# THE COST OF CAPITAL IN THE PRESENCE OF ALTERNATIVE CORPORATION AND PERSONAL TAX REGIMES

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## ABSTRACT

In 1958, Modigliani and Miller initiated an important debate in modern corporate finance literature, when they stated that in the absence of taxes, the cost of capital of a firm is independent of its capital structure. They modified and then corrected their view in 1963 when they stated that the introduction of corporation taxes into their model implied that there is a tax advantage to leverage and therefore taxes influenced the cost of capital. Subsequently, in the 1970s and 1980s, this debate has focused on the interaction of personal taxes and corporation taxes with the cost of capital and on the determination of whether taxes influence the cost of capital at all. This thesis contributes to this debate by addressing the following issues :

(a) Do personal taxes matter at all for calculating the cost of capital ? How sensitive is the influence of personal taxes to differences in the capital structure and pay out policies of the firms ?

(b) How can more realistic features of the tax code be incorporated in the determination of the cost of capital ?

(c) Taxation systems can be classified into 4 main types according to the degree of integration of personal and corporation taxes. These systems are the Classical, Imputation, Two-Rate and the Integrated systems. The cost of capital, in the presence of uncertainty as well as corporation and personal taxes, is derived for each of the above systems in this thesis.

(d) Taxation systems can also be classified into 2 main types according to the taxable base used. These are the Comprehensive Income Tax and the Expenditure Tax systems. The cost of capital, in the presence of uncertainty as well as corporation and personal taxes, is derived under both the above regimes.

(e) Application of the results of (a), (b) and (c) above to address practical issues such as using the cost of capital equation to determine the effect of changes introduced by the 1988 Budget on the cost of capital.

(f) Application of the results of (d) above to contradict the claims made in the Meade Committee regarding the tax neutrality issue. A system that is tax neutral even when uncertainty is taken into account, is proposed.

All the above issues have the common theme of the determination of the appropriate cost of capital in the presence of both uncertainty as well as corporation and personal taxes. The conclusions reached are stated at the end of the relevant chapters.

# THE COST OF CAPITAL IN THE PRESENCE OF ALTERNATIVE PERSONAL AND CORPORATION TAX REGIMES

## CONTENTS

## INTRODUCTION

In 1958, Modigliani and Miller initiated an important debate in modern finance literature when they stated that in the absence of taxes, the cost of capital of a firm is independent of its capital structure. They later modified and then corrected their view in 1963 when they stated that the introduction of corporation taxes into their model implied that there is a tax advantage to leverage and therefore taxes influenced the cost of capital. The influence of taxes was very significant in their model, changing the conclusion from that a firm should be indifferent between debt and equity to that the firm should be 100% debt financed.

Modigliani and Miller considered only simple form of corporation taxes in their model. Subsequently, in the 1970s and 1980s, models which incorporated personal as well as corporation taxes in somewhat elementary fashions were presented by various authors. In 1982, Modigliani presented a comprehensive corporate valuation model which incorporated heterogenous personal as well as corporation taxes in the context of the U.S. tax code. However, as noted by Ashton (1989), the models which were suitable within the U.S. context are discussed and presented as though they are equally suitable under different tax environments, such as that in the U.K.. Notable exceptions to this type of fallacy include Ashton (1989), Franks & Broyles (1979), Kent & Theobald (1980), King (1974,1977) and Rutterford (1988). However, primarily, the existing literature deals with taxes in a simplistic manner, regardless of the fact that inclusion of some major features of the tax system can reverse the conclusions otherwise reached about the cost of capital as well as about the financial policy implications.

The primary objective in this thesis is to incorporate important features of the personal and corporation tax codes, in particular the features relevant in the U.K., in valuation models in order to derive the cost of capital. It then proceeds to demonstrate how some of the variables required for determining the cost of capital can be calculated. It then applies the resulting valuation model to :

(a) demonstrate how to measure the impact of changes in features and rates on the cost of capital, by examining the 1988 Budget changes in particular. The 1988 Budget is chosen because it introduced the most substantial changes to personal taxes in the 1980s.

(b) arrive at the valuation models and the resulting cost of capital equations applicable under each of four tax systems which differ according to the degree of integration of the personal and corporation tax regimes, namely the classical system, imputation system, two rate system and the integrated system.

(c) analyse the cost of capital under the two alternative tax bases considered by the Meade Committee (1978), namely the Expenditure Tax and the Comprehensive Income Tax.

(d) derive a tax system that is tax neutral even when uncertainty and personal taxes are taken into account. This new system is advocated because the use of the more complex valuation model advocated in this thesis demonstrates that the validity of the conclusions reached by the Meade Committee is largely applicable only to simple companies and models.

All the above issues have the common theme of the determination of the appropriate cost of capital in the presence of uncertainty as well as corporation and personal taxes.

We proceed by examining the existing literature on personal taxes and cost of capital in chapter 2. We focus on examining only a few relevant articles but our emphasis is to examine these in depth because they deal with issues which are directly relevant to our thesis. In chapter 3, the general literature on financial policy and cost of capital is analysed. The chapter concludes with a demonstration of the simplistic relationships modelled in the existing literature, against the more complex relationships in the real world. The thesis subsequently aims to incorporate the real world taxation features into the model.

Chapter 4 demonstrates the Modigliani model (1982) in detail and introduces the relevant variables that are needed for practical calculations subsequently. The application of the valuation model under the alternative tax regimes is examined in chapter 5. Subsequent chapters concentrate on the imputation system only because this is the tax system in operation in the U.K.. Thus chapter 6 demonstrates how the variables that are necessary for determining the cost of capital in the UK can be estimated. Chapter 7 demonstrates how the important features of the UK corporation tax code can be incorporated in the valuation model. Chapter 7 ends with the relevant valuation model and the associated cost of capital equation advocated in this thesis. Thus we achieve the first of our two main objectives that we set out to achieve in this thesis.

Our second objective is to use the model we develop in practical context. Thus chapter 8 begins the practical application of the valuation model derived in chapter 7. The cost of capital equation is used to assess the impact of the significant changes introduced in the 1988 Budget on the cost of capital in the U.K. . In chapter 9 we address the critical issue of whether personal taxes matter for determining the cost of capital. We use the model developed in chapter 7 to demonstrate that personal taxes are relevant for valuation purposes.

Chapters 10 and 11 use the cost of capital equations developed in chapter 7 to critically examine the conclusions reached by the Meade Committee (1978). We conclude that the Meade Committee's recommendations are valid only with simple models, such as those that do not include uncertainty or mixed capital structures. A new system that is tax neutral even when uncertainty, mixed capital structures and firms earning super-normal profits are considered, is advocated in chapter 11.

This concludes the examination of the practical applications of the valuation model that incorporates the personal and corporation tax features present in the U.K. The overall conclusion of this thesis, noted in chapter 12, is that the model presented in chapter 7 can be used to provide many useful insights into the various aspects of the cost of capital question.

The conclusions reached within the chapters which contribute either innovative concepts or analysis include the following. The cost of capital equations under the different tax systems are presented in Chapter 5. Chapter 6 concludes with the practical methods for calculating the requisite valuation variables. Chapter 7 concludes with a valuation model and a cost of capital equation which incorporates the personal and the corporation tax features that are relevant in the U.K.. This comprehensive valuation model which is directly relevant to the U.K. tax regime is one of the two major conclusions reached in this thesis. Chapter 8 concludes that the 1988 Budget changes should have changed the cost of capital in the U.K. by 0.5% . Chapter 9 concludes that personal taxes are relevant to the valuation model and that they cannot be ignored in the cost of capital equations. Chapter 10 concludes that the Meade Committee's recommendations are relevant only under simplistic assumptions regarding certainty and capital structures. Chapter 11 presents a new model which is tax neutral under uncertainty and personal and corporation taxes. This is second of the two major results derived in this thesis.

The contributions made in this thesis include the above mentioned conclusions, an in depth derivation of the valuation models used in Modigliani (1982) and in the Meade Report (1978), and suggested corrections to the published literature, as demonstrated in Chapter 11 and in appendix A.

## COST OF CAPITAL & PERSONAL TAXES

### SECTION A
### INTRODUCTION AND HAMADA & SCHOLES (1985)

In this thesis, we examine the role of corporation and personal taxes in valuation models and the resulting cost of capital equations. Our aim is to extend the existing literature by deriving a Capital Asset Pricing Model ("CAPM") based valuation model that includes some of the more interesting features of the U.K. tax code, including personal tax features. The model we achieve should recognise that investors have heterogeneous personal tax characteristics.

There are some authors who have already incorporated personal taxes, in addition to corporate taxes, in valuation models in the literature. the pioneering work in fact was done by Brennan as far back as in 1970. These models which incorporate taxes are examined in detail in this chapter because they are relevant for the subject of our thesis. However, a comparison of the models examined in this chapter, with the model we develop for the U.K. in chapter 7, will show that the models in the existing literature treated taxes in an elementary fashion. This simple treatment as seen in the existing literature could lead to errors in calculating the appropriate cost of capital which our more elaborate model avoids. We note the advantages of our model in chapter 7. We go on to use the more elaborate features of our model in subsequent chapters (chapters 8 to 11). However, it is nevertheless important to look at the more relevant articles from the existing literature which address the issue of personal taxes in valuation models.

We begin by examining in this section a review article by Hamada and Scholes (1985) because this article directly addresses the issue of whether personal taxes are relevant for determining the cost of capital, and because it provides a useful contrast between after (personal) tax and before (personal) tax models. In section B, we examine the role of personal taxes in determining equilibrium in the corporate capital market as analysed by

Miller (1977). This article is selected because it is perhaps the most widely known attempt to use the role of personal taxes to derive the equilibrium value of debt equity ratios. In section C, we examine Modigliani's criticisms of the Miller model and the comprehensive valuation model advocated by him. Modigliani's model is selected for section C because it is regarded in this thesis as the most useful model for incorporating corporate and personal taxes in valuation models. His model is examined only briefly in this section because it will be developed extensively in the rest of this thesis, thereby necessitating a much fuller analysis which is deferred until chapter 4. Modigliani extended the work of other authors, notably Brennan (1970) and Auerbach (1983) and King (1974). Brennan pioneered the work on CAPM models which incorporated personal taxes and his contribution is described in section D of this chapter. Auerbach's and King's contributions are examined in section E. In section F we examine Poterba and Summer (1985), who calculate the cost of capital in the presence of personal taxes by making use of pay out ratios. This article is selected because it illustrates an interesting practical application of a theoretical model incorporating personal tax rates of investors. All the above articles are relevant to our thesis because they address the role of personal taxes on corporate values and discount rates for investment appraisal, and therefore are examined in detail in this chapter.

In the next chapter we examine, in some what less detail, the literature on financial policy and related issues. These issues are examined because they are relevant for an understanding of the theory of finance which underpins this thesis. They are examined in less detail because their influence on the development of our thesis is not as great as that of the articles examined in this chapter. We state our conclusions on the entire survey of literature (chapters 2 and 3) in section E of chapter 3.

The 1985 article by Hamada and Scholes reviews the literature on the impact of taxes on corporate finance. They do so by considering two models: after personal tax model (including the analysis by Miller (1977)) and before personal tax model (including the analysis by Modigliani & Miller (1963) and Miller & Scholes (1978)). A comparison of these two competing models, the after personal tax model and the before personal tax model, focuses on the relevance of personal taxes in corporate finance and are analysed in detail by Hamada and Scholes.

We will focus on the following issues addressed by Hamada and Scholes (1985):

(a) The after (personal) tax theory of corporate valuations

(b) The before (personal) tax theory of corporate valuations

(c) Implications regarding debt versus equity financing

(d) Dividend policy implications

(e) Cost of capital implications

In particular, we focus on (a) and (b) and comment on a fundamental oversight in Hamada and Scholes derivation of the before tax equilibrium.

**(a) The After (Personal) Tax Theory**

Hamada and Scholes incorrectly state that the first after tax equilibrium model which incorporated personal taxes was the Miller (1977) model. The first such model was the Brennan (1970) model, which was further developed by Modigliani (1982). The fact that Brennen pioneered the models incorporating personal taxes was also acknowledged by Haugen and Senbet (1986) in their review article on Corporate Finance and Taxes.

This thesis will concentrate on after tax equilibrium models. Both Miller (1977) and Modigliani (1982) models are alternative versions of after tax models. Miller (1977) and Modigliani (1982) are described in sections B and C below. The version of the model used in this thesis will be the Modigliani model for the reasons stated in section C. These reasons are essentially that Miller's model is theoretically weak (as noted below in analysing investors' ability to launder personal taxes on income from bonds as well as income from equity), and that Modigliani's model is a comprehensive one. Hamada and Scholes' concentration on the Miller model as both the first after tax model and as the most relevant after tax model therefore is incorrect and this basic error limits the usefulness of their analysis.

Hamada and Scholes extend Miller's model by adding the partnership sector of the economy to the analysis described by Miller (see section B below). They state that the equilibrium rates of return must be consistent between the corporate debt, equity and the unincorporated partnership sectors. They use the following notation :

$r_B$ = Before tax rate of interest on taxable government and private bonds

$r_c$ = Pre tax rate of return on 100% equity financed investment in corporate sector

$r_p$ = Pre tax rate of return on 100% equity financed investment in partnerships

$r_o$ = Municipal bond interest rate

$\tau_c$ = Marginal corporate income tax rate

$\tau_B$ = Marginal personal income tax rate of the marginal bondholder

$\tau_s$ = Marginal personal equity income tax rate of the marginal shareholder

$\tau_p$ = Marginal personal income tax rate of the marginal partner

Like Miller (1977), Hamada and Scholes use certainty equivalents to abstract from the impact of risk. All rates of return are stated as certainty equivalents and consequently the returns to risky securities ($r_c$) are adjusted for differences in risk. This unfortunately means that the important role of diversification in risk reduction is not examined by Miller.

However, Hamada & Scholes deviate from Miller by assuming that the personal tax rate on equity income is not zero, but is small and constant, thereby explicitly including the impact of $\tau_s$ in their formulation, along the lines hinted at by Miller (1977). As a result, the equilibrium after tax return on all equity investment is $(1 - \tau_s)(1 - \tau_c) r_c = r_o$, that is, the return net of all taxes from holding corporate equity is equal to the return that investors could earn from tax exempt municipal bonds ($r_o$ , on which investors do not pay any personal taxes). This equality is required otherwise investors, who are interested in maximizing their after tax returns, will not hold that security which generates lower after tax returns than the others. Since it the after personal tax rates that are equalised in this after tax model, the before personal and corporate tax return ($r_c$) that needs to be generated by corporate equity must be greater than $r_o$ , the tax exempt interest rate.

In equilibrium, the net of tax certainty return to the partnership sector must also be equal to $r_o$. This is so in order to prevent the partnership equity from either being dominated or being the dominant security.

The equilibrium $r_B$ is determined by the intersection of the demand curve and the supply curve for debt, as described by Miller (1977) and analysed below. The equilibrium rate is such that the after personal tax return on corporate debt ($r_{B(1-\tau B)}$)) is equal to $r_o$ .

$$r_o = (1 - \tau_p)\, r_p = (1 - \tau_s)\,(1 - \tau_c)\, r_c = (1 - \tau_B)\, r_B. \qquad (1a)$$

Thus in equilibrium the marginal investors, that is the investors who are just indifferent between the after tax return they get from the alternative securities, have tax rates such that

$$(1 - \tau_p) = (1 - \tau_s)\,(1 - \tau_c) = 1 - \tau_B \qquad (1b)$$

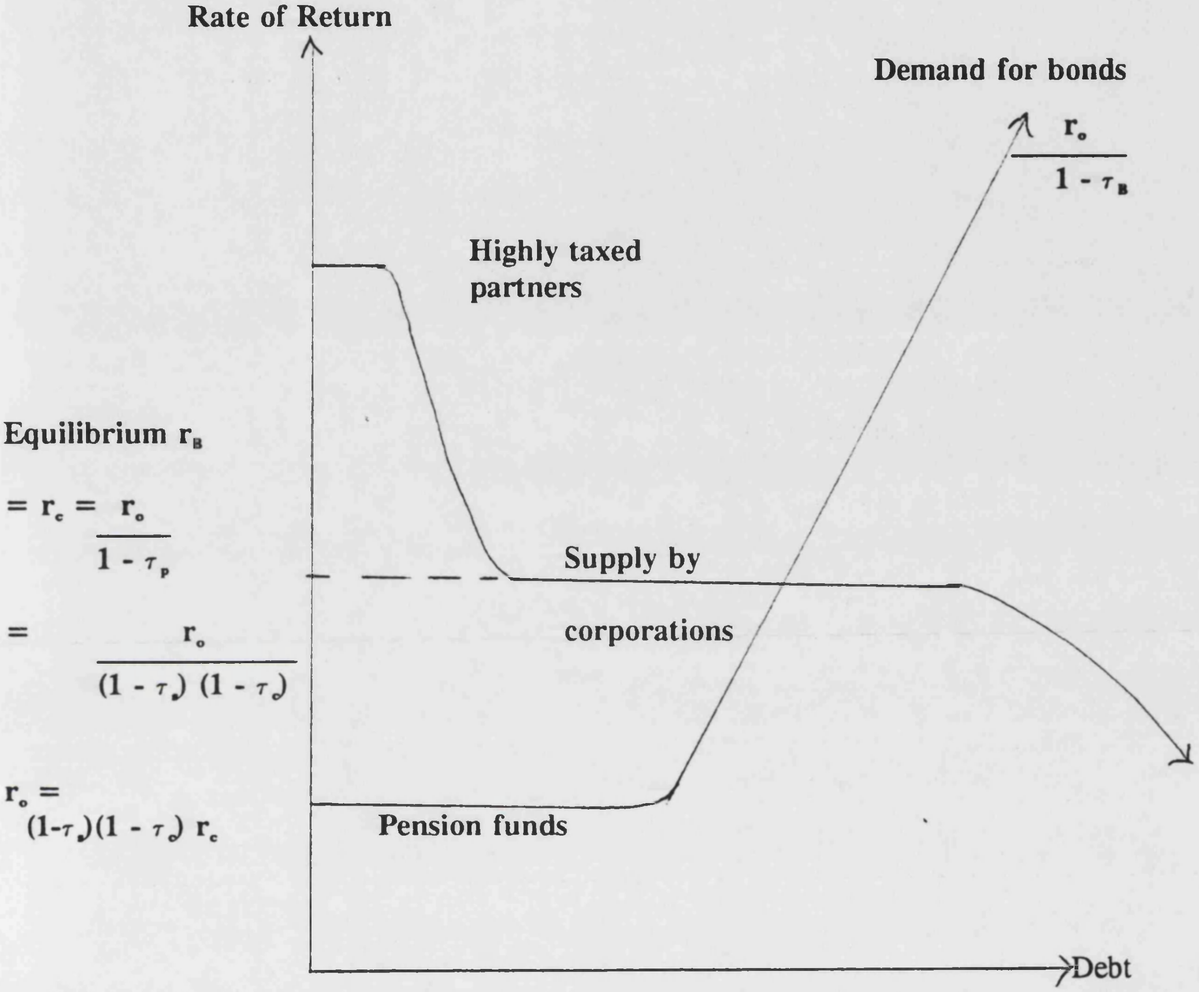
or, by adjustment, we get $\tau_p = \tau_s + \tau_c - \tau_s\,\tau_c = \tau_B$ (1c)

Again, following Miller (1977), the implication of equation (1a) above is that there is an economy-wide equilibrium after tax rate of return, $r_o$, which is common to the after tax risk adjusted returns from the alternative types of investment. This rate of return results from the more general macro equilibrium considerations and is not derived within the model.

Miller's horizontal supply of debt schedule is altered by the presence of partnerships. Highly taxed partners, whose $\tau_p$ is greater than $\tau_s + \tau_c - \tau_s\,\tau_c$ (see (1c) above), would prefer more debt in their capital structure. This is because of the tax deductibility of debt interest, which, at their high tax rates, would reduce their after tax cost of debt to be below the cost of the alternative sources of finance. On the other hand, partners whose $\tau_p$ is less than $\tau_s + \tau_c - \tau_s\,\tau_c$ (see (1c) above) would prefer more equity finance. Thus Hamada and Scholes analysis adds the following to Miller's after tax model :

(i) They include the partnership sector and show how the marginal tax rates and the rate of returns for the partnerships are related to those earned in the corporate sector and on tax exempt municipal bonds,

(ii) They add the demand and the supply of debt by the partnership sector to the Miller model to derive an equilibrium model which includes the partnership sector. This model is illustrated by the following diagram.




**Aggregate debt (partnerships, government and corporate) market equilibrium.**

Hamada and Scholes then describe the before personal tax equilibrium model, and then go on to discuss the implications of the two competing models ( the after personal tax model and the before personal tax model) for capital structure, dividend policy, and the cost of capital.

### (b) The Before (Personal) Tax Theory

The before tax theory assumes that all personal income taxes, to bondholders, shareholders and partners of businesses, can be reduced to zero by making use of the various loopholes in the tax system. As stated previously, Miller & Scholes (1978) were the first to put forward this view. They also assume that the effective personal tax on capital gains is zero. Furthermore, they claim that in the U.S.A., the federal tax statutes contained many easy ways which allow investment at before tax rates of return so that the effective personal tax rate applicable on dividend income is reduced to zero. This arises because interest paid on personal debt is deductible in the U.S.A. as long as it is less than dividend income received. Therefore individuals can take out loans and ensure that the associated interest payments payable cancel out dividends receivable. The proceeds of loan can then be invested in one of the many untaxed vehicles and are allowed to pay the before tax rate of return (like pension, Keogh and IRA plans, and life insurance annuity schemes). The individual thus can reduce his personal tax on dividend income to zero while at the same time (a) staying within income tax rules and (b) without increasing leverage because personal loans are offset by investment in pension or insurance funds.

Therefore investors in the U.S.A. can earn and retain the before tax rate of return on dividends. Investors can use these procedures effectively to reduce all personal taxes on savings at the margin to zero (including personal taxes on partnership income and on corporate bonds). Therefore, personal taxes are not relevant for the determination of equilibrium and they can be ignored.

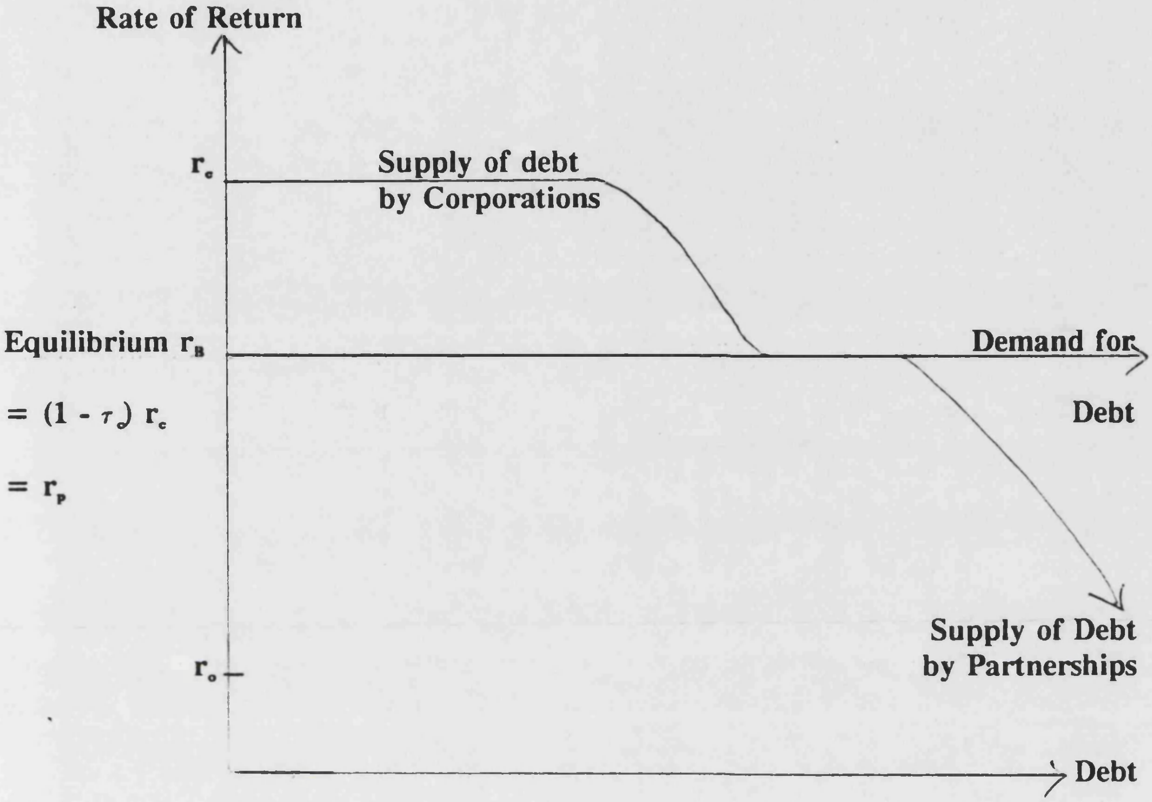
In the before tax model (which means "before 'personal' taxes model"), personal taxes are irrelevant and only corporate taxes need to be considered. Corporate taxes were considered by Modigliani and Miller (1963) in their equilibrium model which implied a corner solution for the corporate debt equity structure. Hamada and Scholes discussion of the before tax model essentially relies on the Modigliani and Miller (1963) model, to which they add (a) the partnership sector and (b) the analysis, given in Miller and Scholes (1978), on the laundering of personal taxes.

They state that in the before personal tax model, the certainty equivalent equilibrium relative rates of return, which ensure that all types of securities (partnership debt and equity, corporate debt and equity, and government debt) are held, would be as follows:

$$r_p = (1 - \tau_c)\, r_c = r_B = \frac{r_o}{(1 - \tau_c)} \qquad (2)$$

Equation (2) above shows that the pre tax rate of return required from the projects in the partnership sector ($r_p$) is the same as the required rate of return on corporate bonds, since both these type of finance sources are similar to the extent that neither bears the burden of any corporation tax. In contrast, the projects financed by corporate equity must earn a rate of return $r_c$ , which is higher than both $r_p$ and $r_B$ , because corporate equity income is subject to corporation tax, and therefore it must be at the level of grossed up rate, in order to yield the same net of corporate tax return. Hence $r_B$ is equal to $r_c(1 - \tau_c)$. Hamada and Scholes have not explained why $r_B$ is equivalent to ro grossed up by the corporation tax rate. Presumably, their reasoning is similar to their analysis in the after tax model in which case they argued that the net cost of corporate debt after taking the benefit of interest tax deductibility, is $r_o$ , the rate offered by the tax exempt institutions. Their reasoning perhaps is that the corporations can "afford" to pay a higher rate to their investors because they can treat the interest payments as a deduction against their corporation tax. We argue below that Hamada and Scholes are mistaken in stating that $r_B$ is equal to $r_o$ grossed up at the corporate tax rate.

The before tax equilibrium as analysed by Hamada and Scholes is depicted in the following diagram, and the derivation of the demand and the supply curves are discussed subsequently.




In the above diagram, the demand curve for corporate debt is a horizontal line at a rate equal to $r_c\,(1 - \tau_c)$. The demand curve is horizontal because in the absence of personal taxes, the after tax return required by the debt holders does not increase with an increase in the level of corporate debt. There is no need to offer higher rates of interest, for example to entice investors in higher tax brackets, because all investors will be willing to invest at this rate in the absence of personal taxes. The rate is equal to the after corporate tax return on equity because equity returns are subject to corporation tax where as debt returns are not. Analogously, the demand for debt is at the rate earned on partnership equity, because neither of these bear any corporation tax, and, in this model, neither bears any personal tax. However, Hamada and Scholes do not explain why the demand for debt is at a level greater than the interest rate on tax exempt bonds, $r_o$.

The supply curve begins with a horizontal section at the rate $r_c$ , the pre tax rate of return earned on corporate equity. The company is indifferent between supplying equity at a cost of $r_c$ (on which investors effectively bear corporation tax, and only earn net return $r_c\,(1 - \tau_c)$) or supplying debt at the same rate. However, because of the costs associated with increasing leverage (see below), the supply curve begins to curve downwards. A second horizontal portion, at the equilibrium rate $r_B$ (which equals the return on partnership equity $r_p$) results from the debt supplied by the partnership sector. Like the corporate sector, the partnership sector too faces costs associated with increasing leverage, and hence the supply curve slopes downwards. The equilibrium is determined by the intersection of the two curves, at a rate $r_B$.

As far as implications for capital structure are concerned, for partnerships, with the rate of return $r_p = r_B$, the capital structure decision is irrelevant. This is because in a world without personal taxes, including personal taxes on partnership income, the cost of debt and of equity for the partnership sector will be the same. However corporations should still prefer 100% debt unless there are market imperfections involved. The reason for debt preference is, of course, that the after corporation tax cost of debt capital is lower because debt is tax deductible. Therefore, as stated above, for the aggregate supply curve of debt to slope downwards, there must be some other leverage related costs. Hamada and Scholes discuss the following conditions which have been mentioned in the existing literature.

(i) bankruptcy costs
(ii) contracting and monitoring costs
(iii) information and signalling costs
(iv) differential flotation costs
(v) incomplete markets, and
(vi) "wasted" tax deductions.

(See chapter 3 section B below for a review of the literature on these costs).

If one or more of these costs exist, the aggregate supply curve for debt will eventually slope downwards as more and more corporate debt is issued. The resulting equilibrium as derived by Hamada and Scholes, is shown in the diagram above.

Two issues can be raised regarding the above version of before tax theory presented by Hamada and Scholes.

(a) The first issue is regarding the laundering of personal taxes on equity income. Miller and Scholes (1978), in common with Modigliani and Miller (1958), ignore the practical difficulties of creating home made leverage. In practice, lending institutions are not willing to lend to individual investors in order to undertake transactions in the capital markets or for the purposes of tax arbitrage. The simple reason for this is because the assets that can be offered as security by the large corporations often exceed, even proportionately, the assets that can be offered by an individual investor.

Moreover, Miller and Scholes ignore the disadvantages of investing in "risk free" pension plans and insurance policies. The mechanisms they outline for laundering personal taxes on equity dividends could also be used to launder interest receipts on investments in corporate debt, because their use is not limited to dividend income only. Thus the personal tax rate on corporate debt interest could be reduced to zero. The point is that if this is possible, and there is no major disadvantage of investing in risk free pension plans and insurance policies, then there is no need to invest in corporate debt in the first place. Taxable individuals who wish to avoid personal taxes on corporate debt income, need not undertake the laundering operation, but simply invest in the risk free pension plans which are able to generate and pay out at the pre-personal tax rates. Corporate debt thus becomes completely dominated by the pension plans as a risk free investment.

Finally, this argument of laundering personal taxes ignores the vast amount of personal tax on investment income collected by the tax authorities in the U.S.A. and in the other countries. Such sums would be nearer zero if personal taxes on investment income could easily be laundered. Therefore, the foundations of the before tax model are not sound.

(b) The second issue concerns the rate of return at which the elastic demand curve for debt is shown by Hamada and Scholes in the before tax model . They do not describe the derivation of the demand curve in their article, but simply show it as an elastic horizontal line and state that at the equilibrium, $r_B = r_o / (1-\tau_c)$ (see equation (2) above) . We shall give six different explanations which demonstrate that the above assertion by Hamada and Scholes is incorrect.

(1) In the after tax model (Miller 1977, see section B below), the demand curve for debt is described as $r_o / (1-\tau^a_{PB})$, where $\tau^a_{PB}$ is the personal tax rate on income from bonds for the marginal investor a. The demand curve is defined by the need to gross up the tax free rate of interest $r_o$ in order to induce the taxable investors in progressively higher tax brackets to hold (that is, to demand) corporate debt. However, Hamada and Scholes are dealing with situations where there is no effective personal income tax. Therefore, the demand curve ( $r_o / (1-\tau^a_{PB})$ ) should simply be a horizontal line at the rate $r_o$, since the rate at which the demand curve needs to be grossed up ( $\tau^a_{PB}$ ) is zero. Therefore the equilibrium rate of return should be at $r_o$, not at $r_o / (1-\tau_c)$, as shown by Hamada and Scholes.

(2) Hamada and Scholes, on page 200, admit while discussing the after tax theory that: "First, if the shareholders' personal taxes on dividends can be laundered via the pension fund / life insurance annuity route, it should also be possible for the bondholders to do the same to eliminate their personal taxes on interest received. If this is true, then the demand curve for corporate bonds in Figure 1 will not be upward sloping, but would remain a horizontal line at $r_o$."
This is precisely what we demonstrated in (1) above. It is puzzling that Hamada and Scholes state this on page 200 but ignore it in their depiction of the before tax equilibrium on page 199, where they clearly state that they are in fact assuming that "investors can use (these) procedures effectively to reduce all personal taxes on savings at the margin to zero".

(3) This point can also be demonstrated by examining the equilibrium equation in the after tax model (equality 1a shown earlier in this section) which is reproduced below.

$r_o = (1 - \tau_p) r_p = (1 - \tau_s) (1 - \tau_c) r_c = (1 - \tau_B) r_B.$ (1a)

If the rates of all the personal taxes in the above equation ( $\tau_p$ , $\tau_s$ and $\tau_B$ ) are set to zero, then the above equalities transform into:

$r_p = (1 - \tau_c) r_c = r_B = r_o$ (2')

Equation 2' therefore represents the correct equilibrium conditions in the before tax model, and not equation (2) of Hamada and Scholes.

(4) In the after personal tax model, the equilibrium rate of interest on corporate bonds, $r_B$, is equal to $r_c$ , the before corporation tax cost of corporate equity. If the personal taxes on equity income are assumed to be zero, then the equilibrium $r_B$ is equivalent to $r_o / (1-\tau_c)$ and to $r_o / (1-\tau_B)$, because the after tax equilibrium is reached when the rate of personal tax on corporate bonds equals the corporation tax rate (Miller Equilibrium). This equilibrium interest rate requires, on the demand side, that the tax exempt interest rate $r_o$ is grossed up for the personal tax payable by the marginal investor on corporate debt, as described above. On the supply side, companies can raise equity at the before corporation tax risk adjusted rate $r_c$, which, after all taxes is equivalent to the rate $r_o$ . Companies will supply debt if the cost of debt is less than $r_c$. Companies would be just indifferent between supplying debt and equity when the equilibrium rate of interest $r_B$ is equal to $r_c$ , which, in equilibrium in the after personal tax model, is equal to $r_o / (1-\tau_c)$. The supply of corporate debt is infinitely elastic at that level of interest rates because no supply side disadvantages of debt are allowed for in these after tax models (note that the downward slope in the supply curve in the diagram for the after tax equilibrium above is entirely due to the partnership sector). Hence the after personal tax equilibrium is at a level represented by horizontal supply curve of corporate debt, which is at interest rate of $r_c$ (which is equal to $r_o / (1-\tau_c)$ .

In the case of before ("personal") tax model, the demand curve should be horizontal at the level $r_o$ , as explained above. If we assume that in this before tax model, as in the after tax model, there are no supply side disadvantages of debt, then companies should be willing to supply as much of debt as they can at an interest rate of $r_B$ equal to $r_c$ ( Modigliani & Miller (1963) corner solution). However, Hamada and Scholes consider supply side disadvantages to debt in the before tax model (see below) and presumably these disadvantages come into effect only at very high levels of debt. These disadvantages should make the supply curve of corporate debt in the before tax model eventually slope downwards to below the level of $r_c$ ( equal to $r_o$ / $(1-\tau_c)$ ) . The before tax equilibrium will be when the supply curve of corporate debt, which initially is infinitely elastic and eventually slopes downwards, reaches the elastic demand curve at the level $r_o$.

This in contrast to the level $r_o$ / $(1-\tau_c)$ stated by Hamada and Scholes as the equilibrium. However we maintain that if the level $r_o$ / $(1-\tau_c)$ is the equilibrium level in the after tax model, which incorporates personal tax disadvantage to debt interest, then the equilibrium in the before tax model, in which we do not incorporate any personal tax disadvantage to debt interest, cannot be at the same level. Since the before tax model assumes away the personal tax disadvantage of debt interest, the equilibrium must be at a higher level of corporate debt and therefore at a lower level of rate of interest. We claim and prove that this lower interest level is $r_o$, at which the demand for debt is infinitely elastic.

(5) Hamada and Scholes state on page 199 that the equilibrium relative rates of return should be such that they enable "all financial instruments to be held". However they go on to depict the tax exempt bonds as a fully dominated security, an investment in which is an "infra-marginal investment by corporations and financial institutions, simply offsetting tax-deductible liabilities." They thus appear to reach equilibrium rates of return which imply that very attractive securities, such as tax exempt bonds, are dominated by other securities. This appears odd in itself and leads to an another oddity examined below.

(6) The above leads to an anomaly in the relative (certainty equivalent) rate of return on corporate equity and the rate of return on tax exempt bonds. The relationship as given by Hamada and Scholes as in equation (2) above is:

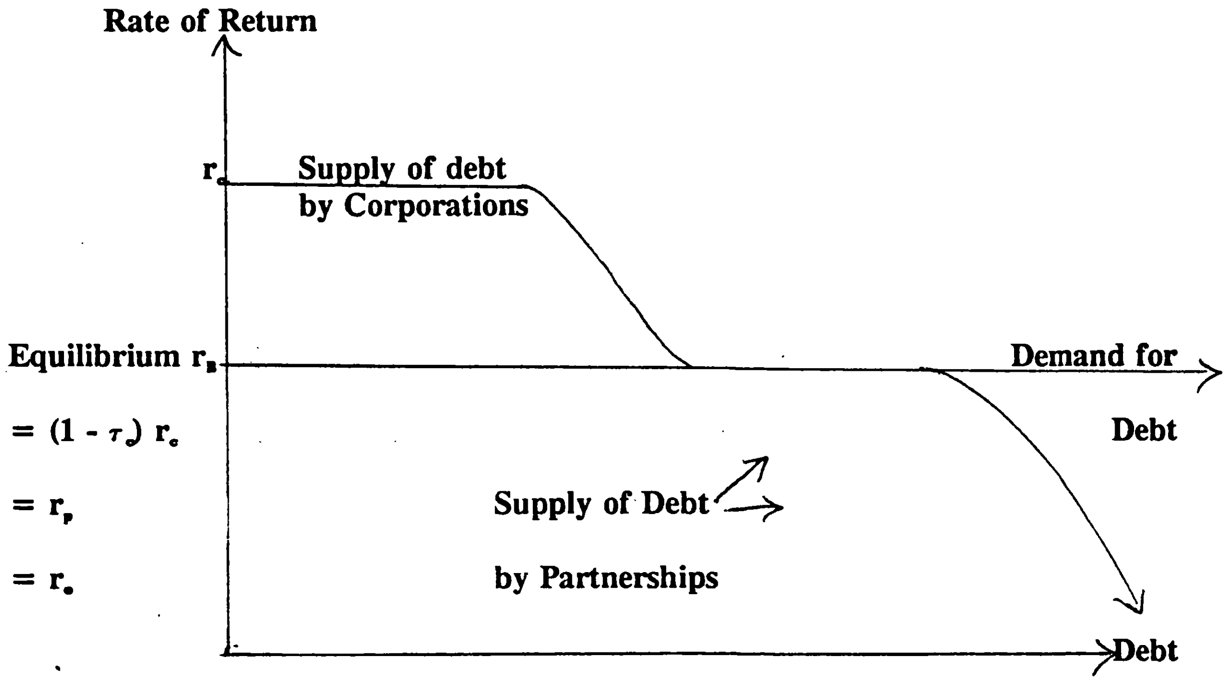
$$(1 - \tau_c)\, r_c \;=\; \frac{r_o}{(1 - \tau_c)} \qquad (2)$$

Rearranging above, we obtain,

$$(1 - \tau_c)\,(1 - \tau_c)\, r_c \;=\; r_o \qquad (3)$$

Equation (3) above shows that (certainty equivalent) pre tax returns to corporate equity, which are subject to corporation tax, must be grossed up twice for corporation tax relative to return on the tax exempt bonds. This again appears odd in itself and is a different result from that obtained in the after tax model, under which corporation equity returns were grossed up only once for corporation tax and not twice.

All the above discrepancies are resolved by correctly interpreting the before personal tax equilibrium along the lines suggested in this thesis above. Therefore we assert that the correct before personal tax equilibrium relationship is given by equation 2' above, which is depicted below.




In the above diagram, the demand for bonds is a horizontal line at $r_o$. The demand curve does not rise because there are no personal taxes. It is at a level equal to $r_o$ because investors demand a certainty equivalent net return equal to $r_o$ from all investments. Thus they receive $r_o$ from tax exempt bonds (by definition), from corporate equity (because corporate equity earns $r_c$ which is subject to corporation tax), from partnership equity, and also from corporate bonds. Hence demand curve is correctly depicted at this level.

The supply of corporate bonds commences at a level equal to $r_c$, the before corporation tax return on equity income. The disadvantages of debt as leverage increases result in the supply curve sloping downwards. When the supply curve reaches the level $r_p$, partnerships begin to supply debt because at that level of interest rates, they are indifferent between supplying debt or equity. The disadvantages associated with debt as partnership leverage increases implies that the supply curve eventually falls again.

The equilibrium is determined by the intersection of the two curves. Equilibrium interest rate is equal to the interest rate on the tax exempt bonds. This before tax equilibrium does not suffer from the six anomalies noted above. It is therefore the correct before personal tax equilibrium.

The after tax and the before tax models are based on two competing theories. They have different implications for the tax rates of the marginal investor, the capital structure, the dividend policy and the cost of capital. With the after tax theory, the rate of personal tax of the marginal investor is close to $\tau_c$. In the before tax theory, the marginal investor driving the equilibrium has a tax rate less than $\tau_c$, and which is equal to zero at the extreme. This occurs where there is complete laundering of personal taxes on interest income, dividend income and partnership income.

The different implications of the above competing theories for corporate financial policies are discussed below.

### (c) Implications for Debt versus equity financing

**After Personal Tax Model:** In this model, the corporations paying the top rate of corporation tax should be indifferent between financing their assets with debt or with equity. They are paying a higher before tax unit cost of debt ($r_B$) with debt capital, but would receive the full value of the tax deductibility of interest payments equal of ($\tau_c$ + $\tau_s$ - $\tau_c \tau_s$) $r_B$. This equals the risk adjusted cost of equity capital. Therefore the market value of such companies facing the marginal corporate tax rate is independent of their capital structure.

Equity financing is better for companies that are not liable for taxation at the top marginal corporate income tax rate. This is because such companies will not be realising as much benefit of tax deductibility of interest payments as the companies subject to the marginal rate of corporation tax. These intra marginal companies should prefer equity finance.

Now assume that the supply curve of corporate debt is not horizontal but slopes downwards because of the presence of leverage related costs. In such a case, the equilibrium may be at a point where $\tau_c$ is below the top marginal corporation tax rate. The companies paying the top corporate tax rate are receiving more benefit for interest-tax deductibility than the marginal companies. The marginal companies are indifferent between debt or equity. Hence the top corporate rate companies should prefer debt capital. And, as explained in the above paragraph, companies with a lower marginal tax rate than that impounded in the pricing of $r_B$ should use equity finance.

**Before Personal Tax Model:** As stated previously, in this model, the interest payments are tax deductible while the equity returns are not. Hence, in the absence of other imperfections, companies that pay corporation taxes should prefer debt finance to equity finance (Modigliani & Miller 1963).

### (d) Dividend Policy Implications

**After Personal Tax Model:** If the personal tax rate on dividend income is higher than the personal tax rate on capital gains, then a company will maximise its after tax wealth by retaining the after tax profits and investing them in, at worst, a zero net present value venture. The company should therefore prefer to buy back its shares rather than pay dividends.

However, for shareholders that pay no tax at all, such as pension funds, or for those shareholders for whom capital gains and income are taxed together such as broker-dealers, the dividend policy does not matter.

In the U.K. corporate shareholders are one class of investors for whom the effective tax rate on capital gains though small, is still positive whereas the tax rate on dividend income is zero. On the basis of this fact alone (that is, ignoring the other implications of the imputation type of taxation regime), U.K. corporate shareholders, in contrast to the individual investors, and the pension fumds and broker-dealers referred to above, should prefer dividends to capital gains.

In the U.S.A. 85% of corporate dividend income is excluded from taxation. With a corporate tax rate of 0.46, the effective tax rate on dividend income is 6.9%. Therefore a company has to decide whether the economy wide $\tau_s$, that is the average personal tax on equity income, is greater than 6.9% or not - if $\tau_s$ greater than 6.9%, then the companies which own shares in other companies should prefer to hold dividend paying stock.

The dividend policy implications thus depend upon the personal tax characteristics of the shareholders as well as upon the taxation regime (U.S.A. , U.K. or some other tax regime). Moreover, the after tax model is not yet fully developed in so far as the tax status ($\tau_s$) of the marginal shareholder has not been identified. Its dividend policy implications are therefore not clear-cut prescriptions.

**Before Personal Tax Model:** As all personal taxes are laundered in this model, the irrelevancy of dividend policy is maintained for individual shareholders, pension funds, non-profit institution and for broker-dealers.

**(e) The Cost of Capital Implications:**

A project's cost of capital is given by the following CAPM equation:

$$E(R_P) = R_F + [\, E(R_M) - R_F \,]\, \beta_P$$

where

| | |
|---|---|
| $E(R_P)$ = | The project's expected rate of return |
| $R_F$ = | The economy's risk free rate |
| $E(R_M)$ = | Expected rate of return on the market portfolio |
| $\beta_P$ = | The beta factor, that is the covariance of the project's rate of return with the market portfolio's rate of return, divided by the variance of the market portfolio's rate of return. |

**The After Personal Tax Model:** In this model, the appropriate risk free rate to use is the after tax risk free rate, which is equal to $r_o$ in the previous discussions. All the rates of returns are the after tax rates of return. Our thesis will develop an after tax model, beginning in chapter 4 to the derivation of the final valuation equation in chapter 7.

**The Before Personal Tax Model:** In this model, Hamada and Scholes state that the appropriate risk free rate to use is the before tax rate of $r_B$ , which according to them is higher than the rate $r_o$. They add that the marginal investor in this case is one who has effectively laundered all the personal taxes on investment income. If this is accepted, then what Hamada and Scholes fail to state is that the before personal tax rate is equal to the effective after personal tax rate for an investor who has laundered his personal taxes. Therefore, as argued above, the equilibrium risk free rate $r_B$ should be equal to $r_o$. Hence

Hamada and Scholes have mis-specified the cost of capital under the before tax model.

We began our review of the literature on taxes and corporate finance above by undertaking an in-depth analysis of one of the more relevant articles on the subject. Our above review of Hamada & Scholes (1985) has contributed to the literature by pointing out possible changes to the variables they specify, and also pointing out that they have not chosen the best of the after tax models then existing.

We will use a model that incorporates personal taxes and, unlike Hamada & Scholes and Miller, which incorporates uncertainty explicitly, as is done by Modigliani (see section C below). Before that, it may be appropriate to review Miller (1977) because:

(a) Hamada and Scholes relied heavily on the analysis developed by Miller, and

(b) although we disagree with the existing literature, the literature generally regards it as "the" model which incorporates personal taxes, and thus is relevant to the subject of our thesis.

## SECTION B
## Model of Miller (1977)

Generally, as demonstrated by Modigliani and Miller (1963), in the absence of personal taxes and assuming perpetual debt, the gain from leverage is given by

$$GL = T_c D$$

where

$D$ = the market value of debt, and

$T_c$ = the marginal corporation tax rate.

This would lead to a corner solution for optimal capital structure because the greater the amount of debt capital, the greater is the benefit from tax deductibility of debt. Hence all firms should be almost 100% debt financed.

However, the observed capital structures do not display anyway near this proportion of debt. Capital structure of companies in the major industrial countries shows the following ratios:

| | 1983 | 1984 |
|---|---|---|
| | % | % |
| U.S.A. | 21.85 | 24.60 |
| Canada | 35.67 | 38.19 |
| U.K. | 32.44 | 28.01 |
| France | 56.56 | n/a |
| Germany | 35.55 | 33.92 |
| Japan | 21.67 | 28.84 |

Note: The table shows the ratio of value of Debt to Debt plus market value of Equity. (Source: J Rutterford (1986) p495.)

The above table shows that in practice, the debt proportions are much more modest than the 100% debt finance suggested by MM . Hence we need to examine some explanations provided to show why companies do not issue more debt.

(i) Increase in debt capital increases the probability of bankruptcy, thereby increasing the expected value of bankruptcy costs, and hence, lowers the expected value of gain from leverage.

(ii) Increase in debt capital generally increases the probability that the company's taxable income will be insufficient to cover the interest payments on debt capital. This may have been particularly true in the case of U.K. companies before the Finance Act 1984, because of the availability of other generous tax shelters. For example, before this Act, U.K. companies were granted other tax shelters such as 100% capital allowances for expenditure on plant and machinery and 75% capital allowances on industrial buildings. As a result of these other tax shelters, many manufacturing companies had tax losses carried forward or had low taxable incomes. The availability of tax deductible interest expense therefore was of no immediate use to them. Therefore, these companies were not in a position to enjoy fully the tax advantages of tax deductible interest payments and consequently debt capital would not be as attractive as it would have been if its tax benefits could be enjoyed immediately.

(iii) As stated earlier, the cost of debt capital may increase as the proportion of debt increases. The reason for the increase in the cost of debt is that with greater amounts of debt, debt holders begin to bear more and more of the risks borne by the equity holders. Hence they demand a greater return. This phenomenon has become very prominent in the capital markets in the 1980s with an increase in leveraged corporate takeovers. These are financed by "mezzanine" debt which is subordinated to senior debt finance. The cost of this debt is considerably higher than the cost of senior debt finance. The higher cost is to compensate the mezzanine investors for the higher risk they are undertaking which is present because of the high debt level of the company. Thus, in practice, high costs may be associated with high debt levels although this phenomenon is not reflected in most of the theoretical models.

(iv) There could be constraints imposed on the company by existing debt holders, or the company may be following the market rules of thumb, which prevent the company from issuing more debt.

(v) The gain from leverage is $T_cD$ if the benefit is discounted at the rate of the cost of debt capital. However, this stream may not be as certain as the normal interest payments on debt capital. For example, the Finance Act 1984 altered the corporate tax provisions, including lowering $T_c$ gradually from 52% to 35%. Hence probably the benefits from tax deductibility of interest should be discounted at a higher rate than the cost of debt capital.

(vi) The tax advantage to debt at the corporate level may be offset by the tax disadvantage at the personal level. The personal taxes on debt income received by bondholders may be taxed at higher rates than the equity income received by shareholders.

These above explanations have been considered by various authors, particularly Miller (1977), in the context of equilibrium and valuations using models that incorporate personal taxes. This article, referred to as the "Miller Equilibrium" also provides first evidence of divergence between the views of the researchers, Miller and Modigliani, who had earlier worked together.

Miller (1977) rejected the bankruptcy costs argument stated in (i) above as an explanation for the absence of 100% debt capital structures and rather concentrated on the role of personal taxes mentioned in (vi) above.

Miller has correctly stated that the magnitude of bankruptcy costs was much too small compared to the potential tax advantage of debt (which could be as high as the corporation tax rate ($t_c$) times the debt level (D)) to explain the low level of debt in the observed capital structures. His conclusion was based on criticism of the many empirical studies of bankruptcy costs, including those by Baxter (1967), Van Horne (1976), Warner (1977) and the managerial behaviour study by Jensen and Meckling (1976). His criticisms include:

(i) The Baxter study related mainly to liquidations of businesses rather than reorganisations and concentrated on the bankruptcies of individuals rather than companies. For the purpose of determining the optimal corporate structure, a study of corporate bankruptcies would have been more relevant. Hence Miller is correct in stating that the Baxter study of bankruptcies is not relevant in the context of corporate debt levels.

(ii) The study by Warner on direct costs of bankruptcy of rail-roads shows these costs to average on 5.3% of the market value of the firm's securities as of the end of the month in which the rail-road filed the petition. Warner goes on to state that if a comparison is made with the value of the firm three years prior to bankruptcy the direct costs average 2.5%. These percentages of bankruptcy costs imply that the magnitude of these costs is relatively small in comparison with the tax savings that can result from increasing the level of debt. The tax savings, in the absence of personal taxes, will be at the rate of corporate tax, which is likely to be in the 30% to 60% range in European countries or in the U.S.A.. Therefore bankruptcy costs alone would seem to be of too small a magnitude to explain why the corporations are willing to forego the much larger tax advantage resulting from issuing more debt.

(iii) Although there are indirect costs of bankruptcy, such as the diversion of the time and energies of the management and the reluctance of customers and suppliers to enter into long term commitments, the companies need not bear the burden of these costs. Miller states quite correctly that the companies could design other forms of debt contracts with lower deadweight direct and indirect costs. Hence it is possible to raise or to increase debt and still keep the potential bankruptcy costs at a very low level.

Therefore Miller correctly rejects the theory that bankruptcy costs disadvantage is of sufficient magnitude to offset the advantage of debt finance. Instead he concentrates on the role of personal taxes as an explanation of why there may be an optimal capital structure for the corporate sector as a whole. Personal taxes influence the optimal leverage through :

(a) the tax advantages of debt finance, and

(b) the market equilibrium with personal taxes.

Both these are discussed below.

**(a) Tax advantage of debt finance**

The gain from leverage, when personal taxes are taken into account in the most simple setting, is:

$$GL = 1 - \frac{(1-T_c)\,(1-T_{ps})}{1 - T_{PB}} \quad D$$

where:

| | | |
|---|---|---|
| GL | = | gain from leverage |
| D | = | level of debt |
| $T_{ps}$ | = | personal income tax rate applicable to income from shares |
| $T_{PB}$ | = | personal income tax rate applicable to income from bonds. |
| $T_c$ | = | corporation tax rate |

In the absence of personal taxes, the above expression states that the gain from leverage is $T_cD$, as before. If personal tax on income from shares is zero, but income from bonds is taxed at a positive rate $T_{PB}$ at personal tax level, then the gain from leverage is reduced and this is reflected by 1-$t_{PB}$ in the denominator. The tax advantage at the corporate level is offset by the tax disadvantage at the personal level. An alternative to income from debt is to own shares and receive income from shares. If income from shares is taxed at the personal level at the rate $T_{ps}$, then it adds to the gain from leverage (because it is a tax that will be avoided if debt is increased and equity is reduced). Hence 1-$T_{ps}$ appears in the numerator above.

Therefore the relative tax on income from bonds and equity at the personal tax level affects the gain from leverage. If the tax rate on income from bonds is the same as that on income from shares, that is, if $T_{PB}$ is equal to $T_{ps}$, then the gain from leverage is as before, that is, $T_cD$. However, if the average personal tax rate on income from shares is less than the tax rate on income from bonds, then the gain from leverage is less than $T_cD$. The reason is that the advantage of tax deductibility at corporate level is partially offset by the tax disadvantage at the personal tax level. Therefore while it is still true that the owners of a levered corporation have the advantage of deducting their interest payments to bond holders in computing their corporate income tax, these interest payments have already been "grossed up" by any differential in the taxes that the bondholders will have to pay at personal tax level on their interest income.

In order to show that the advantage from issuing debt is less than $T_cD$, Miller stated that the tax on income from shares was lower than that on income from bonds because of the following tax code features applicable in the U.S.A. :

(i) Many institutions (non-profit organisations, pension funds, trust funds, etc.) are exempt from taxes. These institutions would prefer high dividend paying stocks, and since they pay no personal taxes, the average personal tax on the income from shares will be lower than the marginal personal tax rate for individual investors. This statement is true, but not sufficient to prove the point because Miller is trying to compare the personal tax rate on income from equity with the personal tax on income from debt. As stated below, to show that the personal tax rate on equity income is low is insufficient to prove that it is *lower* than the tax rate on debt income for the same investor.

(ii) Capital gains are taxed only when realised and if the shares are held till death, then capital gains tax is avoided altogether in the U.S.A. (prior to the Tax Reform Act of 1976). Investors may borrow against unrealised potential gain but the amount they can borrow may be limited to the gain net of potential taxes.

Copeland and Weston (1983 pp 397) have added the following arguments as to why the personal tax rate on income from equity will be lower:

(iii) Gains and losses in a well diversified portfolio of equity investments can offset each other, thereby eliminating the payment of capital gains tax.

(iv) In the U.S.A., the first several hundred dollars of dividend income received by individuals is not taxed.

(v) 85% of dividends received by taxable corporations can be excluded from taxable income, as per the tax code, in the U.S.A..

Most of the above arguments by Miller and by Copeland and Weston are not valid arguments in view of the following:

(i) If tax exempt institutions held bonds, then bonds too will not be taxed at personal tax level. Hence, even though the equity income received by the tax exempt institutions is tax free, there is no RELATIVE personal tax advantage to receiving equity income. Thus Miller's point (i) above is not valid at all.

(ii) Although capital gains are not taxed until the gain is realised, neither is the benefit of capital gain (i.e. higher amount of cash resources) enjoyed until the gain is realised. Even if the investor is able to enjoy the "psychological advantage" of increases in wealth as share prices increase, at the same time he must prudently make a provision for the accompanying potential capital gains tax.

(iii) In relation to Copeland and Weston's argument given in (iii) above, if capital gains are largely offset by capital losses in an equity portfolio, then that portfolio must be very poorly managed indeed. What is relevant is the personal tax applicable on the *net gains* generated in an averagely managed portfolio. Therefore Copeland and Weston's argument is quite irrelevant. If their argument was valid, and capital losses did "eliminate" capital gains in a portfolio, then it would be an extremely powerful argument for not holding any equity investments (since they do not result in gains), rather than an argument for holding equity investment because of lower personal tax rates.

Note that we are not stating that there will be no capital losses on some shares or even on some portfolios as a whole. What is more important is that the reason why investors in general invest in equity is because they *expect* to receive a net gain (risk premium) compared to risk free investments. Therefore, on average, gains are expected, and realised, on equity investments, and, on average, capital gains taxes are payable on equity investments. Therefore, Copeland & Weston's argument is not a relevant one.

(iv) Income from shares in the U.K. has the following effect because of the special income tax rules for dividends where the imputation system of taxes is in use. Under this system, investors receive personal tax credit at the standard rate of personal tax along with the cash dividend income. The companies have to pay Advance Corporation Tax at the corporate level whenever they pay dividends. However, companies which have sufficient corporation tax liabilities arising from their U.K. activities, or companies which receive sufficient U.K. dividend income, suffer no extra tax burden because any ACT paid by the company can be offset against their mainstream corporation tax liability. The only disadvantage is the burden of the timing difference of tax payments, because the tax payments need to be made a few months earlier if dividends are paid. Ignoring this timing difference because it is not material for U.K. income generating companies, we can state that at the corporate level, this imputation system causes no extra burden ( overseas and loss making companies are an exception), and at the personal tax level, there is a distinct advantage (because of the tax credit) compared to income from bonds for individuals in the U.K.. Thus in the U.K., because of the imputation system of taxation, the personal tax rate on dividend income is lower than the personal tax on income from bonds.

(v) In the U.K. dividend income received by companies is termed as Franked Investment Income and is not taxed again in companies that receive dividends. Income from bonds held as assets by companies is taxed. Moreover, in the U.K. since 1982, capital gains tax computations allow an "indexation allowance" which is based on the annual changes in the retail price index. Therefore the base cost of an asset can be increased by this indexation allowance. The amount of capital gain that is taxed is therefore reduced. This is another advantage to income from shares.

In conclusion, it can be said that income from shares is subject to lower rates of personal taxes in the U.K. than income from bonds. However, the reasons why it is so, namely the imputation system, the indexation allowance and the treatment of franked income, all of which are specific to the tax code in the U.K., are different from those put forward originally by Miller or by Copeland and Weston. These authors were therefore incorrect in their interpretations of the tax codes they looked at. Therefore in the U.K. we can accept that the personal income tax rate on equity income is lower than the personal tax rate on debt income. This point will be fully illustrated with explicit calculations of weighted averages of the tax rates in chapter 6. However, even though we agree that the personal tax rate on equity income is lower than income from bonds in the U.K., it can not be assumed to be zero, as is done by Miller (Miller states that he assumes zero tax on equity for the sake of simplicity only but goes on to state that the effective rate at the margin on equity income is likely to be so close to zero that the assumption of zero rate is not so wide off the mark). Miller presents arguments which attempt to justify that the tax rate on equity income is low or zero, while ignoring a much more obvious and compelling argument which shows that the personal tax rate on equity income cannot be close to zero. The latter is shown by even a casual observation of the vast amounts of taxes on dividend income collected by the tax authorities in the U.S.A., and even in the U.K., and is sufficiently compelling to conclude that the tax rate on equity income is not zero. However Miller chooses to ignore this evidence and instead he incorrectly assumes that the tax rate on equity income is low or zero.

We continue to examine how Miller uses the above differences in the personal tax rates to derive a market equilibrium for the optimal level of debt in the capital structures.

**(b) The Market Equilibrium with taxes**

Miller assumes the following:

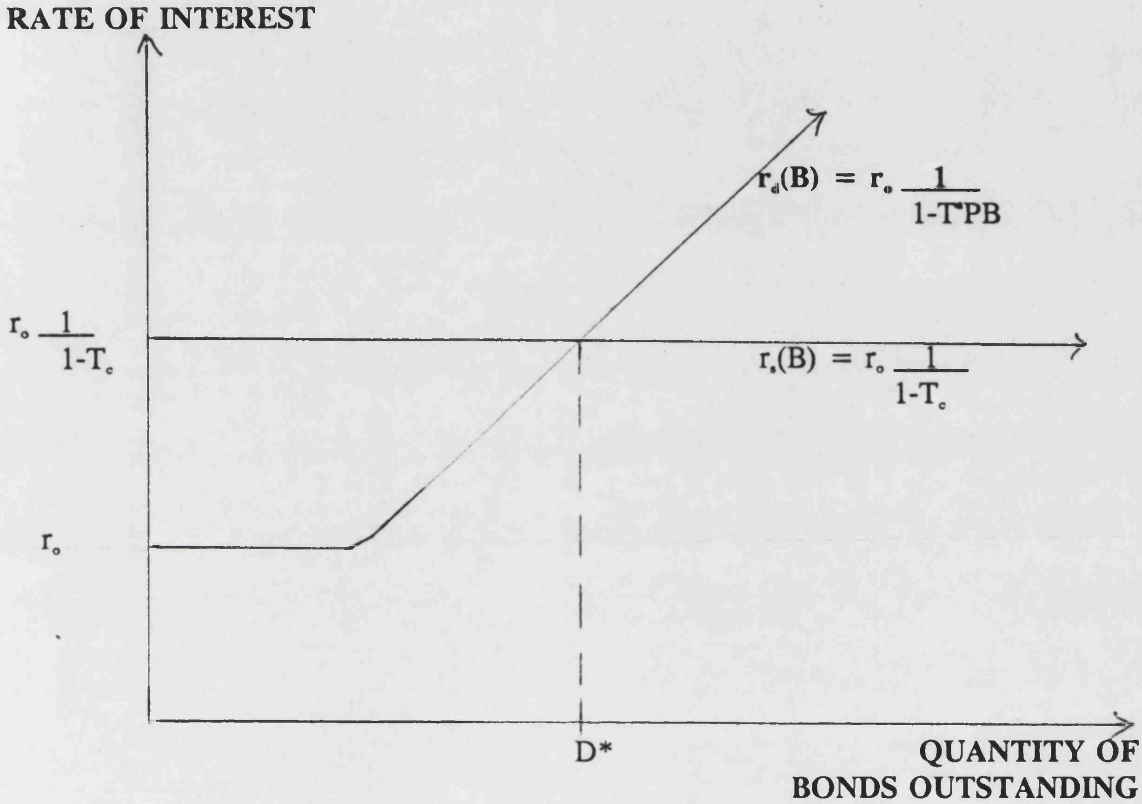
(i) personal tax rate on income from shares is low and for simplicity, that it is zero,

(ii) all bonds are riskless (for simplicity), and

(iii) there are no transaction costs, flotation costs or surveillance costs.

The equilibrium in the market for bonds under the above assumptions is shown in the following diagram.

EQUILIBRIUM IN THE MARKET FOR BONDS




In the above diagram, $r_o$ represents the minimum interest rate at which corporations could issue bonds, which is equal to the exogenously determined interest rate on tax exempt municipal bonds. The supply curve $r_s(B)$ is assumed by Miller to be infinitely elastic at the rate $r_o$, grossed up by the corporation tax rate. The rationale for this assumption of infinite elasticity of supply at this level is that with interest rates below this level, the effective after tax cost of debt will be attractive enough for unlevered companies and they will increase the supply of debt. Above the rate $r_o / (1 - T_c)$, where $T_c$ is the corporation tax rate, all corporations will find equity to be the cheaper source of finance because the after tax cost of debt will be greater than $r_o$, the exogenously determined rate. Therefore the supply curve is infinitely elastic at a rate equal to $r_o / (1 - T_c)$. This argument is not valid if one considers costs associated with the supply of debt, and this criticism is elaborated upon when discussing Modigliani (1982) in the next section.

The demand curve for bonds is labelled $r_d(B)$. The intercept $r_o$, which is the rate of interest on fully tax-exempt bonds (such as tax exempt municipal bonds in the U.S.A.), is the minimum rate at which there can be effective demand for the corporate bonds. The horizontal portion of this curve $r_d(B)$ represents the demand for fully taxable corporate bonds (that is fully taxable at the personal level by the investors) by fully tax exempt institutions. This is so because only these investors would be indifferent between holding taxable corporate bonds or tax exempt municipal bonds - there is no taxation at personal level for tax-exempt institutions so they would be indifferent between these two types of bonds (note the assumption that the bonds are riskless). It is, however, disadvantageous for investors who have to pay personal tax to hold taxable corporate bonds at this rate of interest; they would prefer municipal bonds. (One important restriction that could be considered here is the results if the supply of municipal bonds were limited. Miller does not consider this.)

To entice personally taxable investors into the market for corporate bonds, the rate of interest on taxable corporate bonds should be increased so that the after personal tax return of the bondholder is at least $r_o$. In order to do this, the rate of interest must be grossed up by the fraction $1 / 1\text{-}T^{*}_{PB}$, where $T^{*}_{PB}$ is the personal income tax rate on income from bonds of the marginal investor. Since the personal income tax rate is progressive, the interest rate demanded on taxable bonds has to keep rising as investors move into higher and higher tax brackets. Therefore the curve slopes upwards.

Equilibrium in the corporate debt market is determined when the two curves intersect - which is when the rate of interest is $r_o / (1\text{-}T_c)$, the interest rate on tax-exempt municipal bonds grossed up by the corporate tax rate. Therefore D* represents the optimal amount of debt for the corporate sector as a whole.

However, there is no optimal debt-equity ratio for a SINGLE firm. Companies that follow a low-leverage strategy will find a market among investors taxed at high rates and vice versa. This is because Miller's analysis includes heterogeneous investors in terms of the personal taxes that they bear. Thus investors with high taxes will hold tax exempt bonds and those with low taxes will hold corporate bonds (because these offer higher rates of return). Once such a clientele is established, an individual firm cannot increase its value by increasing leverage and the value of leverage to an individual firm is zero. What will happen when a firm changes its leverage is that its clientele will change, so that it is only the tax characteristics of its clientele that will be different, leaving its market value unchanged. Therefore the value of the firm, in equilibrium, is independent of its leverage, despite the deductibility of interest payments in computing corporate income tax and although there is an overall optimal level of debt for the corporate sector as a whole.

In the above equilibrium, the market interest rate on corporate debt is grossed up by the taxes of the marginal bondholder, whose personal tax rate in equilibrium is equal to the corporation tax rate. Therefore infra-marginal investors - that is, tax exempt institutions and individuals in low tax brackets, benefit from what might be called the bondholders' surplus, that is, they receive a higher post tax income than would be necessary to induce them to invest. On the other hand, if these low tax investors wanted to borrow instead, then they would have to pay a market rate of interest which is higher than $r_o$ because the equilibrium market rate under this model , is $r_o$ grossed up by the corporation tax rate.

Miller's model thus enables one to derive the optimal debt equity ratio for the corporate sector as a whole, while using the influence of the personal tax rates and while correctly ignoring the effect of bankruptcy costs on equilibrium. This represents a significant contribution although, as has been noted in various places above, Miller's reasoning has not always been sound. A better model (because it fully takes into account risk and diversification, and because it does not attempt to contrive zero tax rate on equity income) for analysing the role of personal taxes in determining not only the tax advantages of debt but also in determining the valuation of projects and the cost of capital, is Modigliani (1982) described in section C below.

Prior to examining the Modigliani model, we complete this section by examining how some of Miller's assumptions can be relaxed in order to obtain more realistic models. The following factors can be considered:

(i) positive tax rate on income from shares

(ii) riskiness in debt finance

(iii) bankruptcy costs

(iv) a borrowing rate for individuals which is higher than the firm's borrowing rate

(v) non-debt tax shelters

(vi) portfolio considerations: investors who may want to invest in one type of firm because of clientele effect may nevertheless wish to hold securities of other types of firms from the diversification point of view.

In a more realistic setting, when all the above are taken into account, the firm benefits from leverage. In particular, regarding (iv) above, the firms normally borrow at a much lower rate than individuals and therefore corporate debt is valuable to individuals because it enables leverage to take place at a lower cost (that is, at a lower rate of interest). Factor (v) above was considered by De Angelo and Masulis (1980 - see chapter 3). Factor (vi) is extensively considered by Modigliani (1982 - see section C below) and Auerbach (see section E below). The effects of the remaining factors [(i) to (iii)] are therefore considered below.

**(i) Positive Tax Rate on Income from Shares**

If there is tax on income from shares, then there are two effects, demand effect and supply effect. the demand effect is that investors demand less of equity. Therefore they demand more debt and increase the equilibrium interest rate. The supply effect is that the horizontal line rises to a rate of interest represented by $r_o$ / $(1-T_c)$ $(1-T_{ps})$, where $T_{ps}$ is the personal tax rate on equity income. The equilibrium interest rate is grossed up not only by the corporation tax rate but also by the personal tax rate on equity income. The equilibrium rate will be higher, because we are increasing the burden of taxes on equity income, equity financing will be less attractive, and the companies will therefore be willing to pay a comparatively higher rate of interest on corporate debt. Therefore equilibrium market interest rate ($T^{a}_{PB}$) is higher than it would be if personal taxes on equity were zero. If the "equilibrium" interest rate is so high that the implicit personal tax rate (implicit in ($T^{a}_{PB}$) may be greater than the top rate of personal tax, in which case there is no market equilibrium. In such a scenario, the after tax interest rate on corporate debt will be so attractive compared to the after tax rate on equity income that no one would wish to hold common shares because of their relatively low expected financial returns.

**(ii) Riskiness in Debt Finance**

Risk can be incorporated into the analysis by reinterpreting all the before-tax interest rates as risk-adjusted or certainty-equivalent rates. With risky debt, however, the present value of interest tax shield is lowered because of uncertainty as to whether there is sufficient taxable income to fully benefit from the tax shield. Hence to entice firms to issue risky debt, the risk-adjusted supply price would have to be less than $r_o$ / $(1-T_c)$.

**(iii) Bankruptcy costs**

As in the above case, in order to induce firms to issue debt in the presence of bankruptcy costs, the risk adjusted supply rate would have to be lower than $r_o$ / $(1-T_c)$. Therefore part of the costs are shifted to the bond buyers in the form of lower risk-adjusted rates of interest in equilibrium.

The above adjustments modify equilibrium interest rate reached but most of Miller's results essentially remain unchanged.

The Miller Equilibrium model, with its implications that while there is an optimal level of debt for the corporate sector as a whole, there is no optimal debt-equity ratio for a single firm, has been commented upon by many financial economists concerned with the impact of taxation on corporate financial structure. This model also represents a divergence in the view of Miller and Modigliani - with Modigliani taking a different view, which is described in the next section.

From the point of view of our thesis, we would like to include explicitly risk in our recommended model. We do so in the model we derive in chapters 4 to 7. Therefore we do not rely particularly on Miller (1977), but instead on Modigliani (1982) which we believe is better at presenting a more relevant and comprehensive model, as discussed in the next section.

## SECTION C

### Debt, Dividend Policy, Taxes, Inflation and Market Valuation

Modigliani (1982) noted that the following points were stated in the literature to explain why the companies did not adopt 100% debt finance, as was implied by the MM (1963) hypothesis with corporate taxes:

**(a) On supply side**

(i) Bankruptcy costs reduce the expected value of cash flows

(ii) Agency costs to protect creditors increases with extra debt. (Jensen & Meckling [1976] Chen & Kim [1979]).

(iii) For firms with true growth opportunities, there is the cost of foregone valuable opportunities or moral hazard (Myers [1977]).

(iv) Increased probability that income will be insufficient to fully utilise the tax shelter (Brennan and Schwartz [1978]; De Angelo and Masulis [1980]; Cordes and Sheffrin [1981]).

**(b) On demand side**

Miller Equilibrium (1977). As described in section B above, Miller argues that on the basis that the supply of debt is infinitely elastic and assuming that the top marginal rate of personal tax exceeds the corporation tax rate, then in equilibrium, the market value of leverage to an individual firm must be zero. Modigliani disagrees with Miller's conclusion.

Modigliani's comprehensive model is described in this section. Modigliani begins by noting the following weaknesses in Miller's analysis:

(i) Miller dismissed factors limiting the supply of debt. Modigliani states that the supply of debt has associated costs. Therefore at equilibrium, the marginal benefit of debt (tax advantages) is offset by the marginal cost of debt (bankruptcy etc.); and therefore in equilibrium, debt must be valuable (that is, tax advantages must be positive) to an individual firm. Therefore leverage is a serious issue of financial policy.

(ii) Miller assumed that the rate of return on tax-exempt securities (municipal bonds) was exogenously determined.

(iii) Miller's model implied a counterfactual equality between the ratio of tax exempt to fully taxed interest and its relationship to the corporate tax rate. This relationship was not supported by the interest and tax rates actually applicable in the U.S.A. over any period of time(Gordon and Malkiel) [1981]; Skelton [1980]).

(iv) Basically, Miller had failed to take account of the role of diversification, in a world of uncertainty and risk aversion, in determining the market equilibrium.

Therefore Modigliani develops a model in the mean-variance framework, using models developed earlier by Brennan (1970), Elton and Gruber (1978) and Auerbach and King (1981). This Modigliani model will be used extensively in this thesis and therefore it will be critically analysed in Chapter 4. Instead, we concentrate in this chapter on the conclusions reached by Modigliani in his article. Therefore, in order to limit repetition, the description of the model here is brief and only introduces the various concepts used by Modigliani.

Modigliani assumes:
(i) That although the cash flows from projects are stochastic, the cash flows associated with debt are both permanent and riskless
(ii) Dividend flows too are permanent and riskless
(iii) The value of shares increases directly with retained earnings, which are stochastic because the returns from projects are assumed to be stochastic.
(iv) Investors expect earnings in real, as opposed to nominal, terms
(v) Taxes are levied on nominal income.

An investor will expect income in the form of dividend income and capital gains on his equity investments. His real expected cash flows, net of corporate and personal taxes, under the above assumptions, are given by the following equation for the expected return on equity :

$$y_i^m = n_i^m \{ [ \mu_i - \tau_c (\mu_i - RD_i) - RD_i + pD_i - \Delta_i ] \theta_g^m + \Delta_i \theta_p^m - \tau_g^m pS_i \}$$
$$= n_i^m [ (\mu_i^* - r_c D_i) \theta_g^m + \Delta_i (\theta_p^m - \theta_g^m) - \tau_g^m pS_i ] \qquad \text{[Eqn II.1]}$$

Here $\tau_c$ = corporate tax rate

$\tau_g$ = capital gain rate

$\tau_p$ = personal income tax rate

$\theta_x$ = $1 - \tau_x$ where x = c (Corporate), g (Gains) or p (Personal) tax. $\theta_x$s therefore are proportions of cash flow after deduction of the respective tax.

$\mu$ = cash flow (EBIT)

$\mu^*$ = $\theta_c\mu$, that is earnings of a company (before interest deduction), net of corporation tax at the statutory rate $\tau_c$.

[M] = variance covariance matrix of tax adjusted cash flows

S = market value of equity

D = net corporate debt (market value)

V = S + D = market value of the firm

$\Delta$ = dividend payment

R = nominal rate of interest

p = rate of inflation

r = R-p = real rate of interest

$r_p$ = $\theta_p$ (R-p) = real interest rate after personal taxes

$r_c$ = $\theta_c$ (R-p) = real interest rate after corporate taxes

$n_i^m$ = The proportion of equity of company i held by the investor m.

In the first equality of equation II.1, Modigliani obtains retained earnings by deducting the corporation tax ($\tau_c$), interest payment ($RD_i$), and dividends ($\Delta_i$) from earnings before tax and interest ($\mu_i$). The real value of debt decreases with inflation, and this gain to the equity holders is reflected by the term $pD_i$, which forms part of the retained earnings. This increase in retained earnings is subject to capital gains tax - therefore the net of tax ($\theta_g^m$) cash flow is considered. Modigliani assumes, like most other researchers, that capital gains are taxed as they accrue. In chapter 6, we show how the more realistic assumption of taxation of gains when realised can be incorporated in the valuation model.

Similarly, dividends net of personal taxes are denoted by $\Delta_i \theta_p^m$. As the earnings are stated in real terms, the equity investor gains because of inflation since the debt obligation is fixed in nominal terms. The term $\tau_g^m pS_i$ measures the personal capital gains tax on the inflation portion of the rise in the nominal value of shares, since investors who own shares in the U.S.A. are subject to gains tax on the nominal gains, with no allowance for inflation. This latter term would need to be altered for the U.K. (since the inflation indexation allowance introduced in 1982) and this will be done in the subsequent chapters.

The second equality of equation II.1 rearranges and simplifies the terms.

In addition to investments in risky equity, whose return net of taxes is given by the above expression, the investor m will hold the remaining portion of his investible wealth $w^m$, in risk free assets. Risk free assets yield interest income which is subject to personal income tax. The total expected portfolio return to the mth individual, with wealth $w^m$, is the sum of his net interest income on risk free assets and on the risky assets, and is given by the following expression:

$$y^m = (w^m - \Sigma_i n_i^m S_i) r_p^m + \Sigma_i n_i^m [ (\mu_i^* - r_c D_i) \theta_g^m + \Delta_i (\theta_p^m - \theta_g^m) - \tau_g^m pS_i ] \qquad \text{[Eqn II.2]}$$

Equation II.2 has two terms, the first one measures the real net of personal tax interest income from wealth in risk free securities, and the second term is net income from risky securities.

Modigliani proceeds by defining investors' utility in the mean variance framework, that is the utility increases with the mean expected return on the investors portfolio, as described above, and decreases with the variance of his portfolio return because the investors are risk averse. Modigliani then derives the market equilibrium conditions and simplifies them in order to derive equations for the valuation of securities.

We derive the valuation equation in detail in chapter 4. Here we simply note that the resulting valuation equation and then go on to analysing its policy implications. The value of equity ( $S_i$ ) derived from above equation is:

$$S_i = \frac{(\mu_i^* - r_c D_i)\,\theta_g}{r_p + \beta_i \pi + p\tau_g} + LD_i \qquad - \frac{\Delta_i\ \theta_g - \theta_p}{r_p + p\tau_g}$$

where the new variables are defined as:

$L \quad = \quad$ leverage $= \quad 1 - \dfrac{r_c\ \theta_g}{r_p + p\tau_g}$

$\pi \quad = \quad$ Market Risk Premium

$\beta_i \quad = \quad$ See below

Modigliani defines asset beta factors as:

$$\beta_i = \mathrm{cov}\left( \frac{\mu_i^*}{V_i^*}, \frac{\mu^*}{V^*} \right)$$

**The above definition of beta factors given by Modigliani is incorrect, even within the context of the specific definitions used in his model. The covariance term should be divided by the variance of the returns on the market as a whole, as proved in appendix A1. This omission is examined in appendix A1. This omission has not been picked up during the reviews of this Journal of Finance article or in the Erratum in a subsequent Journal of Finance (June 1993, page 1041). In the following chapters, the correct definition of the beta factors is used.**

The above model, which includes personal taxes, is then used by Modigliani to state the following implications for financial policy:

(a) **In the absence of inflation**

(i) Leverage: Taking the benefit of diversification into account, the value of leverage, *L*, is independent of the supply of bonds and depends instead on the weighted average tax factors $\theta_p$, $\theta_g$ and $\theta_c$, which represent net of personal, capital gains and corporate tax cash flows respectively. Using estimates from U.S.A. tax codes in 1982, Modigliani calculates the value of leverage to be around 1 / 3 of the value of permanent debt. This is lower than the value of leverage when only the corporate taxes are taken into account. The tax advantage of leverage is reduced because personal taxes on debt interest income are higher than personal taxes on equity income in the U.S.A. .

There are two main differences in the value of leverage derived by Modigliani from that derived by Miller. Modigliani's above analysis states that leverage is valuable at the margin for an individual firm. Secondly, the tax rates implicit in the above valuation equation are weighted averages of the marginal tax rates of investors. In contrast, the Miller model is driven by the tax rates of the marginal investor. Modigliani's analysis, rather than Miller's analysis, is more likely to be correct because:

(a) Leverage must provide tax related value to an individual firm which offsets the non-tax disadvantages of leverage (bankruptcy costs etc), otherwise firms will not be levered,

(b) the marginal tax rates implied by Miller analysis are not borne out in reality.

(ii) Dividend Policy: $\theta_g$ can be taken to be larger than $\theta_p$ in the U.S.A.. Therefore, the payment of dividends reduces the market value of the firm. However, in the U.K., $\theta_g$ cannot be assumed to be larger than $\theta_p$ because of the U.K. tax code features, namely the imputation system and the franked investment income provisions, described previously. hence payment of dividends in the U.K. should add to the value of the company and this is shown in chapters 8 and 9 when we use tax rates applicable in the U.K. .

(b) **Impact of Inflation**

(i) On interest rates: Modigliani agrees with the conclusions reached by Feldstein (1976) and Summers (1981a) that the nominal interest rate should not rise only by p, the inflation rate, as stated by Fisher's Law, but by $p / \theta_p$, which is the relevant rate in the presence of personal taxes. This is so because in the presence of personal taxes, an investor who receives ( before tax ) compensation for inflation at the rate p, will be worse off in after tax terms, as compared to the no inflation case. He will be worse off because he has to pay personal taxes on the nominal interest income that he receives. Therefore, to fully compensate a taxed investor, the nominal interest rate should rise by $p / \theta_p$ , if the inflation rate is p.

(ii) On leverage: Inflation has a large positive effect on the gain from leverage mainly because debt is fixed in nominal terms; therefore the real burden of debt decreases with inflation.

(iii) On the market value of a stock: The impact of inflation on S, the market value of stock, through $\mu^*$ will be negative and will conflict with the positive impact of inflation through leverage and through the real interest rate $r_p$. The negative impact of the former (through taxation of paper profits) is likely to be the dominant effect.

(c) **Implication of uncertain tax savings, dividends and earnings**

The model described above assumes that the cash flows associated with interest payments, taxes and with dividends are certain and consequently, these cash flows are discounted at the after tax risk free rate. Modigliani considers the case where the interest and dividend payments and their tax consequences are stochastic. In order to do so, he treats the following as given:

$d_i = D_i / \mu_i^* =$ the debt-equity ratio

$\delta_i = \Delta_i / \mu_i^* =$ the pay out ratio

The definition of debt equity ratio is unusual, since Modigliani is defining debt balance as a proportion of the income generated. This unusual definition is however necessary in order to keep the model tractable. The valuation expression now becomes

$$V_i = \mu_i^* [\theta_g + d_i (r_p + p\tau_g - r_c\theta_g) - \delta_i (\theta_g - \theta_p)] \;/\; (r_p + p\tau_g + \beta_i'\pi)$$

In the above valuation equation, all of the cash flows in the numerator are divided by the risk ( and tax ) adjusted discount rate. This follows from the assumption that all types of cash flows to the investor are stochastic. In this model, the tax parameters ($\theta_g$, $\theta_p$) are averages of the individual investors' tax rates weighted by their share in the market ( as in the previous model ), $\beta_i'$ is a similarly weighted average of the $\beta_i$ of the individual portfolios, and $\pi$ is the spread between the overall after tax market return on equity and the average after tax interest rate. The gain from leverage ($L^*$) now decreases, relative to the previous model (see ai above). The reason is that debt earnings are stochastic, in contrast to their being certain cash flows in the previous model. Therefore the tax benefits are discounted at the risk adjusted discount rate, instead of the risk free discount rate as in the previous model. Therefore, the discounted value of leverage is lower in this model.

$$L^* = \frac{dV_i}{dD_i} = \frac{L\,(r_p + p\tau_g)}{ß_i\pi + r_p + p\tau_g}$$

The gain from leverage in the uncertainty case is about one third the gain in the certainty case, using U.S.A. tax rates and assuming inflation is zero. The effect of dividend policy is also the same - in the uncertainty case, the disadvantage of dividends is again about one third of that in the certainty case (using the tax code applicable in the U.S.A.). The reason again is because in the second model, we discount the cash flows associated with dividends at a risk adjusted discount rate, instead of the risk free discount rate that was appropriate in the first model, where we assumed the dividend cash flows to be non-stochastic.

If inflation is present and increasing, then the gain from leverage in the uncertainty case increases at a slower rate in comparison with the certainty case. Modigliani's estimate of the benefit from leverage in the uncertainty case (of approximately 0.10) is very low. However, his estimate is not inconsistent with the estimate provided by Masulis (1981).

### (d) Individual Portfolio composition

The mean variance approach implies that the optimum portfolio of every investor will include the same fraction of every firm's equity in the absence of taxes, and with uniform risk aversion. This is the accepted result of the CAPM model without taxes.

This is no longer true when investors are taxed at different rates of personal taxes. This proposition has also been stated by Black (1974, 1973), Elton and Gruber (1978), Gordon and Malkiel (1981) and Auerbach and King (1983). By rearranging the earlier valuation equations, one gets the proportion of a firm's value held by an investor m as:

$$n^m = \frac{\Lambda}{\gamma^m} \left\{ T_1^m\, 1 + \frac{1}{\Lambda} [M]^{-1} \left[ (\mu^* - r_c D)\, T_2^m - T_3^m\, \Delta \right] \right\}$$

where

$$T_1^m = (r_p^m + p\tau_g^m / r_p + p\tau_g) \;/\; (\theta_g^m)^2$$

$$T_2^m = (\theta_g^m - \{ (r_p^m + p\tau_g^m) / (r_p + p\tau_g)\, \theta_g \} ) \;/\; (\theta_g^m)^2$$

$$T_3^m = [ (\theta_g^m - \theta_p^m) - (r_p^m + p\tau_g^m) / (r_p + p\tau_g)\, (\theta_g - \theta_p) ] \;/\; (\theta_g^m)^2$$

The above equations show that the investor's shareholding is inversely related to his risk aversion, and but is influenced by the ratio of his personal tax factors ( referenced by superscript m ) to the average tax factors of the market as a whole.

The implications of taxation for portfolio composition are as follows:

(i) When all investors are taxed equally or if there is no personal tax, then every investor holds a proportion of the market portfolio, inversely related to the investor's risk aversion.

(ii) If individuals are taxed differently, but (a) gains and income are taxed at the same rate for the same individual (ie. $\theta_g^m = \theta_p^m$) and (b) there is no inflation (or only real interest is taxed), then each portfolio consists of a proportion of the market but that proportion depends on the investor's tax bracket, $\theta^m$, relative to the average tax rate (implicit in $\Lambda$). The proportion held by investor m, that is $n^m$, increases with investors' tax brackets. The reason for this is that a higher tax rate means an equal decline in the return from either equity or debt, but it also involves a decline, ceteris paribus, in the variance of the after tax income from equity, which makes equity relatively more attractive. The variance of after tax equity income declines because as greater proportion of income is leaked in the form of personal tax, the remaining cash flow is proportionately reduced and therefore it has a lower variance.

(iii) If both individuals and sources of income are taxed differently, investors will hold different proportions of the equity of any given firm depending on both their tax rates and their risk aversion. However, if two investors, say m and m', have identical tax rates, then the proportion of their portfolio represented by shares will depend upon their risk aversion - ie. $n_i^m / n_i^{m'} = \gamma^{m'} / \gamma^m$. Therefore, in principle, every investor could secure an optimal portfolio by combining positive or negative debt with a share of a single fund which held securities in the relative quantities appropriate to his tax bracket.

Besides the effect of differential tax rates between individuals and between income sources, Modigliani also describes the effect of the firm's characteristics and of inflation on portfolio composition and on the extent of variation in optimal portfolios of different investors.

The main conclusions of the Modigliani model, which includes personal taxes, are summarised below:

(a) Leverage is valuable but the value of leverage is small if the market regards the tax savings associated with debt interest as stochastic rather than as a sure perpetuity.

(b) Inflation should increase the value of leverage.

(c) The payment of dividends will reduce the value of the firm (using U.S.A. tax code) but again the effect could be small if the market regards the associated taxes as uncertain rather than as certain perpetuities.

(d) Differences in tax rates between investors and between types of income will result in clientele effects. Moving from the lowest brackets to the highest, one should find a steady rise in the share of the portfolio invested in stocks with low dividend yield (and consequently high growth) and with relatively low equity betas (due mainly to low leverage). The reduction in the dividend yield of stocks and in their leverage referred to above results from the increase in tax disadvantage of dividends, and decrease in the relative tax advantage of debt, as the personal tax rates increase. However, the differences between the portfolios of the highest and the lowest tax brackets is much less than that implied by Miller's comer type solution. Modigliani's results (but not Miller's) are supported by the observation that the portfolios of investors do not consist either of only equity or of only debt; hence Modigliani's model is more realistic and better than the Miller model.

The main contribution made by Modigliani is to include risk reduction, diversification and inflation in a model to obtain the after personal tax returns in a very rich equilibrium model. We accept this model as "the" after tax model and develop it further in this thesis. Thus, in chapter 4, we derive the model equation by equation, pointing out couple of areas where we feel the derivation could be somewhat improved. In chapter 5 and subsequent chapters, we develop the model further until we obtain our model in equation 7.22 in chapter 7.

Modigliani has relied essentially on the pioneering work done by Brennan (1970) in order to derive his model. Brennan's contribution is examined next.

## SECTION D

### Model of Brennan (1970)

The pioneering work for incorporating personal taxes to estimate the cost of equity capital using CAPM was done by Brennan (1970). This fact was surprisingly not noted by Hamada and Scholes in their review article (section A above). The foundations of the Modigliani (1982) model were laid by Brennan. Therefore it is appropriate to briefly examine this pioneering contribution to the issue of personal taxes and cost of capital.

Brennan (1970) criticised Farrar and Selwyn (1967) for relying on measuring net income received after all taxes to derive market equilibrium. He stated that such an approach ignored the opportunities available to investors to trade in financial assets and sell those securities which resulted in lower post tax income. Therefore proper consideration of personal taxes required proper valuation principles and models. Brennan therefore used the CAPM mean-variance approach.

Brennan assumed that the investors' utility depended on the mean and variance of the after tax returns on their portfolios. Brennan assumed that dividends were certain and known. Therefore the only uncertainty related to the end of the period value of securities, which would affect only capital gains, and hence only capital gains tax entered the calculation of the after tax variance, as is shown below:

$$S_i^2 = \sum_{j=1}^{n} \sum_{k=1}^{n} S_{jk} X_{ji} X_{ki} (1 - T_{gi})^2$$

where: $S_i^2$ = portfolio variance

$S_{jk}$ = covariance between securities j and k

$X_{ji}$, $X_{ki}$ = proportion of security owned by investor i

$T_{gi}$ = capital gains tax faced by investor i.

Brennan derives the following after tax model incorporating :

(1) known and therefore non stochastic dividends,

(2) the after tax variance of returns, which is reduced by capital gains taxes applicable

(3) the investors' ability to borrow and lend at the risk free rate

(4) and tax deductibility of personal borrowing.

This model can be written as:

$$R_j - r = H \operatorname{cov}(\tilde{R}_j, \tilde{R}_m) + T(\delta_j - r)$$

where: $R_j$ = expected return on an asset

$\tilde{R}_j$ = rate of return on an asset given by $\dfrac{\tilde{\pi}_j + d_j - P_j}{P_j}$ where $\tilde{\pi}_j$ is the unknown share price at the end of the period and $d_j$ and $P_j$ are the known dividend and current price respectively

$r$ = risk free rate of interest

Therefore $R_j - r$ = excess return

$$H = \frac{\left[\sum_{i=1}^{m} \dfrac{w_1}{(1 - t_{gi})^2}\right]^{-1}}{1 - T_g} \qquad w_1 = \frac{1}{2} \frac{U_1'}{U_2''}$$

$\operatorname{Cov}(\tilde{R}_j, \tilde{R}_m)$ = covariance between pre personal tax return on this security and market portfolio

$T$ = $(T_d - T_g) / (1 - T_g)$ where $T_d$ and $T_g$ are weighted averages of the personal tax on dividend income and capital gains tax

$\delta_j$ = prospective dividend yield ($d_j / P_j$).

Brennan's model is essentially a CAPM model which incorporates the average personal tax characteristics (in variables H and T). The expected excess return on a risky project j (that is, $R_j - r$) is equal to the risk of the project (measured by the covariance of the return on the project with the return on the market portfolio) multiplied by the factor reflecting the risk aversion of the market as well as the tax characteristics of the participants. The required excess return includes the term T ($\delta_j$ - r). This measures the adverse impact due to taxes on dividend income being higher than the taxes on capital gains. Therefore, increasing dividend yield, through the impact of dividend taxes which are higher than capital gains taxes, will increase the risk premium demanded by investors.

Brennan's contribution to the subject includes the incorporation of the weighted averages of the tax rates of the investors in the valuation model. He was also able to consider the application of different taxes on different types of income. His model was used by Modigliani(1982). The Brennan model is not derived in detail here in order to avoid repetition because the Modigliani model will be derived in detail in chapter 4. However, two errors noted in the derivation, one made by Brennan and the other made by Modigliani, are noted below.

**The variable H should be multiplied by M, the market value of all the securities. This error is made by Brennan in his summation,** but this error is not repeated in Modigliani (1982), where the correct formulation is used. However, as stated in section C above, Modigliani incorrectly states the CAPM beta factors to be the covariance of the rate of return to the company with the market rate of return. He did not divide the covariance by the variance of the market rate of return. The measure of risk used by Brennan ( Cov (Rj, Rm) also does not include division by the variance term, and it is incorrect to do so. **These errors are proved in appendices A1 and A2.**

The contribution made by this review of literature thus also includes pointing out the above errors in published literature, including the errors made by Modigliani and by Brennan in defining risk factors.

## SECTION E

### Models of M A King and A J Auerbach

In section B above, the Miller model which considered the impact of personal taxes on financial policy under certainty was introduced. The direct relevance for the cost of capital calculation is limited because Miller was working under the assumption of certainty, or using certainty equivalents. Thus Miller failed to properly take account of diversification and its impact on equilibrium values. In section C above, we considered Modigliani's equilibrium model with personal taxes, uncertainty and other factors thereby deriving a very rich equity valuation model. However, his interest was primarily in considering the impact of financial policy rather than directly deriving the cost of capital measures. However, in the subsequent chapters, we wish to derive cost of capital measures after adapting his model. The existing literature also includes some articles on the derivation of cost of capital in the presence of personal taxes, notably those by A J Auerbach and M A King. Their contributions which will be examined below are in King (1974), where the focus is on the cost of capital issue, and in Auerbach (1983), which is a comprehensive survey article again focusing on the cost of capital question.

King (1974) showed that in a world of perfect certainty, the cost of capital in the presence of personal taxes would still be the rate of interest (i) on debt provided that there were no constraints on the firm's financial policy. We use the notation used in the original article so that we can make criticisms using notation which is consistent with the original article and therefore avoid confusion.

The cost of capital would be i regardless of the method of the financing of investment. However, because of tax differentials, investors could profit at the expense of the tax authorities - therefore there are legal constraints on firms to prevent tax avoidance. Taking these constraints into account, the cost of capital becomes a function of the optimal financial policy. On the assumption that the tax system is based upon economic depreciation allowances, that there is no inflation in the capital goods sector and that individuals and firms can borrow at the same fixed market rate of interest, the cost of capital in the presence of these constraints is as shown below. (As explained above, the notation used is same as in the original article)

| Source of funds | Cost of capital |
|---|---|
| Retentions | $\dfrac{(1 - m)}{(1 - z)\,(1 - t')} \,.\, i$ |
| Debt | i |
| New Issues | $\dfrac{(1 - m))}{\theta\,(1 - t')} \,.\, i$ |

where: i = the market rate of interest i

m = the marginal rate of income tax

z = effective capital gains tax

$t'$ = rate of tax which the company would pay if no profits were distributed

$\theta$ = the opportunity cost of retained earnings in terms of net dividends foregone, ie. the net amount which shareholders could receive if one unit of retained earnings were distributed.

King stressed that constraints were necessary both to prevent tax arbitrage and to require the firm to move away from the cheapest source of finance, which is debt capital. In the absence of these constraints, only one source of finance would be used, thereby resulting in corner solution with only debt capital being used. This result is consistent with Modigliani & Miller (1963) where they included uncertainty in the model as well.

The literature on taxation and the cost of capital has been surveyed by Auerbach (1983). The article is both interesting and fairly comprehensive and most of the points dealt therein are discussed below. They begin by calculating the cost of capital under simple scenario, assuming no taxes and uncertainty. They subsequently add on corporate and then personal taxes, and then inflation and finally uncertainty, and analyse how cost of capital is derived under different assumptions.

### (a) No taxes and Certainty

In a one-good two-period certainty model without taxation, the cost of capital to a company is the rate of return required by the investors (r) , and is equal to interest rate under the assumption of certainty. This cost of capital holds regardless of the shareholders' preferences because of the Fisher Separation Theorem.

In a multi-period context, the cost of capital is the interest rate applicable for each period. Therefore to obtain the present value of a cash flow arising in period T, we need to discount that cash flow by $\sum_{j=1}^{T} (1 + r_j)$, where $r_j$ is the rate of interest applicable in the period j.

The above results break down if borrowing or lending rates for individuals differ from the rate facing the corporation. Greater corporate access to capital markets, the legal distinction introduced by limited liability and the presence of taxation can cause differences in effective borrowing rates. The existence of more than one interest rate destroys the separation between firm policy and individual preferences and the unanimity of owners with respect to the investment decisions and introduces scope for the firm to influence its value and cost of capital through financial policy.

### (b) Corporate Taxes and Certainty

The cost of capital in the presence of corporate taxes is defined as the rate of return the firm must earn on investments before taxes and still be able to meet the investors required rate of after corporate tax return. This is given in this setting by $r / (1 - T_c)$ where $T_c$ is the rate of corporation tax.

### (c) Personal Taxes and Certainty

Most early work on taxation and the cost of capital ignored personal taxation. This is justifiable only if all forms of personal income arising out of companies are taxed at the same rate, because then r can be reinterpreted as the rate of time preference, gross of tax. However, in addition to inflation corrections, such an outcome would require a full integration of corporate and personal income taxes, as was pointed out by King (1977). Besides requiring such an integration, other features of the tax system that make the consideration of personal taxes relevant include:

(i) personal income taxes exhibit progressive marginal rates whereas most corporation tax systems have only one fixed rate

(ii) capital gains are generally taxed at a lower rate than dividends and on a realised basis rather than on an accruals basis

(iii) corporations can deduct interest payments as expenses but not dividends

(iv) in most countries, no price level adjustments are made to account for the fact that nominal interest payments and capital gains include inflation premia that do not represent real income to the recipients (an exception is the U.K. CGT indexation allowance introduced in 1982 ).

As has been said, opportunities for unlimited arbitrage at government expense among households and/or corporations in different tax brackets are implied by the above personal tax characteristics. Many of the results that are obtained largely depend upon how the constraints to avoid these problems are modelled.

Auerbach calculated the cost of capital under the assumption of certainty and personal taxes. On the assumption that the assets being considered are consols (ie. there is no depreciation allowance complication) and that capital gains are taxed on an accruals basis, the cost of capital for an all equity investment in a setting of certainty, is:

$$\bar{x} = \frac{r}{(1 - T_c)\,[\,1 - (p\,\theta + (1 - p)\,c)\,]}$$

where: $\bar{x}$ = cost of capital for the company

$r$ = the after tax rate of return required by equity investor

$p$ = dividend pay out ratio

$\theta$ = personal income tax rate

$c$ = capital gains tax rate.

and note $i$ = interest rate on risk free debt

Essentially, the second term in the denominator is a weighted average of the personal taxes applicable to income that is paid out as dividends and income that is retained ( which converts into capital gains ) respectively.

If the firm issued debt capital, then the weighted average cost of capital ("WACC") at the margin would be:

$$\bar{x}' \quad = \quad bi + (1 - b)\,\bar{x} \text{ (as above)}$$

where i is the return to debt holders before personal taxes.

Note that in these formulae, r is the total after tax return to equity holders. If all individuals faced the same personal tax then $i(1 - \theta) = r$. Firms would prefer debt finance as long as $(1 - \theta) > (1 - T_c)\,[1 - (p\,\theta + (1 - p)\,c)\,]$.

Analysis similar to above led Stiglitz (1973) to the following conclusions:

(i) firms should use debt finance at the margin.

(ii) firms should not pay any dividends (as c is less than $\theta$) because of extra taxes

(iii) investors should realise no capital gains before the terminal year, if gains are taxed when realised rather than when accrued.

All three suggestions are contradicted in reality. Therefore constraints or a consideration of uncertainty needs to be added to the model. Among the various constraints considered by Auerbach (1983), one that is relevant for cost of capital is the explanation he gives for the payment of dividends. This involves an explanation of the "tax capitalisation view" (the "new view") tested by Poterba and Summers (section F below) and it also touches upon the concept that the value of retentions depends upon what the firm does with the retained earnings. This latter point is very important to understand before one can advocate that the firms should not be paying any dividends.

### (d) The New View ( Tax Capitalisation )

Under this view, referred to above, a firm may pay dividends because of its inability to turn retained earnings into equivalent capital gains. Stiglitz assumed that an investor could convert retained earnings into equivalent capital gains by either selling the assets of the firm or the shares in the firm. If the owner can sell the assets, then he will realise the assets' replacement cost. However, sale of shares to another investor need not realise the same value. Therefore there is no arbitrage mechanism to equate the market value of shares to the replacement value of assets if the firm's owner cannot sell the underlying assets but is restricted to selling only the shares in the firm to another taxable investor. In terms of Tobin's q (the ratio of a firm's market value to the replacement cost of its assets), the long run value of q at the margin of new investment need not equal unity. One main reason for this is that the investors may capitalise the potential tax liabilities associated with the future payment of dividends. The company may avoid the payment of dividends and the associated taxes by retaining earnings now and thereby compensating the investors through capital gains instead. However the only way in which the company can get cash out of the corporate sector into the hands of the investors is through the payment of taxable dividends at some stage in the future. At that time, the associated dividends taxes will have to be paid. This may be anticipated by the market participants so that the future tax liability is capitalised by the investors. This would imply that the share price will not rise by the full extent of the retained earnings, but by a smaller amount, the difference representing the potential tax liability. Thus Tobin's q need not equal one.

In terms of the mechanisms available to the firm to make q equal to one, the normal mechanism for equating q to one is through the issue of new shares, that is, if q is greater than one, the firm could increase the wealth of its shareholders by issuing more shares. The firm could increase the shareholders' wealth by repurchasing shares from them if q were less than one.

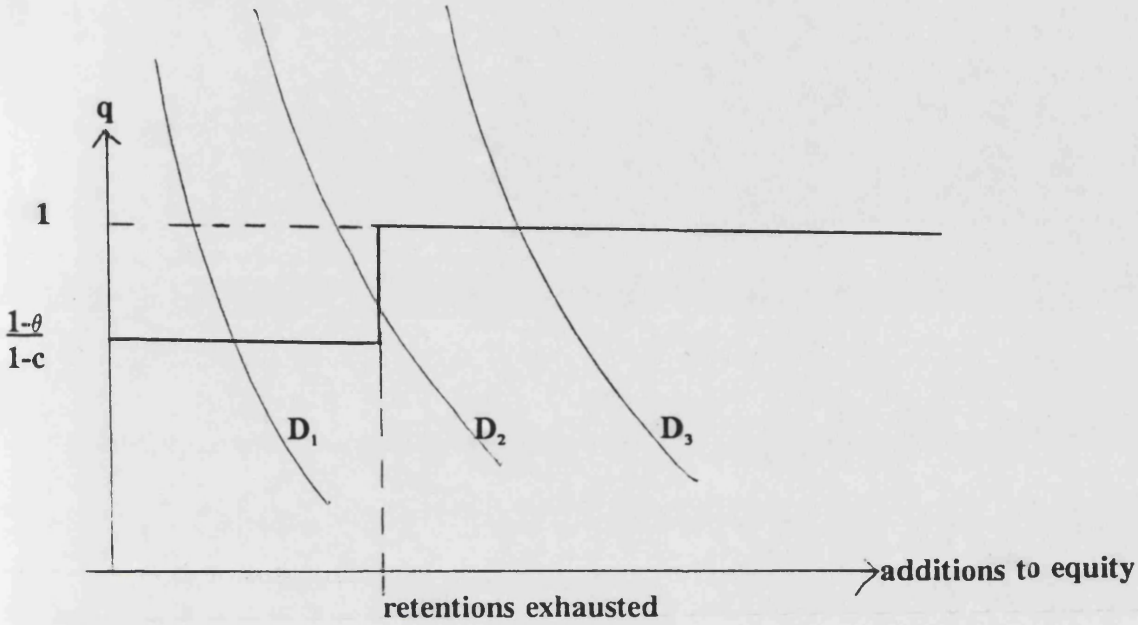
However, share repurchases are illegal in certain countries or this facility is restricted and may attract tax at income tax rates rather than capital gains taxes. Without this facility, the value of q can fall to as low as $(1 - \theta)/(1 - c)$, the ratio of the after-personal tax on dividends to after capital gains tax value of a dollar paid to the investor. At this value of q, firms are just indifferent between paying dividends and retaining, since the dividends tax on a pound of dividends equals the after tax loss in the value of a dollar retained $(1 - q)(1 - c)$.

Hence, there is a range of values for q - between 1 and $(1 - \theta)/(1 - c)$. At the border of this range the firm will :

(a) find it desirable to issue new shares (when $q > 1$), or

(b) find it desirable to pay out dividends (when q falls to $(1 - \theta)/(1 - c)$).

The equilibrium value of q will depend upon the demand for equity. This is illustrated in the diagram below. If demand is low ( curve $D_1$ in the diagram), the equilibrium q will be $(1 - \theta)/(1 - c)$, and the firm will meet this demand by retaining some earnings, paying out the rest as dividends. If demand is high ($D_3$), higher than that which could be met by 100% retention, then q will rise to 1 and firms will issue new equity. This is shown in the following diagram.




The cost of capital with this "new view" depends upon whether

(a) the demand for equity will remain in the same zone in the future as in the present, or

(b) the demand for equity will change from any one zone now to a different one in the future (that is from $D_1$ to $D_2$, or from $D_2$ to $D_3$).

For the case of (a), the cost of capital is given by

$$\frac{r}{(1 - T_c)(1 - c)} .$$

Therefore, if condition (a) holds, then even if the firm is paying dividends (ie. demand for equity is $D_1$ and it will remain in this zone in future too), the cost of capital is independent of the tax on dividends. This view implies that the personal taxes on dividends therefore are not relevant in determining the cost of capital. One implication of this view, examined by Poterba and Summers, is that a change in the dividend tax rate should have no effect on incentive to invest - its effect will be like a wealth tax. Their tests found that British data did not support this view. Therefore we do not accept this view and in our valuation model developed in chapters 4 to 7, we assume that dividend taxes do influence the cost of capital.

In reality, firms do not either pay dividends or issue new shares. They often contradict the predictions of this model (and of the substantial body of knowledge in the field of finance) by issuing new shares and maintaining/raising dividends in the <u>same</u> year. This implies that companies have a preference for paying dividends that this hypothesis of tax capitalisation cannot fully account for. The dividend puzzle therefore remains an unresolved issue in corporate finance.

Next, Aeurbach introduces positive rates of price inflation and assesses the cost of capital under alternative scenarios, including the assumption of inflation.

**(e) Personal Taxes, Inflation and Certainty**

The key factors affecting the cost of capital in the presence of inflation are the basing of depreciation allowances on historical cost; the taxation of nominal, rather than real, capital gains (in some countries); and the full taxation of nominal interest payments received and full deductibility of those made. Auerbach continues to assume that there are no depreciation complications by considering investment in consols. If prices rise at some constant rate $\pi$, the real costs of equity and debt capital are as given below:

$$\text{Real Cost of Equity} = \frac{r - \pi}{(1 - T_c)\,[\,1 - (p\,\theta + (1 - p)\,c\,]} + \frac{\pi\,c}{(1 - T_c)\,[\,1 - (p\,\theta + (1 - p)\,c\,]}$$

The second term measures the cost of paying capital gains tax on nominal gains, which is in accordance with the tax codes of many countries, including the U.S.A..

$$\text{Real Cost of Debt} = i - \frac{\pi}{1 - T_c}$$

$$= \frac{r - \pi}{1 - \theta} + \pi \left( \frac{1}{1 - \theta} - \frac{1}{1 - T_c} \right)$$

The cost of debt may increase or decrease, according to whether the loss from paying taxes on the inflation premium at personal level exceeds the gain from its deductibility at the corporate level.

One needs more assumptions for further implications. We can assume that $\theta < T_c$ and that interest rate rises with leverage, leading to an interior solution for the cost of capital. In such a case, increase in the rate of inflation for a given real return demanded by investors, would raise the cost of equity capital, lower that of debt, and upset the equality between the marginal cost of equity and the marginal cost of debt, thereby leading to an increase in the optimal debt equity ratio when more capital is raised. Under this approach, inflation increases the benefits of leverage (if $\theta < T_c$) but we assume that inflation does not affect the costs of increasing leverage.

### (f) Uncertainty and the Cost of Capital

So far, Auerbach has dealt with the cost of capital issue under the assumption of certainty. Auerbach notes that the cost of capital is given by the CAPM equation when uncertainty is present. He stresses that the cost of capital derived by CAPM depends upon only the risk characteristics of the investment and not on how it is financed. This is the same as is stated in the basic MM 1958 theorem. However, he recognises the various ways in which MM assumptions may break down and that in the presence of uncertainty, the firm's financial policy may influence its valuation and the cost of capital. The most relevant of these are when uncertainty and taxation are both included in the cost of capital model. This is described below.

### (g) Uncertainty and Taxes

Auerbach notes two factors through which the cost of capital may be affected when uncertainty and taxation jointly affect a firm's valuation.

The first is through financial policy: consideration of uncertainty by itself (basic CAPM) implies that financial policy does not matter. Still, paradoxically, financial policy may matter when uncertainty and taxes are considered jointly. This is because investors who prefer to hold only equity for tax purposes may wish to hold debt for diversification purposes - a portfolio of equity holdings may be too risky for their risk preferences. Auerbach and King (1983) showed that such investors would also tend to concentrate on less risky types of equity for a given amount of risk. Thus, a high tax paying investor may personally gain if a firm in which he holds equity chooses to borrow less, because now the firm's equity becomes less risky and he may therefore reduce his own debt holdings and purchase more shares of this now less risky company.

The second way in which uncertainty and taxes combine to affect valuation is because uncertainty reduces the expected value of interest tax shields. (De Angelo & Masulis - 1980).

Finally, Auerbach notes that there is still ambiguity about the empirical importance of the effects of different taxes and of inflation on the cost of capital and states that creative empirical approaches and richer models of behaviour may be necessary for future insights to be gained.

On the basis of data on firms' earnings and their previous investment and financial behaviour, Auerbach (1984) concluded that firms perceived a higher cost of capital when issuing new shares than when making retentions, and that the cost of capital varied significantly across firms having different estimated tax clienteles, in accordance with the theory. This clearly indicates the importance of personal taxation to cost of capital calculations. Therefore we are motivated to develop a model that would take account of the different tax rates and other aspects of the U.K. tax code applicable to investors.

Auerbach and King (1983) have provided a more comprehensive model where they consider uncertainty and taxation together. This analysis includes a CAPM adjusted for taxes, constraints and heterogeneous investors. It is essentially the approach Modigliani (1982) adopted and therefore it also provides the basis for our model developed in chapters 4 to 7.

## SECTION F

### Poterba and Summers (1985)

The article by Auerbach summarises the literature on the cost of capital literature in 1983 by providing a comprehensive analysis which included uncertainty, inflation and personal taxes. Although Auerbach discussed the interaction of uncertainty and personal taxes for the cost of capital, he did not proceed to explicitly derive the cost of capital formulae for cases where personal taxes affected the required return under uncertainty. Such a derivation has been "tentatively" suggested by Poterba and Summers (1985) who follow the Litzenberger and Ramaswamy (1979) style of CAPM equation, where the cost of capital is also affected by pay out ratios. Poterba and Summers (1985), in considering the impact of dividend taxes, also calculated measures for cost of capital incorporating personal taxes. However, their model did not consider clientele effects or the firm's use of debt finance. Their "main" model also did not explicitly consider uncertainty, but they suggested simple substitution for incorporating uncertainty into their cost of capital measures. This cost of capital measure, used to discount the pre-tax cash flows, for the "traditional view" model is as follows :

$$\text{Cost of Capital} = \frac{e\,(\alpha)}{[\,(1 - m)\,\alpha + (1 - z)\,(1 - \alpha)\,]\,(1 - T_c)}$$

| | | |
|---|---|---|
| where: $e\,(\alpha)$ | = | the risk adjusted return demanded by an investor given by $R_F + (R_M - R_F)\,\beta$. Presumably $\beta$ is also dependent on the pay out ratio $\alpha$ because the "traditional view" model is used. |
| $m$ | = | dividend income tax rate |
| $z$ | = | effective capital gains tax rate |
| $T_c$ | = | corporation tax rate |
| $R_F$ | = | after tax risk free rate |
| $R_M$ | = | after tax return to the market portfolio. |

The above notation is as given in the original article. What Poterba and Summers have not stated is how ß(beta) is determined in this after tax model. The following questions arise: is it the covariance of the after-all-taxes return with the market portfolio? How are these after-all-taxes returns calculated? Or is it beta determined by the covariance of before-personal-taxes returns sufficiently accurate in this after tax model? Hamada and Scholes (1985 - see section A above) have noted this difficulty. Therefore, the usefulness of this model is limited because uncertainty has not been incorporated comprehensively in the model.

In sections A to E above, we have examined in depth what we consider to be the more relevant of the literature on personal taxation and cost of capital. All of the above articles are relevant to the subject examined in this thesis. Some of these, in particular the Modigliani model, are relevant to the development of our thesis.

However, none of the above models can provide a satisfactory answer to the question: how should U.K. companies calculate or modify their discount rate to take account of U.K. corporation and personal taxes? Clearly, the models that are rooted in the classical system of taxation, such as tax system of the U.S.A., are clearly misleading and inappropriate for the U.K.. Moreover, these models also ignore some of the more relevant features of corporation tax code (features which we incorporate in our model in chapter 7). It is this gap in the existing literature that we are motivated to fill in this thesis.

The development of our model nevertheless is not completely divorced from the existing literature. Thus, in chapter 4, we develop, equation by equation, the valuation model presented by Modigliani (section C above). In chapter 5, we examine how the valuation model, developed by Modigliani under the "classical" system of taxation, would need to be modified under different tax structures suggested by King. In chapter 6, and appearing for the first time in finance literature, we propose a method for calculating some of the more difficult variables that are used in the valuation model. In chapter 7, we add details of the corporation tax system which are very important to the valuation equation. We present our valuation model as equation 7.22 in that chapter. In subsequent chapters, we apply our model and check how useful it is in different applications.

Next, however, we briefly examine the literature on financial policy aspects which are relevant to an understanding of the financial underpinning of our subject. We sum up the conclusions to the entire survey of literature undertaken in this chapter and in the next chapter in section E of the next chapter.

## CORPORATE FINANCIAL POLICY & COST OF CAPITAL

In the previous chapter, we surveyed some of the literature which focused specifically on the influence of personal taxes on the cost of capital. In this chapter we examine some of the literature which focuses on other important aspects of finance literature, namely the financial policy issues of leverage and dividend policy. We also examine the implications of these and related issues for determining cost of capital because we will be developing cost of capital measures in the subsequent chapters of this thesis. We also survey the literature on international CAPM model, because ignoring the international dimension can lead to an incomplete understanding of the opportunities, players, relevant taxes and cost of capital. We stress the importance of the international dimension in the concluding section of this chapter as well.

This chapter is divided into the following sections:

| Section | |
|---|---|
| A | Dividend Policy |
| B | Debt Policy |
| C | Implications for Capital Budgeting and the Cost of Capital |
| D | International Models |
| E | Conclusions to the Survey of Literature |

In sections A and B respectively we examine the two important financial policy issues, the dividend policy and the debt policy issues, that are unresolved in the literature, particularly with regard to the extent of their influence on the cost of capital. In section C we examine how these controversial issues are handled in practice in calculating the cost of capital. Section D examines the international CAPM models because consideration of CAPM without considering its international dimension can lead to a limited view of the variables that influence the cost of capital. This point is further emphasised in the concluding section (section E) of the chapter.

## SECTION A
## DIVIDEND POLICY

### A ALTERNATIVE THEORIES ON DIVIDEND POLICY & VALUATION

The first important question in relation to dividend policy is whether or not firms can alter their value by changing their dividend payment policy. This question is also important for the determination of the appropriate cost of capital. If value of firms is independent of the level of dividends, then the personal tax rates on dividends are also not relevant for the cost of capital. Hence dividend policy issues are also relevant directly from the point of view of our thesis as well.

Whether companies should pay dividends is a matter of great controversy in finance literature. The main views, analysed in six subsections include:

**(i) Dividends are irrelevant**

Modigliani and Miller showed in a 1961 article that, in a world without transaction costs and taxation, dividend policy is irrelevant. They stated that in order to study the impact of dividend policy, it was essential that the firm's investment programme remained the constant under different payout ratios. Similarly, the firms borrowing policy is held constant. Under these conditions, a firm can increase its dividend pay out ratio only by resorting to issuing new shares to finance its investment programme, which is exogenously determined. The extra cash received by the old shareholders will be balanced by the capital loss in the value of their shares. Therefore, the shareholders' wealth remains unchanged - provided that the new shares issued were fairly priced. Hence increasing dividend pay out does not increase the value of the firm.

Therefore firms should undertake all projects that yield a positive net present value, and any surplus cash available after undertaking these investments should be paid out as dividends. The dividend decision thus is a by-product of the investment decision. Shareholders who need cash can sell part of their holdings to generate cash - but their overall wealth will not diminish if the firm had reduced the dividend to finance positive NPV projects. Hence, dividend policy is irrelevant in perfect capital markets.

**(ii) Dividend Pay out is related positively to firm's value** (risk reduction reasons): Modigliani and Miller's view refuted analytically the traditional belief which was fairly widespread at the time - that firms with high pay out ratios have higher values. For example, M J Gordon (1959) argued that paying more dividends now reduced the risk that investors faced. Hence investors would prefer firms with higher pay out ratios. The investors perceived high payout firms to have lower risk because investors who received dividends could not lose out on the cash they received ("bird in hand"), where as if the firm retained large amounts of earnings, then it was subject to risk that some unforseen disaster could reduce the value of the firm in future.

Against this, Brennan (1971) stated that Gordon was considering changes in investment policy as well as changes in dividend policy. Copeland and Weston (1983) state the argument against this bird-in-hand version of dividend policy succinctly by emphasising the principles of project evaluation under uncertainty. They state that the risk of the firm is determined by the riskiness of the cash flows from its projects. Therefore an increase in dividend pay out will result in a drop in ex-dividend price of the shares. It will not reduce the riskiness of the firm because that remains determined by the riskiness of the projects.

Brealey and Myers (1984) have noted some market imperfections, inefficiencies and restrictions which may result in investors preferring stocks with high pay out ratios. For example, some investors (e.g. certain trusts) are required by law to invest in shares that have a proven track record of dividend payments. Secondly, transaction costs may prevent investors from realising capital gains frequently - hence they may prefer shares with high dividend pay out. Thirdly, dividends could be viewed as signals issued by the management for the investors - a dividend rise would be viewed as indicating that the firm's future cash flows are likely to be higher and hence have a positive impact on the firm's value.

However, excluding such exceptions or short term phenomena, this view that investors prefer cash dividends is not given much credibility in finance literature. It is dismissed by S Bhattacharya (1979) for the reasons given above. On the other hand, in practice, companies tend to maintain dividends and will raise capital by issuing new securities if they need to, rather than cut their dividend.

There is one possible argument in favour of the view that investors can rationally prefer cash dividends which has not yet been fully dealt with in the literature. The investors may perceive dividends as being less risky not because they have cash in hand, but because the mechanism that gives rise to dividends is less volatile than the mechanism which gives rise to capital gains. Dividends are risky in so far as there is riskiness attached to the cash flows generated by the firm's projects. Retained earnings (which give rise to the capital gains for an investor who intends to sell the securities) are also subject to the same riskiness. However, only retained earnings (in the form of capital gains) are subject to the future volatility in the capital markets where the changes in the demand for and the supply of capital assets shifts the equilibrium prices more or less continuously. In other words, capital asset prices can change even though there is no change in the distribution of cash flows expected to be generated by the projects. If the above argument were true, then investors could demand a greater return for bearing any extra expected systematic risk attached to retained earnings.

Finance literature has so far ignored this hypothesis that the factors determining the variability of project returns (and hence the differential variability of dividends and capital gains) may be <u>supplemented</u> by <u>more</u> factors which affect the variability of share prices, and hence capital gains only. Stated in another way, the variability of stock prices is too large to be justified by riskiness and variability of future dividends or project returns only. This point has been taken up by R J Shiller (1981) who argues that there is excess volatility in the capital markets. It has also been recognised by Lawrence Summers (1985):

> *".... Surely work on volatility raises, even if it does not resolve, major issues for our understanding of financial markets. To what extent do fluctuations in stock prices reflect changes in risk premia, safe rates of return, expected future cash flows, or other factors?"*

Therefore if retained earnings are subject to fluctuations quite unrelated to the valuation of the future cash flows to be generated by the companies' projects, then investors may quite correctly value dividends more highly than retained earnings.

Another way of looking at why dividends might be preferred to capital gains is to recognise that retained earnings do not directly lead to an increase in share price. An implicit assumption under CAPM is that one pound of earnings retained in the company will lead to an increase in the market value of the company's shares by one pound. However, the work on Tobin's q (the ratio of market value of assets to their replacement cost) recognises that for an investor who is unable to sell the underlying assets, the gain from retained earnings may be of a ratio of less than one (Auerbach, 1983 - see chapter 2). Under these circumstances, the investor will be better off with returns in the form of dividends than in the form of retained earnings, which generate a smaller capital gain.

Moreover, the value of retained earnings depends largely on what the company does with the retained earnings. The value of a company will not rise by an amount equivalent to the retained earnings if the company is unable to invest in risky projects that are expected to earn returns equal to the cost of equity capital. Would the mean beta factor for a firm, which retains earnings to invest in safe bank deposit accounts, decrease sufficiently to lower the return expected by shareholders (so that the market value of the firm increases by the amount of retained earnings)? Answers to these types of issues are needed before the hypothesis that dividends are more valuable than capital gains can be dismissed. If the answer is that equity beta value of firms that retain cash does not decrease sufficiently to reflect their lower risk, then retentions in the form of cash will be undervalued. Investors in such firms would prefer dividends ( on which they receive full value), instead of retentions, which are subject to relatively unreasonably high discount rate and hence are undervalued. Such firms should adopt high dividend payout policy.

Another approach to explaining preference for dividends is to concentrate on the risk aversion of the investors in the manner done by Shefrin and Statman (1984). They rely mainly on behaviourial aspects (self control theory, prospect theory, minimum regret theory) to explain why dividends may be preferred. Investors would restrict their consumption to what they receive as current income ( in the form of dividends), but would not like to dip into capital ( which they may need to do if dividends are low and they need to realise some capital gains for maintaining their consumption). Such investors would prefer high dividends.

Theories which rely on non-risk reasons for dividend preference (eg. signalling etc.) are examined in sub section (iv) below.

**(iii) Dividend Payout is related negatively to the firm's value:**

This view is based on the personal tax rates of investors, particularly those who are who are individuals and face high rates of personal income tax. Dividends are more heavily taxed than capital gains for individuals in many countries. Therefore increases in dividend result in a reduction in net return (that is return after deducting all taxes) relative to the pay outs obtained via capital gains. Thus there may be a tax disadvantage to paying dividends to shareholders and therefore increases in dividend pay out ratio should result in a reduction in the value of the company. This view assumes that the personal tax rate on dividends is greater than the personal tax on capital gains, and that a company with surplus cash cannot use mechanisms such as the repurchase of shares, in order to pass its surplus cash to the shareholders in the form of capital gains. Share repurchases are an alternative mechanism to pass surplus corporate cash to the shareholders, but one which will result in capital gains rather than the alternative of paying dividends, which attract income tax.

These assumptions, that is that dividends bear higher taxes and that share repurchases are banned, may not be true for all countries.

In the U.S.A., only some investors would prefer dividends to capital gains. For an individual investor, the value of the company decreases with dividend payment because they are taxed more heavily than capital gains (created by retained earnings or share repurchases) are taxed at lower rates. (J Rutterford, PhD thesis 1986, page 448). Therefore, for these investors, dividends are negatively related to the value of the firm.

Only if personal taxes on dividend income and capital gains are zero would an investor be indifferent between dividends and capital gains. This would be the case for pension funds who form a large and increasing proportion of investors in equities. Moreover, since 85% of dividends received by the corporations in the U.S.A. are exempt from tax, corporate shareholders would in fact prefer dividends to capital gains.

The tax provisions differ in the U.K.. Under the imputation system of taxation in the U.K., the tax credit that is attached to dividends makes them relatively more attractive. On the other hand, indexation allowance for inflation introduced by the Finance Act 1982 for capital gains tax reduces the tax on capital gains. However, individuals paying the standard rate of income tax will prefer dividends because they pay no income tax on these. Their income tax liability is fully met by the tax credit that is imputed to them. Depending upon the tax rates used, only those individuals paying top rates of income tax who may prefer capital gains. Pension funds, who are again a large and rapidly increasing body of investors in the UK, would prefer dividends even if they pay no tax on capital gains. The reason is that they can claim back the tax credit on dividends, and therefore for every £1 of pre-tax income generated by the firm, their net receipt, using 1987 tax rates, is £0.206 higher if companies pay out dividends than if they receive capital gains. For corporate investors, dividends are received as "franked investment income" and are not subject to any further charge. However, capital gains would be subject to a positive (but may be small) capital gains tax. Again the corporations might prefer dividends.

Finance literature, particularly that emanating from the U.S.A., stresses that investors should prefer capital gains to dividends for tax reasons. However, on closer inspection, much of the fuss in the literature as to why corporations do not repurchase shares (which would result in capital gains for the selling shareholders instead of dividend income), is quite unnecessary. The only investors who clearly prefer dividends are individuals in the U.S.A., and <u>only</u> very high income tax rate individuals in the U.K.. All other investors would <u>prefer</u> dividends for tax reasons (or be indifferent) and these investors form the majority of investors on stock markets in the U.S.A. and in the U.K. . This conclusion is reached without considering restrictions on share repurchases, which make share repurchases even less attractive.

The internal Revenue Service in the U.S.A. attempts to discourage share repurchases if their only purpose is tax avoidance. A proportionate repurchase of shares would be taxed as dividends in the U.S.A.. In the U.K. share repurchases were illegal until 1981. The Companies Act 1981 permitted share repurchases for companies if these were allowed by their articles and had the approval of their shareholders. However, the majority of the investors are unlikely to favour share repurchase for reasons given above but even if companies did repurchase shares, the Inland Revenue may treat these as a distribution and tax them as dividends. Hence the claim in the literature that there is tax disadvantage to dividends is not universally valid on closer inspection and instead it depends both upon the tax code and the mix of equity owning investors.

**(iv) Irrelevance of dividends in the presence of personal taxes:**

Black and Scholes (May 1974) argue that even if there is a tax disadvantage to the payment of dividends, no company can increase its value by changing its dividend policy. This is because if companies could increase their price by paying out less, then they would have done so already. Firms are seen as having already established clienteles of investors with preferences which are consistent with the firm's pay out policies. There is therefore nothing to be gained by firms switching their dividend policy.

The irrelevance of dividend policy in the presence of taxes is also examined by Miller and Scholes (1978). As stated in the previous chapter, they claim that in the U.S.A., investors can launder personal taxes on dividend income because they are allowed to deduct interest as an expense up to the amount of dividend income. Thus individuals can borrow, create tax deductions due to interest expense on borrowings, and use these to offset the tax due on dividend income. They also have the opportunity of investing in pension schemes etc. whereby they can earn the pre-tax interest rates. They can invest the proceeds of their borrowings in these schemes and therefore there is no net increase in their leverage. As a result of the above transactions, the investors earn dividends and not effectively pay no income tax on them. Therefore investors can convert dividends into capital gains. Thus investors need not worry about difference in personal tax rates on dividend income and capital gains because taxes can be laundered.

This interest deduction for personal tax purposes is specific to the U.S.A. and is not available to individuals in the U.K.. However, if the investor is a corporate entity, then interest payments on borrowings are tax deductible even in the U.K. (and they are not restricted to dividend income received). However, the reason why corporate investors in the U.K. cannot launder taxes is because they cannot invest to earn the pre-tax rate of interest on investments. Note that although the corporate pension funds could theoretically be used to earn pre tax returns, a company will face difficulties in utilising pension fund assets for the benefit of its shareholders. The recent misuse of pension funds by Robert Maxwell in the U.K. is bound to influence legislation which would make it very difficult for shareholders to appropriate pension fund assets.

The main criticism against the Miller & Scholes hypothesis, even in relation to an individual investor in the U.S.A., is that there are disadvantages and restrictions on investing in pension schemes etc on which investors can earn the pre-tax rates of return (such as liquidity constraints). Otherwise, this route would have been very widespread in practice for all investments by individuals, and not just those investments which are to launder taxes on dividends income. However, Miller and Scholes' laundering route is theoretically not sound and it is not used widely in practice (D Feenberg,1981).

**(v) Theories based on the Information Value of Dividend Pay out:**

Ross (1977) emphasised that what is evaluated in the capital markets is the perceived stream of returns for the firm. One method by which investors may form their perception is by observing the stream of dividends paid out by the firm. A firm which increases dividend payments is therefore seen as signalling that it is doing well, that its future earnings will be higher, and that they will be sufficient to meet debt payments without increasing the probability of bankruptcy.

Bhattacharya (1979) suggests that the benefits of signalling by increasing dividends can offset the tax disadvantage associated with dividends. Therefore firms should aim for an optimal dividend pay out ratio where the marginal benefit is just offset by the marginal cost.

Hakansson (1982) has added the insight that dividends can provide additional information only if at least one of the following holds:
(a) investors have different probability assessments of dividend pay out, or
(b) their time preference patterns for consumption differ, or
(c) the capital markets are incomplete.

Even if these conditions exist, the utility of information provided by dividend needs to be very great in countries such as the U.S.A., because the dividend payment is accompanied by personal taxes which could be a significant disadvantage to the individual investors. Therefore, dividends can be justified on the basis of their information content only if :
(a) the benefit of information is large enough to offset the tax disadvantage, and
(b) there is no other cheaper way of conveying the same information.
Since there is no evidence that either of the above two conditions are met in the U.S.A., the information content hypothesis as a possible explanation for the payment of dividends is not accepted in this thesis.

**(vi) Optimal Dividend Pay out and Agency Costs:**

Another theory for optimal dividend pay out is suggested by Rozeff (1981). In his theory, the costs of increasing dividends are seen as the flotation costs of raising more external finance for investments. The benefit of increasing dividends is the reduction in agency costs incurred when non-managers own part of the equity (ie. there are external shareholders). External investors need to monitor how the management handles their investment in companies, and to ensure that the management does not derive undue benefits at the expense of the shareholders. Therefore they need to incur costs for monitoring the management. One way in which the need to monitor can be reduced is to restrict the "free cash flow" available for utilisation at the discretion of the management. The free cash flow available is restricted if the management are required to pay a large proportion of the surplus cash flow as dividends. Thus the external shareholders can reduce the monitoring costs because the amount of cash flow remaining to be "monitored" is reduced. However, this theory cannot provide suitable answers to the following criticisms:

(a) This would require that agency costs are so high that they can offset the tax disadvantage of dividends for individuals in the US.

(b) There is a more tax efficient alternative of reducing the free cash flow, namely by issuing debt, the companies are required to meet (tax deductible) interest payments. Why not use such tax deductible interest payments to reduce the "free cash flow" in a company?

Therefore, although agency costs were very popular in the literature in the 1980s, they are an inadequate explanation for dividend payments.

There are a number of empirical studies which test the above alternative theories on dividends. The results of these studies are inconclusive, partly because of the difficulty of measuring the yield expected from an investment in a company. The empirical evidence is examined briefly below.

## B EMPIRICAL EVIDENCE

Elton and Gruber (1970) attempted to measure the clientele effect by testing price changes when stocks go ex-dividend (see next paragraph). They concluded that a clientele effect did exist, although their results could be refuted if one considered the presence of broker-dealers, for whom gains and income are taxed as one. Consideration of tax free investors too would contradict their conclusion.

The reason for these contradictions is as follows. Elton and Gruber are measuring the changes in the price of shares when dividends are paid. The essential theory is that on the day that a share goes ex-dividend, its price should fall by the amount of the dividend less the personal taxes on dividends. If shares are held by investors whose tax rates are high, then the decrease in the price of shares will be less than as compared to if the shares were held by investors with lower rates of personal taxes. Their aim is to show that the high dividend yield stocks suffer a fall in share price closer to the amount of dividends being paid. This will imply that these high yield shares are held by the investors who have a low tax rate, because otherwise the gap between dividend amount and the fall in the price of shares would have been larger. However, the presence of broker dealers and tax exempt investors further clouds the issue. These investors can use their own particular tax rates to their advantage by undertaking transactions to gain from the changes in the share price which are influenced by the tax rates of the other investors. Thus the net result is that the share price will change because of transactions undertaken by all three types of investors, namely, the broker dealers, the tax exempt, and the taxable investors whose marginal income tax rate is different from their capital gains tax rate. Using the change in share prices to infer the tax rates of the investors which fall only in the last of the three categories above, is extremely difficult and therefore their conclusions can only be tentatively stated.

Black and Scholes (1974) studied whether the before tax returns on common stock were unrelated to dividend pay out. They used CAPM to control for risk variations in their sample of companies. They concluded that it was not possible to show that the expected returns high yield stocks differed from those on low yield stocks.

Litzenberger and Ramaswamy (1979) used an after tax version of CAPM - an extension of Brennan's model (1970). Their results indicated that there was a strong positive relationship between the before tax expected returns and the dividend yields of common stocks. This implies that in order to give the same after tax return to investors, the before tax return on the high dividend yield stocks has to be higher. This is to compensate the investors for the extra dividend taxes that they have to pay on the dividends. The authors also stated that there was some evidence of clientele effects - the stockholders in high tax brackets chose stocks with low yields and vice versa.

The Litzenberger and Ramaswamy study has been criticised by Miller and Scholes (1982) for its handling of the information effect of dividend announcement. Miller & Scholes conclude that tests using short term measures of dividend yield are inappropriate for testing whether expected returns differ with dividend payout.

More recently, J M Poterba and L H Summers (1984) have used U.K. data to examine the effects of dividend taxes on the investors' relative valuation of dividends and capital gains. They studied the changes in share price valuation when there were changes in tax regimes in the UK - in 1965 (introduction of Capital Gains Tax) and 1973 (introduction of the Imputation System of Taxation). Their findings support the hypothesis that taxes influence the relationship between dividend yields and security returns. Their work thus supports the conclusions on the relevance of dividend taxation reached by Auerbach (see chapter 2), Elton & Gruber and Litzenberger & Ramaswamy. It contradicts the hypothesis of Miller & Scholes.

Poterba and Summers (1985) (see chapter 2) use British data and event study methodology for testing which of the following views of dividend taxation is supported by the data:

(a) The traditional view which argues that dividend taxes are an additional tax on corporate profits. The motivation for corporations to pay dividends must therefore depend upon other explanations such as incentive signalling approach (Ross, etc.). In other words, dividend taxes have a negative effect on firm value which must be offset by some other positive factors for firms to pay dividends.

(b) The tax irrelevance view of Miller and Scholes.

(c) The tax capitalisation hypothesis (see chapter 2) stated by Auerbach (1979), Bradford(1981) and King(1977). This states that the only way for mature companies to pass money through the corporate veil is by paying taxable dividends. The new investors too recognise this fact. They will not pay the selling shareholders the full value of the retained earnings, but will pay a smaller amount, the difference reflecting the "capitalisation" of the potential tax liabilities which must be met when the dividends are "eventually" paid out to the investors from the corporate sector. Under this hypothesis, the retained earnings do not lead to an equivalent increase in the share price, but by a smaller amount, the difference reflecting the extra taxes that will have to be paid on dividends. Thus investors will be indifferent between dividends and capital gains because any differences in taxation of dividends and capital gains are offset by the capitalisation of the differential tax liabilities. If the tax rate on dividends is increased, the tax burden is borne by existing shareholders who are "locked in" in their investment in the corporate sector. Thus changes in dividend taxes should have no effect on the required rates of return.

While commenting on the ability of the companies to pass money to the shareholders, Poterba and Summers state that in the U.K., share repurchases are "explicitly banned". They thus have failed to notice the changes introduced by the Companies Act 1981, which allows companies to purchase their own shares, provided certain conditions are met. The second statutory change which will have to be noted for tests on the U.K. data is the indexation allowance for Capital Gains Tax introduced by the Finance Act 1982. Presumably, its impact on their study is negligible, but this provision would reduce the effective capital gains tax rate for future studies.

Their empirical tests show that the traditional model of dividend taxes ( model (a) above) provides the best explanation for the U.K. data. This model states that dividend taxes are an extra burden on the shareholders. This conclusion will be used throughout in this thesis and the main valuation model derived in chapters 4 to 7 will assume that the dividend taxes do influence the required rates of return. Poterba and Summers conclude therefore that it is important to model and provide theoretical motivation for empirical tests of any positive effects of dividend payments, that is what non-tax reasons are there for companies preferring dividends?

The issue of dividend policy is important as it affects conclusions regarding other aspects of corporate policy - eg. what is the benefit of debt finance and what is the cost of capital (ie. the return expected by investors). Currently, no model in the literature fully explains why firms pay dividends.

The next question to be examined is whether the existing literature explains why firms issue a certain level of debt. There are two reasons for addressing this question. Firstly, debt policy by itself is an important aspect of finance literature. Secondly, in the valuation model we develop, we assume that there are tax advantages to corporate debt. Therefore we need to understand debt policy issues.

## SECTION B
## DEBT POLICY

Modigliani and Miller ("MM") (1958) stated that the debt policy followed by a firm was irrelevant in the absence of taxes and certain other simplifying assumptions. These included the absence of transaction costs and the ability of investors to borrow and lend at the risk free rate. The reason why debt policy did not matter was because the total value of the firm and the weighted average cost of capital ("WACC") remain unchanged as the debt-equity ratio was changed. The reason why these remain unchanged is because of the principle of value conservation. An income stream produced by an asset would be worth the same irrespective of how it was divided into returns to owners of capital as long as there were no leakages of cash flows either in the form of bankruptcy costs or in the form of taxes.

They therefore contradicted the traditional analysis which had maintained that increases in the level of debt up to a certain point would increase the value of the firm primarily because debt capital was cheaper; and therefore increases in the debt level would lower the WACC. The increased probability of bankruptcy would however outweigh the beneficial effect at some specific debt/equity ratio after which point the WACC curve would then increase with leverage. Hence, the traditional analysis maintained that there was an optimal level of debt equity ratio at which the WACC was minimised.

MM claimed that this traditional analysis was incorrect in assuming that increasing leverage initially (that is, at low levels of debt) would leave the cost of equity capital unchanged. This was not true because the cost of equity capital increases proportionately with leverage. Copeland and Weston (1982) show that in the absence of taxes the cost of equity capital is

$$Ke = E + (E - Kd)\frac{B}{S}$$

where E is the cost of capital for an all equity firm

B/S is the debt-equity ratio

Ke and Kd are the cost of equity and debt respectively.

Therefore, in the absence of taxes, Ke increases proportionately with leverage and therefore WACC remains unchanged.

The presence of corporation taxes changes the above results dramatically. Interest on debt is tax-deductible to the corporation and therefore increasing leverage increases the beneficial impact of debt finance through savings in corporation tax. Modigliani and Miller in their subsequent article (1963) stated that in the presence of corporation taxes and interest tax deductibility, the solution for optimal debt structure was a corner solution. Firms should be almost 100% financed with debt to maximise the value of the firm. The after tax cost of capital would be minimised with almost 100% debt.

Assuming that debt capital was perpetual and there was no uncertainty regarding the benefit from the tax shield created by corporate debt, the value of the firm would be:

$$\text{Value of firm} = \text{value if all equity financed} + T_cB$$

where $T_c$ is the marginal corporation tax rate and B is the market value of debt. The cost of equity capital now changes to

$$KE = E + (E - KD)(1 - T_c)\frac{B}{S}$$

and the WACC is

$$WACC = E\left(1 - T_c\frac{B}{B+S}\right)$$

Modigliani (1982) stated that if the cash stream associated with tax saved by interest shield were valued using a risky discount rate, the benefit of leverage would be reduced. Miller (1977) stated that in the presence of personal taxes, the benefit from the interest tax shield would be lower than $T_cB$. These arguments are very relevant to the core of this thesis and were therefore examined in some detail in chapter 2. Here, it is sufficient to note that in the presence of only corporate taxes, MM stated that the benefit from interest tax shield is $T_cB$.

Personal tax is only one of the possible explanations for why the firms are not 100% debt financed. Apart from personal taxes (dealt with in detail in chapter 2) other arguments in the literature point to the disadvantages of increasing debt. These include:

(i) bankruptcy costs

(ii) contracting and monitoring costs

(iii) information and signalling costs

(iv) differential flotation costs

(v) incomplete markets

(vi) wasted tax deductions

**(i) Bankruptcy Costs**

The literature on bankruptcy costs has been dismissed by Miller (1977 - see chapter 2). He argues that the bankruptcy costs calculated are too insignificant to counteract the huge tax advantage of debt finance.

However, a subsequent study by E I Altman (1984) concluded that bankruptcy costs are far from trivial. He studied a sample of 18 industrial firms that went bankrupt over the period 1970-78 and a second sample consisting of seven large more recent bankruptcies. The average total cost of bankruptcy for the 18 firms was 16.7% of the value of the company. The average of the indirect costs encountered by the seven firms which went bankrupt more recently was 17.7% of the value of the firm. These are greater than the costs indicated in previous studies and may be sufficient to act as a counterweight to the tax advantage of debt.

Brealey and Meyers (1984) have used expected bankruptcy costs as an argument to explain differences in leverage across industries. They state that industries where assets consist of intangibles (human capital, technology, brand image) the value of which is dependent upon the company carrying on as a going concern, are likely to lose more value in bankruptcy than companies owning physical assets with a good secondhand market. Hence these former type of companies are likely to borrow less. Therefore, the pharmaceutical industry and service industries are likely to consist of firms with low gearing. On the other hand, firms owning hotels or manufacturing companies are more likely to issue more debt. This assertion that firms which rely on intangible assets borrow less is confirmed by M Long and I Malitz (1983).

**(ii) Contracting and Monitoring Costs**

Jensen and Meckling (1976) state that firms have to incur agency costs whenever they issue debt and equity capital. Therefore firms should choose an optimal debt equity ratio which minimises these costs. Thus firms have an optimal debt equity ratio even in the absence of bankruptcy costs and taxes.

Titman (1981) extended the concept of agency costs to include contracts with customers and employees. The greater the leverage, the greater the probability of bankruptcy and therefore the greater the costs borne by the firm. In a competitive market, it is argued that customers and employees will demand better terms (essentially reflecting risk premium) from highly geared firms.

**(iii) Information and Signalling Costs**

Ross (1976) extended the application of his signalling hypothesis to debt structure. He suggests that greater financial leverage can be used by managers to signal an optimistic future for the firm, since with this view, what is valued in the market is the perceived stream of returns. Firms which increase leverage should benefit as the market perceives them to have greater value. Signalling deals with ex ante expectations. The spate of bankruptcies witnessed in the U.S.A. and the U.K. in the very highly leveraged companies in the late 1980s and the early 1990s, are an ex post observation. Although ex ante expectations, and ex post evidence relating to a particular period which may never be repeated, are clearly distinct, some investors in practice, unduly influenced by the ex post evidence, may find it difficult to associate higher financial leverage with an optimistic future for such firms.

**(iv) Differential Flotation Costs**

Many authors including Myers (1977) have argued that since the flotation costs associated with obtaining finance through either new share issues or retained earnings or new debt issue differ, a company will aim to minimise these costs. Whether firms increase debt or equity would therefore be influenced by differences in flotation costs.

**(v) Incomplete Markets**

If the number of securities available is less than the number of states of nature, then this can influence the capital-structure decision ( Arrow (1964), Diamond (1967), Rubinstein (1973) and Ross (1977) ). This view implies that the firm may chose a debt equity ratio simply to provide the risk return characteristics that its investors want and which they are unable to achieve because of incomplete markets.

### (vi) Wasted Tax Deductions

The benefit from interest tax shields is only useful for companies in a position to utilise them. For example, up to the early 1980s, most manufacturing companies in the U.K. carried forward losses for tax purposes because of the availability of other tax shelters, such as 100% first year allowances on capital expenditure. The benefit of tax shield created by debt is reduced for these companies because of the presence of other tax shields. The company therefore does not benefit immediately from the tax deductibility of interest payments. The after tax cost of debt is close to the before tax cost because the company is not in a position to take advantage of the tax benefits that accompany debt interest payments. Secondly, increasing debt increases the probability that the taxable income of the firm will be insufficient to cover interest payments.

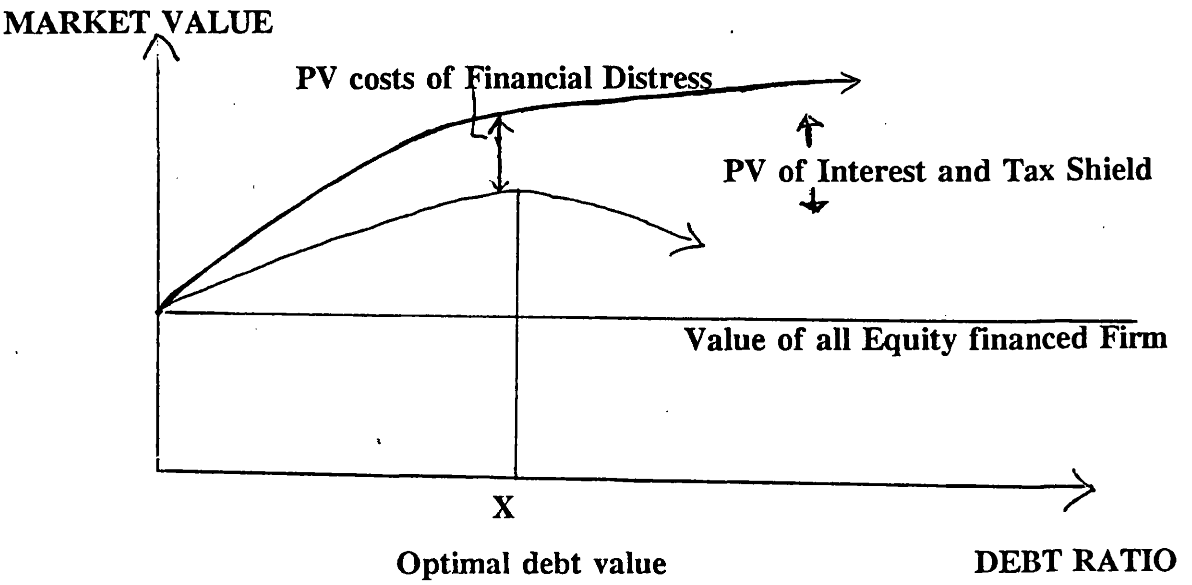
Since the company cannot immediately utilise its tax losses, these may be carried forward to offset future income. Hence the benefit of tax deductions may arise sometime in future but there is a loss of time value of money due to this postponement of benefit. De Angelo and Masulis (1980) suggest that as a result, the supply curve of corporate debt will not be indefinitely horizontal - it will decrease with increasing leverage, thereby reflecting that the tax advantage of leverage decreases with increasing leverage. Thus the firm's may not find it optimal to have 100% debt in their capital structure.

### CONCLUSION

In view of the disadvantages of debt, the value of a levered firm should also be affected by the "costs" of these disadvantages, and therefore the value of such firms should be:

Value of levered firm = Value if all Equity financed + PV of Tax Shield
- PV of costs of financial distress

Firms may have optimal debt level because the latter two vary with leverage as shown below:




In the above diagram, the horizontal line shows the value of the firm if it were all equity financed or if there was no tax benefit to debt finance at the corporate level. The top curved line shows the value of the firm in the presence of corporate tax deductibility of interest payments. It increases initially with increasing leverage. However it does not increase in a straight line because of the impact of the wasted tax deductions. The difference between the top curved line and the horizontal line is the present value of the tax shield. However, there are other disadvantages to debt, such as bankruptcy costs. These increase with leverage. The lower curve is derived by subtracting the negative impact of these costs on the value of the firm as shown by the top curve. The highest point in the lower curve represents the highest value that the firm can achieve after taking into account all the factors that influence debt equity ratios. This determines the point X, which is the optimal debt equity ratio for this firm.

## B Empirical Evidence

If debt is valuable, then increasing debt relatively should increase the value of the shareholders' equity. Masulis (1980, 1983) has found weak evidence that changes in stock prices are positively related to changes in leverage. The two reasons for this are the gain in value induced by any tax shields on debt and a positive information effect from higher leverage.

Whether leverage results in an increase in the cost of equity was examined by Hamada (1972) who found that on average the systematic risk of a levered firm was higher than that of an unleveraged firm.

Whether the overall cost of capital remains unchanged even if leverage changes was examined by MM in their 1958 study of utilities and oil companies. They concluded that the cost of capital was independent of leverage. Their study was criticised by Weston for not including a growth factor in valuation model and for choosing the oil industry where business risk is not homogeneous. Allowing for this, Weston (1963) found that for the same industry, WACC decreased with leverage due to the tax deductibility of interest payments. In 1966, MM used a more elaborate model and reached the same conclusion. The reason why empirical tests on leverage are difficult to conduct have been noted by Copeland and Weston (1983):

(a) Incorporation of anticipated future growth is difficult in the models.
(b) Flotation costs may be inversely related to the size of the firm - therefore there may be economies of scale in the cost of capital.
(c) It is difficult to assume homogeneous business risk for actual companies even in the same industry.
(d) Firms do not change capital structure in isolation. Usually, new investments with changes in business risk accompany capital structure changes.
(e) Much of the empirical work uses cross-section regressions which are likely to have highly correlated residuals across firms.

Therefore evidence on capital structure is not conclusive. How should companies evaluate projects under uncertainty given the lack of clear cut policy implications for dividend policy and for capital structure? This question is taken up in the following section.

## SECTION C
## IMPLICATIONS OF DIVIDEND AND DEBT LEVELS FOR THE COST OF CAPITAL

The implications of dividend policy require consideration of personal taxes and therefore were dealt with in the previous chapter. The valuation formula advocated in this thesis will include a term that specifically includes the impact of the difference in the personal tax rates on dividends from the personaltax rates on the other income. However, presently, in this chapter we restrict ourselves to the inclusion of corporate taxes only. Therefore, although dividend level has an important role in our main valuation model, it is not relevant for the more simplistic valuation methods considered below. On the other hand, corporation taxes have a profound effect on debt policy and therefore debt level is very important in the cost of capital models described below. We look at six different ways in which projects can be valued in practice.

### (a) Adjusted Present Value Approach

In view of the complications introduced by the presence of debt, Brealey & Myers (1984) recommended that the best method for evaluating a project is to follow the adjusted present value approach. This involves the calculation of a base-case NPV on the assumption that the project were to be 100% equity financed, as a first step. The discount rate for the project should therefore reflect only the business risk faced by equity holders. Debt is ignored on the calculation of base-case NPV.

Secondly, one adds the sum of present value of all the side effects of accepting the project to the base-case NPV. These side effects include:

(i) issue costs of raising finance
(ii) any government subsidy or grant that is specific to the project, and
(iii) value of interest tax shield.

The value of interest tax shield is included because the acceptance of the project adds to the debt capacity of the firm, allowing further tax benefits to be obtained. The benefit is given by the following expression which was explained in the previous section:

$$\text{Value of interest tax shield} = T_c{}^*D$$

where D is the market value of debt and $T_c{}^*$ is the **effective** corporate tax rate. The assumption is that debt will be perpetual. $T_c$* will equal the marginal corporate tax rate if the firm was certain of fully utilising the interest tax shield and there was no negative impact (ie. costs of financial distress) associated with increased debt. All other costs and benefits that are associated with debt will be added separately to calculate the adjusted present value of the project.

There is not much guidance on how precisely one can calculate $T_c$*. Obviously it will be less than marginal $T_c$ if the company has other tax shields in use. It will also be less if the company has huge losses for tax purposes brought forward (a quite realistic situation in the UK in the mid 1980s). Presumably, $T_c$* will also be lower, the greater the debt-equity ratio, as the disadvantage of increase in expected value of bankruptcy costs outweighs the benefits of the signalling effects. The calculation of $T_c$* is an area where more empirical work needs to be done.

The calculation of adjusted present value (APV) is summarised below:

APV = Base case PV + sum of PV of all side effects

or APV = Base case PV + $T_c$*D - issue costs + PV of other financing or side effects

**(b) Adjusted Discount Rate**

An alternative to calculating an APV is to calculate an adjusted discount rate which takes into account the benefit of tax shield. One such formula was used by MM in 1963 and 1966. This is shown below.

$$r^* = r\,(1 - T^*L)$$

where r* = adjusted discount rate

r = opportunity cost of capital. It compensates for business risk only.

$T_c$* = the effective tax rate. Its determinants were discussed in (a) above.

L = the project's marginal contribution to the firm's debt capacity as a proportion of the project's PV.

The advantage of this formula is that L can be higher or lower than the firm's overall debt ratio. Its disadvantages include the assumption of permanent debt and the limitation of its use to projects offering level perpetual cash flows. Note that r in this formula is not the cost of equity of a levered firm. Rather, it is the cost of equity in an all equity firm, ie. it is based on only business risk. Also note that this formula does not compensate for other side effects, which will need to be accounted for separately.

The above formula assumes perpetual debt. The calculation of a discount rate when the debt level may vary is discussed below.

### (c) Discount Rate when Future Debt Level is Uncertain

J Miles and R Ezzell (1980) have provided a formula which does not assume permanent debt. However, it assumes that the firm adjusts its borrowing to keep a constant debt proportion. This again is unrealistic as firms do not constantly adjust debt with changes in the market value of equity, although theoretically, it is more sound because it correctly takes into account the changes in the contribution of the project to the firm's debt capacity. They calculate adjusted discount rate as:

$$r^* = r - L\, r_D\, T_c^* \left( \frac{1 + r}{1 + r_D} \right)$$

where $r_D$ = the borrowing rate.

The advantage of this formula over the previous MM formula is that it can be applied to any cash flow pattern or project life.

### (d) Weighted Average Cost of Capital

This approach makes use of statistics which are more readily available, but its usefulness is fairly restricted. The approach is to calculate the adjusted discount rate as

$$r^* = r_D\,(1 - T_c)\,\frac{D}{V} + r_e\,\frac{E}{V}$$

$r_D$ = the firm's current borrowing rate

$r_e$ = expected return on equity (eg. calculated by using CAPM and which compensates equity holders for business and financial risk )

$T_c$ = the marginal corporate tax rate.

D, E and V = market value of debt, equity and the firm respectively.

The advantage of working with the WACC is that it uses $r_e$ which in turn uses equity betas which are easily available (for example through London Business School's Risk Measurement Services in the U.K.). There is controversy as to how to adjust for leverage to get asset betas (see (e) below). Secondly, it uses $T_c$, a known marginal corporate tax rate rather than $T_c$*, which is difficult to estimate in practice. In practice, analysts prefer to work with objective known marginal tax rate.

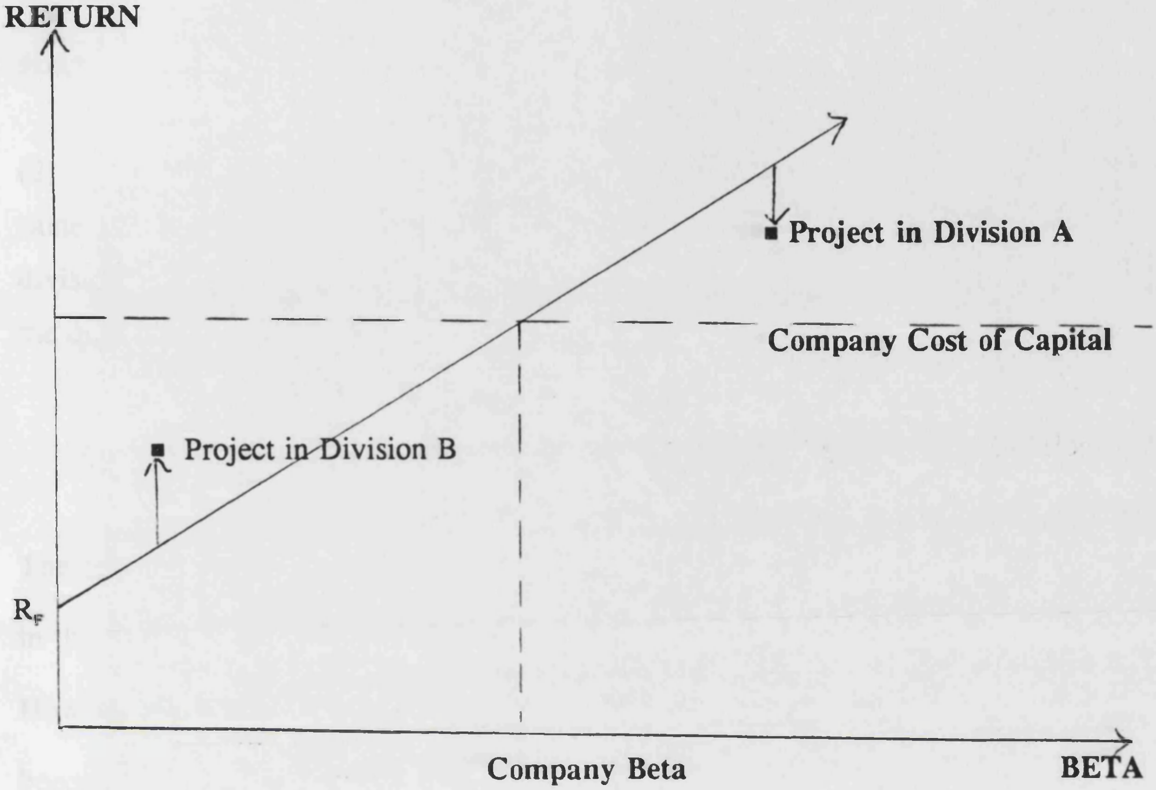
However, it is restrictive in use. It is only suitable for projects that have the same risk as that of the firm's existing activities and where the firm will continue to maintain its debt-equity ratio.

The benefit of interest tax shield should not be double counted in calculating the relevant cash flow to be discounted (that is, the cash flow should be the net operating income [1-$T_c$]). Secondly, the formula should not be used illogically if a project is being financed largely by debt. What is relevant is the long run debt equity ratio which should be used in the formula.

The calculation of WACC for divisions of a company, where the risk differs substantially from the average riskiness of the overall company, is considered next.

### (e) Divisional Cost of Capital

Use of a single WACC as discount rate for projects would lead to an intra-firm mis-allocation of resources as is shown for the following multi-divisional all-equity firm:




In the above diagram, the project in Division A should be rejected because the returns expected from it are below its divisional risk adjusted cost of capital. The project in Division B should be accepted because its returns exceed its risk adjusted divisional cost of capital. However, use of single company wide cost of capital ( horizontal line above) based on company wide beta factor would lead to an opposite and incorrect result.

The calculation of divisional WACC requires the calculation of divisional return to equity which in turn requires an estimate of divisional betas. Therefore we need to calculate betas for divisions which are not quoted on a stock market and therefore do not have readily available market values. The literature contains two approaches for valuing divisional betas:

(1) <u>The Analytical Method</u> M H Gorden and P J Halpern (1974) suggest that the systematic risk of a division can be estimated by assuming that the beta is highly correlated with some observable statistic, such as the ratio of changes in divisional

earnings to changes in total economy wide corporate earnings. The regression equations that result from using this method are not considered sufficiently stable to be used in practice.

(2) Analogy or Pure Play Techniques This involves finding a quoted company in the same line of business and assuming that it has the same business risk as that of our division. The differences in financial risk can be adjusted by unlevering and relevering the quoted company's beta by using the following equations suggested by Hamada:

$$\text{Unlevered Beta} = \frac{\text{Beta of Quoted Company}}{1 + (1-T_c)\ D/E}$$

$$\text{Divisional Beta} = \text{Unlevered Beta} \times (1 + (1-T_c)\ D/E)$$

The debt (D) and equity (E) values used above are those of the pure play quoted company in the first equation and of the multi divisional company in the second equation.

However, Conine (1980) has shown that in the presence of risky debt, the above equations become:

$$\text{Unlevered Beta} = \frac{B_L + B_{debt}\ (1-T)\ D/E + (B_{pref}\ PREF/E)}{1 + D/E\ \ (1-T) + PREF/E}$$

Thus beta factor for the levered equity ($B_L$) , risky debt ($B_{debt}$) and the risky preference shares ($B_{pref}$) are estimated and used to derive the unlevered beta. Divisional Beta is calculated by using the converse of the above equation, but using the equity, debt and preference shares of the multi-divisional company instead.

Fuller and Kerr (1981) in an empirical study conclude that using the adjustments for financial risk proposed by Hamada do not yield better results than not adjusting at all for leverage differences. Conine & Tamarkin (1985) conclude that using the Conine equations could provide superior results because it may not be correct to assume risk-free debt as is done by Hamada. However, they suggest further research before firm conclusions in this area are reached.

Using these betas, it is possible to use CAPM to calculate the cost of capital for divisional equity and then to proceed to calculate a divisional WACC for use in evaluating projects that have risks similar to the average risk of the division's existing projects. Divisional WACCs should produce results superior to evaluations using company wide WACC.

The difficulties encountered in the practical calculations of the company betas are also relevant for the divisional betas. The use of a single rate of discount for multi-period evaluation implies certain assumptions which may not hold in reality. Similarly, there are disagreements regarding the best risk free interest rate and the risk premia to use etc., all of which are very relevant for practical applications using divisional WACC.

## (f) Abnormal Growth Projects and Firms

Copeland and Weston (1983) use WACC and return on all equity firm along the lines of the valuation model used by MM in 1966. They state that if the cash flows generated are going to increase rapidly for a number of years until the project earns a return equal to its cost of capital, then it can be evaluated using the following model:

$$V = \frac{E(NOI_1)(1-T_c)}{e} + T_cD + K\,[E(NOI_1)(1-T_c)]\,T\,\frac{(r - WACC)}{WACC\,(1+WACC)}$$

where

| | | |
|---|---|---|
| $E(NOI_1)(1-T_c)$ | = | expected after tax operating cash flow for the coming year |
| WACC | = | Weighted average cost of capital |
| | = | $e\,[1 - T_cD / (D+S)]$ |
| K | = | rate of reinvestment of earnings |
| $T_c$ | = | Corporation tax rate |
| T | = | the number of years when the firm is earning abnormal returns |
| r | = | the abnormal rate of return, earned for T years, which exceeds WACC. |
| e | = | The cost of equity for an all equity firm |

The first term measures the value of the steady component of income of the firm on the assumption that there is no debt. The second term adds the advantage of debt finance. The third term measures the extra value contributed by the high abnormal growth rate of the company. The amount invested in this high growth project is given by K [E ($NOI_1$)

$(1-T_c)]$. This earns a rate of return r for T years, before reverting to earning its cost of capital. Adding the three components gives the total value of the firm.

## CONCLUSION

The most promising approaches in the literature for valuation of projects are:

(i) The Adjusted Present Value approach of Brealey & Myers (1984), and

(ii) The Divisional Cost of Capital approach where a risk adjusted discount rate is used to evaluate projects. Divisional cost of capital is the cornerstone of the firm's cost of equity capital and of discount rates using CAPM. Secondly, more research on divisional costs of capital and on how the weighted average of divisional capital costs explains the (observed) company wide cost of capital of quoted companies will also give insights into other issues in Corporate Finance especially dividend policy and debt policy. The types of questions that need to be answered include whether a firm's beta decreases if a company retains cash as safe investments, and whether divisional betas increase with leverage of the firm.

By now we have a good idea of the kind of variables and their relationships that are modelled in the articles in the existing literature. We depict these in the concluding section to this chapter. In the concluding section, we also depict our perception of who the relevant players are and how complex the real relationship between project returns and investors are. However, in order to complete our picture, we need to examine the international CAPM models, because, as shown in the next section, the international dimension is very relevant in understanding the complete picture of project returns and their path to the investors.

## SECTION D
## INTERNATIONAL MODELS

One of the main criticisms made at the end of the next section is that the valuation models in the literature fail to include all the important variables in their specifications. The reason for this failure is to keep the models tractable. One dimension in which most of the models described so far fail is in capturing the international opportunities available to the domestic investors. The models that take into account the international dimension are described in this section. Domestic CAPM models have been extended to include many countries, although there is controversy as to what constitutes "many". Do economies differ simply because of different currencies, or because of taxation differences and restrictions on overseas transactions, or because of different consumption bundles? Is exchange risk a real risk, a nominal risk, or a normal "business" risk and is it worth hedging against partially, completely or not at all? Opinions differ on these issues, resulting in different equilibrium models.

The main issues are dealt with in the following order:

(A) What are the benefits and costs of international diversification?

(B) Can multinationals provide such benefits at a low cost?

(C) What are the main international CAPM models?

(D) How can investment projects in overseas countries be evaluated?

The literature on exchange rate determination, although relevant, is too extensive to be included here. Instead, the aim of this section is to introduce some of the relevant variables and opportunities available in the capital markets which are often ignored in the purely domestic models. The methods of determining the cost of capital using these international models are also introduced.

**(A) Benefits and Costs of International Diversification**

R A Cohn and J J Pringle (1973) give the rationale why international diversification might reduce risk. They state that within a domestic market, correlation among securities can be tracked to the fact that returns on the underlying real assets (to which the securities represent claims) are themselves correlated because they depend on certain common domestic economic factors. Hence there is an upper limit to the benefit of domestic portfolio diversification. However, investor diversification across international boundaries should improve investors' risk-reward opportunities to the extent that in different economies the economic activity is less than perfectly correlated. They show that the slope of the capital market line will decrease with logarithmic or exponential utility functions, thereby reducing the risk premium. This should result in a lower cost of capital relative to the case when the investors restrict themselves to the domestic securities only. This is an important benefit of international diversification and therefore it should not be ignored in the specification of the equilibrium valuation models.

The above analysis assumes perfect markets. The following factors are stated by Cohn and Pringle as constituting market imperfections which may prevent or restrict an investor from realising the benefits of international diversification:

(a) interest equalisation tax
(b) exchange controls and other restrictions on capital flows
(c) extra withholding taxes on dividends to foreigners
(d) exchange rate risk
(e) unavailability of and cost of information on foreign securities
(f) political risk.

All these factors, in particular the withholding taxes, the exchange rate risk, and the lack of information about overseas opportunities, reduce the potential benefits of overseas diversification. However, in perfect markets, the diversification benefit would result because of the lower covariance between world wide securities as compared to the domestic securities.

The covariance factors for international portfolios have been studied empirically by Lessard (1976). He states that the main factors which distinguish investing in the international market from investments in the domestic market result from:

(a) the fact that covariances among securities within national markets are much higher than among securities in different markets;
(b) the barriers to international investment including taxation, currency control or even domestic oriented culture; and
(c) the possibility of exchange risk in international investment.

He concentrated on the evidence concerning (a) above.

He concluded that the low correlations between country factors presented possibilities of gains from international diversification in either of the following situations. There is the possibility of gain by the reduction of non-systematic risk by international diversification even if capital markets are integrated. The possibility of gain from international diversification increases if the markets are segmented because some of the risk considered undiversifiable would become diversifiable. However, he cautions that considering covariances alone is insufficient - the expected returns in different markets too must be considered to assess whether diversification will offer benefits. Finally, he states that there must be costs associated with international diversification of sufficient magnitude to make investors bear "unnecessary" risk by sticking mainly to domestic securities only.

The issue of costs associated with international diversification has been addressed by Fisher Black (1974). He developed a model which treats these costs of the barriers to international diversification as a tax on foreign investment. These costs include barriers such as the possibility of expropriation of foreign holdings, direct controls on the import or export of capital, reserve requirements on bank deposits or other assets held by foreigners, lack of information on foreign capital markets and restrictions on the fraction of a business that can be foreign owned. Black treats these costs as a proportional tax. Since the tax is proportional, investors with short positions in foreign assets get a subsidy in this model. This is an incorrect assumption because the costs which Black considered do not permit such a treatment. Some of these costs have to be met regardless of whether one has a net investment overseas or one is in a short position in overseas assets. There is no way that investors in a short position in overseas assets would get a subsidy. Therefore Black could have assumed as an alternative that the costs increase whether the amount of net investment overseas or the short position in overseas assets increases. If the costs of foreign transactions faced by an investor were nil, then his investment portfolio would consist of a risk free asset and the world portfolio. However, in the presence of costs to investment abroad, optimal portfolios tend to be heavy in domestic assets. Black (1974) stated that the costs to overseas investment can be measured by comparing the average return on the minimum variance zero beta portfolio Z and the average short term interest rate across countries and time. If the costs are zero, the expected return on portfolio Z will be the average short term interest rate, and the world CAPM will hold, ie. investors hold portfolio Z and the world market portfolio. Where costs are positive, investors would also hold country portfolios that have minimum variance for a given level of costs faced by the residents of that country.

The costs to international investment derived using Black's framework were tested by I Cooper and E Kaplanis (1985). They have empirically estimated these costs, making use of the fact that investors do not hold an internationally diversified portfolio but one that is biased towards the domestic securities. Their assumptions include the following:

(i) The relevant costs of foreign investment are proportional to net foreign investment. This is reasonable for withholding taxes, safe custody fees and non-interest bearing deposits. However, they note that this is unlikely to be true for information gathering costs (which are more likely to be fixed), exchange control costs or the cost of expropriation risk. Proportionality may however provide the most suitable assumption.

(ii) All investors consume the same bundle of goods and purchasing power parity holds for this bundle. This implies that they do not include exchange risk as a determinant of portfolios.

(iii) Unlimited short sales are allowed, which, because of proportionality of costs assumed in (i) above, implies that investors who are short in foreign holdings, get a benefit instead of bearing deadweight costs.

Cooper and Kaplanis proceed by devising equations to show that investors should hold only a combination of the world market portfolio and zero variance portfolio if the deadweight costs are zero. The investors will deviate from holding the world portfolio to concentrate on the domestic portfolio if there are deadweight costs to overseas investment. The greater the deadweight costs, the greater will be this deviation in favour of domestic portfolio. They estimate the deadweight costs implied by empirical data on fourteen countries for the covariance matrix of returns to broad domestic indices and the market proportions. Further assumptions are necessary, otherwise their system of equations is under-identified. They therefore assume that

either (i) that all deadweight costs depend upon the country of origin of the investor, or

(ii) that deadweight costs depend upon the country of investment only, or

(iii) that the costs of cross investment between any two countries are the same.

The results show that the deadweight costs averaged 5% - 8% for countries with minimum or no exchange controls. In addition, the shadow cost of exchange controls was found to be high and this increased the deadweight costs considerably. The data, however, indicated fairly high costs for Hong Kong and Singapore which may be due to incorrect input of variance data for these economies.

These results concerning deadweight costs are far higher than what they had estimated by casual observations of the costs that could be estimated directly, without the use of the model. The casual observation of costs reveals that the safe custody fees range from only 0.07% to 0.35% of the asset value. Withholding taxes may be taken to be relatively small because they are usually mitigated by double taxation relief. For example, in the U.K. a standard rate tax payer would pay tax only at the difference between his tax rate and the foreign withholding tax rate. Only if the withholding tax rate is greater than the investor's U.K. tax rate is there a cost. For example, pension funds in the U.K. have nil U.K. tax and therefore they may bear the full burden of withholding taxes on foreign investment, to the extent that the double tax treaties do not provide any relief. However, for an average investor, the <u>additional</u> tax burden borne because of withholding taxes may be a fairly small percentage of foreign investment. Hence the quantifiable elements of costs of foreign investment alone are not responsible for the high costs to foreign investment. The explanatory role in their empirical study of the non-quantifiable elements of cost, eg. information costs, must be very high. Another possible reason why the costs are high could be the exchange rate risk, which, however was assumed away in this model.

The deadweight costs described above have implications for the cost of capital. The cost of capital for a U.K. investor investing in USA must be lower than the comparable cost for the US investor - otherwise it will not be worthwhile investing because of the burden of these deadweight costs. Similarly, for those US investors who invest in the U.K.. However, Cooper and Kaplanis in drawing this inference are in danger of not taking a proper account of the diversification and risk reduction motive in international investment. The benefit which an investor realises from an overseas investment may not be in the form of vastly higher returns, but could be in the form of greatly reduced volatility of his portfolio.

After examining the benefits and costs of international diversification, we look at the role of the multinational companies which constantly face such benefits and costs of international diversification.

## (B) Role of Multinationals

B Jacquillat and B Solnick (1978) have compared the returns from multinational corporations (MNCs) with those from internationally diversified portfolios. They found that the total risk associated with a portfolio of MNCs was lower (at 90%) than a portfolio of pure US domestic securities (the risk of which was taken as 100%). In contrast, the existing internationally diversified portfolios had only about 30% to 50% of the risk of the US domestic portfolio. Therefore they concluded that investment in MNCs only partially reduced the risk by diversification and that the potential for further risk reduction by international diversification remained. Multinationals were not the appropriate vehicle from the risk reduction point of view. One possible explanation for this result is that multinationals tend to concentrate on generating high returns, rather than on risk reduction, and secondly they also tend to be in the same industry in the different countries they invest in. Both these factors can imply that they are not the suitable vehicles for the diversification of risk.

Studies have also been conducted to test whether MNCs provide superior returns when compared to domestic firms. The rationale is that the MNCs are providing international portfolio diversification to investors who face high costs if they diversify on their own account. There should be a positive valuation effect when MNCs are compared to domestic firms if MNCs are able to provide this diversification more efficiently. Errunza and Senbet (1984) estimate the excess returns earned by MNCs compared to the domestic firms. Since the domestic CAPM literature yields empirical evidence that indicates that small firms earn positive excess returns and that P/E ratios have some explanatory power, they control for these effects in their tests. They find an association between excess valuation and direct overseas investment, and also find that this relationship increases with increases in the restrictions imposed by the US government on overseas security investments.

A study by A M Fatemi (1984) concludes that MNCs do not provide superior returns when compared to domestic firms on a risk adjusted basis. Indeed, MNCs which are in highly competitive industries provide negative abnormal returns. However, he found that MNCs do have lower risks - both in terms of the absolute variability of returns (total risk) and in terms of having lower beta factors (systematic risk) than the domestic firms.

In summary, the evidence therefore indicates that investment in MNCs only partially reduce risk relative to domestic investment and that there is no conclusive evidence that they provide excess returns.

CAPM literature on returns from foreign investment also includes International CAPM models, which essentially include the risky securities available worldwide in the market portfolio. The main International CAPM models in the literature are examined next.

**(C) International CAPM Models**

One of the earlier international CAPM models was that presented by Solnik (1974). His hypothesis is that security price behaviour is consistent with a single world market concept. The risk pricing relationship he used is:

$$E(r_i) - R_i = B_i (E[r_m] - R_m)$$

where: $r_i$ = the return on a security i

$R_i$ = the risk free rate in the country of security i

$R_m$ = an average riskless rate, calculated by using same weights as those used to calculate the market portfolio

$r_m$ = the return on the world market portfolio

$B_i$ = the international systematic risk coefficient of security i.

He concludes that investors would be indifferent between choosing a portfolio consisting of the original securities from the different countries or a portfolio consisting of the following three funds:

(i) a portfolio of risky stocks of companies in different countries, which is hedged against the exchange risk; this portfolio is referred to as the market portfolio
(ii) a portfolio of bonds, speculative in the exchange risk dimension, and
(iii) the risk free asset of the home country.

The composition of the first two funds is independent of the investor's preferences or nationality. The results of this model show that the return to a security consists of two elements. One of the elements, the risk premium of the security, is determined by the product of its systematic international risk (beta) and the world risk premium. The world risk premium is the excess of the return to the world market portfolio over a world bond rate. The second element of return is the risk free rate, which is defined as the return to the risk free asset in the home country.

The international model by Grauer, Litzenberger and Stehle (GLS) (1976) assumes that the exchange risk is due to only the different stochastic inflation rates. Therefore, the set of relative asset prices in a multi-commodity world is no different from that arising in capital market equilibrium with a single international currency. This is because the introduction of currencies does not alter the real wealth for the same reasons as the price level in a domestic environment does not alter real wealth. Security prices are determined by the relationship between the real rate of return to the security and the return to real aggregate wealth. In this setting, returns on an asset and returns on the world market portfolio can be related in international capital market equilibrium by using a Sharpe-Lintner type CAPM. In the model, the demand for risky and risk free capital does not depend only upon the risk preferences of the nationals of a country. It is more interrelated internationally and depends upon the risk preferences of consumers in the other countries, world real interest rates and the proportion of the world market portfolio represented by the domestic country's assets.

In their survey article, Dumas and Alder (1983) review the literature on the subject. They state that the optimal portfolio strategy for investors would be to hold a combination of

(i) the universal portfolio of risky assets and

(ii) a personalised portfolio which provides a hedge against inflation.

The universal portfolio will consist of stocks in all of the countries. Components of this portfolio could be held short to provide a hedge against the exchange risk. The personalised hedge portfolio will largely consist of domestic treasury bills denominated in home currency as they provide the best hedge against (unanticipated) inflation.

Both the Solnik and GLS models described above advocate the holding of a world portfolio of risky assets. Their market portfolio thus comprises securities from all the countries. The empirical evidence however shows strong bias in favour of domestic securities. Therefore, the model developed by Black (1974), and as further modified by Stulz (see below), is perhaps a fairly realistic model.

Stulz (1981a) modified Black's assumption regarding proportionality of costs to net foreign holdings. Black had assumed that the investors would face costs proportional to their net foreign holdings. Thus investors in short position in foreign holdings would receive a subsidy. Stulz is more realistic in assuming that investors would face costs to international holdings whether they were long or short in international securities. He concludes that in view of these costs, some foreign assets would not be held by domestic investors and that the world market portfolio is inefficient for investors who face barriers to international investment.

Stulz's major contribution, however, is in modifying Breeden's consumption based (domestic) CAPM so that it becomes an international model (1981b). He realistically assumes that the consumption opportunities available to individuals in different countries could differ. He concludes that the expected excess real return on a risky asset in an international portfolio is proportional to the covariance between the domestic rate of return of that asset and changes in the world real consumption rate. In this model, exchange rate risk is not rewarded as the assumption is made that the return on an asset does not depend on unexpected changes in the exchange rate. This model does not include barriers to international investment but Stulz asserts that it is more compatible with the empirical facts than other international pricing models.

In conclusion, it can be stated that there is a significant difference between the results of the international CAPM models, which conclude that international portfolio of securities should be held, and the empirical evidence which shows a significant bias towards domestic securities. The models on costs to international diversifications do not provide sufficient explanations for this divergence. Despite the lack of satisfactory international CAPM models, the literature on evaluating international projects is examined next.

## (D) Evaluation of International Projects

Models of international CAPM have not been universally accepted as being complete nor have they been rigorously tested empirically (one exception is Solnik's model which he tested empirically to find weak support for his International Asset Pricing Model). Therefore, for project appraisal, no one as yet advocates using international beta factors to arrive at the appropriate discount rates for project appraisal. Instead, there is considerable disagreement as to what constitutes risk in an international investment, and therefore there is disagreement as to the cost of capital for foreign investment.

If the international markets were integrated and there were no extra costs to foreign investment, then the relevant risk for a US company considering investment in say the Netherlands, would be the same as for any investment in the same business by a local Netherlands company. However, if markets are segmented, then the relevant risk for the US company is the risk relative to other US investments in the portfolio of the US investors. The two risk measures could differ considerably and therefore could result in two different cost of capital measures.

Secondly there is disagreement as to whether the exchange risk should lead to an increase, no change or decrease (because of the accompanying diversification) in the cost of capital. Brealey and Myers (1984) reject the notion of automatically increasing the cost of capital by certain percentage points whenever an overseas investment is considered. One reason is that the exchange risk, if present, may be offset by diversification benefit. The second reason is that the risk of appropriation etc. can be better incorporated in investment appraisal by decreasing the expected cash flows, rather than by changing the discount rate.

In view of the above, the discount rate for the appraisal of an international project can be calculated by either of the following two methods:

**1st Method**

Step 1 Estimate future cash flow in overseas currency.

Step 2 Convert to domestic currency using the forward exchange rates.

Step 3 Calculate NPV using domestic discount rates.

**2nd Method**

Step 1 Estimate future cash flow in overseas currency.

Step 2 Calculate present value by using discount rate applicable in overseas country which can be calculated by using exchange parity relationships

$$\frac{(1 + r\ (\text{dom}))}{(1 + r\ (\text{overseas}))} = \frac{\text{Forward rate (exch dom/overseas)}}{\text{Spot rate (exch dom/overseas}}$$

i + r (overseas) can be calculated given the domestic discount rate, spot rate and the forecast spot rate.

Step 3 Convert net present value to domestic currency by using the spot exchange rate.

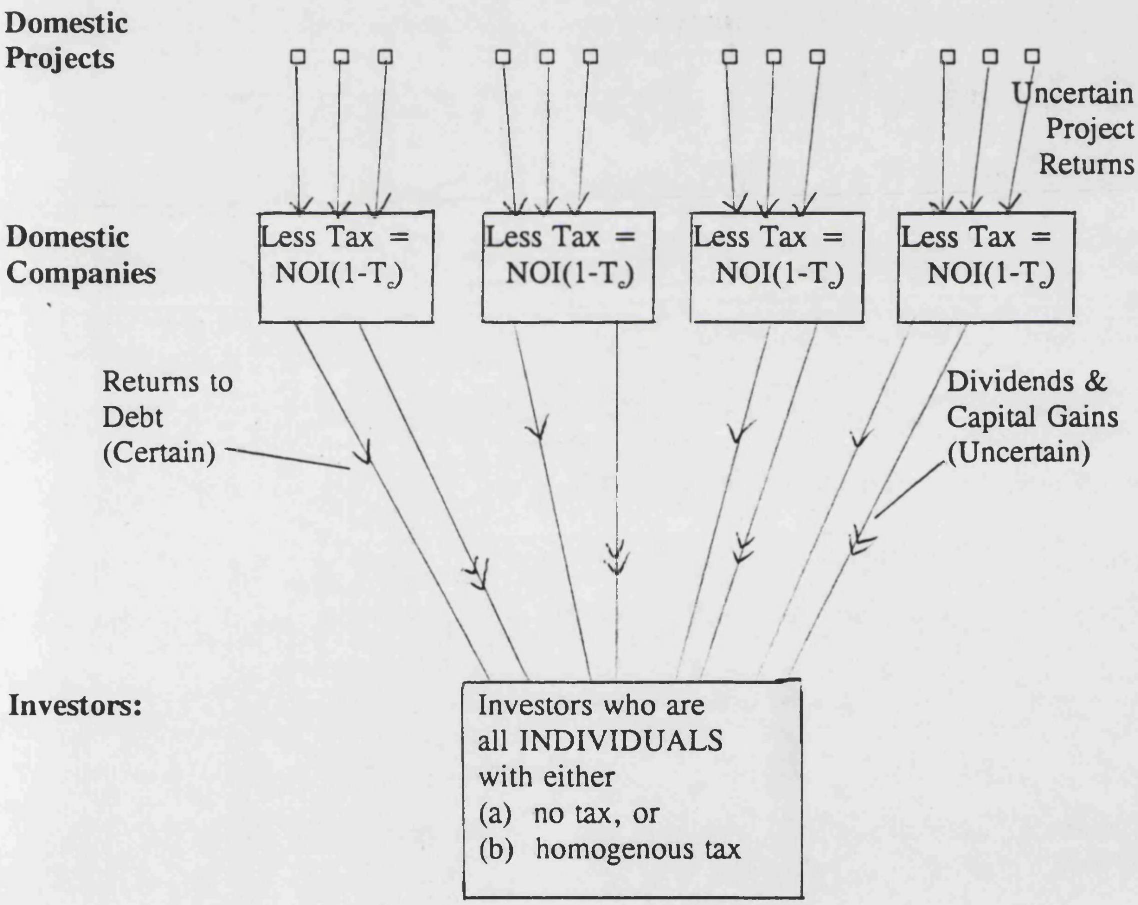
Of the above two, the former is probably the better method for international project appraisal. The main reason for this is because this method correctly defines risk from the investor's point of view. The calculation of the overseas discount rate in step 2 of the second method is artificial and unnecessary. Both approaches, however, represent gross simplifications when real world imperfections, in particular taxes, are introduced. The corporation tax rate may be different in different countries. This will complicate the appraisal of project across different countries. The withholding taxes on cash flows across countries have to be accounted for. Some countries have restrictions on the amount of cash that can be remitted to foreign investors. These restrictions are very important for evaluating highly leveraged transactions, which depend upon sufficient amount of the cash flows generated to be available in the right jurisdiction to service the debt. The forward exchange rates may be poor indicators of the future spot rates. For example, in early 1991, the forecast DM-Sterling exchange rate, which is based on interest rate differentials, is significantly outside the range of exchange rate bounds indicated by the European Exchange Rate Mechanism. Hence, in a practical context, the evaluation of foreign projects can be very complex indeed.

This thesis will examine the role of personal taxation in cost of capital for project appraisal. The vastness of that topic precludes an in-depth extension of the international CAPM models in this thesis to include these factors noted above. The international dimension nevertheless forms an important part of our summary of the literature in the next section.

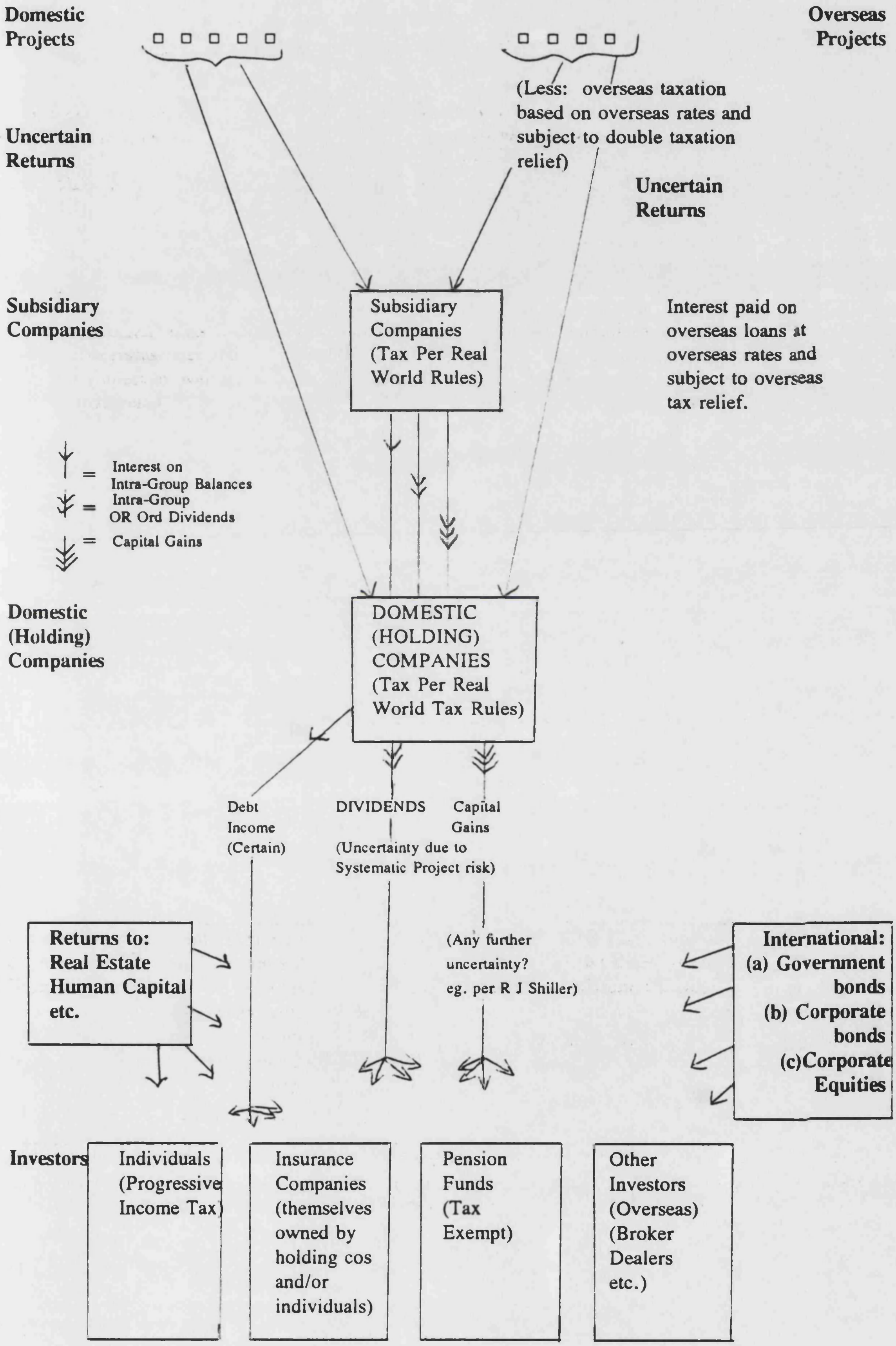
## SECTION E

## CONCLUSION TO THE SURVEY OF THE LITERATURE

In financial modelling, it is necessary to simplify the real world to obtain interesting, useful and manageable models. This is particularly true in the case of the before personal taxes models discussed in the survey, the earliest of which is the Modigliani and Miller (1963) paper described in section B above. The simplified version of the relevant players and variables which most of the before tax models incorporate is shown below:




The above diagram shows that the models generally concentrate only on the domestic capital market and projects. This is true of almost all of the models included in the survey. Thus the literature fails to take account of the opportunities afforded by the international sector for both risk reduction through diversification, and higher returns through attractive investments overseas. These domestic projects generate uncertain returns for the domestic companies. These returns are subject to corporation tax at the statutory rate. This is the assumption made by Miller (1977) although DeAngelo and Masulis (1980) explored the other relevant corporation tax provisions. In chapter 7 below, a much more realistic method of incorporating corporation taxes will be presented. The companies pay out debt interest, dividend income and capital gains to the investors. Most of the models in the literature assume that the investors are individuals, and thereby ignore the fact that the majority of the corporate stock held is by institutions such as pension funds or by other corporate entities. All the before tax models assume that there is no personal taxes faced by the investors or at the most, the taxes are uniform across both the types of investors and the types of income streams. In reality, the tax rules applicable for individuals are quite different from those applicable for pension funds or the companies. Secondly, there are different tax rules for debt interest, dividend income and capital gains. Some of these factors are reflected in some versions of the after tax model, like the Modigliani (1982) model. They are reflected in the valuation model developed in this thesis in chapters 4 to 7. These and the other features and relationships which are relevant in examining capital markets equilibrium are shown in the following diagram. The diagram may be thought of as depicting an after tax international capital market.


Domestic Projects
Overseas Projects
(Less: overseas taxation based on overseas rates and subject to double taxation relief)
Uncertain Returns
Uncertain Returns
Subsidiary Companies
Subsidiary Companies (Tax Per Real World Rules)
Interest paid on overseas loans at overseas rates and subject to overseas tax relief.
= Interest on Intra-Group Balances
= Intra-Group OR Ord Dividends
= Capital Gains
Domestic (Holding) Companies
DOMESTIC (HOLDING) COMPANIES (Tax Per Real World Tax Rules)
Debt Income (Certain)
DIVIDENDS
Capital Gains
(Uncertainty due to Systematic Project risk)
Returns to: Real Estate Human Capital etc.
(Any further uncertainty? eg. per R J Shiller)
International: (a) Government bonds (b) Corporate bonds (c)Corporate Equities
Investors
Individuals (Progressive Income Tax)
Insurance Companies (themselves owned by holding cos and/or individuals)
Pension Funds (Tax Exempt)
Other Investors (Overseas) (Broker Dealers etc.)

The figure, although complex in itself, does not pretend to be comprehensive. However, it does point out some of the players and inter-relationships in the capital markets which are essential for modelling capital market equilibrium. The diagram highlights the role of the overseas sector. These overseas projects yield uncertain returns which are subject to taxation in the overseas countries at the overseas rates and under the tax provisions applicable in their respective countries. There are withholding taxes on cash flows between countries and there is the issue of double taxation relief. Some of these overseas companies may have raised finance overseas, on which they will be paying the overseas nominal interest which could differ considerably from the domestic interest rate, even after allowing for the expected exchange rate movements. The domestic and overseas risky projects may be held directly by the holding company, or more commonly, by a subsidiary company. The subsidiary companies' corporation tax may differ substantially from the statutory tax rate because of many provisions in the tax code, like accelerated depreciation allowances. These subsidiary companies are themselves held by the holding companies. Thus a substantial part of the corporate equity in an economy is held by other corporate entities. In the U.K., there are separate rules for the taxation of cash flows within a corporate group, such as the exemption from taxation of the dividend income received from other U.K. companies. Poterba and Summers (1984, 1985) measured the impact of dividend taxes on corporate values in the U.K. but unless they also took into account the fact that some of the companies in their sample may be owned by other companies (and hence their dividends were free from any taxes at the "personal" tax level), their model would be mis-specified.

These holding companies provide debt interest income, dividend income and capital gains for investors. However, the investor has many other alternative sources of capital income. For example, the investor may invest in real estate, human capital, or overseas government or private bonds, or in overseas quoted equities. A precise market portfolio under CAPM should include all these assets in the market portfolio. Finally, the investor groups are quite diverse, particularly in so far as their "personal" taxation provisions are concerned. Therefore the economists who equate personal investor taxes with the income tax on individuals are very far from a sensible modelling of the actual situation. The investor group includes pension companies which are tax exempt. The investor group includes corporation - thus corporation tax paid by such companies, on their income from other companies, is a "personal " tax at the shareholder level.

Therefore, personal and corporate taxes (among the other factors) in reality are quite complex and incorrect conclusions may be derived if they are treated simplistically. This gap in the existing literature provides motivation for this thesis. We propose to develop a valuation model that incorporates some of the important features of the U.K. tax code and which recognises that corporate equity is owned by investors with varied tax characteristics. Subsequently, we aim to apply the model we develop to analyse interesting propositions.

As noted above, depicting reality is very complicated indeed. Most of the models either concentrate on the overseas aspects, and assume away the tax issues, or vice versa. This thesis will concentrate on the taxation aspects. The overseas dimension will not be addressed in order to keep the model tractable. From the literature surveyed, the model that comes closest to incorporating most of the taxation characteristics mentioned above is the Modigliani (1982) model. Therefore this model will be the starting point of our investigation into the cost of capital in the presence of more realistic personal tax features.

Since we have emphasised that the individuals do not own most of the equity, it is interesting to look at who actually owns the companies. This is shown below:

**Beneficial Share Ownership in the U.K. 1957-1975**

| | **1957** | **1975** |
|---|---|---|
| Individuals | 79.44% | 54.02% |
| Tax Exempt institutions | 5.89% | 21.63% |
| Insurance Companies | 9.78% | 18.01% |
| Overseas | 4.89% | 6.34% |
| | 100% | 100% |

Source: King & Fullerton (1984), page 67

The official surveys of the London Stock Exchange show that institutions have increased their share further. As the above analysis shows, individual investors do not monopolise the ownership of corporate equity. Therefore, if the owners of corporate equity are heterogeneous both in terms of the personal tax codes applicable as well as the tax rates, how can a company determine its cost of capital? The next few chapters provide an answer to this question.

As previously stated, the motivation for this thesis is to develop an appropriate valuation model to enable us to derive the cost of capital for U.K. companies, owned by heterogenous investors, in the presence of the U.K. imputation system and the main features of the U.K. tax code.

In order to do so, we develop the Modigliani model (referred to in chapter 2) by providing detailed derivation in chapter 4. The model developed is applicable under the "classic" system of taxation, such as that prevalent in the U.S.A. We then develop models applicable under the imputation, two rate and the integrated system of taxation in chapter 5. In chapter 6, we provide practical guidance on calculating the variables, including the more difficult ones, which are needed in order to estimate the cost of capital using the model we develop. In chapter 7, we add on useful analysis of the corporation tax code in the U.K. to our valuation model. We thus derive the valuation model in equation 7.22 which meets the criteria set out above.

We use our model presented in chapter 7 for analysis in the subsequent chapters. In chapter 8, we use our model to analyse the difference that the personal tax changes introduced in the 1988 budget made to discount rates that U.K. companies should be using for project appraisal. In chapter 9 we analyse the extent to which personal taxes influence the discount rate. In chapter 10, we apply our model to evaluate the claims made in the study of U.K. taxes by the Meade Committee. In chapter 11, we use our model to help derive a tax system that is tax neutral under uncertainty, mixed capital structures and supernormal profits. All these applications provide ample evidence of the advantages of our model over the simpler models reviewed in this and the previous chapter.

# ASSET PRICING MODEL IN THE PRESENCE OF HETEROGENOUS PERSONAL TAXES

## Section A - Introduction

The previous chapters survey the existing literature in finance in general with particular emphasis on personal taxation. Personal taxes in the context of cost of capital is the central theme of this thesis. The thesis proceeds in this chapter by describing one main CAPM model which includes taxation. The model which is described below is the Modigliani (1982) model referred to in chapter 2. This model is analysed in detail below because it forms the foundation for the valuation equation we derive in chapter 7.

Modigliani (1982) model is chosen because it a comprehensive model which allows for different personal tax rates across different types of investors as well as across different sources of income. This makes the model more realistic than models which assume homogeneous investors and it overcomes one of the deficiencies in most of the prior literature in which all of the investors are assumed to be individuals. Modigliani derives interesting tax factors which are relevant for determining market equilibrium. These tax factors are weighted averages for all investors and therefore these are more plausible than some of the other tax factors, such as the tax rates of the marginal investor, used in the literature. One of the variables used in the model is Earnings Before Interest and Taxation. This is a variable used in practice by most investment banks in their analysis. Modigliani also incorporates inflation in his model, thereby making it fairly comprehensive.

Therefore this model is (a) comprehensive, (b) it includes what may be regarded as realistic practical variables, and (c) it is capable of being modified, extended and it is suitable for useful analytical work. Its suitability for analytical work is illustrated by the work done in the following chapters in general, and the evaluation of the impact of the 1988 budget (in chapter 8) and the critique of the Report by the Meade Committee (in chapters 10 & 11) in particular. Hence, it is considered to be the most suitable model for the purposes of this thesis. Therefore, in this chapter, we analyse the Modigliani

(1982) model equation by equation. The original article by Modigliani is a journal article and therefore it is fairly cryptic. Since the model in this article is fundamental to the analysis in this thesis, it is important to analyse it in detail.

The contributions made by the analysis in this chapter include :

(a) providing a more detailed and a more elegant narrative description of the Modigliani (1982) model. In the Modigliani article, there is little or no narrative description of the model.

(b) providing the links between the various equations of the model. The Modigliani article describes the model in 3 pages only and therefore includes only a minimum number of equations. An understanding of the links between the various equations are necessary in order to gain confidence in the results of the model and also to point out couple of omissions in the Modigliani article. These oversights include (1) the definition of beta factors used by Modigliani and (2) the calculation of weighted averages for tax factors.

(c) providing links between this model and the other CAPM-tax models in the literature. This analysis in this chapter results in pointing out couple of apparent inconsistencies in the analysis by Brennen (1970).

(d) focusing in more detail on the impact of taxes.

This chapter focuses on the Modigliani (1982) model which implicitly applies under the "classical" system of taxation ( as in the U.S.A. ). The following chapter ( Chapter 5 ) revises the model to derive equations which are suitable for the Imputation ( as in the U.K. ), Two-Rate and the Integrated systems of taxation. Chapters 6 and 7 extend the revised model to include more of the corporation tax features found in U.K. tax code. Subsequent chapters use the revised model for further analysis of the impact of taxes on the cost of capital.

Modigliani (1982) model referred to above incorporates the following features:

(i) Investors expect a real return from their investment in equity of the companies, after allowing for fully anticipated inflation. Thus the model is dealing with *real monetary value* (that is, pound value) returns to investors as opposed to *nominal* *rates* of returns. This feature is relevant in describing definitions of the means and variances of returns to the investors and to the market. This is important for reconciling the results of Modigliani (1982) model with the other models in the literature.

(ii) Investors face different rates of personal taxation, both on income from dividends and interest (income tax) and on capital gains (capital gains tax).

(iii) Any cash retained in the company is assumed to result in an equivalent capital gain (that is, a pound of net of tax retained earnings result in a pound of capital gain).

(iv) Investors are risk averse, but the degree of risk aversion may differ and is not constant across investors.

(v) In the first model described below it is assumed that the cash flows associated with debt and with dividends are both riskless and permanent. Therefore, the only random variable in the return expected by the investors relates to capital gains. Capital gains, on the assumption of certain debt-interest and dividends, are dependent on the stochastic earnings before interest and tax (EBIT) generated by the companies.

The following notation, originally used by Modigliani, is used in the model which is described subsequently equation by equation. This notation is the fundamental notation used in this thesis. It may differ from the notations used when other models are described in this thesis (in which case the notations used by those authors have been maintained to ease understanding of comments on their works).

| | | |
|---|---|---|
| $\tau_c$ | = | corporation tax |
| $\tau_g$ | = | capital gains tax |
| $\tau_p$ | = | personal income tax |
| $\theta_x$ | = | $1-\tau_x$ where x = c, g or p above ($\theta_x$ therefore refers to the net of tax flows, that is $\theta_c \mu$ is cash flow $\mu$ net of corporation tax at the rate $\tau_c$) |
| $\mu$ | = | cash flow earnings before interest and tax (EBIT) |
| $\mu^*$ | = | $\theta_c \mu$ ie. EBIT x $(1-\tau_c)$. This does not equal accounting post-tax profit. Instead it is EBIT cash flow net of corporation tax levied on the entire EBIT. It differs from accounting post-tax profit because of different assumptions regarding depreciation, interest expense, and corporation tax deduction for interest expense. |
| [M] | = | variance-covariance matrix of corporation tax-adjusted cash flows, $\mu^*$ |
| S | = | market value of equity |
| D | = | nominal value of corporate debt |
| V | = | S+D = market value of firm |
| $\Delta$ | = | dividend payment |
| R | = | nominal rate of interest |
| p | = | rate of inflation |
| r | = | R-p = real rate of interest |
| $r_p$ | = | $\theta_p$ R-p = real rate of interest after personal taxes |
| $r_c$ | = | $\theta_c$ R-p = real interest payment, net of corporation tax cost of interest payment |
| $n_i^m$ | = | the proportion of equity of firm i held by an investor m |
| $u_1^m$, $u_2^m$ | = | the derivative of the utility function with respect to the mean and variance respectively of the portfolio of investor m |

| | | |
|---|---|---|
| Superscript | = | an investor |
| Subscript | = | a firm |
| Superscript ~ | = | random variable |
| **Bold letter** | = | column vector |

The symbol "x" means multiplication and not any variable x, unless otherwise stated.

The equation numbers on the left hand side refer to equations in this thesis, starting with number 4.1 below. The references to the equations in Modigliani's article (1982) appear on the right hand side and are prefixed M.

The mean and variance of portfolio return to an investor, m, are derived in section B below. These are used in section C to determine the optimal portfolio of risk free and risky equity assets for the investor, m. The results of this are used in section D to determine the market equilibrium for optimal portfolio allocation in the presence of risk and taxes. Market risk premium is determined in section E and is used to calculate a valuation equation for the corporate sector as a whole. This is extended in section F to determine the valuation equation for equity in individual companies. In section G, this model is contrasted with a second model presented by Modigliani.

## Section B - Mean & Variance Of A Portfolio

Modigliani begins by stating the real return expected by an investor who holds some equity in a company i. The real return expected by investor m ($y_i^m$), from holding a fraction $n_i^m$ of the equity of firm i, net of all taxes (corporate and personal) is given by:

$$(4.1)\quad y_i^m = n_i^m \{ [\mu_i - \tau_c (\mu_i - RD_i) - RD_i + pD_i - \Delta_i]\, \theta_g^m + \Delta_i\, \theta_p^m - \tau_g^m\, pS_i \} \qquad \text{(MII.1a)}$$

The above return includes 3 components : (a) retained earnings, (b) cash dividend and (c) taxation of nominal, as opposed to real, capital gains (as for example, in the U.S.A.). These components are explained below.

(a) Retained Earnings : The term within square brackets refers to retained earnings. It consists of total earnings before interest and taxation ($\mu_i$). It is arrived at after deducting (i) (ii) and (iv) below and adding (iii) below to the stochastic EBIT figure ($\mu_i$) :

(i) Interest payments ($RD_i$). These are calculated at the nominal rate of interest on the amount of debt outstanding.

(ii) Corporation Tax. This is calculated as $\tau_c(\mu_i - RD_i)$, because interest payments are assumed to be an allowable expense for corporation tax purposes (which they are in practice in most countries). One assumption implied by this definition of the determination of corporation tax is that depreciation does not affect corporation taxes. Net of interest cash flow earnings ($\mu_i$ - $RD_i$) are the same as the income for corporation tax purposes. Thus Modigliani does not consider (i) tax effects of depreciation, (ii) depreciation itself or (iii) any reinvestment required to maintain operating capacity. These issues are examined and incorporated in our valuation model in chapter 7.

(iii) The term $pD_i$ in the expression is positive in inflationary environment. Equity investors are assumed to anticipate returns in real terms. They are therefore assumed to gain if the rate of inflation is positive because the debt owed by the company is assumed to be fixed in nominal terms. Debt holders therefore lose $pD_i$ in real terms because of inflation. The benefit is transferred to equity investors. Hence the term $pD_i$ due to decline in real value of debt outstanding is <u>added</u> to the return to equity investors.

(iv) $\Delta_i$. This represents dividends paid out to the investors which consequently reduces the retained earnings.

The expression in square brackets above, ie. $[\mu_i - \tau_c (\mu_i - RD_i) - RD_i + pD_i - \Delta_i]$ thus refers to retained real earnings. These retained earnings are assumed to convert immediately to an equivalent capital gain. This assumption will be discussed later. This gain is taxed at personal capital gains tax rate of $\tau_g^m$ which is specific to the investor. Returns net of this tax are therefore denoted by multiplying the above expression by $\theta_g^m$ (which equals ($1-\tau_g^m$) ). As explained earlier, $\theta_g^m$ is a net-of-capital gains tax factor. Another assumption implied in this model is that capital gains are taxed on an accruals basis rather than on a realisation basis. This assumption is unrealistic and it is discussed and corrected in chapter 6. Moreover, the analysis assumes constant marginal rate of capital gains tax.

(b) <u>Cash Dividend</u> : The return to an equity investor in company i includes dividends $\Delta_i$. These dividends are subject to personal income tax at the rate $\tau_p^m$, specific to individual investor m. Net of personal tax dividends are shown by $\Delta_i \theta_p^m$ where $\theta_p^m$ equals ($1-\tau_p^m$). It should be noted that $\theta_p^m$ will also be used to denote net of personal tax debt-interest income in this model when dealing with the income to debt fund providers. This implies that an individual investor m bears the same income tax rate $\tau_p^m$ on income from equity dividends or from debt interest. This equality of tax rates may be true in the U.S.A. but is not true for the U.K. . In the U.K., personal tax applicable on debt interest income is different from the tax rate on post-corporation-tax dividend income (due to imputation system of taxation). We use the correct definition of personal taxes applicable under the imputation system from chapter 5 onwards.

(c) Finally, the term $\tau_g^m\, pS_i$ is deducted from the total return. This is necessary because the equity investor is assumed to not only pay tax on real capital gains, but also on the increase due to inflation (at rate p) on nominal shareholding (S). In the U.S.A., capital gains tax is levied on nominal as opposed to real gains. This increase in nominal value, which is given by $pS_i$, results in a tax cost of $\tau_g^m\, pS_i$, where $\tau_g^m$ is the personal capital gains tax rate for investor m. Note that in the U.K., since 1982, an indexation allowance mitigates this term. This is discussed in more detail, and the model corrected, in the following three chapters when the Modigliani (1982) model is adapted for the U.K. tax structure.

The expression in equation (4.1) thus describes the real return expected by an equity investor. It is given in the Modigliani article by the first equality in MII.1a. It transforms into the second equality in that article as follows:

(4.1) $y_i^m = \quad n_i^m \{ [\mu_i - \tau_c (\mu_i - RD_i) - RD_i + pD_i - \Delta_i]\, \theta_g^m + \Delta_i\, \theta_p^m - \tau_g^m\, pS_i \}$

(4.2) $y_i^m = \quad n_i^m \{ [(\mu_i - \tau_c\, \mu_i) + \tau_c\, RD_i - RD_i + pD_i]\, \theta_g^m - \Delta_i\, \theta_g^m + \Delta_i\, \theta_p^m - \tau_g^m\, pS_i \}$

Note that $\mu_i - \tau_c\, \mu_i = \mu_i^*$ by definition. And then, collecting terms involving $D_i$ and $\Delta_i$ transforms (4.2) above into :

(4.3) $y_i^m = \quad n_i^m \{ [\mu_i^* - D_i (-\tau_c R + R - p) ]\, \theta_g^m + \Delta_i (\theta_p^m - \theta_g^m) - \tau_g^m\, pS_i \}$

The expression in the first of the round brackets refers to the factors that determine the cash flows which relate to debt. The real (net of inflation) cost of debt is R-p. However, debt shelters earnings from bearing corporation tax (through interest payments) and this benefit at the rate $\tau_c R$ is therefore recognised in the equation. $R - \tau_c R = R\theta_c$ , the net of corporation tax interest rate by definition and $R\theta_c - p = r_c$ , the real net of corporation tax interest rate. Therefore the coefficient term of $D_i$ in the round brackets above is $r_c$. This yields :

(4.4) $y_i^m = \quad n_i^m \{ [\mu_i^* - D_i (r_c)] \theta_g^m + \Delta_i (\theta_p^m - \theta_g^m) - \tau_g^m pS_i \}$ (MII.1b)

which is the second equality of equation MII.1b in the Modigliani article. Equation (4.4) above describes the real return to investor m on the risky equity investment in company i. Since the investor m will hold equity (represented by the holding of $n_i^m$ fraction of the equity of company i) in several companies, the expected return from all of the equity investments is given by the summation of R.H.S.of (4.4) above for all companies.

Investor m, with wealth $w^m$, may also hold risk-free assets. The amount invested in risk-free assets is equal to the total wealth less investment in equity, that is $w^m - \Sigma n_i^m S_i$. The investor earns a nominal return R on this risk free investment and bears personal income tax at the rate $\tau_p^m$. Since this risk free asset is denominated in nominal terms, inflation means that in real terms an investor will suffer a loss because their investment is not hedged against inflation. Inflation at rate p therefore results in the investor losing $p(w^m - \Sigma n_i^m S_i)$ of value in real terms by holding risk-free assets. Thus the investors' real return after personal taxes on non equity investment is:

$$(w^m-\Sigma n_i^m S_i) R - (w^m-\Sigma n_i^m S_i) R\tau_p^m - p (w^m-\Sigma n_i^m S_i)$$

$$= (w^m-\Sigma n_i^m S_i) [ (R - R\tau_p^m) - p ]$$

(4.5) $= (w^m-\Sigma n_i^m S_i) [ R (1-\tau_p^m) - p ]$

By definition, $(1-\tau_p^m) = \theta_p^m$, therefore $R(1-\tau_p^m) = R\theta_p^m$. Therefore the term in the square brackets is $R\theta_p^m - p$, which, by definition, is $r_p^m$, that is, the real interest rate after personal taxes. Therefore the return to investor on non equity investment is $(w^m-\Sigma n_i^m S_i)r_p^m$ , which enables expression (4.5) above to be stated as :

(4.6) Real net of tax return on risk free asset = $(w^m-\Sigma n_i^m S_i)r_p^m$

Total return to investor m from risky investment in companies i and the risk-free assets is thus given by adding the summation of R.H.S. of equations (4.4) and the R.H.S. of equation (4.6).

(4.7) $y^m = \quad (w^m-\Sigma_i n_i^m S_i)\, r_p^m + \Sigma_i n_i^m [ (\mu_i^* - r_c D_i)\, \theta_g^m + \Delta_i (\theta_p^m - \theta_g^m) - \tau_g^m pS_i ]$ (MII.2)

Equation (4.7) above is the same as Equation MII.2 in the Modigliani article. This gives the mean return expected by the investor m and it will be used later below in the determination of the optimum portfolio for the investor. In addition to the portfolio mean, the determination of the optimum portfolio requires calculation of the portfolio variance. This is explained below.

**Variance of Portfolio Returns :**

In general, the variance of $\tilde{z}$, a linear combination of fractions $b_i$s of n random variables $\tilde{x}_i$, is given by the following:

(4.8) $\text{Var}(\tilde{z}) = \Sigma_{i=1}^{n} \Sigma_{j=1}^{n} b_i \, b_j \, \text{cov}(\tilde{x}_i, \tilde{x}_j)$

The only stochastic variables in the mean return $y^m$ considered in (4.7) above are the $\mu_i^*$s. Dividends , interest rate and inflation rate are all assumed to be known. In equation (4.7) above, the $\mu_i^*$s influence $y^m$ through $n_i^m$ $(\mu_i^*)$ $(\theta_g^m)$ - all other terms are non-stochastic. Therefore variance of the portfolio return for the mth individual ( $[\sigma_y^m]^2$) can be written as shown below. The symbol "x" means multiplication and not any variable x in the equations that follow in this thesis:

$$[\sigma_y^m]^2 = \Sigma_i \Sigma_j n_i^m \theta_g^m \times n_j^m \theta_g^m \times \text{cov}(\mu_i^*, \mu_j^*)$$

$$= \theta_g^m \theta_g^m \Sigma_i \Sigma_j n_i^m n_j^m \mu_{ij}^* \qquad [\text{ let } \mu_{ij}^* = \text{cov}(\mu_i^*, \mu_j^*)\ ]$$

(4.9) $[\sigma_y^m]^2 = (\theta_g^m)^2 \Sigma_i \Sigma_j n_i^m n_j^m \mu_{ij}^*$ (MII.3)

The above is equation MII.3 of the Modigliani article which gives the portfolio variance of return to investor m ( $[\sigma_y^m]^2$). The portfolio mean and variance are used in section C below to calculate the optimal portfolio for investor m.

## Section C - Optimisation

**Utility Function**

As is common in the literature, Modigliani assumes that an investor's utility is determined by the mean and variance of portfolio returns. Thus the investors' objective function can be written as :

(4.10) $u^m = u^m [\ y^m, (\sigma y^m)^2\ ]$

Let $u_1^m$ and $u_2^m$ represent the derivative of the above utility function (4.10) with respect to the portfolio mean (monetary) return and the portfolio (monetary) variance respectively. In common with the literature, an assumption in this model is that increases in expected return increase utility ( ($u_1^m > 0$), and that increases in variance decrease utility ($u_2^m < 0$). The utility functions could either be quadratic utility functions or at least be capable of being described by two or fewer parameters. Risk aversion measure for the utility function (4.10) above is given for investor m by $\gamma^m$ which is defined below :

(4.11) $\gamma^m = \dfrac{-2\,u_2^m}{u_1^m}$ where the derivatives of the utility function are as described in the above paragraph.

(4.11) above is a standard measure of risk aversion in the literature. If investor m' is more risk averse than investor m", then $\gamma^{m'}$ will be greater than $\gamma^{m''}$. An implicit assumption in this model, which is set in the mean-variance framework, is that higher moments (skewness etc) are not relevant for the analysis. The work on higher moments is limited. Therefore, in common with most of the literature, the analysis in this model is carried on within the mean-variance framework. The utility function in equation (4.10) above is optimised as described below.

**Constrained Optimisation**

Individual m maximises his utility subject to a budget constraint. The budget constraint states that amounts invested in risk-free asset plus equity cannot exceed total wealth $w^m$, that is,

(4.12) $w^m - (w^m - \Sigma n_i S_i) - \Sigma n_i S_i = 0$

Optimisation can proceed by rearranging and substituting in the budget constraint shown in equation 4.12 above directly in the objective function. Alternatively, a Lagrangean can be formed by adding the budget constraint explicitly. Using this approach, the individual's objective is to maximise the following objective function :

(4.13) $L = u^m \{ \bar{y}, (\sigma_y^m)^2 \} - \lambda \, [ \, w^m - (w^m - \Sigma_i n_i^m S_i) - \Sigma_i n_i^m S_i \, ]$

where $\lambda$ is the Lagrangean multiplier on the investor m's wealth constraint. In (4.13) the investor m can maximise his utility, consistent with his risk-return preferences, by choosing the proportions $n_i$'s of the risky assets (and consequently also the risk free asset) that he holds. The investor's utility will be maximized when the derivative of the utility function with respect to each $n_i$ is a stationery point. Therefore, differentiating (4.13) with respect to each and every choice variable, that is $n_i^m$, we obtain equation (4.14). The differentiation is carried out using the chain rule and equations (4.7) and (4.9) above.

(4.14a) $\frac{\delta L}{\delta n_i^m} = ( \delta U/\delta y^m \times \delta y^m / \delta n_i^m ) + ( \delta U/\delta(\sigma^{ym})^2 \times \delta(\sigma^{ym})^2/\delta n_i^m ) + S_i\lambda - S_i\lambda$

(4.14b) $\frac{\delta L}{\delta n_i^m} = u_1^m \times \{ -S_i r_p^m + (\mu_i^* - r_c D_i)\, \theta_g^m + \Delta_i (\theta_p^m - \theta_g^m) - \tau_g^m pS_i \}$

derivative of utility w.r.t. mean $y^m$ — derivative of mean $y^m$ w.r.t. changes in $n_i^m$

$+ \{ u_2^m \times [ 2 (\theta_g^m)^2 \Sigma_{j=1}^n \mu_{ij} n_j^m ] \} \qquad + S_i \lambda - S_i \lambda$

derivative of utility w.r.t. portfolio variance — derivative of variance w.r.t. changes in $n_i^m$

For an optimal point, the above derivative should equal zero.

$\frac{\delta L}{\delta n_i^m} = 0$ at optimum

Therefore $u_1^m$ {derivative of $y^m$ $_{\text{(as in bracketed element of (4.14b))}}$} $+ u_2^m \{ 2 (\theta_g^m)^2 \Sigma_{j=1}^n \mu_{ij} n_j^m \} = 0$

$\Rightarrow \qquad u_1^m$ {derivative of $y^m$} $= -2\, u_2^m (\theta_g^m)^2 \Sigma_{j=1}^n \mu_{ij} n_j^m$

(4.15) $\Rightarrow$ {derivative of $y^m$} $= \frac{-2\, u_2^m}{u_1^m} (\theta_g^m)^2 \Sigma_{j=1}^n \mu_{ij} n_j^m$

As stated in the above discussion of the utility function, the first term after the equality in 4.15 represents risk aversion factor $\gamma^m$. Substituting $\gamma^m$ in (4.15) above, we obtain

(4.16) {derivative of $y^m$} $= \gamma^m (\theta_g^m)^2 \Sigma_{j=1}^n \mu_{ij} n_j^m$

Substituting in the detailed expression for the derivative of $y^m$ from equation (14b) into (4.16), we derive the following relationship :

$$(4.17)\quad -S_i\, r_p^{\ m} + [\, (\mu_i^{\ *} - r_c\, D_i)\, \theta_g^{\ m} + \Delta_i\, (\theta_p^{\ m} - \theta_g^{\ m}) - \tau_g^{\ m}\, pS_i\, ]$$
$$= \gamma^m\, (\theta_g^{\ m})^2\, \Sigma_{j=1}^{n}\, \mu_{ij}\, n_j^{\ m}$$

The above holds for investor m for all companies i. It shows the relationship, at an investor's optimal utility point, between the variables of company i (S,D and $\Delta$), investor m's risk aversion ($\gamma^m$) and the covariance between earnings of company i with the other companies in the economy ($\mu_{ij}$). This expression (4.17) can be made more tractable by the following transformations. Let $\boldsymbol{\mu}^*$ **D and** $\Delta$ be vectors representing $\mu_i^{\ *}$, $D_i$ and $\Delta_i$ for all companies. $\Sigma_{j=1}^{n}\, \mu_{ij}\, n_j^{\ m}$ can be represented for all companies by $[M]n^m$ where $[M]$ is the variance-covariance matrix of $\mu_i^{\ *}$s and $n^m$ is a column vector of proportion of shares held by investor m in different companies. The product of the appropriate row of [M] and column vector $n^m$ is equal to $\Sigma_{j=1}^{n}\, \mu_{ij}\, n_j^{\ m}$ considered above. Equation (4.17) is stated in terms of one company, company i. It holds true for all other companies as well. Therefore making use of the notation described in this paragraph, equation (4.17) transforms into the following equalities for all companies.

$$(4.18)\quad -\mathbf{S}\, r_p^{\ m} + (\boldsymbol{\mu}^* - r_c\, \mathbf{D})\, \theta_g^{\ m} + \Delta\, (\theta_p^{\ m} - \theta_g^{\ m}) - \tau_g^{\ m}\, p\mathbf{S} = \gamma^m\, (\theta_g^{\ m})^2\, [\mathbf{M}]\, \mathbf{n}^m$$

Collecting terms involving **S**, the equity value, we obtain :

$$(4.19)\quad (\boldsymbol{\mu}^* - r_c\, \mathbf{D})\, \theta_g^{\ m} - \Delta\, (\theta_g^{\ m} - \theta_p^{\ m}) - \mathbf{S}\, (r_p^{\ m} + p\tau_g^{\ m}) = \gamma^m\, (\theta_g^{\ m})^2\, [\mathbf{M}]\, \mathbf{n}^m \qquad \text{(MII.4)}$$

The above illustrates how Equation II.4 of the Modigliani model is obtained. This is used to determine the market equilibrium in section D below.

## Section D - Market Equilibrium

To solve for the market equilibrium for investors' portfolio allocation in the presence of risk and uncertainty, Modigliani relies on the work by Brennan (1970) (Brennan was his PhD student). The procedure to obtain market equilibrium can be sub-divided into 4 steps:

(a) calculating and substituting the market risk aversion factors into (4.19)

(b) summation over all investors

(c) describing tax factors

(d) solving for the Equity Value (S)

**(a) Market risk aversion factor ($\Lambda$) :** This is the harmonic mean of the investors' risk aversion factors ($\gamma^m$) , where the latter are weighted by the capital gains tax factors ($(\theta_g^m)^2$). This is shown in Modigliani (1982) as:

$$(4.20) \quad \Lambda = \frac{1}{\Sigma_m \dfrac{1}{\gamma^m (\theta_g^m)^2}}$$

One oversight made by Modigliani and Brennan is that the summations have also to be weighted by the proportion of investor m's wealth in the market. In their present form these equations, like equation (4.20) above, imply that an investor owning only £1 of assets and an investor owning a million pounds of assets have equal weighting in the determination of the market risk aversion. This is incorrect and the correct form is used in the numeric illustration of the calculation of market risk aversion in the next chapter. The market risk aversion factor is also influenced by the capital gains tax rate of the investors. If the gains tax rate increases, then the corresponding $\theta_g^m$ will decrease, thereby reducing the impact of that investor's risk aversion on the market risk aversion factor. In other words, higher rate tax payers have a smaller impact on market risk aversion in the presence of taxes, as compared with models which ignore taxes. Payment of taxes mitigates the risk effect. This tax factor is relevant with the increasing presence of tax-exempt pension funds in the market. Their risk aversion characteristics may therefore be more relevant than those of individuals who pay taxes @ 40% in the U.K..

To obtain the equilibrium conditions, both sides of equation (4.19) are multiplied by $\dfrac{\Lambda}{\gamma^m (\theta_g^m)^2}$, therefore

$$(4.21) \quad \{ (\mu^* - r_c D)\, \theta_g^m - S(r_p^m + p\tau_g^m) - \Delta (\theta_g^m - \theta_p^m) \} \times \frac{\Lambda}{\gamma^m (\theta_g^m)^2}$$

$$= \frac{\Lambda}{\gamma^m (\theta_p^m)^2}\, \gamma^m (\theta_p^m)^2\, [M]\, n^m$$

The factor $\Lambda / \gamma^m (\theta_p^m)^2$ above shows the ratio of the market risk aversion to investor m's risk aversion and it will be used below to help sum equation (4.21) over all investors.

**(b) Summation over all investors :** Equation (4.21) holds for all investors. To obtain market equilibrium, it is summed over all investors while taking into account the following points. In this summation $\mu^*$, $r_c$, D, S, $\Delta$, $\Lambda$ and [M] are common for all investors. Therefore these terms remain unchanged in the summation. Tax factors $\theta_g^m$, $r_p^m$, $\tau_g^m$, $\theta_p^m$ vary across investors and these are summed to obtain the market tax factors. In this summation, due account is taken of the fact that equation (4.21) has the coefficient $\Lambda / \gamma^m(\theta_p^m)^2$ for each investor m. $n_i^m$ varies across investors but $\Sigma_m n_i^m = 1$ for all i since $n_i^m$ refers to the fraction of the equity of company i owned by the investor m. Therefore summing over all types of investors, equation (4.21) transforms into :

(4.22) $(\mu^* - r_c D)\, \theta_g - S(r_p + p\tau_g) - \Delta(\theta_g - \theta_p) = \Lambda\, [M]1$ (MII.5)

$$\text{where } \theta_g = \Sigma_m \theta_g^m \times \frac{\Lambda}{\gamma^m (\theta_g^m)^2}$$

$$r_p = \theta_p R\text{-}p \quad \text{where}$$

$$\theta_p = \Sigma_m \theta_p^m \times \frac{\Lambda}{\gamma^m (\theta_g^m)^2}$$

$$\tau_g = \Sigma_m \tau_g^m \times \frac{\Lambda}{\gamma^m (\theta_g^m)^2}$$

**(c) Tax Factors :** Since equation (4.22) is a summation of equations (4.21) for all investors, it contains market variables instead of variables which are specific to investor m. The market tax terms above are weighted sums of investors' tax rates where the weights represent investors' risk aversion. Again the oversight by Modigliani and by Brennan is that the tax factors need to be weighted by the proportion of market value of assets owned by the investor otherwise the market tax rates will be the distorted. Thus, as stated earlier, the weights $\Lambda / \gamma^m (\theta_p^m)^2$ need to take into account the relative size of the investors' assets. The market tax rates are influenced by the investors' risk aversion through the weights $\Lambda / \gamma^m (\theta_p^m)^2$. The greater the risk aversion of an investor, the lower is the weightage given to their tax rate in the formation of the relevant tax rates for the purposes of valuation in the market.

Although the market tax rate factors above look very complex, they can be used in a practical context by using the following simplified procedure advocated in this thesis:

(i) subdivide investors into 3 main categories: individuals, corporate investors and pension funds. These three categories differ in terms of personal tax characteristics.

(ii) use the proportions of total corporate equity owned by the investors in each of the above 3 categories to simplify calculations.

(iii) The complex tax expressions can be calculated by using (i) and (ii) above and theoretical and some empirical work regarding the degree of substitution between return and variance of investors. These tax factors in respect of the U.K. capital market are calculated in chapter 6. This thesis is perhaps the first place that numerical values have been derived for such variables in the literature.

**(d)Equity Value (S) :** Equation (4.22) above can be rearranged to calculate the equity value (S).

$$(4.23)\quad (\mu^* - \tau_c \mathbf{D})\, \theta_g - \Delta(\theta_g - \theta_p) - \Lambda\, [\mathbf{M}]\, \mathbf{1} \;=\; \mathrm{S}(\mathrm{r_p} + \mathrm{p}\tau_g)$$

$$(4.24)\quad \text{or } \mathbf{S} = \frac{(\mu^* - \mathrm{r_c}\mathbf{D})\, \theta_g - \Delta(\theta_g - \theta_p) - \Lambda\, [\mathbf{M}]\, \mathbf{1}}{\mathrm{r_p} + \mathrm{p}\tau_g}$$

Equation (4.24) gives the market equilibrium value for the vector of equity (S) in terms of a certainty equivalent formulation. This equation will be simplified further in the next section.

Equation (4.24) gives the value of equity as (i) a risk and tax adjusted cash flow divided by (ii) a risk-free but tax adjusted discount rate, where :

**(i) The adjusted cash flow** is corporate earnings less real after corporate-tax debt-interest. These corporate earnings are subject to a capital gains tax deduction which reduces the cash flow to $(\mu^* - \mathrm{r_c}\mathrm{D})\theta_g$. $\Delta(\theta_g - \theta_p)$ is subtracted to reflect any extra tax on dividends and this term will be positive if $\tau_p$ is higher than $\tau_g$. There is also a deduction to compensate for risk. This deduction is based on the product of (a) the sum of covariances (this sum results from the appropriate row of [M] multiplied by **1** vector) and (b) a term incorporating risk preferences of the investors who own the capital assets in the economy. Altogether, this adjusted cash flow expression is a certainty equivalent cash flow appropriately adjusted for the relevant taxes.

**(ii) The discount rate is** the real risk free interest rate after deducting the weighted average of personal income taxes on the investors($\tau_p$). As stated above, this weighted average reflects the tax rates of the investors as well as their risk preferences. The discount rate also includes an element to compensate for taxation of the inflationary element of gain in share prices ($p\tau_g$). This element arises because when we first specified the equity returns to an investor ( equation 1), the cash flows were specified in real terms and it was noted that capital gains tax in most countries is levied on the nominal capital gains.

The next section concentrates on the market risk premium which will be useful in converting equation (4.24) from a certainty equivalent basis to a CAPM based equation.

## Section E - Market Risk Premium

Starting with equation (4.24) of the previous section, the calculation of the market risk premium can be sub-divided into 3 steps :

(a) calculating the total value of all firms

(b) calculating the market risk premium

(c) calculating the total value of aggregate corporate sector using market risk premium

These 3 steps are shown below:

**(a) calculating total value of all firms**

This is done by simply adding a vector representing the value of debt in the companies to both sides of equation (4.24), and rearranging terms on RHS to isolate the impact of leverage. This results in the following equation :

$$(4.25)\quad \mathbf{S} + \mathbf{D} = \frac{(\mu^* - r_c\mathbf{D})\,\theta_g - \Delta(\theta_g - \theta_p) - \Lambda\,[\mathbf{M}]\,\mathbf{1}}{r_p + p\tau_g} + \mathbf{D}$$

$$(4.26)\quad \Rightarrow \mathbf{V} = \frac{\mu^*\,\theta_g - \Delta(\theta_g - \theta_p) - \Lambda\,[\mathrm{M}]\,1}{r_p + p\tau_g} - \frac{r_c\,\mathbf{D}\,\theta_g}{r_p + p\tau_g} + \mathbf{D}$$

$$(4.27) \Rightarrow V = \frac{1}{r_p + p\tau_g} [\mu^* \theta_g - \Delta(\theta_g - \theta_p) - \Lambda [M] 1 ] + D \left(1 - \frac{r_c \theta_g}{r_p + p\tau_g}\right) \quad \text{(MII.7)}$$

The above equation (4.27) gives the total value of the companies. The first term on the R.H.S. is still a certainty equivalent term and the only change from the previous equation concerns the coefficients of the debt term D. The coefficients of the term D (which are the leverage factor L) have been pulled together in the last term of the above equation. The factors that determine its value can be seen more clearly by expanding the components of the term and making the simplifying assumption that the rate of inflation p is zero. The leverage term (L) expands to the following :

$$(4.28) \quad L = 1 - \{ [ (1-\tau_c)R / (1-\tau_p)R ] \times (1 - \tau_g) \}$$
$$(4.29) \quad L = 1 - \{\theta_c \times \theta_g / \theta_p \}$$

Modigliani & Miller (1963) stated that in the absence of personal taxes, the value of leverage was $(1 - \theta_c) = \tau_c$. With the introduction of personal taxes, in the absence of inflation, the value of leverage depends on the ratio of personal income tax and the personal capital gains tax. If the personal capital gains tax rate is lower, then $\theta_g$ exceeds $\theta_p$ , which increases the expression in the curly brackets and this results in a lower benefit of leverage.

Equation (4.27) is used to calculate the market risk premium as shown below.

**(b) calculating the market risk premium**

This is obtained by summing over each individual equation contained in equation (4.27), in which all companies are included and represented by using the vector notation. Previously, we aggregated over all investors who invest in company i in order to derive valuation equations for an individual company. Those equations, however, include variables which are complex. In order to relate those equations with more easily recognisable variables, such as the market risk premium, we need to aggregate over all companies. Therefore, now we aggregate over all companies and the resulting equation is an aggregate equation for the corporate sector as a whole and is shown below :

$$(4.30)\quad V = \frac{1}{r_p + p\tau_g}\left[\mu^* \theta_g - \Delta(\theta_g - \theta_p) - \Lambda \operatorname{var}(\bar{\mu}^*)\right] + L\,D$$

In equation (4.30), the variance of overall market returns (var ($\bar{\mu}^*$)) is obtained because **[M] 1** is summed over all companies. This transformation is arrived at as follows:

the ith row of **[M] 1** $= \Sigma_{j=1}^{n} \mu_{ij}^*$

$= \operatorname{cov}(\mu_i^*, \mu_1) + \operatorname{cov}(\mu_i^*, u_2) + \operatorname{cov}(\mu_i^*, \mu_3)$

By property of covariances, the above is equal to

$\operatorname{cov}(\mu_i^*, \mu_1^* + \mu_2^* + \mu_3^* + \mu_4^* \ldots\ldots \mu_n^*)$

$= \operatorname{cov}(\mu_i^*, \mu^*)$ where $\mu^*$ is the aggregate corporate income (EBIT) net of corporation tax.

The above holds for ith firm and for all the firms. Since we are aggregating over all firms in equation (4.30) above, we have to therefore also aggregate $\Sigma_i \operatorname{cov}(\mu_i^*, \mu^*)$ which is equal to

$\operatorname{cov}(\mu_1^*, \mu^*) + \operatorname{cov}(\mu_2^*, \mu^*) \ldots\ldots + \operatorname{cov}(\mu_n^*, \mu^*)$

By property of covariances this equals

$\operatorname{cov}(\mu_1^* + \mu_2^* + \mu_3^* \ldots\ldots \mu_n^*, \mu^*)$

$= \operatorname{cov}(\bar{\mu}^*, \bar{\mu}^*)$

which is the variance of market returns used in equation (4.30) above. Therefore

$$(4.30)\quad V = \frac{1}{r_p + p\tau_g}\left[\mu^* \theta_g - \Delta(\theta_g - \theta_p) - \Lambda \operatorname{var}(\bar{\mu}^*)\right] + L\,D$$

By transferring $\Delta(\theta_g - \theta_p)$ and L D to LHS we get

$$(4.31)\quad V - L\,D + \frac{\Delta(\theta_g - \theta_p)}{r_p + p\tau_g} = V^* = \frac{\mu^* \theta_g - \Lambda \operatorname{var}(\bar{\mu}^*)}{r_p + p\tau_g} \qquad \text{(MII.8)}$$

The above Equation (4.31) expresses the terms of equation (4.30) as equal to the value of an unlevered stream V*, ignoring any differential tax on capital gains and dividends. V* is the aggregate value of all the companies in the economy after excluding the impact on that valuation of the different rates of taxation of dividends and capital gains and after also excluding the benefit of leverage.

The RHS of equation (4.31) can be rearranged to give the market risk premium ($\pi$) as follows:

(4.32) $$V^* (r_p + p\tau_g) = \mu^* \theta_g - \Lambda \operatorname{var}(\tilde{\mu}^*)$$

(4.33) $$\Rightarrow (r_p + p\tau_g) = \frac{\mu^* \theta_g}{V^*} - \frac{\Lambda \operatorname{var}(\tilde{\mu}^*)}{V^*}$$

(4.34) $$\Rightarrow \frac{\Lambda \operatorname{var}(\tilde{\mu}^*)}{V^*} = \frac{\mu^* \theta_g}{V^*} - (r_p + p\tau_g) = \pi \quad \text{(MII.9)}$$

The market risk premium $\pi$ represents the difference between two terms (a) the market return ($\mu^*/V^*$), adjusted for corporation and personal gains taxes and (b) real interest rate after personal income taxes, adjusted by the inflationary tax factor. Thus, in this model, personal taxes are relevant for determining the market risk premium.

**(c) calculating total value of aggregate corporate sector using market risk premium**

The market risk premium in equation (4.34) above can be used to restate the value of the unlevered stream using a risk adjusted discount rate.

(4.35) $$\pi = \frac{\mu^* \theta_g}{V^*} - (r_p + p\tau_g)$$

(4.36) $$\Rightarrow \pi + (r_p + p\tau_g) = \frac{\mu^* \tau_g}{V^*}$$

(4.37) $$\Rightarrow V^* = \frac{\mu^* \theta_g}{\pi + r_p + p\tau_g}$$

$V^*$ is value of unlevered stream considered in (4.31) where it was stated that

(4.38) $$V - L\,D + \frac{(\theta_g - \theta_p)\Delta}{r_p + p\tau_g} = V^*$$

By substituting for $V^*$ from equation (4.37) into equation (4.38), we get

(4.39) $$V - L\,D + \frac{(\theta_g - \theta_p)\Delta}{r_p + p\tau_g} = \frac{\mu^* \theta_g}{\pi + r_p + \tau_g}$$

(4.40) $$\Rightarrow V = \frac{\mu^* \theta_g}{\pi + r_p + p\tau_g} + L\,D - \frac{(\theta_g - \theta_p)\,\Delta}{r_p + p\tau_g} \quad \text{(MII.10)}$$

Equation (4.40) above gives the total market value of the aggregate corporate sector in terms of (a) the post corporate and personal tax cash flow discounted by the risk and tax adjusted risk premium , (b) the tax benefit of leverage and (c) the contribution to value by the taxation of dividends at an income tax rate which may be different from the capital gains tax rate. Since this contribution is a sure stream (because the dividends as well as the tax rates are certain), it is discounted at the risk free rate. The risk free rate in this model is the post income tax real interest rate , plus a term which adjusts for the taxation of inflationary gains. In the next chapter, this model is revised to incorporate features of the U.K. tax system, such as the equality between the income tax rate and the gains tax rate, the imputation tax credit on dividends, and the indexation allowance for capital gains tax purposes.

## Section F

**Calculating total value of a firm using CAPM betas and market risk premium**

Equations similar to equation (4.40) can be derived for individual companies, where the market risk premium ($\pi$) is replaced by a risk premium specific to each individual company ($\beta_i \pi$). Modigliani defines $\beta_i = Cov(\mu_i^*/V_i^* , \mu^*/V^*)$, which is the covariance between the RATES of return on the unlevered streams for the company i and for the market. **As stated earlier, this definition is incorrect because Modigliani has overlooked dividing this covariance term by the variance of the rate of return to the market.** Thus the correct definition of the market risk premium in this model is $\beta_i = Cov(\mu_i^*/V_i^* , \mu^*/V^*) / Var(\mu^*/V^*)$. This is in accordance with the standard definition of beta factors in the literature, and is mathematically proved in appendix A1.

The valuation equation for an individual firm is derived from equation (4.40) above.

$$(4.41) \quad V_i = \frac{\mu_i^* \theta_g}{r_p + p\tau_g + \beta_i \pi} + L D_i - \Delta_i \frac{(\theta_g - \theta_p)}{r_p + p\tau_g}$$

Equation (4.41) states that the value of unlevered stream in the absence of dividends (ie. $\mu_i^* \theta_g$) is discounted by a risk adjusted discount rate ($r_p + p\tau_g + \beta_i \pi$) where the risk-free rate ($r_p + p\tau_g$) is calculated after deducting the weighted average of personal income taxes. The other influences on the aggregate value are through leverage (L D) or differences in personal tax rates on dividends and capital gains ($\Delta_i (\theta_g - \theta_p)$ ). The discount rate to be applied to these last two items is only $r_p + p\tau_g$ , that is, no risk premium is demanded since these cash flows are assumed to be certain in this model. Equation (4.41) is the main valuation equation derived from the above model by Modigliani. It will be used extensively in the rest of the thesis. Presently, in the next section, Modigliani's second model is presented. The second model is different in that it assumes that dividends as well as the cash flows associated with debt are stochastic. However, to keep the model tractable, Modigliani assumes that these cash flows are always a fixed proportion of the net of tax EBIT. This is described in the next section below.

## Section G - Modigliani's Second Model

This model is derived in Modigliani (1982) as an alternative to the model analysed above. The main changes from the previous model are that Modigliani assumes that the cash flows associated with dividends and debt are variable and that the following are constant:
(a) The payout ratio $\delta_i = \Delta_i/\mu_i^*$. It is ratio of dividends to net of tax EBIT ($\mu_i(1-\tau_c)$) .
(b) The debt proportion, which, for ease of use in calculating returns, is defined as $d_i = D_i/\mu_i^*$. This is an unusual definition with little practical relevance.
Using these assumptions, Modigliani calculates the total return to an investor m, that is the return to an investor owning all of both the debt and the equity of the firm as:

(4.42) $\tilde{y}_i^m = [\,\mu_i - \tau_c(\mu_i - RD_i) - RD_i + pD_i - \Delta_i\,]\,\theta_g^m + \Delta_i\,\theta_p^m - \tau_g^m\, pS_i$ (Equity return)
$+ RD_i - \tau_p^m RD_i - pD_i$ (Debt return)

The return to equity in this model is the same as in the first model. The real return on debt consists of interest rate less personal taxes on interest received, less the fall in real value of debt due to inflation.
Simplifying the flow to equity as was done for the first model, one gets

$$(4.43)\quad \tilde{y}_i^m = [\,\mu_i - \tau_c\,\mu_i + \tau_c\,RD_i) - RD_i + pD_i\,]\,\theta_g^m - \Delta_i\,(\theta_g^m - \theta_p^m) - \tau_g^m\, pS_i + D_i\,(R - \tau_p^m R - p)$$

Multiplying and dividing the terms with $D_i$ and $\Delta_i$ by $\mu_i^*$ (since $D_i/\mu_i^*$ and $\Delta_i/\mu_i^*$ are assumed constant), we get

$$(4.44)\quad \tilde{y}_i^m = [\mu_i^* + \mu_i^*\,\tau_c\,R\,\frac{D_i}{\mu_i^*} - \mu^*\,\frac{RD_i}{\mu_i^*} + \mu^*\,\frac{pD_i}{\mu_i^*}\,]\,\theta_g^m - \mu_i^*\,\frac{\Delta_i}{\mu_i^*}\,(\theta_g^m - \theta_p^m) - \tau_g\, pS_i + \mu_i^*\,\frac{D_i}{\mu_i^*}\,(R - \tau_p^m R - p)$$

Substituting $d_i = D_i/\mu_i^*$ & $\delta_i = \Delta_i/\mu_i^*$ in equation (4.44), we get

$$(4.45)\ \tilde{y}_i^m = [\mu_i^* + \mu_i^* \tau_c Rd_i - \mu^* Rd_i + \mu^* pd_i]\ \theta_g^m - \mu_i^* \delta_i (\theta_g^m - \theta_p^m) - \tau_g pS_i + \mu_i^* d_i [R(1-\tau_p^m) - p]$$

Taking $\mu_i^*$ outside the brackets in equation (4.45), we obtain,

$$\begin{aligned}
\tilde{y}_i^m &= \mu_i^* \{ [1 + \tau_c Rd_i - Rd_i + pd_i]\ \theta_g^m - \delta_i (\theta_g^m - \theta_p^m) + d_i r_p^m \} - \tau_g pS_i \\
&= \mu_i^* \{ \theta_g^m + \theta_g^m (\tau_c R - R + p)\, d_i + d_i r_p^m - \delta_i (\theta_g^m - \theta_p^m) \} - \tau_g pS_i \\
&= \mu_i^* \{ \theta_g^m - \theta_g^m ( R(1-\tau_c) - p)\, d_i + d_i r_p^m - \delta_i (\theta_g^m - \theta_p^m) \} - \tau_g pS_i \\
&= \mu_i^* \{ \theta_g^m - \theta_g^m r_c d_i + r_p^m d_i - \delta_i (\theta_g^m - \theta_p^m) \} - \tau_g pS_i
\end{aligned}$$

$$(4.46) = \mu_i^* \{ \theta_g^m + d_i (r_p^m - \theta_g^m r_c) - \delta_i (\theta_g^m - \theta_p^m) \} - \tau_g pS_i$$

In Modigliani (1982), the only equation given in respect of this second model is equation 4.46 above. The above interpretation and derivation is a contribution of this thesis. This total return to the investor as given by equation (4.46) consists of six elements which are discussed below.

(i) earnings before interest and corporation taxes ($\mu_i$)

(ii) less corporation tax (on all of EBIT - that is, on $\mu_i$)

(iii) less capital gains tax ( implicit in $\theta_g^m$). Capital gains tax is borne on retained earnings only. However, in this expression, personal capital gains tax is initially levied on all earnings after corporation tax ( that is on all of $\mu_i$). Differences in personal taxes on dividends are reflected in the subsequent terms ($\delta_i(\theta_g - \theta_p)$) of the expression in equation (4.46) above.

The above three elements give the term $\mu_i^* \theta_g^m$ which is EBIT net of corporation and personal capital gains tax. This is adjusted by the following terms.

(iv) The term $(r_p^m - r_c \theta_g^m)\, d_i$ inside the curly brackets in equation (4.46) measures the effect of debt on investor income. The term consists of:

(a)income to investor as debt holder

(b)loss of income to investor as an equity holder - the loss resulting from the presence of debt.

(a) is given by $r_p^m$ x $d_i$ x $\mu_i^*$ which is equal to $r_p^m D_i$ , the real interest rate after the personal taxes of the debt holder. (b) is given by $r_c \theta_g^m$ x $d_i$ x $\mu_i^*$ which is equal to $r_c D_i \theta_g^m$. This shows the net of capital gains tax loss of income to equity holders ,net of capital gains tax, when $D_i$ of debt is held.

(v) The term $\delta_i\,(\theta_g^m - \theta_p^m)$ measures the impact of differential rates of tax on dividends and capital gains. Only the difference is considered here since the total earnings are subjected to capital gains tax in (iii) above.

(vi) Finally, the term $\tau_g\, pS_i$, as in the first model, measures the loss of paying capital gains taxes on the inflationary element of any increase in the value of shares. This is included because the investor m is assumed to anticipate real returns, and capital gains taxes are assumed to be levied on *nominal* gains, as they arise, in the model.

The above describes the total return received by an investor holding equity and debt in company i. Using above, and following the procedures described in the previous sections, we obtain the following valuation equation :

$$(4.47)\quad V_i = \frac{\mu_i^*\,[\,\theta_g + d_i\,(r_p + p\tau_g - r_c\,\theta_g) - \delta_i\,(\theta_g - \theta_p)\,]}{r_p + p\tau_g + \bar{\beta}_i\,\pi}$$

This can be considered as an adjusted return divided by a risk adjusted discount rate. The adjusted return consists of:

(i) $\mu_i^*\,\theta_g$. This measures the EBIT earnings of company i subject to the standard rate corporation tax and a weighted average of capital gains tax. Thus earnings net of all taxes are considered. The impact of leverage on taxation, and of the differing tax rates on dividends are considered separately in the other terms discussed below.

(ii) $d_i\,(r_p + p\tau_g - r_c\,\theta_g)$. As stated earlier this term adjusts earnings for the impact of debt capital. It can be sub-analysed into two parts. Firstly, we can look at the income to a debt holder from holding debt in the company. This is reflected in the term $r_p$ which represents the real gain to the investor from holding debt in the company. Secondly we can examine the impact on the equity investor as a shareholder because of inflation in the presence of debt. This results in the term $p\tau_g$ being added in the above equation to reflect the gain to the investor as an equity holder because of a decrease in the real value of debt due to inflation. In addition to this, the term $r_c\,\theta_g$ measures the loss of income to the investor as an equity holder because of the presence of debt. This loss is equal to the real interest rate net of corporation and personal capital gains tax.

(iii) $\delta_i\,(\theta_g - \theta_p)$. In (i) above, all earnings are subject to capital gains tax. The taxes on dividends can differ and this term measures that difference.

The expression in the numerator in equation (4.47) above thus gives the total return after all taxes to investors owning both debt and equity in the company i. Unlike the first model, all the terms are discounted by the risk adjusted discount rate because in this model it is assumed that income streams relating to debt interest and dividends too are stochastic. In the first model, only the earnings cash flow was uncertain.

The discount rate in equation (4.47) consists of the following components: (a) the after personal tax real interest rate plus (b) an inflation adjustment. These two components determine the rate used to discount risk free streams. (c) risk premium calculated as $\beta_i\pi$ where $\pi$ is the market risk premium. Discount rates in this model are similar to discount rates in the first model. The implications of the second model, for value of leverage, tax advantage / disadvantage of dividends etc. have already been discussed in chapter 2.

The statement by Modigliani that the cash flows associated with debt and dividends are truly stochastic in this second model is an exaggerated one. The only independent source of stochasticity in the second model is the randomness associated with the earnings from projects, that is with the $\mu_i$s. Dividends and debt interest vary but only to the extent that they are fixed proportions of $\mu_i^*$ s, that is $\delta_i = \Delta_i/\mu_i^*$ and $d_i = D_i/\mu_i^*$ are fixed proportions in this model. In practice, the dividend pay out ratio term $\delta_i$ is often constant although companies tend to increase their pay out ratio if their earnings temporarily fall (in order to maintain the amount of dividend) and to decrease the pay out ratio if earnings jump (to smooth up the dividend increases over a number of years). The second term, $d_i$ , is of no practical significance and is contrived by Modigliani merely to derive elegant expressions such as in equation 4.46 above. Therefore, the second model does not add any genuine second source of stochasticity in the valuation model, but instead adds some contrived variables. Therefore the first Modigliani model is preferred in this thesis.

The next chapter examines alternative tax systems and their impact on the discount rate and makes use of the analysis developed in this chapter. For the reasons stated above, Modigliani's first model is the one that is extended and developed further in this thesis.

# ALTERNATIVE INTERNATIONAL TAX SYSTEMS

## Section A - The Classical System

The tax systems of various countries can be classified according to the degree of integration between the corporation tax system and the personal tax system. This integration is reflected in some form of relief at the personal tax level for the corporation tax borne by the investor on the income streams that are received. This relief usually involves a reduction in the personal tax borne by the investor on the income received in the form of dividends or capital gains on their investment in companies. Tax systems of commercially sophisticated countries falls into one of the following 4 categories depending upon the nature or degree of such relief:

(1) The Classical System
(2) The Imputation System
(3) The Two Rate System
(4) The integrated System

At one extreme is the classical system under which there is no integration at all. At the other extreme is the integrated system under which the personal and the corporate tax systems are fully integrated. This leaves partial integration as the only alternative, and the remaining two systems are the only two partial integration systems established in practice. These four types of tax systems therefore comprise an exhaustive list of the tax systems based on the degree of integration of the personal and corporate tax systems.

These systems and the valuation equations under each of these four systems are described below.

### I The Classical System

At one extreme in the above classification is the classical system under which there is no integration between the corporation and the personal tax systems. This system is characterised by the following:

(a) the company pays a flat rate of corporation tax on all taxable profits (that is, the same tax rate on retained earnings as well as on dividends)

(b) the shareholders are liable to personal income tax on the dividends that they receive. They do not receive any credit for the corporation tax paid by the company.

(c) the shareholders pay full rate of personal tax on the capital gains that accrue on their corporate investments.

Thus, this system implies that the income generated by the projects undertaken by the company is subject to taxation twice - once at the corporate level and once again at the personal tax level. This double taxation of corporate income is said to impose excess burden on the investors and thereby discourage corporate investment. This argument is examined in more detail in chapter 10 where the Report of the Meade Committee on the burden of taxation on corporate income is examined.

The classical system is used in the U.S.A., Australia, and some European countries such as Denmark, Holland, Luxembourg, Spain and Switzerland. It was tried in the U.K. from 1965 to 1973 but then it was replaced by the imputation system. The Modigliani model (1982) corresponds to the "classical" system of taxation. Thus in equation 4.1 in chapter 4, we deducted both the corporation taxes and the personal taxes at their full rates. The valuation equation (4.41) derived in chapter 4 is therefore an example of a valuation equation under the classical system of taxation. If the valuation equation (4.41) is transformed to express it over a common denominator, then the valuation equation becomes:

$$(5.1)\ V_i = \frac{\mu^* \theta_g + LD(r_p + p\tau_g + \beta_i\pi) - [\Delta_i (\theta_g - \theta_p) + \frac{\beta_i\pi\ \Delta_i (\theta_g - \theta_p)]}{r_p + p\tau_g}}{r_p + p\tau_g + \beta_i\pi}$$

In this equation, the cash flows in the numerator of equation (4.41) are adjusted to enable the entire expression to be discounted by a single risk adjusted discount rate in the denominator. The expression in equation (5.1) above is derived by multiplying and dividing the individual terms in equation (4.41) by ($r_p + p\tau_g + \beta_i\pi$), and simplifying as appropriate. Thus the term LD in equation (4.41) is multiplied by the above expression, and this enables it to be included in the expression in the numerator of equation (5.1) which is divided by a common risk adjusted discount rate term. Similarly, the other terms in the numerator of equation (4.41) are adjusted, although their adjustment includes some more simple algebra. This valuation equation under the classical system (5.1 above) may be contrasted to the equation under the imputation system derived in the next section.

## Section B - The Imputation System

The imputation system of taxation was introduced in some countries in order to relieve the excess burden of double taxation under the classical system of taxation. Thus under the imputation system of taxation, shareholders receive a tax credit along with cash dividends to alleviate some / all of the burden of taxing income at corporate as well as personal level. This tax credit implies that the shareholders are "deemed" to have paid a certain proportion of their personal tax on their dividend income and therefore the effective personal tax on dividend income borne by the shareholders is reduced. The characteristics of the imputation system are as follows:

(a) Company pays full flat rate of corporation tax on all taxable profits (that is on retained earnings as well as on dividends), which is the same as under the classical system.

(b) The shareholders receive a tax credit along with the cash dividend from the company. In the U.K., the tax credit is calculated as a proportion of gross dividend at the rate "s", where s is equal ( presently in the U.K. ) to the standard rate of personal income tax on individuals. If an investor receives net cash dividend of £1, his gross dividend ("G") is calculated by adding the tax credit Gs to the cash amount of £1, thus:

$G = (1 + Gs)$. This transforms into the following relationship:

(5.2) $G - Gs = 1$

(5.3) $G(1 - s) = 1$

(5.4) $G = 1 / (1 - s)$.

For cash dividend of the amount £$\Delta$, 5.4 transforms to:

(5.5) $G = \Delta / (1 - s)$ .

Equations 5.4 and 5.5 will be used below.

An investor with a personal income tax rate of $\tau_p^m$ has total tax liability of $G\tau_p^m$. Part of this liability is met by the tax credit Gs, leaving a net liability borne by the taxpayer of $G\tau_p^m$ - Gs, which can be written as:

(5.6) $G(\tau_p^m - s)$

Expression 5.6 shows that investors whose personal tax rate is equal to the imputation rate (for example, the standard rate tax payers in the U.K.) do not bear any further income tax on their dividend income. An investor has some personal income tax liability if his personal tax rate $\tau_p^m$ exceeds the imputation rate s. If investors, such as pension funds in the U.K., have a zero personal income tax rate, then, in the U.K., they receive a refund of the tax credit (sG) on their dividend income. Thus the imputation tax credit mitigates some of the disadvantage of the double taxation of dividend income. Note that full relief for the corporation tax suffered will occur only when the imputation rate s is equal to the corporation tax rate. Thus, with s below the corporation tax rate in the U.K., the imputation relief is only partial.

In some countries, such as France, the imputation rate (s') is calculated as a percentage of net cash dividend and not as a percentage of the gross dividend. The relationship $s' = s / (1 - s)$ enables one to transform the imputation rate from one system to the other. This relationship is used extensively in chapters 10 and 11.

(c) The capital gains on the disposal of shares are usually subject to a personal capital gains tax in the hands of the investors. This means that even if imputation credit is given at the rate of corporation tax rate on dividend income, the investors will still suffer some double taxation due to personal capital gains tax on the capital gains. This contrasts with the fourth type of taxation system, namely the integrated system, where the burden of double taxation is reduced by the imputation of tax credits on capital gains in addition to the tax credits on dividends. Therefore as far as the treatment of capital gains is concerned, the imputation system in most of the countries is similar to the effect of the classical system of taxes.

The rate of personal income tax applicable to an investor can be calculated by substituting for G from equation 5.5 into equation 5.6, to obtain

(5.7) $[\Delta / (1 - s)]\,(\tau_p^m - s)$ This expression simplifies as follows:

(5.8) $\Delta\,[\,(\tau_p^m - s) / (1 - s)\,]$

The equivalent expression for total corporate sector is :

(5.9) $\Delta\,[\,(\tau_p - s) / (1 - s)\,]$ where $\tau_p$ is a weighted average of the individual $\tau_p^m$s, calculated in the manner explained in Chapter 4, section D. **The expression within the square brackets in 5.9 will be referred to as $\tau_{pDIVI}$, that is , the personal income tax borne on dividends under the imputation system.** The cash flow net of the additional personal income tax on dividends under the imputation system($\theta_{pDIVI}$), is given by the following expression in the square brackets:

(5.10) $\Delta\,[1 - \{(\tau_p - s)/(1 - s)\}\,] = \theta_{pDIVI}$

This expression 5.10 is substituted into the "classical" valuation equation 5.1, to obtain the valuation equation under the imputation system of taxation:

(5.11) $$V_i = \frac{\mu^* \theta_g + LD(r_p + p\tau_g + \beta_i\pi) - \left[\Delta_i\left(\theta_g - \left\{1 - \frac{(\tau_p - s)}{1-s}\right\}\right) + \dfrac{\beta_i\pi\,\Delta_i\left(\theta_g - \left\{1 - \frac{(\tau_p - s)}{1-s}\right\}\right)}{r_p + p\tau_g}\right]}{r_p + p\tau_g + \beta_i\pi}$$

In equation 5.11,

(i) the expression within the square brackets deals with the impact on valuation of the differential taxation of dividends. It contains two main terms. The second of these main terms is the adjustment required to the numerator to enable the entire cash flow to be expressed over a common denominator. The first of the main terms measures the differential impact of taxation, namely $\theta_g - \theta_{pDIVI}$, where the latter is expressed in detail, as shown in expression 5.10 above. As stated in the survey, one possible consequence of the imputation tax credit can be that $\theta_{pDIVI}$ exceeds $\theta_g$. In that case there will be a tax advantage of dividends instead of the tax disadvantage. In chapter 3, it was also noted that the literature, which is based largely on the classical system of taxation, emphasised that the "dividend puzzle" was driven by the heavier burden of taxation on dividend income. The analysis in this paragraph suggests that there may be no dividend puzzle to explain in the case of imputation system of taxation. In chapter 8, it will be shown that presently (in 1992) in the U.K., there is a tax advantage to dividends.

(ii) $\tau_{pDIVI}$ (as defined in expression 5.9) replaces the personal income tax rate in the coefficient relating to personal income tax on dividends only. The other places where personal income tax appears, namely personal income tax levied on debt interest income, the relevant tax rate is still $\tau_p$. In the numerator of equation 5.11, the first term of the discount rate expression is the net of tax real interest rate. Since this term arises in the context of the interest rate on risk free instruments, the relevant tax rate to use is $\tau_p$, not $\tau_{pDIVI}$, even under an imputation system of taxation.

(iii) As a result of the discussion in (i) above, the valuation of companies under an imputation system of taxation as given by equation 5.11 will be higher than the valuation under a classical system of taxation with the same rates of taxation. This implies that the ***effective*** discount rate under an imputation system will be lower than that under a classical system.

The ***effective*** discount rate concept is used extensively in this thesis and is explained below. Consider two companies each of which generates £100 of EBIT (that is, $\mu$). Let us further assume that the discount rate ($r_p + p\tau_g + \beta_i\pi$) is the same for both companies. The value of these companies can still be different, despite same EBIT and discount rate, if they are subject to different tax systems, which result in different amount of cash flow leakages due to taxes. Thus the value of a company which has greater tax burden will be lower than the value of a company which has a smaller amount of tax leakages. This can be rephrased to state that the effective discount rate which is applied to pre tax cash flows (EBIT) for a company in a high tax burden regime is higher than the effective discount rate in a lower tax regime. The effective discount rate can be calculated by dividing EBIT (which is assumed to be at a constant level in perpetuity), by the value of the asset that produces that cash flow. This concept will be used to compare the net impact of different types or values of various alternative tax regimes, and the regimes which result in lower asset value for a given EBIT level, will have a higher effective discount rate and thus deemed to have a higher tax burden in comparison with the regimes which result in higher asset values.

An alternative method of giving relief for the double taxation of dividends is to use a lower rate of corporation tax on corporate earnings which are disbursed as dividends, and a higher rate of corporation tax on retained earnings. This would result in the Two Rate System which is discussed in the next section.

## Section C - The Two Rate System

Under this system, some relief is given for double taxation of dividends by charging a lower rate of corporation tax on distributed profits. Variations of the two-rate have been tried in West Germany, Japan, Finland and Norway.

In order to understand the valuation equation under this system, we need to consider the expected return for investor m under a two rate taxation system. Let the corporation tax on undistributed profits be $\tau_{cu}$ and that on distributed profits be $\tau_{cd}$. Now the expected return to the mth investor from holding a fraction $n_i^m$ of the equity of firm i, net of all taxes, is as follows:

$$(5.12)\quad y_i^m = n_i^m \{ [\mu_i - \tau_{cu} (\mu_i - RD_i - \Delta_i) - RD_i + pD_i - \Delta_i - \Delta_i \tau_{cd} ]\, \theta_g^m + \Delta_i\, \theta_p^m - \tau_g^m pS_i \},$$

where the terms within the square brackets equal retained earnings.

The above equation 5.12 is different from equation 4.1 of the Modigliani model (1982) as follows:

The corporation tax levied is now separated into 2 components :

(a) on retained earnings, that is on $(\mu_i - RD_i - \Delta_i)$, where corporation tax at the rate $\tau_{cu}$ is levied, and

(b) on dividends $\Delta_i$, where corporation tax at the rate of $\tau_{cd}$ is levied. $\tau_{cd}$ is lower than $\tau_{cu}$ in order to reduce the burden on the double taxation of dividends.

The valuation system for this type of tax regime will now be presented. We begin by expanding 5.12 as follows:

$$(5.13)\quad y_i^m = n_i^m \{ [\mu_i - \tau_{cu}\, \mu_i + \tau_{cu} RD_i + \tau_{cu} \Delta_i - RD_i + pD_i - \Delta_i - \Delta_i \tau_{cd} ]\, \theta_g^m + \Delta_i\, \theta_p^m - \tau_g^m pS_i$$

Let (i) $\mu_i^{*u}$ now represent $\mu_i(\theta_{cu})$, that is, cash flow EBIT net of tax at the rate applicable for undistributed earnings, and

(ii) $r_{cu}$ represent $\theta_{cu}$ R-p that is, real rate of interest net of corporation tax on retained earnings.

(iii) In equation 5.13, the terms involving dividend can be simplified as follows :

(5.14) $(\tau_{cu} \Delta_i - \Delta_i - \Delta_i \tau_{cd})\, \theta_g^m + \Delta_i\, \theta_p^m$

$$= \Delta_i\, \theta_p^m - \Delta_i\, \theta_g^m - \Delta_i\, \tau_{cd}\, \theta_g^m + \Delta_i\, \tau_{cu}\, \theta_g^m$$

(5.15) $= \Delta_i\, (\, \theta_p^m - \theta_g^m \{1 + \tau_{cd}\, \theta_g^m - \tau_{cu}\,\}\,)$

Substituting (i), (ii) and (iii) above, the expected return $y_i^m$ in equation 5.13 above can therefore be simplified to:

(5.16) $y_i^m \quad = \quad n_i^m\, [\, (\mu_i^{*u} - \tau_{cu}\, D_i)\, \theta_g^m + \Delta_i\, (\theta_p^m - \theta_g^m \{1 + \tau_{cd} - \tau_{cu}\}) - \tau_g^m\, pS_i\, ]$

Equation 5.16 gives the equity returns expected by the investor m. Investor m may also invest in risk free assets but their return is known and it does not contribute to the variance of the portfolio returns. The variance of the portfolio returns to an investor if dividends and debt cash flows are certain and perpetual, is as follows:

(5.17) $(\sigma_y^m)^2 \;=\; (\theta_g^m)^2\, \Sigma_i\, \Sigma_j\, n_i^m\, \mu_{ij}^{*u}\, n_j^m$

where $\mu_{ij}^{*u}$ is the covariance between net of <u>retained</u> corporation tax cash flows (that is, covariance between $\mu_i^{*u}$ and $\mu_j^{*u}$ as defined above). The relevant point here is that one of the consequences of assuming a certain dividend stream is that the associated corporation tax on dividend income is not relevant for determining the variance of the portfolio. Therefore, under this certain dividend assumption, only one of the two corporate tax rates, namely the tax on retained earnings, is of critical importance for the determination of the corporate values. This important role of the retained earnings corporate tax will be detailed after the valuation equation below.

Allowing for the above, and following the steps illustrated in chapter 4 sections B to F, we obtain a valuation equation for the two rate tax system. If we express the valuation equation as over a common denominator, (as in equation 5.1), then we obtain the following valuation model :

$$(5.18)\quad V_i = \frac{\mu_i^{*u}\,\theta_g + LD\,(r_p + p\tau_g + \beta_i\pi) - \left[\Delta_i\,(\theta_g\,(1+\tau_{cd} - \tau_{cu}) - \theta_p) + \dfrac{\beta_i\pi\,\Delta_i\,(\theta_g\,(1+\tau_{cd} - \tau_{cu}) - \theta_p)}{r_p + p\tau_g}\right]}{r_p + p\tau_g + \beta_i\pi}$$

The differences between the above equation, which is derived for the two rate tax regime, from equation 5.1 which is derived under the classical system of taxation, are as follows:

(i) $\mu_i^{*u}$ now is the net of corporation tax at the rate applicable for retained earnings, that is $\tau_{cu}$ is the relevant corporation tax rate. The relevant tax rate is not a weighted average of the two corporate tax rates. As explained above, since the dividends are certain, the relevant rate for determining retained earnings is $\tau_{cu}$.

(ii) Similarly the $\beta$ factors are now calculated to express the covariance between company's cash flows net of corporation tax rate on retained earnings.

(iii) The disadvantage of paying dividends is lowered relative to the classical Modigliani system (chapter 4) if $\tau_{cd}$ is lower than $\tau_{cu}$. For example, let $\theta_p = 1 - 60\% = 0.40$; $\theta_g = 1 - 30\% = 0.70$. $\tau_{cd} = 0.25$ and $\tau_{cu} = 0.50$. The coefficient of $\Delta_i$ in the first term within the square brackets above measures the tax that has to be paid because dividends are taxed differently from the retained earnings. Under the two rate system, the value of this term is:

$$\begin{aligned} & \theta_g\,(1.00 + \tau_{cd} - \tau_{cu}) - \theta_p) \\ = \quad & 0.70\,(1 + 0.25 - 0.50) - 0.40 \\ = \quad & 0.70\,(0.75) - 0.40 \\ = \quad & 0.525 - 0.40 \quad = \quad 0.125 \end{aligned}$$

Under the classical system, since $\tau_{cd} = \tau_{cu} = 0.50$, and therefore the tax coefficient of $\Delta_i$ is:

$$\begin{aligned} & \theta_g\,(1 + 0.50 - 0.50) - \theta_p \\ = \quad & 0.70 - 0.40 \quad = 0.30 \end{aligned}$$

Therefore, the coefficient is significantly lower under the two rate system than under the classical system. This implies that the tax disadvantage of paying dividends is lower under this two rate system as compared to the classical system. This example has been simplified by ignoring the second term in the expression

( $\beta_i \pi \Delta_i (\theta_g (1 + \tau_{cd} - \tau_{cu}) - \theta_p \;/\; r_p + p\tau_g$ ), which is an expression required simply to enable only one discount rate to appear in the denominator. It does not affect the conclusion reached in the simple example we used above. This example is also simplistic because it uses partial equilibrium analysis, that is, we do not attempt to raise equal amount of tax revenue under the alternatives considered using a more elaborate general equilibrium model.

(iv) The value of leverage in the two rate model is $1 - (\tau_{cu} / r_p + p\tau_g) \theta_g$, where $\tau_{cu}$ is as defined in this section, that is, the real interest rate net of corporation tax (at the rate for undistributed earnings) ( $\theta_{cu}$ R - p). Thus, once again, the relevant corporation tax rate is not a weighted average of the two tax rates as one may intuitively expect. The relevant tax rate is the rate for the retained earnings since it is the retained earnings only that are affected by the marginal changes in leverage in this model. The exact impact of leverage will depend upon the tax rates assumed.

Taking all the above into account, the corporate values will be higher under this two rate system than under the classical system. This is largely as a consequence of (iii) above. The ***effective*** discount rate will therefore be lower under the two rate system than that under the classical system. However, one common point between this system and the classical system is their common treatment of retained earnings. Under both these systems, the retained earnings bear full rate of both the corporate and the personal capital gains tax. Relief for personal capital gains tax is provided by the last one of the four systems, namely the integrated system which is described in the next section.

## Section D - The Integrated System

A system of taxation which would completely eliminate the tax discrimination between dividends and retentions is the integrated system where the personal and corporation tax systems are integrated. Under this system, there would be no corporate taxes per se since any corporation tax paid would be fully imputed to the shareholders. The imputation would be in respect of total corporate taxes, and not just on the proportion relating to dividends. Thus the shareholders would pay further taxes only on the difference between their personal income tax rate and the corporation tax rate. Tax refunds would arise if the personal taxes payable were below the corporation taxes imputed.

The results under this system would be the same if instead of imputing the corporation taxes, the profits of the firm were imputed to the shareholders in proportion to their shareholdings. This imputed income would be subject to personal income tax. Since retentions are also included in the imputed profit, there would be no capital gains tax. This latter approach is easier to incorporate in the model and therefore it is used below to illustrate this integrated system.

This integrated system was considered in Canada and West Germany but it was not implemented in either of them. The attraction of raising revenue by taxing companies as well as the investors appears to be very strong in all countries despite the detrimental impact on the discount rate and therefore on the investment undertaken in the economies. The impact of the alternative methods of taxing returns on investment is considered in detail in chapters 10 and 11 where the Report of the Meade Committee (1978) is examined. We continue here by deriving the valuation equation for companies under this integrated tax regime. The full model is derived in detail to enable one to examine the relevant intermediate variables as well.

Under the integrated system, the expected real return for an investor m owning $n_i$ proportion of shares in company i would be:

$$(5.19)\quad y_i^m = n_i^m \{ [\mu_i - RD_i]\, \theta_p^m + pD_i \}$$

Under this system, the cash flow earnings, less nominal payments of interest, are imputed to the shareholders and taxed at their personal income tax rates. Therefore the term $n_i^m[\mu_i\text{-}RD_i]\theta_p^m$ in equation 5.19 measures the return net of personal taxes. The second term $n_i^m \{pD_i\}$ reflects the gain accruing to shareholders when there is inflation at the rate p and the debt is fixed in nominal terms. For any level of dividends (up to the level of corporate profits ), the expected return will be the same. Hence the level of dividends does not affect the valuation equation.

Under this system, the total expected portfolio return to the mth individual with wealth

$w^m$ is:

$$(5.20a)\; y^m = (w^m - \Sigma_i n_i^m S_i) r_p^m + \Sigma_i n_i^m \{ [\mu_i - RD_i] \theta_p + pD_i \}$$

where $(w^m - \Sigma_i n_i^m S_i) r_p^m$ is the income from the riskless assets and $\Sigma_i n_i^m \{ [\mu_i - RD_i] \theta_p + pD_i \}$ is the income from equity investments, as described in equation 5.19 above. The variance of the portfolio is:

$$(5.20b) \qquad (\theta_p^m)^2 \Sigma_i \Sigma_j n_i^m \mu_{ij} n_j^m$$

where $\mu_{ij}$ is the covariance of $\tilde{\mu}_i$ and $\tilde{\mu}_j$ - the cash flows before <u>any</u> tax and interest. The relevant covariance terms therefore are before any taxes, in contrast to the post corporation tax variables used in the classical model.

The valuation equation under this system will be quite different from the valuation equation under the classical system. The first order conditions for a maximisation are obtained as follows:

$$(5.21)\; \max L = u^m \{ \bar{y}^m, (\sigma_y^m)^2 ] - \lambda [ w^m - (w^m - \Sigma_i n_i^m S_i) - \Sigma_i n_i^m S_i ]$$

$$(5.22)\; \frac{dL}{dn_i^m} = \frac{\delta u^m}{\delta \bar{y}^m} \cdot \frac{d\bar{y}^m}{dn_i^m} + \frac{\delta u^m}{\delta (\sigma_y^m)^2} \cdot \frac{d(\sigma_y^m)^2}{dn_i^m} + \frac{\delta u^m}{\delta n_i^m}$$

$$(5.23) = u_1^m \times \{ - S_i r_p^m + ( [\mu_i - RD_i] \theta_p^m + pD_i) \}$$
$$+ u_2^m \times [2 (\theta_p^m)^2 \Sigma_{j=1}^n \mu_{ij} n_j^m] - \lambda S_i + \lambda S_i$$

Equating the above to zero gives

$$(5.24)\; \mu_i^m \times \{ -S_i r_p^m + ( [\mu_i - RD_i] \theta_p^m + pD_i ) \}$$
$$+ u_2^m \times [2 (\theta_p^m)^2 \Sigma_{j=1}^n \mu_{ij} n_j^m] = 0$$

$$(5.25)\; - S_i r_p^m + (\mu_i - RD_i) \theta_p^m + pD_i = \frac{- 2 u_2^m [ (\theta_p^m)^2 \Sigma_{j=1}^n \mu_{ij} n_j^m ]}{u_1^m}$$

$$(5.26)\; - S_i r_p^m + (\mu_i - RD_i) \theta_p^m + pD_i = \gamma^m [ (\theta_p^m)^2 \Sigma_{j=1}^n \mu_{ij} n_j^m ]$$

Equation 5.26 states the first order conditions for equilibrium for one company i. Similar conditions apply for all companies. Therefore let **S**, $\mu$ and **D** be the vectors of $S_i$ $\mu_i$ and $D_i$ respectively for all companies. The equilibrium condition can thus be stated as:

(5.27) $- \mathbf{S}\, r_p^m + (\mu - R\mathbf{D})\, \theta_p^m + p\mathbf{D} = \gamma^m (\theta_p^m)^2\, [\mathbf{M}]\, \mathbf{n}^m$

where **[M]** and $n^m$ are the variance - covariance matrix and column vector of $n_i^m$ respectively.

(5.28) Let $\Lambda = \dfrac{1}{\Sigma_m \dfrac{1}{\gamma^m (\theta_p^m)^2}}$

Note that since in this system there are no personal capital gains taxes, and the retained earnings bear the personal income tax, the relevant tax factor in equation 5.28 is $\theta_p^m$, not $\theta_g^m$. Multiplying both sides of equation 5.27 by $\dfrac{\Lambda}{\gamma^m (\theta_p^m)^2}$ we get:

(5.29) $[ - \mathbf{S}\, r_p^m + (\mu - R\mathbf{D})\, \theta_p^m + p\mathbf{D} \;\; x \;\; \dfrac{\Lambda}{\gamma^m (\theta_p^m)^2} = \dfrac{\Lambda}{\gamma^m (\theta_p^m)^2}\, \gamma^m (\theta_p^m)^2\, [\mathbf{M}] \mathbf{n}^m$

Summing equation 5.29 over all investors, we get:

(5.30) $- \mathbf{S}\, r_p + (\mu - R\mathbf{D})\, \theta_p + p'\mathbf{D} = \Lambda\, [\mathbf{M}]\, \mathbf{1}$

(since $\Sigma_m n_i^m = 1$ for all i)

where $\theta_p = \Sigma_m \theta_p^m \;\; x \;\; \dfrac{\Lambda}{\gamma^m (\theta_p^m)^2}$

$r_p = \theta_p R - p$ where $\theta_p$ is as defined above, and

(5.31) $p' = \Sigma_m\, p \;\; x \;\; \dfrac{\Lambda}{\gamma^m (\theta_p^m)^2}$

While summing over all investors, terms such as p′ (5.31 above) will remain equal to p, that is, multiplication by $\Lambda/\gamma^m (\theta_p^m)^2$ and summing across all investors does not change the value of terms such as p ( that is, terms which do <u>not</u> vary across investors). The brief mathematical digression below is to prove that terms which do not vary across investors remain the same when summing equations over all investors.

As an example, let the corporate investors be subdivided into 3 types of groups. Assume that 25% of all investors are of type 1, characterised by $\gamma^1$ and $\theta_p^{\,1}$; 25% are of type 2, with $\gamma^2$ and $\theta_p^{\,2}$; and 50% are of type 3 with $\gamma^3$ and $\theta_p^{\,3}$ respectively. Suppose $\gamma^1 = 2.5$; $\gamma^2 = 2.5$; $\gamma^3 = 3$; $\theta_p^{\,1} = 1$; $\theta_p^{\,2} = 0.40$; $\theta_p^{\,3} = 0.65$. Further assume that the proportion of investors of the 3 types are as follows : type 1 investor = ¼; type 2 = ¼; type 3 = ½. Let p, the rate of inflation, equal 5 % . We need to input the above into the equation 5.31 above. To solve further, we need to calculate the value of $\Lambda$, the harmonic mean of the investors' risk aversion measure.

$$(5.32)\quad \Lambda = \frac{1}{\Sigma_m \dfrac{1}{\gamma^m (\theta_p)^2}} = 1 \Big/ \; \frac{1}{\frac{1}{4} \times 2.5 \times 1} + \frac{1}{\frac{1}{4} \times 2.5 \times (.4)^2} + \frac{1}{\frac{1}{2} \times 3 \times (.65)^2}$$

$$= \quad 1 \Big/ \quad \frac{1}{.625} + \frac{1}{.1} + \frac{1}{.63375}$$

$$= \quad 1 / \quad 1.6 + 10 + 1.578 = 1 / 13.178 = 0.07588$$

We substitute this value of $\Lambda$ in equation 5.31.

$$(5.33)\quad p' = 0.05 \times \frac{0.07588}{\frac{1}{4} \times 2.5 \times 1} + 0.05 \times \frac{0.07588}{\frac{1}{4} \times 2.5 \times (.4)^2}$$

$$+ \quad 0.05 \times \frac{0.07588}{\frac{1}{2} \times 3 \times (.65)^2}$$

$$= \quad .05 \times \left\{ \frac{0.07588}{.625} + \frac{0.07588}{.1} + \frac{0.07588}{.63375} \right\}$$

$$= \quad .05 \times \{ 0.121408 + 0.7588 + 0.11973$$

$$= \quad .05 \times 0.9999 = 0.05\ (=p)$$

This demonstrates that the value of a constant ( such as p) does not change when it is summed across investors, even when weights, such as $\Lambda / \gamma^m (\theta_p^{\,m})^2$ , are used for summation. Therefore multiplication by $\Lambda / \gamma^m (\theta_p^{\,m})^2$ and summing across all individuals (a) does <u>not</u> change terms such as p etc. which are common for all investors, and (b) it only changes terms which differ across investors, such as $\theta_p^{\,m}$, which, as described on the previous page , becomes

$$(5.34)\quad \theta_p = \Sigma_m \theta_p^m \times \frac{\Lambda}{\gamma^m (\theta_p^m)^2}$$

Using the above technique for summation, equation 5.30 can be rewritten as:

$$(5.35)\quad - S\, r_p + (\mu - RD)\,\theta_p + pD = \Lambda\,[M]\,1$$

Solving 5.35 above for S, we get

$$(5.36)\quad S = \frac{(\mu - RD)\,\theta_p + pD - \Lambda\,[M]\,1}{r_p}$$

The above expression gives the value of equity as an adjusted return divided by the risk free discount rate. The EBIT figure ($\mu$) is adjusted for interest payments on debt, taxes, the benefits to shareholders as a result of debt being fixed in nominal terms, and riskiness of cash flows. The discount rate is adjusted for the weighted average of investors' personal tax rates, where the weights depend both on the tax characteristics as well as the degree of risk aversion of the shareholders.

As in the case of the previous model in Chapter 4, one can obtain a valuation equation by undertaking the following steps:

(i) calculating total value of the firm

(ii) calculating market risk premium

(iii) calculating total value of aggregate corporate sector, using the market risk premium

(iv) calculating total value of an individual firm and of equity in firms using CAPM betas and market risk premium.

These operations are by now familiar and are carried out below, dispensing as far as possible with any detailed information which would be similar to that used with the earlier model in Chapter 4.

**(i) Total value of the firm**

Adding debt to both sides of equation 5.36, we get:

$$(5.37a)\quad S + D = \frac{(\mu - RD)\,\theta_p + pD - \Lambda\,[M]\,1}{r_p} + \frac{Dr_p}{r_p}$$

(5.37b)
$$V = \frac{\mu\,\theta_p - RD\,\theta_p + pD + Dr_p - \Lambda\,[M]\,1}{r_p}$$

$$= \frac{\mu\,\theta_p - D\,(R\,\theta_p - p - r_p) - \Lambda\,[M]\,1}{r_p}$$

But $R\,\theta_p - p = r_p$ by definition, (see chapter 4 or the notations definitions leaflet). Therefore,

(5.38)
$$V = \frac{\mu\,\theta_p - D\,(r_p - r_p) - \Lambda\,[M]\,1}{r_p}$$

$$= \frac{\mu\,\theta_p - D\,(o) - \Lambda\,[M]\,1}{r_p}$$

(5.39)
$$= \frac{\mu\,\theta_p - \Lambda\,[M]\,1}{r_p}$$

The valuation expression as given by equation 5.39 has the following characteristics. The value of the firm is independent of the level of debt of the firm. Since, given the assumptions of this model, there is no capital gains tax, and the payment of dividends does not give rise to any additional taxes, dividend policy too does not matter in this tax regime. Personal taxes, however, do enter in the calculation of the net of tax cash flow ($\mu\,\theta_p$) as well as the discount rate $r_p$.

That the value of the firm is independent of the debt level in this model can also be concluded by considering the value of leverage under the classical system of taxation. Under the classical system, L, as defined by Modigliani (Chapter 4 Equation MII.7) is:

(5.40)
$$L = \left(1 - \frac{r_c}{r_p + p\tau_g}\,\theta_g\right)$$

Since there is no capital gains tax in the integrated system of taxation, $\theta_g$ will equal one and $p\tau_g$ will be zero.

Therefore

(5.41) $L = 1 - \frac{r_c}{r_p}$

Using our original definitions (Chapter 4, section A), $r_c = R\theta_c - p$ and $r_p = R\theta_p - p$. Under this regime, the corporation pays tax at the personal tax rates of the shareholders, therefore $\theta_c = \theta_p$. Therefore $L = 1 - 1 = 0$: a result which shows that there is no benefit of leverage. This result is consistent with the valuation equation 5.39 above.

Note that in equation 5.38, the coefficient of **D** is ($r_p$ -$r_p$) whose cancellation makes debt irrelevant. It should, however, be stated that the first $r_p$ relates to $R\theta_p - p$ where R is the interest paid on debt. The second $r_p$ first arose in the context of risk-free rate in the economy. Therefore the entire analysis assumes that corporate debt is risk free and therefore interest paid on corporate debt is the same as the economy's risk free rate. The equilibrium, however, would be different if the two interest rates were different. Moreover, debt policy would no longer be irrelevant, despite the integrated system of taxation. There would be an impact of debt on project evaluation and on the cost of capital.

Equation 5.39 can be used to derive the market risk premium as follows.

**(ii) Market Risk Premium**

Summing the vectors in Equation 5.39, that is summing over all companies to obtain totals for aggregate corporate sector, we get

(5.43) $V = \frac{\mu\,\theta_p - \Lambda \operatorname{var}(\tilde{\mu}\,\theta_p)}{r_p}$

where V and $\mu$ are aggregate values for the corporate sector and $\operatorname{var}(\tilde{\mu}\,\theta_p)$ is the variance of the overall market return, net of taxes. Therefore

(5.44) $r_p V = \mu\,\theta_p - \Lambda \operatorname{var}(\tilde{\mu}\,\theta_p)$

$$\Rightarrow \quad r_p = \frac{\mu\,\theta_p}{V} - \frac{\Lambda \operatorname{var}(\tilde{\mu}\,\theta_p)}{V}$$

(5.45) $\Rightarrow \quad \frac{\Lambda \operatorname{var}(\tilde{\mu}\,\theta_p)}{V} = \frac{\mu\,\theta_p}{V} - r_p = \pi$ (the market risk premium)

Thus the market risk premium is the difference between the net of personal tax rate of return on the aggregate market and the net of personal tax interest rate. There is no tax advantage to debt nor any tax disadvantage to dividends in this model, and therefore the value of the unlevered stream (V*) is equal to the value of the aggregate corporate cash flow (V).

We now investigate whether personal taxes can be ignored altogether in this model. Expanding the term $r_p$, we can rewrite equation 5.45 as:

$$(5.46) \quad \pi = \frac{\mu\,\theta_p}{V} - (R\,\theta_p - p)$$

The presence of the term p for inflation in the above equation prevents any further simplification. But even assuming $p = 0$, $\theta_p$ will still appear in the calculation as a scaling factor, that is 5.46 becomes

$$(5.47) \quad \pi = \theta_p\ (\mu/V - R)$$

Therefore the excess of gross corporate rate of return for the aggregate corporate sector over the nominal risk free rate has to be scaled by $\theta_p$ to get the correct market risk premium. Hence personal taxes cannot be ignored in calculating the risk premium (and therefore the discount rates), even under this integrated tax regime.

**(iii) Calculating total value of aggregate corporate sector, using market premium**

Rearranging equation 5.45 we get

$$(5.49) \quad \frac{\mu\,\theta_p}{V} - r_p = \pi$$

$$\Rightarrow \quad \frac{\mu\,\theta_p}{V} = r_p + \pi$$

$$(5.50) \Rightarrow \quad \frac{\mu\,\theta_p}{r_p + \pi} = V$$

that is, the value of aggregate corporate sector is obtained by discounting the net of personal tax aggregate corporate cash flow by a risk adjusted discount rate.

**(iv) Calculating Required Return For An Individual company**

Similarly, for an individual company, the valuation equation under this tax regime is:

$$(5.51)\quad V_i = \frac{\mu_i \theta_p}{r_p + ß_i \pi}$$

$$(5.52)\quad \text{where } ß_i = \operatorname{cov}\left( \frac{\mu_i \theta_p}{V_i}, \frac{\mu \theta_p}{V} \right) \Big/ \operatorname{Var}\left( \frac{\mu \theta_p}{V} \right)$$

that is, $ß_i$ is the covariance between the net of personal tax rate of return for the company and that for the aggregate corporate sector, divided by the variance of the aggregate corporate sector's rate of return.

Given that $\theta_p$ appears in the numerator of the above equation 5.51 may suggest that it can be cancelled out with the $\theta_p$ which appeared as a scaling factor in equation 5.47.
To examine this, we rewrite equation 5.52 as :

$$(5.53)\quad \operatorname{cov}\left(\frac{\mu_i \theta_p}{V_i}, \frac{\mu \theta_p}{V}\right) \Big/ \operatorname{var}\left(\frac{\mu \theta_p}{V}\right) = \theta_p^2 \operatorname{cov}\left(\frac{\mu_i}{V_i}, \frac{\mu}{V}\right) \Big/ \theta_p^2 \operatorname{Var}\left(\frac{\mu}{V}\right)$$

$$= ß_i$$

where $ß_i$ is expressed as the covariance between the before tax returns for the company and for the market. Thus there is no impact of taxes in the beta factor.
Using this, substituting the definition of $r_p$, and using the expression for $\pi$ as given in equation (5.46), equation (5.51) now becomes:

$$(5.54)\quad V_i = \frac{\theta_p \mu_i}{\theta_p R - p + ß_i \cdot \left\{ \theta_p \left( \frac{\mu}{V} - R \right) + p \right\}}$$

Further simplification is not possible unless anticipated inflation (p) is zero. If we make this assumption, then Equation 5.54 becomes

$$(5.55)\quad V_i = \frac{\theta_p \mu_i}{\theta_p R + ß_i \cdot \theta_p \left( \frac{\mu}{V} - R \right)}$$

$$(5.56) = \frac{\theta_p \; \mu_i}{\theta_p \left(R + ß_i \, . \, \left(\frac{\mu}{V} - R\right)\right.}$$

$$(5.57) = \frac{\mu_i}{R + ß_i \; . \; \pi'}$$

where $\pi'$ is the excess of aggregate corporate sector rate of return over the nominal risk free interest rate. In 5.56, $\theta_p$ cancels out - therefore the cash flow EBIT of the firm can be discounted by the nominal risk free rate plus a risk premium. The risk premium too is calculated without taking into account any taxes. Firstly, $\pi'$ in equation 5.57 does not include any taxes. Secondly, as shown in equation 5.53 above, the beta factors can be calculated using the pre-tax figures.

In conclusion it can be stated that under the integrated tax system the valuation equation is

$$(5.58) \; V_i = \frac{\mu_i \, \theta_p}{r_p + ß_i \pi}$$

Assuming that anticipated inflation is zero, the above changes to

$$(5.59) \; V_i = \frac{\mu_i}{R + ß_i \, . \, \pi'}$$

Here $\mu_i$, R and $\pi'$ are all free of any taxes .

Equation 5.47 derived earlier illustrates an interesting point. $\mu/V$ net of corporation tax can be written as ($\mu/V \; \theta_p$) because under this system, the corporation tax is levied at the personal tax rates. Thus, even under this tax regime, it would be incorrect to compare net of corporate tax returns ($\mu\theta_p/V$) with the <u>nominal</u> risk free interest rate. However, that is precisely what is likely to be followed by companies in practice, when they use CAPM which is defined as

(5.48) $R_j - R_F = \beta \, \pi$

where $R_j$ = expected return on project

| | | |
|---|---|---|
| $R_F$ | = | nominal risk free rate |
| $\beta$ | = | relevant beta factor |
| $\pi$ | = | market risk premium |

Since $R_j$ is likely to be compared to the net of corporation tax cash flows, it is considered to be a required rate of return net of corporation tax ($\mu\theta_p/V$). Equation 5.46 emphasises that this rate of return should not be compared to $R_F$ - the nominal rate, but to $R_F\,\theta_p$, ie. risk free rate net of personal taxes. Thus, in the case of integrated tax regime, the excess return on the project should be calculated by subtracting the after tax risk free rate from the rate of return on the project. The main point of the above discussion is that even under the simpler tax regimes, personal taxes need to be correctly interpreted in order to calculate the risk premium and the discount rates. Thus under an integrated tax regime, where the corporation tax is fully imputed to the shareholders, equation 5.59 shows that if the nominal interest rate is used, then it the pre tax cash flows from the project that are relevant.

## Section E - Conclusion

The survey revealed that the valuation models in the literature which incorporated CAPM and corporation and personal taxation were largely concerned with the classical system of taxation. The Modigliani model described in the previous chapter is an example of such a valuation model. The contribution made in this chapter is in sections B, C and D , where for the first time in the literature, the valuation model incorporating heterogeneous personal taxes and uncertainty under the imputation, two rate, and the integrated systems of taxation were presented. The last mentioned model was fully derived in section D.

A comparison of the valuation equations shows that corporate values will be the highest under the integrated system and the lowest under the classical system. The effective discount rate, assuming same tax rates, will be the lowest under the integrated rate system and the highest under the classical system. The imputation and the two rate system fall in between the other two systems. However, both these systems offer relief for double taxation of only one form of corporate returns - namely dividends. It is only the integrated system that completely eliminates the double taxation of corporate earnings.

U.K. follows the imputation system of taxation. In addition to this imputation relief, the U.K. tax system offers some relief against inflation for capital gains tax purposes. Moreover, another important fact is that in common with other countries, a greater proportion of shares of companies in the U.K. are now owned by pension funds, whose "personal" tax characteristics are quite different from those of individuals. The next chapter incorporates all these relevant features and gives a practical illustration of the calculation of the valuation variables for a company in the U.K.. This result is extended in Chapter 7 where the relevant detailed features of the corporation tax system in the U.K. are incorporated to present the final valuation equation advocated in this thesis.

# MEASUREMENT OF VALUATION VARIABLES IN THE U.K.

## Section A

The aim in this chapter is to discuss how the variables that make up the valuation equation for project appraisal in the U.K. can be estimated in practice. For this purpose, in this section A, we chose a version of the model described in chapter 4, and adapt it to the U.K. tax environment. In doing so, we also make use of the analysis of the alternative tax systems described in the previous chapter (chapter 5), because we are adapting a model developed under the classical system into one applicable for the imputation system. Thus, in section A, we examine the relationship between the valuation model for the U.K. and the valuation models derived in the previous 2 chapters. Note that only the two main features of the U.K. tax system are incorporated in the valuation model in this chapter, and that the detailed provisions of the corporation tax code are examined in the next chapter. In sections B and C, the variables of the equation derived in this section A are examined and practical solutions to the measurement problem are suggested.

The Modigliani valuation equation, which is applicable for the classical system of taxes described in the previous chapter, was derived previously in chapter 4 as equation (4.41). This equation, which assumes that the income flows associated with debt and dividends are constant, is reproduced below (Modigliani, 1982, page 260, equation II.12):

$$(6.1)\quad V_i = \frac{\mu_i^* \theta_g}{r_p + p\tau_g + \beta_i \pi} + LD_i \quad \frac{-\Delta_i \ \theta_g - \theta_p}{r_p + p\tau_g}$$

The first term evaluates earnings before taxes net of standard rate corporation tax and personal capital gains tax; the second term measures the impact of leverage in the presence of personal taxes ($L = 1 - r_c \theta_g / (r_p + p\tau_g)$); the third term measures the impact of differences in personal taxes on capital gains and on dividends.

The above valuation equation consists of three terms and therefore is unfamiliar to investment analysts who are more conversant with using a single discount rate for valuation purposes. Unfortunately, if personal taxes have to be incorporated realistically, then the resulting equation will look complex. A single discount rate can still be obtained in the denominator by transforming equation (6.1) into the following :

$$(6.2) \quad V_i = \frac{\mu_i^* \theta_g + LD\,(r_p + p\tau_g + \beta_i\pi) - \dfrac{[\ \Delta_i\,(\theta_g - \theta_p) + \beta_i\pi\ \Delta_i\,(\theta_g - \theta_p)\ ]}{r_p + p\tau_g}}{r_p + p\tau_g + \beta_i\pi}$$

Thus the term " $r_p + p\tau_g + \beta_i\pi$ " gives the single discount rate which is used to discount the cash flow $\mu_i$, after the cash flow is adjusted for the impact of taxes as shown by the expression given in the numerator. The above equation would hold true for the countries which operate under the classical system of taxation described in the previous chapter. Two main changes in the case of U.K. tax system are that in the U.K., there is an indexation allowance for Capital Gains Tax purposes and there is the imputation system of taxation described in the previous chapter. Therefore the real return expected by an investor holding equity in the U.K. would be (equivalent to equation (4.4)):

$$(6.3) \quad y_i^m = n_i^m\ [\ (\mu_i^* - r_c\,D_i)\ \theta_g^m + \Delta_i\,(\theta_{p\ DIVI}^m - \theta_g^m)\ ]$$

where $\theta_{p\ DIVI}^m$ = net of personal taxes dividends received by an individual/corporate/pension fund shareholder from £1 of <u>post</u>-corporation tax income.

Equation 4.4 is reproduced below

$$(4.4) \quad y_i^m = n_i^m\ \{\ [\mu_i^* - D_i\,(r_c)]\ \theta_g^m + \Delta_i\,(\theta_p^m - \theta_g^m) - \tau_g^m\,pS_i\ \} \qquad \text{(MII.1b)}$$

The two changes in equation (6.3) from equation (4.4) are :

(i) The omission of the term $\tau_g^m\,pS_i$ , which took into account personal tax on the inflation element of the increase in share price. In the U.K., the inflationary element of the gain is not taxed, therefore it is appropriate to exclude this term from the equation. This point was not addressed while discussing the imputation system in the last chapter because it relates to the tax provisions in the U.K. tax code only, and is not generic to the imputation system of taxation.

(ii) The term $\theta_p^{mDIVI}$ replaces the term $\theta_p^m$ in calculating the taxes on dividend income. This is because under the imputation system, the tax on dividend income is no longer equal to the personal income tax . In chapter 5, equation (5.10) gave the derivation of the net of tax cash flow from dividend income ( i.e. $\theta_{pDIVI}$ ) under the imputation system. Making these substitutions, and following the steps described in chapter 4 , will result in the following valuation equation:

$$(6.4) \quad V_i = \frac{\mu_i^* \theta_g}{r_p + \beta_i \pi} + LD_i - \frac{\Delta_i \, (\theta_g - \theta_{pDIVI})}{r_p}$$

$$\text{where } L = 1 - \frac{r_c \, \theta_g}{r_p}$$

Equation (6.4) for the U.K. system differs from equation (4.41) under the classical system in two respects which follow directly from the two changes noted on the previous page. The extra taxes borne on dividend income, if any, become the relevant tax in determining the impact of differential taxes on dividend income on valuation. Secondly, the term $p\tau_g$, which arose because capital gains taxes are usually levied on nominal gains, is not applicable to the U.K. where gains taxes are levied on real gains, and hence this term no longer appears in the valuation equation relevant for the U.K. Note that throughout this thesis, we ignore the initial annual exemption from personal capital gains tax, which although very relevant for personal tax planning for many individuals, is less significant for the investors with large shareholdings who own most of the stock held by individuals.

Equation (6.4) can be expressed as a valuation equation in which a single discount rate " $r_p + \beta_i \pi$ " is used by making the adjustments to the numerator, as was previously described in deriving equation (5.1) in the previous chapter. This results in the following equation:

$$(6.5) \quad V_i = \frac{\mu_i^* \theta_g + L\, D_i(r_p + \beta_i \pi) - \left[ \Delta_i \, (\theta_g - \theta_{pDIVI}) + \dfrac{\beta_i \pi \, \Delta_i \, (\theta_g - \theta_{pDIVI})}{r_p} \right]}{r_p + \beta_i \pi}$$

Equation (6.5) gives the valuation equation for valuation of projects under the U.K. system of taxation. Equation (6.5) incorporates the two main features described above which distinguish the U.K. tax system from the U.S.A. (classical) system. The other more detailed features of the U.K. corporation tax system are examined in the next chapter.

In the above equation, there are a number of coefficients by which the cash flows $\mu_i$ , $\Delta_i$ and the debt level $D_i$, are multiplied. These coefficients are $\pi$, $\beta_i$, $\theta_g$, $\theta_p$, $\theta_{PDIVI}$, $r_c$ , $r_p$ and L. The following sections will analyse the calculation of these coefficients, namely, $\pi$, $\beta_i$, $\theta_g$, $\theta_p$, $\theta_{PDIVI}$, $r_c$ , $r_p$ and L in the U.K. context.

## Section B

## Measurement Of Market Risk Premium And Beta Factors

These two measures are based directly on the returns expected by investors on the capital markets,such as the Stock Exchange. We use the studies on the stock market returns in the U.S.A. and in the U.K. to draw inferences about the relevant variables below.

**(1) Market Risk Premium:** $\pi$

The market risk premium in the model which gives the valuation equation (6.5) above is is largely similar to that in the Modigliani model of chapter 4 . The market risk premium in the U.K. valuation model is:

$$(6.7) \quad \pi = \frac{\mu^* \theta_g}{V^*} - r_p$$

where calculation of $\theta_g$ and $r_p$ take into account the different personal taxes as well as the risk-preference characteristics of the shareholders (as shown below). V* is the aggregate value of an unlevered stream, and it is the relevant term because it is refers to only the stochastic component of value. This differs from the market risk premium shown in chapter 4 as equation (4.34) only in so far as the term $p\tau_g$ is now absent in numerator of equation (6.7), since we are now describing the U.K. tax system (which is fully indexed).

The market risk premium has been estimated by R A Brealey (1981) for the U.K. to be 8% . The technique he used was to measure the excess of capital market returns to the shareholders (i.e. sum of capital gains and dividends) over the short term treasury bills rate. He used data from the 1940s up to 1979 in his calculations. The 8% market risk premium was calculated using returns to investors net of personal tax. However, his measure is likely to differ from the measure defined in equation (6.7) because :

(i) Brealey used standard rate of personal income tax for his calculations. In contrast, one of the main themes advocated in this thesis, and which underlies the variables in equation (6.7) above, is that we should be thinking in terms of the weighted averages of the tax rates of all types of relevant investors.

(ii) The formula for calculating the equity return is different from that used in equation (6.7). This can be seen by examining the following equation which attempts to interpret Brealey's estimate of risk premium, in a format compatible with the model developed in this thesis. Let the market risk premium calculated by Brealey be denoted by $\pi^e$ :

$$(6.8) \quad \pi^e = \frac{(\mu^* - r_c D)\,\theta_g^s + \Delta\,(\theta_p^s - \theta_g^s)}{V} - r_p^s = 8\%$$

The numerator in the first term measures the sum of dividends and capital gains, net of the standard rate of personal taxes for individuals (hence the superscript "s"), as used by R A Brealey. In common with the literature in finance, the assumption being made here is that retained earnings result in an equivalent capital gains. In other words, the companies' market capitalisation increases by the amount of its equity earnings which are not distributed. This assumption is made throughout the analysis in this thesis. V in the denominator indicates that the return on equity is calculated over the total value of equity - not over V*, the value of unlevered stream (equation 6.7). Thus Brealey's estimate of return to equity investor differs as it does not calculate the return over the unlevered equity, as is done in the models in the thesis. The models are correct to the extent that under the assumptions of non stochastic contribution to the value of the companies by the tax authorities, it is correct to include only V* in the denominator in order to calculate the rate of return.

There are 3 ways to obtain estimates of the market risk premium advocated in this thesis:

(i) Adjust Brealey's measure

(ii) Calculate by estimating the components of eq 6.7 afresh

(iii) Use Brealey's measure as an approximation

(i) The adjustment required to obtain the market risk premium $\pi$ from Brealey's measure $\pi^e$ is quite complicated and is shown below:

$$(6.9) \qquad \pi = \{ (\pi^{e} \text{ x } V) - \Delta(\theta_p^{s} - \theta_g^{s}) + r_p^{s} V + r_c D \theta_g^{s} \} \text{ x } \frac{\theta_g}{\theta_g^{s} V^*} - r_p$$

In the above transformation, the expression within the curly brackets essentially reduces the term $\pi^{e}$ to $\mu^{*}\theta_g^{s}$ , which is then transformed further by the other variables in the equation. Unfortunately, the full equation is quite complicated and therefore this procedure is not very practicable.

(ii) It is likely to be simpler to estimate $\pi$ afresh except that it will be very difficult to estimate V*, the value of the unlevered stream which is required in the formula in equation (6.7) above. We need to deduct the contribution made by the tax authorities to the value of the company by taxing dividends and capital gains at different rates, and by allowing interest deductibility of debt interest, from V*, in order to calculate V*. However, whereas V, the equity value, is readily available for quoted securities, V* would require judgement in calculating the contribution to value made by the tax authorities since such data has not been calculated as yet for the quoted companies. Thus this measure too is not very practicable.

(iii) Therefore, a practical alternative is to use the estimate made by Brealey as an approximation for the market risk premium in this thesis.

Two further comments on risk premium are pertinent.

(i) R A Brealey stated that the risk premium calculated was not very sensitive to changes in personal tax rates. Hence risk premium calculated using $\theta_g^{s}$, $\theta_p^{s}$ and $r_p^{s}$ may not differ significantly from that calculated using $\theta_g$, $\theta_p$ and $r_p$.

(ii) The entire model relies on variance relating to $\mu^{*}$, the earnings of the company. The return calculated above by R A Brealey, as is common in calculating return on equity, beta factors, etc, are total returns received by shareholders on their equity investments - that is, capital gains plus dividends. In any one year, return on equity in terms of earnings after tax varies considerably from the return on equity in capital market. Therefore the assumption that retained earnings translate into equivalent capital gains is not borne out by the actual price stock market, at least on a short term year by year comparison. One simple explanation for this may be that the rate of return required by the investors at the beginning of the year is different from the required return at the end of the year. This implies that the capital gain earned by the investors over a year comprises retained earnings plus the difference due to capitalising returns at a different discount rate. Thus, while the market risk premium formula is based on the variability of the accounting cash flow returns, we are estimating it using capital market based equity return. This makes our approximation less accurate.

**(2) Beta Factors**

However the above disadvantage of using accounting cash flow earnings to reflect capital market returns, does not carry over to considerations of beta factors. This is so because beta factors based on earnings variations are similar to beta factors based on capital market returns (W H Beaver and J Manegold, 1975) .

The beta factors in the valuation model are defined as :

$$(6.11)\quad \beta_i = \text{cov}\left(\frac{\mu_i^*}{V_i^*}, \frac{\mu^*}{V^*}\right) \Big/ \text{VAR}\left(\frac{\mu^*}{V^*}\right)$$

that is, the covariance between the return (denoted by earnings over the value of unlevered stream) for company i and the return for the market, divided by the variance of the return for the aggregate corporate sector ($\mu^*/V^*$).

Assuming that retained earnings convert into equivalent capital gains, the beta factors ($\beta_i^e$), as calculated by say , the Risk Measurement Service of the London Business School, can be represented by :

$$(6.12)\quad \beta_i^e = \text{cov}\left[\frac{(\mu_i^* - r_c D_i - \Delta_i) + \Delta_i}{V_i^* + LD_i - \Delta_i\dfrac{(\theta_g - \theta_p)}{r_p}}, \frac{(\mu^* - r_c D - \Delta) + \Delta}{V^* + LD - \Delta\dfrac{(\theta_g - \theta_p)}{r_p}}\right] \Big/ \begin{matrix}\text{Market}\\ \text{Variance}\end{matrix}$$

where $(\mu_i^* - r_c D_i - \Delta_i)$ = capital gain; $\Delta_i$ = dividend; $V_i^* + lD_i - \Delta_i (\theta_g - \theta_p/r_p)$ = total equity value = $V_i$. The denominator is equal to V - the total value of equity (and not just V*, the value of unlevered stream). Thus, while it is practical to use published beta factors, such as those produced by the London Business School, a comparison of the above equations 6.11 and 6.12 shows that these published beta factors are not precisely consistent with the definition adopted in our model. This discrepancy is a weakness when using the valuation model in a practical context. However, the other discrepancy, that retained earnings do not generally transform into equivalent capital gains, is not so significant keeping in mind the earlier comment regarding accounting based and market based betas. Hence we proceed with the conclusion that it is practicable to get reasonable estimates for the beta factors advocated in this model.

## Section C

### Measurement Of Other Coefficients ($\theta_g$, $\theta_p$, $\theta_{PDIVI}$, $r_c$ , $r_p$ and L)

**Other Variables :** In order to estimate $\theta_g$, $\theta_p$, $\theta_{PDIVI}$, etc. we need to first estimate $\gamma^m$ and $\Lambda$. As stated in the model, $\gamma^m = -2u_2^m/u_1^m$ and $\Lambda = 1/\Sigma_m \ 1/\gamma^m (\theta_g^m)^2$. $\gamma^m$ is a measure of an investor's risk aversion, and is calculated for all investors. $\Lambda$ is an appropriate harmonic mean which correctly takes into account the personal taxes faced by each of the heterogenous investors. This section examines how all these coefficients can be estimated in practice. The calculations are sub-divided into 4 steps.

**(Step 1) Measurement of Risk Aversion variable: $\gamma^m$**

$\gamma^m$ measures the degree of substitution between risk and return by the investors.
We first calculate the substitution for the market as a whole and then estimate the substitution for individual investors such as individuals, corporations and pension funds. One can calculate the degree of substitution between return and variance for the market as follows.

The real rate of return in the U.S.A. on common stocks and treasury bills averaged 8.4% and 0.1% per year respectively, according to the Ibbotson and Sinquefield study. Their study is subject to all the same difficulties as Brealey (1981). The average risk premium, based on the realised historical data from 1926 to 1981, is therefore approximately 8.3% . Note that this risk premium is similar to that estimated by Brealey for the U.K.. Hence the risk-return characteristics in the U.S.A. may be a good representation for investor behaviour in the U.K. as well. Over the same period, 1926-1981, the standard deviation of common stock in the U.S.A. has been 21.9%, which implies a variance of 479.6%. Treasury bills are risk free investment with nil variance. Therefore, on average, the investors demanded an 8.3% (say, 8%) extra return above the return to treasury bills, for bearing a market variance of 479.6% (say 480%). These estimates are used below to calculate the degree of risk aversion.

Let the total change in utility associated with these changes in return and variance be du. The derivative of utility with respect to return, $u_1$, for an investor who holds the overall market portfolio, is $u_1^{mkt} = du/8$.

Similarly, the derivative of utility with respect to variance, $u_2^{mkt}$, = -du/480.

$$\gamma^{mkt} = \frac{-2\ u_2^{mkt}}{u_1^{mkt}} \qquad \text{by definition}$$

Therefore

$$\gamma^{mkt} = -2 \times \frac{-du}{480} \div \frac{du}{8}$$

$$= 2 \times \frac{du}{480} \times \frac{8}{du} = \frac{16}{480} = 0.0333$$

We use the above ratio of 0.0333, as derived for the overall equity market as a whole, to estimate similar ratios for groups of investors such as individuals, corporations and pension funds.

$\gamma^{Indiv}$

Since the wealth of individuals is lower than that of corporations or pension funds, they are likely to be more risk averse than the corporations or the pension funds. Thus individuals would require more return to compensate them for undertaking similar risk. We assume, based on judgement, that individuals are 50% more risk averse than the overall equity market. In other words, they require an increase in real return of 12% to compensate for bearing a similar amount of variance (480) as the overall market. This assumption is used to calculate $\gamma^{indiv}$ below :

$$\gamma^{indiv} = \frac{-2\ u_2^{indiv}}{u_1^{indiv}}$$

$$= -2 \times \frac{-du}{480} \div \frac{du}{12}$$

$$= +2 \times \frac{du}{480} \times \frac{12}{du} = \frac{24}{480} = 0.05$$

Therefore $\gamma^{indiv}$ (estimated) = 0.05.

$\gamma^{corp}$ It is reasonable to assume that the companies, in their role as investors, exhibit the same degree of risk aversion as the market as a whole, because the market as a whole comprises such companies themselves. Therefore, a reasonable estimate of $\gamma^{corp}$ is 0.0333, which equals the risk aversion factor for the market as a whole.

$\gamma^{pension}$ Pension funds have large resources and therefore may be considered less risk averse than the individuals. Their $\gamma^{pension}$ can be estimated as the balancing figure, as shown below. In order to do so, we need to analyse the proportions of the overall equity in the capital markets owned by individuals, corporations and pension funds.

The proportion of investors in the U.K. stock market as per the Survey of Share Ownership carried out by the Stock Exchange in 1981 are as follows:

| | |
|---|---|
| Individuals | 30% |
| Corporations & Insurance Companies | 30% |
| Pension funds and tax exempt insurance unit trusts, etc. | 40% |

Note that individuals own only 30% of the shares directly. Therefore the other models in the literature which assume that the only relevant personal tax characteristics are those of individuals are inaccurate. ( We noted previously that the annual exemption from personal capital gains tax is ignored.)

The above proportions are used to calculate the risk aversion variable for pension funds as a balancing figure in the following equation :

$$0.0333 = 30\% \times 0.05 + 30\% \times 0.0333 + 40\% \times \gamma^{pension}$$

Therefore $\gamma^{pension}$ = ( 0.0333 - 0.025 ) / .40

$$= 0.0083 \,/\, .40 \;=\; 0.0208$$

Step 2 below uses these separate measures of risk aversion to estimate the harmonic mean, which will be relevant for calculating the tax variables in step 3.

**(Step 2) Calculation of $\Lambda$ :**

Equation 4.20 defines the harmonic mean $\Lambda$ as :

$$\Lambda = 1/\Sigma_m \frac{1}{\gamma^m (\theta_g^{\,m})^2}$$

As stated in chapter 4, the summation over m investors should be weighted by the proportion of investors in the capital markets who possess those tax characteristics. Note that only $\theta_g^{\,m}$, which is based on capital gains tax, is relevant. The reason for this is that the personal income tax rates and the dividend income tax rates apply only to income streams that are non stochastic in this model. On the other hand, capital gains tax is levied on capital gains which are based on retained earnings, which are assumed to be stochastic in the model. Therefore, in the context of risk aversion, where we are concerned with stochasticity, the relevant tax variable is the one related to capital gains tax only.

Capital gains tax rate in the U.K. during most of 1980s was 30% for individuals and companies( essentially insurance companies ), and zero for pension funds.

Therefore $\theta_g^{\,Indiv} = 0.7$ ; $\theta_g^{\,corp} = 0.7$ and $\theta_g^{\,pension} = 1.0$.

$$\Sigma_m \frac{1}{\gamma^m (\theta_g^{\,m})^2} = \frac{1}{\gamma^{indiv} \text{ x } (\theta_g^{\,indiv})^2 \text{ x } 30\%} + \frac{1}{\gamma^{corp} \text{ x } (\theta_g^{\,corp})^2 \text{ x } 30\%}$$

$$+ \frac{1}{\gamma^{pension} \text{ x } (\theta_g^{\,pension})^2 \text{ x } 40\%}$$

$$= \frac{1}{.05 \text{ x } .49 \text{ x } .3} + \frac{1}{.0333 \text{ x } .49 \text{ x } .3} + \frac{1}{.0208 \text{ x } 1 \text{ x } .4}$$

$$= \frac{1}{.0074} + \frac{1}{.0049} + \frac{1}{.0083}$$

$$= 135.14 + 204.08 + 120.48$$

$$= 459.70$$

$$\Lambda = 1 \,/\, 459.70 = 0.002175.$$

This harmonic mean of 0.002175 is used as weight in the calculation of $\theta_g$, $\theta_p$, $\theta_{PDIVI}$ in the following step.

**(Step 3 a ) Calculation Of $\theta_g$ :**

This is a weighted average of personal capital gains tax of individuals ($\theta_g$ = .7), corporations ($\theta_g$ = .7) and pension funds ($\theta_g$ = 1), using weights which incorporate the risk aversion characteristics of the investors, as calculated in steps 1 and 2 above. Eq 4.22 defines $\theta_g$ as :

$$\theta_g \quad = \quad \Sigma_m \, \theta_g^{\,m} \quad x \quad \frac{\Lambda}{\gamma^m \, (\theta_g^{\,m})^2}$$

Substituting the values calculated above, we obtain :

$$\begin{aligned} \theta_g \quad = \quad & 0.7^{indiv} \quad x \quad \frac{0.002175}{0.0074} \\ + \; & 0.7^{corp} \quad x \quad \frac{0.002175}{0.0049} \\ + \; & 1.0^{pension} \quad x \quad \frac{0.002175}{0.0083} \end{aligned}$$

Thus $\theta_g$ is a weighted of $\theta_g^{\,m}$ s as derived below :

| | $\theta_g^{\,m}$ | | Weights | | $\theta_g$ |
|---|---|---|---|---|---|
| = | 0.7 | x | .2940 | = | 0.2058 |
| + | 0.7 | x | .4440 | = | 0.3108 |
| + | 1 | x | .2620 | = | 0.2620 |
| | | | 1.0000 | | 0.7786 |

Therefore, an estimate of $\theta_g$, using the above tax rates and shareholder proportions, calculated in accordance with the formulae in the valuation model, is 0.7786 .

If simple weights, which ignored the risk aversion characteristics, were used to estimate $\theta_g$, the result would be slightly different. Using simple weights, we get $\theta_g$ equal to :

0.7 x 30% + 0.7 x 30% + 1 x 40% =

0.21 + 0.21 + 0.40 = 0.82

Therefore, the use of simple weights results in an overstatement of $\theta_g$ by approximately 4%.

The capital gains tax considered above is an overstatement since capital gains are taxed when they are realised rather than when they accrue. However, given the vast volume of turnover in securities on the stock market, the AVERAGE stockholding period is low. The market value of all the companies on the London Stock Market at March 1987 was £390.8 billion with an average turnover of £275 billion per year. Therefore the turnover period is £390.8 bn / £275 bn = 1.42 years. The average stockholding is 0.71 years or 8.5 months - a very low figure. The results shown by this calculation above are in sharp contrast to comments in the existing literature, in which the authors claim that the average stockholding period is as great as 10 years. Therefore the effective tax rate will only be slightly lower than the nominal tax rate. Of course, this average shareholding includes some investors who hold stocks for a number of years and therefore, in present value terms, pay a much lower effective CGT. Hence the effective average CGT will be lower than the statutory rate, but not as low as is claimed by many authors in the literature, who may not have appreciated the relationship between turnover in shares and the stockholding period considered above.

**( Step 3 b ) Cash flow net of personal tax on interest income ($\theta_p$) :**

The top rate of personal income tax in the U.K. for most of the 1980s was 60%. Companies pay tax on interest income they receive at the corporation tax rate of 35%. Pension funds are tax exempt. These tax rates are used below to calculate $\theta_p$ , the weighted average which incorporates the tax and risk aversion characteristics of the heterogeneous investors. The calculation of $\theta_p$ using the current top rate of income tax of 40%, and consideration of its impact, are dealt with later in chapter 8.

Using U.K. tax rates $\theta_p^{indiv}$ = 1 - 60% = 0.40

$\theta_p^{corp}$ = 1 - 35% = 0.65

$\theta_p^{pension}$ = 1 - 0% = 1.0

| | | | | Weights ( As Above ) | | |
|---|---|---|---|---|---|---|
| $\theta_p$ = | | 0.40 | x | .2940 | = | 0.1176 |
| | + | 0.65 | x | .4440 | = | 0.2886 |
| | + | 1 | x | .2620 | = | 0.2620 |
| | | | | 1.0000 | | 0.6682 |

This weighted average is approximately 5% different from the average using simple weights, which is calculated below.

$$0.40 \times 30\% + 0.65 \times 30\% + 1 \times 40\% =$$

$$0.12 + 0.195 + 0.40 = 0.7150$$

**(Step 3 c ) Cash flow Net Of Dividend Income Tax ($\theta_{PDIVI}$) :**

In the mid 1980s, the tax credit on dividend income was related to the standard rate of income tax, then at the rate of 27%. Thus for every £1 of post corporation tax income distributed, with U.K. imputation system gross dividend was £1 x 100/(100-27) = £1.3698 . Therefore individuals paying 60% tax paid £1.3698 x 60% = 0.8219, <u>less</u> credit of 0.3698 = 0.4521 of net extra payment of tax on every £1 of cash dividend. Corporations paid no extra tax on corporate dividends received, which are treated as franked investment income. Pension funds received a tax credit from Inland Revenue, in addition to the cash dividend. Therefore their "net of tax" dividend is in fact greater than their pre-personal tax dividend, as shown below.

$$\theta^{indiv}_{PDIVI} = 1 - 0.4521 = 0.5479$$

$$\theta^{corp}_{PDIVI} = 1 - 0 = 1.0$$

$$\theta^{pension}_{PDIVI} = 1 - \underset{\substack{\uparrow \\ \text{refund}}}{-0.3698} = 1.3698$$

Therefore the relevant weighted average $\theta_{pDIVI}$ is

| | Weights | |
|---|---|---|
| 0.5279 x | 0.2940 = | 0.1552 |
| 1.00 x | 0.4440 = | 0.4440 |
| 1.3698 x | 0.2620 = | 0.3589 |
| | | 1.05087 |

Thus, mainly due to the positive impact of refunds of tax to pension funds, $\theta_{pDIVI}$ is very close to 1. If simple weights were used, $\theta_{pDIVI}$ would be approximately 4% lower.

0.5479 x 30% + 1.00 x 30% + 1.3698 x 40% =

.1644 + .3000 + .5479 = 1.0123 .

This implies that in fact, for every £1 of cash dividend, on average, the Inland Revenue pays out 1.2 pence of net refund ( that is, the refunds to pension funds exceed the extra personal dividend tax collected from individuals). This fact is extremely important for analysing the rationality of corporate dividend decisions.

Which of the above 2 weightings, and the associated $\theta_{pDIVI}$ , is the relevant one ? In relation to the cash flows to or from Inland Revenue, the simple weightings, which resulted in the average of 1.0123 in the calculations shown above, are relevant. However, when discussing valuation, the risk aversion and the capital gains tax characteristics too become relevant for the determination of the market equilibrium, and hence the more complex weightings calculated in step 3a above become relevant.

**(Step 4 a ) Calculation of L , the advantage of leverage :**

The tax advantage of leverage was defined in equation 6.4 as :

$$L = 1 - (r_c/r_p)\,\theta_g$$

where $r_c = \theta_c R - p$

In order to illustrate the calculation of L, we assume nominal interest rate R = 10% and inflation, p = 4% .

$$r_c = 0.65 \times 10\% - 4\% = 0.065 - .04 = 0.025$$

$$r_p = \theta_p R - p \quad (\theta_p \text{ as calculated in step 3 b, is } 0.6682)$$

$$r_p = (0.6682 \times 10\%) - 4\% = 0.0668 - 0.04 = 0.0268$$

Now L = 1 - (0.025 / 0.0268) x 0.7786 = 0.2737

Given the tax characteristics used above, debt has a significant tax advantage. The inclusion of personal taxes though reduces this advantage when compared to the advantage of 0.35 if no personal taxes are considered.

Two further comments are pertinent here:

(a) The Institute of Fiscal Studies recommends that the Corporation Tax Rate be lowered to 27%. It states that as a result :

(1) there would be improved performance through higher investment

(2) it would remove the tax incentive in favour of debt finance rather than equity.

(1) above is self evident since it is based on the impact of smaller tax leakage which would result in better after tax returns and hence more investment. In relation to (2), even if $\theta_c = \theta_p$ (keeping in mind that $\theta_p$ is the average income, net of personal tax rate faced by investors (companies; tax-exempt institutions; personal) ) , the benefit of leverage is:

$$L = 1 - r_c/r_p\, \theta_g$$
$$= 1 - 1\, \theta_g \qquad = 1 - \theta_g$$

Therefore the presence of capital gains tax means that the benefit of leverage would still be positive -there will still be tax advantage to debt even though personal income tax and the corporation tax rates are equalised.

**(step 4 b ) Calculation of $\theta_g - \theta_{pDIVI}$, the difference between dividends and capital gains taxes**

$\theta_g$ and $\theta_{pDIVI}$ were calculated above as 0.7786 and 1.05087 respectively. Therefore the extra tax payable on dividend income is :

$$0.7786 - 1.05087 = -0.27227 .$$

**This implies that there is a significant tax *advantage* to dividend payments in the U.K..** Companies that pay higher dividends in the U.K. add to the value of the company because of favourable tax treatment. This contrasts with the literature which deals essentially with models which assume that there is tax disadvantage to paying dividends.

**Conclusion :**

The above sections demonstrate that it is possible to obtain practical estimates of the rather esoteric coefficients used in the valuation model shown in equation 6.5.

The advantages of this valuation model over other models in the literature include the following :

(i) Models that do not take personal taxes into account would be invariant to changes in personal taxes. This model is more robust and correctly treats all withdrawals of funds in the form of taxes (both corporate and personal) from gross returns of projects as leakages in determining the net returns available to an investor, and hence in correctly specifying the valuation model.

(ii) Models for U.K. which did take personal taxes and the imputation system into account did not however allow for consideration of differences in taxes on capital gains and on dividend income and the impact of heterogeneous investors and risk aversion. The model shown as equation 6.5 allows for such treatment.

(iii) The model advocated is capable of providing answers to questions such as:

(a)How would the discount rate change if the proportion of shareholders shifted in favour of pension funds?

(b)If the U.S.A. were to introduce the imputation system of taxation, how would the discount rates used by companies be affected?

(c)If investors' risk preference characteristics change, how would the discount rates be affected (through changes in $\gamma^m$ resulting in changes in $\theta_g$, and in $\pi$)?

Thus the model being developed in this thesis is both capable of practical applications and is more powerful than the simpler models in providing answers to relevant questions. The next chapter introduces important features of the corporation tax code in the U.K. in the valuation model (equation (6.5)) developed so far. Subsequent chapters examine the practical applications of our model.

# CORPORATION TAX INCIDENCE & VALUATION MODEL

## Section A - Corporation Tax System Features

In the previous chapter, we critically examined some of the variables that are relevant in the valuation equation (6.5). We interpreted the variables, assessed their relevance and provided practical methodology for calculating these variables. In this chapter, one major variable implicit in the valuation equation (6.5), namely, corporation tax, is examined in detail. Some of the features of the corporation tax system which are relevant to investment appraisal are examined in this section. Section B incorporates these features into the valuation equation. Section C examines how in treating corporation taxes naively, it is possible to overlook one significant factor in investment appraisal, namely capital allowances on the initial investment in physical assets. In the same section, it is emphasised that capital allowances on initial investment are relevant even when examining steady-state perpetual cash flows ( the Modigliani model (1982) examined in Chapter 4 is based on perpetual cash flows). Section D examines an alternative method of treating capital allowances in the valuation process.

The analysis in Chapters 4 to 6 has so far assumed that $\mu_i$ - $D_iR$ , that is cash flow earnings before interest and tax less debt interest, equals the taxable income of the company for the year. Hence corporation tax at the rate $\tau_c$ is levied on $\mu_i$ - $D_iR$, for example in equation (4.1) (chapter 4).

This treatment of corporation taxes overlooks the following relevant features found in the tax systems of many countries:

(i) Some of the cash payments made by the firm will not be allowed as a deduction for corporation tax purposes. For example, these would include entertainment expenses and legal expenses in respect of certain capital items, etc. The magnitude of these items could be small relative to total expenses. However, they may perhaps be more material when compared to net cash flow earnings (EBIT). Let "a" be the proportion of Earnings before interest and taxes that is disallowable for corporation tax purposes. "a" will be used in section B below to calculate the effective corporation tax rate.

There are some companies for which "a" could be material. The taxation rules in the U.K. changed in 1988 to disallow all types of entertainment expenses. For some companies, this change in tax rules has increased their effective corporation tax rate from 35% towards the 38% to 40% range.

The number and magnitude of companies subject to a takeover has increased substantially in the 1980s. Typically, these transactions include legal and other advisory fees. Other occasions when these fees arise is in relation to acquisition of capital assets or in setting up long term finance. The proportion of such fees disallowed for corporation tax purposes is material. For these reasons, such fees will also be included in the proportion "a" referred to above.

(ii) A company may receive some allowances for tax purposes even though it has not incurred any operating cash flow expense in respect of that allowance during the year. The most notable example of this is capital allowances of certain types. In the case of most projects involving purchase of some plant and machinery or industrial buildings, capital allowances are likely to be very material in relation to the cash flow EBIT. Therefore capital allowances will be incorporated into the valuation equation as shown in section B below. In addition to their role as a non cash flow tax allowance, capital allowances are also very important in determining the net effective cost of investment (which generates the EBITs). In section C below, it will be demonstrated that ignoring capital allowances on the initial investment is a very material oversight within the context of the Modigliani model.

Capital allowances include the following:

(a) Initial or first year allowances given as a percentage of eligible capital expenditure, in the year in which the investment is undertaken. In the U.K. 100% first year allowances (FYA) were available until 1983.

Immediate cash benefit is derived with initial allowances only if the company can utilise any 'tax loss' created by initial allowance against

(1) EBIT from the project,

(2) EBIT from other projects of the company or

(3) EBIT of the previous year (as per the U.K. tax rules), in which case there will be a tax repayment.

Unless a company can immediately utilise its initial allowance in any one of ways stated above, it will be at a disadvantage. The disadvantage arises because the company has a tax loss due to the initial allowances but it cannot immediately benefit from this loss by reducing its cash payments to the tax authorities or increasing any tax repayments from the tax authorities. The tax authorities do not refund taxes to companies except in certain limited circumstances, as mentioned in (3) above. Hence when there is no full loss offset provision in the tax code, the benefit of tax allowances to a company is reduced because the company can only carry forward the tax loss and make use of it in a future year. In such a case, the point in time when it benefits from the tax loss is postponed and it loses the time value of money. Hence, unutilised losses imply a reduction in the benefit of tax allowances in present value terms.

(b) Writing Down Allowances: These are usually given on a reducing balance method whereby a given percentage (25% in the U.K.) of written down value for tax purposes can be offset against the profits for the year.

(c) Balancing Allowance/Charge: These arise when an asset is disposed. If the sale proceeds on disposal of an asset exceeds the written down value of that asset for tax purposes, then a balancing charge arises. If, however, the written down value for tax purposes exceeds the sale proceeds, then a balancing allowance is given in the year in which the asset is disposed.

These three types of capital allowances are related to the acquisition and disposal of assets. The initial allowances and the balancing allowance/charge are thus available in the years when an asset is acquired or disposed respectively. The writing down allowances are available in the interweening years. We describe below a method for incorporating the effect of all these capital allowances on the valuation equation.

An initial investment in fixed assets may give rise to capital allowances over a number of years. The amount of allowances in each year may be different, if, for example, the capital allowances are given by the reducing balance method. All of the valuation models described in Chapters 4 to 6 are based on uncertain but non-decreasing and non-increasing perpetual cash flows. Therefore we need to convert the cash flows associated with capital allowances into equivalent constant perpetual flows. This will enable us to incorporate the cash flows associated with capital allowances easily into the models developed in the previous chapters. There are two steps required to convert variable cash flows into constant perpetual cash flows. Firstly, the present value of the variable cash flows can be calculated by discounting them at the appropriate discount rate. The method of discounting decreasing streams such as capital allowances is discussed further in detail in section D of this chapter. The second step is to convert the present value calculated in the first step into a perpetual annual equivalent flow by multiplying the present value by the appropriate discount rate. This annual equivalent of capital allowances and other non cash flow allowances will be denoted by "C" in the valuation equation in section B below.

If the investment appraisal concerns a project where there is no direct investment in physical assets, but instead shares of a company are purchased, then capital allowances will not be material. However there may be a significant element of goodwill on acquisition. Amortisation of goodwill on acquisition is not deductible for corporation tax purposes in the U.K. . However in certain countries, for example West Germany and Italy, goodwill amortisation within a tax group is an allowable expense. The goodwill amounts are usually very material and the appropriate annual equivalent will be included in the coefficient "C" referred to in the previous paragraph.

(iii) Government Grants on Capital Items: These act like initial allowances in the sense that they subsidise the cost of investment. However, their advantage over initial allowances is that there is a cash flow effect in the first year in the case of government grants - that is the company's cash flow improves by the cash received from grant. As noted above, the benefit of initial allowances is reduced if the company's tax position is such that it is unable to take full advantage of the initial allowances.

The government grants on capital items can be converted into perpetual annual equivalent cash flows by using the two-step procedure described above regarding capital allowances. This annual equivalent cash flow of the capital based government grants will be denoted by "G" in the following sections.

(iv) Non-taxable Income. The above illustrates the benefit of depreciation allowances which can be offset against taxable income, as well as government grants on capital expenditure which act to reduce the cost of investment.

There is yet another complication possible - receipt of non-taxable income. An example could be revenue based grants from the government or, if a project involves holding shares in other companies, then the dividends received from other U.K. companies will be non taxable.

An important example of non taxable income in the U.K. is the receipt of net dividends by a *company*, which are treated as "franked investment income" in the U.K.. The only tax borne on this is the standard rate income tax deducted at source under the imputation system. No further corporation tax is levied in the accounts of the dividend receiving company as dividends have already borne corporation tax in the accounts of the dividend paying company. Hence, if the cash flow earnings before interest and tax figure in the accounts of the dividend receiving company includes net cash dividend received as income then this income is exempt from any further corporation tax. Let "f" be the proportion of EBIT represented by franked investment income and Government Revenue Grants. The symbol "f" will be used in the valuation equation in the next section.

(v) Timing Of The Payment Of Corporation Tax: Besides ignoring disallowable expenses, capital allowances and non-taxable income, the models considered in Chapters 4 to 6 assume that the corporation tax is paid at the same time as the earnings are received. Usually in the U.K., corporation tax is paid 9 months after the end of the tax year - which means that tax is paid 15 months after the average time of receipt of EBIT (on the assumption that the earnings arise evenly during the year). If the company pays dividends, then payment of the associated Advance Corporation Tax to the Inland Revenue soon after the payment of dividends means that some corporation tax is paid much earlier than the due date of the mainstream corporation tax liability. Keeping these factors which affect the timing of tax payment in mind, it would seem reasonable to assume that all corporation tax payments in the U.K. are lagged by one year. Other average period may be assumed according to the circumstances in different countries. For example, in Australia, South Africa and West Germany, the revenue authorities collect most of the corporation tax during the tax year itself and therefore there is no time lag in the payment of corporation tax. One way of reflecting the time lag in the model is to discount the corporation tax rate by the risk free discount rate for the appropriate period. Henceforth $\tau_c'$ will reflect $\tau_{cn}/(1+r_p)$ that is, the effective corporation tax rate ($\tau_c'$) is the nominal corporation tax rate ( $\tau_{cn}$ ) discounted by the risk free rate applicable ( $r_p$ ).

The 5 points raised above illustrate some features of the corporation tax systems which are overlooked in the simpler models. Such treatment of corporation taxes is entirely justified in the interest of simplification and tractability of those models. However, where the aim is to understand correctly the impact of taxation on the cost of capital, or where greater accuracy is required, then these features need to be incorporated in the model. This is done in section B below.

## Section B - Valuation Model With Detailed Corporation Tax

The 5 factors stated in the above section imply that the effective corporation tax on the earnings of the company is as given by the following expression :

(7.1) $\tau_c'$ [ EBIT (1 + a - f) - RD - C ] - G where a = proportion of disallowable expenses

f = proportion of non taxable income

RD = interest payments on debt

C = annual equivalent of capital allowances on the initial investment

G = annual equivalent of capital based government grant

$\tau_c$' = the nominal corporation tax rate appropriately discounted for timing of the payment.

The coefficients in expression 7.1 were explained above in section A. In the expression, a and f are the disallowable and non-taxable cash flows respectively stated as proportions of EBIT. Therefore they are multiplied by the cash flow EBIT to arrive at the taxable EBIT. Monetary amounts of the interest payment (RD) and the annual equivalent capital allowances (C) are deducted from the taxable EBIT to arrive at the taxable income. Corporation tax at the discounted rate of $\tau_c$' is levied on this taxable income to arrive at the tax payable. The annual equivalent of the capital based government grant is then deducted to give the net contribution to the government from the company.

To simplify the analysis below, it may be assumed that G is reflected in C at an appropriate rate (that is, $\tau$ x G* = G), where G* is the grossed up amount included in C to reflect capital based grants. With this assumption, the effective corporation tax levied is

(7.2) $\tau_c'$ [ EBIT (1 + a - f) - RD - C ]

where C has been modified as stated above.

If the above expression 7.2 is substituted in the expression for expected return to the mth investor under the classical system of taxation (chapter 4, equation 4.1) then that equation is modified to:

$$(7.3) \quad y_i^m = n_i^m \{ \mu_i - \tau_c' [\mu_i (1 + a - f) - RD_i - C] - RD_i + (pD_i - \Delta_i) \theta_g^m + \Delta_i \theta_p^m - \tau_g^m pS_i \}$$

This simplifies to

$$(7.4)\quad y_i^m = n_i^m \{ [\mu_i^* - \tau_c' \mu_i a + \tau_c' \mu_i f + \tau_c' C - r_c' D_i] \theta_g^m + \Delta_i (\theta_p^m - \theta_g^m) - \tau_g^m pS_i \}$$

$\mu_i^* = (1-\tau_c')\mu_i$ ; $r_c' = R\theta_c' - p$, where $\theta_c' = 1-\tau_c'$. $r_c'$ now refers to the real interest rate net of the discounted corporation tax - that is net of $\tau_c'$ , not $\tau_c$. Therefore there is a change of variable from $\tau_c$ to $\tau_c'$ and since the latter variable is necessarily the smaller of the two, the expected return $y_i^m$ is greater as compared to the Modigliani model.

Since capital allowances, C, can be determined at the outset, and since $D_i$ and $\Delta_i$, debt level and dividend, are constant, the stochastic elements in equation 7.4 above are:

$$(7.5)\qquad \mu_i^* - \tau_c' (\mu_i a - \mu_i f)$$

Let $\mu_i'$ represent the above expression 7.5 . Thus $\mu_i'$ equals :

$$\begin{aligned}(7.6)\quad \mu_i' &= \mu_i^* - \tau_c' (\mu_i a - \mu_i f)\\ &= \mu_i (1 - \tau_c') - \tau_c' (\mu_i a - \mu_i f)\\ &= \mu_i - \mu_i \tau_c' - \tau_c' \mu_i a + \tau_c' \mu_i f\\ &= \mu_i - \tau_c' \mu_i (1 - a + f)\\ (7.7)\qquad &= \mu_i [1 - \tau_c' (1 - a + f) ]\end{aligned}$$

$\mu_i'$ thus differs from $\mu_i^*$ because $\mu_i'$ also takes into account the proportion of EBIT which is disallowed for tax purposes , the proportion of non-taxable income in EBIT, and the timing of payment of the tax liability. This was not done by the expression $\mu_i^*$ in the Modigliani model presented in chapter 4.

Using equation (7.7) to simplify the equation for expected return ((7.4) above), we obtain:

$$(7.8)\quad y_i^m = n_i^m \{ [\mu_i' + \tau_c' C_i - r_c' D_i] \theta_g^m + \Delta_i (\theta_p^m - \theta_g^m) - \tau_g^m pS_i \}$$

The variance of the portfolio return given in equation 7.8 is:

$$(7.9)\quad (\sigma_y^m)^2 = (\theta_g^m)^2 \Sigma_i \Sigma_j n_i^m \mu_{ij}' n_j^m$$

where $\mu_{ij}'$ is the covariance between $\mu_i'$ and $\mu_j'$. This replaces $\mu_{ij}^*$ which was the covariance between $\mu_i^*$ and $\mu_j^*$ in the Modigliani model (chapter 4, section B). Note that equation 7.9 is based on the stated assumption that the cash flows associated with capital allowances are risk free.

Optimising an investor's expected utility in a similar manner to that followed in chapter 4, section C, the resulting first order conditions are:

$$(7.10)\quad (\mu' - r_c'\, \mathbf{D} + \tau_c'\, \mathbf{C})\, \theta_g^m - \mathbf{S}(r_p^m + p\tau_g^m) - \Delta(\theta_g^m - \theta_p^m) = \gamma^m\, (\theta_g^m)^2\, [\mathbf{M}]\, \mathbf{n}^m$$

This is equivalent to equation MII.4 of the Modigliani valuation model (chapter 4, Section C).

The changes from Equation MII.4 are:

(i) Expression $(\mu^* - r_c\, D)$ is replaced by $(\mu' - r_c'D + \tau_c'C)$, where $\mu'$, $r_c'$ and $\tau_c'C$ reflect the changes resulting from the introduction of the corporation tax features discussed above.

(ii) [M] is now the variance covariance matrix of $\mu'$ - not $\mu^*$.

Summing over all individuals, one gets the following equation as equivalent of equation MII.5 (chapter 4) :

$$(7.11)\quad (\mu' - r_c'\mathbf{D} + \tau_c'\mathbf{C})\, \theta_g - \mathbf{S}(r_p + p\tau_g) - \Delta(\theta_g - \theta_p) = \Lambda\, [\mathbf{M}]\, \mathbf{1}$$

Here C is a vector of capital allowances available for all of the firms i. To obtain the valuation equation, we undertake steps which are similar to those undertaken in sections D and E of chapter 4.

Rearranging the terms in equation (7.11), we get:

$$(7.12)\quad -\mathbf{S}\,(r_p + p\tau_g) = -(\mu' - r_c'\mathbf{D} + \tau_c'\mathbf{C})\, \theta_g + \Delta(\theta_g - \theta_p) + \Lambda\, [\mathbf{M}]\, \mathbf{1}$$

Changing signs:

(7.13) $$S\,(r_p + p\tau_g) = (\mu' - r_c'\mathbf{D} + \tau_c'\mathbf{C})\,\theta_g - \Delta(\theta_g - \theta_p) - \Lambda\,[\mathbf{M}]\,\mathbf{1}$$

Extracting $r_c'\mathbf{D}$ we get:

(7.14) $$S\,(r_p + p\tau_g) = (\mu' + \tau_c'\mathbf{C})\,\theta_g - \Delta(\theta_g - \theta_p) - \Lambda\,[\mathbf{M}]\,\mathbf{1} - r_c'\mathbf{D}\,\theta_g$$

Divide throughout by $r_p + p\tau_g$

(7.15) $$S = \frac{(\mu' + \tau_g'\mathbf{C})\,\theta_g - \Delta\,(\theta_g - \theta_p) - \Lambda\,[\mathbf{M}]\,\mathbf{1}}{r_p + p\tau_g} \quad \frac{-r_c'\mathbf{D}\,\theta_g}{r_p + p\tau_g}$$

Add debt to both sides:

(7.16) $$S+D = \frac{(\mu' + \tau_g'\mathbf{C})\,\theta_g - \Delta\,(\theta_g - \theta_p) - \Lambda\,[\mathbf{M}]\,\mathbf{1}}{r_p + p\tau_g} + \mathbf{D} - \frac{r_c'\mathbf{D}\,\theta_g}{r_p + p\tau_g}$$

or

(7.17) $$\mathbf{V} = \frac{(\mu' + \tau_g'\mathbf{C})\,\theta_g - \Delta\,(\theta_g - \theta_p) - \Lambda\,[\mathbf{M}]\,\mathbf{1}}{r_p + p\tau_g} + \mathrm{LD}$$

which is equivalent to equation MII.7 (chapter 4) of the Modigliani model.

Since capital allowances are non-stochastic, the equivalent of equation MII.8 (chapter 4 section E), which shows V*, the value of risky stream for the aggregate corporate sector, is:

(7.18) $$V^* = V - L\,D + \frac{(\theta_g - \theta_p)\,\Delta}{r_p + p\tau_g} - \frac{\tau_c'C\,\theta_g}{r_p + p\tau_g} = \frac{\mu'\,\theta_g - \Lambda\,[M]\,1}{r_p + p\tau_g}$$

In equation (7.18) above, the point to note is that $\tau_c'C$, along with its coefficients, is extracted as a certain stream and only $\mu'\theta_g$ is treated as the risky part.

This implies that the total market value of aggregate corporate sector in a form equivalent to equation MII.10 is :

(7.19) $$V = \frac{\mu'\,\theta_g}{r_p + p\tau_g + \pi} + LD + \frac{\tau_c'C\,\theta_g}{r_p + p\tau_g} - \frac{\Delta\,(\theta_g - \theta_p)}{r_p + p\tau_g}$$

This definition shows that the benefit of tax savings flowing from the capital allowances created by the initial investment in the fixed assets (C) adds to the value of the aggregate corporate sector (V). This is explained fully in section C below.

For an individual firm the valuation equation equivalent to MII.12 (chapter 4 , Section F) is:

$$(7.20)\quad V_i = \frac{\mu_i'\,\theta_g}{r_p + \beta_i\pi + p\tau_g} + L\,D_i + \frac{\tau_c' C_i\,\theta_g}{r_p + p\tau_g} - \frac{\Delta_i\,(\theta_g - \theta_p)}{r_p + p\tau_g}$$

Equation (7.20) above represents a valuation equation for valuing risky assets under the classical system of taxation (chapters 4 and 5) after including some of the main relevant features of the corporation tax systems. The first term represents the value contributed by the risky component of the returns, and both the return component in the numerator and the discount rate in the denominator incorporate the changes proposed above. The second term measures the tax advantage of debt. As $\tau_{c'}$ is smaller than $\tau_c$ , $\theta_{c'}$ will be larger than $\theta_c$. As a consequence, the advantage of leverage will be lower when one is using the discounted rate advocated in this chapter in comparison with the rate used in the Modigliani model. The third term in equation (7.20) shows the contribution of made by the capital allowances on the initial investment in fixed assets. The risk free discount rate is used to value this component because the cash flows associated with taxes are assumed to be certain in this model. This assumption also results in the same risk free discount rate being used to discount the last component in the equation, which capitalises the impact of the differential rate of taxation of dividends.

To summarise, this equation (7.20) includes the following changes from the Modigliani equation (MII.12) which incorporated only a simple corporate tax framework:

(i) $\mu_i'$ differs from $\mu_i^*$ as it incorporates the impact of disallowable expenses and non taxable income.

(ii) An additional term appears: $\dfrac{\tau_c'\ C_i\,\theta_g}{r_p + p\tau_g}$

This term measures the favourable impact of capital allowances $C_i$, evaluated at the effective corporate tax rate $\tau_c$', taken net of applicable capital gains tax at the personal level ($\theta_g$) - all of which ( $\tau_c$' $C_i$ $\theta_g$ ) is discounted at the risk free rate. The magnitude of this component is important and this will be illustrated by the numerical examples in the analysis in the following chapters. This component represents a significant difference from the valuation model discussed in chapter 4 and therefore it is examined in greater detail in the next section below.

(iii) The corporation tax rate used is $\tau_c$' - the effective rate which is lower than the nominal rate in order to allow for the time lag in the payment of corporation tax.

(iv) The beta factor now refers to $ß_i = \text{cov}\left(\frac{\mu_i'}{V_i^*}, \frac{\mu'}{V^*}\right) / \text{Var}\left(\frac{\mu'}{V^*}\right)$

where $\mu_i$' and $\mu$' (for company i and the aggregate market return respectively) are now based on $\mu_i$' concept which differs from $\mu_i^*$. Similarly, $V_i$* and V*, the market value of company i and of the market respectively, are based on equation 7.18 above, which incorporates the changes made to the valuation model in this chapter.

If equation (7.20) is to be expressed over one common risk adjusted discount rate only in the denominator, then the numerator too must be correspondingly adjusted. The procedure followed is the same as in the case of derivation of equations (5.1) and (6.5) in the prevoius chapters. Following the same procedure, we derive the following equation:

$$(7.21)\quad V_i = \frac{\mu_i'\theta_g + LD_i\,[r_p + ß_i\pi] + \tau_c'\,C_i\,\theta_g + \dfrac{[\beta_i\pi\,\tau_c'\,C_i\,\theta_g\;]}{r_p + p\tau_g} - \Delta_i\,(\theta_g - \theta_p) - \dfrac{[ß_i\pi\,\Delta_i\,(\theta_g - \theta_p)\;]}{r_p + p\tau_g}}{r_p + p\tau_g + \pi}$$

The adjustments within the square brackets are the adjustments required simply to express the equation over a common denominator.
The above represents a valuation formula applicable in countries which have the classical system of corporation and personal taxes, such as the U.S.A.. Equation (7.21) reflects the important provisions of the tax system whereas the Modigliani model described in chapter 4 assumed a relatively simple tax regime.

A valuation model under the imputation system such as in the U.K., was described in the previous chapter. The corporation tax features introduced in this chapter can also be incorporated in the valuation equation under an imputation system of taxation. Following the procedure stated in this section, we can derive the valuation equation for companies in the U.K.. The resulting valuation will differ from the valuation equation (7.21) because of the two changes relevant to the U.K. considered in chapter 6, namely the indexation allowance on capital gains (which results in the elimination of the term $p\tau_g$ ), and substitution of the term $\theta_{PDIVI}$ when describing cashflows net of personal tax on dividends. Making these changes to (7.21), the resulting valuation equation for U.K. project appraisal is :

$$(7.22)\quad V_i = \frac{\mu_i'\theta_g + LD_i\,[r_p + ß_i\pi] + \tau_c'\,C_i\,\theta_g + \dfrac{[\beta_i\pi\,\tau_c'\,C_i\,\theta_g\;]}{r_p} - \Delta_i\,(\theta_g - \theta_{PDIVI}) - \dfrac{[ß_i\pi\Delta_i\,(\theta_g - \theta_{PDIVI})\,]}{r_p}}{r_p + ß_i\pi}$$

Equation (7.22) is the valuation equation for valuing risky corporate investments in the U.K.. It differs from the valuation equation derived in (7.21) above to the extent that it takes into account the two features of the U.K. tax system analysed in chapter 6, namely the imputation system of taxation and the taxation of real, as opposed to nominal, capital gains in the U.K. This equation is the main valuation equation advocated in this thesis. This equation is simplified and analysed after its characteristics are highlighted below in a comparison of this model with other models in the literature.

In chapter 4, we started with the Modigliani model which gave a relationship between corporate returns and the value of companies in the presence of corporate and personal taxes. During the course of describing that model, some adjustments were proposed for calculating the aggregate market risk aversion and revisions were proposed in the definition of beta factors. Chapter 5 described how the model could be adapted for the imputation system of taxation in general. Such a model is useful for the U.K. where the imputation system is in use. Chapter 6 described the other changes and the calculation of the coefficients required in order to use the model specifically for the U.K.. The present chapter (chapter 7) described the important features of the corporation tax system which resulted in some requisite changes to the model. All these adjustments and changes to the original Modigliani model are reflected either explicitly or implicitly in the above equation (7.22) . All these changes are improvements upon the original Modigliani model. The changes advocated in this thesis, and the reason why they represent an improvement, are listed below.

(1) The model represented by equation (7.22) ( "recommended model" ) incorporates the correct definition of beta factors. As pointed out in chapter 4, section F, it is incorrect to define beta factors as simply covariance within the context of this model.

(2) The recommended model incorporates the correct weightings in aggregating variables over investors. This was illustrated in chapter 6 in the calculation of $\Lambda$ and the tax variables for the market. The correct weightings, which include the relative capital of the investors, are particularly relevant in capital markets of most of the countries in the world where the presence of numerous but "small" investors may distort the

calculation of market variables otherwise.

(3) The recommended model has been adapted to reflect two of the main features of the tax system in the U.K., namely the imputation system and the taxation of real, as opposed to nominal, capital gains. The Modigliani model is not "incorrect" in this regard. It is simply applicable for the U.S.A., where the classical system (chapter 5) of taxation applies. The contribution made in this thesis is to explicitly state the model in the context of the U.K. tax system and to calculate the tax "advantage" of dividends in the U.K. (chapter 6).

(4) The recommended model points out that the average period for stockholding in the U.K. is substantially lower than the period assumed in the literature. An average period of 10 years mentioned in the literature is simply too long, given the high level of turnover relative to the total capitalisation value of the stock market, even after the market crash of 1987. The contribution made in this thesis is to use the concept of stock turnover (which is used in Accountancy to calculate the stockholding period of physical inventory) in the context of the capital markets. This usage enables one to calculate the effective capital gains tax, and this points out that the high turnover of shares on the stock market implies a much lower stockholding period and a much higher level of the effective capital gain tax than is assumed in the literature.

(5) The recommended model correctly incorporates the benefit of capital allowances on the initial investment in the valuation equation. Its ommission would lead to a material understatement of the project values.

(6) The recommended model incorporates the other relevant features of the corporation tax system (chapter 7, sections A & B) in the valuation equation, thereby making it more precise than the other models in the literature.

(7) The recommended model includes illustration of the calculations of the values of the relevant variables for the U.K..

Thus, within the context of the perpetual cash flow models, it is believed that the recommended model improves upon the existing literature.

The valuation equation 7.22 can be transformed into the following four components (the transformation is detailed in the next chapter):

$$(7.22')\ V_i = \underset{(a)}{\frac{\mu_i' \theta_g}{r_p + \beta_i \pi}} + \underset{(b)}{LD_i} + \underset{(c)}{\frac{\tau_c' C_i \theta_g}{r_p}} - \underset{(d)}{\frac{\Delta_i (\theta_g - \theta_{pDIVI})}{r_p}}$$

The term (a) in the above equation shows the after tax uncertain returns from the project discounted by the risk and tax adjusted discount rate. In the numerator of component (a), the stochastic cash flow from the project is subjected to two taxes, the effective corporation tax and the weighted average of the capital gains tax borne by the investors. The corporation tax rate is the effective tax rate which takes into account the lagged payment of corporate taxes as well as the influence of other tax allowable / disallowable expenditure. The discount rate is the risk adjusted discount rate, after accounting for the weighted average of personal tax borne by the investors, implicit in $r_p$. There is no adjustment for capital gains tax on inflationary element of capital gains because of the indexation allowance available in U.K..

The term (b) measures the impact of corporate leverage on valuation. Our model states that leverage is valuable for firms because of the net tax advantages. The benefit of leverage is dicounted at the risk free rate and is implicit in the term $LD_i$.

The term (c) is the effect of capital allowances and other corporate tax code features on valuation. These benefits have an important impact on valuations and the cost of capital, and this thesis advocates their inclusion in the valuation model, unlike the models in the existing literature which are not so elaborate. Therefore the models in the existing literature may be understating values by a considerable amount in comparison with the above model, which we believe is more accurate.

The term (d) measures the impact of taxing capital gains and dividend income at differential rates. We therefore are implying that dividend policy matters because of the taxes on dividend income are different from the taxes on capital gains. We estimate that in the U.K., dividend taxes in fact add to value because of the benefits of imputation system of taxation. We illustrate how to calculate the relevant weighted averages of the tax factors, given that the investors are heterogenous in terms of their risk characteristics as well as their personal income, capital gains and dividend tax rates.

Addition of these four terms results in the valuation equation advocated in this thesis. As is evident from the above description, the valuation equation is comprehensive, specific to the U.K., and incorporates those elements of the tax code that have not yet been incorporated in other models in the literature.

## Section C - Perpetual Cash flows And Capital Allowances

One of the advantages of the recommended model claimed in the previous section is that it correctly incorporates the benefit of capital allowances. The aim of this section is to prove that within the context of perpetual cash flow models, the simple treatment of corporation taxes in the Modigliani model is incorrect. It misses out the contribution made by the initial capital allowances to the value of a project.

To be fair to the author of the Modigliani (1982) model, it should be mentioned that nowhere does the author state explicitly that the valuation model for valuing corporate cash flows can be used directly for project valuation. Modigliani model was written to illustrate the valuation fo corporate equity and debt. However,

(i) One of the main uses of the valuation models which aim to calculate corporate values is for the purposes of project evaluation. This thesis too advocates the use of a corporate valuation model ( such as the recommended model of the previous section) for project evaluation. For example, consider an investment in a project which has an expected cash flow EBIT of $\mu_i$, an estimated project beta factor of $\beta_i$, which is expected to contribute $\Delta_i$ as dividends to the investors, and which is financed partly by long term debt of the amount $D_i$. The value of this project can be calculated by inputting these variables in the recommended model equation. Thus although Modigliani does not explicitly advocate that his model be used for project valuation, it is nonetheless appropriate in Finance to use such corporate valuation models for project valuation. Hence it is also appropriate to include capital allowances, which are relevant for project appraisal, into these valuation models.

(ii) The point made in this section, namely the correct inclusion of capital allowances on initial investment, is relevant to a certain extent even for the purposes of valuing companies ( as opposed to valuing individual projects).

The proof of relevance of capital allowances on initial investment is subdivided into 3 parts:

(i) Description of EBIT and the assumptions underlying this perpetual cash flow model.

(ii) Capital Allowances in the earlier years of the perpetual cash flow project.

(iii) Capital Allowances in the "Terminal Period" of "perpetual" cash flow model.

**(i) Description of EBIT and the assumptions underlying this perpetual cash flow model.**

The assumption underlying all the models in this thesis is that the company/project being evaluated generates perpetual cash flow EBIT with an expected value of $\mu_i$. This assumption is not as unrealistic as it appears at the first sight because it can be interpreted to mean either that the project is a very long term project or that the project investment is the first of a series of investments and thus the equipment installed will continue to be replaced when it is fully depreciated. This latter assumption is used in this section.

"Cash flow" EBIT equals EBITDA (Earnings Before Interest, Taxes, Depreciation and Amortisation Of Goodwill) less "capital expenditure" required to maintain the operating capacity of this perpetual cash flow project. For simplicity, we assume here that there is no inflation or interest expense and that there are no tax disallowable expenses nor any non-taxable income. We further assume that the only non cash flow related expense is the depreciation expense and that all of the earnings are received in cash during each of the periods in which they arise.

On the basis of these simplifying assumptions, "cash flow" EBIT will equal the taxable income once the capital allowances have reached a "steady state". This statement is explained as follows. Assume that a project has EBITDA (defined above) of £1200, and the accounting depreciation expense is £500 every year. With our simplifying assumptions, the accounting EBIT thus is £700. Since we are assuming no inflation in this section, the capital expenditure required to maintain the operating capacity will be equal to the depreciation expense and therefore capital expenditure in this example is £500. Therefore the "cash flow" EBIT as defined above will also be £700 ( EBITDA of £1200 less capital expenditure of £500). In project evaluation, due to the time value of money, what is relevant is not the accounting income but the cash flows arising. Therefore, in this thesis, both the Modigliani model and the recommended model emphasise that the income variable is "cash flow" EBIT. In this example, the cash flows arising from the project (EBITDA) are reduced by £500 which is spent on replacing the machinery that has depreciated. Thus the operating capacity of the business will be maintained and therefore it is correct to assume that the net cash flow (that is, net of replacement expenditure) will be generated to perpetuity.

In the tax computations, accounting depreciation is not an allowable expense. Therefore a relevant starting point for the tax computations is the income before depreciation, that is EBITDA. From EBITDA, capital allowances can be subtracted as a tax allowable expense. In order to calculate their magnitude we continue further with the above example. Let the project involve purchase of 4 machines, each costing £500. The machines have a four year life. Assume that capital allowances are available over the four year life of the equipment on a straight line basis. Since the gross value of machinery is maintained at £2000 by undertaking the replacement capital expenditure, capital allowances of £500 would be available every year. Thus, in a steady state, the capital allowances will equal the replacement capital expenditure of £500. Furthermore, the taxable income will be £700 (EBITDA of £1200 less capital allowances of £500). This taxable income equals the "cash flow" EBIT. Therefore, in the Modigliani model, it was possible to calculate the cash flow net of corporation taxes as $\mu_i - (\mu_i \tau_c)$, since the "cash flow" EBIT $\mu_i$ is assumed to be equal to the taxable income. ( Note that in this discussion, one of our simplifying assumptions is that there is no debt interest.)

Thus this discussion clarifies (a) the precise meaning of the term "cash flow" EBIT (b) the reason why "cash flow" EBIT is emphasised in the models, (c) the reason why it is correct to assume that cash flow EBIT can continue to perpetuity, and (d) the reason why a perpetual cash flow net of corporation tax can be written as $\mu_i - (\mu_i \tau_c)$. We continue by examining the capital allowances in the early years of the project using the simple example developed above.

**(ii) Capital Allowances in the earlier years of the perpetual cash flow project.**

The first column in the table below gives the year for which we are calculating the capital allowances. The second and third columns show the initial investment and the related capital allowances respectively. The capital allowances are assumed to be on a straight line basis over 4 years. The next column shows the replacement capital expenditure of £500 required to replace the 1/4th of the total machinery that has depreciated. The next 4 columns shows the capital allowances on the replacement capital expenses. The total of all the available capital allowances, that is on the initial expenditure and on replacement expenditure, is shown in the last column. The figures in bold are the capital expenses relating to the capital expenditure in the first 4 years. The figures in italics show the capital allowances arising on the replacement capital expenditure in the future years. The final column shows the total capital allowances available in the year.

| | Initial | Cap. | Cap. | Cap. Allowances On Expd. In | | | | Total |
|---|---|---|---|---|---|---|---|---|
| Year | Expd. | Allow. | Expd. | Yr 1 | Yr 2 | Yr3 | Yr4 | Allowances |
| 1 | 2000 | 500 | 500 | **125** | na | na | na | 625 |
| 2 | - | 500 | 500 | **125** | **125** | na | na | 750 |
| 3 | - | 500 | 500 | **125** | **125** | **125** | na | 875 |
| 4 | - | 500 | 500 | **125** | **125** | **125** | **125** | 1000 |
| 5 | - | - | 500 | *125* | **125** | **125** | **125** | 500 |
| 6 | - | - | 500 | *125* | *125* | **125** | **125** | 500 |
| 7 | - | - | 500 | *125* | *125* | *125* | **125** | 500 |
| 8 | - | - | 500 | *125* | *125* | *125* | *125* | 500 |

The above table shows that once the capital allowances have reached the steady state in year 5, the allowance is indeed equal to the replacement capital expenditure of £500. However, in the earlier years, the total capital allowances exceed the replacement capital expenditure because of the presence of capital allowances on the initial investment. Hence the model needs to take into account the benefit from these allowances. This is not done in the Modigliani model but it is done in the recommended model by the term $\tau_c' C_i \theta_g$. The magnitude of this term is fairly material to the valuations illustrated in the subsequent chapters.

One further point to note is that although the benefit of initial capital allowances occurs in the first four years in the above example, it is "available " in each and every year of our perpetual cash flow model. The reason for this is that in arriving at the term $C_i$ we first calculated the present value of these capital allowances and then calculated an annual equivalent of that present value for a perpetuity. Therefore, since $C_i$ is an annual equivalent, it is a relevant benefit for each and every year to perpetuity, including even those years when there are no original capital allowances on the initial investment.

**(iii) Capital Allowances in the "Terminal Period" of "perpetual" cash flow model.**

The aim of this sub-section is to briefly check whether the result obtained in the previous subsection will be negated by any deficit of allowances at the time the project is eventually terminated. The model is based on perpetual cash flows and therefore there is no terminal year of the project. However, in order to satisfy ourselves of how sound the conclusion in the above subsection is, we assume that the project "terminates" in some future year. Our aim is to find out if the capital allowances in the terminal year are less than the replacement capital expenditure. If they are less, then this will negate some of the benefit of the capital allowances indicated in the previous subsection. We can assume that the situation represented by year 8 in the above table is typical of any subsequent year and therefore examine the situation of capital allowances in that year. If the project were to terminate in that year, there would be unused capital allowances available since all of the expenditure in respect of the machinery existing in that year have not been used up. For example, out of the £500 spent on the machine purchased in year 8, only £125 of allowances have been used up. The conclusion therefore is that the capital allowances in the terminal year will be greater than £500, and not less. Therefore the benefit of the capital allowances considered in the above subsection is not reduced by any reversal of the situation when the project terminates. Thus we have demonstrated that even if we assume that the project terminates, there is no need for us to revise our conclusions. Furthermore, since the models that are analysed in these chapters are the perpetual cash flow models, there is no revision that needs to be made to the valuation equation (7.22) above.

## Section D - Alternative Method For Incorporating Capital Allowances

### INCORPORATING CAPITAL ALLOWANCES BY ADJUSTING COST OF INVESTMENT

The recommended model incorporates the benefit of the capital allowances on initial investment by advocating a new method of calculating an annual equivalent of the tax cash flows. There is an alternative method in the literature for calculating the benefit of such allowances.

The alternative method of incorporating capital allowances is to reduce the cost of the asset by the present value of capital allowances and capital grants. This gives the following expression for the effective price of an investment good ($p_I$) whose market value is $p_I^m$.

(7.30) $$p_I = p_I^m (1 - V)$$

where V is the present value of investment incentives comprising of capital allowances and grants (M A King, 1974, page 24). The terminology in this section is the same as that in the original article. V itself depends on various tax provisions which are represented in the following equation:

(7.31) $$V = g + t'(1 - g)\left(I_v + I_n + \frac{d(1 - I_n)}{e + d}\right)$$

| | | | |
|---|---|---|---|
| where | $g$ | = | rate of capital grant |
| | $t'$ | = | corporation tax rate |
| | $I_v$ | = | rate of investment allowance |

| | | |
|---|---|---|
| $I_n$ | = | rate of initial allowance |
| d | = | annual depreciation allowance on reducing balance method |
| e | = | rate of discount |

In the above equation, the term "g" is the capital grant proportion and therefore it is one of the components of V. To prevent duplication of incentives, the remaining incentives are available only on the proportion of cost of the asset not covered by "g", that is on (1 - g). These include any special investment allowance $I_v$ and any initial allowances $I_n$. Moreover, depreciation allowances ( called "writing down allowances" in the U.K.) are available but again, to prevent duplication, the depreciation allowances ( "d") are available only on the proportion which has not already been covered by the initial allowances ( $1 - I_n$ ). "d" represents the rate of depreciation allowance and in order to calculate the present value of the depreciatin allowance, it has to be discounted as described below.

The present value of capital allowances on a reducing balance method equals d/(e+d) of the capital expenditure eligible for writing down allowances. This can be checked by using a simple example involving the discounting of a perpetuity with a steady negative growth. For example, if £100 is eligible for capital allowances on reducing balance method then:

| | **Written Down Value b/f** | **Allowance @ 25%** |
|---|---|---|
| Yr 1 | 100 | 25 |
| Yr 2 | 75 | 18.75 |
| Yr 3 | 56.25 | 14.065 |
| Yr 4 | 42.185 | 10.546 |
| | | ....etc. |

The stream of depreciation allowances in the third column above is decreasing at a "growth" rate (g) of -25% , which is the rate of depeciation allowance. The present value of such a stream is:

(7.32) $$\frac{d}{e-g} = \frac{d}{e-(-d)} = \frac{d}{e+d}$$

Therefore the present value of the decreasing depreciation allowances is correctly included in equation (7.31) . Thus most types of capital allowances can be handled easily by the formula in equation (7.31). The value of V calculated by that equation can then be used in equation (7.30) to calculate the effective cost of the investment good.

However this formula which is derived in a two period model context, ignores balancing allowances and balancing charges on the termination of the project. It does not specify the tax adjustments required to the discount rate. Its advantage, however, is that where allowances are declining, this method of adjusting the cost of asset can be easily applied.

However, the expression we used in section B above, namely,

$$\tau_c' (\mu_i - RD_i + \mu_i a - \mu_i f - C) \qquad \text{(7.2 above)}$$

gives a comprehensive picture of the applicable corporation tax. It takes into account the disallowable expenses as well as the non taxable income. It takes into account the timing of the payment of corporation tax. It shows all of the relevant tax variables together. Therefore the treatment of corporation tax features in section B above is considered more appropriate and is used in the model in the rest of the thesis. But the point that this section D helps to emphasize is the point already made in section C above, which is that the allowances on the initial investment are very important for determining the desirability or otherwise of an investment.

# Chapter 7

## Conclusion

This chapter completes the development of the valuation model, illustrated by equation (7.22) developed and advocated in section B of this chapter. The advantages of this model over the other models were noted therein. This recommended model is one of the major contributions made in this thesis and it represents achievement of some of the objectives we were motivated to achieve in this thesis.

This recommended model will be applied for analysis in the subsequent chapters. In the next chapter, this equation is used to illustrate the use of the model to evaluate the impact of the changes to the discount rate resulting from the changes in the personal taxes introduced by the 1988 Finance Act.

# IMPACT OF THE 1988 BUDGET

In the previous chapter, equation (7.22), which states the valuation equation for the U.K. incorporates relevant features of the U.K. tax code. We now consider a practical application of the same equation. In this chapter we use that equation and the related valuation model to analyse the impact of the changes in personal taxes announced in the 1988 Budget (as these were substantial changes), on the discount rate which the companies should be using to evaluate projects. If we obtain clear cut and interesting answers, conditional on good estimates of parameters, then the model developed in the previous chapters may be a useful one.

The 1988 budget has reduced the top rate of personal income tax from 60% to 40%. Capital gains realised are to be taxed at the individual's marginal income tax rates instead of at 30% as before.

Most simplified asset pricing models would not imply any changes to the discount rates to be used since they do not incorporate personal taxes. Moreover, they may imply, as the Chancellor erroneously seems to have accepted, that achieving equality between nominal income tax and capital gains tax rates would remove distortions. Such a conclusion ignores the benefit of the imputation system of taxation for dividends in the U.K. as well as the difference between effective and nominal rates of capital gains tax.

The model developed in this thesis, which incorporates personal taxes and has been adapted to the U.K. tax structure (equation (7.22)), can be used to analyse the impact of the 1988 budget on:

(i) the tax advantage/disadvantage of dividends
(ii) the tax advantage of debt
(iii) the discount rate to be used for project appraisal.

The conclusions are that :

(i) Dividends are now even more attractive than retentions in the U.K.. The tax <u>advantage</u> of dividends after the budget is £0.25p compared to £0.19p before the budget for every pound of cash dividend.

(ii) The tax advantage of debt has increased to £0.40p after the budget compared to £0.29p before the budget.

(iii) The discount rate for project appraisal would decrease by 0.5% approximately. The changes in the personal tax rate have had the same impact on the discount rate as a reduction in the interest rate by 0.4% . (According to our model, the interaction of various taxation features, as detailed below, implies that either (a) a reduction of the interest rate by 0.4% or (b) implementing the 1988 Budget proposals, would reduce the effective discount rate for projects by 0.5% approximately.)

Note that this analysis of the impact of the budget on discount rates is only the result of a partial equilibrium analysis in the sense that no attempt is made to determine whether "overall" projects will now be more desirable. An "overall" analysis would include the impact of the budget on expected cash flow from projects - that is if the tax changes had not been made, the government and/or private sector expenditures would have been different and therefore expected cash flows from projects would have been different. This section, like all other literature on project appraisal, however, concentrates only on the impact of budget changes on discount rates.

The analysis backing these and other conclusions is presented below. The analysis are carried out in the following steps:

(a) Discussion of shareholders' behaviour( "investors" versus "speculators"). This is relevant especially for (b) below.

(b) Calculation of effective capital gains tax rate for individuals, corporations and pension funds. (that is $\tau_g^{indiv}$, $\tau_g^{corp}$ and $\tau_g^{pension}$ respectively) under the 1988 Budget.

(c) Calculation of risk aversion measures ($\gamma$) and weightings ($\Lambda$) to be used in aggregating individual tax rates to get overall economy wide averages for $\theta_g$, $\theta_p$ and $\theta_{pDIVI}$.

(d) Calculation of $\theta_g$, the income net of effective capital gains tax.

(e) Calculation of $\tau_p$ and $\theta_p$ , the tax variables relating to personal income tax.

(f) Calculation of $\tau_{pDIVI}$ and $\theta_{pDIVI}$ , the tax variables relating to personal taxation of dividend income in U.K..

(g) Calculation of $r_p$, the net of tax real interest rate.

(h) Calculation of $\Delta_i$ ($\theta_g$ - $\theta_{pDIVI}$), the tax advantage/disadvantage of dividends.

(i) Calculation of $LD_i$, the tax benefit of debt.

(j) Determining the impact of the tax changes in the budget on tax allowable expenses.

(k) Calculation of the change in the effective discount rate.

**(a) Shareholders: "speculators" vs "investors"**

The period for which the shareholders keep their shares is important because capital gains tax is paid on gains when they are realised rather than when they accrue. For example, the present value of tax is likely to be around 40% of the nominal value of tax if the shares are held for ten years. This is because the tax on gains accruing in the early years is not due until the shares have been disposed of. Therefore there is a significant timing benefit because the taxes are delayed until the shares are sold. Hence, it is relevant in calculating the effective capital gains tax to consider whether shareholders are "speculators" - who hold shares only for a short period of time, or whether they are "investors" - who ignore short term movements in share prices and hold shares for a number of years.

If shareholders are largely speculators, the effective CGT rate will be close to the nominal rate as the stockholding period is short. If they are investors, then effective CGT rate will be significantly lower.

The 1988 Budget has introduced changes in capital gains tax as they effect "individuals" (as opposed to corporations and pension funds). Hence initially we will concentrate on individuals.

The taxable gain over the years if an individual realises only $\pi$ percentage of gains each year is as follows:

| | | Realised gain = taxable gain | Gain remaining |
|---|---|---|---|
| Initial Gain £1 | Yr 1 | $\pi$ | $(1 - \pi)$ |
| | Yr 2 | $\pi (1 - \pi)$ | $(1 - \pi)^2$ |
| | Yr 3 | $\pi (1 - \pi)^2$ | $(1 - \pi)^3$ |
| | | ... | ... |

The present value of such a stream of taxable gains is given by:

$$\frac{\pi (1 + i)}{\pi + i}$$

where i is the discount rate. (King M.A., 1977). King estimates $\pi$ to be 10% for individuals, that is individuals realise only 10% of accrued gains each year. As a result, using a discount rate of 5% (as an estimate of real interest rate) we get:

$$\frac{0.1 (1 + i)}{0.1 + 0.05} = \frac{0.105}{0.15} = 0.7 \text{ approx}$$

Therefore, the effective capital gains tax for individuals is approximately 70% of the nominal rate. An estimate of the real interest rate is used instead of a risk adjusted discount rate because the cash flows associated with taxes are non-stochastic. Note that we are ignoring the annual exemption from personal capital gains tax for the reason given in the previous chapter.

Taking the effective rate of CGT as 70% of nominal CGT rate implies that individuals hold shares for approximately 8 years on average (using 5% discount rate). This compares with the 10 year holding period considered in a report by John King and as used by King (1977). 8 years is perhaps more realistic because, as stated in chapter 6 ( Section C, Step 3 a ), for the stock market as a whole the shares are held on average for about 0.7 years. Given the proportion of shares held by the individuals and other market participants considered in chapter 6, the other participants in the stock market, that is the pension funds and the insurance companies, therefore would be holding shares for a very short period indeed (to give the overall average shareholding period of 0.7 years if individuals hold shares on average for 8 years). Hence it will be assumed, in the analysis that follows that the average shareholding periods are:

| | | | |
|---|---|---|---|
| Individuals - "investors" - | i.e. average holding period 8 years | $\Rightarrow$ | effective CGT =70% of nominal rate |
| Pension funds &)- "speculators" - Insurance Cos ) | i.e. average holding period less than 1 year | $\Rightarrow$ | effective CGT = nominal CGT rate |

These holding periods are broadly consistent with the incidence of tax on the various parties. For example, pension funds would be indifferent from tax point of view between accrued and realised gains since they face no CGT. Hence they are more likely to realise gains quickly as compared to individuals, who face a capital gains tax along with realisation.

**(b) Calculation of effective CGT rates**

After the Budget, individuals face CGT at marginal rates equal to marginal income tax rates which are 25% for basic rate payers and 40% for top rate payers. In the following analysis, it is assumed that all individuals are top rate tax payers since this group is likely to dominate basic rate tax payers when the "value" of shares owned is considered. The basic rate tax payers may be significant in numerical terms but are unlikely to be significant in terms of the value of equity owned.

Following the Budget, the effective CGT rates for the investors are:

| | **Nominal Rate (1)** | **Effective Rate % of (1).** | $\tau_g$ |
|---|---|---|---|
| Individuals | 40% | 70% | $\tau_g^{indiv} = 28\%$ |
| Companies (the overwhelming majority of which are insurance companies with special 30% rate) | 30% | 100% | $\tau_g^{corp} = 30\%$ |
| Pension Funds, Charities | 0% | 100% | $\tau_g^{pension} = 0\%$ |

Prior to the Budget, these rates were:

| | | | |
|---|---|---|---|
| Individuals | 30% | 70% | 21% |
| Companies | 30% | 100% | 30% |
| Pension Funds | 0% | 100% | 0% |

A feature of this Budget has therefore been to increase the personal capital gains tax rate. This should have the effect of reducing the after tax cash flows (or equivalently increasing the discount rate) as will be shown later.

The net of gains tax cash flows, $\theta_g$s after the Budget are as follows :

$$\theta_g^{indiv} = 1 - 0.28 = 0.72$$
$$\theta_g^{corp} = 1 - 0.30 = 0.70$$
$$\theta_g^{pension} = 1 - 0 = 1.00$$

Before the budget, these were:

$$\theta_g^{indiv} = 1 - 0.21 = 0.79$$
$$\theta_g^{corp} = 1 - 0.30 = 0.70$$
$$\theta_g^{pension} = 1 - 0 = 1.00$$

Therefore individuals receive less cash flow net of CGT after the Budget than before the budget.

These revised values will be needed to calculate the harmonic mean of the risk aversion factors, $\Lambda$ and the economy-wide $\theta_g$ (which is a weighted average of the investors'$\theta$g), which are considered in sections (c) and (d) below.

**(c) Calculation of risk-aversion measures ($\gamma^m$) and mean of the weightings ($\Lambda$)**

In Chapter 6 the following risk aversion measures were estimated:

$$\gamma^{mkt} = 0.0333$$
$$\gamma^{indiv} = 0.0500$$
$$\gamma^{corp} = 0.0333$$
$$\gamma^{pension} = 0.0208$$

These values of $\gamma^{indiv}$ , $\gamma^{corp}$ and $\gamma^{pension}$ and the values of $\theta_g^m$s (calculated in (b) above) are required to calculate the weightings given to the different groups in market equilibrium.

Calculation of $\Lambda$:

$\Lambda$ was previously defined in chapter 6 as:

$$\Lambda = 1 / \Sigma_m \frac{1}{\gamma^m (\theta_g^m)^2}$$

$$\Sigma_m = \frac{1}{\gamma^{indiv} (\theta_g^{indiv})^2 \text{ x proportion}} + \frac{1}{\gamma^{co} (\theta_g^{co})^2 \text{ x proportion}} + \frac{1}{\gamma^{pension} (\theta_g^{pension})^2 \text{ x proportion}}$$

$$= \frac{1}{.05 \text{ x } (0.72)^2 \text{ x } 0.3} + \frac{1}{0.0333 \text{ x } (0.7)^2 \text{ x } 0.3} + \frac{1}{0.0208 \text{ x } (1)^2 \text{ x } 0.4}$$

$$= \frac{1}{0.0078} + \frac{1}{0.0049} + \frac{1}{0.0083}$$

$$= 128.60 + 204.08 + 120.00$$

$$= 452.68$$

Therefore $\Lambda = \frac{1}{452.68} = 0.0022090 = 0.0022$ approx.

This is the value of $\Lambda$ after the budget.

Calculations of a similar type but using the tax rates applicable before the Budget show that previously $\Lambda$ was 0.00232 .

These calculations above are needed to derive the weights for aggregate values of $\theta_g$, etc. For example

$$\theta_g = \Sigma_m \theta_g^m \times \frac{\Lambda}{\gamma^m (\theta_g^m)^2 \times \text{proportion}}$$

After the Budget, the weights are:

| | | | | | Weights |
|---|---|---|---|---|---|
| Individuals | $\dfrac{\Lambda}{\text{proportion} \times \gamma^{indiv} \times (\theta_g^{indiv})^2}$ | = | $\dfrac{0.0022}{0.0078}$ | = | 0.2840 |
| Companies | $\dfrac{\Lambda}{\text{proportion} \times \gamma^{co} \times (\theta_g^{co})^2}$ | = | $\dfrac{0.0022}{0.0049}$ | = | 0.4510 |
| Pension funds | $\dfrac{\Lambda}{\text{proportion} \times \gamma^{pension} \times (\theta_g^{pension})^2}$ | = | $\dfrac{0.0022}{0.0083}$ | = | 0.2650 |
| | | | Sum of weights | | 1.000 |

These weights will be used to calculate $\theta_g$ and other tax variables, instead of using the simple proportion of shares owned by the individuals, companies and pension funds (that is, instead of using 0.30, 0.30 and 0.40). The reason why these weights differ from the 0.30, 0.30, 0.40 proportions are:

(a) these weights take into account the estimated risk aversion characteristics of the owners of the shares, and

(b) these weights take into account the net of Capital Gains Tax cash flows received by investors. The resulting proportions therefore differ from the simple ownership proportions because the capital gains tax rate differs according to the type of investor. Capital Gains Taxes are the relevant taxes for the calculation of these weights as explained in chapter 6.

Similarly, the pre-budget weights can be calculated. These are:

| | **Pre-Budget Weights** |
|---|---|
| Individuals | 0.2479 |
| Companies | 0.4735 |
| Pension Funds | 0.2786 |
| Sum of weights | 1.0000 |

Note that the weighting given to individuals increases from 0.2479 prior to the budget to 0.2840 because the weights are inversely related to $\theta_g^m$. The decrease in $\theta_g^{indiv}$, due to an increase in the CGT on individuals in the Budget, causes the above change.

The pre-Budget and the post-Budget sets of weights above are used in the calculation of the aggregate economy wide tax factors below.

**(d) Calculation of $\theta_g$**

$$\theta_g = \Sigma_m \theta_g^m \times \frac{\Lambda}{\gamma^m (\theta_g)^2}$$

Using tax rates after the Budget, we obtain the following :

| | | $\theta_g^m$ | | Weights | |
|---|---|---|---|---|---|
| Individuals | = | 0.72 | x | 0.2840 | =0.2044 |
| Companies | = | 0.7 | x | 0.4510 | =0.3157 |
| Pension funds | = | 1.0 | x | 0.2650 | =0.2650 |
| Aggregate $\theta_g$ (post Budget) | | | | = | 0.786 |

Similar calculations can be done for pre budget $\theta_g$.

**Pre Budget $\theta_g$**

| | | | | | |
|---|---|---|---|---|---|
| Individuals | = | 0.79 | x | 0.2479 | =0.1958 |
| Companies | = | 0.7 | x | 0.4735 | =0.3315 |
| Pension funds | = | 1.0 | x | 0.2786 | =0.2786 |
| | | $\theta_g$ (pre Budget) | | | =0.806 |

The above calculations show that due to the increase in CGT for individuals from 30% to 40%, after taking into account the delay in realising gains tax and the risk aversion and the CGT rates of the heterogeneous investors , the post CGT cash flows decrease from 80.6% to 78.6% of the pre tax cash flows for the economy as a whole. This should adversely affect the required discount rate since the leakages in the form of taxes have increased.

**(e) Calculation of $\tau_p$ and $\theta_p$, (Income Tax related variables) :**

Individuals in higher tax brackets pay only 40% tax compared to 60% before the Budget. Hence $\tau_p^{indiv}$ is lower and therefore the after tax flow $\theta_p^{indiv}$ is higher.

Of course, some equity investors are standard rate tax payers, although we consider only higher rate tax payers throughout this thesis. THe reason is, as stated previously, that the higher rate tax payers are likely to dominate equity holdings from the point of view of value of equity. This is supported by estimates based on information from ICD Marketing Services' British Investor Database, which is U.K.'s largest source of household investor information. Their data shows that £3.1 billion of equity is held by 3 million households with small holdings (less than £5,000), whereas well over £15.1 billion is held by households owning more than £5,000 of shares each. We estimate that the former are likely to be standard rate tax payers, whereas the latter are likely to be higher rate tax payers. If this is so, then probably well over 85% of shares are likely to be owned by higher rate tax payers.

**$\theta_p$ after the Budget**

$$\theta_p^{indiv} = 1 - 0.4 = 0.6000$$

$$\theta_p^{co} = 1 - 0.35 = 0.6500$$

$$\theta_p^{pension} = 1 - 0 = 1.0000$$

Using these values and the weights as already calculated in (c), the value of $\theta_p$ after the Budget is:

| | | | | | |
|---|---|---|---|---|---|
| Individuals | .6000 | x | .2840 | = | 0.1705 |
| Companies | .6500 | x | .4510 | = | 0.2932 |
| Pension funds | 1.000 | x | .2650 | = | 0.2652 |
| | | | $\theta_p$ (Post Budget) | = | 0.7289 |

A similar calculation can be made using the pre-budget tax rates, when 60% tax was payable by individuals so that $\theta_p^{indiv}$ = 1 - 60% = 40%. This gives the following average $\theta_p$ :

**$\theta_p$ (Pre Budget)**

| | | | | | |
|---|---|---|---|---|---|
| Individuals | 0.4 | x | .2479 | = | 0.0992 |
| Companies | 0.65 | x | .4735 | = | 0.3078 |
| Pension funds | 1.0 | x | .2786 | = | 0.2786 |
| | **$\theta_p$ (Pre Budget)** | | | | 0.6857 |

Therefore, the cash flows net of income tax are now higher. However, this does not necessarily mean that discount rates will be lower since what is important is how attractive is equity investment relative to risk free asset returns - and the lowering of personal income tax rates implies that the post tax cash flows from risk free asset will also be higher.

Note that although for high rate tax payers the income tax rate has decreased substantially (from 60% to 40%), the impact on $\theta_p$ is more modest - from pre-budget 0.69 to 0.73 post budget. This is so because the tax rates on interest income earned by companies and pension funds are not affected by the 1988 budget.

These values of $\theta_p$ before and after budget will be used to calculate $r_p$ in sub section (g) below. Before that, we calculate the tax variable $\tau_{pDIVI}$.

### (f) Taxation of Dividend Income, $\tau_{pDIVI}$ (and $\theta_{pDIVI}$)

Since dividends are paid out of post corporation tax income, we consider the extra tax payable by a shareholder to whom £1 of cash dividend is distributed.

**Individuals:** Basic rate tax payers pay no extra tax since their tax liability is extinguished by the tax credit they receive. However, as higher rate tax payers are likely to be the dominant group, we continue to concentrate on them.

For every £1 of net cash dividend, the gross dividend, with ACT rate of 25% after the budget, is:

| | | | |
|---|---|---|---|
| £1 x $\frac{100}{100\text{-}25}$ | | = | £1.3333 |
| Tax payable there-on by higher rate tax payer @ 40% | | = | 0.5333 |
| Less: Tax credit = 25% of £1.3333 | | = | 0.3333 |
| | Extra tax payable ($\tau_{pDIVI}^{indiv}$) | = | 0.2000 |

Hence higher rate tax payers pay only 20% of tax for every £1 of cash dividend received. This is lower than the 28% effective rate of capital gains tax calculated in (b) above which would apply to capital gains resulting from retention of earnings. As noted previously, we ignore standard rate tax payers for the reason stated therein.

**Corporations:** Receipt of Franked Investment income bears no additional tax in the receiving company. Therefore $\tau_{pDIVI}^{corp} = 0$.

**Pension funds:** Pension funds can claim back the tax credit they receive along with the cash dividend because they are tax exempt. Therefore $\tau_{pDIVI}^{pension} = -0.3333$, that is, for every £1 of cash dividend, they receive another 33.33p from Inland Revenue. [ Of course, this credit is related to the Advance Corporation Tax that is collected at the corporate level (see below). ]

Using the above figures for $\tau_{p\ DIVI}^{m}$, the figures for $\theta_{p\ DIVI}^{m}$ are:

$$\theta_{pDIVI}^{indiv} = £1 - 0.2000 = 0.8000$$

$$\theta_{pDIVI}^{corp} = £1 - 0 = 1.0000$$

$$\theta_{pDIVI}^{pension} = £1 - -0.3333 = 1.3333$$

Therefore, average $\theta_{pDIVI}$, using the weights previously calculated, is:

| | | | | | |
|---|---|---|---|---|---|
| Individuals | 0.8000 | x | 0.2840 | = | 0.2274 |
| Companies | 1.0000 | x | 0.4510 | = | 0.4510 |
| Pension funds | 1.3333 | x | 0.2650 | = | 0.3536 |
| | $\theta_p$ Post Budget | | | = | 1.032 |

Therefore, on average, using the weights above, Inland Revenue pays out a fraction more than it receives from taxation of dividends. However, it should be noted that all the above relates to post-corporation tax income - and that the Inland Revenue collects the tax credits attached to dividends as Advance Corporation Tax. Hence there is no net cash outflow when ACT collections and dividend taxation are looked at together. Instead, what is implied is that if the Inland Revenue were to collect Corporation Taxes as at present, and set dividend taxation to zero, then, provided there is no switch from capital gains to dividends induced by this move, the Inland Revenue should receive fractionally more revenue than at present. Therefore, the present system of ACT and the related tax credit could be abolished without loss of revenue from the domestic investors. The reason why the Inland Revenue are reluctant to do so is because presently they collect ACT, but do not give any offsetting tax credit in respect of (a) overseas income earned by the companies, and (b) dividends to overseas investors. However, it can be argued that (a) above is in any case an unfair burden on companies with overseas income, and that (b) above can be collected simply as a withholding tax. Therefore there is potential for simplifying the tax system as stated above.

A similar calculation for $\theta_{pDIVI}$ before the budget, using the $\theta_p^{m}{}_{DIVI}$ amounts calculated previously in chapter 6 (page 175), and the weights calculated in section (c) above, is:

$\theta_{pDIVI}$ (Pre Budget):

| | | | | | | |
|---|---|---|---|---|---|---|
| Individuals | 0.5479 | x | 0.2479 | = | 0.1358 |
| Corporations | 1.000 | x | 0.4735 | = | 0.4735 |
| Pension funds | 1.3698 | x | 0.2786 | = | 0.3817 |
| | | $\theta_{pDIVI}$ | | | 0.991 |

Therefore, reduction in the income tax rate for individuals has fractionally improved $\theta_{pDIVI}$ from 0.991 to 1.032.

This benefit is not pronounced because income tax rates do not affect companies. However, pension funds are worse off as standard rate of personal tax (and ACT rate) is reduced from 27% to 25%. This implies that whereas previously for £1 of cash dividend, they would receive 37 pence tax refund; post budget, they receive only 33.3 pence tax refund. Thus pension funds are worse off with reduction in income tax rates AS FAR AS INCOME FROM DIVIDENDS is concerned because of the imputation system.

**(g) Real Rate of Interest after Personal Tax ($r_p$)**

This is defined as $R\theta_p$ - p where R is nominal interest rate (risk free), $\theta_p$ is net of personal income tax flow per (e) above, and p is the rate of inflation. Taking R to be 10% and p to be 4% , the post budget $r_p$ is:

$$\begin{aligned} & 10\% \times 0.7289 - 4\% \\ = \; & 7.289\% - 4\% \\ = \; & 3.289\% \qquad (= 0.03289) \end{aligned}$$

Before the budget, for similar nominal interest rate and inflation rate, $r_p$ was:

$$\begin{aligned} & 10\% \times 0.6857 - 4\% \\ = \; & 6.857\% - 4\% \\ = \; & 2.857\% \qquad (= 0.02857) \end{aligned}$$

Therefore reduction in personal income tax rates increases the post tax real interest rate. This will lead to an increase in discount rate because:

(a) $r_p$ is a component of the discount rate used to discount risky cash flows ($r_p + \beta_i\pi$)

(b) $r_p$ is the discount rate used to discount risk-free cash flows - eg. taxation associated with permanent and steady level of debt, capital allowances (see below) and dividends.

Of course, the reduction in personal taxes improves the tax advantage of dividends (see (h) below) and makes debt more attractive (as is shown in (i) below), but let us abstract from these effects to concentrate only on the after tax risk free discount rate. We can do so by considering an all equity firm which has zero pay out. (Zero pay out is what financial economists recommend for corporations in the U.S.A., and in other countries with a similar "classical" tax structure).

Intuitively, one may argue that since all types of taxes are a drain on the cash flows between the time they arise from projects to the time they are received in investors' pockets, a reduction in any tax should have the same effect - that of reducing the effective discount rate. The effective discount rate should be reduced because since there are fewer leakages of cash flow in the form of taxes, a smaller pre-tax return needs to be earned by the projects and still leave sufficient post tax cash flows to satisfy the investors' required rates of post-tax return. Therefore intuitively, reducing either the corporation taxes or the personal taxes should reduce discount rates. This intuitive reasoning , however, is not correct. This is because factors which are relevant to the determination of the discount rate also include:

(i) whether risk free assets as well as corporations are subject to the particular tax

(ii) what proportions of returns from these two sources are subject to the particular tax.

Above two factors imply that while reduction in personal income taxes does not benefit corporations, reduction in corporation tax does benefit them. This contradiction arises because risk free assets (government bonds or even corporate debt) are not subject to corporation tax. They bear only personal income taxes. Therefore reduction in corporation taxes does not affect returns from risk free assets but only benefits corporate projects - it lowers the required pre tax discount rates.

However, reduction in personal income tax rates benefits cash flows even from risk-free assets. Hence, if corporations cannot benefit from this reduction in income tax rates (for example, an all-equity zero-dividend payout company would bear only capital gains tax at shareholder level and therefore would not benefit from any reduction in income tax) then reduction in income tax is relatively ( that is, relative to investment in risk free assets) disadvantageous for corporate investment. The disadvantage arises because investment in risk free assets becomes more attractive with a reduction in personal income tax rates which are relevant for taxation of interest income. This disadvantage is reflected in an increase in the real after tax risk free discount rate. For a given nominal interest rate, a reduction in income taxes would automatically increase the post tax interest rate.

The above illustrates one major difference between the impact of corporation taxes and personal taxes on the determination of the discount rate, namely that lowering corporation taxes lowers the discount rate, but lowering personal taxes is likely to increase it. The conclusion above is that decreasing personal taxes does not benefit all-equity zero pay out corporations at all. Whether this reduction will benefit companies which are not all equity and which do not have zero pay out will become clearer after considering:

tax advantage/disadvantage of dividends (section (h) below)

tax advantage of debt (section (i) below).

### (h) Tax Advantage/Disadvantage of Dividends

The expression below, taken from the valuation equation 7.22, measures the difference to the post tax cash flows caused by taxing dividend income at a different rate from the tax on capital gains.

$$- \Delta_i (\theta_g - \theta_{pDIVI})$$

A brief reminder that all of the equity cash flows are subject to capital gains tax in the expression $\mu' \theta_g$, therefore what is being considered here is the <u>extra</u> tax when some of the cash flow is taxed as dividends instead of being retained in the company - hence the expression $\theta_g - \theta_{pDIVI}$.

The values of $\theta_g$ and $\theta_{pDIVI}$ were calculated in sections (d) and (f) respectively. Therefore $\theta_g - \theta_{pDIVI}$ after the budget is:

$$\begin{aligned} & - \Delta_i (0.786 - 1.032) \\ = \quad & - \Delta_i (-0.246) \\ = \quad & \Delta_i \times (0.246) \end{aligned}$$

where $\Delta_i$ = constant level of dividends.

Note that the expression is positive, that is for £1 of post corporation tax income, investors get a higher amount after dividend taxation (1.032) than they would after capital gains taxation (0.786).

Prior to the budget, the tax advantage of dividend was:

$$- \Delta_i (\theta_g - \theta_{pDIVI})$$

$$= \quad -\Delta_i\,(0.806 - 0.991)$$

$$= \quad \Delta_i \times (0.185)$$

Therefore, due to the reduction in personal income taxes and an increase in capital gains taxes, the tax advantage of dividends has improved significantly. This is in spite of the fact that CGT may be deferred by individuals and that we have assumed a deferral of approximately 8 years on average for the individual investors.

The reason why there is tax advantage of dividends in the U.K. is because pension funds and insurance companies have a tax preference for dividends. They pay lower personal taxes (refund in case of pension funds) on dividend income in comparison with taxes on capital gains.

Note that simply changing the nominal capital gains tax rate to the income tax rate, as has been done in the 1988 budget, does not imply that $\theta_g = \theta_p$ and that $\theta_g - \theta_p = 0$. Nor does it imply that companies are neutral between retentions and dividends. This lack of neutrality results from the following features:

(i) Effective rates of capital gains tax for individuals at is only 28% (see section (b)) since they do not realise all gains immediately - this is lower than marginal income tax rate of 40%. This would imply tax DISADVANTAGE of dividends for individuals, if (ii) below were not present.

(ii) As has been noted, for the U.K., what is relevant is not $\tau_p$ but $\tau_{pDIVI}$, that is how much additional tax is borne by a £ of post-corporation tax earnings. A comparison of $\tau_g$, the gains tax, with $\tau_{pDIVI}$, the extra tax on dividends is reproduced from sections (b) and (f) above:

**Post budget**

| | $\tau_g$ | $\tau_{pDIVI}$ |
|---|---|---|
| Individuals | 0.28 | 0.20 |
| Companies (including Insurance companies) | 0.30 | 0.00 (franked investment income) |
| Pension funds | 0.00 | -0.33 (credit) |

In each case $\tau_{pDIVI}$ is less than the corresponding $\tau_g$ - hence the tax advantage of dividends. This shows that the Chancellor's claims of tax neutrality brought about by the changes in the 1988 Budget are not true in the wider context discussed above. For a comparison with the pre Budget situation, we examine the pre Budget tax rates.

**Pre budget**

| | $\tau_g$ | $\tau_{pDIVI}$ |
|---|---|---|
| Individuals | 0.21 | 0.4521 |
| Companies (using Insurance companies) | 0.30 | 0.0 |
| Pension funds | 0.0 | -0.3698 (credit) |

Hence by taxing realised gains at income tax rates, the Chancellor has:

(a) reduced the discrepancy between $\tau_g$ and $\tau_{pDIVI}$ for individuals but made dividends more attractive post budget for the higher rate tax payers.

(b) reduced the discrepancy between $\tau_g$ and $\tau_{pDIVI}$ for pension funds by reducing ACT and the basic rate personal tax marginally.

(c) overall has made dividends even more attractive for shareholders as a whole because of the change in the relative advantage of dividends over retentions for higher rate tax paying individuals.

**(i) $LD_i$ - the tax advantage of debt**

The tax advantage of debt is given by L which is:

$$1 - \frac{r_c \theta_g}{r_p + p\tau_g}$$

which, for the U.K., reduces to:

$$\left( 1 - \frac{r_c \theta_g}{r_p} \right)$$

(The term $p\tau_g$ disappears due to CGT indexation as previously noted).

in the above expression $r_c$ is the real interest rate net of corporation tax, $r_p$ is the real interest rate net of personal income tax, and $\theta_g$ is one minus the capital gains tax.

We compare the post Budget and the pre Budget tax advantages of debt. Post budget, $r_c$ is the same as pre budget $r_c$ since the corporation tax rate has not changed. Therefore

$$
\begin{aligned}
r_c &= R\,\theta_p - p \\
&= 10\%\ (.65) - 4\% \\
&= 6.5\% - 4\% \\
&= 2.5\% \qquad (=0.025)
\end{aligned}
$$

From (d) and (g) above, $\theta_g$ and $r_p$ post budget are:

$$\theta_g = 0.786 \qquad \theta_p = 3.289\%$$

therefore

$$
\begin{aligned}
L &= 1 - \frac{2.5}{3.289} \times 0.786 \\
&= 1 - 0.5974 \\
&= 0.4026
\end{aligned}
$$

This implies that if the company were to increase debt by £1, all things being equal, it would increase the value of the company by 40 pence.

Pre budget, $\theta_g$ and $r_p$ per (d) and (g) above were 0.806 and 2.857 respectively. Therefore

$$
\begin{aligned}
L\ (\text{Pre Budget}) &= 1 - \frac{2.5}{2.857} \times 0.806 \\
&= 1 - 0.7053 \\
&= 0.2947
\end{aligned}
$$

Therefore, due to tax changes, the tax advantage of debt has increased from 0.29 to 0.40 - that is, by almost one third of the pre budget level.

Two interesting questions are:

(a) why has debt become even more advantageous?

(b) how will the companies react to it?

In relation to (a) one source of advantage to debt for individuals is reduction in personal income tax paid on debt interest from 60% to 40%. In addition to this obvious advantage, there is one more source of advantage - and that is that retained equity has become less attractive. If debt were not increased, then the cash flows arising to equity holders would be relatively higher, and since we are assuming constant dividends, the retained earnings would be higher. Now retained earnings lead to capital gains, and capital gains tax for higher bracket individuals has been increased by the budget from an effective rate of 21% to 28% (8 year deferment assumed). Hence increasing the capital gains rate for higher tax bracket individuals also makes debt relatively more attractive.

In relation to (b), even before the budget, the question arises of why did the companies not continue to increase debt? After all, then there was a £0.29 tax advantage to debt. The answer is that probably the tax advantage of debt is weighed against other factors which have not been included in the model, eg. bankruptcy costs. Therefore companies in the U.K. probably weighted the tax advantage of debt against these other costs and decided on the interior solution - their own optimal debt level. The literature in this area is not conclusive - the absolute amount of bankruptcy costs which have been studied empirically appears to be too low to counteract this substantial tax advantage. A second explanation is that increasing debt increases probability that interest tax shields will not be utilised due to insufficient taxable income (De Angelo and Masulis (1980)). These arguments are considered earlier in chapters 2 and 3 in the thesis but what is relevant now is that as the tax advantage of debt has increased due to the budget, and therefore so must the optimal level of debt, other things being equal.

This increase in the level of attractiveness of debt will be taken into account when calculating the impact of the Budget changes on the discount rate. The post Budget debt level of debt will be assumed to be higher than the pre Budget debt level for the financing of a typical project taken as an illustrative example. This will be shown to be the main factor contributing to the reduction in the post budget effective discount rate.

### (j) Impact of Budget on Tax Allowable Expenses

The main feature here is the disallowance for all entertainment expenses incurred. Pre-Budget, it was only entertaining overseas customers which was tax deductible. However, it was surprising how high a proportion of entertainment expenses were analysed as being for overseas customers - regardless of the low level of export earnings of many companies.

The impact of this change will be increase 'a' the proportion of disallowable expenses considered in chapter 7 . This will result in a very minor increase in effective discount rate. In the example considered in section (k) below, this factor - the disallowability of entertainment expenses, is ignored as its impact is likely to be negligible for most projects.

The budget has not changed the basic rates for capital allowances. However, change in the after tax risk free discount rate does have an impact on these. Before considering this impact, the calculation of $C_i$, the capital allowances term detailed in Chapter 7, will be simplified.

In Chapter 7, it was shown that the present value of capital allowances available on a reducing balance method is given by

$$\frac{d}{e+d}$$

where d = the rate of Writing Down Allowance (WDA) and e = discount rate. WDA is at the rate of 25% in the U.K..

The simplification proposed is that this present value will be converted to an annual equivalent (AE).

$$\text{Since PV} = \frac{AE}{e} \quad \text{for a perpetuity}$$

$$AE = PV \times e$$

$$= \frac{d}{e+d}\, e = \frac{de}{e+d}$$

Hence we can consider that fraction (de/e+d) of investment expenditure is allowable every year. This fraction is fixed and permanent, like the cash flows associated with taxation of debt and dividends.

As these cash flows are riskless, the appropriate discount rate is $r_p$, the after tax riskless real rate. This is the same rate that is used to discount tax-related cash flows arising in respect of dividends and debt. [Note: dividends are also assumed to be fixed, like debt, in this model.]

The effect of the increase in real after tax interest rate as a result of the Budget is to increase the value of the annual equivalent of the capital allowances calculated above. This increases $C_i$, the capital allowances term which influences the valuation equation 7.22.

However, in the valuation model developed as equation 7.22, the full expression for the impact of capital allowances $C_i$ on the valuation $V_i$ is given by

$$\frac{\tau_c' \, C_i \, \theta_g}{r_p}$$

where

(a) $\tau_c'$ is the effective rate of corporation tax. Since taxes are paid one year late, the effective amount of $\tau_c'$ will be <u>reduced</u> if the discount rate is increased ($r_p$ has increased due to the budget). This counteracts the impact of an increase in $C_i$.

(b) Secondly $\theta_g$ is lower after the Budget compared to before the Budget, again counteracting the impact of an increase in $C_i$.

(c) Finally $r_p$, the discount rate for the entire factor, is higher post budget, again counteracting the impact of an increase in $C_i$.

The net result is a small negative impact on the value of the firm due to the Budget. The net result is due to:

(i) a small increase in $C_i$, but which is more than counteracted by

(ii) (a), (b) and (c) above.

These features will become clearer when an example is considered in section (k) below.

**(k) Calculation of the change in EFFECTIVE DISCOUNT RATE**

Consider a project costing £800, all of which is eligible for capital allowances, which results in expected cash flow of £100 every year. Let the project be financed by £250 of debt before the Budget. Let the permanent stream of dividends be £30 per year.

The above values are reasonable in the sense that the debt ratio is not too high and approximately one half of cash flow after corporation tax available for ordinary shareholders is distributed. The project will show a small net present value. Hence it is a suitable project to consider for analysing the impact of budget changes.

The project can be evaluated using the recommended model shown as equation 7.22 in chapter 7, which is reproduced below.

$$(8.1)\quad V_i = \frac{\mu_i'\theta_g + LD_i\,[r_p + \beta_i\pi] + \tau_c'\,C_i\,\theta_g + \left[\dfrac{\beta_i\pi\,\tau_c'\,C_i\,\theta_g}{r_p}\right] - \Delta_i\,(\theta_g - \theta_{pDIVI}) - \left[\dfrac{\beta_i\pi\,\Delta_i\,(\theta_g - \theta_{pDIVI})}{r_p}\right]}{r_p + \beta_i\pi}$$

The above equation can be expressed alternatively as consisting of the four components identified below

$$(8.2)\quad V_i = \underset{(a)}{\frac{\mu_i'\,\theta_g}{r_p + \beta_i\pi}} + \underset{(b)}{LD_i} + \underset{(c)}{\frac{\tau_c'\,C_i\,\theta_g}{r_p}} - \underset{(d)}{\frac{\Delta_i\,(\theta_g - \theta_{pDIVI})}{r_p}}$$

The four components are:

(a) the valuation of risky cash flows using a risk adjusted discount rate

(b) the benefit of debt

(c) the benefit of capital allowances, discounted at risk-free rate

(d) the advantage/disadvantage of differential taxation of dividends, discounted at risk-free rate.

Let us assume that $\beta_i\pi = 8\%$ represents the average risk premium net of taxes for the U.K.. This 8% rate is based on the average beta factor of 1 ( the *average* beta is one by definition ), and the 8% market risk premium estimated in chapter 6. As stated in chapter 6, this rate has been calculated by Professor Brealey for the U.K. market and also it is reported in Janette Rutterford's *Stock Exchange Investments*. The pre Budget evaluation of the project will now be presented.

**Pre budget**

Before the budget, $\tau_c'$, the effective corporation tax rate is

$$0.35 \times \frac{1}{1 + r_p} \quad \text{(tax paid with 1 year delay)}$$

$$= 0.35 \times \frac{1}{1.02857}$$

$$= 0.3403 = 0.34 \text{ approximately}$$

The effective capital allowances are:

$$C_i = £800 \times \frac{.25 \times .02857}{.25 + .02857} \quad \text{that is } \left(\frac{de}{d+e}\right)$$

$$= £800 \times \frac{0.071425}{0.27857}$$

$$= £800 \times 0.02564$$

$$= £20.51$$

Substituting these rates and using the other relevant rates calculated so far for our model, we get:

$$V_i = \underbrace{\frac{100 \times (1 - .34) \times .806}{.025857 + .08}}_{[\mu_i' \times \theta_g]} + \underbrace{0.2947 \times 250}_{\text{[debt benefit]}} \qquad (8.3)$$

$$+ \underbrace{\frac{0.340 \times 20.51 \times .806}{0.02857}}_{\text{[capital allowances benefit]}} - \underbrace{\frac{30\ (-0.185)}{0.02857}}_{\text{[dividend taxation benefit]}}$$

$$(8.3a) \quad = \quad \frac{53.196}{0.10857} \quad +73.68 \quad + \quad \frac{5.621}{0.02857} \quad + \quad \frac{5.55}{0.02857}$$

$$(8.4) \quad = \quad \underset{(a)}{489.97} \quad \underset{(b)}{+73.68} \quad + \quad \underset{(c)}{196.74} \quad \underset{(d)}{+} \quad 194.26$$

$$= \quad £954.65$$

Since the valuation $V_i$ of £954.65 arises from an expected pre-tax cash inflow of £100 per year, the effective discount rate is:

$$\frac{£100}{£954.65} \quad = \quad 10.47\%$$

In other words, discounting a £100 perpetuity by 10.47% gives the correct present value of £954.65, therefore the pre-Budget effective discount rate for this project is 10.47%. Similar calculations using the rates after the Budget are presented below.

**Post budget**

$r_p$ after the Budget is 0.03289 (post tax risk free rate). The effective corporation tax rate is

$$\tau_c' = 0.35 \quad x \quad \frac{1}{1 + .03289} \quad = \quad .3389$$

$$= \quad .339 \text{ approximately}$$

Calculating the annual equivalent capital allowances $C_i$, gives

$$C_i \quad = \quad £800 \quad x \quad \frac{.25 \text{ x } .03289}{.25 + .03289}$$

$$= \quad £800 \quad x \quad \frac{.008223}{.28289}$$

$$= \quad £800 \quad x \quad 0.029075$$

$$= \quad £23.25$$

This is greater than £20.51 calculated using pre-budget rates, and is as expected from the above discussions.

With regard to the level of debt, as stated in section (i), the level of debt can be expected to increase. Let us assume that due to the increase in post budget tax advantage of debt by one third, the optimal level of debt also increases by one third - from £250 to £330. Therefore $D_i$ after the Budget is £330. (No change is assumed in dividends, reflecting the lack of theoretical underpinnings so far regarding the dividend puzzle).

The valuation of the SAME project using these figures and the post budget taxes would be:

$$V_i = \frac{100 \times (1 - .339) \times .786}{.03289 + .08} + .4026 \times £330 + \frac{0.339 \times 23.25 \times 0.786}{0.03289} - \frac{£30\,(-\,.246)}{0.03289}$$

$$(8.5) \quad V_i = \underset{(a)}{\frac{51.95}{.11289}} + \underset{(b)}{132.86} + \underset{(c)}{\frac{6.195}{0.03289}} + \underset{(d)}{\frac{7.38}{0.03289}}$$

$$(8.6) \quad = 460.18 + 132.86 + 188.36 + 224.38$$

$$= £1005.78$$

This valuation equation implies an effective discount rate of

$$\frac{£100}{£1005.78} = 9.94\%$$

This is a reduction in effective discount rate of 0.5%.

Comparing equations (8.3a) and (8.4) with (8.5) and (8.6) indicates the reasons why the effective discount rate changes.

(a) Risky cash flow (component (a) in equation 8.2): Post Budget, the post corporation and post capital gains tax flow of £51.95 is lower than the pre Budget equivalent of £53.196. This is mainly due to the adverse effect of increase in capital gains tax rate for higher pay tax bracket individuals in the 1988 Budget. Secondly, as the real after tax discount rate has increased due to the Budget, the present value of £460.18 is lower than the present value of £489.97 pre Budget. This would imply an increase in the effective discount rate as a result of the Budget.

(b) Leverage: The main component decreasing the effective discount rate is the increase in leverage due to increase in the tax benefit of debt due to the 1988 Budget changes (see section (i) above).

(c) The present value of benefits of capital allowances is lower at £188.36 compared to £196.74 before the Budget. This is due to the factors listed in section (j) above. This reduction in present value implies a small increase in the effective discount rate.

(d) Dividends are more advantageous post Budget. The present value of relative tax advantage post budget is £224.38 compared to £194.26 before. This too contributes to reducing effective discount rate. If the dividend level increases as a result of increase in tax advantage, then the benefit would be even greater.

Thus, on the basis of the above analysis and the assumptions made therein, investors should react to changes in personal taxes introduced by the 1988 Budget in the same manner as they would react to a cut in interest rate of about half a percent.

The 1988 Budget personal tax changes had to contend with disadvantage of increase in capital gains tax for higher rate tax payers. The impact of the income tax changes would have been more pronounced if capital gains tax had not been increased at the same time.

Thus personal taxes influence corporate variables in many ways as detailed above. These influences can be usefully analysed using the valuation model developed as equation (7.22). After this example of a practical application of the valuation model in the analysis of the 1988 Budget in this chapter, we proceed in the next chapter by questioning whether personal taxes can be completely ignored in the valuation model.

## DO PERSONAL TAXES MATTER ?

We noted earlier in chapter 5 that taxes which are imposed equally on all sources of returns to the investors have no impact on the valuation of assets. The reason for this is that such taxes would have a similar affect on the returns from projects (which usually comprise the numerator of the valuation equation) and the discount rate (which comprises the denominator of the valuation equation). These two equal influences on the valuation would cancel each other and therefore not influence the valuation equation at all. Following a similar line of reasoning, since personal taxes are levied on returns from risk free assets as well as on returns from projects undertaken by companies, would the effective discount rates calculated in chapter 8 be similar to the discount rates obtained where personal taxes are ignored altogether? That is, in practice, can the impact of personal taxes be treated as negligible? This question is central to this thesis and can be answered by comparing the effective discount rate for the project considered in chapter 8 with and without personal taxes.

Valuation equation 8.3 is reproduced below:

$$(9.1)\quad V_i = \underset{(a)}{\frac{\mu_i{}'\,\theta_g}{r_p + \beta_i \pi}} + \underset{(b)}{LD_i} + \underset{(c)}{\frac{\tau_c{}'\,C_i\,\theta_g}{r_p}} - \underset{(d)}{\frac{\Delta_i\,(\theta_g - \theta_{pDIVI})}{r_p}}$$

The term (a) in the above equation shows the after tax uncertain returns from the project discounted by the risk and tax adjusted discount rate. The term (b) measures the impact of corporate leverage on valuation. The term (c) is the effect of capital allowances and other corporate tax code features on valuation. The term (d) measures the impact of taxing capital gains and dividend income at differential rates. Addition of these four terms results in the valuation equation advocated in this thesis.

We need to be precise about what is meant by "ignoring personal taxes" in view of the complications caused by the following two factors:

(a) Heterogeneous Shareholders:

Since the model considers heterogeneous shareholders, "ignoring personal taxes" can be interpreted to mean "ignoring taxes (personal or other) levied at the shareholder level". With this approach, taxes levied on those investors who are shareholders but are not individuals, such as companies and pension funds would therefore also be ignored. Therefore, corporation or capital gains taxes, if any, borne by the companies and pension funds on their income from another corporate entity, are also ignored. In other words, all taxes levied at shareholder level on all types of investors are ignored.

This leads to an anomaly in the treatment of corporation tax. Corporation tax which is levied on cash flows received by a company undertaking the project is taken into account in project appraisal. However, if this company were owned by some corporate shareholders, then any additional corporation tax paid by the latter (on dividends or capital gains) would be ignored if taxes at shareholder level are to be ignored. This is another argument for including all types of taxes in valuation models, so that the taxes are treated consistently between investors.

(b) Imputation System in the U.K.:

This creates complications because part of the corporation tax paid by the companies is imputed to personal tax paid by the shareholders. There are two ways of dealing with shareholder level taxes here:

(i) Assume that the company pays full corporation tax and that all shareholders are basic rate taxpayers. This implies that at shareholder level, there is no additional tax on dividends - that $\theta_{pDIVI}$ is one. Therefore, in order to assume that there are no personal taxes on dividends under the imputation system, one has to assume that all shareholders are standard rate taxpayers as far as income from dividends is concerned. This assumption, of course, is not true for pension funds (zero rate) and individuals in higher rate tax brackets. However, in order to assume $\theta_{pDIVI}$ is one, these assumptions are necessary.

(ii) Alternatively, it can be assumed that the company pays corporation tax at the rate equal to the difference between the corporation tax rate and the rate of imputation. For example if the corporation tax rate is 35% and the imputation rate is 25% of gross dividend, then the effective rate of corporation tax is calculated as follows:

| | |
|---|---|
| Pre-corporation tax income (assume) | = £100 |
| Corporation tax @ 35% | (35) |
| Post-corporation tax income | £65 |

We assume all of the income is distributed as dividends. Therefore gross dividend is

$$65 \times \frac{100}{100-25} = 86.67$$

Therefore if there is to be no personal tax effect, then the effective rate of corporation tax is

$$£100 - £86.67 = £13.33.$$

If this rate of corporation tax is assumed then the net of corporation tax dividend flow (£100 - £13.33 = £86.67) is equal to the gross dividend received under the imputation system. Now it can be assumed that there are no further personal taxes at the shareholder level.

Of these two methods of accounting for the imputation system, method (i) will be used in the following calculations. The main reason for this preference is that the aim of ignoring all personal taxes is to treat $\tau_p$, $\tau_{pDIVI}$ and $\tau_g$ as zero - this is achieved by method (i). Method (ii) would imply that retentions bore a different rate of corporation tax (35%) than dividends (13.33%) - an effect perhaps not intended by those advocating ignoring personal taxes. Hence method (i) is followed.

We proceed by looking at the four components in equation 9.1 one by one, in order to assess their impact on the valuation in the absence of "shareholder" taxes. We use the pre-1988 Budget tax rates for our calculations.

**Component (a): Valuation of risky stream** $\dfrac{\mu_i' \, \theta_g}{r_p + \beta_i \pi}$

The main element in the above that changes is the after tax risk free rate, $r_p$. This has previously been defined as $R\theta_p$ - p. Now, in the absence of personal taxes, $\tau_p$ is zero - therefore $\theta_p$ is one. Hence $r_p$ is simply equal to R - p, where R is the nominal interest rate and p is the inflation rate. Using the 10% and 4% rates used in chapter 8, we get a real interest rate of 6% after (nil) personal taxes.

In chapter 4, it was stated that $\beta_i\pi$, the risk premium, was largely invariant to different rates of personal taxes on the risk free asset and equity returns. Hence the 8% rate considered previously will be used again.

In the numerator of the component (a) above, $\tau_g$ will be zero because at shareholder level, all personal taxes, including capital gains tax $\tau_g$, is assumed to be nil. Therefore $\theta_g$, which is 1 - $\tau_g$ , are equal to one.

The effective corporation tax rate, implicit in $\mu_i'$ , changes marginally as the nominal rate is discounted for one year at the new risk free of 6%. Therefore

$$\tau_c' = .35 \times \frac{1}{1.06} = 0.33$$

Therefore $\theta_c = 1 - 0.33 = 0.67$

Therefore $\mu_i'$ is £100 x 0.67 = £67.00.

The valuation of risky stream therefore is

$$\frac{67 \times 1}{.06 + .08} = \frac{67}{.14} = £478.57$$

This is marginally less than the £489.97 value of component (a) in equation 8.4 in the previous chapter. This shows that ignoring personal taxes in fact lowers the value of the risky stream, primarily because of the increase in the after tax discount rate. This result is will hold for different rates of taxes and returns and is therefore a general conclusion. We continue below by calculating components (b), (c) and (d) under the assumption of no shareholder taxes.

**Component (b): Leverage $LD_i$ where $L = (1 - \frac{r_c \theta_g}{r_p})$**

The real rate of interest after corporation taxes ($r_c$), is the same as before, namely 2.5%. The other variables, $\theta_g$ and $r_p$, are as calculated for component (a) above. Therefore L, the value of leverage, is

$$1 - \frac{.025}{.06} = 1 - 0.4167 = 0.5833$$

This is higher than the value of leverage in the presence of personal taxes. This is consistent with the literature, that is, that the value of leverage decreases when personal taxes are considered because personal taxes on debt income are higher than personal taxes on equity income (particularly due to the lower effective capital gains tax).

Taking the pre-Budget level of debt of £250 considered in chapter 8 section (k), the value of leverage in the absence of personal taxes is

$$0.5833 \times £250 = £145.82 \quad \text{(compared with £73.68 in eq. 8.4)}$$

**Component (c): Capital allowances** $\frac{\tau_c' \times C_i \times \theta_g \times I}{r_p}$

The variables $\tau_c'$, $\theta_g$ and $r_p$ have already been calculated above. The variable $C_i$, the annual equivalent capital allowances were calculated in chapter 8 section (g), as :

$$\frac{de}{d+e}$$

where d is the rate of depreciation allowance and e is the risk free discount rate. Therefore

$$C_i = \frac{.25 \times .06}{.25 + .06} = \frac{0.015}{0.31} = 0.0484$$

With £800 of expenditure eligible for capital allowances, the benefit is

$$\frac{0.0484 \times £800 \times \tau_c'}{r_p}$$

$$= \frac{0.0484 \times £800 \times 0.33}{0.06} = \frac{38.72 \times .33}{0.06} = \frac{12.778}{0.06}$$

$$= £212.96 \quad \text{(compared with £196.74 in equation 8.4)}$$

**Component (d): Tax advantage/disadvantage of dividends** $\dfrac{\Delta_i\,(\theta_g - \theta_{pDIVI})}{r_p}$

Since we are assuming $\theta_g = 1$ and $\theta_{pDIVI} = 1$, in the absence of personal taxes, the value of this component is nil. This component contributes to value only if capital gains are more heavily taxed than dividends, and vice versa but in the absence of personal taxes, this component can neither contribute to or detract from value.

The only exception is if the imputation system of taxation is incorporated in the analysis as per method (ii) discussed in the beginning. In this case, the imputation system of dividend taxation effectively reduces the corporation tax rate applicable to income that is distributed as dividends. However, as stated earlier, we are using method (i) for incorporating the imputation system in our analysis.

Adding the four component of value, (a) to (d) above, we get

$$\begin{aligned} V &= £487.57 + 145.82 + 212.96 + 0 \\ &= £837.35 \end{aligned}$$

Therefore the effective discount rate in the absence of personal taxes is £100/£837.35 = 11.94%.

This compares with the <u>pre budget</u> effective discount rate of 10.47% (chapter 8 section (k)). Therefore, consideration of personal taxes reduces the discount rate by approximately 1.5%.

If the <u>post budget</u> level of debt of £330 is considered, the present value of cash flows from a project undertaken by the company, in the absence of personal taxes, is

$$\begin{aligned} V &= £478.57 + 0.5833 \times £330 + 212.96 + 0 \\ &= 478.57 + 192.49 + 212.96 + 0 \\ &= £884.02 \end{aligned}$$

This value gives an effective discount rate of £100/£884.02 = 11.31%.

This compares with 9.94% effective discount rate in the presence of personal taxes ( chapter 8 section (k)). Once again, the difference is approximately 1.5%.

Hence, ignoring personal taxes is likely to result in overstatement of the discount rate by approximately 1.5%. Therefore ignoring personal taxes would tend to lead to under investment as companies would be demanding a return greater than that required by investors. Since the discount rate when personal taxes are omitted is different from that when the personal taxes are included, we cannot ignore personal taxes if we wish to arrive at a precise valuation equation consistent with our valuation model. Therefore personal taxes are relevant for investment appraisal.

The main reason why the required return with personal taxes is lower is because the real risk free rate after taxes, $r_p$, is lower at 2.9% (using the figures in the above example) when personal taxes are considered, instead of 6% when personal taxes are ignored. This effect is mitigated to the extent that personal taxes also act as a leakage between the instant when the cash flows arise from the projects to the time when they are finally received by the investors. However, the net result is to leave the discount rate lower by approximately 1.5%.

Does this difference, calculated as 1.5% above, vary with the level of debt in the company or its pay out ratio? Answer to this question may indicate the type of companies for which ignoring personal taxes is more erroneous than it is for other companies with different leverage and dividend payout policies. These issues are examined next. In each case, we examine the difference between the discount rates in the presence and in the absence of shareholder taxes.

**(i) Level of Debt**

Since we have partitioned the valuation equation into the 4 distinct components, it is quite easy to examine the impact of leverage on the discount rate. This is so because leverage influences valuation through only one of the components, thus making the analysis easier. We proceed by examining the discount rates at (a) zero and (b) high debt levels. We use the post Budget tax rates in order to illustrate the general points of our analysis using a different set of tax rates from the pre-Budget tax rates considered above.

(a) Difference at zero debt level

Discount rate, post budget, with personal taxes

This can be calculated by simply ignoring $LD_i$, the extra benefit from leverage, which is the second component in the valuation equation. Therefore

$$V = 460.18 + 188.36 + 224.38$$
$$= 872.92$$

The effective discount rate is £100/£872.92 = 11.46%.

Discount rate, post budget, without personal taxes

Using the figures in chapter 8, and again ignoring the second component, $LD_i$ we get

$$V = 478.57 + 212.96$$
$$= 691.53$$

The effective discount rate is £100/£691.53 = 14.46%.

The difference in discount rates at zero debt level is 3% (14.46% - 11.46%). Therefore for companies with zero (or low) debt level, the overstatement of discount rates by ignoring personal taxes is considerably greater than the 1.5% previously calculated.

The reason why this difference in discount rates decreases with increasing debt level is that as the debt level increases, the benefit of leverage without personal taxes is greater than the benefit of leverage with personal taxes. This is consistent with the conclusions of Modigliani (1982) wherein he states that personal taxes reduce the benefit of leverage because the personal income tax on interest earned on debt is usually higher than the personal taxes on capital gains earned by the equity investors. Thus the benefit to leverage at the corporate level is mitigated by the tax disadvantage to leverage at the personal level. Therefore, as the debt level increases, the discount rate without personal taxes falls faster than (and therefore gets closer to) the discount rate with personal taxes. This can be shown by considering an even higher level of debt, say £500, in the valuation equation. The difference in the discount rates should be smaller than 1.5%.

(b) Difference at high debt level (£500)

Discount rate, post budget, with personal taxes

Again, using information previously calculated in chapter 8 section (k), but with debt level of £500, we get

$$\begin{aligned} V &= £460.18 + 0.4026 \times £500 + £188.36 + £224.38 \\ &= £460.18 + £201.30 + £188.36 + £224.38 \\ &= £1074.22 \end{aligned}$$

The effective discount rate is £100/£1074.22 = 9.31%

Discount rate, post budget, without personal taxes

$$\begin{aligned} V &= £478.57 + 0.5833 \times £500 + £212.96 + 0 \\ &= £478.57 + 291.65 + £212.96 + 0 \\ &= £983.18 \end{aligned}$$

The effective discount rate is £100/£938.18 = 10.17%

The overstatement of the discount rate is now only 0.86% (10.17% - 9.31%). Hence, at very high levels of debt, the discrepancy caused by ignoring personal taxes is reduced considerably.

To summarise, by ignoring personal taxes, the discount rate is overstated, but this overstatement decreases with increasing debt level. At zero level of debt, the overstatement is 3%, at a debt level of approximately one third of the initial investment, the overstatement is 1.5%, and when the debt level is approximately two thirds of the total investment, then the overstatement is 0.9%.

**(ii) Level of Dividends**

The overstatement of discount rate clearly varies with the level of the pay out ratio. This is so because the fourth component of valuation, $\Delta_i (\theta_g - \theta_{pDIVI}) / r_p$, enters the valuation equation only when personal taxes are present. At the beginning of this chapter, we concluded that in the absence of personal taxes, $\theta_g = 1 = \theta_{pDIVI}$ and hence this component has a zero value if personal taxes are ignored. Hence, varying the level of dividend, $\Delta_i$, would alter the effective discount rate with personal taxes, but would not affect the discount rate without personal taxes. Hence the overstatement of the discount rate by 1.5% would clearly vary with the level of dividends ($\Delta_j$).

Besides the dividend level of £30 already considered in chapter 8, we shall consider (a) zero dividend and (b) a very high dividend of £60.

(a) Zero dividend

With zero dividend, the post budget valuation equation with personal taxes (eq 8.3) loses the fourth component of value and results in the following changes:

$$V = £460.18 + £132.86 + £188.36 + 0$$
$$= £781.40$$

The effective discount rate is £100/£781.40 = 12.80%

This compares with the effective discount rate, post Budget, in the absence of personal taxes, of 11.31% . Therefore with zero pay out, the discount rate, ignoring personal taxes, is lower than the discount rate with personal taxes.

Note that the above result is specific to the U.K., where the imputation system implies that there is a tax advantage of dividends. In the classical system, such as in the U.S.A., where there is tax disadvantage of dividends, the conclusions would be reversed. The implications for classical system of taxes are stated more clearly after considering the discount rate in the presence of a very high level of dividends (£60).

(b) High level of dividend (£60)

The post budget effective discount rate with personal taxes

Again, using the data from chapter 8, but with dividends at a level of £60 per year, the valuation equation is

$$V = £460.18 + £132.86 + £188.36 + \frac{£60 \times .246}{.03289}$$

$$= £460.18 + £132.86 + £188.36 + £448.76$$

$$= £1230.17$$

The effective discount rate is £100/£1230.17 = 8.13%

Now the error of overstatement, caused by using a discount rate which ignored personal taxes, has increased. The discount rate ignoring personal taxes is 11.31%. Hence overstatement of the discount rate at very high levels of dividends is also greater, at 3.18%.

These derived results can be summarised as below:

| | EFFECTIVE DISCOUNT RATE | | |
|---|---|---|---|
| **LEVEL OF DIVIDENDS** | **No personal taxes** | **With personal taxes** | **DIFFERENCE** |
| (i) | (ii) | (iii) | (iv) |
| High (£60) | 11.31 | 8.13 | 3.18 |
| Moderate (£30) | 11.31 | 9.94 | 1.37 |
| Low (£nil) | 11.31 | 12.80 | (1.49) |

Therefore increasing the level of dividends increases the extent to which ignoring personal taxes overstates the discount rate.

The reason is that in the U.K. there are tax advantages to dividends. Thus increasing the dividend level increases the value of the company, thus reducing the effective discount rate with personal taxes. The discount rate without personal taxes does not change with changes in the level of dividends. Hence, as the level of dividends increases, the constant discount rate in the absence of personal taxes ( 11.31% ) increasingly exceeds the discount rate in the presence of all taxes (since the latter is decreasing from 12.8% to 8.13% in the table above).

Under the classical system of taxation such as in the U.S.A., there is tax disadvantage of dividends, hence the effective discount rate with personal taxes will increase with increases in dividends. Hence the trend in column (iii) would be reversed.

To summarise this chapter, it has been demonstrated that for a U.K. company with one third level of debt and approximately 50% pay out ratio, it is true to say that ignoring personal taxes overstates the discount rate by 1.5%. This overstatement varies both with the level of debt and of dividends. Moreover, this overstatement would differ under a different system of taxation, for example, under the classical system. Therefore, consideration of personal taxes is relevant for the valuation model.

In the last few chapters, we have derived the recommended valuation model (chapters 4 to 7), assessed its relevance (chapter 9), and tested its application for practical analysis (chapter 8). We now proceed in the next chapter by using this valuation model for a much more in-depth application. In the following chapters, we use the analytical tools created so far, to examine the recommendations of the Meade Committee on the taxation system in the U.K.. The Meade Committee utilised the technique of examining the desirability of the alternative tax systems on the basis of their impact on the required rates of return from projects. Their rate of return models were fairly simplistic and they ignored the impact of risk even though they were analysing risky corporate returns. We use our more comprehensive recommended model of equation 7.22 to rectify that deficiency in their analysis, and also to confirm or contradict the validity and robustness of their conclusions and recommendations.

## DISCOUNT RATES UNDER THE MEADE REPORT TAX SYSTEMS

The Meade Committee, whose findings are given in the Meade Report ("The Structure and Reform of Direct Taxation", Institute of Fiscal Studies, 1978), undertook a comprehensive review of direct taxation in the UK. The Meade Report, which thus deals with investment returns under alternative tax regimes, is therefore of importance to this thesis which deals with discount rates for project valuations under the alternative tax regimes, in particular the tax regimes relevant for U.K..

We claim that the valuation model developed in this thesis is useful for examining the impact of taxes on investment returns. In chapter 8, we demonstrated the use of our model for evaluating the impact of tax changes introduced in the 1988 Budget. In chapter 9, we demonstrated the use of our model in determining the relevance of incorporating personal taxes in the valuation model, and examined their impact under alternative levels of corporate debt and dividend payout ratios. We showed that the model developed in this thesis is very useful for those practical applications. We aim to continue to demonstrate the usefulness of our model by examining the conclusions reached in the most comprehensive study of systems of taxation undertaken in the U.K., namely the study by the Meade Committee. The purpose of this examination is to show the usefulness of our model. Although we start with this modest aim, by the stage we complete our analysis of the Meade Report (in chapters 10 and 11), we end up showing how the use of our model would in fact have helped the Meade Report reach more sound conclusions. We also show that our valuation model helps us advocate a somewhat different system of taxation to that advocated in the Meade Report, but which is better at delivering the benefits that the system they advocate is erroneously supposed to deliver.

The Meade Committee analysed two main systems of direct taxation

(i) Comprehensive Income Tax ("CIT")

(ii) Expenditure Tax ("ET")

CIT system is defined as one under which all the income earned by the taxpayer, including income from investments, and including accrued capital gains, form the tax base for personal tax purposes. Similarly, all the profits earned by a company form the tax base (referred to as the P-base in the Meade Report) for corporation tax purposes under a CIT system.

ET system (note that contrary to the rumours, Mr. Steven Spielberg was NOT a member of the Meade Committee) can be stated to be a system under which personal tax is levied on consumption expenditure undertaken by the taxpayer. Under this system, any consumption out of dis-savings would be taxed but any income earned which is invested would not be taxed. At the corporate tax level, the accompanying tax system would be one based on corporate flow of funds instead of on corporate profits. Meade Report states that the post-tax rate of return earned by investors would equal the pre-tax rate of return generated by underlying investments under such a tax system.

The Committee favoured the latter system of taxation. The main reason for advocating an expenditure tax system is that the post-tax rate of return earned by a saver on consumption foregone (while saving), is equal to the pre-tax rate of return generated by the projects financed by that saving. With comprehensive income tax, the post-tax rate of return to savers is reduced by taxes and therefore is lower than the pre-tax rate of return generated by the projects financed by investors' savings. Hence under a comprehensive income tax system, there will be some distortion to the rate of return introduced by personal income tax - thus people may be unwilling to forego present consumption because at the margin, the return they get is lower than the return on investment. Expenditure tax does not suffer from this disadvantage. This fact, and the resulting valuation equations, can be examined by using the model developed in this thesis.

The main technique used for this analysis of the Meade Report is to examine the **"Distortion Ratio"**. The Distortion Ratio is a name given in this thesis to a fundamental concept used in the Meade Report. It compares post-tax rate of return earned by investors to pre-tax rate of return generated by the underlying project. If the two rates of return are equal, then the Distortion Ratio is 1 and this implies that the tax system introduces no distortions in project appraisal. Such a tax system is desirable from the point of view of removing any disincentive to invest that is caused by some of the tax systems. The further the Distortion Ratio moves below one, the greater is the distortion introduced by the tax system. We can state that the tax systems under which the Distortion Ratio is one or nearer one are preferable to tax systems under which the Distortion Ratio is further below one. In common with the Meade Report, we use this criteria extensively in our analysis.

We describe briefly the analysis carried out in the two sections of this chapter.

Section (A)

In section (A), we deal with corporation tax systems under which the government does not intend to raise any incremental revenue from taxes at the corporate level. The purpose of corporation taxes under such systems is to merely act as a fairly efficient vehicle for collecting personal taxes. Thus, along with CIT at the personal tax level, we consider a special case of P-base ("profit base", described below) corporation tax system. Under this special case, **all** corporation taxes are **fully** imputed to the shareholders, consequently imposing no extra tax burden at the corporate level. On the other hand, with ET at the personal level, we state that if the government did not wish to raise any resources from corporation tax, then there is no compatible system of tax at the corporate level that the government could impose. (The only possible exception is perhaps a simple withholding tax at corporate level on the cash flow payments to the shareholders, which could then be credited against the shareholders' personal expenditure tax. We ignore such withholding taxes since they are of no consequence.) Hence, with ET at the personal level, we do not impose any tax at the corporate level if government does not intend to raise any incremental tax revenue at the corporate tax level.

Section (B)

In Section (B) we consider the scenarios under which the government aims to raise incremental revenue from corporation taxes. We derive valuation equations for firms on the assumption that the tax systems present in the economy are those that have been discussed in the Meade Report. Thus we first examine the CIT together with a P-based corporation tax at the corporation tax level, but this time without imputation of 100% of the corporation taxes. Secondly we examine the ET system at the personal tax level together with Corporate-flow-of-funds based taxes at the corporate level (the latter are introduced in the valuation analysis in section (b) because we examine scenarios under which the government intends to raise incremental revenue from corporation taxes in this section). All these tax bases are described later below.

Finally, under both CIT as well as ET systems, we ascertain whether the results we derive in each of the above sections are compatible with the assertions made in the Meade Report. Briefly, the Meade Report asserts that under CIT system, income-based taxes (ie. C.I.T at personal level and P-base tax at corporate level) will always reduce the post-tax rate of return that an investor receives to below the pre-tax rate of return generated by the investment itself. Secondly, the Meade Report asserts that neither an ET system at personal tax level nor a Flow-of-funds base system at corporate tax level will cause any divergence between the post-tax rate of return earned by an investor and the pre-tax rate of return generated by the underlying investment. In section (A), we show that the return to investors expression and the valuation equations that we derive, using the model advocated in this thesis, are compatible with the above assertions made in the Meade Report, particularly when we make the assumptions that there is no debt or uncertainty. We examine the assumptions regarding debt and/or uncertainty made by the Meade Committee in the next chapter.

## Section (A)

## DISCOUNT RATES WHEN CORPORATION TAX SYSTEMS AIM *NOT* TO RAISE INCREMENTAL REVENUE

The characteristic which distinguishes this section from section (B) is that in the cases examined in the present section, the government does not intend to raise incremental revenue from taxes at the corporate level. We examine CIT in sub section (i) and ET in sub section (ii) under the above assumption of section (A).

### (i) CIT

The Meade Report recommends that if the purpose of corporation tax is simply to collect revenue at corporate level because it is easier to do so, then corporation tax may be levied under CIT. However, if the intention is not to raise any additional tax under CIT, but to use corporation tax as a tax "collection" vehicle, then the shareholders should receive a tax credit under the imputation system. This tax credit should be at the rate of 100% of the corporation tax paid - thus there will be no additional burden on investors of corporation tax.

The tax credit on dividends can then be calculated as is normal under an imputation system, such as the UK system, only keeping in mind that the imputation rate is the corporation tax rate ($\tau_c$). Therefore for every £1.00 of dividend paid out of post-corporation tax income, the associated tax credit is ($\tau_c/1-\tau_c$), that is the post tax dividend is grossed up to get $\frac{1}{1-\tau_c}$ the pre tax income, on which corporation tax at $\tau_c$ had been paid. Hence the tax credit for £1 of dividend is $\frac{\tau_c}{1-\tau_c}$.

An investor who receives £1.00 of net dividend (and $\frac{\tau_c}{1-\tau_c}$ of associated tax credit) has to pay personal income tax at the rate $\tau_p$ on gross dividend as follows:

$$\text{personal tax chargeable} = [£1 + \frac{\tau_c}{1-\tau_c}]\,\tau_p$$

$$\text{less: tax credit available} = \frac{\tau_c}{1-\tau_c}$$

therefore, the additional personal tax payable is $= (1 + \frac{\tau_c}{1-\tau_c})\,\tau_p - \frac{\tau_c}{1-\tau_c}$

In the above expressions, total personal tax is calculated by charging personal income tax at the rate $\tau_p$ on the grossed up dividend (which is given by the term in the square brackets). However, the investor receives the benefit of the associated imputation tax credit. Hence, the net additional personal tax borne by the investor is equal to the total personal tax charged minus the tax credit available, as given by the above expression.

This expression simplifies as follows:

$$\left(1 + \frac{\tau_c}{1-\tau_c}\right)\tau_p - \frac{\tau_c}{1-\tau_c}$$

$$= \frac{[(1-\tau_c) + \tau_c]\,\tau_p}{1-\tau_c} - \frac{\tau_c}{1-\tau_c}$$

$$= \frac{\tau_p}{1-\tau_c} - \frac{\tau_c}{1-\tau_c} = \frac{\tau_p - \tau_c}{1-\tau_c}$$

Thus the additional tax that an investor pays is equal to the excess of personal income tax rate over the imputed corporation tax rate, grossed up by the imputed corporation tax rate. This is the additional tax which a shareholder would pay on £1 of cash dividend received.

The extra tax calculated above is consistent with the general formula for calculating extra tax burden when imputation credit is available.

If $\tau_c$ is replaced by s, then the expression $\frac{\tau_p - s}{1-s}$ gives the personal tax due on £1 of cash dividend under any general imputation system where the rate of imputation is s.

**Retained Earnings:** The Meade Report recommends that retained earnings of corporations should be allocated to shareholders and form part of their taxable income. However, along with allocation of earnings, shareholders should also receive a tax credit for corporation tax paid on retained earnings. This imputation, along the lines of imputation credit for dividends, should be at the rate of 100% of the corporation tax rate. Hence, as in the case of £1 of post-corporation tax dividend, the additional personal tax paid by a shareholder who faces a personal tax of $\tau_p$, on £1 of retained earnings allocated to him is: $\frac{\tau_p - \tau_c}{1-\tau_c}$ (as above)

Under CIT there would be no capital gains tax on increases in share prices. Instead, income tax on retained earnings will be calculated as shown above.

However, the Meade Report recognises that due to various reasons such as changes in capitalisation rates, unexpected profitable investments etc, shareholders' capital gains may differ from retained earnings. Hence they recommend that the difference between (a) the capital gain realised and (b) the total of retained earnings which have already been subject to CIT, should be subject to capital gains tax. The tax provisions of this capital gains tax should give relief for inflation, but, on the other hand, should include an interest charge since gains were not taxed when accrued, but only taxed when realised.

In our model, we can ignore such capital gains tax because our model is an equilibrium model on an ex-ante basis. Therefore, on an ex-ante basis, shareholders cannot anticipate that they will make capital gains (or losses) which differ from retained earnings. This assumption that capital gains equal retained earnings is same as that made in other models in the literature, for example in the Modigliani model and is consistent with the assumptions made in the Meade Report. Hence for our model, the personal taxes on dividends and retained earnings is given by $\frac{\tau_p - \tau_c}{1-\tau_c}$.

Using the personal tax expressions derived above, the net of tax return expected by a shareholder m holding $n_i$ shares of a company i, subject to a CIT regime, is given by:

$$[\,10.1] \qquad y_i^m = n_i^m \left\{ \left( \mu_i - \tau_c \left[\mu_i - R\,D_i - C + p\,D_i\right] - RD_i + pD_i - \Delta_i \right) \frac{(1 - \tau_p - \tau_c)}{1-\tau_c} + \Delta_i \frac{(1 - \tau_p - \tau_c)}{1-\tau_c} \right\}$$

The above expression is similar to the expression for expected returns that were presented in the earlier chapters of this thesis. The above expression is not stated in the Meade Report, which instead used simple models only. Hence this thesis shows how the evaluations carried out in the Meade Report can be carried out using a much more comprehensive and rigorous model.

The interpretation and integration of the tax features used in the Meade Report into the valuation model of this thesis is based on our understanding of the Meade Report. The features of the CIT recommended by Meade report which have been incorporated in the above are as follows:

Corporation tax accompanying CIT system should be on profits basis. Nominal interest ($RD_i$) should be deducted and the fall in value of net monetary liabilities ($pD_i$) due to inflation should be added. Capital allowances (C) should be given on an economic depreciation basis. The allowances should be indexed for inflation. More on depreciation allowances is given below.

The retained earning term (within bold round brackets in equation (10.1)) is subject to personal tax at $(\tau_p - \tau_c) / (1-\tau_c)$ as derived above. A similar tax rate applies to the dividend income ($\Delta_i$). Note that the term $-\tau_g\, pS_i$, which previously measured the tax on inflationary increase in value of shares, does not appear in equation 10.1 since only real gains are taxes under CIT.

Equation (10.1) can be simplified as follows:

[Eqn 10.2] $$y_i^m = n_i^m \left\{ (\mu_i(1-\tau_c) - \tau_c\ [-D_i(R-p) - C] - D_i(R-p)\ \frac{(1 - \tau_p - \tau_c)}{1-\tau_c} + \Delta_i \frac{(1 - \tau_p - \tau_c)}{1-\tau_c} - \Delta_i \frac{(1 - \tau_p - \tau_c)}{1-\tau_c} \right\}$$

$$= n_i^m \left\{ (\mu_i(1-\tau_c) - \tau_c\,[-D_i\, r - C] - D_i\, r)\ \frac{(1 - \tau_p - \tau_c)}{1-\tau_c} \right\}$$

In the above equation, the term measuring the impact of taxing dividends at a different rate from retained earnings cancels out because dividends and retained earnings are taxed at the same rate under the proposed CIT. Note also that the relevant interest rate for tax allowable interest expense is r ,the real interest rate, because in effect, under Meade Report proposals, only real interest is an allowable business expense.

Since we are taking the case where $\mu_i$ is a perpetual flow, there is no "economic depreciation" which can be claimed as a capital allowance under CIT. If $\mu_i$ were taken to be decreasing over time, then there would be a fall in present value of expected future cash flows with the passage of time, and thus economic depreciation allowances would arise in that situation. However, given the present model, there are no capital allowances, therefore C=0.

If the personal income tax rate $\tau_p$ is not the same for all investors, then we need to take $\tau_p^m$, the personal tax rate applicable to individual m, into account instead of $\tau_p$.
Incorporating the above two points, equation (10.2) becomes:

$$y_i^m = n_i^m \{ [ \mu_i(1-\tau_c) - D_i r(1-\tau_c) ] \frac{(1-\tau_c-\tau_p^m+\tau_c)}{1-\tau_c} \}$$

$$= n_i^m \{ [ (\mu_i-D_i r) (1-\tau_c) ] \frac{(1-\tau_p^m)}{1-\tau_c} \}$$

[Eqn 10.3] $= n_i^m \{ (\mu_i-D_i r) (1-\tau_p^m) \}$

An interesting feature in the above equation is that the corporation tax term $\tau_c$ disappears since with full imputation, the only relevant tax is $\tau_p$, the personal tax rate.

Following the procedure used in all the earlier chapters, we now consider an investor who invests his wealth, apart from that invested in equity, in bonds. Under CIT the interest income on bonds would be taxed only in respect of real interest. Hence, whereas previously R, the nominal rate was subject to tax; now R-p, the real rate, is subject to personal tax $\tau_p^m$ - hence the real after personal tax interest rate is (R-p) $(1-\tau_p^m)$ which is $r\theta_p^m$.

Hence total expected portfolio return by individual m is:

[Eqn 10.4] $y^m = (w^m - \Sigma_i n_i^m S_i) r\theta_p^m + \Sigma_i n_i^m \{ (\mu_i - D_i r) (\theta_p^m) \}$

The variance of this portfolio return is:

[Eqn 10.5] $(\sigma_y^m)^2 = E(y^m - y^m)^2 = (\theta_p^m)^2 \Sigma_i \Sigma_j n_i^m \mu_{ij} n_j^m$.

This differs from variance under classical, imputation or two rate system because $\mu_{ij}$ replaces $\mu_{ij}^*$ - the covariance of post corporation tax returns. The reason for this is that under the present scenario, the corporation tax rate term is not relevant for determining the market equilibrium. Note also that since imputation at the rate of 100% is given in this system of CIT, the system is closer to the Integrated System described in chapter 5, than it is to the imputation system described in that chapter.

Defining Lagrangean function over utility

$$£ = U^m \{y^m, (\sigma_y^m)^2\}$$

and differentiating w.r.t. the choice variables $n_i^m$, we get

$$\frac{\delta £}{\delta n_i^m} = u_1^m \times \frac{\delta y^m}{\delta n_i^m} + u_2^m \times \frac{\delta (\sigma_y^m)^2}{\delta n_i^m}$$

$$= u_1^m \times \{-S_i\, r\, \theta_p^m + (\mu_i - D_i\, r)\, \theta_p^m\} +$$

$$u_2^m \times \{2(\theta_g^m \theta_p^m)^2 \quad \Sigma_{j=1}^n \mu_{ij}\, n_j^m\}$$

At the optimum the above derivative will equal zero. Therefore the expression simplifies to:

$$-S_i\, r\, \theta_p^m + (\mu_i - D_i r)\, \theta_p^m = \frac{-u_2^m \times 2 \times (\theta_p^m)^2}{u_1^m} \Sigma_{j=1} \mu_{ij}\, n_j^m$$

Replacing by $\gamma = -2 \dfrac{u_2^m}{u_1^m}$ we get

$$-S_i\, r\, \theta_p^m + (\mu_i - D_i\, r)\, \theta_p^m = \gamma\, (\theta_p^m)^2 \quad \Sigma_{j=1}^n \mu_{ij}\, n_j^m.$$

We can sum over all companies in which an investor has an equity to get:

$$(\mu - Dr)\, \theta_p^m - Sr\theta_p^m = \gamma^m (\theta_p^m)^2\, [M]\, n^m$$

Multiplying both sides by $\dfrac{\Lambda}{\gamma^m(\theta_p^m)^2}$ where $\Lambda = \dfrac{1}{\Sigma_m 1/\gamma^m(\theta_p^m)^2}$

we get

$$[\,(\mu\text{-}Dr)\, \theta_p^m - Sr\theta_p^m\,] \frac{\Lambda}{\gamma^m(\theta_p^m)^2} = \Lambda[M]n^m.$$

Note that $(\theta_g^m)^2$ is replaced by $(\theta_p^m)^2$ in case of CIT. Moreover, if we consider how the income earned by shareholders such as insurance companies and pension funds is to be taxed under CIT (see below) then these expressions may be further simplified. For the time being, defining $\theta_p = \Sigma_m\, \theta_p^m \times \dfrac{\Lambda}{\gamma^n(\theta_p^m)^2}$, we sum over all investors to get,

$$(\mu - Dr)\, \theta_p - Sr\theta_p = \Lambda[M]1$$

In order to get the value of a company as a whole, we add and subtract Dr $\theta_p$ on L.H.S. of above equation and simplify :

$$(\mu - Dr)\,\theta_p - Sr\theta_p + Dr\,\theta_p - Dr\,\theta_p = \Lambda[M]1$$

$$\Rightarrow \quad \mu\,\theta_p - \Delta[M]1 = (S+D)(r\theta_p)$$

which gives

[Eqn 10.6] $$\Rightarrow \quad V = \frac{\mu\theta_p - \Lambda[M]1}{r\theta_p}$$

This gives the value of the companies on the basis of risk adjusted cash flow discounted by the real rate of return net of personal taxation.

Interesting points to note are that unlike the previous equations of V, the present equation does not contain any terms involving dividends or leverage. This is so because dividends do not involve any extra tax under CIT in comparison with retention. The leverage term does not appear even though debt interest is an allowable expense for corporation tax purposes under the CIT system examined in the Meade Report. The reason is that corporation tax is fully imputed to the shareholders - hence tax deductibility of debt interest does not add any further benefit to cash flows since corporation tax imposes no real additional burden because it is only a tax collecting vehicle.

We sum equation (10.6) over all companies, and proceed to calculate the market risk premium as follows:

$$V = \frac{\mu\,\theta_p}{r\theta_p} - \frac{\Lambda\,\mathrm{var}(\mu)}{r\theta_p}$$

$$\Rightarrow \quad r\theta_p = \frac{\mu\theta_p}{V} - \frac{\Lambda\,\mathrm{var}(\mu)}{V}$$

$$\Rightarrow \quad \frac{\Lambda\,\mathrm{var}(\mu)}{V} = \frac{\mu\theta_p}{V} - r\theta_p = \pi \qquad (\pi = \text{market risk premium})$$

| | |
|---|---|
| return to market portfolio, net of tax | return to risk free asset, net of tax |

Thus aggregate corporate sector value can be expressed as:

$$V = \frac{\mu\theta_p}{r\theta_p + \pi}$$

For an individual company, the value is given by

[Eqn 10.7] $$V_i = \frac{\mu\theta_p}{r\theta_p + \beta_i \pi}$$ where $\beta_i$ = company's beta

Equation (10.7) gives the formula for the valuation of projects under the CIT regime. The formula implies that valuation is obtained by discounting the cash flows arising from the project, net of only personal income tax, by a risk adjusted discount rate. The cash flows are subject to only personal tax implicit in $\theta_p$ - but there is no distortion by corporation tax, leverage, capital allowances or dividends.

The Meade report states that an investor under CIT will receive a post-tax return lower than the pre-tax return generated by the underlying investment. It can be seen that if the risk premium were zero, then the point made by Meade report is valid.

If risk premium is zero, equation (10.7) becomes:

[10.7a] $$V_i = \frac{\mu_i \theta_p}{r\theta_p} \quad \text{or} \quad \frac{\mu_i}{V_i} = r$$

The above equation shows that the pre-tax return generated by the underlying investment, which is defined as $\mu_i$ divided by $V_i$ , in the absence of a risk premium, becomes the real interest rate. In order to measure distortion introduced by taxes, we need to compare this return generated by the underlying investment with the post-tax return earned by investors. In order to make a fair and consistent comparison with the above return, we should not consider only an investor in equity or an investor only in corporate debt. Instead, we consider the post-tax returns earned collectively by investor(s) who own all of the debt and all of the equity.

Investors who owns all the shares and all the debt would receive the following returns (arising from equity and debt investments):

$$\underset{\text{on shares (Eqn 10.3)}}{(\mu_i - rD_i)\,\theta_p} \quad + \quad \underset{\text{on debt}}{rD_i\,\theta_p}$$

$$= \mu_i\theta_p$$

The investors would have invested $S_i$ for the equity and $D_i$ to acquire the debt of the company i . Note that $S_i$ plus $D_i$ equals $V_i$. The post-tax rate of return earned by the investors is given by dividing their total post-tax return by their total post-tax investment.

This gives the post-tax return to the investor of $\dfrac{\mu_i\theta_p}{V_i}$

Now $\dfrac{\mu_i\theta_p}{V_i} = r\theta_p$ (10.7a) which is lower than r since $\theta_p$ is less than 1 for any positive level of personal tax. Hence the investors get a post-tax rate of return ($r\theta_p$) which is lower than the pre-tax rate of return (r) generated by the investment, due to comprehensive personal income tax. However, there are no further distortions or subsidies resulting from any other aspect of the taxation system.

Having considered CIT when the corporation tax system is not aiming for any additional revenue, we turn now to expenditure tax system under similar assumption for corporation tax system.

**(ii) Expenditure Tax System (when Corporation Tax system does not raise incremental revenue):**

Under an expenditure tax regime, income that is saved is not subject to tax whereas consumption out of dis-saving is subject to tax. Hence expenditure tax is a tax on consumption. An annual proforma computation would take the following form:-

| | | | |
|---|---|---|---|
| A | | Personal Income | x |
| B | Add: | Capital receipts (not just gains but 100% of the proceeds) | x |
| C | | Windfall receipts | x |
| D | Less: | Non-consumption outgoings (acquisition of assets, etc) | (x) |
| E | | Chargeable balance | x |

The Meade Committee has recommended cash flow basis for corporation tax to go along with expenditure tax at the personal level. These may take the form of R, R+F or S basis which are explained after the following table. To enable a clearer understanding of the Corporate flow of funds taxation, the following table is reproduced from the Meade Report.

## CORPORATE FLOW OF FUNDS

| Inflows | Outflows |
|---|---|
| **Real Items** | |
| | R1* Purchase of materials |
| R1 Sale of produce | R2* Wages, salaries & purchase of other services |
| R2 Sale of services | |
| R3 Sale of fixed assets | R3* Purchase of Fixed Assets |
| R | R* |
| **Financial items other than shares of UK resident Corporate Bodies** | |
| F1 Increase in creditors | F1* Decrease in creditors |
| F2 Decrease in debtors | F2* Increase in debtors |
| F3 Increase in overdraft | F3* Decrease in overdraft |
| F4 Decrease in cash balance | F4* Increase in cash balance |
| F5 Increase in other borrowings | F5* Decrease in other borrowings |
| F6 Decrease in other lending | F6* Increase in other lending |
| F7 Interest received | F7* Interest paid |
| F8 Decrease in holding of shares in non-UK corporations | F8* Increase in holding of shares in non-UK corporations |
| F | F* |
| **Share items of corporate bodies resident in the UK** | |
| S1 Increase in own shares issued | S1* Reduction in own shares issued |
| S2 Decrease in holding of shares of other UK companies | S2* Increase in holding of shares in other UK companies |
| S3 Dividends received from other UK companies | S3* Dividends paid |
| S | S* |
| **Tax Items** | |
| T Tax repaid | T* Tax paid |

R + F + S + T (Total Inflows) = R* + F* + S* + T* (Total Outflows)

The table is a corporate flow of funds table in which all inflows are stated in the left hand side table and the outflows are stated in the right hand side column. The corporate cash flows are subdivided into 3 main categories, namely

- those that relate to transactions in real (as opposed to financial) items, such as sale of goods or services, acquisition of assets, etc,
- those that relate to financial items, such as changes in debtors or creditors,
- those that relate to cash flows to or from the shareholders, such as dividends or equity issues.

Tax payments or repayments are not included in the above categories, which form the bases for the flow of funds corporation tax bases.

Briefly, R basis involves taxing excess of receipts from real transactions over payments for real transactions. Real transactions include transactions on income as well as capital account. Hence the increase in investment in plant and machinery, stock, or even buildings and land, etc, would be an allowable expense - in a sense, 100% capital allowances are extended to cover all forms of real capital expenditure.

R+F basis includes financial transactions surplus in addition to the balance on real transactions in the tax base. Financial transactions include not only receipt of interest or payment of interest, but also any changes in net liquidity position - that is, an increase in loan would constitute a financial receipt.

Because of the identity between cash inflows and outflows, R+F basis is the same as S basis. S basis imposes tax only on excess of outgoings to shareholders over amounts received from shareholders. To prevent tax avoidance, transactions in shares of other UK companies are also included in S basis. An R+F or S basis implies that if a company sells goods for £10 for which it had paid £3, and if it keeps the £7 gain as an increase in cash balance, then no tax would be levied. Only when this £7 is paid out as dividend is corporation tax levied.

The advantage of these bases of corporation tax is that none of them distort the rate of return on consumption foregone. This is illustrated later below. However, the Meade report states that if the government does not intend to raise any additional revenue from corporation tax, then no corporation tax should be levied.

Therefore we do not include any corporation tax in the following equation. The calculation of discount rate with expenditure tax system is illustrated below under the assumption of this section that the government does not want to raise any additional revenue from corporation tax .

Let the rate of expenditure tax be $\tau_\epsilon^m$. The net of tax income received by an investor who holds $n_i^m$ shares in company i is given by the following:

$$y_i^m = n_i^m \{ [\mu_i - RD_i + pD_i - \Delta_i] (1-\tau_\epsilon^m) + \Delta_i(1-\tau_\epsilon^m) \}$$

In the above expression, it is assumed that the investor realises investment equal to the capital gain element and spends it on consumption goods - hence the term in square brackets is multiplied by $(1-\tau_\epsilon^m)$. Alternatively, it can be assumed that the company distributes all earnings as dividends which then are spent on consumption goods by the investor.

Similarly, the investor earns r the real interest rate on any investment in bonds. He consumes all his interest income, hence $r(1-\tau_\epsilon)$ gives the amount he can spend on consumption goods out of income from bonds.

The expected portfolio return now is:

$$y^m = (w^m - \Sigma n_i S_i)\, r(1-\tau_\epsilon^m) + \Sigma n_i^m \{ [\mu_i - RD_i + pD_i - \Delta_i] (1-\tau_\epsilon^m) + \Delta_i(1-\tau_\epsilon^m) \}$$

[Eqn 10.8] $\quad = (w^m - \Sigma n_i S_i)\, r\theta_\epsilon^m + \Sigma_i n_i^m \{ (\mu_i - D_i r) (\theta_\epsilon^m) \}$

In the above equation $(1-\tau_\epsilon^m) = \theta_\epsilon^m$.

Now this equation 10.8 is exactly the same as equation 10.4 except that $\theta_p^m$ is replaced by $\theta_\epsilon^m$ due to the fact that now we are dealing with expenditure tax.

By substituting $\theta_\epsilon^m$ for $\theta_p^m$ in appropriate places, we can repeat all the steps till we reach valuation equation 10.7. This equation, under expenditure tax system, is

[Eqn 10.9] $$V_i = \frac{\mu_i \theta_\epsilon}{r\theta_\epsilon + \beta_i \pi}$$

In equation (10.9) the returns generated by the project, net of expenditure tax, are discounted by a risk adjusted discount rate to obtain the valuation. The equation shows that the valuation equation, and hence the discount rate and the pre-tax rates of return, under expenditure tax system are similar to those under CIT. However, the Meade report had stated that the return to the investor under expenditure tax is the same as the return generated by physical investment in the project ; whereas we proved previously that the return to the investor is lower than the return on investment under CIT. Meade report is correct and this point can be easily illustrated by again considering an investment without risk premium.

The valuation equation is reduced to:

$$V_i = \frac{\mu_i \theta_\epsilon}{r\,\theta_\epsilon} = \frac{\mu_i}{r}$$

The pre-tax rate of return on physical investment is $\mu_i/V_i$ = r the real rate of interest, and this result is the same as in the case of CIT.

The main difference between CIT and ET relates to the post-tax rate of return enjoyed by an investor. Under an expenditure tax system, the amount of consumption that an investor has to forego to release $V_i$ for investment is only $\theta_\epsilon V_i$. This is so because when an investor purchases investment worth $V_i$, he gets a credit (see line D on in the previous table) for investment expenditure purposes, thus reducing his liability to expenditure tax. This credit gives a refund of $\tau_\epsilon$ . Therefore the net investment required is:

$$V - \tau_\epsilon V = V(1-\tau_\epsilon) = V\,\theta_\epsilon .$$

Now investor who owns the entire company gets $\mu_i\theta_\epsilon$ as his net return from owning equity and debt of the company. Hence the post-tax rate of return to the investor is

$$\frac{\mu_i\theta_\epsilon}{V_i\theta_\epsilon} = \frac{\mu_i}{V_i} = r$$

Hence the post-tax rate of return which the investor gets equals the pre-tax rate of return generated by the underlying investment and the post-tax rate of return is not distorted under expenditure tax system. This is one of the main conclusions advocated in the Meade Report. We have demonstrated above that the conclusions reached using the simple models in the Meade Report can be reached by using our more comprehensive model. The advantages of our model are that it can help evaluate more complex situations, and help reach useful conclusions using a more restrictive definition of investors thatn that used in the Meade report, as shown below.

In the analysis above, we dealt with discount rates when the government did not raise revenue through corporation tax. We can conclude that under such a scenario, the Meade Report is correct in stating that expenditure tax system does not distort the returns to an investor whereas a CIT system does distort the returns to investors. However, in the expositions above, we assumed simple corporation tax structures which only served as revenue *collecting* vehicles. Next we calculate discount rates when corporation tax rates are such that government raises revenue from this source. We will again examine the two alternative tax regimes, the CIT and the ET, and assess the distortion, if any, caused by the presence of corporation taxes. This is necessary in order to confirm that the conclusions reached in the Meade Report are valid under more realistic scenarios, such as when the corporation tax system is used to *raise* revenue.

## Section (B)

## DISCOUNT RATES WHEN CORPORATION TAX SYSTEMS AIM TO RAISE INCREMENTAL REVENUE

We examine the discount rates under the two alternative tax systems, as we did in section (A) above, but now we also introduce effective corporation tax in the valuation equation.

### (i) COMPREHENSIVE INCOME TAX (CIT)

When the government intends to raise revenue from corporation tax, where the personal tax system is based on CIT, then it has a choice of a number of different corporation tax regimes it can implement. Of these, the two most prominent ones are :

(a) Partial Imputation System

(b) S-base Flow-of-funds System

These are examined in detail below.

### (a) Imputation system where the rate of imputation credit is lower than the full rate of corporation tax (Partial Imputation System - (a) above)

This is the simplest method. The only change from the system detailed in section (a), is that instead of allowing credit for the full rate of corporation tax (on dividends and retained earnings), the tax credit is at a lower rate. Instead of the 100% imputation credit considered in section (a), the percentage of credit is lower , say j%. This means that corporation tax raises net revenue instead of merely serving as a vehicle for *collecting* revenue. Unfortunately this small change in specification results in changes in the tax advantage of leverage, the risk premium, portfolio variance and harmonic mean ($\Lambda$) of risk aversion terms, and the final valuation equation. Hence the entire procedure from section (a) is repeated to get the correct intermediate variables as well as a correct final equation.

*Imputation Rate*: In section (a), we assumed that for every £1 of dividend, the investor received credit at the rate of $\tau_c$, the corporation tax rate. Let us now take the credit an investor receives for corporation tax paid as $j\tau_c$ where j is a fraction between 0 and 1, as stated above. With imputation at the rate of $j\tau_c$ the gross "dividend" received is:

$$\frac{1}{1-j\tau_c}$$

"Dividend" is stated in inverted commas because like in the previous case of CIT, the imputation applies not only to cash dividends but also to the retained earnings. These retained earnings are allocated to the shareholders under a CIT system.

For £1 of gross "dividend", the tax credit is $= j\tau_c$

For $\frac{1}{1-j\tau_c}$ of "dividend", the tax credit is $= j\tau_c \frac{1}{1-j\tau_c}$

$$= \frac{j\tau_c}{1-j\tau_c}$$

The personal tax on gross dividend is $= \left[\frac{1}{1-j\tau_c}\right]\tau_p = \frac{\tau_p}{1-j\tau_c}$

A part of the personal tax levied on investors is deemed to be met by the imputation tax credit that they receive. They need to pay over only the remaining part or, in some cases, receive tax repayment if the total tax due from them is less than the imputed tax credit.

The remaining part of tax payable by investors is therefore $= \frac{\tau_p}{1-j\tau_c} - \frac{j\tau_c}{1-j\tau_c}$

$$= \frac{\tau_p - j\tau_c}{1-j\tau_c}$$

Thus the investor receives £1 cash (or capital gain resulting from retained earnings) on which he has to meet the above extra tax. His net of tax return is:

$$1 - \frac{\tau_p - j\tau_c}{1-j\tau_c} = \frac{1-j\tau_c - \tau_p + j\tau_c}{1-j\tau_c} = \frac{1-\tau_p}{1-j\tau_c} \tag{10.9a}$$

The above expression gives the income net of the tax borne at personal level by shareholders under the tax system described in this section. As before, $1-\tau_p$ is $\theta_p$ - similarly let $1-j\tau_c = \theta_j$. Hence the above expression (10.9a) becomes $\theta_p / \theta_j$ and this represents the income net of tax at personal level in the equations below.

In order to determine a valuation equation, we need to begin by deriving an expression for returns to an investor. We follow the procedures used previously in this thesis. In order to determine the return to the investor, we need to define the corporation tax regime which will determine the corporate level taxes levied on the cash flows from the projects. The Meade report states that the corporation tax system compatible with CIT (which is a profit based tax at the personal tax level) is the P-system (that is, a profits tax rather than a flow of funds tax). The corporation tax system described in chapter 4 is an example of a P-system of taxation. Using that system of corporate taxes, the expected return $y_i^m$ of investor m from holding $n_i^m$ equity shares in company i is given by:

$$y_i^m = n_i^m \{ (\mu_i - \tau_c [\mu_i - RD_i - C + pD_i] - RD_i + pD_i - \Delta_i ) \frac{(\theta_p^m)}{\theta_j} + \frac{\Delta_i(\theta_p^m)}{\theta_j} \}$$

[Eqn 10.10] $$= n_i^m \{ (\mu_i - \tau_c \mu_i - \tau_c[- C + pD_i - RD_i ] - D_i (R-p) ) \frac{\theta_p^m}{\theta_j}$$

$$- \frac{\Delta_i \theta_p^m}{\theta_j} + \frac{\Delta_i (\theta_p^m)}{\theta_j}$$

In equation 10.10, the dividend terms cancel out, since the tax treatment of dividends is no different from the tax treatment of capital gains resulting from retained earnings. Both bear same rate of corporation tax, bear the (same) marginal income tax rate, and get the same imputation credit. Hence the level of dividends will again have no impact on the discount rates.

The term C, representing capital allowances, can be ignored because under P-system, capital allowances will be restricted to equal economic depreciation allowances. However, economic depreciation is zero because $\mu_i$ represents the average return expected to perpetuity. Hence the above expression simplifies to

$$y_i^m = n_i^m \{ (\mu_i^* - \tau_c [-D_i (R-p)] -D_i r) \frac{\theta_p^m}{\theta_j} \}$$

$$= n_i^m \{ (\mu_i^* - \tau_c D_i r -D_i r) \frac{\theta_p^m}{\theta_j} \}$$

$$= n_i^m \{ (\mu_i^* - D_i r [1-\tau_c]) \frac{\theta_p^m}{\theta_j} \} \qquad [ 1-\tau_c = \theta_c \text{ (as before)}]$$

[Eqn 10.11] $$y_i^m = n_i^m \{ (\mu_i^* - D_i r \theta_c) \frac{\theta_p^m}{\theta_j} \}$$

The above shows that the return on equity is influenced by the corporation tax rate, personal tax rate and imputation proportion implicit in $\theta_c$, $\theta_p^m$ and $\theta_j$ respectively.

We obtain the return expected by the investor from his portfolio by adding the return from his holding of bonds to the summation of the returns from holding equities :

$$y^m = (w^m - \Sigma n_i^m S_i) \ r\theta_p^{\ m} + \Sigma_i \ n_i^m \{ (\mu_i^* - D_i \ r \ \theta_c) \frac{\theta_p^{\ m}}{\theta_j}$$

Since the only stochastic element above is $\mu_i^*$ (as before), the portfolio variance is

$$\text{var} \quad (\sigma_y^{\ m})^2 = E(y^m - y^m)^2 = \frac{(\theta_p^{\ m})^2}{\theta_j} \ \Sigma_i \ \Sigma_m \ n_i \ n_j \ \mu_{ij}^*$$

where $\mu_{ij}^*$ is the covariance of $\mu_i^*$ with $\mu_j^*$. Note that now the tax factors $\theta_p^{\ m}$ and $\theta_j$ are also relevant in determining the portfolio variance. The greater the value of p, the larger is the proportion of corporation tax which is imputed as tax credit, which means the smaller is $\theta_j$. This leads to an increase in the portfolio variance - hence increasing p increases portfolio variance.

Defining utility over mean and variance of the portfolio returns as before, we differentiate the Lagrangean with respect to the choice variables $n_i^m$ to get:

$$L = U^m \{y^m, (\sigma_y^{\ m})^2\}$$

$$[\ 10.12] \qquad \frac{\delta L}{\delta n_i^{\ m}} = u_1^{\ m} \ x \ \{ -S_i \ r \ \theta_p^{\ m} + (\mu_i^* - D_i \ r \ \theta_c) \frac{\theta_p^{\ m}}{1\text{-}j\tau_c} \ \}$$

derivative of $y^m$ w.r.t. $n_i$

$$+ \ u_2^{\ m} \ x \ \{ 2 \frac{(\theta_p^{\ m})^2}{\theta_j} \ \sum_{j=1} \ \mu_{ij}^* \ n_j^m \}$$

derivative of $(\sigma_y^{\ m})^2$ w.r.t. $n_i$

At the optimum, the above should equal zero. This yields:

$$u_1^{\ m} \ \{\text{derivative of } y^m\} = -2 \ u_2^{\ m} \ \frac{(\theta_p^{\ m})^2}{\theta_j} \ \Sigma \ \mu_{ij}^* \ n_j^m$$

$$\Rightarrow \qquad \{\text{derivative of } y^m\} = \frac{-2 \ u_2^{\ m}}{u_1^{\ m}} \ \frac{(\theta_p^{\ m})^2}{\theta_j} \ \Sigma \ \mu_{ij}^* \ n_j^m$$

Using $\gamma^m = \dfrac{-2\, u_2^m}{u_1^m}$ and **bold letters** to denote vectors $\mu_i$, $\mathbf{S}_i$ & $\mathbf{D}_i$ for all firms

$$\text{[Eqn 10.13]} \quad \Rightarrow \quad -\,\mathbf{S}\, r\theta_p^m + (\mu^* - \mathbf{D}\, r\theta_c)\,\frac{\theta_p^m}{\theta_j} = \gamma^m\,\frac{(\theta_p^m)^2}{\theta_j}\,[M]\,n^m$$

To add tax factors across individual shareholders, we need to define

$$\Lambda = \frac{1}{\Sigma_m \dfrac{1}{\gamma^m \dfrac{(\theta_p^m)^2}{\theta_j}}}$$

Note that in previous chapters, $\Lambda$ is defined using $(\theta_g)^2$ that is, it is based on the capital gains tax rates. However, under the present set up, the relevant tax factors are $\theta_p^m / \theta_j$ which need to be multiplied by the risk aversion factors $\gamma^m$.

Multiplying both sides of eq 10.13 by $\dfrac{\Lambda}{\gamma^m(\theta_p^m/\theta_j)^2}$, we get

$$\text{[10.14]} \qquad \left\{ -\mathbf{S}\, r\theta_p^m + (\mu^* - \mathbf{D}\, r\theta_c)\,\frac{\theta_p^m}{\theta_j} \right\} \frac{\Lambda}{\gamma^m \dfrac{(\theta_p^m)^2}{\theta_j}} = \Lambda\,[M]\,n^m$$

Summing over all individuals m, we use the following definition for the average of net of personal tax rates

$$\theta_p = \Sigma_m\, \theta_p^m \times \frac{\Lambda}{\gamma^m \dfrac{(\theta_p^m)^2}{\theta_j}}$$

Using this, eq 10.14 simplifies to

$$-\mathbf{S}\, r\,\theta_p + (\mu^* - \mathbf{D}\, r\theta_c)\,\frac{\theta_p}{\theta_j} = \Lambda\,[M]\,1$$

Rearrangement gives the value of S which is the vector of equity values

$$\frac{(\mu^* - \mathbf{D}\, r\theta_c)\,\dfrac{\theta_p}{\theta_j} - \Lambda\,[M]\,1}{r\,\theta_p} = \mathbf{S}$$

To get the value of the firm and of leverage, we add **D** to both sides to get

$$\mathbf{S} + \mathbf{D} = \{ (\mu^* - \mathbf{D}\, r\theta_c) \frac{\theta_p}{\theta_j} - \Lambda\,[\mathrm{M}]\,\mathbf{1} \} / r\theta_p + \mathbf{D}$$

$$\Rightarrow \quad \mathbf{V} = \frac{\mu^* \theta_p/\theta_j - \Lambda[\mathrm{M}]1}{r\theta_p} - \frac{\mathbf{D} r\theta_c\ \theta_p/\theta_j}{r\theta_p} + \mathbf{D}$$

$$\Rightarrow \quad \mathbf{V} = \frac{\mu^* \theta_p/\theta_j - \Lambda[\mathrm{M}]1}{r\theta_p} + \mathbf{D}\,(1 - \theta_c/\theta_j)$$

[Eqn 10.15] $\Rightarrow \quad \mathbf{V} = \dfrac{\mu^* \theta_p/\theta_j - \Lambda[\mathrm{M}]1}{r\theta_p} + \mathrm{DL}$ where $\mathrm{L} = (1 - \theta_c/\theta_j)$

The above equation shows that the value of the firm under CIT with corporation tax is equal to the certainty equivalent return, adjusted for corporate, personal and imputation tax factors, discounted by the net of personal tax real interest rate (as shown by the first term) - plus the benefit of leverage (shown by DL). The certainty equivalent is reflected in the numerator, where the risky net corporate cash flows ($\mu^*\ \theta_p/\theta_j$) are adjusted by a factor representing risk adjustment ($\Lambda$[M]1). Since risk is accounted for in the numerator, the adjusted cash flows can be discounted by the risk free rate in the denominator. The second term (DL) arises because the imputation, which determines $\theta_j$, is not at the rate of 100% under the tax regime under consideration.

Thus leverage, which was not present in the case of CIT with 100% imputation (equation 10.6), now becomes relevant. The implication is that when corporation tax is not fully offset by imputation, then leverage has a positive impact on value. Leverage is beneficial as long as $\theta_c$ is less than $\theta_j$ - which means as long as less than 100% of corporation tax is imputed as a tax credit to the shareholders.

Secondly, the leverage term is independent of $\theta_p$ - the personal tax factor. This is because the rate of personal tax applicable to interest income, dividends and capital gains is the same (provided imputation is taken into account in $\theta_j$). Hence personal tax is not relevant - leverage depends crucially on the proportion of corporation tax imputed as tax credit - the greater this proportion j, the lower is the value of leverage.

This can be illustrated by a simple example. Let corporation tax rate $\tau_c = .4$. If j = .5,
$L = 1 - [\,(1-.4) / (1-.5 \times .4)$

$= 1 - [.6 / .8] \quad = 1 - .75 \quad = .25$, the benefit of leverage.

If j increases to .9, $L = 1 - \dfrac{.6}{1 - .9 \times .4} \quad = 1 - \dfrac{.6}{1 - .36}$

$= 1 - \dfrac{.6}{.64} = 1 - .94 = 0.06$, that is, increasing j has decreased the benefit of leverage.

We add equation 10.15 for all companies to get statistics for the aggregate corporate sector :

$$V = \frac{\mu^* \theta_p/\theta_j - \Lambda \operatorname{var} \mu^*}{r\theta_p} + LD$$

We rearrange to get V*, the unleveraged equity value, as shown below:

[Eqn 10.16] $$V - LD = V^* = \frac{\mu^* \theta_p/\theta_j - \Lambda \operatorname{var} \mu^*}{r\theta_p}$$

Equation 10.16 can be rearranged to get the market risk premium ($\pi$) as:

$$r\theta_p = \frac{\mu^* \theta_p/\theta_j}{V^*} - \frac{\Lambda \operatorname{var} \mu^*}{V^*}$$

[ 10.17] $$\Rightarrow \quad \frac{\Lambda \operatorname{var} \mu^*}{V^*} = \frac{\mu^* \theta_p/\theta_j}{V^*} - r\theta_p = \pi,$$ the market risk premium.

The risk premium is the difference between the rate of return on levered stream $\mu/V^*$, adjusted by corporate, personal and imputation tax factors implicit in $\mu^*$, $\theta_p$ and $\theta_j$ respectively, and the net of personal tax real rate of interest.

Previously, the market risk premium was calculated as $\dfrac{\mu\theta_p}{V^*} - r\theta_p$. The expression now differs because we multiply $\dfrac{\mu\theta_p}{V^*}$ by $\dfrac{\theta_c}{\theta_p}$ to get $\dfrac{\mu^* \theta_p/\theta_j}{V^*}$.

As long as $\theta_c/\theta_p$ is less than one, which it will be since j is less than 100% , the numerator of the risk premium term is smaller than in the case of 100% imputation. This is so because the pound return on equity decreases as the amount of imputation credit decreases, because more and more of the returns are drained off by the corporation tax.

The denominator of the rate of return to unleveraged equity term, that is, V*, in this subsection is also different from V* in section (a), since V* now is $\dfrac{\mu^* \theta_p/\theta_j - \Lambda \operatorname{var} \mu^*}{r\theta_p}$.

We note that in comparison with V* in section (a), the equity return term $\mu^* \theta_p/\theta_j$, the harmonic average term $\Lambda$ and even the market variance term, var $\mu^*$, are different. $\mu^*\theta_p/\theta_j$ is smaller in comparison with the corresponding term of section (a), while $\Lambda$ is greater but is largely offset by a decrease in the variance of $\mu^*$ (variance ($\mu^*$) = var[$\mu(\theta_c)$] = $\theta_c^2$ var $\mu$. Since $\theta_c$ is less than one, variance $\mu^*$ is less than var $\mu$). Hence V*, the value of the unleveraged stream, is likely to be less than the value when the corporation tax system was not intended to raise additional revenue (section (a)).

Since both the numerator $\mu^* \theta_p/\theta_j$ and the denominator V* are lower than the corresponding figures in section (a), their ratio, which gives the rate of return on unleveraged market equity, is likely to be similar to that in section (a). The exact comparison will depend upon the numerical values of the variables affecting $\Lambda$ (ie, on $\theta_j$), affecting $\mu^*$ (ie, on $\theta_c$) and on the ratio $\theta_c/\theta_p$. For the present, we can take that the market risk premium in the present case is likely to be similar to the market risk premium in the case of 100% imputation.

Having calculated the market risk premium, we can calculate the value of the aggregate corporate sector using equation 10.17 and then equation 10.16 as follows:

$$\frac{\mu^* \theta_p/\theta_j}{V^*} - r\,\theta_p = \pi$$

$$\Rightarrow \quad \frac{\mu^* \theta_p/\theta_j}{V^*} = r\,\theta_p + \pi$$

[Eqn 10.18] $$\Rightarrow \quad V^* = \frac{\mu^* \theta_p/\theta_j}{r\,\theta_p + \pi}$$

[Eqn 10.19] $$\Rightarrow \quad V = \frac{\mu^* \theta_p/\theta_j}{r\,\theta_p + \pi} + LD$$

Equation 10.19 can be expressed for an individual company's value $V_i$ by using beta factors $b_i$s as follows:

[Eqn 10.20] $$V_i = \frac{\mu^* \theta_p/\theta_j}{r\,\theta_p + \pi\,ß_i} + LD_i$$

In comparison with the 100% imputation case of equation 10.7 , we note two changes. Firstly, the leverage term LD adds to value, where L is given by $1 - \theta_c/\theta_j$. Secondly, the numerator of the risky return is $\mu^* \theta_p/\theta_j$ which is equal to $\mu\, \theta_p\, \theta_c/\theta_j$, that is, it is less than $\mu\theta_p$ by a factor of $\theta_c/\theta_j$. This will reduce the value of the company in comparison with equation 10.7 .

Of these two effects, the latter will dominate. This can be seen by differentiating a modified version of equation 10.20 with respect to $\theta_c/\theta_j$ to get

$$V_i = \frac{\mu\, \theta_p\, (\theta_c/\theta_j)}{r\theta_p + ß_i\pi} + D\,(1 - \theta_c/\theta_j) \quad \text{(from Eqn 10.15)}$$

$$\frac{\delta V_i}{\delta(\theta_c/\theta_j)} = \frac{\mu\, \theta_p}{r\theta_p + ß_i\pi} - D$$

From above it is clear that as long as the capitalised value of net of personal tax equity cash flow is greater than the value of debt, the above expression will be positive. In other words, as long as debt is not the dominant security, a change in $\theta_c/\theta_j$ will change the value of the company in the same direction. A decrease in $\theta_c/\theta_j$ will lead to a decrease in $V_i$. For a given $\theta_c$, $\theta_c/\theta_j$ decreases when $\theta_j$ increases. $\theta_j$ increases when $1\text{-}j\tau_c$ $(=\theta_j)$ increases. $(1\text{-}j\tau_c)$ increases when j decreases. Hence, when j, the proportion of corporate tax allowed as a tax credit decreases, the value of the firm will decrease. This analysis confirms the intuition that as the government takes out greater proportion of corporation tax for itself, the absolute value of the firm will decrease.

Two further useful insights are provided by the above analysis. Firstly, the negative impact of decreasing the proportion of corporate tax allowed as a tax credit is mitigated partially by the increasing benefit of leverage. The lower the j and higher the leverage, the greater is this benefit. Secondly, for companies with very high debt ratio, this leverage effect may be the dominant effect - therefore for highly geared companies, decreasing j may result in increasing the value of the firm.

However, for normal firms, the conclusion is that decreasing j decreases value, but at a slightly lower rate than if leverage is not present.

Since the valuation equation gives a lower value, the required discount rate under this subsection must be higher than the corresponding discount rate with 100% imputation. In section (a), it was shown that the rate of return to a saver under CIT with 100% imputation is lower than the return on investment, because of the presence of a personal tax structure. Since the results of this section show that the value of the company is even lower if the imputation rate decreases below 100%, the implications are that the distortion due to taxes is even greater when CIT is combined with corporation tax system intended for raising revenue.

**(b) CIT and S-base**

After the above examination of the CIT system of taxation that is used with the partial imputation system at the corporate level, we examine CIT system together with an S-base corporation tax system. This type of tax structure will have the following features:

(i) Comprehensive Income Tax system at personal level, including taxation of gains as they accrue at personal income tax rates, and

(ii) A flow of funds based tax at corporate level (S-base). The following shows a corporate tax based on S level. The tax base under this form of taxation comprises the following:

| | | |
|---|---|---|
| S. Basis | Dividends paid out to the shareholders | x |
| Less: | New share capital raised (net) | (x) |
| Add: | Net purchase of shares of other UK companies (anti-avoidance step) | x |
| Less: | Dividends received from other UK companies (anti-avoidance step) | (x) |
| | CORPORATION TAX S-BASE | x |

(iii) The third characteristic is that there is no imputation - the relationship between personnel and corporate tax structures is that described by the "classical system" (chapter 5).

The advantage of this system, in comparison with the previous system based on corporation tax on P-basis described in the subsection above, is that with an S-base there is no distortion introduced by corporation tax in the rate of return to the saver. This is because S-base is one of the flow-of-funds bases of corporation tax which has this characteristic advantage over P-basis.

Before evaluating the required return under such a system, we need to distinguish tax on a "tax-exclusive basis" and tax on a "tax inclusive basis". The two bases are interrelated and we will use this characteristic extensively in our analysis. If £100 of cash dividend leads to £60 of tax, then the tax on tax-exclusive basis (referred to as $\tau_{cE}$ - ie. corporate tax on tax-exclusive basis) is 60%. On an inclusive basis (referred to as $\tau_{ci}$) the rate of tax is £60/£160 = 37.5%. The two bases are related as follows:

[ 10.21] $$\tau_{ci} = \frac{\tau_{cE}}{1 + \tau_{cE}} \qquad \text{Similarly} \quad \tau_{cE} = \frac{\tau_{ci}}{1 - \tau_{ci}}$$

Similarly, to get a net of corporate tax cash flow, $\theta_{ci}$, the following holds:

$$\theta_{ci} = 1 - \tau_{ci} = 1 - \frac{\tau_{cE}}{1+\tau_{cE}} = \frac{1 + \tau_{cE} - \tau_{cE}}{1+\tau_{cE}}$$

[Eqn 10.22] $\Rightarrow$ $$\theta_{ci} = \frac{1}{1+\tau_{cE}}$$

Therefore reducing cash flow by tax on exclusive basis $\left(\frac{1}{1+\tau_{cE}}\right)$ is the same as multiplying by $\theta_{ci}$. These relationships will be used below.

The return expected by the investor from holding equity may now be written as:

[Eqn 10.23] $$y_i^m = n_i^m \{ (u_i^m - \tau_{cE}(\Delta_i - S_n) - RD_i + pD_i - \Delta_i)\left(\frac{1}{1+\tau_{cE}}\right)(1-\tau_p) + \Delta_i(1-\tau_p) \}$$

In the above, the term $\tau_{cE}(\Delta_i - S_n)$ gives the tax due on S-basis where $S_n$ represents the proceeds of new share issue. We assume for simplicity that $S_n = 0$. Hence the relevant tax is $\tau_{cE}\Delta_i$, that is, a tax on dividends.

The first round brackets give the expression for retained earnings. The second round brackets $\left(\frac{1}{1+\tau_{cE}}\right)$, is introduced to reflect Meade's view of the market valuation on the on the stock exchange under S-basis. The report argues that £1 of retention will not give rise to a £1 of capital gain because the investors will capitalise the potential tax liability associated with getting money out of the corporation in the form of dividends. The argument is that investors realise that whenever future dividends were to be paid out of the company, there would be an associated corporation tax liability under the S-basis. They will capitalise this tax liability in share prices - hence a £1 of retained earnings would lead to $\frac{1}{1+\tau_{cE}}$ of capital gain. This view of tax capitalisation has also been advocated by Auerbach (1979) (see chapter 2). The empirical evidence, though sketchy, refutes it (Poterba & Summers, 1984). However, this term is retained to calculate the discount rate on the basis of Meade's recommendations.

Finally, the first two brackets are multiplied by $(1-\tau_p)$ to reflect the fact that under CIT capital gains are taxed on an accruals basis at the marginal income tax rate. The final term in the expression merely shows cash dividends net of associated personal tax. Equation 10.23 can be simplified as follows:

$$y_i^m = n_i^m \{ (\mu_i^m - D_i r)\frac{(1-\tau_p)}{1+\tau_{cE}} - \Delta_i(1+\tau_{cE})\frac{(1-\tau_p)}{1+\tau_{cE}} + \Delta_i(1-\tau_p) \}$$

$$= \quad n_i^m \{ (\mu_i^m - Dr)\frac{\theta_p}{1+\tau_{cE}} - \Delta_i\theta_p + \Delta_i\theta_p \}$$

[Eqn 10.24] $\Rightarrow$ $y_i^m = n_i^m \{ \mu_i^m \theta_{ci} \theta_p - Dr\theta_{ci} \theta_p \}$ (using Eqn 10.22)

In equation 10.24 above the dividend term $\Delta$ disappears. Thus, even though dividends bear corporation tax (at the rate of $\theta_{cE}$) whereas retained earnings do not bear corporation tax, dividend policy is irrelevant and dividends do not bear any tax disadvantage. This result arises because of the assumption that while retained earnings do not lead to an immediate corporation tax liability, the market capitalises the POTENTIAL tax liability. (Note that throughout this thesis, we assume that tax rates remain constant at all times, that is, there is no uncertainty generated by possible changes in tax rates.) Thus, even in the presence of extra taxes borne by dividends, dividend policy can be shown to be irrelevant if the above assumption is made regarding valuation in the capital markets.

The expected return on portfolio of the investor m now is:

$$y^m = (w^m - \Sigma_{ni} S_i) r\theta_p + \Sigma_i n_i^m \{\mu_i^m \theta_{ci} \theta_p - Dr \theta_{ci} \theta_p \}$$

Variance of this portfolio $= E(y^m - y^m)^2 = (\theta_p \theta\tau_{ci})^2 \Sigma_i \Sigma_j n_i \mu_{ij} n_j$.

We set up a Lagrangean which yields the following equation :

$$-S_i r\theta_p + \mu_i \theta_p \theta_{ci} - D_i r \theta_p \theta_{ci} = \Lambda[M]n^m$$

$$\Rightarrow \quad S_i = \frac{\mu_i \theta_p \theta_{ci} - \Lambda[M]n^m}{r\theta_p} - \frac{D_i r \theta_p \theta_{ci}}{r\theta_p}$$

Adding and subtracting debt $D_i$ yields:

$$S_i + D_i = \frac{\mu_i \theta_{ci} \theta_p - \Lambda[M]n^m}{r\theta_p} - D_i \theta_{ci} + D_i$$

$$\Rightarrow \quad V_i = \frac{\mu_i \theta_{ci} \theta_p - \Lambda[M]n^m}{r\theta_p} + D_i (1 - \theta_{ci})$$

$$[10.25] \quad \Rightarrow \quad V_i = \frac{\mu_i \theta_{ci} \theta_p - \Lambda[M]n^m}{r\theta_p} + D_i \tau_{ci} \qquad [\theta_{ci} = 1-\tau_{ci}]$$

Hence leverage is still important with a S-based tax system and this question has not been addressed by Meade Committee and their results overlook this point. Leverage is important because

(a) at personal level there is no distinction between taxation of income from debt interest, dividends or accrued capital gains, but

(b) at corporate level, debt interest bears no extra tax whereas

 (i) dividends bear full actual corporation tax, and

 (ii) retained earnings suffer from potential corporate tax liability.

Debt faces no potential corporate level tax and therefore debt is advantageous from the tax point of view in comparison with equity even though there is no direct corporation tax on retained equity earnings.

The above equation (10.25) leads to the following equation for valuation using CAPM betas:

$$[\text{Eqn } 10.25a] \quad V_i = \frac{\mu_i \theta_{ci} \theta_p}{r\theta_p + ß_i\pi} + D_i \tau_{ci}$$

In the absence of a risk premium and debt, the above reduces to

$$[\text{Eqn } 10.25b] \quad V_i = \frac{\mu \theta_{ci} \theta_p}{r\theta_p}$$

Now for a company to raise $V_i$ in cash, it need only issue $\theta_{ci} V_i$ of shares. This should be sufficient for it to raise $V_i$ of cash because along with $\theta_{ci} V_i$ of cash, the company will be saving $\tau_{ci} V_i$ of corporation tax payable on any dividends it distributes (as stated above, new share issues are allowed as a deduction in calculating corporation tax). This gives:

| $\theta_{ci} V_i$ | + | $\tau_{ci} V_i$ | = | $V_i$ |
|---|---|---|---|---|
| new shares | | corporation tax saved elsewhere | | total cash raised |

The return to a saver who owns the entire firm is $u_i^m \theta_{ci} \theta_p$ from equation 10.25b . Note that we have assumed that there is no debt).

The rate of return therefore is $$\frac{u_i^m \theta_{ci} \theta_p}{\theta_{ci} V_i} = \frac{u_i^m \theta_p}{V_i}$$

Hence the rate of return is distorted only by personal tax (implicit in $\theta_p$) under CIT - the use of flow of funds based S-basis for corporation tax introduces no further distortion. This point is stressed by Meade.

The conclusion is that under CIT, taxes at the personal tax level cause a distortion in the rate of return to an investor. The distortion increases if a profit based tax is levied at the corporate level. If, instead, CIT is accompanied by a flow of funds tax at the corporate level, then any distortion at the personal tax level is not increased. However, note that we have assumed that there is no debt or risk premium in order to reach the last conclusion. Our conclusion would be different in the presence of debt and risk premium, and these scenarios are discussed in chapter 11.

After considering CIT in detail above, we look at expenditure tax system when the corresponding corporation tax system is intended for raising revenue.

**(ii) EXPENDITURE TAX SYSTEM (when Corporation Taxes raise incremental revenue)**

Denoting expenditure tax by $\tau_\epsilon$ as before, the expected return to an investor from owning $y_i^m$ of equity is:

[10.26] $$y_i^m = n_i^m \{ [\mu_i - \tau_{cE}(\Delta_i - S_n) - \Delta_i - RD_i + pD_i] \frac{(1-\tau_\epsilon^m)}{1+\tau_{cE}} + \Delta_i (1-\tau_\epsilon^m \}$$

Note that $S_n$ refers to the annual equivalent of capitalised values of the investment in new shares, so that it can be treated as a cash flow to perpetuity (like the other cash flows in the equation).

The above assumes an S-based corporation tax system and is similar to equation 10.23 - except that $\tau_\epsilon^m$ - the expenditure tax, replaces $\tau_p^m$ the personal income tax. Assuming $S_n$, the new shares issued is zero, the above expression reduces to:

[Eqn 10.27] $y_i^m = n_i^m \{ \mu_i \theta_{ci} \theta_\epsilon^m \ - \ D_i r \theta_{ci} \theta_\epsilon^m \}$

Note that the above equation again is similar to equation 10.24 , except that $\theta_\epsilon^m$ replaces $\theta_p^m$ everywhere. Therefore, following the same procedure as before, we get the following valuation equation when expenditure tax system at personal tax level is combined with S-based corporation tax system:

[Eqn 10.28] $$V_i = \frac{\mu_i \theta_{ci} \theta_\epsilon}{r\theta_\epsilon + ß_i \pi} + D_i \tau_{ci}$$

Equation 10.28 is similar to equation 10.25(a). It shows that leverage is still valuable as it was under CIT.

The main point made by Meade was that the return to a saver is not distorted at all if an expenditure tax system at personal level is combined with a flow of funds based tax system (like the S-base used here). This is proved to be true below but only by assuming that there is neither any risk premium nor any debt.

In such a no-debt no-risk case, the return to a saver who owns the entire firm is given by $\mu_i \theta_{ci} \theta_\epsilon^m$ (equation 10.27). To obtain the cash inflow equal to $V_i$, the firm need only issue $\theta_{ci} V_i$ of new shares, making up the difference from corporation tax saved, as already shown.

Now for an individual to purchase $\theta_{ci} V_i$ of new shares, he need only give up $\theta_\epsilon^m (\theta_{ci} V_i)$ of consumption, since purchase of qualifying assets leads to a reduction in expenditure tax. This has been shown in section (a) and relies on line D in the table in that section.

Hence consumption foregone = $\theta_\epsilon^m \theta_{ci} V_i$

Return earned = $\mu_i \theta_{ci} \theta_\epsilon^m$

Therefore the rate of return = $\mu_i \theta_{ci} \theta_\epsilon^m / \theta_\epsilon^m \theta_{ci} V_i$

= $\mu_i / V_i$

which is the rate of return on the physical investment generated by the project. Hence the rate of return to a saver in terms of consumption foregone is not distorted by personal or corporation tax if expenditure tax and flow of funds taxes are used at the respective levels.

This chapter has shown the valuation equations (derived using the features of the recommended valuation equation 7.22) and the rates of return to savers under some of the tax systems examined in the Report of the Meade Committee. This chapter demonstrates that the valuation model developed in this thesis is robust and can be used to analyse complex issues addressed in the literature. The next chapter will demonstrate that the recommended valuation model is capable of pointing out how erroneous conclusions can be reached if the much simpler models, like those used by the Meade Committee, are utilised to address complex issues. To reiterate, note that we assumed the absence of debt and of risk premium in this chapter in order to demonstrate the validity of the conclusions of the Meade report. In the next chapter, we extend the analysis of the Meade report by including debt and uncertainty, which provide far more relevant and more realistic scenarios, in the analysis.

# RATES OF RETURN WITH DEBT AND UNCERTAINTY -EXTENSION OF MEADE REPORT ANALYSIS

We begin with a brief overview of the sections in this chapter.

Section (A)

The contribution made in chapter 10 was to provide valuation equations under the alternative tax systems described in the Meade Report. As stated in Chapter 10 section (b), the underlying assumptions in the Meade Report's analysis include the assumption of certainty as well as the assumption of no mixed capital structures (that is, companies are either all equity or all debt). Section (A) of this chapter extends the analysis of the Meade Report into a world where uncertainty and/or debt are present. The main concept used throughout section (A) is that of the "Distortion Ratio" previously described as the Ratio of the "rate of return to the investor" to the "rate of return generated by the underlying physical investment". We assess the impact of the CIT and the ET systems on this Ratio, and state how the conclusions derived in the Meade Report need to be revised in the light of our analysis. In Section (A), we define the "rate of return to an investor" as the "rate of return to an equity holder". In section (B) we derive the Distortion Ratio when we define the "rate of return to investor" to imply the "rate of *total* return to *all* investors (including equity holders as well as debt holders)".

Section (B)

In section (B), we analyse how the "Distortion Ratio" (defined above) behaves when we compare the "rate of *total* return to *all* investors" to the "rate of return generated by the underlying physical investment" when the taxation system in the economy is CIT or the ET system. Out of the 3 different types of Corporate-flow-of-funds bases under ET, the particular tax base which is advocated by the Meade Report as being the ultimate objective for the corporation tax system is the S-base. We show, however, that an S-base corporation tax system does not lead to equality between the "rate of total return to all investors" and the "rate of return on physical investment", when uncertainty and debt are present. This point seems to have not been investigated in the Meade Report.

Section (C)

Therefore, we propose a new tax base, which we call the I-base, in section (C). If the I-base corporation tax system applies, then the "rate of total return to all investors" will equal the "rate of return on physical investment". We also show that this I-base system has certain advantages over the S-base and the R-base corporation tax systems discussed in the Meade Report.

Section (D)

Finally, in Section (D) we analyse the behaviour of the "Ratio" (described above) when the companies undertake projects that generate positive net present values (NPV). In the real world, companies do (hopefully) undertake projects that earn positive NPVs. It is therefore important to check whether the Ratios derived on the basis that companies earn zero NPVs, are also valid in the case of positive NPV projects.

The reason why the Meade Report did not consider positive NPV projects is because it largely assumed that the economy under consideration was in some sort of a long run equilibrium, with investments realising only their cost of capital. However, we consider that it important to consider positive NPV scenarios when recommending a tax system that supposedly will apply under more realistic conditions, such as large periods, sectors of the economy or individual companies which can and do earn returns in excess of their cost of capital.

## SECTION (A)
## MEADE REPORT WITH DEBT AND UNCERTAINTY

In reaching the conclusions in chapter 10, for example the neutrality of the expenditure tax system, it was stressed that these conclusions applied when debt and uncertainty (that is, when the discount rate includes a risk premium) were ignored. Now we look at if and how the conclusions of the Meade Report may be modified with the introduction of debt and uncertainty.

The aim of this section therefore is to seek answers to the following questions: (i) If debt and uncertainty are present, first one by one and then together, will the rate of return to an investor under comprehensive tax system be less than the rate of return on physical investment, and (ii) is the expenditure tax system neutral in the sense that the return to the saver equals the return on physical investment, even when debt and uncertainty are present?

We begin by considering the no-debt no-uncertainty case. Then subsequently, step by step, debt and uncertainty will be added. We consider the rates of return under the two main alternative tax systems (CIT and ET), as we did in chapter 10. The equations in this chapter are numbered beginning with number 11.29, in order to clearly distinguish from the equations in chapter 10, which are referred to very frequently in this chapter.

## (A) (1) COMPREHENSIVE INCOME TAX

### (A)(1)(i) No debt and no uncertainty

In this case, the return that an equity holder earns is defined by modifying equation 10.11 for 100% equity capital structure.

$$y_i^m = n_i^m \left\{ (\mu_i^* - D_i\, r\, \theta_c) \frac{\theta_p^m}{\theta_j} \right\}$$

If Debt is zero, the above modifies to the following equation 11.29, which presents the return to an equity holder owning all of the shares in any all equity company i:

[Eqn 11.29] $y_i^m = \mu_i^* \ \theta_p^m/\theta_j$

In equation 11.29, $\mu_i^*$ is the net of corporation tax rate cash flow from the asset, $\theta_p^m$ is the factor net of personal tax rate for investor m (ie. $\theta_p^m = 1-\tau_p^m$ where $\tau_p^m$ is the personal tax rate of investor m). $\theta_j$ is defined as $(1-j\tau_c)$ where j is the proportion of corporation tax which is imputed as a tax credit in the hands of the shareholder.

If we analyse $\mu_i^*$ into $\mu_i \theta c$ , then equation 11.29 may be restated as:

[Eqn 11.30] $y_i^m = \mu_i\, \theta_p^m\, \theta_c/\theta_j$

Now the owner of all the shares in the company i would have invested $V_i$, the value of the company, which is given by equation 10.20 in chapter 10, reproduced below:

[ 10.20] $$V_i = \frac{\mu^*\, \theta_p/\theta_j}{r\, \theta_p + \pi\, ß_i} + LD_i$$

where subscripts i refer to a company i. With debt ($D_i$) equal to zero and with $\pi ß_i$ ( the risk premium) equal to zero in the absence of uncertainty, the above simplifies to

[11.31] $$V_i = \frac{\mu_i^* \theta_p/\theta_j}{r\theta_p} = \frac{\mu_i \theta_c/\theta_j}{r}$$

Dividing equation 11.30 by equation 11.31 gives the rate of return earned after taxes by the equity holder.

[11.32] $$\frac{\mu_i \theta_p^m \theta_c/\theta_j}{\mu_i \theta_c/\theta_j / r} = r\theta_p^m$$

Thus an investor receives the real rate of interest, r, less the personal taxes applicable to him. Now we need to compare this rate of return with the rate of return on physical investment to evaluate the distortions caused by CIT.

The total investment generates a pre-tax return of $\mu_i$. The cost of this investment, in equilibrium, must be $V_i$. If the physical investment (for example, capital equipment) producing this pre-tax return $\mu_i$ cost more than $V_i$, then it would be unprofitable to use it - its rate of return would be below the cost of capital. If the capital equipment cost less than $V_i$, then it would be offering super-normal profits which would be eliminated by competition in the product market. Hence the cost of investment will be $V_i$ and the rate of return on physical investment is

[11.33] $\mu_i/V_i$

Substituting equation 11.31 for $V_i$ we get

[11.34] Rate of return on physical investment $= \dfrac{\mu_i}{\mu_i\theta_c/\theta_j/r} = \dfrac{r}{\theta_c/\theta_j}$

This rate of return on physical investment is different from the return to equity holders in equation 11.32 - hence under CIT, without debt and uncertainty, there is distortion in the return to investor. It is caused by personal and corporate taxes. This distortion may be expressed by the **Ratio** :

$$\frac{\text{rate of return to equity holder}}{\text{rate of return on physical investment}} = \textbf{DISTORTION RATIO}$$

[11.35] $$= \frac{r\theta_p^m}{r / \theta_c/\theta_j} = \theta_p^m \theta_c/\theta_j$$

equation 11.35 states that the rate of return received an individual investor under CIT is the rate of return on physical investment reduced by personal income tax and by the corporation tax which is not imputed as a tax credit to shareholders (ie. $\theta_c/\theta_j$).

We can calculate the magnitude of this distortion under assumed tax rates. Assuming Corporation tax rate of 35%, rate of imputation of tax credit (=j) of 60%, and marginal personal income tax of 25% for all shareholders (individuals, corporations and pension funds), the Ratio is

$$(1 - .25) \quad \text{x} \quad \frac{(1 - .35)}{(1 - .6 \text{ x } .35)}$$

$$= \quad .75 \quad \text{x} \quad \frac{.65}{.79} = \quad .75 \quad \text{x} \quad .82278 \quad = \quad 0.6171$$

A Ratio of 1 would imply no distortion by taxes - the lower is the Ratio, the greater the distortion. This procedure for calculating the Distortion Ratio will be used frequently in subsequent analysis.

Let us now introduce uncertainty, keeping the no debt assumption for the time being.

**(A)(1)(ii) No debt, uncertainty**

With uncertainty, the value of the company will be lower, given the same cash flows, because the investors will demand a risk premium to compensate for the risk. This value is given by equation 11.36 which is obtained by modifying equation 10.20 for the no debt case:

$$[10.20] \qquad V_i = \frac{\mu^* \theta_p/\theta_j}{r\,\theta_p + \beta_i\,\pi} + LD_i$$

modified for the no debt case:

$$[11.36] \qquad V_i = \frac{\mu^* \theta_p/\theta_j}{r\,\theta_p + \beta_i\,\pi} = \frac{\mu_i\theta_p\,\theta_c/\theta_j}{r\,\theta_p + \beta_i\,\pi}$$

Hence the rate of return that an investor who owns all the equity of the company earns is given by dividing equation 11.30 by equation 11.36:

$$\frac{\mu_i \theta_p^{\,m} \theta_c / \theta_j}{\mu_i \theta_p \theta_c / \theta_j \,/\, r\theta_p + \beta_i \pi}$$

$$[11.37] \quad = \quad (\theta_p^{\,m}/\theta_p)\ (r\theta_p + \beta_i\pi) \quad [= \theta_p^{\,m} r + (\theta_p^{\,m}/\theta_p)(\beta_i\pi)]$$

Similarly the rate of return on physical investment can be calculated by dividing $\mu_i$, the pre-tax return, by the cost of investment given in equation 11.36:

$$\frac{\mu_i}{\mu_i\, \theta_p\, \theta_c / \theta_j \,/\, (r\theta_p + \beta_i\pi)}$$

$$[11.38] \quad = \quad \frac{(r\theta_p + \beta_i\pi)}{\theta_p\, \theta_c / \theta_j} \qquad (= \text{Rate of return on physical investment})$$

This differs from equation 11.37 - therefore there is still distortion due to taxes. This distortion can be calculated again by taking the Ratio of the rate of return to equity investor to the rate of return on physical investment that is, equation 11.37 divided by equation 11.38.

$$\frac{\theta_p^{\,m} (r\theta_p + \beta_i\pi)}{\theta_p} \div \frac{(r\theta_p + \beta_i\pi)}{\theta_p\, \theta_c / \theta_j}$$

$$= \quad \frac{\theta_p^{\,m} (r\theta_p + \beta_i\pi)}{\theta_p} \times \frac{\theta_p\, \theta_c / \theta_j}{(r\theta_p + \beta_i\pi)}$$

$$[11.39] \quad = \quad \theta_p^{\,m}\, \theta_c / \theta_j$$

This is the same Ratio as in the no-uncertainty case (equation 11.35). Hence the distortion under uncertainty is no different from the distortion under certainty as long as there is no debt. If we again used corporation tax rate of 0.35, personal income tax rate of 0.25, and the proportion of imputation allowed of 60% (j=60%) as before, we come to the same Distortion Ratio of 0.6171 .

Two points to note are as follows. Firstly, above we have retained the no debt assumption. Secondly, the analysis does not imply that uncertainty has no effect on valuation or on the rate of return to investor. Uncertainty lowers the values $V_j$s (compare equation 11.36 with equation 11.31) and it increases the rate of return to equity investor (compare equation 11.37 with equation 11.32). Equation 11.37 shows that the investor now earns risk premium, $\beta_i\pi$, as adjusted by personal tax factors $\theta_p^m/\theta_p$. However, the rate of return on physical investment is also similarly increased because changes in $V_i$ also affect the rate of return on physical investment in the same way. As explained above, $V_i$ will be the equilibrium cost of investment in a competitive environment. Hence changing $V_i$ also affects the rate of return on physical investment. In this case of changing from certainty to uncertainty case, the two effects on rates of return (a) to the equity investors and (b) on the physical investment, cancel out, leaving the Ratio of Distortion introduced by taxes unchanged at $\theta_p\ \theta_c/\theta_j$. As this expression is independent of $\beta_i\pi$, there is no impact of uncertainty in this Ratio, that is, using the tax rates of our example, the investor would still earn a rate of return on equity investment which is 0.6171 times the rate of return on physical investment.

This Ratio however changes when debt is introduced in addition to uncertainty.

**(A)(1)(iii) Debt, no uncertainty**

Initially, we will revert back to the no uncertainty case. With the presence of debt, the return that an investor who owns all the equity of a firm gets is given by equation 10.11.

$$[10.11] \qquad y_i^m = n_i^m \left\{ \frac{(\mu_i^* - D_i\, r\, \theta_c)\, \theta_p^m}{\theta_j} \right\}$$

Where $n_i^m$ , the proportion of shares owned by the investor, equals 100%, the above equals $(\mu_i^* - D_i\ \ r\theta_c)\ \theta_p^m/\theta_j$ , which can be simplified as follows :

$$= (\mu_i\, \theta_c - D_i\, r\theta_c)\, \theta_p^m/\theta_j$$

$$[11.40] \qquad = (\mu_i\, \theta_c/\theta_j - D_i\, r\theta_c/\theta_j)\, \theta_p^m$$

The value of equity in a firm with debt is $V_i - D_i$, where $V_i$ is as given by equation 10.20 adjusted for $\beta_i\pi = 0$ (no uncertainty). Therefore the value of equity is

$$[11.40b] \quad V_i - D_i = \frac{\mu_i^*\, \theta_p/\theta_j}{r\theta_p + (\beta_i\pi{=}0)} + LD_i - D_i$$

$$= \frac{\mu_i\, \theta_p\ \theta_c/\theta_j}{r\theta_p} + D_i(L-1)$$

$$[11.41] \qquad = \frac{\mu_i\, \theta_c/\theta_j + rD_i\, (L-1)}{r}$$

Now L, the benefit of leverage, under CIT is defined as $1-\theta_c/\theta_j$ in chapter 10. Substituting in equation 11.41,

$$V_i - D_i = S_i = \frac{\mu_i\,\theta_c/\theta_j + rD_i\,(1 - \theta_c/\theta_j - 1)}{r}$$

$$[11.42] \qquad = \frac{\mu_i\,\theta_c/\theta_j - rD_i\,\theta_c/\theta_j}{r} = \frac{(\mu_i - rD_i)\,\theta_c/\theta_j}{r}$$

The rate of return to an equity investor therefore is given by dividing equation 11.40 by equation 11.42:

$$\text{Rate of return to equity} = \frac{(\mu_i\,\theta_c/\theta_j - D_i\,r\,\theta_c/\theta_j)\,\theta_p^{\,m}}{(\mu_i - rD_i\,)\,\theta_c/\theta_j\,/\,r}$$

$$= \frac{(\mu_i - D_i\,r)\;\;\theta_c/\theta_j\,\theta_p^{\,m}}{(\mu_i - D_i\,r)\;\;\theta_c/\theta_j} \quad x \quad r$$

$$[11.43] \qquad = r\theta_p^{\,m}$$

This is exactly the same as the rate of return an investor would receive without debt and uncertainty. The reason for this similarity is that in the absence of uncertainty, the return to equity remains the same regardless of the debt level - equity return is determined by the real rate of interest adjusted for personal tax factors, regardless of the level of debt. Since there is no risk, equity return does not include a premium which varies with debt level. Hence the rate of return to an equity investor is the same as the rate of return without debt and uncertainty.

How about the rate of return on physical investment? The concept of the rate of return on physical investment has been defined in the discussion on equations 11.33 and 11.34. Even in the presence of debt, the pre tax return on physical investment is $\mu_i$. The total cost of investment is $V_i$, which per equation 10.20 (already used on the previous page) is

$$[11.44] \qquad V_i = \frac{\mu_i^{*}\,\theta_p/\theta_j}{r\theta_p + (\beta_i\pi{=}0)} + LD_i$$

Therefore the return on physical investment $= \mu_i/V_i$

$$= \frac{\mu_i}{[\mu_i\,\theta_c\,\theta_p/\theta_j + LD_i\,r\theta_p\,]\,/\,r\theta_p}$$

$$[11.45] \quad = \quad \frac{\mu_i . r . \theta_p}{\mu_i \, \theta_p \, [\theta_c/\theta_j + \frac{rLD_i}{\mu_i}]} \quad = \quad \frac{r}{\theta_c/\theta_j + \frac{rLD_i}{\mu_i}}$$

By comparing equation 11.45 with equation 11.34, we find that the rate of return on physical investment has decreased - this follows from the fact that $V_i$, the value or cost of investment would be higher for levered companies in comparison with unlevered companies. In our analysis, the assumption is that the cost of the investment is equal to the value generated by the resulting cash flows. (One scenario under which this assumption would hold is if all the companies in an industry were levered to the same extent - which is not unrealistic.) Thus, for levered companies, the cost of investment $V_i$ is given by equation 11.44 - resulting in the comparatively lower rate of return on physical investment given in equation 11.45.

Since the rate of return on physical investment is lower in the presence of debt, we would expect the Distortion Ratio (of the rate of return to saver over rate of return to physical investment) to be higher. Dividing equation 11.43 by equation 11.45 we get

$$\frac{r\theta_p^m \times (\theta_c/\theta_j + L\, rD_i/\mu_i)}{r}$$

$$[11.45a] \quad = \quad \theta_p^m (\theta_c/\theta_j + L\, rD_i/\mu_i)$$

For any positive level of debt, this Distortion Ratio is greater than $\theta_p^m \, \theta_c/\theta_j$ of the no-debt case. Using the tax figures considered earlier and assuming further that r=5%, $\mu_i$=50 and market value of debt $D_i$ = 10,

the Ratio is $.75( .82278 + (1-.82278) \quad \frac{.05 \times 100}{50} )$

$$= \quad .75\,(.82278 + 0.017722) \quad = \quad .75 \times .84050$$

$$= \quad 0.6304$$

This shows a marginal improvement over the 0.6171 obtained earlier. Hence increasing debt can reduce the tax distortions marginally as far as the return on equity is concerned. This effect is accentuated when both debt and uncertainty are present, as shown below.

**(A)(1)(iv) Debt and uncertainty**

We have concluded that uncertainty without debt had no impact on the Distortion Ratio.

Debt without uncertainty reduces the distortion by bringing the Distortion Ratio closer to one. Uncertainty in the presence of debt accentuates the impact of debt on the Ratio. Thus the Ratio will increase more rapidly, as is shown below. The return that an equity investor (who owns all of the equity) earns in the presence of debt and uncertainty is given by equation 10.11

$$[10.11] \qquad y_i^m = n_i^m \left\{ \frac{(\mu_i^* - D_i\, r\, \theta_c)\, \theta_p^m}{\theta_j} \right\}$$

Modifying for the $n_i^m = 100\%$ case and substituting $\mu_i\theta_c$ for $\mu_i^*$, we get

$$[\text{Eqn } 11.46] \quad \text{Return to equity investor} = (\mu_i - D_i r)\, \theta_p^m\, \theta_c/\theta_j$$

The cost of equity investment is given by $S_i = V_i - D_i$ where $V_i$ is given by equation 10.20

$$[11.47] \qquad S_i = V_i - D_i = \frac{\mu_i\theta_p\, \theta_c/\theta_j}{r\theta_p + \pi\beta_i} + LD_i$$

The rate of return to equity investor can then be calculated by dividing equation 11.46 by equation 11.47. The rate of return on physical investment can again be calculated by dividing the pre-tax return $\mu_i$ by the cost of investment (which, due to the assumption of competition in the goods market, is equal to $V_i$). Hence the rate is

$$[11.48] \qquad \text{Rate of return on physical investment} = \frac{\mu_i}{\dfrac{\mu_i\, \theta_p\, \theta_c/\theta_j}{r\theta_p + \pi\beta_i} + LD_i}$$

The Distortion Ratio can again be calculated by dividing the rate of return to the equity investor by equation 11.48. This Ratio will show the distortion caused by taxes under CIT to the return that an equity investor can earn. The Ratio can simply be expressed by using $S_i$ to represent equation 11.47 and $V_i$ to represent the denominator in equation 11.48.

$$\text{Therefore Ratio} = \frac{(\mu_i - D_i\, r)\, \theta_p^m\, \theta_c/\theta_j}{S_i} \div \frac{\mu_i}{V_i}$$

$$[11.49] \qquad = \frac{(\mu_i - D_i\, r)\, \theta_p^m\, \theta_c/\theta_j}{S_i} \times \frac{V_i}{\mu_i}$$

Equation 11.49 can be modified somewhat for ease of comparison with the no debt case, to

[11.50] $$\theta_p^m \theta_c/\theta_j \quad x \quad \frac{\mu_i - D_i r}{\mu_i} \quad x \quad \frac{V_i}{S_i} \qquad \text{Note: } \frac{V_i}{S_i} = \frac{S_i}{S_i} + \frac{D_i}{S_i}$$

In this equation the first term $\theta_p^m \theta_c/\theta_j$ is the Ratio in the absence of debt. The presence of debt modifies this Ratio by multiplying it by (a) the proportion of pre-tax cash flow ($\mu_i$) which is available to equity holders ($\mu_i$-$D_i r$), ignoring tax factors $\theta_p^m$ and the factor $\theta_c/\theta_j$ already separated in the first term, and (b) $V_i/S_i$,that is, the value of the firm divided by the value of equity.

When uncertainty increases, it does not affect the first two terms $\theta_p^m \theta_c/\theta_j$ or $\mu_i$-$D_i r$ / $\mu_i$ since they relate to tax parameters or expected cash flows. However, in the third term, increasing uncertainty will <u>decrease</u> the value of equity $S_i$, relative to the value of $V_i$ (as stated in the note above, $V_i/S_i$ can alternatively be written as $S_i/S_i + D_i/S_i$). Hence increasing uncertainty will increase the third term $V_i/S_i$. This will result in an increase in the entire Ratio. This can be seen with the help of an example.

Let us continue with an investment which yields pre-tax cash flow of £50 ($=\mu_i$), the real rate of interest is 5% (=r), $\theta_p$, the net of <u>average</u> personal tax rate of all shareholders, is 0.80 ( that is, the average personal tax is 0.20), $\theta_c$ equals 0.65, $\theta_j$ equals 0.79, debt $D_i$ is £100 and that the market risk premium is 8%, and the shares have ß = 1. Given this, the value of the company is

$$V_i = \frac{£50 \times 0.80 \times 0.65/0.79}{0.05 \times 0.80 + 0.08 \times 1} + L \times 100$$

$$= \frac{32.9112}{.12} + 0.17722 \times 100$$

$$= £274.26 + £17.722$$

$$= £291.982$$

Therefore $S_i$ = £291.982 - 100 = £191.982.

Using these values of $V_i$ and $S_i$ in the Ratio, we calculate the Distortion Ratio :

$$\theta_p^m \quad x \quad \frac{\theta_c}{\theta_j} \quad x \quad \frac{\mu_i - D_i r}{\mu_i} \quad x \quad \frac{V_i}{S_i}$$

$$= \quad 0.75 \quad \times \quad \frac{.65}{.79} \quad \times \quad \frac{50 - 100 \times 0.5}{50} \times \quad \frac{291.982}{191.982}$$

$$= \quad 0.75 \quad \times \quad .82278 \times \quad \frac{45}{50} \quad \times \quad 1.52088$$

$$(11.50a) \qquad = \qquad 0.8447$$

Hence, under CIT, with the data assumed above, the equity investors earn a rate of return which is 84% of the rate of return on physical investment. This is less than 100%, but nevertheless is more than the 63% which was the Distortion Ratio obtained in the absence of uncertainty, or 61.7%, which was the Ratio in the absence of debt and uncertainty. However, it should be stated once again that uncertainty merely accentuates the effect of the presence of debt. If we have uncertainty without debt, then as we proved earlier, the Ratio remains at 61.7%. It is only when we introduce debt that this rises to 63%. Uncertainty accentuates this rise (in this example to 0.84). The implications of this for the conclusions reached in the Meade Report will be stated after we seek an answer to the question - can this Ratio ever be equal or be greater than 1 under a CIT system?

For example, if the level of debt is changed to £150, the value of the firm now is

$$V_i \quad = \quad \frac{£50 \times 0.80 \times .65/.79}{.05 \times 0.80 + 0.08 \times 1} + (0.17722 \times £150)$$

$$= \quad £274.26 + £26.583 \quad = \quad £300.843$$

The value of equity now is £300.843 - £150 = £150.843 = $S_i$. Hence now the debt equity ratio is 1:1. With this level of debt, the Ratio of the rate of return to the investor compared to physical-investment-return is

$$\theta_p^m V_i \times \frac{\theta_c}{\theta_j} \times \frac{\mu_i - D_i r}{\mu_i} \times \frac{V_i}{S_i}$$

$$= \quad .75 \quad \times \quad \frac{.65}{.79} \quad \times \quad \frac{50 - .05 \times 150}{50} \quad \times \quad \frac{300.843}{150.843}$$

$$= \quad .75 \quad \times \quad .82278 \times \quad 42.5/50 \quad \times \quad 1.9944$$

$$= \quad 1.0461$$

Here the Ratio is greater than one. Therefore it is possible even under CIT for the rate of return to equity investor to exceed the rate of return on physical investment. The presence of corporate and personal taxes has not caused the return to equity holder to be lower.

The reason is that equity holders, by undertaking financial risk, can increase the expected return to equity. The more the uncertainty and/or the more the debt (with uncertainty), the greater is the financial risk borne by equity holders. As a compensation for bearing this risk, the return to equity increases. As shown in the above example, this can increase sufficiently to offset the adverse impact of corporate and personal taxes. Hence it is possible for the remaining equity investors to earn a rate of return greater than the return on physical investment even, under CIT. How does this conclusion compare with Meade Report's conclusions?

The Meade Report states that both the personal and the corporation taxes (unless fully imputed) will, under a CIT system, cause the rate of return to investors to fall below the return earned by investment. In fact this is one of the Report's two main conclusions - the other main conclusion being that under expenditure tax system, taxes do not introduce any distortion. These conclusions were illustrated by a number of examples in Meade Report - examples that considered debt finance as well as equity finance. However, Meade Report did not consider mixed capital structures - they either considered all equity companies or considered cases where 100% of cash flows earned were paid to service the debt finance. Moreover, Meade Report did not consider the impact of uncertainty. Therefore, the conclusions of Meade Report are valid only in the limited context of their specific assumptions, that is, within a certain world with simple capital structures (100% equity or 100% debt). In relation to complex structures, Meade Report only notes that there will be differences but does not consider the question any further. Its references to mixed capital structures are limited to what is stated on page 66 of the Meade Report:

"We now assume that the company is paying a rate of interest on its debt which just absorbs the available profit on the investment, so that there is no net profit subject to corporation tax. (To the extent that higher or lower rates of interest are paid, the difference would affect the rate of return to shareholders on their own investment)."

Reference to mixed capital structures in the Meade report is limited to the above. However, in reality, the vast majority of the companies have mixed capital structures. Therefore investors who purchase shares on a Stock Exchange are investing in levered companies. If the level of leverage is sufficiently high or, as will be shown later, if the average ß of the portfolio is sufficiently high, then the conclusions of the Meade Report are inapplicable to those investors. Hence Meade Report might have been more relevant if it had considered debt and uncertainty together, as considered in this section.

Of course, if the interest rate paid to debt holders is below the rate of return earned by the investment, then again the rate of return earned by an investor (in this case a debt holder) will differ from the rate of return on physical investment, quite irrespective of the tax factors. For debt-holders in the real world, this criterion ( that is, whether they earn interest at a rate below that yielded by the investment) is not a valid criterion because even in the absence of <u>all</u> taxes, the rate of return to debt holders is below the rate of return earned by the investments they help finance (with uncertainty). This is so because the returns to debt-holders are certain (within limits) whereas the returns on investments they help finance are uncertain. Hence the equity return on investments has to be sufficiently high to compensate the equity investors for uncertainty and risk, and this rate of return will therefore naturally be higher than that paid out to fixed-interest debt-holders.

Therefore, it seems that the main criteria that the Meade Report has used as its objective in distinguishing between the alternative tax systems, is not always a useful one, even ignoring tax effects. The Meade Report emphasises the equality between rates of return to investors with rates of returns earned by the investments (page 37). It states,

"It is indeed the characteristic feature of an expenditure tax as contrasted with an income tax that, at any given constant rate of tax, the former will make the rate of return to the saver on his reduced consumption equal to the rate of return which can be earned on the investment which his savings finances, whereas the income tax will <u>reduce</u> the rate of return to the saver below the rate of return which the investment will yield.

In later chapters of this Report we shall frequently use this phenomenon as a base criterion. We shall treat a tax regime as being equivalent to an expenditure tax if, at a constant rate of tax, it leaves the yield to the saver equal to the yield on the investment. Although we shall use this as our test, there is no implication that this is the sole reason why an expenditure tax regime may be preferred to an income tax regime; there are many other considerations discussed in (that) chapter and in (chapter 6).

The equality between the rate of yield to saver and the yield on the investment is, however, more than just a test of the existence of an expenditure tax regime, since it does mean that there is no marginal tax incentive inducing taxpayers to substitute at the margin present consumption for future consumption simply because the future yield on savings has been reduced below the true yield."

This quote above indicates the importance of equality of return to investor and the return earned by physical investment to the Meade Report. However, as has been shown

(i) the equity return to a saver, given the presence of debt and uncertainty, need not be below the rate of return earned by the physical investment even under an income tax regime

(ii) the rate of return to a debt holder will differ from the rate of return earned on the physical investment, in the presence of uncertainty

and it will be shown below that

(iii) in the presence of debt and uncertainty, even under an expenditure tax system, the rate of return to an equity saver need not be equal to the rate of return earned by the investment.

Hence Meade Report's basic criteria does not appear to be sound. How can the conclusions of Meade Report be held valid, given the above criticisms? There are two possible solutions:

(a) The Meade Report's objective could be reinterpreted to mean that the aim is not to achieve the equality of return to a saver and return on physical investment *per se*, but rather that the aim is to simply minimise the distortions to the rates of returns caused by personal and corporation taxes. With this basic objective, most of Meade Report's conclusions would still be valid - although of course it does not give the neat result that only (and always) with the expenditure tax system, the return to a saver equals the return on investment.

(b) Meade Report's conclusions may be <u>largely</u> valid if one does not look at equity investors and debt investors separately, but one considers the total return earned by all savers who have invested in a company, and compares it with the rate of return on physical investment. In order to examine this view, total returns on assets, in the presence of debt and uncertainty, are examined in section (B) of this chapter, after we have examined the Distortion Ratio under different levels of debt and uncertainty, and the Distortion Ratio under the Expenditure Tax system.

In this sub-section it has been stressed that the reason why uncertainty has an impact is because it accentuates the impact of leverage - that is, under an income tax regime, the equity holders earn a higher return mainly because they undertake financial ( that is, leverage) risk . This can be seen by examining the expression for the Ratio once again:

[11.51] Distortion Ratio $= \theta_p \times \dfrac{\theta_c}{\theta_j} \times \dfrac{\mu_i - rD_i}{\mu_i} \times \left[1 + \dfrac{D_i}{S_i}\right] =$ say, E.

Let this expression for the Distortion Ratio be E. In order to see how E is affected by uncertainty, we differentiate it with respect to $\pi\beta_i$ (which represents the risk premium arising because of uncertainty). Therefore

$$\frac{\delta E}{\delta(\pi\beta_i)} = \frac{\delta E}{\delta S_i} \times \frac{\delta S_i}{\delta(\pi\beta_i)} \quad \text{(since } \pi\beta_i \text{ influences E only through } S_i\text{.)}$$

$$= \frac{\delta\left(\theta_p^m . \frac{\theta_c}{\theta_j} \times \frac{\mu_i - rD_i}{\mu_i}\right)\left(1 + \frac{D_i}{S_i}\right)}{\delta S} \times \frac{\delta\left(\frac{\mu_i\theta_p\theta_c/\theta_j}{r\theta_p + \beta_i\pi} + (1-L)D_i\right)}{\delta\beta_i\pi}$$

$$= \frac{-D_i\left(\theta_p^m \frac{\theta_c}{\theta_j} \frac{\mu_i - rD_i}{\mu_i}\right)}{S_i^2} \times - \frac{\mu_i\theta_p\frac{\theta_c}{\theta_j}}{(r\theta_p + \beta_i\pi)^2}$$

[11.52]

$$= \frac{D_i \left(\theta_p^{m} \dfrac{\theta_c}{\theta_j} \dfrac{\mu_i - rD_i}{\mu_i}\right)}{S_i^2} \quad x \quad \frac{\mu_i \, \theta_p \, \dfrac{\theta_c}{\theta_j}}{(r\theta_p + \beta_i \pi)^2}$$

In the above partial derivative, if the level of debt is zero,that is, $D_i = 0$, then the entire derivative is zero and therefore risk premium has no influence on the Ratio expression E. Hence the above clearly proves that uncertainty has an impact on the Ratio only through the presence of debt, or leverage.

The second point to note from the above derivative (11.52) is that if $D_i$ is positive, since all other parameters and expected cash flow $\mu_i$ are positive, the numerator in equation 11.52 will be positive. Since the denominator has squared expressions only, it will be positive. Hence the entire derivative will be positive - implying that if $\pi\beta_i$ increases, then the Ratio expression will also increase. Hence if one is considering investments with greater "business risk", that is, with higher beta factors, then one can improve the Ratio still further. To continue with the numerical example used above, with the debt level again at £100 but the beta factor is now increased to 2, the value of company $V_i$ and equity $S_i$ are as shown below:

$$V_i = \frac{\mu_i \text{ x } \theta_p \text{ x } \theta_c/\theta_j}{r\theta_p + \pi\beta_i} + LD_i$$

$$= \frac{50 \text{ x } 0.8 \text{ x } 0.82278}{0.05 \text{ x } 0.8 + 0.08 \text{ x } 2} + (0.17722 \text{ x } £100)$$

[11.53] $$= \frac{32.9112}{0.04 + 0.16} + 17.722$$

$$= \frac{32.9112}{0.20} + 17.722$$

$$= 164.556 + 17.722 = £182.278$$

Therefore $S_i = V_i - D_i = £182.278 - £100 = £82.278$.

Now we can calculate the Distortion Ratio as

$$\theta_p^m \cdot \frac{\theta_c}{\theta_j} \cdot \frac{\mu_i - rD_i}{\mu_i} \cdot \frac{V_i}{S_i}$$

$$= 0.75 \times 0.82278 \times 0.9 \times \frac{182.278}{82.278}$$

$$= 0.55538 \times 2.21539$$

$$= 1.2304$$

This shows that where a high asset beta company is also levered, then the shareholders of that company, even under a comprehensive income tax system, earn a rate of return in excess of the rate of return earned by the physical investment. Meade Report states that such a result can only come about if there are double reliefs being given against the same tax, for example 100% first year allowances against corporation tax on investment and tax deductibility of interest payments against corporation tax on loans used to finance that investment. However, we have shown above that the rate of return earned by equity investors can exceed the rate of return on physical investment even in the absence of two reliefs being given against the same tax. What we have shown is that high degree of business risk, but only when combined with financial risk resulting from leverage, can push the return to the remaining equity investors to levels not considered possible in the Meade Report.

The reason why business risk by itself does not cause the Ratio to change is that increasing business risk increases both the rate of return to equity investor as well as the rate of return on physical investment. Since both these rates of return are affected proportionately, their Ratio does not change, as was shown previously. Therefore it is financial risk that is the more relevant risk.

Another point to note is that since increasing business risk seems to increase the return to equity holders as well as the return on physical investment, it appears as though one should keep on increasing risk ad infinitum and be better off. This is not true. When we say that increasing business risk increases the rate of return on equity what we mean precisely is that because of greater business risk, equity investors will undertake the investment only if the expected return is high enough to compensate them for that increased risk. Hence higher expected return is simply a compensation for more risk. Similarly, increasing risk does not "improve" the rate of return on physical investment. What increasing risk implies is that because of greater risk, the value (that is, cost in the perfectly competitive market) of physical investment (for example, in capital equipment) must be sufficiently <u>low</u> to offer the investors an adequate rate of return on this capital equipment to compensate them for risk. It is only in this sense - that the value (that is, the cost) of capital equipment is lower whenever greater uncertainty is attached to the cash flows that it generates - that the rate of return on physical investment is greater.

The previous example shows that increasing the business risk in the presence of leverage can result in a Distortion Ratio which is greater than 1, but note the magnitudes of the components of the discount rate shown in the denominator (0.04 and 0.16) in equation 11.53. 0.04 is the real rate of interest after deduction of the weighted average personal income tax rate. The real interest rate for this century has averaged below 2%. Hence a rate of 4% after personal taxes is likely to be high - it is very unlikely to be a low estimate. The risk premium of 16% is based on an asset beta of 2 and market risk premium of 8%. In the U.K. market risk premium of 8% is considered reasonable (Brealey (1981); J Rutterford (1985)). An asset beta of 2 is of course high but then we intend to consider a high business risk company. Hence an equity investor is assumed to demand 4% as compensation for real interest rate foregone and 16% as risk premium - the risk premium is 4 times the required real interest rate. Even if asset beta is taken to be 1, the risk premium of 8% is twice the real interest rate of 4%, and as discussed above, the real interest rate is not understated if we take a figure of 4%. The importance of comparing risk premium with the real interest rate component lies in that if the risk premium is significant in comparison with the real interest rate, then uncertainty is fairly important in the valuation and the case discussed in this section, of debt and uncertainty, is therefore fairly important. On the other hand, if risk premium is very small compared to the real interest rate, then uncertainty is relatively unimportant and this subsection is of little relevance - it would merely point to a possible flaw in the comprehensiveness of the Meade Committee's Report but the magnitude of the flaw would be irrelevant. Therefore if risk premium is relatively small, then the analysis under certainty assumption would be the relevant one. The above comparison, however, shows that risk premium is the dominant component of the return to the equity investors.

We wish to examine how important are the conclusions drawn so far in this section. In order to do so, we summarise the factors that influence how significant is the omission of the combined debt and uncertainty case in Meade Report:

(a) Does the Meade Report emphasise the equality of the rate of return to saver with the return earned by an investment? The answer to this factor is a resounding yes because the Meade Report places great emphasis on this. Hence the omission of combined debt and equity case could be important.

(b) How widespread are mixed financial structures? An overwhelming majority of corporate as well as non-corporate businesses have both debt and equity in their capital structure - hence the points made in this subsection are applicable to a large number of companies. Hence the omission in the Meade Report means that the capital structure of a large proportion of the companies in the real world is ignored.

(c) How large is the magnitude of premium for uncertainty compared to certainty case? As shown above, based on current literature in the U.K. the magnitude of risk premium is significant - therefore uncertainty cannot be ignored as being of little consequence.

(d) How drastically are the conclusions changed if mixed capital structures are considered? Taking debt and uncertainty together into account, the Meade Report <u>cannot</u> substantiate a number of its points, namely

(i) that under CIT the rate of return to the investors is less than the return on investment,

(ii) that the only way investors may earn more than the return on investment is if two forms of relief are given against the same tax,

(iii) that under CIT, the only reason why debt holders may earn a return below the

rate of return earned by the investment is due to taxes,
(iv) that an expenditure tax system properly implemented would always ensure that the return to an equity saver equals return on investment, and
(v) that if investment is financed by debt, then under the expenditure system, the debt holders earn a return equal to the return on investment.

Each and every one of these five claims above is incorrect ((i) - (iii) as shown in this subsection and (iv) and (v) will be shown when expenditure tax is discussed below). Hence it is argued that the omission of debt and uncertainty case may be a significant omission in the Meade Report.

If, however, Meade Report's conclusions are restated to simply mean that expenditure tax causes less distortion than CIT, then this weaker conclusion will nonetheless be valid.

Finally, in this sub section, we prove that the Distortion Ratio calculated in the case of combined debt and uncertainty (in this sub section) is consistent with the Ratio formulae calculated in the earlier subsections of this section, provided appropriate assumptions are made.

For example, if the level of debt is reduced to zero, then the Distortion Ratio formula $\theta_p^m$ . $\theta_c/\theta_j$ . $\mu_i$-r$D_i/\mu_i$ . $V_i/S_i$ reduces to $\theta_p^m$ . $\theta_c/\theta_j$ . $\mu_i/\mu_i$ . $V_i/V_i$ (because $S_i$ = $V_i$). This simplifies further to $\theta_p^m$ . $\theta_c/\theta_j$. This is the same as the Ratio calculated in equation 11.39 where we started off with the no debt assumption right from the beginning. Hence our derivation of the Distortion Ratio in the two subsections is consistent and there are no obvious errors in the derivation.

Secondly, let us investigate the consistency of the formulae for the case when there is no uncertainty but there is debt. In this case, the formulae can be transformed as follows:

$$E = \theta_p^m \cdot \frac{\theta_c}{\theta_j} \cdot \frac{\mu_i - rD_i}{\mu_i} \cdot \frac{V_i}{S_i}$$

$$= \quad \theta_p^{\,m} \cdot \frac{\theta_c}{\theta_j} \cdot \frac{\mu_i\text{-}rD_i}{\mu_i} \cdot \frac{[\mu_i\theta_p\,\theta_c/\theta_j + LD_i\,(r\theta_p)]}{[\quad r\theta_p \quad]} \qquad [..] = V_i$$

$$x \quad \frac{[\quad r\theta_p \quad]}{[\mu_i\theta_p\,\theta_c/\theta_j + (L\text{-}1)\,D_i\,(r\theta_p\,)]} \qquad [..] = \text{reciprocal of } S_i$$

$$= \quad \theta_p^{\,m} \cdot \frac{\theta_c}{\theta_j} \cdot \frac{\mu_i\text{-}rD_i}{\mu_i} \cdot \frac{\mu_i\,\theta_c/\theta_j + LD_ir}{r} \cdot \frac{r}{\mu_i\,\theta_c/\theta_j + (L\text{-}1)\,D_ir}$$

$$= \quad \theta_p^{\,m} \cdot \frac{\theta_c}{\theta_j} \cdot \frac{\mu_i\text{-}rD_i}{\mu_i} \cdot (\mu_i\,\theta_c/\theta_j + LD_ir) \cdot \frac{1}{\mu_i\,\theta_c/\theta_j - \theta_c/\theta_j\,D_ir}$$

(since $L\text{-}1 = 1\text{-}\theta_c/\theta_j\,\text{-}1$)

$$= \quad \theta_p^{\,m} \; x \; \frac{\theta_c}{\theta_j} \; x \; \frac{\mu_i\text{-}rD_i}{\mu_i} \; x \; \frac{\mu_i\,\theta_c/\theta_j + LD_ir}{\theta_c/\theta_j\,(\mu_i\text{-}D_ir)}$$

$$= \quad \theta_p^{\,m} \; x \; \frac{(\mu_i\,\theta_c/\theta_j + LD_ir)}{\mu_i}$$

$$= \quad \theta_p^{\,m} \; x \; \frac{[\mu_i\,\theta_c/\theta_j}{\mu_i} + \frac{LD_ir}{\mu_i} \; ]$$

[11.54] $$\theta_p^{\,m} \quad x \quad [\; \frac{\theta_c}{\theta_j} + \frac{LD_ir}{\mu_i} \;]$$

This is the same as equation 11.45a which we calculated having commenced with the assumption of debt without uncertainty. Hence the Distortion Ratio formula used in this subsection is consistent with the formulae in the earlier subsections.

To summarise, in this sub section, we considered how the presence of combined debt and uncertainty enables us to refute some of the conclusions reached in the Meade Report in the case of CIT.

We summarise the results reached so far in this entire section which examined the CIT case. In the same table, we add on our results from incorporating debt and uncertainty under the ET system, so that our results for the two systems can be compared easily. We discuss the derivation of our results under the ET system after this table.

In the following table, we state the values of the variables that we use to calculate the Ratios. These values, although realistic as per the U.K. tax code, have been used merely to demonstrate the relative magnitude of the Distortion Ratio under alternative scenarios. The direction of the increases or decreases in the Ratio that are illustrated do not depend upon the values that we assume.

The table is divided into two parts. in the first part, we illustrate the Ratios we derive in this section, that is under C.I.T. system. The first column lists the alternative assumptions regarding debt and uncertainty that we made. The second column lists the expressions for Ratio that we derived under those assumptions. The third column shows the values of the expressions using the data we assumed above. It shows that in the no debt no uncertainty case, the Ratio is considerably below 1, as claimed in the Meade Report. Adding uncertainty without debt does not change the Ratio at all. Adding debt without uncertainty results in the Ratio increasing marginally. However, adding debt and uncertainty together results in the Ratio increasing to 0.8447. What is interesting is that if we increase the debt level from the assumed level of £100 to £200, the Ratio increases to 1.0461, thereby appearing to contradict the assertion in the Meade Report (using our definition of investors) that the Ratio under C.I.T. is below 1. If we keep debt level at the assumed level of £100, and instead change the beta factor from 1 to 2 (that is, we consider a high risk company), the Ratio increases to 1.2304 , that is, it is considerably over 1. This reinforces the assertion made in this thesis, namely that Meade Committee should have considered debt and uncertainty together using our assumptions in their model if they wished to use Distortion Ratio as the criterion to distinguish the desirability of the alternative tax systems.

The second part of the table shows the Ratios obtained under the E.T. system considered in the next sub section of this chapter. The conclusions of that sub section can be summarised by stating that as long as we do not consider debt and uncertainty together, the Ratio is 1, as claimed in the Meade Report. With debt and uncertainty, the Ratio exceeds 1, and the Ratio varies directly with the level of debt or risk. These latter Ratios could not be derived using the simple models considered by the Meade Committee.

**DISTORTION RATIO OF**
**RATE OF RETURN TO EQUITY SHAREHOLDER**
**TO RATE OF RETURN ON PHYSICAL INVESTMENT**

**Based on**

| | | | |
|---|---|---|---|
| $\theta_p^m$ | = 0.75 | $\theta_p$ | = 0.80 |
| $\tau_c$ | = 0.35 | j | = 60% |
| $\mu_i$ | = 50 | r | = 5% |
| $\pi$ | = 8% | ß | = 1 |
| $D_i$ | = £100 | $\tau_{ci}$ | = 0.17722 |

| | | Expression | Ratio |
|---|---|---|---|
| **(1)** | **COMPREHENSIVE INCOME TAX** | | |
| (i) | No debt, no uncertainty | $\theta_p^m \theta_c/\theta_j$ | 0.6171 |
| (ii) | No debt, uncertainty | $\theta_p^m \theta_c/\theta_j$ | 0.6171 |
| (iii) | Debt, no uncertainty | $\theta_p^m [\theta_c/\theta_j + L\, rD_i/\mu_i]$ | 0.6304 |
| (iv) | Debt and uncertainty | $\theta_p^m \cdot \theta_c/\theta_j \cdot \mu_i\text{-}rD_i/\mu_i \cdot V_i/S_i$ | 0.8447 |
| | with debt = £200; ß = 1 | $\theta_p^m \cdot \theta_c/\theta_j \cdot \mu_i\text{-}rD_i/\mu_i \cdot V_i/S_i$ | 1.0461 |
| | with debt = £100; ß = 2 | $\theta_p^m \cdot \theta_c/\theta_j \cdot \mu_i\text{-}rD_i/\mu_i \cdot V_i/S_i$ | 1.2304 |
| **(2)** | **EXPENDITURE TAX** | | |
| (i) | No debt, no uncertainty | 1.0000 | 1.0000 |
| (ii) | No debt, uncertainty | 1.0000 | 1.0000 |
| (iii) | Debt, no uncertainty | 1.0000 | 1.0000 |
| (iv) | Debt and uncertainty | $\dfrac{1}{1 - \dfrac{D_i\, ß_i \pi}{\theta_\epsilon [\mu_i\text{-}rD_i]}}$ | 1.2857 |
| | with debt = £200; ß = 1 | ditto | 2.0000 |
| | with debt = £100; ß = 2 | ditto | 1.8000 |

## Section (A)(2) EXPENDITURE TAX SYSTEM

### (A)(2)(i) No debt, no uncertainty

We need to be very precise about the terms we use when describing the expenditure tax system because there is a difference between the market value of the company and the amount of consumption given up by the saver to finance that investment, which results from the structure of expenditure taxes. Therefore it will be helpful to review the expenditure tax rates of return briefly, aiming for greater clarity in discussing the rates of return than was done in chapter 10.

Let $S_i$ represent the market value of shares issued. (Note, this represents a change from terminology used in equation 11.32 where $V_i$ [$=S_i$ since it was assumed that there was no debt] referred to total cash inflow from issuing shares.) Under expenditure tax system, in order to purchase $S_i$ of shares, an investor need forego only $\theta_\epsilon{}^m S_i$ of consumption, since purchase of shares is an "allowable" expense under the expenditure tax system. From this investment, the investor gets a return given by equation 10.27 , which when modified for no-debt and $n_i^m = 100\%$ ownership, is

[11.55] $$y_i^m = \mu_i \theta_{ci} \theta_\epsilon{}^m$$

Therefore rate of return to equity holder $= \dfrac{\mu_i \theta_{ci} \theta_\epsilon{}^m}{\theta_\epsilon{}^m S_i}$

[11.56] $$= \frac{\mu_i \theta_{ci}}{S_i}$$

We wish to compare this with the return on physical investment to judge whether there is any distortion caused by taxes. Now the cost of investment that can be financed by issuing $S_i$ of shares is

[11.57] $$S_i + \text{corporation tax saved by issuing } S_i \text{ shares}$$

Under the expenditure tax system, if the rate of corporation tax on a tax-<u>exclusive</u> basis is $\tau_{cE}$, then equation 11.57 is $S_i + \tau_{cE} S_i \Rightarrow S_i (1+\tau_{cE})$

$$[11.58] \qquad \Rightarrow \qquad S_i \,.\, \frac{(1+\tau_{cE})}{1}$$

Equation 10.22 (chapter 10), showed that $\frac{1}{1+t_{cE}} = \theta_{ci}$ where $\theta_{ci}$ is $1-\tau_{ci}$ and $\tau_{ci}$ is the corporation tax rate but calculated on a tax-<u>inclusive</u> basis. Therefore equation 11.58

$$= S_i \,.\, \frac{1}{\theta_{ci}}$$

$$[11.59] \qquad = \qquad \frac{S_i}{\theta_{ci}}$$

Equation 11.59 gives the cost of investment that can be financed by issuing $S_i$ of shares - it is essentially the value of shares issued grossed up to reflect the corporation tax saved by issuing shares. This $S_i/\theta_{ci}$ amount of physical investment yields $\mu_i$ before taxes - hence

the rate of return on physical investment is $\mu_i \div S_i/\theta_{ci} = \mu_i \times \theta_{ci}/S_i$

$$[11.60] \qquad = \qquad \mu_i\theta_{ci}/S_i$$

Equation 11.60 is the same as equation 11.56 which showed the rate of return to equity investor in the absence of debt and uncertainty. Hence the Ratio of the rate of return to equity over the rate of return on physical investment is one. This is the same as the conclusion reached in chapter 10, but using different terminology and using the procedure developed in this section to evaluate the Distortion Ratio.

**(A)(2)(ii) No debt, uncertainty**

If we introduce uncertainty without introducing debt, then as we saw in the case of CIT, there should be no change to the above conclusion. This can be seen clearly by noting that introducing uncertainty would decrease the value $S_i$ (since investors require a higher rate of return) but that this decrease in the value of $S_i$ affects both equations 11.60 and 11.56 in a similar manner. Hence both the rate of return to equity investor and the rate of return on physical investment would change by the same proportion, leaving the Distortion Ratio unchanged.

**(A)(2)(iii) Debt, no uncertainty**

The introduction of debt under CIT resulted in a marginal improvement in the Distortion Ratio. Will the results be similar under Expenditure Tax regime?

To answer this question, we first calculate the return to an equity investor who owns 100% of the equity shares. This is given in equation 10.27 by

$$y_i \qquad \mu_i \theta_{ci} \theta_\epsilon^m - D_i r \theta_{ci} \theta_\epsilon^m$$

$$[11.61] \qquad = \qquad (\mu_i \theta_{ci} - D_i r \theta_{ci}) \theta_\epsilon^m$$

$$[11.62] \qquad = \qquad (\mu_i - D_i r) \theta_{ci} \theta_\epsilon^m$$

Equation 11.61 looks similar to equation 11.40 under the CIT case, except that $\theta_{ci}$ now represents the applicable corporation tax instead of $\theta_c/\theta_j$, and $\theta_\epsilon^m$ represents personal expenditure tax instead of $\theta_p^m$. It appears odd that the two equations are similar when the two tax systems have different characteristics including (i) under CIT, retained earnings too are subject to corporation tax, and (ii) under CIT, debt interest is specifically an allowable deduction. We therefore firstly resolve this apparent anomaly.

Regarding (i), it was explained in chapter 10 that since investors capitalise potential corporation tax liability on future dividends, retained earnings will result in a capital gain of $1-\tau_{ci}$ - hence the impact on the return received by equity investor is similar in both cases. Regarding (ii), although debt interest is not specifically stated to be an allowable expense for corporation tax purposes under ET, it still has the effect of reducing the retained equity earnings. The cash flows which are paid out as debt interest are not subject to corporation tax directly (as on dividends) or indirectly (as on retained equity earnings) under ET. Hence, debt interest payments are <u>in effect</u> an allowable expense for corporation tax under ET too. Therefore it is reasonable for equation 11.61 to be similar to equation 11.40.

We make use of the above discussion of the tax features in order to derive the valuation equation below. We do so because it would save us from deriving the valuation model again in its entirety. Instead, we simply note that since equation 11.61 under E.T. is similar to equation 11.40 under C.I.T., the resulting valuation equations too must be similar. The resulting valuation equation under C.I.T. isa given by equation 11.40b . If we make the appropriate adjustments for taxes under E.T. as noted above, then the valuation equation for the value of equity under E.T. can be derived. The value of shares owned by the equity holder under E.T. would thus be given by

$$S_i = V_i\text{-}D_i = \frac{\mu_i\,\theta_{ci}\,\theta_\epsilon}{r\theta_\epsilon + 0\ (=\beta_i\pi)} + LD_i - D_i \quad [\ \beta_i\pi = 0 \text{ since no uncertainty }]$$

[11.63] $$= \frac{\mu_i\,\theta_{ci} + (L\text{-}1)\,D_i r}{r}$$

Under E.T. system, the amount of consumption given up by the equity holder when he purchases $S_i$ of shares is only $\theta_\epsilon{}^m S_i$ . Therefore, the consumption foregone is :

$$= \theta_\epsilon{}^m \left[ \frac{\mu_i\theta_{ci} + (L\text{-}1)\,D_i r}{r} \right]$$

But L = $\tau_{ci}$ under ET (per equation 10.25 in chapter 10), therefore

$$\Rightarrow \theta_\epsilon{}^m \left[ \frac{\mu_i\theta_{ci} + (\tau_{ci}\text{-}1)\,D_i r}{r} \right] \Rightarrow \theta_\epsilon{}^m \left[ \frac{\mu_i\theta_{ci} - \theta_{ci}\,D_i r}{r} \right]$$

[11.64] $$\Rightarrow \frac{\theta_\epsilon{}^m\,\theta_{ci}\,(\mu_i\text{-}D_i r)}{r}$$

Now to get the rate of return to equity investor, we simply divide equation 11.62 by 11.64 to get

$$(\mu_i\text{-}r)\,\theta_{ci}\,\theta_\epsilon{}^m \div [\ (\mu_i\text{-}r)\,\theta_{ci}\,\theta_\epsilon{}^m / r\ ]$$

[11.65] $$= r$$

Hence the rate of return to an equity investor is the real rate of interest, a result which confirms the claim made in the Meade Report, but only under the present assumption of no uncertainty.

Does this differ from the return on physical investment? Physical investment requires capital equipment the cost of which is met by cash raised by issuing $S_i$ of equity and $D_i$ of debt. Issuing $S_i$ of equity raises $S_i/\theta_{ci}$ cash as shown in equation 11.59 . Issuing $D_i$ of debt will raise $D_i$ of cash only - because debt finance has no tax effect under S-based corporation tax system. Therefore the rate of return on physical investment is given by

$$[11.66] \qquad \mu_i \ / \ [\frac{S_i}{\theta_{ci}} + D_i]$$

Using the definition of $S_i$ as given by equation 11.63 , we get rate of return on physical investment as

$$\mu_i \div [\{\frac{\mu_i\theta_{ci} + (L-1)\, D_i r}{r} \div \theta_{ci}\} + D_i] \qquad \{...\} = S_i/\theta_{ci}$$

$$= \mu_i \div [\{\frac{\mu_i\theta_{ci} + (\tau_{ci}-1)\, D_i r}{r} \div \theta_{ci}\} + D_i]$$

$$= \mu_i \div [\{\frac{\mu_i\theta_{ci} - \theta_{ci}\, D_i r}{r} \times \frac{1}{\theta_{ci}}\} + D_i]$$

$$= \mu_i \div [\{\frac{(\mu_i - D_i r)\,\theta_{ci}}{r} \times \frac{1}{\theta_{ci}}\} + D_i]$$

$$= \mu_i \div [\frac{\mu_i - D_i r + D_i r}{r}]$$

$$[11.67] \qquad = \mu_i \div \mu_i/r = \mu_i \times r/\mu_i = r$$

Hence the Distortion Ratio in the case of debt without uncertainty too is r/r = 1. Hence expenditure tax combined with flow of funds corporation tax do not cause any distortion, even in the presence of debt (without uncertainty).

Earlier we noted under CIT that debt without uncertainty improved the Ratio marginally from 0.6170 to 0.6304. There is no such improvement under ET system. We examine the reasons why is there this difference between the ET and the CIT case. The short answer is that since we are considering the no uncertainty case, the return on equity should be similar to the return on debt and it should not increase with leverage, since without uncertainty, increasing leverage leads to no increase in equity risk. Therefore the

Ratio should <u>not</u> improve, even marginally, under CIT. The reason why it improves marginally under CIT is because the cost of physical investment is defined in terms of the value of the cash flows it generates. Changing the debt level changes the value of the cash flows (by $D_i \tau_c$ where $\tau_c$ is the applicable corporation tax rate). Hence under CIT, the Ratio improves marginally only because of the inability to attach a unique value to the cost of investment (in common with the literature in Finance), when that investment may be undertaken by companies with differing levels of debt. ET system, on the other hand, does not suffer from this disadvantage. True, even under ET system, there is tax-advantage of debt and the value of the company would improve with debt Ratio. But this effect is neutralised by the corporation tax advantage of issuing shares under a flow of funds corporation tax system - thus resulting in a unique cost of investment in the no-uncertainty case. These points can be explained with the help of examples.

Let us consider the previous example again - that is, $\mu_i$ = £50; $\theta_c/\theta_j$ = 0.8228; $\theta_p$ = .80; $\theta_p^m$ = .75; r = 0.05 - and see how the Ratio changes as debt level changes from (a) zero to (b) £100 under CIT.

With zero debt level, the return to equity holder who owns all of the company is $\mu_i \theta_p^m \theta_c/\theta_j$ = £50 x .75 x .8228 = £30.8542. In order to receive this, the equity holder pays the following for all the shares of the company :

$$S_i = V_i - D_i = \frac{\mu_i \theta_p \theta_c/\theta_j}{r\theta_p + \beta_i \pi} + LD_i - D_i$$

which, without uncertainty and debt,is :

= 50 x .8 x .8228 / .05 x .8 = £32.91 / .04 = £822.78.

This value results in the rate of return to equity holder of £30.8545 / £822.78 = 0.0375 ($r\theta_p^m$). Since there is no debt, the cost of the capital equipment invested in must also be £822.78, giving the rate of return, with a physical investment as per our example, of £50 / £822.78 = 0.0608 (= r $\div$ $\theta_c/\theta_p$). The Distortion Ratio, as also calculated in the beginning of this chapter, is 0.0375 / 0.0608 = 0.6171.

These figures, together with similar figures for when debt = £100, are shown in the table below. With debt = £100, column (v) shows that $V_i$, the value of the firm, increases by benefit of tax deductibility of debt. Now if we continue with the assumption of a competitive environment in the market for capital goods, then the cost of capital equipment invested in must equal this Vi. As a result, as shown in the following table, the rate of return on physical investment decreases marginally from 0.0608 to 0.0595. That the cost of machinery invested in depends upon the value of cash flows resulting from it is a sound assumption. The weakness in analysis however, occurs because different debt equity Ratios, in the presence of corporation tax under CIT, produce different values for the cash flow. If we assumed that the cost of investment would be £822.78 irrespective of which type of company purchased the investment - a fairly realistic assumption because in reality capital equipment values do not change with the capital structure of the purchaser - then the rate of return on physical investment would remain at 0.0608, regardless of the level of debt. Then under CIT too the Ratio would remain the same (though not at level = 1) under the following cases: (i) no debt, no uncertainty (ii) no debt, uncertainty (iii) debt, no uncertainty.

This is illustrated in the following table. Row (a) shows the figures that result when we assume zero debt. Row (b) shows the figures in case we assume debt of £100. The "difference" row shows the relevant differences between the preceding two rows. It demonstrates that the cost of physical investment is different in the second case because of the impact of leverage on the valuation of equity.

**CIT: CALCULATION OF RATIO & COMPONENTS**

**No Uncertainty Debt = (a) zero (b) £100**

| | Level Of Debt | Cash return to Equity Holder | Value of $S_i$ | Rate of return on Equity | Vi | Cost of machinery | Rate of return on physical inv | Ratio | Level Of Debt | |
|---|---|---|---|---|---|---|---|---|---|---|
| | (i) | (ii) | (iii) | (iv) | (v) | (vi) | (vii) | (viii) | (i) | |
| | | | | (ii) ÷ (iii) | (i) + (iii) = (v) | | $\mu_i \div$ (vi) | (iv) ÷ (vii) | | |
| (a) | zero | £30.8542 | £822.78 | 0.0375<br>$= r x \theta_p^m$ | 822.78 | 822.78<br>$= r \div \theta_c / \theta_j$ | 0.0608 | 0.6171 | Zero | (a) |
| (b) | £100 | £27.7688 | £740.50 | 0.0375 | 840.50 | 840.50 | 0.0595 | 0.6304<br>$= r \div [\theta_c / \theta_p + L(D_i r/\mu)]$ | £100 | (b) |
| Difference | | £3.0854 | £82.28 | 0.0375 | 17.72<br>$D_i L$ | 17.72 | | | Difference | |

Assumptions: $\mu_i = £50$; $\theta_c / \theta_j = 0.8228$; $\theta_p = .80$; $\theta_p^m = .75$; $r = 0.05$

Similar calculations for ET case show how, in spite of positive benefit of leverage, the amount of cash invested in physical investment remains unchanged. This is another advantage of ET system - the value of those assets whose value is determined on the basis of the expected value of future cash flows, will remain unchanged regardless of the capital structure of the companies that are bidding for that asset.

We analyse the components of Ratio under E.T. assuming no uncertainty in the following table. In that table, we take figures which are compatible with the previous example, that is, $\mu_i$ = £50; r = 0.05 and tax parameters $\theta_\epsilon = 0.80$, $\theta_\epsilon^m = 0.75$, $\theta_{ci} = 0.8228$, $\tau_{ci} = .1772$, with zero debt, the value of shares issued under ET will be £822.78 (same as in previous table column (iii)). However this value of equity $S_i$ neither represents the consumption foregone by the equity investor, nor is it the cash available to the company for the purchase of capital equipment. The consumption foregone by the equity investor is given by $\theta_\epsilon^m S_i$, and shows that because of the tax-deductibility of investment under ET, the amount of consumption foregone (to acquire £822.78 of equity) is only £617.085. Cash available for investment is given by $S_i/\theta_{ci}$, which implies that because of corporation tax saved whenever new shares are issued under ET, a share issue of £822.78 results in cash "inflow" of £1000 (£822.28 from equity investor and £177.72 from saving due to reduction of corporate taxes). The rates of return in each case are equal to 'r', giving a Distortion Ratio of 1.

Now if instead a part of the investment was financed using £100 debt, then the value of the firm increases by $D_i\ \tau_{ci}$ (£17.722) to £840.502 as it does in the case of CIT. The difference however, under ET, is that the cash available for investment in the capital equipment does not change - it remains constant at £1000 even though debt level and $V_i$ have changed. This is so because issuing equity too has corporation tax effects. For every £ of debt issued, the amount of equity that need not be issued is given by

$$\delta S_i/\delta D_i = \frac{\delta}{\delta D_i}\ y.\frac{\mu_i\theta_\epsilon\theta_{ci}}{r\theta_\epsilon} + \tau_{ci} D_i - D_i$$

$$= \tau_{ci}-1 = -(1-\tau_{ci}) = -\theta_{ci}$$

The above shows that if debt level is increased by £1 which is used to repay £1 of equity, then the equity value decreases by only $1-\tau_{ci}$ ( and not by the full £1), because increasing leverage has an upward impact on the equity value by the tax shield (that is by $\tau_{ci}$)

Now, for £$S_i$ of equity not issued, cash foregone under ET = $S_i/\theta_{ci}$
for £1 of equity not issued, cash foregone under ET = $(S_i/\theta_{ci} . 1/S_i) = 1/\theta_{ci}$
for £$\theta_{ci}$ of equity not issued, cash foregone under ET= $1/\theta_{ci} . \theta_{ci}$ = 1

Hence issuing £1 of debt means giving up £1 of cash available for physical investment from equity ( £$\theta_{ci}$ decrease directly due to decrease in equity value, and the balance due to the decrease in the benefit which is available at the corporation tax level whenever equity is issued under ET system). Therefore the total amount of cash available for investment under expenditure tax is constant at £1000. As a consequence, the rate of return on physical investment under ET is constant even if debt level changes (no uncertainty) and therefore the Ratio remains constant.

This analysis has shown quite clearly

(a) that there is tax advantage to debt as far as the value of the company, $V_i$, is concerned under both the tax systems (CIT and ET)

(b) the reason why we see a marginal improvement in the Distortion Ratio under CIT when debt increases (no uncertainty case), as explained in the previous table

(c) the reason why we do not see any change in the Distortion Ratio under ET when debt increases, as explained above and in the following table, and finally

(d) the advantage of ET system for the valuation of capital equipment purchased by companies with different debt-equity Ratios. The advantage is that the value of the capital equipment remains unchanged under ET system even though the purchasing companies have different debt equity ratios. Therefore capital equipment has unique and unambiguous value. In contrast, under the C.I.T. system, the value of the capital equipment depended upon the debt equity ratio of the purchasing company, which is not an entirely satisfactory result.

The following table shows the figures used in the ET analysis.

## EXPENDITURE TAX: CALCULATION OF RATIO AND COMPONENTS

**No Uncertainty Debt = (a) zero (b) £100**

| | Debt | Cash Return To Equity Holder | Value of $S_i$ | Consumption Foregone | Rate of Return on Equity | $V_i$ | Corp Tax saved when Equity Issued |
|---|---|---|---|---|---|---|---|
| | (i) | (ii) | (iii) | (iv) | (v) | (vi) | (vii) |
| | | | | (iii)x$\theta_\epsilon^m$ | (ii)÷(iv) | (i)+(iii) | (iii).$\tau_{cE}$ ($\tau_{cE}$=.21539) |
| (a) | zero | 30.8542 | 822.78 | 617.085 | 0.05 | 822.78 | 177.22 |
| (b) | £100 | 27.7688 | 740.50 | 555.376 | 0.05 | 840.50 | 159.49 |
| Difference | | 3.0854 | 82.28 | 61.708 | nil | 17.72 =$D_i.\tau_{ci}$ | |

| | Debt | Cash Raised From Equity | Cash Raised From Debt | Total Cash Raised = Cost of investment in Machinery | Rate of Return on Physical Investment | Ratio |
|---|---|---|---|---|---|---|
| | (i) | (viii) | (ix) | (x) | (xi) | (xii) |
| | | (iii)+(vii)=$S_i/\theta_{ci}$ | =(i) | (viii)+(ix) | $\mu_i$÷(x) | (v)÷(xi) |
| (a) | zero | £1000 | - | £1000 | 0.05 | 1.0 |
| (b) | £100 | £900 | £100 | £1000 | 0.05 | 1.0 |
| Difference | | | | nil | nil | nil |

Assumptions: $\mu_i$ = £50; $\theta_c/\theta_j$ = 0.8228; $\theta_p$ = .80; $\theta_p^m$ = .75; r = 0.05

In conclusion to this sub section, we note that under ET, considering mixed capital structures without uncertainty does not change the conclusions of the analysis of Meade Report. Similarly, considering uncertainty without debt too did not show any results that were different from the no debt, no uncertainty case. However, it would still be premature to conclude that neither debt nor uncertainty has an impact on the Ratio under

ET, because the result could be different when debt and uncertainty are considered together, as they are in the following sub section.

### (A)(2)(iv) Debt and Uncertainty

With debt present, the return on equity for an investor owning all shares in the company is given by modifying equation 10.27 to

Return to equity investor $= \mu_i\theta_{ci}\theta_\epsilon^{\,m} - D_i r\theta_{ci}\theta_\epsilon^{\,m}$

[11.68] $= (\mu_i - D_i r)\,\theta_{ci}\,\theta_\epsilon^{\,m}$

Therefore the rate of return on consumption foregone by the equity investor is

$$\frac{(\mu_i - D_i r)\,\theta_{ci}\,\theta_\epsilon^{\,m}}{\theta_\epsilon^{\,m} S_i}$$

[11.69] $$= \frac{(\mu_i - D_i r)\theta_{ci}}{S_i}$$

As described earlier, the cash raised from issuing $S_i$ of equity is $S_i/\theta_{ci}$, which, along with $D_i$ of debt, give $S_i/\theta_{ci} + D_i$ of cash available for investment. This amount of investment results in expected cash flow of $\mu_i$ - thereby giving the rate of return on physical investment of $\mu_i / (S_i/\theta_{ci} + D_i)$

$$= \mu_i \div \frac{S_i + D_i\,\theta_{ci}}{\theta_{ci}}$$

[11.70] $$= \frac{\mu_i\,\theta_{ci}}{S_i + D_i\theta_{ci}}$$

The Ratio can be calculated by simply dividing equation 11.68 by equation 11.69 to get

$$\frac{(\mu_i - D_i r)\theta_{ci}}{S_i} \div \frac{\mu_i\theta_{ci}}{S_i + D_i\theta_{ci}} = \frac{(\mu_i - D_i r)\theta_{ci}}{S_i} \times \frac{S_i + D_i\theta_{ci}}{\mu_i\theta_{ci}}$$

[11.71] $$= \frac{\mu_i - D_i r}{\mu_i} \times \frac{S_i + D_i\,\theta_{ci}}{S_i}$$

By substituting in the expression for $S_i$ in equation 11.71, the Ratio can be expressed as shown in equation 11.72 below. The details of the transformations are given in appendix B1.

[11.72] Distortion Ratio = $$\frac{1}{1 - \dfrac{D_i\ \beta_i\pi}{\theta_\epsilon\ [\mu_i\text{-}rD_i]}}$$

Note that if either $D_i$ (debt level), or $\beta_i\pi$ (risk premium based on uncertainty) or both $D_i$ and $\beta_i\pi$ are zero, the last term in the denominator equals zero, and the Ratio becomes 1. That was the result we encountered in the last 3 subsections above. However, when both $D_i$ and $\beta_i\pi$ are positive, the denominator is less than one - leading to a Ratio in excess of 1. Hence, when both debt and uncertainty are present, the Ratio is not equal to one, but the equity investor, by undertaking financial risk, will earn a rate of return on equity which exceeds the rate of return on physical investment. For a majority of companies, both debt and uncertainty are present. Hence, for a majority of companies, the definition of expenditure tax which the Meade Report used ( that is, expenditure tax system is one in which the return to saver equals return to investment) is inappropriate.

Using $\beta=1$ and $D_i$=£100 debt, the Distortion Ratio is 1.2857 as shown in Appendix B1. Here, although ET itself causes no distortion, the Ratio is greater than 1 under ET since equity investors undertake financial leverage.

If we use a higher level of debt (say $D_i$=£200) the Distortion Ratio increases to 2:

$$\frac{1}{1 - \dfrac{200 \times 0.08}{.8\ (50\text{-}.05 \times 200)}} = \frac{1}{.5} = 2$$

Hence the remaining equity investors in highly geared companies can earn a return double that of physical investment, even under ET system. In the presence of debt and uncertainty, the Distortion Ratio is more sensitive to changes in debt under ET system (note from the previous table that it changes from 1.2857 to 2) than it is under CIT (in which case the change was from a Ratio of 0.8447 to only 1.0461).

Similarly, The Distortion Ratio also varies directly with beta factor of the companies. If a company has beta factor equal to 2, then the Distortion Ratio under ET changes from 1.2857 to 1.80. The Ratio with beta factor of 2 can be calculated using the expression from the previous table, and substituting in the assumed values, as follows:

$$\frac{1}{1 - \frac{100 \times 1 \times .08 \times 2}{.8[50-(100x.05)]}} = \frac{1}{1-.444} = \frac{1}{.5555} = 1.80$$

The contrasts between the Distortion Ratio's behaviour under CIT and ET can be summarised as follows:

| | **Expenditure Tax** | **Comprehensive Income Tax** |
|---|---|---|
| 1 | Although the table on page 303 shows ET system results in a Ratio = 1 in most cases, the more relevant case from the point of view of the real world is case (iv) where both debt and uncertainty are present. In this case, the Ratio is always greater than 1. | In case (iv) CIT gives Ratios which are closer to one than those given by ET. |
| 2 | Under ET, the distortions are not caused by taxes, they are caused by financial leverage. Hence if the objective is to minimise tax induced distortions then ET is better. | The distortions (as illustrated by Ratio differing from 1) are largely caused by taxes in case of CIT. |
| 3 | Even under uncertainty, the Ratio is independent of $\theta_\epsilon^m$ that is, the Ratio does not vary with the personal expenditure tax rate of the investor. | In all cases, $\theta_p^m$ is part of the Ratio. Hence under CIT the Ratio will vary with the personal income tax rate of the investor |
| 4 | Under ET, the corporation tax revenue is collected only on dividends, NOT on capital gains (Capital gains appear to be taxed under ET system because of the presence of the capital gains tax factor in the valuation | Under CIT, the government receives corporation tax on dividends as well as retained earnings - hence government revenue will be higher, other things remaining equal. |

equation under ET. However, that tax factor is present because capital markets anticipate the POTENTIAL tax on future dividends which will be generated by the company, and hence the capital gain is lower than in the absence of taxes. However, there is no immediate tax revenue based on capital gains to the government under ET system).

With debt and uncertainty, CIT is likely to be better than ET if the objective is to move the Distortion Ratio closer to 1 . This is in contrast to Meade Report's conclusions.

To end this entire section, in which we extended the analysis of the Meade Report to evaluate CIT and ET systems when debt and/or uncertainty are present, we summarise our criticisms of the Meade Report.

It is incorrect to take return to debt holders as equal to the rate of return on physical investment. With uncertainty, debt holders face lower risk in comparison with the underlying investment. Hence one should expect debt holders to get a lower return than the return on underlying investment - whatever the tax system, whether it be CIT or ET. Therefore to define a system of ET as one in which return to all savers equals return on investment is not meaningful in the real world with uncertainty and with mixed capital structures.

Equity investors in the real world can undertake two types of risk - business risk and financial risk. Business risk affects both the rate of return to equity investor and the rate of return on physical investment proportionately - hence by itself, business risk does not alter any of the conclusions of the Meade Report. By itself, debt (in a certain world) too does not alter any of the conclusions of Meade Report.

However, when debt and uncertainty together are present, then equity investors can undertake financial risk. In 1958, Modigliani and Miller stated that increasing financial risk increases the rate of return to the remaining equity investors. This fact is very relevant to the real world capital structures and hence the Meade Report overlooks the following point. ***If the increased return due to financial leverage is combined with a tax system that reduces returns (such as CIT) - then the two effects may cancel each other out to some extent - so that the final net rate of return to equity holder may be near the rate of return on physical investment. If the increased return due to financial leverage is combined with a tax system that has no tax-induced distortions, such as an ET system, then the final rate of return to equity holder will always be greater than the rate of return on the underlying physical investment.***

## SECTION (B)
## TOTAL RETURN ON ASSETS

We will now investigate whether the conclusions reached in the Meade Report are more valid if we do not take the return on equity and debt separately, but instead take the total return to all investors into account, but if we define all investors to include debt and equity holders, excluding tax authorities. More specifically, under expenditure tax system, if we add the return to equity holders and the return to debt holders, will the Distortion Ratio equal 1? Will the Distortion Ratio, using the total return to investors concept, always be less than 1 in the case of CIT? These questions are examined for the CIT and the ET systems in the presence of both debt and uncertainty below.

### (B)(1) Comprehensive Income Tax - Distortion Ratio (Total Return Case)

The total return received by investors is $(\mu_i\text{-}rD_i)\,\theta_c/\theta_j\,\theta_p^m$ [return to equity holders] + $rD_i\,\theta_p^m$ [return to debt holders] (see equation 10.11 in chapter 10). The rate of return to investors therefore is

[11.73]
$$\frac{(\mu_i\text{-}rD_i)\,\theta_c/\theta_j\,\theta_p^m + rD_i\,\theta_p^m}{S_i + D_i}$$

$$= \frac{\theta_p^m\,[\,\mu_i\,\theta_c/\theta_j + (1\text{-}\theta_c/\theta_j)\,rD_i\,]}{V_i}$$

The rate of return on physical investment is equal to $\mu_i/V_i$ as before. Hence the Distortion Ratio is

$$\frac{\theta_p^m\,[\,\mu_i\,\theta_c/\theta_j + (1\text{-}\theta_c/\theta_j)\,rD_i\,]}{V_i} \div \frac{\mu_i}{V_i}$$

$$= \frac{\theta_p^m\,[\,\mu_i\,\theta_c/\theta_j + L\,rD_i\,]}{V_i} \quad x \quad \frac{V_i}{\mu_i}$$

$$= \theta_p^m\,[\,\frac{\mu_i}{\mu_i}\,\frac{\theta_c}{\theta_j} + \frac{L\,rD_i}{\mu_i}\,]$$

[11.74]
$$= \quad \theta_p^m\,[\,\theta_c/\theta_j \; + \; L\,rD_i/\mu_i\,]$$

Equation 11.74 is the same Ratio as that shown for return on equity for the debt-without-uncertainty case (equation 11.45a ; subsection (A)(1)(iii) ). It implies that as far as total return is concerned, uncertainty has no impact. The Ratio is dependent on $\theta_p^m$, the personal tax rate of investor, $\theta_c/\theta_j$, the corporation tax effectively charged, and the amount of cash flow $\mu_i$ that is paid out as debt interest ($rD_i$). With debt level of £100 in the example considered, the Ratio is 0.63. This Ratio increases with the level of debt interest. However, Meade Report's conclusions (that under CIT the Distortion Ratio will be less than one) for CIT based on total returns are correct since this Distortion Ratio is likely to be below 1 for realistic levels of debt.

**(B)(2) Expenditure Tax - Distortion Ratio (Total Return Case)**

Under expenditure tax, the total return to an investor who owns all of the equity and debt of the company i is given by $(\mu_i\text{-}rD_i)\,\theta_{ci}\,\theta_\epsilon^m$ [return to equity - see equation 10.27 in chapter 10] + $rD_i\,\theta_\epsilon^m$ [return to debt]. Since purchase of debt is also a qualifying expenditure for ET purposes, the amount of consumption given up by the investor to invest in this company is $\theta_\epsilon^{m}$ $(S_i+D_i)$. Hence the rate of return to investor is

$$[11.75] \qquad \frac{(\mu_i - rD_i)\,\theta_{ci}\,\theta_\epsilon^{m} + rD_i\,\theta_\epsilon^{m}}{\theta_\epsilon^{m}\,(S_i+D_i)}$$

As shown before, the rate of return on physical investment is given by $\mu_i\,/\,(S_i/\theta_{ci}+D_i)$ (eq 11.70). Hence the Distortion Ratio now is

$$[11.76] \qquad \frac{(\mu_i - rD_i)\,\theta_{ci}\;\theta_\epsilon^{m} + rD_i\,\theta_\epsilon^{m}}{\theta_\epsilon^{m}\,(S_i+D_i\,)} \quad \mathrm{x} \quad \frac{(S_i/\theta_{ci} + D_i)}{\mu_i}$$

This Ratio can be simplified, as shown in Appendix 2, to the following

$$[11.77] \qquad \frac{1}{1 + \dfrac{\tau_{ci}\,D_i\,\beta_i\pi}{\theta_\epsilon\,(\mu_i\,\theta_{ci} + rD_i\,\tau_{ci}}}$$

In the above expression, if the fraction in the denominator is equal to zero ( that is, if the second term in the denominator is zero), then the above expression will reduced to 1/1 = 1. The fraction in the denominator of the expression will be zero if at least one of the following variables is zero:

| | | | |
|---|---|---|---|
| (i) | $\tau_{ci}$ | (the benefit of leverage, which equals the tax inclusive corporation tax rate, under the ET system) | |
| (ii) | $D_i$ | (the debt level) | |
| (iii) | $\beta_i$ | (the beta factor of the cash flows) | } resulting from uncertainty |
| (iv) | $\pi$ | (the market risk premium) | } |

The entire fraction will be zero even if only one of these four above is zero, implying that taxes under an ET system caused no distortion to the rate of total return to investors. This indicates that a positive benefit of leverage, a positive debt level as well as uncertainty must all be present if the Ratio of the rate of total return to all investors to the rate of return on physical investment is to be different from one. In the present day U.K.

context, for most corporate as well as non-corporate businesses, these conditions are met. Hence, in the case of a majority of the businesses, if there were an expenditure tax system in the U.K., the rate of total return to all investors would be below the rate of return on physical investment. This is in contrast to the claim made in the Meade Report that these rates of return are the same under an expenditure tax system.

In section (A), we showed that the Ratio is different from one when the equity investors and the debt investors are considered separately. Here we have shown that the Ratio is different from 1 even when we consider all the investors together. Precisely why the Ratio is different is examined below.

Under expenditure tax with debt and uncertainty, when total return to all assets is considered, the investors still get less return than the return on physical investment. The magnitude of the divergence of the Distortion Ratio from 1 can be calculated by using the same example as before. Previously, we used the following variables:

| | |
|---|---|
| $\mu_i$ = £50 | $D_i$ = debt level = £100 |
| $\tau_{ci}$ = 0.17722 | $\beta_{i\pi}$ = 0.08 |
| $\theta_\epsilon$ = 0.8 | r = 0.05 |

Using the above figures, the Ratio is calculated in Appendix B3(a) as 0.9595. This shows that the distortion caused is only marginal, at 4%, given the debt level, beta factor and other variables as above.

The Distortion Ratio will vary with both the level of debt and the risk premium. For higher debt levels, the Ratio is smaller at 0.9237 as is shown in Appendix B3(b). For higher beta level, say $\beta=2$, the Ratio is again smaller, at 0.9222, as calculated in Appendix B3(c). Thus for highly geared companies as well as for high beta companies, the Ratio can be as much as 8% below 1. 8% is not insignificant in the context of this thesis wherein the concern is with discount rates under the various tax structures. Of course, for companies with beta factors less than one or with lower levels of debt than those considered in these examples, the Ratio will be very nearly equal to one (or, of course, if the debt level is zero, the Ratio will be 1). While that is accepted, it is nevertheless true that for the other companies, the divergence from 1 will be significant. Therefore the general claim of the Meade Report that always under ET system, the rate of return to investor equals the rate of return on physical investment, appears to be contradicted, even when we consider total return to all investors, **but only if we define all investors to include debt and equity holders, excluding tax authorities.**

Meade Report's conclusions would be correct if investors are defined to include government, on the basis that the government effectively is an equity investor under the flow of funds tax bases such as the R base or the S base. The Meade Report states on page 233 that:
"In essence, the government would be acquiring an equity stake in the company, sharing both the cost of investment (through free depreciation) and the resulting profit and losses. On the assumption that the rate of return on company activities was on average higher than the rate of return at which the government could borrow, the government would make a net revenue gain."

If we define investors to include government (the "alternative definition") then we should include the contribution made by the government to the cost of investment in the denominator, and taxes in the numerator, and on this basis, the distortion ratio is one, which implies that there is no distortion. Thus the Meade Report would be correct in its assertion that there is no distortion under the S base system.

However, in this thesis we do not use this alternative definition. This is because in our opinion, this alternative definition is not suitable for the analysis of post tax returns that we wish to undertake, and, in our opinion, it may not be the primary definition advocated in the Meade Report. The reasons why we are of this opinion are:

(1) Meade Report itself continues to be interested in net of tax returns to equity or debt investors, excluding government. For example, in its main discussion on the S base, it states on page 235: "The £200 investment with its yield of 10 per cent would be worth £220; but if this sum were in one form or another paid out to the shareholders, it would be subject to corporation tax and ***the shareholders would***

***receive £110, giving a 10 per cent rate of yield on their original £100 contribution.*** " (Our italics).

Thus Meade Report appears to be concerned with showing no distortion on net of tax basis, which would be unnecessary if the alternative definition was being used.

(2) Similarly, in an earlier discussion of the R base, Meade Report appears to be concerned with net of tax returns (page 232 of Meade Report):

"On the ***£100 of independent finance*** which it had to raise in the first place for the purchase of the asset, the company would receive the ***full yield of 10 per cent on its investment***, and this would be true whether the investment had been financed by the issue of new share capital, the raising of a loan or the ploughing back of undistributed profit."

Thus, again when dealing with a flow of funds based tax system, Meade Report appears to be concerned with showing that there is no distortion by using a criterion which is consistent with the way distortion ratio is primarily defined in this thesis.

(3) In contrast, there does not appear to be any example in the Meade Report where the alternative definition is used.

(4) It may be that the primary purpose of mentioning the equity stake of the government was in the context of showing why the government would be raising net revenues under R base, and the purpose may not have been to imply that yields should be measured using an alternative definition.

(5) Net of tax returns are of significant importance to the Meade Report, as is illustrated by the calculation of post tax returns on pages 8, 36-37, 64, 66-69, 252 and 255 of the Meade Report.

(6) In Finance, net of tax returns to equity and debt holders are of interest.

(7) Finally, the criteria specified in the alternative definition can be satisfied by some

very distortionary tax systems. For example, consider a P base tax system under which the government gives no capital allowances but £100 of grant to every company ( which is a contribution and therefore government is an "investor") and taxes companies at 75% tax rate. Such a tax system could well have been regarded as highly distortionary by the Meade Committee, yet, if we used the alternative definition, then the distortion ratio would be one, implying a highly desirable tax system.

**Therefore we are of the opinion that the analysis using the main definition of the distortion ratio is not inconsistent with the analysis in the Meade Report itself and is unlikely to indicate that Meade Report has been misinterpreted in this thesis.** Therefore we continue to focus on the main definition in this chapter.

**Why is the Distortion Ratio less than one?**

When we considered the question of the rate of return to equity holders, the reason why that rate was greater than the rate of return on physical investment was that the equity investors accepted financial (leverage) risk. If, however, we consider the total return to all investors together, that is, add the return to equity holders and the debt holders, then the question of financial (leverage) risk does not arise. (This is so because there is no other class of investors against whom the equity AND debt holders together can "lever" their return). Therefore one would expect that the Meade Report's assertion should be correct when one is considering the total return to all investors, because in this case financial leverage should play no part. We wish to examine precisely what causes the Ratio to be less than 1 in this case.

The main reason why the conclusion reached in the Meade Report differs from our conclusion is because the Meade Report assumes that the rate of return on total assets (equity plus debt financed) is equal to the rate of return to the debt holders, perhaps because they do not incorporate uncertainty and mixed capital structures together explicitly in their analysis. However, we can examine the Distortion Ratio in detail, incorporating risk and uncertainty, since our recommended valuation model is detailed enough to be used for this purpose. We will seek to demonstrate below that the Meade Report's assertions may be incorrect in some settings using our definition of investors. We commence by examining more closely the rate of total return to all investors as given in equation 11.75 , which is reproduced below:

[11.78] $$\frac{(\mu_i\text{-}rD_i)\,\theta_{ci}\,\theta_\epsilon^{m} + rD_i\,\theta_\epsilon^{m}}{\theta_\epsilon^{m}\,(S_i+D_i)}$$

$[(\mu_i\text{-}rD_i)\,\theta_{ci}\,\theta_\epsilon^{m}]$ = return to equity

$[rD_i\,\theta_\epsilon^{m}]$ = return to debt

Note that we have defined

Equation 11.78 can be rearranged as follows:

[11.79] $$\frac{\theta_\epsilon^{m}\,[\mu_i\theta_{ci} - rD_i\,\theta_{ci} + rD_i\,]}{\theta_\epsilon^{m}\,(S_i+D_i)}$$

We cancel $\theta_\epsilon^{m}$ and we substitute the following in the denominator in equation 11.79.

(a) $D_i\,\theta_{ci} + D_i\,\tau_{ci}$ for $D_i$

(b) $(S_i + S_i\,\tau_{cE})\theta_{ci}$ for $S_i$

These substitutions can be justified as follows:

(a) $D_i\,\theta_{ci} + D_i\,\tau_{ci} = D_i\,(1\text{-}\tau_{ci}) + D_i\,\tau_{ci} = D_i - D_i\,\tau_{ci} + D_i\,\tau_{ci} = D_i$ (Note $\theta_{ci} = (1\text{-}\tau_{ci})$)

(b) $(S_i + S_i\,\tau_{cE})\theta_{ci} = S_i$, where $\tau_{cE}$ is the corporation tax rate at the tax-exclusive rate. This equality can be justified intuitively as follows. The total amount of cash raised for investment from equity sources (under the "S-base" corporation tax system being considered) is given by either

(1) $S_i$ (cash from equity holders) + $S_i\,\tau_{cE}$ (corporation tax refund [at tax exclusive rate]) due to issue of $S_i$ of shares, or by

(2) $S_i/\theta_{ci}$

This equality can also be seen by examining column (viii) in the last table. Column (viii) can be derived in two alternative ways, by adding columns (iii) and (vii), or by dividing column (iii) by $\theta_{ci}$. These two alternative methods correspond to the two formulae given in (b) above.

Therefore (1) and (2) are equal (both are equal to the total cash raised from equity)

$$S_i/\theta_{ci} = S_i + S_i\,\tau_{cE}$$

$$\Rightarrow \quad S_i = (S_i + S_i\,\tau_{cE})\,\theta_{ci}$$

Alternatively, we show this mathematically by substituting the formula for calculating the tax-exclusive rate from tax inclusive rate, that is, by substituting $\tau_{cE} = \tau_{ci}/(1-\tau_{ci})$. Replacing $\tau_{cE}$ by $\tau_{ci}/(1-\tau_{ci})$ in $(S_i+S_i\,\tau_{cE})\theta_{ci}$, we get

$$\left(S_i + S_i \frac{\tau_{ci}}{1-\tau_{ci}}\right) \times \theta_{ci}$$

$$= \frac{S_i\,(1-\tau_{ci}) + S_i\,\tau_{ci}}{(1-\tau_{ci})} \times (1-\tau_{ci})$$

$$= S_i - S_i\,\tau_{ci} + S_i\,\tau_{ci}$$

$=$ $S_i$ as claimed above.

Therefore we can substitute (a) and (b) above into equation 11.79 (after cancelling $\theta_\epsilon{}^m$ in the numerator and the denominator of that equation) to get the following:

[11.80] $$\frac{\mu_i\,\theta_{ci} - rD_i(1-\tau_{ci}) + rD_i}{(S_i+S_i\tau_{cE})\,\theta_{ci} + D_i\,\theta_{ci} + D_i\,\tau_{ci}}$$

[11.81] $$= \frac{\theta_{ci}\,\mu_i + rD_i\,\tau_{ci}}{\theta_{ci}\,(S_i + S_i\,\tau_{cE} + D_i) + D_i\,\tau_{ci}}$$

The term within the round brackets in the denominator in equation 11.81 is equal to the total cost of physical investment that is, it is equal to the sum of cash raised from the equity issued, the corporation tax saved because of the equity being issued, and the cash raised from issuing debt. $\mu_i$ is the pre-tax return on investment. Therefore the rate of return on physical investment is

(11.81b) $$\frac{\mu_i}{(S_i + S_i\,\tau_{cE} + D_i)}$$

Hence, <u>if</u> equation 11.81 above had been

[11.82] $$\frac{\theta_{ci}\,\mu_i}{\theta_{ci}\,(S_i + S_i\,\tau_{cE} + D_i)}$$

that is, if it excluded $rD_i\,\tau_{ci}$ from the numerator and excluded $D_i\,\tau_{ci}$ from the denominator, then we would have equated the rate of total return to all investors (11.82) to the rate of return on physical investment (11.81b). Unfortunately, in equation 11.81, we have $rD_i\,\tau_{ci}$ additionally in the numerator and $D_i\,\tau_{ci}$ additionally in the denominator. Equation 11.81 is, thus, not equal to equation 11.82.

The Meade Report assumes a world where the rate of return to the debt holders is the same as the rate of return to equity holders and, it also asserts that these rates would equal the rate of return on physical investment in the case of ET. Consequently, Meade Report would treat the following three expressions as equal to each other :

[11.83]
$$\frac{\theta_{ci}\,\mu_i}{\theta_{ci}\,(S_i+S_i\tau_{cE}+D_i)} = \frac{rD_i\,\theta_{ci}}{D_i\,\theta_{ci}} = r$$

| rate of return on physical investment | rate of return on debt | rate of return on equity |
|---|---|---|

If we temporarily assume that the above equality holds then with this assumption, the Meade Report would conclude that:

[11.84]
$$\frac{\theta_{ci}\,\mu_i + rD_i\,\tau_{ci}}{\theta_{ci}\,(S_i+S_i\tau_c+D_i) + D_i\,\tau_{ci}} = \frac{\theta_{ci}\,\mu_i}{\theta_{ci}\,(S_i+S_i\tau_{cE}+D_i)}$$

But the LHS in equality 11.84 is a transformation of the rate of total return to all investors (see equation 11.81) while the RHS of the equality is a transformation of the rate of return on physical investment (see equation 11.81b). Hence if we assume equation 11.83 holds, then equation 11.84 implies that

[11.85] Rate of total return to all investors = Rate of return on physical investment.

Using our definition of investors, equation 11.85 does not hold. The reason for this lies in the equality shown as equation 11.83 . Meade Report assumes that the rate of return on physical investment will equal the rate of return earned by the debt holders who finance that investment. However, we claim that since the debt holder faces less risk in comparison with the riskiness of the cash flows generated by the physical investment, the first of the two equalities shown in equation 11.83 does not hold. Specifically,

[11.86] $$\frac{\theta_{ci}\ \mu_i}{\theta_{ci}\ (S_i+S_i\tau_{cE}+D_i)} = \text{Rate of return on physical investment} > r = \text{Rate of return of debt holders} = \frac{rD_i\tau_{ci}}{D_i\ \tau_{ci}}$$

This is so because

[11.86a] Riskiness of physical investment > riskiness faced by debt holders.

Since the above two inequalities hold, the addition of the term $rD_i\ \tau_{ci}$ to the numerator and $D_i\ \tau_{ci}$ to the denominator of the fraction $\frac{\theta_{ci}\ \mu_i}{\theta_{ci}\ (S_i+S_i\tau_{cE}+D_i}$ will necessarily lower the fraction. That the fraction will become lower follows intuitively from an examination of equation 11.86. However, to be precise, this is shown formally below.

The aim is to show that the following inequality is true, which will prove that even under ET, the rate of return on physical investment exceeds the rate of total return to all investors.

[11.87] $$\frac{\theta_{ci}\ \mu_i}{\theta_{ci}\ (S_i+S_i\tau_{cE} + D_i)} > \frac{\theta_{ci}\ \mu_i + rD_i\ \tau_{ci}}{\theta_{ci}\ (S_i+S_i\tau_{cE} + D_i) + D_i\ \tau_{ci}}$$

To simplify, let a, b, c and d represent the following:

$\theta_{ci}\ \mu_i = a$ $\qquad rD_i\tau_{ci} = b$ $\qquad \theta_{ci}(S_i+S_i\tau_{cE}+D_i) = c$ $\qquad D_i\ \tau_{ci} = d$

Equation 11.87 thus is $\frac{a}{c} > \frac{a+b}{c+d}$

Since c and d are positive, we can cross multiply without changing direction of the inequality, to get $ac + ad > zc + bc \Rightarrow ad > bc$

Divide by b and a $\frac{d}{b} > \frac{c}{a}$

Taking the reciprocal, and thereby changing the direction of the inequality, we get

$$\frac{b}{d} < \frac{a}{c}$$

Transforming to original variables, we get

[11.88] $$\frac{rD_i\,\tau_{ci}}{D_i\,\tau_{ci}} \quad (=r) < \quad \frac{\theta_{ci}\,\mu_i}{\theta_{ci}\,(S_i+S_i\tau_{cE} + D_i)}$$

We have already shown above in equation 11.86 that as a result of the risk differences, inequality 11.88 holds. This proves formally that the inequality 11.87 is also true, which implies the following:

[11.89] Rate of return on physical investment > Rate of return to all investors.

Hence, our assertion that even under ET the rate of total return to all investors is less than the rate of return on physical investment is correct because we make use of the relationship between risk and expected return as illustrated in equality 11.86. Meade Report's conclusion ignores the relationship between risk and return and therefore it appears that the Meade Report assumes that the equalities shown as equation 11.83 are true.

**The Distortion Ratio when the company earns super-normal profits**

The calculation of Distortion Ratio when companies earn supernormal profits is fully dealt with in section (D) below. The supernormal profits case is introduced here only because the closest that the Meade Report comes to acknowledging that the rate of interest to debt holders may differ from the return on investment is on pages 232-235 of the Report, which deal with supernormal profits.

The context is that on page 235, the Meade Report explains how under an S-based Corporation Tax system, the government will raise revenue. It does so by firstly considering the case of a company that requires £200 of investment. This investment is financed by issuing £100 of equity, and at a tax-exclusive rate ($\tau_{cE}$) of 100%, the company receives £100 of corporation tax refund. Hence it is able to undertake £200 of investment.

The rate of interest on government debt is given to be 10%. The company lasts for only one year, when, with a rate of return on physical investment of 10%, the company has assets worth £220. It sells those assets and pays £110 as dividend and £110 as corporation tax (since the corporation tax is levied at tax-exclusive rate of 100%). The government paid £100 as corporation tax credit when the company started, and received £110 one year later when the company paid its only liquidating dividend. As the interest on government borrowing is 10%, the government has in fact raised no net revenue in present value terms.

However, if at the end of the year, the company's assets were worth more than £220, say £300, then the company makes a supernormal profit. It paid £150 as liquidating dividend, and £150 as corporation tax. The government thus earns additional revenue under S-base corporation tax system whenever the companies make profits in excess of the rate of interest on government debt. We find that the rate of return on physical investment is £300/£200 = 1.5. This is equal to the rate of return received by the investors (£150 ÷ £100 = 1.5). Hence the Distortion Ratio is 1, even though the company is making super-normal profits. The impact of super-normal profits on the Ratio is dealt with in section (D) below. The point of giving this example here, however, is that had the Meade Report continued further with this example, and considered the situation where such an investment was financed partly by debt, then they too would have concluded that the total return to all investors could be less than the rate of return on physical investment. They did not consider mixed capital structures, and therefore they did not use this example to show the point illustrated in the following paragraphs.

Let this same investment be financed by £100 debt. The balance of the cost of investment is made up by £50 equity and £50 refund of corporation tax under the S-basis. The investment realises £300 at the end of the year. The question now is: how much is paid out as interest to the debt holders? The return on investment (net) is 50%. [(£300 - £200/£200] whereas the rate of interest on government debt is 10%. It is quite likely that given this situation, the Meade Committee would have concluded that the rate of interest on corporate debt is likely to be closer to the rate of interest on government debt - for simplicity, let us say it is 10%.

With 10% interest rate, £110 is paid out to the debt holders as principal plus interest. This leaves a balance of £300 - £110 = £190, of which £95 can be paid out to equity holders as liquidating dividend, and £95 is paid as corporation tax. The total (gross) return to all investors is £110 + £95 = £205. The total cost of investment borne by investors is £100 + £50 = £150. Therefore the rate of total return (gross) to all investors is £205/£150 = 1.366. This is less than the gross rate of return on physical investment of 1.5. The Distortion Ratio is thus below 1, even though we are considering ET system. This is because the ET system recommended by the Meade Report does not include the two features which we recommend in section (C) below.

The point of discussing this example is to show that it is not necessary to go through the tedious algebra of the previous subsection in order to show that under ET, the Distortion Ratio can be less than 1. One can make the same point simply by assuming a mixed capital structure within the context of a simple example used in the Meade Report itself. We now show how an ET system can be devised such that the rate of return to all investors is the same as that on physical investment by incorporating the two recommendations made in the next section.

However, if the equity finance contributed by the tax authorities is taken into account, the weighted average return to investors is indeed 1.5 (not 1.3666) and the Distortion Ratio (using alternative definition of distortion ratio discussed previously) is equal to 1. We have, however, noted our opinion on the appropriateness of the alternative definition above.

## SECTION (C)

## Attaining Distortion Ratio = 1 under Expenditure Tax System

### The I-Base Of Taxation

As far as the rate of total return to all investors is concerned, it should be possible to attain a Ratio of 1. This possibility exists because as stated earlier, there is no Distortion due to financial (leverage) risk when we consider "total return to all investors". Therefore what changes need to be introduced to the S-based corporation tax system such that the inequality shown as equation 11.89, becomes an equality? The Meade Report failed to achieve this but can we provide positive analysis that will help any government contemplating imposing an ET system achieve its no distortion objective. To attain a Distortion Ratio equal to one, there are two changes required, one which will adjust the denominator in equation 11.81, and the other which adjusts the numerator in equation 11.81. For ease of reference, equation 11.81, which is a transformation of the expression for the rate of total return to all investors is reproduced below:

$$[11.81] \quad = \quad \frac{\theta_{ci}\,\mu_i + rD_i\,\tau_{ci}}{\theta_{ci}\,(S_i + S_i\,\tau_{cE} + D_i) + D_i\,\tau_{ci}} \quad = \quad \text{rate of total return to all investors}$$

Change No 1: Allow a tax deduction when debt is issued

This change affects the denominator in equation 11.81. If, as in the case of equity issues, the company receives a corporation tax refund at the tax-exclusive rate ($\tau_{cE}$) on the new debt that it issues, then the amount of cash available to the company for investment would be as follows:

| [11.90] | $S_i$ | + | $S_i\tau_{cE}$ | + | $D_i$ | + | $D_i\,\tau_{cE}$ |
|---|---|---|---|---|---|---|---|
| | equity issue | | corporation tax saved because of equity issue | | debt issue | | corporation tax saved because of debt issue |

We aim to derive the above expression (11.90) from the denominator of equation (11.81). We do so because we hope that the following two assertions can be shown to be correct:

(a) expression (11.90) will be a step towards achieving a non-distortionary tax system, and

(b) allowing tax deduction when debt is issued will result in expression (11.90).

The amount spent by investors for the purchase of equity and debt under the provisions for ET as per the Meade Report is as shown in the denominator of equation 11.81 .

$$[11.91] \qquad \theta_{ci}\,(S_i + S_i\,\tau_{cE} + D_i) + \qquad D_i\,\tau_{ci}$$

We aim to simplify equation 11.91 to see if we can get an expression similar to equation 11.90. To do so, we again make use of the relationship between tax exclusive and tax inclusive rates shown below

$$\tau_{cE} = \frac{\tau_{ci}}{1-\tau_{ci}} \quad \Rightarrow \quad \theta_{ci}\,\tau_{cE} = \frac{(1-\tau_{ci})\ \tau_{ci}}{(1-\tau_{ci})}$$

$$\Rightarrow \qquad \tau_{ci} = \theta_{ci}\,\tau_{cE}$$

We therefore replace $\tau_{ci}$ in equation 11.91 by $\theta_{ci}\,\tau_{cE}$ to get:

$$\theta_{ci}\,(S_i+S_i\tau_{cE} + D_i) \;+\; D_i\,\theta_{ci}\,\tau_{cE}$$

[11.92] $$= \quad \theta_{ci}\,(S_i+S_i\tau_{cE} + D_i \;+\; D_i\,\tau_{cE})$$

But the expression inside the brackets in equation 11.92 is the amount available for investment if debt issue also leads to a corporation tax refund, as is shown in equation 11.90. Hence allowing corporation tax refund on debt issue enables us to simplify the denominator of equation 11.81 in a meaningful way.

If only the numerator in equation 11.81 were taken to be $\theta_{ci}\,\mu_i$, then the rate of total return to all investors would be $\theta_{ci}\,\mu_i \;\div$ equation 11.92, which equals:

[11.92a] $$\frac{\theta_{ci}\,\mu_i}{\theta_{ci}\,(S_i+S_i\tau_{cE}+D_i+D_i\tau_{cE})} = \text{rate of return on physical investment}$$

Therefore, allowing a corporation tax refund on debt does not by itself accomplish the desired result. A second change, aimed at removing $rD_i\,\theta_{ci}$ from the numerator of equation 11.81 will accomplish the desired results.

<u>Change No 2: Corporation Tax should be charged on real interest payments to debt holders</u>.

If corporation tax is charged at the tax-exclusive rate $\tau_{cE}$ on real interest payments to debt holders, then we find that the total return to all investors is given by the simple expression $\theta_{ci}\,\mu_i$. Therefore, under such a tax system, the rate of total return to all investors would be equal to the rate of return on physical investment.

In order to see why the total return to all investors is $\theta_{ci}\,\mu_i$, we first need to consider the return expected by an equity investor. This return, under an S-base corporation tax system, is as given by equation 10.26 which is reproduced below

[11.93] $$y_i^m = n_i^m \left\{ [\mu_i - \tau_{cE}(\Delta_i - S_n) - \Delta_i - RD_i + pD_i] \frac{(1-\tau_\epsilon^m)}{1+\tau_{cE}} + \Delta_i\,(1-\tau_\epsilon^m \right\}$$

where $\tau_{cE}$ = corporation tax on S-basis, $\Delta_i$ = dividend, $S_n$ = new shares.

We alter equation (11.93) to reflect the two proposed changes.

Change 1: We subtract $D_i$ from the base on which corporation tax $\tau_{cE}$ is levied. Thus, for tax purposes, $D_i$ is treated like $S_n$.

Change 2: We add $rD_i$, the real interest rate, to the tax base on which corporation tax is levied. Thus, for tax purposes, $rD_i$ is treated like dividends ($\Delta_i$).

**Therefore the return expected by investor m from owning $n_i^m$ shares in company i, given an S-based corporation tax system modified as above, is given by:**

$$y_i^m = n_i^m \left\{ \frac{[\mu_i - \tau_{cE}(\Delta_i + rD_i - S_n - D_n) - \Delta_i - RD_i + pD_i]}{1+\tau_{cE}} (1-\tau_\epsilon^m) + \Delta_i (1-\tau_\epsilon^m \right\} \tag{11.94}$$

In equation 11.94, the square brackets represent the retained equity earnings. These retained equity earnings are the residue after the following items are deducted from the pre tax cash flow $\mu_i$, :

(a) Corporation tax at the rate $\tau_{cE}$ on the tax base shown in round brackets. The rate $\tau_{cE}$ is the rate on a tax-exclusive basis. The tax base, of this modified S-base system, includes payment of dividends ($\Delta_i$) and real debt interest ($rD_i$) to the investors, less any inflow from investors in the shape of either new ($S_n$) equity or new loans ($D_n$). It should be noted that for a particular project, it is only in the initial period that $S_n$ and $D_n$ will have a non-zero value (since investment is likely to be financed only at the very start of the project). Over the remaining life of the project, when it is yielding a return for the investors, $S_n$ and $D_n$ will be zero and the tax will be levied on ($D_i$ + $rD_i$). Alternatively, the annual equivalent of $S_n$ and $D_n$ (to perpetuity) can be considered for the sake of consistency with the other terms in the valuation model.

(b) Dividends. Since tax on dividends at the tax exclusive rate has already been deducted at (a) above, we deduct the full amount of cash dividends.

(c) $RD_i$ is the nominal interest paid to the debt holders.

(d) $pD_i$ is added back. p is the rate of anticipated inflation, and it is added back to take account of the fact that debt holders lose $pD_i$ of value due to inflation, and this value is gained by the equity shareholders.

Subtracting (a) to (c) and adding (d) gives the retained earnings. They are multiplied by $1-\tau_\epsilon^m$ $(=\theta_\epsilon^m)$ where $\tau_\epsilon^m$ is the expenditure tax rate applicable at personal level to the individual m. The underlying assumption is that the gains are realised every period by the investor and the proceeds spent on consumption expenditure. Finally, the entire term is multiplied by

$\frac{1}{(1+\tau_{cE})}$ because, in accordance with the view of the Meade Report, every pound of retained earnings is expected to result in only $(\frac{1}{1+\tau_{cE}})$ of capital gain. The reason for this lower figure of capital gain is that under such a tax system, the market will capitalise potential corporation tax liability (which would result when these retained gains are eventually distributed as dividends, and dividend payments attract corporation tax in this system). This was described more fully in chapter 10.

In addition to retained earnings, the equity investor receives $\Delta_i$ of dividends. It is assumed that these are spent on consumption goods. Therefore $\Delta_i$ is multiplied by $(1-\tau_\epsilon^m)$ in order to arrive at the return to the investor net of the applicable personal expenditure tax.

The above describes equation 11.94 which can be simplified as follows:

(i) replace $1-\tau_\epsilon^m$ by $\theta_\epsilon^m$ and multiply it by the terms inside the square brackets

(ii) collect coefficients of $\Delta_i$ and $D_i$ inside the square brackets

(iii) multiply $\frac{1}{1+\tau_{cE}}$ by terms inside the square brackets

These changes result in the following equation:

[11.95]
$$y_i^m = n_i^m \{ [\mu_i \frac{1}{1+\tau_{cE}} \theta_\epsilon^m - \Delta_i (1+\tau_{cE}) \frac{\theta_\epsilon^m}{(1+\tau_{cE})} - D_i r (1+\tau_{cE}) \frac{\theta_\epsilon^m}{(1+\tau_{cE})} ] + \Delta_i \theta_\epsilon^m \}$$

As shown in detail in chapter 10 , $\frac{1}{1+\tau_{cE}}$ can be replaced by $\theta_{ci}$, where $\theta_{ci}$ is $(1-\tau_{ci})$ where $\tau_{ci}$ is the corporation tax rate on a tax-inclusive basis. We make this substitution in equation 11.95.

$$y_i^m = n_i^m \{ \mu_i\theta_{ci}\theta_\epsilon^m - \Delta_i\theta_\epsilon^m - D_i r\theta_\epsilon^m + \Delta_i\theta_\epsilon^m \} \quad \text{or}$$

[11.96] $$y_i^m = n_i^m \{ \theta_\epsilon^m (\mu_i\theta_{ci} - D_i r) \}$$

Equation 11.96 gives the return to an equity investor who owns $n_i^m$ of the shares. The return to an investor who owns all the shares therefore is

$$y_i^m = \theta_\epsilon^m (\mu_i\theta_{ci} - D_i r).$$

We now add the return earned by an investor who owns all the debt to the above equity return. The return to debt would be $\theta_\epsilon^m (RD_i - pD_i) = \theta_\epsilon^m D_i (R-p) = \theta_\epsilon^m D_i r$. This shows that the debt holder earns nominal interest rate on debt, but loses $pD_i$ of value due to inflation at the rate p because debt is fixed in nominal terms. The return is subject to the personal expenditure tax implicit in $\theta_\epsilon^m$. Thus the total return to an investor who owns 100% of both debt and equity capital of company i, is as follows :

[11.97] $$\theta_\epsilon^m (\mu_i\theta_{ci} - D_i r) + \theta_\epsilon^m D_i r$$

$$= \theta_\epsilon^m (\mu_i\theta_{ci} - D_i r + D_i r)$$

[11.98] $$= \theta_\epsilon^m (\mu_i\theta_{ci})$$

Equation 11.98 implies that the cash flow received by all investors from the company before they pay their own personal expenditure taxes implicit in $\theta_\epsilon^m$, is $\mu_i\theta_{ci}$.

[11.99] Total return from company i to all investors $= \mu_i\theta_{ci}$

Dividing this total return in equation 11.99 by the amount given to the company by all the investors (which is shown in equation 11.92), we obtain the rate of return to all investors as follows:

Rate of total return to all investors $$= \frac{\theta_{ci}\,\mu_i}{\theta_{ci}\,(S_i + S_i\,\tau_{cE} + D_i + D_i\,\tau_{cE})} \qquad [11.99b]$$

$$= \frac{\mu_i}{S_i + S_i\,\tau_{cE} + D_i + D_i\,\tau_{cE}} =$$ Rate of return on physical investment [Eqn 11.92a]

Thus the rate of total return to all investors can equal the rate of return on physical investment if the two proposed changes are made. To summarise the entire discussion on the total return to all investors and expenditure tax, we note the following.

**With an S-based corporation tax under an expenditure tax system, the rate of total return to all investors is less than the rate of return on physical investment if debt and uncertainty are present. If we make two changes**

**(i) allow tax deduction on issue of debt, and**

**(ii) charge corporation tax on the payments to debt holders,**

**then the rate of total return to all investors will equal the rate of return on physical investment.** This equality will be demonstrated with the help of the simple example considered previously.

**Simple Example revisited**

The £200 of investment, under the revised S-based tax system, can be financed by issuing £40 of debt and £60 of equity, with the remaining £100 being received as a refund of corporation tax at the rate of 100% on a tax-exclusive basis.

If the interest rate is 10% (as considered previously) then out of the £300 gross return at the end of the year, £44 would be repaid to the debt holders. This would, at 100% tax-exclusive corporation tax rate, lead to a corporation tax charge of £44 in respect of this payment to debt holders.

This would leave £300 - £88 = £212 available for equity holders. Thus the shareholders could receive £106 as liquidating dividend, paying £106 balance as the corporation tax in respect of the dividend.

The total return received by investors, taking equity and debt holders together, is £106 + £44 = £150. Their total investment was £60 + £40 = £100. Hence the gross rate of total return to all investors is £150/£100 = 1.5. This is the same as the gross rate of return on physical investment (£300 ÷ £200 = 1.5).

Hence the tax system proposed does indeed lead to the equality between the rate of total return to investors and the rate of return on physical investment.

**A Comment on the tax bases**

How does the modified tax base proposed above compare with the tax bases described in the Meade Report? While the main concern of this thesis is the rates of returns (discount rates) under alternative tax systems, it is nevertheless pertinent to briefly examine the proposed modified tax base. The aim of this brief examination is to assess:

(a) whether there are any obvious defects in the proposed system, and

(b) whether there are advantages of our proposed system over the other bases discussed in the Meade Report.

In order to do so, it is necessary once again to start with the table describing the corporate flow of funds which was given in chapter 10. This table is shown on page 231 of the Meade Report and is reproduced below.

## CORPORATE FLOW OF FUNDS

| Inflows | Outflows |
|---|---|
| **Real Items** | |
| R1 Sale of produce | R1* Purchase of materials |
| R2 Sale of services | R2* Wages, salaries & purchase of other services |
| R3 Sale of fixed assets | R3* Purchase of Fixed Assets |
| R | R* |
| **Financial items other than shares of UK resident Corporate Bodies** | |
| F1 Increase in creditors | F1* Decrease in creditors |
| F2 Decrease in debtors | F2* Increase in debtors |
| F3 Increase in overdraft | F3* Decrease in overdraft |
| F4 Decrease in cash balance | F4* Increase in cash balance |
| F5 Increase in other borrowings | F5* Decrease in other borrowings |
| F6 Decrease in other lending | F6* Increase in other lending |
| F7 Interest received | F7* Interest paid |
| F8 Decrease in holding of shares in non-UK corporations | F8* Increase in holding of shares in non-UK corporations |
| F | F* |
| **Share items of corporate bodies resident in the UK** | |
| S1 Increase in own shares issued | S1* Reduction in own shares issued |
| S2 Decrease in holding of shares of other UK companies | S2* Increase in holding of shares in other UK companies |
| S3 Dividends received from other UK companies | S3* Dividends paid |
| S | S* |
| **Tax Items** | |
| T Tax repaid | T* Tax paid |

R + F + S + T (Total Inflows) = R* + F* + S* + T* (Total Outflows)

Note that a dotted line has been drawn after items marked F4 and F4* in the above table. The Meade Report discusses the following flow of funds corporation tax bases which make use of the above flow of funds table:

**R-basis** Tax is levied on real inflows less real outflows. Real outflows include not only trading expenses but also expenditure for the purchase of additions to the fixed asset or to stock of goods. This base can be represented as R-R*.

**R+F basis** Under this basis, the inflows from both real and financial transactions would be taxable, with outflows on both real and financial transactions being treated as tax-deductible expenses. This base includes interest received in item F7, and it shows interest paid as a tax-deductible item in F7*. Hence, the advantage of this base is that it could be used for taxing financial institutions as well. This base can be represented as (R+F) - (R*+F*).

**S basis** As shown at the bottom of the table, the total inflow of funds must be equal to the outflow of funds. Thus the following identity holds

[11.100] $R + F + S + T \equiv R^* + F^* + S^* + T^*$ **or**

[11.101] $(R+F) - (R^*+F^*) = (S^*-S) + (T^*-T)$

The left hand side of equation 11.101 is the R+F basis described above. This is equal to the sum of (a) S basis, where it is the outflows (of S* items) that are treated as taxable items less inflows (of S items) that are treated as "tax-deductible" items, and (b) the net tax paid.

Thus, if tax is charged at a tax exclusive rate on the S items (outflows less inflows), then this S basis is the same as R+F basis (shown on LHS of 11.101). Thus under the S basis, payment of dividends (an S* item) results in a corporation tax charge, whereas issue of new equity (an S item) results in a corporation tax refund.

The advantages of the S-base include the following:

(i) It is the simplest base to administer. It requires keeping track of very few items. The only relevant transactions for tax purposes are those involving share capital (of own company, or of other U.K. companies), and dividends received and paid.

(ii) Since this S basis is similar to the R+F basis, as shown by equation 11.101, this S basis can also be applied directly to the financial institutions.

The Meade Committee states in their conclusions on page 518 of the Report :

> "If, on the other hand, the base for personal taxation were shifted in the EXPENDITURE TAX direction, the corporation tax base would appropriately be shifted more completely than at present in the direction of a flow-of-funds base. In this case we would favour the (R+F) base, WITH THE S BASE AS A POSSIBLE ULTIMATE OBJECTIVE."

It is for the reasons stated above that we considered the S-base as the recommended accompanying corporation tax system whenever ET system was discussed in chapters 10 and 11.

The above describes the tax bases as discussed in the Meade Report. How does the modified S-base, which includes the two changes we suggest for achieving a Distortion Ratio equal to one, compare with the bases in the Meade Report?

For ease of reference, we may label the "modified S-base" recommended in this thesis as the "I-base", because it refers primarily to all Investors. That is precisely how the "modified S-base" (that is, the "I-base") operates. It levies corporation tax on outflows to all *investors* (equity and debt) and gives tax allowance on inflows from all investors (equity issues as well as debt issues). Hence an I-base would also tax *outflows* shown under the broken line after items F4 and F4* in the previous table. The corresponding inflows would be tax allowable items.

To be more precise, the table for corporate flow of funds which is shown in the Meade Report is modified. This results in the corporate flow of funds table on the following page.

## CORPORATE FLOW OF FUNDS

| Inflows | | Outflows | |
|---|---|---|---|
| **Real Items** | | | |
| R1 | Sale of produce | R1* | Purchase of materials |
| R2 | Sale of services | R2* | Wages, salaries & purchase of other services |
| R3 | Sale of fixed assets | R3* | Purchase of Fixed Assets |
| R | | R* | |
| **Financial Working Capital (FWC) Items** | | | |
| F1 | Increase in creditors | F1* | Decrease in creditors |
| F2 | Decrease in debtors | F2* | Increase in debtors |
| F3 | Increase in overdraft | F3* | Decrease in overdraft |
| F4 | Decrease in cash balance | F4* | Increase in cash balance |
| F4a | Interest recd on FWC | F4a* | Interest paid on FWC |
| FN | | FN* | |
| **Debt Investment Items** | | | |
| D1 | Increase in debt borrowings | D1* | Decrease in debt borrowings |
| D2 | Decrease in corporate debt lending | D2* | Increase in corporate debt lending |
| D3 | Interest received on corp debt | D3* | Interest paid |
| D4 | Decrease in holding of shares in non-UK corporations | D4* | Increase in holding of shares in non-UK corporations |
| D | | D* | |
| **Share Items Of Corporate Bodies Resident In The UK** | | | |
| S1 | Increase in own shares issued | S1* | Reduction in own shares issued |
| S2 | Decrease in holding of shares of other UK companies | S2* | Increase in holding of shares in other UK companies |
| S3 | Dividends received from other UK companies | S3* | Dividends paid |
| S | | S* | |
| **Tax Items** | | | |
| T | Tax repaid | T* | Tax paid |

| Total Inflows | | Total Outflows |
|---|---|---|
| R + FN + D + S + T | = | R* + FN* + D* + S* + T* |
| Therefore | | |
| (D* + S*) - (D + S) + (T* - T) | = | (R + FN) - (R* + FN*) |

The I-base flow of funds table has redefined items F1 to F4 as financial working capital items because this term best describes what they are. Interest paid (item F7* in the Meade Report table) has been subdivided into two components. The first, referred to as F4a*, is interest paid on overdraft and other short term borrowings. This interest element is therefore included within the definition of financial working capital. The second component, D3*, includes only the interest paid on debt which is financing the business from the long term point of view. Therefore it is included within the long term Debt Investment Items (see below).

Since items F5 to F8 in the Meade Report table are essentially items concerned with net long term debt (after we have separated out all the short term items in F1 to F4a), they have been classified separately as debt investment items. The tax base under I-base corporation tax system where tax is charged at tax-exclusive rates, is:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| I BASE : | D* | + | S* | - | (D | + | S) |
| Corresponding items from equation 11.94 : | $rD_i$ | | $\Delta_i$ | | $D_n$ | | $S_n$ |
| I-base table reference : | (D3*) | | (S3*) | | (D1) | | (S1) |

The I Base includes all items that comprise D*, S*, D and S in the previous table. Some of the more common items within the I Base categories shown above are D3*, S3*, D1 and S1. These items are listed in the third row above. Their corresponding variable in equation 11.94 are listed in the second row above. These references will be useful in subsequent analysis of the I Base.

Note that since the tax base as shown above does not include the tax itself, corporation tax under I-basis will be charged at a tax-exclusive rate.

Guidance can be taken from the Finance literature as to which debts should qualify to be classified under "Debt Investment Items", and which ones can be treated as part of financial working capital. The Finance literature is the relevant literature to use for guidance because the proposed tax system is based upon the concepts used in the Finance literature. Debt finance generally refers to long term and fairly stable investment in the literature dealing with returns to debt and equity investors. Hence only such debt items should qualify for corporation tax refund as item "D1" in the table. The question of definition of debt in finance literature is comprehensively dealt with by J Rutterford (PhD thesis, London 1986). We define debt finance as those debt items that appear in the Balance Sheet as libilities due after more than one year. Although there can be problems in practice in distinguishing debt in such manner, we consider it is appropriate to distinguish between Financial Working Capital and Debt Investment terms for the same reasons as the two are distinguished in the Meade Report as well as in finance literature, namely we treat as debt holders only those providers of debt capital who are long term investors.

Once debt has been classified into Financial Working Capital or Debt Investment Items, there should be no difficulty as far as imposing tax on outflows to debt investors is concerned. Only those outflows, which are in respect of debts on which tax relief had been given when the debt was first issued, would be subject to corporation tax. Hence identifying cash flows to the debt holders which should be subject to tax should not be a problem under the I Base system.

It may appear that I-basis present difficulties when dealing with financial institutions, since what is being proposed is a system where interest **payments** are taxed and interest **receipts** are an allowable expense. At first glance, this appears absurd but this treatment is on exactly the same lines as the treatment of dividends received under an S-base system. In item S3 in the table in chapter 10, the Meade Report recommends that dividends received from other U.K. companies should result in a corporation tax refund. Secondly, the interest received results in a tax refund only after the entire principal lent had earlier been subject to corporation tax as item D2* in the above table. Thirdly, for financial institutions such as banks, interest received and interest paid will be included as items F4a and F4a* respectively, that is, it will be treated as part of their working capital. This means that interest received and paid by banks would receive exactly the same treatment under the I-base system as it would have under the S-base system. Finally, in the case of an R-base system, the Meade Report recognises that financial institutions have to be dealt with separately. Similarly if financial institutions have to be dealt with separately under an I-base system, then that fact alone does not render the proposed I-base system unworkable.

After having defined and discussed the I-base system above, we examine whether the proposed I base system has some advantages over the bases proposed in the Meade Report. The reason why we advocate it is because the proposed I-base has advantages over both the S-base and the R-base systems. These are described below.

(a) **Advantages of I-base over S-base**

(i) The main advantage of I-base over an S-base is that with an I-base, even **in the presence of debt and uncertainty, the rate of total return to all investors is equal to the rate of return on physical investment.** The S-base recommended by the Meade Report does not lead to such an equality, as was shown above.

(ii) **The I-base treats all investors in a similar manner, whether they be equity holders or debt holders.** Cash inflows from either source result in a corporation tax refund and cash outflows to either equity holders or debt holders results in a corporation tax liability. This symmetry is not present under an S-base system.

It may appear that what is being proposed under the I base system will be unduly harsh on the debt holders, namely, that they will not benefit from the tax deductibility of interest payments at the corporate level. However, debt investment will be compensated for this by enjoying corporation tax deduction on the full capital amount when debt is issued. Therefore the proposed I base does not impose any great extra burden on debt investment, but merely shifts the tax preference from the income stream to the capital inflow.

(b) **Advantages of I-base over R-base**

(i) **An I-base, perhaps with a few modifications, can be easily applied for determining the taxable income of financial institutions.** However, an R-base is completely unsuitable for financial institutions, since it does not tax financial transactions at all.

(ii) **The response of share price to an increase in retained equity cash flows is very clearly defined under an I-base.** On the other hand, an R-base system introduces another source of confusion as to how share prices should respond to an increase in retained earnings. This is explained in detail below.

If the corporation tax system is not a flow of funds system, but instead it is based on corporate profits (that is, if we had a P-base system which was discussed in the context of Comprehensive Income Tax system in chapter 10), then the expected impact of retained earnings on share prices is very clear-cut. Since there are no potential *corporate* tax liabilities attached to distribution of retained earnings in such a system, every £1 of retained earnings is expected to result in £1 of capital gain. Of course, changes expected in other parameters such as interest rates would mean that capital gains differ from retained earnings. However, on an ex-ante basis, under a P-base system, £1 of retained earnings is expected to result in £1 of capital gain.

If, instead, the tax system has an S-base, then there is a potential corporation tax liability which depresses the share prices. The Meade Report states that since equity investors anticipate that corporation tax will be payable whenever retained earnings are distributed, they capitalise this potential liability and as a result, for every £1 of retained earnings, the capital gain is only $(1/1+\tau_{cE})$. Therefore the term in the brackets is multiplied by the expression representing retained earnings in order to derive the capital gains accruing to an equity investor. For example, this was done in the case of equation 11.94 which is reproduced below after minor simplifications.

[11.102] $$y_i^m = n_i^m \{ (\mu_i - \tau_{cE} [\Delta_i + rD_i - S_n - D_n] - \Delta_i - RD_i + pD_i)^x \left( \frac{1}{1+\tau_{cE}} \right)^z \theta_\epsilon^m + \Delta_i \theta_\epsilon^m \}$$

where [...] = tax base, $(\ldots)^x$ $(\ldots)^z$ = retained earnings, $\Delta_i$ = dividend.

Under an I-base, the valuation effect would be similar to that under S base. In other words £1 of retained earnings lead to $(1/1+\tau_{cE})$ of capital gain, as is shown in equation 11.102 above. A company can hold retained earnings in a number of different forms, eg. either as an increase in cash balance or as an increase in stock of goods. Whatever the form these retained earnings take, there is no immediate corporation tax charge under an I-base. The tax charge arises only when the retained earnings are distributed to the shareholders. Hence under an I-base system, it is quite clear that every £ of retained earnings should lead to $(1/1+\tau_{cE})$ of capital gain for the investor.

Under an R-base system, however, retained earnings cannot unambiguously be related to £1 or to £$1/1+\tau_{cE}$ of capital gain. Whether the capital gain is £1 or it is $1/1+\tau_{cE}$ depends upon the form in which retained earnings are held, that is, it depends upon whether the retained earnings are held as a real asset (eg. stock of goods) or as a financial asset (eg. cash). This is so because under an R-base system, any cash outflow used to purchase a real asset will result in a tax allowance for corporation tax purposes (see item R1* in the table and note that R-base is (R-R*)). As a result, there is a potential corporation tax liability attached to this real asset, because when this asset is eventually sold, (as item R1 in the table) there will be a corporation tax liability. Hence under an R-base tax, all real assets have an accompanying potential corporation tax liability attached to them. This point is stated quite clearly in the Meade Report on page 236. As a consequence of this valuation effect, there is an immediate tax refund (which increases retained earnings!) but a potential tax liability if the retained earnings are held as a increase in stock. This potential liability means that capital gains should be $(1/1+\tau_{cE})$ times the amount invested in real assets, where $\tau_{cE}$ is the relevant corporation tax rate. However, because of immediate tax refund when this stock is purchased, the company should receive $\tau_c$ $H_i$ of tax refund which increases the retained earnings and which consequently increases the capital gain ($H_i$ = the amount of retained earnings represented by an increase in the stock). As a net result of these two tax effects, the potential tax effect and the immediate tax effect, the final capital gain should be equal to the retained earnings invested in stock.

On the other hand, if the retained earnings are invested in cash, then quite clearly the capital gain in respect of £1 of earnings is £1 of gain because there is no potential tax liability involved. Therefore the magnitude of the capital gain which should result from an increase in retained earnings will depend upon the nature of assets into which these retained earnings are invested. Hence there is considerable scope for misunderstanding the magnitude of the variables involved and consequently, there is scope for incorrect valuation of companies because of the tax effects under an R-base. As it is, equity valuation under existing tax rules is a difficult area and the disadvantage of the R-base system is that it may lead to more confusion in valuation. In contrast, P-base, S-base and I-base tax systems do not suffer from this disadvantage.

**Conclusion to section (C)**

Due to the inability of the S-base corporation tax system to give a Ratio of one when debt and uncertainty are present, we have devised the I-base system. The above discussions show that the I-base is a practicable and robust base which also possesses various advantages over the other bases discussed in the Meade Report.

## Section (D)

## THE DISTORTION RATIO WHEN COMPANIES EARN SUPERNORMAL PROFITS

All of the previous discussion of the Meade Report (with the exception of the simple example considered earlier), has been conducted assuming a competitive environment in which the physical investment is valued according to the value of the cash flows it generates. This implies, if we assume full equilibrium, that all projects earn only a normal rate of return, and relies on the concepts used extensively in economics literature that competition would eliminate supernormal profits.

In Finance literature, however, we are concerned also with projects that earn a positive net present value. The particular firm undertaking the project may enjoy a "competitive advantage" over other firms, which will result in either its costs being lower or its revenues being higher than if that project was undertaken by other firms. Projects which earn positive net present value are also extensively discussed in the business strategy literature, and are very relevant to the real world project appraisal. Therefore, the aim of this section is to assess how far are the Ratios developed in the previous sections, applicable to an environment where firms earn a positive net present value. If we use the terminology of the Meade Report in respect of the simple example (section (C)), the question being addressed in this section is if, and how, does the Distortion Ratio change under different tax systems if the firm undertaking the project earns supernormal profits.

In order to discuss this question, we discuss the following cases:

| | | | |
|---|---|---|---|
| 1 | Comprehensive Income Tax | (a) Return to equity | (b) Return to assets |
| 2 | Expenditure Tax | (a) Return to equity | (b) Return to assets |

**(D)(1) Comprehensive Income Tax With Supernormal Profits**

**(D)(1)(a) Return to equity case, with supernormal profits, under CIT**

Previously, we assumed that under CIT system, the cost of physical investment was equal to $V_i$, the value of cash flows generated by the capital equipment. The simplest way of introducing supernormal profits is to assume that for the particular firm undertaking the project, the cost of capital equipment is not $V_i$ but $\alpha V_i$ where $\alpha$ is a fraction between 0 and 1. In the examples considered below, we will take $\alpha = 0.7$. Now since the cost of physical investment has been reduced from $V_i$ to $\alpha V_i$, the rate of return on physical investment would increase (from $\mu_i/V_i$ to $\mu_i/\alpha V_i$).

Since the physical investment now costs only $\alpha V_i$, the amount of cash required to be invested by equity holders is also proportionately reduced - that is, we need only raise $\alpha V_i$ of cash to undertake this investment. We will make use of the following definitions:

$$
[11.103] \qquad \begin{cases} \text{Cash raised from equity} & = S_n = \alpha S_i \\ \text{Cash raised from debt} & = D_n = \alpha D_i \\ \text{Total cash raised} & = I = S_n + D_n = \alpha S_i + \alpha D_i \end{cases}
$$

$$
\begin{aligned} &= \text{cost of physical investment} \\ &= \alpha(S_i + D_i) = \alpha V_i \end{aligned}
$$

The rate of return for equity investors who have invested $S_n$ in the investment is given by:

[11.104] $$\frac{(\mu_i - rD_n)\,\theta_c/\theta_j\,\theta_p^m}{S_n}$$

In equation 11.104, the numerator represents the cash flow, net of corporation tax (implicit in $\theta_c/\theta_j$), personal income tax (implicit in $\theta_p^m$), and debt interest $rD_n$. Note that debt level is taken as $D_n$, the actual amount of debt issued to finance this investment whose total cost, $\alpha V_i$ is less than the "zero-super-normal profit" cost of $V_i$. Dividing this numerator by $S_n$, the actual amount invested by equity holders, gives the rate of return to equity holders. The rate of return on physical investment, as explained above, is $\mu_i/\alpha V_i$. Hence the Ratio can be calculated by dividing equation 11.104 by $\mu_i/\alpha V_i$ to get:

[11.105] $$\frac{(\mu_i - rD_n)\,\theta_c/\theta_j\,\theta_p^m}{S_n} \quad / \quad \frac{\mu_i}{\alpha V_i}$$

Making use of equation 11.103 to substitute for $D_n$ and $S_n$, we get

$$= \frac{(\mu_i - r\alpha D_i)\,\theta_c/\theta_j\theta_p^m}{\alpha S_i} \quad \text{x} \quad \frac{\alpha V_i}{\mu_i}$$

$$= \frac{(\mu_i - r\alpha D_i)\,\theta_c/\theta_j\,\theta_p^m}{\mu_i} \quad \text{x} \quad \frac{V_i}{S_i}$$

[11.106] $$= \theta_p^m \,.\, \theta_c/\theta_j \,.\, \frac{\mu_i - r\alpha D_i}{\mu_i} \,.\, \frac{V_i}{S_i}$$

Equation 11.106 is very similar to the expression for the Ratio derived in the case of normal profits (eq. 11.50), which is reproduced below:

$$\theta_p^m \,.\, \theta_c/\theta_j \,.\, \frac{\mu_i - rD_i}{\mu_i} \,.\, \frac{V_i}{S_i} \qquad \text{(normal profit case)}$$

The only change is that the numerator in the fraction $(\mu_i - r\alpha D_i)/\mu_i$ is now larger than the numerator $(\mu_i - rD_i)/\mu_i$ since $\alpha$ is a fraction between 0 and 1. Hence the distortion Ratio will be somewhat larger, but not significantly different because all the other terms are the same as before.

The conclusion deduced from the above is that under CIT, the Ratio for return to equity case improves slightly if the firm undertaking the investment is making supernormal profits. However, the improvement is only marginal and there is no need to revise any of the previous conclusions regarding the impact of CIT on the Ratio, even when firms make supernormal profits.

The fact that the improvement in the Distortion Ratio is marginal can be verified by continuing with the same example as in section (A). In that example, under CIT, with debt and uncertainty, and normal profits, the cost of investment, $V_i$, is £291.982. Therefore, the new variables as shown in equation 11.103, have the following values:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| I | = | cost of investment | = | $\alpha V_i$ | = | 0.7 x £291.982 | = £204.387 |
| $D_n$ | = | cash raised from debt | = | $\alpha D_i$ | = | 0.7 x £100 | = 70.000 |
| $S_n$ | = | cash raised from equity | = | $\alpha S_i$ | = | 0.7 x 191.982 | = 134.387 |

Substituting the above figures into equation 11.104, and using the same values as before for the other variables we get the following return to equity under the supernormal profits scenario :

$$\frac{(50 - 0.05 \times 70) \times 0.82278 \times 0.75}{134.387}$$

$$= \quad 46.5 \times 0.82278 \times 0.75 \;/\; 134.387$$

$$= \quad 28.6944 \;/\; 134.387 \quad = \quad 0.2135\ (=21.35\%)$$

The return on equity under the normal profits scenario, when the investment cost $V_i$ (=£291.982) is financed by £100 debt and £191.982 of equity, is as shown below:

$$\frac{(50 - 0.05 \times 100) \times 0.82278 \times 0.75}{191.982}$$

$$= \quad 45 \times 0.82278 \times 0.75 \;/\; 191.982$$

$$= \quad 27.7688 \;/\; 191.982 \quad = \quad 14.46\%$$

The normal return expected on this investment costing £291.982 is 14.46%. However, since the cost of investment has decreased to £204.387 ($\alpha V_i$), the equity investors also earn super-normal profits in addition to the normal return - this pushes up the rate of return to equity to 21.35% as calculated above.

The rate of return on physical investment under supernormal profit scenario is $\mu_i/\alpha V_i$ which equals:

$$50 / 204.387 = 0.2446$$

The Distortion Ratio can be calculated by dividing the rate of return to equity by the rate of return on physical investment, as shown below:

$$0.2135 / 0.2446 = 0.8729$$

This Ratio, of 0.8728 when the firm makes supernormal profit, compares favourably with the Ratio of 0.8447 (eq. 11.50a) calculated when the underlying assumption was of normal profits only. The improvement is only marginal and it does not alter the earlier conclusion that even under CIT, the Ratio can improve with the presence of debt under uncertainty.

We now calculate the Distortion Ratio for the return on asset case, that is we define the return to include total returns to debt and equity investors in the following subsection.

**(D)(1)(b) Return on assets case, with supernormal profits, under CIT**

In the return on assets case, we add the returns to equity and debt holders, and compare this rate with the rate of return on physical investment. The rate of total return to all investors is

$$\frac{[(\mu_i - rD_n)\,\theta_c/\theta_j\theta_p^{m}] + \{rD_n\,\theta_p^{m}\}}{S_n + D_n} \qquad \begin{array}{l}[\ldots] = \text{return to equity as per Eqn 11.104} \\ \{\ldots\} = \text{return to debt}\end{array}$$

[11.107]
$$\frac{\theta_p^{m}\,[\mu_i\,\theta_c/\theta_j - rD_n\,(\theta_c/\theta_j - 1)]}{\alpha S_i + \alpha D_i}$$

$$= \frac{\theta_p^{m}\,[\mu_i\,\theta_c/\theta_j + rD_n\,(1 - \theta_c/\theta_j)]}{\alpha\,(S_i + D_i)}$$

[11.108]
$$= \frac{\theta_p^{m}\,[\mu_i\,\theta_c/\theta_j + rD_n\,L]}{\alpha V_i}$$

The Distortion Ratio is calculated by dividing equation 11.108 by the rate of return on physical investment, as follows:

$$\frac{\theta_p^m\,[\mu_i\,\theta_c/\theta_j + L\,rD_n]}{\alpha V_i} \quad \div \quad \frac{\mu_i}{\alpha V_i}$$

$$= \quad \frac{\theta_p^m\,[\mu_i\,\theta_c/\theta_j + L\,r\alpha D_i]}{\alpha V_i} \quad x \quad \frac{\alpha V_i}{\mu_i}$$

$$= \quad \frac{\theta_p^m\,[\mu_i\,\theta_c/\theta_j + L\,r\alpha D_i]}{\mu_i}$$

[11.109] $$= \quad \theta_p^m\,[\mu_i\,\theta_c/\theta_j + L\,\frac{r\alpha D_i]}{\mu_i}$$

This Ratio can be compared with the Distortion Ratio calculated in section (B) (when there were only normal profits). That Ratio is shown below.

[11.110] $$= \quad \theta_p^m\,[\mu_i\,\theta_c/\theta_j + L\,\frac{rD_i]}{\mu_i}$$

Since $\alpha$ is a positive fraction which is less than one, the second term within square brackets in equation 11.109 is smaller than the corresponding term in equation 11.110. As a result, the entire expression in equation 11.109 is slightly smaller than the expression in equation 11.110. This implies that when return to all investors is considered together, the Ratio in the presence of supernormal profits is marginally lower than the Ratio when only normal profits are considered.

With $\alpha$ = 0.7 as before, the Ratio in equation 11.109 is equal to 0.6264. This is only marginally below the Ratio of 0.63 considered in section (B) when only normal profits were present. Hence, the presence of supernormal profits does not have any significant impact on the Ratio under CIT when the total return to all investors together is considered.

## (D)(2) Expenditure Tax With Supernormal Profits

### (D)(2)(a) Return to equity case, with supernormal profits, under ET

Under expenditure tax, the amount of cash available for investment includes the corporation tax refund available whenever equity is issued, in addition to the cash raised from equity and debt. Therefore, the cash raised for investment is

$S_i + S_i \tau_{cE} + D_i \quad = \quad S_i/\theta_{ci} + D_i$ (see section (B) table column (viii)).

We again assume that the amount of cash required for investment in the supernormal profits case, is only a proportion $\alpha$ of the amount required in the normal profits case.

[11.111] $I$ = cash required for Investment $= \alpha\,(S_i/\theta_{ci} + D_i)$

$= \alpha S_i/\theta_{ci} + \alpha D_i$

The rate of return on physical investment is calculated by dividing $\mu_i$, the pre-tax return, by the cost of investment, which is given by equation 11.111 above. This gives the rate of return on physical investment as:

[11.112] $\mu_i \,/\, \alpha(S_i/\theta_{ci} + D_i)$

The rate of return on consumption foregone by the equity investor is as given by equation 11.69 , except that we need to substitute $D_n$ $(=\alpha D_i)$. Similarly, the amount of equity issued is $S_n$ $(=\alpha S_i)$. Hence the rate of return to equity, in the super-normal profits case is:

[11.113] $(\mu_i - D_n\, r)\, \theta_{ci}/S_n$

[11.114] $= (\mu_i - \alpha D_i\, r)\, \theta_{ci}/\alpha S_i$

The Ratio can now be calculated by dividing equation 11.114 by equation 11.112. This gives

[11.115] $$\frac{(\mu_i - \alpha D_i\, r)\, \theta_{ci}}{\alpha S_i} \quad x \quad \frac{\alpha(S_i/\theta_{ci} + D_i)}{\mu_i}$$

As shown in Appendix B4, this equation 11.115 can be simplified to the following:

[11.116] $$\text{Ratio (supernormal profits case)} \;=\; 1 \;\;x\;\; \frac{\mu_i - \alpha D_i r}{[\mu_i - D_i r] - \dfrac{D_i\, \beta_i \pi}{\theta_\epsilon}}$$

A similar Ratio was calculated when there were only normal profits in Appendix B1. That Ratio is reproduced below:

[11.117] $$\text{Ratio (normal profits case)} \;=\; 1 \;\;x\;\; \frac{\mu_i - D_i r}{[\mu_i - D_i r] - \dfrac{D_i\, \beta_i \pi}{\theta_\epsilon}}$$

The denominators in equations 11.116 and 11.117 are exactly the same. In the numerator if equation 11.116, $\alpha$ is a positive fraction which is less than one. Therefore the entire numerator ($\mu_i$-$\alpha D_i r$) is larger than the corresponding numerator in equation 11.117. Hence, equation 11.116 (supernormal profits case) gives a somewhat larger ratio than equation 11.117.

The ratio in equation 11.117 is always greater than one since the denominator in equation 11.117 will always be less than the numerator (note that the term $D_i\ ß_i\pi/\theta_\epsilon$ is deducted in the denominator which makes it comparatively smaller). This implies, as noted before, that with an S-base corporation tax system under ET, the Ratio for the return on equity case with normal profits is always greater than one. It implies that the Ratio, when supernormal profits are present, will be even greater than one.

Since the Distortion Ratio does not equal one, the claim made in the Meade Report (that the Ratio is always equal to one) is untenable for projects which earn supernormal profits.

**(D)(2)(b) Return on assets case, with supernormal profits, under ET**

Again, we simply add the cash flow received by debt investors ($rD_n$) and the cash invested by debt investors ($D_n$) to the numerator and the denominator of equation 11.115 respectively, in order to calculate the rate of total return to all investors. This gives the following equation:

[11.118]
$$\frac{(\mu_i-\alpha D_i r)\ \theta_{ci} \ + \ rD_n}{\alpha S_i \ + \ D_n}$$

$$= \frac{\mu_i\ \theta_{ci} - \alpha D_i\ r\ \theta_{ci} \ + \ r\alpha D_i}{\alpha S_i \ + \ \alpha D_i} \qquad (\text{since } D_n = \alpha D_i)$$

[11.119]
$$= \frac{\mu_i\ \theta_{ci} \ + \ \tau_{ci}\ \alpha r D_i}{\alpha(S_i \ + \ D_i)} \qquad (= \text{Total rate of return to investor})$$

The rate of return on physical investment is as given by equation 11.112

$$\mu_i\ /\ \alpha\ (S_i/\theta_{ci} + D_i)$$

$$= \frac{\mu_i}{\dfrac{\alpha\ (S_i \ + \ \theta_{ci}\ D_i)}{\theta_{ci}}}$$

[11.120] $= \mu_i\theta_{ci} \,/\, \alpha(S_i + \theta_{ci} D_i)$

Dividing equation 11.119 by equation 11.120 gives the Ratio when we have supernormal profits. This Ratio is as follows.

$$\frac{\mu_i \theta_{ci} + \tau_{ci} \alpha r D_i}{\alpha(S_i + D_i)} \quad x \quad \frac{\alpha(S_i + \theta_{ci} D_i)}{\mu_i \theta_{ci}}$$

$$= \frac{\mu_i \theta_{ci} + \tau_{ci} \alpha r D_i}{\mu_i \theta_{ci}} \quad x \quad \frac{S_i + \theta_{ci} D_i}{S_i + D_i}$$

[11.121] $$= \frac{\theta_{ci} \; (\mu_i + \tau_{ci}/\theta_{ci} \alpha r D_i)}{\theta_{ci} \; \mu_i} \quad x \quad \frac{S_i + \theta_{ci} D_i}{S_i + D_i}$$

We need to compare equation 11.121 with a suitable equation which gives the Ratio when no super-normal profits are present. A suitable equation can be found by transforming the third equation in Appendix B2 as follows:

$$\text{Ratio (normal profits case)} = \frac{\mu_i - rD_i + rD_i/\theta_{ci}}{\mu_i} \quad x \quad \frac{S_i + D_i \theta_{ci}}{S_i + D_i}$$

[11.122] $$= \frac{\mu_i + rD_i \,(1/\theta_{ci} - 1)}{\mu_i} \quad x \quad \frac{S_i + D_i \theta_{ci}}{S_i + D_i}$$

The term within brackets in equation 11.122 can be rearranged as follows:

$$\left(\frac{1}{\theta_{ci}} - 1\right) = \left(\frac{1}{\theta_{ci}} - \frac{\theta_{ci}}{\theta_{ci}}\right) = \frac{(1-\theta_{ci})}{\theta_{ci}} = \frac{(1 - (1-\tau_{ci}))}{\theta_{ci}} = \frac{(1 - 1 + \tau_{ci})}{\theta_{ci}} = \frac{\tau_{ci}}{\theta_{ci}}$$

Making the above substitution into equation 11.122, we get a suitable equation which gives us the Ratio when normal profits are present.

[11.123] $$\frac{\mu_i + \tau_{ci}/\theta_{ci}\, rD_i}{\mu_i} \quad x \quad \frac{S_i + D_i \theta_{ci}}{S_i + D_i}$$

We call equation 11.123 a suitable equation because it is similar to equation 11.121, which gives the Ratio when super normal profits are present. The only difference between the two equations is that in the numerator in equation 11.121, we have $\alpha r D_i$ instead of $rD_i$ in equation 11.123. Since $\alpha$ is a positive fraction which is less than 1, as a consequence, the Ratio in equation 11.121 (the supernormal case) is smaller that the

Ratio in equation 11.123.

We have discussed extensively in section (B) as to why the Distortion Ratio in equation 11.123 is less than one. Since the Ratio in equation 11.121 is smaller than in equation 11.123 (see above paragraph), it follows that the Ratio in equation 11.121 must be even further away from one. This implies that when supernormal profits are present, the rate of total return to all investors is less than the rate of return on physical investment. Hence the claim made in the Meade Report that the Distortion Ratio is one under expenditure tax is even more untenable when we consider projects that earn a supernormal profit. In reality, companies mostly undertake positive NPV projects and consequently enjoy supernormal profits. Therefore the conclusions of the Meade Report are inappropriate for the taxation of companies in reality.

Does the I-Base system recommended in this thesis fare any better in removing tax distortion when supernormal profits are present? In order to answer this, we need to calculate the Distortion Ratio. The first step is to calculate the rate of total return to all investors under an I Base Expenditure Tax system with supernormal profits. We begin with the equation for the rate of return to all investors in the normal profits S base expenditure tax case and make the appropriate adjustments for the I Base and the supernormal profits case. The rate of return to all investors in normal profits S-Base case is given by equation 11.75 which is reproduced below.

$$[11.75] \qquad \frac{(\mu_i - rD_i)\,\theta_{ci}\,\theta_\epsilon^{\,m} + rD_i\,\theta_\epsilon^{\,m}}{\theta_\epsilon^{\,m}\,(S_i + D_i)}$$

One of the tax changes which we recommended for an I Base system is that the payments to debt holders should be taxed. This change results in the following adjustments to the numerator in equation 11.75 above :

$$\begin{aligned}
& (\mu_i\theta_{ci} - rD_i\theta_{ci} - rD_i\tau_{ci})\,\theta_\epsilon^{\,m} + rD_i\,\theta_\epsilon^{\,m} \\
= \quad & (\mu_i\theta_{ci} - rD_i + rD_i\tau_{ci} - rD_i\tau_{ci})\,\theta_\epsilon^{\,m} + rD_i\,\theta_\epsilon^{\,m} \\
= \quad & (\mu_i\,\theta_{ci} - rD_i)\,\theta_\epsilon^{\,m} + rD_i\,\theta_\epsilon^{\,m} \\
= \quad & \mu_i\,\theta_{ci}\,\theta_\epsilon^{\,m}
\end{aligned}$$

The above expression is independent of the value of debt ($D_i$) or the value of equity ($S_i$).

The denominator of equation 10.75 above can be changed to reflect that the amount originally invested by the investors is only a fraction $\alpha$ of the value of equity and debt - the difference is accounted for by the supernormal profit that the investors enjoy. Hence the rate of total return to all investors is :

$$\frac{\mu_i \; \theta_{ci} \; \theta_\epsilon^{\,m}}{\alpha \, \theta_\epsilon^{\,m} \, (S_i + D_i)}$$

[11.124] $$\frac{\mu_i \, \theta_{ci}}{\alpha \, (S_i + D_i)}$$ (Rate of total return to all investors I Base ET system with supernormal profits)

The rate of return on physical investment is calculated by dividing $\mu_i$, the pre-tax return on projects, by the cost of investment. The cost of investment in the supernormal profits case is the cash raised from investors plus the accompanying tax benefits under an I Base system. The cash raised from investors is $\alpha$ ($S_i + D_i$) as stated above. The tax benefit under I Base is available on both the debt and the equity capital raised. The tax benefit thus is $\alpha$ ($S_i + D_i$) $\tau_{cE}$. Therefore the rate of return on physical investment is:

$$\frac{\mu_i}{\alpha \, (S_i + D_i) + \alpha \, (S_i + D_i) \, \tau_{cE}}$$

$$= \frac{\mu_i}{\alpha \, (S_i + D_i) \; (1 + \tau_{cE})}$$

$$= \frac{\mu_i}{\alpha \, (S_i + D_i)} \times \frac{1}{(1 + \tau_{cE})}$$

As noted earlier in equation 10.22, $1 / (1 + \tau_{cE}) = \theta_{ci}$. Making this substitution, the above equation simplifies to :

(11.125) $$\frac{\mu_i \, \theta_{ci}}{\alpha \, (S_i + D_i)}$$ (Rate of return on physical investment : I Base Expenditure Tax system with supernormal Profits)

The Distortion Ratio can be calculated by dividing equation 11.124 by equation 11.125:

$$\frac{\mu_i \theta_{ci}}{\alpha (S_i + D_i)} \Big/ \frac{\mu_i \theta_{ci}}{\alpha (S_i + D_i)}$$

$$= \quad 1$$

This implies that there is no Distortion under the I Base system, even when the companies earn supernormal profits. Hence, our recommended I Base system is robust enough to deal with supernormal profits - which is something that the S Base system recommended in the Meade Report failed to do. **Therefore our I Base system, with its very simple underlying logic of (a) treating inflows from debt or equity holders symmetrically, and (b) treating outflows to debt or equity holders symmetrically, is superior in coping with the real world conditions and is a fully non-distortionary system, which is something that the Meade Report aimed for but failed to achieve.**

This completes our analysis of the Meade Report. The conclusions to this chapter include the following :

(a) The recommended valuation model (equation 7.22) is very useful in analysing the situations when debt, uncertainty and supernormal profits are present. The simpler models used in the Meade Report fail to do so.

(b) The total rate of return to all investors can be below one even under the S Base expenditure tax system. This is a weakness in the recommendations of the Meade Report.

(c) The S base system recommended by the Meade Report needs two adjustments to make it a non distortionary system, namely taxing payments to the debt holders, and allowing tax deductibility when debt finance is raised.

(d) The I Base system recommended in this chapter is non-distortionary, logical, applicable to real world capital structures as well as to positive NPV projects, and therefore may be seen as superior to the tax bases recommended in the Meade Report. It is based on the simple premise of treating debt and equity holders symmetrically.

(e) Distortion Ratios under different assumptions were derived in this chapter. The relationship between the Distortion Ratios is fully examined and the consistency of the formulae obtained were also checked in this chapter. These Distotion

Ratios represent a contribution to the literature and are useful for organisations which aim to devise non-distortionary tax systems.

(f) The complex valuation models used here are necessary to analyse tax issues in order to avoid the mistakes that can result from the use of the simpler models.

# CONCLUSION

We conclude by reviewing the following in three sections:

Section A : Summary of the motivation for this thesis

Section B : Summary of and the conclusions reached in each of the chapters

Section C : Summary of the contributions made in this thesis.

## Section A
## Motivation for this thesis

We began this thesis in chapter 1 by stating that in 1958, Modigliani and Miller initiated an important debate in modern finance literature when they stated that in the absence of taxes, the cost of capital of a firm is independent of its capital structure. They later modified and then corrected their view in 1963 when they stated that the introduction of corporation taxes into their model implied that there is a tax advantage to leverage and therefore corporate taxes influenced the cost of capital. The influence of taxes was very significant in their model, changing the conclusion from that a firm should be indifferent between debt and equity to that the firm should be 100% debt financed.

Modigliani and Miller considered only simple form of corporation taxes in their model. Subsequently, in the 1970s and 1980s, models which incorporated personal as well as corporation taxes in somewhat elementary fashions were presented by various authors. Some of these models which were directly relevant to our thesis were examined in chapters 2 and 3. In 1982, Modigliani presented a comprehensive corporate valuation model which incorporated heterogenous personal as well as corporation taxes in the context of the U.S. tax code. This model was derived in chapter 4 of this thesis. However, as noted by Ashton (1989), the models which were suitable within the U.S. context are discussed and presented as though they are equally suitable under different tax environments, such as that in the U.K.. Notable exceptions to this type of fallacy include Ashton (1989), Franks & Broyles (1979), Kent & Theobald (1980), King (1974,1977) and Rutterford (1988). However, primarily, the existing literature deals with taxes in a simplistic manner, regardless of the fact that inclusion of some major features of the tax system can reverse the conclusions otherwise reached about the cost of capital as well as about the financial policies that the companies should be following.

Since the existing literature did not deal with the issue of personal taxes satisfactorily, the primary objective in this thesis was to incorporate important features of the personal and corporation tax codes, in particular the features relevant in the U.K., in a corporate valuation model in order to derive the cost of capital. A second objective was to assess how robust our model was and examine whether it was relevant for practical decision making.

We achieved our primary objective in chapter 7 when we derived the valuation model that incorporated heterogenous personal as well as corporate taxes within the UK tax code. We not only derived the model but in doing so, in chapter 6, we showed how to calculate some of the more "difficult" variables for our equation. While deriving our model, we also showed how the valuation equation would differ under alternative international tax systems, in chapter 5.

In order to achieve our second objective, we applied the valuation model derived in chapter 7 to :

(a) demonstrate in chapter 8 how to measure the impact of changes in features and rates on the cost of capital, by examining the 1988 Budget changes in particular. The 1988 Budget was chosen because it introduced the most substantial changes to personal taxes in any budget in the 1980s.

(b) demonstrate in chapter 9 that personal taxes are relevant for calculating cost of capital.

(c) analyse cost of capital under the two alternative tax bases considered by the Meade Committee (1978), namely the Expenditure Tax and the Comprehensive Income Tax, in chapter 10.

(d) derive a tax system in chapter 11 that is tax neutral even when uncertainty, debt and personal taxes are taken into account. This new system was advocated because the use of the more complex valuation model advocated in this thesis demonstrates that the validity of the conclusions reached by the Meade Committee is largely applicable only to simple companies and models.

All the above issues had a common theme which motivated our thesis, namely the determination of the appropriate cost of capital in the presence of realistic, heterogenous personal and corporation taxes, and uncertainty.

As shown above, we have achieved both our objectives which we were motivated to achieve in this thesis. We achieved our objectives by undertaking the work which is summarised in the next section.

## Section B
## Summary of and the conclusions reached in each chapter

We introduced the reason why we were motivated in undertaking this research in chapter 1. The primary reason was that the existing literature did not deal adequately with personal taxes in valuation models. Some of the simplistic way in which taxes were treated included not dealing with the U.K. tax code and implicitly assuming that the U.S.A. tax code was the only relevant one, assuming that all direct investors of corporate securities were individuals, and assuming that some personal taxes were zero. We aimed to derive a valuation model that would deal in greater depth with personal and corporate taxes than the existing literature had done.

We proceeded by examining the existing literature on personal and corporate taxes and the cost of capital in chapter 2. We began with a review of Hamada and Scholes (1985) because their article focused directly on the after personal tax and the before personal tax models. We noted that their analysis was somewhat limited by their inappropriate choice of the Miller (1977) model as the after tax model. We also pointed out that they erred in their derivation of the equilibrium rate of interest. We provided our model and showed in six ways as to why we believed Hamada & Scholes had erred. We also examined the Miller (1977) model and stated our reasons for not agreeing that the marginal rate of tax on equity income was zero. We noted, as had been noted by Modigliani, that the relationship between the interest rates and the corporation tax rate that was necessary for Miller Equilibrium was not observed in the U.S.A. in any year. We considered the Modigliani (1982) model to be the most appropriate valuation model in the literature. The reasons include that the model is fairly comprehensive in comparison with the other models in the literature, although it is not as elaborate as the model we present in equation 7.22 in chapter 7. The model incorporates personal taxes, uncertainty and diversification by the investors and heterogenous investors in terms of their personal and capital gains tax rates and their degree of risk aversion. Therefore we considered that the Modigliani model was the most comprehensive model in the literature and examined its implications and conclusions in chapter 2. We noted that Modigliani had made an error in his definition of beta factors and we provided the correct definition of beta factors within the context of his model. We also examined the pioneering article by Brennan who was the first researcher to incorporate personal taxes in CAPM framework. We noted an error in his derivation and gave our suggested correction. We also examined the contributions to the literature made by King, Auerbach and Poterba & Summers. We concluded that there was no valuation model that dealt comprehensively with cost of capital in the U.K. in the presence of U.K. corporation and personal taxes. We therefore aimed to develop such a model in the thesis.

In chapter 3, we examined the literature on dividend and debt policy, the cost of capital and international tax models. We concluded that the literature overstates the case that the companies should not pay any dividends. We concluded that there may be excess volatility on the stock market which is unrelated to the uncertainty of the project returns, along the lines analysed by Shiller. The chapter concludes with a demonstration of the simplistic relationships modelled in the existing literature, against the more complex relationships in the real world. The thesis subsequently aimed to incorporate the real world taxation features into the model.

In chapter 4 we derived the Modigliani model (1982) in detail and introduced the relevant variables that are needed for practical calculations subsequently. The model is not explicitly derived in detail in the literature. We conclude that the model is fundamentally sound, subject to couple of corrections to Modigliani's derivation that we suggest. The model thus can form the basis for the derivation of our model.

The Modigliani model was based on the classical system of taxation as per the U.S.A. tax code. We derive the valuation model under the alternative tax regimes in chapter 5. These include the imputation system of taxation that applies in the U.K.. Subsequent chapters concentrate on the imputation system only because this is the tax system in operation in the U.K.. The chapter concludes with a comparison of discount rates under alternative tax regimes. We concluded that the classical system of taxation, as illustrated by Modigliani (1982) resulted in the highest cost of capital, in contrast to the integrated system (the least cost of capital), or the imputation or the two rate systems.

Chapter 6 demonstrates how the variables that are necessary for determining the cost of capital in the UK can be estimated. This is the first time that a methodology for the calculation of these variables has been proposed. We also calculate the variables that are relevant for the U.K.. The conclusion is that it is possible to obtain useful figures for some of the more esoteric variables that are present in the valuation model.

Chapter 7 demonstrates how the important features of the UK corporation tax code can be incorporated in the valuation model. In this chapter, we also pointed out the importance of capital allowances to perpetual cash flow models and demonstrated how these could be incorporated in the valuation model. Chapter 7 ends with the relevant valuation model and the associated cost of capital equation advocated in this thesis. This equation (7.22) incorporates the personal as well as the corporate tax features applicable to a range of investors. Thus the model that we present is a useful contribution to the literature on the cost of capital in the presence of corporate and personal taxes in the U.K. . This comprehensive valuation model which is directly relevant to the U.K. tax regime is one of the two major conclusions reached in this thesis.

Chapter 8 begins the practical application of the valuation model derived in chapter 7.

The cost of capital equation is used to assess the impact of the significant changes introduced in the 1988 Budget on the cost of capital in the U.K. . We therefore calculate the cost of capital for an average U.K. company using the tax rates that applied before the 1988 budget and then using the tax rates that applied after the budget. We use the valuation model we developed in this thesis. The simpler models that are present in the existing literature are not elaborate enough for this practical application. Chapter 8 concludes that the 1988 Budget changes should have changed the cost of capital in the U.K. by 0.5%. We conclude that the effect would have been even more pronounced if the capital gains tax rate had not been increased in the same budget.

In chapter 9 we address the critical issue of whether personal taxes matter for determining the cost of capital. We use the model developed in chapter 7 to demonstrate that personal taxes are relevant for valuation purposes. We also assess how the relevance of personal taxes to the cost of capital varies with changes in the dividend payout and leverage of the firms. We conclude in chapter 9 that the personal taxes are relevant for the valuation model, that the discount rate is overstated by approximately 1.5% if they are not considered, and that ignoring personal taxes undervalues assets in the U.K.. The relevance of personal taxes varies inversely with the debt equity ratio and directly with the level of dividends under the imputation system of taxation. We also conclude that the relevance of personal taxes varies inversely with the level of dividends under the classical system of taxes.

Chapters 10 and 11 use the cost of capital equations developed in chapter 7 to critically examine the conclusions reached by the Meade Committee (1978). We conclude that the Meade Committee's recommendations are valid only with simple models, such as those that do not include uncertainty and mixed capital structures.

Chapter 10 concludes that the Meade Committee's recommendations are correct and relevant but only under simplistic assumptions regarding certainty and capital structures. We use our sophisticated model to interpret the assertions made in the Meade Report and find that we can use our valuation model and the derivation technique to evaluate the alternative tax systems recommended in the Meade Report.

A new tax system that is tax neutral even when uncertainty, mixed capital structures and firms earning super-normal profits are considered, is advocated in chapter 11. We conclude that the proposed I base system of taxation enjoys certain advantages over the S base (and even the other bases) recommended in the Meade Report. Chapter 11 thus presents a new model which is tax neutral under uncertainty, personal and corporation taxes and under the normal or supernormal profits assumptions. This is second of the two major results derived in this thesis.

The overall conclusion of this thesis is that the model presented in chapter 7 can be used to provide many useful insights into the various aspects of the cost of capital question.

The contributions to the literature made in the thesis in each of the chapters summarised above are given in the next section.

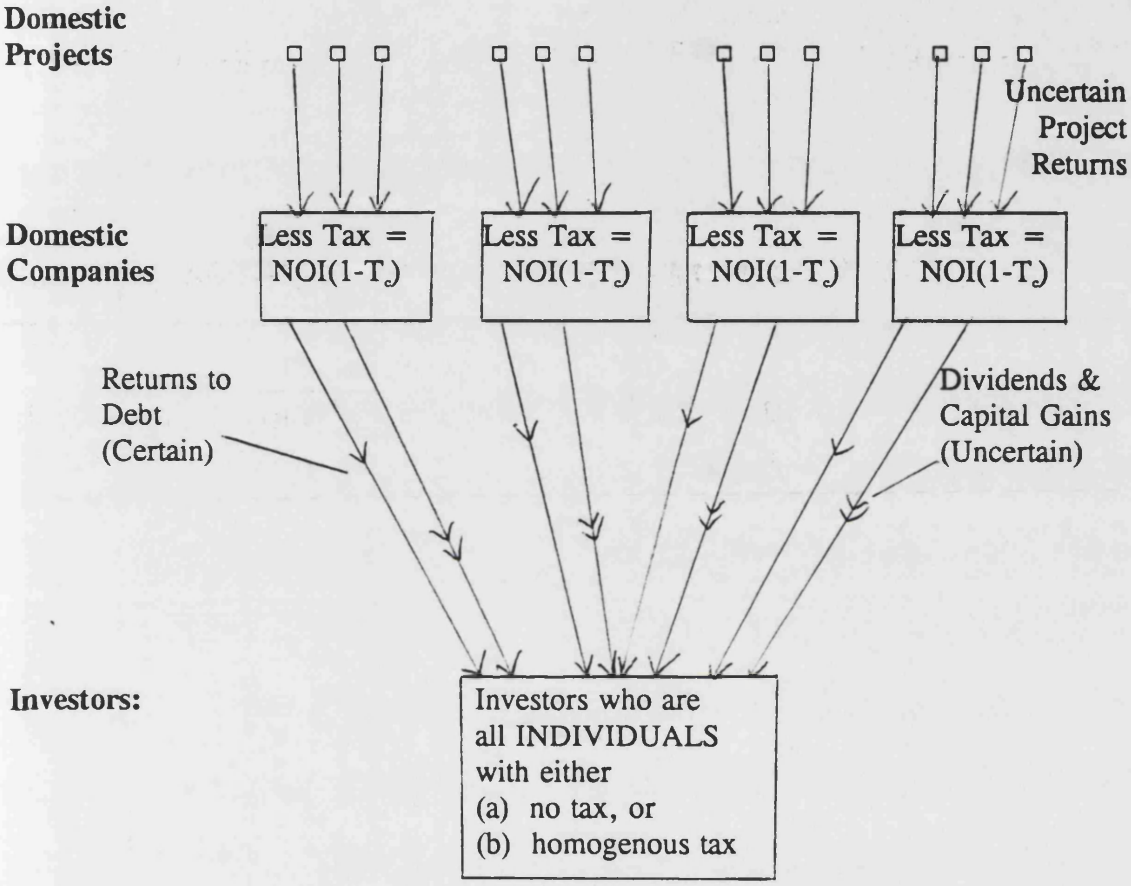
## Section C
## Summary of the contributions made in this thesis

The conclusions reached within the chapters which contribute either innovative concepts or analysis were summarised in the previous section. This section focuses on the new additions to the literature as well as the corrections suggested in the various chapters.

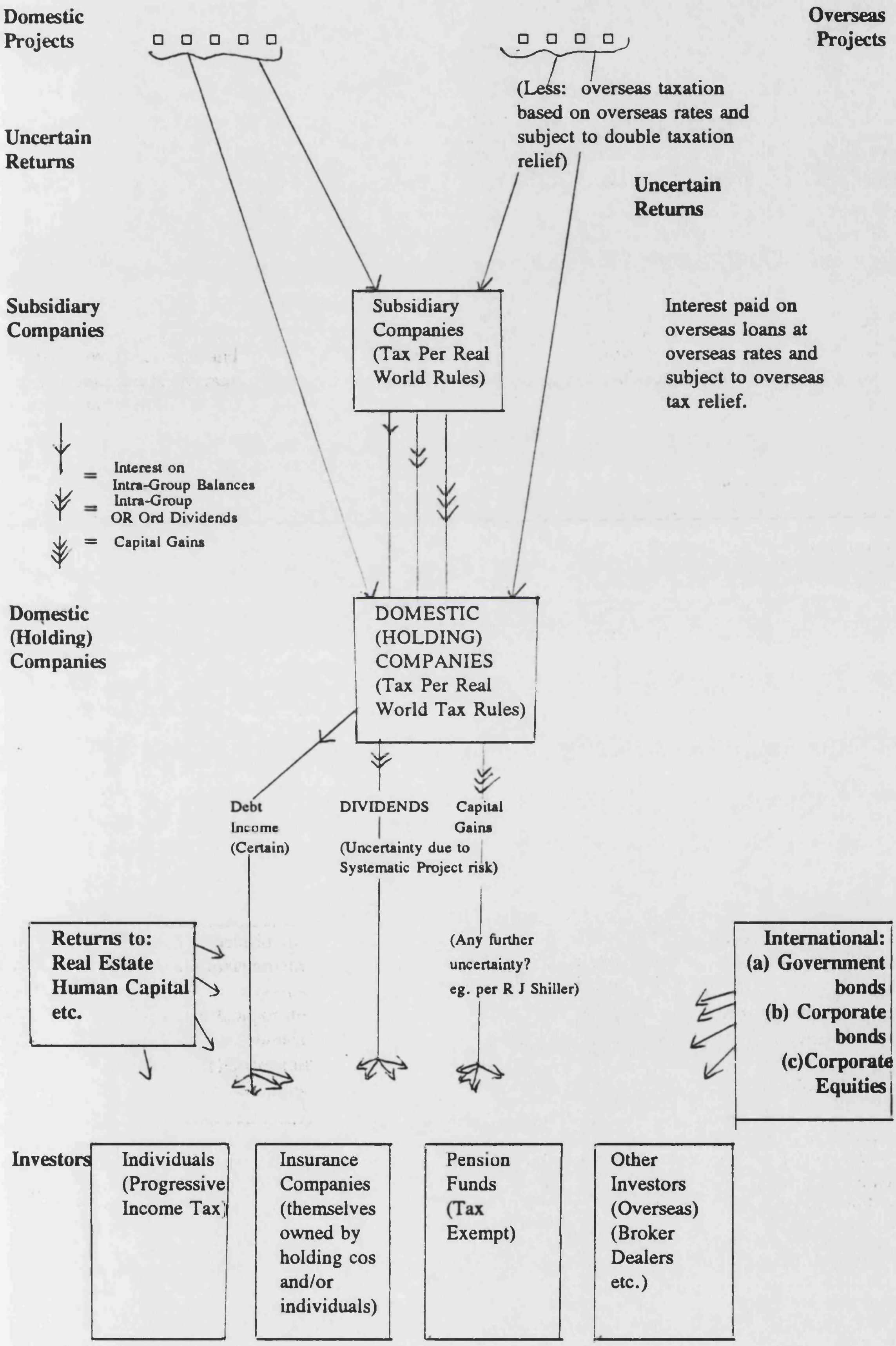
In chapter 2, we extensively examined some of the relevant articles in the literature. We concluded that Hamada and Scholes (1985), in their survey article on the before tax and after tax valuation models, erred in emphasizing that Miller (1977) was the after tax model. We claim that Modigliani (1982) is more relevant and stated the reasons in chapter 4. We also proved (in chapter 2) that the equilibrium interest rate that they calculated was incorrect and derived the correct equilibrium interest rate. This is an important contribution because we provide 6 reasons to demostrate comprehensively why our version is the correct one. We also showed the reasons why we disagreed with Miller's views on the laundering of personal taxes. We showed comprehensively why the arguements put forward by Miller and by Copeland and Weston to show that personal taxes on equity income were low were totally incorrect. Instead, we stated the correct reasons why the equity income taxes could be low but we could do so only by relying on the features of the U.K. tax code, which are usually (but not always) ignored by the writers from U.S.A.. We also corrected the definition of beta factor used by Modigliani in this chapter and in the appendices. This too is a contribution because the misspecification was an error, rather than a misprint, and it has not been picked up in the various reviews subsequently.

In chapter 3, we emphasised that the impact of personal taxes on dividends is very different in the U.K. from that in the U.S.A.. We noted that in the U.K., most investors would have a tax preference for dividends, in contrast to the assertion generally made in the literature that investors should avoid dividends because they are more heavily taxed. We also stated that the literature erroneously assumed all investors to be individuals - in fact individuals owned less than half of the equity on the London Stock Exchange in the 1980s. We concluded chapter 3 by stating that the models in the existing literature were essentially either before tax models or they were models with simplistic assumptions about the ownership structure of corporate equity (see below); and our contribution includes showing the more complex structure that exists in reality (see below).

The simplified version of the relevant players and variables which most of the before tax models incorporate is shown below:




We emphasised that the reality was far more complicated and offered the following alternative:


Domestic Projects
Overseas Projects
(Less: overseas taxation based on overseas rates and subject to double taxation relief)
Uncertain Returns
Uncertain Returns
Subsidiary Companies
Subsidiary Companies (Tax Per Real World Rules)
Interest paid on overseas loans at overseas rates and subject to overseas tax relief.
= Interest on Intra-Group Balances
= Intra-Group OR Ord Dividends
= Capital Gains
Domestic (Holding) Companies
DOMESTIC (HOLDING) COMPANIES (Tax Per Real World Tax Rules)
Debt Income (Certain)
DIVIDENDS
Capital Gains
(Uncertainty due to Systematic Project risk)
(Any further uncertainty? eg. per R J Shiller)
Returns to: Real Estate Human Capital etc.
International: (a) Government bonds (b) Corporate bonds (c)Corporate Equities
Investors
Individuals (Progressive Income Tax)
Insurance Companies (themselves owned by holding cos and/or individuals)
Pension Funds (Tax Exempt)
Other Investors (Overseas) (Broker Dealers etc.)

The above diagram is far more comprehensive and we therefore chose a model that was capable of depicting the more complex but important tax features (Modigliani 1982), and adapted it to the UK structure. In doing so we pointed out the oversights in derivation of beta factors and the valuation model made by Modigliani (1982) and in the pioneering work by Brennan (1970). The correct derivations are proved in appendices A1 and A2 respectively. Our contributions made by the analysis in chapter 4 therefore include :

(a) providing a more detailed and a more elegant narrative description of the Modigliani (1982) model. In the Modigliani article, there is little or no narrative description of the model.

(b) providing the links between the various equations of the model. The Modigliani article describes the model in 3 pages only and therefore includes only a minimum number of equations. An understanding of the links between the various equations are necessary in order to gain confidence in the results of the model and also to point out couple of errors in the Modigliani article. The errors include (1) the definition of beta factors used by Modigliani and (2) the calculation of weighted averages for tax factors.

(c) providing links between this model and the other CAPM-tax models in the literature. This analysis in this chapter results in pointing out couple of errors in the analysis by Brennen (1970).

(d) focusing in more detail on the impact of taxes.

Our contribution in chapter 5 includes providing detailed valuation models, which incorporate heterogenous investors, uncertainty and personal taxes under the imputation, the two rate and the integrated systems of taxation. The last mentioned model was fully derived in section D of the chapter.

The relevant system of taxation for the U.K. is the imputation system. In chapter 6, we derived some of the relevant variables for the U.K. system of taxation, taking into account the more complex ownership structure of corporate equity than is assumed in the literature. Our contributions in chapter 6 therefore includes, for the first time in the literature, providing a methodology for estimating some of the more difficult variables in the valuation equation, including $\Lambda$, the harmonic mean of the risk aversion factors, and $\gamma^{indiv}$, $\gamma^{corp}$ and $\gamma^{pension}$, the measures of risk aversion. We also calculate the relevant weighted averages of tax variables for the U.K., including $\theta_g$, $\theta_p$, $\theta_{PDIVI}$, $\tau_c$, $r_p$ and L (the benefit of leverage). These practical calculations, in particular the former ones, are an innovative and unique contribution to the literature.

We accomplished one of our two main tasks in chapter 7 where we presented the valuation model in equation (7.22) which is reproduced below:

$$(7.22)\quad V_i = \frac{\mu_i'\theta_g + LD_i\,[r_p + \beta_i\pi] + \tau_c'\,C_i\,\theta_g + \dfrac{[\beta_i\pi\,\tau_c'\,C_i\,\theta_g\;]}{r_p} - \Delta_i\,(\theta_g - \theta_{PDIVI}) - \dfrac{[\beta_i\pi\Delta_i\,(\theta_g - \theta_{PDIVI})\,]}{r_p}}{r_p + \beta_i\pi}$$

Equation 7.22 shows the adjustments that need to be made to the cashflow arising from the project in order that they may be discounted by the risk adjusted cost of capital shown in the denominator of the equation. The advantages of the above valuation model over the other models in the literature include:

(1) The model represented by equation 7.22 ( "recommended model" ) incorporates the correct definition of the beta factors. As pointed out in Chapter 4, section F, and proved in appendix A1, it is incorrect to define beta factors as simply covariance within the context of this model.

(2) The recommended model incorporates the correct weightings in aggregating variables over investors. This was illustrated in Chapter 6 in the calculation of $\Lambda$ and the tax variables for the market. The correct weightings, which include the relative capital owned by the investors, are particularly relevant in capital markets of most of the countries in the world where the presence of numerous but "small" investors may distort the market variables.

(3) The recommended model has been adapted to reflect two of the main features of the tax system in the U.K., namely the imputation system and the taxation of real, as opposed to nominal, capital gains. The Modigliani model is not "incorrect" in

this regard. It is simply applicable for the U.S.A., where the classical system (Chapter 5) of taxation applies. The contribution made in this thesis is to explicitly state the model in the context of the U.K. tax system and to calculate the tax "advantage" of dividends in the U.K. (Chapter 6).

(4) The recommended model points out that the average period for stockholding in the U.K. is substantially lower than the period assumed in the literature. An average period of 10 years mentioned in the literature is simply too long, given the high level of turnover relative to the total capitalisation value of the stock market, even after the market crash of 1987. We demonstrate that the stockholding period is less than a year, using our technique which is superior to the methods used in the literature. The contribution made in this thesis is to use the concept of stock turnover (which is used in Accountancy to calculate the stockholding period of physical inventory) in the context of the capital markets. This usage enables one to calculate the effective capital gains tax, and this points out that the high turnover of shares on the stock market implies a much lower stockholding period and a much higher level of the effective capital gain tax than is assumed in the literature.

(5) The recommended model correctly incorporates the benefit of capital allowances on the initial investment in the valuation equation. **Its omission would lead to a material understatement of the project values.**

(6) The recommended model incorporates the other relevant features of the corporation tax system (Chapter 7, sections A & B) in the valuation equation, thereby making it more precise than the other models in the literature.

(7) The recommended model includes illustration of the calculations of the values of the relevant variables for the U.K.. Thus it illustrates a practical methodology for estimating the rather abstract variables that are used in the valuation models in this thesis, as well as in rest of the literature.

Thus, within the context of the perpetual cashflow models, the recommended model improves upon the existing literature.

After the derivation of the recommended model in chapter 7, we proceeded to illustrate its usefulness.

The contribution made in chapter 8 is that we calculate that the changes introduced by the Budget in 1988 reduced the cost of capital for U.K. companies by 0.5%. This conclusion could be reached because we used our recommended valuation model. The simpler models and the before tax models in the literature would have been completely incapable of analysing this impact.

The contributions made in chapter 9 include stating that ignoring personal taxes in the U.K. results in an overstatement of the cost of capital on average by 1.5% approximately. We also contributed a simple sensitivity analysis of the change in this overstatement as the debt level varies. We went on to contribute a simple variation analysis under different dividend level and concluded that increasing dividend level resulted in an increase in the overstatement of discount rate if personal taxes were ignored. This is as shown below.

| | **EFFECTIVE DISCOUNT RATE** | | |
|---|---|---|---|
| **LEVEL OF DIVIDENDS** | **No personal taxes** | **With personal taxes** | **DIFFERENCE** |
| (i) | (ii) | (iii) | (iv) |
| High (£60) | 11.31 | 8.13 | 3.18 |
| Moderate (£30) | 11.31 | 9.94 | 1.37 |
| Low (£nil) | 11.31 | 12.80 | (1.49) |

The contributions made in chapters 10 and 11, include the following

(a) The conclusions reached by the Meade Report are valid only under the assumption of simple corporate capital structures.

(b) The rate of return to an investor under a comprehensive tax system could be greater than the return generated by the project because of financial leverage.

(c) The rate of return to the investors need not equal the rate of return generated by the investment under the expenditure tax system.

(d) The problems noted above occur because the Meade Committee used simple models to analyse complex real world situations; the use of the recommended model would have prevented these errors.

The above four points are summarised in the following table in which we present the Distortion Ratio under alternative assumptions. Derivation of the Distortion Ratio under alternative assumptions is an innovative contribution to the existing literature, where such ratios have not yet been derived. These distortion ratios are shown below.

**DISTORTION RATIO OF**
**RATE OF RETURN TO EQUITY SHAREHOLDER**
**TO RATE OF RETURN ON PHYSICAL INVESTMENT**

Based on

| | | | |
|---|---|---|---|
| $\theta_p^m$ | $=0.75$ | $\theta_p$ | $= 0.80$ |
| $\tau_c$ | $= 0.35$ | j | $= 60\%$ |
| $\mu_i$ | $= 50$ | r | $= 5\%$ |
| $\pi$ | $= 8\%$ | ß | $= 1$ |
| $D_i$ | $= £100$ | $\tau_{ci}$ | $= 0.17722$ |

| | | Expression | Ratio |
|---|---|---|---|
| (1) | COMPREHENSIVE INCOME TAX | | |
| (i) | No debt, no uncertainty | $\theta_p^m\, \theta_c/\theta_j$ | 0.6171 |
| (ii) | No debt, uncertainty | $\theta_p^m\, \theta_c/\theta_j$ | 0.6171 |
| (iii) | Debt, no uncertainty | $\theta_p^m\, [\theta_c/\theta_j + L\ rD_i/\mu_i]$ | 0.6304 |
| (iv) | Debt and uncertainty | $\theta_p^m \cdot \theta_c/\theta_j \cdot \mu_i\text{-}rD_i/\mu_i \cdot V_i/S_i$ | 0.8447 |
| | with debt =£200; ß =1 | $\theta_p^m \cdot \theta_c/\theta_j \cdot \mu_i\text{-}rD_i/\mu_i \cdot V_i/S_i$ | 1.0461 |
| | with debt =£100; ß =2 | $\theta_p^m \cdot \theta_c/\theta_j \cdot \mu_i\text{-}rD_i/\mu_i \cdot V_i/S_i$ | 1.2304 |
| (2) | EXPENDITURE TAX | | |
| (i) | No debt, no uncertainty | 1.0000 | 1.0000 |
| (ii) | No debt, uncertainty | 1.0000 | 1.0000 |
| (iii) | Debt, no uncertainty | 1.0000 | 1.0000 |
| (iv) | Debt and uncertainty | $\dfrac{1}{1 - \dfrac{D_i\, ß_i \pi}{\theta_\epsilon\, [\mu_i\text{-}rD_i]}}$ | 1.2857 |
| | with debt =£200; ß =1 | ditto | 2.0000 |
| | with debt =£100; ß =2 | ditto | 1.8000 |

(e) Our final contribution in chapter 11 is to recommend an I-base tax system. It has the simple underlying logic of taxing cash flows to/from investors symmetrically - whether the investors are debt holders or equity holders. It is proved that such a system would be a non - distortionary system. It is also proved that the expenditure tax system, as advocated by the Meade Report, maintains some distortion because of the way in which the inflows and outflows to the debt holders are treated.

## CORPORATE FLOW OF FUNDS

| Inflows | | Outflows | |
|---|---|---|---|
| **Real Items** | | | |
| R1 | Sale of produce | R1* | Purchase of materials |
| R2 | Sale of services | R2* | Wages, salaries & purchase of other services |
| R3 | Sale of fixed assets | R3* | Purchase of Fixed Assets |
| R | | R* | |
| **Financial Working Capital (FWC) Items** | | | |
| F1 | Increase in creditors | F1* | Decrease in creditors |
| F2 | Decrease in debtors | F2* | Increase in debtors |
| F3 | Increase in overdraft | F3* | Decrease in overdraft |
| F4 | Decrease in cash balance | F4* | Increase in cash balance |
| F4a | Interest recd on FWC | F4a* | Interest paid on FWC |
| FN | | FN* | |
| **Debt Investment Items** | | | |
| D1 | Increase in debt borrowings | D1* | Decrease in debt borrowings |
| D2 | Decrease in corporate debt lending | D2* | Increase in corporate debt lending |
| D3 | Interest received on corp debt | D3* | Interest paid |
| D4 | Decrease in holding of shares in non-UK corporations | D4* | Increase in holding of shares in non-UK corporations |
| D | | D* | |
| **Share Items Of Corporate Bodies Resident In The UK** | | | |
| S1 | Increase in own shares issued | S1* | Reduction in own shares issued |
| S2 | Decrease in holding of shares of other UK companies | S2* | Increase in holding of shares in other UK companies |
| S3 | Dividends received from other UK companies | S3* | Dividends paid |
| S | | S* | |
| **Tax Items** | | | |
| T | Tax repaid | T* | Tax paid |

Total Inflows = Total Outflows

$$R + FN + D + S + T = R^* + FN^* + D^* + S^* + T^*$$

Therefore

$$(D^* + S^*) - (D + S) + (T^* - T) = (R + FN) - (R^* + FN^*)$$

The I-Base tax system (shown on the previous page) under which (D* + S*) - (D + S) represents the tax base, is a system that is tax neutral when uncertainty, complex capital structures and/or supernormal profits projects are examined. Therefore in some ways it is superior to the S-base expenditure tax system recommended by the Meade Report. We derived the I base tax system by making use of the recommended model shown as equation (7.22) above. This reinforces the conclusion that the recommended model represents an improved model for analysing the influence of personal and corporation taxes on the valuation of cash flows from projects. The advantages of the I base system include:

**(a)** **Advantages of I-base over S-base**

(i) The main advantage of I-base over an S-base is that with an I-base, even **in the presence of debt and uncertainty, the rate of total return to all investors is equal to the rate of return on physical investment.** The S-base recommended by the Meade Report does not lead to such an equality, as was shown above.

(ii) **The I-base treats all investors in a similar manner, whether they be equity holders or debt holders.** Cash inflows from either source result in a corporation tax refund and cash outflows to either equity holders or debt holders results in a corporation tax liability. This symmetry is not present under an S-base system.

It may appear that what is being proposed under the I base system will be unduly harsh on the debt holders, namely, that they will not benefit from the tax deductibility of interest payments at the corporate level. However, debt investment will be compensated for this by enjoying corporation tax deduction on the full capital amount when debt is issued. Therefore the proposed I base does not impose any great extra burden on debt investment, but merely shifts the tax preference from the income stream to the capital inflow.

(b) **Advantages of I-base over R-base**

(i) **An I-base, perhaps with a few modifications, can be easily applied for determining the taxable income of financial institutions.** However, an R-base is completely unsuitable for financial institutions, since it does not tax financial transactions at all.

(ii) **The response of share price to an increase in retained equity cash flows is very clearly defined under an I-base.** On the other hand, an R-base system introduces another source of confusion as to how share prices should respond to an increase in retained earnings.

To conclude, the many contributions made in this thesis are varied and include the contributions summarised in this section, an in depth derivation of the valuation models used in Modigliani (1982) and in the Meade Report (1978), the valuation equation derived in chapter 7, the new I-base tax system advocated in chapter 11, as well as the suggested corrections to the published literature suggested in chapters 3, 11 and in appendix A.

The thesis thus adds innovative concepts to the existing literature while achieving its objective of incorporating personal and corporation taxes in a comprehensive valuation model.

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## Definition Of Beta Factor

The purpose of appendices A1 and A2 is to check and clarify any apparent internal inconsistency in the formulae used. Modigliani derives the risk adjustment factors for individual companies in equation (M II.12) by defining $\Lambda[M]1$ in the valuation equation as equivalent of $(\mathbf{V^*}\ \mathbf{ß})\pi$, where **V* and ß** are vectors of the value of the unlevered streams ($V_i$* s) and the beta factors ($ß_i$ s) respectively of the individual companies. Note that there is a misprint in the article where **V** is used instead of the correct variable **V***, but this misprint is not critical to our proof.

The ith row of the matrix $\Lambda[M]1$ is defined by Modigliani as $\Lambda \operatorname{cov}(\mu_i^*, \mu^*)$ in equation (MII.7) in Modigliani (1982). Thus

$$(13.1) \quad \Lambda \operatorname{cov}(\mu_i^*, \mu^*) = (V_i^*\ ß_i)\ \pi$$

$$\text{or } (13.2) \quad \frac{\Lambda \operatorname{cov}(\mu_i^*, \mu^*)}{V_i^*} = ß_i\ \pi$$

Equation (13.2) states that the total risk premium factor on the L.H.S. of equation (13.2) above can be seen as a product of two terms - a beta factor ($ß_i$) and a market risk premium term ($\pi$). The market risk premium term is defined by Modigliani as $\Lambda \operatorname{var}(\mu^*) / V^*$ in equation (M II.9) in Modigliani (1982). Therefore, the beta factor can be calculated by dividing the L.H.S. of equation (13.2) above by the market risk premium (defined above), as shown below:

$$(13.3) \quad ß_i = \frac{\Lambda \operatorname{cov}(\mu_i^*, \mu^*)}{V_i^*} \Big/ \pi$$

$$= \frac{\Lambda \operatorname{cov}(\mu_i^*, \mu^*)}{V_i^*} \Big/ \frac{\Lambda \operatorname{var}(\mu^*)}{V^*}$$

$$= \frac{\operatorname{cov}(\mu_i^*, \mu^*)}{V_i^*} \times \frac{V^*}{\operatorname{var}(\mu^*)} \qquad \text{( next, multiply and divide by V*)}$$

$$= \frac{\text{cov } (\mu_i^* , \mu^*)}{V_i^* \ V^*} \quad x \quad \frac{V^* \ V^*}{\text{var } (\mu^*)}$$

$$(13.4) \qquad = \quad \text{cov } (\mu_i^*/V_i^* , \mu^*/V^*) \quad / \quad \text{var } (\mu^*/V^*)$$

Therefore the correct definition of beta factor is covariance between the unlevered rate of return for the company i, divided by the overall variance of the return on the market. This is the standard definition of beta factor common in the Finance literature. Modigliani erroneously states that the beta factor is simply the covariance term, and omits to divide by the market variance term. As shown above, the correct formulae, even using the specific definitions used in the Modigliani article, should be divided by the market variance term, as is done in rest of the literature. This is unlikely to be a misprint because similar error occurs in Brennan (1970) (see appendix A2), the pioneering work that Modigliani followed.

Therefore, to be doubly sure of the correct definition, we reconcile the derivation of the beta factor in Modigliani with, for example, the derivation by King (lecture notes, London, 1988). King defines CAPM as shown below. The risk factor used by King will be reconciled to the correct formulae for the beta factor which we derived above.

King defines the value of company i as:

$$(13.5) \qquad V_i^k = \frac{u_i^k - C_i / A}{R}$$

where $u_i^k$ is the expected total return (in monetary amount terms), $C_i$ is the covariance between the total return (monetary value) to company i and the return on the market, A is a measure of risk return ratio and R is the total return on the risk free asset. The superscripts k indicate that the variables are as per King's notation. Thus $u_i^k$ is equal to the sum of the original principal ($V_i^*$) and the expected return ($\mu_i^*$). Similarly R is equal to (1 + r), that is the monetary amount of total return on the risk free asset. Making these substitutions, and introducing the appropriate tax factors (eg., $\theta_g$ ), we simplify equation (13.5) as follows:

$$(13.5)\quad V_i^k R + C_i/A = u_i^k$$

$$=> \quad V_i^* (1 + r_p) + C_i/A = u_i^* \theta_g + V_i^*$$

$$=> \quad V_i^* + V_i^* r_p + C_i/A = u_i^* \theta_g + V_i^*$$

$$=> \quad V_i^* r_p + C_i/A = u_i^* \theta_g$$

$$=> \quad V_i^* \left(r_p + \frac{C_i/A)}{V_i^*}\right. = u_i^* \theta_g$$

$$=> (13.6) \quad V_i^* = \frac{u_i^* \theta_g}{\left(r_p + \frac{C_i/A)}{V_i^*}\right.}$$

Equation (13.6) above can be derived from equation (II.12) in Modigliani (1982). Equation (13.6) states that the value of a company i is obtained by discounting its expected cashflow by a risk adjusted discount rate , which includes $(C_i/A)/V_i^*$, the risk premium term. For comparison, equation II.12 from Modigliani (1982) is simplified and reproduced below.

$$(13.6b)\quad V_i - LD_i - \frac{\Delta_i(\theta_g - \theta_p)}{r_p} = V_i^* = \frac{u_i^* \theta_g}{(r_p + \beta_i \pi)} \qquad \text{(M II.12)}$$

A comparison of the two equations implies that the risk premium term $\beta_i\ \pi$ in the Modigliani article is represented by $(C_i/A) / V_i^*$ which we derived from King. Therefore the following holds :

$$(13.7)\quad \frac{C_i / A}{V_i^*} = \beta_i\ \pi$$

We will now substitute in the covariance and variance terms on both sides of equation (13.7) above. We will use the correct definition of the beta factors as per our definition in this thesis. If the above equality still holds, then it implies that our definition and derivation of the beta factors is consistent with the derivation used in some other recent literature, eg., in King.

The terms on the L.H.S. are as follows. $C_i$ is defined as equivalent to cov ($\mu_i$* , $\mu$*) (see note 1 below). 1/A is equivalent to $\Lambda$ of Modigliani (see note 2 below). Making these substitutions, and adding in the correct components of $\beta_i\ \pi$ on the R.H.S., we obtain :

(13.8) $$\text{cov}\ (\mu_i^*\ ,\ \mu^*)\ \ \text{x}\ \ \Lambda\ \ \text{x}\ \ 1/V_i^* = \frac{\text{cov}((\mu_i^*/V_i^*\ ,\ \mu^*/V^*)}{\text{var}\ (\mu^*/V^*)}\ \ \text{x}\ \ \Lambda\ \ \text{x var}\ (\mu^*)\ /\ V^*$$

(13.9) $$\text{cov}\ (\mu_i^*\ ,\ \mu^*)\ \text{x}\ 1/V_i^* = \frac{\text{cov}((\mu_i^*\ ,\ \mu^*)}{V_i^*\ \text{x}\ V^*\ \text{x var}\ (\mu^*/V^*)}\ \ \text{x}\ \ \text{var}\ (\mu^*)\ /\ V^*$$

(13.9b) $$\text{cov}\ (\mu_i^*\ ,\ \mu^*) = \frac{\text{cov}((\mu_i^*\ ,\ \mu^*)\ \text{x}\ V^*\ \text{x}\ V^*}{V^*\ \ \text{x}\ \ \text{var}\ (\mu^*)}\ \ \text{x var}\ (\mu^*)\ /\ V^*$$

(13.10) $$\text{cov}\ (\mu_i^*\ ,\ \mu^*) = \text{cov}((\mu_i^*\ ,\ \mu^*)$$

The L.H.S. of equation (13.10) is equal to the R.H.S.. Hence our derivation, which makes use of the correct definition of beta factors, is consistent with other recent literature on the subject. In the next appendix, we show that the probable reason why this fundamental variable was misdefined in Modigliani (1982) is that a similar error occurs in Brennan (1970).

Note 1

In King $C_i$ = Cov ($u_i^k$ , $u^k$)

(13.11) = Cov ($V_i$ + $u_i$* , V + u*)

In the above equation, the stochastic elements are represented by $u_i$* and u*. $V_i$ and V are based on expectations of $u_i$* and u* respectively, but are fixed in present value terms. Therefore equation (13.11) implies:

$$C_i \; = \; \mathrm{Cov}\,(u_i^* \, , \, u^*)$$

which is the expression used in equation (13.7). We have ignored the precise terms used in the articles for the tax variables because they are not relevant to our problem of the definition of beta factors, in order to facilitate the above derivation.

<u>Note 2</u>

In King, A = $\Sigma_m \, 1/\gamma^m$ where $\gamma^m$ is the investor m's risk aversion measure based on the pound return at the end of the period (that is, it is based on the sum of the initial investment plus the pound value of return). In Modigliani (1982),

$$\Lambda \; = \; 1 \; / \; 1/\Sigma_m \, \gamma^m \, (\theta^m{}_g)^2$$

where $\gamma^m$ is the investor m's risk aversion measure based on the pound return (excluding the initial investment). Since risk aversion in both cases is based on pound amounts ( and not on the rate of return in either one of the cases), then the wealth invariant utility functions used imply that $\gamma^m$ based on end of the period pound returns is the same as $\gamma^m$ based on pound returns (excluding initial investment). Therefore 1/A is equal to $\Lambda$ , as stated in equation (13.8).

## Market Price of Risk In Brennan (1970)

As previously stated, the purpose of appendices A1 and A2 is to clarify and check any apparent internal inconsistency in the formulae used. Brennan (1970) derives the excess return on security j over and above the risk free interest rate r, as shown below.

(13.21) $R_j - r = H\,M\,\Sigma_k\ Q_k \operatorname{cov}(R_j\ R_k) + T(\delta_j - r)$ (B 2.19)

where $R_j$ = Expected return on risky equity security j

$r$ = the risk free interest rate

$H$ = the market risk premium

$M$ = the total market capitalisation of all the securities

$Q_k$ = the share of security k in the total market value M

$T$ = tax factor based on weighted average tax rates and risk aversion

$\delta_j$ = dividend yield on security j

Brennan goes on to define $R_m$, the rate of return on the market portfolio, as

(13.22) $\Sigma_k\ Q_k\,M\,R_k = R_m$

He uses (13.21 and (13.22) to state that the excess return on security j can be expressed as :

(13.23) $R_j - r = H \operatorname{cov}(R_j\ R_m) + T(\delta_j - r)$ (B 2.21)

Brennan defines H as $R_m - r - T(\delta_m - r)$. We will prove below that :

(a) the derivation of H by Brennan is incorrect. Specifically, the R.H.S. of the expression for H should be divided by $Var(R_m) \times M$ .

(b) equation 13.22 is incorrect, and that the rate of return on the market is simply

$$\Sigma_k \; Q_k R_k = R_m$$

As a result of the above two corrections, the valuation equation (13.23) will be adjusted. It will be shown that Brennan missed dividing the risk factor term by $Var(R_m)$. This error in defining a fundamental variable probably carried on into Modigliani (1982). In order to simplify the proof, we shall ignore the tax factors, which are irrelevant in this exercise.

We begin by multiplying both sides of equation (13.21) by $V_j = P_j x_j$ where $P_j$ , $x_j$ , and $V_j$ are the price per share, number of shares and the market capitalisation of security j respectively. As explained above, we ignore the tax terms in the following equations:

$$(13.24) \quad R_j V_j - r V_j = H \; V_j \; M \, \Sigma_k \; Q_k \, cov \, (R_j \; R_k)$$

$R_j$ $V_j$ is equal to the expected rate of return on security j multiplied by its market capitalisation - hence it is equal to the expected pound return on security j, which we denote by $\mu_j$.

$$(13.25) \quad \mu_j - r V_j = H \; M \, \Sigma_k \; Q_k \, cov \, (\mu_j \; R_k)$$

$Q_k$ is the share of company k in the market portfolio and M is the pound value of the total market portfolio. Therefore, $M \; Q_k$ ( $= V_k$ ) represents the market capitalisation of company k. We take M inside the summation and make the above substitution :

$$(13.26) \quad \mu_j - r V_j = H \; \Sigma_k \; V_k \, cov \, (\mu_j \; R_k)$$

(13.27) $\mu_j \ - r V_j \ = \ H \ \Sigma_k \ cov (\mu_j \ \mu_k)$

$\Sigma_k$ cov ($\mu_j$ $\mu_k$) is equal to cov ($\mu_j$ $\mu_m$) by the property of covariances (and as illustrated in chapter 4).

(13.28) $\mu_j \ - r V_j \ = \ H \ cov (\mu_j \ \mu_m)$

We add up equation (13.28) over all companies and obtain :

(13.29) $\mu_m \ - r M \ = \ H \ cov (\mu_m \ \mu_m)$

We divide both sides of equation (13.29) by M, the total capitalisation of the market, to obtain :

(13.30) $\mu_m/M \ - r \ = \ H/M \ var (\mu_m \ \mu_m)$

We multiply and divide the R.H.S. by M, and take the 1/Ms inside the covariance term:

(13.31) $\mu_m/M \ - r \ = \ H \ M \ var (\mu_m/M \ \mu_m/M)$

$\mu_m/M$ is equal to the expected return on the market portfolio, and which is denoted by $R_m$ above. We substitute in $R_m$ and solve for H :

(13.32) $H \ = (R_m \ - r) \ / \ M \ var (R_m \ R_m)$

Equation (13.32) represents the correct derivation of H. In Equation (13.21) above, Brennan omitted to divide by M var ($R_m^2$). This is similar to the error in Modigliani (1982). Note also that in equation (13.22) above, Brennan derives the rate of return on the market portfolio as:

(13.22) $\Sigma_k \ Q_k \ M \ R_k \ = R_m$

However, note that $Q_k$ is defined as a proportion of the market share represented by the security k. The weighted average of rates of returns on different securities is simply the sum of the individual rates of returns multiplied by their respective weights (proportions). The rate of return on the market therefore should be :

$$(13.33) \qquad \Sigma_k \; Q_k R_k \;= R_m$$

Brennan has incorrectly included M, the market's total capitalisation, in the formula in equation 13.22. This error carries on into the final valuation equation (13.23 above). The correct valuation equation should state:

$$(13.34) \qquad R_j - r \;=\; M \;H \;\text{cov}\,(R_j \;R_m) \quad + \;T\,(\delta_j - r)$$

Thus the valuation equation derived by Brennan, and quoted extensively in the subsequent literature, is mis-specified by a substantial factor. The correct valuation equation (13.34 above) includes the term M.

We can substitute the correct definition of H (13.32 above) in the correct valuation equation (13.34), again ignoring the precise tax terms in H.

$$(13.35) \; R_j - r \;= M \;\frac{(R_m \; - \; r)}{M \quad \text{var}\,(R_m \, R_m)} \quad \text{cov}\,(R_j \;R_m) \quad + T\,(\delta_j - r)$$

This simplifies to :

$$(13.36) \; R_j - r \;= (R_m \; - \; r) \;\; \frac{\text{cov}\,(R_j \;R_m)}{\text{var}\,(R_m \, R_m)} \qquad + T\,(\delta_j - r)$$

The first term on the R.H.S. of (13.36) is the market risk premium. The second term is the beta factor, and it uses the correct definition of beta factor which we proved in appendix A1. The product of these two terms gives the correct measure of the risk premium applicable for company j, as quoted extensively in the literature. Therefore, the definition of H that we derived above, as well as the change to the valuation equation that we proposed above, are likely to be correct.

## Expenditure Tax

### Equity Return Ratio with debt and uncertainty

Equation 11.71 gives the ratio as $(\mu_i - rD_i)/\mu_i \ \text{x} \ (S_i + D_i\theta_{ci})/S_i$

Now $S_i$

$$= \frac{\mu_i\theta_\epsilon\theta_{ci}}{r\theta_\epsilon + \beta_i\pi} + D_i\tau_{ci} - D_i$$

$$= \frac{\mu_i\theta_\epsilon\theta_{ci}}{r\theta_\epsilon + \beta_i\pi} - D_i\theta_{ci}$$

Substituting into the expression for ratio, we get

$$\frac{(\mu_i - rD_i)}{\mu_i} \ \text{x} \ \frac{\dfrac{\mu_i\theta_\epsilon\theta_{ci}}{r\theta_\epsilon + \beta_i\pi} - D_i\theta_{ci} + D_i\theta_{ci}}{\dfrac{\mu_i\theta_\epsilon\theta_{ci}}{r\theta_\epsilon + \beta_i\pi} - D_i\theta_{ci}}$$

$$= \frac{(\mu_i - rD_i)}{\mu_i} \ \text{x} \ \frac{\mu_i\theta_\epsilon\theta_{ci}}{r\theta_\epsilon + \beta_i\pi} \ \text{x} \ \frac{r\theta_\epsilon + \beta_i\pi}{\mu_i\theta_\epsilon\theta_{ci} - D_i\theta_{ci}(r\theta_\epsilon + \beta_i\pi)}$$

$$= (\mu_i - rD_i) \ \text{x} \ \theta_\epsilon\theta_{ci} \ \text{x} \ \frac{1}{\theta_{ci}[\mu_i\theta_\epsilon - D_i r\theta_\epsilon - D_i\beta_i\pi}$$

$$= (\mu_i - rD_i) \ \text{x} \ \frac{\theta_\epsilon \ \text{x} \ 1}{\theta_\epsilon(\mu_i - D_i r - \dfrac{D_i\beta_i\pi}{\theta_\epsilon})}$$

$$= 1 \ \text{x} \ \frac{(\mu_i - rD_i)}{(\mu_i - D_i r) - \dfrac{D_i\beta_i\pi}{\theta_\epsilon}}$$

$$= 1 \div \frac{[(\mu_i - D_i r) - \dfrac{D_i\beta_i\pi}{\theta_\epsilon}]}{\mu_i - D_i r}$$

$$= \frac{1}{1 - \dfrac{D_i\beta_i\pi}{\theta_\epsilon(\mu_i - D_i r)}}$$ shown as Equation 11.72

When $D_i = £100$; $\beta_i = 1$; $\pi = 0.08$; $\theta_\epsilon = 0.8$; $\mu_i = 50$; $r = 0.05$

$$\text{Ratio} = \frac{1}{1 - \dfrac{100 \times 1 \times 0.08}{0.8(50 - (100 \times 0.05))}} = \frac{1}{1 - \dfrac{8}{0.8\ (45)}}$$

$$= \frac{1}{1 - 8/36} = \frac{1}{.77778}$$

$= 1.2857$ Thus the ratio exceeds one.

## Expenditure Tax

## Total Return Ratio in the presence of debt and uncertainty

Eqn 11.76 is

$$\frac{(\mu_i - rD_i)\,\theta_{ci}\,\theta_\epsilon^m + rD_i\,\theta_\epsilon}{\theta_\epsilon^m (S_i + D_i)} \quad x \quad \frac{S_i/\theta_{ci} + D_i}{\mu_i}$$

$$= \frac{\theta_\epsilon^m\,\theta_{ci}\,(\mu_i - rD_i + rD_i/\theta_{ci})}{\theta_\epsilon^m (S_i + D_i)} \quad x \quad \frac{S_i + D_i\,\theta_{ci}}{\mu_i\,\theta_{ci}}$$

$$= \frac{\mu_i - rD_i + rD_i/\theta_{ci}}{\mu_i} \quad x \quad \frac{S_i + D_i\,\theta_{ci}}{S_i + D_i}$$

Substituting the value of $S_i = \left[\frac{\mu_i\,\theta_{ci}\,\theta_\epsilon}{(r\theta_\epsilon + \beta_i\pi)} + \tau_{ci}\,D_i\right] - D_i \qquad [\ldots] = V_i$

we get

$$\frac{\mu_i - rD_i\left(1 + \frac{1}{\theta_{ci}}\right)}{\mu_i} \quad x \quad \frac{\frac{\mu_i\,\theta_{ci}\,\theta_\epsilon}{(r\theta_\epsilon + \beta_i\pi)} + \tau_{ci}\,D_i - D_i + D_i\,\theta_{ci}}{\frac{\mu_i\,\theta_{ci}\,\theta_\epsilon}{(r\theta_\epsilon + \beta_i\pi)} + \tau_{ci}\,D_i - D_i + D_i}$$

$$= \frac{\mu_i + rD_i\left(1 - \frac{1}{\theta_{ci}}\right)}{\mu_i} \quad x \quad \frac{\frac{\mu_i\,\theta_{ci}\,\theta_\epsilon}{(r\theta_\epsilon + \beta_i\pi)} - D_i\,\theta_{ci} + D_i\,\theta_{ci}}{\frac{\mu_i\,\theta_{ci}\,\theta_\epsilon + \tau_{ci}\,D_i\,(r\theta_\epsilon + \beta_i\pi)}{(r\theta_\epsilon + \beta_i\pi)}}$$

$$= \frac{\mu_i + rD_i\,(\tau_{ci}/\theta_{ci})}{\mu_i} \quad x \quad \frac{\mu_i\,\theta_{ci}\,\theta_\epsilon}{(r\theta_\epsilon + \beta_i\pi)} \quad x \quad \frac{(r\theta_\epsilon + \beta_i\pi)}{\mu_i\,\theta_{ci}\,\theta_\epsilon + \tau_{ci}\,D_i(r\theta_\epsilon + \beta_i\pi)}$$

$$= (\mu_i + (rD_i\,\tau_{ci}/\theta_{ci})) \quad x \quad \frac{\theta_{ci}\,\theta_\epsilon}{\theta_{ci}\,\theta_\epsilon\left(\mu_i + \frac{\tau_{ci}\,D_i\,r}{\theta_{ci}} + \frac{\tau_{ci}\,D_i\,\beta_i\pi}{\theta_{ci}\,\theta_\epsilon}\right)}$$

$$= 1 \quad x \quad \frac{\left(\mu_i + \frac{rD_i\,\tau_{ci}}{\theta_{ci}}\right)}{\left(\mu_i + \frac{rD_i\,\tau_{ci}}{\theta_{ci}}\right) + \frac{\tau_{ci}\,D_i\,\beta_i\pi}{\theta_{ci}\,\theta_\epsilon}}$$

$$= \quad 1 \div \left[ \frac{\mu_i + \frac{rD_i \tau_{ci}}{\theta_{ci}}}{\mu_i + \frac{rD_i \tau_{ci}}{\theta_{ci}}} + \frac{\frac{\tau_{ci} D_i ß_i \pi}{\theta_{ci} \theta_\epsilon}}{\mu_i + \frac{rD_i \tau_{ci}}{\theta_{ci}}} \right]$$

$$= \quad 1 \div \left[ 1 + \left( \frac{\tau_{ci} D_i ß_i \pi}{\theta_{ci} \theta_\epsilon} \div \frac{\mu_i \theta_{ci} + rD_i \tau_{ci}}{\theta_{ci}} \right) \right]$$

$$= \quad 1 \div \left[ 1 + \left( \frac{\tau_{ci} D_i ß_i \pi}{\theta_{ci} \theta_\epsilon} \times \frac{\theta_{ci}}{\mu_i \theta_{ci} + D_i \tau_{ci} r} \right) \right]$$

$$= \quad 1 \div \left[ 1 + \frac{\tau_{ci} D_i ß_i \pi}{\theta_\epsilon (\mu_i \theta_{ci} + rD_i \tau_{ci})} \right]$$

The above ratio is shown as Equation 11.77.

## Calculation of Ratio

### Expenditure Tax Total Return to all Investors

The formula per equation 11.77 and per Appendix B2 is as follows :

$$\frac{1}{1 + \dfrac{\tau_{ci} D_i \beta_i \pi}{\theta_\epsilon (\mu_i \theta_{ci} + rD_i \tau_{ci})}}$$

(a) We take

| | | | | | |
|---|---|---|---|---|---|
| $\mu_i$ | = | 50 | $D_i$ | = | £100 |
| $\tau_{ci}$ | = | 0.17722 | $\theta_{ci}$ | = | 0.82278 |
| $\beta_i$ | = | 1 | $\pi$ | = | 0.08 |
| $\theta_\epsilon$ | = | 0.8 | r | = | 0.05 |

The ratio is

$$1 \div \left[ 1 + \frac{0.17722 \times 100 \times 1 \times 0.08}{0.8 (50 \times 0.82278 + 0.05 \times 100 \times 0.17722} \right]$$

$$= 1 \div \left[ 1 + \frac{1.41776}{0.8 (41.139 + 0.8861)} \right]$$

$$= 1 \div \left[ 1 + \frac{1.41776}{33.62006} \right]$$

$$= 1 \div [ 1.04217 ]$$

$$= 0.9595$$

As this ratio is less than one, the rate of total return to all investors is less than the rate of return on physical investment, even under expenditure tax system (section (B)(2) of Ch. 11)

(b) For companies with higher leverage, we may take values as in (a) above, except that $D_i$ = £200. Now the ratio is

$$1 \div \left[ 1 + \frac{0.17722 \times 200 \times 1 \times 0.08}{0.8 (50 \times 0.82278 + 0.05 \times 200 \times 0.17722} \right]$$

$$= 1 \div \left[ 1 + \frac{2.83552}{0.8 (41.139 + 1.7722)} \right]$$

$$= 1 \div \left[ 1 + \frac{2.83552}{34.32896} \right]$$

$$= 1 \div [ 1.082598 ]$$

$$= \quad 0.9237$$

This indicates that the distortion under expenditure tax system increases with leverage. The rate of total return to all investors can be 8% below the rate of return on physical investment (section (B)(2) of ch. 11).

(c) For high beta companies, the ratio will be even smaller as the following example shows. We take values as in (a) above, including $D_i$ = £100 but with $\beta_i$ = 2. The ratio now is

$$1 \div [\ 1 + \frac{0.17722 \times 100 \times 1 \times 0.16}{0.8\ (50 \times 0.82278\ + 0.05 \times 100 \times 0.17722} \ ]$$

$$= \quad 1 \div [\ 1 + \frac{2.83552}{0.8\ (41.139\ + 0.8861)}\ ]$$

$$= \quad 1 \div [\ 1 + \frac{2.83552}{33.62006}\ ]$$

$$= \quad 1 \div [\ 1.08434\ ]$$

$$= \quad 0.92222$$

Thus, the higher the beta factor, the lower is the proportion of the rate of return on physical investment that is paid out to all investors (section (B)(2) of ch. 11).

## EXPENDITURE TAX

### Ratio when Projects earn a Supernormal Profit

Equation 11.115 gives the following ratio which is further transformed as shown below:

$$\frac{(\mu_i - \alpha D_i r)\,\theta_{ci}}{\alpha\, S_i} \quad \text{x} \quad \frac{\alpha\,(S_i/\theta_{ci} + D_i)}{\mu_i}$$

$$= \frac{(\mu_i - \alpha D_i r)\,\theta_{ci}}{S_i} \quad \text{x} \quad \frac{(S_i + \theta_{ci} D_i)/\theta_{ci}}{\mu_i}$$

$$= \frac{(\mu_i - \alpha D_i r)\,\theta_{ci}}{S_i} \quad \text{x} \quad \frac{S_i + \theta_{ci} D_i}{\theta_{ci}\ \mu_i}$$

$$= \frac{\mu_i - \alpha D_i r}{\mu_i} \quad \text{x} \quad \frac{S_i + \theta_{ci} D_i}{S_i}$$

[Note: $S_i = \frac{\mu_i \theta_{ci} \theta_\epsilon}{r\theta_\epsilon + \beta_i \pi} + LD_i - D_i$ ]

$$= \frac{\mu_i - \alpha D_i r}{\mu_i} \quad \text{x} \quad \frac{\frac{\mu_i \theta_{ci} \theta_\epsilon}{r\theta_\epsilon + \beta_i \pi} + LD_i - D_i + D_i \theta_{ci}}{\frac{\mu_i \theta_{ci} \theta_\epsilon}{r\theta_\epsilon + \beta_i \pi} + LD_i - D_i}$$

$$= \frac{\mu_i - \alpha D_i r}{\mu_i} \quad \text{x} \quad \frac{\mu_i \theta_{ci} \theta_\epsilon}{(r\theta_\epsilon + \beta_i \pi)} \quad \text{x} \quad \frac{(r\theta_\epsilon + \beta_i \pi)}{\mu_i \theta_{ci} \theta_\epsilon - \theta_{ci} D_i (r\theta_\epsilon + \beta_i \pi)}$$

$$= \mu_i - \alpha D_i r \quad \text{x} \quad \theta_{ci} \theta_\epsilon \quad \text{x} \quad \frac{1}{\theta_{ci} [\mu_i \theta_\epsilon + -D_i r \theta_\epsilon - D_i \beta_i \pi]}$$

$$= \mu_i - \alpha D_i r \quad \text{x} \quad \theta_\epsilon \quad \text{x} \quad \frac{1}{\theta_\epsilon \left\{ [\mu_i - D_i r] - \frac{D_i \beta_i \pi}{\theta_\epsilon} \right\}}$$

$$= 1 \quad \text{x} \quad \frac{\mu_i - \alpha D_i r}{\mu_i - D_i r - \frac{D_i \beta_i \pi}{\theta_\epsilon}}$$

The above ratio is shown as Equation 11.116.

# BIBLIOGRAPHY


Altman E I, 1984 — A Further Empirical Investigation of the Bankruptcy Cost Question, Journal of Finance, September 1984

Arrow K J, 1964 — The Role Of Securities In The Optimal Allocation of Risk Bearing, Review of Economic Studies, 86, 1964

Ashton D J, 1989 — Textbook Formulae and UK Taxation: Modigliani and Miller Revisited, Accounting and Business Research, Vol 19, No. 75, Pgs 207-212, 1989

Auerbach A, 1979 — Wealth Maximisation And The Cost Of Capital, Quarterly Journal Of Economics, 93 (1979)

Auerbach A, 1983 — Taxation, Corporate Financial Policy and the Cost of Capital, Journal of Economic Literature, September 1983

Auerbach A, 1984 — Taxes, Firm Financial Policy and the Cost of Capital: An Empirical Analysis, Journal of Public Economics, 1984

Auerbach and King M A, 1983 — Taxation, Portfolio Choice and Debt-Equity Ratios: A General Equilibrium Model, Quarterly Journal of Economics, November

Baxter N D, 1967 — Leverage, Risk of Ruin, and the Cost of Capital, Journal of Finance, vol 22, pp. 395-404 September 1967

W H Beaver and J manegold, 1975 — The Association between Market determined and Accounting determined measures of systematic risk: some further evidence, JFQA, pp 231-284, June 1975

Bhattacharya S, 1979 — Imperfect Information, Dividend Policy and 'the Bird In Hand' Fallacy, Bell Journal Of Economics, Spring 1979

Black F, 1973 — Taxes & Capital Market Equilibrium Under Uncertainty, Working Paper No. 21B, Chicago , May 1973

| | |
|---|---|
| Black, F 1974 | International Capital Market Equilibrium with Investment Barriers, Journal of Financial Economics, Dec 1974 |
| Black and Scholes, 1974 | The Effect of Dividend Yield and Dividend Policy on Common Stock Prices and Returns, Journal of Financial Economics, May 1974 |
| Bradford D, 1981 | The Incidence And Allocation Effects Of A Tax On Corporate Distributions, Journal Of Public Economics, 15 (1981) |
| Brealey R A and Dimson E,1978 | The Risk Premium on UK Equities, Investment Analyst, December 1978 |
| Brealey R A and Meyers S C, 1984 | Principles of Corporate Finance, 1984 |
| Breeden, 1979 | An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment opportunities, Journal of financial Economics, September 1979 |
| Brennan M J, 1970 | Taxes, Market Valuation and Corporation Financial Policy, National Tax Journal, 1970 |
| Brennan M J, 1971 | A Note on Dividend Irrelevance and the Gordon Valuation Model, Journal of Finance, December 1971 |
| Brennan M J & Schwartz E S, 1978 | Corporate Income Taxes, Valuation, and the problem of optimal Capital Structure, Journal of Business, pp. 103-114, January 1978 |
| Chen A H Y and Kim H, 1979 | Theories of Corporate Debt Policy: A Synthesis, Journal of Finance, May 1979 |
| Cohn R A and Pringle J J, 1973 | Imperfections in International Financial Marketrs: Implications for Risk Premia and the Cost of Capital to Firms, Journal of Finance, March 1973 |

| | |
|---|---|
| Cooper I and Kaplanis E, 1985 | Costs to Crossborder Investment and International Equity Market Equilibrium, London Business School, June 1985 |
| Conine, 1980 | Corporate Debt and Corporate Taxes: An Extension, Journal of Finance, September 1980 |
| Conine & Tamarkin, 1985 | Divisional Cost Of Capital Estimation: Adjusting for leverage, Financial Management, 1985 |
| Copeland T E and Weston J F, 1982, 1983 | Financial Theory and Corporate Policy, 1982 |
| Cordes J J and Sheffrin S M, 1981 | Taxation and the Sectoral Allocation of Capital in the U.S., National Tax Journal, 34:419-432, 1981 |
| De Angelo and Masulis, 1980 | Optimal Capital Structure Under Corporate and Personal Taxes, Journal of Financial Economics, March 1980 |
| Diamond P A, 1967 | The Role Of A Stock Market In A General Equilibrium Model With Technological Uncertainty, American Economic Review, Sept 1967 |
| Dimson E and Marsh P, 1982 | Calculating the Cost of Capital, Long Range Planning, 1982 |
| Dumas & Alder, 1983 | International Portfolio Choice and Corporate Finance: A synthesis, Journal of Finance, June 1983 |
| Elton and Gruber, 1970 | Marginal Stockholder's Tax Rates and the Clientele Effect, The Review of Economics and Statistics, February 1970 |
| Elton & Gruber, 1978 | Taxes and Portfolio Composition, Journal of Financial Economics, 1978 |
| Errunza and Senbet, 1984 | Inretnational Corporate Diversification, Market Valuation and Size-Adjusted Evidence, Journal of Finance, July 1984 |
| Fama E F, 1976 | Foundations of Finance, 1976 |

| | |
|---|---|
| Fama E F and Macbeth J D, 1973 | Risk, Return, and Equilibrium: Empirical Tests, Journal of Political Economy, vol. 81, no. 3 pp.607-636, May/June 1973 |
| Fama E F and Schwert G W, 1977 | Human Capital and Capital Market Equilibrium, Journal of Financial Economics, January 1977 |
| Farrar D E and Selwyn L L, 1967 | Taxes, Corporate Financial Policy and Return to Investors, National Tax Journal, vol 20, pp. 444-454, 1967 |
| Fatemi A M, 1984 | Shareholder Benefits from Corporate International Diversification, Journal of Finance, Dec 1984 |
| Feldstein M, 1976 | Inflation and Income Taxes and the Rate Of Interest: A Theoretical Analysis, American Economic Review, Dec 1976 |
| Feenberg D, 1981 | Does The Investment Interest Limitation Explain The Existence Of Dividends, Journal Of Financial Economics, 9 (1981) |
| Franks J R & Broyles J E ,1979 | Modern Managerial Finance |
| Friend I, Westfield R & Granito M, 1978 | New Evidence on the Capital Asset Pricing Model, Journal of Finance, June 1978 |
| Fuller and Kerr 1981 | Estimating the Divisional Cost of Capital: An Analysis of the Pure-Play Technique, Journal of Finance, December 1981 |
| Gordon M J 1959 | Dividend Earnings and Stock Prices, Review Of Economics and Statistics, May 1959 |
| Gordon R H & Bradford D F, 1980 | Taxation and the Stock Market Valuation of Capital Gains and Dividends, Journal of Public Economics, 1980. |
| Gordon M H and Halpern P J, 1974 | Cost of Capital for a Division of a firm, Journal of Finance, September 1974 |
| Gordon & Malkiel, 1981 | Corporation Finance, in Aaron and Penchman Eds How Taxes Effect Ecomonic Behavior, 1981 |

Grauer F L A, Litzenberger R H & Stehle R E, 1976 — Sharing Rules and Equilibrium in an International Capital Market under Uncertainty, Journal of Financial Economics, June 1976

Hakansson N H , 1982 — To Pay or not to Pay Dividends, Journal of Finance, May 1982

Hamada R S, 1972 — The Effect of the Firm's Capital Structure on the Systematic Risk of Common Stocks, Journal of Finance, May 1972

Hamada & Scholes, 1985 — in "Recent Advances in Corporate Finance", ed. Altman E I and Subramahmanyam

Horne, Van — "Corporate Liquidity and Bankruptcy Costs", Stanford University, Graduate School Of Business, Research Paper No. 205, Undated

Ibbotson R G & Sinquefield R A, 1982 — Stocks, Bonds, Bills, and Inflation: The Past and the Future, Financial Analysts Research Foundation, Charlottesville, Va., 1982

Jacquillat B and Solnick B,1978 — Multinationals are poor Tools for Diversification, Journal of Portfolio Management, Winter 1978

Jensen M C & Meckling W, 1976 — Theory of the Firm: Managerial Behaviour, Agency Costs and Ownership Structure, Journal of Financial Economics, pp.305-360, October 1976

Kent P and Theobald M, 1980 — The Imputation System, Cost of Capital, and Dividend Policy, Accounting And Business Research, Winter 1980, pgs 61-65.

King M A, 1974 — "Taxation and the Cost of Capital", Review of Economic Studies.

King M A, 1977 — Public Policy And The Corporation, London: Chapman & Hall

King M A and Fullerton D, 1984 — The Taxation Of Income From Capital: A Comparative Study Of The U.S., U.K., Sweden And West Germany. Chicago University Press, 1984

Lessard D R, 1976 — World, Country and Industry Relationships in Equity Returns: Implications for Risk Reduction through Intenational Diversifications, Financial Analysts Journal, 1976

| | |
|---|---|
| Lintner J, 1965 | The Valuation of Risky Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets, The Review of Economics and Statistics, February 1965 |
| Litzenberger R H Ramaswamy K, 1979 | The Effect of Personal Taxes and Dividends on Capital Asset Prices: Theory and Emperical Evidence, Journal of Financial Economics, June 1979 |
| Logue D E, 1973 | On the Pricing of Unseasoned Equity Offerings: 1965-1969, Journal of Financial and Quantitative Analysis, January 1973 |
| Long M & Malitz I 1983 | Investment Patterns and Financial Leverage, NBER, January 1983 |
| Markowitz M M, 1952 | Portfolio Selection, Journal of Finance, March 1952 |
| Masulis R, 1980 | Stock Repurchase by Tender Offer: An analysis of the Causes of Common Stock Price Changes, Journal of Finance, May 1980 |
| Masulis R, 1980 | The Effects Of Capital Structure Change On Security Prices: Astudy Of Exchange Offers, Journal Of Financial Economics, June 1980 |
| Masulis R, 1983 | The Impact of Capital Structure Change on Firm Value, Some Estimates, Journal of Finance, 1983 |
| McDonald G, 1981 | Taxation and Corporate Finance and Investment, Accounting and Business Research, Vol. 12, No. 45, Pgs 41-54. |
| Meade Report, 1978 | The Structure & Reform Of Direct Taxation, Institute Of Fiscal Studies, 1978 |
| Merton R C, 1973 | An intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities, Econometrica, Sept 1973 |
| Merton R C, 1980 | On Estimating the Expected Return on the Market: An Exploratory Investigation, Journal of Financial Economics, December 1980 |
| Miles J and Ezzell R, 1980 | WACC, Perfect Capital Markets and Project Life: A Clarification Journal of Financial and Quantitative Analysis, September 1980 |

| | |
|---|---|
| Miller M, 1977 | Debt and Taxes, Journal of Finance, May 1977 |
| Miller M & Modigliani F, 1961 | Dividend Policy, Growth, and Valuation of Shares, Journal of Business, October 1961 |
| Miller M and Modigliani F, 1966 | Some Estimates of the Cost of Capital to the Electric Utility Industry, American Economic Review, June 1966 |
| Miller M and Scholes M, 1978 | Dividends and Taxes, Journal of Financial Economics, December 1978 |
| Miller M & Scholes M 1982 | Dividends and Taxes: Some Empirical Evidence, Journal of Political Economy, 1982 |
| Modigliani F, 1982 | "Debt, Dividend Policy, Taxes, Inflationand Market Valuation" , Journal of Finance, May 1982 |
| Modigliani F and Miller M, 1958 | The Cost fof Capital, Corporate Finance and the Theory of Investment, American Economic Review, June 1958 |
| Modigliani F and Miller M, 1963 | Corporation Income Taxes and the Cost of Capital: A Correction, American Economic Review, June 1963 |
| Mossin J, 1966 | Equilibrium In A Capital Asset Market, Econometrica, Vol 34, No 4 |
| Myers S C, 1974 | Interactions of Corporate Financing and Investment Decisions - Implications for Capital Budgeting, Journal of Finance, March 1974 |
| Myers S C, 1977 | Determinants of Corporate Borrowing, Journal of Financial Economics, November 1977 |
| Myers S C, 1984 | A Simple Model Of Financing And Investment Decisions, Journal Of Finance, May 1984 |
| Myers S C and Turnball , 1977 | Capital Budgeting and the CAPM: Good News and Bad News, S M Jounral Of finance, May 1977 |
| Poterba J M & Summers L H, 1984 | New Evidence that Taxes Affect the Valuation of Dividends, Journal of Finance, Dec 1984 |

Poterba J M & Summers L H, 1985 — The Economic Effect of Dividend Taxation, Recent Advances in Corporate Finance, 1985

Roll R, 1977 — A Critique of the Asset Pricing Theory,s Tests, Journal of Financial Economics, March 1977

Ross S, 1976 — The Arbitrage Theory of Capital Asset Pricing, Journal of Economic Theory, December 1976

Ross S, 1977 — The Determinants of Financial Structure: The Incentive-Signalling Approach, Bell Journal of Economics, Spring 1977

Rozeff, 1981 — Growth, Beta, and Agency Costs as Determinants of Dividend Payout Ratios, w.o. No 81-11, Iowa, June 1981

Rubinstein M E 1973 — Corporate Financial Policy In Segmented Securities Markets, Journal Of Financial and Quantitative Analysis , Dec 1973

Rutterford J, 1986 — Ph.D. Thesis, London, 1986

Rutterford J, 1988 — An International Perspective On The Capital Structure Puzzle, In New Developments In International Finance, Stern J M and Chew D H (Eds) Blackwell Pgs 194-208

Sharpe W F, 1964 — Capital Asset Prices: A theory of Market Equilibrium Under Conditions of Risk, Journal of Finance, September 1964

Shefrin and Statman , 1984 — Explaining Investor Preference for Cash Dividends, Journal of Financial Economics, 1984

Shiller R J 1981 — Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends? , American Economic Review, June 1981

Simkowitz A, and Beedles W L, 1978 — Diversification in a Three Moment World, Journal of Finance and and Quantitative Analysis, Dec 1978

Skelton, 1983 — Banks, Firms and the Relative Pricing Of Tax-Exempt and Taxable Bonds, Journal of Financial Economics, Nov 1983

Solnik, 1974 — An Equilibrium Model of the International Capital Market, Journal of Economic Theory, 1974

Stiglitz, 1973 — Taxation, Corporate Financial Policy, and the Cost of Capital, Journal of Public Ecomomics, Feb 1973

Shultz, 1981 — On the Effects of Barriers to International Investment, Journal of Finance, September 1981

Stulz, 1981b — A model of International Asset Pricing, Journal of Financial Economics, 1981

Summers,(1981a) — The Non Adjustment Of Interest Rates: A Study Of The Fisherian Effect, Symposium in Honor of Arthur Akun, 1981

Summers L, 1985 — On Economics and Finance, Journal of Finance, July 1985

Titman S, 1981 — The Effect Of Capital Structure On A Firm's Liquidation Decision, Unpublished Ph. D. thesis, Graduate School Of Industrial Economics, Carnegie-Mellon University, 1981

Treynor J, 1961 — Toward a Theory of the Market Value of Risky Assets, Unpublished manuscript, 1961

Warner J, 1977 — Bankruptcy, Absolute Priority, and the Pricing of Risky Debt Claims, Journal Of Financial Economics, May 1977

Weston J F, 1963 — A Test Of Cost Of Capital Propositions, Southern economic Journal, Oct 1963

Yu Zhu & Friend I, 1986 — The Effect of Different Taxes on Risky and Risk-free Investment and the Cost of Capital, Journal of Finance, March 1986