CREDIT MARKET IMPERFECTIONS: MACROECONOMIC CONSEQUENCES AND MONETARY POLICY IMPLICATIONS

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Abstract

In this thesis, dynamic general equilibrium models are developed for the analysis of credit market imperfections. The first chapter provides an overview of the thesis and sets out the motivation for the research. In the second chapter, the focus is on house prices. Empirical work is carried out to investigate the co-movement of house prices, housing investment, consumption and monetary policy in the UK. A general equilibrium model is then developed to fit some key patterns in the data. An important feature of the model is that house prices have a direct impact on consumption, because housing serves as collateral against which consumers can borrow. The model is used to analyse how the co-movement of key variables is likely to have changed following financial liberalisation in the 1980s.

The third chapter develops a framework in which entrepreneurs want to borrow from and lend to each other because investment opportunities are always changing. Credit markets do not work perfectly, so borrowing can only take place against collateral. Moreover, monetary policy has real short-run effects due to nominal rigidities. The credit frictions cause productivity shocks to have a large and persistent effect on aggregate output and asset prices, as falls in output are accompanied by a transfer of capital from highly productive borrowers to less productive lenders. But nominal rigidities interact strongly with this mechanism: the more aggressively the monetary authorities stabilise inflation, the larger the output and asset price movements.

The final chapter investigates how monetary policy should be set optimally, in the sense of maximising the welfare of the private sector agents. It is found that optimal monetary policy allows for a temporary rise in inflation following an adverse productivity shock, which will lead to more stable output and asset prices. The interaction of credit frictions and nominal rigidities therefore creates a short-term trade-off between the stabilisation of output relative to its efficient level and the stabilisation of inflation.

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Chapter 1

Overview

What effects do asset price changes have on the economy and how should monetary policy-makers respond? These are the two fundamental questions around which this thesis revolves. Chapter 2 specifically deals with house prices[1]. During the 1990s house prices in the UK, Australia, the US and several continental European countries rose sharply. An important question for monetary policy-makers in this respect was what effect this rise would have on aggregate activity, mainly consumption and housing investment. In the UK, 40% of total household wealth is held in the form of housing, and 80% of all household borrowing is secured on housing. Aside from being quantitatively important to households, housing is of special interest for another reason. Most consumers live in the houses they own and value directly the services provided by their home. So the benefit of an increase in house prices is directly offset by an increase in the opportunity cost of

[1]The material in Chapter 2 is based on joint work with Kosuke Aoki and James Proudman

housing services. An increase in house prices therefore does not generally shift the aggregate budget constraint outwards. It is not obvious, then, that there is a traditional "wealth effect "from housing in the way that we think of a wealth effect arising from a change in the value of households' financial assets.

There are many reasons why house prices and consumption may move together. For example, they may both be driven by the same underlying shock, e.g. a change in the expected future level of income. The mechanism on which I focus in this chapter is that house prices may have a direct impact on consumption via credit market effects. An increase in house prices makes more collateral available to homeowners, which in turn may encourage them to borrow more to finance desired levels of consumption and housing investment. Three observations motivate an examination of such a credit channel. (i) House prices are strongly cyclical, which leads to substantial variation in households' collateral position (or loan to value ratio, or net worth) over the business cycle. (ii) The amount of secured borrowing to finance consumption is highly correlated with this collateral position. (iii) The spread of mortgage rates over the risk-free interest rate varies with the collateral position of each household; and unsecured borrowing rates, which are the marginal source of finance once collateral has been exhausted, are much higher than mortgage rates. So the interest rates borrowers face fall markedly as more collateral becomes available to them.

To analyse these effects, I construct a general equilibrium model, based on the financial accelerator model of Bernanke, Gertler and Gilchrist (1999), hereafter BGG, to capture the essential features of a housing credit channel.

Because of informational frictions, credit markets do not work perfectly, so borrowers pay a premium over and above the risk-free interest rate. Borrowers with higher net worth face a lower external finance premium, so they face lower interest rates. A positive shock to economic activity causes a rise in consumption, housing investment and house prices. This increases homeowners' net worth. The external finance premium falls, which amplifies the initial rise in consumption, housing investment and house prices.

I also consider the implications for monetary policy of recent structural changes in the United Kingdom's retail financial markets: following deregulation in the mortgage market, it has become easier and cheaper for consumers to borrow against housing collateral to finance consumption. I show that cheaper access to home equity means that, for a given house price increase, more additional borrowing will be devoted to consumption relative to housing investment. The response of consumption to an unanticipated change in interest rates will therefore be larger, and the response of house prices and housing investment will be smaller. In other words, whether the financial accelerator has most of its effect on house prices or consumption depends on the degree of deregulation: in a highly deregulated mortgage market, the effect on house prices will be muted, but the effect on consumption will be amplified. This also has implications for the information content of house prices. Empirical models that contain house prices and consumption may have unstable coefficients, even if fundamental shocks (e.g. productivity, government spending and monetary policy shocks) are correctly identified. A given change in house prices is likely to be associated with a larger change in consumption in the post-deregulation period.

Chapter 3 focuses on the interaction between asset prices, macroeconomic quantities and monetary policy, but at a more general level. The asset in question is not housing, but is more generally interpreted as the entire stock of durable, collateralisable productive assets. The model is therefore suitable for thinking about situations when highly leveraged corporate balance sheets are excessively vulnerable to sharp asset price corrections. Examples include the consequences of the early 1990s and the 2001 US recessions, when corporate balance sheets were in a poor state due to excessive borrowing in preceding years, or the 1989 Japanese equity market crash and the resulting slump. A model to analyse the interactions between asset prices, macroeconomic quantities and monetary policy needs to have the following features: a role for credit and a role for monetary policy. To generate a role for credit in the economy, it is necessary to introduce some imperfection in credit markets. Specifically, I assume that enforcement problems exist for financial contracts. To allow monetary policy to influence aggregate real outcomes, I assume that nominal rigidities exist in goods markets, so that product prices cannot adjust intantaneously.

The model economy consists of ex-ante identical entrepreneurs who can produce intermediate goods using capital, inventories and labour. Using the approach of Kiyotaki (1998), I assume that some entrepreneurs are more productive than others, but spells of high productivity do not last, and arrive randomly. While an entrepreneur is highly productive, he will want to invest as much as possible in his own technology. Entrepreneurs with low productivity, on the other hand, would rather invest in the technology of high productivity entrepreneurs, as this generates superior returns.

Let us therefore call the entrepreneurs who currently have high productivity 'producers', and the entrepreneurs with low productivity 'investors'. In principle, investors could lend to producers so that producers end up applying their technology to the entire capital stock. This would be the first-best outcome. But because of credit market imperfections, this outcome cannot be achieved. Both investors and producers hold some capital for production, and output is below its first best. Each period, producers borrow as much as they can from investors, subject to collateral constraints arising from the enforcement problem.

Following a shock that reduces current output and/or the price of capital, the net worth of producers falls by more than the net worth of investors, because producers are highly leveraged. This means that producers can only afford to buy a lower share of the total capital stock for production in the following period. Because capital shifts to those with lower productivity, this reduces expected future returns, which depresses the value of capital today, and exacerbates the initial redistribution of wealth from producers to investors. If the difference in productivity between investors and producers is high enough, output falls further in the subsequent period, as the capital stock is now used much less efficiently. It takes time for the producers to rebuild their share of the wealth distribution to its steady-state level, and output is therefore below its steady-state level for many periods, even if the initial disturbance lasted only a single period.

How is such a mechanism affected by monetary policy? Nominal rigidities can reduce the initial output fall. If monetary policy does not fully offset the inflationary impact of an adverse productivity shock, the output fall will be dampened. If the initial output effect is smaller, the redistribution of capital from producers to investors is also smaller, and the price of capital will not fall as much. The entire credit channel is therefore weakened: output falls less, and the fall is less persistent.

I also analyse how the model is affected when debt contracts are specified in real rather than nominal terms. Nominal contracts dampen the effects of productivity shocks, but amplify the effects of monetary policy shocks. The intuition is straightforward. Consider an adverse productivity shock. Inflation and output move in opposite directions. Output is lower than expected, which causes a transfer of wealth from producers (who are borrowers) to investors (who are lenders), leading to amplification via the credit channel. But inflation is higher than expected. Higher inflation erodes the real value of the debt that producers have to repay, and partly offsets the redistribution of wealth towards investors, thereby weakening the credit channel. So the output fall is smaller and less persistent. Now consider a monetary policy shock: inflation and output move in the same direction. For example, in the case of an unexpected monetary tightening, output will be lower than expected, which will trigger the amplification mechanism via the redistribution of capital from producers to investors. On top of that, lower-than-expected inflation will increase the real value of the debt that producers have to repay. This will cause a further transfer of wealth away from producers, meaning an even less efficient use of capital in future periods, and therefore even greater falls in output, inflation and asset prices.

In Chapter 3 it is shown how systematic monetary policy affects the interaction between asset prices and macroeconomic quantities. I show that monetary policy that aims to stabilise inflation fully and instantaneously exacerbates output fluctuations. Chapter 4 therefore asks the question: what should monetary policy aim to achieve, if the economy is well-described by the model derived in Chapter 3? This is done by assuming that the monetary policy-maker tries to maximise the welfare of private sector agents. This is commonly referred to as a Ramsey problem. There are two frictions in the economy: credit market frictions and nominal rigidities. The policy-maker has a single instrument available, the nominal interest rate, to offset the inefficiencies generated by these frictions. I build intuition for the trade-offs that are created by considering several versions of the model. In particular, I consider a frictionless version, where credit markets operate perfectly and prices are free to adjust. I also consider a flexible-price credit version, where credit market imperfections exist, but where there are no nominal rigidities. The flexible-price solution can also be interpreted as a solution where the monetary policy-maker stabilises prices perfectly, so that the nominal rigidities do not bind.

The frictionless model illustrates in a sense what the policy-maker is trying to achieve, since it is the first-best outcome. Following an exogenous temporary fall in productivity, output should fall, but then return to its steady-state relatively quickly. Asset prices follow the same path as output. In the flex-price credit model output and asset prices fall drastically, creating a fall in net worth and a large shift in the wealth distribution from borrowers to lenders, which results in further and persistent output falls in future periods. The optimal policy is to try and offset some of the initial output fall, by letting inflation rise temporarily following the adverse productivity shock. That will dampen the fall in net worth, which will dampen the fall in asset prices and therefore reduce the efficiency losses associated with a large shift in the wealth distribution towards less productive agents. That may seem puzzling at first sight, because inflation is costly, and there is no reason to dampen the initial output fall when considering the initial period in isolation. But the efficiency loss associated with inflation and the dampened output response are offset by the efficiency gains from preventing the credit mechanism from lowering future output. Output fluctuations create dynamic externalities due to their effect on future output via credit markets, so it is efficient to offset output fluctuations. The dynamic nature of this trade-off between current inflation and future output loss implies that neither the gap between output and its flexible-price level, nor the gap between output and its fully efficient level are adequate descriptions of monetary policy objectives.

So if credit frictions are thought to be a quantitatively important feature of the economy, monetary policy should not stabilise inflation too aggressively following a shock that pushes output and inflation in opposite directions. That policy prescription stands in contrast to the prescription from standard monetary models with nominal rigidities, where it is optimal to stabilise the prices that are subject to nominal rigidities fully and instantaneously. The variability in inflation that has to be tolerated under optimal policy is small: it is in the range of the variability of inflation currently observed in e.g. the US. One might be tempted argue that “price stability ”is therefore still a good approximation of optimal monetary policy. But the reduction in output variability is large, relative to the output variability achieved under price stability. That implies that the costs of stabilising inflation too aggressively can be large too.

Chapter 2

House Prices, Monetary Policy and Consumption: A Financial Accelerator Approach

2.1 Introduction and Related Literature

House prices in the United Kingdom, Australia, the Netherlands, and more recently in the United States, have received a great deal of attention from policy-makers and economic commentators. It is often assumed that if house prices are growing rapidly, consumption growth will be strong too. Recent minutes of the Monetary Policy Committee meetings in the United Kingdom stated: '...the continuing strength in house prices would tend to underpin consumption...' (April 2001). Similarly, the Fed Chairman Alan Greenspan stated 'And thus far this year, consumer spending has indeed risen further, presumably assisted in part by a continued rapid growth in the market value of homes' (Monetary Policy Report to Congress, 18 July 2001). In this paper I study the role that house prices play as collateral for household borrowing. In 2001, the value of housing represented more than 40% of total UK household wealth. While in principle any asset could be used as collateral, housing is by far the easiest asset against which to borrow. Indeed, 80% of all household borrowing in the UK is secured on housing. To further justify our focus on housing as distinct from other assets, it is useful to consider why houses are different. Most consumers live in the houses they own and value directly the services provided by their home. So the benefit of an increase in house prices is directly offset by an increase in the opportunity cost of housing services. An increase in house prices does not generally shift the aggregate budget constraint outwards. Even if one considers finitely-lived households, the capital gain to a last-time seller of a house represents a redistribution away from a first-time buyer, so house price changes can redistribute wealth, but not increase it in aggregate. This contrasts with financial assets: an increase in, say, the value of future dividends on equities due to an increase in productivity shifts the aggregate budget constraint out and can therefore lead to an increase in aggregate consumption. So it is not obvious that there is a traditional "wealth effect "from housing in the way that we think of a wealth effect arising from a change in the value of households' financial assets.

There are many reasons why house prices and consumption may move together. If consumers are optimistic about economic prospects, they are likely to increase their consumption of housing and non-housing goods alike. House prices are also correlated with the volume of housing transactions (see e.g. Stein (1995)). In turn, transactions seem to be correlated with consumption as people buy goods that are complementary to housing, such as furniture, carpets and major appliances. House prices also affect the economy because, in the case in the United Kingdom, they enter directly into the retail price index via housing depreciation, which depends on the level of house prices. The focus of this paper is that house prices may also have a direct impact on consumption via credit market effects. Houses represent collateral for homeowners, and borrowing on a secured basis against ample housing collateral is generally cheaper than borrowing against little collateral or borrowing on an unsecured basis (via a personal loan or credit card). So an increase in house prices makes more collateral available to homeowners, which in turn may encourage them to borrow more, in the form of mortgage equity withdrawal (MEW), to finance desired levels of consumption and housing investment. The increase in house prices may be caused by a variety of shocks, including an unanticipated reduction in real interest rates, which will lower the rate at which future housing services are discounted.

In this paper, I model credit frictions in the consumption/house purchase decision. My motivation is based on three observations for the United Kingdom. (i) House prices are strongly cyclical, which leads to substantial variation in households' collateral position (or loan to value ratio, or net worth) over the business cycle. (ii) The amount of secured borrowing to finance consumption is closely related to this collateral position. (iii) The spread of mortgage rates over the risk-free interest rate varies with the collateral position of each household. Moreover, unsecured borrowing rates, which are the marginal source of finance once collateral has been exhausted, are much higher than mortgage rates. These facts suggest credit frictions may be important in understanding the relationship between interest rates, house prices, housing investment and consumption. There are several empirical studies that support the importance of a credit channel in housing investment and consumption decisions. Muellbauer and Murphy (1993,1995,1997) have argued that when consumers are borrowing-constrained, changes in housing values can change their borrowing opportunities via a collateral effect. These authors find significant effects of households' access to credit on consumption and on housing investment in UK aggregate and regional data. Lamont and Stein (1999) find in US regional data that households with weak balance sheets adjust their housing demand more strongly in the face of income shocks. This is interpreted as consistent with a strong role for borrowing constraints. Iacoviello and Minetti (2000) find evidence for several European countries, including the United Kingdom, that households' aggregate borrowing costs vary with aggregate balance sheet strength.

I therefore propose a general equilibrium model, based on the financial accelerator model of BGG (1999), that describes how this credit market channel may form part of the monetary transmission mechanism. The model focuses on the macroeconomic effects of imperfections in credit markets. Such imperfections generate premia on the external cost of raising funds, which in turn affect borrowing decisions. Within this framework, endogenous developments in credit markets—such as variations in net worth or collateral—work to amplify and propagate shocks to the macroeconomy. A positive shock to economic activity causes a rise in housing demand, which leads to a rise in house prices and so an increase in homeowners' net worth. This decreases the external finance premium, which leads to a further rise in housing demand and also spills over into consumption demand.

I also consider the implications for monetary policy of recent structural changes in the United Kingdom's retail financial markets: following deregulation in the mortgage market, it has become easier and cheaper for consumers to borrow against housing collateral to finance consumption. I show that cheaper access to home equity means that, for a given house price increase, more additional borrowing will be devoted to consumption relative to housing investment. The response of consumption to an unanticipated change in interest rates will therefore be larger, and the response of house prices and housing investment will be smaller. This has important implications for the information content of house prices, because it implies that, even for similar economic shocks, the relationship between house prices and consumption is changing over time.

The paper is organised as follows. Section 2.2 presents some stylised facts and institutional features of the UK housing market. Section 2.3 analyses the empirical relationship between house prices, consumption and monetary policy using a vector autoregression (VAR). Sections 2.4 and 2.5 describe the theoretical model in detail. Section 2.6 presents simulated results for several scenarios of interest. Section 2.7 concludes.

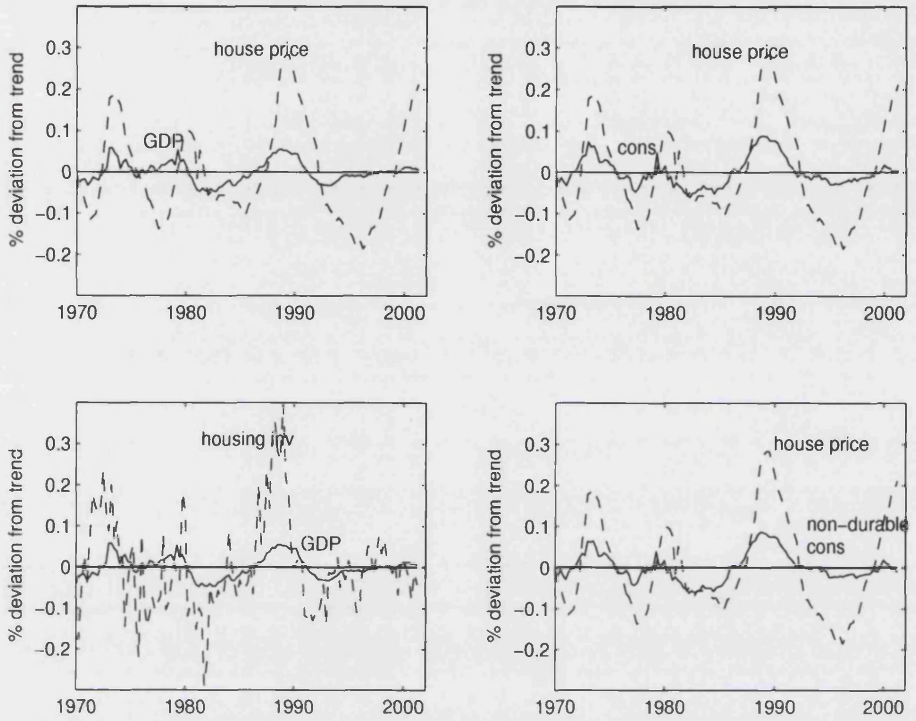

Figure 2.1: Co-movement of housing and macroeconomic variables in the UK

2.2 A brief description of the UK housing market

Figures 2.1 and 2.2 show the cyclical movements of key housing variables[1] (house prices and housing investment), GDP and consumption over the period since 1970. House prices and housing investment co-move closely with each other, and with GDP.

Figure 2.1, panel 2, shows the cyclical movements in house prices and consumption. Breaking down consumption into durables and non-durables,

[1]All variables have been detrended by taking logs and then regressing on a constant, a linear trend and a quadratic trend.

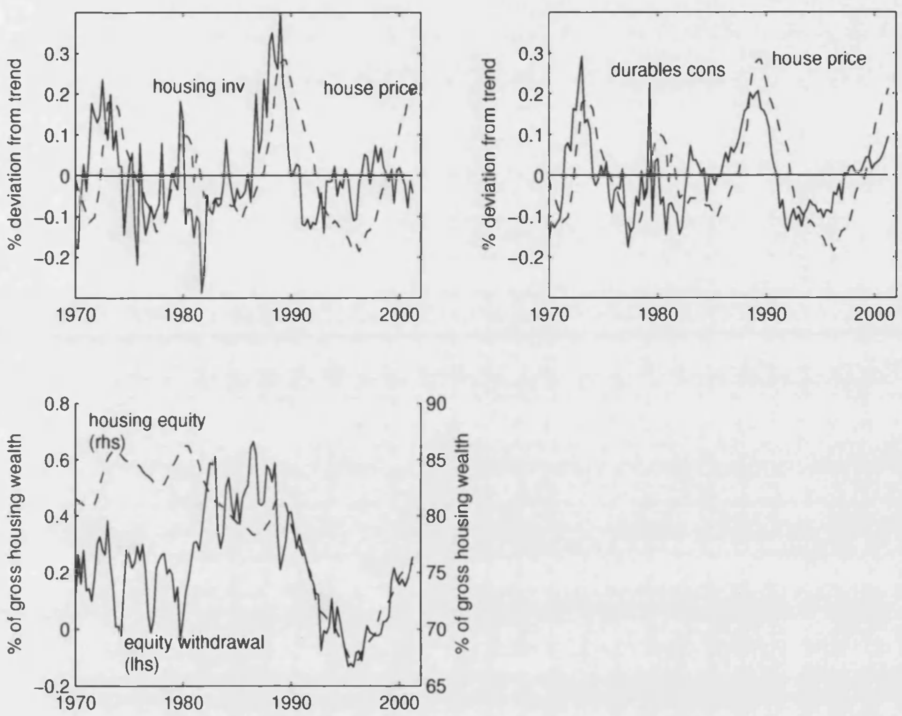

Figure 2.2: Co-movement of housing and macroeconomic variables in the UK (continued)

Table 2.1: Absolute and relative standard deviations of detrended variables

	std.dev.	std.dev. relative to GDP
HP	0.140	5.3
GDP	0.027	1
C	0.035	1.3
HI	0.118	4.4
DC	0.097	3.6
NDC	0.032	1.2

the strongest relationship seems to be that between house prices and consumption of durable goods. This is consistent with a household credit channel, as purchases of durable goods are more likely to be financed by borrowing, and so will be more sensitive to changes in interest rates if there are frictions in the market for credit. If changes in the extent of credit conditions are in turn correlated with fluctuations in house prices—for example if house prices proxy the availability of housing collateral—then this could generate a strong correlation between house prices and durable goods consumption.[2]

Table 2.1 shows the standard deviation of these variables, as well as their relative standard deviation to that of GDP. House prices, housing investment and durables consumption are respectively 5.3, 4.4 and 3.6 times as volatile as GDP, whereas non-durables consumption has a similar standard deviation to GDP.

Part of the motivation for this paper was the changing nature of the credit mechanism over time due to financial deregulation. A series of reg-

[2]Note that a strong correlation between house prices and durable goods consumption could also arise because both goods are 'lumpy', ie they provide services that last several years. So when consumers learn about an increase in their lifetime income, they are likely to increase their immediate demand for durable goods, including housing, more than for non-durable goods. Nevertheless, it is difficult to achieve the observed amplitude of house prices in a calibrated model without credit frictions.

ulatory measures were removed in the UK in the period 1980-1986 to improve competition in the mortgage market: building societies,[3] who then provided the bulk of mortgage finance, were allowed to access wholesale funding markets, and banks, who traditionally provided only a small fraction of mortgages, were allowed to compete directly with building societies in the mortgage market. Other non-bank entrants—particularly department stores, retailers and insurance companies—have also increasingly been able to offer selected retail financial services, including mortgage products. This resulted in increased competition, and encouraged financial innovation. For mortgages in particular, the restrictions in place in the 1970s and early 1980s had the effect of making withdrawal of equity difficult, if not impossible: homeowners generally needed to move house to increase the value of their loan, and even then low loan-to-value restrictions may have limited the extent of the increase (Wilcox (1985)).[4] As competition increased and restrictions were lifted, households have been able to extract equity more easily when house prices rise. The last panel of Figure 2.2 shows the relationship between aggregate net housing equity[5] and secured borrowing for

[3]Building societies in the United Kingdom are mutually owned retail financial institutions.

[4]There is another financial innovation, which we do not consider in this paper, that is likely to have had an effect on the behaviour of house prices. In the 1970s and early 1980s building societies collectively agreed the mortgage and deposit rates they offered, and were reluctant to change rates frequently. When market interest rates were rising, building societies would end up with below-market interest rates. This reduced the supply of deposits, which was their main source of funding (see Pratt (1980) and Wilcox (1985) for an exposition of these mechanisms). Because building societies were also the main provider of mortgages, interest rate rises had a direct effect on the supply of mortgage loans, which is likely to have amplified any effect of interest rates on house prices.

[5]Net housing equity is calculated as the value of the housing stock less the stock of mortgages, as a percentage share of the value of the housing stock.

consumption, or mortgage equity withdrawal (MEW).[6] Prior to the mid-1980s, there was little relationship between housing equity and mortgage equity withdrawal. From the late-1980s, MEW has become more closely linked to movements in net housing equity as new mortgage products allowing refinancing or additional borrowing at ever-lower transaction costs have become available.

2.3 The effect of monetary policy on house prices: some VAR results

As the relationship between consumption and house prices suggests that a household credit channel may be part of the monetary transmission mechanism, I investigate how house prices are affected by monetary policy. I estimate a small vector autoregression (VAR) model to help evaluate and calibrate our theoretical model. Of course, since I will be arguing throughout this paper that the 1980s are likely to have seen an important change in the transmission mechanism, these VAR results must not be taken too literally. I present them for illustrative purposes. The VAR includes quarterly output, real broad money (M4) balances, oil price, GDP deflator, house price, housing investment, durables consumption, non-durables consumption and the 3-month nominal T-bill rate. The oil price and real money balances are included in this system to reduce the price puzzle.[7] The sample period, after

[6]Broadly speaking, the data series constructed by the Bank of England for secured borrowing for consumption (or MEW) is constructed as total net mortgage borrowing (new borrowing less repayments) less investment in housing.

[7]The price puzzle is the finding that inflation rises following a contractionary monetary policy shock. One interpretation of this finding is that supply variables or measures of

adjusting for lags, is 1975:2 to 1999:4 and 6 lags were used.[8] To identify the monetary policy shock, I order the policy rate last in a recursive identification structure. The implied identifying restriction is that the monetary authorities observe contemporaneous variables when setting interest rates, but all variables respond with a lag to monetary policy shocks. The impulse responses accord with our priors about the effects of monetary policy. The impulse response functions to a monetary tightening (ie a positive interest rate shock) are shown in figure 2.3. Real money balances fall in response to an unexpected monetary tightening. Output falls, and the price level falls after some lag. House prices, housing investment and consumption respond negatively to an unexpected monetary tightening. Housing investment responds more quickly than house prices, and falls by more. The peak response in housing investment occurs after two quarters. The peak response to a 50 basis point shock is estimated to be about 180 basis points. The peak response in house prices occurs later, after five quarters, but is smaller at 80 basis points. Durable goods consumption responds more strongly to a monetary tightening than non-durable goods consumption. The estimated effect of a 50 basis point monetary policy shock on durables consumption is about 80 basis points, whereas the response of non-durables consumption is only 10 basis points.[9]

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incipient inflation are not adequately accounted for in the model, so that the systematic monetary policy response to inflationary shocks is erroneously identified as a monetary policy shock. For a discussion of the price puzzle, see, for example, Sims (1992).

[8] I started with 8 lags and tested down using likelihood ratio tests. The null hypothesis of 5 lags against an alternative hypothesis of 6 lags was rejected at the 1% confidence interval.

[9] The standard-error bands on these impulse response functions are large, because by incorporating all variables at once I have sacrificed degrees of freedom. Introducing variables one by one, as in Christiano, Eichenbaum and Evans (1996), reduces the standard-

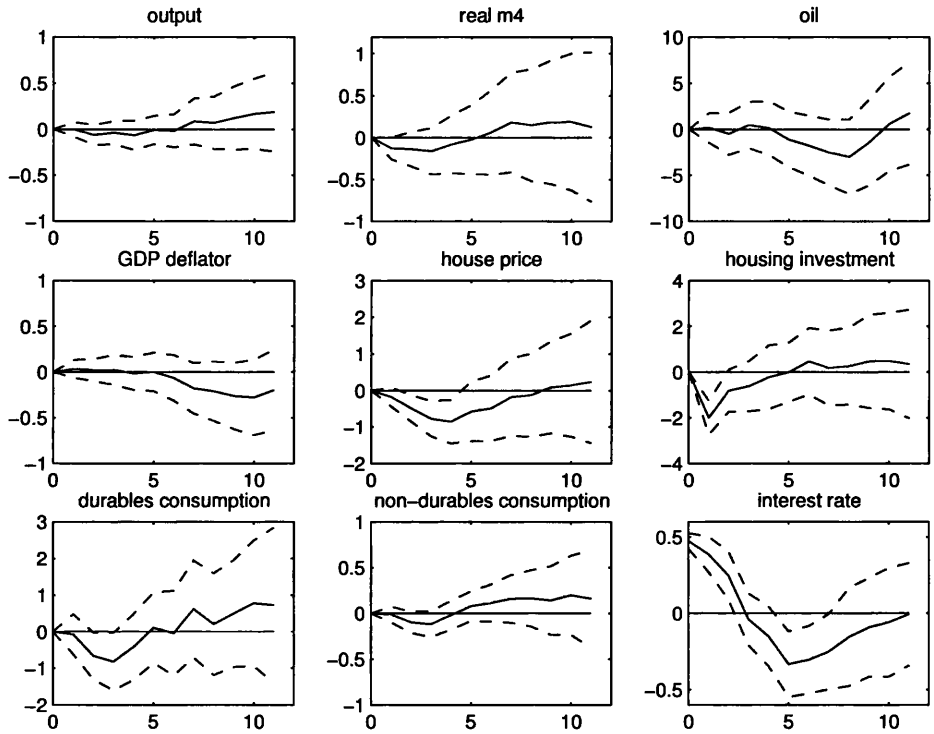

Figure 2.3: Impulse responses to a monetary policy shock

2.4 Modelling the household credit channel

To analyse more formally the implications of financial innovations for the transmission mechanism of monetary policy, a model is needed. Here I sketch the intuitive outline of the model used for the analysis in the subsequent sections. My hypothesis is that house prices play a role because housing is used as collateral to reduce the agency costs associated with borrowing to finance housing investment and consumption. My approach is to apply the BGG model of financial acceleration in the corporate sector to the household sector. The BGG framework links the cost of firms' external finance to the quality of their balance sheet. Because there are parallels between housing investment and business investment, and between house prices and the value of business capital goods, the BGG model provides a useful platform on which to build a model where house prices, housing investment and consumption interact in a general equilibrium framework.

So how should we think of credit frictions in the household sector? Households are exposed to the idiosyncratic risk of fluctuations in the price of their house. On its own, this is not sufficient to generate a credit channel. But personal bankruptcy is associated with significant monitoring costs faced by lenders. Lenders therefore charge a premium over the risk-free interest rate to borrowers. Higher net worth—or lower leverage—reduces the probability of default, and therefore reduces the external finance premium.

In practice, fluctuations in the external finance premium may best be thought of in the following way. When house prices fall, households that are

error bands but leaves patterns of responses broadly unchanged. I report only the more conservative results, based on the full system.

moving home have a smaller deposit (ie net worth) available than they otherwise would for the purchase of their next home. When they have a smaller deposit, they obtain less favourable mortgage interest rates when renegotiating their mortgage, and have less scope for extracting additional equity to finance consumption. Once they have exhausted their collateralised borrowing possibilities, any further borrowing can only be achieved with unsecured credit, which carries much higher interest rates than secured borrowing.[10] Since house prices significantly affect the collateral value of houses, fluctuations in housing prices play a large role in the determination of borrowing conditions that households face.

The main modelling issue is how to generate both consumer borrowing and lending within a general equilibrium framework,[11] without losing tractability and comparability with benchmark macro models. To avoid the complexity inherent in modelling the dynamic optimisation problem of heterogeneous consumers under liquidity constraints, I represent consumer behaviour in a rather stylised way. That is, I think of each household as being a composite of two behavioural types: homeowners and consumers. This separation makes the analysis significantly simpler, but without losing the essence of the financial accelerator mechanism.

On the one hand, 'homeowners' borrow funds to purchase houses from housing producers. Homeowners purchase houses and rent them to con-

[10]For example, the March 2002 quoted average interest rates on variable rate mortgages were 1.65 percentage points above the Bank of England policy rate. Unsecured personal loans and credit cards were charged interest rates of respectively 7.90 and 12.70 percentage points above the policy rate. (MoneyFacts, March 2002.)

[11]Many models of household saving behaviour assume the overlapping generations framework to ensure both borrowing and lending occurs in equilibrium. See, for example, Gourinchas (2000), and Gertler (1999).

sumers. This flow of rental payments within households is captured in the UK national accounts as imputed rents. Homeowners finance the purchase of houses partly with their net worth and partly by borrowing from financial intermediaries. When borrowing from financial intermediaries, homeowners face an external finance premium caused by information asymmetries, just as firms are assumed to do in BGG.

On the other hand, consumers consume goods and housing services. They also supply labour in a competitive labour market. Consumers are assumed to rent housing services from the homeowners. Consumers and homeowners are further linked by a 'transfer' that homeowners pay to consumers.[12] This assumption captures the fact that households use their housing equity to finance consumption as well as housing investment. When house prices increase—and therefore housing equity rises—the household faces the following decision problem. If it were to increases the transfer and hence consumption today, current household utility would go up. But, if transfer payments were kept constant, net worth would increase, reducing the future external finance premium. Thus the household faces a choice between current consumption and a cheaper future finance premium. The optimal allocation—and hence transfer payment—depends on such factors as the elasticity of intertemporal substitution, the sensitivity of the external finance premium with respect to household net worth, and future income uncertainty. In general, there exists a target level of net worth relative to debt (ie leverage), and transfers depend on the deviation of leverage from

[12] Alternatively, one can interpret homeowners as firms who are owned by households and rent houses to the household sector. Then the transfer is equivalent to a dividend paid back to the households.

target. Here I assume a transfer rule that captures the households' decision described above. Transfers are assumed to be increasing in the net worth of the households relative to their debt.

Fluctuations in transfers described in my model can be thought of as borrowing against home equity for consumption (MEW). If one interprets transfers as MEW, then the sensitivity of transfers with respect to home equity will also depend on the transaction costs involved in MEW. Keeping other things constant, if it is less costly to withdraw mortgage equity, MEW becomes more sensitive to households' financial positions and hence to house prices.

In this way, I am able to capture in a parsimonious form the ideas that some elements of the household sector saves while others borrow, and that this process is intermediated through financial markets with credit frictions.

I also assume two types of consumers. Some fraction of consumers have accumulated enough wealth, so that their consumption is well approximated by the permanent income hypothesis (PIH). Their consumption satisfies the standard Euler equation. On the other hand, the consumption of a certain fraction of the population does not. If these latter consumers are impatient (in the sense of Carroll (1997)), or if they are subject to borrowing constraints, their behaviour is similar to rule-of-thumb (ROT) consumers (Campbell and Mankiw (1989)), who spend their current income in each period. Their consumption in each period is equal to their labour income and transfers from the homeowners. To be clear, the ROT consumers are not cut off from all borrowing possibilities, but are assumed to borrow only when the increase in the value of their house gives them access to additional borrowing opportunities. These opportunities in our model are captured by the transfer payment, which should be interpreted as 'borrowing against the value of your house to finance consumption'. The motivation for including the ROT consumers is that, with PIH consumers alone, changes in house prices would not affect the borrowing opportunities for those making non-housing consumption decisions, since PIH consumers are by assumption unconstrained. So collateral values do not affect their borrowing opportunities. In order to create a direct link between housing collateral and non-housing consumption, I therefore introduce ROT consumers.

The rest of the model is standard and broadly follows BGG. I introduce nominal price stickiness in the consumption goods sector so that monetary policy has real effects. Specifically, I assume the Calvo (1983) staggered price setting (see, for example, Woodford (1996), Rotemberg and Woodford (1999), McCallum and Nelson (1999)). House prices are determined by a q-theory of investment with a convex adjustment cost. Monetary policy is assumed to follow a standard Taylor-type feedback rule.

There is large literature, both theoretical and empirical, on consumer behaviour under liquidity constraints.[13] This line of research develops rigorous models of households' optimal behaviour under liquidity constraints and income uncertainty. My model should not be interpreted as an alternative approach to the analysis of consumption and saving under liquidity constraints. Rather, a major challenge for this branch of the literature has

[13]See, for example, Deaton (1991,1992), Carroll (1997), Gourinchas (2000). Although much of the literature focuses on non-durable consumption, Carroll and Dunn (1997) consider the effects of household balance sheet on consumption of both non-durable goods and housing.

been that the solution of household optimisation problems under liquidity constraints and uncertainty is very complex. As a result, the construction of a tractable general equilibrium model is extremely difficult. My approach offers the opportunity to capture many of the implications of this literature for the transmission mechanism of monetary policy in a simple way. I now turn to the derivation of the model.

2.5 The model

2.5.1 Preferences

The treatment of preferences is standard. Consumers consume differentiated consumption goods and housing services. The period-utility of household i is given by

$$\log C_t^i + \xi \log \left(1 - L_t^i\right), \quad \xi > 0 \tag{2.1}$$

where L^i denotes labour, and C_t^i denotes a CES consumption aggregator of form

$$C_t^i = \left[\gamma^{\frac{1}{\eta}} \left(c_t^i\right)^{\frac{\eta-1}{\eta}} + (1-\gamma)^{\frac{1}{\eta}} \left(h_t^i\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{2.2}$$

Here c_t^i is a Dixit-Stiglitz aggregator of differentiated consumption goods, and h_t^i denotes housing services. The differentiated goods are indexed by $z \in (0, 1)$, and the Dixit-Stiglitz aggregator for consumption goods is defined as

$$c_t^i = \left[\int_0^1 c_t^i (z)^{\frac{\varepsilon-1}{\varepsilon}} dz\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.3}$$

The corresponding price index for consumption goods is given by

$$P_{c,t} = \left[\int_0^1 p_t(z)^{1-\varepsilon} \, dz \right]^{\frac{1}{1-\varepsilon}} \tag{2.4}$$

Given a level of composite consumption C_t^i, intra-period utility maximisation implies the following demand functions for each good

$$c_t^i = \gamma \left(\frac{P_{c,t}}{P_t} \right)^{-\eta} C_t^i \tag{2.5}$$

$$h_t^i = (1-\gamma) \left(\frac{P_{h,t}}{P_t} \right)^{-\eta} C_t^i \tag{2.6}$$

where $P_{c,t}$ and $P_{h,t}$ denote prices of consumption goods and rental price of housing, respectively. The composite price index, P_t, is defined as

$$P_t = \left[\gamma P_{c,t}^{1-\eta} + (1-\gamma) P_{h,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{2.7}$$

Demand for each of the consumption goods is given by

$$c_t^i(z) = \left(\frac{p_t(z)}{P_{c,t}} \right)^{-\varepsilon} c_t^i \tag{2.8}$$

I defer intertemporal decision problems to Section 2.5.3 and turn now to the description of house purchase decisions.

2.5.2 House purchase decisions

House purchase decisions of the household sector are made by homeowners. Their problem is modelled in an identical way to the investment decisions of firms in the BGG model. Homeowners purchase houses from housing producers at a price Q_t, and rent houses to their consumers at a rental price $P_{h,t+1}$. Homeowners face an external finance premium, caused by financial market imperfections. Homeowners are risk neutral. As pointed out by BGG and Bernanke and Gertler (1989), this assumption makes both the underlying contract structure and aggregation much simpler. I also assume that homeowners pay transfers to consumers, as discussed later.

At the end of period t, a homeowner purchases a house at nominal price Q_t and rents it to the consumers within their household in the subsequent period $t+1$ at a rental price $P_{h,t+1}$. It finances the purchase of houses partly with its own net worth available at the end of period t, N_{t+1}, and partly by borrowing, b_{t+1}. In real terms, the finance of houses is given by

$$q_t h_{t+1} = N_{t+1} + b_{t+1} \tag{2.9}$$

where $q_t = \frac{Q_t}{P_t}$ is the real price of houses.

Homeowners' demand for houses depends on the expected return on housing and expected marginal financial cost. One unit of housing purchased at time t and rented at time $t+1$ yields the expected gross return, $R_{h,t+1}$, given by

$$E_t\left[R_{h,t+1}\right] = E_t\left[\frac{X_{h,t+1} + (1-\delta)\, q_{t+1}}{q_t}\right] \tag{2.10}$$

where $0 < \delta < 1$ is the depreciation rate of houses and $X_{h,t+1}$ is the rental price relative to the composite price index.

The marginal borrowing cost for a homeowner depends on its financial condition. Following BGG, I assume the existence of an agency problem that makes uncollateralised external finance more expensive than internal finance. The implication of the agency problem is that the external finance premium $f(\cdot)$ can be expressed as a decreasing function of the net worth to asset ratio, $N_{t+1}/q_t h_{t+1}$. I have in mind a costly state verification problem similar to that described in BGG: banks cannot perfectly observe the borrower's ability to repay and banks face an auditing cost to verify repayment ability. The optimal contract will therefore be a debt contract, and when the borrower announces he is unable to repay, the bank takes possession of all the borrower's assets. In the household context, these auditing costs can be interpreted as the costs of legal proceedings to value the borrower's assets and the administration costs of selling the house to realise its collateral value. When there is aggregate uncertainty, the interest payable on the debt contract will be linked to fluctuations in the default probability, which is in turn determined by the leverage of the borrower. The marginal cost of borrowing is given by $f(N_{t+1}/q_t h_{t+1}) R_{t+1}$, $f' < 0$, where R_t is the risk-free real interest rate. The optimality condition for the homeowner's demand for housing is given by:

$$E_t [R_{h,t+1}] = f(N_{t+1}/q_t h_{t+1}) R_{t+1} \tag{2.11}$$

As is shown in BGG, risk neutrality implies that all homeowners choose the same net worth ratio, so equation (2.11) holds for the aggregate level.

The other key aspect is the equation that describes the evolution of net worth of the homeowner. Let V_t denote the value of homeowners at the beginning of period t, net of borrowing costs. It is given by

$$V_t = R_{h,t} q_{t-1} h_t - f\left(N_t / q_{t-1} h_t\right) R_t b_t \tag{2.12}$$

where $R_{h,t}$ is the *ex post* return from housing.

As indicated above, I also assume that homeowners pay transfers, D_t, to consumers in the household. I will discuss D_t in detail later. The homeowner's net worth after he or she pays the transfer is given by

$$N_{t+1} = V_t - D_t \tag{2.13}$$

Note that the price of houses, q_t, may have significant effects on the net worth, as the first term in equation 2.12 can be written as

$$R_{h,t} q_{t-1} h_t = \left(X_{h,t} + (1-\delta)\, q_t\right) h_t \tag{2.14}$$

Thus the price of houses may have strong effects on the net worth and borrowing conditions of households.

Transfers in my model represent the distribution of housing equity (including imputed rent income) between homeowners and consumers.[14] Here I model transfer policy in a simple way, but keeping consistency with underlying economic theory as much as possible. In the economy's steady state—where the leverage ratio is constant—transfers should equal homeowners' rent income minus interest payments and a depreciation allowance.[15]

[14]On the income side of national accounting, imputed rent is counted as households' gross operating surplus.

[15]This condition does hold in the steady state of our model.

In other words, the transfer in steady state is equal to the net return on homeowners' net worth. Consumers can spend this dividend income on consumption.

When house prices increase—and therefore the value of the homeowners, V_t—the household faces the following decision problem. If it increased the transfer and hence consumption today, current household utility would go up. But, if transfer payments were kept constant, net worth would increase, reducing the future external finance premium. Thus the household faces a trade-off between current consumption and a cheaper future premium. The optimal allocation—and hence transfer payment—would depend on factors such as the elasticity of intertemporal substitution, and the slope of function $s(\cdot)$, and future income uncertainty.[16]

I assume a dividend rule that captures the households' decision described above. Transfers are increasing in the net worth of households relative to their assets. That is, the transfer rule is

$$D_t = \chi\left(N_{t+1}/q_t h_{t+1}\right) \tag{2.15}$$

where $\chi' > 0$ and $\chi(\phi) = D$. Here ϕ is the leverage ratio in the steady state, and D is the level of dividend consistent with ϕ.[17]

Of course, in a fully micro-founded model, transfers would also depend on other factors, such as uncertainty about future labour income. However,

[16]The literature on consumption under liquidity constraints studies extensively the implications of labour income uncertainty on optimal consumption-saving decisions. The optimal allocation between transfers (consumption) and retained net worth in our model may have a similar structure, if the model were fully micro-founded.

[17]When I calibrate the model, ϕ is set equal to the average leverage ratio of the UK household sector. This is given by 0.7.

much of our analysis below, in particular the analysis of the effect of house prices on consumption, would go through if we considered a more micro-founded transfer rule.

2.5.3 (Intertemporal) consumption decisions

Now I turn to describe intertemporal consumption decisions. As stated I consider two types of household. A certain fraction of households have accumulated enough wealth so that their consumption decisions are well approximated by the permanent income hypothesis. The other households do not have enough wealth to smooth consumption. If they are facing borrowing constraints or if they are impatient, their marginal propensity to consume out of current income is higher than PIH consumers. I approximate these consumers as rule-of-thumb consumers.[18]

PIH consumers

The assumptions concerning PIH consumers are fairly conventional. The representative PIH consumer can borrow or lend at the (real) riskless rate of return, R_t, and his objective is given by

$$\max E_t \sum_{k=0}^{\infty} \beta^k \left[\log C^p_{t+k} + \xi \log\left(1 - L^p_{t+k}\right)\right] \tag{2.16}$$

[18] An alternative way of getting similar results is to assume patient and impatient consumers, as in Iacoviello (2005). In his model, the impatient consumers behave like our ROT consumers.

Solving the PIH household's problem yields standard first-order conditions for (composite) consumption and labour supply

$$\frac{1}{C_t^p} = \beta E_t \left(\frac{1}{C_{t+1}^p}\right) R_{t+1} \tag{2.17}$$

$$w_t \left(1 - L_t^p\right) = \xi C_t^p \tag{2.18}$$

where w_t is real wage (in terms of composite goods).

Rule-of-thumb consumers

Following Campbell and Mankiw (1989) and others, I assume the ROT consumers consume their current income: that is, the sum of wage income and the transfer paid out by homeowners. In this framework, the ROT consumers have access to mortgage equity withdrawal (MEW), but not to non-secured loans, and the amount they can borrow against the value of their house is represented by the transfer paid out by homeowners. The (composite) consumption of the ROT consumers is given by

$$C_t^r = w_t L_t^r + D_t \tag{2.19}$$

where D_t denotes the transfer they receive from homeowners.[19] The labour supply of the ROT consumers is given by

$$w_t \left(1 - L_t^r\right) = \xi C_t^r \tag{2.20}$$

[19]Bernanke and Gertler (1999) have a similar assumption about entrepreneurs' consumption to generate wealth effects from stock prices.

Let $0 < n < 1$ be a fraction of PIH consumers in the economy. Aggregate consumption is then

$$C_t = nC_t^p + (1-n)\,C_t^r \tag{2.21}$$

and demands for consumption goods and housing services are

$$c_t = \gamma\left(\frac{P_{c,t}}{P_t}\right)^{-\eta} C_t \equiv \gamma X_{c,t}^{-\eta} C_t \tag{2.22}$$

and

$$h_t = (1-\gamma)\left(\frac{P_{h,t}}{P_t}\right)^{-\eta} C_t \equiv (1-\gamma)\,X_{h,t}^{-\eta} C_t \tag{2.23}$$

Aggregate labour supply is defined as

$$L_t = nL_t^p + (1-n)\,L_t^r \tag{2.24}$$

Finally, from (2.18) and (2.20) wage is determined as

$$w_t\,(1-L_t) = \xi C_t \tag{2.25}$$

2.5.4 House producers

House prices are determined by a q-theory of investment. I assume that house producers purchase consumption goods and use them to produce new houses. Investment of I_t units of composite consumption goods yields $h_{t+1} = \Phi\left(I_t/h_t\right) h_t$ units of new housing stock, where $\Phi\left(\cdot\right)$ is assumed to be concave. The assumption of concavity implies convex adjustment costs of housing investment. In equilibrium, the price of housing is given by

$$\frac{q_t}{X_{c,t}} = \Phi'\left(\frac{I_t}{h_t}\right) \tag{2.26}$$

where $X_{c,t}$ is the price of consumption goods relative to the composite price index. As discussed above, changes in house prices will affect the balance sheets of the household sector, and hence their cost of borrowing.

2.5.5 Producers of consumption goods

For simplicity, I assume capital is fixed and labour is the only variable input. I assume a Cobb-Douglas production function of the form

$$y_t(z) = A_t\bar{K}(z)^{\alpha} L_t(z)^{1-\alpha} \tag{2.27}$$

Following the large literature on monetary business cycles, I assume that prices of consumption goods are sticky. Specifically, I assume the Calvo (1983) staggered price setting (see, for example, Woodford (1996), Rotemberg and Woodford (1999)). In each period, only a fraction θ of sellers are allowed to change their prices. The seller indexed z who gets a chance to change his price, chooses his price in order to maximise

$$E_t \sum_{k=0}^{\infty} \theta^k \frac{\Lambda_{t,t+k}}{P_{t+k}} \left[p_t(z)\, y_{t+k}(z) - W_{t+k} L_{t+k}(z)\right] \tag{2.28}$$

subject to its demand condition

$$y_{t+k}(z) = \left(\frac{p_t(z)}{P_{c,t+k}}\right)^{-\varepsilon} Y_{t+k} \tag{2.29}$$

where $\Lambda_{t,t+k}$ is the shareholder's intertemporal marginal rate of substitution. The term Y_t denotes aggregate demand for consumption goods. This consists of consumption demand, investment demand, and government expenditure. That is,

$$Y_t = c_t + I_t + G_t \tag{2.30}$$

The first-order condition for optimal pricing is given by

$$E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} \left[\frac{P_{c,t+k}}{P_{t+k}} \left(\frac{p_t(z)}{P_{c,t+k}} \right)^{-\varepsilon} Y_{t+k} \left\{ \frac{p_t(z)}{P_{c,t+k}} - \frac{\varepsilon}{\varepsilon - 1} mc_{t+k} \right\} \right] = 0 \tag{2.31}$$

where mc_{t+k} is real marginal cost at time $t + k$ in terms of consumption goods, given by

$$\frac{W_{t+k}}{P_{c,t+k}} \left(\frac{y_{t+k}(z)}{A_{t+k}} \right)^{\frac{1}{1-\alpha}} \tag{2.32}$$

2.5.6 Parameterisation

I have aimed to keep the parameterisation of the model fairly standard. The discount rate β equals 0.99. The steady state quarterly real interest rate is therefore $1/\beta$ or 1.01, which implies an annual real interest rate of about 4%. The elasticity of substitution η between consumption and housing services is equal to 1. Together with the parameter γ in the CES consumption aggregator, this pins down the steady-state ratio of imputed rent to total consumption at about 12%, which is consistent with aggregate data.[20] The depreciation rate of housing is set at an annual rate of 2%. The elasticity of

[20] I experimented with a lower value for η, which would imply some degree of complementarity between housing and consumption. However, as long as γ is increased to correct for obtaining reasonable share of imputed rents, there is no significant effect in the model simulations from having changed η.

the price of capital with respect to the investment capital ratio, ψ, is set to 0.5. BGG suggest that a reasonable range for this parameter is 0-0.5. The disutility of labour parameter, ε, is set to 1. The capital share in aggregate production, α, is set to 0.33. Following BGG, I set labour supply elasticity to 3.

The parameter θ governs the stickiness of prices of consumption goods. I set it at 0.75, which implies that the average period between price adjustments is four quarters. There are three shock processes in our model: productivity shocks, demand shocks and monetary policy shocks. Productivity and demand shocks are assumed to be autocorrelated with autoregressive parameters $\rho_A = 0.95$ and $\rho_G = 0.9$ respectively. The monetary authorities are assumed to follow a smoothed feedback rule with autoregressive parameter $\rho_R = 0.9$ and a coefficient on lagged inflation of 0.2, implying a long-run response to inflation of $\lambda_\pi = 2$. The linearised feedback rule has the form

$$\widehat{R}_t = (1 - \rho_R)\lambda_\pi \widehat{\pi}_t + \rho_R \widehat{R}_{t-1} + \widehat{\varepsilon}_{R,t} \tag{2.33}$$

In order to evaluate model covariance of the variables, I also need to specify variances of the shocks. I specify all shocks to have a variance of $(0.01)^2$ which is well within the range used in the literature (e.g Batini, Harrison and Millard (2001), Nelson (2000) and Nelson and Neiss (2003)).

The parameters governing the financial accelerator are similar to those used in BGG. I assume that underlying these parameters is a model of costly state verification, for example the one derived explicitly in BGG. The steady state annual external finance premium is 200 basis points, and the ratio of net worth to capital is 0.7, which is the average historical leverage ratio of UK households. This may seem low at first sight, since households are often thought of as highly leveraged. While it is true that first-time buyers only put down deposits of 0.2 or less, the household sector in aggregate has a much higher net worth ratio reflecting the fact that mature mortgages have been partly paid off and that many households actually own their houses outright. The only thing about these parameters that is important for our results is that the cost of external finance is some upward sloping function of leverage. The individual parameters underlying the financial accelerator mechanism simply act as scaling factors on the overall acceleration. I set the elasticity of the external finance premium with respect to leverage equal to 0.1—higher than the BGG value of 0.05— which, together with the elasticity of the capital price is used to match the empirical relative responses of consumption, house prices and housing investment to a monetary policy shock once the financial accelerator is switched on. The adjustment factor s on the dividend rule is set at 3. This is the estimated average elasticity of mortgage equity withdrawal with respect to the net worth ratio. In other words, if the net worth of the aggregate UK household sector rises by 1%, the amount of equity withdrawn will increase by 3%. In our structural change experiment, I vary this baseline parameter as described in Section 2.6.2. The share of rule-of-thumb consumers is set at 0.5. For the United Kingdom, there is no consensus on this share in the literature, but a reasonable range appears to be 0-0.6 (Bayoumi (1993), Jappelli and Pagano (1989), Campbell and Mankiw (1989)). I use 0.5 as our baseline scenario.

In the next section, I use the model to illustrate the implications for monetary policy of recent financial innovations. To obtain the simulated paths for model variables, the model is first log-linearised, and then solved using the method of King and Watson (1998).

2.6 Model simulations

2.6.1 With and without financial accelerator

So how does the financial accelerator work in our world? A positive shock to the economy causes a rise in housing demand, which leads to a rise in house prices and a rise in homeowners' net worth. This causes decrease in the external finance premium, which leads to a further rise in housing demand and a rise in the transfer paid back to consumers. This rise in the transfer generates a further increase in consumption. As in BGG, credit market frictions amplify and propagate shocks to the economy.

In this section, I present some impulse responses of the model to an expansionary monetary policy shock.[21] Figure 2.4 shows the impulse responses with and without the financial accelerator. In response to a 50 basis points monetary policy shock, a baseline model with the financial accelerator turned off produces peak responses in consumption, house prices and housing investment of 56 basis points, 48 basis points and 111 basis points respectively. Compared to the VAR results in section 2.3, both the house price and housing investment responses are too low. The peak VAR responses of house prices and housing investment to a 50 basis points mon-

[21]Here I set a monetary policy shock as a 50 basis points (annualised) fall in nominal interest rates. This corresponds approximately to a one standard deviation monetary policy shock from the estimated VAR.

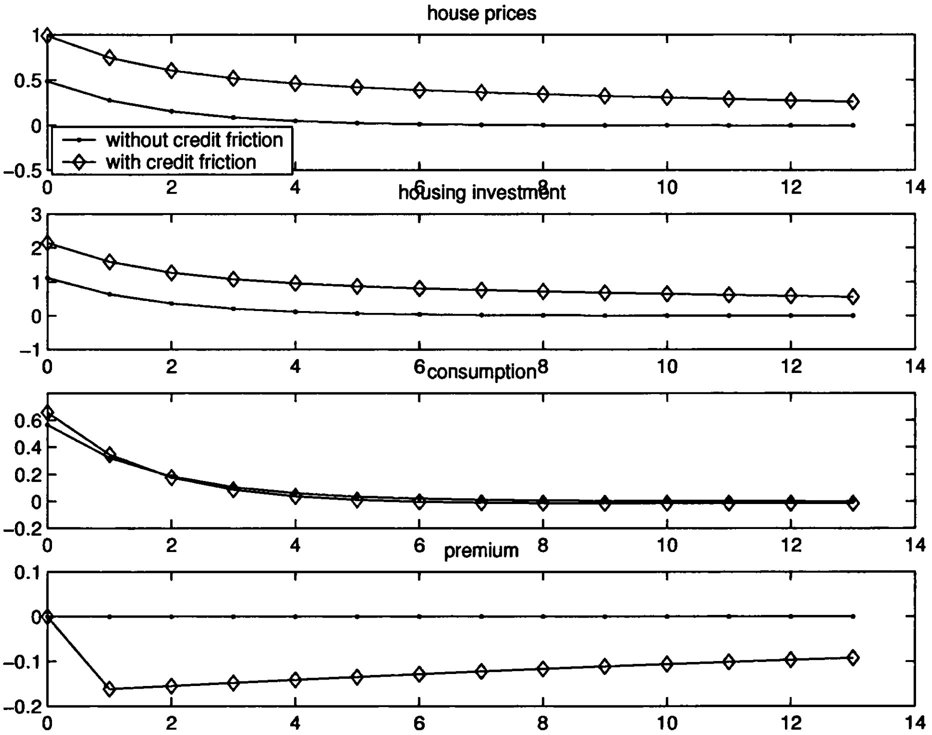

Figure 2.4: Responses to a monetary policy shock: with and without credit frictions

etary policy shock are 80 basis points and 180 basis points respectively. When the financial accelerator is switched on, the model responses of consumption, house prices and housing investment increase to 66 basis points, 99 basis points and 214 basis points respectively, much more in line with the VAR evidence.

2.6.2 Deregulation: Increased access to housing equity

In Section 2.2, I discussed that the transaction cost of extracting equity from housing has fallen, and that product development is likely to reduce them further in coming years: mortgage equity withdrawal and net housing equity have become more closely linked. In this section I examine the implications of this structural change for monetary policy.

In our model, households face a trade-off when house prices rise: they can either withdraw the additional equity for consumption or they can use their stronger balance sheet to lower the rate at which they can borrow. This trade-off is captured by the adjustment parameter on the transfer stream between the house-owning and consuming part of the household. A fall in transaction costs increases the elasticity of the transfer with respect to housing equity.

Figure 2.5 shows the responses of key variables to an unexpected monetary policy loosening when the elasticity of transfer with respect to housing equity is changed from 3 to 30.[22] The net effect of reducing transaction costs on housing investment and house prices is to dampen the response to the policy loosening (from 214 basis points to 110 basis points for housing investment, and from 99 basis points to 45 basis points for house prices). Its effect on consumption is to heighten the response (from 66 basis points to 75 basis points). The intuition is as follows: following the monetary policy shock, households respond to the unexpected increase in house prices. When transaction costs are lower, they use more of the increased housing equity to finance consumption. The balance sheet improvement is therefore smaller and shorter-lasting than it would otherwise have been, and this dampens the positive response of housing investment and house prices. Table 2.2

[22]The estimated elasticity of MEW with respect to net housing equity over the recent period 1990-2000 is 30.

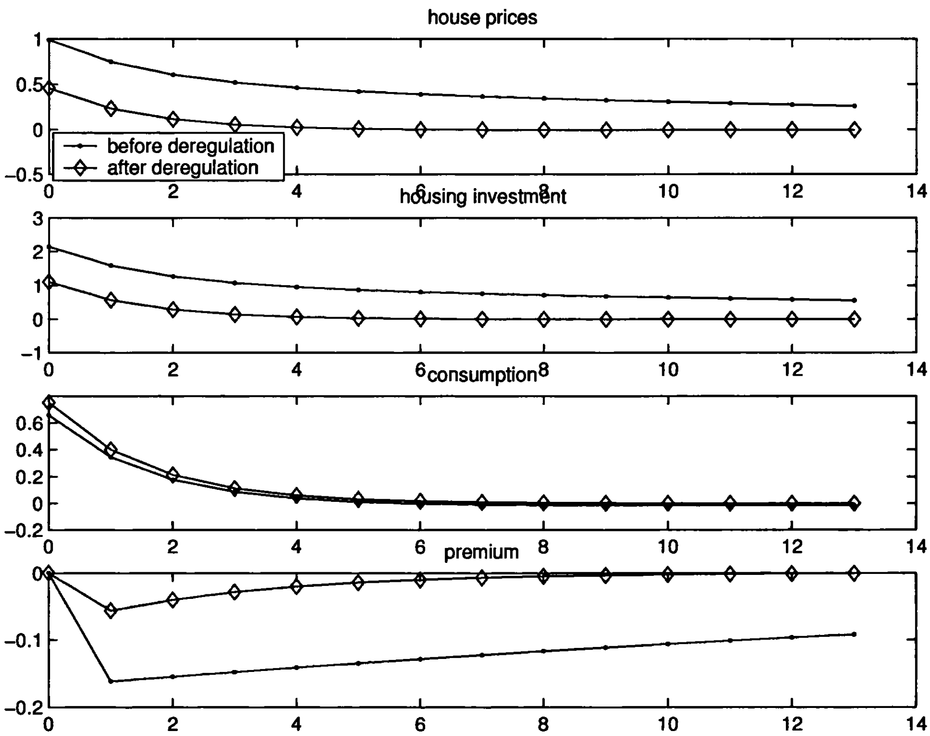

Figure 2.5: Responses to a monetary policy shock: before and after deregulation

Table 2.2: Model responses under different assumptions about the financial accelerator

Peak response of	C_t	q_t	I_t
without financial accelerator	0.56	0.48	1.11
with financial accelerator	0.66	0.99	2.14
before deregulation	0.66	0.99	2.14
after deregulation	0.75	0.45	1.10

Results in percentage deviations from steady state, following a 50bp monetary policy contraction

summarises these findings.

The quantitative impact of deregulation on consumption is therefore to increase the peak response to monetary policy shocks by 14%. The peak house price response falls by about 55%, and the peak housing investment response falls by 49%. A key parameter driving these results is the elasticity of the transfer with respect to housing equity. Although this parameter can be estimated from data as the elasticity of MEW with respect to housing equity, there are many uncertainties surrounding this estimate. MEW is a flow, which has historically been positive as well as negative. To estimate the elasticity of this flow over short subsamples requires taking a stand on the appropriate 'average MEW', as well as the appropriate sub-sample period. Although I characterise my experiment as comparing 'before deregulation' with 'after deregulation', in reality there are probably three sub-periods in the data, corresponding roughly to a decade each. From 1970-79 could be characterised as before deregulation. From 1980-89 is a period of gradual deregulation. The period 1990-2001 could be characterised as after deregulation. Estimates of the elasticity of MEW with respect to housing equity are highly sensitive to the precise sub-period chosen as some of the sub-period averages of MEW are close to zero. And in the early period 1970-79 the relationship between MEW and net housing equity is not statistically significant. To check the robustness of my approach, I therefore also explore an alternative calibration strategy for the elasticity parameter. Rather than estimating it directly, I calibrate it to match the data covariance between consumption and MEW. The data covariance is 0.0050 for the period 1970-79, 0.0083 for the period 1980-89 and 0.035 for the period 1990-2001. To match these covariances in the model, I need to set the elasticity parameter at 1.6, 2.4 and 42 respectively. This is very much in the same range as our estimates of 3 and 30 based on the direct estimation approach. If anything, the direct estimation approach perhaps underestimated the difference between the pre and post-deregulation regimes.[23]

It could also be argued that certain types of structural change in financial markets that accompanied deregulation should change the deep parameters of the financial accelerator mechanism, which have been kept constant in my experiments. The most likely change is that the degree to which uncertainty about individual borrower quality may be reduced as a result of improved *ex ante* monitoring techniques of potential borrowers. This in turn would increase the steady-state leverage ratio. But as shown by Hall and Vila Wetherilt (2002), if increases in the steady-state ratio are due to lower

[23]Another robustness check I performed was to investigate the impact of the curvature of the utility function. The baseline model has log utility, implying an intertemporal rate of substitution of 1, which is on the high side compared to direct estimates from consumption data (Hall (1988)). A lower intertemporal rate of substitution clearly scales down the consumption response to monetary policy shocks, but increases the difference between the pre and post-deregulation regimes.

uncertainty, they would be accompanied by a lower steady-state external finance premium and a lower elasticity of the external finance premium with respect to leverage. Higher leverage would work to increase amplification, but a lower finance premium and a lower elasticity would partly offset this. Overall, Hall and Vila Wetherilt (2002) show that amplification would still increase, but only moderately.[24] What would be the impact for the model? Higher amplification after the structural change would act as a scaling factor for all of the model responses: relative to the 'after deregulation scenario', consumption and house price responses would be slightly higher. The conclusions about the relative movements of consumption (more responsive to monetary policy shocks) would still be valid. The conclusion about house prices (less responsive to monetary policy shocks) would still be valid if the moderate amplification effect from higher leverage is outweighed by the strong dampening effect of better access to borrowing for consumption.

2.7 Conclusion

In this paper I have presented a general equilibrium model, based on the financial accelerator model of Bernanke, Gertler and Gilchrist (1999) that describes how a credit channel may form part of the monetary transmission mechanism. The model focuses on the macroeconomic effects of imperfections in credit markets used by households. Such imperfections generate premia on the external cost of raising funds, which in turn affect borrow-

[24]Hall and Vila Wetherilt (2002) conclude that the amplification effect is moderate even though the uncertainty parameter has halved. The reduction in uncertainty I consider here, due to improved monitoring technology in retail financial markets, is unlikely to have been that large.

ing decisions. Within this framework, endogenous developments in credit markets—such as variations in net worth or collateral—work to amplify and propagate shocks to the macroeconomy. A positive shock to economic activity causes a rise in housing demand, which leads to a rise in house prices and so an increase in homeowners' net worth. This decreases the external finance premium, which leads to a further rise in housing demand and also spills over into consumption demand.

I also consider the implications for monetary policy of structural changes in the United Kingdom's retail financial markets: following deregulation in the mortgage market, it has become easier and cheaper for consumers to borrow against housing collateral to finance consumption. I show that cheaper access to home equity means that, for a given house price increase, more borrowing will be devoted to consumption relative to housing investment. The response of consumption to an unanticipated change in interest rates will therefore be larger, and the response of house prices and housing investment will be smaller. In other words, whether the financial accelerator has most of its effect on house prices or consumption depends on the degree of deregulation: in a highly deregulated mortgage market, the effect on house prices will be muted, but the effect on consumption will be amplified.

My work suggests therefore that empirical models that contain house prices and consumption may have unstable coefficients, even if fundamental shocks (e.g. productivity, government spending and monetary policy shocks) are correctly identified. A given change in house prices is likely to be associated with a larger change in consumption in the post-deregulation period.

2.8 Appendices

2.8.1 Data sources

I used the following data series. Item codes from the National Statistics Office are provided in parentheses where applicable.

Real house prices: DETR house price index deflated by the GDP deflator (YBGB). Real consumption (ABJR). Real durables consumption (AEIW). To construct non-durables consumption, I subtracted durables consumption from total consumption, and also subtracted imputed rents (CCUO), which reflects the consumption of housing services by owner-occupiers. My measure of non-durables consumption therefore excludes consumption of housing services. Mortgage Equity Withdrawal: nominal MEW (Bank of England estimate). Net housing equity: nominal housing stock (National Statistics estimate) minus total secured borrowing by households (Bank of England) divided by the nominal housing stock. Real housing investment (DFEA). Household disposable income (RPQK).

2.8.2 Complete log-linearised model

I log-linearise the model around the steady state with constant prices. In the steady state, the leverage ratio of the household sector is assumed to be ϕ. Furthermore, I normalise the adjustment cost function of housing investment such that the relative price of houses in the steady state is unity. Below, variables with hats denote per cent deviations from the steady state, and variables without time subscripts denote the steady-state values of those.

Aggregate demand

$$\hat{Y}_t = \frac{c}{Y}\hat{c}_t + \frac{I}{Y}\hat{I}_t + \frac{G}{Y}\hat{G}_t \tag{2.34}$$

$$\hat{C}_t = n_p\hat{C}_t^p + (1 - n_p)\,\hat{C}_t^r \tag{2.35}$$

$$\hat{C}_t^p = E_t\hat{C}_{t+1}^p - \hat{R}_t \tag{2.36}$$

$$\hat{C}_t^r = c_w\hat{w}_t + (1 - c_w)\,\hat{D}_t \tag{2.37}$$

$$\hat{c}_t = \hat{C}_t - \eta\hat{X}_{c,t} \tag{2.38}$$

$$\hat{h}_t = \hat{C}_t - \eta\hat{X}_{h,t} \tag{2.39}$$

$$\hat{X}_{c,t} = -\frac{1-\gamma}{\gamma}\frac{X_h^{1-\eta}}{X_c^{1-\eta}}\hat{X}_{h,t} \tag{2.40}$$

$$E_t\hat{R}_{h,t+1} = \hat{R}_{t+1} - v\left\{\hat{N}_{t+1} - \left(\hat{q}_t - \hat{h}_{t+1}\right)\right\} \tag{2.41}$$

$$\hat{R}_{h,t+1} = (1 - \mu)\,\hat{X}_{h,t+1} + \mu\hat{q}_{t+1} - \hat{q}_t \tag{2.42}$$

$$\hat{q}_t = \psi\left(\hat{I}_t - \hat{h}_t\right) + \hat{X}_{c,t} \tag{2.43}$$

Aggregate supply

$$\hat{Y}_t = \hat{A}_t + (1 - \alpha)\,\hat{L}_t \tag{2.44}$$

$$m\hat{c}_t = \hat{w}_t + \frac{1}{1-\alpha}\hat{Y}_t - \frac{1}{1-\alpha}\hat{A}_t + \hat{X}_{c,t} \tag{2.45}$$

$$\hat{w}_t = \hat{C}_t + \xi\hat{L}_t \tag{2.46}$$

$$\hat{\pi}_{c,t} = \kappa_1 m\hat{c}_t + \beta E_t\hat{\pi}_{c,t+1} \tag{2.47}$$

Evolution of state variables

$$\hat{h}_{t+1} = \delta \hat{I}_t + (1-\delta)\,\hat{h}_t \tag{2.48}$$

$$\begin{aligned}\hat{N}_{t+1} &= R_h \hat{V}_t - (R_h - 1)\,\hat{D}_t \\ &= R_h \left[(1+\phi)\,\hat{R}_{h,t} - \phi v \left(\hat{q}_{t-1} + \hat{h}_t\right) + (1+\phi v)\,\hat{N}_t - \phi \hat{R}_t\right] \\ &- (R_h - 1)\,\hat{D}_t \end{aligned} \tag{2.49}$$

$$\hat{D}_t = s\left(\hat{N}_{t+1} - \left(\hat{q}_t - \hat{h}_{t+1}\right)\right) \tag{2.50}$$

Monetary policy and exogenous shocks

$$\hat{R}^n_t = \rho_R \hat{R}^n_{t-1} + r_\pi \hat{\pi}_t + \varepsilon_{R,t} \tag{2.51}$$

where the relationship between nominal and real rates are given by

$$R^n_{t+1} = R_{t+1} + E_t \pi_{t+1} \tag{2.52}$$

$$\hat{G}_t = \rho_G \hat{G}_{t-1} + \varepsilon_{G,t} \tag{2.53}$$

$$\hat{A}_t = \rho_A \hat{A}_{t-1} + \varepsilon_{A,t} \tag{2.54}$$

with

$$n_p \equiv n\frac{C^p}{C}, \quad c_w \equiv \frac{wL^r}{C^w} \tag{2.55}$$

$$v \equiv \frac{f'(\phi)}{f(\phi)}\phi, \quad \mu \equiv \frac{X^h}{X_h - (1-\delta)} \tag{2.56}$$

$$s \equiv \frac{\chi'(\phi)}{\chi(\phi)}\phi, \quad \psi \equiv \frac{\left(\Phi\left(I/h\right)^{-1}\right)'}{\left(\Phi\left(I/h\right)^{-1}\right)''} \tag{2.57}$$

$$\kappa_1 \equiv \left(\frac{1-\theta}{\theta}\right)(1-\theta\beta) \tag{2.58}$$

Equation 2.34 is resource constraint.[25] Equation 2.35 defines aggregate consumption, while equations 2.36 and 2.37 represent consumption of each of the PIH and ROT consumers. Equations 2.38 and 2.39 are the demands for consumption goods and housing services, respectively. Equation 2.40 is an identity. Equations 2.41, 2.42, and 2.43 characterise housing investment demand. They are log-linearised versions of 2.11, 2.10, and 2.26, respectively. Equation 2.41 represents the relationship between the external finance premium and household net worth relative to gross value of housing. A rise in this ratio reduces the cost of external finance. Equation 2.42 defines *ex post* return from housing investment.

Equation 2.44 is the log-linearised Cobb-Douglas production function, under the assumption that capital is fixed. Equations 2.45 and 2.46 jointly characterise labour market equilibrium. Equation 2.47 is a variant of the New Keynesian Phillips curve.

Equation 2.48 is the log-linearised version of the conventional transition equation of housing capital. The evolution of net worth, 2.49, depends on the net return from housing investment minus dividend payments. This is obtained from log-linearised equations of 2.9, 2.12, and 2.13. The dividend rule is given by 2.50, which is the log-linearised version of 2.15. Dividends

[25]Here I omit resources devoted to monitoring costs of the household sector, but this does not significantly change the analysis below. See, also, BGG.

are assumed to be increasing in the ratio of net worth to the gross value of housing.

Equation 2.51 is the monetary policy rule. Following a large literature, I assume the short-term nominal interest rate is the policy instrument. This does not imply that such a rule is an accurate description of monetary policy in the United Kingdom, but it offers a convenient way in which to capture an active monetary policy. Finally, equations 2.53 and 2.54 represent the exogenous processes of technology and government expenditures.

Chapter 3

Imperfect credit markets and the transmission of macroeconomic shocks

3.1 Introduction

This chapter aims to address the following questions. If credit market imperfections are an important feature of the economy, how might they affect the economy's response to shocks? Furthermore, if monetary policy can influence real outcomes in the short run, how do credit market frictions alter the effect of systematic monetary policy?

Any model to address these questions needs to have the following features: a role for credit and a role for monetary policy. To generate a role for credit in the economy, it is necessary to introduce some imperfection so that heterogeneity across agents matters. The model in this paper will feature agents who endogenously become either lenders or borrowers, and who operate in a credit market where enforcement problems exist. In such a setting, the distribution of wealth across agents will affect aggregate outcomes.

To allow monetary policy to influence aggregate real outcomes, there has to be some friction, or non-neutrality, preventing instantaneous adjustment of prices, wages, debt contracts or asset portfolios. My approach is to assume that product prices cannot fully adjust, but the results of the paper do not hinge crucially on this particular choice of non-neutrality.

The model economy consists of ex-ante identical entrepreneurs who can produce intermediate goods using capital, which is in fixed supply (e.g. land), and a variable input. Using the approach of Kiyotaki (1998), I assume that some entrepreneurs are more productive than others, but spells of high productivity do not last, and arrive randomly. While an entrepreneur is highly productive, he will want to invest as much as possible in his own technology. Entrepreneurs with low productivity, on the other hand, would rather invest in the technology of high productivity entrepreneurs, as this generates superior returns. Let us therefore call the entrepreneurs that currently have high productivity 'producers', and the entrepreneurs with low productivity 'investors'. In principle, investors could lend to producers so that producers end up applying their technology to the entire capital stock. This would be the first-best outcome. But it is assumed that there are credit market imperfections, so borrowing is permitted against collateral. The larger the net worth of the borrower, the more capital he can buy. Moreover, since capital serves as collateral as well as a factor of production, an increase in the value of capital will increase the net worth of a producer who already had some capital installed and will therefore allow him to invest more. The model also features workers, who provide labour to entrepreneurs. Workers do not have access to productive technology. They therefore do not hold capital. This also means that they do not hold any collateral, so they are not able to borrow. Finally, while entrepreneurs sell their intermediate goods output in competitive markets, there is a monopolistically competitive sector that buys intermediate inputs and produces diversified final consumption goods. It is assumed that not all final goods producers can adjust the nominal price of their output in each period.

In the baseline model, I assume that some fraction of final goods producers have to set prices one period in advance. Not all prices can therefore adjust instantaneously, and nominal changes can have short-run real effects. In traditional models with this type of price stickiness, most or all of the short-run real effects die out when all agents have been able to change their prices. But in this model, the redistribution of wealth caused by any nominal shock will continue to have real effects even after all prices have adjusted, because the wealth distribution across agents, which affects aggregate outcomes, only returns to its stationary distribution slowly as producers rebuild their share of wealth. Monetary policy therefore works through wealth redistribution as well as through sticky prices, a powerful mechanism emphasised by Fisher (1933).

The effect of the wealth distribution on aggregate output works as follows. Following a shock that reduces current output and/or the price of capital, the net worth of producers falls by more than the net worth of investors, because producers are highly leveraged. This means that producers can only afford to buy a lower share of the total capital stock for production in the following period. Because capital shifts to those with lower productivity, this reduces expected future returns, which depresses the value of capital today, and exacerbates the initial redistribution of wealth from producers to investors. If the difference in productivity between investors and producers is high enough, output falls further in the subsequent period, as the capital stock is now used much less efficiently. The model is therefore able to generate a 'hump-shaped' response of output, i.e. one that gets amplified further following the initial shock. It takes time for the producers to rebuild their share of the wealth distribution to its steady-state level, and output is therefore below its steady-state level for many periods, even if the initial disturbance only lasted a single period.

How does this mechanism interact with monetary policy? Sticky prices reduce the initial redistribution following a productivity shock: when output is temporarily lower, nominal goods prices need to rise for a given systematic monetary policy response that does not fully accommodate the fall in output. But nominal prices cannot rise enough, because they are sticky, so output increases relative to the case where prices are fully flexible. So while the direct effect of an adverse productivity shock is obviously to lower output, the effect of sticky prices is to mitigate this fall somewhat. Since the initial output effect is smaller under sticky prices, the redistribution from producers to investors is also smaller, and the price of capital will fall by less. The entire credit mechanism is therefore weakened.

I also analyse how the model is affected when debt contracts are specified in nominal rather than real terms. Nominal contracts dampen the effects of productivity shocks, but amplify the effects of monetary policy shocks. Following an adverse productivity shock, output is lower than expected, which causes a transfer of capital from producers (who are borrowers) to investors (who are lenders). But inflation is higher than expected. Higher inflation erodes the real value of the debt that producers have to repay, and partly offsets the redistribution of wealth towards investors. So the output fall is smaller and less persistent. Now consider a monetary policy shock: inflation and output move in the same direction. Following a monetary tightening, output will be lower than expected, which will result in the redistribution of capital from producers to investors. On top of that, lower-than-expected inflation increases the real value of the debt that producers have to repay. This causes a further transfer of wealth away from producers, meaning an even less efficient use of capital in future periods, and therefore even greater falls in output, inflation and asset prices.

Relative to the existing literature on monetary policy and credit frictions, the model offers two key insights. First, credit frictions are a potential source of persistence in the output response to shocks. Such endogenous persistence is absent from the workhorse real business cycle and New Keynesian models. [1] Note that the persistence manifests itself as persistent variation in measured aggregate total factor productivity, even when total factor productivity at the level of each firm is actually white noise. And unlike models where total factor productivity is entirely exogenous, in this

[1]King and Rebelo (1999) document the absence of endogenous persistence in the real business cycle model, and Woodford (2003) and many others discuss the absence of endogenous persistence in the baseline so-called Dynamic New-Keynesian models.

model aggregate total factor productivity is driven not only by exogenous shocks to firm-level total factor productivity, but by anything that affects credit and asset prices, such as monetary policy. Second, the fact that credit frictions affect the productive capacity of the economy directly has important consequences for the desirable systematic response of monetary policy to shocks. Systematic monetary policy–stabilising inflation aggressively–can generate large output fluctuations as the efficiency with which capital is employed is affected. A trade-off therefore exists between deviations of output from its efficient level and deviations of inflation from its efficient level. Such a trade-off is not generally present in standard New Keynesian monetary models, unless one considers shocks that hit the price level directly.[2]

The remainder of this paper is organised as follows. Section 3.2 reviews the literature that relates to the questions studied in this chapter. Section 3.3 presents the model in detail. Section 3.4 outlines the competitive equilibrium. Section 3.5 presents quantitative results, section 3.6 presents a variation of the model with nominal debt contracts, and section 3.7 concludes.

[2]In models such as those discussed by Clarida, Gali and Gertler (1999) and Woodford (2003), the level of output that prevails under flexible prices is the appropriate target for monetary policy, and this level can theoretically be achieved as long as there are no direct shocks to the price level. For the case of productivity shocks, there is therefore no trade-off between output fluctuations from their flex-price level and inflation deviations from target. This is not the case if other frictions are added. For example, Erceg, Henderson and Levin (2000) show that a trade-off also exists if both wages and prices are sticky.

3.2 Related literature

There is a vast theoretical and empirical literature that investigates the qualitative and quantitative importance of credit frictions in the propagation of shocks. Gertler (1988) gives a useful overview of the literature up to that date, and Schiantarelli (1996) and Hubbard (1998) specifically review the empirical micro-evidence. Since this chapter is concerned with constructing a theoretical macro-model, I will focus on that particular literature. A first clear statement of how the financial health of borrowers could influence the propagation of shocks was made by Fisher (1933), who emphasised that the fall in inflation following a downturn in the economy could exacerbate the downturn by increasing the real burden of debt faced by borrowers, which would trigger fire sales of assets and bankruptcies. Gurley and Shaw (1955) used the notion of financial capacity of an economy, meaning the extent to which firms are able to borrow, and argued that this was an important aspect of aggregate demand. During the late 1970s and early 1980s, there was much progress in making the theoretical case for credit frictions at the micro-level. Jaffee and Russell (1976) explain credit rationing as the result of unobserved borrower quality. Stiglitz and Weiss (1981) similarly obtain the result that credit rationing can result as an equilibrium when the riskiness of borrowers' projects is unobserved. Townsend (1979) introduced the notion of costly state-verification, the idea that a lender must pay a fixed cost to observe the financial health of the borrower. He showed that a standard debt contract, with a fixed interest rate and liquidation in case of non-repayment, emerges as the optimal financial arrangement. At the macro-level, Scheinkman and

Weiss (1986) showed that in a model where each agent faces exogenous borrowing constraints and idiosyncratic productivity, aggregate output and asset price dynamics will depend on the entire distribution of wealth. Their model also features more volatile asset prices than the complete markets version of that model, but not necessarily an amplification of output effects. Bernanke and Gertler (1989) integrated the costly-state verification idea into an overlapping generations model and showed that such a credit market imperfection could lead increased persistence of the effects of shocks on aggregate output. Carlstrom and Fuerst (1997) and (1998) embed the costly-state verification mechanism into an otherwise standard real business cycle model, and analyse to what extent this modifies the properties of the real business cycle model. They find that the effect of shocks on output can be either amplified or dampened, depending on which sector of the economy the financial constraint applies to. They also find that in their particular set-up there is either amplification or increased persistence, but not both.

Kiyotaki and Moore (1997) also examine the effect of credit market frictions on business cycle dynamics, but do not use the costly-state verification framework. Instead of putting constraints on information, they put constraints on contracting, in the sense that borrowers cannot commit to repay. In their setting, this leads to one period debt contracts being optimal, and the quantity of lending will be limited by the quantity of available collateral. This mechanism is then embedded in a model where there are producers with high and low productivity. In the first-best, all output should be produced by highly productive agents, but with limited commitment, an equilibrium can emerge where some of the capital is owned by less productive agents, and highly productive agents borrow all the way up to the binding collateral limit. Following an adverse shock, there is a redistribution of capital from highly productive agents to less productive agents, and this results in an amplified and persistent drop in output following a small and temporary drop in productivity. Kiyotaki (1998) extends this mechanism by considering a situation where agents are not permanently stuck in a high or low productivity state, but their productivity state changes stochastically. This leads to added richness in the dynamics, as the persistence of the stochastic productivity switching process affects the dynamics of aggregate output. There is some empirical literature that finds evidence for such a mechanism of reallocation across different producers. Eisfeldt and Rampini (2005) find that the amount of capital reallocation across firms is procyclical, and that the dispersion of productivity across firms is countercyclical. These two facts are consistent with a model where capital needs to flow to the producers with the highest productivity, but these flows can more easily happen during cyclical upturns, when informational or contractual frictions are smaller. A second empirical paper that is directly relevant to this framework is Barlevy (2003), who shows that highly productive firms tend to borrow more, again consistent with a framework where credit needs to flow from low to high productivity firms, making highly productive firms highly indebted. Kocherlakota (2000) constructs a useful, highly simplified version of a credit constrained economy to examine what business cycle features can and cannot be explained by such models. He uses a small open economy setting, so that the interest rate is exogenous. He shows that the amount of amplification is related to the share in production of the collateralisable asset, and that the degree of amplification that can plausibly be achieved in his setting is small. All the models discussed so far are real models. There is no role for monetary policy. The models are useful for analysing how credit frictions change business cycle dynamics, but are not useful for policy analysis. Bernanke, Gertler and Gilchrist (1999) introduce the costly-state verification mechanism into a New Keynesian business cycle model, i.e. into a real business cycle framework with nominal rigidities added. They use this model to analyse macroeconomic dynamics resulting from a wide range of shocks, and find that, compared to a version of the model that has no financial frictions, the investment response to shocks is amplified and more persistent, leading to an amplified and more persistent response of aggregate output. Iacoviello (2005) also merges a real model of credit frictions with New Keynesian features, but uses the Kiyotaki and Moore (1997) framework for the credit frictions part. He also focuses particularly on the role of real estate, which in his model can serve as both an input for production and is demanded directly by consumers for its housing services. He therefore reinterprets the switch of capital between low and high productivity agents as a switch of real estate usage between households and firms. Iacoviello (2005) uses the model to analyse a range of issues, including the effect of changing the maximum amount of leverage that is permitted, and the effect of introducing some heterogeneity in the household sector, so that some households borrow while other lend. A final strand of literature that is relevant is the series of papers that examine the effects of limited commitment in financial contracting, but still allow multi-period financial contracts. The only restriction is that there must not be a state of the world in which the borrower would rather default than make the contractual payment. Kehoe and Levine (1993), Kocherlakota (1996) and Alvarez and Jermann (2000) work within this framework, and find that such constraints on contracting lower the equilibrium real interest rate, and generate richer asset price dynamics, since the risk premium depends on idiosyncratic as well as aggregate shocks. The richness of the permissible financial contracts of these models makes it attractive relative to the more stringent restrictions in, e.g. Kiyotaki and Moore (1997) and the papers based on their framework. But this more general limited commitment framework is also more complex, and most of the models in this strand of the literature have been real models, and most have exogenous income processes, limiting their usefulness–for now– for policy analysis. Cooley, Marimon and Quadrini (2004) numerically solve a real model with production and limited commitment in financial contracting. They find that this mechanism causes the output effects of productivity shocks to be amplified by a factor of 6, and the effect lasts many years. The crucial assumption in their paper is that it must not be too costly for entrepreneurs to repudiate the contract, otherwise the economy behaves as if contracts were fully enforceable.

3.3 The environment

The model features a basic credit frictions mechanism due to Kiyotaki (1998), which is extended to allow for endogenous labour supply, monopolistic competition and a role for monetary policy.

There is a continuum of entrepreneurs. They are identical in terms of preferences. Their production technology is also identical, up to a productivity factor, which randomly switches between high (α) and low (γ). Denote those who currently have high productivity 'producers', and those who currently have low productivity 'investors'. The productivity factor follows an exogenous Markov process with probability matrix

$$P = \begin{bmatrix} 1-\delta & \delta \\ n\delta & 1-n\delta \end{bmatrix} \tag{3.1}$$

so the probability of switching from high productivity to low productivity is δ, and the probability of switching from low productivity to high productivity is $n\delta$. This probability matrix implies that from any initial distribution, the distribution will converge to a stationary distribution with a ratio of productive to unproductive agents of n.

Producers maximise life-time utility given by

$$\max_{c_t} E_0 \sum_{t=0}^{\infty} \beta^t \ln c_t \tag{3.2}$$

s.t. budget constraint,

$$c_t + x_t + q_t(k_t - k_{t-1}) + w_t l_t = \frac{y_t}{\varphi_t} + \frac{b_{t+1}}{r_t} - b_t \tag{3.3}$$

production technology,

$$y_t = \alpha \left(\frac{k_{t-1}}{\sigma}\right)^{\sigma} \left(\frac{x_{t-1}}{\eta}\right)^{\eta} \left(\frac{l_t}{1-\eta-\sigma}\right)^{1-\eta-\sigma} \tag{3.4}$$

and borrowing constraint

$$b_{t+1} \leqslant E_t q_{t+1} k_t \tag{3.5}$$

The variable c_t denotes consumption, x_t denotes a non-durable input (eg inventories), k_t denotes durable capital, w_t denotes the wage paid, l_t denotes the quantity of labour employed, b_{t+1} denotes the amount of real borrowing taken out at time t and repayable at time $t+1$, and q_t is the price of capital.

It is assumed that producers do not consume their output directly, but sell it to a monopolistically competitive retailer, who then offers the diversified goods back to producers, investors and workers with a mark-up of φ_t. All variables are denominated in terms of a consumption index. Define a Dixit-Stiglitz (1977) aggregate of a continuum of differentiated goods of type $z \in [0,1]$ each with price $p(z)$

$$c_t = \left[\int_0^1 c_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} \tag{3.6}$$

The corresponding price index, defined as the minimum cost of a unit of the consumption aggregate, is defined as

$$p_t = \left[\int_0^1 p_t(z)^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{3.7}$$

For simplicity, it is assumed that inventories are costlessly created from the consumption aggregate, so that their relative price in terms of consumption is 1.

Following Kiyotaki and Moore (1997), borrowing constraints are interpreted as follows: it is assumed that when an entrepreneur has installed some capital, he invests some specific skill into that capital to generate output. The total value of his project is therefore the next period resale value of the installed capital plus the value of the output that can be generated using his specific skill. But he cannot commit to investing his specific skill: once the capital is in place, he can always choose to walk away. Because of this inability to commit to full repayment, the investor will never lend more than the resale value of capital. It is assumed that, should the value of collateral fall short of what was expected at the time the loan was taken out, the entrepreneur still repays the borrowing in full, because by the time he finds out about the realisation of the aggregate shock, he has already produced, and no longer has the opportunity to walk away.[3] Also following Kiyotaki and Moore (1997), it is assumed that, after the initial uncertainty about aggregate productivity is resolved, agents assume that future aggregate productivity is constant. In other words, their decisions are assumed to be unaffected by aggregate uncertainty.

It is useful to define $u_t \equiv q_t - E_t \frac{q_{t+1}}{r_t}$, the user cost of a unit of capital.

If we assume the borrowing constraint is binding, which will be verified later, we can rewrite the budget constraint as

$$c_t + x_t + u_t k_t + w_t l_t = \frac{\alpha}{\varphi_t} \left(\frac{k_{t-1}}{\sigma}\right)^{\sigma} \left(\frac{x_{t-1}}{\eta}\right)^{\eta} \left(\frac{l_t}{1-\eta-\sigma}\right)^{1-\eta-\sigma} + q_t k_{t-1} - b_t \tag{3.8}$$

To solve this, we break up the problem into two steps. First, given last

[3]He could still have an incentive to walk away if the debt burden exceeds not only the value of his collateral, but exceeds the value of his collateral plus current output. It is assumed that shocks are never that large.

period's capital and intermediate goods, what is the optimal demand for labour?

$$\pi_t = \max_{l_t} \left\{ \frac{\alpha}{\varphi_t} \left(\frac{k_{t-1}}{\sigma}\right)^{\sigma} \left(\frac{x_{t-1}}{\eta}\right)^{\eta} \left(\frac{l_t}{1-\eta-\sigma}\right)^{1-\eta-\sigma} - w_t l_t \right\} \tag{3.9}$$

This leads to the first-order condition

$$w_t = (1-\eta-\sigma)\frac{\alpha}{\varphi_t} \left(\frac{k_{t-1}}{\sigma}\right)^{\sigma} \left(\frac{x_{t-1}}{\eta}\right)^{\eta} \frac{l_t^{-\eta-\sigma}}{(1-\eta-\sigma)^{1-\eta-\sigma}} \tag{3.10}$$

which can also be written in the familiar form

$$w_t l_t = (1-\eta-\sigma)\frac{y_t}{\varphi_t} \tag{3.11}$$

The maximised profit after paying for labour input is therefore

$$\pi_t = (\eta+\sigma)\frac{y_t}{\varphi_t} \tag{3.12}$$

For the second step of the producer's problem, we analyse what combination of capital and inventories he should buy to minimise expenditure, given a desired level of profits.[4]

$$z_t = \min_{k_t, x_t} \{u_t k_t + x_t\} \tag{3.13}$$

$$s.t. E_t \pi_{t+1} \geqslant \overline{\pi} \tag{3.14}$$

[4]The actual level of profits is irrelevant to the optimisation problem given the constant returns to scale technology.

Let λ_t denote the Lagrangean multiplier on the profit constraint. Substituting the optimal level of labour demanded into the production function, the first-order conditions become

$$u_t = E_t \left\{ \lambda_t \left(\frac{\alpha}{\varphi_{t+1}} \right)^{\frac{1}{\eta+\sigma}} w_{t+1}^{-\frac{1-\eta-\sigma}{\eta+\sigma}} \left(\frac{\sigma}{\eta} \right)^{\frac{\eta}{\eta+\sigma}} \left(\frac{x_t}{k_t} \right)^{\frac{\eta}{\eta+\sigma}} \right\} \tag{3.15}$$

$$1 = E_t \left\{ \lambda_t \left(\frac{\alpha}{\varphi_{t+1}} \right)^{\frac{1}{\eta+\sigma}} w_{t+1}^{-\frac{1-\eta-\sigma}{\eta+\sigma}} \left(\frac{\sigma}{\eta} \right)^{-\frac{\sigma}{\eta+\sigma}} \left(\frac{x_t}{k_t} \right)^{-\frac{\sigma}{\eta+\sigma}} \right\} \tag{3.16}$$

This can be simplified to

$$u_t = \frac{\sigma x_t}{\eta k_t} \tag{3.17}$$

This optimal combination of inputs yields the minimised expenditure function

$$z_t = \frac{\eta + \sigma}{\sigma} u_t k_t \tag{3.18}$$

Note that λ_t is the resource cost of another unit of profit, or, in other words, $1/\lambda_t$ is the return on an investment of z_t. For convenience we define this as a new variable:

$$r_t^p \equiv E_t \left\{ \left(\frac{\alpha}{\varphi_{t+1}} \right)^{\frac{1}{\eta+\sigma}} w_{t+1}^{-\frac{1-\eta-\sigma}{\eta+\sigma}} u_t^{-\frac{\sigma}{\eta+\sigma}} \right\} \tag{3.19}$$

In a similar way, we can also calculate the ex post return from having used resources x_{t-1}, k_{t-1} and l_t in the optimal combination given u_{t-1}, w_t and φ_t. This return is equal to:

$$r^p_{t-1} \equiv \left\{ \left(\frac{j}{\varphi_t}\right)^{\frac{1}{\eta+\sigma}} w_t^{-\frac{1-\eta-\sigma}{\eta+\sigma}} u_{t-1}^{-\frac{\sigma}{\eta+\sigma}} \right\} \tag{3.20}$$

In this equation, $j = \alpha, \gamma$ depending on whether the entrepreneur had high or low productivity in the previous period.

Substituting the optimal labour demand and factor demand conditions into the production function, we can now write the budget constraint as

$$c_t + z_t = r^j_{t-1} z_{t-1} + q_t k_{t-1} - b_t \tag{3.21}$$

This can be interpreted as a savings problem with uncertain returns (eg Sargent (1987)). The optimal decision rules for consumption and investment are linear in wealth:

$$c_t = (1-\beta)(r^j_{t-1} z_{t-1} + q_t k_{t-1} - b_t) \tag{3.22}$$

$$z_t = \beta(r^j_{t-1} z_{t-1} + q_t k_{t-1} - b_t) \tag{3.23}$$

3.3.1 Investors

Let lower-case variables with a prime denote the choices of an individual investor. The labour demand conditions facing the agents with low productivity, i.e. the investors, are the same as those for the producers, so the maximised profits after paying the wage bill are

$$\pi_t' = (\eta + \sigma)\frac{y_t'}{\varphi_t} \tag{3.24}$$

The second step of the problem, minimising the expenditure on x_t' and k_t', is solved by maximising

$$\min_{x_t', k_t'} \left(q_t - E_t \frac{q_{t+1}}{r_t} \right) k_t' + x_t' \tag{3.25}$$

$$s.t. \pi_{t+1}' \geqslant \overline{\pi} \tag{3.26}$$

Using our earlier definition of u_t, this problem is again parallel to that faced by producers, except that the rate of return for investors is

$$r_t^i \equiv E_t \left\{ \left(\frac{\gamma}{\varphi_{t+1}} \right)^{\frac{1}{\eta+\sigma}} w_{t+1}^{-\frac{1-\eta-\sigma}{\eta+\sigma}} u_t^{-\frac{\sigma}{\eta+\sigma}} \right\} \tag{3.27}$$

Just as for producers, the decision rule for consumption and investment of investors is therefore also linear in wealth with the same coefficients.

3.3.2 Retailers

Retailers buy output and use a costless technology to turn output goods into differentiated consumption or input goods, which they sell onwards. The separation of producers and retailers is a modelling choice similar to Bernanke, Gertler and Gilchrist (1999) and is chosen to introduce monopolistic competition while maintaining tractable aggregation of producers. If producers operate directly in monopolistically competitive markets, they no longer face constant returns to scale at the firm level, and their optimisation problem will no longer yield the linear decision rules that are needed for tractable aggregation. Per period real profits for the retailers are given by

$$\Pi_t(p_t(z)) = \frac{(p_t(z) - p_t^p)}{p_t} y_t^R(z) \tag{3.28}$$

where p_t^p is the nominal price of output goods, so that $\frac{p_t^p}{p_t} = \frac{1}{\varphi_t}$. In other words, φ_t is the retail sector's average mark-up. Retailer output is denoted $y_t^R(z)$.

Demand for each retailer's output is given by

$$y_t^R(z) = \left(\frac{p_t(z)}{p_t}\right)^{-\theta} Y_t^R \tag{3.29}$$

where Y_t^R is aggregate demand for retail goods, which is given by

$$Y_t^R = \left[\int_0^1 y_t^R(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}} \tag{3.30}$$

In the baseline model, it is assumed that some fraction κ of retailers must set their price, $p_{2,t}(z)$, one period in advance, while the remainder can change their price, $p_{1,t}(z)$ each period. Each type of retailer maximises profits, leading to the following first order conditions:

$$\frac{p_{1,t}(z)}{p_t} = \frac{\theta}{\theta - 1}\frac{1}{\varphi_t} \tag{3.31}$$

$$E_{t-1}\left\{\Lambda_{t-1,t}\frac{Y_t^R}{p_t^{-\theta}}\left[\frac{p_{2,t}(z)}{p_t} - \frac{\theta}{\theta-1}\frac{1}{\varphi_t}\right]\right\} = 0 \tag{3.32}$$

The term $\Lambda_{t-1,t}$ is a discount factor applied at time $t-1$ to profits earned at time t. It is assumed that retailers are owned by workers, so it is the workers' discount factor that is relevant here. The aggregate price level evolves according to:

$$p_t = \left[(1-\kappa)p_{1,t}^{1-\theta} + \kappa p_{2,t}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{3.33}$$

I will end up working with a linearised model, and it is convenient to note already that the first-order conditions for retailer profit maximisation, combined with the evolution of the aggregate price level, once linearised, will give the following pricing equation:

$$\widehat{\pi}_t = E_{t-1}\widehat{\pi}_t - \frac{1-\kappa}{\kappa}\widehat{\varphi}_t \tag{3.34}$$

where $\widehat{x}_t \equiv \frac{x_t - \overline{x}}{\overline{x}}$ denotes proportional deviations from the steady-state.

In an extension of the model, I consider an environment where retailers face opportunities for price changes that arrive randomly, so that price setting follows a discrete time version of the model proposed by Calvo (1983), as described, for example, in Woodford (1995), Yun (1996), Clarida, Gali and Gertler (1999) and many others. The implication is that actual prices can deviate from their optimally chosen level for more than one period following a shock, which allows for richer inflation and mark-up dynamics. The probability for each retailer of being able to reset their price equals $(1-\kappa)$ in each period, and is independent of when the last price change occurred. The retailers who can set a price at time s will maximise the intertemporal objective function:

$$E_s \sum_{t=s}^{\infty} \Lambda_{s,t} \kappa^{t-s} \left[\Pi_t(p_s^*(z)) \right] \tag{3.35}$$

where $\Lambda_{s,t}$ is a discount factor applied in period s to profits expected in period t, and $p_s^*(z)$ is the optimal price chosen. Retailers are owned by workers. It is assumed that retailers (but not entrepreneurs) form a cooperative that redistributes income between those who were able to change their price and those who were not able to do so. This assumption implies that retailers do not face idiosyncratic risk. This in turn implies that all retailers who are able to change their price will set the same price, regardless of their history. This greatly facilitates aggregation across retailers.

The first-order condition for retailers who are able to change their price in period s is:

$$E_s \sum_{t=s}^{\infty} \Lambda_{s,t} \kappa^{t-s} \frac{Y_t^R}{p_t^{-\theta}} \left\{ \frac{p_s^*(z)}{p_t} - \frac{\theta}{(\theta - 1)} \frac{1}{\varphi_t} \right\} = 0 \tag{3.36}$$

The aggregate price level evolves according to:

$$p_t = \left[(1-\kappa) p_t^{*1-\theta} + \kappa p_{t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{3.37}$$

The linearised aggregate pricing condition now becomes:

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} - \frac{(1-\kappa)(1-\beta\kappa)}{\kappa} \widehat{\varphi}_t \tag{3.38}$$

3.3.3 Workers

There is a set of agents in the economy who have no access to productive technology, but who can work for the producers and investors. They derive utility from consumption and leisure, and their objective is to maximise

$$\max_{c_t, l_t} \sum_{t=0}^{\infty} \beta^t \ln \left(c_t - \frac{\chi}{1+\tau} l_t^{1+\tau} \right) \tag{3.39}$$

$$s.t. c_t^w + \frac{b_{t+1}^w}{r_t} = w_t l_t + b_t^w + \Pi_t \tag{3.40}$$

where l_t is the fraction of time spent on work, and Π_t are the profits from the retail sector, which is owned by the workers.

There is a modeling choice that needs to be made here concerning who receives the profits from monopolistic competition. Paying profits to the workers makes the model very tractable, but strictly speaking the workers would not want to own the retailers in equilibrium, because they do not want to save. I am simply not allowing workers to sell their stake in retailer profits. An alternative would be to consider retailers as consuming agents in their own right, i.e. give the retailers a utility function, so that they themselves could consume the profits from their technology of diversifying goods. However, just like the workers, retailers would not want to save in equilibrium due to the low interest rate, and they would not be able to borrow against future profits because there is no collateral. So they would simply consume the profits each period. The model results would therefore be identical, and the modeling choice is simply determined by which story one finds more appealing. Paying the profits back to the entrepreneurs raises different complications. Investors and producers have different opportunity costs of investment. Ideally, producers would not like to own any assets other than their own capital. If the shares in retail profits are not tradable, there is the issue that each type of entrepreneur would value the future profits differently. If the shares in retail profits are tradable, producers would sell them to investors. That means that the entire present value of future profits would shift between producers and investors each time they switch roles, which introduces another asset price, and creates large transfers between producers and investors that have no obvious real world counterpart and may affect the transmission mechanism in unrealistic ways. I derived a version of the model with tradable retail shares, and found that the model behaved similarly in some parameter ranges, but the behaviour changed drastically and in unintuitive ways for small changes in the parameters. I therefore prefer to work with the specification where workers receive the profits from monopolistic competition.

Setting the workers' marginal utility of leisure equal to their marginal utility of consumption, the labour supply decision is

$$w_t = \chi l_t^{\tau} \tag{3.41}$$

It is to be verified later that the interest rate on bonds is below the rate of time preference $1/\beta$. This implies that, near the stationary state, the workers will choose not to hold any bonds, and simply consume their wage and profit income. Their consumption therefore becomes:

$$c_t^w = w_t l_t + \Pi_t \tag{3.42}$$

3.3.4 Monetary authorities

Prices in the economy are set in money terms. As described in Woodford (2003), it is not necessary for agents to have a well-behaved demand for money balances in order for the monetary authorities to have control of the nominal interest rate. All that is necessary is for agents to have some, possibly infinitely small, demand for money balance. I assume such a 'cashless limit' (Woodford (2003)) here, so that money balances, and therefore the central bank's balance sheet, approach zero. Given this assumption, it is a reasonable approximation to omit money from the agents' utility function and budget constraint. A similar approach is used, for example, by Aoki (2001) who also omits money balances from a model that allows the central bank to set nominal interest rates. The central bank simply announces the one-period nominal interest rate R_t, which means that it stands ready to deposit or lend any amount[5] the private sector desires at this rate, subject to a (infinitely small) spread. The spread ensures that the private sector will attempt to clear the loan market first without resorting to the central bank. The influence of the central bank on the market for loanable funds is therefore unrelated to the amount of base money, but instead works via arbitrage with the private market for loanable funds. No private agent would be willing to borrow at a rate higher than that offered by the central bank, and

[5]The central bank does not have better enforcement mechanisms for the collection of loan repayments than does the private sector. It will therefore not lend any funds to a producer who is already at the binding borrowing constraint.

no private agents would deposit funds that receive a lower return than that offered by the central bank. This arbitrage mechanism is similar to the way actual monetary policy operates in countries such as New Zealand, Canada, the United Kingdom and Scandinavian countries, although in practice the spreads are of course not infinitely small. This environment gives rise to an arbitrage condition based on the marginal utility of the investors.

$$E_t \left\{ \beta R_t \frac{P_t}{P_{t+1}} \frac{1}{c^i_{t+1}} \right\} = E_t \left\{ \beta r^i_t \frac{1}{c^i_{t+1}} \right\} \tag{3.43}$$

The central bank is assumed to follow a simple rule for setting monetary policy, [6] by responding to current inflation. There are also random deviations from the rule, which we will interpret as monetary policy shocks.

$$\frac{R_t}{r^i} = \pi_t^{\lambda} \exp(\varepsilon_t^R) \tag{3.44}$$

3.4 Competitive Equilibrium

We now look for a competitive rational expectations equilibrium for this model economy. This will consist of aggregate decision rules for consumption, investment, labour supply and asset holdings, and aggregate laws of motion so that market clearing and individual optimality conditions hold. Because the distribution of wealth between producers and investors directly affects aggregate outcomes, it becomes a state variable. As will be shown,

[6] Sargent and Wallace (1975) showed that if the interest rate follows an exogenous path, the price level is indeterminate. However, McCallum (1981) showed that the price level can be determinate under an interest rate rule if interest rates respond to a nominal variable, such as the price level in his paper, or inflation in my case.

the distribution of wealth can be summarised by the share of wealth owned by producers.[7] Let capital letters denote aggregate variables. The market clearing conditions are that

$$B_t + B'_t + B_t^W = 0 \tag{3.45}$$

$$K_t + K'_t = \overline{K} \tag{3.46}$$

and that labour supply equals labour demand. For the goods market, the following must hold. It is assumed that each retailer buys a single output good, turns it into a single diversified consumption/inventory good and sells it back to producers. The market clearing condition for each good is then

$$y_t(z) = y_t^R(z), \forall z, t \tag{3.47}$$

Recall that aggregate output is given by the sum across all identical output goods produced by the entrepreneurial sector:

$$Y_t + Y'_t = \int_0^1 y_t(z)dz \tag{3.48}$$

[7]In model simulations I will consider a stochastic process for aggregate productivity. Because each entrepreneur's problem collapses to a linear savings problem with log consumption, the fact that future returns are uncertain does not affect the consumption and savings decision. Where uncertainty might affect decision rules is that borrowers may not want to borrow up to the borrowing limit if uncertainty about future asset prices is large. I only consider an approximation of the model where the borrowing constraint binds at all times.

and aggregate demand for retail goods is given by

$$Y_t^R = \left[\int_0^1 y_t^R(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}} \tag{3.49}$$

In general, it is not the case that $Y_t^R = Y_t + Y_t'$, but this will be true in a neighbourhood of the steady state. It is understood that the following condition only applies in such a neighbourhood:

$$C_t + C_t' + X_t + X_t' + C_t^w = \frac{Y_t + Y_t'}{\varphi_t} + \Pi_t \tag{3.50}$$

Aggregate retailers' profits will be equal to

$$\Pi_t = \left(1 - \frac{1}{\varphi_t}\right)(Y_t + Y_t') \tag{3.51}$$

Note that the individual decision rules for consumption and investment are all linear, so that we can simply sum them to obtain aggregate decision rules and laws of motion. Each agent consumes a fraction $1-\beta$ of their wealth and reinvests a fraction β of their wealth.

The following is asserted, to be verified later: I am interested in equilibria near a steady state where the investors hold some capital for their own production. This has two implications. First, investors must then be indifferent between holding capital for production and bonds, so that they equalise the expected return to each

$$r_t^i = r_t \tag{3.52}$$

Second, because we have shown that

$$r_t^p = \left(\frac{\alpha}{\gamma}\right)^{\frac{1}{\eta+\sigma}} r_t^i > r_t^i \tag{3.53}$$

it follows that the borrowing constraint is indeed binding near the steady state, since producers achieve a larger return on their own productive investment than the interest rate they have to pay on the bonds they issue.

Next, it is useful to define aggregate wealth as the quantity of output available for consumption or reinvestment, i.e. after paying the wage bill.

$$W_t = (\eta + \sigma)\,\frac{Y_t + Y_t'}{\varphi_t} + q_t\overline{K} \tag{3.54}$$

We also define the share of wealth held by producers as s_t.

We can now write a law of motion for aggregate wealth as

$$W_{t+1} = \left[r_t^p s_t + r_t^i(1 - s_t)\right]\beta W_t \tag{3.55}$$

Using the Markov-process for the way agents switch between having high and low productivity, the law of motion for the share of wealth can be written as

$$s_{t+1} = \frac{(1-\delta)\widetilde{\alpha}s_t + n\delta\widetilde{\gamma}(1-s_t)}{\widetilde{\alpha}s_t + \widetilde{\gamma}(1-s_t)} \tag{3.56}$$

where $\widetilde{\alpha} = \alpha^{\frac{1}{\eta+\sigma}}$ and similarly for $\widetilde{\gamma}$.

Using the expressions for r_t^i and r_t^p derived earlier, the law of motion for wealth can be written as

$$W_{t+1} = [\widetilde{\alpha} s_t + \widetilde{\gamma}(1-s_t)]\, \varphi_{t+1}^{-\frac{1}{\eta+\sigma}} w_{t+1}^{-\frac{1-\eta-\sigma}{\eta+\sigma}} u_t^{\frac{-\sigma}{\eta+\sigma}} \beta W_t \tag{3.57}$$

Next, we use the aggregate budget constraint, substitute the decision rules for consumption, investment, labour, and use the fact that $W_t = (\eta+\sigma)\left(\frac{Y_t+Y_t'}{\varphi_t}\right) + q_t\overline{K}$. This can be then be written as two equilibrium conditions linking the user cost and the wage to wealth and asset prices:

$$(1-\beta)W_t + \frac{\eta}{\sigma} u_t \overline{K} = \left(W_t - q_t\overline{K}\right) \tag{3.58}$$

and

$$w_t^{\frac{1+\tau}{\tau}} \left(\frac{1}{\chi}\right)^{1/\tau} = \frac{1-\eta-\sigma}{\eta+\sigma}\left(W_t - q_t\overline{K}\right) \tag{3.59}$$

The asset pricing equation is given by

$$q_t = u_t + E_t\left(\frac{q_{t+1}}{r_t}\right) \tag{3.60}$$

combined with the condition

$$r_t = E_t\left\{\widetilde{\gamma}\varphi_{t+1}^{-\frac{1}{\eta+\sigma}} w_{t+1}^{-\frac{1-\eta-\sigma}{\eta+\sigma}} u_t^{\frac{-\sigma}{\eta+\sigma}}\right\} \tag{3.61}$$

We now need to complete the model by adding a set of equations describing the role of monetary policy. Note that the arbitrage equation for nominal bonds, when considered along the certainty-equivalent path, is just a Fisher equation:

$$r_t = \frac{R_t}{E_t \pi_{t+1}} \tag{3.62}$$

Combined with the monetary policy rule, this can be written as

$$r_t = \frac{\pi_t^\lambda \exp(\varepsilon_t^R)}{E_t \pi_{t+1}} \tag{3.63}$$

Note that u_t and w_t can be eliminated using (3.58) and (3.59), and r_t can be eliminated using (3.61). This leaves a system of 4 dynamic equations (3.56),(3.57),(3.60), (3.63) in $\{s_t, W_t, q_t, \pi_t\}$, 3 initial conditions

$$\begin{aligned} W_0 s_0 =& (1-\delta)\left((\eta+\sigma)\frac{Y_0}{\varphi_0} + q_0 K_{-1} - B_0\right) \\ &+ n\delta((\eta+\sigma)\frac{Y_0'}{\varphi_0} + q_0(\overline{K} - K_{-1}) + B_0) \end{aligned} \tag{3.64}$$

$$W_0 = (\eta+\sigma)\left(\frac{Y_0 + Y_0'}{\varphi_0}\right) + q_0 \overline{K} \tag{3.65}$$

$$\widehat{\pi}_0 = E_{-1}\widehat{\pi}_0 - \frac{1-\kappa}{\kappa}\widehat{\varphi}_0 \tag{3.66}$$

I now want to consider an aggregate disturbance to productivity. I achieve this by multiplying α and γ by a productivity disturbance ε_P. The assumed stochastic process for the productivity disturbance is that its log follows an autoregressive process with a normally distributed shock:

$$\widehat{\varepsilon}_{P,t+1} = \rho \widehat{\varepsilon}_{P,t} + \nu_{t+1} \tag{3.67}$$

3.5 Model solution

3.5.1 Dynamics

The system of 5 equations (3.56),(3.57),(3.60), (3.63), (3.67) and 3 initial conditions (3.64), (3.65), (3.66) is solved as follows. First, we take a linear approximation of all the equations, including the initial conditions, around the steady state. The steady state is the level that aggregate variables tend to when there are no aggregate shocks. Associated with these levels for aggregate variables is a stationary wealth distribution summarised by the share of wealth owned by producers, $s_t = \bar{s}$.

Suppressing the expectations notation, the linearised system can be written as

$$AX_{t+1} = BX_t \tag{3.68}$$

$$X_t = \begin{bmatrix} \widehat{q}_t \\ \widehat{\pi}_t \\ \widehat{s}_t \\ \widehat{W}_t \\ \widehat{\varepsilon}_{P,t} \end{bmatrix} \tag{3.69}$$

This system can then be written as:

$$X_{t+1} = FX_t \tag{3.70}$$

where $F = A^{-1}B$.

Using a simple eigenvalue decomposition of $F = P\Lambda P^{-1}$ this can be written as a new system

$$Y_{t+1} = \Lambda Y_t \tag{3.71}$$

where $Y_t = P^{-1}X_t$. This system is 'uncoupled' as Λ is a diagonal matrix containing the eigenvalues of F. I am interested in non-explosive, determinate solutions. Order the eigenvalues in decreasing absolute magnitude, and let n be the number of eigenvalues outside the unit circle. Let P_1^{-1} denote the upper n rows of P^{-1}. For a solution to be non-explosive, it is necessary for $P_1^{-1}X_t$ to be zero for all t. For a solution to be determinate (following Blanchard and Kahn (1980)), it is necessary for $n = 2$ eigenvalues (corresponding to the number of 'jump' variables q_t, π_t) to lie outside the unit circle and for the remaining eigenvalues to lie inside the unit circle. After some re-arranging, the non-explosive condition can then be rewritten as

$$\begin{bmatrix} \widehat{q}_t \\ \widehat{\pi}_t \end{bmatrix} = P_{12}\left(P_{22}\right)^{-1} \begin{bmatrix} \widehat{s}_t \\ \widehat{W}_t \\ \widehat{\varepsilon}_{P,t} \end{bmatrix} \tag{3.72}$$

where P_{12} denotes the first n rows and the left $(5-n)$ columns of P, and P_{22} denotes the bottom right $(5-n) \times (5-n)$ block of P. Given this relationship, the initial response to any shock at time 0 can be found by substituting out the jump variables from the system of initial conditions, which can then be solved for $\widehat{W}_0$, $\widehat{s}_0, \widehat{\varphi}_0$. This then gives the initial response to a shock. From the dynamic system (3.70), again with the jump variables substituted out using (3.72), the remaining dynamic path of all the variables can be computed, noting that $\widehat{\varphi}_t = 0, \forall t \geqslant 1$.

It can be shown that the eigenvalues of this system include, in descending order $e_1, \lambda, \frac{(1-\delta)\widetilde{\alpha}-\widetilde{\gamma}n\delta-(\widetilde{\alpha}-\widetilde{\gamma})s}{\widetilde{\alpha}s+\widetilde{\gamma}(1-s)}, \frac{\eta(1+\tau)}{\tau+\eta+\sigma}, \rho$.[8] So for a monetary policy that satisfies the Taylor principle of reacting to inflation by a factor greater than 1, this system has a non-explosive, determinate solution.

3.5.2 Steady state

The full steady state of the model is given in the appendix. However, it is instructive to consider the expression for the steady-state interest rate:

$$r = \frac{1}{\beta}\left(\frac{\widetilde{\gamma}}{\widetilde{\alpha}s + \widetilde{\gamma}(1-s)}\right) < \frac{1}{\beta} \tag{3.73}$$

Since $s < 1$, the real interest rate is strictly lower than the (inverse of) the rate of time preference. At these low interest rates, workers will not wish to save, so workers choose not to participate in the financial asset market. This proves the earlier assertion that workers simply consume their wage and profit income in each period.

3.5.3 Frictionless model

Before turning to the properties of the full model, I show what the properties of the model would be without binding borrowing constraints. In that case, the efficient allocation would always be reached, in the sense that the most productive agents would always hold the entire capital stock. The full

[8] e_1 represents an eigenvalue which needs to be greater than 1, for which I could not find an analytical solution. In the calibration that I use e_1 is indeed greater than 1.

derivation of the model is given in the appendix. I state here the law of motion for aggregate output:

$$Y_{t+1} = \varepsilon_{P,t+1}^{\frac{\tau+1}{\tau+\eta+\sigma}} (Y_t)^{\frac{\eta(\tau+1)}{\tau+\eta+\sigma}} c \tag{3.74}$$

where c denotes a constant term that is a function of the model parameters. This implies that output dynamics are entirely driven by the exogenous process for aggregate productivity and lagged output. There is no feedback from any net worth or asset price variable in the model. The equations for the asset price and wealth are

$$q_t = \frac{\sigma\beta}{\varphi\overline{K}(1-\beta)} Y_t \tag{3.75}$$

and

$$W_t = \frac{\eta + \sigma - \eta\beta}{\varphi(1-\beta)} Y_t \tag{3.76}$$

So asset prices and entrepreneurial wealth are simply proportional to output.

3.5.4 Calibration

The model contains 13 parameters. Some of the parameters are standard, in the sense that they can be chosen to match key steady-state ratios in the economy. Other parameters, in particular those specific to the credit mechanism, are more difficult to assign values to. The calibration I have chosen is designed to show how the mechanism might work, not how it most likely

Table 3.1: Calibrated parameter values for the baseline model

parameter	assigned value
β	0.99
η	0.1
σ	0.3
τ	0.5
χ	0.29
γ	0.12
κ	0.5
θ	11
λ	1.5
α/γ	1.034
n	0.0073
δ	0.5

does work, as there is little guidance from actual observation in choosing plausible values for these parameters. Table 3.1 shows the parameter values chosen for the baseline model.

The model is calibrated so that each period can be interpreted as one quarter of a year. The discount factor $\beta = 0.99$ is a standard choice in many general equilibrium macromodels (see e.g. Cooley and Prescott (1995)). While in this model such a discount factor will lead to a lower real interest rate compared with models where there is perfect enforcement or commitment, the difference is small under the baseline parameterisation: the steady-state annual real interest rate is just under 4%. The values for $\eta, \sigma, \tau, \chi, \gamma$ were chosen to achieve a capital to output ratio of 10, a labour share in output of 0.6, hours worked of 0.31 as a fraction of total available time, and a wage elasticity of labour supply of 2, values very close again to those in Cooley and Prescott (1995) and subsequent literature. The monetary policy reaction function parameter λ is set at the value used by Taylor (1993), although the reaction function does not have exactly the same form. The rule used in this paper is certainly too simplistic to be realistic, and is used to illustrate the basic mechanisms of the model. The elasticity parameter θ determines a steady-state net mark-up for consumption goods of 0.10, corresponding to the empirical findings by Basu and Fernald (1997). The share of prices that are set one period in advance, κ, is set at 0.5. In the extended model, which features staggered pricing, the probability for each firm of not being able to reset their price is 2/3, implying that firms change their price on average every 3 quarters, in line with the estimates in Sbordone (2002). The extended model also features a more realistic monetary policy rule, which is necessary in order to obtain plausible inflation dynamics. The form of the rule in the extended model is

$$\widehat{R}_t = (1 - \rho_R)\, \lambda_\pi \widehat{\pi}_t + (1 - \rho_R)\, \lambda_\varphi \widehat{\varphi}_t + \rho_R \widehat{R}_{t-1} + \varepsilon_{R,t} \tag{3.77}$$

In other words, monetary policy now responds gradually to inflation, and also responds to the mark-up, which is a proxy for the deviation of output from the level of output that would prevail under flexible prices (when the mark-up is constant). The calibrated values for $\{\lambda_\pi, \lambda_\varphi, \rho_R\}$ are $\{1.5, -2, 0.9\}$.

The crucial parameters for the strength of the credit mechanism are the productivity difference between producers and investors α/γ, the steady-state ratio of productive to unproductive agents n, and the probability of a highly productive agent becoming less productive, δ. The parameters n and α/γ were chosen so that productive agents hold about 2/3 of the capital stock in steady state, the same value as that in Kiyotaki and Moore (1997). But other combinations of these parameters could achieve the same ratio, and generate either more or less persistence. The parameter δ was chosen to be low enough so that the credit mechanism generates substantial persistence, while still producing model responses that appear well behaved.

3.5.5 Response to aggregate productivity shock

In this section I consider the response of the model economy to aggregate productivity shocks. I compare these responses with the responses of a 'flexible price' version of the model (with $\kappa = 0$), and also with the response of the fully efficient model, outlined in section (3.5.3). Figure 3.1 shows the response of output, the price of capital, and aggregate entrepreneurial wealth. The units on the vertical axes are percentage deviations from steady state. The units on the horizontal axes are quarters, with the shock taking place in quarter 1. The productivity shock is a 0.25 per cent fall in aggregate productivity, which lasts only for a single period. In other words, aggregate productivity follows a white noise process. Output in the efficient model falls by about 1.7 times the fall in productivity, which is the combined effect of lower productivity and lower labour inputs. After the shock, output returns fairly quickly to its steady-state value. We know from equation (3.74) that, if productivity follows a white noise process, then the persistence of output, as measured by the autocorrelation coefficient, is equal to $\frac{\eta(\tau+1)}{\tau+\eta+\sigma}$. Using the baseline calibration, this is equal to 0.17. Asset prices and aggregate wealth respond with the same proportional magnitudes as output. For the flexible price model with credit frictions, the initial output response is the same as the efficient response, because all determinants of output other than labour (i.e. last period's borrowing decision, the share of capital held by productive agents, and investment in inventories) are predetermined. But note that the asset price falls more than twice as much. This amplification is due to the following mechanism. In period 1, producers and investors experience an unanticipated loss of output, as well as an unanticipated reduction in the value of producers' collateral. This means that in period 1, producers cannot maintain their share of the capital stock: they can now afford less than the steady-state share, because they buy capital with the reinvested share of output and with collateralised borrowing. This means that capital will be less efficiently used for production from period 2 onwards. Because today's capital price is the present discounted value of all future marginal returns to capital, the price of capital falls by more than in the efficient model, and this fall further exacerbates the reduction in producers' net worth. Output in period 2, rather than rising back towards the steady-state, falls further due to the shift in capital from highly productive to less productive entrepreneurs. After period 2, it takes time for the most productive agents to rebuild their share of wealth, and it therefore takes time for asset prices and output to return to their steady-state values. It is interesting to note that the high degree of amplification is achieved with a plausible parameter value for the capital share and a plausible parameter value for the intertemporal elasticity of substitution (log utility implies a value of 1). Cordoba and Ripoll (2004) find that, in a model where the agents' productivity level is fixed permanently, no substantial amplification can be achieved unless either of these two parameters take on values that

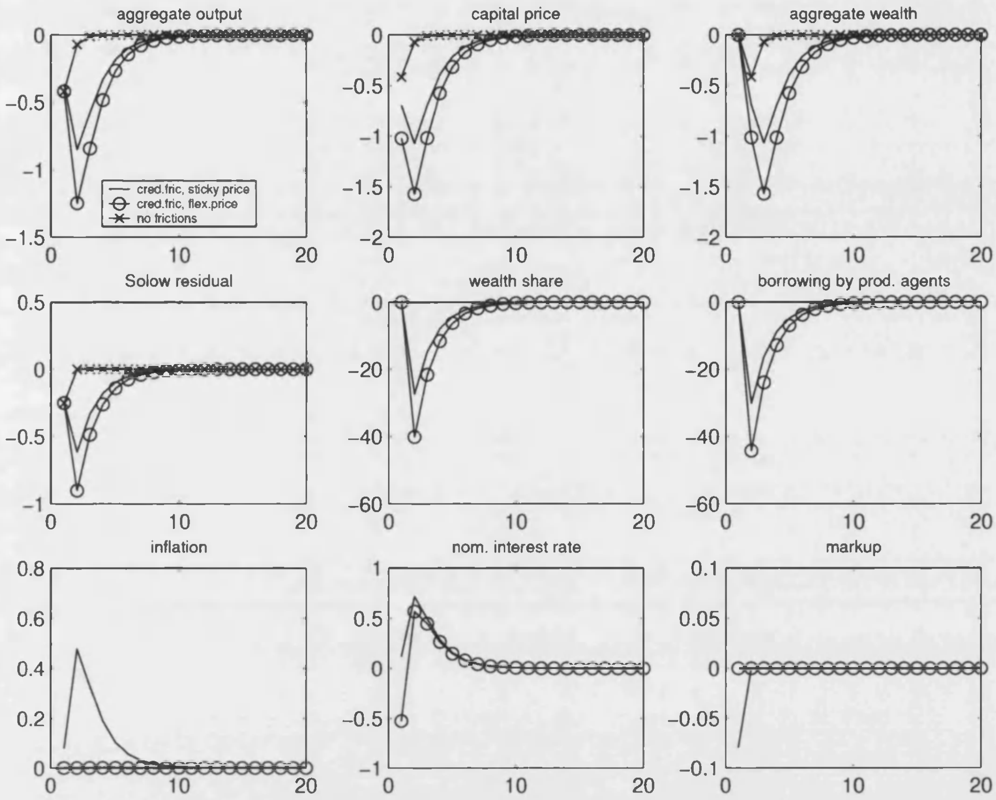

Figure 3.1: Response to productivity shock (baseline model)

are well outside the range usually thought to be plausible, such as capital shares in excess of 0.5, and elasticities of substitition below 0.1.

In the full model, with sticky prices as well, the initial fall in aggregate output is slightly muted relative to the efficient and flexible price models. As output falls, the nominal price level needs to rise for any given monetary policy stance that does not fully accommodate the output fall. But prices are sticky, so they do not rise enough. This causes the real marginal cost of the retail sector to rise, as not all retailers are able to charge their desired mark-up. For the entrepreneurs, however, paying a lower mark-up is beneficial: it increases the value of their output in consumption terms, which in turn increases the amount of labour they want to hire, relative to the amount of labour they would want to hire with constant mark-ups. This mechanism, while appearing perhaps non-standard when described this way, is simply the New Keynesian channel whereby those who cannot change prices change output to meet demand. Ouput is therefore higher than it would have been under flexible prices. So aggregate output falls by less in the period of the shock. This has important consequences for output dynamics in future periods. Because output falls by less, there is a smaller redistribution of wealth from producers to investors. There is therefore a smaller response of asset prices and aggregate wealth, because less of the capital stock shifts from producers to investors during the transmission of the shock. The entire credit - asset price effect has been dampened by the stickiness of prices. The response of inflation, nominal interest rates and the mark-up in the sticky price model are also shown in figure 3.1.

The key difference, relative to standard sticky-price monetary models, is that the flexible price fall in output from period 2 onwards following an adverse productivity shock is no longer fully efficient. This can be seen from the fact that the no-frictions level of output, which also corresponds to a social planner solution in the absence of all frictions, lies strictly above the flexible-price level of output from period 2 onwards.[9]. In standard sticky-

[9]It is important to emphasise that to achieve the first best it is necessary for the path of all variables to match the social planner path, not just output. I am using output deviations here as an indication of whether we are moving further from or closer to an optimal path. A full welfare analysis is carried out in the next chapter.

price monetary models, it is considered desirable for monetary policy to respond aggressively to inflation following a productivity shock, as this will simultaneously reduce inflation and ensure that output follows the same path as a model without price stickiness. In those models, as soon as productivity has returned to its steady-state level, so does the flexible price level of output. But in the credit frictions model considered in this chapter, only the initial fall in output is an efficient response to a change in aggregate productivity. The subsequent further fall, and the slow return to steady state are the result of inefficiencies in the credit market.

How large the dampening effect of sticky prices will be depends on how aggressively monetary policy responds to inflation. As the adverse productivity shock puts upward pressure on inflation, the monetary policy reaction function dictates that the nominal interest rate should rise. The more aggressive the rise in interest rates, the smaller the resulting increase in inflation, and the smaller the reduction in mark-ups. As monetary policy becomes sufficiently aggressive in its response to inflation, the economy's response to productivity shocks approaches that of the flexible price economy, where mark-ups are constant. As monetary policy becomes less aggressive, by responding less strongly to inflation, output fluctuations become smaller. However, in order to ensure determinacy of the equilibrium, monetary policy must react to inflation with a coefficient of at least 1, so aggressiveness cannot be toned down too far.

One further aspect of the model that is worth mentioning is that, even though the level of productivity of each firm is only perturbed for a single period, the measured aggregate level of productivity falls persistently. Panel

4 of figure 3.1 shows the response of the Solow residual, A_t. This is calculated as the total factor productivity in the economy under the assumption that there is no heterogeneity in productivity. When log-linearised, it is equal to

$$\hat{A}_t = \frac{y}{y+y'}\hat{y}_t + \frac{y'}{y+y'}\hat{y'}_t - \eta\hat{X}_{t-1} - (1-\sigma-\eta)\hat{l}_t \tag{3.78}$$

The shift in capital from producers to investors causes measured aggregate productivity to fall further in the period following the shock, and given that the shift in capital is persistent, the fall in aggregate productivity is persistent too. Furthermore, the extent of the fall depends on how monetary policy reacts to the shock. If monetary policy keeps inflation strictly constant, aggregate productivity falls further, relative to the case where monetary policy allows inflation to rise temporarily. The model therefore gives an interesting perspective on the interaction between aggregate productivity, heterogeneity and monetary policy. This is discussed in more detail in the next section.

3.5.6 Response to monetary policy shock

Figure 3.2 shows the model economy's response to a temporary white noise shock to the monetary policy rule, where the model now features staggered prices and the monetary policy rule (3.77)[10]. The shock is calibrated to cause a 0.25 per cent rise in the annualised nominal interest rate. The discussion here is brief, because most of the mechanism is similar to that in the case of a productivity shock. Only the initial phase of the transmission of the

[10]For completeness, the response of this staggered pricing version of the model to productivity shocks is given in figure 3.3 .

disturbance differs. Nominal interest rates rise in response to the shock. Because retailers are unable to lower their prices sufficiently in response to the monetary contraction, their mark-ups rise. Entrepreneurs therefore face a fall in the consumption value of their output, which reduces net worth both via a direct effect of the mark-up and via the consequent reduction in labour inputs. The fall in output is only 10bp, but total wealth is around 70bp. Because of the leverage effect, producers suffer a larger fall in net worth than investors. Their share of total wealth falls by nearly 30 per cent, so the wealth distribution is shifted from those with high productivity to those with low productivity. This lowers return on capital in future periods, which causes a fall in the price of capital today, resulting in a reduction of net worth that is much larger than the reduction of the initial period's output alone. Output in the following period is lower still, because capital is now being used less efficiently. The return to the steady-state happens gradually, as producers rebuild their share of wealth, so that the wealth distribution returns to its stationary distribution. Note that in this case the efficient path of output, as well as the path of output under flexible prices, remains constant, because monetary policy would have no effect in this model absent sticky prices.

It is also interesting to note that aggregate productivity, as measured by the Solow residual, falls in response to a monetary contraction, as capital shifts from high to low productivity agents, and is therefore less efficiently used even for a given level of inputs. This puts an interesting perspective on the Real Business Cycle and monetary policy literature. The RBC tradition is to claim that monetary policy does not explain much of the variation in output, because a large share of the fluctuation can be explained as an endogenous response to exogenous productivity or technology shocks (see e.g. Prescott (1986) and Plosser (1989). But if measured aggregate productivity is not exogenous, but instead is affected by monetary policy shocks, as well as by the systematic response of monetary policy to other shocks, this conclusion in unwarranted. More recently, several authors of the real business cycle tradition have questioned the interpretation of aggregate productivity as strictly determined by technology alone (see e.g. Prescott (1998) and Kehoe and Prescott (2002)). Chari, Kehoe and McGrattan (2004) have suggested that aggregate productivity, rather than being taken as given, is something that needs to be formally explained by a model. They call it the "efficiency wedge". The model I present here is one possible formalisation of a process that makes the efficiency wedge endogenous, and sensitive to monetary policy. It also counters Chari, Kehoe and McGrattan's (2004) claim that credit frictions are unlikely to explain a significant share of business cycle fluctuations. I believe they make this claim because the model of credit frictions they have in mind is one of the BGG (1999) variety, where the credit frictions do not affect aggregate productivity. In the BGG (1999) model, credit frictions primarily affect investment, and while that will of course affect the aggregate capital stock, this effect is small, because the capital stock is so large relative to investment. The distribution of capital among entrepreneurs has no direct effect on aggregate productivity.

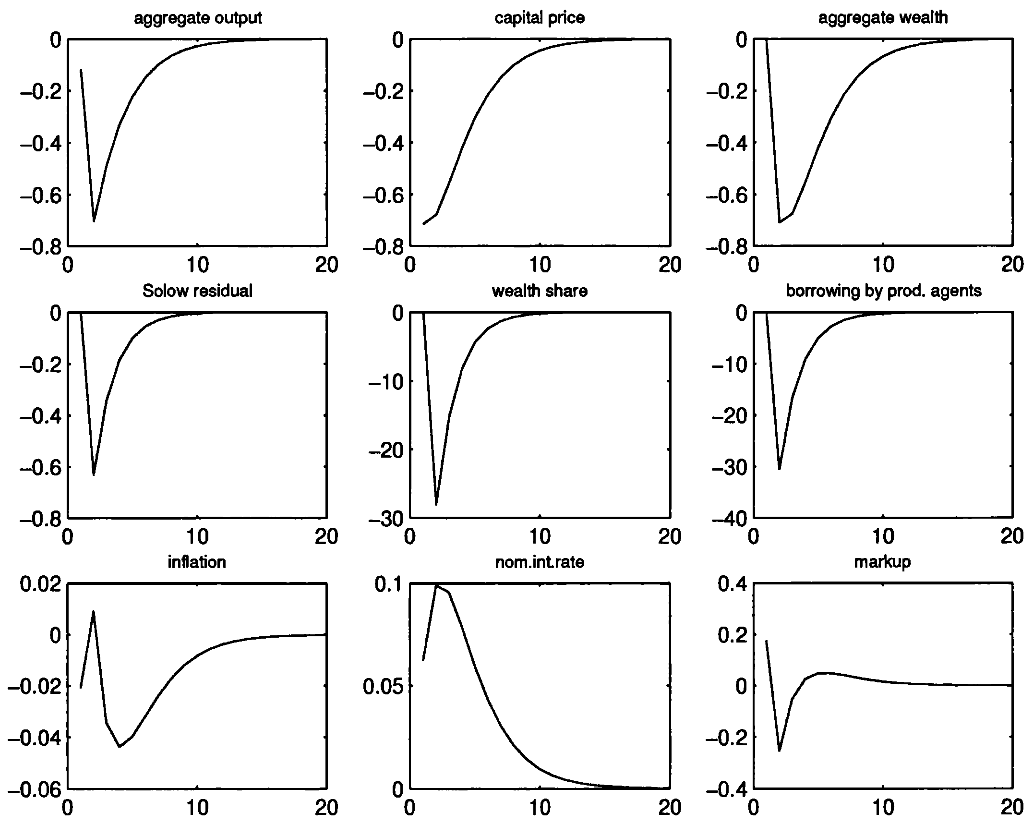

Figure 3.2: Response to monetary policy shock (staggered pricing model)

3.6 An extension: nominal debt contracts

So far the debt contracts between investors and producers were specified in real terms. This section explores how the model properties change when contracts are specified in nominal terms. Real world debt contracts are usually specified in nominal terms, i.e. they are usually not indexed. I will not seek to explain why debt contracts are specified in nominal terms, but will analyse how the economy's response to shocks changes under nominal contracting.

To keep the notation as close as possible to the other versions of the model, I will adopt the following notation. Let $b_{N,t+1}$ equal the nominal amount of debt that is repayable at time $t+1$, but known at time t. I then define $b_{t+1} = \frac{b_{N,t+1}}{E_t p_{t+1}}$ to be the real amount of debt that is expected to be repaid in period $t+1$, but which need not be equal to the ex-post real amount of debt that will actually be repaid. The borrowing constraint, when specified in nominal terms, becomes

$$b_{N,t+1} \leqslant E_t p_{t+1} q_{t+1} k_t \tag{3.79}$$

When linearised and binding, this constraint is exactly equal to the real borrowing constraint. Where the model does change following the introduction of nominal contracts is in the budget constraints of investors and producers. When written in real terms, and using the timing convention for real debt explained above, the budget constraint becomes:

Table 3.2: Comparison of properties of baseline model and model with nominal contracts

	s.d.(output)	s.d.(π)	s.d.(q)
baseline model (prod.shocks)	0.046	0.025	0.061
baseline model (mon.shocks)	0.030	0.017	0.041
nominal contracts (prod.shocks)	0.026	0.013	0.033
nominal contracts (mon.shocks)	0.061	0.035	0.082

Note: table shows the theoretical standard deviations of log-linearised total output, inflation and the price of capital, under the assumption that there is one type of shock only, which is normally distributed with zero mean and 0.01 standard deviation.

$$c_t + x_t + q_t(k_t - k_{t-1}) + w_t l_t = \frac{y_t}{\varphi_t} + \frac{b_{t+1}}{r_t} - b_t \frac{E_{t-1}\pi_t}{\pi_t} \tag{3.80}$$

It is immediately clear that what matters now is whether inflation in period t is equal to its ex-ante expected level. If inflation is higher than expected, the real value of repayments will be lower, which will shift real resources from investors to producers. In terms of the model solution, the only modification is therefore in equation (3.64), which becomes:

$$W_0 s_0 = (1-\delta)\left((\eta+\sigma)\frac{Y_0}{\varphi_0} + q_0 K_{-1} - B_0 \frac{E_{-1}\pi_0}{\pi_0}\right) + n\delta\left((\eta+\sigma)\frac{Y_0'}{\varphi_0} + q_0(\overline{K} - K_{-1}) + B_0 \frac{E_{-1}\pi_0}{\pi_0}\right)$$

To summarise how the model properties change following the introduction of nominal contracts, table 3.2 compares the standard deviation of output, inflation and asset prices with the baseline case with real contracts.

The results illustrate that nominal contracts dampen the effects of productivity shocks, but amplify the effects of monetary policy shocks. The intuition is straightforward. Consider an adverse productivity shock: output is lower than expected, which causes a transfer of wealth from producers (who are borrowers) to investors (who are lenders), creating the output and asset price amplification mechanism due to credit market imperfections discussed extensively in section (3.5.5). But following a productivity shock inflation is higher than expected. In other words, inflation and output move in opposite directions. Higher inflation erodes the real value of the debt that producers have to repay, and partly offsets the effects of the lower-than-expected output and asset price. The result is that, following a productivity shock, the transfer of wealth between producers and investors is smaller in the case of nominal contracts, which means the output and asset price amplification mechanism is weaker. The dampening effect of nominal contracts in the case of supply shocks is also discussed in Iacoviello (2005).

In the case of a monetary policy shock, the economy responds more strongly under nominal contracts. That is because in the case of monetary policy shocks inflation and output move in the same direction. For example, in the case of an unexpected monetary tightening, output will be lower than expected, which will trigger the amplification mechanism via credit market imperfections and cause output and asset price falls to be amplified as wealth is transferred from highly productive producers to less productive investors. On top of that, the lower-than-expected inflation increases the real value of the debt that producers have to repay. This causes a further transfer of wealth away from producers, meaning an even less efficient use of capital in future periods, and therefore even greater falls in output, inflation and asset prices. This is precisely the debt-deflation mechanism described in Fisher (1933). This amplifying mechanism of nominal contracts would operate for any shock that moved inflation and output unexpectedly in the same direction.

3.7 Conclusion

I have outlined a macroeconomic model where credit markets operate less than perfectly due to enforcement problems, and I have used this model to discuss the interaction between aggregate output dynamics, the wealth distribution and the effect of monetary policy. None of the building blocks of the model are new. The idea that monetary policy works through a redistribution of wealth between highly productive and less productive agents is very much in the spirit of Fisher (1933). The notion that the net worth of agents affects the quantity of investment is a common theme in the macro-economic 'credit-channel' literature, reviewed by Gertler (1988) and Bernanke and Gertler (1995). And the idea that the wealth distribution can have a first-order effect on aggregate output via the efficiency with which capital is used was formalised in Kiyotaki and Moore (1997) and Kiyotaki (1998). The contribution of this paper is to put these elements together in an internally consistent, tractable model. The analysis has shown that the credit mechanism can amplify shocks and make them highly persistent, so that small, temporary disturbances to productivity or monetary policy have large and persistent effects on output. The basic mechanism is that, because highly productive agents find it optimal to borrow from less productive agents, they are leveraged. Any aggregate disturbance will affect borrowers' net worth more than lenders' net worth due to leverage, and so will affect the wealth distribution. The most productive agents will end up holding less of the economy's productive resources, which will lower aggregate output and further depresses the current price of capital, exacerbating the shift in the wealth distribution. It takes time for the most productive agents to rebuild their share of wealth, and output therefore deviates from its steady-state level for many periods. Aggregate productivity in this model is endogenous, and is affected by the systematic response of monetary policy to non-policy shocks, as well as by monetary policy shocks. This is relevant to the debate about whether business cycle fluctuations are caused mainly by exogenous factors such as technology, and to what extent better monetary policy can result in less variable output. I have shown that sticky prices not only dampen the output effect of productivity shocks, which is not new, but that they bring the output effect of productivity shocks closer to efficient levels - which is new. This casts new light on the trade-off between output and inflation variability that systematic monetary policy aims to balance. The flexible-price response of the economy to productivity shocks is no longer efficient. And by allowing some inflation variability, monetary policy can achieve lower output variability around the efficient level. These ideas are pursued further in the following chapter, where I consider how monetary policy should optimally react to productivity shocks, given the trade-off created by credit frictions.

3.8 Appendices

3.8.1 Frictionless Model

This section describes the model without nominal rigidities and without credit frictions. All capital is now employed by producers, so we can ignore the distinction between the two types of entrepreneurs, and the single aggregate production function becomes

$$y_t = \alpha \varepsilon_{P,t} \left(\frac{\overline{K}}{\sigma} \right)^{\sigma} \left(\frac{x_{t-1}}{\eta} \right)^{\eta} \left(\frac{l_t}{1 - \sigma - \eta} \right)^{1-\sigma-\eta} \tag{3.81}$$

As before, the demand for labour satisfies

$$w_t l_t = (1 - \sigma - \eta) \frac{y_t}{\varphi_t} \tag{3.82}$$

It will be optimal for workers to consume current labour income and the profits from retailers, although they are no longer prevented from borrowing. Since prices are flexible, retailers will set $\varphi_t = \varphi = \frac{\theta}{\theta - 1}$.

Given labour demand, the problem can be reduced to a savings problem where the only decision is how much to save into variable inputs x_t. We guess that the optimal decision rule is

$$x_t = \eta \beta \frac{y_t}{\varphi} \tag{3.83}$$

which implies, by the budget constraint, that

$$c_t = (\eta + \sigma - \eta \beta) \frac{y_t}{\varphi} \tag{3.84}$$

These decision rules are linear, so they can be added up and will also apply to aggregate variables, denoted with capitals. The interest rate is the marginal return on investment, and can be simplified to

$$r_t = \eta \frac{Y_{t+1}}{Y_t} \tag{3.85}$$

It can be shown that the Euler equations for entrepreneurs and workers are satisfied at the above decision rules for consumption and investment, confirming the initial guess.

The law of motion for aggregate output can be found by substituting factor demands and labour supply into the production function, and is equal to

$$Y_{t+1} = (\alpha \varepsilon_{P,t+1})^{\frac{\tau+1}{\tau+\eta+\sigma}} \left(\frac{\overline{K}}{\sigma}\right)^{\sigma\frac{\tau+1}{\tau+\eta+\sigma}} \left(\frac{1-\eta-\sigma}{\chi}\right)^{\frac{1-\eta-\sigma}{\tau+\eta+\sigma}} (\beta Y_t)^{\eta\frac{\tau+1}{\tau+\eta+\sigma}} \tag{3.86}$$

The price of capital, q_t, can be derived with a user cost type equation, leading to

$$q_t = \frac{\sigma\beta}{\overline{K}(1-\beta)} \frac{Y_t}{\varphi} \tag{3.87}$$

which is also equal to the present discounted value of future $u_t = \sigma\beta\frac{Y_t}{\varphi}$.

By defining wealth as $W_t = (\eta + \sigma)\frac{Y_t}{\varphi} + q_t\overline{K}$, we can use the following an expression for W_t:

$$W_t = \left(\frac{\eta+\sigma-\eta\beta}{1-\beta}\right)\frac{Y_t}{\varphi} \tag{3.88}$$

3.8.2 Steady State Equations

Note that I have used z to denote the wage paid to workers, to avoid confusion with wealth, denoted W.

Implicitly,

$$s = \frac{(1-\delta)s\widetilde{\alpha} + n\delta(1-s)\widetilde{\gamma}}{\widetilde{\alpha}s + \widetilde{\gamma}(1-s)} \tag{3.89}$$

So

$$\begin{aligned} s \;=\; & -\frac{(\widetilde{\gamma} - (1-\delta)\widetilde{\alpha} + n\delta\widetilde{\gamma})}{2(\widetilde{\alpha} - \widetilde{\gamma})} \\ & + \frac{\sqrt{(\widetilde{\gamma} - (1-\delta)\widetilde{\alpha} + n\delta\widetilde{\gamma})^2 - 4(\widetilde{\alpha} - \widetilde{\gamma})n\delta\widetilde{\gamma}}}{2(\widetilde{\alpha} - \widetilde{\gamma})} \end{aligned} \tag{3.90}$$

$$r = \frac{1}{\beta}\frac{\widetilde{\gamma}}{\widetilde{\alpha}s + \widetilde{\gamma}(1-s)} \tag{3.91}$$

To avoid excessively lengthy expressions, it is useful to create some constants

$$a_1 = \frac{\overline{K}}{\beta}\left(\frac{1-\beta}{\eta+\sigma}\frac{r}{r-1} + \frac{\eta}{\eta+\sigma}\frac{1}{\sigma}\right) \tag{3.92}$$

$$a_2 = (\eta+\sigma)\, a_1 + \frac{r}{r-1}\overline{K} \tag{3.93}$$

$$a_3 = 1 + \frac{\tau}{1+\tau}\frac{1-\eta-\sigma}{\eta+\sigma} \tag{3.94}$$

$$a_4 = \frac{\sigma}{\eta+\sigma} \tag{3.95}$$

$$a_5 = \frac{\widetilde{\alpha}K + \widetilde{\gamma}(\overline{K} - K)}{\sigma} \left(\frac{1}{\chi\varphi}\right)^{\frac{1}{1+\tau}\frac{1-\eta-\sigma}{\eta+\sigma}} \left(\frac{1}{1-\eta-\sigma}\right)^{\frac{\tau}{1+\tau}\frac{1-\eta-\sigma}{\eta+\sigma}} \tag{3.96}$$

$$Y + Y' = \left(\frac{1}{a_1\varphi}\right)^{\frac{a_4}{a_3-a_4}} a_5^{\frac{1}{a_3-a_4}} \tag{3.97}$$

$$u = \frac{Y + Y'}{a_1\varphi} \tag{3.98}$$

$$W = a_2 u \tag{3.99}$$

$$z = \chi^{\frac{1}{\tau}} \left[(1-\eta-\sigma)\frac{Y+Y'}{\varphi}\right]^{\frac{\tau}{1+\tau}} \tag{3.100}$$

$$Y = \left(\frac{\widetilde{\alpha}}{z\varphi}\right)^{\frac{1-\eta-\sigma}{\eta+\sigma}} \frac{K}{\sigma} u^{\frac{\eta}{\eta+\sigma}} \tag{3.101}$$

$$Y' = \left(\frac{\widetilde{\gamma}}{z\varphi}\right)^{\frac{1-\eta-\sigma}{\eta+\sigma}} \frac{\overline{K} - K}{\sigma} u^{\frac{\eta}{\eta+\sigma}} \tag{3.102}$$

$$q = \frac{r}{r-1} u \tag{3.103}$$

$$b = qK \tag{3.104}$$

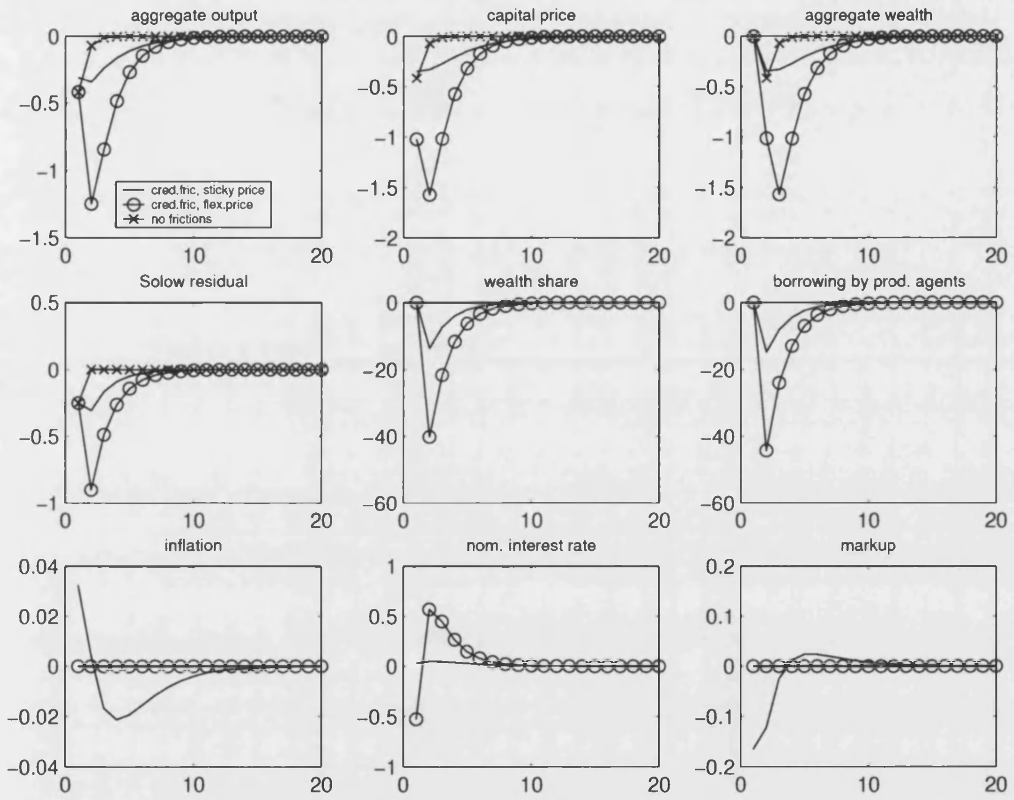

Figure 3.3: Response to productivity shock (staggered pricing model)

Chapter 4

Optimal monetary policy in a model with credit market imperfections

4.1 Introduction

If imperfections in credit markets are an important part of the propagation mechanism of shocks, and monetary policy can have real short-term effects, then what is the implication for how monetary policy should optimally be conducted? That is the question this chapter addresses, using a model–developed in the previous chapter–of an economy with a strong credit channel that amplifies output and asset price fluctuations.

To analyse the implications for monetary policy, I embed this credit propagation mechanism into a model with sticky goods prices, so that monetary policy can have short-run real effects. I then investigate what optimal monetary policy should aim to achieve. This is done by assuming that the monetary policy-maker tries to maximise the welfare of the private sector agents, which is commonly referred to as a Ramsey problem. There are two frictions in the economy: credit market frictions and sticky prices. The policy-maker has a single instrument available, the nominal interest rate, to off-set the inefficiencies generated by these frictions. I build intuition for the trade-offs that are created by considering several versions of the model. In particular, I consider a frictionless version, where credit markets operate perfectly and prices are free to adjust. I also consider a flexible-price credit version, where credit market imperfections exist, but prices are fully flexible (or the monetary policy-maker is assumed to stabilise prices perfectly, which is equivalent). The frictionless model illustrates how the economy should behave absent any inefficiencies, and is therefore the first best. Essentially, following an exogenous temporary fall in productivity, output should fall, but then return to its steady-state relatively quickly. Asset prices follow the same path as output. The flex-price credit model illustrates what happens if the policy-maker cannot or will not take any action to off-set the adverse effects from the credit propagation mechanism. Following an exogenous temporary fall in output, asset prices fall drastically, creating a fall in net worth and a large shift in the wealth distribution from borrowers to lenders, which results in further and persistent output falls in future periods. The optimal policy is to try and offset some of the initial output fall, by letting inflation rise temporarily following the adverse productivity shock. That will dampen the fall in net worth, which will dampen the fall in asset prices and therefore reduce the efficiency losses associated with a large shift in the wealth distribution towards less productive agents. Although inflation is costly, and there is no reason to dampen the initial output fall when considering the initial period in isolation, the efficiency loss associated with inflation and the dampened output response are off-set by the efficiency gains from preventing the credit mechanism from lowering future output. Output fluctuations create dynamic externalities due to their effect on future output via credit markets, so it is efficient to offset output fluctuations.

The combination of credit frictions with sticky prices therefore means that there is a trade-off between output and inflation fluctuations following a productivity shock. Such a trade-off is largely absent in modern models of monetary policy, where stabilising prices does not conflict with stabilising the output gap. Optimal policy in a world of both credit frictions and sticky prices involves using short-run inflation fluctuations to smooth output in an absolute sense, instead of smoothing output relative to its flexible-price level. The end result is that output fluctuations are smaller, even relative to the fully optimal path of output.

Because any output fall has a dynamic effect on future output, the trade-off is also dynamic: it is a trade-off between inflation immediately following the shock, and the fall in *future* output relative to its efficient level. The dynamic nature of this trade-off implies that neither the gap between output and its flexible-price level, nor the gap between output and its fully efficient level are adequate descriptions of monetary policy objectives.

The paper is structured as follows. Section (4.2) surveys the related literature, section (4.3) briefly describes the model, section (4.4) sets out the formal problem that the policy-maker is trying to solve, section (4.5) describes the solution, i.e. how output, inflation and asset prices behave under optimal policy, section (4.6) discusses how the presence of nominal contracts affect the problem. Section (4.7) analysis how sensitive the results are to variations in parameter choices, and section (4.8) concludes.

4.2 Related literature

This chapter relates closely to the modern literature on optimal monetary policy. The investigation into the short-run role of monetary policy was revived when economists sought to provide analytical underpinnings for the short-run Phillips correlation, i.e. the notion that expansionary monetary policy can raise output in the short term, but will only raise prices in the long term. Friedman took a strong stand on this, and argued that monetary policy-makers would never have enough information to use their ability to influence the real economy in the short run to good effect. Lucas (1972) and Phelps et al (1970) outlined an analytical mechanism that would allow monetary expansions to have a short-term real effect, but the mechanism could not be exploited by the central bank to the economy's advantage. The policy prescription, as in Friedman, was not to try and actively influence the short-run fluctuations of the economy, but simply keep the money supply constant. It was not until the so-called New Keynesian literature that a systematic short-term role for monetary policy was put on a modern analytical footing. Fischer (1977) outlined a model of staggered wage contracts where monetary policy could improve outcomes by following a rule for the money supply that responds to shocks. By the mid-to-late 1990s, a number of authors had started to work on what is now known as the Neo-Classical Synthesis or New Keynesian Synthesis. Important contributions were Kimball (1995), Yun (1996), Woodford (1996), King and Wolman (1996), Kerr and King (1996). The basic framework that emerged from these papers is a modern, dynamic version of the IS/LM model which is derived from optimising behaviour. It consists of an aggregate demand relationship, linking output to real interest rates, a Phillips curve based on a staggered pricing mechanism initially due to Calvo (1983), linking inflation fluctuations to output fluctuations, and an equation that specifies how money or interest rates are set. Because the model is derived from optimising behaviour, there is an explicit use of utility or welfare. This allows for a discussion of which policies are optimal in a welfare optimising sense, whereas previous authors were restricted to minimising the variance of output or an ad hoc weighted sum of the variance of output and inflation. Since the precise weight can influence the policy prescription quite dramatically, it is useful to have some theoretical basis for the welfare criterion. The link between the policy-maker's objective function and the utility of private sector agents was first derived in Rotemberg and Woodford (1998).

An important technical contribution was that control theorists developed a recursive way to express optimal policy with forward-looking private sector agents. Miller and Salmon (1985) and Currie and Levine (1985) showed how to formulate in a recursive way the solution to the problem set out by Kydland and Prescott (1977). To be clear, this did nothing to solve the time-inconsistency of optimal plans that Kydland and Prescott had identified.

But it was a way to formulate a fully optimal plan as a rule. Although this particular recursive method then became frequently used in game theory and public finance, its use in monetary policy did not come to the fore until Woodford (1999), who used the method to show that when the central bank can commit to any policy, the fully optimal policy implies that interest rates depend partly on lagged interest rates. Gali, Gertler and Gilchirst (1999) also formulate optimal policy under commitment, and show that it leads to improved outcomes relative to a discretionary monetary policy.

So by the late 1990s several important strands of the literature had been usefully combined to study optimal monetary policy: (a) a theory of nominal rigidities, (b) an optimisation-based formulation of a complete macro-model, (c) a recursive technique for formulating optimal policy under commitment when the private sector agents are forward-looking, and (d) an explicit link between the policy-maker's objective and the welfare of the private sector agents.

Because it is convenient and customary to work with linear or log-linear approximations of models, the issue arose of what the appropriate order of approximation was for welfare analysis. Rotemberg and Woodford (1998) studied the case of a quadratic approximation around an undistorted steady-state. The key insight was that first-order terms in the welfare approximation cancel out in the neighbourhood of an unconstrained optimum, leaving a quadratic approximation to the welfare function that was accurate up to second order. The problem is then reduced to maximising a quadratic objective function subject to linear constraints (since the equations governing the private sector are linearised). The reason why it is sufficient to use first-order, or linear, approximations to the model equations is the following: the first-order approximations to the model equations will contain approximation errors of second order. But welfare is computed by taking the square and cross-products of model variables, so that the approximation errors will be squared too. The second-order accuracy of the welfare approximation is therefore unaffected. This approach is widely used by many authors who study optimal monetary policy in various settings.

The drawback, however, is the restriction that the approximation is only valid if the steady state is undistorted or the distortion is very small, because it is this condition that ensures that the linear terms cancel out of the welfare approximation. Kim and Kim (2003) showed how erroneous welfare conclusions can be drawn with linear approximations to distorted models. Various authors solved the problem by imposing that the steady-state distortion created by monopolistic competition, which is present in all of the models of this class, is off-set by a government subsidy. Some found this solution to be inappropriate, and two alternatives were proposed.

A first method is that of Benigno and Woodford (2003), who take second-order approximations to some of the model equations in order to substitute out the first-order terms in the welfare approximation and replace them by second-order terms. This new approximation to the welfare function is correct up to second-order, for a steady-state distortion of any size, and one need only use first-order approximations to the model equations to evaluate welfare with second-order accuracy. A drawback is that it can be very difficult in some cases to find the appropriate substitution to eliminate the first-order terms in the welfare function.

A second method is that of Schmitt-Grohe and Uribe (2004a). They compute the first-order conditions that govern the optimal plan with respect to the original, non-linearised equations. The resulting system of non-linear first-order conditions facing the policy-maker is then linearised. This will produce second-order accuracy for second moments of any model variable, even when the steady state is distorted. This is the approached I have used. To compute an approximation of welfare, which consists of both first and second moments of model variables, one would have to take a quadratic approximation to the complete resulting system of equations.

Having traced the evolution of the optimal monetary policy literature and the technical advances that were made to get to the present state of the literature, it is useful to highlight two strands of applications of these techniques. One is a series of extensions to the basic framework that give rise to a trade-off between output and inflation variability. The other relevant strand is the literature on the optimal response to asset price fluctuations.

The basic New Keynesian or Neo-Classical Synthesis model has the feature that it is optimal to stabilise inflation fully and instantaneously. There is generally no trade-off between stabilising output and inflation, something which Blanchard (2005) has dubbed "the divine coincidence". This is often thought to be an unsatisfactory feature of the framework, since it leads to an incredibly aggressive policy prescription, namely that inflation should be kept completely constant. Erceg, Henderson and Levin (2000) analyse the case where it is not just goods prices that are subject to nominal rigidities, but wages as well. They show that it is no longer desirable to stabilise inflation fully and instantly. Consider a positive productivity shock. In the absence of frictions, output should rise and the real wage should rise. In the presence of staggered goods prices, it is optimal to keep the price level constant following the shock, so that there is no price dispersion. But since nominal wages are staggered too, keeping the price level constant will mean that real wages do not rise by enough. So the optimum response for each variable cannot be achieved, and it is, in general, optimal to allow some variability in inflation, wage inflation and the output gap. Optimal policy can be approximated reasonably well by the stabilisation of a weighted index of prices and wages. Blanchard and Gali (2005) suggest another source of a trade-off between the deviation of inflation from target and the deviation of output from its first best level. They model, albeit in a reduced-form way, a process for the real wage that only gradually adjusts to the equilibrium real wage. The interaction of this real rigidity with nominal goods price rigidity makes it optimal to stabilise inflation only gradually.

The second relevant set of extensions of the basic New Keynesian model is the literature that explores the optimal monetary policy response to asset price movements. Bernanke, Gertler and Gilchrist (1999) formulated a model which, in addition to nominal rigidities, featured credit frictions, so that there is meaningful feedback from asset prices to the real economy, which may give asset prices a special role in monetary policy. In their model, firms need to borrow from households in order to invest in capital, but households cannot costlessly observe the finances of the firm. This costly-state-verification problem implies that it is optimal for households to lend to firms via intermediated debt contracts, and for intermediaries to monitor and liquidate only those firms which do not repay their debt. The interest rate charged on the debt will vary inversely with the net worth of the firm, because firms with less net worth are more likely to fail following a shock, so households are more likely to have to incur the monitoring cost. In this framework, a positive shock to output is amplified as it results in an increase in net worth, which lowers the cost of borrowing, thereby raising investment demand. Bernanke and Gertler (2001) then use this framework to ask whether monetary policy should respond to asset prices as well as to inflation and the output gap. In practice, this means analysing whether, in the class of ad hoc monetary policy rules, a rule that includes asset prices performs better than one that does not. To stack the cards in favour of finding a strong role of asset prices in monetary policy, the authors add to their model non-fundamental movements in asset prices, or bubbles, which, via the net worth effect, have real effects on investment and output. They find that there is very little benefit to be had, in terms of minimising an ad hoc loss function, from letting monetary policy-makers respond to asset prices. Iacoviello (2005) carries out a similar analysis, based on the credit frictions framework of Kiyotaki and Moore (1997), and also concludes that there is little benefit to be derived from monetary policy that responds directly to asset prices. Faia and Monacelli (2004) extend the analysis of Bernanke and Gertler (2001) by examining a wider class of monetary policy rules, and by evaluating an approximation of welfare, rather than an ad hoc loss function. They find the optimal coefficients on various arguments of a monetary policy rule, and then experiment with changing those coefficients, and analyse the resulting welfare loss. They too find that including asset prices in the monetary policy rule does not improve welfare much.

There is one fundamental shortcoming of these analyses. It is not clear whether this is an interesting question at all, to ask whether optimised coefficients on asset prices in ad hoc monetary policy rules are big; and whether omitting asset prices from ad hoc rules causes large welfare losses. The class of monetary policy rules that are considered is rather arbitrary, and even if changing or restricting those coefficients has only a small effect on welfare, there is potentially a large welfare loss from using the restricted monetary policy rule relative to a fully optimal monetary policy in the sense of a Ramsey solution to a planning problem (see e.g. Ljunqvist and Sargent (2004), chapter 30 for a very general formulation). So it is not clear at all what one can conclude from the statement that asset prices do not have a big coefficient in optimised ad hoc policy rules, and that changes in such coefficients do not have a large effect on welfare or a measure of loss.

Gilchrist and Leahy (2002) use a different methodology to analyse the problem, and get closer to what is probably the more interesting question: if credit frictions are important, does that mean monetary policy makers should try to achieve a significantly different path for macroeconomic variables compared to an economy without credit frictions? Gilchrist and Leahy (2002) analyse the response of macroeconomic variables in a model with credit frictions and nominal rigidities, and compare this with the response of a frictionless economy, which provides the benchmark of what the optimal response of all variables should be, and a New Keynesian economy with nominal rigidities but no credit frictions. They then experiment with some simple policy rules, and see whether the resulting macroeconomic responses get closer to the frictionless response. They conclude that in the case of a gradual productivity increase (which is akin to a demand shock, as the bulk of the productivity increase occurs in the future), it is sufficient for monetary policy simply to respond to inflation. A stronger response to inflation will bring the economy closer to a frictionless economy. But in the case of shocks to net worth, responding more strongly to inflation causes output to deviate further from its optimal path, so there is a short-run policy trade-off between inflation and output variability. A rule that responds to net worth as well as inflation can achieve lower output variability at the expense of higher inflation variability. They conclude, as many others have done, that there is little benefit from monetary policy responding to asset prices, but they speculate that it may well pay to respond to net worth or the spread between risky and risk-free interest rates, although they do not develop this idea any further.

This chapter of the thesis takes the next logical step in the literature, which is to carry out a full quantitative analysis of what paths of macroeconomic variables monetary policy should try to achieve in order to maximise welfare, if there are both nominal rigidities and credit frictions in the economy.

4.3 Overview of the model

In this section I outline the private sector model of the economy, which the policy-maker will face when designing an optimal policy plan. The model is explained in detail in the previous chapter. I summarise it briefly here. The full list of equations is given in the appendix. The economy consists of entrepreneurs, workers, retailers and a central bank.

Entrepreneurs face idiosyncratic variation in their level of total factor productivity, which follows a first-order Markov process. Call the entrepreneurs who have high productivity 'producers' and those with low productivity 'investors'. Entrepreneurs produce output using capital, which is in fixed aggregate supply and does not depreciate, inventories, which are costlessly created from consumption goods and depreciate fully each period, and labour, which is provided by workers. In a first-best world, producers would hold all of the capital stock, and would finance it by borrowing from investors or workers.

But entrepreneurs have no commitment technology to promise to repay, and it is assumed that anonimity prevents long term relationships from being established. So entrepreneurs can finance themselves only by using one-period, non-state contingent real bonds secured against collateral. The fixed capital stock can serve as collateral, ie investors are assumed to be able to confiscate producers' capital if they threaten to repay less than was initially agreed. Investors will then only be willing to lend up to the value that collateral is expected to have when the loan becomes due, so that in equilibrium, producers have no incentive to default. Provided spells of high productivity for any agent do not last too long, and provided that the share of agents who are productive is not too large, producers will not be able to borrow enough to finance the entire capital stock. Investors will hold some capital as well, and the economy will not operate at first best: since investors have lower productivity than producers, it is not efficient for them to hold any capital for production. I consider calibrations of the model where the borrowing constraint binds in the steady state, and linearise the model around this binding steady state.

Workers provide labour to entrepreneurs, and do not have access to productive technology. They are allowed to invest in entrepreneurs' bonds, but choose not to do so in equilibrium: this is because the real interest rate is below the inverse rate of time preference. This is a common feature of heterogeneous agent models with incomplete risk-sharing (e.g. Kehoe and Levine (2001)). In these models, the constrained agents face an upward-sloping consumption path as they are unable to borrow sufficiently to maintain a constant consumption path. This means in turn that the unconstrained agents face a downward-sloping consumption path. Bonds are priced by unconstrained agents. The interest rate that is consistent with a downward-sloping consumption path is below the inverse rate of time preference. In the neighbourhood of the steady state, workers therefore choose not to buy any bonds, and simply consume their income.

Entrepreneurs produce output of a single type, but all agents have preferences over a consumption basket of diversified goods. Retailers have a costless technology to turn output goods into diversified consumption goods. Because preferences are such that the various consumption goods are not perfectly substitutable, retailers have market power, and charge a mark-up on the consumption goods. Retailers are owned by workers, who receive the monopoly profits. It is also assumed that retailers face a cost in changing their price, which is quadratic in the size of the price change, following Rotemberg (1982). This therefore introduces price stickiness into the model, and a potential role for monetary policy. This formulation of nominal rigidities is slightly different from the previous chapter, where prices were staggered. Adjustment cost in prices is convenient to work with in welfare analysis because we can consider equilibria where all agents set the same price. Schmitt-Grohe and Uribe (2004b) use this formulation for welfare analysis. Because the linearised Rotemberg pricing equation is identical to a linearised Calvo pricing equation, the cost of price adjustment can be calibrated to be equivalent to a particular Calvo adjustment frequency.[1] In this model the equivalent of a Calvo probability of keeping prices fixed of 2/3 is to set the cost parameter of price changes $\psi = 5.4$.

The central bank is introduced in the model using Woodford's (2003) cashless limit: it is assumed that money balances are negligible, and the central bank stands ready to lend or borrow at a nominal interest rate it sets directly.

4.4 The optimal policy problem

4.4.1 Objective of the policy-maker

The policy-maker maximises the weighted sum of the welfare of entrepreneurs and of workers. The one-period welfare function is therefore the sum of the utility of all the agents. There is no unique way to sum utilities, but one candidate is

$$W_t = \ln\left(c_t + c_t'\right) + \mu \ln\left(c_t^w - \frac{\chi l_t^{\tau+1}}{\tau+1}\right) \tag{4.1}$$

[1]Although the calibration can be set so that the linearised pricing equations are identical, the welfare effects, and therefore the optimal monetary policy, are not necessarily identical because they are based on the non-linearised versions of the pricing equation.

This formulation uses total consumption across entrepreneurs, who are ex-ante identical. Workers are not identical to entrepreneurs: they face different constraints and have a different utility function, so they are treated separately, and added to the aggregate welfare function using μ, the Pareto weight on workers. This particular welfare function does not give any importance to the distribution of consumption across entrepreneurs, as only total entrepreneurial consumption matters. The distribution of consumption across entrepreneurs matters indirectly, of course, because a reallocation of resources away from highly productive producers lowers total output, and hence total consumption.

This welfare function can be written in terms of the model variables by using the consumption decision rules for entrepreneurs and workers:

$$c_t + c_t' = C_t = (1-\beta)w_t \tag{4.2}$$

$$\begin{aligned} c_t^w &= z_t l_t + \Pi_t \\ &= \left(\frac{1}{\chi}\right)^{\frac{1}{\tau}} z_t^{\frac{1+\tau}{\tau}} + (y_t + y_t')\left(1 - \frac{1}{\varphi_t}\right) - \frac{\psi}{2}(\pi_t - 1)^2 \end{aligned} \tag{4.3}$$

$$l_t = \left(\frac{1}{\chi}\right)^{\frac{1}{\tau}} z_t^{\frac{1}{\tau}} \tag{4.4}$$

In terms of the model variables, the one-period welfare function is then

$$W_t = \ln(1-\beta) + \ln w_t + \mu \ln \frac{1}{u_{w,t}} \qquad (4.5)$$

where u_w is the marginal utility of consumption of workers

$$\frac{1}{u_{w,t}} = \frac{\tau}{\tau+1}\left(\frac{1}{\chi}\right)^{\frac{1}{\tau}} z_t^{\frac{1+\tau}{\tau}} + \left(y_t + y_t'\right)\left(1 - \frac{1}{\varphi_t}\right) - \frac{\psi}{2}\left(\pi_t - 1\right)^2 \qquad (4.6)$$

The policy-maker then solves the dynamic problem of maximising welfare, conditional on being in some given initial state, subject to the private sector model equations outlined in the appendix. This problem takes the form

$$\max \sum_{t=0}^{\infty} \beta^t \left\{W_t - \lambda_t f(x_{1,t-1}, x_{1,t}, x_{2,t}, x_{2,t+1})\right\} \qquad (4.7)$$

where $f(.)$ is a vector of the equations describing the behaviour of the private sector, x_1 is a vector of the natural state variables of the private sector model, $x_{2,t}$ is a vector of non-predetermined private sector variables and λ_t is a vector of Lagrange multipliers. The maximisation is subject to initial conditions $x_{1,-1}$, which are the initial conditions of the natural state variables.

4.4.2 First-order conditions

This Lagrangean formulation leads to the following system of first-order conditions

$$g(x_{1,t-1}, x_{1,t}, x_{2,t}, x_{2,t+1}, \lambda_{t-1}, \lambda_t) \qquad (4.8)$$

which is given in detail in the appendix. The system of first-order conditions is solved by log-linearising it around its steady-state, and then solving the resulting system of linear difference equations using the Schur decomposition as described in Soderlind (1999)[2]. A recent application of this method to an optimal policy problem is Schmitt-Grohé and Uribe (2004a). My model contains large steady-state distortions created by both monopolistic competition and credit frictions which are binding in the steady-state, so I cannot use methods that require an undistorted steady-state. Because I am evaluating only the second moments of model variables, using just a linear approximation to the first-order conditions gives sufficient accuracy.

The natural state variables of the private sector model are the level of borrowing b_{t-1}, the lagged user cost u_{t-1}, and the level of capital held by productive agents k_{t-1}.[3] As discussed in, e.g. Ljungqvist and Sargent (2004), we must be careful how to treat the Lagrange multipliers on the various constraints. The multipliers on equations with a forward-looking element must be treated as additional state variables. This is because these Lagrange multipliers capture the policy-maker's earlier promises upon which private sector expectations were formed. It is this particular treatment of past promises that makes the policy a 'commitment' policy. It is assumed that the policy-maker acts as a Stackelberg leader, and does not re-optimise after the private sector has formed expectations. The remaining Lagrange

[2]Because of the size of my model, I use the Matlab code described in Schmitt-Grohé and Uribe (2004b) to log-linearise the system around its steady-state using analytical derivatives.

[3]There is no unique way to choose state variables. In the previous chapter it was convenient to work with wealth and the share of wealth held by producers as states. In this chapter it is more convenient to write down the equations with fewer substitutions, and use borrowing, user cost and capital as the state variables.

multipliers are treated as non-predetermined, i.e. they can jump freely at period t. Because of the timing convention adopted here for predetermined variables, the Lagrange multipliers that are to be treated as additional state variables show up 'automatically', so to speak, because they appear in the first-order conditions dated $t-1$. The predetermined Lagrange multipliers in this particular system are the multipliers on the borrowing constraint, the Phillips curve, the expected return on investment and the asset-pricing condition for capital, which are the equations of the private sector model that involve expectations of future variables.

4.5 Optimal response to productivity shock

To understand what optimal monetary policy is trying to achieve, it is useful to consider, in addition to the optimal policy solution, two other solutions for the model. First, I consider the solution of the model when there are no credit frictions and prices are fully flexible. This version of the model collapses to a variant of the Brock and Mirman (1972) model, and can be solved analytically, as shown in chapter 3. The equilibrium laws of motion are:

$$Y_{t+1} = \left(\alpha_{t+1}\varepsilon_{P,t+1}\right)^{\frac{\tau+1}{\tau+\eta+\sigma}} \left(Y_t\right)^{\frac{\eta(\tau+1)}{\tau+\eta+\sigma}} c \tag{4.9}$$

where c denotes a constant term. This implies that output dynamics are entirely driven by the exogenous process for aggregate productivity and lagged output. There is no feedback from any net worth or asset price variable in the model. The equations for the asset price and wealth are

$$q_t = \frac{\sigma\beta}{\varphi\overline{K}(1-\beta)} Y_t \tag{4.10}$$

and

$$W_t = \frac{\eta + \sigma - \eta\beta}{\varphi(1-\beta)} Y_t \tag{4.11}$$

So asset prices and entrepreneurial wealth are simply proportional to output.

A second version of the model that is useful for comparison is the model with credit frictions but flexible prices. This can be interpreted either as an economy where there are no impediments or costs to changing prices, or as an economy where the monetary policy maker is concerned only with stabilising inflation, which can be achieved perfectly in this model.

As discussed in the previous chapter, in the credit frictions version of the model, there is substantial feedback from asset prices back to the real economy. When there is an adverse productivity shock, the net worth of highly leveraged producers falls by much more than the net worth of investors, who have no borrowing. The result of this shift of net worth from producers to investors is that producers will hold less capital in the subsequent period. Because producers have higher productivity than investors, a transfer of capital to investors means that capital is less efficiently used in the future. The price of capital, which is the present value of future marginal returns, will fall immediately in anticipation of this future reduction in marginal returns, which exacerbates the initial fall in net worth, and therefore also the shift in net worth from producers to investors. As shown in figure (4.1), this mechanism results in a much larger fall in the price of capital, aggregate wealth, and the share of wealth held by producers. The output response in the initial period is the same as that of the 'no frictions' model, because borrowing and capital holdings are predetermined. But in subsequent periods, the reduction in the efficiency with which capital is used in aggregate results in a further fall in output. It takes time for producers to rebuild their share of the capital stock, and output therefore is away from its steady-state value for around 10 quarters after the shock, a far more persistent response than in the 'no frictions' case. I refer to this version of the model as the flex-price credit model.

Let us now consider the optimal monetary policy or Ramsey solution. This is the model economy with credit frictions and sticky prices, and with a monetary policy maker who maximises the welfare of the private sector agents as outlined in detail in section (4.4.1). As shown in figure (4.1), the initial output fall is smaller than in the frictionless model and the flex-price credit model. That raises two questions: first, why is it efficient to let output fall by less than the frictionless (and therefore fully optimal) model, and second, how is this achieved? The answer to the first question is that it is, in fact, not efficient to dampen the output fall when considering the first period in isolation. Productivity has fallen, and it is therefore efficient for output to fall as much as in the frictionless model. The reason why the policy-maker wants to dampen this initial output fall is because of the consequences it has in future periods. Any reduction in output and asset prices will feed back onto the real economy, and cause an amplified and persistent fall in output in subsequent periods, as illustrated by flexible-

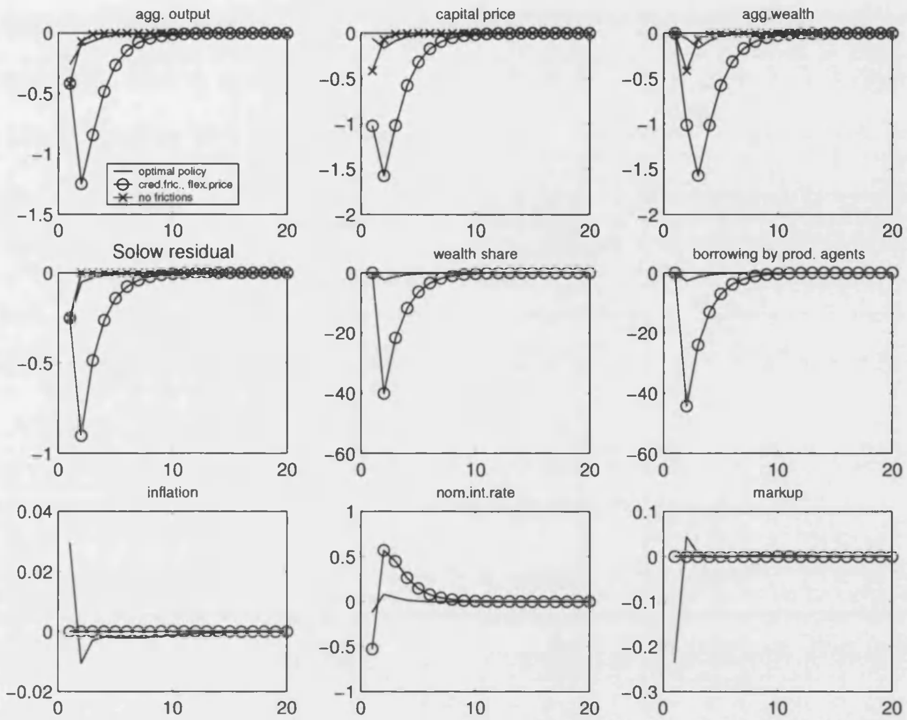

Figure 4.1: Response to adverse productivity shock

price credit model. The only way to dampen the strong effect of this credit mechanism is to dampen the initial fall in output and therefore asset prices, even if, considered in isolation, such initial dampening is inefficient. The policy-maker is therefore trading off the efficiency loss of dampening the initial output and asset price fall against the efficiency gain from limiting the damaging effect of the credit propagation mechanism in subsequent periods.

The second question is how such a dampening of the output response can be achieved. The answer is a short bout of inflation, which results in a fall in the mark-up charged by the retail sector. Because monetary policy is inflationary in the initial period, retailers will want to raise their prices. But price rises are costly, so they do not rise enough to neutralise the monetary policy action. The prices do not rise as much as the marginal costs of retailers, so the retailers' mark-up falls. That effectively transfers resources to the entrepreneurs, because the reduction in the mark-up means the consumption value of entrepreneurs' output is larger, and this will spur them to hire more labour and increase output further. Of course, output still falls, but not by as much as under a policy that instantly and fully stabilises inflation. The story sounds perhaps a little more complicated than it needs to because of the fact that retailers and entrepreneurs have been split into two distinct groups in this model. But essentially, this mechanism just the conventional sticky-price view that if there is an adverse supply shock, and monetary policy does not fully accommodate the output effects of this supply shock, inflation will rise and the output fall will be dampened. The key insight is that, if credit frictions do play an important part in the propagation of shocks, such a dampening of the output fall is efficient. That stands in contrast to a model with only sticky prices, where it would not be efficient to prevent output from falling with a temporary bout of inflation.

In effect, the combination of both credit frictions and sticky prices has resulted in a traditional short-run trade-off between the deviation of output from its efficient level and inflation, albeit with a new twist. The twist lies in the fact that it is not a trade-off between inflation and the output gap in each period, but between inflation now and the output gap later. A trade-off between the output gap and inflation in the short run is largely absent from the New Keynesian literature. This absence of a fundamental trade-off has been dubbed the "divine coincidence" by Blanchard (2005), in reference to the fact that closing the welfare-relevant measure of the output gap coincides with stabilising inflation. Angeletos (2003) also discussed this problem with the New Keynesian models. In my approach, there is no longer any divine coincidence, because stabilising prices does not stabilise output around its efficient level, or even its constrained efficient level. And as shown in figure 4.1, the optimal policy involves allowing inflation to rise briefly following an adverse productivity shock. Woodford (2003) discusses the policy challenges posed by a time-varying gap between the efficient level of output and the level of output under flexible prices. As an illustration, he uses an exogenous, time-varying gap and shows that if such a gap exists, optimal monetary policy will not stabilise prices completely, but tolerate some inflation variability.

The contribution of this chapter is to present an endogenous mechanism in which such a gap arises. Furthermore, the nature of the propagation mechanism due to credit frictions implies that in this model the trade-off is not between inflation and the gap between output and its *current* efficient level, i.e. the level of output without credit frictions or nominal rigidities. Instead, there is a dynamic trade-off between current inflation and the *future* gap between output and its efficient level. This dynamic nature of the trade-off has important consequences for the concept of the output gap. It means that, even if we could measure it accurately, the distance between output and its efficient level is not a useful summary of the objective of monetary policy, in the way that the New Keynesian gap between output and its flexible price level summarises the monetary policy objective. In my model, it is actually feasible to conduct monetary policy in such a way as to close the gap between output and its efficient level completely in each period. But as I argued earlier, this is not generally a good summary of how different variables that enter the welfare function ought to be weighted together. It is fairly intuitive that such a policy would not be optimal in this particular model. It would require output to fall by the full extent of the fall in efficient output in the first period. That would result in a shift in the wealth distribution that is equal to the shift under the flexible price (or full inflation stabilisation) model. This results in a large reduction in the efficiency with which resources are used. After that initial period, monetary policy could bring output back to its steady-state level quickly by running an inflationary policy for a while, so that output exceeds its flexible price level. But that would fail completely to take advantage of the fact that the welfare loss due to the shift in the wealth distribution can actually be reduced quite effectively by having some inflation in the initial period. And indeed, the Ramsey solution shows that it is optimal to exploit this possibility.

Table 4.1 illustrates the trade-off and the desirability of smoothing output and asset price fluctuations. Under optimal or Ramsey policy, inflation variability[4] is non-zero. It is of the same order of magnitude as the variability of actual inflation in low-inflation industrialised countries such as the US[5]. Output variability under optimal policy is much smaller than in the

[4]The theoretical standard deviations and autocorrelations of the model variables were calculated using the method described in Hamilton (1994), p. 265-266.

[5]The standard deviation of US quarterly inflation, on the GDP deflator measure, is 0.25% for the sample period 1983:1-2005:1. (Source: US BEA). The standard deviation of US quarterly output, measured as deviations from a Hodrick-Prescott filtered trend, was 1.11% over the same period.

Table 4.1: Theoretical moments of selected variables

	Ramsey	Frictionless	Flex-price credit
$s.d.(y+y')$	1.157	2.000	6.658
$s.d.(q)$	0.798	2.000	8.974
$s.d.(\pi)$	0.128	0	0
$a.r.(y+y')$	0.431	0.167	0.779
$a.r.(q)$	0.698	0.167	0.806
$a.r.(\pi)$	-0.246	0	0
$s.d.(\varepsilon_{P,t})$	1.000	1.000	1.000

Notes: $s.d.$ denotes standard deviation, expressed in per cent, and $a.r$ denotes first-order autocorrelation.
Moments were calculated for log-linear deviations of aggregate output $(y+y')$, the price of capital (q), and inflation(π).
Each column represents a different version of the model.

flex-price credit model. The reduction in output variability also implies a reduction in asset price variability. Quantitatively, the ability of the monetary policy-maker to affect the real economy in the short run allows most of the adverse effects of credit frictions to be off-set. In the illustrative calibration used here, the standard deviation of output under optimal policy is around one sixth of the standard deviation of output under price stability. In other words, a little inflation variability buys a large reduction in output variability.

Comparing the Ramsey outcome with the frictionless model, we see that aggregate output is more persistent, but less variable, under the Ramsey policy than in the frictionless model. The increased persistence of Ramsey output arises because it is not efficient to off-set the initial output fall entirely, so there is still some persistence from the credit mechanism that prevents output from rising back to its steady-state level as quickly as the frictionless model. This is illustrated in figure (4.1) by the fact that the wealth share of producers still falls under optimal policy.

4.6 Nominal Contracts

I now consider an extension of the model in which debt contracts are set in nominal rather than real terms. The details of the new model equations are given in the appendix. Essentially, the real value of debt that producers have to repay now depends on the level of ex-post inflation, relative to its ex ante expected level. For example, if inflation turns out higher than expected, the real value of debt that needs to be repaid by producers will be lower. That constitutes a reallocation of wealth from investors to producers, which will lead to an output increase as resources are shifted to those who can use them more productively.

What does that imply for the optimal response to productivity shocks? As before, the initial output fall following a productivity shock has adverse consequences on future levels of output via the credit propagation mechanism. So it is optimal for the policy-maker to off-set some of the initial output fall by having an expansionary policy that results in a short burst of inflation immediately following the shock. But in the case of nominal debt contracts, such a burst of otherwise costly inflation has stronger positive side-effects: it helps reallocate wealth back to producers by lowering their real debt burden. So a given burst of inflation has a stronger dampening effect on the credit propagation mechanism. That suggests it would be optimal to allow a slightly stronger burst of inflation under nominal con-

Table 4.2: Theoretical moments of selected variables, baseline vs. nominal contracts

	Ramsey (baseline)	Ramsey (nom.contracts)
$s.d.(y+y')$	1.157	1.135
$s.d.(q)$	0.798	0.711
$s.d.(\pi)$	0.128	0.134
$a.r.(y+y')$	0.431	0.396
$a.r.(q)$	0.698	0.700
$a.r.(\pi)$	-0.246	-0.197
$s.d.(\varepsilon_{P,t})$	1.000	1.000

Notes: *s.d.* denotes standard deviation, expressed in per cent, and *a.r.* denotes first-order autocorrelation.
Moments were calculated for log-linear deviations of aggregate output $(y + y')$, the price of capital (q), and inflation(π).
Each column represents a different version of the model.

tracts, since the benefits are stronger and costs (in terms of the direct cost of changing prices) are unchanged. And such a stronger burst of inflation will achieve a bigger reduction in output volatility, relative to the baseline case with real debt contracts.

Table 4.2 illustrates these effects. It shows the standard deviation of key model variables under optimal monetary policy, both for the baseline case with real debt contracts and for the extended model with nominal debt contracts. Output and asset prices have a lower standard deviation under nominal contracts, while inflation has a higher standard deviation. The standard deviation of asset prices is reduced by 11%, the standard deviation of output is reduced by 2%, and the standard deviation of inflation is increased by 5%. The further stabilisation in output that is achieved under nominal debt contracts is quantitatively small, relative to the stabilisation in output achieved by allowing inflation to deviate from pure price stability.

Table 4.3: Robustness of optimal policy results to parameter changes

	baseline	workers	lab.elast.	nom.rigid.	prod.gap
$s.d.(y + y')$	1.157	1.1337	1.105	1.270	1.427
$s.d.(q)$	0.798	0.597	0.764	1.133	1.139
$s.d.(\pi)$	0.128	0.196	0.160	0.313	0.094
$a.r.(y + y')$	0.431	0.373	0.359	0.558	0.300
$a.r.(q)$	0.698	0.665	0.660	0.676	0.371
$a.r.(\pi)$	-0.246	0.245	-0.347	-0.372	0.220
$s.d.(\varepsilon_{P,t})$	1.000	1.000	1.000	1.000	1.000

Notes: *s.d.* denotes standard deviation, expressed in per cent, and *a.r.* denotes first-order autocorrelation.
Moments were calculated for log-linear deviations of aggregate output $(y + y')$, the price of capital (q), and inflation(π).
Each column represents a different version of the model.

4.7 Robustness Checks

In this section I want to explore the extent to which the quantitative conclusions are sensitive to the particular choice of parameters. I will vary 4 key parameters. I explore the consequences of (a) putting a smaller Pareto weight on workers ($\mu = 0.1$), (b) making labour supply less elastic ($\tau = 1$), (c) making goods prices less sticky ($\psi = 2$), and (d) weakening the credit channel by lowering the productivity gap between producers and investors ($\frac{\alpha}{\gamma} = 1.01$).

The results appear to be robust to even these large parameter changes, with the crucial parameters being the strength of the credit channel and the extent of nominal rigidities, as can be expected, since these are the two frictions the policy-maker is trading off against each other. The changes in the model properties help to firm up the underlying intuition, so I will describe them case by case.

Lowering the welfare weight on workers makes output less variable, but inflation more so. That is because, in order to dampen the effect of the shock on initial output, the expansionary monetary policy dampens output partly by its effect on labour. Since only workers supply labour, if policy is less concerned with worker welfare, it can tolerate greater deviations from the optimal path of labour, meaning it will dampen the output fall more strongly and tolerate higher inflation variability.

Less elastic labour supply implies that output falls by less following an adverse productivity shock, even in the frictionless model. That automatically weakens the credit channel, leading to less output variability. But it also means that monetary policy has to generate more inflation to dampen output by a given amount. In other words, the slope of the short-run Phillips curve has become steeper. So inflation variability is higher.

Lowering price stickiness gives monetary policy less traction, but leaves the strength of the credit channel unchanged. Monetary policy is therefore less able to dampen output responses, and a stronger burst of inflation is needed to dampen output by a given amount. The result is that both output variability and inflation variability under optimal policy are larger.

Finally, weakening the credit channel brings the model closer to the frictionless model. Higher variability of output can be tolerated, because it no longer has strong effects on the efficiency with which capital is used. And inflation variability is smaller, because there is no longer the need to use inflation to dampen the output response to a productivity shock as strongly.

4.8 Conclusion

In this chapter, a quantitative analysis of optimal monetary policy was carried out in a model that features both credit frictions and nominal rigidities. It was shown that, in the case of productivity shocks, the presence of both types of rigidities creates a trade-off between inflation variability and output variability. Stabilising inflation fully and instantaneously results in large deviations of output from its efficient level. Allowing a small temporary rise in inflation following an adverse productivity shock results in output being much closer to its efficient level. A large reduction in output variability can be achieved by allowing only a small amount of inflation variability. Conversely, the cost of stabilising inflation too aggressively can be large.

The intuition for the optimality of some inflation variability is the following. The initial output fall following an adverse productivity shock has adverse welfare effects via the reallocation of capital from high to low productivity entrepreneurs, which will cause further output falls in the future due to the credit channel. To limit this reallocation of capital, it is optimal for monetary policy to dampen the initial output fall. This can be achieved by having a monetary policy that is initially expansionary, which will result in a temporary rise in inflation and a dampening of the output fall. Such a dampening of the output fall is not efficient in isolation, because the optimal level of output has also fallen. But any output fall has a dynamic effect on future output, so there is a trade-off between the rise in inflation immediately following the shock, and the fall in *future* output relative to its efficient level. The dynamic nature of this trade-off implies that neither the gap between output and its flexible-price level, nor the gap between output and its fully efficient level are adequate descriptions of monetary policy objectives.

Of course, real life policy decisions must be made without detailed knowledge of the state of the economy and of the laws of motion of the economy, so even if credit frictions are quantitatively important, the large reduction in output variability may not be achievable. Nevertheless, a realistic policy prescription might be not to stabilise inflation too aggressively following a shock that pushes output and inflation in opposite directions, especially when the financial system is fragile or borrowers' balance sheets are weak.

4.9 Appendices

4.9.1 Model equations of the private sector

The full model is described by the following equations, the derivation of which is discussed in detail in Chapter 3. There are some small changes in notation relative to Chapter 3. The timing convention is that all variables that are decided at date t after the realisation of the period t shock will have the subscript t. Predetermined variables therefore have a subscript $t-1$. The wage of workers is denoted z_t.

Total wealth of entrepreneurs (w_t)

$$w_t = (\eta + \sigma) \frac{y_t + y_t'}{\varphi_t} + q_t \tag{4.12}$$

the share of wealth held by producers (s_t)

$$
\begin{aligned}
s_t w_t = (1-\delta) & \left[(\eta+\sigma) \frac{y_t}{\varphi_t} + q_t k_{t-1} - b_{t-1} \right] \\
& + n\delta \left[(\eta+\sigma) \frac{y_t'}{\varphi_t} + q_t (1-k_{t-1}) + b_{t-1} \right]
\end{aligned} \tag{4.13}
$$

the user cost of capital (u_t)

$$
u_t = \frac{\sigma}{\eta} (\beta w_t - q_t) \tag{4.14}
$$

capital held by producers (k_t)

$$
k_t = \beta \frac{\sigma}{\eta+\sigma} \frac{s_t w_t}{u_t} \tag{4.15}
$$

borrowing constraint (b_t)

$$
b_t = E_t q_{t+1} k_t \tag{4.16}
$$

Phillips curve (π_t)

$$
\begin{aligned}
\pi_t (\pi_t - 1) = \beta E_t & \left[\frac{u_{w,t+1}}{u_{w,t}} \pi_{t+1} (\pi_{t+1} - 1) \right] \\
& + \frac{\theta-1}{\psi} (y_t + y_t') \left(\frac{\theta}{\theta-1} \frac{1}{\varphi_t} - 1 \right)
\end{aligned} \tag{4.17}
$$

return on investment (r_t)

$$
r_t = E_t \left(\varepsilon_{P,t+1}^{\frac{1}{\eta+\sigma}} \gamma^{\frac{1}{\eta+\sigma}} \varphi_{t+1}^{-\frac{1}{\eta+\sigma}} z_{t+1}^{-\frac{1-\eta-\sigma}{\eta+\sigma}} u_t^{-\frac{\sigma}{\eta+\sigma}} \right) \tag{4.18}
$$

pricing equation for capital (q_t)

$$u_t = q_t - E_t \frac{q_{t+1}}{r_t} \tag{4.19}$$

labour market equilibrium in terms of workers' wage (z_t)

$$z_t^{\frac{1+\tau}{\tau}} \left(\frac{1}{\chi}\right)^{1/\tau} = \frac{1-\eta-\sigma}{\eta+\sigma} (w_t - q_t) \tag{4.20}$$

Fisher equation[6] determining the nominal interest rate (R_t)

$$E_t \frac{R_t}{\pi_{t+1}} \frac{1}{(\eta+\sigma) \frac{y'_{t+1}}{\varphi_{t+1}} \varphi + q_{t+1}} = E_t r_t \frac{1}{(\eta+\sigma) \frac{y'_{t+1}}{\varphi_{t+1}} \varphi + q_{t+1}} \tag{4.22}$$

producers' output (y_t)

$$y_t = \alpha^{\frac{1}{\eta+\sigma}} \varepsilon_{P,t}^{\frac{1}{\eta+\sigma}} u_{t-1}^{\frac{\eta}{\eta+\sigma}} (z_t \varphi_t)^{-\frac{1-\eta-\sigma}{\eta+\sigma}} \frac{k_{t-1}}{\sigma} \tag{4.23}$$

investors' output (y'_t)

$$y'_t = \gamma^{\frac{1}{\eta+\sigma}} \varepsilon_{P,t}^{\frac{1}{\eta+\sigma}} u_{t-1}^{\frac{\eta}{\eta+\sigma}} (z_t \varphi_t)^{-\frac{1-\eta-\sigma}{\eta+\sigma}} \frac{(1-k_{t-1})}{\sigma} \tag{4.24}$$

definition of u_w, which is the marginal utility of consumption of workers

[6]This is the standard asset pricing arbitrage condition, based on the marginal utility of consumption of the investors. I have made the following substitution:

$$E_t c'_{t+1} = (1-\beta) \left[(\eta+\sigma) \frac{y'_{t+1}}{\varphi_{t+1}} + q_{t+1} \right] \tag{4.21}$$

$$\frac{1}{u_{w,t}} = \frac{\tau}{\tau+1}\left(\frac{1}{\chi}\right)^{\frac{1}{\tau}} z_t^{\frac{1+\tau}{\tau}} + \left(y_t + y_t'\right)\left(1 - \frac{1}{\varphi_t}\right) - \frac{\psi}{2}\left(\pi_t - 1\right)^2 \qquad (4.25)$$

the aggregate productivity process $(\varepsilon_{P,t})$

$$\log \varepsilon_{P,t} = \rho \log \varepsilon_{P,t-1} + \upsilon_t \qquad (4.26)$$

4.9.2 Lagrangean formulation of the policy-maker's objective

The Lagrangean formulation of the policy-maker's problem is to maximise welfare subject to the equations governing the behaviour of the private sector.

$$\max \sum_{t=0}^{\infty} \beta^t \{W_t \qquad (4.27)$$

$$-\lambda_{1,t}\left\{w_t - (\eta+\sigma)\frac{y_t + y_t'}{\varphi_t} - q_t\right\} \qquad (4.28)$$

$$-\lambda_{2,t}\left\{\begin{array}{c} s_t w_t - (1-\delta)\left[(\eta+\sigma)\frac{y_t}{\varphi_t} + q_t k_{t-1} - b_{t-1}\right] \\ -n\delta\left[(\eta+\sigma)\frac{y_t'}{\varphi_t} + q_t(1-k_{t-1}) + b_{t-1}\right] \end{array}\right\} \qquad (4.29)$$

$$-\lambda_{3,t}\left\{u_t - \frac{\sigma}{\eta}\left(\beta w_t - q_t\right)\right\} \qquad (4.30)$$

$$-\lambda_{4,t}\left\{k_t-\beta\frac{\sigma}{\eta+\sigma}\frac{s_t w_t}{u_t}\right\} \tag{4.31}$$

$$-\lambda_{5,t}\left\{b_t-E_t q_{t+1}k_t\right\} \tag{4.32}$$

$$-\lambda_{6,t}\left\{\begin{array}{c}\pi_t\left(\pi_t-1\right)-\beta E_t\left[\frac{u_{w,t+1}}{u_{w,t}}\pi_{t+1}\left(\pi_{t+1}-1\right)\right]\\ -\frac{\theta-1}{\psi}\left(y_t+y_t'\right)\left(\frac{\theta}{\theta-1}\frac{1}{\varphi_t}-1\right)\end{array}\right\} \tag{4.33}$$

$$-\lambda_{7,t}\left\{r_t-E_t\left(\varepsilon_{P,t+1}^{\frac{1}{\eta+\sigma}}\gamma^{\frac{1}{\eta+\sigma}}\varphi_{t+1}^{-\frac{1}{\eta+\sigma}}z_{t+1}^{-\frac{1-\eta-\sigma}{\eta+\sigma}}u_t^{-\frac{\sigma}{\eta+\sigma}}\right)\right\} \tag{4.34}$$

$$-\lambda_{8,t}\left\{u_t-q_t+E_t\frac{q_{t+1}}{r_t}\right\} \tag{4.35}$$

$$-\lambda_{9,t}\left\{z_t^{\frac{1+\tau}{\tau}}\left(\frac{1}{\chi}\right)^{1/\tau}-\frac{1-\eta-\sigma}{\eta+\sigma}\left(w_t-q_t\right)\right\} \tag{4.36}$$

$$-\lambda_{10,t}\left\{E_t\frac{R_t}{\pi_{t+1}}\frac{1}{\left(\eta+\sigma\right)\frac{y_{t+1}'}{\varphi_{t+1}}+q_{t+1}}-E_t r_t\frac{1}{\left(\eta+\sigma\right)\frac{y_{t+1}'}{\varphi_{t+1}}+q_{t+1}}\right\} \tag{4.37}$$

$$-\lambda_{11,t}\left\{y_t-\alpha^{\frac{1}{\eta+\sigma}}\varepsilon_{P,t}^{\frac{1}{\eta+\sigma}}u_{t-1}^{\frac{\eta}{\eta+\sigma}}\left(z_t\varphi_t\right)^{-\frac{1-\eta-\sigma}{\eta+\sigma}}\frac{k_{t-1}}{\sigma}\right\} \tag{4.38}$$

$$-\lambda_{12,t}\left\{y_t'-\gamma^{\frac{1}{\eta+\sigma}}\varepsilon_{P,t}^{\frac{1}{\eta+\sigma}}u_{t-1}^{\frac{\eta}{\eta+\sigma}}\left(z_t\varphi_t\right)^{-\frac{1-\eta-\sigma}{\eta+\sigma}}\frac{1-k_{t-1}}{\sigma}\right\} \tag{4.39}$$

$$-\lambda_{13,t}\left\{\begin{array}{c}\frac{1}{u_{w,t}}-\frac{\tau}{\tau+1}\left(\frac{1}{\chi}\right)^{\frac{1}{\tau}}z_t^{\frac{1+\tau}{\tau}}-(y_t+y_t')\left(1-\frac{1}{\varphi_t}\right)\\+\frac{\psi}{2}\left(\pi_t-1\right)^2\end{array}\right\}\tag{4.40}$$

given the exogenous process

$$\log\varepsilon_{P,t}=\rho\log\varepsilon_{P,t-1}+v_t\tag{4.41}$$

and the initial conditions k_{-1},b_{-1},u_{-1}.

In the case of nominal debt contracts, inflation expectations are added to the model as a new state variable. It is convenient to create a new variable for this purpose, $\pi_{e,t-1}$, which will be added as a new constraint:

$$-\lambda_{14,t}\left\{\pi_{e,t}-E_t\pi_{t+1}\right\}\tag{4.42}$$

The second constraint changes to reflect that, under nominal debt contracts, ex post inflation redistributes wealth between borrowers and lenders:

$$-\lambda_{2,t}\left\{\begin{array}{c}s_tw_t-(1-\delta)\left[(\eta+\sigma)\frac{y_t}{\varphi_t}+q_tk_{t-1}-b_{t-1}\frac{\pi_{e,t-1}}{\pi_t}\right]\\-n\delta\left[(\eta+\sigma)\frac{y_t'}{\varphi_t}+q_t(1-k_{t-1})+b_{t-1}\frac{\pi_{e,t-1}}{\pi_t}\right]\end{array}\right\}\tag{4.43}$$

4.9.3 First-order conditions for the optimal plan

$$\begin{aligned}w_t:&\frac{1}{w_t}-\lambda_{1,t}-\lambda_{2,t}s_t+\lambda_{3,t}\frac{\beta\sigma}{\eta}+\lambda_{4,t}\frac{\sigma}{\eta+\sigma}\beta\frac{s_t}{u_t}\\&+\lambda_{9,t}\frac{1-\eta-\sigma}{\eta+\sigma}=0\end{aligned}\tag{4.44}$$

$$s_t : -\lambda_{2,t} w_t + \lambda_{4,t} \frac{\sigma}{\eta+\sigma} \beta \frac{w_t}{u_t} = 0 \tag{4.45}$$

$$\begin{aligned}
u_t : & -\lambda_{3,t} - \lambda_{4,t} \frac{\sigma}{\eta+\sigma} \beta \frac{s_t w_t}{u_t^2} \\
& - \lambda_{7,t} \frac{\sigma}{\eta+\sigma} u_t^{\frac{-\eta-2\sigma}{\eta+\sigma}} \varepsilon_{P,t+1}^{\frac{1}{\eta+\sigma}} \gamma^{\frac{1}{\eta+\sigma}} \varphi_{t+1}^{-\frac{1}{\eta+\sigma}} z_{t+1}^{-\frac{1-\eta-\sigma}{\eta+\sigma}} \\
& - \lambda_{8,t} + \beta \lambda_{11,t+1} \frac{\eta}{\eta+\sigma} \alpha^{\frac{1}{\eta+\sigma}} \varepsilon_{P,t+1}^{\frac{1}{\eta+\sigma}} u_t^{\frac{-\sigma}{\eta+\sigma}} (z_{t+1}\varphi_{t+1})^{-\frac{1-\eta-\sigma}{\eta+\sigma}} \frac{k_t}{\sigma} \\
& + \beta \lambda_{12,t+1} \frac{\eta}{\eta+\sigma} \gamma^{\frac{1}{\eta+\sigma}} \varepsilon_{P,t+1}^{\frac{1}{\eta+\sigma}} u_t^{\frac{-\sigma}{\eta+\sigma}} (z_{t+1}\varphi_{t+1})^{-\frac{1-\eta-\sigma}{\eta+\sigma}} \frac{1-k_t}{\sigma} \\
& = 0
\end{aligned} \tag{4.46}$$

$$\begin{aligned}
k_t : & \beta \lambda_{2,t+1} (1-\delta-n\delta) q_{t+1} - \lambda_{4,t} + \lambda_{5,t} q_{t+1} \\
& + \beta \lambda_{11,t+1} \alpha^{\frac{1}{\eta+\sigma}} \varepsilon_{P,t+1}^{\frac{1}{\eta+\sigma}} u_t^{\frac{\eta}{\eta+\sigma}} (z_{t+1}\varphi_{t+1})^{-\frac{1-\eta-\sigma}{\eta+\sigma}} \frac{1}{\sigma} \\
& - \beta \lambda_{12,t+1} \gamma^{\frac{1}{\eta+\sigma}} \varepsilon_{P,t+1}^{\frac{1}{\eta+\sigma}} u_t^{\frac{\eta}{\eta+\sigma}} (z_{t+1}\varphi_{t+1})^{-\frac{1-\eta-\sigma}{\eta+\sigma}} \frac{1}{\sigma} \\
& = 0
\end{aligned} \tag{4.47}$$

$$b_t : -\beta \lambda_{2,t+1} (1-\delta-n\delta) - \lambda_{5,t} = 0 \tag{4.48}$$

$$\begin{aligned}\pi_t :& -\lambda_{6,t}(2\pi_t - 1) + \lambda_{6,t-1}\frac{u_{w,t}}{u_{w,t-1}}(2\pi_t - 1)\\ &+ \frac{1}{\beta}\lambda_{10,t-1}\frac{R_{t-1}}{\pi_t^2}\frac{1}{(\eta+\sigma)\frac{y'_t}{\varphi_t} + q_t} - \lambda_{13,t}\psi(\pi_t - 1)\\ &= 0\end{aligned} \tag{4.49}$$

$$r_t : -\lambda_{7,t} + \lambda_{8,t}\frac{q_{t+1}}{r_t^2} + \lambda_{10,t}\frac{1}{(\eta+\sigma)\frac{y'_{t+1}}{\varphi_{t+1}} + q_{t+1}} = 0 \tag{4.50}$$

$$\begin{aligned}\varphi_t :& -\lambda_{1,t}(\eta+\sigma)\frac{(y_t + y'_t)}{\varphi_t^2} - \lambda_{2,t}\left[(1-\delta)y_t + n\delta y'_t\right]\frac{(\eta+\sigma)}{\varphi_t^2}\\ &- \lambda_{6,t}\frac{\theta}{\psi}\frac{(y_t+y'_t)}{\varphi_t^2} - \frac{1}{\beta}\lambda_{7,t-1}\frac{1}{\eta+\sigma}\varepsilon_{P,t}^{\frac{1}{\eta+\sigma}}\gamma^{\frac{1}{\eta+\sigma}}\varphi_t^{-\frac{1+\eta+\sigma}{\eta+\sigma}}z_t^{-\frac{1-\eta-\sigma}{\eta+\sigma}}u_{t-1}^{-\frac{\sigma}{\eta+\sigma}}\\ &- \frac{1}{\beta}\lambda_{10,t-1}\frac{(\eta+\sigma)\frac{y'_t}{\varphi_t^2}}{\left[(\eta+\sigma)\frac{y'_t}{\varphi_t} + q_t\right]^2}\left(\frac{R_{t-1}}{\pi_t} - r_{t-1}\right)\\ &- \lambda_{11,t}\frac{1-\eta-\sigma}{\eta+\sigma}\alpha^{\frac{1}{\eta+\sigma}}\varepsilon_{P,t}^{\frac{1}{\eta+\sigma}}u_{t-1}^{\frac{\eta}{\eta+\sigma}}z_t^{-\frac{1-\eta-\sigma}{\eta+\sigma}}\varphi_t^{-\frac{1}{\eta+\sigma}}\frac{k_{t-1}}{\sigma}\\ &- \lambda_{12,t}\frac{1-\eta-\sigma}{\eta+\sigma}\gamma^{\frac{1}{\eta+\sigma}}\varepsilon_{P,t}^{\frac{1}{\eta+\sigma}}u_{t-1}^{\frac{\eta}{\eta+\sigma}}z_t^{-\frac{1-\eta-\sigma}{\eta+\sigma}}\varphi_t^{-\frac{1}{\eta+\sigma}}\frac{1-k_{t-1}}{\sigma} + \lambda_{13,t}\frac{(y_t+y'_t)}{\varphi_t^2}\\ &= 0\end{aligned} \tag{4.51}$$

$$
\begin{aligned}
z_t :& -\frac{1}{\beta}\lambda_{7,t-1}\frac{1-\eta-\sigma}{\eta+\sigma}\varepsilon_{P,t}^{\frac{1}{\eta+\sigma}}\gamma^{\frac{1}{\eta+\sigma}}\varphi_t^{-\frac{1}{\eta+\sigma}}z_{t+1}^{-\frac{1}{\eta+\sigma}}u_t^{-\frac{\sigma}{\eta+\sigma}} \\
&-\lambda_{9,t}\left(\frac{1}{\chi}\right)^{\frac{1}{\tau}}\frac{1+\tau}{\tau}z_t^{\frac{1}{\tau}} \\
&-\lambda_{11,t}\frac{1-\eta-\sigma}{\eta+\sigma}\alpha^{\frac{1}{\eta+\sigma}}\varepsilon_{P,t}^{\frac{1}{\eta+\sigma}}u_{t-1}^{\frac{\eta}{\eta+\sigma}}z_t^{-\frac{1}{\eta+\sigma}}\varphi_t^{-\frac{1-\eta-\sigma}{\eta+\sigma}}\frac{k_{t-1}}{\sigma} \\
&-\lambda_{12,t}\frac{1-\eta-\sigma}{\eta+\sigma}\gamma^{\frac{1}{\eta+\sigma}}\varepsilon_{P,t}^{\frac{1}{\eta+\sigma}}u_{t-1}^{\frac{\eta}{\eta+\sigma}}z_t^{-\frac{1}{\eta+\sigma}}\varphi_t^{-\frac{1-\eta-\sigma}{\eta+\sigma}}\frac{1-k_{t-1}}{\sigma} \\
&+\lambda_{13,t}\left(\frac{1}{\chi}\right)^{\frac{1}{\tau}}z_t^{\frac{1}{\tau}} \\
&=0
\end{aligned}
\tag{4.52}
$$

$$
\begin{aligned}
q_t :& \;\lambda_{1,t}+\lambda_{2,t}\left[\left(1-\delta\right)k_{t-1}+n\delta\left(1-k_{t-1}\right)\right]-\lambda_{3,t}\frac{\sigma}{\eta} \\
&+\frac{1}{\beta}\lambda_{5,t-1}k_{t-1}+\lambda_{8,t}-\frac{1}{\beta}\lambda_{8,t-1}\frac{1}{r_{t-1}}-\lambda_{9,t}\frac{(1-\eta-\sigma)}{\eta+\sigma} \\
&+\frac{1}{\beta}\lambda_{10,t-1}\frac{1}{\left[\left(\eta+\sigma\right)\frac{y'_t}{\varphi_t}+q_t\right]^2}\left(\frac{R_{t-1}}{\pi_t}-r_{t-1}\right) \\
&=0
\end{aligned}
\tag{4.53}
$$

$$
R_t : \lambda_{10,t}\frac{1}{\pi_{t+1}\left[\left(\eta+\sigma\right)\frac{y'_{t+1}}{\varphi_{t+1}}+q_{t+1}\right]}=0 \tag{4.54}
$$

$$
\begin{aligned}
y_t :\ & \lambda_{1,t}\frac{(\eta+\sigma)}{\varphi_t} + \lambda_{2,t}\frac{(1-\delta)(\eta+\sigma)}{\varphi_t} \\
& + \lambda_{6,t}\frac{\theta-1}{\psi}\left(\frac{\theta}{\theta-1}\frac{1}{\varphi_t}-1\right) - \lambda_{11,t} + \lambda_{13,t}\left(1-\frac{1}{\varphi_t}\right) \\
& = 0
\end{aligned} \tag{4.55}
$$

$$
\begin{aligned}
y_t' :\ & \lambda_{1,t}\frac{(\eta+\sigma)}{\varphi_t} + \lambda_{2,t}\frac{n\delta(\eta+\sigma)}{\varphi_t} + \lambda_{6,t}\frac{\theta-1}{\psi}\left(\frac{\theta}{\theta-1}\frac{1}{\varphi_t}-1\right) \\
& + \frac{1}{\beta}\lambda_{10,t-1}\frac{\frac{(\eta+\sigma)}{\varphi_t}}{\left[(\eta+\sigma)\frac{y_t'}{\varphi_t}+q_t\right]^2}\left(\frac{R_{t-1}}{\pi_t}-r_{t-1}\right) \\
& - \lambda_{12,t} + \lambda_{13,t}\left(1-\frac{1}{\varphi_t}\right) \\
& = 0
\end{aligned} \tag{4.56}
$$

$$
\begin{aligned}
u_{w,t} :\ & -\frac{\mu}{u_{w,t}} - \lambda_{6,t}\beta\frac{u_{w,t+1}}{u_{w,t}^2}(\pi_{t+1}-1)\pi_{t+1} \\
& + \lambda_{6,t-1}\frac{1}{u_{w,t-1}}(\pi_t-1)\pi_t + \lambda_{13,t}\frac{1}{u_{w,t}^2} \\
& = 0
\end{aligned} \tag{4.57}
$$

The partial derivatives of the Lagrangean with respect to the Lagrange multipliers simply yield the model equations for the private sector (4.12) to (4.25). These also form part of the full system of first-order conditions that the model variables under optimal policy must satisfy.

To implement nominal debt contracts, the first-order condition with respect to inflation is changed to:

$$\begin{aligned}
\pi_t : \; & \lambda_{2,t}(1-\delta-n\delta)\frac{b_{t-1}\pi_{e,t-1}}{\pi_t^2} + \frac{1}{\beta}\lambda_{14,t-1} \\
& - \lambda_{6,t}(2\pi_t - 1) + \lambda_{6,t-1}\frac{u_{w,t}}{u_{w,t-1}}(2\pi_t - 1) \\
& + \frac{1}{\beta}\lambda_{10,t-1}\frac{R_{t-1}}{\pi_t^2}\frac{1}{(\eta+\sigma)\frac{y_t'}{\varphi_t} + q_t} - \lambda_{13,t}\psi(\pi_t - 1) \\
& = 0
\end{aligned} \tag{4.58}$$

And a new first-order condition is added, with respect to the new inflation expectations variable

$$\pi_{e,t} : -\beta\lambda_{2,t+1}(1-\delta-n\delta)\frac{b_t}{\pi_{t+1}} - \lambda_{14,t} = 0 \tag{4.59}$$

4.9.4 Steady state

To solve for the steady state, set $x_t = x_{t-1} = x_{t+1} = \overline{x}$ for all variables. This is a large non-linear system, but some intuitive reasoning can simplify its solution substantially. I explain the approach in some detail because it can be applied to a potentially large set of optimal monetary policy problems, which all have the same basic structure.

It helps to write down a simple but general example. The objective is to maximise a function

$$\max U(x) \tag{4.60}$$

subject to some constraints (the private sector model)

$$f(x) = 0 \tag{4.61}$$

where f represents $n-1$ equations and x has n elements, one of which is the instrument, i.e. is set by the policy-maker.

The first-order conditions of this problem are

$$U_x(x) - \lambda f_x(x) = 0 \tag{4.62}$$

$$f(x) = 0 \tag{4.63}$$

The first block is a system of n equations in $2n-1$ variables (n elements of x and $n-1$ elements of λ). The second block is a system of $n-1$ equations in the n elements of x. Suppose that we have a guess for one of the elements of x. And suppose that, conditional on this guess, we have available an analytical solution for the system of $n-1$ non-linear equations from the private sector model in the remaining $n-1$ variables. Note that there is no deep general reason for such an analytical solution to exist. It is simply the case that in chapter 3, I found such an analytical solution to the steady state of the model for a given level of the policy instrument. We now have a value for all the elements of x. There remains a linear system (4.62) of n equations in the $n-1$ elements of λ, the Lagrange multipliers. To find the solution, we leave out one of the linear equations, and solve the remaining system of $n-1$ linear equations in $n-1$ Lagrange multipliers by

Gaussian elimination. The final step is then to verify the equation that was so far unused: it must hold for the assumed and calculated values of λ and x. The advantage of this approach is that there is no need for a numerical solution, which may be difficult to obtain without a good initial guess for all variables.

Where does the guess for one of the elements of x come from in my particular case? Because changing prices is costly by assumption, it seems reasonable to conjecture that the steady-state solution will involve $\pi_t = \overline{\pi} = 0$. Because I consider only symmetric equilibria where all retailers set the same price, for any inflation rate retailers will be keeping their mark-up at the profit-maximising level, but incurring the cost of price changes for inflation rates other than zero. It could be beneficial from a welfare perspective if some steady-state non-zero inflationary policy could achieve a lower mark-up of prices over costs, which might compensate for the loss of changing prices, but if the mark-up is not affected, inflation (or deflation) can only be costly. If we conjecture then that the solution involves zero inflation and the mark-up equal to its profit-maximising level, the solution of the system as a whole becomes much simpler. In Chapter 3 it was shown that, for any ad-hoc monetary policy with zero steady-state inflation, the steady-state could be found analytically. The first-order conditions involving multipliers represent a system of 14 equations in 13 variables. By leaving out one of these equations, we now have a linear system (conditional on the solution of the non-linear system of model equations) of 13 equations in 13 unknowns, which can trivially be solved using Gaussian elimination. This leaves one final 'free' equation, which must hold if the initial conjecture of zero inflation was correct.

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