Industry Structure and the Dynamics of Competition

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Thesis submitted for the degree of
Doctor of Philosophy
at the University of London

Abstract

This dissertation focusses on the analysis of industrial market structure and related topics in industrial economics. It comprises three self-contained essays on dynamic aspects of industry structure, collusion, and the limits of monopolisation.

The first essay, which is contained in chapter 2, analyses a dynamic game of investment in R&D or advertising, where current investments change future market conditions. It investigates whether underinvestment can be supported in equilibrium by the threat of escalation in investment outlays. When there are no spillovers, or there is full patent protection, underinvestment equilibria are shown to exist even though, by deviating, a firm can get a persistent strategic advantage. When there are strong spillovers and weak patent protection, underinvestment equilibria fail to exist. This implies that weaker patent protection can actually lead to more investment in equilibrium. Furthermore, potential entry is introduced into the model so as to address issues of market structure. It is shown that underinvestment equilibria can be stable with respect to further entry, independently of market size and entry costs. Finally, the "nonfragmentation" result of static stage games (Shaked and Sutton 1987) is proved to hold in this dynamic game. That is, fragmented outcomes can not be supported in any equilibrium, no matter how large the market, and despite the existence of underinvestment equilibria.

The starting point of the essay in chapter 3 is the traditional view in the IO literature, according to which there is a negative relationship between cartel stability and the level of excess capacity in an industry. Recent supergame-theoretic contributions appear to show that this view is ill-founded. Focussing on the issue of enforcement of cartel rules ("incentive constraints"), however, this literature completely ignores firms' "participation constraints". Reverting the focus of attention, the paper restores the traditional view: large cartels will not be sustainable in periods of high excess capacity (low demand). In contrast to the supergame-theoretic literature, it predicts a negative relationship between excess capacity and the collusive price.

The aim of the final essay, contained in chapter 4, is to provide empirically testable predictions regarding the relationship between market size and concentration. In a model of endogenous horizontal mergers, it is shown that concentrated outcomes can not be supported in a free entry equilibrium in large exogenous sunk cost industries. In contrast, very concentrated outcomes may be sustained in endogenous sunk cost industries, no matter how large the market, and even in the absence of mergers. It is shown that these predictions do not depend on any details of the extensive form of the game, even allowing for side payments between firms and endogenous product choice. The results complement those of Sutton (1991) on the stability of fragmented outcomes.

Acknowledgements

My greatest intellectual debt is to my supervisor, John Sutton. In this dissertation, my debt is most apparent: two of the three essays are closely linked to his work. More importantly, however, John has profoundly influenced my views on economics. With his intellectual integrity he has set a standard that is most difficult to live up to. I am very grateful to him.

During my PhD, I have profited enormously from the stimulating research environment at my academic home, the London School of Economics. In many ways, the intellectual influence LSE has exerted on me has been complementary to the academic rigour I previously experienced at the University of Bonn, where I did most of my undergraduate studies. I am indebted to both institutions and their faculties.

I am also thankful to Luis Cabral and Leonardo Felli for numerous discussions and, in particular, their support during my (successful) job search. Furthermore, I received helpful comments from John Moore, Patrick Rey, Jean Tirole, and Adam Ostaszewski, amongst others.

In the last years, I was fortunate to have the chance to interact with a great group of fellow students, officemates, and friends. I would like to single out Alison Oldale, Tommaso Valletti, Andrea Lofaro, Sönje Reiche, Philip Kalmus, and Martin Peitz. Thanks to all of them! I am especially grateful to my friend Markus Brunnermeier, with whom I spent many hours debating economics and the economics profession.

Above all, however, I owe a great debt to my family for giving me a wonderful home, and reminding me, again and again, that success is not everything in life. Finally, I would like to thank Annabel, Rado, Aron, Serge, Jean-Luc, Martin, Monika, Andrew, Holger, Sabine, Ralf, Tanja, and many others, for having been friends.

Earlier versions of chapter 2, entitled *Underinvestment and Market Structure*, were presented in seminars at LSE, Wissenschaftszentrum Berlin, the Universities of East Anglia, Basel, Toulouse, Brussels (ECARE), and Exeter, the Stockholm School of Economics, IAE/CSIC (Barcelona), CEMFI (Madrid), and the University of Pennsylvania. Moreover, I presented the paper at the 1998 Royal Economic Society Conference (Young Economists' Session) at the University of Warwick, the 1998 North American Summer Meeting of the Econometric Society (NASM) in Montreal, the 1998 EARIE Conference in Copenhagen, the 1998 European Summer Meeting of the Econometric Society (ESEM) in Berlin, and the 1998 Jahrestagung des Vereins für Socialpolitik in Rostock.

Chapter 3, *Cartel Stability under Capacity Constraints: The Traditional View Restored*, is a winner of the 1999 EARIE Young Economists' Essay Competition. Earlier versions of the paper were presented at LSE, Royal Holloway College (London), LBS, the University of Essex, the European University Institute (Florence), Universidade Nova de Lisboa (Lisbon), and the University of Toulouse.

Chapter 4, *Monopolisation and Industry Structure*, was partly written while I was visiting the University of Toulouse. I am very grateful for the hospitality of its faculty and students. In particular, I would like to thank Patrick Rey, Aude Schloesing, and Valérie Rabassa.

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Chapter 1

Introduction

Industries differ greatly in the level of concentration: some are dominated by at most a handful of firms, while others come close to the classic notion of competitive markets consisting of many firms without much market power. To explain these cross-industry differences in concentration levels has traditionally been the major focus of the literature on industrial market structure. The main aim of this dissertation is to shed new light on various issues of industry structure and related topics in industrial economics.

For decades, the dominant approach in the study of industry structure has been Bain's (1956) structure-conduct-performance paradigm, according to which structure (concentration) can be explained by the level of certain "barriers to entry", e.g. increasing returns to scale, and high R&D or advertising intensity. However, researchers remarked quite early that firms' conduct has an impact on industry concentration as well. In particular, market structure is affected by the degree of collusive behaviour of industry participants. All other things being equal, a market in which firms collude may sustain more firms than a noncollusive one since collusion will raise profits for a given number of firms in the market. In this dissertation, different aspects of collusion, and their implications on industry structure, are studied. We consider collusion in investment levels in chapter 2, and collusion in output levels in chapter 3.

Since the game-theoretic revolution in industrial economics researchers have become quite uneasy with cross-industry studies. This is for at least two reasons. The first is that equilibrium outcomes delicately depend on variables that are unobservable for empirical researchers, and likely to vary across industries. The second is that many models exhibit a multiplicity of equilibria. Now, many aspects of competition are dynamic by nature and cannot be captured in static models. The drawback of dynamic models, however, is that, in these models, the problem of multiplicity of equilibria is particularly severe: Folk theorem-like results imply that a vast menu of outcomes may be sustained in equilibrium. This clearly represents an embarrassment of riches. The question of interest, therefore, is whether it is nevertheless possible to make empirically testable predictions using dynamic models of competition. We investigate this topic in chapter 2 of this dissertation.

In most models on industrial market structure, the boundaries and characteristics of firms, such as the number of firms' products, are taken as given. Is it impossible to exclude (arbitrarily) concentrated outcomes once firms are free to merge? After all, industry profits would be maximised if firms decided to merge to monopoly. This important question is addressed in chapter 4, where we study the constraints on equilibrium outcomes that can be obtained when firms are allowed to merge or to choose the number of their products.

In this dissertation, there are two unifying themes. The first is the search for empirically testable predictions. The second is the impact of market size (or the level of demand) on equilibrium outcomes. These two are related since market size and demand are not too difficult to measure empirically, and can often be seen as exogenous to the problem studied.

Traditionally, industrial economists have believed that the level of industry concentration is decreasing with the size of the market. After all, an increase in market size raises firms' profits and should thus trigger additional entry, which in turn should reduce concentration. However, Sutton (1991), in his book *Sunk Costs and Market Structure*, shows that the alleged negative size-concentration relationship breaks down in certain groups of industries. In particular, Sutton introduces the distinction between exogenous and endogenous sunk cost industries. Exogenous sunk cost industries are industries in which the level of sunk costs is exogenously given by firms' setup costs; R&D or advertising outlays are insignificant. In endogenous sunk cost industries, on the other hand, the level of sunk costs is endogenously determined by firms' investment decisions; roughly, these are industries, in which R&D or advertising are effective in that an increase in some fixed outlays raises the consumers' willingness-to-pay, or reduces the marginal costs of production. Sutton's predictions are that, in exogenous sunk costs industries, the lower bound to concentration goes to zero as market size tends to infinity. In contrast, in endogenous sunk cost industries, the lower bound to concentration is bounded away from zero, no matter how large the market. That is, fragmented outcomes may be supported as equilibrium outcomes in large exogenous sunk cost industries, but not in endogenous sunk cost industries. The "nonconvergence" or "nonfragmentation" result for endogenous sunk cost industries has formally been shown in the context of static stage games by Shaked and Sutton (1987), and empirically tested by Sutton (1991).

In the following, we give a short overview of the three essays in this dissertation.

In oligopolistic markets, firms often invest mainly in order to get a competitive advantage over their rivals. In doing so, they do not take into account the negative externalities they thereby impose on their competitors. Hence, firms in an oligopolistic industry would be jointly better off if they could somehow coordinate to invest less. In a dynamic infinite-horizon investment game, firms may sustain "underinvestment equilibria", which exhibit lower investment levels than noncollusive equilibria, by threatening to engage in an escalation of investment outlays in case of deviation. In contrast to infinitely repeated games, however, the existence of such tacitly collusive equilibria is not obviously ensured since, by deviating from such an underinvestment situation, a firm may leapfrog its rivals, and thereby get a persistent strategic advantage.

In chapter 2 of this dissertation, we consider a dynamic game of investment in R&D or advertising. In the first part of the essay, we analyse the effects of spillovers and patent protection on the sustainability of underinvestment equilibria. We start by constructing a "noncollusive" benchmark equilibrium. (In infinitely repeated games, the noncollusive equilibrium is simply the infinite repetition of the (unique) equilibrium of the static stage game. In dynamic games, the definition of a noncollusive equilibrium is no longer clear since the stage game may endogenously change over time. However, the benchmark equilibrium we construct has properties which allow us to interpret the equilibrium as the natural generalisation of the familiar noncollusive equilibrium.) We then consider a class of strategies according to which firms do not invest, unless their rivals invest. If a firm deviated in the last period, then all firms revert forever to the noncollusive benchmark equilibrium. That is, firms try to sustain underinvestment (no investment) by the threat of escalation in case of deviation. We show that these trigger strategies do indeed form a Markov perfect equilibrium, provided there are no spillovers (or, alternatively, there is full patent protection) and the discount factor is sufficiently close to unity. In contrast, underinvestment can not be sustained in this way if there are strong spillovers (or weak patent protection), independently of the level of the discount factor. Hence, weaker patent protection may actually lead to more investment in equilibrium precisely because it reduces the incentives to invest in the benchmark equilibrium so that the threat of escalation becomes blunt. This should be of concern for antitrust authorities since welfare is shown to be unambiguously lower in underinvestment equilibria than in the noncollusive benchmark equilibrium.

The second part of the essay is concerned with issues of market structure. We first show that underinvestment equilibria may be stable with respect to further entry, independently of the level of entry costs and the size of the market. This implies that very concentrated outcomes may be sustained in arbitrarily large markets. The main question we address, however, is the following. The static version of the dynamic investment game possesses the nonconvergence property (Shaked and Sutton 1987): the number of firms remains finite even as the size of the market grows without bound. This nonfragmentation result for endogenous sunk cost industries has almost solely been shown in the context of static stage games. The open question is whether it still holds in dynamic investment games. The answer is not obvious since the nonfragmentation result excludes certain outcomes, namely fragmented ones, as being not sustainable in equilibrium. However, as is well known, in dynamic games, many more outcomes may be sustained in equilibrium than in the corresponding static games. Our main result on industry structure is reassuring in that it shows the robustness of the nonconvergence property: in any Markov perfect equilibrium of the dynamic game, the number of firms remains finite, no matter how large the market.

'Cartels tend to break down under the pressure of excess capacity in periods of low demand.' This has, for a long time, been the classical conjecture in the industrial economics literature. The recent supergame-theoretic literature, however, appears to show that the traditional view is theoretically ill-founded, but without providing alternative testable predictions; see, for instance, Brock and Scheinkman (1985). In chapter 3 of this dissertation, we reconsider the traditional view in a (static) model of cartel stability, which focusses on firms' trade off between participation in a cartel (so as to achieve a more collusive outcome) and nonparticipation (so as to take a free ride on the cartel's effort to restrict output). The model follows Selten (1973) and others in assuming that firms do not cheat on cartel agreements once they have joined a cartel; this allows us to focus on firms' participation decisions. In contrast, the supergame-theoretic literature usually assumes that all firms in the industry try to sustain collusion; this approach focusses on firms' incentives to cheat.

The main part of the essay analyses the following two-stage game. At the first stage, firms simultaneously decide whether to join the cartel or rather the "fringe". At the second stage, firms compete in quantities. The difference between cartel and fringe members is that the former set output so as to maximise joint cartel profit, while the latter maximise individual profit. Firms are symmetric and face an exogenous capacity constraint. We show that an increase in capacity, or a decrease in demand, has two effects that reinforce each other. First, for a given cartel size, it has a direct negative impact on equilibrium price. Second, it decreases the equilibrium cartel size, which in turn leads to lower prices. Hence, the model provides a theoretical foundation for the traditional view on the relationship between cartel stability and the level of demand (or capacity). Moreover, it makes a sharp empirical prediction: equilibrium price (or mark up) and demand are positively correlated, price and capacity negatively. In addition, we analyse the case of heterogeneity in firms' capacities. Under some assumption on the cartel's output sharing rule, we show that large firms have more incentives to join the cartel than their smaller rivals. As a result, one should observe a negative cross-sectional relationship between capacity utilisation rates and capacity levels. Finally, we consider a simple dynamic extension of the game, where demand follows an arbitrary stochastic process. In a Markov perfect equilibrium, prices will tend to vary procyclically.

Sutton's (1991) theory on the relationship between concentration and market size is concerned with the stability of *fragmented outcomes.* His predictions relate to the (asymptotic) properties of the *lower bound* to concentration. In the final chapter of this dissertation, we investigate whether it is possible to tighten Sutton's predictions. In particular, we analyse the stability of *concentrated outcomes* and seek to characterise properties of an *upper bound* to concentration.

Economic history provides many examples of cases where firms attempted to monopolise markets through horizontal mergers. Accordingly, in the first part of the chapter, we analyse an endogenous horizontal merger model. Following Sutton (1991), we distinguish between exogenous and endogenous sunk cost industries. In the exogenous sunk cost case, our model consists of three stages. At the first stage, firms decide whether or not to enter the market. At the second stage, firms are allowed to merge; coalition formation is modelled as an open membership game. Finally, the newly formed coalitions (merged entities) compete in prices. In the endogenous sunk cost case, there is an additional investment stage just prior to the output stage. At this stage, the newly formed coalitions decide how much to invest in the quality of their product portfolio. Products are horizontally differentiated in a non-localised fashion; all goods are treated symmetrically. Mergers are conceptually well defined in this setup: the product portfolio of a coalition is the collection of its members' products. In this model, we show that merger to monopoly cannot be sustained in large exogenous sunk cost industries. In fact, in exogenous sunk cost industries, the upper bound to the one-firm concentration ratio goes to zero as market size tends to infinity. This is in contrast to endogenous sunk cost industries, where monopoly may be sustained even in arbitrarily large markets. That is, in endogenous sunk cost industries, the upper bound to concentration does not decrease with market size.

In the second part of chapter 4, we investigate the robustness of our predictions. Using Sutton's (1997) equilibrium concept, we allow for arbitrary coalition formation games, side payments between firms, and monopolisation not only through horizontal mergers but also through product proliferation. In fact, we do not explicitly specify the extensive form of the game. We are, nevertheless, able to achieve powerful results by introducing the possibility of "ex-post entry" (e.g. post-merger entry). We show that our previous predictions carry over to this model. That is, allowing for ex-post entry, our predictions do not depend on any details regarding coalition formation or product selection.

Chapter 2

Underinvestment and Market Structure

2.1 Introduction

A central theme in the literature on investment is whether firms have sufficient incentives to invest. For instance, the rationale for patent protection is to give innovating firms the "right" incentives to engage in R&D. In oligopolistic markets, firms usually invest in order to gain a competitive advantage over their rivals. Because of this "business stealing effect", noncooperative investment levels tend to be higher than the level that maximises firms' joint profits. In a dynamic model of investment, the following question arises therefore naturally: 'Is it possible that firms "underinvest" in equilibrium only because investing more would lead to an escalation of investment outlays by rival firms?' In other words, 'Do tacitly collusive "underinvestment equilibria" exist?'

When analysing this question, there is a subtle but important distinction to be made between infinitely repeated games (supergames) and truly dynamic investment games. The theory of supergames, which has been applied mostly to price or quantity setting games, is well developed. Since oligopolistic interaction has the underlying structure of a prisoner's dilemma game, we know from the Folk Theorem that in supergames tacitly collusive equilibria always exist for a discount factor sufficiently large. By contrast, truly dynamic investment games, in which current actions change future payoffs, are not yet very well understood. In particular, the existence of tacitly collusive (underinvestment) equilibria is not obviously ensured. The reason is that, by deviating, a firm might change future market conditions and, thereby, gain a persistent strategic advantage over its rivals. In their paper on investment in capacity, Fudenberg and Tirole (1983) have given an example of the existence of underinvestment equilibria in dynamic investment games. However, in their continuous-time framework, they have muted, by construction, the above distinction in that a firm can not leapfrog its rivals by deviating.

In R&D- and advertising-intensive industries, *endogenous* industry dynamics play a particularly important role in that current investments in product or process innovation, or in "goodwill", change not only current but also future market conditions. Since investments in R&D or advertising are sunk, these investments have a commitment value. This obviously suggests to model dynamic competition in R&D or advertising as a truly dynamic investment game rather than as an infinitely repeated game since, in the latter, tangible market conditions are assumed to be stationary.

In this paper, we explore the incentives of firms to invest, and to collude, in a dynamic (infinite horizon) game of investment in R&D or advertising. The focus is not on the dynamics of investments as such but rather on the commitment value of investments. We investigate, in particular, the issue of existence of tacitly collusive underinvestment equilibria when firms, by deviating, can get ahead of their rivals and, thereby, gain a considerable strategic advantage.

In our model, the existence of underinvestment equilibria depends crucially on the presence of spillover effects in the appropriation of the benefits from investment. When there are no spillovers or, alternatively, there is full patent protection, underinvestment equilibria exist as long as the investment cost function is sufficiently elastic, and the discount factor sufficiently large. However, when there are strong spillovers and no patent protection, underinvestment equilibria fail to exist, even for discount factors arbitrarily close to unity. This implies that a weakening in the degree of patent protection can actually lead to *more* investment in equilibrium. The reason is that firms have less incentives to invest when they can not fully appropriate the benefits, and this reduction in the incentives to invest destroys the mechanism through which underinvestment can be supported in equilibrium. Our model thus casts doubt on the effectiveness of complete patent protection in fostering investment. This should be of particular concern since, as we show, underinvestment unambiguously reduces welfare.

The issue of existence of underinvestment equilibria in our model raises an important question for the analysis of market structure. The static version of our dynamic investment model satisfies the "nonconvergence property" (see Shaked and Sutton (1987) and Sutton (1991)): as market size becomes large, free entry does *not* lead to a fragmentation of the market.[1] In the limit when market size tends to infinity, the market share of the largest firm is bounded away from zero. This result is based on the "escalation mechanism". The larger the size of the market, the greater are the returns accruing to a firm from raising its investment outlays. This implies, under some conditions, that increases in market size are associated with a rising level of firms' investment outlays. In the limit, at least one and at most a finite number of firms will find it worthwhile to engage in an escalation of R&D or advertising spendings to capture a positive market share. Hence, concentration remains bounded away from zero, no matter how large the market. The escalation mechanism has been successfully tested by Sutton (1991) in his seminal work on advertising-intensive industries.[2]

However, the nonconvergence property has been obtained almost solely in static stage-game models. The open question is whether this result still holds in dynamic models. In a static model, the way to prove the nonconvergence property is to show that there always exists a profitable deviation for some firm in a large and fragmented market. This deviation consists in a sufficient rise in investment outlays so as to capture a positive market share. In a dynamic model, however, such a single deviation might be followed by a severe (and possibly complex) "punishment" strategy by rival firms. What is at issue here is that the existence of underinvestment equilibria (when there are no spillovers) implies that firms do not necessarily engage in an escalation of R&D or advertising spendings precisely because firms will otherwise be punished. But without an escalation mechanism at work, the nonconvergence property breaks down. This is the central question on market structure we address in the present paper. Our result is very reassuring: the nonconvergence property is robust to the existence of underinvestment equilibria in our dynamic investment game.

[1]In Sutton's (1991) terminology, the static version of our model is an "endogenous sunk cost model".

[2]For a recent study on R&D-intensive industries, see Sutton (1998).

The plan of the paper is as follows. In section 3.2, we present the basic two-firm version of our model when there are no spillovers. The equilibrium analysis is given in section 2.3. This is, in section 2.4, followed by a comparison of welfare in the collusive underinvestment equilibria and the noncollusive investment equilibrium. In section 2.5, we introduce spillovers into the model. Then, in section 2.6, we turn to the analysis of market structure, and investigate whether the two-firm underinvestment equilibria are stable with respect to further entry, independently of market size and entry costs. In the following section, we turn to the central question on market structure: 'Does the nonconvergence property hold in our dynamic game, despite the existence of underinvestment equilibria?' Finally, in section 2.8, we conclude briefly.

2.2 The Basic Model

In this section, we present our basic dynamic model without spillovers. There are two firms, each offering a product variety. In each period, firms first decide how much to invest in R&D or advertising. Then, they compete in quantities. Investment is sunk, and persistently raises the consumers' willingness-to-pay for the product variety. By investing more than its rival, a firm can, therefore, get a competitive advantage. Here, we do not allow for entry of a third firm. The topic of potential entry, which is essential for the analysis of market structure, will be taken up in the second part of the paper.

We consider an infinite-horizon game of investment in R&D or advertising. The framework is essentially a dynamic version of the model in Sutton (1991). Time is discrete and indexed by t. There are two firms, $i = 1, 2$, and N consumers, indexed by l. Consumer preferences are defined over a 'quality good', produced in the industry under consideration, and an 'outside good' (or Hicksian composite commodity) whose price and attributes are assumed to be constant. There are two varieties of the quality good on offer, one by each firm. Consumers are assumed to value quality. More specifically, consumer l's utility in period t is given by

$$U^l(x_t^{l,1}, x_t^{l,2}, y_t^l) = \alpha^l \ln\left(\sum\nolimits_{i=1}^{2} u_t^i x_t^{l,i}\right) + y_t^l \tag{2.1}$$

if $\sum_{i=1}^{2} u_t^i x_t^{l,i} > 0$, and $U^l(x_t^{l,1}, x_t^{l,2}, y_t^l) = -\infty$ otherwise. We denote by $x_t^{l,i} \geq 0$ and $y_t^l \geq 0$ the quantities consumed of firm i's variety of the quality good and the outside good, respectively; u_t^i is the quality of firm i's offering in period t, and α^l is a parameter, assumed to be strictly positive, that measures the intensity of consumer l's preferences for the quality good. Consumer income in each period is denoted by m^l. We assume $m^l \geq \alpha^l$, for all consumers l; otherwise we allow for arbitrary heterogeneity of consumers in the α's and in income. The quality index is normalised so that the 'basic version' of the quality good is of quality 1, i.e. $u_t^i \geq 1$. Note that, for $x_t^{l,i} > 0$, utility is strictly increasing in quality u_t^i.

In the quality good industry, firm i's period-t-cost of investment in R&D or advertising is given by

$$F(u_t^i; u_{t-1}^i) = F_0 \left(u_t^i\right)^\beta - F_0 \left(u_{t-1}^i\right)^\beta \tag{2.2}$$

if $u_t^i \geq u_{t-1}^i$, and $F(u_t^i; u_{t-1}^i) = 0$ otherwise, where $F_0 > 0$ and $\beta > 1$ are parameters that measure the effectiveness of R&D or advertising outlays in raising the consumers' willingness-to-pay. That is, we assume that the effectiveness of R&D or advertising outlays are subject to diminishing returns; for simplicity, we do not consider "adjustment costs". In the case of investment in advertising, the quality of firm i's period-t offering can be interpreted as the stock of firm i's "goodwill" accumulated up to period t.[3] Note that $F(u; u) = 0$; that is, investment costs are zero if a firm does not want to raise the quality of its product. We assume that quality does not depreciate. Notice that we do not require $u_t^i \geq u_{t-1}^i$; however, investment costs are sunk. Both firms have constant and strictly positive marginal costs of production, c, that are independent of quality.

The time structure of the game is as follows. In each period, there are two stages. In the first stage, firms 1 and 2 simultaneously decide whether and how much to invest in quality improvement, and incur the fixed investment outlays. In the second stage, the two firms simultaneously decide how much to produce (quantity competition); consumers, taking price as given, decide how much to consume of each product, and prices are such that markets clear. Firm i's second-stage profit in period t is therefore given by $(p_t^i - c)x_t^i$, where p_t^i and x_t^i are price and quantity, respectively; firm i's total profit in period t is then $(p_t^i - c)x_t^i - F(u_t^i; u_{t-1}^i)$.

Consumers are assumed to maximise the discounted value of per-period-utility, taking the sequence of prices and qualities as given. Since, for simplicity, saving and storing are

[3] The goodwill approach to advertising goes back to Nerlove and Arrow (1962).

not allowed, this amounts to consumers maximising per-period-utility myopically. Firms maximise the discounted value of profits. The common discount factor is δ, $0 < \delta < 1$. All parameters of the model, and all moves in past periods and stages, are assumed to be common knowledge.

2.3 Equilibrium Analysis: Escalation and Underinvestment

In this section, we turn to the equilibrium analysis of the basic model without spillovers. We first show the existence of a noncollusive "investment equilibrium" in which firms engage in an escalation of investment outlays. The central result of this section is developed in subsection 2.3.3, where we show the existence of a collusive "underinvestment equilibrium". Underinvestment can be supported in equilibrium by the credible threat of escalation in case of deviation, even though, by deviating, a firm can get a persistent strategic advantage over its rival.

In the equilibrium analysis, we confine attention to Markov strategies that depend on the tangible state only; hence, the relevant solution concept is that of Markov perfect equilibrium (MPE). Recall that every MPE is a subgame perfect equilibrium (SPE), even when strategies are *not* restricted to be Markov. The idea of this approach is that history should influence current actions only if it has a direct effect on the current environment, but not because players *believe* that history matters. Furthermore, the state-space approach greatly simplifies the equilibrium analysis; as Shapiro (1989) notes, it allows us to focus on strategic aspects of commitment. (For further justification of the approach, see Maskin and Tirole (1988).)

At each decision node, the state of the industry can be summarised by the current pair of qualities $(u^1, u^2) \in [1, \infty)^2$. Firm i's (pure) Markov action rule at stage 1 in period t is a mapping $s^i : (u^1_{t-1}, u^2_{t-1}) \longmapsto u^i_t$; at the second stage of the same period its action rule is a mapping $t^i : (u^1_t, u^2_t) \longmapsto x^i_t$. The state-space approach has real bite here in that it eliminates all bootstrap-type action rules in the output stage. Since quantity choice at a given stage 2 does not affect future payoff-relevant variables (qualities), the second stage in any given period can be analysed as a *one-shot game*.

2.3.1 Cournot Competition with Perceived Quality

The important result of this subsection is that, for all pairs of qualities, there exists a unique stage-2 Nash equilibrium in quantities. The associated equilibrium profit is given by equation (2.6), which will serve, in the remainder of the paper, as a reduced-form stage-2 profit function for the dynamic investment game. Below we present some routine calculations; they can easily be skipped by the reader.

Given qualities and prices, a consumer's optimisation problem in period t (stage 2) can be written as

$$\max_{\{x^{l,i}\},y^l} \ \alpha^l \ln(\textstyle\sum_i u^i x^{l,i}) + y^l$$
$$\text{s.t. } \textstyle\sum_i p^i x^{l,i} + y^l \leq m^l$$

where we have normalised the price of the outside good to one, and dropped time indices for convenience. This programme is equivalent to

$$\max_{y^l} \ \alpha^l \ln\left(\max_i \left\{\frac{u^i}{p^i}\right\}\right) + \alpha^l \ln(m^l - y^l) + y^l.$$

Hence, in equilibrium the quality-price ratio u^i/p^i must be the same for all firms i with positive market share. Solving the first-order condition yields $y^l = m^l - \alpha^l$, which is nonnegative by assumption. Total sales in the quality good industry, S, are therefore equal to

$$S \equiv \sum_{i=1}^{2} p^i x^i = \sum_{i=1}^{2} p^i \sum_{l=1}^{N} x^{l,i} = \sum_{l=1}^{N} (m^l - y^l) = \sum_{l=1}^{N} \alpha^l. \tag{2.3}$$

Given its rival's price, firm j's price in equilibrium is given by

$$p^j = \frac{u^j}{u^i} p^i \tag{2.4}$$

for $j \neq i$. Using equation (2.4) and the definition of S yields firm i's price as a function of firms' quantities:

$$p^i = \frac{S}{x^i + (u^j/u^i)x^j}.$$

Thus, given its rival's quantity, firm i sets x^i so as to maximise

$$x^i \left(\frac{S}{x^i + (u^j/u^i)x^j} - c\right). \tag{2.5}$$

Remark that this expression is strictly concave in x^i; it is zero at $x^i = 0$, and tends to $-\infty$ as $x^i \to \infty$. Its first derivative at $x^i = 0$ is strictly positive if $S/c > (u^j/u^i)x^j$. Hence, the first-order condition, which can be written as

$$\frac{S}{x^i + (u^j/u^i)x^j}\left(1 - \frac{x^i}{x^i + (u^j/u^i)x^j}\right) - c = 0,$$

gives a unique interior maximum if $S/c > (u^j/u^i)x^j$. Subtracting the two first-order conditions yields $x^i = x^j = x$. Simple calculations then give quantities, prices and profits in the unique stage-2 Nash equilibrium:

$$x = \frac{u^i/u^j}{(u^i/u^j + 1)^2}\frac{S}{c},$$

$$p^i = c\left(\frac{u^i}{u^j} + 1\right),$$

and

$$\pi^i(u^1, u^2) = S\left(\frac{u^i/u^j}{u^i/u^j + 1}\right)^2 \tag{2.6}$$

for $i, j = 1, 2,\ i \neq j$. Observe that profits in stage 2 depend on the quality *ratio*, and market size, only. Furthermore, equation (2.6) has the nice and intuitive property that a firm's stage-2 profit is increasing in its own quality, and decreasing in its rival's quality; this differs from models of pure vertical product differentiation.

Above we have *assumed* that both firms will have positive market shares in equilibrium. To show uniqueness, we still have to prove that there does not exist an equilibrium with only one firm making (strictly) positive sales. From expression (2.5) it can be seen that firm i's unique best reply to any quantity x^j such that $x^j \geq (u^i/u^j)S/c$ is to set its own quantity equal to zero. If only one firm has a positive market share in equilibrium (firm j, say), then its price is given by $p^j = S/x^j$, and its profit by $S - cx^j$, which is strictly decreasing in x^j. Given that firm i sets its quantity x^i equal to zero, firm j therefore wants to set x^j strictly *below* $(u^i/u^j)S/c$. Hence, there is no (pure strategy) Nash equilibrium in quantities such that only one firm has a positive market share.

Finally, remark that, in equilibrium, each consumer is indifferent between the offerings by the two firms. Consumer l's period-t utility, in equilibrium, is therefore equal to

$$\begin{aligned} U^l(u_t^1, u_t^2) &= \alpha^l \ln\left(\alpha^l \frac{u_t^i}{p_t^i}\right) + m^l - \alpha^l \\ &= \alpha^l \ln\left(\alpha^l \frac{u_t^1 u_t^2}{u_t^1 + u_t^2}\right) - \alpha^l \ln c + m^l - \alpha^l. \end{aligned} \tag{2.7}$$

2.3.2 Dynamic Investment: Escalation

Having solved each period's quantity competition stage, the dynamic game can now be viewed as a simple infinite-horizon investment game in which, in each period, the two firms simultaneously invest in quality, and firm i's payoff in period t is given by

$$\Pi^i(u_t^1, u_t^2) = \pi^i(u_t^1, u_t^2) - F(u_t^i; u_{t-1}^i). \tag{2.8}$$

Infinite-horizon dynamic games, like the present one, are notoriously difficult to analyse since neither do they have a stationary structure (like infinitely repeated games), nor can they be solved by backward induction (like finite-horizon games). However, a class of subgame perfect equilibria can be found in our game by first viewing each firm's sequence of investment decisions as a single-player dynamic optimisation problem, holding the quality of the other player fixed. In this way, we can determine a region in the space of qualities (state variables) such that neither firm wants to invest further, *given that its rival will never invest again.* Since this region is associated with "high" quality levels, we can then, in a backward induction fashion, proceed to determine equilibria for subgames starting at "lower" quality levels.

Suppose that the current quality of firm i's offering is given by u_{-1}^i, with $u_{-1}^i \geq 1$. Holding firm j's quality, u^j, fixed forever, firm i's optimisation problem is then given by

$$\max_{\{u_\tau^i\}} \sum_{\tau=0}^{\infty} \delta^\tau \Pi^i(u_\tau^1, u_\tau^2), \tag{2.9}$$

with $u_\tau^j = u^j$ for all $\tau \geq 0$. Due to the additive separability of the investment cost function, the dynamics are conveniently simple: given that its rival will never invest again, it is optimal for firm i to do all its investment at once, and then cease investing forever.[4] That is, firm i's optimisation problem can be rewritten as

$$\max_{u^i} \frac{S}{1-\delta}\left(\frac{u^i/u^j}{u^i/u^j+1}\right)^2 - F(u^i; u_{-1}^i),$$

and the optimal sequence of qualities is given by $u_\tau^i = u^*(u^j)$ for all $\tau \geq 0$, where $u^*(u^j)$ denotes the solution to the above problem.[5] Note that $u^*(u^j) \geq u_{-1}^i$ since a firm's stage-2

[4]We could get more "interesting" dynamics by allowing for "adjustment costs", for instance. However, this would complicate the analysis unnecessarily and not change the qualitative insights.

[5]Remark that firm i's stationary best-reply, $u^*(\cdot)$, depends on firm i's current quality, u_{-1}^i. For notational convenience, we drop this argument.

[5]Remark that firm i's stationary best-reply, $u^*(\cdot)$, depends on firm i's current quality, u^i

profit is strictly increasing in its own quality, so that it never pays to reduce quality; that is, sunk investments have commitment value. If $u^*(u^j) > u^i_{-1}$, then $u^*(u^j)$ is the solution to[6]

$$\max_{u^i} \frac{S}{1-\delta}\left(\frac{u^i/u^j}{u^i/u^j+1}\right)^2 - F_0\left(u^i\right)^\beta. \tag{2.10}$$

The concept of a best-reply (or reaction) function is a familiar one in the context of static games. Now, in a dynamic game, a best reply is defined relative to the whole sequence of action rules only; the best reply to any given quality level is not well defined since it depends on future actions. What is well defined, however, is a firm's best reply to its rival's quality, given that the rival firm will never invest again. This is exactly how we have constructed $u^*(\cdot)$; we will, therefore, refer to $u^*(\cdot)$ as to the "stationary" best-reply function. In the following, we will characterise both firms' stationary reaction functions; due to symmetry, we can restrict ourselves to firm i's best-reply function.

Lemma 2.1 *If $\beta \geq 2$, firm i's stationary best reply, $u^*(u^j)$, is given by*

$$u^*(u^j) = \max\left\{\widehat{u}(u^j), u^i_{-1}\right\},$$

where u^i_{-1} is firm i's current quality, and $\widehat{u}(u^j)$ is the unique strictly positive solution to the first-order condition of (2.10), i.e.

$$\frac{2S}{1-\delta}\frac{\widehat{u}(u^j)u^j}{(\widehat{u}(u^j)+u^j)^3} - \beta F_0\left(\widehat{u}(u^j)\right)^{\beta-1} = 0. \tag{2.11}$$

(If $\beta = 2$ and $u^j \geq \sqrt{2S/(1-\delta)\beta F_0}$, there is no strictly positive solution to (2.11). In this case, $\widehat{u}(u^j) = 0$.)[7]

Remark that firm i's "interior" stationary best-reply function, $\widehat{u}(\cdot)$, does not depend on the initial quality u^i_{-1}; this is in contrast to $u^*(\cdot)$. The following lemma is straightforward to show. For all proofs that are not given in the text, the interested reader is referred to the appendix.

[6]To see that in this case it is indeed optimal to invest all at once, observe that the dynamic optimization problem (2.9) can be rewritten in the following way:

$$\max_{\{u^i_\tau\}} \sum_{\tau=0}^{\infty}\delta^\tau\left\{S\left(\frac{u^i_\tau/u^j}{u^i_\tau/u^j+1}\right)^2 - (1-\delta)F_0\left(u^i_\tau\right)^\beta\right\} + F_0\left(u^i_{-1}\right)^\beta,$$

and that $u^*(u^j)$ maximizes the expression in curly brackets.

[7]Strictly speaking, the stage-2 reduced-form profit function is not defined for qualities below the minimum quality of 1. For expositional clarity, we extend function (2.6) to all nonnegative qualities u^i.

Lemma 2.2 *There is a unique intersection of the two interior (stationary) best-reply curves in* $(0,\infty)^2$*. This intersection corresponds to a symmetric state,* $(\overline{u},\overline{u})$*, where* $\overline{u}$ *is given by*

$$\overline{u} = \left(\frac{S}{4(1-\delta)\beta F_0}\right)^{\frac{1}{\beta}}. \tag{2.12}$$

Lemma 2.2 implies that if $(u^1_{-1}, u^2_{-1}) \leq (\overline{u},\overline{u})$, then $(\overline{u},\overline{u})$ is the unique intersection of the two stationary reaction curves. The stationary reaction curves are shown in Figure 3.1.

We can now define four regions in the space of qualities. In Region 1, $U^{(1)}$, the qualities of both firms are above their respective interior best-replies:

$$U^{(1)} \equiv \left\{(u^1,u^2) \in [1,\infty)^2 \mid u^i \geq \widehat{u}(u^j),\ i,j=1,2,\ i \neq j\right\}.$$

Graphically speaking, this is the region above the outer envelope of the two interior best-reply curves. Region 2 consists of the pairs of qualities such that firm 1's quality is above $\overline{u}$ and firm 2's quality is below its interior best-reply; that is,

$$U^{(2)} \equiv \left\{(u^1,u^2) \in [1,\infty)^2 \mid u^1 \geq \overline{u},\ u^2 < \widehat{u}(u^1)\right\}.$$

Region 4 is defined as Region 2, but firm indices are reversed. Finally, Region 3 encompasses all states that are below the symmetric intersection:

$$U^{(3)} \equiv \left\{(u^1,u^2) \in [1,\infty)^2 \mid u^i < \overline{u},\ i=1,2\right\}.$$

We are now in the position to determine a MPE of the dynamic investment game, starting from any state of the industry.

Proposition 2.1 *The following set of mappings from the current state,* (u^1_{t-1}, u^2_{t-1})*, to the space of feasible actions,* $[1,\infty)$*, induces a pure strategy for each firm. The induced strategy profile,* Σ^{esc}*, forms a MPE starting from any state.*

(i) If $(u^1_{t-1}, u^2_{t-1}) \in U^{(1)}$*, then* $s^i(u^1_{t-1}, u^2_{t-1}) = u^i_{t-1}$*,* $i=1,2$*. ("No investment.")*

(ii) If $(u^1_{t-1}, u^2_{t-1}) \in U^{(2)}$*, then* $s^1(u^1_{t-1}, u^2_{t-1}) = u^1_{t-1}$*, and* $s^2(u^1_{t-1}, u^2_{t-1}) = \widehat{u}(u^1_{t-1})$*. ("Only firm 2 invests.")*

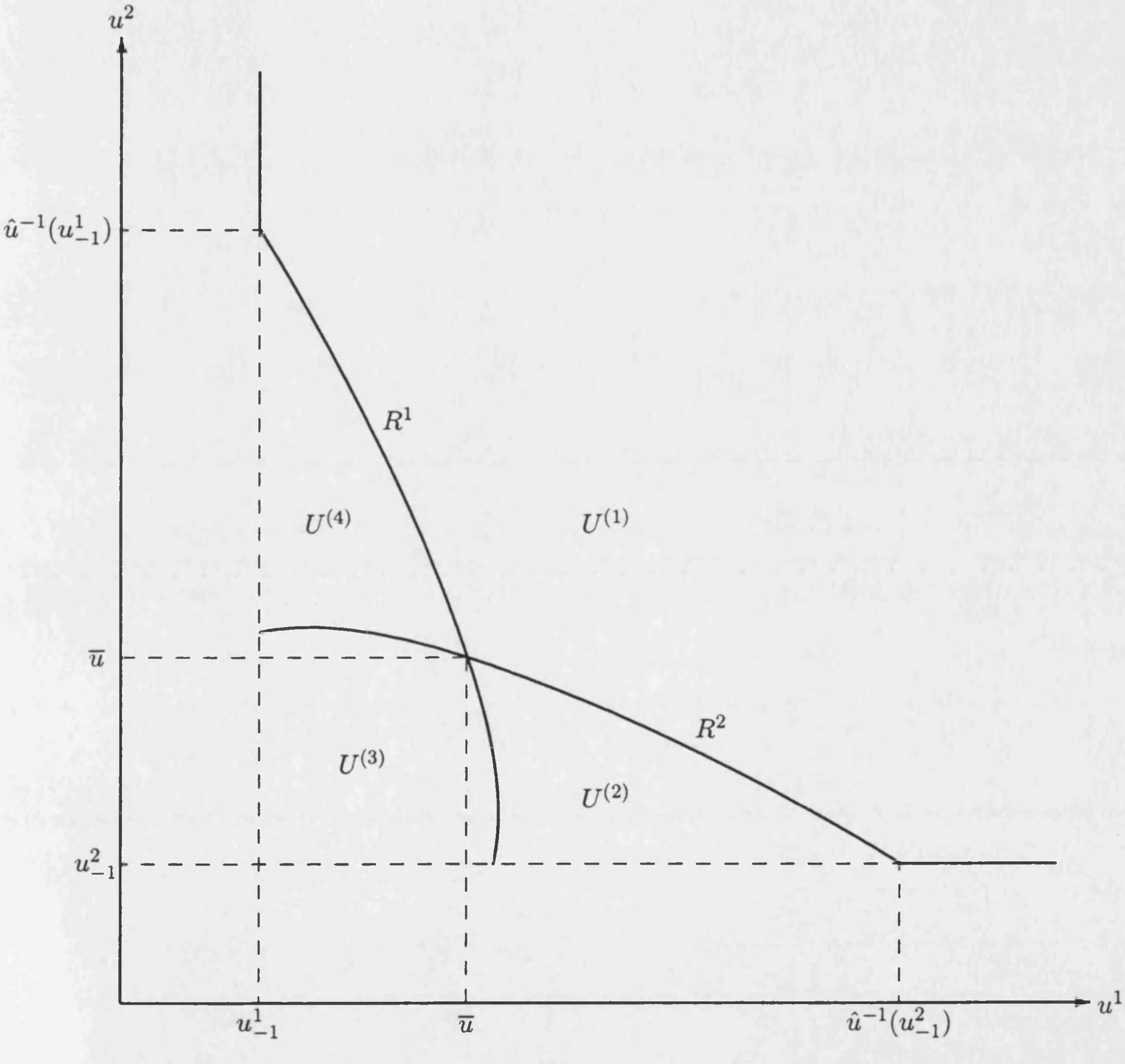

Figure 2.1: Stationary Reaction Curves

(iii) If $(u_{t-1}^1, u_{t-1}^2) \in U^{(3)}$*, then* $s^i(u_{t-1}^1, u_{t-1}^2) = \overline{u}$*,* $i = 1, 2$*. ("Both firms invest up to* $\overline{u}$*.")*

(iv) If $(u_{t-1}^1, u_{t-1}^2) \in U^{(4)}$*, then* $s^1(u_{t-1}^1, u_{t-1}^2) = \widehat{u}(u_{t-1}^2)$*, and* $s^2(u_{t-1}^1, u_{t-1}^2) = u_{t-1}^2$*. ("Only firm 1 invests.")*

Proof. Note that per-period net profits, $\Pi^i(u_t^1, u_t^2)$, are bounded above (by S), and each firm maximises the *discounted* sum of its per-period net profits. This implies that the one-stage deviation principle for infinite-horizon games applies (see Fudenberg and Tirole (1991)): it is impossible to gain by an infinite sequence of deviations when one cannot gain by a single deviation in any subgame.

Observe now that, in each state of the industry, there exists a unique intersection of the two stationary reaction curves. Remark further that, according to Σ^{esc}, the state of the industry will move at once to this unique intersection. Now, recall that the stationary best-reply function, $u^*(\cdot)$, gives the unique best reply, holding the rival firm's quality fixed forever. Furthermore, notice that the unique intersection lies in $U^{(1)}$, so that no firm will invest again *along the equilibrium path.* Thus, by definition of $u^*(\cdot)$, any single deviation that does not induce the nondeviant rival to (dis)invest again can not be profitable. However, according to Σ^{esc}, a deviation can never induce the nondeviant firm to disinvest since investment costs are sunk. Finally, consider a single deviation that induces the nondeviant rival to invest in the following period. Since stage-2 profits are decreasing in the rival's quality, the deviant's payoff along this path would be smaller than the payoff if the nondeviant did not invest again. But if the nondeviant firm did not invest again, then the deviant's payoff would be even higher by not deviating at all. Hence, such a deviation can not be profitable. ■

Comparative statics results are easily obtained. Investment along the equilibrium path is weakly increasing in the discount factor, δ, and the size of the market, S, and weakly decreasing in the cost parameters β and F_0.

Remarks. *(1)* Given the current pair of qualities, the stationary reaction curves of the dynamic game converge to the usual reaction curves of the corresponding static stage game as the discount factor δ goes to zero. Hence, as $\delta \to 0$, strategy profile Σ^{esc} converges (in the space of action rules) to the unique Nash equilibrium of the associated static game. That is, the "investment equilibrium" is simply the dynamic version of the static (noncollusive) equilibrium.

(2) The investment equilibrium has another nice property. Let us denote our dynamic investment game with payoff function (2.6) by Γ. Define the dynamic game Γ' as being equivalent to Γ, but with the following assumption on each firm's information set: starting from an initial state (u^1, u^2), which is common knowledge, each firm observes, in any period, calendar time and its own past moves only; the rival's quality level is unobservable. (This is equivalent to the assumption that, in the initial period, each firm has to "precommit" to the sequence of its future investments. Hence, Γ' is essentially a static game.) It is possible to show that, for any initial state, Γ' possesses a unique Nash equilibrium. Given (u^1, u^2), the unique equilibrium path of Γ' coincides with the equilibrium path induced by Σ^{esc} in the original game Γ. The unique Nash equilibrium of the modified game Γ' is often referred to as the *open-loop* (or *precommitment*) *equilibrium* of the original game Γ; see Fudenberg and Tirole (1991). Notice, however, that the result is due to the absence of adjustment costs in our investment cost function.

(3) Consider the following T-period truncation of the dynamic investment game Γ with payoff function (2.6): after T periods, $T \geq 1$, firms are restricted to choose the null action "no investment", i.e. $u^i_{t+1} = u^i_t$ for all $t \geq T$. The truncated game Γ^T possesses a unique SPE, which coincides with $\sum^{esc}$ (except for the fact that the SPE of the truncated game depends in a degenerate way on the entire history of the game). For more general investment cost functions with adjustment costs, the unique SPE of Γ^T would converge to $\sum^{esc}$ as $T \to \infty$.

Remarks *(1)* and *(3)* show that $\sum^{esc}$ can be interpreted as the noncollusive equilibrium in the dynamic investment game Γ. In the remainder of the paper, the investment equilibrium will, therefore, serve as the benchmark noncollusive equilibrium. We will call "underinvestment equilibrium" any equilibrium that exhibits less investment along the equilibrium path than this benchmark equilibrium.

2.3.3 Dynamic Investment: Underinvestment

Along the equilibrium path, induced by strategy profile Σ^{esc} from proposition 2.1, both firms engage in an "escalation" of R&D or advertising spendings up to the symmetric quality level $\overline{u}$ if the current state is "below" $(\overline{u}, \overline{u})$. In particular, if the current state is (u, u), where $u < \overline{u}$, then the state of the industry will move to $(\overline{u}, \overline{u})$, and stay there forever, even though *both* firms would prefer to stay at (u, u). (Since the stage-2 profit function, (2.6), depends on the ratio of qualities only, the stage-2 profit is the same in both states but, of course, moving to the higher state involves spending on R&D or advertising.) That is, both firms have an incentive to coordinate not to invest at all in order to avoid an escalation of R&D or advertising outlays, which is wasteful from their point of view.

Since we have established the existence of an MPE exhibiting escalation of investment, it might be possible to support "tacitly collusive" MPE, exhibiting little or no investment, by the threat of escalation in case of deviation. Formally, we consider a strategy profile, denoted by Σ^{coll}, that is induced by the following action rules. If $(u^1_{t-1}, u^2_{t-1}) = (u, u)$, then $s^i(u^1_{t-1}, u^2_{t-1}) = u$; if, however, $(u^1_{t-1}, u^2_{t-1}) \neq (u, u)$, then firms revert to strategy profile Σ^{esc}.

However, it is by no means obvious whether such an underinvestment equilibrium exists. Firstly, suppose the discount factor is (approximately) zero. Then, clearly, a deviation to $\widehat{u}(u)$, where $\widehat{u}(u) > u$ by definition of underinvestment, is profitable since the deviant firm does not care about future costs and stage-2 profits. Hence, by continuity of discounted payoffs in δ, there exists a profitable deviation for discount factors sufficiently small.

Secondly, suppose firm i deviates in period t by investing up to quality level u', $u' > \overline{u}$. According to strategy profile Σ^{coll}, the nondeviant will then, in period $t + 1$, react and invest up to $\widehat{u}(u')$, where $\widehat{u}(u') < \overline{u}$; no further investment will occur. (By deviating to $u' \geq \widehat{u}^{-1}(u)$, firm i can even preempt any reaction by its rival.) Along this path, the deviant will make, in *each* period, higher stage-2 profits than in the symmetric underinvestment situation. That is, by deviating, firm i can get ahead of its rival, and ensure that it will always have the higher quality. These additional stage-2 profits have to be compared with the associated investment costs, which occur in period t only. Intuitively then, such a deviation should be profitable for a sufficiently large discount factor. However, the higher is the discount factor, the larger are the returns accruing from investment in R&D or advertising, and hence the higher is the level of investment associated with Σ^{esc}. That is, the larger is δ, the more expensive it is for the deviant firm to ensure itself a persistent strategic advantage over its rival.

The following proposition gives one of the main results of this paper.

Proposition 2.2 *If $\beta \geq 2$, and the discount factor is sufficiently large, underinvestment equilibria exist. In particular, suppose the current state is given by $(u^1_{t-1}, u^2_{t-1}) = (u, u)$, where the quality level u is arbitrary. Then there is a threshold discount factor $\widehat{\delta} \in (0,1)$, such that for all $\delta \in (\widehat{\delta}, 1)$, the path $(u^1_\tau, u^2_\tau) = (u, u)$, for all $\tau \geq t$, can be supported as a MPE, namely by strategy profile Σ^{coll}.*

It is straightforward to see that, under the conditions of of the proposition, asymmetric underinvestment equilibria exist as well; this is a consequence of stage-2 profits being continuous in qualities.

As we have already argued above, the higher is the discount factor δ, the larger is the increase in the discounted sum of stage-2 profits from deviating to a given quality level u', $u' > \overline{u}$. But with increasing δ, the stationary reaction curves move outwards so that deviating to u' for a given quality *ratio* $u'/\widehat{u}(u') > 1$ becomes more and more expensive. Now, when the investment cost function is sufficiently elastic, i.e. β is sufficiently large, then the cost effect dominates the stage-2 profit effect.[8].

Our results are reminiscent of the existence of "early stopping equilibria" in the dynamic model of investment in capacity by Fudenberg and Tirole (1983).[9] In their continuous-time model, firms face linear investment costs and an exogenous upper bound on the feasible flow of investment at each point in time.[10] These extreme assumptions

[8]Actually, the effect of an increase in β on the profits from deviation is rather subtle; there are two opposing effects. On the one hand, an increase in β makes the deviation to a given u' more expensive; on the other hand, it makes the response of the nondeviant rival less aggressive in that it decreases $\widehat{u}(u')$ for a given u'. For any *given* quality ratio $u'/\widehat{u}(u') > 1$, one can show that profits from deviation are first increasing, and then decreasing, in β.

[9]Reynolds (1987,1991) analyses Fudenberg and Tirole's model in a linear-quadratic differential game framework, where capacity depreciates over time.

[10]That is, Fudenberg and Tirole assume the information lag to be extremely short (zero) relative to the speed of investment, in contrast to our model. Their assumption seems to be more reasonable in the

directly imply that a firm can not leapfrog its rival by deviating. Moreover, notice that, in a model of capacity investment, gross profits for both firms can be higher at low capacity levels than at high levels; this is due to the fact that competition in quantities is tougher when both firms have higher capacities. (This is an important difference to models of investment in R&D or advertising, where gross profits will, in general, not be higher in lower states; see the remark below.) Hence, in this model, underinvestment equilibria trivially exist for all discount factors. To see this, consider two points in the state space, "A" and "B", where "A" exhibits lower capacity levels than "B", but higher gross profits (stage-2 profits). Suppose, moreover, that the noncollusive benchmark equilibrium requires firms to invest from "A" to "B". Clearly, "no investment" at "A" can be sustained in equilibrium, independently of the discount factor.[11]

Remark. Consider a model of capacity investment such as Fudenberg and Tirole (1983), but allow for more general cost functions and positive detection lags. We then claim the following. *In a model of investment in capacity, if a firm can leapfrog its rival by deviating, and thereby get forever higher gross profits (stage-2 profits), then, for discount factors sufficiently close to unity, such a deviation must be profitable.*[12] Hence, underinvestment can not be supported. The idea behind this claim is simple. As the discount factor δ goes to one, the stationary reaction curves converge to some "limit curves". (The reason is that firms have no incentives to build huge excess capacities which are worthless.) Thus, investment levels are bounded from above, and get "dwarfed" by the discounted sum of gross profits as $\delta \to 1$. In fact, Fudenberg and Tirole mostly confine attention to the case $\delta = 1$, where investment costs do not enter firms' objective functions.

This is in sharp contrast to our model of investment in R&D or advertising, where the toughness of competition in investment levels is essentially independent of the level of investment. This implies that the stationary reaction curves do not converge to some limit curves as $\delta \to 1$, and investment costs do not get dwarfed. Hence, underinvestment can be

context of capacity investment than in the case of investment in R&D (or advertising).

[11]Notice also that Fudenberg and Tirole's motivation is quite different from ours: they focus on "mobility barriers". In particular, they investigate the validity of the proposed Stackelberg solution in Spence (1979), where an early entrant in a new market can exploit its head start by strategic investment in capacity.

[12]Implicitly, we assume here that the increase in stage-2 profits is bounded away from zero, no matter how large the discount factor.

supported in our model, even though, by deviating, a firm can ensure itself forever higher gross profits. We believe that this difference between investment in capacity and investment in R&D or advertising is an important one. In fact, this difference is closely related to Sutton's (1991) distinction between exogenous and endogenous sunk cost industries, which is crucial for the analysis of industrial market structure; see section 2.7.[13]

In the analysis conducted so far, we have left out the important issue of potential entry. We will generalise the model so as to allow for potential entry and an arbitrary number of active firms in the second part of the paper, namely in sections 2.6 and 2.7. But before turning to the analysis of market structure, we will first analyse welfare in the basic model and, then, introduce spillovers.

2.4 Welfare Analysis

The aim of this section is to compare "welfare" in the symmetric investment equilibrium and in an arbitrary (symmetric) underinvestment equilibrium. This is of great interest since any action by antitrust authorities is justified only if "tacit collusion" indeed reduces welfare.

As a welfare measure, we choose the sum of discounted profits and discounted utility; we will call this measure "net surplus". In our setting, this choice is natural and theoretically well justified since we use quasilinear preferences for this same reason. In particular, consumer utility is linear in "money" (outside numéraire good), and there are no income effects so that all profits can be redistributed to consumers without changing our analysis.

A priori it is not quite obvious whether or not net surplus is lower in an underinvestment equilibrium. Clearly, consumers' utility, prior to any redistribution of profits, is lower in an underinvestment equilibrium, simply because per-period utility is increasing in quality, and prices depend on the ratio of qualities only. However, firms' profits are unambiguously larger under "tacit collusion" since firms engage in less R&D or advertising. Indeed, from

[13]The point is the following. If a firm can already serve the whole market with its capacity, any further increase in its capacity has no impact on the firm's market share; this is the exogenous sunk cost case. In contrast, by outspending its rivals in fixed R&D or advertising outlays, a firm can steal business from its rivals and thus increase its market share, although the investment may not increase industry sales. This is the endogenous sunk cost case.

the viewpoint of a social planner, any R&D (or advertising) outlays by a second firm are wasted, holding prices fixed. (A social planner would set price equal to marginal cost, and one (subsidised) firm only would engage in investment and production.)

Nevertheless, the following proposition shows that the welfare comparison is unambiguous. But before stating and proving the proposition, we want to set the problem formally. Given an initial state (u^1_{-1}, u^2_{-1}) and any sequence of states, $\{(u^1_t, u^2_t)\}_{t=0}^{\infty}$, net surplus along this path is equal to

$$\sum_{t=0}^{\infty} \delta^t \left\{ \sum_{i=1}^{2} \Pi^i(u^1_t, u^2_t) + \sum_{l=1}^{N} U^l(u^1_t, u^2_t) \right\}, \tag{2.13}$$

with $U^l(\cdot)$ and $\Pi^i(\cdot)$ defined in (2.7) and (2.8), respectively. This assumes implicitly that at any stage 2 the industry is in equilibrium, depending on the current state. The equilibrium path associated with the symmetric investment equilibrium is $(u^1_t, u^2_t) = (\overline{u}, \overline{u})$ for all $t \geq 0$; in case of a symmetric underinvestment equilibrium it is $(u^1_t, u^2_t) = (u, u)$ for all $t \geq 0$, where, by definition of underinvestment, $\max\{u^1_{-1}, u^2_{-1}\} \leq u < \overline{u}$.

Proposition 2.3 *In the symmetric investment equilibrium welfare, as measured by (2.13), is higher than in any symmetric underinvestment equilibrium.*

The intuition behind the result is the following. Even in a second best world where two firms compete à la Cournot and the quality level is constrained to be identical for both firms, the noncollusive symmetric "investment" equilibrium exhibits too low a level of investment; this is true despite the presence of a business stealing effect. The reason is that the duopolists capture only a relatively small part of the surplus from R&D or advertising, and thus invest too little. Consequently, the problem of underinvestment is even more severe in any collusive "underinvestment" equilibrium.

2.5 Spillovers and Patents

So far we have assumed that a firm's investment cost function, $F(u^i_t; u^i_{t-1})$, depends on its own quality level only. However, spillover effects are a pervasive phenomenon in many markets. For instance, a firm might be able to copy cheaply its rival's technology. The rationale for patents is, of course, to prevent such free-riding; but, in practice, firms can often "invent around" existing patents. (For a survey on spillovers and R&D, see De Bondt (1996).) Similarly, it might be profitable for a firm to imitate the design and packaging of a rival brand so as to free-ride on the rival's advertising outlays.

To highlight the effects of spillovers on the incentives for firms to invest and to collude, we make the following clearcut assumption: There are no immediate spillovers, but full spillovers after one period. More precisely, a firm can costlessly "copy" its rival's quality of last period. Consequently, firm i's investment cost function, (2.2), is now replaced by

$$F(u_t^i; u_{t-1}^1, u_{t-1}^2) = \begin{cases} F_0 \left(u_t^i\right)^\beta - F_0 \left[\max\left\{u_{t-1}^1, u_{t-1}^2\right\}\right]^\beta & \text{if } u_t^i > \max\left\{u_{t-1}^1, u_{t-1}^2\right\} \\ 0 & \text{otherwise.} \end{cases}$$

This can be thought of, for instance, as firms having one-period patent protection when imitation is virtually costless. Alternatively, there might be no patent protection but a time-lag of imitation.[14]

Intuition might suggest that the presence of spillover effects makes tacit collusion "easier" (supportable for lower discount factors) since, in the long run, firms will always end up in a symmetric state. Hence, by deviating to a quality level above the symmetric investment quality, $\overline{u}$, a firm can no longer ensure itself higher stage-2 profits *ad infinitum* than in any (symmetric) underinvestment equilibrium. The proposition below shows, however, that the opposite result holds.

In the presence of spillovers, a firm's stationary reaction function has to be defined differently since it will generally be optimal for a firm to install at least last period's maximum quality. Firm i's stationary best reply to u_0^j, $u^*(u_0^j)$, is now the solution to the following dynamic optimisation problem:

$$\max_{\{u_\tau^i\}} \sum_{\tau=0}^{\infty} \delta^\tau \Pi^i \left(u_\tau^1, u_\tau^2\right),$$

where $u_\tau^j = \max\left\{u_{\tau-1}^1, u_{\tau-1}^2\right\}$ for $\tau \geq 1$, and $\Pi^i\left(u_\tau^1, u_\tau^2\right) = \pi^i\left(u_\tau^1, u_\tau^2\right) - F(u_\tau^i; u_{\tau-1}^1, u_{\tau-1}^2)$. The interior stationary best reply, $\widehat{u}(u_0^j)$, is defined as the unique positive solution[15] to

[14]This is consistent with there being many paths that lead to a given quality level: by investing in R&D, a firm discovers its own path; but a firm has also the option to copy its rival's path which is protected for only one period.

[15]We assume here, as before, that $\beta \geq 2$. If $\beta = 2$ and $u_0^j \geq \sqrt{2S/\beta F_0}$, then $\widehat{u}(u_0^j) = 0$.

the following programme:

$$\max_{u_0^i} S\left(\frac{u_0^i/u_0^j}{u_0^i/u_0^j+1}\right)^2 - F_0(u_0^i)^\beta + F_0[\max\{u_{-1}^1, u_{-1}^2\}]^\beta + \frac{\delta}{1-\delta}\frac{S}{4}, \tag{2.14}$$

where the last term is the discounted sum of stage-2 profits from $\tau = 1$ onwards that arise when both firms offer the same qualities. Clearly, the stationary best reply is given by $u^*(u_0^j) = \max\{\widehat{u}(u_0^j), u_{-1}^1, u_{-1}^2\}$.[16] As in the case without spillovers, one can show that there is a unique intersection of the interior (stationary) best-reply curves, namely at $(\overline{u}, \overline{u})$. The corresponding symmetric quality level, $\overline{u}$, is now

$$\overline{u} = \left(\frac{S}{4\beta F_0}\right)^{\frac{1}{\beta}}.$$

Note that this symmetric quality level equals the one without spillovers, as given by (2.12), when the discount factor is zero. The four regions in the space of qualities, $U^{(1)}$ to $U^{(4)}$, are defined exactly as before.

Since it is always optimal for firm i to set $u_t^i \geq \max\{u_{t-1}^1, u_{t-1}^2\}$, we have to adapt strategy profiles Σ^{esc} and Σ^{coll}. Strategy profile $\Sigma^{esc'}$ is induced by the following set of action rules:

(i) If $(u_{t-1}^1, u_{t-1}^2) \in U^{(1)}$, then $s^i(u_{t-1}^1, u_{t-1}^2) = \max\{u_{t-1}^1, u_{t-1}^2\}$, $i = 1, 2$.

(ii) If $(u_{t-1}^1, u_{t-1}^2) \in U^{(2)}$, then $s^i(u_{t-1}^1, u_{t-1}^2) = u_{t-1}^1$, $i = 1, 2$.

(iii) If $(u_{t-1}^1, u_{t-1}^2) \in U^{(3)}$, then $s^i(u_{t-1}^1, u_{t-1}^2) = \overline{u}$, $i = 1, 2$.

(iv) If $(u_{t-1}^1, u_{t-1}^2) \in U^{(4)}$, then $s^i(u_{t-1}^1, u_{t-1}^2) = u_{t-1}^2$, $i = 1, 2$.

Analogously to Σ^{coll}, strategy profile $\Sigma^{coll'}$ is defined as follows: If $(u_{t-1}^1, u_{t-1}^2) = (u, u)$, then $s^i(u_{t-1}^1, u_{t-1}^2) = u$, $i = 1, 2$; otherwise firms revert to $\Sigma^{esc'}$.

We can now state and prove another main result of our paper.

Proposition 2.4 *In the presence of spillovers, strategy profile $\Sigma^{esc'}$ forms a MPE. However, (symmetric) underinvestment can not be supported as a MPE (by strategy profile $\Sigma^{coll'}$), independently of the discount factor δ.*

[16]Notice that firm i's stationary best reply, $u^*(u_0^j)$, now depends not only on u_{-1}^i, but also on u_{-1}^j. As before, we drop these arguments for notational convenience.

The nonexistence result for underinvestment equilibria can easily be extended to asymmetric underinvestment. The intuition for proposition 2.4 is as follows. The existence of spillover effects reduces each firm's (noncooperative) incentive to invest, given its rival's quality. This implies that any "noncollusive" investment equilibrium, as supported by strategy profile $\Sigma^{esc'}$, exhibits low quality levels in the long run, relative to the case without spillovers. But any underinvestment equilibrium can only be enforced by the credible threat of escalation. In the presence of spillover effects, however, this threat is rather blunt.

Abstracting from strategic issues, in a world where costless imitation is possible, the individual incentives to get ahead of one's rival are exactly the same as in a "myopic" world with or without spillovers, where the discount factor is equal to zero. Now, clearly, if the discount factor were zero, underinvestment could not be an equilibrium outcome since, by definition of underinvestment, there are always short-run gains from some suitable deviation. However, introducing spillovers in our model is not equivalent to reducing the discount factor (to zero). To see this, notice that the optimal "myopic" deviation from state $(u, u) < (\overline{u}, \overline{u})$ is to invest up to quality $\widehat{u}(u)$. Suppose that, indeed, one firm deviates to $\widehat{u}(u)$ in, say, period t. If $\widehat{u}(u) < \overline{u}$, then, in period $t + 1$, *both* firms will invest further, namely up to quality level $\overline{u}$. That is, the optimal myopic deviation requires to invest in both periods t and $t + 1$, whereas the gain in stage-2 profits is confined to period t. Hence, when $\delta > 0$, the optimal deviation in the presence of spillovers is, by continuity, not identical to the optimal myopic deviation. Furthermore, it is a priori not obvious whether, for *large* δ, a profitable deviation exists at all. (Notice, however, that the case $\widehat{u}(u) < \overline{u}$ for $u < \overline{u}$ would not arise if qualities were *global* strategic substitutes.)

Let us compare the equilibrium investment level when there are spillovers to the investment level when there are no spillovers. Clearly, if firms do not collude in the latter case, then the investment level is higher than in the case with spillovers, holding fixed all parameters. But if firms do underinvest in the absence of spillovers, then the equilibrium investment level can be higher in the absence of spillovers. To see this, suppose the current state is given by (u, u) and choose parameters such that $\overline{u} > u$ in the presence of spillovers. Then, if the discount factor is sufficiently close to unity, there exists an equilibrium in the absence of spillovers such that no firms raises its quality level above u. Another interesting comparison is the following. Suppose the discount factor δ is such that, in the absence of spillovers, underinvestment can be supported in equilibrium. Now, if the size of the market in the presence of spillovers is $1/(1-\delta)$ times the market size in the absence of spillovers, then the quality level $\overline{u}$ is the same in both cases. However, proposition 2.4 implies that underinvestment can not be supported in the presence of spillovers. In particular, a deviation from a quality level u, $u < \overline{u}$, to $\overline{u}$ is profitable if spillovers are present and firms use strategy profile $\Sigma^{coll'}$, but unprofitable if there are no spillovers and firms use Σ^{coll} – *although the path induced by the deviation is the same in both cases.* The reason is that market size was assumed to be larger in the presence of spillovers so that the deviant's gain in the period of deviation is larger.

Our result has important implications for the literature on patents. A recurrent theme throughout the whole literature is that patents give firms higher incentives to invest in R&D, and will hence result in higher equilibrium levels of investment. Now, in a world where technological spillovers are present, one can interpret our model without spillovers as representing the case of infinite patent length and breadth, while the extension with spillovers corresponds to the case of short patent length. As we have shown, a shorter patent length can lead to higher R&D in equilibrium simply because it *reduces* the incentives to invest, and hence destroys the mechanism through which underinvestment can be supported. In light of the welfare analysis conducted in the last section, this suggests that, for any given discount factor, there exists an "optimal" patent length that gives maximal incentives to invest but is just short enough so as to prevent firms from colluding in investment.

Remark. In this section, we have focussed on spillovers in the appropriation of the benefits from investment in R&D or advertising. In particular, we have assumed that spillovers are asymmetric in that technological laggards (or weak brands) profit from the investments of technological (or brand) leaders but *not* vice versa. This is a natural way of modelling spillovers in the present setup, and captures exactly what patent protection is about.

This differs from the way how spillovers are modelled in the literature on R&D cartels and joint ventures in the tradition of d'Aspremont and Jacquemin (1988) and Kamien, Muller, and Zang (1992). In this literature, process innovation is modelled as a static two-stage game. Spillovers directly affect the innovation process and are assumed to be immediate and symmetric: the innovation process of a technological leader benefits as much from the current investments of a technological laggard as the laggard can free-ride on the leader's current effort. When products are substitutes and there are strong positive spillovers, the "cooperative equilibrium" exhibits *higher* investment in R&D than the noncooperative Nash equilibrium, while the opposite result holds when spillovers are negative, or positive but weak. Notice that in these static models, the joint profit maximising "cooperative equilibrium" is *not* a Nash equilibrium.

In a recent paper, Kesteloot and Veugelers (1995) have analysed the standard two-stage model of this literature when it is infinitely repeated. In this setting, tacitly collusive SPE always exist for large discount factors. However, as we have already argued in the Introduction, the supergame framework is not very appropriate for modelling strategic interaction in investment since it fails to capture the commitment value of investment. Kesteloot and Veugelers focus on the question of how the threshold discount factor, $\widehat{\delta}$, above which tacit collusion can be sustained, varies with the magnitude of spillovers. They show that when the strength of the spillover effect is sufficiently high, then an increase in the magnitude of the spillover leads to a rise in $\widehat{\delta}$, i.e. collusion becomes "more difficult" to sustain. This is somewhat in line with our results. However, in the case of strong positive spillovers, welfare in the collusive equilibrium is higher than in the noncollusive one since it exhibits higher levels of investment; this is in contrast to our model. The intuition for their result is the following: the larger are the spillovers, the stronger are the incentives for a firm to invest less and to free-ride on the nondeviant's R&D expenditures. In our model, in contrast, it is always the nondeviant laggard that can free-ride in the following period on the deviant leader.

2.6 Potential Entry

We now turn to the analysis of market structure. In particular, we take up the issue of potential entry that has so far been kept aside in the analysis. The question is whether the high profits the incumbents make while underinvesting will trigger new entry. For this purpose, we extend the basic model by introducing an additional stage in each period at which further entry can occur.

By postulating a sufficiently high entry cost, the modeller could always ensure that it is not profitable for a new firm to enter the market. But for a given entry cost, entry would then still occur in sufficiently large markets. There is, however, an endogenous mechanism which might deter entry: the incumbents' threat of escalation. The aim of this section, therefore, is to investigate whether or not this threat of escalation is credible, and whether it succesfully deters entry, no matter how large the market.This question is of interest for two reasons. First, it relates to the robustness of our two-firm underinvestment equilibrium. Second, it addresses a fundamental issue in the theory of market structure, namely whether or not concentration can be high in large markets.

The basic model is modified as follows. There are three players: the incumbents, firms 1 and 2, and a potential entrant, firm 3. In each period, there are now three stages. At stage 1, the potential entrant decides whether to enter or not if it has not yet decided to do so. If firm 3 decides to enter, it has to pay an entry fee ("setup cost") $\epsilon > 0$. At stage 2, the firms that are present in the market (the two incumbents, and firm 3 if it has decided to enter in this period or before) decide simultaneously whether and how much to invest in R&D or advertising. The potential entrant starts up with "zero" quality; its investment cost function in the period of entry is given by $F^e(u) = F_0 u^\beta$, and in all subsequent periods by (2.2). There are no spillovers. Finally, at stage 3, firms compete simultaneously in quantities. Consumers' utility is given by the natural extension of (2.1) to three varieties of the quality good. As before, all past actions are assumed to be common knowledge.

The equilibrium analysis proceeds along the lines of section 2.3. In period t, the state of the industry is given by the quality triple $(u_t^1, u_t^2, u_t^3) \in \Re^3$, where we adopt the convention that $u_t^3 = -1$ if firm 3 has not yet entered the market, and $u_t^3 = 0$ if firm 3 has entered the market but not yet invested in quality. A pure investment action rule is a mapping s^i : $(u_{t-1}^1, u_{t-1}^2, u_{t-1}^3) \mapsto u_t^i$; a pure output action rule is a mapping $t^i : (u_t^1, u_t^2, u_t^3) \mapsto x_t^i$. As before, the minimum quality (in order to make positive sales) is equal to one; therefore, the initial investment outlays necessary to produce the basic version of the quality good are equal to F_0.

As to the equilibrium analysis of the output stage, it is straightforward to show that there exists a unique pure strategy Nash equilibrium in quantities, given any state (u_t^1, u_t^2, u_t^3). If firm 3 has not yet entered the market, or not invested, then its stage-3 profit is zero, and the incumbents' equilibrium profits are given by (2.6). Otherwise, firm i's stage-3 equilibrium profits are given by

$$\pi^i(u_t^1, u_t^2, u_t^3) = \begin{cases} S\left(\frac{\sum_{k=1}^3 u_t^i/u_t^k - 2}{\sum_{k=1}^3 u_t^i/u_t^k}\right)^2 & \text{if } \sum_{k=1}^3 \frac{u_t^{\min}}{u_t^k} \geq 2 \\ S\left(\frac{u_t^i/u_t^j}{u_t^i/u_t^j+1}\right)^2 & \text{if } \sum_{k=1}^3 \frac{u_t^{\min}}{u_t^k} < 2 \text{ and } u_t^i, u_t^j > u_t^{\min} \ (\ i \neq j) \\ 0 & \text{if } \sum_{k=1}^3 \frac{u_t^{\min}}{u_t^k} < 2 \text{ and } u_t^i = u_t^{\min}, \end{cases} \tag{2.15}$$

where $u_t^{\min} = \min\{u_t^1, u_t^2, u_t^3\}$.[17] Hence, in the three-firm equilibrium there exists a "quality window" such that a firm makes zero sales if its quality is too low relative to its rivals' qualities. But there will always be at least two firms making positive sales in equilibrium; this explains why we did not find any quality window in the two-firm case. Observe that $\pi^i(u_t^1, u_t^2, u_t^3)$ is continuous in all its arguments, despite the quality window.

The resulting subgame *after* entry of firm 3 can, in principle, be analysed analogously to the two-firm investment game, given the stage-3 profit function (2.15). However, entry is endogenous and might be deterred by the incumbents. We do not attempt here to investigate the three-firm case comprehensively. Rather, we focus on the question whether or not the two incumbents can be in a two-firm underinvestment equilibrium, and successfully deter entry by credibly threatening to engage in an escalation of R&D or advertising outlays in case of entry. The following proposition summarises our results.

Proposition 2.5 *There exists a $\widehat{\beta} > 2$ such that if $\beta \in [2, \widehat{\beta}]$, any two-firm underinvestment equilibrium is stable with respect to entry by a third firm. In particular, for δ sufficiently large, there exists a MPE such that $(u_\tau^1, u_\tau^2, u_\tau^3) = (u, u, -1)$ for all τ, with $u \leq \overline{u}$. This is true independently of market size and entry costs.*

We have thus shown that the same mechanism that supports underinvestment in equilibrium can be sufficient to deter further entry. The proposition illustrates that concentration can be high even in very large markets. Remark that, if $\beta \in [2, \widehat{\beta}]$, the (symmetric) investment equilibrium is also stable with respect to entry by a third firm but, of course, even without the threat of further escalation.

[17]For a general proof of the n-firm case, see the proof of Lemma 2.3 in the Appendix.

2.7 Market Structure and Nonconvergence

In the early literature on industrial market structure, the alleged negative relationship between market size and concentration, even though noted by some authors, has not received much attention. From a theoretical viewpoint, such a negative relationship was considered to be quite obvious: for a given level of "barriers to entry", an increase in market size should raise the profitability of incumbent firms, and thus trigger new entry, which would lead to a fall in concentration. However, the empirical evidence from cross-sectional studies was found to be rather weak.

It is only quite recently that the size-structure relationship has become a major focus of research. In his landmark book, "Sunk Costs and Market Structure", Sutton (1991) shows that the alleged size-structure relationship breaks down in certain groups of industries. In particular, Sutton makes the important distinction between "exogenous" and "endogenous" sunk cost industries. In exogenous sunk cost industries, the only sunk costs involved are the exogenously given setup costs; R&D and advertising outlays are insignificant. In endogenous sunk cost industries, on the other hand, the equilibrium level of sunk costs is endogenously determined by firms' investment decisions. Roughly, these are industries in which advertising or R&D "works" in that investments in some fixed outlays raise the consumers' willingness-to-pay (or reduces marginal costs of production). Sutton's predictions are that in exogenous sunk cost industries, the lower bound to concentration tends to zero as the market becomes large, whereas in industries for which the endogenous sunk cost model applies, the lower bound to concentration is bounded away from zero.[18]

The "fragmentation" result for exogenous sunk cost industries can be illustrated by reference to a simple two-stage game. At stage 1 ("entry stage"), firms decide simultaneously or sequentially whether or not to enter the industry. If they decide to do so, they have to pay an entry fee $\epsilon > 0$. At stage 2 ("output stage"), the firms that have entered the market compete in prices or quantities, according to some static oligopoly model. Firm i's stage-2 equilibrium payoff can be summarised by some reduced-form profit function $S\pi^i(n(S,\epsilon))$, where S denotes market size, and $n(S,\epsilon)$ the number of entrants. It is assumed that the number of potential entrants, $n_0(S,\epsilon)$, is sufficiently large; that is,

[18]This result follows from an exercise in comparative statics with respect to market size. Notice that it is *not* assumed that market size increases *over time*.

$n_0(S, \epsilon) > n(S, \epsilon)$ ("free entry"). For a wide class of standard oligopoly models describing competition in the output stage, the equilibrium number of firms in the market tends to infinity as the market becomes large, i.e. $n(S, \epsilon) \to \infty$ as $S \to \infty$, and the market share of each firm converges to zero.[19]

In the endogenous sunk cost model, there is a further stage in which active firms make sunk investments in, say, R&D or advertising. The resulting game consists of three stages: the entry stage, the investment stage, and the output stage. The "nonfragmentation" or "nonconvergence" result states that, under some general conditions, the market share of the largest firm is bounded away from zero in *any* equilibrium. In some models a stronger result obtains: the number of active firms remains finite in the limit when $S \to \infty$. The reason is that, as the market becomes large, firms engage in an escalation of investment outlays which makes it increasingly expensive for rivals to capture a positive market share. Sutton calls this the "escalation mechanism".[20]

However, the nonconvergence result has been obtained almost solely in *static* stage games.[21] According to Sutton (1998), the open question is whether it still holds in dynamic investment games like ours. What is at issue is that the existence of underinvestment equilibria in our dynamic game implies that firms do not necessarily engage in an escalation of R&D or advertising outlays; but without an escalation mechanism at work, the nonconvergence property can not hold. To make this point clear, let us consider the following example. Suppose that, for a given market size S, there exists a symmetric underinvestment equilibrium in which all $n(S, \epsilon)$ active firms offer quality u in each period, where $n(S, \epsilon)$ is such that any additional entrant would make an overall loss. Now, if this underinvestment equilibrium still holds under free entry when the market becomes large, then we are back in an "exogenous sunk cost world", in which each firm has to pay an

[19]For another class of models (which allow, for instance, for multiproduct firms), multiple equilibria are endemic. The same model may permit fragmented equilibria, in which, for example, each firm offers one product, and concentrated equilibria, in which a single firm is crowding out the product space. The more general fragmentation result refers, therefore, to the *lower bound* to concentration. This has been dubbed the "bounds approach" to concentration. For a discussion, see Sutton (1991).

[20]For a precise statement of the conditions under which the nonconvergence property holds, see Shaked and Sutton (1987) and Sutton (1991). In the case of pure vertical product differentiation and price competition, the finiteness result has been first obtained by Shaked and Sutton (1983).

[21]Two exceptions are in Sutton (1998) and Hole (1997).

exogenous setup cost of $\epsilon + F(u)$. Consequently, the nonconvergence result breaks down in this case. (Actually, one could allow quality u to increase with S, and still get that $n(S,\epsilon) \to \infty$ as $S \to \infty$, unless u increases too fast with S.)

Another way of seeing this point is the following. In a static stage game, the nonconvergence property is proved by showing that in a sufficiently large and fragmented market, there always exists a profitable deviation for some firm. This deviation consists in an escalation of fixed R&D or advertising outlays so as to capture a larger share of the market. Now, in a dynamic game such a deviation might not be profitable since it can trigger an escalation of investment spendings by rival firms, which is detrimental for the deviant firm's profit.

As to the result of section 2.6, this can be seen as an example of nonfragmentation in that two firms are able to deter further entry, no matter how large the market, as long as $\beta \in [2, \widehat{\beta}]$. However, this equilibrium is not unique; there is another equilibrium in which the two firms acquiesce, and further entry takes place.

To address the issue of nonconvergence, we have to modify the basic version of our dynamic investment game. The time structure is as in section 2.6; that is, there are three stages in each period: entry, investment, and quantity competition. There is an initial period (say, 0) before which there are no active firms, i.e. *all* firms are potential entrants in period 0. Entry costs as well as the investment cost functions for a new entrant and for an incumbent are as in section 2.6. The consumers' utility function can be generalised in an obvious way to an arbitrary number of firms offering each a variant of the quality good.

As before, the output stage in each period can be analysed as a one-shot game.

Lemma 2.3 *In any given stage 3, there exists a unique (Markov-)Nash equilibrium in quantities. Suppose there are $n(S)$ active firms. Re-label the firms such that firm 1 offers the highest quality, u^1, and firm $n(S)$ the lowest quality, $u^{n(S)}$. Then, in equilibrium, there is a "quality window" such that firms 1 to $\underline{n}(S)$ only make positive sales, where $\underline{n}(S)$ is the maximum integer z, $z \leq n(S)$, such that $\sum_{i=1}^{z} \left(u^z/u^i\right) > z - 1$. Firm i's stage-3* *equilibrium profit is given by*[22]

$$\pi^i(u^1,...,u^i,...,u^{n(S)}) = \begin{cases} S\left(1 - \frac{\underline{n}(S)-1}{\sum_{j=1}^{\underline{n}(S)}(u^i/u^j)}\right)^2 & \textit{if } i \leq \underline{n}(S) \\ 0 & \textit{otherwise.} \end{cases} \tag{2.16}$$

Notice that the above labelling is not unique when two or more firms offer the same quality; in this case, however, each of these firms produces the same amount in equilibrium.

Using equation (2.16) as the reduced-form stage-3 profit function, we can now focus on the analysis of investment strategies. For technical convenience, we restrict attention to equilibria such that all investment, *along the equilibrium path*, occurs in the initial period, when the market opens.[23] This implies, in particular, that the number of active firms remains constant over time. However, we allow for all the "investment", "underinvestment" and "entry deterring" strategies we have considered earlier as well as for much more complex strategies. Of course, we allow strategies and any equilibrium path to depend on market size.

In a dynamic game, the lower bound to concentration for a given market size might be quite different from that in a static game. The open theoretical question is whether or not the asymptotic properties are the same, namely that the lower bound does not converge to zero as market size becomes large. In the section on potential entry, we have already seen that nonconvergence is a *possible* outcome in our model; what is at issue is whether or not it is a *necessary* outcome in that it occurs in all equilibria. The following proposition gives the central result on market structure.

Proposition 2.6 *The nonconvergence property holds in our dynamic game. In particular, in any MPE, the number of active firms, $n(S)$, remains finite as market size tends to infinity.*

The proof of this proposition is rather lengthy, and can be found in the appendix. Here, we just give a sketch of it. We first assume that there exists an equilibrium such

[22]For notational convenience, we describe the current state by the quality tuple of *active* firms only.

[23]This assumption is not essential; its only purpose is to pin down an (arbitrary) equilibrium path. Notice also that there is no "technological" reason why firms should not do all investment in the first period.

that $n(S) \to \infty$ as $S \to \infty$, and then show that this leads to a contradiction. We pick the firm with the lowest quality in equilibrium and calculate an upper bound on its equilibrium profit. Then, we consider a carefully selected deviation for this firm, which is a function of market size and its rivals' qualities. One can easily calculate the deviant's associated profits in the period of deviation. Since we do not make any restrictions on the "punishment strategies", we can not say much, at this level of generality, about the deviant's profit in the periods *after* deviation. What we do know, however, is that these profits are nonnegative. This is sufficient to show that the deviation is profitable for large markets. Hence, the threat of future escalation does not have enough bite to prevent the deviating firm from escalating its investment outlays so as to capture a larger share of the market. This contradicts the initial assumption.

Our result is reassuring in that it shows the robustness of the nonfragmentation result to the existence of underinvestment equilibria in dynamic games. Remark that we have actually shown a "strong version" of the nonconvergence property to hold: the market share of *all* firms is bounded away from zero, no matter how large the market. Note also that proposition 2.6 does *not* imply that underinvestment equilibria necessarily break down when market size becomes large, as we have already seen in section 2.6.

2.8 Conclusion

In this paper, we have explored a dynamic game of investment in R&D or advertising. It is quite distinct from the applied literature on supergames since, in our model, current investments change future market conditions. From a game-theoretic viewpoint, the model is related to the dynamic game of capacity investment by Fudenberg and Tirole (1983). It differs from their continuous-time framework in that firms can leapfrog their rivals. Therefore, the existence of tacitly collusive equilibria is no longer ensured. In the first part of the paper, we have focussed on the issue of existence of underinvestment equilibria when firms have strong incentives to deviate and, thereby, to persistently improve their strategic position. In the second part, we have introduced potential entry into the model so as to address issues of market structure.

Using a state-space approach, we have shown that when strong spillovers in the appropriation of the benefits from investment are present, underinvestment equilibria fail to exist, while the opposite result holds without spillovers. This implies that a weakening in the degree of patent protection can actually lead to more R&D in equilibrium even though (or, rather, because) it reduces the individual incentives to invest. Furthermore, we have shown that underinvestment should be an issue of concern for antitrust authorities in that it unambiguously reduces welfare. This is especially true since detecting tacit collusion in R&D or advertising is likely to be much more difficult than detecting tacit collusion in price setting.

The existence of underinvestment equilibria has raised the question whether one of the central results on market structure in the I.O. literature, the "nonconvergence property" namely, breaks down in dynamic investment games. What has been at issue is that, in an underinvestment equilibrium, firms do not engage in an escalation of fixed investment outlays; but without an escalation mechanism at work, the nonconvergence property can not hold. Our main result on market structure is very reassuring: the nonconvergence property is robust to the existence of underinvestment equilibria.

2.9 Appendix

Proof of lemma 2.1. The first-order condition of (2.10) is given by

$$\varphi\left(u^i \mid u^j\right) \equiv \frac{2S}{1-\delta}\frac{u^i u^j}{(u^i+u^j)^3} - \beta F_0\left(u^i\right)^{\beta-1} = 0. \tag{2.17}$$

Now, observe that $\varphi\left(u^i \mid u^j\right) \to -\infty$ as $u^i \to \infty$, $\varphi\left(0 \mid u^j\right) = 0$. Furthermore, for $u^i > 0$:

$$\varphi\left(u^i \mid u^j\right) > 0 \Leftrightarrow \frac{2S}{(1-\delta)\beta F_0}u^j > \left(u^i\right)^{\beta-2}\left(u^i+u^j\right)^3.$$

The l.h.s. of the last inequality is independent of u^i, and strictly positive for $u^j > 0$. If $\beta > 2$, then the r.h.s. tends to zero as $u^i \to 0$. If $\beta = 2$, then in the limit as $u^i \to 0$, the l.h.s. is larger than the r.h.s. if and only if $u^j < \sqrt{2S/(1-\delta)\beta F_0}$. Note also that for $\beta \geq 2$, the r.h.s. is strictly increasing in u^i. Therefore, if $\beta > 2$ (or if $\beta = 2$ and $u^j < \sqrt{2S/(1-\delta)\beta F_0}$), there exists a strictly positive $\widehat{u}(u^j)$ such that $\varphi\left(u^i \mid u^j\right) > 0$ if and only if $u^i < \widehat{u}(u^j)$, and $\varphi\left(u^i \mid u^j\right) < 0$ if and only if $u^i > \widehat{u}(u^j)$. Thus, $\widehat{u}(u^j)$ is the unique strictly positive solution of (2.17), and hence of (2.10). (If $\beta = 2$ and $u^j \geq \sqrt{2S/(1-\delta)\beta F_0}$, however, then $\widehat{u}(u^j) = 0$.)

This obviously implies

$$u^*(u^j) = \max\left\{\widehat{u}(u^j), u^i_{-1}\right\}$$

as long as $\beta \geq 2$. ■

Proof of lemma 2.2. Suppose there exists an intersection of the two interior stationary best-reply curves at $(u^1, u^2) \in (0, \infty)^2$. By definition, $\widehat{u}(u^1) = u^2$ and $\widehat{u}(u^1) = u^2$. From lemma 2.1, the associated first-order conditions are given by (2.11), i.e.

$$\frac{2S}{1-\delta}\frac{u^1u^2}{(u^1+u^2)^3} = \beta F_0\left(u^i\right)^{\beta-1}, i = 1, 2.$$

Since the left-hand side is the same for both firms, it follows immediately that $u^1 = u^2$. Simple calculations show that $u^1 = u^2 = \overline{u}$, as given by (2.12). ■

Proof of proposition 2.2. Since the strategy profile Σ^{esc} forms an MPE, it is sufficient to show that there is no single profitable deviation when the current state is given by $(u^1_{t-1}, u^2_{t-1}) = (u, u)$. The proof is organised as follows. We first seek the optimal deviation for any player (due to symmetry, we can confine attention to an arbitrary firm), and then show that the associated net present value of future profits, Π^{dev}, is not larger than the corresponding value in case of nondeviation, Π^{coll}. We distinguish three cases.

Case *(i)*: Firm 1, say, deviates in period t by raising its quality to u', where $u < u' < \overline{u}$; that is, the state moves to (u', u) in period t. According to strategy profile Σ^{coll}, *both* firms will then invest further in period $t+1$, and the state of the industry will be given by $(u^1_\tau, u^2_\tau) = (\overline{u}, \overline{u})$ for all $\tau \geq t+1$. The associated discounted sum of profits for the deviant is equal to

$$\Pi^{dev} = S\left(\frac{u'/u}{u'/u+1}\right)^2 - (1-\delta)F_0(u')^\beta + F_0u^\beta + \frac{\delta}{1-\delta}\frac{S}{4} - \delta F_0\overline{u}^\beta, \tag{2.18}$$

while in case of nondeviation it is given by $\Pi^{coll} = S/[(1-\delta)4]$. Maximising Π^{dev} with respect to u' gives a first-order condition identical to (2.11); hence the condition is sufficient for a maximum. Note, however, that the unique positive solution to (2.11) might be larger than $\overline{u}$. (It is straightforward to show that this is indeed the case when $\overline{u} < (2+\sqrt{5})u$; we are dealing with this case in part (ii) of the proof. Hence, in the following we analyse the case when the reverse inequality holds. By choosing δ sufficiently close to 1 this can always be ensured.) Denote the optimal value of u' by $u'(u)$. Then, from (2.11),

$(1-\delta)F_0[u'(u)]^\beta = (2/\beta)S[u'(u)]^2u/[u'(u)+u]^3$. Substituting $(1-\delta)F_0[u'(u)]^\beta$ and $\overline{u}$ in (2.18) gives

$$\Pi^{dev} = S\left(\frac{u'(u)}{u'(u)+u}\right)^2 - \frac{2S[u'(u)]^2u}{\beta[u'(u)+u]^3} + F_0u^\beta + \frac{\delta}{1-\delta}\frac{S}{4}\left(\frac{\beta-1}{\beta}\right),$$

which is continuous in δ. Now, multiplying both sides by $(1-\delta)$, and taking the limit as δ goes to one, one gets

$$\lim_{\delta\to 1}(1-\delta)\Pi^{dev} = \frac{S}{4}\left(\frac{\beta-1}{\beta}\right) < \frac{S}{4} = \lim_{\delta\to 1}(1-\delta)\Pi^{coll}.$$

Hence, there exists a $\widehat{\delta}^{(i)} < 1$ such that for all $\delta \geq \widehat{\delta}^{(i)}$ deviation is not profitable.

Case *(ii)*: Suppose now that, in period t, firm 1 deviates to a quality u' such that $\widehat{u}^{-1}(u) > u' \geq \overline{u}$. In period $t+1$, firm 2 will then react and raise its quality to $u^*(u') = \widehat{u}(u')$, where $u < \widehat{u}(u') \leq \overline{u}$. Hence, the sequence of states induced by the deviation will be given by $(u_\tau^1, u_\tau^2) = (u', u)$ for $\tau = t$, and $(u_\tau^1, u_\tau^2) = (u', \widehat{u}(u'))$ for $\tau \geq t+1$. The deviant's discounted sum of profits is thus equal to

$$\Pi^{dev} = S\left(\frac{u'/u}{u'/u+1}\right)^2 - F_0(u')^\beta + F_0u^\beta + \frac{\delta}{1-\delta}S\left(\frac{u'/\widehat{u}(u')}{u'/\widehat{u}(u')+1}\right)^2. \qquad (2.19)$$

Maximising this expression with respect to u' yields the first-order condition for optimal deviation:

$$2S\frac{u'u}{[u'+u]^3} - \beta F_0(u')^{\beta-1} + 2S\frac{\delta}{1-\delta}\frac{u'\left[\widehat{u}(u') - u'\frac{d\widehat{u}(u')}{du'}\right]}{[u'+\widehat{u}(u')]^3} = 0, \qquad (2.20)$$

where $\widehat{u}(u')$ is implicitly defined by (2.11), and $d\widehat{u}(u')/du'$ can be obtained by implicit differentiation of (2.11):

$$\frac{d\widehat{u}(u')}{du'} = -\frac{\frac{2S}{1-\delta}\frac{\widehat{u}(u')[\widehat{u}(u')-2u']}{[u'+\widehat{u}(u')]^4}}{\frac{2S}{1-\delta}\frac{u'[u'-2\widehat{u}(u')]}{[u'+\widehat{u}(u')]^4} - \beta(\beta-1)F_0[\widehat{u}(u')]^{\beta-2}}.$$

In order to reduce the dimensionality of the problem, let us define u'_λ such that $\widehat{u}(u'_\lambda) = \lambda u'_\lambda$, where $\lambda \in (0,1]$. For a fixed λ, the first-order condition for the nondeviant's best reply to u'_λ, (2.11), can then be rewritten as

$$\frac{2S}{1-\delta}\frac{u'_\lambda(\lambda u'_\lambda)}{[u'_\lambda+\lambda u'_\lambda]^3} - \beta F_0[\lambda u'_\lambda]^{\beta-1} = 0.$$

Solving for u'_λ gives

$$u'_\lambda = \left(\frac{2S}{(1-\delta)\beta F_0}\frac{1}{\lambda^{\beta-2}(1+\lambda)^3}\right)^{\frac{1}{\beta}}. \qquad (2.21)$$

This enables us to calculate $d\widehat{u}(u')/du'$ locally at $u' = u'_\lambda$, as a function of λ:

$$\left.\frac{d\widehat{u}(u')}{du'}\right|_{u'=u'_\lambda} = -\frac{\lambda(2-\lambda)}{2\lambda - 1 + (\beta-1)(1+\lambda)}, \tag{2.22}$$

which is strictly negative for $\lambda \in (0,1]$ and $\beta \geq 2$: the higher is the deviant's quality, the less will be invested by its rival.

We can interpret firm 1's "optimal deviation problem" as a choice of λ. The deviant's first-order condition, (2.20), for the optimal λ, denoted by λ_δ, can now be written as

$$2S\frac{u'_{\lambda_\delta}u}{[u'_{\lambda_\delta}+u]^3} - \beta F_0(u'_{\lambda_\delta})^{\beta-1} + 2S\frac{\delta}{1-\delta}\frac{u'_{\lambda_\delta}\left[\lambda_\delta u'_{\lambda_\delta} - u'_{\lambda_\delta}\left.\frac{d\widehat{u}(u')}{du'}\right|_{u'=u'_{\lambda_\delta}}\right]}{[\lambda_\delta u'_{\lambda_\delta} + u'_{\lambda_\delta}]^3} = 0,$$

where u'_{λ_δ} and $d\widehat{u}(u')/du'|_{u'=u'_{\lambda_\delta}}$ are given by (2.21) and (2.22), respectively. Multiplying both sides by $(1-\delta)$, taking the limit as δ goes to one, and simplifying, one gets

$$\xi(\lambda_1) \equiv \lambda_1^\beta + \lambda_1^{\beta-1} - \frac{1+\beta}{\beta}\lambda_1 - \frac{\beta-2}{\beta} = 0, \tag{2.23}$$

where $\lambda_1 = \lim_{\delta\to 1}\lambda_\delta$. This is the first-order condition for the optimal λ as a function of β in the limit when $\delta \to 1$. Since the sign of the coefficients in (2.23) changes once if $\beta \geq 2$, Descartes' sign rule tells us that (2.23) has exactly one (strictly) positive root.[24] Now, $\xi(1) = 1/\beta$, $\xi(0) = (2-\beta)/\beta \leq 0$ if $\beta \geq 2$, and $\xi'(0) < 0$. Hence, if $\beta \geq 2$, there exists exactly one $\lambda_1 \in (0,1]$ such that $\xi(\lambda_1) = 0$. Since an increase in u' corresponds to a *decrease* in λ, and $\xi(0) \leq 0$ and $\xi(1) > 0$, (2.23) defines indeed a maximum! The optimal choice of λ, in the limit when $\delta \to 1$, is therefore the unique $\lambda_1 \in (0,1]$ satisfying (2.23).[25] It is straightforward to show that λ_1 is strictly increasing in β,[26] and that $\lambda_1 \to 1$ as

[24]It is straightforward to generalize Descartes' sign rule, which has been developed for polynomials, to the case when the powers are not necessarily integers, but (more generally) rational numbers. To see this, define $\xi(x) \equiv a_0 + a_1x^{b_1} + ... + a_nx^{b_n}$, where $b_i = p_i/q_i$ and $p_i, q_i \in N$. Suppose q is the smallest common denominator of the b_i's. Then, $\xi(x)$ can be rewritten as a polynomial: $\xi(x) = a_0 + a_1y^{\tilde{b}_1} + ... + a_ny^{\tilde{b}_n}$, where $y \equiv x^{1/q}$ and $\tilde{b}_i \equiv qp_i/q_i \in N$. As to irrational β's, one can show that, in our case, $\xi(x)$ has exactly one sign change at some positive x for any real (rational or irrational) $\beta \geq 2$.

[25]Here, we abstract from the lower bound on λ_1, which is given by $u/\widehat{u}^{-1}(u)$.

[26]Implicit differentiation of (2.23) gives

$$\frac{d\lambda_1}{d\beta} = -\frac{(\ln\lambda_1)(\lambda_1^\beta + \lambda_1^{\beta-1}) + (\lambda_1 - 2)/\beta^2}{\beta\lambda_1^{\beta-1} + (\beta-1)\lambda_1^{\beta-2} - (\beta+1)/\beta}.$$

Clearly, the numerator of the r.h.s. expression is negative for $\lambda_1 \leq 1$. As to the denominator, (2.23) implies

$\beta \to \infty$.[27].

Substituting u' in (2.19) by u'_{λ_1}, as given by (2.21), and substituting $\widehat{u}(u')$ by $\lambda_1 u'_{\lambda_1}$, multiplying both sides of (2.19) by $(1-\delta)$, and taking the limit as $\delta \to 1$, yields

$$\begin{aligned}
\lim_{\delta\to 1}(1-\delta)\Pi^{dev} &= \frac{S}{(1+\lambda_1)^2}\left(1-\frac{2}{\beta\lambda_1^{\beta-2}(1+\lambda_1)}\right) \\
&= \frac{S}{(1+\lambda_1)^2}\left(1-\frac{2}{\beta+1+(\beta-2)/\lambda_1}\right) \\
&= \frac{S}{(1+\lambda_1)^2}\left(\frac{(\beta-1)\lambda_1+\beta-2}{(\beta+1)\lambda_1+\beta-2}\right) \equiv \widehat{\Pi}^{dev}(\lambda_1,\beta),
\end{aligned}$$

where the second equality follows from the definition of λ_1 in equation (2.23). Observe that $\partial\widehat{\Pi}^{dev}(\lambda_1,\beta)/\partial\lambda_1 < 0$. To find a suitable lower bound on λ_1, let us define

$$\eta(\lambda) \equiv \lambda^2 - \frac{\lambda}{\beta(\beta-1)} - \frac{\beta-2}{\beta}.$$

If $\beta \geq 2$, there is a unique strictly positive $\widehat{\lambda}_1$ such that $\eta(\widehat{\lambda}_1) = 0$; it is given by $\widehat{\lambda}_1 = (\beta-1)/\beta$. Furthermore, we have $\eta(\lambda) \geq \xi(\lambda)$ for all $\lambda \in (0,1]$, where $\xi(\lambda)$ is defined as in (2.23). Hence, $\widehat{\lambda}_1 \leq \lambda_1$ and $\widehat{\Pi}^{dev}(\widehat{\lambda}_1,\beta) \geq \widehat{\Pi}^{dev}(\lambda_1,\beta) = \lim_{\delta\to1}(1-\delta)\Pi^{dev}$.

Remark that if $\beta = 2$, then $\widehat{\lambda}_1 = 1/2$, and $\widehat{\Pi}^{dev}(1/2,2) = 4S/27 < S/4 = \lim_{\delta\to1}(1-\delta)\Pi^{coll}$. One can show that the total derivative of $\widehat{\Pi}^{dev}(\widehat{\lambda}_1(\beta),\beta)$ with respect to β is positive: the higher is the elasticity of the investment cost function, the higher is the upper bound on the profits from deviation. Finally note that $\widehat{\lambda}_1(\beta) \to 1$ as $\beta \to \infty$, and thus $\widehat{\Pi}^{dev}(\widehat{\lambda}_1(\beta),\beta) \to_{\beta\to\infty} S/4 = \lim_{\delta\to1}(1-\delta)\Pi^{coll}$. Hence, for all $\beta \geq 2$, $\lim_{\delta\to1}(1-\delta)\Pi^{dev} < \lim_{\delta\to1}(1-\delta)\Pi^{coll}$.

Because of continuity in δ, there exists therefore, for any $\beta \geq 2$, a threshold value $\widehat{\delta}^{(ii)} < 1$ such that for all $\delta \geq \widehat{\delta}^{(ii)}$, deviation is not profitable.

Case *(iii)*: Finally, suppose the deviant firm (firm 1, say) preempts any reaction by its rival. That is, in period t, firm 1 chooses a quality level u' such that $\widehat{u}(u') \leq u$; in the induced subgame, the state of the industry will then be given by $(u^1_\tau, u^2_\tau) = (u', u)$ for all $\tau \geq t$.

that $\lambda_1^{\beta} + \lambda_1^{\beta-1} - (1+\beta)\lambda_1/\beta = (\beta-2)/\beta \geq 0$ if $\beta \geq 2$, and hence $\beta\lambda_1^{\beta-1} + (\beta-1)\lambda_1^{\beta-2} - (1+\beta)/\beta > \lambda_1^{\beta-1} + \lambda_1^{\beta-2} - (1+\beta)/\beta \geq 0$ if $\beta \geq 2$. That is, the denominator is positive, and hence $d\lambda_1/d\beta > 0$, for $\beta \geq 2$.

[27]To see this, suppose otherwise that $\lambda_1 \to k < 1$ as $\beta \to \infty$. Then, from (2.23), it follows that $\xi(\lambda_1) \to -k-1 < 0$ as $\beta \to \infty$. But this contradicts the definition of λ_1.

Since $\widehat{u}^{-1}(u) > u^*(u)$ (where the inverse of $\widehat{u}(\cdot)$ is defined over the decreasing part of $\widehat{u}(\cdot)$ only), the deviant firm chooses u' such that $\widehat{u}(u') = u$ so that its rival is *just* preempted. That is, the optimal preemptive deviation, u', is implicitly defined by

$$\phi(u') \equiv \frac{2S}{1-\delta}\frac{uu'}{(u+u')^3} - \beta F_0 u^{\beta-1} = 0. \tag{2.24}$$

Now, $\phi(u) > 0$ if and only if $u < \overline{u}$ (which is, of course, the relevant case of underinvestment, and can always be ensured by choosing δ sufficiently large), $\lim_{u'\to\infty}\phi(u') = -\beta F_0 u^{\beta-1} < 0$, and $\phi'(u') < 0$ for all $u' > u/2$. Thus, if $u < \overline{u}$, there exists a unique u', $u' > u$, such that $\phi(u') = 0$. Define $\psi(u') \equiv u'/(u+u')$, and note that $\psi(u') \in (1/2, 1)$, and $\lim_{u'\to\infty}\psi(u') = 1$. Equation (2.24) can now be rewritten, and solved for u':

$$u' = \left(\frac{2S}{(1-\delta)\beta F_0}\frac{\psi(u')}{u^{\beta-2}}\right)^{1/2} - u.$$

The discounted sum of profits from deviation is then equal to

$$\Pi^{dev} = \frac{S}{(1-\delta)}\left[\frac{\left(\frac{2S}{(1-\delta)\beta F_0}\frac{\psi(u')}{u^{\beta-2}}\right)^{1/2} - u}{\left(\frac{2S}{(1-\delta)\beta F_0}\frac{\psi(u')}{u^{\beta-2}}\right)^{1/2}}\right]^2 - F_0\left[\left(\frac{2S}{(1-\delta)\beta F_0}\frac{\psi(u')}{u^{\beta-2}}\right)^{1/2} - u\right]^\beta + F_0 u^\beta,$$

and, hence,

$$\lim_{\delta\to 1}(1-\delta)\Pi^{dev} = \begin{cases} -\infty & \text{if } \beta > 2 \\ 0 & \text{if } \beta = 2 \\ S & \text{otherwise.} \end{cases}$$

If $\beta \geq 2$, there exists therefore a $\widehat{\delta}^{(iii)} < 1$ such that for all $\delta \geq \widehat{\delta}^{(iii)}$, $\Pi^{dev} < \Pi^{coll}$. ■

Proof of proposition 2.3. Instead of directly comparing welfare in the two "classes" of equilibria under consideration, we opt for a more instructive proof. Let us define the symmetric "second best quality" as the common quality level, u^*, that maximises our welfare measure, (2.13), under the assumption that both firms compete à la Cournot at the output stage. Given any quality level $\widetilde{u}$, with $\max\{u^1_{-1}, u^2_{-1}\} \leq \widetilde{u}$, consider the sequence of quality pairs $(u^1_t, u^2_t) = (\widetilde{u}, \widetilde{u})$ for all $t \geq 0$. From (2.7), (2.8) and (2.12) it follows directly that the sum of discounted consumer utility along this path is given by

$$\sum_{t=0}^{\infty}\delta^t\sum_{l=1}^{N}U^l(u^1_t, u^2_t) = \frac{1}{1-\delta}\sum_{l=1}^{N}\left\{\alpha^l\ln\left(\frac{\alpha^l}{2}\widetilde{u}\right) - \alpha^l\ln c + m^l - \alpha^l\right\},$$

and the sum of discounted net profits by

$$\sum_{t=0}^{\infty}\delta^t\left\{\sum_{i=1}^{2}\Pi^i(u_t^1,u_t^2)\right\}=2\left[\frac{1}{1-\delta}\frac{S}{4}-F_0\widetilde{u}^\beta\right]+F_0\left(u_{-1}^1\right)^\beta+F_0\left(u_{-1}^2\right)^\beta.$$

Maximising net surplus (i.e. the sum of the two expressions above) with respect to $\widetilde{u}$ yields the following first-order condition:

$$\gamma(\widetilde{u})\equiv\frac{S}{1-\delta}\frac{1}{\widetilde{u}}-2\beta F_0\widetilde{u}^{\beta-1}=0,$$

where we have used the fact that $\sum_l \alpha^l=S$. It is easy to see that $\gamma'(\widetilde{u})<0$ for all $\widetilde{u}$, i.e. net surplus is strictly concave in $\widetilde{u}$. Furthermore, $\gamma(1)>0$ if $\overline{u}\geq 1$ as assumed, and $\gamma(\widetilde{u})\to-\infty$ as $\widetilde{u}\to\infty$. Therefore, the second best symmetric quality level is uniquely defined by $\gamma(u^*)=0$, and is equal to

$$u^*=\left(\frac{S}{2(1-\delta)\beta F_0}\right)^{\frac{1}{\beta}}.$$

Since $u^*>\overline{u}>u$, it follows immediately that welfare is higher in the symmetric investment equilibrium than in any symmetric underinvestment equilibrium. ■

Proof of proposition 2.4. The proof of the first assertion is similar to that of proposition 2.1. As to the second assertion, we have to show that there exists a single profitable deviation in state (u,u). We assume, of course, that $u<\overline{u}$. If $\widehat{u}(u)\geq\overline{u}>u$, then a deviation by, say firm 1, to $\widehat{u}(u)$ is profitable by definition of $\widehat{u}(\cdot)$ since then, according to $\Sigma^{coll'}$, the nondeviant's only reaction will be to costlessly copy the deviant's quality in the period after deviation. That is, the state will move from $(\widehat{u}(u),u)$ to $(\widehat{u}(u),\widehat{u}(u))\in U^{(1)}$, and stay there forever. Now, it is easy to see that

$$\begin{aligned}\widehat{u}(u)\geq\overline{u}\quad&\Leftrightarrow\quad 2S\frac{u\overline{u}}{(u+\overline{u})^3}-\beta F_0\overline{u}^{\beta-1}\geq 0\\&\Leftarrow\quad \overline{u}\in[u,(2+\sqrt{5})u].\end{aligned}$$

Hence, if $\overline{u}\leq(2+\sqrt{5})u$, then in state (u,u) a deviation to $\widehat{u}(u)$ is profitable. Let us now turn to the case where $\overline{u}\geq(2+\sqrt{5})u$ and consider a period-t deviation by firm 1 to quality level $\overline{u}$. The deviation induces the following sequence of states: $(u_t^1,u_t^2)=(\overline{u},u)$, and $(u_\tau^1,u_\tau^2)=(\overline{u},\overline{u})\in U^{(1)}$ for all $\tau\geq t+1$. Using simple algebra, one can show that $\Pi^{dev}>\Pi^{coll}$ if and only if

$$S\left(\frac{\overline{u}/u}{1+\overline{u}/u}\right)^2-F_0\overline{u}^\beta+F_0u^\beta+\frac{\delta}{1-\delta}\frac{S}{4}>\frac{1}{1-\delta}\frac{S}{4}$$

$$\begin{aligned}
&\Leftrightarrow \quad S\left(\frac{\overline{u}/u}{1+\overline{u}/u}\right)^2 > \frac{S}{4}\left(\frac{\beta+1}{\beta}\right) - F_0 u^\beta \\
&\Leftarrow \quad \beta > \frac{1}{\left[4\left(\frac{\overline{u}/u}{1+\overline{u}/u}\right)^2 - 1\right]}.
\end{aligned}$$

For a given β, the l.h.s. of the last inequality is independent of u and $\overline{u}$, while the r.h.s. is strictly decreasing in the ratio $\overline{u}/u$, for $\overline{u} \geq u$. Since $\overline{u} \geq (2+\sqrt{5})u$, we are done if we can show that

$$\beta > \frac{1}{\left[4\left(\frac{2+\sqrt{5}}{1+2+\sqrt{5}}\right)^2 - 1\right]} = 0.61803.$$

But this inequality holds by assumption. Thus, if $\overline{u} \geq (2+\sqrt{5})u$ a deviation to $\overline{u}$ is profitable. ■

Proof of proposition 2.5. Suppose that, at date t, the state of the industry is given by $(u, u, -1)$, where $1 \leq u < \overline{u} = [S/(4(1-\delta)\beta F_0]^{1/\beta}$. We want to show that there exists an MPE such that $(u^1_\tau, u^2_\tau, u^3_\tau) = (u, u, -1)$ for all $\tau \geq t$. Consider the following threat by the two incumbents: if entry is observed in any period, then firms 1 and 2 immediately engage in R&D or advertising, and "jump" to the two-firm symmetric investment equilibrium quality level $\overline{u}$, as given by (2.12). We have to show that this threat is indeed credible, and that it deters entry by firm 3.

The proof strategy is as follows. We do *not* fully specify a strategy profile and show that it forms indeed an MPE. Rather, we refer to the existence result of (possibly mixed strategy) MPE in infinite-horizon games[28], and prove that in all mixed strategy equilibria there cannot possibly be a profitable deviation from the proposed equilibrium *path*. Notice that firms do not randomise along the proposed equilibrium path.

Suppose that, in period t, firm 3 enters the market, i.e. the state of the industry, at the start of stage 2 in period t, is given by $(u, u, 0)$. We want to show that there exists an MPE, starting from this subgame, that supports the path $(u^1_\tau, u^2_\tau, u^3_\tau) = (\overline{u}, \overline{u}, 0)$ for all $\tau \geq t$.

Suppose firm 3 deviates from this path in an arbitrary period, say t, and invests up to quality level $u' > 0$. In the following subgame, starting from $(\overline{u}, \overline{u}, u')$, we know that there exists a (possibly mixed strategy) MPE, which might involve further investment, but

[28] See, for instance, Fudenberg and Tirole (1991).

certainly no "disinvestment". We want to calculate an upper bound on firm 3's discounted stage-3 profits from this deviation. Since a firm's stage-3 profit is decreasing in its rivals' qualities, firm 3's payoff is clearly maximal if its rivals will never invest again. Given that firms 1 and 2 will never invest above quality level $\overline{u}$, we want to determine firm 3's optimal deviation $u' > 0$. Note first that in this case it is optimal to invest up to u' in period t, and then cease investing forever. Neglecting the quality window for a moment, firm 3's optimisation programme in period t can thus be written as

$$\max_{u'} \frac{S}{1-\delta}\left(\frac{2u'/\overline{u}-1}{2u'/\overline{u}+1}\right)^2 - F_0\left(u'\right)^\beta. \tag{2.25}$$

The first-order condition is given by

$$\frac{8S}{1-\delta}\frac{(2u'-\overline{u})\overline{u}}{(2u'+\overline{u})^3} - \beta F_0\left(u'\right)^{\beta-1} = 0.$$

Defining $\lambda \equiv u'/\overline{u}$, and using (2.12), the first-order condition can be simplified to

$$\psi(\lambda) \equiv 32\frac{2\lambda-1}{(2\lambda+1)^3} - \lambda^{\beta-1} = 0. \tag{2.26}$$

Observe that $\psi(0) = -32$, $\psi(1) = 5/27$, and $\psi(\lambda) \to -\infty$ as $\lambda \to \infty$. Therefore, (2.26) has at least two strictly positive roots: $\lambda_1 \in (0,1)$ and $\lambda_2 \in (1,\infty)$. Equation (2.26) can be rewritten as follows

$$-8\lambda^{\beta+2} - 12\lambda^{\beta+1} - 6\lambda^\beta - \lambda^{\beta-1} + 64\lambda - 32 = 0.$$

In this expression, the signs of the coefficients change twice; from Descartes' sign rule we thus know that there are either 0 or 2 positive roots. Hence, equation (2.26) has exactly two positive roots: $\lambda_1 \in (0,1)$ corresponds to a local minimum of (2.25), $\lambda_2 \in (1,\infty)$ to a local maximum. This implies that the solution to (2.25) is either $u' = 0$ ("no-investment") or $u' = \lambda_2\overline{u}$ ("investment"). Neglecting the sunk entry fee, the entrant's net present value of future profits from no-investment is zero, while from investment it is equal to

$$\begin{aligned} & \frac{S}{1-\delta}\left(\frac{2\lambda_2-1}{2\lambda_2+1}\right)^2 - F_0\lambda_2^\beta\frac{S}{4(1-\delta)\beta F_0} < 0 \\ \Leftrightarrow \quad & \beta < \frac{8\lambda_2}{4\lambda_2^2-1}, \end{aligned} \tag{2.27}$$

where we used the fact that, from (2.26), $\lambda_2^\beta = \lambda_2 \cdot 32(2\lambda_2-1)/(2\lambda_2+1)^3$. Now, if $\beta = 2$, then $\lambda_2 = 1.1557$, and the corresponding sum of discounted profits is negative; if $\beta \to \infty$, then $\lambda_2 \to 1$, and (2.27) is violated, i.e. the net present value of profits is positive. Thus, there exists a $\widehat{\beta} > 2$, such that if $\beta \in [2, \widehat{\beta}]$, firm 3 can not gain by deviating from the proposed equilibrium path.

Remark that the quality window does not change the above argument. Since $\pi^i(u_t^1, u_t^2, u_t^3)$ is continuous in all arguments and (weakly) decreasing in u_t^j, $j \neq i$, the upper bound on firm 3's payoff from deviation can still be calculated by assuming that the two incumbents will not invest above $\overline{u}$. But given that the incumbents' quality is $\overline{u}$, they will make positive sales in equilibrium, independently of firm 3's quality.[29]

Now, we have to show that neither of the two incumbents has an incentive to deviate from the proposed path $(u_\tau^1, u_\tau^2, u_\tau^3) = (\overline{u}, \overline{u}, 0)$ for all $\tau \geq t$, once firm 3 has entered the industry. But this follows immediately from proposition 2.1 and the fact that an incumbent's profit is (weakly) decreasing in firm 3's quality.

Hence, we have shown that, if $\beta \in [2, \widehat{\beta}]$, there exists an MPE that supports the sequence of states $(u_\tau^1, u_\tau^2, u_\tau^3) = (\overline{u}, \overline{u}, 0)$ for all $\tau \geq t$ starting from state $(u, u, 0)$. Along this path firm 3's discounted profit is equal to zero. Firm 3 will, therefore, optimally not enter in the first place, and save the entry fee ϵ. That is, if $\beta \in [2, \widehat{\beta}]$, the equilibrium path is given by $(u_\tau^1, u_\tau^2, u_\tau^3) = (u, u, -1)$ for all τ. Conditional on firm 3 not entering, this path can be supported as an MPE by strategy profile Σ^{coll} from section 2.3. ■

Proof of lemma 2.3. The analysis of the $n(S)$-firm case proceeds analogously to that of the 2-firm case (see subsection 2.3.1). Denote by I the set of firms with positive equilibrium market share, i.e. $I \equiv \{i = 1, ..., n(S) \mid x^i > 0\}$. Since each consumer chooses the variant of the quality good with the highest quality-price ratio, all firms with positive sales must exhibit the same quality-price ratio in equilibrium. That is, firm j's equilibrium price is given by

$$p^j = \frac{u^j}{u^i} p^i$$

for $i, j \in I$, $i \neq j$. Using the definition of total sales, $S = \sum_{i \in I} p^i x^i$, one obtains

$$p^j = \frac{u^j S}{\sum_{i \in I} u^i x^i}.$$

[29]One might think that firm 3 would gain if it invested heavily so that one of the incumbents falls out of the quality window. However, firm 1, say, would only make zero sales with quality $\overline{u}$ if firm 2 invested up to $u'' > \overline{u}$ such that $\overline{u}/u' + \overline{u}/u'' \leq 1$, assuming of course that $u' > \overline{u}$. But it is easy to see that then $\pi^3(\overline{u}, u'', u') < \pi^3(\overline{u}, \overline{u}, u')$.

Firm j's stage-3 profit can then be written as

$$x^j \left(\frac{u^j S}{\sum_{i\in I} u^i x^i} - c \right).$$

This expression is strictly concave in x^j, equal to zero at $x^j = 0$, and tends to $-\infty$ as $x^j \to \infty$. Its first derivative is strictly positive at $x^j = 0$ if and only if $u^j S > c \sum_{i\in I} u^i x^i$. Thus, the following first-order condition yields a unique interior maximum if $u^j S > c \sum_{i\in I} u^i x^i$:

$$\frac{S}{\sum_{i\in I} u^i x^i} - \frac{x^j u^j S}{\left(\sum_{i\in I} u^i x^i\right)^2} = \frac{c}{u^j}. \tag{2.28}$$

This gives x^j as a function of "weighted" aggregate output:

$$x^j = \frac{\sum_{i\in I} u^i x^i}{u^j} \left(1 - \frac{c \sum_{i\in I} u^i x^i}{u^j S} \right). \tag{2.29}$$

Summing (2.28) over all firms with positive market share, one gets

$$\sum_{i\in I} u^i x^i = \frac{S\,(\#I - 1)}{c\,\sum_{i\in I} \frac{1}{u^i}},$$

where $\#I$ denotes the number of elements in I. Inserting the r.h.s. expression into (2.29), we obtain firm j's equilibrium output, price and profit:

$$x^j = \frac{S\,(\#I - 1)}{c\,\sum_{i\in I} \frac{u^j}{u^i}} \left(1 - \frac{\#I - 1}{\sum_{i\in I} \frac{u^j}{u^i}} \right),$$

$$p^j = c \frac{\sum_{i\in I} \frac{u^j}{u^i}}{\#I - 1},$$

and

$$\pi^j(u^1, ..., u^{n(S)}) = S \left(1 - \frac{\#I - 1}{\sum_{i\in I} \frac{u^j}{u^i}} \right)^2$$

provided that $\sum_{i\in I} \frac{u^j}{u^i} > \#I - 1$, and $x^j = \pi^j = 0$ otherwise.[30]

It remains to show that, in *any* equilibrium, $I = \{1, ..., \underline{n}(S)\}$ and, hence, $\#I = \underline{n}(S)$. First, notice if $\sum_{i\in I\cup\{j\}} \frac{u^j}{u^i} > \#(I \cup \{j\}) - 1$, then $x^j > 0$, i.e. $j \in I$, in equilibrium; otherwise firm j could profitably deviate by producing $x^j = (S/c) \left(\#I / \sum_{i\in I\cup\{j\}} (u^j/u^i) \right) \times \left(1 - \left(\#I / \sum_{i\in I\cup\{j\}} (u^j/u^i) \right) \right)$. We now prove that there can not be an equilibrium in which a product of some quality has zero market share while another offering of lower

[30]Remark that, in equilibrium, the condition for positive output is equivalent to the condition for an interior solution, $u^j S > c \sum_{i\in I} u^i x^i$.

quality makes positive sales. That is, there are no firms k and l, $k < l$ such that $k \notin I$ and $l \in I$. To see this, suppose otherwise. From $k \notin I$, it follows that $\sum_{i \in I \cup \{k\}} (u^k/u^i) \leq \#I$, and from $l \in I$ that $\sum_{i \in I} (u^l/u^i) > \#I - 1$. It is easy to show that these two inequalities lead to a contradiction. This completes the proof. ■

Proof of proposition 2.6. Suppose that there exists an equilibrium such that $n(S) \to \infty$ as $S \to \infty$. Below, we will prove that, for S sufficiently large, there will then exist a profitable deviation for some firm, contradicting the existence of such an equilibrium.

Consider firm $n(S)$. Remark first that $n(S) = \underline{n}(S)$; otherwise, if $n(S) > \underline{n}(S)$, firm $n(S)$'s stage-3 profit would be nil in each period, and firm $n(S)$'s profitable deviation would be not to enter the market. Now, oberserve that firm $n(S)$'s discounted sum of profits, prior to entry, is bounded above by

$$B(S) \equiv \frac{S}{(1-\delta)[n(S)]^2} - F_0 \left(u_S^{n(S)}\right)^\beta - \epsilon,$$

where we allow quality $u_S^{n(S)}$ to depend directly on S. For notational convenience, we will henceforth drop the subscript S. In equilibrium, clearly, $B(S) \geq 0$, and hence

$$\frac{(1-\delta)\epsilon}{S} + (1-\delta) F_0 \frac{\left(u^{n(S)}\right)^\beta}{S} \leq \frac{1}{[n(S)]^2}.$$

Since, by assumption, $n(S) \to \infty$ as $S \to \infty$, it follows that

$$S \left(u^{n(S)}\right)^{-\beta} \to_{S \to \infty} \infty. \tag{2.30}$$

Now, consider the investment stage in an arbitrary period. Suppose that firm $n(S)$ deviates and invests up to quality level $u' > u^{n(S)}$, where u' is allowed to depend on S. Then, a sufficient condition for this deviation to be profitable is given by

$$\frac{S}{(1-\delta)[n(S)]^2} < S \left(1 - \frac{n(S) - 1}{\sum_{i=1}^{n(S)-1} \frac{u'}{u^i} + 1}\right)^2 - F_0 \left(u'\right)^\beta + F_0 \left(u^{n(S)}\right)^\beta.$$

The expression on the l.h.s. is an upper bound on the discounted sum of stage-3 profits from nondeviation. The first term on the r.h.s. is a lower bound on stage-3 payoffs from deviation, and the remaining terms correspond to investment costs. (Actual payoffs from deviation might be higher for two reasons: firstly, the deviant firm might get positive stage-3 profits in future periods as well, and not only in the period of deviation, and secondly, the deviation might induce low-quality firms (such as firm $n(S)-1$) to fall out of the quality window.)

Let us now consider the following deviation:

$$u' = \left(Su^{h(n(S))}\right)^{\frac{1}{\beta+1}},$$

where $u^{h(n(S))}$ is the harmonic mean of firm $n(S)$'s rival qualities, i.e.

$$u^{h(n(S))} \equiv \frac{n(S)-1}{\sum_{i=1}^{n(S)-1} \frac{1}{u^i}}.$$

Notice that $u^{h(n(S))} \geq u^{n(S)} \geq 1$, and that $\left(Su^{h(n(S))}\right)^{\frac{1}{\beta+1}} > u^{n(S)}$ for S sufficiently large. Furthermore, $u^{n(S)}/\ u^{h(n(S))} \to 1$ as $S \to \infty$ since firm $n(S)$'s stage-3 profit is positive in equilibrium. This, in conjunction with (2.30), implies that $\lim_{S\to\infty} S\left[u^{h(n(S))}\right]^{-\beta} = \infty$. The sufficient condition for the deviation to be profitable can be written as

$$\begin{aligned}\frac{1}{(1-\delta)[n(S)]^2} \quad < \quad & \left(1 - \frac{1}{\left(S\left[u^{h(n(S))}\right]^{-\beta}\right)^{\frac{1}{\beta+1}} + \frac{1}{n(S)-1}}\right)^2 \\ & -S^{-1}F_0\left(Su^{h(n(S))}\right)^{\frac{\beta}{\beta+1}} + S^{-1}F_0\left(u^{n(S)}\right)^{\beta}.\end{aligned}$$

Remark that the l.h.s. of this inequality converges to zero as market size tends to infinity. Furthermore, it is straightforward to see that

$$\lim_{S\to\infty}\left\{\left(1 - \frac{1}{\left(S\left[u^{h(n(S))}\right]^{-\beta}\right)^{\frac{1}{\beta+1}} + \frac{1}{n(S)-1}}\right)^2 - S^{-1}F_0\left(Su^{h(n(S))}\right)^{\frac{\beta}{\beta+1}}\right\} = 1$$

since $\lim_{S\to\infty} S\left[u^{h(n(S))}\right]^{-\beta} = \infty$. Hence, for S sufficiently large, the r.h.s. of the above inequality is larger than the l.h.s.; that is, there exists a profitable deviation in large markets. ∎

Chapter 3

Cartel Stability under Capacity Constraints: The Traditional View Restored

3.1 Introduction

For a long time, economists have believed that cartels will tend to break down under the pressure of low demand and high excess capacity. In the second edition of his famous work, "Industrial Market Structure and Economic Performance", Scherer (1980) states:

> There is evidence that industries characterized by high overhead costs are particularly susceptible to pricing discipline breakdowns when a cyclical or secular decline in demand forces member firms to operate well below designed plant capacity.[1]

Examples of industries mentioned by Scherer include chemicals, steel, aluminium, cement and mining. According to this view, we should thus expect to find a negative correlation between the level of excess capacity and the degree of collusion.[2] This, in turn, should reinforce the tendency of prices to be high during booms and low during recessions.

[1] See Scherer (1980), page 206.

[2] See Phlips (1995), page 152, for another statement of this traditional view.

However, recent supergame-theoretic contributions appear to show that the traditional view is theoretically ill-founded. For instance, Brock and Scheinkman (1985) analyse an infinitely repeated price-setting game, in which each firm faces the same capacity constraint. They investigate the situation where all firms in the industry try to sustain "full collusion", i.e. the monopoly price, by the threat of an infinite Nash reversion in case of deviation. In particular, Brock and Scheinkman analyse how the threshold discount factor, above which the monopoly price can be sustained in equilibrium, varies with the level of capacity in the industry. They show that an increase in the industry level of capacity has two opposing effects. On the one hand, it will make cheating more profitable in terms of current profits; on the other, it implies that a more severe punishment can be inflicted on a deviating firm. As a result, the threshold discount factor is generally a nonmonotonic function of industry capacity. Therefore, the model does not give any sharp predictions as to the relationship between the level of capacity (or demand) and the equilibrium price under collusion.

For all its merits, the following two questions arise regarding such a supergame-theoretic approach. First, in supergames, there exists a continuum of more or less collusive equilibria, among which the literature usually selects a particular one. However, there may be other, equally plausible, equilibria with quite different properties.[3] Second, the supergame-theoretic approach to collusion focusses on the problem of enforcement of collusive behaviour, that is, on firms' "incentive constraints". What this approach leaves out, are firms' "participation constraints": it can not explain why many real world cartels do not comprise all firms in the industry, the OPEC being a famous example.[4]

There is another strand in the literature on cartel stability, which takes a quite different route. The seminal papers in this literature are Selten (1973) and d'Aspremont et al.

[3] In particular, the literature mostly confines attention to the "best" symmetric subgame perfect equilibrium, which requires firms to inflict the worst possible punishment on any deviator. The well-documented cases of price wars do not appear to support the existence of such severe punishments. See, for instance, Genesove and Mullin (1998) on the Sugar Institute, and Levenstein (1997) on the Bromine Cartel.

[4] The continuum of equilibria in supergames allows us to select equilibria such that only a subset of firms collude. But mere selection of equilibria does not explain why the number of colluding firms is larger in some situations than in other. In fact, the literature usually selects the best symmetric subgame perfect equilibrium. By selecting a *symmetric* equilibrium, it is simply assumed that, in equilibrium, either all firms participate in the cartel or none. Clearly, the *best* symmetric equilibrium is such that all participate.

(1983). These papers investigate cartel stability in static models. By their very nature, these models leave unexplained why cartel members do not cheat on a cartel agreement; they may, therefore, be viewed as models of explicit rather than implicit collusion. In contrast to the supergame-theoretic literature, these papers focus on firms' "participation constraints". At the heart of this literature lies the trade off between participation and nonparticipation in a cartel: on the one hand, a firm has an incentive to join the cartel so as to achieve a more collusive outcome; on the other, it has an incentive to stay out of the cartel so as to take a free ride on the cartel's effort to restrict output.[5]

The main aim of the present paper is to develop a theoretical foundation of the traditional view on the relationship between cartel stability and capacity levels. The paper follows the tradition of Selten (1973) and d'Aspremont et al. (1983) in focussing on the issue of participation in a cartel. As in all static models of cartel stability, it is simply assumed that cartel rules on pricing or output setting can be enforced.[6] Alternatively, when joining the cartel, firms are assumed to delegate output decisions to some "cartel manager". This leaves the study of the interaction between incentive and participation constraints for future research.

We consider a two-stage game in which firms, each being subject to the same capacity constraint, first decide whether or not to join the cartel, and then compete in quantities.[7] Since cartel members internalise the externalities they impose on each other, they will tend to produce less than fringe firms, which take a free ride on the cartel's effort to restrict output. Now, the size of the cartel and, hence, the degree of collusion, is determined by the relative incentives to join the cartel rather than the fringe. Intuitively, if the level of demand is rather high, or the industry level of capacity rather low, then free-riding on

[5]Here, "participation constraints" refer to firms' incentive to join the cartel rather than the "fringe", whereas "incentive constraints" refer to the incentives of cartel members only to cheat on the cartel agreement.

[6]This applies also to models of adverse selection in cartel formation, such as Roberts (1985) and Kihlstrom and Vives (1992). In these models, colluding firms may not truthfully announce their costs; cheating at the output stage is not considered.

[7]The main reason why we consider quantity rather than price competition is the nonexistence of pure strategy equilibria in games of price competition under capacity constraints. The notion of a market price is not well-defined in these games. Hence, it is not quite clear what is meant by the relationship between demand or capacity and the collusive equilibrium price.

the cartel is not very profitable since fringe firms will face a binding capacity constraint in equilibrium. Therefore, the lower (higher) is the level of demand (capacity), the more attractive it is to join the fringe, and the more unstable the cartel becomes. The model thus predicts a positive (negative) relationship between the equilibrium cartel size and the level of demand (capacity). This reinforces the positive (negative) relationship between the collusive equilibrium price and demand (capacity) for a given degree of collusion. Introducing heterogeneity in capacities, we show that firms with large capacities have stronger incentives to join the cartel then small firms. The model thus predicts that firm sizes and capacity utilisation rates are negatively correlated across firms.

In their paper, Brock and Scheinkman have studied the comparative statics of cartel stability with respect to the level of capacity; all exogenous variables are constant over time. Following the paper by Rotemberg and Saloner (1986), there also exists a large literature on collusion over the business cycle where the level of demand is assumed to follow some dynamic process. Most of these papers do not consider capacity constraints. This literature investigates, in particular, whether collusive prices tend to vary procyclically or rather countercyclically. However, the results appear to depend quite delicately on the time-series properties of the assumed process. To address the issue of cartel stability over the business cycle, we analyse a very simple dynamic extension of the basic two-stage game. We show that collusive prices will tend to vary procyclically, independently of the assumed stochastic process. This, again, restores the traditional view.

3.2 A Simple Model of Cartel Formation and Cournot Competition

Consider the following two-stage game of cartel formation and quantity competition. There are n (identical) firms, each seeking to maximise its own profit; let N denote the set of firms. Each firm in the industry produces the same homogenous good. Firm i's cost of producing quantity Q_i is given by

$$C(Q_i) = \begin{cases} cQ_i & \text{if } Q_i \in [0, K] \\ \infty & \text{if } Q_i > K; \end{cases}$$

that is, all firms face a common unit cost of c up to the capacity level K.

Market demand can be represented by the (twice continuously differentiable) inverse demand function $P(Q/S)$ with $P'(\cdot) < 0$, where $Q \equiv \sum_{i=1}^{n} Q_i$ is total industry output, and S is a measure of market size (or the level of demand). An increase in the size of the market is understood to mean a replication of the population of consumers, leaving the distribution of incomes and tastes unchanged. We put the following, rather mild, restriction on the shape of the demand curve:

$$P'(q) + qP''(q) < 0 \text{ for all } q \in (0, nK/S], \tag{3.1}$$

which implies that, under quantity competition, each firm's best-reply function is downward-sloping; that is, quantities are strategic substitutes. Inequality (3.1) holds, for instance, if demand is concave, i.e. $P''(\cdot) \leq 0$. To exclude the trivial case in which production is not viable, we posit $P(0) > c$. For notational convenience, we define output and capacity levels per unit of market size: $q_i \equiv Q_i/S$, $i = 1, ..., n$, and $k \equiv K/S$.

The timing of the game is as follows. At the first stage ("participation stage"), firms simultaneously decide whether to join the cartel (or coalition) M or the "fringe" $N \backslash M$. Formally, each firm i, $i \in N$, selects a zero-one variable z_i:

$$z_i = \begin{cases} 1 & \text{iff firm } i \text{ joins the cartel } M \\ 0 & \text{iff firm } i \text{ joins the fringe } N \backslash M. \end{cases}$$

In modelling cartel formation as a noncooperative simultaneous-move game, we follow Selten (1973).

At the second stage ("output stage"), firms simultaneously set quantities; that is, each firm i chooses q_i, $q_i \in [0, \infty)$, as a function of the vector of participation decisions $\mathbf{z}$, $\mathbf{z} = (z_1, ..., z_n)$. The difference between fringe firms and cartel members is the following. Each fringe firm i, $i \in N \backslash M$, sets q_i so as to maximise its own profit. In contrast, each cartel member i, $i \in M$, is constrained (by cartel rules) to set q_i so as to maximise the cartel's joint profit. This is the key assumption of the paper. One way to make cartel firms' behaviour consistent with individual profit maximisation is the following. By joining the cartel, each firm has to delegate its output decision to some "cartel manager" whose objective it is to maximise the cartel's joint profit. Alternatively, one might assume that cartel members can write a binding contract which prevents them from deviating.[8] All

[8]Another way of interpreting the assumption of joint profit maximisation of cartel members is the

this is to say that we simply assume that cartel rules (which stipulate that firms set output so as to maximise the cartel's joint profit) can be enforced; this allows us to focus on firms' participation decisions, and follows Selten (1973), d'Aspremont et al. (1983), and others.[9]

How a given joint cartel output q_M, $q_M \equiv \sum_{i \in M} q_i$, is divided among cartel members does not affect joint cartel profit; this is due to the assumption of constant returns to scale. Since firms are symmetric, we assume throughout an equal output sharing rule: $q_i = q_j$, $\forall i, j \in M$. Side payments between firms are not allowed. (Alternatively, we could directly assume that each cartel member receives the same profit in equilibrium, independently of how cartel output is divided among firms.)

3.3 Equilibrium Analysis

In this central section of the paper, we seek the (pure strategy) subgame perfect equilibrium (SPE) of the simple two-stage game. We proceed by backward induction. First, we take the size of the coalition as given, and show existence and uniqueness of a Nash equilibrium in quantities. We then investigate how changes in capacity and the size of the cartel affect the stage-2 Nash equilibrium. In the third subsection, we first determine necessary and sufficient conditions for a cartel size to be "stable", i.e. to be supportable in an SPE. We then show the existence of an SPE, and investigate the set of cartel sizes than can be

following. The output stage of the two-stage game is simply the "reduced form" of a quantity-setting supergame in which cartel members sustain joint profit maximisation using trigger strategies. According to these trigger strategies, cartel members will revert forever to the static (noncollusive) Nash equilibrium in case of deviation by a cartel member. The difference to the usual supergame models is that firms that choose not to participate in the cartel do not get punished. If firms do not discount future payoffs, then joint profit maximisation by cartel members can be sustained in an SPE of this supergame as long as joint profit maximisation gives each firm a higher per-period payoff than it could get in the static (noncollusive) Nash equilibrium. The latter condition does not necessarily hold. Hence, under this interpretation of the output stage, we would have to modify firms' payoffs slightly. (This would not change the results: one can show that, in equilibrium, cartel size is always such that each cartel member makes at least the same profit as under noncollusion.) Given the appropriate modification of payoffs, the participation stage simply "selects" (induces) an SPE of the ensuing quantity-setting supergame, which has quite different properties than the SPE usually considered in the literature.

[9]Notice that any renegotiation-proof contract between cartel members must involve joint profit maximisation if renegotiation is allowed between cartel members only.

supported in equilibrium. Finally, we turn to the main question of this paper, namely the effects of capacity and market size on equilibrium cartel size and, in particular, equilibrium price.

3.3.1 Equilibrium Analysis for a Given Cartel Size

In this subsection, we take firms' participation decisions, described by the vector $\mathbf{z} = (z_1, ..., z_n)$, as given. Suppose there are $m \equiv \#M \in \{0, 1, ..., n\}$ cartel members and $n - m \equiv \#N \backslash M$ fringe members. We now seek the associated Nash equilibrium in quantities.

Let $g(x, q) \equiv P(q) - c + xP'(q)$. Since $P'(q) < 0$, there exists a unique $x(q)$, $x(q) \in (-\infty, \infty)$, such that $g(x(q), q) = 0$. From inequality (3.1), it follows that $x'(q) = -[P'(q) + x(q)P''(q)]/P'(q) < 0$ for all $x(q) \in [0, q]$. Define $\overline{x}(q) \equiv \max\{0, x(q)\}$. Then, a vector of (normalised) quantities, $((q_i)_{i \in N})$, forms a stage-2 Nash equilibrium if and only if

$$q_i = \min\{\overline{x}(q), k\}, \ i \in N \backslash M, \tag{3.2}$$

and

$$q_M \equiv \sum_{i \in M} q_i = \min\{\overline{x}(q), mk\}, \tag{3.3}$$

where $q \equiv \sum_{i \in N} q_i$ is industry output (per unit of market size). To see this, notice that each fringe firm's profit function is strictly concave in its own output; similarly, the cartel's joint profit function is strictly concave in joint cartel output. Equations (3.2) and (3.3), combined with the cartel's output sharing rule, uniquely determine each firm's output for a given value of q. It follows immediately that all $n - m$ fringe firms produce the same quantity in equilibrium.

Define the function $h(q) \equiv (n - m) \min\{\overline{x}(q), k\} + \min\{\overline{x}(q), mk\} - q$. An output vector $((q_i)_{i \in N})$ is then an equilibrium if and only if $h(q) = 0$. Observe that $h(0) > 0$ since $P(0) > c$, and $\lim_{q \to \infty} h(q) = -\infty$. Moreover, $h'(q)|_{h(q)=0} < 0$, where this derivative exists (otherwise, both the right- and left-hand derivatives are negative), since $\min\{\overline{x}(q), mk\}|_{h(q)=0} < q$. Hence, $h(q)$ has a unique nonnegative root. This shows that there exists a unique Nash equilibrium in quantities. In equilibrium, each firm produces a positive output, i.e. $\overline{x}(q) = x(q) > 0$.

Denote by $q_f(m;k)$ and $q_c(m;k)$ a fringe firm's and a cartel member's equilibrium output, respectively; the cartel's joint equilibrium output is $q_M(m;k) = mq_c(m;k)$. We have $q_f(1;k) = q_f(0;k)$ since $q_M(1;k) = q_f(1;k)$. In the following characterisation of equilibrium quantities, we therefore restrict attention to $m \in \{1,...,n\}$. Depending on the values of k and m, three different cases can arise.

Case (i): If

$$P((n-m+1)k) - c + kP'((n-m+1)k) \leq 0, \tag{3.4}$$

then capacity constraints are nonbinding for all firms in equilibrium. In this case, equilibrium quantities are given by

$$q_M(m;k) = q_f(m;k) = x((n-m+1)q_f(m;k)) \in (0,k]. \tag{3.5}$$

Equilibrium profits are

$$\pi_c(m;k) = [P((n-m+1)q_f(m;k)) - c] \frac{q_f(m;k)}{m} \tag{3.6}$$

and

$$\pi_f(m;k) = [P((n-m+1)q_f(m;k)) - c]\, q_f(m;k). \tag{3.7}$$

Case (ii): If

$$P((n-m+1)k) - c + kP'((n-m+1)k) > 0 \tag{3.8}$$

and

$$P(nk) - c + mkP'(nk) \leq 0, \tag{3.9}$$

then fringe firms only face a binding capacity constraint. Hence, $q_f(m;k) = k$, and

$$q_M(m;k) = x(q_M(m;k) + (n-m)k) \in (k, mk]. \tag{3.10}$$

Equilibrium profits are then given by

$$\pi_c(m;k) = [P(mq_c(m;k) + (n-m)k) - c]\, q_c(m;k) \tag{3.11}$$

and

$$\pi_f(m;k) = [P(mq_c(m;k) + (n-m)k) - c]\, k. \tag{3.12}$$

Notice that this case can only arise if cartel size $m \geq 2$.

Case (iii): Capacity constraints are binding for all firms in equilibrium if and only if

$$P(nk) - c + mkP'(nk) > 0. \tag{3.13}$$

Then, each firm sets its output equal to its capacity level. Equilibrium profit is

$$\pi_c(m;k) = \pi_f(m;k) = [P(nk) - c]\, k. \tag{3.14}$$

Let us summarise the results in the following proposition.

Proposition 3.1 *There exists a unique Nash equilibrium in quantities. In equilibrium, $\pi_c(m;k) < \pi_f(m;k)$ if $P(nk) - c + mkP'(nk) < 0$, and $\pi_c(m;k) = \pi_f(m;k)$ otherwise.*

In equilibrium, fringe firms are at least as well off as cartel members. This follows from the fact that fringe firms maximise their own profit whereas cartel members maximise joint cartel profit, whereby they internalise the externality they impose on fellow cartel firms.

In this paper, the (m,k)-space plays a crucial role. The following lemma gives a useful result on the division of this space into different regions.

Lemma 3.1 *There exist real-valued functions $m_c(k)$ and $m_f(k)$, with $dm_c(k)/dk < 0$ and $dm_f(k)/dk > 0$, such that*

- *cartel members face a binding capacity constraint if and only if cartel size m, $m \in [1,n]$, is such that $m < m_c(k)$;*
- *fringe firms face a binding capacity constraint if and only if $m > m_f(k)$ for $m \in [1,n)$.*

Moreover, if k such that $m_c(k) > 0$, then $m_f(k) < 1$.

The proof can be found in the appendix. See figure 3.1 for an illustration. Lemma 3.1 implies, in particular, that if a cartel member faces a binding capacity constraint when the cartel size is m, then the capacity constraint is binding for all smaller cartel sizes m', $m' \in \{1,...,m-1\}$.

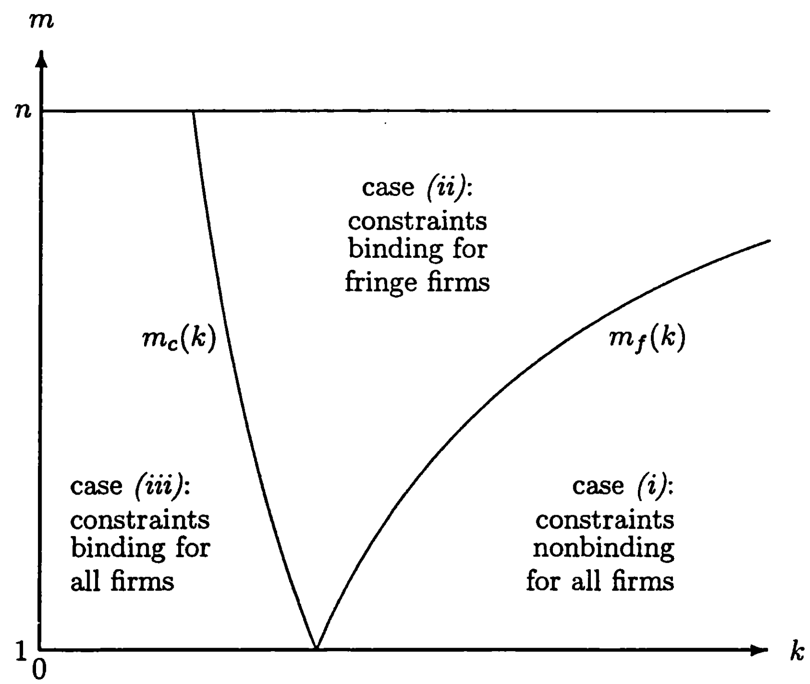

Figure 3.1: The Division of the (m, k)-space: The Functions $m_c(k)$ and $m_f(k)$.

3.3.2 Comparative Statics

The main aim of this paper is to investigate how the equilibrium price of the two-stage game varies with capacity K and demand S. There are two potential channels of influence. For any given coalition structure, capacity per unit of market size, k, has a direct impact on the stage-2 equilibrium price. But k can also influence the equilibrium cartel size, which in turn affects the stage-2 equilibrium price. Therefore, having established existence and uniqueness of a Nash equilibrium in quantities, we now want to investigate the comparative statics of this stage-2 equilibrium price with respect to k and m. In subsection 3.3.4, we will analyse how the equilibrium value of m varies with k.

Observe that equilibrium output, $q_c(m;k)$ and $q_f(m;k)$, and equilibrium profit, $\pi_c(m;k)$ and $\pi_f(m;k)$, are continuous functions of k, and of cartel size m if one treats m as a continuous variable. The following result shows how equilibrium output, and hence equilibrium price, varies with cartel size.

Lemma 3.2 *Total industry output, $q_M(m;k) + (n-m)q_f(m;k)$, is weakly decreasing in cartel size m. Joint cartel output, $q_M(m;k)$, and each fringe firm's output, $q_f(m;k)$, are weakly increasing in cartel size m.*

The proof can be found in the appendix; the intuition for the result is as follows. Decompose the effect of an increase in cartel size m into a (converging) sequence of output changes – neglecting, for ease of exposition, the existence of capacity constraints. As the first step, let the ("old" and "new") cartel members "myopically" adjust their output decisions, holding fixed the output level of the remaining fringe firms. Since cartel members optimally internalise the externalities they impose on each other, this will lead to a lower industry output, and hence a higher price. As the second step, let the remaining fringe firms myopically change their output, keeping constant the cartel's output. As a response to the higher price, fringe firms will increase their output. This, in turn, will induce the cartel firms to reduce their output (step 3) since quantities are strategic substitutes, and so on. In equilibrium, the remaining fringe firms will have increased their output. However, this joint output increase will be less than the decrease in output by the (old and new) cartel members since each firm's best-reply function has a slope larger than -1. In the absence of capacity constraints, joint cartel output is equal to each fringe firm's output.

Hence, joint cartel output will increase as well.

Since industry output is a (weakly) decreasing function of the size of the coalition, the equilibrium price is (weakly) increasing in m. The following corollary is then immediate.

Corollary 3.1 *A fringe firm's equilibrium profit, $\pi_f(m;k)$, and joint cartel profit, $m\pi_c(m;k)$, are weakly increasing functions of cartel size m.*

The relationship between an individual cartel member's output (and profit) and cartel size is, however, generally nonmonotonic. This is a well-known result from the literature on horizontal mergers under Cournot competition without capacity constraints; see Salant, Switzer, and Reynolds (1983). The reason is that an increase in cartel size will induce the remaining fringe firms to increase their output, provided that they are not capacity constrained. Since quantities are strategic substitutes, each cartel member's output and profit may decrease. If, however, fringe firms face a binding constraint in equilibrium, then an increase in cartel size will not have this adverse effect on a cartel member's profit.

Lemma 3.3 *If $m_c(k) > 1$, then $\pi_c(m;k)$ is constant on $[0, m_c(k)]$, and strictly increasing in m on $(m_c(k), n)$.*

Proof. First, notice that $\pi_c(m;k) = k[P(nk) - c]$ for all $m \in [0, \max\{m_c(k), 1\}]$. Second, if $m \in (m_c(k), n)$, $m_c(k) > 1$, then $\pi_c(m;k)$ is given by (3.11). Using the envelope theorem, we get

$$\frac{\partial}{\partial m}\pi_c(m;k) = -(k - q_c(m;k))q_c(m;k)P'(mq_c(m;k) + (n-m)k),$$

which is strictly positive since $k > q_c(m;k)$ if $m > m_c(k) > 1$. ■

We now turn to comparative statics – for a given cartel size – with respect to k.

Lemma 3.4 *For a given cartel size m, equilibrium price $P(q_M(m;k) + (n-m)q_f(m;k))$ is a (weakly) decreasing function of the capacity level per unit of market size, k.*

Proof. As in the proofs of proposition 3.1 and lemma 3.2, we have to distinguish between three cases; again, we confine attention to $m \in [1, n]$.

Case (i): Since capacity constraints are nonbinding for all firms, equilibrium output is unaffected by a marginal change in k.

Case (ii): Note first that $\partial q_M(m;k)/\partial k > -(n-m)$. Taking the derivative of the equilibrium price with respect to k yields

$$\frac{dP(q_M(m;k)+(n-m)k)}{dk} = P'(q_M(m;k)+(n-m)k)\left[\partial q_M(m;k)/\partial k + n - m\right] < 0.$$

Case (iii): When all firms face a binding constraint, equilibrium output is equal to $P(nk)$. We thus have $dP(nk)/dk = nP'(nk) < 0$. ■

We have thus established that an increase in the level of capacity or a decrease in demand (market size), will lower price for a given size of the coalition. From lemma 3.2 we already know that an increase in the size of the cartel will increase price. In order to determine the overall effect on equilibrium price, we, therefore, need to analyse the effect of changes in K and S on the equilibrium cartel size. But before doing so, we have to determine the conditions that need to hold for a cartel size to be supportable in an SPE of the two-stage game; this is the purpose of the next subsection.

3.3.3 Stable Cartel Size

So far, we have analysed firms' stage-2 output decisions for a given size of the coalition M. We now turn to the analysis of the participation decision stage (stage 1) at which firms simultaneously decide whether to join the cartel M or rather the fringe $N\backslash M$. The equilibrium cartel size is determined by the tension between a firm's incentive to free ride on the cartel's effort to restrain output (by staying out of the cartel) and to achieve a more collusive outcome (by joining the cartel).

The following lemma gives the necessary and sufficient conditions for a cartel size to be sustainable in an SPE of the two-stage game. It turns out that these conditions are identical to the notions of internal and external stability, which are used in the (not explicitly game-theoretic) literature on static cartel stability. The seminal contribution in this literature is d'Aspremont et al. (1983); other papers include Donsimoni (1985), Donsimoni, Economides, and Polemarchakis (1986), and Shaffer (1995).

Lemma 3.5 *A cartel of size m can be supported in an SPE of the two-stage game if and only if the following two conditions hold:*

(internal stability) $m = 0$ or

$$\pi_c(m;k) \geq \pi_f(m-1;k) \text{ for } m \in \{1,...,n\} \tag{3.15}$$

and

(external stability) $m = n$ or

$$\pi_c(m+1;k) \leq \pi_f(m;k) \text{ for } m \in \{0,...,n-1\}. \tag{3.16}$$

Proof. The condition of internal stability implies that no firm has an incentive to leave the cartel. Similarly, the condition of external stability ensures that no firm can profitably deviate at stage 1 by joining the cartel. Hence, these conditions are sufficient. Conversely, if one of these conditions is not satisfied, then there exists a profitable deviation for some firm. Hence, the conditions are necessary for an SPE. ■

In the following, we will say that a cartel size is *stable* whenever it can be supported in an SPE. Having defined the necessary and sufficient conditions for an equilibrium cartel size, we want to be sure that the set of equilibria is nonempty for all possible parameter values. For this purpose, we introduce some notation. Let $[x]$ denote the integer part of x, and define the step function

$$\overline{m}_c(k) \equiv \begin{cases} n & \text{if } m_c(k) \in (n,\infty) \\ [m_c(k)] & \text{if } m_c(k) \in [1,n] \\ 1 & \text{otherwise.} \end{cases}$$

Notice that $\overline{m}_c(k)$ is weakly decreasing in k.

Proposition 3.2 *The set of cartel sizes that can be supported in an SPE, $M^*(k)$, has at least two elements. In particular, $\{0,...,\overline{m}_c(k)-1\} \subset M^*(k)$; hence, noncollusion ($m = 0$) can always be supported in equilibrium. Moreover, there exists at least one other stable cartel size $m^*(k)$, $m^*(k) \in \{\overline{m}_c(k),...,n\}$.*

Proof. To see that $\{0,...,\overline{m}_c(k)-1\} \subset M^*(k)$, notice first that $\pi_f(0;k) = \pi_c(1;k)$. Hence, $m = 0$ is a stable cartel size. Furthermore, $\pi_c(m;k) = \pi_f(m-1;k)$ for all $m \in \{1,...,\overline{m}_c(k)\}$. This proves the assertion. To show that there exists a stable cartel size $m^*(k)$, $m^*(k) \in \{\overline{m}_c(k),...,n\}$, we can simply apply the algorithm by d'Aspremont et al. (1983). Let us start with a cartel of size $m = \overline{m}_c(k)$. It is internally stable since $\pi_c(\overline{m}_c(k);k) = \pi_f(\overline{m}_c(k)-1;k)$; if it is externally stable, then the algorithm stops since we have found a stable cartel. Otherwise, we continue with a cartel of size $m = \overline{m}_c(k)+1$, which is internally stable since the cartel of size $m = \overline{m}_c(k)$ was found to be externally unstable. If $m = \overline{m}_c(k) + 1$ is externally stable, then the algorithm stops; otherwise, it continues with $m = \overline{m}_c(k) + 2$, and so on. By induction, the algorithm reaches size $m = n$ if and only if $m = n - 1$ was found to be externally unstable. But, then, $m = n$ must be internally stable, and it is externally stable by definition; hence, it is stable. This concludes the proof. ■

In order to conduct comparative statics, ideally, we want to have uniqueness of a stable cartel size. Unfortunately, there is no hope of finding a unique equilibrium cartel size, as proposition 3.2 indicates. Notice, however, that the equilibrium price is the same for all $m \in \{0, ..., \overline{m}_c(k)\}$. Arguably, *this* kind of multiplicity should not be a cause of concern. Below, we will further characterise the set of stable cartel sizes; however, a full characterisation for a general demand function is beyond the scope of this paper.

Due to the multiplicity of equilibria, we will focus on the maximum and minimum sustainable cartel sizes. From proposition 3.2, we already know that the empty cartel is the smallest stable cartel. The following result gives an instructive and useful result regarding the largest stable cartel.

Lemma 3.6 *All cartel sizes above the maximum stable cartel size, $\overline{m}^*(k)$, are internally unstable; that is,*

$$\pi_c(m; k) < \pi_f(m-1; k) \text{ for all } m \in \{\overline{m}^*(k) + 1, ..., n\}.$$

Proof. Suppose $\overline{m}^*(k) = n - 1 - l$, $l \in \{0, 1, ..., n-2\}$. Since n is externally stable by definition, it must be internally unstable. Assume now that $l \geq 1$. Since n is internally unstable, $n - 1$ is externally stable; hence, $n - 1$ must be internally unstable. The proof proceeds in the same fashion. ■

Proposition 3.2 leaves the possibility open that all stable cartel sizes are "degenerate" cartels. Indeed, for all $m \in \{1, ..., \overline{m}_c(k)\}$, the equilibrium price is the noncollusive one (when $m = 0$). Using lemma 3.6, we are now in the position to further characterise the set of stable cartel sizes. The following result shows that "nondegenerate" cartels exist for all k such that $m_c(k) \in [1, n)$.

Proposition 3.3 *If $m_c(k) \in [1,n)$, then the maximum sustainable cartel size, $\overline{m}^*(k)$, is strictly larger than $m_c(k)$, i.e. $\overline{m}^*(k) \geq \overline{m}_c(k)+1$; moreover, if $m_c(k) \in (1,n)$, then $\overline{m}_c(k)$ is externally unstable.*

Proof. Notice first that $\pi_c(\overline{m}_c(k);k) = \pi_f(\overline{m}_c(k);k)$. Furthermore, if $m_c(k) \in (1,n)$, then $\pi_c(\overline{m}_c(k)+1;k) > \pi_c(\overline{m}_c(k);k)$ by lemma 3.3. It follows immediately that $\pi_c(\overline{m}_c(k)+1;k) > \pi_f(\overline{m}_c(k);k)$. Hence, $\overline{m}_c(k)$ is externally unstable, and $\overline{m}_c(k)+1$ internally stable, if $m_c(k) \in (1,n)$. By continuity, $\overline{m}_c(k)+1$ must also be internally stable if $m_c(k) = 1$. Suppose now the assertion is false; that is, suppose there exists a capacity level k such that $m_c(k) \in [1,n)$ and $\overline{m}^*(k) < \overline{m}_c(k)+1$. Then, $\overline{m}_c(k)+1$ is internally unstable by lemma 3.6. But this can not be true as we have just shown. ■

Proposition 3.3 implies that if $m_c(k) \in (n-1,n)$, then there exists an equilibrium such that all firms participate in the cartel, i.e. $\overline{m}^*(k) = n$, although no firm faces a binding capacity constraint. Nevertheless, even in this case, the existence and level of capacity constraints have a major impact on the equilibrium outcome in that a further rise in capacity or a fall in demand can cause the collapse of collusion. This will become clear from the analysis below. Before we turn to the analysis of changes in capacity or demand on the equilibrium cartel size, let us make two further remarks on stability. First, suppose the smallest stable cartel in $\{\overline{m}_c(k),...,\overline{m}^*(k)\}$ is strictly larger than $\overline{m}_c(k)$. Then, $\overline{m}_c(k)$ and all cartel sizes below the smallest stable one are externally unstable. The argument is similar to that in the proof of proposition 3.3. Second, recall that, from proposition 3.2, if k is sufficiently small such that $m_c(k) \geq n$, then all cartel sizes can be supported in equilibrium, i.e. $\overline{m}^*(k) = n$.

3.3.4 The Effects of Capacity and Demand on Cartel Size and Price

We now want to investigate how the equilibrium cartel size varies with k, that is, with capacity level K and market size S. Consider a stable cartel size, say $m^*(k)$. Since $\pi_c(m;k)$ and $\pi_f(m;k)$ are continuous in k, a marginal change in k will have no effect on $m^*(k)$ unless the condition for internal stability, (3.15), or the condition for external stability, (3.16), hold with equality, that is, unless $\pi_c(m^*(k);k) = \pi_f(m^*(k)-1;k)$ or $\pi_c(m^*(k)+1;k) = \pi_f(m^*(k);k)$ hold.

Therefore, we will, in turn, analyse the effect of a small change in k on equilibrium profits for cartel and fringe members when these conditions hold with equality. The proof of the following lemma is quite involved and can be found in the appendix.

Lemma 3.7 *Suppose cartel size m, $m \in \{\overline{m}_c(k)+1, ..., n\}$, and capacity k are such that the condition for internal stability is binding, i.e. $\pi_c(m;k) = \pi_f(m-1;k)$. Then,*

$$\frac{\partial}{\partial k}\{\pi_c(m;k) - \pi_f(m-1;k)\} \leq 0, \tag{3.17}$$

where this derivative exists; otherwise, the inequality holds for both the derivatives from the right and the left. That is, a marginal increase in k either implies that the condition for internal stability continues to be binding or that it will be violated.

A direct consequence of lemma 3.7 is the following result.

Corollary 3.2 *Suppose cartel size m, $m \in \{\overline{m}_c(k), ..., n-1\}$, is such that the condition for external stability is binding, i.e. $\pi_c(m+1;k) = \pi_f(m;k)$. Then,*

$$\frac{\partial}{\partial k}\{\pi_c(m+1;k) - \pi_f(m;k)\} \leq 0,$$

where this derivative exists; otherwise, the inequality holds for both the derivatives from the right and the left. That is, a marginal increase in k either implies that the condition for external stability continues to be binding or that it will become slack.

Proof. This follows directly from lemma 3.7, substituting m for $m-1$, and $m+1$ for m. ■

The idea behing lemma 3.7 is the following. If a firm decides to join the fringe rather than the cartel, its equilibrium output will tend to be higher since it will take a free ride on the cartel's attempt to restrict output. For very high levels of capacity, a firm will be unconstrained, independently of whether or not it joins the cartel. If it is initially indifferent, then a small increase in capacity has no effect on the relative incentives to join the cartel. Now, for moderate levels of capacity k, a firm will face a binding capacity constraint if it decides to join the fringe, but not if it becomes a member of the cartel. In this case, then, an increase in capacity will make it relatively more attractive to join the fringe since the rise in k relaxes the fringe's constraint. Notice, however, that inequality

(3.17) does not hold if $m = m_c(\overline{k}) \in \{2, ..., n\}$ at some capacity level $\overline{k}$. For all $k \leq \overline{k}$, we have $\pi_c(m; k) = \pi_f(m-1; k)$, whereas for capacity levels k, $k > \overline{k}$, such that $m > m_c(k) > m - 1$, we have $\pi_c(m; k) > \pi_f(m - 1; k)$; see proposition 3.3. That is, the right-hand derivative of $\pi_c(m; \overline{k}) - \pi_f(m - 1; \overline{k})$ with respect to k is strictly positive. The reason is that at capacity $\overline{k}$ and cartel size $m - 1$, fringe firms *and* cartel members are constrained so that a rise in capacity will induce *all* firms to increase output. In contrast, at capacity $\overline{k}$ and cartel size m, cartel members are just unconstrained so that fringe firms only will raise output as k increases.

Lemma 3.7 and corollary 3.2 show that an increase in capacity (or a decrease in demand) can cause a hitherto stable cartel size m to become internally unstable but not externally unstable, given that $m > \overline{m}_c(k)$. This suggests that stable cartels tend to become smaller the higher is the level of capacity per unit of market size. Due to the multiplicity of equilibria, we focus on the maximum stable cartel size, $\overline{m}^*(k)$. (From proposition 3.2, we already know that the smallest stable cartel is the empty one.)

We are now in the position to state and prove one of the main results of this paper.

Proposition 3.4 *The maximum stable cartel size $\overline{m}^*(k)$, $\overline{m}^*(k) \in \{1, ..., n\}$, is weakly decreasing in capacity K, and weakly increasing in market size S.*

Proof. Suppose that the assertion of the proposition is false. Then, there exist some capacity levels k_1 and k_2, $k_2 > k_1$, such that $\overline{m}^*(k_1) \equiv m_1 < m_2 \equiv \overline{m}^*(k_2)$. By proposition 3.2, $m_1 \geq \overline{m}_c(k_1)$; moreover, $\overline{m}_c(k_1) \geq \overline{m}_c(k)$, for all $k \in [k_1, k_2]$, since $\overline{m}_c(k)$ is weakly decreasing in k. Hence, $m_2 > \overline{m}_c(k)$ for all $k \in [k_1, k_2]$. From lemma 3.6, we know that m_2 is internally instable at k_1, but it is, by assumption, internally stable at k_2. Hence, there exists a capacity level k^*, $k^* \in [k_1, k_2]$, such that $\pi_c(m_2; k^*) = \pi_f(m_2 - 1; k^*)$, and

$$\frac{\partial}{\partial k} \{\pi_c(m_2; k^*) - \pi_f(m_2 - 1; k^*)\} > 0,$$

where this derivative exists (otherwise, the inequality must hold for the corresponding derivative from the left). But this contradicts lemma 3.7. Hence, the assertion of the proposition can not be false. ■

Proposition 3.4 restores the traditional view in that it shows that large cartels tend to break down in periods of high (excess) capacity and low demand. Although proposition

3.4 states an important result, eventually, we are more interested in price rather than in the size of the cartel. This is not only for the obvious reason that price directly affects consumer surplus, whereas cartel size has an impact on consumer surplus only via price (and output). More importantly, in empirical applications, we can hope to observe or measure price, but usually not the size of a coalition.

The following proposition states the central prediction of this paper, which only involves variables that are, in principle, measurable.

Proposition 3.5 *The maximum and minimum sustainable equilibrium prices, $\overline{p}^*(k)$ and $\underline{p}^*(k)$, are weakly decreasing in capacity K, and weakly increasing in market size S.*

Proof. Observe that, given a size m of the cartel, there exists a unique equilibrium price (proposition 3.1). Moreover, this equilibrium price is weakly increasing in m, given k (lemma 3.2). Hence, for any k, the maximum stable equilibrium price $\overline{p}^*(k)$ is the unique equilibrium price given the maximum stable cartel size. Now, the maximum sustainable cartel size is (weakly) decreasing in k (proposition 3.4). Furthermore, the equilibrium price is weakly increasing in m, given k (lemma 3.2), and weakly decreasing in k, given m (lemma 3.4). Hence, $\overline{p}^*(k)$ is weakly decreasing in k. Similarly, for any k, the minimum sustainable cartel size $\underline{p}^*(k)$ is the unique equilibrium price given the minimum stable cartel size. From proposition 3.2, we know that the empty cartel ($m = 0$) can be supported in an SPE for all values of k. Hence, $\underline{p}^*(k)$ is the unique equilibrium price given $m = 0$. The assertion of the proposition then follows directly from lemma 3.4. ■

3.3.5 Robustness of Results

Quantity Competition. An important question is whether our results hinge on the assumption of quantity competition at the output stage. It would, in particular, be interesting to study the case of price competition. However, as is well known, pure strategy equilibria fail to exist in games of price competition under capacity constraints so that the notion of a market price is not well defined. Now, following the seminal contribution by d'Aspremont et al. (1983), much of the literature on cartel stability in static models assumes that the cartel acts as price leader, whereas all firms in the fringe are price takers (when deciding upon output); see, for instance, Donsimoni (1985) and Donsimoni, Economides, and Polemarchakis (1986).[10] It is quite straightforward to show that the main conclusions of our Cournot model carry over to the price leadership model. To see this, notice that, for any price above marginal cost, each fringe firm sets its output equal to its capacity. Given the aggregate supply of the fringe, the cartel faces a residual demand curve, and sets price so as to maximise its joint profit. But in this case, price and quantity setting are equivalent. Hence, we are back in the Cournot world, namely in the subcase where fringe firms face a binding capacity constraint. In contrast to the previous Cournot model, however, an empty cartel ($m = 0$) may *not* be supported in equilibrium. This is due to the fact that fringe firms are assumed to take price as given; that is, they neglect the effect of their output decisions on price. Most other previous results go through. In particular, the maximum stable cartel size, $\overline{m}^*(k)$, is weakly decreasing, and the maximum sustainable equilibrium price, $\overline{p}^*(k)$, is even *strictly* decreasing in k.[11]

Cartel Formation. Following Selten (1973), we have modelled cartel formation as a noncooperative simultaneous-move game. Let us denote this game by Γ. To investigate the robustness of our predictions, we now want to study what happens if we assume that firms decide sequentially, rather than simultaneously, whether or not to join the cartel. That is, suppose that, at the participation stage, first firm 1 selects z_1, then firm 2 selects z_2, and so on; the labelling of firms is arbitrary. Denote the new game by Γ'. It is straightforward to show that there exists a pure strategy subgame perfect equilibrium (SPE) in the sequential cartel formation game Γ'. Moreover, any cartel size m that can be supported in an SPE of Γ' can also be supported in an SPE of Γ. To see this, notice that the conditions for internal and external stability of cartel size m, equations (3.15) and (3.16), are necessary conditions for m to be sustainable in an SPE of Γ', since, otherwise, firm n could profitably deviate. Indeed, the sequential-move game selects the "most efficient" equilibrium of the simultaneous-move game in that it solves the "coordination problem" in the formation of the cartel. Hence, the maximum sustainable equilibrium size in Γ, $\overline{m}^*(k)$, can always be supported in Γ'. Moreover, we prove the following result in the appendix.

[10]These models invite an obvious criticism: Why are firms in the fringe so "naive" not to take into account the effect of their output decision on price but, at the same time, so "sophisticated" to compute the exact price effect of their participation decision? Putting this theoretical issue aside, these models might nevertheless give a good empirical description of the nature of competition in some markets.

[11]See the proof of lemma 3.4.

Proposition 3.6 *If the maximum sustainable equilibrium size in the simultaneous cartel formation game* Γ, $\overline{m}^*(k)$, *is such that* $\pi_c(\overline{m}^*(k); k) > \pi_f(\overline{m}^*(k) - 1; k)$, *then* $\overline{m}^*(k)$ *is the unique equilibrium cartel size in the sequential cartel formation game* Γ'. *In the unique equilibrium of* Γ', $z_i = 0$ *if* $i \in \{1, ..., n - \overline{m}^*(k)\}$, *and* $z_i = 1$ *if* $i \in \{n - \overline{m}^*(k) + 1, ..., n\}$.

It is possible to show that the condition of the proposition is satisfied for "almost all" k such that $m_c(k) \in (1, n)$. This result is reassuring in that it shows the robustness of our predictions, which are even sharper under the assumption of sequential cartel formation.

We have modelled cartel formation as an open membership game; that is, cartel members can not prevent other firms from joining the cartel. However, under quantity competition, profit per cartel member is not monotonic in the number of cartel members. Does this imply that, in equilibrium, a cartel member may be better off by restricting membership? The following proposition gives a reassuring answer.

Proposition 3.7 *In equilibrium, a cartel member is never better off by restricting cartel membership. Formally, if* m^* *is a stable cartel size, then*

$$\pi_c(m^*; k) \geq \pi_c(m^* - l; k) \text{ for any } l \in \{1, ..., m^* - 1\}.$$

Proof. Fix $l \in \{1, ..., m^* - 1\}$. For a given cartel size, a fringe firm is always at least as well off as a cartel member, i.e. $\pi_c(m^* - l; k) \leq \pi_f(m^* - l; k)$. From corollary 3.1, the profit of a fringe member is (weakly) increasing in the size of the cartel; accordingly, $\pi_f(m^* - l; k) \leq \pi_f(m^* - 1; k)$. Finally, we have $\pi_f(m^* - 1; k) \leq \pi_c(m^*; k)$ by stability of m^*. This proves the assertion. ■

3.4 Heterogeneity in Capacity

Following Brock and Scheinkman (1985), we have assumed so far that all firms face the same capacity constraint. The aim of this section is to sharpen the predictions of the basic two-stage game. In particular, we address the following question. Allowing for heterogeneity in capacity levels among firms, do "larger" firms have stronger incentives to join the cartel than their "smaller" rivals? If the answer were yes, then this would be reinsuring for two reasons. First, it would reinforce the tendency for fringe firms to face a binding capacity constraint in equilibrium, and for cartel firms not to be constrained. As we have shown, it is in this case that a change in demand will have a major impact on the equilibrium cartel size. Second, due to their free-riding behaviour, fringe firms tend to be much larger than cartel members in terms of production. However, economists usually think of "fringe firms" as being rather small. The pure existence of capacity constraints alleviates this "problem" somewhat. But if firms with large capacities had stronger incentives to join the cartel than firms with small capacities, then cartel firms would tend to be large, and fringe firms would tend to be small. Such an equilibrium outcome would thus be consistent with the common notion of small fringe firms. Below, we show that the answer to the question is indeed yes. Moreover, we show that the model predicts that firm size and the degree of capacity utilisation are negatively correlated across firms.

We generalise the model of section 3.2 in that we allow for arbitrary heterogeneity in capacity levels among firms. Each firm i, $i \in N$, is equipped with a capacity per unit of market size of k_i, $k_i \in (0, \infty)$. Inequality (3.1) is now supposed to hold for all $q \in [0, \sum_{i \in N} k_i]$. Since firms are no longer symmetric, we have to chose an output sharing rule for the cartel that reflects the different firm sizes. The most natural assumption appears to be that each cartel member's share α_i of joint cartel output, and hence of joint profit, is equal to its share of joint cartel capacity, $\alpha_i \equiv k_i / k_M$, where $k_M \equiv \sum_{i \in M} k_i$.[12] All other features of the basic model remain unchanged.

Turning to the equilibrium analysis of the generalised two-stage game, we proceed again by backward induction. We first seek the stage-2 Nash equilibrium, given any vector $\mathbf{z}$ of participation decisions. Using the same notation as in subsection 3.3.1, a vector of quantities, $((q_i)_{i \in N})$, forms a Nash equilibrium if and only if

$$q_i = \min\{\overline{x}(q), k_i\}, \ i \in N \backslash M,$$

and

$$q_M \equiv \sum_{i \in M} q_i = \min\{\overline{x}(q), k_M\},$$

where $q \equiv \sum_{i \in N} q_i$ is industry output, and $\overline{x}(q)$ the backward reaction mapping. Let

[12] There is evidence that cartels use such a rule in setting output quotas; see Davidson and Deneckere (1990) and Scherer (1980).

$h(q) \equiv \sum_{i \in N \backslash M} \min\{\overline{x}(q), k_i\} + \min\{\overline{x}(q), k_M\} - q$. As before, there exists a unique nonnegative q such that $h(q) = 0$. Hence, there is a unique Nash equilibrium in quantities. In equilibrium,

$$q_i = \begin{cases} \overline{x}(q) & \text{if } k_i \geq \overline{x}(q) \\ k_i & \text{otherwise,} \end{cases} \quad i \in N \backslash M,$$

and

$$q_M = \begin{cases} \overline{x}(q) & \text{if } k_M \geq \overline{x}(q) \\ k_M & \text{otherwise.} \end{cases}$$

Denote by $q_i((z_i, \mathbf{z}_{-i}); \mathbf{k})$ firm i's equilibrium output, where $\mathbf{z}_{-i} = (z_1, ..., z_{i-1}, z_{i+1}, ..., z_n)$ are the participation decisions of i's rivals, and $\mathbf{k} = (k_1, ..., k_n)$ gives the distribution of capacities. Similarly, $\pi_i((z_i, \mathbf{z}_{-i}); \mathbf{k})$ denotes firm i's equilibrium profit; the cartel's joint equilibrium profit is $\pi_M(\mathbf{z}; \mathbf{k})$.

Before analysing firms' participation decisions, we consider some useful properties of the stage-2 equilibrium. The proof of the following results can be found in the appendix.

Lemma 3.8 *A fringe firm's equilibrium profit $\pi_i(\mathbf{z}; \mathbf{k})$, $i \in N \backslash M$, is weakly decreasing in k_j, $j \neq i$, and weakly increasing in k_i. Similarly, joint cartel profit is weakly decreasing in rival capacity, and weakly increasing in its own joint capacity.*

Inducing backwards, let us now turn to firms' participation decisions. From the definition of subgame perfection, it follows that the vector $\mathbf{z}$ can be supported in an SPE if and only if

$$\pi_i((1, \mathbf{z}_{-i}); \mathbf{k}) \geq \pi_i((0, \mathbf{z}_{-i}); \mathbf{k}) \text{ if } z_i = 1 \text{ ("internal stability")},$$

and

$$\pi_i((1, \mathbf{z}_{-i}); \mathbf{k}) \leq \pi_i((0, \mathbf{z}_{-i}); \mathbf{k}) \text{ if } z_i = 0 \text{ ("external stability")}.$$

This implies that noncollusion, i.e. $z_i = 0$ for all $i \in N$, can again be sustained in equilibrium since $\pi_i((1, 0, ..., 0); \mathbf{k}) = \pi_i((0, 0, ..., 0); \mathbf{k})$. The following lemma considers firms' incentives to join the cartel. For the proof, the reader is referred to the appendix.

Lemma 3.9 *Fix an arbitrary vector $\mathbf{z}$ of participation decisions, and consider the two largest fringe firms i and j, $z_i = z_j = 0$, with $k_i \geq k_j$. Then,*

$$\pi_i((1, \mathbf{z}_{-i}); \mathbf{k}) < \pi_i((0, \mathbf{z}_{-i}); \mathbf{k}) \Longrightarrow \pi_j((1, \mathbf{z}_{-j}); \mathbf{k}) < \pi_j((0, \mathbf{z}_{-j}); \mathbf{k}).$$

That is, the larger firm has the stronger incentive to join the cartel.

Note that this result is not trivial. To see this, suppose firm i's capacity is twice that of firm j's. Then, firm i's share of cartel output in case of firm i's deviation will be *less* than twice the share of firm j in case firm j deviates. The following lemma considers the incentives for firms to leave the cartel.

Lemma 3.10 *Fix an arbitrary vector* $\mathbf{z}$ *of participation decisions such that* $k_h \geq k_l$ *for all* $h \in M$ *and* $l \in N \backslash M$. *Consider any two cartel members* i *and* j, $z_i = z_j = 1$, *with* $k_i \geq k_j$. *Then,*

$$\pi_j((1, \mathbf{z}_{-j}); \mathbf{k}) \geq \pi_j((0, \mathbf{z}_{-j}); \mathbf{k}) \Longrightarrow \pi_i((1, \mathbf{z}_{-i}); \mathbf{k}) \geq \pi_i((0, \mathbf{z}_{-i}); \mathbf{k}).$$

That is, the smaller firm has the stronger incentive to leave the cartel.

Proof. The proof is similar to that of the previous lemma. ■

We are now in the position to state and prove the main result of this section.

Proposition 3.8 *There always exists a nonempty stable cartel* M^* *such that* $k_i \geq k_j$ *for all* $i \in M^*$ *and* $j \in N \backslash M^*$.

Proof. We proceed by applying the algorithm in the proof of proposition 3.2 in decreasing order of capacity. Let us start with the participation decision of the largest firm. Since $\pi_i((1, 0, ..., 0); \mathbf{k}) = \pi_i((0, 0, ..., 0); \mathbf{k})$, a cartel consisting of the largest firm only is internally stable. If the next largest firm has no incentive to join the cartel, then, from lemma 3.9 no smaller firm has an incentive to do so. Hence, the cartel consisting of the largest firm is externally stable. In this case, the algorithm stops since we have found a nonempty stable cartel. Suppose now that the second largest firm has an incentive to join the cartel. But this means that it has no incentive to leave the cartel, nor has, from lemma 3.10, the largest firm. The cartel consisting of the two largest firms is thus internally stable. If the next largest firm does not want to join the cartel, then it is externally stable as well. In this case, the algorithm stops; otherwise, it continues in the same fashion. The algorithm is finite since either it stops before all firms have decided to join the cartel or it does not, in which case the cartel consisting of all firms in the industry is found to be stable. ■

Proposition 3.8 thus shows that there always exists an equilibrium in which the cartel consists of the largest firms in the industry, and the fringe of the smallest.[13] The mechanism behind lemmas 3.9 and 3.10, and proposition 3.8, may be more general. A large firm has more incentives to restrict output than a small firm. The reason is that the output restriction leads to a higher price which is more beneficial to larger firms. It is this mechanism that drives, for instance, the prediction of Ghemawat and Nalebuff (1990), namely that, in declining industries, large firms will start decreasing their output first.

Proposition 3.8 shows that, in equilibrium, cartel members tend to be larger than fringe firms, given the assumed output sharing rule. We already know that a fringe firm's unconstrained equilibrium output is larger than that of a cartel member. Hence, it appears to be quite "likely" that, in equilibrium, fringe firms face a binding capacity constraint, while cartel members are unconstrained. Moreover, the model makes the following empirical prediction.

Proposition 3.9 *Consider an equilibrium such that $k_i \geq k_j$ for all $i \in M$ and $j \in N \backslash M$. There exists a nonpositive cross-sectional correlation between firm size (as measured by capacity) and the degree of capacity utilisation.*

Proof. Given the assumed output sharing rule of the cartel, all cartel members produce at the same degree of capacity utilisation. All fringe firms have at least the same degree of capacity utilisation, namely for two reasons. First, their unconstrained equilibrium output is larger than that of a cartel member. Second, fringe firms tend to be smaller. Hence, any fringe firm that is smaller than a cartel member produces at a weakly larger degree of capacity utilisation. Since the unconstrained equilibrium output is the same for all fringe firms, the assertion holds also when comparing fringe firms of different sizes. ■

Hence, firms with larger capacities will tend to have lower capacity utilisation rates.

[13] A similar result has been shown by Donsimoni (1985) in the context of a (non-game-theoretic) model of price leadership with a competitive fringe. In her model, demand is assumed to be linear, and firms differ in the slope of their linearly increasing marginal cost curve; there are no capacity constraints.

3.5 Cartel Stability over the Business Cycle

Oligopolistic price and quantity setting over the business cycle has been a major topic of research in theoretical and empirical IO. At the heart of this literature lies the question whether prices in oligopolistic markets tend to vary procyclically or rather countercyclically. Standard oligopoly and monopoly models usually predict either a positive or no correlation between prices and demand. The seminal contribution by Rotemberg and Saloner (1986) was to show that this relationship can be reversed in collusive equilibria because firms tend to have a higher incentive to deviate in states of high demand. Their paper triggered a large and still growing literature which has been mainly concerned with the robustness of Rotemberg and Saloner's predictions; see, for instance, Haltiwanger and Harrington (1991), Kandori (1991), and Bagwell and Staiger (1997).[14] The literature commonly uses an infinitely repeated game where demand follows some (deterministic or stochastic) process; using trigger strategies, all firms in the industry together try to sustain the joint profit maximising price subject to the constraint that no firm has an incentive to deviate. That is, these papers focus on firms' "incentive constraints" for maximum sustainable collusion, and neglect the "participation constraints". Most papers do not consider capacity constraints.[15] Results appear to depend quite delicately on the time-series properties of the assumed process, and the level of the discount factor.

In our static two-stage game, we have investigated the comparative statics of cartel stability and the collusive equilibrium price with respect to capacity K and market size (or demand) S. One might think that this model does not lend itself easily to the analysis of collusion over the business cycle. There is, however, an extremely simple dynamic extension of the model in which the issue can be investigated in a meaningful way; it goes as follows.

Time is discrete, and indexed by t ($t = 1, 2, ...$); the time horizon is infinite. There are n firms whose objective it is to maximise the sum of discounted profits. In each period, the dynamic game consists of the two stages described in section 3.2: the participation

[14] The influential paper by Green and Porter (1984) is often cited in this context. This is somewhat misleading since their model deals with demand fluctuations that are even ex post unobservable.

[15] An exception is the paper by Staiger and Wolak (1992), where firms have to build capacity from scratch in each period.

stage and the output stage. If a firm decides to join the cartel in a given period, then it delegates its output decision for that period to some cartel manager who seeks to maximise the cartel's joint profit in that period. As before, the cartel is assumed to share profit equally. In contrast to the static model, demand (market size) S now follows an arbitrary (discrete-time) stochastic process; capacity level K is assumed to be constant over time. At the start of each period, before firms decide upon joining the cartel, the realisation of S becomes common knowledge. Notice that the only tangible state variable just prior to the participation decisions is thus given by the current realisation of S; at the start of the output stage, the tangible state can be described by the tuple $(S, \mathbf{z})$, where $\mathbf{z} = (z_1, ..., z_n)$ is again the vector of participation decisions. It is then immediate to see that, in a given period, a vector of participation and output decisions that can be sustained in an SPE of the static two-stage game can also be sustained in a Markov perfect equilibrium of the dynamic game (for the same realisation of S). (Recall that Markov perfection is a refinement of subgame perfection.) Hence, the comparative statics results with respect to S carry over from the static to the dynamic model. From propositions 3.4 and 3.5, it follows in particular that the minimum and maximum sustainable cartel sizes and prices are (weakly) procyclical, in the sense of a nonnegative correlation between S and these variables.[16]

It is interesting to compare our results with those of Rotemberg and Saloner (1986). They consider an infinitely repeated price setting game where demand follows an i.i.d. process, and firms face constant marginal cost; there are no capacity constraints. Rotemberg and Saloner investigate the dynamic properties of a particular equilibrium, namely the *symmetric* SPE which attains the highest profit (by the threat of an infinite Nash reversion). To the extent that the static monopoly price tends to be procyclical, there are two opposing forces at work.[17] On the one hand, collusive prices will tend to be procyclical since monopoly price is positively correlated with demand. On the other, the gain from cheating is positively related to the state of demand. (Due to the i.i.d. assumption,

[16] It should be noted, however, that this does not necessarily imply that equilibrium price varies procyclically. For instance, if there is some exogenous "regime" shift from a collusive equilibrium ($m \geq 2$) to a noncollusive equilibrium ($m = 0$), then the equilibrium price might fall despite an increase in demand.

[17] Note, however, that in Rotemberg and Saloner's leading example the monopoly price is independent of the state of demand.

the expected future loss from punishment is the same in each state.) In the case of intermediate discount factors, collusive prices will be procyclical in periods of low demand, and countercyclical in high demand states. The reason is that the monopoly price can be sustained only when demand is low. In high demand states, collusive profits are optimally constant so as to keep firms just indifferent between deviating and sticking to the collusive price; to achieve this, prices must be set the lower, the higher is demand.[18]

In the dynamic extension of our model, an increase in demand has two effects as well; however, in contrast to Rotemberg and Saloner's paper, they reinforce each other. For a given degree of collusion (i.e. cartel size), equilibrium price is procyclical; see lemma 3.4. Now, the maximum degree of collusion is itself procyclical (proposition 3.4), which reinforces the positve correlation between price and demand (lemma 3.2).

The (theoretical and empirical) study of different kinds and causes of price wars is a very active area of research, even though there does not appear to be a generally accepted definition of a "price war" in the literature.[19] Let us (somewhat loosely) call a "price war" a situation in which some or all firms in the industry cease "colluding". In this sense, price wars never take place in the model by Rotemberg and Saloner. In our model, however, two quite different kinds of price wars can occur along the equilibrium path. First, a sufficient fall in demand will lead to (more) excess capacity in the industry, which in turn causes a cartel to become unstable. In equilibrium, some or all firms will leave the cartel, and price will collapse. Second, due to the possible multiplicity of equilibria, an exogenous "regime" shift from a more to a less collusive equilibrium can occur. Such a price war might be caused by some change in beliefs of industry participants.

3.6 Conclusion

The main aim of this paper has been to develop a theoretical foundation of the traditional view in industrial organisation, according to which there exists a negative relationship between the level of excess capacity and cartel stability. In contrast to the supergame-theoretic approach to collusion, we have focussed on firms' incentives to participate in a

[18]For large discount factors, the monopoly price can be sustained in each state so that prices will tend to vary procyclically; for small discount factors, collusion can not be sustained in any state.

[19]An outstanding example is Levenstein's work on the Bromine Cartel; see Levenstein (1996,1997).

cartel, rather than on the issue of enforcement of cartel rules. The basic two-stage game predicts a positive (negative) correlation between the equilibrium cartel size and the level of demand (resp. capacity). This reinforces the tendency of prices to be low in periods of high excess capacity. In a simple dynamic extension of the model, we have shown that collusive equilibrium prices will vary procyclically, independently of the assumed stochastic process for demand. Allowing for arbitrary heterogeneity in capacity among firms, the analysis has revealed that smaller firms have stronger incentives to take a free ride on the cartel's effort to restrict output. Hence, the model predicts that firm size and the degree of capacity utilisation tend to be negatively correlated across firms.

Although the model has been cast as a model of cartel formation, it can also be seen as a contribution to the still underdeveloped literature on endogenous horizontal mergers.[20] If mergers are driven by market power rather than efficiency considerations, then the model predicts merger waves to occur during booms.

Several issues have been left open for future research. First, following Selten (1973), we have modelled cartel formation as a noncooperative simultaneous-move game; we have shown that our results are even sharper in the case of sequential cartel formation. It may be fruitful to consider different ways of modelling cartel formation, or to allow for more general coalition structures. Second, as in Brock and Scheinkman (1985), each firm's capacity level was assumed to be exogenously given. Endogenising capacity decisions may provide new insights, as Davidson and Deneckere (1990) and Staiger and Wolak (1992) have shown in the context of supergames. Third, it would be interesting to study the interaction of firms' incentive and participation constraints, thereby combining the supergame-theoretic literature with the present approach.

3.7 Appendix

Proof of lemma 3.1. Let $r(m,k) \equiv P(nk) - c + mkP'(nk)$ and $s(m,k) \equiv P((n-m+1)k) - c + kP'((n-m+1)k)$. At cartel size m, $m \in [1,n]$, a cartel member (fringe firm) faces a binding capacity constraint if and only if $r(m,k) > 0$ ($s(m,k) > 0$). Note that $dr(m,k)/dm < 0$, $dr(m,k)/dk < 0$, $ds(m,k)/dm > 0$, and $ds(m,k)/dk < 0$, for all real

[20]The seminal paper in this literature is Kamien and Zang (1990).

numbers $m \in [1, n]$ and $k \in (0, \infty)$. Hence, if k is such that $r(n, k) \le 0 \le r(1, k)$, then $m_c(k)$, $m_c(k) \in [1, n]$, is uniquely defined by $r(m_c(k), k) = 0$. We thus have $r(m, k) > 0$ if and only if $m > m_c(k)$. From the implicit function theorem, it follows that $dm_c(k)/dk < 0$ in this domain. Outside this domain, the function $m_c(k)$ can be extended in an arbitrary way as long as it satisfies $dm_c(k)/dk < 0$ for all $k \in (0, \infty)$. Similarly, if k is such that $s(1, k) \le 0 \le s(n, k)$, then $m_f(k)$, $m_f(k) \in [1, n]$, is uniquely defined by $s(m_f(k), k) = 0$; we have $dm_f(k)/dk > 0$. Outside this domain, $m_f(k)$ can be extended in an obvious way. Finally, observe that $r(1, k) = s(1, k)$, which implies the last assertion of the lemma. ■

Proof of lemma 3.2. As in subsection 3.3.1, we have to distinguish between three cases, depending on whether the fringe firms and cartel members face a binding capacity constraint. Since $q_f(0; k) = q_f(1; k) = q_M(1; k)$, we restrict attention to real numbers $m \in [1, n)$.

Case (i): Suppose capacity constraints are nonbinding for all firms, i.e. condition (3.4) holds. Then, equilibrium output is such that $q_c(m; k) = q_f(m; k)/m$, where $q_f(m; k)$ is defined by (3.5). Implicit differentiation yields

$$\frac{\partial q_f(m; k)}{\partial m} = \frac{q_f}{n - m + 2} \cdot \frac{P' + q_f P''}{P' + \frac{n-m+1}{n-m+2} q_f P''} \in \left(0, \frac{q_f}{n - m + 1}\right).$$

Hence, a fringe firm's output and joint cartel output are increasing in m. Differentiating total industry output with respect to m gives

$$\frac{\partial}{\partial m}(n - m + 1) q_f(m; k) = -q_f + (n - m + 1)\frac{\partial q_f}{\partial m} < 0;$$

that is, total industry output is negatively related to cartel size.

Case (ii): Suppose now that capacity constraints are binding for fringe firms but nonbinding for cartel members. Then, $q_f(m; k) = k$, which is independent of m, and $q_c(m; k)$ is defined by (3.10). Implicit differentiation of joint cartel output gives

$$\frac{\partial}{\partial m} q_M(m; k) = k \frac{P' + q_M P''}{2P' + q_M P''} \in (0, k);$$

joint cartel output is thus an increasing function of m. To see that total industry output is negatively related to m, note that

$$\frac{\partial}{\partial m}\left\{q_M(m; k) + (n - m)k\right\} = \frac{\partial}{\partial m} q_M - k < 0.$$

Case (iii): If capacity constraints are binding for all firms, then a marginal change in m has no effect on output. ■

Proof of lemma 3.7. In the following, we distinguish between different cases, depending on whether fringe firms face binding capacity constraints when cartel size is m or $m-1$, and whether cartel members are constrained when cartel size is $m-1$. Notice that the assumption $m \geq \overline{m}_c(k)$ implies that $m > m_c(k)$, i.e. cartel members are unconstrained when cartel size is m. For expositional clarity, we suppose below that $m, m-1 \neq m_c(k), m_f(k)$ so that the derivative of profit with respect to k exists. Let us first discuss the cases where the derivative does not exist. Suppose $m-1 = m_c(k)$. Since we have assumed that $m \in [\overline{m}_c(k)+1, n]$, this is possible only if $m_c(k) \in [1, n-1]$. Now, from the proof of lemma 3.3, we know that $\pi_c(m;k) > \pi_f(m-1;k)$ if $m-1 = m_c(k) \in (1, n-1]$; but this contradicts the assumption $\pi_c(m;k) > \pi_f(m-1;k)$. By continuity, $\pi_c(m;k) \geq \pi_f(m-1;k)$ if $m-1 = m_c(k) = 1 = m_f(k)$. Hence, if indeed $\pi_c(2;k) = \pi_f(1;k)$ at this value of k, then the left-hand derivative of $\pi_c(2;k) - \pi_f(1;k)$ with respect to k must be strictly negative. The sign of the derivative from the right at this point will be dealt with below in case *(b),* where $m-1 > m_c(k)$ and $m > m_f(k) > m-1$. Suppose now that $m = m_f(k)$ ($m-1 = m_f(k)$). Since $m_f(k)$ is strictly increasing in k, the derivate from the left will be dealt with in the case where $m > m_f(k)$ (resp. $m-1 > m_f(k)$), and the derivative from the right in the case where $m < m_f(k)$ (resp. $m-1 < m_f(k)$).

Case (a): Suppose $m-1 > m_c(k)$ and $m < m_f(k)$. This implies that capacity constraints are nonbinding for both fringe and cartel members when there are $m-1$ or m cartel members. Then, obviously, a marginal increase in the capacity level k has no effect on profits.

Case (b): Suppose that $m-1 > m_c(k)$ and $m > m_f(k) > m-1$. In this case, capacity constraints do not matter when cartel size is $m-1$ so that $\partial\pi_f(m-1;k)/\partial k = 0$. When there are m cartel members, equilibrium profit is given by (3.11). Using the envelope theorem, we get $\partial\pi_c(m;k)/\partial k = (n-m)q_c(m;k)P'(mq_c(m;k) + (n-m)k) \leq 0$, and hence $\partial\left\{\pi_c(m;k) - \pi_f(m-1;k)\right\}/\partial k \leq 0$.

Case (c): Suppose now that $m-1 > m_c(k)$ and $m-1 > m_f(k)$. Then, equilibrium profits are, from (3.11) and (3.12),

$$\pi_c(m;k) = \left[P(mq_c(m;k) + (n-m)k) - c\right] q_c(m;k)$$

and

$$\pi_f(m-1;k) = [P((m-1)q_c(m-1;k) + (n-m+1)k) - c]\, k,$$

where $q_c(m;k)$ is implicitly defined by (3.10), and $q_c(m-1;k)$ is defined analogously. Let us now make two observations, namely

$$\begin{aligned} \frac{\partial q_c(m-1;k)}{\partial k} &= -\frac{n-m+1}{m-1} \cdot \frac{P' + (m-1)q_c(m-1;k)P''}{2P' + (m-1)q_c(m-1;k)P''} \\ &< -\frac{(n-m+1)^2 k}{(m-1)[(m-1)q_c(m-1;k) + 2(n-m+1)k]} \end{aligned} \tag{3.18}$$

and

$$mq_c(m;k) \leq (m-1)q_c(m-1;k) + k. \tag{3.19}$$

To see inequality (3.18), observe first that inequality (3.1) implies

$$(m-1)q_c(m-1;k)P'' < -\frac{(m-1)q_c(m-1;k)}{(m-1)q_c(m-1;k) + (n-m+1)k} P',$$

where both P' and P'' have to be evalued at $(m-1)q_c(m-1;k) + (n-m+1)k$. We then get

$$\frac{P' + (m-1)q_c(m-1;k)P''}{2P' + (m-1)q_c(m-1;k)P''} > \frac{(n-m+1)k}{(m-1)q_c(m-1;k) + 2(n-m+1)k},$$

using the fact that the ratio on the l.h.s. is decreasing in $(m-1)q_c(m-1;k)P''$. Inequality (3.18) follows immediately.

To see inequality (3.19), note that, by assumption, we have $q_c(m;k) \leq k$ and $\pi_c(m;k) = \pi_f(m-1;k)$ so that $P(mq_c(m;k) + (n-m)k) \geq P((m-1)q_c(m-1;k) + (n-m+1)k)$, and hence the result follows.

Let us now take the derivative of $\Delta\pi(k) \equiv \{\pi_c(m;k) - \pi_f(m-1;k)\}$ with respect to k:

$$\begin{aligned} \Delta\pi'(k) &= (n-m)q_c(m;k)P'(mq_c(m;k) + (n-m)k) \\ &\quad - [P((m-1)q_c(m-1;k) + (n-m+1)k) - c] \\ &\quad - P'((m-1)q_c(m-1;k) + (n-m+1)k)k \\ &\quad \times \left[(m-1)\frac{\partial q_c(m-1;k)}{\partial k} + n - m + 1\right], \end{aligned}$$

where we have made use of the envelope theorem. Now, using (3.18) and the fact that, by assumption, m and k are such that $\pi_c(m;k) = \pi_f(m-1;k)$, we get

$$\Delta\pi'(k) \;\leq\; [P((m-1)q_c(m-1;k) + (n-m+1)k) - c]$$

$$\times \left[-\frac{(n-m)k}{mq_c(m;k)} - 1 \right.$$
$$\left. + \frac{(n-m+1)^2 \left[k + \frac{(m-1)q_c(m-1;k)}{(n-m+1)} \right] k}{(m-1)^2 q_c^2(m-1;k) + 2(m-1)(n-m+1)kq_c(m-1;k)} \right].$$

Using (3.19), and $(m-1)q_c(m-1;k) \geq k$, one can show that the expression on the r.h.s. is negative. Thus, we obtain $\partial \{\pi_c(m;k) - \pi_f(m-1;k)\}/\partial k < 0$ (where this derivative exists).

Case (d): Suppose $m-1 < m_c(k) < m$ and $m-1 > m_f(k)$. Equilibrium profits are then given by $\pi_c(m;k) = [P(mq_c(m;k) + (n-m)k) - c]\, q_c(m;k)$ and $\pi_f(m-1;k) = [P(nk) - c]\, k$, where $q_c(m;k)$ is implicitly defined by (3.10). Since $m > m_c(k)$ by assumption, we must have $q_c(m;k) < k$. Then, joint cartel profit must be strictly higher at a joint cartel output of $mq_c(m;k)$ than at mk. That is, $[P(mq_c(m;k) + (n-m)k) - c]\, mq_c(m;k) > [P(nk) - c]\, mk$, but this contradicts the assumption $\pi_c(m;k) = \pi_f(m-1;k)$. Hence, this case can not occur. ■

Proof of proposition 3.6. Suppose we are at the start of "substage" l, $l \in \{1, ..., n\}$, i.e. just prior to firm l's participation decision. Define $z(l) \equiv \sum_{i=1}^{l} z_i$ for $l \in \{1, ..., n\}$, and $z(0) \equiv 0$. Assume $z(l-1) \geq \overline{m}^*(k) - (n-l+1)$, i.e. if firms l to n all choose to join the cartel, then the resulting cartel size is at least $\overline{m}^*(k)$. Then, there exists a unique SPE of the ensuing subgame such that $z(l-1) + \sum_{i=l}^{n} z_i^* = \min\{\overline{m}^*(k), z(l-1)\}$, where z_i^* is firm i's participation decision along the equilibrium path. In equilibrium,

$$z_l^* = \begin{cases} 1 & \text{if } z(l-1) = \overline{m}^*(k) - (n-l+1) \\ 0 & \text{if } z(l-1) > \overline{m}^*(k) - (n-l+1). \end{cases}$$

The proposition follows by setting $l = 1$. We prove the assertion by induction.

Assume $l = n$. If $z(n-1) \geq \overline{m}^*(k)$, then $z_n^* = 0$ since all cartel sizes above $\overline{m}^*(k)$ are internally unstable (lemma 3.6), i.e. $\pi_f(m;k) > \pi_c(m+1;k)$ for all $m \in \{\overline{m}^*(k)+1, ..., n\}$. If, however, $z(n-1) = \overline{m}^*(k) - 1$, then $z_i^* = 1$ since $\pi_c(\overline{m}^*(k);k) > \pi_f(\overline{m}^*(k)-1;k)$ by assumption. Hence, the assertion holds for $l = n$. Assume now that the assertion is satisfied for all $l \in \{\widehat{l}+1, ..., n\}$, where $\widehat{l} \in \{1, ..., n-1\}$. We now want to prove that it is then still satisfied for $l = \widehat{l}$. Now, if $z(\widehat{l}-1) \geq \overline{m}^*(k)$, then clearly $z_{\widehat{l}}^* = 0$ since $\pi_f(m;k) > \pi_c(m+1;k)$ for all $m \in \{\overline{m}^*(k)+1, ..., n\}$. If $z(\widehat{l}-1) = \overline{m}^*(k) - (n-\widehat{l}+1)$, then firm $\widehat{l}$'s payoff from joining the cartel is $\pi_c(\overline{m}^*(k);k)$, whereas from joining the fringe it is *at most* $\pi_f(\overline{m}^*(k) - 1; k)$ recalling that $\pi_f(m; k)$ is strictly increasing in m. By assumption, $\pi_c(\overline{m}^*(k); k) > \pi_f(\overline{m}^*(k) - 1; k)$. Hence, $z^*_{\widehat{l}} = 1$ in this case. If, however, $z(\widehat{l} - 1) \in \{\overline{m}^*(k) - (n - \widehat{l}), \overline{m}^*(k) - 1\}$, then firm $\widehat{l}$'s profit from joining the cartel is $\pi_c(\overline{m}^*(k); k)$; the payoff from joining the fringe is $\pi_f(\overline{m}^*(k); k)$, which is strictly more. In equilibrium, $z^*_{\widehat{l}} = 0$ in this case. This proves the assertion. ■

Proof of lemma 3.8. Suppose first that $k_i \geq \overline{x}(\mathbf{z}; \mathbf{k})$. Then, an increase in k_i has no effect on $q(\mathbf{z}; \mathbf{k})$, and hence none on firm i's profit. A rise in k_j, $j \neq i$, weakly increases $q(\mathbf{z}; \mathbf{k})$, as can be seen by examining the implicit function $h(q(\mathbf{z}; \mathbf{k})) = 0$. Assuming the derivatives exist,

$$\begin{aligned}
\frac{d\pi_i(\mathbf{z}; \mathbf{k})}{dk_j} &= \frac{d}{dq} \{\overline{x}(q(\mathbf{z}; \mathbf{k})) [P(q(\mathbf{z}; \mathbf{k})) - c]\} \frac{dq(\mathbf{z}; \mathbf{k})}{dk_j} \\
&= \{\overline{x}'(q(\mathbf{z}; \mathbf{k})) [P(q(\mathbf{z}; \mathbf{k})) - c] + \overline{x}(q(\mathbf{z}; \mathbf{k})) P'(q(\mathbf{z}; \mathbf{k}))\} \frac{dq(\mathbf{z}; \mathbf{k})}{dk_j} \leq 0,
\end{aligned}$$

since $\overline{x}'(q(\mathbf{z}; \mathbf{k})) < 0$. Suppose now that $k_i < \overline{x}(\mathbf{z}; \mathbf{k})$. Again assuming the derivatives exist, we have

$$\begin{aligned}
\frac{d\pi_i(\mathbf{z}; \mathbf{k})}{dk_i} &= \frac{d}{dk_i} k_i [P(q(\mathbf{z}; \mathbf{k})) - c] \\
&= [P(q(\mathbf{z}; \mathbf{k})) - c] + k_i P'(q(\mathbf{z}; \mathbf{k})) \frac{dq(\mathbf{z}; \mathbf{k})}{dk_i} \\
&\geq [P(q(\mathbf{z}; \mathbf{k})) - c] + k_i P'(q(\mathbf{z}; \mathbf{k})) > 0,
\end{aligned}$$

where the first inequality follows from $dq(\mathbf{z}; \mathbf{k})/dk_i \in (0, 1]$, and the second from the fact that $k_i < \overline{x}(\mathbf{z}; \mathbf{k})$. Similarly,

$$\frac{d\pi_i(\mathbf{z}; \mathbf{k})}{dk_j} = k_i P'(q(\mathbf{z}; \mathbf{k})) \frac{dq(\mathbf{z}; \mathbf{k})}{dk_j} \leq 0.$$

The same analysis applies to the cartel's joint profit by simply changing the indices. ■

Proof of lemma 3.9. Suppose first that neither firm i nor firm j face a binding capacity constraint if $z_i = z_j = 0$; hence, $q_i((0, \mathbf{z}_{-i}); \mathbf{k}) = q_j((0, \mathbf{z}_{-j}); \mathbf{k}) = \overline{x}(q(\mathbf{z}; \mathbf{k}))$, where $q(\mathbf{z}; \mathbf{k}) \equiv \sum_{i \in N} q_i((z_i, \mathbf{z}_{-i}); \mathbf{k})$ is aggregate industry output. This implies that both firms make the same profit in the fringe, i.e. $\pi_i(\mathbf{z}; \mathbf{k}) = \pi_j(\mathbf{z}; \mathbf{k})$. But then, firm i has a (weakly) higher incentive to deviate since $\pi_i((1, \mathbf{z}_{-i}); \mathbf{k}) \geq \pi_j((1, \mathbf{z}_{-j}); \mathbf{k})$. To see this notice that $\pi_i((1, \mathbf{z}_{-i}); \mathbf{k}) = k_i / \left(k_{M(\mathbf{z})} + k_i\right) \pi_M((1, \mathbf{z}_{-i}); \mathbf{k})$, where $k_{M(\mathbf{z})}$ is the cartel's joint capacity given $\mathbf{z}$, i.e. excluding firms i and j. Moreover, $\pi_M((1, \mathbf{z}_{-i}); \mathbf{k}) \geq$

$\pi_M((1, \mathbf{z}_{-j}); \mathbf{k})$ since the cartel's joint profit is weakly increasing in its own capacity, and weakly decreasing in the capacity of a fringe firm; see lemma 3.8.

Suppose now that both firms i and j face a binding capacity constraint if $z_i = z_j = 0$; hence, $q_i((0, \mathbf{z}_{-i}); \mathbf{k}) = k_i$, and $q_j((0, \mathbf{z}_{-j}); \mathbf{k}) = k_j$. Since i and j are the largest fringe firms, all other fringe firms are then constrained as well. We need to show that if

$$\frac{k_i}{k_{M(\mathbf{z})} + k_i}\pi_M((1, \mathbf{z}_{-i}); \mathbf{k}) - k_i\,[P(q(\mathbf{z}; \mathbf{k})) - c] < 0,$$

then

$$\frac{k_j}{k_{M(\mathbf{z})} + k_j}\pi_M((1, \mathbf{z}_{-j}); \mathbf{k}) - k_j\,[P(q(\mathbf{z}; \mathbf{k})) - c] < 0. \tag{3.20}$$

Comparing the two inequalities, one sees that it is sufficient to show the following inequality:

$$\frac{\pi_M((1, \mathbf{z}_{-i}); \mathbf{k})}{k_{M(\mathbf{z})} + k_i} \geq \frac{\pi_M((1, \mathbf{z}_{-j}); \mathbf{k})}{k_{M(\mathbf{z})} + k_j}. \tag{3.21}$$

Notice that firms i and j continue to face a binding constraint in the fringe if the other firm deviates and joins the cartel. To see this notice that, from the implicit function $h(q(\mathbf{z}; \mathbf{k})) = 0$, such a deviation will weakly decrease industry output $q(\mathbf{z}; \mathbf{k})$, and hence weakly increase $\overline{x}(q(\mathbf{z}; \mathbf{k}))$. In order to show inequality (3.21), we adopt a differential approach. If $q_M((1, \mathbf{z}_{-i}); \mathbf{k}) = \overline{x}(q((1, \mathbf{z}_{-i}); \mathbf{k}))$, then one can prove that

$$\begin{aligned}
&\frac{d}{dk_i}\left(\frac{\pi_M((1, \mathbf{z}_{-i}); \mathbf{k})}{k_{M(1,\mathbf{z}_{-i})}}\right)\Bigg|_{k_i + k_j = const.} \\
\geq\ & \frac{k_{M(1,\mathbf{z}_{-i})}\left(-\overline{x}(q((1, \mathbf{z}_{-i}); \mathbf{k}))P'(q((1, \mathbf{z}_{-i}); \mathbf{k}))\right) - \pi_M((1, \mathbf{z}_{-i}); \mathbf{k})}{\left(k_{M(1,\mathbf{z}_{-i})}\right)^2} \\
=\ & \frac{\left(k_{M(1,\mathbf{z}_{-i})} - \overline{x}(q((1, \mathbf{z}_{-i}); \mathbf{k}))\right)[P(q((1, \mathbf{z}_{-i}); \mathbf{k})) - c]}{\left(k_{M(1,\mathbf{z}_{-i})}\right)^2} \geq 0
\end{aligned}$$

which implies inequality (3.21). If, instead, $q_M((1, \mathbf{z}_{-i}); \mathbf{k}) = k_{M(1,\mathbf{z}_{-i})} < \overline{x}(q((1, \mathbf{z}_{-i}); \mathbf{k}))$, then this derivative is zero, since aggregate output is independent of how capacity is distributed between a fringe firm and the cartel if both face a binding capacity constraint.

Finally, suppose that firm j, but not firm i, faces a binding capacity constraint if $z_i = z_j = 0$; hence, $q_i((0, \mathbf{z}_{-i}); \mathbf{k}) = \overline{x}(q(\mathbf{z}; \mathbf{k}))$, and $q_j((0, \mathbf{z}_{-j}); \mathbf{k}) = k_j$. First, observe that firm j's incentive to deviate, as represented by the l.h.s. of (3.20), is weakly decreasing in k_i for $k_i \geq \overline{x}(q(\mathbf{z}; \mathbf{k}))$; the first term in (3.20) is weakly decreasing in k_i, and the second term is independent of k_i for $k_i \geq \overline{x}(q(\mathbf{z};\mathbf{k}))$. Second, notice that firm i's incentive to deviate,

$$\frac{k_i}{k_{M(\mathbf{z})} + k_i}\pi_M((1,\mathbf{z}_{-i});\mathbf{k}) - \overline{x}(q(\mathbf{z};\mathbf{k}))\left[P(q(\mathbf{z};\mathbf{k})) - c\right],$$

is strictly increasing in k_i for $k_i \geq \overline{x}(q(\mathbf{z};\mathbf{k}))$. To see this, observe that the ratio of capacities is strictly increasing in k_i, whereas all other terms are independent of k_i for $k_i \geq \overline{x}(q(\mathbf{z};\mathbf{k}))$. Hence, it is sufficient to prove that firm i has a stronger incentive to join the cartel than firm j if $k_i = \overline{x}(q(\mathbf{z};\mathbf{k}))$. But this has already been shown. ■

Chapter 4

Monopolisation and Industry Structure

4.1 Introduction

For a long time, a major topic in the literature on industrial market structure has been to explain differences in concentration across industries by reference to a small number of explanatory variables. The agenda of this traditional strand of the literature was strengthened by the finding that the ranking of industries by concentration tends to be very similar from one country to another.[1] This regularity appeared to show that the underlying pattern of technology and tastes strongly constrains equilibrium structure. Much of the old empirical work on cross-sectional differences in concentration was rooted in Bain's (1956) structure-conduct-performance paradigm according to which structure (concentration) is determined by certain "barriers to entry". A typical study in this literature seeked to explain structure by regressing observed concentration measures on proxies for barriers to entry such as scale economies, advertising and R&D intensity etc. This approach, however, was strongly criticised even prior to the game-theoretic revolution in industrial organisation (IO). In particular, researchers remarked that many of the right-hand side variables were endogenous; advertising and R&D intensity, for instance, should depend on market concentration. The econometric response to the endogeneity problem consisted

[1]See, for instance, Bain (1966).

in estimating a system of simultaneous equations. However, this line of research did not prove to be entirely successful; see Schmalensee (1989).

The introduction of formal game-theoretic modelling into IO increased the unease of many researchers with cross-industry studies. The dilemma of the game-theoretic approach appears to be two-fold. First, equilibrium outcomes often depend delicately on features of the model that are unobservable to an empirical researcher, and likely to vary from one industry to another. Second, even if we can pin down the specification of the game, there remains the problem that many models have multiple equilibria. The response of many researchers to this dilemma has been to focus on single-industry studies so as to rely on specific features of that industry to motivate assumptions. A quite different response of other researchers has been not to give up on cross-industry studies, but rather to seek for more robust mechanisms that hold good across a broad range of industries. An outstanding example of the latter line of research is the "bounds approach to concentration", developed by Sutton (1991) in his book *Sunk Costs and Market Structure.* The idea of the bounds approach to concentration is to divide the space of outcomes into those outcomes that can be sustained as equilibrium outcomes in a broad class of admissible models and those that can not.

Sutton (1991) applies the bounds approach to the study of the relationship between concentration and market size. Quite surprisingly, this relationship did not receive much attention in the early literature, even though the relative size of an industry appears to be exogenous (at least as a first approximation). From a theoretical viewpoint, a negative size-structure relationship was considered to be obvious: for a given level of barriers to entry, an increase in market size should raise the profitability of incumbent firms and thus trigger new entry, which would lead to a fall in concentration. However, the empirical evidence for a negative relationship was found to be rather weak.

Sutton shows that the alleged negative relationship between market size and concentration breaks down in certain groups of industries. In particular, he introduces the important distinction between "exogenous" and "endogenous" sunk cost industries. In exogenous sunk cost industries, the only sunk costs involved are the exogenously given setup costs; R&D and advertising outlays are insignificant. In endogenous sunk cost industries, on the other hand, the equilibrium level of sunk costs is endogenously determined by firms' investments decisions. Roughly, these are industries in which advertising or R&D are effective in that investments in some fixed outlays raise consumers' willingness-to-pay, or reduce marginal costs of production. Sutton's predictions are that, in exogenous sunk cost industries, the lower bound to concentration (i.e. the lower bound to the set of "rationalisable" outcomes) tends to zero as the market becomes large, whereas in industries for which the endogenous sunk cost model applies the lower bound to concentration is bounded away from zero, no matter how large the market. That is, in endogenous sunk cost industries, very fragmented outcomes (in the sense of low one-firm concentration ratios) can not be supported as equilibrium outcomes in large markets; such outcomes can not be excluded in exogenous sunk cost industries.[2] An empirical test of Sutton's predictions can be found in Sutton (1991), and Robinson and Chiang (1996).

Sutton's predictions, although robust, may not appear entirely satisfactory in that they are quite "weak": they refer to the stability of fragmented outcomes in large markets only. But such a criticism would miss the point of the bounds approach to concentration. Nevertheless, the important open question, raised by Bresnahan (1992) and others, is whether or not it is possible to make tighter predictions regarding the size-structure relationship.

The aim of this paper is to investigate in what kind of industries it is possible to sustain *concentrated* outcomes, and in what kind of industries it is not. The question addressed in this article is thus: 'Is there an *upper bound to concentration*?' In fact, an inspection of Sutton's dataset reveals that most datapoints are relatively close to the estimated lower bound.[3] Moreover, the datapoints do not "fill" the space above the lower bound. That is, there appear to be limits to monopolisation of industries. Notice that identifying an upper bound does not mean providing a certain maximum concentration measure but rather finding testable comparative statics results. This obviously requires first identifying a trade off firms may face in their attempts to monopolise markets.

The economic history of the U.S. at the turn of the century provides many examples of "attempts of monopolisation". At a time when mergers were not yet scrutinised by

[2]The "nonfragmentation" or "nonconvergence" result for endogenous sunk cost models has been formally shown by Shaked and Sutton (1987).

[3]See Figure 5.4 on page 118 in Sutton (1991).

antitrust authorities, firms attempted to monopolise industries by horizontal mergers. The most famous case is probably that of the formation, in 1901, of the United States Steel Corporation by the consolidation of twelve steel producers. In their study of the U.S. Steel case, Parsons and Ray (1975) convincingly argue that it was primarily motivated by the successful attempt to gain market power. An important feature of the U.S. Steel case (as well as of many other merger cases) was the steady decline of U.S. Steel's market share. For instance, U.S. Steel's market share of steel ingot and casting production decreased from 65.4% in 1902 to 54% in 1911, and 38.9% in 1929.[4] According to Stigler (1950), the observed (albeit relatively slow) decline in market share was due to new entry and the expansion of fringe firms.[5] The lesson one can learn from the U.S. Steel merger case is thus the following. Firms have an incentive to monopolise the market in order to gain market power. In the absence of antitrust authorities, this can be accomplished by horizontal mergers. However, due to new entry and the free-riding behaviour of outsiders, it appears to be difficult to persistently monopolise an industry.

To study the limits of monopolisation, we investigate, in the first part of the paper, the market structure that would emerge if firms were free to merge in the absence of any antitrust laws. As Stigler (1950) pointed out, the resulting market structure is not necessarily a monopoly since firms face a trade off between participation in a merger (so as to achieve a less competitive outcome) and nonparticipation (so as to take a free ride on the merging firms' effort to restrict output). We consider the linear-demand model due to Shubik and Levitan (1980), in which goods are horizontally differentiated; all products are treated symmetrically, and competition is non-localised. The degree of substitutability between goods is summarised by a one-dimensional parameter, σ. Prior to mergers, each firm is equipped with the knowledge to produce one distinct product; the product portfolio of post-merger coalitions is the collection of products offered by its members.

[4]This reduction in market share was slowed down by U.S. Steel's aggressive purchase of ore deposits. The corresponding rise in market price for ore sharply reduced the profitability of entry. This case is a nice example of how backward integration reduces the profitability of new entry into the downstream market.

[5]Parsons and Ray (1975) describe a revealing example of free-riding behaviour of fringe firms. In 1930, National Steel possessed just 2.5% of the industry's ingot capacity; but in the 1930's, its ingot capacity expansion was one quarter of the industry total expansion during the 1930's. In many years, its steel production subsidiary's capacity utilisation rate was about double the industry average rate.

Following Sutton (1991), we distinguish between exogenous and endogenous sunk cost industries. In the exogenous sunk cost case, the endogenous horizontal merger game consists of three stages. At the first stage, firms decide whether or not to enter the industry; if a firm decides to enter it has to pay some entry fee. At the second stage, the firms that have decided to enter form "coalitions". Finally, at the third stage, the newly formed coalitions compete in prices. We show that, for a given number of firms in the industry, merger to monopoly obtains as long as products are sufficiently good substitutes. As the number of firms increases, the interval of values of σ, for which merger to monopoly obtains, shrinks and eventually vanishes in the limit. That is, for a given value of σ, monopoly does not emerge endogenously if the number of firms is sufficiently large. Moreover, in any equilibrium, the market share of the largest coalition converges to zero as the initial number of firms tends to infinity. We assume that the industry is in a long-run free-entry equilibrium; that is, the number of potential entrants is sufficiently large so that, in equilibrium, further entry is unprofitable. An increase in the size of the market relative to the level of entry costs raises the profitability of entry, and will thus lead to a larger number of entering firms. In the limit as market size (relative to setup costs) goes to infinity, the number of firms increases without bound. But this implies that it is impossible to sustain concentrated outcomes in large markets. *In exogenous sunk cost industries, the upper bound to concentration goes to zero as market size tends to infinity.*

To analyse the endogenous sunk cost case, we use the "quality-augmented" linear-demand model by Sutton (1997,1998). In this case, the model consists of four stages. First, firms decide whether or not to enter the market. Second, firms form coalitions. Third, the coalitions decide how much to invest in the quality of the goods in their portfolio. Finally, the coalitions compete in prices. Even in the absence of mergers, the number of entering firms in the long-run free-entry equilibrium remains finite, no matter how large the market. This is nothing but an application of the nonconvergence result for endogenous sunk cost industries. In fact, if products are sufficiently good substitutes (or investment in quality is sufficiently effective), then only one firm can be supported in equilibrium. Thus, not even in large markets is it in general possible to exclude concentrated outcomes. That is, *in endogenous sunk cost industries, the upper bound to concentration does not decrease with the size of the market.*

In the second part of the paper, we investigate the robustness of our main predictions. For this purpose, we use a recent equilibrium concept by Sutton (1997), which is defined not in the space of strategies but in the space of (observable) outcomes. This equilibrum concept involves two rather weak assumptions, which are both implied by subgame perfection. Moreover, the extensive form of the game is not specified explicitly. In particular, we allow firms to make side payments and to monopolise markets not only through mergers but also through product proliferation. The key feature of the extensive form is the following: there is some penultimate stage (before firms compete in prices) at which new firms can enter the market. This formalises the notion of "ex-post entry" (e.g. post-merger entry), which is, according to Stigler (1950), an empirically powerful force preventing the monopolisation of industries, as exemplified by the U.S. Steel case. We show that, in exogenous sunk cost industries, the upper bound to concentration does indeed go to zero as market size (relative to setup costs) tends to infinity. In contrast, monopoly outcomes may be sustained in endogenous sunk cost industries, no matter how large the market. Hence, *allowing for ex-post entry, the predictions of this paper hold independently of any details regarding coalition formation or product selection.*

4.2 Related Literature

This paper is closely related to several strands in the IO literature. First, it belongs to the game-theoretic literature on industrial market structure, and to the literature on the relationship between market size and concentration in particular. The seminal works in this literature are Sutton's books *Sunk Costs and Market Structure* (Sutton 1991) and *Technology and Market Structure* (Sutton 1998), dealing with advertising-intensive and R&D-intensive industries, respectively, and combining theory, econometric tests and case studies.[6] Much of Sutton's work is concerned with the stability of fragmented outcomes in large markets. The instability of such outcomes in industries in which firms can effectively raise consumers' willingness-to-pay, by investing in some fixed outlays, was first shown in the context of pure vertical product differentiation by Shaked and Sutton (1983), extending earlier work by Gabszewicz and Thisse (1980). More general conditions for non-

[6]Important empirical articles are Bresnahan and Reiss (1990), and Berry (1992).

convergence were proposed by Shaked and Sutton (1987). In these works, nonconvergence is analysed in the context of static stage games. The robustness of these results to the existence of collusive underinvestment equilibria in dynamic investment games has been shown by Nocke (1998). The present article develops this literature further in that it investigates the stability of concentrated outcomes in large markets. It follows Sutton (1991) in distinguishing between exogenous and endogenous sunk cost industries, and builds upon the insights of this earlier work: whether or not merger to monopoly obtains depends upon the number of firms the market could support in the absence of mergers; it is this number that lies at the heart of the analysis in Sutton's book.

Second, in this paper, firms attempt to monopolise markets by horizontal mergers. The IO literature on horizontal mergers can roughly be divided into two strands: exogenous and endogenous mergers. The first strand is mainly concerned with the profitability and welfare consequences of an exogenous horizontal merger by two or more firms; important papers include Salant, Switzer and Reynolds (1983), Perry and Porter (1985), Deneckere and Davidson (1985), Levin (1990), and Farrell and Shapiro (1990).[7] Most of these papers analyse mergers in a homogenous goods industry under Cournot competition. The important contribution by Salant, Switzer and Reynolds (1983) was to show that mergers tend to be unprofitable in such a setting, provided there are no efficiency gains and the merger does not lead to an almost complete monopolisation. Deneckere and Davidson (1985) showed that this result does no longer hold under Bertrand competition (with differentiated products). In fact, the first part of the present paper uses essentially the same multiproduct demand system as Deneckere and Davidson. The advantage of such a setting is that mergers are conceptually well defined; in contrast, in a homogenous goods model with constant returns-to-scale technology, as in Salant, Switzer and Reynolds (1983) and Kamien and Zang (1990), a merger by m firms is equivalent to a reduction in the number of players by $m-1$.

The present paper is even more closely connected to the still underdeveloped literature on endogenous horizontal mergers, which starts from Stigler's (1950) insight that firms may not want to participate in a merger since they may prefer to take a free ride on the merging

[7]There are also some papers on the effects of exogenous horizontal mergers on collusion; see Davidson and Deneckere (1984), and Compte, Jenny and Rey (1996).

firms' effort to restrict output. Hence, the private profitability of a given merger (relative to no merger at all) is in general not sufficient for a merger to occur in equilibrium. Using a homogenous goods Cournot model with constant returns-to-scale technology, Kamien and Zang (1990) were the first to formally analyse endogenous horizontal mergers.[8] In such a setting, clearly, merger to monopoly, although privately profitable (if the alternative is no merger at all), does not emerge in equilibrium if the number of firms, n, is sufficiently large. To see this, notice that, by not participating in the merger of its $n-1$ rivals, a firm can ensure itself the duopoly profit. If the monopoly profit is k times the duopoly profit, then merger to monopoly obtains only if $n \leq [k]$, where $[k]$ is the integer part of k. From Salant, Switzer and Reynolds' (1983) analysis, it is well-known that a merger that falls (significantly) short of monopoly is not privately profitable. Not surprisingly, then, for n sufficiently large, mergers do not occur in equilibrium.[9] The main differences between Kamien and Zang (1990) and the present article are the following. First, we analyse a multiproduct demand system in which mergers are not merely a reduction in the number of players. Second, we consider the case of price competition in which mergers tend to be privately profitable; see Deneckere and Davidson (1985). Indeed, we show that mergers will occur in equilibrium, even in the limit as n tends to infinity. The open question, addressed in this paper, is whether concentrated outcomes can endogenously occur in equilibrium. Third, and most importantly, we introduce several ingredients which allow us to make empirically testable predictions: the degree of horizontal product differentiation (σ), the distinction between exogenous and endogenous sunk cost industries, and market size relative to setup costs (by assuming free but costly entry). Finally, in the second part of the paper, we show that our predictions do not depend on the details of the coalition formation game if we introduce the notion of ex-post entry.

Some of the insights of the endogenous horizontal merger literature have been anticipated by the literature on cartel stability and explicit collusion; see, for instance, Selten (1973), d'Aspremont et al. (1983), and Nocke (1999). In fact, these models are formally

[8]Later work includes Kamien and Zang (1991,1993) and Gowrisankaran (1999).

[9]Kamien and Zang (1990) show that if merged entities are allowed to partly "demerge" after the merger (by forming independent subunits), then mergers tend to be more profitable. But again, merger to monopoly will not obtain if n is sufficiently large.

equivalent to endogenous horizontal merger models. Conversely, the results of the present article may also be of significance for the literature on cartel stability. Finally, this paper is related to the literatures on endogenous (noncooperative) coalition formation (e.g. Hart and Kurz 1983, Bloch 1996, Yi 1997) and multiproduct oligopoly (e.g. Champsaur and Rochet 1989, Shaked and Sutton 1990).

4.3 Endogenous Horizontal Mergers and the Upper Bound to Concentration

In this section, we investigate whether concentrated outcomes can be sustained as equilibrium outcomes in large markets, using a model of endogenous horizontal mergers. Coalition formation is modelled as a noncooperative open membership game; post-merger entry is not considered. The robustness of the predictions is analysed in the next section, where we allow for ex-post entry, endogenous product selection, and more general models of coalition formation. In the following, we distinguish between exogenous and endogenous sunk cost industries.

4.3.1 Monopolisation in Exogenous Sunk Cost Industries

We first analyse the limits of monopolisation in exogenous sunk cost industries, where R&D and advertising outlays are insignificant; the only kind of sunk costs involved are exogenously given by firms' setup costs.

4.3.1.1 The Model

We consider an industry offering a potentially infinite number of substitute goods to S identical consumers. A consumer's utility is given by

$$U(\mathrm{x}; M) = \sum_{k=1}^{\infty} \left(x_k - x_k^2\right) - 2\sigma \sum_{k=1}^{\infty} \sum_{l<k} x_k x_l + M, \tag{4.1}$$

where x_k is consumption of substitute good k, and M is consumption of the outside good whose price is normalised to one. Let Y denote income and p_k the price of good k. Then, $M = Y - \sum_k p_k x_k$. The utility function is taken from Sutton (1997,1998), and can also be found, albeit in slightly different form, in Shubik and Levitan (1980), Deneckere and

Davidson (1985), and Shaked and Sutton (1990).[10] It defines utility over the domain of $\mathbf{x}$ for which all the marginal utilities $\partial U(\mathbf{x}; M)/\partial x_k$ are nonnegative. The form of the utility function ensures that a consumer's inverse demand for any good is linear. Income Y is assumed to be sufficiently large, $Y \geq 1/8\sigma$, so that $Y > \sum_k p_k x_k$ in equilibrium. The parameter σ, $\sigma \in (0,1)$, measures the degree of substitutability between products. In the limit as $\sigma \to 1$, goods become perfect substitutes; in the limit as $\sigma \to 0$, products become independent. All goods (other than the outside good) are treated symmetrically.

We consider the following three-stage game. There are n_0 potential entrants, each of which has the knowledge to produce one distinct substitute good. At the first stage, these n_0 firms decide (either simultaneously or sequentially) whether or not to enter the industry. Entry costs in the industry are given by ϵ, $\epsilon > 0$. Since we want to confine attention to free-entry equilibria, we assume that n_0 is sufficiently large, $n_0 > [S - 8\epsilon(1-\sigma)]/(8\epsilon\sigma)$, so that in any equilibrium of the game there is at least one nonentering firm. At the second stage, the firms that have decided to enter at the previous stage simultaneously decide which "coalition" to join. All firms that have decided to join the same coalition then merge. Formally, firm k selects $z_k = i$, $i \in Z$, if it decides to join coalition M_i. A coalition structure $\left\{(M_i)_{i \in Z}\right\}$ is thus an endogenous partition of the set of entrants, induced by the vector of participation decisions, $\mathbf{z}$. Since there is an infinite number of coalitions firms can join (almost all of which will be empty in equilibrium), arbitrary coalition structures are allowed to emerge in equilibrium. This coalition formation game is sometimes called an open membership game; see Yi (1997). At the third and final stage, the newly formed coalitions, each offering the products of its "members", compete simultaneously in prices. Each coalition faces a constant marginal cost of production, c; to simplify notation, we assume $c = 0$.

Each of the n_0 firms is assumed to act so as to maximise its profit; the same applies to the merged entities at the price-setting stage of the game. The members of each coalition share the coalition's profit. Since firms are symmetric, we assume, for simplicity, that profit is shared equally.

[10] Shubik and Levitan (1980), Deneckere and Davidson (1985), and Shaked and Sutton (1990) all consider price competition, whereas Sutton (1997,1998) analyses quantity competition using these preferences.

4.3.1.2 Equilibrium Analysis

We now seek the pure strategy subgame perfect equilibrium (SPE) of the three-stage merger game.

Price-Setting Stage. We solve for the SPE of the merger game by inducing backwards. Let us, therefore, start with the third stage at which the merged entities simultaneously compete in prices. Suppose that n firms have entered the industry at the first stage, relabel these firms as firms 1 to n, and let $N \equiv \{1,...,n\}$. We take firms' merger decisions, described by the vector $\mathbf{z} = (z_1,...,z_n)$, as given. Let m_i denote the number of members of coalition M_i, i.e. $m_i \equiv \#\{k \mid z_k = i\}$. The following lemma summarises equilibrium behaviour at the price-setting stage; the proof can be found in the appendix.

Lemma 4.1 *For any vector of merger decisions* $\mathbf{z}$, *there exists a unique Nash equilibrium in prices. In equilibrium, each coalition sets the same price for all of its products. Coalition* M_i*'s equilibrium price is given by*

$$p^*_{M_i} = \frac{1-\sigma}{[2(1-\sigma)+(2n-m_i)\sigma]\left[1-\sigma\sum_{j\in I}\frac{m_j}{2(1-\sigma)+(2n-m_j)\sigma}\right]}, \tag{4.2}$$

while its equilibrium profit is

$$S\pi^*_{M_i}(\mathbf{z}) = S\frac{(1-\sigma)\left[1-\sigma+(n-m_i)\sigma\right]}{2\left[1-\sigma+n\sigma\right]\left[2(1-\sigma)+(2n-m_i)\sigma\right]^2\left[1-\sigma\sum_{j\in I}\frac{m_j}{2(1-\sigma)+(2n-m_j)\sigma}\right]^2}. \tag{4.3}$$

Regarding price, several interesting comparative statics results can be obtained. First, for a given coalition structure, the equilibrium price is strictly increasing in the size of the coalition. The reason is that an increase in the price of a product exerts a positive externality on the demand of all other products. Each coalition internalises the externality of a price change on its own products; the more products a merged entity possesses, the higher is, therefore, the price of its products. Second, for a given number of own products, the coalition's price is strictly increasing in industry concentration in the following sense. Consider any two coalitions, M_j and M_l, say, where $m_j \geq m_l$. Then, any increase in $m_j - m_l$ that leaves $m_j + m_l$ unchanged, raises the equilibrium price of coalition M_i, $i \neq j,l$. To see this, observe that the function $\xi(m) \equiv m/[2(1-\sigma)+(2n-m)\sigma]$ is strictly convex. The intuition is that an increase in the average price of rival goods, due to an increase in concentration, induces a coalition to raise its own price as well since prices are strategic complements. Third, if $\sigma \to 1$, then each coalition's price converges to the competitive price, which is equal to zero; this limit case is the famous "Bertrand paradox". If $\sigma \to 0$, then goods become independent, and the price converges to the monopoly price of 1/2.

As to equilibrium profit, there are two important observations to make. The first is that a coalition's profit per product is decreasing in the number of its products, holding the coalition structure fixed. This is a consequence of the fact that merged entities with several products internalise the effect of a price change of one of its products on its other products. The second is that a coalition's profit per product is increasing in industry concentration for a given size of the coalition.

Merger Stage. Let us now induce backwards and analyse the second stage of the game. A vector of merger decisions, $\mathbf{z}^*$, can be sustained in an SPE of the subgame in which n firms enter if and only if

$$\pi^*_{M_{z^*_k}}(z^*_k, \mathbf{z}^*_{-k}) \geq \pi^*_{M_{z_k}}(z_k, \mathbf{z}^*_{-k}) \text{ for all } z_k \in Z,\ k \in \{1, ..., n\}. \tag{4.4}$$

We do not attempt here to give a general characterisation of firms' merger decisions. Instead, we focus on the conditions under which concentrated outcomes can emerge in equilibrium. For the proof of the following result, the interested reader is referred to the appendix.

Proposition 4.1 *If $n \in \{2,3\}$, then merger to monopoly can be supported in an SPE for all $\sigma \in (0,1)$. If $n \geq 4$, then there exists a $\widehat{\sigma}(n)$, $\widehat{\sigma}(n) \in (0,1)$, such that merger to monopoly is sustainable if and only if $\sigma \in [\widehat{\sigma}(n), 1)$, where $\widehat{\sigma}(n)$ is given by*

$$\widehat{\sigma}(n) = \frac{2\left(n^2 - 6n + 7\right) + 2\sqrt{n^4 - 8n^3 + 27n^2 - 44n + 28}}{(n-1)(4n-7)}. \tag{4.5}$$

To understand proposition 4.1, notice first that merger to monopoly from duopoly is always an equilibrium outcome: a monopolist can make at least the same profit per product as a duopolist (by mimicking the duopolists' pricing decisions), and strictly more whenever products are not independent (by raising the price slightly in order to internalise the externalities). This argument breaks down when there are more than two firms (products). Clearly, profit per product is higher under merger to monopoly than under the completely fragmented market structure where each firm offers one product only. However, if a firm deviates unilaterally, it can take a free ride on the $n-1$ merging rivals. The profit of such a free rider is strictly higher than the profit (per product) prior to the merger game, and may be higher or lower than profit under monopoly. Merger to monopoly will occur if and only if products are sufficiently good substitutes since, in this case, price competition is sufficiently tough so as to drive profits down whenever firms do not merge to monopoly.[11]

The higher is the number of entrants, n, the more "difficult" is merger to monopoly. The reason is that each firm's merger decision becomes less "decisive" as n increases; since, for a given market structure, a free-riding firm is always better off than a merging firm, firms have less incentives to merge for higher n.[12] Indeed, it is possible to show that $\widehat{\sigma}'(n) > 0$ for $n \geq 4$. Moreover, we get the following result.

Corollary 4.1 *For any $\sigma \in (0,1)$, there exists a finite threshold value $\widehat{n}(\sigma)$ such that merger to monopoly is sustainable in an SPE if and only if $n < \widehat{n}(\sigma)$.*

Proof. All we need to show is that $\lim_{n\to\infty} \widehat{\sigma}(n) = 1$. Since $\widehat{\sigma}'(n) > 0$ and $\widehat{\sigma}(n) \in (0,1)$ for all $n \geq 4$, it follows that $\lim_{n\to\infty} \widehat{\sigma}(n) \leq 1$. Suppose the assertion is false. Then there exists a $k \in (0,1)$ such that $\lim_{n\to\infty} \widehat{\sigma}(n) < k$. Using (4.5), this can be shown to lead to a contradiction. ■

The corollary shows that it is impossible to sustain merger to monopoly in markets with a sufficiently large number of firms; this holds for any degree of substitutability between products. In the following proposition, we further tighten our predictions; the proof is relegated to the appendix.

Proposition 4.2 *For any $(\sigma, \gamma) \in (0,1)^2$, there exists a finite $n(\sigma;\gamma)$ such that, for all $n \geq n(\sigma;\gamma)$, the market share of each coalition is bounded above by γ in any equilibrium of the merger game.*

Proposition 4.2 may suggest that mergers will not occur in the limit as the number of firms tends to infinity. This is not true, however, as the following proposition shows.

[11] Monopoly profit is also decreasing in the degree of substitutability, σ, since consumers value variety. In the limit as $\sigma \to 1$, industry profit under monopoly converges to to the (standard) monopoly profit of a homogenous goods industry; under all other market structures, industry profit goes to zero as $\sigma \to 1$.

[12] With a continuum of firms, no firm would want to merge since its own decision does not affect market price, and a firm is clearly worse off by reducing output.

Proposition 4.3 *For any $n \geq 2$, the completely fragmented market structure, in which each nonempty coalition has one member only, can not be sustained in equilibrium. That is, mergers will occur in any equilibrium, provided at least two firms enter in equilibrium.*

Proof. This is essentially a corollary of Theorem 1 in Deneckere and Davidson (1985). All we need to show is that if $z_k = k$ for all $k \in N$ (after relabelling post-merger coalitions), then firm 1, say, can profitably deviate by joining coalition M_2, for instance. This deviation clearly increases the prices of all products in the industry. Decompose the price effect of the proposed deviation into two steps. First, let the outsiders (coalitions M_3 through M_n) raise their prices to their new equilibrium values. This will benefit the members of M_2 since it raises the demand for M_2's products. Second, let the coalition M_2 raise the prices of its two products to their new equilibrium values. By definition, this price increase must be profitable for the members of M_2. We have thus shown that the proposed deviation by firm 1 is profitable. ■

It is worth pointing out that proposition 4.3 obtains under the maintained hypothesis of no post-merger entry.

Entry Stage. We now turn to the determination of n, the number of pre-merger entrants, as a function of market size, S, and entry costs, ϵ. Since we are unable to characterise equilibrium in all ensuing subgames, it is impossible to determine the equilibrium number of entering firms, $n^*(S/\epsilon)$. It is, however, possible and useful to compute a lower bound on this number, and to study the limit behaviour of this bound as market size relative to setup costs tends to infinity. For this purpose, it is important to notice that the equilibrium profit (per product) of an entrant, $S\pi_k^*(n)$, is bounded above by the profit under merger to monopoly, $S\overline{\pi}(n)$, and bounded below by the profit in the absence of mergers, $S\underline{\pi}(n)$. From (4.3), these bounds on profits are given by

$$S\overline{\pi}(n) = S \cdot \frac{1}{8\left[1 - \sigma + n\sigma\right]},$$

and

$$S\underline{\pi}(n) = S \cdot \frac{(1-\sigma)\left[1 - \sigma + (n-1)\sigma\right]}{2\left[1 - \sigma + n\sigma\right]\left[2(1-\sigma) + (n-1)\sigma\right]^2}.$$

The upper bound on the number of entrants, $\overline{n}(S/\epsilon)$, is given by the maximum integer n such that $S\overline{\pi}(n) \geq \epsilon$; that is, $\overline{n}(S/\epsilon)$ is the integer part of $\left[S - 8\epsilon(1-\sigma)\right]/(8\epsilon\sigma)$, which is, by assumption, less than the number of potential entrants, n_0. To calculate the lower bound, observe that $S\underline{\pi}(n)$ is strictly decreasing in n, and strictly increasing in S. Hence, the number of entering firms in the absence of mergers, $\underline{n}(S/\epsilon)$, is the maximum integer n such that $S\underline{\pi}(n) \geq \epsilon$. Moreover, $\underline{n}(S/\epsilon)$ is strictly increasing in S/ϵ, and $\lim_{S/\epsilon\to\infty} \underline{n}(S/\epsilon) = \infty$. Since $n^*(S/\epsilon) \geq \underline{n}(S/\epsilon)$, it follows that $\lim_{S/\epsilon\to\infty} n^*(S/\epsilon) = \infty$ This, in conjunction with propositions 4.2 and 4.3, establishes the following proposition.

Proposition 4.4 (1.) *For any $(\sigma, \gamma) \in (0,1)^2$, there exists a finite $(S/\epsilon)(\sigma;\gamma)$ such that, for all $S/\epsilon \geq (S/\epsilon)(\sigma;\gamma)$, the market share of each coalition is bounded above by γ in any equilibrium of the game.* (2.) *For any S/ϵ sufficiently large, mergers occur in any equilibrium.*

Proposition 4.4 states the central prediction for exogenous sunk cost industries. It is impossible to sustain very concentrated outcomes (in the sense of high concentration ratios) in large exogenous sunk cost industries. More precisely, the *upper bound to concentration* converges to zero as market size goes to infinity.

4.3.2 Monopolisation in Endogenous Sunk Cost Industries

We now turn to the analysis of the limits of monopolisation in endogenous sunk cost industries, where the level of sunk costs is endogenously determined by firms' investment decisions.

4.3.2.1 The Model

The endogenous sunk cost model differs from the exogenous sunk cost model analysed in the previous subsection in that firms can invest in R&D or advertising so as to increase the consumers' willingness-to-pay for their products. As before, there are n_0 potential entrants, each equipped with the know how to produce one distinct substitute good, and S identical consumers. Using our previous notation, the utility function is now given by

$$U(\mathbf{x}; M; \mathbf{u}) = \sum_{k=1}^{\infty} \left(x_k - \frac{x_k^2}{u_k^2} \right) - 2\sigma \sum_{k=1}^{\infty} \sum_{l<k} \frac{x_k}{u_k} \frac{x_l}{u_l} + M, \tag{4.6}$$

where u_k, $u_k \in [1, \infty)$, is the perceived quality of substitute good k. If $u_k = 1$ for all k, then (4.6) reduces to the utility function of the exogenous sunk cost model, (4.1). It is easy to verify that an increase in u_k strictly increases utility whenever $x_k > 0$; that is, consumers value quality.

The timing of the game is as follows. At the first stage, the n_0 potential entrants decide whether to enter the market; the entry cost is denoted by ϵ. Again, we assume n_0 to be sufficiently large. At the second stage, the firms that have decided to enter at the previous stage play the same simultaneous-move coalition formation game as in the exogenous sunk cost case. At the third stage, the newly formed coalitions simultaneously choose the qualities of their products by investing in fixed R&D or advertising outlays. The cost of achieving quality u_k, $u_k \in [1, \infty)$, for good k is given by

$$F(u_k) = F_o u_k^{\beta}, \tag{4.7}$$

where the parameter β is the elasticity of the investment cost function; we assume $\beta > 2$. However, a coalition may decide not to invest at all in the quality of good k; in this case, it cannot offer the good at the output stage. At the final stage, the coalitions simultaneously compete in prices; production costs are assumed to be zero.

4.3.2.2 Equilibrium Analysis

We now seek to characterise the pure strategy subgame perfect equilibrium (SPE) of the four-stage game. Let us first consider the final price-setting stage. We take as given firms' previous entry, merger and quality decisions.

Let $y_k(\mathbf{p}; \mathbf{u}) \equiv d_k(\mathbf{p}; \mathbf{u})/u_k$ be normalised demand for good k; similarly, $q_k \equiv p_k u_k$ denotes the normalised price of k. Equilibrium behaviour at the price-setting stage can now be characterised as follows.

Lemma 4.2 *Deleting all products with zero sales in equilibrium, coalition M_i's average normalised equilibrium price is given by*[13]

$$\overline{q}^*_{M_i} = \frac{(1-\sigma)\overline{u}_{M_i} + \sigma \sum_{j} \overline{m}_j (\overline{u}_{M_i} - \overline{u}_{M_j}) - (1 - \sigma + \overline{n}\sigma) \sum_j \frac{\sigma \overline{m}_j (\overline{u}_{M_i} - \overline{u}_{M_j})}{2(1-\sigma) + (2\overline{n} - \overline{m}_j)\sigma}}{[2(1-\sigma) + (2\overline{n} - \overline{m}_i)\sigma] \left[1 - \sum_j \frac{\sigma \overline{m}_j}{2(1-\sigma) + (2\overline{n} - \overline{m}_j)\sigma}\right]}, \tag{4.8}$$

[13]Strictly speaking, equation (4.8) only holds if there are no products, which make *just* zero sales but constrain equilibrium.

where the $\overline{m}_j$'s and $\overline{n}$ are the numbers of products with positive sales in equilibrium, and $\overline{u}_{M_i}$ is the average over the u_k's with positive sales, $k \in M_i$. Equilibrium price of good k is

$$q_k^* = \overline{q}_{M_i}^* + \frac{u_k - \overline{u}_{M_i}}{2}, \tag{4.9}$$

provided sales are positive.

The proof of lemma 4.2 is rather lengthy and can be found in the appendix. Indeed, in the endogenous sunk cost model, characterisation of equilibrium is more difficult than in the exogenous sunk cost model. The reason is that products of sufficiently low quality make zero sales in equilibrium; the equilibrium price of these products is not uniquely defined (except possibly for those goods that make *just* zero sales). Moreover, it is possible that a good of a certain quality is not produced in equilibrium while a good of a lower quality, owned by a different coalition, makes positive sales; this is due to "portfolio effects".

Our aim is to investigate whether it is possible to sustain concentrated outcomes in large endogenous sunk cost industries. Unfortunately, it is very hard to solve for equilibria at the investment stage; the following problems arise.

- A multiproduct coalition may find it optimal to invest in a subset of its products only. The equilibrium number of products it offers depends on the substitutability of products (σ), the effectiveness of R&D and advertising (β), and on the details of the coalition structure. For instance, a monopolist may offer less products than a coalition that has less members but faces rivals.

- Even for a fixed number of products, it is not possible to solve the first-order conditions for quality explicitly, unless the coalition structure is symmetric. Moreover, the first-order conditions are not sufficient conditions for a global profit maximum; boundary solutions are endemic.

- Multiple equilibria at the investment stage may arise even under symmetric coalition structures.

In contrast to the exogenous sunk cost case, the size of a coalition and the sizes of rival coalitions are no longer sufficient to summarise a coalition's equilibrium profit in the ensuing subgame: the profits of coalitions of equal size may differ since equilibria at the investment stage are often asymmetric.

Instead of solving the game, we therefore seek to find a lower bound on the one-firm concentration ratio that can arise in equilibrium. Clearly, any equilibrium market structure can not be more fragmented (in terms of the market share of the largest coalition) than the market structure that would arise in the absence of mergers.

Most-Fragmented Market Structure. To find a lower bound on concentration, we assume that firms are not allowed to merge; that is, each entrant k is constrained to set $z_k = k$ at the second stage. Let us now start by considering the pricing stage, where each firm offers one product. We take the vector of qualities, $\mathbf{u}$, as given. Suppose $\mathbf{q}^*$ forms a Nash equilibrium in (normalised) prices. Then, if product k makes strictly positive sales, then any product l with $u_l \geq u_k$ makes positive sales as well. To see this, notice that firm l, offering quality $u_l \geq u_k$, can ensure itself positive sales and, hence, positive profits by setting price q_l such that $u_l - q_l = u_k - q_k^*$. Relabel firms in decreasing order of quality, i.e. $u_k \geq u_{k+1}$ for all $k \in \{1, ..., n-1\}$. Suppose there are $\overline{n}$ products with positive sales. From (4.8), the equilibrium price of good k, $k \leq \overline{n}$, is given by[14]

$$q_k^*(\overline{n}) = \frac{(1-\sigma)\left[2(1-\sigma)+(2\overline{n}-1)\sigma\right]u_k + \overline{n}\sigma\left[1-\sigma+(\overline{n}-1)\sigma\right](u_k - \overline{u}_N)}{\left[2(1-\sigma)+(\overline{n}-1)\sigma\right]\left[2(1-\sigma)+(2\overline{n}-1)\sigma\right]},$$

where $\overline{u}_N \equiv (1/\overline{n})\sum_{l=1}^{\overline{n}} u_l$ is the average quality of products with positive sales. Equilibrium output can be computed as

$$y_k^*(\overline{n}) = \frac{\left[1-\sigma+(\overline{n}-1)\sigma\right]}{2(1-\sigma)\left[1-\sigma+\overline{n}\sigma\right]} \cdot q_k^*(\overline{n}).$$

The equilibrium number of products with positive sales, $\overline{n}^*$, is uniquely defined as the maximum integer $\overline{n}$, $\overline{n} \leq n$, such that $q_{\overline{n}}^*(\overline{n}) > 0$. Except for the prices of products with zero sales, the equilibrium is unique.

At the third stage, firms (coalitions) simultaneously decide how much to invest in quality. Due to symmetry, any equilibrium is such that $u_k \in \{0, \overline{u}\}$ for all k. That is, each firm spends the same amount on quality, given that it invests at all. Suppose $\widetilde{n}$, $\widetilde{n} \leq n$, firms invest in quality. The equilibrium quality level, $\overline{u}(\widetilde{n})$, is then implicitly defined by

[14] Again, this equation implicitly assumes that there is no product with zero sales that constrains equilibrium.

the first-order condition

$$S\frac{[1-\sigma+(\widetilde{n}-1)\sigma]\left[2+3(\widetilde{n}-2)\sigma+(\widetilde{n}^2-5\widetilde{n}+5)\sigma^2\right]}{[1-\sigma+\widetilde{n}\sigma]\,[2(1-\sigma)+(2\widetilde{n}-1)\sigma]\,[2(1-\sigma)+(\widetilde{n}-1)\sigma]^2}\overline{u}(\widetilde{n})-\beta F_0\overline{u}^{\beta-1}(\widetilde{n})=0,$$

which yields

$$\overline{u}(\widetilde{n})=\left(\frac{S\,[1-\sigma+(\widetilde{n}-1)\sigma]\left[2+3(\widetilde{n}-2)\sigma+(\widetilde{n}^2-5\widetilde{n}+5)\sigma^2\right]}{\beta F_0\,[1-\sigma+\widetilde{n}\sigma]\,[2(1-\sigma)+(2\widetilde{n}-1)\sigma]\,[2(1-\sigma)+(\widetilde{n}-1)\sigma]^2}\right)^{\frac{1}{\beta-2}}. \quad (4.10)$$

It is possible to show that, for the same number of products with positive quality, firms invest more in quality in the completely fragmented market structure than under monopoly; this is due to the "business stealing effect" of investment, which is internalised by a monopolist. The total profit of a firm with quality $\overline{u}(\widetilde{n})$ is of the form $\Pi_k^*(\widetilde{n})=\phi(\widetilde{n};\beta,\sigma,S,F_0)\cdot\gamma(\widetilde{n};\beta,\sigma)$, where $\phi(\widetilde{n};\beta,\sigma,S,F_0)>0$, and

$$\gamma(\widetilde{n};\beta,\sigma)\equiv\beta(1-\sigma)\,[2(1-\sigma)+(2\widetilde{n}-1)\sigma]-2\left[2+3(\widetilde{n}-2)\sigma+(\widetilde{n}^2-5\widetilde{n}+5)\sigma^2\right].$$

It is easy to verify that $\gamma(1;\beta,\sigma)>0$ and $\lim_{\widetilde{n}\to\infty}\gamma(\widetilde{n};\beta,\sigma)=-\infty$. Since $\gamma(\widetilde{n};\beta,\sigma)$ is a quadratic function of $\widetilde{n}$, it has a unique root $\widetilde{n}^*$ such that $\partial\gamma(\widetilde{n}^*;\beta,\sigma)/\partial\widetilde{n}<0$, which is given by

$$\widetilde{n}^*=\frac{1}{2\sigma}\left[(\beta-3)-(\beta-5)\sigma+\sqrt{(\beta-1)^2-2\left(\beta^2-3\beta+3\right)\sigma+\left(\beta^2-4\beta+5\right)\sigma^2}\right].$$

We claim that the (maximum) equilibrium number of firms that invest in quality is given by $\min\{[\widetilde{n}^*],n\}$. To see this, notice first that qualities of rival products (with positive sales) are strategic substitutes and that $\overline{u}(\widetilde{n})$ is decreasing in $\widetilde{n}$. Hence, if a firm finds it unprofitable to invest in quality if $[\widetilde{n}^*]$ firms have quality level $\overline{u}\left([\widetilde{n}^*]+1\right)$, then no firm will find it profitable if the same number of firms offer quality $\overline{u}\left([\widetilde{n}^*]\right)$. That is, firms offering zero quality in equilibrium have no incentive to deviate. Firms offering positive quality levels in the candidate equilibrium can not profitably deviate either since their quality levels are given by the first-order conditions, which satisfy the second-order conditions. Notice, however, that more concentrated equilibria may exist.[15]

We now turn to the analysis of the first stage at which firms simultaneously decide whether or not to enter the market. Since $\phi(\widetilde{n};\beta,\sigma,S,F_0)\to\infty$ as $S\to\infty$, it follows from

[15] A pathological multiplicity arises if $\widetilde{n}^*$ is an integer. In this case, $\min\{[\widetilde{n}^*]-1,n\}$ can also be sustained in equilibrium.

the analysis of the investment stage that the maximum equilibrium number of entering firms, n^*, is given by $[\widetilde{n}^*]$ in the limit as market size S tends to infinity, provided that $\widetilde{n}^*$ is not an integer. If $\widetilde{n}^*$ is an integer, then the limit number of entrants is $\widetilde{n}^* - 1$.

In endogenous sunk cost industries, the equilibrium number of entering firms is bounded, no matter how large the market. Even in the absence of mergers, concentration in endogenous sunk cost industries can not get arbitrarily small by increasing the size of the market. This implies that proposition 4.4 can not hold in endogenous sunk cost industries. Our result is nothing else but an instance of a fundamental result in the analysis of industrial market structure: the "nonconvergence property", due to Shaked and Sutton (1987), according to which fragmented outcomes can not supported as equilibrium outcomes in endogenous sunk cost industries. The question of interest, not addressed by Shaked and Sutton, is whether it is possible to sustain *arbitrarily* concentrated outcomes.

The limit number of entering firms is decreasing in σ for two reasons. First, the higher is σ, the less variety is offered by the market, and hence the less consumers spend on the goods on offer in this industry, holding prices fixed. Second, the larger the degree of substitutability between goods, the tougher is price competition, and thus the lower are profit margins. The number of entrants is increasing in β since the more convex the investment cost function is, the less firms will outspend each other in equilibrium. An instructive way of expressing the comparative statics results is the following. The limit number of entrants, as market size tends to infinity, is one if and only if $\sigma \in (\sigma_1(\beta), 1)$; it is equal to n, $n \geq 2$, if and only if $\sigma \in (\sigma_n(\beta), \sigma_{n-1}(\beta)]$, where

$$\sigma_n(\beta) \equiv \frac{(2n-3)\beta - 6(n-1) + \sqrt{(2n+1)^2\beta^2 - 4(2n^2+5n+1)\beta + 4(n^2+6n+1)}}{2\left[(2n-1)\beta + 2(n^2-3n+1)\right]}.$$

We can now analyse two limit cases. First, if $\beta \to \infty$, then $\sigma_n(\beta) \to 1$, and for no value of σ is the limit number of entrants finite. Indeed, this limit can be interpreted as the "exogenous sunk cost case", where every active firm chooses quality level 1 in equilibrium, and the number of firms grows without bound as market size increases.[16] Second, if $\beta \to 2$, then $\sigma_n(\beta) \to 0$, and for all values of σ the limit number of entering firms is one. This limit may be dubbed the "natural monopoly case".

[16]More precisely, we consider the number of entrants as both S and β tend to infinity in such a way that $S/\beta \to \infty$ and $(S/\beta)^{2/(\beta-2)} \to 1$.

Let us summarise our results in the following proposition.

Proposition 4.5 *If firms are not allowed to merge, the equilibrium number of firms remains finite, no matter how large the market. In particular, if goods are sufficiently good substitutes (σ close to 1) or investment is sufficiently effective (β close to 2), only one firm will enter the market, even as market size (relative to setup costs) tends to infinity.*

If firms are allowed to merge, then more concentrated outcomes will emerge in equilibrium, and more firms will enter the market, than in the absence of mergers. We have shown that the most fragmented market structure in endogenous sunk cost industries may involve arbitrarily concentrated outcomes in large markets. The empirical prediction for endogenous sunk cost industries can thus be summarised as follows.

Corollary 4.2 *In endogenous sunk cost industries, arbitrarily high one-firm sales concentration ratios may be supported in equilibrium, even in the limit as market size tends to infinity. That is, the upper bound to concentration does not decrease with market size.*

It is important to point out that this prediction is *not* a mere consequence of the nonconvergence result, according to which it is impossible to sustain arbitrarily fragmented market structures in large endogenous sunk cost industries. Our corollary states that it is possible to support arbitrarily concentrated outcomes; this holds independently of the size of the market and the level of setup cost.

4.4 Ex-Post Entry and the Limits to Concentration

The main predictions of the last section, regarding the relationship between market size and the upper bound to concentration, have been derived under quite special assumptions. First, we have modelled coalition formation, i.e. mergers, as an open membership game. Second, we have assumed that multiproduct firms can only emerge through mergers; firms are not allowed to choose the number of products they would like to offer. Third, we have derived our results under the assumption of no post-merger entry.

Due to the first assumption, our previous analysis leaves open the question whether the "instability" of concentrated outcomes is a consequence of coordination failures, which may or may not occur under different assumptions on coalition formation. After all, for a given number of firms in the industry, joint profits are maximised under monopoly. The second assumption implies that the only way to sustain a concentrated outcome is through mergers. This is clearly both unrealistic and restrictive, especially in the presence of antitrust laws. The third assumption in our previous analysis leaves out the possibility that a merger induces new entry once the merger has occurred. However, as Stigler (1950) pointed out, post-merger entry is an empirically important force that prevents firms from monopolising markets through mergers.

The aim of this section is to investigate the robustness of our previous predictions. For this purpose, we relax the three assumptions mentioned above. In fact, we show that the possibility of "ex-post entry" prevents the emergence of concentrated outcomes in large exogenous sunk cost industries, independently of any details regarding coalition formation or product selection. To show that our predictions do not depend on the details of the model, we do neither specify explicitly the extensive form of the game nor the strategy space of players. Instead, we apply a recent equilibrium concept, due to Sutton (1997), which is defined not in the space of strategies, but in the space of outcomes. This equilibrium concept involves two rather weak assumptions, "viability" (no firm makes losses) and "stability" (there is one smart agent who would fill a profitable opportunity in the market), both of which are implied by subgame perfection.

4.4.1 The Model

There are n_0 firms that can take actions at certain specified stages. Each firm's action space is denoted by A. Actions may include entry decisions, the choice of the number of products, merger decisions, the choice of product quality (in the case of endogenous sunk cost industries), takeover bids, and so on. Firm i's actions in the entire game are summarised by the vector $\mathbf{a}_i$, $\mathbf{a}_i \subset A$. Each firm may decide not to enter the market, i.e. to choose the "null action", denoted by $\mathbf{a}_i = \emptyset$. The outcome of the game can then be described by the n_0-tuple $(\mathbf{a}_1, \mathbf{a}_2, ..., \mathbf{a}_{n_0})$. Suppose n, $n \in \{1, ..., n_0\}$, firms decide to enter the market, i.e. to choose a non-null action Then, deleting all inactive firms und relabelling the remaining active firms, yields the n-tuple

$$\mathbf{a} \equiv (\mathbf{a}_1, \mathbf{a}_2, ..., \mathbf{a}_n),$$

which is referred to as a *configuration.*

The total payoff (profit) of firm i from the set of actions $\mathbf{a}_i$, when rivals' actions are given by $\mathbf{a}_{-i} \equiv (\mathbf{a}_1, ..., \mathbf{a}_{i-1}, \mathbf{a}_{i+1}, ..., \mathbf{a}_n)$, is written as

$$\overline{\Pi}(\mathbf{a}_i; \mathbf{a}_{-i}).$$

If firm i decides not to enter the market ($\mathbf{a}_i = \emptyset$), then its payoff is zero:

$$\overline{\Pi}(\emptyset; \mathbf{a}_{-i}) = 0.$$

The function $\overline{\Pi}(\mathbf{a}_i; \mathbf{a}_{-i})$ summarises not only the final-stage profits but also possible costs from taking the set of actions $\mathbf{a}_i$ (e.g. costs from entering a new product) as well as payments between firms (resulting from merger or takeover decisions).

To exclude nonviable markets, we assume that there is some action $\mathbf{a}_0$, $\mathbf{a}_0 \neq \emptyset$, such that

$$\overline{\Pi}(\mathbf{a}_0; \emptyset) > 0.$$

Furthermore, it is assumed that the number of potential entrants, n_0, is sufficiently large such that if all firms choose to enter the market, then there is at least one that makes a negative profit. The idea is that entering the market requires a minimum setup cost of ϵ, and the sum of final-stage payoffs is bounded above by $(n_0 - 1)\epsilon$.

All we have to specify about the extensive form of the game is the following. There are T stages at which firms can enter the market and take actions; associated with these actions are certain costs and payments between firms. Additionally, at stage $T + 1$, firms engage in some kind of price (or quanitity) competition. All payoffs are summarised by the reduced-form payoff function $\overline{\Pi}(\cdot; \cdot)$. Firm i is free to take actions at any date t, $t \in \{t_i, t_i + 1, ..., T\}$, where $t_i \in \{1, ..., T\}$ is firm i's "date of arrival". The important feature of the extensive form is that there is some penultimate stage, T, at which firms take actions simultaneously, and new firms can enter the market, before firms engage in price competition. We do not allow for actions that are effectively conditioning on the outcome of this penultimate stage.

A configuration $\mathbf{a}^*$ is called an *equilibrium configuration* if the following two conditions are satisfied:

(i) (viability) For all firms i,

$$\overline{\Pi}(\mathbf{a}_i^*; \mathbf{a}_{-i}^*) \geq 0.$$

(ii) (stability) There is no set of actions $\mathbf{a}_{n+1}$ such that entry is profitable. That is,

$$\overline{\Pi}(\mathbf{a}_{n+1}; \mathbf{a}^*) \leq 0.$$

Condition (i) requires that no firm makes a loss in equilibrium, while condition (ii) says that if there is a profitable opportunity in the market, then there is some smart agent who will fill it. It may be worth pointing out that both conditions are consistent with boundedly rational agents who do not fully maximise their payoffs. Moreover, both conditions are implied by subgame perfection. To see this, notice that if the viability condition was not satisfied in a candidate SPE, then a firm could profitably deviate by choosing the null action ("do not enter"), and make zero profit. Similarly, if the stability condition was not satisfied, then an inactive firm could profitably deviate by entering the market at stage T. We thus have the following "inclusion" property.

Proposition 4.6 (Sutton 1997) *Any outcome that can be supported in an SPE in pure strategies is an equilibrium configuration.*

The concept of an equilibrium configuration has bite for empirical applications if the conditions of viability and stability can be expressed in the space of observable outcomes. This is indeed the case if firms' actions merely consist in the choice of the number of products (exogenous sunk cost industries) or product quality (endogenous sunk cost industries). If we want to allow for side payments between firms, however, then we have to re-formulate these conditions in the space of observables. Let us now apply the equilibrium concept to study the limits of concentration in exogenous and endogenous sunk cost industries.

Exogenous Sunk Cost Industries. In the case of exogenous sunk cost industries, a profile of firms' actions, i.e. a configuration, $\mathbf{a}$, induces a profile of products (a coalition structure)

$$\mathbf{m} \equiv (m_1, m_2, ..., m_l),$$

$l \in \{1, ..., n\}$, where m_i gives the number of products in firm (or coalition) i's portfolio. The demand structure is as in section 4.3.1, and firms (coalitions) are assumed to compete in prices at the ultimate stage. Suppose the total number of products offered in the industry is given by $\overline{m} \equiv \sum_{j=1}^{l} m_i$. Then, provided it has chosen to enter the market, firm i's profit from the final price competition stage is given by $S\Pi\left(m_i; \mathbf{m}_{-i}\right)$, which can be derived from equation (4.3) in section 4.3.1. The setup cost per product is denoted by ϵ.

Suppose now that $\mathbf{a}^*$ forms an equilibrium configuration, which induces the profile of products, $\mathbf{m}^*$. We may express the viability and stability conditions for exogenous sunk cost industries in the space of observables (i.e. in the space of profiles of product numbers) as follows:

(i') For all $i \in \{1, ..., l\}$,

$$S\Pi\left(m_i^*; \mathbf{m}_{-i}^*\right) - m_i^* \epsilon \geq 0.$$

(ii') There does not exist an m_{n+1}, $m_{n+1} \in \{1, 2, ...\}$, such that

$$S\Pi\left(m_{n+1}; \mathbf{m}^*\right) - m_{n+1}\epsilon > 0.$$

Condition (ii') is slightly weaker than (ii) in that we restrict the actions of an additional entrant to the choice of the number of its products. To understand condition (i'), notice that $S\Pi\left(m_i^*; \mathbf{m}_{-i}^*\right) - m_i^*\epsilon$ is an upper bound on firm i's total payoff under the assumption that the implicit price (in case of coalition formation: the profit share), or the explicit price (in case of takeovers: the takeover bid), of acquiring a product from another firm is at least the setup cost per product, ϵ. (The rationale for this assumption is that, otherwise, there would be some firm which sells a product to a rival below its cost. This assumption, however, is somewhat stronger than the viability condition, which only requires that a firm makes no loss on its combined activities.) Conditions (i') and (ii') coincide with (i) and (ii) if firms' actions merely consist in selecting the number of their products. In the equilibrium analysis below we show that these two rather weak requirements are powerful enough to obtain strong empirical predictions.

Endogenous Sunk Cost Industries. The application of the equilibrium concept to the case of endogenous sunk cost industries proceeds similarly to the case of exogenous sunk cost industries. A configuration $\mathbf{a}$ induces a profile of qualities

$$\mathbf{u} \equiv \left(\mathbf{u}_1, \mathbf{u}_2, ..., \mathbf{u}_l\right),$$

$l \in \{1, ..., n\}$, where $\mathbf{u}_i$ is the vector of qualities offered by firm (coalition) i. Firm i's final stage profit is denoted by $S\Pi(\mathbf{u}_i; \mathbf{u}_{-i})$. We use the demand system of section 4.3.2 and assume that firms compete in prices, so that $S\Pi(\mathbf{u}_i; \mathbf{u}_{-i})$ is the Nash equilibrium profit as described in section 4.3.2. The cost of investment is given by equation (4.7); the entry cost per product is again ϵ.

Suppose now that $\mathbf{a}^*$ forms an equilibrium configuration, which induces the profile of qualities $\mathbf{u}^*$. Denote by M_i the set of products in firm i's portfolio at the end of the game. The viability and stability conditions can then be expressed in the space of observables as follows:

(i") For all i, $i \in \{1, ..., l\}$,

$$S\Pi(\mathbf{u}_i^*; \mathbf{u}_{-i}^*) - \sum_{k \in M_i} \left(F_o u_k^\beta + \epsilon\right) \geq 0.$$

(ii") There does not exist a vector of qualities, $\mathbf{u}_{n+1}$, $\mathbf{u}_{n+1} \neq \mathbf{0}$, such that

$$S\Pi(\mathbf{u}_{n+1}; \mathbf{u}^*) - \sum_{k \in M_{n+1}} \left(F_o u_k^\beta + \epsilon\right) > 0.$$

Again, if firms' actions merely consist in the choice of product qualities (and the number of products), then these two conditions coincide with (i) and (ii).

4.4.2 Equilibrium Configurations

In the following, we study equilibrium configurations in exogenous and endogenous sunk cost industries, respectively. We investigate, in particular, whether concentrated outcomes can be sustained as equilibrium configurations in large markets.

4.4.2.1 Exogenous Sunk Cost Industries

Before turning to the equilibrium analysis of the exogenous sunk cost case, let us introduce some further notation. We denote by $S\pi(m_i; \mathbf{m}_{-i})$ firm i's final-stage profit *per product*, i.e. $\pi(m_i; \mathbf{m}_{-i}) \equiv \Pi(m_i; \mathbf{m}_{-i})/m_i$. From equation (4.3),

$$\pi(m_i; \mathbf{m}_{-i}) = \frac{(1-\sigma)\left[1-\sigma+(\overline{m}-m_i)\sigma\right]}{2\left[1-\sigma+\overline{m}\sigma\right]\left[2(1-\sigma)+(2\overline{m}-m_i)\sigma\right]^2\left[1-\sigma\sum_{j=1}^{l}\frac{m_j}{2(1-\sigma)+(2\overline{m}-m_j)\sigma}\right]^2}.$$

For a given numer of products, industry profits are clearly maximised under monopoly. Does this imply that monopoly will endogenously emerge in equilibrium? Not necessarily, as we have seen in the section on endogenous horizontal mergers. The reason is that a firm, by staying out of a coalition, may be better off than by joining. In fact, we have shown (abusing notation slightly) that

$$\pi(1;\overline{m}-1) > \pi(\overline{m};0) \text{ for } \overline{m} \text{ sufficiently large.} \tag{4.11}$$

In an open membership game, firms will, therefore, not endogenously merge to monopoly if the number of firms in the industry is sufficiently large.

This "inefficient" outcome may be viewed as being due to some coordination failure. One may, therefore, think that if firms were allowed to renegotiate on coalition formation, and make side payments, monopoly could be achieved. However, any renegotiation should be modelled explicitly, and it is a priori not clear whether such renegotiation would lead to an efficient outcome. Firms, anticipating renegotiation, would have even less incentives to merge prior to renegotiation. More importantly, renegotiation has bite only if it takes place after all entry has occured. The present model formalises this idea: there is a penultimate stage at which firms may renegotiate earlier agreements and, simultaneously, new entry may occur.

In the first part of the paper, we have shown that (4.11) holds. Below, we prove a stronger result:

$$\pi(1;\overline{m}) > \pi(\overline{m};0) \text{ for } \overline{m} \text{ sufficiently large.} \tag{4.12}$$

Ex-post entry in conjunction with this claim are sufficient to imply that monopoly will not occur in large markets. To see this, suppose that $(\overline{m},0)$ is sustainable as an equilibrium configuration. Viability and stability require

$$\pi(\overline{m};0) \geq \epsilon/S$$

and

$$\pi(1;\overline{m}) \leq \epsilon/S, \tag{4.13}$$

which imply

$$\pi(\overline{m};0) \geq \pi(1;\overline{m}). \tag{4.14}$$

Let $\widehat{m}(S/\epsilon)$ denote the maximum integer such that $\pi(1;\widehat{m}(S/\epsilon)) \geq \epsilon/S$. Since $\pi(1;\overline{m})$ is strictly decreasing in $\overline{m}$, and $\lim_{\overline{m}\to\infty} \pi(1;\overline{m}) = 0$, we have $\widehat{m}(S/\epsilon) \to \infty$ as $S/\epsilon \to \infty$. Configuration $(\overline{m}, 0)$ satisfies the stability condition (4.13) if $\overline{m} \geq \widehat{m}(S/\epsilon)$. From equation (4.12), it then follows that $\pi(1;\overline{m}) > \pi(\overline{m};0)$ in sufficiently large markets. But this is in contradiction to (4.14).

Let us now show that equation (4.12) does indeed hold. It is straightforward to compute that

$$\pi(\overline{m};0) = \frac{1}{8\left[1-\sigma+\overline{m}\sigma\right]}$$

and

$$\pi(1;\overline{m}) = \frac{(1-\sigma)\left[1+(\overline{m}-1)\sigma\right]\left[2+\overline{m}\sigma\right]^2}{2[1+\overline{m}\sigma]\left[4+4(\overline{m}-1)\sigma-\overline{m}\sigma^2\right]^2}.$$

Taking the limit as $\overline{m}$ tends to infinity, we obtain

$$\lim_{\overline{m}\to\infty} \{\pi(\overline{m};0) - \pi(1;\overline{m})\} < 0,$$

which proves the claim.

We have thus shown that it is possible to exclude monopoly outcomes in large exogenous sunk cost industries. In fact, we are able to obtain a much stronger result: the upper bound to concentration goes to zero as market size tends to infinity.

Proposition 4.7 *For any $(\sigma, \gamma) \in (0,1)^2$, there exists a threshold level $\left(\overline{S/\epsilon}\right)(\sigma;\gamma)$ such that for all $S/\epsilon \geq \left(\overline{S/\epsilon}\right)(\sigma;\gamma)$, the market share of the largest firm is bounded above by γ in any equilibrium configuration.*

The proof of proposition 4.7 is similar to that of proposition 4.2, and can be found in the appendix. The proposition shows that our previously derived predictions regarding exogenous sunk cost industries are robust; they do not depend on the details of the extensive form of the game, provided we allow for ex-post entry.

4.4.2.2 Endogenous Sunk Cost Industries

In the rather specific endogenous horizontal merger model of section 4.3.2, we have shown that, in endogenous sunk cost industries, it is possible to sustain very concentrated outcomes, even monopoly, no matter how large the market. We are now in the position to show that this prediction carries over to the current setting.

Proposition 4.8 *If products are sufficiently good substitutes (σ close to 1), or investment in quality enhancement sufficiently effective (β close to 2), monopoly can be sustained in an equilibrium configuration. This holds independently of the level of market size relative to setup costs, provided the market is not too small so as to support at least one firm.*

Proof. Using the notation of section 4.3.2, suppose the candidate monopolist offers one product, which is of quality $\overline{u}(1)$. In section 4.3.2, we have shown that if $\sigma \in (\sigma_1(\beta), 1))$, then, for any level of market size and setup costs, entry by a firm, which is restricted to offer only one product, is unprofitable. Moreover, the monopolist makes positive profits, provided market size (relative to setup costs) is not too small. The proof of the extension of the result to the case of a multiproduct entrant proceeds as follows. The first step consists in showing that a multiproduct entrant optimally chooses the same quality for all of its products. The second and final step consists in observing that the final stage profit *per product* is decreasing in the number of own products, holding quality fixed. ■

Although the possibility of ex-post entry works against the emergence of concentrated outcomes, there are several reasons why, in the present model, monopoly may be sustained in equilibrium for a larger set of parameter values than in the model of section 4.3.2. First, in the present model, entry deterrence through quality investment (and product proliferation) is consistent with the concept of an equilibrium configuration. In contrast, in the model of section 4.3.2, entry deterrence is not consistent with subgame perfection since all firms invest simultaneously in quality. Second, the present equilibrium concept does not require the candidate monopolist to maximise profits. In particular, the monopolist may overinvest in quality relative to the profit-maximising level. Third, in the earlier model, a multiproduct firm can only emerge through mergers, and mergers are, potentially, subject to coordination failures. In the present setup, such coordination failures are muted; for instance, the candidate monopolist may simply select the number of products so as to deter entry.

4.5 Conclusion

The aim of this paper has been to sharpen the predictions of the game-theoretic literature on industrial market structure. In his book on the relationship between market size and concentration, Sutton (1991) showed that *fragmented outcomes* can in general be sustained in large exogenous sunk cost industries, but not in large endogenous sunk cost industries. The question addressed in this paper has been whether it is possible to make predictions as to in what kind of industries it is possible to sustain *concentrated outcomes*, and in what kind of industries it is not. Using an endogenous horizontal merger model with free but costly entry, we have shown, in the first part of the paper, that it is impossible to sustain concentrated outcomes in large exogenous sunk cost industries. More precisely, the upper bound to the one-firm concentration ratio goes to zero as market size (relative to setup costs) tends to infinity. In contrast, in endogenous sunk cost industries, where firms can invest in some fixed R&D or advertising outlays to increase the (perceived) quality of their products, concentrated outcomes can be sustained even in the absence of mergers, no matter how large the market. In the second part of the paper, we have shown that the same results obtain independently of the details of the extensive form of the game, and allowing for side payments between firms and endogenous product choice, provided one allows for ex-post entry.

We believe that the predictions of this paper are robust. For instance, it is possible to show that the conclusions of the paper do not hinge on the assumption of price competition. In fact, under quantity competition, the incentive to take a free ride on rivals' effort to restrict output is larger than under price competition, so that it is more difficult to obtain concentrated outcomes in exogenous sunk cost industries. The possible emergence of very concentrated outcomes in large endogenous sunk cost industries is not affected. More research on the robustness of our results is needed. Most importantly, the predictions of the paper should be tested empirically. We plan to conduct an empirical analysis, starting with Sutton's (1991) data set of the food and drink sector.

4.6 Appendix

Proof of lemma 4.1. Since $U(\mathbf{x}; Y - \sum_k p_k x_k)$ is strictly concave in $\mathbf{x}$, there exists a unique utility-maximising consumption bundle, given any price vector $\mathbf{p}$. That is, each consumer has a well-defined demand function for good k, $d_k(\mathbf{p})$; market demand is $D_k(\mathbf{p}) = Sd_k(\mathbf{p})$. Recall that, by assumption, $Y > \sum_k p_k d_k(\mathbf{p})$ in equilibrium. Hence, if $d_k(\mathbf{p}) > 0$, demand for good k is implicitly defined by the first-order condition

$$1 - 2(1-\sigma)d_k(\mathbf{p}) - 2\sigma \sum_{l=1}^{n} d_l(\mathbf{p}) - p_k = 0.$$

Relabel products in increasing order of price, i.e. $p_k \leq p_{k+1}$ for all $k \in \{1, ..., n-1\}$. Define the integer $\overline{n}(\mathbf{p})$ as follows. If $p_1 > 1$, then $\overline{n}(\mathbf{p}) = 0$; otherwise, let $\overline{n}(\mathbf{p})$ be the largest integer v, $v \leq n$, such that $(1-\sigma)(1-p_v) - v\sigma\left(p_v - (1/v)\sum_{l=1}^{v} p_l\right) \geq 0$. Demand (per consumer) for good k can now be written as

$$d_k(\mathbf{p}) = \frac{(1-\sigma)(1-p_k) - \overline{n}(\mathbf{p})\sigma\left(p_k - \frac{1}{\overline{n}(\mathbf{p})}\sum_{l=1}^{\overline{n}(\mathbf{p})} p_l\right)}{2(1-\sigma)\left[1-\sigma+\overline{n}(\mathbf{p})\sigma\right]} \tag{4.15}$$

if $k \leq \overline{n}(\mathbf{p})$, and $d_k(\mathbf{p}) = 0$ otherwise. Notice that, although $\overline{n}(\mathbf{p})$ takes only integer values, $d_k(\mathbf{p})$ is continuous in $\mathbf{p}$.

Let $\mathbf{p}_{M_i}$ denote the vector of coalition M_i's prices, and $\mathbf{p}_{-M_i}$ the price vector of its rivals. Coalition M_i sets the prices of its products so as to maximise its profit:

$$\max_{\{p_k\}_{k \in M_i}} \sum_{k \in M_i} p_k D_k(\mathbf{p}_{M_i}, \mathbf{p}_{-M_i}).$$

Since the demand function possesses (a finite number of) kinks, a firm's best-reply function is not continuous everywhere. Denote by $\mathbf{p}_{-k} \equiv (p_1, ..., p_{k-1}, p_{k+1}, ..., p_n)$ the vector of prices of all goods other than k. Let us now make three observations. First, there exists a $\overline{p}(\mathbf{p}_{-k})$ such that $d_k(p_k, \mathbf{p}_{-k}) > 0$ if and only if $p_k < \overline{p}(\mathbf{p}_{-k})$. It is easy to see that $\overline{p}(\mathbf{p}_{-k}) > 0$ for all nonnegative $\mathbf{p}_{-k}$, which implies that each firm make positive profit in equilibrium. Second, each product makes positive sales in equilibrium. To see this, suppose that good k, $k \in M_i$, makes zero sales. But then, M_i could raise its profit by setting p_k slightly below $\overline{p}(\mathbf{p}_{-k})$, holding all other prices fixed. Third, coalition M_i's profit is continuous in $\mathbf{p}$; it is strictly concave in $\mathbf{p}_{M_i}$ for any $\mathbf{p}_{-M_i}$, provided that prices are such that $p_k < \overline{p}(\mathbf{p}_{-k})$ for all $k \in M_i$. These observations together imply that the set of first-order conditions is necessary and sufficient for $\mathbf{p}^*$ to form a Nash equilibrium.[17] Hence, equilibrium price p_k^*, $k \in M_i$, is implicitly defined by

$$1 + \frac{2\sigma}{1-\sigma}\sum_{l \in M_i} p_l^* + \frac{\sigma}{1-\sigma}\sum_{j \notin M_i} p_j^* = 2\left(\frac{1-\sigma+n\sigma}{1-\sigma}\right)p_k^*. \tag{4.16}$$

[17]In the following, we suppress the dependence of strategies on $\mathbf{z}$ for notational simplicity.

Since the left-hand side of (4.16) is independent of k, it follows that $p_k^* = p_{M_i}^* < \overline{p}(\mathbf{p}_{-k}^*)$ for all $k \in M_i$. That is, a merged entity sets the same price for each of its products.

We can now rewrite the first-order condition as follows

$$p_{M_i}^* = \frac{1-\sigma+\sigma\sum_{j\in Z} m_j p_{M_j}^*}{2(1-\sigma)+(2n-m_i)\sigma}. \tag{4.17}$$

Multiplying both sides with m_i, and summing over all coalitions, gives

$$\sum_{j\in Z} m_j p_{M_j}^* = \frac{(1-\sigma)\sum_{j\in Z}\frac{m_j}{2(1-\sigma)+(2n-m_j)\sigma}}{1-\sigma\sum_{j\in Z}\frac{m_j}{2(1-\sigma)+(2n-m_j)\sigma}}. \tag{4.18}$$

Inserting (4.18) into (4.17) yields the (unique) equilibrium price of coalition M_i's products, as given by equation (4.2).

Using (4.15) and (4.2), we can now calculate the market demand per product of coalition M_i as

$$D_{M_i}(\mathbf{p}^*) = S\frac{1-\sigma+(n-m_i)\sigma}{2(1-\sigma)\left[1-\sigma+n\sigma\right]}\cdot p_{M_i}^*. \tag{4.19}$$

It is straightforward to verify that $\sum_k p_k^* d_k(\mathbf{p}^*) < 1/8\sigma$, so that the assumption on income indeed ensures that income is higher than the consumer's equilibrium expenditure on the n substitute goods.[18] ■

Proof of proposition 4.1. Let $S\pi\,(m;n-m)$ denote the profit per product of a coalition with m members, facing a single nonempty rival coalition with $n-m$ members. Merger to monopoly can be sustained in equilibrium if and only if

$$\pi(n;0) \geq \pi(1;n-1).$$

Using (4.3), this condition can be rewritten as

$$\begin{aligned} &\left[4(1-\sigma)^2+4n\sigma(1-\sigma)+3(n-1)\sigma^2\right]^2 \\ -4(1-\sigma)\left[1-\sigma+(n-1)\sigma\right]\left[2(1-\sigma)+(n+1)\sigma\right]^2 &\geq 0, \end{aligned}$$

which simplifies to

$$\phi(n,\sigma) \equiv (n-1)^2\left[4n-7\right]\sigma^2+4(n-1)\left[-n^2+6n-7\right]\sigma+4\left[-n^2+4n-3\right] \geq 0.$$

[18] Note that expenditure is maximised under merger to monopoly, in which case $\sum_k p_k^* d_k(\mathbf{p}^*) = n/\left[8(1-\sigma+n\sigma)\right]$.

It is easily checked that $\phi(2,\sigma) = \sigma^2+4\sigma+4$ and $\phi(3,\sigma) = 20\sigma^2+16\sigma$. Hence, if $n \in \{2,3\}$, then merger to monopoly is sustainable for all $\sigma \in (0,1)$. If $n \geq 4$, then $\phi(n,0) < 0$, and $\phi(n,\sigma)$ has a unique positive root, $\widehat{\sigma}(n)$, given by

$$\widehat{\sigma}(n) = \frac{2\left(n^2-6n+7\right)+2\sqrt{n^4-8n^3+27n^2-44n+28}}{(n-1)(4n-7)}.$$

Note that $\widehat{\sigma}(n) \in (0,1)$ for all $n \geq 4$. That is, if $n \geq 4$, then merger to monopoly can be supported for all $\sigma \in [\widehat{\sigma}(n), 1)$. ■

Proof of proposition 4.2. Suppose the assertion is false. Then, there exist an increasing sequence $\{n^k\}_{k=1}^{\infty}$ of numbers of active firms and a sequence of coalition M_i's number of products, $\{m_i^k\}_{k=1}^{\infty}$, such that M_i's market share, γ_i^k, as measured by the relative number of its products, m_i^k/n^k, is bounded below by γ, i.e. $\gamma_i^k \geq \gamma$ for all k, and such that $\lim_{k\to\infty}\gamma_i^k = \gamma_i^{\infty}$. (Notice that it is always possible to find a convergent subsequence since $\gamma_i^k \in [\gamma, 1]$.) For this to be an equilibrium, a member of M_i should have no incentive to deviate and form a coalition on its own. Formally,

$$\frac{\left[1-\sigma+n^k(1-\gamma_i^k)\sigma\right]}{\left[2(1-\sigma)+n^k(2-\gamma_i^k)\sigma\right]^2\left[\Psi^k\right]^2} \geq \frac{\left[1-\sigma+n^k(1-1/n^k)\sigma\right]}{\left[2(1-\sigma)+n^k(2-1/n^k)\sigma\right]^2\left[\Psi^k+\Phi_i^k\right]^2},$$

where

$$\Psi^k \equiv 1-\sigma\sum_{j\in Z}\frac{n^k\gamma_j^k}{2(1-\sigma)+n^k(2-\gamma_j^k)\sigma},$$

and

$$\begin{aligned}\Phi_i^k \;&\equiv\; \frac{\sigma n^k\gamma_i^k}{2(1-\sigma)+n^k(2-\gamma_i^k)\sigma}-\frac{\sigma n^k\left[\gamma_i^k-1/n^k\right]}{2(1-\sigma)+n^k(2-\gamma_i^k+1/n^k)\sigma}\\ &-\frac{\sigma}{2(1-\sigma)+n^k(2-1/n^k)\sigma}.\end{aligned}$$

This condition can be rewritten as

$$\left(\frac{1-\sigma+n^k(1-\gamma_i^k)\sigma}{1-\sigma+n^k(1-1/n^k)\sigma}\right)\left(\frac{2(1-\sigma)+n^k(2-1/n^k)\sigma}{2(1-\sigma)+n^k(2-\gamma_i^k)\sigma}\right)^2 \geq \left(\frac{\Psi^k}{\Psi^k+\Phi_i^k}\right)^2. \tag{4.20}$$

Observe that $\lim_{k\to\infty}\Phi_i^k = 0$ since $n^k \to \infty$, and $\gamma_i^k \to \gamma_i^{\infty}$, as $k \to \infty$. Hence, if Ψ^k is bounded away from zero, the right-hand side of equation (4.20) converges to 1 as $k \to \infty$, whereas the left-hand side converges to $4(1-\gamma_i^{\infty})/(2-\gamma_i^{\infty})^2 < 1$. That is, if Ψ^k is bounded away from zero, then for n^k sufficiently large, the above inequality can not hold – a contradiction.

If $\lim_{k\to\infty}\Psi^k = 0$, however, the right-hand side of (4.20) may not converge to 1. Notice that this case occurs if and only if there exists a firm j such that $\gamma_j^\infty = 1$, and hence $\gamma_k^\infty = 0$ for all $k \neq j$. Now, if $i \neq j$, we are done. The interesting case is when $i = j$, i.e. $\gamma_i^\infty = 1$, so that the left-hand side of (4.20) converges to zero. In fact, the right-hand side of the equation converges to zero as well, provided that firm 1 is a monopolist, $\gamma_i^k = 1$, for k sufficiently large; but we already know from corollary 4.1 that monopoly can not be sustained in equilibrium for n sufficiently large. It, therefore, remains to show that the right-hand side of equation (4.20) is bounded away from zero if $\gamma_i^k \leq (n^k-1)/n^k$, and hence if $\Psi^k > 0$, for sufficiently large k. To show this, remark first that $\phi(\gamma) \equiv n\gamma/\left[2(1-\sigma)+n(2-\gamma)\sigma\right]$ is increasing and convex in γ, which implies, first, that Ψ^k is decreasing in the industry level of concentration (where a rise in the level of concentration is defined as a transfer of a certain number of products from some firm to a weakly larger one) and, second, that Φ_i^k is increasing in γ_i^k. Let us define

$$\underline{\Psi}^k \equiv 1 - \frac{(n^k-1)\,\sigma}{2(1-\sigma)+(n^k+1)\sigma} - \frac{\sigma}{2(1-\sigma)+(2n^k-1)\sigma},$$

and

$$\overline{\Phi}_i^k \equiv \frac{(n^k-1)\,\sigma}{2(1-\sigma)+(n^k+1)\sigma} - \frac{(n^k-2)\,\sigma}{2(1-\sigma)+(n^k+2)\sigma} - \frac{\sigma}{2(1-\sigma)+(2n^k-1)\sigma}.$$

If firm i is the largest firm, and $\gamma_i^k \leq (n^k-1)/n^k$, we thus have $\Psi^k \geq \underline{\Psi}^k > 0$, $0 < \Phi_i^k \leq \overline{\Phi}_i^k$, and hence

$$\frac{\Psi^k}{\Psi^k+\Phi_i^k} \geq \frac{\underline{\Psi}^k}{\underline{\Psi}^k+\overline{\Phi}_i^k}.$$

It is straightforward to check that the right-hand side of this inequality is bounded away from zero. Hence, for k sufficiently large, equation (4.20) does not hold. This completes the proof. ■

Proof of lemma 4.2. Suppose that income is sufficiently large so that $Y > \sum_k p_k x_k$ in equilibrium. Then, if $d_k(\mathbf{p};\mathbf{u}) > 0$, demand per consumer for good k is given by the first-order condition

$$u_k - 2(1-\sigma)d_k(\mathbf{p};\mathbf{u})/u_k - 2\sigma\sum_{l=1}^{n} d_l(\mathbf{p};\mathbf{u})/u_l - p_k u_k = 0.$$

Relabel firms such that $u_k(1-p_k) \geq u_{k+1}(1-p_{k+1})$ for all $k \in \{1,...,n-1\}$. Define the integer $\overline{n}(\mathbf{p};\mathbf{u})$ in the following way. If $p_1 > 0$, then $\overline{n}(\mathbf{p};\mathbf{u}) = 0$; otherwise, let $\overline{n}(\mathbf{p};\mathbf{u})$ be the largest integer v, $v \leq n$, such that

$$(1-\sigma)u_v(1-p_v) + v\sigma\left[u_v(1-p_v) - \frac{1}{v}\sum_{l=1}^{v} u_l(1-p_l)\right] \geq 0.$$

Demand (per consumer) for k can then be written as

$$d_k(\mathbf{p};\mathbf{u}) = \frac{(1-\sigma)u_k(1-p_k) + \overline{n}(\mathbf{p};\mathbf{u})\sigma\left[u_k(1-p_k) - \frac{1}{\overline{n}(\mathbf{p};\mathbf{u})}\sum_{l=1}^{\overline{n}(\mathbf{p};\mathbf{u})} u_l(1-p_l)\right]}{2(1-\sigma)\left[1-\sigma+\overline{n}(\mathbf{p};\mathbf{u})\sigma\right]} \cdot u_k$$

if $k \leq \overline{n}(\mathbf{p};\mathbf{u})$, and $d_k(\mathbf{p};\mathbf{u}) = 0$ otherwise. To simplify the algebraic expressions, let us define $y_k(\mathbf{p};\mathbf{u}) \equiv d_k(\mathbf{p};\mathbf{u})/u_k$, $q_k \equiv p_k u_k$, $\overline{u}_N(\mathbf{q};\mathbf{u}) \equiv (1/\overline{n}(\mathbf{q};\mathbf{u}))\sum_{l=1}^{\overline{n}(\mathbf{q};\mathbf{u})} u_l$, and $\overline{q}_N(\mathbf{q};\mathbf{u}) \equiv (1/\overline{n}(\mathbf{q};\mathbf{u}))\sum_{l=1}^{\overline{n}(\mathbf{q};\mathbf{u})} q_l$. We thus get the following (normalised) demand function for good k, $k \leq \overline{n}(\mathbf{q};\mathbf{u})$,

$$y_k(\mathbf{q};\mathbf{u}) = \frac{(1-\sigma)u_k + \overline{n}(\mathbf{q};\mathbf{u})\sigma\left[u_k - \overline{u}_N(\mathbf{q};\mathbf{u})\right] - (1-\sigma)q_k - \overline{n}(\mathbf{q};\mathbf{u})\sigma\left[q_k - \overline{q}_N(\mathbf{q};\mathbf{u})\right]}{2(1-\sigma)\left[1-\sigma+\overline{n}(\mathbf{q};\mathbf{u})\sigma\right]},$$

which is continuous in $\mathbf{q}$ and $\mathbf{u}$.

Coalition M_i's best reply to $\mathbf{q}_{-M_i}$ is given by the solution of the following optimisation programme:

$$\max_{\{q_k\}_{k\in M_i}} \sum_{k\in M_i} y_k(\mathbf{q};\mathbf{u})q_k.$$

In the following, we attempt to characterise equilibrium. Suppose $\mathbf{q}^*$ forms a Nash equilibrium.[19] If $y_k(0,\mathbf{q}_{-k}) > 0$, define the threshold price $\overline{q}_k(\mathbf{q}_{-k})$ such that $y_k(0,\mathbf{q}_{-k}) > 0$ if and only if $q_k < \overline{q}_k(\mathbf{q}_{-k})$; otherwise, let $\overline{q}_k(\mathbf{q}_{-k}) = 0$.[20] Observe that M_i's profit is strictly concave in q_k on $(0, \overline{q}_k(\mathbf{q}_{-k}))$, holding all other prices fixed. Hence, if $q_k^* \in (0, \overline{q}_k(\mathbf{q}_{-k}^*))$, then q_k^* is implicitly defined by the first-order condition

$$(1-\sigma)u_k + \overline{n}\sigma(u_k - \overline{u}_N) - (1-\sigma)q_k^* - \overline{n}\sigma(q_k^* - \overline{q}_N^*) + \overline{m}_i\sigma\overline{q}_{M_i}^* - (1-\sigma+\overline{n}\sigma)q_k^* = 0, \quad (4.21)$$

where $\overline{m}_i$ is the number of M_i's products with positive sales, and $\overline{q}_{M_i}^*$ the average normalised price of these products. Clearly, $\overline{n} = \sum_{j\in Z}\overline{m}_j$. Taking averages over M_i's products with positive sales, yields

$$\overline{q}_{M_i}^* = \frac{(1-\sigma)\overline{u}_{M_i} + \sigma\sum_j \overline{m}_j(\overline{u}_{M_i} - \overline{u}_{M_j}) + \sigma\sum_j \overline{m}_j\overline{q}_{M_j}^*}{2(1-\sigma) + (2\overline{n} - \overline{m}_i)\sigma}. \quad (4.22)$$

[19] In the remainder, we suppress dependence of strategies on $(\mathbf{z},\mathbf{u})$ for notational simplicity. Moreover, we abstract from the problem that products with zero sales may nevertheless constrain equilibrium.

[20] The function $\overline{q}_k(\cdot)$ varies across k for two reasons. First, different products are produced by different coalitions. Second, goods produced by the same coalition may differ in quality.

Multiplying both sides with $\sigma\overline{m}_i$, and summing over all coalitions, gives

$$\sigma\sum_j \overline{m}_j\overline{q}^*_{M_j} = \frac{\sum_j \frac{\sigma\overline{m}_j\left[(1-\sigma+\overline{n}\sigma)\overline{u}_{M_j}-\sum_i\sigma\overline{m}_i\overline{u}_{M_i}\right]}{2(1-\sigma)+(2\overline{n}-\overline{m}_j)\sigma}}{1-\sum_j\frac{\sigma\overline{m}_j}{2(1-\sigma)+(2\overline{n}-\overline{m}_j)\sigma}}. \tag{4.23}$$

We obtain M_i's average equilibrium price, (4.8), by inserting (4.23) into (4.22). Using (4.21), we finally get the (normalised) equilibrium price of good k, $k\in M_i$, provided it makes positive sales:

$$q^*_k = \overline{q}^*_{M_i} + \frac{u_k-\overline{u}_{M_i}}{2}.$$

■

Proof of proposition 4.7. For stability condition (ii') to hold, we must have $\overline{m}\to\infty$ as $S/\epsilon\to\infty$. Hence, it suffices to show that, for large $\overline{m}$, the market share of any firm is bounded above by γ. The proof proceeds along the lines of that of proposition 4.2.

Suppose the assertion is false. Then, there exist an increasing sequence $\{\overline{m}^k\}_{k=1}^{\infty}$ of number of products and a sequence of the number of firm i's products, $\{m_i^k\}_{k=1}^{\infty}$, such that firm i's market share γ_i^k, as measured by the relative number of its products, $m_i^k/\overline{m}^k$, is bounded below by γ, i.e. $\gamma_i^k\geq\gamma$, and such that $\lim_{k\to\infty}\gamma_i^k\equiv\gamma_i^\infty$. For $(\overline{m}^k\gamma^k)$ to be sustainable in an equilibrium configuration, we must have

$$\pi\left(\overline{m}^k\gamma_i^k;\overline{m}^k\gamma_{-i}^k\right)\geq\pi\left(1;\overline{m}^k\gamma^k\right),$$

where γ_{-i}^k is the vector of market shares of firm i's rivals. Let us reformulate this inequality as

$$\frac{\pi\left(\overline{m}^k\gamma_i^k;\overline{m}^k\gamma_{-i}^k\right)}{\pi\left(1;\overline{m}^k\gamma^k\right)} = \left(\frac{\Psi^k+\Omega^k}{\Psi^k}\right)^2\Theta_i^k\geq 1, \tag{4.24}$$

where

$$\begin{aligned}
\Psi^k &\equiv 1-\sigma\sum_j\frac{\overline{m}^k\gamma_j^k}{2(1-\sigma)+\overline{m}^k(2-\gamma_j^k)\sigma},\\
\Omega^k &\equiv -\frac{\sigma}{2(1-\sigma)+(2\overline{m}^k+1)\sigma}\\
&\quad+\sigma\sum_j\left(\frac{\overline{m}^k\gamma_j^k}{2(1-\sigma)+\overline{m}^k(2-\gamma_j^k)\sigma}-\frac{\overline{m}^k\gamma_j^k}{2(1-\sigma)+\overline{m}^k\left(2\left(1+\frac{1}{\overline{m}^k}\right)-\gamma_j^k\right)\sigma}\right),
\end{aligned}$$

and

$$\Theta_i^k\equiv\frac{\left[1-\sigma+\overline{m}^k(1-\gamma_i^k)\sigma\right]\left[1-\sigma+(\overline{m}^k+1)\sigma\right]\left[2(1-\sigma)+(2\overline{m}^k+1)\sigma\right]^2}{\left[1-\sigma+\overline{m}^k\sigma\right]^2\left[2(1-\sigma)+\overline{m}^k(2-\gamma_i^k)\sigma\right]^2}.$$

It is straightforward to check that

$$\lim_{k\to\infty} \Theta_i^k = \frac{4(1-\gamma_i^\infty)}{(2-\gamma_i^\infty)^2} < 1.$$

Notice that $\Theta_i^k, \Psi^k \geq 0$, with the inequalities being strict under all market structures other than monopoly. Let $\lambda(\overline{m}; m_j) \equiv -m_j/[2(1-\sigma)+(2\overline{m}-m_j)\sigma]$. Since $\eta(m_j) \equiv \partial\lambda(\overline{m}; m_j)/\partial\overline{m}$ is convex in m_j, the candidate equilibrium market structure that maximises Ω^k for a given $\overline{m}^k$, is monopoly; that is,

$$\Omega^k \leq -\frac{\sigma}{2(1-\sigma)+(2\overline{m}^k+1)\sigma} + \left(\frac{\sigma\overline{m}^k}{2(1-\sigma)+\overline{m}^k\sigma} - \frac{\sigma\overline{m}^k}{2(1-\sigma)+\left(\overline{m}^k+2\right)\sigma}\right).$$

It is easy to see that the right-hand side of this inequality converges to zero as $\overline{m}^k \to \infty$; hence, $\lim_{k\to\infty}\Omega^k = 0$. If Ψ^k does not converge to zero, i.e. if there is no firm j with $\gamma_j^\infty = 1$, one obtains

$$\lim_{k\to\infty}\left(\frac{\Psi^k+\Omega^k}{\Psi^k}\right)\Theta_i^k = \frac{4(1-\gamma_i^\infty)}{(2-\gamma_i^\infty)^2} < 1,$$

which is in contradiction to equation (4.24).

If $\lim_{k\to\infty}\Psi^k = 0$, however, $\left(\Psi^k+\Omega^k\right)/\Psi^k$ may not converge to 1. This case occurs if and only if there is a firm j such that $\gamma_j^\infty = 1$, and $\gamma_l^\infty = 0$ for all $l \neq j$. Accordingly, suppose $\gamma_i^\infty = 1$. Since we have already shown in the text that monopoly can not be sustained as an equilibrium configuration in large markets, let us assume, moreover, that $\gamma_i^k \leq \left(\overline{m}^k - 1\right)/\overline{m}^k$. Under this assumption,

$$\Psi^k \geq \underline{\Psi}^k \equiv 1 - \frac{\left(\overline{m}^k-1\right)\sigma}{2(1-\sigma)+(\overline{m}^k+1)\sigma} - \frac{\sigma}{2(1-\sigma)+(2\overline{m}^k-1)\sigma},$$

and

$$\begin{aligned}\Omega^k \leq \overline{\Omega}^k \quad \equiv \quad & -\frac{\sigma}{2(1-\sigma)+(2\overline{m}^k+1)\sigma} \\ & +\sigma\left(\frac{1}{2(1-\sigma)+(2\overline{m}^k-1)\sigma} - \frac{1}{2(1-\sigma)+(2\overline{m}^k+1)\sigma}\right) \\ & +\sigma\left(\frac{\overline{m}^k-1}{2(1-\sigma)+\left(\overline{m}^k+1\right)\sigma} - \frac{\overline{m}^k-1}{2(1-\sigma)+\left(\overline{m}^k+3\right)\sigma}\right),\end{aligned}$$

since $\lambda(\overline{m}; m_j)$ is concave in m_j, and $\partial\lambda(\overline{m}; m_j)/\partial\overline{m}$ convex in m_j, respectively. Accordingly,

$$\frac{\Psi^k+\Omega^k}{\Psi^k} \leq \max\left\{\frac{\underline{\Psi}^k+\overline{\Omega}^k}{\underline{\Psi}^k}, 1\right\}.$$

It is easily verified that the right-hand is bounded from above. Hence, if $\gamma_i^\infty = 1$, we obtain

$$\lim_{k\to\infty}\left(\frac{\Psi^k+\Omega^k}{\Psi^k}\right)\Theta_i^k = 0.$$

Again, this is in contradiction to equation (4.24). ■

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