MONETARY POLICY
UNDER A NEW KEYNESIAN PERSPECTIVE

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DECLARATION

I declare the work presented in this thesis is my own except in two chapters based on joint work with Paul Castillo and Vicente Tuesta.

Chapter 2 rely on the paper "Inflation Premium and Oil Price Volatility" joint with Mr. Castillo and Mr. Tuesta.

Chapter 4 is based on the paper "Non-Homothetic Preferences and the Asymmetric Effects of Monetary Policy" joint with Mr. Castillo.

I was responsible for about 50 per cent of the work involved in both papers.

Carlos Montoro

ABSTRACT

This thesis studies monetary policy in a dynamic general equilibrium framework with nominal price rigidities. It analyses monetary policy in a non-linear environment and explores issues concerning optimal monetary policy.

The introductory chapter sets out the motivation of the thesis and puts it into the framework of the existing literature.

Chapter 2 provides a New Keynesian framework to study the interaction among oil price volatility, firms' pricing behaviour and monetary policy. We show that when oil is difficult to substitute in production, firms find optimal to charge higher relative prices as a premium in compensation for the risk that oil price volatility generates on their marginal costs.

Chapter 3 uses the model laid out chapter 2 to investigate how monetary policy should react to oil shocks. The main result is oil price shocks generate a trade-off between inflation and output stabilisation when oil has low substitutability in production. Therefore it becomes optimal to the monetary authority to react partially to oil shocks and some inflation is desirable.

In chapter 4 we extend a New Keynesian model considering preferences that exhibit intertemporal non-homotheticity. We show that under this framework the intertemporal elasticity of substitution becomes state dependent, which induces asymmetric shifts in aggregate demand in response to monetary policy shocks

In chapter 5 we extend the New Keynesian Monetary Policy literature relaxing the assumption that decisions are taken by a single policymaker, considering instead a Monetary Policy Committee (MPC) whose members have different preferences between output and inflation stabilisation. We show that under this

framework, the interest rate behaves non-linearly upon the lagged interest rate and expected inflation.

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Finally, I dedicate this thesis to my son, Carlos-Andres, who born during this project and has been my major inspiration.

CHAPTER 1

INTRODUCTION

As Clarida, Gali and Gertler (1999) point out, during the last years has increased the interest on the study of how to conduct monetary policy. During the last 25 years it's been great advances on the study of the effects of monetary policy on short-term aggregate activity stressing the role of nominal price rigidities. Moreover, the literature has also incorporated the techniques of dynamic stochastic general equilibrium (DSGE) models initiated by real business cycle analysis. This thesis studies monetary policy in a dynamic general equilibrium framework with nominal price rigidities, considering a non-linear environment.

Since the seminal papers of Kydland and Prescott (1982) and King et al. (1988), the use of DSGE models has relied mostly on linear approximations of the solution of the model. Those approximations are useful to characterise certain aspects of the dynamic properties of the models, such as the local existence and determinacy of equilibrium and the characterisation of the second moments of endogenous variables. However, as Kim and Kim (2003) point out, first-order approximation techniques are not well suited to handle questions such as welfare comparisons for different policy rules. Also, since linear approximations satisfy the certainty equivalence property, they are also inappropriate to measure the effects of risk on the equilibrium of endogenous variables. They are also inappropriate to analyse non-linear behaviour, such as asymmetric responses to monetary policy shocks.

The thesis contains two parts. The first part introduces dynamic models aimed at understanding three key issues that have been in the monetary policy debate in recent times. The first is the role of the volatility of oil shocks on the dynamics of inflation. The second is on how should be the optimal monetary policy reaction to oil shocks. The third is on the asymmetric effects of monetary policy in a general equilibrium framework. The thesis approaches these three topics in chapters 2, 3 and 4 respectively. The second part analyses how the appointment of a committee collectively in charge of monetary policy decisions can generate a non-linear behaviour on interest rate reaction function of the central bank.

The relationship between risk and the level of variables has been widely studied in finance. For instance, in the basic capital asset pricing model (CAPM), the volatility of assets returns generates a premium over the return of the risk free asset. However, the study of this relationship has been almost absent among macroeconomists, since they rely mostly on certainty-equivalent linear-approximations for the solution of the models. The exception is Obstfeld and Rogoff (1998), who study in a DSGE model the effects of exchange rate volatility on the level of the exchange rate under different exchange rate regimes. The purpose of chapter 2 is to contribute in this area of macroeconomics, studying the relationship between oil prices volatility and the level of inflation.

Chapter 2 analyses the interaction among oil prices volatility, pricing behaviour of firms and monetary policy in a microfounded New Keynesian framework. We show that when oil is difficult to substitute in production, firms find optimal to charge higher relative prices as a premium in compensation for the risk that oil price volatility generates on their marginal costs. Overall, in general equilibrium, the interaction of the aforementioned mechanisms produces a positive and meaningful relationship between oil price volatility and average inflation, which we denominate *inflation premium.*

Obstfeld and Rogoff (1998), rely on simplifying assumptions about the dynamics of the model in order to have closed form solutions for the risk premium in their model. In contrast to them, we characterise analytically this relationship in a fully dynamic model by using the perturbation method to solve the rational expectations equilibrium of the model up to second order of accuracy. The solution we obtain implies that the *inflation premium* is higher in economies where: a) oil has low substitutability and b) the Phillips Curve is convex. We also show that the larger the reaction of the central bank to the output fluctuations generated by oil shocks, the greater the inflation premium. Finally, we also provide some quantitative evidence that the calibrated model for the US with an estimated active Taylor rule produces a sizable *inflation premium*, similar to level observed in the US during the 70's.

In chapter 2 we show how oil price volatility generates an inflation premium and also how monetary policy can affect this link. In chapter 3 we take a step forward to answer the question about how the central bank should respond to oil shocks.

In chapter 3 we extend Benigno and Woodford (2005) to obtain a second order approximation to the expected utility of the representative household of the model laid out chapter 2. The main result is that oil price shocks generate a trade-off between inflation and output stabilisation when oil has low substitutability in production. Therefore, it becomes optimal to the monetary authority to stabilise partially the effects of oil shocks on inflation and some inflation is desirable. We also find, in contrast to Benigno and Woodford (2005), that this trade-off remains even we eliminate the effects of monopolistic distortions from the steady state.

Chapter 2 and 3 have analysed two kind of non-linear effects that shocks can have in a dynamic stochastic general equilibrium model, which are the effects of shocks volatility on both the mean of endogenous variables and the welfare of the representative agent. However, there is another type of non-linear effect that is important to study, which is the asymmetric effects of monetary policy shocks on the endogenous variables.

In chapter 4 we extend a New Keynesian model considering preferences that exhibit intertemporal non-homotheticity. We show that introducing this feature generates a state-dependent intertemporal elasticity of substitution, which induces asymmetric shifts in aggregate demand in response to monetary policy shocks. This effect, in combination with a convex Phillips curve, generates in equilibrium asymmetric responses in output and inflation to monetary policy shocks similar to those observed in the data. In particular, a higher response of both output and inflation to policy shocks when the economy growth is temporarily high than when it is temporarily low.

The previous chapters studied the relationship between non-linearities in a New-Keynesian model and monetary policy under the implicit assumption that there exists a single policymaker. However, this is not the case for most central banks, since policy decisions are taken mostly collectively. In chapter 5 we analyse the effects of relaxing this assumption on the interest rate reaction of the central bank.

In chapter 5 we extend the New Keynesian Monetary Policy literature considering that monetary policy decisions are taken collectively in a committee. We introduce a Monetary Policy Committee (MPC), whose members have different preferences between output and inflation stabilisation and have to vote on the level of the interest rate.

The model presented in chapter 5 helps to explain interest rate smoothing from a political economy point of view, where MPC members face a bargaining problem on the level of the interest rate. Under this framework, the interest rate behaves non-linearly upon the lagged interest rate and expected inflation. Our approach can reproduce both features documented by the empirical evidence on interest rate smoothing: a) the modest response of the interest rate to inflation and output gap; and b) the dependence on lagged interest rate. Features that are difficult to reproduce altogether in standard New Keynesian models. It also provides a theoretical framework on how disagreement among policymakers can slow down the adjustment on interest rates and on menu costs in interest rate decisions. Furthermore, a numerical exercise shows that this inertial behaviour of the interest rate is internalised by the economic agents through an increase in expected inflation.

To sum up, the thesis provides a rigourous treatment of the key issues concerning non-linearities in general equilibrium New Keynesian models. We have analysed the effects in general equilibrium of volatility of shocks in the dynamics of the model. We have studied the optimal monetary policy reaction to a specific, but considerably important, type of shock. Also, we have analysed how monetary policy can generate asymmetric effects in general equilibrium and what are the effects of non-linear interest rate reaction functions in the dynamics of the model. These are important questions that traditional log-linear solutions are not able to analyse and some new techniques are needed.

In this thesis we apply new modern numerical techniques to solve for DSGE models exhibiting non-linear behaviour. In chapter 2 and 4 we apply the perturbation method [1] to solve, both analytically and numerically, for the effects of volatility of shocks on the level of variables and for the asymmetric responses to shocks. When solving analytically for the second order solution of the models, we follow a methodology similar to the one proposed by Sutherland (2002), which consist on using the first order solution of the model to obtain the terms of the second order solution. In section 5 we use the collocation method [2], to solve for the non-linear reaction

[1]The perturbation method was developed by Judd(1998), Collard and Julliard (2001) and Schmitt-Grohe and Uribe (2004). It consists on approximating the solution of the model with a taylor approximation around the steady-state of order higher than one.

[2]The collocation method consists on finding a function that approximates the solution at a

function of the central bank.

Finally, I must mention that in this thesis I have benefited from working with Paul Castillo and Vicente Tuesta on a series of joint papers, two of which form a basis of the second and fourth chapters of this thesis. The paper that form the basis for the second chapter is a joined work with both of them, whereas the fourth chapter is based on a joint work with Paul Castillo only. In both papers I worked together with my co-authors on the discussion of the models and the interpretation of the results, whilst I obtained analytically the second order solution of the models and I made the simulation exercises.

finite number of specified points. See Judd(1998) and Miranda and Fackler (2002) for discussion on collocation methods

CHAPTER 2

THE EFFECTS OF INFLATION VOLATILITY ON INFLATION

2.1 Introduction

In an influential paper, Clarida, Gali and Gertler (2000, from now on CGG), advanced the idea that the high average levels of inflation observed in the USA during the 1970s could be explained mainly by the failure of monetary policy to properly react to higher expected inflation. In addition, they pointed out that oil price shocks played a minor role in generating those levels of inflation. CGG based their conclusions on the estimations of monetary policy reaction functions for two periods: pre- and post-Volcker[1]. Their estimations show that during the 1970s, on average, the FED allowed the real short term interest rate to decline as expected inflation rose. Whereas, during the post-Volcker period became more active, by raising the real interest rate in response to higher inflation expectations. Cogley and Sargent (2002) and Lubik and Schorfheide (2004) find similar evidence.

This evidence, however, is not conclusive. In a series of papers, Sims and Zha (2005), Canova, Gambetti and Pappa (2005), Primiceri (2004), Gordon (2005) and Leeper and Zha (2003) find weak evidence of a substantial change in the reaction function of monetary policy after the Volcker period[2]. In particular, they find evidence that the fall on both the aggregate volatility and the average inflation is

[1]It refers to the appointment of Paul Volcker as Chairman of the Federal Reserve System of the USA.

[2]Orphanides (2001) shows that when real time data are used to estimate policy reaction functions, the evidence of a change in policy after 1980 is weak.

related to a sizeable reduction of the volatility of the main business cycle driving forces[3]. Moreover, they highlight that in order to estimate the reaction function of the central bank, it is necessary to consider changes in the variance of structural shocks. Otherwise, these estimations may be biased towards finding significant shifts in coefficients in the monetary policy rule.

Motivated by this recent evidence, in this chapter we provide an analytical and tractable framework that can be used to study the relationship between structural shocks volatility, in particular oil price shocks, and the average level of inflation. In doing so, we use a standard microfounded New Keynesian model with staggered Calvo pricing where the central bank implements its policy following a Taylor rule. We modify this simple framework considering oil as a production input for intermediate goods. A key assumption in our set up is that oil is difficult to substitute in production, thus we use a constant elasticity of substitution (CES) production function with an elasticity lower than one as a prime of our model. Under this assumption, oil price shocks generate an endogenous trade-off between stabilising inflation and output gap, thus a policy of zero inflation cannot be achieved at zero cost. This trade-off emerges when we allow for a distorted steady-state along the CES production function[4].

Then, we solve the rational expectations equilibrium of this model up to second order of accuracy using the perturbation method developed by Schmitt-Grohé and Uribe (2004). The second-order solution has the advantage of incorporating

[3]The literature has also associated oil prices to periods of recession. Bernanke, Gertler and Watson (1997) argue that monetary policy played a larger role during the 1970s in explaining the negative output dynamics. On the other hand, Hamilton (2001) and Hamilton and Herrada (2004) find out that the results of previous authors rely on a particular identification scheme and, on the contrary, they find that a contractionary monetary policy played only a minor role on the contractions in real output, oil prices being the main source of shock.

[4]Blanchard and Galí (2006) find that with a Cobb-Douglas production function, oil price shocks do not generate a trade-off between the stabilisation of inflation and output gap. In order to generate the trade-off, they rely on a reduced form of real rigidities in the labour market. In chapter 3 we characterise this trade-off from the quadratic approximation of the welfare of the representative agent

the effects of shocks volatility on the equilibrium, which are absent in the linear solution. We implement this method both analytically and numerically[5]. The former allows us to disentangle the key determinants of the relationship between volatility of oil price shocks and the average level of inflation, and the latter allows us to quantify the importance of each mechanism.

Using a similar model, CGG concluded that oil prices are notable to generate high average levels of inflation, unless monetary policy is passive. Instead our results give an important role to oil price volatility along an active monetary policy. In our set up, oil prices play a central role on inflation determination and on the trade-off faced by the central bank The key difference between CGG and our set up is that we use a second-order solution for the rational expectations equilibrium, instead of a log-linear one.

The second-order solution, by relaxing certainty equivalence, allows us to establish a link between the volatility of oil price shocks and the average level of inflation, absent in a log-linear model. We define this extra level of average inflation as the time varying *level of inflation premium*[6]. Moreover, the analytical solution allowed us to identify and to disentangle the sources of *inflation premium* in general equilibrium[7].

There are many novel results to highlight. First, the solution up to second order shows that oil price volatility produces an extra level of inflation by altering the way in which forward- looking firms set their prices. In particular, when oil has low substitutability, marginal costs are convex in oil prices, hence its price volatility

[5]As part of our contribution, we use a novel strategy for the analytical solution. In contrast with other papers in which the perturbation method is applied directly to the non-linear system of equations, instead we first approximate the model up to second order and then we apply the perturbation method to this approximated model.

[6]The extra level of inflation generated by volatility is similar to the effect of consumption volatility on the level of average savings as in the literature of precautionary savings.

[7]We are not aware of any other paper in the literature that has obtained and developed the concept of inflation premium in general equilibrium.

increases the expected value of marginal costs.

Second, oil price volatility, by generating inflation volatility, induces price-setters to be more cautious to future expected marginal costs. In particular, their relative price becomes more sensitive to marginal costs, amplifying the previous channel.

Third, relative price dispersion, by increasing the amount of labour required to produce a given level of output, increases average wages. So, relative price dispersion amplifies the effect of expected marginal costs over average inflation.

Fourth, we find that, in general equilibrium, the weight that the central bank puts on output fluctuations is a key determinant for positive level of inflation premium. As a result, we show that the larger the endogenous responses of a central bank to output fluctuations, the greater the level of inflation premium. This finding is consistent with the fact that, in the model, oil price shocks generate an endogenous trade-off between stabilising inflation and output gap. Hence a benevolent central bank would choose to put a positive weight on output gap stabilisation and would generate inflation premium.

Finally, we also evaluate the implications of the model with numerical exercises calibrated for the US economy. For the calibration, we consider that oil price shocks have exhibited a change in their volatility across the pre- and post-Volcker periods. Our results are broadly consistent with predictions of the analytical solution. Remarkably, we are able to generate a level of inflation premium similar to the one observed during the 1970s in the USA even when an active monetary policy, as in CGG, is in place. Also, we show in our simulated exercise that the convexity of the Phillips curve accounts for 59 percent of the inflation premium in the pre-Volcker period, whereas the effects of oil price volatility on marginal costs accounts for another 45 percent. Overall, we find that the model can track quantitatively the average values of inflation fairly well. We check the robustness of our results with alternative estimated Taylor rules, yet the qualitative results do not change. Hence, this chapter provides support to the empirical findings of Sims and Zha (2005) that second moments of shocks might be important to understand the change in macroeconomic behaviour observed in the US economy without relying on an accommodative monetary policy.

Closer to our work is the recent paper by Evans and Hnatkovska (2005), who evaluate the role of uncertainty in explaining differences in asset holdings in a two-country model. Also, in chapter 4 we build up a model with non-homothetic preferences and show how asymmetric responses of output and inflation emerge from the interaction of a convex Phillips curve and a state-dependent elasticity of substitution in a standard New Keynesian model. Finally, Obstfeld and Rogoff (1998) develop an explicit stochastic New Open Economy model relaxing the assumption of certainty equivalence. Based on simplified assumptions, they obtain analytical solutions for the level exchange rate premium. Differently from Obstfeld and Rogoff (1998) and the aforementioned authors, in this chapter we perform both a quantitative and analytical evaluation of the second-order approximation of the New Keynesian benchmark economy in order to account for the level of inflation premium generated by oil shocks volatility.

The plan of this chapter is as follows. Section 2.2 presents some stylised facts for the US economy on the relationship between oil price volatility and the level of inflation. Also, this section presents an informal explanation of the link between oil price volatility and the inflation mean. In section 2.3 we outline a benchmark New Keynesian model augmented with oil as a non-produced input and we discuss its implications for monetary policy. Section 2.4 explains the mechanism at work in generating the level of inflation premium and we also find the analytical solution of inflation premium. In section 2.5 we report the numerical results. In the last section we draw conclusions.

2.2 Motivation

2.2.1 Average Inflation and Oil Price Volatility

Inspection of US inflation data seems to suggest that the average inflation rate and the volatility of oil prices followed a similar pattern during the last 30 years. Figure 2.1 plot in the left hand axis, with a solid line the annual inflation rate of the US, measured by the non-farm business sector deflator (LXNFI), and in the right hand axis, with a dotted line, the real oil price in log [8]. As the figure shows, both the volatility of the real oil prices and the average quarterly annualised inflation rate has increased during the first half of the sample, 1970.1-1987.2, and has fallen in the second half, 1987.3-2005.2. In the first sub-sample, the standard deviation of real oil prices reached 0.57 and the average level of inflation 5.5 percent, whereas during the second sub-sample, the same statistics fall to 0.20 and 2.1 percent, respectively.

Interestingly, also the dynamics of inflation seems to closely mimics that of oil prices. Thus, in the first sub-sample we observe a persistent initial increase in inflation vis-a-vis and increase in oil prices following the oil price shock in 1974. Instead, from 1980 on we observe a steadily decline in inflation accompanied by a persistent drop in oil prices. For the second sub-sample, we observe also a close co-movement between inflation and oil prices; from early nineties until 1999 it is observed a downward trend in both oil prices and inflation, whereas from 2000 on we observe a markedly upward trend in oil and a moderate increase in inflation.

In a nutshell, the data seems to suggest that the change in oil prices volatility has some information on the behaviour of the inflation mean from the 1970s on. This causal evidence motivates the development of the model and the mechanism that we highlight in the coming sections in order to generate a link between average inflation and oil price volatility.

[8]We obtain the data from the Haver USECON database (mnemonics are in parentheses).

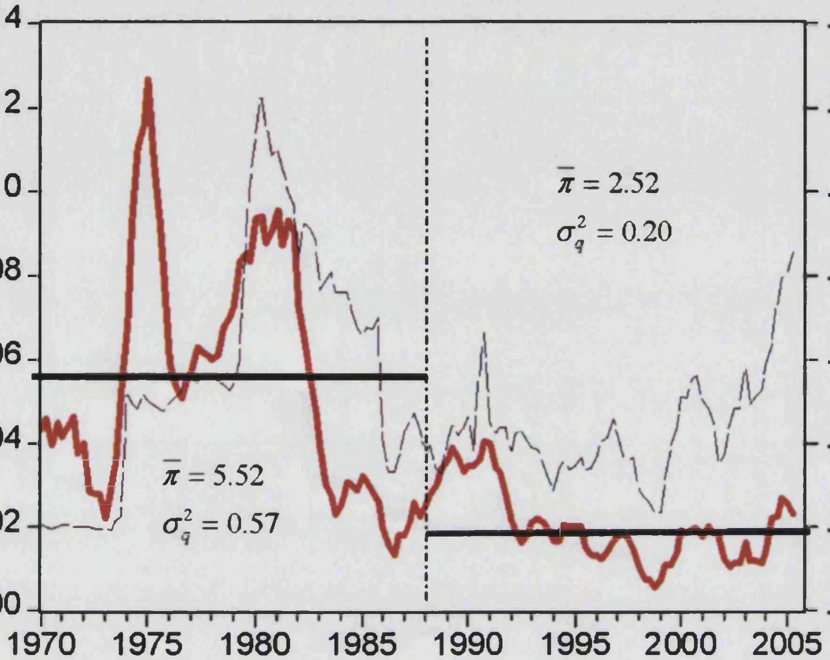

Figure 2.1: US inflation and oil prices

2.2.2 The link between average inflation and oil price volatility

As mentioned in the introduction the goal of this paper is to study the link between the volatility of oil price shocks and the average level of inflation in general equilibrium. Though, before moving to a fully general equilibrium analysis, in this section we provide the intuition of how the mechanism operates in a simple way. For that purpose, we use a simple two period price setting partial equilibrium model.

Consider that some firms producing a differentiated good set prices one period in advance. They face a downward sloping demand function of the type, $Y_t(z) = \left(\frac{P_t^*(z)}{P_t}\right)^{-\varepsilon} Y$, where ε represents the elasticity of substitution across goods and Y aggregate output, which we assume is fixed[9]. Under these assumptions, the optimal pricing decision of a particular firm z for time t is given by mark-up over the expected next period marginal cost,

$$\frac{P_t^*(z)}{P_{t-1}} = \mu E_{t-1}\left[\Psi_t MC_t\right] \tag{2-1}$$

where μ, MC_t and $\Psi_t = \frac{\Pi_t^{\varepsilon+1}}{E_{t-1}\Pi_t^{\varepsilon}}$ denote the mark-up, firm´s marginal costs and a measure of the responsiveness of the optimal price to future marginal costs, respectively. A second order Taylor expansion of the expected responsiveness to marginal cost is:

$$E_{t-1}\left[\Psi_t\right] = E_{t-1}\left[\pi_t + \frac{1}{2}\left(2\varepsilon+1\right)\pi_t^2\right] \tag{2-2}$$

$E_{t-1}\Psi_t$ is convex function on expected inflation, that means that inflation volatility increases the weight that a firm put on expected marginal costs. Furthermore, let's assume the following marginal cost function:

$$MC_t = \phi_1 q_t + \frac{\phi_2}{2} q_t^2 \tag{2-3}$$

where q_t represents the real price of oil, $\phi_1 > 0$ measures the linear effect of oil over the marginal cost and $\phi_2 > 0$ accounts for the impact of oil price volatility on marginal costs. When $\phi_2 > 0$, marginal costs are convex in oil prices, thus expected marginal costs become an increasing function of the volatility of oil prices.[10].

Different forms of aggregation of sticky prices in the literature show that the inflation rate is proportional to the optimal relative price of firms, given by equation (2-1). Hence, when marginal cost are convex, both the optimal relative price and inflation are increasing in oil price volatility. Interestingly, other channels

[9]This assumption helps to highlight the channels by which supply shocks as oil prices affect inflation. In section 2.4 we consider a fully general equilibrium model that deals with both sources of inflation fluctuations.

[10]In section 2.4 we show that when the production function is a CES with an elasticity of substitution between labour and oil lower than one, then the marginal cost are convex on oil prices, that is $\phi_2 > 0$.

amplify this effect. For instance, to the extent that oil price volatility increases inflation volatility, price setters react by increasing the weight they put on marginal costs, Ψ_t, when setting prices. As equation (2-2) shows, up to second order, this weight depends not only on the level of expected inflation but also on its volatility. Yet, are those second order effects important? Two special features of oil prices, its high volatility and its low substitutability with other production factors, make those second order effects quantitative sizable. Hence, a linear approximation that omits the role of oil price and inflation volatility would be very inaccurate in capturing the dynamics of inflation. We will overcome this restriction by using the perturbation method, which allows to obtain the second order solution of the rational expectations equilibrium of the model.

In the next section we formalise the previous informal link by obtaining a second order rational expectations solution of a New Keynesian general equilibrium model with oil prices. We use this model to show under which conditions both the marginal cost of firms become a convex function of oil price shocks. We also show how relative price distortions and monetary policy might amplify the effect of uncertainty, inducing a meaningful level of inflation premium.

2.3 A New Keynesian model with oil prices

The model economy corresponds to the standard New Keynesian Model in the line of CGG (2000). In order to capture oil shocks we follow Blanchard and Gali (2005) by introducing a non-produced input M, represented in this case by oil. Q denotes the real price of oil which is assumed to be exogenous.

2.3.1 Households

We assume the following period utility on consumption and labour

$$U_t = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\nu}}{1+\nu}, \tag{2-4}$$

where σ and ν represent the coefficient of risk aversion and the inverse of the elasticity of labour supply, respectively. The optimiser consumer takes decisions subject to a standard budget constraint which is given by

$$C_t = \frac{W_t L_t}{P_t} + \frac{B_{t-1}}{P_t} - \frac{1}{R_t}\frac{B_t}{P_t} + \frac{\Gamma_t}{P_t} + \frac{T_t}{P_t} \tag{2-5}$$

where W_t is the nominal wage, P_t is the price of the consumption good, B_t is the end of period nominal bond holdings, R_t is the nominal gross interest rate , Γ_t is the share of the representative household on total nominal profits, and T_t are transfers from the government[11]. The first order conditions for the optimising consumer´s problem are:

$$1 = \beta E_t \left[R_t \left(\frac{P_t}{P_{t+1}}\right) \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \right] \tag{2-6}$$

$$\frac{W_t}{P_t} = C_t^{\sigma} L_t^{\nu} = MRS_t \tag{2-7}$$

Equation $(2-6)$ is the standard Euler equation that determines the optimal path of consumption. At the optimum the representative consumer is indifferent between consuming today or tomorrow, whereas equation $(2-7)$ describes the optimal labour supply decision. MRS_t denotes the marginal rate of substitution between labour and consumption. We assume that labour markets are competitive and also that individuals work in each sector $z \in [0,1]$. Therefore, L corresponds to the aggregate labour supply:

$$L = \int_0^1 L_t(z)dz \tag{2-8}$$

[11]In the model we assume that the government owns the oil´s endowment. Oil is produced in the economy at zero cost and sold to the firms at an exogenous price Q_t. The government transfers all the revenues generated by oil to consumers represented by T_t

2.3.2 Firms

Final Good Producers

There is a continuum of final good producers of mass one, indexed by $f \in [0,1]$ that operate in an environment of perfect competition. They use intermediate goods as inputs, indexed by $z \in [0,1]$ to produce final consumption goods using the following technology:

$$Y_t^f = \left[\int_0^1 Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{2-9}$$

where ε is the elasticity of substitution between intermediate goods. Then the demand function of each type of differentiated good is obtained by aggregating the input demand of final good producers

$$Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon} Y_t \tag{2-10}$$

where the price level is equal to the marginal cost of the final good producers and is given by:

$$P_t = \left[\int_0^1 P_t(z)^{1-\varepsilon} dz \right]^{\frac{1}{1-\varepsilon}} \tag{2-11}$$

and Y_t represents the aggregate level of output.

$$Y_t = \int_0^1 Y_t^f df \tag{2-12}$$

Intermediate Goods Producers

There is a continuum of intermediate good producers. All of them have the following CES production function

$$Y_t(z) = \left[(1-\alpha)\left(L_t(z)\right)^{\frac{\psi-1}{\psi}} + \alpha\left(M_t(z)\right)^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}} \tag{2-13}$$

where M is oil which enters as a non-produced input, ψ represents the intratemporal elasticity of substitution between labour-input and oil and α denotes the share of oil in the production function. We use this generic production function in order to capture the fact that oil has few substitutes[12], in general we assume that.ψ is lower than one. The oil price shock, Q_t, is assumed to follow an $AR(1)$ process in logs,

$$\log Q_t = \log \overline{Q} + \rho \log Q_{t-1} + \varepsilon_t \tag{2-14}$$

where $\overline{Q}$ is the steady state level of oil price. From the cost minimisation problem of the firm we obtain an expression for the real marginal cost given by:

$$MC_t(z) = \left[(1-\alpha)^{\psi}\left(\frac{W_t}{P_t}\right)^{1-\psi} + \alpha^{\psi}\left(Q_t\right)^{1-\psi}\right]^{\frac{1}{1-\psi}} \tag{2-15}$$

where $MC_t(z)$ represents the real marginal cost, W_t nominal wages and P_t the consumer price index. Note that since technology has constant returns to scale and factor markets are competitive, marginal costs are the same for all intermediate firms, i.e. $MC_t(z) = MC_t$. On the other hand, the individual firm´s labour demand is given by:

$$L_t^d(z) = \left(\frac{1}{1-\alpha}\frac{W_t/P_t}{MC_t}\right)^{-\psi} Y_t(z) \tag{2-16}$$

Intermediate producers set prices following a staggered pricing mechanism

[12]Since oil has few substitutes an appealing functional form to capture this feature is the CES production function. This function offers flexibility in the calibration of the degree of substitution between oil and labour. Some authors that have included oil in the analysis of RBC models and monetary policy, have omitted this feature. For example, Kim and Loungani (1992) assume for the U.S. a Cobb-Douglas production function between labour and a composite of capital and energy. Given that they calibrate their model considering that oil has a small share on output, they found that the impact of oil in the U.S. business cycle is small . Notice that when $\psi = 1$, the production function collapses to the standard Cobb-Douglas function as the one used by Blanchard and Gali (2005): $Y_t(z) = (L_t(z))^{1-\alpha} M_t^{\alpha}$.

a la Calvo. Each firm faces an exogenous probability of changing prices given by $(1-\theta)$. The optimal price that solves the firm's problem is given by

$$\left(\frac{P_t^*(z)}{P_t}\right) = \frac{\mu E_t\left[\sum_{k=0}^{\infty} \theta^k \zeta_{t,t+k} MC_{t+k} F_{t,t+k}^{\varepsilon+1} Y_{t+k}\right]}{E_t\left[\sum_{k=0}^{\infty} \theta^k \zeta_{t,t+k} F_{t,t+k}^{\varepsilon} Y_{t+k}\right]} \tag{2-17}$$

where $\mu \equiv \frac{\varepsilon}{\varepsilon-1}$ is the price markup, $\zeta_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}}$ is the stochastic discount factor, $P_t^*(z)$ is the optimal price level chosen by the firm, $F_{t,t+k} = \frac{P_{t+k}}{P_t}$ the cumulative level of inflation and Y_{t+k} is the aggregate level of output.

Since only a fraction $(1-\theta)$ of firms changes prices every period and the remaining one keeps its price fixed, the aggregate price level, the price of the final good that minimise the cost of the final goods producers, is given by the following equation:

$$P_t^{1-\varepsilon} = \theta P_{t-1}^{1-\varepsilon} + (1-\theta)\left(P_t^*(z)\right)^{1-\varepsilon} \tag{2-18}$$

Following Benigno and Woodford (2005), equations $(2-17)$ and (2-18) can be written recursively introducing the auxiliary variables N_t and D_t (see appendix A2 for details on the derivation):

$$\theta\left(\Pi_t\right)^{\varepsilon-1} = 1-(1-\theta)\left(\frac{N_t}{D_t}\right)^{1-\epsilon} \tag{2-19}$$

$$D_t = Y_t\left(C_t\right)^{-\sigma} + \theta\beta E_t\left[\left(\Pi_{t+1}\right)^{\epsilon-1} D_{t+1}\right] \tag{2-20}$$

$$N_t = \mu Y_t\left(C_t\right)^{-\sigma} MC_t + \theta\beta E_t\left[\left(\Pi_{t+1}\right)^{\epsilon} N_{t+1}\right] \tag{2-21}$$

Equation $(2-19)$ comes from the aggregation of individual firms prices. The ratio N_t/D_t represents the optimal relative price $P_t^*(z)/P_t$. Equations (2-19), (2-20) and (2-21) summarise the recursive representation of the non- linear Phillips curve. Writing the optimal price setting in a recursive way is necessary in order to implement both numerically and algebraically the perturbation method.

2.3.3 Monetary Policy

The central bank conducts monetary policy by targeting the nominal interest rate in the following way

$$R_t = R_{t-1}^{\phi_r} \left[\overline{R} \left(\frac{E_t \Pi_{t+1}}{\overline{\Pi}} \right)^{\phi_\pi} \left(\frac{Y_t}{\overline{Y}} \right)^{\phi_y} \right]^{1-\phi_r} \tag{2-22}$$

where, $\phi_\pi > 1$ and $\phi_y > 0$ measure the response of the nominal interest rate to expected future inflation and output, respectively. Also, the degree of interest rate smoothing is measured by $0 \leq \phi_r \leq 1$. The steady state values are expressed without time subscript and with and upper bar.

2.3.4 Market Clearing

In equilibrium labour, intermediate and final goods markets clear. Since there is neither capital accumulation nor government sector, the economywide resource constraint is given by

$$Y_t = C_t \tag{2-23}$$

The labour market clearing condition is given by:

$$L_t^s = L_t^d \tag{2-24}$$

Where the demand for labour comes from the aggregation of individual intermediate producers in the same way as the labour supply:

$$
\begin{aligned}
L^d &= \int_0^1 L_t^d(z)dz = \left(\frac{1}{1-\alpha}\frac{W_t/P_t}{MC_t}\right)^{-\psi} \int_0^1 Y_t(z)dz \qquad (2\text{-}25)\\
L^d &= \left(\frac{1}{1-\alpha}\frac{W_t/P_t}{MC_t}\right)^{-\psi} Y_t\Delta_t
\end{aligned}
$$

where $\Delta_t = \int_0^1 \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon} dz$ is a measure of price dispersion. Since relative prices differ across firms due to staggered price setting, input usage will differ as well. Implying that it is not possible to use the usual representative firm assumption. Therefore, the price dispersion factor, Δ_t appears in the aggregate labour demand equation. Also, from (2-25) we can see that higher price dispersion increases the labour amount necessary to produce a given level of output.

2.3.5 The Log Linear Economy

To illustrate the effects of oil in the dynamic equilibrium of the economy, we take a log linear approximation of equations (2-6), (2-7),(2-15), (2-19), (2-20), (2-21), (2-22) and (2-25) around the deterministic steady-state[13]. We denote variables in steady state with upper bar (i.e. $\overline{X}$) and their log deviations around the steady state with lower case letters (i.e. $x = \log(\frac{X_t}{X})$). After, imposing the goods and labour market clearing conditions to eliminate real wages and labour from the system, the dynamics of the economy is determined by the following equations:

$$mc_t = \chi\,(\nu + \sigma)\,y_t + (1-\chi)\,q_t \qquad (2\text{-}26)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa mc_t \qquad (2\text{-}27)$$

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}\left(r_t - E_t \pi_{t+1}\right) \qquad (2\text{-}28)$$

[13]See appendix A1 for the derivation of the steady-state of the economy.

$$r_t = \phi_r r_{t-1} + (1 - \phi_r)(\phi_\pi E_t \pi_{t+1} + \phi_y y_t) \tag{2-29}$$

$$q_t = \rho q_{t-1} + \eta \sigma_q e_t \tag{2-30}$$

where, $\chi \equiv \frac{1-\alpha^F}{1+v\psi\alpha^F}$, $\alpha^F \equiv \alpha^\psi \left(\frac{\overline{Q}}{\overline{MC}}\right)^{1-\psi}$, $\kappa \equiv \frac{1-\theta}{\theta}(1-\theta\beta)$; and $\overline{Q}$, and $\overline{MC}$, represent the steady-state value of oil prices and of marginal costs, respectively.

Interestingly, the effects of oil prices on marginal costs, equation (2-26), depends crucially on both the share of oil in the production function, α, and the elasticity of substitution between oil and labour,ψ. Thus, when α is large, χ is small making marginal costs more responsive to oil prices. Also, the smaller the ψ, the greater the impact of oil on marginal costs. It is important to note that even though the share of oil in the production function, α, can be small, its impact on marginal cost, α^F, can be magnified when oil has few substitutes (that is when ψ is low) [14]. Note also that a permanent increase in oil prices, that is an increase in $\overline{Q}$, makes marginal cost of firms more sensitive to oil price shocks given its effect over α^F. Finally, when $\alpha = 0$, the model collapses to a standard close economy New Keynesian model without oil.

The model also has a key implication for monetary policy. Notably, it delivers an endogenous trade-off for the central bank when stabilising inflation and output gap. We denote output gap by x_t and it is defined as the difference between the sticky price level of output and its corresponding efficient level, $x_t = y_t - y_t^E$, where y_t^E denotes the log deviations of the efficient level of output. In this economy, the efficient allocation is achieved when $\overline{MC} = 1$, since this equilibrium corresponds to one where intermediate firms are perfectly competitive. Therefore, when the equilibrium is efficient we have that $\alpha^F \neq \alpha^E$, where, $\alpha^E = \alpha^\psi \left(\overline{Q}\right)^{1-\psi}$. Using the previous definition of output gap, the economy can be represented by two equations in terms of the efficient output gap, x_t and inflation, π_t (see appendix A3 for

[14]For example, considering an oil share in the order of 1%. and an elasticity of substitution of 0.6, and assuming $\overline{Q} = \overline{W/P} = \overline{MC}$, gives $\alpha^F = (0.01)^{0.56} = 6\%$. This share would be even higher if we consider a higher steady state value of the oil price,$\overline{Q}$.

details),

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - r_t^E\right) \quad (2\text{-}31)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_y x_t + \mu_t \quad (2\text{-}32)$$

where $\mu_t = \frac{\kappa_q}{\kappa_y}\left(\frac{1}{(1-\alpha^E)\alpha^F}\right)\left(\alpha^F - \alpha^E\right) q_t$, $\kappa_q = (1-\chi)$ and $\kappa_y = \chi(\nu + \sigma)$. In our model the endogenous trade off emerges from the combination of a distorted steady state and a CES production function[15]. When the elasticity of substitution between oil and labour is equal to one, the Cobb-Douglas case as in Blanchard and Gali, the trade off disappears. Hence, in that case, the flexible and efficient level of output only differ by a constant term, which in turn implies that $\alpha^E = \alpha^F$. In addition, when monopolistic competition distortion is eliminated, using a proportional subsidy tax, as in Woodford (2003), the trade-off is inhibited, since again $\alpha^E = \alpha^F$. The existence of this endogenous trade off implies that is optimal for the central bank to allow higher levels of inflation in response to supply shocks.

The special features of oil, such as high price volatility and low substitutability in production, induce the volatility of oil prices to have non trivial second order effects that the log-linear representation described by equations (2-26) to (2-30) does not takes into account[16]. These second order effects are crucial elements in establishing the link between oil price volatility and inflation premium. The next section provides a log-quadratic approximation of the economy around its steady-state to study the link between oil price volatility and inflation.

[15]Benigno and Woodford (2005), in a similar model but without oil price shocks, have found an endogenous trade-off by combining a distorted steady state with a government expenditure shock. In their framework, the combination of a distorted steady state along with a non-linear aggregate budget constraint due to government expenditure is crucial for the existence of this endogenous trade-off. Analogously to Benigno and Woodfords finding, in our model the combination of a distorted steady state and the non-linearity of the CES production function delivers a trade-off when considering an efficient level of output such that eliminate monopolistic distortions. However, in chapter 3 we demonstrate that we still have this trade-off even when monopolistic distortions are eliminated

[16]In a log-linear representation certainty equivalence holds, thus uncertainty does not play any role.

2.4 Inflation Premium in General Equilibrium

2.4.1 The second order representation of the model

In this sub-section we present a log-quadratic (Taylor series) approximation of the fundamental equations of the model around the steady state. A detailed derivation is provided in Appendix A2. The second-order Taylor-series expansion serves to compute the equilibrium fluctuations of the endogenous variables of the model up to a residual of order $O\left(\|q_t, \sigma_q\|^2\right)$, where $\|q_t, \sigma_q\|$ is a bound on the deviation and volatility of the oil price generating process around its steady state[17]. Up to second order, equations (2-26) - (2-29) are replaced by the following set of log-quadratic equations:

Aggregate Supply	
Marginal Costs	
$mc_t = \kappa_y y_t + \kappa_q q_t + \frac{1}{2}(1-\chi)\chi^2 \frac{1-\psi}{1-\alpha^F}\left((\nu+\sigma) y_t - q_t\right)^2 + \chi v \widehat{\Delta}_t + O\left(\|q_t, \sigma_q\|^3\right)$	$2-i$
Price dispersion	
$\widehat{\Delta}_t = \theta \widehat{\Delta}_{t-1} + \frac{1}{2}\varepsilon \frac{\theta}{1-\theta}\pi_t^2 + O\left(\|q_t, \sigma_q\|^3\right)$	$2-ii$
Phillips Curve	
$v_t = \kappa mc_t + \frac{1}{2}\kappa mc_t\left(2(1-\sigma) y_t + mc_t\right) + \frac{1}{2}\varepsilon\pi_t^2 + \beta E_t v_{t+1} + O\left(\|q_t, \sigma_q\|^3\right)$	$2-iii$
where we have defined the auxiliary variables:	
$v_t \equiv \pi_t + \frac{1}{2}\left(\frac{\varepsilon-1}{1-\theta} + \varepsilon\right)\pi_t^2 + \frac{1}{2}(1-\theta\beta)\pi_t z_t$	$2-iv$
$z_t \equiv 2(1-\sigma) y_t + mc_t + \theta\beta E_t\left(\frac{2\varepsilon-1}{1-\theta\beta}\pi_{t+1} + z_{t+1}\right) + O\left(\|q_t, \sigma_q\|^2\right)$	$2-v$
Aggregate Demand	
$y_t = E_t y_{t+1} - \frac{1}{\sigma}\left(r_t - E_t\pi_{t+1}\right) - \frac{1}{2}\sigma E_t\left[(y_t - y_{t+1}) - \frac{1}{\sigma}(r_t - \pi_{t+1})\right]^2 + \left(\|q_t, \sigma_q\|^3\right)$	$2-vi$

Table 2.1: Second order Taylor expansion of the equations of the model

Equation (2-i) is obtained taking a second-order Taylor-series expansion of the real marginal cost equation, and using the labour market equilibrium condition

[17]Since we want to make explicit the effects of changes in the volatility of oil prices in the equilibrium of the endogenous variables, we solve the policy functions as in Schmitt-Grohe and Uribe (2004) in terms of q_t and σ_q. This is different to the approach taken by other authors, for example Woodford (2003), who consider the policy function in terms of the shocks (e_t).

to eliminate real wages. $\widehat{\Delta}_t$ is the log-deviation of the price dispersion measure Δ_t, which is a second order function of inflation and its dynamics is represented by equation (2-ii). Importantly, the second order approximation adds two new ingredients in the determination of marginal costs. The first one is related to the convexity of marginal costs respect to oil prices. From this expression, when, $\psi < 1$, marginal costs become a convex function of oil prices, hence, the volatility of oil prices increases expected marginal costs. This is an important channel through which oil price volatility generates higher inflation rates. Notice, however that when the production function is a Cobb-Douglas, $\psi = 1$, this second order effect disappears, and the marginal cost equation does not depends directly on the volatility of oil prices, but only indirectly through its effects on $\widehat{\Delta}_t$. In this particular case, marginal costs are given by,

$$mc_t = \kappa_y y_t + \kappa_q q_t + \chi v \widehat{\Delta}_t$$

the second new ingredient is associated to the indirect effect of oil price volatility through $\widehat{\Delta}_t$. From equation (2-25), it is clear that as price dispersion increases, the required number of hours to produce a given level of output also rises. Thus, this higher labour demand increases real wages, and consequently marginal costs. This effect is higher when the elasticity of labour supply, $\frac{1}{v}$ is lower and when the participation of oil in production is higher.

Equations (2-iii), (2-iv) and (2-v) in turn represent the second order version of the Phillips curve, and equation (2-vi) is the quadratic representation of the aggregate demand which includes the negative effect of the real interest rate on consumption and the precautionary savings effect. The second order representation of the aggregate demand considers, additional to the linear approximation, the effect of the volatility of the growth rate of consumption on savings. Indeed, when the volatility of consumption increases, consumption falls, since households increase their savings for precautionary reasons. Next we further simplify the model economy by witting it as a second order two equation system of output and inflation. This canonical second order representation of the economy with oil allows us to discuss in a simple way the determinants of the inflation premium.

2.4.2 Determinants of Inflation Premium

Since the second order terms of the equations (2-i) - (2-vi) depend on the first order solution of the model, we can use the latter to express the second order terms as quadratic functions of the oil process as in Sutherland (2002). Then, we replace equations (2-i),(2-ii),(2-iv) and (2-v)in (2-iii), and the policy rule of the central bank in equation (2-vi), to write the model as second order system of two equations on inflation, output and the oil price[18]:

$$\pi_t = \kappa_y y_t + \kappa_q q_t + \beta E_t \pi_{t+1} + \frac{1}{2}\omega_v \sigma_q^2 + \frac{1}{2}\left(\Omega_{mc} + \Omega_\pi + \Omega_v\right) q_t^2 + O\left(\left\|q_t, \sigma_q\right\|^3\right) \quad (2\text{-}33)$$

$$y_t = E_t\left(y_{t+1}\right) - \frac{1}{\sigma}\left(\left(\phi_\pi - 1\right) E_t \pi_{t+1} + \phi_y y_t\right) + \frac{1}{2}\omega_y \sigma_q^2 + O\left(\left\|q_t, \sigma_q\right\|^3\right) \quad (2\text{-}34)$$

where κ_y and κ_q were defined in the previous section.

We represent the second order terms as function of σ_q^2, q_t^2 and the *"omega"* coefficients $\{\Omega_{mc}, \Omega_\pi, \Omega_v, \omega_v, \omega_y\}$, which are defined in appendix A.2. Each of these "omega" coefficients represent the second order term in the equations for the marginal costs (subscript mc), the Phillips Curve (subscript π) , the auxiliary variable v_t (subscript v) and the aggregate demand (subscript y). Given $\{q_t\}$, the rational expectations equilibrium for $\{\pi_t\}$ and $\{y_t\}$ is obtained from, equations (2-33) and (2-34).

The *"omega"* coefficients are the sources of inflation premium in general equilibrium and capture the interaction between the nonlinearities of the model and

[18] To make the analysis analytically tractable , we have eliminated state variables such us the lagged nominal interest rate by setting the smoothing parameter in the Taylor rule equal to zero. Similarly, we assume an small initial price dispersion, that is $\widehat{\Delta}_{t_o-1} \approx 0$ up to second order. However, in the next section, the numerical exercises consider the more general specification of the model.

the volatility of oil price shocks. Coefficients denoted by capital omega (Ω) represent the time variant components of the inflation premium, whereas coefficients denoted by small omega (ω) are time invariant and depend on the unconditional variance of oil prices. Note that if the aforementioned coefficients were equal to zero the model would collapse to a standard version of a New Keynesian model in log linear form. In what follows we provide economic interpretation to the determinants of the inflation premium.

The **coefficient** Ω_{mc} captures the direct effect of oil price volatility on marginal costs and its indirect effect through the labour market. Let's consider first its direct effect. When oil has few substitutes, $\psi < 1$, marginal costs are convex in oil prices, hence expected marginal costs become an increasing function of oil price volatility. To compensate the increase in expected marginal costs generated by oil price volatility, forward looking firms react by optimally charging higher prices. This response of firms, in turn, leads to higher aggregate inflation when prices are sticky [19]. Interestingly, the increase on marginal costs and inflation in response to oil price volatility is larger when the elasticity of substitution between oil and labour is small.

Additionally, oil price volatility affects marginal cost indirectly, through its effects on the labour market. Since oil price volatility generates inflation volatility, which is costly because it increases relative price distortions, efficiency in production falls as the volatility of oil prices increases. In particular, firms require, at the aggregate level, more hours of work to produce the same amount of output. Hence, the demand for labour rises, making labour more expensive and increasing marginal cost even further. Then, the increase in marginal costs through both effects, the direct and indirect, lead to an increases on aggregate inflation.

[19]This mechanism can be understood by observing equation (2-i), where $\frac{\partial^2 mc_t}{\partial q_t^2} = (1-\chi)\chi^2 \frac{1-\psi}{1-\alpha^F}$. When $\psi < 1(\psi > 1)$, $\frac{\partial^2 mc_t}{\partial q_t^2} < 0(> 0)$

We illustrate these mechanisms in figure 2.2. In panel (a) we plot the relation between Ω_{mc} and the parameter ψ. We see that Ω_{mc} increases exponentially as ψ decreases. Also, the steady state oil price affects the impact of oil prices in marginal costs: the higher the oil price in steady-state, *ceteris paribus*, also the higher the effect of oil price volatility on marginal costs. According to this, in economies where oil is more difficult to substitute in production, or when the oil price level is relatively high, oil price volatility would be more important in the determination of the dynamics of inflation. Similarly, in panel (b) we plot the relation between Ω_{mc} and the elasticity of labour supply $(1/v)$. We see that a more elastic labour supply increase the effects of oil price volatility. This latter effect works through the indirect impact of oil price volatility on the labour market.

On the other hand, the **coefficient** Ω_π accounts for the effects of oil price volatility on the way price setters weight future marginal costs. When prices are sticky and firms face a positive probability of not being able to change prices, as in the Calvo price-setting model, the weight that firms assign to future marginal cost depends on both future expected inflation and future expected inflation's volatility. Oil price volatility by raising inflation volatility induces prices setters to put a higher weight on future marginal costs. Hence, oil price volatility not only increases expected marginal costs but also make relative price of firms more responsive to those future marginal costs.

Panel (c) shows that when the elasticity of substitution of goods ε increases, it increases the effect of inflation volatility on the price of individual firms and Ω_π increases. Similarly, panel (d) shows that lower price stickiness θ makes the Phillips curve stepper and also more convex, then the effects of inflation volatility on Ω_π increases.

The **coefficients** Ω_v **and** ω_v accounts for the time variant and constant effects of inflation volatility on the composite of inflation v_t. This mechanism is similar to that of Ω_π, however both coefficients are quantitatively small. Finally, the **coefficient** ω_y is negative and accounts for the standard precautionary savings effect, by which the uncertainty that oil price volatility generates induces households to increase savings to buffer future states of the nature where income can be low.

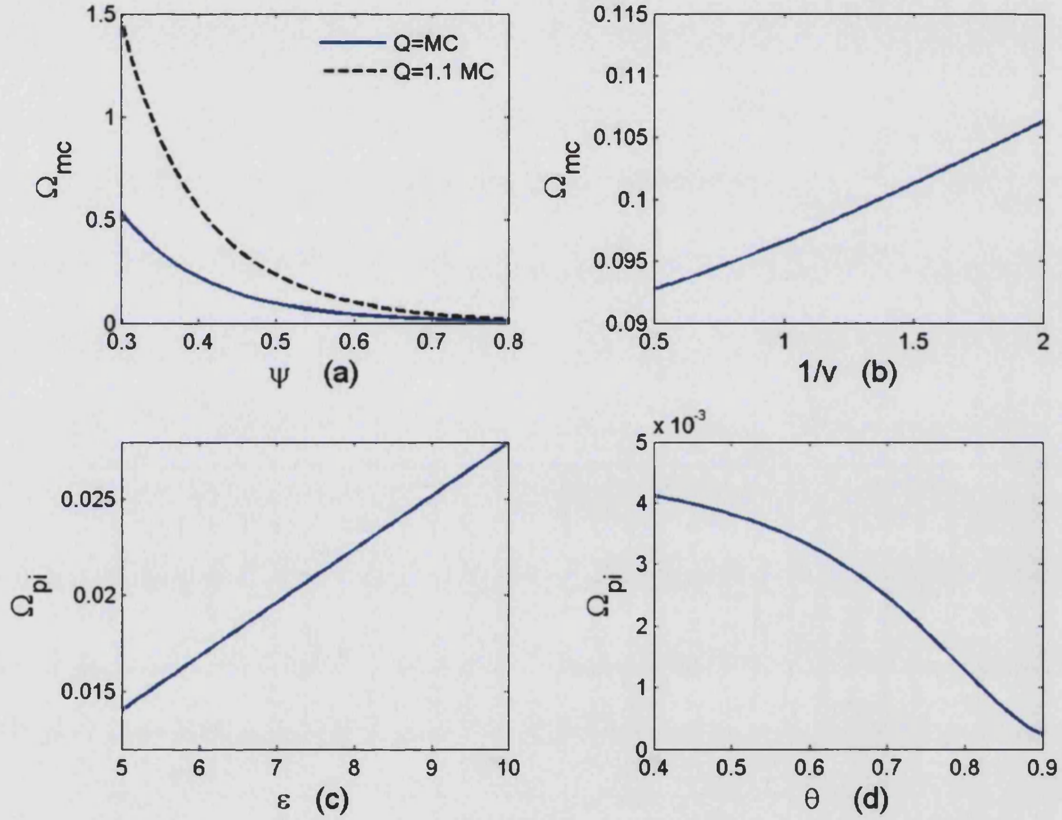

Figure 2.2: Inflation premium components (Uses benchmark calibration presented in section 2.5). (a) Effects of elasticity of substitution (ψ) on Ω_{mc}. (b) Effects of labour supply elasticity ($1/v$) on Ω_{mc}. (c) Effects of elasticity of substitution of goods (ε) on Ω_π. (d) Effects of elasticity of price stickiness (θ) on Ω_π.

2.4.3 The analytical solution for inflation premium

We use the perturbation method, implemented by Schmitt-Grohe and Uribe (2004)[20], to obtain the second order rational expectations solution of the model. The second order solution makes explicitly the potential effects of oil price´s volatility and the dynamics of endogenous variables. As we mentioned before, we define inflation premium as the extra level of inflation that arises in equilibrium once the second order solution is considered[21]. Also, different from other papers which apply perturbation methods directly to the non-linear system of equations, we first approximate the model up to second order and then apply the perturbation method[22]. Our proposed approach has the advantage that makes easier to obtain clear analytical results for the sources of the level of inflation premium.

The rational expectations second order solution of output and inflation, in log-deviations from the steady state, can be written as quadratic polynomials in both the level and the standard deviation of oil prices:

$$y_t = \frac{1}{2}a_o\sigma_q^2 + a_1 q_t + \frac{1}{2}a_2 (q_t)^2 + O\left(\|q_t, \sigma_q\|^3\right) \tag{2-35}$$

$$\pi_t = \frac{1}{2}b_o\sigma_q^2 + b_1 q_t + \frac{1}{2}b_2 (q_t)^2 + O\left(\|q_t, \sigma_q\|^3\right) \tag{2-36}$$

where the a's and b's are the unknown coefficients that we need to solve for and

[20]The perturbation method was originally developed by Judd (1998) and Collard and Julliard (2001). The fixed point algorithm proposed by Collard and Julliard introduces a dependence of the coefficients of the linear and quadratic terms of the solution with the volatility of the shocks. In contrast, the advantage of the algorithm proposed by Schmitt-Grohe and Uribe is that the coefficients of the policy are invariant to the volatility of the shocks and the corresponding ones to the linear part of the solution are the same as those obtained solving a log linear approximated model, which makes both techniques comparable.

[21]It is important to remark that this extra level of average inflation is part of the dynamic rational expectations equilibrium up to second order, and it can not be interpreted as a part of the steady state equilibrium. This second order effect on the level inflation is similar to the effect of the volatility of consumption on savings that is known in the literature as precautionary savings.

[22]Since a second order Taylor expansion is an exact approximation up to second order of any non-linear equation, having the system expressed in that way would give the same solution as the system in its non-linear form.

$O\left(\|q_t,\sigma_q\|^3\right)$ denotes terms on q and σ_q of order equal or higher than 3[23]. Notice that the linear terms ($a_1 q_t$ and $b_1 q_t$) correspond to the policy functions that we would obtain using any standard method for linear models (i.e. undetermined coefficients), whereas the additional elements account for the effects of uncertainty (premium) on the equilibrium variables.

The quadratic terms in the policy function of inflation have two components: $\frac{1}{2}b_o\sigma_q^2$, which is constant and $\frac{1}{2}b_2\left(q_t\right)^2$, which is time varying. The analytical solution obtained with the perturbation method implies the following expression for the overall expected level of inflation premium

$$E(\pi) = \frac{1}{2}\left(b_o + b_2\right)\sigma_q^2$$

which can be expressed as:

$$E(\pi) = \frac{1}{2}\frac{1}{\Lambda_0}\left[\phi_y\left(\Omega_{mc} + \Omega_\pi + \Omega_v\right)(1+\Psi) + \phi_y\omega_v + \sigma\kappa_y\omega_y\right]\sigma_q^2 \tag{2-37}$$

for $\Lambda_0 = (\phi_\pi - 1)\kappa_y + (1-\beta)\phi_y > 0$ and $\Psi > 0$ defined in the appendix A.2. According to this closed form, the inflation premium is proportional to the oil price volatility and depends on a linear combination of the *"omega's"* coefficients. Moreover, these sources of inflation premium interact with monetary policy to determine the sign and size of the premium. Under a Taylor rule, inflation premium will be positive if monetary policy reacts also to fluctuations in output due to oil shocks. From equation (4.11), the inflation premium will be positive when :

$$\phi_y > -\omega_y\sigma\kappa_y / \left[\omega_v + \left(\Omega_{mc} + \Omega_\pi + \Omega_v\right)(1+\Psi)\right] > 0 \tag{2-38}$$

since ω_y is negative, the right hand side is positive. When the coefficient of output fluctuations in the Taylor rule, ϕ_y, is positive and above this threshold, then the inflation premium is always positive. The higher ϕ_y, the higher the inflation premium. Therefore, when the central bank reacts also to output fluctuations it

[23]Schmitt-Grohe and Uribe (2004) show that the quadratic solution does not depend neither on σ_q nor on $q_t\sigma_q$. That is, they show that the coefficients in the solution for those terms are zero.

also generates, in equilibrium, an inflation premium. Yet, if the central bank cares only about inflation and does not react to output fluctuations, that is $\phi_y = 0$, then the inflation premium would be negative and small. Although oil price volatility is an important determinant of inflation, the previous result shows that in general equilibrium, the reaction of the central bank turns out to be crucial. A central bank that reacts only to inflation can fully eliminate the effects of oil price volatility on inflation raising output volatility. However, this type of reaction would come at a considerable cost, since output fluctuations are inefficient when they are generated by oil price shocks.

In figure 2.3, we depict the relation between the level of inflation premium an the parameter ϕ_y. There is a small positive threshold for ϕ_y such that the premium becomes positive. Also, the higher the reaction to output fluctuations, the higher the premium. Remarkably, the existence of the inflation premium depends crucially on the existence of a trade-off between inflation and output. When the central bank does not face this trade-off, it is always possible to find a policy rule where the inflation premium is zero. The previous implication steams from the fact that the second order solution depends upon the log-linear one[24]. Therefore, in order to observe a positive inflation premium a necessary condition is the existence of an endogenous trade-off for the central bank. Moreover, as shown in the previous section, such trade-off exists when the elasticity of substitution between oil and labour is lower than one.

2.5 Some Numerical Experiments

In this section we explore the ability of the model to explain high average levels of inflation in periods of high volatility of oil prices. To obtain the numerical results

[24]In a log-linear solution, when the central bank does not face a meaningful trade-off between stabilising inflation and output, the optimal policy implies both zero inflation and output gap.

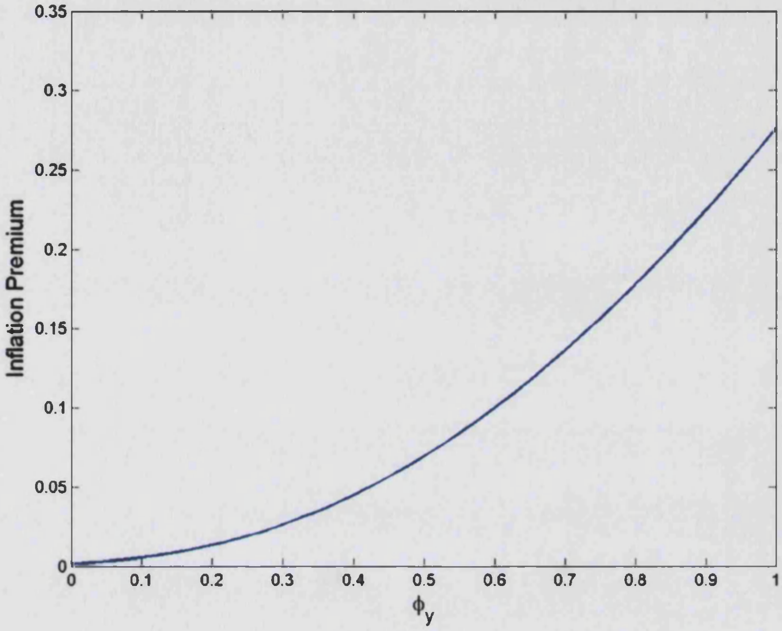

Figure 2.3: Inflation premium and the output parameter (ϕ_y) in the policy rule

we use the method developed by Schmitt-Grohe and Uribe (2004), which provides second order numerical solutions to non-linear rational expectations models .

2.5.1 Calibration

To calibrate the model we choose standard parameter values in the literature. We set a quarterly discount factor, β, equal to 0.99 which implies an annualised rate of interest of 4%. For the coefficient of risk aversion parameter, σ, we choose a value of 1 and the inverse of the elasticity of labour supply, v, is calibrated to be equal to 0.5, similar to those used in the *RBC* literature and consistent with the micro evidence. We choose a degree of monopolistic competition, ε, equal to 11, which implies a firm mark-up of 10% over the marginal cost. The steady state level of oil price, $\overline{Q}$ is set equal to the inverse of the mark-up in order to isolate the effect of the share of oil in the production function. The elasticity of substitution between oil and labour, ψ, is set equal to 0.6 and we use modest value for $\alpha = 0.01$, so that the share of oil prices in the marginal cost is around 6%[25]. The probability of the Calvo lottery is set equal to 0.66 which implies that firms adjust prices, on average, every three quarters. Finally, the log of real oil price follows an $AR(1)$ stochastic process with $\rho_q = 0.95$ and standard deviation, $\sigma_\varepsilon = 0.14$ for the first sample and $\rho_q = 0.82$ and standard deviation, $\sigma_\varepsilon = 0.13$ for the second one. These processes imply standard deviations for real oil prices of 0.46 and 0.22 in each sample, respectively. Our benchmark monetary policy rule is the estimated by CGG for the post-Volcker period.. We also perform robustness exercises by comparing the results of this benchmark rule with those obtained with the estimated rules by Orphanides (2001) and Judd and Rudebush (1998)[26]. The coefficients of the alternative policy rules analysed are presented in the following table:

	CGG	Taylor	Orphanides	Judd-Rudebush
ϕ_r	0.79	0.00	0.79	0.72
ϕ_π	2.15	1.53	1.80	1.54
ϕ_y	0.93	0.77	0.27	0.99

Table 2.2: Alternative Policy Rule Coefficients

[25]We consider a conservative calibration for the share of oil in production. Other authors have considered a larger share of oil in production or costs. For example, Atkenson and Kehoe (1999) use a share of energy in production of 0.043 and Rotemberg and Woodford (1996) a share of energy equal to 5.5% of the labour costs..

[26]Importantly, we have used the same Taylor type rule for the overall sample. Values $\phi_\pi > 1$ and $\phi_y > 0$ are consistent with recent estimation using bayesian methods by Rabanal and Rubio-Ramirez (2005). Although the previous authors find that from 1982 on, both parameters are estimated to be higher with respect to the overall sample.

2.5.2 Explaining the U.S. Level of Inflation Premium with Oil Price Shocks

In this section we evaluate how the model does at capturing the conditional mean of the key macro variables, in particular of inflation. In Table 2.3 we report the means of inflation, output gap and nominal interest rates compared with the values observed in the data based on our benchmark parameterization[27]. Notice that by comparing the sub-samples we observe an important change in means and volatilities in inflation, GDP gap, and interest rates across sub-samples (columns 3 and 5 of table 2.3). Thus, quarterly inflation standard deviation has decreased from 0.8% to 0.3% and the mean has moved from 1.4% to 0.5%, between the pre-Volcker and post-Volcker periods, respectively. Similarly, the three-month T-bill has decreased in both means and volatilities. Finally, GDP gap has decreased in volatility (from a standard deviation of 2.8% to 1.3%) and has experimented and increase in its mean (from -0.20% to 0.26%).

To clarify, the simulations that follow are a first step at exploring whether the mechanisms we have just have emphasised have potential for explaining the inflation-premium. In the model, we interpret oil price shocks as the main driven force of the inflation premium, although we are aware that in order to closely match the moments of other macro variables, additional shocks might be necessary. Thus, by performing these numerical exercises we intend to confront the data to the mechanism previously described. We do so by generating the unconditional mean of inflation, output and interest rates implied by the calibrated model for the pre and

[27]We use the data from the Haver USECON database (mnemonics are in parentheses). Our measure of the price level is the non-farm business sector deflator (LXNFI), the measure of GDP corresponds to the non-farm business sector output (LXNFO), we use the quarterly average daily of the 3-month T-bill (FTB3) as the nominal interest rate, and our measure of oil prices is the Spot Oil Prices West Texas Intermediate (PZTEXP). We express output in per-capita terms by dividing LXNFO by a measure of civilian non-institutional population aged above 16 (LNN) and oil prices are deflated by the non-farm business sector deflator.

post Volcker periods. The only difference in the calibration between these two periods is the assumption on the data generating process of oil prices. We fit an AR (1) process for oil prices in each period and find that both the persistence and the variance of oil price shocks have fallen from the first to the second period.

	Pre-Volcker		Post-Volcker	
	Simulated	Observed	Simulated	Observed
Mean Inflation	1.09	1.38	0.19	0.53
Mean Output Gap (HP)	-1.35	-0.20	-0.23	0.26
Mean Nominal Interest Rate	1.08	7.65	0.18	5.36
Standard Deviation Inflation	1.91	0.80	0.75	0.29
Standard Deviation Output Gap (HP)	2.02	2.79	0.56	1.33
Standar Deviation Nominal Interest Rate	1.64	2.84	0.45	1.44
Standard Deviation Real Oil Price	0.46	0.57	0.22	0.21

Al variables are quarterly, except the nominal interest rate which is annualised.

Table 2.3: Unconditional Moments Generated by the Benchmark Model

The key result to highlight from table 2.3 is that we are able to generate a positive level of inflation premium that allows the model to mimic the average inflation level in the US in the pre-Volcker and post Volcker periods without relying on different monetary policy regimes across periods. Remarkably, the model can match very closely the mean of inflation for the two sub-periods. Thus, inflation mean during the first period is 1.38% while the model delivers a value of 1.09%. Similarly, for the second period we observe a mean inflation of 0.53% and the model predicts a value of 0.19%. The model is much less successful at matching the moments of the nominal interest rate and to a less extent those of output. Yet, the model does a fairly good job at matching qualitatively changes in average levels of inflation, output and interest rates across sub-samples.

2.5.3 Decomposition of the Determinants of Inflation Premium

As described in the previous section, in general equilibrium, the determinants of inflation premium can be de-composed in four components: those coming from the non-linearity (convexity) of the Phillips curve (Ω_π), the non-linearity of the marginal costs (Ω_π), the auxiliary variable v_t (ω_v and Ω_v) and the precautionary savings effect (ω_y). We show in table 2.4 the decomposition of inflation premium across samples by these determinants. Worth noting is that the convexity of the Phillips curve with respect to oil prices, accounts for roughly 59 and 55 percent of the inflation premium in the pre and post Volcker periods, respectively. The second determinant in importance is the convexity of the marginal cost with respect to oil that accounts for 45 and 48 percent, respectively. For instance, out of this effect, the level of inflation premium attributed to price distortions represents about 50 percent in each sample. Finally, the precautionary savings effect is negative and almost negligible.

	CGG	
	Pre-Volcker	Post-Volcker
Convexity Phillips curve (Ω_π)	58.9	55.4
Marginal costs (Ω_{mc})	45.2	48.2
Indirect effect: price dispersion	27.4	24.8
Direct effect: convexity respect to oil prices	17.9	23.4
Auxiliary variable v_t (ω_v and Ω_v)	-3.9	-2.9
Precautionary Savings (ω_y)	-0.3	-0.6
Total	100.0	100.0

Table 2.4: Inflation Premium - Effects Decomposition

2.5.4 Comparing Different Monetary Policy Rules

We now evaluate how monetary policy can affect the level of inflation premium. We do so by comparing the benchmark specification (CGG) with the estimated taylor rules suggested by Orphanides (2001) and Judd and Rudebush (1998). Table 2.5 shows that Orphanides´s generate a smaller average inflation in both subsamples. This finding is explained by the smaller weight assigned on output in the

Orphanides´s rule with respect to the CGG´s rule. This result is consistent with threshold for the parameter ϕ_y from our analytical results, equation (2-38)

Notice also, that the smaller average level of inflation is consistent with a smaller mean level of the nominal interest rate. Hence, the aggressiveness of the central bank towards inflation determines how the premium is distributed between inflation and output means. The more aggressive at fighting inflation the central bank is, the smaller the level of inflation premium and the larger the reduction of average output. Note also that Rudebush´s rule delivers an excessive inflation premium during the pre-Volcker period (6.38%). This result is basically explained by the higher weight over output fluctuations that this rule implies.

	CGG		Orphanides		Judd-Rudebush	
	Pre-Volcker	Post-Volcker	Pre-Volcker	Post-Volcker	Pre-Volcker	Post-Volcker
Mean Inflation	1.09	0.19	0.19	0.05	6.38	0.64
Mean Output Gap (HP)	-1.35	-0.23	-0.57	-0.15	-3.49	-0.35
Mean Nominal Interest Rate	1.08	0.18	0.19	0.05	6.37	0.63
S.D Deviation Inflation	1.91	0.75	1.01	0.54	3.34	1.00
S.D Output Gap (HP)	2.02	0.56	2.22	0.68	1.73	0.43
S.D Nominal Interest Rate	1.64	0.45	0.82	0.28	2.91	0.62
S.D Real Oil Price	0.46	0.22	0.46	0.22	0.46	0.22

Table 2.5: Alternative Policy Rules

2.6 Conclusions

Traditionally New Keynesian log-linear models have been used to match second order moments. However, they have the limitation that their solution implies certainty equivalence, neglecting any role of uncertainty and volatility over the level of inflation. To the extent that uncertainty is important in real economies, a second order solution of the New Keynesian model is required to improve their fit to the data. In particular, this type of solution provides a link between volatility of shocks and the average values of endogenous variables offering a non-conventional way to analyse business cycles. In this chapter we have taken this approach and we show how the interaction between volatility and the convexity of both the marginal costs and the Phillips curve improves the ability of a standard New Keynesian model to explain the history of inflation in the USA.

The second order solution allows us to provide an additional element to the explanation suggested by CGG for the high inflation episode during the 70s. Our hypothesis puts at the centre of the discussion the volatility of supply shocks, in particular oil price shocks. Contrary to what a linear solution implies, a second order solution establishes the link between volatility of oil prices and expected inflation, what we called *inflation premium*. In this chapter we show that a calibrated version of our model can match very closely the inflation behaviour observed in the USA during both the pre-Volcker and post-Volcker periods. In particular we show that the high volatility of oil price shocks during the 70s implied an endogenous high level of *inflation premium* that can account for the high average inflation levels observed in US during that period. The analytical solution obtained by implementing the perturbation method shows that the existence of the *inflation premium* depends crucially on, first, the convexity of both the marginal costs and the Phillips curve and second, the response of the monetary authority. In particular, the reaction of the central bank determines in equilibrium how higher volatility generated by oil price shocks is distributed between a higher average inflation and lower growth rate. Moreover, in order to observe a positive *inflation premium* it is required that the central bank partially reacts to supply shocks.

In addition, a standard result of the New Keynesian models is that they can not generate an endogenous trade-off for monetary policy. Therefore, in those models zero inflation and zero output gap is the optimal response of the Central Bank, consequently zero *inflation premium* becomes optimal. In this chapter, we show that this result, denominated by Blanchard and Gali as the "Divine Coincidence" holds only under rather special assumptions: when the steady state coincides with the efficient one (i.e. when there is no distorted steady state) or when the production function has an elasticity of substitution equal to 1. Instead, we show that for the general case, allowing for a distorted along with a CES production function, oil price shocks are able to generate an endogenous cost push shock making the central bank problem a meaningful one.

This endogenous cost-push shock generates a trade-off in means for the central bank. In this case the central bank can not reduce the average level of inflation without sacrificing output growth. We show that the optimal policy implies to partially accommodate oil price shocks and to let, on average, a higher level of inflation. Thus our results imply that the inflation behaviour in the U.S. during the 70s not only might reflect a perfectly consistent monetary policy but an optimal one.

Our results can be extended in many directions. First, it will be worth to explore the effect of openness in *inflation premium.* Second, the analytical perturbation method strategy proposed in the chapter can be used to capture the effects of change in a monetary policy regime over inflation. Third, it will be worth also to explore the implications of other source of shocks in the determination in the level of *inflation premium.* Finally, the estimation of a non-linear Phillips curve considering the effects of oil price volatility on inflation will be an issue to work in.

A1 Appendix: Equations of the Model

A1.1 The system of equations

Using the the market clearing conditions that close the model, the dynamic equilibrium of the model described in section 3 is given by the following set of 10 equations:

AGGREGATE SUPPLY	
Marginal Costs	
$MC_t = \left[(1-\alpha)^{\psi}(W_t/P_t)^{1-\psi} + \alpha^{\psi}(Q_t)^{1-\psi}\right]^{\frac{1}{1-\psi}}$	A1-i
Labour market	
$\frac{W_t}{P_t} = Y_t^{\sigma} L_t^{v}$	A1-ii
$L_t = \left(\frac{1}{1-\alpha}\frac{W_t/P_t}{MC_t}\right)^{-\psi} Y_t \Delta_t$	A1-iii
Price dispersion	
$\Delta_t = (1-\theta)\left(\frac{1-\theta(\Pi_t)^{\varepsilon-1}}{1-\theta}\right)^{\varepsilon/(\varepsilon-1)} + \theta\Delta_{t-1}(\Pi_t)^{\varepsilon}$	A1-iv
Phillips Curve	
$\theta(\Pi_t)^{\varepsilon-1} = 1-(1-\theta)\left(\frac{N_t}{D_t}\right)^{1-\varepsilon}$	A1-v
$N_t = \mu Y_t^{1-\sigma} MC_t + \theta\beta E_t\left[(\Pi_{t+1})^{\varepsilon} N_{t+1}\right]$	A1-vi
$D_t = Y_t^{1-\sigma} + \theta\beta E_t\left[(\Pi_{t+1})^{\varepsilon-1} D_{t+1}\right]$	A1-vii
AGGREGATE DEMAND	
$1 = \beta E_t\left[\left(\frac{Y_{t+1}}{Y_t}\right)^{-\sigma}\frac{R_t}{\Pi_{t+1}}\right]$	A1-vii
MONETARY POLICY	
$R_t = \overline{R}\left(\frac{E_t\Pi_{t+1}}{\overline{\Pi}}\right)^{\phi_\pi}\left(\frac{Y_t}{\overline{Y}}\right)^{\phi_y}$	A1-ix
OIL PRICES	
$Q_t = \overline{Q}Q_{t-1}^{\rho}\exp(\eta\sigma_q e_t)$	A1-x

Table A1.1: Equations of the model

The first block represents the aggregate supply, which consists on the marginal costs, the labour market equilibrium and the Phillips curve, which has been written recursively using the auxiliary variables N_t and D_t. The aggregate demand block is represented with the Euler equation and Monetary Policy block is given by the Taylor rule. The last equation describes the dynamics of oil prices. We use this set of ten non-linear equations to obtain numerically the second order solution of the model.

A1.2 The deterministic steady state

The non-stochastic steady state of the endogenous variables is given by: where

Inflation	$\overline{\Pi} = 1$
Auxiliary variables	$\overline{N} = \overline{D} = \overline{Y}/\left(1-\theta\beta\right)$
Interest rate	$\overline{R} = \beta^{-1}$
Marginal costs	$\overline{MC} = 1/\mu$
Real wages	$\overline{W/P} = \tau_y \frac{1}{\mu}\left(1-\alpha^F\right)^{\frac{1}{1-\psi}}$
Output	$\overline{Y} = \tau_y \left(\frac{1}{\mu}\right)^{\frac{1}{\sigma+\nu}} \left(1-\alpha^F\right)^{\frac{1+\psi\nu}{\sigma+\nu}\frac{1}{1-\psi}}$
Labour	$\overline{L} = \tau_l \left(\frac{1}{\mu}\right)^{\frac{1}{\sigma+\nu}} \left(1-\alpha^F\right)^{\frac{1-\sigma\psi}{\sigma+\nu}\frac{1}{1-\psi}}$

Table A1.2: The steady state

$$\alpha^F = \alpha^{\psi}\left(\frac{\overline{Q}}{\overline{MC}}\right)^{1-\psi} = \alpha^{\psi}\left(\mu\overline{Q}\right)^{1-\psi}$$

α^F is the share of oil in the marginal costs, τ_y and τ_l are constants[28]. Notice that the steady state values of real wages, output and labour depend on the steady state ratio of oil prices with respect to the marginal cost. This implies that permanent changes in oil prices would generate changes in the steady state of this variables. Also, as the standard New-Keynesian models, the marginal cost in steady state is equal to the inverse of the mark-up ($\overline{MC} = 1/\mu = \left(\varepsilon - 1\right)/\varepsilon$). Since monopolistic competition affects the steady state of the model, output in steady state is below the efficient level. We call to this feature a distorted steady state.

[28]More precisely: $\tau_y = \left(\frac{1}{1-\alpha}\right)^{\frac{\psi}{1-\psi}\frac{1+v}{\sigma+v}}$ and $\tau_l = \left(\frac{1}{1-\alpha}\right)^{\frac{\psi}{1-\psi}\frac{1-\sigma}{\sigma+v}}$.

A1.3 The flexible price equilibrium

The flexible price equilibrium of the endogenous variables is consistent with zero inflation in every period (i.e. $.\Pi_t^F = 1$). In this case marginal costs are constant, equal to its steady state value, and the other variables are affected by the oil shock.

Inflation	$\Pi_t^F = 1$
Interest rate	$1/R_t^F = E_t\left(\frac{1-\alpha^F(Q_{t+1}/\overline{Q})^{1-\psi}}{1-\alpha^F(Q_t/\overline{Q})^{1-\psi}}\right)^{-\sigma\gamma}$
Marginal costs	$MC_t^F = 1/\mu$
Real wages	$W_t^F/P_t^F = \tau_y\frac{1}{\mu}\left(1-\alpha^F\left(Q_t/\overline{Q}\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}$
Output	$Y_t^F = \tau_y\left(\frac{1}{\mu}\right)^{\frac{1}{\sigma+\nu}}\left(1-\alpha^F\left(Q_t/\overline{Q}\right)^{1-\psi}\right)^{\frac{1+\psi\nu}{\sigma+\nu}\frac{1}{1-\psi}}$
Labour	$L_t^F = \tau_l\left(\frac{1}{\mu}\right)^{\frac{1}{\sigma+\nu}}\left(1-\alpha^F\left(Q_t/\overline{Q}\right)^{1-\psi}\right)^{\frac{1-\sigma\psi}{\sigma+\nu}\frac{1}{1-\psi}}$

Table A1.3: The flexible price equilibrium

Notice that the flexible price equilibrium is not efficient, since there are distortions from monopolistic competition in the intermediate goods market (i.e. $MC_t^F > 1$).

A2 Appendix: The second order solution of the model

A2.1 The recursive AS equation

We divide the equation for the aggregate price level (2-18) by $P_t^{1-\varepsilon}$ and make $P_t/P_{t-1} = \Pi_t$

$$1 = \theta\left(\Pi_t\right)^{-(1-\varepsilon)} + (1-\theta)\left(\frac{P_t^*(z)}{P_t}\right)^{1-\varepsilon} \tag{A2.1}$$

Aggregate inflation is function of the optimal price level of firm z. Also, from equation (2-17) the optimal price of a typical firm can be written as:

$$\frac{P_t^*(z)}{P_t} = \frac{N_t}{D_t}$$

where, after using the definition for the stochastic discount factor: $\zeta_{t,t+k} = \beta C_{t+k}^{-\sigma}/C_t^{-\sigma} P_t/P_{t+k}$, we define N_t and D_t as follows:

$$N_t = E_t\left[\sum_{k=0}^{\infty} \mu(\theta\beta)^k F_{t,t+k}^{\varepsilon} Y_{t+k} C_{t+k}^{-\sigma} MC_{t+k}\right] \tag{A2.2}$$

$$D_t = E_t\left[\sum_{k=0}^{\infty} (\theta\beta)^k F_{t,t+k}^{\varepsilon-1} Y_{t+k} C_{t+k}^{-\sigma}\right] \tag{A2.3}$$

N_t and D_t can be expanded as:

$$N_t = \mu Y_t C_t^{-\sigma} MC_t + E_t\left[\Pi_{t+1}^{\varepsilon} \sum_{k=0}^{\infty} \mu(\theta\beta)^k F_{t+1,t+1+k}^{\varepsilon} Y_{t+1+k} C_{t+1+k}^{-\sigma} MC_{t+1+k}\right] \tag{A2.4}$$

$$D_t = Y_t C_t^{-\sigma} + E_t\left[\Pi_{t+1}^{\varepsilon-1} \sum_{k=0}^{\infty} (\theta\beta)^k F_{t+1,t+1+k}^{\varepsilon-1} C_{t+1+k}^{-\sigma} Y_{t+1+k}\right] \tag{A2.5}$$

where we have used the definition for $F_{t,t+k} = P_{t+k}/P_t$.

The Phillips curve with oil prices is given by the following three equations:

$$\theta\left(\Pi_t\right)^{\varepsilon-1} = 1 - (1-\theta)\left(\frac{P_t^*(z)}{P_t}\right)^{1-\varepsilon} \tag{A2.6}$$

$$N_t = \mu Y_t^{1-\sigma} MC_t + \theta\beta E_t\left(\Pi_{t+1}\right)^{\varepsilon} N_{t+1} \tag{A2.7}$$

$$D_t = Y_t^{1-\sigma} + \theta\beta E_t\left(\Pi_{t+1}\right)^{\varepsilon-1} D_{t+1} \tag{A2.8}$$

where we have reordered equation (A2.1) and we have used equations (A2.2) and (A2.3) evaluated one period forward to replace N_{t+1} and D_{t+1} in equations (A2.4) and (A2.5).

A2.2 The second order approximation of the system

The second order approximation of the Phillips Curve

The second order expansion for equations ($A2.6$), ($A2.7$) and ($A2.8$) are:

$$\pi_t = \frac{(1-\theta)}{\theta}(n_t - d_t) - \frac{1}{2}\frac{(\varepsilon-1)}{1-\theta}(\pi_t)^2 + O\left(\|q_t, \sigma_q\|^3\right) \tag{A2.9}$$

$$n_t = (1-\theta\beta)\left(a_t + \frac{1}{2}a_t^2\right) + \theta\beta\left(E_t b_{t+1} + \frac{1}{2}E_t b_{t+1}^2\right) - \frac{1}{2}n_t^2 + O\left(\|q_t, \sigma_q\|^3\right) \tag{A2.10}$$

$$d_t = (1-\theta\beta)\left(c_t + \frac{1}{2}c_t^2\right) + \theta\beta\left(E_t e_{t+1} + \frac{1}{2}E_t e_{t+1}^2\right) - \frac{1}{2}d_t^2 + O\left(\|q_t, \sigma_q\|^3\right) \tag{A2.11}$$

Where we have defined the auxiliary variables a_t,b_{t+1},c_t and e_{t+1} as:

$$a_t \equiv (1-\sigma)\, y_t + mc_t \qquad b_{t+1} \equiv \varepsilon\pi_{t+1} + n_{t+1}$$
$$c_t \equiv (1-\sigma)\, y_t \qquad e_{t+1} \equiv (\varepsilon - 1)\,\pi_{t+1} + d_{t+1}$$

Subtract equations ($A2.10$) and ($A2.11$), and using the fact that $X^2 - Y^2 = (X-Y)(X+Y)$, for any two variables X and Y :

$$\begin{aligned} n_t - d_t &= (1-\theta\beta)(a_t - c_t) + \frac{1}{2}(1-\theta\beta)(a_t - c_t)(a_t + c_t) \\ &\quad + \theta\beta E_t (b_{t+1} - e_{t+1}) + \frac{1}{2}\theta\beta E_t (b_{t+1} - e_{t+1})(b_{t+1} + e_{t+1}) \\ &\quad - \frac{1}{2}(n_t - d_t)(n_t + d_t) + O\left(\|q_t, \sigma_q\|^3\right) \end{aligned} \tag{A2.12}$$

Plugging in the values of a_t, b_{t+1}, c_t and e_{t+1} into equation ($A2.12$), we obtain ($A2.13$)

$$\begin{aligned} n_t - d_t &= (1-\theta\beta)\, mc_t + \frac{1}{2}(1-\theta\beta)\, mc_t\,(2(1-\sigma)\, y_t + mc_t) \qquad (\\ &\quad + \theta\beta E_t (\pi_{t+1} + n_{t+1} - d_{t+1}) + \frac{1}{2}\theta\beta E_t (\pi_{t+1} + n_{t+1} - d_{t+1})((2\varepsilon - 1)\,\pi_{t+1} + n_{t+1} - \\ &\quad - \frac{1}{2}(n_t - d_t)(n_t + d_t) + O\left(\|q_t, \sigma_q\|^3\right) \end{aligned}$$

Taking forward one period equation ($A2.9$), we can solve for $n_{t+1} - d_{t+1}$:

$$n_{t+1} - d_{t+1} = \frac{\theta}{1-\theta}\pi_{t+1} + \frac{1}{2}\frac{\theta}{1-\theta}\frac{(\varepsilon-1)}{1-\theta}(\pi_{t+1})^2 + O\left(\|q_t, \sigma_q\|^3\right) \tag{A2.14}$$

replace equation ($A2.14$) in ($A2.13$) and make use of the auxiliary variable $z_t = (n_t + d_t) / (1 - \theta\beta)$

$$\begin{aligned} n_t - d_t &= (1-\theta\beta)\, mc_t + \frac{1}{2}(1-\theta\beta)\, mc_t \left(2(1-\sigma)\, y_t + mc_t\right) \qquad \text{(A2.15)} \\ &+ \frac{\theta}{1-\theta}\beta\left[E_t\pi_{t+1} + \left(\frac{\varepsilon-1}{1-\theta}+\varepsilon\right)E_t\pi_{t+1}^2 + (1-\theta\beta)\, E_t\pi_{t+1}z_{t+1}\right] \\ &- \frac{1}{2}\frac{\theta}{1-\theta}(1-\theta\beta)\,\pi_t z_t + O\left(\|q_t,\sigma_q\|^3\right) \end{aligned}$$

Notice that we use only the linear part of equation ($A2.14$) when we replace $n_{t+1} - d_{t+1}$ in the quadratic terms because we are interested in capture terms only up to second order of accuracy. Similarly, we make use of the linear part of equation ($A2.9$) to replace $(n_t - d_t) = \frac{\theta}{1-\theta}\pi_t$ in the right hand side of equation ($A2.15$).

Replace equation ($A2.15$) in ($A2.9$):

$$\begin{aligned} \pi_t &= \kappa mc_t + \frac{1}{2}\kappa mc_t \left(2(1-\sigma)\, y_t + mc_t\right) \qquad \text{(A2.16)} \\ &+ \beta\left[E_t\pi_{t+1} + \left(\frac{\varepsilon-1}{1-\theta}+\varepsilon\right)E_t\pi_{t+1}^2 + (1-\theta\beta)\, E_t\pi_{t+1}z_{t+1}\right] \\ &- \frac{1}{2}(1-\theta\beta)\,\pi_t z_t - \frac{1}{2}\frac{(\varepsilon-1)}{1-\theta}(\pi_t)^2 + O\left(\|q_t,\sigma_q\|^3\right) \end{aligned}$$

for

$$\kappa \equiv \frac{(1-\theta)}{\theta}(1-\theta\beta)$$

where z_t has the following linear expansion:

$$z_t = 2(1-\sigma)\, y_t + mc_t + \theta\beta E_t\left(\frac{2\varepsilon-1}{1-\theta\beta}\pi_{t+1} + z_{t+1}\right) + O\left(\|q_t,\sigma_q\|^2\right) \qquad \text{(A2.17)}$$

Define the following auxiliary variable:

$$v_t = \pi_t + \frac{1}{2}\left(\frac{\varepsilon-1}{1-\theta}+\varepsilon\right)\pi_t^2 + \frac{1}{2}(1-\theta\beta)\,\pi_t z_t \qquad \text{(A2.18)}$$

Using the definition for v_t, equation ($A2.16$) can be expressed as:

$$v_t = \kappa mc_t + \frac{1}{2}\kappa mc_t\left(2(1-\sigma)\, y_t + mc_t\right) + \frac{1}{2}\varepsilon\pi_t^2 + \beta E_t v_{t+1} + O\left(\|q_t,\sigma_q\|^2\right) \quad \text{(A2.19)}$$

which is equation (4.3) in the main text.

Moreover, the linear part of equation (A2.19) is:

$$\pi_t = \kappa mc_t + \beta E_t(\pi_{t+1}) + O\left(\|q_t, \sigma_q\|^2\right)$$

which is the standard New Keynesian Phillips curve, inflation depends linearly on the real marginal costs and expected inflation.

The MC equation and the labour market equilibrium

The real marginal cost (2-15) and the labour market equations (2-7 and 2-25) have the following second order expansion:

$$mc_t = \left(1-\alpha^F\right) w_t + \alpha^F q_t + \frac{1}{2}\alpha^F\left(1-\alpha^F\right)(1-\psi)(w_t - q_t)^2 + O\left(\|q_t, \sigma_q\|^3\right) \quad \text{(A2.20)}$$

$$w_t = \nu l_t + \sigma y_t \quad \text{(A2.21)}$$

$$l_t = y_t - \psi(w_t - mc_t) + \widehat{\Delta}_t \quad \text{(A2.22)}$$

Where w_t and $\widehat{\Delta}_t$ are, respectively, the log of the deviation of the real wage and the price dispersion measure from their respective steady state. Notice that equations $(A2.21)$and $(A2.22)$ are not approximations, but exact expressions.

Solving equations $(A2.21)$ and $(A2.22)$ for the equilibrium real wage:

$$w_t = \frac{1}{1+\nu\psi}\left[(\nu+\sigma)y_t + \nu\psi mc_t + v\widehat{\Delta}_t\right] \quad \text{(A2.23)}$$

Plugging the real wage in equation $(A2.20)$ and simplifying:

$$\begin{aligned} mc_t &= \chi(\sigma+v)y_t + (1-\chi)(q_t) + \chi v\widehat{\Delta}_t \\ &+ \frac{1}{2}\frac{1-\psi}{1-\alpha^F}\chi^2(1-\chi)\left[(\sigma+v)y_t - q_t\right]^2 + O\left(\|q_t, \sigma_q\|^3\right) \end{aligned} \quad \text{(A2.24)}$$

where $\chi \equiv \left(1-\alpha^F\right)/\left(1+v\psi\alpha^F\right)$ .This is the equation (4.1) in the main text. This expression is the second order expansion of the real marginal cost as a function of output and the oil prices.

The price dispersion measure

The price dispersion measure is given by

$$\Delta_t = \int_0^1 \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon} dz$$

Since a proportion $1-\theta$ of intermediate firms set prices optimally, whereas the other θ set the price last period, this price dispersion measure can be written as:

$$\Delta_t = (1-\theta) \left(\frac{P_t^*(z)}{P_t} \right)^{-\varepsilon} + \theta \int_0^1 \left(\frac{P_{t-1}(z)}{P_t} \right)^{-\varepsilon} dz$$

Dividing and multiplying by $(P_{t-1})^{-\varepsilon}$ the last term of the RHS:

$$\Delta_t = (1-\theta) \left(\frac{P_t^*(z)}{P_t} \right)^{-\varepsilon} + \theta \int_0^1 \left(\frac{P_{t-1}(z)}{P_{t-1}} \right)^{-\varepsilon} \left(\frac{P_{t-1}}{P_t} \right)^{-\varepsilon} dz$$

Since $P_t^*(z)/P_t = N_t/D_t$ and $P_t/P_{t-1} = \Pi_t$, using equation $(2-11)$ in the text and the definition for the dispersion measure lagged on period, this can be expressed as

$$\Delta_t = (1-\theta) \left(\frac{1-\theta (\Pi_t)^{\varepsilon-1}}{1-\theta} \right)^{\varepsilon/(\varepsilon-1)} + \theta \Delta_{t-1} (\Pi_t)^{\varepsilon} \qquad \text{(A2.25)}$$

which is a recursive representation of Δ_t as a function of Δ_{t-1} and Π_t.

Benigno and Woodford (2005) show that a second order approximation of the price dispersion depends solely on second order terms on inflation. Then, the second order approximation of equation (A2.25) is:

$$\widehat{\Delta}_t = \theta \widehat{\Delta}_{t-1} + \frac{1}{2} \varepsilon \frac{\theta}{1-\theta} \pi_t^2 + O\left(\|q_t, \sigma_q\|^3\right) \qquad \text{(A2.26)}$$

which is equation $(2-ii)$ in the main text. Moreover, we can use equation $(A2.26)$ to write the infinite sum:

$$\begin{aligned}
\sum_{t=t_o}^{\infty} \beta^{t-t_o} \widehat{\Delta}_t &= \theta \sum_{t=t_o}^{\infty} \beta^{t-t_o} \widehat{\Delta}_{t-1} + \frac{1}{2} \varepsilon \frac{\theta}{1-\theta} \sum_{t=t_o}^{\infty} \beta^{t-t_o} \frac{\pi_t^2}{2} + O\left(\|q_t, \sigma_q\|^3\right) \\
(1-\beta\theta) \sum_{t=t_o}^{\infty} \beta^{t-t_o} \widehat{\Delta}_t &= \theta \widehat{\Delta}_{t_o-1} + \frac{1}{2} \varepsilon \frac{\theta}{1-\theta} \sum_{t=t_o}^{\infty} \beta^{t-t_o} \frac{\pi_t^2}{2} + O\left(\|q_t, \sigma_q\|^3\right)
\end{aligned}$$

Dividing by $(1-\beta\theta)$ and using the definition of κ :

$$\sum_{t=t_o}^{\infty}\beta^{t-t_o}\widehat{\Delta}_t=\frac{\theta}{1-\beta\theta}\widehat{\Delta}_{t_o-1}+\frac{1}{2}\frac{\varepsilon}{\kappa}\sum_{t=t_o}^{\infty}\beta^{t-t_o}\frac{\pi_t^2}{2}+O\left(\|q_t,\sigma_q\|^3\right) \qquad \text{(A2.27)}$$

The discounted infinite sum of $\widehat{\Delta}_t$ is equal to the sum of two terms, on the initial price dispersion and the discounted infinite sum of π_t^2.

The IS

Similarly, the second order expansion of the IS is:

$$y_t=E_t y_{t+1}-\frac{1}{\sigma}\left(r_t-E_t\pi_{t+1}\right)-\frac{1}{2}\sigma E_t\left[\left(y_t-y_{t+1}\right)-\frac{1}{\sigma}\left(r_t-\pi_{t+1}\right)\right]^2+\left(\|q_t,\sigma_q\|^3\right) \qquad \text{(A2.28)}$$

Replacing the linear solution of y_t inside the quadratic part of equation $(A2.28)$:

$$y_t=E_t y_{t+1}-\frac{1}{\sigma}\left(r_t-E_t\pi_{t+1}\right)-\frac{1}{2}\sigma E_t\left[y_{t+1}+\frac{1}{\sigma}\pi_{t+1}-E_t\left(y_{t+1}+\frac{1}{\sigma}\pi_{t+1}\right)\right]^2+\left(\|q_t,\sigma_q\|^3\right) \qquad \text{(A2.29)}$$

where $E_t\left[y_{t+1}+\frac{1}{\sigma}\pi_{t+1}-E_t\left(y_{t+1}+\frac{1}{\sigma}\pi_{t+1}\right)\right]^2$ is the variance of $\left(y_{t+1}+\frac{1}{\sigma}\pi_{t+1}\right)$.

A2.3 The system in two equations

Since the quadratic terms of the second order Taylor expansions of the equations depend on the linear solution, we can use the latter to solve for the formers. Let´s assume the linear solution for output, inflation and the auxiliary variable z_t[29]

$$\begin{aligned} y_t &= a_1 q_t + O\left(\left\|q_t, \sigma_q\right\|^2\right) \\ \pi_t &= b_1 q_t + O\left(\left\|q_t, \sigma_q\right\|^2\right) \\ z_t &= c_1 q_t + O\left(\left\|q_t, \sigma_q\right\|^2\right) \end{aligned}$$

Additionally, we have the transition process for the oil price:

$$q_t = \rho q_{t-1} + \eta \sigma_q e_t$$

where $e\tilde{\ } iid\,(0,1)$ and $\eta = \sqrt{1-\rho^2}$.

The AS

Replacing the equation for the price dispersion in the equation for the marginal costs, the latter can be expressed as:

$$mc_t = \chi\,(v+\sigma)\,y_t + (1-\chi)\,q_t + \chi v \widehat{\Delta}_t + \frac{1}{2}\widetilde{\Omega}_{mc} q_t^2 + O\left(\left\|q_t, \sigma_q\right\|^3\right) \tag{A2.30}$$

where $\widetilde{\Omega}_{mc} = (1-\chi)\,\chi^2 \frac{1-\psi}{1-\alpha^F}\left((\nu+\sigma)\,a_1 - 1\right)^2 + \varepsilon \frac{\theta}{1-\theta}\,(b_1)^2$.

Similarly, the Phillips curve equation can be expressed as:

$$v_t = \kappa m c_t + \beta E_t v_{t+1} + \frac{1}{2}\Omega_\pi q_t^2 + O\left(\left\|q_t, \sigma_q\right\|^3\right) \tag{A2.31}$$

where $\Omega_\pi = \varepsilon\,(b_1)^2 + \kappa\left[\chi\,(v+\sigma)\,a_1 + (1-\chi)\right]\left[2\,(1-\sigma)\,y_t + \chi\,(v+\sigma)\,a_1 + (1-\chi)\right]$. We have used the linear solution of output and inflation to express Ω_π in terms of a_1 and b_1.

[29]From the linear expansion of the definition of z_t we can solve for c_1, where $c_1 = \frac{1}{1-\theta\beta\rho}\left\{\left[2\,(1-\sigma) + \chi\,(\sigma+v)\right] a_1 + (1-\chi) + \theta\beta\frac{2\varepsilon-1}{1-\theta\beta}\rho b_1\right\}$

Replace the equation for the marginal costs in the second order expansion of the Phillips Curve and iterate forward, the Phillips curve can be expressed as the discounted infinite sum:

$$v_t = \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left\{ \kappa_y y_t + \kappa_q q_t + \kappa \chi v \widehat{\Delta}_t + \frac{1}{2} \kappa \widetilde{\Omega}_{mc} q_t^2 + \frac{1}{2} \Omega_\pi q_t^2 \right\} + \left(\|q_t, \sigma_q\|^3 \right) \quad \text{(A2.32)}$$

where $\kappa_y = \kappa \chi (\sigma + \nu)$ and $\kappa_q = \kappa (1 - \chi)$. Make use of equation $(A2.27)$, the discounted infinite sum of $\widehat{\Delta}_t$, v_t becomes:

$$\begin{aligned} v_t &= \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left\{ \kappa_y y_t + \kappa_q q_t + \frac{1}{2} \varepsilon \chi v \pi_t^2 + \frac{1}{2} \kappa \widetilde{\Omega}_{mc} q_t^2 + \frac{1}{2} \Omega_\pi q_t^2 \right\} \quad \text{(A2.33)} \\ &+ \frac{\kappa \chi v \theta}{1 - \beta \theta} \widehat{\Delta}_{t-1} + \left(\|q_t, \sigma_q\|^3 \right) \end{aligned}$$

Assuming that we depart from an initial state where the price dispersion is small, that is $\widehat{\Delta}_{t-1} \simeq 0$ up to second order, then equation (A2.33) can be expressed recursively as[30]:

$$v_t = \kappa_y y_t + \kappa_q q_t + \frac{1}{2} \varepsilon \theta \beta \frac{\pi_t^2}{2} + \frac{1}{2} \kappa \widetilde{\Omega}_{mc} q_t^2 + \frac{1}{2} \Omega_\pi q_t^2 + \beta E_t v_{t+1} + \left(\|q_t, \sigma_q\|^3 \right) \quad \text{(A2.34)}$$

Let´s consider the total second order terms coming from the marginal costs:

$$\Omega_{mc} q_t^2 = \varepsilon \chi v \pi_t^2 + \kappa \widetilde{\Omega}_{mc} q_t^2 \quad \text{(A2.35)}$$

then, $\Omega_{mc} = \varepsilon \chi v (b_1)^2 + \kappa \widetilde{\Omega}_{mc}$.

The auxiliary variable v_t is also affected by second order terms:

$$v_t = \pi_t + \frac{1}{2} \widetilde{\Omega}_v q_t^2 \quad \text{(A2.36)}$$

where $\widetilde{\Omega}_v = \left[\left(\frac{\varepsilon - 1}{1 - \theta} + \varepsilon \right) b_1^2 + (1 - \theta \beta) b_1 c_1 \right]$. $E_t v_{t+1}$ becomes:

$$\begin{aligned} E_t v_{t+1} &= E_t \pi_t + \frac{1}{2} \widetilde{\omega}_\pi E_t q_{t+1}^2 \quad \text{(A2.37)} \\ &= E_t \pi_{t+1} + \frac{1}{2} \widetilde{\Omega}_v \left(\rho^2 q_t^2 + \eta^2 \sigma_q^2 \right) \end{aligned}$$

[30] We make the assumption that the initial price dispersion is small to make the analysis analytically tractable. However, in the numerical exercise we work with the general case and the results are quantitatively similar.

Replacing equations (A2.35), (A2.36) and (A2.37) in (A2.34), we obtain the equation (2-33) in the text:

$$\pi_t = \kappa_y y_t + \kappa_q q_t + \beta E_t \pi_{t+1} + \frac{1}{2}\left(\Omega_{mc} + \Omega_\pi + \frac{1}{2}\Omega_v\right) q_t^2 + \frac{1}{2}\omega_v \sigma_q^2 + \left(\|q_t, \sigma_q\|^3\right) \quad \text{(A2.38)}$$

where $\Omega_v = -\widetilde{\Omega}_v \left(1 - \beta\rho^2\right)$ and $\omega_v = \widetilde{\omega}_v \beta\eta^2$. $\Omega_{mc}, \Omega_\pi, \Omega_v$ and ω_v are respectively the second order terms coming from the marginal costs, the Phillips Curve and the auxiliary variable v_t.

The aggregate demand

Replace the policy rule (2-29) in the second order expansion of the IS (A2.29), assuming there is not interest rate smoothing (that is $\phi_r = 0$) :

$$\begin{aligned} y_t &= E_t y_{t+1} - \frac{1}{\sigma}\left[\left(\phi_\pi - 1\right) E_t \pi_{t+1} + \phi_y y_t\right] + \qquad \text{(A2.39)} \\ & -\frac{1}{2}\sigma E_t \left[y_{t+1} + \frac{1}{\sigma}\pi_{t+1} - E_t\left(y_{t+1} + \frac{1}{\sigma}\pi_{t+1}\right)\right]^2 + O\left(\|q_t, \sigma_q\|^3\right) \end{aligned}$$

This can be expressed as:

$$y_t = E_t\left(y_{t+1}\right) - \frac{1}{\sigma}\left[\left(\phi_\pi - 1\right) E_t \pi_{t+1} + \phi_y y_t\right] + \frac{1}{2}\omega_y \sigma_q^2 + O\left(\|q_t, \sigma_q\|^3\right) \quad \text{(A2.40)}$$

where:

$$\omega_y \sigma_q^2 = -\sigma E_t \left[a_1 q_{t+1} + \frac{1}{\sigma} b_1 q_{t+1} - E_t\left(a_1 q_{t+1} + \frac{1}{\sigma} b_1 q_{t+1}\right)\right]^2 \quad \text{(A2.41)}$$

Similar to the previous sub-section, the IS risk premium can be written as a function of the linear solution of inflation and output:

$$\omega_y = -\sigma\left(a_1 + \frac{1}{\sigma} b_1\right)^2 < 0 \quad \text{(A2.42)}$$

Note that the risk premium component of the IS is negative, capturing precautionary savings due to output and inflation volatility.

A2.4 The perturbation method

The policy functions of the second order solution for output and inflation can be written in the following form:

$$\begin{aligned} y_t &= \frac{1}{2}a_o\sigma_q^2 + a_1 q_t + \frac{1}{2}a_2 (q_t)^2 + O\left(\|q_t, \sigma_q\|^3\right) \\ \pi_t &= \frac{1}{2}b_o\sigma_q^2 + b_1 q_t + \frac{1}{2}b_2 (q_t)^2 + O\left(\|q_t, \sigma_q\|^3\right) \end{aligned} \tag{A2.43}$$

where the a's and b's are the unknown coefficients that we need to solve for and $O\left(\|q_t, \sigma_q\|^3\right)$ denotes terms on q and σ_q of order equal or higher than 3. We express the dynamics of the oil price as:

$$q_t = \rho q_{t-1} + \eta\sigma_q e_t \tag{A2.44}$$

where the oil shock has been normalised to have mean zero and standard deviation of one, i.e. $e\tilde{\ }iid\,(0,1)$ .Also, we set $\eta = \sqrt{1-\rho^2}$ in order to express $V(q_t) = \sigma_q^2$.

In order to solve for the 6 unknown coefficients, we use the following algorithm that consist in solving recursively for three systems of two equations. This allow us to obtain algebraic solutions for the unknown coefficients. We follow the following steps:

1. We replace the closed forms of the policy functions (A2.43) and the transition equation for the shock (A2.44) in the equations for the AS (A2.38) and the AD (A2.40).

2. **Solve for a_1 and b_1:** we take the partial derivatives with respect to q_t to the two equations of step 1, then we proceed to evaluate them in the non-stochastic steady state (i.e. when $q_t = 0$ and $\sigma_q = 0$). Then, the only unknowns left are a_1 and b_1 for two equations. We proceed to solve for a_1 and b_1 as function of the deep parameters of the model.

$$\begin{aligned} a_1 &= -\left[(\phi_\pi - 1)\rho\right]\kappa_q\frac{1}{\Lambda_1} < 0 \\ b_1 &= \left[\sigma(1-\rho) + \phi_y\right]\kappa_q\frac{1}{\Lambda_1} > 0 \end{aligned}$$

3. **Solve for a_2 and b_2:** similar to step 2, we take successive partial derivatives with respect to q_t and q_t to the two equations of step 1 and we evaluate them at the non-stochastic steady state. Then, we solve for the unknowns a_2 and b_2.

$$\begin{aligned} a_2 &= -\left[(\phi_\pi - 1)\,\rho^2\right](\Omega_\pi + \Omega_{mc})\,\frac{1}{\Lambda_2} < 0 \\ b_2 &= \left[\sigma\left(1-\rho^2\right) + \phi_y\right](\Omega_\pi + \Omega_{mc})\,\frac{1}{\Lambda_2} > 0 \end{aligned}$$

4. **Solve for a_0 and b_0:** similar to steps 2 and 3, we take successive partial derivatives with respect to σ_q and σ_q to the two equations of step 1 and we evaluate them at the non-stochastic steady state. Then, we solve for the unknowns a_0 and b_0. The solution for the coefficients is given by:

$$\begin{aligned} a_o &= -(\phi_\pi - 1)\left[\left(b_2\eta^2 + \omega_\pi\right) - \sigma(1-\beta)\left(a_2\eta^2 + \omega_y\right)\right]\frac{1}{\Lambda_0} \\ b_o &= -b_2\eta^2 + \left[\phi_y\left(b_2\eta^2 + \omega_\pi\right) + \sigma\kappa_y\left(a_2\eta^2 + \omega_y\right)\right]\frac{1}{\Lambda_0} \end{aligned}$$

where we have defined the following auxiliary variables:

$$\begin{aligned} \Lambda_0 &= (\phi_\pi - 1)\,\kappa_1 + (1-\beta)\,\phi_y \\ \Lambda_1 &= (\phi_\pi - 1)\,\rho\kappa_y + (1-\beta\rho)\left[\sigma(1-\rho) + \phi_y\right] \\ \Lambda_2 &= (\phi_\pi - 1)\,\rho^2\kappa_y + \left(1-\beta\rho^2\right)\left[\sigma(1-\rho)^2 + \phi_y\right] \end{aligned}$$

where Λ_0, Λ_1, and Λ_2 are all positive.

The Inflation premium

The inflation premium is given by:

$$E(\pi) = \frac{1}{2}(b_o + b_2)\,\sigma_q^2$$

replace the solution for b_o:

$$b_o + b_2 = b_2\rho^2 + \frac{\phi_y\left(b_2\eta^2 + \omega_\pi\right) + \sigma\kappa_y\left(a_2\eta^2 + \omega_y\right)}{\Lambda_0} \tag{A2.45}$$

Replace the solution of a_2 and the definition of η, and collect for b_2 :

$$b_o + b_2 = \frac{1}{\Lambda_0}\left\{b_2\left[\rho^2\Lambda_o + \left(\phi_y - \sigma\kappa_y\frac{\phi_\pi - 1}{\sigma\left(1-\rho^2\right)+\phi_y}\right)\left(1-\rho^2\right)\right] + \phi_y\omega_\pi + \sigma\kappa_y\omega_y\right\} \tag{A2.46}$$

After some algebra, it can be expressed as:

$$b_o + b_2 = \frac{1}{\Lambda_0}\left\{\frac{b_2\phi_y}{\sigma\left(1-\rho^2\right)+\phi_y}\left[\Lambda_2 + 2\left(1-\beta\rho^2\right)\sigma\left(1-\rho\right)^2\right] + \phi_y\omega_\pi + \sigma\kappa_y\omega_y\right\} \tag{A2.47}$$

Replace the definition for b_2 :

$$b_o + b_2 = \frac{1}{\Lambda_0}\left\{\phi_y\left(\Omega_\pi + \Omega_{mc} + \Omega_v\right)\left(1+\Psi\right) + \phi_y\omega_v + \sigma\kappa_y\omega_y\right\} \tag{A2.48}$$

where $\Psi = 2\left(1-\beta\rho^2\right)\sigma\left(1-\rho\right)^2/\Lambda_2$. Ψ is positive and very small for ρ close to 1

A3 Appendix: Endogenous Trade-off

From equation (A2.24), we can derive linearly the marginal cost as function of output and oil price shocks, as follows:

$$mc_t = \frac{\left(1-\alpha^F\right)(\sigma+v)}{1+v\psi\alpha^F} y_t + \alpha^F \frac{(1+v\psi)}{1+v\psi\alpha^F} q_t + O\left(\left\|q_t, \sigma_q\right\|^2\right) \tag{A3.1}$$

This equation can be also written in terms of parameters κ_y and κ_q, defined previously in the main text, as follows:

$$mc_t = \frac{\kappa_y}{\kappa} y_t + \frac{\kappa_q}{\kappa} q_t + O\left(\left\|q_t, \sigma_q\right\|^2\right) \tag{A3.2}$$

Under flexible prices, $mc_t = 0$. Condition that defines the natural level of output in terms of the oil price shock :

$$y_t^F = -\frac{\kappa_q}{\kappa_y} q_t + O\left(\left\|q_t, \sigma_q\right\|^2\right) \tag{A3.3}$$

Notice that in this economy the flexible price level of output does not coincide with the efficient one since the steady state is distorted by monopolistic competition The efficient level of output is defined as the level of output with flexible prices under perfect competition, we use equation $(A3.2)$ to calculate this efficient level of output under the condition that $\mu = 1$ as follows:

$$y_t^E = -\frac{\alpha^E}{\left(1-\alpha^E\right)} \frac{\left(1-\alpha^F\right)}{\alpha^F} \frac{\kappa_q}{\kappa_y} q_t + O\left(\left\|q_t, \sigma_q\right\|^2\right) \tag{A3.4}$$

Where $\alpha^E = \alpha^\psi \left(\overline{Q}\right)^{1-\psi}$. This parameter can be also expressed in terms of the participation of oil under flexible prices as follows:

$$\alpha^E = \alpha^F \mu^{\psi-1}$$

Notice that when there is no monopolistic distortion or when $\psi = 1$ we have that $\alpha^E = \alpha^F$ and $y_t^E = y_t^F$.

Using the definition of efficient level of output, we can write the marginal costs equation in terms an efficient output gap, x_t. Where $x_t = \left(y_t - y_t^E\right)$ in the following way

$$mc_t = \frac{\kappa_y}{\kappa}\left(y_t - y_t^E\right) + \frac{1}{\kappa}\mu_t + O\left(\|q_t, \sigma_q\|^2\right) \tag{A3.5}$$

Where

$$\mu_t = \kappa_y \left(1 - \frac{\alpha^F}{\left(1-\alpha^F\right)} \frac{\left(1-\alpha^E\right)}{\alpha^E}\right) y_t^E$$

Using equations (A3.5) and (2-27), the Phillips curve can be written as follows:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_y x_t + \mu_t + O\left(\|q_t, \sigma_q\|^2\right) \tag{A3.6}$$

This equation corresponds to equation $(2-31)$ in the main text. We can further write μ_t in terms of the oil price shocks using the definition of the efficient level of output:

$$\mu_t = \frac{\kappa_q}{\kappa_y}\left(\frac{\alpha^F - \alpha^E}{\left(1-\alpha^E\right)\alpha^F}\right) q_t$$

The dynamic IS equation can also be written in terms of the efficient output gap.

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - r_t^E\right) + O\left(\|q_t, \sigma_q\|^2\right) \tag{A3.7}$$

where r_t^E is the natural interest rate, the real interest rate consistent with y_t^E:

$$r_t^E = \sigma\left(1-\rho\right) y_t^E + O\left(\|q_t, \sigma_q\|^2\right)$$

which in turn can be written as follows:

$$r_t^E = -\sigma\left(1-\rho\right) \frac{\alpha^E}{\left(1-\alpha^E\right)} \frac{\left(1-\alpha^F\right)}{\alpha^F} \frac{\kappa_q}{\kappa_y} q_t + O\left(\|q_t, \sigma_q\|^2\right)$$

Notice that when there is no monopolistic distortion or when $\psi = 1$ we have that $\alpha^E = \alpha^F$, which implies that there is no an endogenous trade off.

$$\mu_t = 0 \;\forall t$$

CHAPTER 3

OIL SHOCKS AND OPTIMAL MONETARY POLICY

3.1 Introduction

Oil is an important production factor in economic activity, because every industry uses it to some extent. Moreover, since oil cannot easily be substituted by other production factors, economic activity is heavily dependent on its use. Furthermore, the oil price is determined in a weakly competitive market; there are few large oil producers dominating the world market, setting its price above a perfect competition level. Also, its price fluctuates considerably due to the effects of supply and demand shocks in this market[1].

The heavy dependence on oil and the high volatility of its price generates a concern among the policymakers on how to react to oil shocks. Oil shocks have serious effects on the economy because they raise prices for an important production input and for important consumer goods (gasoline and heating oil). This causes an increase in inflation and subsequently a decrease in output, generating also a dilemma for policymaking. On one hand, if monetary policy makers focus exclusively on the recessive effects of oil shocks and try to stabilise output, this would generate inflation. On the other hand, if monetary policy makers focus exclusively on neutralising the impact of the shock on inflation through a contractive monetary

[1]For example during the 1970s and through the 1990s most of the oil shocks seemed clearly to be on the international supply side, either because of attempts to gain more oil revenue or because of supply interruptions, such as the Iranian Revolution and the first Gulf war. In contrast, in the 2000s the high price of oil is more related to demand growth in the USA, China, India and other countries.

policy, some sluggishness in the response of prices to changes in output would imply large reductions in output. Therefore, policymakers are confronted with a trade-off between stabilising inflation and output. But, what exactly should be the optimal stabilisation of inflation and output? Which factors affect this trade-off? To our knowledge there is not been a formal study on this topic.

To answer these questions we extend the literature on optimal monetary policy including oil in the production process in a standard New Keynesian model. In doing so, we extend Benigno and Woodford (2005) to obtain a second-order approximation to the expected utility of the representative household when the steady state is distorted and the economy is hit by oil price shocks. We include oil as a non-produced input as in Blanchard and Gali (2005), but differently from those authors we use a constant-elasticity-of-substitution (CES) production function to capture the low substitutability of oil. Then, a low elasticity of substitution between labour and oil indicates a high dependence on oil[2].

The analysis of optimal monetary policy in microfounded models with staggered price setting using a quadratic welfare approximation was first introduced by Rotemberg and Woodford (1997) and expounded by Woodford (2003) and Benigno and Woodford (2005). This method allows us to obtain a linear policy rule derived from maximising the quadratic approximation of the welfare objective subject to the linear constraints that are first-order approximations of the true structural equations. This methodology is called linear-quadratic (LQ). The advantage of this approach is that it allows us to characterise analytically how changes in the production function and in the oil shock process affect the monetary policy problem. Moreover, in contrast to the Ramsey policy methodology, which also allows a correct calculation of a linear approximation of the optimal policy rule, the LQ approach is useful to evaluate not only the optimal rules, but also to evaluate and rank sub-optimal monetary policy rules.

[2]In contrast, Blanchard and Gali (2005) use a Cobb-Douglas production function, in which the elasticity of substitution is equal to one.

A property of standard New Keynesian models is that stabilising inflation is equivalent to stabilising output around some desired level, unless some exogenous cost-push shock disturbances are taken into account. Blanchard and Gali (2005) called this feature the "divine coincidence". These authors argue that this special feature comes from the absence of non-trivial real imperfections, such as real wage rigidities. Similarly, Benigno and Woodford (2004, 2005) show that this trade-off also arises when the steady state of the model is distorted and there are government purchases in the model.

We found that, when oil is introduced as a low-substitutable input in a New Keynesian model, a trade-off arises between stabilising inflation and the gap between output and some desired level. We call this desired level the "'efficient level". In this case, because output at the efficient level fluctuates less than it does at the natural level, it becomes optimal to the monetary authority to react partially to oil shocks and therefore, some inflation is desirable. Moreover, in contrast to Benigno and Woodford (2005), this trade-off remains even when the effects of the monopolistic distortions are eliminated from the steady state.

This trade-off is generated because oil shocks affect output and labour differently, generating a wedge between the effects on the utility of consumption and the disutility of labour. The lower the elasticity of substitution in production, the higher this wedge and also the greater the trade-off. In contrast, in the case of a Cobb-Douglas production function, there is no such a trade-off because this wedge is zero. Then, in the Cobb-Douglas case stabilising output around the natural level also implies stabilising output around its efficient level.

Also, the substitutability among production factors affects both the weights on the two stabilisation objectives and the definition of the welfare-relevant output gap. The lower the elasticity of substitution, the higher the cost-push shock generated by oil shocks and the higher the weight on output stabilisation relative to inflation stabilisation. Moreover, when the share of oil in the production function is higher, or the steady-state oil price is higher, the size of the cost-push shock increases.

Section 3.2 presents our New Keynesian model with oil prices in the production function. Section 3.3 includes a linear quadratic approximation to the policy problem. Section 3.4 uses the linear quadratic approximation to the problem to solve for the different rules of monetary policy and make some comparative statics to the parameters related to oil. The last section concludes.

3.2 A New Keynesian model with oil prices

The model economy corresponds to the standard New Keynesian Model in the line of CGG (2000). In order to capture oil shocks we follow Blanchard and Gali (2005) by introducing a non-produced input M, represented in this case by oil. Q will be the real price of oil which is assumed to be exogenous. This model is similar to the one used in chapter 2, except that we additionally include taxes on sales of intermediate goods and oil to analyse the distortions in steady state.

3.2.1 Households

We assume the following utility function on consumption and labour of the representative consumer

$$U_{t_o} = E_{t_o} \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\nu}}{1+\nu} \right] \tag{3-1}$$

where σ represents the coefficient of risk aversion and ν captures the inverse of the elasticity of labour supply. The optimiser consumer takes decisions subject to a standard budget constraint which is given by

$$C_t = \frac{W_t L_t}{P_t} + \frac{B_{t-1}}{P_t} - \frac{1}{R_t}\frac{B_t}{P_t} + \frac{\Gamma_t}{P_t} + \frac{T_t}{P_t} \tag{3-2}$$

where W_t is the nominal wage, P_t is the price of the consumption good, B_t is the end of period nominal bond holdings, R_t is the nominal gross interest rate , Γ_t is the share of the representative household on total nominal profits, and T_t are net transfers from the government[3]. The first order conditions for the optimising consumer's problem are:

$$1 = \beta E_t \left[R_t \left(\frac{P_t}{P_{t+1}} \right) \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \right] \tag{3-3}$$

$$\frac{W_t}{P_t} = C_t^{\sigma} L_t^{\nu} = MRS_t \tag{3-4}$$

Equation $(3-3)$ is the standard Euler equation that determines the optimal path of consumption. At the optimum the representative consumer is indifferent between consuming today or tomorrow, whereas equation $(3-4)$ describes the optimal labour supply decision. MRS_t denotes for the marginal rate of substitution between labour and consumption. We assume that labour markets are competitive and also that individuals work in each sector $z \in [0, 1]$. Therefore, L corresponds to the aggregate labour supply:

$$L = \int_0^1 L_t(z) dz \tag{3-5}$$

3.2.2 Firms

Final Good Producers

There is a continuum of final good producers of mass one, indexed by $f \in [0, 1]$ that operate in an environment of perfect competition. They use intermediate goods as inputs, indexed by $z \in [0, 1]$ to produce final consumption goods using the following

[3] In the model we assume that the government owns the oil endowment. Oil is produced in the economy at zero cost and sold to the firms at an exogenous price Q_t. The government transfers all the revenues generated by oil to consumers represented by T_t^q. There are also a proportional tax on sale revenues (τ^y) and a proportional taxes on oil sales (τ^q). Then, total net transfers are $T_t = T_t^q - \tau^y Y_t - \tau^q M_t$.

technology:

$$Y_t^f = \left[\int_0^1 Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{3-6}$$

where ε is the elasticity of substitution between intermediate goods. Then the demand function of each type of differentiated good is obtained by aggregating the input demand of final good producers

$$Y_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon} Y_t \tag{3-7}$$

where the price level is equal to the marginal cost of the final good producers and is given by:

$$P_t = \left[\int_0^1 P_t(z)^{1-\varepsilon} dz\right]^{\frac{1}{1-\varepsilon}} \tag{3-8}$$

and Y_t represents the aggregate level of output.

$$Y_t = \int_0^1 Y_t^f df \tag{3-9}$$

Intermediate Goods Producers

There is a continuum of intermediate good producers. All of them have the following CES production function

$$Y_t(z) = \left[(1-\alpha)\left(L_t(z)\right)^{\frac{\psi-1}{\psi}} + \alpha\left(M_t(z)\right)^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}} \tag{3-10}$$

where M is oil which enters as a non-produced input, ψ represents the intratemporal elasticity of substitution between labour-input and oil and α denotes the share of oil in the production function. We use this generic production function in order to capture the fact that oil has few substitutes, in general we assume that.ψ is lower than one. The oil price shock, Q_t, is assumed to follow an $AR(1)$ process in logs,

$$\log Q_t = \log \overline{Q} + \rho \log Q_{t-1} + \varepsilon_t \tag{3-11}$$

where $\overline{Q}$ is the steady state level of oil price. From the cost minimisation problem of the firm we obtain an expression for the real marginal cost given by:

$$MC_t(z) = \left[(1-\alpha)^{\psi}\left(\frac{W_t}{P_t}\right)^{1-\psi} + \alpha^{\psi}\left((1+\tau^q) Q_t\right)^{1-\psi}\right]^{\frac{1}{1-\psi}} \tag{3-12}$$

where $MC_t(z)$ represents the real marginal cost, W_t nominal wages and P_t the consumer price index, and τ^q is a proportional tax on oil sales. Notice that marginal costs are the same for all intermediate firms, since technology has constant returns to scale and factor markets are competitive, i.e. $MC_t(z) = MC_t$. On the other hand, the individual firm's labour demand is given by:

$$L_t^d(z) = \left(\frac{1}{1-\alpha}\frac{W_t/P_t}{MC_t}\right)^{-\psi} Y_t(z) \tag{3-13}$$

Intermediate producers set prices following a staggered pricing mechanism a la Calvo. Each firm faces an exogenous probability of changing prices given by $(1-\theta)$. A firm that changes its price in period t chooses its new price $P_t(z)$ to maximise:

$$E_t \sum_{k=0}^{\infty} \theta^k \zeta_{t,t+k} \Gamma\left(P_t(z), P_{t+k}, MC_{t+k}, Y_{t+k}\right)$$

where $\zeta_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}}$ is the stochastic discount factor. The function:

$$\Gamma\left(P(z), P, MC, Y\right) = \left[(1-\tau^y)\, P(z) - P\, MC\right] \left(\frac{P(z)}{P}\right)^{-\varepsilon} Y$$

is the after-tax nominal profits of the supplier of good z with price $P_t(z)$, when the aggregate demand and aggregate marginal costs are equal to Y and MC, respectively. τ^y is the proportional tax on sale revenues, which we assume constant and equal to $\overline{\tau^y}$.The optimal price that solves the firm's problem is given by

$$\left(\frac{P_t^*(z)}{P_t}\right) = \frac{\overline{\mu} E_t \left[\sum\limits_{k=0}^{\infty} \theta^k \zeta_{t,t+k} MC_{t,t+k} F_{t+k}^{\varepsilon+1} Y_{t+k}\right]}{E_t \left[\sum\limits_{k=0}^{\infty} \theta^k \zeta_{t,t+k} F_{t+k}^{\varepsilon} Y_{t+k}\right]} \tag{3-14}$$

where $\overline{\mu} \equiv \frac{\varepsilon}{\varepsilon-1} / \left(1-\overline{\tau^y}\right)$ is the price markup, $P_t^*(z)$ is the optimal price level chosen by the firm and $F_{t+k} = \frac{P_{t+k}}{P_t}$ the cumulative level of inflation. The optimal price solves equation $(3-14)$and its determined by the average of expected future marginal costs as follows:

$$\left(\frac{P_t^*(z)}{P_t}\right) = \mu E_t\left[\sum_{k=0}^{\infty}\varphi_{t,t+k}MC_{t,t+k}\right] \tag{3-15}$$

where

$$\varphi_{t,t+k} = \frac{\theta^k\zeta_{t,t+k}F_{t+k}^{\varepsilon+1}Y_{t+k}}{E_t\left[\sum_{k=0}^{\infty}\theta^k\zeta_{t,t+k}F_{t+k}^{\varepsilon}Y_{t+k}\right]} \tag{3-16}$$

Since only a fraction $(1-\theta)$ of firms changes prices every period and the remaining one keeps its price fixed, the aggregate price level, the price of the final good that minimise the cost of the final goods producers, is given by the following equation:

$$P_t^{1-\varepsilon} = \theta P_{t-1}^{1-\varepsilon} + (1-\theta)\left(P_t^*(z)\right)^{1-\varepsilon} \tag{3-17}$$

Following Benigno and Woodford (2005), equations $(3-14)$ and (3-17) can be written recursively introducing the auxiliary variables N_t and D_t (see appendix B.2 for details on the derivation):

$$\theta\left(\Pi_t\right)^{\varepsilon-1} = 1-(1-\theta)\left(\frac{N_t}{D_t}\right)^{1-\epsilon} \tag{3-18}$$

$$D_t = Y_t\left(C_t\right)^{-\sigma} + \theta\beta E_t\left[\left(\Pi_{t+1}\right)^{\epsilon-1}D_{t+1}\right] \tag{3-19}$$

$$N_t = \mu Y_t\left(C_t\right)^{-\sigma}MC_t + \theta\beta E_t\left[\left(\Pi_{t+1}\right)^{\epsilon}N_{t+1}\right] \tag{3-20}$$

Equation $(3-18)$ comes from the aggregation of individual firms prices. The ratio N_t/D_t represents the optimal relative price $P_t^*(z)/P_t$. These three last equations summarise the recursive representation of the non linear Phillips curve.

3.2.3 Market Clearing

In equilibrium labour, intermediate and final goods markets clear. Since there is neither capital accumulation nor government sector, the economy-wide resource constraint is given by

$$Y_t = C_t \tag{3-21}$$

The labour market clearing condition is given by:

$$L_t^s = L_t^d \tag{3-22}$$

Where the demand for labour comes from the aggregation of individual intermediate producers in the same way as for the labour supply:

$$\begin{aligned} L^d &= \int_0^1 L_t^d(z)dz = \left(\frac{1}{1-\alpha}\frac{W_t/P_t}{MC_t}\right)^{-\psi} \int_0^1 Y_t(z)dz \\ L^d &= \left(\frac{1}{1-\alpha}\frac{W_t/P_t}{MC_t}\right)^{-\psi} Y_t\Delta_t \end{aligned} \tag{3-23}$$

where $\Delta_t = \int_0^1 \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon} dz$ is a measure of price dispersion. Since relative prices differ across firms due to staggered price setting, input usage will differ as well, implying that is not possible to use the usual representative firm assumption, therefore, the price dispersion factor, Δ_t appears in the aggregate labour demand equation. We can also use (3-17) to derive the law of motion of Δ_t

$$\Delta_t = (1-\theta)\left(\frac{1-\theta\left(\Pi_t\right)^{\varepsilon-1}}{1-\theta}\right)^{\varepsilon/(\varepsilon-1)} + \theta\Delta_{t-1}\left(\Pi_t\right)^{\varepsilon} \tag{3-24}$$

Note that inflation affects welfare of the representative agent through the labour market. From (3-24) we can see that higher inflation increases price dispersion and from (3-23) that higher price dispersion increases the labour amount necessary to produce certain level of output, implying more disutility on (3-1).

3.2.4 Monetary Policy

We abstract from any monetary frictions assuming that the central bank can control directly the risk-less short-term interest rate R_t.

3.2.5 The Log Linear Economy

To illustrate the effects of oil in the dynamic equilibrium of the economy, we take a log linear approximation of equations (3-1), (3-4),(3-11),(3-12),(3-18),(3-19),(3-20) and (3-23) around the deterministic steady-state. We denote variables in steady state with over bars (i.e. $\overline{X}$) and their log deviations around the steady state with lower case letters (i.e. $x_t = \log(\frac{X_t}{\overline{X}})$). After, imposing the goods and labour market clearing conditions to eliminate real wages and labour from the system, the dynamics of the economy is determined by the following equations,

$$l_t = y_t - \delta\left[(\sigma + v)\, y_t - q_t\right] \tag{3-25}$$

$$mc_t = \chi\,(\nu + \sigma)\, y_t + (1 - \chi)\, q_t \tag{3-26}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa mc_t \tag{3-27}$$

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}\left(r_t - E_t \pi_{t+1}\right) \tag{3-28}$$

$$q_t = \rho q_{t-1} + \xi_t \tag{3-29}$$

where $\overline{\alpha} \equiv \alpha^{\psi}\left(\frac{(1+\tau^q)\overline{Q}}{\overline{MC}}\right)^{1-\psi}$, $\delta \equiv \psi\chi\frac{\overline{\alpha}}{1-\overline{\alpha}}$, $\chi \equiv \frac{1-\overline{\alpha}}{1+v\psi\overline{\alpha}}$ and $\kappa \equiv \frac{1-\theta}{\theta}\left(1-\theta\beta\right)$. $\overline{Q}$, and $\overline{MC}$ represent the steady-state value of oil prices and of the marginal cost, respectively. $\overline{\alpha}$ corresponds to the share of oil on marginal costs in steady state, δ and $(1-\chi)$ accounts for the effects oil prices in labour and marginal costs, respectively; and κ is the elasticity of inflation respect to marginal costs.

Interestingly, the effects of oil prices on marginal costs, equation (3-26), depends crucially on the share of oil in the production function, α, and on the elasticity of substitution between oil and labour,ψ. Thus, when α is large, χ is smaller making marginal costs more responsive to oil prices. Also, when ψ is lower, the impact of oil on marginal costs is larger. It is important to note that even though the share of oil in the production function, α, can be small, its impact on marginal cost, $\overline{\alpha}$, can be magnified when oil has few substitutes (that is when ψ is low). Moreover, a permanent increase in oil prices, that is. an increase in $\overline{Q}$, would make marginal cost of firms more sensitive to oil price shocks since it increases $\overline{\alpha}$. In the case that $\alpha = 0$, the model collapses to a standard close economy New Keynesian model without oil.

If we replace equation (3-26) in (3-27) we obtain the traditional New Keynesian Phillips curve.

$$\pi_t = \kappa_y y_t + \kappa_q q_t + \beta E_t \pi_{t+1} \tag{3-30}$$

where $\kappa_y = \kappa\chi(v+\sigma)$ and $\kappa_q = \kappa(1-\chi)$. We define the natural rate of output as the level of output such inflation is zero in all periods, this is given by $y_t^n = -\frac{\kappa_q}{\kappa_y} q_t$ Then the Phillips curve can be written as deviations of output from its natural level:

$$\pi_t = \kappa_y (y_t - y_t^n) + \beta E_t \pi_{t+1}$$

3.2.6 Distortions in steady state

The details of the steady state of the variables is in appendix B.1. In steady state we have two distortions: the first one is the monopolistic distortion and the second one comes from the Oil market. Related to the first distortion, because intermediate goods producers set prices monopolistically, the price they charge is higher than the marginal cost, and the monopolistic distortion is given by:

$$\overline{MC} = \frac{1-\overline{\tau}}{\varepsilon/(\varepsilon-1)} = \frac{1}{\overline{\mu}} \leq 1 \tag{3-31}$$

Let's denote the steady state distortion caused by monopolistic competition by

$$\Phi = 1 - \frac{1-\overline{\tau}}{\varepsilon/(\varepsilon-1)}$$

where Φ measures the monopolistic distortion, when taxes on sales can eliminate this distortion we have that $\Phi = 0$. In a competitive equilibrium the marginal rate of substitution between consumption and leisure must equal the marginal product of labour. However, monopolistic distortions generates a wedge between this two, given by Φ_L

$$\begin{aligned}\Phi_L &= 1 - \frac{\overline{V}_L}{\overline{U}_C}\frac{\partial \overline{L}}{\partial \overline{Y}} \\ &= 1 - (1-\overline{\alpha})(1-\Phi)(1-\delta(\sigma+v))\end{aligned} \tag{3-32}$$

Note that in this economy since labour is not the only input in the production function, then $\Phi_L \neq \Phi$ the wedge in the labour market is not the same as the distortion in marginal costs. Also, eliminating the monopolistic distortion (Φ) doesn't eliminate this wedge. The effect of the monopolistic distortion on Φ_L can be eliminated with a subsidy (negative tax rate) such that $\Phi_L = 0$.

Similarly, the oil market distortion affects the share of oil in the steady state marginal costs:

$$\overline{\alpha} = \alpha^{\psi} \left(\frac{\left(1+\tau^{Q}\right) \overline{Q}}{\overline{MC}} \right)^{1-\psi}$$

Since in this economy firms are price takers for oil, its price can also be distorted from a competitive equilibrium. Again, this distortion can be eliminated with a tax (or subsidy) such that $\left(1+\tau^{Q}\right) \overline{Q}/\overline{MC}$ equals to the one from a competitive equilibrium. In general, when the oil price is to high respect to marginal cost, the policy to eliminate this distortion is to subsidise the use of oil ($\tau^{q} < 0$), since such high price increases the costs of firms and reduces output and consumption below the optimal.

3.3 A Linear-Quadratic Approximate Problem

In this section we present a second order approximation of the welfare function of the representative household as function of purely quadratic terms. This representation allow us tho characterise the policy problem using only a linear approximation of the structural equations of the model and also to rank sub-optimal monetary policy rules.

Since the model has a distorted steady state, a standard second order Taylor approximation of the welfare function will include linear terms, which would lead to an inaccurate approximation of the optimal policy in a linear-quadratic approach. We use then the methodology proposed by Benigno and Woodford (2005), which consists on eliminating the linear terms of the policy objective using a second order approximation of the aggregate supply.

3.3.1 Second order Taylor expansion of the model

In this sub-section we present a log-quadratic (Taylor-series) approximation of the fundamental equations of the model around the steady state, a detailed derivation is provided in Appendix B. The second-order Taylor-series expansion serves to compute the equilibrium fluctuations of the endogenous variables of the model up to a residual of order $O\left(\|\xi\|^2\right)$, where $\|\xi_t\|$ is a bound on the size of the oil price shock. Up to second order, equations (3-25) to (3-28) are replaced by the following set of log-quadratic equations:

Labour Market	
$l_t = y_t - \delta\left[(v+\sigma) y_t - q_t\right] + \frac{\chi}{1-\alpha}\widehat{\Delta}_t + \frac{1}{2}\frac{1-\psi}{1-\alpha}\delta\chi^2\left[(v+\sigma) y_t - q_t\right]^2 + O\left(\|\xi\|^3\right)$	$3-i$
Aggregate Supply	
Marginal Costs	
$mc_t = \chi(v+\sigma) y_t + (1-\chi) q_t + \frac{1}{2}\frac{1-\psi}{1-\alpha}(1-\chi)\chi^2\left[(v+\sigma) y_t - q_t\right]^2 + \chi v\widehat{\Delta}_t + O\left(\|\xi\|^3\right)$	$3-ii$
Price dispersion	
$\widehat{\Delta}_t = \theta\widehat{\Delta}_t + \frac{1}{2}\varepsilon\frac{\theta}{1-\theta}\pi_t^2 + O\left(\|\xi\|^3\right)$	$3-iii$
Phillips Curve	
$v_t = \kappa mc_t + \frac{1}{2}\kappa mc_t\left(2(1-\sigma) y_t + mc_t\right) + \frac{1}{2}\varepsilon\pi_t^2 + \beta E_t v_{t+1} + O\left(\|\xi\|^3\right)$	$3-iv$
where we have defined the auxiliary variables:	
$v_t \equiv \pi_t + \left(\frac{\varepsilon-1}{1-\theta} + \varepsilon\right)\pi_t^2 + \frac{1}{2}(1-\theta\beta)\pi_t z_t$	$3-v$
$z_t \equiv 2(1-\sigma) y_t + mc_t + \theta\beta E_t\left(\frac{2\varepsilon-1}{1-\theta\beta}\pi_{t+1} + z_{t+1}\right) + O\left(\|\xi_t\|^2\right)$	$3-vi$
Aggregate Demand	
$y_t = E_t y_{t+1} - \frac{1}{\sigma}\left(r_t - E_t\pi_{t+1}\right) - \frac{1}{2}\sigma E_t\left[(y_t - y_{t+1}) - \frac{1}{\sigma}\left(r_t - \pi_{t+1}\right)\right]^2 + O\left(\|\xi\|^3\right)$	$3-vii$

Table 3.1: Second order Taylor expansion of the equations of the model

Equations (3-i) and (3-ii) are obtained taking a second-order Taylor-series expansion of the aggregate labour and the real marginal cost equation2, after using the labour market equilibrium to eliminate real wages. $\widehat{\Delta}_t$ is the log-deviation of the price dispersion measure Δ_t, which is a second order function of inflation (see appendix B.2 for details) and its dynamic is represented with equation (3-iii).

We replace the equation for the marginal costs (3-ii) in the second order expansion of the Philips curve and iterate forward. Then, replace recursively the price dispersion terms from equation (3-iii) to obtain the infinite sum of the Phillips curve only as a function of output, inflation and the oil shock:

$$\begin{aligned} v_{t_o} &= \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left\{ \begin{array}{c} \kappa_y y_t + \kappa_q q_t + \frac{1}{2}\varepsilon\left(1+\chi v\right)\pi_t^2 \\ +\frac{1}{2}\kappa\left[c_{yy} y_t^2 + 2c_{yq} y_t q_t + c_{qq} q_t^2\right] \end{array} \right\} \\ &\quad + \left(1-\theta\right)\chi v \widehat{\Delta}_{t_o-1} + \left(\|\xi_t\|^3\right) \end{aligned} \tag{3-33}$$

where c_{yy}, c_{yy} and c_{yy} are defined in the appendix.

3.3.2 A second-order approximation to utility

A second order Taylor-series approximation to the utility function, expanding around the non-stochastic steady-state allocation is:

$$U_{t_o} = \overline{Y}\overline{u}_c \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left(\Phi_L y_t + \frac{1}{2} u_{yy} y_t^2 + u_{yq} y_t q_t + u_{\Delta}\widehat{\Delta}_t \right) + t.i.p. + O\left(\|\xi_t\|^3\right) \tag{3-34}$$

where $y_t \equiv \log\left(Y_t/\overline{Y}\right)$ and $\widehat{\Delta}_t \equiv \log \Delta_t$ measure deviations of aggregate output and the price dispersion measure from their steady state levels, respectively. The term "t.i.p." collects terms that are independent of policy (constants and functions of exogenous disturbances) and hence irrelevant for ranking alternative policies. Φ_L is the wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labour generated by the monopolistic distortion, defined in the previous section. The coefficients: u_{yy}, u_{yq} and u_{Δ} are defined in the appendix B.2

We use equation (3-iii) to substitute in our welfare approximation the measure of price dispersion as a function of quadratic terms of inflation. Also, we use the second order approximation of the AS (equation 3-33) to solve for the infinite discounted sum of the expected level of output as function of purely quadratic terms. Then, as in Beningno and Woodford (2005) we replace this last expression in (3-34). We can rewrite (3-34) as:

$$U_{t_o} = -\Omega \left[E_{t_o} \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left(\frac{1}{2}\lambda \left(y_t - y_t^*\right)^2 + \frac{1}{2}\pi_t^2 \right) - T_{t_o} \right] + t.i.p. + O\left(\|\xi_t\|^3\right) \quad \text{(3-35)}$$

where $\Omega = \overline{Y}\overline{u}_c\lambda_\pi$ and $T_{t_o} = \frac{\Phi_L}{\kappa_y} v_{t_o}$, λ_π is defined in the appendix. λ measures the relative weight between a welfare-relevant output gap and inflation. y_t^* is the efficient output, the level of output that maximises our measure of welfare when inflation is zero. The values of λ and y_t^* are given by:

$$\lambda = \frac{\kappa_y}{\varepsilon}\left(1 - \sigma\psi\overline{\alpha}\right)\gamma \quad \text{(3-36)}$$

$$y_t^* = -\left(\frac{1+\psi v}{\sigma + v}\right)\left(\frac{\alpha^*}{1-\alpha^*}\right) q_t \quad \text{(3-37)}$$

where α^* is the efficient share in steady state of oil in the marginal costs, given by:

$$\alpha^* = \frac{\overline{\alpha}}{1+\eta} \quad \text{(3-38)}$$

Both γ and η are function of the deep parameters of the model and are defined in the appendix. Note that the natural rate of output can be written in a similar way as the efficient output:

$$y_t^n = -\left(\frac{1+\psi v}{\sigma + v}\right)\left(\frac{\overline{\alpha}}{1-\overline{\alpha}}\right) q_t$$

3.3.3 The linear-quadratic policy problem

The policy objective U_{t_o} can be written on terms of inflation and the welfare-relevant output gap defined by x_t:

$$x_t \equiv y_t - y_t^*$$

Benigno and Woodford (2005) show that maximisation of U_{t_o} is equivalent to minimise the following lost function L_{t_o} subject to a predeterminated value of

v_{t_o} :

$$L_{t_o} \equiv E_{t_o} \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left(\frac{1}{2} \lambda x_t^2 + \frac{1}{2} \pi_t^2 \right) \tag{3-39}$$

Also, because the objective function is purely quadratic, a linear approximation of v_{t_o} suffices to describe the initial commitments, given by $v_{t_o} = \pi_{t_o}$.

We are interested in evaluating monetary policy from a timeless perspective: optimising without regard of possible short run effects and avoiding possible time inconsistency problems. Then, from a timeless perspective the predetermined value of π_{t_o} must equal $\pi_{t_o}^*$, the optimal value of inflation at t_o consistent with the policy problem. Thus, the policy objective consists on minimise (3-39) subject to the initial inflation rate:

$$\pi_{t_o} = \pi_{t_o}^* \tag{3-40}$$

and the Phillips curve for any date from t_o onwards:

$$\pi_t = \kappa_y x_t + \beta E_t \pi_{t+1} + u_t \tag{3-41}$$

Note that we have expressed (3-41) in terms of the welfare relevant output gap, x_t. u_t is a "cost-push" shock, that is proportional to the deviations in the real oil price:

$$\begin{aligned} u_t &\equiv \kappa_y \left(y_t^* - y_t^n \right) \\ &= \varpi q_t \end{aligned}$$

where

$$\varpi \equiv \kappa_y \left(\frac{1 + \psi v}{\sigma + v} \right) \left[\frac{\overline{\alpha}}{1 - \overline{\alpha}} - \frac{\alpha^*}{1 - \alpha^*} \right]$$

In this model a "cost-push" shock arises endogenously since oil generates a trade-off between stabilising inflation and deviations of output from an efficient level, different from the natural level. In the next section we characterise the conditions under which oil shocks preclude simultaneous stabilisation of inflation and the welfare-relevant output gap.

3.4 Optimal monetary response to oil shocks from a timeless perspective.

In this section we use the linear-quadratic policy problem defined in the previous section to evaluate optimal and sub-optimal monetary policy rules under oil shocks. This policy problem can be summarised to maximise the following Lagrangian:

$$\mathcal{L}_{t_o} \equiv -E_{t_o}\left\{\begin{array}{c}\sum_{t=t_o}^{\infty}\beta^{t-t_o}\left[\frac{1}{2}\lambda x_t^2+\frac{1}{2}\pi_t^2-\varphi_t\left(\pi_t-\kappa_y x_t-\beta E_t\pi_{t+1}-u_t\right)\right] \\ +\varphi_{t_o-1}\left(\pi_{t_o}-\pi_{t_o}^*\right)\end{array}\right\} \tag{3-42}$$

where $\beta^{t-t_o}\varphi_t$ is the Lagrange multiplier at period t.

The second order conditions for this problem are well defined for $\lambda \geq 0$, which is the case for plausible parameters of the model[4]. Then, as Benigno and Woodford (2005) show, since the loss function is convex , then randomisation of monetary policy is welfare reducing and there are welfare gains when using monetary policy rules.

Under certain circumstances the optimal policy involves complete stabilisation of the inflation rate at zero for every period, that is complete price stability. These conditions are related to how oil enters in the production function. These conditions are summarised in the following proposition:

Proposition 3.1. *When the production function is Cobb-Douglas the efficient level of output is equivalent to the natural level of output.*

In the case of a Cobb-Douglas production function, the elasticity of substitution between labour and oil is unity (i.e. $\psi = 1$). In this case $\eta = 0$ and the share of oil on the marginal costs in the efficient level is equal to the share in the

[4]More precisely, we are interested on study the model when $0 < \psi \leq 1$ and σ not too high. Since λ is positive for $\psi \leq 1$ and $\sigma < (\overline{\alpha}\psi)^{-1}$, which is a very high value for the threshold since $\overline{\alpha}$ is lower than one and small.

distorted steady state, equal to α (that is $\alpha^* = \overline{\alpha} = \alpha$) Then, the efficient level of output is equal to the natural level of output.

In this special case of the CES production function, fluctuations in output caused by oil shocks at the efficient level equals the fluctuations in the natural level. Then, stabilisation of output around the latter also implies stabilisation around the former. This is a special case in which the "divine coincidence" appears . Therefore, setting output equal to the efficient level also implies complete stabilisation of inflation at zero.

In this particular case there is not trade-off between stabilising output and inflation. However, in a more general specification of the CES production function this trade-off appears, as it is established in the next proposition:

Proposition 3.2. *When oil is difficult to substitute in production the efficient output respond less to oil shocks than the natural level, which generates a trade-off.*

When oil is difficult to substitute the elasticity of substitution between inputs is lower than one (that is $\psi < 1$). In this case $\eta < 0$ and the efficient share of oil on marginal costs is lower than in the steady state (that is $\alpha^* < \overline{\alpha}$), which causes that the efficient output fluctuates less than the natural level (that is $|y_t^*| < |y_t^n|$). Then, in this case it is not possible to have both inflation zero and output at the efficient level at all periods.

It is important to mention that **we have this trade-off even in the case when the effects of monopolistic distortions on welfare are eliminated (that is when $\Phi_L = 0$).** This is because oil shocks affects differently consumption and leisure in the welfare function. When there is an oil price shock, output (and hence consumption) decreases because of the effects on marginal costs. Similarly, labour (and hence leisure) also decreases because of lower aggregate demand. Since the elasticity of substitution is lower than one, labour decreases less than the decrease in output, generating a wedge between the utility of consumption and the disutility of labour. The lower the elasticity, the lower the effect on labour and the higher the relative effect on production, and the higher this wedge. The efficient level of output is the one that minimises the effects of oil fluctuations on welfare, which is different from the natural level of output.

Figure 3.1 shows the effect on α^* and $\overline{\alpha}$ and on y^* and y^n of the elasticity of substitution. As mentioned in proposition 1, when $\psi = 1$ then $\alpha^* = \overline{\alpha} = \alpha$. Similarly, as in proposition 2, when $\psi < 1$ it increases both α^* and $\overline{\alpha}$, but α^* is lower than $\overline{\alpha}$. Also, for $\psi < 1$ the efficient output fluctuates less than the natural level of output an oil price shock of unity.[5]

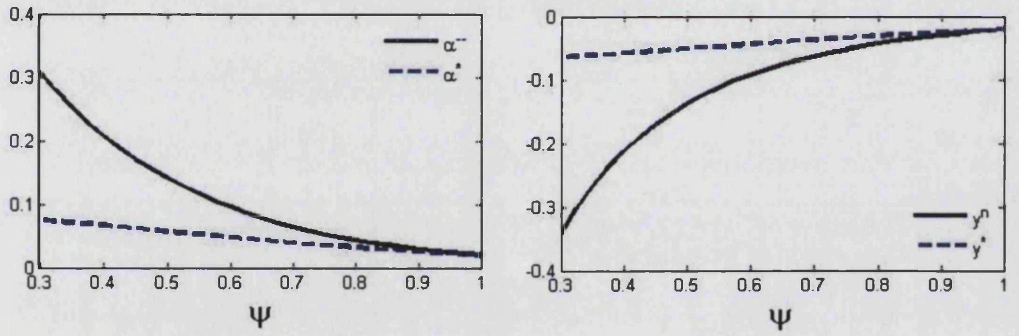

Figure 3.1: (a) Steady state and efficient share of oil on marginal costs. (b) Natural and efficient level of output.

It is also important to analyse how the production function affects λ, the weight between stabilising the welfare relevant output-gap and inflation. The next two propositions summarise behaviour of λ.

Proposition 3.3. *When the production function is Cobb-Douglas, the relative weight in the loss function between welfare-relevant output gap and inflation stabilisation (λ) becomes* $\frac{\kappa_y}{\varepsilon}(1 - \sigma\alpha)$

[5]As benchmark calibration we use the same values as in chapter 2. Those values are: $\beta = 0.99, \sigma = 1, v = 0.5, \varepsilon = 11, \overline{Q} = \overline{(MC)}, \psi = 0.6, \alpha = 0.01, \rho = 0.94$ and $\sigma_\varepsilon = 0.14$

In the case of a Cobb-Douglas production function the coefficient $\gamma = 1$ and $\lambda = \frac{\kappa_y}{\varepsilon}(1-\sigma\alpha)$. This is similar to the coefficient found for many authors for the case of a closed economy[6], which is the ratio of the effect of output on inflation in the Phillips curve and the elasticity of substitution across goods over, but multiplied by the additional term $(1-\sigma\alpha)$.

The term $(1-\sigma\alpha)$ captures the effects of oil shocks in inflation through costs, which is independent of the degree of substitution. When the weight of oil in the production function (α) is higher, the effects of oil shocks in marginal costs and inflation are more important. Then, the more important becomes to stabilise inflation over output.

Proposition 3.4. *The lower the elasticity of substitution between oil and labour, the higher the weight in the loss function between welfare-relevant output gap and inflation stabilisation (λ).*

When the elasticity of substitution ψ is lower, both the coefficient γ and the term $(1-\sigma\overline{\alpha}\psi)$ in λ becomes higher, then λ becomes higher. As mention in proposition 3.2, when the elasticity of substitution is smaller, an oil shock affects more output than labour. Then, as inflation affects welfare through the labour market, lower ψ implies lower relative effect on inflation respect to output and therefore, higher λ.

The next graphs shows the effects on λ of the elasticity of substitution for three different values of α. λ takes its lowest value when $\psi = 1$ and increases exponentially for lower ψ. Also, higher α reduces λ , which means a higher weight on inflation relative to output fluctuations in the welfare function.

[6]See for example Woodford (2003) and Benigno and Woodford (2005).

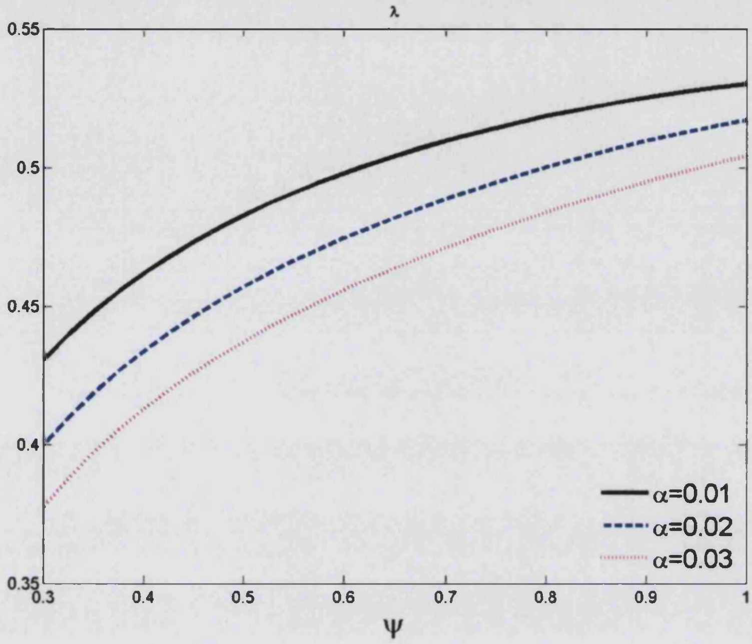

Figure 3.2: Relative weight between output and inflation stabilisation (λ).

3.4.1 Optimal unconstrained response to oil shocks

When we solve for the Lagrangian (3-42), we obtain the following first order conditions that characterise the solution of the optimal path of inflation and the welfare-relevant output gap in terms of the Lagrange multipliers:

Proposition 3.5. *The optimal unconstrained response to oil shocks is given by the following conditions:*

$$\begin{aligned} \pi_t &= \varphi_{t-1} - \varphi_t \\ x_t &= \frac{\kappa_y}{\lambda}\varphi_t \end{aligned}$$

where φ_t is the Lagrange multiplier of the optimisation problem, that has the following law of motion :

$$\varphi_t = \tau_\varphi \varphi_{t-1} - \phi q_t$$

for $\phi \equiv \frac{\tau_\varphi}{1-\beta\tau_\varphi\rho}\varpi$, *and satisfies the initial condition:*

$$\varphi_{t_o-1} = -\phi \sum_{k=0}^{\infty} \tau_\varphi^k q_{t-1-k}$$

where $\tau_\varphi = Z - \sqrt{Z^2 - \frac{1}{\beta}} < 1$ *and* $Z = \left((1+\beta) + \frac{\kappa_y^2}{\lambda}\right)/(2\beta)$.

The proof is in the appendix. From a timeless perspective the initial condition for φ_{t_o-1} depends on the past realisations of the oil prices and it is time-consistent with the policy problem.

Also, we define the impulse response of a shock in the oil price in period t (ξ_t) in a variable z in $t+j$ as the unexpected change in its transition path. Then the impulse is calculated by:

$$I_t(z_{t+j}) = E_t[z_{t+j}] - E_{t-1}[z_{t+j}]$$

and the impulse response for inflation and output gap for the optimal policy is:

$$I_t^{opt}(\pi_{t+j}) = \left(\frac{\rho^{j+1} - \tau_\varphi^{j+1}}{\rho - \tau_\varphi} - \frac{\rho^j - \tau_\varphi^j}{\rho - \tau_\varphi}\right)\phi\xi_t \tag{3-43}$$

$$I_t^{opt}(x_{t+j}) = -\frac{\kappa_y}{\lambda}\left(\frac{\rho^{j+1} - \tau_\varphi^{j+1}}{\rho - \tau_\varphi}\right)\phi\xi_t \tag{3-44}$$

See appendix B.3 for details on the derivation.

Figure 3.3 shows the optimal unconstrained impulse response functions to an oil price shock of size one for different values of the elasticity of substitution (psi) for inflation, welfare-relevant output gap, the nominal interest rate and inflation. Inflation and the nominal interest rate are in yearly terms. The benchmark case is a value of $\psi = 0.6$, similar to the one used in chapter 2. In this graph we can see that after an oil shock the optimal response is an increase of inflation and a reduction of the welfare-relevant output gap, and consequently also of output. The nominal interest rate also increases to partially offset the effects of the oil shock on inflation. Inflation after 8 quarters become negative as the optimal unconstrained

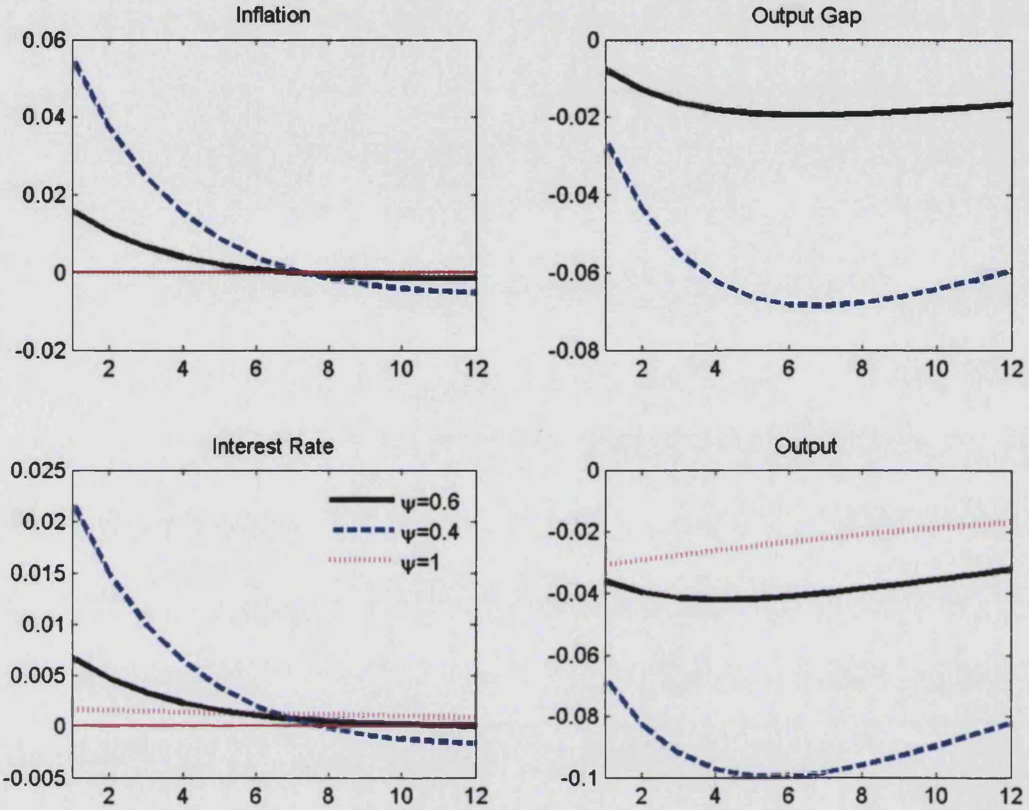

Figure 3.3: Impulse response to an oil shock under optimal monetary policy.

plan is associated to price stability [7]. To summarise, the optimal response to an oil shock imply an effect on impact on inflation that dies dies out very rapidly and a more persistent effect on output.

A reduction in the elasticity of substitution from 0.6 to 0.4 magnifies the size of the cost push shock, and increases both λ and $\overline{\alpha}$. Then, the impact on all the variables increases exponentially, being inflation initially the more affected variable. However, after 8 quarters the response is magnified on the welfare relevant output gap. In contrast, when the elasticity of substitution is unity, since there is no such

[7]Note: see Woodford (2003) for a discussion on optimal monetary policy rules

a trade-off, both inflation and welfare-relevant output gap are zero in every period. There is also a reduction on output caused by the oil shock and the increase on the interest rate needed to maintain zero inflation.

3.4.2 Evaluation of suboptimal rules - the non-inertial plan

We can use our linear-quadratic policy problem for ranking alternative sub-optimal policies. One example of such policies is the optimal non-inertial plan. By a non-inertial policy we mean on in which the monetary policy rule depends only in the current state of the economy. In this case, if the policy results in a determinate equilibrium, then the endogenous variables depend also on the current state.

If the current state of the economy is given by the cost push shock, which has the following law of motion:

$$u_t = \rho u_{t-1} + \varpi \xi_t$$

where ξ_t is the oil price shock and ϖ is defined in the previous section. A first order general description of the possible equilibrium dynamics can be written in the form:

$$\pi_t = \overline{\pi} + f_\pi u_t \tag{3-45}$$

$$x_t = \overline{x} + f_x u_t \tag{3-46}$$

$$\varphi_t = \overline{\varphi} + f_\varphi u_t \tag{3-47}$$

where we need to determine the coefficients: $\overline{\pi}, \overline{x}, \overline{\varphi}, f_\pi, f_x$ and f_φ. To solve for the optimal non-inertial plan from a timeless perspective we need to replace (3-45),(3-46) and (3-47) in the Lagrangian (3-42) and solve for the coefficients that maximise the objective function. The results are summarised in the following proposition:

Proposition 3.6. *The optimal non-inertial plan from a "timeless perspective" is given by $\pi_t = \overline{\pi} + f_\pi u_t$ and $x_t = \overline{x} + f_x u_t$, where*

$$\overline{\pi} = 0 \quad f_\pi = \frac{\lambda(1-\rho)}{\kappa_y^2 + \lambda(1-\beta\rho)(1-\rho)}$$
$$\overline{x} = 0 \quad f_x = \frac{\kappa_y}{\kappa_y^2 + \lambda(1-\beta\rho)(1-\rho)}$$

Note that in the optimal non-inertial plan the ratio of inflation/output gap is constant and equal to $\frac{\lambda(1-\rho)}{\kappa_y}$. The higher the weight in the loss function for output fluctuations relative to inflation fluctuations, the higher the inflation rate. Also, the more persistent the oil shocks, the lower the weight on inflation relative to the welfare-relevant output-gap.

Similar the the optimal case, the impulse response functions for inflation and output are defined by:

$$\begin{aligned} I_t^{ni}(\pi_{t+j}) &= f_\pi \varpi \rho^j \xi_t \\ I_t^{ni}(x_{t+j}) &= f_\pi \varpi \rho^j \xi_t \end{aligned}$$

Figure 3.4 shows the optimal non-inertial plan to an unitary oil price shock. In this case, the ratio of inflation to the welfare-relevant output gap is constant. For the benchmark case ($\psi = 0.6$) the response of inflation is lower than in the unconstrained optimal plan, but the effect on output is higher. Also, the effects on both variables are more persistent than in the unconstrained plan.

Furthermore, under the optimal non-inertial plan, when ψ decreases from 0.6 to 0.4 the impact on all the variables increases. This is due to the magnifying effect of ψ on the cost-push shock. Also, the reduction of ψ raises λ, which increases more the effect on inflation relatively more than that on on output. As in the unconstrained case, when $\psi = 1$ the trade-off disappears. In that case, inflation is zero in every period and output reduces.

Both exercises, the optimal unconstrained plan and the optimal non-inertial plan, show that to the extent that economies are more dependent on oil, in the sense that oil is difficult to substitute, the impact of oil shocks on both inflation and output is greater. Also, in this case, monetary policy should react by raising more the nominal interest rate and allowing relatively more fluctuations on inflation than on output.

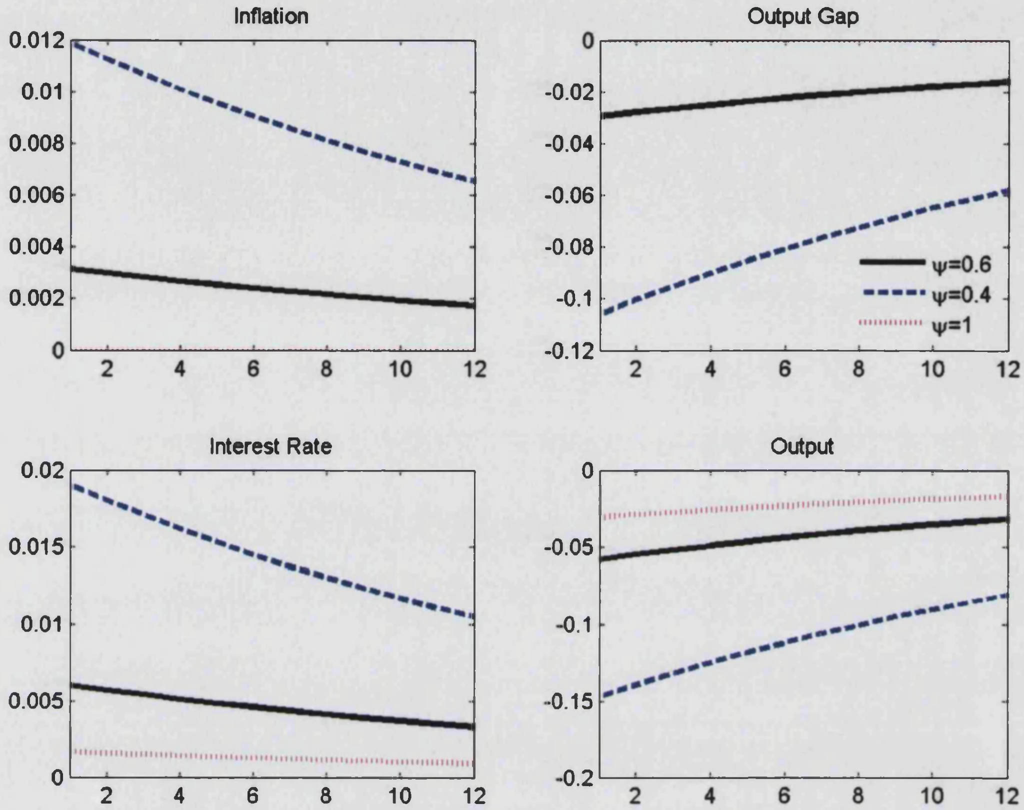

Figure 3.4: Impulse response to an oil shock under the optimal non-inertial plan.

3.5 Conclusions

This chapter characterises the utility-based loss function for a closed economy in which oil is used in the production process, there is staggered price setting and monopolistic competition. As in Benigno and Woodford (2005), our utility based-loss function is a quadratic on inflation and the deviations of output from an efficient level, which is the welfare-relevant output gap.

We found that this efficient level differs from the natural level of output when the elasticity of substitution between labour and oil is different from one. This generates a trade-off between stabilising inflation and output in the presence of oil shocks. Also, the cost-push shocks involved in this trade-off are proportional to oil shocks. The lower this elasticity of substitution, the higher the size of the cost-push shock. We also find, in contrast to Benigno and Woodford (2005), that this trade-off remains even when the effects of monopolistic distortions on the steady state are eliminated.

Furthermore, the relative weight between the welfare-relevant output gap and inflation on the utility-based loss function depends inversely to this elasticity of substitution. On the contrary, the higher the share of oil in the production function, the relative weight is smaller.

These results show that to the extent that economies are more dependent on oil, in the sense that oil is difficult to substitute in production, the impact of oil shocks on both inflation and output is higher. Also, in this case the central bank should allow more fluctuations on inflation relative to output due to oil shocks.

Moreover, these results shed light on how technological improvements which reduces the dependence on oil, also reduce the impact of oil shocks on the economy. This could also explain why oil shocks have lower impact on inflation in the 2000s in contrast to the 1970s. Since oil has become easier to substitute with other renewable resources, the impact of oil shocks has been dampened. An observation that accords with the theoretical model provided in this chapter.

B1 Appendix: The deterministic steady state

The non-stochastic steady state of the endogenous variables for $\overline{\Pi} = 1$ is given by:

Interest rate	$\overline{R} = \beta^{-1}$
Marginal costs	$\overline{MC} = 1/\overline{\mu}$
Real wages	$\overline{W/P} = \frac{1-\overline{\alpha}}{\overline{\mu}} \left(\frac{1-\overline{\alpha}}{1-\alpha}\right)^{\frac{1}{1-\psi}}$
Output	$\overline{Y} = \left(\frac{1-\overline{\alpha}}{\overline{\mu}}\right)^{\frac{1}{\sigma+\nu}} \left(\frac{1-\overline{\alpha}}{1-\alpha}\right)^{\frac{1+\psi\nu}{\sigma+\nu}\frac{1}{1-\psi}}$
Labor	$\overline{L} = \left(\frac{1-\overline{\alpha}}{\overline{\mu}}\right)^{\frac{1}{\sigma+\nu}} \left(\frac{1-\overline{\alpha}}{1-\alpha}\right)^{\frac{1-\sigma\psi}{\sigma+\nu}\frac{1}{1-\psi}}$

Table B1.1: The deterministic steady state

where

$$\overline{\alpha} = \alpha^{\psi} \left(\frac{\overline{Q}}{\overline{MC}}\right)^{1-\psi} = \alpha^{\psi} \left(\overline{\mu}\overline{Q}\right)^{1-\psi}$$

$\overline{\alpha}$ is the share of oil in the marginal costs. Notice that the steady state values of real wages, output and labour depend on the steady state ratio of oil prices with respect to the marginal cost. This implies that permanent changes in oil prices would generate changes in the steady state of this variables. Also, as the standard New-Keynesian models, the marginal cost in steady state is equal to the inverse of the mark-up

$$\begin{aligned} \overline{MC} &= \overline{\mu}^{-1} = \left[\frac{(\varepsilon-1)(1-\tau^{y})}{\varepsilon}\right]^{-1} \\ &\equiv 1-\Phi \end{aligned}$$

Since monopolistic competition affects the steady state of the model, output in steady state is below the efficient level. We call to this feature a distorted steady state and Φ accounts effects of the monopolistic distortions in steady state.

Since the technology has constant returns to scale, we have that:

$$\begin{aligned}\frac{\overline{V}_L}{\overline{U}_C}\frac{\overline{L}}{\overline{Y}} &= \left(\frac{\overline{W/P}}{\overline{MC}}\frac{\overline{L}}{\overline{Y}}\right)\overline{MC} \\ &= (1-\overline{\alpha})(1-\Phi)\end{aligned}$$

the ratio of the marginal rate of substitution multiplied by the ratio labour/output is a proportion $(1-\overline{\alpha})$ of the marginal costs. This expression helps us to obtain the wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labour:

$$\begin{aligned}\frac{\overline{V}_L}{\overline{U}_C}\frac{\partial\overline{L}}{\partial\overline{Y}} &= \left(\frac{\overline{V}_L}{\overline{U}_C}\frac{\overline{L}}{\overline{Y}}\right)\left(\frac{\partial\overline{L/L}}{\partial\overline{Y/Y}}\right) \\ &= (1-\overline{\alpha})(1-\Phi)(1-\delta(\sigma+v)) \\ &\equiv 1-\Phi_L\end{aligned}$$

where $1-\Phi_L$ accounts for the effects of the monopolistic distortions on the wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labour.

B2 Appendix: The second order solution of the model

B2.1 The recursive AS equation

We divide the equation for the aggregate price level (3-17) by $P_t^{1-\varepsilon}$ and make $P_t/P_{t-1}=\Pi_t$

$$1=\theta(\Pi_t)^{-(1-\varepsilon)}+(1-\theta)\left(\frac{P_t^*(z)}{P_t}\right)^{1-\varepsilon} \tag{B2-1}$$

Aggregate inflation is function of the optimal price level of firm z. Also, from equation (3-14) the optimal price of a typical firm can be written as:

$$\frac{P_t^*(z)}{P_t}=\frac{N_t}{D_t}$$

where, after using the definition for the stochastic discount factor: $\zeta_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}}$, we define N_t and D_t as follows:

$$N_t = E_t\left[\sum_{k=0}^{\infty} \mu \left(\theta\beta\right)^k F_{t,t+k}^{\varepsilon} Y_{t+k} C_{t+k}^{-\sigma} MC_{t+k}\right] \tag{B2-2}$$

$$D_t = E_t\left[\sum_{k=0}^{\infty} \left(\theta\beta\right)^k F_{t,t+k}^{\varepsilon-1} Y_{t+k} C_{t+k}^{-\sigma}\right] \tag{B2-3}$$

N_t and D_t can be expanded as:

$$N_t = \mu Y_t C_t^{-\sigma} MC_t + E_t\left[\Pi_{t+1}^{\varepsilon} \sum_{k=0}^{\infty} \mu \left(\theta\beta\right)^k F_{t+1,t+1+k}^{\varepsilon} Y_{t+1+k} C_{t+1+k}^{-\sigma} MC_{t+1+k}\right] \tag{B2-4}$$

$$D_t = Y_t C_t^{-\sigma} + E_t\left[\Pi_{t+1}^{\varepsilon-1} \sum_{k=0}^{\infty} \left(\theta\beta\right)^k F_{t+1,t+1+k}^{\varepsilon-1} C_{t+1+k}^{-\sigma} Y_{t+1+k}\right] \tag{B2-5}$$

where we have used the definition for $F_{t,t+k} = P_{t+k}/P_t$.

The Phillips curve with oil prices is given by the following three equations:

$$\theta \left(\Pi_t\right)^{\varepsilon-1} = 1 - (1-\theta)\left(\frac{P_t^*(z)}{P_t}\right)^{1-\varepsilon} \tag{B2-6}$$

$$N_t = \mu Y_t^{1-\sigma} MC_t + \theta\beta E_t \left(\Pi_{t+1}\right)^{\varepsilon} N_{t+1} \tag{B2-7}$$

$$D_t = Y_t^{1-\sigma} + \theta\beta E_t \left(\Pi_{t+1}\right)^{\varepsilon-1} D_{t+1} \tag{B2-8}$$

where we have reordered equation (B2-1) and we have used equations (B2-2) and (B2-3) evaluated one period forward to replace N_{t+1} and D_{t+1} in equations (B2-4) and (B2-5).

B2.2 The second order approximation of the system

The MC equation and the labour market equilibrium

The real marginal cost (3-12) and the labour market equations (3-4 and 3-23) have the following second order expansion:

$$mc_t = (1-\overline{\alpha})\, w_t + \overline{\alpha} q_t + \frac{1}{2}\overline{\alpha}\,(1-\overline{\alpha})\,(1-\psi)\,(w_t - q_t)^2 + O\left(\|\xi_t\|^3\right) \tag{B2-9}$$

$$w_t = \nu l_t + \sigma y_t \tag{B2-10}$$

$$l_t = y_t - \psi\left(w_t - mc_t\right) + \widehat{\Delta}_t \tag{B2-11}$$

Where w_t and $\widehat{\Delta}_t$ are, respectively, the log of the deviation of the real wage and the price dispersion measure from their respective steady state. Notice that equations $(B2-10)$ and $(B2-11)$ are not approximations, but exact expressions. Solving equations $(B2-10)$ and $(B2-11)$ for the equilibrium real wage:

$$w_t = \frac{1}{1+\nu\psi}\left[\left(\nu+\sigma\right)y_t + \nu\psi mc_t + v\widehat{\Delta}_t\right] \tag{B2-12}$$

Plugging the real wage in equation $(B2-9)$ and simplifying:

$$\begin{aligned} mc_t &= \chi\left(\sigma+v\right)y_t + \left(1-\chi\right)\left(q_t\right) + \chi v\widehat{\Delta}_t \\ &+\frac{1}{2}\frac{1-\psi}{1-\overline{\alpha}}\chi^2\left(1-\chi\right)\left[\left(\sigma+v\right)y_t - q_t\right]^2 + O\left(\|\xi_t\|^3\right) \end{aligned} \tag{B2-13}$$

where $\chi \equiv \left(1-\overline{\alpha}\right)/\left(1+v\psi\overline{\alpha}\right)$. This is the equation $(3-ii)$ in the main text. This expression is the second order expansion of the real marginal cost as a function of output and the oil prices. Similarly, we can express labour in equilibrium as a function of of output and oil prices:

$$l_t = y_t - \delta\left[\left(v+\sigma\right)y_t - q_t\right] + \frac{\chi}{1-\overline{\alpha}}\widehat{\Delta}_t + \frac{1}{2}\frac{1-\psi}{1-\overline{\alpha}}\delta\chi^2\left[\left(v+\sigma\right)y_t - q_t\right]^2 + O\left(\|\xi_t\|^3\right) \tag{B2-14}$$

for:

$$\delta \equiv \psi\chi\frac{\overline{\alpha}}{1-\overline{\alpha}}$$

where δ measures the effects of oil shocks on labour.

The price dispersion measure

The price dispersion measure is given by

$$\Delta_t = \int_0^1 \left(\frac{P_t\left(z\right)}{P_t}\right)^{-\varepsilon} dz$$

Since a proportion $1-\theta$ of intermediate firms set prices optimally, whereas the other θ set the price last period, this price dispersion measure can be written as:

$$\Delta_t = (1-\theta)\left(\frac{P_t^*(z)}{P_t}\right)^{-\varepsilon} + \theta\int_0^1 \left(\frac{P_{t-1}(z)}{P_t}\right)^{-\varepsilon} dz$$

Dividing and multiplying by $(P_{t-1})^{-\varepsilon}$ the last term of the RHS:

$$\Delta_t = (1-\theta)\left(\frac{P_t^*(z)}{P_t}\right)^{-\varepsilon} + \theta\int_0^1 \left(\frac{P_{t-1}(z)}{P_{t-1}}\right)^{-\varepsilon}\left(\frac{P_{t-1}}{P_t}\right)^{-\varepsilon} dz$$

Since $P_t^*(z)/P_t = N_t/D_t$ and $P_t/P_{t-1} = \Pi_t$, using equation $(3-8)$ in the text and the definition for the dispersion measure lagged on period, this can be expressed as

$$\Delta_t = (1-\theta)\left(\frac{1-\theta(\Pi_t)^{\varepsilon-1}}{1-\theta}\right)^{\varepsilon/(\varepsilon-1)} + \theta\Delta_{t-1}(\Pi_t)^{\varepsilon} \qquad \text{(B2-15)}$$

which is a recursive representation of Δ_t as a function of Δ_{t-1} and Π_t.

Benigno and Woodford (2005) show that a second order approximation of the price dispersion depends solely on second order terms on inflation. Then, the second order approximation of equation (B2-15) is:

$$\widehat{\Delta}_t = \theta\widehat{\Delta}_{t-1} + \frac{1}{2}\varepsilon\frac{\theta}{1-\theta}\pi_t^2 + O\left(\|\xi_t\|^3\right) \qquad \text{(B2-16)}$$

which is equation $(3-iii)$ in the main text. Moreover, we can use equation $(B2-16)$ to write the infinite sum:

$$\begin{aligned}\sum_{t=t_o}^{\infty}\beta^{t-t_o}\widehat{\Delta}_t &= \theta\sum_{t=t_o}^{\infty}\beta^{t-t_o}\widehat{\Delta}_{t-1} + \frac{1}{2}\varepsilon\frac{\theta}{1-\theta}\sum_{t=t_o}^{\infty}\beta^{t-t_o}\frac{\pi_t^2}{2} + O\left(\|\xi_t\|^3\right)\\ (1-\beta\theta)\sum_{t=t_o}^{\infty}\beta^{t-t_o}\widehat{\Delta}_t &= \theta\widehat{\Delta}_{t_o-1} + \frac{1}{2}\varepsilon\frac{\theta}{1-\theta}\sum_{t=t_o}^{\infty}\beta^{t-t_o}\frac{\pi_t^2}{2} + O\left(\|\xi_t\|^3\right)\end{aligned}$$

Dividing by $(1-\beta\theta)$ and using the definition of κ :

$$\sum_{t=t_o}^{\infty}\beta^{t-t_o}\widehat{\Delta}_t = \frac{\theta}{1-\beta\theta}\widehat{\Delta}_{t_o-1} + \frac{1}{2}\frac{\varepsilon}{\kappa}\sum_{t=t_o}^{\infty}\beta^{t-t_o}\frac{\pi_t^2}{2} + O\left(\|\xi_t\|^3\right) \qquad \text{(B2-17)}$$

The discounted infinite sum of $\widehat{\Delta}_t$ is equal to the sum of two terms, on the initial price dispersion and the discounted infinite sum of π_t^2.

The second order approximation of the Phillips Curve

The second order expansion for equations $(B2-6)$, $(B2-7)$ and $(B2-8)$ are:

$$\pi_t = \frac{(1-\theta)}{\theta}(n_t - d_t) - \frac{1}{2}\frac{(\varepsilon-1)}{1-\theta}(\pi_t)^2 + O\left(\|\xi_t\|^3\right) \qquad \text{(B2-18)}$$

$$n_t = (1-\theta\beta)\left(a_t + \frac{1}{2}a_t^2\right) + \theta\beta\left(E_t b_{t+1} + \frac{1}{2}E_t b_{t+1}^2\right) - \frac{1}{2}n_t^2 + O\left(\|\xi_t\|^3\right) \qquad \text{(B2-19)}$$

$$d_t = (1-\theta\beta)\left(c_t + \frac{1}{2}c_t^2\right) + \theta\beta\left(E_t e_{t+1} + \frac{1}{2}E_t e_{t+1}^2\right) - \frac{1}{2}d_t^2 + O\left(\|\xi_t\|^3\right) \qquad \text{(B2-20)}$$

Where we have defined the auxiliary variables a_t,b_{t+1},c_t and e_{t+1} as:

$$\begin{array}{ll} a_t \equiv (1-\sigma)\, y_t + mc_t & b_{t+1} \equiv \varepsilon\pi_{t+1} + n_{t+1} \\ c_t \equiv (1-\sigma)\, y_t & e_{t+1} \equiv (\varepsilon-1)\,\pi_{t+1} + d_{t+1} \end{array}$$

Subtract equations $(B2-19)$ and $(B2-20)$, and using the fact that $X^2 - Y^2 = (X-Y)(X+Y)$, for any two variables X and Y :

$$\begin{aligned} n_t - d_t &= (1-\theta\beta)(a_t - c_t) + \frac{1}{2}(1-\theta\beta)(a_t - c_t)(a_t + c_t) \qquad \text{(B2-21)} \\ &\quad + \theta\beta E_t (b_{t+1} - e_{t+1}) + \frac{1}{2}\theta\beta E_t (b_{t+1} - e_{t+1})(b_{t+1} + e_{t+1}) \\ &\quad - \frac{1}{2}(n_t - d_t)(n_t + d_t) + O\left(\|\xi_t\|^3\right) \end{aligned}$$

Plugging in the values of a_t, b_{t+1}, c_t and e_{t+1} into equation $(B2-21)$, we obtain: $(B2-22)$

$$\begin{aligned} n_t - d_t &= (1-\theta\beta)\, mc_t + \frac{1}{2}(1-\theta\beta)\, mc_t \left(2(1-\sigma)\, y_t + mc_t\right) \qquad (\\ &\quad + \theta\beta E_t (\pi_{t+1} + n_{t+1} - d_{t+1}) + \frac{1}{2}\theta\beta E_t (\pi_{t+1} + n_{t+1} - d_{t+1})\left((2\varepsilon-1)\,\pi_{t+1} + n_{t+1} -\right. \\ &\quad - \frac{1}{2}(n_t - d_t)(n_t + d_t) + O\left(\|\xi_t\|^3\right) \end{aligned}$$

Taking forward one period equation $(B2-18)$, we can solve for $n_{t+1} - d_{t+1}$:

$$n_{t+1} - d_{t+1} = \frac{\theta}{1-\theta}\pi_{t+1} + \frac{1}{2}\frac{\theta}{1-\theta}\frac{(\varepsilon-1)}{1-\theta}(\pi_{t+1})^2 + O\left(\|\xi_t\|^3\right) \qquad \text{(B2-23)}$$

replace equation $(B2-23)$ in $(B2-22)$ and make use of the auxiliary variable $z_t = (n_t + d_t) / (1 - \theta\beta)$

$$\begin{aligned} n_t - d_t &= (1-\theta\beta)\, mc_t + \frac{1}{2}(1-\theta\beta)\, mc_t \left(2(1-\sigma)\, y_t + mc_t\right) \\ &+ \frac{\theta}{1-\theta}\beta \left[E_t\pi_{t+1} + \left(\frac{\varepsilon-1}{1-\theta} + \varepsilon\right) E_t\pi_{t+1}^2 + (1-\theta\beta)\, E_t\pi_{t+1}z_{t+1}\right] \\ &- \frac{1}{2}\frac{\theta}{1-\theta}(1-\theta\beta)\, \pi_t z_t + O\left(\|\xi_t\|^3\right) \end{aligned} \tag{B2-24}$$

Notice that we use only the linear part of equation $(B2-23)$ when we replace $n_{t+1} - d_{t+1}$ in the quadratic terms because we are interested in capture terms only up to second order of accuracy. Similarly, we make use of the linear part of equation $(B2-18)$ to replace $(n_t - d_t) = \frac{\theta}{1-\theta}\pi_t$ in the right hand side of equation $(B2-24)$. Replace equation $(B2-24)$ in $(B2-18)$:

$$\begin{aligned} \pi_t &= \kappa mc_t + \frac{1}{2}\kappa mc_t \left(2(1-\sigma)\, y_t + mc_t\right) \\ &+ \beta \left[E_t\pi_{t+1} + \left(\frac{\varepsilon-1}{1-\theta} + \varepsilon\right) E_t\pi_{t+1}^2 + (1-\theta\beta)\, E_t\pi_{t+1}z_{t+1}\right] \\ &- \frac{1}{2}(1-\theta\beta)\, \pi_t z_t - \frac{1}{2}\frac{(\varepsilon-1)}{1-\theta}(\pi_t)^2 + O\left(\|\xi_t\|^3\right) \end{aligned} \tag{B2-25}$$

for

$$\kappa \equiv \frac{(1-\theta)}{\theta}(1-\theta\beta)$$

where z_t has the following linear expansion:

$$z_t = 2(1-\sigma)\, y_t + mc_t + \theta\beta E_t \left(\frac{2\varepsilon-1}{1-\theta\beta}\pi_{t+1} + z_{t+1}\right) + O\left(\|\xi_t\|^3\right) \tag{B2-26}$$

Define the following auxiliary variable:

$$v_t = \pi_t + \frac{1}{2}\left(\frac{\varepsilon-1}{1-\theta} + \varepsilon\right)\pi_t^2 + \frac{1}{2}(1-\theta\beta)\, \pi_t z_t \tag{B2-27}$$

Using the definition for v_t, equation $(B2-25)$ can be expressed as:

$$v_t = \kappa mc_t + \frac{1}{2}\kappa mc_t \left(2(1-\sigma)\, y_t + mc_t\right) + \frac{1}{2}\varepsilon\pi_t^2 + \beta E_t v_{t+1} + O\left(\|\xi_t\|^3\right) \tag{B2-28}$$

which is equation $(3-iv)$ in the main text.

Moreover, the linear part of equation (B2-28) is:

$$\pi_t = \kappa mc_t + \beta E_t(\pi_{t+1}) + O\left(\|\xi_t\|^3\right)$$

which is the standard New Keynesian Phillips curve, inflation depends linearly on the real marginal costs and expected inflation.

Replace the equation for the marginal costs (B2-13) in the second order expansion of the Phillips curve (B2-28)

$$\begin{aligned} v_t &= \kappa_y y_t + \kappa_q q_t + \kappa\chi v\widehat{\Delta}_t + \frac{1}{2}\varepsilon\pi_t^2 + \\ &+\frac{1}{2}\kappa\left[c_{yy}y_t^2 + 2c_{yq}y_tq_t + c_{qq}q_t^2\right] + \beta E_t v_{t+1} + O\left(\|\xi_t\|^3\right) \end{aligned} \tag{B2-29}$$

where the coefficients coefficients of the linear part are given by

$$\begin{aligned} \kappa_y &= \kappa\chi(\sigma+\nu) \\ \kappa_q &= \kappa(1-\chi) \end{aligned}$$

and those of the quadratic part are:

$$\begin{aligned} c_{yy} &= \chi(\sigma+\nu)\left[2(1-\sigma)+\chi(\sigma+\nu)\right] + (1-\psi)\frac{\chi^2(1-\chi)(\sigma+\nu)^2}{1-\overline{\alpha}} \\ c_{yq} &= (1-\chi)\left[2(1-\sigma)+\chi(\sigma+\nu)\right] - (1-\psi)\frac{\chi^2(1-\chi)(\dot{\sigma}+\nu)}{1-\overline{\alpha}} \\ c_{qq} &= (1-\chi)^2 + (1-\psi)\frac{\chi^2(1-\chi)}{1-\overline{\alpha}} \end{aligned}$$

Equation B2-29 is a recursive second order representation of the Phillips curve. However, we need to express the price dispersion in terms of inflation in order to have a the Phillips curve only as a function of output, inflation and the oil shock. Equation B2-29 can also be expressed as the discounted infinite sum:

$$v_{t_o} = \sum_{t=t_o}^{\infty}\beta^{t-t_o}\left\{\kappa_y y_t + \kappa_q q_t + \kappa\chi v\widehat{\Delta}_t + \frac{1}{2}\varepsilon\pi_t^2 + \frac{1}{2}\kappa\left[c_{yy}y_t^2 + 2c_{yq}y_tq_t + c_{qq}q_t^2\right]\right\} + \left(\|\xi_t\|^3\right)$$

after making use of equation B2-17, the discounted infinite sum of $\widehat{\Delta}_t$, v_{t_o} becomes

$$v_{t_o} = \sum_{t=t_o}^{\infty}\beta^{t-t_o}\left\{\kappa_y y_t + \kappa_q q_t + \frac{1}{2}\varepsilon(1+\chi v)\pi_t^2 + \frac{1}{2}\kappa\left[c_{yy}y_t^2 + 2c_{yq}y_tq_t + c_{qq}q_t^2\right]\right\} + \frac{\chi v\theta}{1-\beta\theta}\widehat{\Delta}_{t_o-1} \tag{B2-30}$$

which is equation (3-34) in the main text.

B2.3 A second-order approximation to utility

The expected discounted value of the utility of the representative household

$$U_{t_o} = E_{t_o} \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left[u\left(C_t\right) - v\left(L_t\right) \right] \tag{B2-31}$$

The first term can be approximated as:

$$u\left(C_t\right) = \overline{C}\overline{u}_c \left\{ c_t + \frac{1}{2}\left(1-\sigma\right) c_t^2 \right\} + t.i.p. + O\left(\|\xi_t\|^3\right) \tag{B2-32}$$

Similarly, the second term:

$$v\left(L_t\right) = \overline{L}\overline{v}_L \left\{ l_t + \frac{1}{2}\left(1+\nu\right) l_t^2 \right\} + t.i.p. + O\left(\|\xi_t\|^3\right) \tag{B2-33}$$

Replace the equation for labour in equilibrium in B2-33:

$$v\left(L_t\right) = \overline{L}\overline{v}_L \left\{ v_y y_t + \frac{1}{2} v_{yy} y_t^2 + v_{yq} y_t q_t + v_{\Delta} \widehat{\Delta}_t \right\} + t.i.p. + O\left(\|\xi_t\|^3\right) \tag{B2-34}$$

where:

$$\begin{aligned}
v_y &= 1 - \delta\left(v + \sigma\right) \\
v_{yy} &= \left(1+v\right)\left(1 - \delta\left(v+\sigma\right)\right)^2 + \frac{1}{2}\frac{1-\psi}{1-\overline{\alpha}} \chi^2 \delta \left(\sigma + v\right)^2 \\
v_{yq} &= \left(1+v\right)\delta\left(1 - \delta\left(v+\sigma\right)\right) - \frac{1}{2}\frac{1-\psi}{1-\overline{\alpha}} \chi^2 \delta^2 \left(\sigma + v\right) \\
v_{\Delta} &= \frac{\chi}{1-\overline{\alpha}}
\end{aligned}$$

We make use on the following relation:

$$\overline{L}\overline{v}_L = \left(1-\Phi\right)\left(1-\overline{\alpha}\right)\overline{Y}\overline{u}_c \tag{B2-35}$$

where $\Phi = 1 - \frac{1}{\mu} = 1 - \frac{1-\overline{\tau}}{\varepsilon/(\varepsilon-1)}$ is the steady state distortion from monopolistic competition. Replace the previous relation, equation B2-32 and B2-34 in B2-31, and make use of the clearing market condition: $C_t = Y_t$

$$U_{t_o} = \overline{Y}\overline{u}_c \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left(u_y y_t + \frac{1}{2} u_{yy} y_t^2 + u_{yq} y_t q_t + u_{\Delta} \widehat{\Delta}_t \right) + t.i.p. + O\left(\|\xi_t\|^3\right) \tag{B2-36}$$

where

$$
\begin{aligned}
u_y &= 1-(1-\Phi)(1-\overline{\alpha})v_y=\Phi_L \\
u_{yy} &= 1-\sigma-(1-\Phi)(1-\overline{\alpha})v_{yy}=1-\sigma-(1-\Phi_L)v_{yy}/(1-\delta(v+\sigma)) \\
u_{yq} &= -(1-\Phi)(1-\overline{\alpha})v_{yq}=-(1-\Phi_L)v_{yq}/(1-\delta(v+\sigma)) \\
u_{\Delta} &= -(1-\Phi)(1-\overline{\alpha})v_{\Delta}=-(1-\Phi)\chi
\end{aligned}
$$

where we make use of the following change of variable:

$$\Phi_L=1-(1-\Phi)(1-\overline{\alpha})(1-\delta(v+\sigma)) \tag{B2-37}$$

where Φ_L is the effective effect of the monopolistic distortion in welfare through the of output. Notice that when we eliminate the monopolistic distortion, i.e. $\Phi=0$, Φ_L is not necessarily equal to zero.

Replace the present discounted value of the price distortion (B2-17) in B2-36:

$$U_{t_o}=\overline{Y}\overline{u}_cE_{t_o}\sum_{t=t_o}^{\infty}\beta^{t-t_o}\left(u_yy_t+\frac{1}{2}u_{yy}y_t^2+u_{yq}y_tq_t+\frac{1}{2}u_{\pi}\pi_t^2\right)+t.i.p.+O\left(\|q_t\|^3\right) \tag{B2-38}$$

where

$$u_{\pi}=\frac{\varepsilon}{\kappa}u_{\Delta}=-(1-\Phi)\chi\frac{\varepsilon}{\kappa}$$

Use equation B2-30, the second order approximation of the Phillips curve, to solve for the expected level of output:

$$
\begin{aligned}
\sum_{t=t_o}^{\infty}\beta^{t-t_o}y_t &= -\frac{1}{\kappa_y}\sum_{t=t_o}^{\infty}\beta^{t-t_o}\left\{\kappa_qq_t+\frac{1}{2}\varepsilon(1+\chi v)\pi_t^2+\frac{1}{2}\kappa\left[c_{yy}y_t^2+2c_{yq}y_tq_t+c_{qq}q_t^2\right]\right\} \\
&\quad+\frac{1}{\kappa_y}\left(v_{t_o}-\chi v(1-\theta)\widehat{\Delta}_{t_o-1}\right)+\left(\|\xi_t\|^3\right)
\end{aligned} \tag{B2-39}
$$

Replace equation B2-39 in B2-38 to express it as function of only second order terms:

$$U_{t_o}=-\Omega E_{t_o}\sum_{t=t_o}^{\infty}\beta^{t-t_o}\left(\frac{1}{2}\lambda_y(y_t-y_t^*)^2+\frac{1}{2}\lambda_{\pi}\pi_t^2\right)+T_{t_o}+t.i.p.+O\left(\|q_t\|^3\right) \tag{B2-40}$$

which is equation B2-35 in the text, where:

$$\begin{aligned}
\lambda_y &= \Phi_L \frac{\kappa}{\kappa_y} c_{yy} - u_{yy} \\
\lambda_\pi &= \Phi_L \frac{\varepsilon(1+\chi v)}{\kappa_y} - u_\pi \\
y_t^* &= -\frac{\Phi_L \frac{\kappa}{\kappa_y} c_{yq} - u_{yq}}{\Phi_L \frac{\kappa}{\kappa_y} c_{yy} - u_{yy}} q_t
\end{aligned}$$

additionally we have that$\Omega = \overline{Y}\overline{u}_c$ and $T_{t_o} = \overline{Y}\overline{u}_c \frac{\Phi_L}{\kappa_y} v_{t_o}$

Make use of the following auxiliary variables:

$$\begin{aligned}
\omega_1 &= (1-\sigma)\Phi_L + \chi(\sigma+v) \\
\omega_2 &= \chi(\sigma+v)\left[\frac{1-\chi}{1-\overline{\alpha}} + (1-\Phi_L)\frac{\sigma\psi\overline{\alpha}}{1-\sigma\psi\overline{\alpha}}\right] \\
\omega_3 &= \Phi_L \sigma\overline{\alpha}
\end{aligned}$$

then, λ_y, λ_π and y_t^* can be written as a function of ω_1, ω_2 and ω_3

$$\begin{aligned}
\lambda_y &= \omega_1 + (1-\psi)\omega_2 \\
\lambda_\pi &= \frac{\varepsilon}{\kappa_y(1-\sigma\psi\overline{\alpha})}\left[\omega_1 + (1-\psi)\omega_3\right] \\
y_t^* &= -\frac{1-\chi}{\chi(\sigma+v)}\left[\frac{\omega_1 - (1-\psi)\frac{\chi}{1-\chi}\omega_2}{\omega_1 + (1-\psi)\omega_2}\right] q_t
\end{aligned}$$

using the definitions for χ, y_t^* can be expressed as:

$$y_t^* = -\left(\frac{1+\psi v}{\sigma+v}\right)\left(\frac{\overline{\alpha}}{1-\overline{\alpha}+\eta}\right) \tag{B2-41}$$

where

$$\eta \equiv \frac{(1-\psi)(1-\overline{\alpha})\omega_2}{(1-\chi)\omega_1 - (1-\psi)\chi\omega_2}$$

Denote α^*, the efficient share in steady state of oil in the marginal costs, where

$$\alpha^* = \frac{\overline{\alpha}}{1+\eta}$$

then y_t^* is

$$y_t^* = -\left(\frac{1+\psi v}{\sigma+v}\right)\left(\frac{\alpha^*}{1-\alpha^*}\right) q_t \tag{B2-42}$$

Note from the definition for η that when $\psi = 1$, then $\eta = 0$, $\alpha^* = \overline{\alpha} = \alpha$ and $y_t^* = y_t^n$. For a Cobb-Douglas production function the efficient level of output equals the natural level. Also, when $\psi < 1$, then $\eta > 0$, $\alpha^* < \overline{\alpha}$ and $|y_t^*| < |y_t^n|$. For elasticity of substitution between inputs lower than one the efficient level fluctuates less to oil shocks than the natural level. Also note that even when Φ_L is equal to zero, which summarises the effect of monopolistic distortions on the wedge between the marginal rate of substitution and the marginal product of labour, η is still different than zero for $\psi \neq 1$. This indicates that the efficient level of output still diverges from the natural level even we eliminate the effects of monopolistic distortions.

In the same way, the natural rate of output can be expressed as:

$$y_t^n = -\left(\frac{1+\psi v}{\sigma + v}\right)\left(\frac{\overline{\alpha}}{1-\overline{\alpha}}\right) q_t \tag{B2-43}$$

Similarly, we can simplify $\lambda = \lambda_y / \lambda_\pi$ as:

$$\lambda = \frac{\lambda_y}{\lambda_\pi} = \frac{\kappa_y \left(1 - \sigma\psi\overline{\alpha}\right)}{\varepsilon}\gamma$$

where we use the auxiliary variable:

$$\gamma \equiv \left[\frac{\omega_1 + (1-\psi)\,\omega_2}{\omega_1 + (1-\psi)\,\omega_3}\right]$$

Note that when $\psi = 1$, then $\gamma = 1$ and when $\psi < 1$, then $\gamma = 1$ since $\omega_2 > \omega_3$.

B3 Appendix: Optimal Monetary Policy

B3.1 Optimal response to oil shocks

The policy problem consists in choosing x_t and π_t to maximise the following Lagrangian:

$$\mathcal{L} = -E_{t_o}\left\{\sum_{t=t_o}^{\infty} \beta^{t-t_o}\left[\frac{1}{2}\lambda x_t^2 + \frac{1}{2}\pi_t^2 - \varphi_t\left(\pi_t - \kappa_y \widehat{y}_t - \beta E_t \pi_{t+1} - u_t\right)\right] + \varphi_{t_o-1}\left(\pi_{t_o} - \pi_{t_o}^*\right)\right\}$$

where $\beta^{t-t_o}\varphi_t$ is the Lagrange multiplier associated with the constraint at time t

The first order conditions with respect to π_t and y_t are respectively

$$\pi_t = \varphi_{t-1} - \varphi_t \quad \text{(B3-1)}$$

$$\lambda x_t = \kappa_y \varphi_t \quad \text{(B3-2)}$$

and for the initial condition:

$$\pi_{t_o} = \pi^*_{t_o}$$

where $\pi^*_{t_o}$ is the initial value of inflation which is consistent with the policy problem in a "timeless perspective".

Replace conditions B3-1 and B3-2 in the Phillips Curve:

$$\beta E_t\varphi_{t+1} - \left[(1+\beta)\lambda + \kappa_y^2\right]\varphi_t + \lambda\varphi_{t-1} = \lambda u_t \quad \text{(B3-3)}$$

this difference equation has the following solution[8] :

$$\varphi_t = \tau_\varphi\varphi_{t-1} - \tau_\varphi \sum_{j=0}^{\infty} \beta^j \tau_\varphi^j E_t u_{t+j} \quad \text{(B3-4)}$$

where τ_φ is the characteristic root, lower than one, of B3-3, and it is equal to

$$\tau_\varphi = Z - \sqrt{Z^2 - \frac{1}{\beta}}$$

for $Z = \left((1+\beta) + \frac{\kappa_y^2}{\lambda}\right)/(2\beta)$. Since the oil price follows an AR(1) process of the form:

$$q_t = \rho q_{t-1} + \xi_t$$

and the mark-up shock is: $u_t = \varpi q_t$, then u_t follows the following process:

$$u_t = \rho u_{t-1} + \varpi\xi_t \quad \text{(B3-5)}$$

Solution to the optimal problem Taking into account B3-5, equation B3-4 can be expressed as:

$$\varphi_t = \tau_\varphi\varphi_{t-1} - \phi q_t \quad \text{(B3-6)}$$

[8]See Woodford (2003), pp. 488-490 for details on the derivation.

where:

$$\phi = \frac{\tau_\varphi}{1-\beta\tau_\varphi\rho}\varpi$$

Initial condition Iterate backward equation (B3-6) and evaluate it at $t_o - 1$, this is the timeless solution to the initial condition φ_{t_o-1} :

$$\varphi_{t_o-1} = -\phi\Sigma_{k=0}^{\infty}\left(\tau_\varphi\right)^k q_{t_o-1-k} \tag{B3-7}$$

which is a weighted sum of all the past realisations of oil prices.

Equations (B3-1), (B3-2), (B3-6) and (B3-7) are the conditions for the optimal unconstrained plan presented in proposition 3.5. **Impulse responses** An innovation of ξ_t to the real oil price affects the current level and the expected future path of the lagrange multiplier by an amount:

$$E_t\varphi_{t+j} - E_{t-1}\varphi_{t+j} = -\frac{\rho^{j+1} - \left(\tau_\varphi\right)^{j+1}}{\rho - \tau_\varphi}\phi\xi_t$$

for each $j \geq 0$. Given this impulse response for the multiplier. (B3-1) and (B3-2) can be used to derive the corresponding impulse responses for inflation and output gap:

$$\begin{aligned} E_t\pi_{t+j} - E_{t-1}\pi_{t+j} &= \left[\frac{\rho^{j+1} - \left(\tau_\varphi\right)^{j+1}}{\rho - \tau_\varphi} - \frac{\rho^{j} - \left(\tau_\varphi\right)^{j}}{\rho - \tau_\varphi}\right]\phi\xi_t \\ E_t y_{t+j} - E_{t-1}y_{t+j} &= -\frac{\kappa_y}{\lambda}\frac{\rho^{j+1} - \left(\tau_\varphi\right)^{j+1}}{\rho - \tau_\varphi}\phi\xi_t \end{aligned}$$

which are expressions that appear in the main text.

B3.2 The optimal Non-inertial plan

We want to find a solution for the paths of inflation and output gap such that the behaviour of endogenous variables is function only on the current state. That is:

$$\pi_t = \overline{\pi} + f_\pi u_t \tag{B3-8}$$

$$x_t = \overline{x} + f_x u_t \tag{B3-9}$$

$$\varphi_t = \overline{\varphi} + f_\varphi u_t \tag{B3-10}$$

where the coefficients $\overline{\pi}, \overline{y}, \overline{\varphi}, f_\pi, f_x$ and f_φ are to be determined

Replace (B3-8), (B3-9) and (B3-10) in the Lagrangian and take unconditional expected value:

$$
\begin{aligned}
-E\left(L_{t_o}\right) &\equiv E\left\{E_{t_o}\sum_{t=t_o}^{\infty}\beta^{t-t_o}\left[\begin{array}{c}\frac{1}{2}\lambda\left(\overline{x}+f_x u_t\right)^2+\frac{1}{2}\left(\overline{\pi}+f_\pi u_t\right)^2 \\ -\left(\overline{\varphi}+f_\varphi u_t\right)\left(\begin{array}{c}(1-\beta)\overline{\pi}-\kappa_y\overline{x} \\ +(1-\beta\rho)f_\pi u_t-u_t-\kappa_y f_x u_t\end{array}\right)\end{array}\right]\right\} \\
&\quad +E\left(\left(\overline{\varphi}+f_\varphi u_{t_o-1}\right)\left[\overline{\pi}+f_\pi u_{t_o}\right]\right) \qquad \text{(B3-11)}
\end{aligned}
$$

suppressing the terms that are independent of policy and using the law of motion for u_t, this can be simplified as:

$$
\begin{aligned}
-E\left(L_{t_o}\right) &\equiv \frac{1}{2(1-\beta)}\left(\lambda\overline{x}^2+\overline{\pi}^2\right)-\frac{1}{2(1-\beta)}\overline{\varphi}\left((1-\beta)\overline{\pi}-\kappa_y\overline{x}\right) \\
&\quad +\frac{1}{2}\frac{\sigma_u^2}{1-\beta}\left(\lambda f_x{}^2+f_\pi{}^2\right)-\frac{1}{2}\frac{\sigma_u^2}{1-\beta}f_\varphi\left((1-\beta\rho)f_\pi-1-\kappa_y f_x\right) \\
&\quad +\rho\sigma_u^2 f_\varphi f_\pi
\end{aligned}
$$

the problem becomes to find $\overline{\pi}, \overline{y}, \overline{\varphi}, f_\pi, f_x$ and f_φ such that maximise the previous expression. Those coefficients are:

$$
\begin{aligned}
\overline{\pi} &= \overline{x}=\overline{\varphi}=0 \\
f_\pi &= \frac{\lambda(1-\rho)}{\lambda(1-\beta\rho)(1-\rho)+\kappa_y^2} \\
f_x &= -\frac{\kappa_y}{\lambda(1-\beta\rho)(1-\rho)+\kappa_y^2} \\
f_\varphi &= \frac{\lambda}{\lambda(1-\beta\rho)(1-\rho)+\kappa_y^2}
\end{aligned}
$$

which is the solution to the optimal non-inertial plan given in proposition 3.6.

CHAPTER 4

THE ASYMMETRIC EFFECTS OF MONETARY POLICY IN GENERAL EQUILIBRIUM

4.1 Introduction

There exists a fair amount of empirical research reporting asymmetric effects of monetary policy for the USA and for most of the industrialised countries. Monetary policy seems to have asymmetric effects on output and inflation depending not only on the state of the economy, whether the output gap is positive or negative or whether inflation is high or low, but also depending on the sign and size of the monetary policy shock. On the theoretical side, the literature on asymmetric effects of monetary policy can be broadly categorised into two groups: those that emphasise that the asymmetric effects of monetary policy come from the convexity of the supply curve and those that consider that asymmetry is generated by non-linear effects of monetary policy on aggregate demand, also denominated *pushing-on-a-string* type.

Although there is a lot of theoretical work explaining asymmetric responses of output and inflation, as we discuss in more detail in the next section, most of this work has been done within partial equilibrium frameworks[1]. Furthermore, these theories are capable of explaining only one source of asymmetry: namely either convex supply curves or *pushing-on-a-string* type of asymmetry. To the best of our knowledge there is no general equilibrium model that can generate asymmetries coming from both sources simultaneously.

[1]See for instance Jackman and Sutton (1982), Ball and Mankiw (1994).

In that sense, this chapter contributes to the theoretical literature on asymmetric effects of monetary policy by proposing a new set-up where asymmetric effects emerge naturally in a New Keynesian DSGE model. Our approach has the advantage of generating asymmetric effects in a very simple way, which come both from shifts in aggregate demand *(pushing-on-a-string* type of theory) and from a convex supply curve. In particular, our model can generate responses of output and inflation to monetary policy shocks that are stronger when the economy is above potential, in line with the empirical evidence reported by Thoma (1994) and Weiss (1999) for the USA. This contrasts with the asymmetric effects generated by models that only consider a convex Phillips curve, where monetary policy is more effective to affect output in recessions than in booms.

We introduce into, an otherwise standard, New Keynesian model preferences that exhibit non-homotheticity[2] and solve for its dynamic equilibrium using a perturbation method that allows us to obtain a higher order solution that is more accurate than the traditional linear approximated solution. We are able to characterise analytically the non-linear behaviour of the solution, the asymmetry, and to establish the implications that non-homotheticity has in the dynamic equilibrium of the model.

We introduce intertemporal non-homotheticity by considering that there exists a subsistence level of consumption. This assumption make the intertemporal elasticity of substitution (IES) state-dependent. The intuition of the mechanism that generates the asymmetry in the model is straightforward. On one hand, when the subsistence level of consumption is positive the IES changes with the level of income of the household, therefore in a boom (recession) when consumption levels move further away (closer to) from the subsistence level, the IES is higher (lower),

[2] A general non-homothetic utility function is defined as a set of preferences that exhibits non-linear Engel's curves, i.e. the expenditure on good i increases non-linearly with income. Non-homotheticity is intertemporal, when real income affects the profile of consumption across time, and intratemporal, when real income affects consumption allocation across different goods over time

therefore making consumption more (less) responsive to changes in the real interest rate. With intertemporal non-homotheticity the path of consumption across time is affected by the path of income[3]. This mechanism generates asymmetric shifts in aggregate demand to monetary shocks.

Another study that have analysed the effects of subsistence in general equilibrium is Ravn et. al. (2004). They include a specific subsistence point to each variety of good. Similarly to our work, they obtain a procyclical price elasticity of demand, which generates countercyclically markups in equilibrium.

Our specification of intertemporal non-homotheticity, as a constant subsistence level, can also be interpreted as an extreme form of external habit formation, where the reference level of consumption remains constant. Models with external habit formation have proven to be useful in accounting for empirical regularities of asset prices. For instance, Campbell and Cochrane (1999) show that introducing a time-varying subsistence level to a basic isoelastic power utility function allows solving for a series of puzzles related to asset prices such as: the equity premium puzzle, countercyclical risk premium and forecastability of excess of stocks.

Moreover, non-homothetic preferences have the advantage of being able to reproduce consumer behaviour that is closer to what is observed empirically. In particular, it offers an explanation of why agents seem to have different degrees of elasticity of substitution depending on the state of their wealth and income, consistent with what is reported in micro-empirical studies for countries like the USA and India[4].

This chapter extends the literature in many directions; first we show that introducing non-homothetic preferences over time in a standard general equilibrium New Keynesian model can generate patterns of asymmetry observed in the data that

[3]The effect of intertemporal non-homotheticity is in some sense similar to the effects of borrowing constraints on consumption, since with borrowing constraints the optimal path of consumption is also affected by the level of income.

[4]See Atkeson and Ogaki (1996)

is consistent with both a convex supply curve and asymmetric shifts in aggregate demand. Second, the chapter provides another argument in favour of using higher order approximation to the solutions of general equilibrium dynamics models. This is, linear solutions are not only inaccurate to measure welfare as reported by Kim and Kim (2003), but also in measuring the dynamics of the model, in particular where non-linear behaviour is important around the steady state, as with non-homothetic preferences.

We find that the key parameters determining the asymmetry in the response of output and inflation are the subsistence level of consumption, which generates asymmetric shifts in aggregated demand; and the price elasticity of demand for individual goods, which determines the degree of convexity of aggregate supply. Also, we find that it is important to differentiate between states of output that are generated by demand shocks, from those that are generated by supply shocks. We find in our model that monetary policy is more effective to affect output in a boom than in a recession (positive asymmetry) when the degree of intertemporal non-homotheticity is high. Moreover,the asymmetric effects on on output are higher when the deviations from the steady state come from supply shocks instead of demand shocks. On the other hand, the sign of the asymmetric effects of monetary policy on inflation will depend on the type of the shock: when the state is driven by demand shocks, the asymmetric effects on inflation are positive, but they become negative when when the state is driven by supply shocks.

We also have found that the way the central bank responds to output has implications for the asymmetric response of output. In the benchmark case, when the central bank uses a log-linear Taylor rule, we find that the higher the coefficient of output in the interest rate rule, the lower the degree of asymmetric response of output and inflation.

The remaining part of this chapter is organised as follows. In the next section we review the theoretical and empirical literature on asymmetric effects of monetary policy. In section 4.3, we present the model used to analyse asymmetry. Section 4.4 discusses in detail the effects of non-homotheticity in generating asymmetric effects of monetary policy. Finally the last section presents some concluding remarks.

4.2 Literature review

4.2.1 Theoretical Literature

Theoretical Literature

Within the group of theories that consider a convex supply curve as the main factor generating asymmetric responses are theories related to wage stickiness, which emphasise that nominal wages are sticky to cuts but not to increases[5]; theories that highlight the role of capacity constraints that make the marginal cost of firms more responsive to aggregate demand changes when the economy is closer to its short-term fixed level of production capacity; and theories of menu costs that consider that adjustment costs are state-dependent.

For instance, Ball and Mankiw (1994) propose a theoretical model to explain asymmetric adjustment in prices. They assume that firms face menu cost of adjusting prices and that inflation is positive every period. They further assume that this menu cost is paid only when the firm chooses to change its price within periods. Since inflation is positive, when shocks are negative, inflation brings their relative price closer to its optimal level. Therefore, firms find optimal to adjust their prices less frequently or only when shocks are relatively big. In contrast, when

[5]For models of wage rigidity based on optimal contract and fairness considerations, see Steinar (1992 and 2002). For a model of wage stickiness based on loss aversion, see Elsby (2004).

the shock is positive, inflation has the opposite effect on relative prices, it moves it further away from its optimum, consequently firms react by changing prices more frequently. When inflation is zero, their model implies a symmetric adjustment of prices.

Within the second group of theories, those denominated *pushing-on-a-string*, we have papers like that of Jackman and Sutton (1982) who propose a partial equilibrium model, where changes in the short-term interest rate generate asymmetric effects in aggregate demand when borrowing constraints are binding for a mass of consumers. They show that it is optimal for consumers at some point in their life to choose to borrow up to the limit of their borrowing constraints to smooth consumption. Consequently, they show that when some individuals are liquidity-constrained, the response of aggregate consumption to changes in interest rates involves an asymmetry between increases and decreases. The increase in the interest rate may be strongly contractionary, these effects follow from the redistribution of income between (liquidity-constrained) monetary debtors and (unconstrained) creditors brought about by interest rate changes. Also, contractionary monetary policy shocks can lead to rationing in the credit market, increasing the strength of the monetary shock through the credit channel, since a positive monetary policy shock has a different effect on the credit market, theories of credit rationing imply that negative monetary shocks have stronger effects than positive shocks[6].

4.2.2 Empirical Literature

We can organise the empirical literature historically into two categories; the early studies, which focus mainly on studying the asymmetric effects generated by monetary policy shocks depending on the sign and size of the shock, and those more recent ones, which focus on state-dependent asymmetry. The early studies used a simple extension of the methodology used by Barro (1978) to test for effects of

[6]For models of credit rationing see Jafee and Stiglitz (1990).

anticipated versus unanticipated monetary policy shocks. The more recent studies use the Markov switching time series process developed by Hamilton (1988) and the logistic smooth transition vector autoregression model described in Terasvirta and Anderson (1992).

Sign and size asymmetry

De Long and Summers (1988) and Cover (1992) are amongst the earliest papers reporting asymmetric effects of monetary policy, using a simple two-stage estimation process and innovations to money growth rate as a measure of the stance of monetary policy. They find that, for the USA, positive innovations to money growth rate have no effect on output, whereas negative innovations have a significant negative effect on output.

Using a similar approach, but using instead as a policy instrument the Federal Funds Rate, Morgan (1993) finds results in the same direction of Cover's results. This is that an increase in the Federal Funds Rate has significant negative effects on economic activity, whilst a cut in interest rates has no effect. Karras and Stokes (1996) extend Cover's methodology, allowing not only for asymmetric effects on output but also on inflation. They also test for asymmetric effects on the components of aggregate demand consumption and investment. Karras and Stokes (1996) confirm the findings of Cover (1992) and find that negative policy shocks have stronger effects than positive shocks.

On the other hand, Ravn and Sola (1996), using an extension of the methodology used by Cover that distinguished between small and big monetary policy shocks, find that the evidence reported by Cover is not robust for the sample period. Instead they conclude that asymmetry is not related to the sign of the shock, as Cover reported, but to its size. They conclude that for the USA during the period 1948-1987 unanticipated small changes in money supply are non-neutral whereas big unanticipated shocks and anticipated shocks are neutral. For a sample of industrial countries, Karras(1996) reports similar evidence to that reported by Cover for the

USA, in general the effects of money supply and the interest rate shocks on output tend to be asymmetric; monetary contractions tend to reduce output more than monetary expansions tend to raise it.

State-Dependent Asymmetry

Thoma (1994) extends the previous work on asymmetric effects of monetary policy shocks considering the existence of nonlinearities in the relationship between money and income. First, using rolling causality tests he finds that the causality relationship between income and money becomes stronger when activity declines and weaker when it increases, suggesting the existence of a non-linear response of income to monetary policy shocks. Following Cover (1992) and Morgan (1993), he distinguishes money shocks into positive and negative ones, but in contrast with the previous authors he also allows for a state-dependent response of output to positive and negative shocks. Using data for the USA that covers the period from January 1959 to December 1989 of M1, three-months treasury bills, consumer price index and industrial production, he finds that negative monetary shocks have stronger effects on output during high-growth periods than during low-growth periods, whilst the effects of positive monetary shocks do not vary over the business cycle.

More recently, using data for the USA as well, Weiss (1999) applied non-linear vector autoregression approach tests for asymmetric effects of monetary policy. This approach has the advantage of allowing a more flexible specification to test for which variable is important in generating the asymmetry. Using quarterly data from 1960 to 1995 of the industrial production index, the consumer price index and M1, he finds that negative monetary policy shocks have stronger effects on output when the initial state of the economy is high growth than when the initial state of the economy is negative growth. In particular, he estimates that one standard deviation shock to money growth rate generates, after twelve quarters, a cumulative reduction of 0.15 percent in output when the initial state of the economy is negative growth and 3.06 percent when the initial state is positive growth. However, he does not find any difference between the effects of positive versus negative shocks. In this sense, his results contradict Cover's findings. One possible explanation of this contradiction might be that the early papers on asymmetric effects of monetary policy do not control for the state of the economy when estimating asymmetric responses. Therefore, negative shocks are perceived as having stronger effects than positive shocks in those papers because negative shocks occur more frequently when the economy is in a high-growth state, the phase in which monetary policy seems to be more effective. On the contrary, positive shocks tend to occur during negative growth states, where monetary policy seems to be less effective, according to more recent papers.

Other studies, such as Caballero and Engel (1992), find that asymmetries in the response of output to demand shocks depend not only on the level of output but also on the level of inflation. For developing economies, Agenor (2001) using a VAR methodology also reports asymmetric responses of output and inflation to monetary policy shocks. Holmes and Wang (2002) find that negative monetary shocks have a more potent effect on output than positive shocks and that inflation renders monetary policy less effective, using data for the United Kingdom.

Overall the empirical evidence strongly suggests the existence of asymmetric effects of monetary policy on output and inflation. The earlier studies highlighted that negative monetary shocks have stronger effects than positive shocks, whereas more recent studies find that monetary shocks tend to be more effective in booms than in recessions. Our reading of the empirical evidence is that this is consistent with the fact that there exists more than one source of asymmetry in real economies, as the theoretical works emphasise, and that these different sources of asymmetry are working simultaneously. As we have shown in this chapter, introducing intertemporal non-hometheticity can generate asymmetric effects similar to those reported by recent studies, that is, that monetary policy is more effective in booms, through the interaction of a convex supply curve and a state-dependent IES.

4.3 The Model

The economy is populated by a continuum of agents of mass one who consume a set of differentiated goods and supply labour to firms. Each firm produces a different type of consumption good with a constant returns to scale technology that uses labour as production factor. We assume that the production of each good uses the same type of labour; that is, labour markets are integrated and there is only one wage that clears the labour market [7]. This assumption allows as to obtained a simplified version of the Phillips curve, whilst maintaining qualitatively the dynamics of the model.

We introduce intertemporal non-homotheticity by considering a subsistence level of consumption. In this case, the IES of consumption is not constant, but it changes prociclically. When the output deviation is negative, i.e. the economy is in a recession, consumption is relatively closer to the subsistence level and reacts less to changes in the interest rate than in a boom. Therefore, the IES is lower in a recession than in an expansion, and consequently consumption becomes less responsive to changes in the real interest rate than in an expansion.

Since goods are differentiated, firms have some degree of monopolistic power to set prices. Prices are set to maximise the present discounted value of profits. Following Calvo (1983) we assume that prices are staggered. Staggered price adjustment generates price inflexibility in equilibrium and makes monetary policy effective to control aggregate demand, and consequently to affect prices and output in the short run. Also, we assume that monetary policy is set choosing the nominal interest rate according to a Taylor rule.

[7]This assumption is different from Woodford's (2003), who assumes that each good uses a differentiated skill labour to generate strategic complementarity in pricing decisions.

4.3.1 Households

A typical household in the economy receives utility from consuming a variety of consumption goods and disutility from working. Preferences over consumption and labour effort for each household are represented by the following utility function:

$$\mathcal{U}_t = \sum_{s=0}^{\infty} \beta^s \left[U\left(C_{t+s}\right) - V\left(L_{t+s}\right) \right] \tag{4-1}$$

where $\beta \in [0, 1]$ represents the discount factor. $U(C)$ and $V(L)$ corresponds to the utility and disutility flow in each period that come from consumption and labour, respectively. We assume the following functional forms:

$$U\left(C_t\right) = \frac{(C_t - \underline{C})^{1-\sigma}}{1-\sigma} \quad V\left(L_t\right) = \frac{(L_t)^{1+v}}{1+v} \tag{4-2}$$

where σ is a parameter associated to the coefficient of risk aversion and v is the inverse of the elasticity of labour supply. $\underline{C}$ represents a subsistence level of consumption. Under this type of preferences, the coefficient of relative risk aversion is state-dependent, given by:

$$CRRA = \frac{-U_{cc}\left(C_t\right) C_t}{U_c\left(C_t\right)} = \sigma \left(\frac{C_t}{C_t - \underline{C}} \right)$$

Notice that when $\underline{C} = 0$ the model collapses to the standard model with isoelastic preferences. This parameter, $\underline{C}$, allows us to control for the degree of intertemporal non-homotheticity in the model, the higher $\underline{C}$ the higher the degree of intertemporal non-homotheticity. We normalize the subsistence level as a proportion ψ of the steady state level of consumption $(\overline{C})$, that is:

$$\underline{C} = \psi \overline{C}$$

The intertemporal elasticity of substitution (IES) is also state-dependent, which can be approximated by:

$$IES \approx \overline{\sigma}^{-1} \left(\frac{1}{1 - \frac{\psi}{1-\psi} y_t} \right)$$

where $\overline{\sigma}^{-1} \equiv \frac{1-\psi}{\sigma}$ is the steady state IES and y_t is the log-deviation of income around its steady state[8]. When income is above (below) its steady state, the IES is higher (lower). C_t is the Dixit-Stiglitz aggregator over all varieties of consumption goods.

$$C_t = \left[\int_0^1 C_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{4-3}$$

where $\varepsilon > 1$ is the elasticity of substitution across varieties of consumption goods. Since preferences over type of consumption goods are homothetic, the household problem can be solved in two stages. In the first stage, we solve for the optimal allocation of consumption across type of goods, given a total level of consumption C_t. In the second stage we solve for the intertemporal allocation of consumption and labour. The solution of the intratemporal allocation of consumption is given by the following set of equations:

$$C_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon} C_t \tag{4-4}$$

where P_t is the consumer price index, defined as :

$$P_t = \left[\int_0^1 P_t(z)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{4-5}$$

In the second stage the optimiser household takes decisions subject to a standard budget constraint which is given by

$$C_t = \frac{W_t L_t}{P_t} + \frac{B_{t-1}}{P_t} - \frac{1}{R_t} \frac{B_t}{P_t} + \frac{\Gamma_t}{P_t} \tag{4-6}$$

where W_t is the nominal wage, P_t is the price consumer price index, B_t is the end of period nominal bond holdings, R_t is the nominal gross interest rate and Γ_t is the share of the representative household on total nominal profits. The first order conditions for the optimising consumer´s problem are:

$$1 = \beta E_t \left[R_t \left(\frac{P_t}{P_{t+1}} \right) \left(\frac{C_{t+1} - \underline{C}}{C_t - \underline{C}} \right)^{-\sigma} \right] \tag{4-7}$$

[8]Here we have assumed a closed economy without capital nor government expenditures.

$$\frac{W_t}{P_t} = (C_t - \underline{C})^{\sigma} (L_t^s)^v = MRS_t \tag{4-8}$$

Equation $(4-7)$ is the Euler equation that determines the optimal path of consumption. Equation $(4-8)$ describes the optimal labour supply decision. MRS_t denotes for the marginal rate of substitution between labour and consumption. We assume that labour markets are competitive and also that individuals work in each sector $z \in [0,1]$. Therefore, L^s corresponds to the aggregate labour supply:

$$L_t^s = \int_0^1 L_t^s(z)dz \tag{4-9}$$

4.3.2 Firms

Each variety of consumption good is produced in an environment of monopolistic competition using a (linear) constant returns to scale technology that uses labour as production factor.

$$Y_t(z) = A_t L_t^d(z) \tag{4-10}$$

where A_t is a stochastic variable that represents the state of technology and $L_t^d(z)$ represents the demand for labour for producing consumption good of variety z. Furthermore, we assume that technology evolves over time following an autoregressive stochastic process of order 1.

$$\ln A_t = \rho_a \ln a_{t-1} + \xi_t^a \tag{4-11}$$

where $\xi_t^a \backsim N(0, \sigma_a^2)$.

Under this specification of technology, the real marginal cost of a typical firm can be expressed as:

$$MC_t(z) = \frac{W_t/P_t}{A_t} \tag{4-12}$$

The marginal cost is increasing an real wages, W_t/P_t, and decreasing on the level of technology. Notice that marginal costs are the same for the production of each variety of good, since technology has constant returns to scale and factor markets are competitive, i.e. $MC_t(z) = MC_t$.

Firms set prices following a staggered pricing mechanism *a la Calvo*. Each firm faces an exogenous probability of changing prices given by $(1-\theta)$. The optimal price that solves the firm's problem is given by

$$\left(\frac{P_t^*(z)}{P_t}\right)=\frac{\mu E_t\left[\sum\limits_{k=0}^{\infty}\theta^k\zeta_{t,t+k}MC_{t+k}F_{t,t+k}^{\varepsilon+1}Y_{t+k}\right]}{E_t\left[\sum\limits_{k=0}^{\infty}\theta^k\zeta_{t,t+k}F_{t,t+k}^{\varepsilon}Y_{t+k}\right]} \tag{4-13}$$

where $\mu=\frac{\varepsilon}{\varepsilon-1}$ is the price markup, $\zeta_{t,t+k}=\beta^k\left(\frac{C_{t+k}-\underline{C}}{C_t-\underline{C}}\right)^{-\sigma}\frac{P_t}{P_{t+k}}$ is the stochastic discount factor, $P_t^*(z)$ is the optimal price level chosen by the firm, $F_{t,t+k}=\frac{P_{t+k}}{P_t}$ the cumulative level of inflation and Y_{t+k} is the aggregate level of output.

Since only a fraction $(1-\theta)$ of firms changes prices every period and the remaining one keeps its price fixed, the aggregate price level, the price of the final good that minimise the cost of the final goods producers, is given by the following equation:

$$P_t^{1-\varepsilon}=\theta P_{t-1}^{1-\varepsilon}+(1-\theta)\left(P_t^*(z)\right)^{1-\varepsilon} \tag{4-14}$$

Following Benigno and Woodford (2005), equations $(4-13)$ and (4-14) can be written recursively introducing the auxiliary variables N_t and D_t (see appendix C for details on the derivation):

$$\theta\left(\Pi_t\right)^{\varepsilon-1}=1-(1-\theta)\left(\frac{N_t}{D_t}\right)^{1-\varepsilon} \tag{4-15}$$

$$D_t=Y_t\left(C_t-\underline{C}\right)^{-\sigma}+\theta\beta E_t\left[\left(\Pi_{t+1}\right)^{\varepsilon-1}D_{t+1}\right] \tag{4-16}$$

$$N_t=\mu Y_t\left(C_t-\underline{C}\right)^{-\sigma}MC_t+\theta\beta E_t\left[\left(\Pi_{t+1}\right)^{\varepsilon}N_{t+1}\right] \tag{4-17}$$

Equation $(4-15)$ comes from the aggregation of individual firms prices. The ratio N_t/D_t represents the optimal relative price $P_t^*(z)/P_t$. These three last equations summarise the recursive representation of the non linear Phillips curve.[9]

[9] Writing the optimal price setting in a recursive way is necessary in order to implement numerically or algebraically the perturbation method.

4.3.3 Monetary Policy

Monetary policy is implemented by a central bank setting the nominal interest rate according to a Taylor rule specified in the following way:

$$R_t = \overline{R}\left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\phi_\pi}\left(\frac{Y_t}{\overline{Y}}\right)^{\phi_y}\exp\left(-e_t\right) \tag{4-18}$$

The steady state values are expressed without time subscript and with an upper bar. $\phi_\pi > 1$ and $\phi_y > 0$ are the coefficients of the rule, and ϵ_t represents an exogenous monetary policy shock. Under this policy rule the central bank increases the nominal interest rate when inflation is positive and when domestic output its above its steady state. The exogenous monetary policy shock evolve according to the following stochastic autoregressive process

$$e_t = \rho_e e_{t-1} + \xi_t^e \tag{4-19}$$

where $\xi_t^e \backsim N\left(0, \sigma_e^2\right)$.

4.3.4 Market Clearing

In equilibrium labour and each variety of good market clear. Since there is no capital accumulation nor government sector, the economywide resource constraint is given by:

$$C_t = Y_t \tag{4-20}$$

The labour market clearing market condition is given by:

$$L_t^s = L_t^d \tag{4-21}$$

where the labour demand comes from the aggregation of the producers of each type of good:

$$\begin{aligned} L_t^d &= \int_0^1 L_t^d(z)dz = \frac{1}{A_t}\int_0^1 Y_t(z) \\ L_t^d &= \frac{Y_t\Delta_t}{A_t} \end{aligned} \tag{4-22}$$

where $\Delta_t = \int_0^1 \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon} dz$ is a measure of price dispersion. Since relative prices differ across firms due to staggered price setting, input usage will differ as well, implying that is not possible to use the usual representative firm assumption, therefore, the price dispersion factor, Δ_t appears in the aggregate labour demand equation.

4.3.5 Steady-State

We define the steady-state equilibrium as a competitive equilibrium where the shocks, ξ_t^ϵ and ξ_t^a, are zero. In this equilibrium all endogenous variables remain constant. Under these assumptions, the steady-state level of output is given by:

$$\overline{Y} = \left(\frac{(1-\psi)^{-\sigma}}{\mu}\right)^{\frac{1}{v+\sigma}}$$

We assume a zero steady state of inflation is zero in the policy rule, then the real interest rate is given by:

$$\overline{R} = \frac{1}{\beta}$$

4.4 Asymmetric Effects of Monetary Policy

As we discuss in the introduction of this chapter, monetary policy can have asymmetric effects on output and inflation depending on either the state of the economy or the sign of the monetary policy shock. We define the former as *state-asymmetry*, when the response is different in a recession from an expansion, and the latter as *sign-asymmetry*, when the size and sign of the monetary policy shock affect the response. We argue that, in equilibrium, both types of asymmetry come from the interaction of two different sources: the non-linear, generally convex, form of the Phillips curve as well as the non-linear response of the aggregate demand to the interest rate.

We focus in this chapter on state-asymmetry in which the response of output and inflation changes with the state of the economy, i.e. the deviation of output with respect to its steady state value. However, since the state of the economy depends on the source of the shocks, it is not straightforward to analyse how the state of the economy influence the effectiveness of monetary policy. The methodology we use, based on the perturbation method helps us to disentangle both sources of asymmetry, since we can solve for state-asymmetry after controlling for the type of this shock. Also, **we define as positive (negative) asymmetry when monetary policy has more (less) effect in an expansion than in a recession.**

In the next sub-section we solve the second order Taylor expansion of the model and we analyse the implications of intertemporal non-homotheticity in the aggregate demand and the aggregate supply. Then, we solve analytically for the asymmetric effects of monetary policy in equilibrium and we do some comparative statics of the solution on the parameters of the model.

4.4.1 Second order approximation of the structural equations

We present in table 4.1 a log-quadratic (Taylor-series) approximation of the fundamental equations of the model around the steady state, a detailed derivation is provided in Appendix C. The second-order Taylor-series expansion serves to compute the equilibrium fluctuations of the endogenous variables of the model up to a residual of order $O\left(\|\xi_t\|^2\right)$, where $\|\xi_t\|$ is a bound on the size of the shocks $\xi_t = [\xi_t^a, \xi_t^e]$. We denote variables in steady state with upper bars (that is $\overline{X}$) and their log deviations around the steady state with lower case letters (that is $x_t \equiv \ln \frac{X_t}{\overline{X}}$).

Notice that we make following change of variable $\overline{\sigma} \equiv \sigma / (1 - \psi)$, where $\overline{\sigma}^{-1}$ denotes the IES in steady state [10]. Equation $(4 - i)$ is the second-order approximation of the Euler equation (4-7), which represents the aggregate demand. The term $\frac{1}{2}\overline{\sigma}^{-1}\frac{\psi}{1-\psi}\left(y_t^2 - E_t y_{t+1}^2\right)$ captures the non-linear effect of non-homothetic

[10] In all the following analysis we replace $\overline{\sigma}^{-1} = \sigma^{-1}(1 - \psi)$, which is the intertemporal elasticity of substitution in steady state with subsistence, and then we change σ^{-1}endogenously as ψ changes to keep $\overline{\sigma}^{-1}$constant. Doing this allows us to compare the effects of ψ on asymmetry without considering the effects caused by the change on the steady state IES.

Aggregate Demand		
	$y_t = E_t y_{t+1} - \overline{\sigma}^{-1}(r_t - E_t\pi_{t+1}) + \frac{1}{2}\frac{\psi}{1-\psi}\left(y_t^2 - E_t y_{t+1}^2\right) + \omega_y + O\left(\lVert\xi_t\rVert^3\right)$	$4-i$
Aggregate Supply		
Phillips Curve		
	$v_t = \kappa mc_t + \frac{1}{2}\kappa mc_t\left(2\left(1-\overline{\sigma}\right)y_t + mc_t\right) + \frac{1}{2}\varepsilon\pi_t^2 + \beta E_t v_{t+1} + O\left(\lVert\xi_t\rVert^3\right)$	$4-ii$
Auxiliary Variables		
	$v_t \equiv \pi_t + \frac{1}{2}\left(\frac{\varepsilon-1}{1-\theta} + \varepsilon\right)\pi_t^2 + \frac{1}{2}\left(1-\theta\beta\right)\pi_t z_t$	$4-iii$
	$z_t = 2\left(1-\sigma\right)y_t + mc_t + \theta\beta E_t\left(\frac{2\varepsilon-1}{1-\theta\beta}\pi_{t+1} + z_{t+1}\right) + O\left(\lVert\xi_t\rVert^2\right)$	$4-iv$
Price dispersion		
	$\widehat{\Delta}_t = \theta\widehat{\Delta}_{t-1} + \frac{1}{2}\varepsilon\frac{\theta}{1-\theta}\pi_t^2 + O\left(\lVert\xi_t\rVert^3\right)$	$4-v$
Marginal Costs		
	$mc_t = (v+\overline{\sigma})y_t - (v+1)a_t - \frac{1}{2}\overline{\sigma}\frac{\psi}{1-\psi}y_t^2 + v\widehat{\Delta}_t + O\left(\lVert\xi_t\rVert^3\right)$	$4-vi$
Monetary Policy		
	$r_t = \phi_\pi E_t\pi_{t+1} + \phi_y y_t$	$4-vii$
Exogenous disturbances		
	$a_t = \rho_a a_{t-1} + \xi_t^a$	$4-viii$
	$\epsilon_t = \rho_\epsilon \epsilon_{t-1} + \xi_t^\epsilon$	$4-ix$

Table 4.1: Second order Taylor expansion of the equations of the model

preferences on aggregate demand, which makes output to respond non-linearly to changes in the interest rate. More precisely, since the IES changes prociclically, the interest rate affects more aggregate demand in a boom than in a recession. The term $\omega_y \equiv -\frac{1}{2}\overline{\sigma}E_t\left[\left(y_t - y_{t+1}\right) - \frac{1}{\overline{\sigma}}\left(r_t - \pi_{t+1}\right)\right]^2 < 0$ is independent of policy and captures the precautionary savings effects of shocks volatility on consumption.

Equation $(4-ii)$ is the second-order approximation of the aggregate supply, which uses the auxiliary variables v_t (defined by $4-iii$) and z_t (which has a first order approximation in $4-iv$). v_t is a quadratic function on π_t and z_t and in linear terms in $v_t = \pi_t$. Equation $(4-v)$ represents the dynamics of the price dispersion measure, which is a second order function of inflation.

The first two terms in equation $(4-ii)$ capture the effects of marginal costs on inflation. $\kappa \equiv \frac{1-\theta}{\theta}\left(1-\theta\beta\right)$ is the slope of the Phillips curve with respect to the marginal costs. Note that under Calvo price setting inflation is a quadratic function of marginal costs. The third term in equation $(4-ii)$, $\frac{1}{2}\varepsilon\pi_t^2$, captures the effect of inflation volatility on the response of relative prices to marginal costs. More precisely, when inflation volatility is higher, firms put a higher weight on marginal costs when setting prices, increasing the level of inflation[11]. In the overall, these two effects make that inflation is a convex function on marginal costs under Calvo price setting. In contrast, other forms of modelling price rigidities such as the Rotemberg (1982) type with adjustment costs to changing prices, can give the same solution in linear terms, but imply a concave function of inflation on marginal costs[12].

The equation for the marginal costs $(4 - vi)$ is obtained taking the second order expansion of the real marginal costs and using the labour market equilibrium condition to eliminate real wages. Marginal costs are affected, in linear terms, positively by output fluctuations and negatively by productivity. Additionally, there are two second order terms: the first term captures the effect of intertemporal non-homothetic preferences on real wages, lower IES reduces the income effect on the labour supply and hence lower real wages. The second term captures the effect of price dispersion on real wages: since higher price dispersion increases the labour amount necessary to produce a given level of output, it also increases marginal costs.

After replacing the marginal costs $(4 - vi)$ and the dynamics of the price dispersion $(4 - vi)$ on $(4 - ii)$, under certain assumptions[13], we can express the

[11]Also, we have seen in chapter 2 that this term is important in generating a risk premium on inflation.

[12]This is because in the Rotemberg (1982) setup the adjustment costs are quadratic on price deviations. Then, higher price deviations from its steady state level are relatively more costly to the firms than small deviations, which induces firms to respond relatively less when deviations on marginal costs are higher.

[13]More precisely, when the initial price dispersion is small, that is $\Delta_{t_o-1} \simeq 0$ up to second order. This assumption make the analysis analytically tractable, without changing qualitatively the results.

Phillips curve as:

$$\begin{aligned} v_t &= \kappa\left[(v+\overline{\sigma})\,y_t-(v+1)a_t\right] \\ &+\frac{1}{2}\kappa\left[(1+v)^2(y_t-a_t)^2-(1-\overline{\sigma})^2y_t^2\right]-\frac{1}{2}\kappa\left[\overline{\sigma}\frac{\psi}{1-\psi}y_t^2\right] \\ &+\frac{1}{2}\varepsilon\,(1+v)\,\pi_t^2+\beta E_t v_{t+1}+O\left(\|\xi_t\|^3\right) \end{aligned} \tag{4-23}$$

We can use equation $(4-i)$ and (4-23) together with the definition of the auxiliary variables v_t and z_t to solve analytically for the asymmetric effects of monetary policy in general equilibrium.The two sources of asymmetry previously analysed, the state-dependent IES and the convex PC, interact in equilibrium to determine the degree of asymmetry of output and prices. In the next section, we solve analytically for the dynamic equilibrium of the economy using a second order-approximated solution. This approach allows to disentangle the asymmetric responses of output and inflation controlling for the source of the shock, demand or supply shocks.

4.4.2 Solving asymmetric response analytically

We use the perturbation method, developed by Judd (1998), Collard and Julliard (2001) and Schmitt-Grohe and Uribe (2004), to find a second order approximation of the solution of the model. This method consists in obtaining the coefficients of a taylor expansion of the solution of the model near the steady state using a system of equations that come from the differentiation of the equilibrium conditions of the model. The implementation of this method is discussed in appendix C. We approximate the policy functions for output and inflation as second order polynomials on the state variables $s=[a,e]$, the productivity and monetary policy shocks, respectively. Furthermore, the former represents supply shocks and the latter demand shocks:

$$\begin{aligned} y &= b + b_a a + b_e e + b_{ae} ae + \frac{1}{2} b_{aa} a^2 + \frac{1}{2} b_{ee} e^2 + \mathcal{O}\left(\|s\|^3\right), \\ \pi &= d + d_a a + d_v v + d_{ae} ae + \frac{1}{2} d_{aa} a^2 + \frac{1}{2} d_{ee} e^2 + \mathcal{O}\left(\|s\|^3\right) \end{aligned} \tag{4-24}$$

where y and π are output and inflation in log-deviations from the steady state. We assume initially a log-linear policy rule of the form

$$i_t = \phi_y y_t + \phi_\pi \pi_t - e$$

The coefficients of the first order terms $\{b_a, b_e, d_a, d_e\}$ are equal to those of the log-linearised solution of the model. The second order solution only adds additional terms to the log-linearised solution, $\{b_{ae}, b_{aa}, b_{ee}, d_{ae}, d_{aa}, d_{ee}\}$, preserving the existing terms. Additionally, b and d are constants that depend on the variance of the shocks, as it is shown in and Schmitt-Grohe and Uribe (2004).

The marginal responses of output and inflation due to unexpected changes to the interest rate are given by:

$$\begin{aligned} \frac{\partial y}{\partial e} &= b_e + b_{ee} v + b_{ae} a \\ \frac{\partial \pi}{\partial e} &= d_e + d_{ee} v + d_{ae} a \end{aligned}$$

The state-asymmetry effects of monetary policy can be seen by the coefficients of the the quadratic terms (b_{ee} and d_{ee}) and the crossed terms (b_{ae} and d_{ae}), because they take into account both supply shocks and demand shocks in the marginal response. Therefore, the quadratic terms b_{ee} and d_{ee} take into account the asymmetry effects when the economy is away from the steady state because of demand shocks, and the crossed terms b_{ee} and d_{ee} because of supply shocks.

In order to analyse the effects on impact of the state of the economy in the impulse response, it is convenient to write the marginal response of output and inflation in the following form:

$$\begin{aligned}\frac{\partial y}{\partial e} &= b_e\left(1+\sigma_y^d y^d+\sigma_y^s y^s\right) \\ \frac{\partial \pi}{\partial e} &= d_e\left(1+\sigma_\pi^d y^d+\sigma_\pi^s y^s\right)\end{aligned} \tag{4-25}$$

where σ_y^d and σ_π^d are the elasticities of the impulse response with respect to output when its deviations are caused by demand shocks, and σ_y^s and σ_π^s are the elasticities of the impulse response with respect to output when its deviations are caused by supply shocks. For instance, the impulse response of output when the output are deviations caused by demand shocks are equal to $\pm 4\%$ is given by $b_v\left(1 \pm \sigma_y^d 4\%\right)$. These elasticities are defined by:

$$\begin{array}{ll}\sigma_y^d = \frac{b_{ee}}{b_e}\frac{1}{b_e} & \sigma_y^s = \frac{b_{ae}}{b_e}\frac{1}{b_a} \\ \sigma_\pi^d = \frac{d_{ee}}{d_e}\frac{1}{b_e} & \sigma_\pi^s = \frac{d_{ae}}{d_e}\frac{1}{b_a}\end{array}$$

This uses the fact that from the log-linearised solution of the model, output is equal to $y \approx b_e v + b_e a$. We define $y^d = b_e e$ and $y^s = b_a a$ as the deviations of output due to demand shocks and supply shocks, respectively.

We solve for the state-asymmetry elasticities applying the perturbation method to the second order Taylor approximation of the equations of the model. The solution for both types of state-asymmetry elasticities $\{\sigma_y^d, \sigma_\pi^d\}$ and $\{\sigma_y^s, \sigma_\pi^s\}$ is given by the intersection of two linear equations, one that comes from the expansion of the IS and the other that comes from the expansion of the Phillips Curve. See appendix C for the derivation. It is useful to separate the effects in this form, because this allows us to disentangle the asymmetric effects that come either from the aggregate demand and the aggregate supply.

For the special case that the shocks are uncorrelated (that is $\rho_a = \rho_e = 0$), these expressions are summarised by:

$$\sigma_y^i = -\frac{\kappa(v+1)}{\overline{\sigma}+\phi_y}\phi_\pi\sigma_\pi^i + \frac{\psi}{1-\psi}\frac{1}{1+\phi_y\overline{\sigma}^{-1}} \qquad (IS\_as^i) \qquad (4\text{-}26)$$

$$\sigma_y^i = \sigma_\pi^i + \frac{\psi}{1-\psi}\frac{1}{1+v\overline{\sigma}^{-1}} + \Omega^i \qquad (PC\_as^i) \quad (4\text{-}27)$$

where the index $i = \{d, s\}$ indicates if the output deviations are given by demand shocks ($i = d$) or supply shocks ($i = s$). Ω^d and Ω^s are defined in the appendix and capture the non-linearity of the Phillips Curve . These schedules are named as IS_as^i and PC_as^i, because they come from the second order expansion of the IS and the Phillips curve, respectively.The elasticities are found in equilibrium by the intersection of both equations.

The IS_as^i schedule has a negative slope equal to $\frac{\kappa(v+1)}{\overline{\sigma}+\phi_y}\phi_\pi$and the PC_as^i schedule has a positive slope equal to one. Moreover, when $\psi = 0$ the intercept is zero for the IS_as^i schedule and equal to Ω^i for the PC_as^i, which can be either positive or negative. The state-asymmetry elasticities solution is given by the intersection of these two curves. In the next sub-section we analyse the equilibrium in two cases, for $\psi = 0$ and $\psi > 0$ and we do some comparative statics with respect to some parameters.

4.4.3 Comparative statics

The model is parameterised using values that are standard in the literature. We set a quarterly discount factor, β, equal to 0.99 which implies an annualised rate of interest of 4%. For the coefficient of risk aversion parameter, σ, we choose a value of 1 and the inverse of the elasticity of labour supply, v, is calibrated to be equal to 1. We choose a degree of monopolistic competition, ε, equal to 7.88, which implies a firm mark-up of 15% over the marginal cost. The probability of not adjusting prices θ is set to 0.66 which implies that a typically firms changes prices every three quarters. We set the parameters of the Taylor rule ϕ_π and ϕ_y to

1.5 and 0.5, respectively. We assume the same distribution for both productivity and monetary policy shocks, with standard deviation of 0.1 and $\rho_a = \rho_e = 0.6$ for the impulse response. Finally, the subsistence consumption level was set to 0.8 of the steady-state level of output, similar to the values used in the habit formation models.

We make some comparative statics for the state-asymmetry elasticities calculated for the special case that the shocks are uncorrelated using expressions (4-26) and (4-27). Then, we use the second order solution of the model to solve numerically for the impulse response to a monetary policy shock conditional on the initial state of the economy.

The comparative statics for the state-asymmetry elasticities are summarised in following table 4.2.

Parameters			Elasticities			
ψ	ε	ϕ_y	σ_y^d	σ_π^d	σ_y^s	σ_π^s
0	7.88	0.50	−1.3	1.6	1.6	−1.6
0	10	0.50	−0.8	2.1	2.1	−2.1
0.8	7.88	0.50	1.7	2.7	3.7	−1.0
0.8	7.88	0	2.3	3.2	4.3	−0.2

Elasticities calculated for the case of uncorrelated shocks.

Table 4.2: Comparative Statics Results

Intertemporal Homothetic preferences

When the subsistence level is zero, that is. $\psi = 0$, the solution of the model converges to the case of isoelastic preferences on consumption. In this case, the non-linearities of the model come uniquely from the Phillips curve, since the aggregate demand respond linearly to the interest rate. In the case of isoelastic preferences, when the state is given by demand shocks, inflation responds more $\left(\sigma_\pi^d > 0\right)$ and output responds less $\left(\sigma_y^d < 0\right)$ in an expansion than in an recession. However we have the opposite effect when the state is given by supply shocks: in that case the response of inflation is lower $(\sigma_\pi^s < 0)$ and output is higher $\left(\sigma_y^s > 0\right)$ in an expansion than in an recession. This effects are solely driven by the convexity of the Phillips curve implied by the Calvo price-setting.

The difference in the asymmetry depending on source of output deviations can be seen in figure 4.1 . In the graph on the left, an expansion driven by demand shocks implies a demand on the right of the steady state, it intersects the PC in an area where it is steeper, therefore a movement in demand due to monetary policy will have more effect in prices and less effect in output that when the economy is in an expansion than in a recession. On the other hand, when the expansion is driven by supply shocks, the demand intersects a Phillips curve that is on the right from its steady state schedule, which implies that it intersects the PC in an area where it is flatter. Therefore, when the state is driven by supply shocks monetary policy have the opposite asymmetric effects than when the state is driven by demand shocks: that is, monetary policy affects more output (and less inflation) in an expansion than in a recession.

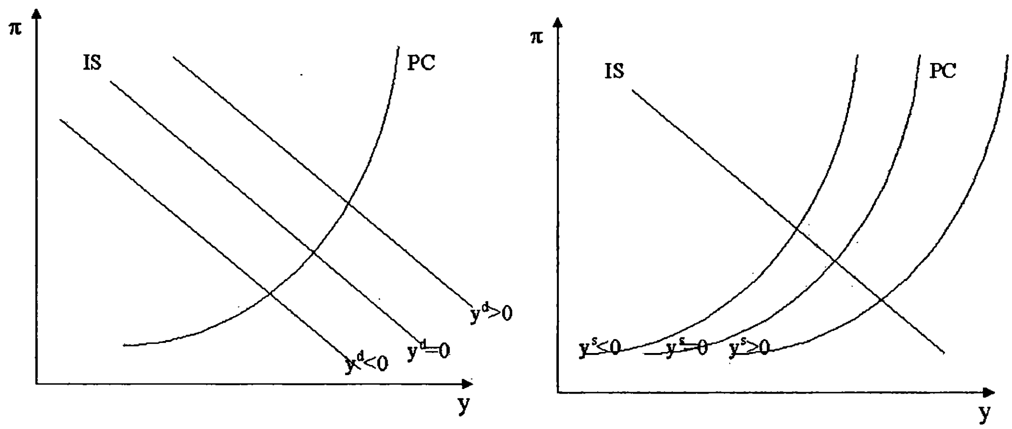

Figure 4.1: State-dependent asymmetric effects of monetary policy in the IS-PC equilibrium. a) Output deviations driven by demand shocks. b) Output deviations driven by supply shocks.

The second row in table 4.1 shows that an increase in the price elasticity of demand of individual goods (ε) increases the convexity of the Phillips curve. This is because ε increases the responsiveness individual firm's optimal prices to marginal costs [14]. Then, for a more convex Phillips curve, the state-asymmetry elasticities increase for all the cases.

Intertemporal Non-homothetic preferences

When we introduce a subsistence level, that is when $\psi > 0$, the aggregate demand responds non-linearly to the interest rate, affecting the asymmetric effects of monetary policy. We can see in the third row of table 4.1, that the introduction of non-homothetic preferences generates that output responds more to monetary policy in an expansive part of the cycle generated by demand shocks (that is $\sigma_y^d > 0$), in contrast with the opposite result when preferences are homothetic. The subsistence level also reinforces the asymmetric effects generated by the Phillips curve, increasing both σ_π^d and σ_y^s.

Alternative policy rule: strict inflation target.

An strict inflation target eliminates the output term in the Taylor rule, that is $i_t = \phi_\pi \pi_t$, and has more asymmetric effects on output and inflation than the traditional one. A Taylor rule that puts some weight in the output deviations partially offsets the asymmetric effects of monetary policy on demand. Therefore, in a Taylor rule that only considers inflation (i.e. $\phi_y = 0$) the asymmetric effects of monetary policy are higher in both output and inflation. As shown in the fourth row in table 4.1, eliminating the output term in the Taylor rule in our baseline model increases the asymmetry on both output and inflation, for both sources of deviations.

State-dependent impulse responses to a monetary policy shock.

[14]See chapter 2 for a more detailed discussion on the effects of ε on the convexity of the Phillips curve

The next two figures show the state-dependent impulse response to a monetary policy shock, when the shocks exhibit some persistence, conditioned on the source of output deviations[15]. In order to capture the difference between the peak and the (bottom) of the cycle, we have calculated the impulse responses when deviations of output from the steady state were ±4%.

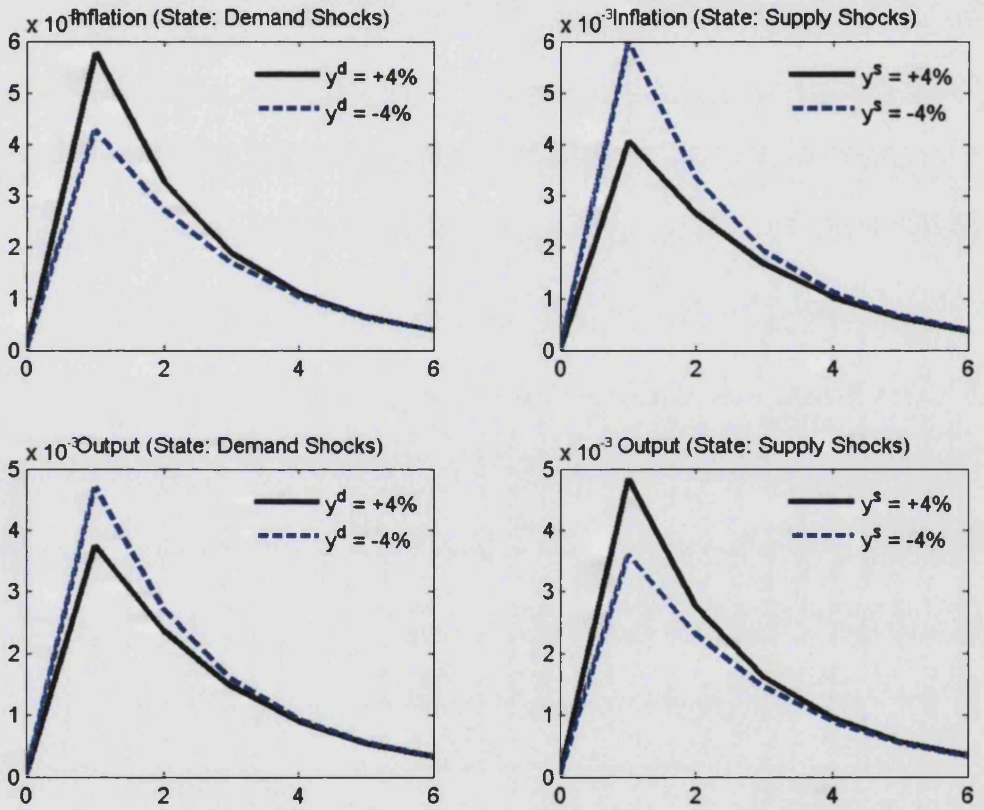

Figure 4.2: State-dependent impulse response to a monetary policy shock , the case of homothetic preferences ($\psi = 0$).

Figure 4.2 analyses the case of isoelastic preferences ($\psi = 0$). In this case

[15]The state-dependent impulse responses are calculated numerically from the second order solution of the model, conditional to an initial value of output deviations equal to $y_{t-1} = \pm 4\%$, and considering the definitions for the deviations of output due to demand shocks ($y^d = b_e e$) and supply shocks ($y^s = b_a a$).

asymmetric monetary policy effects are generated solely by the non-linearity of the Phillips curve. According to this graph, when the deviations in output are generated by demand shocks, inflation respond more and output respond less in an expansion than in a recession, which is consistent with a convex Phillips curve. On the other hand, we have exactly the opposite effect when output deviations are generated by supply shocks.

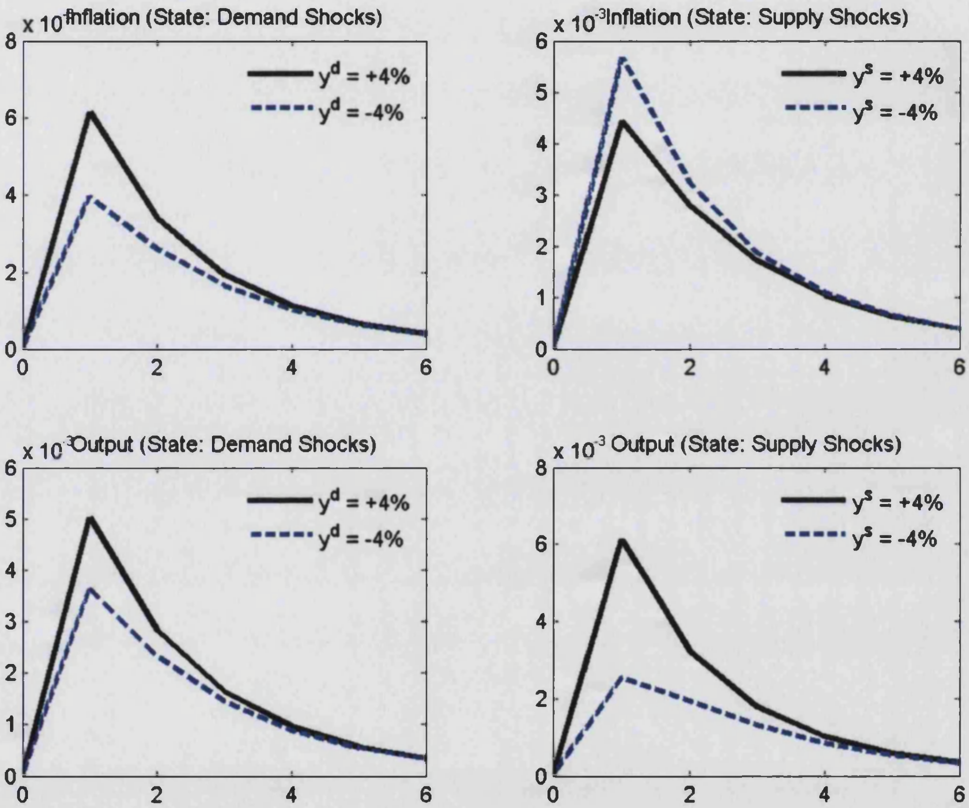

Figure 4.3: State-dependent impulse response to a monetary policy shock , the case of non-homothetic preferences ($\psi = 0.8$).

Figure 4.3 shows the case of non-homothetic preferences ($\psi = 0.8$). In contrast to the previous case, output reacts more to monetary policy in an expansion than in a recession when the state is driven by demand shocks. Also, the asymmetric effects on output are amplified when deviations come from supply shocks. This is generated by the interaction of both sources of non-linearities: the convexity of the Phillips curve and the asymmetric shifts of aggregate demand. Also, the asymmetric effects on inflation are not qualitatively changed, since they are mostly captured by the non-linearity of the Phillips curve.

4.5 Conclusions

Empirical studies for the USA and other developed countries have reported that monetary policy seems to have stronger effects on output and prices when the economy is growing fast than when it is in a recession. This pattern of asymmetric response in output and inflation however cannot be explained by only the existence of a convex supply curve, which predicts the opposite asymmetric response for output. In this chapter we show that it is possible to generate asymmetric responses of output and inflation similar to those observed in the data by incorporating in an otherwise standard New Keynesian model a subsistence level for consumption that generates a state-dependent IES.

We find that the interaction of these two mechanisms is key in generating in equilibrium asymmetric responses in output and inflation that match the empirical evidence. On one hand, when consumption is relatively closer to the subsistence level, as in a recession, the IES is lower, therefore consumption reacts less to changes in the interest rate than in an expansion. This generates an asymmetric response of aggregate demand to monetary policy shocks. On the other hand, the convexity of the Phillips curve implies that output reacts less to demand shocks when output is initially low.

We further differentiate between state generated by demand shocks versus supply shocks. We found that monetary policy is more effective to affect output in a boom than in a recession (positive asymmetry) when the degree of intertemporal non-homotheticity is high. Moreover, this asymmetry is higher when the deviations from the steady state come from supply shocks instead of demand shocks. On the other hand, the sign of the asymmetric effects of monetary policy on inflation will depend on the type of the shock: when the state is driven by demand shocks the asymmetry in inflation is positive, however when the state is driven by supply shocks, the asymmetry in inflation is negative.

This chapter provides a framework for analysis of the asymmetric effects of monetary policy, considering an elasticity of the impulse response respect to the state of the economy. This analysis can be expanded to other factors that can contribute to explain asymmetric effects of monetary policy, such as borrowing constraints and adjustment cost to investment, that have not been analysed in this chapter. The introduction of non-homotheticity in the preferences of consumption over time can be considered as a proxy of these other sources of asymmetric effects of monetary policy on demand. However, solving some of these problems can involve non-differentiabilities what would prevent the implementation of the perturbation method. Therefore, in the cases of non-differentiability it would be necessary to apply other kinds of methods, like collocation methods, in order to find a numerical solution to the policy functions.

C Appendix: The second order approximation of the system

C.1 The second order approximation of the model

The second order approximation of the marginal utility becomes:

$$mu_t = -\overline{\sigma}\left(c_t - \frac{1}{2}\frac{\psi}{1-\psi}c_t^2\right) + O\left(\|\xi\|^3\right) \tag{C.1}$$

where $\overline{\sigma} \equiv \frac{\sigma}{1-\psi}$ is the steady state risk aversion coefficient.

The second order approximation of the IS is:

$$0 = E_t\left(mu_{t+1} - mu_t + r_t - \pi_{t+1}\right) + \frac{1}{2}E_t\left[mu_{t+1} - mu_t + r_t - \pi_{t+1}\right]^2 \tag{C.2}$$

Replacing (C.1) and the clearing market condition in (C.2) and eliminating the terms of higher order than 2:

$$\begin{aligned} y_t &= E_t y_{t+1} - \overline{\sigma}^{-1}\left(r_t - E_t\pi_{t+1}\right) + \frac{1}{2}\frac{\psi}{1-\psi}\left(y_t^2 - E_t y_{t+1}^2\right) \qquad \text{(C.3)} \\ &\quad -\frac{1}{2}\overline{\sigma}E_t\left[\left(y_t - y_{t+1}\right) - \frac{1}{\overline{\sigma}}\left(r_t - \pi_{t+1}\right)\right]^2 + O\left(\|\xi\|^3\right) \end{aligned}$$

which is equation (v-i) in the main text.

The derivation of the second order approximation of the Phillips curve is the same as the one presented in chapter 2, after replacing σ by $\overline{\sigma}$:

$$\begin{aligned} v_t &= \kappa mc_t + \frac{1}{2}\kappa mc_t\left(2\left(1-\overline{\sigma}\right)y_t + mc_t\right) + \frac{1}{2}\varepsilon\pi_t^2 + \beta E_t v_{t+1} + O\left(\|\xi\|^3\right) &\text{(C.4)} \\ v_t &= \pi_t + \frac{1}{2}\left(\frac{\varepsilon-1}{1-\theta} + \varepsilon\right)\pi_t^2 + \frac{1}{2}\left(1-\theta\beta\right)\pi_t z_t &\text{(C.5)} \\ z_t &= 2\left(1-\overline{\sigma}\right)y_t + mc_t + \theta\beta E_t\left(\frac{2\varepsilon-1}{1-\theta\beta}\pi_{t+1} + z_{t+1}\right) + O\left(\|\xi\|^3\right) &\text{(C.6)} \end{aligned}$$

Similarly, price dispersion has the same dynamics as in chapter 2:

$$\widehat{\Delta}_t = \theta\widehat{\Delta}_{t-1} + \frac{1}{2}\varepsilon\frac{\theta}{1-\theta}\pi_t^2 + O\left(\|\xi\|^3\right) \tag{C.7}$$

As in chapter 2, the discounted infinite sum of $\widehat{\Delta}_t$ can be expressed as the sum of two terms, the initial price dispersion and the discounted infinite sum of π_t^2.

$$\sum_{t=t_o}^{\infty} \beta^{t-t_o} \widehat{\Delta}_t = \frac{\theta}{1-\beta\theta} \widehat{\Delta}_{t_o-1} + \frac{1}{2}\frac{\varepsilon}{\kappa} \sum_{t=t_o}^{\infty} \beta^{t-t_o} \frac{\pi_t^2}{2} + O\left(\|\xi\|^3\right) \tag{C.8}$$

The real marginal cost (4-12) and the labour market equations (4-8 and 4-22) have the following second order expansion:

$$mc_t = w_t - a_t \tag{C.9}$$

$$w_t = \nu l_t - mu_t \tag{C.10}$$

$$l_t = y_t - a_t + \widehat{\Delta}_t \tag{C.11}$$

Replace (C.11) and (C.1) in (C.10), we obtain the wage that clears the labour market:

$$w_t = \nu \left(y_t - a_t + \widehat{\Delta}_t\right) + \overline{\sigma} \left(y_t - \frac{1}{2}\frac{\psi}{1-\psi} y_t^2\right) + O\left(\|\xi\|^3\right)$$

Replace w_t in the marginal costs (C.9):

$$mc_t = (v + \overline{\sigma})\, y_t - (v+1) a_t - \frac{1}{2}\overline{\sigma}\frac{\psi}{1-\psi} y_t^2 + v\widehat{\Delta}_t + O\left(\|\xi_t\|^3\right) \tag{C.12}$$

which is equation (4-vi) in the main text.

Replace the marginal costs (C.12) in the Phillips curve equation (C.4) and eliminate the terms of order higher than 2:

$$\begin{aligned} v_t \;=\; & \kappa\left[(v+\overline{\sigma})\, y_t - (v+1) a_t\right] \\ & + \frac{1}{2}\kappa\left[(1+v)^2 (y_t - a_t)^2 - (1-\overline{\sigma})^2 y_t^2\right] - \frac{1}{2}\kappa\left[\overline{\sigma}\frac{\psi}{1-\psi} y_t^2\right] \\ & + \frac{1}{2}\varepsilon\pi_t^2 + \kappa v \widehat{\Delta}_t + \beta E_t v_{t+1} + O\left(\|\xi_t\|^3\right) \end{aligned} \tag{C.13}$$

Iterating forward (C.13), the Phillips curve can be expressed as the discounted infinite sum:

$$v_t = \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left\{ \begin{array}{c} \kappa\left[(v+\overline{\sigma})\, y_t - (v+1) a_t\right] \\ +\frac{1}{2}\kappa\left[(1+v)^2 (y_t - a_t)^2 - (1-\overline{\sigma})^2 y_t^2\right] - \frac{1}{2}\kappa\left[\overline{\sigma}\frac{\psi}{1-\psi} y_t^2\right] \\ +\frac{1}{2}\varepsilon\pi_t^2 + \kappa v \widehat{\Delta}_t \end{array} \right\} + O\left(\|\xi_t\|^3\right) \tag{C.14}$$

Replace equation ($C.8$), the discounted infinite sum of $\widehat{\Delta}_t$, then v_t becomes:

$$\begin{aligned} v_t &= \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left\{ \begin{array}{c} \kappa \left[(v+\overline{\sigma})\, y_t - (v+1) a_t \right] \\ +\frac{1}{2}\kappa \left[(1+v)^2 (y_t - a_t)^2 - (1-\overline{\sigma})^2 y_t^2 \right] - \frac{1}{2}\kappa \left[\overline{\sigma} \frac{\psi}{1-\psi} y_t^2 \right] \\ +\frac{1}{2}\varepsilon (1+v)\, \pi_t^2 \end{array} \right\} \\ &\quad + \frac{\kappa \chi v \theta}{1-\beta\theta} \widehat{\Delta}_{t-1} + O\left(\|\xi_t\|^3 \right) \end{aligned} \tag{C.15}$$

Assuming that we depart from an initial state where the price dispersion is small, that is $\widehat{\Delta}_{t-1} \simeq 0$ up to second order, then equation (C.15) can be expressed recursively as[16]:

$$\begin{aligned} v_t &= \kappa \left[(v+\overline{\sigma})\, y_t - (v+1) a_t \right] \\ &\quad + \frac{1}{2}\kappa \left[(1+v)^2 (y_t - a_t)^2 - (1-\overline{\sigma})^2 y_t^2 \right] - \frac{1}{2}\kappa \left[\overline{\sigma} \frac{\psi}{1-\psi} y_t^2 \right] \\ &\quad + \frac{1}{2}\varepsilon (1+v)\, \pi_t^2 + \beta E_t v_{t+1} + O\left(\|\xi_t\|^3 \right) \end{aligned} \tag{C.16}$$

which is equation (4-23) in the main text.

C.2 The perturbation method

The perturbation method, developed originally by Judd (1998) and implemented to monetary policy by Uribe and Schmitt-Grohe (2004), and Collard and Julliard (2001), consists in obtaining the coefficients of a taylor expansion of the solution of the model near the steady state using a system of equations that come from the differentiation of the equilibrium conditions of the model. For instance, given a set x of endogenous variables $x \in R^m$, one state state variable s, and a system of equations m equations F, that can be expressed in the following form: $F(x, s) = 0$. The perturbation method consist in solving the policy functions $x(s)$ for a system of the form:

$$F(x(s), s) = 0$$

[16]The assumption that the initial price dispersion is small make the analysis analytically tractable, without changing qualitatively the results.

with a taylor expansion around the steady state, i.e. $x(0) = 0$. In the case of only one state variable, the taylor expansion has the following form:

$$x(s) \simeq \sum_{n=0}^{N} \frac{s^n}{n!} x^{(n)}(0) = x(0) + s x'(0) + \frac{s^2}{2} x''(0) + \ldots \frac{s^N}{N!} x^{(N)\prime}(0)$$

For this, we need to solve for $x(0), x'(0), \ldots, x^{(N)}(0)$ for an $N - order$ approximation, around the steady state $s = 0$. The methodology consist in taking successive derivatives to the system of m equations F and evaluate it around the steady state. Then we need to solve for the m coefficients $x^{(n)}(0)$ for each order of approximation $n = 0..N$, that is:

$$\begin{aligned}
0 &= F_1 x'(s) + F_2 \mapsto x'(0) = -\left[F_1(0,0)\right]^{-1} F_2(0,0) \\
0 &= F_{11} x'(s) + F_1 x''(s) + F_{21} x'(s) + F_{22} \\
&\mapsto x''(0) = -\left[F_1(0,0)\right]^{-1} \left[F_{22}(0,0) + \left(F_{12}(0,0) + F_{11}(0,0)\right) x'(0)\right]
\end{aligned}$$

In our model we have two endogenous variables $x = [y, \pi]$, two state variables $s = [a, e]$ and a system of two non-linear equations, the IS and the Phillips curve and two auxiliary variables v_t and z_t. Our second order approximation to the solution of the model is given by:

$$y(a,e) = b + b_a a + b_e e + b_{ae} ae + \frac{1}{2} b_{aa} a^2 + \frac{1}{2} b_{ee} e^2 + \mathcal{O}\left(\|s\|^3\right) \quad \text{(C.17)}$$

$$\pi(a,e) = d + d_a a + d_e e + d_{ae} ae + \frac{1}{2} d_{aa} a^2 + \frac{1}{2} d_{ee} e^2 + \mathcal{O}\left(\|s\|^3\right) \quad \text{(C.18)}$$

C.3 The first and second order solution

We replace the policy functions (C.17) and (C.18) in the IS (C.3), PC (C.4) in the definitions for the auxiliary variables v_t (C.5) and z_t (C.6). We have a recursive system for the policy functions.

To solve for the linear coefficients, take the derivative to the equations of the system with respect to the shock $j = \{a, e\}$. We obtain a system of two equations, one for the IS and the other for the PC: for $j = a$

$$\begin{aligned} \left[\overline{\sigma}\left(1-\rho_a\right)+\phi_y\right] b_a &= -\left(\phi_\pi - \rho_a\right) d_a \\ \left(1-\beta\rho_a\right) d_a &= \kappa\left(\overline{\sigma}+v\right) b_a - \kappa\left(v+1\right) \end{aligned} \tag{C.19}$$

for $j = e$

$$\begin{aligned} \left[\overline{\sigma}\left(1-\rho_e\right)+\phi_y\right] b_e &= -\left(\phi_\pi - \rho_e\right) d_e + 1 \\ \left(1-\beta\rho_e\right) d_e &= \kappa\left(\overline{\sigma}+v\right) b_e \end{aligned} \tag{C.20}$$

Similarly, we take derivatives with respect to i and $j \in \{a, e\}$ and obtain a system of two equations for the 2 unknowns b_{ij} and d_{ij}

$$\left[\overline{\sigma}\left(1-\rho_i\right)\left(1-\rho_j\right)+\phi_y\right] b_{ij} = -\left(\phi_\pi - \rho_i\rho_j\right) d_{ij} + \frac{\psi}{1-\psi}\overline{\sigma} b_i b_j \tag{C.21}$$

$$d_{ij} = \kappa\left(\overline{\sigma}+v\right) b_{ij} + \frac{\psi}{1-\psi}\kappa\overline{\sigma} b_i b_j + \lambda_{ij} b_i b_j \tag{C.22}$$

where

$$\lambda_{ij} = \kappa\left[\left(1+v\right)^2 - \left(1-\overline{\sigma}\right)^2\right] + \left(\varepsilon\left(v+1\right) - \left(\frac{\varepsilon-1}{1-\theta}+\varepsilon\right)\left(1-\beta\rho_a\rho_e\right)\right)\frac{d_i d_j}{b_i b_j} + \chi_{ij}$$

for

$$\begin{aligned} \chi_{ae} &\equiv -\frac{1}{2}\left(1-\theta\beta\right)\left(1-\beta\rho_a\rho_e\right)\left(\frac{d_a g_e + d_e g_a}{b_a b_e}\right) \\ \chi_{ee} &\equiv -\left(1-\theta\beta\right)\left(1-\beta\rho_e^2\right)\left(\frac{d_e g_e}{b_e^2}\right) - \kappa\left(1+v\right)^2\frac{1}{b_e} \end{aligned}$$

and

$$\begin{aligned} g_a &\equiv \frac{1}{1-\theta\beta\rho_a}\left(\left(v+2+\overline{\sigma}\right) b_a + \frac{2\varepsilon-1}{1-\theta\beta}\theta\beta\rho_a d_a - v - 1\right) \\ g_e &\equiv \frac{1}{1-\theta\beta\rho_e}\left(\left(v+2+\overline{\sigma}\right) b_e + \frac{2\varepsilon-1}{1-\theta\beta}\theta\beta\rho_e d_e\right) \end{aligned}$$

where g_a and g_e are the coefficients of the policy function for z_t

We make the following change of variable to express the system (C.21 and C.22)in terms of elasticities:

$$\begin{aligned}\sigma_y^i &\equiv \frac{b_{ie}}{b_e}\frac{1}{b_i}\\ \sigma_\pi^i &\equiv \frac{d_{ie}}{d_e}\frac{1}{b_i}\end{aligned}$$

The system of equations can be expressed as:

$$\begin{aligned}\left[\overline{\sigma}\left(1-\rho_i\right)\left(1-\rho_e\right)+\phi_y\right]\sigma_y^i b_i b_e &= -\left(\phi_\pi-\rho_i\rho_e\right)\sigma_\pi^i b_i d_e+\frac{\psi}{1-\psi}\overline{\sigma}b_i b_e \qquad (C.23)\\ \sigma_\pi^i b_i d_e &= \kappa\left(\overline{\sigma}+v\right)\sigma_y^i b_i b_e+\frac{\psi}{1-\psi}\kappa\overline{\sigma}b_i b_e+\lambda_{ie}b_i b_e\end{aligned}$$

Divide by $b_i b_e$

$$\begin{aligned}\left[\overline{\sigma}\left(1-\rho_i\right)\left(1-\rho_e\right)+\phi_y\right]\sigma_y^i &= -\left(\phi_\pi-\rho_i\rho_e\right)\sigma_\pi^i\frac{d_e}{b_e}+\frac{\psi}{1-\psi}\overline{\sigma}\\ \sigma_\pi^i &= \kappa\left(\overline{\sigma}+v\right)\sigma_y^i\frac{b_e}{d_e}+\frac{\psi}{1-\psi}\kappa\overline{\sigma}\frac{b_e}{d_e}+\lambda_{ie}\frac{b_e}{d_e}\end{aligned}$$

and make use of the relationship from the Phillips curve: $d_e=\frac{\kappa(\overline{\sigma}+v)}{1-\beta\rho_e}b_e$.

$$\begin{aligned}\left[\overline{\sigma}\left(1-\rho_i\right)\left(1-\rho_e\right)+\phi_y\right]\sigma_y^i &= -\left(\phi_\pi-\rho_i\rho_e\right)\sigma_\pi^i\frac{\kappa\left(\overline{\sigma}+v\right)}{1-\beta\rho_e}+\frac{\psi}{1-\psi}\overline{\sigma} \qquad (C.24)\\ \sigma_\pi^i &= \left(1-\beta\rho_e\right)\sigma_y^i+\frac{\psi}{1-\psi}\overline{\sigma}\frac{1-\beta\rho_e}{\left(\overline{\sigma}+v\right)}+\lambda_{ie}\frac{1-\beta\rho_e}{\kappa\left(\overline{\sigma}+v\right)}\end{aligned}$$

For the especial case that the shocks are uncorrelated, that is $\rho_a=\rho_e=0$. This system can be expressed as:

$$\begin{aligned}\left[\overline{\sigma}+\phi_y\right]\sigma_y^i &= -\phi_\pi\kappa\left(\overline{\sigma}+v\right)\sigma_\pi^i+\frac{\psi}{1-\psi}\overline{\sigma} \qquad (C.25)\\ \sigma_\pi^i &= \sigma_y^i+\frac{\psi}{1-\psi}\frac{\overline{\sigma}}{\left(\overline{\sigma}+v\right)}+\lambda_{ie}\frac{1}{\kappa\left(\overline{\sigma}+v\right)}\end{aligned}$$

which is the system (4-26) and (4-27) in the main text

CHAPTER 5

MONETARY POLICY COMMITTEES AND INTEREST RATE SMOOTHING

5.1 Introduction

An existing puzzle in the optimal monetary policy literature is why, in practice, central banks change the interest rate less often than the theory predicts. This feature is called interest rate smoothing and it is well documented for many central banks[1]. For instance, Lowes and Ellis (1997), in a study for different countries, listed as the common patterns in official interest rates set by central banks: they change rarely, they are made in a sequence of steps in the same direction, and they are left unchanged for relatively long periods of time before moving in the opposite direction.

Regarding interest rates reaction functions, Taylor (1993) proposed a policy rule for the interest rate, modelled by a linear combination of output gap and inflation, as a rough description of the monetary policy for the USA during the chairmanship of Alan Greenspan. On the other hand, some authors, such as Judd and Rudebusch (1998), Clarida and others (1999) and Orphanides (2003), have pointed out that, empirically, the monetary policy rule that best captures the data has the following form:

$$i_t = (1-\rho)\left(\bar{i} + \phi_\pi \pi_t + \phi_x x_t\right) + \rho i_{t-1} + \epsilon_t$$

[1]See Sack and Wieland (2000) for a discussion on interest rate smoothing.

where $\bar{i}$ is a constant, interpretable as the steady state nominal interest rate. π_t and x_t are the inflation and output gap, respectively. $\rho \in [0, 1]$ is a parameter that reflects the degree of lagged dependence in the interest rate. In these estimations, interest rate smoothing is present in two ways. Firstly, the estimated coefficients ϕ_π and ϕ_x are typically smaller than the optimal rule would suggest; and secondly, the partial adjustment to movements in π_t and x_t is reflected by the presence of i_{t-1}. In other words, the empirical form of the official interest rate is a weighted average of some desired value that depends on the state of the economy and on the lagged interest rate. Also, the estimates of ρ are on the order of 0.7 or 0.9 for quarterly data, which indicates a very slow adjustment in practice.

The existing literature that explains interest rate smoothing has three branches. The first explanation relies on the effects of uncertainty on the policy decisions. Uncertainty about the structure and the state of the economy can lead to lower response of the interest rate to shocks. An early work by William Brainard (1967) showed that uncertainty on the parameters of the economy's equations reduces policy activism, which means a more cautious response to shocks. In more recent papers the actions taken by policymakers are those with outcomes that they are confident about. For that reason, they delay action until they collect enough information about a shock. On the other hand, Clarida and others (1999) argue that model uncertainty may help to explain the fairly low variability of interest rate in the data. However, they consider that it does not capture the feature of strong lagged dependence in the interest rate.

A second explanation, given by Rotemberg and Woodford (1997) can help to explain the lagged dependence feature. Their argument is based on the effects of the short-term interest rates on the aggregate demand through the effect on long-term interest rates. Being long-term interest rates those that affect aggregate demand. Lagged dependence in short-term interest rates allows the central bank to manipulate long-term rates with more modest movements in the short-term rate than otherwise needed. Therefore, the central bank may care about avoiding excessive volatility in the short-term interest rate in pursuing its stabilisation goal. In the same context, Goodhart (1999) and Woodford (1999) argue that inertial monetary policy makes the future path of short-term interest rates more predictable and increases policy effectiveness. These authors provide a reasonable explanation for lagged dependence on interest rate. However, it is still to be seen if this story can account as well for the empirically modest response of the short-term rate to inflation and output gap.

A third explanation is based on financial markets stability. It considers that large movements in the interest rate are avoided because they destabilise financial markets (Goodfriend 1991). Therefore, by changing policy rates gradually central banks can reduce the likelihood that a change in policy triggers excessive reactions. In a forward-looking environment with rational expectations, concern about the variance of the interest rate induces interest rate smoothing.

Among other explanations, Clarida and others (1999) argue that disagreement among policy makers is another explanation for slow adjustment rates. However, they consider that this story has not yet been well developed and this is where we want to provide an alternative framework. The current literature on interest rate smoothing, as well as most of the literature on optimal monetary policy, relies on the assumption that policy decisions are taken by a single policy maker that maximises some measure of social welfare. However, in real life this is not the case, because in practice monetary policy decisions are taken mostly collectively, in committees.

This chapter intends to explain interest rate smoothing giving more structure to the decision-making process, in which policy decisions are made through a Monetary Policy Committee (MPC), whose members have different preferences. This chapter helps to explain interest rate smoothing from a political economy point of view, in which members of an MPC have a bargaining problem on the interest rate. In this framework, the political equilibrium interest rate is a function of the lagged interest rate and expected inflation. We have found that when the difference between expected inflation and its long run value is relatively high, the interest rate reacts as the optimal monetary policy predicts. However, the smaller the difference, the interest rate reacts less than the optimal or does not react at all.

The literature on Monetary Policy Committees is fairly new and it has focused mainly on how the structure of an MPC can affect the policy decisions. It has two branches, the first branch considers the case of members with different preferences and how this affects expectations formation and policy outcomes[2]. The second branch of the literature of MPCs has focused on the differences in skills among members and how it interacts with different voting rules[3].

Considering the existing literature on MPCs, this research is closer to Riboni's (2003). In Riboni's model, a committee with heterogeneous preferences can work as a substitute of a commitment technology when there is dynamic bargaining among members. In this model, the member in charge of setting the agenda to vote is less willing to deviate from the optimal time-consistent inflation level, because it will reduce her negotiation power next period. This model has a voting mechanism similar to ours, in which there exists an agenda-setter that every period submits a policy to vote, but it differs from us in the type of heterogeneity. Riboni works on heterogeneity in inflation goals, whilst we work on heterogeneity in the relative weights in the preferences between output gap and inflation among members. Also,

[2]Aksoy, De Grauwe and Dewachter (2002) and Von Hagen and Süppel (1994) have worked on the case of a monetary union in which, because of nationality, the members have different goals regarding the level of inflation and output gap. Riboni (2003) and Silbert (2004) show that in a committee with members with different inflation targets, the policymaker's capacity to bring about surprise inflation is reduced. Waller (1989) showed that assigning the task of conducting monetary policy to a committee with staggered membership enhances continuity in expectations formation and reduces inflation.

[3]Gersbach and Hahn (2001) showed that less skilled policymakers in general want to abstain from voting. If a voting record is published, they try to mimic their more skilful colleagues; therefore voting records can be undesirable. Karotkin (1996) analysed the performance of different voting rules in committees in which individual skills differ. Berk and Bierut (2003) introduce the effects of learning on the performance of voting rules. In a new strand of the literature, Gerlach-Kristen (2003b, 2004, 2006a) studies the effects of uncertainty about the state of the economy when the members have the same skills.

Riboni's model is dynamic from a political economy point of view, but its economic structure is static since there are no shocks that affect the economy differently every period. Therefore, it would be difficult to disentangle whether the results of a reduction in inflation come from an effective reduction of the time-inconsistency problem or just that the policy decision is sluggish in equilibrium, that is interest rate smoothing.

Our model relaxes the traditional assumption that monetary policy decisions are made by a single policy maker and introduces strategic decisions in an MPC with heterogeneous preferences. This approach is new in the interest rate smoothing literature and helps to explain this problem through a different channel, from a political economy point of view. It also provides a theoretical framework on how disagreement among policymakers can slow the adjustment on interest rates and on adjustment costs or "menu costs" in interest rate decisions.

Moreover, this model can also reproduce altogether both features of interest rate smoothing, which are the modest response of the interest rate to inflation and output gap and the lagged dependence. These are features that other models fail to reproduce at the same time. In our model, when lagged interest rates are close to the current period optimum, they do not change because it is costly to have an agreement among members. Only when the size of the shocks is such that it is sub-optimal to keep the interest rate, it will be changed. However, in other cases the change will be below the optimal, in the exact size necessary to obtain a coalition for passing the new interest rate, or equal to the optimal, when the expected inflation is high enough that make the status quo sub-optimal.

The structure of this chapter is as follows. The second section presents the benchmark model in the spirit of the New Keynesian monetary economics. The third section introduces the policy decision problem in an MPC with members with heterogeneous preferences and solves the political economy problem. The fourth section presents some stylised facts on the voting process for some MPCs in relation with its effects on interest rate adjustments. The last section concludes.

5.2 Benchmark Model

During the past years, it has been a broad use of theoretical models of monetary policy based on the techniques of general equilibrium theory. On this literature, the New Keynesian approach departs from the real business cycle theory with the explicit incorporation of nominal price rigidities. These models are fairly simple and have some qualitative core features that are suitable to evaluate monetary policy. In order of being able to compare our results with the existing literature, we depart from a baseline framework for the analysis of monetary policy based on a New Keynesian perspective. In this section we develop our benchmark model with a single policymaker, which follows closely Clarida, Gali and Gertler (1999) and Woodford (2003). In the next section we will analyse the policy problem under a Monetary Policy Committee with members with heterogeneous preferences in which the interest rate is determined in a political equilibrium.

We assume a closed economy; all the variables are expressed as log deviations from the steady state. The economic equilibrium in this economy is given by the intersection of the aggregate demand (AD) and the aggregate supply (AS). As in any standard macroeconomic model, the aggregate demand is determined by "IS" and "LM" equilibrium. In our model the "IS" relates the output gap inversely to the real interest rate and the "LM" is represented by the nominal interest rate chosen by the central bank as policy instrument. The aggregate supply (AS) is represented by the Phillips curve, which relates the inflation positively to the output gap. These two equations can be obtained from a standard general equilibrium model with price frictions. We can summarise the economy by two equations, the

"IS" and the "AS", that have the following form[4]:

$$x_t = -\varphi \left[i_t - E_t \pi_{t+1} - r_t^n \right] + E_t x_{t+1} \tag{IS}$$

$$\pi_t = \lambda x_t + \beta E_t \pi_{t+1} + u_t \tag{AS}$$

where π_t and x_t are the period t inflation and output gap i_t and r_t^n are the nominal and the natural interest rate [5]. All the variables are expressed as log-deviations from their long-run level. According to the IS, lower real interest rate and higher future output increases current output. On the other hand, in the Phillips curve the output gap variable captures movements in marginal costs associated with changes in excess demand and the shock u_t captures anything else that might affect expected marginal costs. u_t is usually named as a "cost push" shock and it is related to supply shocks that do not affect the potential output. Moreover, u_t gives a trade-off between inflation and output gap stabilisation. We assume the disturbance term u_t follow:

$$u_t = \rho u_{t-1} + \varepsilon_t$$

where $0 \leq \rho \leq 1$ and ε_t is an i.i.d. random variables with zero mean and variance σ_u^2.

We assume, following much of the literature on optimal monetary policy, that the policy objective is a quadratic function of the target variables x_t and π_t and takes the form of:

$$W = -\frac{1}{2} E_t \left\{ \sum_{s=0}^{\infty} \beta^s \left[\alpha x_{t+s}^2 + \pi_{t+s}^2 \right] \right\} \tag{5-1}$$

[4]The IS equation can obtained from log-linearising the Euler equation from the household's optimal consumption decisions. The Phillips curve can be obtained from aggregating the log-linear approximation of the individual firm pricing decisions. The price friction in this model comes from staggered nominal price setting in the essence Taylor (1979). The most common formulation of staggered price setting in the literature comes from Calvo (1983), in which he assumes that in any given period a firm has fixed probability of keeping its price fixed during the period.

[5]The natural interest rate is defined as the equilibrium real rate of return in the case of fully flexible prices.

where the parameter α is the relative weight on output deviations. This loss function takes potential output and zero inflation rate as the targets for the deviations of output and inflation from the deterministic long-run trend. As we discuss in Chapter 3, during the past years have been some works on deriving the policy problem from first principles. Rotemberg and Woodford (1997) and Woodford (2003) show that an objective function of the form of (5-1) can be obtained as a quadratic approximation of the utility-based welfare[6]. Though, this works rely on some assumptions, like representative agent economy, which can be a restrictive representation of how the preferences over inflation and output gap really are. However, they are useful to establish the policy problem from the welfare criterion. Moreover, Woodford (2003) shows that the weight α is a function of the primitive parameters of the model, such as the slope of the Phillips curve and the degree of monopolistic competition.

5.2.1 The Policy Problem for a Single Policymaker

In this part we assume that the policy decisions are taken by a single policymaker. We further assume the policymaker is unable to commit their future policies; therefore he cannot change the private sector expectations with policy announcements over future policy decisions. In each period the policy maker chooses the policy instrument to maximise the welfare function subject to the IS and the AS. The policymaker's problem can be summarised by maximising the Bellman equation:

$$\max_{\{x_t,\pi_t\}} W_t = -\frac{1}{2}\left[\alpha x_t^2 + \pi_t^2\right] + \beta E_t \overline{W}_{t+1}$$

subject to

$$\begin{aligned} x_t &= -\varphi\left[i_t - E_t\pi_{t+1} - r_t^n\right] + E_t x_{t+1} \\ \pi_t &= \lambda x_t + \beta E_t \pi_{t+1} + u_t \end{aligned}$$

[6]In these works the output gap is included in the welfare function, because the volatility of income reduces welfare. On the other hand, inflation is included because, as firms face uncertainty on the time when they are going to be able to adjust their price, higher aggregate inflation increases the volatility of the individual price and income.

where $E_t\overline{W}_{t+1}$ is taken as given by the Policymaker, since her cannot credibly manipulate beliefs in the absence of commitment. Moreover, in order to obtain tractability on the problem, we focus on the optimum within a simple family of policy rules, which is a linear function of expected inflation.

Proposition 5.1. *The optimal feedback policy for the interest rate, within the family rules mentioned above without commitment, is :*

$$i_t = r_t^n + \phi_\pi E_t \pi_{t+1}$$

where $\phi_\pi = 1 + \frac{(1-\rho)\lambda}{\rho\varphi\alpha} > 1$.

See appendix D for a derivation. According to this policy rule, the nominal interest rate should rise in response to a rise in expected inflation, and that increase should be high enough to increase real rates. In other words, in the optimal rule for the nominal interest rate, the coefficient on expected inflation should exceed unity (that is $\phi_\pi > 1$)[7].

Moreover, in this policy rule, the interest rate is adjusted to perfectly offset shocks that affect the natural interest rate, but to partially offset cost-push shocks (that is $\partial\pi_t/\partial u_t > 0$).. Therefore, when "cost-push" shocks are present, the optimal policy rule incorporates convergence of inflation to its target over time. Also, the relative weight between output and inflation stabilisation is given by the parameter alpha.

[7]In contrast, in the case of a single policymaker that can commit to a policy rule, and if the policy rule is linear on the shocks, the optimal feedback policy rule has the following form:

$$i_t = r_t^n + \gamma_\pi^c E_t \pi_{t+1}$$

where $\gamma_\pi^c = 1 + \frac{(1-\rho)\lambda}{\rho\varphi\alpha^c} \geq \gamma_\pi$ because $\alpha^c = \alpha(1-\beta\rho) < \alpha$. See Clarida and others (1999) for a derivation. Therefore, commitment increases the effectiveness of monetary policy, reducing expected inflation.

5.3 The Policy Problem in a Monetary Policy Committee

The traditional approach on the optimal monetary policy literature relies on the assumption that decisions are taken by a single policymaker. However, in real-life this is not the case, because in practice monetary policy decisions are taken mostly collectively in a committee. In this section we introduce a Monetary Policy Committee (MPC) in charge of the monetary policy decisions. Also, we assume that the members in the MPC differ in their preferences. More precisely, they have different relative weight between output and inflation stabilisation in their policy objectives [8].

We assume the MPC has three members [9], $j = \{1, 2, 3\}$, each one with different preference parameters: $\alpha^1 < \alpha^2 < \alpha^3$. The first (third) member is the most (least) conservative, while the second has moderate preferences over inflation and output gap. Therefore, the aggressiveness in the response of the interest rate to expected inflation decreases with the index of each member.

5.3.1 Bargaining problem

We assume the policy decision is a bargaining problem in the spirit of Baron and Ferejohn (1989), which is closer to how the interest rate is decided in practise by an MPC. In every period the interest rate is determined by the following game: one member, the agenda setter, proposes a new interest rate. Then, the members of the MPC vote. We assume that it is necessary a simple majority to have the new interest rate approved. Then, the new interest rate is implemented if at least two

[8] We work on the heterogeneity in the weights but not on heterogeneity on the targets. Heterogeneity on targets gives different inflation bias among members, whilst the degree of adjustment of the interest rate to shocks is the same for every member. In other words, with heterogeneity on targets the members only differ on the level of the interest rate, and this difference is independent of the type and size of shocks

[9] We assume a committee of three members because this is the minimum odd number of members in order to have a conflict.

out of three members of the MPC approve it, otherwise the last period interest rate is maintained.

In this voting system the status quo is given by last period interest rate, it means that this is the default interest rate if the members do not accept the new interest rate proposed by the agenda setter. Moreover, because the agenda setter makes a take-it-or-leave-it proposal, she has a first mover advantage, which in this setup gives her more bargaining power than to the other MPC's members. Therefore, the agenda setter can strategically set to vote an interest rate that maximises her own utility constrained by the reaction of other members. Denote the identity of the agenda setter by A, her optimisation problem becomes:

$$\max_{\{i_t\}} W_t^A = -\frac{1}{2}\left[\alpha^A x_t^2 + \pi_t^2\right] + \beta E_t \overline{W_{t+1}^A} \tag{5-2}$$

subject to

$$\begin{aligned} x_t &= -\varphi\left[i_t - E_t\pi_{t+1} - r_t^n\right] + E_t x_{t+1} \\ \pi_t &= \lambda x_t + \beta E_t \pi_{t+1} + u_t \end{aligned} \tag{5-3}$$

and to

$$\begin{aligned} W_t^A(i_t) &\geq W_t^A(i_{t-1}) \\ W_t^j(i_t) &\geq W_t^j(i_{t-1}) \text{ for at least one } j \neq A \end{aligned} \tag{5-4}$$

The problem for the agenda setter is similar to the benchmark model, but with an extra constraint. Within an MPC, the agenda setter has to choose an interest rate such that also obtains the majority needed for approval. This problem includes some participation constraints on the behaviour of the other members. According to these participation constraints, the new interest rate should give at least the same utility than the status quo for the agenda setter and at least one additional member.

Since MPC members have different preferences over output and inflation stabilisation, there is a conflict on the size of the adjustment of the interest rate to "cost-push" shocks. For this reason, the political economy solution will depend on the size and direction of the shocks. When shocks affecting the natural rate are big relatively to "cost-push" shocks (u_t), there is no conflict among members since their preferred interest rates are similar. However, in the opposite case, when the "cost-push" shock are big relatively to shocks affecting the natural rate, the MPC's members have different preferences on the policy instrument. In that case, the political economy solution will depend on the state variable i_{t-1} and the shocks. For simplicity, in order to describe easily the mechanism, we will focus in the case where there are no shocks affecting the natural rate, that is we assume $r_t^n = 0$.

5.3.2 MPC members' reaction functions

Since MPC's members cannot credibly manipulate beliefs in the absence of commitment, they take private sector expectations as given when solving their optimisation problem[10]. Therefore, as in the case of section 5.2, the private sector forms beliefs rationally conditional on the MPC's reaction function. Given absence of commitment, member $j's$ preferences are given by

$$W_t^j = -\frac{1}{2}\left[\alpha_j x_t^2 + \pi_t^2\right] + \beta E_t \overline{W}_{t+1}^j$$

where $E_t \overline{W}_{t+1}^j$ are taken as given. Therefore, similar to the case of the previous section, her preferences are maximised by i_t^{j*}, *the member-j optimal rate:*

$$i_t^{j*} = \phi_\pi^j E_t \pi_{t+1}$$

where $\phi_\pi^j = 1 + \frac{(1-\rho)\lambda}{\rho\varphi\alpha^j}$[11]. This optimal rate is similar to the rate in the single policymaker case for $\alpha = \alpha^j$. Moreover, given the ordering of the preference parameter

[10]This assumption also allow us to simplify greatly the problem, since expectations are taken as fixed by the MPC members, the political equilibrium doesn't depend on the rational expectations economic equilibrium. If this were not the case, the fixed point problem would be more difficult to solve and the uniqueness of the equilibrium is not guarantied.

[11]The member-j optimal rate without commitment has the following form: $i_t^{j*} = \phi_\pi^j E_t \pi_{t+1} - \frac{1}{\psi}\frac{\lambda}{\alpha}\frac{1}{\rho}\left(E_t \pi_{t+1} - \rho \pi_t\right)$. However, to get the simplest result as possible, we have assumed the second element is zero, as in the single-policymaker case when expected inflation is a linear combination of the shocks. The results don't change if we include the more general policy rule, but notation gets more complicated.

α^j, the responsiveness of the interest rate to expected inflation diminishes with the index j : that is $\phi_\pi^3 < \phi_\pi^2 < \phi_\pi^1$. Then, the more conservative a MPC member is, the stronger she prefers the interest rate to react to expected inflation.

Conditional on the shocks, the welfare function for every MPC member is strictly concave in the interest rate, which is maximised at the *member-j optimal rate* $i_t = i_t^{j*}$. The concavity comes from the quadratic preferences. Because of this concavity it is possible to define $\overline{i_t^j}$, *the member-j participation rate*, the interest rate that would make member j indifferent between this rate and the status quo interest rate (i_{t-1}):

Proposition 5.2. *Given last period interest rate, i_{t-1}, member j will be indifferent between $\overline{i_t^j}$ and i_{t-1} for*

$$\overline{i_t^j} = 2i_t^{j*} - i_{t-1}$$

See proof in appendix D. The member-j participation rate $(\overline{i_t^j})$ gives to her the same utility than last period rate, that is $W_t^j\left(\overline{i_t^j}\right) - W_t^j(i_{t-1}) = 0$. Figure 5.1 shows the preferences over the interest rate for member j. As we mentioned before, the welfare function is concave on the interest rate and it is maximised at the member-j optimal rate, i_t^{j*}. The graph shows a case where the last period interest rate is lower than the optimal rate (that is $i_{t-1} < i_t^{j*}$). According to this case, the participation rate is higher than the optimal rate. Then, any rate between last period's and the participation rate will give her higher utility than the status quo. That means that member-j will be willing to accept a rate different than the optimal rate in order to be better off than the status quo. We can also generalise the opposite case: when last period's rate is on the right of member-j optimal rate, the participation rate will be on the left of last period's rate and any rate in between will give her higher utility than the status quo.

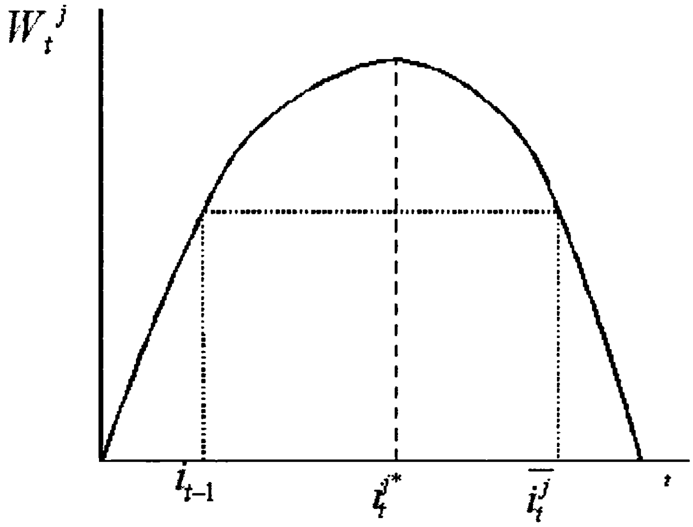

Figure 5.1: Welfare function

5.3.3 The policy problem

The agenda setter has a first mover advantage, because she can influence other member's decisions through the interest rate she sends to vote. In figure 5.2 we show one example of how she can influence the vote of a member j. Let's assume the status quo interest rate (i_{t-1}) is below the agenda setter's optimal rate (i_t^{A*}). The panel on the left (right) shows a case when the agenda setter's optimal rate is lower (higher) than member-j participation rate. In this example the initial interest rate is low and there is an increase in expected inflation (most likely because of a "cost-push" shock). Both members j and A want an increase in the policy rate, but A prefers a higher increase than j. If the agenda setter's optimal rate is not too high, as in the case on the left, member-j will accept it. However, if it is too high, as in the case on the right, it violates member-j participation constraint and the best the agenda setter can do is to set $\overline{i_t^j}$ that makes the constraint binding.

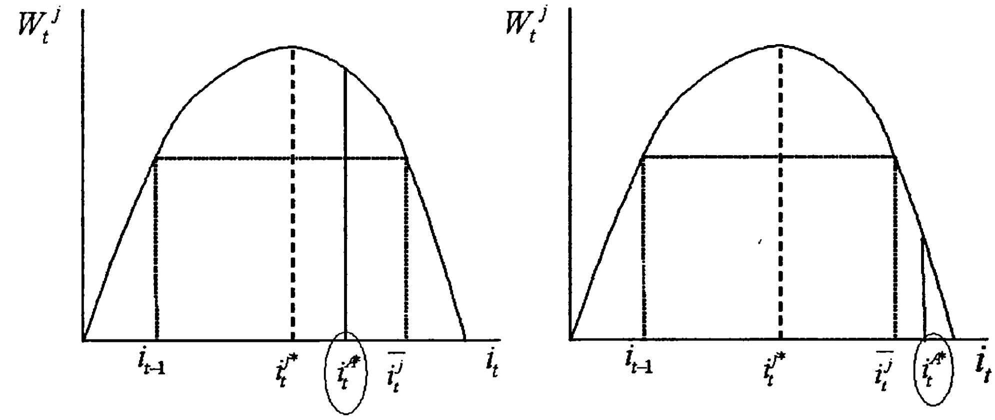

Figure 5.2: a) Policy problem when: $i^{A*} < \overline{i^j}$. b) Policy problem when: $i^{A*} < \overline{i^j}$.

In this subsection we analyse the optimisation problem for the agenda setter and its implications for interest rate smoothing. We show that what matters for interest rate smoothing is the identity of the agenda setter, the degree of heterogeneity of preferences among members and the size of the shocks. In brief, we observe interest rate smoothing only when the agenda setter is either the first or the third member, and not when she is the second member. The following propositions summarise our results taking into account the identity of the agenda setter.

Proposition 5.3. *When the agenda setter is the member with median preferences, member 2, there is no interest rate smoothing*

The policy problem when the agenda setter is the second member satisfies the median voter theorem. In this case, she is always able to form a coalition with either the first or the third member, to support her most preferred rate. Therefore, there is not interest rate smoothing when the agenda setter is the member with median preferences.

Member 1 prefers a more active interest rate to reduce deviations of inflation around its long–run value, while member 3 prefers a less active policy to reduce deviations in output gap. The agenda setter tends to form a coalition with the first member when she needs to adjust the interest rate because of a new shock, for instance an increase in expected inflation. But as the shock vanishes, she tends to form a coalition with the third member to return the interest rate closer to its neutral level.

Therefore, coalitions in the MPC vary with the sign of expected inflation and the state variable i_{t-1}. When expected inflation is positive the agenda setter will look for a coalition with the more conservative member (member 1) if the initial interest rate is too low. However, if the initial interest rate is too high, she forms a coalition with the less conservative member (member 3). A similar analysis applies when expected inflation is negative. Also, when the size of the shocks is too high, both other members of the MPC agree with the agenda setter to change the interest rate as her wish.

Being the agenda setter the member with median preferences would prevent interest rate smoothing from a political economy point of view. However, this is not always the case, since often the most conservative member is appointed as the agenda setter. As Barro and Gordon (1993) have pointed out, assigning the monetary policy decision task to a conservative policy-maker can help to reduce the time inconsistency problem. However, if the decisions are taken in an MPC, it will also induce to interest rate smoothing. We show this in the next proposition:

Proposition 5.4. *When the agenda setter is the more conservative member, member 1,there is interest rate smoothing and the policy function is given by:*

$$
\begin{aligned}
i_t &= i_{t-1} && \textit{when } i_{t-1} \in [i_t^{2*}, i_t^{1*}] \textit{ or } i_{t-1} \in [i_t^{1*}, i_t^{2*}] \\
i_t &= \overline{i_t^2} && \textit{when } i_{t-1} \in [2i_t^{2*} - i_t^{1*}, i_t^{2*}] \textit{ or } i_{t-1} \in [i_t^{2*}, 2i_t^{2*} - i_t^{1*}] \\
i_t &= i_t^{1*} && \textit{otherwise}
\end{aligned}
$$

According to this proposition, the policy function can take three different functional forms. We present the thresholds defining the areas for those functions in terms of the optimal rates for MPC members. These optimal rates are function of expected inflation, which at the end also depend on the shocks and the policy decision. Therefore, the functional form of the policy function depends on last period interest rate and the shocks.[12]. In the first functional form the interest rate doesn't change, in the second one the participation constraint for member 2 is binding and in the third one the interest rate responds the same than member-1's optimal.

In the third area, there will be always a member that will prefer the agenda setter's optimal rate (i_t^{1*}) than the status quo rate (i_{t-1}). The agenda setter can obtain from the voting process the same interest rate that maximises her unconstrained utility, because the participation constraint is not binding for at least one other member. This is possible because she has the first moving advantage in the voting process and the change in expected inflation is such that makes the last period rate sub-optimal for the other members in comparison with i_t^{1*}.

In the second area, the agenda setter sets an interest rate such the participation constraint is binding for one of the members. She chooses to make binding the participation constraint for member 2 because she has the closest preferences to hers. In such area, the agenda setter cannot obtain from the voting process her preferred rate, but she can obtain a rate that maximises her utility subject to the participation constraint of member 2.

The first area defines an area of inaction, where the participation rate of any member does not satisfy the participation constraint of the agenda setter. That means, any rate that satisfy the participation constraint of any other member would make the agenda setter worst off than last period's rate. Then, the agenda setter by any means would prevent to have the interest rate changed. This area is defined when last period's rate is between the optimal rate for members 1 and 2. In this area, the gains from changing the rate are small in comparison to the cost of having

[12]In each row the thresholds on the left correspond to the case when expected inflation is positive ($E_t\pi_{t+1} > 0$), because in that case $i_t^{2*} < i_t^{1*}$. Similarly, the thresholds on the right are for the case when expected inflation is negative.

an agreement, so MPC members would prefer to leave it unchanged[13].

The interest rate reaction function has a piecewise form with 2 thresholds and 3 zones, and the form depends on the sign of future expected inflation. When expected inflation is positive (negative), the individual member's optimal rates are also positive (negative) and $i_t^{1*} > i_t^{2*} > i_t^{3*}$ ($i_t^{1*} > i_t^{2*} > i_t^{3*}$). The reaction function is summarised in figure 5.3. The graph on the left shows, given positive expected inflation, in the bold line the interest rate reaction function and in the light line the unconstrained optimal interest rate at i_t^{1*}. Similarly, the graphs on the right shows, also given positive expected inflation, in the bold the change in the interest rate in period t, and in the light one the optimal change in the unconstrained case, that is $\Delta i_t = i_t^{1*} - i_{t-1}$.

Both graphs show that there is interest rate smoothing when $i_{t-1} \in [2i_t^{2*} - i_t^{1*}, i_t^{1*}]$, because the interest rate change less than the optimum. In this area we have two degrees of interest rate smoothing: when $i_{t-1} \in [i_t^{2*}, i_t^{1*}]$ the interest rate does not change at all and when $[2i_t^{2*} - i_t^{1*}, i_t^{1*}]$ the interest rate changes less than the optimal. In the former case, negotiating in the MPC imposes a menu cost that makes not optimal to do small changes to the interest rate. In the latter, the agenda setter present to vote a change smaller than the optimal, to obtain a coalition with one of the other members, member 2.

In these graphs it is possible to see that the political economy solution can explain both features of interest rate smoothing: the modest response of the interest rate to inflation expectations and the lagged dependence. The reaction function has a smoothing area where the interest rate either has partial adjustment or it is completely fixed. Moreover, the type of smoothing depends on the difference

[13] In this area, the optimal strategy for the agenda setter is to set to vote an interest rate that violates both participation constraints of the other two members, then from the voting process the i_{t-1} is maintained. However, this strategic voting seems unrealistic, because the agenda setter could lose credibility requesting those policies rates. We could also think about a more complex game, where if none of the other members agree with the agenda setter to maintain the rate unchanged, they will have to start again a new meeting which involves a cost. Even a small cost to keep arguing, different from zero, can make MPC members to maintain the rate unchanged.

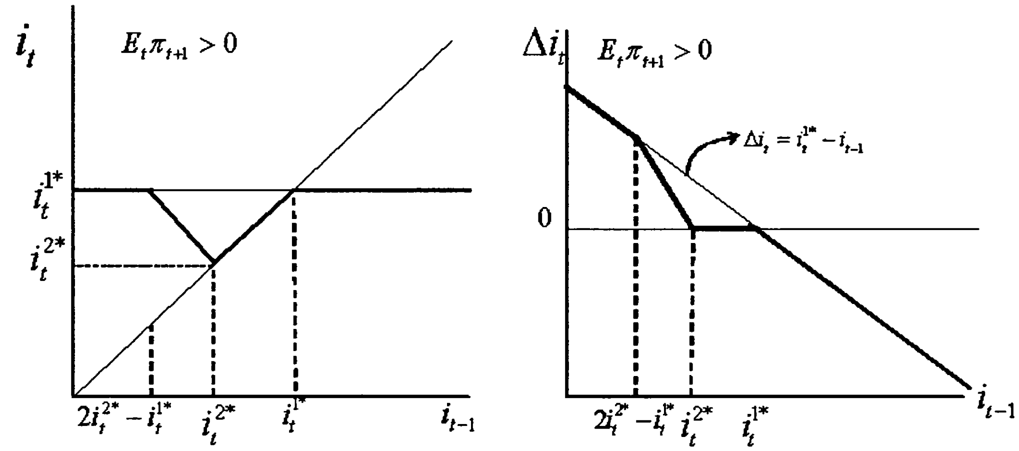

Figure 5.3: Interest rate reaction function: a) i_t, b) Δi_t

between the optimal rate and the lagged interest rate. When the difference between i_t^{1*} and i_{t-1} is small, the interest rate is fixed. However, when this difference takes intermediate values the interest rate change but less than the optimal. When this difference is big enough, the change will be equal to the optimal. Moreover, in the absence of cost-push shocks that give a trade-off between inflation and output volatility, the interest rate reaction function converges to i_t^{1*}, the optimal reaction function for the agenda setter. This is equal to the benchmark case with a single policymaker.

We can obtain a similar result in the opposite case, when the less conservative member is appointed as the agenda setter we have:

Proposition 5.5. *When the agenda setter is the less conservative member, member 3, there is interest rate smoothing and the policy function is given by:*

$$
\begin{array}{ll}
i_t = i_{t-1} & \textit{when } i_{t-1} \in [i_t^{3*}, i_t^{2*}] \textit{ or } i_{t-1} \in [i_t^{2*}, i_t^{3*}] \\
i_t = \overline{i_t^2} & \textit{when } i_{t-1} \in [i_t^{2*}, 2i_t^{2*} - i_t^{3*}] \textit{ or } i_{t-1} \in [2i_t^{2*} - i_t^{3*}, i_t^{2*}] \\
i_t = i_t^{3*} & \textit{otherwise}
\end{array}
$$

The proof follows the same steps as proposition 5.4. This policy function has features similar to the previous case. There is an area where the interest rate is completely fixed and another where there is partial adjustment. Also, the coalitions are made with member 2, who has preferences closer to the agenda setter. However, the direction of the smoothing is different. For example, for positive expected inflation, in the smoothing area the interest rate change more than the optimal for member 3.

In this model we have interest rate smoothing when the agenda setter is either the first or the third member, and the reaction function is non-linear on the lagged interest rate and expected inflation. An important issue in this model is to determine if this non-linear policy rule can guarantee the existence of a rational expectations equilibrium. The following proposition shows that the determination properties of the rational expectations equilibrium are satisfied.

Proposition 5.6. *A sufficient condition for the determinacy of a rational expectations equilibrium with the reaction functions described in propositions (5.4) and (5.5) is that* $\phi^1 < 1 + 2\frac{1+\beta}{\lambda\varphi}$.

The proof is in the appendix D. The intuition behind this is that, as the response in the reaction function to expected inflation is bounded between the optimal response for members 1 and 3. And also, since each of those optimal responses satisfy the conditions for the existence of an equilibrium, this also guarantees the existence of the equilibrium in the context of voting on a MPC. From the political economy equilibrium it can be some sluggishness on the response of the interest rate, but this response always will be high enough in order to control inflation.

5.4 Economic Equilibrium

In this section we solve for the rational expectations equilibrium of inflation and output gap, given the interest rate reaction function of proposition (5.4). However, since the reaction function is non-linear and the solution doesn't have a closed solution, we need to approximate it by a non-linear method.

5.4.1 Methodology

We obtain a numerical solution to the rational expectations problem using a collocation method, which allows us to obtain an approximate solution of the problem with a high degree of accuracy. The collocation method consists on finding a function that approximates the value of the policy functions of the problem at a finite number of specified points[14]. This sub-section describes the procedure we have used. The system of endogenous equations is the following:

$$
\begin{aligned}
x_t &= -\varphi\left[i_t - E_t\pi_{t+1}\right] + E_t x_{t+1} \\
\pi_t &= \lambda x_t + \beta E_t\pi_{t+1} + u_t \\
i_t &= f\left(E_t\pi_{t+1}, i_{t-1}\right)
\end{aligned}
$$

for the IS, the AS and the non-linear reaction function. The system can be written as:

$$F\left(X_t, E_t\left(X_{t+1}\right), S_t\right) = 0 \tag{5-5}$$

where $X_t = [x_t, \pi_t, i_t]$ are the endogenous variables and $S_t = [u_t, i_{t-1}]$ are the state variables, that evolves according to:

$$S_{t+1} = g\left(X_t, e_t\right) = \left[\rho u_{t-1} + e_t, i_{t-1}\right] \tag{5-6}$$

We approximate the expected value of the rational expectations solution of the model as a non-linear function on the states:

$$EX_{t+1} = Z\left(S_t\right) \tag{5-7}$$

which is unknown. The rational expectations equilibrium satisfies:

$$
\begin{aligned}
F\left(X_t, Z\left(S_t\right), S_t\right) &= 0 \qquad (5\text{-}8) \\
S_{t+1} &= g\left(X_t, e_t\right)
\end{aligned}
$$

[14] See Judd (1998) and Miranda and Fackler (2002) for discussion on collocation methods.

The collocation method consists on finding a function of the states, $\Phi(S_t)_{1xn}$, evaluated in at S_{nx1} nodes[15] to approximate $Z(S_t)$ by:

$$Z(S) = \Phi(S)\,C \tag{5-9}$$

where C is a $nx1$ matrix of coefficients. We need to solve for the matrix of coefficients C in (5-9) such that satisfy (5-8). We use linear splines evaluated at 200x200 points as a basis for the projection method. To calculate the expected value we use numerical integration based on Gauss-Legendre quadrature evaluated at 5 points. We select splines as the basis function in order to have enough flexibility in the approximation function to capture the non-linearities of the solution. Similarly, we choose to approximate the expected value of the endogenous variable because it is smoother than the solution for the endogenous variable.

The algorithm has two steps:

Step 1: Since the interest rate reaction function is non-differentiable in the thresholds which makes difficult to apply the numerical methods to solve for (5-9), we use a first guess the following non-linear function for the interest rate:

$$i_t = f(E_t\pi_{t+1}, i_{t-1}) = i_t^{1*} - \frac{\left(i_t^{1*} - i_{t-1}\right)\left(2i_t^{2*} - i_t^{1*} - i_{t-1}\right)}{i_t^{2*} - i_t^{1*}} \exp\left(-\tau\left(\frac{i_t^{2*} - i_{t-1}}{i_t^{2*} - i_t^{1*}}\right)^2\right)$$

where i_t^{1*} and i_t^{2*} are member's 1 and 2 optimal rates, and τ is chosen such that it minimises the approximation error. We select this non-linear form, because it captures many of the properties of the original reaction function: the values of the reaction function at the thresholds and at extreme values are the same. It also preserves the shape of the original reaction function, but it is smoother at the kinks. We compare the original with the smoothed reaction function in the following graph:

As we can see, this smoothed reaction function captures the two characteristics of the original one: lagged dependence and modest response. Features that we want to evaluate in a general equilibrium framework.

[15]The system is evaluated at $n = n_1 * n_2$ nodes, n_1 and n_2 for the state space of u_t and i_{t-1}, respectively.

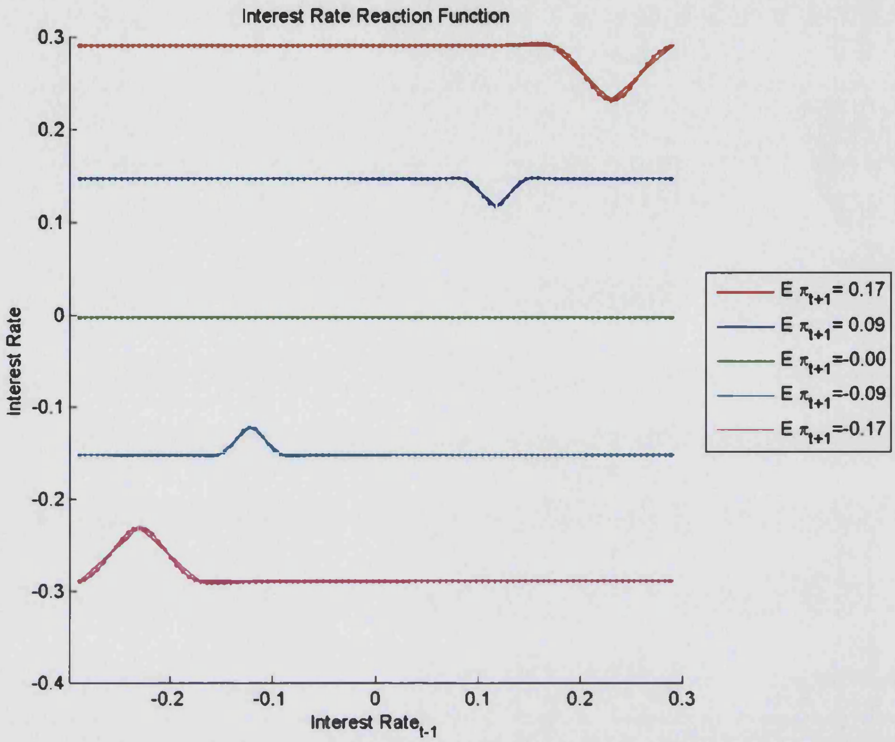

Figure 5.4: Interest rate reaction function (original vs. smoothed function)

Step 2: We use the solution for $Z(S)$ from step 1 as a first guess for the real piece-wise reaction function and estimate it again the policy function using the collocation method.

The algorithm converges after a total of 140 iterations with a degree of tolerance of $10E-8$. We consider the following parameterisation: the discount factor $\beta = 0.98$, the intertemporal elasticity of substitution $\varphi = 1/5$, the slope of the Philips Curve $\lambda = 0.2$, the preference parameters for member 1 and 2 are $\alpha_1 = 0.5$ and $\alpha_1 = 1$, the autocorrelation of the "cost-push" shock is $\rho = 0.75$ and its shock is normally *iid* with mean 0 and standard deviation of 0.01.

5.4.2 Policy functions

In this subsection we describe the solution of the endogenous variables as a function of the state variables, u_t and i_{t-1}, We focus on the effects that the interactions within the MPC have on the interest rate and expected inflation. As we see in the next graphs, the political equilibrium problem generates lagged dependence, lower response to shocks and an increase in expected inflation.

Figure 5.5 shows the policy function for the interest rate. We show in the panel on the left the interest rate as a function of the lagged interest rate for different values of the cost-push shocks. It shows that the interest rate has areas where it is independent of its lagged values, but there are areas where the response depends on its lagged value, when such lagged value is close to the optimal. Also, these areas increase the higher the size of the shocks. Similarly, we show in the panel on the right the interest rate as a function of the cost-push shocks for different values of the lagged interest rate. We observe that there is no interest rate smoothing when the initial interest rate is close to its neutral value (that is $i_{t-1} = 0$). However, there is a lower response when the interest rate is closer to its optimal value.

In the model the MPC takes as given expected inflation because there is a lack of commitment. However, the interactions within the MPC generate interest rate smoothing and the economic agents internalise this, which also has an effect in expected inflation. In the next graph we compare the expected inflation policy function of our the model with that of the single unconstrained policymaker. We show that the inertial behaviour of the interest rate increases expected inflation proportional to the size of the cost-push shock, but independently on the lagged interest rate. **Under our benchmark parameterisation, a cost push shock has an additional effect on expected inflation of 4.5 percent.** This effect is independent of the lagged interest rate, because the solution takes into account the distribution of the shocks, which smoothes the effects of the shocks. As economic agents internalise that the decisions of the MPC have an inertial component, they

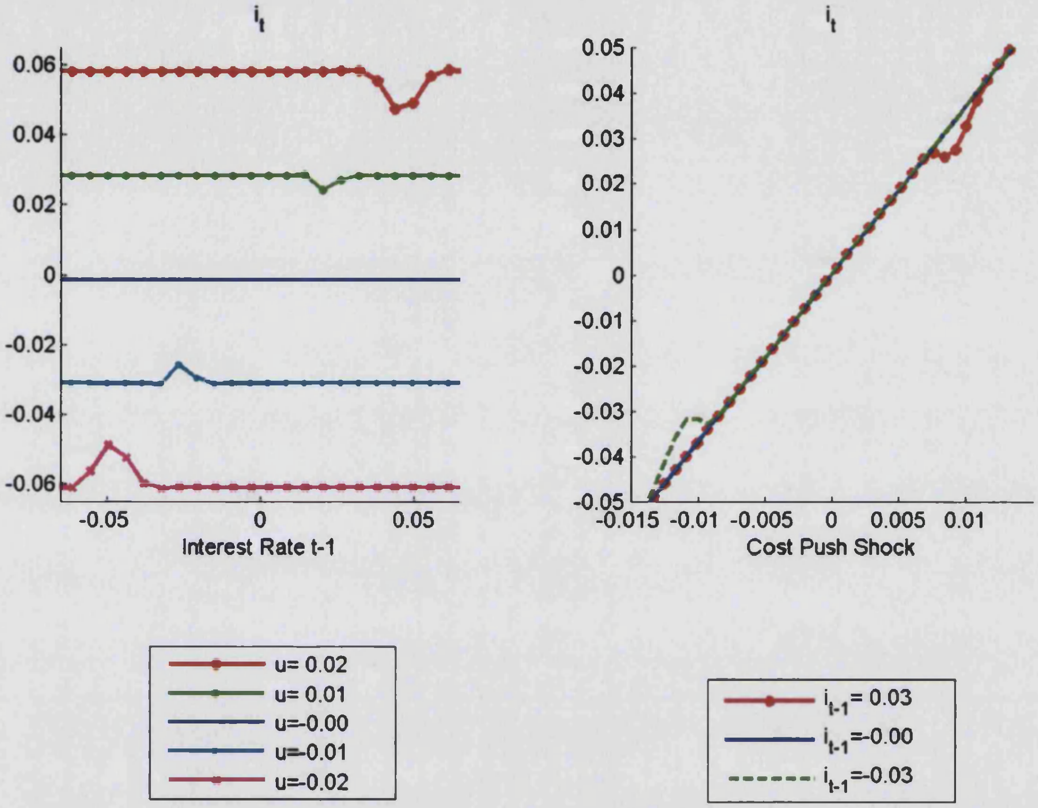

Figure 5.5: Interest rate policy function.

consider this effect in their expectations. Therefore, the more heterogeneous the preferences in an MPC are, the effect cost-push shocks on expected inflation.

5.4.3 Impulse response to "cost-push" shocks.

Figure 5.7 shows the effect of a shock of 1 standard deviation in the cost-push shock, for different values of the initial interest rate and for the case of the unconstrained policymaker. The initial interest rate takes values that can be high (3%), medium (2%) or low (1%). We see that the expected response of the interest rate is different

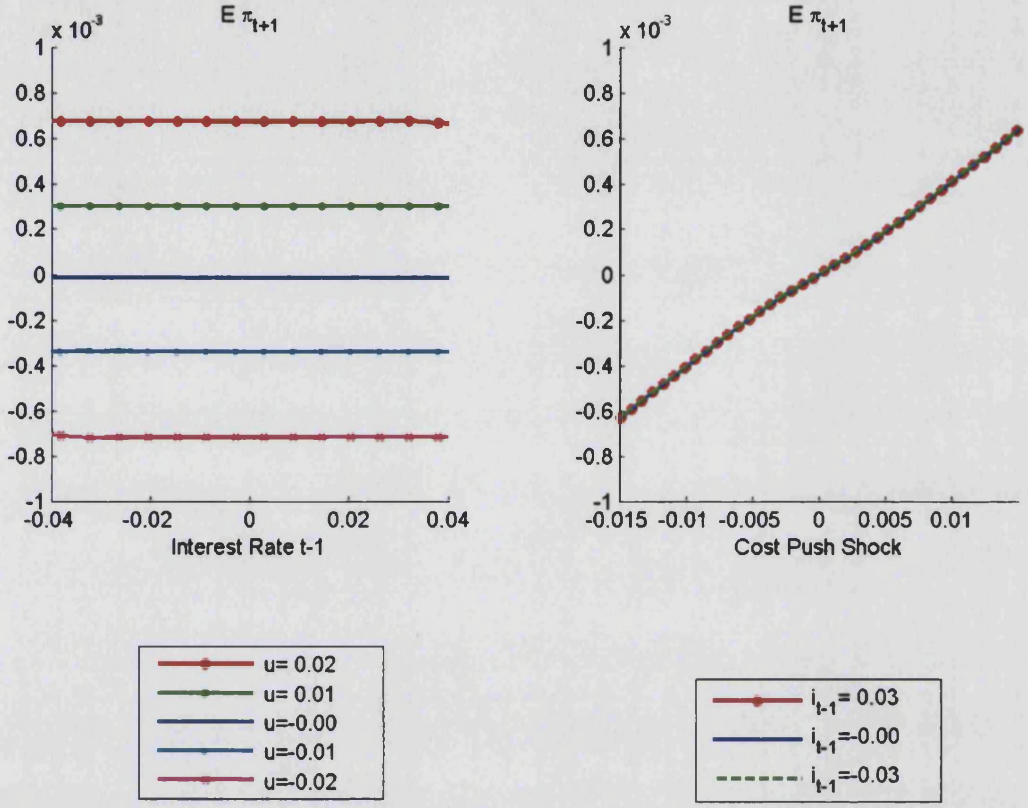

Figure 5.6: Change in expected inflation (benchmark model vs. MPC model)

depending on the starting point. If the interest rate is close to the optimal, it almost doesn't change. However, for the case when the initial interest rate is low, the change is higher and closer to the unconstrained case. For the intermediate value, the new rate is in between. We can also see that this effect is transitory, as in period 2 the response is very similar for the four cases. However, since this is the expected path of the interest rate, it is taking into account that other shocks would arrive in the next period, which reduces the expected effect of interest rate smoothing.

Similarly, figure 5.8 shows the expected responses for inflation and output

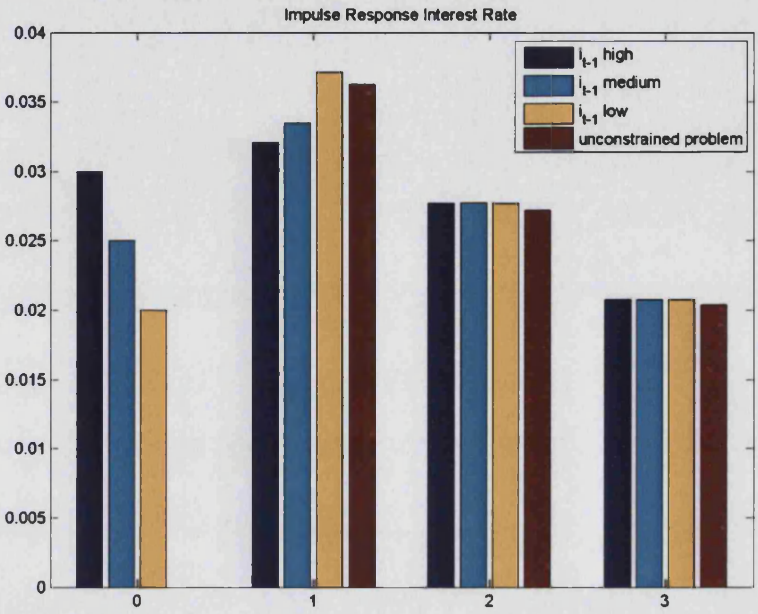

Figure 5.7: Impulse response to a cost-push shock: interest rate

gap. We can observe here the trade-off between output and inflation volatility. The higher the initial interest rate, the more interest rate smoothing and the less volatility of output in relation with inflation.

5.5 Empirical Implications

The model that we develop in section 5.3 has some empirical implications. In this section we analyse if those implications are consistent with what is observed in the data. According to the model, more interest rate smoothing will be observed when the preferences among MPC members are more unequal, the agenda setter has preferences that are not in the median of the MPC members, and the size of the shocks is small. Moreover, this result comes from the assumption that the agenda setter can influence other members and there is an strategic game within the MPC.

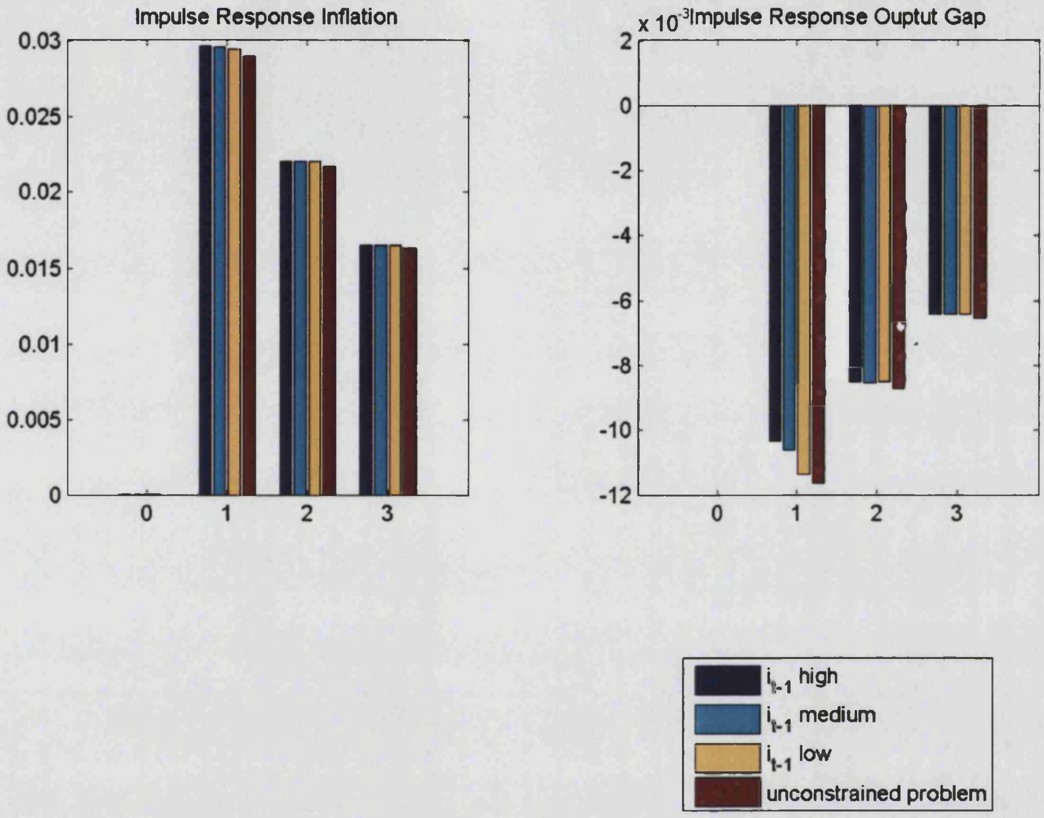

Figure 5.8: Impulse response to a cost-push shock: inflation and output gap

We analysed in this section whether these stylised facts are consistent with the path of the official rates for the USA, UK, EMU, Canada, Sweden and Switzerland, and with the published voting record of the Bank of England.

Stylised fact 1: Agenda setter influence on the other members

When the MPC members vote, they express their own view about the economy. However, we argue that in the voting process, the agenda setter can influence the votes of some members to obtain a policy that is closer to its own optimum. Also, the other members influence the decision of the agenda setter, because she needs the votes of other members to have the policy approved. The final outcome in the voting process is a political equilibrium. There are some open questions about this: Does this strategic behaviour take place? Has the Chairman/Governor/President of the MPC more power and influence than her peers?

Regarding the first question, we can see from the voting record of the MPC at the Bank of England that in almost all cases, from when the MPC started in July 1997 until May 2006, the final policy outcome is the same as the voting record for the Governor[16] . In other words, the agenda setter never loses. This indicates a strategic behaviour from the agenda setter, in order to obtain the coalition needed to have a policy passed.

Also, there is evidence that the person in charge of the MPC meeting has more power and can influence other members' decisions; however the final product is a political equilibrium. Laurence Meyer, Board Governor of the FOCM from 1996 to 2002, remarks on "the chairman's disproportionate influence on FOMC decisions" and on "his efforts to build consensus around his policy recommendations"[17]. Similarly, Sherman Maisel, who was a member of the Board during Burns' chairmanship also points out that "while the influence of the Chairman is indeed great, he does not make policy alone"[18]. Then, the interest rate decisions come from the interaction between the agenda setter ant the other members of the MPC.

Stylised fact 2: Heterogeneity in the preferences

The model relies on the assumption that MPC members have different preferences. This heterogeneity, together with strategic behaviour of the agenda setter,

[16]The exception was the meeting of August 2005, in which the Governor – Mervyn King invited members to vote on the proposition that the repo rate should be reduced by 25 basis point. Five members of the committee vote in favour, whilst the other four members, among them the Governor, preferred to maintain the rate.

[17]Meyer (2004), p.50.

[18]Maisel (1973), p.124.

causes interest rate smoothing. How heterogeneous are the preferences among members? Do they really think differently? We take as an indicator of this heterogeneity the dissenting record of each member with respect to the agenda setter. We construct this indicator using the information of the voting record for the Bank of England, which is available for the period since the MPC was established in July 1997.

Gerlach-Kristen (2003) analyses the voting record of the BoE since the introduction of the MPC. She characterise the MPC member in four groups: the first group, the agenda setter, always vote with the majority; the second group, the "doves", when dissenting always favoured a level of interest rates lower than that set by the majority; the third group, the "hawks", always favoured a tighter monetary policy when dissenting; and the fourth group doesn't show a systematic preference to higher or lower rates. We can classify the members of the third (second) group as those that are more (less) conservative than the agenda setter.

In table 5.1 and 5.2 we classify the MPC members in the four categories as Gerlach-Kristen (2003a) for both, the governorship of Sir Edward George and Mr. Mervyn King. For this classification we consider if the preferred rate when dissenting was higher or lower than the voted rate, and how frequent they dissent. We have considered only those members with at least ten votes in the record and those that show systematic preferences to either lower or higher rates. Also, we have also classified the members as internal or external depending on the way they are appointed[19].

Table 5.1 shows the classification during the Governorship of Sir Edward George from July 1997 to June 2003, and table 5.2 for the Governorship of Mr Mervyn King from July 2003 to June 2006. The members are classified by its

[19]The MPC at the Bank of England was established in June 1997. It has nine members, five full-time Bank executives (the Governor and two Deputy Governors, the Chief Economist and the Markets Director) and four external members, who are appointed for a three-year term by the Chancellor of the Exchequer.

conservativeness degree, being those at the top the more conservative[20]

According to this classification, we can see some differences on MPC members preferences across sub-samples. First, Sir George has been on average closer to the median preferences than Mr. King does. Second, we can see more dispersion among MPC member's preferences during Mr King's governorship than during Sir Edward George's Governorship. Third, the MPC members internally appointed show a tendency to be more conservative than those appointed externally. According to these features, our model predicts under Mr. King's governorship, *ceteris paribus*, more interest smoothing than under Sir King's governorship. Effectively, during Mr King's governorship, the official rate has been maintained 80 percent of the time, in comparison to 68 percent in Sir Edward George's governorship.

[20]For instance, according our classification Sir Budd has been the most conservative during Sir George Governorship, since he has preferred proportionally more times a higher rate than the Governor.

	Frequency of dissents	of which for higher rate	Appointment
The most conservative			
Sir Alan Budd	22.2%	100.0%	External
John Vickers	17.9%	100.0%	Internal
Mervyn King	16.2%	100.0%	Internal
Charles Goodhart	8.3%	100.0%	External
Paul Tucker	7.7%	100.0%	Internal
Sir Edward George (Governor)	**0.0%**	**0.0%**	**Internal**
Charles Bean	2.9%	0.0%	Internal
Kate Barker	11.5%	0.0%	External
Sushil Wadhwani	35.1%	0.0%	External
DeAnne Julius	28.9%	0.0%	External
Christopher Allsopp	29.7%	0.0%	External
Marian Bell	25.0%	0.0%	External
The least conservative			

Table 5.1: Classification MPC members: Sir George's governorship

	Frequency of dissents	of which for higher rate	Appointment
The most conservative			
Sir Andrew Large	25.8%	100.0%	Internal
Paul Tucker	11.4%	100.0%	Internal
Rachel Lomax	2.9%	100.0%	Internal
Mervyn King (Governor)	**0.0%**	**0.0%**	**Internal**
Kate Barker	2.9%	0.0%	External
Richard Lambert	3.0%	0.0%	External
Charles Bean	8.6%	0.0%	Internal
David Walton	9.1%	0.0%	External
Marian Bell	12.5%	0.0%	External
Stephen Nickell	25.7%	11.1%	External
The least conservative			

Table 5.2: Classification MPC members: Mr. King's governorship

Stylised fact 3: Dispersion of preferences and interest rate smoothing

The model predicts that the more heterogeneous the preferences are, if the agenda setter is not the median member, *ceteris paribus* will be more interest rate smoothing. To analyse this fact, we compare the paths of the official interest rate for the European Central Bank (ECB) and the Swiss National Bank (SNB). We expect those economies to have similar paths for interest rate decisions, since the main trading partners for Switzerland are the members of the EMU and those economies are hit by similar shocks. However, the pattern of the official interest rate for the SNB is more dynamic than for the ECB. On average, the changes of the interest rate had a duration of five months for the SNB in comparison to seven months in the ECB. Also, the SNB has changed the interest rate by higher amounts than the ECB, the mode in the change of the interest rate is 0.5 percent for the SNB in contrast to 0.25 percent for the ECB. This would be explained by how the MPCs are formed in both central banks. At the ECB, the Governing Council is formed by the six members of the Executive Board, plus the governors of all the national central banks (NCBs) from the 12 euro area countries, while at the SNB, the Governing Board in charge of monetary policy decisions is formed of only three members.

In table 5.3 we show some rough indicators about the dynamics of the official interest rate for six countries. The first indicator is the average duration of a change in the interest rate; we expect that the easier it is to have an agreement within the MPC, the lower the interest rate smoothing and the more frequent the adjustment in the rate. The second indicator is the mode of the change in the interest rate, the easier it is to have an agreement within the MPC, the higher the changes in the interest rates.

According to the first indicator, Canada and Sweden have the more active central banks, where a change in the interest rate lasts on average two months, followed by the United Kingdom and the USA with three months. While according

Country	Data since:	Change Rate Mode	Avg. duration (Months)	Number of members MPC	Number of meetings per year
Canada	Abr-96	0.25	2.4	6	8
United Kingdom	Jun-97	0.25	3.3	9	12
USA	Ene-96	0.25	3.3	12	8
Switzerland	Ene-00	0.50	5.4	3	12
EMU	Ene-01	0.25	7.0	18	11
Sweden	Jun-94	0.25	2.2	6	8-9

Table 5.3: Dynamics of Official Interest Rate

to the second indicator, Switzerland is more active with a mode in the changes of the interest rate of 0.5 percent, a difference from the other countries whose interest rates usually change by 0.25 percent. Both indicators also suggest that the central bank with more interest rate smoothing is the ECB, which changes the interest rate every seven months on average, at steps of 0.25 percent. As we mentioned before, these results are related to the composition of the MPC. The MPC in Switzerland has only 3 members, and Canada and Sweden 6; in contrast to the MPC in the USA and the EMU, which they have 12 and 18, respectively. The more members an MPC has, the more likely that their the preferences will differ and the more difficult it is to have an agreement.

5.6 Conclusions

This chapter helps to explain the existing puzzle in the optimal monetary policy literature of interest rate smoothing: why in practice do central banks change the interest rate less frequently than the theory predicts? In doing this, we extend the New Keynesian Monetary Policy literature relaxing the assumption that the decisions are taken by a single policy maker, considering instead that monetary policy decisions are taken collectively in a committee. We introduce a Monetary Policy Committee whose members have different preferences between output and inflation stabilisation and have to vote on the level of the interest rate. Also, there is one member in charge of setting the agenda of the meeting, which can be the Chairman/Governor/President of the MPC.

We explain interest rate smoothing from a political economy point of view, in which MPC members face a bargaining problem on the level of the interest rate. In this framework, the interest rate is a non-linear reaction function on the lagged interest rate and the expected inflation. This result comes from a political equilibrium in which there is a strategic behaviour of the agenda setter with respect to the other MPC members in order to maximise his own policy objective.

According to the model, there is not such interest rate smoothing when the agenda setter is the member with median preferences. As in the median voter theorem, she can always get a coalition to have her most preferred (lagged independent) interest rate. However, when the agenda setter is either one of the most or the least conservative members, it will be interest rate smoothing from a political economy point of view. Also, interest rate smoothing is higher when the preferences among the MPC members are more heterogeneous.

The size of the shocks is also important for interest rate smoothing. We find that the interest rate will adjust in the same magnitude as in the single policymaker case when the size of the shocks is high enough. However, when the size of the shocks is of intermediate size, we have found that the interest rate adjusts partially in order to form a coalition between the agenda setter and at least one of the other two members. Also, when the size of the shocks is small, it is preferred to maintain the interest rate unchanged.

We present this explanation of interest rate smoothing as an alternative approach in order to reproduce altogether both features documented by the empirical evidence of interest rate smoothing: the modest response of the interest rate to inflation and the lagged dependence. These are features that other models fail to reproduce at the same time. Our model also provides a theoretical framework on how disagreement among policy makers can slow the adjustment on interest rates and on 'menu costs' in interest rate decisions.

We also present some evidence based on the official interest rate path for five central banks and the voting record at the Bank of England. We show that this information is consistent with the assumptions of the model and with the results. We observe in the data that central banks whose members have more heterogeneous preferences adjust the interest rate less frequently, as in the case of the European Central Bank and the FED. Central banks with fewer members adjust the interest rate more aggressively, as in the case of the Swiss National Bank, the Bank of Canada and the Bank of Sweden. Also, according to the voting records at the Bank of England, there is also evidence of heterogeneity in the voting preferences among the members of the MPC, which is positive related to the degree of interest rate smoothing.

We do some quantitative exercises to show how interest rate smoothing in our model affect the economic equilibrium. We show that interest rate smoothing increases the effects of cost-push shocks on expected inflation by 4.5 percent given our benchmark calibration. As economic agents internalise the inertial component of the MPC decisions, they also consider this effect when forming expectations.

D Appendix: Proof of propositions

D.1 Proof of proposition 5.1:

We divide the proof in two steps: first the policy-maker chooses x_t and π_t to maximise her welfare subject to the aggregate supply. Then, conditional on the optimal values of x_t and π_t, she determines the value of i_t implied by the IS.

The first step of the policymaker's problem is given by maximising the bellman equation:

$$\max_{\{x_t,\pi_t\}} W_t = -\frac{1}{2}\left[\alpha x_t^2 + \pi_t^2\right] + \beta E_t W_{t+1}$$

subject to

$$\pi_t = \lambda x_t + \beta E_t \pi_{t+1} + u_t$$

Since the policymaker cannot credibly manipulate beliefs in the absence of commitment, she takes private sector expectations as given when solving her optimisation problem. Then, conditional on the policymaker's optimal rule, the private sector forms beliefs rationally. Therefore, the policymaker takes $E_t W_{t+1}$ and $\beta E_t \pi_{t+1}$as given in her optimisation problem.

The solution to the first stage problem yields the following optimally condition:

$$x_t = -\frac{\lambda}{\alpha}\pi_t \tag{D-1}$$

According to this condition, whenever inflation is above target, the policymaker contracts demand below capacity by raising the interest rate; and vice versa when it is below target. The aggressiveness of the policymaker depends positively on the gain in reduced inflation per unit of output loss, λ, and inversely on the relative weight placed on output losses α.

In order to obtain the reduced for expression for x_t and π_t, we combine the first order condition with the PC, and then impose that private sector expectations are rational to obtain:

$$\begin{aligned} x_t &= -\varpi u_t \\ \pi_t &= \frac{\alpha}{\lambda}\varpi u_t \end{aligned} \tag{D-2}$$

where $\varpi = \frac{\lambda}{\lambda^2+\alpha(1-\beta\rho)}$ is a decreasing function of the preference parameter α. From the second step, the optimal feedback policy for the interest rate is found by inserting the desired value of x_t in the IS:

$$i_t = r_t^n + \phi_\pi E_t \pi_{t+1}$$

where $\phi_\pi = 1 + \frac{(1-\rho)\lambda}{\rho\varphi\alpha} > 1$.

D.2 Proof of proposition 5.2:

Replace the IS and the AS in the welfare function of member j and operate:

$$W_t^j(i_t) = -\frac{1}{2}\left\{ \begin{array}{c} \alpha^j\left[-\varphi\left(i_t - E_t\pi_{t+1} - r_t^n\right) + E_t x_{t+1}\right]^2 \\ +\left[\lambda\left(-\varphi\left(i_t - E_t\pi_{t+1} - r_t^n\right) + E_t x_{t+1}\right) + \beta E_t\pi_{t+1} + u_t\right]^2 \end{array}\right\} + \beta \overline{W}_{t+1}^j \tag{D-3}$$

Subtract the welfare function evaluated at i_{t-1} :

$$W_t^j(i_t) - W_t^j(i_{t-1}) = -\frac{1}{2}\left\{ \begin{array}{c} \alpha^j\varphi\left(i_t - i_{t-1}\right)\left[\begin{array}{c} \varphi\left(i_t + i_{t-1}\right) \\ -2\left(\varphi\left(E_t\pi_{t+1} + r_t^n + \frac{1}{\varphi}E_t x_{t+1}\right)\right) \end{array}\right] \\ +\lambda\varphi\left(i_t - i_{t-1}\right)\left[\begin{array}{c} \lambda\varphi\left(i_t + i_{t-1}\right) - \\ 2\left(\lambda\varphi\left(E_t\pi_{t+1} + r_t^n + \frac{1}{\varphi}E_t x_{t+1}\right) + \beta E_t\pi_{t+1} + u_t\right) \end{array}\right] \end{array}\right\} \tag{D-4}$$

factorise $\varphi\left(i_t - i_{t-1}\right)$ and rearrange the terms that are similar:

$$W_t^j(i_t) - W_t^j(i_{t-1}) = -\frac{1}{2}\varphi\left(i_t - i_{t-1}\right)\left\{ \begin{array}{c} \left(\alpha^j + \lambda^2\right)\varphi\left(i_t + i_{t-1}\right) \\ -2\left(\alpha^j + \lambda^2\right)\left[\varphi\left(E_t\pi_{t+1} + r_t^n + \frac{1}{\varphi}E_t x_{t+1}\right)\right] \\ -2\lambda\left(\beta E_t\pi_{t+1} + u_t\right) \end{array}\right\} \tag{D-5}$$

Member j optimal rate satisfies:

$$x_t^{*j} = -\frac{\lambda}{\alpha^j}\pi_t^j \tag{D-6}$$

also the optimal rate for member j is

$$i_t^{j*} = r_t^n + E_t\pi_{t+1} - \frac{1}{\varphi}\left(x_t^j - E_t x_{t+1}\right) \tag{D-7}$$

replace (D-7) in (D-5) and factorise the term $(\alpha^j + \lambda^2)$, we obtain:

$$W_t^j(i_t) - W_t^j(i_{t-1}) = -\frac{1}{2}\left(\alpha^j + \lambda^2\right)\varphi\left(i_t - i_{t-1}\right)\left\{\begin{array}{c} \varphi\left(i_t + i_{t-1}\right) \\ -2\left(\varphi i_t^{j*} + x_t^j\right) - 2\frac{\lambda}{\alpha^j+\lambda^2}\left(\beta E_t\pi_{t+1} + u_t\right) \end{array}\right\} \tag{D-8}$$

make use of the AS and (D-6) to eliminate some terms. The condition can be written by:

$$W_t^j(i_t) - W_t^j(i_{t-1}) = -\frac{1}{2}\varphi^2\left(\alpha^j + \lambda^2\right)\left(i_t - i_{t-1}\right)\left\{\left(i_t + i_{t-1}\right) - 2i_t^{j*}\right\} \tag{D-9}$$

We have that $W_t^j(i_t) = W_t^j(i_{t-1})$ when either $i_t = i_{t-1}$ or $i_t = 2i_t^{j*} - i_{t-1} = \overline{i_t^j}$

D.3 Proof of proposition 5.4

Let's analyse the case when $E_t\pi_{t+1} > 0$, the proof for the opposite case is similar. When inflation expectations are positive, we have the following ordering for each member preferred interest rate:

$$i_t^{1*} > i_t^{2*} > i_t^{3*}$$

We will analyse three possible cases: when the agenda setter can set the interest rate equal to her most preferred rate (i_t^{1*}), to the participation rate of either member 2 $(\overline{i_t^2})$ or 3 $(\overline{i_t^3})$, or the status-quo (i_{t-1}). **Case 1:** when member 2 or member 3 accept agenda setter's preferred rate (i_t^{1*})? The utility of member j in comparison with the status quo is:

$$W_t^j\left(i_t^{1*}\right) - W_t^j(i_{t-1}) = -\frac{1}{2}\varphi^2\left(\alpha^j + \lambda^2\right)\left(i_t^{1*} - i_{t-1}\right)\left\{\left(i_t^{1*} + i_{t-1}\right) - 2i_t^{j*}\right\}$$

This is positive for member 2 when $i_{t-1} < 2i_t^{2*} - i_t^{1*} < i_t^{2*} < i_t^{1*}$ or when $i_{t-1} > i_t^{1*}$. Similarly, this is positive for member 3 when $i_{t-1} < 2i_t^{3*} - i_t^{1*} < i_t^{3*} < i_t^{1*}$ or when $i_{t-1} > i_t^{1*}$. Then, since $i_t^{3*} < i_t^{2*}$, when either $i_{t-1} \leq 2i_t^{2*} - i_t^{1*}$ or $i_{t-1} > i_t^{1*}$ at least one member will accept i_t^{1*}. **Case 2:** when the agenda setter will prefer to attract the vote of member 2 with $i = \overline{i_2}$ instead of the vote of member 3 with $i = \overline{i_3}$?

Compare the utility of the agenda setter under both rates:

$$\begin{aligned} W_t^A\left(\overline{i_t^2}\right) - W_t^A\left(\overline{i_t^3}\right) &= -2\varphi^2\left(\alpha^A + \lambda^2\right)\left(i_t^{2*} - i_t^{3*}\right)\left\{\left(i_t^{2*} + i_t^{3*} - i_t^{1*}\right) - i_{t-1}\right\} \\ &= (-)(+)(?) \end{aligned}$$

She will prefer to attract the votes of member 2 with $i = \overline{i_2}$ when $i_{t-1} > i_t^{2*} + i_t^{3*} - i_t^{1*}$, otherwise she will prefer to attract the votes of member 3 with $i = \overline{i_3}$.

The agenda setter will always prefer to set i_t^{1*}. However, when it is not possible to obtain the votes for i_t^{1*}, she can obtain the votes of either member 2 or 3 setting the participation rate. But, we still need to compare if the agenda setter can be better-off with the status quo than with the participation rate. As the agenda setter has the first moving advantage, she can influence the votes of the other members if she prefer to maintain the rate unchanged. **Case 3:** when the agenda setter prefer the status quo to either $\overline{i_2}$ or $\overline{i_3}$? Compare the utility of the agenda setter under both cases:

$$W_t^A\left(\overline{i_t^j}\right) - W_t^A\left(i_{t-1}\right) = -2\varphi^2\left(\alpha^A + \lambda^2\right)\left(i_t^{j*} - i_{t-1}\right)\left\{i_t^{j*} - i_t^{1*}\right\}$$

for $j = 2 : W_t^A\left(\overline{i_t^2}\right) < W_t^A\left(i_{t-1}\right)$ when $i_{t-1} > i_t^{2*}$. Similarly, for $j = 3$: $W_t^A\left(\overline{i_t^3}\right) < W_t^A\left(i_{t-1}\right)$ when $i_{t-1} > i_t^{3*}$.

Then, when $i_t^{2*} < i_{t-1} < i_t^{1*}$: the agenda setter will prefer the status quo to rate necessary to obtain the votes.

In the remaining area $(2i_t^{2*} - i_t^{1*} < i_{t-1} < i_t^{2*})$, since $2i_t^{2*} - i_t^{1*} > i_t^{2*} + i_t^{3*} - i_t^{1*}$, the agenda setter can attract the votes of member 2 setting $i_t = \overline{i_t^2}$. This define four areas of the interest rate reaction function when $E_t\pi_{t+1} > 0$.

D.4 Proof of proposition 5.6

Consider F as the forward operator, the Phillips curve equation and the IS equation can be expressed as:

$$\begin{aligned}(1-\beta F)\,\pi_t &= \lambda x_t + u_t &\text{(D-10)}\\ (1-F)\,x_t &= -\varphi\,(i_t - F\pi_t) &\text{(D-11)}\end{aligned}$$

where i_t is function of expected inflation. Multiply (D-10) by $(1-F)$ and subtract (D-11):

$$(1-F)(1-\beta F)\,\pi_t + \lambda\varphi\,(i_t - F\pi_t) = (1-F)\,u_t \tag{D-12}$$

In order to have a stable rational expectations equilibrium, we need that the roots of F in the left hand side of (D-12) being outside the unit circle.

Let's analyse first, the determinacy for member-j preferred policy rule:

$$i_t^* = \phi^j E_t \pi_{t+1} = \phi^j F\pi_t$$

The condition for determinacy is that root to the problem

$$\lambda\varphi\left(\phi^j - 1\right)F = -(1-F)(1-\beta F) \tag{D-13}$$

being outside the unit circle. The value of ϕ^j at the boundary $F = 1$ is $\phi^j = 1$. Similarly, the value at the boundary $F = -1$ is $\phi^j = 1 + 2\frac{1+\beta}{\lambda\varphi}$. Then, any value of $\phi^j \in \left[1, 1 + 2\frac{1+\beta}{\lambda\varphi}\right]$ satisfies the determinacy condition. As $\phi^1 > \phi^2 > \phi^3 > 1$, a sufficient condition for determinacy is that $\phi^1 < 1 + 2\frac{1+\beta}{\lambda\varphi}$.

To analyse the roots of F in (D-12) for the policy rule in proposition (5.4) or (5.5), note that it is bounded by preferred rate for member 1 and 3,that is: $i_t \in [\phi^3 F\pi_t, \phi^1 F\pi_t]$. Then

$$\begin{aligned}\left[-q(F) + \lambda\varphi\left(\phi^3 - 1\right)F\right]\pi_t &\leq q(F)\,\pi_t + \lambda\varphi\,(i_t - F\pi_t) &\text{(D-14)}\\ &\leq \left[q(F) + \lambda\varphi\left(\phi^1 - 1\right)F\right]\pi_t\end{aligned}$$

where we have defined the polynomial $q(F) \equiv -(1-F)(1-\beta F)$. In the following figure we graph the polynomial $q(F)$ and the two lines $\lambda\varphi(\phi^3 - 1) F$ and $\lambda\varphi(\phi^1 - 1)$, which satisfy the determinacy condition. The intersection of each line with $q(F)$ give the value for the root of F. On the other hand, the root for the policy function i_t is located on the segment of $q(F)$ between both lines. Also, note that any point in that segment satisfies the determinacy condition, and the exact position will depend on the last period interest rate.

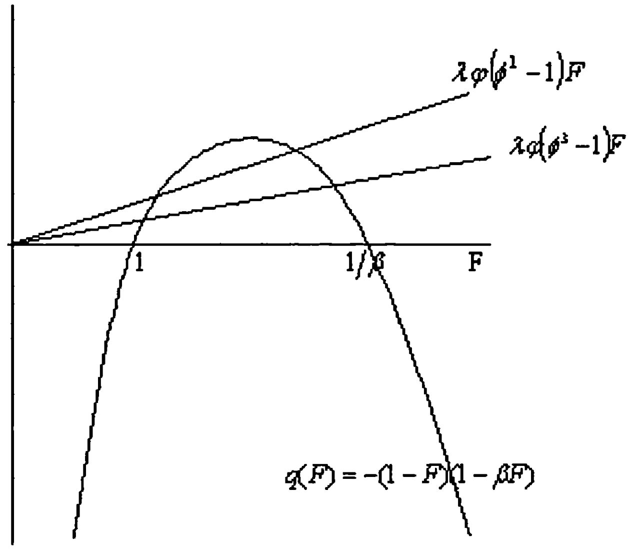

Figure D.1: Determinacy condition

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