Gender discrimination, optimal allocation and partial-pooling Nash equilibrium: essays on insurance markets with a participation option

A thesis submitted in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

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Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

I warrant that this authorization does not, to the best of my belief, infringe the rights of any third party.

Abstract

The thesis is made up of three essays which study three related topics. The first essay examines the welfare effect of a non-discrimination policy, which bans using gender in pricing insurance in the context of motor insurance markets. It comprises two models. The first models comprehensive insurance markets, in which motorists decide whether to buy insurance that offers full coverage. The second model examines third-party insurance markets, in which motorists must be fully insured and agents decide whether to drive. The essay examines the welfare effects of the non-discrimination policy by examining the change of aggregate social welfare before and after the policy is implemented. It shows that in comprehensive insurance market typical adverse selection happens. Aggregate social welfare may increase or decrease. In third-party insurance market, advantageous selection happens. Aggregate social welfare may decrease after the policy implemented.

The second essay endogenizes insurance coverage and finds the optimal allocation which maximizes aggregate social welfare. Agents can now choose whether to drive and whether to buy insurance, and insurers are allowed to offer a menu of cross-subsidizing insurance contracts in competitive insurance markets. The author finds pooling allocation can never maximize aggregate social welfare and the market may end up with too much insurance.

The third essay examines market equilibrium and market efficiency in competitive insurance markets when agents differ in both risk probabilities and risk preferences, and can choose whether to participate in risky activity and whether to buy insurance. With different levels of risk probabilities, risk preferences, and driving benefit, the market may end up with four different separating equilibria, partial-pooling equilibrium, or even no equilibrium. The partial-pooling equilibrium is Pareto efficient under certain conditions. If it is inefficient, taxing insurance breaks the equilibrium and separating equilibrium arises, which leads to Pareto gain.

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Gender discrimination, optimal allocation and partial-pooling Nash equilibrium: essays on insurance markets with a participation option

Chapter 1 Introduction

Since the seminal work of Rothschild and Stiglitz (1976), there has been a substantial amount of research on market equilibrium and market efficiency in insurance markets under asymmetric information. To the author's knowledge, none of these works considers participation option. Agents are assumed to engage in risky activities. They can mitigate risk by taking unobservable precautions and buying insurance against observable losses.

The present work conducts research on three related topics in insurance markets where agents have participation options. The first essay introduces participation option but contains results with contract form exogenous. It examines the welfare effect of a non-discrimination policy, which bans using gender in pricing insurance. The context is motor insurance markets where people can choose whether to drive and whether to buy more than the legal minimum of insurance. It comprises two models. The first models comprehensive insurance markets, in which motorists decide whether to buy insurance that offers full coverage. The second model examines third-party insurance markets, in which motorists must be fully insured and agents decide whether to drive. Agents are risk-averse and are identical except for risk probabilities which are private information. The distribution of risk probabilities of the two types of agents (men and women) differ and this is public information.

As gender is observable before the policy is implemented, the markets are separated with women charged a relatively lower premium. After the policy is introduced, the two markets merge into one market in which gender is in effect unobservable. The essay examines the welfare effects of the non-discrimination policy by examining the change of aggregate social welfare before and after the policy implemented.

The second essay extends the research of the first one by endogenizing insurance coverage. But as the only heterogeneity is in hazard rate, only separating equilibrium can arise. Cross-subsidies are allowed, so policy intervention may raise aggregate social welfare but can not yield strict Pareto gain.

It finds the optimal allocation which maximizes aggregate social welfare. As in standard Rothschild-Stiglitz model, risk-averse agents are identical except for risk probabilities. Furthermore, agents can now choose whether to drive and whether to buy insurance, and insurers are allowed to offer a menu of cross-subsidizing insurance contracts which earns normal profit overall. In such competitive insurance markets, received wisdom has it there is too little insurance available in the market due to the asymmetric information problem and that a pooling allocation maximizes aggregate social welfare. The present analysis finds that with a participation option a pooling allocation can never maximize aggregate social welfare and the market may end up with too much insurance.

The third essay endogenises insurance contract form and allows two types of hidden information. So partial-pooling equilibrium can emerge. It shows new source of gain from taxation of insurance. No cross-subsidization is allowed so Pareto gain realizes by expelling the high-risk types.

It extends the research by assuming agents differ in two dimensions: risk probability and risk preference. In addition, agents have choices now on whether to take risk activity and on whether to buy insurance in competitive insurance markets. With agents differing in both risk probabilities and risk preferences, the single-crossing property of indifference curves of agents may not hold. This gives rise to partial-pooling Nash equilibrium. Due to differing risk preferences, the same financial loss causes different changes in utilities of agents. Along with the participation option, this results in four different types of separating equilibrium.

The essay examines when the partial-pooling Nash equilibrium is Pareto efficient, and, if it is inefficient, whether taxing insurance can drive out the high-risks and lead to a Pareto improvement.

In more detail, the rest of the thesis is structured as follows. Chapter 2 examines the welfare effect of the non-discrimination policy. Section 2.1 introduces the topic and provides a literature review. Section 2.2 specifies the first model. It finds the market equilibrium before and after the non-discrimination policy, and then conducts welfare analysis. Section 2.3 specifies and analyzes the second model. Section 2.4 concludes.

Chapter 3 integrates the participation choice with endogenous contractual form and finds the optimal allocation which maximizes aggregate social welfare. Section 3.1 introduces the topic and provides a literature review. Section 3.2 finds the market equilibrium in an insurance market with adverse selection and participation option.

Section 3.3 examines conditions for such an equilibrium to exist. Section 3.4 introduces the tax-subsidy scheme into the model. Section 3.5 analyzes the tax-subsidy scheme and gives the most important findings of this paper. Section 3.6 and Section 3.7 considers two extreme contingencies: full-insurance pooling contract and over-insurance contract. Section 3.8 concludes.

Chapter 4 examines the market equilibrium and market efficiency in competitive insurance markets when agents differ in both risk probabilities and risk preferences, and can choose whether to participate in the risky activity and whether to buy insurance. Section 4.1 introduces the topic and provides a literature review. Section 4.2 specifies the model. Section 4.3 finds the partial-pooling Nash equilibrium as well as the four separating equilibria. It also finds the conditions of the partial-pooling equilibrium. Section 4.4 analyzes the efficiency of the partial-pooling equilibrium and demonstrates that it is Pareto efficient under certain conditions. Section 4.5 shows that inefficient partial-pooling equilibrium exists and taxing insurance leads to Pareto gain.

Chapter 5 concludes the three essays.

Chapter 2 Gender discrimination and participation option

2.1 Introduction

Statistics show that female motorists on average are less risky than male motorists, especially for young drivers. According to Diamond Insurance, who specialises in covering female drivers, men are convicted of 92% of driving offences and account for 98% of dangerous driving convictions.[1] This is reflected on insurance premium. Other things equal, female motorists pay less than male motorists. However, Anna Diamantopoulou, the then social affairs commissioner of the EC in 2003, considered this a form of gender discrimination and proposed a non-discrimination policy which bans using gender in setting premium.[2]

The proposal met strong opposition in the UK. Insurance companies, the Association of British Insurers (ABI), the British government, and the Financial Services Authority (FSA) were all strongly against the proposal, claiming that the proposed directive forbade best market practice and would cause adverse selection. The FSA estimated that young women drivers were likely to see their premiums rise by between 10% and 30% if the proposal became law. The European Union Committee of House of Lords even estimated an up to 40% rise of young women drivers' car insurance premium.[3] What was even more interesting was that a survey shown most motorists, even male motorists who were supposed to benefit from this proposal, supported the market practice and were against the proposal.[4]

With a British-led rebellion against the proposal, the EC compromised and allowed the insurance industry to opt out from the directive. Insurance companies can continue using gender in pricing insurance as long as they can justify their methods are based on actuarial facts. A regular review of the issue will be held, comprising the industry, anti-discrimination bodies and member states, when insurers have to explain discriminatory pricing policies.[5]

But this policy deserves further research. For instance, will the market suffer from adverse selection problem, or even collapse as some people have predicted, if the

[1] Guardian (4 January 2005), "Women keep deal on lower car insurance".
[2] Financial Times (3 November 2003), "Insurers to fight Brussels over bar on sex discrimination".
[3] Guardian (22 September 2004), "Women drivers to pay more under EU equality plans, warn peers".
[4] Observer (18 April 2004); "Men should pay higher insurance"; Financial Services Review (November 2004), "Equal opportunities".
[5] Financial Times (4 October 2004), "Women drivers to enjoy lower premiums".

policy is carried out? Furthermore, what is the policy's impact on social welfare from a Utilitarian point of view?

As gender is observable prior to the policy, the markets are separated and reach two equilibria, one for women who are charged at a relatively lower premium, the other for men who are charged at a relatively higher premium. After the policy is implemented, the two markets merge into one market in which gender in effect is unobservable.

It is difficult to think that prohibiting using gender in pricing insurance would actually result in complete market collapse. This has not occurred in Denmark, Greece, Luxembourg, the Netherlands and Sweden, where unisex motor insurance has already been established. Actually, as the House of Lords committee noted, this "did not seem to create any particular problems."[6] So most likely there would be a pooling equilibrium in which both genders get the same level of insurance coverage at the same premium.

Other things equal, pooling the risks increases aggregate social welfare as it redistributes to the worse off. But what if motorists are allowed to opt out of insurance market although they still drive? Furthermore, what if motorists must be insured but they are allowed to quit driving? This essay attempts to answer these questions by constructing two related models.

The first model analyzes motorists' decision on whether to buy full coverage insurance where everyone drives regardless of being insured or not. Consider, as an example, comprehensive insurance market where motorists decide whether to buy insurance which offers comprehensive coverage. In the model, we have a number of male and female drivers, each of whom has a risk probability uniformly distributed in a risk range. On average, male motorists are more risky than female motorists and the variance of their risk distribution is greater than the female one. The insurance market is perfectly competitive. Insurers offer an insurance policy of full coverage. An accident causes a financial loss to the motorist. Everyone is risk averse with the same utility function and initial wealth. There is no moral hazard. Everyone drives no matter insured or not. The motorist needs to decide whether to purchase full coverage insurance.

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[6] European Union Committee, House of Lords (22 September 2004), "Sexual Equality in Access to Goods and Services", 27th Report of Session 2003-04

The second model analyzes agents' decision on whether to drive when motorists must be fully insured. Consider, as another example, third-party insurance market where insurance coverage is compulsory for motorists. In Italy, more than 60% of the insurance companies do not offer deductibles at all on third-party insurance products. The remaining firms offer minimal deductibles. Furthermore, the minimum indemnity limit has to be approximately $1 million, which covers virtually all accidents in practice.[7] The setup of the second model is similar to the first one except that people now choose whether to become a motorist instead of choosing whether to get insured. Each person has a reservation utility if they choose not to drive. An accident causes a non-pecuniary cost in addition to a financial loss.

As Crocker and Snow (2000) have noted, "the efficiency and equity effects of risk classification in insurance markets have been a source of substantial debate, both amongst economists and in the public policy arena." Hoy (1982) is the first attempt to analyze the welfare implications of imperfectly categorizing risks in the insurance industry under conditions of asymmetric information. Hoy considers the pure strategy Nash equilibrium of Rothschild and Stiglitz (1976), the anticipatory equilibrium of Wilson (1977), and the Miyazaki-Spence separating equilibrium suggested by Miyazaki (1977) and Spence (1978). He finds the welfare effect of risk classification is ambiguous. Only in the case where the initial equilibrium is of the Nash no-subsidy type is there a strict Pareto-type improvement in welfare.

In contrast to Hoy, Crocker and Snow (1986) examine the efficiency effect of risk classification by comparing the utilities possibilities frontier for the regime where risk categorization is permitted to the one in which it is not. They demonstrate that costless imperfect risk categorization enhances efficiency by showing that the utility possibility frontier after the categorization lies somewhere outside of and nowhere inside of the frontier before the categorization. Both Hoy (1982) and Crocker and Snow (1986) use the standard setup as in Rothschild and Stiglitz (1976), in which there are two risk types.

Hoy, Polborn and Sadanand (2006) explicitly consider the effects of regulations that prohibit the use of information to risk-rate premiums in a life insurance market. Using a dynamic three period model, they show that legislation prohibiting the use of results from genetic screening tests for ratemaking purposes in the life insurance

[7] Buzzacchi and Valletti (2005), "Strategic Price Discrimination in Compulsory Insurance Markets", The Geneva Risk and Insurance Review, 30: 71-97, 2005.

market may increase aggregate social welfare despite the fact that such regulations create adverse selection costs. Their research differs from the previous works not only in the setup of multi-period stages but also in the linear pricing insurance contracts.

Hoy (2006) analyzes the policy effects of restrictions on risk classification with canonical models of Rothschild and Stiglitz (1976) and Wilson (1977). Similar to this paper, Hoy analyzes the aggregate social welfare using a Utilitarian social welfare function. It is actually an extension of Hoy (1982) and has derived some explicit conditions that determine when such regulations are either welfare enhancing or detrimental.

Close to the present essay, de Meza (2002) considers the effects of equal opportunity legislation. After checking scenarios of hidden types and/or hidden actions, he concludes that banning discrimination, when combined with mandatory protection against failure, may well be the best way of effecting redistribution of income. The present essay is inspired by his work on reservation utility and participation option.

Deviated from the standard line of research, Buzzacchi and Valletti (2005) offer an interesting study on the problem of risk classification when insurers do not observe a customer's type but only some other variable correlated to it. They consider an insurance market with no adverse selection where there is full market participation and full insurance coverage. In particular, they study the strategic interaction between imperfectly competitive firms that may decide whether to adopt classification variables. They find that discrimination based on immutable characteristics such as gender is a dominant strategy.

The present essay differs from the above works in three aspects. First, it extends the standard setup of Rothschild and Stiglitz (1976) by assuming a number of agents with risk probabilities distributed in two ranges. Second, it avoids the unsettled issue on the concept of market equilibrium by assuming full insurance coverage. Last and most importantly, it incorporates a participation option by allowing agents to opt out of risky activity.

The rest of the essay is structured as follows. Section 2.2 specifies the first model. It finds the market equilibrium before and after the non-discrimination policy, and then conducts welfare analysis. Section 2.3 specifies and analyzes the second model. Section 2.4 concludes.

2.2 To insure or not to insure

Suppose we have N_M number of men and N_W number of women. For a male driver i, his accident probability is π_i^M, where $\pi_i^M \in [\pi_2, \pi_4]$, $i = 1, \cdots, N_M$. For a female driver j, her accident probability is π_j^W, where $\pi_j^W \in [\pi_1, \pi_3]$, $j = 1, \cdots, N_W$, and $0 < \pi_1 < \pi_2 < \pi_3 < \pi_4 < 1$. People with different accident probabilities are distributed uniformly within the risk probability range. So for any accident probability π^* in the range, the number of female motorists whose accident probability are less than or equal to π^* is $N^* = \int_{\pi_1}^{\pi^*} \frac{N_W}{\pi_3 - \pi_1} d\pi$. Similarly, it is $N^* = \int_{\pi_2}^{\pi^*} \frac{N_M}{\pi_4 - \pi_2} d\pi$ for male motorists. Suppose male motorists have a greater variance of accident probability distribution, hence we assume $\pi_4 - \pi_2 = T(\pi_3 - \pi_1)$, where $T \geq 1$. Accident probability is private information but insurers know the population distributions.

Insurers offer an insurance with coverage q at premium p. Each person has an initial wealth y. An accident causes a financial loss of L to the motorist. The utility function, the same for everybody, is $u(c)$.

Everybody drives no matter with insurance or not. The motorist needs to decide whether to get insured.

2.2.1 Insurance decision

Suppose only comprehensive insurance is available. So $q = L$ if insured, $q = 0$ otherwise. An Individual's expected utility with no insurance is

$$V_1(\pi_i) = (1 - \pi_i) \cdot u(y) + \pi_i \cdot u(y - L)$$

His expected utility with insurance will be

$$V_2(\pi_i, p) = (1 - \pi_i) \cdot u(y - pq) + \pi_i \cdot u(y - L - pq + q)$$

With full coverage, the utility becomes

$$V_2(p) = u(y - pL)$$

The motorist will buy the insurance if

$$D = V_2(p) - V_1(\pi_i) \geq 0$$

Checking the properties of D with respect to p and π, we have

$$\frac{\partial D}{\partial p} = -L \cdot u'(y - pL) < 0, \quad \frac{\partial^2 D}{\partial p^2} = L^2 \cdot u''(y - pL) < 0$$

$$\frac{\partial D}{\partial \pi} = u(y) - u(y - L) > 0, \quad \frac{\partial^2 D}{\partial \pi^2} = 0$$

So the utility difference decreases monotonically in insurance premium p and increases monotonically in risk probability π. At corners,

$D = u(y) - u(y) = 0$ when $p = 0$ and $\pi = 0$,

$D = u(y) - u(y - L) > 0$ when $p = 0$ and $\pi = 1$,

$D = u(y - L) - u(y) < 0$ when $p = 1$ and $\pi = 0$,

$D = u(y - L) - u(y - L) = 0$ when $p = 1$ and $\pi = 1$.

Consider the marginal buyer whose utility difference satisfies

$D = V_2(p) - V_1(\pi_i) = 0$ i.e. $u(y - pL) - (1 - \pi_i) \cdot u(y) - \pi_i \cdot u(y - L) = 0$

This yields

$$\pi_i = \frac{u(y) - u(y - pL)}{u(y) - u(y - L)} \tag{2.2.1}$$

Checking the variation of π with p gives

$$\frac{d\pi}{dp} = \frac{u'(y - pL) \cdot L}{u(y) - u(y - L)} > 0, \quad \frac{d^2\pi}{dp^2} = \frac{-L^2 \cdot u''(y - pL)}{u(y) - u(y - L)} > 0$$

So for a given premium p^*, we can find a unique risk probability π^* which satisfies $D = V_2(p^*) - V_1(\pi^*) = 0$. For the people whose risk probability is equal to or greater than π^*, their utility difference from buying the insurance will be equal to or greater than zero, i.e., $D = V_2(p^*) - V_1(\pi_i) \geq 0$, where $\pi_i \geq \pi^*$. This leads to the following proposition.

Proposition 1: for a given insurance premium, p^*, an Individual customer will buy the insurance if his own risk probability, π_i is greater than or equal to the threshold risk probability π^*, i.e. $\pi_i \geq \pi^*$, where

$$\pi^* = \frac{u(y) - u(y - p^* L)}{u(y) - u(y - L)} \tag{2.2.2}$$

2.2.2 Equilibria when gender is observable

Consider the market for male motorists first. For an insurance premium p_M, denote the risk probability of the marginal buyer in the male motorists market as π_M. Restrict the analysis on interior solutions, i.e., the case where the market is partially covered by insurance before and after the non-discrimination policy implemented, i.e. $\pi_M \in (\pi_2, \pi_4)$.

Equation (2.2.2) gives the threshold risk probability value as

$$\pi_M = \frac{u(y) - u(y - p_M L)}{u(y) - u(y - L)} \tag{2.2.3}$$

In competitive insurance market, insurers offer contracts that break even, i.e. the total premium proceeds insurers receive are equal to the total expected loss from the insureds. This gives

$$p_M L \cdot \int_{\pi_M}^{\pi_4} \frac{N_M}{\pi_4 - \pi_2} d\pi = \frac{1}{2}(\pi_4 + \pi_M) \cdot L \cdot \int_{\pi_M}^{\pi_4} \frac{N_M}{\pi_4 - \pi_2} d\pi$$

or $p_M = \frac{1}{2}(\pi_4 + \pi_M)$ (2.2.4)

Equation (2.2.3) and (2.2.4) give the conditions for the equilibrium in the male motorist insurance market. Similarly, we have the conditions for the female market as

$$\pi_W = \frac{u(y) - u(y - p_W L)}{u(y) - u(y - L)} \tag{2.2.5}$$

$$p_W = \frac{1}{2}(\pi_3 + \pi_W) \tag{2.2.6}$$

where $\pi_W \in (\pi_1, \pi_3)$.

2.2.3 The equilibrium when gender is unobservable

After the non-discrimination policy is implemented, gender is effectively unobservable. The two markets of male and female motorists hence merge into one. In this new merged market, both genders are changed premiums of the same level. Similar to the analysis in Section 2.2.1 Insurance decision, we can find the relationship between the threshold risk probability and a given level of premium as follows

$$\pi_p = \frac{u(y) - u(y - p_p L)}{u(y) - u(y - L)} \tag{2.2.7}$$

where partial coverage condition requires $\pi_p \in (\pi_2, \pi_3)$.

In competitive markets, insurers offer contracts that break even, i.e. the total premiums they receive from both male and female customers are equal to the expected loss from them. This gives

$$\left(\int_{\pi_p}^{\pi_4} \frac{N_M}{\pi_4-\pi_2} d\pi + \int_{\pi_p}^{\pi_3} \frac{N_W}{\pi_3-\pi_1} d\pi\right) \cdot p_p L = \frac{1}{2}(\pi_4+\pi_p) \cdot L \cdot \int_{\pi_p}^{\pi_4} \frac{N_M}{\pi_4-\pi_2} d\pi + \frac{1}{2}(\pi_3+\pi_p) \cdot L \cdot \int_{\pi_p}^{\pi_3} \frac{N_W}{\pi_3-\pi_1} d\pi$$

or

$$\left(p_p - \frac{1}{2}(\pi_4+\pi_p)\right) \cdot \int_{\pi_p}^{\pi_4} \frac{N_M}{\pi_4-\pi_2} d\pi + \left(p_p - \frac{1}{2}(\pi_3+\pi_p)\right) \cdot \int_{\pi_p}^{\pi_3} \frac{N_W}{\pi_3-\pi_1} d\pi = 0 \qquad (2.2.8)$$

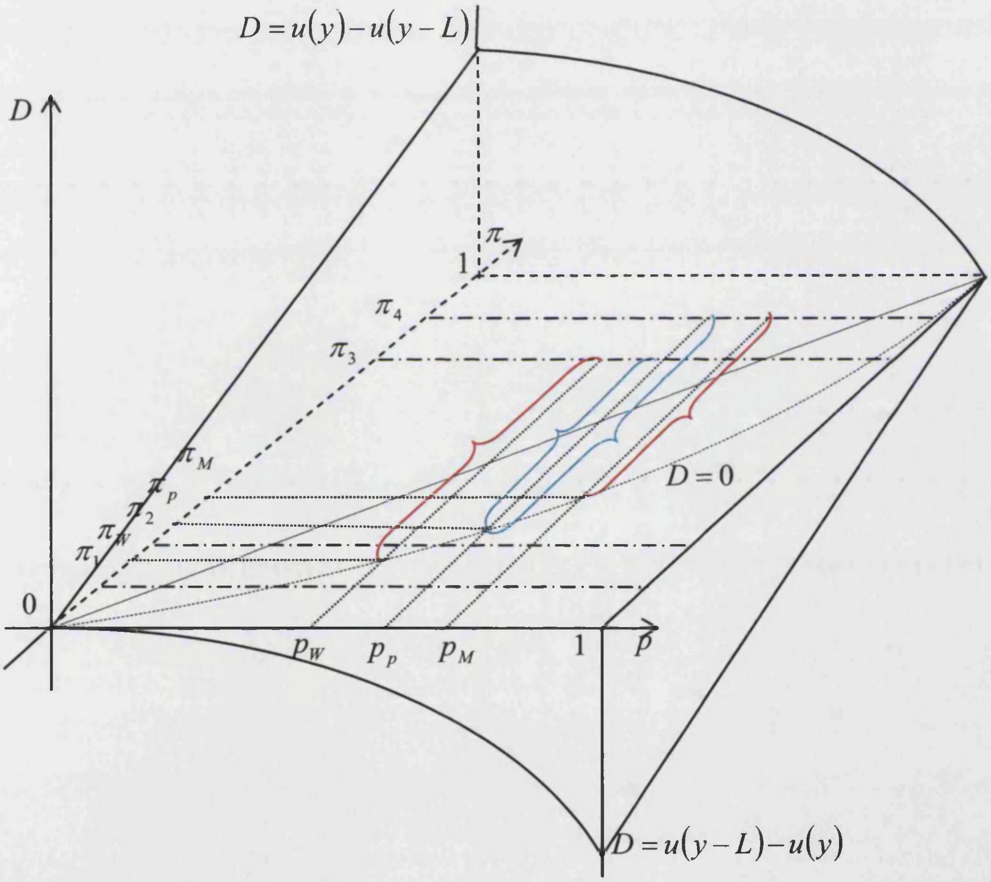


Figure 2.1

Figure 2.1 shows the equilibria when gender is observable and the equilibrium when it is not. When it is observable female motorists with risk probability ranging between $[\pi_W, \pi_3]$ purchase the insurance at the premium p_W while male motorists between $[\pi_M, \pi_4]$ purchase at p_M. When it is unobservable both are charged at p_p. The risk range of female customers is $\lfloor \pi_p, \pi_3 \rfloor$ while it is $\lfloor \pi_p, \pi_4 \rfloor$ for male motorists.

2.2.4 Possibility of multiple equilibria

Take the male motorists market as an example. When gender is observable, from (2.2.4), we have $\frac{dp_M}{d\pi_M} = \frac{1}{2}$. From (2.2.3), we have $\frac{d\pi_M}{dp_M} = \frac{u'(y - p_M L) \cdot L}{u(y) - u(y - L)} > 0$ and $\frac{d^2\pi_M}{dp_M^2} = \frac{-L^2 \cdot u''(y - p_M L)}{u(y) - u(y - L)} > 0$. Taking into consideration that (2.2.4) gives $p_M = \frac{1}{2}\pi_4$ when $\pi_M = 0$ and (2.2.3) gives $\pi_M = 0$ when $p_M = 0$, we can not rule out the possibility of no interior equilibrium or multiple quilibria.[8] Figure 2.2 illustrates the possibility of the existence of multiple equilibria.

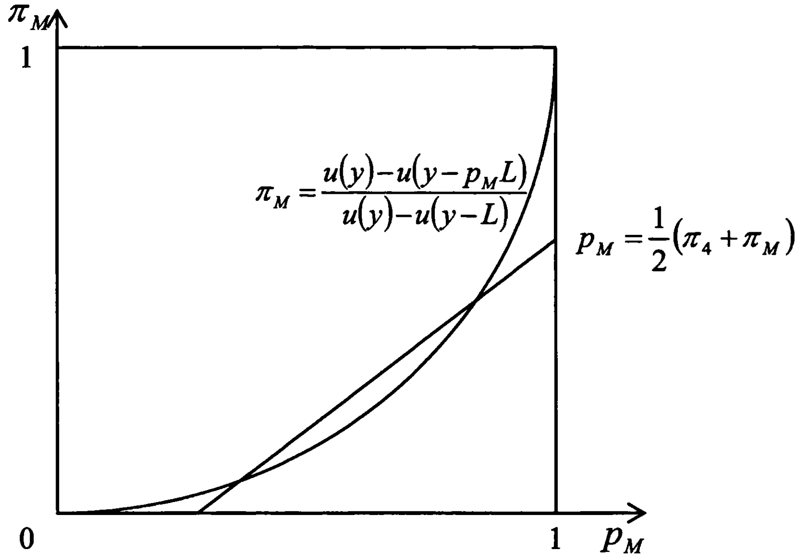


Figure 2.2

When gender is unobservable, from (2.2.8), we have

$$\frac{dp_p}{d\pi_p} = \frac{(1+T)\left[(\pi_4 - \pi_p)^2 + T(\pi_3 - \pi_p)^2\right]}{2\left[\pi_4 - \pi_p + T(\pi_3 - \pi_p)\right]^2} > 0 \text{ and } \frac{d^2 p_p}{d\pi_p^2} = \frac{T(1+T)(\pi_4 - \pi_3)^2}{\left[\pi_4 - \pi_p + T(\pi_3 - \pi_p)\right]^3} > 0$$

From (2.2.7), we have

$$\frac{d\pi_p}{dp_p} = \frac{u'(y - p_p L) \cdot L}{u(y) - u(y - L)} > 0 \text{ and } \frac{d^2\pi_p}{dp_p^2} = \frac{-L^2 \cdot u''(y - p_p L)}{u(y) - u(y - L)} > 0$$

[8] An equilibrium must surely exist but it may involve all motorists insured. Take the premium which makes worst motorist just willing to insure. As risk averse this must be profitable. So premium must fall. Profits may rise as better types enter but when every one in further falls in premium must lower profitability.

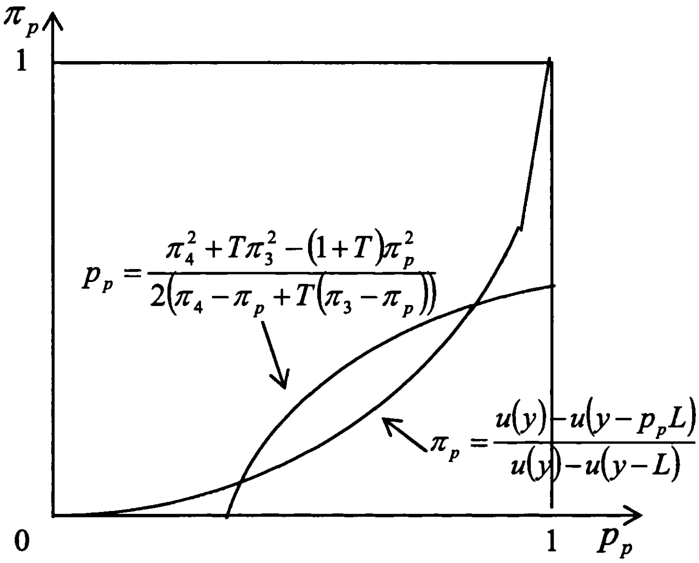


Figure 2.3

Taking into consideration that (2.2.8) gives $p_p = \frac{\pi_4^2 + T\pi_3^2}{2(\pi_4 + T\pi_3)}$ when $\pi_p = 0$ and (2.2.7) gives $\pi_p = 0$ when $p_p = 0$, we can not rule out the possibility of no interior equilibrium or multiple quilibria. Figure 2.3 illustrates the existence of multiple equilibria.

One interesting feature of the pooling equilibrium figure is the possible kink which happens when one or the other group ceases to be present. Figure 2.3 illustrates one possibility when the accident probability is so high that the female motorists cease to be present. Then the market has only male motorists left and hence the line kinks. However, this research focuses on the interior equilibrium and does not go further on analyzing the equilibrium in the kink part.

2.2.5 Welfare property when gender is observable

First consider the male motorists insurance market. An Individual driver with no insurance has the expected utility $V_1(\pi_i) = (1-\pi_i)\cdot u(y) + \pi_i \cdot u(y-L)$ where

$\pi_i \in [\pi_2, \pi_4]$. After purchasing the insurance his utility is fixed at $V_2(p_M) = u(y - p_M L)$. In the partially covered market the threshold risk probability $\pi_M \in (\pi_2, \pi_4)$.

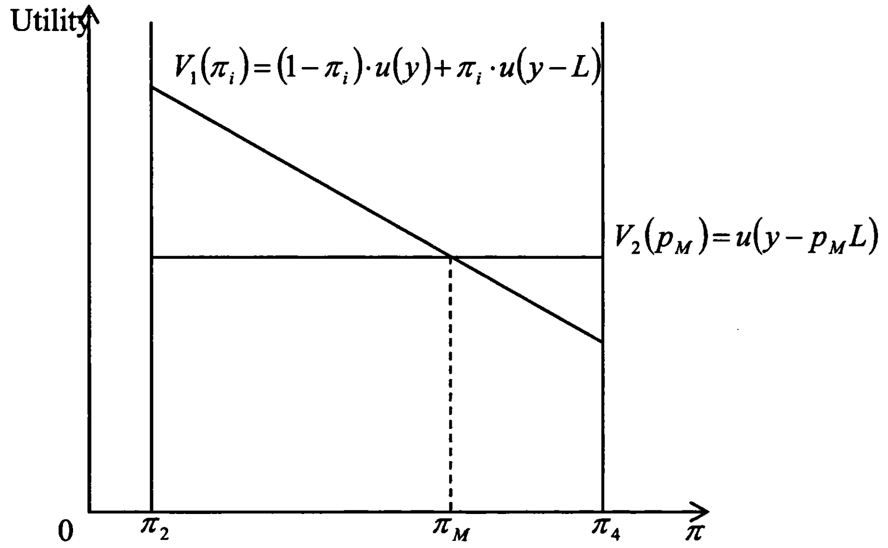


Figure 2.4

Denote the number of the uninsured motorists and the insured as N_1 and N_2 respectively. Then $N_1 = \int_{\pi_2}^{\pi_M} \frac{N_M}{\pi_4 - \pi_2} d\pi$ and $N_2 = N_M - N_1 = \int_{\pi_M}^{\pi_4} \frac{N_M}{\pi_4 - \pi_2} d\pi$.

Similarly, denote the aggregate utility of the uninsured motorists and the insured as AU_1 and AU_2 respectively. Then the aggregate utility of the market is

$$AU = AU_1 + AU_2 \tag{2.2.9}$$

Where

$$AU_1 = \int_{\pi_2}^{\pi_M} \frac{N_M}{\pi_4 - \pi_2} \cdot V_1(\pi_i) d\pi = \frac{N_M}{\pi_4 - \pi_2} \cdot \int_{\pi_2}^{\pi_M} V_1(\pi_i) d\pi \tag{2.2.10}$$

$$AU_2 = V_2(p_M) \cdot N_2 = u(y - p_M L) \cdot \int_{\pi_M}^{\pi_4} \frac{N_M}{\pi_4 - \pi_2} d\pi \tag{2.2.11}$$

Given $V_1(\pi_i) = (1-\pi_i)\cdot u(y) + \pi_i \cdot u(y-L)$ is linear, AU_1 can be expressed as

$$AU_1 = \frac{N_M}{\pi_4 - \pi_2} \cdot \frac{1}{2} (V_1(\pi_2) + V_1(\pi_M))(\pi_M - \pi_2) \tag{2.2.12}$$

where $V_1(\pi_2) = (1-\pi_2)\cdot u(y) + \pi_2 \cdot u(y-L)$ and

$V_1(\pi_M) = (1-\pi_M)\cdot u(y) + \pi_M \cdot u(y-L) = u(y - p_M L)$

Similarly, the aggregate utility of the female motorists market is $AU = AU_1 + AU_2$, where

$$AU_1 = \int_{\pi_1}^{\pi_W} \frac{N_W}{\pi_3 - \pi_1} \cdot V_1(\pi_i) d\pi = \frac{N_W}{\pi_3 - \pi_1} \cdot \int_{\pi_1}^{\pi_W} V_1(\pi_i) d\pi \tag{2.2.13}$$

$$AU_2 = V_2(p_W) \cdot N_2 = u(y - p_W L) \cdot \int_{\pi_W}^{\pi_3} \frac{N_W}{\pi_3 - \pi_1} d\pi \tag{2.2.14}$$

As above, AU_1 is expressed as

$$AU_1 = \frac{N_W}{\pi_3 - \pi_1} \cdot \frac{1}{2} (V_1(\pi_1) + V_1(\pi_W))(\pi_W - \pi_1) \tag{2.2.15}$$

where $V_1(\pi_1) = (1 - \pi_1) \cdot u(y) + \pi_1 \cdot u(y - L)$ and

$V_1(\pi_W) = (1 - \pi_W) \cdot u(y) + \pi_W \cdot u(y - L) = u(y - p_W L)$

2.2.6 Welfare change from implementing the policy

Male drivers benefit from the non-discrimination policy which leads to a decrease of insurance premium. The already insured motorists benefit from the lower cost. Some previously uninsured motorists join the market and gain from the higher utility. Oppositely, female motorists suffer a welfare loss from the increase of insurance premium. The existing customers suffer from the higher cost. Some of them even drop out of the insurance market.

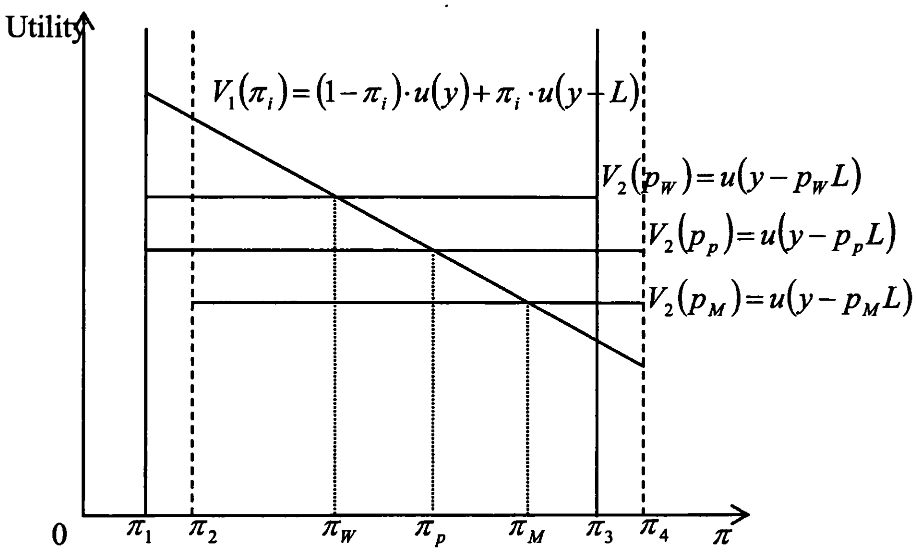


Figure 2.5

The welfare loss is

$$WL=\left(V_2(p_W)-V_2(p_p)\right)\cdot\int_{\pi_p}^{\pi_3}\frac{N_W}{\pi_3-\pi_1}d\pi+V_2(p_W)\cdot\int_{\pi_W}^{\pi_p}\frac{N_W}{\pi_3-\pi_1}d\pi-\frac{N_W}{\pi_3-\pi_1}\cdot\int_{\pi_W}^{\pi_p}V_1(\pi_i)d\pi$$

$$=\frac{N_W}{\pi_3-\pi_1}\left[\left(V_2(p_W)-V_2(p_p)\right)(\pi_3-\pi_p)+V_2(p_W)(\pi_p-\pi_W)-\int_{\pi_W}^{\pi_p}V_1(\pi_i)d\pi\right]$$

or

$$WL=\frac{1}{2}\cdot\frac{N_W}{\pi_3-\pi_1}\left(V_2(p_W)-V_2(p_p)\right)(2\pi_3-\pi_W-\pi_p) \tag{2.2.16}$$

The welfare gain is

$$WG=\left(V_2(p_p)-V_2(p_M)\right)\cdot\int_{\pi_M}^{\pi_4}\frac{N_M}{\pi_4-\pi_2}d\pi+V_2(p_p)\cdot\int_{\pi_p}^{\pi_M}\frac{N_M}{\pi_4-\pi_2}d\pi-\frac{N_M}{\pi_4-\pi_2}\cdot\int_{\pi_p}^{\pi_M}V_1(\pi_i)d\pi$$

$$=\frac{N_M}{\pi_4-\pi_2}\left[\left(V_2(p_p)-V_2(p_M)\right)(\pi_4-\pi_M)+V_2(p_p)(\pi_M-\pi_p)-\int_{\pi_p}^{\pi_M}V_1(\pi_i)d\pi\right]$$

or

$$WG=\frac{1}{2}\cdot\frac{N_M}{\pi_4-\pi_2}\left(V_2(p_p)-V_2(p_M)\right)(2\pi_4-\pi_M-\pi_p) \tag{2.2.17}$$

Whether the aggregate welfare of both markets increases depends on which effect dominates.

2.2.7 Simulation

Specifying the utility function as Constant Relative Risk Averse (CRRA) utility function as

$$u(c)=\frac{c^{1-\theta}}{1-\theta},\ \theta>0$$

In order to check whether the welfare gain dominates the loss, we check $WG-WL$, where

$$WG=\frac{1}{2}\cdot\frac{N_M}{\pi_4-\pi_2}\left(V_2(p_p)-V_2(p_M)\right)(2\pi_4-\pi_M-\pi_p)$$ and

$$WL=\frac{1}{2}\cdot\frac{N_W}{\pi_3-\pi_1}\left(V_2(p_W)-V_2(p_p)\right)(2\pi_3-\pi_W-\pi_p)$$

With the assumptions that $\pi_4-\pi_2=T(\pi_3-\pi_1)$, we only need to check the tendency of

$$\frac{N_M}{T}\left(V_2(p_p)-V_2(p_M)\right)(2\pi_4-\pi_M-\pi_p)-N_W\left(V_2(p_W)-V_2(p_p)\right)(2\pi_3-\pi_W-\pi_p)$$

which for brevity we write as

$$D_1 = \frac{N_M}{T}(V_p - V_M)(2\pi_4 - \pi_M - \pi_p) - N_W(V_W - V_p)(2\pi_3 - \pi_W - \pi_p)$$

where

$$V_W = \frac{(y - p_W L)^{1-\theta}}{1-\theta}, \; p_W = \frac{1}{2}(\pi_3 + \pi_W), \; \pi_W = \frac{y^{1-\theta} - (y - p_W L)^{1-\theta}}{y^{1-\theta} - (y-L)^{1-\theta}}$$

$$V_p = \frac{(y - p_p L)^{1-\theta}}{1-\theta}, \; \pi_p = \frac{y^{1-\theta} - (y - p_p L)^{1-\theta}}{y^{1-\theta} - (y-L)^{1-\theta}}$$

$$\frac{N_M}{T}(\pi_4 - \pi_p)\left(p_p - \frac{1}{2}(\pi_4 + \pi_p)\right) + N_W(\pi_3 - \pi_p)\left(p_p - \frac{1}{2}(\pi_3 + \pi_p)\right) = 0$$

$$V_M = \frac{(y - p_M L)^{1-\theta}}{1-\theta}, \; p_M = \frac{1}{2}(\pi_4 + \pi_M), \; \pi_M = \frac{y^{1-\theta} - (y - p_M L)^{1-\theta}}{y^{1-\theta} - (y-L)^{1-\theta}}$$

For $y = 100$, $\theta = 0.8$, $\pi_1 = 0.1$, $\pi_3 = 0.7$, $\pi_2 = 0.2$, $\pi_4 = 0.8$, $T = 1$ and $N_M = N_W$, we have the following figure depicting the relationship between welfare change and loss level.[9]

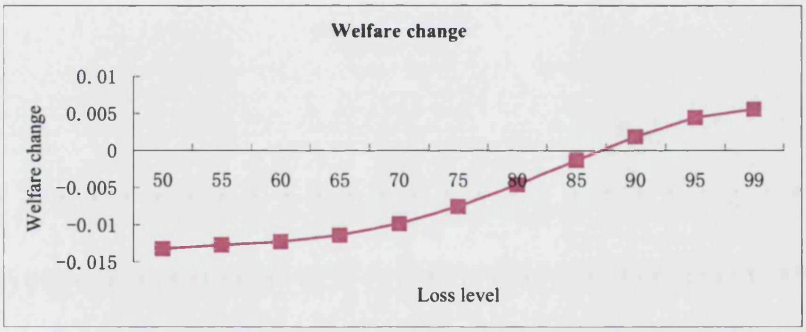


Figure 2.6

Figure 2.6 illustrates that the welfare change is ambiguous which can be either positive or negative with different loss levels and the welfare change increases monotonically in loss level. This is consistent with the intuition: pooling effect increases by redistributing more income with loss level increasing.

[9] Appendix 2.1 provides detailed simulation results.

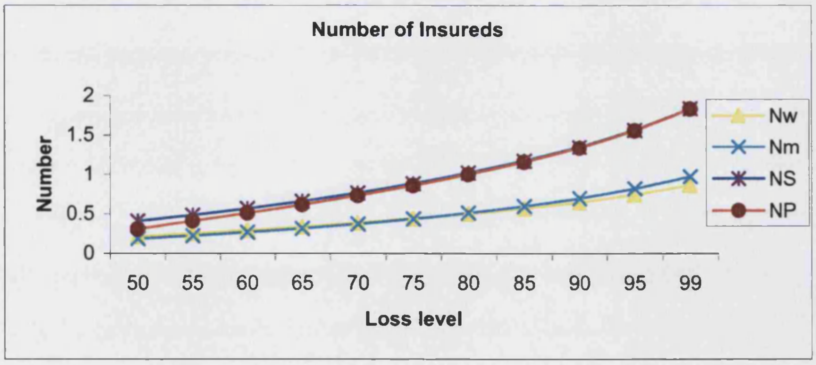


Figure 2.7

Figure 2.7 shows that between loss level of 50 and 99, the number of insureds when gender is unobservable is always less than that when gender is observable. Roughly at the loss level of 87, the aggregate social welfare when gender is unobservable is equal to that when observable, although there are less insureds when unobservable. This illustrates that pooling can generate higher aggregate utility with other things equal.

For $y=100$, $\theta=0.8$, $L=90$, $\pi_1=0.1$, $\pi_3=0.7$, $T=1$ and $N_M=N_W$, we have the following figure depicting the relationship between welfare change and risk range.[10]

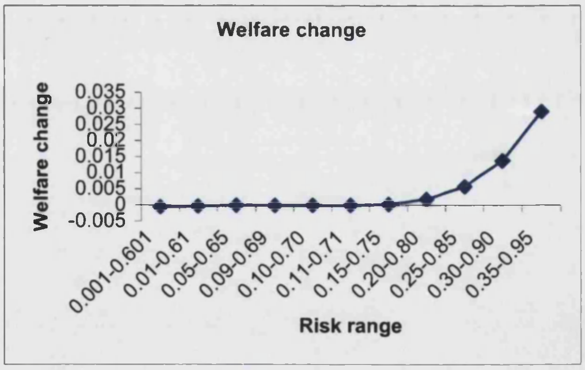


Figure 2.8

[10] Appendix 2.2 provides detailed simulation results.

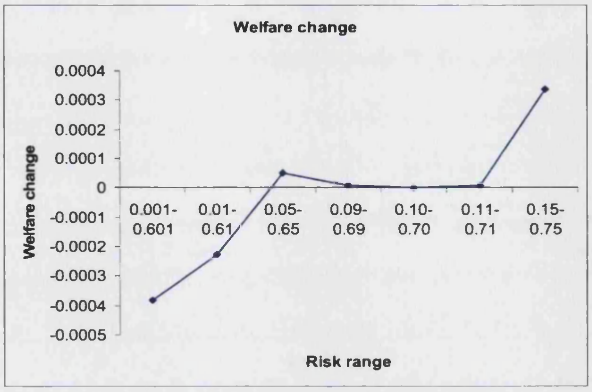


Figure 2.9

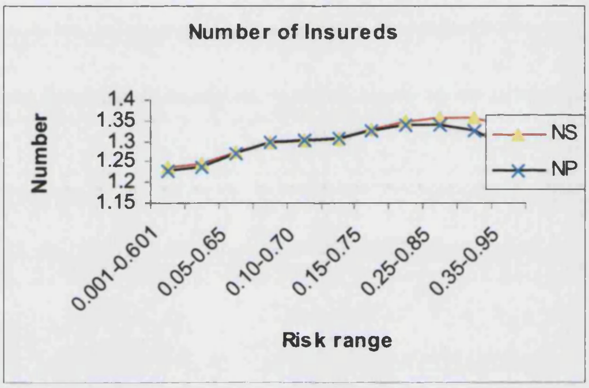
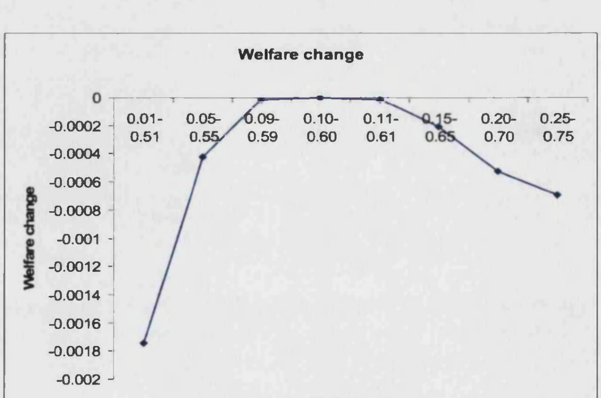


Figure 2.10

Figure 2.8 shows how the welfare change varies with the risk range while Figure 2.9 is a part of it. Figure 2.10 shows the variation of the number of the insureds with risk range. The number of motorists when gender is unobservable is always less than that when gender is observable except the point where both risk range overlap.

For $y=100$, $\theta=0.7$, $L=95$, $\pi_1=0.1$, $\pi_3=0.6$, $T=1$ and $N_M=N_W$, we have the following figure depicting the relationship between welfare change and risk range.[11]



[11] Appendix 2.3 provides detailed simulation results.

Figure 2.11

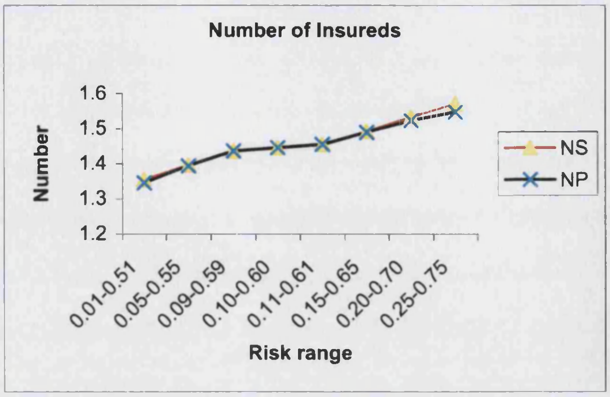


Figure 2.12

Figure 2.11 shows how the welfare change varies with the risk. Figure 2.12 shows the variation of the number of insureds with risk range. The number of motorists when gender is unobservable is always less than that when gender is observable except the point where both risk range overlap.

2.3 To drive or not to drive

Consider now a scenario in which every assumption is the same as section 2 except there is a mandatory requirement that motorists must be fully insured. Instead of choosing whether to get insured, people now choose whether to become a motorist. Everyone has a reservation utility $\overline{V}$ if not a motorist. Accidents cause a non-pecuniary cost C which is not insurable in addition to a financial loss L which is insurable.[12]

2.3.1 Participation decision

A motorist has expected utility

$$V_i(\pi_i, p_i) = (1-\pi_i)\cdot u(y - p_i q) + \pi_i(u(y - p_i q - L + q) - C)$$

When fully insured the utility becomes

$$V_i(\pi_i, p_i) = u(y - p_i L) - \pi_i C$$

An individual will become a motorist and purchase insurance if the utility from be a motorist is greater than the reservation utility:

$$D_2 = V_i(\pi_i, p_i) - \overline{V} \geq 0$$

[12] Externalities could mean that individual accident probabilities depend on the number of motorists but to focus on pure insurance issues this possibility is ignored.

Checking the properties of D_2 with respect to p and π, we have

$$\frac{\partial D_2}{\partial p_i} = -L \cdot u'(y - p_i L) < 0, \quad \frac{\partial^2 D_2}{\partial p_i^2} = L^2 \cdot u''(y - p_i L) < 0$$

$$\frac{\partial D_2}{\partial \pi_i} = -C < 0, \quad \frac{\partial^2 D_2}{\partial \pi_i^2} = 0$$

So the utility difference decreases monotonically in both insurance premium, p, and risk probability, π. To ensure interior solutions, make the further assumption

$$u(y-L) < \overline{V} < u(y) < C + \overline{V}$$

Therefore, for corners,

$D_2 = u(y) - \overline{V} > 0$ when $p = 0$ and $\pi = 0$,

$D_2 = u(y) - C - \overline{V} < 0$ when $p = 0$ and $\pi = 1$,

$D_2 = u(y-L) - \overline{V} < 0$ when $p = 1$ and $\pi = 0$,

$D_2 = u(y-L) - C - \overline{V} < 0$ when $p = 1$ and $\pi = 1$.

The marginal buyer receives the same level of utility from driving as the reservation utility. So we have $D_2 = V_i(\pi_i, p_i) - \overline{V} = 0$, i.e.

$u(y - p_i L) - \pi_i C - \overline{V} = 0$, which gives

$$\pi_i = \frac{u(y - p_i L) - \overline{V}}{C} \tag{2.3.1}$$

Check the variation of π with p gives

$$\frac{d\pi_i}{dp_i} = -\frac{L}{C} \cdot u'(y - p_i L) < 0, \quad \frac{d^2 \pi_i}{dp_i^2} = \frac{L^2}{C} \cdot u''(y - p_i L) < 0$$

So for a given premium p^*, we can find a unique risk probability π^* which satisfies $D_2 = V_i(\pi_i, p_i) - \overline{V} = 0$. For the people whose risk probability is equal to or less than π^*, their utility difference from becoming a motorist with full insurance will be equal to or greater than zero, i.e., $D_2 = V_i(\pi_i, p_i) - \overline{V} \geq 0$, where $\pi_i \leq \pi^*$. This leads to the following proposition.

Proposition 2: for a given insurance premium p^*, an Individual customer will become a motorist and buy the insurance if his own risk probability π_i is lower than or equal to the threshold risk probability π^*, i.e. $\pi_i \leq \pi^*$, where

$$\pi^* = \frac{u(y - p^* L) - \bar{V}}{C} \tag{2.3.2}$$

2.3.2 The equilibria when gender is observable

Consider the male market first. For an insurance premium, p_M, set the risk probability of the marginal buyer in the male motorists market as π_M. Consider the case where the market is partially covered by insurance before the equality policy is implemented, i.e. $\pi_M \in (\pi_2, \pi_4)$.

Equation (2.3.2) gives the threshold risk probability as

$$\pi_M = \frac{u(y - p_M L) - \bar{V}}{C} \tag{2.3.3}$$

In a competitive insurance market, insurers offer contracts that break even. This gives

$$p_M L \cdot \int_{\pi_2}^{\pi_M} \frac{N_M}{\pi_4 - \pi_2} d\pi = \frac{1}{2}(\pi_2 + \pi_M) \cdot L \cdot \int_{\pi_2}^{\pi_M} \frac{N_M}{\pi_4 - \pi_2} d\pi$$

or $p_M = \frac{1}{2}(\pi_2 + \pi_M)$ (2.3.4)

Equation (2.3.3) and (2.3.4) give the conditions for the equilibrium in male motorist insurance market. Similarly, we have the conditions for the female market as

$$\pi_W = \frac{u(y - p_W L) - \bar{V}}{C} \tag{2.3.5}$$

$p_W = \frac{1}{2}(\pi_1 + \pi_W)$, where $\pi_W \in (\pi_1, \pi_3)$ (2.3.6)

2.3.3 The equilibrium when gender is unobservable

After the policy, the two markets of male and female motorists merge into one as gender is now effectively unobservable. Both genders are charged at the same premium. From (2.3.2), we have

$$\pi_p = \frac{u(y - p_p L) - \bar{V}}{C} \tag{2.3.7}$$

where partial coverage condition requires $\pi_p \in (\pi_2, \pi_3)$.

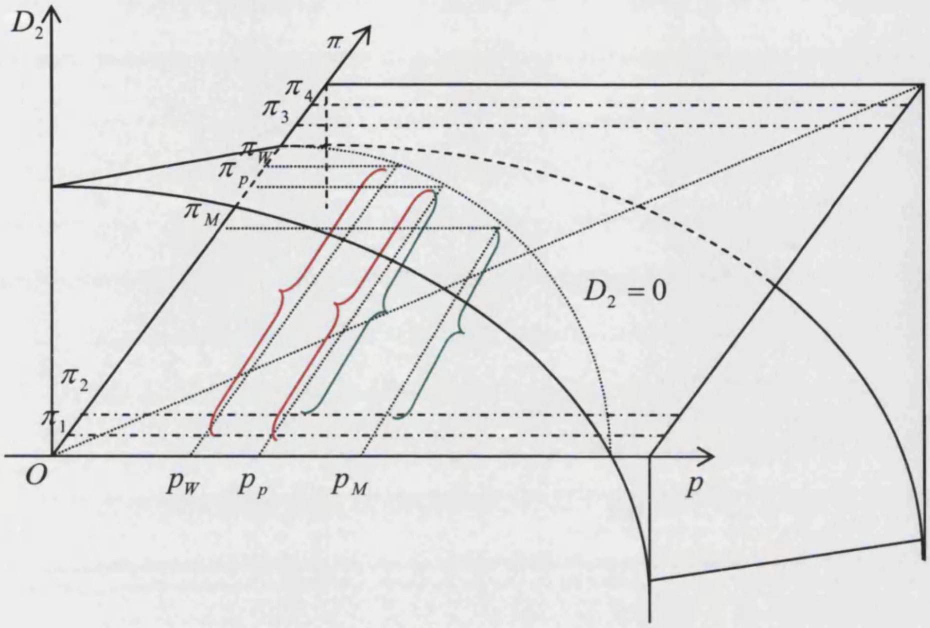


Figure 2.13

For the contract to break even we have

$$\left(\int_{\pi_2}^{\pi_p} \frac{N_M}{\pi_4 - \pi_2} d\pi + \int_{\pi_1}^{\pi_p} \frac{N_W}{\pi_3 - \pi_1} d\pi\right) \cdot p_p L = \frac{1}{2}(\pi_2 + \pi_p) \cdot L \cdot \int_{\pi_2}^{\pi_p} \frac{N_M}{\pi_4 - \pi_2} d\pi + \frac{1}{2}(\pi_3 + \pi_p) \cdot L \cdot \int_{\pi_1}^{\pi_p} \frac{N_W}{\pi_3 - \pi_1} d\pi$$

or
$$\left(p_p - \frac{1}{2}(\pi_2 + \pi_p)\right) \cdot \int_{\pi_2}^{\pi_p} \frac{N_M}{\pi_4 - \pi_2} d\pi + \left(p_p - \frac{1}{2}(\pi_1 + \pi_p)\right) \cdot \int_{\pi_1}^{\pi_p} \frac{N_W}{\pi_3 - \pi_1} d\pi = 0 \qquad (2.3.8)$$

Equation (2.3.7) and (2.3.8) give the conditions for the pooling equilibrium.

Figure 2.13 illustrates the separating equilibria and the pooling equilibrium. In the separating equilibria female motorists with risk probability ranging between $[\pi_1, \pi_W]$ purchase the insurance at the premium p_W while male motorists between $[\pi_2, \pi_M]$ purchase at p_M. In the pooling equilibrium both are charged at p_p. The risk range of female customers is $\lfloor \pi_1, \pi_p \rfloor$ while it is $\lfloor \pi_2, \pi_p \rfloor$ for male motorists.

2.3.4 The possibility of multiple equilibria

Take the male motorists market as an example. When gender is observable, from (2.3.4), we have $\frac{dp_M}{d\pi_M} = \frac{1}{2}$. From (2.3.3), we have

$$\frac{d\pi_M}{dp_M} = -\frac{L}{C} \cdot u'(y - p_M L) < 0, \quad \frac{d^2\pi_M}{dp_M^2} = \frac{L^2}{C} \cdot u''(y - p_M L) < 0$$

Taking into consideration that (2.3.4) gives $p_M = \frac{1}{2}\pi_2$ when $\pi_M = 0$ and (2.3.3) gives $\pi_M = \frac{u(y) - \bar{V}}{C} > 0$ when $p_M = 0$, we can not rule out the possibility of no interior equilibrium but multiple quilibria are impossible. Figure 2.14 illustrates the non-existence of multiple equilibria.

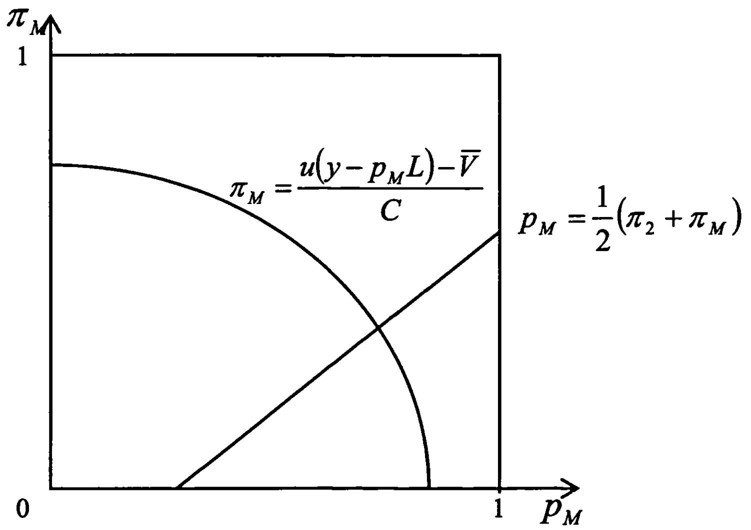


Figure 2.14

When gender is unobservable, from (2.3.8), we have

$$\frac{dp_p}{d\pi_p} = \frac{(1+T)\left[(\pi_p - \pi_2)^2 + T(\pi_p - \pi_1)^2\right]}{2\left[\pi_p - \pi_2 + T(\pi_p - \pi_1)\right]^2} > 0 \text{ and}$$

$$\frac{d^2 p_p}{d\pi_p^2} = -\frac{T(1+T)(\pi_2 - \pi_1)^2}{\left[\pi_p - \pi_2 + T(\pi_p - \pi_1)\right]^3} < 0$$

From (2.3.7), we have

$$\frac{d\pi_p}{dp_p} = -\frac{L}{C} \cdot u'(y - p_p L) < 0, \text{ and } \frac{d^2\pi_p}{dp_p^2} = \frac{L^2}{C} \cdot u''(y - p_p L) < 0$$

Taking into consideration that (2.3.8) gives $p_p = \frac{\pi_2^2 + T\pi_1^2}{2(\pi_2 + T\pi_1)}$ when $\pi_p = 0$ and (2.3.7) gives $\pi_p = \frac{u(y) - \bar{V}}{C} > 0$ when $p_p = 0$, we can not rule out the possibility of no interior equilibrium but multiple quilibria are impossible. Figure 2.15 illustrates the non-existence of multiple equilibria.[13]

[13] The feature of kink applies to this scenario as well. But, again, we focus on the interior solution in this research.

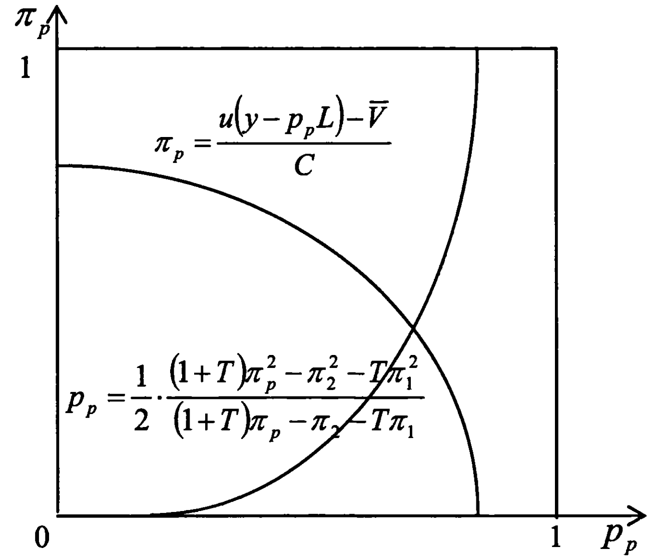


Figure 2.15

2.3.5 Welfare property when gender is observable

First consider the male motorists insurance market. An Individual who chooses not to become a motorist has a reserve utility $\bar{V}$. After becoming a motorist with full insurance his expected utility is $V_i(\pi_i, p_i) = u(y - p_i L) - \pi_i C$, where $\pi \in [\pi_2, \pi_4]$. In the partially covered market the threshold risk probability $\pi_M \in (\pi_2, \pi_4)$.

Set the number of the fully insured motorists and the non-motorists as N_1 and N_2 respectively. Then $N_1 = \int_{\pi_2}^{\pi_M} \frac{N_M}{\pi_4 - \pi_2} d\pi$ and $N_2 = N_M - N_1 = \int_{\pi_M}^{\pi_4} \frac{N_M}{\pi_4 - \pi_2} d\pi$.

Similarly, set the aggregate utility of the motorists and the non-motorists as AU_1 and AU_2 respectively. Then the aggregate utility of the market is

$$AU = AU_1 + AU_2 \tag{2.3.9}$$

Where

$$AU_1 = \int_{\pi_2}^{\pi_M} \frac{N_M}{\pi_4 - \pi_2} \cdot V(\pi_i) d\pi = \frac{N_M}{\pi_4 - \pi_2} \cdot \int_{\pi_2}^{\pi_M} V(\pi_i) d\pi \tag{2.3.10}$$

$$AU_2 = \bar{V} \cdot N_2 = \bar{V} \cdot \int_{\pi_M}^{\pi_4} \frac{N_M}{\pi_4 - \pi_2} d\pi \tag{2.3.11}$$

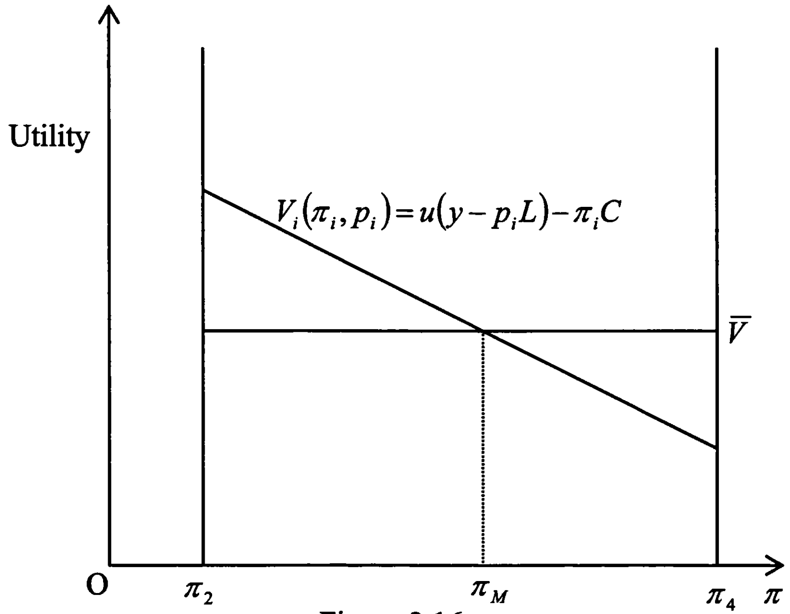


Figure 2.16

Given $V(\pi_i)=u(y-p_iL)-\pi_iC$ is linear, equation (2.3.10) can be expressed as

$$AU_1=\frac{N_M}{\pi_4-\pi_2}\cdot\frac{1}{2}\left(V(\pi_2)+V(\pi_M)\right)\left(\pi_M-\pi_2\right) \qquad (2.3.12)$$

where $V(\pi_2)=u(y-p_ML)-\pi_2C$ and $V(\pi_M)=u(y-p_ML)-\pi_MC=\overline{V}$

Similarly, the aggregate utility of the female motorists market is $AU=AU_1+AU_2$.

Where

$$AU_2=\overline{V}\cdot N_2=\overline{V}\cdot\int_{\pi_W}^{\pi_3}\frac{N_W}{\pi_3-\pi_1}d\pi \qquad (2.3.13)$$

$$AU_1=\int_{\pi_1}^{\pi_W}\frac{N_W}{\pi_3-\pi_1}\cdot V(\pi_i)d\pi=\frac{N_W}{\pi_3-\pi_1}\cdot\int_{\pi_1}^{\pi_W}V(\pi_i)d\pi \qquad (2.3.14)$$

As the above, AU_1 is expressed as

$$AU_1=\frac{N_W}{\pi_3-\pi_1}\cdot\frac{1}{2}\left(V(\pi_1)+V(\pi_W)\right)\left(\pi_W-\pi_1\right) \qquad (2.3.15)$$

where $V(\pi_1)=u(y-p_WL)-\pi_1C$ and $V(\pi_W)=u(y-p_WL)-\pi_WC=\overline{V}$

2.3.6 Welfare change from implementing the policy

For existing motorists, male drivers gain from the policy with lower premiums while female drivers suffer a welfare loss with higher premiums. Furthermore, as a result of the premium change (decrease for man and increase for woman), some relatively safer man would choose to become a motorist while some relatively riskier woman would stop driving.[14]

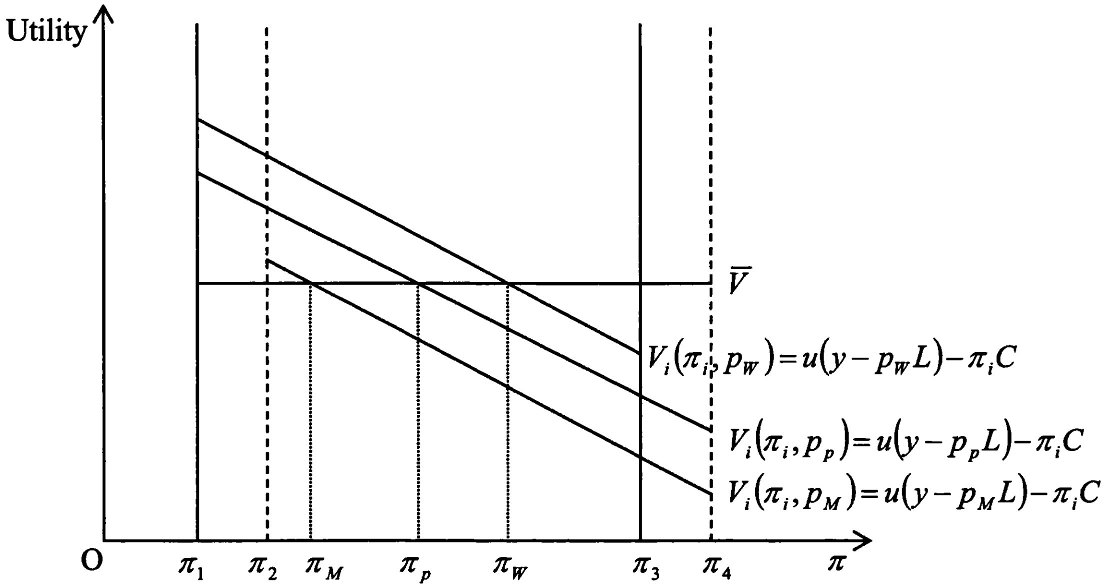


Figure 2.17

The welfare loss hence consists of two parts: loss for existing female motorists from higher premium and loss for former female motorists who drop out of driving:

$$WL = \frac{N_W}{\pi_3 - \pi_1} \cdot \int_{\pi_1}^{\pi_W} V(\pi_i, p_W) d\pi - \frac{N_W}{\pi_3 - \pi_1} \cdot \int_{\pi_1}^{\pi_p} V(\pi_i, p_p) d\pi - \frac{N_W}{\pi_3 - \pi_1} \cdot \overline{V} \cdot (\pi_W - \pi_p)$$

$$= \frac{N_W}{\pi_3 - \pi_1} \begin{bmatrix} \frac{1}{2}(V(\pi_1, p_W) + V(\pi_W, p_W))(\pi_W - \pi_1) - \frac{1}{2}(V(\pi_1, p_p) + V(\pi_p, p_p))(\pi_p - \pi_1) \\ -\overline{V} \cdot (\pi_W - \pi_p) \end{bmatrix}$$

or $$WL = \frac{1}{2} \cdot \frac{N_W}{\pi_3 - \pi_1} \cdot C(\pi_W - \pi_p)(\pi_W + \pi_p - 2\pi_1) \qquad (2.3.16)$$

The welfare gain similarly consists of two parts: gain for existing male motorists from lower premium and gain for new male motorists who now choose to drive:

[14] The whole society gains with the relatively riskier female drivers replaced by relatively safer male drivers. This can be seen as a positive externality brought by the policy. However, this present research focuses on the aggregate welfare change of the man and woman agent groups and hence does not seek to quantify the benefits of this positive externality.

$$WG=\frac{N_M}{\pi_4-\pi_2}\cdot\int_{\pi_2}^{\pi_p}V(\pi_i,p_p)d\pi-\frac{N_M}{\pi_4-\pi_2}\cdot\int_{\pi_2}^{\pi_M}V(\pi_i,p_M)d\pi-\frac{N_M}{\pi_4-\pi_2}\cdot\overline{V}\cdot(\pi_p-\pi_M)$$

$$=\frac{N_M}{\pi_4-\pi_2}\begin{bmatrix}\frac{1}{2}(V(\pi_2,p_p)+V(\pi_p,p_p))(\pi_p-\pi_2)-\frac{1}{2}(V(\pi_2,p_M)+V(\pi_M,p_M))(\pi_M-\pi_2)\\-\overline{V}\cdot(\pi_p-\pi_M)\end{bmatrix}$$

or $$WG=\frac{1}{2}\cdot\frac{N_M}{\pi_4-\pi_2}\cdot C(\pi_p-\pi_M)(\pi_M+\pi_p-2\pi_2) \qquad (2.3.17)$$

Whether the aggregate welfare of both markets increases depends on which effect dominates.

2.3.7 Simulation

Set the utility function as

$$u(c)=\frac{c^{1-\theta}}{1-\theta},\ \theta>0$$

In order to check whether the welfare gain dominates the loss, we check $WG-WL$, where

$$WG=\frac{1}{2}\cdot\frac{N_M}{\pi_4-\pi_2}\cdot C(\pi_p-\pi_M)(\pi_M+\pi_p-2\pi_2)\text{ and}$$

$$WL=\frac{1}{2}\cdot\frac{N_W}{\pi_3-\pi_1}\cdot C(\pi_W-\pi_p)(\pi_W+\pi_p-2\pi_1)$$

With the assumption that $\pi_4-\pi_2=T(\pi_3-\pi_1)$, we only need to check the tendency of D_3:

$$D_3=\frac{N_M}{T}(\pi_p-\pi_M)(\pi_M+\pi_p-2\pi_2)-N_W(\pi_W-\pi_p)(\pi_W+\pi_p-2\pi_1)$$

where

$$p_W=\frac{1}{2}(\pi_1+\pi_W),\ \pi_W=\frac{1}{C}\cdot\left(\frac{(y-p_WL)^{1-\theta}}{1-\theta}-\overline{V}\right)$$

$$\frac{N_M}{T}(\pi_p-\pi_2)\left(p_p-\frac{1}{2}(\pi_2+\pi_p)\right)+N_W(\pi_p-\pi_1)\left(p_p-\frac{1}{2}(\pi_1+\pi_p)\right)=0$$

$$\pi_p=\frac{1}{C}\cdot\left(\frac{(y-p_pL)^{1-\theta}}{1-\theta}-\overline{V}\right)$$

$$p_M=\frac{1}{2}(\pi_2+\pi_M),\ \pi_M=\frac{1}{C}\cdot\left(\frac{(y-p_ML)^{1-\theta}}{1-\theta}-\overline{V}\right)$$

For $y=100$, $\theta=0.8$, $C=5$, $\overline{V}=11$, $\pi_1=0.1$, $\pi_3=0.7$, $\pi_2=0.2$, $\pi_4=0.8$, $T=1$ and $N_M=N_W$, we have figure 2.18 and 2.19 depicting the relationship between welfare change and loss level.[15]

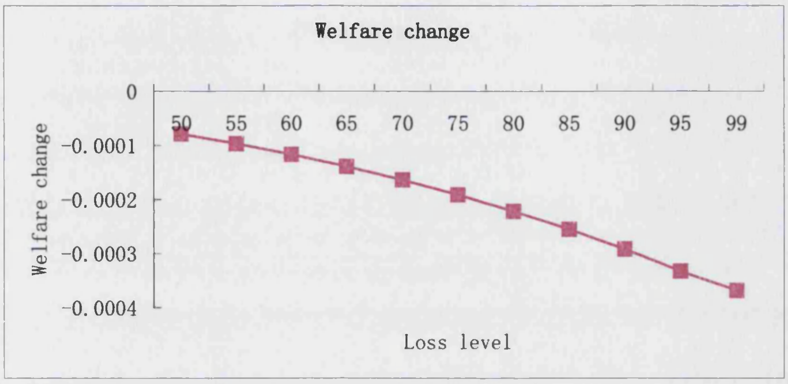


Figure 2.18

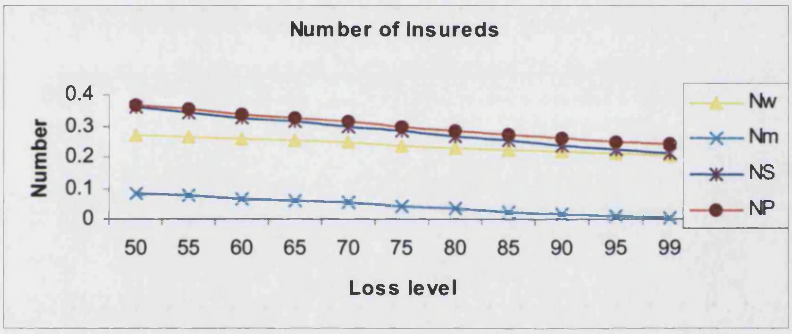


Figure 2.19

Figure 2.18 illustrates that the welfare change decreases monotonically in loss level. Figure 2.19 shows that between loss level of 50 and 99, the number of insureds when gender is unobservable is always greater than that when gender is observable.

For $y=100$, $\theta=0.7$, $L=60$, $C=5$, $\overline{V}=11$, $\pi_1=0.1$, $\pi_3=0.7$, $T=1$ and $N_M=N_W$, we have figure 2.20, 2.21, and 2.22 depicting the relationship between welfare change and the distribution of the risk range of male motorists.[16]

[15] Appendix 2.4 provides detailed simulation results.
[16] Appendix 2.5 provides detailed simulation results.

Figure 2.20

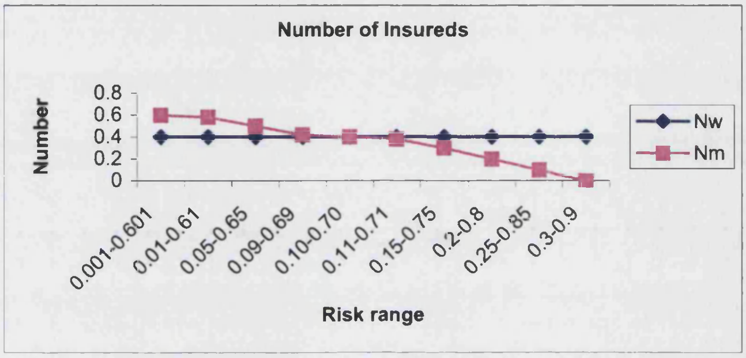


Figure 2.21

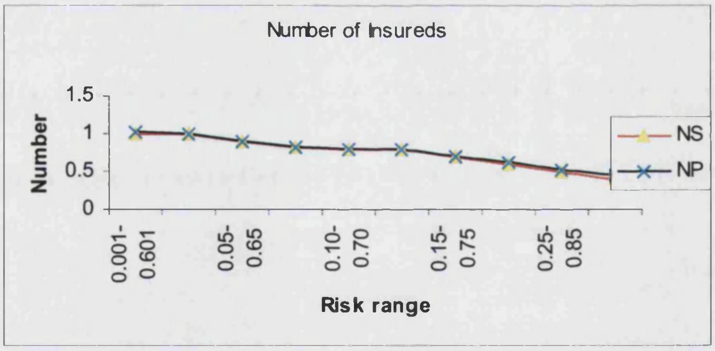


Figure 2.22

Figure 2.20 illustrates that the welfare change first increases in the risk range then decreases. Figure 2.21 shows that when gender is observable, the number of male motorists decreases as the risk range move into greater area. Figure 2.22 shows that the total number of motorists decreases in risk range both when gender is observable and when unobservable while the number of motorists when gender is unobservable is always greater than that when gender is observable except the point where both risk range overlap.

For $y = 100$, $\theta = 0.6$, $L = 80$, $C = 5$, $\overline{V} = 11$, $\pi_1 = 0.2$, $\pi_3 = 0.7$, $T = 1$ and $N_M = N_W$, we have the following figures depicting the similar relationship between welfare change and the distribution of the risk range of male motorists.[17]

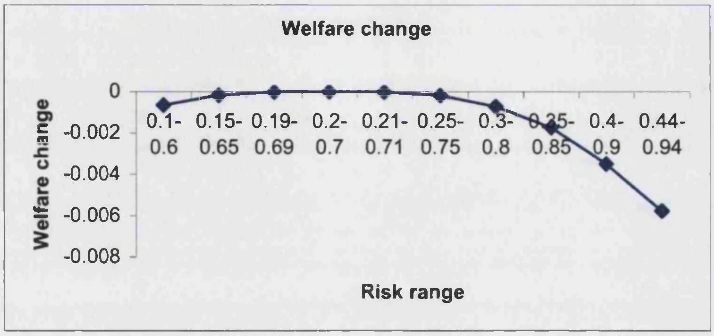


Figure 2.23

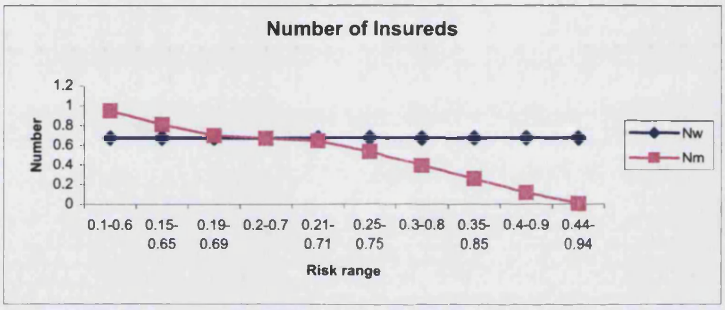


Figure 2.24

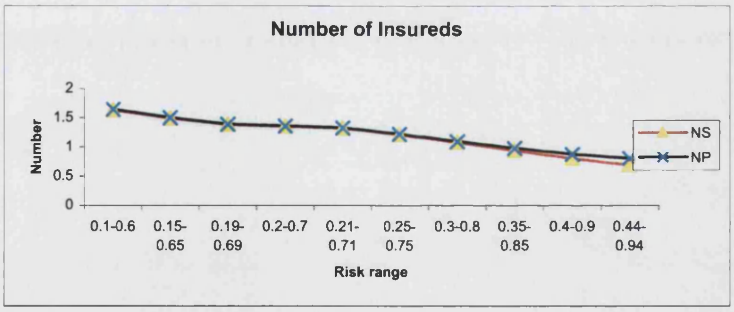


Figure 2.25

2.4 Conclusion

When everyone drives and decides whether to purchase insurance, typical adverse selection happens. High-risk motorists will purchase insurance while the low-risks will not. After the unisex premium policy is implemented, more male motorists purchase insurance with lower premium while some female motorists drop out of the

[17] Appendix 2.6 provides detailed simulation results.

insurance market as the premium increased. Simulations show the total number of the insured is less than before the policy is implemented. The policy has no effect on insurers who always earn normal profit in the competitive industry. The aggregate social welfare may increase or decrease. There is no effect on road safety since everyone drives whether or not insured and there is no moral hazard.

When motorists must be insured and people choose whether to become a motorist, advantageous selection happens. Low-risk people become motorists and purchase insurance while the high-risks will not. After the policy is implemented, more relatively safer men become motorist while some relatively riskier women stop driving. From the simulations, the total number of motorists is greater than before the policy is implemented. The policy has no effect on insurers. Simulations show that the aggregate social welfare decreases after the policy is implemented.

Chapter 3 Optimal allocation and participation option

3.1 Introduction

It has become an established truth that information asymmetry causes efficiency loss. One application is to insurance markets. Traditional research finds that insufficient insurance coverage occurs in an insurance market with adverse selection and hence the insurance policy should be subsidized. The only reason to tax is to allow cross-subsidization: the low-risks pay tax for their insurance policy and the proceeds is used to subsidize the insurance policy purchased by the high-risks. The purpose of this cross-subsidization is still to improve insurance coverage for both risk types. Moreover, the full-insurance pooling contract is the first best allocation and maximizes total social welfare. Does this conclusion still hold when people can opt out of insurance market and even quit the risky activity? Is there always too little insurance in the market equilibrium?

This essay attempts to answer these questions in the context of motor insurance markets. Consider a model with the set up as in the standard Rothschild and Stiglitz model where there are two types of agents who are identical except for risk probability and the form of insurance contracts is fully endogenized. Let us now extend the model by allowing people to choose whether to drive and motorists to choose whether to insure. Motorists enjoy the benefits from driving but may suffer a financial loss from engaging in the risky activity, while non-motorists receive reservation utility which is the same for everyone but have no driving benefits.

If the reservation utility is too low, then both agent types would choose to drive and buy insurance, the outcome would be the same as the Rothschild and Stiglitz model. If, on the contrary, the reservation utility is too high, both types would stop driving. The insurance market therefore disappears.

Let us focus on the more interesting scenario where the low-risks choose to drive and buy insurance while the high-risks choose to stay out of the risky activity. Furthermore, insurers are allowed to offer a menu of cross-subsidizing contracts that earns normal profit overall. Therefore, contracts at the equilibrium must be Pareto efficient and policy intervention may raise aggregate social welfare but can not yield strict Pareto gain.

At the equilibrium, the low-risks enjoy higher utility than the high-risks because of the driving benefits they receive from driving, but their expected income is actually lower than the high-risks because of the expected loss from engaging in the risky activity. Therefore, the low-risks actually have higher marginal utility than the high-risks.

Now let us bring in a balanced budget tax and subsidy scheme which taxes insurance policies and uses the proceeds to subsidize the whole population. Such a scheme effectively redistributes the income from the motorists, i.e. the low-risks, to the non-motorists, i.e. the high-risks. It will have two effects: redistribution effect and efficiency effect.

On redistribution effect, the scheme redistributes the income from the low-risks who have higher marginal utility to the high-risks who have lower marginal utility. Hence the net redistribution effect on the aggregate social welfare is negative. On efficiency effect, the scheme improves the economic efficiency by allowing the low-risks to buy more insurance coverage. As existing research shows, the presence of the high-risks imposes a negative externality on the insurance coverage that the low-risks are allowed to purchase, as insurers have to restrict the coverage to discourage the high-risks from buying the insurance. Taxing insurance makes it more expensive and hence less attractive to the high-risks while subsidizing the whole population raises the reservation utility and hence makes the opt-out option more attractive to the high-risks. So insurers can now raise the insurance coverage to a higher level without attracting the high-risks into buying it. The low-risks can now buy more insurance coverage, which increases their utility.

When the efficiency effect dominates the redistribution effect, taxing insurance and subsidizing the whole population therefore increases the aggregate social welfare. Counter-intuitively, the scheme increases the insurance coverage sold on the market by making the insurance more expensive. However, the present research shows that a pooling allocation with full insurance coverage never maximizes the total welfare.

When the redistribution effect dominates the efficiency effect, taxing the whole population and subsidizing the motorists therefore increases the aggregate social welfare. So, surprisingly, in this case there is too much insurance in the market being sold too expensively and it would increase the total social welfare by decreasing insurance coverage.

Research on information asymmetry has long been a topic of tremendous debates ever since the seminal works by Akerlof (1970), Spence (1973) and Rothschild and Stiglitz (1976). Akerlof (1970) shows that adverse selection happens under asymmetric information. Bad quality goods (lemons) drive good quality goods out of market and the market size generally shrinks. The present essay challenges this conclusion in the context of insurance.

Rothschild and Stiglitz (1976) investigate two interesting issues: the existence of market equilibrium under asymmetric information and the efficiency of such equilibrium. They assume there are two types of agents in the risky activity, who are identical except the probability of having an accident. They find there might be no Nash equilibrium in the market, and if there is, it must be separating equilibrium. As part of the efforts to establish the conditions for market equilibrium to exist, they give the condition of when the market equilibrium is Pareto efficient, but did not go further into finding the allocation that maximizes aggregate social welfare, which is the most desirable outcome from the utilitarian point of view.

Following their work, there have been many papers focusing on the existence problem and the efficiency analysis[18]. Among them, Crocker and Snow provide key efficiency results.

Starting from the setup of Rothschild and Stiglitz, Crocker and Snow (1985a) introduce a tax and subsidy system, which ensures that the Miyazaki equilibrium can always be supported as a Nash equilibrium. They show that more than one Nash equilibrium is open to the regulator, but do not analyze the distributional effects and do not discuss which equilibrium is optimal. Furthermore, some of their conclusions are not robust as they do not allow cross-subsidization. For instance, a Pareto improvement is impossible once cross-subsidization is allowed.

[18] To name a few examples, Wilson (1977) found pooling equilibrium could exist if an "anticipatory equilibrium", a non-Nash equilibrium type of equilibrium, is adopted. Riley (1979) introduced "reactive equilibrium", another form of non-Nash equilibrium, which results in the same allocation as that of Rothschild and Stiglitz for any proportion of the high risks. Engers and Fernandez (1987) generalized the reactive equilibrium. Grossman (1979) proposed "dissembling equilibrium", which sustains the same allocation as Wilson's anticipatory equilibrium, although the mechanism is different. Miyazaki (1977) relaxed the assumptions of Rothschild and Stiglitz by allowing insurers to offer a menu of contracts which allows cross-subsidization, and adopting Wilson's anticipatory equilibrium instead of Nash equilibrium. It shows the only market equilibrium is Pareto efficient separating equilibrium, which is the same as that of Rothschild and Stiglitz (1976) when there is no cross-subsidization, or an interior allocation which maximizes the utility of the low-risks when cross-subsidization is involved. Spence (1978) applied this analysis to the insurance market. Other than the above work which focuses on non-Nash type equilibrium, Cho and Kreps (1987) and Hellwig (1987) give game theoretic foundation for the equilibrium.

Crocker and Snow (1985b) apply the definition proposed by Harris and Townsend (1981) to check the relationship between competitive equilibrium and efficient allocation in an insurance market. They find utility possibility frontier and show that full insurance pooling allocation maximizes aggregate social welfare. However, this present essay shows that pooling allocation is never optimal once the participation option is included. Furthermore, it could be optimal that nobody buys insurance, which is certainly not optimal in their setup.

Crocker and Snow (1986 and 2000) consider the efficiency effects of categorical discrimination in insurance market. They demonstrate costless risk categorization enhances efficiency and they further state that full insurance pooling allocation can never be improved on by the introduction of categorization. This present essay shows that when insurers are allowed to offer cross-subsidizing contracts, the market allocation must be Pareto efficient for there to be a Nash equilibrium. Therefore, the interior solution in Crocker and Snow (1986) does not exist. Moreover, the sufficient condition they claimed to allow costless risk classification to enhance efficiency does not hold either.

Crocker and Snow (2006) show that multidimensional screening reduces the externality cost of adverse selection and enhances the efficiency of insurance contracting. They recognize the impact of cross-subsidization on the existence of pure strategy Nash equilibrium and notice that it would be efficient for the high-risks to subsidize the low-risks under certain parameter values. However, as their model does not consider the participation option, such subsidization can not hold as a Nash equilibrium.

In addition to the above studies which do not consider participation option at all, there are several papers that, to some degree, involve a participation option.

Abadie and Franc (2004) explicitly consider whether it is welfare improving to opt out of public insurance. However, public insurance is very different from the private insurance that this present essay considers. Furthermore, they assume insurers can observe the individual type of risk, which makes their research fundamentally different from this present essay.

Gollier (2003) considers a participation option in the context of insurance demand in a lifecycle model. However, his research is actually on the choice between self-insurance and purchasing an insurance policy from private insurers. The agents always engage in the risky activity.

Kim and Schlesinger (2005) examine the individuals' demand for insurance coverage on the assumption that individuals receive the benefit of some type of potential government assistance that guarantees them a minimum level of wealth. This may seem similar to the reservation utility of this present paper at first sight, but it is actually completely different. In their model, agents always take up risky activity, and there is no government budget constraint. So the government assistance is in effect free insurance.

The structure of this essay is as follows: Section 3.2 finds the market equilibrium in an insurance market with adverse selection and a participation option. Section 3.3 finds conditions for such equilibrium to exist. Section 3.4 introduces the tax-subsidy scheme into the model. Section 3.5 analyzes the tax-subsidy scheme and gives the most important findings of this essay. Section 3.6 and Section 3.7 considers two extreme cases: full-insurance pooling contract and an over-insurance contract. Section 3.8 concludes.

3.2 Market equilibrium

Consider the market for motor insurance. Suppose there are two types of agent, low-risk type with accident probability of π_L and high-risk type with π_H, where $\pi_H > \pi_L$. The proportion of the high-risk type in the population is $\lambda \in (0,1)$. Everyone has an initial wealth W.

The market equilibrium is the result of a two-stage sequential game. In the first stage, insurance companies offer insurance policies. The insurance market is assumed to be perfectly competitive and insurers are allowed to offer a menu of contracts which can cross-subsidize each other but earn zero profit as a whole.

In the second stage, agents decide whether to drive or not. If he chooses not to drive, he receives a reservation utility $u(W)$, which is assumed to be the same for all. If he chooses to drive, he receives a utility benefit of B from driving but with probability $\pi_i, i = H, L$ may suffer a financial loss as a result of an accident of D.[19] Motorists then need to decide whether to purchase the insurance policy which is offered by insurers in the first stage.

[19] Here I ignore an interesting factor: a pecuniary cost related to driving such as the cost to purchase a car. Such a cost tends to move down the budget line of motorists along 45 degree line. The disposable income of drivers will be lower even in the absence of an accident, so the redistribution effect of an income tax will be strengthened, but the analysis will not be fundamentally changed.

3.2.1 Reservation utility and participation decision

Consider the result of the second stage game first. When motorists decide whether to get insured, they need to compare the utility with and without insurance, and buy insurance if it is beneficial.

As Rothschild and Stiglitz (1976) show, the contracts which can stand as a Nash equilibrium in the insurance market must be separating contracts. Pooling contract may disturb the separating equilibrium but can never themselves be a Nash equilibrium. In the competitive insurance market, insurers can not charge more than fair premium, for others can cut the price and earn positive profit. Hence, a menu of cross-subsidizing contracts, in which the profits from profitable contracts offset the loss from contracts of loss, can not constitute a Nash equilibrium, even though such a menu of contracts may disturb the separating equilibrium when it can result in strict Pareto improvement. Therefore, in the competitive insurance market, insurance contracts, if offered, must be separating contracts at fair premium.

Expecting insurance coverage at fair premium, motorists will buy the insurance contracts since such policies increase motorists' utility compared with being uninsured.

For both risk types, if they choose to drive and get insured, the minimum utility they can get is $\underline{U} = B + u(W - D\pi_H)$, for they can always reveal their risk type as high-risk and buy insurance policy at the fair premium. If the reservation utility $u(W) < \underline{U}$, both risk types will choose to drive and get insured. Figure 3.1 illustrates this case.

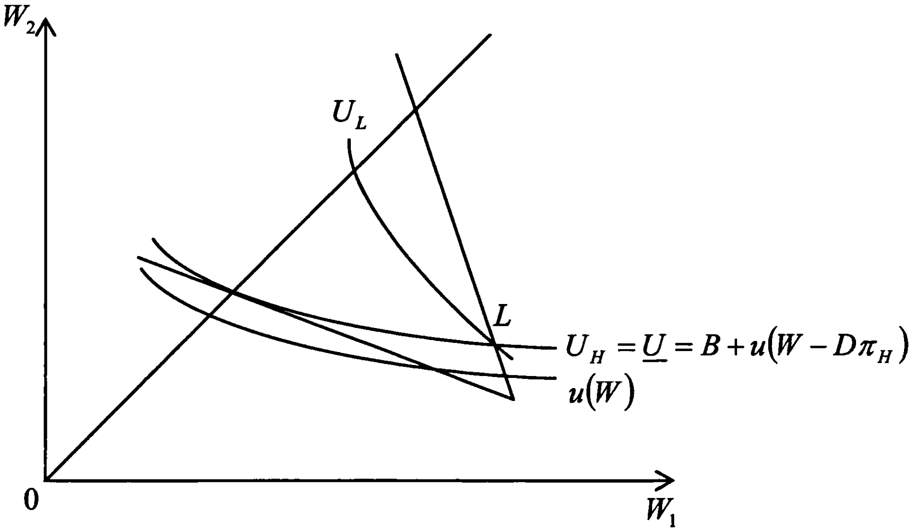


Figure 3.1

In this case, insurers offer full insurance coverage to the high-risks and yet only partial coverage to the low-risks. The insurance premium p is fair for both types: $p_i = \theta_i \cdot D \cdot \pi_i, i = H, L$, where $0 \le \theta_i \le 1$ is the percentage of insurance coverage. The high-risks are fully insured and receive the utility $U_H = B + u(W - D\pi_H)$. The low-risks are partially insured and receive the utility $U_L = B + (1-\pi_L)u(W - \theta_L D\pi_L) + \pi_L u(W - \theta_L D\pi_L - (1-\theta_L)D)$ while $U_H < U_L$. The outcome at the equilibrium is the same as in the standard Rothschild and Stiglitz model since the reservation utility is at so low a level that it does not affect the participation decision of both agent types.[20]

The highest utility motorists can get is $\overline{U} = B + u(W - D\pi_L)$, as the best insurance policy insurers are willing to offer is full coverage contract with fair premium for the low-risks. If the reservation utility $u(W) > \overline{U} = B + u(W - D\pi_L)$, the highest utility agents can receive when they drive, both risk types will stop driving. Figure 3.2 illustrates.

[20] Given our tax and subsidy scheme, which taxes motorists and uses the proceeds to subsidize the population, a small tax and subsidy won't have any effect in this case. But when the tax and subsidy are large enough as a whole, the subsidized reservation utility becomes greater than the utility of the high risks from driving, i.e. $u(W+(1-\lambda)t) > U_H = B + u(W - D\pi_H)$ where t is the tax, the high-risks will opt out of the market and become non-motorists. So one effect of such a policy is that road safety will increase. But it is more difficult to increase the aggregate social welfare in this case compared to the case we are going to further discuss where $\underline{U} < u(W) < \overline{U}$. Consider the marginal case where the subsidized reservation utility is equal to the utility of the high-risks from driving, i.e. $u(W+(1-\lambda)t) = U_H = B + u(W - D\pi_H)$. The high-risks are indifferent before and after the policy implemented but the low-risks are strictly worse off. So there is a strict Pareto loss in this scenario. There is obviously no efficiency effect here and the redistribution effect is purely negative. This is because the low-risks have to pay an extra cost to the high-risks to compensate them for give up driving, before the high-risks can be better off from this subsidy. So besides the redistribution effect and the efficiency effect in the case of $\underline{U} < u(W) < \overline{U}$, this case involves an extra efficiency loss related to this compensation payment.

If the total social welfare from the tax and subsidized allocation is greater than the initial allocation, the optimal allocation when the tax and subsidy are positive should involve more wealth transfer from the low-risks to the high-risks compared to the case $\underline{U} < u(W) < \overline{U}$ to cover the extra compensation cost, but it should be less than the sum of the optimal wealth transfer in the case $\underline{U} < u(W) < \overline{U}$ and the extra compensation cost. Because after having paid the compensation cost, the low-risks have less disposable income and hence higher marginal utility than in the case $\underline{U} < u(W) < \overline{U}$. No insurance can never be the optimal allocation any more because the high-risks can always opt back to be motorists and neutralise the tax and subsidy scheme.

I will leave the technical analysis for future work.

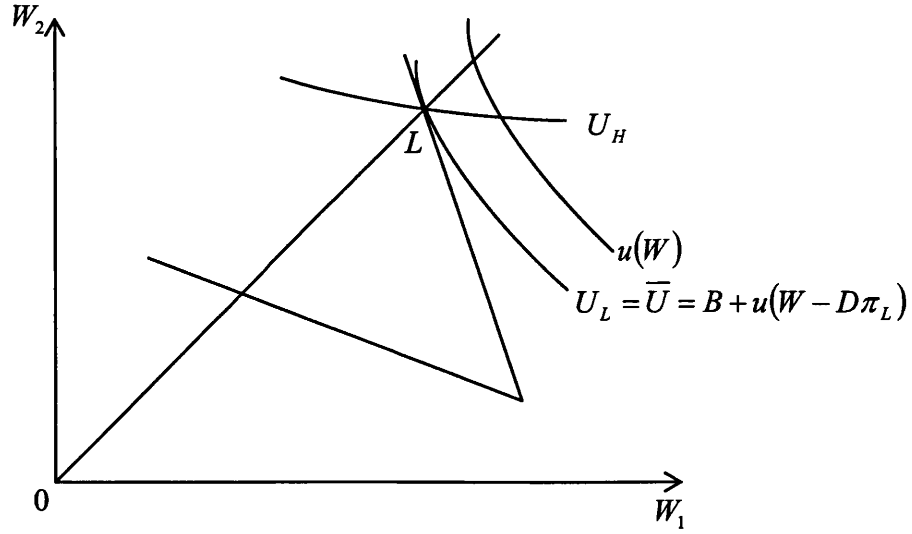


Figure 3.2

If the reservation utility is such that $\underline{U} < u(W) < \overline{U}$, the high-risks will rather opt out of market and receive the reservation utility. The low-risks will choose to become motorists and get insured. Figure 3.3 illustrates this case. The following analysis of this essay focuses on the reservation utility of this range.

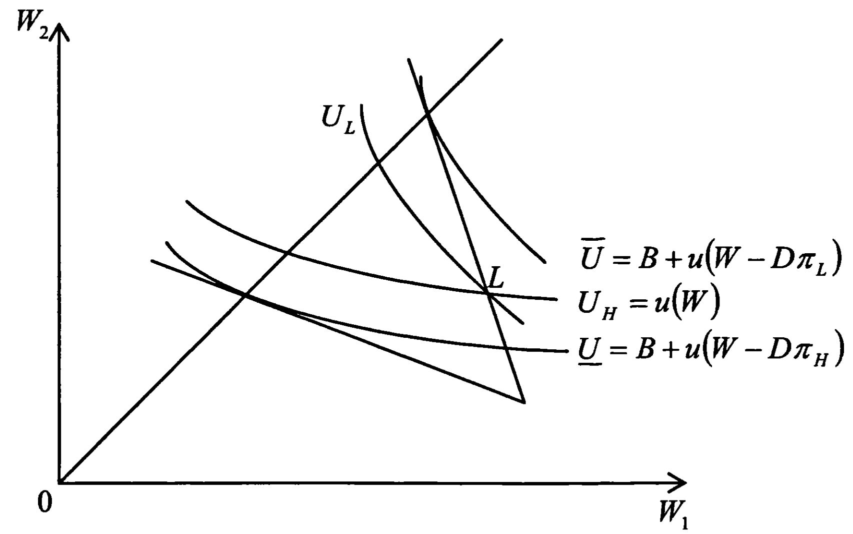


Figure 3.3

3.2.2 Separating equilibrium

Given the result of the second stage game, when $\underline{U} < u(W) < \overline{U}$, insurers will offer a single contract which offers partial coverage at the fair premium of the low-risks. The high-risks will opt out of the insurance market and stop driving while the low-risks will drive and buy the insurance contract.

At the separating equilibrium of the competitive insurance market, insurers offer contracts that maximizes the low-risk type's utility subject to the high-risk types preferring not to choose it (and to stay out of the market).

$$\underset{\theta}{Max}\, U_L = B + (1-\pi_L)u(W - \theta D\pi_L) + \pi_L u(W - \theta D\pi_L - (1-\theta)D)$$

$$\text{s.t. } U_H = B + (1-\pi_H)u(W - \theta D\pi_L) + \pi_H u(W - \theta D\pi_L - (1-\theta)D) \leq u(W)$$

The constraint is binding, therefore $U_H = u(W)$, which gives the optimal θ^*. Figure 3.4 illustrates the market equilibrium.

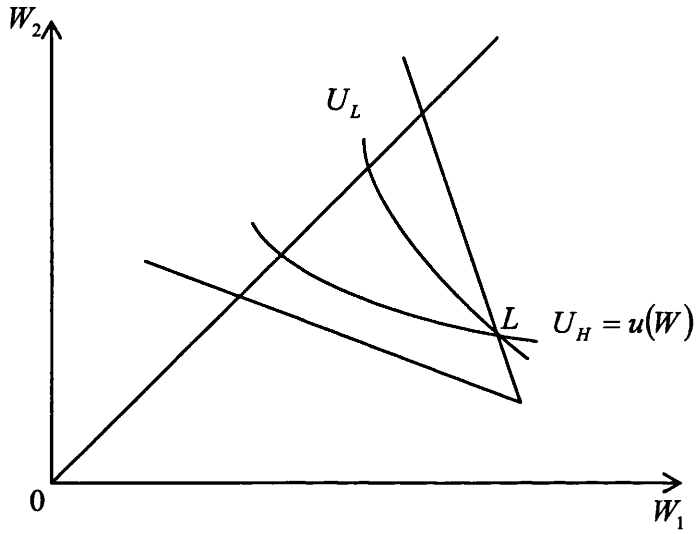


Figure 3.4

At the equilibrium, the high-risks opt out of driving and receive the reservation utility $u(W)$ while the low-risks choose to drive and receive

$$U_L = B + (1-\pi_L)u(W - \theta^* D\pi_L) + \pi_L u(W - \theta^* D\pi_L - (1-\theta^*)D)$$

3.3 Conditions for market equilibrium

The market may not have equilibrium under certain circumstances.

3.3.1 Separating equilibrium and pooling contract

As Rothschild and Stiglitz (1976) show, a pooling contract that Pareto dominates the separating contract can disturb the separating equilibrium but can not itself constitutes a Nash equilibrium. Hence, no Nash equilibrium in pure strategies exists if the low-risks receive greater utility from the pooling contract than from the separating contract.

The optimal pooling contract is found by maximizing the low-risk types' utility while keeping the insurers break even.

$$\underset{\theta_P}{Max}\, U_L = B + (1-\pi_L)u(W - \theta_P D\pi_P) + \pi_L u(W - \theta_P D\pi_P - (1-\theta_P)D)$$

s.t. $\pi_P = \lambda\pi_H + (1-\lambda)\pi_L$

Solving the optimization problem gives the optimal θ_P^*.

The optimal pooling contract offers partial insurance coverage to both risk types ($0 < \theta_P^* < 1$) at pooling premium, $p_P = \lambda p_H + (1-\lambda)p_L = \theta_P D\pi_P$, where $\pi_P = \lambda\pi_H + (1-\lambda)\pi_L$ is the weighted average of each type's accident probability. With the pooling contract, agents receive the utility

$$U_i^* = B + (1-\pi_i)u(W - \theta_P^* L\pi_P) + \pi_i u(W - \theta_P^* L\pi_P - (1-\theta_P^*)L),\ i = H, L$$

Figure 3.5 illustrates the case.

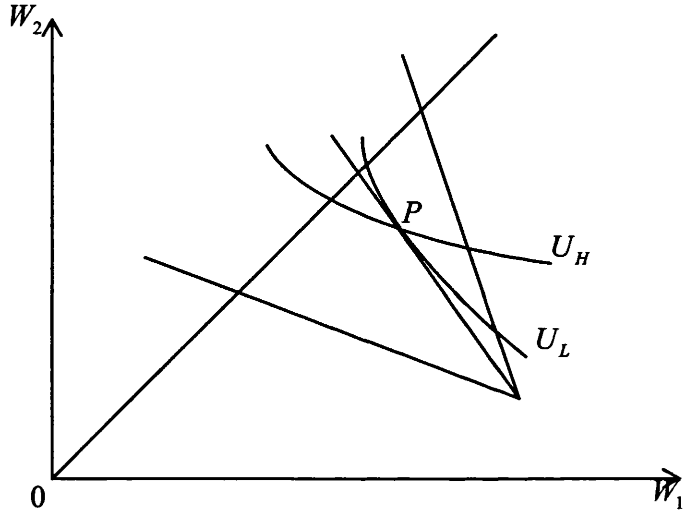


Figure 3.5

So a separating equilibrium arises when

$$U_L^P \le U_L^S \qquad (3.1)$$

where

$$U_L^P = B + (1-\pi_L)u(W - \theta_P^* D\pi_P) + \pi_L u(W - \theta_P^* D\pi_P - (1-\theta_P^*)D)$$

$$U_L^S = B + (1-\pi_L)u(W - \theta_S^* D\pi_L) + \pi_L u(W - \theta_S^* D\pi_L - (1-\theta_S^*)D)$$

Different parameter values may result in different equilibrium. Take the proportion of high-risk types for example. The threshold value of λ^* is found by solving (3.1) as an equation. There is a separating equilibrium when $\lambda \ge \lambda^*$. Figure 3.6 illustrates such a case. When $\lambda \ge \lambda^*$, the low-risks receive higher utility from the separating contract S than from the pooling contract P ($U_L^S > U_L^P$), so the pooling contract can not disturb the separating equilibrium which dominates the pooling allocation.

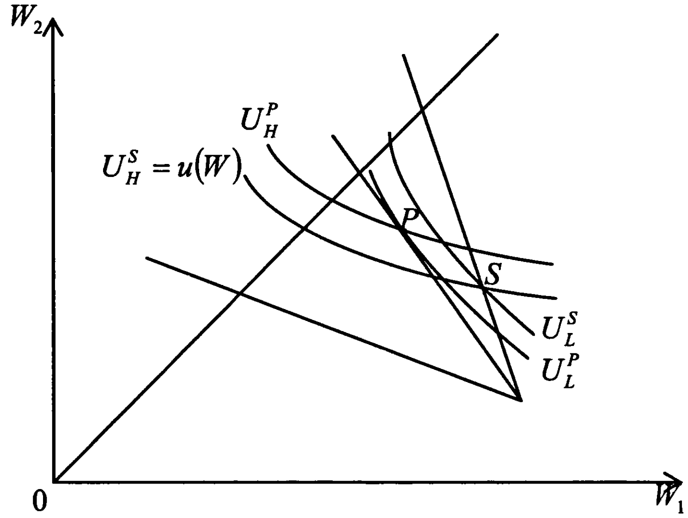


Figure 3.6

3.3.2 Separating equilibrium and cross-subsidization

When insurers are allowed to offer a menu of cross-subsidizing insurance policies that earns normal profit overall, i.e., they are allowed to charge the low-risks greater than fair premium and use the proceeds to subsidize the policies sold to the high-risks, there may exist room for a set of break even policies that leads to a strict Pareto improvement. Figure 3.7 illustrates.

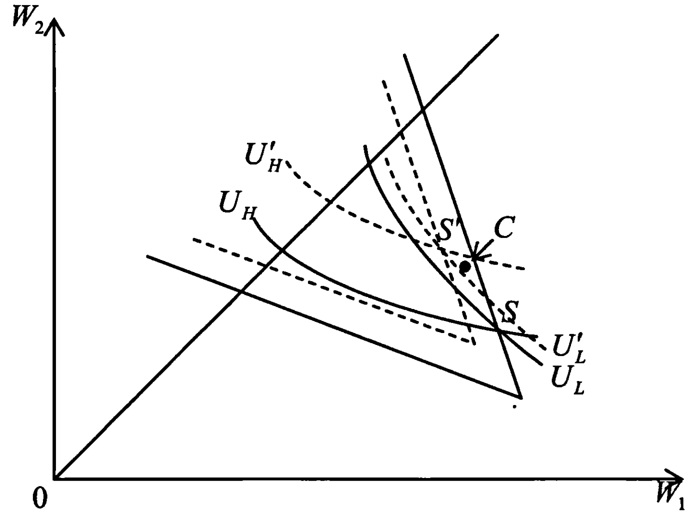


Figure 3.7

The initial no cross-subsidization market equilibrium would be at S. Insurers can then charge the low risks greater than fair premium and use the proceeds to subsidize the high risks. This cross-subsidization will lead to a new allocation S' which Pareto-dominates the previous allocation S.[21] At first sight, this new allocation appears to be an equilibrium at which both types are better off than at S. However, S' can not be Nash equilibrium as insurers can profit from deviating from S' to C. Under such circumstance, there will be no equilibrium. So the allocation resulting from market force must be Pareto efficient to be a Nash equilibrium.

One way to check whether the initial market allocation S is Pareto efficient is by introducing a balanced budget tax and subsidy scheme, which taxes the insurance and uses the proceeds to subsidize the whole population. If such a scheme can achieve Pareto improvement, then there exists room for the cross-subsidizing contracts and the separating allocation can be disturbed by these contracts. If there is no positive tax that can improve the welfare of the low-risks, i.e. the optimal tax for the low-risks is zero, such cross-subsidizing contracts do not exist and the separating equilibrium is indeed a Nash equilibrium.

Suppose a lump sum tax $t \geq 0$ is imposed for drivers per capita. The proceeds are used to subsidize the whole population. $s = (1-\lambda)t$, where s is the subsidy per capita. The net tax for drivers is $t - s = \lambda t$. The reservation utility after the subsidy is then $u(W + (1-\lambda)t)$.

As the market equilibrium must be Pareto efficient when insurers are allowed to offer cross-subsidization contracts, this means $t = 0$ for the following optimal allocation problem:

$$\underset{\theta}{Max} U_L = B + (1-\pi_L)u(W - \theta D\pi_L - \lambda t) + \pi_L u(W - \theta D\pi_L - (1-\theta)D - \lambda t)$$

s.t.

$$U_H = B + (1-\pi_H)u(W - \theta D\pi_L - \lambda t) + \pi_H u(W - \theta D\pi_L - (1-\theta)D - \lambda t) \leq u(W + (1-\lambda)t)$$

$$t \geq 0$$

For brevity, let's set

$$Y = W + (1-\lambda)t$$

$$X = W - \theta D\pi_L - \lambda t$$

$$Z = W - \theta D\pi_L - \lambda t - (1-\theta)D$$

Obviously, $X \geq Z$ and hence $u'(X) \leq u'(Z)$.

The problem can be rewritten as the following

---

[21] At S' the high risks are paid not to take insurance from any company. Enforcement of such a contract may be infeasible or illegal.

$$\underset{\theta}{Max}\, U_L = B + (1-\pi_L)u(X) + \pi_L u(Z)$$

$$\text{s.t. } U_H = B + (1-\pi_H)u(X) + \pi_H u(Z) \le u(Y) \tag{3.2}$$

$$t \ge 0 \tag{3.3}$$

The Lagrangian is

$$L = B + (1-\pi_L)u(X) + \pi_L u(Z) + \delta(u(Y) - B - (1-\pi_H)u(X) - \pi_H u(Z))$$

The Kuhn-Tucker condition gives the necessary conditions

$$\frac{\partial L}{\partial t} = -\lambda(1-\pi_L)u'(X) - \lambda\pi_L u'(Z) + \delta((1-\lambda)u'(Y) + \lambda(1-\pi_H)u'(X) + \lambda\pi_H u'(Z)) \le 0$$

$$t \ge 0 \text{ and } t\frac{\partial L}{\partial t} = 0 \tag{3.4}$$

$$\frac{\partial L}{\partial \theta} = \pi_L D(1-\pi_L)u'(Z) - \pi_L D(1-\pi_L)u'(X)$$
$$+ \delta(D\pi_L(1-\pi_H)u'(X) - D(1-\pi_L)\pi_H u'(Z)) \le 0$$

$$\theta \ge 0 \text{ and } \theta\frac{\partial L}{\partial \theta} = 0 \tag{3.5}$$

$$\frac{\partial L}{\partial \delta} = u(Y) - B - (1-\pi_H)u(X) - \pi_H u(Z) = 0 \tag{3.6}$$

We are interested in positive insurance coverage so we set $\theta > 0$. Therefore $\frac{\partial L}{\partial \theta} = 0$ which gives

$$\delta = \frac{\pi_L(1-\pi_L)(u'(Z) - u'(X))}{\pi_H(1-\pi_L)u'(Z) - \pi_L(1-\pi_H)u'(X)} \ge 0 \tag{3.7}$$

The sufficient condition for the market allocation to be efficient is $t = 0$ at the optimum and $\frac{\partial L}{\partial t} < 0$. The necessary condition is $\frac{\partial L}{\partial t} \le 0$.

Substituting (3.7) into (3.4) gives

$$-\lambda(1-\pi_L)u'(X) - \lambda\pi_L u'(Z) +$$
$$\frac{\pi_L(1-\pi_L)(u'(Z) - u'(X))}{\pi_H(1-\pi_L)u'(Z) - \pi_L(1-\pi_H)u'(X)}((1-\lambda)u'(Y) + \lambda(1-\pi_H)u'(X) + \lambda\pi_H u'(Z)) < 0$$

This is the sufficient condition for that there is no room for government intervention. The condition can be simplified as following

$$\frac{\lambda(\pi_H - \pi_L)}{(1-\lambda)\pi_L(1-\pi_L)} > \frac{(u'(Z) - u'(X))u'(Y)}{u'(Z)u'(X)} \tag{3.8}$$

Since $u'(Y) \leq u'(W) < u'(X) \leq u'(Z)$, and $u'(Z) < u'(W-D)$ if we assume $t < \frac{\theta D(1-\pi_L)}{\lambda}$ (Obviously, $t = 0$ is consistent with this assumption), it always holds that

$$\frac{(u'(W-d)-u'(W))u'(W)}{u'(W)u'(W)} > \frac{(u'(Z)-u'(X))u'(Y)}{u'(Z)u'(X)}$$

Therefore, there exist values of λ large enough to satisfy (3.8). For these values, a Pareto efficient separating market equilibrium exists.

3.4 Cross-subsidization taxation and the aggregate social welfare

Pareto efficiency is not enough if the government's goal is to maximize the aggregate social welfare. Assuming the government is subject to the same information asymmetry constraint as the insurers and uses a balanced tax-subsidy, its problem is:

$$\underset{\theta,t}{Max}\, \lambda U_H + (1-\lambda)U_L$$

$$\text{s.t. } B + (1-\pi_H)u(X) + \pi_H u(Z) \leq u(Y) \tag{3.9}$$

$$B + (1-\pi_L)u(X) + \pi_L u(Z) \geq u(Y) \tag{3.10}$$

Here we do not require $t \geq 0$. The constraint on balanced tax system is already embedded in the equations. The self-selection constraints cannot hold with strict inequality for both risk types at a solution, for then a Pareto improvement would be possible. Hence, either (3.9) or (3.10), or both, must hold in equality.

The Lagrangian is

$$\begin{aligned} L = {} & \lambda U(Y) + (1-\lambda)(B + (1-\pi_L)u(X) + \pi_L u(Z)) + \beta^H (u(Y) - B - (1-\pi_H)u(X) - \pi_H u(Z)) \\ & + \beta^L (B + (1-\pi_L)u(X) + \pi_L u(Z) - u(Y)) \end{aligned}$$

The Kuhn-Tucker condition gives the first-order condition for an interior solution:

$$\beta^H \frac{\partial L}{\partial \beta^H} = \beta^L \frac{\partial L}{\partial \beta^L} = \theta \frac{\partial L}{\partial \theta} = t \frac{\partial L}{\partial t} = 0$$

With the assumption that $\theta > 0$, we have:

$$\begin{aligned} \frac{\partial L}{\partial \theta} = {} & (1-\lambda)\pi_L(1-\pi_L)(u'(Z)-u'(X)) + \beta^H (\pi_L(1-\pi_H)u'(X) - (1-\pi_L)\pi_H u'(Z)) \\ & + \beta^L (1-\pi_L)\pi_L (u'(Z)-u'(X)) = 0 \end{aligned} \tag{3.11}$$

The optimal tax must maximize the aggregate social welfare, so we have

$$\frac{\partial L}{\partial t}=\lambda(1-\lambda)u'(Y)-\lambda(1-\lambda)((1-\pi_L)u'(X)+\pi_L u'(Z))$$
$$+\beta^H((1-\lambda)u'(Y)+\lambda(1-\pi_H)u'(X)+\lambda\pi_H u'(Z)) \tag{3.12}$$
$$+\beta^L(-\lambda(1-\pi_L)u'(X)-\lambda\pi_L u'(Z)-(1-\lambda)u'(Y))=0$$

From (3.11), we have

$$\beta^L=-(1-\lambda)+\beta^H\frac{\pi_H(1-\pi_L)u'(Z)-\pi_L(1-\pi_H)u'(X)}{\pi_L(1-\pi_L)(u'(Z)-u'(X))} \tag{3.13}$$

Substituting (3.13) into (3.12) gives

$$\beta^H=\frac{\pi_L(1-\pi_L)(1-\lambda)u'(Y)(u'(Z)-u'(X))}{(\pi_H-\pi_L)(\lambda u'(X)u'(Z)+(1-\lambda)(1-\pi_L)u'(Y)u'(Z)+(1-\lambda)\pi_L u'(Y)u'(X))} \tag{3.14}$$

(3.13) and (3.14) give

$$\beta^L=\frac{(1-\lambda)}{(\pi_H-\pi_L)(\lambda u'(X)u'(Z)+(1-\lambda)(1-\pi_L)u'(Y)u'(Z)+(1-\lambda)\pi_L u'(Y)u'(X))}$$
$$\times\begin{bmatrix}(1-\lambda)\pi_L(1-\pi_L)u'(Y)(u'(Z)-u'(X))-\lambda(\pi_H-\pi_L)u'(X)u'(Z)\\+\lambda u'(Y)(\pi_H(1-\pi_L)u'(Z)-\pi_L(1-\pi_H)u'(X))\end{bmatrix} \tag{3.15}$$

3.5 The high-risks opt out while the low-risks participate

As Section 3.4 Cross-subsidization taxation and the aggregate social welfare shows, a government that uses a balanced budget tax and subsidy scheme to maximize the aggregate social welfare faces the following problem:

$$\underset{\theta,t}{Max}\,\lambda U_H+(1-\lambda)U_L$$

s.t. $B+(1-\pi_H)u(X)+\pi_H u(Z)\le u(Y)$ (3.9)

$B+(1-\pi_L)u(X)+\pi_L u(Z)\ge u(Y)$ (3.10)

As the self-selection constraints cannot hold with strict inequality for both risk types at a solution, we first consider the most interesting case – the high-risks opt out of the risky activity while the low-risks choose to participate in the risky activity and buy insurance, i.e. condition (3.9) binds whilst (3.10) does not.

At the market equilibrium, the high-risks are indifferent between driving and not driving, and we assume they choose not to drive and receive the reservation utility $u(W)$. The low-risks drive and receive $B+(1-\pi_L)u(X)+\pi_L u(Z)>u(W)$.

Welfare maximization allocation may involve a positive tax on insurance and subsidy from the low-risks to the high-risks, as figure 3.8 illustrates. The laissez-faire market equilibrium is at S. A positive tax on insurance shifts down the indifference curve of the low-risks from U_L to U_L^* but pushes up that of the high-risks from U_H to U_H^*. The optimal allocation under the policy intervention that maximizes the aggregate social welfare may end up at S^*.

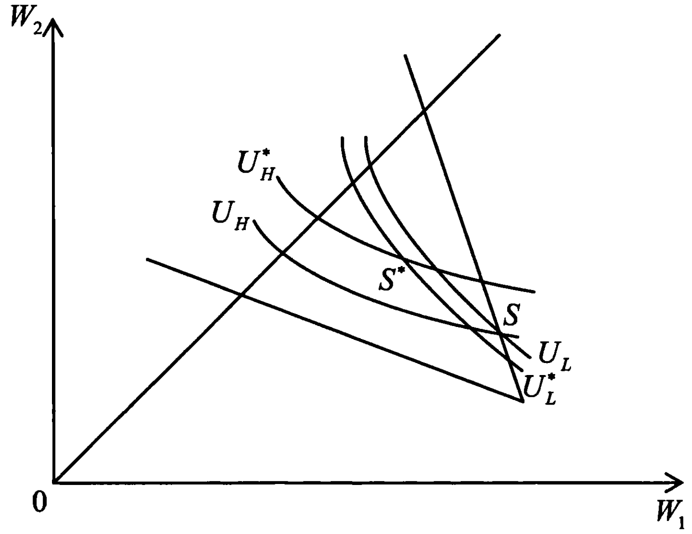


Figure 3.8

Or it may involve a negative tax on the insurance and subsidy from the high risks to the low risks, as figure 3.9 illustrates. The laissez-faire market equilibrium is at S. A negative tax on insurance shifts up the indifference curve of the low-risks from U_L to U_L^* but pushes down that of the high-risks from U_H to U_H^*. The optimal allocation under the policy intervention that maximizes the aggregate social welfare may end up at S^*.

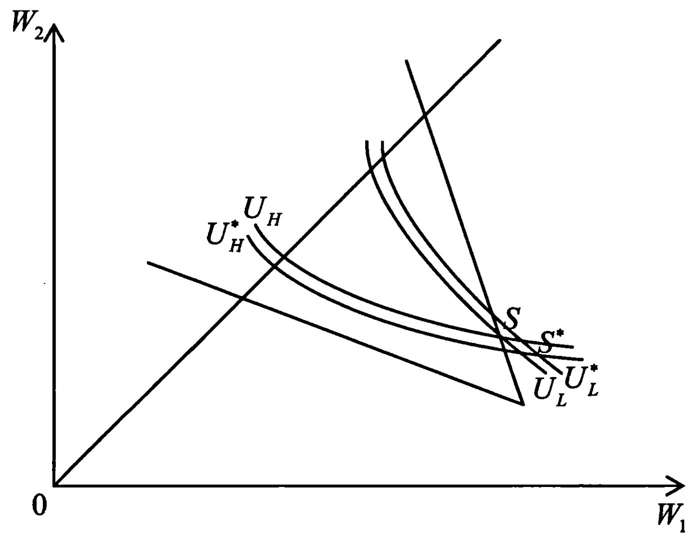


Figure 3.9

In both cases, the high-risks opt out of the risky activity while the low-risks choose to participate in the risky activity and buy insurance, i.e. condition (3.9) binds whilst (3.10) does not, i.e.

$$B+(1-\pi_H)u(X)+\pi_H u(Z)=u(Y)$$

$$B+(1-\pi_L)u(X)+\pi_L u(Z)>u(Y)$$

3.5.1 To tax or to subsidize insurance policies

As the section above discusses, in order to maximize the aggregate social welfare, the tax and subsidy scheme may involve a positive tax on insurance and subsidy from the low-risks to the high-risks or a negative tax (effectively a subsidy) on insurance and subsidy from the high-risks to the low-risks. Therefore, an interesting question is when to tax or subsidize insurance policies.

When condition (3.9) is binding but (3.10) is not, $\beta^L=0$. The welfare maximization problem can be rewritten as

$$\underset{\theta,t}{Max}\,\lambda U_H+(1-\lambda)U_L$$

s.t. $B+(1-\pi_H)u(X)+\pi_H u(Z)\leq u(Y)$

The Lagrangian is simplified into

$$L=\lambda U(Y)+(1-\lambda)(B+(1-\pi_L)u(X)+\pi_L u(Z))+\beta^H(u(Y)-B-(1-\pi_H)u(X)-\pi_H u(Z))$$

And we have

$$\begin{aligned}\frac{\partial L}{\partial t}=&\lambda(1-\lambda)u'(Y)-\lambda(1-\lambda)((1-\pi_L)u'(X)+\pi_L u'(Z))\\&+\beta^H((1-\lambda)u'(Y)+\lambda(1-\pi_H)u'(X)+\lambda\pi_H u'(Z))\end{aligned}\tag{3.16}$$

For $\theta\neq 0$, we have

$$\frac{\partial L}{\partial \theta}=(1-\lambda)\pi_L(1-\pi_L)(u'(Z)-u'(X))+\beta^H(\pi_L(1-\pi_H)u'(X)-(1-\pi_L)\pi_H u'(Z))=0$$

(3.17)

Equation (3.17) gives

$$\beta^H=\frac{\pi_L(1-\pi_L)(1-\lambda)(u'(Z)-u'(X))}{\pi_H(1-\pi_L)u'(Z)-\pi_L(1-\pi_H)u'(X)}\tag{3.18}$$

3.5.1.1 Redistribution effect and efficiency effect

It is more straightforward to analyze the effects of the taxation by rewriting (3.16) as

$$\begin{aligned}
\partial L &= \lambda(1-\lambda)u'(Y)\partial t - \lambda(1-\lambda)((1-\pi_L)u'(X)+\pi_L u'(Z))\partial t \\
&\quad + \beta^H((1-\lambda)u'(Y)+\lambda(1-\pi_H)u'(X)+\lambda\pi_H u'(Z))\partial t \\
&= \lambda\frac{\partial u(Y)}{\partial t}\partial t + (1-\lambda)\left((1-\pi_L)\frac{\partial u(X)}{\partial t}+\pi_L\frac{\partial u(Z)}{\partial t}\right)\partial t \\
&\quad + \beta^H\left(\frac{\partial u(Y)}{\partial t}-(1-\pi_H)\frac{\partial u(X)}{\partial t}-\pi_H\frac{\partial u(Z)}{\partial t}\right)\partial t
\end{aligned} \tag{3.16'}$$

The effect of taxation on total social welfare can be decomposed into two effects: redistribution effect $\lambda\frac{\partial u(Y)}{\partial t}\partial t + (1-\lambda)\left((1-\pi_L)\frac{\partial u(X)}{\partial t}+\pi_L\frac{\partial u(Z)}{\partial t}\right)\partial t$ and efficiency effect $\beta^H\left(\frac{\partial u(Y)}{\partial t}-(1-\pi_H)\frac{\partial u(X)}{\partial t}-\pi_H\frac{\partial u(Z)}{\partial t}\right)\partial t$.

The redistribution effect measures the utility change from transferring wealth between the high-risks and the low-risks, weighted by the ratios of the population respectively. A positive tax and subsidy will transfer wealth from the low-risks to the high-risks, and decrease (increase) the utility of the low-risks (the high-risks) accordingly. A negative tax and subsidy works in the same way but in the opposite direction. Whether the redistribution effect is positive or negative is dependant on whether the weighted utility gain is more than offset by the weighted utility loss or not.

If we rewrite the redistribution effect as $\lambda(1-\lambda)(u'(Y)-((1-\pi_L)u'(X)+\pi_L u'(Z)))\partial t$, we can see that the redistribution effect is actually dependent on whether the marginal utility of the high-risks is greater than the expected marginal utility of the low-risks.

At the equilibrium, the low-risks enjoy higher utility than the high-risks because of the driving benefits they receive from driving. However, their expected income is actually less than the high-risks because of the expected loss from engaging in the risky activity. So the expected marginal utility of the low-risks is greater than the high-risks. As the result, the net effect of the redistribution is negative.[22]

Besides the redistribution effect, such a tax and subsidy scheme also has an efficiency effect: The presence of the high-risks imposes a negative externality on the insurance coverage that the low-risks are allowed to purchase, as insurers have to restrict the coverage to discourage the high-risks from buying the insurance. Taxing insurance makes it more expensive and hence less attractive to the high-risks while subsidizing the whole population raises the reservation utility and hence makes the

[22] This is a feature of the additive utility function.

opt-out option more attractive to the high-risks. So insurers can now raise the insurance coverage to a higher level without attracting the high-risks into buying it. The low-risks can now buy more insurance coverage, which increases their utility.

Techincally, the efficiency effect measures the utility change from relaxing the incentive constraint caused by the information asymmetry. Unlike the redistribution effect, as we can see below, the efficiency effect is positive for positive tax and subsidy. Because a tax allows the low risks greater insurance coverage, the compensating variation is less than the value of the subsidy to the high risks. In this setting, the deadweight cost of taxes is negative.

For interior solutions, i.e., $\theta \neq 1$ and $X \neq Z$, we have from equation (3.18)

$$\beta^H = \frac{\pi_L(1-\pi_L)(1-\lambda)(u'(Z)-u'(X))}{\pi_H(1-\pi_L)u'(Z)-\pi_L(1-\pi_H)u'(X)} > 0$$

As we know, the Lagrangian multiplier measures the marginal change of the objective function caused by the marginal change of the constraint value. Before the insurance coverage reaches full insurance ($\theta = 1$ and $X = Z$), we always have $\beta^H > 0$, so positive tax and subsidy, which increases the reservation utility of the high risks and hence relaxes the incentive constraint, always increases the aggregate social welfare.

The marginal change of the incentive constraint to the marginal change of tax is measured by $\left(\frac{\partial u(Y)}{\partial t} - (1-\pi_H)\frac{\partial u(X)}{\partial t} - \pi_H \frac{\partial u(Z)}{\partial t}\right)$, i.e., $((1-\lambda)u'(Y) + \lambda(1-\pi_H)u'(X) + \lambda\pi_H u'(Z))$, which is clearly positive.

Put together, the efficiency effect is measured by the marginal change of tax, times the marginal change of the incentive constraint to the marginal change of tax, times the marginal change of the objective function to the marginal change of the incentive constraint:

$$\beta^H\left(\frac{\partial u(Y)}{\partial t} - (1-\pi_H)\frac{\partial u(X)}{\partial t} - \pi_H \frac{\partial u(Z)}{\partial t}\right)\partial t$$

The overall effect of taxation on total social welfare depends on whether the redistribution effect is more than offset by the efficiency effect. The next section gives the condition on when the overall effect is positive or negative.

3.5.1.2 Condition on whether to tax or to subsidize the insurance

As the analysis in the section above shows, the tax and subsidy scheme has two effects: the redistribution effect which reduces the aggregate social welfare and the efficiency effect which increases the aggregate social welfare. When the efficiency effect dominates the redistribution effect, taxing insurance and subsidizing the whole population increases the aggregate social welfare. When, on the contrary, the redistribution effect dominates the efficiency effect, taxing the whole population and subsidizing the insurance increases the aggregate social welfare. The following technical analysis examines when to tax or to subsidize the insurance in order to maximize the total social welfare.

Substitute (3.18) into (3.16) and rearrange it, we have

$$\frac{\partial L}{\partial t}=\lambda(1-\lambda)u'(Y)+\frac{\pi_L(1-\pi_L)(1-\lambda)(u'(Z)-u'(X))}{\pi_H(1-\pi_L)u'(Z)-\pi_L(1-\pi_H)u'(X)}((1-\lambda)u'(Y)+\lambda(1-\pi_H)u'(X)+\lambda\pi_H u'(Z))$$
$$-\lambda(1-\lambda)((1-\pi_L)u'(X)+\pi_L u'(Z))$$

or

$$\begin{aligned}\frac{\partial L}{\partial t}=&\frac{(1-\lambda)}{\pi_H(1-\pi_L)u'(Z)-\pi_L(1-\pi_H)u'(X)}\times\\&\begin{bmatrix}(1-\lambda)\pi_L(1-\pi_L)(u'(Z)-u'(X))u'(Y)-\lambda(\pi_H-\pi_L)u'(Z)u'(X)\\+\lambda u'(Y)(\pi_H(1-\pi_L)u'(Z)-\pi_L(1-\pi_H)u'(X))\end{bmatrix}\end{aligned}\tag{3.19}$$

It is straightforward that $\pi_H(1-\pi_L)u'(Z)-\pi_L(1-\pi_H)u'(X)>0$, so the sign of (3.19) is dependent on the sign of

$$\begin{aligned}&(1-\lambda)\pi_L(1-\pi_L)(u'(Z)-u'(X))u'(Y)-\lambda(\pi_H-\pi_L)u'(Z)u'(X)\\&+\lambda u'(Y)(\pi_H(1-\pi_L)u'(Z)-\pi_L(1-\pi_H)u'(X))\end{aligned}\tag{3.20}$$

The necessary efficiency condition of the market equilibrium gives

$$\frac{\lambda(\pi_H-\pi_L)}{(1-\lambda)\pi_L(1-\pi_L)}\geq\frac{(u'(Z)-u'(X))u'(Y)}{u'(Z)u'(X)},\text{ i.e.}$$

$$(1-\lambda)\pi_L(1-\pi_L)(u'(Z)-u'(X))u'(Y)-\lambda(\pi_H-\pi_L)u'(Z)u'(X)\leq 0$$

Clearly, the first part of (3.20) is negative while the second part is positive, so (3.20) could be either positive or negative. We can rewrite (3.19) as

$$\begin{aligned}\frac{\partial L}{\partial t}=&\frac{(1-\lambda)u'(X)u'(Z)}{\pi_H(1-\pi_L)u'(Z)-\pi_L(1-\pi_H)u'(X)}\times\\&\left[\frac{u'(Y)}{u'(X)}(1-\pi_L)(\lambda\pi_H+(1-\lambda)\pi_L)-\frac{u'(Y)}{u'(Z)}\pi_L(\lambda(1-\pi_H)+(1-\lambda)(1-\pi_L))-\lambda(\pi_H-\pi_L)\right]\end{aligned}$$

(3.19')

The sign of (3.19') is dependent on the sign of

$$\frac{u'(Y)}{u'(X)}(1-\pi_L)(\lambda\pi_H+(1-\lambda)\pi_L)-\frac{u'(Y)}{u'(Z)}\pi_L(\lambda(1-\pi_H)+(1-\lambda)(1-\pi_L))-\lambda(\pi_H-\pi_L)$$

(3.20')

(3.20') can be rewritten as

$$\left(\frac{u'(Y)}{u'(X)}-\frac{u'(Y)}{u'(Z)}\right)(1-\pi_L)(\pi_L+\lambda(\pi_H-\pi_L))-\left(1-\frac{u'(Y)}{u'(Z)}\right)\lambda(\pi_H-\pi_L) \qquad (3.21)$$

When $t=0$,

$$Y=W\ ,\ X=W-\theta D\pi_L\ ,\ Z=W-\theta D\pi_L-(1-\theta)D$$

So $Y>X>Z$ and hence $u'(Y)<u'(X)<u'(Z)$. Therefore,

$$\frac{u'(Y)}{u'(X)}-\frac{u'(Y)}{u'(Z)}>0 \text{ and } 1-\frac{u'(Y)}{u'(Z)}>0$$

So $\left.\frac{\partial L}{\partial t}\right|_{t=0}>(<)0$ when

$$\frac{\dfrac{u'(Y)}{u'(X)}-\dfrac{u'(Y)}{u'(Z)}}{1-\dfrac{u'(Y)}{u'(Z)}}>(<)\frac{\lambda(\pi_H-\pi_L)}{(1-\pi_L)(\pi_L+\lambda(\pi_H-\pi_L))} \qquad (3.22)$$

So the optimal tax could be positive or negative, depending on the parameter values. When

$$\frac{\dfrac{u'(Y)}{u'(X)}-\dfrac{u'(Y)}{u'(Z)}}{1-\dfrac{u'(Y)}{u'(Z)}}>\frac{\lambda(\pi_H-\pi_L)}{(1-\pi_L)(\pi_L+\lambda(\pi_H-\pi_L))}$$

We have $\left.\frac{\partial L}{\partial t}\right|_{t=0}>0$, i.e. the aggregate social welfare increases with tax t, so the efficiency effect dominates the redistribution effect. A positive tax on the insurance and subsidy from the low-risks to the high-risks increases the aggregate social welfare. Similarly, when

$$\frac{\dfrac{u'(Y)}{u'(X)}-\dfrac{u'(Y)}{u'(Z)}}{1-\dfrac{u'(Y)}{u'(Z)}}<\frac{\lambda(\pi_H-\pi_L)}{(1-\pi_L)(\pi_L+\lambda(\pi_H-\pi_L))}$$

The redistribution effect dominates the efficiency effect. A negative tax on the insurance and subsidy from the high-risks to the low-risks increases the total social welfare.

3.5.1.3 Numerical examples

Set $u(x) = \ln x$. When $W = 100$, $B = 1$, $D = 90$, $\pi_H = 0.8$, $\pi_L = 0.3$, $\lambda = 0.3$, the left hand side of (3.22) equals 0.8832 and the right hand side is 0.47619, so a positive tax increases social welfare. For example, a positive tax $T = 17.857143$ increases the aggregate social welfare from 9.7815 to 9.8708.[23] Figure 3.10 illustrates. The laissez faire market equilibrium is at S. A positive tax on insurance shifts down the indifference curve of the low-risks from U_L to U_L^* but pushes up that of the high-risks from U_H to U_H^*. The optimal allocation under the policy intervention that maximizes the aggregate social welfare may end up at S^*.

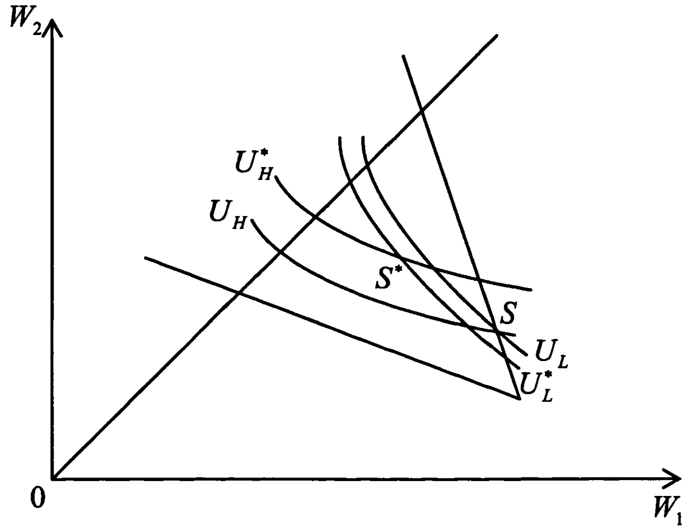


Figure 3.10

When $W = 100$, $B = 1$, $D = 99$, $\pi_H = 0.9$, $\pi_L = 0.3$, $\lambda = 0.83$, the left hand side is 0.79082 and the right hand side is 0.89151, so negative tax increases social welfare. For example, a negative tax $T = -1$ increases the aggregate social welfare from 9.7771 to 9.7844.[24] Figure 3.11 illustrates. The laissez-faire market equilibrium is at S. A negative tax on insurance shifts up the indifference curve of the low-risks from U_L to U_L^* but pushes down that of the high-risks from U_H to U_H^*. The optimal allocation under the policy intervention that maximizes the aggregate social welfare may end up at S^*.

[23] Appendix 3.1 provides detailed simulation results.
[24] Appendix 3.2 provides detailed simulation results.

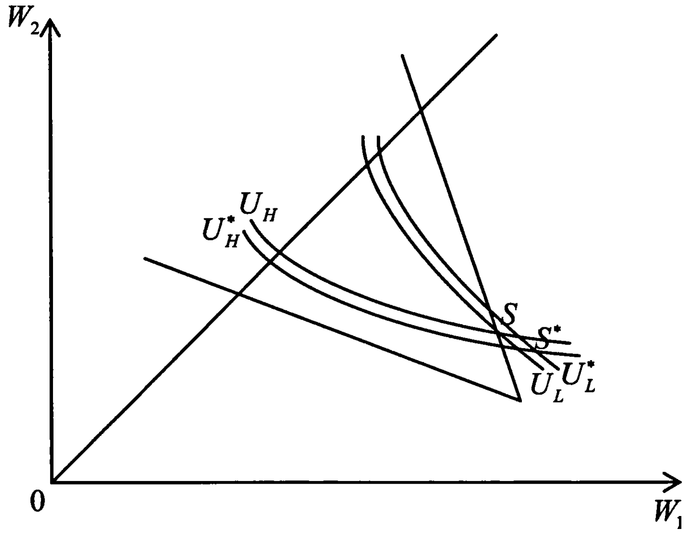


Figure 3.11

3.5.2 The optimal tax level and efficient allocation

Section 3.5.1 finds the condition on whether to tax or to subsidize the insurance in order to maximize the aggregate social welfare. However, it does not answer what is the optimal level of the tax or the subsidy (i.e. negative tax). This section finds the optimal level for an interior solution.

From (3.21), we know that for an interior solution, t must satisfies

$$\left(\frac{u'(Y)}{u'(X)}-\frac{u'(Y)}{u'(Z)}\right)(1-\pi_L)(\pi_L+\lambda(\pi_H-\pi_L))-\left(1-\frac{u'(Y)}{u'(Z)}\right)\lambda(\pi_H-\pi_L)=0, \text{ i.e.}$$

$$\frac{\dfrac{u'(Y)}{u'(X)}-\dfrac{u'(Y)}{u'(Z)}}{1-\dfrac{u'(Y)}{u'(Z)}}=\frac{\lambda(\pi_H-\pi_L)}{(1-\pi_L)(\pi_L+\lambda(\pi_H-\pi_L))} \tag{3.22}$$

When equation (3.22) holds, $\beta^L=0$ and $\dfrac{\partial L}{\partial t}=0$. The solution clearly satisfies the Kuhn-Tucker conditions. Equation (3.22) therefore fixes the optimal allocation and hence the optimal tax and subsidy.

3.5.3 Pooling full insurance coverage is never efficient

When agents can decide whether to participate in the risky activity and whether to buy insurance, full insurance can never maximize the total social welfare.

Full insurance would be optimal only when increasing tax always increases the aggregate social welfare or at least does not reduce the welfare, which implies at the full insurance $\frac{\partial L}{\partial t} \geq 0$, i.e.

$$\frac{\frac{u'(Y)}{u'(X)} - \frac{u'(Y)}{u'(Z)}}{1 - \frac{u'(Y)}{u'(Z)}} \geq \frac{\lambda(\pi_H - \pi_L)}{(1-\pi_L)(\pi_L + \lambda(\pi_H - \pi_L))} \tag{3.23}$$

At full insurance, $X = Z = W - \pi_L D - \lambda t$, so the left hand side of (3.23) is 0 while the right hand side is positive. This violates the condition of $\frac{\partial L}{\partial t} \geq 0$.

Intuitively, under full insurance, $X = Z$, so $\beta^H = \frac{\pi_L(1-\pi_L)(1-\lambda)(u'(Z)-u'(X))}{\pi_H(1-\pi_L)u'(Z) - \pi_L(1-\pi_H)u'(X)} = 0$, i.e., there is no efficiency effect from the taxation: relaxing the incentive constraint has no efficiency effect now since the insurance contract has reached full coverage. All that matters is the redistribution effect. As we know, motorists have higher marginal utility than the non-motorists and hence the redistribution effect is negative. Therefore, the overall effect of the taxation at full insurance must be negative, i.e. $\left.\frac{\partial L}{\partial t}\right|_{X=Z} < 0$.

Clearly, when $t > 0$ increases social welfare, an optimal allocation involves cross subsidization from the low-risks to the high-risks, but only with partial insurance. In the previous simulation example that $u(x) = \ln x$, $W = 100$, $B = 1$, $D = 90$, $\pi_H = 0.8$, $\pi_L = 0.3$, $\lambda = 0.3$, aggregate social welfare first increases with the tax but then decreases before reaching full insurance. A positive tax at about $t = 59$ maximizes the aggregate social welfare.[25] Figure 3.12 illustrates: the laissez faire market equilibrium is at S. A positive tax $t > 0$ on the insurance and subsidy from the low-risks to the high-risks increase the total social welfare, which is maximised at S^* before reaching the full insurance coverage.

[25] Appendix 3.1 provides detailed simulation results.

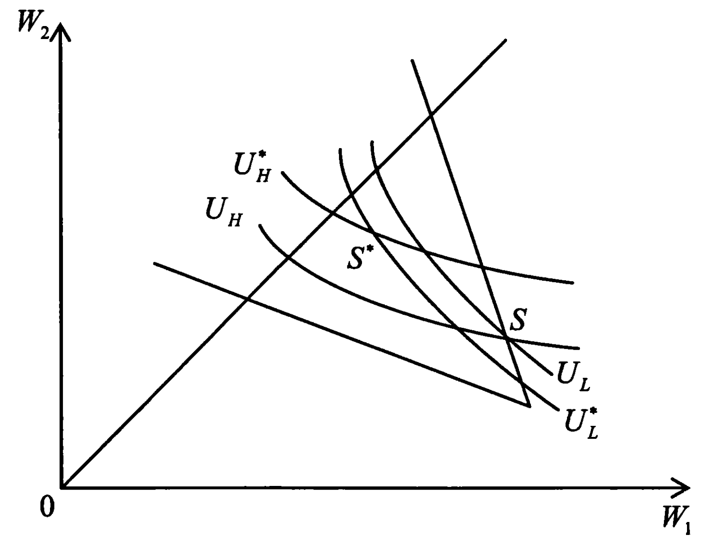


Figure 3.12

3.5.4 Too much insurance

As section 3.5.1 shows, when the redistribution effect dominates the efficiency effect, a negative tax increases the total social welfare, i.e. when

$$\frac{\frac{u'(Y)}{u'(X)}-\frac{u'(Y)}{u'(Z)}}{1-\frac{u'(Y)}{u'(Z)}}<\frac{\lambda(\pi_H-\pi_L)}{(1-\pi_L)(\pi_L+\lambda(\pi_H-\pi_L))}$$

In this case, there is too much insurance being sold too expensively in the market equilibrium. The effect of the negative tax is to reduce the insurance coverage and the insurance premium at the same time, which increases total welfare. Under certain circumstances, the market only needs infinitesimal insurance coverage. In this case, "almost no insurance" is the optimal allocation.[26]

As in the full insurance extreme, "almost no insurance" is optimal when decreasing the tax always increases the social welfare. So at $\theta=0$, $\frac{\partial L}{\partial t}\leq 0$.

[26] As we can see below, reducing insurance increases aggregate social welfare, which is actually highest when there is no insurance at all. However, as the present research set the tax and subsidy scheme on insurance policies, there would be no such a scheme when insurance is completely eliminated. Therefore, "almost no insurance" becomes the optimal allocation. If the tax-subsidy scheme was set on the motoring activity, insurance could be eliminated completely then. Since this only has trivial technical effects on the corner solution, the present research sticks to the current tax and subsidy scheme.

When $\theta = 0$ and $t < -D$, $u'(X) < u'(Z) < u'(Y)$ and $\frac{u'(Y)}{u'(X)} - \frac{u'(Y)}{u'(Z)} > 0$ and

$1 - \frac{u'(Y)}{u'(Z)} < 0$, so $\frac{\partial L}{\partial t} > 0$, which contradicts $\frac{\partial L}{\partial t} \leq 0$. Therefore, "almost no insurance" cannot be optimal. Instead, an interior solution exists.

When $\theta = 0$ and $t > -D$, $u'(X) < u'(Y) < u'(Z)$ and $\frac{u'(Y)}{u'(X)} - \frac{u'(Y)}{u'(Z)} > 0$ and

$1 - \frac{u'(Y)}{u'(Z)} > 0$, so $\frac{\partial L}{\partial t} \leq 0$ implies

$$\frac{\frac{u'(Y)}{u'(X)} - \frac{u'(Y)}{u'(Z)}}{1 - \frac{u'(Y)}{u'(Z)}} \leq \frac{\lambda(\pi_H - \pi_L)}{(1 - \pi_L)(\pi_L + \lambda(\pi_H - \pi_L))} \tag{3.24}$$

It is possible to satisfy (3.24) so "almost no insurance" could be optimal. For instance, when $u(x) = \ln x$, $W = 100$, $B = 1$, $D = 90$, $\pi_H = 0.8$, $\pi_L = 0.3$, $\lambda = 0.8$, the aggregate social welfare increases monotonically with the absolute value of negative tax.[27] Figure 3.13 illustrates: the laissez faire market equilibrium is at S. A negative tax $t < 0$ on the insurance and subsidy from the high-risks to the low-risks increase the total social welfare, which is maximised at S^*, which is infinitesimally close to no insurance at all.

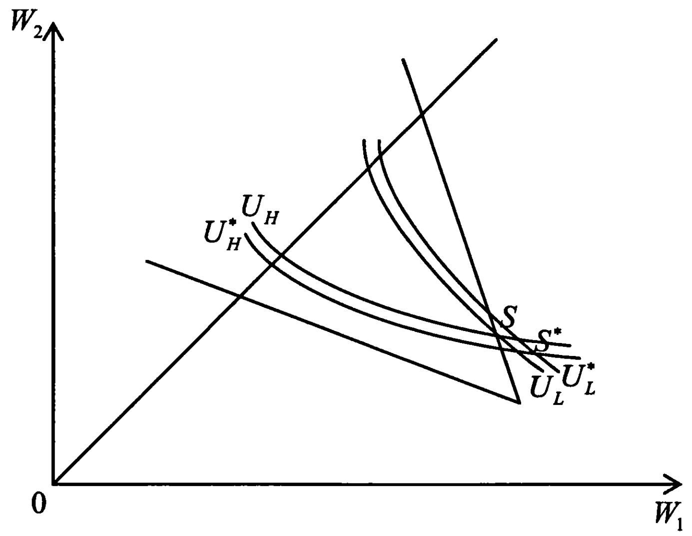


Figure 3.13

[27] Appendix 3.3 provides detailed simulation results.

3.6 Extreme case: utility from driving with full insurance is the same as reservation utility

As we know, (3.9) and (3.10) cannot both hold with strict inequality at a solution. If they both bind, we have

$$B+(1-\pi_H)u(X)+\pi_H u(Z)=u(Y) \tag{3.25}$$

$$B+(1-\pi_L)u(X)+\pi_L u(Z)=u(Y) \tag{3.26}$$

(3.25) and (3.26) give $X=Z$, i.e., full insurance, which in turn gives

$$B+u(W-D\pi_L-\lambda t)=u(W+(1-\lambda)t)$$

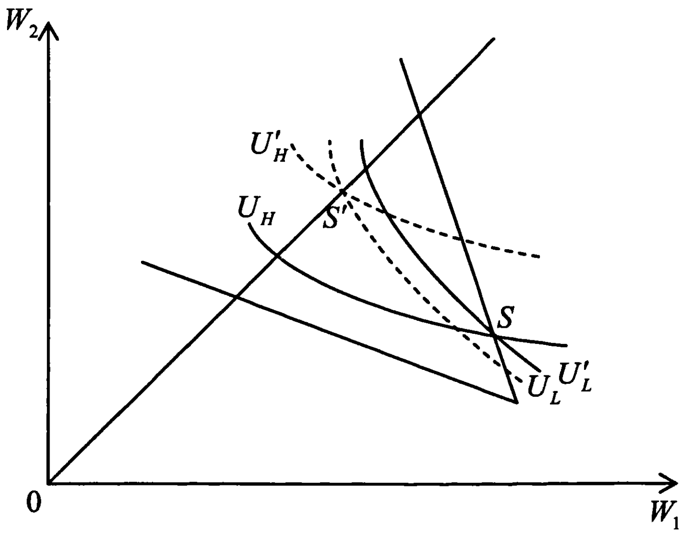


Figure 3.14

Figure 3.14 illustrates the allocation. The utility the low-risks receive from driving is at the same level as the reservation utility after the tax and subsidy scheme is implemented. This is actually just a special case of section 3.5 where $t>0$ is such that it makes $\theta=1$. However, as we have seen in section 3.5.3, full insurance can never maximize total welfare.

3.7 Extreme case: the high-risks opt out while the low-risks are indifferent

Another extreme case happens when (3.10) binds but (3.9) does not. If $B+(1-\pi_H)u(X)+\pi_H u(Z)<u(Y)$, then $B+(1-\pi_L)u(X)+\pi_L u(Z)=u(Y)$ and $\beta^H=0$. So the high-risks will not drive. $\beta^H=0$ implies $u'(Z)=u'(X)$. So the low-risks receive full insurance and are indifferent between driving and not driving.

With full insurance coverage, the low-risks, who are motorists, receive utility $B+u(W-D\pi_L-\lambda t)$ while the high-risks do not drive and receive the reservation utility $u(W+(1-\lambda)t)$. Since $B+(1-\pi_L)u(X)+\pi_L u(Z)=u(Y)$, we have, with full insurance coverage, $B+u(W-D\pi_L-\lambda t)=u(W+(1-\lambda)t)$. So there is a full insurance allocation as figure 3.14 illustrates of section 3.6.

But what is the optimal tax scheme in this case? Does it increase the total welfare if the low risks are taxed to subsidize the high risks and result in an allocation as figure 3.15 illustrates?

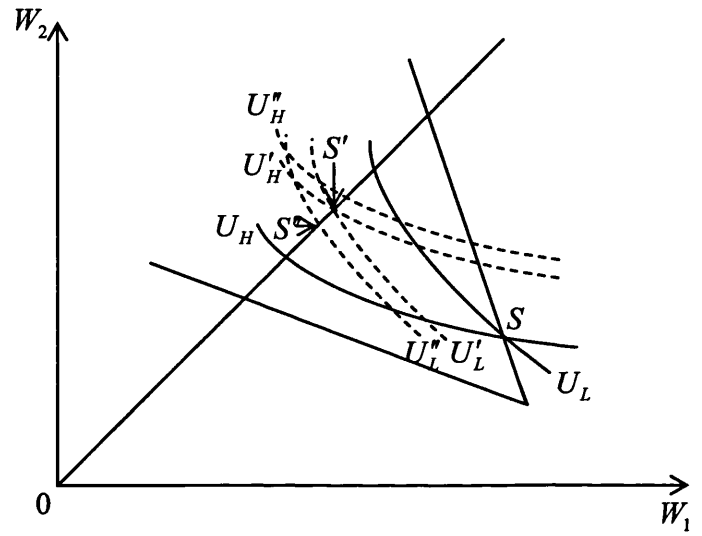


Figure 3.15

In this case, the welfare maximization problem can be rewritten as

$\underset{t}{Max}\,\lambda U_H+(1-\lambda)U_L$

Where $U_H=u(W+(1-\lambda)t)$ and $U_L=B+u(W-D\pi_L-\lambda t)$

Set $L=\lambda U_H+(1-\lambda)U_L$

$$\frac{\partial L}{\partial t}=\lambda(1-\lambda)(u'(W+(1-\lambda)t)-u'(W-D\pi_L-\lambda t))$$

For $t>0$, we have $\frac{\partial L}{\partial t}<0$. Actually, for any $t>-D\pi_L$, we have $\frac{\partial L}{\partial t}<0$. So a positive tax clearly decreases the total social welfare. Actually, a positive tax which moves allocation from S' to S'' violates incentive constraint and will stop the motorists from driving. Clearly, allocations such as S'' can not be efficient.

When $t<-D\pi_L$, we have $\frac{\partial L}{\partial t}>0$: taxing the high-risks to subsidize the low-risks increases the total welfare. It immediately involves (3.9) binding and (3.10) not (that is, $B+(1-\pi_H)u(X)+\pi_H u(Z)=u(Y)$ and $B+(1-\pi_L)u(X)+\pi_L u(Z)>u(Y)$), a scenario we have discussed in section 3.5. Such a tax in effect moves the allocation from the full insurance S' to the partial insurance coverage S^*, as figure 3.16 illustrates.

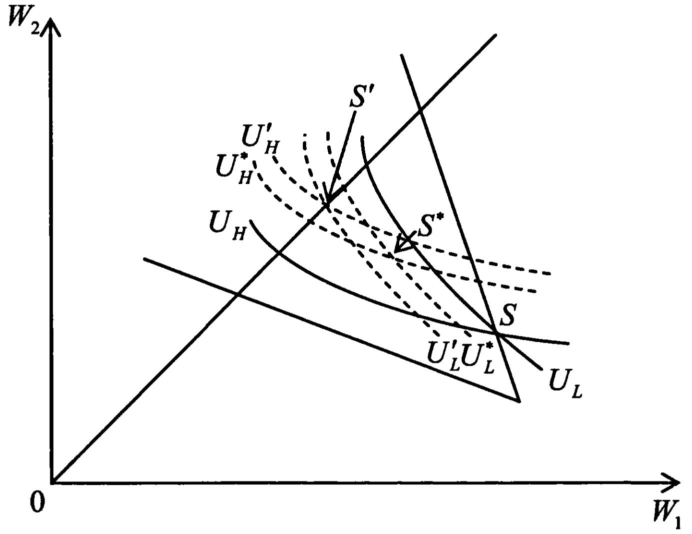


Figure 3.16

3.8 Conclusion

This chapter shows, introducing a participation option into traditional models of adverse selection causes significant changes. Due to their high probability of incurring loss, which makes the reservation utility more attractive, the high-risks drop out of the risky activity. Instead of being stuck with the low-risks, the market is now filled with the good-risks – the "lemon" market has turned into "peach" market. Taxing insurance makes participation less attractive for the high risks, allowing the low risks to extend coverage. Thus the tax has an offsetting benefit to those paying it creating an efficiency gain. When the redistribution effect dominates the efficiency effect, redistributing the wealth from the high-risks to the low-risks is optimal. This requires an insurance subsidy, which paradoxically decreases the insurance coverage in the market equilibrium. For certain parameter values, it would maximize total social welfare to eliminate the whole insurance market. The market equilibrium involves too much insurance.

Another interesting finding is that full insurance pooling allocation never maximizes the aggregate social welfare. Although the high-risks enjoy higher reservation utility in the allocation, there is no efficiency gain at all for the whole society, and there is only redistribution effect which is always negative.

Chapter 4 Partial-pooling Nash equilibrium and participation option

4.1 Introduction

Since Rothschild and Stiglitz (1976) found that a pooling Nash equilibrium can not exist in competitive insurance markets, explaining how pooling can arise has become a notoriously difficult task in insurance studies. Wilson (1977) adopts "anticipatory equilibrium", a different equilibrium concept from Nash equilibrium, and finds a pooling equilibrium in the same setup as Rothschild and Stiglitz. However, his assumption on the strategic behaviour of insurers is inconsistent with the assumption of competitive insurance markets. Wambach (2000) extends the Rothschild-Stiglitz model by introducing unobservable wealth in addition to the differing risks, which, under the assumption of constant relative risk aversion, in effect changes agents' risk preferences. Wambach shows that for large wealth differences, partial risk pooling contracts, in which one type chooses different contracts in equilibrium, are feasible. Furthermore, complete risk pooling contracts can also occur.

Smart (2000) explicitly introduces different risk preference into the Rothschild-Stiglitz model. With the double crossing property of indifference curves, he finds that different risk types can be pooled in Nash equilibrium if differences in risk aversion are sufficiently large. Similar to his work, de Meza (2002) also finds the partial-pooling Nash equilibrium. Their works lay the foundation on equilibrium analysis for the present essay.

In contrast to the works above, de Meza and Webb (2001) find partial-pooling Nash equilibrium assuming heterogeneous risk preferences and hidden action. More risk-averse agents choose higher precautionary effort which leads to different risk probabilities. Risk-tolerant agents are drawn into a pooling equilibrium by the low premiums created by the presence of safer, more risk-averse types. Their welfare analysis shows that taxing insurance drives out the reckless agents, allowing a strict Pareto gain, however administrative costs are necessary for this result.

This present essay introduces participation option and differing risk preferences into the Rothschild-Stiglitz model. Agents not only differ in risk preferences and risk probabilities, they also have choices on whether to take the risk activity and whether to buy insurance in competitive insurance markets where insurers offer single contracts with endogenous contract form.

Because of differing risk preferences, the same financial loss resulted from the risky activity results in different changes in the utilities of agents. This together with the reservation utility from the participation option, and the driving benefit from the risky activity, gives rise to four different separating equilibria, partial-pooling equilibrium, and sometimes no equilibrium at all in the market.

At the partial-pooling equilibrium, the timid high-risks do not drive to avoid the possible financial loss resulted from the accident which is more likely to happen to them and receive the reservation utility, the bold high-risks are attracted into driving and buying insurance because of the low insurance premium resulted from the presence of the timid low-risks, and the timid low-risks, as the bold high-risks, choose to drive and buy insurance.

The essay demonstrates that the partial-pooling Nash equilibrium is Pareto efficient under certain conditions. But if it is inefficient, a tax and subsidy scheme which taxes insurance and uses the proceeds to subsidize the whole population may achieve Pareto improvement: taxing insurance makes it more expensive and hence less attractive to the bold high-risks who would now opt to stop driving and receive the reservation utility which is now higher because of the subsidy. So taxing insurance drives the bold high-risks out of the insurance contract. This will in turn lower the insurance premium for the timid low-risks who will continue driving and buying insurance. The timid high-risks will become better off as well because of the raised reservation utility. Therefore, a Pareto improvement is achieved.

This result complements the case for taxing insurance by extending it to pooling equilibrium where the efficiency problem is the wrong composition of extents.

The essay is organized as follows. Section 4.2 specifies the model. Section 4.3 finds the partial-pooling Nash equilibrium as well as the four separating equilibria. It also finds conditions of the partial-pooling equilibrium. Section 4.4 analyzes the efficiency of the partial-pooling equilibrium and demonstrates that it is Pareto efficient under certain conditions. Section 4.5 shows that inefficient partial-pooling equilibrium exists and taxing insurance leads to Pareto gain.

4.2 The model

There are many insurance companies in the market. Agents are identical except for accident probability and risk aversion, which are unobservable. Accident probability

π_i and risk aversion θ_j each take on one of two values in the population, $ij \in \{L,H\} \times \{B,T\}$, with $0 < \pi_L < \pi_H < 1$ and $\theta_B < \theta_T$. The coefficient of absolute risk aversion is defined as $\theta_j = -\frac{u_j''}{u_j'}$, $j = B,T$, where u represents utility function and u_T always has greater degree of risk aversion than u_B. Furthermore, we assume $\theta_B = 0$, so u_B is a linear utility function to simplify the analysis without losing any general implications. Everyone has an initial wealth W.

So there are four types of agents in the insurance market: N_{LB} agents with low accident probability and low risk aversion L_B (the bold low-risks); N_{LT} agents with low accident probability and high risk aversion L_T (the timid low-risks); N_{HB} agents with high accident probability and low risk aversion H_B (the bold high-risks); N_{HT} agents with high accident probability and high risk aversion H_T (the timid high-risks).

The market equilibrium is the outcome of a two-stage game. In the first stage, each insurance company offers a single insurance policy $\alpha = \{I, p\}$, which specifies insurance indemnity I and premium p. Agents then choose whether to drive and, if drive, whether to purchase insurance. If they do not drive, they receive a reservation utility $u_j(W)$, $j = B,T$. If they drive, they enjoy a utility benefit B from driving but may incur a pecuniary loss D with probability π_i, $i = L,H$.

4.2.1 Insurance demand

The bold low-risks L_B receive the reservation utility $u_B(W)$ if they choose not to drive. If they drive without insurance, they receive $U_{LB} = B + \pi_L u_B(W - D) + (1 - \pi_L)u_B(W)$. Since u_B is assumed to be linear, U_{LB} can be simplified as $U_{LB} = B + u_B(W - \pi_L D)$. If they drive with insurance, they receive $U_{LB} = B + \pi_L u_B(W - D - p + I) + (1 - \pi_L)u_B(W - p)$, which can be simplified as $U_{LB} = B + u_B(W - \pi_L D + \pi_L I - p)$. Hence if the bold low-risks drive, they will buy insurance policy if $p < \pi_L I$. But this policy will not be offered since it surely results in loss to insurers. The best premium they can get is the fair premium $p = \pi_L I$, at which they are indifferent between buying and not buying insurance.

For the timid low-risks L_T, they receive a reservation utility $U_{LT} = u_T(W)$ if they choose not to drive. If they drive without insurance, they receive $U_{LT} = B + \pi_L u_T(W - D) + (1 - \pi_L)u_T(W)$. If they drive with insurance, they receive $U_{LT} = B + \pi_L u_T(W - D - p + I) + (1 - \pi_L)u_T(W - p)$. If they drive with full insurance coverage at a fair premium, i.e., $I = D$ and $p = \pi_L D$, they will receive $U_{LT} = B + u_T(W - \pi_L D)$.

For the bold high-risks H_B, their reservation utility is $u_B(W)$ if they do not drive. They receive $U_{HB} = B + \pi_H u_B(W - D) + (1 - \pi_H)u_B(W)$, i.e., $U_{HB} = B + u_B(W - \pi_H D)$, if they drive without insurance. If they drive with insurance, they receive $U_{HB} = B + \pi_H u_B(W - D - p + I) + (1 - \pi_H)u_B(W - p)$, i.e., $U_{HB} = B + u_B(W - \pi_H D + \pi_H I - p)$. Therefore, when they drive, they will buy insurance if $\pi_H I - p > 0$ and will be indifferent if $\pi_H I - p = 0$.

For the timid high-risks H_T, their reservation utility is $u_T(W)$ if they don't drive. If they drive without insurance, it is $U_{HT} = B + \pi_H u_T(W - D) + (1 - \pi_H)u_T(W)$, and it is $U_{HT} = B + \pi_H u_T(W - D - p + I) + (1 - \pi_H)u_T(W - p)$ if they drive with insurance. Hence they would choose to drive with insurance if $B + \pi_H u_T(W - D - p + I) + (1 - \pi_H)u_T(W - p) > u_T(W)$. If offered full insurance coverage at fair premium, i.e., $I = D$ and $p = \pi_H D$, their utility is $U_{HT} = B + u_T(W - \pi_H D)$.

4.2.2 Participation option and reservation utility

With different levels of reservation utility, agents may choose to drive or not to drive. This paper makes further assumptions of reservation utility as follows.

$$U_{HB} = B + u_B(W - \pi_H D) < u_B(W) < U_{LB} = B + u_B(W - \pi_L D)$$

$$U_{HT} = B + u_T(W - \pi_H D) < u_T(W) < U_{LT} = B + u_T(W - \pi_L D)$$

where $u_B(W)$ is the reservation utility of the bold and $u_T(W)$ is the reservation utility of the timid. The bold high-risks H_B will stop driving if offered insurance coverage at their fair premium, but they will drive with insurance if $B + u_B(W - \pi_H D + \pi_H I - p) > u_B(W)$. The bold low-risks L_B will definitely choose to drive. As we have seen in Section 1.1, the bold low-risks are indifferent between buying and not buying fair insurance when they drive. Since insurance is never better than fair for the low risks, the L_B have no impact on the equilibrium. So we drop out the bold low-risks in the rest of the analysis.

It is straightforward that the timid high-risks H_T will not drive but it is ambiguous whether the timid low-risks L_T will drive. The following equilibrium analysis gives answer for that.

4.3 Market equilibrium

4.3.1 Full pooling allocation can never be a Nash equilibrium

One possible equilibrium is the full pooling allocation: the timid high-risks, the bold high-risks and the timid low-risks are all offered the same insurance contract and hence are pooled into a full pooling allocation. But can it constitute a Nash equilibrium?

The slopes of the indifference curves of the timids are $-\dfrac{(1-\pi_i)}{\pi_i}\dfrac{u_T'(W_1)}{u_T'(W_2)}$, $i = L, H$, where W_1 represents the wealth in good state where there is no accident and W_2 the wealth in bad state where there is an accident, $W_1 \le W$ and $W_2 \ge W - D$. Clearly, the timid low-risks L_T have steeper slope of indifference curve than the timid high-risks H_T at any wealth allocation, so their indifference curves cross only once. Therefore, for any contract that pools both the types, insurers can always find a deviation offering cheaper premium with more deductible, which is attractive to the timid low-risks L_T but not to the timid high-risks H_T. So a full pooling allocation can never constitute a Nash equilibrium.

4.3.2 Separating equilibrium

Another possibility is separating equilibrium: each type of agents is offered a insurance contract different from one another. The following analysis shows there may be four different separating equilibria.

In the models where agents only differ in risk probabilities, indifference curves of different agent types cross only once. However, with differing risk preferences, the difference curves may cross twice. This gives rises to the existence of partial-pooling equilibrium.

Technically, the slopes of the indifference curves of the bolds are $-\frac{(1-\pi_i)}{\pi_i}\frac{u_B'(W_1)}{u_B'(W_2)}$, i.e., $-\frac{(1-\pi_i)}{\pi_i}$, $i=L,H$, since their utility function is assumed linear. It is straightforward that the slope of the indifference curve of the bold low-risks L_B is always steeper than that of the timid low-risks L_T, i.e., $-\frac{(1-\pi_L)}{\pi_L} \leq -\frac{(1-\pi_L)}{\pi_L}\frac{u_T'(W_1)}{u_T'(W_2)}$, $\forall\, W_1 \geq W_2$, $W_1 \leq W$ and $W_2 \geq W-D$. However, it is ambiguous whether or not the slope of the indifference curve of the bold high-risks H_B is steeper than that of the timid low-risks L_T. With different wealth allocation $\{W_1, W_2\}$ in the region that $W_1 \geq W_2$, $W_1 \leq W$ and $W_2 \geq W-D$, $-\frac{(1-\pi_L)}{\pi_L}\frac{u_T'(W_1)}{u_T'(W_2)}$ might be greater or less than $-\frac{(1-\pi_H)}{\pi_H}$, dependent on the wealth allocation and the degree of risk aversion of the timid low-risks L_T. If $-\frac{(1-\pi_H)}{\pi_H} > -\frac{(1-\pi_L)}{\pi_L}\frac{u_T'(W_1)}{u_T'(W_2)}$, $\forall\, W_1 \geq W_2$, $W_1 \leq W$ and $W_2 \geq W-D$, the indifference curves of the timid low-risks L_T and the bold high-risks H_B cross only once. If $-\frac{(1-\pi_H)}{\pi_H} < -\frac{(1-\pi_L)}{\pi_L}\frac{u_T'(W_1)}{u_T'(W_2)}$ for some wealth allocations $\{W_1, W_2\}$ in the region that $W_1 \geq W_2$, $W_1 \leq W$ and $W_2 \geq W-D$, they will cross twice. As $-\frac{(1-\pi_H)}{\pi_H} > -\frac{(1-\pi_L)}{\pi_L}$, $0 < \frac{u_T'(W_1)}{u_T'(W_2)} \leq 1$, $\frac{d}{d\theta}\frac{u_T'(W_1)}{u_T'(W_2)} < 0$, one can always find a θ that is large enough to make $-\frac{(1-\pi_H)}{\pi_H} < -\frac{(1-\pi_L)}{\pi_L}\frac{u_T'(W_1)}{u_T'(W_2)}$ for a given wealth allocation $\{W_1, W_2\}$.

Differing risk preferences also affects equilibrium from another aspect. With different degrees of risk aversion, the utility functions of the bolds and the timids give different scales of utility. Hence the same wealth change may result in different changes in utility. This together with a fixed utility benefit B of driving may have very different effects on the bolds and the timids. As a result, the indifference curve of reservation utility of the H_B may lie above (like I'), cross (like I''), or below (like I''') that of the H_T (I_{HT}^R), as figure 4.1 illustrates.

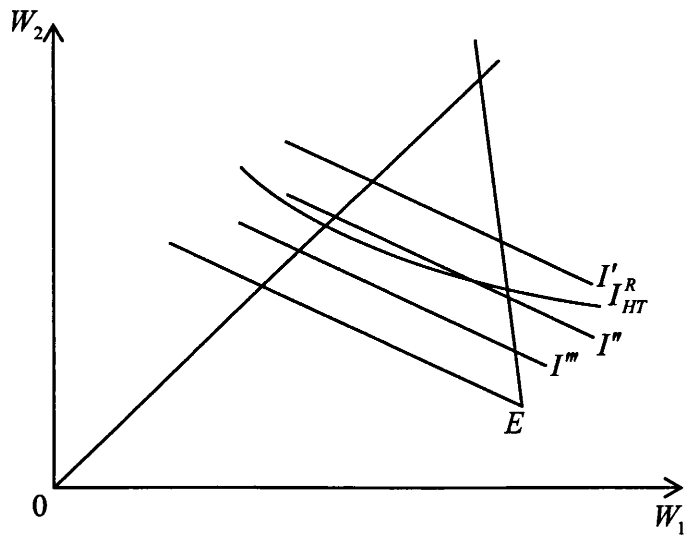


Figure 4.1

4.3.2.1 Separating equilibrium – type 1

Accordingly, there can be four types of separating equilibrium as section 4.3.2.1 – section 4.3.2.4 show. When the indifference curve of reservation utility of the H_B lies below that of the H_T, the reservation utility of the bold high-risks constitutes the binding constraint for the insurance coverage offered to the timid low-risks. The equilibrium allocation is therefore the allocation that maximizes the utility of the timid low-risks whilst keeps the bold high-risks indifferent. When the indifference curves of the bold high-risks and the timid low-risks cross only once, the optimal allocation involves no insurance for the bold high-risk, partial insurance coverage for the timid low-risks and zero profit for the insurers.

In the competitive market, insurers offer contracts that maximise the utility of the timid low-risks subject to the bold high-risks are indifferent between buying the insurance and receiving the reservation utility. Consider the following maximization problem

$$\underset{I_1, p_1}{Max}\, U^1_{LT} = B + \pi_L u_T(W - D - p_1 + I_1) + (1 - \pi_L) u_T(W - p_1)$$

s.t. $p_1 = \pi_L I_1$

$$B + \pi_H u_B(W - D - p_1 + I_1) + (1 - \pi_H) u_B(W - p_1) = u_B(W) \text{ i.e.}$$

$$B + u_B(W - \pi_H D + (\pi_H - \pi_L) I_1) = u_B(W)$$

Set $\{I_1^*, p_1^*\}$ as the solution to the problem.

If the indifference curve of the timid low-riks only cross once that of the bold high-risks, i.e. $-\frac{(1-\pi_H)}{\pi_H} > -\frac{(1-\pi_L)}{\pi_L}\frac{u_T'(W-p_1^*)}{u_T'(W-D-p_1^*+I_1^*)}$, and the bold high-risks are indifferent between buying the insurance contract and receiving the reservation utility, i.e. $B+\pi_H u_T(W-D-p_1^*+I_1^*)+(1-\pi_H)u_T(W-p_1^*)\le u_T(W)$, $\{I_1^*, p_1^*\}$ constitutes a separating equilibrium, which we call type 1 separating equilibrium. In this equilibrium, both the H_B and the H_T will stop driving, and the L_T will drive with partial insurance coverage at their fair premium. Figure 4.2 illustrates the equilibrium.

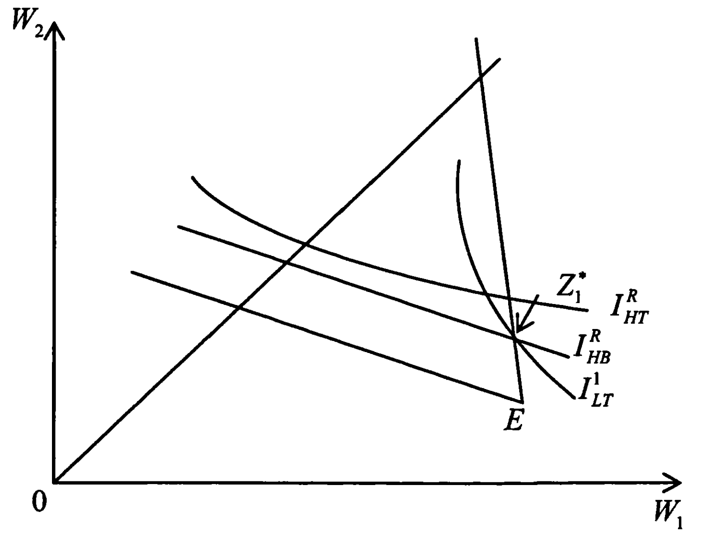


Figure 4.2

4.3.2.2 Separating equilibrium – type 2

As the timid low-risks become more risk averse, their indifference curves become more curved and may double cross the indifference curves of the bold high-risks. Technically, as the degree of risk aversion of the L_T increases,

$-\frac{(1-\pi_L)}{\pi_L}\frac{u_T'(W-p_1^*)}{u_T'(W-D-p_1^*+I_1^*)}$ increases: they are willing to give up more wealth in exchange for a given wealth increase in the bad state. After a critical value, we will have $-\frac{(1-\pi_H)}{\pi_H} < -\frac{(1-\pi_L)}{\pi_L}\frac{u_T'(W-p_1^*)}{u_T'(W-D-p_1^*+I_1^*)}$. There then exists profitable deviation from a type 1 equilibrium a type 2 separating equilibrium in which high risks do not drive and low risks drive with partial coverage. At the type 2 separating equilibrium, insurers offer contract Z_2^* rather than the type 1 separating contract Z_1^*, as it is preferred by the timid low-risks. Also, insurers make more than normal profits from Z_2^*, as it is below the fair offer curve of the timid low-risks. The equilibrium can be defined as the outcome of the following maximization problem.

$$\underset{I_2,p_2}{Max}\, U_{LT}^2 = B + \pi_L u_T(W - D - p_2 + I_2) + (1-\pi_L)u_T(W - p_2)$$

$$\text{s.t.} \quad -\frac{(1-\pi_H)}{\pi_H} = -\frac{(1-\pi_L)}{\pi_L}\frac{u_T'(W-p_2)}{u_T'(W-D-p_2+I_2)}$$

$$B + \pi_H u_B(W - D - p_2 + I_2) + (1-\pi_H)u_B(W - p_2) = u_B(W) \text{ i.e.}$$

$$B + u_B(W - \pi_H D + \pi_H I_2 - p_2) = u_B(W)$$

Set $\{I_2^*, p_2^*\}$ as the solution to the problem.

If the bold high-risks are indifferent between buying the insurance contract and receiving the reservation utility, i.e.

$B + \pi_H u_T(W - D - p_2^* + I_2^*) + (1-\pi_H)u_T(W - p_2^*) \le u_T(W)$, $\{I_2^*, p_2^*\}$ constitutes what we delegate type 2 separating equilibrium. In this equilibrium, the L_T receive a utility of $U_{LT}^2 = B + \pi_L u_T(W - D - p_2^* + I_2^*) + (1-\pi_L)u_T(W - p_2^*)$. The L_T would pay more than their actuarially fair premium to get more insurance coverage.[28] Figure 4.3 illustrates this equilibrium.

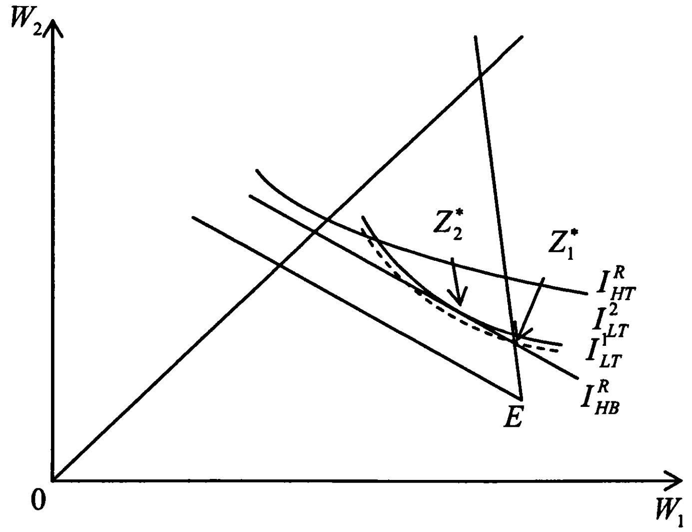


Figure 4.3

[28] One odd feature of this knife-edge equilibrium is the positive profit for insurers, which contradicts the perfect competition assumption. de Meza and Webb (2001) offer a way to smooth this odd feature by adding in "trivial costs" – low heterogeneous costs such as the cost of filling in application forms. Such a modification will remove this odd feature. Since the modification will make the technical analysis very complicated and will not fundamentally change the current analysis, the present research does not add in the "trivial cost".

4.3.2.3 Separating equilibrium – type 3

When the indifference curve of reservation utility of the H_T, I_{HT}^R, lies below that of the H_B, I_{HB}^R, i.e. $B+\pi_H u_T(W-D-p_1^*+I_1^*)+(1-\pi_H)u_T(W-p_1^*)\geq u_T(W)$, we have a case as figure 4.4 illustrates.

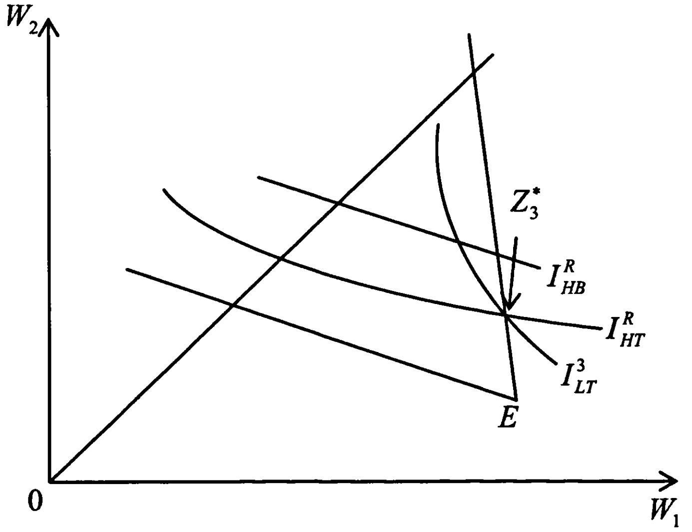


Figure 4.4

In this case, the reservation utility of the timid high-risks constitutes the binding constraint for the insurance offered to the timid low-risks. At the equilibrium allocation, the timid and bold high-risks do not have insurance and the timid low-risks are partially insured. Thus, the double-crossing property does not matter in this case, i.e. no matter whether $-\frac{(1-\pi_H)}{\pi_H}>-\frac{(1-\pi_L)}{\pi_L}\frac{u_T'(W-p_1^*)}{u_T'(W-D-p_1^*+I_1^*)}$, the equilibrium, which we delegate as type 3 separating equilibrium, is solely determined by the reservation utility of the H_T and the offer curve of the L_T, which can be summarized as the following maximization problem.

$$\underset{I_3,p_3}{Max}\, U_{LT}^3=B+\pi_L u_T(W-D-p_3+I_3)+(1-\pi_L)u_T(W-p_3)$$

s.t. $p_3=\pi_L I_3$

$$B+\pi_H u_T(W-D-p_3+I_3)+(1-\pi_H)u_T(W-p_3)=u_T(W)$$

4.3.2.4 Separating equilibrium – type 4

When the indifference curve of the reservation utility of the bold high-risks I_{HB}^{R} crosses that of the timid high risks I_{HT}^{R}, and double-crosses the indifference curve of the timid low-risks I_{LT}^{4}, as illustrated in figure 4.5, i.e.

$$B+\pi_H u_T\left(W-D-p_1^*+I_1^*\right)+\left(1-\pi_H\right)u_T\left(W-p_1^*\right)<u_T(W)$$
$$<B+\pi_H u_T\left(W-D-p_2^*+I_2^*\right)+\left(1-\pi_H\right)u_T\left(W-p_2^*\right)$$

$$-\frac{\left(1-\pi_H\right)}{\pi_H}<-\frac{\left(1-\pi_L\right)}{\pi_L}\frac{u_T'\left(W-p_1^*\right)}{u_T'\left(W-D-p_1^*+I_1^*\right)},$$

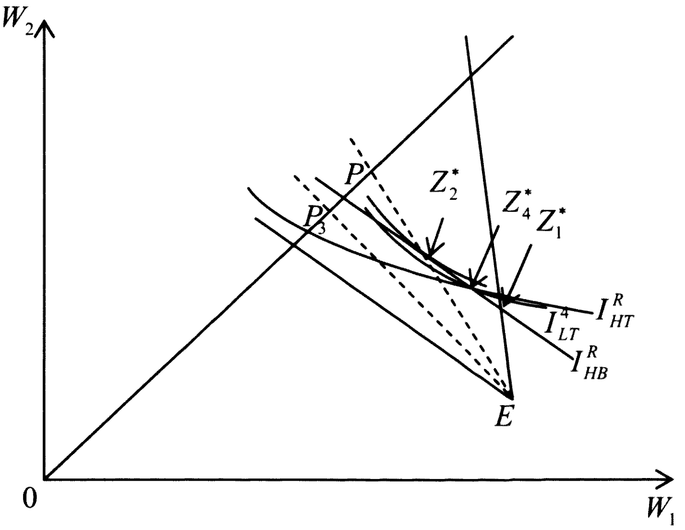


Figure 4.5

We will have a case as figure 4.5 illustrates. The reservation utilities of the bold high-risks and the timid high-risks both constitute binding constraints for the insurance offered to the timid low-risks. The equilibrium contract needs to maximize the utility of the timid low-risks whilst keep both the bold and the timid high-risks indifferent. Hence it can be found by solving the following maximization problem:

$$\max_{I_4,p_4} U_{LT}^{4}=B+\pi_L u_T\left(W-D-p_4+I_4\right)+\left(1-\pi_L\right)u_T\left(W-p_4\right)$$

s.t. $B+\pi_H u_T\left(W-D-p_4+I_4\right)+\left(1-\pi_H\right)u_T\left(W-p_4\right)=u_T(W)$

$B+\pi_H u_B\left(W-D-p_4+I_4\right)+\left(1-\pi_H\right)u_B\left(W-p_4\right)=u_B(W)$ i.e.

$B+u_B\left(W-\pi_H D+\pi_H I_4-p_4\right)=u_B(W)$

When EP, the pooling offer curve of the H_B and the L_T, lies below Z_4^*, and EP_3, the pooling offer curve of the H_T and the L_T, lies below I_{LT}^{4}, the indifference curve of the L_T that passes through Z_4^*, Z_4^* constitutes a separating equilibrium, which we call as type 4 separating equilibrium. Since the position of Z_4^* is independent of N_{HT} and N_{HB}, the number of the H_T and H_B, we can always find a N_{HT} and a N_{HB} that is large enough to make EP lie below Z_4^* and EP_3 lie below I_{LT}^4.

In the following sections, we focus our analysis on the case that

$$-\frac{(1-\pi_H)}{\pi_H} < -\frac{(1-\pi_L)}{\pi_L}\frac{u_T'(W-p_1^*)}{u_T'(W-D-p_1^*+I_1^*)},$$

$$B+\pi_H u_T(W-D-p_2^*+I_2^*)+(1-\pi_H)u_T(W-p_2^*)<u_T(W)$$

Figure 4.6 illustrates such a typical case.

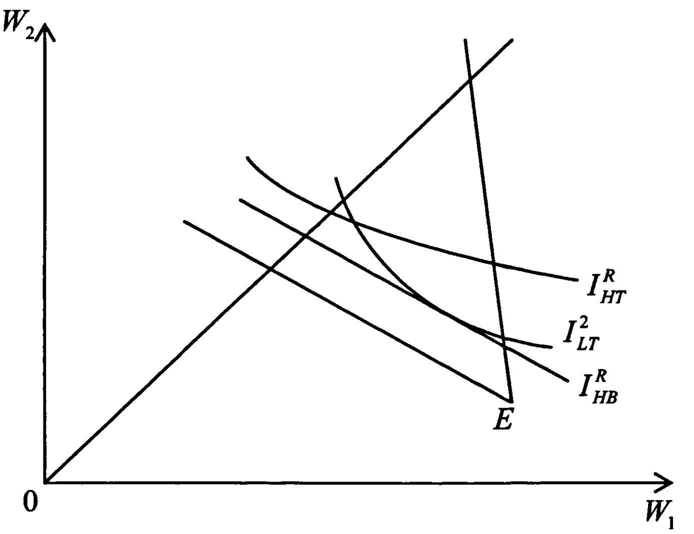


Figure 4.6

4.3.3 Partial-pooling equilibrium

Consider the case that the indifference curve of the reservation utility of the bold high-risks I_{HB}^R lies below that of the timid high-risks I_{HT}^R and double-crosses the indifference curve of the timid low-risks, as Figure 4.7 illustrates. If the pooling offer line EP lies above Z_2^*, the tangent point of the indifference curves of the bold high-risks I_{HB}^R and the timid low risks, insurers in the competitive market will offer insurance contracts along EP which make both the bold high-risks and the timid low-risks better off. Competition will push up the insurance coverage until the timid low-risks are indifferent between buying and not buying the insurance contract. The reservation utility of the bold low-risks constitutes the binding constraint for the partial-pooling insurance offered to both the timid low-risks and the bold high-risks.

At the partial-pooling equilibrium, the timid high-risks opt out of the risky activity, the timid low-risks and the bold high-risks drive and buy the insurance contract Z_P^* which in effect cross-subsidizes the bold high-risks from the timid low-risks. The timid low-risks pay greater than fair premium but get more insurance coverage than Z_2^* in the type 2 separating equilibrium.

The partial-pooling equilibrium can be defined as the following maximization problem.

$$\underset{I_P, p_P}{Max}\, U_{LT}^P = B + \pi_L u_T(W - D - p_P + I_P) + (1-\pi_L)u_T(W - p_P)$$

$$\text{s.t. } p_P = \left(\frac{N_{LT}}{N_{LT}+N_{HB}}\pi_L + \frac{N_{HB}}{N_{LT}+N_{HB}}\pi_H\right)I_P$$

$$B + \pi_H u_T(W - D - p_P + I_P) + (1-\pi_H)u_T(W - p_P) = u_T(W)$$

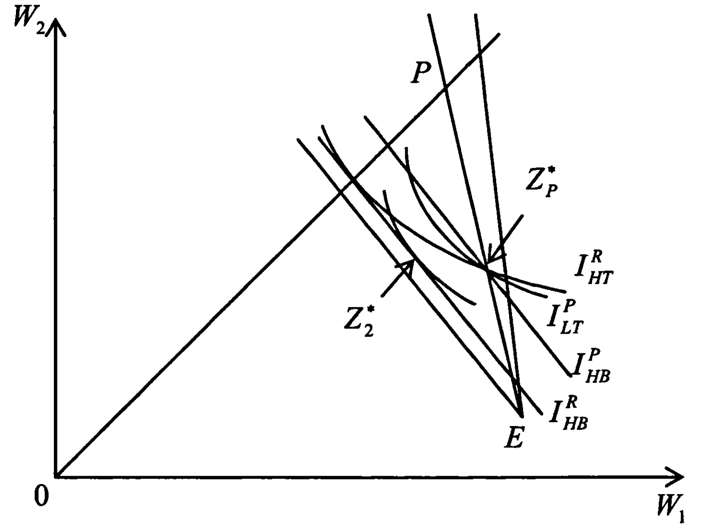


Figure 4.7

Set $\{I_P^*, p_P^*\}$ as the solution to the problem. Then, the L_T receive a utility of $U_{LT}^P = B + \pi_L u_T(W - D - p_P^* + I_P^*) + (1-\pi_L)u_T(W - p_P^*)$ in the partial-pooling equilibrium. The H_B receive from the partial-pooling contract a utility of $U_{HB}^P = B + \pi_H u_B(W - D - p_P^* + I_P^*) + (1-\pi_H)u_B(W - p_P^*)$, which can be simplified as $U_{HB}^P = B + u_B(W - \pi_H D + \pi_H I_P^* - p_P^*)$. It is straightforward that I_{HB}^P, the indifference curve of the H_B passing through the partial-pooling allocation Z_P^*, must lie above I_{HB}^R, their indifferent curve of reservation utility. That is,

$U_{HB}^{P} = B + u_B\left(W - \pi_H D + \pi_H I_P^* - p_P^*\right) > u_B(W)$. So the H_B will drive and buy the insurance as well. Figure 4.7 illustrates the equilibrium.

4.3.4 Conditions of partial-pooling equilibrium

4.3.4.1 Two partial-pooling contracts

There are some conditions for a partial-pooling allocation to be a Nash equilibrium. First, as Figure 4.8 illustrates, EP_3, the pooling offer curve of the L_T and the H_T, must lie below the indifference curve I_{LT}^P of the L_T that passes through Z_P^*, the partial-pooling contract. Otherwise, a deviating contract such as C will break the partial-pooling equilibrium. As the proportion of the H_T is independent of the position of the indifference curve I_{LT}^P, we can always find a N_{HT} that is large enough to make EP_3 lie below I_{LT}^P.

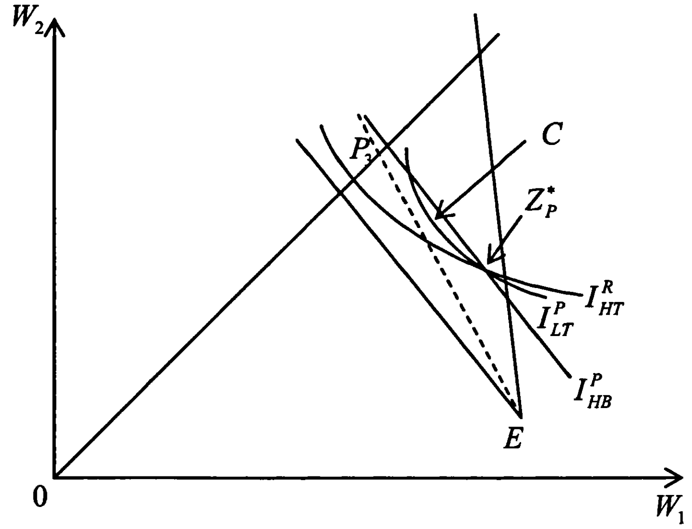


Figure 4.8

Technically, the partial pooling contract of the H_T and the L_T can be found by the maximization problem

$$\underset{I_5, p_5}{Max}\, U_{LT}^5 = B + \pi_L u_T\left(W - D - p_5 + I_5\right) + \left(1 - \pi_L\right) u_T\left(W - p_5\right)$$

$$\text{s.t. } p_5 = \left(\frac{N_{LT}}{N_{LT} + N_{HT}} \pi_L + \frac{N_{HT}}{N_{LT} + N_{HT}} \pi_H\right) I_5$$

Set $\{I_5^*, p_5^*\}$ as the solution to the problem. Then, the L_T receive a utility of $U_{LT}^5 = B + \pi_L u_T\left(W - D - p_5^* + I_5^*\right) + \left(1 - \pi_L\right) u_T\left(W - p_5^*\right)$ in this allocation. The partial-pooling allocation of the H_B and the L_T must dominate this allocation for it to be a Nash equilibrium. Thus, we must have

$$\begin{aligned} U_{LT}^{P} &= B+\pi_L u_T\left(W-D-p_P^*+I_P^*\right)+\left(1-\pi_L\right)u_T\left(W-p_P^*\right)\geq \\ U_{LT}^{5} &= B+\pi_L u_T\left(W-D-p_5^*+I_5^*\right)+\left(1-\pi_L\right)u_T\left(W-p_5^*\right) \end{aligned} \tag{4.1}$$

As $p_5=\left(\frac{N_{LT}}{N_{LT}+N_{HT}}\pi_L+\frac{N_{HT}}{N_{LT}+N_{HT}}\pi_H\right)I_5$ increases in N_{HT},

$U_{LT}^{5}=B+\pi_L u_T(W-D-p_5+I_5)+(1-\pi_L)u_T(W-p_5)$ decreases in p, and

$U_{LT}^{P}=B+\pi_L u_T\left(W-D-p_P^*+I_P^*\right)+\left(1-\pi_L\right)u_T\left(W-p_P^*\right)$ is independent of N_{HT}, there exists a N_{HT} large enough to make (4.1) hold.

4.3.4.2 Partial-pooling equilibrium and separating equilibrium

The L_T must receive greater utility from the partial-pooling allocation than from the separating allocation Z_2^* for the partial-pooling allocation to be a Nash equilibrium.

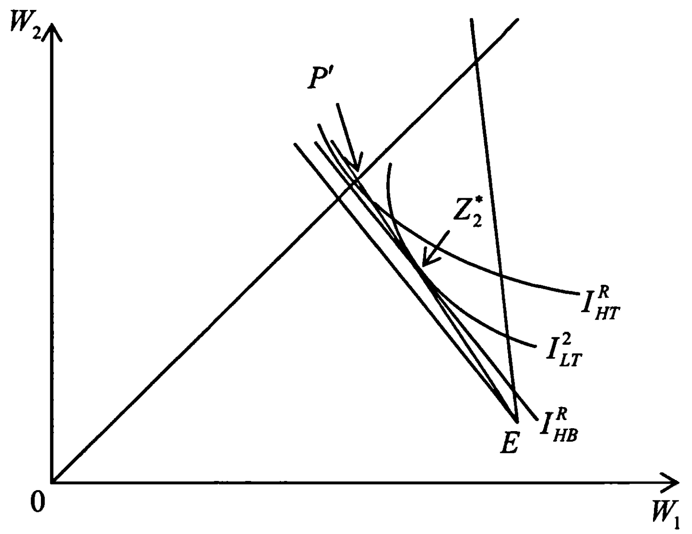


Figure 4.9

As figure 4.9 illustrates, if EP', the pooling offer curve of the L_T and the H_B, intercepts the indifference curve I_{HT}^R of the H_T at a point that is below the indifference I_{LT}^2 of the L_T that passes through the separating contract Z_2^*, the separating equilibrium will dominate the partial pooling equilibrium. That is, we must have

$$U_{LT}^{P}=B+\pi_L u_T\left(W-D-p_P^*+I_P^*\right)+\left(1-\pi_L\right)u_T\left(W-p_P^*\right)\geq$$
$$U_{LT}^{2}=B+\pi_L u_T\left(W-D-p_2^*+I_2^*\right)+\left(1-\pi_L\right)u_T\left(W-p_2^*\right) \tag{4.2}$$

As the position of I_{LT}^2 is independent of N_{HB}, the number of the bold high-risks H_B, there exists a N_{HB} that is small enough to make the pooling offer curve intercept the I_{HT}^R at a point that is above the I_{LT}^2. That is,

$U_{LT}^{2}=B+\pi_L u_T\left(W-D-p_2^*+I_2^*\right)+\left(1-\pi_L\right)u_T\left(W-p_2^*\right)$ is independent of N_{HB}, while

$p=\left(\dfrac{N_{LT}}{N_{LT}+N_{HB}}\pi_L+\dfrac{N_{HB}}{N_{LT}+N_{HB}}\pi_H\right)I$ increases in N_{HB} and

$U_{LT}^{P}=B+\pi_L u_T\left(W-D-p_P^*+I_P^*\right)+\left(1-\pi_L\right)u_T\left(W-p_P^*\right)$ decreases in N_{HB}, so there exists a N_{HB} small enough to make (4.2) hold.

4.3.4.3 No deviation

Furthermore, at the partial-pooling contract Z_P^*, the slope of the indifference curve I_{LT}^P of the L_T must be flatter than that of the H_B to deter potential deviation. That is, we must have

$$-\frac{\left(1-\pi_H\right)}{\pi_H}<-\frac{\left(1-\pi_L\right)}{\pi_L}\frac{u_T'\left(W-p_P^*\right)}{u_T'\left(W-D-p_P^*+I_P^*\right)} \tag{4.3}$$

As $-\dfrac{\left(1-\pi_L\right)}{\pi_L}\dfrac{u_T'\left(W-p_P^*\right)}{u_T'\left(W-D-p_P^*+I_P^*\right)}$, the slope of I_{LT}^P at Z_P^*, increases in θ, there exists a θ large enough to make (4.3) hold.

4.4 Market efficiency

The partial-pooling equilibrium can be Pareto-efficient under certain conditions. One way to check whether it is Pareto-efficient is to check whether government can use a tax-subsidy scheme to achieve Pareto improvement.

Suppose the government has initiated a balanced budget taxation which taxes insurance policies and uses the proceeds to subsidise the whole population. Motorists have to pay a lump sum tax t for the insurance policy they buy. Every one receives a lump sum subsidy s .

4.4.1 From partial-pooling equilibrium to partial-pooling equilibrium

We first consider the case that the tax and subsidy scheme does not break the partial-pooling equilibrium, hence we have $s = \dfrac{N_{LT} + N_{HB}}{N_{LT} + N_{HB} + N_{HT}} t$ and the net payout of motorists is then $t - s = \dfrac{N_{HT}}{N_{LT} + N_{HB} + N_{HT}} t$.

As non-motorists pay no tax and receive subsidy, their reservation utility surely increases with the amount of tax and subsidy. Tax increases the cost of insurance and shifts down the pooling offer curve along the 45 degree line. However, as the non-motorists' reservation utility increases, the insurance coverage the motorists can buy increases as well. Thus, with increased premium and increased indemnity, it is ambiguous whether or not the motorists will be better off under the tax and subsidy scheme.

Figure 4.10 illustrates a possibility of Pareto improvement from the laissez faire partial-pooling equilibrium Z_P^* to the new partial-pooling equilibrium Z_P^T under the policy intervention. The subsidy increases the reservation utility of non-motorists and hence relaxes the restriction on the insurance coverage that motorists are allowed to purchase, while the tax increases the cost of insurance and hence shifts down the offer curve. The following argument shows, under certain conditions, it is impossible for the motorists to receive greater utility from the new partial-pooling contract.

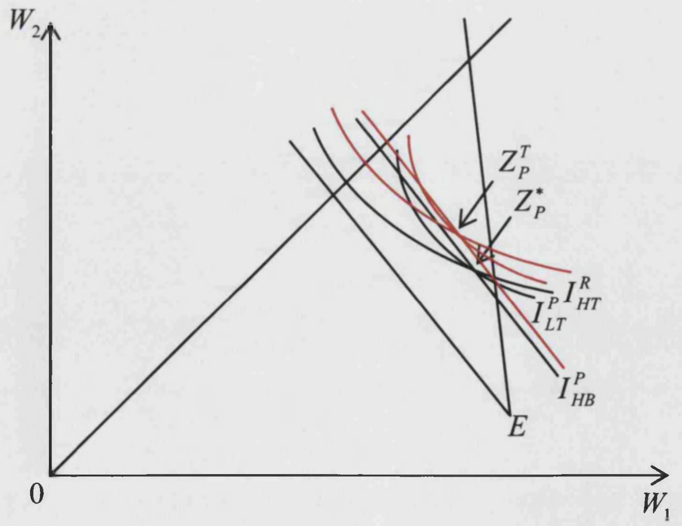


Figure 4.10

The partial-pooling equilibrium is the solution to the following maximization problem

$$\underset{I_P,p_P}{Max}\, U_{LT}^{P} = B + \pi_L u_T\left(W - D - p_P + I_P\right) + \left(1 - \pi_L\right)u_T\left(W - p_P\right)$$

$$\text{s.t. } p_P = \left(\frac{N_{LT}}{N_{LT} + N_{HB}}\pi_L + \frac{N_{HB}}{N_{LT} + N_{HB}}\pi_H\right)I_P$$

$$B + \pi_H u_T\left(W - D - p_P + I_P\right) + \left(1 - \pi_H\right)u_T\left(W - p_P\right) = u_T(W)$$

which is equivalent to the following problem

$$\underset{I_P,p_P}{Max}\, U_{HB}^{P} = B + u_B\left(W - \pi_H D + \pi_H I_P - p_P\right)$$

$$\text{s.t. } p_P = \left(\frac{N_{LT}}{N_{LT} + N_{HB}}\pi_L + \frac{N_{HB}}{N_{LT} + N_{HB}}\pi_H\right)I_P$$

$$B + \pi_H u_T\left(W - D - p_P + I_P\right) + \left(1 - \pi_H\right)u_T\left(W - p_P\right) = u_T(W)$$

After the tax and subsidy scheme implemented, the maximization problem becomes

$$\underset{I_P^T,p_P^T}{Max}\, U_{HB}^{PT} = B + u_B\left(W - \pi_H D + \pi_H I_P^T - p_P^T - \frac{N_{HT}}{N_{LT} + N_{HB} + N_{HT}}t\right)$$

$$\text{s.t. } p_P^T = \left(\frac{N_{LT}}{N_{LT} + N_{HB}}\pi_L + \frac{N_{HB}}{N_{LT} + N_{HB}}\pi_H\right)I_P^T \tag{4.4}$$

$$B + \pi_H u_T\left(W - D - p_P^T + I_P^T - \frac{N_{HT}}{N_{LT} + N_{HB} + N_{HT}}t\right) + \left(1 - \pi_H\right)u_T\left(W - p_P^T - \frac{N_{HT}}{N_{LT} + N_{HB} + N_{HT}}t\right)$$
$$= u_T\left(W + \frac{N_{LT} + N_{HB}}{N_{LT} + N_{HB} + N_{HT}}t\right)$$

Substitute (4.4) into the problem and we get

$$\underset{I_P^T}{Max}\, U_{HB}^{PT} = B + u_B\left(W - \pi_H D + \frac{N_{LT}}{N_{LT} + N_{HB}}\left(\pi_H - \pi_L\right)I_P^T - \frac{N_{HT}}{N_{LT} + N_{HB} + N_{HT}}t\right)$$

s.t.

$$B + \pi_H u_T\left(W - D + \left(\frac{N_{LT}}{N_{LT} + N_{HB}}\left(1 - \pi_L\right) + \frac{N_{HB}}{N_{LT} + N_{HB}}\left(1 - \pi_H\right)\right)I_P^T - \frac{N_{HT}}{N_{LT} + N_{HB} + N_{HT}}t\right)$$
$$+ \left(1 - \pi_H\right)u_T\left(W - \left(\frac{N_{LT}}{N_{LT} + N_{HB}}\pi_L + \frac{N_{HB}}{N_{LT} + N_{HB}}\pi_H\right)I_P^T - \frac{N_{HT}}{N_{LT} + N_{HB} + N_{HT}}t\right)$$
$$= u_T\left(W + \frac{N_{LT} + N_{HB}}{N_{LT} + N_{HB} + N_{HT}}t\right)$$

For brevity, set

$$\lambda_1 = \frac{N_{LT}}{N_{LT} + N_{HB}},\ 1 - \lambda_1 = \frac{N_{HB}}{N_{LT} + N_{HB}}$$

$$\lambda_2 = \frac{N_{HT}}{N_{LT} + N_{HB} + N_{HT}},\ 1 - \lambda_2 = \frac{N_{LT} + N_{HB}}{N_{LT} + N_{HB} + N_{HT}}$$

$$W_P^T = W - \pi_H D + \lambda_1(\pi_H - \pi_L)I_P^T - \lambda_2 t$$

$$W_1^T = W - (\lambda_1\pi_L + (1-\lambda_1)\pi_H)I_P^T - \lambda_2 t$$

$$W_2^T = W - D + (\lambda_1(1-\pi_L) + (1-\lambda_1)(1-\pi_H))I_P^T - \lambda_2 t$$

$$W_R^T = W + (1-\lambda_2)t$$

The problem can then be rewritten as

$$\underset{I_P^T}{Max}\, U_{HB}^{PT} = B + u_B(W_P^T)$$

$$\text{s.t. } B + \pi_H u_T(W_2^T) + (1-\pi_H)u_T(W_1^T) = u_T(W_R^T)$$

The Lagrangian is then

$$L = B + u_B(W_P^T) + \beta(u_T(W_R^T) - B - \pi_H u_T(W_2^T) - (1-\pi_H)u_T(W_1^T))$$

The Kuhn-Tucker condition for an interior solution gives

$$\frac{\partial L}{\partial I_P^T} \le 0,\ I_P^T \ge 0, \text{ and } I_P^T \frac{\partial L}{\partial I_P^T} = 0$$

For an interior solution $I_P^T > 0$, we have $\frac{\partial L}{\partial I_P^T} = 0$

$$\frac{\partial L}{\partial I_P^T} = u_B'(W_P^T)\lambda_1(\pi_H - \pi_L)$$
$$+ \beta((1-\pi_H)u_T'(W_1^T)(\lambda_1\pi_L + (1-\lambda_1)\pi_H) - \pi_H u_T'(W_2^T)(\lambda_1(1-\pi_L) + (1-\lambda_1)(1-\pi_H))) = 0$$

Solving it for β gives

$$\beta = \frac{u_B'(W_P^T)\lambda_1(\pi_H - \pi_L)}{\pi_H u_T'(W_2^T)(\lambda_1(1-\pi_L) + (1-\lambda_1)(1-\pi_H)) - (1-\pi_H)u_T'(W_1^T)(\lambda_1\pi_L + (1-\lambda_1)\pi_H)} > 0,$$

$$\forall\, W_1^T \ge W_2^T \tag{4.5}$$

Differentiate the Lagrangian w.r.t. t gives

$$\frac{\partial L}{\partial t} = -\lambda_2 u_B'(W_P^T) + \beta(u_T'(W_R^T)(1-\lambda_2) + \pi_H u_T'(W_2^T)\lambda_2 + (1-\pi_H)u_T'(W_1^T)\lambda_2) \tag{4.6}$$

Substitute (4.5) into (4.6) gives

$$\frac{\partial L}{\partial t} = -\lambda_2 u_B'(W_P^T) + \frac{u_B'(W_P^T)\lambda_1(\pi_H - \pi_L)(u_T'(W_R^T)(1-\lambda_2) + \pi_H u_T'(W_2^T)\lambda_2 + (1-\pi_H)u_T'(W_1^T)\lambda_2)}{\pi_H u_T'(W_2^T)(\lambda_1(1-\pi_L) + (1-\lambda_1)(1-\pi_H)) - (1-\pi_H)u_T'(W_1^T)(\lambda_1\pi_L + (1-\lambda_1)\pi_H)}$$

or

$$\frac{\partial L}{\partial t} = \frac{u_B'(W_P^T)(\lambda_1(1-\lambda_2)(\pi_H - \pi_L)u_T'(W_R^T) - \lambda_2\pi_H(1-\pi_H)(u_T'(W_2^T) - u_T'(W_1^T)))}{\pi_H u_T'(W_2^T)(\lambda_1(1-\pi_L) + (1-\lambda_1)(1-\pi_H)) - (1-\pi_H)u_T'(W_1^T)(\lambda_1\pi_L + (1-\lambda_1)\pi_H)} \tag{4.7}$$

Clearly, the sign of (4.7) is dependent on the sign of (4.8)

$$\lambda_1(1-\lambda_2)(\pi_H-\pi_L)u_T'(W_R^T)-\lambda_2\pi_H(1-\pi_H)(u_T'(W_2^T)-u_T'(W_1^T)) \tag{4.8}$$

i.e., $\frac{\partial L}{\partial t}>0$ if (4.9) holds

$$\frac{u_T'(W_R^T)}{u_T'(W_2^T)-u_T'(W_1^T)}>\frac{\lambda_2\pi_H(1-\pi_H)}{\lambda_1(1-\lambda_2)(\pi_H-\pi_L)} \tag{4.9}$$

When $t=0$, we have

$$W_1^0=W-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_P^T$$

$$W_2^0=W-D+(\lambda_1(1-\pi_L)+(1-\lambda_1)(1-\pi_H))I_P^T$$

$$W_R^0=W$$

Since $W_1^0>W-D$, $W_2^0<W$, we have

$$\frac{u_T'(W_R^0)}{u_T'(W_2^0)-u_T'(W_1^0)}<\frac{u_T'(W)}{u_T'(W)-u_T'(W-D)}$$

It holds for a large enough λ_2 that $\frac{u_T'(W)}{u_T'(W)-u_T'(W-D)}<\frac{\lambda_2\pi_H(1-\pi_H)}{\lambda_1(1-\lambda_2)(\pi_H-\pi_L)}$.

Therefore, there always exists a λ_2 large enough to make (4.10) hold and hence it is impossible for the tax and subsidy scheme to achieve a Pareto-improving partial-pooling equilibrium.

$$\frac{u_T'(W_R^T)}{u_T'(W_2^T)-u_T'(W_1^T)}<\frac{\lambda_2\pi_H(1-\pi_H)}{\lambda_1(1-\lambda_2)(\pi_H-\pi_L)} \tag{4.10}$$

In fact, section 4.3.4.3 gives $-\frac{(1-\pi_H)}{\pi_H}<-\frac{(1-\pi_L)}{\pi_L}\frac{u_T'(W-p_P^*)}{u_T'(W-D-p_P^*+I_P^*)}$, i.e.,

$\frac{(1-\pi_H)}{\pi_H}>\frac{(1-\pi_L)}{\pi_L}\frac{u_T'(W_1^0)}{u_T'(W_2^0)}$, from which we have $u_T'(W_2^0)-u_T'(W_1^0)>\frac{(\pi_H-\pi_L)u_T'(W_1^0)}{(1-\pi_H)\pi_L}$.

So

$$\lambda_1(1-\lambda_2)(\pi_H-\pi_L)u_T'(W_R^0)-\lambda_2\pi_H(1-\pi_H)(u_T'(W_2^0)-u_T'(W_1^0))<$$

$$\lambda_1(1-\lambda_2)(\pi_H-\pi_L)u_T'(W_R^0)-\lambda_2\pi_H\frac{(\pi_H-\pi_L)}{\pi_L}u_T'(W_1^0)<$$

$$\lambda_1(1-\lambda_2)(\pi_H-\pi_L)u_T'(W_R^0)-\lambda_2\pi_H\frac{(\pi_H-\pi_L)}{\pi_L}u_T'(W_R^0)=$$

$$\left(\lambda_1(1-\lambda_2)-\lambda_2\frac{\pi_H}{\pi_L}\right)(\pi_H-\pi_L)u_T'(W_R^0)<$$

$$(\lambda_1(1-\lambda_2)-\lambda_2)(\pi_H-\pi_L)u_T'(W_R^0)$$

Clearly, when $\lambda_2 > 0.5$, $(\lambda_1(1-\lambda_2)-\lambda_2)(\pi_H-\pi_L)u_T'(W_R^0)<0$. So when $\lambda_2 > 0.5$, i.e., when the H_T are more than half of the population,

$\lambda_1(1-\lambda_2)(\pi_H-\pi_L)u_T'(W_R^0)-\lambda_2\pi_H(1-\pi_H)(u_T'(W_2^0)-u_T'(W_1^0))<0$ and there is no Pareto-improving partial-pooling allocation.

4.4.2 From partial-pooling equilibrium to separating equlibrium

There is another possibility. It could be that the L_T and the H_B get worse off when the tax is imposed. As the tax increases, the pooling offer curve shifts further down. It eventually lies below the indifference curve of the L_T that passes through the subsidized separating contract. Therefore, the pooling equilibrium will be dominated by the separating equilibrium. As the indifference curves of reservation utility of the H_B and the H_T after subsidy moves to different positions, there might be four different separating equilibria that lead to Pareto improvement.

When the H_B stop driving, only the L_T drive and buy the insurance, so the subsidy s is now $s=\dfrac{N_{LT}}{N_{LT}+N_{HB}+N_{HT}}t$ and the net payout of motorists is then

$t-s=\dfrac{N_{HB}+N_{HT}}{N_{LT}+N_{HB}+N_{HT}}t$. Set $\lambda_4=\dfrac{N_{LT}}{N_{LT}+N_{HB}+N_{HT}}$,[29] then

$1-\lambda_4=\dfrac{N_{HB}+N_{HT}}{N_{LT}+N_{HB}+N_{HT}}$, so $s=\lambda_4 t$, $t-s=(1-\lambda_4)t$.[30]

4.4.2.1 From partial-pooling to type 1 separating equilibrium

Let's first consider the case where the indifference curve of the H_T goes up at the same pace as or faster than that of the H_B. When the reservation utility of the H_B after subsidy reaches the same level as in the partial-pooling equilibrium before the taxation, the high-risks are no worse-off. It could be that the L_T are better off. Figure

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[29] Again, from the definitions of λ_1 and λ_3, we have $N_{HB}=\dfrac{(1-\lambda_1)(1-\lambda_3)}{\lambda_1\lambda_3}N_{HT}$ and $N_{LT}=\dfrac{1-\lambda_3}{\lambda_3}N_{HT}$, which give $\lambda_4=\dfrac{N_{LT}}{N_{LT}+N_{HB}+N_{HT}}=\dfrac{\lambda_1(1-\lambda_3)}{\lambda_1(1-\lambda_3)+(1-\lambda_1)(1-\lambda_3)+\lambda_1\lambda_3}$ accordingly.

[30] Recall that we have set $\lambda_1=\dfrac{N_{LT}}{N_{LT}+N_{HB}}$, $1-\lambda_1=\dfrac{N_{HB}}{N_{LT}+N_{HB}}$, $\lambda_2=\dfrac{N_{HT}}{N_{LT}+N_{HB}+N_{HT}}$, $1-\lambda_2=\dfrac{N_{LT}+N_{HB}}{N_{LT}+N_{HB}+N_{HT}}$.

4.11 illustrates such a case where the indifference curve of the L_T passing through the subsidized separating contract Z^T crosses the indifference curve U_{HB}^* twice. The tax and subsidy scheme breaks the laissez faire partial-pooling equilibrium Z_P^* into type 1 separating equilibrium Z_1^T. The L_T and the H_T are better off and the H_B are no worse off. Insurers earn normal profit.

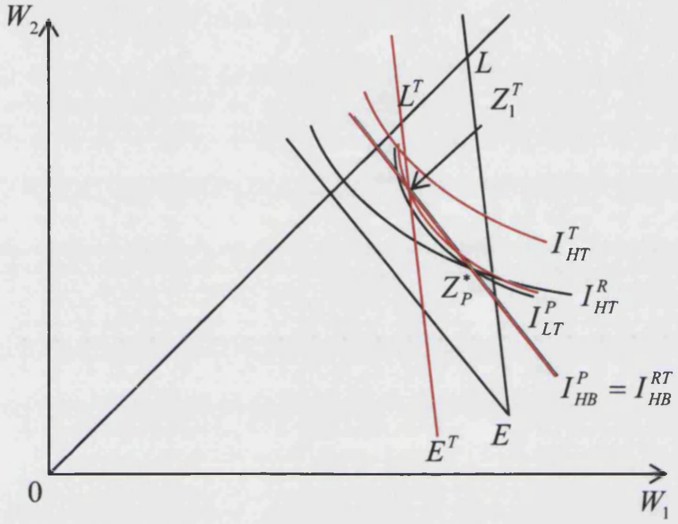

Figure 4.11

The possibility can be formalized as the following maximization problem

$$\max_{I_1^T, p_1^T} U_{LT}^{1T} = B + \pi_L u_T\left(W - D - p_1^T + I_1^T - (1-\lambda_4)t\right) + (1-\pi_L)u_T\left(W - p_1^T - (1-\lambda_4)t\right)$$

s.t. $p_1^T = \pi_L I_1^T$

$$B + u_B\left(W - \pi_H D + (\pi_H - \pi_L)I_1^T - (1-\lambda_4)t\right) = u_B(W + \lambda_4 t)$$

$$p_P = (\lambda_1 \pi_L + (1-\lambda_1)\pi_H)I_P$$

$$B + \pi_H u_T(W - D - p_P + I_P) + (1-\pi_H)u_T(W - p_P) = u_T(W)$$

$$u_B(W + \lambda_4 t) \geq B + u_B(W - \pi_H D + \pi_H I_P - p_P)$$

$$u_T(W + \lambda_4 t) \geq B + \pi_H u_T\left(W - D - p_1^T + I_1^T - (1-\lambda_4)t\right) + (1-\pi_H)u_T\left(W - p_1^T - (1-\lambda_4)t\right)$$

Set $\{I_1^{T*}, p_1^{T*}\}$ as the solution to the above problem. If

$$-\frac{(1-\pi_H)}{\pi_H} > -\frac{(1-\pi_L)}{\pi_L}\frac{u_T'\left(W - p_1^{T*} - (1-\lambda_4)t\right)}{u_T'\left(W - D - p_1^{T*} + I_1^{T*} - (1-\lambda_4)t\right)},$$ the equilibrium is then a

separating equilibrium in which both the high risks do not drive while the L_T drive, buy insurance at their fair premium and pay the tax. If

$$U_{LT}^{1T*} = B + \pi_L u_T\left(W - D - p_1^{T*} + I_1^{T*} - (1-\lambda_4)t\right) + (1-\pi_L)u_T\left(W - p_1^{T*} - (1-\lambda_4)t\right) \geq$$
$$U_{LT}^{P} = B + \pi_L u_T(W - D - p_P + I_P) + (1-\pi_L)u_T(W - p_P)$$

then there is a Pareto improvement.

4.4.2.2 From partial-pooling to type 2 separating equilibrium

Figure 4.12 illustrates another case. The tax and subsidy scheme breaks the laissez faire partial-pooling equilibrium Z_P^* into type 2 separating equilibrium Z_2^T where the indifference curve of the L_T passing through the subsidized separating contract Z_2^T is tangent to the indifference curve U_{HB}^*, i.e.
$-\dfrac{(1-\pi_H)}{\pi_H} < -\dfrac{(1-\pi_L)}{\pi_L}\dfrac{u_T'\left(W - p_1^{T*} - (1-\lambda_4)t\right)}{u_T'\left(W - D - p_1^{T*} + I_1^{T*} - (1-\lambda_4)t\right)}$. The L_T and the H_T are better off and the H_B are no worse off. Insurers earn above normal profit.

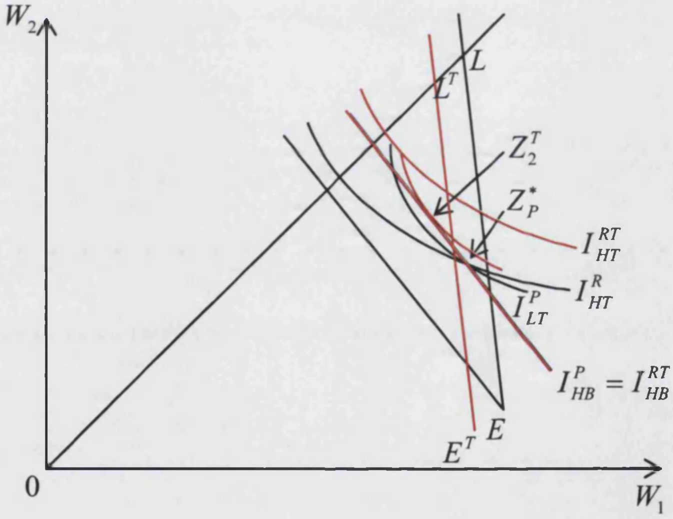

Figure 4.12

The possibility can be defined as the following maximization problem

$$\max_{I_2^T, p_2^T} U_{LT}^{2T} = B + \pi_L u_T\left(W - D - p_2^T + I_2^T - (1-\lambda_4)t\right) + (1-\pi_L)u_T\left(W - p_2^T - (1-\lambda_4)t\right)$$

s.t. $$-\frac{(1-\pi_H)}{\pi_H} = -\frac{(1-\pi_L)}{\pi_L}\frac{u_T'\left(W - p_2^T - (1-\lambda_4)t\right)}{u_T'\left(W - D - p_2^T + I_2^T - (1-\lambda_4)t\right)}$$

$$B + u_B\left(W - \pi_H D + \pi_H I_2^T - p_2^T - (1-\lambda_4)t\right) = u_B(W + \lambda_4 t)$$

$$p_P = (\lambda_1\pi_L + (1-\lambda_1)\pi_H)I_P$$

$$B + \pi_H u_T(W - D - p_P + I_P) + (1-\pi_H)u_T(W - p_P) = u_T(W)$$

$$u_B(W + \lambda_4 t) \geq B + u_B(W - \pi_H D + \pi_H I_P - p_P)$$

$$u_T(W+\lambda_4 t) \geq B+\pi_H u_T\left(W-D-p_2^T+I_2^T-(1-\lambda_4)t\right)+(1-\pi_H)u_T\left(W-p_2^T-(1-\lambda_4)t\right)$$

Again, if

$$U_{LT}^{2T*}=B+\pi_L u_T\left(W-D-p_2^{T*}+I_2^{T*}-(1-\lambda_4)t\right)+(1-\pi_L)u_T\left(W-p_2^{T*}-(1-\lambda_4)t\right) \geq$$

$$U_{LT}^P=B+\pi_L u_T(W-D-p_P+I_P)+(1-\pi_L)u_T(W-p_P)$$

then there is a Pareto improvement.

4.4.2.3 From partial-pooling to type 3 separating equilibrium

When the indifference curve of the H_T goes up slower than that of the H_B, the partial-pooling equilibrium may break into type 3 or type 4 separating equilibrium. Figure 4.13 illustrates a case where the laissez faire partial-pooling equilibrium Z_P^* breaks into type 3 separating equilibrium Z_3^T.

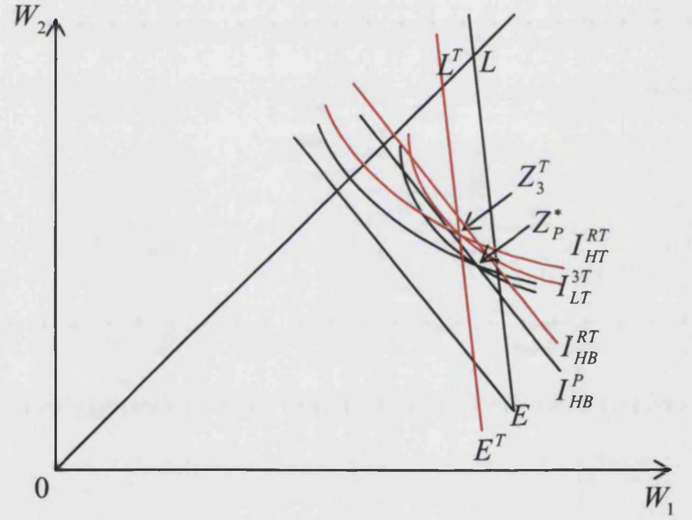

Figure 4.13

The possibility can be defined as the following maximization problem

$$\underset{I_3^T, p_3^T}{Max}\ U_{LT}^{3T}=B+\pi_L u_T\left(W-D-p_3^T+I_3^T-(1-\lambda_4)t\right)+(1-\pi_L)u_T\left(W-p_3^T-(1-\lambda_4)t\right)$$

s.t. $p_3^T=\pi_L I_3^T$

$$B+\pi_H u_T\left(W-D-p_3^T+I_3^T-(1-\lambda_4)t\right)+(1-\pi_H)u_T\left(W-p_3^T-(1-\lambda_4)t\right)=u_T(W+\lambda_4 t)$$

$$p_P=(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_P$$

$$B+\pi_H u_T(W-D-p_P+I_P)+(1-\pi_H)u_T(W-p_P)=u_T(W)$$

$$u_B(W+\lambda_4 t) \geq B+u_B(W-\pi_H D+\pi_H I_P-p_P)$$

$$u_T(W+\lambda_4 t) \leq B+\pi_H u_T\left(W-D-p_1^T+I_1^T-(1-\lambda_4)t\right)+(1-\pi_H)u_T\left(W-p_1^T-(1-\lambda_4)t\right)$$

If $$U_{LT}^{3T*} = B + \pi_L u_T\left(W - D - p_3^{T*} + I_3^{T*} - (1-\lambda_4)t\right) + (1-\pi_L)u_T\left(W - p_3^{T*} - (1-\lambda_4)t\right) \geq U_{LT}^{P} = B + \pi_L u_T(W - D - p_P + I_P) + (1-\pi_L)u_T(W - p_P),$$

there is then a Pareto improvement.

4.4.2.4 From partial-pooling to type 4 separating equilibrium

The tax and subsidy scheme may also lead to type 4 separating equilibrium. Figure 4.14 illustrates such a case where the laissez faire partial-pooling equilibrium Z_P^* breaks to type 4 separating equilibrium Z_4.

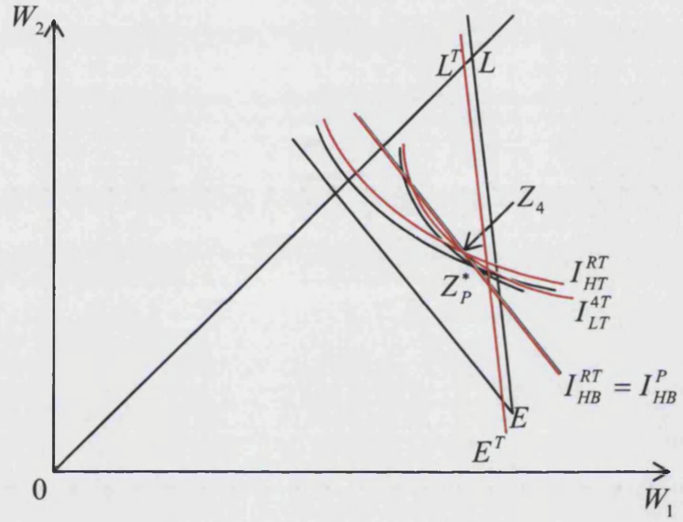

Figure 4.14

The possibility can be defined as the following maximization problem

$$\underset{I_4^T, p_4^T}{Max}\, U_{LT}^{4T} = B + \pi_L u_T\left(W - D - p_4^T + I_4^T - (1-\lambda_4)t\right) + (1-\pi_L)u_T\left(W - p_4^T - (1-\lambda_4)t\right)$$

s.t.

$$B + \pi_H u_T\left(W - D - p_4^T + I_4^T - (1-\lambda_4)t\right) + (1-\pi_H)u_T\left(W - p_4^T - (1-\lambda_4)t\right) = u_T(W + \lambda_4 t)$$

$$B + \pi_H u_B\left(W - D - p_4^T + I_4^T - (1-\lambda_4)t\right) + (1-\pi_H)u_B\left(W - p_4^T - (1-\lambda_4)t\right) = u_B(W + \lambda_4 t)$$

i.e., $B + u_B\left(W - \pi_H D + \pi_H I_4^T - p_4^T - (1-\lambda_4)t\right) = u_B(W + \lambda_4 t)$

$$p_P = (\lambda_1 \pi_L + (1-\lambda_1)\pi_H)I_P$$

$$B + \pi_H u_T(W - D - p_P + I_P) + (1-\pi_H)u_T(W - p_P) = u_T(W)$$

$$u_B(W + \lambda_4 t) \geq B + u_B(W - \pi_H D + \pi_H I_P - p_P)$$

$$-\frac{(1-\pi_H)}{\pi_H} < -\frac{(1-\pi_L)}{\pi_L}\frac{u_T'\left(W - p_4^T - (1-\lambda_4)t\right)}{u_T'\left(W - D - p_4^T + I_4^T - (1-\lambda_4)t\right)}$$

$$B+\pi_H u_T(W-D-p_1+I_1-(1-\lambda_4)t)+(1-\pi_H)u_T(W-p_1-(1-\lambda_4)t)<u_T(W+\lambda_4 t)$$
$$<B+\pi_H u_T(W-D-p_2+I_2-(1-\lambda_4)t)+(1-\pi_H)u_T(W-p_2-(1-\lambda_4)t)$$

Again, there will exist a Pareto improvement if

$$U_{LT}^{4T}=B+\pi_L u_T(W-D-p_4^T+I_4^T-(1-\lambda_4)t)+(1-\pi_L)u_T(W-p_4^T-(1-\lambda_4)t)\geq$$
$$U_{LT}^P=B+\pi_L u_T(W-D-p_P+I_P)+(1-\pi_L)u_T(W-p_P)$$

4.4.2.5 Conditions for efficient partial-pooling equilibrium

In all the above four possibilities, it will involve the H_B opting out of driving and receiving the subsidized reservation utility. However, since $\lambda_2=\frac{N_{HT}}{N_{LT}+N_{HB}+N_{HT}}$ and $\lambda_4=\frac{N_{LT}}{N_{LT}+N_{HB}+N_{HT}}$, for a given N_{LT} and N_{HB}, when $N_{HT}\to\infty$, $\lambda_2\to 1$, and $\lambda_4\to 0$. Hence $u_B(W+\lambda_4 t)\to u_B(W)$ as $t\leq W$. As shown in section 4.3.3, we have $U_{HB}^P=B+u_B(W-\pi_H D+\pi_H I_P^*-p_P^*)>u_B(W)$, so there exists a N_{HT} large enough to make $u_B(W+\lambda_4 t)<B+u_B(W-\pi_H D+\pi_H I_P-p_P)$.

Therefore, under certain conditions, it is impossible to achieve Pareto improvement, and the market equilibrium is hence Pareto efficient.

4.4.3 Efficient partial-pooling Nash equilibrium: an example

One straightforward way to see an efficient partial-pooling Nash equilibrium exists is through numerical examples. Suppose $u_T(W)=-e^{-\theta W}$, $u_B(W)=W$, $\pi_H=0.5$, $\pi_L=0.2$, $\theta=20$, $W=0.2$, $B=0.09$, $D=0.19$, $\lambda_1=0.9$, $\lambda_3=0.5$, $\lambda_2=\frac{\lambda_1\lambda_3}{\lambda_1(1-\lambda_3)+(1-\lambda_1)(1-\lambda_3)+\lambda_1\lambda_3}=0.47368$, $\lambda_4=\frac{\lambda_1(1-\lambda_3)}{\lambda_1(1-\lambda_3)+(1-\lambda_1)(1-\lambda_3)+\lambda_1\lambda_3}=0.47368$,[31]

$\left(\lambda_1=\frac{N_{LT}}{N_{LT}+N_{HB}},1-\lambda_1=\frac{N_{HB}}{N_{LT}+N_{HB}},\lambda_2=\frac{N_{HT}}{N_{LT}+N_{HB}+N_{HT}},\lambda_3=\frac{N_{HT}}{N_{LT}+N_{HT}},1-\lambda_3=\frac{N_{LT}}{N_{LT}+N_{HT}}\right)$.

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[31] From the definitions of λ_1 and λ_3, we can easily find that $N_{HB}=\frac{(1-\lambda_1)(1-\lambda_3)}{\lambda_1\lambda_3}N_{HT}$ and $N_{LT}=\frac{1-\lambda_3}{\lambda_3}N_{HT}$, which give $\lambda_2=\frac{N_{HT}}{N_{LT}+N_{HB}+N_{HT}}=\frac{\lambda_1\lambda_3}{\lambda_1(1-\lambda_3)+(1-\lambda_1)(1-\lambda_3)+\lambda_1\lambda_3}$ and $\lambda_4=\frac{N_{LT}}{N_{LT}+N_{HB}+N_{HT}}=\frac{\lambda_1(1-\lambda_3)}{\lambda_1(1-\lambda_3)+(1-\lambda_1)(1-\lambda_3)+\lambda_1\lambda_3}$ accordingly.

For these parameter values, a partial-pooling allocation constitutes Nash equilibrium. Since $\frac{u_T'(W_R^T)}{u_T'(W_2^T)-u_T'(W_1^T)}=0.17178<\frac{\lambda_2\pi_H(1-\pi_H)}{\lambda_1(1-\lambda_2)(\pi_H-\pi_L)}=0.83333$, $\frac{\partial L}{\partial t}<0$ and there is no Pareto-improving partial-pooling contract available.

The tax-subsidy scheme breaks the partial-pooling equilibrium to type 3 separating equilibrium. The subsidy raises the reservation utility of the H_B to the same level as when they drive and buy insurance ($W+\lambda_4 t=B+(W-\pi_H D+\pi_H I_P-P_P)=0.22078$). The reservation utility of the H_T increases from -0.018316 to -0.012088, but the utility of the L_T decreases from 0.029625 to 0.019481. Clearly, there is no way to make every one better off by breaking the partial-pooling equilibrium into separating equilibrium.[32]

Therefore, the partial-pooling Nash equilibrium is efficient.

4.5 Inefficient market equilibrium

Section 4.4 has demonstrated that the partial-pooling Nash equilibrium can be Pareto efficient under certain conditions, but that does not mean inefficient partial-pooling Nash equilibrium does not exist.

4.5.1 The existence of inefficient market equilibrium

If the parameter values are such that $\frac{u_T'(W_R^T)}{u_T'(W_2^T)-u_T'(W_1^T)}>\frac{\lambda_2\pi_H(1-\pi_H)}{\lambda_1(1-\lambda_2)(\pi_H-\pi_L)}$, there then exist Pareto-improving partial-pooling allocations. Or if one of the four possible separating equilibria exists, there is then room for the tax-subsidy scheme to achieve Pareto improvement.

For instance, the market equilibrium is inefficient if there exists a solution to the following problem

$$\underset{I_2^T,p_2^T}{Max}\,U_{LT}^{2T}=B+\pi_L u_T\left(W-D-p_2^T+I_2^T-(1-\lambda_4)t\right)+(1-\pi_L)u_T\left(W-p_2^T-(1-\lambda_4)t\right)$$

$$\text{s.t.}\quad -\frac{(1-\pi_H)}{\pi_H}=-\frac{(1-\pi_L)}{\pi_L}\frac{u_T'\left(W-p_2^T-(1-\lambda_4)t\right)}{u_T'\left(W-D-p_2^T+I_2^T-(1-\lambda_4)t\right)}$$

$$B+u_B\left(W-\pi_H D+\pi_H I_2^T-p_2^T-(1-\lambda_4)t\right)=u_B(W+\lambda_4 t)$$

[32] Appendix 4.1 provides detailed simulation results.

$$p_P = (\lambda_1\pi_L + (1-\lambda_1)\pi_H)I_P$$
$$B + \pi_H u_T(W - D - p_P + I_P) + (1-\pi_H)u_T(W - p_P) = u_T(W)$$
$$u_B(W + \lambda_4 t) \geq B + u_B(W - \pi_H D + \pi_H I_P - p_P)$$
$$u_T(W + \lambda_4 t) \geq B + \pi_H u_T\left(W - D - p_2^T + I_2^T - (1-\lambda_4)t\right) + (1-\pi_H)u_T\left(W - p_2^T - (1-\lambda_4)t\right)$$
$$U_{LT}^{2T} = B + \pi_L u_T\left(W - D - p_2^T + I_2^T - (1-\lambda_4)t\right) + (1-\pi_L)u_T\left(W - p_2^T - (1-\lambda_4)t\right) \geq$$
$$U_{LT}^{P} = B + \pi_L u_T(W - D - p_P + I_P) + (1-\pi_L)u_T(W - p_P)$$

If there exists a solution to this problem, the tax-subsidy scheme can then break the partial-pooling equilibrium into to type 2 separating equilibrium, which leads to Pareto improvement.

One straightforward way to demonstrate that such a solution exists is, again, through a numerical example. Consider a case where $u_T(W) = -e^{-\theta W}$, $u_B(W) = W$, $\pi_H = 0.4$, $\pi_L = 0.1$, $\theta = 30$, $W = 0.15$, $B = 0.04$, $D = 0.14$, $\lambda_1 = 0.75$, $\lambda_3 = 0.4$, $\lambda_2 = 0.33333$, $\lambda_4 = 0.5$. The *laissez-faire* market equilibrium with these parameter values is a partial-pooling equilibrium.

However, if we impose a tax of $t = 3.976171992 \times 10^{-3}$ and use the proceeds to subsidize the whole population, we can break the *ex ante* partial-pooling equilibrium to an *ex post* type 2 separating equilibrium.

The *ex ante* utility of the H_T, the H_B, and the L_T is $-1.110899654 \times 10^{-2}$, 0.1519880860, and $1.454567626 \times 10^{-2}$ respectively. After the tax-subsidy scheme, their *ex post* utility is $-1.046579896 \times 10^{-2}$, 0.1519880860, and $1.454597154 \times 10^{-2}$ respectively.[33] The H_B is not worse off, but the H_T and the L_T are better off due to this scheme.

Moreover, if $u_T(W) = -e^{-\theta W}$, $u_B(W) = W$, $\pi_H = 0.4$, $\pi_L = 0.1$, $\theta = 30$, $W = 0.15$, $B = 0.04$, $D = 0.14$, $\lambda_1 = 0.75$, $\lambda_3 = 0.733$, a tax of $t = 8.108710298 \times 10^{-3}$ breaks the partial-pooling equilibrium to a type 1 separating equilibrium. The utility of the H_T, the H_B, and the L_T goes from $-1.110899654 \times 10^{-2}$, 0.1519880860, and $1.454567626 \times 10^{-2}$ to $-1.046579896 \times 10^{-2}$, 0.1519880860 and $1.454595041 \times 10^{-2}$ respectively.

[33] Appendix 4.3 provides detailed simulation results.

If $u_T(W) = -e^{-\theta W}$, $u_B(W) = W$, $\pi_H = 0.4$, $\pi_L = 0.1$, $\theta = 30$, $W = 0.15$, $B = 0.04$, $D = 0.14$, $\lambda_1 = 0.75$, $\lambda_3 = 0.4$, a tax of $t = 0.008$ will break the partial-pooling equilibrium to a type 3 separating equilibrium. The utility of the H_T, the H_B, and the L_T goes from $-1.110899654\times10^{-2}$, 0.1519880860, and 1.454567626×10^{-2} to $-9.852796061\times10^{-3}$, 0.154 and 1.565564196×10^{-2} respectively.

If $u_T(W) = -e^{-\theta W}$, $u_B(W) = W$, $\pi_H = 0.4$, $\pi_L = 0.1$, $\theta = 30$, $W = 0.15$, $B = 0.04$, $D = 0.14$, $\lambda_1 = 0.9$, $\lambda_3 = 0.35$, a tax of $t = 6.8703\times10^{-3}$ breaks the partial-pooling equilibrium to a type 4 separating equilibrium. The utility of the H_T, the H_B, and the L_T goes from -1.1109×10^{-2}, 0.15416, and 1.6074×10^{-2} to -9.8042×10^{-3}, 0.15416 and 1.6123×10^{-2} respectively.[34]

The above numerical simulations have clearly demonstrated that the inefficient partial-pooling equilibrium does exist and a tax-subsidy scheme can break it to one of the four possible separating equilibria.

4.5.2 Policy implication

It is widely known that the presence of the high-risks exerts negative externality to the low-risks: their insurance coverage has to be restricted to prevent the high-risks from taking it. When there exist bold high-risks in addition to timid high-risks and potential pooling emerges, the situation becomes even worse for the timid low-risks: in addition to the restriction on insurance coverage, they have to pay more than their actuarially fair premium to get the insurance, because the bold high-risks are now taking the insurance as well.

As having been shown in section 4.5.1, a tax-subsidy scheme can eliminate the externality caused by the partial-pooling of the H_B and the L_T, when the market equilibrium is not Pareto efficient and can break to separating equilibrium. As motorists pay the tax and non-motorists receive the subsidy, the reservation utility of the H_B will exceed their utility from driving after some critical value of tax and subsidy, and hence the H_B will stop driving. With the H_B choose not to drive, the premium rate (premium/indemnity) the L_T pay for insurance could be much cheaper, sometimes even with the tax they have paid. For example, in the case when the

[34] Appendix 4.2, 4.4, 4.5 provide the detailed results of these numerical simulations.

partial-pooling equilibrium can break to a type 3 separating equilibrium, the *ex ante* premium rate is $\frac{P_P}{I_P}=0.1750000001$, but the *ex post* premium rate is only $\frac{(1-\lambda_4)t+P_6}{I_6}=\frac{(1-\lambda_4)t+7.829355446\times10^{-3}}{7.829355446\times10^{-2}}=0.1510897740$ even with the tax included. Clearly, the tax-subsidy scheme can achieve Pareto gain in such cases.

An interesting case happens when insurers earn positive profit in the equilibrium after the tax-subsidy scheme. For insurance, in the case that the scheme breaks the partial-pooling equilibrium to a type 2 separating equilibrium, insurers earn positive profit because the L_T are paying more than their actuarially fair premium for the insurance. If we increase the tax from $t=3.9756\times10^{-3}$ to $t=0.0045$, the utility of the H_T, the H_B and the L_T increases to $-e^{-\theta(W+\lambda_4 t)}=-1.038388701\times10^{-2}$, $W+\lambda_4 t=0.15225$, and $B+\pi_L\left(-e^{-\theta(W-D-P_6+I_6-(1-\lambda_4)t)}\right)+\left(1-\pi_L\right)\left(-e^{-\theta(W-P_6-(1-\lambda_4)t)}\right)=1.474519084\times10^{-2}$ respectively.[35] All consumers are better off except the insurers: the premium they can charge is now closer to the fair one. So this move is clearly not Pareto improvement. Whether government should increase the tax from $t=3.9756\times10^{-3}$ to $t=0.0045$ is dependent on the weight of the welfare being they put on the insurers.

4.6 Conclusion

When agents differ in risk preferences and risk probabilities and have an option whether to take the risky activity and whether to buy insurance, the timid high-risks may choose not to drive, even though under full information driving brings utility benefit. With different levels of risk probabilities, risk preferences, and driving benefit, the market may end up with four different separating equilibria, partial-pooling equilibrium, or even no pure-strategy equilibrium.

The partial-pooling equilibrium is Pareto efficient under certain conditions, notably, when the timid high-risks account for a large proportion of the population. When the partial-pooling equilibrium is inefficient, taxing insurance may create a separating equilibrium. The bold high-risks are driven out of the insurance market and stop driving. Everyone may be better off.

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[35] Appendix 4.6 provides detailed simulation results.

Chapter 5 Conclusion

Chapter 2 examines the welfare effects of a non-discrimination policy which requires unisex premium for insurance. It shows that in comprehensive insurance market, when everybody drives and decides whether to purchase insurance, typical adverse selection happens. High-risk motorists will purchase the insurance while the low-risks will not. After the unisex premium policy is implemented, more male motorists purchase insurance with lower premium while some female motorists drop out of the insurance market with the premium increased. Simulations show the total number of the insured is less than that before the policy is implemented. The policy has no effect on insurers who always earn normal profit in competitive industry. The aggregate social welfare may increase or decrease. There is no effect on road safety since everybody drives whether insured or not.

In third-party insurance market where motorists must be insured and people choose whether to become a motorist, advantageous selection happens. Low-risk people become motorists and purchase insurance while the high-risks will not. After the policy is implemented, more relatively safe men become motorist while some relatively risky women stop driving. Roads are safer. From the simulations, the total number of motorists is greater than that before the policy. The policy has no effect on insurers. Simulations show that the aggregate social welfare decreases after the policy is implemented.

Chapter 3 extends the research and finds the optimal allocation which maximizes aggregate social welfare. Agents are identical except in risk probabilities and they can choose whether to participate in risky activity and whether to buy insurance. Due to their high probability of incurring loss, which makes the reservation utility more attractive, the high-risks now choose not to engage in the risky activity. Instead of being stuck with the low-risks, the market is now filled with the good-risks – the "lemon" market has turned into "peach" market. Moreover, when the redistribution effect dominates the efficiency effect, it would increase the total social welfare by redistributing the wealth from the high-risks to the low-risks and hence decreasing the insurance coverage in the market equilibrium. For certain parameter values, it would increase the total social welfare by reducing insurance – instead of a shrunk market under optimal level, we have got too much insurance.

Another interesting finding is that full insurance pooling allocation never maximizes the aggregate social welfare. Although the high-risks enjoy higher reservation utility in the allocation, there is no efficiency gain at all for the whole society and there is only redistribution effect which is always negative.

Chapter 4 examines the market equilibrium and market efficiency in competitive insurance markets when agents differ in both risk probabilities and risk preferences, and can choose whether to participate in risky activity and whether to buy insurance. It is shown that the timid high-risks will stop driving for reservation utility, even though driving brings utility benefit. With different levels of risk probabilities, risk preferences, and driving benefit, the market may end up with four different separating equilibria, partial-pooling equilibrium, or even no equilibrium.

The partial-pooling equilibrium is Pareto efficient under certain conditions. Particularly, when the timid high-risks account for a large proportion of the population, it is impossible to achieve Pareto improvement. When the partial-pooling equilibrium is inefficient, taxing insurance breaks the equilibrium and separating equilibrium arises. The bold high-risks are driven out of the insurance market and stop driving. Everyone could be better off.

Appendix

Appendix 2.1 Welfare change with loss level

$$D=(V_p-V_M)(2\pi_6-\pi_p-\pi_5)-(V_W-V_p)(2\pi_3-\pi_2-\pi_p)$$
$$V_W=\frac{(Y-P_WL)^{1-\theta}}{1-\theta}$$
$$P_W=\tfrac{1}{2}(\pi_3+\pi_2)$$
$$\pi_2=\frac{Y^{1-\theta}-(Y-P_WL)^{1-\theta}}{Y^{1-\theta}-(Y-L)^{1-\theta}}$$
$$V_p=\frac{(Y-P_pL)^{1-\theta}}{1-\theta}$$
$$(\pi_6-\pi_p)[P_p-\tfrac{1}{2}(\pi_6+\pi_p)]+(\pi_3-\pi_p)[P_p-\tfrac{1}{2}(\pi_3+\pi_p)]=0$$
$$\pi_p=\frac{Y^{1-\theta}-(Y-P_pL)^{1-\theta}}{Y^{1-\theta}-(Y-L)^{1-\theta}}$$
$$V_M=\frac{(Y-P_ML)^{1-\theta}}{1-\theta}$$
$$P_M=\tfrac{1}{2}(\pi_6+\pi_5)$$
$$\pi_5=\frac{Y^{1-\theta}-(Y-P_ML)^{1-\theta}}{Y^{1-\theta}-(Y-L)^{1-\theta}}$$
$$\pi_3=0.7$$
$$\pi_6=0.8$$
$$Y=100$$
$$\theta=0.8$$
$$L=85$$

$$\left\{\begin{bmatrix}D=-1.3111\times10^{-2},V_W=11.639,P_W=0.63311,\pi_2=0.56622,\\ V_p=11.495,P_p=0.71534,\pi_p=0.6545,\\ V_M=11.447,P_M=0.74211,\pi_5=0.68422,\\ \pi_3=0.7,\pi_6=0.8,Y=100.0,L=50.0,\theta=0.8\end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix}D=-1.2618\times10^{-2},V_W=11.55,P_W=0.62225,\pi_2=0.54449,\\ V_p=11.401,P_p=0.69743,\pi_p=0.62487,\\ V_M=11.332,P_M=0.73119,\pi_5=0.66239,\\ \pi_3=0.7,\pi_6=0.8,Y=100.0,L=55.0,\theta=0.8\end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix}D=-1.2176\times10^{-2},V_W=11.465,P_W=0.61024,\pi_2=0.52048,\\ V_p=11.308,P_p=0.68067,\pi_p=0.59519,\\ V_M=11.219,P_M=0.71861,\pi_5=0.63722,\\ \pi_3=0.7,\pi_6=0.8,Y=100.0,L=60.0,\theta=0.8\end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix}D=-1.1288\times10^{-2},V_W=11.385,P_W=0.59697,\pi_2=0.49394,\\ V_p=11.219,P_p=0.66349,\pi_p=0.56357,\\ V_M=11.113,P_M=0.70409,\pi_5=0.60817,\\ \pi_3=0.7,\pi_6=0.8,Y=100.0,L=65.0,\theta=0.8\end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} D=-9.7253\times10^{-3}, V_W=11.311, P_W=0.58229, \pi_2=0.46459, \\ V_p=11.137, P_p=0.64522, \pi_p=0.52912, \\ V_M=11.015, P_M=0.68733, \pi_5=0.57466, \\ \pi_3=0.7, \pi_6=0.8, Y=100.0, L=70.0, \theta=0.8 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} D=-7.4215\times10^{-3}-2.5917\times10^{-20}i, V_W=11.245, P_W=0.56605, \pi_2=0.43209, \\ V_p=11.066, P_p=0.62542+1.3598\times10^{-20}i, \pi_p=0.49117+1.5664\times10^{-20}i, \\ V_M=10.929, P_M=0.66804, \pi_5=0.53609, \\ \pi_3=0.7, \pi_6=0.8, Y=100.0, L=75.0, \theta=0.8 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} D=-4.486\times10^{-3}, V_W=11.191, P_W=0.54799, \pi_2=0.39598, \\ V_p=11.007, P_p=0.60368, \pi_p=0.44906, \\ V_M=10.860, P_M=0.64589, \pi_5=0.49178, \\ \pi_3=0.7, \pi_6=0.8, Y=100.0, L=80.0, \theta=0.8 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} D=-1.2131\times10^{-3}-4.01\times10^{-20}i, V_W=11.150, P_W=0.52773, \pi_2=0.35546, \\ V_p=10.966, P_p=0.57952+1.2692\times10^{-20}i, \pi_p=0.40186+1.4154\times10^{-20}i, \\ V_M=10.811, P_M=0.62042, \pi_5=0.44083, \\ \pi_3=0.7, \pi_6=0.8, Y=100.0, L=85.0, \theta=0.8 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} D=1.9642\times10^{-3}, V_W=11.128, P_W=0.50444, \pi_2=0.30888, \\ V_p=10.948, P_p=0.55198, \pi_p=0.34774, \\ V_M=10.792, P_M=0.5907, \pi_5=0.3814, \\ \pi_3=0.7, \pi_6=0.8, Y=100.0, L=90.0, \theta=0.8 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} D=4.5401\times10^{-3}, V_W=11.136, P_W=0.47571, \pi_2=0.25141, \\ V_p=10.966, P_p=0.51837, \pi_p=0.28141, \\ V_M=10.816, P_M=0.55397, \pi_5=0.30795, \\ \pi_3=0.7, \pi_6=0.8, Y=100.0, L=95.0, \theta=0.8 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} D=5.6718\times10^{-3}, V_W=11.203, P_W=0.43973, \pi_2=0.17945, \\ V_p=11.053, P_p=0.47692, \pi_p=0.19931, \\ V_M=10.918, P_M=0.50854, \pi_5=0.21707, \\ \pi_3=0.7, \pi_6=0.8, Y=100.0, L=99.0, \theta=0.8 \end{bmatrix}\right\}$$

Appendix 2.2 Welfare change with risk range 1

$$D=(V_p-V_M)(2\pi_6-\pi_p-\pi_5)-(V_W-V_p)(2\pi_3-\pi_2-\pi_p)$$
$$V_W=\frac{(Y-P_W L)^{1-\theta}}{1-\theta}$$
$$P_W=\tfrac{1}{2}(\pi_3+\pi_2)$$
$$\pi_2=\frac{Y^{1-\theta}-(Y-P_W L)^{1-\theta}}{Y^{1-\theta}-(Y-L)^{1-\theta}}$$
$$V_p=\frac{(Y-P_p L)^{1-\theta}}{1-\theta}$$
$$(\pi_6-\pi_p)[P_p-\tfrac{1}{2}(\pi_6+\pi_p)]+(\pi_3-\pi_p)[P_p-\tfrac{1}{2}(\pi_3+\pi_p)]=0$$
$$\pi_p=\frac{Y^{1-\theta}-(Y-P_p L)^{1-\theta}}{Y^{1-\theta}-(Y-L)^{1-\theta}}$$
$$V_M=\frac{(Y-P_M L)^{1-\theta}}{1-\theta}$$
$$P_M=\tfrac{1}{2}(\pi_6+\pi_5)$$
$$\pi_5=\frac{Y^{1-\theta}-(Y-P_M L)^{1-\theta}}{Y^{1-\theta}-(Y-L)^{1-\theta}}$$
$$\pi_3=0.7$$
$$\pi_6=0.99$$
$$Y=100$$
$$\theta=0.8$$
$$L=90$$

$$\left\{\begin{bmatrix} D=2.9231\times10^{-2}, \\ V_W=11.128, P_W=0.50444, \pi_2=0.30888, \\ V_p=10.522, P_p=0.65257, \pi_p=0.4396, \\ V_M=10.062, P_M=0.74444, \pi_5=0.53887, \\ \pi_3=0.7, \pi_6=0.95, Y=100.0, L=90.0, \theta=0.8 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} D=1.3954\times10^{-2}, \\ V_W=11.128, P_W=0.50444, \pi_2=0.30888, \\ V_p=10.694, P_p=0.6138, \pi_p=0.40245, \\ V_M=10.355, P_M=0.68781, \pi_5=0.47562, \\ \pi_3=0.7, \pi_6=0.9, Y=100.0, L=90.0, \theta=0.8 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} D=5.9598\times10^{-3}, \\ V_W=11.128, P_W=0.50444, \pi_2=0.30888, \\ V_p=10.832, P_p=0.58079, \pi_p=0.3726, \\ V_M=10.591, P_M=0.63738, \pi_5=0.42475, \\ \pi_3=0.7, \pi_6=0.85, Y=100.0, L=90.0, \theta=0.8 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} D=1.9642\times10^{-3}, \\ V_W=11.128, P_W=0.50444, \pi_2=0.30888, \\ V_p=10.948, P_p=0.55198, \pi_p=0.34774, \\ V_M=10.792, P_M=0.5907, \pi_5=0.3814, \\ \pi_3=0.7, \pi_6=0.8, Y=100.0, L=90.0, \theta=0.8 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} D=3.3895\times10^{-4}, \\ V_W=11.128, P_W=0.50444, \pi_2=0.30888, \\ V_p=11.045, P_p=0.52664, \pi_p=0.32671, \\ V_M=10.969, P_M=0.54661, \pi_5=0.34322, \\ \pi_3=0.7, \pi_6=0.75, Y=100.0, L=90.0, \theta=0.8 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} D=8.9378\times10^{-6}, \\ V_W=11.128, P_W=0.50444, \pi_2=0.30888, \\ V_p=11.112, P_p=0.50864, \pi_p=0.31221, \\ V_M=11.097, P_M=0.51274, \pi_5=0.31548, \\ \pi_3=0.7, \pi_6=0.71, Y=100.0, L=90.0, \theta=0.8 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} D=6.6514\times10^{-6}, \\ V_W=11.128, P_W=0.50444, \pi_2=0.30888, \\ V_p=11.143, P_p=0.50036, \pi_p=0.30566, \\ V_M=11.158, P_M=0.49619, \pi_5=0.30238, \\ \pi_3=0.7, \pi_6=0.69, Y=100.0, L=90.0, \theta=0.8 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} D=5.0789\times10^{-5}, \\ V_W=11.128, P_W=0.50444, \pi_2=0.30888, \\ V_p=11.197, P_p=0.48526, \pi_p=0.29389, \\ V_M=11.273, P_M=0.46375, \pi_5=0.2775, \\ \pi_3=0.7, \pi_6=0.65, Y=100.0, L=90.0, \theta=0.8 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} D=-2.2505\times10^{-4}, \\ V_W=11.128, P_W=0.50444, \pi_2=0.30888, \\ V_p=11.244, P_p=0.47215, \pi_p=0.28385, \\ V_M=11.381, P_M=0.43208, \pi_5=0.25416, \\ \pi_3=0.7, \pi_6=0.61, Y=100.0, L=90.0, \theta=0.8 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} D=-3.8262\times10^{-4}, \\ V_W=11.128, P_W=0.50444, \pi_2=0.30888, \\ V_p=11.253, P_p=0.46949, \pi_p=0.28183, \\ V_M=11.405, P_M=0.42505, \pi_5=0.24909, \\ \pi_3=0.7, \pi_6=0.601, Y=100.0, L=90.0, \theta=0.8 \end{bmatrix}\right\}$$

Appendix 2.3 Welfare change with risk range 2

$$D = (V_p - V_M)(2\pi_6 - \pi_p - \pi_5) - (V_W - V_p)(2\pi_3 - \pi_2 - \pi_p)$$
$$V_W = \frac{(Y - P_W L)^{1-\theta}}{1-\theta}$$
$$P_W = \tfrac{1}{2}(\pi_3 + \pi_2)$$
$$\pi_2 = \frac{Y^{1-\theta} - (Y - P_W L)^{1-\theta}}{Y^{1-\theta} - (Y - L)^{1-\theta}}$$
$$V_p = \frac{(Y - P_p L)^{1-\theta}}{1-\theta}$$
$$(\pi_6 - \pi_p)[P_p - \tfrac{1}{2}(\pi_6 + \pi_p)] + (\pi_3 - \pi_p)[P_p - \tfrac{1}{2}(\pi_3 + \pi_p)] = 0$$
$$\pi_p = \frac{Y^{1-\theta} - (Y - P_p L)^{1-\theta}}{Y^{1-\theta} - (Y - L)^{1-\theta}}$$
$$V_M = \frac{(Y - P_M L)^{1-\theta}}{1-\theta}$$
$$P_M = \tfrac{1}{2}(\pi_6 + \pi_5)$$
$$\pi_5 = \frac{Y^{1-\theta} - (Y - P_M L)^{1-\theta}}{Y^{1-\theta} - (Y - L)^{1-\theta}}$$
$$\pi_3 = 0.6$$
$$\pi_6 = 0.501$$
$$Y = 100$$
$$\theta = 0.7$$
$$L = 95$$

$$\left\{\begin{bmatrix} D = -1.7417\times10^{-3}, V_W = 11.395, P_W = 0.41917, \pi_2 = 0.23835, \\ V_p = 11.551, P_p = 0.38975, \pi_p = 0.21849, \\ V_M = 11.745, P_M = 0.35192, \pi_5 = 0.19383, \\ \pi_3 = 0.6, \pi_6 = 0.51, Y = 100.0, L = 95.0, \theta = 0.7 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} D = -4.2208\times10^{-4}, V_W = 11.395, P_W = 0.41917, \pi_2 = 0.23835, \\ V_p = 11.489, P_p = 0.4016, \pi_p = 0.22641, \\ V_M = 11.594, P_M = 0.38152, \pi_5 = 0.21304, \\ \pi_3 = 0.6, \pi_6 = 0.55, Y = 100.0, L = 95.0, \theta = 0.7 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} D = -1.3004\times10^{-5}, V_W = 11.395, P_W = 0.41917, \pi_2 = 0.23835, \\ V_p = 11.415, P_p = 0.41543, \pi_p = 0.23578, \\ V_M = 11.436, P_M = 0.41158, \pi_5 = 0.23316, \\ \pi_3 = 0.6, \pi_6 = 0.59, Y = 100.0, L = 95.0, \theta = 0.7 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} D = -1.1298\times10^{-5}, V_W = 11.395, P_W = 0.41917, \pi_2 = 0.23835, \\ V_p = 11.374, P_p = 0.42303, \pi_p = 0.241, \\ V_M = 11.354, P_M = 0.42680, \pi_5 = 0.24360, \\ \pi_3 = 0.6, \pi_6 = 0.61, Y = 100.0, L = 95.0, \theta = 0.7 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} D=-2.0753\times 10^{-4}, V_W=11.395, P_W=0.41917, \pi_2=0.23835, \\ V_p=11.283, P_p=0.43960, \pi_p=0.25252, \\ V_M=11.183, P_M=0.45766, \pi_5=0.26533, \\ \pi_3=0.6, \pi_6=0.65, Y=100.0, L=95.0, \theta=0.7 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} D=-5.2832\times 10^{-4}, V_W=11.395, P_W=0.41917, \pi_2=0.23835, \\ V_p=11.154, P_p=0.46279, \pi_p=0.26901, \\ V_M=10.955, P_M=0.49716, \pi_5=0.29431, \\ \pi_3=0.6, \pi_6=0.7, Y=100.0, L=95.0, \theta=0.7 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} D=-6.9674\times 10^{-4}, V_W=11.395, P_W=0.41917, \pi_2=0.23835, \\ V_p=11.004, P_p=0.48880, \pi_p=0.28806, \\ V_M=10.707, P_M=0.53788, \pi_5=0.32575, \\ \pi_3=0.6, \pi_6=0.75, Y=100.0, L=95.0, \theta=0.7 \end{bmatrix}\right\}$$

Appendix 2.4 Welfare change with loss level 2

$$D_3 = (\pi_p - \pi_M)(\pi_M + \pi_p - 2\pi_2) - (\pi_W - \pi_p)(\pi_W + \pi_p - 2\pi_1)$$

$$P_W = \tfrac{1}{2}(\pi_1 + \pi_W)$$

$$\pi_W = \tfrac{1}{C}\left[\tfrac{(y-P_W L)^{1-\theta}}{1-\theta} - V\right]$$

$$(\pi_p - \pi_2)\left[P_p - \tfrac{1}{2}(\pi_2 + \pi_p)\right] + (\pi_p - \pi_1)\left[P_p - \tfrac{1}{2}(\pi_1 + \pi_p)\right] = 0$$

$$\pi_p = \tfrac{1}{C}\left[\tfrac{(y-P_p L)^{1-\theta}}{1-\theta} - V\right]$$

$$P_M = \tfrac{1}{2}(\pi_2 + \pi_M)$$

$$\pi_M = \tfrac{1}{C}\left[\tfrac{(y-P_M L)^{1-\theta}}{1-\theta} - V\right]$$

$$R_1 = \tfrac{(y-L)^{1-\theta}}{1-\theta}$$

$$R_2 = V$$

$$R_3 = \tfrac{y^{1-\theta}}{1-\theta}$$

$$R_4 = C + V$$

$$\pi_1 = 0.1$$

$$\pi_2 = 0.2$$

$$y = 100$$

$$\theta = 0.8$$

$$L = 45$$

$$C = 5$$

$$V = 11$$

$$\left\{\left[\begin{array}{c} P_W = 0.18437, \pi_W = 0.26875, P_M = 0.22893, \\ R_4 = 16.0, V = 11.0, \pi_M = 0.25786, R_3 = 12.559, R_2 = 11.0, \\ P_p = 0.19703, y = 100.0, R_1 = 11.144, D_3 = -6.2498\times10^{-5}, \\ \pi_p = 0.26567, \pi_2 = 0.2, \pi_1 = 0.1, \theta = 0.8, L = 45.0, C = 5.0 \end{array}\right]\right\}$$

$$\left\{\left[\begin{array}{c} P_W = 0.16262, \pi_W = 0.22523, P_M = 0.20133, \\ R_4 = 16.0, V = 11.0, \pi_M = 0.20266, R_3 = 12.559, R_2 = 11.0, \\ P_p = 0.16852, y = 100.0, R_1 = 5.0, D_3 = -3.6748\times10^{-4}, \\ \pi_p = 0.22184, \pi_2 = 0.2, \pi_1 = 0.1, \theta = 0.8, L = 99.0, C = 5.0 \end{array}\right]\right\}$$

$$\left\{\left[\begin{array}{c} P_W = 0.16409, \pi_W = 0.22818, P_M = 0.20321, \\ R_4 = 16.0, V = 11.0, \pi_M = 0.20641, R_3 = 12.559, R_2 = 11.0, \\ P_p = 0.17056, y = 100.0, R_1 = 6.8986, D_3 = -3.3191\times10^{-4}, \\ \pi_p = 0.22463, \pi_2 = 0.2, \pi_1 = 0.1, \theta = 0.8, L = 95.0, C = 5.0 \end{array}\right]\right\}$$

$$\left\{\left[\begin{array}{c} P_W = 0.16596, \pi_W = 0.23192, P_M = 0.20559, \\ R_4 = 16.0, V = 11.0, \pi_M = 0.21117, R_3 = 12.559, R_2 = 11.0, \\ P_p = 0.17313, y = 100.0, R_1 = 7.9245, D_3 = -2.9122\times10^{-4}, \\ \pi_p = 0.22822, \pi_2 = 0.2, \pi_1 = 0.1, \theta = 0.8, L = 90.0, C = 5.0 \end{array}\right]\right\}$$

$$\left\{\begin{bmatrix} P_W = 0.16786, \pi_W = 0.23573, P_M = 0.20801, \\ R_4 = 16.0, V = 11.0, \pi_M = 0.21602, R_3 = 12.559, R_2 = 11.0, \\ P_p = 0.1757, y = 100.0, R_1 = 8.5939, D_3 = -2.5433 \times 10^{-4}, \\ \pi_p = 0.23193, \pi_2 = 0.2, \pi_1 = 0.1, \theta = 0.8, L = 85.0, C = 5.0 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} P_W = 0.1698, \pi_W = 0.2396, P_M = 0.21047, \\ R_4 = 16.0, V = 11.0, \pi_M = 0.22095, R_3 = 12.559, R_2 = 11.0, \\ P_p = 0.17830, y = 100.0, R_1 = 9.1028, D_3 = -2.2087 \times 10^{-4}, \\ \pi_p = 0.23575, \pi_2 = 0.2, \pi_1 = 0.1, \theta = 0.8, L = 80.0, C = 5.0 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} P_W = 0.17177, \pi_W = 0.24355, P_M = 0.21298, \\ R_4 = 16.0, V = 11.0, \pi_M = 0.22596, R_3 = 12.559, R_2 = 11.0, \\ P_p = 0.18091, y = 100.0, R_1 = 9.5183, D_3 = -1.9056 \times 10^{-4}, \\ \pi_p = 0.23969, \pi_2 = 0.2, \pi_1 = 0.1, \theta = 0.8, L = 75.0, C = 5.0 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} P_W = 0.17378, \pi_W = 0.24756, P_M = 0.21553, \\ R_4 = 16.0, V = 11.0, \pi_M = 0.23106, R_3 = 12.559, R_2 = 11.0, \\ P_p = 0.18354, y = 100.0, R_1 = 9.8718, D_3 = -1.6313 \times 10^{-4}, \\ \pi_p = 0.24374, \pi_2 = 0.2, \pi_1 = 0.1, \theta = 0.8, L = 70.0, C = 5.0 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} P_W = 0.17583, \pi_W = 0.25165, P_M = 0.21812, \\ R_4 = 16.0, V = 11.0, \pi_M = 0.23624, R_3 = 12.559, R_2 = 11.0, \\ P_p = 0.18619, y = 100.0, R_1 = 10.181, D_3 = -1.3835 \times 10^{-4}, \\ \pi_p = 0.24791, \pi_2 = 0.2, \pi_1 = 0.1, \theta = 0.8, L = 65.0, C = 5.0 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} P_W = 0.17791, \pi_W = 0.25581, P_M = 0.22076, \\ R_4 = 16.0, V = 11.0, \pi_M = 0.24151, R_3 = 12.559, R_2 = 11.0, \\ P_p = 0.18886, y = 100.0, R_1 = 10.456, D_3 = -1.1605 \times 10^{-4}, \\ \pi_p = 0.25219, \pi_2 = 0.2, \pi_1 = 0.1, \theta = 0.8, L = 60.0, C = 5.0 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} P_W = 0.18002, \pi_W = 0.26005, P_M = 0.22343, \\ R_4 = 16.0, V = 11.0, \pi_M = 0.24687, R_3 = 12.559, R_2 = 11.0, \\ P_p = 0.19156, y = 100.0, R_1 = 10.706, D_3 = -9.6063 \times 10^{-5}, \\ \pi_p = 0.25657, \pi_2 = 0.2, \pi_1 = 0.1, \theta = 0.8, L = 55.0, C = 5.0 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} P_W = 0.18218, \pi_W = 0.26436, P_M = 0.22616, \\ R_4 = 16.0, V = 11.0, \pi_M = 0.25232, R_3 = 12.559, R_2 = 11.0, \\ P_p = 0.19428, y = 100.0, R_1 = 10.934, D_3 = -7.8250 \times 10^{-5}, \\ \pi_p = 0.26107, \pi_2 = 0.2, \pi_1 = 0.1, \theta = 0.8, L = 50.0, C = 5.0 \end{bmatrix}\right\}$$

Appendix 2.5 Welfare change with risk range 3

$$D_3 = (\pi_p - \pi_M)(\pi_M + \pi_p - 2\pi_2) - (\pi_W - \pi_p)(\pi_W + \pi_p - 2\pi_1)$$

$$P_W = \tfrac{1}{2}(\pi_1 + \pi_W)$$

$$\pi_W = \tfrac{1}{C}\left[\frac{(y-P_W L)^{1-\theta}}{1-\theta} - V\right]$$

$$(\pi_p - \pi_2)\left[P_p - \tfrac{1}{2}(\pi_2 + \pi_p)\right] + (\pi_p - \pi_1)\left[P_p - \tfrac{1}{2}(\pi_1 + \pi_p)\right] = 0$$

$$\pi_p = \tfrac{1}{C}\left[\frac{(y-P_p L)^{1-\theta}}{1-\theta} - V\right]$$

$$P_M = \tfrac{1}{2}(\pi_2 + \pi_M)$$

$$\pi_M = \tfrac{1}{C}\left[\frac{(y-P_M L)^{1-\theta}}{1-\theta} - V\right]$$

$$R_1 = \frac{(y-L)^{1-\theta}}{1-\theta}$$

$$R_2 = V$$

$$R_3 = \frac{y^{1-\theta}}{1-\theta}$$

$$R_4 = C + V$$

$$\pi_1 = 0.1$$

$$\pi_2 = 0.001$$

$$y = 100$$

$$\theta = 0.7$$

$$L = 60$$

$$C = 5$$

$$V = 11$$

$$\left\{\begin{bmatrix} P_W = 0.22146, \pi_W = 0.34292, P_M = 0.18222, R_4 = 16.0, \\ \pi_M = 0.36344, V = 11.0, R_3 = 13.27, P_p = 0.19867, R_2 = 11.0, \\ y = 100.0, \pi_p = 0.35489, D_3 = -1.7119\times10^{-4}, \\ R_1 = 10.081, \pi_2 = 0.001, \pi_1 = 0.1, \theta = 0.7, L = 60.0, C = 5.0 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} P_W = 0.22146, \pi_W = 0.34292, P_M = 0.18579, R_4 = 16.0, \\ \pi_M = 0.36159, V = 11.0, R_3 = 13.27, P_p = 0.20096, R_2 = 11.0, \\ y = 100.0, \pi_p = 0.35369, D_3 = -1.4153\times10^{-4}, \\ R_1 = 10.081, \pi_2 = 0.01, \pi_1 = 0.1, \theta = 0.7, L = 60.0, C = 5.0 \end{bmatrix}\right\}$$

$$\left\{\begin{bmatrix} P_W = 0.22146, \pi_W = 0.34292, P_M = 0.20166, R_4 = 16.0, \\ \pi_M = 0.35332, V = 11.0, R_3 = 13.27, P_p = 0.21066, R_2 = 11.0, \\ y = 100.0, \pi_p = 0.34861, D_3 = -4.3925\times10^{-5}, \\ R_1 = 10.081, \pi_2 = 0.05, \pi_1 = 0.1, \theta = 0.7, L = 60.0, C = 5.0 \end{bmatrix}\right\}$$

$$\left\{\left[\begin{array}{c} P_W = 0.22146, \pi_W = 0.34292, P_M = 0.2175, R_4 = 16.0, \\ \pi_M = 0.34501, V = 11.0, R_3 = 13.27, P_p = 0.21944, R_2 = 11.0, \\ y = 100.0, \pi_p = 0.34399, D_3 = -1.7846\times10^{-6} + 7.8107\times10^{-24} i, \\ R_1 = 10.081, \pi_2 = 0.09, \pi_1 = 0.1, \theta = 0.7, L = 60.0, C = 5.0 \end{array}\right]\right\}$$

$$\left\{\left[\begin{array}{c} P_W = 0.22146, \pi_W = 0.34292, P_M = 0.22146, R_4 = 16.0, \\ \pi_M = 0.34292, V = 11.0, R_3 = 13.27, P_p = 0.22146, R_2 = 11.0, \\ y = 100.0, \pi_p = 0.34292, D_3 = 5.4956\times10^{-23} + 4.2661\times10^{-23} i, \\ R_1 = 10.081, \pi_2 = 0.1, \pi_1 = 0.1, \theta = 0.7, L = 60.0, C = 5.0 \end{array}\right]\right\}$$

$$\left\{\left[\begin{array}{c} P_W = 0.22146, \pi_W = 0.34292, P_M = 0.22542, R_4 = 16.0, \\ \pi_M = 0.34083, V = 11.0, R_3 = 13.27, P_p = 0.22340, R_2 = 11.0, \\ y = 100.0, \pi_p = 0.34190, D_3 = -1.8091\times10^{-6} - 1.9729\times10^{-22} i, \\ R_1 = 10.081, \pi_2 = 0.11, \pi_1 = 0.1, \theta = 0.7, L = 60.0, C = 5.0 \end{array}\right]\right\}$$

$$\left\{\left[\begin{array}{c} P_W = 0.22146, \pi_W = 0.34292, P_M = 0.24122, R_4 = 16.0, \\ \pi_M = 0.33244, V = 11.0, R_3 = 13.27, P_p = 0.23019, R_2 = 11.0, \\ y = 100.0, \pi_p = 0.33831, D_3 = -4.7308\times10^{-5}, \\ R_1 = 10.081, \pi_2 = 0.15, \pi_1 = 0.1, \theta = 0.7, L = 60.0, C = 5.0 \end{array}\right]\right\}$$

$$\left\{\left[\begin{array}{c} P_W = 0.22146, \pi_W = 0.34292, P_M = 0.26094, R_4 = 16.0, \\ \pi_M = 0.32188, V = 11.0, R_3 = 13.27, P_p = 0.23589 + 4.6048\times10^{-21} i, R_2 = 11.0, \\ y = 100.0, \pi_p = 0.33528, D_3 = -2.1031\times10^{-4} - 9.639\times10^{-22} i, \\ R_1 = 10.081, \pi_2 = 0.2, \pi_1 = 0.1, \theta = 0.7, L = 60.0, C = 5.0 \end{array}\right]\right\}$$

$$\left\{\left[\begin{array}{c} P_W = 0.22146, \pi_W = 0.34292, P_M = 0.28062, R_4 = 16.0, \\ \pi_M = 0.31124, V = 11.0, R_3 = 13.27, P_p = 0.23717 + 1.6546\times10^{-20} i, R_2 = 11.0, \\ y = 100.0, \pi_p = 0.33460 - 9.4339\times10^{-21} i, D_3 = -5.6969\times10^{-4} - 2.8637\times10^{-21} i, \\ R_1 = 10.081, \pi_2 = 0.25, \pi_1 = 0.1, \theta = 0.7, L = 60.0, C = 5.0 \end{array}\right]\right\}$$

$$\left\{\left[\begin{array}{c} P_W = 0.22146, \pi_W = 0.34292, P_M = 0.30025, R_4 = 16.0, \\ \pi_M = 0.30051, V = 11.0, R_3 = 13.27, P_p = 0.23219, R_2 = 11.0, \\ y = 100.0, \pi_p = 0.33724, D_3 = -1.3397\times10^{-3}, \\ R_1 = 10.081, \pi_2 = 0.3, \pi_1 = 0.1, \theta = 0.7, L = 60.0, C = 5.0 \end{array}\right]\right\}$$

Appendix 2.6 Welfare change with risk range 4

$$D_3 = (\pi_p - \pi_M)(\pi_M + \pi_p - 2\pi_2) - (\pi_W - \pi_p)(\pi_W + \pi_p - 2\pi_1)$$
$$P_W = \tfrac{1}{2}(\pi_1 + \pi_W)$$
$$\pi_W = \tfrac{1}{C}\left[\tfrac{(y - P_W L)^{1-\theta}}{1-\theta} - V\right]$$
$$(\pi_p - \pi_2)\left[P_p - \tfrac{1}{2}(\pi_2 + \pi_p)\right] + (\pi_p - \pi_1)\left[P_p - \tfrac{1}{2}(\pi_1 + \pi_p)\right] = 0$$
$$\pi_p = \tfrac{1}{C}\left[\tfrac{(y - P_p L)^{1-\theta}}{1-\theta} - V\right]$$
$$P_M = \tfrac{1}{2}(\pi_2 + \pi_M)$$
$$\pi_M = \tfrac{1}{C}\left[\tfrac{(y - P_M L)^{1-\theta}}{1-\theta} - V\right]$$
$$R_1 = \tfrac{(y-L)^{1-\theta}}{1-\theta}$$
$$R_2 = V$$
$$R_3 = \tfrac{y^{1-\theta}}{1-\theta}$$
$$R_4 = C + V$$
$$\pi_1 = 0.2$$
$$\pi_2 = 0.05$$
$$y = 100$$
$$\theta = 0.6$$
$$L = 80$$
$$C = 5$$
$$V = 11$$

$$\left\{\left[\begin{array}{c} P_W = 0.37048, \pi_W = 0.54097, P_M = 0.33956, \\ R_4 = 16.0, V = 11.0, \pi_M = 0.57912, R_3 = 15.774, R_2 = 11.0, \\ P_p = 0.35318, y = 100.0, R_1 = 8.2861, D_3 = -6.3967\times 10^{-4} + 6.5107\times 10^{-24} i, \\ \pi_p = 0.56241, \pi_2 = 0.1, \pi_1 = 0.2, \theta = 0.6, L = 80.0, C = 5.0 \end{array}\right]\right\}$$

$$\left\{\left[\begin{array}{c} P_W = 0.37048, \pi_W = 0.54097, P_M = 0.35505, \\ R_4 = 16.0, V = 11.0, \pi_M = 0.5601, R_3 = 15.774, R_2 = 11.0, \\ P_p = 0.36226, y = 100.0, R_1 = 8.2861, D_3 = -1.6087\times 10^{-4} + 4.6623\times 10^{-22} i, \\ \pi_p = 0.55119, \pi_2 = 0.15, \pi_1 = 0.2, \theta = 0.6, L = 80.0, C = 5.0 \end{array}\right]\right\}$$

$$\left\{\left[\begin{array}{c} P_W = 0.37048, \pi_W = 0.54097, P_M = 0.3674, \\ R_4 = 16.0, V = 11.0, \pi_M = 0.5448, R_3 = 15.774, R_2 = 11.0, \\ P_p = 0.36892 - 1.0082\times 10^{-20} i, y = 100.0, R_1 = 8.2861, D_3 = -6.5005\times 10^{-6} + 9.5005\times 10^{-21} i, \\ \pi_p = 0.54291 + 1.3654\times 10^{-20} i, \pi_2 = 0.19, \pi_1 = 0.2, \theta = 0.6, L = 80.0, C = 5.0 \end{array}\right]\right\}$$

$$\left\{\left[\begin{array}{c} P_W = 0.37048, \pi_W = 0.54097, P_M = 0.37048, \\ R_4 = 16.0, V = 11.0, \pi_M = 0.54097, R_3 = 15.774, R_2 = 11.0, \\ P_p = 0.37048 - 2.0471\times 10^{-20} i, y = 100.0, R_1 = 8.2861, D_3 = 1.741\times 10^{-20} + 1.8899\times 10^{-20} i, \\ \pi_p = 0.54097 + 2.7712\times 10^{-20} i, \pi_2 = 0.2, \pi_1 = 0.2, \theta = 0.6, L = 80.0, C = 5.0 \end{array}\right]\right\}$$

$$\left\{\left[\begin{array}{c} P_W = 0.37048, \pi_W = 0.54097, P_M = 0.37356, \\ R_4 = 16.0, V = 11.0, \pi_M = 0.53712, R_3 = 15.774, R_2 = 11.0, \\ P_p = 0.37200 - 4.0219\times 10^{-20} i, y = 100.0, R_1 = 8.2861, D_3 = -6.5515\times 10^{-6} + 3.6361\times 10^{-20} i, \\ \pi_p = 0.53907 + 5.4423\times 10^{-20} i, \pi_2 = 0.21, \pi_1 = 0.2, \theta = 0.6, L = 80.0, C = 5.0 \end{array}\right]\right\}$$

$$\left\{\left[\begin{array}{c} P_W = 0.37048, \pi_W = 0.54097, P_M = 0.38585, \\ R_4 = 16.0, V = 11.0, \pi_M = 0.52171, R_3 = 15.774, R_2 = 11.0, \\ P_p = 0.37755, y = 100.0, R_1 = 8.2861, D_3 = -1.6766\times 10^{-4}, \\ \pi_p = 0.53214, \pi_2 = 0.25, \pi_1 = 0.2, \theta = 0.6, L = 80.0, C = 5.0 \end{array}\right]\right\}$$

$$\left\{\left[\begin{array}{c} P_W = 0.37048, \pi_W = 0.54097, P_M = 0.40116, \\ R_4 = 16.0, V = 11.0, \pi_M = 0.50232, R_3 = 15.774, R_2 = 11.0, \\ P_p = 0.38307, y = 100.0, R_1 = 8.2861, D_3 = -7.052\times 10^{-4}, \\ \pi_p = 0.52521, \pi_2 = 0.3, \pi_1 = 0.2, \theta = 0.6, L = 80.0, C = 5.0 \end{array}\right]\right\}$$

$$\left\{\left[\begin{array}{c} P_W = 0.37048, \pi_W = 0.54097, P_M = 0.4164, \\ R_4 = 16.0, V = 11.0, \pi_M = 0.4828, R_3 = 15.774, R_2 = 11.0, \\ P_p = 0.3865, y = 100.0, R_1 = 8.2861, D_3 = -1.7261\times 10^{-3}, \\ \pi_p = 0.52088, \pi_2 = 0.35, \pi_1 = 0.2, \theta = 0.6, L = 80.0, C = 5.0 \end{array}\right]\right\}$$

$$\left\{\left[\begin{array}{c} P_W = 0.37048, \pi_W = 0.54097, P_M = 0.43158, \\ R_4 = 16.0, V = 11.0, \pi_M = 0.46316, R_3 = 15.774, R_2 = 11.0, \\ P_p = 0.38724, y = 100.0, R_1 = 8.2861, D_3 = -3.4890\times 10^{-3}, \\ \pi_p = 0.51995, \pi_2 = 0.4, \pi_1 = 0.2, \theta = 0.6, L = 80.0, C = 5.0 \end{array}\right]\right\}$$

$$\left\{\left[\begin{array}{c} P_W = 0.37048, \pi_W = 0.54097, P_M = 0.44367, \\ R_4 = 16.0, V = 11.0, \pi_M = 0.44735, R_3 = 15.774, R_2 = 11.0, \\ P_p = 0.38548, y = 100.0, R_1 = 8.2861, D_3 = -5.7596\times 10^{-3}, \\ \pi_p = 0.52218, \pi_2 = 0.44, \pi_1 = 0.2, \theta = 0.6, L = 80.0, C = 5.0 \end{array}\right]\right\}$$

Appendix 3.1 Aggregate social welfare increases with positive T

The high risks don't drive while the low risks drive before and after the taxation

The constraint on the high risks is binding, ie, $v(\pi_H,\alpha_H)=v(\pi_H,\alpha_L)$

$u(x)=\ln x$, $W=100$, $B=1$, $D=90$, $\pi_H=0.8$, $\pi_L=0.3$, $\lambda=0.3$,

$$
\begin{gathered}
U_H=B+\ln(W-D\pi_H)\\
U_1=\ln W\\
U_L=B+\ln(W-D\pi_L)\\
B+(1-\pi_H)\ln(W-\theta_1 D\pi_L)+\pi_H\ln[W-\theta_1 D\pi_L-(1-\theta_1)D]=\ln W\\
U_S=B+(1-\pi_L)\ln(W-\theta_1 D\pi_L)+\pi_L\ln[W-\theta_1 D\pi_L-(1-\theta_1)D]\\
\pi_P=\lambda\pi_H+(1-\lambda)\pi_L\\
\pi_L(1-\pi_P)(W-\theta_p D\pi_P)-(1-\pi_L)\pi_P[W-\theta_p D\pi_P-(1-\theta_p)D]=0\\
U_p=B+(1-\pi_L)\ln(W-\theta_p D\pi_P)+\pi_L\ln[W-\theta_p D\pi_P-(1-\theta_p)D]\\
U_2=\ln(W+(1-\lambda)T)\\
B+(1-\pi_H)\ln(W-\theta_2 D\pi_L-\lambda T)+\pi_H\ln[W-\theta_2 D\pi_L-(1-\theta_2)D-\lambda T]=\ln(W+(1-\lambda)T)\\
U_T=B+(1-\pi_L)\ln(W-\theta_2 D\pi_L-\lambda T)+\pi_L\ln[W-\theta_2 D\pi_L-(1-\theta_2)D-\lambda T]\\
E_1=\tfrac{\lambda(\pi_H-\pi_L)}{(1-\lambda)\pi_L(1-\pi_L)}\\
E_2=\tfrac{D}{W}\\
\theta_T=1-\left(1-\tfrac{1-\pi_H}{1-\pi_L}\right)\lambda\\
D_1=U_1-U_H\\
D_2=U_L-U_1\\
D_3=U_S-U_p\\
D_4=E_1-E_2\\
D_5=\theta_T-\theta_2\\
D_6=U_T-U_S\\
D_7=U_2-U_1\\
D_8=U_T+U_2-U_S-U_1
\end{gathered}
$$

$$
\left\{\begin{bmatrix}
\theta_1=0.30595,\theta_2=1.0636,\theta_T=0.78571,\theta_p=0.59933,\\
D_1=0.27297,D_2=0.68529,E_1=1.0204,\\
D_3=4.3134\times10^{-2},E_2=0.9,D_4=0.12041,D_5=-0.27793,\\
D_6=-0.22628,D_7=0.39878,D_8=0.17250,\\
\pi_P=0.45,U_1=4.6052,U_2=5.0039,U_H=4.3322,\\
U_L=5.2905,U_S=5.1763,U_T=4.95,U_p=5.1332
\end{bmatrix}\right\}T=70
$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 1.013, \theta_T = 0.78571, \theta_p = 0.59933, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 1.0204, \\ D_3 = 4.3134\times10^{-2}, E_2 = 0.9, D_4 = 0.12041, D_5 = -0.22731, \\ D_6 = -0.20228, D_7 = 0.37981, D_8 = 0.17753, \\ \pi_P = 0.45, U_1 = 4.6052, U_2 = 4.9850, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 4.974, U_p = 5.1332 \end{bmatrix}\right\} T = 66$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 1.0, \theta_T = 0.78571, \theta_p = 0.59933, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 1.0204, \\ D_3 = 4.3134\times10^{-2}, E_2 = 0.9, D_4 = 0.12041, D_5 = -0.21429, \\ D_6 = -0.19634, D_7 = 0.37478, D_8 = 0.17844, \\ \pi_P = 0.45, U_1 = 4.6052, U_2 = 4.9799, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 4.9799, U_p = 5.1332 \end{bmatrix}\right\} T = 64.953$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.95165, \theta_T = 0.78571, \theta_p = 0.59933, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 1.0204, \\ D_3 = 4.3134\times10^{-2}, E_2 = 0.9, D_4 = 0.12041, D_5 = -0.16594, \\ D_6 = -0.17510, D_7 = 0.35557, D_8 = 0.18047, \\ \pi_P = 0.45, U_1 = 4.6052, U_2 = 4.9607, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.0012, U_p = 5.1332 \end{bmatrix}\right\} T = 61$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.93961, \theta_T = 0.78571, \theta_p = 0.59933, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 1.0204, \\ D_3 = 4.3134\times10^{-2}, E_2 = 0.9, D_4 = 0.12041, D_5 = -0.15389, \\ D_6 = -0.17001, D_7 = 0.35066, D_8 = 0.18065, \\ \pi_P = 0.45, U_1 = 4.6052, U_2 = 4.9558, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.0063, U_p = 5.1332 \end{bmatrix}\right\} T = 60$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.93361, \theta_T = 0.78571, \theta_p = 0.59933, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 1.0204, \\ D_3 = 4.3134\times10^{-2}, E_2 = 0.9, D_4 = 0.12041, D_5 = -0.14790, \\ D_6 = -0.16750, D_7 = 0.34819, D_8 = 0.18069, \\ \pi_P = 0.45, U_1 = 4.6052, U_2 = 4.9534, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.0088, U_p = 5.1332 \end{bmatrix}\right\} T = 59.5$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.92763, \theta_T = 0.78571, \theta_p = 0.59933, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 1.0204, \\ D_3 = 4.3134\times10^{-2}, E_2 = 0.9, D_4 = 0.12041, D_5 = -0.14192, \\ D_6 = -0.16502, D_7 = 0.34572, D_8 = 0.18070, \\ \pi_P = 0.45, U_1 = 4.6052, U_2 = 4.9509, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.0113, U_p = 5.1332 \end{bmatrix}\right\} T = 59$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.92167, \theta_T = 0.78571, \theta_p = 0.59933, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 1.0204, \\ D_3 = 4.3134 \times 10^{-2}, E_2 = 0.9, D_4 = 0.12041, D_5 = -0.13596, \\ D_6 = -0.16256, D_7 = 0.34324, D_8 = 0.18067, \\ \pi_P = 0.45, U_1 = 4.6052, U_2 = 4.9484, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.0137, U_p = 5.1332 \end{bmatrix}\right\} T = 58.5$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.91573, \theta_T = 0.78571, \theta_p = 0.59933, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 1.0204, \\ D_3 = 4.3134 \times 10^{-2}, E_2 = 0.9, D_4 = 0.12041, D_5 = -0.13002, \\ D_6 = -0.16013, D_7 = 0.34075, D_8 = 0.18062, \\ \pi_P = 0.45, U_1 = 4.6052, U_2 = 4.9459, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.0162, U_p = 5.1332 \end{bmatrix}\right\} T = 58$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.88042, \theta_T = 0.78571, \theta_p = 0.59933, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 1.0204, \\ D_3 = 4.3134 \times 10^{-2}, E_2 = 0.9, D_4 = 0.12041, D_5 = -9.4705 \times 10^{-2}, \\ D_6 = -0.14607, D_7 = 0.3257, D_8 = 0.17963, \\ \pi_P = 0.45, U_1 = 4.6052, U_2 = 4.9309, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.0302, U_p = 5.1332 \end{bmatrix}\right\} T = 55$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.8228, \theta_T = 0.78571, \theta_p = 0.59933, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 1.0204, \\ D_3 = 4.3134 \times 10^{-2}, E_2 = 0.9, D_4 = 0.12041, D_5 = -3.7091 \times 10^{-2}, \\ D_6 = -0.12449, D_7 = 0.3001, D_8 = 0.17561, \\ \pi_P = 0.45, U_1 = 4.6052, U_2 = 4.9053, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.0518, U_p = 5.1332 \end{bmatrix}\right\} T = 50$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.76659, \theta_T = 0.78571, \theta_p = 0.59933, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 1.0204, \\ D_3 = 4.3134 \times 10^{-2}, E_2 = 0.9, D_4 = 0.12041, D_5 = 1.9128 \times 10^{-2}, \\ D_6 = -0.10500, D_7 = 0.27384, D_8 = 0.16884, \\ \pi_P = 0.45, U_1 = 4.6052, U_2 = 4.879, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.0713, U_p = 5.1332 \end{bmatrix}\right\} T = 45$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.71162, \theta_T = 0.78571, \theta_p = 0.59933, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 1.0204, \\ D_3 = 4.3134 \times 10^{-2}, E_2 = 0.9, D_4 = 0.12041, D_5 = 7.4094 \times 10^{-2}, \\ D_6 = -8.7395 \times 10^{-2}, D_7 = 0.24686, D_8 = 0.15947, \\ \pi_P = 0.45, U_1 = 4.6052, U_2 = 4.852, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.0889, U_p = 5.1332 \end{bmatrix}\right\} T = 40$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.65779, \theta_T = 0.78571, \theta_p = 0.59933, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 1.0204, \\ D_3 = 4.3134\times10^{-2}, E_2 = 0.9, D_4 = 0.12041, D_5 = 0.12793, \\ D_6 = -7.1516\times10^{-2}, D_7 = 0.21914, D_8 = 0.14762, \\ \pi_P = 0.45, U_1 = 4.6052, U_2 = 4.8243, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.1048, U_p = 5.1332 \end{bmatrix}\right\} T = 35$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.60499, \theta_T = 0.78571, \theta_p = 0.59933, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 1.0204, \\ D_3 = 4.3134\times10^{-2}, E_2 = 0.9, D_4 = 0.12041, D_5 = 0.18073, \\ D_6 = -5.7227\times10^{-2}, D_7 = 0.19062, D_8 = 0.13339, \\ \pi_P = 0.45, U_1 = 4.6052, U_2 = 4.7958, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.1191, U_p = 5.1332 \end{bmatrix}\right\} T = 30$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.55314, \theta_T = 0.78571, \theta_p = 0.59933, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 1.0204, \\ D_3 = 4.3134\times10^{-2}, E_2 = 0.9, D_4 = 0.12041, D_5 = 0.23258, \\ D_6 = -4.4419\times10^{-2}, D_7 = 0.16127, D_8 = 0.11685, \\ \pi_P = 0.45, U_1 = 4.6052, U_2 = 4.7664, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.1319, U_p = 5.1332 \end{bmatrix}\right\} T = 25$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.50216, \theta_T = 0.78571, \theta_p = 0.59933, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 1.0204, \\ D_3 = 4.3134\times10^{-2}, E_2 = 0.9, D_4 = 0.12041, D_5 = 0.28355, \\ D_6 = -3.3002\times10^{-2}, D_7 = 0.13103, D_8 = 9.8026\times10^{-2}, \\ \pi_P = 0.45, U_1 = 4.6052, U_2 = 4.7362, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.1433, U_p = 5.1332 \end{bmatrix}\right\} T = 20$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.48057, \theta_T = 0.78571, \theta_p = 0.59933, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 1.0204, \\ D_3 = 4.3134\times10^{-2}, E_2 = 0.9, D_4 = 0.12041, D_5 = 0.30514, \\ D_6 = -2.8517\times10^{-2}, D_7 = 0.11778, D_8 = 8.9266\times10^{-2}, \\ \pi_P = 0.45, U_1 = 4.6052, U_2 = 4.7230, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.1478, U_p = 5.1332 \end{bmatrix}\right\} T = 17.857143$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.46198, \theta_T = 0.78571, \theta_p = 0.59933, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 1.0204, \\ D_3 = 4.3134\times10^{-2}, E_2 = 0.9, D_4 = 0.12041, D_5 = 0.32374, \\ D_6 = -2.4821\times10^{-2}, D_7 = 0.10616, D_8 = 8.1339\times10^{-2}, \\ \pi_P = 0.45, U_1 = 4.6052, U_2 = 4.7113, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.1515, U_p = 5.1332 \end{bmatrix}\right\} T = 16$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.39283, \theta_T = 0.78571, \theta_p = 0.59933, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 1.0204, \\ D_3 = 4.3134\times10^{-2}, E_2 = 0.9, D_4 = 0.12041, D_5 = 0.39289, \\ D_6 = -1.2446\times10^{-2}, D_7 = 6.1095\times10^{-2}, D_8 = 4.8649\times10^{-2}, \\ \pi_P = 0.45, U_1 = 4.6052, U_2 = 4.6663, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.1638, U_p = 5.1332 \end{bmatrix}\right\} T = 9$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.35394, \theta_T = 0.78571, \theta_p = 0.59933, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 1.0204, \\ D_3 = 4.3134\times10^{-2}, E_2 = 0.9, D_4 = 0.12041, D_5 = 0.43177, \\ D_6 = -6.4434\times10^{-3}, D_7 = 3.4401\times10^{-2}, D_8 = 2.7958\times10^{-2}, \\ \pi_P = 0.45, U_1 = 4.6052, U_2 = 4.6396, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.1698, U_p = 5.1332 \end{bmatrix}\right\} T = 5$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.31549, \theta_T = 0.78571, \theta_p = 0.59933, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 1.0204, \\ D_3 = 4.3134\times10^{-2}, E_2 = 0.9, D_4 = 0.12041, D_5 = 0.47022, \\ D_6 = -1.1956\times10^{-3}, D_7 = 6.9756\times10^{-3}, D_8 = 5.7800\times10^{-3}, \\ \pi_P = 0.45, U_1 = 4.6052, U_2 = 4.6121, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.1751, U_p = 5.1332 \end{bmatrix}\right\} T = 1$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.3069, \theta_T = 0.78571, \theta_p = 0.59933, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 1.0204, \\ D_3 = 4.3134\times10^{-2}, E_2 = 0.9, D_4 = 0.12041, D_5 = 0.47881, \\ D_6 = -1.1748\times10^{-4} - 9.5581\times10^{-24} i, D_7 = 6.9976\times10^{-4}, \\ D_8 = 5.8227\times10^{-4} - 9.5581\times10^{-24} i, \\ \pi_P = 0.45, U_1 = 4.6052, U_2 = 4.6059, U_H = 4.3322, U_L = 5.2905, \\ U_S = 5.1763, U_T = 5.1762, U_p = 5.1332 \end{bmatrix}\right\} T = 0.1$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.30604, \theta_T = 0.78571, \theta_p = 0.59933, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 1.0204, \\ D_3 = 4.3134\times10^{-2}, E_2 = 0.9, D_4 = 0.12041, D_5 = 0.47967, \\ D_6 = -1.1727\times10^{-5} - 2.4151\times10^{-25} i, D_7 = 6.9998\times10^{-5}, D_8 = 5.827\times10^{-5}, \\ \pi_P = 0.45, U_1 = 4.6052, U_2 = 4.6052, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.1763, U_p = 5.1332 \end{bmatrix}\right\} T = 0.01$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.30596, \theta_T = 0.78571, \theta_p = 0.59933, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 1.0204, \\ D_3 = 4.3134\times10^{-2}, E_2 = 0.9, D_4 = 0.12041, D_5 = 0.47976, \\ D_6 = -1.1725\times10^{-6} - 2.1499\times10^{-26} i, D_7 = 7.0000\times10^{-6}, D_8 = 5.8274\times10^{-6}, \\ \pi_P = 0.45, U_1 = 4.6052, U_2 = 4.6052, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.1763, U_p = 5.1332 \end{bmatrix}\right\} T = 0.001$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.30594, \theta_T = 0.78571, \theta_p = 0.59933, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 1.0204, \\ D_3 = 4.3134\times10^{-2}, E_2 = 0.9, D_4 = 0.12041, D_5 = 0.47978, \\ D_6 = 1.1725\times10^{-6} + 2.0962\times10^{-26} i, D_7 = -7.0\times10^{-6}, D_8 = -5.8275\times10^{-6}, \\ \pi_P = 0.45, U_1 = 4.6052, U_2 = 4.6052, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.1763, U_p = 5.1332 \end{bmatrix}\right\} T = -0.001$$

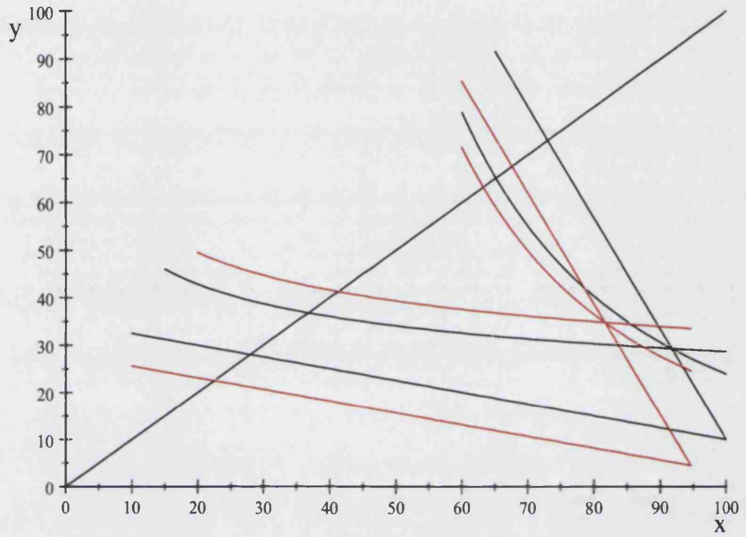

Figure 3.10 Positive tax increases total social welfare (Scientific Workplace driven)

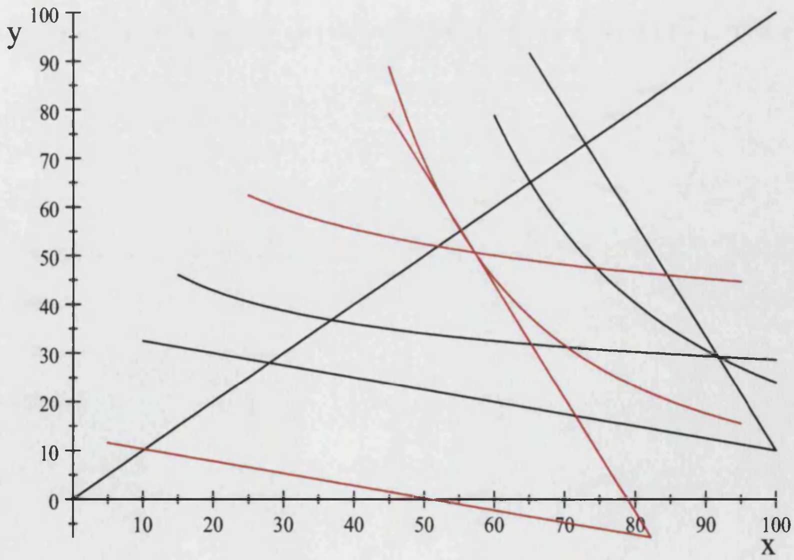

Figure 3.12 Full insurance coverage is not optimal (Scientific Workplace driven)

Appendix 3.2 Aggregate social welfare increases with negative T 1

The high risks don't drive while the low risks drive before and after the taxation

The constraint on the high risks is binding, ie, $v(\pi_H, \alpha_H) = v(\pi_H, \alpha_L)$

$u(x) = \ln x$, $W = 100$, $B = 1$, $D = 99$, $\pi_H = 0.9$, $\pi_L = 0.3$, $\lambda = 0.83$, $T = -1$

$$
\begin{gathered}
U_H = B + \ln(W - D\pi_H) \\
U_1 = \ln W \\
U_L = B + \ln(W - D\pi_L) \\
B + (1-\pi_H)\ln(W - \theta_1 D\pi_L) + \pi_H \ln[W - \theta_1 D\pi_L - (1-\theta_1)D] = \ln W \\
U_S = B + (1-\pi_L)\ln(W - \theta_1 D\pi_L) + \pi_L \ln[W - \theta_1 D\pi_L - (1-\theta_1)D] \\
\pi_P = \lambda\pi_H + (1-\lambda)\pi_L \\
\pi_L(1-\pi_P)(W - \theta_p D\pi_P) - (1-\pi_L)\pi_P[W - \theta_p D\pi_P - (1-\theta_p)D] = 0 \\
U_p = B + (1-\pi_L)\ln(W - \theta_p D\pi_P) + \pi_L \ln[W - \theta_p D\pi_P - (1-\theta_p)D] \\
U_2 = \ln(W + (1-\lambda)T) \\
B + (1-\pi_H)\ln(W - \theta_2 D\pi_L - \lambda T) + \pi_H \ln[W - \theta_2 D\pi_L - (1-\theta_2)D - \lambda T] = \ln(W + (1-\lambda)T) \\
U_T = B + (1-\pi_L)\ln(W - \theta_2 D\pi_L - \lambda T) + \pi_L \ln[W - \theta_2 D\pi_L - (1-\theta_2)D - \lambda T] \\
E_1 = \tfrac{\lambda(\pi_H - \pi_L)}{(1-\lambda)\pi_L(1-\pi_L)} \\
E_2 = \tfrac{D}{W} \\
\theta_T = 1 - \left(1 - \tfrac{1-\pi_H}{1-\pi_L}\right)\lambda \\
D_1 = U_1 - U_H \\
D_2 = U_L - U_1 \\
D_3 = U_S - U_p \\
D_4 = E_1 - E_2 \\
D_5 = \theta_T - \theta_2 \\
D_6 = U_T - U_S \\
D_7 = U_2 - U_1 \\
D_8 = U_T + U_2 - U_S - U_1
\end{gathered}
$$

$$
\left\{\begin{bmatrix}
\theta_1 = 0.46857, \theta_2 = 0.45492 + 7.6927\times10^{-21}i, \theta_T = 0.28857, \theta_p = 0.34473, \\
D_1 = 1.2164, D_2 = 0.6476, E_1 = 13.950, \\
D_3 = 0.55104, E_2 = 0.99, D_4 = 12.960, D_5 = -0.16635 - 7.6927\times10^{-21}i, \\
D_6 = 8.9328\times10^{-3} - 2.0239\times10^{-21}i, D_7 = -1.7014\times10^{-3}, \\
D_8 = 7.2313\times10^{-3} - 2.0239\times10^{-21}i, \\
\pi_P = 0.798, U_1 = 4.6052, U_2 = 4.6035, U_H = 3.3888, \\
U_L = 5.2528, U_S = 5.1719, U_T = 5.1809, U_p = 4.6209
\end{bmatrix}\right\}
$$

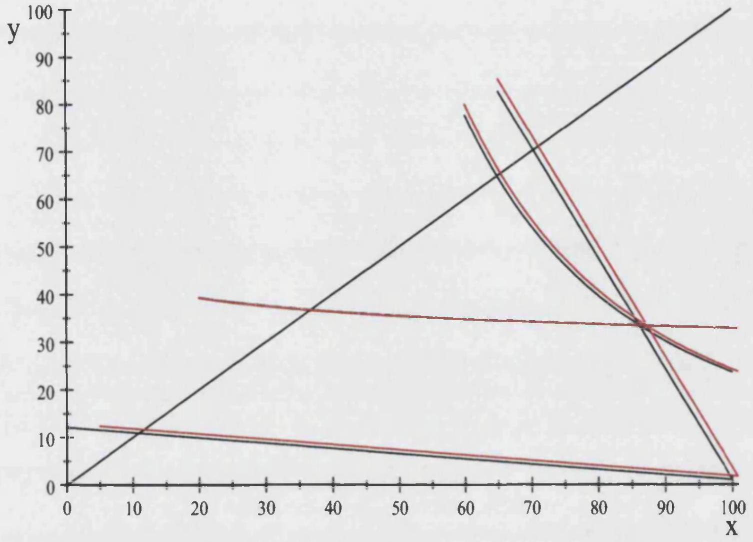

Figure 3.11 Negative tax increases total social welfare (Scientific Workplace driven)

Appendix 3.3 Aggregate social welfare increases with negative T 2

The high risks don't drive while the low risks drive before and after the taxation

The constraint on the high risks is binding, ie, $v(\pi_H,\alpha_H)=v(\pi_H,\alpha_L)$

$u(x)=\ln x$, $W=100$, $B=1$, $D=90$, $\pi_H=0.8$, $\pi_L=0.3$, $\lambda=0.8$,

$$
\begin{gathered}
U_H = B+\ln(W-D\pi_H)\\
U_1=\ln W\\
U_L=B+\ln(W-D\pi_L)\\
B+(1-\pi_H)\ln(W-\theta_1 D\pi_L)+\pi_H\ln[W-\theta_1 D\pi_L-(1-\theta_1)D]=\ln W\\
U_S=B+(1-\pi_L)\ln(W-\theta_1 D\pi_L)+\pi_L\ln[W-\theta_1 D\pi_L-(1-\theta_1)D]\\
\pi_P=\lambda\pi_H+(1-\lambda)\pi_L\\
\pi_L(1-\pi_P)(W-\theta_p D\pi_P)-(1-\pi_L)\pi_P[W-\theta_p D\pi_P-(1-\theta_p)D]=0\\
U_p=B+(1-\pi_L)\ln(W-\theta_p D\pi_P)+\pi_L\ln[W-\theta_p D\pi_P-(1-\theta_p)D]\\
U_2=\ln(W+(1-\lambda)T)\\
B+(1-\pi_H)\ln(W-\theta_2 D\pi_L-\lambda T)+\pi_H\ln[W-\theta_2 D\pi_L-(1-\theta_2)D-\lambda T]=\ln(W+(1-\lambda)T)\\
U_T=B+(1-\pi_L)\ln(W-\theta_2 D\pi_L-\lambda T)+\pi_L\ln[W-\theta_2 D\pi_L-(1-\theta_2)D-\lambda T]\\
E_1=\tfrac{\lambda(\pi_H-\pi_L)}{(1-\lambda)\pi_L(1-\pi_L)}\\
E_2=\tfrac{D}{W}\\
\theta_T=1-\left(1-\tfrac{1-\pi_H}{1-\pi_L}\right)\lambda\\
D_1=U_1-U_H\\
D_2=U_L-U_1\\
D_3=U_S-U_p\\
D_4=E_1-E_2\\
D_5=\theta_T-\theta_2\\
D_6=U_T-U_S\\
D_7=U_2-U_1\\
D_8=U_T+U_2-U_S-U_1
\end{gathered}
$$

$$
\left\{\begin{bmatrix}
\theta_1=0.30595,\theta_2=-7.0264\times10^{-2},\theta_T=0.42857,\theta_p=0.21693,\\
D_1=0.27297,D_2=0.68529,E_1=9.5238,\\
D_3=0.22645,E_2=0.9,\\
D_4=8.6238,D_5=0.49884,\\
D_6=0.15841,D_7=-5.1293\times10^{-2},D_8=0.10711,\\
\pi_P=0.7,U_1=4.6052,U_2=4.5539,U_H=4.3322,\\
U_L=5.2905,U_S=5.1763,U_T=5.3347,U_p=4.9498
\end{bmatrix}\right\}T=-25
$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = -1.1053\times10^{-2}, \theta_T = 0.42857, \theta_p = 0.21693, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 9.5238, \\ D_3 = 0.22645, E_2 = 0.9, \\ D_4 = 8.6238, D_5 = 0.43962, \\ D_6 = 0.13645, D_7 = -4.2908\times10^{-2}, D_8 = 9.3542\times10^{-2}, \\ \pi_P = 0.7, U_1 = 4.6052, U_2 = 4.5623, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.3127, U_p = 4.9498 \end{bmatrix}\right\} T = -21$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = -3.6299\times10^{-3}, \theta_T = 0.42857, \theta_p = 0.21693, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 9.5238, \\ D_3 = 0.22645, E_2 = 0.9, \\ D_4 = 8.6238, D_5 = 0.4322, \\ D_6 = 0.13363, D_7 = -4.1864\times10^{-2}, D_8 = 9.1763\times10^{-2}, \\ \pi_P = 0.7, U_1 = 4.6052, U_2 = 4.5633, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.3099, U_p = 4.9498 \end{bmatrix}\right\} T = -20.5$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = -6.5912\times10^{-4}, \theta_T = 0.42857, \theta_p = 0.21693, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 9.5238, \\ D_3 = 0.22645, E_2 = 0.9, \\ D_4 = 8.6238, D_5 = 0.42923, \\ D_6 = 0.13249, D_7 = -4.1447\times10^{-2}, D_8 = 9.1046\times10^{-2}, \\ \pi_P = 0.7, U_1 = 4.6052, U_2 = 4.5637, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.3088, U_p = 4.9498 \end{bmatrix}\right\} T = -20.3$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 8.2659\times10^{-4}, \theta_T = 0.42857, \theta_p = 0.21693, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 9.5238, \\ D_3 = 0.22645, E_2 = 0.9, \\ D_4 = 8.6238, D_5 = 0.42774, \\ D_6 = 0.13193, D_7 = -4.1239\times10^{-2}, D_8 = 9.0687\times10^{-2}, \\ \pi_P = 0.7, U_1 = 4.6052, U_2 = 4.5639, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.3082, U_p = 4.9498 \end{bmatrix}\right\} T = -20.2$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 2.3125\times10^{-3}, \theta_T = 0.42857, \theta_p = 0.21693, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 9.5238, \\ D_3 = 0.22645, E_2 = 0.9, \\ D_4 = 8.6238, D_5 = 0.42626, \\ D_6 = 0.13136, D_7 = -0.04103, D_8 = 9.0327\times10^{-2}, \\ \pi_P = 0.7, U_1 = 4.6052, U_2 = 4.5641, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.3076, U_p = 4.9498 \end{bmatrix}\right\} T = -20.1$$

$$\left\{\begin{bmatrix}\theta_1=0.30595,\theta_2=3.7986\times10^{-3},\theta_T=0.42857,\theta_p=0.21693,\\ D_1=0.27297,D_2=0.68529,E_1=9.5238,\\ D_3=0.22645,E_2=0.9,\\ D_4=8.6238,D_5=0.42477,\\ D_6=0.13079,D_7=-4.0822\times10^{-2},D_8=8.9966\times10^{-2},\\ \pi_P=0.7,U_1=4.6052,U_2=4.5643,U_H=4.3322,\\ U_L=5.2905,U_S=5.1763,U_T=5.3071,U_p=4.9498\end{bmatrix}\right\}T=-20$$

$$\left\{\begin{bmatrix}\theta_1=0.30595,\theta_2=7.8379\times10^{-2},\theta_T=0.42857,\theta_p=0.21693,\\ D_1=0.27297,D_2=0.68529,E_1=9.5238,\\ D_3=0.22645,E_2=0.9,\\ D_4=8.6238,D_5=0.35019,\\ D_6=0.10137,D_7=-3.0459\times10^{-2},D_8=7.0913\times10^{-2},\\ \pi_P=0.7,U_1=4.6052,U_2=4.5747,U_H=4.3322,\\ U_L=5.2905,U_S=5.1763,U_T=5.2777,U_p=4.9498\end{bmatrix}\right\}T=-15$$

$$\left\{\begin{bmatrix}\theta_1=0.30595,\theta_2=0.15354,\theta_T=0.42857,\theta_p=0.21693,\\ D_1=0.27297,D_2=0.68529,E_1=9.5238,\\ D_3=0.22645,E_2=0.9,\\ D_4=8.6238,D_5=0.27503,\\ D_6=6.9948\times10^{-2},D_7=-2.0203\times10^{-2},D_8=4.9745\times10^{-2},\\ \pi_P=0.7,U_1=4.6052,U_2=4.5850,U_H=4.3322,\\ U_L=5.2905,U_S=5.1763,U_T=5.2462,U_p=4.9498\end{bmatrix}\right\}T=-10$$

$$\left\{\begin{bmatrix}\theta_1=0.30595,\theta_2=0.22937,\theta_T=0.42857,\theta_p=0.21693,\\ D_1=0.27297,D_2=0.68529,E_1=9.5238,\\ D_3=0.22645,E_2=0.9,\\ D_4=8.6238,D_5=0.19921,\\ D_6=3.6259\times10^{-2},D_7=-0.01005,D_8=2.6209\times10^{-2},\\ \pi_P=0.7,U_1=4.6052,U_2=4.5951,U_H=4.3322,\\ U_L=5.2905,U_S=5.1763,U_T=5.2125,U_p=4.9498\end{bmatrix}\right\}T=-5$$

$$\left\{\begin{bmatrix}\theta_1=0.30595,\theta_2=0.29057,\theta_T=0.42857,\theta_p=0.21693,\\ D_1=0.27297,D_2=0.68529,E_1=9.5238,D_3=0.22645,E_2=0.9,\\ D_4=8.6238,D_5=0.13801,\\ D_6=7.4746\times10^{-3},D_7=-2.002\times10^{-3},D_8=5.4726\times10^{-3},\\ \pi_P=0.7,U_1=4.6052,U_2=4.6032,U_H=4.3322,\\ U_L=5.2905,U_S=5.1763,U_T=5.1838,U_p=4.9498\end{bmatrix}\right\}T=-1$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.30441, \theta_T = 0.42857, \theta_p = 0.21693, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 9.5238, \\ D_3 = 0.22645, E_2 = 0.9, \\ D_4 = 8.6238, D_5 = 0.12416, \\ D_6 = 7.527\times10^{-4}, D_7 = -2.0002\times10^{-4}, D_8 = 5.5268\times10^{-4}, \\ \pi_P = 0.7, U_1 = 4.6052, U_2 = 4.6050, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.177, U_p = 4.9498 \end{bmatrix}\right\} T = -0.1$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.30579, \theta_T = 0.42857, \theta_p = 0.21693, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 9.5238, \\ D_3 = 0.22645, E_2 = 0.9, \\ D_4 = 8.6238, D_5 = 0.12278, \\ D_6 = 7.5323\times10^{-5}, D_7 = -0.00002, D_8 = 5.5323\times10^{-5}, \\ \pi_P = 0.7, U_1 = 4.6052, U_2 = 4.6052, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.1764, U_p = 4.9498 \end{bmatrix}\right\} T = -0.01$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.30593, \theta_T = 0.42857, \theta_p = 0.21693, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 9.5238, \\ D_3 = 0.22645, E_2 = 0.9, \\ D_4 = 8.6238, D_5 = 0.12264, \\ D_6 = 7.5329\times10^{-6}, D_7 = -2.0\times10^{-6}, D_8 = 5.5329\times10^{-6}, \\ \pi_P = 0.7, U_1 = 4.6052, U_2 = 4.6052, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.1763, U_p = 4.9498 \end{bmatrix}\right\} T = -0.001$$

$$\left\{\begin{bmatrix} \theta_1 = 0.30595, \theta_2 = 0.30596, \theta_T = 0.42857, \theta_p = 0.21693, \\ D_1 = 0.27297, D_2 = 0.68529, E_1 = 9.5238, \\ D_3 = 0.22645, E_2 = 0.9, \\ D_4 = 8.6238, D_5 = 0.12261, \\ D_6 = -7.5330\times10^{-6}, D_7 = 2.0000\times10^{-6}, D_8 = -5.5330\times10^{-6}, \\ \pi_P = 0.7, U_1 = 4.6052, U_2 = 4.6052, U_H = 4.3322, \\ U_L = 5.2905, U_S = 5.1763, U_T = 5.1763, U_p = 4.9498 \end{bmatrix}\right\} T = 0.001$$

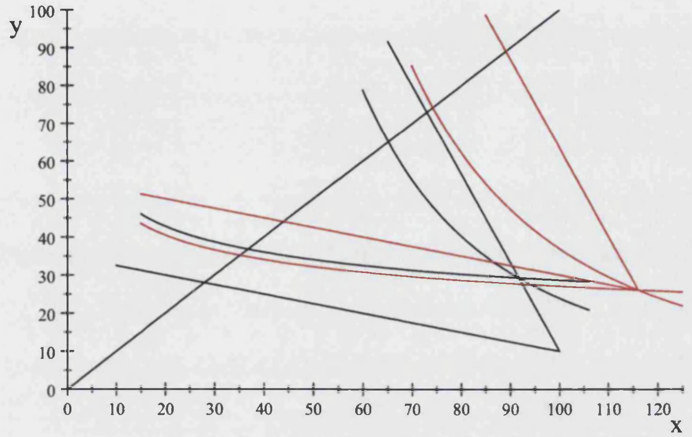

Figure 3.13 Almost no insurance (Scientific Workplace driven)

Appendix 4.1 Numerical examples of efficient partial-pooling Nash equilibrium

$u_T(W) = -e^{-\theta W}$, $u_B(W) = W$, $\pi_H = 0.5$, $\pi_L = 0.2$, $\theta = 20$, $W = 0.2$, $B = 0.09$,

$D = 0.19$, $\lambda_1 = 0.9$, $\lambda_3 = 0.5$, $\lambda_2 = \frac{\lambda_1\lambda_3}{\lambda_1(1-\lambda_3)+(1-\lambda_1)(1-\lambda_3)+\lambda_1\lambda_3} = 0.47368$,

$\lambda_4 = \frac{\lambda_1(1-\lambda_3)}{\lambda_1(1-\lambda_3)+(1-\lambda_1)(1-\lambda_3)+\lambda_1\lambda_3} = 0.47368$, $\pi_1 = \lambda_1\pi_L + (1-\lambda_1)\pi_H$, $\pi_3 = (1-\lambda_3)\pi_L + \lambda_3\pi_H$

$\left(\lambda_1 = \frac{N_{LT}}{N_{LT}+N_{HB}}, 1-\lambda_1 = \frac{N_{HB}}{N_{LT}+N_{HB}}, \lambda_2 = \frac{N_{HT}}{N_{LT}+N_{HB}+N_{HT}}, \lambda_3 = \frac{N_{HT}}{N_{LT}+N_{HT}}, 1-\lambda_3 = \frac{N_{LT}}{N_{LT}+N_{HT}}, \lambda_4 = \frac{N_{LT}}{N_{LT}+N_{HB}+N_{HT}}\right)$

$U_1 = B + (W - \pi_H D)$, $U_2 = B - e^{-\theta(W-\pi_H D)}$, $U_3 = -e^{-\theta W}$, $U_4 = B - e^{-\theta(W-\pi_L D)}$

$$D_1 = \pi_H D - B$$
$$D_2 = U_3 - U_2$$
$$D_3 = U_4 - U_3$$

Solution is: $\{[D_1 = 0.005, D_2 = 1.4141\times10^{-2}, D_3 = 6.9152\times10^{-2}]\}$

$$P_1 = \pi_L I_1$$
$$B + (W - \pi_H D + (\pi_H - \pi_L)I_1) = W$$
$$D_4 = \frac{1-\pi_H}{\pi_H} - \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-P_1)}}{e^{-\theta(W-D-P_1+I_1)}}$$

Solution is: $\{[D_4 = 0.87512, I_1 = 1.6667\times10^{-2}, P_1 = 3.3333\times10^{-3}]\}$

$$B + (W - \pi_H D + \pi_H I_2 - P_2) = W$$
$$\frac{1-\pi_H}{\pi_H} = \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-P_2)}}{e^{-\theta(W-D-P_2+I_2)}}$$
$$U_5 = B + \pi_L\left(-e^{-\theta(W-D-P_2+I_2)}\right) + (1-\pi_L)\left(-e^{-\theta(W-P_2)}\right)$$
$$\frac{1-\pi_3}{\pi_3} = \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-\pi_3 I_3)}}{e^{-\theta(W-D-\pi_3 I_3+I_3)}}$$
$$U_6 = B + \pi_L\left(-e^{-\theta(W-D-\pi_3 I_3+I_3)}\right) + (1-\pi_L)\left(-e^{-\theta(W-\pi_3 I_3)}\right)$$
$$B + \pi_H\left(-e^{-\theta(W-D-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4+I_4)}\right) + (1-\pi_H)\left(-e^{-\theta(W-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4)}\right) = -e^{-\theta W}$$
$$P_4 = (\lambda_1\pi_L + (1-\lambda_1)\pi_H)I_4$$
$$U_7 = B + \pi_L\left(-e^{-\theta(W-D-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4+I_4)}\right) + (1-\pi_L)\left(-e^{-\theta(W-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4)}\right)$$
$$D_5 = \frac{1-\pi_H}{\pi_H} - \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4)}}{e^{-\theta(W-D-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4+I_4)}}$$
$$D_6 = U_7 - U_6$$
$$D_7 = U_7 - U_5$$

$$\text{Solution is: } \left\{ \begin{bmatrix} D_5 = 0.39614, D_6 = 4.7858\times10^{-3}, D_7 = 2.8268\times10^{-2}, \\ I_2 = 0.12069, I_3 = 0.15164, I_4 = 9.5465\times10^{-2}, \\ P_2 = 5.5343\times10^{-2}, P_4 = 2.1957\times10^{-2}, \\ U_5 = 1.3575\times10^{-3}, U_6 = 2.4839\times10^{-2}, U_7 = 2.9625\times10^{-2} \end{bmatrix} \right\}$$

$I_P = 9.5465\times10^{-2}$, $P_P = 2.1957\times10^{-2}$

$$\frac{\theta e^{-\theta W}}{\theta e^{-\theta(W-D+(\lambda_1(1-\pi_L)+(1-\lambda_1)(1-\pi_H))I_P)} - \theta e^{-\theta(W-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_P)}} = 0.17178 < \frac{\lambda_2\pi_H(1-\pi_H)}{\lambda_1(1-\lambda_2)(\pi_H-\pi_L)} = 0.83333$$

So $\dfrac{u_T'(W_R)}{u_T'(W_2)-u_T'(W_1)} = 0.17178 < \dfrac{\lambda_2\pi_H(1-\pi_H)}{\lambda_1(1-\lambda_2)(\pi_H-\pi_L)} = 0.83333$ and there is no Pareto-improving partial-pooling equilibrium.

The following simulation shows the L_T are worse off when the tax breaks the partial-pooling equilibrium into separating equilibrium. Therefore, the market equilibrium is Pareto efficient.

$$W + \lambda_4 t = B + (W - \pi_H D + \pi_H I_P - P_P)$$
$$B + (W - \pi_H D + (\pi_H - \pi_L) I_5 - (1-\lambda_4)t) = W + \lambda_4 t$$
$$P_5 = \pi_L I_5$$
$$U_8 = B + \pi_L\left(-e^{-\theta(W-D-P_5+I_5-(1-\lambda_4)t)}\right) + (1-\pi_L)\left(-e^{-\theta(W-P_5-(1-\lambda_4)t)}\right)$$
$$D_8 = \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-P_5-(1-\lambda_4)t)}}{e^{-\theta(W-D-P_5+I_5-(1-\lambda_4)t)}} - \frac{1-\pi_H}{\pi_H}$$
$$D_9 = -e^{-\theta(W+\lambda_4 t)} - \left(B + \pi_H\left(-e^{-\theta(W-D-P_5+I_5-(1-\lambda_4)t)}\right) + (1-\pi_H)\left(-e^{-\theta(W-P_5-(1-\lambda_4)t)}\right)\right)$$

$$\text{Solution is: } \left\{ \begin{bmatrix} t = 4.3859\times10^{-2}, D_8 = 1.3247, D_9 = -2.6249\times10^{-2}, \\ I_5 = 0.16286, P_5 = 3.2573\times10^{-2}, U_8 = 2.6214\times10^{-2} \end{bmatrix} \right\}$$

$$W + \lambda_4 t = B + (W - \pi_H D + \pi_H I_P - P_P)$$
$$P_6 = \pi_L I_6$$
$$B + \pi_H\left(-e^{-\theta(W-D-P_6+I_6-(1-\lambda_4)t)}\right) + (1-\pi_H)\left(-e^{-\theta(W-P_6-(1-\lambda_4)t)}\right) = -e^{-\theta(W+\lambda_4 t)}$$
$$U_9 = B + \pi_L\left(-e^{-\theta(W-D-P_6+I_6-(1-\lambda_4)t)}\right) + (1-\pi_L)\left(-e^{-\theta(W-P_6-(1-\lambda_4)t)}\right)$$

Solution is: $\{[t = 4.3859\times10^{-2}, I_6 = 0.13299, P_6 = 2.6599\times10^{-2}, U_9 = 1.9481\times10^{-2}]\}$

$\frac{P_P}{I_P} = 0.23$, $\frac{(1-\lambda_4)t+P_6}{I_6} = \frac{(1-\lambda_4)4.3859\times10^{-2}+2.6599\times10^{-2}}{0.13299} = 0.37358$

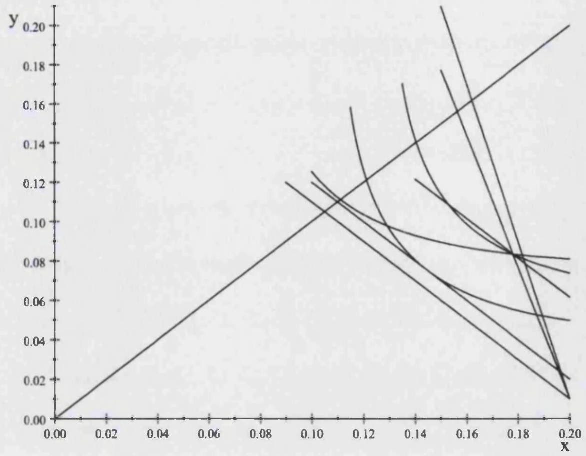

Figure 4.15 Laissez faire partial-pooling equilibrium (Scientific Workplace driven)

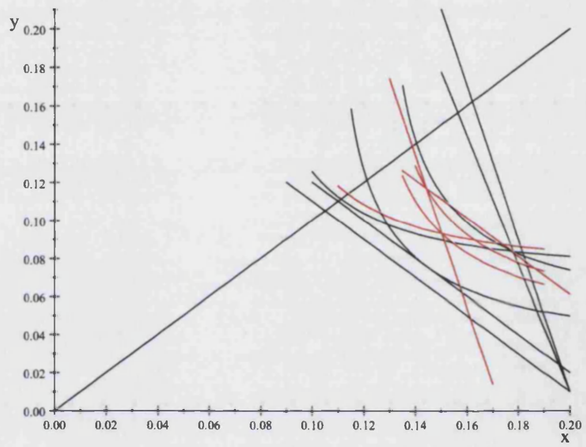

Figure 4.16 From partial-pooling to type 3 separating equilibrium (Scientific Workplace driven)

Appendix 4.2 From partial-pooling equilibrium to type 1 separating equilibrium

$u_T(W) = -e^{-\theta W}$, $u_B(W) = W$, $\pi_H = 0.4$, $\pi_L = 0.1$, $\theta = 30$, $W = 0.15$, $B = 0.04$,

$D = 0.14$, $\lambda_1 = 0.75$, $\lambda_3 = 0.733$, $\lambda_2 = \frac{\lambda_1\lambda_3}{\lambda_1(1-\lambda_3)+(1-\lambda_1)(1-\lambda_3)+\lambda_1\lambda_3}$, $\lambda_4 = \frac{\lambda_1(1-\lambda_3)}{\lambda_1(1-\lambda_3)+(1-\lambda_1)(1-\lambda_3)+\lambda_1\lambda_3}$,

$\pi_1 = \lambda_1\pi_L + (1-\lambda_1)\pi_H$, $\pi_3 = (1-\lambda_3)\pi_L + \lambda_3\pi_H$

$\left(\lambda_1 = \frac{N_{LT}}{N_{LT}+N_{HB}}, 1-\lambda_1 = \frac{N_{HB}}{N_{LT}+N_{HB}}, \lambda_2 = \frac{N_{HT}}{N_{LT}+N_{HB}+N_{HT}}, \lambda_3 = \frac{N_{HT}}{N_{LT}+N_{HT}}, 1-\lambda_3 = \frac{N_{LT}}{N_{LT}+N_{HT}}, \lambda_4 = \frac{N_{LT}}{N_{LT}+N_{HB}+N_{HT}}\right)$

$U_1 = B + (W - \pi_H D)$, $U_2 = B - e^{-\theta(W-\pi_H D)}$, $U_3 = -e^{-\theta W}$, $U_4 = B - e^{-\theta(W-\pi_L D)}$

$$D_1 = \pi_H D - B$$
$$D_2 = U_3 - U_2$$
$$D_3 = U_4 - U_3$$

Solution is: $\{D_1 = 0.016, D_2 = 8.4969\times10^{-3}, D_3 = 3.4202\times10^{-2}\}$

$$P_1 = \pi_L I_1$$
$$B + (W - \pi_H D + (\pi_H - \pi_L)I_1) = W$$
$$D_4 = \frac{1-\pi_H}{\pi_H} - \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-P_1)}}{e^{-\theta(W-D-P_1+I_1)}}$$

Solution is: $\{D_4 = 0.83154, I_1 = 5.3333\times10^{-2}, P_1 = 5.3333\times10^{-3}\}$

$$B + (W - \pi_H D + \pi_H I_2 - P_2) = W$$
$$\frac{1-\pi_H}{\pi_H} = \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-P_2)}}{e^{-\theta(W-D-P_2+I_2)}}$$
$$U_5 = B + \pi_L\left(-e^{-\theta(W-D-P_2+I_2)}\right) + (1-\pi_L)\left(-e^{-\theta(W-P_2)}\right)$$
$$\frac{1-\pi_3}{\pi_3} = \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-\pi_3 I_3)}}{e^{-\theta(W-D-\pi_3 I_3+I_3)}}$$
$$U_6 = B + \pi_L\left(-e^{-\theta(W-D-\pi_3 I_3+I_3)}\right) + (1-\pi_L)\left(-e^{-\theta(W-\pi_3 I_3)}\right)$$
$$B + \pi_H\left(-e^{-\theta(W-D-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4+I_4)}\right) + (1-\pi_H)\left(-e^{-\theta(W-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4)}\right) = -e^{-\theta W}$$
$$P_4 = (\lambda_1\pi_L + (1-\lambda_1)\pi_H)I_4$$
$$U_7 = B + \pi_L\left(-e^{-\theta(W-D-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4+I_4)}\right) + (1-\pi_L)\left(-e^{-\theta(W-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4)}\right)$$
$$D_5 = \frac{1-\pi_H}{\pi_H} - \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4)}}{e^{-\theta(W-D-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4+I_4)}}$$
$$D_6 = U_7 - U_6$$
$$D_7 = U_7 - U_5$$

Solution is:

$$\left\{\left[\begin{array}{c} D_5 = 1.467137501\times10^{-2}, D_6 = 1.011665336\times10^{-2}, \\ D_7 = 1.564035298\times10^{-3}, I_2 = 8.027468436\times10^{-2}, \\ I_3 = 9.180834600\times10^{-2}, I_4 = 7.994704889\times10^{-2}, \\ P_2 = 1.610987374\times10^{-2}, P_4 = 1.399073356\times10^{-2}, \\ U_5 = 1.298164096\times10^{-2}, U_6 = 4.429022896\times10^{-3}, \\ U_7 = 1.454567626\times10^{-2} \end{array}\right]\right\}$$

$I_P = 7.994704889\times10^{-2}$, $P_P = 1.399073356\times10^{-2}$

$$\frac{\theta e^{-\theta W}}{\theta e^{-\theta(W-D+(\lambda_1(1-\pi_L)+(1-\lambda_1)(1-\pi_H))I_P)} - \theta e^{-\theta(W-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_P)}} = 0.1801027864 < \frac{\lambda_2\pi_H(1-\pi_H)}{\lambda_1(1-\lambda_2)(\pi_H-\pi_L)} = 2.196254682$$

$$W + \lambda_4 t = B + (W - \pi_H D + \pi_H I_P - P_P)$$

$$B + (W - \pi_H D + (\pi_H - \pi_L)I_5 - (1-\lambda_4)t) = W + \lambda_4 t$$

$$P_5 = \pi_L I_5$$

$$U_8 = B + \pi_L\left(-e^{-\theta(W-D-P_5+I_5-(1-\lambda_4)t)}\right) + (1-\pi_L)\left(-e^{-\theta(W-P_5-(1-\lambda_4)t)}\right)$$

$$D_8 = \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-P_5-(1-\lambda_4)t)}}{e^{-\theta(W-D-P_5+I_5-(1-\lambda_4)t)}} - \frac{1-\pi_H}{\pi_H}$$

$$D_9 = -e^{-\theta(W+\lambda_4 t)} - \left(B + \pi_H\left(-e^{-\theta(W-D-P_5+I_5-(1-\lambda_4)t)}\right) + (1-\pi_H)\left(-e^{-\theta(W-P_5-(1-\lambda_4)t)}\right)\right)$$

$$U_9 = -e^{-\theta(W+\lambda_4 t)}$$

$$U_{10} = W + \lambda_4 t$$

$$D_{10} = U_8 - 1.454567626\times10^{-2}$$

$$D_{11} = U_9 - \left(-e^{-\theta W}\right)$$

$$D_{12} = W + \lambda_4 t - (B + (W - \pi_H D + \pi_H I_P - P_P))$$

Solution is:

$$\left\{\left[\begin{array}{c} t = 8.108710298\times10^{-3}, D_8 = 3.950942783\times10^{-3}, \\ D_9 = 3.887488849\times10^{-4}, D_{10} = 2.741518324\times10^{-7}, \\ D_{11} = 6.431975761\times10^{-4}, D_{12} = -3.069603385\times10^{-32}, \\ I_5 = 8.036236766\times10^{-2}, P_5 = 8.036236766\times10^{-3}, \\ U_8 = 1.454595041\times10^{-2}, U_9 = -1.046579896\times10^{-2}, \\ U_{10} = 0.1519880860 \end{array}\right]\right\}$$

$\frac{P_P}{I_P} = 0.1750000001$, $\frac{(1-\lambda_4)t+P_7}{I_7} = \frac{(1-\lambda_4)8.108710298\times10^{-3}+8.036236766\times10^{-3}}{8.036236766\times10^{-2}} = 0.176162817$

Appendix 4.3 From partial-pooling equilibrium to type 2 separating equilibrium

$u_T(W) = -e^{-\theta W}$, $u_B(W) = W$, $\pi_H = 0.4$, $\pi_L = 0.1$, $\theta = 30$, $W = 0.15$, $B = 0.04$,

$D = 0.14$, $\lambda_1 = 0.75$, $\lambda_3 = 0.4$, $\lambda_2 = \frac{\lambda_1\lambda_3}{\lambda_1(1-\lambda_3)+(1-\lambda_1)(1-\lambda_3)+\lambda_1\lambda_3} = 0.33333$,

$\lambda_4 = \frac{\lambda_1(1-\lambda_3)}{\lambda_1(1-\lambda_3)+(1-\lambda_1)(1-\lambda_3)+\lambda_1\lambda_3} = 0.5$, $\pi_1 = \lambda_1\pi_L + (1-\lambda_1)\pi_H$, $\pi_3 = (1-\lambda_3)\pi_L + \lambda_3\pi_H$

$\left(\lambda_1 = \frac{N_{LT}}{N_{LT}+N_{HB}}, 1-\lambda_1 = \frac{N_{HB}}{N_{LT}+N_{HB}}, \lambda_2 = \frac{N_{HT}}{N_{LT}+N_{HB}+N_{HT}}, \lambda_3 = \frac{N_{HT}}{N_{LT}+N_{HT}}, 1-\lambda_3 = \frac{N_{LT}}{N_{LT}+N_{HT}}, \lambda_4 = \frac{N_{LT}}{N_{LT}+N_{HB}+N_{HT}}\right)$

$U_1 = B + (W - \pi_H D)$, $U_2 = B - e^{-\theta(W-\pi_H D)}$, $U_3 = -e^{-\theta W}$, $U_4 = B - e^{-\theta(W-\pi_L D)}$

$$D_1 = \pi_H D - B$$
$$D_2 = U_3 - U_2$$
$$D_3 = U_4 - U_3$$

Solution is: $\{[D_1 = 0.016, D_2 = 8.4969\times10^{-3}, D_3 = 3.4202\times10^{-2}]\}$

$$P_1 = \pi_L I_1$$
$$B + (W - \pi_H D + (\pi_H - \pi_L)I_1) = W$$
$$D_4 = \frac{1-\pi_H}{\pi_H} - \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-P_1)}}{e^{-\theta(W-D-P_1+I_1)}}$$

Solution is: $\{[D_4 = 0.83154, I_1 = 5.3333\times10^{-2}, P_1 = 5.3333\times10^{-3}]\}$

$$B + (W - \pi_H D + \pi_H I_2 - P_2) = W$$
$$\frac{1-\pi_H}{\pi_H} = \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-P_2)}}{e^{-\theta(W-D-P_2+I_2)}}$$
$$U_5 = B + \pi_L\left(-e^{-\theta(W-D-P_2+I_2)}\right) + (1-\pi_L)\left(-e^{-\theta(W-P_2)}\right)$$
$$\frac{1-\pi_3}{\pi_3} = \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-\pi_3 I_3)}}{e^{-\theta(W-D-\pi_3 I_3+I_3)}}$$
$$U_6 = B + \pi_L\left(-e^{-\theta(W-D-\pi_3 I_3+I_3)}\right) + (1-\pi_L)\left(-e^{-\theta(W-\pi_3 I_3)}\right)$$
$$B + \pi_H\left(-e^{-\theta(W-D-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4+I_4)}\right) + (1-\pi_H)\left(-e^{-\theta(W-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4)}\right) = -e^{-\theta W}$$
$$P_4 = (\lambda_1\pi_L + (1-\lambda_1)\pi_H)I_4$$
$$U_7 = B + \pi_L\left(-e^{-\theta(W-D-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4+I_4)}\right) + (1-\pi_L)\left(-e^{-\theta(W-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4)}\right)$$
$$D_5 = \frac{1-\pi_H}{\pi_H} - \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4)}}{e^{-\theta(W-D-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4+I_4)}}$$
$$D_6 = U_7 - U_6$$
$$D_7 = U_7 - U_5$$

Solution is:

$$\left\{\begin{bmatrix} D_5 = 1.467137501\times10^{-2}, D_6 = 8.547349737\times10^{-4}, D_7 = 1.564035298\times10^{-3}, \\ I_2 = 8.027468436\times10^{-2}, I_3 = 0.1089480599, I_4 = 7.994704889\times10^{-2}, \\ P_2 = 1.610987374\times10^{-2}, P_4 = 1.399073356\times10^{-2}, \\ U_5 = 1.298164096\times10^{-2}, U_6 = 1.369094128\times10^{-2}, U_7 = 1.454567626\times10^{-2} \end{bmatrix}\right\}$$

$I_P = 7.994704889\times10^{-2}$, $P_P = 1.399073356\times10^{-2}$

$$\frac{\theta e^{-\theta W}}{\theta e^{-\theta(W-D+(\lambda_1(1-\pi_L)+(1-\lambda_1)(1-\pi_H))I_P)} - \theta e^{-\theta(W-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_P)}} = 0.1801 < \frac{\lambda_2\pi_H(1-\pi_H)}{\lambda_1(1-\lambda_2)(\pi_H-\pi_L)} = 0.53333$$

$$W + \lambda_4 t = B + (W - \pi_H D + \pi_H I_P - P_P)$$
$$B + (W - \pi_H D + (\pi_H - \pi_L) I_5 - (1-\lambda_4)t) = W + \lambda_4 t$$
$$P_5 = \pi_L I_5$$
$$U_8 = B + \pi_L\left(-e^{-\theta(W-D-P_5+I_5-(1-\lambda_4)t)}\right) + (1-\pi_L)\left(-e^{-\theta(W-P_5-(1-\lambda_4)t)}\right)$$
$$D_8 = \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-P_5-(1-\lambda_4)t)}}{e^{-\theta(W-D-P_5+I_5-(1-\lambda_4)t)}} - \frac{1-\pi_H}{\pi_H}$$
$$D_{10} = -e^{-\theta(W+\lambda_4 t)} - \left(B + \pi_H\left(-e^{-\theta(W-D-P_5+I_5-(1-\lambda_4)t)}\right) + (1-\pi_H)\left(-e^{-\theta(W-P_5-(1-\lambda_4)t)}\right)\right)$$

Solution is:

$$\left\{\begin{bmatrix} t = 3.976171992\times10^{-3}, D_8 = -0.5051449427, D_{10} = 1.027812533\times10^{-2}, \\ I_5 = 6.658723997\times10^{-2}, P_5 = 6.658723997\times10^{-3}, \\ U_8 = 1.401476008\times10^{-2} \end{bmatrix}\right\}$$

$$W + \lambda_4 t = B + (W - \pi_H D + \pi_H I_P - P_P)$$
$$B + (W - \pi_H D + \pi_H I_6 - P_6 - (1-\lambda_4)t) = W + \lambda_4 t$$
$$\frac{1-\pi_H}{\pi_H} = \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-P_6-(1-\lambda_4)t)}}{e^{-\theta(W-D-P_6+I_6-(1-\lambda_4)t)}}$$
$$U_9 = B + \pi_L\left(-e^{-\theta(W-D-P_6+I_6-(1-\lambda_4)t)}\right) + (1-\pi_L)\left(-e^{-\theta(W-P_6-(1-\lambda_4)t)}\right)$$
$$D_9 = -e^{-\theta(W+\lambda_4 t)} - \left(B + \pi_H\left(-e^{-\theta(W-D-P_6+I_6-(1-\lambda_4)t)}\right) + (1-\pi_H)\left(-e^{-\theta(W-P_6-(1-\lambda_4)t)}\right)\right)$$
$$U_{10} = -e^{-\theta(W+\lambda_4 t)}$$
$$U_{11} = W + \lambda_4 t$$
$$D_{11} = U_9 - 1.454567626\times10^{-2}$$
$$D_{12} = U_{10} - \left(-e^{-\theta W}\right)$$
$$D_{13} = U_{11} - (B + (W - \pi_H D + \pi_H I_P - P_P))$$

Solution is:

$$\left\{\begin{bmatrix} t = 3.976171992\times10^{-3}, \\ D_9 = 4.422579505\times10^{-4}, D_{11} = 2.952837045\times10^{-7}, \\ D_{12} = 6.431975761\times10^{-4}, D_{13} = -2.366582716\times10^{-30}, \\ I_6 = 8.027468436\times10^{-2}, P_6 = 1.213370175\times10^{-2}, \\ U_9 = 1.454597154\times10^{-2}, U_{10} = -1.046579896\times10^{-2}, U_{11} = 0.1519880860 \end{bmatrix}\right\}$$

$\frac{P_P}{I_P} = 0.1750000001$, $\frac{(1-\lambda_4)t+P_6}{I_6} = \frac{(1-\lambda_4)3.976171992\times10^{-3}+1.213370175\times10^{-2}}{8.027468436\times10^{-2}} = 0.1759183217$

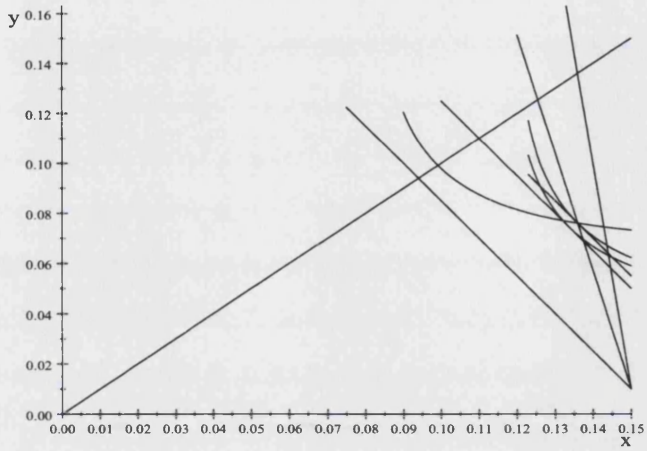

Figure 4.17 Laissez faire partial-pooling equilibrium (Scientific Workplace driven)

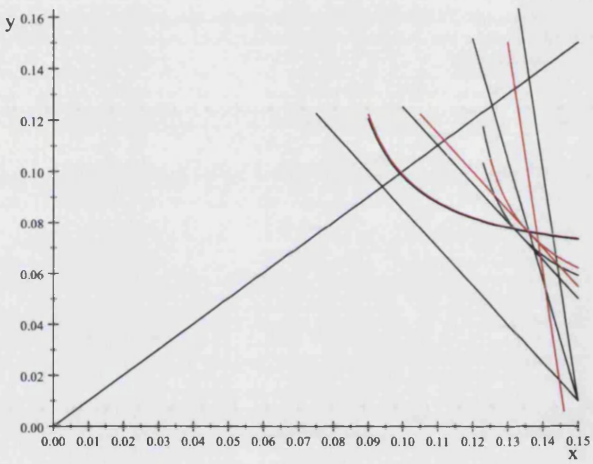

Figure 4.18 From partial-pooling to type 2 separating equilibrium (Scientific Workplace driven)

Appendix 4.4 From partial-pooling equilibrium to type 3 separating equilibrium

$u_T(W) = -e^{-\theta W}$, $u_B(W) = W$, $\pi_H = 0.4$, $\pi_L = 0.1$, $\theta = 30$, $W = 0.15$, $B = 0.04$,

$D = 0.14$, $\lambda_1 = 0.75$, $\lambda_3 = 0.4$, $\lambda_2 = \frac{\lambda_1\lambda_3}{\lambda_1(1-\lambda_3)+(1-\lambda_1)(1-\lambda_3)+\lambda_1\lambda_3} = 0.33333$,

$\lambda_4 = \frac{\lambda_1(1-\lambda_3)}{\lambda_1(1-\lambda_3)+(1-\lambda_1)(1-\lambda_3)+\lambda_1\lambda_3} = 0.5$, $\pi_1 = \lambda_1\pi_L + (1-\lambda_1)\pi_H$, $\pi_3 = (1-\lambda_3)\pi_L + \lambda_3\pi_H$

$\left(\lambda_1 = \frac{N_{LT}}{N_{LT}+N_{HB}}, 1-\lambda_1 = \frac{N_{HB}}{N_{LT}+N_{HB}}, \lambda_2 = \frac{N_{HT}}{N_{LT}+N_{HB}+N_{HT}}, \lambda_3 = \frac{N_{HT}}{N_{LT}+N_{HT}}, 1-\lambda_3 = \frac{N_{LT}}{N_{LT}+N_{HT}}, \lambda_4 = \frac{N_{LT}}{N_{LT}+N_{HB}+N_{HT}}\right)$

$U_1 = B + (W - \pi_H D)$, $U_2 = B - e^{-\theta(W-\pi_H D)}$, $U_3 = -e^{-\theta W}$, $U_4 = B - e^{-\theta(W-\pi_L D)}$

$$D_1 = \pi_H D - B$$
$$D_2 = U_3 - U_2$$
$$D_3 = U_4 - U_3$$

Solution is: $\{[D_1 = 0.016, D_2 = 8.4969\times10^{-3}, D_3 = 3.4202\times10^{-2}]\}$

$$P_1 = \pi_L I_1$$
$$B + (W - \pi_H D + (\pi_H - \pi_L)I_1) = W$$
$$D_4 = \frac{1-\pi_H}{\pi_H} - \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-P_1)}}{e^{-\theta(W-D-P_1+I_1)}}$$

Solution is: $\{[D_4 = 0.83154, I_1 = 5.3333\times10^{-2}, P_1 = 5.3333\times10^{-3}]\}$

$$B + (W - \pi_H D + \pi_H I_2 - P_2) = W$$
$$\frac{1-\pi_H}{\pi_H} = \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-P_2)}}{e^{-\theta(W-D-P_2+I_2)}}$$
$$U_5 = B + \pi_L\left(-e^{-\theta(W-D-P_2+I_2)}\right) + (1-\pi_L)\left(-e^{-\theta(W-P_2)}\right)$$
$$\frac{1-\pi_3}{\pi_3} = \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-\pi_3 I_3)}}{e^{-\theta(W-D-\pi_3 I_3+I_3)}}$$
$$U_6 = B + \pi_L\left(-e^{-\theta(W-D-\pi_3 I_3+I_3)}\right) + (1-\pi_L)\left(-e^{-\theta(W-\pi_3 I_3)}\right)$$
$$B + \pi_H\left(-e^{-\theta(W-D-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4+I_4)}\right) + (1-\pi_H)\left(-e^{-\theta(W-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4)}\right) = -e^{-\theta W}$$
$$P_4 = (\lambda_1\pi_L + (1-\lambda_1)\pi_H)I_4$$
$$U_7 = B + \pi_L\left(-e^{-\theta(W-D-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4+I_4)}\right) + (1-\pi_L)\left(-e^{-\theta(W-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4)}\right)$$
$$D_5 = \frac{1-\pi_H}{\pi_H} - \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4)}}{e^{-\theta(W-D-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4+I_4)}}$$
$$D_6 = U_7 - U_6$$
$$D_7 = U_7 - U_5$$

Solution is:

$$\left\{\begin{bmatrix} D_5 = 1.467137501\times10^{-2}, D_6 = 8.547349737\times10^{-4}, D_7 = 1.564035298\times10^{-3}, \\ I_2 = 8.027468436\times10^{-2}, I_3 = 0.1089480599, I_4 = 7.994704889\times10^{-2}, \\ P_2 = 1.610987374\times10^{-2}, P_4 = 1.399073356\times10^{-2}, \\ U_5 = 1.298164096\times10^{-2}, U_6 = 1.369094128\times10^{-2}, U_7 = 1.454567626\times10^{-2} \end{bmatrix}\right\}$$

$I_P = 7.994704889\times10^{-2}$, $P_P = 1.399073356\times10^{-2}$

$\frac{\theta e^{-\theta W}}{\theta e^{-\theta(W-D+(\lambda_1(1-\pi_L)+(1-\lambda_1)(1-\pi_H))I_P)}-\theta e^{-\theta(W-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_P)}} = 0.1801 < \frac{\lambda_2\pi_H(1-\pi_H)}{\lambda_1(1-\lambda_2)(\pi_H-\pi_L)} = 0.53333$

$W+\lambda_4 t = (B+(W-\pi_H D+\pi_H I_P-P_P))$, Solution is: $t = 3.976171992\times10^{-3}$

Set $t = 0.008$

$$B+(W-\pi_H D+(\pi_H-\pi_L)I_5-(1-\lambda_4)t) = W+\lambda_4 t$$
$$P_5 = \pi_L I_5$$
$$U_8 = B+\pi_L\left(-e^{-\theta(W-D-P_5+I_5-(1-\lambda_4)t)}\right)+(1-\pi_L)\left(-e^{-\theta(W-P_5-(1-\lambda_4)t)}\right)$$
$$D_8 = \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-P_5-(1-\lambda_4)t)}}{e^{-\theta(W-D-P_5+I_5-(1-\lambda_4)t)}}-\frac{1-\pi_H}{\pi_H}$$
$$D_{10} = -e^{-\theta(W+\lambda_4 t)}-\left(B+\pi_H\left(-e^{-\theta(W-D-P_5+I_5-(1-\lambda_4)t)}\right)+(1-\pi_H)\left(-e^{-\theta(W-P_5-(1-\lambda_4)t)}\right)\right)$$

Solution is: $\left\{\begin{bmatrix} D_8 = -1.231000601\times10^{-2}, D_{10} = -1.768029866\times10^{-3}, \\ I_5 = 0.08, P_5 = 0.008, U_8 = 1.603666982\times10^{-2} \end{bmatrix}\right\}$

$$P_6 = \pi_L I_6$$
$$B+\pi_H\left(-e^{-\theta(W-D-P_6+I_6-(1-\lambda_4)t)}\right)+(1-\pi_H)\left(-e^{-\theta(W-P_6-(1-\lambda_4)t)}\right) = -e^{-\theta(W+\lambda_4 t)}$$
$$U_9 = B+\pi_L\left(-e^{-\theta(W-D-P_6+I_6-(1-\lambda_4)t)}\right)+(1-\pi_L)\left(-e^{-\theta(W-P_6-(1-\lambda_4)t)}\right)$$
$$U_{10} = -e^{-\theta(W+\lambda_4 t)}$$
$$U_{11} = W+\lambda_4 t$$
$$D_9 = U_9-1.454567626\times10^{-2}$$
$$D_{10} = -e^{-\theta(W+\lambda_4 t)}-\left(-e^{-\theta W}\right)$$
$$D_{11} = W+\lambda_4 t-(B+(W-\pi_H D+\pi_H I_P-P_P))$$

Solution is:

$$\left\{\begin{bmatrix} D_9 = 1.109965703\times10^{-3}, D_{10} = 1.256200477\times10^{-3}, D_{11} = 2.011914004\times10^{-3}, \\ I_6 = 7.829355446\times10^{-2}, P_6 = 7.829355446\times10^{-3}, \\ U_9 = 1.565564196\times10^{-2}, U_{10} = -9.852796061\times10^{-3}, U_{11} = 0.154 \end{bmatrix}\right\}$$

$\frac{P_P}{I_P} = 0.1750000001$,

$\frac{(1-\lambda_4)t+P_6}{I_6} = \frac{(1-\lambda_4)t+7.829355446\times10^{-3}}{7.829355446\times10^{-2}} = 0.1510897740$

Appendix 4.5 From partial-pooling equilibrium to type 4 separating equilibrium

$u_T(W) = -e^{-\theta W}$, $u_B(W) = W$, $\pi_H = 0.4$, $\pi_L = 0.1$, $\theta = 30$, $W = 0.15$, $B = 0.04$,

$D = 0.14$, $\lambda_1 = 0.9$, $\lambda_3 = 0.35$, $\lambda_2 = \frac{\lambda_1\lambda_3}{\lambda_1(1-\lambda_3)+(1-\lambda_1)(1-\lambda_3)+\lambda_1\lambda_3} = 0.32642$,

$\lambda_4 = \frac{\lambda_1(1-\lambda_3)}{\lambda_1(1-\lambda_3)+(1-\lambda_1)(1-\lambda_3)+\lambda_1\lambda_3} = 0.60622$, $\pi_1 = \lambda_1\pi_L + (1-\lambda_1)\pi_H$, $\pi_3 = (1-\lambda_3)\pi_L + \lambda_3\pi_H$

$\left(\lambda_1 = \frac{N_{LT}}{N_{LT}+N_{HB}}, 1-\lambda_1 = \frac{N_{HB}}{N_{LT}+N_{HB}}, \lambda_2 = \frac{N_{HT}}{N_{LT}+N_{HB}+N_{HT}}, \lambda_3 = \frac{N_{HT}}{N_{LT}+N_{HT}}, 1-\lambda_3 = \frac{N_{LT}}{N_{LT}+N_{HT}}, \lambda_4 = \frac{N_{LT}}{N_{LT}+N_{HB}+N_{HT}}\right)$

$U_1 = B + (W - \pi_H D)$, $U_2 = B - e^{-\theta(W-\pi_H D)}$, $U_3 = -e^{-\theta W}$, $U_4 = B - e^{-\theta(W-\pi_L D)}$

$$D_1 = \pi_H D - B$$
$$D_2 = U_3 - U_2$$
$$D_3 = U_4 - U_3$$

Solution is: $\{[D_1 = 0.016, D_2 = 8.4969\times10^{-3}, D_3 = 3.4202\times10^{-2}]\}$

$$P_1 = \pi_L I_1$$
$$B + (W - \pi_H D + (\pi_H - \pi_L)I_1) = W$$
$$D_4 = \frac{1-\pi_H}{\pi_H} - \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-P_1)}}{e^{-\theta(W-D-P_1+I_1)}}$$

Solution is: $\{[D_4 = 0.83154, I_1 = 5.3333\times10^{-2}, P_1 = 5.3333\times10^{-3}]\}$

$$B + (W - \pi_H D + \pi_H I_2 - P_2) = W$$
$$\frac{1-\pi_H}{\pi_H} = \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-P_2)}}{e^{-\theta(W-D-P_2+I_2)}}$$
$$U_5 = B + \pi_L\left(-e^{-\theta(W-D-P_2+I_2)}\right) + (1-\pi_L)\left(-e^{-\theta(W-P_2)}\right)$$
$$\frac{1-\pi_3}{\pi_3} = \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-\pi_3 I_3)}}{e^{-\theta(W-D-\pi_3 I_3+I_3)}}$$
$$U_6 = B + \pi_L\left(-e^{-\theta(W-D-\pi_3 I_3+I_3)}\right) + (1-\pi_L)\left(-e^{-\theta(W-\pi_3 I_3)}\right)$$
$$B + \pi_H\left(-e^{-\theta(W-D-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4+I_4)}\right) + (1-\pi_H)\left(-e^{-\theta(W-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4)}\right) = -e^{-\theta W}$$
$$P_4 = (\lambda_1\pi_L + (1-\lambda_1)\pi_H)I_4$$
$$U_7 = B + \pi_L\left(-e^{-\theta(W-D-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4+I_4)}\right) + (1-\pi_L)\left(-e^{-\theta(W-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4)}\right)$$
$$D_5 = \frac{1-\pi_H}{\pi_H} - \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4)}}{e^{-\theta(W-D-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4+I_4)}}$$
$$D_6 = U_7 - U_6$$
$$D_7 = U_7 - U_5$$

Solution is:

$$\left\{\begin{bmatrix} D_5 = 0.23157, D_6 = 1.1074\times10^{-3}, D_7 = 3.0922\times10^{-3}, \\ I_2 = 8.0275\times10^{-2}, I_3 = 0.11194, I_4 = 7.4685\times10^{-2}, \\ P_2 = 1.6110\times10^{-2}, P_4 = 9.7091\times10^{-3}, \\ U_5 = 1.2982\times10^{-2}, U_6 = 1.4966\times10^{-2}, U_7 = 1.6074\times10^{-2} \end{bmatrix}\right\}$$

$I_P = 7.4685\times10^{-2}$, $P_P = 9.7091\times10^{-3}$

$$\frac{\theta e^{-\theta W}}{\theta e^{-\theta(W-D+(\lambda_1(1-\pi_L)+(1-\lambda_1)(1-\pi_H))I_P)}-\theta e^{-\theta(W-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_P)}} = 0.16451 < \frac{\lambda_2\pi_H(1-\pi_H)}{\lambda_1(1-\lambda_2)(\pi_H-\pi_L)} = 0.43077$$

$$W+\lambda_4 t = B+(W-\pi_H D+\pi_H I_P - P_P)$$
$$B+(W-\pi_H D+(\pi_H-\pi_L)I_5-(1-\lambda_4)t) = W+\lambda_4 t$$
$$P_5 = \pi_L I_5$$
$$U_8 = B+\pi_L\left(-e^{-\theta(W-D-P_5+I_5-(1-\lambda_4)t)}\right)+(1-\pi_L)\left(-e^{-\theta(W-P_5-(1-\lambda_4)t)}\right)$$
$$D_8 = \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-P_5-(1-\lambda_4)t)}}{e^{-\theta(W-D-P_5+I_5-(1-\lambda_4)t)}}-\frac{1-\pi_H}{\pi_H}$$
$$D_{10} = -e^{-\theta(W+\lambda_4 t)}-\left(B+\pi_H\left(-e^{-\theta(W-D-P_5+I_5-(1-\lambda_4)t)}\right)+(1-\pi_H)\left(-e^{-\theta(W-P_5-(1-\lambda_4)t)}\right)\right)$$

Solution is: $\left\{\begin{bmatrix} t = 6.8703\times10^{-3}, D_8 = -0.17123, D_{10} = 3.1220\times10^{-4}, \\ I_5 = 7.6234\times10^{-2}, P_5 = 7.6234\times10^{-3}, U_8 = 1.6113\times10^{-2} \end{bmatrix}\right\}$

$$W+\lambda_4 t = B+(W-\pi_H D+\pi_H I_P - P_P)$$
$$B+(W-\pi_H D+\pi_H I_6 - P_6-(1-\lambda_4)t) = W+\lambda_4 t$$
$$\frac{1-\pi_H}{\pi_H} = \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-P_6-(1-\lambda_4)t)}}{e^{-\theta(W-D-P_6+I_6-(1-\lambda_4)t)}}$$
$$U_9 = B+\pi_L\left(-e^{-\theta(W-D-P_6+I_6-(1-\lambda_4)t)}\right)+(1-\pi_L)\left(-e^{-\theta(W-P_6-(1-\lambda_4)t)}\right)$$
$$D_9 = -e^{-\theta(W+\lambda_4 t)}-\left(B+\pi_H\left(-e^{-\theta(W-D-P_6+I_6-(1-\lambda_4)t)}\right)+(1-\pi_H)\left(-e^{-\theta(W-P_6-(1-\lambda_4)t)}\right)\right)$$

Solution is:

$$\left\{\begin{bmatrix} t = 6.8703\times10^{-3}, D_9 = -2.1144\times10^{-3}, \\ I_6 = 8.0275\times10^{-2}, P_6 = 9.2396\times10^{-3}, U_9 = 1.6155\times10^{-2} \end{bmatrix}\right\}$$

$$W + \lambda_4 t = B + (W - \pi_H D + \pi_H I_P - P_P)$$
$$B + (W - \pi_H D + \pi_H I_7 - P_7 - (1 - \lambda_4)t) = W + \lambda_4 t$$
$$B + \pi_H\left(-e^{-\theta(W - D - P_7 + I_7 - (1-\lambda_4)t)}\right) + (1 - \pi_H)\left(-e^{-\theta(W - P_7 - (1-\lambda_4)t)}\right) = -e^{-\theta(W + \lambda_4 t)}$$
$$U_{10} = B + \pi_L\left(-e^{-\theta(W - D - P_7 + I_7 - (1-\lambda_4)t)}\right) + (1 - \pi_L)\left(-e^{-\theta(W - P_7 - (1-\lambda_4)t)}\right)$$
$$U_{11} = -e^{-\theta(W + \lambda_4 t)}$$
$$U_{12} = W + \lambda_4 t$$
$$D_{11} = U_{10} - 1.6074 \times 10^{-2}$$
$$D_{12} = U_{11} - \left(-e^{-\theta W}\right)$$
$$D_{13} = U_{12} - (B + (W - \pi_H D + \pi_H I_P - P_P))$$

Solution is:

$$\left\{\left[\begin{array}{c} t = 6.8703 \times 10^{-3}, \\ D_{11} = 4.8581 \times 10^{-5}, D_{12} = 1.3048 \times 10^{-3}, D_{13} = -3.0568 \times 10^{-30}, \\ I_7 = 7.6733 \times 10^{-2}, P_7 = 7.8230 \times 10^{-3}, \\ U_{10} = 1.6123 \times 10^{-2}, U_{11} = -9.8042 \times 10^{-3}, U_{12} = 0.15416 \end{array}\right]\right\}$$

$$\frac{P_P}{I_P} = 0.13$$

$$\frac{(1-\lambda_4)t + P_7}{I_7} = \frac{(1-\lambda_4)6.8703 \times 10^{-3} + 7.8230 \times 10^{-3}}{7.6733 \times 10^{-2}} = 0.13721$$

Appendix 4.6 Further tax

$u_T(W)=-e^{-\theta W}$, $u_B(W)=W$, $\pi_H=0.4$, $\pi_L=0.1$, $\theta=30$, $W=0.15$, $B=0.04$,

$D=0.14$, $\lambda_1=0.75$, $\lambda_3=0.4$, $\lambda_2=\frac{\lambda_1\lambda_3}{\lambda_1(1-\lambda_3)+(1-\lambda_1)(1-\lambda_3)+\lambda_1\lambda_3}=0.33333$,

$\lambda_4=\frac{\lambda_1(1-\lambda_3)}{\lambda_1(1-\lambda_3)+(1-\lambda_1)(1-\lambda_3)+\lambda_1\lambda_3}=0.5$, $\pi_1=\lambda_1\pi_L+(1-\lambda_1)\pi_H$, $\pi_3=(1-\lambda_3)\pi_L+\lambda_3\pi_H$

$\left(\lambda_1=\frac{N_{LT}}{N_{LT}+N_{HB}}, 1-\lambda_1=\frac{N_{HB}}{N_{LT}+N_{HB}}, \lambda_2=\frac{N_{HT}}{N_{LT}+N_{HB}+N_{HT}}, \lambda_3=\frac{N_{HT}}{N_{LT}+N_{HT}}, 1-\lambda_3=\frac{N_{LT}}{N_{LT}+N_{HT}}, \lambda_4=\frac{N_{LT}}{N_{LT}+N_{HB}+N_{HT}}\right)$

$U_1=B+(W-\pi_H D)$, $U_2=B-e^{-\theta(W-\pi_H D)}$, $U_3=-e^{-\theta W}$, $U_4=B-e^{-\theta(W-\pi_L D)}$

$$D_1=\pi_H D-B$$
$$D_2=U_3-U_2$$
$$D_3=U_4-U_3$$

Solution is: $\{[D_1=0.016, D_2=8.4969\times10^{-3}, D_3=3.4202\times10^{-2}]\}$

$$P_1=\pi_L I_1$$
$$B+(W-\pi_H D+(\pi_H-\pi_L)I_1)=W$$
$$D_4=\frac{1-\pi_H}{\pi_H}-\frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-P_1)}}{e^{-\theta(W-D-P_1+I_1)}}$$

Solution is: $\{[D_4=0.83154, I_1=5.3333\times10^{-2}, P_1=5.3333\times10^{-3}]\}$

$$B+(W-\pi_H D+\pi_H I_2-P_2)=W$$
$$\frac{1-\pi_H}{\pi_H}=\frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-P_2)}}{e^{-\theta(W-D-P_2+I_2)}}$$
$$U_5=B+\pi_L\left(-e^{-\theta(W-D-P_2+I_2)}\right)+(1-\pi_L)\left(-e^{-\theta(W-P_2)}\right)$$
$$\frac{1-\pi_3}{\pi_3}=\frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-\pi_3 I_3)}}{e^{-\theta(W-D-\pi_3 I_3+I_3)}}$$
$$U_6=B+\pi_L\left(-e^{-\theta(W-D-\pi_3 I_3+I_3)}\right)+(1-\pi_L)\left(-e^{-\theta(W-\pi_3 I_3)}\right)$$
$$B+\pi_H\left(-e^{-\theta(W-D-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4+I_4)}\right)+(1-\pi_H)\left(-e^{-\theta(W-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4)}\right)=-e^{-\theta W}$$
$$P_4=(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4$$
$$U_7=B+\pi_L\left(-e^{-\theta(W-D-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4+I_4)}\right)+(1-\pi_L)\left(-e^{-\theta(W-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4)}\right)$$
$$D_5=\frac{1-\pi_H}{\pi_H}-\frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4)}}{e^{-\theta(W-D-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_4+I_4)}}$$
$$D_6=U_7-U_6$$
$$D_7=U_7-U_5$$

Solution is:

$$\left\{\begin{bmatrix} D_5 = 1.467137501\times10^{-2}, D_6 = 8.547349737\times10^{-4}, D_7 = 1.564035298\times10^{-3}, \\ I_2 = 8.027468436\times10^{-2}, I_3 = 0.1089480599, I_4 = 7.994704889\times10^{-2}, \\ P_2 = 1.610987374\times10^{-2}, P_4 = 1.399073356\times10^{-2}, \\ U_5 = 1.298164096\times10^{-2}, U_6 = 1.369094128\times10^{-2}, U_7 = 1.454567626\times10^{-2} \end{bmatrix}\right\}$$

$I_P = 7.994704889\times10^{-2}$, $P_P = 1.399073356\times10^{-2}$

$$\frac{\theta e^{-\theta W}}{\theta e^{-\theta(W-D+(\lambda_1(1-\pi_L)+(1-\lambda_1)(1-\pi_H))I_P)} - \theta e^{-\theta(W-(\lambda_1\pi_L+(1-\lambda_1)\pi_H)I_P)}} = 0.1801 < \frac{\lambda_2\pi_H(1-\pi_H)}{\lambda_1(1-\lambda_2)(\pi_H-\pi_L)} = 0.53333$$

$$\begin{gathered} W + \lambda_4 t = B + (W - \pi_H D + \pi_H I_P - P_P) \\ B + (W - \pi_H D + (\pi_H - \pi_L) I_5 - (1-\lambda_4)t) = W + \lambda_4 t \\ P_5 = \pi_L I_5 \\ U_8 = B + \pi_L\left(-e^{-\theta(W-D-P_5+I_5-(1-\lambda_4)t)}\right) + (1-\pi_L)\left(-e^{-\theta(W-P_5-(1-\lambda_4)t)}\right) \\ D_8 = \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-P_5-(1-\lambda_4)t)}}{e^{-\theta(W-D-P_5+I_5-(1-\lambda_4)t)}} - \frac{1-\pi_H}{\pi_H} \end{gathered}$$

Solution is: $\left\{\begin{bmatrix} t = 3.9756\times10^{-3}, D_8 = -0.5052, \\ I_5 = 6.6585\times10^{-2}, P_5 = 6.6585\times10^{-3}, U_8 = 1.4014\times10^{-2} \end{bmatrix}\right\}$

$$\begin{gathered} W + \lambda_4 t = B + (W - \pi_H D + \pi_H I_P - P_P) \\ B + (W - \pi_H D + \pi_H I_6 - P_6 - (1-\lambda_4)t) = W + \lambda_4 t \\ \frac{1-\pi_H}{\pi_H} = \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-P_6-(1-\lambda_4)t)}}{e^{-\theta(W-D-P_6+I_6-(1-\lambda_4)t)}} \\ U_9 = B + \pi_L\left(-e^{-\theta(W-D-P_6+I_6-(1-\lambda_4)t)}\right) + (1-\pi_L)\left(-e^{-\theta(W-P_6-(1-\lambda_4)t)}\right) \\ D_9 = -e^{-\theta(W+\lambda_4 t)} - \left(B + \pi_H\left(-e^{-\theta(W-D-P_6+I_6-(1-\lambda_4)t)}\right) + (1-\pi_H)\left(-e^{-\theta(W-P_6-(1-\lambda_4)t)}\right)\right) \end{gathered}$$

Solution is:

$$\left\{\begin{bmatrix} t = 0.0039756, D_9 = 4.426049417\times10^{-4}, \\ I_6 = 8.027468436\times10^{-2}, P_6 = 1.213427374\times10^{-2}, U_9 = 1.454575315\times10^{-2} \end{bmatrix}\right\}$$

$\frac{P_P}{I_P} = 0.1750000001$, $\frac{(1-\lambda_4)t+P_6}{I_6} = \frac{(1-\lambda_4)0.0039756+1.213427374\times10^{-2}}{8.027468436\times10^{-2}} = 0.1759218844$

$W + \lambda_4 t = B + (W - \pi_H D + \pi_H I_P - P_P)$, Solution is : $t = 3.9756\times10^{-3}$

Set $t = 0.0045$

$$\begin{gathered} B + (W - \pi_H D + (\pi_H - \pi_L) I_5 - (1-\lambda_4)t) = W + \lambda_4 t \\ P_5 = \pi_L I_5 \\ U_8 = B + \pi_L\left(-e^{-\theta(W-D-P_5+I_5-(1-\lambda_4)t)}\right) + (1-\pi_L)\left(-e^{-\theta(W-P_5-(1-\lambda_4)t)}\right) \\ D_8 = \frac{1-\pi_L}{\pi_L}\frac{e^{-\theta(W-P_5-(1-\lambda_4)t)}}{e^{-\theta(W-D-P_5+I_5-(1-\lambda_4)t)}} - \frac{1-\pi_H}{\pi_H} \\ D_{10} = -e^{-\theta(W+\lambda_4 t)} - \left(B + \pi_H\left(-e^{-\theta(W-D-P_5+I_5-(1-\lambda_4)t)}\right) + (1-\pi_H)\left(-e^{-\theta(W-P_5-(1-\lambda_4)t)}\right)\right) \end{gathered}$$

$$\text{Solution is: } \left\{ \begin{bmatrix} D_8 = -0.45164258, D_{10} = 8.466574595 \times 10^{-3}, \\ I_5 = 6.833333333 \times 10^{-2}, P_5 = 6.833333333 \times 10^{-3}, \\ U_8 = 1.434577182 \times 10^{-2} \end{bmatrix} \right\}$$

$$B + \left(W - \pi_H D + \pi_H I_6 - P_6 - (1-\lambda_4)t\right) = W + \lambda_4 t$$

$$\frac{1-\pi_H}{\pi_H} = \frac{1-\pi_L}{\pi_L} \frac{e^{-\theta(W-P_6-(1-\lambda_4)t)}}{e^{-\theta(W-D-P_6+I_6-(1-\lambda_4)t)}}$$

$$U_9 = B + \pi_L\left(-e^{-\theta(W-D-P_6+I_6-(1-\lambda_4)t)}\right) + (1-\pi_L)\left(-e^{-\theta(W-P_6-(1-\lambda_4)t)}\right)$$

$$D_9 = -e^{-\theta(W+\lambda_4 t)} - \left(B + \pi_H\left(-e^{-\theta(W-D-P_6+I_6-(1-\lambda_4)t)}\right) + (1-\pi_H)\left(-e^{-\theta(W-P_6-(1-\lambda_4)t)}\right)\right)$$

$$D_{11} = U_9 - 1.454575315 \times 10^{-2}$$

$$D_{12} = W + \lambda_4 t - \left(B + \left(W - \pi_H D + \pi_H I_P - P_P\right)\right)$$

$$D_{13} = -e^{-\theta(W+\lambda_4 t)} - \left(-e^{-\theta\left(W+\lambda_4 3.9756\times10^{-3}\right)}\right)$$

$$\text{Solution is: } \left\{ \begin{bmatrix} D_9 = 1.25731309 \times 10^{-4}, D_{11} = 1.994376892 \times 10^{-4}, \\ D_{12} = 2.61914004 \times 10^{-4}, D_{13} = 8.200174529 \times 10^{-5}, \\ I_6 = 8.027468436 \times 10^{-2}, P_6 = 1.160987374 \times 10^{-2}, \\ U_9 = 1.474519084 \times 10^{-2} \end{bmatrix} \right\}$$

When $t = 3.9756 \times 10^{-3}$

The H_B: $B + \left(W - \pi_H D + \pi_H I_P - P_P\right) = 0.1519880860$,

The H_T: $-e^{-\theta(W+\lambda_4 t)} = -e^{-\theta\left(W+\lambda_4 3.9756\times10^{-3}\right)} = -1.046588876 \times 10^{-2}$

The L_T: $B + \pi_L\left(-e^{-\theta(W-D-P_6+I_6-(1-\lambda_4)t)}\right) + (1-\pi_L)\left(-e^{-\theta(W-P_6-(1-\lambda_4)t)}\right) = 1.454575315 \times 10^{-2}$

When $t = 0.0045$

The H_B: $W + \lambda_4 t = 0.15225$

The H_T: $-e^{-\theta(W+\lambda_4 t)} = -1.038388701 \times 10^{-2}$

The L_T: $B + \pi_L\left(-e^{-\theta(W-D-P_6+I_6-(1-\lambda_4)t)}\right) + (1-\pi_L)\left(-e^{-\theta(W-P_6-(1-\lambda_4)t)}\right) = 1.474519084 \times 10^{-2}$

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