Essays in Empirical Macroeconomics

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Declaration

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Abstract

This thesis contains three chapters. The first two chapters are essays on monetary economics. The last chapter is an essay on general equilibrium asset pricing.

In chapter 1, I study the behavior of disaggregated prices in response to economic shocks. I suggest a production chain model with nominal rigidities to replicate some stylized facts about data. I argue, first, that the input-output linkages in production can create heterogeneity in the response of sectoral prices to aggregate shocks. Second, a realistic calibration of this multi-sector model to the US data can create 5 times more real rigidities in response to nominal shocks, compared to an equivalent homogeneous economy with intermediate inputs. Finally, the model implies that upstream industries would respond faster to aggregate shocks than downstream industries.

In chapter 2, I study the effect of imperfect commitment of a central bank on inflationary outcomes. I present a model in which monetary authority is a committee with churning of members who have finite terms. Older and younger generations of Monetary Policy Committee (MPC) members decide on policy by engaging in a bargaining process. I show that this set-up gives rise to a continuous measure of the degree of monetary authority's commitment. The model suggests that lowering the churning rate or increasing the tenure time improves welfare.

Chapter 3 (joint work with Aytek Malkhozov) focuses on the asset pricing implications of a real-business-cycle model with recursive preferences and a general shock structure that allows for news shocks. We show that introducing recursive preferences and anticipated shocks into a canonical DSGE model can produce large premia and low risk-free rates without compromising the model's ability to fit the key macroeconomic variables. We illustrate how this class of dynamic stochastic general equilibrium (DSGE) models can be solved using higher order perturbation methods.
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Preface

This thesis presents three essays in empirical macroeconomics. The first two chapters broadly fall under monetary economics. The final chapter deals with modeling asset prices in a general equilibrium setting.

The first chapter of this thesis focuses on reconciling the high frequency of price changes at the micro level, and the apparent rigidity at the aggregate level that has been the subject of considerable debate in macroeconomics recently.

Much applied work in monetary economics relies on models in which nominal rigidities are the key friction generating monetary non-neutrality. However, studies calibrating models based on newly available micro-data on the frequency of price changes conclude that nominal rigidities cause very little monetary non-neutrality. This conclusion poses a serious challenge for our understanding of the transmission of monetary policy. It is therefore of great importance for monetary economics to explore whether richer models can be consistent with both of the stylized facts above.

The literature has enriched the basic monetary models to bring them closer to the reality, for instance by including intermediate inputs and heterogeneity between goods in several dimensions. In chapter 1, I explore how a production chain structure in the economy can translate small rigidities at the micro level into large rigidities at the aggregate level. I argue that along a production chain the marginal cost of firms depends on the prices of their material inputs, i.e. the prices of other firms' output in the economy. Therefore, despite individual prices adjusting relatively quickly to changes in their own marginal cost, the accumulation of small lags along the production chain will lead to large lags at the aggregate level. I formalize this idea in a model with a multi-sector economy.

An important implication of this model is that upstream industries (such as crude materials and agricultural products) would be the first to respond to aggregate shocks, whereas downstream industries (such as consumption goods) responds much more slowly. I show that this broad pattern is supported by the data.

The second chapter looks at how the institutional design of central banks can influence monetary policy outcomes. Despite the popularity of committee-based central banks, and the usually complicated structure of these committees, the literature on monetary policy has mostly focused on a single, infinitely-lived central banker. In reality monetary policy committees are composed of several members whose tenures overlap and enter and exit the committee at different times. Chapter 2 proposes a model where the set-up of the monetary policy committee is closer to reality. In particular I assume that the members of the monetary policy committee have finite,
overlapping tenures and study the implications of this institutional design on monetary outcomes.

The reason that the outcomes would be different under such a committee is that the incoming ("young") and existing ("old") policy makers face different expectations. An incoming committee member has not had the opportunity to influence the expectations about his performance. Therefore, he takes expectations about future path of policy as given and hence faces a worse trade-off between inflation and output gap. On the other hand, a committee member who has served before has had the opportunity to commit to certain future (state-contingent) policies. I assume that the differences between these two groups of committee members are resolved through a bargaining process, and I study the welfare implications of different compositions of monetary committees.

The main implication of the model is that slower the replacement rate of the committee members (referred to as the churning rate), the closer the monetary outcomes would be to the optimal policy under commitment.

The third chapter (joint work with Aytek Malkhozov) studies the asset pricing implications of the "long-run risks" class of asset pricing models, in a real business cycle (RBC) setting. Whereas the standard time-separable utility model parsimoniously links the returns of all assets to per capita consumption growth through the Euler equation of consumption, per capita consumption growth covaries too little with the returns of most classes of financial assets, creating the familiar asset pricing puzzles. Several generalizations of essential features of the model have been proposed to mitigate its poor performance. In particular, Bansal and Yaron (2004) introduce a long-run risk state variable that simultaneously drives aggregate consumption growth and aggregate dividend growth. In conjunction with Epstein-Zin preferences, the long-run risk state variable has a rich set of pricing implications.

In this chapter, we investigate whether long-run risk can arise endogenously by including a richer structure of news about productivity shocks in an RBC model. Anticipated shocks are news about movements in the productivity process that materialize in the future. We show that anticipated shocks can generate a significant amount of long-run risk. This is because shocks to expected productivity growth will translate into shocks to expected consumption growth which are priced when preferences are defined recursively à la Epstein and Zin (1989). Hence, enriching the standard RBC model along two dimensions (incorporating news and assuming recursive preferences) generates much larger premia compared to the standard RBC model.

The methodological contribution of this paper is also noteworthy. We show how to solve a large class of asset pricing models with Epstein-Zin preferences using a second order approximation. This method significantly improves the computational
efficiency in models with a large number of state variables, compared to the often used alternative, i.e. value function iteration.

A calibration of our benchmark model matches important macro indicators such as consumption growth level and volatility and investment volatility as a share of output volatility. The model also matches the financial data well. The implied level of risk free rate and its volatility are in line with the data. Expected premia over risk free rate are on average 4.5% annually. These results are even more remarkable given that we do not resort to extreme values of risk aversion.
1 Price Setting in a Model with Production Chains: Evidence from Sectoral Data

1.1 Introduction

Reconciling the high frequency of price changes at the micro level, and the apparent rigidity at the aggregate level has been the subject of considerable debate in macroeconomics recently. It has been shown that a model of price adjustment with some type of nominal rigidity, such as menu-costs or Calvo type rigidities, calibrated to match the frequency of individual price changes, fails to deliver aggregate nominal rigidities that are consistent with typical VAR studies (See, for example, Golosov and Lucas (2007) for an empirical documentation of this fact).1

In a recent development, Boivin, Giannoni, and Mihov (2009) (henceforth BGM) offer an explanation for the apparent discrepancy: They decompose the fluctuations in prices into aggregate versus sector-specific components, and show that even disaggregated prices appear sticky in response to aggregate shocks whereas they are flexible in response to sector-specific disturbances. Therefore, the observed flexibility of disaggregated prices, as reported by Bils and Klenow (2005), Nakamura and Steinsson (2008a), and others, is not necessarily at odds with the results of typical VAR studies.

Furthermore, BGM show that there is significant heterogeneity in the speed of response to aggregate shocks, such as monetary policy shocks, whereas the speed of response of disaggregated prices to own sector specific disturbances are similar across sectors. For instance they report a 11% standard deviation of price adjustment (relative to the price level before the shock) across all sectors six months after a monetary policy shock has occurred (average adjustment over the same period is 6%), whereas following a sector specific shock nearly all sectors respond fully within the first 6 months.

The different nature of response of firms to aggregate versus idiosyncratic shocks can be an explanation for the discrepancy in the frequency of price adjustment at the micro and macro level. But what mechanism causes such difference? In this paper I explore a possible explanation for this observation. I argue that the existence of a structure which amplifies the small nominal rigidities at the firm level, could deliver large nominal rigidities at the aggregate level. The particular structure that I have in mind is a production chain. Along a production chain, the marginal cost of firms

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1Many papers, beginning with Caplin and Spulber (1987), argue this point theoretically. Caballero and Engel (2007) offer a very useful discussion of this literature.
depends on the prices of their material inputs, i.e. the prices of other firms in the economy. Therefore, despite individual prices adjusting relatively quickly to changes in their own marginal cost, the accumulation of small lags along the production chain will lead to large lags at the aggregate level. I formalize this idea in a model with a multi-sector economy where firms face Calvo-type nominal rigidities.

An important implication of the model is that there is heterogeneity across sectors in response to aggregate shocks, based on where a sector is situated along the production chain. On the other hand all sectors respond to their own sectoral shocks quickly. This is precisely in line with the findings of BGM. Furthermore, the model would suggest that upstream industries (such as crude materials and agricultural products) would be the first to respond to aggregate shocks, whereas downstream industries (such as consumption goods) responds much more slowly. I will show that this broad pattern is supported by the data.

The appealing feature of this model is that it can substantially add to the aggregate nominal rigidity without making prices too sticky at micro-level. Nakamura and Steinsson (2008b) emphasize this fact in a multi-sector menu cost model. They show that adding input-output linkages will substantially increase the aggregate price rigidity. I show in this paper, that heterogeneity in the degree of intermediate inputs use increases the real rigidities even further. Furthermore, a realistic calibration of the model shows that heterogeneity in “inherent” stickiness (captured here by Calvo adjustment frequency) is reinforced by heterogeneity in the material inputs share. Under the most general (and realistic) calibration, the model can produce real rigidities which are five times larger than the equivalent economy, with intermediate inputs.

A related paper by Mackowiak and Wiederholt (2007) develops a model to address the differential response to aggregate vs. idiosyncratic shocks. In their model price setting firms decide what to pay attention to, subject to a constraint on the information flow. When idiosyncratic conditions are more variable or more important than aggregate conditions, firms pay more attention to idiosyncratic conditions. This implies prices react fast and by large amounts to idiosyncratic shocks, but only slowly and by small amounts to idiosyncratic shocks, which is consistent with the results found in BGM. The model I present here obtains the same conclusions but by relying on the real-side features of the economy.

Carvalho (2006) shows that monetary shocks tend to have larger and more persistent real effects in heterogeneous economies, when compared to identical-firms economies with similar degrees of nominal and real rigidity.

These results are not typical and crucially depend on assumptions about information structure. For instance, Woodford (2002) assumes that firms pay little attention to aggregate conditions, if these signals are noisy. Mankiw and Reis (2002) develop a different model in which information disseminates slowly. In their model, prices respond with equal speed to all disturbances.
The paper is organized as follows. In section 2, I present the model and discuss the solution method. In section 3, I calibrate the model. I start from a very special case of the model: A two sector example, in which one good is purely an intermediate good and the other purely a consumption good. In developing this special example, I would like to isolate the effect I am interested in, i.e. the differential response of sectoral prices to sector-specific vs. aggregate shocks. I will then present a more realistic, 6-sector calibration of the US economy. I relax the assumption of symmetry across sectors in the make-up of consumption and intermediate input goods. I also add heterogeneity in the frequency of price adjustment across sectors. I will examine two of the model implications discussed above, using this calibrated version. The first implication relates to the different nature of the response to aggregate versus idiosyncratic shocks, as observed empirically by BGM. The second implication, is the ability of the model to create real rigidities in response to monetary policy shocks. I compare the fully calibrated model with equivalent homogeneous economies. I show that the production chain effect reinforces the heterogeneity in price adjustment frequencies effect. In section 4, I present some empirical evidence supporting the third implication of the model. The model suggests that “up-stream” industries respond faster to aggregate shocks than “down-stream” industries. Using disaggregated data from manufacturing, I find a significant negative relationship between the position down the production chain and the speed of response to monetary policy and oil price shocks. I conclude in section 6.

1.2 Model

1.2.1 Households

The model is a multi-sector version of the workhorse New Keynesian model with monopolistic competition presented in Woodford (2003), chapter 2, or Walsh (2003), chapter 5 (among many others). The economy is populated by identical, infinitely lived households of measure 1 and an infinite number of firms in a J-sector economy. The representative household maximizes a lifetime utility function, specified as follows:

\[
E_t \sum_{\tau=0}^{\tau=\infty} \beta^\tau \left[ \frac{C_{t+\tau}^{1-\sigma}}{1-\sigma} - \chi \frac{L_{t+\tau}^{1+\eta}}{1+\eta} \right]
\]

where \( E_t \) denotes the expectations operator conditional on information known at time \( t \), \( C_t \) denotes the household consumption of a composite consumption good and \( L_t \) denotes the household supply of labor. Households own the firms in this economy which means that they receive the profits earned by the firms. Markets are complete and therefore the household’s budget constraint may be written as:

\[
C_t + E_t [\Delta_{t,t+1} B_{t+1}] \leq \frac{W_t L_t}{P_t} + \frac{B_t}{P_t} + \sum_{j=1}^{J} \Pi_t^j
\]
where $B_{t+1}$ is the stochastic payoff of securities purchased at time $t$, $\Delta_{t,t+1}$ is the stochastic discount factor, $W_t$ is the wage at time $t$ and $\Pi_j^t$ denotes total real profits earned by sector $j$. Wages are assumed to be flexible (I will discuss the implications of relaxing this assumption in section 1.3.1).

The household’s composite consumption good is an aggregator over the variety of all the goods available in the economy:

$$C_t = \prod_{j=1}^{J} \left( \varepsilon^j \right)^{-\varepsilon^j} \left( \Pi_j^t \right)^{\varepsilon^j}$$

where $C_j^t$ denotes the household’s consumption of the good produced by sector $j$ and $\varepsilon_j$’s are a vector of weights associated with each sector in the consumption basket of the household and they satisfy $\sum_{j=1}^{J} \varepsilon^j = 1$. The Cobb-Douglas functional form assumed is a special case of CES aggregator with a unit elasticity of substitution. In this specification, I follow Bouakez, Cardia, and Ruge-Murcia (2009). The advantage of this specification is that the weights $\varepsilon^j$ are equal to the household’s sectoral expenditures shares, which can be easily obtained from the sectoral break-up of Personal Consumption Expenditure, reported by BEA.

Let $P_j^t$ be the price of the good produced by sector $j$, and $P_t$ the aggregate price level in period $t$, defined as

$$P_t = \prod_{j=1}^{J} \left( P_j^t \right)^{\varepsilon^j}$$

Then, the cost minimization problem of the household implies that the household’s demand for the good produced by sector $j$, $C_j^t$, is given by:

$$C_j^t = \varepsilon^j \left( \frac{P_j^t}{P_t} \right)^{-1} C_t$$

(1.1)

Note that the definition above for the aggregate price level also implies that $\sum_{j=1}^{J} P_j^t C_j^t = P_t C_t$.

Once the household has decided the cost minimizing composition of its consumption basket, given the consumption of the aggregate good, it will choose the optimal total consumption expenditure and labor supply. The first order conditions are standard:

$$\Delta_{t,t+\tau} = \beta^\tau \frac{C_t^{t+\tau}}{C_t}$$

---

4 The Cobb-Douglas assumption for consumption aggregator and intermediate input aggregator is not essential. All the results will be the same to the first order if a CES aggregator with a non-unity elasticity of substitution is used instead.
Note that the last equation is implied by the assumption about flexible wages.

1.2.2 Firms

The $J$ differentiated goods in the economy are produced by one of the $J$ monopolistically competitive sectors. Each sector itself is composed of a continuum of firms of measure one, who produce goods that are imperfect substitutes. These goods are aggregated by a competitive sector into sector $j$'s output. In the interest of brevity, the firm level analysis is included in appendix (A.2). \(^5\) Firms are indexed by $z$. The representative firm in sector $j$, has a production technology as follows:

$$y_j^j(z) = \left( A_j^j L_j^j(z) \right)^{\rho_j} M_j^j(z)^{1-\rho_j}$$

where $A_j^j$ is the sector-specific stochastic level of technology. $M_j^j(z)$ is an intermediate input, itself a CES aggregator of all the goods produced in the economy. \(^6\) These goods are combined to form the sector-specific intermediate input according to

$$M_j^j(z) = \left[ \prod_{i=1}^{J} \left( \frac{1}{z^i_j} \right)^{c_i^j} \left( m_{i,t}^j(z) \right)^{-c_i^j} \right]$$

where $m_{i,t}^j(z)$ is the quantity of input $i$ purchased by firm $z$ in sector $j$. $c_i^j$ is the weight of input $i$ in sector $j$. Define the price of the intermediate input for industry $j$ as:

$$X_j^j = \left[ \prod_{i=1}^{J} (P_i^j)^{c_i^j} \right]$$

Given that goods from different sectors are imperfect substitutes in the production function of firms, the demand for each good by other firms depends on its price. Isomorphically to the consumer's problem, cost minimization by firm $z$ in sector $j$ implies that its demand for the goods produced by sector $i$ is determined by:

$$m_{i,t}^j(z) = c_i^j \left( \frac{P_i^j}{X_j^j} \right)^{-1} M_j^j(z) \quad (1.2)$$

\(^5\) The assumption that each sector is made up of a large number of firms is needed for two reasons: First, for the purposes of calibration, I would like to be able to use the model where $J$, the number of sectors, is not necessarily very large. If sectors were populated by a single firm, the assumption that sectors take the aggregate prices in the economy as given will become hard to justify. The second reason is that an infinite number of firms existing within each sector allows for deriving sector specific Philips curves.

\(^6\) In my notation, I use superscripts to refer to the recipient industry and subscripts to the donor industry. So for instance, $M_j^j(z)$ refers to inputs used by firm $z$ in sector $j$ and $m_{i,t}^j(z)$, refers to inputs produced by sector $i$ and used by firm $z$ in sector $j$. 
Given the definition of $X_i^j$ it can be shown that $\sum_{i=1}^{J} P_i^j m_{i,j}(z) = X_i^j M_i^j(z)$.

**Market Clearing** Imposing a market clearing condition for each firm, and each sector and using demand functions (1.1) and (1.2) one can show that:

$$y_i^j(z) = \left( \frac{p_i^j(z)}{P_i^j} \right)^{-\theta} Y_i^j$$

where $Y_i^j = C_i^j + \sum_{i=1}^{J} \int_{0}^{1} m_{i,j}(z')dz'$ and $\theta$ is the elasticity of substitution between goods within the same sector. See appendix (A.2) for the proof.

A few points are worth noting: First, following Basu (1995) I have used a “round-about” model of intermediate goods, in that all goods could potentially be used as an intermediate input and a consumption good. Second, the assumption that the elasticity of substitution between goods is the same for consumption and for production, means that the price elasticity of demand for a good does not depend on its use, and therefore there is no distinction, from a producer’s point of view, in the two uses for its output and hence no price discrimination based on the product’s use. Also note that a reasonable choice for $\theta$ would imply that $\theta \geq 1$. This implies that the elasticity of substitution between goods within a sector is at least as large as that for goods from different sectors, which is a desirable assumption.

Second, note that whereas all the firms within a sector are identical in the steady state, firms in different sectors are heterogeneous in a number of dimensions: 1) their production functions differ in the intensity with which they use different factors of production, 2) the combination of goods used as material inputs can potentially differ and 3) they differ in the level of their technology. Therefore in the steady state the relative prices of goods produced by firms within a sector will always be 1, whereas goods from different sectors will in general have different prices even in the steady state.

Finally, the concept of “production chain” in this paper is related to the difference in the production function of different sectors. In particular, the higher is the $s_j$ (the share of labor in production) the lower would be the dependence on other firms’ output in production. I rank industries along the production chain according to their corresponding $s_j$’s. The higher the $s_j$ the earlier in the chain an industry would be.

Note that there is a close relationship between this definition of a production chain and one in which the chain is defined along a temporal dimension. By the latter, I mean a model which assumes upstream firms’ output can only be used by more downstream firms with a time lag. Such a model will assume that quantities are fixed and therefore
prices adjust to clear the markets in the period following production. In the model that I present here, the assumption is that prices may be fixed after the realization of a shock, and therefore quantities have to adjust in order to clear the market. Thus, the two models are analytically analogous. Yet, the assumption I have chosen allows for a comparison between the results presented here and the New Keynesian literature.

Firms’ Price Setting  Firms face price rigidities of the form described by Calvo (1983). Specifically, in each period $1 - \omega_j$ fraction of the firms in sector $j$ get to adjust their prices, whereas the remaining $\omega_j$ fraction do not. Those firms who do adjust their prices do so to maximize the expected discounted value of current and future profits, discounted both by the stochastic discount factor and by the probability of survival of the current price. Therefore, the firm’s problem can be written as maximizing:

$$\max_{m^i_{t+\tau}(z), p^i_t(z), L^i_t(z)} E_t \sum_{\tau=0}^{\infty} \omega_j^{\tau} \Delta_{t,t+\tau} \Pi^i_{t+\tau}(z)$$

subject to the production function and total demand for the good produced by firm $z$. Period profits of firm $z$ in sector $j$ as a function of the price it sets for its output is defined as:

$$\Pi^i_{t}(z) = p^i_t(z) y^i_t(z) - W_t L^i_t(z) - X^i_t M^i_t(z)$$

Substituting the demand for a firm’s output and optimal choice of inputs, the firm’s problem can be written as choosing the optimal price $\hat{p}^i_t(z)$, to maximize

$$E_t \sum_{\tau=0}^{\infty} \omega^{\tau} (j) \Delta_{t,t+\tau} \left[ \hat{p}^i_t(z) \left( \frac{\tilde{p}^i_t(z)}{p^j_{t+\tau}} \right)^{-\theta} - \Psi^j_{t+\tau} \left( \frac{\tilde{p}^i_t(z)}{p^j_{t+\tau}} \right)^{-\theta} \right] Y^j_{t+\tau}$$

where $\Psi^j_{t+\tau}$ is the nominal marginal cost of a firm in sector $j$. The cost-minimization of the firm implies that $\Psi_t^j = \frac{1}{s_j} \left( \frac{W_t}{A_t^j} \right)^{s_j} \left( X^j_t \right)^{1-s_j} \left( \frac{s_j}{1-s_j} \right)^{1-s_j}$. The interpretation is that the nominal marginal cost is a weighted average of the effective wage and the price of the intermediate good. The higher $1 - s_j$ (the further down the production chain an industry is), the higher the dependence on price of intermediate inputs. An industry “inherits” the stickiness of its suppliers through the dynamics of $X^j_t$.

1.2.3 Monetary Policy and Shocks

The monetary authority acts so as to make nominal GDP follow a random walk with drift in logs. Denote the nominal GDP by $S_t = P_t C_t$. Then,

$$\log S_t = \log S_{t-1} + \nu_t$$

where

$$\nu_t = \rho \nu_{t-1} + \epsilon_{\nu,t}$$
1 PRICE SETTING WITH PRODUCTION CHAINS

and $\epsilon_{e,t}$ are white noise innovations with variance $\sigma^2_e$. $\rho_v$ is strictly smaller than 1. The stochastic level of technology in each sector follows a random walk process:

$$\ln(A^j_t) = \ln(A^j_{t-1}) + \epsilon^j_{A,t}$$

where $\epsilon^j_{A,t}$ are sector specific white noise innovations, uncorrelated across sectors and with variance $\sigma^2_{A,j}, \epsilon^j_{A,t}$ and $\epsilon_{e,t}$ are independent processes.

1.2.4 Linearized Steady State

Log-linearizing the optimal pricing decision of a firm around a zero inflation, zero output growth steady state, the price setting dynamics implies a Phillips curve relation for each sector $j$ such that

$$\pi^j_t = \beta E_t \pi^j_{t+1} + \kappa^j_p \left[ \varphi^j_t - p^j_t \right]$$

where $\pi^j_t = p^j_t - p^j_{t-1}$ is the change in sector $j$'s (log) price from $t - 1$ to $t$. $\varphi^j_t$ is the deviation of the nominal marginal cost from its steady state. The derivation is presented in the appendix (A.2).

1.3 Calibration

In calibrating the model, I begin by choosing some benchmark parameters which will remain fixed throughout all the calibration exercises presented below. Table (1.1) shows the choice of these parameters. For the consumer's preferences, I assume log utility in consumption and a linear disutility of labor ($\sigma = 1, \eta = 0$). Assuming log utility allows for the existence of a balanced growth path with non-stationary technology shocks in a multi-sector setting (see Ngai and Pissarides (2007)). The assumption on linear labor disutility can be interpreted as indivisible labor with lotteries, following Hansen (1985). To calibrate the discount rate, I choose an annual interest rate of 3% which corresponds to a monthly value for $\beta = 0.9975$.

I choose $\theta = 8$ for the elasticity of substitution between goods within a sector. This value for $\theta$ places it in the middle of the range used in the literature. Nakamura and Steinsson (2008b) use $\theta = 4$. This rather low estimate for $\theta$ allows them to have a higher implied intermediate input share in the production function (see the calibration of intermediate input shares below) and thus create greater real rigidities. Carvalho (2006) uses $\theta = 5$ and $\theta = 11$ as a lower and upper bound and Golosov and Lucas (2007) use $\theta = 7$. The choice of $\theta = 8$ implies a markup of $\mu = 1.14$, which if interpreted as profits, is a realistic estimate for the U.S. economy. Estimates of mark-ups typically fall in the 10 – 20 percent range, implying values of $\theta$ in the 6 – 10 range.\(^7\) Also note

\(^7\)See Rotemberg and Woodford (1993) and Basu and Fernald (1997).
that $\theta = 8$ is larger than the elasticity of substitution between goods from different sectors (assumed to be 1), which is a reasonable assumption.

To calibrate the characteristics of monetary policy shocks, I estimate an $I(1)$ model for the quarterly U.S. nominal GDP during the period 1948 to 2008. The estimate for the standard deviation of nominal GDP growth corresponds to a quarterly value for $\sigma_v = 0.004$ (monthly $\sigma_v = 0.0025$) and $\rho_v = 0.50$, which are in line with estimates in the literature. I choose the variance of the sector specific productivity shock $\sigma_A = 0.01$, to match the median estimate of the unconditional (monthly) variance of the idiosyncratic shock found in the BGM FAVAR exercise across the PPI prices.

In the remainder of this section, I will go through four calibration exercises. To make the intuition clear I first calibrate the model to an “extreme” two-sector production chain, where one good is solely used as an intermediate input and the other entirely as a consumption good. In the subsequent three calibration exercises, I gradually build a 6-sector version of the U.S. economy: In the second exercise, I calibrate the production share of the intermediate goods in each sector using the BEA’s Input-Output (IO) use table, but assume that sectors are homogenous along all other dimensions. Next, I add heterogeneity in the parameter describing the Calvo frequency of price adjustment across sectors, and finally I allow for varying intensity with which a good is used for consumption versus as an input for production, again using the IO use table. In terms of the notation introduced earlier, these intensities correspond to calibrating the $\zeta^i$ and $\varepsilon^j$ shares.

At the end of this section, I present a version of the model in which the production technologies are characterized by decreasing returns to scale.

### 1.3.1 A Two Sector Example

Here I develop a special example of the economy described above. This economy is composed of two sectors. Sector 1 only uses labor in its production function ($s_1 = 1, Y^1_t = A^1_t L_t$), and sector 2 only uses material inputs which are solely composed of sector 1 goods ($s_2 = 0, \zeta^2_t = 1, Y^2_t = A^1_t Y^1_t$). Finally, the consumption basket is entirely composed of good 2 ($\epsilon^2 = 1, C_t = Y^2_t$). These specific set of assumptions give rise to a production function which can be graphically represented as in figure (1.1). The log-linearized model can be represented by two Phillips curves, a wage setting equation,
the stochastic path of nominal aggregates and the aggregate production equation.

\[ \pi^1_t = \beta E_t \pi^1_{t+1} + \kappa^1_p \left( w_t - p^1_t - a^1_t \right) \]

\[ \pi^2_t = \beta E_t \pi^2_{t+1} + \kappa^2_p \left( p^1_t - p^2_t - a^2_t \right) \]

\[ w_t - p^2_t = \sigma c_t + \eta_t \]

\[ c_t + p^2_t - (c_{t-1} + p^2_{t-1}) = \nu_t \]

\[ c_t = a^1_t + a^2_t + l_t \]

Figure (1.2) shows the response of prices for the two sectors to a shock to monetary policy (panel (A)) and to their own idiosyncratic TFP shock (panel (B)). Sector 2 responds more slowly to a monetary policy shock because its marginal cost is the slow-moving price of sector 1 output. On the other hand, the responses of each sector to a shock in its sector specific technology \( a^2_t \) are indistinguishable. This is not surprising given that the two sectors are identical except for their position in the production chain.

Note that the relative speed of response to an aggregate shock remains the same regardless of the assumption about wage rigidity. To see this more clearly, note that \( Y^1_t = A^1_L L_t \) and \( Y^2_t = A^1_Y Y^1_t \). Therefore, even if sector 2's price is flexible compared to wages, since the marginal cost in sector 2 follows the price of sector 1 output, the prices in sector 2 inherit the sluggishness in the response of sector 1 through the marginal cost movements and thus, would be the slower sector to respond to an aggregate shock. This intuition holds in all the exercises presented below: although wage rigidity affects the overall amount of real rigidities created in response to a monetary policy shock, it does not affect the order in which sectors respond to aggregate shocks. Thus, in the interest of brevity, I only present the results under the assumption of flexible wages in the main text, but the same exercises are repeated for a model with staggered wage setting à-la Erceg, Henderson, and Levin (2000) in appendix (A.1).

1.3.2 The Multi-Sector Model

I calibrate the multi-sector model to a 6-sector version of the US economy. The sectors are Agriculture, Mining, Utilities, Construction, Manufacturing and Services. These sectors correspond to the most aggregated industry classification in the BLS IO table. I start by calibrating the sector shares to the US IO matrix. Given the Cobb-Douglas form assumed for the production function, the input share in production will be proportional to expenditure share \( 1 - s_j = \mu \frac{M^j}{P^j Y^j} \). The expenditure shares

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8I exclude government, and some other categories of services (including trade, finance and health care). The latter are excluded for the lack of data on the frequency of adjustment in prices, which will be used to calibrate the Calvo adjustment parameters of each sector in the full calibration in case 3. The largest two omissions are Financial services and Business services, which together amount to 40% of total value added.
are readily available from the IO use table. The corresponding labor shares for each sector are reported in table (1.2). The final column in table (1.2) shows the estimates of $s$ for some of the sectors in Bouakez, Cardia, and Ruge-Murcia (2009), who estimate the parameters of a similar multi-sector model. 9

**Case 1: Heterogeneity in $s_i$** In this exercise, the only source of heterogeneity between the sectors is the intensity with which they use intermediate inputs versus labor. Therefore, only the $s$ column in table (1.2) is relevant. I calibrate the (monthly) Calvo price stickiness in all sectors $\omega_i = 0.85$ which is close to the corresponding median frequency of price adjustment reported by Nakamura and Steinsson (2008a) for intermediate goods. This value implies a duration of 7.6 months which is close to the slightly larger than the average duration of price rigidity reported by Carvalho (2006) (6.6 months), using data from Bils and Klenow (2005). The response of this economy to a shock to the nominal GDP process $(\nu_t)$ is shown in panels (A)-(C) of figure (1.3). As would be predicted by the model, Utilities, the sector earliest in the chain (characterized by the largest labor share) responds first whereas Manufacturing, the latest industry in the chain, is the slowest.

The differences in the speed of response means that the existence of production chain creates short-run relative price effects. This non-neutrality caused by monetary policy can be measured in several ways. I look at the maximum relative price across all the sectors at all horizons in response to a MP shock in the first row of table (1.3). Note that without any heterogeneity in labor shares, this metric would be equal to zero. I also report the maximum standard deviation between prices at any horizon $t$ (row 2). This standard deviation is another way of measuring the extent to which relative prices deviate from 1 at each time. Thus, this measure would also be equal to zero in the absence of heterogeneity in sectoral characteristics. In other words, these two measures would not be useful for measuring monetary non-neutrality in a 1-sector model.

To measure the non-neutrality of nominal shocks for a single sector economy, or in order to compare the non-neutrality of the multi-sector economy with an equivalent economy, I report two measures of non-neutrality for the overall economy. First, I report the conditional variance of consumption's response to a MP shock. An alternative measure following Midrigan (2006) and Nakamura and Steinsson (2008b), which I also report, is the variance of real value-added output when the model is simulated with purely nominal aggregate shocks (respectively in rows 3 and 4 of table (1.3)).

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9 Bouakez, Cardia, and Ruge-Murcia (2009) assume the following production function for the firms in sector $j$:

$$ y^j = (z^j_i n^j_i)^{\omega_j} (k^j)^{\alpha_j} (H^j)^{\gamma_j}, $$

where $z^j_i$ is a sector-specific productivity shock, $k^j$ is capital, $H^j$ is material inputs, and $\omega_j + \alpha_j + \gamma_j = 1$. They estimate the production function parameters using the yearly data on nominal expenditures on capital, labor and material inputs for each sector collected by Dale Jorgenson for the period 1958 to 1996.
The relative price effects are not uniform across sectors either. Panel (A) of figure (1.7) shows the deviations of relative sectoral prices from their steady state level in response to a MP shock. This is captured by plotting sectoral prices relative to that of utilities. The figure shows that the relative prices are not large, around 7%. Also, it is intuitive that the largest relative price effect is between manufacturing and utilities sectors with the largest difference in their intermediate input shares.

I now look at price responses to technology shocks. The sectoral price responses to a productivity shock in their own sector are demonstrated in panel (B) of figure (1.3). The non-stationary productivity shocks cause permanent relative price effects, and the larger the labor share of a sector, the larger the effect of a one standard deviation shock on its final price. This property naturally follows the assumption that technology is labor augmenting.

Panel (C) of figure (1.3) shows the response of sectoral prices to a common productivity shock. The aggregate productivity shock can be thought of as a common component to technology shocks across different sectors. The response to an aggregate productivity shock is nearly identical to the response of sectoral prices to a monetary policy shock. This result justifies the BGM classification of shocks into aggregate vs. idiosyncratic regardless of whether they are supply-side or demand-side shocks.

The fact that an aggregate productivity shock leaves the relative prices unaffected in the long-run is because increases in productivity are “shared” among sectors through uses of intermediate inputs. Given the Cobb-Douglas structure of the production functions across sectors, it can be shown that an aggregate technology shocks leaves relative prices unchanged in the long-run (See appendix (A.3)).

Case 2: Heterogeneity in $s_i$ and $\omega_i$. In this exercise I add heterogeneity in the Calvo price adjustment parameter across sectors in addition to varying $s_i$. The $\omega_i$ are matched to the PPI-based frequency of price adjustment reported by Nakamura and Steinsson (2008a). I match their products to the larger NAICS categories included in the definition of industries in the IO table, provided by the BEA. The frequencies of adjustment for each sector is the median frequency of adjustment of all the categories within that sector. The calibrated values are reported in table (1.2). In this calibration, only columns corresponding to $s$ and $\omega$ are relevant.

The responses to the shocks discussed in the previous calibration exercise are reproduced for the new calibration and are presented in figure (1.4). Note that the heterogeneity in the response to a monetary policy shock substantially increases. The real effects of monetary policy are summarized in table (1.3). Compared to the previous case, where $\omega_i$ were constant across sectors, the real effect of monetary policy has increased substantially.
Furthermore, how fast an industry responds to an aggregate shock is determined by a combination of the size of $\omega_i$ and the position of the sector in the chain. Utilities is still the fastest sector to respond but agriculture and manufacturing are no longer the slowest industries. Note that despite having the highest frequency of price adjustment, agricultural prices respond more slowly than either utilities or mining, because agriculture has a high share of intermediate inputs which affect its marginal cost.

In the same way, the relative price effects are not only a function of differences in intermediate input shares, but also affected by differences in $\omega_i$. Panel (B) of figure (1.7) shows the largest deviation of relative prices compared to the steady state is now between utilities and services, mainly due to the sticky nature of services prices. The figure shows that the relative price effect is large reaching around 45%.

The response to sector specific productivity shocks are shown in panel (B). First note that the long-run response to these shocks is not different from the previous case. This is to be expected, because the only difference between case (1) and case (2) calibrations is the heterogeneity in $\omega_i$ which should not affect the long-run response. Also note that responses cross. The reason is that the short run response is driven by the heterogeneity in $\omega_i$, whereas the long-run responses reflect the heterogeneity in $s_i$. To the extent that $\omega_i$ and $s_i$ are not perfectly correlated, the short term and long-term ordering of prices may differ.

**Case 3: Heterogeneity in $\varepsilon^j$ and $\zeta_i^j$** Up to now, I have assumed that all sectors are used with equal weights in the consumption and intermediate good baskets. Empirically, this is unrealistic. In this section, I calibrate the $\varepsilon^j$ and $\zeta_i^j$ weights to the IO matrix. The Cobb-Douglas form assumed for consumption- and intermediate-good aggregator would imply that $\varepsilon^j$ is the expenditure share of good $C^j$ in total consumption expenditure. Therefore, $\varepsilon^j$ are readily available by taking each sector’s share in the “Personal Consumption Expenditure” column of the IO matrix.

The $\zeta_i^j$ denotes the share of sector $i$ in the intermediate-input of sector $j$. So potentially, we would have $n \times n$ different values. In the interest of tractability, I will make the simplifying assumption that $\zeta_i^j = \zeta_i^k = \zeta_i$, for all $i,j$ and $k$. This means that the composition of the intermediate good is the same for all sectors (across the recipient sectors) but in the composition of the intermediate input different sector outputs are used with different intensity ($\zeta_i \neq \zeta_j$). I compute $\zeta_i^j$ as the expenditure on intermediate inputs purchased from sector $i$ as a share of total intermediate input expenditure for sector $j$. I then compute $\zeta_i = \sum_j \lambda_j \zeta_i^j$, where $\lambda_j$ is the weight of sector $j$ in the economy.

The calibrated values for $\varepsilon^j$ and $\zeta_i$ are shown in table (1.2). Services form a large share of consumption whereas manufacturing is the largest share of the intermediate
good. Using the new calibrated $\epsilon^j$ and $\zeta^j$, and keeping $s_i$ and $\omega_i$ as before, I again simulate the model subject to the three different shocks discussed above. Figure (1.5) shows the result. The relative speed of response has not changed compared to the previous case. However, the overall real rigidity caused by a monetary policy shock is affected, as this economy puts a higher weight on two of the stickiest sectors of the economy: Services (because of a low Calvo price adjustment frequency) and manufacturing (a sector at the end of the production chain). Compare the cumulative response of the GDP to a monetary shock in the fully calibrated model (last column) with a perfectly homogeneous economy, in which $s = 0.38$ and $\omega = 0.62$, both equal to the weighted average of the same values in the heterogeneous economy. Table (1.3) shows that the realistically calibrated heterogenous model creates around five times more rigidity compared to the “equivalent” homogeneous economy (0.38 c.f. 0.07).

### 1.3.3 Decreasing Returns to Scale

In the previous section we assumed that the production function featured constant returns to scale to labor and material inputs. In this section, I will relax this assumption by assuming that the production function is $y_j(z) = \left(\frac{A_j L_j(z)}{L_j(z)^{\alpha_j}} \right)^{\theta_j} M_j(z)^{\gamma_j}$. Sbordone (2002) and Gali, Gertler, and Lopez-Salido (2001) derive the New Keynesian Phillips curve for a model with decreasing returns to scale. Following their approach, it can be shown that within a sector, the marginal cost of each firm can be written as a function of the average marginal cost in that sector. Therefore, when written in terms of log-deviations from the steady state it can be shown that:

$$\varphi^j(z) = \varphi^j - A(p^j(z) - p\bar{j})$$

where $\varphi^j(z)$ is the nominal marginal cost of firm $z$ in sector $j$, $\varphi^j$ is the average nominal marginal cost in sector $j$ and $p^j(z)$ and $p\bar{j}$ represent the price of firm $z$’s output and the average price level in sector $j$ respectively. This implies that firms with high price and thus low output, have lower marginal cost. The full derivation of the Phillips curve under decreasing returns to scale is included in appendix (A.2).

To calibrate this model, we use the Cobb-Douglas properties of the production function. It is still true that

$$\alpha_j = \frac{WL_j}{P_j Y_j} \text{ and } \gamma_j = \frac{XM_j}{P_j Y_j}$$  \hspace{1cm} (1.4)

Both of these ratios are available in the data. The wage spending is obtained from the GDP by industry tables published by the BEA. The wage bill is calculated as the

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10My conjecture is that under extreme assumptions about the composition of intermediate input, this result may be reversed.
share of compensation for employees to gross output for each industry. The material inputs expenditures are obtained as before. The degree of returns to scale, $\alpha + \gamma$, depends not only on the sum of wage bill and material inputs expenditure but also on $\mu$, the mark-up. It turns out that a calibration of $\theta = 8$ will imply increasing returns to scale for some of the industries. Therefore, for calibrating the model with decreasing returns I assume $\theta = 10$. This calibration for $\theta$ will ensure that the total returns to scale for all industries will be smaller than 1. The calibrated shares of labor ($\alpha$) and intermediate inputs ($\gamma$) are shown in table (1.4). The DRS shows the total degree of returns to scale ($\alpha + \gamma$). Calibration of all other parameters are as in Case (3) above, i.e. sectors not only differ in their labor and intermediate inputs shares, but also in their frequency of price adjustment (Calvo parameter) and the weights of different sectoral outputs in the consumption basket and intermediate inputs baskets. Table (1.5) shows the degree non-neutrality caused by nominal shocks.

1.3.4 Discussion of results

Creating real rigidities caused by nominal shocks has been one of the challenges of monetary models in DSGE context. The New Keynesian models create notoriously little rigidity for reasonable levels of micro-rigidities assumed. However, through previous research (Carvalho (2006), Nakamura and Steinsson (2008b) and many others) we have learned that richer, more realistic models of an economy will go a long way in increasing the real rigidities caused by nominal shocks. The model presented in this paper includes some of the ingredients found in previous papers as important – existence of intermediate inputs and heterogeneity in the frequency of price adjustment – and adds new ones: heterogeneity in the production functions across sectors and the possibility of decreasing returns to scale. A realistic calibration of the model to a 6-sector version of the US economy shows that these two assumptions are featured in the data.

Nakamura and Steinsson (2008b) report the variance of HP-filtered log U.S. real GDP for the period 1988-2006 to be $0.81 \times 10^{-4}$. Table (1.3) shows that the benchmark homogeneous economy model (one representative sector) with intermediate inputs equal to the average in the economy produces less than a 10th of the variance in output observed in the data. The most realistic calibration of the model under constant returns to scale assumptions is presented in Case (3), where sectors differ in the relative intensity with which they use factors of production, their relative sizes in consumption and intermediate input baskets and the frequency of price adjustment. This model implies a volatility for real GDP which is about 45% of the fluctuations in the real GDP in the data. Finally, in a model with DRS this share rises to about 55%.

To conclude, the exercises above demonstrate that departure from the simplified
model of one sector homogeneous economy is an important step in capturing the quantitative effects of nominal shocks on the output in the short-run. Furthermore, this class of multi-sector models are important in understanding the short-run relative price effects of monetary policy and the optimal monetary policy response as discussed for instance in Aoki (2001).

1.4 Empirical Evidence

The relevance of the production chain as a mechanism for amplifying the micro-level nominal rigidities in the economy is consistent with the findings of a few papers on the frequency of price adjustment. BGM report much faster responses for the PPI index to monetary policy shocks compared to the CPI. Nakamura and Steinsson (2008a) report that the frequency of price change is strongly related to the stage of processing. Although this fact could be evidence for different intrinsic factors for price stickiness, such as higher variance of idiosyncratic shocks at the crude material level or lower costs of price change, it is also consistent with the lower speed of response to shocks as predicted in the production chain model. In this section, I will explore this issue.

One implication of the model presented above was that ceteris paribus, upstream industries respond faster to aggregate shocks compared to downstream industries. I test this prediction against the data by considering the response of prices to two types of aggregate shocks: A monetary policy shock and an oil supply shock. I regress the cumulative response of disaggregated prices of around 150 industries in manufacturing to these shocks on a measure of their position in the production chain, which I will define below, as well as other explanatory factors. I find a significant and negative relationship between the position in the chain and the speed of response at different horizons.

In this section, I will present the empirical evidence. First I will discuss the identification of the shocks, and the implied impulse responses of sectoral prices. I will then present the reduced form regressions.

1.4.1 Identification of Shocks and Impulse Responses

I use two measures of identified monetary policy shocks: First, the Romer and Romer (2004) measure of monetary policy shocks, which uses a narrative approach based on the detailed examination of the Federal Reserve's meeting minutes. The second is monetary policy shocks as identified by the FAVAR method in BGM. A summary of the assumptions for this method is included in appendix (A.4). In an isomorphic fashion, I use two estimates for oil supply shocks, both due to Lutz Kilian. The first approach is similar to the Romer and Romer (2004) analysis, in that oil supply shocks are identified by examining historical events and their effects on oil prices. The second
is a VAR approach based on co-movements of changes in oil production, real oil prices and global economic activity (Kilian 2009). I embed this VAR approach into a factor augmented framework, similar to the one used by BGM to identify monetary policy shocks. A more detailed description of this identification scheme is also included in appendix (A.4).

Under the narrative approach, in order to find the impulse response of each price series to the identified shocks I proceed as follows. The response to the two historical measures of monetary policy and oil price shocks can be computed directly. In particular, I run the following regressions:

\[
\Delta p_{it} = a_{i0}^{MP} + \sum_{k=1}^{11} a_{ik}^{MP} D_{kt} + \sum_{j=1}^{24} b_{ij}^{MP} \Delta p_{i,t-j} + \sum_{j=1}^{48} c_{ij}^{MP} S_{t-j}^{MP} + e_{it}^{MP}
\]

\[
\Delta p_{it} = a_{i0}^{O} + \sum_{k=1}^{11} a_{ik}^{O} D_{kt} + \sum_{j=1}^{24} b_{ij}^{O} \Delta p_{i,t-j} + \sum_{j=1}^{64} c_{ij}^{O} S_{t-j}^{O} + e_{it}^{MP}
\]

where \(p_{it}\) is log of individual PPI price series described in the previous section and indexed by \(i\), \(D_{kt}\) are monthly dummies, \(\Delta p_{i,t-j}\) are lags of inflation for the price series being analyzed and \(S_{t-j}^{MP}\) and \(S_{t-j}^{O}\) are the measures of monetary policy and oil price shocks, respectively. In the two regressions above, superscripts \(MP\) and \(O\) refer to monetary policy and oil regressions, respectively.

In the monetary policy regression I use exactly the same number of lags as Romer and Romer (2004). They use 24 lags of monthly inflation series, and 48 lags of the shock series to analyze the effect of their measure of monetary policy shocks on the price index for finished goods. The regression is performed on monthly data and the regression dates are 1976:1 to 1996:12. For the oil supply shock, I try different lag specifications.

In analyzing inflation response, Kilian (2008) uses 4 lags of the inflation series and 8 lags of oil price, on a quarterly basis. Given that the oil shock and price series are both available in monthly frequency, I use the monthly data in the oil-supply shock regression to be consistent with the monetary policy shock regression. I use the same number of lags for the inflation series (24 months), but longer lags for the oil-price shock to capture the notion that an oil-price shock might take longer to affect production and prices. The results presented in the paper are robust to changes in these horizons, and use of quarterly data as in Kilian (2008). The regression uses data from 1976:1 to 2004:9. The impulse response of prices to monetary (oil) shocks can be directly computed using \(b_{ij}^{MP}\) and \(c_{ij}^{MP}\) (\(b_{ij}^{O}\) and \(c_{ij}^{O}\)) coefficients.

To find the impulse response of prices to monetary and oil price shocks using a VAR, I follow closely the FAVAR approach in BGM. Briefly, this amounts to extract-
ing a number of latent factors from a large data-set, including all the sectoral prices, as well as major series describing the state of the US economy. In the case of monetary policy shocks, the federal funds (FF) rate is added to the latent factors. The VAR is composed of the latent factors as well as the FF rate, with a recursive identification assumption which imposes that the FF rate can respond to all factors within a month, but not vice-versa. Monetary policy shocks are identified this way, and the corresponding impulse responses for each price series can be computed. The approach is described in more detail in appendix (A.4).

A similar approach is used for identifying oil price shocks. Following Kilian (2009), I impose that the monthly change in the global oil production, a measure of global real economic activity and real oil prices are the three observable factors. The identification assumption, as discussed in Kilian (2009) is that production does not respond, within a month, to changes in real economic activity and real oil prices, and that economic activity cannot respond, within the same month, to changes in real oil prices, whereas real oil prices can respond to shocks to all the factors. The identification scheme is discussed in more detail in appendix (A.4).

Finally, I need to construct a measure for the position of an industry in the production chain. I define the position of sector $i$ in the production chain as

$$pos_i = \frac{\text{total final use of } y_i}{y_i}$$

i.e. the position of the industry $i$ in the chain is determined by how intensively it is used as a final good as a share of that industry's total output. The higher this ratio, the further "downstream" is the corresponding industry.

1.4.2 Data

The data for the FAVARs are exactly the same as those used in the BGM exercise. This is a balanced panel of 653 monthly series, for the period running from 1976:1 to 2005:6. The choice of the initial date reflects the fact that a significant number of the disaggregated producer price indices start in 1976:1. All data have been transformed to induce stationarity. The original and transformed data are posted by the authors on the World Wide Web.\footnote{http://www2.gsb.columbia.edu/faculty/mgiannoni/research.html}

To find the impulse response of prices to monetary policy shocks identified by Romer and Romer (2004), I directly take their measure for monetary policy which is also available on the Web.\footnote{http://elsa.berkeley.edu/~dromer/} This measure documents monthly shocks to monetary policy from 1969:1 to 1996:12. Therefore, the regressions based on this measure of monetary policy use monthly data from 1976:1 to 1996:12. I append 24 months of...
zero inflation to the disaggregated price data (starting from 1974:2), in order to avoid throwing away the first 24 months of price data needed for the AR structure of the regression.

To identify oil supply shocks in the FAVAR framework, as well as the panel describing the economy, I need the monthly index of real activity which is available from Lutz Kilian’s website.\textsuperscript{13} The oil production and real oil price data are also readily available from the Department of Energy’s Energy Information Administration. The three series required to repeat the Kilian exercise are available from 1974:1-2006:10, and therefore the entire BGM panel can be used in this framework.\textsuperscript{14}

Finally, for the identification of oil supply shocks using the historical measure, I use the monthly historical oil supply shocks identified by Kilian (2008) based on episodes of political turmoil in the Middle East. This data is available from 1973:1 to 2004:9. Therefore the impulse responses based on this measure are calculated using monthly data on prices and oil supply shocks from 1976:1 to 2004:9. All sectoral inflation between 1973 and 1976 are assumed to be zero, but this will only affect the estimates using the first few months of data.

1.4.3 Regressions

As a first pass at the data, I use the Bureau of Labor Statistics classification of PPI commodity data by their stage of processing. This classification covers 1893 commodity categories classified into three stages of processing: “Crude materials for further processing”, “Intermediate materials, supplies and components” and “Finished goods”.

Figures (1.8) to (1.11) show the impulse response of price indices for each of these three broad categories to the four aggregate shocks discussed above. For the two monetary policy shocks, the relative speed of response of the different price categories strongly support the prediction of the model: The response of the final goods is much slower than that of the intermediate goods and the crude materials are the fastest to respond. The response of prices to oil shocks, particularly when identified in the FAVAR also suggests the same order in the speed of response. However, it seems that crude prices are much more volatile and far fewer lags are needed to estimate their response to oil price shocks.

For more conclusive evidence, I now use the responses of the 153 industries used by BGM in their sectoral regressions. I regress the response of disaggregated prices to shocks above at different horizons on $\mathbf{p}_{it}$, defined above. The model would predict

\begin{footnotesize}
\textsuperscript{13}http://www-personal.umich.edu/~lkilian/rea.txt
\textsuperscript{14}Kilian is able to start his series from January 1973 because he uses the Barsky and Kilian (2001) estimates of oil prices, before the start of the series published by the Energy department from January 1974. Given that the price data used in the FAVAR start in 1975, I do not use this extension.
\end{footnotesize}
a negative relationship between the speed of response of an industry to an aggregate shock, and the measure $pos_i$. Under the assumption of neutrality of money, all prices should respond by the same amount to a monetary policy shock in the long-run. Therefore the magnitude of the response of a price series at any time horizon is a valid measure of its speed of response.\textsuperscript{15}

On the other hand, oil prices can have also long-run level effects. Thus, the magnitude of response cannot be used as an indicator of the speed of response. To control for this level effect, I either control for the long-run effect, or control for the level of energy use. I construct an index for the energy use which is the total expenditure on energy as a share of total expenditure on intermediate inputs for each industry. Controlling for this index of energy use, should allow us to separate the long-run level effect of oil-price shocks on sectoral prices from the short-run transition effects. Thus, I expect the coefficient on the index of energy use to be negative.

Furthermore, BGM find that other factors, such as the standard deviation of the sector specific shocks, or the degree of competition in an industry can affect the dispersion in the response to monetary policy shocks. Therefore, I also include those variables in my regressions. In particular, I will use the following specifications for the cross-industry price responses:

\begin{align*}
IR^{\text{h}}_{i,h} &= \alpha + \beta_1 POS_i + \beta_2 s.d.(x_i) + \beta_3 \text{profit} + \beta_4 \rho(x_i) + \varepsilon_i \\
IR^O_{i,h} &= \alpha + \beta_1 POS_i + \beta_2 s.d.(x_i) + \beta_3 \text{profit} + \beta_4 \rho(x_i) + \beta_5 \text{energy} + \varepsilon_i \\
IR^O_{i,h} - IR^O_{i,m} &= \alpha + \beta_1 POS_i + \beta_2 s.d.(x_i) + \beta_3 \text{profit} + \beta_4 \rho(x_i) + \varepsilon_i
\end{align*}

where $IR^{\text{h}}_{i,h}$ ($IR^O_{i,h}$) is the log of price level in industry $i$, $h$ periods after an expansionary monetary policy shock (positive oil price shock); $pos_i$ is the share of final use of industry $i$ output; $s.d.(x_i)$ is the standard deviation of the inflation series, $\rho(x_i)$ is the persistence of the inflation series, $\text{profit}$ is the level of profits as a share of output, a measure of competitiveness in industry $i$ which BGM find significant and finally, $\text{energy}$ is the total energy input as a share of total inputs.

The specification is similar to that in the cross-sectional analysis by BGM. I control for all factors that they find significant in explaining cross-sectional dispersion in response to aggregate shocks, and argue the position in the chain still has some explanatory power. There is one small difference however: In BGM, $s.d.(x_i)$ and $\rho(x_i)$ etc.
are replaced by the $s.d.(e_i)$ and $\rho(e_i)$, where $e_i$ is the VAR error term. These estimates, though consistent, suffer from generated regressor bias and therefore, we need to correct the standard errors. I use $s.d.(x_i)$ and $\rho(x_i)$ (properties of the inflation series) as instruments for $s.d.(e_i)$ and $\rho(e_i)$. Table (1.6) shows that these are indeed strong instruments, particularly in the case of $s.d.(x_i)$. Note that I use $IR_{t,h}^Q - IR_{t,m}^Q$ as the dependent variable in regression (1.7) as opposed to using $IR_{t,h}^Q$ as the dependent variable and controlling for $IR_{t,m}^Q$ on the right hand side. Again, this is in order to avoid generated regressor problem, as $IR_{t,m}^Q$ are also estimated in the VAR.

Given the specification above, the model suggests that $\beta_1$ is negative. BGM find a positive estimate for $\beta_2$ and a negative estimate for $\beta_3$, both statistically significant. A positive $\beta_2$, although not predicted by the pricing model presented in this paper, could suggest some form of menu-cost pricing: firms with highly volatile idiosyncratic shocks need to adjust their prices often; therefore, they will also respond faster to aggregate shocks. A negative $\beta_3$ suggests that in those sectors with higher profit levels (associated with less competitive sectors) prices respond more slowly. Finally, I expect $\beta_5$ in the first oil regression to be positive.

Tables (1.7) to (1.8) show the regression results. First, note that the estimates of the effect of position in the chain on the speed of chain ($pos$) is negative, and significant in almost all the regressions presented. Furthermore, the estimates are quite close, despite the fact that the dependent variables across different regressions represent responses to different shocks (or at least the same shocks identified with different strategies).

Looking at table (1.7), the estimates for $pos$ can be interpreted as the effect of moving an industry from the “end” of production chain to the “beginning” of chain. These estimates say that an industry would respond between 20 — 40 per cent faster if it was moved from the end of the chain to the beginning. This effect is economically significant. Of course, as found in BGM, the effect of one unit larger standard deviation of idiosyncratic shocks is several orders of magnitude larger.

Table (1.8) confirms the same intuition for response to oil supply shocks. In the first two regressions of each panel I use $IR_{t,9}^Q - IR_{t,12}^Q$ as a measure of speed of response of prices between months 9 and 12. As explained earlier, the purpose of this choice of variable is twofold. First, I need to control for the long-run level response to an oil price shock. Secondly, to avoid a generated regressor problem, I use the difference in the 9-month and 12-month responses as my preferred measure of independent variable. In regressions (3) — (4) instead I use the share of energy use in total intermediate inputs to control for the long-run effects.

Here again the estimates of the coefficient on $pos$ are all negative and mostly significant, albeit slightly smaller than the estimates obtained from the monetary policy
responses. The estimates of the energy coefficient are quite small in the regressions based on the FAVAR impulse responses, but are significant and have the correct sign in the historical-based regression. Overall, the coefficient on energy is less robust in alternative specifications of the horizon at which the regression is performed, compared to controlling for the long-run response. This might be an indication that the energy index formed this way does not fully capture the extent of the energy use, or the relationship between the speed of response and the degree of energy use is not correctly specified.

Overall, these results lend support to the hypothesis that the position of an industry in the chain can affect the speed of its price response to aggregate shocks through the dependence of the industry's marginal cost on other prices in the economy.

1.5 Conclusions

Several recent papers have argued that there is significant heterogeneity in the behavior of prices across different sectors, and a literature has emerged to identify the sources of this heterogeneity. This paper belongs to this strand of research. In particular, this paper asks whether the existence of a production chain structure in the economy can be an important source of heterogeneity in the response of sectoral prices to shocks.

I present a multi-sector version of an otherwise typical New Keynesian model with intermediate inputs. In the benchmark model, the sectors in the economy differ on how intensively they use intermediate inputs, which determines their position in the production chain: those industries who mainly use labor as their input to production are classified as upstream, whereas those heavily dependent on intermediate inputs from other sectors for their production are classified as downstream industries. I discuss three implications of this model.

First, I argue that the input-output linkages in production can create heterogeneity in the response of sectoral prices to aggregate shocks. The model suggests that if there are small nominal rigidities, industries at the end of the chain "inherit" these rigidities from their suppliers and hence respond more slowly to aggregate shocks. Whereas in response to idiosyncratic shocks, the first order effect of a change in productivity comes into effect immediately and thus is reflected in the price, regardless of the position in the chain. I argue that the implications of this model are consistent with the facts in BGM. Their paper suggests, based on empirical work, that prices respond only slowly to aggregate whereas the response to sector-specific shocks are fast.

Second, I argue that in a realistic calibration of this multi-sector model to the US data, heterogeneity in the frequency of price adjustments can reinforce the heterogeneity in response to aggregate shocks, caused by the position in the chain, to
produce large rigidities in response to monetary shocks. The fact that introducing intermediate goods increases real rigidities has been pointed out by others, and most recently by Nakamura and Steinsson (2008b). Furthermore, Carvalho (2006) shows that heterogeneity in sectoral frequency of price adjustment also increases rigidities. Using data on the sectoral frequency of price adjustment, I show that differences across sectors in the intensity of intermediate input use, reinforces the heterogeneity in sectoral price adjustment frequencies. So an equivalent “average” economy might be underestimating the real rigidities quite substantially.

Finally, the model implies that upstream industries would respond faster to aggregate shocks, compared to “downstream” industries. I test this prediction against the data, by looking at the response of 150 industries in manufacturing to two types of aggregate shocks: A monetary policy shock and an oil supply shock. I find a significant and negative relationship between the position in the chain and the speed of response at different horizons. These evidence support the view that production chain can be an important mechanism for propagation of aggregate shocks and explaining the heterogeneity across sectors in response to these shocks.
A Appendix

A.1 Results in the Presence of Wage Rigidity

Here, I have reproduced the graphs (1.3) - (1.5) in the presence of wage rigidity. In modelling wage rigidity, I follow Erceg, Henderson, and Levin (2000) staggered wages set-up. The assumptions about households’ problem are altered slightly to allow for this set-up. In particular, I assume a continuum of monopolistically competitive households, each of which supplies a differentiated labor service to the production sector. Under these assumptions, Erceg, Henderson, and Levin (2000) show that a wage-setting equation analogous to the price-setting Phillips curve can be derived:

\[ \omega_t = \beta E_t \omega_{t+1} + \kappa_w [m_{rs_t} - \zeta_t] \]

where \( \omega_t = w_t - w_{t-1} \) is the wage inflation at time \( t \), \( \kappa_w = \frac{(1 - \varphi^w \beta)(1 - \varphi^w)}{\varphi^w} \) is a constant related to the stickiness of wages (\( \varphi^w \)), and \( \zeta_t \) is the real wage. I calibrate the probability of the nominal wage stickiness such that \( \varphi^w = 0.85 \). This calibration implies that wages are more rigid than all of the sectoral prices.

The important point to note is that the ordering of sectoral responses do not change in the presence of wage rigidity. This is due to the intuition provided in the two-sector case: Industries further down the change inherit the stickiness of earlier ones, regardless of the source (wage or price stickiness).
A.2 Derivation of the Sectoral Phillips Curve

**Deriving the demand curves** Each sector in the economy is composed of an infinite number of firms of mass 1, indexed by $z$. These firms produce imperfectly substitutable goods. In this sense, each sector can be thought of as a classical New Keynesian economy. Denote by $C^j_t$, the household’s consumption of the good produced by sector $j$ at time $t$. Then,

$$C^j_t = \left[ \int_0^1 (c^j_t(z))^{\theta-1}_\theta dz \right]^{\frac{1}{\theta-1}}$$

where $\theta$ is the elasticity of substitution between differentiated goods within sector $j$.

Define the price of sector $j$’s good, $P^j_t$, as an index over individual firms’ prices:

$$P^j_t = \left[ \int_0^1 (p^j_t(z))^{1-\theta}_\theta dz \right]^{\frac{1}{1-\theta}}$$

Then the demand for $c^j_t(z)$ is determined by

$$c^j_t(z) = \left( \frac{p^j_t(z)}{P^j_t} \right)^{-\theta} C^j_t$$

A similar set of steps can be taken to determine the demand for intermediate inputs. Remember that the representative firm in sector $j$, has a production technology as follows:

$$y_t^j(z) = \left( A^j_t L^j_t(z) \right)^{\sigma_j} M^j_t(z)^{1-\sigma_j}$$

where $M^j_t(z) = \left[ \prod_{i=1}^j \left( c^j_t(z) \right)^{\alpha_{ij}} \left( m^j_{t,i}(z) \right)^{\alpha_{ij}} \right]$ denotes the intermediate input of firm $z$, in sector $j$. $m^j_{t,i}(z)$ denotes firm $z$’s demand of intermediate goods from sector $i$ (superscripts refer to the recipient industry). Firm $z$’s demand for imperfectly substitutable goods in sector $i$ are formed as follows:

$$m^j_{t,i}(z) = \left[ \int_0^1 m^j_{t,i}(z,z')^{\alpha_j}_{\alpha_j} dz' \right]^{\frac{1}{\theta-1}}$$

where $m^j_{t,i}(z,z')$ is the quantity of input produced by firm $z'$ in sector $i$, purchased by firm $z$ in sector $j$. Given the price of the intermediate good $X^j_t$, the demand function $m^j_{t,i}(z,z')$ is determined by

$$m^j_{t,i}(z,z') = \left( \frac{p^j_t(z')}{P^j_t} \right)^{-\theta} m^j_{t,i}(z)$$

Define the total output of each sector as the sum of consumption goods plus the
sum of intermediate input goods produced by that sector. In other words,

\[ Y_i^j = C_i^j + \sum_{i=1}^{J} \int_0^1 m_{i,j}^j(z)dz \]

Market clearing for the output of each firm implies,

\[ y_t^j(z) = c_t^j(z) + \sum_{i=1}^{J} \int_0^1 m_{i,j}^j(z',z)dz' \]

\[ = \left( \frac{p_t^j(z)}{p_t^j(z)} \right)^{-\theta} C_t^j + \sum_{i=1}^{J} \int_0^1 \left( \frac{p_t^j(z)}{p_t^j(z)} \right)^{-\theta} m_{i,j}^j(z')dz' \]

\[ = \left( \frac{p_t^j(z)}{p_t^j(z)} \right)^{-\theta} Y_t^j \]

The second line uses the demand functions introduced above, and the last equation uses the definition of \( Y_t^j \). This last equation is used for deriving the Phillips curve.

**Log-linearization and the Phillips Curve**

Consider firm \( z \) in sector \( j \). The firm's profit maximization problem involves picking a desired price, \( \bar{p}_t^j(z) \), at time \( t \) that maximizes

\[ E_t \sum_{\tau=0}^{\infty} \omega_t^j \Delta_{t,t+\tau} \left[ \bar{p}_t^j(z) y_t^j(z) - W_{t+\tau} \Delta_{t+\tau}^j(z) - X_t^j M_t^j(z) \right] \]

Given the form of the demand function derived in appendix (A.2), the firm's objective function can be re-written in terms of optimal price set by the firm and aggregate variables as follows:

\[ E_t \sum_{\tau=0}^{\infty} \omega_t^j \Delta_{t,t+\tau} \left[ \bar{p}_t^j(z) \left( \frac{\bar{p}_t^j(z)}{\overline{p}_t^{j+1}} \right)^{-\theta} - \Psi_t^j \left( \frac{\bar{p}_t^j(z)}{\overline{p}_t^{j+1}} \right)^{-\theta} Y_t^j \right] \]

where \( \Psi_t^j = \left( \frac{W_t}{A_t^j} \right)^{s_j} \left( X_t^j \right)^{1-s_j} \left( \frac{s_j}{1-s_j} \right)^{1-s_j} \) is a firm's nominal marginal cost and is constant for all firms within sector \( j \), and hence independent of index \( z \).

The first order condition for this maximization problem is

\[ \bar{p}_t^j(z) = \left( \frac{\theta}{\theta - 1} \right) \frac{E_t \sum_{\tau=0}^{\infty} \omega_t^j \Delta_{t,t+\tau} Y_t^j}{E_t \sum_{\tau=0}^{\infty} \omega_t^j \Delta_{t,t+\tau} Y_t^j} \left( \frac{P_{t+\tau}^j}{P_t^j} \right)^{\theta} \]
therefore, dividing both sides by $P_t^j$

$$\frac{\tilde{p}_t^j(z)}{P_t^j} = \left(\frac{\theta}{\theta - 1}\right) \frac{E_t \sum_{\tau=0}^{\infty} \omega_j^j \Delta_{t,\tau+r} Y_t^{\theta} \psi_{t+r}(P_t^j)^{\theta}}{E_t \sum_{\tau=0}^{\infty} \omega_j^j \Delta_{t,\tau+r} Y_t^{\theta} (P_t^j)^{\theta} P_t^j}$$ (1.8)

Denote the desired reset price relative to the average industry price by $Q_t^j = \frac{\tilde{p}_t^j(z)}{P_t^j}$.

This relative price is stationary and thus equation (1.8) can be log-linearized to obtain:

$$\eta_t^j + \xi_t^j = (1 - \omega^j \beta^j) E_t \sum_{\tau=0}^{\infty} \omega_j^j \beta^j \left[ \varphi_{t+r}^j \right]$$ (1.9)

where small letter variables denote deviations from steady state. Equation (1.9) can be rewritten

$$\eta_t^j + \xi_t^j = (1 - \omega^j \beta^j) \left[ \varphi_t^j \right] + \omega^j \beta^j E_t \left[ \eta_{t+1}^j + \xi_{t+1}^j \right]$$ (1.10)

At any time, $\omega^j$ fraction of firms in sector $j$ adjust their prices and choose the exact same reset price $\tilde{p}_t^j(z)$. Thus, the price level in sector $j$ is a weighted average of the optimal price and last period's price:

$$p_t^j = (1 - \omega^j) \tilde{p}_t^j(z) + \omega^j p_{t-1}^j$$

Re-arranging this expression yields:

$$0 = (1 - \omega^j) \eta_t^j - \omega^j \xi_t^j$$ (1.11)

Combining (1.10) and (1.11) to eliminate $\eta_t^j$ yields:

$$\xi_t^j = \beta E_t \pi_{t+1}^j + \kappa_p^j \left[ \varphi_t^j - \tilde{p}_t^j \right]$$

where $\kappa_p^j = \frac{(1 - \omega^j \beta^j)(1 - \omega^j)}{\omega^j}$. This is the same as equation (1.3) in the text.

Note that aggregate inflation can be obtained by summing the sectoral inflation, weighted by the relative steady state coefficients.

$$\pi_t = \epsilon_p \pi_t = \beta \epsilon_p E_t \pi_{t+1} + \epsilon_p (\Phi_t - P_t) \kappa_p$$ (1.12)

where $\Phi_t = diag(\varphi_t)$, is a diagonal matrix of nominal marginal costs, $P_t = diag(p_t)$ is a diagonal matrix of the sectoral prices, $\pi_t$ is the vector of sectoral inflation rates, $\epsilon_p$ is the vector of elasticities in the aggregate price expression, or the vector of weights associated with the price of each sector in the steady state. $\kappa_p$ is a vector of the sector specific Phillips curve slope parameters. Note that if the sectors are homogeneous, equation (1.12) above would collapse to the usual Phillips curve.

When firms face decreasing return to scale in their production, their marginal
cost depends on the level of output. Thus firms within the same sector face different marginal costs if their prices (and hence output levels) are different. As assumed in the text, let $y_t^j(z) = \left( A_t^j L_t^j(z) \right)^{\alpha_j} M_t^j(z)^{\gamma_j}$. It can be shown that the log-linearized marginal cost for firm $z$ in sector $j$ is equal to

$$
\varphi_t^j(z) = \frac{\alpha_j}{\alpha_j + \gamma_j}(w_t - a_t^j) + \frac{\gamma_j}{\alpha_j + \gamma_j} x_t + \left( \frac{1}{\alpha_j + \gamma_j} - 1 \right) y_t^j(z)
$$

$$
= \frac{\alpha_j}{\alpha_j + \gamma_j}(w_t - a_t^j) + \frac{\gamma_j}{\alpha_j + \gamma_j} x_t - \theta \left( \frac{1}{\alpha_j + \gamma_j} - 1 \right) \left( p_t^j(z) - p_t^j \right)
$$

$$
= \varphi_t^j - A \left( p_t^j(z) - p_t^j \right)
$$

where $\varphi_t^j$ in the last line is the average marginal cost in sector $j$ and $A$ is a constant. Thus (1.9) above can be re-written as:

$$
q_t^j + p_t^j = (1 - \omega^j \beta) \sum_{\tau=0}^{\infty} \omega_t^j \beta^\tau \left[ \varphi_{t+\tau}^j(z) \right]
$$

(1.13)

$$
= (1 - \omega^j \beta) \sum_{\tau=0}^{\infty} \omega_t^j \beta^\tau \left[ \varphi_{t+\tau}^j - A(q_t^j + p_t^j - p_{t+\tau}^j) \right]
$$

(1.14)

Equation (1.14) can be re-written as

$$
(1 + A) (q_t^j + p_t^j) = (1 - \omega^j \beta) \left[ \varphi_t^j + A(p_t^j) \right] + \omega^j \beta (1 + A) \sum_{\tau=0}^{\infty} \omega_t^j \beta^\tau [\varphi_{t+\tau}^j + p_{t+\tau}^j]
$$

Replace for $q_t^j = \frac{\omega_t^j}{1 - \omega^j} \pi_t^j$ from equation (1.11) to obtain

$$
\pi_t^j = \beta E_t \pi_{t+1}^j + \kappa_p^j \left[ \varphi_t^j - p_t^j \right]
$$

$$
= \beta E_t \pi_{t+1}^j + \frac{\kappa_p^j}{1 + A} \left[ \varphi_t^j - p_t^j \right]
$$

where $\kappa_p^j = \frac{(1 - \omega^j \beta)(1 - \omega^j)}{\omega_t^j} \beta$ as in the case with constant returns, $A = \theta \left( \frac{1}{\alpha_j + \gamma_j} - 1 \right)$ and $\varphi_t^j = \frac{\alpha_j}{\alpha_j + \gamma_j} (w_t - a_t^j) + \frac{\gamma_j}{\alpha_j + \gamma_j} x_t$. 
A.3 Long-run response of prices to an aggregate technology shock

First, I prove that $x_{LR} = -a$, where $x^{LR}$ is the long-run response of the price of intermediate input, and $a$ is the size of aggregate technology shock, where the term "aggregate technology shock" implies $a_i = a_j = a, \forall i$. The proof is by contradiction. Let $x^{LR} = -b$. Then, for each sector $i$ the long-run price response implied is:

$$p_i^{LR} = s_j(w - a_j) + (1 - s_j)x^{LR}$$
$$= -s_ja - (1 - s_j)b$$

The second line follows because $\eta = 0$ implies perfectly elastic labour supply and therefore $w = 0$. The definition of $x$ implies:

$$x^{LR} = \sum \zeta_i p_i^{LR} = -b + \sum \zeta_i s_i(b - a)$$
$$= -b$$

The second line follows from our assumption about the long-run response of $x$. This must be true for any vector $s$ and $\zeta$, which implies $b = a$ and $x^{LR} = -a$. It immediately follows that

$$p_i^{LR} = s_j(w - a_j) + (1 - s_j)x^{LR}$$
$$= -a, \forall i$$
A.4 Monetary Policy and Oil Price Shock FAVAR

The empirical framework is based on the factor-augmented vector auto-regression model (FAVAR) described in BGM and originally due to Bernanke, Boivin, and Eliasz (2005) (BBE). The main feature is to extract a few key variables or "latent factors" from a large set of economic variables, in order to summarize the movements of the macroeconomy. This strategy is particularly useful for eliminating the identification problems associated with small size VARs. As it is largely documented in this literature, a VAR specified based on an information set smaller than that of the policy maker will be potentially misspecified. The FAVAR framework addresses this problem by using a large information set from which factors are extracted.

Furthermore, the FAVAR model allows for decomposing fluctuations in all variables into common and idiosyncratic movements. BGM use this feature to establish their stylized facts about the responses of disaggregated prices to aggregate versus idiosyncratic shocks. As the methodology for factor decomposition and identification of monetary policy shocks is based on BGM, I only provide a brief description of the assumptions here and refer the interested reader to BGM for more details.

**Identifying monetary policy shocks** The assumption is that the economy is affected by a vector of factors, $C_t$, which are common to all variables entering the data set. To estimate this vector of common components I follow BGM. I impose that one of these factors is the federal funds rate, as we are interested in identifying monetary policy shocks. The rest of the common dynamics are captured by a $K \times 1$ vector of unobserved factors $F_t$. These unobserved factors may reflect general economic conditions such as "economic activity" or the level of "productivity", which are captured by a wide range of economic variables. The dynamics of $C_t$ are given by

$$C_t = \Phi(L)C_{t-1} + v_t$$  \hspace{1cm} (1.15)

where

$$C_t = \begin{bmatrix} F_t \\ R_t \end{bmatrix}$$

and $\Phi(L)$ is a lag polynomial. The error term $v_t$ is i.i.d. with mean zero and covariance matrix $Q$. Equation (1.15) defines a VAR in $C_t$, except that the $F_t$ are unobservable. I follow a similar strategy to BGM to extract these factors in a two-step principal components approach. In the first step, principal components are extracted from the entire data set. In the second step, the federal funds rate is appended to the estimated factors, so that the VAR described in (1.15) can be estimated.

To identify a monetary policy shock, again, I follow the strategy described in BGM. Specifically, I assume that the federal funds rate may respond to contemporaneous
fluctuations in estimated factors, but that none of the latent factors of the economy can respond within a month to unanticipated changes in monetary policy. This is the FAVAR version of the standard VAR identification schemes of monetary policy shocks. Note that this identification assumption implies that all the variables (including price series) are allowed to respond to monetary policy immediately, insofar this is a response only to the monetary policy shock directly and not through changes in other latent factors. In all of the simulations presented here I extract 5 latent factors. The results are similar with 7, 9 and 10 extracted factors.

Identifying oil price shocks To identify oil price shocks in VAR, I use the identification scheme proposed by Kilian (2009), and embed it in the factor augmented framework. He proposed a VAR model based on monthly data for $z_t = (\Delta prod_t, rea_t, rpo_t)$ where $\Delta prod_t$ is the percent change in global crude oil production, $rea_t$ denotes an index of monthly global real economic activity in industrial commodity markets based on data for dry cargo bulk freight rates, and $rpo_t$ refers to the real price of oil. The structural VAR representation and the reduced form representations are:

$$A_0 z_t = \alpha + \sum_{i=1}^{24} A_i z_{t-i} + \varepsilon_t$$

$$z_t = \beta + \sum_{i=1}^{24} B_i z_{t-i} + \varepsilon_t$$

where $\varepsilon_t$ denotes the vector of serially and mutually uncorrelated structural innovations and $\varepsilon_t = A_0^{-1} \varepsilon_t$. The identification assumptions are a set of linear restrictions on $A_0^{-1}$ which uniquely map $\varepsilon_t$ to $\varepsilon_t$. Kilian postulates the following:

$$e_t = \begin{pmatrix} e_{t, \Delta prod} \\ e_{t, rea} \\ e_{t, rpo} \end{pmatrix} = \begin{bmatrix} x & 0 & 0 \\ x & x & 0 \\ x & x & x \end{bmatrix} \begin{pmatrix} \varepsilon_{t, oil supply shock} \\ \varepsilon_{t, aggregate demand shock} \\ \varepsilon_{t, oil demand shock} \end{pmatrix}$$

Kilian motivates the restrictions on $A_0^{-1}$ as follows: Crude oil supply shocks are defined as unpredictable innovations to global oil production. Crude oil supply is assumed not to respond to innovations to the demand for oil within the same month. That exclusion restriction is plausible because, in practice, oil-producing countries will be slow to respond to demand shocks, given the costs of adjusting oil production and the uncertainty about the state of the crude oil market. Innovations to global real economic activity that cannot be explained based on crude oil supply shocks will be referred to as shocks to the aggregate demand shocks. The model imposes the exclusion restriction that increases in the real price of oil driven by shocks that are specific to the oil market will not lower global real economic activity immediately, but with a delay of at least a month. This restriction is consistent with the sluggish
behavior of global real economic activity after each of the major oil price increases in the sample. Finally, innovations to the real price of oil that cannot be explained based on oil supply shocks or aggregate demand shocks by construction will reflect changes in the demand for oil as opposed to changes in the global demand. The latter structural shock will reflect in particular fluctuations in precautionary demand for oil driven by uncertainty about the availability of future oil supplies.

So to identify oil supply shocks using this identification scheme, I follow a similar set of steps to those taken in identifying monetary policy shocks. Specifically, I extract 5 latent factors from the large data-set describing the economy and impose three additional observable factors, i.e. $z_t = (\Delta prod_t, rea_t, rpo_t)$. I consider the response of all disaggregated price series to an impulse in the global oil supply. The regression results presented in panel (A) of table (1.8) are based on these impulse responses.
Table 1.1: Calibrating the Benchmark Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta = 0.9975$</td>
</tr>
<tr>
<td>Intertemporal elasticity of substitution</td>
<td>$\sigma = 1$</td>
</tr>
<tr>
<td>Inverse Frisch elasticity of labor supply</td>
<td>$\eta = 0$</td>
</tr>
<tr>
<td>Elasticity of substitution for goods within a sector</td>
<td>$\theta = 8$</td>
</tr>
<tr>
<td>Speed of mean reversion of the shock to nominal GDP growth</td>
<td>$\rho_v = 0.50$</td>
</tr>
<tr>
<td>St. deviation of nominal GDP growth</td>
<td>$\sigma_v = 0.002$</td>
</tr>
<tr>
<td>St. deviation of the idiosyncratic productivity shock</td>
<td>$\sigma_A = 0.01$</td>
</tr>
</tbody>
</table>

This table describes the benchmark monthly calibration of the multi-sector model. The parameter values presented in this table will be used throughout all exercises.
Table 1.2: Calibrating the Multi-sector Economy

<table>
<thead>
<tr>
<th>Industry</th>
<th>$M.X/P.Y$</th>
<th>$s$</th>
<th>$\omega$</th>
<th>$\zeta$</th>
<th>$\varepsilon$</th>
<th>BCR ($s$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturing</td>
<td>0.67</td>
<td>0.24</td>
<td>0.87</td>
<td>0.70</td>
<td>0.51</td>
<td>n.a.</td>
</tr>
<tr>
<td>Agriculture</td>
<td>0.65</td>
<td>0.26</td>
<td>0.15</td>
<td>0.08</td>
<td>0.02</td>
<td>0.26</td>
</tr>
<tr>
<td>Mining</td>
<td>0.50</td>
<td>0.43</td>
<td>0.42</td>
<td>0.09</td>
<td>0.00</td>
<td>0.24</td>
</tr>
<tr>
<td>Construction</td>
<td>0.50</td>
<td>0.43</td>
<td>0.88</td>
<td>0.02</td>
<td>0.00</td>
<td>0.39</td>
</tr>
<tr>
<td>Services</td>
<td>0.44</td>
<td>0.50</td>
<td>0.92</td>
<td>0.06</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>Utilities</td>
<td>0.41</td>
<td>0.53</td>
<td>0.56</td>
<td>0.05</td>
<td>0.07</td>
<td>n.a.</td>
</tr>
</tbody>
</table>

This table describes the sectoral heterogeneity as described in cases 1 to 3 of the calibration exercise. In case 1, $s$ (the share of labor in production function) varies across sectors. In case 2, $\omega$ (the frequency of price adjustment) is also heterogeneous. In case 3, $\zeta$ and $\varepsilon$ (the weights in the intermediate input aggregator and consumption basket) also vary.
Table 1.3: Real Effects of Shocks

<table>
<thead>
<tr>
<th></th>
<th>Hom. Econ.</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. relative price</td>
<td>0.0%</td>
<td>7.9%</td>
<td>43.3%</td>
<td>39.1%</td>
</tr>
<tr>
<td>Max. std. of the prices</td>
<td>0.0%</td>
<td>3.3%</td>
<td>20.3%</td>
<td>17.7%</td>
</tr>
<tr>
<td>Cumulative response of consumption</td>
<td>$0.68 \times 10^{-2}$</td>
<td>$1.11 \times 10^{-2}$</td>
<td>$2.07 \times 10^{-2}$</td>
<td>$3.10 \times 10^{-2}$</td>
</tr>
<tr>
<td>$Var(C) \times 10^4$</td>
<td>0.07</td>
<td>0.15</td>
<td>0.24</td>
<td>0.38</td>
</tr>
</tbody>
</table>

This table summarizes the short-run relative price effects or the extent of short-run monetary non-neutrality in each of the models described in the text. The measure in first row is the maximum relative price across all the sectors at all horizons. Without any heterogeneity in labor shares, this metric would be equal to zero. The second measure (row 2) reports the maximum standard deviation between prices at any horizon $t$. Rows (3) and (4) report measure of the non-neutrality which are relevant even in a 1-sector economy. Row (3) reports the conditional variance of consumption’s response to a MP shock. Row (4) reports the variance of real value-added output when the model is simulated with purely nominal aggregate shocks.
Table 1.4: Calibrating the Multi-sector Economy: Decreasing Returns to Scale

<table>
<thead>
<tr>
<th>Industry</th>
<th>(\alpha)</th>
<th>(\gamma)</th>
<th>(DRS)</th>
<th>(\omega)</th>
<th>(\zeta)</th>
<th>(\varepsilon)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturing</td>
<td>0.25</td>
<td>0.74</td>
<td>0.99</td>
<td>0.87</td>
<td>0.70</td>
<td>0.51</td>
</tr>
<tr>
<td>Agriculture</td>
<td>0.15</td>
<td>0.72</td>
<td>0.87</td>
<td>0.15</td>
<td>0.08</td>
<td>0.02</td>
</tr>
<tr>
<td>Mining</td>
<td>0.56</td>
<td>0.56</td>
<td>0.78</td>
<td>0.42</td>
<td>0.09</td>
<td>0.00</td>
</tr>
<tr>
<td>Construction</td>
<td>0.38</td>
<td>0.56</td>
<td>0.94</td>
<td>0.88</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>Services</td>
<td>0.37</td>
<td>0.49</td>
<td>0.86</td>
<td>0.92</td>
<td>0.06</td>
<td>0.40</td>
</tr>
<tr>
<td>Utilities</td>
<td>0.18</td>
<td>0.46</td>
<td>0.64</td>
<td>0.56</td>
<td>0.05</td>
<td>0.07</td>
</tr>
</tbody>
</table>

This table describes the calibration of the model where decreasing returns to scale are assumed. \(\alpha\) and \(\gamma\) denote the share of labor and intermediate inputs in the production function \(y^i(z) = \left( A^i L_i(z) \right)^{\alpha_j} \left( M_i(z) \right)^{\gamma_j}\) for each industry respectively. The column \(DRS\) is the total returns to scale \(\alpha + \gamma\). \(\omega, \zeta\) and \(\varepsilon\) are calibrated as in table (1.2).
## Table 1.5: Real Effects of Nominal Shocks

<table>
<thead>
<tr>
<th></th>
<th>DRS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. relative price</td>
<td>29.9%</td>
</tr>
<tr>
<td>Max. std. of the prices</td>
<td>10.9%</td>
</tr>
<tr>
<td>Cumulative response of consumption</td>
<td>$4.30 \times 10^{-2}$</td>
</tr>
<tr>
<td>$Var(C) \times 10^4$</td>
<td>0.47</td>
</tr>
</tbody>
</table>

This table summarizes the short-run relative price effects for the model with decreasing returns to scale. The measures presented are the same as in table (1.3). The 1st row is the maximum relative price across all the sectors at all horizons, the 2nd row reports the maximum standard deviation between prices at any horizon $t$. Row (3) reports the conditional variance of consumption's response to a MP shock. Row (4) reports the variance of real value-added output when the model is simulated with purely nominal aggregate shocks.
Table 1.6: Validity of Instruments

<table>
<thead>
<tr>
<th>Dependent variable: $s.d.(x)$</th>
<th>Dependent variable: $\rho(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s.d.(e)$</td>
<td>$\rho(e)$</td>
</tr>
<tr>
<td>1.00*</td>
<td>0.28*</td>
</tr>
<tr>
<td>(0.00)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>constant</td>
<td>constant</td>
</tr>
<tr>
<td>0.00*</td>
<td>0.50*</td>
</tr>
<tr>
<td>(0.00)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Observations</td>
<td>Observations</td>
</tr>
<tr>
<td>154</td>
<td>154</td>
</tr>
<tr>
<td>$R^2$</td>
<td>$R^2$</td>
</tr>
<tr>
<td>1.00</td>
<td>0.15</td>
</tr>
</tbody>
</table>

This table shows the validity of $s.d.(x_i)$ and $\rho(x_i)$ (properties of the data) as good instruments for the $s.d.(e_i)$ and $\rho(e_i)$ (properties of the estimated VAR errors). Using $s.d.(e_i)$ and $\rho(e_i)$ in the second stage regressions would result in incorrect standard errors. This table shows that $s.d.(x_i)$ and $\rho(x_i)$ are strong instruments, particularly in the case of $s.d.(x_i)$.
Table 1.7: Speed of Price Responses to Monetary Policy Shocks

<table>
<thead>
<tr>
<th>Dependent variable: $IR_{12}^{MP}$</th>
<th>Panel A: FAVAR</th>
<th>Panel B: Romer &amp; Romer (2004)</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>(1)</td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td>(3)</td>
</tr>
<tr>
<td></td>
<td>(4)</td>
<td>(4)</td>
</tr>
<tr>
<td>$-1.03^*$</td>
<td>$-2.45^*$</td>
<td></td>
</tr>
<tr>
<td>$(0.05)$</td>
<td>$(0.07)$</td>
<td></td>
</tr>
<tr>
<td>$-1.33^*$</td>
<td>$-2.87^*$</td>
<td></td>
</tr>
<tr>
<td>$(0.07)$</td>
<td>$(0.13)$</td>
<td></td>
</tr>
<tr>
<td>$-1.07^*$</td>
<td>$-2.78^*$</td>
<td></td>
</tr>
<tr>
<td>$(0.18)$</td>
<td>$(0.12)$</td>
<td></td>
</tr>
<tr>
<td>$-1.3^*$</td>
<td>$-2.83^*$</td>
<td></td>
</tr>
<tr>
<td>$(0.07)$</td>
<td>$(0.09)$</td>
<td></td>
</tr>
<tr>
<td>$p_{os}$</td>
<td>(0.09)</td>
<td></td>
</tr>
<tr>
<td>$-0.41^*$</td>
<td>$-0.30^*$</td>
<td></td>
</tr>
<tr>
<td>$(0.09)$</td>
<td>$(0.08)$</td>
<td></td>
</tr>
<tr>
<td>$-0.33^*$</td>
<td>$-0.22^*$</td>
<td></td>
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<tr>
<td>$(0.08)$</td>
<td>$(0.07)$</td>
<td></td>
</tr>
<tr>
<td>$-0.27^*$</td>
<td>$-0.20^*$</td>
<td></td>
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<tr>
<td>$(0.07)$</td>
<td>$(0.07)$</td>
<td></td>
</tr>
<tr>
<td>$-0.21^*$</td>
<td>$-0.24^*$</td>
<td></td>
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<tr>
<td>$(0.07)$</td>
<td>$(0.07)$</td>
<td></td>
</tr>
<tr>
<td>$s.d.(x)$</td>
<td>(0.09)</td>
<td></td>
</tr>
<tr>
<td>$19.70^*$</td>
<td>$70.6^*$</td>
<td></td>
</tr>
<tr>
<td>$(0.024)$</td>
<td>$(0.25)$</td>
<td></td>
</tr>
<tr>
<td>$18.6^*$</td>
<td>$69.2^*$</td>
<td></td>
</tr>
<tr>
<td>$(4.25)$</td>
<td>$(5.40)$</td>
<td></td>
</tr>
<tr>
<td>$24.1^*$</td>
<td>$69.6^*$</td>
<td></td>
</tr>
<tr>
<td>$(5.40)$</td>
<td>$(8.8)$</td>
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<tr>
<td>$p_{rofit}$</td>
<td>(0.09)</td>
<td></td>
</tr>
<tr>
<td>$-1.05^\dagger$</td>
<td>$-0.34$</td>
<td></td>
</tr>
<tr>
<td>$(0.43)$</td>
<td>$(0.40)$</td>
<td></td>
</tr>
<tr>
<td>$-0.94^\dagger$</td>
<td>$(0.46)$</td>
<td></td>
</tr>
<tr>
<td>$(0.40)$</td>
<td>$(0.46)$</td>
<td></td>
</tr>
<tr>
<td>$\rho(x)$</td>
<td>(0.13)</td>
<td></td>
</tr>
<tr>
<td>$0.35^*$</td>
<td>$-0.10$</td>
<td></td>
</tr>
<tr>
<td>$(0.13)$</td>
<td>$(0.27)$</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.34</td>
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</tr>
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<td>0.38</td>
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</tr>
</tbody>
</table>

* : Significant at 5%  †: Significant at 10%

This table presents the results of the regression $IR_{12}^{MP} = \alpha + \beta_1 POS_i + \beta_2 s.d.(x_i) + \beta_3 profit + \beta_4 \rho(x_i) + \varepsilon_i$. Panel (A) presents the results where the dependent variable are the impulse response of sectoral prices to a monetary policy shock identified in a FAVAR model, as explained in the text. Panel (B) presents the regression results where impulse responses are computed in response to a monetary policy shock identified using the Romer and Romer (2004) measure of monetary policy shocks. The dependent variables in both cases are measured as the percentage of price decline 12 months after the shock occurs relative to the pre-shock level.
Table 1.8: Speed of Price Responses to Oil Supply Shocks

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>Panel A: FAVAR</th>
<th>Panel B: Historical</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$IR_{O}^O - IR_{12}^O$</td>
<td>$IR_{O}^O$</td>
</tr>
<tr>
<td>constant</td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>0.07</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>pos</td>
<td>-0.23*</td>
<td>-0.22*</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>s.d.(x)</td>
<td>1.51</td>
<td>12.3*</td>
</tr>
<tr>
<td></td>
<td>(2.4)</td>
<td>(3.4)</td>
</tr>
<tr>
<td>energy</td>
<td>0.0*</td>
<td>0.0*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.06</td>
<td>0.06</td>
</tr>
</tbody>
</table>

* : Significant at 5% † : Significant at 10%

This table presents the following regression results: $IR_{O,h}^O = \alpha + \beta_1 POS_i + \beta_2 s.d.(x_i) + \beta_3 profit + \beta_4 rho(x_i) + \beta_5 energy + \epsilon_i$ and $IR_{O,h}^O - IR_{O,m}^O = \alpha + \beta_1 POS_i + \beta_2 s.d.(x_i) + \beta_3 profit + \beta_4 rho(x_i) + \epsilon_i$. Panel (A) presents the results where the dependent variable are the impulse response of sectoral prices to an oil supply shock identified in a FAVAR model, as explained in the text. Panel (B) presents the regression results where impulse responses are computed in response to an oil supply shock identified using Kilian’s historical measure of oil supply shocks. The dependent variables are either the price response between 9 months and 12 months after the shock, or the percentage of price change 12 months after the shock relative to the pre-shock level.
Figure 1.1: A Special Two-sector Model

\[ Y(1) = A(1) \times L \]

\[ Y(2) = A(2) \times Y(1) \]

The structure of the economy in the stylized two-sector model.
Figure 1.2: A Special Two Sector Economy Example

Panel A: Response to an expansionary MP shock

Panel B: Response of sectoral prices to sector-specific tech. shock

This figure shows the impulse response of the sectoral prices in a special two sector economy to
an expansionary MP shock (panel (A)) and a positive technology shock (panel (B)). As argued,
the position of a sector along the production chain only matters in response to aggregate
shocks.
Figure 1.3: Heterogeneity in $s_i \omega_i = 0.85 \epsilon_i = \zeta_i = (1/6)$.

This figure shows the impulse response of sectoral prices to a monetary policy shock (A), idiosyncratic technology shocks (B) and an aggregate technology shock (C), under Case (1). Sectors only differ in their share of intermediate input use. Frequency of prices adjustment and the weights in the intermediate input and consumption baskets are identical.
Figure 1.4: Heterogeneity in $s_t, \omega_t$. $\varepsilon_i = \zeta_i = \frac{1}{\theta}$.

Panel A: Response of sectoral prices to an expansionary MP shock

Panel B: Response of sectoral prices to sector-specific tech. shock

Panel C: Response of sectoral prices to an aggregate tech. shock

This figure shows the impulse response of sectoral prices to a monetary policy shock (A), idiosyncratic technology shocks (B) and an aggregate technology shock (C), under Case (2). Sectors differ in their share of intermediate input use and frequency of prices adjustment. The weights in the intermediate input and consumption baskets are identical.
Figure 1.5: Heterogeneity in $s_i$, $\omega_i$, $\varepsilon_i$ and $z_i$.

Panel A: Response of sectoral prices to an expansionary MP shock

Panel B: Response of sectoral prices to sector-specific tech. shock

Panel C: Response of sectoral prices to an aggregate tech. shock

This figure shows the impulse response of sectoral prices to a monetary policy shock (A), idiosyncratic technology shocks (B) and an aggregate technology shock (C), under Case (3). Sectors only differ in their share of intermediate input use, frequency of prices adjustment and the weights in the intermediate input and consumption baskets.
Figure 1.6: Equivalent Homogeneous Economy: $\sigma_i = 0.38, \omega_i = 0.62, \xi_i = \zeta_i = \frac{1}{6}$.

This figure shows the impulse response of sectoral prices to a monetary policy shock (A), idiosyncratic technology shocks (B) and an aggregate technology shock (C).
Figure 1.7: Relative Price Effects of Monetary Policy Shock (sectoral prices relative to utilities)

This figure shows the prices of all sectors relative to that of utilities in response to a monetary policy shock in the three cases analyzed in the text. In cases (2) and (3) where frequency of price adjustment also varies across sectors, manufacturing sees the largest relative price effect.
Figure 1.8: The Impulse Response of PPI Aggregates to a MP Shock (FAVAR).

This figure shows the impulse response of three PPI aggregates to a MP shock identified using the FAVAR method.

Figure 1.9: Response of PPI Aggregates to a MP Shock Identified by Romer and Romer (2004).

This figure shows the impulse response of three PPI aggregates to a MP shock identified using the Romer and Romer (2004) measure of monetary policy shocks.
Figure 1.10: Response of PPI Aggregates to an Oil Supply Shock (FAVAR).

This figure shows the impulse response of three PPI aggregates to an oil supply shock identified in a FAVAR model, as explained in the text.
Figure 1.11: Response of PPI Aggregates to an Oil Supply Shock (Kilian’s narrative approach).

This figure presents the impulse responses of three PPI aggregates to an oil supply shock identified using Kilian’s historical measure.
Figure 1.12: Heterogeneity in $s_t$. $\omega_i = 0.85$, $\varepsilon_i = \zeta_i = \frac{1}{6}$.

Panel A: Response of sectoral prices to an expansionary MP shock

Panel B: Response of sectoral prices to sector-specific tech. shock

Panel C: Response of sectoral prices to an aggregate tech. shock

This figure shows the impulse response of sectoral prices to a monetary policy shock (A), idiosyncratic technology shocks (B) and an aggregate technology shock (C), under Case (1) and wage rigidity. Sectors only differ in their share of intermediate input use. Frequency of prices adjustment and the weights in the intermediate input and consumption baskets are identical.
This figure shows the impulse response of sectoral prices to a monetary policy shock (A), idiosyncratic technology shocks (B) and an aggregate technology shock (C), under Case (2) and wage rigidity. Sectors differ in their share of intermediate input use and frequency of prices adjustment. The weights in the intermediate input and consumption baskets are identical.
Figure 1.14: Heterogeneity in $s_i$, $\omega_i$, $\varepsilon_i$ and $\varsigma_i$.

This figure shows the impulse response of sectoral prices to a monetary policy shock (A), idiosyncratic technology shocks (B) and an aggregate technology shock (C), under Case (3) and with wage rigidity. Sectors differ in their share of intermediate input use, frequency of prices adjustment and the weights in the intermediate input and consumption baskets.
2 Optimal Monetary Policy with Overlapping Generations of Policy Makers

2.1 Introduction

The responsibility for the conduct of monetary policy in many countries lies with a committee. Major central banks, most notably the Federal Reserve, the European Central Bank, the Bank of England, the Bank of Japan, and the Bank of Sweden operate under Monetary Policy Committees (MPCs). These MPCs consist of a small number of individuals who are assigned for a certain period of time and reach their decisions in a variety of manners. However, the literature on monetary policy has often focused on a single, infinitely-lived central banker. This paper proposes a model where members of MPC have finite, overlapping tenures and studies the implications of departing from the standard institutional set-up assumed in the previous literature.

I start by assuming a two-member MPC, where each member is in office for two terms, and at each point in time there are two MPC members in office. Furthermore, I assume each MPC member sits on the committee with an "older" MPC member in their first year, and with a "younger" MPC member in their second year. Since there is an overlap between the tenures of different MPC members I refer to this model as a model of overlapping generations of MPC members. Each MPC member has a loss function which spans over his two-period tenure, and penalizes him for deviations of inflation and output gap from their respective targets. The set-up is otherwise standard to the New Keynesian literature (see Clarida, Gali, and Gertler (1999), McCallum and Nelson (2004), and Woodford (2003), Ch.7). The inflation-output gap trade-off is governed by a Phillips curve which can be derived from a variety of price rigidity models, e.g., a staggered pricing model of Calvo (1983) or Taylor (1980) and a price adjustment cost model of Rotemberg (1982).

I assume that each MPC member is individually able to commit to a path of future state-contingent policies. The overlapping structure of MPC member tenures will imply that the equilibrium outcome will be different from the optimal policy under commitment. A young MPC member would like to optimize in their first period and commit to a strong inflation response thereafter. On the other hand, an old MPC member has already made state-contingent plans in his first period of tenure.

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16 See Blinder (1998) and Gerlach-Kristen (2008) for a discussion on different decision making procedures in MPCs.
18 See, for instance, Clarida, Gali, and Gertler (1999), and Woodford (2003), Ch.7.
and thus does not want to re-optimize. This is the source of disagreement between the overlapping generations of MPC members. Put differently, an MPC member can commit to their own future policies but cannot influence the behavior of their successors. Therefore the incoming MPC members do not have access to a commitment technology in the first period of their tenure. Under rational expectations, the lack of such a technology leads the incoming MPC members to find it optimal to choose policy sequentially in their first period.

Further, I assume that differences between the choices of the old and the young MPC members are resolved through a utilitarian bargaining mechanism (Mas-Colell, Whinston, and Green (1995), Chapter 12); namely by maximizing the sum of objective functions of both (old and young) MPC members. I will show that this solution coincides with averaging the desired inflation rates proposed by each MPC member.

I extend the analysis to a larger MPC in which a fixed proportion \((1 - \alpha)\) of MPC members retire in each period and are replaced by young members. Therefore, in each period a proportion \(\alpha\) of the MPC are old members and the remaining \((1 - \alpha)\) share are young members. I refer to this process as "churning" and to \((1 - \alpha)\) as the churning rate. Again, MPC members reach decisions under a utilitarian bargaining mechanism, but now the share of each group in the MPC determines their "weight" in the bargaining process. An MPC in which old policy members never retire, is not different from a single infinitely lived MPC member who possesses a commitment technology. On the other hand, an MPC in which churning is complete, i.e. the entire committee is replaced by new members in each period, is equivalent to a single MPC member acting under discretion.\(^{19}\) Any churning rate in between these two extreme examples results in paths for inflation and output which are neither like those under discretionary monetary policy nor under full commitment, but something in between. These intermediate results are similar to the concepts of "quasi-commitment" suggested by Schaumburg and Tambalotti (2007), "loose commitment" by Debortoli and Nunes (2007) or "imperfect credibility" by Kara (2003). Whereas these studies assume an exogenous stochastic process which determines when and how often past promises are ignored, the present paper suggests an institutional reason which justifies imperfect credibility and imposes the frequency with which promises are broken.

The model implies that slower churning rates in MPC increase social welfare. This means that at any point in time the majority of MPC members should be old. The larger the proportion of old members in the MPC, the closer are monetary outcomes to optimal policy under commitment. The important question that arises is to what extent the institutional set-up (in particular, the size of churning) influences changes

\(^{19}\) As Blinder (1998) points out, if all members of an MPC are identical it does not matter whether decisions are made by an individual or by a committee. What differentiates monetary outcomes under a committee vs. under an individual policy maker in this model, is that MPC members differ in the starting date of their tenure.
in welfare. I show that this relation is quite sensitive to calibration. In particular, the results found in Schaumburg and Tambalotti (2007) who find that small departures from discretionary monetary policy bridge most of the gap in terms of welfare between discretionary and commitment policy, do not hold in general. I find that under the benchmark calibration, gains from commitment increase linearly with the proportion of old MPC members. This sensitivity to calibration is consistent with the results in Debortoli and Nunes (2007), who also offer a model of "loose commitment" similar to that of Schaumburg and Tambalotti (2007), and apply their theoretical model to a fiscal policy application. They find that in their application average allocations are substantially closer to discretion. When the probability of keeping promises is decreased from 1 to 0.75, most variables move more than half of the distance towards discretion.

2.1.1 Related Literature

This paper is related to a few strands in the literature. The most obvious is the literature on MPCs. As noted by Blinder (1998) some form of heterogeneity must distinguish MPC members from each other for policy under an MPC to be different from outcomes under an individual policy maker. A number of papers assume differences in preferences, for instance different weights to the twin objectives of inflation and output stabilization (see Aksoy, De Grauwe, and Dewachter (2002), Hefeker (2003) and Sibert (2003)). Hahn and Gersbach (2001) consider differences in skill and Gerlach-Kristen (2006) studies an MPC with different information. To the best of my knowledge no other paper analyzes MPCs where members have overlapping tenures.

The present paper does not aim to justify the existence of MPCs (for an example see Gerlach-Kristen (2006)); rather, it takes the overlapping structure of MPC as given. However, it has implications for how different decision making procedures affect welfare. Gerlach-Kristen (2006) presents a model in which MPC members receive different signals about the economy and studies the performance of different decision making procedures in this economy. She concludes that setting the interest rate equal to the average of the rates favored by individual MPC members coincides with the optimal procedure if the committee members receive signals of equal quality. Thus, "averaging" is favored over a voting procedure, whereby the median of the rates proposed by the MPC members is chosen. In the set-up proposed in this paper, the result of a voting procedure is trivial: if the majority of MPC members are old, equilibrium under voting will be identical to policy under commitment. On the other hand, if young members are in majority, the voting procedure will result in discretionary policy. Therefore, voting could be superior or inferior to a utilitarian bargaining in our model.

20See Blinder (1998) for a general discussion of the literature on MPCs.
A second related strand of literature is that of imperfect credibility or quasi-commitment. Since the seminal work by Kydland and Prescott (1977), time consistency of a policy with or without a commitment technology is well understood. More recently, a literature has emerged which abandons the assumption that policy is either conducted under full commitment or under period-by-period optimization. These papers build models that can support a continuum of policies under varying degrees of commitment by the central bank.

Schaumburg and Tambalotti (2007) offer a model of “quasi-commitment” by the central bank. In their model the monetary policy authority is assumed to formulate optimal commitment plans but is tempted to renege on them, and succumbs to this temptation with a constant exogenous probability known to and internalized by the private sector. By varying this probability of re-optimization the authors investigate the welfare effect of a marginal increase in credibility. The present model suggests a mechanism which gives rise to this exogenous probability based on an observable institutional setting.

In a similar paper, Kara (2003) assumes an additional mechanism for departure from perfect commitment. He uses the term “imperfect commitment” to refer to the possibility that the central bank will renege on its promises, as it is the case in Schaumburg and Tambalotti (2007). However, he allows for a difference in the actual probability with which the central bank re-optimizes and the probability of re-optimization perceived by the public. If the two probabilities are different (i.e. the private sector expects the commitment to last shorter than actually intended by the MPC member), the bank is said to have an “imperfect credibility”. Kara (2003) examines a case in which the monetary authority has both imperfect commitment and imperfect credibility and shows that in such a situation, an optimizing monetary authority will choose a less history dependent rule than in the case with perfect credibility.

Finally, a parallel discussion of policy commitment is widely studied in the fiscal policy literature, particularly in the context of optimal capital taxation policy. Judd (1985) shows that under certain conditions, the optimal tax on capital in a deterministic steady state is zero (this result holds in a variety of cases). He assumes that the government has access to a commitment technology. In the absence of a commitment net technology agents could enforce a given equilibrium using a trigger strategy. The public and the government follow a particular belief and action strategy which is optimal and revert to the sub-optimal, non-strategic equilibrium, should the other party deviate. This mechanism is used in Chari and Kehoe (1990). Kurozumi (2008) applies the mechanism suggested by Chari and Kehoe (1990) in a monetary policy context. These strategies rely on an infinite horizon objective function and therefore, will not be sustainable in the model presented in this paper, since the tenure of each MPC
member is finite. I take for granted that individual MPC members can commit to their own actions but cannot guarantee the actions of their successors. Therefore, it would be optimal for the successor to re-optimize.

Alternatively, Persson, Persson, and Svensson (2006) and Lucas and Stokey (1983), suggest a mechanism that makes the commitment solution to be time-consistent. Each government should leave its successor with a carefully chosen maturity of nominal and indexed debt for each contingent state of nature and at all maturities. This strategy recovers the optimal policy under commitment without access to such technology. Their suggestion involves a partial commitment, namely to honor previous debt, but no commitment about taxes. In the context of monetary policy this solution will not be operational since there are no state variables that could link two consecutive periods. In other words, there are no instruments which would allow an MPC member to affect the decision of his successor. The new MPC member could reset inflation and output gap instantaneously and in effect re-optimize the problem.

The contribution of this paper is twofold: First, the model studies a departure from the standard New Keynesian framework assumptions, which brings the model closer to what we observe in reality. Namely, I relax the assumption of a single infinitely-lived MPC member (or an MPC whose objective function is infinitely long), and replace it by an overlapping structure for the MPC members, as we observe in reality. Second, the model proposes a mechanism which gives rise to imperfect commitment by the monetary authority, as opposed to imposing an exogenous probability with which previous promises are broken.21

The normative implication of the model is a particular institutional structure: The model suggests that churning rate in the MPC must be slow, whether decisions are made through a voting procedure or through (utilitarian) bargaining. Furthermore, the model suggests that if churning rate is high, bargaining is preferred to voting. On the positive side the model suggests that welfare gains from commitment are sensitive to calibration. Under the benchmark calibration (Woodford 1999) welfare gains are close to linear in the churning rate of committee.

The remainder of this paper is organized as follows: Section 2 provides the theoretical model and derives optimal monetary policy under commitment and discretion as benchmarks. Section 3 introduces the overlapping generations of MPC members and discusses the solution to the model under utilitarian bargaining. It also extends the results to an n-member committee. Section 4 compares utilitarian bargaining with voting. Section 5 presents the model calibration and discusses the impulse responses of inflation and output gap under varying degrees of credibility. Section 6 concludes.

2.2 Optimal Monetary Policy: The New Keynesian Framework

This section briefly reviews the canonical New Keynesian framework for monetary policy design problem used in the literature (see, for instance, Clarida, Gali, and Gertler (1999) and Woodford (1999)), and presents the solution under discretion and perfect commitment as benchmarks.

2.2.1 Optimal monetary policy and stabilization bias

A representative household maximizes utility in an economy with monopolistically competitive firms and some form of nominal rigidity. The private sector’s forward-looking behavior is the essence of the New Keynesian model and the source of the time-inconsistency problem. Consider an economy with a continuum of infinitely lived private agents and a monetary policymaker. The private agents produce and consume a continuum of differentiated, imperfectly substitutable goods. The agents price their goods in an environment of monopolistic competition, and in the presence of nominal rigidities of the form described by Calvo (1983). The supply side of the economy is obtained by log-linearizing the first order condition of the firm’s profit maximization problem. This gives rise to an equation often referred to as the New Keynesian Phillips Curve (NKPC) and takes the following form:

\[ \pi_t = \beta E_t \pi_{t+1} + k x_t + v_t \]  \hspace{1cm} (2.1)

\[ v_t = \rho v_{t-1} + \epsilon_t \] \hspace{1cm} (2.2)

where \( x_t \) is the output gap, or the deviation of output from its natural level, \( \pi_t \) is the average price level inflation from time \( t-1 \) to time \( t \), and \( v_t \) is a cost-push shock, representing a variety of supply shocks in the economy. \( v_t \) is assumed to follow an AR(1) process with an autoregressive coefficient of \( \rho \), and is subject to innovations \( \epsilon_t \) with the standard deviation of \( \sigma^2 \). \( \beta \) is the subjective discount factor of the representative consumer, and \( k \) is a constant and a function of the structural parameters of the model. It is clear from the NKPC (2.1) that inflation is a forward-looking variable and depends positively on future expectations about inflation and current output gap.

The demand side of the economy is described by a dynamic IS equation, which is simply a log-linear version of the Euler equation of the representative consumer

\[ x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1} - \pi^n_t) \] \hspace{1cm} (2.3)

where \( i_t \) is the nominal interest rate controlled by the central bank and \( \pi^n_t \) is the natural (real) rate of interest. The natural interest rate is the rate that would prevail in an equilibrium with flexible prices. The parameter \( \sigma > 0 \) is the intertemporal elasticity of substitution in consumption.
The monetary authority minimizes a quadratic objective function subject to the NKPC (2.1) and the IS equation (2.3). As shown in Woodford (2003), Ch.6 and Erceg and Levin (2006), this loss function can be obtained as a second-order approximation of the representative household's utility function with Calvo (1983) or Taylor (1980) style staggered pricing of monopolistically competitive firms.\footnote{This result is based on certain assumptions including an infinite horizon model, and therefore, does not hold in the overlapping generations model assumed here, where MPC members' loss function spans a finite period. See Debortoli and Nunes (2007).} The period objective function depends on the variance of the output gap, \( x_t^2 \), and the variance of inflation, \( \pi_t^2 \), in the economy. Specifically, the monetary authority minimizes:

\[
L = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \pi_t^2 + \lambda x_t^2 \right]
\]  

(2.4)

Note that in this specification the monetary authority's inflation and output gap targets are both assumed to be zero. Targeting the natural rate of output eliminates the traditional inflation bias à la Barro and Gordon (1983). The monetary authority will not attempt to push the economy beyond its potential through inflation surprises. For example, the monopolistic competition distortions could be removed and efficient level of output restored using subsidies, eliminating the need for monetary policy to target an output level higher than the natural rate. Nevertheless, as argued extensively in Clarida, Gali, and Gertler (1999) and Woodford (1999) the ability of the monetary authority to commit to a policy will still deliver a better outcome than under discretion, as it gives the monetary authority an extra policy instrument. This additional instrument is the ability to influence expectations through promises about future policies. By committing to future policies a monetary authority can internalize the effect of its decisions on expectations and thus gain an extra policy tool. In the absence of a commitment technology, the monetary authority takes agents' expectations as given which will result in sub-optimal outcomes. This inefficiency is often referred to as the "stabilization bias".

\subsection{Optimal response under discretion}

In the absence of a commitment technology, the monetary authority ignores the effect of its actions today on the formation of expectations. In effect, the monetary authority re-optimizes the objective function in every period, taking the expectations about future values of inflation as given. Thus, the dynamic optimization problem above breaks into an infinite number of contemporaneous optimization problems, or one-shot games. This is what we refer to as the \textit{discretionary outcome}. The problem therefore simplifies to minimizing

\[
L_t = (\pi_t^2 + \lambda x_t^2)
\]
subject to (2.1) and (2.3). Note that since \( i_t \) does not appear in the objective function, the constraint (2.3) can be ignored. We can obtain the optimal solutions for \( \pi_t \) and \( x_t \) by minimizing the monetary authority’s loss function, subject only to the NKPC (2.1), and then recover the optimal path for \( i_t \) using the IS relationship.

The first order condition of this optimization problem is

\[
\pi_t + \frac{\lambda}{k} x_t = 0 \tag{2.5}
\]

Combining (2.5) with the NKPC (2.1), we obtain the following expressions for \( \pi_t \) and \( x_t \):

\[
\pi_t = \frac{\lambda}{\kappa^2 + \lambda(1-\beta\rho)} v_t \tag{2.6}
\]

\[
x_t = -\frac{k}{\kappa^2 + \lambda(1-\beta\rho)} v_t \tag{2.7}
\]

Note that both inflation and output gap are functions of the current period cost push shock. In other words the monetary authority brings back inflation to its zero level immediately. The impulse response is shown in figure (2.1).

### 2.2.3 Optimal response under commitment

When the monetary authority has access to a commitment technology, it has the ability to influence expectations by committing to a certain form of policy behavior in the future. This implies that the problem can no longer be treated as a static optimization problem. The Lagrangian \( \mathcal{L}_c \) can be written as

\[
\mathcal{L}_c = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \pi_t^2 + \lambda x_t^2 + 2\varphi_t (\pi_t - \beta E_t \pi_{t+1} - k x_t - v_t) \right] \tag{2.8}
\]

where \( \varphi_t \) is the Lagrange multiplier on constraint (2.1) in period \( t \geq 0 \). The first-order condition for the optimal policy is then given by differentiating the infinite period objective function (2.8) with respect to \( \pi_t \) and \( x_t \):

\[
\pi_t - \varphi_t + \varphi_{t-1} = 0 \text{ and } \lambda x_t + k \varphi_t = 0 \quad \forall t \geq 0
\]

together with the initial condition that \( \varphi_{-1} = 0 \), which indicates no previous commitment in period zero. Substituting out the Lagrange multipliers yields an inflation rate rule that implements the optimal policy

\[
\pi_0 = -\frac{\lambda}{k} x_0 \text{ and } \pi_t + \frac{\lambda}{k} [x_t - x_{t-1}] = 0 \quad \forall t > 0
\]
Woodford (1999) argues that optimal commitment policy lacks continuity in its form and thereby “fails to be time consistent only if the central bank considers ‘optimality’ at each point in time” (p. 293). He thus proposes its variant from a timeless perspective, which specifies the equilibrium to be optimal in all periods. This requires that the policy at time zero is also that associated with the commitment policy. Therefore the optimal policy from a timeless perspective is implemented by an inflation rate rule of the following form:

\[ \pi_t + \frac{\lambda}{k} [x_t - x_{t-1}] = 0 \quad \forall t \geq 0 \quad (2.9) \]

The inflation rate rule (2.9), together with the NKPC (2.1), implies the following second order difference equation in \( x_t \):

\[ x_{t+1} - \left( \frac{\beta + 1 + k^2/\lambda}{\beta} \right) x_t + \frac{1}{\beta} x_{t-1} = \frac{k}{\beta \lambda} v_t \quad (2.10) \]

The stationary solution to this difference equation is:

\[ x_t = \mu_1^c x_{t-1} - \frac{k}{\lambda \beta [\mu_2^c - \rho]} v_t \quad (2.11) \]

where \( \mu_1^c < 1 \) and \( \mu_2^c > 1 \) are the roots to the characteristic equation below:

\[ \mu^2 - \left( \frac{\beta + 1 + k^2/\lambda}{\beta} \right) \mu + \frac{1}{\beta} = 0 \]

Combining the solution for \( x_t \) (2.11) and (2.9), obtains the solution for \( \pi_t \):

\[ \pi_t = \frac{\lambda}{k} (1 - \mu_1^c) x_{t-1} + \frac{1}{\beta [\mu_2^c - \rho]} v_t \quad (2.12) \]

### 2.3 Overlapping Generations of MPC members

This section introduces a committee with overlapping generations of MPC members. First it analyzes the structure of a two member MPC. It defines the bargaining mechanism and derives the equilibrium path of inflation, output gap and interest rates. It then generalizes the solution to an \( n \)-member MPC and derives the equilibrium policy.

#### 2.3.1 A 2-member MPC

Consider the following set-up. Monetary policy is set by an MPC which comprises of two members in each period. Each MPC member is in office for two periods. The first period of his tenure overlaps with the 2\textsuperscript{nd} period of his predecessor’s term in office. In the second period of his term, an MPC member shares the office with his successor. So during each period, the terms of two “generations” of MPC members overlap. In
this paper I refer to an MPC member as “young” while serving their first term, and as “old” while serving their second (or higher) terms.

The objective function of each MPC member only spans across his tenure while in office. Specifically, consider the two MPC members who share an office in period $t$. I label the loss function of each MPC member by the date at which he begins his term in office. The objective function of an MPC member who begins his term at time $t-1$ is

$$L_{t-1,0} = -\beta^{t-1}E_{0}\left[\left(\pi_{t-1}^2 + \lambda \pi_{t-1} \pi_{t} + \beta(\pi_{t+1}^2 + \lambda \pi_{t+1})\right)\right]$$

(2.13)

where $L_{t-1,0}$ denotes the time-zero expectation of the objective function of an MPC member whose term begins at time $t-1$. Similarly, the time-zero expectation of the objective function of an MPC member whose term begins at time $t$ is:

$$L_{t,0} = -\beta^{t}E_{0}\left[\left(\pi_{t}^2 + \lambda \pi_{t} \pi_{t+1} + \beta(\pi_{t+1}^2 + \lambda \pi_{t+1})\right)\right]$$

(2.14)

The old MPC member would like to minimize $L_{t-1,0}$ subject to the Phillips curve relationships in periods $t-1$ and $t$; i.e.:

$$\pi_{t-1} = k\pi_{t-1} + \beta E_{t-1}\pi_{t} + \nu_{t-1} \quad \text{and}$$

$$\pi_{t} = k\pi_{t} + \beta E_{t}\pi_{t+1} + \nu_{t}$$

(2.15)

(2.16)

whereas the “young” MPC member would like to minimize $L_{t,0}$ subject to:

$$\pi_{t} = k\pi_{t} + \beta E_{t}\pi_{t+1} + \nu_{t} \quad \text{and}$$

$$\pi_{t+1} = k\pi_{t+1} + \beta E_{t+1}\pi_{t+2} + \nu_{t+1}$$

(2.17)

(2.18)

One can impose the contraints each MPC member faces into their objective functions, given that these constraints should always be weakly binding. This means that

$$L_{s,0} = -\beta^{s}E_{0}\left\{\left[\pi_{s}^2 + \lambda\left(\frac{\pi_{s} - \beta E_{s}\pi_{s+1} - u_{s}}{k}\right)^2\right] + \beta E_{s}\left[\pi_{s+1}^2 + \lambda\left(\frac{\pi_{s+1} - \beta E_{s+1}\pi_{s+2} - u_{s+1}}{k}\right)^2\right]\right\}$$

(2.19)

where $L_{s,0}$ defines the objective function of the life-time maximization problem of an MPC member whose career begins at time $s$, and it already takes into account the constraints that each MPC member faces in his lifetime. The two MPC members in office at any point in time decide what the inflation and output gap in that period should be \(^{23}\) through a bargaining process.

The source of disagreement between the old and the young MPC members in office

\(^{23}\)As it is customary in such problems, we assume that the monetary authority decides the inflation level desired and the corresponding $\pi_{t}$ can be uniquely determined from the IS equation.
at anytime is their access to a commitment technology. I have assumed that each MPC member can only commit to their own future policies. Therefore, they have access to a commitment technology in all but the first period of their tenure. This assumption may be justified in the real world, in that policy makers are unknown before they are elected, and there is larger uncertainty about their future policies. However, once they occupy an office and announce their future plans, the uncertainty is reduced to a large extent.

Take the two MPC members in office at time $t$. In the set-up proposed above, the lack of a commitment technology for the incoming MPC member leads him to choose policy sequentially in his first period. The young MPC member would maximize (2.14) subject to (2.17) and (2.18) taking private agents' expectations about $\pi_t^*$ as given. On the other hand, the old MPC member maximizes (2.13) subject to (2.15) and (2.16), since this policy maker has already served in $t-1$, he has had the ability to commit to his time $t$ policy and therefore influence $E_{t-1}\pi_t$. This MPC member can thus take advantage of this additional endogenous variable to offer a better trade-off between inflation and output gap. This is the source of disagreement between the two MPC members, which needs to be resolved through bargaining.

Next, we explore the bargaining process and its equilibrium solution to the monetary design problem.

### 2.3.2 The Bargaining Process

The two MPC members in office at any time period $t$ decide what the inflation and output gap at that period will be, through a bargaining process. This can be thought of as a time zero bargaining. In other words, all future MPC members meet at time zero and agree on a state-contingent plan. I make two important assumptions for the following analysis. First, only the MPC members in office at time $t$ can decide on the monetary outcomes of period $t$. This is a crucial assumption; since all MPC members taking office from time $t$ onwards affect monetary outcomes at time $t$ by influencing the expectations of agents about future. Therefore, potentially all MPC members could enter into a “grand bargain” and decide for a state-contingent plan for the paths of inflation and output gap. The optimal outcome under this scenario, would be equivalent to the commitment solution. By limiting the bargaining process to the two incumbants at each time, excludes this “grand bargain”.

The second underlying assumption is that we analyze the solution from a timeless perspective (Woodford 1999). In other words, the time periods in the analysis that follows are sufficiently far away from time zero, that the effect of the initial period can be neglected. Next, we explore the bargaining mechanism and its equilibrium solution to the monetary policy design problem.
The Bargaining Problem  Mas-Colell, Whinston, and Green (1995) define a bargaining problem among $I$ agents by its two elements: a utility possibility set $U \subset \mathbb{R}^I$ and a threat point, or status quo point $u^* \in U$. The set $U$ represents the allocations of utility that can be settled on if there is cooperation among the different agents. The point $u^*$ is the outcome that will occur if there is a breakdown of cooperation. Cooperation requires the unanimous participation of all agents and thus in equilibrium will imply that: $U \geq u^* t$ where $t$ is a unity vector of length $I$.

**Definition 1** A bargaining solution is a rule that assigns a solution vector $f(U, u^*) \in U$ to every bargaining problem $(U, u^*)$.

In our particular problem, $U_t = (L_{t-1}, L_t)'$. In other words, the bargaining possibility set at time $t$ is comprised of the objective functions of the young and the old MPC members, occupying office at time $t$.

Let the status quo point be $u^* = (-1, -1)$. The negative value for status quo may be interpreted as the social costs of indecision or irreconcilable differences within the monetary authority and the ensuing public embarrassment. Note that the objective functions of both participants are always weakly positive, and therefore, abandoning the bargaining process is never optimal for either party.

The Utilitarian Solution and the Stationary Equilibrium  In this paper I will consider a particular bargaining mechanism, namely the utilitarian solution. Define the utilitarian solution $f_t(U)$ such that it maximizes $\sum u_t$ on $U$. Given that $U$ is convex, the solution is uniquely defined. I focus on this bargaining solution for two reasons: the additivity makes the maximization very tractable. The second reason is more conceptual. In an infinite horizon context, an agent minimizes a weighted average of the losses in all periods. The utilitarian solution to the bargaining problem above proposes a similar method, in that it aims to minimize a weighted average of the losses accrued to each party (or all parties, in the case of a committee).

In the case of an MPC which comprises of two members, $f_t(U)$ maximizes the sum of the two utility payoffs from each party. Therefore, the two MPC members choose $\pi_t$ and $x_t$ to find:

$$f_t(U) = \max_{\pi_t, x_t} [L_{t-1,0} + L_{t,0}]$$

(2.20)

where $L_{t-1}$ and $L_t$ are defined as in (2.19).

The first order condition of the maximization problem with respect to $\pi_t$ is as follows:

---

24 We will explore a particular solution to the bargaining problem, namely the utilitarian solution. It is easy to show that the utilitarian solution to a bargaining problem satisfies the property of independence of utility origins (IUO). This means that the bargaining solution does not depend on absolute scales of utility. Therefore, our choice of $u^*$ does not matter, and in fact, we can suppress the term $u^*$ in the definition of $f$. 

---
Note that the first order condition (2.21) is an average of the two first order conditions under commitment and discretion (c.f. first order conditions (2.5) and (2.9)). Now, combining the first order condition (2.21) and substituting it into the NKPC (2.1) yields a second order difference equation for $x_t$:

$$x_{t+1} = \left(\frac{\beta/2 + 1 + k^2/\lambda}{\beta}\right) x_t + \frac{1}{2\beta} x_{t-1} = \frac{k}{\beta \lambda} v_t$$

Its characteristic polynomial

$$\mu^2 - \left(\frac{\beta/2 + 1 + k^2/\lambda}{\beta}\right) \mu + \frac{1}{2\beta} = 0$$

has roots $\mu_1$ and $\mu_2$ inside and outside the unit circle respectively. The corresponding solution for $x_t$ and $\pi_t$ are:

$$x_t = \mu_1 x_{t-1} - \frac{k}{\lambda \beta [\mu_2 - \rho]} v_t$$

$$\pi_t = \frac{\lambda}{k} (1 - \mu_1) x_{t-1} + \frac{1}{\beta [\mu_2 - \rho]} v_t$$

Comparing the roots $\mu_1 < \mu_1^c$ and $\mu_2 < \mu_2^c$. This implies that the equilibrium response of the output gap to a cost-push shock is less persistent compared to optimal policy under commitment, and closer to the response under discretion. The equilibrium response of inflation, output gap and interest rates under a two member committee are displayed in figure (2.2) with a dashed lines with plus signs. We see that the response of all three variables is roughly half-way between the response under commitment and discretion. The initial response of inflation and output gap are smaller than that under discretion but larger compared to optimal policy under commitment. As argued above, the persistence of all three variables are less compared to the policy under commitment.

### 2.3.3 n-member Committee

The results of previous section can be extended to a more generalized setting. In this section, I relax two assumptions about the set-up in section 2.3.1. First, the MPC is composed of $n \geq 2$ number of members. Second, MPC members serve a $T$-period term. Here, an MPC member is considered “young” if and only if he is serving his first term. Note that it is only the first term which is special in the optimal policy under commitment. If policy was timeless, as suggested by Woodford (1999), or the tenure of an MPC member was long enough that the effect of time zero could be ignored, then nothing would distinguish different MPC members from each other, regardless
of when they started their term. However, when tenures are finite the incoming MPC members find it optimal to implement the discretionary policy in their first term. This is the source of disagreement between young and old policy makers.

Assume that at any period $t$, the MPC is made-up of $n$ members. Further assume that in each period $n_y$ of the MPC members retire and are replaced by young ones. Therefore, at any period $t$ the MPC is composed of $n_y$ young member and $n_o = n - n_y$ old members. Note that the number of incoming and outgoing members should be equal for the size of the committee to remain constant. I refer to $\frac{n_o}{n} = \alpha$ and define the churning rate to be $1 - \alpha$. A constant churning rate $1 - \alpha$ and a constant committee size requires that $n_y$ MPC members retire in each period and are replaced by young ones. All the members of the MPC at time $t$ should retire by $t + T$, therefore we must have that $n_y = n / T$. Moreover, not all of the old MPC members begin their careers together. The only distinction we need to make is between those members whose terms start at time $t$ and those members who have started their term earlier. The bargaining solution will maximize the sum of all MPC members' utilities. Again, denote the loss function of an MPC member by a subscript referring to the date their tenures begin. Thus, $L_{t,0}$ denotes the time zero expectation of the loss function of an MPC member whose term begins at time $j$. For determining monetary policy at time $t$, all MPC members whose terms began from $t - T + 1$ to MPC members beginning their tenure at $t$ enter the bargaining process. The utilitarian solution to this problem is defined as:

$$f_t^*(U) = \max_{\pi_t, x_t} \sum_{i=1}^{T-1} \omega_i L_{t-i,0} + n_y L_{t,0}$$

(2.25)

where $L_{s,0}$ is the time zero expectation of the loss function of an MPC member whose term began at time $s$ and is defined as

$$L_{s,0} = -\beta^s E_0 \sum_{i=0}^{T-1} [\pi_{s+i}^2 + \lambda(\pi_{s+i} - \beta E_s \pi_{s+i+1} - u_{s+i})^2]$$

and $\omega_i$ are the number of old MPC members whose terms begin at time $t - i$. Note that $\sum \omega_i = n_o$. Differentiating equation (2.25) with respect to $\pi_t$ yields the following first order condition:

$$n_o[\pi_t + \frac{\lambda}{k} x_t - \frac{\lambda}{k} x_{t-1}] + n_y[\pi_t + \frac{\lambda}{k} x_t] = 0$$

(2.26)

Equation (2.26) shows that the first order condition under a utilitarian bargaining solution is a weighted average of the first order conditions of the old and the young MPC members. Rewrite equation (2.26) as

$$\alpha[\pi_t + \frac{\lambda}{k} x_t - \frac{\lambda}{k} x_{t-1}] + (1 - \alpha)[\pi_t + \frac{\lambda}{k} x_t] = 0$$

(2.27)
where \( \alpha = \frac{n_y}{n} \), or the churning rate. Combining first order condition (2.26) and substituting it into the NKPC (2.1) yields a second order difference equation for \( x_t \)

\[
x_{t+1} - \left( \frac{\alpha \beta + 1 + k^2/\lambda}{\beta} \right) x_t + \frac{\alpha}{\beta} x_{t-1} = \frac{k}{\beta \lambda} v_t
\]  

(2.28)

This difference equation is stable with constant coefficients if and only if the ratio \( \alpha \) of old MPC members to total is constant at all times. This is the requirement the churning rate is constant. The characteristic polynomial of this difference equation is

\[
\mu^2 - \left( \frac{\alpha \beta + 1 + k^2/\lambda}{\beta} \right) \mu + \frac{\alpha}{\beta} = 0
\]  

(2.29)

and has roots \( \mu_1^2 \) and \( \mu_2^2 \) inside and outside the unit circle respectively. The solution for \( x_t \) is:

\[
x_t = \mu_1^2 x_{t-1} - \frac{k}{\lambda \beta (\mu_2^2 - \rho)} v_t
\]  

(2.30)

It can be shown that \( \frac{\partial \mu_1}{\partial \alpha} > 0 \) \( \forall \alpha \in [0,1] \). Furthermore, note that when \( \alpha = 1 \), the solution coincides with that under commitment and when \( \alpha = 0 \) the solution coincides with period-by-period optimization. The immediate conclusion is that the lower the churning rate (higher \( \alpha \)) the closer the outcomes will be to optimal policy under commitment.

This conclusion can be cast in terms of the duration of the MPC members’ service as well. Note that we defined the churning rate as \( 1 - \alpha = \frac{n_y}{n} = \frac{1}{T} \). As \( T \) increases towards infinity, \( \alpha \) increases towards 1. This is reasonable; we would expect that the longer the duration of an MPC member’s term in office, the closer the solution would be to that under full commitment. In the limit, if policy members never retire \( (T \to \infty \text{ or } \alpha = 1) \) then the outcomes would be identical to those under commitment. On the other hand, if \( T = 1 \) (or \( \alpha = 0 \)), each MPC member will be in office for 1 term only, clearly delivery the solution under commitment.

2.4 Utilitarian Bargaining vs. Voting

Blinder (1998) classifies MPCs into individualistic vs. collegial. In individualistic committees decisions are reached through voting: positions are offered and debated and once all committee members put forth their case, they vote. On the other hand, in a collegial committee a consensus is built and recalcitrant members are persuaded to go along (usually by the chair person).

The model presented here could be interpreted as a collegial committee whereby a compromise solution is reached. Inflation is chosen as the weighted average of inflation levels favored by MPC members. What would be the result if decisions were made through voting instead? In other words, how would an overlapping generations
structure for MPC members affect the monetary outcomes if the MPC was more “individualistic” as opposed to “collegial” in Blinder’s words?

Gerlach-Kristen (2006) compares decision making procedures in a model where there is uncertainty about potential output. She compares “averaging” procedure where the interest rate is set equal to the mean of the rates favored by the individual MPC members with a voting procedure which implements the median of these rates. She concludes that averaging coincides with the optimal procedure if the committee members are “equally skilled” in the sense that the signals they receive in the economy are of equal quality, however voting can lead to better decisions than averaging if abilities vary between policy makers.

In the model presented in this paper, a voting procedure has a trivial outcome. If the majority of the committee members are old, the voting outcome will be the same as policy under commitment and else, voting will result in discretionary monetary policy. Thus, whether voting is a welfare improving decision making procedure compared to bargaining depends on the composition of the committee. What is clear is that under both procedures low churning rate is preferred. However, as we will numerically show in the next section, the gains from low churning rate are continuous under bargaining whereas they are discrete under voting. These results suggest that if churning rate in the MPC is low, voting is preferred to bargaining since voting will recover the outcome under commitment but bargaining will not. On the other hand, if churning rate is high bargaining is the preferred decision making procedure. The bargaining outcome will be strictly preferred to the discretionary policy as long as the churning rate is less than 100%.

2.5 Calibration and Simulation Results

In the benchmark calibration for the model’s parameters I follow Woodford (1999). This calibration is summarized in table (2.1). The model is assumed to refer to quarterly variables, with interest rates and inflation measured as annualized percentages. All parameter values are reasonably standard, with the possible exception of the relative weight on the output gap in the monetary authority’s loss function. This relatively low value is derived from a micro-founded model which approximates the loss function as a second order expansion of the utility function of the representative consumer. It is therefore consistent with the rest of the structural parameters. I also report results for values of \( \lambda \) more commonly found in the optimal monetary policy literature. In measuring the loss to the society, one metric would be the unconditional expectation of expression (2.4).\(^{25}\) I follow Schaumburg and Tambalotti (2007) and

\(^{25}\)This is the metric chosen by King and Wolman (1999), Rotemberg and Woodford (1999), Rudebusch and Svensson (1999), and Walsh (2003).
report the change in welfare associated with different levels of credibility as a fraction of the total difference in welfare between discretion and commitment.\footnote{This method eliminates the need to specify the standard deviation of the cost-push shock.}

Table (2.1) also compares the benchmark calibration with another set of parameters commonly used in the literature, suggested by McCallum and Nelson (2004). These authors suggest that the actual value of $\kappa$ probably lies between 0.01 and 0.1 which is consistent with the estimates in Gali and Gertler (1999). The value of $\lambda$ which is related to the monetary authority preferences is more subjective. The range suggested by McCallum and Nelson (2004) includes the benchmark parameter.

2.5.1 Impulse Responses to Independent Shocks

Impulses are normalized to produce an annualized one percentage point increase in inflation on impact, for given expectations. Given the forward looking nature of the model, the actual increase in inflation is a function of the forecasted response of policy to the shock.

We assume that the economy starts in the steady state, with zero inflation and no output gap. Figure (2.1) shows the path of inflation, output gap and interest rate under commitment and discretion. With the benchmark calibration, this just replicates the results in Woodford (1999). Under discretion the monetary authority moves its instrument with the shock, returning the economy to the steady state as soon as the effects of the shock have faded. Given an i.i.d. impulse, this implies that the economy is driven into a sharp recession, accompanied by high inflation, but for only one period. This is the policy that each young MPC member would like to choose in any period following the period of the shock.

Under commitment instead, the monetary authority exploits the possibility of influencing inflation expectations in its favor in the period of the shock, by promising a protracted mild recession, accompanied by deflation. This can be accomplished with a very limited movement in the interest rate.\footnote{We are assuming that there are no shocks to the natural interest rate.} It is clear why absent of a commitment technology this path for the policy variable is time inconsistent: the monetary authority would want to return to zero inflation and output gap as soon as the shock has disappeared. Note that in the absence of new shocks and inflation bias, the steady state values of the endogenous variables under discretion are consistent with those under optimal commitment.

These two extreme results are obtained with an MPC also, when monetary policy is set by voting among MPC members. As noted in section 2.4 if the level of inflation is decided upon by a simple majority vote, then the paths of inflation, output gap and interest rate will be those under optimal policy with commitment if a majority of
MPC members are old. On the other hand, when the majority of MPC members are young the equilibrium paths of variables of interest will be those under discretionary monetary policy.

Now consider the response of variables when monetary policy is set by an MPC. As argued in section 2.3.3 this is equivalent to choosing an inflation level equal to the average of inflation levels favored by all MPC members. Consider, first, the impulse response of the two-member committee studied in section 2.3.1. Figure (2.2) presents these impulse responses (with a dashed-plus line). The paths of inflation, output gap and interest rates under a two member committee is the same as that under a committee of any size with $\alpha = \frac{1}{2}$.

Note that the inflation response is roughly halfway between the response under commitment and discretion. This is unlike the results in Schaumburg and Tambalotti (2007) who conclude that relatively low levels of credibility are enough to produce qualitative responses of the economy very close to the ones obtained under commitment. Furthermore, the contraction required to bring back inflation to the steady state level, is also halfway between that under discretion and commitment. Figure (2.2) also shows the monetary outcome under a committee with three quarters of its members being old. As we see the results are closer to the response under commitment. Very roughly, the responses of all variables are three-quarters of the way between discretionary and commitment response. We will quantify this more accurately in section 2.6.

The response of nominal interest rate is similar. The optimal response to a positive cost-push shock is raising nominal interest rates. Under discretion, and when shocks are i.i.d, interest rates are raised heavily in the period of the shock, but return to their steady state value immediately after. Under commitment, the initial rise in interest rates is much lower, but the return to the steady state is very persistent. These two patterns are reflected in the response of interest rates under a committee. Under a two-member committee, or when $\alpha = 1/2$, the initial hike in interest rates is roughly halfway between the discretionary and commitment responses. When $\alpha = 3/4$, the response is much closer to that under commitment.

### 2.5.2 Impulse Responses to Persistent Shocks

This section analyzes the response of the model when the economy is hit by a persistent shock ($\rho = 0.80$). Figure (2.3) shows this response under commitment and discretion. It is apparent that the path of the variables under discretion is much closer to commitment with a persistent shock, since the exogenous persistence causes a similar persistence in inflation and output gap under discretion that is desirable under commitment.
The initial rise in inflation and output gap is much higher under a persistent shock, compared to an i.i.d. shock. This is because of the forward looking nature of the model. Although the initial shock is just as large as the one studied in the i.i.d case, the prolonged cost-push shock implies a much higher cost in terms of output gap, and higher initial inflation. Figure (2.4) shows the response of these variables for a committee with $\alpha = 0.5$ (dashed-plus line) and $\alpha = 0.75$ (dashed-cross line) proportion of old members. The shape of all inflation responses are similar, with initial inflation hike being the highest under discretionary monetary policy. Furthermore, the paths of inflation with $\alpha = 0.5$ and $\alpha = 0.75$ are distributed quite evenly between the path of inflation under commitment and discretion.

The path for output-gap shows a different pattern. Under discretion, the response to a cost-push shock is always proportional to the current period output gap. Since the shock is monotonically decreasing in size, the output gap also increases monotonically towards its steady state. However, under commitment, and to a lesser degree under committees with $\alpha = 0.5$ and $\alpha = 0.75$, there is some persistence in the response of output gap to a cost-push shock. This means that the response to a persistent shock becomes hump-shaped and the recession worsens before it gets better. The same non-monotonicity is reflected in the response of interest rates. In reality, the monetary authority would control interest rates and thus the hump-shaped pattern in interest rates would also be reflected in the output-gap.\(^{28}\)

2.6 Welfare Analysis

An important question that arises is to what extent the institutional set-up of a MPC influences the level of welfare. Schaumburg and Tambalotti (2007) answer this question in the context of their quasi-commitment technology and conclude that only small deviations from discretion are needed in order to obtain welfare levels very close to optimal policy under commitment. I analyze the same question by looking at the changes in society’s loss when proportion of old members in the MPC moves away from $\alpha = 1$ (equivalent to policy under commitment) to $\alpha = 0$ (equivalent to policy under discretion). The loss to society is calculated as the unconditional expectation of variances in inflation and output gap, as in (2.4). The variation of this loss function with $\alpha$ is demonstrated in figure (2.5). One can see that unlike the conclusion reached by Schaumburg and Tambalotti (2007) in this model, and under the benchmark calibration, the gains from commitment are close to linear with the proportion of old members in MPC.

It turns out that this conclusion is quite specific to this calibration. Consider the alternative calibration suggested by McCallum and Nelson (2004) in table (2.1).

\(^{28}\)Since we have assumed away demand shocks, setting inflation or interest rate as monetary targets are equivalent.
Specifically, assume that $\lambda = 0.1$ and $k = 0.01$. As figure (2.7) shows the variations in the loss function become a convex function of $\alpha$. In other words, the committee must be much closer to an old member majority, before most benefits of commitment are obtained. This convexity is consistent with the findings by Debortoli and Nunes (2007). In the fiscal policy application they analyze, they find that a small departure from full commitment moves most variables substantially towards discretion.

Another metric for measuring the effect of a varying $\alpha$ is to plot the volatility of output gap against inflation volatility associated with the optimal policy for different levels of $\alpha$. Figure (2.6) shows the efficient frontier under the benchmark calibration. Moving away from fully discretionary monetary policy, i.e. the case when all MPC members begin their terms simultaneously will reduce both standard deviation of output gap and inflation. Beyond a certain $\alpha$, decreases in inflation volatility require volatilities in output-gap. Compare this efficient frontier, with the one in figure (2.8) which corresponds to a higher $\lambda = 1$ (and $k = 0.01$). Given the higher weight associated with inflation volatility, moving towards the optimal point, i.e. commitment policy, involves decreasing inflation volatility substantially at the cost of higher output-gap volatility.

2.7 Conclusions

This paper presents a model in which monetary authority is modeled as a committee with churning of members who have finite terms and are replaced by new committee members. Older and younger generations of MPC members decide on monetary policy by engaging in a utilitarian bargaining procedure. A MPC composed of entirely old members replicates the monetary outcomes under a single central banker who sets policy under commitment. The other extreme is a committee fully composed of young or incoming members who replicate the results under discretion. Any combination in between will generate results that are often referred to as imperfect credibility in the literature. Thus, this model provides a justification for imperfect credibility of the monetary authority, which has hitherto been exogenously assumed by other papers.

The model has a normative implication: slower replacement rates of MPC members results in improved welfare. Under a voting procedure the change in welfare is discrete. If the majority of MPC members are old, the equilibrium policy will be that under commitment and else, it will be discretionary. However, the change is continuous if bargaining is the decision making procedure. In this case, the lower the churning rate, the closer the expected loss will be to the commitment policy.

Furthermore, calibrating the model reveals that the rate of change of welfare with the degree of commitment is highly sensitive to the parameters used, particularly the subjective weight of inflation vs. output-gap volatility and the slope of the NKPC.
This result explains the apparent discrepancy between previous studies which had reported convexity or concavity of welfare gains in the degree of commitment. Under the benchmark calibration (Woodford 1999) welfare gains are roughly linear in the degree of commitment or the share of old members in the MPC.
Table 2.1: Calibration of Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>Intertemporal elasticity of substitution in consumption</td>
<td>1.5</td>
<td>-</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>output-gap elasticity of inflation</td>
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<td>0.01, 0.1</td>
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<tr>
<td>$\lambda$</td>
<td>relative weight on the output gap in the welfare function</td>
<td>0.048</td>
<td>0.001, 0.1</td>
</tr>
<tr>
<td>$\rho$</td>
<td>autoregression parameter for $ut$</td>
<td>0</td>
<td>0.80</td>
</tr>
</tbody>
</table>

This table presents the benchmark calibration of the model, following Woodford (1999) in column (1) and the alternative calibration suggested by McCallum and Nelson (2004) in column (2).

Figure 2.1: Dynamic Response to a Cost-push Shock

This figure shows the dynamic responses to a one standard deviation, uncorrelated cost push shock under commitment (dotted line) and discretion (dashed line). The parameters are calibrated according to the benchmark presented in table (2.1).
Figure 2.2: Dynamic Response to a Cost-push Shock: Committee

This figure shows the dynamic responses to a one standard deviation, uncorrelated cost push shock in an MPC with various majorities. The parameters are calibrated according to the benchmark presented in table (2.1). The dashed-plus line corresponds to $\alpha = 0.5$, the dashed-cross line corresponds to $\alpha = 0.75$. Commitment and discretion responses are represented by the dotted line and the dashed line respectively. The path of inflation and output-gap in a two person committee ($\alpha = 0.5$) is roughly halfway between commitment and discretion. The responses under $\alpha = 0.75$ are much closer to commitment outcomes.
Figure 2.3: Dynamic Response to a Persistent Cost-push Shock

This figure shows the dynamic responses to a one standard deviation cost push shock with persistence ($\rho = 0.8$) under commitment (dotted line) and discretion (dashed line). Other parameters are calibrated according to the benchmark presented in table (2.1).
Figure 2.4: Dynamic Response to a Persistent Cost-push Shock: Committee

This figure shows the dynamic responses to a one standard deviation, persistent cost push shock ($\rho = 0.8$) in an MPC with various majorities. The other parameters are calibrated according to the benchmark presented in table (2.1). The dashed-plus line corresponds to $\alpha = 0.5$, the dashed-cross line corresponds to $\alpha = 0.75$. Commitment and discretion responses are represented by the dotted line and the dashed line respectively. The path of inflation and output-gap in a two person committee ($\alpha = 0.5$) is roughly halfway between commitment and discretion. The responses under $\alpha = 0.75$ are much closer to commitment outcomes.
This figure shows the variation of expected loss to society with \( \alpha \), or the proportion of old MPC members under benchmark calibration. The two solid lines show the expected loss under commitment and discretion. When \( \alpha = 0 \) (churning is 100\%) the loss to society is the same as in discretionary monetary policy. When \( \alpha = 1 \) the loss to society is the same as under commitment.
Figure 2.6: Efficient frontier - Benchmark Calibration

This figure shows the efficient frontier under the benchmark calibration. Each point corresponds to a value of $\alpha$ between 0 and 1. The y-axis shows the standard deviation of output-gap and the x-axis the variation of standard deviation of inflation. The parameters are calibrated according to the benchmark case in table (2.1).
This figure shows the variation of expected loss to society with $\alpha$, or the proportion of old MPC members under the alternative McCallum and Nelson (2004) calibration ($\lambda = 0.1$, $k = 0.01$). The two solid lines show the expected loss under commitment and discretion. When $\alpha = 0$ (churning is 100%) the loss to society is the same as in discretionary monetary policy. When $\alpha = 1$ the loss to society is the same as under commitment.
This figure shows the efficient frontier under the benchmark calibration. Each point corresponds to a value of $\alpha$ between 0 and 1. The y-axis shows the standard deviation of output gap and the x-axis the variation of standard deviation of inflation. The parameters are calibrated according to the alternative calibration suggested by McCallum and Nelson (2004) $\lambda = 0.1, k = 0.01$. 

Figure 2.8: Efficient Frontier - McCallum and Nelson (2004) Calibration
3 Asset Prices in a News Driven RBC Model

3.1 Introduction

In this paper we study the hypothesis that time varying expectation of consumption growth – long-run consumption risk – can be generated by a richer structure of news about productivity shocks. This has important asset pricing implications. A growing literature, pioneered by Bansal and Yaron (2004), suggests that shocks to expected consumption growth, or long-run risk, can generate the premia we observe in the data. These studies take consumption dynamics as given. The natural question that follows is whether our standard real-business-cycle (RBC) models can give rise to such dynamics in consumption growth. We attempt to answer this question by studying the asset pricing implications of a production economy.

Our theoretical premise is a standard RBC model augmented along two dimensions. First, we assume recursive preferences as suggested by Epstein and Zin (1989), Epstein and Zin (1991) and Weil (1989). This preference specification allows us to separate risk aversion from elasticity of intertemporal substitution (EIS). More importantly, investors with Epstein-Zin preferences also demand a premium for holding assets correlated with shocks to expected consumption growth, as well as the standard shocks to realized consumption growth. Following Bansal and Yaron (2004), the riskiness associated with expected consumption growth is what the literature often refers to as “long-run risk”. More formally, long-run risk can be defined as stochastic conditional expectation of consumption.

The second departure of our model from the standard RBC model is incorporating “anticipated shocks”. Our model is driven by productivity shocks. These shocks have an unanticipated component, as is often assumed in macroeconomic literature, but also an anticipated component. Anticipated shocks are news about movements in the productivity process that materialize in the future. Anticipated shocks are important because they can generate a significant amount of long-run risk. As we will discuss in detail, long-run risk in consumption can arise in two ways. First, through consumption smoothing and second, exogenously, through anticipated shocks. Shocks to expected productivity growth will translate into shocks to expected consumption growth. Epstein-Zin preferences allow us to price these risks and hence generate much larger premia compared to a standard RBC model with constant relative risk aversion (CRRA) preferences.

It is worth emphasizing the intuition for anticipated shocks. Anticipated shocks are important because they generate significant amount of long-run risk, which from
Bansal and Yaron (2004), we know translate into realistic asset pricing implications. These shocks create expected movements in consumption. Consider the following example: In an endowment economy, an agent receives some news at time \( t \), about an increase in his consumption in several periods \( t + 2 \). With power utility preferences, the stochastic discount factor depends only on realized consumption growth and is not affected by the news. Therefore the pricing of a realized return on a generic asset \( (R_{t+1}) \) is not affected. In other words, shocks to expected consumption growth do not affect asset returns. Whereas with recursive preferences, the shock to expected consumption growth affects the return on wealth which enters the stochastic discount factor. This simple example shows how allowing for news shocks will create long-run risk in consumption, which combined with non-recursive preferences can significantly increase the price of risk in the economy.

We solve the model using the standard macroeconomic technique of approximation around the non-stochastic steady state, often referred to as perturbation method. A first order approximation eliminates any premia as returns to all assets are the same to the first order. Therefore, we use a second order approximation in order to capture the effect of risks on asset prices. We do so by drawing on Schmitt-Grohe and Uribe (2004) and their framework for approximating a general class of DSGE models to the second order. We show how their framework can be applied to solve models with recursive preferences. This approach is well adapted to solve problems with a large number of state variables and is computationally efficient.

Shocks to the predicted component of consumption and dividend growth were explicitly modeled in Bansal and Yaron (2004). However, extending it to the general framework of anticipated shocks and quantifying their impact on asset prices, as we do, is novel. We follow Schmitt-Grohe and Uribe (2008) by assuming that shocks may be predicted up to 3 quarters in advance. The authors estimate these shocks in an RBC model using Bayesian methods and show that anticipated shocks explain the majority of business cycle frequency movements of the main macroeconomic indicators. We study the impact of such shocks on pricing of risk in the economy.

We argue that using the claim on dividend payments by the firm as the theoretical counterpart to equity might be misleading. Assumptions about the industrial organization of the goods and, importantly, labor markets will be reflected in dividend flows and thus will affect equity prices. It would be desirable to separate these two aspects. Thus in this paper we take the view that using the claim on future consumption flows as the risky asset of interest is a more appropriate tool to measure the price of risk in a production economy. So in referring to "equity premia" we mean the excess return of holding the consumption claim over the risk-free rate.

Our benchmark model matches important macro indicators such as consumption growth level and volatility and investment volatility as a share of output volatility.
The model also matches the financial data well. The implied level of risk free rate and its volatility are in line with the data. Expected premia over risk free rate are on average 4.5% annually. This is still about 2% less than that observed in the data, but much higher than previously achieved in general equilibrium models with long-run risk. These results are even more remarkable given that our parameter values fit well within the range regarded as “reasonable” in the literature. We use a risk aversion parameter of $\gamma = 10$ and an EIS of $\psi = 1.5$. These are the values used in the benchmark calibration of Bansal and Yaron (2004).

Our analysis is related to previous work in a few strands of literature. The first group are papers which study the impact of anticipated shocks or news on the dynamics of macroeconomic variables. Beaudry and Portier (2006) find that long-run movements in TFP explain a large fraction of business cycle fluctuations and cause standard business cycle co-movements. Moreover, these authors argue that anticipated shocks are highly correlated with the component in the innovations to stock prices which are orthogonal to TFP shocks. Jaimovich and Rebelo (2009) propose a more comprehensive model of news driven business cycles. Finally, Schmitt-Grohe and Uribe (2008) estimate an RBC model allowing for anticipated shocks at different horizons. We borrow their suggested shock structure and their quantitative estimates for the sizes of these shocks.

Various calibrations of long-run risk models assign different relative importance to news about expected growth and volatility or their correlation thus putting emphasis on different channels through which risk premia can be increased. Bansal and Yaron (2004), Bansal, Kiku, and Yaron (2006) and Bansal, Kiku, and Yaron (2007) and Backus, Routledge, and Zin (2008) present the results of different calibrations. Beeler and Campbell (2009) provide a valuable summary and discussion.

We know of two previous attempts to understand whether long-run consumption risk is a realistic and reasonable feature in a production economy with endogenous consumption. Kaltenbrunner and Lochstoer (2007) answer in the affirmative by pointing out that in the simple RBC model i.i.d. shocks to productivity growth generate predictable movements in consumption growth. Croce (2008) introduces long-run productivity risk and studies its implications for the endogenous consumption dynamics and asset prices. Nevertheless, despite creating some endogenous persistent movements in consumption neither paper achieves the asset pricing implications presented in Bansal and Yaron (2004).

In terms of methodology, our paper is related to those using approximation techniques to study recursive preferences in DSGE models. Tallarini (2000) is among the first to separate RRA from EIS in an RBC model in order to reconcile macroeconomic and asset pricing facts. Rudebusch and Swanson (2008a) study the term premium on
nominal long-term bonds in a DSGE model with Epstein-Zin preferences. They are interested in the size as well as time-variation in premia, therefore they approximate their model to the 3rd order around the non-stochastic steady state. They do so using the algorithm of Swanson, Anderson, and Levin (2006). Binsbergen van, Fernández-Villaverde, Koijen, and Rubio-Ramírez (2008) perform likelihood-based inference in a DSGE model with Epstein-Zin preferences. To do so, they solve their model using a multi-step perturbation technique. One advantage of our technique is that we use the code provided by Schmitt-Grohe and Uribe (2004) for a generic DSGE model and show how approximating this class of models with Epstein-Zin preferences can be obtained by only obtaining the first-order conditions to the social-planner’s problem. Finally, Backus, Routledge, and Zin (2007) use a DSGE model with recursive preference to explain the lead of asset prices over the business cycle. They use a log-linear approximation to do so. In a companion note to the present paper, Malkhozov and Shamloo (2009) demonstrate the relation between Backus, Routledge, and Zin (2008) approximation to a second order perturbation method.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 describes the structure of anticipated shocks. Section 4 briefly discusses our solution technique. Section 5 discusses our benchmark calibration and its main results. Section 6 presents the impulse responses of macro and financial variables to different types of shocks. Here, we also discuss in detail why the claim on dividend is not a suitable counterpart to equity in reality. In section 7 we offer some discussion on the role of EIS, stationary vs. non-stationary shocks, and news shocks in our model. Section 8 concludes.

3.2 Model

Our setup is a variant of the Kydland and Prescott (1982) RBC model with one good, physical capital, endogenous labor input, and shocks to productivity. We extend the standard model by assuming recursive preferences, adding frictions in the form of adjustment costs to investment and allowing for news shocks. We exploit the second welfare theorem and find the equilibrium allocations by solving the social planner problem.

3.2.1 Preferences

The representative consumer maximizes a utility function defined recursively:

\[ \max_{C_t, N_t} U_t \]
where
\[ U_t = \left( u_t \left( \frac{1}{\psi} + \beta(E_t(U_{t+1}^{1-\gamma}))^{\frac{1-\psi}{1-\gamma}} \right) \right)^{\frac{1}{1-\psi}} \]
\[ u_t = (1 - N_t)^{\gamma}C_t \]

The period utility \( u_t \) is multiplicative in consumption \((C_t)\) and leisure \((1 - N_t)\), reflecting the complementarity of leisure (see for example Eichenbaum, Hansen, and Singleton (1988)). This specification has two advantages over a utility separable in consumption and leisure. First, it ensures that period utility is always positive. Second, as emphasized by Rudebusch and Swanson (2008a), under separable utility the value function is not proportional to wealth, and therefore, \( \gamma \) could not be interpreted as the coefficient of relative risk aversion. Whereas our specification ensures that the household’s value function \( V_t = MaxU_t \) is proportional to \( W_t^{1-\gamma} \), making \( \gamma \) a direct measure of risk of aversion.\(^{30}\)

Unlike CRRA utility function, Epstein-Zin preferences allow us to separate the EIS from the coefficient of relative risk aversion (see Epstein and Zin (1989)). The parameter \( \gamma \) stands for the agent’s relative risk aversion and \( \psi \), for his EIS. This separation has an important implication for the agent’s preferences towards the early resolution of uncertainty. In the power utility case investor is indifferent towards the timing of resolution of uncertainty, if \( \gamma > 1/\psi \) \((\gamma < 1/\psi)\) the investor prefers early (late) resolution of uncertainty. Intuitively, with \( \gamma > 1/\psi \) the agent’s propensity to smooth consumption across states is greater than his propensity to smooth consumption across time.

### 3.2.2 Technology

The consumption good is produced according to a constant returns to scale neoclassical production function
\[ Y_t = Z_t (A_t N_t)^{1-\alpha} K_t^\alpha \]
where \( K_t \) is the stock of capital, \( N_t \) is the labor hours and \( Y_t \) is the output. \( Z_t \) and \( A_t \) represent the stationary and non-stationary components of the TFP respectively. We will describe their dynamics below.

The law of motion of capital is given by
\[ K_{t+1} = K_t \cdot [(1 - \delta) + \phi(I_t/K_t)] \]
where \( I_t = Y_t - C_t \) and \( \phi \) is a positive, concave function, capturing the fact that capital

\(^{30}\)For more details see Rudebusch and Swanson (2008a).
adjustments are costly. We follow Jermann (1998) in specifying $\phi$ as below:

$$\phi(I_t/K_t) = \left( \frac{\alpha_1}{1 - 1/\tau} (I_t/K_t)^{1-1/\tau} + \alpha_2 \right)$$  \hspace{1cm} (3.4)

where $\alpha_1$ and $\alpha_2$ are parameters. These parameters are set such that the adjustment cost is zero on the balanced growth path; i.e. $\phi(I/K) = I/K$ and $\phi'(I/K) = 1$.\(^{31}\) Note that $q_t = \frac{1}{\phi'(I_t/K_t)}$ is the marginal rate of transformation between new capital and consumption, or Tobin’s $q$. Since $q_t = (I/K)^{-1/\tau} (I_t/K_t)^{1/\tau}$ we can interpret $\tau$ as the elasticity of investment-capital ratio with respect to Tobin’s $q$.

We now introduce our specification for the technology shocks. The stationary ($Z_t$) and non-stationary ($A_t$) components of the TFP follow:

$$\ln A_{t+1} - \ln A_t = x^1_{t+1}$$
$$\ln Z_{t+1} = x^2_{t+1}$$

where $x^1$ and $x^2$ are stationary shocks. We let the two shocks to be the first two elements of a first order auto-regressive vector (VAR(1)) $x_t$.

$$x_{t+1} = H_0 + H_1 x_t + H_2 \epsilon_{t+1}$$  \hspace{1cm} (3.5)

where $H_0$ is $(n \times 1)$, $H_1$ is $(n \times n)$, $H_2$ is $(n \times n_e)$ and $\epsilon_t$ is a vector of normally distributed innovations, $\epsilon_t \sim N(0, I_{n_e})$. We assume all innovations are perfectly observable. As we will demonstrate when we introduce anticipated shocks, the other elements of the vector of state variables $x_t$ include the anticipated components of the productivity shocks. This general specification nests several models as special cases: the standard growth model, the long-run productivity risk model (Croce 2008), a model with anticipated shocks (Schmitt-Grohe and Uribe 2008) as well as models with transitory shocks around a time trend in productivity growth (Kaltenbrunner and Lochstoer 2007).

3.2.3 Equilibrium

The social planner’s problem can be summarized by the Bellman equation

$$V(K_t, x_t) = \max_{C_t, N_t} (U_t) \text{ or }$$

$$V_t = \max_{C_t, N_t} \left( (1 - N_t)^{\gamma} C_t^{1-\frac{1}{\delta}} + \beta(E_t(V_{t+1}^{1-\gamma}))^{\frac{1-\frac{1}{\delta}}{1-\gamma}} \right) \frac{1}{1-\gamma}$$  \hspace{1cm} (3.6)

where $V_t$ is the problem’s continuation value or simply the value function. The maximization problem is subject to the production function (3.2), law of motion of capital

\(^{31}\)In particular, we set $\alpha_1 = (I/K)^{1/\tau}$ and $\alpha_2 = \frac{1}{1-\tau} (I/K)$
(3.3) and the exogenous dynamics (3.5). It follows that the Euler equation and the optimal consumption-leisure trade-off can be written as (3.7) and (3.8) (see appendix (A.1) for the derivation):

\[ E_t \left[ M_{t+1} \left( \frac{\partial K_{t+2}}{\partial C_{t+1}} \right)^{-1} \frac{\partial K_{t+1}}{\partial K_{t+2}} \right] = 1 \]  

(3.7)

\[ (1 - N_t) \frac{\partial K_{t+1}}{\partial N_t} - \eta C_t \frac{\partial K_{t+1}}{\partial C_t} = 0 \]  

(3.8)

where \( M_{t+1} \) is the stochastic discount factor, defined as:

\[ M_{t+1} = \beta \left( \frac{V_{t+1}}{E_t \left( V_{t+1}^{1-\gamma} \right)^{1-\gamma}} \right)^{1/\psi - \gamma} \left( \frac{C_{t+1}}{C_t} \right)^{-1/\psi} \left( \frac{1 - N_{t+1}}{1 - N_t} \right)^{\eta(1-1/\psi)} \]  

(3.9)

We assume perfect labor markets, and therefore wages are equal to the marginal product of labor. Hence, it is optimal for the representative firm to employ the level of labor chosen by the social planner and supplied by the representative agent.

### 3.2.4 Financial implications

The focus of our analysis are equilibrium asset prices. In equilibrium the return on any asset \( i \), \( R_{t+1}^i \), satisfies

\[ E_t \left( M_{t+1} R_{t+1}^i \right) = 1 \]

We define the one-period risk-free rate as

\[ R_t^f = E_t \left( M_{t+1} \right)^{-1} \]

We consider two assets. First, a claim on the aggregate consumption stream and second, a claim on the dividends paid out by the firms who own the capital stock. The dividend stream at time \( t \) is defined as output net of investment and wages

\[ D_t = Y_t - W_t N_t - I_t \]

Let \( P_t^c \) and \( P_t^d \) represent the price of the consumption claim and the dividend claim respectively. Then, the total returns on the assets can be defined as the sum of period flow and capital gain:

\[ R_{t+1}^c = \frac{P_{t+1}^c + C_{t+1}}{P_t^c} \]

\[ R_{t+1}^d = \frac{P_{t+1}^d + D_{t+1}}{P_t^d} \]

Alternatively, the return on firm equity can be computed from the profit maximization problem of the representative firm (for instance, see Kaltenbrunner and Lochstoer
Previous attempts, such as Kaltenbrunner and Lochstoer (2007) and Croce (2008) have interpreted equity in the data as the claim on dividend stream paid by the firm. This interpretation is, to say the least, questionable as a direct counterpart for the stockmarket. Indeed, it has proved difficult for these papers to match the dynamics of the firm dividends to the actual stockmarket aggregate dividend in a model which reasonably describes the economy. This poor result could arise even in a model which prices the risks in the economy accurately.

We will discuss the co-movements of the dividend stream (as defined above) with macro aggregates implied by this model and show that in general they are very poor. We interpret this result as evidence against the use of dividend claim as the counterpart to stocks. We take the view that this restricted definition of equity has been a major obstacle in the previous attempts to reproduce the Bansal and Yaron (2004) results in a DSGE model. Thus, in this paper we restrict our focus to obtaining a realistic price of risk in the economy. One possible approach is to look at the risk premium and Sharpe ratio of the most generic asset - i.e. the consumption claim.32

3.3 Introducing Anticipated Shocks

In this section we specify our model by introducing several anticipated shocks. Our framework is that of Schmitt-Grohe and Uribe (2008). Jaimovich and Rebelo (2009) have a similar model in which they include news shocks. The focus of these papers is to reproduce the correct macro aggregate co-movements and in particular neither paper studies the asset pricing implications.

3.3.1 Long run risk and anticipated shocks

From the stochastic discount factor (3.9), we can see that (assuming that labor supply is inelastic) there are two priced factors in this economy. The first is the realized consumption growth which is the usual risk factor in the Consumption CAPM. The second is the deviation of value function from its expected value, which will occur

\[ R_{t+1}^d = \frac{1/\phi' \left( \frac{I_{t+1}}{K_{t+1}} \right) + d_{t+1}}{1/\phi' \left( \frac{I_t}{K_t} \right)} \]

where

\[ d_{t+1} = \alpha Y_{t+1} - \delta \frac{I_{t+1}}{K_{t+1}} + \frac{\phi' \left( \frac{I_{t+1}}{K_{t+1}} \right)}{\phi' \left( \frac{I_t}{K_t} \right)} \]

It can be shown that the value of consumption claim represents aggregate wealth when leisure does not enter the utility function. The calibration we present in this paper assumes an inelastic labor supply and therefore, satisfies the condition above.
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if there are shocks to expected consumption growth. Under power utility, only the former factor is priced.

We refer to stochastic conditional expectation of a certain variable as long-run risk. Anticipated shocks are not the only way to create long-run risk in consumption. Indeed Kaltenbrunner and Lochstoer (2007) show that persistent time variation in expected consumption growth is a salient feature of RBC models even with unanticipated i.i.d. productivity shocks provided the propensity to smooth consumption is not too high (EIS not too low). In other words consumption long-run risk arises endogenously. An obvious way to amplify the effect is to increase the persistence in productivity shocks, which gives rise to persistent future movements in consumption.

Our framework allows for persistence in TFP shocks. The main contribution of our paper is to introduce shocks that will increase productivity not immediately but only one or several periods in the future. They lead to an expected increase in consumption at the period when technology improvement is realized. The relative importance of substitution and income effects will determine the sizes of the immediate response of consumption to the shock and the response after the eventual productivity increase.

We see that the mechanism through which anticipated shocks give rise to high premia is closely related to long-run risk and crucially depends on the recursive preferences assumption.

3.3.2 Structure of shocks

The stationary and non-stationary TFP shocks are autoregressive and subject to anticipated ($e^{1}, e^{2}, e^{3}$) and unanticipated ($e^{0}$) innovations. This specification is taken from Schmitt-Grohe and Uribe (2008).

\[ x_{t}^{1} = (1 - \rho_{A})x_{t-1}^{1} + \varepsilon_{A,t}^{1} + \varepsilon_{A,t-1}^{1} + \varepsilon_{A,t-2}^{1} + \varepsilon_{A,t-3}^{1} \]
\[ x_{t}^{2} = (1 - \rho_{Z})x_{t-1}^{2} + \varepsilon_{Z,t}^{0} + \varepsilon_{Z,t-1}^{0} + \varepsilon_{Z,t-2}^{0} + \varepsilon_{Z,t-3}^{0} \]

The agent learns about innovation $\varepsilon_{t}$ at date $t$ and it affects the productivity at the same date. Innovations $\varepsilon_{t-1}^{0}, \varepsilon_{t-2}^{1}, \varepsilon_{t-3}^{2}$ are anticipated one, two and three periods ahead - they affect date-$t$ productivity, but are in period $t-1$, $t-2$, and $t-3$ information sets respectively. Therefore, at date $t$ the agent learns about 4 shocks $\varepsilon_{t}, \varepsilon_{t}^{0}, \varepsilon_{t}^{1}$ and $\varepsilon_{t}^{2}$, affecting productivity immediately and in one, two and three periods ahead. We assume all shocks are independent.

Additional lagged innovations can easily be incorporated in to the recursive formulation of the problem by increasing the number of state variables. The matrices $H_{0}, H_{1}$, and $H_{2}$ of the VAR(1) representation of the shocks are given in the appendix (A.4).
3.4 Perturbation Methods Solution

We solve the model introduced in the previous section by expanding the value function, policy functions, and the laws of motion for the state variables around the non-stochastic steady state. This is the standard practice in macroeconomic literature since the closed form solution of the model is known at this particular point in the state-space. We opt for perturbation methods as opposed to numerical methods such as value function iteration or projection for two reasons: First, we would like to investigate a problem with a large number of state variables. The high dimensionality of the problem makes numerical methods computationally infeasible. Furthermore, perturbation methods are attractive as they obtain (approximate) analytical expressions for state-evolution equations and policy functions. Caldara, Fernández-Villaverde, Rubio-Ramírez, and Yao (2008) document the accuracy of perturbation methods and compare them with alternative computational approaches.

We follow the procedure suggested by Schmitt-Grohe and Uribe (2004) for finding a second-order approximation to the solution of a general class of discrete-time rational expectations models. Their method applies to a large class of DSGE models in macroeconomics. We will briefly review this method below. Let the set of equilibrium conditions of the model we wish to approximate be written as

\[ E_t f (y_{t+1}, y_t, x_{t+1}, x_t) = 0 \]  

where \( E_t \) denotes the mathematical expectations operator conditional on information available at time \( t \). The vector \( x_t \) of predetermined variables is of size \( n_x \times 1 \) and the vector \( y_t \) of non-predetermined variables is of size \( n_y \times 1 \). The state vector \( x_t \) can be partitioned as \( x_t = [x_t^e; x_t^d]^T \). The vector \( x_t^e \) consists of endogenous predetermined state variables and the vector \( x_t^d \) of exogenous state variables. Specifically, we assume that \( x_t^d \) follows the exogenous stochastic process given by

\[ x_{t+1} = \Lambda x_t^2 + \eta \sigma \epsilon_{t+1}; \]

where both the vector \( x_t^2 \) and the innovation \( \epsilon_t \) are of order \( n \times 1 \). The vector \( \epsilon_t \) is assumed to be independently and identically distributed, with mean zero and variance/covariance matrix \( I \).\(^{33}\) The scalar \( \sigma \) and the \( n \times n \) matrix \( \eta \) are known parameters.\(^{34}\)

The solution to the recursively defined model is given by the equilibrium policy

\(^{33}\)It is not necessary to specify the exact distribution of the vector of shocks. For obtaining an \( n^{th} \) order approximation, we only need to specify the first \( n \) moments.

\(^{34}\)\( \sigma \) can be thought of as a parameter scaling the size of uncertainty. Without loss of generality we can assume that \( \sigma \) is either 0, at the non-stochastic steady state, or equal to 1 otherwise.
function for $y_t$ and the laws of motion for $x_t$:

\[
y_t = g(x_t, \sigma) \quad (3.11)
\]

\[
x_{t+1} = h(x_t, \sigma) + \sigma \eta_{t+1} \quad (3.12)
\]

We wish to approximate functions $g$ and $h$ around the non-stochastic steady state, where $\sigma = 0$ and $x_t = \bar{x}$. We define the non-stochastic steady state as vectors $(\bar{x}; \bar{y})$ such that

\[
f(\bar{y}, \bar{y}, \bar{x}, \bar{x}) = 0
\]

Thus, finding a second order approximation to the solution equations (3.11) and (3.12) involves solving for elements of the (matrix) derivatives of $g$ and $h$ with respect to $x$ and $\sigma$, in the Taylor expansion below:

\[
g(x_t, \sigma) = g(\bar{x}, 0) + g_x(\bar{x}, 0)(x - \bar{x}) + g_{\sigma}(\bar{x}, 0)\sigma
\]

\[
+ \frac{1}{2}(x - \bar{x})^T g_{xx}(\bar{x}, 0)(x - \bar{x})
\]

\[
+ \frac{1}{2}(x - \bar{x})^T g_{x\sigma}(\bar{x}, 0)\sigma
\]

\[
+ \frac{1}{2}\sigma g_{\sigma x}(\bar{x}, 0)(x - \bar{x})^T
\]

\[
+ \frac{1}{2}g_{\sigma\sigma}(\bar{x}, 0)\sigma^2
\]

\[
h(x_t, \sigma) = h(\bar{x}, 0) + h_x(\bar{x}, 0)(x - \bar{x}) + h_{\sigma}(\bar{x}, 0)\sigma
\]

\[
+ \frac{1}{2}(x - \bar{x})^T h_{xx}(\bar{x}, 0)(x - \bar{x})
\]

\[
+ \frac{1}{2}(x - \bar{x})^T h_{x\sigma}(\bar{x}, 0)\sigma
\]

\[
+ \frac{1}{2}\sigma h_{\sigma x}(\bar{x}, 0)(x - \bar{x})^T
\]

\[
+ \frac{1}{2}h_{\sigma\sigma}(\bar{x}, 0)\sigma^2
\]

Schmitt-Grohe and Uribe (2004) offer an algorithm for finding the elements of these matrixes. This is achieved by differentiating (3.10) with respect to all elements of $x$ and the scalar $\sigma$ and evaluating the derivatives at the non-stochastic steady state. These obtain a system of equations, the solution to which are the elements of matrix derivatives of $g$ and $h$. We refer the reader to these authors' original paper for more details.

Schmitt-Grohe and Uribe (2004) show that $g_{\sigma} = h_{\sigma} = g_{\sigma x} = h_{\sigma x} = 0$. The fact that $g_{\sigma} = h_{\sigma} = 0$ implies that the first order approximation is not affected by the volatility of the shock. In other words volatility does not matter to the first order. This implies that to the first-order the unconditional mean of all variables are equal
Moreover, \( g_{\sigma x} = h_{\sigma x} = 0 \) implies that a second-order approximation can only produce constant risk premia. Note that the only non-zero term involving \( \sigma \) is \( \frac{1}{2} g_{\sigma \sigma}(\bar{x}, 0) \sigma^2 \), which is constant over time. Therefore, a second-order approximation cannot produce time-varying risk premia. Furthermore, it can be shown that the risk aversion parameter \( \gamma \) does not affect the non-stochastic steady state value (or the vector \( \bar{x} \)). Neither does it enter the expressions for \( h_x \) or \( h_{xx} \) evaluated at the non-stochastic steady-state. \( \gamma \) only enters in the term \( \frac{1}{2} g_{\sigma \sigma}(\bar{x}, 0) \sigma^2 \). This means that, to the second order, risk aversion only affects the difference between the stochastic steady state and the non-stochastic steady states but not the dynamics of the variables. This result is intuitive given that variance of shocks only affect the difference between the stochastic and non-stochastic steady state and therefore \( \gamma \), which measures the sensitivity of agents to this risk, should also only show up in the terms where variance does.

We can apply perturbation of any order to any transformation of the variables. As it is standard in macroeconomics, we obtain a second order approximation to the logarithm of variables. The resulting policy functions are different from what would have obtained if the linearization was performed on levels. But the results are the same, up to the second order.

Since labour augmenting productivity process \( A_t \) has a unit root, the model economy is growing. In order to find a local approximation to the model solution around a particular point we need to transform the problem into a stationary one. For any variable \( X_t \) inheriting the unit root, define \( \tilde{X}_t \) as the variable normalized by the unit-root technology shock, so that \( \tilde{X}_t = \frac{X_t}{A_{t-1}} \). Note that the utility function (3.1), the production function (3.2) and the capital stock (3.3) are all homogeneous of degree one in \( K_t \) and \( A_t \). As a consequence \( V_t \) is also homogeneous of degree one. Therefore, \( \tilde{V}_t = \frac{V_t}{A_{t-1}} = V \left( \tilde{K}_t, \tilde{x}_t \right) \). We obtain the stationary equilibrium by re-writing equations (3.2) to (3.8) in terms of the new stationary variables. See appendix (A.2) for details.\(^\text{35}\)

\(^{35}\)A technical note on the compatibility of our model with Schmitt-Grohe and Uribe (2004) solution is in order. In models with recursive utility, unlike those with time-additive preferences, the value function \( V_t \) (as well as the expression \( E_t \left( V_{t+1}^{1+\gamma} \right) \)) appear in the Euler equation, and therefore have to be approximated. This has been problematic for several previous attempts to apply standard perturbation techniques to models with recursive preferences. We treat \( \tilde{V}_t \) and \( E_t \left( V_{t+1}^{1+\gamma} \right) \) as two
3.5 Calibration and Results

The model produces macroeconomic aggregates such as output, consumption and investment, in addition to the standard financial moments. In this section, we present a calibrated version of the model in section 3.2, with one source of exogenous variation: the non-stationary TFP shock. We calibrate the break-down of the variance of this shock into different horizons of anticipation according to Schmitt-Grohe and Uribe (2008), which to the best of our knowledge is the only study which estimates a DSGE model with news shocks. We compare the results with the data and show that the model is capable of producing large premia and low risk-free rates. Finally, we argue that including capacity utilization might improve the co-movements of macroeconomic variables.

We borrow our shock structure and the relative sizes of the innovations from the estimated RBC model in Schmitt-Grohe and Uribe (2008). However, it is worthwhile to emphasize that there are important differences between the model we consider and their estimated model, which makes it hard for us to compare our results to these authors'. First, we differ in our preference specifications. We assume Epstein-Zin preferences to study the effects of long-run risk, whereas Schmitt-Grohe and Uribe (2008) assume habit formation in consumption and leisure. Second, the authors introduce capacity utilization, which plays an important role in creating the correct co-movements of macro aggregates. These effects will be discussed in section 3.6. Third, (in the current version of the paper) we ignore labor supply decisions by assuming a perfectly inelastic labor supply. Despite these modeling differences, their exercise is uniquely useful to us for disciplining our calibration of anticipated shocks.

3.5.1 Calibration

Parameters Panel (A) of table (3.1) reports the values of the parameters which are constant across all models. All the parameters are calibrated at quarterly frequency. This is the frequency often used in analyzing the business cycle properties of macroeconomic aggregates. We set \( \alpha \), the share of capital in the production function and depreciation rate of capital to standard values in business cycle literature (see Christiano, Eichenbaum, and Evans (2005)). The mean technology growth rate is set such that the output growth of the economy is around 2% annually. In this paper, control variables in vector \( \mu_t \) and add two additional equilibrium conditions: these are the definition of the value function and the value of \( E_t \left( \tilde{V}_{t+1}^{s} \right) \), both evaluated in the non-stochastic steady state.

Our model and solution method allow for endogenous labor supply. In general, including labor allows for an extra dimension along which agents can smooth consumption and hence makes it a harder task to match the observed premia. On the other hand, calibrating the employment dynamics jointly with the asset pricing implications of our model is an interesting exercise. In an extension to this work we intend to investigate this issue further. The only other asset pricing paper we know of in which labor enters recursive preferences is Bakus, Routledge and Zin (2007).
we assume a perfectly inelastic labor supply, i.e. we set $\eta = 0$ and fix the size of the labor force at 1.

Panel (B) describes the calibration of some parameters in the benchmark case. We set $\tau$, or the parameter governing the amount of adjustment costs to match the relative volatility of investment to output. We find that unlike typical calibrations of RBC models, investment is not too volatile relative to output in our model. Thus, in our calibration we set this cost to be zero (corresponding to an infinitely large value for $\tau$). This is also the finding by Kaltenbrunner and Lochstoer (2007). In their model with non-stationary TFP shocks (which the authors refer to as LRRH) $\tau$ is set at 18. This elasticity is so high that the resulting investment volatility is virtually the same as if $\tau$ was assumed to be infinitely large.

We choose $\gamma = 10$ for our benchmark calibration, similar to Bansal and Yaron (2004). We reproduce our results for $\gamma = 7.5$ - the other value considered in that paper - and show that lower risk aversion only reduces premia and raises the risk free rate (which can be brought down by re-calibrating the discount factor), and very little else (see table (3.4) for the results). The fact that macroeconomic time series are unaffected by the coefficient of relative risk aversion has been pointed out by Tallarini (2000).

Finally, we calibrate the discount factor $\beta$ is calibrated as the inverse of steady state risk-free rate.

Shocks Our model is (potentially) driven by two exogenous forces: the stationary productivity shock ($Z_t$) and the non-stationary productivity shock ($A_t$). In our benchmark calibration we set the volatility of stationary shocks, $Z_t$, equal to zero. As we will demonstrate in detail in section 3.8.1, these shocks only contribute to higher output volatility without raising consumption volatility and thus creating too little volatility of consumption relative to output compared to the data. This also means that stationary shocks have virtually no effect on premia.

The non-stationary TFP shock is subject to anticipated ($e_A^1, e_A^2, e_A^3$) and unanticipated ($e_A^0$) innovations. We use the estimated properties of the non-stationary TFP shocks described in the estimation exercise of Schmitt-Grohe and Uribe (2008). The authors also include investment specific productivity shocks and government shocks in their model. However, after estimating the full information model, they show that productivity shocks (stationary and non-stationary) account for 98% of output growth volatility and 100% of consumption growth volatility. This suggests that ignoring investment specific and government shocks in our model should have little effect on our predictions about business cycle properties of macroeconomic time-series.
3 ASSET PRICES IN A NEWS DRIVEN RBC MODEL

Table (3.2) reproduces the estimated variances for all of the components of the non-stationary shock, reported in Schmitt-Grohe and Uribe (2008). These estimates are obtained using a Bayesian estimation method. We weigh all the variances by a factor $\kappa$ to achieve the level of consumption volatility observed in the data. The three columns in this table show the mean, 5% and 95% estimates of each parameter.

3.5.2 Results

Table (3.3) contains the main results of our benchmark calibration exercise. The empirical moments, except for the unconditional mean of consumption growth rate, are taken from Kaltenbrunner and Lochstoer (2007) and pertain to annual U.S. sample from 1929-1998. The unconditional mean of per capita consumption growth rate is the annualized mean, obtained using BEA quarterly data on per capita consumption from 1948Q1-1998Q4.

Looking at the macro variables in panel (A), we observe that the model matches the volatility of consumption, but volatility of output is slightly higher than in the data. Investment volatility (as a share of output volatility) is also very close to the data. The accurate volatility of investment means that we do not need to include adjustment costs. Adjustment costs are often incorporated into RBC models as a way to dampen the volatility of investment in response to shocks to levels observed in the data and also create hump-shaped responses to monetary and TFP shocks (see, for instance, Christiano, Eichenbaum, and Evans (2005)). But in our model, we achieve the correct investment volatility without adjustment costs.37

In general we believe that real frictions such as capital adjustment costs are a realistic feature of the economy and are necessary to explain a number macroeconomic stylized facts. However, in this paper we show that some of the implications of including real frictions in an RBC model can also be achieved by assuming a richer shock structure instead. A good example would be the volatility of investment. It is well understood that the volatility of investment obtained from an RBC without adjustment costs is too low compared to the date. This was the original motivation for including adjustment costs. Including anticipated shocks allows investment to respond to future shocks as well the realized one, therefore decreasing the volatility of investment for a given exogenous productivity volatility assumed.

The model also matches the financial data relatively well. The level of risk free rate is in line with the data, but its volatility is slightly lower. There is no empirical counterpart for the consumption claim and therefore its risk premium. However, the

37Adjustment costs are useful for other features they generate in an RBC model. They give agents an incentive to respond immediately to news about future fundamentals (see, e.g., Jaimovich and Rebello, 2007) and are necessary to create the correct comovements in response to investment specific shocks (see, e.g., Christiano, Eichenbaum and Evans, 2001).
Sharpe-ratio of this asset, which represents the price of one unit of risk associated with the consumption claim, is higher than the Sharpe-ratio of stockmarket returns. This implies that our model has no difficulty in assigning a price to consumption risk which is of the same order of magnitude as in the data. We view this as the major achievement of our model.

Note that in Bansal and Yaron (2004) specification, achieving a high variance of excess returns is possible since the volatility of dividend growth is calibrated. In the data, dividend growth volatility is much higher than consumption growth volatility (see table (3.5)). One way to reconcile this issue, is to interpret the stock-market as leveraged consumption claim, a view often taken in the consumption-based asset pricing literature. See for example the discussion in Barro (2005). Adopting this view, we can choose the leverage parameter in order to match the stock-market volatility and magnify the consumption claim premium accordingly. We choose to look directly at Sharpe-ratios.

The model is less successful in explaining the co-movements of macroeconomic time series. In the model, consumption and output do not co-move as strongly as in the data. Introducing capacity utilization and increasing adjustment costs will increase this co-movements as output also responds to future news about productivity. The same is true for the co-movements of consumption and investment. As we will discuss in detail below, the response of consumption is hump-shaped. Therefore, anything that would generate the same type of response in investment and output will increase the co-movements of these two variables with consumption.

3.6 The Dynamic Effects of Anticipated Shocks

In this section we present the impulse response of the variables of interest to different shocks in the economy. First, we study the impulse response of macro aggregates and compare them to the standard RBC model. We then look at the response of financial variables. In particular we show explicitly, why the claim on the stream of dividends paid out by firms (with the Cobb-Douglas technology specified in the model) is not a suitable theoretical counterpart for the empirical market returns.

3.6.1 Response of macroeconomic variables

Figure (3.1) displays the impulse responses to two types of non-stationary TFP shock, namely an unanticipated and a one-quarter anticipated shock to the non-stationary component of total factor productivity ($\varepsilon^0_A$, shown in solid and $\varepsilon^1_A$, shown in crossed line respectively). All variables are measured in percent deviation from the de-trended non-stochastic steady state.
The response to the 1-quarter anticipated non-stationary shock is shown with a
crossed line in figure (3.1). TFP remains at the steady state level until period 2
and stays high thereafter. Nevertheless, consumption increases in anticipation of the
increase in TFP next period. Note that output does not increase. This is because TFP
has not changed and capital is also fixed. Given that there are no adjustment costs,
it is optimal to increase investment when productivity rises. However, due to higher
consumption and constant output, there are less resources available for investment.
Therefore, capital stock falls slightly and then rises (this pattern will be reflected in
dividend flows, as we will observe later).

The solid line in figure (3.1) shows the response to the unanticipated non-stationary
TFP shock. Note that the different final level of $A_t$ in response to anticipated and
unanticipated innovations is due to the different variances of the two shocks. The
responses are qualitatively the same as in the anticipated case. Output rises in pe­
riod 1, when TFP rises and stays higher permanently. Consumption also gradually
moves towards a higher level. Unlike the anticipated case however, capital increases
monotonically as period zero capital is fixed and output is higher at period 1 already.

Figure (3.2) shows the response to a three-quarter anticipated stationary TFP
shock (crossed line) and the response to an unanticipated stationary TFP shock (solid
line). Even though in calibrating our benchmark model we eliminate stationary shocks,
it is interesting to analyze the response of variables of interest to such shocks. In re­
sponse to the anticipated shock, consumption rises immediately, even though the TFP
increase has not materialized. Between the announcement and the actual increase in
productivity, output is roughly constant, decreasing only slightly due to the decreas­
ing capital stock and capital decreases for the same reason as above: The absence
of adjustment costs do not bring forward future needs to increase investment. But
higher consumption, without an increase in output results in a temporary reduction
in investment and capital stock levels. The capital stock rises slowly after the shock
is materialized.

Note that this model does not produce the hump-shaped response of output in
response to stationary TFP shocks, which is typically found in VAR studies (but it
does create a hump-shaped response in consumption). Other authors mainly focusing
on business cycle responses of aggregate variables, such as Schmitt-Grohe and Uribe
(2008) and Jaimovich and Rebelo (2009) obtain these responses by including adjust­
ment costs and capacity utilization. Adjustment costs in investment imply that if

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38The specification of adjustment costs in this paper penalizes the level of investment (for instance,
see Chari, Kehoe and McGratten (2000)). Christiano, Eichenbaum and Evans (2001) claim that it is
difficult to account for the hump-shaped response of investment with this type of adjustment costs
and thus, these authors and many others in the RBC literature use an adjustment cost function which
penalizes the change in investment.
the optimal level of investment is about to rise in the future, agents increase invest­
ment today in order to arrive in the period with high productivity with a high level
of capital stock, therefore, avoiding adjustment costs of rapid changes in investment.
Allowing for variable capacity utilization also implies that output rises upon receiving
the news about higher future productivity and in response to higher consumption and
investment demand. Therefore allowing for these features can generate hump-shaped
responses in output and investment as well as consumption.

3.6.2 Response of financial variables

The bottom two panels of figures (3.1) and (3.2) show the response of premia on the
consumption claim and risk free rates to the four shocks we analyzed. In response
to shocks to the non-stationary TFP, the premia on the consumption claim rises.
Note that this occurs for both anticipated and unanticipated shocks at the same time.
Indeed, we expect this to be the case as anticipated shocks to asset prices should
be arbitrated away. The magnitude of the effect is much larger for the anticipated
shock, for it has a larger standard deviation and thus a larger final adjustment to
consumption level.

Anticipated shocks are a natural channel through which we can explain the asset
prices lead of the business cycle. This pattern has been reported in a number of
empirical studies. Most recently this has been documented by Backus, Routledge, and
Zin (2007) who show that both equity prices and term spreads lead the business cycle.
Because asset prices are forward-looking, news about the future are incorporated in
them but might not be reflected in the macroeconomic quantities as the anticipated
shocks are yet to realize. Prices may contain more information than the history of
macro variables. This is the simplest theoretical way to account for the predictive
power of financial variables.

The risk free rate also rises as all the agents try to borrow against future wealth to
increase consumption today. In response to an anticipated shock to the non-stationary
TFP, the major increase in the risk free rate occurs a period before the TFP shock
is realized. The increase in TFP will bring about a large increase in consumption,
and thus risk-free rate increases as all agents try to bring forward part of this wealth
effect.

The response to the stationary TFP shock is qualitatively similar. Again, the
excess return on the consumption claim increases immediately for the unanticipated
shock and at the time of news release for the anticipated shock. The risk free rate rises
immediately in response to the unanticipated shock. In response to the anticipated
shock, the risk free rate only rises moderately once the news arrives but the large
increase occurs the period before consumption increases due to the realization of the
shock.
3.6.3 Behavior of dividend flows

In this section we analyze the behavior of dividend flows. Dividends are calculated as firms' excess output after paying wages and investment expenditure:

\[ D = rK - I = C - wL \]  

(3.13)

This is the specification used in previous papers which considered asset prices in a production economy (see Kaltenbrunner and Lochstoer (2007) and Croce (2008)). These papers are less successful in generating the correct size, volatility and co-movements of dividends with other variables than in generating the key features of macro variables. We show that given the specification above the claim on dividend flows is not a good measure for stock market returns and thus it is difficult to generate asset pricing implications similar to those in Bansal and Yaron (2004), where the authors assume the properties of dividend growth. A major difficulty arises from the fact that dividend flows, as defined in (3.13), are negative in many periods, i.e. they flow from households to firms. This finding is counter-factual. As emphasized before, it is important to isolate the assumptions about the production function and industrial organization of markets from dividends. This allows us to focus on pricing of risk in general as opposed to a particular type of asset.

**Impulse response of dividends** Figure (3.3) shows the response of de-trended dividend flows in response to the four types of TFP shocks analyzed above \((\epsilon_A^0, \epsilon_A^1, \epsilon_2^0\) and \(\epsilon_2^1\), clockwise from the top). In response to an unanticipated shock to TFP (both stationary and non-stationary), dividend flows fall. Given the definition of dividends from (3.13) above, a mechanical explanation for this decrease is the following: in response to an increase in TFP investment grows whereas capital \((K)\) is fixed. More intuitively, dividends are cut back to fund investment. In the periods after the shock, dividends will increase again as the return to the extra capital accumulated is paid out to investors and dividends eventually go back to their steady state.

An alternative explanation is that in the presence of high EIS, the substitution effect dominates the income effect and therefore the agent increasingly wants to make use of the higher TFP in the future (permanently, in the case of \(A_2\) shocks and temporarily in the case of \(Z_t\) shocks). Thus, he increases investment which diverts resources away from dividend payments. When the EIS is high enough (as is the case in our calibration), in fact dividend payments can be negative; i.e. the household diverts resources to the firm in order to fund future investment.

The panels on the right in figure (3.3) show the response of dividend flows to an anticipated TFP shock. When the news of a future shock to TFP arrives, the agent increases his consumption at the cost of investment. This means that resources are transferred from the firm to the household to fund consumption. On the other hand,
when the shock is materialized, the resources have to be transferred back to the firm in order to fund the increase in investment.

These dynamics are counter-factual. The assets held by stockmarket participants almost never have negative dividend flows. In general, we argue that the claim on the dividends paid out by a representative firm is not a good counterpart for equity in the real world. One of the conceptual difficulties of studying equity premia in an RBC model has been the definition of a counterpart for equity in the model. The common approach is to look at the marginal product of capital or the return on the dividend claim. We argue that this has some undesirable implications. Most importantly, it does not allow us to separate the asset pricing part of the model from the employment and wages part (which is not the focus of such papers). This comes about because dividends are defined as consumption minus wages, and therefore a model generating realistic consumption dynamics but failing on employment and wages, will inevitably produce unrealistic dividends and thus asset pricing implications.

A good alternative approach is to explicitly model equity and debt as in Gomes and Schmid (2009). The authors embed an endogenous capital structure model in a simplified production economy in order to study the lead of asset prices and spreads over the business cycle. We aim to incorporate capital structure in our framework as a future extension. In this paper, we circumvent the imperfect modeling of the labor market by focusing on the most generic asset i.e. the consumption claim.

**Correlation of dividend and consumption growth** In their benchmark model, Bansal and Yaron (2004) achieve asset pricing implications that are much closer to what is observed in the data than previous attempts. They assume an endowment economy and thus are able to calibrate the process for consumption and dividend growth according to the salient features of these two series in the data. In order to generate the results in their paper in a general equilibrium setting, a model should be able to obtain the correct moments for consumption growth and dividend growth processes as closely as possible. Table (3.5), column 2, summarizes five important statistics in the data from Bansal and Yaron (2004). Column 3 summarizes the counterparts these authors generate in their calibrated model (under their Case I calibration, without stochastic volatility).

Columns 4 and 5 show the counterparts of these moments generated under our benchmark calibration. The "dividend claim" column reports these values when equity is interpreted as a claim on all future dividends paid by the firm. The "consumption claim" column reports the statistics for the same model, but when equity is interpreted as the claim on future consumption instead.
The properties of the consumption process are clearly unaffected by the choice
between the two models. The properties of what is interpreted as dividend varies sig-
nificantly. Volatility of firm dividend growth is much closer to the data compared to
the volatility of consumption growth. This is reflected in the low volatility of excess
returns in table (3.3). On the other hand, dividend growth shows very little auto-
correlation and more importantly, close to no correlation with consumption growth.

3.7 Discussion

3.7.1 The role of EIS

The pricing of long-run risk in Bansal and Yaron (2004) economy can be understood by
looking at assets as both intertemporal and interstate consumption smoothing devices.
Bhamra and Uppal (2006) provide a good intuition for the respective role of RRA
and EIS in portfolio-consumption choice: A positive shock to expected consumption
growth increases wealth to consumption ratio, which adjusts through movements in
wealth since consumption is exogenous. This adjustment depends on the size of the
EIS. If the substitution effect dominates the wealth effect, i.e. EIS>1, the agent would
like to hold more of the asset, thus driving prices up. Otherwise (when EIS<1) the
agent prefers bringing the increase in consumption forward, depressing prices.

How does this matter for risk premia? Shocks to expected consumption growth af-
fect expected future returns to wealth. The agent with RRA>1 wants to hedge against
these changes in the investment opportunity set (and bet on them if RRA<1). Notice
that RRA and 1/EIS are comparable measures of propensity to smooth consumption
across states and time respectively. Therefore if RRA=1/EIS (CRRA) the changes
in wealth-consumption ratio exactly offset the hedging demand. With Epstein-Zin
preferences there can be a wedge between RRA and 1/EIS which will translate into
premia. As an example, consider an agent with EIS and RRA>1, exposed to a pos-
itive shock to expected consumption growth. The intertemporal substitution effect
drives up asset prices. The hedging demand effect would imply that the agent wants
his portfolio to depreciate. Therefore a premium is required for the agent to hold the
asset in equilibrium (See (Malkhozov 2009) for a formal argument). If consumption
and dividends are correlated the results for the pricing of aggregate risk carry forward
to the risk premium for the claim on aggregate dividends.

Figure (3.4) summarizes these effects. Each panel displays how the main moments
in the economy vary as we increase \( \psi \) from 0.1 to 2. The rest of the parameters are
held constant as in our benchmark calibration. The top right panel shows that as \( \psi \)
increases the risk premium on consumption claim rises. Moreover, the importance of
the long-run risk increases with increased wedge between \( \gamma \) and 1/\( \psi \) as the agent is
willing to resolve intertemporal risk sooner. Overall the two effects do not cancel each other and the risk premium for the consumption claim (aggregate wealth) goes up. These effects are demonstrated in figure (3.4).

The second panel in figure (3.4) shows that the risk-free rate also falls with the EIS. This is intuitive: as EIS increases the agent cares less about the volatility of his consumption across time. Thus, he does not have a strong desire to borrow in order to bring forward future increases in his consumption, and therefore driving down the risk free rate.

The volatility of consumption growth decreases monotonically with EIS. This can be understood by looking at the response of consumption growth to a shock to the permanent component of TFP. As emphasized by Kaltenbrunner and Lochstoer (2007), in response to a positive shock to the non-stationary TFP, consumption growth jumps to a higher level, increasing towards its higher steady state value gradually. The higher the EIS the lower this initial jump, and thus the lower the unconditional volatility of consumption growth. In other words, in response to a permanent TFP shock, the agents with higher EIS choose to have small realized consumption growth but large expected consumption growth.

We must distinguish between the unconditional volatility of consumption growth and the conditional volatility of consumption growth. The former is the variable displayed in the panel 3 of figure (3.4). The latter is the variable emphasized in Kaltenbrunner and Lochstoer (2007) in the context of asset pricing implications. Also note that the monotonicity of the variation in consumption growth volatility with EIS is very specific to the our assumptions about the shocks. In general consumption growth volatility can vary non-monotonically with EIS.

The increase in autocorrelation of consumption with higher values of EIS is consistent with the intuition above. With higher values of EIS, and in response to TFP shocks, the agents make smaller adjustment to consumption initially, but expect a large adjustment in consumption in the long-run. Another way of looking at this is that with low values of EIS the wealth effect dominates and the agent increases his consumption immediately rather than invest to take advantage of the positive technology shock. Consumption growth is not spread through time but occurs (close to) instantaneously with a volatility close to that of the productivity shock. Therefore the unconditional volatility of consumption is high and the persistence of consumption growth is low. Raising EIS decreases the short-run component of risk and increases the long-run component.

Correlation of output and consumption growth also increases. As we observed in the impulse responses, the response of output is delayed. In response to anticipated shocks, in fact output does not respond until the shock is realized. Therefore, the more
delayed the response of output to TFP shocks, the higher the correlation between consumption and output. We argued that by increasing EIS the expected growth in consumption becomes larger relative to the instantaneous consumption adjustment. This is consistent with the rising correlation between output and consumption growth.

### 3.8 The role of anticipated shocks

In this section, we discuss the relative effect of anticipated and unanticipated shocks to TFP on macro and financial variables. We have introduced news shocks in the RBC setting in order to account for the strong empirical evidence that most of the changes to productivity are anticipated one or several quarters in advance. The motivation for allowing news shocks is twofold. First, it is a reasonable generalization, especially when studying asset prices, that the information set of the agents contains more information than the current and the past realizations of the productivity shocks. Furthermore, there is strong empirical evidence which shows that anticipated shocks are important in explaining macroeconomic and financial variables. In a VAR study Beaudry and Portier (2006) find that technology shocks are first captured by stock prices before they affect productivity. Schmitt-Grohe and Uribe (2008) find that anticipated (stationary and non-stationary) shocks explain close to 70% of the variance of output growth, over 80% of the variance in consumption growth, and close to 50% of the variance in investment growth. As explained in detail in section 3.5, the authors estimate the relative importance of different shocks within an RBC model. In this paper we take their suggested estimates on the relative importance of these shocks and ask how introducing anticipation (or enlarging the agents' information set) in an otherwise simple RBC framework affects quantities, prices and price-quantity co-movements compared to the model with unanticipated shocks only.

We answer this question using two measures. First, we look at the variance decomposition (for trended variables), or level decomposition (for stationary variables). Second, following Schmitt-Grohe and Uribe (2008), we look at the pure "anticipation effect", by changing the information set of households to separate their actions into response to news (anticipation effect) and response to the realization of a shock.

**Decomposing the effects of anticipated shocks** Table (3.6) shows the contribution of each of the four shocks in our calibration to variables of interest. Columns (1) and (2) show the share of variance of consumption and output growth attributable to each of the four shocks. The one-quarter ahead anticipated shock explains about 60% of the variance of output and consumption. Column (3) shows the reduction in risk-free rate compared to a non-stochastic economy, attributable to each shock. Again, $\epsilon_A$ has the largest effect on risk-free rate. Finally, column (4) breaks down the excess return of the consumption claim into its components.
Overall, the anticipated shocks explain about 95% of consumption and output volatility in our calibration. They also reduce the risk-free rate by over 300 basis points relative to the non-stochastic case and account for 97% of the premia generated by the model.

Anticipation effect To understand the importance of anticipated shocks in creating long-run risk and their effect on financial variables, we find it useful to decompose anticipated shocks into an “anticipation” and “realization” effect, as put by Schmitt-Grohe and Uribe (2008). The anticipation effect is changes in behavior triggered by the arrival of a news about future without any physical variable being affected. The realization effect occurs when the shock materializes. We observed some of these effects qualitatively when studying the impulse response of variables of interest to anticipated and unanticipated shocks. For instance in response to news about a future increase in TFP capital falls whereas it starts rising when TFP in fact increases. Thus, the anticipation effect is a fall in capital whereas the realization effect is an increase in capital.

Following Schmitt-Grohe and Uribe (2008) we decompose these two effects by eliminating the anticipation channel, and comparing all variable in a “no anticipation” world to those of the benchmark economy. The difference must be due to the pure anticipation effect. To do so, we change the information set of households such that they can only learn about shocks on the period that they occur. In other words, we replace the law of motion of the non-stationary TFP shock as follows:

$$\Delta \ln A_t = x_{t+1}^1 = \lambda + \nu_{A,t}$$

but now replace the variance of $\nu_{A,t}$ with the sum of all of its anticipated and unanticipated components. In other words,

$$\sigma_{A,t}^2 = (\sigma_0^A)^2 + (\sigma_1^A)^2 + (\sigma_2^A)^2 + (\sigma_3^A)^2$$

This economy is identical to the baseline model with anticipated shocks introduced earlier, except for the timing with which the agents learn about the shocks.\textsuperscript{39} Table (3.7) reproduces all the results in table (3.3) under the no-anticipation calibration.

We observe that without anticipated shocks the volatility of consumption rises, and therefore volatility of investment falls. The volatility of output is the same as in the benchmark model given that the variance of productivity shocks are kept constant. We also observe, that without anticipated shocks, the risk-free rate rises and the excess return of consumption claim falls by 0.8%. The intuition for these two results are the following: by eliminating anticipated shocks in our model, we shut down a main source

\textsuperscript{39}For more details, we refer the reader to section 6 of Schmitt-Grohe and Uribe (2008) paper
of expected consumption growth risk – or long-run risk in consumption. Thus, there is less incentive for buffer-stock savings and risk-free rates rise. The fall in premia could be due to lower amount of risk, or, lower price of a unit of risk. As observed from table (3.7), the amount of risk associated with this asset, i.e. the volatility of consumption growth, in fact rises slightly compared to the benchmark case. Thus, all of the fall must be due to a lower price of risk. The lower price of risk can also be explained by the absence of anticipated shocks. To understand this, consider a special example: In an endowment economy where consumption is subject to i.i.d. shocks, shocks to consumption growth are entirely “realized”; i.e. the expected consumption growth is always the same as in the steady state. This model has no long-run risk.

A production economy generated long-run risk in a few ways: consumption smoothing always implies that shocks to today’s productivity show up as persistent consumption growth. Furthermore, persistence in TFP itself creates some movement in the expected consumption growth. Anticipated shocks, as explained before, are yet another avenue for generating expected consumption growth. By eliminating anticipated shocks, shocks to consumption do not have a “news” component and are therefore less undesirable. This intuition is confirmed by the lower premia.

The largest difference is the correlations between output and consumption growth. As explained earlier, in our specification there is no reason to bring future increases in output forward, and thus in response to news about future shocks output remains roughly constant until the realization of the shock. Whereas consumption rises immediately upon the arrival of the news. Thus, allowing for anticipated shocks weakens the correlation between consumption and output growth.

### 3.8.1 Stationary TFP Shocks

In the Schmitt-Grohe and Uribe (2008) calibration stationary TFP shocks play an important role. They account for 66% percent of output growth fluctuations and about 40% of consumption growth fluctuations. An important question is how eliminating them will affect our results. Table (3.9) reproduces the results in table (3.3) for a model with stationary as well as non-stationary TFP shocks. The non-stationary shocks are calibrated as before. We calibrate the stationary TFP shocks according to Schmitt-Grohe and Uribe (2008) estimates. They are reproduced in table (3.8). We multiply all variances (stationary and non-stationary) with a factor $\kappa'$ in order to keep the standard deviation of consumption growth constant.

We observe that the volatility of output growth has increased nearly twofold, even though the volatility of consumption is the same. This means that the ratio of consumption to output volatility is counterfactually low. The risk free rate is not much different despite the additional risk that the economy is exposed to. And surprisingly,
3 ASSET PRICES IN A NEWS DRIVEN RBC MODEL

premiums are lower. Given these findings, and our current specification we find that a model without stationary shocks will fit the data better and hence stationary shocks are excluded from our benchmark calibration.

3.8.2 Alternative Preferences

Habit formation has been advanced as an important alternative to the standard preferences specification in both consumption-based asset pricing and RBC models. We take the view that the Epstein-Zin utility specification imposes less structure on the preferences of the agent than the habit formation specification, as it only assumes constant EIS and RRA coefficients. Models with habit formation claim a partial success in the Lucas-tree setup: they achieve high equity risk premia at the cost of volatile interest rates. However, simply introducing habit formation in a production economy does not generate the desired asset pricing implications. The agent has a strong desire for smoothing consumption yet he can achieve this without generating fluctuations in equity returns (see Boldrin, Christiano, and Fisher (2001)).

In the context of a news-driven business cycle Schmitt-Grohe and Uribe (2008) and Jaimovich and Rebelo (2009) both assume habit formation utility functions (albeit with some differences) in order to match the macro-economic co-movements. In particular the preferences are designed in such a way to mitigate the wealth effects of an anticipated productivity shock and generate a boom in investment in response to it. The Epstein-Zin preferences address this issue, at least partially, since the EIS coefficient is set to be greater than 1.

A related work by Rudebusch and Swanson (2008a), Rudebusch and Swanson (2008b) compares the performance of DSGE models with habit formation vs. recursive preferences and nominal long-run risks for the pricing of bonds. The authors find that the model with Epstein-Zin preferences produces better joint results for the macroeconomic variables and the bond yields.

3.9 Conclusions

In this paper we study the asset pricing implications of a stochastic growth model with recursive preferences and a general shock structure, which allows for news shocks. The aim is to investigate whether asset prices obtained in an endowment economy with long-run risk (such as Bansal and Yaron (2004)) can be replicated in a production economy framework.

The model is subject to stationary and non-stationary TFP shocks, each of which have an unanticipated component as well as anticipated components at 1, 2, and 3 quarter horizons. We solve the model by a second order perturbation technique. We
use the framework introduced by Schmitt-Grohe and Uribe (2004) and show how the
generic code they develop for approximating DSGE models can be applied to models
with recursive preferences.

Our benchmark calibration can match the main macroeconomic moments in the
data such as volatility of consumption, output, and investment growth. It also pro­
duces financial moments close to those observed in the data: low risk free rates, with
volatility close to that observed in the data; risk premia of around 4.5% annually
(compared to 6.5% in the data). We achieve these results without resorting to un­
usual parameter values for EIS or risk aversion. In our benchmark calibration, we
choose an EIS of 1.5 and a risk aversion coefficient of 10 which corresponds to the
benchmark calibration of Bansal and Yaron (2004).

Both the volatility of excess returns and the risk premium of consumption claim
are lower for consumption claim than it is empirically observed for equity. However,
the stock-market is not the empirical counterpart of the consumption claim. For the
latter, the model is successful in generating a high Sharpe-ratio.

Schmitt-Grohe and Uribe (2008) show that news shocks are important in explain­
ing business cycle movements of the main macroeconomic variables. In this paper,
we discuss their asset pricing implications. We show that news shocks increase the
endogenous long-run risk in consumption. Under Epstein-Zin preferences these risks
will be priced, therefore creating much larger premia compared to a case where all
shocks are unanticipated.

From our results, we conclude that sufficient long-run risk can endogenously arise
in a production economy with the right ingredients. The importance of anticipated
shocks is recently emphasized in the macro literature. The contribution of this paper
is to note that these shocks can be thought of as a source of long-run risk and have
important asset pricing implications.
Appendix

A.1 Equilibrium Conditions

The first order conditions with respect to consumption and labor are respectively:

\[(1 - N_t)\eta^{(1-1/\psi)}C_t^{-1/\psi} = -\beta E_t \left( V_{t+1}^{1-\gamma} \right)^{\gamma-1/\psi} \frac{\partial K_{t+1}}{\partial C_t} \] (3.14)

\[-\eta(1 - N_t)\eta^{-\eta/\psi-1}C_t^{1-1/\psi} = -\beta E_t \left( V_{t+1}^{1-\gamma} \right)^{\gamma-1/\psi} \frac{\partial K_{t+1}}{\partial N_t} \] (3.15)

The envelope condition with respect to capital implies

\[V_{Kt} = \beta V_t^{1/\psi} E_t \left( V_{t+1}^{1-\gamma} \right)^{\gamma-1/\psi} E_t \left( V_{t+1}^{\gamma} V_{Kt+1} \frac{\partial K_{t+1}}{\partial K_t} \right) \]

where

\[\frac{\partial K_{t+1}}{\partial l_t} = \zeta K^{1-\zeta} I^{1-\zeta} K^{-\zeta} = \phi' \left( \frac{I_t}{K_t} \right) \]

\[\frac{\partial I_t}{\partial C_t} = -1 \]

\[\frac{\partial I_t}{\partial N_t} = \frac{\partial Y_t}{\partial N_t} = (1 - \alpha) Z_t A_t^{1-\alpha} N_t^{-\alpha} K_t^\alpha \]

\[\frac{\partial K_{t+1}}{\partial K_t} = (1 - \delta) + \frac{\zeta - 1}{\zeta} K^{-1} I + \frac{1}{\zeta} K^{1-\zeta} I^{1-\zeta} \left( \zeta K^{1-\zeta} I^{1-\zeta} \frac{\partial Y_t}{\partial K_t} + (1 - \zeta) K^{1-\zeta} I \right) \]

\[\frac{\partial Y_t}{\partial K_t} = \alpha Z_t (A_t N_t)^{1-\alpha} K_t^{\alpha-1} \]

Combining first order conditions (3.14) and (3.15), we obtain the condition for optimal consumption-leisure trade-off

\[-(1 - N_t) \frac{\partial K_{t+1}}{\partial N_t} = \eta C_t \frac{\partial K_{t+1}}{\partial C_t} \]

Finally, the Euler equation can be derived by combining first order conditions (3.14) and (3.15), and transversality conditions

\[E_t \left[ \beta \left( \frac{V_{t+1}}{E_t \left( V_{t+1}^{1-\gamma} \right)^{\gamma-1/\psi}} \right)^{1/\psi-\gamma} \left( \frac{C_{t+1}}{C_t} \right)^{-1/\psi} \left( \frac{1 - N_{t+1}}{1 - N_t} \right)^{(1-1/\psi)} \left( \frac{\partial K_{t+2}}{\partial C_{t+1}} \right)^{-1} \frac{\partial K_{t+1}}{\partial C_t} \frac{\partial K_{t+2}}{\partial K_{t+1}} \right] = 1 \]
A.2 Stationary Form and the Steady State

Defining the stationary version of a variable $X_t$ as $\bar{X}_t$, where $\bar{X}_t = \frac{X_t}{A_{t-1}}$ and defining the stationary version of the value function as $\bar{V}_t$, where $\bar{V}_t = V(\bar{K}_t, \bar{A}_t, Z_t, \mu_t)$ we can re-define the the equilibrium point of the model in terms of the stationary variables. Since the value function is homogeneous of degree one in $K_t$ and $A_t$

$$V(K_t, A_t, Z_t, x_t^{(-1,-2)}) = A_{t-1}V(K_t, A_t, Z_t, x_t^{(-1,-2)})$$

$$\frac{V_t}{A_{t-1}} = \max_{C_t, N_t} \left( \left( (1 - N_t)^{\eta} \frac{C_t}{A_{t-1}} \right)^{1-\gamma} + \left( \frac{1}{A_{t-1}} \right)^{1-\gamma} (\beta(E_t(V_{t+1}^{1-\gamma})))^{1-\gamma} \right)^{1-\gamma}$$

$$\bar{V}_t = \max_{\bar{C}_t, \bar{N}_t} \left( \left( (1 - N_t)^{\eta} \bar{C}_t \right)^{1-\gamma} + \bar{A}_t^{1-\gamma} (\beta(E_t(\bar{V}_{t+1}^{1-\gamma})))^{1-\gamma} \right)^{1-\gamma}$$

The optimality conditions can be rewritten as

$$E_t \left[ M_{t+1} \left( \frac{\partial K_{t+2}}{\partial C_{t+2}} \right)^{-1} \frac{\partial \bar{K}_{t+1}}{\partial C_t} \frac{\partial \bar{K}_{t+2}}{\partial C_t} \right] = 1$$

$$(1 - N_t) \frac{\partial \bar{K}_{t+1}}{\partial N_t} - \eta \bar{C}_t \frac{\partial \bar{K}_{t+1}}{\partial \bar{C}_t} = 0$$

where

$$M_{t+1} = \bar{A}_t^{-1/\psi} \beta \left( \frac{\bar{V}_{t+1}}{E_t(\bar{V}_{t+1}^{1-\gamma})} \right)^{1/\psi-\gamma} \left( \frac{\bar{C}_{t+1}}{\bar{C}_t} \right)^{-1/\psi} \left( \frac{1 - N_{t+1}}{1 - N_t} \right)^{\eta(1-1/\psi)}$$

The capital evolution equation and investment become

$$\bar{K}_{t+1} \bar{A}_t = (1 - \delta) \bar{K}_t - \bar{K}_t \left( \frac{1}{\xi} \bar{K}^{\xi-1} \bar{I}^{1-\xi} \left( \frac{\bar{I}_t}{\bar{K}_t} \right) \bar{K}^{1-\xi} \right)$$

$$\bar{I}_t = Z_t \left( \bar{A}_t N_t \right)^{1-\alpha} \bar{K}^\alpha - \bar{C}_t$$

Finally, the non-stationary technology shock is normalized as below, whereas the other stationary state variables remain unchanged.

$$\ln \bar{A}_{t+1} = x_{t+1}^1$$

$$\ln Z_{t+1} = x_{t+1}^2$$

$$x_{t+1} = H_0 + H_1 x_t + H_2 \epsilon_{t+1}$$
The solution for the non-stochastic steady state can be expressed in closed form

\[ N = \text{const} \]
\[ x = (I - H_1)^{-1} H_0 \]
\[ \tilde{A} = \exp(x^1) \]
\[ Z = \exp(x^2) \]
\[ \tilde{K} = (\tilde{A}N) \left[ \frac{\beta^{-1} \tilde{A}^{1/\psi} - (1 - \delta)}{\alpha Z} \right] \frac{1}{\alpha - 1} \]
\[ \tilde{Y} = Z (\tilde{A}N)^{1-\alpha} \tilde{K}^\alpha \]
\[ \tilde{I} = (\tilde{A} - (1 - \delta))\tilde{K} \]
\[ \eta = -\tilde{C}^{-1}(1 - N)(1 - \alpha)Z\tilde{A}^{1-\alpha}N^{-\alpha}\tilde{K}^\alpha \]
\[ \tilde{V} = \left[ \frac{(1 - N)^{\eta\tilde{C}}}{1 - \beta\tilde{A}^{1-\frac{1}{\psi}}} \right]^{\frac{1}{1-\psi}} \]
\[ \tilde{Q} = \tilde{V}^{1-\gamma} \]
A.3 Scaling the Value Function

In practice value function can take very large or very small values. These would be magnified when evaluating the term $E_t \left( \tilde{V}_{t+1}^{1-\gamma} \right)$ which appears in the Euler equation. This can be problematic when numerically evaluating the model. To avoid this issue, we scale the value function such that $\tilde{V}$ takes a reasonable value in the non-stochastic steady state. We note that pre-multiplying $u_t^{1-\frac{1}{\phi}}$ by a constant $\Lambda$ scales the value function by $\Lambda^{\frac{1}{\phi}}$. Moreover this does not have any effect on equilibrium quantities or prices.

$$\tilde{V}_t = \max_{\tilde{C}_t, N_t} \left( \Lambda \left( (1 - N_t) \tilde{c}_t \right)^{1-\frac{1}{\phi}} + \tilde{A}_t^{1-\frac{1}{\phi}} \beta(E_t(\tilde{V}_{t+1}^{1-\gamma}))^{\frac{1-\frac{1}{\phi}}{1-\gamma}} \right)^{1-\frac{1}{\phi}}$$

Therefore, we choose $\Lambda$ such that at the steady state $\tilde{V} = \tilde{V}^{1-\gamma} = 1$. 
A.4 The Structure for Anticipated Shocks

Anticipated productivity shocks can be represented as a VAR(1). We present only the non-stationary shocks, as is the case in our benchmark calibration. Adding the stationary shocks is very similar, but are omitted for parsimonious presentation.

\[ \ln A_{t+1} - \ln A_t = x_{t+1}^1 \]

\[ x_{t+1} = H_0 + H_1 x_t + H_2 \varepsilon_{t+1} \]

where

\[ x_t = \begin{pmatrix} x_t^1 \\ \varepsilon_{A,t}^1 \\ \varepsilon_{A,t}^2 \\ \varepsilon_{A,t}^3 \\ \varepsilon_{A,t-1}^2 \\ \varepsilon_{A,t-1}^3 \end{pmatrix} \]

where \( \varepsilon_{A,t}^1, \varepsilon_{A,t}^2, \) and \( \varepsilon_{A,t}^3 \) are respectively the 1-quarter-, 2-quarter- and 3-quarter-ahead anticipated component of the non-stationary TFP shock. Also, the autoregressive matrices are defined as:

\[
H_0 = \begin{pmatrix} (1 - \rho_A) \lambda_A \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad H_1 = \begin{pmatrix} \rho_A \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad H_2 = \begin{pmatrix} \sigma_A^0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}
\]
Table 3.1: Model Calibration

<table>
<thead>
<tr>
<th>Panel A: Basic Parameters Calibration (Quarterly)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>α</td>
</tr>
<tr>
<td>δ</td>
</tr>
<tr>
<td>λ</td>
</tr>
<tr>
<td>η</td>
</tr>
<tr>
<td>N</td>
</tr>
<tr>
<td>ψ</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Panel B: Calibration of the Benchmark Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>β</td>
</tr>
<tr>
<td>τ</td>
</tr>
<tr>
<td>γ</td>
</tr>
</tbody>
</table>

This table reports our calibrated parameters. The parameter values in panel (A) are kept constant throughout the models. The parameters in panel (B) pertain to our benchmark calibration. The time unit is a quarter.
This table reports Schmitt-Grohe and Uribe (2008) estimates of the variance and autocorrelation coefficient of the non-stationary TFP shock. Their results are based on 4 million elements of a MCMC chain of draws from the posterior distribution. In our calibration we weight all the variances by a factor $\kappa$ to obtain a variance in consumption which is the same as in the data.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>5 percent</th>
<th>95 percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_A$</td>
<td>0.14</td>
<td>0.0</td>
<td>0.27</td>
</tr>
<tr>
<td>$\sigma^0_A$ (%)</td>
<td>0.59</td>
<td>0.05</td>
<td>1.4</td>
</tr>
<tr>
<td>$\sigma^1_A$ (%)</td>
<td>2.3</td>
<td>1.6</td>
<td>3.0</td>
</tr>
<tr>
<td>$\sigma^2_A$ (%)</td>
<td>1.3</td>
<td>0.2</td>
<td>2.4</td>
</tr>
<tr>
<td>$\sigma^3_A$ (%)</td>
<td>1.1</td>
<td>0.1</td>
<td>2.0</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>1.52</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>
### Table 3.3: Benchmark Calibration Results

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>Benchmark Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average consumption growth</td>
<td>$E[\Delta c]$ (%)</td>
<td>1.9</td>
</tr>
<tr>
<td>Volatility of consumption growth</td>
<td>$\sigma(\Delta c)$ (%)</td>
<td>2.72</td>
</tr>
<tr>
<td>Relative volatility of consumption to output</td>
<td>$\sigma(\Delta c)/\sigma(\Delta y)$</td>
<td>0.52</td>
</tr>
<tr>
<td>Relative volatility of investment to output</td>
<td>$\sigma(\Delta i)/\sigma(\Delta y)$</td>
<td>3.0</td>
</tr>
</tbody>
</table>

**Panel A: Macroeconomic Variables**

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>Benchmark Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average risk-free rate</td>
<td>$E[r_f]$ (%)</td>
<td>0.86</td>
</tr>
<tr>
<td>Volatility of risk-free rate</td>
<td>$\sigma[r_f]$ (%)</td>
<td>0.97</td>
</tr>
<tr>
<td>Average excess returns</td>
<td>$E[r_E - r_f]$ (%)</td>
<td>6.33</td>
</tr>
<tr>
<td>Volatility of excess returns</td>
<td>$\sigma[r_E - r_f]$ (%)</td>
<td>19.42</td>
</tr>
<tr>
<td>Sharpe ratio of consumption claim</td>
<td>$E[r_E - r_f]/\sigma[r_E - r_f]$</td>
<td>0.33</td>
</tr>
</tbody>
</table>

**Panel B: Financial Variables**

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>Benchmark Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation of output and consumption growth</td>
<td>$corr(g_Y, g_C)$</td>
<td>0.49</td>
</tr>
<tr>
<td>Correlation of output and investment growth</td>
<td>$corr(g_Y, g_I)$</td>
<td>0.67</td>
</tr>
<tr>
<td>Correlation of consumption and investment growth</td>
<td>$corr(g_C, g_I)$</td>
<td>-0.16</td>
</tr>
</tbody>
</table>

This table reports key annualized moments for the benchmark calibration discussed in the text. TFP shocks are non-stationary. The empirical moments are taken from Kaltenbrunner and Lochstöer (2007). The unconditional mean of per capita consumption growth rate is the annualized mean, obtained using BEA quarterly data on per capita consumption from 1948Q1-1998QIV.
Table 3.4: Benchmark Calibration with $\gamma = 7.5$

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>Benchmark</th>
<th>Benchmark w/ $\gamma = 7.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Macroeconomic Variables</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average consumption growth</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[\Delta c]$ (%)</td>
<td>1.9</td>
<td>1.9</td>
<td>1.9</td>
</tr>
<tr>
<td>Volatility of consumption growth</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(\Delta c)$ (%)</td>
<td>2.72</td>
<td>2.75</td>
<td>2.75</td>
</tr>
<tr>
<td>Relative volatility of consumption to output</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(\Delta c)/\sigma(\Delta y)$</td>
<td>0.52</td>
<td>0.47</td>
<td>0.47</td>
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<tr>
<td>Relative volatility of investment to output</td>
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<tr>
<td>$\sigma(\Delta i)/\sigma(\Delta y)$</td>
<td>3.0</td>
<td>3.0</td>
<td>3.1</td>
</tr>
<tr>
<td><strong>Panel B: Financial Variables</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average risk-free rate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[r_f]$ (%)</td>
<td>0.86</td>
<td>0.82</td>
<td>1.62</td>
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<tr>
<td>Volatility of risk-free rate</td>
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<td></td>
</tr>
<tr>
<td>$\sigma[r_f]$ (%)</td>
<td>0.97</td>
<td>0.70</td>
<td>0.70</td>
</tr>
<tr>
<td>Average risk premia</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[r_E - r_f]$ (%)</td>
<td>6.33</td>
<td>4.53</td>
<td>3.34</td>
</tr>
<tr>
<td>Volatility of risk premia</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma[r_E - r_f]$ (%)</td>
<td>19.42</td>
<td>5.00</td>
<td>5.00</td>
</tr>
<tr>
<td>Sharpe ratio of consumption claim</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[r_E - r_f]/\sigma[r_E - r_f]$</td>
<td>0.33</td>
<td>0.91</td>
<td></td>
</tr>
<tr>
<td><strong>Panel C: Growth Rate Correlations</strong></td>
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</tr>
<tr>
<td>Correlation of output and consumption growth</td>
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<td></td>
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</tr>
<tr>
<td>$corr(g_Y, g_C)$</td>
<td>0.49</td>
<td>0.15</td>
<td>0.15</td>
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<tr>
<td>Correlation of output and investment growth</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$corr(g_Y, g_I)$</td>
<td>0.67</td>
<td>0.95</td>
<td>0.94</td>
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<tr>
<td>Correlation of consumption and investment growth</td>
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<tr>
<td>$corr(g_C, g_I)$</td>
<td>0.40</td>
<td>-0.16</td>
<td>-0.16</td>
</tr>
</tbody>
</table>

This table reports key annualized moments for an alternative calibration with $\gamma = 7.5$; all other parameters are as in the benchmark calibration discussed in the text. TFP shocks are non-stationary.
Table 3.5: Dividend and Consumption Growth

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data</th>
<th>BY model</th>
<th>Dividend claim</th>
<th>Cons. claim</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volatility of consumption growth $\sigma(g)$</td>
<td>2.93</td>
<td>2.72</td>
<td>2.45</td>
<td>2.45</td>
</tr>
<tr>
<td>Autocorrelation of consumption growth $AC(1)$</td>
<td>0.49</td>
<td>0.48</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>Volatility of dividend growth $\sigma(g_d)$</td>
<td>11.5</td>
<td>10.96</td>
<td>10.9</td>
<td>2.45</td>
</tr>
<tr>
<td>Autocorrelation of dividend growth $AC_d(1)$</td>
<td>0.21</td>
<td>0.33</td>
<td>0.01</td>
<td>0.20</td>
</tr>
<tr>
<td>Corr. of cons. and dividend growth $corr(g, g_d)$</td>
<td>0.55</td>
<td>0.31</td>
<td>-0.08</td>
<td>1</td>
</tr>
</tbody>
</table>

This table compares the properties of dividends flows in the data and its co-movement with consumption, with two possible model counterparts: 1) If a claim on future dividend payments by firms is considered equity; and 2) if a claim on future stream of consumption is considered as equity. Column 3, reports the moments achieved in Bansal and Yaron (2004) model, in Case I.
This table shows the variance decomposition of consumption and output growth volatility, as well as the amount of risk free rate and premia generated, due to each of the four anticipated shocks. The first row reports these values under the benchmark calibrations. Columns 3 and 4, marked as % of benchmark, report the share of variance in consumption and output growth volatility due to each shock.

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_{\Delta c}$ (%)</th>
<th>$\sigma_{\Delta y}$ %</th>
<th>$E[rf]$ (%)</th>
<th>$E[r_E - rf]$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>-</td>
<td>-</td>
<td>0.82</td>
<td>4.53</td>
</tr>
<tr>
<td>$\varepsilon_A^0$</td>
<td>5</td>
<td>4</td>
<td>0.20</td>
<td>0.15</td>
</tr>
<tr>
<td>$\varepsilon_A^1$</td>
<td>62</td>
<td>62</td>
<td>2.21</td>
<td>2.87</td>
</tr>
<tr>
<td>$\varepsilon_A^2$</td>
<td>20</td>
<td>20</td>
<td>0.47</td>
<td>0.89</td>
</tr>
<tr>
<td>$\varepsilon_A^3$</td>
<td>13</td>
<td>14</td>
<td>0.44</td>
<td>0.62</td>
</tr>
</tbody>
</table>
Table 3.7: Model without Anticipated Shocks

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>Benchmark Model</th>
<th>No Anticipation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Macroeconomic Variables</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average consumption growth</td>
<td></td>
<td>1.9</td>
<td>1.9</td>
</tr>
<tr>
<td>$E[(\Delta c)]%$</td>
<td></td>
<td>1.9</td>
<td>1.8</td>
</tr>
<tr>
<td>Volatility of consumption growth</td>
<td></td>
<td>2.72</td>
<td>2.75</td>
</tr>
<tr>
<td>Relative volatility of consumption to output</td>
<td></td>
<td>0.52</td>
<td>0.47</td>
</tr>
<tr>
<td>Relative volatility of investment to output</td>
<td></td>
<td>0.52</td>
<td>0.47</td>
</tr>
<tr>
<td><strong>Panel B: Financial Variables</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average risk-free rate</td>
<td></td>
<td>0.86</td>
<td>0.82</td>
</tr>
<tr>
<td>Volatility of risk-free rate</td>
<td></td>
<td>0.97</td>
<td>0.70</td>
</tr>
<tr>
<td>Average excess returns</td>
<td></td>
<td>6.33</td>
<td>4.53</td>
</tr>
<tr>
<td>Volatility of excess returns</td>
<td></td>
<td>19.42</td>
<td>5.00</td>
</tr>
<tr>
<td>Sharpe ratio of consumption claim</td>
<td></td>
<td>0.33</td>
<td>0.91</td>
</tr>
<tr>
<td><strong>Panel C: Growth Rate Correlations</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correlation of output and consumption growth</td>
<td></td>
<td>0.49</td>
<td>0.15</td>
</tr>
<tr>
<td>$corr(g_Y, g_C)$</td>
<td></td>
<td>0.49</td>
<td>0.15</td>
</tr>
<tr>
<td>Correlation of output and investment growth</td>
<td></td>
<td>0.67</td>
<td>0.95</td>
</tr>
<tr>
<td>$corr(g_Y, g_I)$</td>
<td></td>
<td>0.67</td>
<td>0.95</td>
</tr>
<tr>
<td>Correlation of consumption and investment growth</td>
<td></td>
<td>0.40</td>
<td>-0.16</td>
</tr>
<tr>
<td>$corr(g_C, g_I)$</td>
<td></td>
<td>0.40</td>
<td>-0.16</td>
</tr>
</tbody>
</table>

This table reports key annualized moments when we assume, as discussed in the text, that shocks are not anticipated. TFP shocks are non-stationary. The empirical moments are taken from Kaltenbrunner and Lochstoer (2007). For comparison, results from the benchmark calibration are reproduced here.
Table 3.8: Calibrating the Stationary TFP Shocks

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>5 percent</th>
<th>95 percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_z$</td>
<td>0.89</td>
<td>0.87</td>
<td>0.89</td>
</tr>
<tr>
<td>$\sigma_0^Z$ (%)</td>
<td>2.7</td>
<td>2.4</td>
<td>3.1</td>
</tr>
<tr>
<td>$\sigma_1^Z$ (%)</td>
<td>0.56</td>
<td>0.05</td>
<td>1.3</td>
</tr>
<tr>
<td>$\sigma_2^Z$ (%)</td>
<td>0.56</td>
<td>0.05</td>
<td>1.3</td>
</tr>
<tr>
<td>$\sigma_3^Z$ (%)</td>
<td>3.0</td>
<td>2.5</td>
<td>3.6</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>1.21</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

This table reports Schmitt-Grohe and Uribe (2008) estimates of the variance and autocorrelation coefficient of the stationary TFP shocks. Their results are based on 4 million elements of a MCMC chain of draws from the posterior distribution. In our calibration we weight all the variances by a factor $\kappa$ to obtain a variance in consumption which is the same as in the data.
Table 3.9: Model with Stationary and Non-stationary Shocks

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>Benchmark Model</th>
<th>Model w/ $Z(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Macroeconomic Variables</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average consumption growth $E[\Delta c]$ (%)</td>
<td>1.9</td>
<td>1.9</td>
<td>2.22</td>
</tr>
<tr>
<td>Volatility of consumption growth $\sigma(\Delta c)$ (%)</td>
<td>2.72</td>
<td>2.75</td>
<td>2.77</td>
</tr>
<tr>
<td>Relative volatility of consumption to output $\sigma(\Delta c) / \sigma(\Delta y)$</td>
<td>0.52</td>
<td>0.47</td>
<td>0.24</td>
</tr>
<tr>
<td>Relative volatility of investment to output $\sigma(\Delta i) / \sigma(\Delta y)$</td>
<td>3.0</td>
<td>3.0</td>
<td>2.8</td>
</tr>
<tr>
<td><strong>Panel B: Financial Variables</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average risk-free rate $E[r_f]$ (%)</td>
<td>0.86</td>
<td>0.82</td>
<td>0.83</td>
</tr>
<tr>
<td>Volatility of risk-free rate $\sigma[r_f]$ (%)</td>
<td>0.97</td>
<td>0.70</td>
<td>0.74</td>
</tr>
<tr>
<td>Average excess returns $E[r_E - r_f]$ (%)</td>
<td>6.33</td>
<td>4.53</td>
<td>3.13</td>
</tr>
<tr>
<td>Volatility of excess returns $\sigma[r_E - r_f]$ (%)</td>
<td>19.42</td>
<td>5.00</td>
<td>4.25</td>
</tr>
<tr>
<td>Sharpe ratio of consumption claim $E[r_E - r_f] / \sigma[r_E - r_f]$</td>
<td>0.33</td>
<td>0.91</td>
<td>0.89</td>
</tr>
<tr>
<td><strong>Panel C: Growth Rate Correlations</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correlation of output and consumption growth $\text{corr}(g_Y, g_C)$</td>
<td>0.49</td>
<td>0.15</td>
<td>0.33</td>
</tr>
<tr>
<td>Correlation of output and investment growth $\text{corr}(g_Y, g_I)$</td>
<td>0.67</td>
<td>0.95</td>
<td>0.97</td>
</tr>
<tr>
<td>Correlation of consumption and investment growth $\text{corr}(g_C, g_I)$</td>
<td>0.40</td>
<td>-0.16</td>
<td>-0.04</td>
</tr>
</tbody>
</table>

This table reports key annualized moments for the economy subject to both stationary and non-stationary TFP shocks, each at 4 different anticipation horizons. The empirical moments are taken from Kaltenbrunner and Lochstoer (2007). For comparison, results from the benchmark calibration are reproduced here.
Figure 3.1: Macroeconomic Impulse Responses to $A_t$

Impulse responses to a one-standard-deviation innovation in the anticipated and unanticipated component of the non-stationary TFP ($\varepsilon_A^0$: Solid line and $\varepsilon_A^1$: Crossed line).
Figure 3.2: Macroeconomic Impulse Responses to $Z_t$

Impulse responses to a one-standard-deviation innovation in the anticipated and unanticipated component of the stationary TFP ($\varepsilon^0_z$: Solid line and $\varepsilon^3_z$: Crossed line).
Figure 3.3: Dividend Impulse Responses

Impulse response of dividends to one-standard-deviation anticipated and unanticipated shock to the stationary and non-stationary TFP shocks ($\varepsilon_A^0$, $\varepsilon_{\mu}^0$, $\varepsilon_z^0$ and $\varepsilon_z^3$, clockwise from the top).
Figure 3.4: Variation of Macroeconomic and Financial Variables with EIS

Variation of macroeconomic and financial variables with EIS
4 Future Research

The research presented in this thesis offers considerable scope for future developments.

Chapter 1 of this thesis contributes to a growing literature, which tries to reconcile the high frequency of price changes at the micro level with the apparent rigidity at the aggregate level. The research in this area has shown that there are significant heterogeneity in the frequency of price changes between different sectors, for a variety of reasons. I believe disentangling the deep determinants of this heterogeneity into "intrinsic" vs. "extrinsic" price sources of stickiness is an important issue. By intrinsic price stickiness I refer to a measure of how costly it is for a firm to change its prices, literally. In other words, some measure of how efficient the price-changing technology of a firm is. By the "extrinsic" component of price stickiness I refer to a measure of how the environment external to a firm's technology of price change would affect the optimal frequency with which the firm chooses to change its price. For instance, factors such as the market structure of an industry, or, as proposed by the model I introduced earlier, the position of a firm in the production chain, might affect the frequency with which a firm changes its price. Most papers in the literature use the observed "frequency of price change" as a measure of how sticky the price of a certain good is. But this frequency could be affected both by intrinsic and extrinsic factors. Disentangling these two effects would help us predict how changes in industrial organization or business practices may affect price stickiness in an economy and thus would provide guidance for policy.

Reconciling macroeconomic co-movements and asset pricing implications of DSGE models is an important direction of future research. Chapter 3 of this thesis is a contribution in this direction. We see three dimensions along which Chapter 3 can be extended in order to improve the macroeconomic and asset pricing predictions of this class of models.

First, is defining an appropriate model counterpart for the equity market. One way to deal with this issue would be to introduce a realistic representation of the dynamic capital structure choices faced by the firms, as in Gomes and Schmid (2009). Such setup allows for more realistic counterparts for the equity and debt markets. Furthermore, this framework would allow us to study additional asset pricing implication for credit spreads.

Second, is incorporating a more realistic model of the labor market. In the current version of the model, the assumptions about the labor market imply constant shares of profits and wages in the output. This implication is counterfactual. A better model of the labor market will also affect asset prices in our model. Dividends are defined as consumption less wages in the economy. Therefore, even if we achieve realistic
consumption dynamics but counterfactual labor market predictions, the aggregate dividend process and therefore the asset prices will be erroneous.

Finally, we believe that estimation would be an interesting direction of future research. Solving DSGE models with perturbation methods has an important advantage over other numerical procedures, and that is their computational tractability. This solution technique will allow us to take the model directly to the data and estimate the importance of the news shocks at various horizons using jointly macroeconomic and asset pricing time series.
References


BARRO, R. J. (2005): “Rare Events and the Equity Premium,”.


REFERENCES


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