"PUBLIC SECTOR PRICING" : THEORETICAL ANALYSIS IN A DYNAMIC MACROECONOMIC MODEL

by

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ABSTRACT

This thesis examines the role of public policy, with specific reference to public sector pricing, in an economy where all markets do not necessarily clear. The discussion focuses on three non-Walrasian market situations termed, in the nomenclature of Malinvaud and others, as: Keynesian Unemployment, Classical Unemployment and Repressed Inflation. In chapter 1, there is a brief summary of the literature in this area. The framework of the model used, is described in some detail in chapter 2.

In chapter 3, the role of the traditional instruments of taxation and public expenditures is analysed in the framework of the present model. The "non-traditional" instrument of public sector pricing throws up some interesting results. Among other things, my results indicate that public sector pricing can be designed to effectively influence the level of aggregate income and employment.

The method of financing the government deficit is the subject of chapter 4. My results indicate that the bond-financed multiplier of Blinder and Solow or Tobin and Buiter is simply a special case of the multiplier in my model, when the public sector enterprise prices its output at marginal cost. Equally important, I establish that the Blinder-Solow result "the long-run multiplier for bond-financed deficit spending exceeds that for money-financed deficit spending" is not necessarily true. Furthermore, the stability and convergence properties of the system are shown to rest on the choice of public sector prices.

The characterisation of optimal public sector prices is dealt with in chapters 5 and 6, viewed from a different perspective in each of the chapters. In chapter 5, optimal pricing rules are derived which are explicit and readily operational. Finally, in chapter 6, we characterise the dynamic time-path of optimal public sector prices.

To my parents and Sai Nath

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CONTENTS

Chapter 1

INTRODUCTION

This thesis aims to analyse the effectiveness of public sector production as an instrument of fiscal policy in a dynamic quantity constrained macroeconomic model.

In the basic Walrasian models, market clearing is taken as an axiom through the actions of a ghost "auctioneer", and is not based on any microeconomic analysis of price making behaviour. However, real world markets with the exception, perhaps, of the stock market do not conform to these characteristics. In most cases, the assumption of competitive market clearing does not hold even as an approximation, there being several factors explaining this. First, some prices are subject to institutional constraints where prices are fixed by legislation, for example, minimum wages. Secondly, the prices of many products are currently set in the framework of imperfect competition. Product differentiation and advertising have partially replaced price competition. In such a context, prices are no longer determined by the equality of demand and supply, and price response to changes in conditions of supply and demand may be substantially modified. Lastly, exchange relations in labour markets are contractual in nature, and the "spot-market paradigm" does not apply since most contracts are of longer term.

Having recognised the weakness of the above approach, one of the most basic insights of the non-Walrasian school is that, in the short-run, adjustments will be made through quantities as well as prices. This is the central idea of Keynes (1936) where the level of activity takes on the role of an adjustment variable in much the same way as the interest rate in the money market or other price variable. While Walrasian models, by definition, cover only the case where all markets clear, non-Walrasian models enable a study, both at the microeconomic and macroeconomic levels, the consequences of numerous schemes of price and expectations formation. Agents here form expectations not only on price signals but on quantity signals as well. Each agent may have his own scheme of expectations, which may be be "rational" or otherwise, so that this cover a large number of specifications.

As an immediate corollary, an important characteristic of non-Walrasian models is that they yield a large array of equilibrium concepts by considering more general price mechanisms that allow for quantity and mixed price-quantity adjustments in the short-run. In this sense, they generalise traditional microeconomic concepts and this, in turn, results in a similar generality at the macroeconomic level. This approach seems more general than the new classical method which assumes market clearing or the traditional Keynesian approach which considers states of excess supply only. Non-Walrasian models, by their very nature, endogenously generate a multiplicity of subregimes, which renders them an excellent tool to synthesize apparently conflicting theories, to show the limits of each, and to actually generalize by introducing new possibilities.

This class of models seems to best explain the wide-spread occurance of short and medium term unemployment of labour. The involuntary nature of the unemployment points to a rationing of the available labour supply in the sense that a large proportion of households are unsuccessful in finding the jobs that they seek at the prevailing market wages and prices. The consequences as well as causes of this type of market failure are not confined to the particular market in which it is observed to occur. The writings of Clower (1985) and Leijonhufvud (1986, 1973) recognise the "general equilibrium" aspects of the rationing. Malinvaud (1977) succinctly states "rationing on the labour market is related to, and dependent on, rationing in the goods market."

The schematic occurance of events (in a stylised form, for purposes of analytical simplicity) is as follows: an insufficient demand in the goods market leads to a correspondingly small level of demand in the labour market from profit maximising private entrepreneurs who would not produce more than is demanded, i.e. a rationing of the labour supply, and the resulting low incomes lead to a low aggregate demand and hence, rationing of suppliers in the product market. In the wake of the articles by Leijonhufvud and Clower, the 1970s saw the significant contributions of Barro and Grossman (1971), Drèze (1975), Benassy (1975) and Malinvaud (1977) to analyse a wide range of policy issues.

The quantity constrained equilibrium concept was first used in a simple general equilibrium model by Barro and Grossman (1971). Two rigorous formalisations of this are by Drèze (1975) and Benassy (1975). In both, prices ar taken as fixed, and agents maximise utility or profits taking as given not only prices but also the rationing of the quantities which they may buy or sell. The two formulations are equivalent with regard to the levels of trade. The essential difference arises from the fact there is no excess supply or demand in the Drèze framework. This is a draw-back of the Drèze analysis since the original motivation was to model trade under conditions in which demand did not match supply. A thorough investigation of these models is provided in Drazen (1980).

Plan of thesis

The innovative feature in this thesis is the extension of government activity to include production of goods for private consumption. Within this, there are two objectives. First is to examine how the standard resuts concerning fiscal policy in conventional macroeconomic theory are extended, and perhaps modified by the new policy instrument of public enterprise production. This is the subject of chapter 3, where three non-Walrasian market situations are studied in some depth. Furthermore, the traditional approach to macroeconomic theory postulates ad hoc behavioural relationships while the present work is based on an overlapping generations model with well specified microeconomic foundations, some important examples of the former being Blinder and Solow (1973), Christ (1967, 1968, 1979), Tobin and Buiter (1976) and a host of others.

The present model, discussed in detail in chapter 2, has two assets, money and bonds. Though money earns no interest, it is assumed to provide liquidity services which is reflected in real balances being an argument of the individual utility function. The introduction of bonds as an additional financial asset breaks the rigid "proportionality" relationship between money and real output that would occur in the absence of the second asset, bonds. To model bonds, in any serious way, requires taking account of the inter-temporal consumption and saving behaviour of individuals. For this, an overlapping generations framework seemed the most natural choice, which also facilitates an investigation into the dynamics of the government budget constraint.

The two most significant contributions on the financing of the government deficit are by Blinder and Solow (1973) and Tobin and Buiter (1976), written as a counter to monetarists claiming the inefficacy of fiscal policy. It seemed in place to review the results of the previous authors, in the extended context of the current model, and to examine the role of public sector pricing, in particular. This is studied in chapter 4, and it is established that public secor pricing is a key element both in the efficacy of policy as well as in the stability of the system.

The second objective seen as a further development of quantity constrained models is the focus on "optimal" public sector pricing, optimal in the sense of maximising social welfare. There are two levels at which this problem is addressed. In an economic environment where policy decisions are centralised, the appropriate pricing rules would take into consideration all macroeconomic multiplier effects. This appoach would be in line with the work of Drèze (1984), where he derives optimal pricing rules, as a second-best solution, for the public sector firm operating under a budget constraint in an economy facing Keynesian unemployment. The background to the Drèze results is the classic work of Boiteux (1956): though no rationing situation is assumed, the presence of a budget constraint in itself points to a departure from Pareto efficiency. In chapter 5, we derive optimal pricing rules for the public sector enterprise in an economic setting similar to that of the previous authors but in the framework of an overlapping generations model, and which takes account of the financing mechanism of the government budget.

At a second level of analysis of optimal pricing rules, the use of an overlapping generations model permits us to characterise the time-path of optimal public sector prices. This is the subject of chapter 6. The pricing rules indicate that the method of financing the budget deficit needs to be taken into account explicitly since the associated costs would actually differ. As a topic for future research, it is intended to investigate the role of public sector prices, which may be viewed as indirect taxes, in comparison with lump-sum taxes.

Chapter 2

A Model with Supply and Demand Constraints

(1) The Frame-work of Analysis

In this chapter, we introduce and build the frame-work of the model used in our analysis.

We adopt the Benassy formulation (1975) where the actual or transacted volume of output and employment is governed by the short side of the market. This is a choice guided by two considerations. First, the Keynesian view of the economy as capable of experiencing significant and sustained unemployment appears to be best explained in terms of models of quantity constrained equilibrium in a way in which other models cannot. Second, by its simplicity, Benassy's concept allows models to be tractable at the macro-economic level.

Three different non-Walrasian situations are analysed, each characterised by identifying the set of markets in disequilibrium. The agents in the economy are: the public sector, private firm and the household-consumer. The public and private firms supply two disjoint sets of consumption goods, G and Y, respectively using labour as the sole variable input, supplied by households. Both Y and G are perishable, and cannot be carried over from one period to another. The total number of commodities is 5: labour, L, money, M, bonds, B, and the goods G and Y listed above, resulting in 4 relative prices. By Walras Law, we can suppress the market clearing condition for any one of the 5 commodities, which in our model is bonds.

Throughout the analysis, we maintain that the markets for the government produced good, G, and money are always cleared so that disequilibrium arises because either individually the market for labour and for the output of the private sector fails to clear, or both fail to clear.

The types of disequilibria discussed are: a) both markets exhibit excess supply, a situation termed, in the literature, as "Keynesian" unemployment, b) alternatively, both the product and labour markets experience excess demand, in which event, the economy is said to have "Repressed" inflation, c) the commodity market (Y) is in excess demand while the labour market has excess supply in a state of "Classical" unemployment, and finally, the degenerate case of d) with excess demand in the labour market combined with excess supply in the product market. In the absence of inventories, the last case violates profit maximising behaviour of private firms so that we need consider only the first three listed above. A formal statement of the market clearing conditions for the non-Walrasian cases considered here is postponed to the next section, after a fuller description of individual behaviour.

The problem is posed in the intertemporal framework of an overlapping generations (OLG) model. An innovative feature of the present analysis is the introduction of financial assets, money and bonds, whereby a generation saves for its future consumption. Money plays the role of both a medium of exchange as well as a store of value, a role similar to that in the work of Barro and Grossman (1971,1976) and Malinvaud (1977). A major drawback of the original versions, from a fiscal policy viewpoint, is that only one asset is considered - money. This implies that a comparison of its results with the standard ones based on an IS-LM model is not possible since a distinction cannot be made between monetary and fiscal policy changes. All changes in the government deficit have to be money financed, and all changes in the money supply have to be accompanied by a dificit of corresponding size. In order to break this rigid relationship, I introduce the possibility of bond financing which in turn means introducing in the model an additional good, bonds. More about this point at a later stage.

A factor in favour of using an overlapping generations model is that it provides an excellent alternative to assuming an ad hoc demand for assets. The demand for bonds/money is the result of inter-temporal optimisation by households, rather than by putting them in a single-period utility function as in Hool's (1980) extension of the model of Barro and Grossman or the Malinvaud model to include bonds. Gale (1983) uses an OLG framework but suffers the limitations mentioned above of having a single asset. Rankin (1984), excellent in its underlying microeconomic foundations, works with a 2-asset overlapping generations model but the focus of analysis is different.

Another major difference arises in that, in the present approach, government spending is confined not merely to what is commonly termed in the literature as "wasteful" expenditure but also extends to the public production of a good for private consumption. Cuddington, et. al. (1980) make a similar attempt in that government engages in both productive and "unproductive" ventures but a comparison of the results cannot be made since the private and publicly produced goods are no different for consumption purposes, pre-empting the need for rules of optimal public sector pricing.

The Model

We now discuss in some detail the behaviour of each of the agents in the economy.

(2.1) Private Firm

The production function for the private sector firm is:

$$Y = f(l_y),\ f'>0,\ f'' \leq 0 \qquad (1)$$

where l_y = labour input of Y.

We assume that firms do not hold inventories or invest, so that net sales are equal to output, and hence firms are not required to form expectations. Also, all profits are distributed to workers, a scheme which we can think of as a form of profit-sharing agreement. Ofcourse, there are other possible schemes of income distribution, for instance, the Kaldorian one, which are centered on the different uses of wage and profit income. But for reasons of simplicity, and also since the focus of this thesis is not to construct a macroeconomic theory of income distribution, the present assumption appears to be a reasonable choice.

The unconstrained or notional product supply, $Y_{s,n}$ and labour demand, $l_{d,n}$, are a solution to the firm's objective of maximising profits:

$$\text{Max. } \Pi = p_y Y - w l_y \qquad (1.1)$$

s.t.

$$f(l_y) \geq Y$$

where,

p_y = price of Y

w = wage rate

$Y_{s,n}$ and $l_{d,n}$ are, hence, functions of the real wage, and given as:

$$l_{d,n} = f'^{-1}(w/p_y) \qquad (2.1)$$

and,

$$Y_{s,n} = f[f'^{-1}(w/p_y)] \qquad (2.2)$$

Next, consider the case where the firm's output is constrained by deficient product demand to $\underline{Y}$, say, but is not constrained by the available labour supply. Then, the "effective" supply of Y, $Y_{s,e}$, i.e. the level of output taking account of the demand constraint, is a solution to profit maximising with the additional constraint $Y_s \leq \underline{Y}$. And the resulting demand for labour is at a lower level than in (2.1), and also less than the available labour supply. In this case,

$$Y_{s,e} = \underline{Y} \qquad (3.1)$$

and,

$$l_{d,e} = f^{-1}(\underline{Y}) = f[w/(\ p_y - \mu)] \leq l_{d,n} = f^{-1}(w/p_y) \qquad (3.2)$$

$$l_{d,e} \leq l_{s,n} \qquad (3.3)$$

where

$l_{s,n}$ = notional labour supply of the household

$l_{d,e}$ = effective labour demand of the firm

and,

μ = multiplier associated with $\underline{Y}$, and is interpreted as the marginal gain associated with relaxing $\underline{Y}$ or increasing aggregate demand for Y. Equation (3.2) indicates that at the profit maximising labour demand under rationing, marginal product of labour exceeds the real wage, resulting in lower labour demand than in the unconstrained situation as depicted in Fig.1 below. This situation is typically a feature of Keynesian unemployment, and constitutes the basis for arguments from some quarters favouring real wage cuts to reduce unemployment.

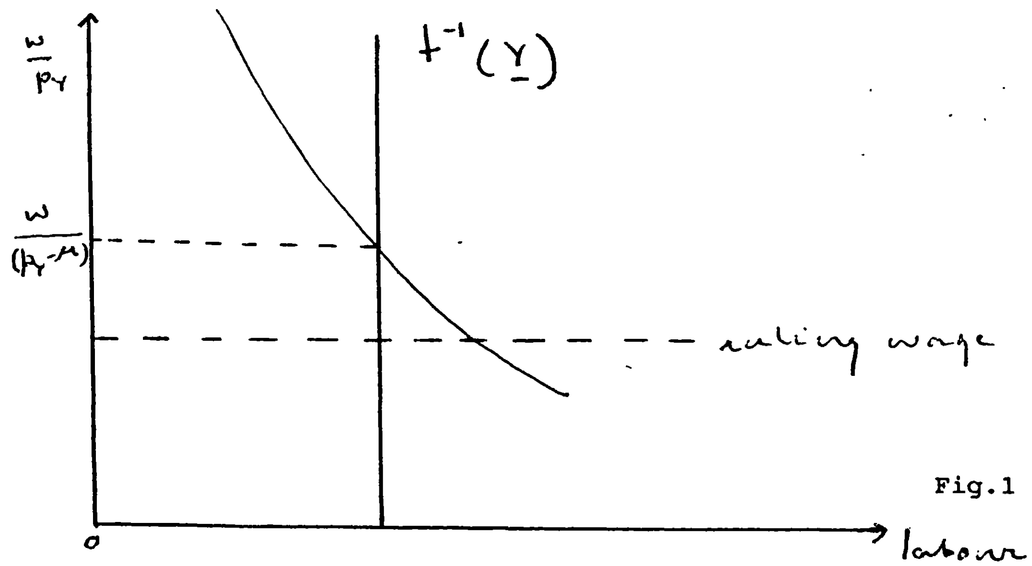

Fig.1

Finally, if labour supply is the limiting factor on private production, and rationed to, say, $\underline{l}$ then the effective supply of Y subject to this input constraint is:

$$Y_{s,e} = f(\underline{l}) \tag{4.1}$$

and,

$$l_{d,n} \geq \underline{l} \tag{4.2}$$

Therefore, Y_a and $l_{a,y}$, the transacted or actual volume of production and employment in the private sector respectively, covering the three possibilities above, may be written as:

$$Y_a = \min\{ f[f'^{-1}(w/p_y)], f(\underline{l}), \underline{Y} \} \qquad (5.1)$$

$$= \text{transacted amount of } Y$$

and,

$$l_{a,y} = f^{-1}(Y_a) \qquad (5.2)$$

$$= \text{actual employment in the private sector}$$

As an illustration, we shall later consider the specific form of the production function given as:

$$Y = f(l_y) = (l_y)^k \qquad (6)$$

where k is a positive constant. So, $f'>0$ and $f'' \leq 0$ if $0< k \leq 1$, denoting a positive but decreasing or constant marginal product of labour, and for the particular case of $k=1$, we have a CRS production technology. This function will be used widely in subsequent analysis.

(2.2) Public Sector Firm

A novel feature of the public sector here is its production of a privately consumed commodity G (that actually enters the individual utility function), relaxing the often assumed bounds to the government's role of providing services that are of no direct utility to consumers.

The government produces G subject to its production function, and to its revenue constraint where the production function is given as:

$$G = Q(l_g),\ Q'>0,\ Q''\leq 0 \qquad (7.1)$$

and

$$l_g = \text{labour input of } G \qquad (7.2)$$

The profit maximising labour demand for G would be such that:

$$p_g = w/Q'(l_g) \qquad (7.3)$$

where the right-hand side is the cost of a marginal unit of G.

However, one of the important questions facing a public sector enterprise is whether or not to follow marginal cost pricing, and to examine the situations warranting deviations therefrom at the price of efficiency. It is this latter problem that is analytically interesting, and dealt with here. We now consider a specific form of the production function to illustrate this point. Suppose that the production function of G takes the specific form of:

$$G = Q(l_g) = (l_g)^{1/\theta} \qquad (8)$$

where $\theta > 0$ so that $Q' > 0$. In the particular case of $\theta = 1$, public sector production is characterised by constant returns to scale. Labour demand is then solved for simply as: $l_{d,g} = G$ and, if now the government undertaking opts for marginal cost pricing, then p_g is simply equated to w, the prevailing wage since marginal product of labour in the public sector is unity. Similarly, we could just as well have considered alternative pricing rules as also different technologies of $\theta \gtrless 1$, representing decreasing or increasing returns.

But at this stage, we limit ourselves to this brief introduction, intended merely as a "preview", as it were, to the analytics of public sector pricing.

It is assumed that the market for the government produced good is always cleared, and that there are no capacity constraints on the provision of G so that the demand for labour is $l_g = Q^{-1}(G)$. Alternatively, we could have a situation where the public sector does not supply all that is demanded of its output as implicitly assumed above. This would effectively imply a rationing of G. Then, in the Keynesian unemployment case, over-all, we would have an environment where the consumer faces a constraint in his demand for G and none in his demand for Y while his labour supply is restricted by aggregate labour demand. At a subsequent stage, Classical Unemployment is considered where there is rationing of both labour supply and the demand for the private good so that in some sense the effects of rationing of product demand is dealt with. But, if we choose to restrict the consumption of G , then, both Y and G demand will be rationed while in the Keynesian Unemployment regime only G would be rationed and Y is available to the extent demanded. Therefore, rationing of G as well implies four alternative possibilities in the goods markets itself where Y may or may not be rationed and likewise, G may or may not be rationed. Allowing for the labour market, we have an additional set of possible market situations. For simplicity, and also because the thrust of this paper is to capture the effects of rationing of the private sector output, attention is focussed on the case set out originally - G unrationed with Y rationed or otherwise.

In the sections to follow, we deal with deriving the rules for the optimal pricing of G and related issues. The analysis dealt with now is true for any p_g, the public sector price.

The specification of the revenue constraint follows from the role the government plays in this economy. Not only does the government produce a good for private consumption, its functions extend beyond to incurring expenditure Z on purchases of the private good, Y, for its other activities and also raising lump-sum taxes, T. These other activities are items such as the provision of parks, defense, intelligence on the African National Congress in the case of some "friendly countries" of South Africa, and such other. With reference to the individual consumer, it is assumed that public expenditures of this kind do not enter his utility function so that it has often been treated as "waste" in the literature.

On the financial side, the government issues bonds, B_t, at date t each priced at q_t. There is no private sector debt so that government is the sole agent issuing bonds which are of single-period duration. The maturity value is one unit of money so that the interest rate r_t is $(1/q_t)-1$. Taxes, money supply increases and new bond issues are used, together or in some combination, to finance the government expenditures Z, any deficits incurred in the supply of G and to redeem the existing bond stock, B_{t-1}. Therefore, the government budget constraint is:

$$\Phi = -(w_t l_{g,t} - p_{g,t} G_t) - (Z_t - T_t) + M_t - M_{t-1} + q_t B_t - B_{t-1} = 0 \qquad (9)$$

which may be re-written as:

$$w_t l_{g,t} - p_{g,t} G_t + (1-q_t) B_{t-1} + Z_t - T_t = (M_t - M_{t-1}) + q_t (B_t - B_{t-1}) \qquad (10)$$

Changes in the outstanding bond stock are represented on the R.H.S. of (10) while the current interest payments are given on the L.H.S.

Under the assumptions made so far, total labour demand, L_d, is simply the aggregation across the demands of the private and public sector firms:

$$L_d = l_{d,y} + l_{d,g} \qquad (11)$$

(2.3) Households

Consumers live for two periods. There are no bequests. A household born in period t lives for 2 periods, t and t+1, and seeks to maximise life-time utility defined as:

$$U = U(C_t, C_{t+1}) \qquad (12)$$

where

$C_t = (x_t, g_t, m_t, b_t)$

= consumption demand of young of generation t

$C_{t+1} = (x_{t+1}, g_{t+1})$

= demand when old of generation t

x = consumption demand for Y, the good produced by the private firm

g = consumption demand for G, the Government produced good

m = demand for money balances

b = demand for bonds

The subscripts, t and t+1, refer to the first and second periods of life of a generation born in time t, say. All generations are assumed to be identical in tastes, and their resource endowments of labour, $\underline{L}$. Though money earns no interest, it is assumed to provide "liquidity" services which is reflected in having real money balances as an argument of the individual utility function. Feenstra (1986) shows that it may be exactly derivable from explicit models of transactions demand, under suitable assumptions.

Labour does not enter the utility function because we assume the household has no utility for leisure, and desires to work as much as possible. Households have a labour endowment $\underline{L}$ in period 1 and zero in period 2. This implies that total wage income is $w\underline{L}$ if there are no constraints on labour supply. Since all profits are re-distributed, households own all production as well as supply labour in the first period of their life. These assumptions greatly simplify analysis of the 2nd period for generation t: in period 2, generation t is not interested in the labour market conditions since it has zero labour endowment and zero profit income. Therefore, the household does not need to form expectations of future labour rationing because this becomes irrelevant. In the second period when the household retires from work, consumption expenditures are met out of net savings accrued from the active life of period one. Following these assumptions, the total gross income, K, defined as the sum of private profits and wage earnings, is:

$$K = \Pi + w\underline{L} = p_y Y + w l_g \qquad (13)$$

if the household faces no labour supply rationing.

In the unconstrained situation, the consumer chooses C_t

and C_{t+1} so as to maximise (12) subject to:

$$K^d_t \equiv K_t - T_t = p_{y,t}x_t + p_{g,t}g_t + m_t + q_t b_t \qquad (13)'$$

and,

$$(p^e_{y,t+1})\; x_{t+1} \;=\; m_t + b_t \qquad (14)$$

where

K_t = total income in period t

K^d_t = disposable income

$p^e_{y,t+1}$ = expected price of good Y in time t+1.

T_t = lump-sum taxes at time t.

b_t = number of bonds

q_t = issue price per bond

Constraint (14) excludes g_{t+1} because it is assumed that the consumer demands G only when he is young, e.g. college education that is provided by, say, the State university system. In terms of expectations formation, all that is required of generation t is to form expectations on the 2nd period price level, $p^e_{t+1,y}$, at time t since $g^t_{t+1} = 0$ by assumption. Eliminating b_t, constraints (13) and (14) are combined as the budget constraint below:

$$K_t - T_t = p_{y,t}x_t + p_{g,t}g_t + (q_t p^e_{y,t+1})x_{t+1} + (1-q_t)m_t \qquad (15)$$

The optimal consumption demands may be written as:

$$x_t = x[K^d_t, p_{y,t}, p_{g,t}, p^e_{y,t+1}, q_t] \qquad (16.1)$$

$$g_t = g[K^d_t, p_{y,t}, p_{g,t}, p^e_{y,t+1}, q_t] \qquad (16.2)$$

$$m_t = m[K^d{}_t, p_{y,t}, p_{g,t}, p^e_{y,t+1}, q_t] \qquad (16.3)$$

$$b_t = b[K^d{}_t, p_{y,t}, p_{g,t}, p^e_{y,t+1}, q_t] \qquad (16.4)$$

The rationale for holding money which is non-interest bearing when there exists the choice of investing all savings in interest yielding bonds is justified on the type of arguments used by Brock(1974), Hicks (1935). The theory of the demand for money has two main reasons: risk and transactions costs. But since both of these introduce considerable complications, I assume the common short-cut of supposing that real money balances provide utility. The rationale for money demand underlying this assumption is closer to transactions demand theories than to asset demand ones: money is seen as reducing the loss of leisure otherwise involved in purchasing goods, or reducing the necessary sacrifice of consumption itself.

(2.4)Alternative Rationing Mechanisms

(i) Next, we investigate the effects on consumer demand and labour supply of rationing in the private product and labour markets. When there is excess supply in the labour market, the household faces a rationing of its desired level of employment to $\underline{l}$, say, because now the firm's unconstrained labour demand determines the actual volume of employment since:

$$l_{d,n} = f'^{-1}(w/p_y) \leq l_{s,n} = \underline{L} - l_{d,g} \qquad (17)$$

where $l_{s,n}$ is the notional labour supply of the (unconstrained) household to the private firm net of governnment's labour demand, $l_{d,g}$. Though the going wage rate is common to both the sectors, the fact that the government is not constrained means it gets priority in meeting its labour demand, leaving the private sector to be content with the residual labour supply of $\underline{L} - l_{d,g}$.

The optimal consumption levels are a solution to the household maximising its utility function (12) subject to (15) and the additional constraint (17) as well. The labour supply constraint on households implies that their effective labour supply to the private firm is restricted to some level $\underline{l}$, as determined by the latter, and employment-constrained total income of the household, $K^{d''}$, is now given by:

$$K^{d''} = \Pi + w(\underline{l} + l_g) - T \leq \Pi + w\underline{L} - T \qquad (18)$$

where $\underline{l}$ is the constrained employment level in the private sector. At the lower income level, it is to be expected that the constrained demands are smaller correspondingly. But since total income, be it $K^{d''}$ or K^d, is in both cases exogenous to the consumer (because labour supplied is either $\underline{L}$ or $\underline{l}$, does not enter the utility function, and we do not have a labour supply function), the functional form of the Walrasian and constrained demand functions for all goods other than Y are the same. However, the excess demand for Y implies rationing of consumer demand; specifically, I assume, throughout the analysis, that production of Y is at least sufficient to meet the demands of government and the old who are not rationed in any way. But suppose that the young are uniformly curtailed in their demand for Y to some level x*, say. Neary and Stiglitz (1983), hereafter referred to as NS, give a comprehensive and excellent discussion of the properties of constrained demand and supply functions, the results of which will be drawn upon in our subsequent analysis.

(ii) Lastly, we consider the effects of constraints on consumption demand. If excess demand for Y rations consumption demand to $\underline{X}$, say, then the income available for expenditure on the other goods is the residual disposable income, K^{d*}, given by:

$$K^{d*} = K^d - p_y\underline{X} \qquad (19)$$

The optimal consumption levels of g_t and m_t are determined as before by maximising the utility function subject to the budget constraint (15), and the additional constraint $x_t = \underline{X}$, at the residual income $K^{d*} < K^d$. Note, the arguments in the optimal solution for consumption demands now includes $\underline{X}$, since the consumption demands are given by:

$$g_t = \bar{g}[K^{d*}_t, p^e_{y,t+1}, p_{g,t}, q_t; \underline{X}] \qquad (20.1)$$

$$m_t = \bar{m}[K^{d*}_t, p^e_{y,t+1}, p_{g,t}, q_t; \underline{X}] \qquad (20.2)$$

$$b_t = \bar{b}[K^{d*}_t, p^e_{y,t+1}, p_{g,t}, q_t; \underline{X}] \qquad (20.3)$$

where the upper bar $\bar{g}$, $\bar{b}$ and $\bar{m}$ indicate constrained demands. As mentioned earlier, the Neary and Stiglitz results on the spill-over effects of such constrained demands are drawn upon in our subsequent analysis.

With reference to the consumption demand of the old, they simply consume an amount equal to the value of their outstanding assets at the beginning of the period:

$$x^{t-1}_t = (b_{t-1} + m_{t-1})/p_{y,t} \qquad (21),$$

and are, by assumption, never rationed in their demand.

(2.5) Properties of the demand/supply functions

At this point, I discuss the properties of the demand functions (16) and (20) since they are crucial in determining the behaviour of the model. Denote $x_{t+1} = C_f$ as the demand for future consumption, and substitute this along with (16) into the budget constraint, (15). Differentiating this with respect to income yields:

$$1 = p_y x_k + p_g g_k + (q_t p^e_{y,t+1}) C_{f,k} + (1-q_t) m_k \qquad (22)$$

The time subscripts on current consumption demands are suppressed to avoid notational clutter, and $x_k = \partial x/\partial K$, $g_k = \partial g/\partial K$, etc. If all goods are assumed strictly "normal", then

$$0 < p_y x_k,\ p_g g_k,\ (p^e_{y,t+1} q_t) C_{f,k},\ (1-q_t) m_k < 1 \qquad (23)$$

We consider the price derivatives next. From (15), it is clear that a change in q_t brings about a change both in the "price" of future consumption, q_t, and of money, $(1-q_t)$. Denote these prices as v_t and r_t, respectively, without constraining $r_t = 1-v_t$. The derivatives x_q, g_q and m_q are decomposed to give:

$$x_q = x_v - x_r, \qquad (24.1)$$

$$g_q = g_v - g_r, \qquad (24.2)$$

$$m_q = m_v - m_r \qquad (24.3)$$

where x_q, g_q, m_v, etc. are interpreted as the usual price derivatives.

These can, therefore, be split using the Slutsky equation as:

$$x_v = x_v^c - C_f x_k \qquad x_r = x_r^c - m x_k \qquad (25.1)$$

$$g_v = g_v^c - C_f g_k \qquad g_r = g_r^c - m g_k \qquad (25.2)$$

$$m_v = m_v^c - c_f m_k \qquad m_r = m_r^c - m m_k \qquad (25.3)$$

where the superscript c refers to a compensated derivative. So, (25) may be re-written as:

$$x_q = [x_v^c - x_r^c] - [C_f - m]x_k \qquad (26)$$

$$g_q = [g_v^c - g_r^c] - [C_f - m]g_k \qquad (27)$$

$$m_q = [m_v^c - m_r^c] - [C_f - m]m_k \qquad (28)$$

The RHS terms correspond to the net substitution and net income effects. The most common signs assumed for x_q, g_q and m_q would be that all three be positive, indicating a negative dependence of both commodity consumption and money demand on the interest rate. We see under what conditions this occurs. If desired money holdings are less than total desired savings, i.e. if $m < C_f$, then (continuing to assume all goods are normal) both net income effects are negative. Next, if all goods are net substitutes, so x_v^c, x_r^c, g_v^c, g_r^c, $m_v^c > 0$, then the net substitution effect on money demand is positive (because own price effect of money, m_r must be negative), while that on consumption is ambiguous. If, instead, money is a net complement to current consumption, i.e. x_r^c and $g_r^c < 0$, the net substitution effect on consumption is positive. The upshot of this is that a variety of configurations of signs of x_q, g_q and m_q is possible. Given this, we shall consider the "familiar" case, where x_q, g_q, $m_q > 0$, in subsequent ananlysis although that there are other possibilities should not be lost sight of. A sufficient condition for this would be the assumption of homotheticity of the instantaneous utility fuction to help ensure that the demand for both goods declines.

As we shall see, the sign of the expression, $x_k m_q - m_k x_q$, is equally important for some of the subsequent analysis. From (26) and (27) we have, since income effects cancel,

$$x_k m_q - m_k x_q = (x_k m_v^c - m_k x_v^c) - (x_k m_r^c - m_k x_r^c) \qquad (29)$$

For an intuition of what the sign of this term would be, suppose that U(.) is weakly separable in x_{t+1} and $[x_t, g_t, m_t]$. Then we have that (Deaton and Muellbauer, 1980):

$$x_v = \eta\, x_k C_{f,k} \quad \text{and} \quad m_v = \eta\, m_k C_{f,k} \qquad (30)$$

where η is a multiplier.

Using these expressions in (29), the first term drops out, leaving:

$$x_k m_q - m_k x_q = m_k x_r^c - x_k m_r^c \qquad (31)$$

Recall that since m_r is negative, then it must be the case that (31) is positive (continuing to assume all goods are normal) except in case there is a sufficiently strong degree of net complementarity between money and current consumption, i.e. $x_r^c < 0$, $g_r^c < 0$.

If instead, U(.) is assumed to be additive, the same conclusion about the sign of (31) holds. But here the assumption of normality of all goods imposes the additional property that they are all net substitutes. Consider secondly the constrained money demand function in (20). Exactly analogous arguments indicate that, if all goods are normal,

$$0 < (1-q_t)\bar{m}_k < 1 \qquad (32)$$

while the sign of $\bar{m}_q$ is uncertain: we shall, as above, illustrate with the usual assumption in which case it is positive. The quantity constrained $\partial\bar{m}/\partial(p_y x)$ may be considered the outcome of income and substitution effects. The income effect is evidently negative, while the substitution effect reinforces this if money and current consumption are net substitutes, but counteracts it if they are net complements. In general, we shall assume that $\partial\bar{m}/\partial(p_y x) < 0$. If U(.) is weakly separable in x_t, it is easy to show that:

$$\partial\bar{m}/\partial(p_y x) = -\bar{m}_k \qquad (33)$$

Variations in $\underline{X}$ have an income effect since more of income is committed to the fixed expenditure, and the change in $\underline{X}$ implies a move to a different part of the indifference surface. Though this latter effect disappears with the assumption of separability, it provides a useful comparison, although restrictive, in that x_t and m_t cannot be net complements consistently with being normal in such a case.

(3) <u>An Illustration with a particular utility function</u>

A specific form of utility function of the CES type is now used as an example:

$$U(x_t, g_t, x_{t+1}, m_t) = [\, x_t^{\rho} + g_t^{\rho} + m_t^{\rho} + x_{t+1}^{\rho} \,]^{1/\rho} \qquad (34)$$

Since preferences are invariant with respect to a monotonic transformation, I use:

$$\underline{U} = [\ x_t^{\rho} + g_t^{\rho} + m_t^{\rho} + x_{t+1}^{\rho}\ ] \qquad (35)$$

Define $\sigma = \rho/(\rho-1)$ and let:

$$p_1 = p_{y,t},\ p_2 = p_{g,t},\ p_3 = (1-q_t),\ p_4 = q_t p^e_{y,t+1} \text{ and } P = \sum_{i=1-4} (p_i)^{\sigma} \qquad (36)$$

The elasticity of substitution for this CES function is given as:

$$\hat{\sigma} = \frac{1}{(1-\rho)}$$

where $-\infty \leq \rho \leq +1$ implying $-1 \leq \hat{\sigma} \leq +\infty$. It follows that: $0 \leq \sigma \leq +\infty$ since σ is nothing other than $(1 + \hat{\sigma})$.

(i) No Rationing

When there are no constraints on individual choice in any of the markets, then utility as given by (35) is maximised subject to equation (15), and the optimal consumption demands are:

$$x_t = \left[\frac{p_1}{P}\right]^{\sigma-1} K^d_t \qquad (37)$$

$$g_t = \left[\frac{p_2}{P}\right]^{\sigma-1} K^d_t \qquad (38)$$

$$x_{t+1} = \left[\frac{p_4}{P}\right]^{\sigma-1} K^d_t \qquad (39)$$

$$= \frac{(m_t+b_t)}{p_4} \qquad (39)'$$

The second equality in (39)' is due to constraint (14) above implying that the old simply consume an amount equal to the value of their savings since there are no bequests. Note that money does not earn any interest, and therefore, rules out a term such as

$(1+r_t)m_t$ in the expression for total savings of the individual.

$$m_t = \left[\frac{p_3}{P}\right]^{\sigma-1} K_t^d \qquad (40)$$

and,

$$\frac{1}{\delta} = \frac{1}{\rho}\left[\frac{K^d}{P}\right]^{\rho-1} \qquad (41)$$

The multiplier δ is the multiplier associated with the individual budget constraint, and is interpreted as the marginal utility of income.

Suppose we now consider the special case of a Cobb-Douglas utility function where $\sigma = 0$. Then the demand for each of the goods is given as:

$$p_{y,t}x_t = p_{g,t}g_t = q_t p^e_{y,t+1})x_{t+1} = (1-q_t)m_t = \frac{1}{4} K^d{}_t \qquad (42)$$

But from the last two inequalities of (42) above, and using the consumption equation of the old of generation t given in (21), we have:

$$q_t\ (m_t + b_t) = p^e_{y,t+1}\ x_{t+1} = \frac{1}{4} K^d{}_t \qquad (43)$$

Equation (43) implies a demand for bonds of:

$$b_t = \left[\frac{1}{q_t} - \frac{1}{(1-q_t)}\right]\frac{K^d{}_t}{4} \qquad (44)$$

This bond demand function indicates, importantly, the absence of any negative net wealth effects in that total income increases with a rising bond stock. Also, we observe that bond demand varies inversely with q_t, as one would expect, since q_t is the unit price at which the government sells bonds.

(ii) Consumption Demand Constraints

Next, considering the case when the young consumer's demand for Y is rationed to $\underline{X}$, say, the changes in optimal consumption demands (37) to (40), and in (41) are:

$$x_t = \underline{X} \tag{45}$$

and a different income term, K^{d*}, as in equation (19) above:

$$K^{d*} = K^d - p_y\underline{X} \tag{46}$$

K^{d*} is substituted for K^d as the new "income" term, and the expression for P in the demand rationing situation changes to:

$$\underline{P} = p_2^{\sigma} + p_3^{\sigma} + p_4^{\sigma} < P \tag{47}$$

It is immediately clear that changes in p_y have only an income effect on demand and no substitution effects. Similarly, it is easily established that the marginal utility of income, denoted by δ, falls. This is intuitively clear since it merely indicates the fact that the consumer can no longer achieve his desired consumption with his earnings, reflecting the decline in the marginal utility of income from δ to $\underline{\delta}$.

(iii) Constraints on Labour Supply

The effects of labour supply rationing to some level, $\underline{l}$, in the private firm is the reduction of income to $K^{d''}$, as mentioned earlier in equation (18). Here, we have both income and substitution effects if there are price changes in Y. Obviously, changes in the constraint level $\underline{l}$ generate income effects on consumption demands.

Lastly, a little bit of detail on the role of money and expectations before studying the nature of the short-run equilibrium to be covered in the next chapter.

(4) <u>Money and Expectations</u>

Since there are no credit markets and no form of assets other than money and government bonds, the supply of money is from the old of generation t-1, and the expansion of money supply to balance the government revenue constraint.

For analytical simplicity, I assume rational expectations. However, different assumptions on expectations and price formation have significantly different effects on output and employment (Benassy,1986). Given our assumptions that the old do not work, future wages and labour market conditions are not relevant, and it is not necessary to form expectations on them. The government produced good is consumed only by the young, ruling out the need for anticipating their future prices. Hence, the important element here is the expected future price of Y, $p^e_{y,t+1}$, in respect of which consumers are assumed to hold rational expectations.

This completes the discussion of the individual economic agents of our model, and their behaviour in different economic environments. The nature of the short-run equilibrium, and the corresponding policy prescriptions are studied, regime by regime, in the chapter to follow.

Chapter 3

Short-run Equilibrium and Policy

(3.1) Principal Findings

Here we show that the public sector provides a viable control measure not only in the Keynesian unemployment regime but also in situations of Classical unemployment. Depending on the consumption demand relations, and the importance of the state owned enterprises in the economy, private production and employment decisions can be quite effectively influenced by the choice of public sector prices. A rigorous analysis of this, and other results is the subject of this chapter.

Our principal results on policy changes indicate that:

1) Changes in taxes are not necessarily contractionary. This unconventional result follows from the fact that money and commodity demands are determined by the level of disposable income and not by gross income. As a consequence, taxes have both a contractionary and an expansionary impact, leading to amgibuity in their final effect. But if "too high a degree of complementarity between money and current consumption" is ruled out, then a policy of tax cuts is expansionary for the most commonly assumed demand responses.

2) In the current model, aggregate income is determined by the value of private sector output and total labour incomes in the public sector. This fact leads to some interesting results with respect to the effect of changes in public sector prices, as a policy instrument, to influence private and public sector production and employment

levels in the two sectors and hence, aggregate income and employment in the economy. Typically, a lower price for the public sector good, for instance, leads to a corresponding increase in its demand and therefore, an increase in labour income in the public sector. However, the effects on the demand for the other goods, money and the privately supplied good, also require to be taken account of to assess the overall macroeconomic consequences of the price change. In the analysis that follows, a detailed investigation of the conditions under which a price change would be expansionary is spelt out.

3) Even in the context of a situation characterised by "Classical" unemployment, where the traditional instruments of government fiscal policy are ineffective, it is verified that changes in public sector price may well proove effective in reducing the rate of unemployment.

The framework of the analysis used in deriving these results remains the same as in the previous chapter. The prices, p_y and w, are fixed exogenously. But q_t is assumed to be determined in the money market as equating money supply and demand. The policy or control variables assumed throughout our paper are the level of government spending Z, lump-sum taxes T, public sector prices p_g, changes in the money supply M_t and bond stocks B_t - any four of which may be chosen independently. Assuming Z and p_g are always chosen independently, leaves us with the following policy instruments, or financing mechanisms:

Tax financing, M_t and B_t constant

Money financing, T_t and B_t constant

Bond financing, T_t and M_t constant

The policy framework used here, in the first instance, is in the context of bond-financing i.e., treating M_t, T_t, Z_t and $p_{g,t}$ as exogeneously determined control variables, while B_t is implicitly determined by the government budget constraint, equation (10), chapter 2 above. With a population stationary in size, the market clearing conditions and the corresponding set of equilibria Y, l_y and q for the three disequilibrium regimes discussed earlier are analysed below, following a brief description of the Walrasian equilibrium.

(3.2) Walrasian Equilibrium

The Walrasian equilibrium is described by the set of prices (p_o,w_o,q_o) at which all 3 markets viz. the private product, labour and money markets clear simultaneously. The equilibrium conditions are:

$$f'^{-1}(w_o/p_o) = l_{o,y} \qquad (1)$$

$$l_{o,y} + l_{d,g} = \underline{L} \qquad (2)$$

Equation (1) indicates that private sector labour demand is determined solely by the Walrasian real wage, w_o/p_o, while (2) implies that $l_{o,y}$ is exactly equal to the available labour supply, $\underline{L}-l_{d,g}$. Equations (3) and (4) below indicate that in both the private product and money markets, demand exactly matches supply at the Walrasian interest rate, q_o, and real wage, w_o/p_o.

$$f[f'^{-1}(w_o/p_o)] = Y_o = x_t^t + x_t^{t-1} + Z_t \qquad (3)$$

$$m = m(K_o,q_o,p_o,w_o,p_g) \qquad (4)$$

where p_g is a policy variable, and

$$Y_o = f(l_{o,y})$$

$$K_o = p_oY_o + w\underline{L}$$

$$= \text{aggregate income}$$

To sum up, equations (1)-(4) describe the Walrasian equilibrium in terms of the interest rate, q_o, and real wage, w_o/p_o. Note that the assumption of an exogenous labour supply, absence of disutility of labour, an independently determined public sector price p_g as a government policy variable and rational expectations on $p^e_{y,t+1}$ result in the Walrasian real wage and output, Y_o, being endogenously determined.

(3.3) <u>Keynesian Unemployment Equilibrium</u>

In this regime, private product price, p_y and wages, w, are fixed exogenously, and moreover at levels at which there is excess supply in both the labour and private product markets. The equilibrium (Y, l_y) are determined by the short side of the market, while q_t equates money demand and supply. Therefore, Y, l_y and q, are given as a solution to (5), (6) and (7) below:

$$Y_{k,t} = x^t_t + x^{t-1}_t + z_t \qquad (5)$$

$$l_{y,t} = f^{-1}(Y_{k,t}) \qquad (6)$$

$$M_t = m[\, K^d_{k,t}, q_t, p_t] \qquad (7)$$

For the general utility function (12) assumed in chapter 2 and the corresponding set of optimal demands, we rewrite the private product market clearing equation above as:

$$Y_{k,t} = x[K^d_{k,t}, q_t, p_t] + A_t + Z_t \qquad (5)'$$

where

$$x^t_t = x[K^d_{k,t}, q_t; p_t] \text{ by equation (16.1), chapter 2}$$

$$x^{t-1}_t = (m_{t-1} + b_{t-1})/p_1 \text{ by equation (21), chapter 2}$$

$$A_t \equiv (b_{t-1} + m_{t-1})/p_1$$

$$K^d_{k,t} = \pi + w(\underline{l}_k + l_g) - T \text{ by equation (18), chapter 2}$$

$$= \text{ total disposable income in the K.U. regime}$$

$$\underline{l}_k = \text{constraint on labour supply in K.U. regime}$$

$$p_t = p_1, p_2, p^e_{y,t+1}$$

From equation (5)', we see that the level of employment does not appear independently in the consumption demand. This is due to our assumptions that labour has no disutility, the old do not work and consume out of their savings, and that all profits are redistributed immediately. Therefore, equations (5)' and (6) can be solved recursively for the equilibrium level of output and employment in the private firms.

We now consider the overall effects of policy variable changes in the product and money markets, depicting the goods and money market equations in terms of the traditional "IS-LM" diagram, Fig.1 below, where (Y,q) are the endogenous variables. The translation of our earlier analysis into this framework is to establish a synthesis with the earlier generation of Keynesian models of the IS-LM type, as in Rankin (1984) and Benassy (1986) to cite some of the previous authors.

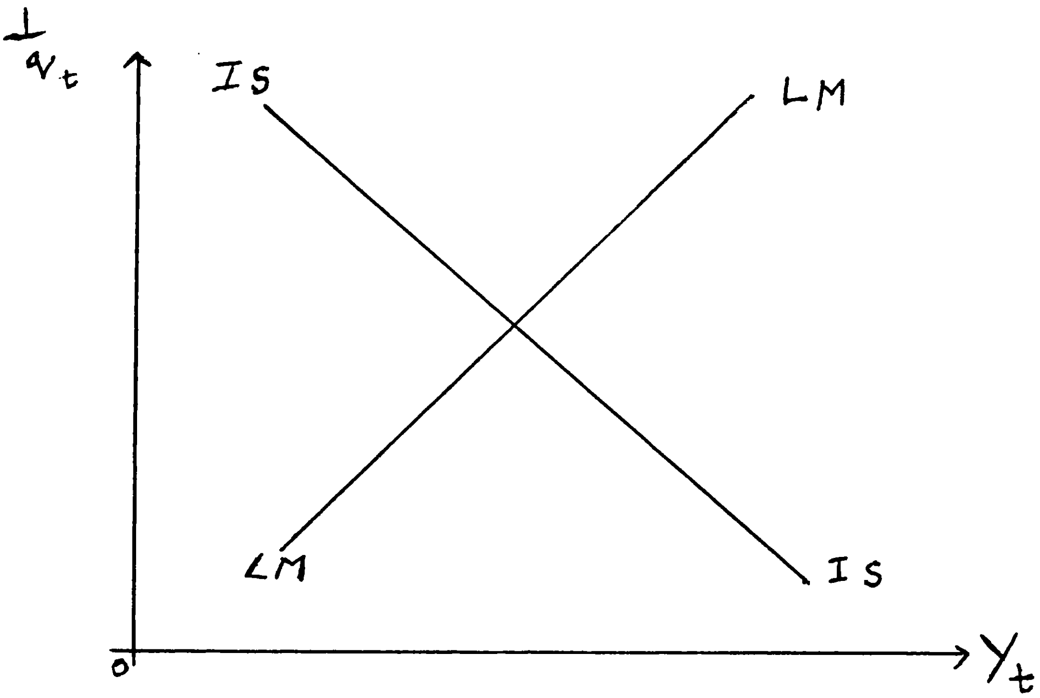

Fig.1

From the earlier discussion on the properties of the consumption demand functions, x_t, m_t, etc., the slopes of the two curves are established. For illustrative purposes, I consider the particular case when public sector production is characterised by constant returns to scale i.e, θ=1 when l_g = G (see equation 8, chapter 2). Recalling the definition of disposable income as $K^d_k = p_y Y_k + wl_g - T$, and using the demand function for G, (see equation (16.2) in chapter 2), we obtain an expression for K^d_k as an implicit function of the variables (Y,q):

$$K^d_k = p_y Y_k + w\ g(K^d_k, q, p) - T = k[Y_k, q; \underline{l}_k, T, p] \qquad (8)$$

$$\text{where } K_y = dK/dY > 0 \text{ and } K_q > 0, \qquad (9)$$

and we continue to assume p_y,w as exogenous, rational expectations on $p^e_{y,t+1}$, and p_g a given policy parameter. Therefore, the product and money market equations (5) and (7) may now be rewritten in terms of the endogenous variables (Y,q) as:

$$Y_k = x[K^d_k(Y_k, q; p, \underline{l}_k), q; p] + A + Z \qquad (10)$$

and,

$$M = m[K_k^d(Y_k, q; p, l_k), q; p] \qquad (11)$$

Making use of the income derivatives K_y and K_q in (9), and the properties of the demand functions, we obtain:

$$dY_k = (x_k K_y)dY_k + (x_k K_q)dq + (x_q)dq \qquad (12)$$

so that $\partial Y_k/\partial q > 0$, establishing a negatively sloped IS. The mechanism underlying this response is the fact that x_q is positive because of the savings effect of a decline in the interest rate (see pages 19, 20, chapter 2) and also because K_q is positive as seen earlier in (9) since $g_q > 0$ and all consumption demands are normal, by assumption.

Similarly, from the money market equilibrium condition and using the properties of K_k^d, and of the money demand function, we derive:

$$(m_k K_y)dY_k + (m_k K_q)dq + (m_q)dq = 0 \qquad (13)$$

indicating a positively sloped LM.

At this point, it has to be pointed out that alternative assumptions on the technology in public sector production of $\theta \gtrless 1$ would lead to a different response in product demand to changes in the interest rate and hence, a correspondingly different slope of the IS or LM curves. Also, I draw attention to the special case of the Cobb-Douglas utility function when both x_q and g_q are zero, resulting in the supply of the privately produced good, Y, being invariant to changes in the interest rate, and the IS curve in (12) is vertical. But the LM curve continues to be positively sloped since the price of money is (1-q). I have suppressed the bond market equation by Walras Law. As shown earlier in chapter 2 with the specific example of a Cobb-Douglas utility function, it is easily verified here as well that there is a similar direct relationship between the bond stock and aggregate income, K, and the absence of a negative net wealth effect.

(3.4) Policy variable changes in the Keynesian regime

The comparative statics differ from the usual results in some important respects: though an increase in Z and M, for instance, lead to rightward shifts in the IS and LM curves respectively, more detailed analysis of the policy variable changes in T, Z and p_g reveal some notable differences. Also, it is important to bear in mind that the comparative static results derived below are seen as holding at a given point of time t for fixed level of bond stock, B_{t-1}. This is so because an equilibrium (q_t, y_t) indicates a corresponding value of bond stock, B_t, and since there is no presumption that B_t equals B_{t-1}, this in turn, implies that the IS will be shifting over time.

(i) An increased government demand for private output

From the government budget constraint, equation (10) in chapter 2, a rise in government expenditures Z, for example, will affect B_t and hence, both aggregate income and interest q in the next equation. With respect to the changes in the level of private sector production, we derive that increased government demand is expansionary as shown below. For the utility and production funcions assumed here, we obtain by totally differentiating (10) and (11) the effects of increased government spending Z on private sector production in (14) below, and hence, on aggregate income since changes in private sector output and total income are directly related.

$$\frac{\partial Y}{\partial Z} = \frac{[m_k K_q + m_q]}{H} \tag{14}$$

where H = determinant of AD+BC > 0, since A,B,C and D are defined as in equations (14)' below:

$$\left.\begin{array}{l} A = (1-x_k K_y) = 1-\text{ marginal propensity to consume } Y \\ B = (x_k K_q + x_q) > 0 \\ C = m_k K_y = \text{ marginal propensity to demand money} \\ D = (m_k K_q + m_q) > 0 \end{array}\right| \qquad (14)'$$

As argued earler, B and D, are the composite effect of changes in the interest rate, q, on commodity and money demand, repectively and are both positive since x_k, m_k, x_q and $m_q > 0$. The numerator of (14) is nothing but D, and denotes the total effect of interest rate changes on money demand, comprising the income effect, $m_k K_q$, and substitution effect, m_q, the latter term being positive (see equations (24)-(28), chapter 2). Since all goods are assumed normal, money demand increases/decreases with income (equation (23), chapter 2) so that $m_k > 0$. With reference to the response of aggregate income, K_y or K_q, we consider the particular case where total income K is defined as in (8) above. Therefore, $m_k K_q + m_q$ is positive, and by earlier analysis, the sign of the denominator determinant H is positive so that $\partial Y/\partial Z > 0$, establishing that increases in government expenditures Z stimulate private production. Note, however, that the multiplier (14) is smaller than the "simple" Keynesian multiplier of $1/(1-x_k)$, as a result of the presence of financial assets competing with current consumption of goods.

As a **corollary**, we conclude that steping up government demand has expansionary effects on aggregate income since increases in private sector production lead to an increase in aggregate income. As an illustration, I consider the particular case of the Cobb-Douglas utility function, and constant returns to scale production function in public sector production.

An example

Writing α for the proportion of income spent on X, β for the proportion spent on public output, and γ for the proportion held in money, and setting $A \equiv Z_t + (B_{t-1}+M_{t-1})/p_1$, the key equations of the model are:

$$p_1Y = p_1A + \alpha K^d \qquad (A)$$

$$K^d = p_1Y + (w/p_g)\beta K^d - T \qquad (B)$$

$$M = \gamma K^d/(1-q) \qquad (C)$$

So,

$$K^d[1- \alpha - (w/p_g)\beta] = p_1A - T \qquad (D)$$

The multiplier is:

$$\partial K^d/\partial Z = 1/\{1 - [\alpha+(w/p_g)\beta]\}$$

If $\alpha = \beta = 1/4$, the multiplier is

$$\partial K^d/\partial Z = 1/[1 - 1/4(1+w/p_g)]$$

and for public sector pricing at marginal cost, we have:

$\partial K^d/\partial Z = 2$, and for $p_g=.5w$, the multiplier is 4.

(ii) Changes in lump-sum taxes

Unlike the standard text-book results, we observe ambiguity in the effects of tax changes. This is because taxes have a dual effect insofar as they have both a contractionary and an expansionary impact on private sector production Y, given that money and commodity demands are a function os disposable income and not of gross income in our model. The result of a change in lump-sum taxes, T, on private firm output Y is given by:

$$\frac{\partial Y}{\partial T} = \frac{[ED - FB]}{H} \qquad (15)$$

where B and D continue to be defined as in (14)′ while E and F are:

$$E = x_k k_T < 0$$
$$F = m_k k_T < 0$$

since $m_k, x_k > 0$ by assumption of normality, and $K_T = -1$ simply follows from the definition of disposable income as in equation (8) above. Hence, we may express:

$$E = -(1-A) \text{ and}$$
$$F = -C$$

so that:

$$\frac{\partial Y}{\partial T} = \frac{[ED - FB]}{H} = \frac{-[(1-A)D - CB]}{H} \qquad (15)'$$

Now, (1-A) is nothing other than $x_k K_y = x_k$, the marginal propensity to consume the private good Y since $K_y = 1$ follows from the definition of aggregate income K. For the tax multiplier to be negative, we need that $(1-A)D - CB > 0$. It is not obvious that this will be the case unless we rule out, as discussed in equation (31), chapter 2, too high a degree of complementarity between money and current consumption. We are then able to claim unequivocally, for the most commonly assumed demand responses, that $\partial Y/\partial T<0$: a decrease in lump-sum taxes stimulates private production, the opposite being the case for increased taxation.

(iii) Balanced Budget Multiplier

The "balanced budget multiplier" is usually less than unity. However, our analysis yields a multiplier of exactly 1, a result due to the absence of an investment demand function in our model. Adding equations (14) and (15)' above results in:

$$\frac{\partial Y}{\partial T} + \frac{\partial Y}{\partial Z} = \frac{D}{H} + \frac{[ED - FB]}{H}$$

$$= \frac{D(1+E)}{H} - \frac{FB}{H} = \frac{[AD + BC]}{H} = 1 \qquad (16)$$

since as indicated earlier, (1+E) = A and F = - C. However, the presence of an interest sensitive investment function would modify the government spending multiplier, $\partial Y/\partial Z$; the value of the matrix H would be different since it would then include the additional influence of interest rate changes on investment, and hence, dampen the expansion in income, Y.

An important feature to note is that results (i) and (iii) are wholly independent of particular parameter values.

(iv) Changes in the price of the public sector good

The specific questions posed in this chapter are - What would be the effect of changes in public sector prices on aggregate income? In section (i), I gave an example with a Cobb-Douglas utility function and CRS in public sector production, where we found that lowering of the public sector price from $p_g=w$ to $p_g=.5w$ brought about an increase in the value of the multiplier $\partial K^d/\partial Z$. In the general case, would it be expansionary to drop price? and if so under what conditions would this be true? How is private production affected by a price change p_g? The subject of "optimal" public sector pricing, its relation to marginal cost and related issues is deferred to Chapter 5.

In order to answer this and related questions, it is useful first to derive the effect on privately supplied output, Y:

$$\frac{\partial Y}{\partial p_g} = \frac{[ED - FB]}{H} \qquad (17)$$

The expression for E and F is different now, reflecting the effects of changes in p_g on consumption and money demand:

$$E = x_k K_{p,g} + x_{p,g} \qquad (18)$$

$$F = m_k K_{p,g} + m_{p,g} \qquad (18)'$$

Once again, as we might expect, there are both income, $x_k K_{p,g}$ and $m_k K_{p,g}$, and substitution, $x_{p,g}$ and $m_{p,g}$, effects on commodity and money demands respectively, where $K_{p,g} \equiv K_g g_{p,g} < 0$ since $K_g > 0$ and $g_{p,g} < 0$ i.e. negative own-price effect of G. Therefore, normality of demands implies both $x_k k_{p,g} <0$ and $m_k k_{p,g} <0$, following a decline in K via the higher p_g, as in this instance. The terms $x_{p,g}$ and $m_{p,g}$ in E and F (equations (18) and (18)') respectively depend on whether x and m are complements or substitutes with g, in the former situation re-inforcing the income effects, or counter-acting it in the case of substitutes. Three possibilities are envisaged, as detailed below:

a) x and g are net substitutes while m and g are net complements:

$E > 0$

$F < 0$, then $ED - FB > 0$ and hence, $\partial Y/\partial p_g > 0$

b) alternatively, x and g are net complements with m and g as net substitutes in which event,

$E < 0$, and $F > 0$, resulting in $\partial Y/\partial p_g < 0$.

The intuition for (a) and (b) is straight-forward - since production of Y is demand determined, any changes in demand for x as in this case via changes in p_g, call for corresponding adjustments in production levels of Y.

(c) Lastly, if:

(i) $E,F > 0$

or,

(ii) $E,F < 0$

then the sign of $ED - FB$ is uncertain. However, (i) is ruled out since it implies that both x and m are complements with G, but by homogenity requirements: $\Sigma_{i=0} S_{i,j} = 0$, implying that all goods cannot be complements. The ambiguity, therefore, reduces to only case (ii) of both x and m being substitutes for G, in which event further assumptions about the relative magnitudes of each term are required to sign the effects of changes in public sector prices on the privately produced output.

Returning to the question posed at the outset about the effects on aggregate income, we recall the definition of aggregate income K as: $K = p_y Y + wl_g(G)$. With reference to the effects on the public sector, an increase in price would lead to a fall in the demand for G, the publicly supplied good, and hence, in labour demand l_g and total wage income in the public sector, wl_g. However, as seen in the analysis above the effects on private output Y depends on the demand relations between the goods Y, G and money. If the demand relations are as analysed in case a) then $\partial Y/\partial p_g$ is expansionary, and we do not have an unambiguous answer as to the multiplier effects on aggregate income K and total employment since there is the countervailing effect of a lower wage income in the public sector. Only if the expansionary effect on Y outweighs the effect of reduced employment and income in the public enterprise, could a higher price in the public sector be considered to have an expansionary impact on the economy. On the other hand, in case b) when a decline in public sector price leads to an increased demand for both Y and G, then unequivocally, a lower price p_g would lead to a higher level of aggregate income whilst a price increase in such a situation would be contractionary.

I go back to the particular case (see section (i)) of the Cobb-Douglas utility function, and constant returns to scale in the production of G, where it is readily verified that a rise in the public sector price reduces the value of the multiplier through a fall in G/K. Also, it was shown taking particular values of p_g that a lowering of price resulted in higher aggregate income.

To summarize the comparative statics, changes in tax or public sector prices have both contractionary and expansionary effects on consumption demands, resulting in ambiguity in the behaviour of private sector production. The economy-wide effects of changes in public sector prices on aggregate income and level of employment are dependent on the relative significance of the two sectors, and the structure of demand relations. However, under quite plausible and fairly commonly assumed demand responses, something definite can be said about the effects of changes in public sector prices and taxes on private production. On the other hand, the effects of increased government spending is unambiguously expansionary, and quite independent of any particular parameterisation.

(3.5) Classical Unemployment

The private firm's Walrasian product supply and labour demand now determine output and employment in the private sector:

$$l^d_{c,y} = f'^{-1}(w_t/p_{y,t}) \qquad (19)$$

as in equation (2.1), chapter 2

and,

$$Y_c = f[f'^{-1}(w_t/p_{y,t})] \qquad (20)$$

as in equation (2.2), chapter 2

$$M_t = m[K^{d*}_c, q_t; p_t] \qquad (21)$$

where,

$$K^{d*}_c = p_{t,t}Y_c + w_tG[K^{d*}_c, q_t; p_t] - p_{y,t}x^* - T_t$$

= disposable income net of expenditures on the rationed good, x^*. (See equation (19), chapter 2)

x^* = the ration amount of Y, obtained from the income-expenditure identity as $x^* = Y_c - A_{t-1}$.

The income derivatives are $\partial K_c^d/\partial Y > 0$ and $\partial K_c^d/\partial q > 0$.

Clearly, by (19) and (20), output and employment are solely determined by the exogenous real wage, and independent of any demand management policies, resulting in the vertical IS as depicted in Fig.2. Using the properties of income, K_c^d, and of the money demand function above, we obtain an upward sloping LM, as shown in Fig.2.

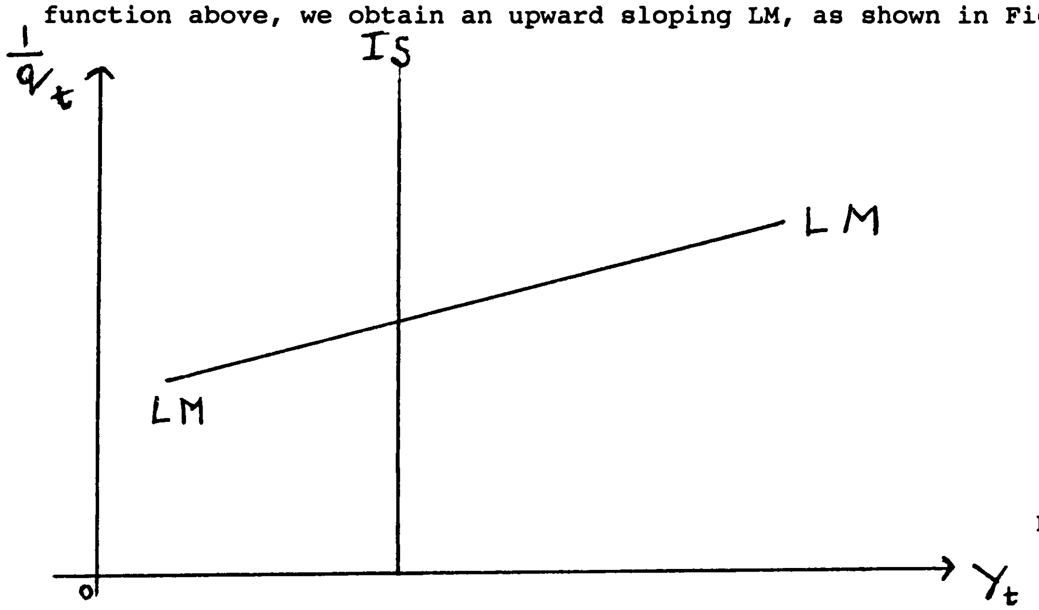

Fig.2

(3.6) Policy changes in the CU regime

The government budgetary measures on G or T are ineffective in influencing the level of output and labour input in the private sector. It is possible, though, that changes in p_g influence the net excess supply of labour: an increased product demand for G, for example, implies higher employment in the government sector with labour demand in the private sector unchanged since the real wage remains invariant with respect to changes in public sector prices. This point is illustrated below using once again the specific example of a Cobb-Douglas utility function, and a constant returns to scale production function in the public sector:

An Example

Private output is determined by the real wage, and that available to the first generation is, in terms of expenditure, (p_yY-A). This implies that the 'surplus' to be spent on the public sector good, money and future consumption, is $K-(p_yY-A) = A-T + wl_g$. It follows that the demand for the publicly produced good is:

$$p_gG = \frac{\beta}{(1-\alpha)} (A-T+wl_g) \qquad (22)$$

and the demand for money is:

$$(1-q)M = \frac{\gamma}{(1-\alpha)} (A-T+wl_g) \qquad (22)'$$

where α, β and γ are the proportions of income spent on the private good Y, public good G and money respectively. From the first of the two equations above,

$$p_gG \left[1 - \frac{\beta}{(1-\alpha)} \frac{w}{p_g} \right] = \frac{\beta}{(1-\alpha)} [A - T] \qquad (23)$$

Equation (23) indicates that a fall in the public sector price raises G, and hence, both employment and total private sector income, confirming the results noted in the general case. However, limits to the extent to which p_g can be varied to generate a larger volume of employment, in any given period, is set by the available residual income K_c^{d*} since x* and p_y are both pre-determined by assumption. Furthermore, the increased aggregate incomes could well exacerbate the net excess demand for the privately produced good in the subsequent period, resulting in inflationary effects, a perhaps unavoidable cost of a lower rate of unemployment.

(3.7) Repressed Inflation

Private sector output and employment are now both supply determined so that

$$l_{y,r} = \underline{L} - l_g \qquad (24)$$

$$Y_r = \bar{y}(l_{y,r}) = f(\underline{L} - l_g) \qquad (25)$$

where the upper bar y indicates labour constrained output

The money market clearing equation is represented by:

$$M_t = \bar{m}[K_r^d(Y_r, q; \underline{X}), q; p] \qquad (26)$$

where,

$$K_r^d = \pi + wL - p_y\underline{X} - T = k[Y_r, q; \underline{X}]$$

= disposable income net of spending on $\underline{X}$, the rationed amount of Y.

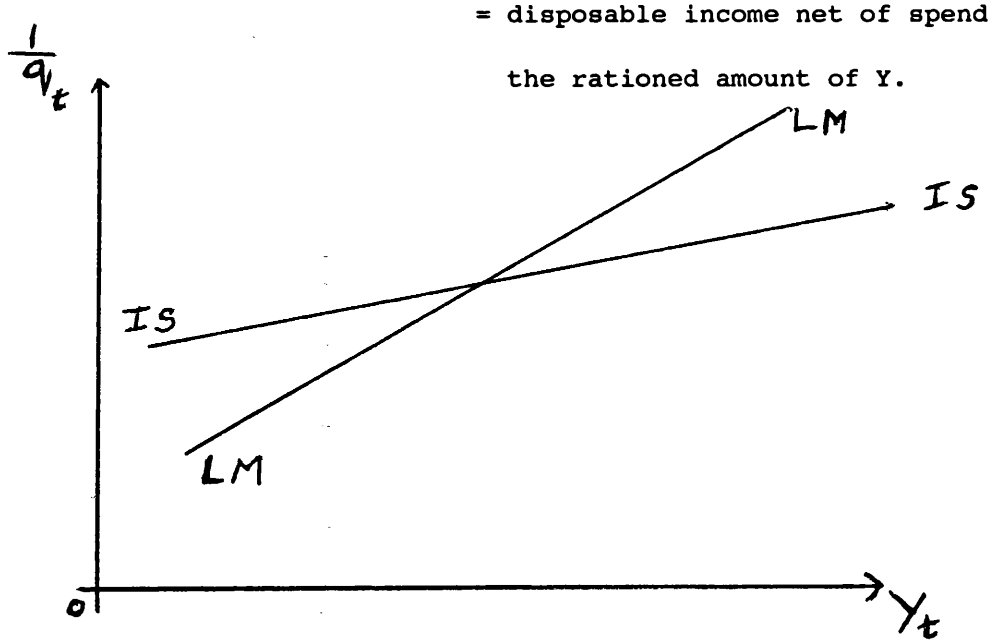

Fig.3

Given the excess demand for labour, there is full employment, and private firm production is at a higher level of output than in the Classical Unemployment regime. This is because in the latter case, real wage configuration is such that $(w/p_y)_C > (w/p_y)_R$ i.e., real wages are higher in the Classical regime, accounting for the net excess supply of labour, than in the Repressed Inflation case, characterised by an excess demand for labour (and goods). Assuming constant returns to scale in public sector production, equation (25) is totally differentiated to obtain:

$$dY_r = f'(\underline{L}-G)[-G_k K_y dY_r - (g_k K_q + g_q)dq]$$

or,

$$dY_r/dq = - f'G_q/[1+f'G_k K_y] < 0 \qquad (27)$$

implying an upward sloping IS curve. The intuition for this follows from the assumption that the labour supply available to the private sector is net of the labour demand in the public sector which increases with an increase in the demand G. By assumption, since the consumption of the government supplied good increases with an increase in q: $g_q>0$, it follows that output and labour demand, l_g, in the public sector exhibits a corresponding rise. This, in turn, results in a reduced availability of labour for the private firm since $l_y=\underline{L}-l_g$, and hence, a smaller volume of private sector output, Y_r.

explain more.

The case of a stable RI equilibrium is depicted in Fig.3. It is readily verified that the LM curve has a positive slope as shown.

(3.8) Regime Classification

We now classify the regimes in terms of the real wage, $\omega_t \equiv w_t/p_{y,t}$, and wealth, $A_{t-1} \equiv M_{t-1} + B_{t-1}$, illustrating for the production and utility functions assumed here. The choice of the two variables, real wage and wealth, to depict the regime configuration is guided by two factors:

a) The level of wealth in the economy is determined by the operations of the government to finance its budget since both bond issue and money creation is determined solely by the government. Therefore, the level of wealth A is a public policy instrument which can be used to influence aggregate income and employment.

b) Real wages seems the natural choice since it enables comparison with the policy results of earlier authors like Malinvaud, and others.

On the C-R boundary, and continuing to assume CRS in the production of G, it must be true that:

$$f'[\underline{L}-G_t] = (w_t/p_{y,t})_o \qquad (28)$$

and it follows that the C-R boundary is invariant with respect to the level of wealth A_{t-1} as shown in Fig.4.

On the R-K boundary, (assuming $\theta = 1$ i.e. constant returns to scale in the production of the publicly supplied good G) the goods and money market equations are:

$$f(\underline{L}-G_t) = x_t[K_{r,t},q_t,p_t] + A_{t-1} + z_t \qquad (29)$$

and,

$$M_t = m[K_{r,t}, q_t, p_t] \qquad (30)$$

Private firm output is at its maximum, and equal to aggregate demand on the K-R. Also note that A_{t-1} does not enter (30) since only the young demand money to provide for their non-working second period, and it is easily established that K-R has a negative slope with intercept given by the wealth level of the old, A_{t-1}. In this result, I have assumed that an increase in real wage, $\omega_t \equiv w_t/p_{y,t}$, leads to an increase in aggregate income so that K_ω is positive. However, this need not necessarily be the case since, as mentioned earlier, while an increase in real wage increases labour income in the public sector, it brings about simultaneously a reduction in employment levels and income in the private firm. The rest of the analysis maintains this assumption of $K_\omega > 0$ purely for ease of exposition. Totally differentiating (29), we have:

$$d\omega_t/dA_{t-1} = -\ 1/[f'G_k K_\omega + x_k K_\omega] \qquad (31)$$

Finally, with respect to the K-C boundary, the two market equilibrium conditions are:

$$f[f'^{-1}(w_t/p_{y,t}) = x_t[K_{k,t}, q_t, p_t] + A_{t-1} + Z_t \qquad (32)$$

and,

$$M_t = m[K_{k,t}, q_t, p_t] \qquad (33)$$

where $Y = h(w/p_y)$, $h'<0$, is substituted for Y in aggregate income $K_{k,t}$ since the economy is now on the border of the Classical and Keynesian regimes. Once again note that A_{t-1} does not appear in the money market equation for the reasons mentioned earlier. Totally differentiating (32), we obtain:

$$[h'(\omega_t) - x_k K_w] d\omega_t = dA_{t-1} \qquad (34)$$

so that the K-C boundary is downward sloping.

Putting together the three different boundaries discussed above, we have the regime configuration of Fig.4 below.

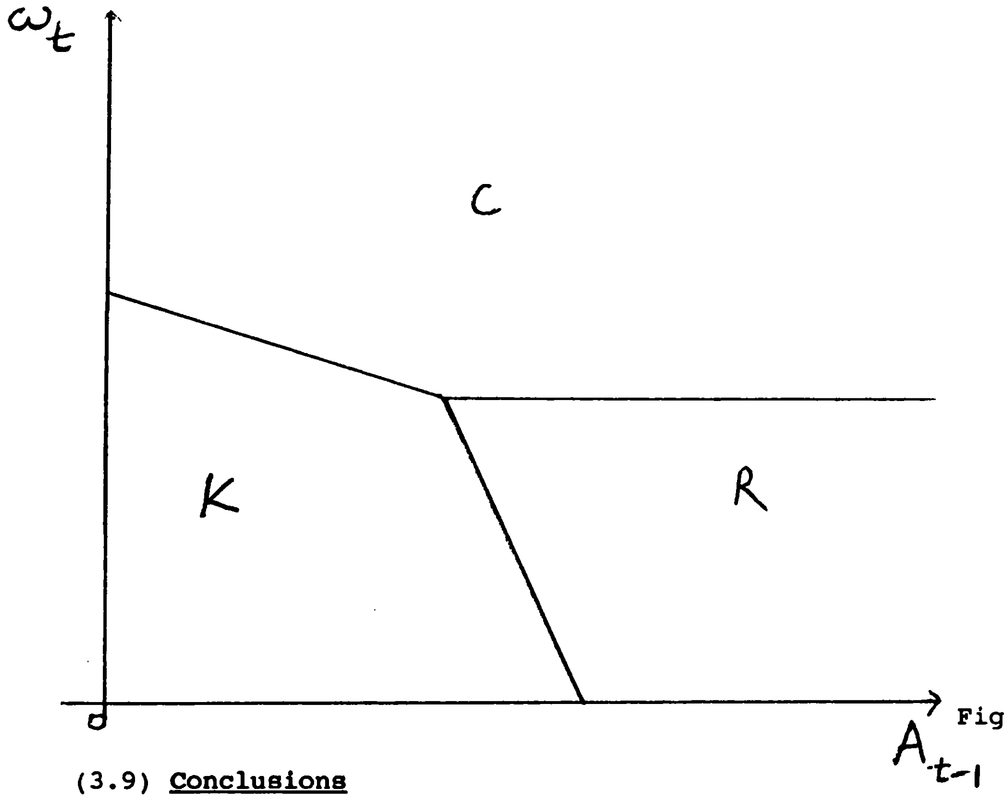

Fig. 4

(3.9) <u>Conclusions</u>

By enlarging its activities to include the production of goods for private consumption, the government has a new and effective tool in its public sector pricing policy. The traditional measures for Keynesian unemployment are increased government expenditures and a reduction of taxes, which have been discussed here, and found to conform, by and large, to the standard text-book analysis. But the novel element, worth noting in this treatment of government policy measures, is the choice of public sector prices. The nature of the underlying demand relationships plays a key role in policy decisions of what constitutes an appropriate public sector price: if the goods relate as in case (a) above where the privately and publicly supplied goods, Y and G respectively, are net substitutes while money and G are net complements, then p_g is to be raised. On the other hand, in a situation described by (b), where Y and G are net complements while money and G are net substitutes, then the policy prescription is to lower p_g, the public sector price.

Even in this relatively simplistic diagram, Fig.4, some of the results obtained are rather similar in essence to the results of Malinvaud (1977). In terms of policy recommendations, the solution in the Keynesian regime is, unequivocally, increased aggregate demand via increases in A or an appropriately chosen public sector pricing policy, whilst for a wage-price configuration resulting in Classical Unemployment the key lies in wage regulation. A larger volume of employment follows a lower wage (and vice-versa) as depicted by a move towards W, the Walrasian equilibrium. On the other hand, the Keynesian market equilibrium equations indicate that higher levels of wealth increase aggregate demand and hence, generate larger total labour employment and higher aggregate incomes. The absence of a labour supply function in this version of the model would appear to curtail the role of wage manipulataion as a policy measure to buoy aggregate demand in the K.U. situation. But relaxing our initial labour supply assumptions to allow for labour/leisure decisions as household choice variables does not alter the message so far. Only, we would have built up an explicit argument, as in Malinvaud (1977) and Benassy (1984), for an increase in wages.

In a situation of Classical Unemployment, where the traditional budgetary measures bearing on government spending Z or taxes T fail to influence private sector output Y and employment, the presence of public sector production and a suitable pricing policy may well prove effective in curtailing, to some extent, the excess supply of labour. Through induced changes in the level of employment for public sector production via its product price changes, the government enterprise can potentially reduce the overall rate of unemployment in the economy.

To sum up, the upshot of this analysis is that introduction of public sector production provides an additional fiscal policy tool to administer aggregate demand and employment, which is quite besides the existing instruments of government spending and taxation. In particular with respect to Classical unemployment, it is commonplace that public spending and taxes are ineffective but what we have now demonstrated is a potentially viable fiscal policy measure in public sector pricing. This aspect becomes particularly significant in the context of a large number of developing countries where state owned enterprises account for a substantial share of the gross domestic product.

Chapter 4

"Does Fiscal Policy Matter? " - the role of public sector prices

(4.1) Background

So far we may appear to have side-stepped the issue of how the government chooses to finance its deficit. But as mentioned earlier, the government has three available options which it can use singly or in some combination: new bond issue, taxes and printing money. Each has its own different consequences for the economy, but the last two as alternative financial mechanisms have been the subject of extended debate and controversy. To pose the problem in its historical perspective, a quick overview of some of the literature is presented. In this chapter, I try to gain some insights into the effects of extending government activities to include production for the market, the choice of alternative public sector prices, and therefore, the consequences in terms of modifying the earlier results.

(4.2) Brief review of literature

Monetarists (Friedman, 1956, 1959, and others) assert the inefficacy of bond financed fiscal policy claiming the preponderance of "negative net wealth effects" negating or even reversing the initial positive effects of increased aggregate demand. As against this, Blinder and Solow (abbreviated as BS) suggest that it is an "empirical question whether the subsequent wealth effects of bond-financed deficits, while less expansionary than money-financed deficits in the short run (Friedman's 'first round), are actually more expansionary in the long run." A review of the work of Blinder and

Solow (1973),(1974) and Tobin and Buiter (1976), both of which were undertaken in the framework of fix price IS-LM based models, seems in place. But in line with most of the literature, these two studies confine public expenditure to activities which are termed as "waste" in that they do not enter the individual utility function, and also there is no government production for private consumption.

Suppose government spending is permanently increased from an initial position of budget balance. Let us refer to this change in the time path of government spending as the Blinder-Solow rule. Tobin and Buiter define a new variable "government outlay", $\underline{Z}_t$, as:

$$\underline{Z}_t = p_{y,t} Z_t + (1-q_t) B_{t-1} \qquad (1)$$

i.e. the sum of spending and interest payments. Their rule, hereafter referred to as the TB rule, is to permanently increase $\underline{Z}_t$ rather than Z_t, implying that Z_t is first raised and then lowered since interest payments increase over time under bond finance, as one would expect. If a Steady State is to be achieved starting from an arbitrary initial point, when prices and wages are taken as fixed, a standard modification to the assumption of lump-sum taxes is to set $T=\tau Y$, where τ is the constant average and marginal rate of tax, as a necessary device to automatically close the government deficit since taxes now change with income changes. For the BS rule, convergence is rather less likely under pure bond financing than under pure money financing of the deficit; but given convergence, both long-run spending multipliers are greater than one, with the bond-financing multiplier exceeding the money-financing one. The money financed multiplier is $1/T'(Y)$ where $T(Y)$ is the tax function while the bond-financed multiplier is $[1+(1+T')\partial B/\partial Z]/T'(Y)$. This last result, the authors claim, lays to rest any monetarist doubts about the efficacy of fiscal policy in situations where the system converges and is stable. The TB rule, on the other hand, generates identical long-run multipliers with a value of $1/\tau$.

In the context of my model with the added dimension of public sector production and explicit treatment of different regimes, I now investigate the effects, regime by regime, of the alternative methods of bond- and money-financing of the government deficit. From the government budget constraint, equation (10), chapter 2, it is clear that a non-zero budget balance, taking account of the balance in public sector production, implies changing stocks of bonds and/or money. The pertinent questions to pose are: what is the likely evolution over time of the economy and particularly, what additional dimensions does public sector production feature in this evolution, is convergence more likely under bond-financing than under money-financing, and if the system does converge, is the solution stable, and what are the likely values of the corresponding policy multipliers? Given the focus of this research, the area of principal interest is to examine how the inclusion of public sector production alters and extends the results of the previous authors and how it modifies the conclusions drawn with repect to p_g, the public sector price.

In order to answer these, a clear elucidation of the price, wage and asset dynamics is a pre-requisite. The formation of prices/wages has to be endogenised under alternative specifications, and the effects of the simultaneous changes in the three variables studied. But for the time being, I choose to abstract from this degree of complexity in order to keep the analysis sufficiently simple to focus attention solely on the asset dynamics of a non-zero government budget - this is perceived as a better clue to isolating the effects of a changing bond/money stock on the evolution of the economy. Accordingly, some simplifying assumptions on price and wage behaviour are made in the tradition of the earlier studies by Barro and Grossman, Tobin and Buiter and others, both to single out the effects of the time-path of assets, and furthermore to render our results on a comparable basis with the results of the afore-mentioned authors. Hence for the purposes of the current research, fixed prices are assumed but it is planned, as a topic for future work, to allow for more realism by incorporating endogenous price formation.

It is useful that we start by considering the static equilibrium equations and the government budget constraint under both the TB and BS definitions. Our results indicate that:

a) with bond-financing under the TB rule, the BS and the TB multipliers are simply special cases of our value of the multiplier when the public sector prices its output at marginal cost.

b) the convergence and stability results of the earlier authors are modified; with public sector production as an additional feature, it is not certain even under money financing that the above results hold unambiguously since the choice of prices by the public sector now plays a crucial role.

c) it is no longer clear as to whether "the long-run multiplier for bond-financed deficit spending exceeds that for money-financed deficit spending" as in Blinder-Solow; while money-financed government spending is unequivocally expansionary, the effects of bond-financed public debt on private output is ambiguous.

d) the multiplier effects are smaller since there are now other competing demands.

(4.3))K.U.Regime

Pure Bond Financing

(4.3.1) Tobin-Buiter definition of the Government budget

Following the convention, we set taxes proportional to aggregate income K in order to close the system, so that $T = \tau K$. As a social security measure for the old, interest incomes on bonds, B, are assumed to be tax exempt.

The static equilibrium equations are defined by (2), (3) and (4), respectively the private national income, the "public" national income and money market equilibrium equations. The budget balance requirement under the TB rule with $\underline{Z}$ defined as in equation (1), in a regime of pure bond financing, is (5) below:

$$Y_t = x[K_t, q_t; p_t] + M_{t-1} + B_{t-1} + Z_t \qquad (2)$$

where $K_t = (1-\tau)[p_{y,t}Y_t + w_t l_{g,t}]$ is aggregate disposable income (see equation (13), chapter 2)

and p_t = vector of prices $(p_{y,t}, w_t, p_{g,t})$ with $(p_{y,t}, w_t)$ fixed and p_g a policy variable.

$$l_{g,t} = G_t = g[K_t, q_t; p_t] \qquad (3)$$

$$M_t = m[K_t, q_t; p_t] \qquad (4)$$

$$\Delta B_{t-1} = [\underline{Z}_t - (p_{g,t} - w_t)G_t - \tau K_t]/q_t \qquad (5)$$

In equation (3), we have considered the particular case of θ=1 i.e. constant returns to scale in public sector production (see chapter 2). Purely for expositional ease, this simplification is maintained through the rest of this chapter. The symbol Δ denotes the forward difference of a variable, for instance $\Delta Y_{t-1} = Y_t - Y_{t-1}$. Note that (5) is a first-order difference equation describing the time path of bond stocks, driving the economy from one instantaneous equilibrium to another.

In order to examine if the BS or TB results continue to hold, or are modified in the context of the current model, it is necessary to determine:

a) the equivalent multipliers for this model, and

b) also the stability properties since long-run multipliers are of interest only if the system under consideration is itself not unstable.

With this aim in mind, we derive $\partial Y_t/\partial B_{t-1}$ and $\partial q_t/\partial B_{t-1}$.

For a given level of government expenditure, $\underline{Z}$, an increase in bonds, unaccompanied by an increase in money supply, leads to an increase in the interest rate as verified below:

$$\frac{\partial q_t}{\partial B_{t-1}} = - \frac{q_t[m_k K_y(1-\tau)]}{D} \qquad (6)$$

where

$$D = \left[p_y[1-x_K K_y(1-\tau)] + \frac{m_K K_y(1-\tau)[p_y X_q + B_{t-1}]}{M_q}\right] \qquad (6)'$$

Drawing on our earlier assumptions on demand responses (see equations (22)-(33), chapter 2), the terms X_q, $M_q > 0$ are the total effect of interest rate changes on commodity and money demands, defined respectively as:

$$X_q = (1-\tau)x_k K_q + x_q \quad \text{and} \quad M_q = (1-\tau)m_k K_q + m_q \qquad (6)''$$
$$G_q = (1-\tau)g_k K_q + g_g$$

Therefore, the determinant $D > 0$ and hence, $\partial q_t/\partial B_{t-1} < 0$ or $\partial(1/q_t)/\partial B_{t-1} > 0$, confirming the rise in interest rate with an increasing bond stock.

As mentioned earlier, the effects of an increase in bond stock on private sector output has been the subject of debate, and the arguments are principally that:

a) The higher interest rates result in a cut in interest sensitive private expenditures, and hence a lowering of the traditional multiplier of 1/(1-marginal propensity to spend). There is little, if any, controversy on this aspect.

b) What is at issue, however, is the strength of the "wealth effects". Friedman and other monetarists held the view that these wealth effects are sufficiently large to offset or even reverse the initial expansionary effects of a government spending program through an increased demand for real money balances jacking up the initial first round rise in interest rates even further. As is now quite well recognised and rather widely accepted, the extent of these negative net wealth effects is a matter for empirical resolution (Blinder and Solow, Tobin and Buiter, Turnovsky and others).

In the context of our model, we derive:

$$\frac{\partial Y_t}{\partial B_{t-1}} = \frac{q_t M_q}{D} > 0 \qquad (7)$$

The second term in D as in (6)' above contains $(p_y X_q + B_{t-1})/M_q$. The increase in the interest rate leads to a decline in both the private product and money demands, since X_q and M_q are positive. But as against this, the consumption demand of the old for the privately produced good, Y, increases to the full extent of the increase in bond issue, B_{t-1}, since there are no bequests and ${g^{t-1}}_t$ = 0 by assumption. This expansion in the consumption demand of the elderly compensates for the reduced consumption of the young, and the net effect is increased total demand for the private firm's output which is demand determined in the Keynesian Unemployment regime.

A further reason for the unambiguous positive multiplier of bond financing is that wealth effects in our model are not contractionary. Interest incomes on bonds bought by the young accrue to them when old with a one-period lag. Since interest incomes of the old are tax exempt and there are no bequests, they fail to exert the possible negative effects, observed in the BS and TB models, simply because they do not fall into any interest-sensitive expenditure category.

We are now ready to investigate the stability properties of our model. It turns out that this model is indeed stable with the TB rule under bond financing. Totally differentiating (5) and making use of (6) and (7), the stability equation evaluated at the steady state is given by:

$$\frac{\partial \Delta B_{t-1}}{\partial B_{t-1}} = - \frac{\partial Y_t}{\partial B_{t-1}} \cdot C \qquad (8)$$

where,

$$C = \{(1-\tau)(p_g-w))[g_yM_q-G_qm_y] + \tau K_y[1-K_qm_k(1-\tau)/M_q]\} \qquad (8)'$$

Stability requires, therefore, that {.} be positive. For a sufficiently large rate of tax, τ, the second term on the RHS of (8)' becomes positive. Also, by analogy of equation (31), chapter 2, we have $g_yM_q-G_qm_y = m_yG_r^c-G_ym_r^c$ which must be positive except when $G_r^c<0$ i.e., there is too high a degree of complementarity between money and the public sector product and since the own-price effect of money demand is negative, i.e. $m_r^c<0$. Therefore, the bracketed term { . } is positive, provided public sector price is at least equated to marginal cost and $(p_g-w) \geq 0$.

An example

An illustration using the Cobb-Douglas utility function, and constant returns to scale production function in the public sector production, introduced earlier in chapter 3, proves to be informative. The tax function is $T=\tau K$, and the total tax base is:

$$K = \frac{\underline{Z} + qB_{t-1} + M_{t-1}}{\{1 - (1-\tau)[\alpha+(w/p_g)\beta]\}} \qquad (A)'$$

Using the TB definition of government expenditure, we have:

$$\underline{Z}_t = p_1z_t + (1-q_t)B_{t-1}$$

so that,

$$\partial Z/\partial B_{t-1} = - (1-q)/p_1 \qquad (B)'$$

and the government budget constraint gives,

$$q_t \Delta B_{t-1} = \underline{Z} - \tau K + (w-p_g)(1-\tau)\beta K \qquad \text{(C)}'$$

Writing the right hand side as $\Psi(qB_{t-1})$, and differentiating we have

$$\Psi^1 = \frac{[(w-p_g)(1-\tau)\beta - \tau]}{[1 - (1-\tau)(\alpha + \beta w/p_g)]} \qquad \text{(D)}'$$

For $(p_g-w) \geq 0$, the numerator of (D) is unambiguously negative, while the denominator is positive for sufficiently small values of $(1-\tau)$, i.e. large τ or tax rate.

(4.3.2) Some Conclusions about the Keynesian Unemployment Regime

But if {.} above is positive, then the system is stable. This last requirement has important economic implications for policy making:

a) The sufficient condition for {.} to be positive is that $(p_g-w) \geq 0$, and a large rate of tax.

b) Suppose the required pricing is adopted, then the long-run multiplier becomes relevant and is derived as:

$$\frac{\partial Y}{\partial Z} = \frac{1}{[(1-\tau)\ (p_g-w)G_k K_y \ + \ \tau K_y\ ]} \qquad (9)$$

From the value of our multiplier, we find that the Tobin-Buiter or the Blinder-Solow multiplier, ignoring interest payments as a budgetary deficit, is simply a special case of (9) when the public sector prices its output at marginal cost. Otherwise, the multiplier effect on Y is smaller, the reason being that the initial increase in income is now distributed over an increased demand for the privately produced good as well as the other commodities.

c) Furthermore, equation (9) provides an interesting result - the multiplier is larger, the larger w/p_g. Alternatively, since the denominator comprises the sum of two terms: we can think of the first term of [.] as a weighted "indirect tax" or subsidy element $(1-\tau)[.]$ and the second term as the direct tax on income, so that {.} is a measure of the net composite effect of the tax structure: a weighted tax subsidy $(1-\tau)(p_g-w)(.)$ and a direct tax element τ (.). Only if the net effect of these two measures is positive, would an increase in government expenditures be unambiguously expansionary.

(4.4) Blinder-Solow definition of the government budget

The endogenous variables are (Y,q,B,l_g) as before but the budget constraint is re-defined as:

$$\Delta B_{t-1} = [\ Z_t + (1-q_t)B_{t-1}-(p_{g,t}-w_t)G_t-\tau K_t]/q_t \qquad (10)$$

It is not obvious in this case that the long-run multiplier is expansionary, nor is it clear that the system is stable under this rule. Totally differentiating (3), (4) and (10), the long-run multiplier is derived as:

$$\frac{\partial Y}{\partial Z} = \frac{q}{\{\ D'\ \}} \qquad (11)$$

where,

$$D' = \{\tau-M_y(1-\tau)[(B+\tau K_q)/M_q]-(1-q)/k+(p_g-w)(1-\tau)[g_yM_q-m_yG_q]/M_q\} \qquad (11)'$$

and k is defined as the short-run multiplier derived in chapter 3.

The sign of D′ is ambiguous so that we cannot unequivocally say that government spending is expansionary under the BS definition of the bond financing of the government deficit.

Furthermore, the stability condition is also difficult to ascertain since:

$$\partial \Delta B_{t-1}/\partial B_{t-1} = -\{.\} \tag{12}$$

in the neighbourhood of equilibrium, where {.} is nothing but the expression D′ defined in (11)′ above . Therefore, this leads to the conclusion that, with pure bond financing, the question of stability is an open one under the BS rule.

Example

As an illustration, we return to the example of the Cobb-Douglas case and CRS production function in the public sector enterprise. With Z constant rather than $\underline{Z}$, we find that Ψ^1 is replaced by:

$$\frac{-\tau}{[(1-(1-\tau)(\alpha+\beta w/p_g)]} + (1-q) + (w-p_g)(1-\tau)\beta \ \frac{\partial Y}{\partial B_{t-1}} - \frac{\partial q}{\partial B_{t-1}}$$

It follows that for $p_g = w$ and τ small that Ψ^1 is positive, and hence that it is unstable.

In the next two sections, we investigate money financing of the government budget constraint.

(4.5) Pure Money Financing

The economy is described by the following set of equations, the first three of which are similar to (2) - (4) above. But the government budget constraint is now different:

$$Y_t = x(K_t, q_t) + M_{t-1} + B_{t-1} + Z_t \qquad (13)$$

$$l_{g,t} = G(K_t, q_t) \qquad (14)$$

$$M_t = m(K_t, q_t) \qquad (15)$$

$$\Delta M_{t-1} = Z_t + B_{t-1} + (p_{g,t} - w_t)G_t - \tau K_t \qquad (16)$$

We can assume a unique solution, and solve for the Steady State values of Y_t, q_t, l_g at the exogenous level of $Z_t = \underline{Z}$. As in the previous section, it is important that we investigate the stability properties of the system. Substituting for $\partial Y_t / \partial M_{t-1}$ and $\partial q_t / \partial M_{t-1}$, stability of the system at the Steady State requires that:

$$\frac{\partial \Delta M_{t-1}}{\partial M_{t-1}} = - \frac{\partial Y_t}{\partial M_{t-1}} \cdot C_1 \qquad (17)$$

where,

$$C_1 = \{[(p_g - w)(1-\tau)g_y + \tau K_y](M_q + X_q + \tau K_q) \qquad (17)'$$
$$+ [(p_g - w)(1-\tau)G_q + \tau K_q][\tau + (1-\tau(1-x_y-m_y)]\}$$

Since {.} is clearly positive under our sign assumptions, and provided public sector prices at least cover marginal cost, and the very likely case that $(1-x_y-m_y) > 0$, we then merely require that $\partial Y_t / \partial M_{t-1} > 0$. However, the possibility that $(1-x_y-m_y)$ is negative cannot be ruled out all together, and arises because of our use of single period bonds rather than perpetuities with a fixed coupon rate. Note also that interest payments may vary even with a fixed bond stock. Assuming the more likely case of $(1-x_y-m_y) > 0$, it is straightforward to verify that the effects on Y of a money supply increase is expansionary, so that we are assured of a stable equilibrium rule with pure money financing of the government deficit. Once again, we note the crucial role public sector prices play in meeting the stability conditions.

Finally, with pure money financing it is important to note that, at the Steady State, there is no difference between the TB and BS definitions of the government budget constraint since $\underline{Z}_t=Z_t$ when $\Delta M_{t-1}=0$. Therefore, it is not necessary to carry out a separate analysis for each of the two definitions.

(4.6) Summary of results for the Keynesian regime

To sum up, the results here for the Keynesian regime are, to some extent, similar to the results of Blinder and Solow (1973) and that of Tobin and Buiter (1976). For a change in government spending, there is a convergent expansion of the economy with the TB rule for both methods of financing, but conditional on the public sector following some bounds on its product pricing. With the Blinder-Solow definition of the government budget, it is rather less likely that convergence is achieved with bond-financing, and this is irrespective of the choices of the public sector enterprise. In cases where the system does converge, the multiplier is smaller than in the earlier studies.

(4.7) Repressed Inflation Regime

The equilibrium equations are:

$$Y_t = f(l_{y,t}) = f(\underline{L}-l_{g,t}) \qquad (18)$$

$$l_{g,t} = G_t = \bar{g}[K_{r,t}, q_t; p_t] \qquad (19)$$

$$M_t = \bar{m}[K_{r,t}, q_t; p_t] \qquad (20)$$

and the government budget constraint with bond financing is:

$$\Delta B_{t-1} = [Z_t + (1-q_t)B_{t-1} - (p_{g,t}-w_t)G_t - \tau K_t]/q_t \qquad (21)$$

By definition, this is a regime characterised by excess demand in both the labour and private product markets. The consumption of the privately supplied good Y by the young is rationed to some level $\underline{X}$, say, and is determined by the income-expenditure identity:

$$\underline{X}_t = Y_t - B_{t-1} - M_{t-1} - Z_t \tag{22}$$

Since there is no money demand rationing, and also because the supply of the public enterprise is demand determined, both $\bar{m}_x \equiv \partial\bar{m}/\partial(p_y\underline{x})$ and $\bar{g}_x \equiv \partial\bar{g}/\partial(p_y\underline{x})$ are negative. These cross effects considerably complicate the final impact of a bond or money supply expansion on the interest rate and private sector output. An increase in B_{t-1} enables increased consumption by the old at time t, but it also implies a tightening of the consumption rations for the young or a smaller $\underline{X}$. Given that $\bar{g}_x < 0$, this last effect results in an increased demand for the government supplied good and hence, a larger labour demand in the public sector, leading to lower labour availability of labour for the private firm reducing its production level of Y.

However, the presence of money demand, with its added dimension, confuses the story so far since the reduction of the consumption ration $\underline{X}$ affects the demand for money balances as well. Taking into account the combined increase in demand for both money balances and G, it is not possible to detemine whether bond financed public spending is expansionary or otherwise without further structure to the model since the expression
for $\partial Y_t/\partial B_{t-1}$ is:

$$\frac{\partial Y_t}{\partial B_{t-1}} = \frac{f'[\,\bar{g}_x\bar{m}_q - \bar{m}_x\bar{g}_q]}{D''} \tag{23}$$

where,

$$D'' = M_q\ [1 + f'(\bar{g}_y K_y(1-\tau)+\bar{g}_x)] - f'G_q[\bar{m}_k K_y(1-\tau) + \bar{m}_x] \qquad (23)'$$

at the quantity constrained levels of M and G, and the sign of D" is ambiguous.

For similar reasons the effects of money financing is also uncertain. Therefore, we cannot say anything definitive in this regime about the stability properties of equilibrium, be it bond or money financed public spending.

We move on now to the Classical unemployment regime, and examine the effects, if any, of government policy decisions.

(4.8)Classical Unemployment

Private sector output and employment are now both solely determined by the exogenous real wage. Recall that K_c is the sum of incomes generated in the private and public firms so that: $K_c = f(w/p_y) + wl_g$. Considering the particular case of constant returns to scale in public sector production, $l_g = G = \bar{g}[K_c, q; \underline{x}]$, the demand for G and hence, l_g, can be potentially influenced by public policy through bringing about the desired changes in prices and interest rates. From the money market market equilibrium condition and the government budget constraint with money financing, we derive:

$$\frac{\partial q_t}{\partial M_{t-1}} = \frac{(1+\bar{m}_x)}{[(p_g - w)G_q + M_q]} \qquad (24)$$

where M_q and G_q are as defined in (6)'' but at the constrained demands. For similar reasons as in the Repressed Inflation regime $\partial\bar{m}/\partial(p_y\underline{x}) \equiv \bar{m} < 0$ since the Classical Unemployment regime is also characterised by demand rationing in the private product market, and the sign of (24) is ambiguous.

Nevertheless, it is interesting to investigate the possible channel by which government fiscal policy could prove effective, assuming that $(1+\bar{m}_x)>0$. Provided (p_g-w) is positive, we have the very plausible economic situation of a monetary expansion leading to a fall in the interest rate. This in turn, induces a bigger demand for the public sector good (since $g_q>0$) and, hence, a larger volume of employment in the public enterprise. But now since the real wage is exogenously given in this regime, by assumption, the production level of Y and hence, employment in the private firm remains unaffected throughout this monetary exercise of the government. Therefore, the final impact of a money supply increase is to generate higher aggregate employment in the economy, the sufficient condition for this being public sector prices that at least cover marginal cost: $(p_g-w)\geq 0$.

(4.9) Conclusion

The present analysis indicates that only some of the conclusions of the earlier authors, Blinder and Solow or Tobin and Buiter, can be extended to the case where government activities include public sector production, and to its financing of deficits that cover deficits incurred on its production account as well. Furthermore, the stability as well as multiplier effects depend on the choice of public sector prices. The financing mechanism considered is bond- or money-financing. Our results do not lend support to the Blinder-Solow assertion that the long-run bond financed multiplier exceeds the money financed multiplier.

Chapter 5

Public Sector Prices

(5.1) Introduction

The ruling price in the private sector is self selecting, as it were, in that it meets the objective of maximising profit/revenue/sales of the private firm. But in the context of a public enterprise, the "optimality" of its price-setting is governed by somewhat more complex criteria. Much has been said and written about this problem since different market structures and pre-specified societal objectives quite naturally suggest different rules for optimal public sector prices. These include marginal cost pricing and its various second-best extensions to considerations of equity, financial constraints and macroeconomic policy. Atkinson and Stiglitz (1980) and D.Bös (1986) have an excellent discussion of these various models.

We can broadly classify the literature along the following lines. One approach assumes that the economy is competitive and markets clear, while the other recognises the possibility of an environment where prices do not reflect economic scarcity. In both situations, it is possible that the public sector operates under a budget constraint which may or may not be binding. This requirement on public sector production points to a departure from economic efficiency. In such a context, the aim of optimal pricing rules is to cope with this source of economic inefficiency to achieve Pareto Optimality in resource utilisation in an economy that is otherwise competitive. On the other hand, in the case where the public firm, faced with a budget constraint, operates in an economy with market imperfections and quantity rationing, the rules of pricing are rather different. The traditional rules of marginal cost pricing have now to be adapted to reflect both market imperfections as well as the economic inefficiency arising from a binding budget limit.

Yet another dimension that optimal pricing has attempted is to reflect societal judgement on income distribution in its pricing schedule. This may again lead to deviations from marginal cost pricing, with the distributional characteristics (Feldstein, 1972) entering the determination of prices.

To sum up, one might say that the model underlying the traditional marginal cost pricing rule focuses on efficiency of resource use in a competitive environment while the other models in the literature are extensions of this, and which consider other additional criteria as well, either singly or simultaneously, namely:

a) distributional effects

b) deficit financing implications

c) macroeconomic effects

Also, the economic environment considered was broadened to allow for non-competitive market settings. In the next section (5.2), a brief overview of some of the more important early works in this area is presented. This is followed in section (5.3) by a characterisation at the steady state of the rules of optimal public sector prices that takes account of macroeconomic effects, as in Drèze (1982, '84), in an economy that features excess supply in both the private product and labour markets.

(5.2) Brief Review of Select Earlier Literature

The neoclassical analysis of Boiteux (1956) considers a second-best model where the public sector is constrained to produce under a budget constraint in a competitive economy. The goal of the government or public authority in this setting is to achieve a Pareto Optimal allocation in which households maximise utility, private firms their profits and the public undertaking its objective function, each subject to its own respective constraints. The instruments at the government's disposal are the level of public production, prices and household incomes. The private and public firms produce the same product but with different technologies so that the choice of prices is applicable to both types of firms. Denoting V as the indirect utility function of the individual consumer, and Π, Φ as the private and public sector profits respectively, the principal proposition for optimal public sector prices based on the first-order necessary conditions is given below:

(5.2.1) Boiteux Proposition on optimal public sector prices

At an interior solution, there exists a multiplier ρ such that:

$$\Sigma_h \lambda^h (\partial v^h/\partial r^h) \ [x^h - \Sigma_j \theta^{hj} (d\Pi^j/dp) - \beta^h d\Phi/dp] = \rho (d\Phi/dp) \qquad (1)$$

where,

x = vector of l consumption goods with prices p

p = (p1,...,pl)

w = wage rate assumed fixed and treated as numeraire

λ^h = welfare weights of household h

θ^{hj} = share of household h in profits of firm j, Π^j

β^h = share of household h in public sector profits Φ

r^h = income of household h, defined as a share θ^h of private profits plus a share β^h of public sector profits.

ρ = multiplier, interpreted as the shadow price of government net revenue

If the budget constraint on the public sector $\Phi \geq b$ is not binding so that $\Phi > b$, the multiplier $\rho = 0$. Then, the optimality condition (1) implies public sector pricing at marginal cost, if the marginal social utility of income $\lambda^h(\partial v^h/\partial r^h)$ is assumed to be uniform over all households. The derivation of this result follows from the fact that (1) is now rewritten as:

$$\Sigma_h x^h - \Sigma_j y^j - (d\Phi/dp) = z - (d\Phi/dp) = 0 \qquad (1.1)$$

since

$\partial\Pi/\partial p = y$ by Hotelling's lemma,

$\Sigma_h x^h - \Sigma_j y^j - z = 0$ from the product market clearing equation,

and,

$d\Phi/dp = z$ when $p_g = w\partial l_z/\partial z$

where $d\Phi/dp$ is equal to:

$$\frac{d\Phi}{dp} = \frac{z + (p - w\partial l^z/\partial z)[\Sigma_h(\partial x^h/\partial p + \partial x^h/\partial r^h \Sigma_j \theta^{hj} y^j) - \Sigma_j \partial y^j/\partial p]}{1 - (p - w\partial l^z/\partial z)\Sigma_h B^h \partial x^h/\partial r^h}$$

However, when the welfare weights are not uniform, and distributional considerations are taken account of explicitly, then (1) clearly points to a departure from marginal cost pricing.

Alternatively, consider the case when the revenue constraint on the public sector is binding, then $\rho>0$, and we have a different interpretation of the optimality condition, an interpretation in terms of a cost-benefit rule. The left-hand side of (1) is the cost to the household of an increase in price, taking account of the direct costs as well as the indirect effects on household income via the effect of the price increase on both private and public sector profits. In this situation, condition (1) requires that the marginal costs to the consumer of a price change, as measured by the left-hand side of (1), be proportional to the marginal benefit to the public sector, $d\Phi/dp$, where ρ, the shadow value of government revenue, is the factor of proportionality adjusted to satisfy the constraint.

To sum up, the only situation where marginal cost pricing is justified is when distributional considerations are either ignored or the existing income distribution is considered satisfactory, and additionally, only if public sector revenues carry no premium in that the multiplier ρ attached to the public budget is zero. In alternative circumstances, there is no clear-cut directive for marginal cost pricing, rather the rules for optimal pricing suggest a cost-benefit determination of the macroeconomic effects of any price change.

As mentioned earlier, the Boiteux analysis, significant and seminal as it is, is limited to a competitive market environment and it is the subsequent work of Drèze (1982, 1984) that addresses the problem of markets not clearing and quantity rationing whilst deriving optimal pricing rules for the public sector. Clearly in an economic situtation where prices do not reflect economic scarcities (especially of labour, for instance, as in several countries currently), it becomes particularly important to devise different and practical rules of pricing which take account of this type of market imperfections. The questions to pose are: How should public sector prices be determined? and in particular, is price to be equated to or set below marginal cost to stimulate market demand? and starting from an arbitrary initial point, what are the directions of welfare improving price changes? The next section presents a brief description of the salient features of the way in which Drèze tackles these issues.

(5.2.2) <u>The Drèze Proposition</u>

Drèze (1982, 1984) makes a significant departure from the approach of the earlier works in that he derives pricing rules for the public sector operating under a budget constraint but in an economy where quantity rationing prevails. He deals with the specific instance of Keynesian unemployment. The framework of his model is similar to the Boiteux model in some respects in that there are l privately produced goods supplied by j number of private firms and k publicly supplied goods, both of which are demanded by the H consumers of this economy. However, the similarity ends here in that the publicly and privately supplied goods are disjoint, and hence, the vector of prices for the two sets of goods are different. More importantly, excess supply in the labour and product markets are explicitly modeled so that Drèze features the reservation wages of labour in his results.

Moreover, the novelty of the Drèze work lies in the use of a general equilibrium framework that takes account of the macroeconomic effects of price changes in the public sector on consumers, private firms and the government undertaking, and the corresponding welfare implications. Maintaining the notation used by the Drèze, and denoting Φ, Π and L as net public sector revenue, profits of the private firm and aggregate employment respectively, the first order necessary conditions are:

At an interior solution, there exists a multiplier $\rho \geq$ such that:

$$\rho d\Phi/dp_z=\Sigma_h \lambda^h(\partial v^h/\partial r^h)[z^h-\theta^h(d\Pi/dp_z)-\beta^h(d\Phi/dp_z)-(w-w_r^h)(\partial l^h/\partial L)dL/dp_z] \quad (2)$$

A close look at (2) would indicate that it is very similar to the terms in the optimality condition of Boiteux, equation (1) above, except for the last term on the RHS of (2) – $(w-w_r^h)(\partial l^h/\partial L)dL/dp_z$ which arises here because of market disequilibrium. This expression evaluates the welfare effects of a change in total employment: "If increasing p_z leads to less employment ($dL/dp_z<0$), some households see their labour supply further constrained ($(\partial l^h/\partial L)(dL/dp_z) < 0$), at a loss of welfare (per unit of labour time) equal to the difference between the foregone market wage and the reservation wage w_r^h."(Drèze, 1984). Quite obviously, we would not expect this expression in the Boiteux optimality condition; given his assumptions of a competitive market structure, the market and reservation wages are equal.

The total derivatives $d\Phi/dp_z$, $d\Pi/dp_z$ and dL/dp_z are the "multiplier effects" of changes in public sector prices, p_z, on public sector net revenues Φ, on private profits Π, and aggregate employment L. The multiplier effects here refer to the reaction of household demand to the change in p_z, which in turn entails adjustments in private profits and employment. These latter second-round effects induce a further response in household behaviour, and so forth so that the final impact on L and Π, following the multiplier process through is denoted as: dL/dp_g and $d\Pi/dp_g$. These two multiplier terms are reflected in the net revenue accruing to the public sector, $d\Phi/dpg$, due to the interaction between the public and private supply and aggregate employment.

Therefore, the interpretation of (2) for $\rho>0$, is that it reduces to a cost-benefit analysis of adjustments in the public sector price, where the benefits and costs are evaluated in a more comprehensive manner than in (1) insofar as "all multiplier effects" are taken into account.

Addressing the question of the "reform problem" as discussed in Guesnerie (1977, 1981) for the case where the budget constraint of the public sector is not binding, and $\rho=0$, it is necessary to search for price changes which improve welfare, i.e. price changes dp_z at which $(d\Lambda/dp_g)dp_g>0$ where Λ is the Lagrangian of the Drèze optimisation problem. These "infinitesimal" price changes are in the direction of welfare-improving price changes. And if the budget constraint is binding, then additionally, these price changes dp_z are such that they satisfy: $(d\Phi/dp_z)dp_z = 0$ indicating that the budget constraint is satisfied, and also that $(d\Lambda/dp_z)dp_z>0$.

I do not give the details of the formulae simply because of the tedium of reproducing the algebra here.

(5.3) Optimal Pricing with constraints on the private sector

Against this backdrop of some of the literature briefly reviewed here, I address the same problem of optimal public sector pricing but in the context of the present model. The private sector is assumed to encounter quantity rationing. I adapt the Drèze model, and the principal new elements are: a) a change in the budget constraint, to allow for bond/money financing of the government deficits, and b) to characterise rules for optimal public sector prices in the dynamic context of an over-lapping generations model. As seen earlier in chapter 4, the method of financing the government budget constraint is important both for determining the effects of the different policy instruments as well as for the stability properties of the model. The use of an over-lapping generations model, on the other hand, appears to be the natural framework for examining the dynamics of a changing government budget constraint, and also to analyse the optimal time-path of prices as shown in chapter 6, the next chapter.

The goals of the government and the public firm are assumed to coincide, and the concern here is only with the normative issues of deriving optimal behavioural rules for the public firm. I concentrate on second-best pricing rules for the public sector operating under a budget constraint, and that attempts to take account of the macroeconomic effects of its pricing. The two alternative financing mechanisms of endogenous money and bond creation are studied, the former type of financing in this chapter and in the subsequent chapter, I consider bond financing. The decision not to consider equity and welfare issues in any great detail, as in Drèze or Bös, at this stage is primarily to keep the analysis simple. Moreover, our results do incorporate distributional considerations, albeit in a rather stylised fashion. In our economy at any given time, there is a representative individual of each generation so that equity here would simply be in terms of the welfare representation of the old and the young of that time.

The constraints on product supply by private firms results in their producing at a level at which the marginal product of labour exceeds the real wage: $w/p_y \leq f'(ly)$ [see Chapter 2].

The problem of the government is to maximise an appropriately chosen social welfare function using suitable policy variables. In the particular context of an over-lapping generations model, we may take as the social welfare function the welfare function, S, (Samuelson 1958, 1975) given below. Then we have:

$$S = \Sigma_{t=0,\infty} \delta^t W_t \quad (3)$$

where,

$$W_t = U_1(C_t^t) + \delta' U_2(C_{t+1}^t) \quad (4)$$

W_t is the life-time utility of the generation born in period t, deriving utility U_1 from consumption of C_t when young at time t, and utility U_2 consuming C_{t+1} when old in the subsequent time of t+1. In the many consumer case, the individual rate of time preference δ' may not necessarily be the same as the social rate of discount δ but in the context of our model, the two discount rates are assumed to coincide, and $0<\delta<1$.

Now as seen in chapter 2, C_t^t is the vector of demands of the young of generation t: $C_t^t = (x_t, g_t, m_t, b_t)$ each element of which is a function of the product prices $p_{t,y}$ and $p_{t,g}$, wages w_t, individual after tax income $K_t^d \equiv \Pi_t + wL_t^d - T_t = p_{y,t}Y_t + w_t l_{g,t} - T_t$, interest rate q_t as well as the expected price of Y, $p_{y,t+1}^e$. On the other hand, the consumption of the old C_t^{t-1} at time t, is a function only of their savings which equals the previous period's bond and money stocks $(b_{t-1}+m_{t-1})$, and among the current prices, just the price of Y, $p_{y,t}$. This implies that we can rewrite (4) in terms of V' defined as the indirect utility function:

$$V' = v'[\, p_t,\ p_{y,t+1}^e, \underline{K}_t \,] \qquad (5)$$

where p_t is the vector of current prices. It is important to note that L_d, total labour demand, does not enter V' separately but only via the disposable income term $\underline{K}_t$. The reason for this lies in our having assumed that labour does not enter the utility function.

Some further transformations enable us to express (5) entirely in terms of current prices and incomes. Suppose now that all price expectations are rational and unit elastic; by the assumption of rationality, $p_{y,t+1}^e = p_{y,t+1}$ and unit elastic expectations is satisfied for $p_{y,t+1} = \beta p_{y,t}$. Then, a likely candidate to consider is $p_{y,t+1}^e = \beta\, p_{y,t}$, so that the indirect utility function is a function of only $[p_t, \underline{K}_t, \beta]$, and

$$V = v\,[\, p_t, \underline{K}_t;\ \beta] \qquad (6)$$

In deriving optimal pricing rules, we consider two alternative financing mechanisms of the government deficit: money financing and bond financing. First, we deal with pricing rules under pure money financing so that both T_t and $B_t = B_{t-1} = 0$, and consider the case when the economy is at the Steady State. I assume the existence of the Steady State since the models of Benassy are very similar to my own, and Benassy proves the existence of the Steady State in like models under similar assumptions.

(5.3.1) Optimal Pricing

The government determines its optimal policy by maximising the welfare function S*, subject to its budget constraint at the steady state. It is readily verified that the steady state social welfare function implied by the indirect utility function (6) and welfare function (3)
above is given by:

$$S^* = v[p,K,\beta]\delta/(1-\delta) \qquad (7)$$

The need to consider the social welfare function at the steady state arises from the fact that in the context of an over-lapping generations model, there is neither a "representative point" of time nor a "representative generation" as is often possible to assume in most of the literature whilst determining optimal policies in a single period model.

From individual utility maximisation, consumption demand is set by the vector of prices, p_t, and disposable income, K_t. Since the supply of both the private and public goods, Y_t and G_t respectively, are demand determined in this Keynesian unemployment regime, we may express the supply of Y and G as functions of the variables which determine their demands so that $Y_t = Y(p_t, K_t, \beta)$, and $G_t = G(p_t, K_t, \beta)$. With reference to the labour demand in the two sectors, it is worth recalling that both are technology determined so that $l_{d,y} = f^{-1}(Y) = l_y(p_t, K_t, \beta)$ and $lg = Q^{-1}(G) = l_g(p_t, K_t, \beta)$.

However, unlike most other models, the labour and private product market clearing conditions are not treated as constraints in the exercise solving for the "optimal" public sector price. The reason for this lies in our use of an overlapping generations model with a single representative individual of each generation in this economy featuring excess supply in both markets. There is only one member of the young generation who supplies labour at any instant of time, and hence, both the amount of labour supplied and the labour demand constraint coincide. The extension to many goods, many consumers and producers, has been done elsewhere using by now fairly standard techniques but all these earlier models are strictly confined to a single period time-frame. In the dynamic context of the present model using overlapping generations, any such extension to several goods and individual agents should be relatively straightforward since the solution techniques could, by and large, be the same as those used by previous authors, and therefore, it would appear that no new important insights are to be gained by this extension of the (my) model.

Now, coming to the government budget constraint, two major changes are made. I drop the assumption of a binding zero-budget constraint so that Φ is no longer necessarily equated to zero. Suppose we were to continue with the original assumption of $\Phi=0$, then this, in effect, renders irrelevant the whole issue of "optimal" public sector prices since the only price consistent with this budget requirement at the Steady State, where $M_t=M_{t-1}$, is marginal cost pricing. Therefore, it becomes essential to modify the government budget constraint for otherwise one would simply be begging the question of optimal pricing.

Similarly, the reason for considering pure money financing, and where there are no lump sum taxes is the following. Suppose indeed $\Phi=0$, and there are lump sum taxes, then $\Phi = T + (p_gG-wl_g) = 0$ at the Steady State. The obvious solution to the optimal choice of public sector prices is $p_g=0$ for this would appear to maximise individual utility. The optimal lump-sum tax T is equated to the labour costs of public production so that $T=wl_g$. Also, it is important to note that there are no distributional costs of levying this tax, that needs taking account of, since there is only a single young consumer paying the tax at any given point of time in this model. However, some considered thinking leads to the conclusion that there are both economic and other arguments against persuing this policy as discussed below.

In more general situations where such lump-sum taxes are not feasible, then the issue of public sector pricing becomes pertinent. A very plausible case against the introduction of such taxes would be, for instance, the costs of setting up the requisite administrative machinery. Furthermore, the recent experience in the U.K. is a pointer to the hefty political costs of introducing the poll tax which quite clearly appears to be taking its toll on the ruling government. As a follow on, the next question arises: are governments willing to pay such a high political cost when a more painless and standard alternative exists of financing the operations of the state-owned enterprise by charging a price for the goods produced by the enterprise?

Furthermore, the comparative statics of chapter 3 indicate that, under commonly assumed demand responses, lump-sum taxes have contractionary effects on aggregate income and employment. On the other hand, as analysed in chapter 3, it is possible to levy a (positive) price without political detriment. Also, by an appropriately chosen pricing policy, along the lines detailed in chapter 3, it would be possible to even increase aggregate demand and reduce the rate of labour unemployment! Therefore, the "solution" of setting T=wlg with a zero price for the public sector good seems myopic public policy.

Solving the model

Therefore, the government determines the welfare optimal public sector price by maximising the following Lagrangian function:

$$\text{Max } L = V(p_g, K(p_g)) + \alpha_1[\, p_g G - w l_g] \qquad (8)$$

The α_1 is the associated dual variable of the government budget constraint $\Phi = p_g G - w l_g$, and Φ is not necessarily zero.

From the first-order necessary conditions, the optimal public sector pricing rule, taking account of the multiplier effects on aggregate income, K, is:

$$L_p = V_p + V_k K_p + \alpha_1 \left[\, p_g - \frac{w\partial l_g}{\partial G} \right] \frac{\partial G}{\partial p_g} + \alpha_1 G = 0$$

$$= -\alpha G + \alpha K_p + \alpha_1 [p_g - w\partial l_g/\partial G]\partial G/\partial p_g + \alpha_1 G = 0 \qquad (9)$$

using Roy's identity.

Interpretation of optimality condition (9)

The interpretation of (9) is along the lines of Drèze in terms of the cost-benefit analysis of a price change in the public sector. Using Roy's identity and denoting α as the individual marginal utility of income, the optimality condition is rewritten as:

$$\alpha\left[G - \frac{d\Pi}{dp_g} - w\,\frac{dL}{dp_g}\right] = \alpha_1\,\frac{d\Phi}{dp_g} \qquad (10)$$

since $V_k = \alpha$, and the second term on the left-hand side of (10) follows from the definition of K as $K = \Pi + wL$. For $\alpha_1 > 0$, the marginal revenue to the public sector of, say, an increase in p_g is given by the right-hand side. The total marginal cost to the consumer of the price increase is given by the left-hand side. The price increase alters the real income of the household, both directly and indirectly. The direct effects are through the consumption effect equivalent to a loss of nominal income of G, while the indirect effects are through the income effects on profit and wage income. Therefore, the optimality condition (10) can be viewed as requiring the marginal costs of a price increase to the consumer to be proportional to the marginal net benefits to the public enterprise where α_1 is the factor of proportionality.

The analysis here is close to that of Bös in determining the relationsip between p_g and marginal cost. By dividing through by α_1, (9) is rewritten as:

$$L_p = (1-\lambda)G + \lambda(dK/dp_g) + \left[p_g - w\,\frac{\partial l_g}{\partial G}\right]\frac{\partial G}{\partial p_g} = 0 \qquad (10)'$$

where $\lambda \equiv \alpha/\alpha_1 \geq 0$.

There are two alternative possible scenarios now.

Case (i): $\alpha<\alpha_1$ i.e., the government would like to levy a lump-sum tax

From (10)', we have:

$$p_g = w \frac{\partial l_g}{\partial G} - (1-\lambda) \frac{G}{\partial G/\partial p_g} - \lambda \frac{\partial K/\partial p_g}{\partial G/\partial p_g} \qquad (11)$$

The first term on RHS of (11) is clearly positive, and if $(1-\lambda)>0$, then the second term is also positive since the own price effect of p_g is negative. Lastly, if the goods Y and G are substitutes, and if the effect of the price change in p_g on the the private sector good outweighs the own price effect, then the overall effect on aggregate income K is expansionary: $\partial K/\partial p_g > 0$. In this situation, the optimal public sector price is greater than marginal cost $w\partial l_g/\partial G$. But if the two good are complements, then we do not have a definite answer on theoretical grounds alone about the magnitude of p_g relative to marginal cost. In short, the net effect depends on the relative magnitude of the effects on private firm profits, $\partial \Pi/\partial p_g$, following a change in public sector price vis-a-vis the income and employment effects since, by definition, $K=\Pi+wL = p_yY+wl_g$. However, under the most commonly assumed demand responses of the private sector output and the publicly produced good being substitutes, we have a clear-cut pricing rule that public sector price be set above marginal cost. Furthermore, we have explicit and measurable indicators of the factors determing the mark-up of price over marginal cost.

An Example:

I consider the Cobb-Douglas utility function to illustrate circumstances where the effect of a public sector price change on private output is zero or sufficiently small to be ignored, then effectively a change in price affects only the level of output in the public sector. Additionally, if there is constant returns to scale, θ=1, in public sector production, then (10)′ yields:

$$\frac{t}{p_g} = \frac{1}{\eta_g} + \frac{\alpha}{(\alpha-\alpha_1)} \qquad (11)'$$

where η_g = own price elasticity of demand of the public sector good G, and t is defined as $t \equiv (p_g-w)/p_g$. The "distributional" factors are captured by the term $\alpha/(\alpha-\alpha_1)$ on RHS of (11)′, while the market demand factors are reflected in the term $1/\eta_g$ with the cross price elasticity of price changes in the public sector assumed to be zero on Y, the privately supplied good.

The detailed derivation of (11)′ is given in the appendix. This formulation of the optimal pricing rule facilitates comparison with the optimal tax literature, e.g. Ramsey (1927) and Boiteux (1956). The earlier work by Atkinson and Stiglitz (1980) draws parallels between the rules of optimal pricing and optimal taxation for the specific case considered by them. In like manner, I make a comparison of the Ramsey tax rules and the optimal pricing rule of equation (11)′ above. Note, in particular, that the "distributional" term and $1/\eta$ do not occur multiplicatively as in Atkinson and Stiglitz, for instance. Here, the two terms occur in additive form so that we are able to separate out the influence of each term.

The first term of (11)′ indicates the familiar Ramsey result, that the "tax" should be inversely proportional to the elasticity of demand, and supports "what the market will bear" view in the present context of the optimal choice of prices: the extent of the mark-up over marginal cost should vary inversely with the elasticity of demand.

The second term, $\alpha/(\alpha-\alpha_1)$, modifies the principle of charging a mark-up based on the inverse of the demand elasticity.

i) A larger weightage on α, for given α_1, implies a lower mark-up of price. This is intuitively clear because in this situation a higher premium is attached to individual utility and hence, this calls for a smaller price mark-up.

ii) On the other hand, an increase in the social value of government revenue, i.e. an increase in α_1 is associated with a larger mark-up of public sector price, which is as one would expect.

iii) Next, I consider the situation where the government budget constraint is not binding and the shadow price of government net revenue is zero: $\alpha_1=0$, then,

$$\left[\frac{t}{p_g}\right] = \frac{1}{\eta_g} + 1 \qquad (11)''$$

Suppose our Cobb-Douglas utility function takes the simple form: U = YG, for which it is well known that the own price elasticity of demand for both goods Y and G is $\eta_y = \eta_g = -1$. It follows, therefore, that equation (11)'' is satisfied for public sector pricing at marginal cost: $p_g=w$.

(iv) Finally, only for $\alpha=0$ does (11)' yields the pure Ramsey rule,

$$\left[\frac{t}{p_g}\right] = \frac{1}{\eta_g} \qquad (11)'''$$

I have considered above the special case of constant returns to scale in public sector production. It is worth noting that the results derived here do not require us to assume the absence of income effects as assumed in the Atkinson and Stiglitz results. Furthermore, it can be shown under even weaker assumptions, of not requiring CRS in public sector production, that, by and large, the principal results derived above carry over. Suppose we relax this requirement so that the income effects are given by $K_p=w(\partial l_g/\partial G)(\partial G/\partial p_g)$, then the optimality result of (11)' remains unchanged

$$\left[\frac{t}{p_g}\right] = \frac{1}{\eta_g} - \frac{\alpha}{(\alpha_1-\alpha)} \qquad (12)$$

except that t is now defined as $t \equiv [p_g-w(\partial l_g/\partial G)]$.

This completes the discussion of optimal pricing rules in the special case of interdependent consumption demands. However, in the general case, where the effect of an increase in p_g on complementary private and publicly supplied goods leads to a reduction in total income $dK/dp_g<0$, then it becomes a matter for empirical resolution whether $p_g \gtreqless w$; theoretical analysis at this level draws attention to the issues to take account of in public sector pricing decisions.

<u>Case (ii)</u> $\alpha=\alpha_1$

Reverting to the general case depicted in the optimality rule of (10)', suppose $\alpha_1=\alpha$, i.e. the social marginal utility of income is the same as that of the individual, then $\lambda=1$ and for a fixed given private sector price p_y, the pricing rule reduces to:

$$p_g = - p_y \frac{\partial Y/\partial p_g}{\partial G/\partial p_g} \qquad (13)$$

i.e. set public sector price p_g such that the ratio of prices is equal to the rate of substitution dY/dG, and we have the standard efficiency criterion.

The two remaining issues to deal with are the reform problem, i.e. identifying the direction of welfare-improving price changes, and finally, the effects of price changes vis- a-vis inflation in the model. Consider the case when the budget constraint is not binding, $\alpha_1=0$, then the welfare improving price changes are identified as dp_g such that:

$$[G - d\Pi/dp_g - wdL/dp_g]\ dp_g < 0 \qquad (14)$$

This can be readily interpreted by relating it to the effects on inflation of the price change. The total value of production is given by p_yY+p_gG which we define as N for notational brevity, and dividing through by N we can express (13) as:

$$Gdp_g/N - (d\Pi/\Pi)(\Pi/N) - w(dL/L)(L/N) < 0 \qquad (15)$$

The first term is the inflation due to the price increase in p_g, and the second term is a measure of the change in income due to the change in profit income while the last term is a measure of the change in wage income following the changes in rate of employment dL/L. The combined effect of these different elements determine the direction of welfare improving changes in the public sector price.

Conclusions

This ends the discussion of optimal pricing in the steady state, and in chapter 6, I derive the rules for optimal inter-temporal pricing.

To sum up the results of the present chapter, if the shadow price of government revenue is positive, then we have a pricing rule similar to that of Drèze where we take account of the multiplier effects on aggregate income and employment. Differences between the two arise in that in my optimality result, equation (10), wage income is expressed in terms of the market wage, and there are no terms which include the reservation wage as in Drèze. This should be expected because, by assumption, labour does not have any disutility and hence, this effectively implies a reservation wage of zero.

In the situation of a binding government budget constraint where the shadow price of government resources exceeds the private marginal utility of income to the individual, it is possible to establish explicit guidelines for public sector prices in relation to marginal cost - which is one of the questions posed at the very outset. If the publicly produced and private firm output are substitutes, then the pricing rule tells us that price should be above marginal cost provided the overall effect of any price change in the public sector good on aggregate income is expansionary. In a consumption demand environment where changes in p_g have only an insignificant effect on the level of total private output, I develop the conditions for pricing at or above marginal cost, and this is illustrated with the specific example of a Cobb-Douglas utility function. The two limiting situations are considered: if $\alpha_1=0$ implying a non-binding government budget constraint, then the optimal rule suggests marginal cost pricing. Only in the event of $\alpha=0$, would the optimal price follow the pure Ramsey dictate of charging what the market will bear as determined by the inverse of the price elasticity of demand. In the general case, it is only when the two goods are complements that there is ambiguity, and then we can not say categorically what the relationship between price and marginal cost should be in the public sector. In all other situations, we have fairly clear-cut guidelines for determing the optimal public sector price, its relation to marginal cost, and also there are explicit indicators of the extent of the price mark-up, if any.

Technical Appendix

Derivation of equation (10):

Legrangian for the problem is given by:

$$L = V[p_g, K(p_g)] + \alpha_1[p_g G - w\partial l_g/\partial G] \qquad (1)$$

and maximising with respect to p_g, the first-order condition is:

$$L_p = V_p + K_p + \alpha_1[p_g - w\partial l_g/\partial G]\partial G/\partial p_g + \alpha_1 G = 0 \qquad (2)$$

Since aggregate income K is defined as:

$$K = \Pi + w\ L = p_y Y + w\ l_g(G)$$

$$\partial K/\partial p_g \equiv K_p = \partial\Pi/\partial p_g + w\ \partial l_g/\partial G \qquad (3)$$

Substituting (3) in (2), and using Roy's identity, yields:

$$-\alpha G + \alpha[\partial\Pi/\partial p_g + w\partial l_g/\partial G] + \alpha_1[(p_g - w\partial l_g/\partial G)\partial G/\partial p_g + \alpha_1 G = 0 \qquad (4)$$

Rearranging terms, (4) is rewritten as:

$$\alpha[G - \partial\Pi/\partial p_g - w\partial l_g/\partial G] = \alpha_1\ \partial\Phi/\partial p_g \qquad (5)$$

Derivation of equation (11)':

If independent demands,

$$K_p = w\ (\partial l_g/\partial g)\partial G/\partial p_g \qquad (1)$$

and substituting (1) in the first-order condition yields

$$L_p = (\alpha-\alpha_1)G + \alpha(w\partial l_g/\partial G)\partial G/\partial p_g + \alpha_1 t\ \partial G/\partial p_g = 0 \qquad (2)$$

where $t \equiv (p_g - w\ \partial l_g/\partial G)$.

Using the definition of t, and dividing through by G, equation (2) is re-written as:

$$L_p = (\alpha_1-\alpha) + \alpha(1-t/p_g)(-\eta) - \alpha_1\eta(t/p_g) \qquad (3)$$

where the own price elasticity of demand for G is denoted:

$$\eta = -\ (\partial G/\partial p_g)p_g/G$$

Dividing (3) by $(\alpha_1-\alpha)$, and re-arranging yields:

$$t/p_g = 1/\eta + \alpha/(\alpha-\ \alpha_1) \qquad (4)$$

Chapter 6

OPTIMAL INTERTEMPORAL PRICING

(6.1) Introduction

Finally, in this chapter, the discussion focuses on pricing rules using the method of analysis of Diamond (1973) and subsequently, followed by Atkinson and Sandmo (1980). Rather than to start out by considering a representative generation, and deriving optimal pricing rules at the Steady State, the approach taken is different now. The financing of the government deficit is assumed to be through bond issue and tax revenues. We study the recursive behavioural relations generated by the changing bond stock financing a non-zero government budget. Having done this, it is only in the next step that stationarity is assumed, and the rules of optimal pricing characterised at the Steady State. The rationale for this alternative approach has its roots in the Growth literature, where there is the "Golden Rule of Savings" derived at the Steady State, and the savings rules that look at the dynamic time path of the optimal level of savings, Solow (1987) and Dixit (1976).

(6.2) Pricing and allocation

We introduce the notion of the state valuation function, $H(p_{g,t-1}, B_{t-1}, T_{t-1})$ to represent the maximal level of welfare, discounted to time t, that can be attained given the initial values of the variables inherited in time t. The government maximises the social welfare function $V(p_g, K(p_g, T))$, as given in equation (6), chapter 5, by choosing $p_{g,t}$ and T_t subject to its budget constraint:

$$B_t = [B_{t-1}+w_t l_{g,t}-p_{g,t}G_t-T_t]/q_t \qquad (1)$$

We apply the principle of optimality of dynamic programming, as in Diamond(1973) and Atkinson and Sandmo (1980). In view of the stationarity of the problem:

$$H(t) \equiv H(p_{g,t-1},B_{t-1},T_{t-1})$$

$$= \max\{V(t) + \mu\ H(t+1)(p_{g,t},B_t,T_t)\} \qquad (2)$$

where B_t in H(t+1) is given by equation (1), and $0<\mu<1$.

The first-order necessary conditions for optimality are set out below in terms of the choice of $p_{g,t}$ and T_t. The sub-scripts i=1-3 refer to differentiation of H with respect p_g, B and T respectively.

$$-V_{p,g} = \mu\ H_1(t+1) + \mu\ H_2(t+1)\partial B_t/\partial p_{g,t} \qquad (3)$$

and,

$$-V_T = \mu\ H_3(t+1) + \mu\ H_2(t+1)\partial Bt/\partial Tt \qquad (4)$$

The second set of equations are those obtained by differentiating the recursion relation (2) with respect to the state variables $p_{g,t-1}$, B_{t-1} and T_{t-1} respectively:

$$H_1(t) = \mu\ H_2(t+1)(1/q_t)\partial B_{t-1}/\partial p_{g,t-1} \qquad (5)$$

$$H_2(t) = \mu\, H_2(t+1)(1/q_t) \tag{6}$$

$$H_3(t) = \mu\, H_2(t+1)(1/q_t)\partial B_{t-1}/\partial T_{t-1} \tag{7}$$

Assuming that an optimum policy exists and that it converges to a steady state (see chapter 4), we turn to the interpretation of these results in the next section.

(6.3) Interpretation of the results

Focusing on the steady state properties, from (6) we observe that $H_2(t)=H_2(t+1)$ or:

$$\mu = q = 1/(1+r) \tag{8}$$

At this point it is worth recalling some of the behavioural and macroeconomic relations mentioned earlier. From the bond market equilibrium equation, B_t is equated to b_t, but b_t is the savings of the old of generation t to finance its consumption when old. Also, from the individual budget constraint, we have:

$$K_t^d = p_{y,t}X_t + p_{g,t}G_t + q_t b_t \tag{9}$$

and,

$$b_t = p_{y,t+1}\, X_{t+1} \tag{9'}$$

The definition of disposable income is:

$$K_t^d = p_{y,t}Y_t + wl_{g,t} - T_t \tag{10}$$

while the private product market clearing equation is:

$$p_{y,t}Y_t = p_{y,t}X_t + B_{t-1} \qquad (11)$$

Therefore, from equations (10) and (11), we have:

$$\underline{K}_t = p_{y,t}X_t + B_{t-1} + w_t l_{g,t} - Tt \qquad (12)$$

so that equations (9) and (12), yields:

$$q_t b_t = B_{t-1} + (w_t l_{g,t} - p_{g,t}\, G_t) - T_t \qquad (13)$$

The optimality condition (3) is recalled below:

$$- V_{p,g} = \mu\, H_1(t+1) + \mu\, H_2(t+1)\partial B_t/\partial p_{g,t}$$

and from equation (5) above, we substitute for $H_1(t+1)$ as:

$$H_1(t+1) = \mu\, H_2(t+2)\ (\ 1/q_{t+1})(\partial B_t/\partial p_{g,t}) \qquad (14)$$

so that equation (3) is rewritten as:

$$-V_{p,g} = \mu\ [\mu\ H_2(t+2)(1/q_{t+1})(\partial B_t/\partial p_{g,t}\ ] + \mu\ H_2(t+1)\partial B_t/\partial p_{g,t} \qquad (15)$$

and at the steady state, (15) is re-written as:

$$-V_{p,g}/H_2 = q\ \partial B/\partial p_g + \partial[(wl_g - p_g G) + B_{t-1} - T_t\ ]/\partial p_g \qquad (16)$$

since $\mu = q$, and the second term on RHS of (16) is derived by substituting (13) above.

Therefore, our optimal public sector price satisfies:

$$(\alpha/H_2)[G-\Pi_{p,g-wdL/dp_g}] = q\partial B/\partial p_g + [(w\partial l_g/\partial G - p_g)\partial G/\partial p_g - G] \quad (17)$$

The term α/H_2 is the private marginal utility of income in terms of the 'shadow price' of government revenue, H_2. This has been "referred to in the optimum tax literature as the 'net social marginal valuation of income' (Atkinson and Stiglitz, 1980)". The term on the right side of the optimality condition is the change in government revenue arising from the response of individual consumption demand and the consequent changes in labour employment and income in the public sector.

Next, we need to determine the value of α/H_2. From the second optimality condition (4) and the first order condition (7), we have:

$$H_3(t+1) = \mu\, H_2(t+2) \left[\frac{1}{q_{t+1}} \frac{\partial B_t}{\partial T_t}\right] \quad (18)$$

and substituting in (4) yields:

$$L_T = V_T + \mu\left[\mu\, H_{2(t+2)} \frac{1}{q_{t+1}} - \frac{\partial B_t}{\partial T_t}\right] +$$

$$+ \mu\, H_2(t+1)\frac{1}{q_t}\left[w \frac{\partial l_g}{\partial G} - p_g\right]\frac{\partial G_t}{\partial T_t} - (\mu/ q_t)H_2(t+1) = 0 \quad (19)$$

Therefore, at the steady state where $H_2(t)=H_{2(t+1)}=H_2(t+j)=H_2$, and also $q_t= q_{t+j} = \mu$ for all j, equation (19) implies, using Roy's identity, that:

$$\left[\frac{\alpha}{H_2}\right] = q\ (\partial b/\partial T) + \left[w \frac{\partial l_g}{\partial G} - p_g\right] \frac{\partial G}{\partial T} - 1 \quad (20)$$

(6.4) Conclusion

A comparison of equations (10), chapter 5, and (20) indicates that the valuation of marginal costs to the consumer is rather similar in both approaches. However, the marginal net benefits to the public enterprise are appraised differently: the latter method requires taking account of the changes in the financial instruments as well, in this instance, changes in the bond stock. Also, we have in equation (20) a well defined rule for computing α/H_2, the social valuation of the individual's marginal utility of income.

As a topic of future research, it is intended to extend the above analysis in a dynamic context to examine the relative merits of lump-sum taxes vis.a.vis. charging prices above marginal cost for the public sector good. This analysis should be relatively straightforward since there is already a wealth of literature on the similarities between indirect taxes and pricing in a static context, and likewise there is by now fairly standard work on the comparison of direct taxes and indirect taxes.

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