^n) \right]$$

$$- \beta^t \lambda_t \left[ u_{c,t} - \beta u_{c,t+1} \left( 1 + \frac{1 - \tau_{t+1}^n}{1 - \zeta \tau_{t+1}^n} (f_{k,t+1} - \delta^k) \right) \right]$$

$$\tau_t < 1 \text{ for all } t = 0, 1, \ldots$$

$$P_0, k_0 \text{ given}$$

The first-order conditions of this problem are:

$$\tau_t^v = - \mu_{1,t} \left[ f_{p,t} P_t + \frac{1 - \zeta}{1 - \zeta \tau_t^v} (f_{k,t} - \delta^k) k_t \right] - \lambda_{t-1} u_{c,t} \left( f_{k,t} - \delta^k \right) \frac{1 - \zeta}{1 - \zeta \tau_t^v} = 0$$

$$\lambda_t^v = \mu_{1,t} f_{n,t} n_t - \mu_{3,t} f_{n,t} = 0$$

$$c_t = u_{c,t} - \mu_{2,t} - \mu_{3,t} \frac{u_{n,t} u_{c,t}}{u_{c,t}} - \lambda_t u_{c,t} + \lambda_{t-1} u_{c,t} \left( 1 + \frac{1 - \tau_{t+1}^v}{1 - \zeta \tau_{t+1}^v} (f_{k,t} - \delta^k) \right) = 0$$

$$g_t = u_{g,t} - \mu_{2,t} - \mu_{1,t} = 0$$

$$n_t = u_{n,t} + \mu_{2,t} f_{n,t} + \mu_{1,t} \left[ \tau_t^v (f_{n,t} + f_{n,t} n_t) + \tau_t^r \frac{1 - \zeta}{1 - \zeta \tau_t^r} f_{k,n,t} k_t + f_{p,n,t} P_t \right] + \mu_{3,t} \left[ \frac{u_{n,t} u_{c,t}}{u_{c,t}} + f_{n,n,t}(1 - \tau_t^n) \right] + \lambda_{t-1} u_{c,t} \frac{1 - \tau_{t+1}^v}{1 - \zeta \tau_{t+1}^v} f_{k,n,t} = 0$$

$$k_{t+1} = - \mu_{2,t} + \beta \mu_{2,t+1} \left[ (1 - \delta^k) + f_{k,t+1} + \beta f_{k,t+1} \right] + \beta \mu_{2,t+1} \times \left[ \tau_{t+1}^v (f_{k,n,t+1} n_{t+1}) + \tau_{t+1}^v \left( \frac{1 - \zeta}{1 - \zeta \tau_{t+1}^v} (f_{k,t+1} - \delta^k) + \frac{1 - \zeta}{1 - \zeta \tau_{t+1}^v} f_{k,k,t+1} k_{t+1} + f_{p,k,t+1} P_{t+1} \right) \right] + \beta \mu_{3,t+1} \left[ f_{k,n,t+1}(1 - \tau_{t+1}^n) + \lambda_t \left( \lambda_{t-1} u_{c,t} \frac{1 - \tau_{t+1}^v}{1 - \zeta \tau_{t+1}^v} f_{k,k,t+1} \right) = 0 \right]$$

$$P_{t+1} = - \mu_{2,t} + \beta u_{p,t+1} + \beta \mu_{2,t+1} \left[ (1 - \delta^P) + f_{p,t+1} - \mu_{1,t} + \beta u_{p,t+1} \right] + \beta \mu_{2,t+1} \times \left[ (1 - \delta^P) + \tau_{t+1}^v (f_{p,m,t+1} n_{t+1}) + \tau_{t+1}^v \left( \frac{1 - \zeta}{1 - \zeta \tau_{t+1}^v} f_{k,p,t+1} k_{t+1} + f_{p,p,t+1} P_{t+1} \right) \right] + \beta \mu_{3,t+1} \left[ f_{p,m,t+1}(1 - \tau_{t+1}^n) + \lambda_t \left( \lambda_{t-1} u_{c,t} \frac{1 - \tau_{t+1}^v}{1 - \zeta \tau_{t+1}^v} f_{k,k,t+1} \right) = 0 \right]$$

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4.7.2 Figures

STEADY-STATE EFFECTS OF PARAMETERS

Figure A1: Steady-state effects of the elasticity of output w.r.t. private capital

Figure A2: Steady-state effects of disutility of working
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Figure A3: Steady-state effects of utility of public capital

Figure A4: Steady-state effects of utility of government consumption
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EXOGENOUS CHANGES IN INSTRUMENTS

Figure A5: Steady-state effects of exogenous changes in public capital

Figure A6: Steady-state effects of exogenous changes in labour tax

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4.7.3 Taxes and public spending in the G7 countries

Figure A7: Taxes and allocation of public spending in the G7 countries
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![Graphs showing tax rates and public investment over government consumption for the United Kingdom, United States, and average 18 OECD countries.](image)

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Chapter 5

Corporate tax competition and the supply of public capital

5.1 Introduction

Over the past 30 years, there has been a downward trend in two distinct government policy tools. On the one hand, statutory corporate tax rates have gone down in the majority of OECD countries from around 45% to 30%. On the other hand, public investment has declined from an average of 4.5% of GDP to below 3% of GDP. As a consequence, public capital stock has fallen by 10% of GDP (see Figure 5.1 below). In contrast, government consumption has increased during this period.

The decline in statutory corporate tax rate is a well documented phenomenon. It is usually attributed to international tax competition and a higher degree of capital and profit mobility. In contrast, the literature examining the downward trend in public investment is scarcer and far less compelling. In a way, the decline of public investment and public capital stock is a puzzle. Bénassy-Quéré, Gobalraja, and Trannoy (2007), among others, show for instance that the location of multinational firms does not entirely depend on national tax policies but also on ‘public infrastructure’, partly because of its positive effect of the productivity of private capital. Under these circumstances, the relationship displayed in Figure 5.1 could appear counter-intuitive: in a more competitive

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1This chapter was written in co-authorship with François Pouget from Dauphine University.
2See, for instance, Krogstrup (2004).
3Some frequent explanations for the decline of public investment include: the increase of privatization, the increase of private-public partnerships, the smaller role of the government or, in the case of Europe, the need for fiscal stringency. Some of these explanations are not very convincing as argued by Mehrotra and Väliä (2006). First, under national accounts, the investment undertaken by public enterprizes counts as private investment. Only investment recorded and financed from the budget counts as public investment. Second, private and public partnership is a very recent phenomenon that could not account for the pattern observed since the 1970s. Furthermore, government consumption has increased during the same period for most OECD countries. See Balassone and Franco (2000).
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Figure 5.1: Corporate taxation and allocation of public spending in OECD countries

environment we would indeed expect countries to increase their stock of public capital (at the expense of public consumption) in order to attract more private investment.

We argue that these two phenomena are related. Firstly, we claim that there is an intrinsic relation between corporate tax and public investment, beyond the simple identity of the government budget constraint. On the one hand, if the tax rate is high, governments spend more on public investment, relative to government consumption. The intuition for this is the following. The existence of public capital creates rents for the firms. Part of these rents is appropriated by the government through the corporate taxation. In a way corporate taxation can be seen as a return on public investment. If the tax rate is at high levels, a government that cares about revenue (or cares about the distortions of raising revenue) tends to favor public investment, at the expenses of government consumption. On the other hand, the level of tax rate also depends positively on the level of public capital. The higher the level of public capital, the higher the rents for the firm. The firm is, therefore, able to support a higher tax burden on its profit. Both policy variables respond positively to each other and the two are jointly determined.

Given the endogenous relation between corporate tax rate and public investment, we argue that the increase in the international tax competition, that has been exogenously driving the corporate tax rate down over the past years, caused, as a side effect, the reduction of public investment.

To make our case, we first build a model where the decision-maker decides on a corporate tax policy, but also chooses how to allocate its public resource. In this respect, the government has two alternatives: it can either invest (and therefore increase the stock of public capital) or allocate its tax receipts into "unproductive" government consumption. Additionally, we consider an element of tax competition to assess the short and long-run macroeconomic implications of a greater degree of corporate tax competition. We, then, perform an empirical analysis for 21 OECD countries for the period between 1966 and 2002.

We set up a simpler version of the model in Chapter 4 that illustrate the interdep-
dence between the corporate tax rate and productive spending and develop more deeply
the element of international tax competition. In our two-country model, governments
can enlarge their tax base by deciding on a more accommodating corporate tax rate or
by increasing the stock of productive public capital (or public infrastructures). Because
the exact source of tax competition does not affect the main mechanism of the model, we
focus on profit shifting as opposed to capital mobility. Due to the growing internalization
of the corporate sector, particularly in Europe, multinational companies have increased
their ability to change the location of their declared profit in response to tax rates dif­
f erentials for tax avoidance purposes. Our simulations indicate that, following a decline
of 15% in tax rate (driven by increasing competition), public investment in steady state
diminishes between 0.2% and 0.4% of output. This leads to a drop in the steady-state
public capital stock over output ranging from 4 to 11 percentage points. We also perform
simulation on the transition between steady states and find that the short-run impact
on public investment can be up to three times larger than the long-run effect. We also
create different scenarios and challenge the robustness of the relationship. In all cases,
international tax competition reduces the share of public spending allocated to public
investment, therefore reducing the stock of public capital.

In the empirical part we estimate two endogenous policy functions of corporate tax
rate and public investment that also respond to their foreign counterpart. Evidence con­
firms the endogeneity and the complementarity between the two tools: tax rate increases
with the level of public investment and public investment increases with the tax rate.
We find that a decline in tax rate of 15%, reduces public investment by 0.6% to 1.1% of
GDP. Further evidence suggests that both tools are driven by competition, particular­
te corporate tax rate.

The next section of this chapter introduces the theoretical model by presenting the
main assumptions and mechanisms in a partial equilibrium setting. In Section 5.3 we cal­
ibrate the model and present the quantitative results. The empirical analysis is presented
in the Section 5.4 while Section 5.5 concludes.

5.2 Model

The general equilibrium model consists of two countries denoted A and B. National
governments decide on a tax rate levied on the benefits of the corporate sector and

\footnote{Bartelsman and Beetsma (2003) performed an empirical analysis based on OECD countries and
estimate in their baseline scenario that 65% of the additional revenue from a unilateral tax increase is
lost due to a decrease in the reported profit to the national tax authorities. See also Huizinga and Laeven
(2008) who have calculated that the average semi-elasticity of reported profits with respect to the top
statutory tax rate. In particular, Germany appears to have lost considerable tax revenues due to profit
mobility -see Weichenrieder (2009). For other contributions on international tax competition and profit
shifting, see Kind, Midelfart, and Schjelderup (2005), and Elitzur and Mintz (1996).}
allocate their tax receipts either to “productive” public investment or public consumption. The corporate sector is introduced through a single representative multinational firm producing a homogeneous good in both countries.

Capital is perfectly mobile between the two countries and the firm can borrow at a world interest rate. Since the two national tax bases are not consolidated, the corporate sector has the ability to shift profit in order to reduce its overall tax burden. However, these operations entail some costs. We assume perfect foresight and no uncertainty.

5.2.1 Households

In each country \( i \in \{A; B\} \), a representative household derives its utility from both private consumption and public spending. The instantaneous utility function at time \( t \) is given by

\[
U_i^t = \ln c_i^t + \xi \ln g_i^t + \gamma \ln P_i^t. \tag{5.1}
\]

The utility derived from public spending depends first on government consumption, \( g_i^t \), which covers all current expenditures with no direct productive purposes. Additionally, the household’s utility depends on the stock of public capital, denoted \( P_i^t \). This stock represents a wide range of public infrastructures, such as roads or bridges, that are valued by the representative household but also used in the production process (see below). Therefore, in line with Keen and Marchand (1997), our model relies on a clear-cut distinction between productive and non-productive government spending. Parameters \( \xi \) and \( \gamma \) tell us that the representative household can valuate differently these two categories of public spending.

In each country, the representative household takes public variables as given and maximizes the present discounted value of the lifetime utility of private consumption: 

\[
\bar{U}(c_i^t) = \sum_{t=0}^{\infty} \beta^t \ln c_i^t, \quad \beta \text{ being the discount factor.}
\]

The household’s budget constraint is therefore described by

\[
c_i^t + I_i^t = w_i^t + r_i^k B_i^t + \tau_i^t - \bar{t}. \tag{5.2}
\]

In each period, household’s resources are either consumed (\( c_i^t \)) or saved by holding shares of the private sector (\( I_i^t \)). We assume that the representative household supplies one unit of labour inelastically and wage rate is set at \( w_i^t \). Total net resources depend also on the total amount of private capital owned by the household, denoted \( B_i^t \), which yields a gross return of \( r_i^k \) and whose law of motion (assume that the depreciation rate of private capital is \( \delta \)) is

\[
B_{i+1}^t = (1 - \delta) B_i^t + I_i^t. \tag{5.3}
\]

The household receives also dividends earned by the private sector: \( \tau_i^t \) (which will be defined later on). Besides, a lump sum tax on personal income, \( \bar{t} \), is levied in order to
finance public policy. Note that this specific tax rate will be considered exogenous in this model. Maximizing $\bar{U}(c_t^i)$ subject to (5.2) gives us the consumption pattern of the representative household, which is determined by the following Euler condition (we define $r_t = r_t^k - \delta$ as the net interest rate):

$$c_{t+1}^i = c_t^i(1 + r_{t+1})^\beta.$$  

(5.4)

5.2.2 Corporate sector

A single multinational firm operating in the two countries represents the private sector. It produces a homogeneous private good according to the following production function:

$$y_t^i = F(k_t^i, P_t^i, n_t^i) = k_t^{\alpha P_t^\theta} n_t^{(1-\alpha-\theta)}.$$  

(5.5)

The labor input, $n_t^i$, is considered to be immobile between the two countries. By contrast, capital is perfectly mobile and $k_t^i$ describes the total quantity of capital used in country $i$. Public capital stock is included in the production function and, therefore, increases the marginal productivity of capital. $P_t^i$ is considered as given by the firm. The production technology is identical in the two countries.

A source-based corporate tax is applied on the declared profit of the representative firm in the two countries. Therefore, the aggregated net profit of the corporate sector is as follows:

$$\Pi_t^{Tot} = (1 - \tau^A_t)\Gamma_t^A + (1 - \tau^B_t)\Gamma_t^B - r_t(k_t^A + k_t^B) - \psi(S_t)$$

with: $$\begin{cases}
\Gamma_t^A = F(k_t^A, P_t^A) - n_t^A w_t^A - \delta k_t^A - s_t \\
\Gamma_t^B = F(k_t^B, P_t^B) - n_t^B w_t^B - \delta k_t^B + s_t 
\end{cases}$$

(5.6)

$\Gamma_t^i$ represents the declared profits of the firm in country $i$, and therefore its corporate tax base. We assume that the firm can deduct capital depreciation from the taxable profits. We define $s_t > 0$ (respect. $< 0$) the total amount of profit shifted from country A to country B (respect. from B to A). These profit manipulations are costly to the firm since national tax authorities seek to prevent tax evasion (for instance, transfer pricing distortions have to be justified). The function $\psi(s_t)$ capturing this cost is convex:

$$\psi(0) = 0, \psi'(s_t) > 0 \text{ and } \psi''(s_t) > 0.$$  

Following Kolmar and Wagener (2007), we use the following functional form: $\psi(S_t) = b(s_t)^2$.

$^5$Modeling this way implies that the statutory tax rate is equivalent to the effective marginal tax rate. If we allow the firm to deduct the financial cost of capital, the effective marginal tax rate would then be zero. This alternative is less realistic and it does not change the mechanism of the model. The relation between public capital and tax rate depends mainly on the statutory tax rate.

$^6$This cost should be interpreted as the probability of being audit by the authorities, not being able to justify the transfer prices, and consequently being fined. We, therefore, assume that the marginal cost of tax evasion increases with the total amount of profit shifted.
By maximizing after-tax profits (5.6) with respect to \( k^*_i, w^*_i \) and \( s_t \), we obtain the equations describing the behaviour of the corporate sector. The allocation of capital in each country depends on the following first-order condition:

\[
F_K(k^*_i, P^i_t, n^*_i) = v^*_i + \delta \quad \text{with:} \quad v^*_i = \frac{\tau_t}{1 - \tau_t}.
\]  

(5.7)

The total amount of capital used in country \( i \) is such that its marginal productivity equals the gross cost of capital (which includes the cost of depreciation). Net cost of capital in a given country, \( v^*_i \), increases with interest rates and corporate tax rate. Besides, because of the perfect mobility of capital, a unique interest rate applies in the two countries. When the government increases the total stock of public capital, \( P^i_t \), this automatically increases \( k^*_i \) due to its positive effect on marginal productivity of capital.

As one unit of labour is inelastically supplied in the two countries, the firm's decision on labor consists on the choice of the wage rate according to the following condition:

\[
F_n(k^*_i, g^*_i, n^*_i) = w^*_i.
\]  

(5.8)

At last, the firm's decision on paper profit responds to the tax rate differential. Because \( \psi_s(s_t) \), profit will be shifted from \( A \) to \( B \) if \( \tau_A - \tau_B > 0 \). Profit-shifting flows are decreasing with the marginal cost associated to these operations:

\[
\psi_s(s^*_t) = \tau_A^t - \tau_B^t \iff s^*_t = \frac{\tau_A - \tau_B}{2b}.
\]  

(5.9)

5.2.3 Government

The objective function of the government is given by (5.10).

\[
V(P^i_t, g^*_i) = \sum_{t=0}^{\infty} \beta^t (\xi \ln g^i_t + \gamma \ln P^i_t).
\]  

(5.10)

The purpose of the government is to maximize the present discounted value of the household lifetime utility derived from public spending. In our model, the decision maker aims to increase public spending ultimately and, therefore, behaves like a leviathan. This assumption should be seen as a shortcut. The alternative would be to have two types of distortive taxation and a decision-maker maximising the consumer's utility as in Chapter 4. However, as we want to focus on the element of competition, approaching this issue through the optimal dynamic taxation theory, would add unnecessary complications.

Public resources in country \( i \) depend on the personal and corporate income tax revenue. Corporate tax revenue \( R^i_\tau \) depends on a statutory tax rate and the corporate tax
base (i.e. the declared profit of the firm in country $i$)

\[ g^i_t + p^i_t = \ell + R^t_i(P^i_t, \tau^i_t, \tau^i_1), \]

with:

\[ R^t_i(P^i_t, \tau^i_t, \tau^i_1) = \tau^i_t[F(k^i, P^i_t, n^i_t) - \delta k^i_t - w^i_t \pm s^i_t]. \] (5.11)

The second constraint the government faces is the law of motion equation of public capital stock ($\delta_p$ is the rate of depreciation)

\[ P^i_t = (1 - \delta_p)P^i_{t-1} + p^i_t. \] (5.12)

We consider that the governments anticipate the outcome of their choice on the decisions of the private sector. In this sense governments know that both their decision on tax rate and public capital affect the firm’s choice of capital (5.7), labour (5.8) and profit shifting (5.9) and, therefore, the corporate revenue. Public decision consists of the choice of a statutory tax rate, $\tau^i_t$ and a decision on public resources allocation between public investment and government consumption. Each government decides simultaneously and non-cooperatively. The Lagrangian associated with the government allocation problem is

\[ L = \sum_{t=0}^{\infty} \beta^t \{ \xi \ln [\ell + R^t_i(P^i_t, \tau^i_t, \tau^i_1) - p^i_t] + \gamma \ln P^i_t - \lambda_t [P^i_{t+1} - (1 - \delta_p)P^i_t - p^i_t] \}. \] (5.13)

Not surprisingly, the government chooses $\tau^i_t$ in order to maximize its corporate tax revenue:

\[ \frac{\partial R^t_i(P^i_t, \tau^i_t, \tau^i_1)}{\partial \tau^i_t} = 0. \] (5.14)

Since a corporate tax rate policy is decided simultaneously and non-cooperatively by the two countries, tax equilibrium between A and B is, thus, the outcome of a Nash game. Using (5.14) we obtain the reaction functions of the two countries.

\[ \left\{ \begin{array}{l}
\tau^A_t = \tau^B_t + \frac{\partial R^A_t(P^A_t)}{\partial \tau^A_t} \\
\tau^B_t = \tau^A_t + \frac{\partial R^B_t(P^B_t)}{\partial \tau^B_t}
\end{array} \right. \] (5.15)

A corporate tax policy stance has two major determinants. Firstly, each government attempts to maximize the revenue of its “productive” tax base, denoted $\bar{\Gamma}(.)$. This consists of the tax base that would be only determined through the allocation of capital

\[ \bar{\Gamma}(\tau^i_t; k^i_t; P^i_t) = \tau^i_t \left\{ (\theta + \alpha)P^{i+\alpha}_{t+\alpha} \left[ \frac{\alpha(1 - \tau^i_t)}{\tau^i_t + \delta_p(1 - \tau^i_t)} \right]^{1-\alpha} - \delta P^i_t \left[ \frac{\alpha(1 - \tau^i_t)}{\tau^i_t + \delta_p(1 - \tau^i_t)} \right]^{1-\alpha} \right\}. \] (5.16)

As one can observe on Figure 5.2, the revenue derived from this fraction of the tax base follows the pattern of a traditional Laffer curve with respect to the corporate tax rate.
and is maximized for $\tau_{t}^{\max}$. When $\tau_{t}^{i} > \tau_{t}^{\max}$, any corporate tax hike would entail a net loss, because the marginal revenue would be offset by the shrinking of the tax base.

The second determinant of a corporate tax policy is easily observed in (5.15): tax rate in a given country responds to its partner’s tax rate. These strategic interactions describe a race to the bottom phenomenon which is entirely dependent on profit mobility. When $b$ is low, the multinational firm can engage profit shifting operations forcing the two countries to compete more (see Figure 5.3). On the other hand, if profit shifting operations were no longer affordable ($b \rightarrow +\infty$), strategic interactions would disappear and corporate tax rates would be set at $\tau_{t}^{\max}$.

Having described in details the nature of tax competition in this model, we can now analyze the determination of the stock of public capital in our model, given by the following first-order condition:

\[
\frac{\xi}{g_{t}^{i}} = \beta \left[ \frac{\gamma}{P_{t+1}^{i}} + \frac{\partial R_{t+1}^{i}(P_{t+1}^{i}, \tau_{t+1}^{i}, \tau_{t+1}^{\star})}{\partial P_{t+1}^{i}} \frac{\xi}{g_{t+1}^{i}} + (1 - \delta_p) \frac{\xi}{g_{t+1}^{i}} \right].
\]  

(5.17)

When maximizing (5.13) with respect to $p_{t}^{i}$ we obtain: $\lambda_{t} = \xi / g_{t}^{i}$, so that the Lagrange multiplier can be interpreted as the marginal cost of public investment in $t$ (in terms of households foregone utility of consumption of the public good). The right hand side represents the discounted benefits of investing on public capital. It is composed of the direct benefit of public capital on the representative household utility ($\gamma / P_{t+1}^{i}$). The second component of the benefit refers to the anticipated effect of public capital stock on the tax revenue: investing more on public capital, will drive the multinational firm to install more capital, thus bringing extra revenues in the future. This revenue may then be used to supply a general public good to the population. The third component reflects the fact that public capital is a durable good so these two effects carry on to the following periods after depreciation is accounted for.
Using (5.4) and re-writing (5.17) at the steady state, we obtain:

\[
\frac{\partial R^i(P^i, \tau^i, \tau^j)}{\partial P^i} + \frac{\gamma g^i}{\xi P^i} = r + \delta_p. \tag{5.18}
\]

The marginal benefit of an increase of public capital stock depends on its positive effect on corporate tax revenue and the marginal utility that the representative household derives from this public policy (which is of course decreasing with \(P^i\)). The gross marginal cost increases with the interest rate and the depreciation rate of public capital stock.

We can observe by the first-order conditions (5.14) and (5.17) that the two instruments used by the decision maker in order to collect corporate tax revenue are interdependent. We illustrate this partial equilibrium relationship for country \(A\) on the Figure 5.4 below. Except for extreme values of \(\tau^A\), the stock of public capital is increasing with the statutory tax rate. This pattern directly depends on how strong is the impact of public capital on total corporate tax revenue (\(\partial R^i(P^i, \tau^i, \tau^j)/\partial P^i\)), which obviously declines when tax rate takes lower values. Note that total capital stock remains positive even when tax rate is equal to zero (indeed, as we can clearly see on (5.18), public capital stock does provide a satisfaction to the representative household besides increasing future tax revenue, so, it does not disappear even in the absence of corporate taxation).

On the other hand, tax rate depends positively on the level of public capital. The higher the public capital, the higher the rents, so the higher the governments will set their tax rate. Nevertheless, tax rate appears to be less reactive to public capital stock. In our model the tax policy stance relies mostly on the level of tax competition and on the partner country’s tax rate.

The endogenous relation between public capital and corporate tax rate that we described above is not model specific. Under the "leviathan" assumption, the government uses corporate tax rate to maximize tax receipts and public investment partially to maximize future tax revenue. If we had assumed a perfectly benevolent government using two

Figure 5.4: Government’s first order conditions
types of distortionary taxes to raise revenue, as in the previous chapter, a very similar intuition would hold. Firstly, the government would use a combination of taxes to minimize the distortions needed to raise a certain amount of revenue. Public capital would, nevertheless, still create rents and it would guarantee that part of the future revenue would be non-distortionary. Thus, if competition drives the corporate tax rate down, given a certain level of rents, the amount of non-distortionary revenue raised by the government would go down. This would reduce the attractiveness of government investment relative to government consumption, so a benevolent government would optimally shift its expenditure from one to the other.

5.2.4 Market clearing

In order to close the model we need three additional conditions. First we have the market clearing condition for both capital and goods markets:

\[
\begin{align*}
    b_i^A + b_i^B &= k_i^A + k_i^B \\
    y_i^A + y_i^B &= c_i^A + c_i^B + g_i^A + g_i^B + p_i^A + p_i^B + I_i^A + I_i^B + b(s_i)^2
\end{align*}
\]  

(5.19)

Total capital used by the firm equals to the amount of capital held by the households. Total production in the two countries must equal to the total private and public consumption, private and public investment and the cost of profit shifting. Finally, we need a final equation to pin down the consumption level of each country.

\[
c_i^A + I_i^A = w_i^A + r_i b_i^A - \bar{c} + T_i^A. \tag{5.20}
\]

5.3 Quantitative analysis

5.3.1 Calibration

In this section we analyze the quantitative implications of corporate tax competition for public capital and public investment. The model is calibrated for an annual frequency. Table 5.1 shows the values of the parameters and the implied steady state values for key variables.

The calibration of the first four parameters is quite standard. The discount factor is such that the annual real interest rate is 3.5%. In line with Kamps (2006), the annual rate of depreciation of public capital is 5%. Its private counterpart is set at 8%. The elasticity of output with respect to private capital is 0.26. The parameter $\theta$ is more controversial.

---

7We defined the dividend paid in country $i$ as the total declared profit minus the interest rate payment on existing capital: $T_i^A = [(1 - r_i^A)(y_i^A - w_i^A - \delta k_i^A - s_i) - r_i k_i^A]$. 

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Table 5.1: Calibration and steady state values in the benchmark case

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta ) Depreciation rate (private capital)</td>
<td>0.08</td>
</tr>
<tr>
<td>( \delta_p ) Depreciation rate (public capital)</td>
<td>0.05</td>
</tr>
<tr>
<td>( \alpha ) Elasticity of output (private capital)</td>
<td>0.26</td>
</tr>
<tr>
<td>( \theta ) Elasticity of output (public capital)</td>
<td>0.08</td>
</tr>
<tr>
<td>( \beta ) Discount Factor</td>
<td>0.966</td>
</tr>
<tr>
<td>( \tau ) Relative preference for public cap.</td>
<td>0.182</td>
</tr>
<tr>
<td>( \lambda ) Cost of profit shifting</td>
<td>0.625</td>
</tr>
</tbody>
</table>

Steady-state variables

<table>
<thead>
<tr>
<th>Variables</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption / output ( c/y )</td>
<td>0.587</td>
</tr>
<tr>
<td>Investment / output ( I/y )</td>
<td>0.16</td>
</tr>
<tr>
<td>Government consumption / output ( g/y )</td>
<td>0.225</td>
</tr>
<tr>
<td>Corporate tax rate ( \tau )</td>
<td>0.3</td>
</tr>
<tr>
<td>Corporate tax revenue / output ( R/y )</td>
<td>0.054</td>
</tr>
</tbody>
</table>

Estimates of the elasticity of output with respect to public capital range from 0 to 0.80. We set the value to 0.08 following a meta-analysis study of Bom and Ligthart (2008).

The last three coefficients are calibrated in order to obtain realistic steady state values for some variables. The relative preference for the two types of public goods, \( \gamma / \xi \), is such that public capital stock as a share of output in equilibrium is 0.55. The lump sum tax \( \lambda \) is such that the government consumption in the economy is close to 22 percent of output. As we do not have any estimates of the cost parameter of profit shifting, \( \lambda \) is set such that the corporate tax rate equilibrium is 30 percent. These three values are in line the evidence on OECD countries shown in the introduction.

5.3.2 Steady-state effects of competition

Starting from this baseline calibration, we now illustrate the consequences of tax competition on public capital stock and other key variables in the economy. Figure 5.5 illustrates how the tax rate equilibrium depends on the cost of profit shifting. We observe that when profit shifting becomes more affordable, a race to the bottom occurs. Not surprisingly in the extreme case of perfect profit mobility, tax rate is driven to zero.

The Figure 5.6 below shows how the steady state stocks of public capital and public investment over output respond to changes in the tax rate (driven by the decline in \( \lambda \)). Under the benchmark scenario, a change of the statutory tax rate from 45% to 30% percent leads to a decline of public capital stock of 11% of output and a decline of public investment of 0.4% of output.

The overall effect of increasing competition can be decomposed in two: the revenue and substitution effects. On the one hand, a decline in the tax rate automatically reduces total tax receipts, thus reducing the level of public investment, as well as government consumption via the budget identity. On the other hand, reduction of the tax rate makes public investment less attractive in relation to government consumption, as discussed in
the previous section. The overall decline might be, however, over-estimated because of the influence of the revenue effect. In reality, this effect is indeed likely to play a minor role since the total tax revenue derived from corporate taxation has remained relatively stable despite the fall of the statutory tax rate. In order to isolate the substitution effect in our analysis, we artificially control for the revenue effect by changing $\bar{t}$ such that total revenue is kept constant (see the dash lines in Figure 5.6). The decline of public capital and public investment would be slightly less than a half, 4% and 0.2% of output respectively.

Figure 5.7 illustrates the revenue and substitution effects. As tax rate decreases, corporate tax revenue goes down (left-hand side). In the case of extreme competition, corporate taxation disappears. The substitution effect is visible in the ratio between public investment and the general public good. As tax rates are driven to lower levels, we indeed observe a shift in the composition of public spending in favour of government consumption.

Figure 5.8 depicts the steady state response of some macroeconomic variables to changes in competition. The stock of private capital as well as private consumption go up with the increase in the tax competition. One might also expect that the race to the bottom of corporate tax rate always has a positive effect on total output. However, when
tax rates reach very low levels, tax competition turns out to be counter-productive to the total output. The hump-shape curve displayed on the left-hand side is explained by the fact that public capital stock increases the marginal productivity of private capital. This productivity deteriorates when tax competition reduces the stock of public capital. When tax competition is strong, this negative effect cannot be compensated by the increase in private capital. One can, therefore, observe that there exists a threshold tax rate under which corporate tax competition is harmful to production.8

5.3.3 Dynamic effects of competition

Our model allows us to study the transition dynamics. We analyse how the main variables of the model respond to an increase in corporate tax rate competition from $b = 0.705$ (which implies a tax rate of 35%) to $b = 0.625$ (baseline scenario with 30% tax rate). One can see from the Figure 5.9 that tax rate declines immediately close to its new steady-state value once the shock occurs. Furthermore, public investment also drops sharply in response, then picks up and converges to the new steady state. Simulations show that the short-run effect is roughly three times higher than the long-run effect. In this particular case, a decline of the tax rate by 5% has an immediate impact on public

8Note that this threshold value is highly dependent on the value of the parameter $\theta$. We do not explore this issue further as our model is not indicated for welfare analysis.
investment of 0.41% while the long run effect is only 0.14%. This outcome occurs because the government readjusts his optimal stock public capital in response to the decline of the tax rate. Not surprisingly, a greater degree of tax competition has a positive effect on the stock of private capital and on output.

5.3.4 Robustness analysis

Alternative calibration

Having described the main effects of corporate tax competition on our benchmark model, we now consider different realistic scenarios. For all of them we analyze the evolution of the public capital stock, public investment and government consumption. We observe that the main conclusion of the first section is confirmed: corporate tax competition has a negative impact on the stock of public capital. The quantitative prediction is also quite robust. Public capital stock over GDP falls between 8% and 15% of output and public investment between 0.3% and 0.6% of output under the alternative scenarios. The substitution effect accounts for close to half of the total effect.
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Figure 5.10: Robustness analysis - substitution effect

Table 5.2: Effects of competition under alternative parameterizations

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( \tau = 45% )</th>
<th>( \tau = 30% )</th>
<th>( \tau = 45% )</th>
<th>( \tau = 30% )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta = 0.12 )</td>
<td>77.2%</td>
<td>62.6%</td>
<td>14.6%</td>
<td>5.2%</td>
</tr>
<tr>
<td>( \theta = 0.04 )</td>
<td>56.5%</td>
<td>48.4%</td>
<td>8.1%</td>
<td>2.6%</td>
</tr>
<tr>
<td>( \alpha = 0.30 )</td>
<td>65.0%</td>
<td>52.5%</td>
<td>12.4%</td>
<td>4.4%</td>
</tr>
<tr>
<td>( \alpha = 0.20 )</td>
<td>67.2%</td>
<td>57.4%</td>
<td>9.8%</td>
<td>3.4%</td>
</tr>
<tr>
<td>( \gamma / \xi = 0.22 )</td>
<td>76.1%</td>
<td>63.7%</td>
<td>12.4%</td>
<td>4.0%</td>
</tr>
<tr>
<td>( \gamma / \xi = 0.14 )</td>
<td>55.7%</td>
<td>46.0%</td>
<td>9.7%</td>
<td>3.8%</td>
</tr>
<tr>
<td>( \bar{t} = 0.30 )</td>
<td>75.1%</td>
<td>63.7%</td>
<td>11.4%</td>
<td>4.3%</td>
</tr>
<tr>
<td>( \bar{t} = 0.18 )</td>
<td>57.1%</td>
<td>46.2%</td>
<td>10.8%</td>
<td>3.4%</td>
</tr>
</tbody>
</table>
Since the value of the parameter $\theta$ has involved a lot of controversies, we test alternative values for the contribution of public capital stock on private output (scenarios 1 and 2). For this reason, and despite the fact that our calibration is in line with the recent estimations given in the literature, we simulate two extreme cases. When $P^t$ has a minor effect on the output, the stock of public capital is lower at the steady state and exhibits a lower variability in the tax rates. By contrast, when $\theta$ is relatively high, we observe that tax competition entails a larger drop of public capital stock.

Allowing different values for $\alpha$ (scenarios 3 and 4) affects the substitutability between the private and the public capital. Not surprisingly, when the production process relies more on private capital, we observe a greater decline of public capital stock. Scenarios 5 and 6 describe the effect of a change of the relative preferences of the society for the two public policy dimensions. Without a doubt, the relative preferences for public capital $\gamma/\xi$ has a relatively high impact on the level of public capital stock but less on its pattern. Different preferences do not affect the main mechanism of our model. Analyzing the impact of the variation of the exogenous tax rate leads to the same conclusions. The total stock of public capital increases with $t$, whose real value is a major determinant of the scope of government. The relationship between the corporate tax rate and the stock of public capital is robust to changes in the value of the parameters.

### Asymmetric countries

Our final exercise is to analyse the steady-state values when asymmetries between the two countries are introduced. We consider two cases: asymmetries in $\theta$ and in $\alpha$. The respective results are shown in appendix.

When the production of a country relies more on public capital (country A in this simulation), the government accumulates more public capital than country B. This generates higher rents for the firm in country A, so the government sets a higher tax rate. In this case, there is profit shifting from country A to county B. If the level of competition is low, then the country with high $\theta$ has a higher output than its partner, but as competition increases and tax rate declines, the negative effect in public capital is very strong and total output also declines. This suggests that for countries that depend more on public capital, corporate tax competition might induce significant welfare costs.

We now turn to the analysis of the case with different $\alpha$'s. Country A, where $\alpha$ is higher, has a higher capital stock and higher output than its partner. The government sets a higher tax rate for this country, so there is profit shifting to country B. It is also interesting to notice that the decline in public capital is much stronger for country A. In the absence of tax competition, it has a higher public capital stock as a share of output, but when competition is more intense it is lower than of the country B.
5.4 Empirical evidence

5.4.1 Estimation strategy

To access the validity of the main mechanism of our model, we estimate policy functions for the statutory corporate tax rate ($\text{tax}_{it}$) and for public investment ($\text{inv}_{it}$) in the spirit of Devereux, Lockwood, and Redoano (2008):

\[
\begin{align*}
\text{tax}_{it} &= \alpha_1 \text{inv}_{it} + \alpha_2 \text{tax}_{it}^w + \alpha_3 X_{it} + \epsilon_i + \epsilon_{it} \\
\text{inv}_{it} &= \beta_1 \text{tax}_{it} + \beta_2 \text{inv}_{it}^w + \beta_3 X_{it} + \nu_i + \mu_{it}
\end{align*}
\] (5.21)

We use the statutory tax rate and not effective marginal tax rate. It is clear from the model that public investment depends on the statutory tax rate (the true rate of return of generating one extra unit of rents) and not on the effective marginal tax rate.\(^9\)

Apart from the endogeneity between the two tools, we also consider the international competition element. The statutory tax rate also responds to the tax rate of the rest of the world ($\text{tax}_{it}^w$).\(^10\) Public investment depends on the statutory tax rate, but we also allow it to respond to the level of public investment of foreign countries ($\text{inv}_{it}^w$). Although we do not model this element explicitly with our model, we include it to make the setting more realistic. Moreover, it allows us to identify the effect of public investment on the tax rate. In the absence of this element, it is hard to find another possible instrument for public investment. $X_{it}$ is a vector of control variables. We estimate each equation separately using instrumental variables estimation. The system is exactly identified: each equation has one omitted variable that is used as an instrument for the endogenous variable in the other equation. For the instruments to be valid, it is crucial that the corporate tax rate does not respond to foreign public investment and public investment does not react to the foreign tax rate. Although we cannot a priori justify this assumption based on existing evidence, we can test the validity of the instruments after the estimation.

The estimation of these reaction functions suffers from more problems of endogeneity. The tax rate and public investment of the rest of the world might react to domestic developments in the respective variables. Furthermore, some of the controls might also be endogenous to the tax rate or to public investment. To minimize these problems, we compute a 3 year non-overlapping averages. Each time observation corresponds to 3 years averages. We, then, estimate these equations with all controls that might be endogenous,

\(^9\)It should be noted that the dependence of public investment on the statutory tax rate exists regardless of the level of the effective marginal tax rate. Also, this relation persists in a closed economy or in the absence of international competition.

\(^10\)It is not our purpose to find out if the response to the foreign tax rate is due to competition for profits or for private investment.
as well as the foreign variables entering in lags. Although we cut the sample size to one third, it still allows us to be much more confident that our estimator will be consistent.

The corporate tax and public investment of the rest of the world are weighted averages of the variables for all other countries in the sample.

\[
tax_{it}^{rw} = \sum_{j=-1}^{\infty} w_{jt} tax_{jt}^{rw}
\]
\[
inv_{it}^{rw} = \sum_{j=-1}^{\infty} w_{jt} inv_{jt}^{rw}
\]

In the reaction functions we include public investment instead of public capital. Firstly because the decision variable of governments is public investment. Secondly, this way we avoid problems of non-stationarity, because both tax rate and public investment are bound between 0 and 1 and, therefore, cannot have unit roots. Similarly to Devereux, Lockwood, and Redoano (2008) we do not include lagged dependent variables.\(^{11}\)

### 5.4.2 Data

We estimate the policy functions using a panel of 21 OECD countries. The variable corporate tax rate was taken from Michigan World Tax Database, and public investment was taken from Kamps (2006) and expanded with OECD data until 2005.

For robustness purposes, we use three different weights to calculate the variables for the rest of the world: uniform weights \((W_1)\), the openness of the economy \((W_2)\) and the population \((W_3)\). The correlations between the three measures within a country range from 0.80 to 0.95 for both variables.

We use the following control variables: government consumption, the fiscal surplus, the degree of openness, GDP growth, the level of private capital, population growth, a dummy for election year, the % of left wing votes and a dummy if the country joined the EMU2 after 1999. Summary statistics and the source of each variable can be found in Table A5.1 in appendix.

### 5.4.3 Estimation

We estimate the policy functions using IV estimation. Given that we only have 21 countries, we model the country's specific error as fixed effect. In the estimations, we

---

\(^{11}\)In this way we can still have consistent estimates of the short-run coefficients without introducing technical complications of estimating equations with lagged dependent variables in panel data. For instance, we would have to estimate the equations in differences (Arellano and Bond), which would be problematic because there would be many zeros on the left hand side. If we ignored the bias generated by the lagged dependent variable and estimated the system in levels, we would have multicollinearity problems between the \((tax_{it-1})\) and \((tax_{it-1}^{rw})\).
also include country specific time trends. We consider government consumption, the fiscal surplus, the degree of openness, GDP growth, the level of private capital as potentially endogenous, so they enter the equation in lags (previous non-overlapping 3 year's average).

We estimate an unrestricted and a restricted model. The unrestricted model includes

Table 5.3: Estimation results: corporate tax rate

<table>
<thead>
<tr>
<th></th>
<th>W1</th>
<th>W2</th>
<th>W3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{1t}$</td>
<td>2.999</td>
<td>2.478</td>
<td>15.224</td>
</tr>
<tr>
<td>(0.78)</td>
<td>(1.35)</td>
<td>(0.54)</td>
<td>(1.02)</td>
</tr>
<tr>
<td>$T_{ax_{t-1}}$</td>
<td>0.385***</td>
<td>0.410***</td>
<td>0.342</td>
</tr>
<tr>
<td>(3.14)</td>
<td>(3.74)</td>
<td>(1.05)</td>
<td>(4.18)</td>
</tr>
<tr>
<td>$Gov_{cons_{t-1}}$</td>
<td>0.892*</td>
<td>0.858***</td>
<td>2.165</td>
</tr>
<tr>
<td>(1.92)</td>
<td>(3.20)</td>
<td>(0.68)</td>
<td>(2.34)</td>
</tr>
<tr>
<td>$Budget_{t-1}$</td>
<td>0.038</td>
<td>0.637</td>
<td>0.136</td>
</tr>
<tr>
<td>(0.17)</td>
<td>(0.47)</td>
<td>(0.85)</td>
<td></td>
</tr>
<tr>
<td>$GDP_{g_{t-1}}$</td>
<td>0.007</td>
<td>0.212</td>
<td>0.068</td>
</tr>
<tr>
<td>(0.03)</td>
<td>(0.34)</td>
<td>(0.28)</td>
<td></td>
</tr>
<tr>
<td>$Open_{t-1}$</td>
<td>0.137**</td>
<td>0.128**</td>
<td>0.180</td>
</tr>
<tr>
<td>(2.05)</td>
<td>(2.12)</td>
<td>(0.94)</td>
<td>(1.92)</td>
</tr>
<tr>
<td>$K_{t-1}$</td>
<td>0.628</td>
<td>12.967</td>
<td>2.379</td>
</tr>
<tr>
<td>(0.13)</td>
<td>(0.46)</td>
<td>(0.64)</td>
<td></td>
</tr>
<tr>
<td>$Pop_{gt}$</td>
<td>-4.266***</td>
<td>-4.079***</td>
<td>-7.945</td>
</tr>
<tr>
<td>(-2.83)</td>
<td>(-4.11)</td>
<td>(-0.87)</td>
<td>(-3.65)</td>
</tr>
<tr>
<td>$Election_{t}$</td>
<td>-0.255</td>
<td>-0.709</td>
<td>-0.464</td>
</tr>
<tr>
<td>(-0.17)</td>
<td>(-0.22)</td>
<td>(-0.29)</td>
<td></td>
</tr>
<tr>
<td>$Left_{t}$</td>
<td>0.071</td>
<td>0.225</td>
<td>0.086</td>
</tr>
<tr>
<td>(0.82)</td>
<td>(0.59)</td>
<td>(1.05)</td>
<td></td>
</tr>
<tr>
<td>$Emu_{t}$</td>
<td>-0.735</td>
<td>1.391</td>
<td>-0.36</td>
</tr>
<tr>
<td>(-0.38)</td>
<td>(0.25)</td>
<td>(-0.19)</td>
<td></td>
</tr>
</tbody>
</table>

| Country trends | Yes | Yes | Yes | Yes | Yes | Yes |
| Country dummies | Yes | Yes | Yes | Yes | Yes | Yes |
| Observations | 245 | 245 | 245 | 245 | 245 | 245 |
| Countries | 21 | 21 | 21 | 21 | 21 | 21 |
| R2 | 0.67 | 0.69 | 0.41 | 0.70 | 0.63 | 0.66 |
| Underidentification test# | 4.505 | 17.689 | 0.360 | 14.290 | 25.577 | 38.122 |
| Underidentification test | 0.034 | 0.014 | 0.548 | 0.046 | 0.000 | 0.000 |
| Sargan test$ | -1.160 | -2.015 | -1.803 |
| Sargan test | -0.979 | -0.918 | -0.937 |

Notes: Each observation corresponds to a 3 year average. The subscript t-1 denotes the observation of the previous 3 years. The estimation runs from 1966-1969 to 1999-2002. It includes the following countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom and United States. The equations are estimated using Instrumental Variables fixed effects estimation. In columns (1) the equation is exactly identify with $Inv_{t}^{c}$ as instrument for $Inv_{t}$. In columns (2) the non-significant variables are excluded from the equation but added as additional instruments. The t statistics are in parentheses. *, **, *** - statistically significant at the 10, 5, and 1 per cent. # The underidentification test is an LM test of whether the instruments are correlated with the endogenous regressors. The null hypothesis that the equation is underidentified. The test statistic is to be compared to a chi-square with degrees of freedom equal to the number of instruments. The p-value is in brackets. $ The null hypothesis of the Sargan overidentification test is that the instruments are uncorrelated with the error term and that the excluded instruments are correctly excluded from the estimated equation. Under the null, the test statistic is distributed as chi-squared in the number of overidentifying restrictions. The p-value is in brackets.
Table 5.4: Estimation results: public investment

<table>
<thead>
<tr>
<th></th>
<th>W1</th>
<th>W2</th>
<th>W3</th>
<th>W4</th>
<th>W5</th>
<th>W6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax</td>
<td>0.062**</td>
<td>0.039*</td>
<td>0.066</td>
<td>0.069*</td>
<td>0.077*</td>
<td>0.075**</td>
</tr>
<tr>
<td></td>
<td>(1.98)</td>
<td>(1.73)</td>
<td>(1.57)</td>
<td>(1.9)</td>
<td>(1.92)</td>
<td>(2.00)</td>
</tr>
<tr>
<td>Inv</td>
<td>0.315*</td>
<td>0.356**</td>
<td>0.233</td>
<td>0.231</td>
<td>0.126</td>
<td>0.125</td>
</tr>
<tr>
<td></td>
<td>(1.71)</td>
<td>(2.13)</td>
<td>(1.38)</td>
<td>(1.48)</td>
<td>(0.44)</td>
<td>(0.44)</td>
</tr>
<tr>
<td>Govcon</td>
<td>-0.164**</td>
<td>-0.142***</td>
<td>-0.173***</td>
<td>-0.172***</td>
<td>-0.166***</td>
<td>-0.161***</td>
</tr>
<tr>
<td></td>
<td>(-4.11)</td>
<td>(-4.11)</td>
<td>(-3.91)</td>
<td>(-4.06)</td>
<td>(-3.81)</td>
<td>(-3.82)</td>
</tr>
<tr>
<td>Budget</td>
<td>-0.043**</td>
<td>-0.0462***</td>
<td>-0.043**</td>
<td>-0.044**</td>
<td>-0.042**</td>
<td>-0.042**</td>
</tr>
<tr>
<td></td>
<td>(-2.47)</td>
<td>(-2.84)</td>
<td>(-2.44)</td>
<td>(-2.45)</td>
<td>(-2.25)</td>
<td>(-2.28)</td>
</tr>
<tr>
<td>GDP</td>
<td>-0.009</td>
<td>-0.010</td>
<td>-0.016</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.29)</td>
<td>(-0.31)</td>
<td>(-0.51)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Open</td>
<td>-0.014</td>
<td>-0.017</td>
<td>-0.018*</td>
<td>-0.017</td>
<td>-0.018*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-1.27)</td>
<td>(-1.38)</td>
<td>(-1.70)</td>
<td>(-1.57)</td>
<td>(-1.75)</td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>-0.711</td>
<td>-0.571</td>
<td>-0.757*</td>
<td>-0.712*</td>
<td>-0.944**</td>
<td>-0.865**</td>
</tr>
<tr>
<td></td>
<td>(-1.63)</td>
<td>(-1.53)</td>
<td>(-1.67)</td>
<td>(-1.72)</td>
<td>(-2.14)</td>
<td>(-2.11)</td>
</tr>
<tr>
<td>Pop</td>
<td>0.552***</td>
<td>0.467***</td>
<td>0.581***</td>
<td>0.578***</td>
<td>0.590***</td>
<td>0.567***</td>
</tr>
<tr>
<td></td>
<td>(3.16)</td>
<td>(3.10)</td>
<td>(2.90)</td>
<td>(3.11)</td>
<td>(2.96)</td>
<td>(2.96)</td>
</tr>
<tr>
<td>Election</td>
<td>0.015</td>
<td>0.012</td>
<td>0.038</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.06)</td>
<td>(0.18)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Left</td>
<td>-0.015</td>
<td>-0.013</td>
<td>-0.016</td>
<td>-0.015</td>
<td>-0.016</td>
<td>-0.015</td>
</tr>
<tr>
<td></td>
<td>(-1.47)</td>
<td>(-1.43)</td>
<td>(-1.52)</td>
<td>(-1.49)</td>
<td>(-1.53)</td>
<td>(-1.47)</td>
</tr>
<tr>
<td>Emu</td>
<td>-0.068</td>
<td>-0.05</td>
<td>0.019</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.27)</td>
<td>(-0.18)</td>
<td>(0.07)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Country trends: Yes, Yes, Yes, Yes, Yes, Yes
Country dummies: Yes, Yes, Yes, Yes, Yes, Yes
Observations: 245, 245, 245, 245, 245, 245
Countries: 21, 21, 21, 21, 21, 21
R2: 0.70, 0.73, 0.69, 0.68, 0.66, 0.67
Underidentification test: 19.840, 32.897, 11.449, 15.437, 14.140, 15.711
Sargan test: -2.865, -2.865, -0.151, -0.304, -0.581, -0.985

Notes: Each observation corresponds to a 3 year average. The subscript t-1 denotes the observation of the previous 3 years. The estimation runs from 1966-1969 to 1999-2002. It includes the following countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom and United States. The equations are estimated using Instrumental Variables fixed effects estimation. In columns (1) the equation is exactly identify with Tax as instrument for Tax. In columns (2) the non-significant variables are excluded from the equation but added as additional instruments. The t statistics are in parentheses. *, **, *** - statistically significant at the 10, 5, and 1 per cent. # The underidentification test is an LM test of whether the instruments are correlated with the endogenous regressors. The null hypothesis that the equation is underidentified. The test statistic is to be compared to a chi-square with degrees of freedom equal to the number of instruments. The p-value is in brackets. $ The null hypothesis of the Sargan overidentification test is that the instruments are uncorrelated with the error term and that the excluded instruments are correctly excluded from the estimated equation. Under the null, the test statistic is distributed as chi-squared in the number of overidentifying restrictions. The p-value is in brackets.

all controls. We, then, remove the non-significant variables and add them as additional controls. We test the under-identification of each equation and, in the case of the restricted models, we perform the Sargan over-identification test.

Table 5.3 and 5.4 shows the results. All specifications have considerable good fit with
an $R^2$ above 0.65. Except for the unrestricted specification for the corporate tax rate using $W_2$, all regressions pass the underindentification test, suggesting that in general, the rest of the world variables are valid instruments for the corresponding domestic variables. Also, in all restricted specifications we conclude from the Sargan test that we do not reject the null hypothesis that the instruments are valid.

There are two important results. Firstly, there is evidence of the endogeneity between the two variables, particularly from the corporate tax rate to the public investment. Corporate tax rate also responds positively to public investment but it is only statistically significant when we use $W_3$ as weights. This result is consistent with our model, as the reaction function of tax rate was positive, but very flat in the stock of public capital. On the other hand, the result that public investment increases with the statutory tax rate is quite robust to different weighting procedures. The coefficient ranges from 0.04 to 0.07. For an exogenously driven reduction of 15% of the tax rate, public investment goes down between 0.6% to 1.1%.

The second result is that there is evidence for international competition particularly in the corporate tax rate. A country’s tax rate responds close to 0.4% to an increase of 1% in the tax rate of the rest of the world. This is in line with values reported by Devereux, Lockwood, and Redoano (2008). For public investment, the coefficient of response to the foreign public investment is lower - between 0.2 and 0.3, but is only significant if we use uniform weights.

With respect to the control variables, government consumption, openness and population growth are, in general, significant in both equations. Private capital and fiscal surplus are only significant for public investment.

## 5.5 Conclusion

The strong downward trend of the statutory corporate tax rates represents one of the most striking aspects of international competition between governments. The main objective of this paper is to point out that other consequences of corporate tax rate competition have been overlooked.

Keen and Marchand (1997) argued that tax competition might lead to “too many business centers and airports but not enough parks or libraries”. In fact, this statement might be inconsistent with the general decline of public capital stock that has taken place over the last two decades among many OECD countries.

By contrast, we find a negative relationship between tax competition and public capital stock. The key difference between these two results is the following. In their setting,
PART II. TAXATION AND PUBLIC CAPITAL

the relation between tax rate and public capital comes from international competition. In the presence of competition, there will be a bias in favour of public investment. In our model, this relation exists even in the absence of competition. Public capital stock increases tax revenue. Because of this positive externality there is always a bias in favour of public capital. As competition drives tax rate down, this reduces the externality of public capital and governments have an incentive to reduce their supply. The robustness analysis we performed provides a strong evidence that the central mechanism of our paper remains valid. Besides, this link appears to be significant for most countries where the share of public capital stock has, indeed, decreased.

The general equilibrium analysis appears to be extremely helpful since it allows us to assess quantitatively the effects of competition. We show that tax competition leads to a reduction of both tax rate and public investment. If tax rate goes down by 15%, public investment in steady state goes down between 0.2% and 0.4% of GDP. The short run impact is three times stronger. Our empirical estimates point to slightly higher values: between 0.6% and 1.1% of GDP. Further empirical evidence indicates that there is international competition in both corporate tax rate and public investment.

Although tax competition is likely to have a negative effect on the overall supply of public goods, the traditional view considers that tax competition favours the private sector. This is indeed what we found in the baseline scenario and it is explained by the fact that a race to the bottom reduces the net cost of capital. Nevertheless, we found that it exists a threshold tax rate under which tax competition has a negative effect on total output. This threshold depends crucially on the elasticity of output with respect to public capital.

We believe that our analysis is particularly relevant for the European Union countries where enlargement is likely to put more pressure on tax rates and therefore could reinforce the downward trend of public capital stock in western European countries.
5.6 Appendix

ASYMMETRIC COUNTRIES

Figure A5.1: Asymmetric countries (elasticity of output with respect to public capital)

Country A: $\theta = 0.12$; country B: $\theta = 0.04$
Figure A5.2: Asymmetric countries (elasticity of output with respect to private capital)

Country A: $\alpha = 0.30$; country B: $\alpha = 0.22$
CHAPTER 5. TAX COMPETITION AND THE SUPPLY OF PUBLIC CAPITAL

Data

Table A5.1: Summary statistics and sources

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Mean</th>
<th>Sd</th>
<th>Max</th>
<th>Min</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inv</td>
<td>Public investment (% GDP)</td>
<td>3.499</td>
<td>1.504</td>
<td>10.09</td>
<td>0.770</td>
<td>Kamps (2006)</td>
</tr>
<tr>
<td>Tax</td>
<td>Top bracket corporate tax</td>
<td>38.21</td>
<td>8.730</td>
<td>56.41</td>
<td>7.148</td>
<td>Michigan World Tax Database</td>
</tr>
<tr>
<td>Govcons</td>
<td>Gov. consumption (% GDP)</td>
<td>17.66</td>
<td>4.511</td>
<td>30.14</td>
<td>7.325</td>
<td>OECD-Main Economic Indicators</td>
</tr>
<tr>
<td>Budget</td>
<td>Budget surplus (% GDP)</td>
<td>-2.212</td>
<td>3.851</td>
<td>18.00</td>
<td>-15.71</td>
<td>IMF-IFS</td>
</tr>
<tr>
<td>GDPg</td>
<td>GDP growth</td>
<td>2.778</td>
<td>2.639</td>
<td>12.48</td>
<td>-7.283</td>
<td>IMF-IFS</td>
</tr>
<tr>
<td>Popg</td>
<td>Population growth</td>
<td>0.660</td>
<td>0.569</td>
<td>3.799</td>
<td>-4.526</td>
<td>WB - WDI</td>
</tr>
<tr>
<td>Open</td>
<td>Openness (% GDP)</td>
<td>54.77</td>
<td>29.29</td>
<td>184.2</td>
<td>7.416</td>
<td>WB - WDI</td>
</tr>
<tr>
<td>Capital</td>
<td>Private capital (% GDP)</td>
<td>2.512</td>
<td>0.541</td>
<td>3.818</td>
<td>1.255</td>
<td>Kamps (2006)</td>
</tr>
<tr>
<td>Left</td>
<td>Left party votes (% total)</td>
<td>37.96</td>
<td>14.15</td>
<td>67.6</td>
<td>0</td>
<td>Comparative parties dataset</td>
</tr>
<tr>
<td>Election</td>
<td>Dummy for election year</td>
<td>0.316</td>
<td>0.465</td>
<td>1</td>
<td>0</td>
<td>Comparative parties dataset</td>
</tr>
</tbody>
</table>

Note: the comparative party dataset was created by Duane Swank and is available on http://www.mu.edu/polisci/Swank.htm.

Figure A5.3: Corporate tax rate and allocation of spending in OECD countries
PART II. TAXATION AND PUBLIC CAPITAL

[Charts showing statistical data for different countries over various years, including statutory corporate tax rates, government consumption as a percentage of GDP, and public investment as a percentage of GDP.]
Part III

Other essays
Chapter 6

Labour market flows: Facts from the United Kingdom

6.1 Introduction

The behaviour of flows between employment, unemployment and inactivity drive movements in aggregate indicators, such as the employment and unemployment rate. They are critical to our understanding of labour market dynamics and business cycle fluctuations. Furthermore, worker gross flows and transition rates lie at the heart of state-of-the-art models of unemployment, anchored in the Mortensen and Pissarides (1994) search and matching framework.

The objective of this chapter is to establish a number of key facts about the properties of the UK labour market flows, by examining data from the Labour Force Survey over the past fifteen years. In so doing, it extends the work by Bell and Smith (2002) and provides a systematic study of worker gross flows based on UK data, along the lines of the pioneer work of Blanchard and Diamond (1990) for the United States.1

One significant aspect of the data is the differences in conditional transition probabilities. For example, the job-separation probability is 1% if the person was previously employed, 5% if inactive and 11% if unemployed. The job-finding rate is 20% if the person has been unemployed for two periods, but it is 48% if the person was previously employed. Because of this, if we compute the annual flows based on the quarterly transition probabilities and compare them with the actual ones, calculated directly from Labour Force Survey, the implied annual transitions overshoots by 50% the actual transitions and alters

1 There are some studies on the UK labour market flows, notably classical studies by Nickell (1982) and Pissarides (1986) or more recently Burgess and Turon (2005), but they only consider inflows and outflows of unemployment using claimant count data. Bell and Smith (2002), on the other hand, use Labour Force Survey data but their sample only runs until 2000 and they restrict their analysis to the size and cyclical properties of gross flows, job-to-job flows and job separations by reason.
PART III. OTHER ESSAYS

the relative pattern of the flows. When doing the reverse exercise – extrapolating the quarterly flows from the observed annual flows – one underestimates the true transitions by roughly one half.

The second main contribution of this paper is that it adds to the debate recently revived by Shimer (2007) regarding the relative importance of job-finding and separation rates for fluctuations in unemployment. It provides evidence for the United Kingdom using different decomposition methods proposed in the literature. The additional interest, relative to Petrongolo and Pissarides (2008) is that my sample covers a complete business cycle: the expansion between 1993 to 2001, the slowdown and the beginning of the current recession. I find that the job-separation rate is as important as the job-finding rate, particularly in the early 1990s recession and in the current one. This gives strength to the point made by Davis, Faberman, and Haltiwanger (2006) that changes in the job separation rate explain most of the variation in unemployment during sharp recessions.

Finally, I go on to analyse particular elements of the labour market that can be useful for economists in other areas of research. Given the size of the flows from and into inactivity I have explored in more detail their role over the business cycle. In particular, I have disaggregated the inactive into two subgroups: those that want a job (and therefore can be considered marginally close to the labour market) and those that do not want a job and evaluated the differences between them. In the last few years the United Kingdom experienced structural changes in the level of education of the labour force. Therefore, it seems important to examine the size and the behaviour of labour market flows by education. I have also provided evidence on job-to-job flows and on-the-job search, on the causes of employment separations and on the differences between the public and private sectors.

These stylised facts are of interest to policymakers and macroeconomists alike. For policymakers they can help improve the monitoring of business cycles, the detection of turning points and the assessment of labour market tightness. For macroeconomists, this paper can be seen as a reference for the calibration of a number of parameters, and also provide a guideline of the empirical features that theoretical models should ideally have.

6.2 Preliminary concepts

6.2.1 Labour market dynamics

In order to analyse labour market dynamics I make use of some fundamental equations that describe the evolution of the stock of employed $E$ and the stock of unemployed $U$. The pool of inactive is denoted as $I$. Adding the three pools gives us the working-age
population $W$, while summing employment and unemployment corresponds to the labour force $L$. The unemployment rate is defined as $u = \frac{U}{L}$ and the participation rate as $p = \frac{L}{W}$.

Total employment evolves according to the following equation:

$$E_{t+1} = E_t + N_t^{UE} + N_t^{IE} - N_t^{EU} - N_t^{EI}. \tag{6.1}$$

where $N$ is the gross flows between the pools indicated by the superscript. If we normalise this equation by the total working-age population, we get the following equation that focuses on the total gross flows as the determinant of changes in the employment rate.

$$\frac{E_{t+1} - E_t}{W_t} = \frac{N_t^{UE}}{W_t} - \frac{N_t^{EU}}{W_t} - \frac{N_t^{EI}}{W_t}. \tag{6.2}$$

Alternatively, (2) may be written in terms of transition probabilities rates ($\Lambda$):

$$\frac{E_{t+1} - E_t}{E_t} = \Lambda_t^{UE} \frac{u_t}{1-u_t} + \Lambda_t^{IE} \frac{(1-p_t)}{p_t(1-u_t)} - \Lambda_t^{EU} - \Lambda_t^{EI}. \tag{6.3}$$

We can perform a similar decomposition of the changes in unemployment:

$$U_{t+1} = U_t - N_t^{UE} + N_t^{EU} - N_t^{UI} + N_t^{IU}. \tag{6.4}$$

Again, we can either focus on the gross flows or on the transition rates:

$$\frac{U_{t+1} - U_t}{W_t} = -\frac{N_t^{UE}}{W_t} + \frac{N_t^{EU}}{W_t} - \frac{N_t^{UI}}{W_t} + \frac{N_t^{IU}}{W_t}. \tag{6.5}$$

$$\frac{U_{t+1} - U_t}{U_t} = \Lambda_t^{EU} \frac{1-u_t}{u_t} + \Lambda_t^{IU} \frac{(1-p_t)}{p_t u_t} - \Lambda_t^{UE} - \Lambda_t^{UI}. \tag{6.6}$$

Some authors like Blanchard and Diamond (1990) or Davis (2006) focus on gross flows, while others, such as Shimer (2007) or Fujita and Ramey (2009) give more emphasis to transition rates. The two perspectives are complementary in the analysis of the labour market and the interest in one or the other depends ultimately on the theoretical model one has in mind. Thus I explore both of them.

### 6.2.2 Labour Force Survey

The data are constructed from the Labour Force Survey (LFS). The LFS is a quarterly survey of households living at private addresses in Great Britain. Its random sample design, is based on the Postcode Address File, a list prepared by the Post Office with all addresses which receive fewer than 25 articles of mail a day. The LFS panel samples around 60,000 households for five successive quarters. The households are interviewed
face-to-face when first included on the survey, and by telephone thereafter. The respondents are asked about the household’s characteristic, education, labour market status, economic activity, as well as other elements. The sample is split into five waves. Every quarter one wave of approximately 12,000 leaves the survey and a new wave enters. In this way, we can observe the changes in the labour market status of 80% of the households that took part in the survey and, therefore, obtain the gross labour market flows.

Although the quarterly survey starts in 1992, the 5 waves only run since the first quarter of 1993, so my sample is restricted to the period between 1993:2 and 2008:4. There is a break in the survey in 1996 as before, it did not include Northern Ireland. As Northern Ireland represent less than 3% of the working-age population of the United Kingdom, the break does not affect the size of the gross flows as a percentage of the working-age population or in hazard rates. The constructed flows series are weighted using the Office for National Statistics (ONS) census population weights.

Estimating gross flows on the basis of survey data has two shortcomings: they suffer from non-response bias, and response-error bias. For the LFS, the non-contact rate is around 5% while the refusal rate ranges between 10% and 15%. The response error bias is a more serious problem because, in longitudinal data the errors are cumulative and lead to an overestimation of flows. There is no practical way to deal with response-error bias. We should bear in mind that the results might be biased upwards, particularly in the flows between unemployment and inactivity. Nevertheless there is no reason to believe that the response-error bias affects the cyclical properties of the gross flows.

6.3 Worker gross flows in the United Kingdom

6.3.1 Average gross flows

Figure 6.1 summarises the average quarterly worker flows over the 1996-2008 period. It reports the total number of people that changed status in thousands (t), as a percentage of the working-age population (p) and as a transition probability or hazard rate (h).

Over the sample period there was an average 60,000 net increase in employment every quarter. Substantial quarterly gross flows hide, however, behind this value. An average of 870,000 people move out of employment every quarter, approximately 60% of whom go into inactivity. An average of around 930,000 people move into employment, where the split is broadly similar between unemployment and inactivity. In addition to the 2.7% of the total working-age population that join the pool of employed, there is an additional 2.1% that change employer every quarter.
CHAPTER 6. LABOUR MARKET FLOWS

Figure 6.1: Average quarterly worker flows, Labour Force Survey, 1996-2008

<table>
<thead>
<tr>
<th>Into W.A.P.</th>
<th>Out of W.A.P.</th>
</tr>
</thead>
<tbody>
<tr>
<td>E - 40 (t)</td>
<td>E - 69 (t)</td>
</tr>
<tr>
<td>U - 24 (t)</td>
<td>U - 2 (t)</td>
</tr>
<tr>
<td>I - 130 (t)</td>
<td>I - 86 (t)</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>766 (t)</td>
<td>517 (t)</td>
</tr>
<tr>
<td>2.1 (p)</td>
<td>1.4 (p)</td>
</tr>
<tr>
<td>2.9 (h)</td>
<td>1.9 (h)</td>
</tr>
</tbody>
</table>

Employment
26972 (t)
74.5 (p)

Unemployment
1612 (t)
4.5 (p)

Inactivity
7630 (t)
21 (p)

Note: the worker flows are expressed as total number of people in thousands (t), as a percentage of the working-age population (p) and as a hazard rate (h). The two boxes show the movements in and out of the working-age population. The statistics are for the period starting in 1996 to include Northern Ireland.

Demographic change represents a very small fraction of worker turnover, as shown in the two boxes within the chart. Only a minority of young people (less than 16 years of age) joining the working-age population enter the labour force directly. Similarly, more than half of the people that reach retirement age (65 plus for men, 60 plus for women) are already inactive. For this reason, I exclude from the analysis new entries and exits from the working-age population.

How do these numbers compare to those for the United States? Table 6.1 compares the quarterly figures for the United Kingdom with the monthly values for the United States taken from Bleakley, Ferris, and Fuhrer (1999). If we interpret the size of the gross flows between unemployment, employment and inactivity as a proxy for labour market flexibility, one could be tempted to say that the labour market in the United Kingdom is much less flexible than in the United States. While 6.8% of the population change status every quarter in the United Kingdom, in the United States 6.5% change status every month. In my opinion, a comparison between these values can be misleading for two reasons.

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Table 6.1: Gross flows for the United States and the United Kingdom

<table>
<thead>
<tr>
<th></th>
<th>Monthly US</th>
<th>Quarterly US</th>
<th>Quarterly UK</th>
<th>Annual UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E \rightarrow U$</td>
<td>0.8</td>
<td>1.4*</td>
<td>1.0</td>
<td>1.4</td>
</tr>
<tr>
<td>$E \rightarrow I$</td>
<td>1.7*</td>
<td>5.1*</td>
<td>1.3</td>
<td>2.9</td>
</tr>
<tr>
<td>$U \rightarrow E$</td>
<td>1.0</td>
<td>1.6*</td>
<td>0.8*</td>
<td>2.0</td>
</tr>
<tr>
<td>$U \rightarrow I$</td>
<td>0.8</td>
<td>1.3*</td>
<td>0.8</td>
<td>1.4</td>
</tr>
<tr>
<td>$I \rightarrow E$</td>
<td>1.5</td>
<td>4.5*</td>
<td>1.3</td>
<td>3.1</td>
</tr>
<tr>
<td>$I \rightarrow U$</td>
<td>0.6</td>
<td>0.8*</td>
<td>1.0</td>
<td>1.2</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>6.5</td>
<td>15.0*</td>
<td>6.8</td>
<td>11.6</td>
</tr>
</tbody>
</table>

Note: gross flows in percentage of the working-age population. The columns with an * are extrapolated from a different frequency denoted by the superscript (monthly (m), quarterly (q) or annual (a)), by allowing for multiple transitions and assuming constant transition probabilities.

First, because there might exist multiple transitions within the quarter. Suppose someone is unemployed in the first month, then moves to employment in the second, and then back to unemployment. While a monthly survey would pick up all transitions, the quarterly survey would not detect any. It is possible to overcome the problem of multiple transitions by calculating for the United States the quarterly probabilities implied by the monthly rates or, conversely for the United Kingdom, computing the monthly transition probabilities that generate the observed quarterly probabilities. This is what the literature typically does. The results are also shown in the second and third column of Table 6.1. After correcting for multiple transitions, the total implied quarterly flows in the United States are around 15% of the working-age population, twice the value for the United Kingdom. A big part of the flows, however, are accounted by the flows between inactivity and employment.

But for this comparison to be correct, we are implicitly assuming that: first, there is no heterogeneity on the labour force in terms of transition probabilities and second, that there is no duration dependency. If these assumptions do not hold, the surveys at different frequencies are implicitly putting different weights on the unemployed. One way to test if these two assumptions hold is to look at the conditional transition probabilities. Both duration dependence and heterogeneity would reflect on different conditional probabilities. I computed the average conditional probabilities in the LFS, based on three period flows ($N^{ij}$):

$$
\Lambda_{ij|E_{t-2}} = \frac{N_{ij|E_{t-2}}}{N_{E_{t-1}}}, \quad \Lambda_{ij|U_{t-2}} = \frac{N_{ij|U_{t-2}}}{N_{U_{t-1}}}, \quad \Lambda_{ij|I_{t-2}} = \frac{N_{ij|I_{t-2}}}{N_{I_{t-1}}}.
$$

We can see in Table 6.2 the substantial differences in conditional probabilities. The probability of separation from employment to unemployment is 1% if the person was previously employed, 11% if he was previously unemployed and 5% if he was inactive.

---

2See Appendix for details on the calculations.
Table 6.2: Conditional transition probabilities, Labour Force Survey

<table>
<thead>
<tr>
<th>Unconditional</th>
<th>Conditional on</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_{t-2}$</td>
</tr>
<tr>
<td>$E \rightarrow U$</td>
<td>1.3</td>
</tr>
<tr>
<td>$E \rightarrow I$</td>
<td>1.9</td>
</tr>
<tr>
<td>$U \rightarrow E$</td>
<td>28.0</td>
</tr>
<tr>
<td>$U \rightarrow I$</td>
<td>18.2</td>
</tr>
<tr>
<td>$I \rightarrow E$</td>
<td>6.4</td>
</tr>
<tr>
<td>$I \rightarrow U$</td>
<td>4.8</td>
</tr>
</tbody>
</table>

Note: averages between 1996 and 2008.

The job-finding rate is 49% if two quarters earlier the person was employed, 25% if the person was inactive and 21% if the person was unemployed. In addition, one inactive person is between 3 to 6 times more likely to return to the labour force, if it has only been inactive for one period.

The are two possible explanations for these differences. On the one hand, they can be due to history dependence. Ruhm (1991), for instance, finds that displaced workers face higher unemployment rates for at least four years. Also, Stevens (1997) shows that the effects of displacements in earnings are quite persistent, because of additional job losses in the years following the displacement. The results show that this feature is not only present in job separations, but it is common to all movements between the three states. On the other hand, it might simply indicate a selection effect. The unemployed who have not found a job in the last quarter are more likely to have lower ability and therefore lower transition probability. The differences in conditional probabilities just reflect a changing composition composition of unemployed and not history dependence.

Whatever the cause of the differences of the conditional probabilities is, their existence implies that the extrapolation of flows at a different frequency than the one which the survey was carried, is biased. Therefore, one should be cautious when comparing results from surveys carried out at different frequencies, which often happens between the United States, United Kingdom and other European economies.³

In order to show how possible misleading this can be, I compute the annual transition probability for the LFS directly, by looking at the flows between the first and the fifth survey, and the annual rate extrapolated from the quarterly rates, by assuming equal conditional probabilities. We can see from the last two columns of Table 6.1 that the result is quite different. The annual transitions, calculated through the quarterly rates tend to overestimate the true values, particularly on the employment-inactivity transitions. The

³Petrongolo and Pissarides (2008), for example, compare the United Kingdom based on a quarterly survey and Spain based on a bi-annual survey.
PART III. OTHER ESSAYS

total gross flows, estimated using the quarterly transition probabilities, are 18.4% of working-age population as opposed to the consistently calculated value of 11.6%. When doing the reverse exercise – extrapolating the quarterly flows from the observed annual flows – we underestimate the true transitions by roughly one half.

6.3.2 Evolution of labour market stocks and flows

The first row in Figure 6.2 displays the evolution of the employment rate, unemployment rate and inactivity rate in the United Kingdom over the past 30 years. The vertical line signals the beginning of the flows sample. We can see that the sample covers one complete business cycle. The unemployment rate fell until 2001, it was relatively stable until 2005 and has increased since, particularly in the last two quarters. The inactivity rate has a small downward trend, but compared to historical standards it can be considered relatively flat.

The second and third rows in Figure 6.2 show the flows between the three pools, as a percentage of the working-age population and as hazard rates. Most of the action over

Figure 6.2: Labour market stocks, gross flows and hazard rates

Note: the flows series are a four-quarter moving average to remove seasonality and high frequency movements.
the sample has been driven by the flows into and out of unemployment. For instance, at the beginning of the sample, 2.9% of the working-age population moved into employment every quarter, but by the end of the sample it was less than 2.5% of the working-age population were entering the pool of employed. The reduction of the inflows to employment comes exclusively from unemployment: every quarter, the gross flows from unemployment to employment fell by 0.4% of the working-age population which corresponds to, roughly, 150,000 people. Separations from employment to unemployment have also fallen, from 1.4% to 0.9% of the working-age population in 2007 but they have picked up sharply since the beginning of the recession. By contrast, flows between employment and inactivity have remained broadly stable across the sample period.

Although the picture of the gross flows and hazard rates is very similar for employment and inactivity, this is not the case for unemployment. While the actual number of people that moved out of unemployment fell, shrinking the pool of unemployed, the probability of moving out of unemployment increased sharply between 1996 and 2005.

6.3.3 Cyclical properties of labour market flows

The literature on worker flows defines the cyclicality of flows as their correlation with the level of economic activity. I estimate it by running a ordinary least square regression of the log of each variable on season dummy variables, a linear trend and the unemployment rate. This follows Baker (1992), who undertakes a similar procedure to analyse the cyclical movements of unemployment duration. In the working paper version of this paper I use simple correlation coefficients and the results are identical whether I use unemployment or employment rate as the cyclical indicator. Additionally, I used detrended GDP or capacity utilisation but, although the results pointed in the same direction, the correlations were weaker and in many cases insignificant. I have also tried different detrending methods: using an Hodrick-Prescott filter with a smoothing parameter of $10^5$ is very close to linear detrending, so the results are similar. Conversely, when I used a Baxter-King bandpass filter and an HP filter with the usual quarterly data parameter (1600), the correlations were low and not significant. These filters remove too much of the variation of the series.

Inflows and outflows of all pools are countercyclical. In economic downturns, as the labour market gets looser, there are more movements between the three states. In particular, most of the action occurs in the unemployment pool. More of the unemployed find a job or stop searching for one. Also, more of the inactive start looking for a job and more workers lose theirs. Flows between employment and inactivity do not seem to have a cyclical component.

The separation rate from employment to unemployment, and the transition probability from inactivity to unemployment are strongly countercyclical, while the job finding rate is strongly procyclical. In other words, recessions are periods when it is harder for
Table 6.3: Cyclical variation of labour market flows and hazard rates

<table>
<thead>
<tr>
<th>Gross flows</th>
<th>Hazard rates</th>
<th>Gross flows</th>
<th>Hazard rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>→ E</td>
<td>0.016</td>
<td>(1.33)</td>
<td>-0.007</td>
</tr>
<tr>
<td>E →</td>
<td>0.027**</td>
<td>(2.42)</td>
<td>0.036*</td>
</tr>
<tr>
<td>→ U</td>
<td>0.078*</td>
<td>(9.91)</td>
<td>0.086*</td>
</tr>
<tr>
<td>U →</td>
<td>0.057**</td>
<td>(7.13)</td>
<td>-0.072*</td>
</tr>
<tr>
<td>→ I</td>
<td>0.026**</td>
<td>(2.02)</td>
<td>0.024</td>
</tr>
<tr>
<td>I →</td>
<td>0.029**</td>
<td>(2.04)</td>
<td>0.036*</td>
</tr>
</tbody>
</table>

Note: The cyclicity of the series is the coefficient on unemployment rate in a regression of the series in logs on season quarter dummies, time trend and unemployment rate. T-statistics are in brackets. * denotes significant at 1% and ** significant at 5%.

an unemployed individual to find a job, an employed person is more likely to lose their job and an inactive person is more likely to start looking for one.

In terms of magnitude, job-finding rate seems to fluctuate as much as the job-separation rate. Also quite responsive is the hazard rate between inactivity and unemployment. It seems that job-finding and separation rates are equally important determinants of unemployment fluctuations. Nevertheless, given the ongoing debate on the relative importance of each transition rate, I make a more careful analysis in the next section.

6.4 What drives unemployment in United Kingdom?

What dictates the cyclical behaviour of unemployment: hires or separations? The seminal work on labour market flows by Blanchard and Diamond (1990) and Davis and Haltiwanger (1992) set the ‘conventional wisdom’ that recessions are mainly driven by high job loss rates. In two recent papers, Shimer (2007) and Hall (2006) have challenged this view by presenting evidence that cyclical unemployment dynamics are largely driven by a time-varying job-finding rate and that the separation rate is very close to being acyclical. These two papers had a very strong impact on the field. On the one hand, many researchers have used this evidence to develop models that incorporate constant job destruction rates (for instance Blanchard and Gali (2010) or Gertler and Trigari (2009)). On the other hand, other researchers put forward more evidence that opposes their claims. Davis, Faberman, and Haltiwanger (2006) provide new empirical evidence in support of the view that a recession starts out with a wave of separations. Fujita and Ramey (2009) and Elsby, Michaels, and Solon (2009) argue that both job-separation rate and job-finding rate play a significant role in unemployment fluctuations.

The UK evidence is also controversial. Pissarides (1986) finds that, for the period between 1967 and 1983, almost all changes in unemployment can be accounted for by changes in the job-finding rate. In contrast, Burgess and Turon (2005) claim that between
1967 and 1998 the unemployment dynamics arise mostly from shocks to inflows. More recently, Petrongolo and Pissarides (2008) find that, using claimant count data, the job-separation rate accounts for one third of unemployment volatility. However, using LFS data they conclude that the job-separation rate is as important as the job-finding rate.

The ongoing debate about the roles of hires and separations give emphasis to the use of different methodological approaches and data sources across researchers. To evaluate the contribution of job-finding and job-separation rates in the United Kingdom I use two decompositions of unemployment that have been proposed in the recent literature: Shimer (2007) and Fujita and Ramey (2009). I also correct the data for time aggregation, by applying the discrete correction method proposed by Elsby, Michaels, and Solon (2009), that ignores movements out of a given state and back, which occur within one week.\(^5\)

### 6.4.1 Unemployment decompositions

The starting point for all unemployment decompositions is the equation of the steady-state unemployment \(u_{t}^{ss}\):\(^6\)

\[
\frac{A_{EU} + U_{E}}{A_{EU} + \Lambda_{EU} + U_{E}^{'}},
\]

Shimer (2007) isolates the effect of the job-finding rate by constructing a counterfactual unemployment rate if job-separation were always to be at its sample average (denoted \(u_{t}^{s}\)). Similarly, he constructs the series for the unemployment rate, if the job-finding rate were to be at its sample average (denoted \(u_{t}^{*}\)).

\[
\frac{A_{EU} + \Lambda_{EU} + U_{E}^{'} - \Lambda_{EU} + \Lambda_{EU}^{*}}{A_{EU} + \Lambda_{EU} + U_{E}^{'}},
\]

Shimer’s decomposition has faced some criticism because the steady-state approximation is non-linear in the two hazard rates. In this sense, if we chose different values for \(\bar{s}\) and \(\bar{f}\) instead of the sample average we would get different answers.

Fujita and Ramey (2009) propose a more natural decomposition. By linearizing steady-state unemployment around the previous period steady-state \(u_{t-1}^{ss}\), we get the following expression:

\[
\frac{u_{t}^{ss} - u_{t-1}^{ss}}{u_{t-1}^{ss}} = \frac{A_{EU} - \Lambda_{EU} - A_{EU}^{*} - \Lambda_{EU}^{*}}{A_{EU} - \Lambda_{EU} - A_{EU}^{*} - \Lambda_{EU}^{*}}.
\]

\(^5\)We can write each one of the nine quarterly transition probabilities as a non-linear function of all nine monthly or weekly transition probabilities. We can then retrieve them by solving a system of nine equations. See details in Appendix.

\(^6\)This expression is only correct when we ignore the flows in and out of inactivity. I do this to be easier to compare with the results from existing literature. Furthermore, both Shimer (2007) and Petrongolo and Pissarides (2008) conclude that, including the flows in and out of inactivity do not change the relative importance of the job-finding and job-separation rates.
which is simply breaking down the percentage change of the steady-state unemployment rate into percentage changes on both job-finding and job-separation rates. We can restate this expression as $du_{t}^s = du_{t}^f + du_{t}^s$, where

$$
du_{t}^s \equiv \frac{u_{t-1}^{ss} - u_{t-1}^{ss}}{u_{t-1}^{ss}}, \quad du_{t}^f \equiv -(1 - u_{t-1}^{ss}) \frac{\Lambda_{t-1}^{UE} - \Lambda_{t-1}^{FE}}{\Lambda_{t-1}^{UE}} \quad \text{and} \quad du_{t}^s \equiv (1 - u_{t-1}^{ss}) \frac{\Lambda_{t-1}^{FE} - \Lambda_{t-1}^{EU}}{\Lambda_{t-1}^{EU}}.
$$

(6.10)

The variance of the percentage change of the steady-state equilibrium unemployment is the sum of the covariance between $du_{t}^s$ and $du_{t}^f$ and the covariance between $du_{t}^s$ and $du_{t}^s$:

$$
\text{Var}(du_{t}^s) = \text{Cov}(du_{t}^s, du_{t}^f) + \text{Cov}(du_{t}^s, du_{t}^s).
$$

(6.11)

6.4.2 Claimant count data

One way to assess the robustness of the results is to repeat the exercise using data generated at a monthly frequency. I use data on the claimant count unemployment outflows and inflows to calculate a proxy for job-finding and job-separation rates. This data, provided by ONS, covers the unemployed that are claiming unemployment benefits. It is a proxy for two reasons. First, people registered in the claimant count are only a subset of the unemployed. Second, despite constituting the large majority, claimant account flows include not only flows between unemployment and employment but also include flows between unemployment and inactivity. With this data we can go back to 1989, which allows us to also capture the early 1990s recession. Figure 6.3 shows a comparison between the unemployment rate, the monthly job-finding rate and the monthly job-separation rate based on the LFS and the ones calculated from the claimant count (three month average for the quarter).

The claimant count job-finding and job-separation rates are always higher than the monthly transition probability calculated from the LFS: the job-finding rate between 5 to 10 percentage points and the job-separation rate between 0.2 and 0.6 percentage points. As mentioned above, claimant count flows data also include flows into and from inactivity. These flows can bias the job-finding and separation-rates upwards. The unemployed registered in the claimant count are a subset of total unemployment and have a more effective search mechanism but they are also more likely to lose their jobs and therefore have higher transition rates. In addition, the discrepancy in the results might be attributed to the time aggregation correction of the quarterly LFS data. As shown before, extrapolating monthly transition probabilities based on quarterly series might generate a downward biased series. Putting these issues aside, the correlations between the series are quite high: 0.99 between the unemployment rates, 0.90 between the job-separation rates and 0.91 between the job-finding rates.

7This is a discrete time version of the decomposition Elsby, Michaels, and Solon (2009).
Table 6.4 displays the relative importance of job-separation rate using the LFS and claimant count data, for the two methodologies and the different frequencies. If we look at weekly transition probabilities, using LFS data the job-separation rate accounts for around 50% of unemployment volatility. Using claimant count data, for the same sample the values are lower – between 38% to 48%. However, when we include the early 1990s recession the the importance of job-separation increases, particularly using Shimer decomposition. This evidence gives strength to the point made by Davis, Faberman, and Haltiwanger (2006) that changes in the job-separation rate explain most of the variation in unemployment during sharp recessions, whereas fluctuations of the job-finding rate are the focal element during mild recessions.

These values are in line with the ones reported by Petrongolo and Pissarides (2008). Using LFS data they find that job-separation rate has the same contribution to unemployment volatility as the job-finding rate – around one third each – while the rest is accounted for the flows between unemployment and inactivity. They also find that, when using claimant count the job-separation rate seems less important.8

Table 6.4: Relative importance of job-separation rate

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Shimer F &amp; R</td>
<td>Shimer F &amp; R</td>
<td>Shimer F &amp; R</td>
</tr>
<tr>
<td>Quarterly</td>
<td>0.516</td>
<td>0.619</td>
<td>0.424</td>
</tr>
<tr>
<td>Monthly</td>
<td>0.453</td>
<td>0.593</td>
<td>0.384</td>
</tr>
<tr>
<td>Weekly</td>
<td>0.429</td>
<td>0.582</td>
<td>0.439</td>
</tr>
</tbody>
</table>

Note: for Shimer, the value is the ratio between the standard deviation of the linearly detrended counterfactual steady-state unemployment rate $u_t^*$ and the sum of the standard deviations of both $u_t^*$ and $u_s^*_t$. For Fujita and Ramey, the value correspond to ratio between the covariance between $du_t^*$ and $du_s^*_t$ and the variance of $du_s^*_t$. The series are previously seasonally adjusted using the X12 Census programme.

8In terms of methodology, there are three differences of this work relative to Petrongolo and Pissarides (2008). First they compute the transition probabilities for all people above 15, while I exclude people out of retirement age (60 plus for women and 65 plus for men). Because they include them, their transition probabilities are lower than mine, particularly out of inactivity. Second, they compute the continuous transition probabilities. Third, they include in the decomposition the transitions between unemployment and inactivity.
6.5 Other perspectives on the UK labour market

6.5.1 Job-to-job flows

Many economists think that on-the-job search and job-to-job flows are important elements of business cycles. For instance, Krause and Lubik (2007), building on the Pis- sarides (1994) on-the-job search model, concluded that on-the-job search and job-to-job transitions greatly amplify shocks to the economy.

One advantage of the LFS, relative to the US surveys, is that it allows us to calculate job-to-job flows. It asks the respondent what year and month it started the current job, making it possible to compute the length of current job tenure. I count as a job-to-job transitions the cases where an individual is employed in the first quarter and still employed in the second quarter but with a job tenure of less than three months. We should bear in mind that this measure of job-to-job flows includes people that changed job directly as well as individuals that had non-measured spells of unemployment or inactivity. In other words, it includes individuals that moved out of employment and back into employment within the quarter.

The first graph of Figure 6.4 plots the job-to-job flows as a percentage of the working population. Job-to-job flows increased from 1996 to 2001, but have fallen since. As one expects, there are substantial differences in the transition probabilities among employees engaged in on-the-job search and the ones that are not searching. If a worker is searching for a job, the probability of changing job in any given quarter is, on average, 14%. If he is not searching, the probability is only 2%. Each quarter, on average, 7% of workers are searching for a different job. This is higher than the value of 5.2% found by Pissarides and Wadsworth (1994). All in all, roughly one third of all the job changers were previously searching for a job.

Evidence from the United States by Fallick and Fleischman (2004) suggests that job-to-job flows are procyclical. We observe the same pattern in the United Kingdom as we

\[ \text{Figure 6.4: Job-to-job flows} \]

\[ \text{Note: the flows series are a four-quarter moving average to remove seasonality and high frequency movements.} \]
can see in Table 6.5. Job-to-job transition probabilities are strongly procyclical as well as actual job-to-job flows.

Some on-the-job search theories have different predictions for the cyclicality of the number of employees searching for a different job. For instance, the stylised model presented in Pissarides (2000) predicts that increasing productivity leads to more people searching for jobs. Conversely, Nagypál (2008) argues that workers undertake less on-the-job search when they face lower unemployment risk, as is the case of expansions. In the United Kingdom, these effects seem to cancel out such that the number of employees searching for a different job is not related to the business cycle.

<table>
<thead>
<tr>
<th>Table 6.5: Cyclical variation of job-to-job flows</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job-to-job flows</td>
</tr>
<tr>
<td>Job-to-job hazard rate if searching</td>
</tr>
<tr>
<td>Job-to-job hazard rate if not searching</td>
</tr>
<tr>
<td>Employees searching</td>
</tr>
</tbody>
</table>

Note: the cyclicality of the series is the coefficient on unemployment rate in a regression of the series in logs on season quarter dummies, time trend and unemployment rate. T-statistics are in brackets. * denotes significant at 1%.

6.5.2 Outflows from employment by reason

Are separations from employment driven by firms or workers? The LFS allows us to split the cause of employment separations into three categories: involuntary separations, resignations, or other reasons. The first category includes dismissals, termination of temporary employment contracts or redundancies, which are involuntary from the worker’s point of view. Resignations include cases where the worker resigned, and also where they took voluntary redundancy. Finally, other reasons encompasses giving up work for health, family or personal reasons or taking early retirement. Roughly half of total separations from employment are due to other reasons and the other half is due to resignations and involuntary separations in equal shares.

The graphs in Figure 6.5 plot, for the three types of job separations, the share caused by each reason. The flows from employment to unemployment are dominated by involuntary separations. They account between 40% and 45% of total employment to unemployment flows. In the beginning of the sample resignations only accounted for 20% of total employment to unemployment flows, in 2007 that value was close to 30% but it has fallen sharply since.

As expected, other reasons accounts for more than 70% of the employment to inactivity flows, with the remaining being split equally between involuntary separations and
Figure 6.5: Employment outflows by reason

![Graph showing employment outflows by reason.](image)

Note: the flows series are a four-quarter moving average to remove seasonality and high frequency movements.

resignations. Finally, only a minority of the job-to-job flows are caused by involuntary separations. Around 50% of job-to-job flows are due to other reasons and 35% are due to resignations.

Table 6.6 shows that, as expected, involuntary separations are strongly countercyclical, while resignations are very procyclical. In economic slowdowns less people quit, which partially counterbalances the fact the more people lose their jobs. Separations by other reasons are acyclical which is consistent with the incidence of personal reasons having a weaker relationship with the business cycle. When we disaggregate the separations even more, we see that the counter-cyclicality of the involuntary separations is mainly driven by the employment-unemployment flows while the procyclicality of resignations is much stronger in the job-to-job transitions.

Table 6.6: Cyclical variation of employment outflows hazard rates by reason

<table>
<thead>
<tr>
<th>Reason</th>
<th>Total</th>
<th>Employment to</th>
<th>Employment to</th>
<th>Job-to-job</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>separations</td>
<td>unemployment</td>
<td>inactivity</td>
<td></td>
</tr>
<tr>
<td>Involuntary separations</td>
<td>0.046* (3.58)</td>
<td>0.128* (5.70)</td>
<td>0.021 (1.13)</td>
<td>-0.040** (-2.42)</td>
</tr>
<tr>
<td>Resignations</td>
<td>-0.116* (-7.65)</td>
<td>-0.006 (-0.37)</td>
<td>-0.082* (-3.61)</td>
<td>-0.169* (-8.20)</td>
</tr>
<tr>
<td>Other reasons</td>
<td>-0.005 (-0.37)</td>
<td>0.111* (6.67)</td>
<td>0.003 (0.13)</td>
<td>-0.050* (-3.06)</td>
</tr>
</tbody>
</table>

Note: the cyclicity of the series is the coefficient on unemployment rate in a regression of the series in logs on season quarter dummies, time trend and unemployment rate. T-statistics are in brackets. * denotes significant at 1% and ** significant at 5%.

### 6.5.3 Disaggregating inactivity

Given the magnitude of the flows in and out of inactivity, researchers have asked if some of the inactive should be considered as unemployed. Flinn and Heckman (1983) analysed conditional and unconditional transition probabilities between the two states and concluded that they are essentially different. In the United Kingdom, however, Joyce, Jones, and Thomas (2003) found that many subgroups of the inactive have the same transition probability to employment as the unemployed. Blanchard and Diamond...
disaggregate the pool of inactive into two well-defined sub-groups: those that want a job ($I^w$) and those that do not want a job ($I^o$). The inactive that want a job can be considered marginally close to the labour market, and consequently more likely to go into the labour force. The LFS also allows for this distinction.

The first graph in Figure 6.6 shows the two series as a percentage of the working-age population. It is evident that the relative size of the two groups has changed over the sample. While the pool of inactive that want a job have a negative trend similar to the unemployment rate, the pool of inactive that do not want a job has increased over the sample period. On average, the number of inactive that do not want a job is almost three times higher than the number of inactive that want a job.

Figure 6.6: Inactivity by sub-groups

![Graph showing inactivity by sub-groups]

Note: the flows series are a four-quarter moving average to remove seasonality and high frequency movements.

Table 6.7 reports the transition probabilities between the four groups. The inactive that want a job are twice as likely to join the labour force, and almost four times more likely to join the pool of unemployed than the inactive that do not want a job. Additionally, every quarter, 11% of the unemployed move into inactivity but still want a job while only 6% move to inactivity and do not want a job. There are also relatively high transition probabilities between the two groups of inactive. Around 21% of the inactive that want a job abandon their intentions by the following quarter. It seems that this state is a limbo between inactivity and the labour force.

Table 6.7: Transition matrix, Labour Force Survey (% per quarter)

<table>
<thead>
<tr>
<th>To:</th>
<th>From: Employment</th>
<th>Unemployment</th>
<th>Inactive (want)</th>
<th>Inactive (out)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Employment</td>
<td>96.7</td>
<td>26.8</td>
<td>8.4</td>
<td>5.5</td>
</tr>
<tr>
<td>Unemployment</td>
<td>1.4</td>
<td>55.8</td>
<td>10.1</td>
<td>2.9</td>
</tr>
<tr>
<td>Inactive (want)</td>
<td>0.7</td>
<td>11.3</td>
<td>60.6</td>
<td>10.8</td>
</tr>
<tr>
<td>Inactive (out)</td>
<td>1.2</td>
<td>6.2</td>
<td>20.9</td>
<td>80.8</td>
</tr>
</tbody>
</table>

Note: averages between 1993:1 to 2008:4.
Table 6.8 exhibits the cyclical properties of the gross flows and hazard rates between the two groups of the inactive and the labour force. The outflows from inactivity to the labour force are countercyclical, independent of the subgroup of inactive we consider. However the converse is not true. Whereas the flows from the labour force to inactivity (out) are not related to the cycle, the flows between the labour force and the inactive that want a job are countercyclical. In recessions, more people leave the labour force but still want a job.

Taking the evidence as a whole, there seems to exist a closer link between the pool of the inactive that want a job and the labour force, particularly unemployment.

<table>
<thead>
<tr>
<th>Gross flows</th>
<th>Hazard rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L \rightarrow I^w$</td>
<td>0.081* (6.80) 0.045* (3.84)</td>
</tr>
<tr>
<td>$L \rightarrow I^o$</td>
<td>0.000 (0.02) -0.001 (-0.06)</td>
</tr>
<tr>
<td>$I^w \rightarrow L$</td>
<td>0.057* (4.14) 0.058* (4.21)</td>
</tr>
<tr>
<td>$I^o \rightarrow L$</td>
<td>0.031 (1.70) 0.038** (2.21)</td>
</tr>
<tr>
<td>$I^w \rightarrow I^o$</td>
<td>0.020 (1.66) 0.021** (2.34)</td>
</tr>
<tr>
<td>$I^o \rightarrow I^w$</td>
<td>0.050 (3.64) 0.057* (3.99)</td>
</tr>
</tbody>
</table>

Note: the cyclicity of the series is the coefficient on unemployment rate in a regression of the series in logs on season quarter dummies, time trend and unemployment rate. $T$-statistics are in brackets. * denotes significant at 1% and ** significant at 5%.

6.5.4 Flows by education

Previous studies on labour market flows have paid relatively little attention to differences by levels of education. To explore such differences, I divide the working-age population into three groups depending on the level of education: higher education (Education 1), A-levels and GCSE or equivalent (Education 2) and below GCSE (Education 3). There has been a significant change in the UK economy over the past decade, with the share of working-age population with higher education increasing from 20% in 1997 to above 30% in 2008. Over the same period, the share of the working-age population in the lowest education category fell from around 30% to below 20%.

There are striking differences across the three education groups with respect to employment, unemployment and inactivity rates, as one can see in the first row of Figure 6.7. The average employment rate among the most educated is 89%, as opposed to 56% for individuals in the lowest education category. Both the unemployment rate and inactivity rate are monotonically decreasing in the level of education. Individuals in the lowest education category face an average unemployment rate of 11%, almost four times higher than the average unemployment rate of those with higher education. The average inactivity rates are, in increasing order of level of education, 37%, 19% and 10%.
The difference between education categories extends to transition probabilities, as shown in the remaining figures of Figure 6.7. For example, the average separation rates from employment to unemployment are 0.9%, 1.4% and 1.9% as we go down the education ranking. The job-finding rate also presents significant differences. By the end of the sample, individuals in the highest education category are twice as likely to find a job than individuals in the lowest education group.

Another interesting fact is that, after 2001, the employment rate has fallen for all groups, with the exception of the more educated. In other words, it has fallen for 70% of the working-age population. The aggregate employment rate has, therefore, been largely supported by the increase in the share of the working-age population in the highest education category.

Table 6.9 presents the coefficient of the regression of each transition probability with the unemployment rate. The cyclical properties of most transition probabilities are quite robust across levels of education. The job-finding rate is highly procyclical and the separation rate from employment to unemployment is countercyclical. The probability
of moving from inactivity to unemployment is countercyclical at all levels of education. From the magnitude of the coefficients we can see that individuals with higher education face less cyclical fluctuations in transition probabilities, than the individuals with lower education.

Table 6.9: Cyclical variation of labour market hazard rates by education

<table>
<thead>
<tr>
<th></th>
<th>Education 1</th>
<th>Education 2</th>
<th>Education 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E \rightarrow U$</td>
<td>0.082* (2.71)</td>
<td>0.101* (6.20)</td>
<td>0.103* (4.86)</td>
</tr>
<tr>
<td>$E \rightarrow I$</td>
<td>0.026 (1.18)</td>
<td>-0.002 (-0.10)</td>
<td>-0.018 (-0.82)</td>
</tr>
<tr>
<td>$U \rightarrow E$</td>
<td>-0.071* (-3.66)</td>
<td>-0.110* (-7.29)</td>
<td>-0.096* (-5.09)</td>
</tr>
<tr>
<td>$U \rightarrow I$</td>
<td>-0.036 (-1.13)</td>
<td>-0.017 (-1.12)</td>
<td>-0.042** (-2.25)</td>
</tr>
<tr>
<td>$I \rightarrow E$</td>
<td>0.016 (0.66)</td>
<td>-0.025 (-1.19)</td>
<td>-0.040 (-1.47)</td>
</tr>
<tr>
<td>$I \rightarrow U$</td>
<td>0.089* (2.88)</td>
<td>0.133* (5.64)</td>
<td>0.149* (8.21)</td>
</tr>
</tbody>
</table>

Note: the cyclicity of the series is the coefficient on unemployment rate in a regression of the series in logs on season quarter dummies, time trend and the unemployment rate of the respective category. T-statistics are in brackets. * denotes significant at 1% and ** significant at 5%.

6.5.5 Public and private sector employment

LFS employment can be decomposed into public and private sector. Public sector employment is composed mainly of employment from local and central government and health authorities, but it also includes employment by public companies, universities, armed forces, charities and other organizations. As in other OECD countries, the level of public employment in the United Kingdom is quite high at around 25% of total employment. Figure 6.8 shows the public and private sector employment rates and the different transition probabilities.

The behaviour of the two employment rates differ significantly. The private sector employment rate grew from 53% of the working age population in 1996 to just under 57% in 2002 before falling back somewhat. By contrast, the public sector employment rate was relatively stable until 2002, at around 17% of the working age population, but had risen close to 19% by the end of the sample.

On average, the separation rate to unemployment is 2.5 times higher in the private sector than in the public sector. If one is unemployed, the average probability of finding a public sector job is 3.6%, while for a private sector job it is 22% - six times higher. The probability of moving from inactivity to private sector employment is also, roughly, six times higher than for the public sector.

---

*As the LFS is a household survey, the split is based on individuals self reporting whether they work in the public sector and is therefore they are prone to misclassification error.
Table 6.10 summarises the cyclical properties of private and public sector employment rates and their transition probabilities. While private sector employment is naturally procyclical, public sector employment is countercyclical. Previous studies, for instance Algan, Cahuc, and Zylberberg (2002), have also identified this positive correlation between public sector employment and the unemployment rate in several OECD economies.

The most puzzling element of Table 6.10 is the fact that we cannot identify why private and public sector employment rates have opposite cyclical behaviours. Job separation rates are strongly countercyclical for both sectors. Moreover, job finding rates go up in expansions for both sectors although the cyclicity is stronger for private sector employment. This dual behaviour of private and public sector employment cannot also be explained by flows between the two sectors or with inactivity, as they all seem unrelated to the business cycle.

Table 6.10: Cyclical variation of labour market flows and hazard rates

<table>
<thead>
<tr>
<th>Private sector</th>
<th>Public sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross flow</td>
<td>Hazard rate</td>
</tr>
<tr>
<td>( E^p )</td>
<td>-0.013* (10.58)</td>
</tr>
<tr>
<td>( E^p \to U )</td>
<td>0.067* (6.09) 0.080* (7.67)</td>
</tr>
<tr>
<td>( E^p \to I )</td>
<td>-0.020 (-1.25) -0.006 (-0.41)</td>
</tr>
<tr>
<td>( U \to E^p )</td>
<td>0.048* (3.80) -0.091* (-10.59)</td>
</tr>
<tr>
<td>( I \to E^p )</td>
<td>-0.013 (-0.76) -0.008 (-0.48)</td>
</tr>
<tr>
<td>( E^p \to E^q )</td>
<td>0.008 (0.44) 0.021 (1.21)</td>
</tr>
</tbody>
</table>

Note: * denotes significant at 1% and ** significant at 5%. T-statistics in parenthesis.
PART III. OTHER ESSAYS

6.6 Conclusion

The objective of this chapter was to describe the main developments in, and establish a number of key facts about, the recent history of the UK worker gross flows. It provided a picture of a wide range of information about worker gross flows from different angles, which is essential to understand the UK labour market.

The findings of this paper can be summarised as follows:

- In each quarter, 7% of the working-age population change status between inactivity, employment and unemployment and 2.1% of the working-age population change their employer.

- Gross flows in and out of the three pools are countercyclical. In expansions, as the labour market becomes tighter there are fewer movements between the three pools.

- Employment to unemployment flows are countercyclical, as well as the job-separation rate. Unemployment to employment flows are countercyclical too, but the job-finding rate is procyclical.

- The job-finding rate and job-separation rate are equally important determinants of unemployment fluctuations. Job-separation rate is particularly relevant during recessions.

- Every quarter, 7% of all employees are searching for a different job. They are seven times more likely to change jobs than the ones not searching. Job-to-job transition probability is strongly procyclical, but the number of employees searching for a different job is acyclical.

- Resignations are strongly procyclical, involuntary separations (layoffs) are countercyclical and separations for other reasons are acyclical. Involuntary separations dominate the employment to unemployment flows while 70% of all employment to inactivity flows occur because of other reasons. Only 15% of the job-to-job flows are driven by involuntary separations.

- The inactive that want a job are twice as likely to move into the labour force and four times more likely to move into unemployment than the inactive that do not want a job.

- There are substantial differences in the employment, unemployment and inactivity rates of different education categories, as well as in transition probabilities. Individuals in the lowest education category face a three times higher unemployment and inactivity rate, twice as high separation rate and half the job-finding rate, than individuals in the highest education category.
Close to 25% of total employment is public sector employment which, unlike private sector employment, is countercyclical. The separation rate is twice as high in the private sector as in the public sector. In contrast, both the unemployed and inactive are six times more likely to find a job in the private sector than in the public sector.

In addition to these findings, the most striking aspect of the data is that condition transition probabilities are substantially different across different histories. Because of these differences, estimating annual gross flows or transition probabilities based on the quarterly transition probabilities overestimates the actual ones. This finding has three significant implications.

First, one should be cautious when comparing results from surveys carried out at different frequencies, which often happens between the United States, United Kingdom and other European economies. Having the transition probabilities at a given frequency is not enough to characterise a labour market. We have to know also how they change at different frequencies. Second, we should reassess some of numbers used in calibrating macroeconomic models. For instance, Blanchard and Gali (2010) calibrate the quarterly separation rate for the United States as 12%, which seems to be too high. We should not extrapolate the information from monthly to quarterly frequency and, instead, we should try to get these values directly. Finally, one should be careful when applying standard procedures like time aggregation correction, that rely on the implicit assumption of equality of conditional transition probabilities.
6.7 Appendix

MULTIPLE TRANSITIONS AND DIFFERENT FREQUENCIES

Suppose that we have nine transition probabilities, calculated at a given frequency $i$: $\Lambda_{EE}^i, \Lambda_{EU}^i, \Lambda_{UE}^i, \Lambda_{UU}^i, \Lambda_{UI}^i, \Lambda_{IU}^i, \Lambda_{II}^i$ and $\Lambda_{II}^i$. If we consider that the transition probabilities of each individual are constant across time, we can compute the implied transition probabilities at any frequency (yearly, quarterly, monthly or weekly), using one of the following system of equations.

\[
\Lambda_{ij}^q = \sum_k \sum_l \sum_n \Lambda_{ik}^q \Lambda_{kl}^q \Lambda_{ln}^q \Lambda_{nj}^q, \quad i, j, k, l, n = \{E, U, I\} \quad (A6.1)
\]

\[
\Lambda_{ij}^m = \sum_k \sum_l \Lambda_{ik}^m \Lambda_{kl}^m \Lambda_{lj}^m, \quad i, j, k, l = \{E, U, I\} \quad (A6.2)
\]

\[
\Lambda_{ij}^w = \sum_k \sum_l \sum_n \Lambda_{ik}^w \Lambda_{kl}^w \Lambda_{ln}^w \Lambda_{nj}^w, \quad i, j, k, l, n = \{E, U, I\} \quad (A6.3)
\]

When we use LFS data we have data on quarterly flows between the three states: employment, unemployment and inactivity. To solve for the annual rates we can compute the value directly from equation (A6.1). To compute the monthly transition probabilities we need to solve the non-linear system of equation (A6.2) for the monthly rates and (A6.3) for the weakly rates. To calculate the gross flows, we just multiply the transition probabilities by the stocks of employment, unemployment or inactivity.

When we use the claimant count data, there are only the transition rates between employment and unemployment, so we get a 4 equations system instead of 9.
Chapter 7

Short and long-run determinants of sovereign credit ratings

7.1 Introduction

Sovereign credit ratings are a condensed assessment of a government’s ability and willingness to repay its public debt on time. Such measures of the probability of default are particularly relevant for international financial markets, economic agents and governments. First, sovereign ratings are a key determinant of the interest rates a country faces in the international financial market and therefore of its borrowing costs. Second, sovereign ratings may have a constraining impact on the ratings assigned to domestic banks or companies. Third, some institutional investors have lower bounds for the risk they can assume in their investments. Consequently, they choose their bond portfolio composition taking into account the credit risk perceived by the rating notations. Therefore, it is important, both for governments and for financial markets, to understand what factors rating agencies put more emphasis on, when attributing a rating score.

We perform an empirical analysis of foreign currency sovereign debt ratings, using data from the three main rating agencies: Fitch Ratings, Moody’s, and Standard & Poor’s (S&P). We have compiled a panel data set on sovereign debt ratings, macroeconomic data and qualitative variables for a wide range of countries starting in 1995. The use of panel data is appealing because it allows examining not only how the agencies attribute a rating, but also how they decide on upgrades and downgrades.

Our main contribution to the existing literature is methodological. The fact that a country’s rating does not vary much across time raises some econometric problems. On

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1This chapter was written in co-authorship with António Afonso and Philipp Rother from the European Central Bank.

2For instance, the European Central Bank when conducting open market operations can only take as collateral bonds that have at least a single A rating, attributed by at least one rating agency.
the one hand, fixed effects estimation only informs us on how the agency decides on up­
grades and downgrades, because the country dummy captures the average rating. On the 
other hand, random effects estimation is inadequate because of the correlation between 
the country specific error and the regressors. We salvage the random effects approach by 
modelling the country specific error, which in practical terms implies adding time-averages 
of the explanatory variables as additional time-invariant regressors. This setting allows 
us to make a distinction between short and long-run determinants of sovereign ratings.

Regarding the empirical modelling strategy, we follow the two main strands in the 
literature. We use linear regression methods on a linear transformation of the ratings 
and we also estimate our specification using both ordered probit and random effects 
ordered probit methods. The latter is the best procedure for panel data as it considers 
the existence of an additional normally distributed cross-section error. This approach 
allows us to determine the cut-off points throughout the rating scale, as well as to test 
whether a linear quantitative transformation of the ratings is a good approximation.

The results show that four core variables have a consistent short-run impact on 
sovereign ratings: the level of GDP per capita, real GDP growth, the public debt level 
and the government balance. Government effectiveness, as well as the level of external 
debt and external reserves are important long-run determinants. A dummy reflecting 
past sovereign defaults is also found significant. Fiscal variables seem more important 
determinants than previously found in the literature.

The paper is organised as follows. In Section 7.2 we give an overview of the rating 
systems and review the related literature. Section 7.3 explains our methodology. In 
Section 7.4 reports the estimation and prediction results, as well as some country specific 
analysis. Section 7.5 summarises the paper's main findings.

7.2 Rating systems and literature

Sovereign ratings are assessments of the relative likelihood of default. The rating agencies 
look at a wide range of elements, from solvency factors affecting the capacity to repay the 
debt, to socio-political factors that might influence the willingness to pay of the borrower, 
and assess the risk of default using a code. Although these agencies do not use the same 
qualitative codes, in general, there is a correspondence between each agency rating level 
as shown in Table 7.1.

An earlier study on the determinants of sovereign ratings by Cantor and Packer (1996) 
concluded that the ratings can be largely explained by a small set of variables: per capita 
income, GDP growth, inflation, external debt, level of economic development, and default
<table>
<thead>
<tr>
<th>Characterization of debt and issuer (source: Moody's)</th>
<th>Rating (S&amp;P)</th>
<th>Moody's</th>
<th>Fitch</th>
<th>Linear transformation</th>
<th>Scale 21</th>
<th>Scale 17</th>
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<tbody>
<tr>
<td>Highest quality</td>
<td>AAA</td>
<td>Aaa</td>
<td>AAA</td>
<td>21</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td></td>
<td>AA+</td>
<td>Aa1</td>
<td>AA+</td>
<td>20</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>High quality</td>
<td>AA</td>
<td>Aa2</td>
<td>AA</td>
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<td>15</td>
<td></td>
</tr>
<tr>
<td></td>
<td>AA-</td>
<td>Aa3</td>
<td>AA-</td>
<td>18</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>Strong payment capacity</td>
<td>A+</td>
<td>A1</td>
<td>A+</td>
<td>17</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>A2</td>
<td>A</td>
<td>16</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A-</td>
<td>A3</td>
<td>A-</td>
<td>15</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>Strong payment capacity</td>
<td>BBB+</td>
<td>Baa1</td>
<td>BBB+</td>
<td>14</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BBB</td>
<td>Baa2</td>
<td>BBB</td>
<td>13</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BBB-</td>
<td>Baa3</td>
<td>BBB-</td>
<td>12</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Likely to fulfil obligations, ongoing uncertainty</td>
<td>BB+</td>
<td>Ba1</td>
<td>BB+</td>
<td>11</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BB</td>
<td>Ba2</td>
<td>BB</td>
<td>10</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BB-</td>
<td>Ba3</td>
<td>BB-</td>
<td>9</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>High credit risk</td>
<td>B+</td>
<td>B1</td>
<td>B+</td>
<td>8</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>B2</td>
<td>B</td>
<td>7</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B-</td>
<td>B3</td>
<td>B-</td>
<td>6</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Very high credit risk</td>
<td>CCC+</td>
<td>Caa1</td>
<td>CCC+</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>CCC</td>
<td>Caa2</td>
<td>CCC</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>CCC-</td>
<td>Caa3</td>
<td>CCC-</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Near default with possibility of recovery</td>
<td>CC</td>
<td>Ca</td>
<td>CC</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>C</td>
<td>DDD</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Default</td>
<td>D</td>
<td>DD</td>
<td>D</td>
<td>1</td>
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</tr>
</tbody>
</table>
eral, a good predictive power, it faces some critiques. As ratings are a qualitative ordinal measure, traditional estimation techniques on a linear representation of the ratings are not adequate. First, they implicitly assume that the difference between any two adjacent categories is always equal. Besides, even if this is true, in the presence of elements in the top and bottom category, the coefficient estimates are still biased, even in large samples.

To overcome this critique, another strand of the literature uses ordered response models, for instance, Hu, Kiesel, and Perraudin (2002), Bissoondoyal-Bheenick (2005) and Depken, LaFountain, and Butters (2006). Although ordered probit should be considered the preferred estimation procedure, it is not entirely satisfying. The ordered probit asymptotic properties do not generalise for small samples, so it is problematic to estimate it using only a cross-section of countries. It is, therefore, imperative to maximize the number of observation by having a panel data, but when doing so, one has to be careful. Indeed, the generalization of ordered probit to panel data is not simple, because of the country specific effect. Furthermore, within this framework, the need to have many observations makes it harder to perform robustness analysis by, for instance, partitioning the sample.

7.3 Methodology

7.3.1 Linear regression framework

Our starting point is the straightforward generalization of the a cross-section specification to panel data,

\[ R_{it} = \beta X_{it} + \lambda Z_{it} + \alpha_i + \mu_{it}, \]

where we have: \( R_{it} \) - quantitative variable, obtained by a linear transformation; \( X_{it} \) and \( Z_i \) are vectors of explanatory variables. The index \( i \) (\( i = 1, ..., N \)) denotes the country, the index \( t \) (\( t = 1, ..., T \)) indicates the period and \( \alpha_i \) stands for the individual effects for each country \( i \). Additionally, we assume that the disturbances \( \mu_{it} \) are independent across countries and across time.

There are three ways to estimate this equation: pooled OLS, fixed effects or random effects estimation. Under standard conditions all estimators are consistent and the ranking of the three methods in terms of efficiency is clear: a random effects approach is preferable to the fixed effects, which is preferable to pooled OLS. What we mean by standard conditions is whether or not the country specific error is uncorrelated with the regressors \( E(\alpha_i|X_{it}, Z_i) = 0 \). If this is the case one should opt for the random effects estimation, while if this condition does not hold, both the pooled OLS and the random effects estimation give inconsistent estimates and fixed effects estimation is preferable.
In our case, it seems natural that the country specific effect is correlated with the regressors, so one may be tempted to say that the "fixed effects estimation" is the best strategy.\footnote{In several studies, like Depken, LaFountain, and Butters (2006) and Mora (2006), the random effects estimator is rejected by the Hausman test. We confirm this result by doing some exploratory regressions. We estimated equation (7.1) using random effects and performed the Hausman test: the Chi-Square statistic is very high and the null hypothesis of no correlation is rejected with p-value of 0.000.} This conclusion is flawed. As there is not much variation of a country's rating over time, the dummies included in the regression capture the country's average rating, while the other variables only capture movements in the ratings across time. Although statistically correct, a fixed effects regression is partially stripped of meaning.

There are two ways of rescuing a random effects approach when there is correlation between the country specific error and the regressors. One is to do the Hausman-Taylor IV estimation, but we would need instruments that are uncorrelated with $\alpha_i$, which are not easy to find. We opt for a different approach that consists on modelling the error term $\alpha_i$. This approach, introduced by Mundlack (1978) and described in Wooldridge (2002), is usually applied when estimating non-linear models, as IV estimation proves to be a herculean task. As we shall see, the application to our case is quite successful. The idea is to give an explicit expression for the correlation between the error and the regressors, stating that the expected value of the country specific error is a linear combination of time-averages of the regressors $\bar{X}_i$.

$$E(\alpha_i|X_{it}, Z_i) = \eta \bar{X}_i. \quad (7.2)$$

If we modify our initial equation (7.1), with $\alpha_i = \eta \bar{X}_i$ we get

$$R_{it} = \beta X_{it} + \lambda Z_i + \eta \bar{X}_i + \epsilon_i + \mu_{it}, \quad (7.3)$$

where $\epsilon_i$ is an error term by definition uncorrelated with the regressors. In practical terms, we eliminate the problem by including a time-average of the explanatory variables as additional time-invariant regressors. We can rewrite (7.3) as:

$$R_{it} = \beta (X_{it} - \bar{X}_i) + (\eta + \beta) \bar{X}_i + \lambda Z_i + \epsilon_i + \mu_{it}. \quad (7.4)$$

This expression is quite intuitive. $\delta = \eta + \beta$ can be interpreted as a long-term effect (e.g. if a country has a permanent high inflation what is the effect on the rating), while $\beta$ is a short-term effect (e.g. if a country manages to reduce inflation this year by one point what is the impact on the rating). This intuitive distinction is useful for policy purposes as it can tell what a country can do to improve its rating in the short to medium-term. Alternatively, we can interpret $\delta$ as the coefficient of the cross-country determinants of the credit rating. We estimate equation (7.4) by random effects. The way we modelled
the error term can be considered successful if the coefficients $\eta$ are significant and if the Hausman test indicates no correlation between the regressors and the new error term.4

7.3.2 Ordered response framework

We also estimate the determinants of sovereign debt ratings under a limited dependent variable framework. The ordered probit is a natural approach for this type of problem, because the rating is a discrete variable and reflects an order in terms of probability of default. Each rating agency makes a continuous evaluation of a country's creditworthiness, embodied in an unobserved latent variable $R_{it}^*$. The latent variable has a linear form and depends on the same set of variables as before,

$$R_{it}^* = \beta(X_{it} - \bar{X}_i) + (\eta + \beta)\bar{X}_i + \lambda Z_i + \epsilon_i + \mu_{it}. \tag{7.5}$$

Because there is a limited number of rating categories, there are several cut-off points to draw up their boundaries. The final rating is given by

$$R_{it} = \begin{cases} 
AAA(AAa) & \text{if } R_{it}^* > c_{16} \\
AA + (Aa1) & \text{if } c_{16} > R_{it}^* > c_{15} \\
AA(AA2) & \text{if } c_{15} > R_{it}^* > c_{14} \\
\ldots & \text{if } \ldots \\
< CCC + (Caa1) & \text{if } c_1 > R_{it}^*
\end{cases} \tag{7.6}$$

The parameters of equation (7.5) and (7.6), notably $\beta$, $\delta$, $\lambda$ and the cut-off points $c_1$ to $c_{16}$ are estimated using maximum likelihood. As we are working with panel data, the generalization of ordered probit is not straightforward, because instead of having one error term, we now have two. Wooldridge (2002) describes two approaches to estimate the parameters. One "quick and dirty" possibility is to assume we only have one error term that is serially correlated within countries. We can then do the standard ordered probit estimation and use a robust variance-covariance matrix to account for the serial correlation. The second possibility is a random effects ordered probit estimation, which considers both errors $\epsilon_{it}$ and $\mu_{it}$ to be normally distributed, and maximizes the log-likelihood accordingly. The second approach should be considered the best one, but it has as a drawback the quite cumbersome calculations involved.5

4An alternative way would be to estimate $\beta$ using fixed effects and regress the country dummies on the time averages of the regressors to estimate $\eta$. We do not follow such method because it cannot be generalized to ordered response models.

5In STATA, this procedure was created by Rabe-Hesketh, Pickles, and Taylor (2000) and substantially improved by Frechette (2001a) and Frechette (2001b). We use such procedures in our calculations.
CHAPTER 7. DETERMINANTS OF SOVEREIGN CREDIT RATINGS

7.3.3 Explanatory variables

Building on the evidence from the existing literature, we identify a set of variables that may determine sovereign ratings, aggregated in four main areas.

**Macroeconomic variables**

*GDP per capita (+)*: richer economies are expected to have more stable institutions to prevent government over-borrowing and to be less vulnerable to exogenous shocks.

*Real GDP growth (+)*: higher real growth strengthens the government’s ability to repay outstanding obligations.

*Unemployment (-)*: a country with lower unemployment tends to have more flexible labour markets. In addition, lower unemployment reduces the fiscal burden of unemployment and social benefits while broadening the base for labour taxation.

*Inflation (+/-)*: on the one hand, it reduces the real stock of outstanding government debt in domestic currency, leaving more resources to cover foreign debt obligations. On the other hand, it is symptomatic of problems at the macroeconomic level.

**Government variables**

*Government debt (-)*: a higher stock of outstanding government debt implies a higher interest burden and should correspond to a higher risk of default.

*Fiscal balance (+)*: large fiscal deficits absorb domestic savings and also suggest macroeconomic disequilibria. Persistent deficits may signal problems with the institutional environment for policy makers.

*Government effectiveness (+)*: high quality of public service delivery, competence of bureaucracy and lower corruption should improve the ability to service debt obligations.

**External variables**

*External debt (-)*: the higher the external indebtedness, the higher the risk for additional fiscal burden, either directly due to a sell-off of foreign government debt or indirectly because of the need to support over-indebted domestic borrowers.

*Foreign reserves (+)*: higher (official) foreign reserves should shield the government from having to default on its foreign currency obligations.

*Current account balance (+/-)*: a higher current account deficit could signal an economy’s tendency to over-consume, undermining long-term sustainability. Alternatively, it could reflect rapid accumulation of investment, which should lead to higher growth and improved sustainability over the medium term.

**Other variables**

*Default history (-)*: past sovereign defaults may indicate a great acceptance of reducing the outstanding debt burden via a default.
European Union (+): countries that join the European Union (EU) improve their credibility as their economic policy is restricted and monitored by other member states.

Regional dummies (+/-): some groups of countries of the same geographical location may have common characteristics that affect their rating.

7.4 Empirical analysis

7.4.1 Data

We build a ratings database with sovereign foreign currency rating, attributed by the three main rating agencies, S&P, Moody’s and Fitch Ratings. We cover a period 1970-2005. The rating of a particular year is the rating attributed at 31st of December. We group the ratings in 17 categories by putting together the few observations below B-, which are given the value one, while AAA observations receive the value 17. In 2005, there are 130 countries with a rating, though only 78 have a rating attributed by all three agencies.

Given data availability for the explanatory variables, our estimations only cover the period 1995-2005. Fiscal balance, current account and government debt are in percentage of GDP, foreign reserves enter as percentage of imports and external debt as percentage of exports. The variables inflation, unemployment, GDP growth, fiscal balance and current account enter as a 3-year average, reflecting the agencies’ approach to take out the effect of the business cycle when deciding on a sovereign rating. The external debt variable is taken from the World Bank and is only available for non-industrialised countries, so for industrialised countries we attribute the value zero, which is equivalent to having a multiplicative dummy. We include a dummy variable indicating a past default and a variable measuring the number of years since it last occurrence. This variable captures the recovery of credibility after a default. As for the dummy variable for EU, we consider that the rating agencies anticipated the EU accession. Thus we test the contemporaneous variable, as well as up to three leads. We find that for Moody’s and S&P the variable enters with two leads, while for Fitch we find no anticipation of EU accession. Regarding the regional dummies, only the dummies for Industrialised countries and for Latin America and Caribbean countries were significant. Overall, we have an unbalanced panel with 66 countries for Moody’s, 65 for S&P and 58 for Fitch, with an average of 8 yearly observations per country. Each country experienced, on average, either one or two changes in its rating.6

6See appendix for a full list of variables used in the estimations as well as their specification and data sources.
7.4.2 Linear panel results

We focus the discussion on the random effects estimations (see Table 7.2). We report the results for each rating agency of a restricted and an unrestricted model. While the unrestricted model incorporates all variables, the restricted model contains only the variables which have a statistically significant impact. The restricted models are quite robust to alternative exclusion procedures. The explanatory power of the models is very high with R-square values around 0.95 in both restricted and unrestricted versions. We can also assess how successful our specification is. First, in most of the cases, the short and long-run coefficients of the explanatory variables are quite different, which implies that, if we did not include the additional regressors, we would be misSpecifying the model.7 Second, the models pass the Hausman test, which suggests that the country specific error is now uncorrelated with the regressors. In other words, if we do not include the time averages the model would suffer from an omitted variables problem, which would make OLS and random effects inconsistent.

The restricted models (columns 2, 4, and 6 in Table 7.2) reveal a homogenous set of explanatory variables across agencies. On the real side, the short-run coefficients of GDP per capita and GDP growth rates are significant for all three companies, but do not seem to have a long-run effect. An increase of 2 percentage points of GDP growth improves the rating by around 0.17 notches for Moody’s and S&P, while an increase of 6 percent of GDP per capita improves the rating by 0.1 notches.

Regarding the fiscal variables, the coefficient of the government debt-to-GDP ratio as a difference from the average is significant for all three agencies. S&P and Fitch put more emphasis on this variable: a 10 percentage point decline improves the rating by 0.3 notches (0.15 notches for Moody’s). On the other hand, Moody’s puts more emphasis on the government balance: a 3 percentage point decrease in the deficit raises its rating by 0.2, compared to 0.1 in the other two agencies. Given their interdependence, one should not see these effects in isolation but rather together, which implies a high overall effect of fiscal policies on the ratings. Finally, the government effectiveness indicator is an important determinant of the rating in the long run. An improvement of 1 point in the World Bank indicator translates into an improvement of 2 notches. The cross-country difference between the 10th and 90th percentile of the average government effectiveness indicator between countries is 2.5 points. Thus, it captures elements that account for 5 notches difference between ratings.

7We perform the formal significance test of the coefficients of the time averages of the explanatory variables by estimating equation (7.3). Average per capita GDP and government effectiveness are always significant at 5% for all agencies. In addition, average unemployment is significant for Moody’s, average government debt is significant for S&P and the average reserves-to-imports is significant for S&P and Fitch. If we exclude all the time invariant regressors the remaining coefficients change slightly and some lose their significance. None of the models without the additional variables pass the Hausman test.
<table>
<thead>
<tr>
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<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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<td><strong>Constant</strong></td>
<td>3.431</td>
<td>8.291</td>
<td>4.347</td>
<td>7.421***</td>
<td>4.409</td>
<td>7.179***</td>
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<td>1.789***</td>
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<td>1.697***</td>
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<td>(8.03)</td>
<td>(7.12)</td>
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<tr>
<td><strong>GDP growth</strong></td>
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<td>8.768***</td>
<td>8.125***</td>
<td>8.256***</td>
<td>3.855</td>
<td>4.110*</td>
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<td><strong>(0.46)</strong></td>
<td>(0.20)</td>
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<td>0.055**</td>
<td>0.056***</td>
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<td><strong>(0.52)</strong></td>
<td>(2.53)</td>
<td>(2.73)</td>
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<td>-0.073*</td>
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<td><strong>Inflation</strong></td>
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<td>-0.145**</td>
<td>-0.235***</td>
<td>-0.229***</td>
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<td><strong>(-1.79)</strong></td>
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<td>(-6.17)</td>
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<td><strong>Inflation Avg.</strong></td>
<td>-0.360**</td>
<td>-0.347**</td>
<td>-0.427***</td>
<td>-0.353**</td>
<td>-0.150</td>
<td></td>
</tr>
<tr>
<td><strong>(-1.84)</strong></td>
<td>(-2.00)</td>
<td>(-2.65)</td>
<td>(-2.44)</td>
<td>(-0.66)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Gov Debt</strong></td>
<td>-0.014**</td>
<td>-0.014**</td>
<td>-0.033**</td>
<td>-0.022**</td>
<td>-0.027**</td>
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<tr>
<td><strong>(-2.38)</strong></td>
<td>(-2.53)</td>
<td>(-6.61)</td>
<td>(-7.32)</td>
<td>(-3.82)</td>
<td>(-7.30)</td>
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<tr>
<td><strong>Gov Debt Avg.</strong></td>
<td>-0.001</td>
<td>-0.014</td>
<td>-0.010</td>
<td>-0.012</td>
<td>-0.007</td>
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<tr>
<td><strong>(-1.43)</strong></td>
<td>(-2.24)</td>
<td>(-1.34)</td>
<td>(-1.97)</td>
<td>(-0.69)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Gov Balance</strong></td>
<td>7.740***</td>
<td>0.991***</td>
<td>4.387**</td>
<td>4.411**</td>
<td>4.371</td>
<td></td>
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<tr>
<td><strong>(2.77)</strong></td>
<td>(2.54)</td>
<td>(1.97)</td>
<td>(2.01)</td>
<td>(1.37)</td>
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<tr>
<td><strong>Gov Balance Avg.</strong></td>
<td>7.893</td>
<td>5.144</td>
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<tr>
<td><strong>(0.80)</strong></td>
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<td>(0.59)</td>
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<tr>
<td><strong>Gov Effectiveness</strong></td>
<td>0.242</td>
<td>0.370**</td>
<td>0.362**</td>
<td>0.787**</td>
<td>0.887**</td>
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<tr>
<td><strong>(1.18)</strong></td>
<td>(2.36)</td>
<td>(2.47)</td>
<td>(5.44)</td>
<td>(5.34)</td>
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<tr>
<td><strong>Gov Effectiveness Avg.</strong></td>
<td>1.906***</td>
<td>2.470***</td>
<td>2.370***</td>
<td>2.758***</td>
<td>2.155**</td>
<td>2.741**</td>
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<tr>
<td><strong>(4.06)</strong></td>
<td>(6.80)</td>
<td>(4.91)</td>
<td>(7.75)</td>
<td>(4.23)</td>
<td>(7.47)</td>
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<tr>
<td><strong>External Debt</strong></td>
<td>-0.004*</td>
<td>-0.004*</td>
<td>-0.003</td>
<td>-0.003</td>
<td>-0.005**</td>
<td>-0.005**</td>
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<td>(-1.95)</td>
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<td>(-1.51)</td>
<td>(-2.97)</td>
<td>(-2.76)</td>
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<tr>
<td><strong>External Debt Avg.</strong></td>
<td>-0.004**</td>
<td>-0.004**</td>
<td>-0.006</td>
<td>-0.007**</td>
<td>-0.010**</td>
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<td>(-1.81)</td>
<td>(-2.18)</td>
<td>(-2.53)</td>
<td>(-3.34)</td>
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<tr>
<td><strong>(-3.67)</strong></td>
<td>(-4.84)</td>
<td>(-2.18)</td>
<td>(-2.18)</td>
<td>(-1.16)</td>
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</tr>
<tr>
<td><strong>Current Account Avg.</strong></td>
<td>-3.321</td>
<td>0.123</td>
<td>2.955</td>
<td></td>
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<td></td>
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<tr>
<td><strong>(-0.78)</strong></td>
<td></td>
<td>(0.03)</td>
<td>(0.63)</td>
<td></td>
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<tr>
<td><strong>Reserves</strong></td>
<td>1.423**</td>
<td>1.710***</td>
<td>0.064</td>
<td>-0.100</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(3.63)</strong></td>
<td>(4.61)</td>
<td>(0.19)</td>
<td>(0.23)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Reserves Avg.</strong></td>
<td>1.475</td>
<td>1.254</td>
<td>1.909**</td>
<td>1.988**</td>
<td>3.090***</td>
<td>2.987***</td>
</tr>
<tr>
<td><strong>(1.60)</strong></td>
<td>(1.43)</td>
<td>(2.06)</td>
<td>(2.28)</td>
<td>(3.59)</td>
<td>(3.78)</td>
<td></td>
</tr>
<tr>
<td><strong>Def 1</strong></td>
<td>-1.998***</td>
<td>-2.075***</td>
<td>-1.307***</td>
<td>-1.337***</td>
<td>-1.523***</td>
<td>-1.391***</td>
</tr>
<tr>
<td><strong>(-6.87)</strong></td>
<td>(-8.11)</td>
<td>(-5.23)</td>
<td>(-6.74)</td>
<td>(-4.13)</td>
<td>(-4.60)</td>
<td></td>
</tr>
<tr>
<td><strong>Def 2</strong></td>
<td>-0.015</td>
<td></td>
<td>-0.018</td>
<td>0.075</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(-0.32)</strong></td>
<td></td>
<td>(0.33)</td>
<td></td>
<td>(1.15)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>EU (2)</strong></td>
<td>1.598***</td>
<td>1.650***</td>
<td>0.415**</td>
<td>0.418**</td>
<td>0.507**</td>
<td>0.554**</td>
</tr>
<tr>
<td><strong>(6.63)</strong></td>
<td>(6.69)</td>
<td>(2.41)</td>
<td>(2.48)</td>
<td>(2.03)</td>
<td>(2.40)</td>
<td></td>
</tr>
<tr>
<td><strong>IND</strong></td>
<td>2.289***</td>
<td>3.157***</td>
<td>2.831***</td>
<td>3.438***</td>
<td>2.781***</td>
<td>2.634***</td>
</tr>
<tr>
<td><strong>(2.89)</strong></td>
<td>(4.61)</td>
<td>(3.03)</td>
<td>(4.69)</td>
<td>(2.61)</td>
<td>(3.55)</td>
<td></td>
</tr>
<tr>
<td><strong>LAC</strong></td>
<td>-0.903*</td>
<td></td>
<td>-0.459</td>
<td>-0.718</td>
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<td></td>
</tr>
<tr>
<td><strong>(-1.93)</strong></td>
<td></td>
<td>(0.94)</td>
<td></td>
<td>(1.29)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** The coefficient of the variable with Avg. corresponds to the long-run coefficient ($\beta + \eta$) while the one without Avg. corresponds to the short-run coefficient $\beta$. White diagonal standard errors & covariance (d.f. corrected). The t statistics are in parentheses. *, **, *** - statistically significant at the 10, 5, and 1 per cent. The null is that RE estimation is consistent and therefore preferable to fixed effects. The test statistic is to be compared to a Chi-Square with 13 and 11 degrees of freedom respectively (the number of time-varying regressors). The p-value is in brackets.
CHAPTER 7. DETERMINANTS OF SOVEREIGN CREDIT RATINGS

The external debt-to-exports ratio and the reserves-to-imports ratio are also significant. Increases in external debt drive the rating down in the short and long run. The difference between the 10th and 90th percentile of the cross-country average external debt ratio is around 300, which corresponds to a cross-country difference of 3 notches for Fitch, 2 notches for S&P, and 1.2 notches for Moody’s. External reserves are significantly positive, in the long-run for S&P and Fitch and in the short-run for Moody’s. The difference between the 10th and 90th percentile of the average reserves-to-import ratio is 0.4, so they account for a 1.2 notch cross-country difference for Fitch and a 0.8 notches for S&P. The current account balance has a negative impact in the short run. A current account deficit seems to be an indicator for the willingness of foreigners to cover the current account gap through loans and foreign investment. In this situation, a higher current account deficit is associated with either higher credit-worthiness or good economic prospects of the economy and consequently a higher sovereign rating.

EU and industrialised country dummies are also significant for all agencies. If a country has previously defaulted on its debt, it is permanently penalized by 1 to 2 notches. Beyond the set of core variables, the agencies appear to employ a limited number of additional variables. Fitch relies on the smallest set of additional variables, comprising government effectiveness and foreign currency reserves as deviation from the average. By contrast, Moody’s and S&P look at more factors, with a large degree of homogeneity between these two agencies. In particular, inflation is found to have a significantly negative impact, although to a relatively small extent.

Finally, the impact of the unemployment on the rating illustrates the importance of distinguishing between short and long-run impacts. While the average (structural) level of unemployment has a significant negative impact on the rating by Moody’s, the short-run deviation from the average enters positively and significantly in the S&P model. Unemployment in the short run can be driven by re-adjustments of economic activity that might improve economic performance in the future. Also, structural reforms that raise unemployment in the short run but improve fiscal sustainability or economic prospects in the long run could explain this finding.

Differentiation across sub-periods and ratings levels

The separation of the overall sample into different sub-samples allows to assess broadly the robustness of the empirical models. The results for the sub-periods 1996-2000 and 2001-2005 are in line with those for the full estimation period, although the significance

---

8We performed additional analysis from different perspectives. For instance, we used the information on credit rating outlooks but no relevant improvement on the fit of the models occurred. In addition, we assessed whether different exchange rate regimes added information to the rating determination, estimated the model with the average rating of the three agencies, and also pooled the data for the three agencies, but the results were quite similar.
PART III. OTHER ESSAYS

level of the individual coefficients is lower. The most noteworthy element is that the current account balance was more important for the early period, while external reserves were more important in the later.

As a further test of the robustness of the results, the sample was split into two groups according to the ratings level: regressions were run separately for high-rated countries with grades BBB+ and below and those above this grade. The results for the separate regressions according to ratings levels again confirm the overall results from the full sample. Still, low rating levels are more affected by external debt and external reserves while inflation plays a bigger role for high rating levels. The short-run coefficient of inflation is bigger for the high ratings group, where an increase of 5 percentage points in inflation reduce the rating by 0.2 notches.

7.4.3 Ordered probit results

Ordered probit models should give additional insight into the determinants of sovereign ratings. As discussed, they generate estimates of the threshold values between rating notches allowing an assessment of the shape of the ratings curve. Given the data requirements, we only apply them to the full sample, which appears appropriate in view of the overall robustness of the empirical results to the use of sub-samples.

The results from the ordered probit estimations validate the findings highlighted above (see Table 7.3 for the random effects ordered probit). The core variables identified in the linear regressions also show up with the same sign. In addition, the ordered probit models suggest the significance of more explanatory variables, particularly for Fitch. Finally, for the current account variable, the restricted specification for Moody's shows a negative sign for deviations from the average, but a positive sign for the average. Similar sign switches also come out for S&P. This result confirms our priors. In the short run, a higher current account deficit is associated with either higher credit-worthiness or good economic prospects of the economy, but if the countries run permanent current account deficits, it negatively affects their ratings.

The estimated threshold coefficients reported in the second part of Table 7.3 suggest that the linear specification, assumed for the panel regression, is broadly acceptable. Nevertheless, the econometric tests at the bottom of the tables reveal additional insights. For the restricted model of Moody's, the test does not reject the null hypothesis of equal distances between thresholds, but the significance level is close to 10%. Indeed the estimated thresholds point to a relatively large jump between the ratings for BBB-

\footnote{The choice of the threshold reflects practical considerations. While market participants generally divide bond issuers into investment-grade and non-investment grade at the threshold of BBB-, this threshold would result in a relatively small number of observations for low ratings.}
and BBB. Countries close to the non-investment grade rating are given a wider range before they actually cross that threshold. For Fitch, the hypothesis of equal distances is rejected, as the thresholds for higher ratings are further apart than those of the lower ratings. In this case the kink lies at the A rating. For S&P, above investment grade, the distances between thresholds first decline and then increase, making the transition to the highest grades more difficult.

### Table 7.3: Random effects ordered probit estimation

<table>
<thead>
<tr>
<th></th>
<th>Moody's</th>
<th>S&amp;P</th>
<th>Fitch</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP per capita</td>
<td>3.422*** (9.40)</td>
<td>3.240*** (9.14)</td>
<td>2.686*** (9.02)</td>
</tr>
<tr>
<td>GDP per capita Avg.</td>
<td>0.475*** (2.75)</td>
<td>1.117*** (3.84)</td>
<td>0.614*** (6.03)</td>
</tr>
<tr>
<td>GDP growth</td>
<td>6.464** (2.06)</td>
<td>5.979* (2.30)</td>
<td>7.729*** (1.93)</td>
</tr>
<tr>
<td>GDP growth Avg.</td>
<td>-9.387** (2.04)</td>
<td>-8.43* (1.93)</td>
<td>-6.083 (2.60)</td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.016 (0.50)</td>
<td>0.152*** (4.57)</td>
<td>0.135*** (3.01)</td>
</tr>
<tr>
<td>Unemployment Avg.</td>
<td>-0.078*** (-4.40)</td>
<td>0.002 (0.10)</td>
<td>-0.073*** (-4.40)</td>
</tr>
<tr>
<td>Inflation</td>
<td>-0.623*** (-4.01)</td>
<td>-0.532*** (-6.11)</td>
<td>-0.949*** (-3.41)</td>
</tr>
<tr>
<td>Inflation Avg.</td>
<td>-0.03*** (-4.61)</td>
<td>-0.085*** (-6.11)</td>
<td>-0.088*** (-3.41)</td>
</tr>
<tr>
<td>Gov Debt</td>
<td>-0.03*** (-8.99)</td>
<td>-0.027*** (-8.80)</td>
<td>-0.031*** (2.77)</td>
</tr>
<tr>
<td>Gov Debt Avg.</td>
<td>13.898*** (3.74)</td>
<td>10.937*** (2.77)</td>
<td>10.187*** (2.07)</td>
</tr>
<tr>
<td>Gov Balance</td>
<td>6.757* (1.84)</td>
<td>8.873*** (2.40)</td>
<td>22.364*** (3.07)</td>
</tr>
<tr>
<td>Gov Balance Avg.</td>
<td>0.223 (0.64)</td>
<td>0.707** (2.08)</td>
<td>1.761*** (2.42)</td>
</tr>
<tr>
<td>Gov Effectiveness</td>
<td>3.679*** (13.46)</td>
<td>3.547*** (15.44)</td>
<td>4.606*** (16.30)</td>
</tr>
<tr>
<td>Gov Effectiveness Avg.</td>
<td>2.246*** (15.44)</td>
<td>4.606*** (16.30)</td>
<td>2.722*** (11.37)</td>
</tr>
<tr>
<td>External Debt</td>
<td>-0.004** (-2.29)</td>
<td>-0.002*** (-2.21)</td>
<td>-0.002 (6.30)</td>
</tr>
<tr>
<td>External Debt Avg.</td>
<td>-3.11*** (-3.11)</td>
<td>-12.863*** (-6.40)</td>
<td>-12.863*** (-10.39)</td>
</tr>
<tr>
<td>Current Account</td>
<td>-8.57*** (-3.62)</td>
<td>-4.899** (-5.94)</td>
<td>2.772 (1.24)</td>
</tr>
<tr>
<td>Current Account Avg.</td>
<td>5.24*** (2.21)</td>
<td>18.69*** (1.73)</td>
<td>5.769*** (7.21)</td>
</tr>
<tr>
<td>Reserves</td>
<td>2.246*** (4.37)</td>
<td>0.705 (5.82)</td>
<td>-0.549 (0.42)</td>
</tr>
<tr>
<td>Reserves Avg.</td>
<td>3.416*** (0.88)</td>
<td>3.365*** (6.94)</td>
<td>2.520*** (5.57)</td>
</tr>
<tr>
<td>Def 1</td>
<td>-3.101*** (-12.18)</td>
<td>-1.789*** (-8.98)</td>
<td>-2.077*** (-9.25)</td>
</tr>
<tr>
<td>EU</td>
<td>2.197*** (9.04)</td>
<td>0.324 (8.90)</td>
<td>0.336 (1.55)</td>
</tr>
<tr>
<td>IND</td>
<td>3.554*** (7.71)</td>
<td>3.923*** (9.08)</td>
<td>5.848*** (8.18)</td>
</tr>
<tr>
<td>LAC</td>
<td>-1.767*** (-7.08)</td>
<td>-1.711*** (-8.86)</td>
<td>-0.901*** (-6.38)</td>
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</tbody>
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Table 7.3 (cont.): Random effects ordered probit estimation

<table>
<thead>
<tr>
<th></th>
<th>Moody's</th>
<th>S&amp;P</th>
<th>Fitch</th>
</tr>
</thead>
<tbody>
<tr>
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<td>(1)</td>
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<td>(3)</td>
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<td>4.12</td>
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<td>4.94</td>
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<td>Cut5</td>
<td>5.94</td>
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<td>22.72</td>
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<td>Cut15</td>
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<td>20.26</td>
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<td>Cut16</td>
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<tr>
<td>LogLik</td>
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<tr>
<td>Equal differences</td>
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<td>52.21 (0.000)</td>
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<td>Jump &amp;</td>
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<td>[9-10]</td>
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<td>Different Slopes</td>
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<td>[13-14, 15-16]</td>
<td>15-16</td>
</tr>
<tr>
<td>Test*</td>
<td>18.22 (0.149)</td>
<td>12.22 (0.510)</td>
<td>19.23 (0.116)</td>
</tr>
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</table>

Notes: The coefficient of the variable with Avg. corresponds to the long-run coefficient \( \beta + \eta \), while the one without corresponds to the short-run coefficient \( \beta \). The t statistics are in parentheses. *, **, *** - statistically significant at the 10, 5, and 1 per cent. $ The null is that the differences between categories is equal for all categories. The test statistic is to be compared to a Chi-Square with 14 degrees of freedom. & Identifies two cut points that have a irregular difference. # Identifies a cluster of categories that seem to have a higher slope (increase difficulty in transition between adjacent notches). $ The null is that, excluding the jump point, within the two identified clusters the slopes are equal. The test statistic is to be compared to a Chi-Square with either 13 degrees of freedom (if only a jump or different slopes was identified) or 12 degrees of freedom (if both where identified). The p-value is in brackets. The correspondence between the ratings and the cut-off points is specified in (7.6).

### 7.4.4 Prediction analysis

Our prediction analysis focuses on two elements: the prediction for the rating of each individual observation in the sample, as well as the prediction of movements in the ratings through time. For the random effects estimations we can have two predictions, with or without the country specific effect, \( \epsilon_i \), and we can write the corresponding estimated versions of (7.4) as:

\[
\hat{R}_{it} = \hat{\beta}(X_{it} - \bar{X}_i) + \hat{\delta}X_i + \hat{\lambda}Z_{it} + \hat{\epsilon}_i,
\]

\[
\hat{R}_{it} = \hat{\beta}(X_{it} - \bar{X}_i) + \hat{\delta}X_i + \hat{\lambda}Z_{it}.
\]

We can then estimate each country specific effect by taking the time average of the estimated residual for each country. As a result we can include or exclude this additional information that comes out of the estimation. We also present the predicting results using OLS estimation. For the linear models we compute the fitted value and then rounded it to the closest integer between 1 and 17. For both ordered probit and the random effects
## Chapter 7. Determinants of Sovereign Credit Ratings

### Table 7.4: Summary of prediction errors

<table>
<thead>
<tr>
<th>Estimation procedure</th>
<th>Obs.</th>
<th>Prediction error (notches)</th>
<th>% Correctly predicted</th>
<th>% Within 1 notch*</th>
<th>% Within 2 notch*</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Moody’s</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>557</td>
<td>0 5 12 42 88 209 141 58 2 0 0 0</td>
<td>37.5%</td>
<td>78.6%</td>
<td>96.6%</td>
</tr>
<tr>
<td>RE with εi</td>
<td>557</td>
<td>0 0 1 17 78 361 91 8 1 0 0 0</td>
<td>64.8%</td>
<td>95.2%</td>
<td>99.6%</td>
</tr>
<tr>
<td>RE without εi</td>
<td>557</td>
<td>0 6 15 49 92 188 141 53 12 1 0 0</td>
<td>33.8%</td>
<td>75.6%</td>
<td>93.9%</td>
</tr>
<tr>
<td>Ordered Probit</td>
<td>557</td>
<td>0 4 14 35 99 259 86 46 10 0 0 0</td>
<td>46.5%</td>
<td>79.7%</td>
<td>94.3%</td>
</tr>
<tr>
<td>RE Ordered Probit</td>
<td>557</td>
<td>0 8 23 59 100 244 71 34 11 1 0 0</td>
<td>43.8%</td>
<td>75.6%</td>
<td>92.3%</td>
</tr>
<tr>
<td><strong>S&amp;P</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RE with εi</td>
<td>565</td>
<td>0 3 15 34 104 318 147 41 6 0 0 0</td>
<td>38.4%</td>
<td>82.6%</td>
<td>95.8%</td>
</tr>
<tr>
<td>RE without εi</td>
<td>565</td>
<td>0 5 12 39 98 210 133 52 10 0 0 0</td>
<td>38.2%</td>
<td>79.1%</td>
<td>95.2%</td>
</tr>
<tr>
<td>Ordered Probit</td>
<td>565</td>
<td>0 10 14 28 99 262 118 23 10 1 0 0</td>
<td>46.4%</td>
<td>84.8%</td>
<td>93.8%</td>
</tr>
<tr>
<td>RE Ordered Probit</td>
<td>565</td>
<td>1 12 13 41 115 218 130 29 6 0 0 0</td>
<td>38.6%</td>
<td>81.9%</td>
<td>94.3%</td>
</tr>
<tr>
<td><strong>Fitch</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>481</td>
<td>1 3 6 32 87 106 113 43 0 0 0 0</td>
<td>40.7%</td>
<td>82.3%</td>
<td>97.9%</td>
</tr>
<tr>
<td>RE with εi</td>
<td>481</td>
<td>0 1 2 4 63 339 71 1 0 0 0 0</td>
<td>70.5%</td>
<td>98.3%</td>
<td>99.4%</td>
</tr>
<tr>
<td>RE without εi</td>
<td>481</td>
<td>1 3 7 39 93 174 108 57 1 0 0 0</td>
<td>36.2%</td>
<td>77.5%</td>
<td>97.5%</td>
</tr>
<tr>
<td>Ordered Probit</td>
<td>481</td>
<td>1 0 16 32 91 209 95 31 6 0 0 0</td>
<td>43.5%</td>
<td>82.1%</td>
<td>95.2%</td>
</tr>
<tr>
<td>RE Ordered Probit</td>
<td>553</td>
<td>1 3 25 53 115 191 121 36 8 0 0 0</td>
<td>34.5%</td>
<td>77.2%</td>
<td>93.3%</td>
</tr>
</tbody>
</table>

*Note: * prediction error within +/- 1 notch. **prediction error within +/- 2 notches.*

Ordered probit we fit the value of the latent variable, by setting the error term to zero, and match it up to the cut-off points to determine the predicted rating. Table 7.4 presents an overall summary of the prediction errors, using the restricted specifications.

The random effects model including the estimated country effect is the method with the best fit. On average for the three agencies, it correctly predicts 70 per cent of all observations and more than 95 per cent of the predicted ratings lie within one notch (99 per cent within two notches). This is expected, as the estimated country errors capture factors like political risk, geopolitical uncertainty and social tensions that are likely to systematically affect the ratings, therefore, they act as a correction for these factors.

This additional information cropping up from the random effects estimation with the country specific effect can be useful if we want to work with countries that belong to our sample. But if we want to make an out-of-sample prediction we do not have this information. In that case, only the random effects estimation excluding the country error is comparable to the other specifications. We can see that, in general, both ordered probit and random effects ordered probit have a better fit than the pooled OLS and random effects. Overall, the simple ordered probit seems the best method as far as prediction in levels is concerned as it predicts correctly around 45 per cent of all observations and more then 80 per cent within one notch.

Let us now turn to how the models perform in predicting changes in ratings. Table 7.5 presents the total number of sample upgrades (downgrades), the predicted number of upgrades (downgrades) and the number of upgrades (downgrades) that where correctly predicted by the several models. Over the sample period, on average, there was a change of rating every six years for Moody’s and every five years for S&P and Fitch. A country was twice more likely to be upgraded than downgraded.10

---

10This analysis is, in a way, limited as it does not capture upgrades/downgrades across multiple grades
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Table 7.5: Prediction of rating changes

<table>
<thead>
<tr>
<th>Model</th>
<th>Sample Upgrades</th>
<th>Predicted Upgrades</th>
<th>Upgrades correctly predicted at time ( t )</th>
<th>Sample Downgrades</th>
<th>Predicted Downgrades</th>
<th>Downgrades correctly predicted at time ( t + 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS</td>
<td>60</td>
<td>95</td>
<td>23</td>
<td>20</td>
<td>34</td>
<td>55</td>
</tr>
<tr>
<td>RE with ( e_t )</td>
<td>60</td>
<td>87</td>
<td>28</td>
<td>17</td>
<td>34</td>
<td>51</td>
</tr>
<tr>
<td>RE without ( e_t )</td>
<td>60</td>
<td>89</td>
<td>23</td>
<td>16</td>
<td>34</td>
<td>51</td>
</tr>
<tr>
<td>Ordered Probit</td>
<td>60</td>
<td>127</td>
<td>31</td>
<td>25</td>
<td>34</td>
<td>72</td>
</tr>
<tr>
<td>RE Ordered Probit</td>
<td>60</td>
<td>101</td>
<td>23</td>
<td>23</td>
<td>34</td>
<td>65</td>
</tr>
<tr>
<td>Moody's</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>79</td>
<td>79</td>
<td>32</td>
<td>17</td>
<td>41</td>
<td>50</td>
</tr>
<tr>
<td>RE with ( e_t )</td>
<td>79</td>
<td>79</td>
<td>31</td>
<td>14</td>
<td>41</td>
<td>52</td>
</tr>
<tr>
<td>RE without ( e_t )</td>
<td>79</td>
<td>90</td>
<td>34</td>
<td>15</td>
<td>41</td>
<td>61</td>
</tr>
<tr>
<td>Ordered Probit</td>
<td>79</td>
<td>102</td>
<td>38</td>
<td>14</td>
<td>41</td>
<td>64</td>
</tr>
<tr>
<td>RE Ordered Probit</td>
<td>79</td>
<td>90</td>
<td>31</td>
<td>15</td>
<td>41</td>
<td>68</td>
</tr>
<tr>
<td>S&amp;P</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>68</td>
<td>74</td>
<td>28</td>
<td>19</td>
<td>25</td>
<td>35</td>
</tr>
<tr>
<td>RE with ( e_t )</td>
<td>68</td>
<td>67</td>
<td>25</td>
<td>19</td>
<td>25</td>
<td>34</td>
</tr>
<tr>
<td>RE without ( e_t )</td>
<td>68</td>
<td>89</td>
<td>24</td>
<td>20</td>
<td>25</td>
<td>53</td>
</tr>
<tr>
<td>Ordered Probit</td>
<td>69</td>
<td>115</td>
<td>30</td>
<td>24</td>
<td>25</td>
<td>71</td>
</tr>
<tr>
<td>RE Ordered Probit</td>
<td>69</td>
<td>154</td>
<td>43</td>
<td>29</td>
<td>26</td>
<td>77</td>
</tr>
<tr>
<td>Fitch</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>68</td>
<td>74</td>
<td>28</td>
<td>19</td>
<td>25</td>
<td>35</td>
</tr>
<tr>
<td>RE with ( e_t )</td>
<td>68</td>
<td>67</td>
<td>25</td>
<td>19</td>
<td>25</td>
<td>34</td>
</tr>
<tr>
<td>RE without ( e_t )</td>
<td>68</td>
<td>89</td>
<td>24</td>
<td>20</td>
<td>25</td>
<td>53</td>
</tr>
<tr>
<td>Ordered Probit</td>
<td>69</td>
<td>115</td>
<td>30</td>
<td>24</td>
<td>25</td>
<td>71</td>
</tr>
<tr>
<td>RE Ordered Probit</td>
<td>69</td>
<td>154</td>
<td>43</td>
<td>29</td>
<td>26</td>
<td>77</td>
</tr>
</tbody>
</table>

The models correctly predict between one third and one half of both upgrades and downgrades. This is quite satisfactory for two reasons: first, the rating agencies also have a forward looking behaviour that is absent from our models and second, other qualitative factors not captured in our variables may play an important role.

The most noticeable difference between the models is not the number of corrected predicted changes, but the total number of predicted changes. In fact, the ordered probit and random effects ordered probit predict substantially more changes than the OLS and random effects counterparts. For instance, for S&P, while both OLS and random effects predict around 79 upgrades and 50 downgrades, the ordered probit model predicts 102 upgrades and 64 downgrades. This gives strength to the idea that rating agencies smooth the ratings, along the lines discussed in Altman and Rijken (2004). It also suggests that linear methods might be better in capturing the inertia of rating agencies than ordered response models.

7.4.5 Examples of specific country analysis

In Table 7.6 we show the rating for some European and emerging countries in 1998 and 2005. Then, we use the estimated short-run coefficients of the random effects ordered probit, together with the values for the relevant variables to disaggregate the overall prediction change in the rating of each agency into the contributions of the different blocks of explanatory variables: macroeconomic performance, government performance, external elements and the EU. The upper and lower bounds are computed by adding and subtracting one standard deviation to the point estimate of the coefficients, or multiple upgrades/downgrades within a year. Although this could be important to analyse particular cases, such as, for instance, currency crises, the cases of multiple upgrades/downgrades are relatively small compared to the full sample.
### CHAPTER 7. DETERMINANTS OF SOVEREIGN CREDIT RATINGS

#### Table 7.6: Case studies

<table>
<thead>
<tr>
<th>European Union countries</th>
<th>Portugal</th>
<th>Spain</th>
<th>Greece</th>
<th>Italy</th>
<th>Ireland</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moodys Rating</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S&amp;P Rating</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fitch Rating</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>As2 (15)</td>
<td>Aa2 (15)</td>
<td>Aaa (17)</td>
<td>As1 (14)</td>
<td>Aas (17)</td>
<td>Aaa (17)</td>
</tr>
<tr>
<td>0.53</td>
<td>0.73</td>
<td>0.93</td>
<td>1.69</td>
<td>1.98</td>
<td>2.28</td>
</tr>
<tr>
<td>-0.69</td>
<td>-0.48</td>
<td>-0.23</td>
<td>0.27</td>
<td>0.65</td>
<td>1.03</td>
</tr>
<tr>
<td>0.09</td>
<td>0.12</td>
<td>0.15</td>
<td>0.22</td>
<td>0.31</td>
<td>0.39</td>
</tr>
<tr>
<td>Overall change</td>
<td>-0.07</td>
<td>0.39</td>
<td>0.86</td>
<td>2.19</td>
<td>2.95</td>
</tr>
<tr>
<td>S&amp;P Rating</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fitch Rating</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>As (10)</td>
<td>Aa (10)</td>
<td>Aaa (12)</td>
<td>BB (9)</td>
<td>BBB- (10)</td>
<td>BBB+ (12)</td>
</tr>
<tr>
<td>0.72</td>
<td>1.36</td>
<td>2.01</td>
<td>0.12</td>
<td>0.52</td>
<td>0.91</td>
</tr>
<tr>
<td>-0.30</td>
<td>0.60</td>
<td>1.49</td>
<td>0.55</td>
<td>1.26</td>
<td>1.97</td>
</tr>
</tbody>
</table>

| Note: The block contributions were calculated using the changes in the variables multiplied by the short-run coefficients estimated by random effects ordered probit, and then aggregated. The only exception was unemployment, for which we used the long-run coefficient. The upper and lower bounds were calculated using plus and minus one standard deviation. The quantitative rating scale is in brackets. | 203 |
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Let us compare, for instance, Portugal and Spain. In 1998 they both had an AA (Aa2) rating but in 2005 while Spain had been upgraded to AAA (Aaa) by all agencies, Portugal had been downgraded by S&P. For Portugal, the positive contribution of the macroeconomic performance was overshadowed by the negative government developments: the worsening of the budget deficit since 2000, the upward trend in government debt and the decline in the World Bank government effectiveness indicator. As for Spain, the good macroeconomic performance was the main cause of the upgrade, especially the reduction of structural unemployment since the mid nineties and the increase of GDP per capita due to the persistent high growth.

The new European Union member states Slovakia, the Czech Republic, Hungary, Slovenia and Poland, have been upgraded, in some cases more than two notches. The good macroeconomic performance, especially in Slovakia and the Czech Republic, plays a major role, but there was also an important credibility effect of joining the European Union, mostly visible for Moody’s.

As a final example we report the results for five emerging economies that have also been upgraded: Brazil, Mexico, Malaysia, Thailand and South Africa. For Brazil, Malaysia and Thailand the main positive contribution came from the external area specially the reduction of external debt and the increase in foreign reserves. This effect is stronger for Fitch. For Mexico and South Africa the contributions are balanced.

7.5 Conclusion

In this chapter we have studied the determinants of global sovereign debt ratings using ratings from the three main international rating agencies for the period 1995-2005. Overall, our results point to a good performance of the estimated models across agencies and across time.

Regarding the methodological approach, we have used linear regression methods and limited dependent variable models, by means of an ordered probit and random effects ordered probit estimations. The latter is the best estimation procedure using panel data, as it considers the existence of an additional cross-country error term. We have also employed a new specification that consists of including time averages of the explanatory variables as additional time-invariant regressors. On the one hand, it allows us to correct the problem of correlation between the country specific error and the regressors. On the other hand, it allows us to distinguish between short-run and long-run effects of a variable on the sovereign rating level, which improves the economic interpretation of the results.

Our results show that a set of core variables have a short-run impact on a country’s credit rating: per capita GDP, real GDP growth, government debt and government deficit.
Government effectiveness, external debt, foreign reserves and sovereign default dummies are important determinants of the cross-country dimension of the ratings, and therefore, only have a long-run impact. Moreover, the importance of fiscal variables appears stronger than in the previous literature.

The models correctly predict the rating of 40% of the sample and more than 75% of the predicted ratings lie within one notch of the observed value. They also correctly predict between one third and one half of upgrades and downgrades. In our opinion this is quite satisfactory given that the empirical approach used here necessarily neglects two sources of information that are known to enter the decision of the rating agencies. On the one hand, rating agencies generally state that they cover several qualitative variables in addition to quantitative data in the rating process. On the other hand, rating agencies base their decision, to some extent, on projected economic developments. Thus, a more comprehensive model could also incorporate the agencies' expectations regarding the relevant explanatory variables.

Although incorporating forward-looking behaviour of agencies into an econometric model seems important to study particular episodes of sudden and repeated changes in ratings, we think it is not essential for our purposes. First, because most of the countries do not have frequent changes in their ratings, timing is not a fundamental issue. Second, even if the behaviour of agencies were strictly forward-looking, they still base their projections on current information, which should be captured in our modelling. All in all, we believe that such attempt to incorporate expectations would remain tentative.
## 7.6 Appendix

Table A7.1: Some previous related studies

<table>
<thead>
<tr>
<th>Reference</th>
<th>Data</th>
<th>Explanatory variables</th>
<th>Agencies</th>
<th>Methodology</th>
</tr>
</thead>
<tbody>
<tr>
<td>Borio and Packer (2004)</td>
<td>Panel 1996-2003, 52 countries</td>
<td>Per capita GDP, GDP growth, inflation, corruption perception index, political risk index, years since default, frequency of high inflation periods, government debt-to-GDP ratio, debt-to-exports ratio, others.</td>
<td>S&amp;P Moody’s</td>
<td>Linear transformation of the data. OLS regression of average credit rating including year dummies as regressors.</td>
</tr>
</tbody>
</table>
### Table A7.2: Data sources

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Source</th>
<th>Codename</th>
</tr>
</thead>
<tbody>
<tr>
<td>Per Capita GDP</td>
<td>Per capita nominal GDP in US dollars (logs)</td>
<td>IMF (WEO)</td>
<td>NGDPDPC</td>
</tr>
<tr>
<td>GDP Growth</td>
<td>Annual growth rate of real GDP</td>
<td>IMF (WEO)</td>
<td>NGDP_R</td>
</tr>
<tr>
<td>Unemployment Rate</td>
<td>Unemployment Rate</td>
<td>IMF (WEO)</td>
<td>LUR</td>
</tr>
<tr>
<td>Inflation</td>
<td>Annual growth rate of Consumer Price Index</td>
<td>IMF (WEO)</td>
<td>PCPI</td>
</tr>
<tr>
<td>Government balance</td>
<td>General government balance as percentage of GDP</td>
<td>IMF (WEO)</td>
<td>GGB, NGDP</td>
</tr>
<tr>
<td>Government Effectiveness</td>
<td>Aggregate Governance Indicators 1996-2006</td>
<td>WB(AGI)</td>
<td></td>
</tr>
<tr>
<td>External Debt</td>
<td>Total debt as share of exports of goods and services</td>
<td>WB (GDF)</td>
<td></td>
</tr>
<tr>
<td>Current Account</td>
<td>Current account balance as percentage of GDP</td>
<td>IMF (WEO)</td>
<td>BCA, NGPD</td>
</tr>
<tr>
<td>Reserves</td>
<td>Reserves to Imports ratio</td>
<td>IMF (WEO, IFS)</td>
<td>BM, .01.DS$Z.F.</td>
</tr>
<tr>
<td>DEF 1</td>
<td>Dummy: 1 if country has defaulted since 1980</td>
<td>S&amp;P</td>
<td></td>
</tr>
<tr>
<td>DEF 2</td>
<td>Years since last default</td>
<td>S&amp;P</td>
<td></td>
</tr>
<tr>
<td>EU</td>
<td>Dummy: 1 If country belongs to European Union</td>
<td>WB</td>
<td></td>
</tr>
<tr>
<td>IND</td>
<td>Dummy: 1 if Industrial Countries</td>
<td>WB</td>
<td></td>
</tr>
<tr>
<td>LAC</td>
<td>Dummy: 1 if Latin America and Caribbean</td>
<td>WB</td>
<td></td>
</tr>
<tr>
<td>Investment</td>
<td>Gross capital formation as percentage of GDP</td>
<td>IMF (WEO)</td>
<td>NLR, NGDP_R</td>
</tr>
<tr>
<td>Oil balance</td>
<td>Oil trade balance as percentage of GDP</td>
<td>IMF (WEO)</td>
<td>TBO, NGPD</td>
</tr>
<tr>
<td>Government Expenditure</td>
<td>General government total expenditure as percentage of GDP</td>
<td>IMF (WEO)</td>
<td>GGEI, NGDP</td>
</tr>
<tr>
<td>Government Interest</td>
<td>General government interest expenditure as percentage of GDP</td>
<td>IMF (WEO)</td>
<td>GGRG, NGDP</td>
</tr>
<tr>
<td>Expenditure</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Government Revenue</td>
<td>General government total revenue as percentage of GDP</td>
<td>IMF (WEO)</td>
<td>GGENL, NGDP</td>
</tr>
<tr>
<td>Trade openness</td>
<td>Total Exports plus total Imports as percentage of GDP</td>
<td>IMF (WEO)</td>
<td>BM, BX NGPD</td>
</tr>
<tr>
<td>Exports Growth</td>
<td>Annual growth rate of real exports</td>
<td>IMF (WEO)</td>
<td>NX.R</td>
</tr>
<tr>
<td>Domestic Credit Growth</td>
<td>Annual growth rate of Domestic credit</td>
<td>IMF (IFS)</td>
<td>.3.12.$$Z.F.$$</td>
</tr>
<tr>
<td>Interest over Exports</td>
<td>Interest paid as percentage of total exports of goods and services</td>
<td>WB (GDF)</td>
<td></td>
</tr>
<tr>
<td>Reserves over total debt</td>
<td>Reserves as percentage of total debt</td>
<td>WB (GDF)</td>
<td></td>
</tr>
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**Notes:** WEO - World Economic Outlook; AGI - Aggregate Governance Indicators; GDF - Global Development Finance; IFS - International Financial Statistics; WB - World Bank; IMF - International Monetary Fund; JP - Jaimovich and Panizza (2006); TI - Transparent International.
Figure A7.1: Number of countries rated and rating categories

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Chapter 8

Closing Remarks

This thesis applied several research approaches to understand the effects of different types of government spending and their role in achieving optimality. Chapter 2 tells us that the government should be concerned about queues for public sector jobs, which can be an indicator that the wages are higher than optimal. Chapter 4 describes how governments should decide on the allocation of spending between general consumption and investment. Chapters 3 and 5 set variants of the baseline models in order to draw some positive conclusions and guide the empirical study.

Although the analysis contributed to overcome some of the theoretical gaps that exist in the literature, there are further areas where more research is needed. These are summarised in the subsequent paragraphs. Following Part I, several interesting exercises can be done:

- One could include distortionary taxation in the model with public sector employment and analyse the optimal policy. The introduction of distortionary taxes creates an additional cost of raising revenue, so we should expect a lower level of vacancies and wages, relative to the first best. It also creates a problem of time inconsistency, as the government can promise higher wages in the future to attract workers but it has the incentive to lower the current wage. Finally, the optimal business cycle policy of procyclical wages should be amplified, as it requires lower tax burden in recessions.

- One could also include nominal rigidities in the model with public sector employment with the objective of understanding how the government's nominal wage can affect the marginal costs in the private sector and consequently the nominal private sector wage and inflation. In addition, one could explore the interdependencies between monetary and fiscal policy.

- As there are large differences in the public sector wage premium for workers with different education levels, it is appealing to develop a model that distinguishes
between high-skilled and low-skilled workers. High-skilled workers usually have a wage gap and low-skilled workers have a premium of working in the public sector, so there might be two types of inefficiencies: recruitment problems for high-skilled workers and long queues for low-skilled workers.

- Another interesting extension would be to endogenise some of the labour market friction parameters, particularly the separation rate. We should expect public sector wage to affect negatively the separation rate in the public sector and positively in the private sector. This creates yet another transmission channel of public sector wages.

Finally, following the main hypothesis that the macroeconomic effects of government spending depend on the type of expenditure we are considering, two other areas seem important to explore:

- Along the lines of Blanchard and Perotti (2002), to study the effects of the different components of spending using time series data for the United States. We can start by examining the size, volatility and comovement of the different types of expenditure, and then use a VAR approach to identify the effects of the different types of expenditure.

- There were two types of spending left out in the analysis: transfers and interest payments. It would be interesting to analyse the different redistributive effects of transfers and interest payment from a theoretical point of view.
Bibliography


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