Competition in Decentralized Electricity Markets

Three Papers on Electricity Auctions

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Abstract

This thesis consists of three self-contained papers on the analysis of electricity auctions written over a period of twelve years. The first paper models price competition in a decentralized wholesale market for electricity as a first-price, sealed-bid, multi-unit auction. In both the pure and mixed-strategy equilibria of the model, above marginal cost pricing and inefficient despatch of generating units occur. An alternative regulatory pricing rule is considered and it is shown that offering to supply at marginal cost can be induced as a dominant strategy for all firms. The second paper analyses strategic interaction between long-term contracts and price competition in the British electricity wholesale market, and confirms that forward contracts will tend to put downward pressure on spot market prices. A 'strategic commitment' motive for selling forward contracts is also identified: a generator may commit itself to bidding lower prices into the spot market in order to ensure that it will be despatched with its full capacity. The third paper characterizes bidding behavior and market outcomes in uniform and discriminatory electricity auctions. Uniform auctions result in higher average prices than discriminatory auctions, but the ranking in terms of productive efficiency is ambiguous. The comparative effects of other market design features, such as the number of steps in suppliers' bid functions, the duration of bids and the elasticity of demand are analyzed. The paper also clarifies some methodological issues in the analysis of electricity auctions. In particular we show that analogies with continuous share auctions are misplaced so long as firms are restricted to a finite number of bids.

Contents

Declaration

Sections 2 and 3 of this thesis contain work carried out conjointly with Professor Nils-Henrik von der Fehr, Department of Economics, University of Oslo. Section 4 of the thesis contains work carried out conjointly with Professor Nils-Henrik von der Fehr and Dr. Natalia Fabra, Department of Economics, Universidad Carlos III de Madrid. I was responsible for at least fifty percent (50%) of the conjoint work contained in Sections 2 and 3 of the thesis, and thirty-three and a third percent (33.33%) of the conjoint work in Section 4 of the thesis.

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1 Introduction

This thesis consists of three self-contained papers on the analysis of electricity auctions written over a period of twelve years from 1992 to 2004. The paper in Section 2, "Spot Market Competition in the UK Electricity Industry", was written jointly with Nils-Henrik von der Fehr.[1] It models price competition in a decentralized wholesale market for electricity as a first-price, sealed-bid, multi-unit auction. In this model, firms simultaneously submit offer prices at which they are willing to supply their capacities for each generating unit, units are ranked according to their offer prices, demand is realized and the system marginal price determined by the by the offer price of the marginal operating unit. A key result is that pure-strategy equilibria do not always exist in the model. The reason is basically the same as that in standard oligopoly models of capacity-constrained price competition. Since, when demand is sufficiently large, a firm is unable to serve the whole market at the competitive price, there is an incentive to bid above marginal cost and the competitive outcome cannot be an equilibrium. It can then be shown that for a range of demand distributions no other pure strategy combinations constitute an equilibrium either. In both the pure and mixed-strategy equilibria of the model, above marginal cost pricing and inefficient despatch of generating units occur. We also consider an alternative regulatory pricing rule, and show that offering to supply at marginal cost can be induced as a dominant strategy for all firms, thereby securing efficient despatch.

The paper in Section 3 was previously entitled "Long-Term Contracts and Imperfectly Competitive Spot Markets: A Study of the UK Electricity Industry" and written jointly with Nils-Henrik von der Fehr.[2] It analyses

[1] The paper was originally published as University of Oslo Department of Economics Memorandum No. 9, 1992. An abridged version was published in the *Economic Journal* in 1993, and reprinted in Paul L. Joskow and Michael Waterson (eds) *Empirical Industrial Organization*, Edward Elgar publishing Ltd, 2004; and in Ray Rees (ed) *The Economics of Public Utilities*, Edward Elgar publishing Ltd, (forthcoming 2005).

[2] It was published as University of Oslo Department of Economics Memorandum No. 14, 1994.

strategic interaction between long-term contracts and price competition in the British electricity wholesale market. As in Section 2, the price mechanism is modelled as a first-price, sealed-bid auction, and we demonstrate that forward contracts, or "contracts for differences," will put a downward pressure on spot market prices. In addition, a 'strategic commitment' motive for selling a large number of contracts is identified: a generator may thereby commit itself to bidding lower prices into the spot market in order to ensure that it will be despatched with its full capacity. In the resulting asymmetric equilibrium, the generator which has not contracted forward bids high in order to ensure high prices, but sells less output.

The paper in Section 4 was written jointly with Natalia Fabra and Nils-Henrik von der Fehr.[3] Motivated by the introduction of a new auction format in the England and Wales electricity market, and the auction design debates in California, it characterizes bidding behavior and market outcomes in uniform and discriminatory electricity auctions under a variety of assumptions concerning firms' costs and capacities, demand elasticities, the auction bid format and the number of suppliers in the market. The aim was to gain an improved understanding of how different auction formats affect the degree of competition and overall welfare in decentralized electricity markets. The uniform auction is outperformed in consumer surplus terms by the discriminatory auction, but uniform auctions can be more efficient. The overall welfare ranking of the auctions is thus ambiguous. The paper also addresses some methodological issues in the analysis of electricity auctions. First, it demonstrates that the set of equilibrium outcomes in uniform and discriminatory auctions is essentially independent of the number of admissible steps in suppliers' offer-price functions, so as long as this number is finite. This reduces the complexity involved in the analysis of multi-unit auctions. Second, we demonstrate that the 'implicitly collusive' equilibria found in the

[3]The paper will appear in the *Rand Journal of Economics* in 2006 under the title "Designing Electricity Auctions." An earlier version of the paper, entitled "Designing Electricity Auctions: Uniform, Discriminatory and Vickrey," was presented to the IDEI's Conference on "Wholesale Markets for Electricity," University of Toulouse, 22-23 November 2002.

uniform auction when offer prices are infinitely divisible are unique to this formulation of the auction (i.e. to share auctions), and do not arise when offer-price functions are discrete. Hence the concerns expressed in the literature that uniform auctions may lead to 'collusive-like' outcomes even in potentially competitive periods when there is considerable excess capacity, are likely misplaced.

When we first began to study the new electricity market introduced in England and Wales in 1990, it was not widely understood that this market was a first-price, sealed-bid, multi-unit auction and thus could be analyzed using tools from auction theory. The early literature tended to use traditional models from IO theory to describe this market,[4] and a major part of the purpose of our 1992/3 paper was simply to model the market rules as they were given to us, albeit in admittedly long and near-impenetrable documents produced by the then National Grid Company. Although some researchers have continued to make use of Cournot or other standard IO models for this purpose, by the late 1990's it was no longer possible to dispute the intimate connection between decentralized electricity wholesale markets and auction theory. As Paul Klemperer noted in 2001, "von der Fehr and Harbord were seen as rather novel in pointing out that the new electricity markets could be viewed as auctions. Now, however, it is uncontroversial that these markets are best understood through auction theory, and electricity market design has become the province of leading auction theorists."[5]

That decentralized electricity wholesale markets are auctions, and hence best understood through auction theory, became so well accepted that much of the debate in Britain in the late 1990's concerning the reform of the electricity trading arrangements focused on the merits and demerits of different auction formats.[6] The analysis in Fabra, von der Fehr and Harbord (2004)

[4] As described in Armstrong, Cowan and Vickers (1994), Chapter 9, for example.
[5] Klemperer (2001).
[6] Harbord and McCoy (2000), Klemperer (2002) and Wolfram (1999) contain discussions of this debate. See also Kahn et. al. (2001) for an account of a similar debate that took place in California.

(in Section 4 of this thesis) was at least partly motivated by a perceived need to have rigorous and tractable economic models which would permit a comparison of market performance under different auction rules in light of this debate.

If controversy remains, it is now confined to the type of auction theory to be applied to these markets. Since 1992 there have been two leading contenders. The discrete, multi-unit auction approach taken in our own papers, and the continuous auction approach of Wilson (1979) and Klemperer and Meyer (1989), originally adopted in Green and Newbery (1992). The continuous auction, or "supply function," approach was initially popular and has recently been taken up again by Holmberg (2005) and Wilson (2005). We have commented extensively on the distinction to be drawn between these two approaches in von der Fehr and Harbord (1993) and Fabra, von der Fehr and Harbord (2004), and in the methodological essay Fabra, von der Fehr and Harbord (2002). Our basic result, that discrete or finite bid functions largely eliminate the collusive equilibrium problem which characterizes continuous auction models was first reported in von der Fehr and Harbord (1993), and extended and elaborated on in Fabra, von der Fehr and Harbord (2004). This result was subsequently discovered independently by Kremer and Nyborg (2004).

In 1993 when our first paper in this area was published, there were almost no other papers available on this topic.[7] Now a bibliography of articles on electricity auctions from around the world would number in the hundreds, if not thousands.[8] I have therefore made no attempt to revise our earlier papers to take account of this ever-expanding literature. The paper in Section 4 to a large degree updates and extends the analysis presented in Section 2 in any event, and relatively little further progress has been made

[7] The notable exception was Green and Newbery (1992).

[8] According to "Google Scholar," our 1993 *Economic Journal* paper has been cited in well over 100 subsequent articles, but there are many notable omissions from the list, and these are just the analyses which cite our 1993 paper. Our early survey paper (von der Fehr and Harbord, 1998) cited over 90 academic articles, excluding reports and documents published by regulatory authorities.

in the analysis of electricity contract markets which is the subject of the paper in Section 3.

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2 Spot Market Competition in the UK Electricity Industry

2.1 Introduction

At the core of the recently deregulated and privatized UK electricity industry is the wholesale spot market.[9] Before each period that the market is open, the generating companies (the generators) submit minimum prices at which they are willing to supply power. On the basis of these "offer prices", the National Grid Company, which plays a central role as coordinator and is responsible for running the transmission grid, draws up a least-cost plan of generating units for despatch in the next period. This "rank order", together with demand, determines which units will actually be despatched. Payments to supplying units, or 'sets', are based on a "system marginal price" determined as the offer price of the marginal operating unit in every period.

The particular organization of the electricity spot market makes standard oligopoly models inadequate as analytical tools. We propose instead to model this market as a sealed-bid multiple-unit auction. In the first stage of the model, firms simultaneously submit offer prices at which they are willing to supply their (given) capacities. As in the UK industry, firms (generators) can submit different offer prices for each individual plant or generating set, i.e. firms offer step-supply functions. Sets are then ranked according to their offer prices (i.e. a supply function is constructed). In the final stage, demand is realized and the system marginal price is determined by the intersection of demand and supply, that is by the offer price of the marginal operating unit.

It turns out that pure-strategy equilibria do not always exist in such a model. The reason is basically the same as that in standard oligopoly models of capacity constrained price competition (Kreps and Scheinkman, 1986). Since, when demand is sufficiently large, a firm is unable to serve the whole

[9] For details on the UK electricity industry, new and old, see Vickers and Yarrow (1991), Green (1991) and James Capel & Co.(1990).

market at the competitive price, there is an incentive to raise bids above marginal cost, and thus the competitive outcome cannot be an equilibrium. It can then be shown that for a range of demand distributions no other pure strategy combinations constitute an equilibrium either. We believe that this result does not necessarily reflect an inadequacy of our modelling approach, but rather suggests that there is an inherent price instability in the present regulatory set up. Indeed, empirical evidence would seem to confirm that experimentation and abrupt changes in pricing strategies is a feature of the new industry.

This particular result (the nonexistence of equilibrium in pure strategies) also casts some doubt on the relevance of the model analyzed by Richard Green and David Newbery (1992) (see also Bolle, 1990 and Newbery, 1991). These authors argue that the "step-length", i.e. the size of individual generating sets, is small enough to justify approximating the step-supply schedules by smooth (i.e. continuously differentiable) functions, thus applying the supply function framework developed by Klemperer and Meyer (1989). As we demonstrate however, the particular types of equilibria they derive do not generalize to a model where sets are of positive size.

Although theirs is a seemingly very useful contribution, it remains "an open question whether the bidding strategies of the firms will differ significantly if they are forced to provide a step function, or whether they are allowed to provide a smooth schedule" (Green and Newbery, 1992, footnote 2).[10]

Nevertheless the most important result, inefficient pricing, turns out to be robust to alternative forms of modelling. Indeed, we find an even stronger tendency than Green and Newbery towards above marginal-cost pricing. Thus the conjecture that the Bertrand outcome is unlikely in the present institutional set-up of the UK electricity industry, even if there is no collusive behavior, seems to be strongly supported. In addition, our model

[10] Green and Newbery also assume downward sloping demand curves, whereas completely inelastic demand would seem to be more appropriate for the UK industry. Bolle (1990) proves that in the latter case, no equilibrium exists in the supply function model.

suggests that high-cost sets may be bid in at lower offer prices than lower-cost sets and thus be despatched before these more efficient units. Hence despatching may be inefficient in the sense that overall economic generation costs are not minimized.

An important advantage of our framework is that it makes it possible to model explicitly the role of the grid company (the auctioneer), and then use insights from the auction literature to study the effects of different pricing rules, i.e. the rules determining the prices paid to different supplying units. With a pricing rule like the one used in the present UK electricity industry, the game corresponds to a first-price sealed-bid auction. However, by letting the system marginal price be determined by the offer price of one of the marginal non-operating sets, the game corresponds to a (generalized) second-price sealed-bid auction, and in this case offering to supply at marginal cost can be shown to be a dominant strategy for each firm. This result is in accord with what is typically found in the literature on optimal auctions where it is well known that second-price (or Vickrey) auctions lead to higher revenues for the auctioneer than do first-price auctions (Myerson, 1981, or Maskin and Riley, 1989).

The remainder of the paper is organized as follows. Our auction model of the U.K. electricity spot market is presented in Section 2.2 and then analyzed in Section 2.3. In Section 2.4 we consider an alternative regulatory pricing rule. Section 2.5 contains a short summary and conclusions.

2.2 The Model

Before the actual opening of the market, N independent firms simultaneously submit offer prices at which they are willing to supply electricity from each of their generating units, or sets. On the basis of these bids, the market organizer (or 'auctioneer') draws up a ranking of units, i.e. a market supply curve is constructed. When the market opens, demand is determined as a random variable independent of price, and the auctioneer, by calling suppliers into operation, equates demand and supply. Operating units, i.e.

units actually supplying output, are paid the system marginal price which is equal to the offer price of the marginal operating unit.

It is assumed that generators have constant marginal costs, $c_n \geq 0, n = 1, 2, ..., N$, at production levels below capacity, while production above capacity is impossible. We let the index n rank firms according to their marginal costs, i.e. $c_n \leq c_{n+1}$. The total capacity of generator n is given by $k_n, n = 1, 2, \ldots, N$. The capacity of generator n consists of m_n sets, where k_{ni} is the capacity of the i'th set, $i = 1, 2, ..., m_n$, and $\sum_i k_{ni} = k_n$. M denotes the total number of sets, i.e., $M = \sum_n m_n$. Generators can submit different bids for each of their sets. If two or more sets (of any firm) are offered at the same price, they are equally likely to be called into operation.

We consider a game G with $N+1$ players: N suppliers and Nature. The *move order* is as follows:

Stage 1: The suppliers simultaneously submit offer prices, $p_{ni} \leq \overline{p}, i = 1, 2, ..., m_n, n = 1, 2, \ldots, N$ at which they are willing to supply electricity from each of their generating units.

Stage 2: Sets are ranked according to their bids such that if the bid of the set with rank r is p^r and that of the set with rank $\widehat{r}$ is $p^{\widehat{r}}$ and $p^r < p^{\widehat{r}}$ then $r < \widehat{r}$. If m sets are offered at the same price $\widetilde{p}$, then these sets are designated numbers $r, r+1, ..., r+m-1$, with (marginal) probabilities $1/m$, for some $r \in \{1, 2, ..., M-m+1\}$.

Stage 3: Nature chooses demand $d \in [\underline{d}, \overline{d}] \subseteq [0, K]$, $K = \sum_n k_n$ according to some probability distribution $G(d)$. The auctioneer, by calling suppliers into operation, equates demand and supply. Despatched units are paid the market-clearing price, which is equal to the offer price of the marginal operating unit.[11]

Note that to assume $d \in [0, K]$ is without loss of generality since supply is limited to K and thus demand will have to be rationed if it increases

[11] More precisely, let $K_0 = 0$ and $K_r = \sum_{j=1}^{r} k^j, r = 1, 2, ..., M$ where k^r is the capacity of the set with rank r. Let $\rho = \max\{r \mid K_{r-1} < d\}$.Then all sets with rank $r = 1, 2, ..., \rho-1$ get paid $p^\rho k^r$ while set ρ gets $p^\rho[d - K_{\rho-1}]$. Let s_n be the actual supply of firm n, i.e. $s_n = \sum_{r=1}^{\rho-1} \delta_n(r) k^r + \delta_n(\rho)[d - K_{\rho-1}]$, where $\delta_n(r)$ is 1 if the set with rank r belongs to firm n, and zero otherwise. The payoff to firm n is then $s_n[p^\rho - c_n]$.

beyond K. In particular, $G(d)$ will typically have an atom at $d = K$, reflecting the fact that rationing may occur with positive probability. In addition, the present U.K. electricity supply industry is characterized by significant excess capacity, and this is likely to remain true for the foreseeable future. Hence $d < K$ would appear to be the relevant case.

All players are assumed to be risk neutral and hence aim to maximize their expected payoff in the game. All aspects of the game, as well as the players' marginal costs and capacities and the probability distribution $G(d)$, are assumed to be common knowledge.

Firms' offer prices are constrained to be below some threshold level $\overline{p} < \infty$, since otherwise, in cases when there is a positive probability that all sets will be called into operation, expected payoffs could be made infinitely large. A natural interpretation of $\overline{p}$ is that it is a regulated maximum price, either officially, or as perceived by the generators (i.e. firms believe that the regulatory authorities will effectuate price regulation if the price rises above $\overline{p}$).[12] An alternative interpretation is that $\overline{p}$ is a reservation price, below which demand is completely inelastic.

The model may be interpreted as a first-price, sealed-bid, multiple-unit auction with a random number of units in which all units are sold simultaneously (McAfee and McMillan, 1987; , Hausch, 1986). It is 'sealed-bid' because of the simultaneous move structure and 'first-price' in the sense that the market price is determined by the marginal successful supplier. This interpretation is particularly convenient for analyzing alternative pricing rules.

2.3 Analysis

In this section we characterize the Nash equilibria of the model presented in Section 2.2. Most of the discussion will centre on the duopoly case, for which we are able to derive explicit results. Apart from being the relevant case for the UK electricity industry, explicit formulae for optimal strategies are difficult to derive in the more general oligopoly case. Hence our discussion

[12] In the England and Wakes pool, system marginal price cannot exceed the value of lost load (approx. £2 per kWh).

of oligopoly in this section is in most cases limited to pointing out where and how the duopoly results generalize.

Before discussing particular equilibrium outcomes, we present a basic result characterizing the types of pure-strategy equilibria that can occur in the model.

Proposition 2.1 *In a pure-strategy equilibrium (generically) at most one generator will determine system marginal price with positive probability.*

Remark: By genericity is here meant that firms have different marginal costs. If firms have identical marginal costs there may exist Bertrand-type equilibria where firms submit offer prices equal to the marginal costs of each set, and in which more than one firm owns sets which with positive probability may become the marginal operating unit.

The intuition underlying Proposition 2.1 is straightforward. A player which owns a set that has a positive probability of becoming the marginal operating unit, will always want to increase the bid of that set by some small amount towards the next higher bid, since that does not affect the ranking, but increases the generator's payoff in the event that this is the marginal set. On the other hand, it cannot be optimal to submit a bid equal to or just above that of a set of another player, since as long as the bid is above marginal cost (which it will be) profits can be increased by undercutting the rival slightly, thereby increasing the probability of being called into operation, without significantly reducing the price received in any state. These two opposing forces destroy any candidate for a pure-strategy equilibrium in which two or more generators both have sets which with positive probability will become the marginal operating unit.

Proposition 2.1 implies that the types of pure-strategy equilibria that may exist are very restricted and, furthermore, it rules out the existence of pure-strategy equilibria for a wide range of demand distributions. From this it follows that the types of equilibria found by Green and Newbery (1992) in their model, do not generalize to the case where individual generating sets are of positive size. The reason that such equilibria exist in the supply function framework is that when individual sets are of size zero (the cost and supply functions are continuously differentiable everywhere), the effect on the system marginal price from a bid of any individual set is negligible, and thus the first part of the above argument does not apply.

Below we consider circumstances under which pure-strategy equilibria will exist, as well as presenting examples of mixed-strategy equilibria when pure-strategy equilibria are non-existent. The existence, multiplicity and the type of equilibria will be seen to depend crucially on the support of the demand distribution. We therefore distinguish between three cases: 'low-demand periods' in which a single generator can supply the whole of demand; 'high-demand periods' in which neither generator has sufficient capacity to supply the entire market; and 'variable-demand periods' in which there is positive probability for both the event that a single generator can supply the whole of demand, and the event that both generators will have to be called into operation, irrespective of their bids.

2.3.1 Low-demand periods

This case corresponds to the standard Bertrand model of oligopoly in the sense that there is a unique equilibrium in which both generators offer to supply at a price equal to the marginal cost of the least efficient generator:

Proposition 2.2 *If $Pr(d < min\{k_1, k_2\}) = 1$, there exist pure-strategy equilibria, in all of which the market clearing price equals the marginal cost of the least efficient generator, c_2, and only generator 1 produces.*[13]

Remark: When $k_1 > k_2$, such equilibria continue to exist when $Pr(d \leq k_1) = 1$, but other equilibria may exist also (see the next section).

[13] To avoid non-existence, we impose the tie-breaking rule that firm 1 is called into operation with probability 1 whenever the firms' offer prices tie at c_2. This captures the idea that the most efficient firm marginally underbids its rival, while simplifying the description of the equilibrium.

The argument in the proof is identical to that of the standard Bertrand model. Since, with probability 1, demand can be met by a single generator, there will be competition to become the single operating generator. In particular, a generator will always undercut its rival so long as its rival's bids are above its own marginal costs. Thus any equilibrium must have the most efficient generator (generator 1) submitting offer prices for a capacity sufficient to meet demand, at or below the marginal cost of the least efficient generator. Since in this range, generator 1's profit is increasing in its own offer price, these bids must equal c_2. We conclude that in low-demand periods, the system marginal price is bounded above by the marginal costs of the less efficient generator.[14]

2.3.2 *High-demand periods*

We now consider the case when, with probability 1, both generators will be called into operation. Since the high-pricing generator will be operating for sure, and in equilibrium generators never submit equal bids (see Proposition 2.1), its profit will be increasing in its own offer price. Thus, the extreme opposite to the result of the previous section holds; whereas in low-demand periods the system marginal price equals the marginal cost of the least efficient generator, in high-demand periods it always equals the highest admissible price.

Proposition 2.3 *If $Pr(d > max\{k_1 k_2\}) = 1$, all pure-strategy equilibria are given by offer-price pairs (p_1, p_2) satisfying either $p_1 = \overline{p}$ and $p_2 \leq b_2$ or $p_2 = \overline{p}$ and $p_1 \leq b_1$, for some $b_i < \overline{p}, i = 1, 2$.*

Remark: If $k_1 > k_2$ ($k_2 > k_1$), then all (p_1, p_2) *such that* $p_1 = \overline{p}$ *and* $p_2 \leq b_2$ ($p_2 = \overline{p}$ *and* $p_1 \leq b_1$) continue to be equilibria for all $G(\cdot)$ such that $Pr(d > k_2) = 1$ ($Pr(d > k_1) = 1$). That is, a sufficient condition for the

[14] A similar result can be shown to hold in the oligopoly model. If, with probability 1, demand is less than the total capacity of the n most efficient generators ($n < N$), then in an equilibrium system marginal price cannot exceed the marginal cost of the n+1st most efficient generator.

existence of this type of equilibrium is that, with probability one, demand is greater than the capacity of the smaller firm.

The intuition for the result is straightforward. The high-bidding generator will always determine the system marginal price by Proposition 2.1. Therefore its payoff is increasing in its own offer prices and profit maximization requires bidding at the highest admissible price. The low-bidding generator is indifferent between offer prices lower than that of the high-bidding generator. However, to ensure that the high-bidding generator does not deviate, the low-bidding generator has to bid low enough so that the high-bidding generator's payoff from undercutting is less than the payoff earned in equilibrium. Thus the upper bound on the low-bidding generator's offer price.

In all of the equilibria characterized by Proposition 2.3, the system marginal price equals the highest admissible price. The low-bidding generator is despatched with full capacity while the high-bidding generator supplies the residual demand. It follows that both generators prefer equilibria where they act as the low-bidding generator, since the received price is the same while a generator's output is greater in the equilibrium in which it is ranked first.

Note that some of these equilibria involve inefficient despatch: the high-cost generator may be the generator with the lowest bid and thus will be despatched with its total capacity, while the low cost generator is only despatched with part of its capacity. In such equilibria, generation costs are not minimized.[15]

2.3.3 Variable-demand periods

We turn now to the third case in which either generator may set system marginal price with positive probability independently of its bid (i.e. rank). In the England and Wales pool generators bid daily, and (depending on the

[15] It is easy to see that in the oligopoly case we get a corresponding result. Whenever demand is such that the highest-bidding generator determines the system marginal price with probability 1, any vector of offer prices such that the highest-pricing generator submits the maximum admissible price, while the rest bid sufficiently below this, will be an equilibrium

season) their bids may consequently be constant for time periods in which demand is expected to be high (morning and afternoon) and periods in which it will be low (night time).[16] This can be modelled as if the generators face a single period in which demand could be low or high with some probability.

It turns out that pure strategy equilibria do not exist in this case and hence equilibria are in 'mixed strategies.' Under their mixed strategies, in equilibrium both generators randomize over their price bids from an interval bounded below by the least efficient generator's marginal costs, and above by the highest admissible price. Expected prices still exceed the marginal costs of generation, however what the market price will be is the result of a random process and cannot be predicted exactly.

The non-existence of pure-strategy equilibria follows from observing that bid pairs like those in Proposition 2.3 cannot constitute equilibria in this case since the low-bidding generator will now always wish to increase its bid; in doing so it thereby increases the system marginal price in the event that it becomes the marginal operating generator. We therefore have the following result:

Proposition 2.4 *If $\bar{d} - \underline{d} > max\{k_1, k_2\}$, where $[\underline{d}, \bar{d}]$ is the support of the demand distribution $G(d)$, then there does not exist an equilibrium in pure strategies.*

This result follows directly from Proposition 2.1. Since the range of possible demand distributions exceeds the capacity of the largest generator, it follows that for any strategy combination there is a positive probability that sets of either generator will be the marginal operating unit. We can then apply the result of Proposition 2.1; there cannot exist pure-strategy equilibria for which more than one generator has a positive probability of determining the system marginal price.

Characterization of mixed-strategy equilibria in the general model is

[16]This is different in the Scandinavian pool, in which different price bids may be submitted for each of the 24 hourly periods that the market is open. The variable-demand case is consequently of less relevance for this market.

cumbersome, and in the remainder of this section we consider a simple example. In the example, it is assumed that for all $n, m_n = 1$, i.e. each generator owns only one set, or can submit only one offer price for the whole of its capacity, and we characterize the mixed-strategy equilibria for the duopoly case.[17] We are able to show that there exists a unique mixed-strategy equilibrium, and we derive the explicit form of the two players' strategies. In particular, we find that the lowest price in the support of the players' strategies is strictly greater than the marginal cost of the least efficient generator, and that this lowest price is an increasing function both of the highest possible price $\overline{p}$, the probability that both firms will be operating (i.e. demand), and the marginal cost of the least efficient generator.

The analysis is considerably simplified by restricting attention to the following special case: All firms are assumed to have equal capacities normalized to 1, and demand is discrete and distributed on $\{1, 2\}$ with probabilities $\pi_n = Pr(d = n), n = 1, 2$, with $Pr(d = n) > 0$ and $\sum_n \pi_n = 1$. Since the main results carry over to the more general model, we concentrate on this special case.

Assume $N = 2$ and define $\pi_1 \equiv \pi$. Without loss of generality, normalize c_1, to zero and let $c_2 = c$. The assumption on the support of the demand distribution in Proposition 2.4 now corresponds to the case where $0 < \pi < 1$, i.e. the events that one and two generators are called into operation both occur with positive probability. Define:

$$F_1(p) = \begin{cases} \ln(e \cdot \frac{p-c}{\overline{p}-c}), & \text{when } \pi = \frac{1}{2} \\ \frac{\pi-1}{2\pi-1}[\frac{p-c}{\overline{p}-c}]^{\frac{1-2\pi}{\pi}} + \frac{\pi}{2\pi-1} & \text{when } \pi \neq \frac{1}{2} \end{cases} \tag{2.1}$$

$$F_2(p) = \begin{cases} \ln(e \cdot \frac{p-c}{\overline{p}+[e-1]c}), & \text{when } p < \overline{p} \text{ and } \pi = \frac{1}{2} \\ \frac{\pi-1}{2\pi-1}[\frac{p}{\overline{p}+[\alpha(\pi)-1]c}]^{\frac{1-2\pi}{\pi}} + \frac{\pi}{2\pi-1}, & \text{when } p < \overline{p} \text{ and } \pi \neq \frac{1}{2} \\ 1, & \text{when } p = \overline{p} \end{cases} \tag{2.2}$$

$$\text{where } \alpha(\pi) = [\frac{\pi}{1-\pi}]^{\frac{\pi}{2\pi-1}}$$

[17] Analyses of mixed strategies in models with a similar structure to the model presented here can be found in Shilony (1977), Varian (1980), and Padilla (1992). See Brandenburger (1992) for a discussion of the interpretation of mixed-strategy equilibria as equilibria in beliefs.

$$p^m = \begin{cases} \frac{\overline{p}-c}{e} + c, & \text{when } \pi = \frac{1}{2} \\ [\overline{p} - c]\left[\frac{\pi}{1-\pi}\right]^{\frac{\pi}{1-2\pi}} + c & \text{when } \pi \neq \frac{1}{2} \end{cases} \tag{2.3}$$

where $ln(e) = 1$. We can then prove the following result.

Proposition 2.5 *There exists a unique mixed-strategy Nash equilibrium in which player i's strategy is to play $p\varepsilon[p^m, \overline{p}]$ according to the probability distribution $F_i(p), i = 1, 2$, where $F_i(p)$ are given by (2.1) and (2.2) and $p^m > 0$ is given by (2.3). Further, $F_1(p) \geq F_2(p)$.*

In equilibrium, players strike a balance between two opposing effects. On the one hand a high bid results in a high system marginal price - and hence payoff - in the event that the generator is marginal. On the other hand, bidding high reduces the probability of being despatched. The latter effect is less important the smaller is π, since then it is very likely that both generators will be despatched. Conversely, when the probability that both generators will be operating is low (i.e. π is large), less probability mass is placed on higher prices. To say this another way, the incentive to raise one's bid is checked by the likelihood of ending up as the higher pricing generator and not being called into operation: when π is small, there is a substantial probability that a generator will be operating even if it offers to supply only at a very high price. Thus, for small π both generators will tend to submit high bids and visa versa. Indeed, the following is easily demonstrated:

$$\lim_{\pi \to 1} \quad p^m = c, \tag{2.4}$$

$$\lim_{\pi \to 0} \quad p^m = \overline{p}. \tag{2.5}$$

Note that the limit in (2.4) corresponds to the case discussed in the 'low-demand periods' subsection, while the limit in (2.5) corresponds to the 'high-demand' case.

The high-cost generator's strategy profile first-order stochastically dominates the strategy profile of the low-cost generator (i.e. $F_2(p) \leq F_1(p)$).

Thus in expected terms, the high-cost generator will submit higher bids than the low-cost generator. A lower bound for the probability that the high-cost generator submits a bid *below* that of the low-cost generator can be established by considering the probability that $p_2 < p_1 - c$. When $\pi = \frac{1}{2}$, this reduces to

$$\Pr(p_2 < p_1 - c_2) = \frac{1}{2}[1 - \ln(1 + \frac{e}{\alpha - 1})]^2 \qquad (2.6)$$

where $\alpha \equiv \overline{p}/c$. If $\alpha = 5(10)$, i.e. $\overline{p}$ is 5 (10) times the marginal cost of the high-cost generator, this probability equals 12% (27%). Thus, although the typical outcome is that the high-cost generator prices above the low-cost generator, there is a potentially significant positive probability that the high-cost generator submits the lowest price offer and thus becomes the only operating generator. Therefore we may conclude that, as in the high-demand periods case discussed above, the regulatory rule, as it is modelled here, is not ex-post efficient (we discuss below how the rule may be adjusted to ensure efficient despatch).

We may use this model to consider the question of how an increase in the number of independent generators will effect (average) pool prices, by briefly extending the analysis here to the oligopoly model (i.e. $N > 2$). In order to do so we assume that firms have equal marginal costs and without further loss of generality, these are normalized to zero. We consider a symmetric model since this is the only case in which it is possible to characterize equilibria in any detail. We continue to assume that there is a positive probability that all units will be called into operation, i.e. $\pi_N > 0$, since otherwise, given the symmetry assumption, only the perfectly competitive outcome would be possible. We then obtain the following result:

Proposition 2.6 *Assume* $c_n = 0, n = 1, ...N$. *Then there exists a unique symmetric mixed-strategy equilibrium for the game in which each player plays prices* $p \in [p^m, \overline{p}]$ *according to the probability distribution* $F(p)$ *where* $F(p)$ *is the solution to:*

$$F'(p) = \frac{\alpha(F(p))}{p\beta(F(p))} \equiv \Omega(p, F(p)), \tag{2.7}$$

and

$$\begin{aligned} \alpha(q) &\equiv \textstyle\sum_{i=1}^{N} \pi_i b(i-1; N-1, q) \\ \beta(q) &\equiv \textstyle\sum_{i=1}^{N-1} \pi_i B_q(i; N-1, q) \end{aligned} \tag{2.8}$$

In (2.8), $b(i; N, q)$ is the density function of the binomial probability distribution with parameters N and q, $B(i; N, q)$ is one minus the corresponding cumulative binomial probability distribution, and $B_q = \partial B / \partial q$. Furthermore, $F(p^m) = 0, F(\overline{p}) = 1$, and $p^m > 0$.

From the uniqueness of the solution to (2.7) and (2.8), it follows that p^m is decreasing in $\overline{p}$. Note that $B_q(i; N-1, q)$ is always decreasing in i for sufficiently small q. For larger $q, Bq(i; N-1, q)$ is increasing (decreasing) in i for small (large) i. $b(i-1; N-1, q)$, as a function of i, is shaped as an inverse V. Thus, reducing π_i for small i and increasing π_i for larger i, typically increases $\Omega(p, F(p))$ for given p. Therefore one would expect that p^m is larger the more probability weight there are on p_i for large i's. We have the following limiting results:

$$\lim_{\pi_1 \to 1} p^m = 0 \tag{2.9}$$

$$\lim_{\pi_N \to 1} p^m = \overline{p} \tag{2.10}$$

The question of particular interest here is how the number of suppliers in the market will affect the price structure. There are in general two different ways of analyzing this. We could either think of a situation where, for a given level of demand, additional firms are introduced into the market, i.e. total capacity is increased, or a situation in which existing firms are split up into smaller units, i.e. a given total capacity is divided between a larger number of firms. If the question of primary interest is the organization of the deregulated structure of an existing industry, the latter approach seems the most natural and this is what will be considered here. We analyze a particular example, where $\pi_i = 1/N$, by comparing the outcome for different N's. By substituting for p_i and solving (2.7), we obtain the following:

Result: When $\forall n$, $c_n = 0$, and $\forall i$, $p_i = 1/N$,

$$\begin{aligned} F(p) &= \frac{1}{N-1} - \ln(e^{N-1} - \frac{p}{\overline{p}}), \\ p^m &= \overline{p} - e^{1-N}, \text{and} \\ Ep &= \frac{\overline{p}}{N-1} - [1 - e^{1-n}]. \end{aligned} \tag{2.11}$$

Thus, both p^m and Ep are decreasing in N. That is, prices will tend to be lower on average in a more fragmented industry. The intuition for this may be explained as follows: By increasing its offer price a generator reduces the probability that it will receive a positive payoff. On the other hand, submitting a high offer price increases, in expected terms, the system marginal price. The system marginal price effect, however, benefits the generator only when it happens to be the marginal generator, an event which is less likely the more firms there are in the industry.

This intuition also suggests that in the more general model with multi-unit firms, prices will tend to be higher than in the model in which these same units act independently. As indicated above, raising the offer price of one unit will have an external effect on other units in that it increases the expected system marginal price. A generator which controls many units will internalize part of this externality and will thus have an additional incentive to increase its offer prices. In particular, this "coordination incentive" is stronger the more units an owner controls. It therefore seems reasonable to conclude that for a given number of generating sets in the industry, the system marginal price will be a decreasing function of the number of owners, or generators controlling the sets, i.e. the industry concentration ratio.

2.4 An Alternative Payoff Rule

As we have shown in Section 2.3, firms will in general choose bids greater than their marginal costs, and thus the system marginal price will tend to exceed the marginal costs of each of the operating units. Furthermore, since less efficient sets may submit lower offer prices than more efficient sets, inefficient despatching may result. It is therefore an interesting question whether the regulatory rule can be modified so as to induce truthful revelation of costs and, as a result, efficient despatching. In this section we show that by extending an insight on optimal auctions due to Vickrey (1961), such a modification is indeed possible.

The electricity market game G may be interpreted as a first-price, sealed-bid, multiple-unit auction with a random number of units. In particular, the system marginal price is determined by the offer price of the marginal operating set, and thus a firm's bids will determine the price received in the event that one of its sets is the marginal operating unit. The fundamental insight of Vickrey (1961) was that by making the price received by a firm independent of its own offer price, marginal cost pricing can be induced as a dominant strategy for all firms. The reason for this is that in such a set-up a firm can only influence its own payoff to the extent that it affects the probability of being called into operation. A firm will prefer to be operating for all realizations of demand such that its payoff is positive, and will prefer not to operate whenever its payoff is negative. Therefore, offering to supply at a price equal to marginal cost becomes a dominant strategy because it maximizes the probability of being called into operation whenever the firm's payoff is expected to be non-negative.

In a standard Vickrey auction, price is determined by the marginal unsuccessful, i.e. non-operating, player. To generalize this result, we must construct a mechanism which is both incentive compatible and individually rational. This can be done by letting the price paid to firm n be determined by the intersection of demand with the residual (i.e. net of the capacity of firm n) supply curve. Consider therefore a slight variation of the game analyzed in Section 2.3 above where the only change involves the payoff rule: the intersection of the demand and the supply curves determines which units will be called into operation. All operating units are paid firm-specific prices determined by the intersection of the demand and the respective residual supply curves if such an intersection exists, and set equal to $\widehat{p} \geq \max\{c_n\}$ otherwise. Call this game $\widehat{G}$. Then the following result holds:

Proposition 2.7 *The game $\widehat{G}$ has a unique dominant strategy equilibrium in which $p_{ni} = c_n$, $n = 1, 2, ..., N$*

Remark: Other Nash equilibria typically exist. However, since offering to supply at marginal cost (weakly) dominates all other strategies, we consider this the natural 'focal' point, and thus base our discussion solely on this equilibrium.

In $\widehat{G}$, as opposed to G, despatch is efficient since firms are always despatched in order of increasing marginal cost. Thus, our alternative regulatory rule leads to minimization of real generation costs.[18]

In addition to technical efficiency, one might ask how total (expected) payments to the generators compare in the two auctions. Denote by EC total expected payments in G, and by $\widehat{EC}$ total expected payments in $\widehat{G}$, respectively. It is easy to verify that revenue equivalence holds when valuations are drawn from the same distribution (if we let $\widehat{p} = \overline{p}$) as we would expect from the Revenue-Equivalence Theorem (Vickrey 1961; McAfee and McMillan 1987). For example, in the symmetric oligopoly model considered in Section 2.3, $EC = N\pi_N\overline{p}$ (since players receive the same payoff whichever of the prices in the support of their strategies they play, and, in particular, the profit from playing $\overline{p}$ is $\pi_N\overline{p}$). On the other hand, $\widehat{EC} = N\pi_N\widehat{p}$ (since payments are zero when some firms do not operate and $\widehat{p}$ to each of the N firms otherwise). It turns out to be difficult to establish the sign of $EC - \widehat{EC}$

[18] Efficiency considerations in electricity supply industries are complicated considerably by the network characteristics of such industries, and being able to rank generating units according to production costs is only a necessary condition for short-run efficiency. For a treatment of efficiency and optimal pricing in electrical networks, see Bohn, Caramanis, and Schweppe (1984).

in the general model. However, in the duopoly case one can show that EC is never smaller than $\widehat{EC}$.This is obvious in the cases discussed in sections 2.3.1 and 2.3.2. In the case analyzed in section 2.3.3, EC can be found by considering $\phi_2(\overline{p})$ and $\phi_1(p)$ as $p \rightarrow \overline{p}$, where $\phi_i(p)$ is the profit of firm i from playing p, i = 1,2, from which it follows that EC=$\pi K c + [1-\pi]2\overline{p}$, where

$$K = \begin{cases} \alpha \ln(1 + \frac{\alpha}{\alpha + [e-1]}), & \text{when } \pi = \frac{1}{2} \\ \alpha \frac{\pi - 1}{2\pi - 1} \left\{ 1 - [\frac{\alpha}{\alpha + [\alpha_K - 1]}]^{\frac{1-2\pi}{\pi}} \right\} & \text{when } \pi \neq \frac{1}{2} \end{cases}$$

and

$$\alpha = \frac{\overline{p}}{c},\ \alpha_K = \left[\frac{\pi}{1-\pi}\right]^{\frac{\pi}{2\pi-1}}$$

and $\widehat{EC} = \pi c + [1-\pi]2\overline{p}$. Now, $K \geq 1$, and $K = 1$ when $\alpha = 1$ and is increasing in α. For given α, K is maximized at $\pi = 1/2$ and $K \leq e - 1$. We summarize the duopoly result in the following proposition:

Proposition 2.8 *When $N = 2$, $\widehat{EC}$ is a lower bound for EC.*

Such an improved pricing performance echoes the result in the optimal-auction literature that second-price sealed-bid auctions yield higher payoffs to the auctioneer than do first-price sealed-bid auctions (McAfee and McMillan, 1987; Myerson, 1981; and Maskin and Riley, 1989). Thus, some of the first-price/second-price comparison results found in the auction literature extend to this setting as well.

We conclude that (disregarding collusion and long-term contracts) an institutional set-up which induces firms to make offer prices equal to marginal costs is perfectly possible even when firms are capacity-constrained, something which seems not to have been appreciated in the literature. As such, it also shows that applying results from standard oligopoly models (such as those found in for example Kreps and Scheinkman, 1983 and Tirole, 1988, ch. 5) can be misleading as a description of the outcome of competition in the U.K. electricity market.

2.5 Conclusion

In this paper price competition in the deregulated wholesale market for electricity for England and Wales has been analyzed as a first-price, sealed-bid, multiple unit auction. In doing so, we have demonstrated that under the existing institutional set-up there is likely to be both inefficient despatching and above marginal cost pricing, even in the absence of collusion and long term contracts. While these points have been argued elsewhere (see for instance, Vickers and Yarrow,1991 or Green,1991), the arguments have been largely informal and usually based upon standard models of oligopoly pricing, and hence somewhat inconclusive. A major purpose of the present paper has been to address these issues in a formal model specifically designed to capture the essential elements of new electricity pricing system in the United Kingdom.

Green and Newbery (1992) is the only other model specifically designed to study the bidding behavior of the generators under the new UK system.[19] While the conclusions from the two models concur in many respects, our results cast some doubt upon the type of equilibrium analysis employed by Green and Newbery, i.e. Klemperer and Meyer's (1989) "supply function equilibrium" model. This is because the equilibria found under the assumption that firms submit smooth, i.e. continuously differentiable, supply functions do not appear to generalize to the case where supply functions must be discrete 'step functions', even when the 'step-length' can be made very small. Indeed, they found that for a wide range of demand distributions, pure strategy (i.e. supply function) equilibria will not exist in this case. It is therefore reassuring to find that Green and Newbery's most significant conclusion for policy purposes, viz. above marginal cost pricing, is also a property of the model analyzed here, and hence does not depend upon the particular assumptions they impose.

While the analysis presented here would appear to be useful in providing

[19] While the model of Bolle (1990) is very close that of Green and Newbery (1992) in many respects, its purpose is somewhat more general.

a framework for studying pricing behavior in the deregulated UK electricity industry, the importance of its conclusions is limited by the extent to which it does not take into account opportunities for collusive behavior between the generators, nor the effects of long-term contracts between suppliers and purchasers (or third parties). These problems call for further research.

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2.6 Appendix: Proofs

Proof of Proposition 2.1: Assume by way of contradiction that more than one firm has sets which with positive probability will become the marginal operating unit, and, thus determine the system marginal price. Then, since the support of the demand distribution is an interval, there must exist two sets with rank r and $r+1$, for some $r \in \{1, 2, ..., M-1\}$, belonging to two different firms, both of which will become the marginal operating unit with positive probability. Call the firm that owns the set ranked r firm n and the other firm $\widehat{n}$. Note that since firms can secure non-negative profits by bidding at marginal cost, one must have $p^r \geq c_n$ and $p^{r+1} \geq c_{\widehat{n}}$.$p^r < p^{r+1}$ cannot be part of an equilibrium since by increasing the bid of set r towards p^{r+1}, firm n will increase its profit. Increasing the bid in this a way does not affect the ranking but does increase the system marginal price in the event that the r'th set becomes the marginal operating unit. $p^r = p^{r+1}$ cannot be an equilibrium either, since if $c_n \neq c_{\widehat{n}}$, at least one firm can increase its profit by undercutting. For example, if $c_n < c_{\widehat{n}}$, $\quad p^r = p^{r+1} > c_n$, by the argument above, and thus firm n can increase its profit by undercutting firm $\widehat{n}$ by an arbitrarily small amount thereby strictly increasing its chance of being called into operation without affecting the (expected) system marginal price. QED.

Proof of Proposition 2.2: In any equilibrium firm 1 will determine the system marginal price with probability one. Assume otherwise, i.e. that firm 1 has bid in so many of it sets at high prices that some of the sets of firm 2 have a positive probability of becoming the marginal operating unit. Since firm 2 can secure non-negative profits by bidding at marginal cost, firm 2 will not bid in these sets below c_2. But then firm 1 can increase its profit by undercutting such firm 2 sets by some arbitrary amount, since this increases the (expected) amount supplied by firm 1 without affecting the system marginal price in any event.

Next we show that the system marginal price will not exceed c_2. Assume otherwise. Then since firm 1 always determines the system marginal price, it must have bid in sets at a price greater than c_2. But then firm 2 can increase its profit by undercutting firm 1.

Lastly, since firm 1's profit is increasing in bids on the sets that may become the marginal operating unit, the only candidate for equilibrium involves these being bid in at c_2. (This will be an equilibrium if one imposes the tie-breaking rule that if firms tie at c_2, firm 1 is despatched with probability 1.) QED.

Proof of Proposition 2.3: From Proposition 2.1 it follows that only sets of one firm will determine the system marginal price. Without loss of generality, assume that these belong to firm 2. Then since the profits of firm 2 are increasing in its own bids, all these sets will be bid in at $\overline{p}$. Furthermore, firm 1 must bid in all its sets at offer prices strictly less than $\overline{p}$. Now, for this to be an equilibrium, firm 1 must bid low enough so that firm 2 cannot increase its profit by undercutting. Firm 1 bidding at or below firm 2's marginal cost is sufficient to guarantee this. (Since firm 1 never becomes the marginal firm, it will be willing to do so even if $c_2 < c_1$.) QED.

Before considering the proofs of Propositions 2.5 and 2.6, we derive some properties of the general oligopoly model's mixed-strategy equilibria when firms have equal capacities and demand is distributed discretely as assumed in the main text and under the assumption that any number of units will be called into operation with positive probability. We start by proving that no firm submits offer prices below its marginal cost. We then show that $\overline{p}$ is always part of some player's strategy. Lastly, we prove the following two results: There can be no mass point at any price in a player's mixed strategy, with the possible exception of $\overline{p}$, i.e. no price less than p is played with positive probability. And if p^m is the smallest price in the support of any player's strategy, then all prices $p \in [p^m, \overline{p}]$ are in the support of at least two players' strategies'.

Note that for any equilibrium (mixed) strategy profile there exist infinitely many (generically) equivalent strategy profiles which differ only on sets of measure zero. We do not make any distinction between such strategies.

Lemma A.2.1 *(Lower bounds for the offer prices)* *In any equilibrium and for all $n = 1, ..., N$, $p_n \geq \max\{c_n, c_2\}$, (remember that c_2 is the marginal cost of the second-most efficient firm).*

The result is rather obvious and follows from the observation that a player, by offering a price below his marginal cost, has a positive probability of obtaining a non-positive payoff which could be avoided by choosing a higher offer price. From this, and the fact that no player will choose an offer price below and bounded away from the lowest price ever chosen by anyone else, one concludes that the system marginal price will not fall below the marginal cost of the second-most efficient firm.

Lemma A.2.2 *(Upper support)* *If $\pi_N > 0$, at least one player will have $\overline{p}$ as part of his equilibrium strategy.*

Proof. Let $\widehat{p}$ be the highest price in the support of any players strategy and assume that $\widehat{p} < \overline{p}$. Let n be one of the players which has $\widehat{p}$ as part of his strategy (in particular, if one player plays $\widehat{p}$ with positive probability, let n be him). Playing $\widehat{p}$ yields him an expected payoff of $\widehat{p}\pi_N$ while playing $\overline{p}$ yields $\overline{p}\pi_N > \widehat{p}\pi_N$. QED. ∎

Lemma A.2.3 *(No interior mass points)* *In equilibrium no offer price $p < \overline{p}$ will be played with positive probability by any player. Furthermore, if $\overline{p}$ is played with positive probability by some player, no other player will play p with positive probability.*

Proof. We start by showing that if $\widehat{p} > c_2$ is (believed to be) played with positive probability by some player, $\widehat{p}$ is not played with positive probability by any other player. Let $\widehat{p} > \max\{c_2, c_{\widehat{n}}\}$ be an offer price which is played with positive probability by a player $\widehat{n}$ and assume that only $\widehat{n}$ plays $\widehat{p}$ with probability greater than zero (the argument below extends in a straightforward manner to the case where more than one player plays $\widehat{p}$ with positive probability). Then if a player n, for which $c_n < \widehat{p}$ (by lemma 2.1 such a player exists), plays $\widehat{p}$ an element in his expected payoff is

$$\Pr(p_{\widehat{n}}=\widehat{p})\sum_{i=1}^{N-1}\Pr([p_{n_{i+1}}<\widehat{p}]\cap[p_{n_{i+2}}>\widehat{p}]|p_{\widehat{n}}=\widehat{p})[\widehat{p}-c_n]\{\frac{\pi_i}{2}+\pi_{i+1}\} \quad (A.1)$$

where

$$\Pr(p_{n_0}<\widehat{p}|p_{\widehat{n}}=\widehat{p})\equiv\Pr(p_{n_{N+1}}>\widehat{p}|p_{\widehat{n}}=\widehat{p})\equiv 1$$

This is the expected payoff in the event that there is a tie at $\widehat{p}$. Given a tie, player n is ranked below $\widehat{n}$ with probability 1/2 and gets a payoff of $\widehat{p}$ whenever he or $\widehat{n}$ is the marginal firm. With probability 1/2 n is ranked above $\widehat{n}$ and receives $\widehat{p}$ only when he is the marginal firm. If n plays $\widehat{p}-\varepsilon$ for some $\varepsilon>0$, then in the limit, as $\varepsilon\to 0$, the corresponding element in his expected payoff is

$$\Pr(p_{\widehat{n}}=\widehat{p})\sum_{i=1}^{N-1}\Pr([p_{n_{i+1}}<\widehat{p}]\cap[p_{n_{i+2}}>\widehat{p}]|p_{\widehat{n}}=\widehat{p})[\widehat{p}-c_n]\{\pi_i+\pi_{i+1}\} \quad (A.2)$$

The difference between (A.1) and (A.2) is that the latter corresponds to the case where n is always ranked below $\widehat{n}$ whenever $\widehat{n}$ plays $\widehat{p}$ since $\forall\varepsilon>0,\widehat{p}-\varepsilon<\widehat{p}$. All other elements in the sum which constitute player n's expected payoff from playing $\widehat{p}$ and $\widehat{p}-\varepsilon$, respectively, can be made arbitrarily close by choosing ε small enough. Thus, there exists $\varepsilon>0$, such that playing $\widehat{p}-\varepsilon$ yields a strictly higher payoff than playing $\widehat{p}$ and, therefore, playing $\widehat{p}$ with positive probability cannot be part of an equilibrium strategy for player n.

We next show that if $\widehat{p}<\overline{p}$ is played with positive probability by some player n, any offer price exceeding $\widehat{p}$ by any other player will be bounded away from $\widehat{p}$. Consider the payoff to player n who plays $\widehat{p}+\varepsilon$ for some $\varepsilon>0$. Then in the limit, as $\varepsilon>0$, the element in his expected payoff corresponding to (A.1) is

$$\Pr(p_{\widehat{n}}=\widehat{p})\sum_{i=1}^{N-1}\Pr([p_{n_{i+1}}<\widehat{p}]\cap[p_{n_{i+2}}>\widehat{p}]|p_{\widehat{n}}=\widehat{p})[\widehat{p}-c_n]\pi_{i+1} \quad (A.3)$$

The difference between this and (A.1) is that n is always ranked above $\widehat{n}$ whenever $\widehat{n}$ plays $\widehat{p}$ since $\forall\varepsilon>0,p_n=\widehat{p}+\varepsilon>\widehat{p}$, i.e. there is never a tie. All other corresponding elements in the sum which constitute player

n's expected payoffs from playing $\widehat{p}$ and $\widehat{p}+\varepsilon$, respectively, can be made arbitrarily close by choosing ε small enough. Thus there exists $\overline{\varepsilon}>0$ such that for all $\varepsilon \in (0,\overline{\varepsilon})$ playing $\widehat{p}$ yields a strictly higher payoff than playing $\widehat{p}+\varepsilon$, and therefore any offer price which forms part of n's strategy and is not less than $\widehat{p}$, must exceed $\widehat{p}+\varepsilon$.

Since the above result must hold for all $n \neq \widehat{n}$, it follows that player $\widehat{n}$ would gain by playing $\widehat{p}+\varepsilon$ instead of $\widehat{p}$, for some $0<\varepsilon<\overline{\varepsilon}$. This contradicts the assumption that $\widehat{p}$ is part of his equilibrium strategy and completes the proof. QED. ■

Lemma A.2.4 *(No holes)* *If $\widehat{p}$ is part of any equilibrium strategy, then for any interval $S \subset (\widehat{p},\overline{p}], \forall p \in S$, S are part of at least two players equilibrium strategies.*

Proof: We first show that there cannot exist any interval $S \subset (\widehat{p},\overline{p}]$, such that no player has elements in S as part of his strategy. Assume, for a contradiction, that such an interval S exists, and let $p_{\inf}=\inf\{p|p\in S\}$ and $p_{\sup}=\sup\{p|p\in S\}$. Then, for the player $\widehat{n}$ with $p_{\inf}-\varepsilon$, for some $\varepsilon>0$, as part of his strategy, playing $\widehat{p}=p_{\inf}+\widehat{\varepsilon}\in S$ instead of $p_{\inf}-\varepsilon$ yields in the limit as $\varepsilon\to 0$, an increase in expected payoff of

$$\begin{aligned}\sum_{i=1}^{N}\pi_i \Pr([p_{n_{i+1}} &< \widehat{p}]\cap[p_{n_{i+1}}>\widehat{p}]|p_{\widehat{n}}=\widehat{p})\widehat{\varepsilon}= \qquad \text{(A.4)}\\ \sum_{i=1}^{N}\pi_i \Pr([p_{n_{i+1}} &< \widehat{p}]\cap[p_{n_{i+1}}>\widehat{p}]|p_{\widehat{n}}\in[p_{\inf},p_{\sup}])\widehat{\varepsilon}>0\end{aligned}$$

where

$$\Pr([p_{n_0}<\widehat{p}]|p_{\widehat{n}}\in[p_{\inf},p_{\sup}])\widehat{\varepsilon}=\Pr([p_{n_{i+1}}>\widehat{p}]|p_{\widehat{n}}\in[p_{\inf},p_{\sup}])\equiv 1$$

A contradiction. By applying a similar argument we can show that it is not possible that only one player has elements in S as part of his equilibrium strategy. If $\widehat{n}$ is the only player with $\widehat{p}=p_{\sup}-\widehat{\varepsilon}\in S$ as part of his strategy, then by playing $p_{\sup}+\varepsilon, \varepsilon>0$, instead of $\widehat{p}$ this yields in the limit as $\varepsilon\to 0$, an increase in his expected payoff equal to that in (A.4). QED.

Proof of Proposition 2.5: From Lemmas A.2.1-A.2.4 we know that there is at most one player who plays $\overline{p}$ with positive probability (this will be player 2, a result which follows from the observation that the argument below leads to a contradiction if one makes the assumption that firm 1 plays $\overline{p}$ with positive probability), that no player plays any price $p < \overline{p}$ with positive probability, and that if p^m is the smallest price played by any player, both players' mixed strategies have full support on $[p^m, \overline{p}]$. Then, 2's expected payoff from playing $\overline{p}$ is

$$\Phi_2(\overline{p}) = [1-\pi][\overline{p}-c] \tag{A.5}$$

Prices below c cannot be in the support of his equilibrium strategy since

$$\forall p \le c : \Phi_2(p) \le \Phi_2(c) = \pi * 0 + [1-\pi][Ep_1 - c] < [1-\pi][\overline{p}-c] \tag{A.6}$$

Thus, the smallest offer price which is in the support of 2's strategy must be strictly greater than his marginal cost. Furthermore, it is clear that if p^m is the smallest price in the support of 2's strategy, then 1 never offers anything less than p^m, and vice versa

Let $F_1(p) \equiv \Pr(p_1 \le p)$ and $f_1(p) \equiv F'(p)$. Then the expected payoff to player 2 of playing $p \in [p^m, \overline{p}]$ is

$$\Phi_2(p) = \pi[1-F_1(p)][p-c]+[1-\pi]F_1(p)[p-c]+[1-\pi]\int_p^{\overline{p}}[p-c]f_1(p)dp \tag{A.7}$$

The first two elements in the sum are the expected payoff when firm 2 is the marginal firm, i.e. determines the market price, and one and two firms are active respectively. The third element is the expected payoff given that both firms are called into operation and firm 2 has the lower price. From (A.7)

$$\Phi_2'(p) = \pi\{1 - f_1(p)[p-c]\} + [1-2\pi]F_1(p) \tag{A.8}$$

Using the fact that in equilibrium $\forall p \in [p^m, \overline{p}), \Phi_2'(p) = 0$, one gets

$$f_1(p) - \frac{1-2\pi}{\pi}\frac{F_1(p)}{p-c} = \frac{1}{p-c} \tag{A.9}$$

This and the fact that F,(p) = 1, imply the following unique solution to (A 9):

$$F_1(p) = \begin{cases} \ln(e\frac{p-c}{\overline{p}-c} & ,\text{when } \pi = \frac{1}{2} \\ \frac{\pi-1}{2\pi-1}[\frac{p-c}{\overline{p}-c}]^{\frac{1-2\pi}{\pi}} + \frac{\pi}{2\pi-1} & ,\text{when } \pi \neq \frac{1}{2} \end{cases} \quad \text{(A.10)}$$

$$p^m = \begin{cases} \frac{\overline{p}-c}{e} + c & ,\text{when } \pi = \frac{1}{2} \\ [\overline{p} - c][\frac{\pi}{1-\pi}]^{\frac{\pi}{1-2\pi}} + c & ,\text{when } \pi \neq \frac{1}{2} \end{cases} \quad \text{(A.11)}$$

where $\ln(e) \equiv 1$. A similar reasoning gives

$$F_2(p) = \begin{cases} \ln(e\frac{p}{\overline{p}[e-1]c} & ,\text{when } p < \overline{p} \text{ and } \pi = \frac{1}{2} \\ \frac{\pi-1}{2\pi-1}[\frac{p}{\overline{p}+[\alpha_\pi-1]c}]^{\frac{1-2\pi}{\pi}} + \frac{\pi}{2\pi-1} & ,\text{when } p < \overline{p} \text{ and } \pi \neq \frac{1}{2} \end{cases} \quad \text{(A.12)}$$

$F_2(\overline{p}) = 1$,

where

$$\alpha_\pi \equiv [\frac{\pi}{1-\pi}]^{\frac{\pi}{2\pi-1}}$$

QED.

Proof of Proposition 2.6: Note that it follows from lemma A.2.2 that no symmetric mixed strategy equilibrium can contain mass points at any offer price. Furthermore, from lemmas 2.3 and 2.4, it follows that in any symmetric mixed strategy equilibrium, if p^m is the smallest offer price, all $p \in [p^m, p]$ are in the support of the players' strategies. Let $\Phi_n(p)$ be the expected payoff to firm n of choosing offer price p. Then one has the following

$$\Phi_n(p) = \sum_{i=1}^{N} \pi_i \{\Pr([p_{n_{i+1}} < p] \cap [p_{n_{i+1}} > p] | p_n = p)p + \int_p^{\overline{p}} pdFn_i(p)\} \quad \text{(A.13)}$$

where

$$F_{n_i}(p) = \Pr(p_{n_i} \leq p | p_n \leq p) = \sum_{j=i-1}^{N-1} \binom{N-1}{j} q^j [1-q]^{N-1-j} \quad \text{(A.14)}$$

$$\Pr([p_{n_{i-1}} < p] \cap [p_{n_{i+1}} > p | p_n = p]) = \binom{N-1}{i-1} q^{i-1} [1-q]^{N-1} \quad \text{(A.15)}$$

$$q \equiv F(p) \equiv \Pr(p_k \leq p), \ k \neq n \tag{A.16}$$

The first part of each element of the sum in (A.13) represents the payoff in the event that firm n is the marginal supplier, while the second part is the payoff given that firm n supplies but is not the marginal operating unit. The sum is over all possible demand realizations.

In a mixed strategy equilibrium, it (generically) must be the case that for any two points $\widehat{p}$ and $\widetilde{p}$ in the support of the players' strategies, $\Phi_n(\widehat{p}) = \Phi_n(\widetilde{p})$. Thus,

$$0 = \Phi_n'(p) = \alpha(q) - pq'\beta(q) \tag{A.17}$$

where $q' \equiv f(p) \equiv F'(p)$ and

$$\alpha(q) \equiv \sum_{i=1}^{N} \pi_i b(i-1; N-1, q) \tag{A.18}$$

$$\beta(q) \equiv \sum_{i=1}^{N-1} \pi_i B_q(i; N-1, q) \tag{A.19}$$

$$b(i-1; N-1, q) \equiv \binom{N-1}{i-1} q^{i-1}[1-q]^{N-1} \tag{A.20}$$

and

$$B(i; N-1, q) \equiv \sum_{j=i}^{N-1} \binom{N-1}{j} q^j [1-q]^{N-1-j} \tag{A.21}$$

$b(i; N, q)$ is the density function of the binomial probability distribution with parameters N and q, while $B(i; N, q)$ is one minus the corresponding cumulative binomial probability distribution. Note that for $N \geq 2$ and $q \in [0,1), b(1-1; N-1, q) \geq 0$ for all i, with strict inequality for at least one i, and $B_q(i; N-1, q) > 0$ .Thus $\forall q \in [0,1)$

$$\alpha(q) > 0, \text{ and} \tag{A.22}$$

$$\beta(q) > 0 \tag{A.23}$$

Consider the differential equation

$$q' = \frac{\alpha(q)}{p\beta(q)} \equiv \Omega(p,q) > 0 \tag{A.24}$$

Define

$$K^1 = \max\{\alpha(q)|q \in [0,1]\} > 0, \tag{A.25}$$

$$K_1 = \min\{\alpha(q)|q \in [0,1]\} > 0, \tag{A.26}$$

$$K^2 = \max\{\beta(q)|q \in [0,1]\} > 0, \tag{A.27}$$

and

$$K_2 = \min\{\beta(q)| \in [0,1]\} > 0 \tag{A.28}$$

Since $\Omega(p,q)$ and $\Omega_q(p,q)$ are continuous for $p > 0$ and $q \geq 0$, and

$$\forall q \in [0,1] : \Omega(p,q) \geq K/p, \tag{A.29}$$

where $K = K^1/K_2$, for every $p^m \geq 0$ (A.24) has a unique solution $F(p)$ where $F(p^m) = 0$ (see Sydsaeter (1984), p. 25). Since the solution $F(p)$ is continuous in p^m and $f(p) = q\prime > 0$, there exists a $p^m < \overline{p}$ such that $F(\overline{p}) = 1$. Next, one has that

$$\forall q \in [0,1] : \Omega(p,q) \geq K^{\%} \log(\overline{p}/p^m) \tag{A.30}$$

where $K^{\%} = K_1/K^2 < \infty$. Therefore,

$$1 = F(\overline{p}) \geq \int_{p^m}^{\overline{p}} K^{\%}\frac{1}{p}dp = K^{\%} \log(\overline{p}/p^m) \tag{A.31}$$

from which it follows that $p^m > 0$. QED.

Proof of Proposition 2.7: Observe that the payoff to a particular operating firm is independent of its own offer price. Fix the strategies of firms $m \neq n$, and consider the best response of firm n. Let one of firm n's offer prices be $p_{ni} = \widetilde{p}$. First, for realizations of demand for which set i of firm n is operating and gets positive payoff, i.e. $\widetilde{p} < P$ and $P > c_n$, the firm would have been equally well off offering $p_{ni} = c_n$. Second, for realizations of demand for which set i is not operating and the system marginal price exceeds its marginal cost, i.e. $\tilde{p} \geq P > c_n$, firm n would have been strictly better of offering $p_{ni} = c_n$, For realizations of demand such that the system marginal price is below its marginal costs, firm n is at least as well off by offering $p_{ni} = c_n$ than any other price. Thus, $p_{ni} = c_n$ is a (weakly) dominant strategy for firm n. QED.

3 Spot Market Competition and Long-Term Contracts in the British Electricity Market

3.1 Introduction

von der Fehr and Harbord (1993) analyzed the noncooperative equilibria of electricity spot markets in the absence of contracts, and demonstrated that generators will have a strong incentive to bid above short-run generation costs. Green and Newbery (1992) adopted a different analytical approach, but reached broadly similar conclusions. In this paper we extend this analysis to include the effects of long-term financial contracts, such as those traded in the electricity supply industry in England and Wales between generators and electricity distribution companies. Our purpose is to explore the incentives that financial contracts give for altering bidding behavior in the pool and to analyse the potential functioning of the contract market.

The existing literature on the interaction between long-term contracts and imperfectly competitive spot markets has concentrated on futures contracts (see the survey by Anderson, 1990). A general finding of this literature is that there may be a strategic motives for trading futures contracts which are distinct from the traditional hedging and speculative motives. The strategic motives vary depending upon the market structure and the nature of the underlying commodity or good. However, a fairly robust conclusion seems to be that the presence of futures has a pro-competitive effect: i.e. trade in futures contracts tends to increase production above the level that would prevail in its absence, thus reducing prices and ameliorating the efficiency losses due to imperfect competition. This is the conclusion reached in Cournot oligopoly models where firms compete in quantities for example (Eldor and Zilcha, 1990 and Allaz, 1990). In these Cournot-type models futures can act as a commitment to supply large volumes of output through their effect on firms' marginal revenues. An increase in the number of futures contracts shifts out a firm's reaction function and allows it to achieve the advantage of Stackelberg leadership.

Unfortunately, the assumptions in this literature on types of long-term contracts, market structure, and the organization of transactions make these analyses not directly applicable to the deregulated electricity supply industries in the UK and elsewhere.[20] The electricity spot market in England and Wales is organized as a daily reverse auction in which the generators submit offer prices on their available capacity, generating units are ranked according to their offer prices (i.e. a supply schedule is constructed), and half-hourly market prices are determined by the offer prices of the marginal generating units. That is, the electricity pool operates as a uniform, first-price, multi-unit auction and does not correspond to a standard Cournot or Bertrand spot market game, as typically discussed in the literature on futures contracts. In addition, long-term contracts typically take the form of options - i.e. they are purely financial contracts unrelated to the purchase or sale of electricity in the spot market, rather than futures contracts per se.

In this paper we characterize spot and contract market equilibria in a model which is explicitly designed to take account of these characteristics of electricity spot and contract markets. We find that the existence of long-term contracts tends to put downward pressure on spot market prices through their effects on the generators' bidding strategies. For this reason the incentive of generators would seem to be to reduce the sale of contracts below what would otherwise be the case without this strategic effect. However, we also identify a strategic motive which may work in the opposite direction: by selling a large number of contracts, a firm can effectively commit itself to bidding low prices and thus ensuring that it will be despatched with its full capacity. A significant result of our analysis, therefore, is that options contracts may have strategic commitment value for generators in the electricity spot market.[21]

[20] Powell (1993) provides a discussion of the role of contracts in the British electricity spot market based on the futures approach, using a Cournot model of generator interaction. See also Helm and Powell (1992).

[21] There is now a considerable theoretical literature on the commitment value of contracts. See in particular Aghion and Bolton (1987) and Dewatripont (1988) for early analyses; and Bensaid and Gary-Bobo (1991) and Green (1990) and the references cited therein.

By way of background, in the next section we provide a brief overview of the contractual structure of the England and Wales industry at the time of privatization. In Section 3.3 we then present the formal model of price-setting by the duopoly generators, which is subsequently analyzed in Sections 3.4, 3.5 and 3.6. Section 3.7 concludes. All proofs are relegated to an appendix.

3.2 Contracts in the British Electricity Market

Option contracts, or 'contracts for differences', are a fundamental feature of the new electricity market in Britain. At privatization in 1991 both of the major generators in England and Wales, National Power and PowerGen, were endowed with a portfolio of contracts for differences with the regional electricity companies within a pricing and contractual framework set down by the government. National Power's total generation capacity at privatization was approximately 29,500 MW, and 84% (24,800 MW) was 'covered' by contracts for differences with regional electricity companies. All of these contracts had expired by 31st March 1993, and most, if not all, have been replaced with new contracts. The situation of PowerGen is similar. Of a total capacity in January 1991 of 18,800 MW, PowerGen had contracts for differences with regional distribution companies amounting to 86.5% (16,200 MW), 80% of which had expired by 31st March 1993. Again the majority of these contracts have been replaced.[22] From these numbers it should be clear that contracts for differences have played a very significant role in the England and Wales electricity market.

Contracts for differences are written in a variety of forms. Contracts may be 'one-way' or 'two-way,' specify single or multiple prices to apply to different periods of the day, contain minimum or maximum take provisions, and they may or may not be related to the actual availability of generating plant (i.e. 'firm' or 'non firm' contracts). In their most basic form all

[22] See James Capel & Co. (1990) and Holmes and Plaskett (1991) for a more complete description than is given here. Since the expiry of the initial 'vesting' contracts information concerning the contractual liabilities of the privatized electricity companies has not been in the public domain.

contracts specify a strike price and a (megawatt) quantity to which they apply. Under a one-way contract, when the electricity spot (or pool) price exceeds the specified strike price, then the holder of the contract receives a 'difference payment' equal to the difference between the strike price and the pool price multiplied by the specified quantity. Under a two-way contract a negative difference payment is made whenever the pool price is less than the contract strike price. In the England and Wales electricity market at privatization, the vast majority of contracts for differences sold by the generators to the regional electricity (distribution) companies were one-way contracts. The important point however is that these contracts are not related to any physical trade in electricity, and the market for contracts is not necessarily limited to participants in the industry.

Trade in long-term contracts of one to five years in duration has generally occurred via auction or direct negotiation between the major generators, electricity distribution companies and large consumers. There have also been attempts to organize more liquid short term markets, with only limited success to date. A market in short-term contracts with a duration of one month - Electricity Forward Agreements (EFA's) - was created in Britain in 1993, under which trade is carried out through a broker. However trade in this market has never been brisk.

We analyse spot market competition between duopoly generators for an extremely simple contractual form, and focus our attention on one-way contracts. In section 3.6 and Appendix A we discuss how our results are modified when other types of contracts are considered.

3.3 The Model

We consider a model which abstracts from some of the more detailed features of electricity contracts while still being able to shed some light on the interaction between the market for contracts and the electricity spot market. We focus on standardized 'one-way option contracts' of the following form: a contract is for one unit (e.g. a megawatt hour) and commits the contract seller to pay any positive difference between the pool price and the strike price to the holder of the contract. We assume that the duopoly generators are net sellers of such contracts. We later extend the analysis to other forms of contracts as well: in particular contracts which give the generators the right to claim any positive difference between the contract strike price and the electricity pool price, and two-way, or fixed-price, contracts, which in this set-up are identical to futures. The formal analysis is very similar for all three forms of contracts, and since the first form was initially the most common in the England and Wales electricity market, we concentrate attention on this, relegating the analysis of the other contract types to an appendix.

Our analysis is limited to one 'type' of contract; that is, we assume that the contract strike price is given exogenously, and then consider how many contracts the generators would like to sell. Since our main interest is in the interaction between the contract market and the electricity spot market, such a limited scope seems natural. A complete analysis of the market for contracts would require both a full specification of demand (by consumers and electricity distribution companies), as well as allowing for the presence of multiple contract types. However, our model does allow us to evaluate how different types of contracts affect the outcome of competition in the spot market, and, thus, how this feeds back on the generators' incentives to sell particular types of contracts.

Our model of competition in the electricity spot market is based on the approach developed in von der Fehr and Harbord (1993), but we make a number of further simplifying assumptions here. Most importantly, whereas in the more general model firms are allowed to submit step supply curves (i.e. different bids for individual generating units), here they are constrained to a single bid for the whole of their capacity, i.e. it is as if each firm owns only a single unit. All of our major results generalize straightforwardly to the case where generators submit step supply functions so this assumption is not restrictive.

The details of the model are as follows. We consider a two-stage duopoly game. In the first stage firms (generators) compete in the market for long-term contracts, and in the second stage price competition in the spot market takes place. We thus have:

Stage 1: The generators simultaneously decide how many contracts to sell, where x_i is the number of contracts sold by generator $i, i = 1, 2$.

Stage 2.1: Offer prices at which the generators are willing to supply output, are submitted, where $p_i \in (-\infty, \overline{p}]$ is the offer price of generator $i, i = 1, 2$. The capacity of generator i is denoted $k_i, i = 1, 2$. W.l.o.g. we assume $k_1 = k \leq 1$,and $k_2 = 2 - k$.

Stage 2.2: The generators are ranked according to their offer prices, such that generator i is ranked before generator j if $p_i < p_j$. If $p_1 = p_2$, the generators are ranked first with equal probability $(= 1/2)$.

Stage 2.3: Demand, d, is realized. d is a random variable with distribution function $G(d)$, where supp $G(\cdot) = [a, b], 0 \leq a \leq b \leq 2$.

Stage 2.4: The firms are despatched to match supply. Let i be the generator ranked first. If $d \leq k_i$, only i supplies. If $d > k_i$, i is despatched with its total capacity while the generator ranked second produces $d - k_i$.

Marginal costs are constant and equal for both firms, and are normalized to zero.

A system marginal price, p^S, is determined as the offer price of the marginal operating generator. A generator i which is despatched with quantity $y_i \in [0, k_i]$, earns $p^S \cdot y_i$. From this, payouts on its stock of long-term contracts is subtracted: $x_i \cdot max\{p^S - q, 0\}$, where q is the contract strike price. Throughout we assume $q \in (0, \overline{p})$, which seems reasonable given that the system marginal price will never fall below 0 and, by assumption, is bounded from above by $\overline{p}$.

3.4 Spot Market Competition

In this section we analyse second-stage spot-market competition, after the generators have already sold contracts in the amounts x_1 and x_2, respectively, at a given strike price q. We assume that the firms have equal capacities, i.e. $k = 1$. Our results will depend importantly on the distribution of demand. In particular, we distinguish between three cases:

Low-demand period: $suppG(\cdot) \subseteq [0, 1]$, i.e. only the generator ranked first will be producing;

High-demand period: $suppG(\cdot) \subseteq (1, 2]$, i.e. both generators will be producing; and

Variable-demand period: $S_1, S_2 \in suppG(\cdot)$, such that $S_1 \in [0, 1]$ and $S_2 \in (1, 2]$, i.e. there is positive probability for both the event that only one generator produces <u>and</u> the event that both firms will be called into operation.

In the following subsections we consider these cases separately. In the first, which we call 'low-demand periods', competition to become the lowest pricing firm is so fierce that the competitive outcome results irrespective of whether or not firms have entered into long-term contracts. In the second case - 'high-demand periods' - contracts do matter, but only when firms have sold sufficiently large numbers of them. Contracts reduce the incentive of the generators to submit offer prices above the contract strike price, and when the number of contracts is large enough, the pool price is equal to the contract strike price rather than the highest admissible price $\overline{p}$. In the third case, 'variable-demand periods', we again find that the equilibrium pool price is lower the larger the number of contracts the firms have sold and the lower the contract strike price.

3.4.1 Low-demand periods

As can readily be established, for the first two cases (low-demand and high-demand periods), there is no loss of generality in confining attention to degenerate distribution functions, i.e. we let d be non-stochastic (see von der Fehr and Harbord, 1992). In this sub-section, therefore, it is assumed that demand is determinate and so low that only one firm will be despatched, that is, $Pr(d = \overline{d} \in (0, 1)) = 1$. Under this assumption, it turns out that the competitive outcome prevails (as in the standard Bertrand model) whether or not the generators have entered into any contracts for differences. In particular, we may easily prove the following result:

Proposition 3.1 *If $d \in [0,1]$, there is a unique Nash equilibrium in the second-stage game in which $p_1 = p_2 = 0$.*

Since total demand can be supplied by a single generator, the higher-pricing firm receives no payments from the pool. Its profits will therefore be negative if it has sold long-term contracts and the pool price is above the contract strike price, and zero otherwise. In order to avoid this outcome, there is strong competition to become the lower-pricing firm, and the end result is that offer prices are brought down to marginal cost.

The competitive outcome result generalizes to any distribution function G such that $G(1) = 1$, as well as to cases in which firms are asymmetric ($k < 1$ and $G(k) = 1$). Furthermore, the argument does not depend on the type of the contract, i.e. the value of q (as long as $q \in (0, \overline{p})$), nor on the quantity of contracts held by each firm. Indeed, the proposition could easily be extended to a model which allowed for multiple contract types. We conclude that in low-demand periods, when there is zero probability that both firms will be operating, long-term contracts have no effect upon the outcome of spot-market competition.

3.4.2 High-demand periods

We turn now from low-demand periods to the polar case in which both generators are called into operation with probability one, in particular $Pr(d = \overline{d} \in (1,2]) = 1$. By an argument similar to that of the previous section, it can be straightforwardly demonstrated that there is no equilibrium in which $p_1 = p_2 > 0$. $p_1 = p_2 \leq 0$ cannot be an equilibrium either since then either firm could secure positive profits by deviating and offering to supply at a nonnegative price $p_i \in (0, q)$. Thus, any equilibrium of the second-stage game must involve firms charging different prices. Order firms such that $x_1 \leq x_2$, i.e. generator 1 has a stock of contracts not exceeding that of generator 2. Consider first the case where the number of contracts held by each firm is small, in particular, $x_2 \leq d - 1$. We then have the following Nash equilibrium.

Proposition 3.2 *Assume $x_1 \leq x_2 < d-1$. Then a pure-strategy Nash equilibrium of the second-stage spot-market-competition game has the following form: $p_i = \overline{p}$ and $p_j \leq b_j$ for some $b_j < \overline{p}$,$i, j = 1, 2, i \neq j$.*

Remark: As should be clear from the argument in the proof, equilibria where $p_i = \overline{p}$ and $p_j \leq b_j$ continue to exist as long as $x_i \leq d - 1$.

Since, by assumption, the residual demand facing the higher-pricing firm exceeds its stock of long-term contracts, the higher-pricing firm's profit is increasing in its own offer price. Hence given that a firm is going to bid higher than its competitor, it will choose the highest admissible offer price. Now, since the higher-pricing firm supplies less to the pool than the lower-pricing firm, undercutting the lower-pricing firms' offer price will be profitable if the gain from selling a larger volume exceeds the loss from a reduced price. In equilibrium, therefore, the lower-pricing firm must submit an offer price low enough so that such deviations are rendered unprofitable.

Note that although there exists a continuum of equilibria, in each of them the system marginal price equals $\overline{p}$ since the higher-pricing firm is the marginal operating firm with probability 1.[23] We may conclude that when long-term contracts cover a sufficiently small part of the generators' respective (residual) output capacities, then there exist a multiplicity of equilibria, but in each of them the system marginal price is equal to the maximum admissible price, and, therefore, the market price is unaffected by the presence of long-term contracts.[24] Note that this conclusion is independent of the type of contract and could be generalized so as to allow for multiple contract types; only the quantity of contracts sold by the individual generators matters.

[23] In the non-generic case where $x_2 = d - 1$, there are additional equilibria, involving $p_i = p\prime < \overline{p}$and p_j satisfying the constraints of Proposition 2 where $p\prime$ replaces $\overline{p}$.

[24] A more detailed exposition of the spot market equilibria without contracts is given in von der Fehr and Harbord (1992).

We consider next the case where both generators hold a large number of contracts:

Proposition 3.3 *Assume that* $x_1 \geq ([d-1]\overline{p} - q) / [\overline{p} - q]$ *and* $x_2 > d - 1$. *Then any set of strategies* $\{p_1, p_2\}$, *with* $p_1 \leq p_2$, *constitute a Nash equilibrium of the second-stage spot-market-competition game if and only if they have the following form:* $p_1 \leq [d-1]q$ *and* $p_2 = q$.

If a generator has contracted for a greater volume of output than the residual demand it faces in the pool, its profit will be decreasing as the system marginal price, or pool price, increases above the contract strike price. In particular, because the higher-pricing firm determines the system marginal price whenever its stock of contracts is sufficiently large, its profits will be decreasing in its own offer price whenever that exceeds the contract strike price. Since, by assumption, firm 2 has sold more contracts than its residual demand, it follows that as the higher-pricing firm, it will never bid above the contract strike price. On the other hand, below the contract strike price the higher-pricing firm's profit is increasing in its own offer price. Thus, any equilibrium where firm 2 bids above firm 1 must have firm 2 bidding at the strike price. To ensure the existence of such an equilibrium, two conditions must be fulfilled. First, firm 1's bid must be low enough so that undercutting by firm 2 is unprofitable. Second, firm 1 must not want to deviate by bidding above the offer price of firm 2. The latter is ensured by the condition that firm 1's stock of long-term contracts is sufficiently large.

Hence in this case we again find a multiplicity of equilibria, each of which now has firm 2 offering to supply at a price equal to the option strike price and firm 1 offering a price less than or equal to $[d-1]q$. If $x_1 > d-1$, there are corresponding equilibria where firm 1 is the higher pricing firm and bids q. In all of these equilibria the system marginal price is equal to the contract strike price, so the existence of long-term contracts places downward pressure on prices in this case. Moreover, the type of contracts matters; the lower the contract strike price, the lower is the pool price.

In general, when $([d-1]\overline{p}-q)/[\overline{p}-q] \leq x_1 \leq d-1 < x_2$, there are two types of equilibria corresponding to those of propositions 2 and 3, respectively. If firm 1 is the higher pricing firm, system marginal price is equal to the maximum admissible price $\overline{p}$; when firm two is the higher pricing firm it is equal to the contract strike price.

Summing up the results of this and the preceding section, we may conclude the following. If either of the events {demand can be covered by one firm} or {demand cannot be covered by one firm} occur with probability one, then there exist pure-strategy equilibria with the following characteristics: In low-demand periods ($d < 1$), price equals marginal costs. In "moderately" high-demand periods ($1 < d < 1 + x_i, i = 1, 2.$), the system marginal price equals the long-term contract strike price, while in "very" high-demand periods ($d > 1 + x_i, i = 1, 2.$), the system marginal price equals the highest admissible price. Thus only when both firms will be operating with probability 1 and the highest pricing firm operates at very low capacity (less than the quantity covered by its long-term contracts) will the existence of contracts put downward pressure on the spot market price.

3.4.3 Variable-demand periods

Finally, to complete the analysis of spot-market equilibria, we turn to the case where both the event that one firm will be operating <u>and</u> the event that both firms will produce have positive probability. We start by showing that when the distribution of contracts is sufficiently asymmetric, pure-strategy equilibria exist. Define $\alpha(q) = [E(d \mid d > 1) - 1] - \{Pr(d \leq 1)E(d \mid d \leq 1) + Pr(d > 1)[2 - E(d \mid d > 1)]\}q/\{\Pr(d > 1)[\overline{p} - q]\}$. Then we may prove:

Proposition 3.4 *Assume* $0 < Pr(d \leq 1) < 1$. *Then if* $max\{x_1, x_2\} < E(d \mid d \leq 1)$ *or* $min\{x_1, x_2\} > \alpha(q)$, *no-pure strategy Nash equilibria of the second-stage spot market competition game exists. If* $x_i > E(d \mid d \leq 1)$ *and* $x_j < \alpha(q)$, $p_i = q$ *and* $p_j = \overline{p}$ *constitute the only pure-strategy equilibrium where* $p_i \leq p_j, i, j = 1, 2, i \neq j$.

Proposition 3.4 may be explained intuitively as follows. If the lower-pricing firm bids below the contract strike price, options will not be exercised when only one firm is producing. It follows that the lower-pricing firm's profit is increasing in its own bid for all offer prices below the contract strike price, and thus, in equilibrium, it never bids in this range. Furthermore, a firm's profit is always increasing in its own offer price if it holds sufficiently few contracts. Thus, a pure strategy equilibrium cannot exist in which the lower-pricing firm holds few contracts since in that case it would always want to increase its bid towards the offer price of the higher-pricing firm. By a similar argument, it follows that the higher-pricing firm must hold few contracts since otherwise it would always want to reduce its bid towards that of the lower-pricing firm. If the lower-pricing firm holds sufficiently many contracts and the higher-pricing firm sufficiently few, an equilibrium exists in which the two firms bid at the contract strike price and the highest admissible price, respectively. Otherwise, no pure-strategy equilibrium exists.

For the range of parameter values for which pure-strategy equilibria do not exist, we consider equilibria in mixed strategies. We do so by analyzing the specific example in which $Pr(d = 1) = \pi$ and $Pr(d = 2) = 1 - \pi$, i.e. there are only two events; either only the first-ranked firm is despatched with its whole capacity, or both firms produce at full capacity. (Note that in this example, pure-strategy equilibria cannot exist.) This assumption greatly simplifies notation without reducing the generality of the analysis to any significant degree.

W.l.o.g. let firm 2 be the firm with more long-term contracts, i.e. $x_2 \geq x_1$. Let $F_i(p)$ represent the (cumulative) frequency with which firm i plays offer prices $p \in [0, \overline{p}], i = 1, 2$, i.e. $F_i(p) = P_r(p_i \leq p)$. Profits of firm i may then be written:

$$\begin{aligned}\phi_i(p) &= \pi\{[1-F_j(p)][p-x_i\cdot\max\{p-q,0\}] \\ &\quad -\int_0^p x_i\max\{r-q,0\}dF_j(r)\} \\ &\quad +[1-p]\{F_j(p)[p-x_i\cdot\max\{p-q,0\}] \\ &\quad +\int_p^{\overline{p}}[r-x_i\cdot\max\{r-q,0\}]dF_j(r)\}\end{aligned} \tag{3.1}$$

where $i,j=1,2,i\neq j$. Profits equal the sum of the expected payoff in the events that only one firm and both firms will be called into operation, respectively. When only one firm is despatched, firm i is paid from the pool only when it has the lowest offer price, and then at its own bid. Similarly, when both firms are called into operation, a firm is paid at its own bid whenever it has the highest offer price, and at the competitor's (expected) bid otherwise. In either event, and whether it produces or not, a firm will have to honour its contracts whenever the system marginal price exceeds the contract strike price.

In the appendix we prove the following proposition:

Proposition 3.5 *Assume $Pr(d=1)=\pi, Pr(d=2)=1-\pi$, and $x_1\leq x_2<1$. Then there exists a unique Nash equilibrium of the second-stage spot-market-competition game in which firm $i, i=1,2$, plays prices $p\in[p^m,\overline{p}]$ according to the probability distribution $F_i(p)$. $p^m>0$ and $F_1(p)=F_2(p)$ when $p<q$, while $F_1(p)\leq F_2(p)$ for $p\geq q$.*

In equilibrium, players strike a balance between two opposing effects. On the one hand, high bid results in a high system marginal price, and payoff, in the event that the firm becomes the marginal operating firm. On the other hand, bidding high reduces the chance of becoming the lowest-pricing firm, and thus being despatched with a large capacity, or indeed any capacity at all. In equilibrium these two effects are balanced for all prices in the support of the players' strategies; an interval from a price strictly above marginal cost up to the highest admissible price.

On average, i.e. in expected terms, the firm with more long-term contracts prices lower in the spot market. In particular, firms play prices below the contract strike price with equal probability, while firm 1's strategy first-order stochastically dominates that of firm 2 for higher prices. The underlying intuition for this result is that the gain from a high system marginal price is less the more contracts a firm has sold. At the margin, the effect on profits from an increase in the pool price equals one times the net supply to the pool, i.e. total output less the contracted quantity. Therefore, the greater is the number of contracts the smaller is the incentive to bid high.

From the formulae for $F_1(\times), F_2(\times)$, and p^m (given in the appendix), a number of comparative static results can be derived. The lower bound on the support of the mixed strategies, p^m, is decreasing in the number of long-term contracts held by the firm with fewer contracts and increasing in the contract strike price. A higher contract strike price, q, also leads to more frequent play of prices above the strike price and less frequent play of lower prices. Furthermore, the larger the number of contracts held by the firm with fewer contracts (firm 1), the more likely it is that firms play high offer prices, while the opposite is the case for the firm with more contracts (firm 2).

In general, it is difficult to derive explicit comparative static results for the expected pool price. For the specific example $\pi = 1/2$, however, one gets

$$Ep = \overline{p}\{1 - e^{-1}[\frac{q}{\overline{p}}]^{x_1}\} - \frac{x_1 + x_2}{2}[\overline{p} - q]. \tag{3.2}$$

In this example therefore, the expected pool price is decreasing in the number of contracts held by the firm with most contracts. The pool price may increase or decrease in the number of contracts held by the firm with fewer contracts depending on the parameters of the model. The pool price is increasing (decreasing) in the contract strike price if firms hold sufficiently many (few) contracts.

3.5 Competition for Contracts

As noted in section 3.3, a full analysis of the first-stage game in which the generators compete in the market for long-term contracts, would require modelling the demand for contracts by electricity consumers and distribution companies as well as spelling out second-stage equilibria in the presence of multiple types of contracts. Our scope here is more limited; we want to explore how spot-market competition affects firms' incentives to sell a particular type of contract. We do that by fixing the contract strike price and considering how the generators' second-stage profits vary with the number of contracts sold. In order to abstract from other incentives to sell contracts (e.g. extracting hedging premiums etc.) we make a fairly natural "arbitrage" assumption: revenues from sales of contracts equal expected payouts. Such an assumption is consistent with atomistic price-taking and risk neutral buyers (we restrict attention to the case where neither of the generators is a net buyer of options, i.e. $x_i \geq 0, i = 1, 2$). While this simplification has the merit of allowing us to focus exclusively on the incentive to sell contracts arising from how long-term contracts affect spot-market competition, it is probably unrealistic as far as the England and Wales industry is concerned. In particular, the 12 RECs in England and Wales are few and large enough to make concentration on the buyer side an important issue. In the conclusion, we comment briefly on how the presence of strategic buyers may affect the viability of the market for long-term contracts.

As demonstrated in the previous section, the existence of long-term contracts does not affect spot market competition in low-demand periods when supp $G(d) \subset [0, 1]$, i.e. when only one firm will be producing for sure, and thus there are no strategic incentives arising from the existence of contracts in this case. In the rest of this section we concentrate on the analytically simpler (and empirically more interesting) case when demand is greater than the capacity of any individual firm, i.e. supp $G(d) \subset (1, 2]$.

As shown in section 3.4, when supp $G(d) \subset (1, 2]$, there is a multiplicity of equilibria in most cases. In particular, there exist sets of equilibria in which either one of the generators is the higher-pricing firm, determining the system marginal price. We deal with this multiplicity problem in the following way: We assume that one or the other pure-strategy equilibria will be played in the second stage game. Since the generators are symmetric ex ante, i.e. prior to the contracting stage, it seems reasonable to assume that they have equal probability of playing the roles of high-pricing and low-pricing firm, respectively, and thus calculate (expected) payoffs as the mean of profits in the two cases. It turns out that our results are robust to any alternative formulation in which (expected) payoffs are calculated as some weighted average of profits in the two types of equilibria. Thus, if one is willing to believe that one or the other of these equilibrium outcomes is a reasonable prediction for second-stage spot-market competition, this approach would seem to have some merit.

Throughout the rest of this section, w.l.o.g. we assume that d is determinate (generalizing to the stochastic case would basically involve substituting Ed for d in the formulae below). Then, when $p_i < p_j = \overline{p}$, profits, disregarding any revenues from selling contracts, are given by

$$\phi_i = [1 - x_i]\overline{p} + x_i q, \tag{3.3}$$

$$\phi_j = [d - 1 - x_j]\overline{p} + x_j q, \tag{3.4}$$

while when $p_i < p_j = q$,

$$\phi_i = q, \tag{3.5}$$

$$\phi_j = [d - 1]q. \tag{3.6}$$

Thus, from propositions 3.2 and 3.3 one gets;

$$x_1, x_2 \leq d - 1 : E\phi_i = [\frac{1}{2}d - x_i]\overline{p} + x_i q, i = 1, 2, and \tag{3.7}$$

$$x_1, x_2 > d-1 : E\phi_i = \frac{1}{2}dq, i = 1, 2.$$

The case when $x_i < d-1 < x_j$ presents specific problems. As noted in the discussion in section 3.4, when $x_i < \{[d-1]\overline{p} - q\}/[\overline{p} - q]$ there are only equilibria where i is the higher pricing firm and the system marginal price equals $\overline{p}$. Thus, in this case we get:

$$\phi_i = [d-1-x_i]\overline{p} + x_i q, \tag{3.8}$$

$$\phi_j = [1-x_j]\overline{p} + x_j q \tag{3.9}$$

When $d-1 > x_i \geq \{[d-1]\overline{p} - q\}/[\overline{p} - q]$, there are two types of equilibria, one in which firm i prices higher at $\overline{p}$, and one in which j is the higher pricing firm and offers to supply at a price equal to q. In this case also, we assume that payoffs are given by the mean of the profits in the two different equilibria:

$$E\phi_i = \frac{1}{2}[d-1-x_i]\overline{p} + \frac{1}{2}[1+x_i]q, and \tag{3.10}$$

$$E\phi_j = \frac{1}{2}[1-x_j]\overline{p} + \frac{1}{2}[d-1+x_j]q. \tag{3.11}$$

Define $d(q) \equiv \{[d-1]\overline{p} - q\}/[\overline{p} - q]$. Then the following payoff matrix, showing profits including proceeds from sales of contracts, summarizes the discussion above (given that $d(q) > 0$. If $d(q) \leq 0$, the first row and the first column do not apply.) In cells with two entries, the upper is the expected payoff to firm 1 and the lower the expected payoff to firm 2. In cells with one entry, this gives the payoff to firm $i, i = 1, 2$:

	$x_1 \in [0, d(q))$	$x_1 \in [d(q), d-1]$	$x_1 \in (d-1, 1]$
$x_2 \in [0, d(q))$	$\frac{1}{2}d\overline{p}$	$\frac{1}{2}d\overline{p}$	$\overline{p}$, $[d-1]\overline{p}$
$x_2 \in [d(q), d-1]$	$\frac{1}{2}d\overline{p}$	$\frac{1}{2}d\overline{p}$	$\frac{1}{2}p + \frac{1}{2}[d-1]q$, $\frac{1}{2}[d-1]p + \frac{1}{2}q$
$x_2 \in (d-1, 1]$	$[d-1]\overline{p}$, $\overline{p}$	$\frac{1}{2}[d-1]\overline{p} + \frac{1}{2}q$, $\frac{1}{2}p + \frac{1}{2}[d-1]q$	$\frac{1}{2}dq$

It is clear that we cannot have equilibria in which both generators have sold contracts in excess of the residual demand facing the higher-pricing firm, i.e. $x_1, x_2 \geq d-1$. If both generators sell that many contracts, the spot market price will be held at the contract strike price. But then a generator can benefit from unilaterally reducing its sales of contracts since this would lead to a higher spot market price (equal to the highest admissible price) in the event that this generator is the higher-pricing firm.

Assume $d(q) \leq 0$, or $q \geq [d-1]\overline{p}$. In this case the first column and first row do not apply. From the discussion above, it then follows that there can only exist equilibria in which both generators hold few contracts. In fact, there is a continuum of such equilibria in which $x_1, x_2 \in [0, d-1]$. In all of these, contracts are sufficiently few not to influence the spot market price, which equals $\overline{p}$ whichever generator is the higher-pricing firm.

When $d(q) > 0$, and $q < [d-1]\overline{p}$, matters are different. In this case also, there exists a continuum of equilibria in which contracts are few enough not to affect spot market prices, in particular, $x_1, x_2 \in [d(q), d-1]$. However, there also exist equilibria in which generators hold asymmetric contract positions, i.e. $x_i \in [0, d(q))$ and $x_j \in (d-1, 1], i \neq j, i, j = 1, 2$. In these equilibria, the generator with fewer contracts, i, always acts as the higher-pricing firm, pricing at $\overline{p}$, and earns a smaller payoff than generator j since i is despatched with lower output. Generator i cannot increase its profits by selling more contracts; although this would lead to generator i acting as the lower-pricing firm more often, the spot-market price would fall to the contract strike price. Since $q < [d-1]p$, the loss from lower spot prices will not be outweighed by higher output. Note that, because of this strategic effect, when $q < [d-1]\bar{p}$ there are no equilibria in which both generators hold very few contracts, or $x_i \leq d(q), i = 1,2$; then a generator would want to deviate to a large contract position to obtain the gains from committing to become the lower-pricing firm.

We may summarize the above discussion as follows: Since long-term contracts, if held in large enough quantities, place downward pressure on spot market prices, there is a strong disincentive to sell such contracts. However, selling a sufficiently large number of long-term contracts can serve as a commitment to becoming the lower-pricing firm in the second-stage price-competition game, and thus earning higher profits. Such a commitment is only credible for contracts with strike prices that are low enough, because given that one generator has sold many contracts of this type, its competitor will wish to sell few, and hence accept becoming the higher-pricing firm, in order not to depress the spot-market price by a large amount.

There is now a large literature on the commitment value of contracts with third parties (c.f. Dewatripont 1988, Green 1990 and Bensaid and Gary-Bobo 1991 and the references cited therein). Most of this literature has been concerned with the issue of renegotiation, and whether or not contracts can serve as commitment devices when they may be (costlessly) renegotiated at various stages during the play of the game. In our model of the electricity spot market however, in which contracts with third parties can serve as commitment devices, this issue does not arise. This is because, in the first place, the second-stage price competition game is one of simultaneous moves, and hence no opportunity for renegotiating contracts occurs. And secondly, it is not clear that even if such an opportunity did exist, it would have any effect. Because the contract purchasers (the electricity distribution companies) are also purchasers of electricity from the pool, and hold difference contracts to hedge against the risk of high pool prices, under most circumstances they would be unwilling to renegotiate their contracts if this simply had the effect of permitting pool prices to be bid up by one of the generators (the relevant case) [25]. A distribution company which has full contract coverage will be indifferent between all pool prices higher than the contract strike price, and hence will never have any incentive to renegotiate; and a distribution company which is undercovered will strictly prefer not to renegotiate.

Hence only in the case of a contract purchaser who has purchased more contracts than needed for purely hedging purposes, and who would therefore obtain a net profit from higher pool prices, is there any scope for renegotiation to occur. This case however, is an empirically unimportant one, and as such not of particular interest. We conclude that in the empirically important cases, the 'strategic commitment' equilibria of the two-stage game are probably immune to renegotiation. This means that the electricity spot market is one example of a market in which contracts with (interested) third parties would appear to have strategic commitment value, despite the generally negative tenor of the conclusions arrived at in the theoretical literature. As such, it is of particular interest.

One of the simplifying assumptions we have made in the above analysis, is to restrict the firms to a single opportunity to trade in long-term contracts before the spot market opens. However it has been argued elsewhere that oligopolists may want to revise their contract positions if trade is permitted to occur more than once. Allaz and Vila (1986) show in a model in which Cournot oligopolists trade in futures that the accumulated futures positions will increase over time and the perfectly competitive outcome sometimes attained. In our model there is no such tendency. Indeed, the only strategic

[25]Renegotiation would have to occur after the generator which has sold no contracts has (publicly) submitted a (low) price offer, but before the other generator has made a bid. A contract holder not simultaneously in the market for electricity, would expect to receive no difference payments in this case (since the generator with a large number of contracts would also bid low), and hence would be willing to renegotiate his contract(s) in order to permit that generator to make a higher bid. This would be Pareto improving for both the contract-selling generator and the contract holder, and hence contracts would serve as ineffectual commitment devices. As the text argues however, this is not the case when the contract holder is also a purchaser of electricity in the spot market. Bensaid and Gary-Bobo (1991) contains a lucid discussion of the renegotiation issue in a context not too dissimilar to the one considered here. See also Green (1990).

effect we find tends to induce firms to sell a large volume of contracts early. However as soon as one firm has acquired the dominant position, its competitor, for strategic reasons, will wish to reduce its own volume of contracts by as much as possible.

3.6 Other Forms of Contracts

In the preceding sections we have considered contracts which hedge purchasers against unexpectedly high pool prices, and we have assumed throughout that the generators were net sellers of such contracts. It is straightforward to generalize our analysis to other forms of contracts, and in Appendix A we show how spot market outcomes will be affected by the presence of such contracts. In this section we give a brief overview of the results derived in Appendix A.

In principle generators can buy contracts to hedge against unexpectedly low pool prices. Such contracts for differences would, in this setting, be equivalent to European put options, and give the holder a right to claim the difference between the strike price and the pool price whenever the former exceeds the latter. As we demonstrate, this increases firms' incentives to bid low, since part of the negative effect on payoffs from low bids will be offset by contracts. It turns out that, as with European call option contracts, in most cases the offer prices, and thus, the system marginal price, are unaffected by the number of contracts held. However, if the generators hold sufficiently many contracts, equilibrium outcomes may be altered. In low-demand periods, the increased incentive to bid low may make undercutting profitable even when prices are below marginal cost (if net supply to the pool, i.e. output net of the contracted quantity, is negative), and thus render pure-strategy equilibria non-existent. In high-demand periods (and, indeed, variable-demand periods), fiercer price competition may make the competitive (Bertrand-type) outcome an equilibrium. We thus conclude that this form of contract, if anything, tends to put a downward pressure on bids, and hence on pool prices.

Even though the general conclusion is the same for the two types of one-way contracts, equilibria do differ depending on what sort of contracts generators have sold or bought. This is because there is a basic difference in incentives in the two cases. When generators sell contracts which involve the payout of differences in periods when the pool price exceeds the contract strike price, their incentive to increase bids in the range above the strike price is reduced. As we have seen, this effect tends to make equilibrium outcomes where the pool price is very high, more unlikely. On the other hand, if generators have hedged against low pool prices by buying call-option contracts, it is the incentive to bid in the range below the strike price which is affected; in particular, firms tend to become more competitive when bidding low. As a result, the competitive outcome where the pool price equals marginal cost, is more likely.

When we consider two-way contracts, which in this context are equivalent to futures, both effects are present at the same time, and hence firms incentive to reduce their bids is increased over the whole range of admissible offer prices. In other words, since with two-way contracts generators will be hedged against the downward risk of low prices (as with put-option contracts) and will have to pay out differences whenever the pool price rises above the contract strike price (as with call-option contracts), the incentive to bid low is even stronger in this case than in any of the corresponding one-way contract cases. The result is that the competitive outcome is more likely, even in high-demand periods, and generally offer prices are below what they would otherwise have been had firms signed no contracts at all.

3.7 Concluding Remarks

Our analysis has identified a number of important effects that the existence of long-term options contracts may have on the British electricity spot market. In particular we have shown that there are critical quantities of contracts that must be held by the generators for contracts to have any effect on electricity spot prices. In most cases, when contracts are held in large enough quantities, the effect is to reduce spot prices to the contract strike prices. However in the variable-demand case, with contracts held in sufficiently asymmetric quantities, the effect was the opposite. Our broad conclusion is that when contracts exert any influence at all upon bidding strategies, it is to keep spot prices lower than they would otherwise be. Interestingly, this finding is consistent with the evidence presented in Helm and Powell (1992) suggesting a marked increase in pool prices during the spring of 1991 when a proportion of the initial portfolio of contracts expired (see section 3.2).

In addition, in considering the two-stage game (section 3.5) in which the generators first choose the quantity of contracts to sell, and then compete in the spot market, we have found that for at least certain parameter values, there is a strategic incentive to sell a large quantity of contracts to commit to a low-pricing strategy in the second-stage game. Thus contracts may have commitment value, and hence be profitable, even if sold for a low price. This conclusion relates our analysis to a growing literature on the 'commitment value of contracts with third parties'. The asymmetric equilibria which we have identified for the two-stage game, in which only one firm sells (a large quantity of) contracts in the first stage in order to become the low pricing firm in the second stage, are clearly examples of such a commitment effect in operation. While it is not possible to say anything in the abstract about the likelihood of observing such strategic commitment effects in practice in the electricity spot market, (in particular because the generators' contract portfolios are not public information), this would nevertheless appear to be the first positive example of a market in which strategic commitments (via contracts with third parties) may have an influence on the outcome of competition. As such it is of particular interest.

We find that the strategic incentive for selling contracts, viz. a commitment to offer prices below the contract strike price, exists only for contracts with low strike prices. This result may be related to the discussion of whether a viable market for contracts may survive the expiry of the transitional contracts arrangements in March 1993 (see e.g. Helm and Powell (1992) and Powell (1993)). While our model is obviously too simplified and abstract to provide a satisfactory answer to this question, it does at least identify some effects which may be of importance. In particular there appear to be strong disincentives for generators to sell long-term contracts, and hence we would not expect to see both generators holding large contract portfolios. Contracts place downward pressure on spot-market prices, a pressure which is stronger the lower are strike prices and the larger the number of contracts held. On the other hand, there may exist a strategic incentive for selling contracts with low strike prices, which would lead to generators to hold very asymmetric quantities of low-strike-price contracts.

In addition to the effects identified by our formal analysis, there are a number of other features which will be of importance in determining how the market for long-term contracts will evolve in the future. If electricity buyers are willing to pay risk premia in order to hedge against the volatility of spot prices, this will of course make generators more willing to sell contracts. One the other hand, problems of developing adequate standardized contracts, may lead to levels of transactions cost which prevent the opening of markets for many types of contracts (relating to coverage, time of day, season etc.) because they become too "thin". Furthermore, the fact that long-term contracts, if the generators have sold sufficiently many, may lead to lower spot-market prices, suggests that electricity buyers may be willing to pay a premium on contracts in order to reduce the cost of purchases in the spot market. Although this effect could lead to a more viable market for long-term contracts, it should be noted that there is a strong externality at play; purchasers of electricity would like others to buy, and thus pay the premium on, contracts[26]. All in all it seems doubtful that whether the fact that there is concentration on the buyers' side will overcome any disincentive for generators to sell contracts.

[26]This point has been made by Powell (1993).

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3.8 Appendix A: Other Forms of Contracts

In this appendix we extend the analysis of this paper to other forms of contracts. We begin by considering the case in which the generators hedge by purchasing one-way contracts which give payouts to the generators whenever the pool price falls below a specified strike price. We then consider two-way contracts, where, in effect, generators have sold part of their capacity forward.

A.1. One Way Put Option Contracts

In this section we consider spot-market equilibria for the case where the generators have bought contracts which give them the right to sell electricity at a specified strike price. This type of contract is formally equivalent to a European put option. The profit of a generator who has bought z_i contracts at a strike price v and supplies y_i units of electricity to the pool at the pool price p^s, is (net of any lump-sum payments to the sellers of contracts):

$$\Phi_i = p^S y_i + z_i \cdot max\{v - p^S, 0\}, i = 1, 2. \tag{A.1}$$

We assume throughout that generators are net buyers of contracts but do not buy more contracts than their output capacity, i.e. $z_i \in [0, k_i], i = 1, 2$. We also limit attention to cases where $v \in (0, \overline{p})$. As in the previous sections we assume $k_1 = k_2 = 1$, and we distinguish between low-demand, high-demand, and variable-demand periods.

Put-option contracts make firms less reluctant to bid low since the downward risk is partly covered, i.e. a minimum price is secured on part of the output capacity. As we show below, the result is that if equilibrium bids differ from those that would prevail in the case when firms purchase no contracts at all, they will be lower when firms hold these types of contracts. In some cases, when firms have purchased a large number of contracts, the reduced incentive to bid high which tends to make undercutting the rival attractive, may lead to non-existence of pure-strategy equilibria.

A. Low-Demand Periods

In low-demand periods only one firm will be producing. W.l.o.g. we assume demand to be non-stochastic. We can then prove the following proposition:

Proposition A.3.1 *Assume $d \in [0,1]$. If $max\{z_1, z_2\} < d$, then there exists a unique pure-strategy equilibrium of the second-stage spot-market game where $p_1 = p_2 = 0$. If $max\{z_1, z_2\} > d$, no pure strategy-equilibrium exists.*

Proof. Payoffs are given by $\Phi_i = p_i d + z_i \cdot max\{v - p_i, 0\}$ and $\Phi_j = z_j \cdot max\{v - p_i, 0\}$ if $p_i < p_j$, and $\Phi_i = \frac{1}{2}p_i d + z_i \times max\{v - p_i, 0\}, i = 1, 2$, if $p_1 = p_2$. Consider first the case where $p_i < p_j$. Note that when $p_i < v$ and firm $_i$ is a net supplier to (buyer from) the pool, i.e. $d - z_i > 0, (d - z_i < 0)$, its payoff is increasing (decreasing) in its own offer price. When $p_i \geq v$, firm i's payoff is always increasing in p_i. It follows that $p_i < p_j$ can never be an equilibrium. If $p_1 = p_2$, deviating to a slightly lower (higher) price is always profitable as long as prices are above (below) marginal cost. Thus there cannot exist equilibria where $p_1 = p_2 \neq 0$. The proposition then follows by observing that when $p_1 = p_2 = 0$, neither firm will benefit by deviating to a higher price, while the gain to firm i from deviating to a price $p < 0$, $p[d - z_i]$, is positive if and only if $d - z_i < 0$. ■

Remark: In the non-generic case where $z_1 = d(z_2 = d)$, all strategy combinations such that $0 = p_1 \leq p_2 (0 = p_2 \leq p_1)$ are equilibria.

As discussed above, put-option contracts strengthen firms' incentive to reduce their spot-market bids. Therefore it is no surprise that in low-demand periods, the perfectly competitive outcome can still be an equilibrium even when firms hold such contracts. When the volume of such contracts becomes sufficiently large however, the incentive to reduce offer prices leads to the non-existence of pure-strategy equilibria. Observe that firms' equilibrium profits (when such exist) are increasing in both the strike price and the number of contracts held ($\Phi_i = z_i v, i = 1, 2$). As we have seen however, there is no strategic incentive to buy put-option contracts in low-demand periods.

B. High-Demand Periods

We continue to assume d to be non-stochastic, but now let $d\hat{I}(1,2]$, i.e. both firms will be producing for sure. Order firms such that $p_1 \leq p_2$. Then we have:

Proposition A.3.2 *Assume* $1 < d \leq 2$. *Then, generically, all second-stage spot-market equilibrium strategy combinations* $\{p_1, p_2\}$ *such that* $p_1 < p_2$, *must satisfy* $p_2 = \overline{p}$ *and* $p_1 \leq b_1$, *where* $b_1 = \overline{p}[d-1]$ *if* $\overline{p}[d-1] > v$ *and* $b_1 = \{\overline{p}[d-1] - z_2 v\}/[1-z_2]$ *otherwise.*

Proof. If $p_1 < p_2$, payoffs are given by $\Phi_1 = p_2 + z_1 \cdot max\{v - p_2, 0\}$ and $\Phi_2 = p_2[d-1] + z_2 \cdot max\{v - p_2, 0\}$. The payoff to firm 2 is always increasing in its own offer price when p_2 exceeds v. Furthermore, when $p_2 \leq v$, firm 2's payoff is non-decreasing (decreasing) in p_2 when $d-1-z_2 \geq 0 (< 0)$. It follows that an equilibrium candidate must have $p_2 = \overline{p}$. The proposition then follows by observing that if, and only if, the conditions on p_1 are satisfied, firm 2 does not want to deviate by undercutting firm 1. ■

In these equilibria, firms' profits are unaffected by the existence of long-term contracts; indeed, options are never exercised. However when firms have purchased large quantities of such contracts there may exist other equilibria in which firms offer to supply at the same price. In particular:

Proposition A.3.3 *Assume* $1 < d \leq 2$. *Then, there never exists equilibria of the second-stage spot-market game where* $p_1 = p_2 \neq 0$.$p_1 = p_2 = 0$ *is an equilibrium if and only if* $min\{z_1, z_2\} \geq [\overline{p}/v][d-1]$.

Proof. Assume $p_1 = p_2$. The gain to firm i from undercutting firm j by an arbitrarily small amount is given by $p_j[1 - \frac{1}{2}d]$ which is positive if $p_j > 0$. On the other hand, if firm i deviates by raising its price slightly above p_j, its gain, $p_j[\frac{1}{2}d - 1]$, is positive if $p_j < 0$. It follows that there cannot exist equilibria where $p_1 = p_2 \neq 0$. When $p_1 = p_2 = 0$, firm i gains $p_i[d - 1 - z_i]$ if it deviates to a price $p_i < v$,and $p_i[d-1] - z_i v$ if it deviates to a price $p_i \geq v$. Then deviation is unprofitable if and only if the condition in the proposition is fulfilled. ■

Competitive equilibria do not exist in high-demand periods unless firms have sold many contracts. In the competitive equilibrium, profits are given by $\Phi_i = z_i v, i = 1, 2$. In the asymmetric equilibria, profits are $\Phi_1 = \overline{p}$ and $\Phi_2 = \overline{p}[d-1]$, respectively. When the competitive equilibrium exists, this gives higher payoffs to firm 2 than it would get as the higher-pricing firm in an asymmetric equilibrium. By invoking a forward-induction argument, we may then rule out asymmetric equilibria where $p_i \leq b_i$ and $p_j = \overline{p}$ when $z_j > d-1$ (by selling a large amount of contracts, a firm signals that it does not expect the asymmetric equilibrium with itself as the higher pricing firm to be played). This leaves us with a unique equilibrium when $min\{z_1, z_2\} \geq [\overline{p}/v][d-1]$, When this condition is not satisfied, we have two types of equilibria, in which firms 1 and 2 are the higher-pricing firm, alternately. We conclude that in high-demand periods put-option contracts will lead to lower bids if firms have signed large numbers of such contracts.

C. Variable-Demand Periods

We turn now to the case when both the event that only a single firm will be despatched and the event that both firms will be producing occur with positive probabilities, i.e. $0 < Pr\{d < 1\} < 1$. In this case we have the following result:

Proposition A.3.4 *Assume $[\overline{p}/v][E(d|d>1)-1] \leq z_i \leq E(d|d \leq 1), i = 1, 2$. Then there exist a unique pure-strategy equilibrium of the second-stage spot-market game in which $p_1 = p_2 = 0$.*

Proof. W.l.o.g. assume $p_1 \leq p_2$. Then, if $p_1 < p_2$, payoffs are given by:

$$\begin{aligned} \Phi_1 &= Pr(d \leq 1)\{p_1 E(d|d \leq 1) + z_1 \cdot max\{v - p_1, 0\}\} \quad \text{(A.2)} \\ +Pr(d &\geq 1)\{p_2 + z_1 \cdot max\{v - p_2, 0\}\} \end{aligned}$$

$$\begin{aligned} \Phi_2 &= Pr(d \leq 1) \times z_2 \cdot max\{v - p_1, 0\} \quad \text{(A.3)} \\ &\quad +Pr(d \geq 1)\{p_2[E(d|d \geq 1) - 1] + z_2 \cdot max\{v - p_2, 0\}\} \end{aligned}$$

while if $p_1 = p_2$, payoffs are:

$$\begin{aligned} \Phi_i &= Pr(d \leq 1) \cdot p_i \cdot \frac{1}{2} E(d|d \leq 1) \quad \text{(A.4)} \\ +Pr(d &> 1) \cdot p_i \cdot \frac{1}{2} E(d|d > 1) + z_i \cdot max\{v - p_i, 0\}, i = 1, 2. \end{aligned}$$

Note that if $p_i > v$, firm i's profits are always increasing in its own offer price. Furthermore, if $E(d|d \leq 1) - z_i > 0(< 0)$, profits of the lower-pricing firm are increasing (decreasing) in its own offer price. Thus, there cannot exist equilibria in which $p_1 < p_2$. The proposition then follows by observing that deviation from $p_1 = p_2 \neq 0$ is always profitable, while deviation from $p_1 = p_2 = 0$ is unprofitable if and only if the conditions on the z_i's are satisfied. ■

We conclude that in variable-demand periods, the competitive equilibrium may prevail only if firms have purchased put-option contracts. However, if the quantities of contracts held are sufficiently large, no pure-strategy equilibrium will exist. We do not characterize mixed-strategy equilibria for this model, but, as in the call-option contracts model, it can be shown that in such equilibria bids will on average be lower the larger are the quantities of contracts held by firms.

A.2. Two Way Contracts

In this section we turn to the case when firms have entered into two-way contracts, giving both a right and an obligation to sell electricity at a specified strike price. Two-way contracts are formally equivalent to futures in this setting. The profit to a firm who has sold t_i contracts at a strike price w and is despatched with y_i units of output is

$$\Phi_i = p^S[y_i - t_i] + wt_i. \tag{A.5}$$

Thus two-way contracts effectively reduce output-capacity of a firm as far as competition in the spot-market is concerned. The incentive to bid high is now reduced for two reasons; the downward risk from low prices is partly covered because some of the capacity is sold at a pre-determined price. Furthermore, if the system marginal price exceeds the contract strike price, generators have to pay out differences on their contracts. Thus we expect offer prices to be even lower in this than in either of the models where firms enter into one-way contracts. As in the other models, we assume $w \in [0, \overline{p}], \overline{t}_i \in [0, k_i], i = 1, 2$, and $k_1 = k_2 = 1$.

A. Low-Demand Periods

In low-demand periods we get the same result as in the case of one-way put-option contracts; the competitive outcome is the only equilibrium candidate, however, because of the stronger incentive to reduce bids, this will only be an equilibrium if firms have entered into limited numbers of contracts. By an analogous proof to that of Proposition 3.5, one can prove the following result:

Proposition A.3.5 *Assume $d \in [0,1]$. If $max\{t_1, t_2\} < d$, then there exists a unique pure-strategy equilibrium of the second-stage spot-market game where $p_1 = p_2 = 0$. If $max\{t_1, t_2\} > d$, no pure-strategy equilibrium exists.*

B. High-Demand Periods

In high-demand periods, when both firms will be producing, the results resemble those for one-way call-option contracts in that the asymmetric equilibria in which one firm bids at the highest admissible price can only exist when firms hold few contracts. In contrast to that model however, here having system marginal price equal to the contract strike price can never be an equilibrium outcome. Instead, the stronger incentive to undercut caused by the put-option part of the two-way contracts, makes the competitive equilibrium prevail if firms hold large enough quantities of such contracts.

Order firms such that $p_1 \leq p_2$. We may summarize (without proof) the above discussion in two propositions:

Proposition A.3.6 *Assume $1 < d \leq 2$. Then if $min\{t_1, t_2\} < d-1$, all pure-strategy second-stage spot-market equilibrium combinations $\{p_i, p_j\}$ must satisfy $p_i \leq b_i$ and $p_j = \overline{p}$, where $b_i = \overline{p}[d-1-t_j]/[1-t_j]$.*

Remark: If $t_i < d-1 < t_j$, there continues to exist equilibria where $p_i = \overline{p}$, and $p_j \leq b_j$.

Proposition A.3.7 *Assume $1 < d \leq 2$. Then if $min\{t_1, t_2\} > d-1$, there exists a unique pure-strategy equilibrium of the second-stage spot-market game where $p_1 = p_2 = 0$.*

C. Variable-Demand Periods

As in the model of put-option contracts, in variable-demand periods, i.e.

$0 < Pr(d \leq 1) <, 1$ a pure-strategy equilibrium may only exist if firms have signed contracts. In particular, if the amounts of contracts are not excessive, the competitive equilibrium exists and is unique:

Proposition A.3.8 *If* $E(d \mid d \geq 1) - 1 \leq t_i \leq E(d \mid d \leq 1), i = 1, 2$, *there exists a unique pure-strategy equilibrium of the second-stage spot-market game where* $p_1 = p_2 = 0$. *Otherwise, no pure-strategy equilibrium exists.*

3.9 Appendix B: Proofs

Proof of Proposition 3.1: W.l.o.g. let $p_1 \leq p_2$. Then if $p_1 < p_2$, profits are given by $\Phi_1 = p_1 d - x_1 \cdot max\{p_1 - q, 0\}$ and $\Phi_2 = -x_2 \cdot max\{p_1 - q, 0\}$, while if $p_1 = p_2$, profits are $\Phi_i = \frac{1}{2} p_i d - x_i \cdot max\{p_i - q, 0\}$. Existence of $p_1 = p_2 = 0$ as an equilibrium is straightforward. To prove uniqueness, we first observe that $p_1 < 0$ cannot be part of an equilibrium since non-negative profits can be secured by offering to supply at a price equal to marginal cost, i.e. zero. Furthermore, there is no equilibrium in which both generators submit positive offer prices, since if $p_1 > 0$, firm 2 can obtain an increase in profits by undercutting firm 1 by some arbitrarily small amount. Lastly, there cannot exist an equilibrium with $p_1 = 0$ and $p_2 > 0$ either, since generator 1's profit is strictly increasing in p_1 on $[0, q)$. QED.

Proof of Proposition 3.2: Without loss of generality let $p_1 \leq p_2$. Then if $p_1 < p_2$ profits are given by $\Phi_1 = p_2 - x_1 \cdot max\{p_2 - q, 0\}$ and $\Phi_2 = p_2[d-1] - x_2 \cdot max\{p_2 - q, 0\}$, while if $p_1 = p_2$ profits are $\Phi_i = \frac{1}{2} p_i d - x_i \cdot max\{p_i - q, 0\}, i = 1, 2$. Note first that $p_1 = p_2$ cannot be an equilibrium since deviating to a slightly lower (higher) price is always profitable as long as $p_1 = p_2 > 0 (\leq 0)$. If $p_2 > p_1$, firm 2's profit is increasing in p_2 on $(p_1, \overline{p}]$, thus $p_2 = \overline{p}$. For $p_2 = \overline{p}$ to be part of an equilibrium, firm 2's payoff from undercutting firm 1's offer price must not be greater than its equilibrium profits, i.e. if $p_1 = b_1$, then $p[d-1-x_2] + qx_2 \geq b_1 - x_2 \cdot max\{b_1 - q, 0\}$. It follows that either $b_1 = p[d-1-x_2] + x_2 q \leq q$, or $q < b_1 = p[d-1-x_2]/[1-x_2]$. QED.

Proof of Proposition 3.3: Let $p_1 \leq p_2$. Since $x_2 > d - 1 > 0$, firm 2's profit is strictly decreasing in its own offer price on $(max\{p_1, q\}, p]$. By an argument similar to that given in the proof of proposition 3.2, $p_2 = p_1$ cannot be an equilibrium, thus $p_1 < q$, and since firm 2's profit is increasing on $[0, q], p_2 = q$. Again by a similar argument to that in the proof of proposition 3.2, $p_1 \leq [d-1]q$ in order to make it unprofitable for firm 2 to undercut firm 1. Lastly, when the condition on x_1 is fulfilled, firm 1 will not deviate to a price greater than q since $q > [d - 1 - x_1]\overline{p} + x_1 q$, where the former is 1's equilibrium profits and the latter the maximum obtainable payoff from deviation. QED.

Proof of Proposition 3.4: W.l.o.g. let $p_1 \leq p_2$. Then if $p_1 < p_2$, profits are given by:

$$\begin{aligned} E\Phi_1 &= Pr(d \leq 1)\{E(d|d \leq 1)p_1 - x_1 \cdot max\{p_1 - q, 0\}\} \quad \text{(B.1)} \\ +Pr(d &\geq 1)\{p_2 - x_1 \cdot max\{p_2 - q, 0\}\}, \end{aligned}$$

$$\begin{aligned} E\Phi_2 &= -Pr(d \leq 1)x_2 \cdot max\{p_1 - q, 0\} \quad \text{(B.2)} \\ +Pr(d &\geq 1)\{[E(d|d \geq 1) - 1]p_2 - x_2 \cdot max\{p_2 - q, 0\}\}, \end{aligned}$$

while if $p_1 = p_2$,

$$E\Phi_i = Pr(d \leq 1)\{\frac{1}{2}E(d|d \leq 1)p_i - x_i \cdot max\{p_i - q, 0\}\} \quad \text{(B.3)}$$

$$+Pr(d \geq 1)\{1/2[E(d|d \geq 1)]p_i - x_i \cdot max\{p_i - q, 0\}\}, i = 1, 2.$$

It is straightforward to show that $p_1 = p_2$ cannot constitute an equilibrium since if $p_1 = p_2 > 0 (\leq 0)$, a deviation to a lower (higher) price is always profitable. Then if $p_1 < p_2$, firm 1's expected profit is increasing on $(-\infty, min\{q, p_2\})$. It follows that we cannot have $p_1, p_2 < q$ in equilibrium. Furthermore, if $x_1 < E(d \mid d \leq 1)$, firm 1's expected profit is increasing on $[q, p_2)$ also, and no pure-strategy equilibrium can exist. Assume then that $x_1 > E(d|d \leq 1)$, in which case we must have $p_1 = q$. Now, if $x_2 > E(d|d \geq 1) - 1$, firm 2's profit is decreasing on $(q, p]$, and thus equilibrium cannot exist. If, on the other hand, $x_2 < E(d|d \geq 1) - 1$, we must have $p_2 = \overline{p}$. To prove the existence of $\{q, p\}$ as an equilibrium, we must check that firm 2 would not want to deviate by undercutting firm 1. The condition that $x_2 < a(q)(\leq E(d|d \geq 1) - 1)$ ensures this. QED.

Proof of Proposition 3.5: We treat the two cases $p < q$ and $p \geq q$ separately. Noting that for all $p \in supF_i(\times), \phi_i(p) =$ constant, differentiating $\phi_i(p)$ and solving yields:

$$\begin{array}{rl} F_i'(p) - \frac{1-2\pi}{\pi}\frac{F_i(p)}{p} = \frac{1}{p} & when \quad p < q \\ F_i'(p) - [1 - x_j]\frac{1-2\pi}{\pi}\frac{F_i(p)}{p} = \frac{1-x_j}{p} & when \quad p \geq q \end{array} \tag{B.4}$$

The (unique) solution to this is:

$$F_i(p) = \begin{cases} \ln A_i(p), & \pi = \frac{1}{2} \\ B_i p^{\frac{1-2\pi}{\pi}} + \frac{\pi}{2\pi-1}, & \pi \neq \frac{1}{2} \end{cases} \text{ when } p < q \tag{B.5}$$

$$F_i(p) = \begin{cases} [1 - x_j]\ln(C_j p), & \pi = \frac{1}{2} \\ D_i p^{[1-x_1]\frac{1-2\pi}{\pi}} + \frac{\pi}{2\pi-1}, & \pi \neq \frac{1}{2} \end{cases} \text{ when } p \geq q$$

where A_i, B_i, C_i, and D_i are constants to be determined.

Assume that $F_2(\cdot)$ does not have a mass point at $\overline{p}$. (It can be proved that at most one firm plays $\overline{p}$ with positive probability. By going through similar calculations as those below, one can then show that the opposite assumption, i.e. $F_1(\cdot)$ does not have a mass point at $\overline{p}$ leads to a contradiction.) Using the facts $F_2(\overline{p}) = 1$, $F_2(p)$ must be continuous at $p = q$, and $F_2(p^m) = 0$, where is p^m is the lower bound on the support of $F_2(\cdot)$, one gets $F_2(\cdot)$ and p^m as functions of the exogenous parameters. Furthermore, from the facts that $F_1(\cdot)$ must have the same support as $F_2(\cdot)$ and be continuous at $p = q$, straightforward calculations establish that:

$$F_1(p) = \begin{cases} \ln(e^{\frac{p}{q}}[\frac{q}{\overline{p}}]^{1-x_1}), & \pi = \frac{1}{2} \\ \frac{\pi-1}{2\pi-1}{}_{i}[\frac{p}{q}]^{\frac{1-2\pi}{\pi}}[\frac{q}{\overline{p}}]^{1-x\frac{1-2\pi}{\pi}} + \frac{\pi}{2\pi-1}, & \pi \neq \frac{1}{2} \end{cases} \text{ when } p < q \tag{B.6}$$

$$F_1(p) = \begin{cases} \ln(e[\frac{p}{q}]^{1-x_2}[\frac{q}{\overline{p}}]^{1-x_1}), & \pi = \frac{1}{2} \\ \frac{\pi-1}{2\pi-1}[[\frac{p}{q}]^{1-x_2}[\frac{q}{\overline{p}}]^{1-x_1}]^{\frac{1-2\pi}{\pi}} + \frac{\pi}{2\pi-1}, & \pi \neq \frac{1}{2} \end{cases} \text{ when } p \geq q$$

$$F_1(\overline{p}) = 1$$

$$F_2(p) = \begin{cases} \ln(e\frac{p}{q}[\frac{q}{\overline{p}}]^{1-x_1}), & \pi = \frac{1}{2} \\ \frac{\pi-1}{2\pi-1}{}_i[\frac{p}{q}]^{\frac{1-2\pi}{\pi}}[\frac{q}{\overline{p}}]^{1-x\frac{1-2\pi}{\pi}} + \frac{\pi}{2\pi-1}, & \pi \neq \frac{1}{2} \end{cases} \text{ when } p < q \tag{B.7}$$

$$F_2(p) = \begin{cases} \ln(e[\frac{p}{\overline{p}}]^{1-x_1}), & \pi = \frac{1}{2} \\ \frac{\pi-1}{2\pi-1}[\frac{p}{q}]^{(1-x_2)\frac{1-2\pi}{\pi}} + \frac{\pi}{2\pi-1}, & \pi \neq \frac{1}{2} \end{cases} \text{ when } p \geq q$$

$$p^m = \begin{cases} e^{-1}q^{x_1}\overline{p}^{1-x_1} & \pi = \frac{1}{2} \\ [\frac{\pi}{1-\pi}]^{\frac{\pi}{1-2\pi}}q^{x_1}\overline{p}^{1-x_1} & \pi \neq \frac{1}{2} \end{cases} \tag{B.8}$$

QED.

4 Designing Electricity Auctions

4.1 Introduction

Electricity wholesale markets differ in numerous dimensions, but until recently all have been organized as uniform, first-price auctions. Recent experience - and the perceived poor performance - of some decentralized electricity markets however, has led certain regulatory authorities to consider adopting new auction designs. In England and Wales a major overhaul of the electricity trading arrangements introduced in 1990 has recently taken place, and among the reforms implemented in March 2001, a discriminatory or 'pay-as-bid' auction format was adopted. The British regulatory authority (Ofgem) believed that uniform auctions are more subject to strategic manipulation by large traders than are discriminatory auctions, and expected the new market design to yield substantial reductions in wholesale electricity prices. Similarly, before its collapse, the California Power Exchange commissioned a report by leading auction theorists on the advisability of a switch to a discriminatory auction format for the Exchange's day ahead market, due to the increasing incidence of price spikes in both on- and off-peak periods (see Kahn et al., 2001).

It is well-known that discriminatory auctions are not generally superior to uniform auctions. Both types of auction are commonly used in financial and other markets, and there is now a voluminous economic literature devoted to their study.[27] In multi-unit settings the comparison between these two auction forms is particularly complex. Neither theory nor empirical evidence tell us that discriminatory auctions perform better than uniform auctions in markets such as those for electricity, although this has become controversial.

Wolfram (1999), for instance, argues in favor of uniform auctions for electricity, and Rassenti, Smith and Wilson (2003) cite experimental evidence

[27] See Ausubel and Cramton (2002) and Binmore and Swierzbinski (2000) for the theory and empirical evidence. Archibald and Malvey (1998) and Belzer and Reinhart (1996) discuss the US Treasury's experiments with these auction formats in more detail. See also Kremer and Nyborg (2004).

which suggests that discriminatory auctions may reduce volatility (i.e. price spikes), but at the expense of higher average prices. Other authors have come to opposite conclusions. Federico and Rahman (2003) find theoretical evidence in favor of discriminatory auctions, at least for the polar cases of perfect competition and monopoly, while Klemperer (2001, 2002) suggests that discriminatory auctions might be less subject to 'implicit collusion'.[28] Kahn et al. (2001), on the other hand, reject outright the idea that switching to a discriminatory auction will result in greater competition or lower prices.

In Britain, Ofgem has credited the recent fall in wholesale electricity prices in England and Wales to the new market design, however this too is controversial.[29] Evans and Green (2002) present some supporting evidence,[30] but Bower (2002) and Newbery (2003) argue that the decline in prices is fully explained by the reduction in market concentration brought about by asset divestitures, an increase in imports and market excess capacity. Fabra and Toro (2003) suggest that all of these factors, including the change in market design, are significant in explaining the reduction in wholesale electricity prices.[31]

The purpose of this paper is to address this electricity market design issue in a tractable model designed to capture some of the key features of decentralized electricity markets.[32] We characterize equilibrium market out-

[28] In a model similar to that used in this paper, Fabra (2003) shows that tacit collusion may be easier to sustain in uniform auctions than in discriminatory auctions.

[29] Ofgem reported a 19% fall in wholesale baseload prices from the implementation of the reforms in March 2001 to February 2002, and a 40% reduction since 1998 when the reform process began. Wholesale prices have since risen again so that they are now near their pre-reform levels.

[30] Evans and Green argue that the new trading arrangements may have undermined opportunities for tacit collusion. Sweeting (2004) claims to have found evidence of collusion in the England and Wales market during the late 1990s, although this finding has been challenged by Newbery (2003).

[31] Another contributing explanation for the initial fall in prices may be that Ofgem staked its reputation on the market reforms delivering lower-cost electricity, and for more than a year after their introduction sought to expand its regulatory powers to police 'market abuses' by smaller generators. See Bishop and McSorely (2001) for a discussion.

[32] For a discussion of some methodological issues in modelling electricity markets, which has informed our choice of models, see von der Fehr and Harbord (1998) and Fabra, von der Fehr and Harbord (2002).

comes in a discrete, multi-unit auction model for uniform and discriminatory electricity auctions under a variety of assumptions concerning costs and capacity configurations, bid formats, demand elasticities and the number of suppliers in the market. Our purpose is to gain an improved understanding of how these different auction formats affect suppliers' bidding incentives, the degree of competition and overall welfare in decentralized electricity markets.

Our analysis proceeds by first considering a 'basic duopoly model', similar to the discrete, multi-unit auction described in von der Fehr and Harbord (1993), which is then varied in several directions. In the basic duopoly model, two 'single-unit' suppliers with asymmetric capacities and (marginal) costs face a market demand curve which is assumed to be both perfectly inelastic and known with certainty when suppliers submit their offer prices. By 'single-unit' we mean that each supplier must submit a single price offer for its entire capacity (i.e. its bid function is horizontal). This assumption simplifies the analysis considerably, but in Section 4.4.1 we show that it is largely inessential. The assumption of price-inelastic demand can be justified by the fact that the vast majority of consumers purchase electricity under regulated tariffs which are independent of the prices negotiated in the wholesale market, at least in the short run.[33] However, in order to evaluate some of the possible effects of real-time pricing or demand-side bidding, we then extend the basic model and consider downward-sloping demand functions. We also consider the oligopoly case in order to shed some light on the relationship between market concentration and market performance.

Finally, the assumption that suppliers have perfect information concerning market demand is descriptively reasonable when applied to markets in which offers are 'short-lived', such as in Spain where there are 24 hourly markets each day (see García-Díaz and Marín, 2003). In such markets suppliers can be assumed to know the demand they face in any period with a

[33] See Wolak and Patrick (1997) and Wilson (2002) on this. In most electricity markets large industrial consumers can purchase electricity directly from suppliers or the wholesale market, but their demand comprises only a small fraction of the total volume traded.

high degree of certainty. In markets in which offer prices remain fixed for longer periods, e.g. a whole day, such as in Australia and in the original market design in England and Wales, on the other hand, it is more accurate to assume that suppliers face some degree of demand uncertainty or volatility at the time they submit their offers. Hence we allow for this type of uncertainty in Section 4.4.4.

Under each set of assumptions we characterize suppliers' equilibrium bidding behavior in uniform and discriminatory auctions, and compare the equilibrium outcomes in terms of prices and productive efficiency. Our main insights may be summarized as follows. Equilibrium outcomes in either auction format fall essentially into one of two categories, depending upon the level of demand. In low-demand realizations, prices are competitive in the sense that they cannot exceed the cost of the most efficient non-despatched supplier; in high-demand realizations, on the other hand, prices exceed the cost of even the most inefficient supplier. In high-demand states[34] there are multiple, price-equivalent pure strategy equilibria in the uniform auction, while in the discriminatory auction the equilibrium is in mixed strategies. With certain demand (i.e. short-lived bids), payments to suppliers (or average prices) are lower in the discriminatory auction and numerical examples suggest that the difference can be substantial.[35] The comparison in terms of productive efficiency is ambiguous, however, and depends on parameter values as well as on which pure-strategy equilibrium is played in the uniform auction. The relative incidence of low-demand and high-demand states depends upon structural features of the market, such as the degree of market concentration, and on the market design, in particular the market reserve price and opportunities for demand-side bidding. Structural factors that reduce the incidence of high-demand states affect bidding strategies in the discriminatory, but not in the uniform, auction. Market design changes, on the other hand, affect bidding strategies in both types of auction.

[34] The terms 'state' and 'realization' are used interchangeably throughout this paper.

[35] With uncertain demand (or long-lived bids) payments to suppliers are equal in both auction formats, at least for the case of symmetric firms.

4.2 The Model

In the basic duopoly model two independent suppliers compete to supply the market with productive capacities given by $k_i > 0$, $i = 1, 2$. Capacity is assumed to be perfectly divisible. Supplier i's marginal cost of production is $c_i \geq 0$ for production levels less than capacity, while production above capacity is impossible (i.e. infinitely costly). The suppliers are indexed such that $c_1 \leq c_2$. Without further loss of generality we may normalize suppliers' marginal costs so that $0 = c_1 \leq c_2 = c$. The level of demand in any period, θ, is a random variable which is independent of the market price, i.e. perfectly price inelastic. In particular, $\theta \in [\underline{\theta}, \overline{\theta}] \subseteq (0, k_1 + k_2)$ is distributed according to some known distribution function $G(\theta)$.

The two suppliers compete on the basis of bids, or offer prices, submitted to the auctioneer. The timing of the game is as follows. Having observed the realization of demand, each supplier simultaneously and independently submits a bid specifying the minimum price at which it is willing to supply the whole of its capacity, $b_i \leq P$, $i = 1, 2$, where P denotes the 'market reserve price,' possibly determined by regulation.[36] We let $\mathbf{b} \equiv (b_1, b_2)$ denote a bid profile. On the basis of this profile the auctioneer calls suppliers into operation. If suppliers submit different bids, the lower-bidding supplier's capacity is despatched first. If this capacity is not sufficient to satisfy the total demand θ, the higher-bidding supplier's capacity is then despatched to serve the residual demand, i.e. total demand minus the capacity of the lower-bidding supplier. If the two suppliers submit equal bids, then supplier i is ranked first with probability ρ_i, where $\rho_1 + \rho_2 = 1$, $\rho_i = 1$ if $c_i < c_j$ and $\rho_i = \frac{1}{2}$ if $c_i = c_j$, $i = 1, 2, i \neq j$.[37]

For a given bid profile $\mathbf{b}$, the quantities allocated to each supplier are thus independent of the auction format. The output allocated to supplier i,

[36] P can be interpreted as the price at which all consumers are indifferent between consuming and not consuming, or a price cap imposed by the regulatory authorities. See von der Fehr and Harbord (1993, 1998).

[37] This rationing rule is used solely to ensure the existence of a pure-strategy equilibrium in the standard Bertrand game with asymmetric costs.

$i = 1, 2$, denoted by $q_i(\theta; \mathbf{b})$, is given by

$$q_i(\theta; \mathbf{b}) = \begin{cases} \min\{\theta, k_i\} & \text{if} \quad b_i < b_j \\ \rho_i \min\{\theta, k_i\} + [1 - \rho_i] \max\{0, \theta - k_j\} & \text{if} \quad b_i = b_j \\ \max\{0, \theta - k_j\} & \text{if} \quad b_i > b_j, \end{cases} \tag{4.1}$$

and is solely a function of demand and the bid profile.

The payments made by the auctioneer to the suppliers do depend upon the auction format, however. In the *uniform auction*, the price received by a supplier for any positive quantity despatched by the auctioneer is equal to the highest accepted bid in the auction. Hence, for a given value of θ and a bid profile $\mathbf{b} = (b_i, b_j)$, supplier i's profits, $i = 1, 2$, $i \neq j$, can be expressed as

$$\pi_i^u(\theta; \mathbf{b}) = \begin{cases} [b_j - c_i] \, q_i(\theta; \mathbf{b}) & \text{if } b_i \leq b_j \text{ and } \theta > k_i \\ [b_i - c_i] \, q_i(\theta; \mathbf{b}) & \text{otherwise,} \end{cases} \tag{4.2}$$

where $q_i(\theta; \mathbf{b})$ is determined by (4.1).

In the *discriminatory auction*, the price received by supplier i for its output is equal to its own offer price whenever a bid is wholly or partly accepted. Hence for a given value of θ, and a bid profile $\mathbf{b}$, supplier i's, $i = 1, 2$, profits can be expressed as

$$\pi_i^d(\theta; \mathbf{b}) = [b_i - c_i] \, q_i(\theta; \mathbf{b}), \tag{4.3}$$

where again $q_i(\theta; \mathbf{b})$ is determined by (4.1).[38]

Both suppliers are assumed to be risk neutral and to maximize their expected profits in the auction.

4.3 Equilibrium Analysis: A Tale of Two States

We first characterize the Nash equilibria in weakly undominated strategies of the model described in the previous section and then compare equilibrium outcomes.[39]

[38] Note that the discriminatory auction is essentially a Bertrand-Edgeworth game. See Deneckere and Kovenock (1996).

[39] All derivations of results are relegated to the Appendix.

Lemma 4.1. In any pure-strategy equilibrium, the highest accepted price offer is in the set $\{c, P\}$. Moreover, in the discriminatory auction, in a pure-strategy equilibrium all accepted units are offered at the same price.

Based on this ancillary result, we can prove the main result of this section, namely that equilibrium outcomes essentially fall into one of two categories, depending upon the level of demand:

Proposition 4.1. There exists $\widehat{\theta} = \widehat{\theta}(c, k_1, k_2, P)$ such that:

(i) (low demand) if $\theta \leq \widehat{\theta}$, in the unique pure-strategy equilibrium the highest accepted price offer is c.[40]

(ii) (high demand) if $\theta > \widehat{\theta}$, all suppliers are paid prices that exceed c. A pure-strategy equilibrium exists in the uniform auction, with the highest accepted offer price equal to P, but not in the discriminatory auction.

As is easily seen, in low-demand realizations the equilibrium outcome is both unique and identical across the two auction formats. In the pure-strategy equilibrium, both suppliers submit offer prices equal to c (i.e. the cost of the inefficient supplier) but only the most efficient supplier produces. Hence the equilibrium outcomes in both auctions are competitive in the sense that prices are constrained by the cost of the least efficient supplier. They are also cost efficient, i.e. overall generation costs are minimized.

In high-demand realizations the equilibrium outcomes are very different. In the uniform auction, any pure-strategy equilibrium involves one supplier bidding at the market reserve price P, while the other supplier submits an offer price sufficiently low so as to make undercutting unprofitable (c.f. von der Fehr and Harbord, 1993). The precise nature of the equilibrium depends upon parameter values. There are three possible cases: (a) if $\widehat{\theta}_2 \leq \theta \leq \widehat{\theta}_1$,

[40]This result describes the standard Bertrand-like equilibrium with asymmetric firms. In low-demand states the two types of auction are strategically equivalent, since only one supplier ever produces and supplies the entire market. It is well-known that the Bertrand equilibrium relies on at least one firm using a weakly-dominated strategy, i.e. bidding at cost (Mas-Colell, Whinston and Green, 1995), a consequence of the strategy space being continuous. We ignore this issue here, but show in the Appendix that with asymmetric firms there also always exist outcome-equivalent equilibria in which the higher-cost firm plays a mixed strategy and never plays his own cost with positive probability.

or $k_1 \leq \theta \leq k_2 + \frac{c}{P}k_1$, only equilibria in which $b_1 < b_2 = P$ exist; (b) if $\widehat{\theta}_1 < \theta \leq \widehat{\theta}_2$, or $\frac{P}{P-c}k_2 < \theta \leq k_1$ only equilibria in which $b_2 < b_1 = P$ exist; and (c) if $\theta > \max\left\{\widehat{\theta}_1, \widehat{\theta}_2\right\}$, or $\theta > \max\left\{k_1, k_2 + \frac{c}{P}k_1\right\}$ both types of pure-strategy equilibria exist. Note that in Case (a) the equilibrium outcome is always cost efficient, while in Case (b) it is always inefficient. In Case (c) cost efficiency depends on which equilibrium is played.[41]

In the discriminatory auction only mixed-strategy equilibria exist in high-demand states. In particular, there exists a unique equilibrium in which the two suppliers mix over a common support which lies above the cost of the inefficient supplier and includes the market reserve price, i.e. $b_i \in (c, P]$, $i = 1, 2$. This mixed strategy equilibrium is not efficient in general, as there is a positive probability that the inefficient supplier will submit the lowest offer price.

The relative likelihood of low-demand versus high-demand states depends upon structural characteristics of the industry and on the strictness of the regulatory regime. Straightforward calculations show that

$$\widehat{\theta} = \begin{cases} k_1 & \text{if} \quad k_1 \leq \frac{P}{P-c}k_2 \\ \frac{P}{P-c}k_2 & \text{if} \quad k_1 > \frac{P}{P-c}k_2 \end{cases} \tag{4.4}$$

From this expression it follows that, for a given ratio of supplier capacities, the incidence of low-demand states is increasing in aggregate capacity. The incidence of low-demand states is also greater when suppliers are more symmetrically sized; more precisely, given c,P and K, with $k_1 + k_2 = K$, $\widehat{\theta}$ is maximized at $k_1 = \frac{P}{P-c}k_2$, which involves perfect symmetry if $c = 0$. Further, cost asymmetry tends to make low-demand states more likely, since the loss in profit from undercutting the inefficient rival relative to serving residual demand is smaller the higher is his cost. Finally, since pricing monopolistically and serving residual demand is more profitable the higher is

[41] There is also a continuum of mixed-strategy equilibria in the uniform auction in high-demand realizations. However, since each of these equilibria, (i) involve the higher-cost firm playing a weakly-dominated strategy with positive probability, and (ii) is pay-off dominated by either of the pure-strategy equilibria, we do not consider them further here. See the Appendix for the details.

the market reserve price, the incidence of high-demand states is greater the higher is P. If we think of the market reserve price as a regulatory price cap, it follows that stricter regulation can improve market performance, not only because market power is reduced in high-demand states, but also because the likelihood of high-demand states occurring is lowered.

In comparing market performance across the two auction formats we consider both total generation costs and the average price paid to suppliers. For auction format $f = d, u$, let C^f and R^f denote equilibrium levels of total generation costs and payments to suppliers, respectively, and let b_i^f and q_i^f denote supplier i's equilibrium offer price and output, respectively. We have $C^f = \sum_i c_i q_i^f$, $f = u, d$, $R^d = \sum_i b_i^d q_i^d$ in the discriminatory auction, and $R^u = p^u \sum_i q_i^u = p^u \theta$, where $p^u = \max_i \{b_i^u \mid q_i^u > 0\}$ is the market price, in the uniform auction. From Proposition 1 the following result is immediate:

Proposition 4.2. Market performance:

(i) $R^d = R^u$ if $\theta \leq \widehat{\theta}$ and $R^d < R^u$ if $\theta > \widehat{\theta}$.

(ii) $C^d = C^u$ if $\theta \leq \widehat{\theta}$, $C^d > C^u$ if $\widehat{\theta}_2 < \theta \leq \widehat{\theta}_1$, $C^d < C^u$ if $\widehat{\theta}_1 < \theta \leq \widehat{\theta}_2$, and $C^d \gtrless C^u$ otherwise, depending upon whether, in the uniform auction, an equilibrium is played in which Supplier 1 or Supplier 2 submits the higher offer price.

In other words, the discriminatory auction weakly outperforms the uniform auction in terms of payments (or the average price paid) to suppliers. In low-demand realizations the equilibrium outcomes are identical in both auctions. In high-demand realizations, the market price is at its maximum (P) in the uniform auction, while prices in the discriminatory auction are below P with positive probability. Comparison of the auctions in terms of productive efficiency is more complex, however. In low-demand realizations costs are minimized in both auction formats. In high-demand realizations, the comparison is unambiguous in Cases (a) and (b) only. In the uniform auction production costs are minimized in Case (a) and maximized in Case (b), while in the mixed-strategy equilibrium of the discriminatory auction the more efficient supplier is undercut by the inefficient supplier with positive probability. Hence the cost performance in the uniform auction is superior to that of the discriminatory auction in Case (a), but worse in Case (b). In Case (c) the comparison depends upon which pure- strategy equilibrium is played in the uniform auction.

We conclude this section by considering how the performance of the two auction formats depends upon the parameters of the model. A change in parameter values affects outcomes in two distinct ways: first, by altering the relative incidence of high- versus low-demand states, and secondly by affecting the intensity of price competition in high-demand states. The importance of these two effects differ between the two auction formats. In the uniform auction, in high-demand realizations, price always equals the market reserve price, whereas in the discriminatory auction bidding strategies depend on the cost and capacity configuration, as well as on the level of demand and the market reserve price. An increase in the threshold $\widehat{\theta}$ has a profound effect on prices in the uniform auction, as prices jump down from the market reserve price to marginal cost over the relevant range of demand realizations. In the discriminatory auction, however, the effect of an increase in $\widehat{\theta}$ is much less pronounced. Since the equilibrium outcomes in high-demand realizations approach those of low-demand realizations as $\theta \downarrow \widehat{\theta}$, a marginal increase in $\widehat{\theta}$ has no effect on the outcome *per se*.

The different ways in which outcomes are affected by changes in parameter values is illustrated in Figure 1 below. The figure is based on an example in which $[\underline{\theta}, \overline{\theta}] = [0, 1]$, $c = 0$, $P = 1$ and $k_1 = k_2 = \frac{K}{2}$. The two solid lines show (expected) equilibrium prices for different realizations of demand for the two auction formats when $K = 1$. In both formats, price equals $c = 0$ when $\theta \leq \widehat{\theta} = 0.5$. When $\theta > \widehat{\theta}$, price equals $P = 1$ in the uniform auction, whereas it increases gradually with demand in the discriminatory format. The thin lines show the corresponding prices for the case $K = 1.2$, in which the critical threshold is now $\widehat{\theta} = 0.6$. Whereas the increase in the relative incidence of low-demand realizations is the same in both auction formats, the

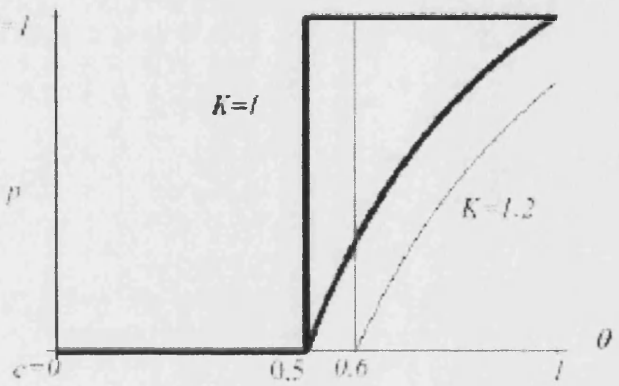

Figure 1: Expected Equilibrium Prices for Different Demand Realizations

effects on prices differ: in the uniform auction, prices jump from $P = 1$ to $c = 0$ for some demand realizations; in the discriminatory auction, the effect on prices is smoother but applies to a wider range of demand realizations.

Because of this fundamental differences in the way in which the equilibrium outcomes are affected, it is not possible in general to specify how a change in a particular parameter affects the relative performance of the two auction formats. In particular, changes in relative performance depend critically upon the distribution of demand G. In order to illustrate the possible effects, as well as the potential order of magnitudes involved, we proceed by considering a series of numerical examples. We maintain the parametrization introduced above, with the added assumption that $G(\theta) = \theta$, and define $k_1 + k_2 = K \geq 1$, with $k_1 \geq k_2$. Then expected payments to suppliers taken over all possible demand realizations (which are equal to expected profits in this case), become $ER^d = \frac{K}{2}\frac{[1-k_2]^2}{k_1}$ and $ER^u = \frac{1}{2}[1-k_2][1+k_2]$, respectively. Table 1 presents numerical results for different values of total installed capacity K for the case in which individual capacities are symmetric, i.e. $k_1 = k_2 = \frac{K}{2}$:

At $K = 1$, total expected payments are 33% lower in the discriminatory

K	1	1.2	1.4	1.6	1.8	2
ER^d	.250	.160	.090	.040	.010	0
ER^u	.375	.320	.255	.180	.095	0
$\frac{ER^d}{ER^u}$	.667	.500	.353	.222	.105	na

Table 1: Increasing Installed Capacity

auction. In the uniform auction, a similar reduction in average prices would require an excess capacity of 40% (i.e. $K = 1.4$).[42] In both auctions, increasing the size of the players reduces both average prices and revenues. The pro-competitive effect on bidding strategies in the discriminatory auction is strong enough in this example so that its relative performance improves the higher is the capacity margin.

In Table 2 we present results for different distributions of a given total capacity $K = 1$.

k_1	.5	.6	.7	.8	.9	1
k_2	.5	.4	.3	.2	.1	0
ER^d	.250	.300	.350	.400	.450	.5
ER^u	.375	.420	.455	.480	.495	.5
$\frac{ER^d}{ER^u}$	.667	.714	.769	.833	.909	1

Table 2: Increasing Capacity Asymmetry

A more asymmetric distribution of capacities implies poorer performance in both types of auction, although the effect is stronger in the discriminatory auction. Reducing the size of the smaller supplier increases the incidence of high-demand states. In the discriminatory auction, the larger supplier faces a larger residual demand and hence has more to gain from submitting higher offer prices. Given this, the smaller supplier responds by increasing its offer prices also. Overall the result is that reallocating capacity from the larger to the smaller supplier (e.g. via capacity divestitures) improves the relative performance of the discriminatory auction over the uniform auction.

[42] Since in both auctions the level of demand served in equilibrium is fixed at θ, expected revenues can be taken as a proxy for the expected (average) price paid by consumers.

Finally, we consider how changes to the market reserve price P affect performance in the two auctions. Using the same example, we fix total capacity so $K = 1$ and consider symmetric firms, i.e. $k_1 = k_2 = 0.5$.[43] Table 3 below presents the numerical results. Reducing the market reserve price reduces equilibrium price (and hence revenues) in both types of auction without affecting the comparison of their relative performance. This is because equilibrium revenues are proportional to the reserve price P in both auctions when $c = 0$.

P	1	.9	0.75	.5	.25	0
ER^d	.250	.225	.188	.125	.063	0
ER^u	.375	.334	.281	.188	.094	0
$\frac{ER^d}{ER^u}$	.667	.667	.667	.667	.667	na

Table 3: Reducing the Market Reserve Price

4.4 Extensions and Variations

In the preceding sections we have analyzed electricity auctions for an asymmetric duopoly assuming that each supplier could submit only a single offer price for its entire capacity, and that demand was both known with certainty at the time offer prices were submitted and perfectly inelastic. In the following subsections we relax each of these assumptions in turn.

4.4.1 Multiple bids

We first extend the analysis by allowing suppliers to submit upward-sloping step offer-price functions instead of constraining them to submit a single bid for their entire capacity. An offer-price function for supplier i, $i = 1, 2$, is then a set of price-quantity pairs (b_{in}, k_{in}), $n = 1, ..., N_i$, $N_i < \infty$. For each pair, the offer price b_{in} specifies the minimum price for the corresponding capacity increment k_{in}, where $b_{in} \in [0, P]$ and $\sum_{n=1}^{N_i} k_{in} = k_i$, $i = 1, 2$. The following lemma states that the *equilibrium outcomes* - but not

[43] This implies that the incidence of high versus low demand states is unaffected by changes in the market reserve price P in this example.

the equilibrium *pricing strategies* - are essentially independent of the number of admissible steps in each supplier's bid function (and whether the 'step sizes' are choice variables for suppliers). This implies that our comparisons between auction types remain valid in this setting.

Lemma 4.2. (Multiple-unit suppliers) (i) Uniform auction: the set of (pure-strategy) equilibrium outcomes is independent of the number of steps in each supplier's bid function (in particular, whether $N_i = 1$ or $N_i > 1$). (ii) Discriminatory auction: for low-demand realizations, there is a unique equilibrium outcome independent of the number of units per supplier. For high-demand realizations, there exists a set of mixed strategies that constitute an equilibrium independently of the number of units per supplier; when $N_1 = N_2 = 1$, these strategies constitute the unique equilibrium.[44]

The existence of a unique, competitive equilibrium outcome in the uniform auction is in stark contrast to analyses which assume continuously differentiable bid functions, i.e. $N_i = \infty$. As first shown by Wilson (1979), and further developed by Back and Zender (1993) and Wang and Zender (2002), in the uniform auction with continuous supply functions there exists a continuum of pure-strategy equilibria, some of which result in very low revenues for the auctioneer (or high payments to suppliers in procurement auctions). The latter are characterized by participants offering very steep supply functions which inhibit competition at the margin: faced with a rival's steep supply function, a supplier's incentive to price more aggressively is offset by the large decrease in price (the 'price effect') that is required to capture an increment in output (the 'quantity effect'). Since the 'price effect' always outweighs the 'quantity effect' for units of *infinitesimal size*, extremely collusive-like equilibria can be supported in the continuous uniform auction, even in a one-shot game.[45]

[44] The equilibrium offer-price functions, however, do depend upon the number of units or admissible bids. For instance, there can be payoff-irrelevant units which are offered at higher prices so long as sufficiently many units are priced at marginal cost.

[45] This type of equilibrium cannot be supported in a discriminatory auction. Klemperer

Discreteness of the bid functions rules out such equilibria however. When suppliers are limited to a finite number of price-quantity bids, a positive increment in output can always be obtained by just slightly undercutting the price of a rival's unit. Since the 'price effect' no longer outweighs the 'quantity effect', the collusive-like equilibria found in the continuous auction cannot be implemented. This observation casts some doubt on the relevance of applying the continuous share auction model to electricity markets in which participants are limited to a small number of offer prices per generating unit. The collusive-like equilibria obtained under the assumption that bid functions are continuous do not generalize to models in which offer increments are of positive size, no matter how small these are (see also Kremer and Nyborg, 2004). We conclude that the equilibrium outcomes for the two types of auction are independent of the number of admissible steps in the offer-price functions, so as long as this number is finite. Hence the characterization of the equilibrium outcomes provided in Proposition 1 would remain unchanged if we had instead assumed that suppliers submit offer-price functions rather than a single offer price for their whole capacity.

It is tempting to draw the conclusion that limiting the number of allowable bids in a uniform-price electricity auction would therefore improve market performance. Strictly speaking, our analysis does not support such a conclusion. What we have shown is that (i) moving from a continuous to a discrete-bid auction potentially improves market performance by eliminating the 'collusive-like' equilibria in the uniform auction, but (ii) market performance in a discrete-bid auction is independent of the number of allowable bids, so long as this number is finite. It could be argued, however, that since limiting the number of bids does not effectively restrict agents' opportunities, it might be desirable in the interests of market simplicity and transparency. Indeed, in equilibrium players may optimally choose not to differentiate their bids even when they are able to do so.

(2002) provides a particularly clear discussion.

4.4.2 *Price-elastic demand*

Our next variation on the basic duopoly model considers the case of price-elastic demand. For this purpose we let the market demand function be represented by $D(p,\theta)$, which is assumed to satisfy the following standard assumptions: as a function of p, D is continuous and bounded; there exists a price $\overline{p}(\theta) > 0$ such that $D(p,\theta) = 0$ if and only if $p \geq \overline{p}(\theta)$; D is decreasing in p, $\forall p \in [0, \overline{p}(\theta)]$; and pD is concave in p, $\forall p \in [0, \overline{p}(\theta)]$.

Given a downward-sloping demand function, in either auction format the output allocated to supplier i, $q_i(\mathbf{b},\theta)$, as a function of the offer price profile $\mathbf{b} = (b_i, b_j)$, becomes:

$$q_i(\mathbf{b},\theta) = \begin{cases} \min\{D(b_i,\theta), k_i\} & \text{if} \quad b_i < b_j \\ \rho_i \min\{D(b_i,\theta), k_i\} \\ +\rho_j \min\{\max\{0, D(b_i,\theta) - k_j\}, k_i\} & \text{if} \quad b_i = b_j \\ \min\{\max\{0, D(b_i,\theta) - k_j\}, k_i\} & \text{if} \quad b_i > b_j, \end{cases}$$

for $i = 1,2$. Note that independently of the payments made to suppliers in either auction format, it is implicitly assumed that consumers are charged the market-clearing price, i.e. the highest accepted offer price. Obviously, this leads to the market (auctioneer) running surpluses in the discriminatory auction. Assuming that such surpluses are dealt with via lump-sum transfers, total surplus (i.e. the sum of supplier profits and consumer surplus) will be determined solely by the market-clearing price and the allocation of output between suppliers.

From the above assumptions it follows that market demand is a continuous and decreasing function of price and that, whenever $D(c_i) > k_j$, $j \neq i$, there exists a unique price p_i^r that maximizes a supplier's profits from serving the residual demand, i.e. $p_i^r(\theta) = \arg\max_p \{p \min[D(p,\theta) - k_j, k_i]\}$. The price p_i^r will be referred to as the 'residual monopoly price' of supplier i.

We further assume that the parameter θ defines a family of demand functions such that if $\theta_1 < \theta_2$, $D(p,\theta_1) < D(p,\theta_2)$. Intuitively, θ is a shift parameter that affects the position, but not the slope, of the demand function (at least not to the extent that demand functions corresponding to different θ's cross). It follows that $p_i^r(\theta)$ is increasing in θ.

Let $P_i^r = \min\{p_i^r, P\}$ be the effective residual monopoly price of supplier i. Then it should be clear that the argument of Lemma 1 goes through as before, with P_1^r and P_2^r substituted for P. Furthermore, we can extend the result of Proposition 1 that there exists a unique threshold $\widehat{\theta}$ such that equilibrium outcomes are of the low-demand and high-demand type, respectively, depending upon whether the shift parameter θ is below or above the threshold. The performance comparison across auction formats is also essentially the same, with the following caveat: since the consumer price is generally lower in the discriminatory auction there is an allocative efficiency gain due to the corresponding increase in consumption.

Our main purpose of this section, however, is to relate the critical threshold $\widehat{\theta}$ to the price elasticity of demand. To this end we use the following definition: for two demand functions D^1 and D^2 with $D^1(p,\theta) = D^2(p,\theta)$ at $p = c$, the demand function D^1 is said to be more elastic than the demand function D^2 if $D^1(p,\theta) < D^2(p,\theta)$ for all $p \geq c$. If we let p_i^{rt} denote the residual monopoly price of supplier i corresponding to the demand function D^t, it follows that $p_i^{r1} < p_i^{r2}$ if D^1 is more elastic than D^2. The following result is then immediate:

Proposition 4.3. The critical threshold $\widehat{\theta}$ is non-decreasing in the elasticity of the demand function D.

In other words, the price elasticity of demand affects market performance in two distinct ways. First, given a high-demand realization, the distortion due to the exercise of market power is smaller when demand is more price-elastic (i.e. the residual monopoly price is lower). Second, the incidence of high-demand realizations is reduced the more elastic is the demand curve. With a downward-sloping demand function, the gain from exercising market power relative to residual demand is less and hence there is more incentive to compete for market share by undercutting the rival, leading to a higher incidence of competitive outcomes.

We conclude this section by considering a numerical example. We maintain the assumptions introduced in the example considered in Section 4.3 above - with $k_1 = k_2 = k$ - and in addition assume that $D(p, \theta) = \theta - \beta p$. It follows that $\widehat{\theta} = k$ and that (for β sufficiently small) $P_1^r = P_2^r = \frac{\theta - k}{2\beta}$ for $\theta < k + 2\beta$ and $P_1^r = P_2^r = P = 1$ otherwise. Expected payments to suppliers become $ER^d = \int_k^{k+2\beta} \frac{1}{2\beta} [\theta - k]^2 \, d\theta + 2 \int_{k+2\beta}^1 [\theta - \beta - k] \, d\theta$ and $ER^u = \int_k^{k+2\beta} \frac{1}{4\beta} [\theta - k] [\theta + k] \, d\theta + \int_{k+2\beta}^1 [\theta - \beta] \, d\theta$, respectively. In Table 4 we present results for different values of the slope of the demand function:[46]

β	0	.025	.05	.075	.100	.125	.15
ER^d	.250	.226	.203	.183	.163	.146	.130
ER^u	.375	.350	.327	.304	.282	.260	.240
$\frac{ER^d}{ER^u}$	.667	.646	.621	.602	.578	.562	.542

Table 4: Increasing the Elasticity of Demand

As expected, a more elastic demand reduces payments to suppliers. In this example, the relative incidence of low-demand and high-demand states $(\widehat{\theta})$ is not affected, although more elastic demand does reduce the effective residual monopoly price. In the discriminatory auction we have the additional effect that bidding becomes more aggressive in high-demand states. Consequently, the relative performance of the discriminatory auction increases with the elasticity of demand here.[47,48]

4.4.3 *Oligopoly*

Our next variation on the basic duopoly model considers the case of oligopoly. This allows us to generalize some of the insights from the duopoly model as well as analyze the impact of changes in the number of suppliers

[46] Note that, for β sufficiently small, β approximates the price elasticity of demand at the peak (i.e. $\theta = 1$) evaluated at the maximum admissible price $P = 1$.

[47] As pointed out above, the revenue comparison tends to understate the performance of the discriminatory auction relative to that of the uniform auction as far as consumer prices (and, indeed, consumer surplus) is concerned.

[48] The difference in total payments between the two auction formats in the case of perfectly inelastic demand ($\beta = 0$) corresponds to the difference between the cases $\beta = 0$ and $\beta = 0.15$ in the uniform auction.

on profits and pricing behavior.

Accordingly we now consider S suppliers, where k_s is the capacity and c_s the marginal cost of supplier s, $s = 1, 2, ..., S$. Suppliers are ordered by efficiency, so that $0 = c_1 \leq c_2 \leq ... \leq c_S = c$. As before, the types of equilibria which arise in the different auction formats depend upon the value of the market demand θ relative to suppliers' individual and aggregate capacities. In particular, we have the following result:

Proposition 4.4 There exists $\widehat{\theta_s^-}$ and $\widehat{\theta_s^+}$, $\widehat{\theta_s^-} \leq \widehat{\theta_s^+}$, such that, for $s = 1, 2, ..., S$,

(i) if $\theta \leq \widehat{\theta_s^-}$, in any equilibrium the highest accepted price offer is at or below c_s;

(ii) if $\theta > \widehat{\theta_s^+}$, in any equilibrium suppliers are paid prices that are at least equal to c_s and strictly above c_s if $s = S$ or $c_s < c_{s+1}$, $s = 1, 2, ..., S-1$;

(iii) $\theta_s^- = \widehat{\theta_s^+} = \widehat{\theta}_s$ if $k_s \geq \max_{i<S} k_i$.

In other words, we have a series of demand threshold pairs, each pair corresponding to the cost of a particular supplier. When demand is below the lower of these two thresholds, equilibrium prices are limited by the cost of the corresponding supplier; when demand is above the upper threshold, equilibrium prices always exceed the cost of that same supplier. A sufficient condition for the two thresholds to be equal is that the capacity of the corresponding supplier is at least as large as that of any more efficient supplier.

To demonstrate that the two thresholds may in fact differ, and hence that there may be a range of demand outcomes for which competitive and non-competitive equilibria coexist, consider the following example. Let $S = 3$, $c_1 = 0$, $c_2 = 0.5$, $c_3 = 1$, $k_1 = 1$, $k_2 = 1$, and $k_3 = 0.25$. Furthermore, let $P = 1.75$ and $\theta = 1.5$. Then it is easily verified that the following equilibria exist in the uniform auction: $\{b_1 = 1, b_2 = 0.5, b_3 = 1\}$ and $\{b_1 = 0, b_2 = 1.75, b_3 = 1\}$. Note that the first of these equilibria is competitive in the sense that price is limited by the cost of the inefficient supplier, whereas the second equilibrium is not. Note further that the both equilibria are inefficient in the sense that overall generation costs are not minimized: in particular, when the market outcome is competitive, inefficient dispatch nevertheless results.

In the discriminatory auction, no pure-strategy equilibria exists so long as $\theta > \widehat{\theta_1^-}$. To see this, note that in any equilibrium in which more than one supplier is despatched, profits of lower-pricing suppliers are strictly increasing in their offer prices below the offer price of the marginal supplier. Furthermore, for the marginal supplier, undercutting is always profitable so long as competing offer prices are sufficiently close. These opposing forces destroy any candidate pure-strategy equilibrium. We consequently have a similar dichotomy to that observed in the duopoly case, in which the comparison of outcomes between the two auction formats generally depends on which equilibrium is played in the uniform auction.

We end this section by considering the relationship between market structure and market performance. We take as our starting point a generalization of the 'two-state' result of the duopoly section, which follows as a corollary of the above equilibrium characterization:

Corollary 4.1. There exists $\widehat{\theta}^-$ and $\widehat{\theta}^+$, $\widehat{\theta}^- \leq \widehat{\theta}^+$, such that

(i) (low demand) if $\theta \leq \widehat{\theta}^-$, in any equilibrium the highest accepted price offer is at or below c;

(ii) (high demand) if $\theta > \widehat{\theta}^+$, in any equilibrium suppliers are paid prices that exceed c;

(iii) $\widehat{\theta}^- = \widehat{\theta}^+ = \widehat{\theta}$ if $k_S \geq \max_{j<S} k_s$.

In low-demand realizations prices are limited by costs, whereas in high-demand realizations they are not. Low-demand equilibria are competitive in the sense that prices are limited by the cost of less efficient, non-despatched suppliers. However, unlike in the duopoly case, low-demand equilibria are not necessarily cost efficient. In the uniform auction there may exist pure-strategy equilibria in which less efficient suppliers are ranked before more efficient suppliers, while in the mixed-strategy equilibria of the discriminatory auction such outcomes occur with positive probability.

To highlight the relationship between market concentration and performance, we focus on the symmetric case, in which we readily obtain the following result that corresponds directly with the results obtained in the duopoly case:

Proposition 4.5. In the oligopoly model with symmetric suppliers, in particular, $k_s = \frac{K}{S}$, $s = 1, 2, ..., S$:

(i) (low demand) if $\theta \leq \widehat{\theta} = \frac{S-1}{S}K$, $R^d = R^u = 0$.

(ii) (high demand) if $\theta > \widehat{\theta} = \frac{S-1}{S}K$, $R^d = PS\left[\theta - \frac{S-1}{S}K\right] < P\theta = R^u$.

Market structure affects equilibrium outcomes differently in the two auction formats. In both formats, the threshold that determines whether demand is 'low' or 'high' is increasing in the number of suppliers. In other words, pricing at marginal cost is more likely in a more fragmented industry. However, in the discriminatory auction (as opposed to the uniform auction), market structure also affects bidding strategies in high-demand realizations. In the discriminatory auction suppliers play symmetric mixed strategies, and in equilibrium these strategies strike a balance between a 'price' and a 'quantity' effect: lowering the price offer reduces the price received, but increases the likelihood of undercutting rivals and hence gaining a larger market share. For a given level of demand, the 'quantity effect' is more important the larger is the number of competitors. Hence in the discriminatory auction price competition will be more intense the less concentrated is the market structure.

To illustrate the above points, we again consider the numerical example introduced above, with the specification that $k_s = \frac{K}{S}$ with $K = 1$ and $c_s = 0$, $s = 1, 2, ..., S$. Expected payments to suppliers become $ER^d = \frac{1}{2S}$ and $ER^u = \frac{2S-1}{2S^2}$, respectively. Numerical values for different numbers of suppliers are given in the following table:

A more fragmented industry structure improves the performance of both

Table 5: Increasing the Number of Suppliers

S	2	3	4	5	10	100	∞
ER^d	.250	.167	.125	.100	.050	.005	0
ER^U	.375	.278	.219	.180	.095	.010	0
$\frac{ER^d}{ER^u}$	.667	.600	.571	.556	.526	.503	.5

auctions, as well as the relative performance of the discriminatory auction in this example. For a given number of suppliers, the difference in payments between the two auctions roughly corresponds to the effect of doubling the number of suppliers in the uniform auction.

4.4.4 Long-lived bids

Our final variation on the basic duopoly model considers the case in which suppliers face time-varying, or stochastic, demand. This is of particular relevance to electricity markets in which suppliers submit offer-prices that remain fixed for twenty-four or forty-eight market periods, such as in Australia and the original market in England and Wales. We therefore assume here that price offers must be made before the realization of demand (i.e. θ) is known. It is easy to verify that our previous analysis is robust to this change in the timing of decisions so long as the largest possible demand realization is low enough, or the lowest possible demand realization is large enough. For instance, when demand never exceeds the critical threshold $\widehat{\theta}$ defined in Proposition 1 equilibria correspond to those analyzed for low-demand realizations. The introduction of demand variability adds a new dimension to the problem only when *both* low and high demand realizations occur with positive probability. We therefore assume that demand θ takes values in the support $[\underline{\theta}, \overline{\theta}] \subseteq (0, k_1 + k_2)$, with $\underline{\theta} < \widehat{\theta} < \overline{\theta}$, according to some (commonly known) distribution function $G(\theta)$.

The equilibria of both the uniform and discriminatory auctions now differ significantly from the case in which demand is known with certainty before bids are submitted. Demand uncertainty, or variability, upsets all candidate pure-strategy equilibria in both types of auction (see von der Fehr and Harbord, 1993 and García-Díaz, 2000). We therefore consider equilibria in mixed strategies. For both the uniform and discriminatory auctions there exist unique mixed-strategy equilibria, and it is possible to derive explicit formulae for the suppliers' strategies:[49]

Lemma 4.3. Assume $[\underline{\theta}, \overline{\theta}] \subseteq (0, k_1 + k_2)$, with $\underline{\theta} < \widehat{\theta} < \overline{\theta}$. Then there does not exist an equilibrium in pure strategies in either auction. In the unique mixed-strategy equilibrium suppliers submit bids that strictly exceed c.

In a mixed-strategy equilibrium in either type of auction, suppliers must strike a balance between two opposing effects: on the one hand, a higher offer price tends to result in higher equilibrium prices; on the other hand, pricing high reduces each suppliers' expected output, *ceteris paribus*. The first effect is less pronounced in the uniform auction than in the discriminatory auction. In the uniform auction, a higher offer price translates into a higher market price only in the event that the offer price is marginal, while in the discriminatory auction pricing higher always results in the supplier increasing the expected price it receives, conditional on being despatched. Consequently, there is a tendency for suppliers to price less aggressively in the discriminatory auction compared to a uniform auction. This intuition is confirmed in the symmetric case (i.e. when $k_1 = k_2 = k$ and $c_1 = c_2 = 0$), in which the equilibrium mixed-strategy distribution function in the discriminatory auction first-order stochastically dominates the corresponding distribution function in the uniform auction, i.e. $F_i^u(b) \geq F_i^d(b)$.[50]

We have not been able to characterize in detail the relationship between the model parameters and suppliers' equilibrium strategies in the general

[49] We are only able to characterize the mixed-strategy equilibria with long-lived bids by restricting attention to single-unit suppliers. See Anwar (1999) who shows that the equilibria derived under this assumption may not survive when we allow more complicated bidding strategies to be used.

[50] The result follows from the observation that $F_i^u(b) < F_i^d(b)$ implies $\pi_i^u > \pi_i^d$, whereas in the symmetric case, $\pi_i^u = \pi_i^d$.

case. In the case of symmetric capacities, however, we can show that in the limit, as $\overline{\theta} \to k$ (or $k \to \overline{\theta}$), so that demand is always less than the capacity of a single supplier, the mixed-strategy equilibrium outcome in either auction approaches the equilibrium outcome for a low-demand realization, with price equal to the marginal cost of the higher-cost supplier. Similarly, as $\underline{\theta} \to k$ (or $k \to \underline{\theta}$), so that demand always exceeds the capacity of a single supplier, the equilibrium outcomes approach those for a high-demand realization. Further, in the uniform auction the limiting equilibrium outcome is efficient, i.e. the more efficient supplier produces at capacity and the less efficient supplier supplies the residual demand. This is in contrast to the model with non-stochastic demand, in which there exist both efficient and inefficient pure-strategy equilibria in high-demand realizations in the uniform auction.[51] This suggests that the uniform auction performs better in efficiency terms than the discriminatory auction, although we have not been able to demonstrate that this result holds generally. Revenue comparisons also prove difficult, except in the symmetric case, where it is easily demonstrated that (in expected terms) total payments to suppliers are the same in both auction formats.

We end this section by comparing market performance under short-lived and long-lived bids, respectively. This comparison is difficult in the general case and hence we limit our attention to the symmetric case. Let ER_s^f and ER_l^f denote expected total supplier payments in auction format $f = d, u$ in the case of short-lived and long-lived bids, respectively. We obtain the following result:

Proposition 4.6. In the symmetric duopoly model, $ER_l^u < ER_s^u$, $ER_l^d = ER_s^d$ and $ER_l^u = ER_l^d$.

In other words, while there is no difference in the discriminatory auction, in the uniform auction long-lived bids outperform short-lived bids. With

[51] The fact that with uncertain demand the efficient outcome is unique might be viewed as a justification for treating this as a natural 'focal point' in the certain-demand case also.

short-lived bids, the poor performance of the uniform auction is caused by the extreme equilibrium outcome for high-demand realizations, in which suppliers are paid the market reserve price. This equilibrium is supported by the inframarginal supplier bidding sufficiently low so as to discourage undercutting by the high-bidding, price-setting supplier. With long-lived bids, however, the low-bidding supplier determines the market price in low-demand realizations, and hence has an incentive to increase its offer price. As a result, incentives for undercutting and competing for market share are increased, leading to more aggressive bidding and lower prices overall in the uniform auction.

4.5 Conclusions

In this paper we have characterized equilibrium pricing behavior in uniform and discriminatory auctions in a multi-unit auction model reflecting some key features of decentralized electricity markets. Equilibria in the two auction formats have been compared in terms of both average prices paid to suppliers and productive efficiency. In the case of certain demand (i.e. short-lived bids), we found that uniform auctions yield higher average prices than discriminatory auctions. Comparison of the auctions in terms of productive efficiency is more complex, however, as it depends on which equilibrium is played in the uniform auction as well as on parameter values. When demand is uncertain (or bids are long-lived), at least in the perfectly symmetric case, expected payments to suppliers are the same in both auction formats.

Our theoretical model is obviously highly stylized, and while it does lead to a number of qualitative results, it does not allow us to draw conclusions about their quantitative importance. Nevertheless, numerical examples suggest that some of the effects identified may be significant. For example, moving from a uniform to a discriminatory auction format in the certain demand case may have a similar effect on average prices to either a doubling of the number of suppliers or increasing the capacity of two symmetric duopolists by almost 40%. However, under the restrictive assumption that firms are symmetric,[52] moving from a uniform auction with long-lived bids (as in the original England and Wales market) to a discriminatory auction with short-lived bids (as under NETA) has no impact on expected prices. This suggests that reduced market concentration and increased total capacity may have been as responsible for the initial reduction in England and Wales wholesale electricity prices in 2001/2 as any change in the market design, although our model is obviously too specialized to decide this issue.

A key determinant of market performance in our analysis is the relative incidence of low-demand and high-demand states, and this does not depend upon the auction format. Rather, it depends on other market design issues and on structural features of the market. In particular, the incidence of high-demand states is lower when there is more excess capacity in the industry, the market structure is more fragmented, suppliers have symmetric capacities, demand is price elastic and the market reserve price is low. These factors affect not only the relative incidence of low and high-demand states, but may also influence bidding strategies. Changes in total capacity, the capacity distribution and market structure (i.e. 'structural factors') have no effect on prices in the uniform auction *in high-demand states*, but can lead to more vigorous price competition in the discriminatory auction. Regulatory interventions to change the market rules, on the other hand, affect bidding strategies in both types of auction. A reduction in the market reserve price reduces average market prices in both auctions. Measures that increase the elasticity of demand (e.g. the introduction of demand-side bidding) have similar effects. A change from short-lived to long-lived bids, however, which makes the demand state uncertain when suppliers' submit their bids, may have a greater effect on prices in the uniform auction.

Our analysis allows us to make the following comments on regulatory policy with respect to the design of electricity auctions:

- *Auction format* The uniform auction is always weakly outperformed

[52] And assuming that the support of the demand distribution includes both high and low demand realizations.

by the discriminatory auction with respect to total revenues in our set-up. Thus our analysis suggests that a regulator who is only concerned with the minimization prices should prefer the discriminatory format. However, if the regulator assigns positive weights to both productive efficiency and consumer surplus, the auction ranking will depend on the specific weights assigned to each, and on industry data.

- *Bid format* Long-lived bids outperform short-lived bids in the uniform auction. In particular, bids that cover a whole day or longer periods lead to lower average prices than bids which vary hourly or half-hourly. There is no corresponding effect in the discriminatory action. However, in both types of auction, a single-bid format performs as well as formats in which suppliers are allowed to make multiple bids (e.g. different bids for equal-cost capacity units). Our analysis therefore provides some support for the view that simplifying bid formats - both with regard to duration and structure - is likely to improve market performance.

- *Market reserve price* Reserve prices, or price caps, in most electricity markets are intended to reduce the incidence of high price spikes. A lower market reserve price obviously affects prices in events in which the price cap binds. However, it also affects prices indirectly via its effect on competition, i.e. by reducing the number of high-demand periods and intensifying competition in high-demand periods in the discriminatory auction.[53]

- *Demand-side measures* Measures to stimulate the price responsiveness of demand directly improve allocative efficiency and increase supply security. They also result in more competition via similar effects to those achieved by reducing the market reserve price.

[53] An important caveat is that we are only considering short-run comparative static effects, and ignoring longer run investment or entry incentives. In particular, price caps may deter investment in peaking capacity, which in some power systems is a major problem.

From a methodological point of view, the paper has also contributed to the analysis of multi-unit electricity auctions in a number of ways.[54] First, we have shown that the set of equilibrium outcomes in uniform and discriminatory auctions with short-lived bids is essentially independent of the number of admissible steps in suppliers' offer-price functions, so as long as this number is finite. This reduces the complexity involved in the analysis of multi-unit auctions as it allows us to focus on the single-unit case with no significant loss in generality. Secondly, we have demonstrated that the 'implicitly collusive' equilibria found in the uniform auction when offer prices are infinitely divisible are unique to this formulation of the auction (i.e. to share auctions), and do not arise when offer-price functions are discrete. Hence the concerns expressed in the literature that uniform auctions may lead to 'collusive-like' outcomes even in potentially competitive periods when there is considerable excess capacity, are likely misplaced.[55]

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4.6 Appendix: Proofs

Proof of Lemma 4.1

Let p denote the highest accepted price offer and let $b_i = p$. Clearly, we must have $p \geq c_i$. Let $c_p = \max_{c_j \leq p} c_j$ and $c^p = \{\min_{c_j > p} c_j$ if $p < c$; and P otherwise$\}$. Suppose $p > c_p$. Then, for $j \neq i$ with $c_j < p$, we must have $b_j \leq p$ (with strict inequality if $c_j = c_i$) since otherwise supplier j could gain by matching (undercutting) b_i But then i's profit is strictly increasing in b_i on $[p, c^p]$, proving the first part of the result. Lastly, in the discriminatory auction, in a pure-strategy equilibrium we cannot have $b_j < p$, given that supplier j's profit is strictly increasing in b_j up to p. *Q.E.D.*

Proof of Proposition 4.1

Consider first the possibility of a pure-strategy equilibrium in which the highest accepted offer price equals c. Profits to Supplier i are given by $[c - c_i] \min\{\theta - K_{i-1}, k_i\}$, where $K_i = \sum_{j=1}^{i} k_j$, $i = 1, 2$ and $K_0 = 0$, while the profits from deviating to a higher price is at most $[P - c_i] \max\{\theta - K_{-i}, 0\}$, where $K_{-i} = \sum_{j \neq i} k_j$. A necessary (and, indeed, sufficient) condition for such an equilibrium to exist consequently is $[c - c_i] \min\{\theta - K_{i-1}, k_i\} - [P - c_i] \max\{\theta - K_{-i}, 0\} \geq 0$. Given that, for $\theta \geq K_{-i}$, the left-hand side of this expression is non-increasing in θ, there exists a unique $\widehat{\theta}_i$ such that the condition is satisfied iff $\theta \leq \widehat{\theta}_i$. Existence of the equilibrium then requires $\theta \leq \min \widehat{\theta}_i \equiv \widehat{\theta}$.

Consider next the possibility of an equilibrium in which supplier i submits the highest accepted price offer $b_i = P$. Clearly, for such an equilibrium to exist we must have $\theta - K_{-i} > 0$. By the argument in the proof of Lemma 1, it follows that i's equilibrium profits are $[P - c_i][\theta - K_{-i}]$. Obviously, any profitable deviation by i would involve undercutting the competitor so as to increase output (with a consequent fall in price). If the competitor prices at cost, the maximum gain from undercutting is given by $[c_j - c_i] \min\{\theta - K_{i-1}, k_i\}$ when $\theta \in (K_{j-1}, K_j]$. Consequently, a necessary condition for such an equilibrium to exist is that $[P - c_i][\theta - K_{-i}] -$

$[c_j - c_i] \min\{\theta - K_{i-1}, k_i\} \geq 0$. By the monotonicity of the left-hand side of the condition, it follows that the condition is satisfied iff $\theta \geq \widehat{\theta}_i$, implying that a monopolistic pure-strategy equilibrium can exist only if $\theta \geq \widehat{\theta}$.

The existence of a monopolistic pure-strategy equilibrium in the uniform auction when $\theta \geq \widehat{\theta}_i$ for some i is straightforward and involves Supplier i pricing at P while the competitor prices sufficiently low so as to make undercutting by i unprofitable. In the discriminatory auction, by the result in Lemma 1 that in a pure-strategy equilibrium all accepted units are offered at the same price, it follows that there cannot exist an equilibrium in which accepted price offers exceed c, since then at least one supplier could increase output by (marginally) undercutting its competitor. When $\theta \geq \widehat{\theta}_i$, Supplier i's rival knows that a price offer of c being undercut is a probability-zero event, and hence will surely price above c also.

For further reference, we register the following results. Noting that we must have $\widehat{\theta}_1 \geq k_2$, $\widehat{\theta}_1$ is implicitly defined by the equation $c \min\left\{\widehat{\theta}_1, k_1\right\} = P\left[\widehat{\theta}_1 - k_2\right]$. It follows that $\widehat{\theta}_1 = \frac{P}{P-c}k_2$ if $\widehat{\theta}_1 \leq k_1$ and $\widehat{\theta}_1 = k_2 + \frac{c}{P}k_1$ if $\widehat{\theta}_1 > k_1$. This may alternatively be stated as $\widehat{\theta}_1 = \frac{P}{P-c}k_2$ if $\frac{P}{P-c}k_2 \leq k_1$ and $\widehat{\theta}_1 = k_2 + \frac{c}{P}k_1$ otherwise. Similar reasoning leads to the result that $\widehat{\theta}_2 = k_1$. Consequently, $\widehat{\theta} = \frac{P}{P-c}k_2$ if $\frac{P}{P-c}k_2 \leq k_1$ and $\widehat{\theta} = k_1$ otherwise. *Q.E.D.*

Mixed-strategy equilibria in the basic model

We now characterize mixed-strategy equilibria in both auction formats. We first consider the existence of mixed-strategy equilibria in low-demand realizations (i.e. for $\theta < \widehat{\theta} = \min\left\{\widehat{\theta}_1, \widehat{\theta}_2\right\}$) in both auction formats. We then consider the uniform auction for high-demand realizations in which there are multiple pure-strategy equilibria (i.e. $\theta \geq \max\left\{\widehat{\theta}_1, \widehat{\theta}_2\right\}$). Lastly we characterize mixed-strategy equilibria in the discriminatory auction for all high-demand realizations (i.e. $\theta > \widehat{\theta} = \min\left\{\widehat{\theta}_1, \widehat{\theta}_2\right\}$).

Low demand: Both auction formats

Assume $\theta < \widehat{\theta} = \min\left\{\widehat{\theta}_1, \widehat{\theta}_2\right\}$. Let $\underline{b}_i$ and $\overline{b}_i$ denote the infimum and supremum, respectively, of the support of supplier i's strategy. We first note that $\underline{b}_1 = \underline{b}_2 \geq c$. This follows from the facts that $b_i \geq c_i$ and that profits are strictly increasing in the bid whenever it is the lowest. We next observe that supplier i obtains zero profits if $\overline{b}_i > \overline{b}_j$. The same is true if $\underline{b}_1 = \underline{b}_2 < \overline{b}_1 = \overline{b}_2 = \overline{b}$ and either no-one plays $\overline{b}$ with positive probability or, if some player does (there is at most one), this is supplier 2. It follows that at least one player earns zero profits in any mixed-strategy equilibrium. If $c > 0$, this is not supplier 1, who can always guarantee positive profits by bidding below c; so $\overline{b}_1 \leq \overline{b}_2$. Furthermore, if $c > 0$, $\overline{b}_1 = c$, since otherwise supplier 2 could obtain positive profits by undercutting.

Consequently, if $c > 0$, there exist mixed-strategy equilibria in which supplier 1 bids $b_1 = c$ with probability 1 and supplier 2 (who obtains zero profits and is consequently indifferent between any bid at or above c) mixes over the range $[c, b')$ for any $b' \in (c, P)$, according to some strategy $F_2(b) = \Pr(b_2 \leq b)$ that satisfies $F_2(b) \geq 1 - \frac{c}{b}$, so as to deter supplier 1 from raising his bid. Given supplier 2's strategy, supplier 1's profit from bidding $b > c$ is $F_2(b) \cdot 0 + [1 - F_2(b)] \cdot b\theta \leq c\theta$.

Note that the set of mixed-strategy equilibria are the same in both auctions and that outcomes (outputs, profits and costs) are identical to those of the pure-strategy equilibrium. Note further that while the pure-strategy equilibrium involves Supplier 2 playing a weakly-dominated strategy (i.e. bidding at c), in any mixed-strategy equilibrium supplier 2 plays an undominated strategy almost surely.

If $c = 0$, $\min\{\overline{b}_1, \overline{b}_2\} = 0$, since otherwise either supplier could obtain positive profits by undercutting. It follows that there does not exist a mixed-strategy equilibrium in this case.

High Demand: Uniform auction

Assume $\theta \geq \max\left\{\widehat{\theta}_1, \widehat{\theta}_2\right\} = \max\left\{k_1, k_2 + \frac{c}{P}k_1\right\}$. Let $F_i^u(b) = \Pr\{b_i \leq b\}$ denote the equilibrium mixed-strategy of supplier i, $i = 1, 2$, with density $f_i^u(b) = F_i'^u(b)$, and let S_i^u be the support of F_i^u. Furthermore, let $S^u = (\max\{\inf S_1^u, \inf S_2^u\}, \min\{\sup S_1^u, \sup S_2^u\})$. Note first that F_i^u cannot have a mass point on S^u. To see this, suppose, for contradiction, that F_i^u

has a mass point at some $b' \in S^u$. Then, for some interval $[b', b' + \varepsilon)$, $\varepsilon > 0$, i's competitor would be better off by offering to supply at a price just below b' than to offer prices in this interval. But then i's profit would be strictly increasing on $[b', b' + \varepsilon)$, contradicting the assumption that b' is in the support of i's strategy. A similar argument establishes that S_i^u is an interval (i.e. without 'holes'). Furthermore, since P must be in the support of at least one supplier's strategy, we have $S^u = S_1^u \cap S_2^u = (\underline{b}, P)$. We want to demonstrate that any mixed-strategy equilibrium has the form

$$\begin{aligned}
F_1^u(b) &= \begin{cases} A_1 \left[\frac{b-c}{P-c}\right]^{\frac{\theta-k_1}{k_1+k_2-\theta}} & for \quad \underline{b} < b < P \\ 1 & for \quad b = P \end{cases} \\
F_2^u(b) &= \begin{cases} A_2 \left[\frac{b}{P}\right]^{\frac{\theta-k_2}{k_1+k_2-\theta}} & for \quad \underline{b} < b < P \\ 1 & for \quad b = P \end{cases} \\
\underline{b} &= c
\end{aligned}$$

where either (i) $A_1 = 1$ and $0 < A_2 \leq 1$ or (ii) $0 < A_1 \leq 1$ and $A_2 = 1$.

On $(\underline{b}, P)$, strategies must satisfy the following differential equations:

$$\begin{aligned}
F_2^u(b)\,[\theta - k_2] - f_2^u(b)\, b\, [k_1 + k_2 - \theta] &= 0, \\
F_1^u(b)\,[\theta - k_1] - f_1^u(b)\,[b - c]\,[k_1 + k_2 - \theta] &= 0.
\end{aligned}$$

On the interior of the support of the mixed strategies the net gain from raising the bid marginally must be zero. The first elements on the left-hand side of the above expressions represents the gain to a supplier from the resulting increase in the price received in the event that the rival bids below. The second element represents the loss from reducing the chance of being despatched at full capacity instead of serving the residual demand only (the difference being, for supplier i, $k_i - [\theta - k_j] = k_1 + k_2 - \theta$). The above expressions may alternatively be written:

$$\begin{aligned}
f_2^u(b) - \frac{1}{b}\frac{\theta - k_2}{k_1 + k_2 - \theta} F_2^u(b) &= 0, \\
f_1^u(b) - \frac{1}{b - c}\frac{\theta - k_1}{k_1 + k_2 - \theta} F_1^u(b) &= 0,
\end{aligned}$$

and have solutions

$$F_1^u(b) = \widehat{A}_1 [b-c]^{\frac{\theta-k_1}{k_1+k_2-\theta}}$$
$$F_2^u(b) = \widehat{A}_2 b^{\frac{\theta-k_2}{k_1+k_2-\theta}},$$

with $\widehat{A}_i > 0$, $i = 1, 2$.

Since at most one supplier can play P with positive probability (i.e., either $\Pr(b_1 = P) = 0$ or $\Pr(b_2 = P) = 0$), we have either (i) $\lim_{b\to P} F_2^u(b) \leq \lim_{b\to P} F_1^u(b) = 1$, implying $\widehat{A}_1 = \left[\frac{1}{P-c}\right]^{\frac{\theta-k_1}{k_1+k_2-\theta}}$ and $\widehat{A}_2 \leq \left[\frac{1}{P}\right]^{\frac{\theta-k_2}{k_1+k_2-\theta}}$ or (ii) $\lim_{b\to P} F_1^u(b) \leq \lim_{b\to P} F_2^u(b) = 1$, implying $\widehat{A}_1 \leq \left[\frac{1}{P-c}\right]^{\frac{\theta-k1}{k_1+k_2-\theta}}$ and $\widehat{A}_2 = \left[\frac{1}{P}\right]^{\frac{\theta-k_2}{k_1+k_2-\theta}}$.

Note that, because there are no mass points on $(\underline{b}, P)$ and $\lim_{b\to c} F_1^u(b) = 0$, we must have $\underline{b} = c$. Since $\lim_{b\to c} F_2^u(b) = \widehat{A}_2 c^{\frac{\theta-k_2}{k_1+k_2-\theta}} > 0$, while F_2^u cannot have a mass point at c, it follows that for a mixed-strategy equilibrium to exist it must involve, with positive probability, Supplier 2 offering to supply at prices below his own cost (note that this implies that there does not exist a mixed-strategy equilibrium in weakly undominated strategies). The only constraint that $F_2(b)$ must satisfy for $b \leq c$ follows from the condition that undercutting by Supplier 1 must be unprofitable; one solution satisfying this constraint is given by the above first-order condition, but a continuum of other solutions exist as well.

In a mixed-strategy equilibrium profits become:

$$\pi_1^u = P\left\{\Pr(b_2 = P)\, k_1 + [1 - \Pr(b_2 = P)][\theta - k_2]\right\},$$
$$\pi_2^u = [P-c]\left\{\Pr(b_1 = P)\, k_2 + [1 - \Pr(b_1 = P)][\theta - k_1]\right\}.$$

Note that, for the class of equilibria in which $\lim_{b\to P} F_1^u(b) = 1$ (implying that $A_1 = 1$ and $\Pr(b_1 = P) = 0$) total industry profits are maximized in the limiting case $\Pr(b_2 = P) = 1$ (which corresponds to $A_2 = 0$), where $\pi_1^u = Pk_1$ and $\pi_2^u = [P-c][\theta - k_1]$. This is the same as in the corresponding pure-strategy equilibrium in which Supplier 2 is bidding high, implying that profits in this pure-strategy equilibrium dominate those in any mixed-strategy equilibrium. Moreover, industry profits are minimized in the case $\Pr(b_2 = P) = 0$ (which corresponds to $A_2 = 1$), where $\pi_1^u = P[\theta - k_2]$ and $\pi_2^u = [P - c][\theta - k_1]$. Corresponding results hold for the other class of mixed-strategy equilibria.

High Demand: Discriminatory auction

Assume $\theta > \min\left\{\widehat{\theta}_1, \widehat{\theta}_2\right\} = \widehat{\theta}$. From the proof of Proposition 1, there are two cases to consider; $\frac{P}{P-c}k_2 \leq k_1$, in which case $\widehat{\theta} = \frac{P}{P-c}k_2$, and $\frac{P}{P-c}k_2 > k_1$, in which case, $\widehat{\theta} = k_1$.

Let $F_i^d(b) = \Pr\{b_i \leq b\}$ denote the equilibrium mixed strategy of supplier i and let S_i^d be the support of F_i^d. Standard arguments (see above) imply that $S = (\underline{b}, P) \subseteq S_1^d$, $S_2^d \subseteq [\underline{b}, P]$ and that F_i^d and F_j^d do not have mass points on $[\underline{b}, P)$. We want to show that there exists a unique equilibrium with,

$$F_1^d(b) = \begin{cases} \frac{\min\{\theta,k_2\}}{\min\{\theta,k_1\}+\min\{\theta,k_2\}-\theta}\frac{b-\underline{b}}{b-c} & \text{for} \quad b < P \\ 1 & \text{for} \quad b = P \end{cases},$$

$$F_2^d(b) = \begin{cases} \frac{\min\{\theta,k_1\}}{\min\{\theta,k_1\}+\min\{\theta,k_2\}-\theta}\frac{b-\underline{b}}{b} & \text{for} \quad b < P \\ 1 & \text{for} \quad b = P \end{cases},$$

where $\underline{b} = c + [P - c]\frac{\theta - k_1}{\min\{\theta,k_2\}}$ if $Pk_2 > [P - c]k_1$and $\underline{b} = P\frac{\theta - k_2}{\min\{\theta,k_1\}}$ if $Pk_2 \leq [P - c]k_1$ (note that, in both cases, $\underline{b} \geq c$).

Suppliers' profits may be written

$$\begin{aligned} \pi_1^d(b) &= b\left\{F_2^d(b)\max\{\theta - k_2, 0\} + \left[1 - F_2^d(b)\right]\min\{\theta, k_1\}\right\}, \\ \pi_2^d(b) &= [b - c]\left\{F_1^d(b)\max\{\theta - k_1, 0\} + \left[1 - F_1^d(b)\right]\min\{\theta, k_2\}\right\}. \end{aligned}$$

A necessary condition for supplier i to be indifferent between any price in S_i^d is that, for all $b \in S_i^d$, $\pi_i^d(b) = \overline{\pi}_i^d$, implying

$$\begin{aligned} F_1^d(b) &= \frac{[b - c]\min\{\theta, k_2\} - \overline{\pi}_2^d}{[b - c][\min\{\theta, k_1\} + \min\{\theta, k_2\} - \theta]}, \\ F_2^d(b) &= \frac{b\min\{\theta, k_1\} - \overline{\pi}_1^d}{b[\min\{\theta, k_1\} + \min\{\theta, k_2\} - \theta]}, \end{aligned}$$

where we have used the fact that $\max\{\theta - k_i, 0\} = \theta - \min\{\theta, k_i\}$.

Observe that the boundary condition $F_1^d(\underline{b}) = F_2^d(\underline{b}) = 0$ implies

$$\begin{aligned} \pi_1^d &= \underline{b} \min\{\theta, k_1\}, \\ \pi_2^d &= [\underline{b} - c] \min\{\theta, k_2\}. \end{aligned}$$

Furthermore, we have

$$\lim_{b \to P} \left[F_1^d(b) - F_2^d(b) \right] = \frac{P - \underline{b}}{\min\{\theta, k_1\} + \min\{\theta, k_2\} - \theta} \left[\frac{\min\{\theta, k_2\}}{P - c} - \frac{\min\{\theta, k_1\}}{P} \right].$$

If $k_1 < \frac{P}{P-c} k_2$, in which case $\theta > k_1$, we cannot have $\lim_{b \to P} F_2^d(b) = 1$ since this would imply $\lim_{b \to P} F_1^d(b) > 1$. Consequently, we have the boundary condition $\lim_{b \to P} F_1^d(P) = 1$, which implies

$$\pi_2^d = [P - c] [\theta - k_1],$$

and, together with the condition $F_1^d(\underline{b}) = 0$,

$$\underline{b} = c + [P - c] \frac{\theta - k_1}{\min\{\theta, k_2\}} \geq c.$$

If, on the other hand, $k_1 > \frac{P}{P-c} k_2$, in which case $\theta > \frac{P}{P-c} k_2$, we have the boundary condition $\lim_{b \to P} F_2^d(P) = 1$, which implies

$$\pi_1^d = P [\theta - k_2],$$

and, together with the condition $F_2^d(\underline{b}) = 0$,

$$\underline{b} = P \frac{\theta - k_2}{\min\{\theta, k_1\}} \geq c.$$

Note that, in both cases, $\underline{b} \to c$ as $\theta \to \widehat{\theta}$, and so, in the limit, $\pi_1 = c \left[\widehat{\theta} - k_2 \right]$ and $\pi_2 = 0$.

In the case $k_1 < \frac{P}{P-c} k_2$ (similar results are obtained in the alternative case), equilibrium profits, expected costs and expected revenues may be written:

$$\pi_1^d = ck_1 + [P - c] [\theta - k_1] \frac{k_1}{\min\{\theta, k_2\}} \text{ and } \pi_2^d = [P - c] [\theta - k_1]$$

$$\begin{aligned} EC^d &= \Pr\{b_1 \leq b_2\} c [\theta - k_1] + \Pr\{b_1 > b_2\} c \min\{\theta, k_2\} \\ ER^d &= \pi_1^d + \pi_2^d + EC^d \end{aligned}$$

where

$$\Pr\{b_1 \leq b_2\} = \int_{\underline{b}}^{P} F_1^d(b)dF_2^d(b) + 1 - \frac{k_1}{k_1 + \min\{\theta, k_2\} - \theta} \frac{P - \underline{b}}{P}$$

With some algebra,

$$\int_{\underline{b}}^{P} F_1^d(b)dF_2^d(b) = \frac{k_1 \min\{\theta, k_2\}}{[k_1 + \min\{\theta, k_2\} - \theta]^2} \frac{\underline{b}}{c} \left[\frac{P - \underline{b}}{P} - \frac{\underline{b} - c}{c} \ln\left(\frac{P - c}{\underline{b} - c}\frac{\underline{b}}{P}\right)\right]$$

In the limit,

$$\begin{aligned} \lim_{c \to 0} \Pr\{b_1 \leq b_2\} &= 1 - \frac{1}{2}\frac{k_1}{\min\{\theta, k_2\}} \geq \frac{1}{2}, \\ \lim_{c \to P} \Pr\{b_1 \leq b_2\} &= 1, \end{aligned}$$

and hence

$$\frac{1}{2} \leq \Pr\{b_1 \leq b_2\} \leq 1,$$

$$\begin{aligned} c[\theta - k_1] &\leq EC^d \leq \frac{c\min\{\theta, k_2\} + c[\theta - k_1]}{2}, \\ \pi_1^d + \pi_2^d + c[\theta - k_1] &\leq ER^d \leq \pi_1^d + \pi_2^d + \frac{c\min\{\theta, k_2\} + c[\theta - k_1]}{2}. \end{aligned}$$

Furthermore, we know that we cannot have $ER^d = P\theta$, since this would require both suppliers playing P with positive probability. Thus, $ER^d < P\theta$.

Proof of Lemma 4.2

Verifying that the arguments of Lemma 1 and Proposition 2 go through with multiple bids is straightforward. Below we want to demonstrate that, in the discriminatory auction, the best response to a rival offering all of his capacity at the same price according to an equilibrium distribution function is to bid a flat bid function also. Under the assumption that $b_{jn} = b_j$, $n = 1, ..., N_j$, with b_j chosen according to the distribution function F_j, supplier i's expected profits may be written

$$\begin{aligned} \pi_i(\mathbf{b}_i) = & \sum_{n=1}^{N_{i1}} [b_{in} - c_i] \left\{ F_j(b_{in}) \min\left\{ k_{in}, \max\left\{ \theta - k_j - \sum_{m=1}^{n-1} k_{im}, 0 \right\}\right\} \right. \\ & \left. + [1 - F_j(b_{in})] k_{in} \right\}, \end{aligned}$$

where we have defined $\sum_{m=1}^{0} k_{im} \equiv 0$. Suppose $\mathbf{b}_i$ is set optimally, that $N_i > 1$ and that $b_{in} < b_{in+1}$ for some $n = 1, 2, ..., N_i - 1$ (i.e. there are at least two steps in i's bid function). We want to show that this leads to a contradiction. Consider first the case that $\theta > k_j$ and let $\widehat{n}$ be chosen such that $0 < \theta - k_j - \sum_{m=1}^{\widehat{n}-1} k_{im} < k_{i\widehat{n}}$. Clearly such an $\widehat{n}$ exists and is unique. Note that we have $\theta - k_j - \sum_{m=1}^{n-1} k_{im} > k_{in}$ for $n < \widehat{n}$ and $\theta - k_j - \sum_{m=1}^{n-1} k_{im} < 0$ for $n > \widehat{n}$. Supplier i's profit can then be rewritten as,

$$\begin{aligned}
\pi_i(\mathbf{b}_i) &= \sum_{n=1}^{\widehat{n}-1} [b_{in} - c_i] k_{in} \\
&\quad + [b_{i\widehat{n}} - c_i] \left\{ F_j(b_{i\widehat{n}}) \left[\theta - k_j - \sum_{n=1}^{\widehat{n}-1} k_{in} \right] + [1 - F_j(b_{i\widehat{n}})] k_{i\widehat{n}} \right\} \\
&\quad + \sum_{n=\widehat{n}+1}^{N_i} [b_{in} - c_i][1 - F_j(b_{in})] k_{in} \\
&= [b_{i\widehat{n}} - c_i] \{F_j(b_{i\widehat{n}})[\theta - k_j] + [1 - F_j(b_{i\widehat{n}})] k_i\} \\
&\quad + \sum_{n=1}^{\widehat{n}-1} [b_{in} - b_{i\widehat{n}}] k_{in} \\
&\quad + \sum_{n=\widehat{n}+1}^{N_i} \{[b_{in} - c_i][1 - F_j(b_{in})] - [b_{i\widehat{n}} - c_i][1 - F_j(b_{i\widehat{n}})]\} k_{in}.
\end{aligned}$$

The first term in the last expression equals the profit Supplier i would obtain if all of his units were bid in at the same price $b_{i\widehat{n}}$. The second term is clearly negative: it is always profitable to increase offer prices on units that will be despatched with probability 1. The last term is negative also. To see this, note that if F_j is the mixed-strategy corresponding to an equilibrium in which supplier i offers all units at the same price, it must satisfy

$$\begin{aligned}
\pi_i(b_i) &= [b_i - c_i] \{F_j(b_i) \min\{k_i, \max\{\theta - k_j, 0\}\} \\
&\qquad + [1 - F_j(b_i)] \min\{k_i, \theta\}\} = \overline{\pi}_i,
\end{aligned}$$

where $\overline{\pi}_i$ is some constant. Consider two offer prices $\widehat{b} > \widetilde{b}$ on the support of F_j. Then

$$\begin{aligned}
0 &= \left[\widehat{b} - c_i\right] \left\{F_j\left(\widehat{b}\right) \min\left\{k_i, \max\left\{\theta - k_j, 0\right\}\right\} + \left[1 - F_j\left(\widehat{b}\right)\right] \min\left\{k_i, \theta\right\}\right\} \\
&\quad - \left[\widetilde{b} - c_i\right] \left\{F_j\left(\widetilde{b}\right) \min\left\{k_i, \max\left\{\theta - k_j, 0\right\}\right\} + \left[1 - F_j\left(\widetilde{b}\right)\right] \min\left\{k_i, \theta\right\}\right\} \\
&= \left\{\left[\widehat{b} - c_i\right] F_j\left(\widehat{b}\right) - \left[\widetilde{b} - c_i\right] F_j\left(\widetilde{b}\right)\right\} \min\left\{k_i, \max\left\{\theta - k_j, 0\right\}\right\} \\
&\quad + \left\{\left[\widehat{b} - c_i\right]\left[1 - F_j\left(\widehat{b}\right)\right] - \left[\widetilde{b} - c_i\right]\left[1 - F_j\left(\widetilde{b}\right)\right]\right\} \min\left\{k_i, \theta\right\} \\
&\geq \left\{\left[\widehat{b} - c_i\right]\left[1 - F_j\left(\widehat{b}\right)\right] - \left[\widetilde{b} - c_i\right]\left[1 - F_j\left(\widetilde{b}\right)\right]\right\} \min\left\{k_i, \theta\right\},
\end{aligned}$$

where the inequality follows from the observation that $[b - c_i] F_j(b)$ is increasing in b (the inequality is strict if $\theta > k_i$). In the case that $\theta \leq k_j$, supplier i's profits simplify to

$$\pi_i(\mathbf{b}_i) = \sum_{n=1}^{N_i} [b_{in} - c_i][1 - F_j(b_{in})] k_{in},$$

and so we can a apply a similar argument to the one immediately above to demonstrate that profits are maximized for $b_{i1} = b_{i2} = ... = b_{iN_i} = b_i$. We conclude that for supplier i to offer all capacity at a single price is a best response to F_j. *Q.E.D.*

Proof of Proposition 4.4

Let $K_s = \sum_{i=1}^{s} k_i$ be the accumulated capacity of the s most efficient suppliers and $K_s^{-i} = K_s - k_i$, $i \leq s$, the accumulated capacity of the s most efficient suppliers not including supplier i. Note first that accepted price offers cannot exceed c_s if $\theta \leq \min_{i \leq s} \left\{K_s^{-i}\right\}$. To see this, suppose that the highest accepted price offer were indeed $b > c_s$. Since at most one supplier will offer b with positive probability, all other suppliers $i \neq s$, $c_i < b$, will price below b. But then, since $\theta < \min_{i \le s} \{K_s^{-i}\}$ a price offer of b will never be accepted. It follows that $\min_{i \le s} \{K_s^{-i}\}$ is a lower bound for $\widehat{\theta_s^-}$.

Consider next events in which $\theta \ge K_{s-1}$. Then, since supplier s never price below c_s, any supplier $i < s$ who offers $b_i < c_s$ will be accepted with probability 1 and despatched at full capacity. It follows that there cannot exist an equilibrium in which some supplier accepts to be paid a price below c_s. Furthermore, if $c_s < c_{s+1}$, or $s = S$ (so $\theta \ge K_{S-1}$), supplier s will price above c_s with probability 1 and hence suppliers $i < s$ will not accept to be paid prices equal to c_s either. Consequently, K_{s-1} is an upper bound for $\widehat{\theta_s^+}$.

Lastly, we observe that $\min_{i \le s} \{K_s^{-i}\} = K_{s-1}$ if $k_s = \max_{i \le s} k_i$ (or $k_s \ge \max_{i<s} k_i$), in which case we must have $\widehat{\theta_s^-} = \widehat{\theta_s^+}$. *Q.E.D.*

Proof of Lemma 4.3

We start by showing that a pure-strategy equilibrium does not exist in either auction format. To see this, note first that in a pure-strategy equilibrium all effective offer prices (i.e., offers that with positive probability affect the prices suppliers are paid) must be equal; if not, some supplier could profitably increase his offer price towards the next higher bid, thereby increasing profits in the event that this offer is effective without reducing output in any event. Next, observe that this common price cannot exceed c; if it did, some supplier could profitably deviate to a slightly lower price, thereby increasing the expected quantity despatched with only a negligible effect on the expected price. Lastly, bidding at c cannot constitute an equilibrium either, since the supplier with costs equal to c could obtain positive profits in the event that demand exceeds the capacity of his rival by raising his offer price.

We next characterize the unique equilibrium for each auction format.

Uniform auction

Let $F_i^u(b) = \Pr\{b_i \le b\}$ denote the equilibrium mixed-strategy of supplier i, $i = 1, 2$, in the uniform auction, with $f_i^u(b) = F_i^{u\prime}(b)$, and let S_i^u

be the support of F_i^u. Standard arguments imply that $S_1^u \cap S_2^u = [\underline{b}^u, P)$, $\underline{b}^u \geq c$, and that F_1^u and F_2^u do not have mass points on $(\underline{b}^u, P)$.

We focus on the case in which $\underline{\theta} < \min\{k_1, k_2\} \leq \max\{k_1, k_2\} < \overline{\theta}$. Supplier i's profit, when bidding b, may then be written

$$\begin{aligned} \pi_i^u(b) &= F_j^u(b) \int_{k_j}^{\overline{\theta}} [b - c_i][\theta - k_j]\, dG(\theta) \\ &+ \int_b^P \left[\int_{\underline{\theta}}^{k_i} [b - c_i]\, \theta dG(\theta) + \int_{k_i}^{\overline{\theta}} [v - c_i]\, k_i dG(\theta) \right] dF_j^u(v). \end{aligned}$$

The first term on the right-hand side represents supplier i's profits in the event that the rival bids below b, in which case supplier i produces a positive quantity only when demand is above the capacity of the rival. The second term represents supplier i's profits in the event that the rival bids above b. As given by the first element of this term, supplier i will then serve all demand at his own price when his capacity is sufficient to satisfy all of demand. On the other hand, and as given by the second element, supplier i will produce at full capacity and receive a price determined by the rival's bid in the event that demand exceeds his capacity.

On $(\underline{b}^u, P)$, strategies must satisfy the following differential equations:

$$\begin{gathered} F_j^u(b) \int_{k_j}^{\overline{\theta}} [\theta - k_j]\, dG(\theta) + \left[1 - F_j^u(b)\right] \int_{\underline{\theta}}^{k_i} \theta dG(\theta) \\ - [b - c_i]\, f_j^u(b) \left\{ \int_{\underline{\theta}}^{k_i} \theta dG(\theta) + \int_{k_i}^{\overline{\theta}} k_i dG(\theta) - \int_{k_j}^{\overline{\theta}} [\theta - k_j]\, dG(\theta) \right\} = 0 \end{gathered}$$

On the interior of the support of the mixed strategies the net gain from raising the bid marginally must be zero. The first element on the left-hand side represents the gain to a supplier from the resulting increase in the price received in the event that demand exceeds the capacity of the rival and the rival bids below. The second element represents the gain to a supplier from the resulting increase in the price in the event that demand is lower than his capacity and the rival bids above. Lastly, the third term represents the loss from being despatched with a smaller output: in case demand falls below the supplier's capacity the loss of output equals total demand; in case demand exceeds the supplier's capacity the loss equals the difference between being despatched at full capacity and serving residual demand only (i.e., $k_i - [\theta - k_j]$). The above expressions may alternatively be written

$$f_j^u(b) - \frac{\lambda_j}{b - c_i} F_j^u(b) = \frac{\beta_j}{b - c_i},$$

where

$$\lambda_j = \frac{\int_{k_j}^{\overline{\theta}} [\theta - k_j]\, dG(\theta) - \int_{\underline{\theta}}^{k_i} \theta dG(\theta)}{\int_{\underline{\theta}}^{\overline{\theta}} \theta dG(\theta) - \int_{k_i}^{\overline{\theta}} [\theta - k_i]\, dG(\theta) - \int_{k_j}^{\overline{\theta}} [\theta - k_j]\, dG(\theta)}$$

$$\beta_j = \frac{\int_{\underline{\theta}}^{k_i} \theta dG(\theta)}{\int_{\underline{\theta}}^{\overline{\theta}} \theta dG(\theta) - \int_{k_i}^{\overline{\theta}} [\theta - k_i]\, dG(\theta) - \int_{k_j}^{\overline{\theta}} [\theta - k_j]\, dG(\theta)}$$

which have solutions

$$F_j^u(b) = \begin{cases} \beta_j \ln(b - c_i) + \Omega_1^j & \text{for} \quad \lambda_j = 0 \\ \Omega_2^j [b - c_i]^{\lambda_j} - \frac{\beta_j}{\lambda_j} & \text{for} \quad \lambda_j \neq 0 \end{cases}$$

where Ω_1^j, Ω_2^j, $j = 1, 2$, are constants of integration. Note that, if $k_i \leq k_j$, $\beta_i \geq \beta_j$. Furthermore, $\beta_1 = \beta_2$ and $\lambda_1 = \lambda_2$ when $k_1 = k_2$. Also, if $k_i \leq k_j$, $\beta_j \to 0$ as $\underline{\theta} \uparrow k_i$ while $\beta_j + \lambda_j \to 0$ as $\overline{\theta} \downarrow k_j$.

Given the boundary condition $F_j^u(\underline{b}^u) = 0$, these equations yield the mixed-strategy distribution functions for $b \in [\underline{b}^u, P)$:

$$F_j^u(b) = \begin{cases} \beta_j \ln\left(\frac{b - c_i}{\underline{b}^u - c_i}\right) & \text{for} \quad \lambda_j = 0, \\ \frac{\beta_j}{\lambda_j}\left\{\left[\frac{b - c_i}{\underline{b}^u - c_i}\right]^{\lambda_j} - 1\right\} & \text{for} \quad \lambda_j \neq 0. \end{cases}$$

Suppose $\lim_{b\uparrow P} F_2^u(b) \leq \lim_{b\uparrow P} F_1^u(b) = 1$ (in the opposite case, i.e. when $\lim_{b\uparrow P} F_1^u(b) \leq \lim_{b\uparrow P} F_2^u(b) = 1$, a corresponding argument can be applied). Then it is straightforward to verify that $\underline{b}^u$ is given uniquely as

$$\underline{b}^u = \begin{cases} c_2 + [P - c_2]\, e^{-\frac{1}{\beta_1}} & \text{for} \quad \lambda_1 = 0, \\ c_2 + [P - c_2] \left[\frac{\beta_1}{\lambda_1 + \beta_1}\right]^{\frac{1}{\lambda_1}} & \text{for} \quad \lambda_1 \neq 0. \end{cases}$$

Substituting for $\underline{b}^u$, we find

$$F_1^u(b) = \begin{cases} 1 + \beta_1 \ln\left(\frac{b - c_2}{P - c_2}\right) & \text{for} \quad \lambda_1 = 0, \\ \frac{\beta_1}{\lambda_1}\left\{\frac{\lambda_1 + \beta_1}{\beta_1}\left[\frac{b - c_2}{P - c_2}\right]^{\lambda_1} - 1\right\} & \text{for} \quad \lambda_1 \neq 0, \end{cases}$$

while $F_2^u(P) = 1$ and, for $b \in [\underline{b}^u, P)$,

$$F_2^u(b) = \begin{cases} \beta_2 \ln\left(\frac{b - c_1}{[P - c_2]e^{-\frac{1}{\beta_1}} + c_2 - c_1}\right) & \text{for} \quad \lambda_1 = \lambda_2 = 0, \\ \frac{\beta_2}{\lambda_2}\left\{\left[\frac{b - c_1}{[P - c_2]\left[\frac{\beta_1}{\lambda_1 + \beta_1}\right]^{\frac{1}{\lambda_1}} + c_2 - c_1}\right]^{\lambda_2} - 1\right\} & \text{for} \quad \lambda_1, \lambda_2 \neq 0. \end{cases}$$

Equilibrium profits become

$$\pi_1^u = [P - c_1]\left\{\Pr(b_2 < P)\int_{k_2}^{\overline{\theta}} [\theta - k_2]\, dG(\theta) + \Pr(b_2 = P)\int_{\underline{\theta}}^{\overline{\theta}} \min(\theta, k_1)\, dG(\theta)\right\},$$

$$\pi_2^u = [P - c_2]\int_{k_1}^{\overline{\theta}} [\theta - k_1]\, dG(\theta),$$

where

$$\Pr(b_2 < P) = \lim_{b\uparrow P} F_2^d(b).$$

Symmetric Capacities: When $k_1 = k_2 = k$ and $0 = c_1 \leq c_2 = c$, one can show that we must have $\lim_{b \uparrow P} F_2^u(b) \leq \lim_{b \uparrow P} F_1^u(b) = 1$ and so we find

$$\underline{b}^u = \begin{cases} c + [P-c]\, e^{-\frac{1}{\beta}} & \text{for} \quad \lambda = 0 \\ c + [P-c]\left[\frac{\beta}{\lambda+\beta}\right]^{\frac{1}{\lambda}} & \text{for} \quad \lambda \neq 0 \end{cases}$$

$$F_1^u(b) = \begin{cases} 1 + \beta \ln\left(\frac{b-c}{P-c}\right) & \text{for} \quad \lambda = 0 \\ \frac{\beta}{\lambda}\left\{\frac{\lambda+\beta}{\beta}\left[\frac{b-c}{P-c}\right]^{\lambda} - 1\right\} & \text{for} \quad \lambda \neq 0 \end{cases}$$

$$F_2^u(b) = \begin{cases} \begin{cases} \beta \ln\left(\frac{b}{[P-c]e^{\frac{-1}{\beta}}+c}\right) & \text{for} \quad \lambda = 0 \\ \frac{\beta}{\lambda}\left\{\left[\frac{b}{[P-c]\left[\frac{\beta}{\lambda+\beta}\right]^{\frac{1}{\lambda}}+c}\right]^{\lambda} - 1\right\} & \text{for} \quad \lambda \neq 0 \end{cases} & , b \in [\underline{b}^u, P), \\ 1, \; b = P \end{cases}$$

where

$$\lambda = \frac{\int_k^{\overline{\theta}} [\theta - k]\, dG(\theta) - \int_{\underline{\theta}}^k \theta dG(\theta)}{\int_{\underline{\theta}}^{\overline{\theta}} \theta dG(\theta) - 2\int_k^{\overline{\theta}} [\theta - k]\, dG(\theta)}$$

$$\beta = \frac{\int_{\underline{\theta}}^k \theta dG(\theta)}{\int_{\underline{\theta}}^{\overline{\theta}} \theta dG(\theta) - 2\int_k^{\overline{\theta}} [\theta - k]\, dG(\theta)}$$

Furthermore,

$$\pi_1^u = P\left\{\Pr(b_2 < P)\int_k^{\overline{\theta}} [\theta - k]\, dG(\theta) + \Pr(b_2 = P)\int_{\underline{\theta}}^{\overline{\theta}} \min(\theta, k)\, dG(\theta)\right\},$$

$$\pi_2^u = [P-c]\int_k^{\overline{\theta}} [\theta - k]\, dG(\theta).$$

Consequently, at equilibrium the low-cost supplier bids more aggressively than the high-cost supplier; in particular, the strategy of the low-cost supplier stochastically first-order dominates the strategy of the high-cost supplier.

Again, $\beta \to 0$ (while $\lambda \neq 0$) as $\underline{\theta} \uparrow k$. In particular,

$$\lim_{\underline{\theta} \uparrow k} \underline{b}^u = c,$$

$$\lim_{\underline{\theta} \downarrow k} F_1^u(b) = \left[\frac{b-c}{P-c}\right]^{\lambda}$$

$$\lim_{\underline{\theta} \uparrow k} F_2^u(b) = \begin{cases} 0, & b < P \\ 1, & b = P \end{cases}$$

$$\lim_{\underline{\theta} \uparrow k} \pi_1^u = Pk,$$

$$\lim_{\underline{\theta} \uparrow k} \pi_2^u = [P-c]\,[\mathrm{E}\theta - k],$$

where we have used the fact that $\lim_{\underline{\theta} \uparrow k} \int_k^{\underline{\theta}} \theta dG(\theta) = \mathrm{E}\theta$. Consequently, as the probability that demand falls below the capacity of an individual supplier goes to zero, equilibrium approaches something with the flavour of the equilibrium found for high-demand realizations, with the high-cost supplier bidding at P and the low-cost supplier mixing over a range between c and P so as to make undercutting by the high-cost supplier unprofitable.

Also, $\beta \to 1$ and $\lambda \to 1$ as $\overline{\theta} \downarrow k$. In particular,

$$\lim_{\overline{\theta} \downarrow k} \underline{b}^u = c,$$

$$\lim_{\overline{\theta} \downarrow k} F_1^u(b) = 1$$

$$\lim_{\overline{\theta} \downarrow k} F_2^u(b) = 1 - \frac{c}{b},\ b < P$$

$$\lim_{\overline{\theta} \downarrow k} \pi_1^u = c\mathrm{E}\theta$$

$$\lim_{\overline{\theta} \downarrow k} \pi_2^u = 0,$$

where we have used the fact that $\lim_{\overline{\theta} \downarrow k} \int_{\overline{\theta}}^{k} \theta dG(\theta) = \mathrm{E}\theta$. Consequently, as the probability that demand exceeds the capacity of an individual supplier goes to zero, equilibrium approaches something with the flavour of the

Bertrand-like equilibrium found for low-demand realizations, with the low-cost supplier bidding at the cost of the high-cost supplier and the high-cost supplier mixing between c and P (with a mass point at P).

Symmetric costs: When $c_1 = c_2 = 0$ and $k_1 \leq k_2$, we again must have $\lim_{b\uparrow P} F_2^u(b) \leq \lim_{b\uparrow P} F_1^u(b) = 1$ and so

$$\underline{b}^u = \begin{cases} Pe^{-\frac{1}{\beta_1}} & \text{for} \quad \lambda_1 = 0 \\ P\left[\frac{\beta_1}{\lambda_1+\beta_1}\right]^{\frac{1}{\lambda_1}} & \text{for} \quad \lambda_1 \neq 0 \end{cases}$$

$$F_1^u(b) = \begin{cases} 1 + \beta_1 \ln\left(\frac{b}{P}\right) & \text{for} \quad \lambda_1 = 0 \\ \frac{\beta_1}{\lambda_1}\left\{\left[\frac{\lambda_1+\beta_1}{\beta_1}\left[\frac{b}{P}\right]^{\lambda_1} - 1\right]\right\} & \text{for} \quad \lambda_1 \neq 0 \end{cases}$$

$$F_2^u(b) = \begin{cases} \begin{cases} \beta_2 \ln\left(\frac{b}{Pe^{\frac{-1}{\beta_1}}}\right) & \text{for} \quad \lambda_1 = \lambda_2 = 0 \\ \frac{\beta_2}{\lambda_2}\left\{\left[\frac{\lambda_1+\beta_1}{\beta_1}\right]^{\frac{\lambda_2}{\lambda_1}}\left[\frac{b}{P}\right]^{\lambda_2} - 1\right\} & \text{for} \quad \lambda_1, \lambda_2 \neq 0 \end{cases} & , b \in [\underline{b}^u, P) \\ 1, \; b = P & \end{cases}$$

$$\pi_1^u = P\left\{\Pr(b_2 < P)\int_{k_2}^{\overline{\theta}} [\theta - k_2]\, dG(\theta) + \Pr(b_2 = P)\int_{\underline{\theta}}^{\overline{\theta}} \min(\theta, k_1)\, dG(\theta)\right\},$$

$$\pi_2^u = P\int_{k_1}^{\overline{\theta}} [\theta - k_1]\, dG(\theta).$$

Consequently, at equilibrium the smaller supplier bids more aggressively than the larger supplier; in particular, the strategy of the smaller supplier stochastically first-order dominates the strategy of the larger supplier.

In the limit,

$$\lim_{\underline{\theta}\uparrow k_1} F_2^u(b) = 0, \; b < P$$
$$\lim_{\underline{\theta}\uparrow k_1} \pi_1^u = Pk_1$$
$$\lim_{\underline{\theta}\uparrow k_1} \pi_2^u = P\left[\mathrm{E}\theta - k_1\right],$$

where we have used the fact that $\lim_{\underline{\theta}\uparrow k_1} \int_{k_1}^{\overline{\theta}} \theta dG(\theta) = \mathrm{E}\theta$. Consequently, as the probability that demand falls below the capacity of the smaller supplier goes to zero, equilibrium approaches something with the flavour of the high-low bidding equilibrium found for high-demand realizations, with the larger supplier bidding at P and the smaller supplier mixing over a range below P so as to make undercutting by the larger supplier unprofitable.

Symmetric costs and capacities: When $k_1 = k_2 = k$ and $c_1 = c_2 = 0$, we have $F_1^u(b) = F_2^u(b)$ and so we find

$$\underline{b}^u = \begin{cases} Pe^{-\frac{1}{\beta}} & \text{for} \quad \lambda = 0 \\ P\left[\frac{\beta}{\lambda+\beta}\right]^{\frac{1}{\lambda}} & \text{for} \quad \lambda \neq 0 \end{cases}$$

$$F_1^u(b) = F_2^u(b) = \begin{cases} 1 + \beta \ln\left(\frac{b}{P}\right) & \text{for} \quad \lambda = 0 \\ \frac{\beta}{\lambda}\left\{\frac{\lambda+\beta}{\beta}\left[\frac{b}{P}\right]^{\lambda} - 1\right\} & \text{for} \quad \lambda \neq 0 \end{cases}$$

$$\pi_1^u = \pi_2^u = P\int_k^{\overline{\theta}} [\theta - k]\, dG(\theta).$$

Discriminatory auction

Let $F_i^d(b) = \Pr\{b_i \leq b\}$ denote the equilibrium mixed-strategy of supplier i, $i = 1, 2$, in the discriminatory auction, and let S_i^d be the support of F_i^d and $f_i^d(b)$ its density function. Standard arguments imply that $S_1^d \cap S_2^d = [\underline{b}^d, P)$, $\underline{b}^d \geq c$, and that F_1^d and F_2^d do not have mass points on $[\underline{b}^d, P)$.

Again we focus on the case in which $\underline{\theta} < \min\{k_1, k_2\} \leq \max\{k_1, k_2\} < \overline{\theta}$. Supplier i's profit, when bidding b, may then be written

$$\begin{aligned} \pi_i^d(b) &= [b - c_i]\Bigg\{F_j^d(b)\int_{k_j}^{\overline{\theta}} [\theta - k_j]\, dG(\theta) \\ &\quad + \left[1 - F_j^d(b)\right]\left[\int_{\underline{\theta}}^{k_i} \theta dG(\theta) + \int_{k_i}^{\overline{\theta}} k_i dG(\theta)\right]\Bigg\}. \end{aligned}$$

A necessary condition for supplier i to be indifferent between any price in S^d is that, for all $b \in S^d$, $\pi_i^d(b) = \overline{\pi}_i^d$, implying

$$F_j^d(b) = \frac{\int_{\underline{\theta}}^{\overline{\theta}} \theta dG(\theta) - \int_{k_i}^{\overline{\theta}} [\theta - k_i]\, dG(\theta) - \frac{\overline{\pi}_i^d}{b-c_i}}{\int_{\underline{\theta}}^{\overline{\theta}} \theta dG(\theta) - \int_{k_i}^{\overline{\theta}} [\theta - k_i]\, dG(\theta) - \int_{k_j}^{\overline{\theta}} [\theta - k_j]\, dG(\theta)}.$$

Observe that the boundary condition $F_j^d(\underline{b}^d) = 0$ implies

$$\overline{\pi}_i^d = \left[\underline{b}^d - c_i\right] \left[\int_{\underline{\theta}}^{\overline{\theta}} \theta dG(\theta) - \int_{k_i}^{\overline{\theta}} [\theta - k_i]\, dG(\theta)\right],$$

and so

$$F_j^d(b) = \frac{\int_{\underline{\theta}}^{\overline{\theta}} \theta dG(\theta) - \int_{k_i}^{\overline{\theta}} [\theta - k_i]\, dG(\theta)}{\int_{\underline{\theta}}^{\overline{\theta}} \theta dG(\theta) - \int_{k_i}^{\overline{\theta}} [\theta - k_i]\, dG(\theta) - \int_{k_j}^{\overline{\theta}} [\theta - k_j]\, dG(\theta)} \frac{b - \underline{b}^d}{b - c_i}.$$

We have

$$F_1^d(b) \gtrless F_2^d(b) \iff \frac{b - c_1}{b - c_2} \gtrless \frac{\int_{\underline{\theta}}^{\overline{\theta}} \theta dG(\theta) - \int_{k_1}^{\overline{\theta}} [\theta - k_1]\, dG(\theta)}{\int_{\underline{\theta}}^{\overline{\theta}} \theta dG(\theta) - \int_{k_2}^{\overline{\theta}} [\theta - k_2]\, dG(\theta)}.$$

Suppose $F_1^d(b) > F_2^d(b)$ (in the opposite case a corresponding argument to the following may be applied). Then we cannot have $\lim_{b \uparrow P} F_2^d(b) = 1$ since this would imply $\lim_{b \uparrow P} F_1^d(b) > 1$. Consequently, we have the boundary condition $\lim_{b \uparrow P} F_1^d(P) = 1$, which implies

$$\overline{\pi}_2^d = [P - c_2] \int_{k_1}^{\overline{\theta}} [\theta - k_1]\, dG(\theta),$$

and, together with the condition $F_1^d(\underline{b}^d) = 0$,

$$\underline{b}^d = c_2 + [P - c_2] \frac{\int_{k_1}^{\overline{\theta}} [\theta - k_1] \, dG(\theta)}{\int_{\underline{\theta}}^{\overline{\theta}} \theta dG(\theta) - \int_{k_2}^{\overline{\theta}} [\theta - k_2] \, dG(\theta)}.$$

Equilibrium profits become

$$\pi_1^d = [P - c_1] \left\{ \Pr(b_2 < P) \int_{k_2}^{\overline{\theta}} [\theta - k_2] \, dG(\theta) + \Pr(b_2 = P) \int_{\underline{\theta}}^{\overline{\theta}} \min(\theta, k_1) \, dG(\theta) \right\},$$

$$\pi_2^d = [P - c_2] \int_{k_1}^{\overline{\theta}} [\theta - k_1] \, dG(\theta),$$

where

$$\Pr(b_2 < P) = \lim_{b \uparrow P} F_2^d(b) = \frac{P - c_2}{P - c_1} \frac{\int_{\underline{\theta}}^{\overline{\theta}} \theta dG(\theta) - \int_{k_1}^{\overline{\theta}} [\theta - k_1] \, dG(\theta)}{\int_{\underline{\theta}}^{\overline{\theta}} \theta dG(\theta) - \int_{k_2}^{\overline{\theta}} [\theta - k_2] \, dG(\theta)}.$$

Symmetric capacities: When $k_1 = k_2 = k$ and $0 = c_1 < c_2 = c$, $F_1^d(b) > F_2^d(b)$ and so we find

$$\underline{b}^d = c + [P - c] \frac{\int_{k}^{\overline{\theta}} [\theta - k] \, dG(\theta)}{\int_{\underline{\theta}}^{\overline{\theta}} \theta dG(\theta) - \int_{k}^{\overline{\theta}} [\theta - k] \, dG(\theta)}$$

$$\pi_1^d = [P - c] \int_{k}^{\overline{\theta}} [\theta - k] \, dG(\theta) + c \left[\int_{\underline{\theta}}^{k} \theta dG(\theta) + \int_{k}^{\overline{\theta}} k dG(\theta) \right],$$

$$\pi_2^d = [P - c] \int_{k}^{\overline{\theta}} [\theta - k] \, dG(\theta).$$

Consequently, at equilibrium the low-cost supplier bids more aggressively than the high-cost supplier; in particular, the strategy of the low-cost supplier first-order stochastically dominates that of the high-cost supplier.

In the limit, we find

$$
\begin{aligned}
\lim_{\underline{\theta}\uparrow k} \underline{b}^d &= c + [P - c]\,\frac{\mathrm{E}\theta - k}{k}, \\
\lim_{\underline{\theta}\uparrow k} F_1^d(b) &= \frac{k}{2k - \mathrm{E}\theta}\,\frac{b - \underline{b}^d}{b - c} \\
\lim_{\underline{\theta}\uparrow k} F_2^d(b) &= \begin{cases} \frac{k}{2k - \mathrm{E}\theta}\frac{b - \underline{b}^d}{b} &, \ b < P \\ 1 &, \ b = P \end{cases} \\
\lim_{\underline{\theta}\uparrow k} \pi_1^d &= [P - c]\,[\mathrm{E}\theta - k] + ck, \\
\lim_{\underline{\theta}\uparrow k} \overline{\pi}_2^d &= [P - c]\,[\mathrm{E}\theta - k]\,.
\end{aligned}
$$

Consequently, when the probability that demand falls below the capacity of any individual supplier goes to zero, equilibrium approaches the mixed-strategy equilibrium for high-demand realizations.

Furthermore,

$$
\begin{aligned}
\lim_{\overline{\theta}\downarrow k} \underline{b}^d &= c, \\
\lim_{\overline{\theta}\downarrow k} F_1^d(b) &= 1, \\
\lim_{\overline{\theta}\downarrow k} F_2^d(b) &= \begin{cases} 1 - \frac{c}{b}, \ b < P \\ 1, \ b = P \end{cases} \\
\lim_{\overline{\theta}\downarrow k} \pi_1^d &= c\mathrm{E}\theta, \\
\lim_{\overline{\theta}\downarrow k} \pi_2^d &= 0.
\end{aligned}
$$

Hence, as the probability that demand exceeds the capacity of an individual supplier goes to zero, equilibrium approaches the Bertrand-like equilibrium for low-demand realizations, with the low-cost supplier bidding at the cost of the high-cost supplier and the high-cost supplier mixing between c and P (with a mass point at P).

Symmetric costs: When $c_1 = c_2 = 0$ and $k_1 < k_2$, $F_1^d(b) > F_2^d(b)$ and so we find

$$\underline{b}^d = P\frac{\int_{k_1}^{\overline{\theta}} [\theta - k_1]\, dG(\theta)}{\int_{\underline{\theta}}^{\overline{\theta}} \theta dG(\theta) - \int_{k_2}^{\overline{\theta}} [\theta - k_2]\, dG(\theta)}$$

$$\pi_1^d = P\int_{k_1}^{\overline{\theta}} [\theta - k_1]\, dG(\theta)\, \frac{\int_{\underline{\theta}}^{\overline{\theta}} \theta dG(\theta) - \int_{k_1}^{\overline{\theta}} [\theta - k_1]\, dG(\theta)}{\int_{\underline{\theta}}^{\overline{\theta}} \theta dG(\theta) - \int_{k_2}^{\overline{\theta}} [\theta - k_2]\, dG(\theta)}$$

$$\pi_2^d = P\int_{k_1}^{\overline{\theta}} [\theta - k_1]\, dG(\theta)$$

In the limit,

$$\lim_{\underline{\theta}\uparrow k_1} \underline{b}^d = P\frac{\mathrm{E}\theta - k_1}{\mathrm{E}\theta - \int_{k_2}^{\overline{\theta}} [\theta - k_2]\, dG(\theta)}$$

$$\lim_{\underline{\theta}\uparrow k_1} F_1^d(b) = \frac{\mathrm{E}\theta - \int_{k_2}^{\overline{\theta}} [\theta - k_2]\, dG(\theta)}{k_1 - \int_{k_2}^{\overline{\theta}} [\theta - k_2]\, dG(\theta)}\frac{b - \underline{b}^d}{b}$$

$$\lim_{\underline{\theta}\uparrow k} F_2^d(b) = \begin{cases} \frac{k_1}{k_1 - \int_{k_2}^{\overline{\theta}} [\theta - k_2] dG(\theta)}\frac{b - \underline{b}^d}{b} & , \quad b < P \\ 1 & , \quad b = P \end{cases}$$

$$\lim_{\underline{\theta}\uparrow k_1} \pi_1^d = P[\mathrm{E}\theta - k_1]\frac{k_1}{\mathrm{E}\theta - \int_{k_2}^{\overline{\theta}} [\theta - k_2]\, dG(\theta)}$$

$$\lim_{\underline{\theta}\uparrow k_1} \pi_2^d = P[\mathrm{E}\theta - k_1]$$

Again, when the probability that demand falls below the capacity of any individual supplier goes to zero, equilibrium approaches the mixed-strategy equilibrium for high-demand realizations.

Furthermore,

$$\lim_{\overline{\theta}\downarrow k_2} \underline{b}^d = P\frac{\int_{k_1}^{k_2}[\theta - k_1]\, dG(\theta)}{\mathrm{E}\theta}$$

$$\lim_{\underline{\theta}\uparrow k_1} F_1^d(b) = \frac{\mathrm{E}\theta}{\mathrm{E}\theta - \int_{k_1}^{k_2}[\theta - k_1]\, dG(\theta)}\frac{b - \underline{b}^d}{b}$$

$$\lim_{\underline{\theta}\uparrow k} F_2^d(b) = \begin{cases} \frac{b-\underline{b}^d}{b} & , \quad b < P \\ 1 & , \quad b = P \end{cases}$$

$$\lim_{\overline{\theta}\downarrow k_2} \pi_1^d = P\int_{k_1}^{k_2}[\theta - k_1]\, dG(\theta)\frac{\mathrm{E}\theta - \int_{k_1}^{\overline{\theta}}[\theta - k_1]\, dG(\theta)}{\mathrm{E}\theta}$$

$$\lim_{\overline{\theta}\downarrow k_2} \pi_2^d = P\int_{k_1}^{k_2}[\theta - k_1]\, dG(\theta)$$

Consequently, as the probability that demand exceeds the capacity of the larger supplier goes to zero, equilibrium approaches the mixed-strategy equilibrium for high-demand realizations, with the smaller supplier bidding more aggressively.

Symmetric capacities and costs: When $k_1 = k_2 = k$ and $c_1 = c_2 = 0$, $F_1^d(b) = F_2^d(b)$ and so we find

$$\underline{b}^d = P\frac{\int_k^{\overline{\theta}}[\theta - k]\, dG(\theta)}{\int_{\underline{\theta}}^{\overline{\theta}}\theta dG(\theta) - \int_k^{\overline{\theta}}[\theta - k]\, dG(\theta)},$$

$$F_1^d(b) = F_2^d(b) = \frac{\int_{\underline{\theta}}^{\overline{\theta}}\theta dG(\theta) - \int_k^{\overline{\theta}}[\theta - k]\, dG(\theta)}{\int_{\underline{\theta}}^{\overline{\theta}}\theta dG(\theta) - 2\int_k^{\overline{\theta}}[\theta - k]\, dG(\theta)}\left[1 - \frac{P}{b}\frac{\int_k^{\overline{\theta}}[\theta - k]\, dG(\theta)}{\int_{\underline{\theta}}^{\overline{\theta}}\theta dG(\theta) - \int_k^{\overline{\theta}}[\theta - k]\, dG(\theta)}\right],$$

$$\pi_1^d = \pi_2^d = P\int_k^{\overline{\theta}}[\theta - k]\, dG(\theta).$$

Q.E.D.

Proof of Proposition 4.6

Uniform auction format: With short-lived bids total payments to suppliers equal zero for low-demand realizations and $P\theta$ for high-demand realizations, and so overall expected payments equal $ER_s^u = PE\{\theta \mid \theta \geq k\}\, G(k)$. With long-lived bids, for given demand realization θ, total payments equal

$2P \max\{\theta - k, 0\}$, and so in expected terms we have $ER_l^u = 2P[E\{\theta \mid \theta \geq k\} - k]\,G(k)$. From these expressions we find

$$ER_l^u - ER_s^u = P[E\{\theta \mid \theta \geq k\} - 2k]\,G(k) < 0.$$

Discriminatory auction format: With short-lived bids total payments to suppliers equal zero for low-demand realizations and $2P[\theta - k]$for high-demand realizations, and so overall expected payments equal

$ER_s^d = 2P[E\{\theta \mid \theta \geq k\} - k]\,G(k)$. With long-lived bids, for given demand realization θ, total payments equal $2P \max\{\theta - k, 0\}$, and so in expected terms we have $ER_l^d = 2P[E\{\theta \mid \theta \geq k\} - k]\,G(k) = ER_s^d$. *Q.E.D.*