JOB CREATION, JOB DESTRUCTION AND
EMPLOYMENT REALLOCATION. THEORY AND EVIDENCE.

Thesis submitted for the degree of
Doctor of Philosophy (PhD)
by
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registered at
the London School of Economics.

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Abstract

This thesis consists of four essays on the determinants, the dynamics and the policy implications of simultaneous job creation and destruction in labour markets. Firstly, it proposes and solves a stochastic search model with endogenous job separation and it shows that the amplitude and time variation of job reallocation depend crucially upon the arrival rate of exogenous firing permissions. Tighter firing restrictions, albeit not directly relevant for differences in average unemployment rates, dramatically reduce the relative volatility of job creation and destruction. A parameterization of the model can rationalise cross-country differences in the cyclical behaviour of job creation and destruction.

Secondly, it brings together aggregate data on job reallocation and labour market policy for nine OECD countries. It shows that long term unemployment and job reallocation are negatively correlated and that job reallocation is lower in countries that offer limited benefit for a limited period of time.

Thirdly, it studies the role of time-consuming search in generating the size distribution of firms and the dynamics of firm-level turnover. It solves a dynamic matching model where the joint distribution of wages and employment results from interacting idiosyncratic shocks, firm-level asymmetries in job creation and destruction and time-consuming search on the part of workers. Theoretical results offer a structural interpretation of existing empirical evidence on firm-size wage differentials and point out novel empirical implications.

Finally, it measures the relation between job flows and establishment size applying econometric techniques best suited for analysing the dynamics of large cross-sections. Using a balanced panel from the Mexican Manufacturing sector it finds no evidence of small establishments converging toward the mean, thus no evidence of convergence.
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Introduction

Throughout the seventies and the eighties, empirical and theoretical scholars working on the labour market concentrated on the determinants and the dynamics of net employment changes, the difference in the aggregate stock of employment in a given period. From the early nineties, the availability of large data-sets on employment changes at the establishment level has shifted the interest from net to gross employment changes. Gross job creation indicates an increase in the demand for labour at some units in the sample (usually establishments but sometimes firms) and gross job destruction a decrease. Changes in the aggregate stock of employment, the more conventional concept of labour demand in aggregate models, are then obtained from the differences in job creation and destruction. Particularly influential have been the data gathered by Davis and Haltiwanger (1990;1992) for the United States, but similar data-sets have soon become available in most OECD countries. It is now clear that aggregate net employment changes are associated with large and simultaneous flows of gross job creation and destruction, at any phase of the cycle. Furthermore, the process of job destruction is as responsive to economic shocks as the process of job creation is.

This thesis aims at contributing to the growing macroeconomic field that studies the determinants, the dynamics and the policy implications of simultaneous job creation and destruction in labour markets. As new data-set become available, new puzzling facts are being discovered and new questions need to be
answered. How do firing restrictions affect the dynamics of job creation and
destruction? Is there any relationship between job reallocation, the sum of job
creation and destruction, unemployment and its average duration? Do small
firms play a key role in the job generation process, as a first look at the data
would indicate? Why is a given worker likely to increase his wage when he moves
to a larger firm? These are some of the key questions addressed throughout the
thesis. As a contribution to a fast developing literature, the thesis does not offer
a complete overview of the new field. Conversely, the thesis selects particular
issues that appear puzzling and, when possible, tries to rationalise them in a
theoretically consistent way.

Throughout the thesis I will think about the process of job creation and
destruction in terms of the matching of job seekers with hiring firms, along the line
of the most recent development of the search equilibrium literature, also defined
as the flow approach to employment determination (Mortensen and Pissarides
1994; Bertola and Caballero 1994; Blanchard and Diamond 1990, 1992). The
flow approach to employment determination grew out of the theory of search
equilibrium, following the success of the latter in suggesting simple ways to model
non-competitive labour markets\(^1\). The concept of the *matching function* and its
eyear empirical success played a key role in the development of this approach.
The explicit assumption made in the simplest model is that all jobs are alike but
their matching is costly. The main matching cost is usually modelled as a time
cost. As a consequence, firms with jobs to fill and unemployed job seekers have to
spend time to find each other. The matching function is used to derive the speed
of job formation and, in its simplest form, is modelled as a stable relationship,
explicitly dependent on the number of unemployed job seekers and the number
of vacant jobs in hiring firms.

\(^1\) Early search models that focused on labour markets are Phelps (1970) and Mortensen
and Diamond (1989).
Within the standard matching literature (Pissarides, 1990), employment dynamics is entirely driven by job creation (the meeting between a job seeker and a vacant job) and by the flow of workers out of unemployment, while job destruction (the separation between existing job matches) is exogenously fixed and constant at any phase of the cycle. Mortensen and Pissarides (1994) have extended the traditional matching approach by assuming heterogeneity in the firm specific productivity. Firms endogenously choose the value of the labour product at which labour shedding takes place and job destruction now plays an important role in the transmission mechanism of aggregate disturbances to the dynamics of employment.

Chapter I, *Job Flow Dynamics and Firing Restrictions*, focuses on the dynamics of job destruction and creation. Davies and Haltiwanger showed that in the American manufacturing sector, job creation is pro-cyclical and job destruction is counter-cyclical. But in the United States the dramatic rise in job destruction during net recessions does not find a symmetric rise in job creation during net expansion. As a consequence job reallocation, the sum of job creation and job destruction, moves counter-cyclically. Empirically, the dynamic asymmetry between job creation and destruction is observed only in Canada and Britain. Conversely, continental Europe job flows appear much more symmetric and job creation is as volatile as job destruction. Chapter I argues that differences across countries in employment protection legislation, albeit not directly relevant to differences in average employment levels, can theoretically account for differences in the dynamics of job creation and destruction. Chapter I introduces employment protection legislation and firing restrictions in the Mortensen and Pissarides model (1994). Firms face idiosyncratic uncertainty in the value of their labour product and endogenously choose the productivity at which existing jobs are no longer profitable. However, to actually shed labour, firms need an exogenous firing permission. Even though firing restrictions have ambiguous ef-
fect on equilibrium unemployment, they dramatically affect the dynamics of job creation and job destruction. The tighter the firing restriction, the less volatile is job destruction and the higher the correlation between job reallocation and net employment changes. The Chapter also shows that the traditional modeling of employment protection legislation, in the form of a simple fixed cost to be incurred when labour shedding takes place, does not affect job flow dynamics. Finally, a parameterization of the model helps rationalise cross-country differences in the cyclical behaviour of job creation and destruction.

Chapter II, *Gross Job Reallocation and Labour Market Policy*, is jointly written with Jozeph Konings and Christopher Pissarides. The aim of the Chapter is to draw together the international data compiled by the OECD (1994a) with a view to understanding the relation between unemployment and job reallocation and the role of labour market policy in the determination of job reallocations. The theoretical literature does not give sufficient reasons for supposing that more job reallocation is better than less, or vice versa. Furthermore there is no evidence of a firm relationship between overall unemployment and job reallocation. Chapter II presents some evidence that low job reallocation is associated with more long-term unemployment. Since the latter is bad, in terms of the loss of skill of the unemployed, Chapter II supposes that policies that restrict job reallocation harm the ability of the market to turn over its unemployment stock quickly. Gross Job Reallocation and Labour Market Policy looks at three kinds of policies: direct restrictions on the firm’s ability to fire employees, “passive” policies, which we measure by income support for the unemployed, and “active” policies, which we measure by the amount of money spent per unemployed worker on measures designed to speed the transition from unemployment to employment. We show that, particularly when we restrict the attention to job reallocation by incumbent firms (i.e. excluding the role of entry and exit), job reallocation is slower in countries that impose restrictions on the dismissal of labour and in countries
that offer income support to the unemployed for long periods of time.

Chapter III, *Wages and the Size of Firms in a Dynamic Matching Model*, is jointly written with Giuseppe Bertola. The existence of wage differentials across observationally equivalent workers is well known in the literature. In particular, different data sets in different countries systematically find evidence of an "employer size-wage effect": firms with relatively higher employment levels pay higher wages. Chapter III uses the idea, already noted in the literature (Pissarides, 1990 and Burdett and Mortensen, 1989) that slow search and imperfect matching can rationalise wage differentials across identical workers. The chapter builds on the matching model of job creation and destruction recently proposed by Bertola and Caballero (1994), where well defined firms with downward sloping labour demand are affected by idiosyncratic shocks. The theoretical approach of the chapter is complex enough to treat profits, employment, and wages as jointly endogenous interrelated variables and the model is realistic enough to allow a quantitative exploration of the employer size-wage effect. Even though wages are decreased by higher employment along a given labour demand schedule, firms with stronger labour demand pay higher wages at any given level of employment, find it optimal to post more vacancies, grow faster, and are larger in size on average. Thus, the wage-size effect is present in the model-generated data, but it does not reflect a positive effect of size on wages, nor does it conflict with the standard assumption of decreasing marginal returns to labour. Rather, firm size proxies in the model's wage regressions for unobservable business conditions, which are also positively correlated to profits and employment growth. Furthermore, the dynamic mechanism we focus on also offers distinctive empirical implications. The model implies that wage dispersion is higher among small firms. Finally, the model predicts that after controlling for employment levels, wages should be higher in faster-growing firms.

While the implementation of the model proposed in Chapter III allows an
analytic solution, the specification of the idiosyncratic process remains rather artificial. In Chapter III, the asymmetry between positive and negative shocks to the unobservable component of individual firm's labour demand is extreme. Every now and then, hiring firms are hit by catastrophic idiosyncratic shocks that cause immediate job destruction and dramatic wage cuts. The following chapter, *A Numerical Approximation of the General Model*, relaxes this extreme assumption and offers a numerical approximation of the most general specification. The aim of the chapter is to show that the results of Chapter III do not depend on the particular specification of the idiosyncratic shock and are consistent with more general implementations.

Chapter V, *Job Flows and Plant Size Dynamics: Traditional Measures and Alternative Econometric Techniques* focuses on the relation between job flows and firm size. If we define firm size as employment in the base year, job creation and job destruction are substantially greater among small firms and net employment changes are a decreasing function of firm size. Several authors (OECD, 1994a; Davis et al. 1995) have pointed to the regression fallacy associated with the relationship between job flows and net employment changes. Chapter V assesses the traditional measures used to correct the regression bias and argues that any definition of firm size that arbitrary forces each unit in the sample into a pre-defined size category, will ignore the flows of jobs between size categories. Chapter V argues that to estimate properly firm size convergence, avoiding the regression fallacy, and to follow accurately employers between size categories, it is necessary to apply the methodology recently introduced by Quah (1993a, 1994) in the empirics of economic growth. Using a balanced panel for the Mexican manufacturing sector I show how conventional results may change when cross-sectional firm dynamics is estimated non-parametrically. I find no evidence of small firms systematically creating more jobs than larger firms and, thus, no evidence of convergence to the mean for the sample as a whole. Finally, I show how
cross-sectional dynamics varies across industries and how it is linked to gross and net flows in each sector. I observe convergence to the mean in relatively stable sectors and asymmetric dynamic behaviour between expanding and declining industries.
Chapter 1

Job Flow Dynamics and Firing Restrictions

1.1 Introduction

The measurement of job creation and job destruction for the U.S. manufacturing sector (Davis and Haltiwanger 1990; 1992) threw new evidence on labour market dynamics. Following Davis and Haltiwanger’s research, empirical studies on employment changes at the establishment level have been performed in most OECD countries. (OECD, 1994a). North American job destruction features wider fluctuations than job creation and causes job reallocation, the sum of job creation and destruction, to move counter-cyclically. By contrast, no asymmetry between time variation in job creation and destruction is apparent in continental European data.

Bentolila and Bertola (1990) and Bertola (1990) argue that high firing costs in Europe and differences in employment protection legislation between countries may explain differences in the dynamics of employment even though they do not necessarily explain low employment on average. This paper follows this line of research and argues that differences in employment protection legislation can

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1 Examples of country studies are Contini et al. (1992) for Italy, Boeri and Cramer (1993) for Germany, Baldwin et al., (1994) for Canada, Konings (1995a) and Blanchflower and Burgess (1994) for UK. OECD (1994a) has tried to standardize studies as much as possible.
theoretically be responsible for observed differences in the cyclical behaviour of job creation and destruction. In a stochastic search model with endogenous job separation, I show that the amplitude and time variation of job reallocation depend crucially upon the arrival rate of exogenous firing permissions. Finally, a parameterization of the model helps rationalise cross-country differences in the cyclical behaviour of job creation and destruction.

The paper builds on the recent matching framework set forth by Mortensen and Pissarides (1994; 1993) and Mortensen (1994). Mortensen and Pissarides have extended the traditional matching approach (Pissarides 1985; 1990; and Mortensen 1991) by assuming an economy populated with a continuum of jobs that differ in the value of a firm’s specific productivity. The idiosyncratic risk for existing jobs is modeled as a jump process characterized by a Poisson arrival frequency and a drawing from a common distribution of productivity. Negative shocks induce job destruction but the firm endogenously chooses the value of labour product to which correspond instantaneous job destruction. Job creation comes from the posting of costly vacancies that are slowly matched to unemployed job seekers. In Mortensen Pissarides (hereafter MP 1994), the asymmetry between hiring and firing technologies rationalises the observed asymmetry between time variation of job creation and job destruction in U.S. manufacturing flows (Davis and Haltiwanger, 1990). In its original form, the MP model cannot rationalise the cyclical behaviour of job flows in continental Europe.

With respect to the MP (1994) model, the innovation of the paper is the modeling of employment protection legislation and job security provisions. Firing and job destruction are no longer instantaneous but can be costly and lengthy. The simplest and widely modeled form of employment protection legislation is a fixed firing cost to be incurred by the firm when firing takes place (Bentolila and Bertola 1990 and Bentolila and Saint-Paul 1994 in partial equilibrium models of labour demand; Burda 1992; Millard 1994 and Millard and Mortensen 1994
in search-equilibrium models). In this paper I focus on a different form of job security provisions and I consider an economy in which firing requires an exogenous firing permission. I show that the traditional fixed firing costs and the more complex firing permission have similar steady state effects, in the sense that they both reduce the job finding rate and have an ambiguous effect on unemployment. Conversely, the dynamic effects of different forms of employment protection legislation vary substantially. A simple fixed firing cost, as in Millard (1994), does not reduce the asymmetry between time variation in job creation and destruction. In the alternative formalization of this paper, tighter firing restrictions smooth out the increase in job destruction during recession and tend to make the dynamics of job destruction symmetric to the dynamics of job creation.

Finally, with respect to the MP (1994) model, where wages are the outcome of a bilateral bargaining, I assume that wages are set by the firms at the workers reservation utility or that the firm continuously extracts all the surplus from the match. The rest of the assumptions are introduced in Section (1.3.2), and they are totally in line with the recent matching literature. The next section briefly looks at the existing empirical evidence on the cyclical behaviour of job flows. Section (1.3.1) discusses the modeling of job security provisions and firing constraints. Section (1.4) presents and solves the steady state model. In section (1.4) the aggregate conditions are fixed while they stochastically jump between "good" and "bad" times in sections (1.5) and (1.6). Section (1.7) presents a parameterization of the model that helps to rationalise cross-country evidence on the cyclical behaviour of job flows.

1.2 A Brief Look at the Evidence

Empirically, job creation (destruction) is defined as the sum of positive (negative) employment changes at the establishment level in a given time interval and in a specific industry. If the industry is representative of the entire economy, we
have a measure of aggregate job creation (destruction). If we divide the num-
ber of jobs created (destroyed) by total employment, we obtain the job creation
(destruction) rate. The sum of job creation and destruction is called job reallo-
cation and is a measure of employment reshuffle across establishments. Finally,
the difference between job creation and destruction is the traditional measure of
net employment changes.

Interest in the dynamic behaviour of aggregate job creation and destruction
surged when Davis and Haltiwanger compiled establishment data for the U.S.
manufacturing sector. Davis-Haltiwanger (1990; 1992) showed that substantial
rates of job creation and destruction coexist at any phase of the cycle, even within
narrowly defined sectors. Job creation and job destruction are negatively but not
perfectly correlated, indicating that significant job creation (destruction) persists
during net recessions (boom). If we take net employment changes as a measure of
the cycle, job creation is pro-cyclical and job destruction is counter-cyclical. But
in the United States, the increase in job destruction during recessions appears
much more pronounced than the increase in job creation during net expansions.
As a result, job reallocation moves counter-cyclically.²

Following the Davis and Haltiwanger research, measurement of employment
changes at the establishment level has been carried out in most OECD countries.
Table (1.1) reports Spearman’s correlations for nine countries. Similar to the
U.S. experience, job creation is pro-cyclical, job-destruction is counter-cyclical,
and job creation is negatively correlated with net employment changes.

Remarkable differences exist in the amplitude of fluctuations of job creation
and destruction. We focus on three simple statistics in Table (1.1) and Table (1.2).

²Since job reallocation (JR) is the difference between job creation (JC) and job destruction
(JD), it follows that

\[ COV(JR, NET) < 0 \rightarrow VAR(JD) > VAR(JC), \]

where NET is the difference between JC and JD.
The first column of Table (1.2) reports the relative variance of job destruction and job creation and shows that the U.S. evidence, where the ratio is greater than two is confirmed only in the UK, and to a lesser extent in Canada and Norway. In most Continental Europe countries the same ratio is close to one or substantially less than one (as in the case of France and Sweden). To differences in the relative variances of Table (1.2) correspond differences in the cyclical behaviour of job reallocation in Table (1.1). The U.S. evidence, where job reallocation fluctuates counter-cyclically, finds similarity in the United Kingdom, where the correlation is negative and significant and in Canada, where even if negative, the correlation is not significantly different than zero.

The last piece of evidence is reported in Table (1.2), which reports the coefficients of variation of job creation and destruction. Two findings are worth pointing out. Firstly, both job creation and destruction appear, proportionally to their mean, more volatile in Anglo saxon countries than in Continental Europe, with the exception of Norway. Secondly, in Continental Europe there is similar time variation in job creation and destruction, whereas the opposite seems true in North-American countries. Overall, there is a clear dichotomy in the cyclical behaviour of job flows. On one side we find the North-American and British experience, where job destruction is more volatile than job creation and job reallocation moves counter-cyclically. On the other side we find the Continental Europe experience, where job reallocation tends to be acyclical and the fluctuations in job creation and destruction are less pronounced.

1.3 Concepts and Notation

1.3.1 Employment Protection Legislation

Employment protection legislation is a form of employment regulation which relates to employers' freedom to dismiss workers. OECD (1994b) argues that
# Table 1.1: Job Flows Over the Cycle

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>0.90 (0.00)</td>
<td>-0.988 (0.00)</td>
<td>-0.745 (0.00)</td>
<td>-0.519 (0.003)</td>
<td>1973-88</td>
</tr>
<tr>
<td>Canada</td>
<td>0.82 (0.00)</td>
<td>-0.89 (0.00)</td>
<td>-0.47 (0.08)</td>
<td>-0.252 (0.38)</td>
<td>1973-86</td>
</tr>
<tr>
<td>UK</td>
<td>0.85 (0.00)</td>
<td>-0.99 (0.00)</td>
<td>-0.78 (0.00)</td>
<td>-0.95 (0.00)</td>
<td>1973-86</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.97 (0.00)</td>
<td>-0.97 (0.00)</td>
<td>-0.88 (0.00)</td>
<td>0.03 (0.90)</td>
<td>1986-91</td>
</tr>
<tr>
<td>Germany</td>
<td>0.93 (0.00)</td>
<td>-0.96 (0.00)</td>
<td>-0.78 (0.01)</td>
<td>-0.04 (0.89)</td>
<td>1977-90</td>
</tr>
<tr>
<td>Norway</td>
<td>0.79 (0.00)</td>
<td>-0.84 (0.00)</td>
<td>-0.35 (0.01)</td>
<td>-0.13 (0.89)</td>
<td>1977-86</td>
</tr>
<tr>
<td>Italy</td>
<td>0.79 (0.00)</td>
<td>-0.81 (0.00)</td>
<td>-0.28 (0.42)</td>
<td>-0.13 (0.88)</td>
<td>1984-93</td>
</tr>
<tr>
<td>France</td>
<td>0.97 (0.00)</td>
<td>-0.85 (0.01)</td>
<td>-0.83 (0.01)</td>
<td>0.74 (0.05)</td>
<td>1984-92</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.97 (0.00)</td>
<td>-0.95 (0.01)</td>
<td>-0.86 (0.01)</td>
<td>0.49 (0.25)</td>
<td>1984-92</td>
</tr>
</tbody>
</table>

(a) Spearman correlation between Job Creation ($JC$) and Net employment Changes ($NET$)
(b) Spearman correlation between Job Destruction ($JD$) and Net employment Changes ($NET$)
(c) Spearman correlation between Job Creation ($JC$) and Job Destruction ($JD$)
(d) Spearman correlation between Job Reallocation ($JR$) and Net employment Changes ($NET$)

Marginal significance in parenthesis

Source: United States, Davis, Halliwanger ans Shuh (1994);
United Kingdom, Konings (1995a); Canada, Baldwin et al. (1994);
Norway, Salvanes (1995); Germany, Boeri and Cramer (1993);
Italy, R&P (1995); France, Lagarde et al. (1994);
Sweden, OECD (1994a); Denmark, Albaek and Sorensen (1995)
employers’ freedom to dismiss may be restricted by: -(i) a requirement to give several months’ notice to the worker before dismissal become effective; and/or to provide severance payments upon dismissal; -(ii) a requirement for prior warnings or written justification to the person to be dismissed (or to a third party); -(iii) a requirement for authorisation from a third party before dismissal can take place, or a requirement that rehabilitative measures be attempted before a worker is dismissed; -(iv) provisions for appeal against unfair dismissal.

The multiple dimensions of employment protection are difficult to model in an simple way. Most of the work in the area, notably Bentolila and Bertola (1990), collapses the multidimensional aspects of employment protection legislation into a simple fixed firing cost, to be incurred by the firm when separation takes place. Firing is always possible, but at an exogenously fixed cost $F$. This simplification has the advantage of being analytically simple and, at least conceptually, empirically observable. In this direction, several quantitative measures
of employment protection have been proposed in the literature (OECD 1994b; Grubb and Wells 1993). Most indices report at one extreme of the scale the American experience, where firing can take place any time at no cost. At the other extreme we usually find the continental Europe experience, with countries like Italy, France and Germany at the very top of the scale. Britain and Canada are often cited as mid-way examples.

With respect to the classification above, the fixed cost rule used in the economic literature can be a good first approximation for the severance payments and the period of notice components of employment protection legislation (-i and -ii above). The problem with this simplification is that it fails to capture the existence of other types of firing restrictions and more complicated firing procedures. In most European countries, before firing can take place a discussion with union representative is often necessary and, in extreme cases, a full agreement with government representatives must be reached (Emerson, 1988). In general, we can say that these complicated procedures introduce a stochastic component in the employment protection legislation. Ex-ante, firms face uncertainty over firing costs along two dimensions: the actual costs of firing and the actual time of shedding. Firstly, European firms discuss with the worker the amount of severance payments to be paid to the employees. Secondly, firms do not know exactly the moment in which the discussions with the unions will end. Furthermore, as in the case of Germany, mass firing requires a given notice period, which can be exogenously lengthened by a government intervention. Finally, the existence of a “just clause” rule in most European legislation, allow the worker to appeal against unfair dismissal and can result in reinstatement of the dismissed worker. The traditional indicators of firing costs may capture the uncertainty over total firing costs, but they definitely fail to capture the uncertainty over the actual timing of labour shedding.

The only indicator that tries to measure the restrictiveness of procedural
Table 1.3: Importance of Procedural Constraints

<table>
<thead>
<tr>
<th>Country</th>
<th>Strictness</th>
<th>Country</th>
<th>Strictness</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>0.4</td>
<td>Sweden</td>
<td>2.0</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.5</td>
<td>France</td>
<td>2.0</td>
</tr>
<tr>
<td>Canada</td>
<td>0.6</td>
<td>Germany</td>
<td>2.5</td>
</tr>
<tr>
<td>Denmark</td>
<td>1.0</td>
<td>Italy</td>
<td>3.0</td>
</tr>
<tr>
<td>Norway</td>
<td>1.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Ranking from least restrictive to most restrictive

ILO classifies regulatory constraint as
- insignificant (scored 0);
- minor (for termination of regular contracts),
insignificant or minor (for fixed-term contracts) (both scored 1);
- serious (scored 2); fundamental (scored 3)

Source: OECD (1994b)

Obstacles to the implementation of no-fault dismissal is the index compiled by ILO, which classified regulatory constraints as insignificant, minor, serious or fundamental. The index, originally compiled by ILO for European countries has only recently been completed by OECD (1994b) and it is shown in Table (1.3) for the same countries for which I have flows data in Tables (1.1) and (1.2).

Comparison of Tables (1.1), (1.2) and (1.3) shows that the countries that experience asymmetric behaviour in the dynamics of job creation and job destruction are the countries with insignificant firing constraints. Conversely, continental Europe countries, with symmetric behaviour in the dynamics of job creation and destruction have serious or fundamental firing constraints. Boeri (1995), using data for 8 OECD economies, argues that job reallocation is counter-cyclical only in the U.S., but in his data set Britain is missing and data from Canada range from 1978 to 1988, whereas the most recent data compiled by Baldwin et al. (1994) range from 1973 to 1988.

The easiest way to capture the effects of procedural constraints in an aggregate model is to assume that a firm can accomplish firing only when it is
granted an exogenous firing permission. In this paper I assume that a firm with low productivity is obliged to pay the worker until firing permission arrives. At the micro-level these firing permissions may well depend on the firm’s bargaining strength over the workers and government’s representatives. Nevertheless, as long as we are only interested in aggregate dynamics, differences in bargaining strength across firms should balance-out and we can assume that a firm gets a firing permission at an *exogenously* given rate and it actually fires when productivity is low and permissions arrive.

More formally, I assume that the arrival rate of firing permissions is a Poisson process with average waiting time equal to $1/s$. A job in good business conditions is an operational job, while a job in bad business conditions without firing permission is an *idle job*. An economy with no firing restrictions is an economy with an average waiting time of firing permissions equal to zero. On the other hand, the longer the waiting time (i.e. the lower the arrival rate $s$), the tighter the firing restrictions and the higher are the degrees of job security provisions. As I show in Section (1.4), the average value of an idle job is a measure of the expected firing costs. As a consequence, the firing restrictions rule can potentially capture the traditional effects of simpler fixed firing cost rules. In Section (1.4) and in appendix (A) I show that this way of modelling firing restrictions, albeit with no immediate empirical counterpart, has very similar steady state properties to the simple fixed cost rule. Furthermore, Bertola (1990) points out that economies with different degrees of job security provisions tend to have very different dynamic behaviour. With respect to job flow dynamics, Section (1.5) shows that only the firing permission dramatically affects the dynamic behaviour of the system. In this sense, the traditional fixed firing cost is not a good candidate for rationalising differences in the cyclical behaviour of job flows.
1.3.2 The model

I consider an economy populated by a continuum of risk-neutral workers of fixed quantity, normalised to one for simplicity. Workers can be either employed or unemployed and, if unemployed, are actively searching for jobs. For simplicity, I rule out on the job search. Each firm has only one job which can be filled and producing or vacant. A filled job can be either fully operational or idle, depending on whether the firm is actually waiting for firing permissions. Following the empirical literature I define job creation as the moment in which a vacant job meets an unemployed worker. Similarly, job destruction takes place when an idle job gets a firing permission, separates and leaves the market.

As in MP (1994), each job is characterized by a fixed irreversible technology and produces at the productivity level $p + \sigma \epsilon$. The productivity is made up of an aggregate component $p$, common to every job and a job specific component $\epsilon$. The stochastic process regulating the idiosyncratic component of the productivity $(\epsilon)$ is Poisson with arrival rate equal to $\lambda$. In the event of a change in $\epsilon$, the new value of the job specific productivity is a drawing from a fixed distribution $F(\epsilon)$, with finite upper support $\epsilon_u$, lower support $\epsilon_l$ and no point mass, other than at the upper support $\epsilon_u$. This way of modeling implies a memoryless but persistent idiosyncratic productivity. The persistence of any given productivity $\epsilon$ is $1/\lambda$.

I follow the earlier literature by assuming that new firms have the option to select the best productivity in the market, and create jobs at the upper support $p + \sigma \epsilon_u$. Following an idiosyncratic shock, however, the firm has no choice over its productivity. Filled job are said to be fully operative if the idiosyncratic productivity is above some critical value $\epsilon_d$, while they are said to be idle if the job specific productivity is below $\epsilon_d$. Operational jobs turn idle at rate $\lambda F(\epsilon_d)$ while idle jobs get firing permissions and leave the market at rate $s$. Finally, idle

---

\(^3\)In the paper $\sigma$ is simply a normalising parameter useful for the simulations of section (1.7); it is common to every job and it will not play any specific role.
jobs are subject to idiosyncratic uncertainty and can return fully operational at rate $\lambda(1 - F(\epsilon_d))$.

Vacant firms and unemployed workers meet at rate $m(u, v)$, where $m$ is a first-degree homogeneous matching function and $u$ and $v$ are the number of vacancies and the number of unemployed normalised by the labour force. Vacancies are filled at the rate

$$q(\theta) = \frac{m(u, v)}{v}; \quad \theta = \frac{v}{u}, \quad \frac{\partial q(\theta)}{\partial \theta} < 0,$$

where $\theta$ is a measure of market tightness from the firm's point of view. Similarly, workers find job at rate

$$\theta q(\theta) = \frac{m(u, v)}{u}; \quad \frac{\partial \theta q(\theta)}{\partial \theta} > 0.$$

Apart from the firing constraints, we depart from the standard MP (1994) framework in the wage-setting behaviour. To simplify the analysis of the effects of employment protection legislation on job flow dynamics, I assume that employers capture all the rents associated with a job-worker match by paying workers the common alternative value of their time, $b$. As Diamond (1971) has shown, this outcome is an equilibrium in a wage setting game played among employers when workers have only the power to accept or reject offers and workers search sequentially at some positive costs. Given this outcome, workers have no incentive to search on the job and their parameters, other than $b$, do not affect the equilibrium. Alternatively, if I allowed a continuously renegotiated Nash bargain between the firm and the worker, the wage would certainly be higher than the worker reservation utility in operational jobs, where the surplus from the match is positive. But the presence of firing restrictions would force the firm to pay the worker even when the job is idle and the worker's participation constraint is binding. This would force idle firms to offer the worker his reservation utility $b$, exactly as in the present model. Thus, a continuously renegotiated bargain would only affect the wage of operational jobs, leaving unchanged the behaviour
of idle jobs, the distinctive feature of this model. To keep track of such bargains would be analytically tedious and would not change the qualitative results of the paper.

1.4 Steady State

The asset valuation of a filled job, conditional on an idiosyncratic productivity $\epsilon$ is

$$r J(\epsilon) = p + \sigma \epsilon - b + \lambda \left[ \int_{\epsilon}^{\infty} J(x) dF(x) - J(\epsilon) \right] + s \left[ \max(0, J(\epsilon)) - J(\epsilon) \right], \quad (1.1)$$

where $J(.)$ is the value of a job, $r$ is the exogenous interest rate, $p + \sigma \epsilon - b$ are operational profits at idiosyncratic productivity $\epsilon$. Apart from the flow-term $p + \sigma \epsilon - b$, (1.1) involves two capital gain terms. At rate $\lambda$ the firm loses its current asset value $J(\epsilon)$ and draws a new $\epsilon$ from the productivity distribution. At rate $s$ firing permissions arrive and the firm gets an option to destroy the job. Since a destroyed job has zero value, the max operator in (1.1) captures the idea that a firm will keep running a job as long as its value is positive. It follows that an operational job is a positively valued job that ignores firing permissions while an idle job is a negatively valued job that destroys the job when permissions arrive. Differentiating (1.1) with respect to $\epsilon$ it shows that $J(\cdot)$ is a piece-wise increasing function of $\epsilon$ and its derivative reads

$$J_t(\epsilon) = -\frac{\sigma}{r + \lambda}, \quad \forall \epsilon : J(\epsilon) > 0, \quad (1.2)$$

and

$$J_t(\epsilon) = -\frac{\sigma}{r + \lambda + s}, \quad \forall \epsilon : J(\epsilon) < 0. \quad (1.3)$$

If we define the reservation productivity $\epsilon_d$ as

$$J(\epsilon_d) = 0,$$
making use of (1.2) and (1.3), after an integration by parts, the expected value of a job in (1.1) reads

$$
\int_{\xi_l}^{\xi_u} J(x) dF(x) = \frac{\sigma}{r + \lambda} \int_{\xi_u}^{\xi_l} (1 - F(z)) dz - \frac{\sigma}{r + \lambda + s} \int_{\xi_l}^{\xi_u} F(z) dz.
$$

The last term of (1.4) is the (negative) value of an idle job and is a measure of expected firing costs. As the average waiting time goes to zero ($s \to \infty$), the second term on the right hand side of (1.4) vanishes, firing is always possible and it is accomplished as soon as the value of the job is negative. To obtain the cut off value $\epsilon_d$, below which the firm will accept firing permission, we make use of (1.4) and we evaluate (1.1) at $J(.) = 0$. The reservation productivity solves

$$
p + \sigma \epsilon_d - b = -\frac{\lambda \sigma}{r + \lambda} \int_{\xi_d}^{\xi_u} (1 - F(z)) dz + \frac{\lambda \sigma}{r + \lambda + s} \int_{\xi_l}^{\xi_u} F(z) dz.
$$

Equation (1.5) is one of the key equations of the model and uniquely determines the reservation productivity as a function of the parameters $r, \lambda, p, s, b, \sigma$ and the productivity distribution $F(\epsilon)$. The left hand side of (1.5) is the profit from the marginal operational job. In an economy with no firing constraints ($s \to \infty$), the second term on the right hand side vanishes, the marginal profit is negative and there is voluntary labour hoarding in equilibrium. When firing is instantaneous ($s \to \infty$) but hiring is costly, the firm will hoard labour up to the level in which current losses compensate savings of hiring costs if conditions improve. The presence of firing delays increases, through the last term in (1.5), the value of the marginal profits. As the average waiting time for firing permissions increase, a job will be kept running in bad times for a longer period of time because of exogenous constraints and there will be institutional labour hoarding. Since the firm anticipates firing restrictions when conditions are bad, in (1.5) the firm reduces the extent of voluntary labour hoarding. As $s$ falls it is possible that firing restrictions become so high that the firm will accept firing permissions at a positive profit per period.
Differentiating (1.5) with respect to s,
\[ \sigma \frac{\partial \epsilon_d}{\partial s} = \frac{\lambda \sigma}{r + \lambda} (1 - F(\epsilon_d)) \frac{\partial \epsilon_d}{\partial s} + \frac{\lambda \sigma}{r + \lambda + s} F(\epsilon_d) \frac{\partial \epsilon_d}{\partial s} - \frac{\lambda \sigma}{(r + \lambda + s)^2} \int_{\epsilon_d} F(z) \, dz \]
and rearranging, yields
\[ \sigma \frac{\partial \epsilon_d}{\partial s} \frac{(r + \lambda) r + s (r + \lambda F(\epsilon_d))}{(r + \lambda)(r + \lambda + s)} = - \frac{\lambda}{(r + \lambda + s)^2} \int_{\epsilon_d} F(z) \, dz. \] (1.6)

Thus \( \frac{\partial \epsilon_d}{\partial s} \leq 0 \): an increase in the average waiting time of permission (fall in s) increases the productivity at which the firm accepts firing permissions. This is consistent with the firm anticipating long waiting time when conditions worsen.

The reservation productivity falls with \( p \), the common productivity. Differentiating (1.5) with respect to \( (p - b) \) yields
\[ 1 + \sigma \frac{\partial \epsilon_d}{\partial (p - b)} = \frac{\lambda \sigma}{r + \lambda} (1 - F(\epsilon_d)) \frac{\partial \epsilon_d}{\partial (p - b)} + \frac{\lambda \sigma}{r + \lambda + s} F(\epsilon_d) \frac{\partial \epsilon_d}{\partial (p - b)} \] (1.7)
and rearranging, yields
\[ \sigma \frac{\partial \epsilon_d}{\partial (p - b)} \frac{(r + \lambda) r + s (r + \lambda F(\epsilon_d))}{(r + \lambda)(r + \lambda + s)} = -1. \] (1.8)

Thus \( \frac{\partial \epsilon_d}{\partial p} \leq 0 \): as the productivity increases the firm will find it profitable to keep a job operational for a higher range of productivities. The effect of other parameters on the reservation productivity is ambiguous. Higher discount rate \( r \) reduces the flow of income from the job and makes labour hoarding less profitable. This would reduce \( \epsilon_d \). But simultaneously, the higher discount rate reduces expected firing costs and makes autonomous labour hoarding profitable. Similar arguments hold for changes in the arrival rate of idiosyncratic shocks. Higher \( \lambda \) corresponds to an increase in the arrival rate of productivity shocks. On the one hand the reservation productivity tends to decrease since the firm expects the duration of adverse conditions to be shorter. At the same time the probability of facing a firing procedure is higher and the net effect depends mainly on the distribution \( F(.) \).
Job creation comes through the posting of vacancies. When creating a job we assume the existing technology is fully flexible and the productivity distribution is common knowledge. Job creation takes place at the upper support of the distribution ($\epsilon_u$). A posted vacancy yields an asset return of $-c$ per period, $c$ being the constant cost of hiring, and a probability $q(\theta)$ of being filled with a job created at the upper support of the distribution. The vacancy asset valuation is

$$rV = -c + q(\theta) [J(\epsilon_u) - V].$$

(1.10)

With free entry into the job market there are, in equilibrium, zero expected profits ($V = 0$) (Pissarides 1990) and the value of a job equals the expected searching costs:

$$J(\epsilon_u) = \frac{c}{q(\theta)},$$

(1.11)

where the value of a job at the upper support of the distribution is obtained subtracting (1.5) to (1.1) and reads

$$J(\epsilon_u) = \frac{\epsilon_u - \epsilon_d}{r + \lambda}.$$  

(1.12)

(1.11) is the job creation condition and uniquely determines the vacancy unemployment ratio $\theta$ as a function of the parameters $r, \lambda, c$, the matching function $q(.)$, the upper support of the distribution $\epsilon_u$ and the reservation productivity $\epsilon_d$.

Differentiating (1.11) with respect to common productivity $p$, yields

$$- \frac{\partial \epsilon_d}{\partial p} \frac{1}{r + \lambda} = \frac{q'(\theta)c}{q(\theta)^2} \frac{\partial \theta}{\partial p},$$

(1.13)

and, making use of the facts that $\frac{\partial \epsilon_d}{\partial p} < 0$ and $q'(.) < 0$, $\frac{\partial \theta}{\partial p} > 0$. Higher common productivity, increasing the flow of future profits, increases job creation at given unemployment. Conversely, higher job security provisions reduce the expected

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4It can easily be checked that if $c = 0$, $J(\epsilon_u) = 0$ from (1.11) and $\epsilon_d = \epsilon_u$.  

34
value of a job and reduce the profitability of new jobs. Job creation at given
unemployment falls. Differentiating (1.11) with respect to s,

$$\frac{\partial \epsilon_d}{\partial s} \frac{1}{r + \lambda} = -\frac{cq'(\theta)}{q(\theta)^2} \frac{\partial \theta}{\partial s},$$

making use of $\frac{\partial \epsilon_d}{\partial s} < 0$ (1.14) implies that $\frac{\partial \theta}{\partial s} > 0$.

To close the model we need to introduce unemployment. With a fixed labour
force, a worker can be either unemployed or employed. If employed, a worker can
be attached to a fully operational ($\epsilon \geq \epsilon_d$) or to an idle job $\epsilon < \epsilon_d$. Normalizing
variables in terms of a constant labour force, the relationship among different
labour force status is

$$u + n_j + n_i = 1,$$

where $u$ is the unemployment rate, $n_i$ is the employed idle capacity and $n_j$ is the
employed operational rate. Let us consider unemployment. In an interval $dt$, the
outflow rate corresponds to the number of matches per unemployed times the
number of unemployed; while the inflow rate corresponds to the fraction of work­
ers in the idle state that obtained firing permission. Unemployment dynamics
reads

$$\dot{u} = sn_i(t) - \theta q(\theta)u(t),$$

where $\theta q(\theta)$ is the job finding rate. If job creation (job destruction) is defined
as the sum of all positive (negative) employment changes, as in the empirical
literature, (1.16) defines unemployment variation as the difference between job
destruction and job creation. Simultaneously there are a number of fully opera­
tional jobs that are hit by a shock below the reservation productivity and enter
the idle state. The outflow from the idle state corresponds to the idle jobs that
have obtained firing permissions plus those idle jobs that, hit by a positive pro­
ductivity shock, return to be fully operational. The inflow into the idle state is
given by the operational jobs hit by a shock below the reservation productivity.
The change in the idle rate is
\[
\dot{n}_i = \lambda F(\epsilon_d)n_j(t) - [s + \lambda(1 - F(\epsilon_d))]n_i(t).
\] (1.17)

In steady state equilibrium, the unemployment rate and the employment composition between idle and operational jobs is constant. From (1.16) and (1.17) it follows that unemployment and the idle rate are constant if the inflow rate is equal to the outflow rate. Steady state idle rate is
\[
n^* = \frac{\theta q(\theta)}{s} u^*.
\] (1.18)

Making use of (1.18), equilibrium unemployment is
\[
u^* = \frac{\lambda F(\epsilon_d)}{\lambda F(\epsilon_d) + \frac{s+\lambda}{s}\theta q(\theta)}.
\] (1.19)

The steady state system is recursive and it reduces down to four equations. (1.5) uniquely determines the reservation productivity \(\epsilon_d\) while (1.11), given \(\epsilon_d\), uniquely determines the vacancy unemployment ratio \(\theta\). Given \(\theta\) and \(\epsilon_d\), (1.18) and (1.19) simultaneously determine unemployment and the idle rate. Finally, given the unemployment rate, \(\theta\) determines vacancies\(^5\).

If firing is unrestricted \((s \to \infty)\), the idle rate in (1.18) tends to zero and equilibrium unemployment in (1.19) coincides with equilibrium unemployment in more standard matching models (MP 1994; Pissarides 1990). As the average waiting time increases, firing restrictions affect both job creation and job destruction decision (i.e. \(\epsilon_d\) and \(\theta\)) and they have an ambiguous impact on unemployment. Differentiating (1.19) with respect to \(s\), it is obvious that the overall
\(^5\)The system (1.18) and (1.19) is fully stable and the convergence to the steady state equilibrium is monotonic. From the characteristic equation of the homogeneous system (1.18) and (1.19),
\[
r^2 + r(s + \lambda + \theta q(\theta)) + \theta q(\theta)(s + \lambda) + s\lambda F(\epsilon_d) = 0,
\] (1.20)
it follows that both roots have negative real parts and the system is stable. Furthermore the convergence is monotonic since in (1.20)
\[
\Delta = (s - \lambda - \theta q(\theta))^2 + 4s\lambda(1 - F(\epsilon_d)) \geq 0.
\] (1.21)
results depends on the particular values of the parameters and on the form of the productivity distribution.

Lower firing restrictions increase the job finding rate, $\theta q(\theta)$, through their positive effect on market tightness ($\frac{\partial \theta}{\partial s} > 0$). Steady state job reallocation is $2sn^* = 2\theta q(\theta)u^*$ and it depends on firing costs in a direct and indirect way. Stricter job security provisions lower the job hiring rate and negatively affect job reallocation. Simultaneously, higher firing costs indirectly affect job reallocation through their ambiguous effect on unemployment and the overall results depend upon parameters of the model. On the other hand higher common productivity reduces both unemployment and the idle rate$^6$.

The distinctive prediction of the model is that higher firing delays (lower $s$) reduce the job finding rate; the effect on job reallocation is likely to be negative, but overall ambiguous. In appendix (A) I show that, as long as we look across steady-state equilibria, the comparative static of modeling firing delays are qualitatively similar to the comparative static of traditional fixed firing cost rules, as in Millard and Mortensen (1994); the only difference being that stricter firing restrictions have ambiguous effect on the job reallocation rate while the latter in the Mortensen Millard (1994) paper is unambiguously reduced by higher firing tax. Even though the prediction that higher firing costs reduce both job creation and destruction is common to many search models, empirical evidence is controversial (Bertola and Rogerson, 1996)$^7$. In Chapter (2) there is some evidence of a negative relationship between job reallocation and long-term unemployment. Countries with high long-term unemployment tend to have lower job reallocation and, as long as we exclude the role of firms' entry and exit, higher job security

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$^6$This prediction should not be expected to hold empirically, especially when considering long-run growth. In the spirit of the model and in the rest of the paper $p$ is a cyclical variable. Section (1.7) shows that when we let $p$ be a cyclical variable its fluctuations results in countercyclical movements of unemployment.

$^7$Bertola and Rogerson (1996) argue that the fact that countries with different degrees of job security provisions have similar rates of job reallocation should not be surprising once we realize that countries with high job security provisions have also centralized wage-setting institutions.
provisions. Nevertheless, the main interest of this paper concerns the effect of firing restrictions on the cyclical behaviour of job creation and destruction. From the next section I turn explicitly to dynamics.

1.5 Job Flows and Cyclical Shocks

1.5.1 The General Model

To study the cyclical behaviour of the model proposed in Section 2 I need an explicit driving force. In this paper I assume that job dynamics is driven by a single aggregate disturbance and I let the state of the economy be described by a realization of a first order Markov process. Aggregate conditions move stochastically between \( n \) states, indexed by net common productivity \( x_i = (p-b)_i \) with \( x_i > x_{i+1} \). Aggregate shocks are described by the elements \( \pi_{ij} \) of a \( nxn \) stochastic matrix that contains the probabilities that the aggregate productivity jumps from state \( i \) to state \( j \). From the analysis in the previous section it is clear that for each aggregate productivity the system is characterized by the pair \( \{ \epsilon_{di}, \theta_i \} \). In this section I describe the methodology for solving for pairs \( \{ \epsilon_{di}, \theta_i \} \), \( i = 1 \cdots n \). The techniques applied in this section were first introduced by Mortensen (1994), but they have to be slightly modified to solve the model of this paper. In what follows, the two steps procedure for obtaining the reservation productivities is very similar to Mortensen (1994), apart for the presence of firing restrictions \( s \), while the methodology for obtaining the market tightness is specific to the model of this paper.

The comparative static results of the previous section let us infer that, in general, since \( x_i > x_{i+1} \), \( \epsilon_{di} \leq \epsilon_{di+1} \) and \( \theta_{i+1} > \theta_i \). Since we assume that cyclical shocks are anticipated, we need to set out an aggregate state contingent value function for each job \( \epsilon \). The value of a job, conditional on aggregate state pro-
ductivity $x_i$ and idiosyncratic productivity $\epsilon$ now reads

$$ rJ_i(\epsilon) = x_i + \sigma \epsilon + \lambda \left[ \int_{c_i}^{c_u} J_{i}(z) dF(z) - J_i(\epsilon) \right] + \sum_{j \neq i} \pi_{ij} [J_j(\epsilon) - J_i(\epsilon)] $$

$$ + s \left[ \max(J_i(\epsilon), 0) - J_i(\epsilon) \right] \quad i, j = 1, \ldots, n, \quad (1.22) $$

where at rate $\lambda$ a job specific shock arrives, at rate $\pi_{ij}$ an aggregate state switch occurs, while at rate $s$ the firm gets firing permission. (1.22) can easily be written as

$$ (r + \lambda + \sum_{j \neq i} \pi_{ij} + s)J_i(\epsilon) = x_i + \sigma \epsilon + \lambda \int_{c_i}^{c_u} J_i(z) dF(z) + \sum_{j \neq i} \pi_{ij} J_j(\epsilon) $$

$$ + s \left[ \max(J_i(\epsilon), 0) \right] \quad i, j = 1, \ldots, n. \quad (1.23) $$

After dividing both terms by $(r + \lambda + \sum_{j \neq i} \pi_{ij} + s)$, the right hand side of (1.23) is a mapping that satisfies Blackwell's sufficient conditions for a contraction. The system in (1.23) has to be solved for the vector of reservation productivities. Each $\epsilon_{di}$, if it exists, is defined as $J(\epsilon_{di}) = 0$, and I let $R_d$ be a column vector containing the $n$ reservation productivities. Differentiation of (1.23) with respect to $\epsilon$ shows that each value function $J_i(\epsilon)$ is a piece-wise linear increasing function in $\epsilon$, with $n$ kinks at values corresponding to elements of the vector $R_d$. The $n$ consecutive kink points at $\epsilon_{di} < \epsilon_{di+1}$, together with $\epsilon_0 < \epsilon_1$ and $\epsilon_n < \epsilon_u$, divide the productivity distribution into $n + 1$ interval of the form $[\epsilon_k - \epsilon_{k+1})$, with $\epsilon_0 = \epsilon_{lo}$ and $\epsilon_{n+1} = \epsilon_u$. In the first stage of the solution the derivative of each $J_i(\epsilon)$ for values of $\epsilon$ in the interval $[\epsilon_k - \epsilon_{k+1})$, is obtained as a solution to the linear system

$$ (r + \lambda + \sum_{j \neq i} \pi_{ij} + \Phi s) \frac{\partial J_i}{\partial \epsilon} = \sigma \sum_{j \neq i} \pi_{ij} \frac{\partial J_j}{\partial \epsilon} \quad i, j = 1, \ldots, n; \quad \epsilon_k \leq \epsilon < \epsilon_{k+1}, \quad (1.24) $$

where $\Phi$ is an indicator function taking the value 1 if $\epsilon_{di} > \epsilon_{k+1}$ and zero otherwise. Let $D$ be an $(n, n+1)$ matrix whose general element $d_{ik}$ gives the partial derivative
of $J_i(\epsilon)$ in the interval $[\epsilon_k, \epsilon_{k+1})$. The elements of $R_d$ are then obtained in the second stage as a solution to the following non-linear system in the $n$ reservation productivities:

$$x_i + \sigma \epsilon_{d_i} = -\lambda E[J_i(\epsilon_1, \ldots, \epsilon_n)] - \sum_{j \neq i} \pi_{ij} J_j(\epsilon_{d_i}) \quad i = 1, \ldots, n. \quad (1.25)$$

In (1.25) $E(J_i)$ is obtained integrating by parts (1.23) for each $J_i$ using the partial derivatives $d_{ij}$ obtained in the first stage. $E(J_i)$ is defined as

$$E(J_i) = \sum_{p=0}^{r_p} d_{i,p+1} \int_{\epsilon_p}^{\epsilon_{p+1}} (1 - F(x)) \, dx - \sum_{p=0}^{r_p} d_{i,p+1} \int_{\epsilon_p}^{\epsilon_{p+1}} F(x) \, dx \quad (1.26)$$

Job creation takes place at the upper support of the productivity distribution. Depending upon the state of the system $x_i$, vacancies will be created so as to eliminate all possible rents. For each state $i$ of the economy, an expression similar to (1.11) of Section 1.4 holds:

$$J_i(\epsilon_u) = \frac{c}{q(\theta_i)} \quad i = 1, \ldots, n. \quad (1.27)$$

To obtain the market tightness $\theta_i$ from (1.27) it is necessary to obtain an expression for the value of the job at the upper support of the distribution $J_i(\epsilon_u)$. Each $J_i(\epsilon_u)$, using the vector $R_d$ obtained from (1.25) is one of the solutions to the following linear system:

$$(r + \lambda + \sum_{k \neq i} \pi_{ik}) J_i(\epsilon_u) = \sigma(\epsilon_u - \epsilon_{d_i}) + \sum_{k \neq i} \pi_{ik} (J_k(\epsilon_u) - J_i(\epsilon_{d_i})) \quad (1.28)$$

and

$$(r + \lambda + \sum_{k \neq j} \pi_{kj}) J_j(\epsilon_u) - (r + \lambda + \sum_{k \neq j} \pi_{kj} + \phi_2) J_j(\epsilon_{d_j}) = \sigma(\epsilon_u - \epsilon_{d_i})$$

$$+ \sum_{k \neq j,i} \pi_{kj} (J_k(\epsilon_u) - J_k(\epsilon_{d_i})) + \pi_{ji} J_j(\epsilon_u) \quad \text{for } j \neq i. \quad (1.29)$$

In (1.29) $\phi_2$ is an indicator function taking the value of 1 if $\epsilon_{d_j} > \epsilon_{d_i}$. The system (1.29) and (1.28) has to be solved recursively for each $i$, starting from
In (1.28) and (1.29) the unknowns are \( J_i(\epsilon_u), (J_k(\epsilon_u) - J_k(\epsilon_{d_i})) \) for \( k < i \) and \( J_k(\epsilon_{d_i}) \) for \( k > i \), while \( J_k(\epsilon_u) \), for \( k > i \), enters the system as a parameter. Each \( J_i(\epsilon_u) \) can then be substituted into (1.27) to obtain the corresponding \( \theta_i \). Since \( x_i > x_{i+1} \) it will, in general, be true that \( J_i(\epsilon_u) > J_{i+1}(\epsilon_u) \) and, from (1.27), \( \theta_i > \theta_{i+1} \).

### 1.5.2 The Dynamics of Job Destruction in a Special Case

With respect to the MP (1994) model, the introduction of firing restrictions has important effects on the dynamics of job destruction and on the cyclical behaviour of job reallocation. Conversely, the dynamics of job creation is completely in line with the MP (1994) model: as conditions improve new vacancies have to be matched to unemployed job seekers and the resulting rise in job creation is time consuming due to the presence of the matching function. In this section, to illustrate analytically how the dynamics of job destruction is affected by firing restrictions, I let the aggregate productivity parameter \( p \) take only a high value \( p^* \) when the economy is booming and a low value \( p \) when the economy is in recession, and I shall indicate with \( \mu \) the instantaneous transition rate from one aggregate state to the other. Since job destruction is completely driven by the reservation productivity, I start describing the equations of the marginal jobs.

Applying the methodology of the previous section to this simpler case and indicating with \( (\epsilon_d) \) and \( (\epsilon_d^*) \) the marginal productivity in bad and good times, the reservation productivity in recession solves

\[
p + \sigma \epsilon_d - b = -\lambda E[J] - \mu J^*(\epsilon_d),
\]

where

\[
J^*(\epsilon_d) = \frac{p^* + \sigma \epsilon_d - b + \lambda E[J^*]}{r + \lambda + \mu}.
\]

Conversely, the reservation productivity in a boom solves

\[
p^* + \sigma \epsilon_d^* - b = -\lambda E[J^*] - \mu J(\epsilon_d^*),
\]
where

$$J(c^*_d) = \frac{p + \sigma c^*_d - b + \lambda E[J]}{r + \lambda + \mu + s}.$$ (1.33)

Equations (1.32) and (1.30) form a system of two equations in $c_d$ and $c^*_d$.

To understand properly the separate effect of cyclical shocks (through the switching parameter $\mu$), and firing restriction (through the arrival rate $s$), I consider each of them separately.

Firstly, let the average waiting time for firing restriction be zero. In equation (1.32) the second term in the right hand side vanishes. Qualitatively, the introduction of cyclical shocks in an environment without firing restriction does not affect the equation of the marginal job during a boom. When $(s \to \infty)$ in (1.32), as the economy switches from boom to recession the marginal job will immediately be destroyed. Conversely, with $s \to \infty$ in (1.30), the probability that common productivity jumps from $p$ to $p^*$ increases the option value of the marginal job in recession. This effect is reflected in the second term of the right hand side in (1.30).

Now let $s$ take a positive and finite value, permission arrive at random and firing restrictions apply. The reservation productivity in recession in (1.30) is not directly affected by the firing restrictions $s$. Obviously, if the state of the economy switches to boom the marginal firm will get a positive asset value and will ignore firing permissions if they arrive. Conversely, the introduction of $s$ directly affects the reservation productivity during a boom in (1.32). If the economy switches from boom to recession the last term in (1.32) is positive as long as the marginal job must wait for a firing restriction and can not destroy a negatively valued asset. This last effect is crucial and it is what makes the dynamics of job destruction in the model different from the dynamic implied by a simple fixed firing cost to be incurred when job destruction takes place. In appendix (B) I show that with a simple fixed firing cost, in the spirit of Millard (1994), a marginal job in

---

*Equation (1.32) when $s \to \infty$ is qualitatively analogous to equation (1.5) of section (1.4).*

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a boom will immediately be destroyed when the economy switches from boom to recession. Thus, in an institutional world with a simple fixed firing cost, all jobs whose marginal productivity lie between the two reservation productivities will be destroyed when the economy switches from boom to recession, exactly as in the world with no firing delays considered by MP (1994).

Solving for the market tightness in recession and in boom, from a system analogous to (1.28) and (1.29), the model is determined by the couple \((\epsilon_d, \theta)\) in a recession and \((\epsilon_d^*, \theta^*)\) in a boom. To see how the model works consider first what happens when the aggregate productivity switches from \(p\) to \(p^*\). Since \(\theta^* > \theta\), firms open up more vacancies and since \((\epsilon_d^* < \epsilon_d)\), firms keep operational more existing jobs. On impact neither job destruction nor job creation jumps, since new vacancies take time to be matched to unemployed workers. As new matching starts to take place, the fall in unemployment induces a fall in job creation and an increase in job destruction until there is convergence to a new steady-state, or until there is a new cyclical shock. When aggregate productivity falls from \(p^*\) to \(p\), the dynamics of job creation follows an opposite pattern to that after an increase in \(p\): vacancies fall on impact, but job creation takes time to fall. As for job destruction, the same reasoning applied to the marginal job in (1.32) holds for all jobs whose productivity lie between \(\epsilon_d^*\) and \(\epsilon_d\). On impact there is an increase in idle jobs that wait for firing permission. As the matching of new vacancies and unemployment take time when there is a rise in \(p\), so the shedding of new idle jobs is time consuming when there is a fall in \(p\). In a world with firing restrictions, *job destruction not only is costly, but also time consuming* and, as will be clear from the simulations in section (1.7), depending on the value of \(s\), its dynamic behaviour may well be symmetric to the dynamic behaviour of job creation. In structural terms, the hiring and firing technologies are governed by the parameters \(s, c, p - b\) and by the matching function \(q\). On the one hand the flow cost \(c\) in the hiring technology plays a role similar to the operational
profits $p + c - b$ (generally negative) in the firing technology. As new vacancies have to pay the flow cost $c$, so an idle job must suffer operational losses when it waits for the firing permission. On the other hand, the average waiting time $1/s$ plays a similar role to the matching function $q$, in the sense that they both act as a stochastic filter and introduce a time lag between the moment in which the firms takes a decision (to post a vacancy or to accept firing permissions) and the moment in which a job-worker pair matches or separates.

The analysis so far has concentrated only on the direct effect of $s$ on the marginal productivity (1.31) and (1.33). In general, the presence of firing restrictions affect the marginal productivity in (1.31) and (1.33) also through its indirect effect on the average value of the jobs. For completeness of exposition, the expected value of a job in recession is

$$E[J] = \frac{\sigma}{r + \lambda} \int_{c_d}^{c_u} (1 - F(x))dx - \frac{\sigma(r + \lambda + 2\mu)}{(r + \lambda + \mu)(r + \lambda + s) + \mu(r + \lambda)}$$

$$\int_{c_d}^{c_u} F(x)dx - \frac{\sigma}{r + \lambda + s} \int_{c_l}^{c_u} F(x)dx. \quad (1.34)$$

Similarly, the expected value of a job during a boom reads

$$E[J^*] = \frac{\sigma}{r + \lambda} \int_{c_d}^{c_u} (1 - F(x))dx + \frac{\sigma(r + \lambda + 2\mu + s)}{(r + \lambda + 2\mu)(r + \lambda) + s(r + \lambda + \mu)}$$

$$\int_{c_d}^{c_u} F(x)dx - \frac{\sigma}{r + \lambda + s} \int_{c_l}^{c_u} F(x)dx. \quad (1.35)$$

In (1.34) $s$ reduces the average value of a job during recession through the second and third term of (1.34). When $s \to \infty$ the second and the third term vanishes, a job in recession is never idle and it is operative only in the interval between $(\epsilon_u - \epsilon_d)$.

Similarly, with no firing restrictions, the third term in (1.35) vanishes and the expected value of a job in the boom becomes

$$\lim_{s \to \infty} E[J^*] = \frac{\sigma}{r + \lambda} \int_{c_d}^{c_u} (1 - F(x))dx + \frac{\sigma}{r + \lambda + \mu} \int_{c_d}^{c_u} (1 - F(x))dx.$$
The indirect effects of firing restrictions on the expected value of the jobs are similar to the steady-state effects of firing restrictions described in section (1.4) and have no distinctive dynamic effects.

### 1.6 Job Flow Determination

Both $\epsilon_i$ and $\theta_i$ are forward-looking jumping variables, independent of history. In general, as the aggregate state switches from $x_i$ to $x_j$, both $\epsilon_i$ and $\theta_i$ will jump, on the impact, to their new values $\epsilon_j$ and $\theta_j$. On the contrary, employment is a sticky variable and to implement the model we need to specify its dynamic behaviour at discrete time $t = 1, \ldots, n$. For this purpose it is necessary to keep track of the entire distribution of employment at each reservation productivity. If $N_t$ is a measure of employment at time $t$, then $N_t = I_t + O_t$, where $I_t$ indicates the idle jobs waiting for firing permission and $O_t$ are the operational jobs that will ignore the arrival of firing permission. Following Mortensen (1994), we assume that the aggregate shock is completely revealed at the beginning of each period. In the time interval between $t$ and $t+1$, $\epsilon_d(x_t)$ and $\theta_t(x_t)$ are state variables determined at the beginning of time $t$ and constant throughout. If $O_t(\epsilon)$ is a measure of operational jobs at idiosyncratic productivity $\epsilon$ its law of motion is

$$O_{t+1}(\epsilon) = (1 - \lambda)O_t + \lambda F'(\epsilon)(I_t + O_t), \quad \epsilon_d(x_t) < \epsilon < \epsilon_u,$$

while the law of motion of idle jobs $I_t(\epsilon)$ is

$$I_{t+1}(\epsilon) = (1 - \lambda)I_t + \lambda F'(\epsilon)(I_t + O_t) - sI_t(\epsilon), \quad \epsilon_i(x_t) \leq \epsilon \leq \epsilon_d(x_t)$$

where the difference between (1.36) and (1.37) is that an idle job is destroyed if firing permission arrives. From the laws of motion (1.36) and (1.37) I can calculate job flows between $t$ and $t+1$. The empirical definition of job creation is the sum of all positive employment changes in a given period. Since, in the model, only the unemployed people are actively searching, job creation between
If the matching function is log-linear with matching elasticity of unemployment $\alpha$, $q(\theta_t)\theta_t$ from the job creation condition (1.27) is

$$q(\theta_t)\theta_t = kJ_t(\epsilon_u)^{1-\alpha},$$

(1.39)

where $k$ is a scale parameter and $J_t(\epsilon_u)$ is the time $t$ value of the job at the upper support of the distribution obtained from (1.29). Similarly, the empirical definition of job destruction is the sum (in absolute value) of all negative employment changes. Endogenously, negative employment change comes from those idle jobs that get firing permission. If we then assume that there is an exogenous turnover rate of $\delta$, the job destruction is

$$JD_t = sI_t + \delta N_t,$$

(1.40)

where $sI_t$ is job destruction via the firing permission and $\delta N_t$ is job destruction by natural turnover $^9$.

To evaluate (1.40) we have to keep track over time of the switch in the composition of employment between operational and idle jobs. If we define $I_{in,ft}$ as the inflow into the idle state at time $t$, it follows that:

$$I_{in,ft} = \lambda F(\epsilon_d(x_t))(N_t - I_t) + \phi_3 \int_{\epsilon_d(x_{t-1})}^{\epsilon_d(x_t)} O(z)dz,$$

(1.41)

where $\phi_3$ is an indicator function taking value 1 if $\epsilon_d(x_t) \geq \epsilon_d(x_{t-1})$. Jobs flow into the idle state for two reasons: either an idiosyncratic shock below the current reservation productivity hits the job or the aggregate state worsens and makes idle all jobs whose productivity lies between the two values. Vice-versa, if we

$^9$In the asset equations describing the value of a match (1.1) and (1.23), the presence of the natural turnover works exactly as the interest rate $r$ and, for simplicity, it has been so far neglected.
define $I_{\text{out}}$ as the outflow from the idle state between $t$ and $t+1$:

$$I_{\text{out}} = (\lambda(1 - F(\epsilon_d(x_t))) + s) I_t + \Phi_4 \int_{\epsilon_d(x_{t-1})}^{\epsilon_d(x_t)} I_t(z) dz,$$  \hspace{1cm} (1.42)

where $\Phi_4$ is an indicator function that takes value 1 if $\epsilon_d(x_t) < \epsilon_d(x_{t-1})$. Jobs leave the idle state for three reasons: a positive idiosyncratic shock makes jobs fully operational, firing permission arrives or a positive aggregate shock makes all jobs between the two reservation productivities fully operational. Given the flows (1.38)-(1.42), obviously:

$$N_{t+1} = JC_t - JD_t + N_t,$$  \hspace{1cm} (1.43)

and

$$I_{t+1} = I_{\text{infl}} - I_{\text{out}} + I_t.$$  \hspace{1cm} (1.44)

Let us consider a positive aggregate shock that switches the system from $x_i$ to $x_j$ and assume that $\theta_j > \theta_i$ and $\epsilon_j < \epsilon_i$. The intuition goes as follows. In (1.38) job creation increases as new vacancies are matched to unemployed through the matching function $q$. On impact, from $I_{\text{out}}$, the outflow from the idle state jumps since all idle jobs whose productivity lay between the two reservation productivities are now fully operational. The number of idle workers jumps downward and, for a given arrival rate $s$, job destruction next period will fall. The process continues as long as a new steady state with job creation equal to job destruction is reached or a new aggregate shock arrives. Let us now repeat the experiment for a negative aggregate shock from $x_j$ to $x_k$, and assume that $\theta_k < \theta_j$ and $\epsilon_k > \epsilon_j$. Job creation falls as the number of vacancies opened falls, while impact, the number of idle jobs jumps upward, since all operational jobs whose productivity lies between the two reservation productivities are now idle. For a given $s$ in (1.40), job destruction next period will increase. Intuitively, in a boom (recession) job creation rises (falls) and job destruction falls (rises). Furthermore, the response of the two flows following a state switch should be symmetric. As new vacancies
have to be matched to unemployed workers as conditions improve, so new idle workers need firing permission to destroy the job. The next section simulates the solution of the general model for different values of firing permission $s$.

### 1.7 Model Simulation

To implement the general stochastic model of the previous sections it is necessary to specify the process for $\{x_t\}$. The Markov chain is determined by the state space of $x_t$, $\chi$, and the transition probability matrix $\Pi$. In this section, as in Christiano (1990), we adopt the following three state model:

$$
\Pi = \begin{bmatrix}
\phi & \gamma & 1 - \phi - \gamma \\
\Psi & 1 - 2\Psi & \Psi \\
1 - \phi - \gamma & \gamma & \phi
\end{bmatrix},
$$

and

$$
\chi = \begin{cases}
-x \\
0 \\
x
\end{cases}.
$$

The Wold representation corresponding to this Markov chain is

$$
x_t = \rho x_{t-1} + e_t, \quad (1.45)
$$

where $e_t$ is mean 0 with variance $\sigma^2$ and is uncorrelated with $x_{t-1}$. Furthermore,

$$
\rho = 2\phi + \gamma - 1; \quad \kappa = 1 + .5\gamma/\Psi, \quad (1.46)
$$

where $\kappa$ is kurtosis, and

$$
\text{var}(x_t) = \frac{\chi^2}{\kappa}; \quad \sigma^2 = \text{var}(x_t)(1 - \rho^2). \quad (1.47)
$$

To determine this model, values must be assigned to four parameters, $\phi, \gamma, \Psi$ and $x$. The simulations in this section follow the lines of a recent paper by Millard and Mortensen (1994), who calibrate the MP model for the U.S. and U.K. economies under the assumption that the only difference between the two countries lies in the policy parameters and in the workers’ bargaining strength.
Thus, the baseline parameters values should be taken as representative of every country, independent of its labour market policies. In this direction the United States turns out to be the country with insignificant firing restrictions and I use parameter values very similar to MP (1993), where they solve the MP (1994) model without explicitly considering wage bargaining. As to the Markov chain parameters, I set $\phi = 0.933$, $\gamma = 0.067$ and $\Psi = 0.017$. As to the aggregate shock I let $x$ be 0.008. These parameters imply a value of $\rho = 0.933$ - slightly less than what is generally used in the real business cycle literature - and a value of $\kappa = 2.97$, so as to approximate the kurtosis of the normal distribution. The productivity distribution is uniform over the interval $[-1,1]$, the arrival rate of the idiosyncratic shock $\lambda$ is set to 0.081, while $\sigma$, the dispersion of the productivity distribution is set to 0.037. The matching function, as in (1.39) is log linear with matching elasticity of unemployment $\alpha$ equal to 0.25 and coefficient $k$ equal to 1. The real interest is set to 0.02. These parameter values are very similar to those chosen by MP (1993) and they are summarised in Table 1.4.

The most problematic parameter to set is $s$. From the Beveridge curve for the steady state model (1.19) it is clear that, given a value of $\lambda$ equal to 0.08, a value of $s$ of the order of 1 should not affect too much equilibrium unemployment directly. Numerical solutions show that a value of $s$ of 1.2 implies an average equilibrium unemployment of 5.8% 10. The corresponding correlation between job reallocation and net employment changes is, on average, $-0.5$, the relative variance of job destruction and creation is 4 while the coefficients of variation are respectively 0.7 for job destruction and 0.4 for job creation. These values are in line with the statistics of job flows in United States, Canada and Britain reported in Tables (1.1) and (1.2) and I take them to be representative of an economy with insignificant firing restrictions.

---

10All the simulations of this paper have been obtained with a Gauss programme written by the author. The programme is available under request.
Table 1.4: Baseline Parameter Values

<table>
<thead>
<tr>
<th>Variables</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matching Elasticity</td>
<td>$\alpha$</td>
<td>0.250</td>
</tr>
<tr>
<td>friction parameter</td>
<td>$k$</td>
<td>5</td>
</tr>
<tr>
<td>net common price</td>
<td>$x_1$</td>
<td>0.008</td>
</tr>
<tr>
<td>net common price</td>
<td>$x_2$</td>
<td>0.0</td>
</tr>
<tr>
<td>net common price</td>
<td>$x_3$</td>
<td>-0.008</td>
</tr>
<tr>
<td>interest rate</td>
<td>$r$</td>
<td>0.020</td>
</tr>
<tr>
<td>natural turnover</td>
<td>$\delta$</td>
<td>0.020</td>
</tr>
<tr>
<td>idiosyncratic shock rate</td>
<td>$\lambda$</td>
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</tr>
<tr>
<td>price dispersion</td>
<td>$\sigma$</td>
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</tr>
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<td>price distribution</td>
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<td>$\epsilon_u$</td>
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</tr>
<tr>
<td>lower support</td>
<td>$\epsilon_{lo}$</td>
<td>-1</td>
</tr>
<tr>
<td>Markov chain probability</td>
<td>$\phi$</td>
<td>0.933</td>
</tr>
<tr>
<td>Markov chain probability</td>
<td>$\gamma$</td>
<td>0.017</td>
</tr>
<tr>
<td>Markov chain probability</td>
<td>$\Psi$</td>
<td>0.067</td>
</tr>
<tr>
<td>firing restrictions (max)</td>
<td>$s$</td>
<td>1.20</td>
</tr>
<tr>
<td>firing restrictions (min)</td>
<td>$s$</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Source: Christiano (1990), Mortensen (1994), Mortensen Pissarides (1993) and author calculations.

Summary statistics for time series simulations are summarised in Table (1.5). For different values of firing restrictions I simulated 150 time series of 64 periods each. Job creation is pro-cyclical (correlation $Jc$-Net) and job destruction is counter cyclical (corr. $Jd$-Net) for different values of $s$ ranging from 1.2 to 0.2. As firing restrictions increase, both job creation and destruction fall, and, from the range of values of $s$ chosen in Table (1.5), equilibrium unemployment is approximately constant. This result is similar to Mortensen and Millard (1994), who find that firing costs are responsible for less than 1 percent of the UK unemployment. The effect of linear firing costs on labour demand in a partial equilibrium model have recently been analysed by Bentolila and Saint-Paul (1994). They argue that labour demand is likely to increase only if firing restrictions are sufficiently
Table 1.5: Simulation Statistics

<table>
<thead>
<tr>
<th>s = 1.20</th>
<th>s = 1.00</th>
<th>s = 0.80</th>
<th>s = 0.60</th>
<th>s = 0.40</th>
<th>s = 0.20</th>
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</thead>
<tbody>
<tr>
<td>corr. JC-NET</td>
<td>0.617</td>
<td>0.607</td>
<td>0.592</td>
<td>0.598</td>
<td>0.644</td>
</tr>
<tr>
<td>corr. JD-NET</td>
<td>-0.814</td>
<td>-0.735</td>
<td>-0.704</td>
<td>-0.607</td>
<td>-0.592</td>
</tr>
<tr>
<td>corr. JC-JD</td>
<td>-0.157</td>
<td>-0.018</td>
<td>0.078</td>
<td>0.200</td>
<td>0.190</td>
</tr>
<tr>
<td>corr. JR-NET</td>
<td>-0.502</td>
<td>-0.367</td>
<td>-0.267</td>
<td>-0.120</td>
<td>-0.007</td>
</tr>
<tr>
<td>$\sigma_{JD}^2/\sigma_{JC}^2$</td>
<td>4.051</td>
<td>2.422</td>
<td>1.676</td>
<td>1.030</td>
<td>0.594</td>
</tr>
<tr>
<td>$\sigma_{JC}/\bar{JC}$</td>
<td>0.377</td>
<td>0.344</td>
<td>0.328</td>
<td>0.315</td>
<td>0.281</td>
</tr>
<tr>
<td>$\sigma_{JD}/\bar{JD}$</td>
<td>0.750</td>
<td>0.588</td>
<td>0.489</td>
<td>0.395</td>
<td>0.301</td>
</tr>
<tr>
<td>$\bar{JC}$ rate</td>
<td>2.776</td>
<td>2.728</td>
<td>2.720</td>
<td>2.634</td>
<td>2.598</td>
</tr>
<tr>
<td>Unem.</td>
<td>0.058</td>
<td>0.058</td>
<td>0.058</td>
<td>0.057</td>
<td>0.059</td>
</tr>
<tr>
<td>Dur. Unem.</td>
<td>2.035</td>
<td>2.046</td>
<td>2.063</td>
<td>2.091</td>
<td>2.218</td>
</tr>
<tr>
<td>Idle</td>
<td>0.022</td>
<td>0.026</td>
<td>0.033</td>
<td>0.042</td>
<td>0.063</td>
</tr>
<tr>
<td>pers. unem.</td>
<td>0.610</td>
<td>0.731</td>
<td>0.793</td>
<td>0.881</td>
<td>0.905</td>
</tr>
</tbody>
</table>

high. In this respect the results in Table (1.5) seem to confirm the Bentolila and Saint-Paul finding, even though their model is a partial equilibrium model.

The most important result in Table (1.5), in line with the spirit of Bertola (1990) is that firing restrictions, albeit not responsible for lower employment levels, dramatically affect labour market dynamics, through their effect on the relative volatility of job creation and job destruction. The relative variance of job destruction to job creation $\frac{\sigma_{JD}^2}{\sigma_{JC}^2}$ falls dramatically as firing restrictions increase. To falls in the relative variance correspond differences in the cyclical where behaviour of job reallocation, which rises from negative to positive values. Overall, as the average waiting time for firing restrictions increases, the statistics in Table (1.5) replicate the dynamics of economies with serious or fundamental firing restrictions, as indicated in Table (1.1) and (1.2).

A controversial result of Table (1.5) is the correlation between job creation
and job destruction, which is not as negative as the one found in Table (1.1) and (1.2). This result is due to a too strong responsiveness of job creation to unemployment. Similar correlation is found in MP (1993). Mortensen (1994) introduces voluntary quits and obtains a negative correlation between job creation and job destruction. Other statistics of Table (1.5) show that as firing restrictions increase the average duration of unemployment increases. Firing restrictions obviously have a strong effect on average idle capacity, the average fraction of jobs waiting for firing permission. Finally Table (1.5) agrees with the econometric evidence on the effect of firing costs on the persistence of employment (Alogoskoufis and Manning, 1988). Employment adjusts more slowly with relatively high firing costs.

1.8 Conclusions

This paper has taken seriously the recent empirical studies on job creation and destruction collected by the OECD (1994a). Focusing on the cyclical properties of these flows, job creation is pro-cyclical while job destruction is counter cyclical. Huge differences exist in the relative volatility of the two flows. This paper has offered a model that for different values of firing restrictions, implies both facts.

When firing permissions are continuously available, job destruction is instantaneous while job creation takes time and job reallocation moves counter cyclically. As firing is restricted to be costly and time consuming, the asymmetry between job flows disappears and job reallocation is uncorrelated with net employment changes. This paper has argued that this mechanism is behind the cross-country variation in the cyclical behaviour of job flows.

Another implication of the model is that reasonable firing restrictions do not imply higher equilibrium unemployment, but they reduce both job creation and job destruction. Since these flows are equal in equilibrium, the effect on unemployment is ambiguous. It has already been recognized (Bertola 1990; Ben-
tolila and Bertola 1990) that higher firing costs do not bias downward average employment, but they significantly affect labour market dynamics. In this paper, we have shown a way in which firing restrictions affect the volatility of job destruction and creation, an aspect of employment dynamics.

Several directions should be taken from this paper. Firstly, following Burda and Wyplosz (1994) for Europe and Mortensen (1994) for the U.S., it is necessary to investigate the effect of firing restrictions on worker flows. Secondly, Hopenhayn and Rogerson (1993) simulate a general equilibrium model with simultaneous job creation and destruction and they find that a tax on job destruction has a sizable negative impact on total employment. With respect to the approach of this paper, Hopenhayn and Rogerson explicitly consider the effects of firing restrictions on labour supply decisions, completely neglected in this paper. Future research should try to model labour supply in a dynamic matching models. The last direction of research would address the question of the optimal level of job reallocation and the relationship between job reallocation and economic performance.
Appendix: Firing Tax in Steady State

In this section I model the behaviour of the firm under the assumption that firing costs take the form of a simple fixed firing tax of $-F$ to be incurred when separation takes place. The model is in the spirit of Mortensen and Millard (1994). In this case, the forward looking asset equation for a job at productivity $\epsilon$ reads

$$rJ(\epsilon) = p + \sigma\epsilon - b + \lambda (E(J) - J(\epsilon)).$$  \hspace{1cm} (I)

A firm hit by a negative idiosyncratic shock will keep running a job as long as its marginal value is greater than the fixed firing cost $-F$. Under this rule the marginal productivity solves

$$J(\epsilon_d^F) = -F,$$  \hspace{1cm} (II)

where $\epsilon_d^F$ is the marginal productivity under the fixed firing rule. The average value of the job is

$$E[J] = \int_{\epsilon_l}^{\epsilon_u} J(x)dF(x) + \int_{\epsilon_d^F}^{\epsilon_u} J(x)dF(x),$$  \hspace{1cm} (III)

where

$$\int_{\epsilon_d^F}^{\epsilon_u} J(x)dF(x) = J(\epsilon_u) - J(\epsilon_d^F)F(\epsilon_d) - \frac{\sigma}{r + \lambda} \int_{\epsilon_d}^{\epsilon_u} F(x)dx.$$  \hspace{1cm} (IV)

Making use of (II), the value of a job at the upper support of the distribution reads

$$J(\epsilon_u) = -F + \frac{\sigma}{r + \lambda} \int_{\epsilon_d}^{\epsilon_u} dx,$$

and substituting this expression into (IV), the average value of a job reads

$$E[J] = \frac{\sigma}{r + \lambda} \int_{\epsilon_d}^{\epsilon_u} (1 - F(x)) dx - F.$$  \hspace{1cm} (V)

From (V) it is clear that, given the reservation productivity, firing costs reduce the average value of the job.

Making use of (II) and (V), the reservation productivity solves

$$p + \sigma\epsilon_d^F - b = -\frac{\lambda\sigma}{r + \lambda} \int_{\epsilon_d}^{\epsilon_u} (1 - F(x)) dx - rF.$$  \hspace{1cm} (VI)
The right hand side of (VI) is negative and there is labour hoarding in the firm’s optimal policy. Differentiating (VI) with respect to \( F \) yields

\[
\frac{\sigma}{r + \lambda \frac{\partial \epsilon_0}{\partial F}} = -r. \tag{VII}
\]

Thus higher firing costs reduce the reservation productivity and induce the firm to hold on to less profitable jobs.

Firms post vacancies and, conditional upon finding an unemployed worker, they create a job at the upper support of the distribution \( J(\epsilon_u) \). Free entry in equilibrium implies that

\[
J(\epsilon_u) = \frac{c}{q(\theta)}. \tag{VIII}
\]

In order to solve (VIII) for \( \theta \) we need an expression for \( J(\epsilon_u) \), the value of a job at the upper support of the distribution. If we evaluate (I) at \( \epsilon_u \) and subtract it from (VI), \( J(\epsilon_u) \) reads

\[
J(\epsilon_u) = \frac{\sigma(\epsilon_u - \epsilon_0^F)}{r + \lambda} - F. \tag{IX}
\]

Differentiating (VIII) with respect to \( F \), making use of (IX) and (VII), yields

\[
\frac{-\lambda F(\epsilon_0^F)}{r + \lambda F(\epsilon_0^F)} = -\frac{c q(\theta)}{q(\theta)^2} \frac{\partial \theta}{\partial F}. \tag{X}
\]

Since \( q(\theta) < 0 \), job creation falls with the increase in the firing tax. Thus higher firing taxes reduce the job finding rate \( \theta q(\theta) \).

Job creation is \( \theta q(\theta)u \), and job destruction is \( \lambda F(\epsilon_d)(1 - u) \) and unemployment is obtained as a solution to

\[
\dot{u} = \lambda F(\epsilon_d^F)(1 - u) - \theta q(\theta)u. \tag{XI}
\]

Unemployment is constant when job creation equals job destruction and reads

\[
u = \frac{\lambda F(\epsilon_d^F)}{\lambda F(\epsilon_d^F) + \theta q(\theta)}. \tag{XII}
\]

Since higher firing costs affect both the job creation and job destruction decision, they have an ambiguous impact on equilibrium unemployment. Differentiating
(XII) with respect to $F$, it is clear that the overall effect depends on the parameters of the model and the distribution of productivity $F(c^F_d)$. Total job reallocation is $2\lambda F(c^F_d)(1 - u)$ and the derivative with respect to $F$ depends on the ambiguous effect of firing costs on unemployment. Nevertheless, if we define job reallocation as the sum, in absolute value, of employment changes, over total employment, it follows that

$$JR = \lambda F(c^F_d).$$

Differentiating (XIII) with respect to $F$, and making use of (VII), higher firing costs unambiguously reduce the job reallocation rate.

**B Appendix: Job Destruction, Firing Tax and Cyclical Shocks**

In this section I extend the model of the previous section to allow the aggregate productivity $p$ to fluctuate stochastically between an high value $p^*$ and a low value $p$, and I indicate with $\mu$ the switching probability between the two values. The model is in the spirit of Millard (1994). If we indicate with $e^F_d$ and $e^{F*}_d$, the reservation productivity in bad and good times, the value of a job in recession, $J(e)$, for $e \geq e^F_d$ solves

$$(r + \lambda + \mu)J(e) = p + \sigma e - b + \lambda E(J) + \mu J^*(e).$$

Similarly, a value of a job during a boom, $J^*(e)$, for $e \geq e^{F*}_d$ solves

$$(r + \lambda + \mu)J^*(e) = p^* + \sigma e - b + \lambda E(J^*) + \mu J(e).$$

Finally, for $e^{F*}_d < e < e^{F}_d$, the value of a job during a boom solves

$$(r + \lambda + \mu)J^*(e) = p^* + \sigma e - b + \lambda E(J^*) - \mu F.$$}

Proceeding in the same way as in section (1.5), the expected value of a job during recession is

$$E(J) = \frac{\sigma}{r + \lambda} \int_{e^F_d}^{e^*} (1 - F(x))dx - F,$$
and the expected value of a job during the boom is

\[ E(J^*) = \frac{\sigma}{r + \lambda} \int_{\epsilon_d^*}^{\epsilon_u} (1 - F(x))dx + \frac{\sigma}{r + \lambda + \mu} \int_{\epsilon_d^*}^{\epsilon_u} (1 - F(x))dx - rF. \]  

(XVIII)

Using (XVII) into (XIV) and evaluating (XIV) at \( J(\epsilon_d) = -F \), yields

\[ p + \sigma \epsilon_d - b = -\frac{\lambda \sigma}{r + \lambda} \int_{\epsilon_d^*}^{\epsilon_u} (1 - F(x))dx - \frac{\mu \sigma (\epsilon_d^* - \epsilon_d)}{r + \lambda + \mu} - rF. \]  

(XIX)

Proceeding similarly for \( J(\epsilon_d^{F*}) = -F \) yields

\[ p + \sigma \epsilon_d^{F*} - b = -\frac{\lambda \sigma}{r + \lambda} \int_{\epsilon_d^{F*}}^{\epsilon_u} (1 - F(x))dx - \frac{\lambda \sigma}{r + \lambda + \mu} \int_{\epsilon_d^{F*}}^{\epsilon_u} (1 - F(x))dx - rF. \]  

(XX)

Proceeding as in section (1.5) it is possible to obtain market tightness in boom \( \theta^* \) and in recession \( \theta \). Consider first what happens when the aggregate productivity switches from \( p \) to \( p^* \). On the one hand, firms open up more vacancies, on the other hand firms hold on to more existing jobs (\( \epsilon_d^{F*} < \epsilon_d^* \)). On impact neither job destruction nor job creation jumps, since new vacancies take time to be matched to unemployed workers. As matching takes place, the fall in unemployment induces a fall in job creation and an increase in job creation until there is convergence to a new steady-state, or until there is a new cyclical shock.

When aggregate productivity falls from \( p^* \) to \( p \) the dynamics of job creation follows an opposite pattern to the one after an increase in \( p \): vacancies fall on impact, but job creation takes time to fall. Conversely, since all jobs whose productivity lies between \( \epsilon_d^{F*} \) and \( \epsilon_d^* \) will immediately be destroyed, there will be an immediate spike in job destruction. This increase in job destruction has no counterpart in the behaviour of job destruction when \( p \) increases, or in the behaviour of job creation during net expansion. Thus, with a fixed firing cost rule, the variance in job destruction is bound to be higher than the variance in job creation, exactly as in the MP (1994) model.
Chapter 2

Job Reallocation and Labour Market Policy

2.1 Introduction

The popularity of the notion of "labour market flexibility" in the policy debate in Europe and the interest in sectoral reallocations as a source of the business cycle in the U.S., have led to the accumulation of statistical information on job reallocations in several OECD countries. The manufacturing data gathered by Davis and Haltiwanger in the United States have been particularly influential. They found that a large number of jobs close down each quarter and an equally large number open up, apparently for specific reasons unrelated to sector or economy-wide performance. When the OECD (1994a) compiled comparable data for several of its members it found that the U.S. experience was by no means exceptional, though it also found that job reallocations elsewhere were on average not as frequent.

Concurrently with the collection of data for the OECD, a number of authors have developed theoretical models to explain the processes of job creation and job destruction. A natural way to think about job creation is in terms of the matching of job seekers with hiring firms, along the lines of the equilibrium search literature. The search literature, however, had only a rudimentary discussion of
job destruction and several suggestions have been put forward about the factors underlying the destruction process. In the analysis of Mortensen and Pissarides (1994) jobs differ according to productivity and job destruction takes place when the productivity of a job, following a shock, drops below a reservation value. Thus job destruction in their model follows the same principles as job creation in more conventional matching models with match productivity differentials. In both cases jobs are independent "islands" that are subjected to both idiosyncratic and common shocks and the key variable that determines whether they are active or not is a unique reservation productivity.

Our interest in this paper is to draw together the international data compiled by the OECD (1994a) with a view to understanding the role of labour market policy in the determination of job reallocations. Of course, we have no strong theoretical reasons for supposing that more job reallocation is better than less, or vice versa. We also do not have evidence yet of a firm relationship between overall unemployment and job reallocation. But we present evidence that low job reallocation is associated with more long-term unemployment. Since the latter is bad, in terms of the loss of skill of the unemployed and the disenfranchisement of those who suffer it, the supposition is that policies that restrict job reallocation are not good for the ability of the market to turn over its unemployment stock quickly.

In section (2.2) we give some definitions and briefly describe the job reallocation data. In section (2.3) we discuss the relation between unemployment and job reallocation in the context of the flow approach to labour markets. Finally, in section (2.4), we look at the relation between labour market policy and job reallocations, with the help of simple figures for ten OECD countries with comparable data.
2.2 Preliminaries

Gross job reallocation is normally defined as the sum of the absolute value of the change in employment in each unit in the sample (normally an establishment but sometimes a company) expressed as a proportion of total employment. More specifically, the job creation \((JC)\) rate is defined as the sum of all increases in employment expressed as a proportion of total employment, and job destruction as the sum of all decreases in employment, again expressed as a proportion of total employment.

Note that because in each case we are dividing by total employment, not just employment in either expanding or contracting establishments, the figure obtained for \(JC\) is not the average expansion rate of expanding establishments and the one for \(JD\) is not the average contraction rate of contracting establishments. If, say, exactly half of establishments expanded, then to find the average expansion rate, \(JC\) has to be doubled.

The difference between \(JC\) and \(JD\), gives the rate of net employment change. Their sum gives the gross job reallocation rate. Because of our definitions, the gross reallocation rate is the average change (positive or negative) experienced by the typical establishment in the sample, expressed as a proportion of mean employment for each establishment. To express it as a proportion of beginning-of-period employment, one can use the transformation \(2(JC + JD)/(2 - JC - JD)\), so if, say, gross reallocation on our definition was 0.2, using beginning-of-period employment would make it 0.22.

Our analysis in this paper compares average job reallocation rates for the OECD countries that have comparable data in order to say something about the role of labour market policy. Since job reallocation rates are highly sensitive to the phase of the cycle that the economy is in, for the comparison to be meaningful the economies have to be either in about the same phase of the cycle over the
sample period or the period has to be long enough to average across cycles. Our sample period is for 1982-89 when the economies covered were coming out of recession and productivity growth was positive. The only exception amongst the OECD countries with comparable data was New Zealand, which experienced a deep recession with large negative productivity growth during this period. We decided to drop New Zealand from the sample. We also decided to make no effort to bring Japan into the sample (the only major OECD economy left out) because its peculiar job tenure arrangements make it difficult to compare its job reallocation rate with that of other OECD countries.

Table (2.1) gives the job reallocation rates for the ten countries in our sample. It also splits job reallocation according to whether the reallocation of jobs was due to contraction or expansion of existing establishments (continuing establishments) or to new entry or exit. The reason for the split is that much of what we shall have to say about policy relates more to large established units rather than to small new ones. Another reason that one might want to split the sample is that the theory of job creation and job destruction as it applies to continuing

<table>
<thead>
<tr>
<th>Country</th>
<th>Notation</th>
<th>Total Job Reallocation</th>
<th>Continuing Establishments</th>
<th>Entry and Exit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>B</td>
<td>14.4</td>
<td>8.9</td>
<td>5.5</td>
</tr>
<tr>
<td>Canada</td>
<td>C</td>
<td>26.3</td>
<td>20</td>
<td>6.3</td>
</tr>
<tr>
<td>Denmark</td>
<td>DK</td>
<td>29.8</td>
<td>18.7</td>
<td>11.1</td>
</tr>
<tr>
<td>Finland</td>
<td>FL</td>
<td>22.4</td>
<td>15.1</td>
<td>7.3</td>
</tr>
<tr>
<td>France</td>
<td>F</td>
<td>27.1</td>
<td>12.9</td>
<td>14.2</td>
</tr>
<tr>
<td>Germany</td>
<td>G</td>
<td>16.5</td>
<td>12.1</td>
<td>4.4</td>
</tr>
<tr>
<td>Italy</td>
<td>I</td>
<td>23.4</td>
<td>15.7</td>
<td>7.7</td>
</tr>
<tr>
<td>Sweden</td>
<td>S</td>
<td>29.1</td>
<td>17.6</td>
<td>11.5</td>
</tr>
<tr>
<td>U.K.</td>
<td>UK</td>
<td>15.3</td>
<td>8.7</td>
<td>6.6</td>
</tr>
<tr>
<td>U.S.A.</td>
<td>US</td>
<td>24.6</td>
<td>18.9</td>
<td>5.7</td>
</tr>
</tbody>
</table>

Source: OECD (1994a) and Appendix.
establishments is often different from the one that applies to entry and exit.

The Table shows that gross job reallocation rates across the ten OECD countries range from low of about 14 percent for Belgium to a high of 29 percent for Sweden. When entry and exit are removed the range drops to about 9 percent for the UK and Belgium to 20 percent for Canada. The new entry and exit figures show some peculiarities, with France having approximately twice as high a figure as the rest of the sample, with the exception of Sweden. In contrast, when only continuing establishment are considered, France and Germany have broadly a similar figure for the low reallocation countries of Europe (UK and Belgium) and high reallocation countries of North America. The North American countries do emerge as countries with more reallocation, as conventional wisdom would lead us to believe, but not by much when compared, for example to Sweden (which might have a high reallocation rate because of its limited-duration job subsidization programmes).

If there are large net changes in employment in the sample, the conventional definition of gross job reallocation can give rise to some peculiarities. For example, imagine a situation where no establishment in the sample expands but all establishments contract by 5 percent. Then, the gross job reallocation rate will be 5 percent, though there has been no job reallocation within the sample. Contrast this with a situation where 3 percent of workers leave from half the establishments and get jobs with the other half. In the latter case the gross job reallocation rate will be 3 percent, lower than in the former case, though in the latter there has been a genuine reallocation of 3 percent of the jobs.

For this reason, a more satisfactory definition of job reallocation is what is often called the “excess” job reallocation, defined as the average of gross reallocation minus net employment change for each year in the sample (or, alternatively, as twice the average of either \(JC\) or \(JD\) whichever is the smaller). Unfortunately we do not have enough data for the countries in our sample to compute the net
Table 2.2: Gross and Excess Job Reallocation

<table>
<thead>
<tr>
<th>Country</th>
<th>Source</th>
<th>JR</th>
<th>Excess</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>Boeri and Cramer (1993)</td>
<td>15.94</td>
<td>14.23</td>
</tr>
<tr>
<td>U.K.</td>
<td>Konings (1995a)</td>
<td>7.18</td>
<td>2.81</td>
</tr>
<tr>
<td>U.S.</td>
<td>Davis and Haltiwanger (1992)</td>
<td>20.43</td>
<td>15.62</td>
</tr>
<tr>
<td>Canada</td>
<td>Baldwin et al. (1994)</td>
<td>20.52</td>
<td>17.77</td>
</tr>
<tr>
<td>Italy</td>
<td>Contini et al. (1992)</td>
<td>23.06</td>
<td>21.96</td>
</tr>
</tbody>
</table>

reallocation rate. For the five countries that we have data, the relation between gross and excess reallocation turns out to be linear with positive intercept and slope less than one, and correlation coefficient 0.97. Table (2.2) gives the gross and excess reallocation rates for the five countries.

2.3 Unemployment and Job Reallocation

A number of different and often contradictory views about the relation between job reallocation and unemployment have been expressed. The current interest in job reallocation has been partly stimulated by the interest in the “sectoral shifts hypothesis”, especially in the United States. This is the view, first put forward by Lillien (1982), that the business cycle in the United States is largely driven by reallocation shocks; that is, shocks that shift real demand from some sectors of the economy to other sectors of the economy and which on aggregate might be neutral. A faster pace of reallocation, according to this view, requires more intersectoral labour mobility: if there are inertia to mobility, unemployment results in the contracting sectors that might last sufficiently long to mirror the cyclical persistence of unemployment in the real economy.

Although intense testing of this view has rejected it as the dominant explanation of the business cycle, even in the United States where unemployment persistence is a lot less than it is in Europe, if there is any truth in this hypoth-
thesis we should expect to observe a positive association between unemployment and gross job reallocation. For it the pace of allocative shocks is faster, gross job reallocation rates should be higher at the same time that unemployment is higher.

Contrary to this view, it is often stated that large rates of job reallocation indicate a "flexible" labour market that is better able to adapt to new conditions. By implication, the allocation of resources in a labour market that has more job reallocation should be better and so unemployment should be less.

Unfortunately, neither economic theory nor empirical work is yet in a position to shed light on the relation between gross job reallocation and the allocation of resources in the labour market. For example one question we do not have an answer is whether individuals participating in a market with more job reallocation should expect to find a better quality match during their job search. Future work will undoubtedly shed light on this and other related questions. But in the absence of a theoretical framework that could shed light on the welfare implications of more or less job reallocation, it is difficult to evaluate the welfare effects of policy measures that influence job reallocation.

For this reason we follow here a different route. We outline first a way of thinking about unemployment, derived from the flow approach to the labour market, that shows that there should not necessarily be a relation between gross reallocation and unemployment, though it is unlikely that there should be no relation between gross job reallocation on the one hand and either unemployment or its duration on the other. We then look at our cross-section of OECD countries and discover that there is a relation between the duration of unemployment and gross job reallocation. We draw some tentative conclusions about the process of job search in the labour market and the contribution of job reallocation to it, before we proceed to evaluate the effects of policy measures on job reallocation.

Looking at employment flows first, we follow the empirical literature and
define the rate of job creation \((JC)\) and the rate of job destruction \((JD)\) during a year by

\[
JC = \frac{\text{Number of Jobs created}}{\text{Total Employment}},
\]

and

\[
JD = \frac{\text{Number of Jobs destroyed}}{\text{Total Employment}}.
\]

If there is an exogenous rate of labour force growth of \(n\), employment flows in the steady state have to satisfy,

\[JC - JD = n.\]

Gross job reallocation is conventionally defined as

\[JR = JC + JD.\]

Let us now look at unemployment flows. In the steady state, the mean duration of unemployment is defined as

\[D = \frac{\text{Total unemployment}}{\text{Outflow from unemployment}}.\]

If the rate of unemployment is to remain constant during periods of population growth, the number of unemployed workers has to grow at rate \(n\). Writing total unemployment as \(U\), we therefore have,

\[
\text{Unemployment Inflow} - \text{Unemployment Outflow} = nU.
\]

The unemployment inflow is made up of workers who lost their jobs because of job destruction and of some other workers, mainly those quitting their jobs to enter unemployment and new labour force entrants. We can therefore write the above formula for unemployment in equilibrium in the form,

\[
\text{Job Destruction} + \text{Other inflow} - \text{Outflow} = nU.
\]
Straight forward manipulation of this formula gives,

\[ JD + \frac{\text{other inflow}}{\text{employment}} - \frac{u}{(1-u)D} = \frac{nu}{1-u}, \]

where \( u \) is the rate of unemployment. Since from the definition of job reallocation we know \( JD = (JR - n)/2 \), we can write the above formula in the form,

\[ JR = \frac{2u'}{D} + \text{other terms}, \]

where \( u' \) is the ratio of unemployment to employment and the other terms depend on the rate of labour-force growth and the other inflow into unemployment as a proportion of employment.

This formula shows that there is an equilibrium relation between the rate of unemployment, the gross job reallocation rate and the mean duration of unemployment but this relation depends also on other factors. As an example of what might cause the difference in job reallocation rates across countries, suppose that the two countries have the same unemployment rate, say 8 percent, but one has population growth rate of 2.5 per cent and the other 1.5 per cent. Then, the formula above says that job reallocation in the country with the faster growth should be 2.35 percentage points higher than in the country with the lower growth rate. Country differences in the flow into unemployment other than those caused by job destruction can also produce differences in job reallocation rates at given rate and duration of unemployment. Since (in the absence of reliable data) such differences are likely to be larger than differences in population growth rates, we would expect this factor to be a more important cause of distortion in the relation between unemployment, its duration and the job reallocation rate.

Having noted that, however, it would be surprising if there were no relation between the three variables in what is essentially a formula between five variables, one of which (the labour force growth) is not likely to differ much across the OECD. In an international cross-section, we might well find that all three are
related, or that the two are related and the third is following its own path. But complete independence between unemployment, its duration and the job reallocation rate is unlikely.

In Figures (2.1) and (2.2) we plotted the gross job reallocation rates against an OECD-adjusted definition of unemployment for the ten countries in our sample. There is a small negative correlation, derived from the negative association between gross job reallocation in continuing establishments and unemployment.

There is, however, a stronger correlation between gross job reallocation and the duration of unemployment. Figures 2.3 and 2.4 show that countries with less job reallocation have more long-term unemployment. The relation is again stronger for the job reallocation that is due to continuing establishments than for
Figure 2.2: Job Reallocation by Continuing Firms and Unemployment
the whole economy. As in the case of total unemployment, there is virtually no relation between entry and exit and gross job reallocation.

Thus, in our decomposition shown in the formula above, the correlation appears to be mainly between the gross reallocation rate and the duration of unemployment, with very little correlation between the gross reallocation rate and unemployment. Of course, the formula above does not suggest any explanation for the observed relations. A possible explanation for the correlation between gross job reallocation and long-term unemployment runs along the following lines.

We think of the process that allocate worker to jobs as taking place in a large hiring hall. Workers search for jobs with given intensity, they are prepared to accept jobs on the basis of a variety of reservation wages and firms choose
Figure 2.4: Job Reallocation by Continuing Firms and Long-Term Unemployment
which workers to hire on the basis of the expected productivity of the match and the wage rate. If the job reallocation rate is small, not many new jobs and also not many previously employed workers enter the hiring hall. The unemployed workers have fewer jobs to search but there is also less competition for them, because of the smaller inflow of workers into the hall. Our finding suggest that unemployed are less likely to find a job when the inflow of both job vacancies and job seekers is down. In the absence of the active job matching induced by large job reallocation rates, the unemployed are more likely to enter long-term unemployment.

If this way of looking at the matching process is correct, doubts can be cast on the “insider-outsider” explanation of the persistence of unemployment and on the view that the unemployed cannot compete for jobs with employed for newly-unemployed job seekers. Insider-outsider theory in this context would imply that the already unemployed are not active participants in a matching round generated by entry of new jobs and new workers. This does not appear to be the case. The competition theory (Burgess, 1993) claims that the outflow from unemployment is virtually independent of the number of job vacancies in the market, which is also inconsistent with the view expressed above.

The process described is, however, consistent with a purely random matching game when the number of job vacancies is less than the number of job seekers (or when there are increasing returns to scale in matching) and even more so with the matching ideas recently put forward by Coles (1992). In his model pre-existing unemployed benefit more from newly created job vacancies than from ones that already existed, because they searched some or all of the already existing ones in the past without success.

Since long term unemployment is wasteful in terms of the loss of skill and the disenfranchisement of those who suffer it, the lower long-term associated with higher turnover is one beneficial effect we can identify at this level of analysis.
Another way of looking at the theoretical relations behind the correlations found between unemployment, its duration and job reallocations is to think of the job reallocation rate as largely determined by a vector of variable $X$, the unemployment rate as largely determined by another vector $Y$, and the duration of unemployment determined by both $X$ and $Y$. Such a formulation justifies the observed correlation between the job reallocation rate and long-term unemployment reported here, the correlation between the rate of unemployment and the long-term unemployment previously found by several studies and also the absence of a close correlation between unemployment and gross job reallocation.

The analysis in the next section identifies policy variables that belong to the set $X$, that is, variables that might explain the co-movement between job reallocation and long-term unemployment for given rate of unemployment.

### 2.4 Job Reallocation and Labour Market Policy

We look at three kinds of labour market policy and, rather briefly, at what might be described as industrial policy. The labour market policies that we look at are direct restrictions on the firm’s ability to fire employees, “passive” policy, which we measure by income support to the unemployed, and active “policy”, which we measure by the amount of money spent per unemployed worker on measures designed to speed the transition from unemployment to employment. Industrial policy refers to subsidization of industrial production or employment.

#### 2.4.1 Employment Protection Legislation

We refer to restrictions on the firm’s ability to dismiss employees as “employment protection legislation”. Our measure of such legislation derives from the OECD, where an index is constructed showing the sum of weeks’ notice and weeks’ compensation that has to be given to dismissed employees. In our sample and for the
period of our analysis this index ranged from virtually zero for the United States to 7 for Belgium and Italy.

The obvious link between employment protection legislation and the gross job reallocation rate is that restrictions on dismissals impose a shadow price on the firm, leading to a drop on dismissal. Because the entry into unemployment is as a consequence reduced, there is less exit, that is, less job creation. Alternatively, looking at it from the firm's point of view, a shadow price on dismissal should lead to higher labour costs and so lower demand for labour. Either way, employment protection legislation should lead to less job reallocation. This link, which has featured in the labour demand literature several times (see for example the survey by Nickell, 1986), was also explored more recently in models with explicit job creation and job destruction by Millard and Mortensen (1994) and in chapter (1).

The negative correlation between employment protection legislation and job reallocation is clearly visible in our sample, especially when entry and exit of firms is excluded from the sample (Figure (2.6)). Since restrictions on dismissals apply mainly to large firms, the fact that there is no relation whatsoever between entry and exit on the one hand and employment protection legislation on the other (not shown in the Figures) is not surprising. The simple correlation coefficient between the gross job reallocation rate of continuing establishments and the OECD index of employment protection legislation is $-0.57$.

2.4.2 Passive Policy Measure

Next we consider the relation between unemployment compensation, the main determinant of the generosity of passive policy measures, and gross job reallocation. In the model of Mortensen and Pissarides (1994) more generous unemployment compensation reduces the cost of unemployment and raises the wages of labour. The implication on impact is that there is less job creation and more job destruc-
Figure 2.5: Job Reallocation and Employment Protection Legislation
Figure 2.6: Job Reallocation by Continuing Firms and Empl. Prot. Leg.
tion, i.e. the Beveridge curve shifts out. But the economy eventually settles down to a higher-unemployment equilibrium, where job creation and job destruction are equal to each other. Whether they equalise at higher rates or lower ones, when compared with the previous steady state is not possible to say without knowledge of parameter values (although it should be noted that in simpler versions of the model, when the wage rate is independent of the rate of unemployment, higher unemployment benefit always leads to higher job destruction in the steady state and so to more reallocation). So, although the generosity of the unemployment insurance system unambiguously raises unemployment, it can either reduce or lower gross job reallocation.

In Figures (2.7) and (2.8) we plot gross job reallocation against the summary
Figure 2.8: Job Reallocation by Continuing Firms and Unem. Ins., Overall Index
index for the generosity of the unemployment insurance system in the OECD constructed by Michael Burda (1988). There is a clear negative relation, with the simple correlation coefficient a strong 0.6. Interestingly, even new entry and exit are negatively related to the generosity index, though with a smaller correlation coefficient of 0.39. On closer examination of the relation between gross job reallocation and the two main components of the generosity index, the level of unemployment benefit and the duration of benefit entitlement, an interesting contrast emerges. The relation between the level of benefit and job reallocation is positive, but that between job reallocation and the duration of benefit is strongly negative. Figures (2.9) and (2.10) shows the two relations for continuing establishments.
Figure 2.10: Job Reallocation by Continuing Firms and Benefit Duration
Our models of job creation and job destruction are not yet in a position to tell us why there is this contrast between the level of benefits on the one hand and their duration on the other. Simulations with the level of benefits in the Mortensen-Pissarides model shows that the economy settles at slightly higher job turnover rate when the level of benefits is increased indefinitely. The analysis of limited duration benefits is a lot more complicated because we lose the stationarity of the optimal strategies. In partial models of search, the prospect of benefit exhaustion leads to a decline in the reservation wage during search and therefore to an increased probability that the worker will be willing to accept a job quickly. Since the jobs that are likely to be accepted in this rather desperate state are not likely to be good long-term jobs, we would expect job destruction to be more frequent. Put differently, in countries where workers know that the state will support them indefinitely they spend more time looking for regular stable jobs; if support is expected to run out they would be prepared to take irregular jobs on a short term basis. When employers realise that attitude they are more likely to bring on to the market the irregular short-term jobs in the latter case than in the former.

2.4.3 Active Policy Measures

Much has been written recently on the advantages of active labour market policies versus passive (OECD 1993; 1994a). Active measures include the subsidization of employment, the subsidization of training, the running of a state employment service and the provision of help to unemployed job seekers, in the forms, for example, of guidance how to fill in job application forms. Thus, spending on active measures either make the unemployed more employable or they help the job seeking activities. Passive measure simply provide income support.

There is some evidence that active measures reduce overall unemployment but the evidence with regard to job reallocations is mixed. One of the difficulties
in making international comparisons of the effects of active labour market policies is how to deal with Sweden. Because Sweden spends far more on active labour market policies than other OECD countries do, any international comparison involving a small number of countries is bound to be distorted by Sweden. If we are not careful in drawing inferences from the comparison we might end up building an argument entirely on the comparison of two points, one for Sweden and one for the rest of the OECD.

This problem shows up in our comparison too. In Figures (2.11) and (2.12) we plot average job reallocation rates against two measures of active policies, the average spending per unemployed worker as a proportion of output per head and the ratio of active to passive spending. Because Sweden is way above all other
Figure 2.12: Job Reallocation by Continuing Firms and Active Labour Market Policy
countries on both measures and because it has a high job reallocation rate, the correlation coefficients between each of our measures and the job reallocation rate are both positive and 0.25. But if Sweden is excluded from the comparison it is clear from the Figures that the relation between active policy and job reallocations is, if anything, negative, though weak.

In view of this, we cannot infer anything about the relation between job reallocation and active measures from our small sample. Indeed, one is likely to learn more about the contribution of active policy to job reallocation from detailed study of Swedish labour markets than from an international comparison. For example, job reallocation rates are likely to be positively affected by active measures if the jobs that are subsidize to hire unemployed are not regular long-term jobs, or if workers are dismissed when the subsidy ends. But they might reduce job reallocations if the subsidization stops firms from closing down jobs that are hit by negative shocks.

2.4.4 Industrial Policy

Finally, we examine the role of subsidies to industry. Our source for the data is the statistical office of the European Union, so we have data only for the member countries in our sample and for the United States. It has recently been argued by Leonard and van Audenrode (1993) that subsidies to industry slow down the process of job renewal by supporting ailing plants. This should imply strong negative correlation between job reallocation and industrial subsidies, at least for continuing plants. There is some evidence for this in our sample for continuing plants, with a correlation coefficient between the two for the seven countries -0.25 (Figures (2.13) and (2.14)). But the relation is lost when we consider total reallocation (since only established ailing plants are likely to be subsidized, the relevant comparison is with continuing establishments). Also, it should be noted that in such a small sample, the relation is driven by the two
countries in the Leonard-van Andenrode study, Belgium and the United States, the former with a lot of subsidies and very low job reallocation and the later with virtually no subsidies and high job reallocation. Thus, although there are strong theoretical argument that providing subsidies to ailing establishments leads to less job destruction, and so to less job reallocation, there is no evidence in our sample that the industrial subsidies in the European Union have been directed at such establishments.
Figure 2.14: Job Reallocation by Continuing Firms and Industrial Policy
2.5 Conclusions

The international data on job creation and job destruction show large variations across countries. We have used this variation for a sample of ten OECD countries to make some inferences about the connection between gross job reallocation on the one hand and aggregate economic performance and labour market policy on the other.

The connection between job reallocation and unemployment in the international domain is rather loose but there is a strong connection between reallocation and long-term unemployment. Countries with less job reallocation experience longer durations of unemployment, presumably because in those countries the employed do not easily relinquish their jobs to enter unemployment and give the unemployed a chance to replace them. Since long-term unemployment is not good for the skills and the morale of those who suffer it, policy measures that restrict job reallocation will have negative impact on the functioning of labour markets in this connection.

When we examined the relation between gross reallocation and policy we found two strong correlations and some other looser ones. Employment protection legislation, in the form of restrictions on the dismissal of employees, slows down both job creation and job destruction, and so leads to longer durations of unemployment. The indefinite availability of unemployment compensation also slows down the reallocation of jobs. The mechanism is probably the elimination of low productivity unstable jobs that the long-term availability of income support is likely to bring about.

In contrast, the level of unemployment benefit seems to exert a mild positive influence on job reallocation, though not a very important one. Spending on active labour market policies, perhaps surprisingly, does not appear to exert a significant influence on job turnover, though it should be pointed out that when
it comes to using OECD data to say something about active policy conclusions are always dependent on how one treats Sweden. As it turns out, Sweden has a high job reallocation rate but our summary data cannot identify active policy spending as the reason.

Finally, industrial subsidies appear to slow down job reallocation, though a warning should be sounded here too. We have data for this comparison for only seven of our countries and the comparison is dominated by the experience of two countries, the United States with no subsidies and high turnover and Belgium with a lot of subsidies and low turnover.
2.A Some Further Evidence

Throughout the discussion in the chapter we concentrate on job reallocation by continuing establishments arguing that the theory of job creation and destruction as it applies to continuing establishments is often different from the one that applies to entry and exit. While this is true, there is also a more pressing empirical justification for concentrating on continuing establishments only. OECD (1994a) points out that the definitions of birth and death vary widely across countries. For example, births can appear for any of the three reasons: (i) the creation of a new business from scratch, (ii) the take-over of an existing business by an entrepreneur and (iii) the reallocation of an existing business into another area or industry. Presumably we would like to concentrate on (i), but different countries have different definitions. Similar problems affect the definition of deaths.

All the results in the text are based upon bivariate correlations between reallocation rates and alternative policy measures. In the empirics of economic growth Levine and Renelt (1992) have shown that many results from cross-country regressions of growth are not robust to changes in the conditioning set of variables in the regression.

To test the robustness of the result I performed a set of simple regressions in the spirit of Bell (1994). Firstly I estimated bivariate regressions on the relationship studied in the chapter and record the significance of the correlation. I then ran regressions in which I include a single conditioning variable and test whether the coefficient on the policy variable remains significant (and of the same sign). So, for example, to test the robustness of the employment protection legislation (EPL) effect I ran regressions that include EPL and total unemployment as independent regressor, EPL and the replacement ratio, etc. I only include one conditioning variable because we begin with only 8 degrees of freedom. There are eight independent variables, so I estimated a total of 64 regressions. Bivari-
ate results are in Table (2.A.1) and multivariate results in Table (2.A.2). The t-statistics in Table (2.A.1) at 10% significance are all consistent with those reported in the text, but the effect of employment protection legislation and overall unemployment insurance is no longer significant at 5%. Table (2.A.2) checks the robustness of results in Table (2.A.1) and confirms that the relation between long term unemployment and job reallocation is a robust one, as well as the relation between job reallocation and the benefit duration. Conversely, the effect of employment protection legislation and unemployment insurance does not pass the test of Table (2.A.2).

2.B Definitions and Sources

2.B.1 Cross Country Comparisons

Job Reallocation. Data come from OECD (1994a) Employment Outlook, chapter 3. They are drawn from national, primarily administrative sources that differ in their methods of collection, in their employment coverage and sectoral classification. The information refers to establishments except for Canada, Italy and the U.K., where data refer to firms. An attempt was made by the OECD to
Table 2.A.2: Multivariate Regressions for Job Reallocation

<table>
<thead>
<tr>
<th>Variable</th>
<th>Num. of times coeff same as Table(2.A.1)</th>
<th>Num. of times coeff. is 10% sig.</th>
<th>Num. of times coeff. is 5% sig.</th>
<th>Highest t.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment</td>
<td>7 of 7</td>
<td>0 of 7</td>
<td>0 of 7</td>
<td>1.24</td>
</tr>
<tr>
<td>Long Term. Unemp.</td>
<td>7 of 7</td>
<td>7 of 7</td>
<td>6 of 7</td>
<td>3.22</td>
</tr>
<tr>
<td>Active Policy</td>
<td>4 of 7</td>
<td>0 of 7</td>
<td>0 of 7</td>
<td>0.8</td>
</tr>
<tr>
<td>Unempl. Insurance</td>
<td>5 of 7</td>
<td>1 of 7</td>
<td>1 of 7</td>
<td>3.84</td>
</tr>
<tr>
<td>Firing Costs</td>
<td>6 of 7</td>
<td>4 of 7</td>
<td>1 of 7</td>
<td>2.45</td>
</tr>
<tr>
<td>Benefit Durat.</td>
<td>7 of 7</td>
<td>7 of 7</td>
<td>7 of 7</td>
<td>6.37</td>
</tr>
<tr>
<td>Industr. Subs.</td>
<td>3 of 7</td>
<td>0 of 7</td>
<td>0 of 7</td>
<td>0.45</td>
</tr>
<tr>
<td>Replacement Ratio</td>
<td>6 of 7</td>
<td>2 of 7</td>
<td>2 of 7</td>
<td>3.9</td>
</tr>
</tbody>
</table>

Source: OECD (1994a) and Appendix.

standardize as much as possible. For the U.S. we used the manufacturing rates computed by Davis and Haltiwanger (1990) adjusted to make them comparable to rates for the entire economy. The transformation was based on a comparison between Canadian job flows for the overall economy (OECD, 1994) with Canadian flows for the manufacturing sector only (Baldwin et al. 1994). This led to multiplying the job flow rate for manufacturing by 1.2. We did not use the U.S. figures in OECD (1994a) because they are based on a much smaller data set with two-year frequency. The two-year frequency biased the picture in favour of entry and exit.


2.B.2 Time Series

Chapter 3

Wages and the Size of Firms in a Dynamic Matching Model

3.1 Introduction

Wage differentials across observationally equivalent workers are both sizable and persistent in reality, and empirical work has given stylized-fact status to several relationships between them and employer characteristics. Our work in this chapter is motivated by what is perhaps the strongest such stylized fact, namely, the employer size-wage effect: Brown and Medoff (1989), Krueger and Summers (1988), and many other contributions find, in a variety of data sets, that firms (or plants) with higher employment levels pay higher wages.

Different compensation for apparently identical workers is inconsistent with textbook models of labour market equilibrium, where wage differentials should be arbitrated away by employment changes. A positive association between wages and employment is perhaps even more puzzling from the point of view of such models: if anything, higher wages should be associated to lower employment in a static model of downward-sloping labour demand. Since wage and employment observations are simultaneously determined in equilibrium, however, the empirical evidence can potentially be rationalized in terms of labour-demand movements along upward-sloping labour supply relationships at the firm (or plant)
level. If labour mobility is costly and/or time consuming, firms offering higher wages are able to attract more numerous (homogeneous) workers in steady state; hence, higher employment levels should indeed be associated to high wages in cross-section. Bertola and Ichino (1995) offer a very simple illustration of the basic idea that positive business-condition shocks should lead to increased employment, higher profitability, and higher wages if higher pay is needed to finance costly worker mobility.

In reality, a large part of worker mobility costs reflects matching difficulties and search unemployment. The idea that imperfect matching or “search” can rationalize wage differentials across identical workers has already noted in the literature. Burdett and Mortensen (1989) propose a model where random matching only slowly draws workers from low-pay to high-pay jobs and, in equilibrium, ex-ante symmetric constant-return-to-scale “firms” earn similar profits through different combinations of wage and employment levels. The association of high wages to a specific firm’s circumstances, however, is lost in a constant-return environment, where jobs paying the same wage could just as well be spread in the economy at large instead of being lumped together in a specific “firm.”

Most search models (e.g. Pissarides 1990, 1994; Mortensen and Pissarides, 1994) similarly focus on matching and wage determination issues at the level of individual jobs in constant-return environments where “firms” can hardly be defined or identified while, of course, observable characteristics of real-life firms do appear empirically relevant: Dickens and Katz (1987), Katz and Summers (1989), and other references in Blanchflower et al. (1996) find that persistently high wages are associated to larger profits, or more generally to indicators of employers’ “ability to pay.”

This paper proposes a model of matching frictions and wage-setting institutions aimed at offering a structural interpretation of the accumulated body of empirical evidence: our theoretical viewpoint is complex enough to treat profits,
employment, and wages as jointly endogenous and interrelated variables. We build upon the earlier work of Bertola and Caballero (1994), who propose a model of search where individual jobs' productivity is determined by standard downward-sloping labour demand functions within firms affected by idiosyncratic labour-demand shocks.

Section 3.2 lays out the basic structure of the Bertola and Caballero model, correcting a minor algebraic error in the original derivations. While employment reduction can be instantaneous in the model, hiring is costly and time-consuming in an environment where unemployed workers and unfilled vacancies are only slowly matched to each other. If higher employment decreases labour’s productivity and wages are continuously renegotiated so as to split in fixed proportions the surplus afforded by existing employment relationships in a search environment, the hiring process results in a pattern of declining wage rates at larger (and growing) establishments. Once extended to allow for a more flexible and realistic patterns of exogenous dynamic events, however, the model is capable of reproducing the wage-size effect.

The model is qualitatively realistic enough to allow a quantitative exploration of the empirical phenomena which motivate our work. In Section 3.3 we show that establishment size has a positive coefficient in wage regressions run on data generated by the model. Even though wages are decreased by higher employment along a given labour-demand schedule, firms with stronger labour demand pay higher wages at any given level of employment, find it optimal to post more vacancies, grow faster, and have larger size on average. Thus, the wage-size effect is present in model-generated data, but it does not reflect a positive effect of size per se on wages, nor does it conflict with the standard assumption of decreasing marginal returns to labour. Rather, firm size proxies in the model’s wage regressions for (unobservable) business conditions, which are also positively correlated to profits and employment growth. In fact, including the latter variables in wage
regression also yields significant coefficients which, however, are similarly hard to interpret in structural terms.

Section 3.4 reviews in some detail existing work on firm- or plant-level wage regression. Profit and size effects are well established in the literature, and we argue that both are qualitatively consistent with our theoretical perspective. Our dynamic approach to wage-determination issues, however, suggests that profits and size should both be included in wage regressions, and that neither variable's coefficient should be given structural interpretations. Further, our modeling perspective implies that employment growth should also be significant in wage regressions, and in the concluding Section 3.5 we briefly discuss how future empirical work might try and use real data to assess the relevance of the theoretical mechanisms we focus on.

3.2 Firm-level Dynamics in a Matching Model

We start with a brief outline of the technology and market structure which our model shares with other matching models and, particularly, with that proposed by Bertola and Caballero (1994). The object of interest is a stationary labour market, inhabited by a continuum of firms of fixed total mass and by an also fixed and continuously divisible amount of inelastically supplied labour. Normalizing the total measure of both sets to unity, we index firms by \( f \in [0, 1] \); since labour is assumed homogeneous in quality, it will not be necessary in what follows to explicitly index individual workers.

The marginal revenue product of labour at an individual firm is a function \( \pi(l, \eta) \) of its current employment level and of \( \eta \), a shifter of its labour demand. While a larger \( l \) decreases \( \pi(\cdot, \cdot) \), larger values of \( \eta \) are associated to higher labour demand, and \( \eta \) follows exogenous idiosyncratic stochastic processes taking a finite number of values across the continuum of firms. As long as the idiosyncratic process \( \{\eta\} \) is Markov in levels, the shadow value of firm \( f \)'s employment is
a function $A(\cdot)$ of the same state variables $l$ and $\eta$. By its definition as the present discounted value of labour's marginal contribution to profits, this function satisfies a standard asset-pricing relationship in the form

$$rA(\cdot) = \pi(\cdot) - w(\cdot) - \frac{\partial w(\cdot)}{\partial l} l + \frac{\partial A(\cdot)}{\partial l} \dot{l} + E_f[\Delta A(\cdot)];$$  \hspace{1cm} (3.1)

to simplify notation, we let $\cdot$ stand for the $(l, \eta)$ pair of relevant state variables, and we let firm and time indexes be implicit on the individual firm's current employment level $l$ and on its time derivative $\dot{l}$.

In equation (3.1), $r \geq 0$ denotes the rate of return on the firm's operation, and $w(\cdot)$ denotes the wage paid to its employees: the latter is allowed to depend on $\eta$ and $l$ and, to the extent that wages do endogenously depend on employment in equilibrium, the marginal cost of employing an additional worker is appropriately written as the derivative of its wage bill with respect to employment. The last two terms in equation (3.1) conveniently decompose the sources of "capital gains" along the firm's dynamic path. When the firm does not experience a state transition, the shadow value of its labour evolves only as a consequence of employment dynamics, hence the term $\dot{A}(\cdot) = (\partial A(\cdot)/\partial l) \dot{l}$. The final term, $E_f[\Delta A(\cdot)]$, represents the state-dependent expectation of the "capital gain" (or loss) resulting from possible positive (or negative) shocks to $\eta$ and from possible employment jumps associated to such events.

The endogenous dynamics of individual-firm employment levels indexed by $l$ interact with each other in the aggregate labour market. An individual firm can increase its employment level by posting a number (or measure) $v_f$ of vacancies. The rate at which each vacancy is matched with an unemployed worker depends on the overall tightness of the labour market, or on the ratio of aggregate vacancies to aggregate unemployment

$$V \equiv \int_0^1 v_f \, df$$

to aggregate unemployment

$$U = 1 - \int_0^1 l_f \, df.$$
Under the convenient and fairly realistic assumption that the matching technology has constant returns to scale and a Cobb-Douglas functional form, the probability intensity per unit time that any open vacancy is matched to an unemployed worker is $\xi (V/U)^{\nu} \equiv \vartheta$, for some $-1 < \nu < 0$ and $\xi > 0$, and is constant in the steady-state equilibrium of interest. Accordingly, when firm $f$ posts $v_f > 0$ vacancies, its employment level evolves according to

$$\dot{i}_f = \vartheta v_f.$$  \hspace{1cm} (3.2)

With probability intensity $\vartheta$, each open vacancy is filled and becomes a marginal job of value $A(\cdot)$. If a firm finds it optimal to post vacancies, then each one's marginal payoff $\vartheta A(\cdot)$ in terms of additions to the firm's employment stock must be equal to its marginal cost which, as in Bertola-Caballero, we parameterize as $cv_f$, with $c > 0$ indexing the rate at which marginal search costs increase as an individual firm posts larger and larger numbers of vacancies.\(^1\) Thus, we have

$$A(\cdot) = \frac{c}{\vartheta} v_f$$

for all $f$ such that $v_f > 0$. This and (3.2) make it possible to write (3.1) in the form

$$rA(\cdot) = \pi(\cdot) - w(\cdot) - \frac{\partial w(\cdot)}{\partial l} l + \frac{\vartheta^2}{c} A(\cdot) \frac{\partial A(\cdot)}{\partial l} + E_f [\Delta A(\cdot)], \quad \forall f \text{ s.t. } A(\cdot) > 0.$$

The value of additional employment, of course, need not be strictly positive for all levels of $\eta$ and $l$. When $A(\cdot, \cdot) \leq 0$, it is obviously not optimal to post costly vacancies: employment, accordingly, is either constant or declines through voluntary quits (which we neglect for simplicity) or firing decisions. While hiring is modeled as a time-consuming process (and employment never jumps upwards), firms are allowed to shed labour instantaneously: if the $\{\eta\}$ process has discrete

\(^1\)Quadratic vacancy-posting costs ensure that it is never optimal to post infinitely many vacancies, and that employment levels never jump upwards.
downward increments, employment may fall by a finite amount $\Delta l$ if a state transition is such as to make it profitable for the firm to fire. In the aftermath of such an event, the firm must be indifferent between retaining and firing employees at the margin if it does employ $l > 0$. Accordingly, the shadow value of a filled job at a firm which posts no vacancies and has positive employment must be equal to the cost $-F \leq 0$ of shedding one additional unit of labour:

$$A(\cdot) = -F;$$

(3.4)

in this case, equation (3.1) reads

$$-rF = \pi(\cdot) - w(\cdot) - \frac{\partial w(\cdot)}{\partial l} l + E_{f} [\Delta A(\cdot)], \quad \forall f \text{ s.t. } v_{f} = 0, A(\cdot) = -F, l > 0.$$

(3.5)

As long as $F > 0$, however, the firm may find it optimal \textit{not} to react to a state transition. If

$$-F < A(\cdot) < 0,$$

the shadow value of labour is negative (making it pointless to post costly vacancies) at the same time as it is larger than the marginal cost of employment reduction (making labour shedding suboptimal as well); the firm then finds it optimal to choose inaction, a situation familiar from, e.g., Bentolila and Bertola (1990). In this case, equation (3.1) features no “capital gains” terms other than those reflecting state-transition expectations: hence,

$$rA(\cdot) = \pi(\cdot) - w(\cdot) - \frac{\partial w(\cdot)}{\partial l} l + E_{f} [\Delta A(\cdot)],$$

$$\forall \text{ f s.t. } v_{f} = 0, -F < A(\cdot) < 0, l > 0.$$

(3.6)

Turning next to wage determination, let all workers be risk neutral, exert constant search effort, and enjoy an income-equivalent flow $z$ from leisure and unemployment benefits when not working. For workers living in the steady-state economy under consideration, the present discounted value of future labour
income depends on whether they are unemployed or matched to a firm. Unemployed workers’ human capital takes a fixed value $J^u$ in steady state and, since each unemployed worker is matched to a random open vacancy with probability intensity $\theta V/U$, we have

$$rJ^u = z + \theta \frac{V}{U}(E_u[J] - J^u)$$ (3.7)

if $E_u[J]$ is the average value of jobs for which vacancies are posted.

The wages and human capital of employed workers will be functions $w(\cdot)$ and $J(\cdot)$ of the employing firm’s circumstances—namely, of its business conditions index $\eta$ and of its employment level $l$—and satisfy the asset-valuation relationship

$$rJ(\cdot) = w(\cdot) + \dot{J}(\cdot) + E_f[\Delta J].$$ (3.8)

Here, as in equation (3.1), the capital-gain component of total return consists of a continuously evolving term, reflecting employment growth for unchanged business conditions, and of the expectation of discontinuous jumps in the event of a state transition.

Since forming matches is costly, existing job/worker relationships may afford a surplus, which we take to be split according to a continuously renegotiated Nash bargain. At firms where the marginal value of employment is zero or negative, there is no surplus to be split, and the workers (if any) which the employer finds it optimal not to fire are just indifferent to the outside option:

$$J^u = J(\cdot) \quad \forall \ f \text{ s.t. } A(\cdot) \leq 0, \ l > 0.$$ (3.9)

As to firms which are posting vacancies, the option of opening a new one is always open to the employer, and has zero value; for workers, the outside option (never taken in equilibrium) is a voluntary quit into unemployment. Denoting with $\beta$ the employees’ bargaining share, at firms where the marginal value of employment is positive we have

$$\beta A(\cdot) = (1 - \beta)(J(\cdot) - J^u) \quad \text{for all firms where } A(\cdot) > 0.$$ (3.10)
Accordingly, workers' human capital functions may be expressed in terms of firms' shadow-value functions:

\[ J(\cdot) = \frac{\beta}{1-\beta} A(\cdot) + J^u, \]
\[ \dot{J}(\cdot) = \frac{\beta}{1-\beta} \dot{A}(\cdot) = \frac{\beta}{1-\beta} \frac{\partial A(\cdot)}{\partial l}. \]  

(3.11)

This completes the model's basic structure. To proceed, one may insert the expressions in (3.11) in the worker's asset-valuation relationship (3.8), and consider the shadow-value dynamics implied by employers' optimal hiring, firing and inaction policies: first, however, more needs to be said on the dynamic structure of the business-conditions process \{\eta\}.

### 3.2.1 Specialization and Solution Method

Like Bertola and Caballero's (1994) model (and unlike other matching models, where each job is managed in isolation), the framework outlined above features well-defined firm "sizes." Hence, it is well suited to a study of wage-size relationships and, in particular, to the role of trade frictions (or "matching") in their rationalization.

We follow that earlier model in taking labour's marginal revenue product be a linear function of employment,

\[ \pi(l, \eta) = \eta - \sigma l. \]  

(3.12)

Bertola and Caballero worked under the assumption that \( \eta \) would take only two values, but the model's wage-size implications deserve to be explored under a richer and more empirically realistic structure of exogenous shocks. Let \( \eta \) take \( n \) possible values \( \eta_i \), with \( \eta_1 > \eta_2 > \ldots > \eta_n \); denoting with \( \delta_i \) the probability intensity of a transition out of state \( i \) among these, let the row vector \( \bar{p}_k = [p_{kj}] \) collect the probabilities of reaching each of the other states upon a transition out of state \( k \) (with \( \sum_{j=1}^{n} p_{kj} \equiv 1 \), and \( p_{kk} \equiv 0 \)).
Under these parametric assumptions, the marginal shadow value of an additional worker for a hiring firm, from equation (3.1), reads

\[ rA(\cdot) = \eta - \sigma l - w(\cdot) - \frac{\partial w(\cdot)}{\partial l} l + A(\cdot) \frac{\partial A(\cdot)}{\partial l} \frac{\vartheta^2}{c} \]

\[ + \delta_{i\tilde{p}i} (\max \{-F, A(l, \tilde{\eta}) - A(\cdot)\}). \] (3.13)

The \( A(l, \tilde{\eta}) \) functions on the right hand side represent vectors of shadow values, and the max. operator selects, for each productivity value, the maximum between the shadow value of employment and the marginal cost of firing. This appropriately accounts for the fact that the firm will fire if the marginal value of labour at the current employment level \( l \) is lower than the marginal firing cost \(-F\).

The wage paid by such a firm, in turn, must satisfy the asset-valuation relationship obtained by inserting expressions from (3.13) into (3.8), and recognizing that the firm's optimal hiring behavior straightforwardly determines its shadow-value dynamics in the absence of a state transition:

\[ rA(\cdot) = \frac{1 - \beta}{\beta} (w(\cdot) - rJ^u) + A(\cdot) \frac{\partial A(\cdot)}{\partial l} \frac{\vartheta^2}{c} \]

\[ + \delta_{i\tilde{p}i} \frac{1 - \beta}{\beta} \left( \max \left[ J^u + \frac{\beta}{1 - \beta} A(l, \tilde{\eta}), J^u \right] - (J^u + \frac{\beta}{1 - \beta} A(\cdot)) \right). \] (3.14)

The wage-size relationship can be characterized rather sharply when the marginal cost of firing is zero and inaction is never optimal. With \( A(\eta, l) = -F = 0 \) for all matches where the workers' participation constraint is binding at \( J^u \), subtracting (3.14) from (3.13) yields a simple differential equation for the \( w(l, \eta) \) wage function at hiring firms:

\[ \eta_i - \sigma l - \frac{1}{\beta} w(\cdot) - \frac{\partial w(\cdot)}{\partial l} l + \frac{1 - \beta}{\beta} rJ^u = 0, \] (3.15)

solved by

\[ w(\cdot) = \beta \eta_i + (1 - \beta) rJ^u - \frac{\beta}{1 + \beta} \sigma l + C^i l^{-1/\beta}, \] (3.16)

for \( C^i \) a constant of integration.
The linear solution which obtains from (3.16) when $C^n = 0$ highlights the key insight to be gained from our model's framework of analysis. As in Bertola and Caballero, the wage paid by a hiring firm is declining in its size: this is intuitive, since labour's marginal productivity is inversely related to employment, but appears to be at odds with the stylized empirical evidence summarized in the Introduction above. Firm-level wages, however, are also increasing in the idiosyncratic shifter $\eta$: at any given employment level, firms with higher productivity pay higher wages. This effect, which could not be captured as clearly by a model with only two productivity levels, follows immediately from the fact that highly productive firms share their good fortune with their employees (in Nash-bargaining fashion) as long as the latter are scarce, which in turn depends on the costly and time-consuming nature of the matching process.

3.2.2 Employment Dynamics in a Special Case

Clearly, the model's implications are consistent with empirical evidence if, as is intuitively obvious, firms with higher productivity and higher wages also tend to have larger employment levels. As in the job-level matching model of Cabrales and Hopenhayn (1995), it would be desirable to allow for proper persistence in the Markov process followed by the $\eta$ indicators. Unfortunately, however, the model does not admit an analytic solution for labour's shadow values and employment dynamics under general and realistic configurations of the idiosyncratic shock process. Inserting the linear form of (3.16) in (3.13) with $F = 0$, one obtains a differential equation for the marginal shadow value of labour at a hiring firm:

$$ rA(\cdot) = (\eta_l - \sigma l) - (h_i + 2k_i l) + A(\cdot) \frac{\partial A(l, \eta_l)}{\partial l} \frac{\varphi^2}{c} + \delta_i \tilde{p}_i (\max [0, A(l, \eta_l)] - A(\cdot)), \quad (3.17) $$
where
\[ h_i = \beta \eta_i + (1 - \beta) r J^u, \quad k_i = -\frac{\beta \sigma}{1 + \beta}. \tag{3.18} \]

This nonlinear and nonhomogeneous differential equation features the product of the solution function’s level and derivative: like the similar equation encountered by Sutherland (1992) and his references in a different context, its nonlinear solutions can only be studied numerically. In this direction, chapter (4) solves a numerical approximation of the nonlinear solution of equation (3.17).

The equation has a linear and economically meaningful solution, however, for a specification of transition probabilities which, while still rather restrictive, is considerably more general than the one considered by Bertola and Caballero. Specifically, let all negative \( \eta \) transitions bring the firm to the lowest-productivity state, and let the other states be reachable only from the latter so that, in our notation, \( p_{in} = 0 \): the \( \tilde{p}_k \) probability vectors attach unit weight to \( j = n \) (with \( \eta_n \) the lowest possible value of \( \eta \)) for \( k > n \), while \( \tilde{p}_n \) may feature non-zero transition probabilities to all the higher productivity states.

In this special case, higher-productivity (hiring) firms enjoy different levels of productivity, but their employment level is irrelevant to its outlook upon state transitions: whenever \( \eta \) changes, all such firms fire some of their workers and turn into (firing) firms with poor productivity and a marginal value of labour equal to 0. This simplifies considerably the capital-loss component of equation (3.17) and validates its linear solution in the form

\[ A(\cdot) = a_i + b_i l, \]

\[ b_i \equiv b(\delta_i) = c(r + \delta_i) - \frac{1}{2} \left( \frac{c^2(r + \delta_i)^2}{\theta^4} + \frac{4c(1 - \beta)\sigma}{\theta^2(1 + \beta)} \right) < 0, \]

\[ a_i \equiv a(\eta_i, \delta_i) = \frac{\eta_i - h_i}{r + \delta_i - \frac{\sigma^2}{c} b_i}. \tag{3.19} \]

We can take advantage of this simple characterization of a hiring firm’s wage function when considering the relationship between the wage and employment.
level of a firm which is not hiring—i.e., in the case under consideration, of a firm which has productivity indicator $\eta_n$ and marginal employment value $A(l_{\text{min}}, \eta_n)$. Evaluating expression (3.5) under our parametric assumptions and indicating with $l_{\text{min}}$ employment of a firm that is not hiring, we obtain
\begin{equation}
\eta_n - \sigma l_{\text{min}} - w(\cdot) - \frac{\partial w(\cdot)}{\partial l} l_{\text{min}} + \delta_n \bar{p}_n A(l_{\text{min}}, \bar{\eta}) = 0, \tag{3.20}
\end{equation}
where, consistently with previously introduced notation, the column vector $A(l_{\text{min}}, \bar{\eta})$ collects the marginal shadow values of this firm's employment stock upon transitions to other (hiring) states, which occur with probability intensity $\delta_n$, and the row vector $\bar{p}_n$ contains the transition probabilities from state $n$ to state $k$.

To obtain an expression for the marginal labour cost term in (3.20), consider that when the worker's participation constrain is binding there is no surplus to be split from the match—or, that from $A(l_{\text{min}}, \eta_n) \leq 0$ it follows that $J^u = J^n$. Workers already employed by a firm in state $n$, however, can look forward to human-capital gains: if their employer experiences a positive productivity shock, the workers' human capital jumps upwards to a fraction of the newly positive shadow value of employment. In other words, it must be the case that the present discounted value of labour earnings for a currently unemployed worker equals the wage paid by bad firms plus the expected capital gain from being employed by a hiring firm, without going through the time-consuming matching process, upon a positive productivity shock. In flow terms, this equilibrium requirement reads
\begin{equation}
r J^u = w(l_{\text{min}}, \eta_n) + \frac{\beta}{1 - \beta} \delta_n \bar{p}_n A(l_{\text{min}}, \bar{\eta}). \tag{3.21}
\end{equation}
In steady state, the left-hand side of this condition is constant: by total differentiation, and recalling that $A(l, \eta_i) = a_i + b_i l$ for $i = 1, \ldots, n - 1$, we obtain
\begin{equation}
\frac{\partial w(l, \eta_n)}{\partial l} \equiv k_n = -\frac{\beta}{1 - \beta} \delta_n \bar{p}_n \frac{\partial A(l, \bar{\eta})}{\partial l} = -\frac{\beta}{1 - \beta} \delta_n \bar{p}_n \bar{b}, \tag{3.22}
\end{equation}
where $\bar{b} = [b_i]$ is a column vector collecting the slopes of the various hiring states' marginal shadow value functions. Like (3.16), this simple relationship
offers insights of some generality. Since wages paid by hiring firms are negatively related to their employment level, workers hoarded by a firing firm can look forward to larger capital gains when their employer's labour force is smaller—hence, in equilibrium, they require a smaller wage flow, and wages paid by labour-hoarding firms are a positively sloped function of their employment level.\footnote{This effect was neglected by Bertola and Caballero, who treat the wage rate offered by labour-hoarding firms as a constant in their derivations.}

Using (3.22) and (3.21) into (3.20) employment at not hiring firms reads

\[
I_{\text{min}} = \frac{\eta_n - h_n + \delta_n \tilde{p}_n a(\tilde{\eta}, \tilde{\delta})}{\sigma + 2k_n},
\]

(3.23)

where \(a(\tilde{\eta}, \tilde{\delta})\) is a column vector of coefficients from (3.19) and

\[
h_n = r J^u - \frac{\beta}{1 - \beta} \delta_n \tilde{p}_n a(\tilde{\eta}, \tilde{\delta}).
\]

(3.24)

Figure 3.1 illustrates the character of the firms' optimal policy under parametric assumptions, listed in Table 3.1, which are of course meant to be suggestive rather than fully realistic. We choose to let most firms be small and subject to frequent, but not very pronounced labour demand fluctuations. Fewer and fewer firms feature higher and higher labour demand levels, and are increasingly unlikely to suffer a negative shock which, in their case, would cause a veritable collapse. As we highlight in the next section, this structure of productivity shocks appears consistent with cross-section evidence on job flows and firm size.\footnote{Bertola and Caballero (1994), discuss in some detail rough calibration criteria for other parameters.} The downward-sloping linear functions plotted in Figure 3.1 depict \(A(\cdot)\), the marginal shadow value of labour, as a function of firm size for different realizations of the idiosyncratic shifter \(\eta_n\). Since vacancies are posted only when \(A(\cdot) > 0\), the horizontal intercepts of the various marginal value schedules in the Figure correspond to the maximum size attainable by firms of each type. For future reference, let
Table 3.1: Baseline parameter values

<table>
<thead>
<tr>
<th>Variable</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matching Function:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>matching elasticity</td>
<td>$\nu$</td>
<td>-0.400</td>
</tr>
<tr>
<td>constant</td>
<td>$\xi$</td>
<td>1.000</td>
</tr>
<tr>
<td>Labor demand slope (all $i$)</td>
<td>$\sigma$</td>
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<tr>
<td>Labor demand intercepts</td>
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<td></td>
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<tr>
<td></td>
<td>$\eta_1$</td>
<td>1.627</td>
</tr>
<tr>
<td></td>
<td>$\eta_2$</td>
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</tr>
<tr>
<td></td>
<td>$\eta_6$</td>
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</tr>
<tr>
<td></td>
<td>$\eta_n$</td>
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</tr>
<tr>
<td>Transition intensities</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\delta_1$</td>
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</tr>
<tr>
<td></td>
<td>$\delta_2$</td>
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</tr>
<tr>
<td></td>
<td>$\delta_3$</td>
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<td>$\delta_4$</td>
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<td>$\delta_6$</td>
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<td></td>
<td>$\delta_n$</td>
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</tr>
<tr>
<td>Distribution across hiring states of positive shocks</td>
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<tr>
<td></td>
<td>$p_{\tau_1}$</td>
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<tr>
<td></td>
<td>$p_{\tau_2}$</td>
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<td>$p_{\tau_3}$</td>
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</tr>
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<td></td>
<td>$p_{\tau_4}$</td>
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</tr>
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<td></td>
<td>$p_{\tau_5}$</td>
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</tr>
<tr>
<td></td>
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<td>search cost slope</td>
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<td>interest rate</td>
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</tr>
<tr>
<td>employees’ bargaining share</td>
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<tr>
<td>Unemployed income</td>
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<tr>
<td>Firing cost</td>
<td>$F$</td>
<td>0.000</td>
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<tr>
<td>Equilibrium values:</td>
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<td></td>
</tr>
<tr>
<td>Market tightness</td>
<td>$\theta$</td>
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</tr>
<tr>
<td>Unemployed welfare</td>
<td>$rJ^U$</td>
<td>0.864</td>
</tr>
</tbody>
</table>
Figure 3.1: Labor's marginal asset valuation
\( \bar{l}_i \) denote the "long-run size" employment level of firms in state \( i \), defined by the condition

\[
A(\bar{l}_i, \eta_i) = 0. \tag{3.25}
\]

Along the horizontal axis we find the marginal shadow value when \( A(l, .) \leq F \), where \(-F \equiv 0\) under our parameterization.

Recalling that not only vacancy posting and job creation, but also equilibrium wages are linearly related to \( A(\cdot) \), the Figure offers a fairly complete characterization of individual firms' policies for given values of \( \vartheta \) and \( J^u \). To actually solve for the equilibrium value of \( V/U \) and \( J^u \) (already reported in Table 1) we need to consider the distribution of firm size for each \( \eta_i \).

### 3.2.3 Firm Size Distribution and Market Equilibrium

In this section we characterize the firm size distribution under the simple probabilistic structure of shocks considered in the previous section. In steady state, the probability mass of firms which flows into each productivity state must balance with the probability mass of firms that flows into the other states. If we indicate with \( \pi \) the ergodic distribution over the productive states, by virtue of our simple structure of productivity changes, this equilibrium requirements reads

\[
\delta_n p_{ni} \pi_n = \delta_i \pi_i \quad \forall i > n, \tag{3.26}
\]

and

\[
\sum_{i=1}^{n-1} \delta_i \pi_i = \delta_n \pi_n, \tag{3.27}
\]

for the lowest labour demand (not hiring) state. The expressions in (3.26) and (3.27) form a homogeneous, rank-deficient system of \( n \) linear equations which, together with the summing-up condition \( \sum_i \pi_i = 1 \), easily yields a solution for \( \pi \).

Since all firms in the poorest business conditions have the same employment level \( l^{\min} \), the joint distribution of employment and business conditions has a point mass of size \( \pi_n \) at the \((l^{\min}, \eta_n)\) point. In what follows we shall let \( f(l, \eta) \)
be the joint density of employment and business conditions within the hiring subsets of the model's state space. For a firm with productivity parameter $\eta^i$ and employment $l_i$,

$$i = \frac{g^2}{c} A(l, \eta_i) = \frac{g^2}{c} (a_i - b_i l);$$

hence, the density must satisfy the simple Kolmogorov forward balance conditions

$$\frac{\partial f(l, \eta_i)}{\partial l} i = -\delta_i f(l, \eta_i): \tag{3.28}$$

in words, transitions into state $(l, i)$ because of hiring (a positive $\dot{l}$) must balance transitions out of it due to shocks into the firing state. The differential equation (3.28) has solutions in the form

$$f(l, \eta_i) = \lambda_i (a_i + b_i l)^{-\dot{\varepsilon}_i}, \quad \text{for} \quad \varepsilon_i = \frac{\delta_i c}{g^2 b_i}, \tag{3.29}$$

and the constant of integration $\lambda_i$ is determined by the summing-up condition

$$\lambda_i \int_{l_{\min}}^{l_i} (a_i + b_i l)^{-\varepsilon_i} dl = \pi_i, \tag{3.30}$$

where $\pi_i$ is one of the solution to (3.26) and (3.27) and $l_i$, as defined in (3.25), is the maximum size attainable by a firm in state $i$.

To complete the solution we need to calculate the aggregate variables implied by the individual policies. For a constant $c$ and $g$, each firm’s vacancies are linearly related to its labour’s marginal value $A(l, \eta)$, which in turn is linear in employment. It follows that

$$v(\cdot) = (a_i + b_i l) \frac{g}{c}, \tag{3.31}$$

and aggregate vacancies are obtained by summing up across all such individual policies

$$V = \frac{g}{c} \sum_{i=1}^{n-1} \int_{l_{\min}}^{l_i} \lambda_i (a_i + b_i l)^{-\varepsilon_i+1} dl. \tag{3.32}$$

Similarly, aggregate employment is obtained by summing up the employment of all hiring and firing firms and is equal to

$$L = \sum_{i=1}^{n-1} \int_{l_{\min}}^{l_i} \lambda_i l (a_i + b_i l)^{-\varepsilon_i} dl + \pi_n l_{\min}. \tag{3.33}$$
An aggregate consistent equilibrium is obtained when the \( \theta \) used to construct individual policies is consistent with the aggregate outcomes in (3.32) and (3.33) and yields a value of \( J^n \) equal to the outside (unemployment) option featured in the dynamic labour demand problem's solution:

\[
r J^u = r J^n = z + \theta \frac{V}{1 - L} (E_u[J] - J^u).
\] (3.34)

By virtue of (3.10) and (3.31), the capital gain term in (3.34) reads

\[
E_u[J] - J^u = \frac{\theta V}{c U} \frac{\beta}{1 - \beta} \frac{1}{1 - \pi_n} \sum_{i=1}^{n-1} \int_{I_{\text{min}}}^{I_i} \lambda_i A(\eta_i, l)(a_i + b_i l)^{-\psi + 1} dl.
\] (3.35)

In (3.35) \( 1 - \pi_n \) is the probability that an unemployed worker meets a firm of quality \( i \), conditional on finding a vacancy posting firm. As in Bertola and Caballero (1994), a simple search routine may be used to compute the model's fixed-point equilibrium.\(^4\)

Figure 3.2 plots the density of employment across all hiring firms, "slicing" each portion of the density according to the business-conditions indices which may be consistent with each employment level; each such portion of the density function reaches zero at the point where firms with the corresponding \( \eta_i \) find it optimal to stop hiring. Figure 3.3 plots the cumulative distribution function of employment, and accounts for the discrete mass of employment located at labour-hoarding firms. Both Figures are drawn for the same parameter values as Figure 3.1.

Table 3.2 reports gross job flows by firm size obtained by Monte Carlo data drawn from the long run distribution plotted in figure 3.3. Our baseline parameters in Table 3.1 imply a structure of job flows consistent with the empirical evidence on job flows and firm size reported in OECD (1994a). Net employment

\(^4\)We recall at this point that all our derivations set \( C_i = 0 \) in the key equation 3.16. Thus, the iterative procedure finds one economically sensible equilibrium of the model but, since the bilateral-bargaining structure of the model does not guarantee uniqueness and Pareto-optimality of market equilibria, others may exist.
Figure 3.2: Probability densities of employment at hiring firms

Table 3.2: Job Flows by Firm Size

<table>
<thead>
<tr>
<th>Firm Size</th>
<th>Job Creation</th>
<th>Job Destruction</th>
<th>Job Reallocation</th>
<th>Net Employment Change</th>
<th>Employment Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>0(N) - 8(N)</td>
<td>0.262</td>
<td>0.071</td>
<td>0.333</td>
<td>0.192</td>
<td>0.335</td>
</tr>
<tr>
<td>8(N) - 15(N)</td>
<td>0.166</td>
<td>0.089</td>
<td>0.255</td>
<td>0.076</td>
<td>0.176</td>
</tr>
<tr>
<td>15(N) - 24(N)</td>
<td>0.107</td>
<td>0.080</td>
<td>0.188</td>
<td>0.027</td>
<td>0.181</td>
</tr>
<tr>
<td>24(N) - 30(N)</td>
<td>0.071</td>
<td>0.073</td>
<td>0.145</td>
<td>-0.002</td>
<td>0.064</td>
</tr>
<tr>
<td>30(N) - ∞</td>
<td>0.039</td>
<td>0.064</td>
<td>0.104</td>
<td>-0.025</td>
<td>0.245</td>
</tr>
</tbody>
</table>
Figure 3.3: Cumulative distribution of employment, all firms
changes (the difference between job creation and destruction) are negatively related to firm size, which is of course unsurprising in a steady-state situation. The firm size distribution generated by our model and parameters is roughly consistent with the evidence reported in the OECD study: for example, the smallest size category accounts for a third of employment in the Table, and the employment share of firms with less than 20 employees ranges from 27% in Canada to more than 40% in New Zealand in the OECD tables. More interestingly, we see in the Table that job reallocation (the sum of job creation and destruction) declines sharply as a function of size, which is also consistent with available evidence: of course, our model and parameters generate employment stability for large firms in the form of unrealistically large, if unlikely, “catastrophic” job destruction or perhaps plant-closure events; still, it is comforting to find that, on average, the model-generated job flows accord so well with qualitative evidence.

3.3 Empirical Implications

Figure 3.4 illustrates the model’s implications for the wage-size relationship by superimposing Monte Carlo data, drawn from the long-run distribution of employment and business conditions, over the theoretical wage functions of equation (3.16). Clusters of points appear at the \( w(l^{min}, \eta_n), l^{min} \) point, and along the downward-sloping wage loci identified by equation (3.16) for the various values of \( \eta \) which induce hiring. Wages are clearly decreasing with employment for given \( \eta \); hence, a regression where \( \eta \) were controlled for would yield a negative (labour-demand originated) relationship between wages and employment. If \( \eta \) is not observable, however, the model generates a positive cross-sectional relationship between wages and employment.

Table 3.3 reports the standardized slope coefficient of a regression (which also includes a constant) of wages on employer size, and thick upward-sloping line in Figure 3.4 plots predicted values from the same OLS regression. Since our model
Figure 3.4: Wage functions and a Monte Carlo sample of observations

Table 3.3: Regression of wages on firm size

<table>
<thead>
<tr>
<th>Variable</th>
<th>Standardized estimate</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>0.4768</td>
<td>19.48</td>
</tr>
</tbody>
</table>
Table 3.4: Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor d.</td>
<td>1.00</td>
<td>0.95</td>
<td>0.74</td>
<td>0.87</td>
<td>0.87</td>
<td>0.54</td>
</tr>
<tr>
<td>Wage</td>
<td>0.95</td>
<td>1.00</td>
<td>0.48</td>
<td>0.91</td>
<td>0.91</td>
<td>0.26</td>
</tr>
<tr>
<td>Employ.</td>
<td>0.74</td>
<td>0.48</td>
<td>1.00</td>
<td>0.45</td>
<td>0.45</td>
<td>0.92</td>
</tr>
<tr>
<td>Job cr.</td>
<td>0.87</td>
<td>0.91</td>
<td>0.45</td>
<td>1.00</td>
<td>1.00</td>
<td>0.23</td>
</tr>
<tr>
<td>Vac.</td>
<td>0.87</td>
<td>0.91</td>
<td>0.45</td>
<td>1.00</td>
<td>1.00</td>
<td>0.23</td>
</tr>
<tr>
<td>Profits</td>
<td>0.54</td>
<td>0.26</td>
<td>0.92</td>
<td>0.23</td>
<td>0.23</td>
<td>1.00</td>
</tr>
</tbody>
</table>

features homogeneous labour and uniform market power, this regression corresponds to what a researcher might find in data where all worker characteristics are controlled for, but wage differentials are generated by dynamic heterogeneity across firms under diminishing returns to labour.

The model-generated data feature a sizable wage-size effect but, of course, the upward sloping regression line does not imply a causal relation of firm size to wages. Rather, it reflects the interplay of exogenous shocks to labour demand and the labour-supply constraints introduced by slow matching. How should data generated by the wage equation (3.16) be approached by an empirical researcher? Among firms with similar employment levels, wages are importantly affected by the labour-demand heterogeneity indexed by \( \eta \) in our model. While \( \eta \) is not directly observable, such variables as job creation and profits are also endogenously determined by \( \eta \). For a given \( l \), higher \( \eta \) is associated to faster job creation by equation (3.31), and to a larger excess of revenue over labour-related costs: by virtue of (3.12) and the structure of hiring costs, each firm’s thusly defined “operating surplus” reads

\[
\Phi(l, \eta_i) = \eta_i l - \frac{\sigma}{2} l^2 - w(l, \eta_i) l - \frac{cv^i(t)^2}{2}. \tag{3.36}
\]

Table 3.4 reports the correlation matrix of four observable variables (wages, employer size, job creation and profits) with each other and the unobservable
Table 3.5: Regression of wages on profits

<table>
<thead>
<tr>
<th>Variable</th>
<th>Standardized estimate</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profits</td>
<td>0.2632</td>
<td>8.928</td>
</tr>
</tbody>
</table>

Table 3.6: Regression of wages on firm size and profits

<table>
<thead>
<tr>
<th>Variable</th>
<th>Standardized estimate</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>1.515</td>
<td>83.17</td>
</tr>
<tr>
<td>Prof.</td>
<td>-1.130</td>
<td>-62.01</td>
</tr>
</tbody>
</table>

labour demand index $\eta$. Other than with size, wages are also significantly correlated with profits, which are significant in the wage regressions reported in Table 3.5 and Table 3.6.

The dynamic nature of the model has further insightful implications for regressions on simulated data. We see in Table 3.4 that wages are most closely correlated with vacancies and job creation: by (3.31) and (3.16), vacancies and wages are both linearly related to employment levels at hiring firms, and only the mass of not-hiring firms prevents their correlation from being unitary. Table 3.7 regresses wages versus vacancies, and finds a much higher $R^2$ than that obtained in regressions with employer size and/or firm profits. The significance of job creation in a cross-section regression persist also when we control for firm size, as reported in Table 3.8.

Finally Table 3.9 regresses wages on all the observable variables. Each coefficient is significant and, with respect to Table 3.7, $R^2$ increases further.
Table 3.7: Regression of wages on vacancies / job creation

<table>
<thead>
<tr>
<th>Variable</th>
<th>Standardized estimate</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vac.</td>
<td>0.9093</td>
<td>165.7</td>
</tr>
</tbody>
</table>

Table 3.8: Regression of wages on firm size and vacancies

<table>
<thead>
<tr>
<th>Variable</th>
<th>Standardized estimate</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>0.08615</td>
<td>16.24</td>
</tr>
<tr>
<td>Vac.</td>
<td>0.8706</td>
<td>164.1</td>
</tr>
</tbody>
</table>

Table 3.9: Regression of wages on profits, size and jc.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Standardized estimate</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>0.2573</td>
<td>49.41</td>
</tr>
<tr>
<td>Prof.</td>
<td>-0.1678</td>
<td>-32.22</td>
</tr>
<tr>
<td>Vac.</td>
<td>0.8330</td>
<td>160.0</td>
</tr>
</tbody>
</table>
3.4 Discussion

It would be misleading, of course, to give structural interpretations to the signs or sizes of regression coefficients in Table 3.9: all regressors are endogenous, and their significance spuriously reflects their role of proxies (to various degrees) for the unobservable labour-demand parameter \( \eta' \). Here we review in more detail the evidence on wage/size differentials and the various theoretical approaches to their interpretation. We seek to evaluate the overall realism of the modeling approach we propose, discussing ways in which it incorporates and generalizes earlier theoretical models, and pointing out novel empirical implications.

3.4.1 Evidence and Theoretical Interpretations

Krueger and Summer (1988), using panel observations of individual workers, find that industry dummies in individual wage equations are always significant, even when controlling for individual effects, quality of working conditions and union threats. They conclude that the traditional explanation for wage differentials, based on the idea of compensating differentials does not find sufficient support in the data. Alternatively, efficiency wage theories (Akerlof, 1982) seem particularly appropriate for rationalizing the existence of persistent and pervasive wage differentials.

In a world with homogeneous workers and decreasing marginal returns to labour, the wage-size effect must necessarily be connected to a firm’s ability to pay. Traditional efficiency wage models (Weiss, 1966 and Mellow, 1982) explicitly take into account this possibility.

A positive association between wage paid and employer size was noted by Krueger and Summers (1988), confirmed by many papers on wage differentials (e.g. Even and Macpherson, 1994; Schmidt and Zimmermann, 1991; Morissette, 1993), and exhaustively documented by Brown and Medoff (1989) who consider
six possible explanations for the size-wage effect. They find support for the idea that large employers hire higher quality workers. Nevertheless, their fixed effect estimates in individual wage equations suggest strongly that the size-wage effect cannot be explained by a pure size-labour quality differential. Unfortunately, the other explanations considered by Brown and Medoff fail to account for the remaining effect. Possible mechanisms include the fact that large firms pay higher wages to compensate for inferior working conditions (Masters, 1969; Stafford, 1980); large firms face higher risk of unionization (Podgursky, 1986) or they share with workers their above normal profits (Weiss, 1966; Mellow, 1982); they pay higher wages to offset the lower applicant-to-job vacancy ratio (Weiss and Landau, 1984) or they pay higher wages to reduce costly monitoring (Oi, 1983). The ability to pay argument, particularly relevant for the issues of this paper, is rejected by Brown and Medoff on the basis that large firms do not have more rigid product demand. This, of course does not exclude the possibility for large firms of being, on average, more profitable.

A second strand of literature on wage differentials refers to a persistent correlation between wages and various measures of an employers's ability to pay. Blanchflower et al. (1996) merge data on individual wages with industrial data on profits and show that in various wage equations industrial profits are persistently significant.\(^5\) The major contribution of their paper is to show that a rise in sector's profitability leads to a long-run increase in wages. These test, they conclude, produce clear evidence for the fact that pay determination appears to exhibit elements of rent-sharing.

\(^5\)The fact that wages are positively correlated with various measure of employer ability to pay is not new in the literature (Dickens and Katz 1987, Katz and Summers 1989).
3.4.2 Wage-differential Implications of Employment Dynamics

Our modeling perspective does not deny the relevance of intuitive "ability to pay" mechanisms in wage determination, and is quite consistent with the empirical evidence supporting its real-life relevance. Rather, a dynamic approach to wage and firm size determination organizes intuitive insights from previous literature in a coherent structural approach. In fact, the model does incorporate in a fashion the "ability to pay" idea underlying the empirical role of profits in cross-section wage regressions, and a measure of profits is significant when regressions are run on model-generated data. Thus, the Blanchflower et al. findings are readily rationalized by the model.

Our structural approach, however, points out that such correlations need not be causal and—perhaps more importantly—that firms size, as well as firm profitability, is in general related to wages. The model interprets rent-sharing phenomena in the tradition of bilateral-bargaining approaches to wage determination. In an environment where matching is costly and time consuming, rents reflect scarcity of suitable labour for a growing firm. Inasmuch as wages are determined by the standard surplus-sharing rule (3.10) in a matching environment, part of such "adjustment rents" is transferred to incumbent workers.

As firms reach their long-run desired employment levels, however, the marginal surplus from additional employment declines: while firms with stronger labour demand (and higher profits) are willing to pay more for the labour they hire, and do so in equilibrium, wages are a decreasing function of employment along the adjustment path associated to a given labour demand schedule. However, the "compensating differentials" idea of other empirical approaches to wage-size relationships is also not extraneous to our model, and it can explain more persistent wage differentials. In the model, a particular (and also dynamic) sort of compensating differentials plays a role—namely, those deriving from the dif-
ferent outlook offered to employees by firms at different points in the range of productivity states. To see this, consider (3.9): a (large) hiring firm, upon reaching its statically optimal employment level $\tilde{\ell}$ such that $A(\tilde{\ell}, \eta) = 0$, would offer its workers the same human capital level $J^u$ as a small (firing) firm—but not the same wage level: things can only get better for a small firing firm and (in the simple case we consider) things can only become worse for a firm which has stopped hiring. At small, labour-hoarding firms, $rJ^u$ is obtained through a combination of a low wage $w(l^{min}, \eta_h)$ plus an expectation of positive capital gains. Large firms, conversely, can offer no capital gains once they reach their long run position, and it must be true that

$$w(\tilde{\ell}, \eta) > w(l^{min}, \eta_h) \quad \forall i > n.$$  

(3.37)

Thus, wage size differentials exist between firms which are not hiring (or are in their “long run” positions) if their outlook on future exogenous events is different; in general, a positive association between wages and labour-demand strength (hence employment) can be expected whenever a stationary process disturbs individual employers’ demand schedules, and mobility is costly for workers. The model proposed here also features adjustment-associated rents: along firm-specific productivity “cycles,” because of slow adjustment in the hiring process, a growing firm enjoy pure economic rents $A(l, \eta_h) > 0$ (and its employees are faced by prospects of negative capital gains if, as in the special case which we can solve explicitly, things can only get worse for them). Wage size differentials can be interpreted in terms of firm-rent effects during the adjustment process, but only as a compensating differential effect when looking across states.

### 3.5 Conclusion

The dynamic mechanism we focus on combines elements of standard theoretical insights, and is consistent with empirical findings put forward in the literature in
their support. It also offers distinctive empirical implications. In the specialized parameterization that allows explicit solution, wage dispersion is higher among small firms (some of which are growing fast, while others are in the model’s “firing” or inactive state) than among large firms (which, inasmuch as the outlook for future developments is uniformly negative for all, offer similar wages to their employees). The next chapter considers a more complex case where transitions need not always occur from hiring to firing, or vice versa. When transitions between different hiring states are possible, of course, large firms may face heterogeneous expectations of further productivity developments (reintroducing wage differentiation among them). Unfortunately, the model does not admit analytic solutions in this case (and, when more than one “bad” transition is possible, the possibility arises that firms will optimally choose inaction, further complicating the model’s algebra).

The extension of the next chapter, however, does not change the basic qualitative predictions of the model in an important respect. In a dynamic environment, wages should be positively related to employment levels—to the extent that both size and wages are correlated to the (unobservable) current position of various firm’s labour demand. The model does suggest a better proxy than employment for explaining firm level wage dispersion and “firm quality” effects: vacancy posting is univocally determined by the marginal value of labour for individual firms (which determines wages in rent-sharing fashion); in turn, vacancy posting determines the rate of employment growth at the individual-firm level. Hence, after controlling for employment levels, wages are predicted to be higher in faster-growing firms. To our knowledge, there is no evidence on whether this is or is not the case in the data. This is not surprisingly, since datasets with information both on worker characteristics and individual firms panel dynamics are hard if not impossible to come by. The statistical significance of employment growth in firm-level wage equations, however, would be a natural test of our
theoretical perspective's empirical relevance.
Chapter 4

Wages and the Size of Firms: a numerical approximation to the general model

4.1 Introduction

Chapter 3 showed that the marginal value function $A(.)$ is the solution to

$$r A(\cdot) = (\eta_i - \sigma l) - (h_i + 2k_i l) + A(\cdot) \frac{\partial A(l, \eta_i)}{\partial l} \sigma^2 c$$

$$+ \delta_i \overline{p_i} \left(\max [0, A(l, \eta) - A(\cdot)]\right), \quad (4.1)$$

where

$$h_i = \beta \eta_i + (1 - \beta) r J^n, \quad k_i = -\frac{\beta \sigma}{1 + \beta}.$$ 

Equation (4.1) is a non-linear, non-homogeneous differential equation that cannot be solved analytically. Nevertheless I showed that the general equation has a linear meaningful solution for a particular specification of transition probabilities. In chapter 3 I assumed that all negative $\eta$ transitions bring the firm to the lowest productivity state and I let the other states be reachable only from the latter so that, consistent with our notation, $p_{in} = 1$ for $i > n$ while $\overline{p_n}$ featured non-zero transition probabilities to all higher productivity states. Furthermore, under these parametric assumptions I explicitly solved for the joint steady-state...
distribution of employer size and wages. Finally, regressions on montecarlo data
drawn from the long-run distribution showed that the model offered a structural
interpretation of existing evidence on firm-size wage differential.

Although analytically convenient, the parametric assumption of the previous
chapter is not fully realistic and the reader may find himself uncomfortable with
the asymmetry between negative and positive transitions of the productivity pa-
rameter $\eta$. In section (4.2) I relax this simplifying assumption and I work out a
numerical approximation to the solution of the general equation (4.1). Section
(4.3) compares the value functions under different parametric assumptions while
section (4.4) simulates the behaviour of a firm under the general model of the
next section and it shows that all the qualitative results of the specific case solved
in the previous chapter continue to hold.

4.2 Solving the General Model: An example

In what follows I assume that there are only three productivity states $\eta_1 >
\eta_2 > \eta_3$. Transitions among these states occur with probability intensities $\delta_i$, for
$i = 1, 2, 3$, while the transition probability vector $\vec{p}_i$ attaches positive weight to $p_{ij}$
for $i, j = 1, 2, 3$ with, for notational convenience, $p_{ii} = 0$. The lowest productivity
states $\eta_3$ is the non-hiring states and $l_3$ is the amount of labour accumulated by
these firms. Conversely, the highest productivity $\eta_1$ represents a hiring state. In
this general problem a firm in the intermediate state, $\eta_2$, may be hiring or firing
depending on the quantity of labour already accumulated.

Let us consider two points $l$ and $l + dl$ in the region between $l_3$ and $\bar{l}_2$, where
$\bar{l}_2$, by definition, solves

$$ A(\bar{l}_2, \eta_2) = 0. \quad (4.2) $$

At a point $l$ between $l_3$ and $\bar{l}_2$, $A^i(l, \cdot) > 0$ for $i = 1, 2$ and it is optimal to hire
at idiosyncratic productivities $\eta_1$ and $\eta_2$. By virtue of (4.1), the marginal value
functions evaluated at $l$ solves
\[
(r + \delta_1)A^i(l,.) = \eta_i - \sigma l - (h_i + 2k_i l) + \frac{\vartheta^2}{c} A^i(l,.) \frac{\partial A^i(l,.)}{\partial l} + \delta_{ij}p_{ij}A^j(l,.) \tag{4.3}
\]
for $i = 1, 2$ and $j = 2, 1$ respectively.

By definition of partial derivatives, it must be true that for $dl$ sufficiently small
\[
A^i(l,.) = A^i(l + dl,.) - \frac{\partial A^i(l,.)}{\partial l} dl; \quad i = 1, 2. \tag{4.4}
\]
Using (4.4) into (4.3) yields
\[
(r + \delta_1) \left( A^i(l + dl,.) - \frac{\partial A^i(l,.)}{\partial l} \right) = (\eta_i - \sigma l) - (h_i + 2k_i l)
+ \frac{\vartheta^2}{c} \left( A^i(l + dl,.) - \frac{\partial A^i(l,.)}{\partial l} dl \right) \frac{\partial A^i(l,.)}{\partial l}
+ \delta_{ij} p_{ij} \left( A^i(l + dl,.) - \frac{\partial A^i(l,.)}{\partial l} \right). \tag{4.5}
\]
Evaluating (4.3) at $l + dl$, subtracting the result from (4.5), and rearranging yields
\[
\frac{\vartheta^2}{c} dl \left( \frac{\partial A^i(l,.)}{\partial l} \right)^2 - \left( (r + \delta_1) dl + \frac{\vartheta^2}{c} A^i(l + dl,.) \right) \left( \frac{\partial A^i(l,.)}{\partial l} \right)
- (\sigma + 2k_i) dl + \frac{\vartheta^2}{c} A^i(l + dl) \frac{\partial A^i(l + dl)}{\partial l} = 0. \tag{4.6}
\]
for $i = 1, 2$ and $j = 2, 1$.

For given values of $\frac{\partial A^i(l+dl,.)}{\partial l}$ and $A^i(l + dl,.)$ for ($i = 1, 2$), (4.6) forms a non-linear system in $\frac{\partial A^i(l,.)}{\partial l}$, $i = 1, 2$. If we evaluate (4.6) at $l + dl = l_2$, by virtue of (4.2), the last term in (4.6) vanishes for $i = 2$. Furthermore, for each $l \geq l_2$, $A^i(l, \eta_1)$ solves
\[
(r + \delta_1)A^i(l,.) = \eta_1 - \sigma l - (h_1 + 2k_1 l) + \frac{\vartheta^2}{c} A^i(l,.) \frac{\partial A^i(l,.)}{\partial l}. \tag{4.7}
\]
Equation (4.7) is a differential equation that coincides with equation (17) of the previous chapter and its linear solution reads

\[ A(l, \eta_1) = a_1 + b_1 l \quad \forall l \geq l_2. \]  

(4.8)

By virtue of (4.8) it follows that

\[ A^1(l_2, \eta_2) = a_1 + b_1 l_2, \]

and

\[ \frac{\partial A^1(l_2, \eta_1)}{\partial l} = b_1, \]

where \( b_1 \) and \( a_1 \) are defined in equation (19) of chapter 3. To actually solve (4.6) at \( l_2 \) we firstly need an expression for \( l_2 \). By virtue of (4.2) and (4.8), (4.3) evaluated at \( l_2 \) reads

\[ \eta_2 - \sigma l_2 - (h_2 + 2k_2 l_2) + \delta_2 \left( a_1 + b_1 l_2 \right) = 0. \]  

(4.9)

We thus have sufficient conditions for solving (4.6) at \( l + dl = l_2 \). Moving backward and proceeding recursively, making use of (4.4), it is possible to obtain values for the marginal value functions at a sequence of points \( \{l_k, A^1(l_k, \eta_1), A^2(l_k, \eta_2)\} \). Finally, an approximation of \( l^{\text{min}} \equiv l_{k^*} \), is obtained at a point in the sequence that satisfies

\[ \eta_3 - \sigma l_{k^*} - w(.) - \frac{\partial w}{\partial l} l_{k^*} + \delta_3 \sum_{i=1}^{2} p_{3i} A^i(l_{k^*}, \eta_i) \simeq 0, \]  

(4.10)

where

\[ w(l_{k^*}, \eta_3) = r Ju - \frac{\beta}{1 - \beta} \delta_3 \sum_{i=1}^{2} p_{3i} A^i(l_{k^*}, \eta_i), \]  

(4.11)

and

\[ \frac{\partial w(l_{k^*}, \eta_3)}{\partial l} = -\frac{\beta}{1 - \beta} \delta_3 \sum_{i=1}^{2} p_{3i} \frac{\partial A^i(l_{k^*}, \eta_i)}{\partial l}. \]  

(4.12)
4.3 A Simple Example

To implement the methodology sets forth in the previous section I use the structural parameters listed in Table (4.1). Section (4.2) showed how to approximate the marginal policy function for given values of the equilibrium quantities $\theta$ and $J^u$. As I argued in more details in the previous chapter, the policy function of the general model is not analytically representable. Even though it is possible to derive a numerical approximation of the policy function for given values $\theta$ and $J^u$, it is impossible to calculate analytically the steady-state distribution and the equilibrium values of $\theta$ and $J^u$ consistent with individual policies. With this limitation in mind, I consider the results of the numerical approximation.
Figure 4.2: Marginal Value Functions
Figure (4.1) shows the marginal value function of the best productivity in the market $A_1(l, \eta_1)|_{p_{12}>0}$, in the interval between the labour hoarding value $l_3$ and the maximum amount of labour accumulated by the intermediate firms $l_2$. The same figure reports the marginal value functions for the same parameters of Table (4.1) when the switching probability between the highest and the intermediate productivity ($p_{12}$) is zero. The latter parameterization is the one worked out in chapter 3 and the dotted line in Figure (4.1) is the linear solution $A_1(\cdot)|_{p_{12}=0} = a_1 + b_1 l$. Conversely, the bold line of Figure (4.1) is $A_1(\cdot)|_{p_{12}>0}$, the marginal value function of the best productivity $\eta_1$ when, conditional on changing the idiosyncratic productivity, there is a non-null probability of reaching the intermediate state $\eta_2$. Intuitively, as accumulated labour tends to $l_2$ the marginal value function $A_1(\cdot)|_{p_{12}>0}$ converges to $A_1(\cdot)|_{p_{12}=0}$: as the marginal value functions of the intermediate state $A_2(\cdot)$ tends to zero the capital gain associated to reaching the intermediate state becomes smaller and, eventually, tends to zero, exactly as in the parameterization in which any productivity change results in a catastrophic switch to the lowest productivity state $\eta_3$.

Figure (4.2) reports the marginal value functions $A_1(\cdot)$ and $A_2(\cdot)$ between the labour hoarding size $l_3$ and the maximum firm size in the market $l_1$. The figure shows once again the functions $A_1(\cdot)|_{p_{12}>0}$ and $A_1(\cdot)|_{p_{12}=0}$ between $l_3$ and $l_2$, and it adds the linear solution $A_1(\cdot)|_{p_{12}=0} = A_1(\cdot)|_{p_{12}>0}$ in the interval between $l_2$ and $l_3$. When accumulated labour is higher than $l_2$, following an idiosyncratic change $\eta_i$, it makes no difference whether the new value is $\eta_3$ with probability one or it is $\eta_2$ or $\eta_3$ with positive probability $p_{12}$ and $p_{13}$.

The lower part of figure (4.2) shows the marginal value functions $A_2(\cdot)|_{p_{21}=0}$ and $A_2(\cdot)|_{p_{21}>0}$. Since a change in the idiosyncratic state may result not only in a capital loss to the lowest $\eta_3$, but also in a pure capital gain to $\eta_1$, the slope of $A_2(\cdot)|_{p_{21}>0}$ is greater than the slope of $A_2(\cdot)|_{p_{21}=0}$. Furthermore, the firm maximum size $l_2|_{p_{21}>0}$ is greater than $l_2|_{p_{21}=0}$. Even though the marginal
Figure 4.3: Wage Functions

Productivity \((\eta_2 - \sigma l)\) is close to its marginal cost \((w() + \frac{\partial w}{\partial l})\), the prospect of a positive capital gain makes the firm willing to accumulate more labour.
Table 4.1: Baseline parameter values

<table>
<thead>
<tr>
<th>Variable</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matching Function:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>matching elasticity</td>
<td>$\nu$</td>
<td>-0.400</td>
</tr>
<tr>
<td>constant</td>
<td>$\xi$</td>
<td>1.000</td>
</tr>
<tr>
<td>Labor demand slope (all $i$)</td>
<td>$\sigma$</td>
<td>0.325</td>
</tr>
<tr>
<td>Labor demand intercepts</td>
<td>$\eta_1$</td>
<td>1.895</td>
</tr>
<tr>
<td></td>
<td>$\eta_2$</td>
<td>1.692</td>
</tr>
<tr>
<td></td>
<td>$\eta_n$</td>
<td>1.462</td>
</tr>
<tr>
<td>Transition intensities</td>
<td>$\delta_1$</td>
<td>0.150</td>
</tr>
<tr>
<td></td>
<td>$\delta_2$</td>
<td>0.180</td>
</tr>
<tr>
<td></td>
<td>$\delta_n$</td>
<td>0.270</td>
</tr>
<tr>
<td>Distribution across hiring states of positive shocks</td>
<td>$p_{11}$</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>$p_{12}$</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td>$p_{13}$</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td>$p_{21}$</td>
<td>0.22</td>
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<tr>
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<td></td>
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</tr>
<tr>
<td>search cost slope</td>
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</tr>
<tr>
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</tr>
<tr>
<td>employees' bargaining share</td>
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</tr>
<tr>
<td>Unemployed income</td>
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</tr>
<tr>
<td>Firing cost</td>
<td>$F$</td>
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</tr>
<tr>
<td>Equilibrium values:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market tightness (assumed)</td>
<td>$\theta$</td>
<td>2.321</td>
</tr>
<tr>
<td>Unemployed welfare (assumed)</td>
<td>$rJ^U$</td>
<td>0.660</td>
</tr>
</tbody>
</table>
4.4 The Wage-Size Effect in the General Model

Figure (4.3) reports the wage functions. As long as the marginal cost of firing is zero the presence of positive switching probabilities $p_{12}$ and $p_{21}$ do not affect the linear solution worked-out in the previous chapter. With respect to the model with $p_{12} = p_{21} = 0$, the interesting feature of the parameterization of Table (4.1) is that large firms with $l > \bar{l}_2$ fire an amount of labour $\Delta l = l - \bar{l}_2$ when their productivity switch to $\eta_2$. As a consequence, a mass-points of non-hiring, low-wage firms will concentrate on $\bar{l}_2$. In general, as it is shown in section (4.2) of chapter (3), even though firms in their long-run positions offer the same human capital $J_u$, the wage paid by these firms will be different and it will depend on the capital gains faced by each firm in its long-run position. In this direction, and consistent with the notion of this chapter, it is necessary to study in some details the wages $w(l_3, \eta_3)$, $w(\bar{l}_2, \eta_2)$ and $w(\bar{l}_1, \eta_1)$. Firstly, the largest firms in the market, namely firms in $\bar{l}_1$, do not face any positive capital gain and, by virtue of (21) of chapter 3, $w(\bar{l}_1, \eta_1)$ reads

$$w(\bar{l}_1, \eta_1) = rJ_u. \quad (4.13)$$

Conversely, firms in $\bar{l}_2$ and $l_3$ face some prospect of capital gains and it must be true that

$$w(\bar{l}_1, \eta_1) > w(\bar{l}_2, \eta_2) \quad (4.14)$$

and

$$w(\bar{l}_1, \eta_1) > w(l_3, \eta_3). \quad (4.15)$$

Furthermore, the wage paid by non non-hiring firms at $l_3$ will be in general different, and likely smaller, than the wage paid at $\bar{l}_2$. By virtue of (21) of chapter 3

$$w(\bar{l}_2, \eta_2) - w(l_3, \eta_3) = \frac{\beta}{1 - \beta} \delta_3 \left( p_{31} A^1(l_3, \eta_1) + p_{32} A^2(l_3, \eta_2) \right) - \frac{\beta}{1 - \beta} \delta_2 p_{21} A^1(\bar{l}_2, \eta_1) \quad (4.16)$$
is positive unless \( \delta_3 p_{31} \ll \delta_2 p_{21} \) so as to over-compensate the fact that in (4.16) \( A^2(l_3, \eta_2) > 0 \) and \( A^1(l_3, \eta_1) > A^1(l_2, \eta_1) \). If we assume \( \delta_3 p_{31} \sim \delta_2 p_{21} \) it follows that

\[
\omega(l_1, \eta_1) > \omega(l_2, \eta_2) > \omega(l_3, \eta_3). \tag{4.17}
\]

Equations (4.16) and (4.17) imply that our model prospective results in a wage-size effect even when the firms reach their long-run position. But if we compare the results in (4.16) and (4.17) with those of section (4.2) of chapter 3, a difference emerges. In the parameterization of chapter 3 large firms can offer no capital gains to their employees and the model implies a wage-differential between small and large firms in their long-run position, independently on the size and the intensity of productivity shocks. Conversely, in the general specification of this chapter, firms of intermediate size \( l_2 \) face prospects of positive and negative capital gains (depending whether \( \eta_2 \) switches to \( \eta_1 \) or to \( \eta_3 \) ) and, although unlikely, it is conceptually possible that a parameterization of the model overturns the second inequality in (4.17). Thus in the general specification, when looking across firms’ long-run positions, by virtue of (4.14) and (4.15), it will be true that the largest firms in the market offer the highest wage; but simultaneously, when we consider the set of all possible long-run position, the relationship between wage and size will not necessarily be monotonic.

Furthermore, it is still true that during the firm optimal adjustment path positive idiosyncratic shock are associated with larger employment levels and higher wages. This is confirmed by figure (4.4), which simulates the behaviour of a single firm that hires according to the parameterization of Table (4.1) and the policy functions of Figures (4.1) and (4.2). Given the steady-state nature of our market, the time-series observations of single firm between \( t = 1 \) and \( t = T \), for sufficiently large \( T \), replicate the cross-section behaviour of \( T \) firms in the market. From the cluster of points in Figure (4.4) two point masses are clearly distinguishable, in correspondence to \( l_3 \) and \( l_2 \) and, overall, the cross-section ob-
observations imply a sizable wage-size effect. This is confirmed by the regression in Table (4.2), where the coefficient on firm size is positive and significant. Similarly to the analysis of the text, it is possible to obtain regressions of wages on firm size, firm vacancies and profits. The results are qualitatively analogous to those in the text and Table (4.3) confirms that vacancies, profits and size have significant effect on wages. Overall, the interpretation of Tables (4.2) and (4.3) is the same as the one given in Chapter 3.
Figure 4.4: The Wage-Size Effect
Chapter 5

Job Flows and Plant Size Dynamics: Traditional Measures and Alternative Econometric Techniques

5.1 Introduction

In the last few years, much of the discussion on the labour market focused on the process of job formation and destruction and great emphasis has been given to the relation between job flows and firm size, both in the policy debate (OECD 1994a) and among academic scholars (Davis et al. 1995).

If we define firm size as employment in a base year, across countries, two statistical regularities hold. Job creation and destruction are substantially greater among small firms and net employment changes are a decreasing function of establishment size. These findings stimulated the policy emphasis on the crucial role of small firms in the process of job formation. Unfortunately, several statistical problems are associated with these findings. Among others, Davis et al. (1994; 1995) point out the regression fallacy associated with the relationship between net employment changes and firm size and they suggest an alternative approach based on the notion of long run optimal size. When firm size is measured as the average employment across all years in the sample, Davis et al. show that in the
U.S. manufacturing sector there is no clear relationship between net job creation and firm size.

In this paper I firstly assess the traditional measures and I argue that any definition of firm size that arbitrary forces each unit in the sample into a pre-defined size category, will ignore the flows of jobs between size categories. Furthermore, when firm size is defined as the average employment across all years in the sample, a positive relationship between firm size and net job creation may simply indicate that initially smaller firms created jobs throughout the period and end up relatively larger.

Clearly, to estimate properly firm size convergence avoiding the regression fallacy and to follow accurately employers between size categories, it is necessary to apply an alternative econometric technique. Fortunately, methodology for this purpose has recently been introduced by Quah (1993a; 1993b) in the context of economic growth, and applied by Lamo and Koopmans (1995) in the study of plant distribution in the Chemical sector and by Konings (1995b) in a paper that studies the evolution of plant size in the British manufacturing sector. Using a balanced panel for the Mexican manufacturing sector, I show how conventional results may change when firm size dynamics is estimated non parametrically. Overall, I find no evidence of small firms systematically creating more jobs than larger firms and, thus, no evidence of convergence to the mean for the sample as a whole. I show how distribution dynamics varies across industries and how it is linked to gross and net flows in each sector. I observe convergence to the mean in relatively stable sectors and asymmetric dynamic behaviour between expanding and declining industries.

The paper proceeds as follows. Section (5.2) briefly describes the data and measures job flows for the Mexican manufacturing industry. Section (5.3) assesses the traditional methodology for studying the relationship between job flows and establishment size. Section (5.4) describes an alternative technique based on a
direct estimate of the dynamics of the entire firm-size distribution. Section (5.5) applies the methodology at the aggregate and at the industry level while section (5.6) briefly summarizes and concludes the paper.

5.2 Measurement Criteria

5.2.1 The Data

The dataset I will use is a panel of 2021 continuing establishment over the period of 1984 to 1990 (7 years). The source of data are administrative records of the “Annual Industrial Survey” of the Mexican manufacturing industry. On average, it covers between 70 and 80 percent of the industry in terms of production and employment. The average establishment size in the sample is 220 employees and entry and exit of establishments are not observed. Each establishment is assigned to an industry at the level of the Mexican Census classification. These industries have been aggregated to the “Raga” level, which corresponds to the classification used in the Input Output table of 1985. The number of industries is 47, which can be aggregated to 10, as I do in section 5.5. Within the sample, large establishments correspond to the manufacturing population while small firms are randomly sampled. The under-representation of small firms may apparently be a problem for the issue of this paper, but I do not see why the results of this paper should be affected by the fact that smaller establishments are randomly sampled. I will come back to this point later in the paper.

5.2.2 Notation and Definitions

Let \( x_{it} \) be the size of establishment \( i \) at time \( t \), which, as we outline in the next section, can be measured as employment at time \( t \), as employment between \( t \) and \( t+1 \) or as average employment in all years in the sample. The growth rate of
establishment \(i\) time \(t\), \(g_{it}\), is then defined as

\[
g_{it} = \frac{n_{it} - n_{it-1}}{x_{it}},
\]

(5.1)

where \(n_{it}\) and \(n_{it-1}\) are employment for establishment \(i\) at time \(t\) and \(t - 1\) respectively. If \(x_{it}\) is measured as the average employment between \(t\) and \(t - 1\), (5.1) is similar to the growth rate used by Davis-Haltiwanger (1990) and (1992), with deaths (births) corresponding to the left (right) endpoint. In the present paper the interval will be somewhat smaller since we do not observe deaths and births. The gross job creation and destruction rate are related to the size weighted frequency distribution of firm growth rates in the following way. Let job creation in sector \(j\) at time \(t\) be defined as

\[
JC_{jt} = \sum_{j \in I} g_{it} \frac{x_{it}}{X_{jt}} \forall i : g_{it} > 0,
\]

(5.2)

where \(X_{jt}\) represents the size of sector \(J\) and \(I\) is the set of all establishment in sector \(j\) at time \(t\). Job destruction rate, \(JD_{jt}\), is defined analogously for declining establishments. Gross job reallocation in sector \(j\) at time \(t\), \(JR_{jt}\), is simply the sum of gross job creation and destruction while the difference between the two, \(NET_{jt}\) is the traditional measure of net employment changes. Since we do not observe jobs, vacancies and entry and exit, the measurement criteria adopted underestimate the true measures of job creation and destruction. For the first two problems there is little one can do about it. The problem with entry and exit is potentially more serious. However, Hamermesh (1993) and OECD (1994a) estimate the relative importance of the various flows of jobs and conclude that the contribution to net and gross employment changes of continuing firms accounts for roughly 70 percent of the gross flow of jobs. With this coefficient in mind, we proceed to the calculation of the flows.
Table 5.1: Job Flows in the Mexican Manufacturing Sector

<table>
<thead>
<tr>
<th>Year</th>
<th>Job Creation Jc</th>
<th>Job Destruction Jd</th>
<th>Net Employment Change Net</th>
<th>Job Reallocation Jr</th>
</tr>
</thead>
<tbody>
<tr>
<td>1984-85</td>
<td>7.49</td>
<td>3.6</td>
<td>3.89</td>
<td>11.09</td>
</tr>
<tr>
<td>1985-86</td>
<td>4.65</td>
<td>6.47</td>
<td>-1.82</td>
<td>11.19</td>
</tr>
<tr>
<td>1986-87</td>
<td>4.29</td>
<td>5.18</td>
<td>-0.89</td>
<td>9.48</td>
</tr>
<tr>
<td>1987-88</td>
<td>4.66</td>
<td>4.1</td>
<td>0.56</td>
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</tr>
<tr>
<td>1988-89</td>
<td>7.2</td>
<td>4.08</td>
<td>3.11</td>
<td>11.29</td>
</tr>
<tr>
<td>1989-90</td>
<td>6.77</td>
<td>4.86</td>
<td>1.91</td>
<td>11.64</td>
</tr>
</tbody>
</table>

Pearson Correlat. \( \rho(JC, NET) \) 0.932 (0.006) \( \rho(JD, NET) \) -0.859 (0.028) \( \rho(JR, NET) \) 0.4 (0.421)

Marginal significance in parenthesis

5.2.3 A Brief Look at the Aggregate Flows

Figure (5.1) plots aggregate manufacturing employment over time. The second half of the eighties is a period of sustained net job creation with overall employment growth equal to 7 percent between 1984 and 1990. Table (5.1) reports the time series introduced in the previous section and Figure (5.2) plots the series against time. Values for net employment changes indicate that between 1984 and 1990 employment fluctuates substantially, with more than 3 percent employment growth in 1984 and 1988 and almost 2 percent fall in 1985. Correlation values in Table (5.1) show that employment changes are strongly correlated with both job creation and destruction. Job reallocation, with the exception of 1986 and 1987 is approximately constant, and does not show any increase during the recession, as in U.S. data compiled by Davis and Haltiwanger (1990).
Figure 5.1: Manufacturing Employment 1984-1990
Figure 5.2: Aggregate Job Flows: 1984-1990
5.3 Job Flows and Firm Size. The Traditional Approach: Assessment and Measures.

The definition of gross job flows in (5.2) depends clearly on the way establishment size is defined. Different size definitions yield different results. In this section I show how the results are affected by each definition and I discuss the methodological problems associated with each measure.

When we measure establishment size by employment in the base year, across countries, two statistical regularities hold. Table (5.2) reports the distribution of gross job flows and employment by establishment size for eight OECD countries. Job reallocation declines sharply as a function of size. R&P (1995) argues that the “empirical evidence in favour of the inverse relationship between job turnover and firm size appears to be the only sure result that we have... It shows up in all countries, independently of the data sources and methodology, as well as of the prevailing institutions”. Table (5.2) reports also a remarkable relationship between net employment changes and firm size. For 7 out of 8 countries in the sample net job creation is positive in small firms, while the opposite seems true for large firms. In this section we consider in some details these findings and we point out why the second relationship is based on a common regression fallacy and thus totally uninformative on the role of small firms in the job creation process.

Table (5.3) reports gross flows by establishment size for the Mexican manufacturing sector and confirms the empirical regularities found in the OECD economies of Table (5.2). Job reallocation is more than 23 percent for the smallest size category and declines sharply as a function of size. Furthermore, with an average employment growth of 1 percent, large firms underperform the aggregate economy while, at the same time, small firms grow at an exceptional average rate of 11 percent. The 11 percent growth is entirely driven by the average creation rate of over 17 percent.
Table 5.2: Net Employment Change by Establishment Size Class across the OECD

<table>
<thead>
<tr>
<th>Country</th>
<th>period</th>
<th>Size</th>
<th>Job Reallocation</th>
<th>Net Changes</th>
</tr>
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<td>&lt;20</td>
<td>39.01</td>
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<td>20-99</td>
<td>26.89</td>
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<td>&gt;500</td>
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<td>-0.41</td>
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<td>Canada</td>
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<td></td>
<td></td>
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<td>16.59</td>
<td>-6.86</td>
</tr>
<tr>
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<td>1986-91</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>&lt;20</td>
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<td>20-49</td>
<td>18.20</td>
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<td></td>
<td>500-999</td>
<td>15.3</td>
<td>-2.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000-2499</td>
<td>14.1</td>
<td>-2.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2500-4999</td>
<td>13.3</td>
<td>-2.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5000-9999</td>
<td>11.8</td>
<td>-2.4</td>
</tr>
</tbody>
</table>

Table 5.3: Job Flows by Establishment Size Category

<table>
<thead>
<tr>
<th>Establishment Size as Employment in Base Year</th>
<th>Employees &lt; 20</th>
<th>Employees 20 - 50</th>
<th>Employees 51 - 99</th>
<th>Employees 100 - 500</th>
<th>Employees &gt; 500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Job Creation</td>
<td>17.33</td>
<td>9.07</td>
<td>8.42</td>
<td>6.66</td>
<td>5.1</td>
</tr>
<tr>
<td>Average Job Destruction</td>
<td>5.99</td>
<td>4.9</td>
<td>5.2</td>
<td>4.7</td>
<td>4.71</td>
</tr>
<tr>
<td>Average Job Reallocation</td>
<td>23.33</td>
<td>14.01</td>
<td>13.68</td>
<td>11.3</td>
<td>9.8</td>
</tr>
<tr>
<td>Average Net Change</td>
<td>11.3</td>
<td>4.08</td>
<td>3.1</td>
<td>1.9</td>
<td>0.39</td>
</tr>
</tbody>
</table>

Average weighted by the number of jobs in each year

Recently, a number of scholars, notably Davis et al. (1995), have pointed out the statistical problems linked to the results in Table (5.3). Firstly, higher rates of job creation by small establishments should be treated with particular attention. Given our measure of job flows, it turns out that a firm of size 10 that creates and destroys one job records a reallocation rate of 20%, while if the same two jobs had been created by an establishment of 100 employees, the same figure would be 2%. Furthermore, higher net employment changes by small firms do not imply by themselves that small establishments create proportionally more jobs. Such conclusions should also consider the share of job creation with respect to the employment share by each category. Table (5.4) takes explicitly into consideration the shares and the proportion of jobs created by each category. The fraction of jobs created by each category over the employment share in the same category is a measure independent of the relative size of the category and, consequently, a proper comparison across categories can be made. On average the ratio between the share of jobs created by small firms over their employment share is more than 3, against 0.86 for the establishments with more than 500 employees. Among the two extremes, the relationship falls monotonically. It is necessary to stress the difference between net and gross flows before reaching any
Table 5.4: Proportional Measure of Job Flows by Establishment Size Category

<table>
<thead>
<tr>
<th></th>
<th>Employees &lt; 20</th>
<th>Employees 20 – 50</th>
<th>Employees 51 – 99</th>
<th>Employees 100 – 500</th>
<th>Employees &gt; 500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Employment Share</td>
<td>0.2</td>
<td>1.6</td>
<td>4.5</td>
<td>36.4</td>
<td>58.1</td>
</tr>
<tr>
<td>Share of Job Creation Employment Share (a)</td>
<td>3.15</td>
<td>1.56</td>
<td>1.45</td>
<td>1.13</td>
<td>0.86</td>
</tr>
<tr>
<td>Share Job Destruction Employment Share (b)</td>
<td>1.27</td>
<td>1.06</td>
<td>1.11</td>
<td>1.00</td>
<td>0.98</td>
</tr>
<tr>
<td>Net Proportional Share</td>
<td>1.87</td>
<td>0.50</td>
<td>0.34</td>
<td>0.13</td>
<td>-0.12</td>
</tr>
</tbody>
</table>

(a) Job created by each category over employment share in the same category
(b) Job destroyed by each category over employment share in the same category

Conclusion from this partial result. In Table (5.4) I calculate the same ratio for job destruction and it turns out that small firms also play a more active role in the process of job destruction. Again, the relationship between our proportional measure of job destruction and firm size falls monotonically. Even though small firms more than proportionally create and destroy jobs, the last rows of Table (5.4) shows that the difference between rows 2 and 3 in Table (5.4) is positive for small firms and negative only for the very large firms. Overall, Table (5.4) confirms a more active role of small establishments in the process of net job formation.

Naturally, from the results of Table (5.4), we expect the growth rate of each firm to be negatively correlated to its initial size. Establishment growth rate regressions and studies on the evolution of the size distribution have been at
Table 5.5: Regressions of Establishment Growth Rate on Establishment Size

<table>
<thead>
<tr>
<th>Establishment Size as Employment in Base Year</th>
<th>Dependent Variable: Growth Rate of Establishment i at time t</th>
</tr>
</thead>
</table>
| ln(employment)_{it}                         | (1)  
|                                             | -0.0097         
|                                             | (-6.28)         |
| Time Dummies                                | No              |
| Industry Dummies                            | No              |
| F statistic                                 | F(1, 12124) = 39.461 |
| Number of Observations                      | 12126           |
| t statistics in parentheses                 |                |

The simplest version of a firm growth model is one in which growth rates are independent of initial size, as predicted by the well-known Gibrat's Law. We thus estimate

$$\frac{\ln(x_{i,t}) - \ln(x_{i,t-1})}{\Delta t} = a + b\ln(x_{i,t-1}) + u_{it},$$

(5.3)

where $x_{i,t}$ and $x_{i,t-1}$ are employment in establishment $i$ at time $t$ and $t - 1$, $\Delta t$ is the time interval between successive observations and $u_{it}$ is a white noise. Regressions in Table (5.5) estimate a negative and significant coefficient on initial size, even when we control for sectoral and time characteristics. Regressions in Table (5.5) are only illustrations of well-known results in the Industrial Organization literature and equations in the spirit of (5.3) have been considered as evidence against Gibrat's law. Results in Tables (5.3), (5.4) and (5.5) all point towards the same conclusion and should throw evidence on an important relationship between job creation and small firms.

Despite the evidence provided in Tables (5.3), (5.4) and (5.5) several difficulties remain. The major problem connected with the role of small firms in the job creation process is the statistical fallacy known as the Galton fallacy, or regression to the mean. The regression bias arises in any longitudinal data set.
and has received particular attention in the empirics of economic growth (Quah (1993a) and Friedman (1992). Technically, because of the Galton fallacy, when we regress net employment changes by firms on initial employment, a negative coefficient on initial size, exactly as in Table (5.5), is uninformative about the relationship between initial size and firm growth. Intuitively, results in Tables (5.3), (5.4) and (5.5) can be affected by the regression fallacy for the following reason. If a firm suffers from transitory deviation of employment around its long-term optimum size, temporarily smaller firms will gain jobs during their path to equilibrium, and vice-versa for temporarily larger firms. If establishment size is measured as employment in base year, the existence of temporary deviation would bias upward job creation by small firms and job destruction by large firms. Consequently, when we regress firm growth over initial employment, a negative coefficient on initial size may simply capture the existence of temporary shocks, without telling anything about the underlying relationship between firm size and firm growth.

Davis et al. (1995) propose a measure of establishment size that tries to capture an establishment optimal long-run size. To avoid the regression fallacy, they attribute job flows to a smaller or larger size category calculating the average size across all observations in the sample. In their study for the U.S. manufacturing sector, they do not find any systematic relationship between establishment size and long-run firm size measure.

Table (5.6) computes job flows by firm size measuring establishment size as the average employment over all years in the sample. Table (5.6) is constructed using the same size category as Table (5.4), but assigning an establishment to each category according to its average employment between 1984 and 1990. The last row of Table (5.6) reverses the results of Table (5.3) and shows a positive

1Quah (1993b) shows why a negative coefficient in regressions in Table (5.6) is consistent with a stationary standard deviation in the underlying distribution.
Table 5.6: Proportional Measure of Job Flows by Establishment Size Category

<table>
<thead>
<tr>
<th></th>
<th>Employees &lt; 20</th>
<th>Employees 21 - 50</th>
<th>Employees 51 - 99</th>
<th>Employees 100 - 500</th>
<th>Employees &gt; 500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Employment</td>
<td>0.2</td>
<td>1.6</td>
<td>4.5</td>
<td>31.4</td>
<td>63.0</td>
</tr>
<tr>
<td>Share</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Job Creation</td>
<td>7.62</td>
<td>6.41</td>
<td>6.72</td>
<td>6.37</td>
<td>5.43</td>
</tr>
<tr>
<td>Average Job Destruction</td>
<td>8.52</td>
<td>6.79</td>
<td>6.3</td>
<td>5.2</td>
<td>4.18</td>
</tr>
<tr>
<td>Average Job Reallocation</td>
<td>16.1</td>
<td>13.2</td>
<td>13.07</td>
<td>11.6</td>
<td>9.6</td>
</tr>
<tr>
<td>Average Net Change</td>
<td>-0.009</td>
<td>-0.003</td>
<td>0.003</td>
<td>1.11</td>
<td>1.25</td>
</tr>
<tr>
<td>Share of Job Creation Employment Share</td>
<td>1.26</td>
<td>1.10</td>
<td>1.14</td>
<td>1.09</td>
<td>0.91</td>
</tr>
<tr>
<td>(a)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Share Job Destruction Employment Share</td>
<td>1.73</td>
<td>1.44</td>
<td>1.34</td>
<td>1.10</td>
<td>0.87</td>
</tr>
<tr>
<td>(b)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net Proportional Share</td>
<td>-0.47</td>
<td>-0.34</td>
<td>-0.19</td>
<td>-0.01</td>
<td>0.037</td>
</tr>
</tbody>
</table>

(a) Job created by each category over employment share in the same category
(b) Job destroyed by each category over employment share in the same category
monotonic relationship between establishment size and net employment changes. Similar conclusions hold for the bottom row of Table (5.6), which replicates the calculation of Table (5.4) with the new long-run size. A casual interpretation of the result of Table (5.6) indicates that small firms more than proportionally destroy jobs.

Even though Table (5.6) may partially avoid the regression fallacy, it does not give us a clear answer on the relationship between firm size and job flows. Results in Table (5.6) may simply indicate that firms initially small (large) created (destroyed) jobs throughout the period and ended up relatively large (small). Furthermore, to the extent that the latter interpretation is correct, it seems that substantial dynamics between category is taking place. In all the Tables so far presented, we focused exclusively on within category job creation and this practice, by construction, hides any intradistribution dynamics. This discussion should highlight the fact that the definition of small and large establishments is a relative concept and any such definition is somehow arbitrary. In the next section I consider an alternative econometric technique that allows us to measure job flows and firm size avoiding the Galton fallacy and explicitly considering job flows between size categories.

5.4 Job Flows Between Category and Analysis of Convergence

The analysis of the previous section suggested that substantial intradistribution dynamics may take place and a considerable number of firms are overtaking each other. A first way of looking at the problem in a different way is as follows. Let $n_{it}$ denote employment in establishment $i$ at time $t$, and let us analyse the natural log of relative size ($n_{it}/\bar{n}_t$), where $\bar{n}_t$ is the average employment in the sample at time $t$. Figure (5.3) plots the size distributions in the following way. Arrayed along the horizontal axis are more than 2000 establishments in the sample, sorted...
Figure 5.3: Ranking of Establishments Over Time

in order of increasing 1984 relative size $n_{it}/\bar{n}_t$. The horizontal line at height 0 indicates the average establishment at time $t$.

The bottom chart in Figure (5.3), monotonic by construction, any establishment below (above) the dotted line in the first panel has employment lower (higher) than the average in 1984. Proceeding further vertically upwards in Figure (5.3), we plot additional cross-profile lines at two year intervals (1986, 1988 and 1990). In these graphs, firms overtake each other when succeeding cross-profiles become non-monotone. As suggested by Quah (1994), to understand better these graphs, let us consider two simple experiments. Suppose the cross-section of establishments were only adjusting towards the same steady state without overtaking each other, i.e. converging towards the mean. Then the profile
in Figure (5.3) should maintain its monotonic property and its slope flatten out. This is not what unambiguously happens in Figure (5.3). Suppose, conversely, that establishments of different size were steadily diverging from each other. This time the cross-profile should still maintain its monotonic property, but with an increasing slope over time. As before, this is not what we see in the graph. The only obvious conclusion from Figure (5.3) is that firms continuously overtake each other and the monotonicity property of the first chart is lost over time. In this context, what we really need are econometric methods that allow us to measure job creation between size categories. Methodology for this purpose has been recently introduced by Quah (1993a; 1994) in the empirics of economic growth.

In what follows, let $F_t$ denote the size distribution across establishments at year $t$; Quah suggests that the simplest probability model that can describe the dynamic behaviour of $F_t$ is

$$F_t = T^*(F_{t-1}, u_t),$$

(5.4)

where $T^*$ is an operator that maps a probability measure and a disturbance into another probability measure. Note that carrying out aggregate statistics of $F$, as we did in the previous section, would not suffice since we would hide any intra distribution dynamics. Furthermore, if we are interested in the long-run behaviour of the size distribution we can proceed as follows. If we ignore the disturbance term $u_t$ and we iterate expression (5.4), the size distribution at time $s > t$ can be described as

$$F_{t+s} = (T^*)^s F_t.$$  

(5.5)

Finally, if we let $s$ go to infinity, the long-run (ergodic) distribution of establishment size can be characterised. In this context convergence (towards the mean) might manifest in $F_{t+s}$ tending towards a mass point; alternatively the size distribution partitioning in small and large firms might be described by $F_{t+s}$ being characterized by two points or a bimodal distribution.
Note that the stochastic difference equation (5.5) is untractable. The problem with (5.5) is that as long as $F$ is a continuous variable, there are an infinite number of states. In this paper we focus on the simplest treatment we can have of (5.5) and we simplify the problem by approximating $T^*$ in the following way. We first assume a countable state space for firm size $S = s_1, \ldots, s_r$ and we transform $T^*$ into a simple transition probability matrix $Q$, which makes the difference equation (5.5) tractable. The problem becomes simply

$$F_{t+1} = QF_t,$$

where $Q$ encodes all the relevant information about mobility within the cross section distribution and allows us to study the long-run ergodic size distribution of firms. The framework set forth let us infer both intradistribution dynamics encoded in the matrix $Q$ and its long-run ergodic behaviour through successive iteration.

### 5.5 Results

To avoid the problems of arbitrariness in the size definition, we consider employment size with respect to the average establishment in each year. Category thresholds are determined to make the initial distribution of firms uniform. Since we choose five categories, the initial proportion of establishments in each category is 0.2. Table (5.7) shows the estimate of a one year transition matrix for the total manufacturing sector. In Table (5.7) the upper end of the state 0.175 indicates that in the first category we find all establishments whose employment is less of 17.5 percent than the average establishment in 1984. The mean establishment falls in the fourth category. Given these categories, we estimate the transition probability for each year of observation. We obtain six estimates and, averaging out across time, we obtain the Markov chain of Table (5.7). Obviously most of the probability lies in the main diagonal. This is a simple indication that
employment is highly persistent. Furthermore, entries in the main diagonal are higher in the first and last rows. This follows simply from the fact that firms in those categories can only move in one direction. One of the most important observations from Table (5.7) concerns elements in the third row. The establishments in the third category have a higher probability of becoming smaller than bigger, the contrary to what we would expect in a world where establishments converge towards the mean. Similarly, the probability mass of an establishment moving from the first to the second row is smaller than the probability of falling from the second to the first row. Finally, plants in the fourth category have an higher probability of falling into the third category than moving up. The last row in Table (5.7) reports the ergodic distribution implied by entries in Table (5.7) and it does not show any evidence of establishment size converging to the mean. If anything, there is some evidence of an increasing weight of the smaller category, but no evidence of convergence to the mean by initially small and large establishments, as results in Tables (5.3),(5.4) and (5.5) would predict. Table (5.8) confirms the results of Table (5.7), for a one step transition matrix. What is happening to intradistribution dynamics is a large movement of initially larger establishments toward a smaller size category, with no evidence of a persistent growth by initially smaller firms towards the mean.

Firm size evolution depends primarily upon technological characteristics and market structure typical of the industry in which each establishment operates. After all, once we have realized that there is nothing peculiar in the behaviour of small firms as a whole, we can start looking at what happens within each industry. In what follows we ask the following questions. Is regression to the mean observed in any industrial sector, independently of its employment dynamics? Alternatively, do we observe different size distribution dynamics in different sectors? If convergence is not observed in some sectors, is this phenomenon correlated with some other observable characteristics, such as average firm size, changes in the
Table 5.7: One Year Transition Matrix
Average 1984-1990. Total Manufacturing

<table>
<thead>
<tr>
<th>Upper end of the state</th>
<th>0.175</th>
<th>0.355</th>
<th>0.665</th>
<th>1.37</th>
<th>∞</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.175</td>
<td>0.925</td>
<td>0.0714</td>
<td>0.00322</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.355</td>
<td>0.0847</td>
<td>0.823</td>
<td>0.0890</td>
<td>0.0037</td>
<td>0.00</td>
</tr>
<tr>
<td>0.665</td>
<td>0.0042</td>
<td>0.0883</td>
<td>0.824</td>
<td>0.0823</td>
<td>0.0085</td>
</tr>
<tr>
<td>1.37</td>
<td>0.00123</td>
<td>0.0045</td>
<td>0.0826</td>
<td>0.0849</td>
<td>0.0629</td>
</tr>
<tr>
<td>∞</td>
<td>0.00</td>
<td>0.0008</td>
<td>0.000161</td>
<td>0.0668</td>
<td>0.931</td>
</tr>
<tr>
<td>Ergodic</td>
<td>0.242</td>
<td>0.20</td>
<td>0.196</td>
<td>0.188</td>
<td>0.174</td>
</tr>
</tbody>
</table>

Table 5.8: 5 Years Transition Matrix
Average 1984-1990. Total Manufacturing

<table>
<thead>
<tr>
<th>Upper end of the state</th>
<th>0.175</th>
<th>0.355</th>
<th>0.665</th>
<th>1.37</th>
<th>∞</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.175</td>
<td>0.832</td>
<td>0.151</td>
<td>0.173</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.355</td>
<td>0.218</td>
<td>0.576</td>
<td>0.19</td>
<td>0.015</td>
<td>0.00</td>
</tr>
<tr>
<td>0.665</td>
<td>0.0198</td>
<td>0.183</td>
<td>0.587</td>
<td>0.20</td>
<td>0.0099</td>
</tr>
<tr>
<td>1.37</td>
<td>0.0495</td>
<td>0.00173</td>
<td>0.218</td>
<td>0.636</td>
<td>0.124</td>
</tr>
<tr>
<td>∞</td>
<td>0.00</td>
<td>0.0483</td>
<td>0.0217</td>
<td>0.145</td>
<td>0.829</td>
</tr>
<tr>
<td>Ergodic</td>
<td>0.287</td>
<td>0.198</td>
<td>0.203</td>
<td>0.175</td>
<td>0.13</td>
</tr>
</tbody>
</table>
Table 5.9: Mobility and Convergence Across Sectors
Average 1984-1990. Total Manufacturing

<table>
<thead>
<tr>
<th>Industry</th>
<th>Ergodic Distribution</th>
<th>NET (a)</th>
<th>JR (b)</th>
<th>Average Mean (c)</th>
<th>Mean Change (d)</th>
<th>Mobility Index (e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wood</td>
<td>0.17 0.14 0.22 0.24 0.21</td>
<td>0.1</td>
<td>7.5</td>
<td>181.6</td>
<td>0.01</td>
<td>0.217</td>
</tr>
<tr>
<td>Chemical</td>
<td>0.18 0.15 0.20 0.24 0.21</td>
<td>1</td>
<td>7.9</td>
<td>263</td>
<td>0.07</td>
<td>0.151</td>
</tr>
<tr>
<td>Group 2:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Textile</td>
<td>0.17 0.12 0.12 0.22 0.34</td>
<td>-1</td>
<td>7.9</td>
<td>295.1</td>
<td>-0.06</td>
<td>0.175</td>
</tr>
<tr>
<td>Car</td>
<td>0.06 0.12 0.20 0.31 0.30</td>
<td>-1.5</td>
<td>10.7</td>
<td>266.5</td>
<td>-0.1</td>
<td>0.155</td>
</tr>
<tr>
<td>Paper</td>
<td>0.09 0.11 0.09 0.18 0.51</td>
<td>-0.6</td>
<td>8.3</td>
<td>161.9</td>
<td>-0.037</td>
<td>0.184</td>
</tr>
<tr>
<td>Group 3:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beverages</td>
<td>0.17 0.21 0.17 0.20 0.22</td>
<td>1.9</td>
<td>8.2</td>
<td>765.1</td>
<td>0.15</td>
<td>0.187</td>
</tr>
<tr>
<td>Machinery</td>
<td>0.38 0.12 0.21 0.18 0.09</td>
<td>2.7</td>
<td>10.0</td>
<td>667.6</td>
<td>0.209</td>
<td>0.176</td>
</tr>
<tr>
<td>Metal</td>
<td>0.18 0.19 0.23 0.19 0.19</td>
<td>0.8</td>
<td>11</td>
<td>284</td>
<td>0.06</td>
<td>0.227</td>
</tr>
<tr>
<td>Food</td>
<td>0.28 0.23 0.18 0.16 0.12</td>
<td>1.6</td>
<td>9.2</td>
<td>238</td>
<td>0.12</td>
<td>0.162</td>
</tr>
<tr>
<td>Group 4:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non Metalic</td>
<td>0.16 0.25 0.19 0.23 0.15</td>
<td>0.0</td>
<td>9.0</td>
<td>323</td>
<td>0.003</td>
<td>0.162</td>
</tr>
</tbody>
</table>

(*) For upper limit of the initial distribution see Appendix
(a) Net employment change between 1984 and 1990
(b) Job Reallocation
(c) Average Establishment Size
(d) Proportional Change in the Mean
(e) Shorrock Mobility Index

average firm size? Is there any relationship between job flow magnitude and long-run behaviour of the sector?

The appendix reports the upper limit of the categories for 10 sectors for which I estimate the transition probability. Even though there are some differences in the initial size distribution, each sector has the mean in its fourth category. The appendix shows also the transition probability matrix for each sector and the ergodic distribution. The transition probability matrix shows that, as expected, the highest probability lies in the main diagonal, but entries in the other cells are non-zero. As expected, firm employment is highly persistent, but intradistribution dynamics exist.
Table (5.9) looks in more details at the ergodic distribution in each sector and a series of other statistics related to job flows. The first result in Table (5.9) is the difference in the ergodic behaviour across industries. Wood and Chemical industry shows some evidence of convergence, with the highest probability clearly in the fourth category. The same ergodic behaviour is not observed in other industries. Non-Metallic sector registers a different form of convergence, with a bimodal concentration of firms in the second and the fourth category. Textile and Clothing, the Car industry and paper are characterized by a totally different ergodic behaviour, with the mode concentrated in the highest category. Finally food, metal, machinery and equipment and beverages, show no clear pattern of mobility. Table (5.9) also reports, for each sector, average net employment change, job reallocation, average establishment size, the proportional change in the mean over time and a measure of Shorrocks (in Geweke et al. (1986)) mobility index of persistence, defined as

$$M(P) = \frac{n - \text{tr}(P)}{n - 1},$$

where $M$ is the mobility index, $P$ is the Markov chain, $n$ is the number of categories and $\text{tr}(P)$ is the trace of $P$. A value of $M(P)$ of 0 indicates an absolute persistence in the process, whereas a value of $n/n - 1$ indicates the highest possible mobility.

Table (5.9) shows that for the converging sectors average net employment change is approximately constant and the proportional change in the mean is slightly positive. If we take a simple average between groups of sectors, job reallocation for Wood and Chemical sectors and the Shorrocks index are the smallest between all groups. Conversely, the second group (textile, car and paper) has a downward shifting of the mean and a negative average net employment change. Both job reallocation and the mobility indices are, on average, higher than the corresponding values for the converging sectors. On the other hand,
group 3 is characterized by a substantial increase in the mean, roughly 15 percent. Note that the mobility index for group 3, on average .19, is the highest among the three groups of sectors identified. Similarly for job reallocation, with an average value of .096.

Table (5.9) suggests that firm size distribution dynamics in each sector is linked to the dynamic behaviour of the sector as a whole. Relatively stable sectors, with small changes in both the mean and total employment (group 1) converge to the mean with relatively little intradistribution dynamics. On the other hand, declining sectors, such as those in group 2, experience a mass concentration into the highest size categories. Finally, expanding sectors (group 3) do not show any particular tendency in the size distribution, but, as a group, they are characterised by the highest mobility level, both in term of job reallocation and the Shorrock index. These results suggest a remarkable asymmetric behaviour in the size distribution between expanding and declining sectors, and some evidence of convergence for relatively stable sectors.

5.6 Conclusions

In the last decade, much emphasis in the policy debate has been given to the role of small firms in the process of job creation. From an empirical standpoint, when we measure firm size as employment in a base year, it is true that small firms more than proportionally create jobs. In this paper I discussed the problem of how to define establishment size and I measured the relation between job flows and establishment size with different plant size definitions. I argued that the traditional measures suffer from the Galton fallacy and they are uninformative on the relationship between job flows and establishment size. Applying non parametric techniques best suited for analysing the dynamics of a large cross-section, I did not find any long-run movement of initially small establishments toward the mean, thus no evidence of convergence. Furthermore, applying the
analysis at the industry level, I find an interesting asymmetric behaviour in the
dynamics of the expanding and declining sector. Konings (1995b) in a paper
that studies the evolution of plant size in the British manufacturing industry
finds similar results. The next step would be to apply the same methodology
to other datasets and control to what extent the results of this paper represent
a more general result. If these empirical results should be further confirmed,
important implications for industry dynamics would naturally follow.

5.A Appendix: Transition Matrices at the Industry Level

In this section I report the one year transition matrices for the 10 industries of
the Mexican manufacturing sector.

<table>
<thead>
<tr>
<th>Upper end of the state</th>
<th>Time</th>
<th>Stationarity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.225</td>
<td>0.390</td>
</tr>
<tr>
<td>0.225</td>
<td>0.939</td>
<td>0.052</td>
</tr>
<tr>
<td>0.390</td>
<td>0.068</td>
<td>0.842</td>
</tr>
<tr>
<td>0.670</td>
<td>0.005</td>
<td>0.115</td>
</tr>
<tr>
<td>1.38</td>
<td>0.00</td>
<td>0.0103</td>
</tr>
<tr>
<td>∞</td>
<td>0.00</td>
<td>0.000</td>
</tr>
<tr>
<td>Ergodic</td>
<td>0.287</td>
<td>0.238</td>
</tr>
</tbody>
</table>

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Table 5.A.2: Beverages Industry. First Order Transition Matrix

<table>
<thead>
<tr>
<th>Upper end of the state</th>
<th>0.310</th>
<th>0.645</th>
<th>0.950</th>
<th>1.32</th>
<th>∞</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.310</td>
<td>0.929</td>
<td>0.066</td>
<td>0.011</td>
<td>0.00</td>
</tr>
<tr>
<td>0.645</td>
<td>0.062</td>
<td>0.823</td>
<td>0.115</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.950</td>
<td>0.005</td>
<td>0.128</td>
<td>0.763</td>
<td>0.11</td>
<td>0.00</td>
</tr>
<tr>
<td>1.32</td>
<td>0.00</td>
<td>0.0202</td>
<td>0.075</td>
<td>0.82</td>
<td>0.084</td>
</tr>
<tr>
<td>∞</td>
<td>0.00</td>
<td>0.000</td>
<td>0.000</td>
<td>0.008</td>
<td>0.923</td>
</tr>
<tr>
<td>Ergodic</td>
<td>0.174</td>
<td>0.217</td>
<td>0.178</td>
<td>0.205</td>
<td>0.226</td>
</tr>
</tbody>
</table>

Table 5.A.3: Textile and Clothing. First Order Transition Matrix

<table>
<thead>
<tr>
<th>Upper end of the state</th>
<th>0.175</th>
<th>0.355</th>
<th>0.640</th>
<th>1.32</th>
<th>∞</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.175</td>
<td>0.931</td>
<td>0.066</td>
<td>0.002</td>
<td>0.00</td>
</tr>
<tr>
<td>0.355</td>
<td>0.082</td>
<td>0.832</td>
<td>0.081</td>
<td>0.003</td>
<td>0.00</td>
</tr>
<tr>
<td>0.640</td>
<td>0.00</td>
<td>0.072</td>
<td>0.826</td>
<td>0.098</td>
<td>0.002</td>
</tr>
<tr>
<td>1.32</td>
<td>0.0058</td>
<td>0.027</td>
<td>0.058</td>
<td>0.861</td>
<td>0.0793</td>
</tr>
<tr>
<td>∞</td>
<td>0.00</td>
<td>0.000</td>
<td>0.000</td>
<td>0.0528</td>
<td>0.947</td>
</tr>
<tr>
<td>Ergodic</td>
<td>0.172</td>
<td>0.127</td>
<td>0.128</td>
<td>0.226</td>
<td>0.347</td>
</tr>
</tbody>
</table>

Table 5.A.4: Wood Industry. First Order Transition Matrix

<table>
<thead>
<tr>
<th>Upper end of the state</th>
<th>0.27</th>
<th>0.490</th>
<th>0.815</th>
<th>1.41</th>
<th>∞</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.27</td>
<td>0.923</td>
<td>0.0772</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.49</td>
<td>0.0917</td>
<td>0.755</td>
<td>0.108</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.815</td>
<td>0.00</td>
<td>0.0664</td>
<td>0.846</td>
<td>0.0873</td>
<td>0.00</td>
</tr>
<tr>
<td>1.41</td>
<td>0.00</td>
<td>0.00</td>
<td>0.075</td>
<td>0.87</td>
<td>0.054</td>
</tr>
<tr>
<td>∞</td>
<td>0.00</td>
<td>0.005</td>
<td>0.005</td>
<td>0.056</td>
<td>0.932</td>
</tr>
<tr>
<td>Ergodic</td>
<td>0.172</td>
<td>0.145</td>
<td>0.229</td>
<td>0.245</td>
<td>0.21</td>
</tr>
</tbody>
</table>
Table 5.A.5: Paper Industry. First Order Transition Matrix  
Average 1984-1990. Paper Industry  

<table>
<thead>
<tr>
<th>Upper end of the state</th>
<th>Time Stationarity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.285</td>
</tr>
<tr>
<td>0.285</td>
<td>0.884</td>
</tr>
<tr>
<td>0.49</td>
<td>0.094</td>
</tr>
<tr>
<td>0.705</td>
<td>0.00</td>
</tr>
<tr>
<td>1.26</td>
<td>0.00</td>
</tr>
<tr>
<td>∞</td>
<td>0.00</td>
</tr>
<tr>
<td>Ergodic</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Table 5.A.6: Chemical Industry. First Order Transition Matrix  
Average 1984-1990. Chemical Industry  

<table>
<thead>
<tr>
<th>Upper end of the state</th>
<th>Time Stationarity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.21</td>
</tr>
<tr>
<td>0.21</td>
<td>0.925</td>
</tr>
<tr>
<td>0.4</td>
<td>0.079</td>
</tr>
<tr>
<td>0.745</td>
<td>0.059</td>
</tr>
<tr>
<td>1.5</td>
<td>0.0017</td>
</tr>
<tr>
<td>∞</td>
<td>0.00</td>
</tr>
<tr>
<td>Ergodic</td>
<td>0.184</td>
</tr>
</tbody>
</table>

Table 5.A.7: Non-metallic Industry. First Order Transition Matrix  
Average 1984-1990. Chemical Industry  

<table>
<thead>
<tr>
<th>Upper end of the state</th>
<th>Time Stationarity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.11</td>
</tr>
<tr>
<td>0.11</td>
<td>0.923</td>
</tr>
<tr>
<td>0.28</td>
<td>0.0479</td>
</tr>
<tr>
<td>0.62</td>
<td>0.00</td>
</tr>
<tr>
<td>1.59</td>
<td>0.00</td>
</tr>
<tr>
<td>∞</td>
<td>0.00</td>
</tr>
<tr>
<td>Ergodic</td>
<td>0.16</td>
</tr>
</tbody>
</table>
### Table 5.A.8: Metallic Industry. First Order Transition Matrix

**Average 1984-1990. Chemical Industry**

<table>
<thead>
<tr>
<th>Upper end of the state</th>
<th>0.19</th>
<th>0.355</th>
<th>0.750</th>
<th>1.51</th>
<th>∞</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.19</td>
<td>0.879</td>
<td>0.117</td>
<td>0.0038</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.355</td>
<td>0.107</td>
<td>0.753</td>
<td>0.137</td>
<td>0.0035</td>
<td>0.00</td>
</tr>
<tr>
<td>0.750</td>
<td>0.0075</td>
<td>0.104</td>
<td>0.768</td>
<td>0.12</td>
<td>0.00</td>
</tr>
<tr>
<td>1.51</td>
<td>0.00</td>
<td>0.0068</td>
<td>0.138</td>
<td>0.773</td>
<td>0.0819</td>
</tr>
<tr>
<td>∞</td>
<td>0.00</td>
<td>0.003</td>
<td>0.00</td>
<td>0.0776</td>
<td>0.919</td>
</tr>
<tr>
<td>Ergodic</td>
<td>0.18</td>
<td>0.19</td>
<td>0.232</td>
<td>0.193</td>
<td>0.196</td>
</tr>
</tbody>
</table>

### Table 5.A.9: Machinery and Equipment Industry. First Order Transition Matrix

**Average 1984-1990. Machinery and Equipment Industry**

<table>
<thead>
<tr>
<th>Upper end of the state</th>
<th>0.145</th>
<th>0.35</th>
<th>0.675</th>
<th>1.42</th>
<th>∞</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.145</td>
<td>0.973</td>
<td>0.0269</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.35</td>
<td>0.075</td>
<td>0.808</td>
<td>0.117</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.67</td>
<td>0.00</td>
<td>0.06</td>
<td>0.832</td>
<td>0.105</td>
<td>0.00</td>
</tr>
<tr>
<td>1.42</td>
<td>0.005</td>
<td>0.00</td>
<td>0.12</td>
<td>0.81</td>
<td>0.0632</td>
</tr>
<tr>
<td>∞</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.13</td>
<td>0.870</td>
</tr>
<tr>
<td>Ergodic</td>
<td>0.385</td>
<td>0.126</td>
<td>0.217</td>
<td>0.182</td>
<td>0.09</td>
</tr>
</tbody>
</table>

### Table 5.A.10: Car Industry. First Order Transition Matrix

**Average 1984-1990. Car Industry**

<table>
<thead>
<tr>
<th>Upper end of the state</th>
<th>0.14</th>
<th>0.255</th>
<th>0.565</th>
<th>1.24</th>
<th>∞</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.14</td>
<td>0.886</td>
<td>0.114</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.255</td>
<td>0.0616</td>
<td>0.836</td>
<td>0.102</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.565</td>
<td>0.00</td>
<td>0.062</td>
<td>0.846</td>
<td>0.092</td>
<td>0.00</td>
</tr>
<tr>
<td>1.24</td>
<td>0.00</td>
<td>0.00</td>
<td>0.59</td>
<td>0.877</td>
<td>0.0837</td>
</tr>
<tr>
<td>∞</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.066</td>
<td>0.934</td>
</tr>
<tr>
<td>Ergodic</td>
<td>0.065</td>
<td>0.122</td>
<td>0.201</td>
<td>0.312</td>
<td>0.301</td>
</tr>
</tbody>
</table>

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Chapter 6

Conclusions

The study of the determinants of large flows of job creation and destruction in labour markets is a fast growing field, both empirically and theoretically. Empirically, the availability of large data-sets at the establishment level in most OECD countries threw new light on labour market dynamics (OECD 1994a). Theoretically, the seminal works by Mortensen and Pissarides (1994) and Bertola and Caballero (1994) offered structural models for thinking about labour markets characterised by simultaneous flows of job creation and destruction. Slow search from the part of workers, asymmetries in hiring and firing technologies and the idea of jobs (firms) as independent islands hit by idiosyncratic shocks are the key features of the new models. This thesis selected four particular issues and tried to rationalise them along the lines of the most recent development of the search equilibrium literature.

Chapter I, *Job Flow Dynamics and Firing Restrictions*, focused on the cyclical properties of job creation and destruction. In each country, job creation is pro-cyclical while job destruction is counter cyclical but, across countries, huge differences exist in the relative volatility of the two flows. Chapter I introduced firing delays in the Mortensen Pissarides model (1994) and solved a search model that rationalises cross-country differences in the cyclical behaviour of job flows. When firing permissions are continuously available, job destruction is instanta-
neous while job creation takes time and job reallocation moves counter cyclically, exactly as in North-American data. Conversely, as firing is restricted to be costly and time-consuming, the asymmetry between job flows disappears, exactly as in continental Europe data. Chapter I argued that this simple mechanism is behind the cross-country differences in the cyclical behaviour of job flows.

From the work of Chapter I, several directions of research should be taken. Firstly, following Burda and Wyplosz (1994) for Europe and Mortensen (1994) for the U.S., it is necessary to investigate the effect of firing restrictions on worker flows. Hopenhayn and Rogerson (1993) simulate a general equilibrium model with simultaneous job creation and destruction and they find that a tax on job destruction has a sizable negative impact on total employment. With respect to the approach of Chapter I, Hopenhayn and Rogerson explicitly consider the effects of firing restrictions on labour supply decisions. Future researches in the matching literature should model in more details the labour supply decisions of unemployed job seekers. Finally, in order to obtain clear policy predictions, the welfare implications of firing delays and the optimal level of job reallocation should be carefully studied.

The international data on job creation and job destruction compiled by OECD (1994a) showed large variations across countries. Chapter II, Job Reallocation and Labour Market Policy, used this variation for a sample of ten OECD countries to make some inferences about the connection between gross job reallocation on the one hand and aggregate economic performance and labour market policy on the other.

The connection between job reallocation and unemployment in the international domain is rather loose but there is a strong connection between job reallocation and long-term unemployment. Countries with less job reallocation experience longer durations of unemployment. Since long-term unemployment is not good for the skills and the morale of those who suffer it, policy measures that
restrict job reallocation will have negative impact on the functioning of labour markets in this connection. Chapter II examined also the relation between gross job reallocation and policy and it found one strong correlation and some other looser ones. The indefinite availability of unemployment compensation slows down the reallocation of jobs. The mechanism is probably the elimination of low productivity unstable jobs that the long-term availability of income support is likely to bring about. In Chapter II there is also some evidence that employment protection legislation, in the form of restrictions on the dismissal of employees, slows down both job creation and job destruction, and so leads to longer durations of unemployment. In contrast, the level of unemployment and active labour market policies are positively but weakly associated with job turnover.

With respect to employment protection legislation, Bertola and Rogerson (1996) and Boeri (1995), with a slightly different data-set and explicitly considering the role of entry and exit of firms, do not find a significant effect of firing costs on the level of job reallocation. Future research should include in the sample a higher number of countries and should pay as much attention as possible to methodological differences among data-sets.

Chapter III and IV, *Wages and the Size of Firms in a Dynamic Matching Model*, studied the role of a costly and time-consuming matching process in generating the size distribution of firms and the dynamics of firm-level turnover. The chapters offered a realistic extension of the work by Bertola and Caballero (1994), where well defined firms with downward sloping labour demand are affected by idiosyncratic shocks. Chapters III and IV focused on a dynamic mechanism that combines elements of standard theoretical insights, and is consistent with empirical findings put forward in the literature in their support. The various specifications of the model solved in Chapters III and IV offer a basic qualitative prediction. In a dynamic environment, wages should be positively related to employment levels—to the extent that both size and wages are correlated to

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the (unobservable) current position of various firm's labour demand. However, the model does suggest a better proxy than employment for explaining firm level wage dispersion and "firm quality" effects: vacancy posting is univocally determined by the marginal value of labour for individual firms (which determines wages in rent-sharing fashion); in turn, vacancy posting determines the rate of employment growth at the individual-firm level. Hence, after controlling for employment levels, wages are predicted to be higher in faster-growing firms. To our knowledge, there is no evidence on whether this is or is not the case in the data and future empirical researches should try to test these predictions.

Chapter V, *Job Flows and Plant Size Dynamics: Traditional Measures and Alternative Econometric Techniques*, focused on the role of small firms in the process of job creation and destruction. From an empirical standpoint, when we measure firm size as employment in a base year, small firms more than proportionally create jobs. Chapter V argued that the traditional measures suffer from the Galton fallacy and they are uninformative on the relationship between job flows and establishment size. Using a sample of Mexican manufacturing plants, Chapter V applied non parametric techniques recently proposed by Quah (1993a,b) and it did not find any long-run movement of initially small establishments toward the mean, thus no evidence of convergence. Furthermore, applying the analysis at the industry level, Chapter V found an interesting asymmetric behaviour in the dynamics of the expanding and declining sectors. Konings (1995a), in a paper that studies the evolution of plant size in the British manufacturing industry, finds similar results. Future researches should apply the methodology to other data-sets and should control to what extent the results of Chapter V represent an empirical regularity.
References


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