The London School of Economics and Political Science

Volatility and Correlation in Financial Markets: Econometric Modeling and Empirical Pricing

Runquan Chen

A thesis submitted to the Department of Finance of the London School of Economics for the degree of Doctor of Finance

May 2009

Declaration

I certify that the thesis I have presented for examination for the MPhil/PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

I warrant that this authorization does not, to the best of my belief, infringe the rights of any third party.

Abstract

This thesis is an empirical study of the volatility and correlation in financial markets, and consists of two parts: The first part is on econometric modeling of the volatility and correlation (Chapter 1). The second part is on the pricing implication of the correlation and volatility as risk factors (Chapter 2 and 3). The thesis begins with proposing a regime-switching multivariate GARCH model of volatilities and correlation. We incorporate the Markov-switching mechanism into the Constant Conditional Correlation model (CCC). The proposed model allows us to capture the different dynamics in both the volatilities and correlations in different regimes. It is particularly useful in examining the contemporaneous relationship between the unobservable volatility and correlation processes. We apply our model to the stock market index paired with two bond market indexes.

Then in the second chapter, we estimate the risk premium for the average correlation in the cross-section of the US stock market. The average correlation is the cross-sectional average of the correlations between each pair of stocks in the stock market. We find there is a negative and statistically significant risk premium for the average correlation controlling for other factors such as Fama-French's SMB and HML as well as the liquidity factor and momentum factor.

The third chapter focuses on the risk-return relation using a set of variance-related risk measures. We combine two lines of literature both of which find significant forecasting power of some risk measures for stock market returns: variance risk premium literature and studies on average variance-correlation decomposition .We illustrate how these two approaches can be related to each other, and empirically evaluate the relative importance of two approaches.

Acknowledgements

I would like to thank my doctoral supervisor Dr. Andrew Patton. My work was funded by the Dorothy Hodgkins Postgraduate Awards (DHPA).

Contents

Introduction

The market risk of financial assets is generally measured by the second moment of their returns. For a portfolio with multiple assets, its risk is jointly determined by the variances of individual assets and the correlations between each pair of assets. For an investor with a typical risk-averse utility, he will optimize his asset holdings taking into account of the correlations and variances of all the assets in his portfolio. This thesis is an empirical study on the econometric modeling and empirical pricing of the volatility and correlation in financial markets.

In chapter 1, we study the correlation and volatilities of the bond and stock markets in a regime-switching bivariate GARCH model. There are a few early attempts of incorporating the Markov-switching (MS) mechanism into GARCH-type models, such as Cai (1994), Hamilton and Susmel (1994), Gray (1996), and Haas, Mittnik and Paolella (2004). The only literature combining GARCH with regime-switching in a multivariate context is Hass and Mittnik (2007). In this paper, we extend the univariate Markov-Switching GARCH of Haas, Mittnik and Paolella (2004) into a bivariate MS-GARCH model with Conditional Constant Correlation (CCC) specification within each regime, though the correlation may change across regimes. Our model allows separate state variable governing each of the three processes: bond volatility, stock volatility and bond-stock correlation. This specification allows us to study both the intertemporal and contemporaneous relationships among the tree latent variables, which is not feasible in the traditional GARCH setup. Our main findings include: A separate state variable for the bond-stock correlation is needed while the two volatility processes could largely share a common state variable. The "low-to-high" switching in stock volatility is more likely to be associated with the "high-to-low" switching in bond-stock correlation while the "low-to-high" switching in bond volatility is likely to be associated with the "low-to-high" switching in correlation. The bond-stock correlation is significantly lower when the stock market volatility is in its high regime, but higher when the bond volatility is in its high regime.

Then in Chapter 2, we estimate the risk premium for the correlation risk in the US equity market. The risk premium for the variance of market index in equity market has been studied By Harvey and Siddique (2000), and more recently by Ang, Hodrick, Xing and Zhang (2006). Using the average correlation instead of the variance of market index as the aggregate risk measure, this project is motivated by the latest empirical finding that the cross-sectional average correlation and average variance might have very different pricing implications in the equity market. For example, Pollet and Wilson (2009) show that the average variance has negative correlation with future market returns while the average correlation positively predicts future market returns. By studying the option data, Driessen, Maenhout and Vilkov (2009) indirectly show that the (average) correlation risk is priced while the (average) variance risk is not. Recognizing that the average correlation might be a better measure for the aggregate market risk, we are of the first in studying the pricing implication of the average correlation in the cross-section of the equity market. We find that the estimate of correlation risk premium is significantly negative after controlling for the Fama-French.s SMB and HML factors as well as the liquidity and momentum factors. The annual return spread between the high correlation-beta stock portfolio and the low correlation-beta stock portfolio varies between -2% and -6% controlling for other risk factors.

Finally, in Chapter 3 we extensively examine the risk-return relation by employing a set of variance-related risk measures, which are recently found important in forecasting future market returns. These risk measures are mainly developed in two separate lines of literature. Among the first line of literature, Pollet and Wilson (2009) decompose the market variance into the average correlation and average variance, and find the two components can positively and negatively forecast future market returns respectively. The second line of literature, represented by Bollerslev, Tauchen and Zhou (2009), finds that the variance risk premium, defined as the difference between implied variance and realized variance, can significantly forecast future market returns. We illustrate that these two approaches can be related to each other and empirically evaluate the relative importance of the two approaches. Our evidence indicates that two components of variance risk premium (implied market variance and realized market variance) contain very similar information to that of the average correlation and average variance respectively. A combination of the average variance and implied market variance provides the best forecast of future stock market returns, beating the performance of variance risk premium or the average correlation-variance combination, and stays significant after controlling for traditional risk factors. Our findings indicate that the variance-correlation decomposition can be the potential driving force of the forecastability of the variance risk premium.

Regime Switching in Volatilities and Correlation between Stock and Bond markets*

Runquan Chen

London School of Economics

First version: 29 Nov, 2007.

This version: 1 May, 2009

Abstract

This paper studies the correlation and volatilities of the bond and stock markets in a regime-switching bivariate GARCH model. We extend the univariate Markov-Switching GARCH of Haas, Mittnik and Paolella (2004) into a bivariate Markov-switching GARCH model with Constant Conditional Correlation (CCC) specification within each regime, though the correlation may change across regimes. Our model allows separate state variable governing each of the three processes: bond volatility, stock volatility and bond-stock correlation. We find that a separate state variable for the correlation is needed while the two volatility processes could largely share a common state variable, especially for the 10-year bond paired with S&P500. The "low-to-high" switching in stock volatility is more likely to be associated with the "high-to-low" switching in correlation while the "low-to-high" switching in bond volatility is likely to be associated with the "low-to-high" switching in correlation. The bond-stock correlation is significantly lower when the stock market volatility is in the high regime, but higher when the bond volatility is in its high regime.

Keywords: Regime-Switching, GARCH, DCC, CCC, Bond-Stock Market Correlation

*Helpful comments and suggestions were received from my supervisor Andrew Patton, Markus Haas, and seminar participants at FMG seminar. Contact address:Department of Finance, London School of Economics. Houghton Street, London WC2A 2AE, UK. R.chen1@lse.ac.uk

1 Introduction

The correlation between bond and stock markets plays an important role in asset allocation as well as risk management. In tranquil time, investors would choose to invest more in equity markets to seek higher returns while they might "flee" to bond markets in turbulent market condition. So accurate modeling of the bond-stock correlation can provide investors with better diversification or hedging benefit. The most common econometric approaches in modeling correlations of multiple assets are the multivariate versions of the general autoregressive conditional heteroskedasticity (GARCH) type models of Engle (1982) and Bollerslev (1986)[1]. The GARCH type models have a fixed persistence level for the covariance process throughout the whole sample. However, there is evidence that volatility is less auto-correlated and has larger response to a shock when the volatility is in a higher level. For example, Hamilton and Susmel (1994) distinguish a low-, moderate-, and high-volatility regime in weekly stock returns, with the high-volatility regime being associated with economic recessions. However, very few papers have studied regime-switching in correlation. Because of the hedging relation between the bond and stock markets, bond-stock returns might have distinct relation in tranquil and turbulent market conditions. So allowing regime-switching in both volatility and correlation might provide better insight into the dynamic properties of the comovement of the stock and bond markets. Another motivation for using regime-switching GARCH model is that forecast errors are much more costly in high-correlation state than in low-correlation state for a risk averse investor, which is shown by Engle and Collacito (2006). Regime-switching would be better in capture extreme swings in correlation.

In this paper, we investigate the bond-stock correlation in a regime-switching bivariate GARCH model that has separate state variable for each of the three latent processes: bond volatility, stock volatility and bond-stock correlation. The model allows us to study the intertemporal and contemporary relation among the three state variables, such as the correlations in different market conditions characterized by the volatilities of the two markets. In the literature, even not limited to the bond-stock context, there is no study on these kinds of effects. In a study of international stock markets, Haas and Mittnik (2007) estimate a diagonal regime-switching GARCH model[2]. In their specification, all the individual variance and covariance processes share the same latent Markov

[1]For a survey of multivariate GARCH, see Bauwens, Laurent and Rombouts (2006).

[2]The actually sepcification in their paper is BEKK model of Engle and Kroner (1995), which guarantees positive-definetiveness of the covariance matrix.

state variable. Even though it can nest a similar model to ours by increasing the number of states in their model, it is very difficult to recover the regime-switching parameters of the correlation from their model. Regime-switching in covariance could be caused by switching either in variance or in correlation. Our model is able to separate these two effects by allowing a separate latent state variable for each latent process.

Using the returns from a one-year bond, ten-year bond, and the S&P500 index, we study the bond-stock correlations for bonds with different maturities. Our main findings include: First, the contemporary state correlations indicate that a separate state variable for the bond-stock correlation is needed while the two volatility processes could largely share a common state variable, especially for the 10-year bond paired with S&P500. Second, the "low-to-high" switching in stock volatility is more likely to be associated with the "high-to-low" switching in correlation while the "low-to-high" switching in bond volatility is likely to be associated with the "low-to-high" switching in correlation. As a result, the expected bond-stock correlation conditional on stock's high volatility state is significant lower than that conditional on stock's low volatility state. However, the results are opposite for those conditional on bond's volatility states. Since shocks to the bond price must be mainly related shocks to the discount factor, which will move the bond and stock prices in the same direction, the bond-stock correlation will increase as a result. However, when there are shocks to the stock price, they should be mainly related to the cash flow news, which affect the bond and stock prices largely in the opposite direction[3]. As a result, the bond-stock correlation will decrease. Finally, we find that when the bond market is in its high volatility state and the stock market is in its low volatility state, the estimates of bond-stock correlation in both high and low correlation-states are non-negative. But when both the bond and stock markets are in high volatility state, the bond-stock correlation has the highest correlation estimate at its high correlation-state and almost lowest correlation estimate at its low correlation-state. This might be attributed to the relative impacts of shocks to the cash flow and shocks to the discount factor on the pricing of bonds and stocks at different stages of the business cycle. According to the findings of Boyd, Hu and Jagannathan (2005), stocks are dominated by the cash flow effect during recessions while respond mainly to the discount rate news during expansions. As a result, the bond-stock correlation should

[3]Cash flow news ultimately will also affect discount factor indirectly. A positve cash flow news will increase both the growth expectation and the discount factor. The increasing growth expecation and discount factor work in opposite way on the price of stock. When the former dominates the latter, the bond and stock move in opposite way.

be positive during expansions and negative during recessions, which is also consistent with the findings of Anderson, Bollerslev, Diebold, and Vega (2007). However, we also find large swings in the bond-stock correlation between positive and negative values after 2003, which can not be explained by business cycles, and possibly can be driven by the time-varying equity premium.

The paper is laid out as follows: Section 2 presents a review of the recent literature on bond-stock correlation and regime-switching GARCH, while section 3 covers the econometric methodology employed in this paper. In section 4, the data and estimation method is explained while some comparative quantities are also introduced. Section 5 presents the empirical results and section 6 concludes and discusses areas for further research.

2 Literature Review

The main econometric model in this paper is a multivariate extension of univariate Markov-switching (MS) GARCH model of Haas, Mittnik and Paolella (2004). The basic idea of their approach is to assume there are several parallel GARCH processes, and the volatility is switching among these processes. Haas and Mittnik (2007) also propose a multivariate extension of the model. Their extension models the covariance matrix directly, which is governed by a single regime-switching state variable. In our simple bivariate context, we are able to assume both volatilities and correlation have their own state variables. This generalization allows us to answer more interesting questions about the bond-stock correlation. Pelletier (2006) proposes a regime-switching constant correlation model and claim to have better fit than Dynamic Conditional Correlation (DCC) model of Engle (2002). However he does not apply the model to study the bond-stock correlations. We are similar in the correlation part of the model but we further allow regime-switching in each volatility series.

Regime-switching modeling of volatility is relatively new in literature. As mentioned above, GARCH type models are the main approaches in modeling volatility of financial asset returns. However, it has been argued that the close-to-unity of the persistence parameter estimate in many GARCH models could be due to the regime-switching in volatility. For example, Diebold and Inoue (2001) analytically show that non-persistent series with stochastic regime switching can appear to have strong persistence, or long memory. In the volatility context, Mikosch and Starica (2004) show that deterministic shifts in the unconditional variance do indeed drive the estimate of persistence parameter toward unity. A natural step forward is to combine a GARCH type model with Markov-switching mechanism, which has been widely adopted in economics since Hamilton (1989). Cai (1994) and Hamilton and Susmel (1994) are among the first to combine an ARCH model with Markov-switching. The reason why they restrict their attention to ARCH structures within each regime rather than a GARCH-type structure is due to the problem of path-dependence of GARCH, which makes maximum likelihood estimation infeasible (to be detailed later). The first attempt to combine the GARCH with Markov-switching is Gray (1996), which is essentially an approximation. Haas, Mittnik and Paolella (2004) propose an approach where regime-switching is both on parameters and the latent process, and so it is able to avoid the path-dependence problem while keeping the GARCH structure. In this paper, We adopt their approach and make a multivariate extension for our study of bond-stock correlations.

On bond-stock correlations, the past literature is mainly using the GARCH-type models. Cappiello, Engle and Sheppard (2006) propose an extension of the DCC model to allow for different structure parameters for different correlation pairs. They estimate the model using weekly returns on the FTSE All-World equity indices for 21 countries and government bond indices for 13 countries, and find a significant asymmetric affect in the conditional correlation between stock and bond return, and that the correlations tend to decrease following an increase in stock volatility or a negative stock return. De Goeij and Marquering (2004) employ a similar approach to Cappiello, *et al.* (2006). They estimate a diagonal VECH extension of Glosten, Jagannathan and Runkle (1993) model using daily returns on a short-term bond, a long-term bond , and the returns on the S&P 500 and NASDAQ indexes. They find strong evidence of time-varying conditional covariance between stock and bond market returns. Their results indicate that not only variances, but also covariances respond asymmetrically to return shocks. But they do not study the implied correlation dynamics from the VECH framework. Gulko (2002) studies the change in correlation between stock and bond market around period of market crashes. He finds that stock and bond correlations change from weakly positive in normal time to strongly negative during stock market crashes, which means treasure bonds could act as a hedging vehicle against stock market crashes. Li (2000) studies the impacts of various macroeconomic factors on stock and bond correlations. He first uses daily data to construct a non-parametric estimate of the correlation for a given month, the so-called "realized correlation", then regresses the realized correlation on various macro factors. He finds that long-term expected inflation and real interest rate has the largest (positive) impact on the stock-bond correlation. Baur and Lucey (2006) study the daily stock-bond correlations of seven European countries. They first estimate the DCC model of Engle (2002) to obtain a time series of estimated conditional correlations between each stock and bond return series, and then regress these estimated conditional correlations on some factors to study the sources of variation in correlations. Although suffering from econometric problems by using estimated correlations in the second-stage regression, they find that the correlation for US markets are about 0.5 at the normal time and -0.4 at the 1997 crisis. Finally, the recent work of Anderson, Bollerslev, Diebold, and Vega (2007) find that during the expansion the stock-bond correlations are positive albeit small, whereas during the contraction they are negative and large[4], which is largely consistent with our results.

3 Econometric Methodology

3.1 Univariate Markov-switching GARCH (MS-GARCH)

To model the regime-switching behavior in the univariate volatility process, we start with the *standard* two-state Markov-switching GARCH(1,1) model:

$$\begin{aligned} R_t &= \mu_t + \epsilon_t \\ \epsilon_t &\sim N(0, V_t) \\ V_t &= \omega_{s(t)} + \beta_{s(t)} V_{t-1} + \alpha_{s(t)} \epsilon_{t-1}^2 \\ P(s(t) &= i | s(t-1) = j) = P_{ij} \text{ with } i = 0, 1 \text{ and } j = 0, 1 \end{aligned} \tag{1}$$

This model allows the volatility process to have different dynamics with different persistence and level parameters in different regimes. And the latent Markov state variable $s(t)$ determines which regime prevails at each point in time. Model (1) look apparently simple and intuitively straightforward. But because the recursive structure of GARCH, current conditional variance is decided by the complete history of the state variable. So the (conditional) likelihood of observation at time t, $f_t(R_t|\theta; R_{[t-1,0]}, s(0))$, needs to be computed from $f_t(R_t, s(t)..., s(1)|\theta; R_{[t-1,0]}, s(0))$

[4]They calculated the unconditional correlations separately for the expansion period from July 1998 through February 2001 and the contraction period from March 2001 through December 2002. And determination of contraction and expansion could be found in their paper.

through integration on $s(1)$ up to $s(t)$. So as the number of observations increases, the integration dimension increases, which makes Maximum likelihood estimation of this Markov-switching GARCH essentially infeasible in practice. To avoid the path-dependence problem while maintaining the GARCH feature of modeling, we adopt an approach proposed by Haas Mittnik and Paolella (2004). The idea, for a two-state model, is to model two parallel volatility processes, each of which has the GARCH dynamics. The latent Markov state variable determines which *process* is selected for each time. Its trick is switching on both parameters and processes while the standard MS model only switches on parameters. Formally, it is as follows:

$$\begin{aligned} R_t &= \mu_t + \epsilon_t \\ \epsilon_t &\sim N(0, V_{s(t),t}) \\ V_{i,t} &= \omega_i + \beta_i V_{i,t-1} + \alpha_i \epsilon_{t-1}^2 \quad i = 0, 1 \\ P(s(t) &= i|s(t-1) = j) = P_{ij} \quad \text{with } i = 0, 1 \text{ and } j = 0, 1 \end{aligned} \tag{2}$$

The benefit of this approach is to be able to avoid the path-dependence problem so we are able to compute the likelihood function without integration over the whole path of the volatility process. As suggested in Haas, Mittnik and Paolella (2004), the interpretation of β_i and α_i are still the persistence parameters in each regime. But the unconditional level in each regime is not only characterized by the three parameters of the GARCH formula. The transition probability P_{ij}, which determines how often the process is in a regime, also determines the expected volatility level of a regime. The formula for computing the unconditional level of volatility is in the Appendix. Another property of this model is that one of the regimes could be non-stationary, i.e. $\beta_i + \alpha_i >= 1$, but the whole system could still be stationary if the transition matrix together with all the persistence parameters satisfies a certain condition, which is also detailed in the Appendix.

3.2 The main model: Markov-switching CCC-GARCH(MSCCC)

A direct multivariate generalization of model (2) is illustrated in Haas and Mittnik (2007). They base the regime-switching on the BEKK model of Engle and Kroner (1995) model. But their setup is not well-suited for our research purpose. Since volatility may have different dynamics to correlation, we would like to have an explicit process for the correlation, as well as a corresponding state variable. So we adopt the Constant Conditional Correlation (CCC) model of Bollerslev (1990) as the base model, and allow both the individual volatility processes and correlation process to switch among different states. While using the DCC Model as the base model would be more general, we will explain later why we use CCC as base model.

To be specific, we model the covariance of the bond and stock markets as three components: stock market volatility, V_t^s, bond market volatility, V_t^b, and the correlation between two markets, C_t. Let S_t^s be a two-state variable governing the regime-switching of V_t^s, and a two-state variable S_t^b governing that of V_t^b. Then for each of the four combinations of S_t^s and S_t^b (hereinafter called "combined states"), we further split it into two "sub-states" which correspond to high and low correlation regimes within each combined state. Finally, to represent the regime-switching of the whole covariance matrix, we define a general state variable S_t with eight states to govern the regime-switching of the covariance matrix. The GARCH parameters of stock volatility are the same in states $\{1;2;3;4\}$ or $\{5;6;7;8\}$, the bond volatility has the same set of GARCH parameters in $\{1;2;5;6\}$ or in $\{3;4;7;8\}$ .And we do not restrict the low and high correlation level in those four combined states to be equal. So there are eight possible values for correlation. Table 1 provides detailed definition on S_t. This specification allows us to investigate how the regime-switching of two volatility processes responds to each other, and between what levels the correlations would switch within each of the four combined sates.

With the definition of S_t, we formally define our general bivariate Markov-Switching Constant Conditional Correlation model (MSCCC). Let $\mathbf{R}_t \equiv \begin{bmatrix} R_t^s & R_t^b \end{bmatrix}^T$ denote the returns of stock index and bond, either 1-year bond or 10-year bond, and $\boldsymbol{\mu}_t \equiv \begin{bmatrix} \mu_t^s & \mu_t^b \end{bmatrix}^T$ be conditional mean vector, $\boldsymbol{\epsilon}_t \equiv \begin{bmatrix} \epsilon_t^s & \epsilon_t^b \end{bmatrix}^T$ be the residual vector. Then the main model is as follows:

$$
\begin{aligned}
&\mathbf{R}_t = \boldsymbol{\mu}_t + \boldsymbol{\epsilon}_t \\
&\boldsymbol{\epsilon}_t \sim N(\mathbf{0}, \mathbf{V}_{s(t),t}) \\
\text{with } \mathbf{0} = &\begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ and } \mathbf{V}_{s(t),t} = \begin{bmatrix} V^s_{s(t),t} & C_{s(t),t}\sqrt{V^s_{s(t),t}V^b_{s(t),t}} \\ C_{s(t),t}\sqrt{V^s_{s(t),t}V^b_{s(t),t}} & V^b_{s(t),t} \end{bmatrix} \\
&V^k_{i,t} = \omega^k_i + \beta^k_i V^k_{i,t-1} + \alpha^k_i(\epsilon^k_{t-1})^2 \quad i = 1,2...8 \text{ and } k = s(stock), b(bond) \\
&C_{i,t} = C_i \quad i = 1,,2...8 \\
P(s(t) = i | s(t-1) = j) &= P_{ij} \quad \text{with } \; i \text{ and } \mathrm{j} = 1, 2\cdots 8
\end{aligned} \tag{3}
$$

Model (3) is different from Haas and Mittnik (2007) mainly in two ways. First, our base model is the Constant Conditional Correlation model (CCC), which means that we explicitly model the correlation. This provides us direct observation of how correlation interacts with the volatilities in both markets. As mentioned earlier, Haas and Mittnik (2007) use BEKK as their base model. Although their approach is very flexible in that it allows any interaction of the squared terms and cross product, it is very difficult to recover the regime-switching parameters of the correlation from their model. Their model could only tell how the covariance switches among different regimes, instead of the correlation. The covariance could appear to have a jump because of either one or both of the volatility processes have a jump even when the correlation remains constant. Our setup enables us to disentangle these two effects neatly. Secondly, our setup allows us to do two-stage maximum likelihood estimation, which is useful in higher dimension problem or robustness for complex model estimation. The justifications for the two-stage estimation of the model are similar to those of CCC or DCC[5]. During the estimation, for identification purpose we restrict the unconditional stock-volatility level in $\{1;2;3;4\}$ to be lower than that in $\{5;6;7;8\}$, and the bond-volatility level in $\{1;2;5;6\}$ lower than that in $\{3;4;7;8\}$. Then in each of the four "combined states", the correlation level in states labelled in smaller number is higher than that in state labeled by larger number (See Table 1 for full details).

[5]We have not formally derived the stationarity conditions for the joint model. But since the correlation part of the model does not affect stationarity, the stationarity conditions for the joint model are similar to the conditions for the univariate model in Haas, Mittnik and Paolella (2004). Basically if both of the non-stationary states of the two volatility processes are transient and have high probability to switch to the stationary states, then the joint model is stationary.

3.3 Discussion on model specification

A common concern about regime-switching models is the number of regimes for the models. By allowing each latent process to be governed by its own state variable, our joint state variable S_t needs to have eight states to characterize the joint model. Although as mentioned before, our model is much more restricted than the general 8-state model without restrictions since the GARCH parameters in our model are only switching between two sets of values. However, it is still natural to consider smaller models by shrinking the general one, which is a way of assessing the importance of having different state variable for different process. We could restrict S_t in various ways, which would result in different restricted models. For example, the simplest restriction is to let the volatility processes of both markets and the correlation share the same two-regime state variable. Alternatively, we could allow two volatility processes share the same state variable, but the correlation has a separate state variable. The full set of specifications we considered in this paper are detailed in Table 2.

On the choice of the "base model", a more general framework would be building the regime-switching upon the DCC, which allows for time-varying correlation within each correlation regime. But because DCC has a volatility-standardized term in its recursive formula, combining DCC with the regime-switching GARCH would hugely complicate the already complicated model structure. And as we will see in the result section below, three out of the four combined states are only visited in very low frequency. Adding 16 extra DCC parameters would not likely produce a great improvement. And we believe eight correlation states should capture well the variation in the bond-stock correlation. As results from Pelletier (2006) suggest, switching constant correlation is adequate to capture the dependence in the correlation.

4 Estimation and Comparative Statistics

4.1 Data and estimation

We use daily returns from both the stock and bond markets to estimate the model. Stock market returns are computed as daily returns on S&P500 index from WRDS. For the bond market, two series of daily returns are computed from bond yields on 10-year and 1-year US government bond, which are available from federal reserve bank in Chicago. The sample period is from January 5, 1986 to December 29, 2006. Table 3 presents the statistics summary of the two return series.

Before estimation of the volatility models, we filter each return series by an ARMA(p,q) model, with p and q chosen to have the best fit according to Bayesian Information Criterion (BIC). The optimal filtering orders are also presented in Table 3. Because bond returns are computed from bond yield, which itself is computed by some interpolations of "rolling bond" returns, there are some autocorrelations in the bond returns, which does not necessarily mean there are any arbitrage opportunities in the market.

After we estimate the conditional mean by an ARMA model, we can estimate the main Markov-Switching volatility model (3). The estimation are done in a two-step procedure. First, for each of the two time series, a univariate two-regime MS-GARCH model (2) is estimated. Then with ϖ_k^i, β_k^i and α_k^i fixed at the estimates from the previous stage, the rest of the parameters of model (3) are estimated.

With 6 GARCH parameters for each volatility process, 8 correlations and $7 * 8 = 56$ transition probabilities, the general model has 76 free parameters. The two-step procedure could separate the problem in a lower dimension sub-problem, which could provide robustness in estimation. However there are still 64 free parameters in the second stage estimation, and the numerical standard errors of the estimates are difficult to compute. To get a more robust standard error estimate of the general transition matrix, we further fix those transition probabilities with second-stage estimate less than 0.0001 to be zero. This results in a estimation problem with only half of the parameters. The specific maximum likelihood estimation algorithm is standard. It combines a Markov-switching estimation algorithm with GARCH likelihood calculation. The details can be found in the Appendix.

4.2 Comparative statistics

Once we finish the estimation of the joint model, we analyze the intertemporal and contemporary relationships among the three state variables and three latent processes by the following comparative statistics. All of them are mainly derived from the general transition matrix of the main model, and their calculations are detailed in Appendix.

Conditional transition probabilities

As ways of analyzing the information contained in the transition matrix, we compute two types of "conditional transition probability". The first one is $t-$ *conditional transition probability*, which is conditional on time t information. It is the probability of one state variable's regime-switching from t to $t+1$ given the state of another state variable at time t. For example, $\Pr(S^b_{t+1} = high|S^b_t = low; S^s_t = high)$ is the transition probability of bond volatility switching from low to high regimes conditional on the current stock volatility state being in high regime. So this kind of conditional transition probabilities can reveal the forecasting benefit of joint modeling. If this transition probabilities are significant different from the marginal transition probabilities implied by the transition matrix, then by joint modelling, we have better forecasting power for the future state of the considered variable. Due to the focus of the paper, we only compute the $t-$conditional transition probabilities of the bond-stock correlation conditional on volatility states in the bond and stock markets.

The second one is $t+1-$*conditional transition probability*, which is conditional on information of up to time $t+1$. It is the probability of one state variable's regime-switching from t to $t+1$ given that there is a regime-switching from t to $t+1$ in another state variable. For example, $\Pr(S^b_{t+1} = high|S^s_{t+1} = high; \{S^s_t = low\ ;\ S^b_t = low\})$ is the transition probability of bond volatility switching from low to high regime conditional on stock volatility's switching from low to high regimes. These conditional transition probabilities are ways of measuring how two considered state variables jump together in their own directions. While the $t-$ *conditionals* are on intertemporal relationships, $t+1-$ *conditionals* are evaluating the contemporary relationships among the state variables.

Conditional expected correlations

Another type of statistics we are interested in is the expected correlation conditional on the contemporary volatility state in either market. For example, $E(C_t|S^s_t = low)$ is the expected correlation given a low volatility state in stock market. In the result section, we further compute the expected correlations conditional on regimes of both volatility state variables, such as $E(C_t|S^s_t = low; S^b_t = low)$. So these quantities can tell us the bond-stock correlation given all four "combined states", and such have important implications for portfolio diversification. Similar relation between correlation and volatility can not be measured in the traditional GARCH-type models.

As we can see, all above "conditional quantities" can be derived only when we allow separate state variable for each of the three latent processes. So we can not derive similar quantities from the model of Haas and Mittnik (2007). And the empirical results of the paper will be mainly focusing on these aspects of the model.

5 Empirical Results

In this section, we first present the estimation results on the univariate model. Second, the results on the joint model are demonstrated, mainly around the "conditional quantities" introduced previously. Third, we compare the filtered bond-stock correlation from our model with that estimated from the DCC model. Finally, we discuss the results on goodness of fit for various restricted model.

5.1 Results on univariate model estimations

The estimation results of the univariate MS-GARCH model (2) using S&P500 daily returns are presented in Table 4, while Table 5 and 6 show the results for 1-year bond and 10-year bond returns. The parameter estimates of the conditional mean are not shown, which will also applies to other tables of the results. The same model has been estimated by Mittnik and Paolella (2004) on foreign exchange returns. We find similar results on equity return data. First, there is a significant difference in the persistence levels of the GARCH processes of the two regimes. When the volatility process is in the low-volatility regime, the GARCH coefficient β is larger, but far away from unity, which is usually the case when one fits a simple GARCH model to daily stock return data. The ARCH coefficient α is much smaller than that of the high-volatility regime, less than one-tenth of the latter[6]. So in tranquil periods, the volatility remains very stable, and the shock has very small effect on the conditional variance of the next day. But in volatile periods, the impact of today's shock has much larger effect on next day's conditional variance. The total persistence level of each process are measured by the sum of α and β. We could see in the table that in the low-volatility regime, the GARCH process has persistence level smaller than unit, which means it is stationary. But as found in Mittnik and Paolella (2004), the volatile regime has larger-than-unity persistence level, so the GARCH process of the high-volatility regime itself is not stationary. The stationarity

[6] More precisely, the comparable quantity should be the standardized impact coefficient, which is the ARCH coefficient devided by the t-1 conditional variance of the regime. Considering the ratio of unconditional level of the high-volatility regime to that of low-volatility regime is much smaller than the ratio of their ARCH coefficients, the standardized impact of shock in high-volatility regime is still much higher.

of the whole system is determined jointly by the transition matrix and the two sets of persistence parameters in both regimes, which is summarized by the largest eigenvalue of the matrix M, which is explained in Appendix. We can see that all three univariate models are stationary with $eig(M)$ less than one. The stationary probability of low-volatility regime is higher than those found in exchange return, which is mostly less than 0.8. The expected level of volatility in high-volatility regime is about 4 times the expected level of low-volatility regime.

Table 5 and 6 report the estimation results of univariate MS-GARCH on the two bond return series. The difference between the two regimes is similar to that found in stock index return. Higher level of persistence, smaller β estimate and larger α estimate are associated with high-volatility regime The persistence levels in the high-volatility regime of both bond series are larger than unity as well. For 10-year bond returns the difference in β estimates is not significant. The difference in β estimates for one-year bond is small as well, compared to that found in stock market. So for bond market, the volatility dynamics of the two regimes mainly distinguish from each other in their response to shocks. For one-year bond returns, the ratio of expected volatility in the high regime to the low regime is much higher than that of the stock index return or ten-year bond return. Thus the one-year bond would experience much larger jump upon regime-switching than stock return, while 10-year bond behaves more similar to stock return.

5.2 Results on estimations of the joint model

We have studied the difference of the bond and stock markets in the univariate MS-GARCH model in last section. Now we discuss their relation in the joint model. The joint model allows us to see how the two state variables correspond to each other, and how the bond-stock correlation behaves within the 4 "combined states". First, we present the main estimation results of model (3) using two pairs of stock-bond returns in Table 7 and Table 8. The first two blocks in the tables are the regime-switching GARCH parameter estimates for the stock and bond series in the joint estimation. The third block reports the constant correlation levels in each of the eight regimes. The last block is the transition matrix estimate, together with the stationary probabilities of each regime. In both tables, the estimates of GARCH parameters are similar to those from the univariate estimations, but with smaller standard deviations due to efficiency increase by using more data. And here we focus on the results on the joint part of the model. Then in Table 9 to Table 12, some derived statistics are presented to better understand the intertemporal and contemporary relationships among the three latent processes. Finally, we have some discussions on the results of the bond-stock correlations in a simple discounted cash-flow model for the bond and stock prices.

5.2.1 Stationary probabilities and correlations within the combined states

First, we can indirectly see how likely the two volatility processes switch together by comparing the stationary probabilities of the four "combined states". These can be obtained simply by summing up the stationary probabilities of S_t states two by two. For the one-year bond pair, in about 76% of time both the stock and bond return are in low-volatility regime, and in about 3.6% of time both of them are in high-volatility regime. So there is about 20% of time that they are in different volatility regimes. For the 10-year bond pair, the proportions of time that bond and stock volatilities are in the same regime is similar to the one-year bond case. The main difference is that there is a much larger percentage, about 8%, of time when both bond and stock are in high-volatility regimes. This feature of ten-year bonds is a disadvantage for diversification. Large stationary probability of two volatilities in the same regime for both pairs indicates that two volatility state variables correspond to each other to a certain extent, which will be re-confirmed by the following results. We can also see the smoothed probabilities of each state across time in Figure 1 and 2[7].

Next, we compare the levels of correlation within the four "combined states". The two correlation levels in the two "sub-states" of all four "combined states" are all significantly different. These results indirectly suggest a separate state variable is needed for both stock-bond correlation processes. Taking into account of estimation errors, only in the "low-stock-high-bond" state are both of the two correlation estimates non-negative. And in the "high-stock-high-bond" state, the high and low correlation estimates are the highest and lowest respectively among the eight correlation estimates for the one-year bond paired with stock. Similar results are found for the 10-year paired with the stock. We will further discuss these findings in more details later. Finally, for the one-year bond pair, taking into account of standard errors, we could see there are mainly four correlation regimes: one extremely high at about 0.8, one negative at -0.4, and the other two in between are around 0.35 and 0.05. A similar state reduction for correlation could be done for ten-year bond pair, and we examine this formally in section 5.5.

[7] The "smoothed" state probabilities of each state across time in the whole sample is computed as in Kim (1994), which is recited in Appendix.

5.2.2 Conditional transition probabilities

Results on the $t-$conditional transition probabilities We study the intertemporal relations between correlation state variable and the volatility state variables in two market through the t – *conditional transition probabilities* reported in Table 9. Comparing these numbers with the "unconditional transition probabilities" in Table 10 can reveal the forecasting benefit of allowing correlation to have separate state variable. First, we look at the transition probabilities of $C_t < 0$ to $C_{t+1} \geqslant 0$ given different volatility states in different markets. The correlation transition probability is higher when conditional on high volatility state in either bond or stock than conditional on low volatility state. And the difference is more significant for the one-year bond case. Comparing the information contained in different markets, the correlation transition probabilities conditional on low volatility states in two markets are not significant different for both pairs. However, correlation transition probability conditional on high volatility states in one-year bond is larger than that conditional on high volatility state in stock market. And for 10-year bond paired with stock market, conditioning on either market produces similar correlation transition probabilities. So it seems that only correlation between the 1-year bond and stock has very different conditional "low to high" transition probabilities for conditioning on different markets or different volatility states, and the high volatility state of 1-year bond is mostly likely to be followed by a jump up in correlation.

Next, we turn to the transition probabilities of $C_t \geqslant 0$ to $C_{t+1} < 0$ given various volatility states. The correlation transition probability conditional on bond's high volatility state is significant higher than that conditional on bond's low volatility state for both pairs. But the two volatility states in stock market has similar information on the correlation transition probability, which is different from above "high-to-low" case. Similar to the "high-to-low" case, there is no significant difference in conditioning on the low volatility state of either stock or bond market for both pairs, and conditioning on high volatility states in both bond markets has higher correlation transition probabilities than conditioning on high volatility state in stock market.

To summarize, high volatility states of bond markets, especially 1-year bond, imply higher t-conditional transition probabilities of bond-stock correlation for both "low-to-high" and "high-to-low" cases. And the differences between these numbers and those "unconditional transition probabilities" in Table 10 are statistically significant. But there is no significant difference in conditioning on today's high and low volatility states in stock market in terms of predicting regime-switching of the bond-stock correlation.

Results on the $t+1-$conditional transition probabilities The contemporary relations among the three state variables can be measured through the estimates of $t+1-$ *conditional transition probabilities* reported in Table 11. The first row of both panel is on how correlation switches conditional on a regime-switching in bond or stock volatility, which can again be compared with the unconditional ones in Table 10. For both bond pairs, when stock market volatility jumps from low to high, the transition probabilities of correlation switching from high to low is significantly higher than the unconditional ones while "high-to-low" correlation (conditional) transition probabilities are lower than the unconditional ones. However, conditioning on a "low-to-high" jump in bond volatility has the opposite results: the correlation (conditional) transition probability is higher, than the unconditional one for the "low-to-high" switching in correlation, and lower for "high-to-low" case. So the "low-to-high" switching in stock volatility is more likely to be associated with "high-to-low" switching in correlation while the "low-to-high" switching in bond volatility is likely to be associated with "low-to-high" switching in correlation. The second row of both panels evaluate how both of the volatility state variables switches conditioning on a regime-switching in bond-stock correlation, which can be compared with the "unconditional (or marginal) transition probabilities" in Table 4-6. The results are similar to those of first row since they inherit the main properties of the quantities in first row by definition.

The last row of both panels describes how likely the two volatility processes would jump in the same direction together. For ten-year bond, it tends to jump with stock in either direction with high probability. But for one-year bond, it has high probability to jump down with stock, but very low probability to jump up together. So the one-year bond does not always react the information from stock market. But if it does react, probably to extreme shock, it resolves the uncertainty together with stock market. Comparing these numbers to those in Table 4-6 also indicates that only the case of "low-to-high" conditioning in the 1-year bond pair does not provide significant more information than the marginal ones. So the two volatility state variables correspond to each other quite well, especially for the 10-year bond pair.

The contemporary relationship revealed in above results indicate that the correlation state variable has very different switching behavior from the volatility state variables while two volatility processes can largely share the same state variable, especially for the 10-year bond paired with

S&P500.

5.2.3 Conditional expected correlations

An important feature of the general model is that it is able to capture the contemporary relations among the correlation and volatilities in two markets. We quantify these relations through the conditional expected correlations which are reported in Table 12.

First, we compare the expected correlations conditional on a single volatility state variable, which are presented in the last row and column of both panels. For both pairs, the expected correlation conditional on stock's high volatility state is significant lower than that conditional on stock's low volatility state, although the difference is significant only for the 1-year bond pair. But the results are opposite for those conditional on bond volatility states. Judging from the differences in conditioning on high and low volatility-states, it seems that stock market has larger contemporary impact on the bond-stock correlation than bond market. A related observation is that only the expected correlation between 1-year bond and stock conditional on the high volatility state in stock market has a negative point estimate. This reflects that during "flight to safety" period, investors transfer money from stock market more to bonds with shorter maturities than to bonds with longer maturities. This is again due to the fact that long-term bond is more like a stock.

Table 12 also presents results on the "joint-conditional" expected correlations. These results reveal the expected correlations in four "combined states", which is basically to aggregate the eight correlation estimates from Table 7 and Table 8 into four correlation levels. For both pairs, the "high-stock-low-bond" state has the lowest expected correlation while the "high-bond-low-stock" state has the highest one. And in the most volatile market condition characterized as the "high-stock-high-bond" state, the expected correlation is surprisingly low, even lower than the peaceful "low-stock-low-bond" state. In the high stock volatility state, the difference in expected correlations is not significant between conditioning on the high and low volatility states of the bond market. But the difference is significant when the stock market is in low volatility state. In contrast, the difference in the expected correlation is more significant between conditioning on stock's high and low volatility states when the bond market is in the high volatility state, especially for the 1-year bond case. So when the stock market is very volatile, the bond-stock correlation is mainly dominated by the stock market, usually with low correlation level. And the effect of the stock volatility on the bond-stock correlation becomes more prominent when the bond market is also in volatile condition.

5.2.4 Discussions on the bond-stock correlations

Above empirical findings can be explained in a simple discounted cash-flow model for both the bond and stock prices. The bond price is the expected value of the discounted factor while the stock price is co-driven by the future cash flow and discount factor[8]. So the high bond-volatility state is when the expected discount rate has high volatility. But for the stock volatility, it comes from either one or both of the two shocks, cash-flow news and discount factor news. The recent findings of Boyd, Hu and Jagannathan (2005) show that the stock price is dominated by the cash flow effect during recessions while it mainly responds to discount rate news during expansions. Our results are consistent with their findings. In the "high-bond-low-stock" state, the shock must come form discount factor which will make the bond and stock prices move in same direction irrespective of business cycle. This is confirmed by the result in Table 7. Only in the "high-bond-low-stock" state can we have positive point estimates for both the high and low correlation states[9]. So when we average them across the whole sample, we have highest unconditional correlation in the "high-bond-low-stock" state as in Table 12.

However, for the "high-stock-high-bond" state, we have two different situations since the source of the stock volatility is dependent on the business cycle. During expansions, both the bond and stock prices are mainly driven by discount rate news. So we will have highest correlation during expansions; But during recessions, the stock price is mainly dominated by the cash flow effect, which works in the opposite direction for bond price as rising future cash flow will increase the discount rate and lower the bond price. As a result, we have lowest bond-stock correlation during recessions. This is confirmed by the regime correlation estimates in Table 7. And when we average the correlations across the whole sample, the correlation is close to zero in the "high-bond-high-stock" state as in Table 12. Results in the other two "combined states" can be explained similarly by the relative effect of shocks to discount factor and cash flow in the discounted cash-flow model.

[8] Different from discount factor for bond, the discount factor for stock is sum of equity premium and the risk-free rate. But for ease of illustration, we assume they are the same.

[9] Since 10-year bond have stock-like characteristics, we mainly focus on 1-year bond in expaining our findings in the discounted cash flow model.

If we further differentiate the discount factors for the bond and stock prices, then the time-varying equity premium will also help to explain the time-varying bond-stock correlations. This is especially relevant for explaining the bond-stock correlation in sample after 2003 when it frequently swings between large positive values and large negative values, which can not be explained by the business cycle. A structural model is needed to measure the effect of time-varying equity premium on the bond-stock correlation, which is out of the scope of this paper.

5.3 Estimated correlations from the MSCCC model

Although we assume constant correlation within each regime, the correlation estimated from our model has quite rich dynamics. Figure 3 and 4 provides the view of the correlation dynamic for both pair over the past 20 years. Both correlations appear to jump to a negative level when there is a crisis in equity market. As mentioned previously, since after 2003 negative correlations happen very frequently, irrespective of having a stock market crisis or not. We believe it might be because that the time-varying equity premium is becoming more important for the bond-stock correlation after 2003. As a comparison, we plot the MSCCC correlations together with correlations estimated from DCC. As shown in Figure 5.1 and 6.1, the main difference between DCC's correlations and the MSCCC correlations is that the MSCCC correlations reveal more extreme, both positive and negative, correlation level than the DCC. And forecast error in such situation are most costly as pointed out by Engle and Collacito (2006). As another view for comparison, we fit the DCC correlations by an 4-order polynomials of MSCCC correlations in Figure 5.2 and 6.2. We could see DCC correlation usually has downward bias for day with high smoothed correlation, and upward bias for day with low smoothed correlation. Although our model is not developed for forecasting, it does point out potential way of improvement for the traditional GARCH type dynamic correlation models, especially for modeling bond-stock correlation.

5.4 Results on restricted models

Although all above analysis is based on the general model, we also estimate some restricted models specified in Table 4 to compare their goodness of fit[10]. Table 13 reports the goodness-of-fit results

[10] We do not conduct formal tests for the number of regimes. As indicated in Hansen(1992) and McLachlam and Peel (2002), the standard likelihood ratio test is not valid for testing the number of regimes. And test on regime number still remains very difficult even in much simpler setups. A more recent study on regime number test is Cho

on all considered models. We rank all the model according to AIC and BIC. For one-year bond pair, the most general model MSCCC(2,2,8), is the best model according to AIC. But according to BIC, which penalize heavily extra parameters, MSCCC($\overline{2}$,$\overline{2}$,4) is the best model, in which two volatility process share the same Markov state variable but the correlation has a separate one. The same best models are identified under the two criterion for ten-year bond paired with S&P500. So generally according AIC, we do need three separate state variables for volatilities and correlation, which justifies our main model. According to BIC the most general model still rank second for the ten-year bond case, but very poorly for the one-year bond case. The MSCCC($\overline{2}$,$\overline{2}$,4) is always the best under BIC and the second under AIC. So while a separate state variable for correlation is very important, stock and bond volatilities could share the same state variable without too much loss in goodness of fit. This seems to reflect the information could be transmitted across two markets efficiently within a day. The good performance of MSCCC($\overline{2}$,$\overline{2}$,4) could also make it a good candidate as a multivariate regime-switching GARCH model in forecasting exercise since it is quite parsimonious, having only 28 parameters.

5.5 Diagnostic analysis of the standardized residuals

In this section, we carry out the ARCH Lagrange multiplier (LM) test proposed by Engle (1982) on the three residuals standardized by the conditional volatilities of the MSCCC model. As a naive comparison, the same test is implemented for residuals standardized by conditional volatilities of the DCC model. For all the tests, the number of lags is 5. The results are in Table 14. We can see that the our regime-switching model has the p-values smaller than those of the DCC model for almost all the lags for all three series. And only in the case of S&P 500 returns at the first lag, the test statistics is significant only at 10% level for the MSCCC model. In contrast, the DCC-standardized residuals have a significant statistics at 5% for the S&P return at the first lag. From these results, we conclude that the proposed model has good performance in filtering out the correlation in the return volatility.

and White(2007).

6 Concluding Remarks

To study the bond-stock correlation and its relation with volatilities in the two markets, we extend the univariate Markov-Switching GARCH of Haas Mittnik and Paolella (2004) into a bivariate MS-GARCH model with Conditional Constant Correlation (CCC) specification. Our specification allows a separate state variable governing each of the three processes: bond volatility, stock volatility and bond-stock correlation, which is different from other multivariate generalizations of their model. We estimate our model using two pairs of daily returns: S&P500 with one-year bond and S&P500 with ten-year bond, to study the difference in bond-stock correlation for different bond maturities.

From the univariate model estimation, we find both volatility processes switch between a stationary and a non-stationary state while the whole system is till stationary, similar to the findings in Haas Mittnik and Paolella (2004). By allowing different latent process to have separate Markov state variable, the results from the joint model estimation has the following main findings: First, we find that a separate state variable for the bond-stock correlation is needed while the two volatility processes could largely share a common state variable, especially for the 10-year bond paired with S&P500. Second, the "low-to-high" switching in stock volatility is more likely to be associated with the "high-to-low" switching in correlation while the "low-to-high" switching in bond volatility is likely to be associated with the "low-to-high" switching in correlation. As a result, we show that the expected bond-stock correlation conditional on stock's high volatility state is significant lower than that conditional on stock's low volatility state. But the results are opposite for those conditional on bond's volatility state. Finally, we find that when the bond market is in its high volatility state and the stock market is in its low volatility state, the estimates of bond-stock correlation in both high and low correlations states are non-negative. But when both bond and stock markets are in high volatility state, the bond-stock correlation has the highest correlation estimate at its high correlation-state and almost lowest correlation estimate at its low correlation-state. This might be attributed to the relative impacts of shocks to the cash flow and shocks to the discount factor on the pricing of bonds and stocks at different stages of business cycles. But we also find large swings in the bond-stock correlation between positive and negative values after 2003, which can not be explained by the business cycle, and possibly can be driven by the time-varying equity premium.

The proposed model assumes constant correlation within each state. The estimate correlations suggest that smaller number of correlation states is needed if we allow time-varying correlation within each state. So modeling the correlation as two-state regime-switching DCC may be adequate to capture both the structural break in the level of the correlation and the variation within each regime. Future work will consider how to effectively incorporate DCC in this regime-switching GARCH framework in a way that is parsimonious and numerically tractable. A better covariance forecasting model could also be developed in a similar approach. Another interesting research topic will be comparing the filtered states with the macro news announcement. For example, the state-7 should be related to the discount rate news in expansion while state-8 should be related to cash flow news in recession. Finally, our model might also be applied to study the contagion effect in international markets.

Appendix: Details

A1: details of maximum likelihood estimation

The general model could be estimated by maximum likelihood method. The algebra is standard, as in Hamilton (1994). Let $\Theta = \{\omega_i^k, \alpha_i^k, \beta_i^k, P_{ij}, C_i\}$ *with* $i = 1, 2...8$ and $k = s$ or b, be the parameter space of model (3). And $R_{[t,0]}$ is the whole history of return from 0 to t. Then the likelihood of the observed return pairs $\begin{bmatrix} R_t^s & R_t^b \end{bmatrix}'$ at time t can be written as:

$$
\begin{aligned}
& f_t(R_t^s, R_t^b|\Theta; R_{[t-1,0]}^s, R_{[t-1,0]}^b) \\
= & \sum_{i=1}^{N} f_t(R_t^s, R_t^b; S_t = i|\Theta; R_{[t-1,0]}^s, R_{[t-1,0]}^b) \\
= & \sum_{i=1}^{N} f_t(R_t^s, R_t^b|\Theta; S_t = i; R_{[t-1,0]}^s, R_{[t-1,0]}^b) * \Pr(S_t = i|\Theta; R_{[t-1,0]}^s, R_{[t-1,0]}^b) \\
= & \sum_{i=1}^{N} f_t(R_t^s, R_t^b|\Theta; S_t = i; R_{[t-1,0]}^s, R_{[t-1,0]}^b) * [\sum_{j=1}^{N} \Pr(S_t = i|\Theta; S_{t-1} = j) * \Pr(S_{t-1} = j|\Theta; R_{[t-1,0]}^s, R_{[t-1,0]}^b)] \\
= & \sum_{i=1}^{N} \eta_t(i) * [\sum_{j=1}^{N} P_{i,j} * \xi_{t-1|t-1}(j)]
\end{aligned}
$$

where $\eta_t(i) = f_t(R_t^s, R_t^b|\Theta; S_t = i; R_{[t-1,0]}^s, R_{[t-1,0]}^b)$, $P_{i,j} = \Pr(S_t = i|\Theta; S_{t-1} = j)$, and $\xi_{t-1|t-1}(j) = \Pr(S_{t-1} = j|\Theta; R_{[t-1,0]}^s, R_{[t-1,0]}^b)$.

Then $\eta_t(i)$ could be calculated as the usual GARCH likelihood function for the subset of the parameters Θ where $S_t = i$., given the whole history of past returns. And $P_{i,j}$ is just the entries of the transition matrix, which is part of Θ. To compute the remaining $\xi_{t-1|t-1}(j)$, let

$$\xi_{t|t-1}(j) = \Pr(S_t = j|\Theta; R_{[t-1,0]}^s, R_{[t-1,0]}^b)$$

Then $\xi_{t|t}(j)$ is computed as:

$$\begin{aligned}
\xi_{t|t}(j) &= \Pr(S_t = j|\Theta; R^s_{[t,0]}, R^b_{[t,0]}) \\
&= \frac{\Pr(S_t = j; R^s_t, R^b_t|\Theta; R^s_{[t-1,0]}, R^b_{[t-1,0]})}{\Pr(R^s_t, R^b_t|\Theta; R^s_{[t-1,0]}, R^b_{[t-1,0]})} \\
&= \frac{\Pr(R^s_t, R^b_t|\Theta; S_t = j; R^s_{[t-1,0]}, R^b_{[t-1,0]}) * \Pr(S_t = j|\Theta; R^s_{[t-1,0]}, R^b_{[t-1,0]})}{\sum\limits_{j=1}^{N} f_t(R^s_t, R^b_t|\Theta; S_t = j; R^s_{[t-1,0]}, R^b_{[t-1,0]}) * \Pr(S_t = j|\Theta; R^s_{[t-1,0]}, R^b_{[t-1,0]})} \\
&= \frac{\eta_t(j) * \xi_{t|t-1}(j)}{\sum\limits_{i=1}^{N} \eta_t(j) * \xi_{t|t-1}(j)}
\end{aligned}$$

So given $\eta_t(j)$ and $\xi_{t|t-1}(j)$, we could compute $\xi_{t|t}(j)$. Finally, the Markov property of the state variable results in the following relation between $\xi_{t|t-1}$ and $\xi_{t-1|t-1}$:

$$\xi_{t|t-1}(j) = P * \xi_{t-1|t-1}$$

So the likelihood function could be computed by iterating $\xi_{t|t-1}$ and $\xi_{t-1|t-1}$ from t=1 to T. And the maximum likelihood estimator of $\widehat{\Theta}$ is simply as:

$$\widehat{\Theta} = \arg\max_{\Theta} \sum_{t=1}^{T} \log\{f_t(R^s_t, R^b_t|\Theta; R^s_{[t-1,0]}, R^b_{[t-1,0]})\}$$

In this paper, we numerically maximize this log-likelihood function. And standard error of parameter estimates are also obtained numerically.

With the estimate of the transition matrix, the "smoothed probability" $\xi_{t|T}$, of the states at t are computed as in Kim (1993):

$$\xi_{t|T} = \xi_{t|t} \odot \{\widehat{P}' * [\xi_{t+1|T}(\div)\xi_{t+1|t}]\} \tag{4}$$

where sign $\odot$ denotes element-by-element multiplication, and $(\div)$ denotes element-by-element division. The smoothed probabilities could be computed by iterating on [4] backward from t=T to 1.

A2: Stationarity condition and unconditional volatility

This part of appendix is reproduced from Haas, Mittnik and Paolella (2004) Let P_{ij} be the ith row and jth column element of the transition matrix, and α and β are 2 by1 vectors of the ARCH and

GARCH parameters for two states. Then define

$$
\begin{aligned}
M &= \begin{bmatrix} M_{1,1} & M_{1,2} \\ M_{2,1} & M_{2,2} \end{bmatrix} \\
M_{i,j} &= P_{ij}(\beta + \alpha * e(i)'), \qquad i,j = 1,2
\end{aligned}
$$

where $e(i)$ is a vector with ith element equal to one and the rest being zero

Then model (2) is stationary if largest eigenvalue of matrix M, denoted as $eig(M)$ is less than one. If this stationarity condition holds, then the unconditional expectation of two volatility processes is given by

$$E(V) = [I_2, I_2](I_4 - M)^{-1}(\overline{P} \otimes \omega)$$

where I_k is k by k identity matrix, $\overline{P}$ is the unconditional state probability vector to be computed in A3, ω is defined as in model (2). and $\otimes$ is the Kronecker product operator. The proof for above result could be found in Haas Mittnik and Paolella (2004).

A3: Algebra for comparative analysis:

Expected correlation conditional on volatility regime

To compute the expected correlation level conditional on volatility regime, we first need to compute the unconditional probability implied by the transition matrix. Let $N = 8$ be the number of state, P be the N by N transition matrix, and C is N by 1 vector of correlation. The unconditional state probability $\overline{P}$ can be computed as:

$$
\begin{aligned}
\overline{P} &= (A'A)^{-1}A'e_{N+1} \\
\text{where} \quad \underset{(N+1)\times N}{A} &= \begin{bmatrix} I_N - P \\ 1' \end{bmatrix}
\end{aligned}
$$

with I_N being the N by N identity matrix, e_{N+1} being the $(N+1)th$ column of I_{N+1}, and 1 being N by 1 vector of 1s.

With the unconditional state probability vector,and the transition matrix, we can calculate the expected correlation conditional volatility regime as follows, for the case of low volatility regime:

$$
\begin{aligned}
E(C_t|S^k &= low) = \sum_{\{i|S_t=i;S_t^k=low\}} C^i * \Pr(S_t^c = i|S_t^k = low) \\
&= \sum_{\{i|S_t=i;S_t^k=low\}} C^i * \frac{\Pr(S_t^c = i, S_t^k = low)}{\Pr(S_t^k = low)} \\
&= \sum_{\{i|S_t=i;S_t^k=low\}} C^i * \frac{\Pr(S_t^c = i, S_t^k = low)}{\sum_{\{i|S_t=i;S_t^k=low\}} \Pr(S_t^c = i, S_t^k = low)}
\end{aligned}
$$

where $\Pr(S_t^c = i, S_t^k = low)$ is one entry in the N by 1 unconditional state probability $\overline{P}$, and $k = s$ for conditioning on stock volatility regime and $k = b$ for conditioning on bonding volatility regime.

Similar calculations can be done for expected correlation conditioned on both of S_t^s and S_t^b, such as $E(C_t|S^s = low; S^b = low)$.

$t - Conditional$ **and** $t + 1 - conditional$ **transition probabilities**

There are two types of conditional transition probabilities we consider. One is the usual transition probability conditional on observing today's realization on other state variable, such as $\Pr(S_{t+1}^c = low|S_t^c = high; S_t^s = low)$.We call this $t - conditional$ *transition probability.* The second one is conditional on $t + 1$ information. It is the transition probabilities of one process conditioning on there is an regime-switching in another process, such as $\Pr(S_{t+1}^b = low|S_{t+1}^s = high; \{S_t^s = low\ ;\ S_t^b = high\})$.We call this $t + 1 - conditional$ *transition probability.* Both of these quantities can be derived from the general transition matrix.

First, we calculate the $t - conditional$ *transition probability* as follows:

$$
\begin{aligned}
\Pr(S_{t+1}^c &= low|S_t^c = high; S_t^s = low) \\
&= \Pr(S_{t+1} = J \mid S_t = I) \\
with\ \{\omega|S_{t+1}(\omega) &= J\} = \{\omega \mid S_{t+1}^c(\omega) = low\}; \\
\{\omega|S_t(\omega) &= I\} = \{\omega \mid S_t^c(\omega) = high; S_t^s(\omega) = low\ \}
\end{aligned}
$$

We can derive the $t+1-conditional$ *transition probability* in a similar way. For example, the probability of t+1's correlation would be in low regime conditioned on the stock volatility switching from low regime (at t) to high regime (at t+1), and t's correlation is in high regime:

$$
\begin{aligned}
\Pr(S^c_{t+1} &= low|S^s_{t+1} = high; \{S^s_t = low\ ;\ S^c_t = high\}) \\
&= \frac{\Pr(S^c_{t+1} = low\ ; S^s_{t+1} = high \mid S^s_t = low\ ;\ S^c_t = high)}{\Pr(S^s_{t+1} = high \mid S^s_t = low\ ;\ S^c_t = high)} \\
&= \frac{\Pr(S_{t+1} = J \mid S_t = I)}{\Pr(S_{t+1} = V \mid S_t = I)} \\
with\ \{\omega|S_{t+1}(\omega) &= J\} = \{\omega \mid S^c_{t+1}(\omega) = low\ ; S^s_{t+1}(\omega) = high\ \}; \\
\{\omega \mid S_t(\omega) &= I\} = \{\omega \mid S^s_t(\omega) = low\ ;\ S^c_t(\omega) = high\ \} \\
\{\omega \mid S_{t+1}(\omega) &= V\} = \{\omega \mid S^s_{t+1}(\omega) = high\ \}
\end{aligned}
$$

Finally, for both the $t-$ and $t+1-$*conditional transition probability*, we can use the stationary probabilities $\Pr(S_t = i)$ to compute:

$$
\begin{aligned}
\Pr(S_{t+1} &= J|S_t = I) \\
&= [\Pr(S_{t+1} = J|S_t = I_1) * \Pr(S_t = I_1) + \ldots.\Pr(S_{t+1} = J|S_t = I_I) * \Pr(S_t = I_I)]/\Pr(S_t = I) \\
&= \frac{\sum\limits_{i\in I} \left\{ \sum\limits_{j\in J} [\Pr(S_{t+1} = j|S_t = i)] * \Pr(S_t = i) \right\}}{\sum\limits_{i\in I} \Pr(S_t = i)}
\end{aligned}
$$

where $\Pr(S_{t+1} = j \mid S_t = i)$ is the $j\ th$ row and $i\ th$ column of the transition matrix P. And I, J, V are numbers from 1 to 8 So we could compute all the relevant quantities from the transition matrix in a similar way.

Note that by $S^c_{t+1} = low$, we mean those states with significantly negative correlation. So for both bond-stock pairs, this includes $S_t = 2, 6$ and 8, as could be seen in Table 9-10. Even though $\widetilde{C}_4<0$ for ten-year bond, it is not significant less than zero. So we still treat $S_t = 4$ to be the low state for ten-year bond. This would make easier the comparison of bonds with different maturities.

7 Tables and Figures:

Table 1: State variable definitions for the general model

For volatility state variables, 0 indicates low level regime, and 1 indicate regime with high level. For correlation state, the actual level of high or low state in different "general state" are different. So correlation state variable S^c is the same as the general state variable S, both of which have eight states.

S (General State)	S^s (stock vol)	S^b(bond vol)	S^c(Correlation)
1	0	0	high1
2	0	0	low1
3	0	1	high2
4	0	1	low2
5	1	0	high3
6	1	0	low3
7	1	1	high4
8	1	1	low4

Table 2: State specifications of all models

This table specifies different model's restrictions on each of the three individual state variables among various states of the general state variable S. The general model is in the first row. The remaining rows present the more restricted models. The numbers in the same curly bracket represent in which states of S individual state variable has the same value. So the collection of curl bracketed-subsets represents the finest $\sigma-$algebra of each state variable in different model specification.

Model	Num. of Regimes	Restriction on S^s	Restriction on S^b	Restriction on S^c
MSCCC(2,2,8)	8	{1,2,3,4}{5,6,7,8}	{1,2,5,6}{3,4,7,8}	{1}{2}{3}{4}{5}{6}{7}{8}
MSCCC(2,2,4)	4	{1,2,3,4}{5,6,7,8}	{1,2,5,6}{3,4,7,8}	{1,2}{3,4}{5,6}{7,8}
MSCCC($\overline{2}$,2,$\overline{2}$)	4	{1,2,3,4}{5,6,7,8}	{1,2,5,6}{3,4,7,8}	{1,2,3,4}{5,6,7,8}
MSCCC(2,$\overline{2}$,$\overline{2}$)	4	{1,2,3,4}{5,6,7,8}	{1,2,5,6}{3,4,7,8}	{1,2,5,6}{3,4,7,8}
MSCCC($\overline{2}$,$\overline{2}$,4)	4	{1,2}{7,8}	{1,2}{7,8}	{1}{2}{7}{8}
MSCCC($\overline{2}$,$\overline{2}$,$\overline{2}$)	2	{1,2}{7,8}	{1,2}{7,8}	{1,2}{7,8}

Table 3: Descriptive Statistics and Optimal Filtering Orders

This table gives descriptive statistics as well as the optimal ARMA filtering orders (p* and q*) for the returns on the S&P 500 index and 1-year Treasury bond and the 10-year Treasury bond, for the period of January 2, 1986 to December 31 2006. All returns are daily returns in percentages.

	1-year bond	10-year bond	S&P 500
Mean	0.021021	0.032231	0.042042
Std. dev.	0.054204	0.45427	1.0665
Minimum	-0.31801	-2.6762	-20.467
Maximum	0.80969	4.8227	9.0994
Skewness	1.1069	-0.042843	-1.4327
Kurtosis	18.573	7.3358	33.622
Jarque-Bera	54052	4107	2.0667e+005
p*	1	0	0
q*	2	1	0

Table 4: Parameters estimates for the MS-GARCH model: S&P500

This table reports the parameter estimates of the MS-GARCH model for S&P500 daily return series. The sample period is from January 5, 1986 to December 29, 2006. Standard deviation of the estimate is in parenthesis. $eig(M)$ is the stationarity statistics estimate. The estimated model is stationary if $eig(M) < 1$ The unconditional expectation of the each state's probability and volatility level in each regime are also reported.

$$\epsilon_t^s \sim N\left(0,\ V_{S_t^s,t}^s\right),\quad S_t^s = 0 \text{ or } 1$$

$$\begin{cases} V_{0,t}^s = \underset{(0.0015)}{0.0026} + \underset{(0.0083)}{0.9528} * V_{0,t-1}^s + \underset{(0.0071)}{0.0321} * (\epsilon_{t-1}^s)^2 \\ \\ V_{1,t}^s = \underset{(0.1523)}{0.2522} + \underset{(0.0811)}{0.7146} * V_{1,t-1}^s + \underset{(0.2249)}{0.6216} * (\epsilon_{t-1}^s)^2 \end{cases}$$

$$\text{P}=\begin{bmatrix} & \textit{low} & \textit{high} \\ \textit{low} & \underset{(0.0603)}{0.8745} & 0.9217 \\ \textit{high} & 0.1255 & \underset{(0.0501)}{0.0783} \end{bmatrix}$$

loglikelihood=-6806.98; $eig(M)$ =0.9913

P_0^{∞} =0.8803; $E(V_0)$ =0.7781

P_1^{∞} =0.1197; $E(V_1)$ =3.1949

Table 5: Parameters estimates for the MS-GARCH model: one-year bond

This table reports the parameter estimates of the MS-GARCH model for 1-year bond daily return series. The sample period is from January 5,1986 to December 29,2006. Standard deviation of the estimate is in parenthesis. $eig(M)$ is the stationarity statistics estimate. The estimated model is stationary if $eig(M) < 1$ The unconditional expectation of the each state's probability and volatility level in each regime are also reported.

$$\epsilon_t^b \sim N\left(0,\ V_{S_t^b,t}^b\right),\ S_t^b = 0 \text{ or } 1$$

$$\begin{cases} V_{0,t}^b = \underset{(0.0397)^\dagger}{0.1359^\dagger} + \underset{(0.0081)}{0.9546} * V_{0,t-1}^b + \underset{(0.0043)}{0.0219} * (\epsilon_{t-1}^b)^2 \\ V_{1,t}^b = \underset{(0.2127)^\dagger}{0.4251^\dagger} + \underset{(0.0167)}{0.9095} * V_{1,t-1}^b + \underset{(0.0936)}{0.3480} * (\epsilon_{t-1}^b)^2 \end{cases}$$

$$P = \begin{array}{c} \\ low \\ high \end{array} \begin{array}{cc} low & high \\ \end{array}$$

$$\text{P}= \begin{bmatrix} & low & high \\ low & \underset{(0.0232)}{0.8671} & 0.8377 \\ high & 0.1329 & \underset{(0.0602)}{0.1627} \end{bmatrix}$$

loglikelihood=8693.43; $eig(M) = 0.9967$.

P_0^∞ =0.8630; $E(V_0)$ =0.0033

P_1^∞ =0.1370; $E(V_1)$ =0.0241

†: these numbers are multiplied by 10^4.

Table 6: Parameters estimates for the MS-GARCH model: 10-year bond

This table reports the parameter estimates of the MS-GARCH model for 10-year bond daily return series. the sample period is from January 5,1986 to December 29,2006. Standard deviation of the estimate is in parenthesis. $eig(M)$ is the stationarity statistics estimate. The estimated model is stationary if $eig(M) < 1$ The unconditional expectation of the each state's probability and volatility level in each regime are also reported.

$$\epsilon_t^b \sim N\left(0,\ V_{S_t^b,t}^b\right),\quad S_t^b = 0 \text{ or } 1$$

$$\begin{cases} V_{0,t}^b = \underset{(0.0007)}{0.0018} + \underset{(0.0104)}{0.9437} * V_{0,t-1}^b + \underset{(0.0045)}{0.0242} * (\epsilon_{t-1}^b)^2 \\ V_{1,t}^b = \underset{(0.0077)}{0.0123} + \underset{(0.0315)}{0.9202} * V_{1,t-1}^b + \underset{(0.0490)}{0.1314} * (\epsilon_{t-1}^b)^2 \end{cases}$$

$$\text{P}=\begin{bmatrix} & \textit{low} & \textit{high} \\ \textit{low} & \underset{(0.0539)}{0.7268} & 0.8851 \\ \textit{high} & 0.2732 & \underset{(0.0531)}{0.1149} \end{bmatrix}$$

loglikelihood=-2958.43; $eig(M)$ =0.9810

$P_0^\infty = 0.7641$; $E(V_0) = 0.1216$

$P_1^\infty = 0.2359$; $E(V_1)$ =0.4990

Table 7: Parameters estimates for the MSCCC(8) model: S&P500 and 1-year bond

This table reports the parameter estimates of the MSCCC(2,2,8) model for S&P500 and 1-year bond returns. The sample period is from January 5,1986 to December 29,2006. Standard deviation of the estimate is in parenthesis. The unconditional expectation of the each state's probability is also reported.

$$
\begin{cases}
V^s_{0,t} = & 0.0033 & + & 0.9548 & * V^s_{0,t-1} + & 0.0302 & * (\epsilon^s_{t-1})^2 \\
 & (0.0011) & & (0.0065) & & (0.0045) & \\
V^s_{1,t} = & 0.4179 & + & 0.6629 & * V^s_{1,t-1} + & 0.5349 & * (\epsilon^s_{t-1})^2 \\
 & (0.1405) & & (0.0775) & & (0.1395) &
\end{cases}
$$

$$
\begin{cases}
V^b_{0,t} = & 0.1406^{\dagger} & + & 0.9524 & * V^b_{0,t-1} + & 0.0233 & * (\epsilon^b_{t-1})^2 \\
 & (0.0375)^{\dagger} & & (0.0073) & & (0.0038) & \\
V^b_{1,t} = & 0.3177^{\dagger} & + & 0.9306 & * V^b_{1,t-1} + & 0.2551 & * (\epsilon^b_{t-1})^2 \\
 & (0.1579)^{\dagger} & & (0.0129) & & (0.0629) &
\end{cases}
$$

	$(S^s = 0, S^b = 0)$	$(S^s = 0, S^b = 1)$	$(S^s = 1, S^b = 0)$	$(S^s = 1, S^b = 1)$
high corr	0.2839 (0.0271)	0.4323 (0.0539)	0.0569 (0.1005)	0.8264 (0.0831)
low corr	−0.3530 (0.0637)	0.0795 (0.1401)	−0.2444 (0.1307)	−0.4856 (0.1392)

	(low,low,1)	(low,low,2)	(low,high,3)	(low,high,4)	(high,low,5)	(high,low,6)	(high,high,7)	(high,high,8)
(low,low,1)	0.8222 (0.0215)	0.0093 (0.0028)	0.6161 (0.0858)		0.6366 (0.1006)		0.8285 (0.0035)	
(low,low,2)		0.7918 (0.0507)		0.7058 (0.2130)		0.7493 (0.0752)		0.8175 (0.0477)
(low,high,3)	0.1338 (0.0147)				0.3100 (0.1329)			
(low,high,4)		0.0243 (0.0255)		0.0113 (0.1135)		0.0836 (0.0974)	0.1269 (0.0667)	0.1825 (0.0477)
(high,low,5)	0.0440 (0.0173)		0.2247 (0.0844)	0.1174 (0.1071)	0.0534 (0.0478)			
(high,low,6)		0.1180 (0.0373)		0.1655 (0.1847)				
(high,high,7)			0.1592 (0.0215)					
(high,high,8)		0.0566 (0.0225)				0.1671 (0.0831)	0.0446 (0.0485)	
unconditional	0.53044	0.24959	0.084875	0.015237	0.044823	0.038193	0.013515	0.023326

Log Likelihood: 2132.1 AIC:-4112.2 BIC:-3613.2

Table 8: Parameters estimates for the MSCCC(8) model: S&P and 10-year bond

This table reports the parameter estimates of the MSCCC(2,2,8) model for S&P500 and 10-year bond returns. The sample period is from January 5,1986 to December 29,2006. Standard deviation of the estimate is in parenthesis. The unconditional expectation of the each state's probability is also reported.

$$
\begin{cases}
V^s_{1,t} = & \underset{(0.0010)}{0.0028} + \underset{(0.0061)}{0.9597} * V^s_{1,t-1} + \underset{(0.0041)}{0.0263} * (\epsilon^s_{t-1})^2 \\
V^s_{2,t} = & \underset{(0.0958)}{0.2151} + \underset{(0.0619)}{0.7423} * V^s_{2,t-1} + \underset{(0.1183)}{0.5420} * (\epsilon^s_{t-1})^2
\end{cases}
$$

$$
\begin{cases}
V^b_{1,t} = & \underset{(0.0007)}{0.0020} + \underset{(0.0101)}{0.9441} * V^b_{1,t-1} + \underset{(0.0040)}{0.0228} * (\epsilon^b_{t-1})^2 \\
V^b_{2,t} = & \underset{(0.0071)}{0.0125} + \underset{(0.0271)}{0.9259} * V^b_{2,t-1} + \underset{(0.0370)}{0.1140} * (\epsilon^b_{t-1})^2
\end{cases}
$$

	$(S^s = 0, S^b = 0)$	$(S^s = 0, S^b = 1)$	$(S^s = 1, S^b = 0)$	$(S^s = 1, S^b = 1)$
high corr	0.4334	0.5501	0.2978	0.6931
	(0.0326)	(0.0554)	(0.1484)	(0.0691)
low corr	-0.2810	-0.0641	-0.5121	-0.5062
	(0.0399)	(0.1188)	(0.1247)	(0.0965)

	(low,low,1)	(low,low,2)	(low,high,3)	(low,high,4)	(high,low,5)	(high,low,6)	(high,high,7)	(high,high,8)
(low,low,1)	0.7206 (0.0208)		0.7117 (0.0660)		0.5618 (0.2162)		0.9779 (0.0231)	
(low,low,2)	0.0014 (0.0033)	0.7547 (0.0365)	0.0052 (0.0134)	0.6249 (0.1780)				0.7829 (0.2015)
(low,high,3)	0.1948 (0.0208)	0.0151 (0.0048)	0.0286 (0.0380)		0.3842 (0.2158)			
(low,high,4)		0.0976 (0.0397)		0.1351 (0.2499)		0.3673 (0.1508)		0.2171 (0,2015)
(high,low,5)			0.2400 (0.0590)					
(high,low,6)	0.0027 (0.0029)			0.0431 (0.0499)		0.6327 (0.1508)	0.0221 (0.0231)	
(high,high,7)	0.0805 (0.0151)		0.0146 (0.0380)		0.0540 (0.0760)			
(high,high,8)		0.1326 (0.0335)		0.1969 (0.1638)				
unconditional	0.49021	0.23586	0.11267	0.041214	0.027035	0.011066	0.042557	0.039381

Log Likelihood: -9329.29 AIC:18811 BIC:19310

Table 9: $t-$Conditional Transition Probabilities for Correlation

This table reports the transition probabilities of correlation state variable conditioned on observing the volatility state variable. We denote stock-vol, bond-vol and their correlation by S, B and C respectively. So $P(C_{+,-}|S_0) \equiv P(C_{t+1} < 0|C_t \geqslant 0; S^s_t = 0)$, and similar meaning for other notations. Note that by $C < 0$, we means those states with *significantly* negative correlation. So for both bond-stock pairs, this includes $S_t = 2, 6$ and 8. Standard deviations of the estimation are in parenthesis. All these are derived from the general transition matrix in Table 9-10.

1-year bond and S&P500				
$C_- \Rightarrow C_+$	$P(C_{-,+}\|S_0)$	$P(C_{-,+}\|S_1)$	$P(C_{-,+}\|B_0)$	$P(C_{-,+}\|B_1)$
	0.0336	0.1211	0.0403	0.1825
	(0.0219)	(0.0547)	(0.0201)	(0.0477)
$C_+ \Rightarrow C_-$	$P(C_{+,-}\|S_0)$	$P(C_{+,-}\|S_1)$	$P(C_{+,-}\|B_0)$	$P(C_{+,-}\|B_1)$
	0.0242	0.0103	0	0.1394
	(0.0366)	(0.0108)	(-†)	(0.0798)
10-year bond and S&P500				
$C_- \Rightarrow C_+$	$P(C_{-,+}\|S_0)$	$P(C_{-,+}\|S_1)$	$P(C_{-,+}\|B_0)$	$P(C_{-,+}\|B_1)$
	0.1128	0.2500	0.1242	0.2171
	(0.0391)	(0.1346)	(0.0400)	(0.1784)
$C_+ \Rightarrow C_-$	$P(C_{+,-}\|S_0)$	$P(C_{+,-}\|S_1)$	$P(C_{+,-}\|B_0)$	$P(C_{+,-}\|B_1)$
	0.0594	0.0135	0.0039	0.1892
	(0.0474)	(0.0124)	(0.0032)	(0.0937)

†No standard deviations due to the zero constraints on the transition matrix, same as reported in Table 9-10.

Table 10: Correlation transition matrix

This table reports the correlation transition matrix estimates. All these are derived from the general transition matrix in Table 9-10. Note that by $C < 0$, we means those states with significantly negative correlation. So for both bond-stock pairs, this includes $S_t = 2, 6$ and 8. Even though $\widetilde{C}_4 < 0$ for ten-year bond, it is not significant less than zero. So we still treat $S_t = 4$ to be one of state in $\{C \geqslant 0\}$ for ten-year bond. This would make easier the comparison of bonds with different maturities.

1-year bond and S&P500		
	$C \geqslant 0$	$C < 0$
$C \geqslant 0$	0.9770 (0.0359)	0.0509
$C < 0$	0.0230	0.9491 (0.0174)

10-year bond and S&P500		
	$C \geqslant 0$	$C < 0$
$C \geqslant 0$	0.9451 (0.0447)	0.1370
$C < 0$	0.0549	0.8630 (0.0407)

Table 11: $t+1-$Conditional Transition Probabilities

This table reports the transition probabilities of one state variable conditioned on there is a switching in other state variables. We denote stock-vol, bond-vol and their correlation by S, B and C respectively. So $P(C_{+,-}|S_{0,1}) \equiv P(C_{t+1} < 0|C_t > 0; S_t^s = 0; S_{t+1}^s = 1)$, and similar meaning for other notations. Note that by $C < 0$, we means those states with significantly negative correlation. So for both bond-stock pairs, this includes $S_t = 2, 6$ and 8. Standard deviations of the estimation are in parenthesis.

1-year bond and S&P500				
$S/B \Rightarrow C$	$P(C_{+,-}\|S_{0,1})$	$P(C_{+,-}\|B_{0,1})$	$P(C_{-,+}\|S_{0,1})$	$P(C_{-,+}\|B_{0,1})$
	0.1632	0	0	0.3110
	(0.0987)	(-)†	(-)	(0.1996)
$C \Rightarrow S/B$	$P(S_{0,1}\|C_{+,-})$	$P(B_{0,1}\|C_{+,-})$	$P(S_{0,1}\|C_{-,+})$	$P(B_{0,1}\|C_{-,+})$
	0.7163	NA‡	0	0.7999
	(0.2194)	(NA)	(-)	(0.1639)
$S \Leftrightarrow B$	$P(B_{0,1}\|S_{0,1})$	$P(B_{1,0}\|S_{1,0})$	$P(S_{0,1}\|B_{0,1})$	$P(S_{1,0}\|B_{1,0})$
	0.2112	0.8352	0.1551	1
	(0.0929)	(0.0481)	(0.0779)	(-)

10-year bond and S&P500				
$S/B \Rightarrow C$	$P(C_{+,-}\|S_{0,1})$	$P(C_{+,-}\|B_{0,1})$	$P(C_{-,+}\|S_{0,1})$	$P(C_{-,+}\|B_{0,1})$
	0.1416	0	0	0.4951
	(0.0996)	(-)†	(-)	(0.1284)
$C \Rightarrow S/B$	$P(S_{0,1}\|C_{+,-})$	$P(B_{0,1}\|C_{+,-})$	$P(S_{0,1}\|C_{-,+})$	$P(B_{0,1}\|C_{-,+})$
	0.2936	0	0	1
	(0.1395)	(-)	(-)	(-)
$S \Leftrightarrow B$	$P(B_{0,1}\|S_{0,1})$	$P(B_{1,0}\|S_{1,0})$	$P(S_{0,1}\|B_{0,1})$	$P(S_{1,0}\|B_{1,0})$
	0.9813	0.8945	0.3668	0.9872
	(0.0156)	(0.0974)	(0.0516)	(0.0096)

† No standard deviations due to the zero constraints on the transition matrix as in Table 9-10.

‡The conditioning event is an empty set. So there is no estimate for such conditional probability

Table 12: Expected correlation conditioned on volatility regime

This table reports the expected bond-stock correlation conditioned on contemporary volatility states in bond and stock markets. The upper panel is for 1-year bond and the lower panel for 10-year bond. For both panels, the last row and last column are the expected correlation conditioned on only one volatility state variable. Standard deviations of the estimation are in parenthesis.

1-year bond and S&P			
	$S^s = 0$	$S^s = 1$	$E(C_t \vert S^b)$
$S^b = 0$	0.0801 (0.0642)	-0.0817 (0.1580)	0.0645 (0.0582)
$S^b = 1$	0.3786 (0.0524)	-0.0043 (0.1464)	0.2756 (0.0711)
$E(C_t \vert S^s)$	0.1140 (0.0596)	-0.0579 (0.1191)	

10-year bond and S&P			
	$S^s = 0$	$S^s = 1$	$E(C_t \vert S^b)$
$S^b = 0$	0.2013 (0.0694)	0.0626 (0.1602)	0.1944 (0.0703)
$S^b = 1$	0.3856 (0.0904)	0.1167 (0.1427)	0.2922 (0.0934)
$E(C_t \vert S^s)$	0.2336 (0.0673)	0.0995 (0.1229)	

Table 13: Likelihood-based Goodness-of-fit

This table shows the likelihood-based goodness-of-fit for models fitted to two pairs of stock and bond return series. The specifications for each model notation in the first column are detailed in Table 2. "loglike" is the value of the maximum log-likelihood value. For parameter number K and sample size T, we calculated the Akaike information criterion (1973) as $AIC = -2*loglike + 2*K$, and the Bayesian Information criterion as $BIC = -2*loglike + log(T)*k$. For both criteria, the ranking of each model is show in parenthesis. Boldface entries indicate the best model for each criterion.

S&P500 and 1-year bond					
Model	Num. of Regimes	Parameter number	loglike	AIC(rank)	BIC(rank)
MSCCC(2,2,8)	8	76	2132.1	**-4112.2(1)**	-3613.2(5)
MSCCC(2,2,4)	4	28	1972	-3528(6)	-3344.2(6)
MSCCC($\bar{2}$,2,$\bar{2}$)	4	26	1964.55	-3877.1(4)	-3706.4(3)
MSCCC(2,$\bar{2}$,$\bar{2}$)	4	26	1952.82	-3853.6(5)	-3682.9(4)
MSCCC($\bar{2}$,$\bar{2}$,4)	4	28	2083.6	-4111.3(2)	**-3927.4(1)**
MSCCC($\bar{2}$,$\bar{2}$,$\bar{2}$)	2	16	1956.4	-3880.7(3)	-3775.7(2)

S&P 500 and 10-year bond					
Model	Num. of Regimes	Parameter number	loglike	AIC(rank)	BIC(rank)
MSCCC(2,2,8)	8	76	-9329.29	**18811(1)**	19310(2)
MSCCC(2,2,4)	4	28	-9579.99	19216(3)	19400(4)
MSCCC($\bar{2}$,2,$\bar{2}$)	4	26	-9645.61	19343(6)	19514(6)
MSCCC(2,$\bar{2}$,$\bar{2}$)	4	26	-9627.6	19307(5)	19478(5)
MSCCC($\bar{2}$,$\bar{2}$,4)	4	28	-9402.46	18861(2)	**19045(1)**
MSCCC($\bar{2}$,$\bar{2}$,$\bar{2}$)	2	16	-9610.36	19253(4)	19358(3)

Table 14: ARCH-LM tests

This table reports the P-values of the ARCH-LM tests on the residuals standardized by the conditional volatilities of MSCCC model and DCC model. For each of the three return series, we report the test statistics for lag of 5.

S&P 500					
Q	1	2	3	4	5
MSCCC	0.089	0.174	0.305	0.448	0.582
DCC	0.033	0.064	0.138	0.225	0.335
1-year bond					
Q	1	2	3	4	5
MSCCC	0.846	0.741	0.645	0.795	0.540
DCC	0.619	0.574	0.775	0.350	0.050
10-year bond					
Q	1	2	3	4	5
MSCCC	0.639	0.871	0.461	0.136	0.202
DCC	0.428	0.686	0.450	0.081	0.109

Figure 1: state probabilities of correlation: 1-year bond and S&P500

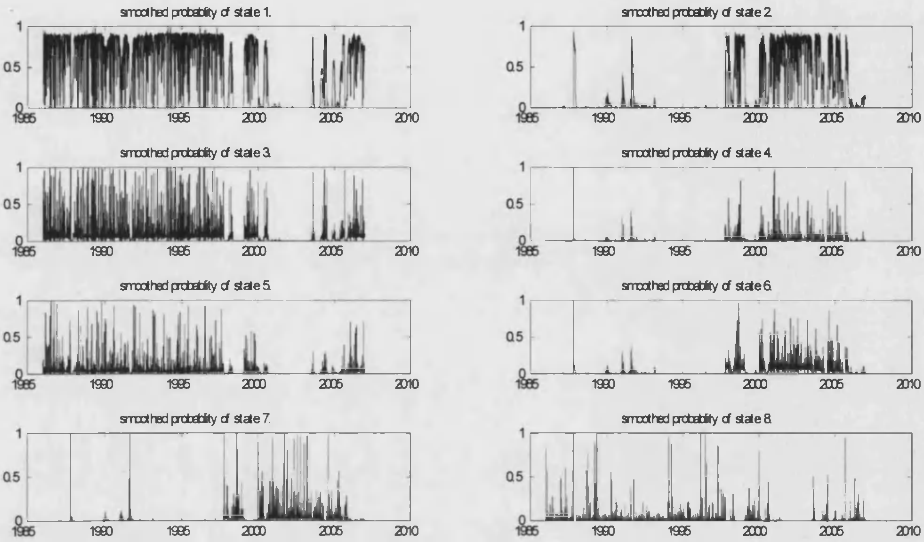


Figure 2: state probabilities of correlation: 10-year bond and S&P500

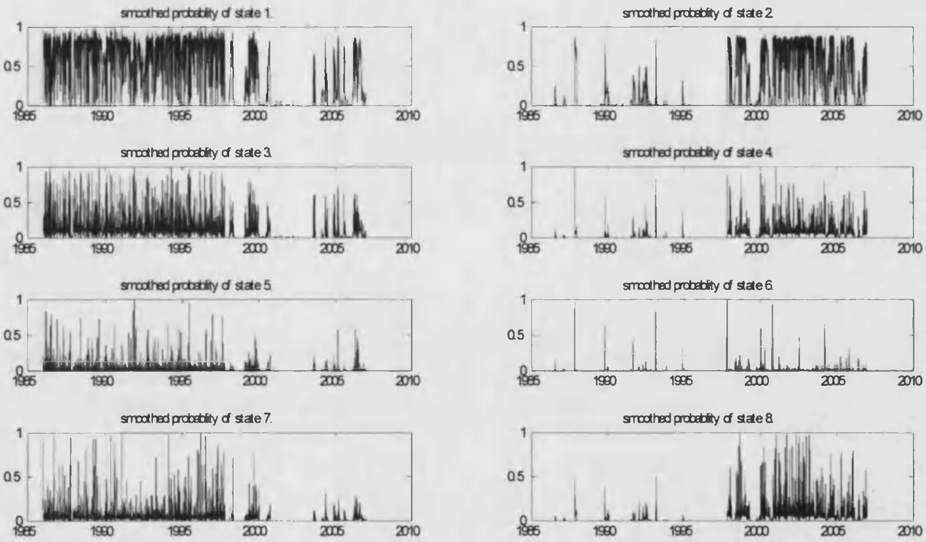


Figure 3: smoothed correlation: 1-year bond and S&P500

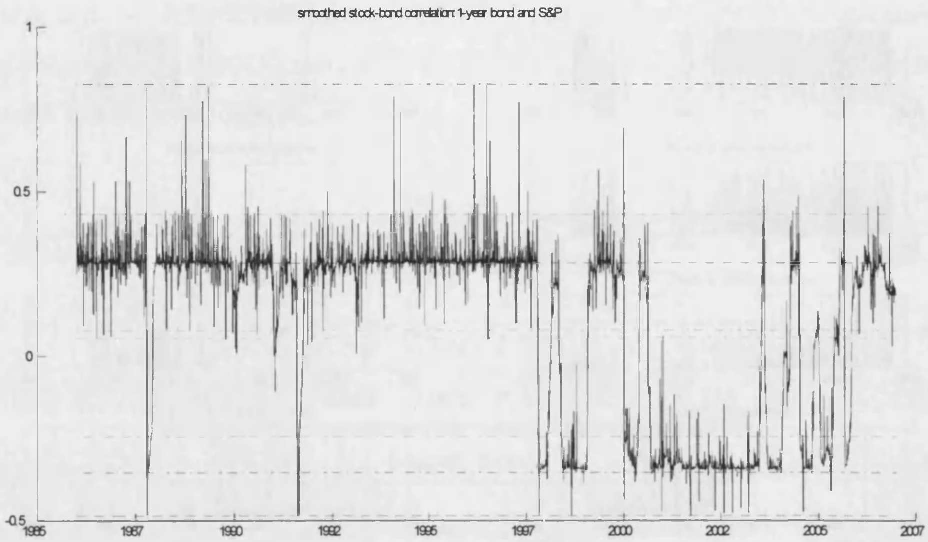


Figure 4: smoothed correlation: 10-year bond and S&P

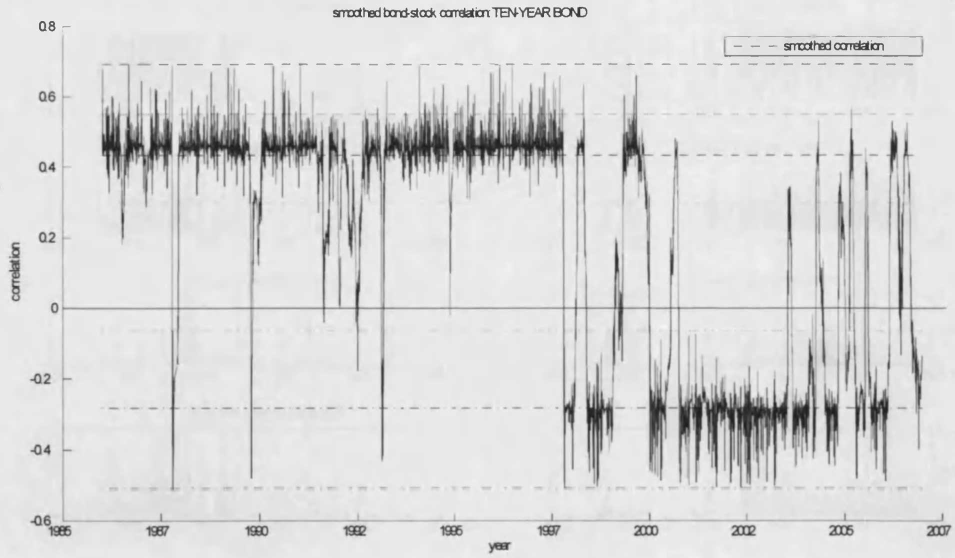


Figure 5.1: MSCCC VS DCC—1-year bond and S&P(1)

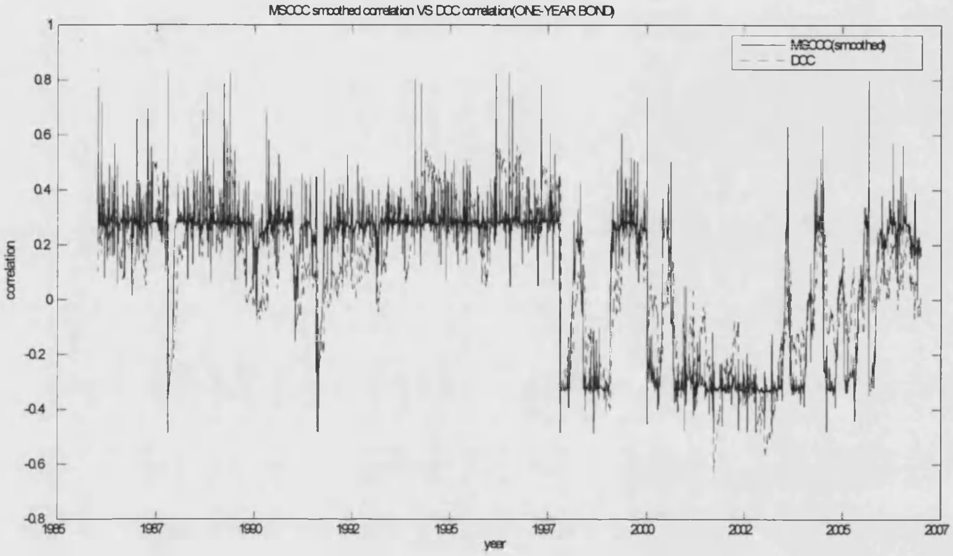


Figure 5.2: MSCCC VS DCC—1-year bond and S&P(2)

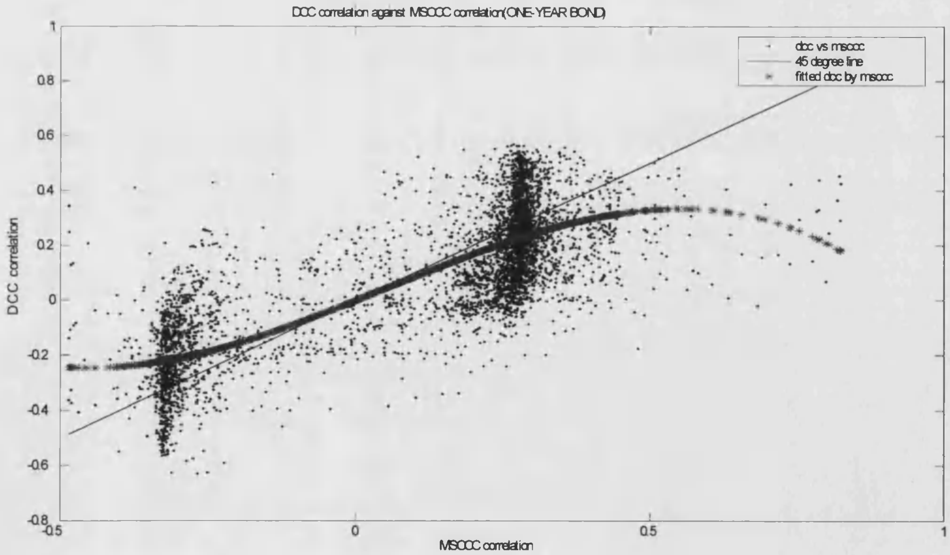


Figure 6.1: MSCCC VS DCC—10-year bond and S&P(1)

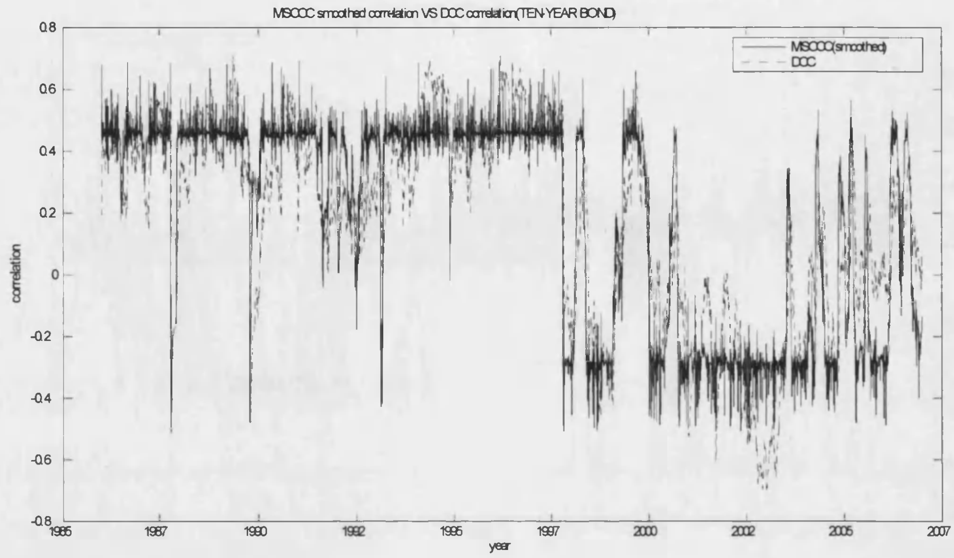
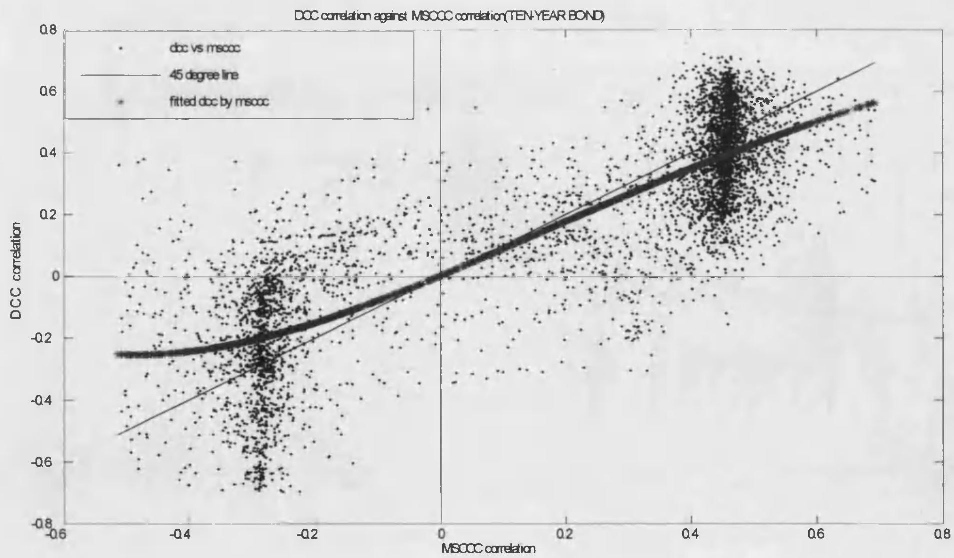


Figure 6.2: MSCCC VS DCC—10-year bond and S&P(2)



References

[1] Akaike, H., 1973, "Information Theory and an Extension of the Maximum Likelihood Principle", In B. N. Petrov and F. Csaki (eds.), 2nd *International Symposium on Information Theory.* Budapest: Akademiai Kiado.

[2] Anderson, T. G., T. Bollerslev, F. X. Diebold and C. Vega, 2007, "Real-time Price Discovery in Global Stock, Bond and Foreign Exchange Markets", *Journal of International Economics*, 73, 251-277.

[3] Baur, D. and B. M. Lucey, 2006, "Flight-to-Quality or Contagion? An Empirical Analysis of Stock-Bond Correlations", working paper, Trinity College Dublin.

[4] Bauwens, L., S. Laurent and J. V. K. Rombouts, 2006, "Multivariate GARCH Models: A Survey", *Journal of Applied Econometrics*, 21, 79-109.

[5] Boyd, J., J. Hu and R. Jagannathan, 2005, "The Stock Market's Reaction to Unemployment News: Why Bad News Is Usually Good For Stocks", *Journal of Finance*, 60, 649–672.

[6] Bollerslev, T., 1986, "Generalized Autoregressive Conditional Heteroskedasticity", *Journal of Econometrics*, 31, 307–327.

[7] Bollerslev, T., 1990,"Modelling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized ARCH Model", *Review of Economics and Statistics*, 72, 498–505.

[8] Cai, J., 1994, "A Markov Model of Switching-Regime ARCH", *Journal of Business and Economic Statistics*, 12, 309–316.

[9] Cappiello, L., R. Engle and K. Sheppard, 2006, "Asymmetric Dynamics in the Correlations of Global Equity and Bond Returns", *Journal of Financial Econometrics*, 4(4), 537-572.

[10] Cho, J.S. and H. White, 2007, "Testing for Regime Switching", *Econometrica*, 75 (6), 1671–1720.

[11] Connolly, R., C. Stivers and L. Sun, 2005, "Stock Market Uncertainty and the Stock-bond Return Relation", *Journal of financial and Quantitative Analysis*, 40(1),161-194.

[12] De Goeij, P. and W. Marquering, 2004, "Modeling the Conditional Covariance between Stock and Bond Returns: A Multivariate GARCH Approach", *Journal of Financial Econometrics,* 2(4), 531-546.

[13] Diebold, F. X. and A. Inoue, 2001, "Long Memory and Regime Switching", *Journal of Econometrics*, 105, 131–159.

[14] Engle, R., 1982, "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation", *Econometrica*, Vol. 50, 987-1008.

[15] Engle, R., 2002, "Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models", *Journal of Business and Economic Statistics*, 20(3), 339–350.

[16] Engle, R. and R. Colacito, 2006, "Testing and Valuing Dynamic Correlations for Asset Allocation.", *Journal of Business and Economic Statistics,* 24(2), 238-253.

[17] Engle, R. F. and K. F. Kroner, 1995, "Multivariate Simultaneous Generalized ARCH", *Econometric Theory*, 11,122-150.

[18] Glosten, L., R. Jagannathan, and D. Runkle, 1993, " Relationship Between the Expected Value and the Volatility of the Nominal Excess Return on Stock", *Journal of Finance*, 48, 1779-1801.

[19] Gray, S. F., 1996, "Modeling the Conditional Distribution of Interest Rates as a Regime-Switching Process", *Journal of Financial Economics*, 42, 27–62.

[20] Gulko, L., 2002, "Decoupling", *Journal of Portfolio Management*, Spring, 59-66.

[21] Hamilton, J. D., 1989, "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle", *Econometrica* 57, 357–384

[22] Hamilton, J. D., 1990, "Analysis of Time Series Subject to Changes in Regime", *Journal of Econometrics* 45, 39–70.

[23] Hamilton, J. D., 1994, *Time Series Analysis,* Princeton Univ. Press.

[24] Hamilton, J. D. and R. Susmel, 1994, "Autoregressive Conditional Heteroskedasticity and Changes in Regime", *Journal of Econometrics* 64, 307–333.

[25] Hansen, B. E., 1992, "The Likelihood Ratio Test under Nonstandard Conditions: Testing the Markov Switching Model of GNP.", *Journal of Applied Econometrics* 7, S61-S82.

[26] Haas, M. and S. Mittnik, 2007, "Multivariate Regime Switching GARCH with an Application to International Stock Markets", Working paper.

[27] Haas, M., S. Mittnik, and M. S. Palolella, 2004, "A New Approach to Markov-Switching GARCH Models," *Journal of Financial Econometrics*, 2(4), 493-530.

[28] Li, L., 2000, "Macroeconomic Factors and the Correlation of Stock and Bond Returns", Yale International Center for Fiance, Working Paper 02-46.

[29] McLachlan, G. J. and D. Peel, 2002, *Finite Mixture Models.* New York: John Wiley & Sons.

[30] Maheu, J. M. and T. H. McCurdy, 2000, "Identifying Bull and Bear Markets in Stock Returns", *Journal of Business and Economic Statistics*, Vol. 18, No. 1, pp. 100-112.

[31] Mikosch, T. and C. Starica, 2004, "Nonstationarities in Financial Time Series, the Long-Range Dependence, and the IGARCH Effects", *Review of Economics and Statistics* 86, 378–390.

[32] Pelletier, D., 2006, "Regime Switching for Dynamic Correlations", *Journal of Econometrics* , 131(1-2), 445-473.

Correlation Risk and Expected Equity Returns*

Runquan Chen

London School of Economics

This version: 1 May, 2009.

Abstract

The cross-sectional average correlation of equity returns represents the general diversification benefit of the whole equity market for investors. We first show that the average correlation is time-varying, and correlated with some macro variables. Then we estimate the risk premium for exposure to the correlation risk in the cross-section of US equities. We find that the estimate of correlation risk premium is significantly negative after controlling for the Fama-French's SMB and HML factors as well as the liquidity and momentum factors. The annual return spread between the high correlation-beta stock portfolio and the low correlation-beta stock portfolio varies between -2% and -6% controlling for other risk factors. The results are more significant in the sample period before 1983.

Keywords: Factor model, Correlation Risk, Variance Risk

J.E.L. Code: G12

*Helpful comments and suggestions were received from Andrew Patton and Michela Verardo. Contact address:Room Rz16b, Financial Markets Group, London School of Economics, Houghton Street, WC2A 2AE Email: R.Chen1@lse.ac.uk

1 Introduction

When the cross-sectional average of the correlations among equity returns is high, the variance of a diversified portfolio will be large as rising average correlation implies diminishing benefit from diversification. As a result, the cross-sectional average correlation of the stock market plays an important role in portfolio optimization for risk-averse investors. Stocks with higher pay-offs when the average correlation is high become more attractive to investors than stocks with lower pay-offs in high correlation states. In equilibrium, all other things equal, investors would pay a premium for stocks that have higher pay-offs in high correlation states. So these stocks should have lower expected returns than stocks with lower exposure to the correlation factor. In other words, the average correlation should have a negative risk premium in the market with a risk-averse representative agent.

The main goal of this paper is to construct an average correlation measure for the US equity market, and test whether this risk measure is priced in the cross-section of the equity market with a negative risk premium according to standard asset pricing theories. We are the first in rigorously estimating the risk premium of the average correlation constructed from the whole cross-section of the US equities. The risk premium for the variance of equity market index has been studied recently by Ang, Hodrick, Xing and Zhang (2006). But they choose the VIX index from the option market as their variance risk measure[1]. A more direct study of variance risk premium is Harvey and Siddique (2000). Their co-skewness measure is essentially stock return's beta on market variance. But they only find weak evidence of pricing for co-skewness[2]. If we decompose the market variance into average correlation and average variance, high correlation exposure should correspond to high co-skewness with the market return holding constant the average variance. Recently, Pollet and Wilson (2009) show the average variance of the whole market has negative correlation with future market returns while the average correlation positively predicts future market index returns. Similarly, Guo and Savickas (2008) also find idiosyncratic volatility negatively predicts future market index returns. These results suggest that only the average correlation part of the market variance has

[1] This choice means their findings are not directly on the market variance risk because VIX is the risk-adjusted expectation of the market variance, and contains information about the time-varying variance risk premium in option market. So their results could be mainly driven by the time-varying variance risk premium.

[2] Using several measures of co-skewness, they find that the premium estimate for co-skewness risk is only significant for stocks with short return history, usually less then 60 months.

the "correct" sign of risk premium on the index level[3]. The weak results of Harvey and Siddique (2000) could be due to the opposite pricing implications of the two components of market variance, average correlation and average variance. In this paper, we propose that the average correlation might be a more accurate and cleaner measure of the aggregate risk level of whole stock market. By focusing the average correlation, we expect the results of our cross-sectional study might be stronger. However, from a theoretical point of view, if we treat the average correlation as a proxy for some unobservable macro information, then the fact that it both predicts future market return and future market variance makes our study more interesting. Under the I-CAPM of Chen (2003), average correlation should have a negative risk premium for its role of positively predicting future market variance but a positive risk premium for its role of positively predicting future market return. As a result, the *net* pricing effect of average correlation will depend on A) its relative forecasting power for future market return and market variance, and B) the risk-aversion level of the representative agent. For a given level of risk aversion, if the average correlation has a much larger forecasting power for future market returns than for the market variance, the aggregate risk premium will become more positive. On the other hand, if the relative forecasting power is fixed, then for a very risk-averse agent , the average correlation's role of predicting future market variance will dominate, and the net correlation risk premium will be negative. This is because an asset with pay-offs positively correlated with average correlation, and so also positively correlated with high future market variance, could act as a hedging against adverse states with poor diversification opportunity. The results of this paper can be interpreted as an evaluation on the relative importance of these two opposite pricing effects.

In this paper, we adopt the General Method of Moment (GMM) approach of Hansen (1982) to estimate the corelation risk premium and correlation betas jointly. The GMM approach allows us to avoid the errors-in-variable (e.i.v.) problem. Using data from 1963 to 2006, we find that there is a significantly negative correlation risk premium in the cross-section of the US equity market. The results are robust to controlling for Fama-French (FF) (1993)'s SMB and HML factors, which are pay-offs on long-short spreads constructed by sorting stocks according to market capitalization and book-to-market ratio, as well as the momentum factor of Carhart (1997), and the liquidity factor

[3]The reason why average variance negatively predicts future market return has been extensivly studied in both empirical and theoretical literature in recent years. But there is still no concensus. And this is beyond the scope of this paper.

of Pastor and Stambaugh (2003). The results of the sub-sample before 1983 are more significant than those of sub-sample after 1983. Even though there is a high correlation between the liquidity factor and the correlation factor, at about -0.4, the average correlation does contain an independent source of risk. The annual return spread between the portfolio of high and low correlation-beta varies from -2% to -6% controlling for other risk factors.

Our paper is also related to the study on downside beta of Ang, Chen, and Xing (2006). They relate the expected equity return to market-beta conditional on the sign of the market return[4]. They show that the cross-section of stock returns reflects a premium for bearing downside risk, but fail to find a significant discount for stocks that have high covariance with upside movements of the market. Driessen, Maenhout and Vilkov (2009) use data on S&P100 index options, as well as options on all index components, to study the pricing of correlation risk in the option market. They find indirect evidence of priced correlation risk by showing that index variance risk is priced while individual variance risk is not. They suggest that correlation risk may be priced in the cross-section of stock returns. In this analysis, we directly study the pricing of the correlation risk in cross-section of the equity market. Our paper is also related to the contagion literature. A sudden increase in the average correlation could also be considered as contagion effect if the shock comes from a subset of the market instead of representing some market-wide information. Forbes and Rigobon (2002) show that heteroskedasticity would bias the estimate of empirical correlation. They find there is no significant increase in cross-country correlation during several financial crises once they adjust for the higher volatility during the crises. To rule out that our measure of average correlation would suffer from the same problem, we also compute the adjusted average correlation, similar to the "conditional correlation" of Forbes and Rigobon (2002). We find our results are robust to this adjustment.

The paper is organized as follows. We first illustrate how we construct the correlation risk measure and show its empirical features in section 2. In section 3 we form portfolios basing on stocks' correlation betas, and indirectly evaluate the significance of correlation risk through post-ranking alpha estimates. In section 4, we formally estimate the correlation risk premium using GMM for the returns of ten stock portfolios sorted on correlation-betas. Finally Section 5 concludes and discusses areas for further research.

[4]This measure is different from co-skewness of Harvey and Siddique (2000) because co-skewness is unconditional on direction of the market.

2 Constructing the Correlation Risk Proxy

2.1 Description of the data

The sample period we examine is from July 1963 to December 2006. The starting date is the same as the Fama-French (1992, 1993) papers. This can make our results comparable to other studies that examine other factors in explaining the cross-section of expected stock returns.

The first data set we use is the CRSP daily stock data. We use the daily returns of stocks of NYSE and AMEX to compute the average correlation risk measure as well as the average variance. The same data set in monthly frequency is used to estimate the factor loadings and construct the portfolio deciles. To be included in the regression sample, we require the stocks satisfy certain rules which are detailed in the following section. The second data set consists of monthly observations of the Fama-French factors (SMB and HML), the momentum factor (MOM), and the liquidity factor (LIQ) of Pastor and Stambaugh (2003).

Finally, we also want to examine what macroeconomic factors might explain the variation in the average correlation. From the Federal Reserve Board we get data on the composite rates of long term (over 10 years) bonds and rates of the three-month treasury bill, and compute the term spread as the difference between these two series. We get data on credit spread, which is the yield spread between corporate bonds with credit rating of BAA and AAA, and data on the Industrial Production Index from the FRED database of the Federal Reserve Bank of st. Louis. The US Business Cycle dates are from the NBER.

2.2 Constructing the average correlation

We first calculate the average correlation and average variance from the US equity market. Later in this section, we will investigate the relationship between these two series. We construct the monthly average correlation and average variance series using the daily returns of cross-section of stocks for each month in the sample. To be specific, for each month, we compute the sample correlation for each pair of stocks using their daily returns. The variance of each stock is computed as the sum of squared daily returns within the month[5]. Then we construct the average correlation as the equal-weighted or value-weighted average of the sample correlations of each pair of stocks,

[5] So the variance series are on monthly horizon. We treat the daily correlation within a month as the proxy for the realized correlation in the month.

and the average variance as the equal-weighted or value-weighted average of individual variances of all the stocks in the sample. The value-weighted average for variance is computed in standard way while we need to define how we do the value-weighted average of the correlation. Let $\rho_t^{i,j}$ be the sample correlation of asset i and j for month t, and Cap_t^i the capitalization of firm i. Then the value-weighted average correlation is:

$$corr_t^V = \frac{1}{M_t}\sum_{i=1}^{N_t-1}\sum_{j=i+1}^{N_t}\left(Cap_t^i Cap_t^j\right)\rho_t^{i,j}\text{with } M_t = \sum_{i=1}^{N_t-1}\sum_{j=i+1}^{N_t} Cap_t^i Cap_t^j \text{for } t = 1, 2....T \qquad (1)$$

where N_t is the number of eligible stocks in month t.

The value-weighting scheme here is the same as Pollet and Wilson (2009). The scheme tries to reflect the magnitude of the effect of correlation risk on the aggregate volatility of the market portfolio. To make the components used in the calculation more stable, we clean the data every year as follows: A stock is only included in our sample for month t if it meets all the following conditions: (1) it is listed in NYSE or AMEX; (2) It is a common stock; (3) it has less than 3 daily observations missing in any month, and has 12 months of observations in this year[6]; (4) the last price of the year is larger than 5 and smaller than 1000. As a robustness check, we also construct both the value-weighted and equal-weighted average correlation series using only the 500 largest firms.

As mentioned earlier, Forbes and Rigobon (2002) show that heteroskedasticity would bias the estimate of the empirical correlation. Under some assumptions, they show that high volatility would bias the correlation upward. As a robust check, we also compute the "adjusted correlation", similar to the "conditional correlation" of Forbes and Rigobon (2002). Let $corr_t$ be the unadjusted correlation, computed in the standard way, then the adjusted correlation is:

$$\begin{aligned} corr_t^* &= corr_t\sqrt{\frac{1+\delta_t}{1+\delta_t * corr_t^2}} \\ \text{with } \delta_t &= \frac{V_t}{V_0} - 1 \qquad (2) \end{aligned}$$

where V_t is the average volatility across all stocks at month t, and V_0 is the mean of the average variance across time.

[6] We apply this rule to keep the number of stocks included for computation of average correlation for each month more stable.

Note that because we do the adjustment at the aggregate level, this adjustment is not exact under the assumptions of Forbes and Rigobon (2002), though it does reflect the main idea of their paper.

2.3 The correlation risk factor: innovations in the average correlation

Once we have constructed the average correlation series, we need a model to filter the correlation series to get the innovations as the correlation risk factor. But there is no consensus about which model to use in obtaining the innovations. In theory, the chosen model should reflect the information set of the representative agent. By examining the dependence structure of the average correlation series, we choose the following $AR(5)$ model, which has the best in-sample fit according to the Bayesian Information Criterion (BIC) for all the auto-regressive models with orders of less than 10.

$$corr_t = a + b_1 corr_{t-1} + b_2 corr_{t-2} + b_3 corr_{t-3} + b_4 corr_{t-4} + b_5 corr_{t-5} + \epsilon_t^c \tag{3}$$

After estimating (3), we define the correlation risk factor used in this paper as $C_t \equiv \widehat{\epsilon_t^c}$, which is the residual estimate from the above regression. The main results are obtained by using the value-weighted average of equity correlations. The equal-weighted average will be used as a robustness check.

2.4 Empirical features of the average correlation

First we can see the dynamics of the value-weighted and equal-weighted average correlation in Figure 1A, together with the US business cycle. The average correlation ($corr_t$) spikes up in any financial crisis. And once it spikes up, it takes a long time to decay, especially for the value-weighted average correlation. Two $corr_t$ series are tracking each other quite closely, and the sample correlation between them is 0.76. But the value-weighted $corr_t$ has a higher mean than the equal-weighted one, reflecting the fact that the large firms move together more closely.

To see the relative contribution of the $corr_t$ and average variance to the market variance, we report their correlations in Table 1. For both the value-weighted $corr_t$ and equal-weighted $corr_t$, their correlations with market variance are lower than those with average variances. And the correlation between $corr_t$ and average variance is quite low especially for the equal-weighted ones. Since the correlation between market variance and $corr_t$ is only about 0.5, our results could be quite different from those of variance risk or co-skewness risk studies.

We also compute the correlation between $corr_t$ and other risk factors extensively studied in the literature. As shown in Table 2, $corr_t$ is negatively correlated with almost all the factors while it has the largest correlation (in absolute value) with the liquidity factor. The correlation (in absolute value) between market factor and $corr_t$ is quite large as well. Considering the $corr_t$ series are strongly auto-correlated, as shown in Figure 2, this might indicate that $corr_t$ has strong predictive power for market return, which is consistent with Pollet and Wilson (2009). To examine what macroeconomic information $corr_t$ can proxy for, we regress the $corr_t$ on the macro variables including GDP, CPI, term spread, and credit spread. From Table 3, we can see that only credit spread has weakly significant explanatory power for the value-weighted $corr_t$. But equal-weighted $corr_t$ is more significantly explained by CPI and GDP.

Apart from the contemporary relation with other factors, we also study the intertemporal relation among the average correlation, average variance and liquidity. The reason why we choose the other two series is due to their direct economic connections with the average correlation. A change in average variance reflects an information shock, while the liquidity level represents how deep the market is for a given information shock. If the information shock is large, we expect the correlation to increase. And if the liquidity is low, we expect the correlation to have further increase for a given information shock. We perform the vector auto-regression (VAR) analysis on the three variables. The number of lags is 5, chosen by likelihood ratio test. Figure 3 is the impulse response functions (IRF) from the VAR estimates. We standardize each series by its standard deviation. We can see from the middle panel of Figure 3 that both average correlation and liquidity have a sizable response to a shock to average variance. But a shock to average correlation or liquidity has limited effect on the other two variables. The Granger Causality Tests results in Table 4 confirm these findings.

Finally, Figure 1B compares the unadjusted average correlation and adjusted average correlation defined as (2). The sample correlation between them is 0.9569, which is evidence that the adjustment does not have a significant effect, and our proxy reflects the real variation in general diversification effect of the whole market. Also, our results from IRF analysis seem to be in favour of contagion story, which is different from Forbes and Rigobon (2002)[7].

[7] Of course this is not direct evidence on contagion effect, since an increase in average volatility could be from news affecting the fundamentals of the whole market.

3 Sorting Portfolios by Correlation Betas

To implement the GMM estimation in the next section, we group all the eligible stocks into portfolios according to their sensitivities to the innovations of the average correlation, the correlation betas. Let $r_{i,t}$ denote stock $i's$ excess return, C_t the innovation of the average correlation, and F_t a vector of other factors. Then the correlation beta, β_i^c, of each stock is defined as in the following regression:

$$r_{i,t} = \beta_i^0 + \beta_i^F F_t + \beta_i^c C_t + e_{i,t} \tag{4}$$

At the end of each month, starting from January 1968, we sort stocks into ten portfolios basing on their historical correlation beta estimates. Each month's historical betas are obtained from the regression (4) using data in the past 60 month in a rolling-window way. The post-formation monthly returns on these portfolios are then linked across months and years to form a single return series for each decile portfolio. These returns of decile portfolios will form the basis for the GMM estimation of correlation risk premium in next section. Our sample includes all common shares traded on the NYSE and AMEX. As a common practice in large cross-sectional studies of equity returns, we also exclude stocks with prices below \$5 or above \$1000. To be include into the portfolios of a month, a stock also has to have at least 30 monthly returns up to last month.

The factors, F_t, considered in this paper include the market factor, MKT_t, which is the market return in excess of the 3-month treasure bill rate, Fama-French's HML_t and SMB_t, the momentum factor, MOM_t, and the liquidity factor, LIQ_t. So β_i^c will capture the stock's comovement with the average correlation in the market that is distinct from its comovement with these factors. The same factors will be included in the model for GMM estimation in the next section.

Before we formally estimate the correlation risk premium, we first regress the returns of the 10 portfolios sorted by correlation-betas on the vectors F_t, and study the intercept estimate α_j, the "excess return", in the following regression:

$$r_{j,t} = \alpha_j + \beta_j^F F_t + e_{j,t} \quad \text{for } j = 1, 2, 3....10 \tag{5}$$

If a stock's sensitivity to the innovations of the average correlation is related to its expected return, the returns on the 10 beta portfolios should have significantly different alpha estimates. As a benchmark case, we report in Table 5 the alpha estimates of (5) for the ten beta portfolios when F_t

only includes MKT_t, HML_t and SMB_t. If the correlation risk premium is negative, there should be a monotonic decreasing pattern in alpha estimates. Since stocks with high correlation betas provide some sort of insurance against adverse market environment of high average correlation, investors would pay a premium for these stocks in equilibrium. This implies that stocks with higher correlation betas should have lower expected returns after controlling for other risk factors. But from the results in Table 5, it seems that there is no clear pattern in alpha estimates although we do find some alpha estimates significantly different from zero. Overall, we can not find strong evidence in favour of priced correlation risk only from these post-ranking alpha estimates. Results of alpha estimates for F_t including more factors also do not show any clear pattern, and are not reported in the paper.

In the next section, we formally estimate the correlation risk premium with the ten return series of portfolios sorted by correlation-betas. Note that we always include the same set of risk factors in F_t during portfolio-sorting stage and the following estimation stage.

4 GMM Estimation of the Correlation Risk Premium

In the previous section, we try to infer the correlation risk premium through the spreads in the expected returns of the decile portfolios. Now we formally estimate the correlation risk premium using all 10 portfolios sorted on correlation-betas. The two-pass regression procedure of Fama-MacBeth (1973) is widely used in risk factor studies due to its easy implementation. But using the beta estimates as observations in the second-stage regression produces the classical errors-in-variable problem. Shanken (1992) develops a correction procedure to correct for the errors-in-variable problem. But his method relies on the assumption of i.i.d. errors in the time series regression, which is very strict considering the vast literature on conditional heteroskedasticity modelling of financial returns. To completely avoid the e.i.v. problem and make minimum assumptions on the distribution of the errors term, we adopt the GMM framework of Hansen (1982) to estimate the correlation betas and correlation risk premium (λ_C) in one pass. The same approach has been used by Pastor and Stambaugh (2003) in their study of liquidity risk pricing.

In this section, we first describe the GMM estimation procedure, and then present the main empirical results. Finally, we do robustness test on the results by using alternative correlation risk measures.

4.1 The GMM estimation procedure

The basic idea of the GMM procedure is to use the pricing relation constraints as moment conditions to jointly estimate the correlation betas of all ten portfolios as well as the correlation risk premium in the factor models. First, define the multivariate regression:

$$r_t = \beta_0 + BF_t + \beta^c C_t + e_t \tag{6}$$

where r_t is a 10×1 vector containing the excess returns on the tested portfolios, and F_t is a $k \times 1$ vector of the realizations of k traded factors. B is a $10 \times k$ coefficient matrix, while β_0 and β^c are 10×1 vectors of intercepts and correlation-betas respectively. According to the standard non-arbitrage pricing theory, portfolios are priced by their returns' sensitivities to the factors, as follows:

$$E(r_t) = B\lambda_F + \beta^c \lambda_C \tag{7}$$

To impose the pricing constraint on the parameters in equation (6), we take expectations of both sides of equation (6), and substitute β_0 using equation (7). This gives us:

$$\beta_0 = \beta^c[\lambda_C - \mu_C] \ with \ \ \mu_C \equiv E(C_t) \tag{8}$$

Equation (8) results from the fact that F_t is a vector of traded factors such that their risk premiums λ_F are equal to their expectations $E(F_t)$. But the correlation risk factor is not tradable, so its risk premium is not equal to its expectation[8]. In the following estimation, the mean of the correlation factor is zero by construction because C_t is the innovation of the level process. To apply the GMM method, we first define the moments to be equal to zero from the multivariate regression with constraint (8). Let $\theta \equiv (B, \beta^c, \lambda_C)$ be the unknown parameters to be estimated, then the sample moments are defined as $g(\theta) = (1/T)\sum_{t=1}^{T} f_t(\theta)$ with :

[8]The liquidity factor we use is not tradable either. So for the case of including liquidity factor, the constraint (8) becomes: $\beta_0 = \beta^c[\lambda_C - \mu_C] + \beta^L[\lambda_L - \mu_L] \ with \ \ \mu_C \equiv E(C_t)$ and $\mu_L \equiv E(LIQ_t)$, where β^L is liquidity beta and λ_L is the liquidity risk premium.

$$
\begin{aligned}
f_t(\theta) &= h_t \otimes e_t && (9)\\
\text{with } h_t^{'} &= \begin{pmatrix} 1 & F_t^{'} & C_t \end{pmatrix}, \\
e_t &= r_t - \beta^c \lambda_C - BF_t - \beta^c C_t
\end{aligned}
$$

The moments in (9) reflect the pricing constraint (8) through the definition of e_t in the last line of (9). Now we define the GMM estimator as:

$$\theta_{GMM} = \arg\min_{\theta} g(\theta)^{'} W g(\theta)$$

where W is a consistent estimator of the optimal weighting matrix, which is estimated as the inverse of $(1/T)\sum_{t=1}^{T} f_t(\widetilde{\theta}) f_t(\widetilde{\theta})'$ where $\widetilde{\theta}$ is a consistent estimator of θ. During the actual estimation, we can choose what factors to include in F_t as controls, which would be same set of factors used during the portfolio sorting stage.

4.2 Empirical results of the GMM estimation

The main results are reported in Tables 6, 7 and 8, for different sets of risk factors F_t. The focus of this paper is the estimate of the correlation risk premium, λ_C, with different sets of controlling variables. A negative and significant estimate will be consistent with the standard asset pricing theory. To show the actual magnitude of the annual compensation for exposure to the correlation risk, we report the "10-1" return spread, which goes long on decile 10 (stocks with highest correlation betas) and short on decile 1 (stocks with lowest correlation betas). We also report the "9-2" return spread defined in a similar way. The GMM procedure also provides us the correlation beta estimate β^c for each portfolio. In each table, we also divide the sample into two sub-samples defined as the periods before and after Jan. 1983[9].

We first discuss the case in which F_t only includes MKT_t, HML_t and SMB_t. The estimation results are reported in Table 6. In the full sample, the estimate of correlation risk premium λ_C is negative and significant at 10% level; the annual "10-1" and "9-2" spreads are -2.07% and -4.08% respectively, both of which are significant at 10% level. However, the "9-2" spread is larger and

[9]This is the same as Pastor and Stambaugh (2003). And this sub-sampling date is also adopted in many macro studies as the US monetary policy has experienced important structural changes and unusual operating procedures around 1983 (see Bagliano and Favero, 1998, and Clarida et al, 2000).

more significant than the "10-1" spread. Since the return spread is the product of λ_C and the difference in correlation betas, this suggests that the correlation beta estimates are not monotonic along the portfolio deciles, which is the case in the results. This also explains why there is no monotonic pattern in the post-ranking alpha estimates in the results of the previous section[10]. From the results in the two sub-samples, we can see that the estimates of the correlation risk premium as well as portfolio spreads are more significant for the period before 1983 than for the period after 1983. There are also more significant estimates of β^c in the first sub-sample.

Next, we report in Table 7 the results of the estimations when F_t includes MKT_t, HML_t, SMB_t, and the momentum factor MOM_t. Comparing the results with those in Table 6, we can see that the estimates of the correlation risk premium as well as the portfolio spreads are even more significant when we control for the momentum factor. But the correlation beta estimates are still not very significant or monotonic across the portfolios. The relative strength of the results in two sub-samples is similar to the previous case: the first sub-sample has stronger results.

Finally, we further add the liquidity factor in F_t and estimate the correlation risk premium in a six-factor model. Since the liquidity factor has the highest (absolute) correlation with the average correlation among all considered factors, inclusion of the liquidity factor might "explain away" some of the variation in the expected returns and reduce the significance level of the correlation premium estimates. This is confirmed by the results reported in Table 8. Estimates of the correlation risk premium and portfolio spreads are still significant, but less so than those in Table 7. However, their significance levels are comparable to those shown in Table 6 where F_t includes only the market factor and FF factors. But as in the previous two tables, the correlation beta estimates are not very significant.

Overall, we find a negative and significant correlation risk premium in the full sample and the first sub-sample. The return spread between high correlation-beta stocks and low correlation beta stocks is about -2% to -6% annually, depending on the controlling factors and samples. This magnitude is comparable to the liquidity premium found in the literature. For example, Pastor and Stambaugh (2003) estimate a return spread of 7.5% between the high liquidity-sensitive stock portfolio and the low liquidity-sensitive stock portfolio. Acharya and Pedersen (2005) find that there is a difference between the highest and lowest liquidity portfolio return corrected for the other risk

[10] So it is likely that our results can not pass a stricter test which penalises the lack of monotonicity in beta estimates, such as Patton and Timmermann (2008)

factors, of 4.6% per year. So the correlation risk premium found in this paper is also economically significant. As mentioned in the introduction section, if we view the average correlation as a proxy for some unobservable macro variables, the exact sign of the risk premium of the average correlation depends on the representative agent's risk aversion and the average correlation's relative forecasting power on market returns and market variance. The above results indicate that the average correlation's role as a risk forecaster dominates its role as a forecaster for future market return.

The results are generally less significant in the second sub-sample, which is difficult to explain. If investors' investment in stock markets is becoming more and more diversified, we should expect to find stronger result in the more recent sample period. One explanation is that the stock market index is not an accurate proxy for the "market portfolio" of the CAPM or ICAPM. So the average correlation we calculate by using only the US equity returns does not fully reflect the time-varying diversification opportunity that US investors face in the actual world. This obviously becomes more true in the second sub-sample as global asset allocation is becoming an increasingly common practice in the investment industry. So a stock with high sensitivity to the local aggregate asset correlation might have low sensitivity to the global aggregate asset correlation. If this is possible, other empirical studies focusing on local risk measures might suffer similar problems. Another explanation is related to the results on the correlation-beta estimates. Considering both the full sample and the two sub-samples, we do not find a monotonic pattern in correlation-beta estimates. Since the portfolios are sorted on correlation-betas estimated from the most recent 60 months, the fact that the monotonic pattern in beta breaks down in the following month indicates that there is large variation in the correlation betas. A larger variation in the correlation-beta in the second sub-sample could produce the weaker results in the second sub-sample. To explore these two possibilities, we would need to modify our data and estimation procedure, which we leave for future research.

4.3 Robustness tests

Our robustness tests mainly consist of using different correlation risk measures. First, we repeat the whole estimation procedure using the equal-weighted average correlation. Since the value-weighted average correlation might place too much emphasis on the correlation between large firms, using equal-weighted average correlation might give us different results. We only report the results for the case of F_t containing all of the six factors we consider, presented in Table 9. The results are very similar to those using the value-weighted average correlation both in the full sample and in the two sub-samples. The correlation risk premium estimate is still significantly negative.

As a further robustness check, we use only the 500 biggest companies to construct the average correlation, both value-weighted and equal-weighted. Again, we only report the results of estimation in a six-factor model. The value-weighted case is presented in Table 10 while Table 11 presents the equal-weighted case. The results in both tables are very similar to those in Table 8 and Table 9. There is evidence of a significantly negative correlation risk premium. The significance of the estimates is stronger in the sample before 1983. The "9-2" spread is mostly larger than "10-1" spread, and more statistically significant. But only very few correlation beta estimates are significant. We do not find a monotonic pattern in beta estimates. The above results suggest that our main findings in the previous section are not affected by the method and type of stocks included in constructing the average correlation series.

Finally, we also conduct a test on the significance of return difference in the decile portfolios sorted on correlation-betas estimated in the six factors model. As we can see in Table 8, we have significant estimates of the product of beta-difference and risk premium (lambda). However, since most of the beta estimates are not significant, the significance of the lambda estimates (and also the product estimates) might not be reliable. To have another check on the result of Table 8, we construct the returns of the long-short portfolios, with "10-1" portfolio consisting of longing 10th decile stocks and shorting 1st decile stocks, and similar definition for the "9-2" portfolio. Note that the decile portfolios are sorted in the same way as we do for Table 8. Then, we regress the two "long-short" portfolio returns on the same set of risk factors used in sorting the decile portfolio but excluding the correlation risk factor. Finally, we get the estimates of the intercept (alpha) of the regressions for both of the two "long-short" portfolios. If the correlation risk is priced we should expect a significant estimate of alpha, which should be similar to the estimate of the product of the beta-difference and lambda. The results are reported in Table 12, which should be compared with the results of Table 8. We can see the results are much weaker than those of Table 12. And we only find marginally significant alpha estimate for the "9-2" portfolio, although the signs of all the estimates are still negative, consistent with standard asset pricing theories' prediction.

5 Concluding Remarks

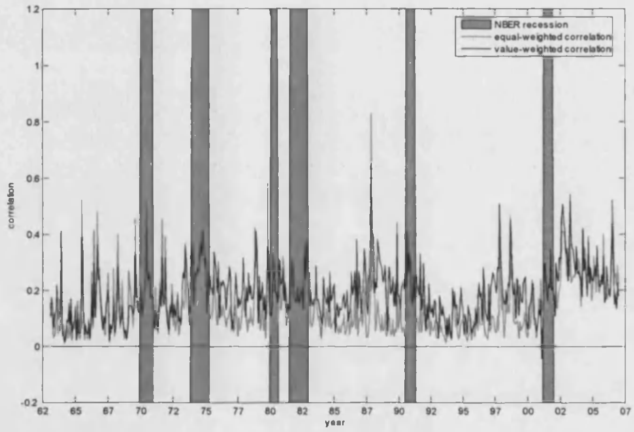
The average correlation in equity markets is an important risk factor investors face when making investment decisions. Recent empirical findings suggest that the average correlation instead of market index variance might be a better proxy for the general risk level in the whole market. In this paper, we investigate the pricing implication of the average correlation in the US equity market. We estimate the correlation risk premium in a GMM framework, which jointly estimates the correlation betas and correlation risk premium to avoid the classical errors-in-variable problem of the two-pass regression approach. Using the cross-section of US equity returns from 1963 to 2006, we find that there is a significantly negative correlation risk premium in the cross-section of the US equity market in the full sample. This result is robust to controlling for Fama-French (FF) (1993)'s SMB and HML factors, as well as Carhart (1997) momentum factor and the liquidity factor of Pastor and Stambaugh (2003). The return spread between the high correlation-beta stock portfolio and the low correlation-beta stock portfolio is about -2% to -6% annually, depending on the samples and controlling factors. The magnitude is comparable to the premium found for other risk factors in the literature. The results are stronger in the sub-period before 1983. There are mainly two issues in this analysis that could be further explored: First, the results in the sub-sample after 1983 are less significant. This might be due to the increasing globalization of investment industry, which makes the average correlation constructed using only US equities an inaccurate measure to represent the time-varying diversification opportunities investors actually face today. Another potential problem is that we fail to find monotonic estimates of the correlation-betas. This might be due to the large variation of the correlation-beta across time. Future research could investigate the correlation risk premium by constructing a global aggregate correlation series, and estimate the premium in a time-varying beta model .

Appendix: graphs and tables

Figure 1: Average correlations

This figure shows the average correlations from Jan. 1963 to Dec. 2006. Panel A shows the value-weighted and equal-weighted average correlations, together with NBER recession periods indicated by vertical lines. Panel B shows the unadjusted average correlations and the adjusted ones according to (2). Both are value-weighted average.

Panel A: Value-weighted and equal-weighted average correlations



Panel B: Adjusted and unadjusted average correlations

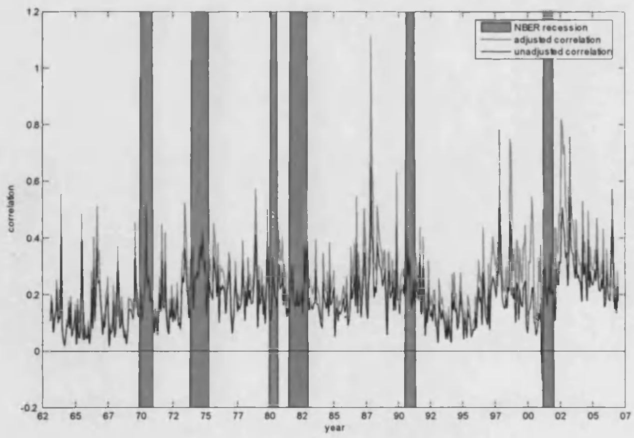
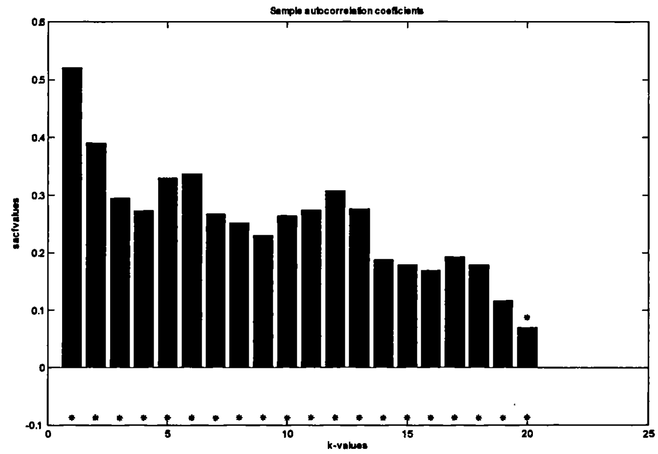


Figure 2 Sample autocorrelation function

This picture plots the sample autocorrelation of the average correlation and variance of market index. The average correlation are value-weighted average of daily correlations for each pair of return across all assets for each month. Variance of the market index and the monthly realized variance of the value-weighted market index. Sample period is from Jan. 1961 to Dec. 2006.

Panel A:: SACF of Average correlation



Panel B: SACF of market variance

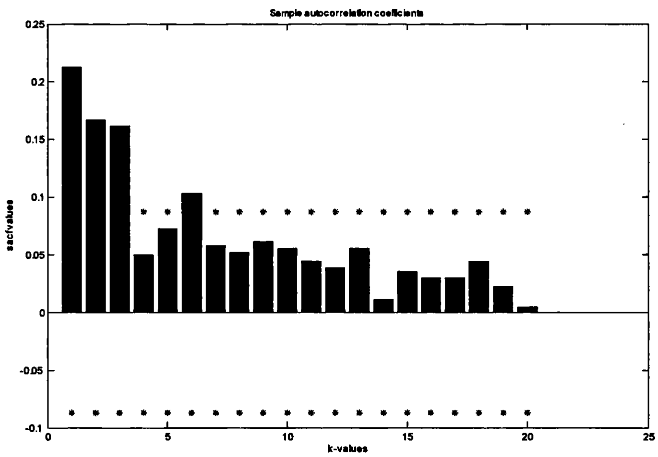


Figure 3: Impulse response function: average correlation with average variance

This figure plots the impulse response functions from the estimations of a vector-auto regression of order 5 (VAR(5)) for the three series: average correlation, average variance and liquidity factor. And the "average" here means value-weighted average.

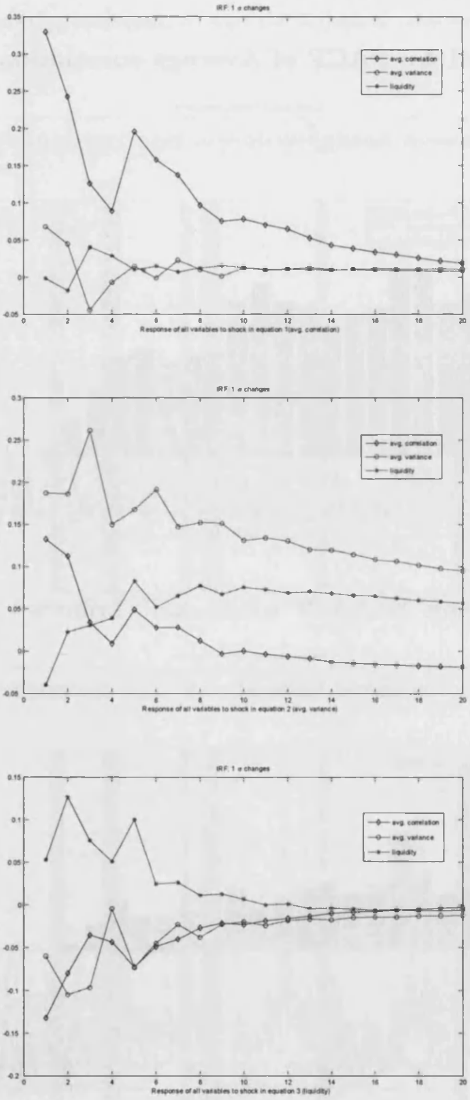


Table 1: Correlation between Market Variance, Average Variance and Average Correlation

This table reports the correlation between the market (index) variance, average correlation and average variance. The market variance σ_M^2 is computed as the sum of squared daily returns of valued-weight market index within each month. The average variance σ_A^2 is the average of variance of all individual stock while the variance of each stock is computed as the sum of squared daily returns within each month. The average correlation ρ_A is the average of daily correlation for each pair of stocks within each month. Both value-weighted average and equal-weighted average are computed for σ_A^2 and ρ_A. The table also reports the mean, minimum and maximum of each series.

Value-weighted average						
	σ_M^2	σ_A^2	ρ_A	mean	min.	max
σ_M^2	1	0.723	0.520	0.00154	0.0000718	0.05760
σ_A^2	0.723	1	0.293	0.0087	0.0024	0.0926
ρ_A	0.520	0.293	1	0.1951	-0.0423	0.6474
Equal-weighted average						
	σ_M^2	σ_A^2	ρ_A	mean	min	max
σ_M^2	1	0.560	0.537	0.00154	0.0000718	0.05760
σ_A^2	0.560	1	0.242	0.0165	0.0062	0.0885
ρ_A	0.537	0.242	1	0.1476	0.0129	0.8273

Table 2: Correlation between the Average Correlation and Other Risk Factors

This table summarizes the correlation between the correlation risk proxy and other factors, which include Fama-French's SMB and HML factors, liquidity factor of Pastor and Stambaugh (2002), and the momentum factor.

Value-weighted average correlation					
	Market	SMB	HML	Momentum	Liquidity
Average Correlation	-0.282	-0.252	-0.007	-0.008	-0.415
Equal-weighted average correlation					
	Market	SMB	HML	Momentum	Liquidity
Average Correlation	-0.385	-0.166	0.134	-0.171	-0.437

Table 3: Results on Regressions of the Average Correlations on Macro Variables

This table reports the result of regressions of the average correlations on selected macro variables. The upper table is for value-weighted correlations and lower table for equal-weighted correlations. The t-statistics are in parentheses

Value-weighted average correlation					
Intercept	GDP	CPI	Term Spread	Credit Spread	adj. R^2
0.1723	-0.6958	-2.5203	0.0042	0.0357	0.032
(6.7897)	(-0.6484)	(-0.8838)	(0.4845)	(1.7968)	

Equal-weighted average correlation					
Intercept	GDP	CPI	Term Spread	Credit Spread	adj. R^2
0.1359	-1.5663	-6.3166	0.0034	0.0208	0.034
(5.4790)	(-1.7177)	(-2.4320)	(0.3112)	(1.0373)	

Table 4: Granger Causality Tests

This table reports the F statistics of Granger causality tests in a vector-auto regression of order 5 (VAR(5)) for the three series: average correlation, average variance and liquidity factor. And the "average" here means value-weighted average. The p-values are in parentheses.

Shock variables	**Response variables**		
	Average Correlation	Average Variance	Liquidity Factor
Average Correlation	34.29	1.29	0.29
	(0.000)	(0.262)	(0.913)
Average Variance	4.86	267.07	3.38
	(0.000)	(0.000)	(0.005)
Liquidity Factor	1.16	1.08	5.67
	(0.323)	(0.367)	(0.000)

Table 5

Alphas of Value-Weighted Portfolios Sorted on Correlation Betas

For each month between 1968 and 2006, eligible stocks are sorted into 10 portfolios according to their historical correlation betas estimated in the regression below using monthly returns in the immediately past 5 years. Eligible stocks are defined as ordinary common shares traded on NYSE, AMEX, and NASDAQ with at least three years of monthly returns, and with the stock prices between \$5 and \$1,000. The monthly post-ranking portfolio returns are then linked across months and years to form series of post-ranking returns for each decile. The table reports the decile portfolios' post-ranking alphas which is the intercept estimate by regressing portfolios' excess returns only on F_t. All alphas are multiplied by 100. The t-statistics are in parentheses.

$$\begin{aligned} r_t &= \alpha + BF_t + \beta^c C_t + e_t \\ \text{with } F_t &= [MKT_t \;\; SMB_t \;\; HML_t] \end{aligned}$$

	alpha									
	1	2	3	4	5	6	7	8	9	10
Jan. 1968-Dec 2006	-0.11	0.00	-0.07	0.01	-0.05	0.02	-0.01	0.13	-0.10	-0.19
	(0.33)	(0.10)	(-1.01)	(0.22)	(-0.74)	(0.29)	(-1.41)	(1.70)	(-1.33)	(-1.90)
Jan. 1968-Dec.1982	0.05	-0.04	-0.08	-0.06	0.01	-0.12	0.13	0.15	-0.09	-0.02
	(-0.58)	(-0.38)	(-0.73)	(-0.56)	(0.13)	(-1.25)	(1.41)	(1.67)	(-0.82)	(-0.12)
Jan. 1983-Dec 2006	-0.23	0.03	-0.06	0.09	-0.06	0.13	-0.25	0.12	-0.10	-0.27
	(-1.66)	(0.26)	(-0.73)	(1.11)	(-0.72)	(1.57)	(-2.31)	(1.14)	(-1.06)	(-2.16)

Table 6

Correlation Risk Premium Estimate and its Contribution to Equity Expected Return in a Four-factor Model

—value-weighted correlation as correlation risk proxy—

This table reports the GMM estimates of the correlation risk premium and the correlation beta of each decile portfolio, as well as the contribution of correlation risk to the expected return on the "10-1" spread and "9-2" spread. The model includes four factors: the market factor MKT_t, the FF's SMB_t and HML_t, and the value-weighted correlation factor C_t. Stocks are sorted into 10 portfolios by their historical correlation betas at end of each month. The premium λ_c is the GMM estimate using post-ranking returns on all 10 portfolios. β^c is the correlation beta estimate of each portfolio. The contribution of correlation risk premium to the portfolio's expected return, $\lambda_c \times \triangle\beta^c$, is multiplied by 1200, so that it is the annual percentage return. The t-statistics are in parentheses.

$$
\begin{aligned}
r_t &= \beta_0 + BF_t + \beta^c C_t + e_t \\
\text{with } F_t &= [MKT_t \;\; SMB_t \;\; HML_t]
\end{aligned}
$$

	β^c										λ_c	$\lambda_c \times \triangle\beta^c$	
	1	2	3	4	5	6	7	8	9	10		10-1	9-2
Jan. 1968-Dec 2006	0.005	-0.003	0.006	-0.003	0.000	-0.001	0.006	-0.008	0.010	0.014	-0.127	-2.07	-4.08
	(0.71)	(-0.49)	(1.10)	(-0.61)	(0.04)	(-0.23)	(1.06)	(-1.24)	(1.50)	(1.44)	(-1.85)	(-1.81)	(-3.32)
Jan. 1968-Dec. 1982	-0.018	-0.001	0.005	0.004	0.002	0.015	-0.006	-0.019	0.043	0.011	-0.034	-2.43	-3.70
	(-1.14)	(-0.09)	(0.42)	(0.36)	(0.15)	(1.31)	(-0.53)	(-1.64)	(3.39)	(0.52)	(-2.15)	(-2.16)	(-3.47)
Jan. 1983-Dec 2006	0.006	0.000	0.002	-0.003	-0.000	-0.003	0.006	-0.002	0.003	0.009	-0.324	-1.93	-2.24
	(0.82)	(0.05)	(0.59)	(-0.86)	(-0.00)	(-0.79)	(0.92)	(-0.64)	(0.71)	(0.89)	(-1.01)	(-0.90)	(-1.15)

Table 7

Correlation Risk Premium Estimate and its Contribution to Equity Expected Return in the Five-factor Model

—value-weighted average correlation as correlation risk proxy—

This table reports the GMM estimates of the correlation risk premium and correlation beta of each decile portfolio, as well as the contribution of correlation risk to the expected return on the "10-1" spread and "9-2" spread. The model includes four factors: the market factor MKT_t, the FF's SMB_t and HML_t, the momentum factor MOM_t, and the value-weighted correlation factor C_t. Stocks are sorted into 10 portfolios by their historical correlation betas at end of each month. The premium λ_c is the GMM estimate using post-ranking returns on all 10 portfolios. β^c is the correlation beta estimate of each portfolio. The contribution of correlation risk premium to the portfolio's expected return, $\lambda_c \times \triangle\beta^c$, is multiplied by 1200, so that it is the annual percentage return. The t-statistics are in parentheses.

$$\begin{aligned} r_t &= \beta_0 + BF_t + \beta^c C_t + e_t \\ \text{with } F_t &= [MKT_t \;\; SMB_t \;\; HML_t \;\; MOM_t] \end{aligned}$$

	β^c										λ_c	$\lambda_c \times \triangle\beta^c$	
	1	2	3	4	5	6	7	8	9	10		10-1	9-2
Jan. 1968-Dec 2006	0.007	-0.008	0.004	-0.003	0.000	-0.002	0.007	-0.006	0.012	0.013	-0.125	-1.63	-6.25
	(0.97)	(-1.19)	(0.60)	(-0.79)	(0.20)	(-0.53)	(1.17)	(-1.13)	(1.65)	(1.44)	(-2.02)	(-1.05)	(-4.61)
Jan. 1968-Dec. 1982	-0.006	-0.004	-0.002	0.002	0.005	0.014	-0.006	-0.017	0.044	0.018	-0.041	-2.50	-4.77
	(-0.43)	(-0.27)	(-0.19)	(0.22)	(0.47)	(1.22)	(-0.51)	(-1.51)	(3.44)	(0.86)	(-2.37)	(-1.90)	(-3.99)
Jan. 1983-Dec 2006	0.011	-0.008	0.006	-0.006	-0.000	-0.009	0.011	-0.004	0.006	0.128	-0.164	-0.60	-5.51
	(1.08)	(-0.98)	(0.81)	(-1.25)	(-0.09)	(-1.29)	(1.41)	(-0.77)	(0.89)	(1.33)	(-2.03)	(-0.29)	(-2.67)

Table 8

Correlation Risk Premium Estimate and its Contribution to Equity Expected Return in the Six-factor Model

—value-weighted average correlation as correlation risk proxy—

This table reports the GMM estimates of the correlation risk premium and correlation beta of each decile portfolio, as well as the contribution of correlation risk to the expected return on the "10-1" spread and "9-2" spread. The model includes four factors: the market factor MKT_t, the FF's SMB_t and HML_t, the momentum factor MOM_t, the liquidity factor LIQ_t, and the value-weighted correlation factor C_t. Stocks are sorted into 10 portfolios by their historical correlation betas at end of each month. The premium λ_c is the GMM estimate using post-ranking returns on all 10 portfolios. β^c is the correlation beta estimate of each portfolio. The contribution of correlation risk premium to the portfolio's expected return, $\lambda_c \times \Delta\beta^c$, is multiplied by 1200, so that it is the annual percentage return. The t-statistics are in parentheses.

$$\begin{aligned} r_t &= \beta_0 + BF_t + \beta^c C_t + e_t \\ \text{with } F_t &= [MKT_t \;\; SMB_t \;\; HML_t \;\; MOM_t \;\; LIQ_t] \end{aligned}$$

	β^c										λ_c	$\lambda_c \times \Delta\beta^c$	
	1	2	3	4	5	6	7	8	9	10		10-1	9-2
Jan. 1968-Dec 2006	0.007	-0.003	0.009	-0.007	0.001	-0.005	0.015	-0.007	0.013	0.009	-0.132	-0.66	-5.48
	(0.84)	(-0.41)	(1.13)	(-1.37)	(0.27)	(-0.85)	(1.69)	(-1.00)	(1.81)	(0.89)	(-1.88)	(-0.36)	(-3.51)
Jan. 1968-Dec. 1982	-0.008	-0.000	0.006	0.003	-0.001	0.008	0.004	-0.013	0.050	0.007	-0.045	-1.74	-5.51
	(-0.45)	(-0.01)	(0.48)	(0.26)	(-0.14)	(0.63)	(0.32)	(-1.04)	(3.44)	(0.32)	(-2.08)	(-1.07)	(-3.16)
Jan. 1983-Dec 2006	0.010	-0.001	0.009	-0.012	0.001	-0.012	0.015	-0.007	0.003	0.009	-0.170	0.52	-2.17
	(0.88)	(-0.16)	(1.06)	(-1.76)	(0.25)	(-1.55)	(1.54)	(-1.06)	(0.46)	(0.75)	(-1.80)	(0.21)	(-0.73)

Table 9: Robustness Check (1)-equal-weighted correlation

Correlation Risk Premium Estimate and its Contribution to Equity Expected Return in the Six-factor Model

—equal-weighted correlation as correlation risk proxy—

This table reports the GMM estimates of the correlation risk premium and correlation beta of each decile portfolio, as well as the contribution of correlation risk to the expected return on the "10-1" spread and "9-2" spread. The model includes four factors: the market factor MKT_t, the FF's SMB_t and HML_t, the momentum factor MOM_t, the liquidity factor LIQ_t, and the equal-weighted correlation factor C_t. Stocks are sorted into 10 portfolios by their historical correlation betas at end of each month. The premium λ_c is the GMM estimate using post-ranking returns on all 10 portfolios. β^c is the correlation beta estimate of each portfolio. The contribution of correlation risk premium to the portfolio's expected return, $\lambda_c \times \triangle\beta^c$, is multiplied by 1200, so that it is the annual percentage return. The t-statistics are in parentheses.

$$\begin{aligned} r_t &= \beta_0 + BF_t + \beta^c C_t + e_t \\ \text{with } F_t &= [MKT_t \ \ SMB_t \ \ HML_t \ \ MOM_t \ \ LIQ_t] \end{aligned}$$

	β^c										λ_c	$\lambda_c \times \triangle\beta^c$	
	1	2	3	4	5	6	7	8	9	10		10-1	9-2
Jan. 1968-Dec 2006	-0.022	-0.003	0.008	0.002	-0.004	-0.015	-0.015	-0.019	-0.000	-0.002	-0.047	-2.30	-0.38
	(-2.09)	(-0.32)	(0.87)	(0.23)	(-0.50)	(-1.61)	(-1.38)	(-1.81)	(-0.02)	(-0.22)	(-1.68)	(-1.88)	(-0.40)
Jan. 1968-Dec 1982	-0.020	-0.013	0.001	-0.003	-0.000	-0.000	0.010	-0.033	0.022	-0.003	-0.083	-3.50	-7.05
	(-1.65)	(-1.21)	(0.12)	(-0.32)	(-0.04)	(-0.08)	(1.20)	(-2.86)	(1.69)	(-0.20)	(-3.34)	(-1.98)	(-3.73)
Jan. 1983-Dec 2006	-0.026	0.000	0.015	0.020	-0.006	-0.021	-0.025	-0.018	-0.026	-0.012	-0.007	-0.24	0.46
	(-1.74)	(0.00)	(1.12)	(2.15)	(-0.51)	(-1.68)	(-1.79)	(-1.29)	(-1.90)	(-0.79)	(-0.19)	(-0.34)	(0.37)

Table 10: Robustness Check (2)-Average correlation of 500 largest firms

Correlation Risk Premium Estimate and its Contribution to Equity Expected Return in the Six-factor Model

—value-weighted correlation as correlation risk proxy—

This table reports the GMM estimates of the correlation risk premium and correlation beta of each decile portfolio, as well as the contribution of correlation risk to the expected return on the "10-1" spread and "9-2" spread. The model includes four factors: the market factor MKT_t, the FF's SMB_t and HML_t, the momentum factor MOM_t, the liquidity factor LIQ_t, and the value-weighted correlation factor C_t, constructed from the 500 largest firms. Stocks are sorted into 10 portfolios by their historical correlation betas at end of each month. The premium λ_c is the GMM estimate using post-ranking returns on all 10 portfolios. β^c is the correlation beta estimate of each portfolio. The contribution of correlation risk premium to the portfolio's expected return, $\lambda_c \times \Delta\beta^c$, is multiplied by 1200, so that it is the annual percentage return. The t-statistics are in parentheses.

$$\begin{aligned} r_t &= \beta_0 + BF_t + \beta^c C_t + e_t \\ \text{with } F_t &= [MKT_t \;\; SMB_t \;\; HML_t \;\; MOM_t \;\; LIQ_t] \end{aligned}$$

	β^c										λ_c	$\lambda_c \times \Delta\beta^c$	
	1	2	3	4	5	6	7	8	9	10		10-1	9-2
Jan. 1968-Dec 2006	0.010	-0.003	0.007	-0.010	-0.003	0.001	0.018	-0.003	0.023	0.012	-0.113	-0.47	-7.33
	(0.90)	(-0.42)	(0.88)	(-1.60)	(-0.60)	(0.08)	(2.26)	(-0.43)	(2.89)	(1.12)	(-1.71)	(-0.228)	(-2.92)
Jan. 1968-Dec. 1982	0.018	-0.004	0.001	0.002	-0.003	0.011	0.001	-0.003	0.042	0.011	-0.083	1.51	-9.41
	(1.19)	(-0.43)	(0.10)	(0.23)	(-0.39)	(1.03)	(0.14)	(-0.30)	(4.43)	(0.62)	(-2.71)	(0.69)	(-4.73)
Jan. 1983-Dec 2006	0.020	-0.001	0.010	-0.007	-0.001	-0.010	0.010	0.002	-0.000	0.011	-0.154	3.31	-0.30
	(1.66)	(-0.18)	(1.36)	(-1.42)	(-0.43)	(-1.58)	(1.46)	(0.30)	(-0.10)	(1.24)	(-2.14)	(1.61)	(-0.17)

Table 11: Robustness Check (3)-Average correlation of 500 firms

Correlation Risk Premium Estimate and its Contribution to Equity Expected Return in the Six-factor Model

—equal-weighted correlation as correlation risk proxy—

This table reports the GMM estimates of the correlation risk premium and correlation beta of each decile portfolio, as well as the contribution of correlation risk to the expected return on the "10-1" spread and "9-2" spread. The model includes four factors: the market factor MKT_t, the FF's SMB_t and HML_t, the momentum factor MOM_t, the liquidity factor LIQ_t, and the equal-weighted correlation factor C_t, constructed from the 500 largest firms. Stocks are sorted into 10 portfolios by their historical correlation betas at end of each month. The premium λ_c is the GMM estimate using post-ranking returns on all 10 portfolios. β^c is the correlation beta estimate of each portfolio. The contribution of correlation risk premium to the portfolio's expected return, $\lambda_c \times \Delta\beta^c$, is multiplied by 1200, so that it is the annual percentage return. The t-statistics are in parentheses.

$$
\begin{aligned}
r_t &= \beta_0 + BF_t + \beta^c C_t + e_t \\
\text{with } F_t &= [MKT_t \;\; SMB_t \;\; HML_t \;\; MOM_t \;\; LIQ_t]
\end{aligned}
$$

	β^c										λ_c	$\lambda_c \times \Delta\beta^c$	
	1	2	3	4	5	6	7	8	9	10		10-1	9-2
Jan. 1968-Dec 2006	0.002	-0.010	0.007	-0.000	0.000	-0.002	0.004	-0.001	0.004	0.007	-0.198	-2.26	-6.34
	(0.52)	(-1.96)	(1.98)	(-0.17)	(0.02)	(-0.71)	(0.97)	(-0.20)	(0.85)	(1.24)	(-2.69)	(-1.25)	(-4.04)
Jan. 1968-Dec. 1982	-0.010	-0.013	-0.003	0.008	0.004	-0.003	0.004	-0.012	0.014	0.015	-0.068	-3.97	-4.49
	(-1.20)	(-1.78)	(-0.48)	(1.05)	(0.87)	(-0.40)	(0.65)	(-1.51)	(1.71)	(1.31)	(-1.80)	(-2.86)	(-3.12)
Jan. 1983-Dec 2006	0.006	-0.016	0.006	0.008	-0.005	-0.002	0.000	0.000	-0.007	-0.002	0.012	-0.28	0.25
	(0.93)	(-2.17)	(1.18)	(1.65)	(-1.00)	(-0.41)	(0.00)	(0.11)	(-1.24)	(-0.33)	(0.12)	(-0.22)	(0.22)

Table 12: Robustness Check (4)-Alpha Estimates of the "long-short" Portfolios

This table reports the intercept estimates of the regressions of "long-short" portfolios' returns on five risk factors, which are the market factor MKT_t, the FF's SMB_t and HML_t, the momentum factor MOM_t, the liquidity factor LIQ_t. Stocks are sorted into 10 portfolios by their historical correlation betas at end of each month. The correlation betas are the coefficient estimates on equal-weighted correlation factor C_t in the regression of Table 8. The "10-1" portfolio consists of longing the 10th decile stocks and shoring the 1st decile stocks. And the "9-2" portfolio is longing the 9th and shorting the 2nd stocks. We report the results for the full sample and two sub-samples. The t-statistics are in parenthesis.

	10-1	9-2
Jan. 1968-Dec 2006	0.0004	-0.0024
	(-0.28)	(-2.01)
Jan. 1968-Dec. 1982	-0.0007	-0.0028
	(-0.32)	(-1.90)
Jan. 1983-Dec 2006	-0.0001	-0.0018
	(-0.05)	(-1.14)

References

[1] Acharya, V., and L. Pedersen, (2005), "Asset pricing and liquidity risk", *Journal of Financial Economics*, 77, 375-410.

[2] Andrews, D. W. K.,(1993), "Tests for parameter instability and structural change with unknown change point", *Econometrica*, 61, 821–856.

[3] Andrews, D.W. K.,(2003), "Tests for parameter instability and structural change with unknown change point: a corrigendum", *Econometrica*, 71, 395-398.

[4] Andrews, D. W. K., and W. Ploberger, (1994), "Optimal tests when a nuisance parameter is present only under the alternative", *Econometrica*, 62, 1383-1414.

[5] Ang, A., J. Chen and Y. Xing, (2006), "Downside Risk", *Review of Financial Studies*, 19, 1191-1239.

[6] Ang, A., R. Hodrick, Y. Xing and X. Zhang, (2006), "The Cross-Section of Volatility and Expected Returns", *Journal of Finance*, 61, 259-299.

[7] Bagliano, F. C. and C. A. Favero, (1998), "Measuring monetary policy with VAR models: An evaluation", *European Economic Review*, 42, 1069–1112.

[8] Bollerslev, T. , G. Tauchen and H. Zhou, (2009), "Expected Stock Returns and Variance Risk Premium", *Review of Financial Studies,* forthcoming.

[9] Campbell, J., (1993), "Intertemporal asset pricing without consumption data", *American Economic Review*, 83(3), 487-512.

[10] Campbell, J., (1996), "Understanding risk and return", *Journal of Political Economy*, 104, 298-345.

[11] Carhart, M., (1997), "On Persistence in Mutual Fund Performance", *Journal of Finance,* 52, 57-82.

[12] Chen, J., (2003), "Intertemporal CAPM and the cross-section of stock returns", Working paper, University of Southern California.

[13] Chen, R. and X. Hao, (2009), "Forecasting Stock Market Returns and the Risk-return Trade-off", Working paper, London School of Economics.

[14] Clarida, R., J. Gali and M. Gertler, (2000), "Monetary policy rules and macroeconomic stability: evidence and some theory", *Quarterly Journal of Economics*, 115, 147–180.

[15] Driessen, J., P. J. Maenhout and G. Vilkov, (2009), "The Price of Correlation Risk: Evidence from Equity Options", *Journal of Finance*, forthcoming.

[16] Forbes, K. J. and R. Rigobon, (2002),"No Contagion, Only Interdependence: Measuring Stock Market Co-movements", *Journal of Finance*, 57(5), 2223-2261.

[17] Fama, E. and J. MacBeth, (1973), "Risk, return, and equilibrium: Empirical tests", *Journal of Political Economy*, 81, 607-636.

[18] Fama, E. and K. French, (1989), "Business conditions and expected returns on stocks and bonds", *Journal of Financial Economics*, 25, 23-49.

[19] Fama, E. and K. French, (1992), "The cross-section of expected stock returns", *Journal of Finance*, 47, 427-465.

[20] Fama, E. and K. French, (1993), "Common risk factors in the returns on bonds and stocks", *Journal of Financial Economics*, 33, 3-56.

[21] Fama, E. and K. French, (1995), "Size and book-to-market factors in earnings and returns", *Journal of Finance*, 50, 131-156.

[22] Guo, H. and R. Savickas, (2008), "Average Idiosyncratic Volatility in G7 Countries", *Review of Financial Studies*, 21(3):1259-1296.

[23] Harvey, C. R. and A. Siddique (2000) "Conditional skewness in asset pricing tests", *Journal of Finance*, 55(3), 1263-1295.

[24] Hansen, L. P., (1982), "Large Sample Properties of Generalized Method of Moments Estimators", *Econometrica*, 50, 1029-1054.

[25] Merton, R. (1973), "An intertemporal capital asset-pricing model", *Econometrica*, 41, 867-887.

[26] Moskowitz, J. T., (2003), "An Analysis of Covariance Risk and Pricing Anomalies", *Review of Financial Studies*, 16(2), 417-457.

[27] Pastor, L. and R. F. Stambaugh (2003), " Liquidity Risk and Expected Stock Return", *Journal of Political Economy*, 111, 642-685.

[28] Patton, A. and A. Timmermann (2008), "Portfolio Sorts and Tests of Cross-Sectional Patterns in Expected Returns", working paper..

[29] Pollet, J. M. and M. Wilson (2009) "Average correlation and stock market returns", *Journal of Financial Economics*, forthcoming.

[30] Shanken, J., (1990), "Intertemporal asset pricing: an empirical investigation", *Journal of Econometrics*, 45, 99-120.

[31] Shanken, J. (1992), "On the estimation of beta-pricing models", *Review of Financial Studies*, 5, 1-34.

[32] Singleton, K. J. (2006) "Empirical Dynamic Asset Pricing: Model Specification and Econometric Assessment", Princeton University Press.

[33] Vassalou, M. and Y. Xing, (2004), "Default Risk in Equity Return", *Journal of Finance*, 59(2), 831-868.

Forecasting Stock Market Returns and the Risk-Return Trade-Off

Runquan Chen
London School of Economics

Xiaoxia Hao
London School of Economics

This version: 1 May, 2009.

Abstract

We forecast stock market returns using a set of variance measures. As argued by previous studies, the stock market index can be an inaccurate proxy for the true market portfolio, thus the cross-sectional average correlation of stock returns would capture the variation in aggregate risk better than the variance of stock market index. In this paper, we first study the separate forecasting power of cross-sectional average correlation and cross-sectional average variance, and identify a significant positive risk/correlation-return relation at relatively short horizons. On the other hand, recent research found variance risk premium could also predict future movement of stock market returns. While many attribute this forecastability to unique information contained in variance risk premium, such as changing risk aversion and risk premium required to compensate stochastic volatility, we relate the variance risk premium to the average correlation and average variance. Our evidence indicates that two components of variance risk premium contain very similar information to that of average correlation and average variance. A combination of average variance and VIX index provides the best forecast of future stock market returns, beating the performance of variance risk premium and average correlation/average variance, and stays significant after controlling for traditional risk factors.

Keywords: variance risk premium, stock market return predictability, average correlation, average variance, VIX.

1 Introduction

The intertemporal risk-return relation is an age old question, and there is ongoing debate about how much, or even in which direction, variance could move stock prices. A large empirical literature has estimated a linear relation between the conditional mean and variance of return on the aggregate stock market. Following the ICAPM model derived in Campbell (1993), excess stock market return depends on market variance and the covariance with the changing future investment opportunities:

$$E_t(r_{m.t+1}) - r_{f,t+1} + \frac{V_{mm,t}}{2} = \gamma V_{mm,t} + \theta V_{mh,t} \tag{1}$$

where $V_{mm,t}$ is the market variance, $V_{mh,t}$ is covariance of market return with revisions in expected future returns and γ is the coefficient of relative risk aversion and θ is a function of parameters from the model[1]. Under certain conditions, the hedge component $V_{mh,t}$ can be omitted or represented as a linear function of market variance $V_{mm,t}$. Then the equation collapses to the test of variance in mean relation for stock market return.

Studies that have tried to determine this empirical relation between risk and return for broad stock market indices produce mixed evidences. French, Schwert, and Stambaugh (1987) measures the expected stock market returns conditional on predetermined predictive variables and models the volatility as a GARCH process. They find a negative relation. Campbell (1987), Breen, Glosten, and Jagannathan (1989), Pagan and Hong (1991), Glosten, Jaganathan, and Runkle (1993), Whitelaw (1994), Goyal and Santa-Clara (2003), Lettau and Ludvigson (2002), Brandt and Kang (2004), also find a negative or insignificant relation using different methodologies to capture the first and second moment variations of stock market returns. For example, Whitelaw (1994) models the conditional mean and variance as functions of lagged instrumental variables and estimates correlation between the fitted values of the two. Lettau and Ludvigson (2002) increase the forecasting power by using more accurate measures. Market variance is given by quarterly realized volatility. They also construct a consumption wealth ratio proxy called "CAY", together with other traditional forecasting variables as conditioning information, to predict market return and variance and then evaluate the risk-return relation. Instead of employing the idea of conditioning on state

[1] In the first three cases in Campbell (1993) paper, $\theta = \gamma - 1$, while in the last case when the intercept of consumption growth is a linear function of the expected return on the market, $\theta = \gamma - 1 - \frac{\omega\psi}{\sigma}$. Fore details, refer to Campbell(1993).

variables, Brandt and Kang (2004) assume conditional return and variance are latent and follow a bivariate Gaussian first-order VAR, and estimate the relation using a filtering algorithm. Contrary to the above results, Bollerslev, Engle and Wooldridge (1988), Bali and Peng (2006) and Ghysels, Santa-Clara and Valkanov (2005) find a positive relation between conditional mean and variance. More recent studies such as Bali and Peng (2006) use intraday data to evaluate risk-return trade-off at a daily frequency. Ghysels, Santa-Clara and Valkanov (2005) claim to be able to extract more information from higher frequency data using mixed data sampling regression so as to give a more accurate estimate of risk-return relation.

Those conflicting findings are not a coincidence. As Harvey (2001) points out, the sign of risk-return relation depends on the variance model being used. He finds that relation between the conditional mean and conditional volatility is sensitive to whether one uses nonparametric density estimation, or GARCH estimation, or forming an estimator based on monthly or quarterly variance of daily returns. The relation is significantly negative for all the estimators that depend on conditioning information. Furthermore, as Lettau and Ludvigson (2002) prove, when variables that forecast both mean and variance are used, this relation is negative as well. On the other hand, recent variance estimators based on high frequency return data all identify a significant positive relation.

Such confusion lies in the fact that conditional mean and conditional variance are both unobservable. While we can replace the conditional mean by ex post stock market return and add an error term on the right hand side of (1), the conditional variance doesn't have a directly observable ex post measure. The parametric structures that conditioning information models and GARCH models put on to the conditional variance could influence the evaluation of risk-return trade-off severely. Fortunately, the recent realized volatility literature has shed light on the measurement of second moment of stock market return. As demonstrated in literature (Anderson, Bollerslev, Diebold and Ebens (2001); Barndorff-Nielsen and Shephard (2004), Meddahi (2002)), the model-free realized volatility measure constructed from intraday data affords a much more accurate ex post measure of actual return variation than those from the fitted value of second moment in regressions or GARCH model, and is also better than the traditional sample variance based on daily or lower frequency returns. In this paper, we use intraday stock return data to construct our market variance measures. Consistent with results of Bali and Peng (2006) and Ghysels, Santa-Clara and Valkanov (2005), we find a positive risk-return relationship with our realized risk measures.

Even though the realized volatility based on intraday data could provide an accurate model-free estimate of stock market variance, another approximation error might arise if we start from ICAPM model. The market variance component $V_{mm,t}$ of equation (1) is the variance of true market portfolio by definition. Since the true market portfolio is unknown, most empirical papers use a broad stock market index as proxy, e.g. S&P500, CRSP equal-weighted or value-weighted indices. If the variance of returns for the true market portfolio is only weakly related to variance of a particular stock market index, then as Pollet and Wilson (2009) argue, the cross-sectional average correlation of the stock market is a better measure of aggregate market risk, and has a positive risk-return relation. And the other component of the market index's variance, the cross-sectional average variance, can have a negative relation with future returns of the market index[2]. They show in their empirical study that it is indeed the case. Average correlation can predict subsequent quarterly stock market excess returns and the relation is significantly positive. The average variance have a negative relation with future excess returns. This paper has the same findings, which we believe can potentially reconcile the mixed findings in the risk-return literature: On one hand, the positive risk-return relation implied by most theoretical models can be supported by a positive relation between stock market return and aggregate risk represented by cross-sectional average correlation. On the other hand, the negative relation identified by previous research might be due to the dominant negative effect of average variance component, which is a correction for the part of stock market variance that is unrelated to aggregate risk.

The decomposition of stock market variance into average correlation and average variance makes both components significant in forecasting future returns with opposite signs. In this paper, we propose another decomposition which we believe is closely related and also found important in forecasting future market returns. In particular, Bollerslev, Tauchen and Zhou (2009) find the

[2]They derive in their paper a new version ICAPM with unobservable market portfolio. The stock market index's variance could be decomposed into two parts: the pairwise correlation between any two stocks (average correlation) and cross-sectional average of the variance of individual stocks (average variance). While higher aggregate risk can be revealed by higher average correlation between stocks, the changes in average variances can work in both directions: higher average variance implies higher risk of the stock market, but lower covariance between stock market and the true market portfolio. Thus the average variance of the stock market can be negatively related to future excess returns if the latter effect dominates.

variance risk premium, defined as the difference between implied and realized variances ($VRP = IV - RV$), is able to explain more than fifteen percent of the future variation in quarterly excess returns on the market portfolio over the sample period of 1990 to 2005. They interpret their results as the evidence of pricing of the variance risk in the market. In this paper, instead of interpreting the variance risk premium as an independent priced factor, we examine its two components (IV and RV) separately. This decomposition is also motivated by the empirical evidence from Drissen, Maenhout and Vilkov (2009). They find exposure to the aggregate correlation risk accounts for a substantial part of the cross-sectional variation in average excess returns of the index options, while the individual variance risk is not significantly priced. If only the correlation risk is priced in index option, VIX index, a proxy for implied variance (IV), would act as a good measure for the risk-neutral expectation of average correlation ($IV \simeq VIX \simeq E^Q(AC)$), thus is related to average correlation effect. On the other hand, the realized variance component (RV) is the product of average correlation and average variance. And the empirical evidence indicates that variation in the stock market volatility is dominated by changes in average variance[3], thus realized variance (RV) is more related to average variance effect. As a result, we believe the positive average correlation–return relation is closely related to the positive forecasting power of implied variance, while the negative average variance-return relation might be related to the negative sign of realized variance in forecasting future returns. If the forecastability of variance risk premium measure can be dominated by the correlation-variance decomposition, it might indicate that the variance risk premium measure does not represent an independent risk factor priced in the market.

We are the first in relating and empirically comparing these two decompositions in forecasting future market returns, both of which have attracted lots of attention in both theoretical and empirical literature recently. By comparing the forecasting performance of different combinations of components of the two decompositions (variance risk premium, implied variance, realized variance, average variance and average correlation), we are able to provide empirical evidence on whether there is a different channel by which variance risk premium is related to future stock market return variations. Our results indeed show that variance-correlation decomposition, instead of the variance risk premium, is driving the forecasting power of considered risk measure. The combination of

[3]We can see from Table 2 that the correlation between equal-weighting average variance and the realized variance of stock market is 0.88, while the correlation between equal-weighting average correlation and the realized variance of stock market is only 0.13.

average variance and VIX index provides the best forecast of future stock market returns, beating the performance of variance risk premium and average correlation/average variance, and stays significant after controlling for traditional risk factors. Different from previous literature, our analysis use intraday returns in constructing our risk measure. We construct the 'realized' average correlation and average variance measures from 5-minute firm specific returns for all the stocks in S&P 500 index. The realized measures could provide us with more accurate estimates of variances and covariances.

The paper is structured as follows. Section 2 introduces the theory of correlation-variance decomposition and the definition of variance risk premium. Section 3 describes data and estimation. Section 4 presents the main empirical results. Section 5 includes the discussion of possible economic explanations of our empirical findings. Section 6 concludes.

2 Theories on Risk-return Trade-off

In this section, we introduce the theories behind the two decompositions we are going to explore empirically later. First we show how the variance-correlation decomposition of the stock market variance arises within ICAPM when the market portfolio is unobservable. Then we briefly illustrate the theory for variance risk premium which induces our second decomposition.

2.1 ICAPM with unobservable market portfolio

Campbell (1993) gives the following expression for a portfolio i,

$$E_t(r_{i,t+1}) - r_{f,t+1} + \frac{V_{ii,t}}{2} = \gamma V_{im,t} + \theta V_{ih,t} \tag{2}$$

As in Pollet and Wilson (2009) , we decompose the stock market return when the true market portfolio is unobservable. Let stock market be an asset s, and the true market portfolio is asset m. Then (2) becomes the following equation,

$$\begin{aligned} E_t(r_{s,t+1}) - r_{f,t+1} + \frac{V_{ss,t}}{2} &= c_0 + c_1 V_{sm,t} && (3) \\ &= c_0 + c_1 Cov_t(r_{s,t+1}, r_{m,t+1}) && (4) \end{aligned}$$

Replace the conditional covariance of $r_{s,t+1}$ and $r_{m,t+1}$ with covariance in levels and then decompose the covariance into two parts. The true market portfolio is composed of stock market (asset s) and an unobservable part (asset u).

$$
\begin{aligned}
E_t(r_{s,t+1}) - r_{f,t+1} + \frac{\sigma_{s,t}}{2} &\simeq c_0 + c_1 Cov_t(R_{s,t+1}, R_{m,t+1}) \\
&= c_0 + c_1 Cov_t(R_{s,t+1}, w_s R_{s,t+1} + (1-w_s) R_{u,t+1}) \\
&= c_0 + c_1 (w_s Var_t(R_{s,t+1}) + (1-w_s) Cov_t(R_{s,t+1}, R_{u,t+1}))
\end{aligned}
\tag{5}
$$

Furthermore, assume there are N identical stocks in the stock market, $R_{s,t+1} = \frac{1}{N}\sum_i R_{i,t+1}$, then we have $\sigma_{s,t} = Var_t(R_{s,t+1}) \simeq \sigma_t \rho_t$, where ρ_t is the average correlation between any pair of stocks and σ_t is the average variance of all stocks. $Cov_t(R_{s,t+1}, R_{u,t+1})$ can also be represented by the combination of ρ_t and σ_t, after a linear approximation[4]. Finally we obtain the following equation,

$$
\begin{aligned}
E_t(r_{s,t+1}) - r_{f,t+1} &\approx \phi_0 + \left(\frac{c_1 E\left(\sigma_t^2\right)}{\beta(1-\theta)} - \frac{E(\sigma_t^2)}{2}\right)\rho_t + \left(\frac{c_1 (E\left(\rho_t\right) - \theta)}{\beta(1-\theta)} - \frac{E\left(\rho_t\right)}{2}\right)\sigma_t^2 \\
&= \phi_0 + \phi_1 \rho_t + \phi_2 \sigma_t
\end{aligned}
\tag{6}
$$

Now the expected stock market excess return is a linear combination of average correlation and average variance. For plausible parameter values, ϕ_1 is positive, and ϕ_2 is negative. The positiveness of ϕ_1 is straight-forward since average correlation is after all a risk measure. The negativeness of ϕ_2 is less intuitive. The key mechanism is that a sudden increase in average variance can be accompanied with a decrease in the covariance between stock market and the unobservable true market portfolio. We have more detailed discussion on this in section 5.

2.2 Variance risk premium and time varying risk-return relation

The variance risk premium is defined as the difference between realized variance and the implied variance, as proposed in Bollerslev, Tauchen and Zhou (2009). The implied variance is the risk-neutral expectation of realized variation between time t and $t+1$ conditional on information available until time t. As shown by Carr and Wu (2009), this implied variance can be approximated by the value of the options.

[4]The rigirous derivation of the model could be found in the Pollet and Wilson (2009) paper.

$$E_t^Q(RV_{t,T}) = \frac{2}{T-t}\int_0^{\infty} \frac{C_t(T,K)}{B(t,T)K^2}dK + \varepsilon \tag{7}$$

In practice, the implied variance (IV_t) is usually expressed in a "model free" discrete time fashion as follows,

$$\begin{aligned} IV_t &\equiv \sum_{i=1}^{m}\left[\frac{C_t(t+1,\frac{K_i}{B(t,t+1)}) - C_t(t,\frac{K_I}{B(t,t)})}{K_i^2} - \frac{C_t(t+1,\frac{K_{I-1}}{B(t,t+1)}) - C_t(t,\frac{K_{i-1}}{B(t,t)})}{K_{i-1}^2}\right]\Delta K \quad (8) \\ &\longrightarrow 2\int_{K_0}^{K_m}\frac{C_t(t+1,\frac{K}{B(t,t+1)}) - C_t(t,\frac{K}{B(t,t)})}{K^2}dK \qquad as\ m \to \infty \\ &\longrightarrow E_t^Q(RV_{t,t+1}) \qquad as\ K_m \to \infty\ and\ K_0 \to 0 \end{aligned}$$

where $B(t,T)$ denotes the time-t price of a bond paying one dollar at T, $C_t(T,K)$ denotes the time-t value of an out-of-the-money option with strike price $K > 0$ and $T \geq t$, and ε denotes the approximation error. In (8) $\Delta K = (K_m - K_0)/m$ and $K_i = K_0 + i\Delta K$. m denotes the number of strike prices spanned in the discrete version. Previous studies has shown that with few different strikes, the risk-neutral expectation of realized variance can be well approximated.

Another component of variance risk premium is realized variance $RV_{t-1,t}$. The realized variance between $t-1$ and t interval can be defined as follows,

$$\begin{aligned} RV_{t-1,t} &\equiv \sum_{j=1}^{m}[p_{t-1+\frac{j}{m}} - p_{t-1+\frac{j-1}{m}}]^2 = \sum_{j=1}^{m} r(t-1+\frac{j}{m},\frac{1}{m})^2 \qquad (9) \\ &\longrightarrow \nu^2(t-1,t) \qquad as\ m \to \infty \end{aligned}$$

where p_t denotes the logarithmic price, m is the number of price observations between $t-1$ and t, and $\nu^2(t-1,t)$ denotes the ex post volatility over $[t-1,t]$, which is usually the object of interest in financial econometrics. However, microstructure noise could restrict the use of highest feasible frequency data as it would bias the estimate. It has been found by previous empirical research that returns sampled at 5 minute interval has the right balance between avoiding microstructure noise contamination and preserving the accuracy of realized variance measure[5].

[5]See Hansen and Lunde (2006) and Ait-Sahalia, Mykland and Zhang (2005) for details.

Finally, the variance risk premium is usually defined as the difference between the risk-neutral and objective expectations of quadratic variation over the sample interval, $E_t^Q(RV_{t,T}) - E_t^P(RV_{t,T})$. In practice, this definition can be slightly different. Bollerslev, Gibson and Zhou (2006) extract the variance risk premium parameter λ from moment restrictions for IV_t and $E_t^P(RV_{t,t+1})$ within a one-factor stochastic volatility model framework, and in turn relate λ to a set of macro-finance state variables. Carr and Wu (2009) measure the variance risk premium as difference between IV_t and ex post $RV_{t,t+1}$. Bollerslev and Zhou (2007) use the measure $VRP_t = IV_t - RV_{t-1,t}$. In this paper, we adopt a similar measure with Bollerslev, Tauchen and Zhou (2009). The reason we didn't subtract ex post realized variance from implied variance is because the focus of our paper is the forecasting of future stock market return, thus we need to use information available at time t instead of the ex post measure. As demonstrated in Bollerslev, Tauchen and Zhou (2009), Bollerslev, Gibson and Zhou (2009) and Carr and Wu (2009), variance risk premium is significantly negative, and closely related to investor risk aversion and a set of macroeconomic variables. Thus they claim it is an independent factor market prices heavily. In this paper, we construct the same variance risk premium measure. But more importantly, we do the decomposition and include the IV_t and $RV_{t-1,t}$ in our study to detect their connection to correlation risk and individual variance risk.

As we mentioned in the introduction, if only correlation risk is priced in index option, then $IV_t \simeq VIX_t \simeq E_t^Q(\rho_{t+1})$, and $RV_{t-1,t} = \sigma_{s,t} \simeq \sigma_t\rho_t$. If σ_t dominates the changes in $RV_{t-1,t}$ and has a negative relation with expected return, $VRP_t = IV_t - RV_{t-1,t}$ would play very similar role as $\phi_1\rho_t + \phi_2\sigma_t$ of (6), with positive ϕ_1, and negative ϕ_2. As IV_t is a forward-looking measure, it could provide more information about the correlation risk than ρ_t in predictive regressions. We would examine the relation between IV_t, $RV_{t-1,t}$, σ_t and ρ_t in detail in the next sections.

3 Data and Estimation

3.1 Data description

To construct the risk measures used in this analysis, we use four sets of data. First, for the left-hand side of (6), we use natural log of stock market returns minus three-month T-bill returns. The weekly and monthly stock market returns are constructed from the daily excess market returns based on CRSP value-weighted portfolio and S&P 500. The daily stock market returns are from CRSP and T-bill rates are obtained from website of Federal Reserve Bank of St. Louis.

Second, we rely on high-frequency data for all the stocks in S&P 500 index from TAQ. The raw data consists of quotes of all the stocks in S&P 500 during the sample period from 1995 to 2006. As in Andersen, Bollerslev, Diebold and Ebens (2001), the use of equally-spaced five-minute returns strikes a satisfactory balance between the accuracy of the continuous-record asymptotics underlying the construction of realized volatility measures and the confounding influences from market microstructure friction. We construct five-minute returns of the mid-quotes[6] for each of the 500 stocks in S&P 500. Every day from 1995 to 2006, we estimate the first two components on the right-hand side of (6) from the five-minute returns of all the stocks in S&P 500.

Similar to the approximation in Pollet and Wilson (2009), the variance of the stock market is given by

$$\sigma_{s,t}^2 = \sum_{j=1}^{N} \sum_{i=1}^{N} w_{j,t} w_{i,t} \rho_{ij,t} \sigma_{j,t} \sigma_{i,t} \tag{10}$$

Assume the symmetry of stocks, $\sigma_{j,t}\sigma_{i,t} = \overline{\sigma_t}^2 = \frac{1}{N}\sum_{j=1}^{N} \sigma_{j,t}^2$, then we have

$$\sigma_{s,t}^2 = \overline{\sigma_t}^2 \sum_{j=1}^{N} \sum_{i=1}^{N} w_{j,t} w_{i,t} \rho_{ij,t} = \overline{\sigma_t}^2 \overline{\rho_t} \tag{11}$$

Thus we use the equal-weighted average variance and value-weighted average correlation in the estimation of expression (6). As a robustness check, we also use the equal-weighted average variance and equal-weighted average correlation as predictors. For each stock j in trading day t, we calculate the realized variance as the summation of five-minute squared returns:

$$\widehat{\sigma}_{j,t}^2 = \sum_{d=1}^{m} r_j(t-1+\frac{d}{m})^2 \tag{12}$$

where m is the number of five-minute returns for the stock covering the normal trading hours from 9:30am to 4:00pm, together with the close-to-open overnight return. And the empirical proxy for the equal-weighted and valued-weighted average variance are as follows:

$$\begin{aligned} AV_t^{ew} &= \frac{1}{500}\sum_{j=1}^{500} \widehat{\sigma}_{j,t}^2 \\ AV_t^{vw} &= \sum_{j=1}^{500} w_{j,t} \widehat{\sigma}_{j,t}^2 \end{aligned} \tag{13}$$

[6] Here we use quotes from all markets (NBBO).

The correlation between any two stocks i and j is $\widehat{\rho}_{ij,t} = \frac{\widehat{\sigma}_{ij,t}}{\widehat{\sigma}_{j,t}\widehat{\sigma}_{i,t}}\widehat{\rho}_{ij,t} = \frac{\widehat{\sigma}_{ij,t}}{\widehat{\sigma}_{j,t}\widehat{\sigma}_{i,t}}$ where $\widehat{\sigma}_{ij,t} = \sum_{d=1}^{m} r_j(t-1+\frac{d}{m})r_i(t-1+\frac{d}{m})$. Then we can calculate the equal-weighted and value-weighted average correlation as

$$
\begin{aligned}
AC_t &= \frac{1}{500(500+1)}\sum_{j=1}^{500}\sum_{i\neq j}\widehat{\rho}_{ij,t} \qquad (14)\\
AC_t &= \sum_{j=1}^{500}\sum_{i\neq j} w_{j,t}w_{i,t}\widehat{\rho}_{ij,t}
\end{aligned}
$$

To illustrate, we plot the time series of market index variance, average correlation and average variance on a daily basis in Figure 1. As in Pollet and Wilson, our measure of average variance does not exhibit a pronounced upward time trend. And average variance and average correlation don't always move together, especially after 2003. In contrast, the time series of average variance and market variance exhibit very similar patterns and follow each other closely. This again reveals that the variation in market variance is mainly caused by changes in average variance.

Third, when it comes to the variance risk premium, we utilize daily and monthly data of the VIX index to quantify the risk-neutral implied variance measure. The VIX index is based on the highly liquid S&P 500 index options along with the model free approach expressed in (8) to replicate the implied variance of the index over a fixed 30-day period. The data is available from the Chicago Board of Options Exchange (CBOE).

The realized variance term in variance risk premium is the summation of the five-minute squared return of a fund tracking S&P 500 composite index[7]. Although using the high-frequency return of the tracking fund increases efficiency, it might also induce bias. So we also use the summation of daily squared return of the S&P 500 index itself as an alternative measure. The difference between the estimated implied variance and realized variance is our measure of variance risk premium. Figure 2 depicts the time series of implied volatility and realized volatility, we can see the difference between the two is almost always positive, indicating there is a systematic variance risk premium.

Finally, we combine the variance measures with some other predictors such as the dividend price ratio, the default spread (i.e., the yield spread between BAA-rated and AAA-rated bonds), the term spread (i.e., the yield spread between long term and short term Treasury securities), the

[7] We don't have intra-day trading data for the index. So we rely on the intra-day data of a fund which closely track the index. The fund we use has ticker "SPY" in TAQ.

consumption-wealth ratio (cay) from Lettau and Ludvigson (2002), and stochastically detrended risk-free rate (RREL), which is used in many previous papers including Guo and Whitelaw (2006), and is defined as

$$RREL_t = r_{f,t} - \frac{1}{12}\sum_{k=1}^{12} r_{f,t-k}$$

The definitions of all the above predictors are given in Table 1.

3.2 Estimation framework

To analyze the relative importance of the two decompositions and various risk measure, we estimate the following predictive regressions which include various combinations of the average correlation, average variance, the variance risk premium, implied market variance and realized market variance.

$$\begin{aligned}
r^s_{t+1} - r^f_{t+1} &= \phi_0 + \phi_1 VRP_t + \varepsilon_{t+1} \qquad (15)\\
r^s_{t+1} - r^f_{t+1} &= \phi_0 + \phi_1 VIX_t + \phi_2 RV_t + \varepsilon_{t+1}\\
r^s_{t+1} - r^f_{t+1} &= \phi_0 + \phi_1 VIX_t + \phi_2 AV_t + \phi_3 AC_t + \varepsilon_{t+1}\\
r^s_{t+1} - r^f_{t+1} &= \phi_0 + \phi_1 VIX_t + \phi_2 AV_t + \phi_3' X_t + \varepsilon_{t+1}\\
r^s_{t+1} - r^f_{t+1} &= \phi_0 + \phi_1 VRP_t + \phi_2 AV_t + \phi_3' X_t + \varepsilon_{t+1}
\end{aligned}$$

Note we also include in the regression $X_{t-h,t}$ as control variables which are DP, TMSP, DFSP, RREL and CAY.

We compare the forecasting power of VRP_t alone and the separate effect of VIX_t and RV_t. Further decompose RV_t into AV_t and AC_t, we combine VRP_t or VIX_t with them to see which part of the information is overlapping. Control variables that are found to be useful in predicting market returns are included. The estimation method is OLS with t-statistics corrected for heteroscedasticity using Newey-West standard errors.

4 Empirical Results

4.1 Summary statistics

The basic summary statistics are given in Table 2. The mean excess return on the CRSP value-weighted portfolio over the sample is 5.46 percent annually. The sample mean for the implied volatilities is 37.31, and the means for the two realized variances measures are 24.89 and 25.62, correspondingly, the means for variance risk premium are 12.71 and 11.63. The means for equal-weighted and value-weighted average variances are 139.39 and 167.84, much higher than market realized variances, which means a large portion of individual risk has been diversified in the market portfolio. All of the measures have been scaled whenever appropriate. The value-weighted average correlations between any pair of stocks have a mean of 0.22 and it is higher than the mean 0.16 of equal-weighted correlations. As for the correlation matrix, implied volatility, realized variance, and variance risk premium are all negatively correlated with the contemporaneous excess returns[8]. The correlation between the implied volatility and the excess returns is as large as -0.51. Average correlation and average variance also exhibit negative correlations with excess returns. The realized market volatility is highly correlated to the average variance, the sample correlation is 0.87, compared to 0.26 with the average correlation.

4.2 Results of the monthly forecasting regressions

In the monthly univariate regressions reported in Panel A of Table 3, most of the variables have an insignificant impact on the changes of future stock market returns at monthly frequency. The only exception is dividend ratio, which is negatively correlated to the next period excess market return, with a t-statistics of -3.02 and adjusted R^2 of 3.17 percent. We can see that the coefficients for two variance risk premium measures are both positive. The average variance (AV_t) has a negative relation with future return, while this relation is positive for average correlation (AC_t), though both variables stay insignificant. In multiple regressions of monthly data, we first regress excess return of the S&P 500 index on VIX_t together with RV_t. Now VIX_t becomes significant in predicting market return with a positive sign and t-statistics of 2.02, while realized variance still remains insignificant. As we argued in the second section, realized variance contains two parts, average correlation and average variance, which work in opposite directions in forecasting future

[8]Note that these are contemporaneous correlations, which is different from the forecasting relations we focus on.

market returns. This might make coefficient of the realized variance term insignificant. As a result, we further split realized variance into average variance and average correlation. VIX_t becomes more significant. The t-statistics increases to 3.57 and overall they account for 3.58 percent of the variation in monthly returns, much higher than the 0.73 percent that VRP_t alone provides. Another interesting feature is that, the average variance has a significant negative relation with the future returns as expected[9], but the forecasting power of average correlation disappears after we control for the VIX_t. This result is very intuitive. It is evidence that VIX_t contains information about the correlation between any two stocks in the market, yet it has better forecasting power than average correlation perhaps due to the fact that it is forward looking. If we exclude the average correlation from above regression, the adjusted R^2 is even higher at 4.16% monthly. As we have showed in the univariate regression, traditional predictors such as PD ratio are also useful in forecasting returns. Combining PD ratio with VIX_t and AV_t produces the adjusted R^2 of 6.05, and three predictors all remain significant. Although $RREL$ is not significant in the univariate regression, it provides marginal improvement in the multiple regression, resulting the best adjusted R^2 of 6.37. We also run the regressions on VRP_t and PD ratio and $RREL$. The results further confirm that VIX_t and AV_t combination provides a better specification than VRP_t in capturing the second-moment-associated forecasting power. Since few of traditional predictors have implications over monthly horizon, the predictability implied variance and average variance provide seems rather impressive. And above result seems to lend more support for the variance-correlation decomposition, with VIX_t as a measure of average correlation than variance risk premium theory.

4.3 Results of the quarterly forecasting regressions

Panel A of Table 4 is the univariate regressions of quarterly stock market returns on all the predictors. Variance risk premium remain a strong predictor. It explains 15.41 percent changes in future returns, higher than R^2s of its two components respectively. These results are in line with findings in Bollerslev, Tauchen and Zhou (2009). As for the traditional predictors, PD ratio and the consumption wealth ratio (CAY) from Lettau and Ludvigson (2002) are also highly significant, and the relative risk-free rate ($RREL$) is marginally significant with a t-statistics of 1.94. The

[9] In the columns (2)-(5), we try different combination of equal-weighted and value-weighted measures and the results don't change significantly. In the following part of this paper, we only report the results from best combination of equal-weighted or value-weighted measures.

default spread and term spread are insignificant and provide little information for future returns.

Furthermore, in the multiple regressions of Panel B of Table 4, VIX_t and AV_t combination beats VRP_t with an adjusted R^2 as high as 26.47 percent. Together with traditional predictors, they still perform better than variance risk premium. Best adjusted R^2 we could get is 43.32 percent. In the column (10) of Panel B[10], we can see the useful predictors are VIX_t, AV_t, CAY, and $RREL$. We can interpret VIX_t and AV_t, as variables related to market variance term in traditional ICAPM. Referring to (1) in section one, we can see that there are also a hedge component, which is omitted in Pollet and Wilson (2009) when they examine the average variance and average correlation effect. The extra predictability $RREL$ and CAY provide here could be attributed to the hedge component of expected return. This conforms with the empirical results from Guo and Whitelaw (2006). In their study, $RREL$ and CAY are the two state variables used to calculate hedge component, and they also proves that these two variables contain little information about the variance term. Consistent with the results from the monthly regression, VIX_t and AV_t remain significant and exhibit a stronger forecasting performance than variance risk premium measure. Although our results can not reject the theory in Bollerslev, Tauchen and Zhou (2009), they do imply that there might be another channel through which VRP_t is useful in predicting stock market returns.

To summarize, our results from both monthly and quarterly forecasting regressions indicate that the variance-correlation decomposition is driving the forecasting power on the stock market returns more than the variance risk premium explanation. And it is robust after controlling for other traditional forecasting variables. But theoretically why the variance-correlation decomposition works well is still an open question. Pollet and Wilson (2009) is only one of the possible explanations we are going discuss in next section.

4.4 Results with alternative sampling frequency

In this section, we repeat the quarterly multivariate forecasting regressions with realized proxies sampling at every 25 minutes. If the trading frequencies of some S&P 500 stocks are substantially lower than 5 minutes, our previous results would be biased. For realized variance, sampling every 5 minutes might be less a concern. However, the average correlation series could be significantly

[10] Although the column (9) provides us the best adjusted R-square, the result from column (10) is comparatively as good as that in column (9), and also the four independent variables are all highly significant. We emphasis on column (10) because of its connection with the results from Guo and Whitelaw (2006).

downward-biased. After re-computing the average variance and average correlation series with the 25 minutes sampling frequency, we repeat the multivariate regressions using the quarterly data. The main results are reported in Table 5. We labelled the regressions in the same way as in Panel B of Table 4 such that the two set of results can be compared. As we can see, the results of both sampling frequencies are very similar. The results for monthly regressions are also similar for the two sampling frequencies, and not reported[11].

5 Discussions

In this section, we discuss possible theoretical explanations for the positive average-correlation(VIX)-return and negative variance-return relation we found in the data, including Pollet and Wilson (2009) which motivates our study.

5.1 Average Correlation/Average Variance Decomposition

In Pollet and Wilson (2009) setting, stock market is only part of the true market portfolio. According to ICAPM, the expected return of stock market is proportional to its covariance with the true market portfolio, which can be divided into two parts, the first part is the stock market return variance itself, and the second part is the covariance between stock market and non-stock market returns. The average-correlation-return relation is in the same direction as aggregate risk-return relation since all the assets co-move with the true market portfolio. Average correlation only affects stock market return through its own variance term. On the other hand, a positive shock in average variance not only increases the aggregate stock market risk but also reduce the covariance between stock market and the non-stock market returns. The later effect makes stock market more 'immune' to the uncertainties in the rest of the true market portfolio, i.e., makes stock market safer, and earns a lower expected return. If this effect dominates, the average-variance-return relation could works in the opposite direction to the true risk-return trade-off. As a result, average correlation alone better captures the aggregate risk. A positive average-correlation-return relation indicates that investors require higher expected return when the aggregate risk increases. A negative average-variance-return relation comes from the fact that investors would accept a lower expected return if

[11] When sampling every 25 minutes, the means of the average variance remain the same while the value-weighted and equal-weighted average correlation increase by 0.01 and 0.02 respectively.

stock market is less exposed to the systematic non-stock market risk.

5.2 Asymmetric correlation: an empirical explanation

The critical assumption of Pollet and Wilson (2009) is that stock market is only part of the true market portfolio. In fact, many investors might only trade within stock market, and the evidence of reduction in covariance between stock market and non-stock market returns when average variance increases is only partial in their empirical studies[12].

Aside from the above story, an increase in average correlation can be seen as a market-wide reduction in diversification benefits, which means higher correlation is more expensive for investors. Empirical evidence from Ang and Chen (2001) and Hong, Tu and Zhou (2007) indicates that the correlation in stock market is higher during the downward movement than that during upward movement. This might be due to the fact that the diversification/hedging benefit is more important in the downside moves. Also, we know the market variance is quite persistent. A change in stock market variance could only be due to the changes in its two components, average correlation and average variance. If a higher average correlation is more associated with downside moves than upside moves of prices, and the aggregate market variance doesn't change drastically, then it would be the case for a higher average variance to be more related to upside moves than downside moves of prices. This will induce a positive relation between average correlation and future market return, and a negative relation for average variance.

5.3 Hedge correlation, pursue volatility: a behavioral explanation

One may argue that a high average correlation not necessarily implies a lower average variance. We can see from Table 2 that the contemporaneous correlation between AC_t and AV_t is -0.16, but the correlation between VIX_t and AV_t is 0.73. Since VIX_t is a forward looking measure, this high positive correlation can not fully reject the possibility of the explanation in last section. However, in our empirical study, after controlling for VIX_t, why is there a significant negative relation between the average variance and expected returns? One answer might be that investors tend to treat correlation as the main risk in stock market index. After they hedged against correlation, or in other words, controlled their exposure to possible deterioration in diversification benefits, their preference

[12] They found some reduction in stock-bond return covariance, but the argument is not complete as long as we don't know exactly what the non-stock part of true market portfilo is.

would become option-like, and they would pursue volatility to speculate and gamble, and derive additional non-wealth utility from it. There are financial news that describes the increasing number of high frequency algorithm traders as investors who don't care about dividend and fundamental but pursue volatility instead. Our findings thus can be seen as an evidence of such kind of behavior.

Another possible behavioral reason for the presence of negative average-variance-return relation is that investors might derive extra utility from realized gains (sell a stock when its price exceeds the purchase price), as proposed in the theoretical model of Barberis and Xiong (2008). In their model, investors would like to purchase a stock even with negative expected returns as long as its volatility is high because high volatility means "it will always come back". High volatility makes the "realized gains" in the future more likely. Even their model is more suitable for individual stock pricing, here in our case, we could think investors have realized utility preference after they hedge away the correlation risk in market index. Then when average volatility is high, stock index will experience excessive buying and price will increase, which results in a lower expected return.

In a word, we think the negative average-variance-return relation identified in our study is a very interesting phenomenon. The explanation of this still remains an open question. To relate average variance and average correlation to different types of information or certain trading behavior could be the future direction of our research.

6 Conclusion

The identification of the risk-return relation is a challenge in empirical asset pricing. Pollet and Wilson (2009) find that if we decompose stock market variance into average variance and average correlation, we are able to have a clearer result on the risk-return relation. On the other hand, the recent variance risk premium literature also finds significant risk-return relation by using option data. These two approaches might be related, which can be inferred from the results of Drissen, Maenhout and Vilkov (2009), who find that variance risk premium is mainly attributed to correlation risk instead of variance risk. We empirically study the relative importance of these two approaches in terms of forecasting market index returns. Among all combinations of different parts of market variance (both under P-measure and Q-measure) we considered, average variance coupled with VIX_t is the best in forecasting market returns. While average variance negatively forecasts future market returns, the VIX_t is positively related to future market returns. If we treat VIX_t

as a better predictor for aggregate correlation in the market, our result is consistent with Pollet and Wilson (2009). More importantly, our results suggest the forecasting power for the market return is driven more by variance-correlation decomposition than variance risk premium. And this is true for both monthly and quarterly regressions. Interesting future research would be to build a asset pricing model which can incorporate the main findings in this paper, especially the negative relation between the average variance and future market return, on which the existing theories still have difficulties in providing a convincing explanation.

7 Appendix: Details

Figure1: Daily Average Correlations and Average Variance

This figure plots the daily realized market variance, average variance and average correlation among the components of S&P 500. Both average are equal-weighted averages.

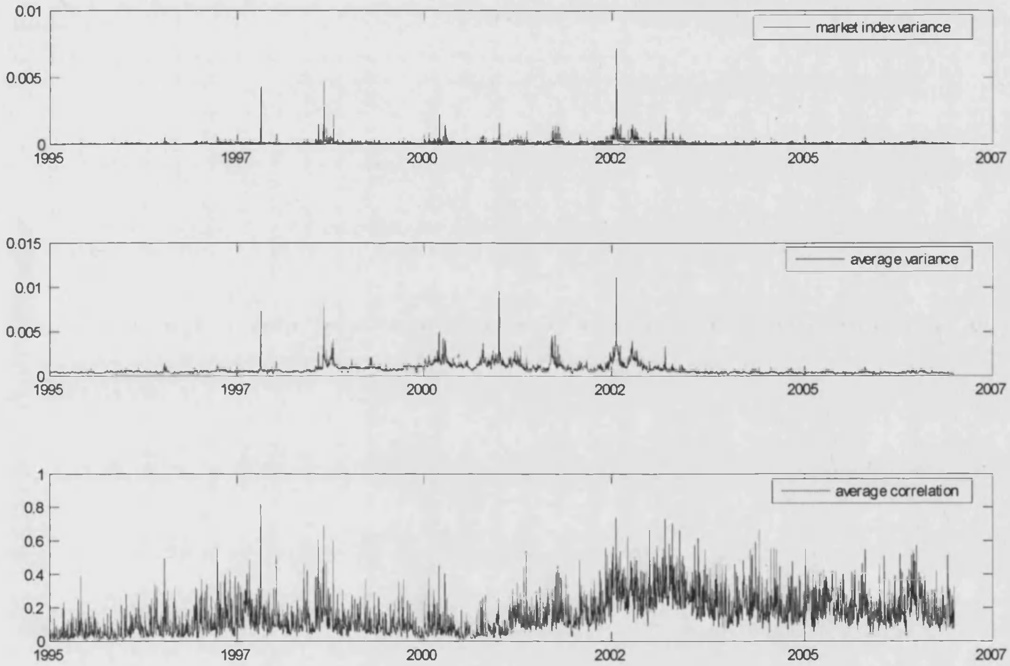


Figure2: Monthly Implied Variance and Realized Variance

This figure plot the monthly observation of the realized market variance, implied market variance and their difference, the variance risk premium. The

realized market variances are scaled up by 10000 while the Implied market variance is computed as : $(VIX/100)^2 \times 30/365$

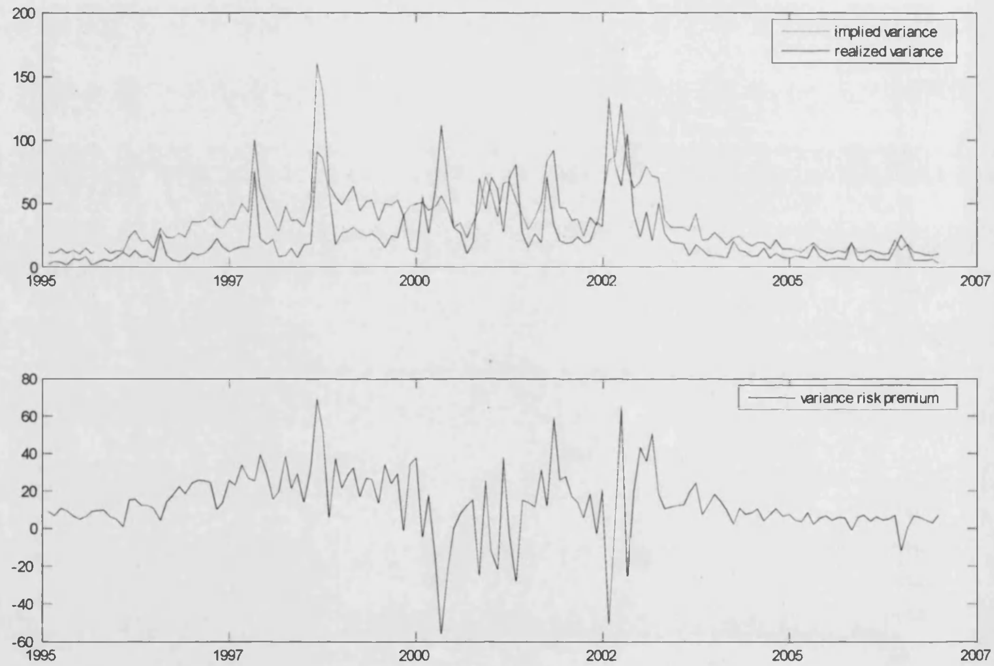


Table 1 Definitions for All Variables

$R_{mt} - R_{ft}$: Annualized return of S&P500 in excess of 3-month T-bill rate (multiplied by 100)
VIX_t: Implied S&P500 variance over one month (multiplied by 10000)
RV_t^1 : Realized S&P 500 variance over one month constructed by daily returns (multiplied by 10000)
RV_t^2 : Realized S&P 500 variance over one month constructed by 5-min returns(multiplied by 10000)
VRP_t^1: Monthly variance risk premium, as difference between VIX_t and RV_t^1
VRP_t^2: Monthly variance risk premium, as difference between VIX_t and RV_t^2
AV_t^{vw} : sum of daily value-weighted average variance over one month(multiplied by 10000)
AV_t^{ew} : sum of daily equal-weighted average variance over one month(multiplied by 10000)
AC_t^{vw} : average of daily value-weighted average correlation over one month.(multiplied by 10000)
AC_t^{ew} : average of daily equal-weighted average correlation over one month.(multiplied by 10000)
PD: log price dividend ratio of the value-weighted market index (multiplied by 12)
DFSP: default spread, defined as the difference between BAA and AAA bond yields (multiplied by 1200)
TMSP: term spread, defined as the difference between 10-year and 3-month treasury yields (multiplied by 1200)
RREL: detrended risk-free rate, defined as 1-month treasure yield minus its past 12 month average(multiplied by 1200)
CAY: consumption-wealth ratio, defined as in Lettau and Ludvigson (2002) (multiplied by 100)

Table 2. Summary Statistics for Key Variables

This table report the summary statistics for all the variables at quarterly frequency during the period of Jan. 1995 to Dec. 2006. All the numbers are annualized when appropriate. The definition of each variable is as in Table 1.

	$R_{mt}-R_{ft}$	VIX_t	VRP_t^1	VRP_t^2	RV_t^1	RV_t^2	AV_t^{vw}	AV_t^{ew}	AC_t^{vw}	AC_t^{ew}	PD	DFSP	TMSP	RREL	CAY
	Summary Statistics														
Mean	5.46	37.31	12.71	11.63	24.89	25.62	139.39	167.84	0.22	0.16	79.24	9.98	15.72	0.07	-0.49
Std. Dev.	32.32	27.53	13.71	13.05	20.58	19.57	95.42	106.21	0.08	0.08	2.87	2.56	13.89	0.74	1.46
Skewness	-0.65	1.83	1.18	1.21	1.75	1.38	1.26	1.16	0.42	0.50	0.78	0.85	0.42	-1.02	0.19
Kurtosis	3.45	7.11	6.68	6.48	6.78	4.75	3.68	3.46	3.21	2.49	3.10	2.84	2.15	4.04	2.36
	Correlation Matrix														
$R_{mt}-R_{ft}$	1.00	-0.51	-0.26	-0.36	-0.50	-0.47	-0.45	-0.44	-0.18	-0.16	-0.17	-0.19	0.01	0.05	-0.04
VIX_t		1.00	0.69	0.76	0.87	0.89	0.67	0.73	0.38	0.20	0.17	0.24	0.06	-0.26	-0.04
VRP_t^1			1.00	0.92	0.24	0.34	0.11	0.14	0.14	0.02	-0.10	-0.03	0.09	-0.14	0.08
VRP_t^2				1.00	0.39	0.39	0.15	0.22	0.34	0.24	-0.10	0.10	0.20	-0.11	0.05
RV_t^1					1.00	0.97	0.82	0.87	0.41	0.26	0.28	0.34	0.00	-0.25	-0.11
RV_t^2						1.00	0.84	0.88	0.30	0.13	0.30	0.27	-0.07	-0.30	-0.10
AV_t^{vw}							1.00	0.97	-0.02	-0.16	0.61	0.17	-0.21	-0.23	-0.25
AV_t^{ew}								1.00	0.13	-0.01	0.53	0.34	-0.06	-0.32	-0.20
AC_t^{vw}									1.00	0.96	-0.13	0.61	0.54	-0.16	-0.13
AC_t^{ew}										1.00	-0.26	0.61	0.53	-0.07	-0.05
PD											1.00	0.08	-0.20	-0.30	-0.63
DFSP												1.00	0.50	-0.39	-0.11
TMSP													1.00	-0.47	0.12
RREL														1.00	-0.17
CAY															1.00

Table 3. Forecast Regressions for Monthly Stock Market Return

This table reports the forecast regression of monthly S&P500 returns on various predictive variables. The sample period extends from January,1995 to December,2006. Newey-West robust t-statistics with 4 lags are reported in parentheses. All the regressors are defined in Table1. Coefficients of AC_t^{vw} and AC_t^{ew} are multiplied by 10000.

Panel A Monthly Simple Regressions

Regressors	beta est.	t-stat	R^2(%)	R^2_adj(%)
VIX_t	0.199	(1.34)	1.02	0.32
RV_t^1	0.057	(0.30)	0.08	-0.63
RV_t^2	0.176	(0.95)	0.83	0.13
VRP_t^1	0.378	(1.80)	1.43	0.73
VRP_t^2	0.131	(0.621)	0.18	-0.53
AV_t^{vw}	-0.040	(-0.90)	0.75	0.05
AV_t^{ew}	-0.032	(-0.84)	0.60	-0.10
AC_t^{vw}	0.004	(0.76)	0.56	-0.15
AC_t^{ew}	0.004	(0.68)	0.34	-0.36
PD	-2.754	(-3.02)	3.85	3.17
DFSP	-2.168	(-1.24)	1.22	0.52
TMSP	-0.153	(-0.41)	0.17	-0.54
RREL	4.874	(0.88)	0.52	-0.19

Panel B: Monthly Multiple Regression

Model	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
VIX_t	0.430	0.636	0.620	0.628	0.625	0.585			0.577	0.659
	(2.02)	(3.57)	(3.68)	(3.87)	(4.04)	(3.35)			(3.93)	(4.34)
RV_t^1	-0.302									
	-1.26									
VRP_t^1							0.350	0.419		
							(1.61)	(1.83)		
AV_t^{vw}		-0.142	-0.142			-0.135				
		(-2.94)	(-3.00)			(-2.96)				
AV_t^{ew}				-0.128	-0.128				-0.087	-0.092
				(-3.07)	(-3.06)				(-1.76)	(-1.78)
AC_t^{vw}		-0.002		0.000						
		(-0.42)		(0.00)						
AC_t^{ew}			-0.002		0.000					
			(-0.46)		(0.07)					
PD							-2.682	-2.466	-2.471	-2.257
							(-3.03)	(-2.50)	(-2.19)	(-1.96)
RREL								5.089		7.203
								(0.99)		(1.54)
R^2(%)	1.80	5.62	5.61	5.61	5.61	5.51	5.08	5.57	8.03	9.00
R^2_adj (%)	0.40	3.58	3.57	3.57	3.57	4.16	3.72	3.53	6.05	6.37

Table 4. Forecast Regressions for Quarterly Stock Market Return

This table reports the forecast regression of quarterly S&P500 returns on various predictive variables. The sample period extends from January,1995 to December,2006. Newey-West robust t-statistics with 3 lags are reported in parentheses. All the regressors are defined as in Table1.Coefficients of AC_t^{vw} and AC_t^{ew} are multiplied by 10000.

Panel A: Quarterly Simple Regression

Regressors	beta est.	t-stat	R^2(%)	R^2_adj(%)
VIX_t	0.301	(2.03)	6.54	4.46
RV_t^1	0.101	(0.45)	0.41	-1.80
RV_t^2	0.157	(0.69)	0.97	-1.23
VRP_t^1	0.983	(4.09)	17.25	15.41
VRP_t^2	0.957	(3.68)	14.14	12.94
AV_t^{vw}	-0.064	(-1.48)	3.57	1.43
AV_t^{ew}	-0.056	(-1.42)	3.42	1.28
AC_t^{vw}	0.006	(1.08)	2.45	0.28
AC_t^{ew}	0.006	(1.08)	1.84	-0.34
PD	-5.659	(-4.94)	25.13	23.47
DFSP	-2.665	(-1.27)	4.50	2.38
TMSP	-0.140	(-0.28)	0.35	-1.87
RREL	10.065	(1.94)	5.30	3.20
CAY	7.214	(4.16)	10.71	8.73

Panel B: Quarterly Multiple Regression

Model	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
VIX_t	1.004	0.941	0.901	1.024	1.000	0.807			0.804	0.920
	(3.68)	(5.61)	(6.15)	(5.44)	(5.84)	(6.05)			(4.52)	(6.02)
RV_t^1	-1.063									
	-2.65									
VRP_t^1							0.871	1.044		
							(3.50)	(3.93)		
AV_t^{vw}		-0.246	-0.245			-0.219				
		(-4.60)	(-4.78)			(-5.27)				
AV_t^{ew}				-0.247	-0.245				-0.150	-0.183
				(-5.13)	(-5.24)				(-2.36)	(-3.91)
AC_t^{vw}		-0.006		-0.003						
		(-1.28)		(-0.62)						
AC_t^{ew}			-0.006		-0.002					
			(-1.33)		(-0.51)					
PD							-5.712		-4.604	
							(-5.40)		(-3.42)	
TMSP							-0.485		-0.529	
							(-1.49)		(-1.79)	
RREL								15.475		12.816
								(3.52)		(3.70)
CAY								7.854		6.42
								(4.80)		(4.04)
R^2(%)	17.39	31.55	31.29	36.00	35.80	29.67	41.88	38.15	48.25	47.29
R^2_adj (%)	11.63	26.78	26.50	31.53	31.32	26.47	37.82	33.83	43.32	42.27

Table 5. Robustness Check: Alternative Sampling Frequency

In estimation of AV_t^{vw}, AV_t^{ew}, AC_t^{vw} and AC_t^{ew}, we sample the equity returns every 25 minutes. The result of quarterly multiple regressions related to the 4 variables are reported in the following table. For ease of comparison with Panel B of Table 4, we labelled the regressions in the same way.

Quarterly Multiple Regression (25min)

Model	(2)	(3)	(4)	(5)	(6)	(9)	(10)
VIX_t	0.924	0.904	1.024	0.987	0.924	0.763	0.865
	(5.43)	(6.08)	(5.43)	(6.04)	(6.61)	(4.53)	(6.03)
AV_t^{vw}	-0.252	-0.253			-0.246		
	(-4.23)	(-4.62)			(-5.36)		
AV_t^{ew}			-0.262	-0.262		-0.151	-0.181
			(-4.69)	(-4.97)		(-2.03)	(-3.69)
AC_t^{vw}	-0.006		-0.004				
	(-1.15)		(-0.78)				
AC_t^{ew}		-0.006		-0.004			
		(-1.40)		(-0.90)			
PD						-4.440	
						(-2.81)	
TMSP						-0.560	
						(-1.81)	
RREL							12.755
							(3.55)
CAY							5.948
							(3.57)
R^2(%)	31.97	31.20	35.59	35.61	34.87	46.72	45.20
R^2_adj (%)	26.15	26.40	31.10	31.12	31.91	41.65	39.98

References

[1] Ait-Sahalia, Y., P. A. Mykland, and L. Zhang, 2005, "Ultra High Frequency Volatility Estimation with Dependent Microstructure Noise," *NBER Working Paper.*

[2] Anderson, T. G., T. Bollerslev, F. X. Diebold, and H. Ebens, 2001, "The Distribution of Realized Stock Return Volatility, " *Journal of Financial Economics*, Vol 61, 43-76.

[3] Andersen, T.G., T. Bollerslev, F.X. Diebold and P. Labys, 2000, "Market Microstructure Effects and the Estimation of Integrated Volatility," Work in Progress, Northwestern University, Duke University and University of Pennsylvania.

[4] Ang, A. and J. S. Chen, 2001, "Asymmetric Correlations of Equity Portfolio," EFA 2001 Barcelona Meetings.

[5] Barberis, N. and W. Xiong, 2008, "Realization Utility," Working Paper.

[6] Bali, T. G. and L. Peng, 2006, "Is there a Risk-Return Trade-off? Evidence from High-Frequency Data," *Journal of Applied Econometrics*, Vol 21, 1169-1198.

[7] Barndorff-Nielson, O. and N. Shephard, 2004, "Econometric Analysis of Realized Covariation: High Frequency Based Covariance, Regression and Correlation in Financial Economics," *Econometrica*, (2004), Vol 72, 885-925.

[8] Bollerslev, T., M. Gibson, and H. Zhou, 2009, " Dynamic Estimation of Volatility Risk Premia and Investor Risk Aversion from Option-Implied and Realized Volatilities," *Journal of Econometrics*, forthcoming

[9] Bollerslev, T., R. F. Engle and J. M. Wooldridge, 1988, "A Capital Asset Pricing Model with Time-Varying Covariances," *Journal of Political Economy*, Vol 96, 116-131.

[10] Bollerslev, T., G. Tauchen and H. Zhou, 2009, "Expected Stock Returns and Variance Risk Premium", *Review of Financial Studies,* forthcoming.

[11] Brandt, M. W. and Q. Kang, 2004, "On the Relation Between the Conditional Mean and Volatility of Stock Returns: A Latent VAR Approach," *Journal of Financial Economics,* 72, 2004, 217-257.

[12] Breen, W., L. R. Glosten and R. Jagannathan, 1989, " Economics Significance of Predictable Variations in Stock Index Returns," *Journal of Finance*, Vol 44, 1177-89.

[13] Campbell, J. Y., 1987, "Stock Returns and the Term Structure,", *Journal of Financial Economics*, Vol 18, 373-399.

[14] Campbell, J. Y., 1991, "A Variance Decomposition for Stock Returns," *Economic Journal*, Vol. 101, 157-179.

[15] Campbell, J. Y. 1993, "Intertemporal Asset Pricing Without Consumption Data," *American Economic Review*, Vol. 83, 487-512.

[16] Campbell, J. Y., A. W. Lo and A. C. MacKinlay, 1997, "The Econometrics of Financial Markets," Princeton University Press.

[17] Campbell, J. Y. and J. H. Cochrane, 1999, "By Force of Habit: A Consumption Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy*, Vol. 107, 205-251.

[18] Carr, P. and L. Wu, 2009, "Variance Risk Premia," *Review of Financial Studies*, 2009, 22(3).

[19] Cochrane, J. H., 2001, "Asset Pricing," Princeton University Press, Princeton.

[20] Cochrane, J. H., 2006, "The Dog that Did Not Bark: A Defence of Return Predictability," mimeo.

[21] Driessen, J., P. J. Maenhout and G. Vilkov, 2009, "The Price of Correlation Risk: Evidence from Equity Options", *Journal of Finance*, forthcoming.

[22] French, K. R., G. W. Schwert and R. E. Stambaugh, 1987, "Expected Stock Returns and Volatility", *Journal of Financial Economics*, 19, 3-29.

[23] Ghysels, E., P. Sanda-Clara and R. Valkanov, 2002, "The MIDAS touch: Mixed Data Sampling Regression Models," Working Paper, UNC and UCLA.

[24] Ghysels, E., P. Santa-Clara and R. Valkanov, 2005, "There is a risk-return trade-off after all," *Journal of Financial Economics*, Vol. 76, 509-548.

[25] Ghysels, E., P. Sanda-Clara and R. Valkanov, 2006, "Predicting Volatility: Getting the Most out of Return Data Sampled at Different Frequencies," *Journal of Econometrics* 131, 59-95.

[26] Glosten, L. R., R. Jagannathan and D. E. Runkle, 1993, "On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks," *Journal of Finance*, Vol 48, 1779-1801

[27] Goyal, A. and P. Santa-Clara, 2003, "Idiosyncratic Risk Matters!," *Journal of Finance*, Vol 58, 975-1008.

[28] Guo, H. and R. F. Whitelaw, 2006, "Uncovering the Risk-Return Relation in the Stock Market," *Journal of Finance*, Vol 61, 1433-1463.

[29] Hansen, P. R., and A. Lunde, 2006, "Realized Variance and Market Microstructure Noise," *Journal of Business and Economic Statistics,* Vol 24, 127-161.

[30] Harvey, C. R., 2001, "The Specification of Conditional Expectations," *Journal of Empirical Finance*, Vol 8, 573-637.

[31] Heston, S., 1993, "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options," *Review of Financial Studies*, Vol. 6, 327-343.

[32] Hong, Y., J. Tu and G. Zhou, 2007, "Asymmetries in Stock Returns: Statistical Tests and Economic Evaluation", *Review of Financial Studies* Vo.20, 1547–1581.

[33] Lettau, M. and S. C. Ludvigson, 2002, "Measuring and Modelling Variation in the Risk-Return Trade-off," in Ait-Sahalia and Hansen (Eds.) Handbook of Financial Econometrics, North-Holland, Holland.

[34] Meddahi, N., (2002), "Theoretical Comparison Between Integrated and Realized Volatility," *Journal of Applied Econometrics*, Vol. 17, 479-508.

[35] Newey, W. K. and K. D. West, 1987, "A Simple Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, Vol 55, 703-708.

[36] Pangan, A. R. and Y.S. Hong, 1991, " Nonparametric Estimation and the Risk Premium," in Nonparametric Models in Econometrics and Statistics: Proceedings of the Fifth International Symposium in Economic Theory and Econometrics," pp.51-75. Cambridge University Press, Cambridge.

[37] Pollet, J. M. and M. Wilson, 2009, "Average Correlation and Stock Market Returns," *Journal of Financial Economics*, forthcoming.

[38] Roll, R., 1977, "A Critique of the Asset Pricing Theory's Tests: Part I,'" *Journal of Financial Economics*, Vol. 4, 129-176.

[39] Rosenberg, J. V. and R. F. Engle, 2002, " Empirical Pricing Kernels," *Journal of Financial Economics*, Vol 54, 341-372.

[40] Whitelaw, R. F., 1994, "Time Variations and Covariations in the Expectations and Volatility of Stock Market Returns," *Journal of Finance*, Vol 49, 515-41.