TESTING FRACTIONAL INTEGRATION IN MACROECONOMIC TIME SERIES

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ABSTRACT

This thesis concentrates on testing fractional (and seasonally fractional) integration and cointegration in macroeconomic time series.

Fractional integration has recently emerged in the literature as an alternative plausible way of modelling economic series, and here we focus mainly on some empirical applications of a testing procedure suggested by Robinson (1994c) for testing unit roots and other nonstationary hypotheses in raw time series. These tests, described in Chapter 2, are asymptotically most powerful against fractional alternatives, have asymptotic critical values given by a chi-squared distribution, and allow great flexibility in the choice of null and alternative hypotheses, which can entail one or more integer or fractional roots of arbitrary order anywhere on the unit circle in the complex plane. In Chapter 2 we also make some simulations, comparing the size-corrected versions of the tests with those based on asymptotic critical values, and other existing unit root tests.

The tests of Robinson (1994c) are applied in Chapter 3 to an extended version of the data set used by Nelson and Plosser (1982). These are fourteen U.S. macroeconomic variables in annual data, and we focus here on cases where the root is located at zero frequency.

In Chapter 4 we concentrate on seasonality. Robinson's (1994c) tests are now applied to quarterly U.K. and Japanese consumption and income series, using the same data as in Hylleberg, Engle, Granger and Yoo (HEGY, 1990) and Hylleberg, Engle, Granger and Lee (HEGY, 1993). We test for the presence of unit or fractional roots, not only at zero but also at seasonal frequencies.

A multivariate version of the tests, based on the score, likelihood-ratio and Wald principles is obtained in Chapter 5 and some simulations, based on Monte Carlo experiments, are carried out at the end of the chapter.

The multivariate tests of Chapter 5 are applied in Chapter 6 to some pairs of macroeconomic variables claimed to be cointegrated by many authors. Using the same data as in Engle and Granger (1987) and Campbell and Shiller (1987), we analyze the relationship between U.S. consumption and income, prices and wages, GNP and money and stock prices and dividends. A testing procedure to investigate if these pairs of variables are fractionally cointegrated is also described and applied in Chapter 6.

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CHAPTER 1

1. INTRODUCTION

The purpose of this section is two-fold. First we motivate and define what we understand by long memory and fractional integration, then we go on to summarize some results concerning estimation and testing in the context of long memory processes.

1.1. GENERAL INTRODUCTION

It is broadly accepted that one feature of macroeconomic variables is that the level of the series evolves or changes with time, although in a rather smooth fashion. A common practice to explain and model these smooth movements was to assume that the series fluctuate around a deterministic trend, via a polynomial and/or a trigonometric function of time, which are fitted by linear regression techniques. A second way came after Nelson and Plosser's (1982) influential work, who following the work and ideas of Box and Jenkins (1970), argued that these fluctuations in the level were better explained by means of the so-called unit roots, or in other words, that the change in level is "stochastic" rather than "deterministic". Both "schools" try to model this persistent trend-cycle behaviour of the data although from a different perspective.

Mandelbrot (1969) and Mandelbrot and co-authors discussed a third way of explaining these fluctuations in the level. He argued that while many macroeconomic series exhibit a persistent trend-cyclical behaviour for a stretch of the data, when the same data is examined for a longer period, the persistent behaviour tends to disappear. The same type of phenomenon was observed in other areas, notably in hydrology, and called the Hurst effect, in honour of the hydrologist Hurst, (Hurst (1951), (1957)), who, studying the records in the level of the river Nile, noticed that kind of pattern in its behaviour. In particular, he noticed that the autocorrelations took far longer to decay to zero than the exponential rate associated with the autoregressive moving average (ARMA) class of models. These kind of processes are called long memory, due to their ability to display significant dependence between distant observations in time.

We can give two definitions of long memory. Given a discrete covariance stationary time series process, say $\{x_t\}$, with autocovariance function $E(x_t-Ex_t)(x_{t-j}-Ex_t) = \gamma_j$, according to McLeod and Hipel (1978), the process is long memory if

$$\lim_{T\to\infty} \sum_{j=-T}^{T} |\gamma_j|$$

is infinite. A second way to characterize this type of processes is in the frequency domain. For that purpose, suppose that $\{x_t\}$ has an absolutely continuous spectral distribution, so that it has a spectral density function, denoted $f(\lambda)$, and defined as

$$f(\lambda) = \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \gamma_j \cos \lambda j, \qquad -\pi < \lambda \leq \pi$$

Thus, we can say that x_t displays the property of long memory if the spectral density function has a pole at some frequency λ in the interval $[0,\pi]$. One model capable of explaining this feature is the fractional Gaussian noise model, analyzed in Mandelbrot and Van Ness (1968), and characterized by having an autocovariance function defined as

$$\gamma_j = \frac{1}{2}\gamma_0(|j+1|^{2d+1} - 2|j|^{2d+1} + |j-1|^{2d+1}), \qquad j = 1,2,\ldots,$$

where $0 < d < 1/2$. Another model, very popular among econometricians, is the so-called fractionally integrated model. A popular technique to analyze this model is through the fractional difference ∇^d, where

$$\nabla^d = (1 - L)^d = \sum_{k=0}^{\infty} (-1)^k \binom{d}{k} L^k,$$

and L is the lag operator. To illustrate this in case of a scalar time series x_t, $t=1,2,\ldots$, suppose that u_t is an unobservable covariance stationary sequence with spectral density that is bounded and bounded away from zero at any frequency, and

$$(1 - L)^d x_t = u_t \qquad t = 1,2,\ldots \tag{1}$$

The process u_t could itself be a stationary and invertible ARMA sequence, when its autocovariances decay exponentially, however, they could decay much slower than exponentially. When $d = 0$ in (1), $x_t = u_t$ and thus, x_t is 'weakly autocorrelated', also termed 'weakly dependent'. If $0 < d < 1/2$, x_t is still stationary, but its lag-j autocovariance γ_j decreases very slowly, like the power law j^{2d-1} as $j\to\infty$ and so the γ_j are non-summable. We say then that x_t has long memory given that its spectral density $f(\lambda)$ is unbounded at the origin. It may also be shown that these kind of processes satisfy[1]

$$\gamma_j \sim c_1 \, j^{2d-1}, \qquad as \;\; j \to \infty \qquad for \quad |c_1| < \infty \tag{2}$$

and

$$f(\lambda) \sim c_2 \, \lambda^{-2d}, \qquad as \;\; \lambda \to 0^+ \qquad for \;\; 0 < c_2 < \infty. \tag{3}$$

where the symbol ~ means that the ratio of the left hand side and the right hand side tends to 1, as $j \to \infty$ in (2), and as $\lambda \to 0^+$ in (3). Conditions (2) and (3) are not always equivalent but Zygmund (1995, Cap.V, Sect.2), and more generally Yong (1974) give conditions under which both expressions are equivalent. Finally, as d in (1) increases beyond 1/2 and through 1 (the unit root case), x_t can be viewed as becoming 'more nonstationary' in the sense, for example, that the variance of the partial sums increases in magnitude. This is also true for $d > 1$, so a large class of nonstationary process may be described by (1) with $d \geq 1/2$. Processes like (1) with positive non-integer d are called fractionally integrated processes and when u_t is ARMA(p,q), x_t has been called a fractional ARIMA(p,d,q) process. These kind of models provide a type of flexibility in modelling low frequency dynamics not achieved by non-fractional ARIMA models. They were introduced by Granger and Joyeux (1980), Granger (1980, 1981) and Hosking (1981), (although earlier work by Adenstedt (1974) and Taqqu (1975) shows an awareness of the representation), and were justified theoretically by Robinson (1978) and Granger (1980). They observed that if the individual series follow AR(1) processes, i.e.,

$$x_{i,t} = \alpha_i \, x_{i,t-1} + u_{i,t}, \qquad i = 1,\ldots N, \quad t = 1,2,\ldots,$$

then the aggregate series

$$x_t = \sum_{i=1}^{N} x_{i,t}$$

can exhibit long memory if, for instance, α_i are drawn from a Beta B(p,q) distribution for certain values of p and q.

So far we have considered processes which have (or have after taking appropriate differences) a singularity in the spectrum at zero frequency. However, $f(\lambda)$ might also display poles at any other frequency in $(0,\pi]$. Gray et al. (1989, 1994) generalized (1) to allow persistent cycle behaviour and considered the

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[1] Condition (2) is satisfied by the fractional ARIMA(0,d,0) case. However, including ARMA components, it is required all γ_j to be eventually non-negative.

Gegenbauer process

$$(1-2\eta L+L^2)^{\frac{d}{2}}\, x_t \;=\; u_t \qquad t = 1,2,... \qquad (4)$$

which is stationary if either $|\eta|<1$ and $d<1$ or $|\eta|=1$ and $d<1/2$, and the spectrum is infinite at $\lambda = \arccos(\eta)$. By analogy with the fractional ARIMA(p,d,q) process, (4) can be generalized to include autoregressive and moving average components in u_t.

A further parametric long memory process suggested by Porter-Hudak (1990) is the seasonal fractionally integrated process given by

$$(1-L^s)^d\, x_t \;=\; u_t, \qquad t = 1,2,..., \qquad (5)$$

with $d \in (-1/2,1/2)$, where s is the seasonal period and u_t may be represented as an ARMA(p,q) process. When $d > 0$, the spectrum is unbounded at frequencies $\lambda_j=2\pi j/s$ for $j=0,1,2,...,s/2$, so the model contains a persistent trend and s/2 persistent cyclical components. Hence, this process shows a behaviour at seasonal frequencies similar to that of the fractional ARIMA process at zero frequency, and thus, much nonstationary behaviour may be modelled at seasonal frequencies allowing $d \geq 1/2$. Therefore, there is some interest in estimating the fractional differencing parameter d. This is important, not only because it reflects the degree of strong dependence in a series, but also because rates of convergence of some statistics that are relevant for statistical inference depend on d. In the following section we review and discuss some aspects concerning estimation and testing in long memory series, and in particular, in fractionally integrated series.

1.2. GENERAL RESULTS ON ESTIMATION AND TESTING

In the previous section we have discussed the role that the parameter d plays, since that parameter gives an indication of the strength of dependence in the time series. Hence, it appears that one important point is how can we estimate d in a given stretch of data.

There are two main approaches to estimate the parameter d. The first approach is parametric, i.e., the model is specified up to a finite number of parameters of which d is one. The second is semi-parametric and is based on the limiting relationships (2) or (3). The methods presented below require that d must belong to the stationary region, so that if the time series is nonstationary, then an appropriate number of differences have to be taken before proceeding to the estimation.

Starting with parametric methods, d is estimated jointly with all the other parameters that specify the model, and the analysis can be carried out in the frequency or in the time domain. In the frequency domain, it is assumed that the spectral density function, $f(\lambda,\theta)$, is known up to a certain parameter vector θ ($d \in \theta$), where θ_0 denotes the true value, and the estimation procedure consists in estimating θ by some Gaussian methods. Fox and Taqqu (1986) assumed Gaussianity of the process, and minimized the Whittle function (an approximation to the exact likelihood function)

$$\frac{1}{4\pi}\int_{-\pi}^{\pi}\left(\log f(\lambda,\theta) + \frac{I(\lambda)}{f(\lambda,\theta)}\right)d\lambda \qquad (6)$$

where $I(\lambda)$ is the periodogram of the process x_t, defined as

$$I(\lambda) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T} x_t\, e^{it\lambda}\right|^2 .$$

The estimate is shown to be consistent and asymptotically normal under appropriate conditions, which are satisfied by fractional models as in (1) with $0 < d < 1/2$. Another estimate with the same asymptotic behaviour is obtained if (6) is replaced by a sum over the Fourier frequencies, i.e., minimizing

$$\frac{1}{2T}\sum_{j=1}^{T-1}\left(\log f(\lambda_j,\theta) + \frac{I(\lambda_j)}{f(\lambda_j,\theta)}\right), \quad with \ \lambda_j = \frac{2\pi j}{T} .$$

Sowell (1992a) analyzed in the time domain the exact maximum likelihood estimates of the parameters of a fractional ARIMA model, using recursive procedures that allow quick evaluation of the likelihood function. A limitation of his procedure is that the roots of the AR polynomial cannot be multiple and the theoretical mean parameter must be either zero or known. Although the time and the frequency domain ML estimators are asymptotically equivalent, their finite sample properties differ, and the Monte Carlo analyses carried out in Sowell (1992a) show that the time domain ML estimator gives better finite-sample properties than the frequency domain ML estimator when the mean of the process is known. Cheung and Diebold (1994) show, however, that the finite-sample efficiency of a discrete version of the approximate (Whittle) frequency domain ML relative to exact time domain ML rises dramatically when the mean is unknown and it has to be estimated.

In addition, Dahlhaus (1989) also assumes Gaussianity but considers the exact likelihood function and minimizes

$$\frac{1}{2T}\log|T_T(f(\theta))| + \frac{1}{2T}(x_T-\mu_T)'T_T(f(\theta))^{-1}(x_T-\mu_T)$$

where $T_T(f(\theta))$ is a TxT matrix with (r,s) element:

$$\{T_T(f(\theta))\}_{(r,s)} = \int_{-\pi}^{\pi} f(\lambda,\theta)\, e^{i(r-s)\lambda} d\lambda \qquad for \quad r,s, = 1,\ldots T,$$

μ_T estimates consistently the mean μ_0 and T denotes the sample size. He proves that his estimate and the one studied by Fox and Taqqu (1986) are both not only asymptotically normal but also asymptotically efficient in the sense of Fisher, i.e. their asymptotic variance is equal to the inverse of the information matrix $\Gamma(\theta_o)$:

$$T^{\frac{1}{2}}(\hat{\theta}-\theta_o) \to_d N(0,\Gamma(\theta_o)^{-1}).$$

It is worth pointing out that all these parametric estimates have the same asymptotic properties of $T^{1/2}$-consistency and asymptotic normality, and when x_t is actually Gaussian, asymptotic efficiency. Finally, Giraitis and Surgailis (1990) relax the Gaussianity assumption and analyze the Whittle estimate for linear processes, showing that it is $T^{1/2}$-consistent and asymptotically normal, although the estimate is no longer asymptotically efficient, while Hosoya (1997) extends the previous analysis to a multivariate framework.

However, on estimating with parametric approaches, the correct choice of the model is important; if it is misspecified, the estimates of d are liable to be inconsistent. In fact, misspecification of the short run components of the series can invalidate the estimation of its long run behaviour. Thus, there might be some advantages in estimating d on the basis of semi-parametric approaches. They are called semi-parametric models because they parameterize only the long-run characteristics of the series. There is a price to be paid in terms of efficiency in not using a correct parametric model, but when the sample size is large the greater robustness of semi-parametric models-based procedures is relevant.

Before considering some semi-parametric estimates discussed in the literature, we should mention an estimate (Hurst (1951)) that is based on the so-called adjusted rescaled range, or "R\S" statistic, and defined as

$$R\backslash S = \frac{\max_{1\le j\le T}\sum_{t=1}^{j}(x_t-\overline{x}) - \min_{1\le j\le T}\sum_{t=1}^{j}(x_t-\overline{x})}{\left(\frac{1}{T}\sum_{t=1}^{T}(x_t-\overline{x})^2\right)^{\frac{1}{2}}}$$

where $\overline{x}$ is the sample mean of the process x_t. The specific estimate of d (Mandelbrot and Wallis (1968)) is given by:

$$\frac{\log(R\backslash S)}{\log T} - \frac{1}{2}.$$

Its properties were analyzed in Mandelbrot and Wallis (1969), Mandelbrot (1972, 1975) and Mandelbrot and Taqqu (1979). Beran (1994) provides a neat explanation of how to implement the R\S procedure. Finally, Lo (1991) modified the R\S statistic to be robust to weak dependence.

Several methods of estimating semi-parametrically the fractional differencing parameter d were examined in a number of papers by Robinson (1994a, 1994b, 1995a, 1995b) which we are to describe. Using the time domain, Robinson (1994a) suggested the log autocovariance estimate, which is based on taking logs in expression (2),

$$\log\gamma_j \sim \log c_1 + (2d-1)\log j, \qquad as \quad j \to \infty,$$

and substituting

$$\hat{\gamma}_j = \frac{1}{T}\sum_{t=1}^{T-j}(x_t-\overline{x})(x_{t+j}-\overline{x}), \qquad j = 0,1,\ldots T\text{-}1$$

for γ_j. The OLS regression of log $\hat{\gamma}_j$ on log j then leads to the estimate

$$\hat{d} = \frac{1}{2}\left(1+\frac{\sum_{j=T-r}^{T-1}\log\hat{\gamma}_j(\log j-\overline{\log j})}{\sum_{j=T-r}^{T-1}(\log j-\overline{\log j})^2}\right), \quad where\ \overline{\log j} = \frac{1}{r}\sum_{j=T-r}^{T-1}\log j$$

and r is a large integer less than T. A disadvantage of this estimate is that even if the γ_j are all positive for large j, some $\hat{\gamma}_j$ can be negative, especially when γ_j is close to zero. An alternative procedure described in the same article is the minimum distance autocovariance estimate, which is implicitly defined by

$$(\hat{d}, \hat{c}_1) = arg\min_{d,c_1}\sum_{j=T-r}^{T-1}(\hat{\gamma}_j - c_1 j^{2d-1})^2,$$

for $d \in (0,1/2)$ and $c_1 \in R$.

Semi-parametric estimates based on the frequency domain are the log-periodogram estimate proposed by Geweke and Porter-Hudak (1983) and modified by Künsch (1986) and Robinson (1995a); the averaged periodogram estimate proposed by Robinson (1994b), and the quasi maximum likelihood estimate (Robinson (1995b)). The first of these estimates is based on the regression model like

$$\log I(\lambda_j) = C - 2d\log\lambda_j + \epsilon_j \qquad (7)$$

$$where \quad \lambda_j = \frac{2\pi j}{T} \quad j=1,...m, \quad \frac{m}{T} \to 0,$$

$$C \sim \log\left(\frac{\sigma^2}{2\pi} f(0)\right), \quad \epsilon_j = \log\left(\frac{I(\lambda_j)}{f(\lambda_j)}\right),$$

and the estimate of d is just the OLS estimate of d in (7). Unfortunately, it has not been proved that this estimate is consistent for d, but Robinson (1995a) modifies the former regression introducing two alterations:

- use a pooled periodogram instead of the raw periodogram, and
- introduce a trimming number p, so that frequencies λ_j, j=1,...p, are excluded from the regression, where p tends to infinity slower than m, so that p/m tends to zero.

So, the final regression model is

$$Y_k^{(J)} = C^{(J)} - 2d\log\lambda_k + U_k^{(J)}$$

$$where \quad Y_k^{(J)} = \log\left(\sum_{j=1}^{J} I(\lambda_{k+j-J})\right) \qquad k = p+J, p+2J, ...m,$$

where J controls the pooling and p controls the trimming. Assuming Gaussianity, he proves the consistency and asymptotic normality of this estimate in a multivariate framework.

The average periodogram estimate of Robinson (1994b) is based on the limiting relation (3). The estimate employs an average of the periodogram near zero frequency,

$$\hat{F}(\lambda_m) = \frac{2\pi}{T}\sum_{j=1}^{m} I(\lambda_j),$$

and suggesting the estimator

$$\frac{1}{2} - \log\left(\frac{\hat{F}(q\lambda_m)}{\hat{F}(\lambda_m)}\right) 2\log q \quad where \quad \lambda_m = \frac{2\pi m}{T}, \quad \frac{m}{T} \to 0$$

for any constant $q \in (0,1)$. He proves the consistency of this estimate under very mild conditions, and Lobato and Robinson (1996a) shows the asymptotic normality for $0 < d < 1/4$, and the non-normal limiting distribution for $1/4 < d < 1/2$.

Finally, the quasi maximum likelihood estimate in Robinson (1995b) is basically a "local Whittle estimate" in the frequency domain, considering a band of frequencies that degenerates to zero. The estimate is implicitly defined by:

$$argmin_d \left(\log \hat{C}(d) - 2d\frac{1}{m}\sum_{j=1}^{m} \log \lambda_j \right)$$

$$for\ d \in (-\frac{1}{2}, \frac{1}{2}), \quad \hat{C}(d) = \frac{1}{m}\sum_{j=1}^{m} I(\lambda_j)\,\lambda_j^{2d}, \quad \lambda_j = \frac{2\pi j}{T}, \quad \frac{m}{T} \to 0.$$

Under finiteness of the fourth moment and other conditions, Robinson (1995b) proves the asymptotic normality of this estimate, which is more efficient than the former ones (Robinson, 1995a, 1994b). Multivariate extensions of these estimation procedures can be found in Lobato (1995).

All estimation methods presented so far concentrate on cases where the pole in the spectrum occurs at zero frequency. Hidalgo and Yajima (1996) suggest two semi-parametric estimates of d when $f(\lambda) \sim C\,|\lambda - \lambda_0|^{-2d}$ as $\lambda \to \lambda_0$ and $\lambda_0 \in [0,\pi]$. These estimates, which have explicit though complicated solutions, are shown to be asymptotically normal, achieving an optimal rate of convergence and being as efficient as the others suggested in the literature. Finally, Hidalgo (1996) proposes an estimator of λ_0, which is asymptotically normal, showing that d can be estimated as well as when the singularity λ_0 is known.

Up to now, we have given a brief discussion of estimation methods in the context of long memory processes. Our next step is to describe some of the most relevant literature on testing in this context. Testing with long memory is an area of research that is attracting growing interest. Tests for white noise against stationary fractional alternatives were developed in Davies and Harte (1987) and Robinson (1991). In the former, they propose tests of white noise against the fractional Gaussian noise alternative. In Robinson (1991), a Lagrange Multiplier (LM) test is described under the standard assumptions which, under the null hypothesis of white noise will have an asymptotic chi-squared distribution. The alternatives are of the class

$$x_t = \sum_{j=1}^{\infty} \phi_j(\theta)\, x_{t-j} + u_t, \tag{8}$$

where the u_t in (8) are the disturbances in a linear regression model satisfying: $E(u_t \mid B_t) = 0$, and $E(u_t^2 \mid B_t) = \sigma^2$, where B_t is the σ-field of events generated by u_s, $s < t$. The coefficients $\phi_j(\theta)$ are uniquely defined functions of the vector θ, such that $\phi_j(\theta) = 0$ for all $j \geq 1$ if and only if the null hypothesis $\theta = 0$ holds. Thus, under the null, x_t is white noise. The partial derivatives at $\theta = 0$, $\phi_j = (\partial/\partial\theta)\phi_j(\theta)$ must be square-summable and also

$$\sum_{j=1}^{\infty} \phi_j \phi_j' \; > \; 0,$$

meaning that the matrix is positive definite. The square-summability condition on the ϕ_j is weak enough to include long memory alternatives, as fractional Gaussian noise, and in particular the ARIMA(0,d,0) class.

Wu (1992) and Agiakloglou and Newbold (1993) examine LM tests of ARMA(p,q) models against fractional ARIMA(p,d,q) alternatives. In the latter, they suggest two variants of a LM test which are identical in spirit to the tests for additional autoregressive or moving-average parameters of Godfrey (1979). They show that the tests have low power when the orders (p,q) are over-specified in the ARMA representation. Lobato and Robinson (1996b) also propose a LM test for testing that a vector process is weakly correlated against alternatives which might be fractionally integrated. The test is non-parametric and they apply the LM principle to the objective function used by Robinson (1995b), obtaining a simply-computed test that is likely to have good efficiency properties. They give some conditions under which the statistic has a limiting null χ_p^2 distribution.

Beran (1992) analyzes for long memory series a goodness-of-fit test proposed by Milhoj (1981) in the frequency domain. This test is an extension of the Box-Pierce (1970) statistic, taking into account all the computable correlations. They show that the asymptotic distribution under the null hypothesis is the same as in the weak autocorrelation case.

Hidalgo and Yajima (1996) consider semiparametric tests for weak dependence (i.e., $d = 0$) against the alternative of long memory ($d > 0$) when the singularity or pole of the spectrum is left unknown. These tests are based on the limiting distributions of the estimates obtained in Hidalgo and Yajima (1996). Finally, Hidalgo and Robinson (1996) propose a Wald test for structural break at a known period of time (say τ) in a linear regression model

$$y_t = \beta_t' z_t + x_t \qquad \beta_t = \begin{Bmatrix} \beta_1 & t = 1,2,\ldots\tau \\ \beta_2 & t = \tau+1,\ldots T \end{Bmatrix},$$

with stochastic and nonstochastic regressors z_t, and x_t being a Gaussian long memory process. Existing tests for structural breaks based on x_t being white noise or weakly dependent will not hold.

Unlike most of these previous procedures, Robinson (1994c) establishes a very general framework in which many long memory as well as nonstationary models can be considered as null or alternative hypotheses. The model he considers is

$$y_t = \beta' z_t + x_t \qquad t = 1,2,\ldots \tag{9}$$

$$\rho(L;\theta)\, x_t = u_t \qquad t = 1,2,\ldots. \tag{10}$$

where y_t and the (kx1) vector z_t in (9) are observable and β is an unknown (kx1) vector. The elements of z_t are assumed to be non-stochastic (such as polynomials in t), and u_t in (10) is a covariance stationary sequence with zero mean and weak parametric autocorrelation. $\rho(L;\theta)$ is a prescribed function of the backshift operator and the (px1) vector θ, of form

$$\rho(L;\theta) = (1-L)^{\gamma_1+\theta_{i_1}} (1+L)^{\gamma_2+\theta_{i_2}} \prod_{j=3}^{h} (1-2\cos w_j L + L^2)^{\gamma_j+\theta_{i_j}} \tag{11}$$

for given γ_j, j=1,...h, where for each j, $\theta_{i_j} = \theta_l$ for some l, and for each l there is at least one j such that $\theta_{i_j} = \theta_l$; thus, $h \geq p$.

The null hypothesis is

$$H_o: \theta = 0, \tag{12}$$

where there is no loss of generality in using the vector of zeros instead of an arbitrary given vector, and the test statistic will be a LM test based on the frequency domain. Given the functional form chosen for ρ in (11), we can consider several cases of fractional integration under the null and alternative hypotheses. Thus, fractional integration of the form as in (1) can be tested if $\rho(L;\theta) = (1 - L)^{d+\theta}$; cyclic behaviour as in (4) if $\rho(L;\theta) = (1 - 2\cos wL + L^2)^{d+\theta}$ for $0 < w < \pi$; seasonally fractional integration as in (5) if $\rho(L;\theta) = (1 - L^s)^{d+\theta}$, and so on. (Note that in the first two cases, h = p = 1, and in the third one, if s = 4, h = 3 and p = 1. However, we could also consider cases with p > 1, for instance, $\rho(L;\theta) =$

$(1 - L)^{d_1+\theta_1} (1 + L)^{d_2+\theta_2} (1 + L^2)^{d_3+\theta_3})$.

The tests of Robinson (1994c) are asymptotically locally most powerful when directed against fractional alternatives, and have asymptotically critical values given by a chi-squared distribution. They constitute the basis of this thesis. In Chapter 2 we describe the tests, justify their null and local limit distributions and make some simulations studying the finite sample behaviour of sized-corrected versions of the tests. Given the great flexibility allowed by Robinson's (1994c) tests for testing different forms of nonstationarity, we use them in Chapter 3 to analyze an extended version of the data set used by Nelson and Plosser (1982). These are historical annual data of fourteen U.S. macroeconomic variables and have been widely analyzed in the literature. We concentrate in this chapter on processes of form as in (1), i.e. fractionally integrated processes with the singularity in the spectrum occurring at zero frequency, and we model the stationary disturbances u_t in (7) not only as white noise or AR processes, but also including the Bloomfield exponential spectral model. Chapter 4 begins by reviewing the literature on seasonality, and different versions of Robinson's (1994c) tests are later applied to some U.K. and Japanese quarterly data analyzed in Hylleberg, Engle, Granger and Yoo (1990) and Hylleberg, Engle, Granger and Lee (1993) respectively. A conclusion drawn in this chapter is that seasonal fractional integration might be another viable way of modelling the nonstationary seasonal component of the series. Multivariate versions of Robinson's (1994c) tests, based on the score, likelihood-ratio and Wald principles are described in Chapter 5. They are shown to be relevant to analyze the interrelationships between different variables, and some Monte Carlo experiments comparing results on finite samples are carried out at the end of this chapter. Finally, these multivariate tests are applied in Chapter 6 to some pairs of economic variables claimed by many authors to be cointegrated. Fractional cointegration is defined and a testing procedure for this hypothesis, based on Robinson's (1994c) tests, is also suggested and applied in this chapter.

CHAPTER 2

ROBINSON'S (1994c) UNIVARIATE TESTS

In this chapter we describe Robinson's (1994c) univariate tests for testing unit roots and other nonstationary hypotheses. We present the tests, their limiting distributions and make some simulations comparing the size-corrected versions of the tests with the non-corrected ones and other existing unit roots tests.

2.1 INTRODUCTION

Versions of the score, likelihood-ratio and Wald principles have been much used in testing for a unit root in a time series against AR alternatives that are stationary or explosive. The test statistics often have nonstandard null and local asymptotic distributions and typically, critical values have to be calculated numerically on a case-by-case basis. However, the AR model is merely one of many models that nest a unit root. We can test H_0 (1.12) in (1.10) with $\rho(L;\theta) = (1 - L)^{1+\theta}$ instead of the AR alternatives described by $\rho(L;\theta) = (1 - (1+\theta)L)$. Robinson (1994c) stresses that the "nonstandard" asymptotic behaviour of commonly used unit roots tests is a consequence of the AR alternative, and provides a different and unified treatment of testing unit roots (and many other hypotheses) as a "standard" problem, in the sense that the test statistics will have an asymptotic χ^2_p null distribution, where p is the number of restrictions tested. Also his tests will be efficient when x_t is Gaussian and more generally, more efficient than other statistics that are also based on sample second moments of x_t. We start first by mentioning some of the most salient features of the tests.

As mentioned in Chapter 1, the tests will allow great flexibility in the choice of the null and alternative hypotheses, which can entail one or more integers or fractional roots of arbitrary order anywhere on the unit circle in the complex plane. This will permit us to test a great variety of model specifications, including seasonal and cyclic behaviours of any stationary and nonstationary degree. Note that under H_0 (1.12), (1.10) becomes

$$\rho(L)\,x_t = u_t \qquad t = 1,2,... \tag{1}$$

with

$$\rho(L) = \rho(L;\theta=0) = (1-L)^{\gamma_1}(1+L)^{\gamma_2}\prod_{j=3}^{h}(1-2\cos w_j L+L^2)^{\gamma_j}$$

for given h, for given distinct real numbers w_j, j=3...h on the interval $(0,\pi]$, and for given real numbers γ_j, j=1,...h. We can briefly indicate some null hypothesized models of interest:

a) "I(1)": $\rho(L) = (1 - L)$. Then x_t given in (1) is a random walk when u_t is a white noise sequence.

b) "I(2)": $\rho(L) = (1 - L)^2$.

c) "Cyclic I(1)": $\rho(L) = (1 - 2 \cos w L + L^2)$, for $0 < w < \pi$.

d) "Quarterly I(1)": $\rho(L) = (1 - L^4) = (1 - L)(1 + L)(1 + L^2)$.

e) "1/f noise": $\rho(L) = (1 - L)^{1/2}$ which is of interest since x_t in (1) is then a fractionally differenced process that is "just nonstationary" opposed to stationary when $\rho(L) = (1 - L)^d$ with $d < 1/2$.

f) "$1/f^{1/2}$ noise": $\rho(L) = (1 - L)^{1/4}$, etc.

Furthermore, x_t need not be observable but can be the errors in a multiple regression model as in (1.9), where the elements of z_t are assumed to be nonstochastic, such as polynomials in t, to include the null hypothesis of a unit root with drift, for example. The limiting null and local distributions of the test statistics will be unaffected by the presence of such regressors. In contrast, asymptotic distributions of test statistics for a unit root null for x_t in (1.9) against AR alternatives seem to be dependent on characteristics of the z_t sequence (see, eg. Schmidt and Phillips, 1992).

The initial discussion of the tests assumes that the u_t in (1) are white noise, so the only nuisance parameters are β and the variance of u_t. Unlike tests based on AR alternatives, the tests of Robinson (1994c) cannot be robustified to allow for weak nonparametric autocorrelation in u_t. (Tests against fractional alternatives with nonparametric autocorrelation under the null would have negligible efficiency relative to parametric autocorrelation). Thus, he includes an extension to the case of weak parametric autocorrelation in u_t, of quite general form to cover stationary and invertible ARMA behaviour and the exponential spectrum model of Bloomfield (1973) (see (12) below), as well as autocorrelations that decay fairly slowly.

The test statistics are derived via score principle, and though undoubtedly the same asymptotic behaviour can be expected of Wald and likelihood-ratio tests, he uses score tests with the usual computational motivation that they entail estimation only under the null hypothesis (1.12). Efficient estimates of fractional models have been studied (see Chapter 1), but they require numerical optimization, have not been very widely used and are not featured in the most widely used time series software packages.

The tests are expressed in the frequency domain. There exists time domain versions of the tests, but the preference here of the frequency domain approach is because of its comparative elegance; the ease with which it accommodates autocorrelation corrections for u_t; and the natural way in which it exploits the fast Fourier transform in case of long time series.

The following section describes the test statistic for the case of white noise u_t and present null and local limit distribution theory in this case. Section 3 does the same, extending the tests for weakly autocorrelated u_t, and finally Section 4 uses Monte Carlo simulations to study finite sample behaviour of sized-corrected versions of the tests.

2.2 SCORE TEST UNDER WHITE NOISE

Robinson (1994c) shows that a score statistic for testing H_0 (1.12) in the model given by (1.9); (1.10) with

$$x_t = 0 \qquad \text{for } t \leq 0 \tag{2}$$

under the presumption that u_t in (1.10) is a sequence of zero mean uncorrelated random variables with unknown variance σ^2 takes the form

$$\bar{R} = \frac{T}{\bar{\sigma}^4}\bar{a}'\bar{A}^{-1}\bar{a},$$

$$\bar{a} = -\int_{-\pi}^{\pi} \psi(\lambda) I_{\bar{u}}(\lambda) d\lambda; \quad \bar{A} = \sum_{l=1}^{T-1}\left(1-\frac{l}{T}\right)\psi_l\psi_l'; \quad \psi(\lambda) = Re\left(\frac{\partial}{\partial\theta}\log\rho(e^{i\lambda};0)\right)$$

$$\bar{\sigma}^2 = \frac{1}{T}\sum_{t=1}^{T}\bar{u}_t^2, \quad \bar{u}_t = \rho(L)(y_t - \bar{\beta}'z_t), \quad \bar{\beta} = \left(\sum_{t=1}^{T} w_t w_t'\right)^{-1}\left(\sum_{t=1}^{T} w_t \rho(L) y_t\right),$$

where $I_{\bar{u}}$ is the periodogram of the $\bar{u}_t$ sequence; $w_t = \rho(L)z_t$ and ψ_l is given by expanding $\psi(\lambda)$ above as $\sum_{l=1}^{\infty}\psi_l \cos l\lambda$; He approximates $\bar{R}$ by

$$\tilde{R} = \frac{T}{\tilde{\sigma}^4}\tilde{a}'\tilde{A}^{-1}\tilde{a} = \tilde{r}'\tilde{r}, \qquad \tilde{r} = \frac{T^{1/2}}{\tilde{\sigma}^2}\tilde{A}^{-1/2}\tilde{a}, \tag{3}$$

$$\textit{where} \quad \tilde{a} = \frac{-2\pi}{T}\sum_j^{*} \psi(\lambda_j) I_{\tilde{u}}(\lambda_j),$$

$$\tilde{A} = \sum_{j=1}^{T-1}\left(1 - \frac{l}{T}\right)\psi_l\psi_l' \quad or \quad 2\Psi \quad or \quad \frac{2}{T}\sum_j^{*}\psi(\lambda_j)\psi(\lambda_j)' \tag{4}$$

$$with \;\; \Psi = \frac{1}{2\pi}\int_{-\pi}^{\pi} \psi(\lambda)\,\psi(\lambda)'d\lambda,$$

in which $\lambda_j = 2\pi j/T$ and the sums on * are over $\lambda_j \in$ M, where M = $\{\lambda$: $-\pi < \lambda < \pi$, $\lambda \notin (\rho_l - \lambda_1, \rho_l + \lambda_1)$, l=1,...,s$\}$, such that ρ_l, l=1,...,s < ∞ are the distinct poles of $\psi(\lambda)$ on $(-\pi,\pi]$. Thus, he uses a discrete approximation to the integral $\bar{a}$, omitting the contribution from the finitely many λ_j in an open λ_1-neighborhood of any of the ρ_l.

Note that by Parseval's equality,

$$\Psi = \frac{1}{2}\sum_{l=1}^{\infty} \psi_l \psi_l',$$

so that asymptotic equivalence of the first two formulas in (4) readily follows. Sometimes a simple closed form is available for Ψ; for example, $\Psi = \pi^2/12$ ($\psi_l = -l^{-1}$), when $\rho(L;\theta) = (1 - L)^{d+\theta}$. More generally, for example when $\rho(L;\theta)$ has complex zeros, a simple formula may be unavailable, and the first expression in (4) may be cumbersome to calculate if the ψ_l are not of simple form, in which case the final option in (4) may be preferred.

Theorems 1 and 2 below describe the null and local limit distributions respectively. Theorem 1 is a large-sample justification for rejecting H_0 at the 100α% level when $\tilde{R} > \chi^2_{p,\alpha}$ where P($\chi^2_p > \chi^2_{p,\alpha}$) = α. It also justifies one-sided tests when p=1: H_0 is rejected in favour of H_1: $\theta > 0$ ($\theta < 0$) at the 100α% level, when $\tilde{r} > z_\alpha$, ($\tilde{r} < -z_\alpha$), where the probability that a standard normal variate exceeds z_α is α. The proofs of the theorems are given in Robinson (1994c).

Theorem 1

Let $\{u_t$, t=0,±1,...$\} \in$ F, where F is the class of sequences $\{v_t$, t=0,±1,...$\}$ of stationary random variables satisfying $E(v_t | B_{t-1}) = 0$ and $E(v_t^2 | B_{t-1}) = \sigma^2$ almost surely, where $0 < \sigma^2 < \infty$ and B_t is the σ-field of events generated by v_s, $s \le t$.

Let $\{z_t$, t=0,±1,...$\} \in$ G, where G is the class of (kx1) vector sequences $\{z_t$, t=0,±1,...$\}$ such that $z_t = 0$ for $t \le 0$ and $\sum_{t=1}^{T} w_t w_t'$ is positive definite for sufficiently large T.

Let $\rho(L;\theta) \in$ H, where H is the class of functions $\rho(z;\theta)$ such that $\rho(0;\theta)=1$ for all θ and $\psi(\lambda)$ as defined above has finitely many poles ρ_l, l=1,...,r, on $(-\pi,\pi]$ such that $\| \psi(\lambda) \|$ is monotonically increasing as $\lambda \to \rho_{l-}$ and as $\lambda \to \rho_{l+}$, for l=1,...,r, and there exist disjoint intervals S_l, l=1...,r such that $\cup_{l=1...r} S_l \in (-\pi,\pi]$, $\rho_l \in S_l$, $\rho_l \notin S_k$ for $l \neq k$, and

$$sup_{\lambda \in S_k - (\rho_k - \delta, \rho_k + \delta)} \left(|\lambda - \rho_k| \left\| \psi(\lambda) - \psi\left(\lambda \pm \frac{1}{2}\delta\right) \right\| \right) = O(\delta^\eta), \qquad as \quad \delta \to 0,$$

for k=1,...,r and some $\eta>1/2$, where $\|\cdot\|$ denotes Euclidean norm.

Then, under H_0 defined by (1.10) and (1.12), the condition

$$0 < \det(\Psi) < \infty \text{[1]} \qquad (5)$$

is sufficient for $\tilde{r}$ in (3) $\rightarrow_d N(0,I_p)$, as $T\rightarrow\infty$, where I_p is the p-rowed identity matrix.

The class F imposes a martingale difference assumption on the white noise u_t, which is substantially weaker than the Gaussianity used in motivating the test statistic and in particular requires a second moment condition that is clearly minimal. Class G imposes a mild lack-of-multicollinearity assumption on the w_t, that is satisfied by, for example, z_t with elements that are polynomials in t. Finally, class H includes technical assumptions on ψ that are costless, but required to justify approximating integrals by sums.

Theorem 2 below justifies optimality of $\tilde{R}$ in the sense of providing an asymptotically most powerful test against local alternatives of form

$$H_1\text{:} \;\; \theta = \theta_T =^{(def)} \delta T^{-\frac{1}{2}}, \qquad (6)$$

where δ is any non-null (px1) vector.

Theorem 2

Let $\{u_t, t=0,\pm1,...\} \in F$, let $\{z_t, t=0,\pm1,...\} \in G$ and let

$$y_{tT} = \beta' z_t + x_{tT}, \qquad (7)$$

where

$$\rho(L;\theta_T)x_{tT} = u_t, \quad t\geq 1, \quad x_{tT} = 0, \quad t\leq 0, \qquad (8)$$

where θ_T satisfies (6) and $\rho(L;\theta) \in J$, where J is the subclass of H such that for all $\rho \in J$, $\xi(z;\theta) = \{\rho(z)/\rho(z;\theta)\}\{(\partial/\partial\theta) \log \rho(z;\theta)\}$ is continuous in θ at $\theta = 0$ for almost all z such that $|z| = 1$, and for a neighborhood S of $\theta = 0$,

$$\int_{-\pi}^{\pi} sup_{\theta \in S} \|\xi(e^{i\lambda};\theta)\|^2 d\lambda < \infty.$$

Let $\tilde{a}$, $\tilde{A}$, and $\tilde{\sigma}^2$ now be defined in terms of x_{tT} rather than x_t. Then,

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[1] Note that the right-side inequality in (5) is not satisfied by the AR alternative $\rho(L;\theta) = (1 - (1+\theta)L)$, but is satisfied by "fractional" alternatives $\rho(L;\theta) = (1-L)^{d+\theta}$ for any real d, for example.

condition (5) is sufficient for $\tilde{r} \to_d N(-\Psi^{1/2}\delta, I_p)$, as $T \to \infty$.

Class J entails a strengthening of the restrictions on ρ, but it is readily checked in case of (1.11). Theorem 2 implies that under local alternatives, $\tilde{R} \to_d \chi_p^2(\delta'\Psi\delta)$, indicating a noncentral χ_p^2 distribution with noncentrality parameter $\delta'\Psi\delta$ which is optimal under Gaussianity of u_t. In non-Gaussian environments the test is no longer fully efficient, but it is still the most efficient test based on quadratic functions of the data.

2.3 SCORE TEST UNDER WEAK PARAMETRIC AUTOCORRELATION

The test developed in the preceding section can be robustified to allow weak parametric autocorrelation in u_t. Let u_t be covariance stationary with spectral density of form $f(\lambda,\tau,\sigma^2) = (\sigma^2/2\pi)g(\lambda,\tau)$, $-\pi < \lambda \leq \pi$, where g is a known function of λ and the unknown (qx1) vector τ, such that τ and σ^2 are not a priori related. Note that σ^2 is generally no longer the variance of u_t, but rather the variance of the innovation sequence in a normalized Wold representation for u_t.

By extending the argument in Section 2, Robinson (1994c) shows that an approximate score statistic for testing (1.12) in (1.9); (1.10) and (2) is

$$\hat{R} = \frac{T}{\hat{\sigma}^4}\hat{a}'\hat{A}^{-1}\hat{a} = \hat{r}'\hat{r}, \qquad \hat{r} = \frac{T^{1/2}}{\hat{\sigma}^2}\hat{A}^{-1/2}\hat{a}, \tag{9}$$

$$\textit{where} \quad \hat{a} = \frac{-2\pi}{T}\sum_j^{*} \psi(\lambda_j)g(\lambda_j;\hat{\tau})^{-1}I_{\hat{u}}(\lambda_j),$$

$$\hat{A} \ \textit{is either} \ 2(\Psi - \Phi\,\Xi^{-1}\Phi') \ \textit{or}$$

$$\frac{2}{T}\sum_j^{*} \psi(\lambda_j)\psi(\lambda_j)' - \frac{2}{T}\sum_j^{*} \psi(\lambda_j)\hat{\epsilon}(\lambda_j)'\left(\frac{1}{T}\sum_j^{*} \hat{\epsilon}(\lambda_j)\hat{\epsilon}(\lambda_j)'\right)^{-1}\frac{1}{T}\hat{\epsilon}(\lambda_j)\psi(\lambda_j)'. \tag{10}$$

$$\Phi = \frac{1}{2\pi}\int_{-\pi}^{\pi}\psi(\lambda)\epsilon(\lambda)'d\lambda, \quad \Xi = \frac{1}{2\pi}\int_{-\pi}^{\pi}\epsilon(\lambda)\epsilon(\lambda)'d\lambda, \quad \epsilon(\lambda) = \frac{\partial}{\partial\tau}\log g(\lambda;\tau),$$

$$\hat{\epsilon}(\lambda_j) = \frac{\partial}{\partial\tau}\log g(\lambda;\hat{\tau}), \quad \hat{\tau} = \arg\min_{\tau\in T^*}\sigma^2(\tau), \quad \sigma^2(\tau) = \frac{2\pi}{T}\sum_{j=1}^{T-1} g(\lambda_j;\tau)^{-1}I_{\hat{u}}(\lambda_j),$$

and $\hat{\sigma}^2 = \sigma^2(\hat{\tau})$, where T^* is a compact subset of q-dimensional Euclidean space.

Some technical assumptions are made on g in the statements of Theorems 3 and 4; their principal practical implications being that though u_t is capable of exhibiting a much stronger degree of autocorrelation than stationary and invertible ARMA processes, its spectrum must be bounded and bounded away from zero.

Though q can be arbitrarily large, we assume it is finite, thus treating only parametric alternatives. As explained in Robinson (1994c), a unit root test against fractional alternatives with nonparametric autocorrelation under the null would have negligible efficiency relative to parametric autocorrelation.

The following theorem strictly relaxes the conditions of Theorem 1, so that, for example, we continue to require finiteness of only second moments of u_t.

Theorem 3

Let $\{u_t, t=0,\pm1,...\}$ be such that $u_t = \sum_{j=0}^{\infty} \alpha_j \epsilon_{t-j}$, $t=0,\pm1,...$, where $\{\varepsilon_t, t = 0,\pm1,...\} \in F$ and $\sum_{j=1}^{\infty} j^{1/2} |\alpha_j| < \infty$.

Let $\{z_t, t=0,\pm1,...\} \in G$, let $\rho \in H$, and let $g \in L$ where L is the class of functions $g(\lambda;\bar{\tau})$ on $(-\pi,\pi]\mathrm{x}T^*$ such that $g(\lambda;\bar{\tau}) = \left|\sum_{j=0}^{\infty} \alpha(j;\bar{\tau}) e^{ij\lambda}\right|^2$, $\alpha(0;\bar{\tau}) \equiv 1$, $\alpha(j;\tau) = \alpha_j$, $j=0,1,..$, where τ is the true value of $\bar{\tau}$; τ is an interior point of T^*, $g(\lambda;\bar{\tau}) \neq g(\lambda;\tau)$ for $\bar{\tau} \in T^* - \{\tau\}$; for all λ, $g(\lambda;\bar{\tau})$ is bounded away from zero on a neighborhood S of τ; $g(\lambda;\bar{\tau})$ is continuous in $(\lambda,\bar{\tau})$ for $\bar{\tau} \in S$ and has first and second derivatives with respect to $\bar{\tau}$ that are also continuous in $(\lambda,\bar{\tau})$ for $\bar{\tau} \in S$; $g(\lambda,\tau)$ and $(\partial/\partial\tau)\, g(\lambda;\tau)$ satisfy a Lipschitz condition in λ of order $\eta > 1/2$. Let also (1.9) and (2) be true.

Then, under H_0 defined by (1.10) and (1.12), the condition

$$0 < \det(\Psi - \Phi\Xi^{-1}\Phi') < \infty \tag{11}$$

is sufficient for $\hat{r} \rightarrow_d N(0,I_p)$, as $T\rightarrow\infty$.

Theorem 2 can likewise be extended.

Theorem 4

Let $\{u_t, t=0,\pm1,...\}$ be as in Theorem 3, let $\{z_t, t=0,\pm1,...\} \in G$, and let (7) and (8) hold where θ_T satisfies (6) and $\rho(L;\theta) \in J$. Let $\hat{a}$, $\hat{A}$ and $\hat{\sigma}^2$ now be defined in terms of x_{tT} rather than x_t.

Then condition (11) is sufficient for $\hat{r} \rightarrow_d N(-(\Psi - \Phi\Xi^{-1}\Phi')^{-1/2}\delta, I_p)$, as $T\rightarrow\infty$.

The most obvious choice of a time series model for u_t satisfying the conditions above is a stationary and invertible ARMA, where relatively simple formulas for g and ε are available. Thus, in the pure AR case,

$$g(\lambda;\tau) = \left|1 - \sum_{j=1}^{q} \tau_j e^{ij\lambda}\right|^{-2}, \qquad \epsilon_l(\lambda) = \left(2\left(\cos l\lambda - \sum_{j=1}^{q} \tau_j \cos(l-j)\lambda\right) g(\lambda;\tau)\right),$$

where $\varepsilon_l(\lambda)$ corresponds to the l^{th} element of $\varepsilon(\lambda)$. However, there are some grounds for preferring the exponential spectrum model of Bloomfield (1973):

$$g(\lambda;\tau) = \exp\left(2\sum_{j=1}^{q} \tau_j \cos j\lambda\right), \qquad -\pi < \lambda \leq \pi. \tag{12}$$

Like the stationary AR case, this has exponentially decaying autocorrelations, and he showed that (12) was remarkably successful at fitting practical data. This expression leads to a neat version of our frequency domain test statistic. In fact,

$$\epsilon_l(\lambda) = (2\cos l\lambda),\ \ \Xi = 2I_q,\ \ \Phi = (\psi_1,\ldots,\psi_q),\ \ and\ \ \Psi - \Phi\,\Xi^{-1}\Phi' = \frac{1}{2}\sum_{l=q+1}^{\infty} \psi_l\psi_l'.$$

Unlike in the AR model, $\varepsilon(\lambda)$, and thus Ψ - $\Phi\Psi^{-1}\Phi'$ are free of the nuisance parameter vector τ, and therefore, expression (10) simplifies.

2.4 FINITE SAMPLE PERFORMANCE AND COMPARISON

In this section we examine the finite-sample behaviour of sized-corrected versions of Robinson's (1994c) tests by means of Monte Carlo simulations, and compare the results obtained here with those in Section 8 in Robinson (1994c), where his tests based on asymptotic critical values were performed and compared with a number of leading unit root tests. Robinson (1994c) stresses large-sample theory and suggests only large-sample approximate critical values. We have considered it convenient in this chapter to attempt a size correction version of his tests in order to study more deeply its finite-sample behaviour.

In Table 2.1 we have calculated the empirical size of $\tilde{r}$ in (3) for different sample sizes, T = 25, 50, 100, 200 and 500, based on 10,000 replications. In the upper part of this table we give the critical values of $\tilde{r}$ when β=0 is correctly assumed, (i.e., $y_t = x_t$), while in the lower part, we give the critical values of the test statistic with unknown β and z_t = (1,t)'. In both cases we take u_t as a Gaussian white noise process with zero mean and variance 1, generated by the routines GASDEV and RAN3 of Press, Flannery, Teukolsky and Vetterling (1986)[2]. We observe that the empirical distributions are similar in both cases, with a negative mean, positively skewed, and with Kurtosis greater than 3, but as we increase the sample size, the values approximate to those given by the Normal distribution, with

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[2] The Fortran codes used in this section require only slight modifications of the program described in Appendix 4.3 in Chapter 4.

the three statistics (mean,skewness and kurtosis), improving with T in both cases. We also note in this table that for most of the quantiles, both the lower and the upper tail critical values are smaller than those given by the Normal distribution. Thus, when testing H_0 (1.12) against H_a: $\theta < 0$, the test statistics based on the asymptotic critical values will reject the null more often than those based on the size-corrected values; however, when testing (1.12) against H_a: $\theta > 0$, the test statistics based on the asymptotic critical values will not reject the null as often as the size-corrected ones.

Tables 2.2-2.9 correspond to Tables 2-9 in Robinson (1994c). In this article, Robinson's (1994c) tests based on asymptotic critical values were performed jointly with seven existing tests that had a random walk null hypothesis. We present the same results here, adding those of the size-corrected tests based on the empirical distributions obtained in Table 2.1.

The null model consists of (1.9); (1) and (2), with $\rho(L) = (1 - L)$, and the u_t in (1) correctly assumed to be white noise. The test denoted S1 in this section is a test of $\tilde{r}$ in (3) with $\beta=0$ and $\rho(L;\theta) = (1 - L)^{1+\theta}$, and the test S2 is the corresponding test with unknown β and $z_t = (1,t)'$, both based on the asymptotic critical values of the Normal distribution; $S1^*$ and $S2^*$ are the size-corrected versions of S1 and S2 tests respectively. The $\hat{\rho}$ and $\hat{\tau}$ tests are due to Fuller (1976) and to Dickey and Fuller (1979), and they assume that $\beta=0$ and are designed to be particularly sensitive to AR alternatives, $\rho(L;\theta) = (1 - (1+\theta)L)$; Likewise, $\hat{\rho}_\tau$ and $\hat{\tau}_\tau$ tests of Fuller (1976) and Dickey and Fuller (1979) take $z_t = (1,t)'$ in (1.9) but assume that the second element of β is zero. The $\tilde{\rho}$ and $\tilde{\tau}$ tests are due to Schmidt and Phillips (1992) and they result from application of a version of the score principle to (1.9); (1.10) and (2) with $\rho(L;\theta) = (1 - (1 +\theta)L)$, $z_t = (1,t)'$; The F test from Robinson (1993) is an exact test under Gaussianity when $\beta=0$ in (1.9) and was shown to be consistent against fractional and AR alternatives. For the seven tests directed against AR alternatives, finite-sample critical values derived from the tables of Fuller (1976) and Schmidt and Phillips (1992) for the $\hat{\rho}$, $\hat{\tau}$, $\hat{\rho}_\tau$, $\hat{\tau}_\tau$, $\tilde{\rho}$, and $\tilde{\tau}$ tests, and from the standard F tables for the F test, were used. As explained in Robinson (1994c), all these tests have asymptotic validity with respect to the same null hypothesis: $y_t = x_t$; $(1-L)x_t = u_t$; u_t white noise.

Because each of the tests is motivated by either fractionally differenced or

AR alternatives, the performances of all tests is evaluated against data generated by both types of model. For both the fractional alternative $\rho(L;\theta) = (1 - L)^{1+\theta}$, and the AR alternative $\rho(L;\theta) = (1 - (1+\theta)L)$, the values $\theta = 0, \pm.05, \pm.1, \pm.2, \pm.3, \pm.5, \pm.7$ and $\pm.9$ are used, and thus, covering the null unit root model as well as stationary, less nonstationary, and more nonstationary fractional alternatives and stationary and explosive AR alternatives. We use sample sizes of T = 25, 50, 100 and 200, and generate Gaussian series, with 5,000 replications of each case. The finite-sample critical values of Fuller (1976) and Schmidt and Phillips (1992) are all apparently based on Gaussian series.

Tables 2.2 and 2.3 contain Monte Carlo rejection frequencies for one-sided tests against fractional alternatives $\theta > 0$, with nominal sizes 5% and 1% respectively. Tables 2.4 and 2.5 correspond, with AR alternatives. Tables 2.6 and 2.7 cover fractional, and Tables 2.8 and 2.9 AR alternatives with $\theta < 0$. Tables 2.2-2.5 omit the F test, because this test covers only alternatives $\theta < 0$.

The first thing that we observe in these tables is that the sizes of $S1^*$ and $S2^*$ are closer to the nominal ones than those of S1 and S2. This is observed for all sample sizes and when directed against both $\theta > 0$ and $\theta < 0$. The sizes of S1 and S2 were too small when directed against $\theta > 0$, but too large when directed against $\theta < 0$. Using the size-corrected versions $S1^*$ and $S2^*$, the sizes increase for positive θ, and decrease for negative θ. This is what we should expect in view of the empirical distributions in Table 2.1, where the critical values were smaller than those given by the Normal distribution. When directed against $\theta > 0$, the sizes range between 4.5% and 5.1% at the 5% level, and between 0.8% and 1.2% at the 1% level; For $\theta < 0$, they range between 3.9% and 5.2% at 5%, and between 0.5% and 1% at the 1% level. Results here are competitive with those obtained in the remaining tests.

Looking again at Table 2.2, the improvement in size observed in $S1^*$ and $S2^*$ relative to S1 and S2, is associated with some superior rejection frequencies in all cases and all sample sizes. These rejection frequencies are also higher for $S1^*$ and $S2^*$ than for the other tests, except in some cases when θ and T are small. We observe that when T = 25 and $\theta = .05$, the highest rejection frequency is obtained for $\tilde{\tau}$, with a rejection probability of .090, compared with .073 for $S1^*$ and $S2^*$, and .028 and .026 for S1 and S2 respectively. Also with T = 25, if $\theta = .1$ $\hat{\tau}$ and $\tilde{\tau}$ beat

$S1^*$ and $S2^*$, however, if $\theta = 0.2$, only $\hat{t}$ is slightly better, and for all other values of θ, $S1^*$ and $S2^*$ give the highest rejection frequencies. With T > 25, $S1^*$ and $S2^*$ outperform the other tests at all values of θ.

Table 2.3 presents a similar picture with higher rejection frequencies for the size-corrected tests over the others except when T and θ are small, with $\hat{t}$ and $\tilde{t}$ performing slightly better in some cases. The efficiency of S1 and S2 in these two tables appears to assert itself as well with $S1^*$ and $S2^*$, observing higher rejection frequencies in $S1^*$ and $S2^*$ over the others for small departures from the null, especially when T is large.

Tables 2.4 and 2.5 correspond to the tests directed against AR alternatives and $\theta > 0$. Again we observe higher rejection frequencies in $S1^*$ and $S2^*$ relative to S1 and S2, though they are smaller than in the remaining tests, which is not at all surprising given that Robinson's (1994c) tests are not efficient with respect to AR alternatives. Comparing $S1^*$ and $S2^*$ with the tests directed against these alternatives, we observe in Table 2.4 that when T = 25, $S1^*$ and $S2^*$ behaves better than $\hat{t}_\tau$ for $\theta = .05$, and they outperform $\tilde{\rho}$ and $\tilde{t}$ for $\theta \geq .2$. We also observe that when T = 100, $S1^*$ and $S2^*$ are as good as the others for $\theta \geq 0.3$, and when T = 200 for $\theta \geq .2$. Similar results are obtained in Table 2.5, with higher rejection frequencies for the size-corrected tests over the non-corrected ones, and competitive results with respect to the other tests when T and θ are large.

Performing the one-sided tests against $\theta < 0$, (in Tables 2.6-2.9), the sizes of S1 and S2 were too large. Using the size-corrected versions $S1^*$ and $S2^*$, the sizes decrease, especially when T is large. When T = 25, sizes are now too small, with 4.1% for $S1^*$ and 3.9% for $S2^*$ at the 5% level, and 0.6% and 0.5% at the 1% level; however, as T increases, they approximate to the nominal ones, and thus, with T > 50, they range between 4.8% and 5.2% at the 5% level, and between 0.8% and 1.0% at the 1% level.

The smaller sizes observed in these tables in $S1^*$ and $S2^*$ relative to S1 and S2 are also associated with smaller rejection frequencies and thus, $S1^*$ and $S2^*$ in Tables 2.6 and 2.7 are beaten not only by S1 and S2 but also by the remaining tests (especially $\hat{\rho}$, $\hat{t}$ and $\hat{\rho}_\tau$) when T is small, even for the fractional data. However as T increases, S1, S2, $S1^*$ and $S2^*$ give higher rejection frequencies than the remaining tests, showing again the efficiency property of Robinson's (1994c) tests, especially with T = 200.

Finally in Tables 2.8 and 2.9, we again observe smaller rejection frequencies in $S1^*$ and $S2^*$ relative to S1 and S2, which must be due to the smaller size of the size-corrected tests. $S1^*$ and $S2^*$ are beaten in practically all cases by the other tests, and this might be due to the lack of efficiency of Robinson's (1994c) tests when directed against AR alternatives, and the lower size of $S1^*$ and $S2^*$ relative to the other tests.

In Table 2.10 we have calculated the empirical distributions for $\tilde{R}$ in (3), again with T = 25, 50, 100, 200 and 500, and for the two cases of $\beta = 0$ a priori, and of unknown β with $z_t = (1,t)'$. The critical values are similar in both cases, and as we should expect, increasing the sample size, the values approximate to those of the χ_1^2 distribution. We note in this table that at 90% and 95% percentiles, the critical values are greater than those given by the χ_1^2 distribution. Therefore, when testing the null (1.12) against the alternative: H_a: $\theta \neq 0$ at the 10% and 5% significance level using the asymptotic critical values, the null hypothesis will be rejected more often than when using the size-corrected critical values.

Table 2.11 concerns two-sided tests based on S1 and $S1^*$ (its size-corrected version), and the test $T1^*$, which denotes $\tilde{R}$ in (3) with $\beta = 0$ using the empirical distribution in Table 2.10, for the same (fractional Gaussian) process used in Tables 2.2, 2.3, 2.6 and 2.7, but for $\theta = 0, \pm.05, \pm.1, \pm.2$ and $\pm.3$, with T = 100 and 200 and nominal sizes of 10%, 5%, 1% and 0.1%. Results for S1 are taken from Table 10 in Robinson (1994c). Looking at S1, the sizes are closer to the nominal ones than in previous tables, though they are too large at 10% and 5%. Using the size-corrected versions $S1^*$ and $T1^*$, the sizes are smaller and they approximate even more to the nominal ones. They range between 9.8% and 10% at the 10% level; between 4.7% and 5.3% at the 5% level; between 0.8% and 1.2% at 1%, and are exactly 0.1% at the 0.1% level. Comparing the rejection frequencies in the S1 test with the size-corrected versions $S1^*$ and $T1^*$, we observe that for nominal sizes of 10% and 5% level, they are slightly higher in $S1^*$ than in S1 for $\theta > 0$, however, for $\theta < 0$, S1 gives higher rejection probabilities than the size-corrected tests. These rejection frequencies decrease in $S1^*$ and $T1^*$ with respect to S1 for positive θ but increase for negative θ, correcting slightly the bias observed in S1 where higher rejection frequencies were observed for negative θ than for positive ones. Using smaller nominal sizes, results are not very conclusive: S1 is the best when α = 0.1%, with $\theta > 0$ and T = 200, and with $\theta < 0$ and both sample sizes. $S1^*$ gives the highest rejection frequencies when α = 1% with $\theta > 0$ and T = 200 and with $\theta < 0$ and T = 100, and also when α = 0.1%, $\theta > 0$ and T = 100; Finally $T1^*$ beats S1 and $S1^*$ when α = 1% with $\theta > 0$ and T = 100, and with $\theta < 0$ and T = 200.

As in Robinson (1994c), we also extended the analysis to cover corrections for AR autocorrelation in u_t and departures from Gaussianity in u_t. However, in order to save space, we have decided not to include the results here. (Note that in doing so, we should also include the empirical distributions of the tests, which are different for each of the parameterizations in the AR representation and are also different for the different distributional assumptions in u_t). Robinson's (1994c) Table 11 reports results of the two-sided S1 as in Table 10 but replacing Gaussianity by a t_3-distribution for the white noise u_t. His results were competitive with the Gaussian ones, with the sizes closer to the nominal ones. Using the size-corrected versions $S1^*$ and $T1^*$, our results were similar to those in Robinson (1994c), though the sizes were slightly smaller and also the rejection frequencies were smaller than in S1.

Attempting AR-corrections to u_t, Robinson's (1994c) Tables 12 and 13 report two-sided tests with u_t generated as white noise (AR(0)) and AR(2) of form $u_t = u_{t-1} - .5\, u_{t-2} + \varepsilon_t$, and the white noise ε_t being generated as N(0,1) and t_3. His results indicated that the sizes were too large in all cases. Using the size-corrected versions of the tests, the sizes were much smaller and thus, closer to the nominal ones. On the other hand, because of these smaller sizes, we also obtained smaller rejection frequencies in practically all cases.

Finally, we should also mention here that in the empirical work carried out in Chapters 3, 4 and 6, we rely on the asymptotic critical values given by the Normal (or χ^2) distribution, motivated mainly by the different models considered in (1.9); the different models used for describing the disturbances u_t; and the different functions $\rho(L;\theta)$ used in (1.10), especially in Chapter 4. Note that for each of these cases, the empirical distributions are different. Furthermore, Robinson (1994c) stresses the large-sample theory in justifying the tests, and therefore, we have considered more convenient for the remaining work the use of the large sample approximate critical values rather than the size-corrected ones.

TABLE 2.1

Critical values on finite samples of $\tilde{r}$ in (3) with $\beta = 0$

Perc.\ T	25	50	100	200	500
0.1%	-2.934	-2.985	-2.852	-3.034	-2.963
0.5%	-2.677	-2.690	-2.586	-2.560	-2.563
1.0%	-2.566	-2.539	-2.406	-2.385	-2.387
2.0%	-2.432	-2.314	-2.234	-2.176	-2.155
2.5%	-2.392	-2.254	-2.173	-2.106	-2.072
5.0%	-2.190	-2.037	-1.933	-1.873	-1.804
10.0%	-1.941	-1.764	-1.644	-1.548	-1.403
20.0%	-1.618	-1.408	-1.261	-1.167	-1.086
30.0%	-1.352	-1.129	-0.985	-0.868	-0.791
40.0%	-1.107	-0.890	-0.741	-0.615	-0.543
50.0%	-0.865	-0.650	-0.495	-0.374	-0.286
60.0%	-0.621	-0.407	-0.238	-0.132	-0.028
70.0%	-0.329	-0.122	0.038	0.142	0.252
80.0%	0.008	0.211	0.370	0.477	0.596
90.0%	0.537	0.741	0.876	0.993	1.080
95.0%	1.024	1.171	1.331	1.414	1.502
97.5%	1.440	1.610	1.762	1.784	1.869
98.0%	1.565	1.709	1.879	1.937	1.966
99.0%	1.895	2.104	2.172	2.301	2.275
99.5%	2.268	2.505	2.506	2.661	2.579
99.9%	2.895	3.402	3.316	3.436	3.193
Mean:	-0.768	-0.567	-0.424	-0.320	-0.235
Skewness:	0.569	0.518	0.434	0.367	0.266
Kurtosis:	3.145	3.394	3.211	3.298	3.107

Critical values on finite samples of $\tilde{r}$ in (3) with unknown β and $z_t = (1,t)'$

Perc.\ T	25	50	100	200	500
0.1%	-2.934	-3.036	-2.842	-3.081	-2.967
0.5%	-2.697	-2.693	-2.587	-2.557	-2.572
1.0%	-2.581	-2.553	-2.425	-2.386	-2.373
2.0%	-2.439	-2.313	-2.237	-2.181	-2.153
2.5%	-2.380	-2.252	-2.167	-2.113	-2.072
5.0%	-2.192	-2.046	-1.929	-1.890	-1.811
10.0%	-1.940	-1.756	-1.636	-1.554	-1.483
20.0%	-1.620	-1.409	-1.258	-1.157	-1.086
30.0%	-1.352	-1.131	-0.979	-0.868	-0.793
40.0%	-1.113	-0.891	-0.738	-0.612	-0.540
50.0%	-0.869	-0.658	-0.494	-0.370	-0.277
60.0%	-0.618	-0.402	-0.241	-0.137	-0.023
70.0%	-0.339	-0.118	0.037	0.142	0.256
80.0%	0.015	0.229	0.361	0.472	0.601
90.0%	0.547	0.742	0.876	0.998	1.077
95.0%	1.029	1.172	1.350	1.418	1.495
97.5%	1.442	1.623	1.751	1.789	1.862
98.0%	1.558	1.735	1.853	1.905	1.985
99.0%	1.950	2.097	2.171	2.308	2.272
99.5%	2.242	2.509	2.509	2.674	2.585
99.9%	2.823	3.435	3.381	3.469	3.126
Mean:	-0.768	-0.566	-0.421	-0.320	-0.235
Skewness:	0.564	0.521	0.430	0.367	0.266
Kurtosis:	3.114	3.454	3.199	3.326	3.313

TABLE 2.2

Rejection frequencies for Upper-Tailed 5% Test and Fractional x_t.

T = 25

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{t}$	$\hat{\rho}_\tau$	$\hat{t}_\tau$	$\tilde{\rho}$	$\tilde{t}$
0	.016	.016	**.049**	**.045**	.053	.049	.046	.044	.048	.063
.05	.028	.026	**.073**	**.073**	.076	.082	.067	.064	.072	.090
.1	.047	.047	**.114**	**.113**	.107	.130	.093	.084	.109	.126
.2	.120	.117	**.233**	**.227**	.181	.241	.149	.132	.193	.220
.3	.230	.225	**.385**	**.377**	.265	.362	.227	.192	.306	.335
.5	.516	.511	**.662**	**.657**	.424	.574	.379	.306	.544	.578
.7	.772	.775	**.871**	**.870**	.563	.703	.524	.393	.735	.763
.9	.909	.916	**.949**	**.953**	.662	.785	.633	.465	.859	.873

T = 50

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{t}$	$\hat{\rho}_\tau$	$\hat{t}_\tau$	$\tilde{\rho}$	$\tilde{t}$
0	.023	.023	**.050**	**.051**	.054	.050	.053	.052	.056	.061
.05	.063	.063	**.117**	**.116**	.080	.092	.088	.080	.096	.102
.1	.125	.124	**.191**	**.188**	.113	.148	.126	.116	.152	.164
.2	.323	.324	**.439**	**.437**	.197	.288	.237	.195	.321	.332
.3	.583	.579	**.672**	**.671**	.290	.435	.360	.285	.495	.508
.5	.906	.902	**.943**	**.946**	.455	.651	.565	.423	.769	.778
.7	.991	.992	**.995**	**.995**	.585	.771	.694	.508	.914	.919
.9	.999	1.000	**.999**	**.999**	.681	.836	.746	.525	.973	.975

T = 100

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{t}$	$\hat{\rho}_\tau$	$\hat{t}_\tau$	$\tilde{\rho}$	$\tilde{t}$
0	.030	.030	**.049**	**.046**	.049	.050	.046	.046	.052	.057
.05	.100	.101	**.144**	**.141**	.086	.111	.092	.087	.109	.116
.1	.233	.232	**.322**	**.312**	.140	.187	.156	.138	.199	.209
.2	.631	.628	**.703**	**.700**	.240	.358	.309	.244	.442	.454
.3	.897	.896	**.931**	**.930**	.338	.516	.483	.361	.670	.679
.5	.998	.997	**.999**	**.999**	.498	.715	.718	.513	.915	.920
.7	1.000	1.000	**1.000**	**1.000**	.626	.823	.779	.559	.986	.988
.9	1.000	1.000	**1.000**	**1.000**	.705	.872	.796	.553	.998	.998

T = 200

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{t}$	$\hat{\rho}_\tau$	$\hat{t}_\tau$	$\tilde{\rho}$	$\tilde{t}$
0	.030	.030	**.046**	**.045**	.055	.052	.051	.050	.054	.054
.05	.168	.169	**.214**	**.209**	.096	.114	.112	.100	.134	.135
.1	.447	.449	**.519**	**.514**	.152	.214	.193	.160	.254	.256
.2	.910	.911	**.931**	**.930**	.272	.416	.400	.307	.572	.574
.3	.995	.995	**.997**	**.997**	.375	.588	.611	.430	.818	.818
.5	1.000	1.000	**1.000**	**1.000**	.529	.780	.814	.574	.982	.983
.7	1.000	1.000	**1.000**	**1.000**	.639	.866	.838	.580	.999	.999
.9	1.000	1.000	**1.000**	**1.000**	.719	.906	.826	.565	1.000	1.000

*: $S1^*$ and $S2^*$ are sized-corrected S1 and S2 tests respectively.

TABLE 2.3

Rejection frequencies for Upper-Tailed 1% Test and Fractional x_t.

T = 25

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{t}$	$\hat{\rho}_\tau$	$\hat{t}_\tau$	$\tilde{\rho}$	$\tilde{t}$
0	.004	.003	**.012**	**.009**	.011	.009	.008	.008	.010	.014
.05	.009	.010	**.019**	**.018**	.017	.019	.013	.013	.017	.022
.1	.015	.016	**.035**	**.033**	.024	.039	.020	.019	.029	.039
.2	.051	.050	**.099**	**.091**	.048	.108	.042	.041	.069	.089
.3	.124	.120	**.193**	**.182**	.077	.213	.076	.074	.143	.166
.5	.374	.362	**.460**	**.446**	.153	.441	.171	.164	.350	.385
.7	.652	.644	**.736**	**.729**	.239	.610	.270	.254	.573	.604
.9	.840	.847	**.880**	**.880**	.317	.717	.366	.336	.743	.771

T = 50

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{t}$	$\hat{\rho}_\tau$	$\hat{t}_\tau$	$\tilde{\rho}$	$\tilde{t}$
0	.007	.006	**.008**	**.008**	.013	.010	.009	.008	.011	.013
.05	.020	.020	**.028**	**.027**	.020	.028	.019	.019	.028	.032
.1	.057	.057	**.064**	**.065**	.032	.059	.033	.034	.061	.066
.2	.195	.193	**.229**	**.232**	.062	.159	.082	.076	.157	.168
.3	.428	.426	**.461**	**.463**	.099	.296	.149	.137	.318	.330
.5	.832	.831	**.864**	**.860**	.180	.554	.296	.270	.626	.639
.7	.975	.975	**.978**	**.979**	.268	.705	.407	.365	.836	.841
.9	.997	.998	**.998**	**.997**	.358	.790	.471	.419	.930	.934

T = 100

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{t}$	$\hat{\rho}_\tau$	$\hat{t}_\tau$	$\tilde{\rho}$	$\tilde{t}$
0	.009	.009	**.008**	**.008**	.011	.010	.008	.008	.012	.013
.05	.039	.039	**.050**	**.048**	.021	.032	.022	.020	.033	.034
.1	.123	.125	**.143**	**.144**	.035	.080	.045	.041	.074	.076
.2	.468	.467	**.504**	**.498**	.078	.234	.123	.110	.245	.251
.3	.805	.805	**.837**	**.835**	.121	.397	.210	.186	.475	.482
.5	.993	.993	**.996**	**.997**	.209	.641	.398	.346	.822	.826
.7	1.000	1.000	**1.000**	**1.000**	.292	.773	.498	.429	.954	.955
.9	1.000	1.000	**1.000**	**1.000**	.387	.840	.547	.467	.989	.989

T = 200

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{t}$	$\hat{\rho}_\tau$	$\hat{t}_\tau$	$\tilde{\rho}$	$\tilde{t}$
0	.009	.009	**.009**	**.008**	.011	.013	.010	.010	.011	.011
.05	.070	.070	**.067**	**.067**	.024	.044	.029	.027	.044	.047
.1	.270	.270	**.281**	**.279**	.044	.098	.064	.059	.114	.117
.2	.822	.821	**.822**	**.822**	.093	.292	.166	.147	.372	.378
.3	.988	.988	**.987**	**.988**	.144	.482	.287	.249	.656	.663
.5	1.000	1.000	**1.000**	**1.000**	.234	.725	.504	.409	.945	.946
.7	1.000	1.000	**1.000**	**1.000**	.311	.832	.582	.475	.994	.994
.9	1.000	1.000	**1.000**	**1.000**	.404	.878	.596	.498	1.000	1.000

: S1 and S2* are sized-corrected S1 and S2 tests respectively.

TABLE 2.4

Rejection frequencies for Upper-Tailed 5% Test and AR x_t.

T = 25

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{t}$	$\hat{\rho}_\tau$	$\hat{t}_\tau$	$\tilde{\rho}$	$\tilde{t}$
0	.016	.016	**.049**	**.045**	.053	.049	.046	.044	.048	.063
.05	.014	.015	**.045**	**.046**	.105	.123	.047	.044	.046	.060
.1	.014	.014	**.045**	**.047**	.288	.325	.049	.049	.046	.059
.2	.135	.173	**.268**	**.297**	.709	.691	.391	.378	.240	.268
.3	.879	.890	**.912**	**.918**	.963	.956	.939	.941	.901	.907
.5	.982	.985	**.984**	**.985**	.993	.991	.991	.992	.985	.987
.7	.999	.999	**.999**	**1.000**	1.000	.999	1.000	1.000	.999	.999
.9	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000

T = 50

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{t}$	$\hat{\rho}_\tau$	$\hat{t}_\tau$	$\tilde{\rho}$	$\tilde{t}$
0	.023	.023	**.050**	**.051**	.054	.050	.053	.052	.056	.061
.05	.022	.022	**.050**	**.048**	.202	.251	.049	.048	.047	.052
.1	.232	.247	**.334**	**.343**	.730	.711	.417	.409	.309	.319
.2	.924	.926	**.942**	**.944**	.970	.963	.954	.954	.933	.934
.3	.999	.999	**.999**	**.999**	1.000	.999	.999	.999	.999	.999
.5	1.000	1.000	**1.000**	**1.00**	1.000	1.000	1.000	1.000	1.000	1.000
.7	1.000	1.000	**1.000**	**1.00**	1.000	1.000	1.000	1.000	1.000	1.000
.9	1.000	1.000	**1.000**	**1.00**	1.000	1.000	1.000	1.000	1.000	1.000

T = 100

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{t}$	$\hat{\rho}_\tau$	$\hat{t}_\tau$	$\tilde{\rho}$	$\tilde{t}$
0	.030	.030	**.049**	**.046**	.049	.050	.046	.046	.052	.057
.05	.087	.091	**.122**	**.121**	.602	.594	.211	.208	.144	.153
.1	.937	.937	**.944**	**.944**	.972	.968	.961	.963	.945	.946
.2	.999	.999	**.999**	**.999**	1.000	1.000	1.000	1.000	1.000	1.000
.3	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000
.5	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000
.7	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000
.9	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000

T = 200

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{t}$	$\hat{\rho}_\tau$	$\hat{t}_\tau$	$\tilde{\rho}$	$\tilde{t}$
0	.030	.030	**.046**	**.045**	.055	.052	.051	.050	.054	.054
.05	.833	.833	**.850**	**.852**	.938	.926	.902	.902	.864	.865
.1	.999	.999	**.999**	**.998**	1.000	1.000	.999	.999	.999	.999
.2	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000
.3	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000
.5	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000
.7	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000
.9	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000

: S1 and S2* are sized-corrected S1 and S2 tests respectively.

TABLE 2.5

Rejection frequencies for Upper-Tailed 1% Test and AR x_t.

T = 25

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{t}$	$\hat{\rho}_\tau$	$\hat{t}_\tau$	$\tilde{\rho}$	$\tilde{t}$
0	.004	.003	**.012**	**.009**	.011	.009	.008	.008	.010	.014
.05	.003	.003	**.010**	**.009**	.020	.036	.008	.009	.010	.013
.1	.004	.003	**.007**	**.008**	.058	.178	.009	.010	.009	.012
.2	.069	.105	**.154**	**.172**	.481	.595	.238	.232	.108	.126
.3	.839	.860	**.882**	**.892**	.956	.940	.918	.918	.843	.855
.5	.979	.980	**.979**	**.980**	.992	.989	.989	.989	.978	.980
.7	.999	.999	**.999**	**.999**	1.000	.999	1.000	1.000	.999	.999
.9	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000

T = 50

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{t}$	$\hat{\rho}_\tau$	$\hat{t}_\tau$	$\tilde{\rho}$	$\tilde{t}$
0	.007	.006	**.008**	**.008**	.013	.010	.009	.008	.011	.013
.05	.005	.006	**.009**	**.009**	.044	.129	.011	.010	.009	.011
.1	.163	.177	**.216**	**.230**	.541	.627	.280	.274	.184	.196
.2	.908	.913	**.927**	**.930**	.965	.956	.941	.941	.910	.912
.3	.999	.999	**.999**	**.999**	1.000	.999	.999	.999	.999	.999
.5	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000
.7	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000
.9	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000

T = 100

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{t}$	$\hat{\rho}_\tau$	$\hat{t}_\tau$	$\tilde{\rho}$	$\tilde{t}$
0	.009	.009	**.008**	**.008**	.011	.010	.008	.008	.012	.013
.05	.039	.042	**.046**	**.049**	.298	.495	.098	.093	.055	.056
.1	.928	.929	**.934**	**.934**	.969	.962	.952	.953	.930	.931
.2	.999	.999	**.999**	**.999**	1.000	1.000	1.000	1.000	.999	.999
.3	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.00	1.000
.5	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.00	1.000
.7	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.00	1.000
.9	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.00	1.000

T = 200

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{t}$	$\hat{\rho}_\tau$	$\hat{t}_\tau$	$\tilde{\rho}$	$\tilde{t}$
0	.009	.009	**.009**	**.008**	.011	.013	.010	.010	.011	.011
.05	.807	.808	**.812**	**.812**	.922	.910	.869	.869	.821	.822
.1	.999	.999	**.999**	**.998**	1.000	.999	.999	.999	.999	.999
.2	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000
.3	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000
.5	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000
.7	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000
.9	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000

: S1 and S2* are sized-corrected S1 and S2 tests respectively.

TABLE 2.6

Rejection frequencies for Lower-Tailed 5% Test and Fractional x_t.

T = 25

θ	S1	S2	S1*	S2*	$\hat{\rho}$	$\hat{t}$	$\hat{\rho}_\tau$	$\hat{t}_\tau$	$\tilde{\rho}$	$\tilde{t}$	F
-.9	.983	.981	**.868**	**.838**	1.000	.998	.933	.879	.905	.886	.310
-.7	.938	.934	**.705**	**.664**	.964	.955	.773	.675	.740	.709	.260
-.5	.819	.816	**.485**	**.466**	.710	.682	.507	.416	.484	.446	.179
-.3	.566	.565	**.237**	**.229**	.327	.306	.244	.203	.232	.203	.116
-.2	.418	.417	**.142**	**.139**	.198	.181	.158	.132	.147	.128	.088
-.1	.279	.275	**.073**	**.071**	.109	.095	.095	.086	.084	.072	.068
-.05	.224	.217	**.057**	**.055**	.079	.071	.069	.067	.066	.057	.058
0	.175	.173	**.041**	**.039**	.056	.049	.051	.052	.049	.041	.047

T = 50

θ	S1	S2	S1*	S2*	$\hat{\rho}$	$\hat{t}$	$\hat{\rho}_\tau$	$\hat{t}_\tau$	$\tilde{\rho}$	$\tilde{t}$	F
-.9	1.000	1.000	**.999**	**.999**	1.000	1.000	1.000	1.000	1.000	1.000	.443
-.7	.998	.998	**.989**	**.986**	1.000	1.000	.997	.991	.989	.986	.376
-.5	.976	.975	**.902**	**.890**	.932	.925	.888	.840	.866	.853	.254
-.3	.763	.761	**.551**	**.536**	.496	.484	.479	.420	.461	.439	.147
-.2	.542	.539	**.312**	**.301**	.278	.268	.260	.221	.252	.237	.108
-.1	.297	.295	**.132**	**.131**	.130	.125	.120	.114	.114	.108	.071
-.05	.196	.195	**.082**	**.075**	.085	.081	.081	.083	.077	.070	.059
0	.117	.117	**.044**	**.042**	.053	.051	.054	.057	.048	.043	.051

T = 100

θ	S1	S2	S1*	S2*	$\hat{\rho}$	$\hat{t}$	$\hat{\rho}_\tau$	$\hat{t}_\tau$	$\tilde{\rho}$	$\tilde{t}$	F
-.9	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000	.594
-.7	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000	.502
-.5	1.000	1.000	**.999**	**.999**	.996	.996	.998	.994	.995	.994	.339
-.3	.960	.960	**.905**	**.905**	.679	.667	.768	.712	.754	.741	.174
-.2	.772	.768	**.622**	**.622**	.380	.370	.439	.388	.435	.420	.115
-.1	.387	.387	**.245**	**.250**	.158	.154	.178	.161	.175	.167	.072
-.05	.209	.213	**.122**	**.124**	.096	.091	.100	.095	.106	.101	.058
0	.097	.097	**.052**	**.052**	.049	.047	.056	.057	.057	.053	.047

T = 200

θ	S1	S2	S1*	S2*	$\hat{\rho}$	$\hat{t}$	$\hat{\rho}_\tau$	$\hat{t}_\tau$	$\tilde{\rho}$	$\tilde{t}$	F
-.9	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000	.768
-.7	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000	.672
-.5	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000	.459
-.3	1.000	.999	**.998**	**.998**	.820	.815	.951	.927	.933	.931	.213
-.2	.959	.958	**.920**	**.918**	.496	.487	.666	.608	.631	.626	.135
-.1	.561	.562	**.432**	**.423**	.199	.198	.256	.224	.249	.245	.088
-.05	.265	.267	**.179**	**.171**	.108	.105	.124	.112	.122	.119	.067
0	.085	.085	**.051**	**.048**	.048	.048	.053	.052	.050	.049	.045

: S1 and S2* are sized-corrected S1 and S2 tests respectively.

TABLE 2.7

Rejection frequencies for Lower-Tailed 1% Test and Fractional x_t.

T = 25

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{t}$	$\hat{\rho}_\tau$	$\hat{t}_\tau$	$\tilde{\rho}$	$\tilde{t}$	F
-.9	.793	.788	**.604**	**.535**	.981	.976	.724	.600	.677	.639	.055
-.7	.607	.595	**.396**	**.339**	.799	.775	.464	.340	.425	.385	.055
-.5	.364	.356	**.195**	**.178**	.391	.368	.216	.153	.197	.166	.036
-.3	.156	.152	**.072**	**.059**	.120	.108	.078	.053	.069	.057	.022
-.2	.092	.089	**.035**	**.031**	.057	.050	.041	.032	.038	.031	.018
-.1	.049	.047	**.014**	**.012**	.025	.022	.022	.018	.017	.013	.013
-.05	.035	.034	**.009**	**.008**	.016	.014	.016	.015	.011	.008	.010
0	.024	.024	**.006**	**.005**	.011	.010	.011	.011	.007	.006	.010

T = 50

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{t}$	$\hat{\rho}_\tau$	$\hat{t}_\tau$	$\tilde{\rho}$	$\tilde{t}$	F
-.9	.994	.994	**.983**	**.975**	1.000	1.000	.999	.998	.992	.991	.096
-.7	.957	.955	**.898**	**.872**	.995	.995	.961	.929	.933	.924	.080
-.5	.761	.756	**.614**	**.585**	.745	.733	.657	.576	.619	.598	.050
-.3	.348	.348	**.213**	**.197**	.239	.230	.202	.167	.193	.178	.029
-.2	.164	.166	**.086**	**.079**	.098	.095	.090	.076	.081	.074	.022
-.1	.059	.057	**.028**	**.026**	.037	.035	.035	.029	.028	.025	.018
-.05	.033	.033	**.013**	**.010**	.019	.020	.020	.018	.017	.015	.013
0	.018	.017	**.008**	**.006**	.013	.013	.013	.012	.010	.009	.013

T = 100

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{t}$	$\hat{\rho}_\tau$	$\hat{t}_\tau$	$\tilde{\rho}$	$\tilde{t}$	F
-.9	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000	.137
-.7	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000	.110
-.5	.990	.990	**.988**	**.984**	.955	.953	.978	.963	.961	.959	.075
-.3	.749	.746	**.700**	**.685**	.426	.415	.499	.432	.482	.465	.037
-.2	.384	.381	**.322**	**.309**	.172	.164	.200	.166	.194	.184	.020
-.1	.101	.101	**.083**	**.080**	.049	.046	.059	.051	.057	.054	.013
-.05	.041	.041	**.030**	**.030**	.024	.024	.029	.027	.028	.027	.010
0	.015	.015	**.010**	**.009**	.012	.010	.012	.013	.012	.012	.008

T = 200

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{t}$	$\hat{\rho}_\tau$	$\hat{t}_\tau$	$\tilde{\rho}$	$\tilde{t}$	F
-.9	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000	.201
-.7	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000	.157
-.5	1.000	1.000	**1.000**	**1.000**	.999	.998	1.000	1.000	.999	.999	.099
-.3	.984	.985	**.985**	**.985**	.630	.623	.825	.768	.773	.771	.046
-.2	.776	.777	**.745**	**.746**	.272	.266	.405	.335	.360	.355	.029
-.1	.220	.221	**.186**	**.186**	.063	.065	.092	.073	.082	.079	.019
-.05	.067	.067	**.057**	**.055**	.027	.027	.033	.027	.031	.030	.011
0	.013	.013	**.009**	**.008**	.010	.010	.010	.010	.010	.009	.009

: S1 and S2* are sized-corrected S1 and S2 tests respectively.

TABLE 2.8

Rejection frequencies for Lower-Tailed 5% Test and AR x_t.

T = 25

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{\tau}$	$\hat{\rho}_\tau$	$\hat{\tau}_\tau$	$\tilde{\rho}$	$\tilde{\tau}$	F
-.9	.978	.974	**.836**	**.803**	1.000	.999	.926	.863	.901	.881	.309
-.7	.703	.694	**.328**	**.291**	.928	.913	.403	.303	.412	.370	.231
-.5	.422	.418	**.130**	**.130**	.599	.567	.170	.128	.173	.145	.181
-.3	.299	.293	**.085**	**.076**	.350	.325	.109	.085	.099	.084	.155
-.2	.210	.211	**.052**	**.052**	.161	.141	.066	.061	.064	.054	.108
-.1	.183	.183	**.042**	**.038**	.097	.087	.055	.054	.055	.045	.083
-.05	.175	.175	**.038**	**.039**	.071	.062	.052	.053	.050	.041	.066
0	.175	.173	**.041**	**.039**	.056	.049	.051	.052	.049	.041	.047

T = 50

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{\tau}$	$\hat{\rho}_\tau$	$\hat{\tau}_\tau$	$\tilde{\rho}$	$\tilde{\tau}$	F
-.9	1.000	1.000	**.999**	**.998**	1.000	1.000	1.000	1.000	1.000	1.000	.449
-.7	.922	.920	**.768**	**.738**	1.000	1.000	.933	.883	.931	.924	.350
-.5	.613	.607	**.362**	**.341**	.975	.969	.505	.410	.539	.516	.265
-.3	.393	.390	**.188**	**.181**	.782	.772	.247	.194	.270	.247	.212
-.2	.217	.213	**.088**	**.085**	.332	.326	.102	.092	.101	.092	.152
-.1	.150	.150	**.058**	**.058**	.147	.145	.067	.065	.063	.057	.114
-.05	.124	.125	**.047**	**.044**	.083	.081	.057	.058	.050	.046	.077
0	.117	.117	**.044**	**.042**	.053	.051	.054	.057	.048	.043	.051

T = 100

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{\tau}$	$\hat{\rho}_\tau$	$\hat{\tau}_\tau$	$\tilde{\rho}$	$\tilde{\tau}$	F
-.9	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000	.595
-.7	.997	.997	**.991**	**.989**	1.000	1.000	1.000	1.000	1.000	1.000	.462
-.5	.891	.888	**.750**	**.745**	1.000	1.000	.982	.959	.983	.980	.366
-.3	.630	.630	**.451**	**.442**	.998	.998	.743	.642	.783	.770	.309
-.2	.285	.284	**.153**	**.155**	.772	.770	.243	.192	.277	.264	.220
-.1	.162	.159	**.085**	**.086**	.317	.316	.106	.092	.115	.106	.164
-.05	.109	.107	**.054**	**.058**	.124	.124	.067	.065	.068	.063	.099
0	.097	.097	**.052**	**.052**	.049	.047	.056	.057	.057	.053	.047

T = 200

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{\tau}$	$\hat{\rho}_\tau$	$\hat{\tau}_\tau$	$\tilde{\rho}$	$\tilde{\tau}$	F
-.9	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000	.768
-.7	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000	.629
-.5	.995	.995	**.986**	**.984**	1.000	1.000	1.000	1.000	1.000	1.000	.511
-.3	.899	.895	**.823**	**.810**	1.000	1.000	1.000	.999	.997	.997	.421
-.2	.470	.470	**.359**	**.348**	.999	.998	.728	.625	.765	.876	.313
-.1	.221	.221	**.132**	**.127**	.759	.760	.245	.191	.271	.266	.218
-.05	.118	.119	**.072**	**.067**	.239	.240	.091	.073	.090	.089	.133
0	.085	.085	**.051**	**.048**	.048	.048	.053	.052	.050	.049	.045

*: S1* and S2* are sized-corrected S1 and S2 tests respectively.

TABLE 2.9

Rejection frequencies for Lower-Tailed 1% Test and AR x_t.

T = 25

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{\tau}$	$\hat{\rho}_\tau$	$\hat{\tau}_\tau$	$\tilde{\rho}$	$\tilde{\tau}$	F
-.9	.756	.746	**.552**	**.478**	.989	.984	.693	.563	.659	.611	.056
-.7	.224	.215	**.102**	**.084**	.601	.568	.143	.087	.137	.116	.045
-.5	.087	.085	**.033**	**.028**	.207	.186	.044	.027	.039	.031	.040
-.3	.051	.048	**.017**	**.017**	.090	.078	.022	.015	.020	.016	.032
-.2	.030	.031	**.008**	**.008**	.029	.027	.013	.011	.009	.007	.022
-.1	.026	.026	**.005**	**.005**	.018	.015	.012	.011	.007	.006	.016
-.05	.024	.024	**.006**	**.005**	.014	.011	.010	.011	.007	.005	.013
0	.024	.024	**.006**	**.005**	.011	.010	.011	.011	.007	.006	.010

T = 50

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{\tau}$	$\hat{\rho}_\tau$	$\hat{\tau}_\tau$	$\tilde{\rho}$	$\tilde{\tau}$	F
-.9	.991	.991	**.972**	**.958**	1.000	1.000	.999	.998	.993	.992	.099
-.7	.566	.562	**.405**	**.374**	.994	.992	.984	.577	.679	.658	.075
-.5	.192	.192	**.095**	**.084**	.716	.710	.188	.133	.208	.191	.055
-.3	.083	.084	**.041**	**.036**	.334	.332	.071	.050	.076	.066	.042
-.2	.033	.034	**.014**	**.013**	.087	.083	.023	.017	.023	.021	.029
-.1	.022	.021	**.009**	**.009**	.034	.034	.014	.011	.013	.011	.025
-.05	.018	.017	**.006**	**.004**	.019	.018	.011	.010	.011	.010	.015
0	.018	.017	**.008**	**.006**	.013	.013	.013	.012	.010	.009	.013

T = 100

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{\tau}$	$\hat{\rho}_\tau$	$\hat{\tau}_\tau$	$\tilde{\rho}$	$\tilde{\tau}$	F
-.9	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	.000	.135
-.7	.955	.952	**.935**	**.922**	1.000	1.000	1.000	1.000	.997	.997	.093
-.5	.514	.511	**.447**	**.426**	.999	.999	.839	.746	.851	.840	.074
-.3	.220	.218	**.184**	**.170**	.925	.923	.371	.285	.419	.401	.066
-.2	.061	.059	**.045**	**.044**	.319	.314	.070	.050	.080	.075	.046
-.1	.027	.027	**.018**	**.016**	.084	.082	.026	.019	.027	.025	.035
-.05	.018	.017	**.012**	**.011**	.027	.026	.013	.013	.016	.015	.020
0	.015	.015	**.010**	**.009**	.012	.010	.012	.013	.012	.012	.008

T = 200

θ	S1	S2	**S1***	**S2***	$\hat{\rho}$	$\hat{\tau}$	$\hat{\rho}_\tau$	$\hat{\tau}_\tau$	$\tilde{\rho}$	$\tilde{\tau}$	F
-.9	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000	.199
-.7	1.000	1.000	**1.000**	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000	.140
-.5	.925	.924	**.908**	**.909**	1.000	1.000	1.000	1.000	1.000	1.000	.109
-.3	.587	.587	**.547**	**.551**	1.000	1.000	.986	.963	.970	.968	.087
-.2	.149	.151	**.135**	**.138**	.924	.922	.361	.261	.386	.379	.072
-.1	.049	.049	**.036**	**.037**	.323	.323	.068	.045	.075	.071	.045
-.05	.021	.021	**.016**	**.018**	.059	.057	.020	.016	.019	.018	.027
0	.013	.013	**.009**	**.008**	.010	.010	.010	.010	.010	.009	.009

*: $S1^*$ and $S2^*$ are sized-corrected S1 and S2 tests respectively.

TABLE 2.10

Finite sample critical values of $\tilde{R}$ in (3) with $\beta = 0$

Perc.\ T	25	50	100	200	500
0.001%	6.47E6	1.57E6	5.95E6	2.62E6	2.77E6
0.005%	1.03E4	9.10E5	7.23E5	3.48E5	5.84E5
0.01%	3.88E4	2.64E4	2.34E4	1.51E4	2.05E4
0.02%	0.001	0.001	7.71E4	6.76E4	7.71E4
0.025%	0.002	0.001	0.001	9.81E4	0.001
0.05%	0.009	0.006	0.005	0.004	0.004
0.1%	0.038	0.026	0.020	0.018	0.018
0.2%	0.155	0.112	0.086	0.076	0.072
0.25%	0.242	0.177	0.138	0.117	0.118
0.3%	0.361	0.262	0.200	0.172	0.172
0.5%	0.988	0.731	0.607	0.532	0.517
0.7%	2.353	1.552	1.344	1.228	1.172
0.75%	2.762	1.883	1.629	1.498	1.428
0.8%	2.900	2.268	1.983	1.832	1.760
0.9%	3.900	3.416	3.153	2.969	2.828
0.95%	4.976	4.428	4.234	4.089	3.953
0.975%	5.847	5.491	5.190	5.093	5.139
0.98%	6.093	5.957	5.546	5.564	5.459
0.99%	6.835	7.102	6.588	6.753	6.602
0.995%	7.523	8.001	7.577	8.290	7.989
0.999%	9.244	11.578	11.039	11.935	10.293
Mean:	1.553	1.302	1.164	1.099	1.061

Finite sample critical values of $\tilde{R}$ in (3) with unknown β and $z_t = (1,t)'$

Perc.\ T	25	50	100	200	500
0.001%	4.19E6	1.93E6	6.41E6	1.71E6	6.94E7
0.005%	9.09E5	6.77E5	9.80E5	4.82E5	3.21E5
0.01%	4.66E4	2.34E4	2.57E4	1.99E4	1.66E4
0.02%	0.002	9.24E4	9.00E4	7.65E4	6.80E4
0.025%	0.003	0.001	0.001	0.001	0.001
0.05%	0.011	0.006	0.005	0.004	0.004
0.1%	0.040	0.028	0.020	0.019	0.017
0.2%	0.162	0.120	0.084	0.075	0.071
0.25%	0.250	0.175	0.133	0.117	0.114
0.3%	0.359	0.255	0.197	0.173	0.171
0.5%	0.997	0.734	0.599	0.529	0.515
0.7%	2.015	1.562	1.327	1.231	1.173
0.75%	2.370	1.881	1.624	1.485	1.422
0.8%	2.782	2.264	1.965	1.833	1.749
0.9%	3.922	3.405	3.110	2.987	2.824
0.95%	4.950	4.512	4.246	4.096	4.007
0.975%	5.800	5.495	5.253	5.146	5.120
0.98%	6.110	5.972	5.558	5.567	5.439
0.99%	6.938	6.932	6.491	6.680	6.622
0.995%	7.551	8.023	7.554	8.334	8.074
0.999%	9.114	11.804	11.436	12.061	10.175
Mean:	1.554	1.305	1.158	1.101	1.061

TABLE 2.11

Rejection frequencies of Two-Sided S1 and S1*, and T1* tests with Gaussian u_t.

Two-sided S1 test

Size	10%		5%		1%		.1%	
T	100	200	100	200	100	200	100	200
-.3	.960	1.000	.904	.997	.581	.963	.116	.710
-.2	.772	.959	.609	.905	.236	.640	.019	.200
-.1	.387	.561	.231	.395	.048	.130	.002	.013
-.05	.215	.268	.112	.154	.018	.033	.001	.001
0	.127	.115	.063	.057	.011	.012	.001	.001
.05	.140	.186	.084	.125	.028	.050	.009	.016
.1	.249	.450	.117	.356	.096	.219	.038	.108
.2	.632	.910	.555	.872	.407	.783	.259	.650
.3	.897	.995	.859	.992	.767	.983	.638	.959

Two-sided S1* test

Size	10%		5%		1%		.1%	
T	100	200	100	200	100	200	100	200
-.3	.907	.998	.822	.995	.580	.986	.318	.696
-.2	.627	.928	.477	.864	.225	.652	.079	.183
-.1	.242	.442	.147	.320	.049	.136	.010	.011
-.05	.127	.191	.065	.118	.013	.036	.012	.002
0	.098	.099	.053	.051	.012	.009	.001	.001
.05	.150	.229	.083	.148	.028	.044	.006	.011
.1	.303	.522	.203	.416	.091	.223	.026	.098
.2	.699	.934	.595	.899	.431	.765	.231	.584
.3	.928	.998	.878	.996	.777	.982	.594	.938

T1* test

Size	10%		5%		1%		.1%	
T	100	200	100	200	100	200	100	200
-.3	.942	.999	.869	.997	.592	.962	.103	.596
-.2	.710	.952	.554	.891	.234	.634	.015	.121
-.1	.310	.517	.190	.362	.053	.126	.002	.007
-.05	.168	.247	.089	.141	.014	.032	.002	.000
0	.097	.100	.052	.047	.011	.008	.001	.001
.05	.100	.168	.061	.116	.025	.050	.009	.013
.1	.208	.434	.155	.360	.086	.234	.034	.107
.2	.593	.907	.528	.865	.417	.776	.259	.605
.3	.876	.996	.839	.993	.766	.983	.629	.946

: S1 and T1* are sized-corrected S1 and T1 tests respectively.

CHAPTER 3

FRACTIONAL INTEGRATION IN MACROECONOMIC TIME SERIES

In this chapter we use Robinson's (1994c) tests described in Chapter 2 for testing fractional integration in macroeconomic time series when the root is located at zero frequency. We will apply a particular form of these tests to an extended version of the fourteen macroeconomic variables used by Nelson and Plosser (1982). A reduced version of this chapter is Gil-Alaña and Robinson (1997).

3.1 INTRODUCTION

Specialized members of fractionally integrated stochastic processes play a considerable role in modelling macroeconomic behaviour. For the purpose of the present chapter, we define an I(d) process x_t, $t = 0,\pm1,...$, as (1.1) and (2.2). The macroeconometric literature stresses the cases $d = 0$ and $d = 1$, and much controversy in macroeconomics has revolved around the question of the suitability of I(1) models, also termed unit root or difference-stationary models, for describing raw time series. These models are in the class of so-called nonstationary stochastic trend models, which typically imply that the mean and variance increase without bound over time, the precision of the forecast error is unbounded, and the effect of shocks persists. Another approach to modelling nonstationarity consists of so-called trend-stationary models, where the raw series is described as an I(0) process plus a deterministic trend (often a linear function of time). Here, the mean of the series is described by the trend function, the variance of the forecast errors remains finite, and shocks have only a transitory effect. The issue of stochastic versus deterministic trend models has considerable implications for our understanding of the economy, and economic planning. In particular, real GNP having a unit root or stochastic trend supports the real business cycle hypothesis, since it is widely accepted that shocks that result in permanent increases in the level of real GNP can only plausibly be interpreted as permanent productivity improvements. In the context of stochastic trends, any shock to the economic system will have a permanent effect, so a policy action will be required to bring the variable back to its original long term projection. On the other hand, in trend-stationary models, fluctuations will be transitory and therefore there exists less need for policy action, since the series will in any case return to its trend sometime in the future.

Unit roots, and linear time trends, each constitute extremely specialized models for nonstationarity, but each has the advantage of conceptual and computational simplicity, and they are naturally thought of as rival models because a unit root without or with a drift implies a constant or linear trend function, the distinction then being in the disturbance terms. The appropriate treatment of trends in economic time series is important. There is evidence that removal of an estimated (typically linear) deterministic trend from time series that are in fact integrated can lead to spurious cyclical behaviour in the detrended series. Chan et al. (1977) studied both inappropriate detrending of integrated series and inappropriate differencing of trending series, and showed that the former produced spurious variation in the detrended series, while the latter produced spurious variation in the differenced series at high frequencies. These results have been amplified by Nelson and Kang (1981, 1984) and Durlauf and Phillips (1988).

Despite the interest aroused in unit root models by Box and Jenkins (1970) and Dickey and Fuller (1979), the deterministic trend approach tended to prevail in macroeconomics until Nelson and Plosser (1982) reported strong evidence of unit roots in U.S. historical annual time series. They considered fourteen macroeconomic series, starting from 1860 through 1909 and ending in 1970, analysing the logged series in all but one of these cases. Let y_t, t = 1,2,... be the series to be studied. The unit root model tested by Nelson and Plosser (1982) was essentially

$$(1 - L)y_t = \alpha + u_t, \qquad t = 1,2,..., \tag{1}$$

where

$$\phi(L)u_t = \epsilon_t, \qquad t = 1,2,..., \tag{2}$$

in which ϕ is a k-th. degree polynomial, all of whose zeroes lie outside the unit circle and ε_t is a white noise sequence. In the terminology of Box and Jenkins (1970), (1) and (2) constitute an ARIMA(k,1,0) model, with drift when $\alpha \neq 0$. Nelson and Plosser (1982) nested (1) in

$$(1 - \rho L)y_t = \mu + \gamma t + u_t, \qquad t = 1,2,... \tag{3}$$

Thus (1) corresponds to the null hypothesis

$$H_o\!: \ \rho = 1 \ \ and \ \ \gamma = 0 \tag{4}$$

in (3), whereas $|\rho| < 1$ corresponds to a (linear) trend-stationary model. We can impose the same initial condition on y_0 in (1) and (3), on taking $\alpha = \mu + \gamma$. For various k in (2), Nelson and Plosser (1982) tested for a unit root, using tests of Dickey and Fuller (1979), Fuller (1976). These tests, based on t-ratios, are not approximately t-distributed under the null, but Dickey and Fuller tabulated the null distribution. The tests failed to reject the unit root null (1) in all series except unemployment rate.

The paper of Nelson and Plosser (1982) has led to much subsequent research. Some of it has involved applying similar methodology to Nelson and Plosser's (1982) to other macroeconomic series, for example non-U.S. series, and some to criticism of their methodology and application of modified or alternative approaches. We attempt only a brief and partial summary of this literature.

Starting with the same model as Nelson and Plosser (1982), Stock (1991) provided asymptotic confidence intervals for the largest autoregressive root when this root is close to one, motivated by concern that reporting only test outcomes or point estimates fails to convey adequate information about sample uncertainty or the range of models consistent with the data. When applied to the Nelson and Plosser (1982) data set, his main conclusion was that the confidence intervals were typically wide, containing $\rho = 1$ for all series except unemployment and bond yield, but typically also values significantly different from one. Another theme has involved the replacement of (2) by alternative or more general models for the stationary disturbance u_t. The tests used by Nelson and Plosser (1982) lose validity if u_t is not autoregressive (AR), as remarked by Schwert (1987) who found that Dickey-Fuller critical values can be misleading even for large sample sizes in case of a mixed ARIMA process. He applied tests of Said and Dickey (1984, 1985) to monthly and quarterly series based on a mixed autoregressive moving average (ARMA) model for u_t with positive moving average order. (These tests approximate the ARMA by an AR.) Also Schwert (1987), Stock and Watson (1986) and Perron (1988) employed tests of Phillips (1987), Phillips and Perron (1988) which, more generally, are valid in case of nonparametric autocorrelation; these tests employ a nonparametric estimate of the spectral density of u_t at zero frequency, for example a weighted autocovariance estimate. All these authors obtained results very similar to those obtained by Nelson and Plosser (1982). Choi (1990) dealt with disturbance autocorrelation using feasible generalized least squares, coming to rather different conclusions.

Kwiatkowski et al. (1992) observed that taking the null hypothesis to be I(1), rather than I(0), might itself have led to a bias in favour of the unit root hypothesis; they proposed an I(0) test which formulates the null as a zero variance in a random walk, and applied it to the Nelson and Plosser (1982) data. They concluded that for many of these series the hypothesis of trend-stationarity could not be rejected. In the same line, Leybourne and McCabe (1994) proposed a similar test for a unit root, where the null was an AR(k) process and the alternative was an integrated ARMA (ARIMA) model with AR order k and unit MA order. Their test differs from that of Kwiatkowski et al. (1992) in its treatment of autocorrelation under the null hypothesis, its critical values appearing more robust to certain forms of autocorrelation.

Campbell and Mankiw (1987) and Cochrane (1988) studied the problem in terms of measures of persistence in macroeconomic series. Campbell and Mankiw (1987) considered the sum of the Wold decomposition weights for the differenced series, which will be zero under trend-stationarity, and estimated this using ARIMA models and nonparametric spectral methods. Their analysis suggested that shocks in U.S. GNP are largely permanent, consistent with the stochastic differencing advocated by Nelson and Plosser (1982). Cochrane (1988) proposed a nonparametric variance ratio statistic and came to empirically different conclusions. Other measures of persistence also suggested by Cochrane (1987, 1988) are based on the spectral density of the differenced series at zero frequency, but Quah (1992) argued that such measures did not identify the magnitude of the permanent component, unless this is a random walk.

Related work has been done by Christiano and Eichenbaum (1990). The tests referred to so far are motivated by their asymptotic statistical properties, but Bhargava (1990) applied tests of Bhargava (1986) with finite sample optimality properties to test for a unit root in quarterly U.S. GNP, finding that it is the inability to capture the complex deterministic trend component that can cause non-rejection. Bayesian procedures have also been employed. Sims (1988) and Sims and Uhlig (1991) used Bayesian arguments to criticize classical unit root testing methodology in abstract. Also DeJong and Whiteman (1989, 1991, for example) conducted empirical research with flat-prior Bayesian techniques and challenged unit root findings in many cases, including Nelson and Plosser's (1982) series. Schotman and Van Dijk (1991) analyzed from a Bayesian viewpoint the random walk hypothesis for real exchange rates, and came to different conclusions from those reached by the classical tests. However Phillips (1991), using objective ignorance priors rather than flat priors, obtained results closely related to those obtained by the classical methods: seven of Nelson and Plosser's (1982) series showed evidence of stochastic trends. Phillips (1991) found that flat priors on the AR coefficients were informative, contrary to their apparent intent, and unit and explosive roots were downweighted in the posterior distribution. Among other authors working with Bayesian procedures, DeJong (1992) and Zivot and Phillips (1994) showed respectively the importance of choice of prior in distinguishing between difference- and trend-stationary, and trend determination with the possibility of structural breaks.

In fact, the implications of structural change on unit root tests which take no account of this possibility has itself been a major focus of attention since Perron (1989, 1993) found that the 1929 crash and the 1973 oil price shock are a cause of non-rejection of the unit root hypothesis, and that when these are taken into account, a deterministic trend model is preferable. This question has been pursued by authors such as Christiano (1992), Krol (1992), Serletis (1992), Demery and Duck (1992), Mills (1994) and Ben-David and Papell (1995), the first author arguing that the date of the break should be treated as unknown, and suggesting that tests for a structural break are themselves biased in favour of non-rejection, and by means of tests based on bootstrap critical values, coming to different conclusions from Perron (1989). Zivot and Andrews (1992) allowed the structural break to be endogenous, finding less conclusive evidence against unit roots than did Perron (1989). Stock (1994) applied a Bayesian procedure that consistently classifies the stochastic component of a time series as I(1) or I(0), applying it to Nelson and Plosser's (1982) data with both linear detrending and piecewise linear detrending, supporting their conclusions in the former, but not the latter, case.

There has been a growing literature which studies the source of nonstationarity in macroeconomic series in terms of fractionally differenced time series. We can replace the alternative (3) by

$$(1-L)^d y_t = \mu + \gamma t + u_t, \quad t = 1,2,... \tag{5}$$

so (1) results when $d = 1$, $\gamma = 0$. On the other hand, if $\mu = \gamma = 0$, if u_t is an I(0) series, and if $0 < d < 1/2$, then y_t is a covariance stationary I(d) series, having autocovariances which decay much more slowly than those of an ARIMA process, in fact so slowly as to be non-summable; thus, if we first-difference y_t, the unit root null corresponds to $d = 0$, but the close alternatives are very different from those in (3). Models such as (5) provide a type of flexibility in modelling low frequency dynamics not achieved by non-fractional ARIMA models. In empirical applications, Diebold and Rudebusch (1989), Haubrich and Lo (1989) and Sowell (1992b) obtained estimates and tests using nonparametric and parametric methods based on differenced quarterly data, while Cheung and Lai (1992) appear to have estimated d from undifferenced data. Sowell's (1992b) model nested both a deterministic trend and a unit root with drift, neither being rejected as a model for postwar US quarterly real GNP. Hauser et al. (1992) and Mills (1992) have discussed the relevance of fractional models in measuring persistence, while Koop (1991a) proposed a Bayesian fractional approach.

Conspicuous features of many of the methods used in the empirical work described above, and of the bulk of all available methods for testing for unit roots (for a review see Diebold and Nerlove, 1989) are the nonstandard nature of the null asymptotic distributions which are involved, and the absence of Pitman efficiency theory. Many of these tests can be viewed as resulting from implementation of the Wald, likelihood ratio (LR), or Lagrange multiplier (LM) rules. Such rules are frequently motivated by the desirable properties of a null chi-squared asymptotic distribution, and Pitman efficiency, but such properties are not automatic, rather depending on what might be called a degree of "smoothness" in the model across parameters of interest, in the sense that limit distributions do not change in an abrupt way with small changes in the parameters. They do not hold in case of unit root tests against AR alternatives such as (3) - as the work of Dickey and Fuller (1979) and numerous subsequent authors indicates, the null asymptotic distribution is nonstandard, and while local alternatives can be considered this does not seem to lead here to a neat optimality theory (though Elliott et al. (1996) show how the tests can be improved). This is associated with the radically variable long-run properties of AR processes around the unit root. Under (3), with, for simplicity, $\mu = \gamma = 0$ and u_t Gaussian white noise, for $|\rho| > 1$ u_t is explosive, for $|\rho| < 1$ u_t is covariance and strictly stationary and is I(0) (indeed strongly mixing with exponentially decaying mixing numbers), and for $\rho = 1$ it is nonstationary but non-explosive. Some of the other procedures that have been used in unit root testing are not derived by the Wald, LR, or LM rules, but many of these seem, therefore, if anything more ad hoc.

The present chapter applies to an extended version of the data set used by Nelson and Plosser (1982), and uses a particular form of Robinson's (1994c) tests for testing unit roots and other nonstationary hypotheses when the root is located at zero frequency. As mentioned in Chapter 2, the tests do possess the standard properties of efficiency and have a null asymptotic chi-squared distribution. This is due to the fact that they are directed against fractional alternatives, which turn out to be a "smoother" class than the AR ones. Salient features of the tests, when compared with those directed against AR alternatives, are described in the following section. The empirical work is in Section 3, and Section 4 contains some concluding comments. The FORTRAN codes used to obtain the tests in this chapter are given in an appendix at the end of Chapter 4.

3.2 L.M. TESTS AGAINST FRACTIONAL ALTERNATIVES

Despite the extent to which it has been stressed in the literature, the AR dynamics in (3) is merely one out of any number of ways of nesting the unit root (1). The literature on long memory or fractional processes, which is of quite long standing and has become rather extensive of late suggests a rival class of alternatives, the I(d) class with fractional d, as defined in (1.1) and (2.2). Following discussions of Bhargava (1986), Schmidt and Phillips (1992) of parameterization of unit root models, let us first take (1.9) where, following Robinson (1994c), x_t is an I(d) process given as in (1.1) and (2.2). (1.1) can be compared to the AR class

$$(1 - \rho L)x_t = u_t, \qquad t = 1,2,..., \tag{6}$$

advocated by Bhargava (1986) and others in the regression setting (1.9). Trivially (1.1) and (6) give an I(0) x_t when d = 0 and $\rho = 0$, respectively, while the I(1), or unit root, hypothesis corresponds to

$$H_o: d = 1 \tag{7}$$

in (1.1) and

$$H_o\colon\ \rho\ =\ 1 \qquad (8)$$

in (6). Fractional and AR departures from (7) and (8) have very different long run implications. In (1.1), x_t is nonstationary but non-explosive for all $d \geq 1/2$. As d increases beyond 1/2 and through 1, u_t can be viewed as becoming "more nonstationary", but it does so gradually, unlike in case of (6) around (8). The dramatic long-run change in (6) around $\rho = 1$ has the attractive implication that rejection of (8) can be interpreted as evidence of either stationarity or explosivity. However, rejection of the null does not necessarily warrant acceptance of any particular alternative, and even when unit root tests are derived by either the Wald, LR or LM criteria against AR alternatives, they can still be expected to be consistent against many of the numerous other possible types of departure (see Robinson (1993)). Tests against (7), proposed by Robinson (1994c), can at the very least be regarded as a useful diagnostic tool to supplement tests directed against such alternatives as AR ones.

There is also interest in other hypotheses within the class (1.1) such as d = 2, (which is also in the class of tests against AR alternatives, in this case AR(2) ones, see eg. Johansen, 1992). Robinson's (1994c) approach to deriving tests (via the LM criterion) against (7) applies equally to any real null hypothesized value of d, and the same, standard, null and local limit distribution theory obtains. (The I(d) class comprises many stationary, nonstationary, invertible and non-invertible processes.) This is in sharp contrast to asymptotic theory for statistics directed against AR alternatives, where, for example, different null theory obtains for I(2) than for I(1). Often when we construct a test of a nonstationary hypothesis against AR alternatives we have to contemplate the possible occurrence of a somewhat new, nonstandard, null limit distribution, the approximation of which may require a new piece of numerical work. As well as any integer, the null d can be fractional, for example d = 1/2, which is of interest in that it represents the boundary between stationarity and nonstationarity in the I(d) class. It may be that the immense econometric stress on so specialized a form of nonstationarity as unit root behaviour owes something to the even more long-standing popularity of stationary AR models, and that this behaviour deserves to be less at the forefront when other classes of model are contemplated. Thus, in the present chapter we report also tests of other hypothesized values of d. We could also test for null stationary d values, indeed Robinson (1991) earlier proposed analogous tests of d = 0.

Observing $\{(y_t,z_t),t=1,2,...T\}$ in (1.9); (1.10); and (2.2), with

$$\rho(L;\theta) = (1 - L)^{d+\theta} \tag{9}$$

we want to test the null hypothesis (1.12) for a given real number d. We make use of the test statistic $\hat{r}$ in (2.9), which includes $\tilde{r}$ in (2.3) as a particular case with $g \equiv 1$. In Chapter 2 we showed that

$$\hat{r} \to_d N(0,1) \quad as \quad T \to \infty. \tag{10}$$

and thus, an approximate one-sided 100α%-level of (1.12) against alternatives

$$H_1\!: \ \theta > 0 \tag{11}$$

is given by the rule:

$$Reject \ H_o \ \ if \ \ \hat{r} > z_\alpha \tag{12}$$

where the probability that a standard normal variate exceeds z_α is α. Conversely, an approximate one sided 100α%-level test of (1.12) against alternatives

$$H_1\!: \ \theta < 0 \tag{13}$$

is given by the rule:

$$Reject \ H_o \ \ if \ \ \hat{r} < -z_\alpha\,. \tag{14}$$

As mentioned in the previous chapter, these tests will be efficient, in the Pitman sense that against local alternatives, $\hat{r}$ has an asymptotic normal distribution with variance 1 and mean which cannot (when u_t is Gaussian) be exceeded in absolute value by that of any rival regular statistic. Of course, this efficiency property holds only in respect of fractional alternatives, and not AR alternatives, for example. We believe that as in other standard large-sample testing situations, Wald and LR test statistics against fractional alternatives will have the same null and local limit theory as our LM tests (unlike in case of AR alternatives). Sowell (1992b) employed essentially such a Wald testing procedure. Wald and LR tests require an efficient estimate of d, and while such estimates can be obtained, the LM tests seem computationally more attractive. As usual, the LM, Wald and LR tests will have differing finite sample properties. However, as mentioned in Chapter 2, we use the asymptotical critical values given by the Normal distribution, instead of the finite-sample critical values obtained in that chapter. The reason for this is mainly because

Robinson's (1994c) tests allow a great variety of model specifications, each with a different empirical distribution in finite samples. As specified below, the model will allow different regressors, different models for the disturbances u_t, and in Chapter 4, we will also allow different functions $\rho(L;\theta)$ in (1.10), each with a different empirical distribution. Thus, we have decided to use the large-sample approximate critical values, rather than the size-corrected ones.

3.3 EMPIRICAL RESULTS

The extended version of the annual data set of fourteen U.S. macroeconomic variables analyzed by Nelson and Plosser (1982) ends in 1988; as with their data, the starting date is 1860 for consumer price index and industrial production; 1869 for velocity; 1871 for stock prices; 1889 for GNP deflator and money stock; 1890 for employment and unemployment rate; 1900 for bond yield, real wages and wages; and 1909 for nominal and real GNP and GNP per capita. As in Nelson and Plosser (1982), all the series except the bond yield are transformed to natural logarithms. Plots of the series are given in Figure 3.1 and we observe that all except unemployment and velocity increase over the sample period, with two possible structural breaks due to the 1929 crash and World War II in 1945.[1] Figure 3.2 contains plots of sample autocorrelations and Figure 3.3 of estimates of the spectral density function[2], observing in all except unemployment a slow decay in the former and a peak around zero frequency in the latter, suggesting nonstationary or at least fractionally integrated behaviour. The first fourteen sample autocorrelations for each series are plotted in Table 3.1, while the autocorrelations of the first differences are plotted in Table 3.2. Qualitatively, these results are similar to those in Tables 2 and 3 of Nelson and Plosser (1982): in Table 3.1, except for unemployment the autocorrelations start at around 0.96 and then decay slowly, which could be consistent even with the simple random walk hypothesis, whereas in Table 3.2 we still see significant autocorrelations, especially at lag 1, with also some apparent slow decay and/or oscillation in some cases, which could be indicative of fractional

[1] The presence of a possible structural break on the data will be studied in Appendix 3.1 at the end of the chapter.

[2] They are estimates of the standardized spectral density function, using Barlett, Tukey and Parzen lag windows of size T-1.

integration of greater than or less than a unit root.

Denoting any of the series y_t, we employ throughout the model (1.9); (1.10); (2.2) and (9) with $z_t = (1,t)'$, $t \geq 1$, $z_t = (0,0)'$. Thus, under H_0 (1.12),

$$y_t = \beta_1 + \beta_2 t + x_t, \qquad t = 1,2,..., \tag{15}$$

$$(1 - L)^d x_t = u_t, \qquad t = 1,2,..., \tag{16}$$

and we treat separately the cases $\beta_1 = \beta_2 = 0$ a priori, β_1 unknown and $\beta_2 = 0$ a priori, and (β_1, β_2) unknown. We will model the I(0) process u_t to be both white noise and to have parametric autocorrelation. Our findings can be briefly summarized as follows. When u_t is white noise, the unit root null is seldom rejected, but a greater degree of integration, d, is sometimes more plausible. When u_t is AR, the tests are suggestive of smaller d's. When u_t follows the Bloomfield (1973) exponential model (2.12) the range of plausible d-values tends to be narrowed for any given series, though the plausibility region varies significantly across series.

We start with the assumption that u_t in (16) is white noise. Thus when d = 1, for example, the differences $(1 - L)y_t$ behave, for t > 1, like a random walk when $\beta_2 = 0$, and a random walk with drift when $\beta_2 \neq 0$. However we report test statistics not merely for the case of d = 1 in (16) but for d = 0.50 (0.25) 2.25, thus including also a test for stationarity (d = 0.5) and for I(2) (d = 2), as well as other possibilities.

The test statistic reported in Table 3.3 (and also in Tables 3.9-3.14) is the one-sided one given by $\hat{r}$ in (2.9), so that significantly positive values of this, see (12), are consistent with (11), whereas significantly negative ones, see (14), are consistent with (13). A notable feature of Table 3.3 (i), in which u_t is taken to be white noise (when the form of $\hat{r}$ significantly simplifies) and $\beta_1 = \beta_2 = 0$ a priori, is the fact that we cannot reject the unit root hypothesis in any of them, while in three (real GNP, real wages and money stock) we cannot reject the null when d = 0.5 or d = 0.75. However, in each of these three series, and in the GNP deflator and wages, we also observe some lack of monotonic decrease of $\hat{r}$ as d increases, for the smaller values of d. Such monotonicity is a characteristic of any reasonable statistic, given correct specification and adequate sample size, because, for example, we would wish that if (1.12) is rejected against (11) when d = 0.75, an even more significant result in this direction would be obtained when d = 0.5. However in the event of misspecification (which in this specialized model can be due to a departure from white noise in u_t, to y_t having a drift, or to both) monotonicity is not necessarily to be expected: frequently misspecification inflates both numerator and denominator of $\hat{r}$, to varying degrees, and thus affects $\hat{r}$ in a complicated way. Computing $\hat{r}$ for a range of d values is thus useful in revealing possible misspecification, though monotonicity is by no means necessarily strong evidence of correct specification. Looking at the nine series where there is monotonicity in $\hat{r}$ in Table 3.3 (i), industrial production and unemployment rate are consistent with $d = 0.75$, while bond yield is the only one in which we cannot reject the null with $d = 1.25$. The departures from monotonicity in Table 3.3 (i) are nowhere so great as to result in contradictory verdicts of tests.

Tables 3.3 (ii) and (iii) give results with, respectively, $\beta_2 = 0$ a priori (no time trend in the undifferenced regression), and both β_1 and β_2 unrestricted, still with white noise u_t. In every case in both tables, $\hat{r}$ is monotonic, and moreover, while there are sometimes large differences in the values of $\hat{r}$ across Tables 3.3 (ii) and (iii) for the same series/d combination, the conclusions suggested by both seem very similar, that on the whole the extreme nonstochastic trends are inappropriate. The most nonstationary series seem to be the consumer price index and money stock, where $d > 1.5$ is suggested and $d = 1$ is rejected. We also reject the unit root hypothesis in the GNP deflator, nominal GNP and wage series, against more nonstationary alternatives. Notice that these five series are a subset of the ones in which the lag-1 autocorrelation was significant in Table 3.2, so the lack of allowance for even I(0) autocorrelation in u_t could be the cause of rejection. The other results could all be consistent with a unit root. The results here are in line with those of DeJong et al. (1992) who did not reject the unit root hypothesis in most series when ignoring the possibility of disturbance autocorrelation. In our case, most of the series could also be fractionally integrated for some $d > 1$, except for industrial production and unemployment rate; these are the only series in which we cannot reject the null with $d = 0.75$ (throughout Table 3.3).

In Table 3.4 we report results of the tests for the same null and alternative hypotheses as in Table 3.3, but using the time domain version. Robinson (1994c) shows that the one-sided test statistic for this case of white noise u_t is

$$\dot{r} = \frac{T^{\frac{1}{2}}}{\dot{\sigma}^2} \dot{A}^{-\frac{1}{2}} \dot{a}, \qquad (17)$$

where

$$\dot{A} = \sum_{l=1}^{T-1}\left(1-\frac{l}{T}\right)l^{-2}; \quad \dot{a} = \sum_{l=1}^{T-1} l^{-1}C_{\tilde{u}}(l); \quad C_{\tilde{u}}(l) = \frac{1}{T-l}\sum_{t=1}^{T-l}\tilde{u}_t\tilde{u}_{t+l}; \quad \dot{\sigma}^2 = C_{\tilde{u}}(0),$$

and $\tilde{u}_t$ as described in Chapter 2. It is known that in finite samples the time and frequency domain versions of the tests might differ substantially, however, looking at Table 3.4 we see that though the values differ analytically in some cases, qualitatively the same conclusions hold, with non-rejections occurring practically at the same values of d in both tables, especially when we include an intercept or a linear time trend in the model.

In Table 3.5 we report sample autocorrelations of estimates of x_t in (15) and (16), obtained by selecting, for each series, the value of d which produces the most insignificant $\hat{r}$ in Table 3.3 (iii), using the OLS estimate of β_1 and β_2 based on that differenced model. While the autocorrelations are generally lower than those of Table 3.1, and indicate a somewhat faster rate of decay, they are again significant and persistent. In Table 3.6 we report sample autocorrelations r_j of the d differences of the estimated x_t used in Table 3.5. Notice that for nine of the series the r_1's are smaller in Table 3.6 than in Table 3.2, often much smaller, while four are the same; for r_{13}, seven are smaller and six are the same.

The bond yield is the only unlogged series (as in Nelson and Plosser, (1982)), but we also computed the tests in both domains for the logged bond yield, (in Table 3.7), and there was no qualitative change; in both cases (and across Tables 3.3 and 3.4 ((i)-(iii)) there was similar evidence of somewhat greater than unit root integration.

In view of Tables 3.3 and 3.4 ((ii) and (iii)) there is some interest in a joint test for

$$H_o:\ \theta = 0 \quad and \quad \beta_2 = 0. \tag{18}$$

This possibility is not addressed by Robinson (1994c), but we can derive an LM test of (18) against the alternatives,

$$H_1:\ \theta \neq 0 \quad or \quad \beta_2 \neq 0, \tag{19}$$

as follows. To be slightly more general, consider the regression model (1.9) with the vector partitions $z_t = (z_{At}', z_{Bt}')'$, $\beta = (\beta_A', \beta_B')'$, and we want to test H_0: $\theta = 0$ and $\beta_B = \beta_{Bo}$. Then an LM statistic may be shown to be $\hat{r}^2$ plus

$$\sum_{t=1}^{T} \hat{u}_t w_{Bt}' \left(\sum_{t=1}^{T} w_{Bt} w_{Bt}' - \sum_{t=1}^{T} w_{Bt} w_{At}' \left(\sum_{t=1}^{T} w_{At} w_{At}' \right)^{-1} \sum_{t=1}^{T} w_{At} w_{Bt}' \right)^{-1} \sum_{t=1}^{T} \hat{u}_t w_{Bt} \qquad (20)$$

with $(w_{At}', w_{Bt}')' = w_t = (1 - L)^d z_t$,

$$\hat{u}_t = (1 - L)^d y_t - (\tilde{\beta}_A', \beta_{Bo}') w_t, \quad \tilde{\beta}_A = \left(\sum_{t=1}^{T} w_{At} w_{At}' \right)^{-1} \sum_{t=1}^{T} w_{At} (1 - L)^d y_t,$$

$\hat{\sigma}^2 = T^{-1} \sum_{t=1}^{T} \hat{u}_t^2$, and $\hat{r}^2$ calculated as described in Chapter 2 but using the $\hat{u}_t$ just defined. If the dimension of z_{Bt} is q_B, then we would compare (20) with the upper tail of the $\chi^2_{1+q_B}$ distribution. In case of testing (18) against (19) in model (15) we have $q_B = 1$, $z_{At} = 1$, $z_{Bt} = t$ for $t \geq 1$. In Table 3.8 we present the statistic (20) for the same d values as before. In each series except industrial production (where the test picks up an effect not immediately noticeable from Tables 3.3 (ii) and (iii)) we find non-rejection values of d. These are similar to those in Tables 3.3 (ii) and (iii), but with a narrowing-down effect (so far as number of non-rejections is concerned) in some of the series, but even the reverse effect (possibly indicative of the loss of power due to the extra degree of freedom) in a couple of others. For unemployment rate, a relatively attractive model in view of Tables 3.3 and 3.8, has $\beta_2 = 0$ and $d = 0.75$, whereas this hypothesis is rejected in all the other series. We do not reject the null when $d = 1$ is paired with $\beta_2 = 0$, in less than half the cases in which it was rejected in Tables 3.3 (ii) and (iii), suggesting the importance of the trend term in a number of these cases. Notice that even for $d \geq 2$ the null hypothesis is less strongly rejected than for small d; this accords with the similarity in the corresponding statistics between Tables 3.3 (ii) and (iii). It could also relate to the fact that whereas $(1-L)^d t$ tends to zero for $d > 1$ as t increases, it continues to trend with t for $d < 1$ (whereas $(1-L)^d t^d$ tends to a non-zero constant for all d). Except in the one case of industrial production, the conclusion seems to be that when an appropriate differencing order is used, the time trend is unimportant.

In connection with the power properties of Robinson's (1994c) tests, it must be stressed that it is only in a local sense that they are optimal, and doubtless they could be bettered against non-local departures of interest by some point optimal procedure. In view of this there is some satisfaction in the fact that the null is always decisively rejected in Tables 3.3 and 3.4 ((ii) and (iii)) for $d \geq 2$ and $d = 0.5$. On the other hand, these significant results might be due in large part to unaccounted-for I(0) autocorrelation in u_t, even bearing in mind the monotonicity of $\hat{r}$ in d achieved in these tables. Thus we also fitted non-seasonal, seasonal, and mixed seasonal/non-seasonal AR in u_t, as anticipated in Section 2, for the same d values and the same cases of no regressors, an intercept and a linear time trend as in Tables 3.3 and 3.4. When modelling with no regressors, the monotonic decrease in $\hat{r}$ with respect to d did not occur very often among the different specifications for the disturbances. Including an intercept or an intercept and a time trend, results were similar in both cases, with unemployment as the less nonstationary series, and consumer prices and money stock as the most nonstationary ones when modelling u_t as seasonal and non-seasonal AR, but observing again a lack of monotonicity in $\hat{r}$ with respect to d in practically all series when u_t was a mixed seasonal and non-seasonal AR process.

In Table 3.9 we concentrate on non-seasonal AR(k) u_t, with k=1,2,...5, and present results only for a subset of the values obtained, choosing for each series a single k across all d. An alternative approach would be to pick a k for each series/d combination, on some basis. This is what Nelson and Plosser (1982) did, but they considered only a single d. We have preferred to choose the k for each series which produces the smallest value of $|\hat{r}|$, across d. This enables better comparison with Table 3.3 and indicates the strongest support for any one hypothesis, while also having a tendency to be accompanied by relatively small $|\hat{r}|$ throughout, thereby providing an impression of relatively lower power. Results are similar for the three cases of no regressors, an intercept and a time trend, with non-rejections occurring practically always when $d \leq 1.50$. Looking at Table 3.9 (iii), which is the most interesting case in view of monotonicity in the value of $\hat{r}$ with respect to d, we see that k = 1 or 2 in eight cases, whereas k = 5 in only one. It is striking that in many of the series the non-rejection d's tend, in Table 3.9 (iii), to be smaller by about 0.5-0.75 than those in Table 3.3 (iii), indicating how the AR model is somewhat confounded with the fractional one in finite samples, and the delicacy of modelling in this situation. (We used Yule-Walker estimates of the AR coefficient, which entail AR roots that are automatically less than one in absolute value, but can be arbitrarily close to one.) We find that when $d \geq 1.75$ the null is rejected in all series, and there are numerous rejections with d = 1.25 and 1.5. The strongest evidence of nonstationarity is found in the GNP deflator and consumer prices. The unit root hypothesis is now rejected only in case of real GNP and industrial production; they are series in which it was not rejected in Table 3.3 (iii), $\hat{r}$ there being positive in these cases, whereas it is negative throughout the unit root column in Table 3.9 (iii). Moreover, when d = 0.75 the null is now never rejected, and when d = 0.5 is only rejected in cases of consumer prices, money stock and velocity, while $\hat{r}$ is even negative for several of the other series.

AR modelling of I(0) processes is very conventional, but there exist many other types of I(0) process, including ones outside the stationary and invertible ARMA class. As we saw in Chapter 2, one that seems especially relevant and convenient in the context of the present tests is that proposed by Bloomfield (1973), in which g is given by (2.12). Like the stationary AR(k), this has exponentially decaying autocorrelations. Formulae for Newton-type iterations for estimating the τ_j are very simple (involving no matrix inversion), updating formulae when k is increased are also simple, and we can replace $\hat{A}$ in (2.9) by the population quantity

$$\sum_{j=k+1}^{\infty} j^{-2} = \pi^2/6 - \sum_{j=1}^{k} j^{-2},$$

which indeed is constant with respect to the τ_j (unlike what happens in the AR case). Using (2.12), the τ_j in $\hat{a}$ were estimated by a Gauss-Newton iteration, convergence being achieved within about seven iterative steps throughout. We again tried k = 1,...,5 for each series/d combination. Overall, there is a somewhat larger proportion of rejections of the higher d than for white noise or AR u_t. As a much more striking comparison with the AR case, when d = 0.5 the null is now rejected in the great proportion of series, and when d = 0.75 in around half. Perhaps this is due to the stationarity of the Bloomfield process for all real values of τ_j, so that it may be less inclined to try to model the nonstationary part than the AR process. We do not report all the results here, but first present, in Table 3.10, ones for the same k values as in Table 3.9 (iii), to facilitate comparison between the two I(0) models. The results are indicative of a somewhat greater degree of nonstationarity, and are definitely less ambiguous, than those just discussed, in all but two cases the non-rejection d's forming a proper subset of those in Table 3.9 (iii); the exceptional time series are money stock, where there is one extra non-rejection value in Table 3.10, and wages, where there is one fewer but they are 1 and 1.25 rather than 0.5, 0.75 and 1. Moreover, in seven of the series there is only one d where the null is not rejected; these d-values are quite variable across the series, being 0.5 for industrial production, 0.75 for real GNP, real per capita GNP and employment, 1 for nominal GNP and velocity, and 1.25 for consumer prices. These results for the Bloomfield model also entail a greater proportion of rejections than those based on white noise u_t in Table 3.3 (iii), despite the additional parameters; we attribute this to smaller $\hat{\sigma}$'s. We also give, in Table 3.11, results for the Bloomfield model when we choose k on the same basis as in Table 3.9. Though nine of the k's differ from those in Table 3.10, the results are very similar in both numbers of rejections and favourable d-values, the only somewhat exceptional case that may deserve mention being industrial production, where the null is now rejected when $d = 0.5$.

3.4 FINAL COMMENTS

The conclusions suggested by the tests of Robinson (1994c), carried out on the extended version of Nelson and Plosser's (1982) data, vary substantially across the fourteen series and across various models for the I(0) process u_t. When u_t is taken to be white noise, the unit root hypothesis is rejected in as many as five series, in each of which a somewhat greater (but less than I(2)) degree of nonstationarity is indicated, while even when the unit root is not rejected there is also evidence of possible fractional differencing. With AR u_t there tend to be fewer rejections, and the evidence points to a substantially smaller degree of nonstationarity, though this may be due in large part to competition with the autoregression in describing the nonstationarity. The results using the Bloomfield u_t are perhaps the most interesting, because of the many rejections and strong evidence in favour of single values of d in a number of series, most of which are 0.75 or 1. Attempting to summarize the conclusions for individual series from the various statistics, we are left with the impression that consumer prices and money stock are the most nonstationary, followed by the GNP deflator and wages, whereas unemployment rate, followed by industrial production, seem closest to stationarity.

It would be worthwhile proceeding to get point estimates of d, perhaps especially in the Bloomfield case. However, not only would this be computationally more expensive, but it is then in any case confidence intervals rather than point estimates which should be stressed, while available rules of inference seem to require preliminary integer differencing to achieve stationarity and invertibility. The approach used in this chapter generates simply-computed diagnostics for departures from any real d. It is not at all surprising that, when fractional hypotheses are entertained, some evidence supporting them appears, because this might happen even when the unit root model is highly suitable. However, even though our practice of computing test statistics for a wide range of null hypotheses does lead to ambiguous conclusions, often the bulk of these hypotheses are rejected, suggesting that the optimal local power properties of the tests, shown by Robinson (1994c), may be supported by reasonable performance against non-local alternatives. It is the known efficiency property of the tests which really distinguishes them from much other work on testing for unit roots (and indeed fractional roots).

The frequency domain seems to be unpopular with many econometricians, and it is important to stress that our frequency domain formulation of the test statistics has nothing to do with nonparametric spectral estimation. We have also reported results of the time domain version of the tests, (see also Robinson (1991)) for some cases but our preference here for the frequency domain set-up of Robinson (1994c) is motivated by the somewhat greater elegance of formulae it affords, especially when the Bloomfield model is used. Though the results in both domains for white noise u_t using Nelson and Plosser's (1982) data are similar, in general, in finite samples the time and frequency domain versions of the tests will differ from each other, in some cases possibly considerably. Under the null, the difference is $O_p(T^{-1/2})$, but substantial differences could appear when the null hypothesis is seriously in error, because of the great degree of non-circularity of nonstationary processes. It is not known in general to what extent this could lead to different testing conclusions. Some attempt has been made to study the problem analytically, but it is complicated and one may need to resort to Monte Carlo simulations.

APPENDIX 3.1

Following work of Perron (1988) and other authors mentioned in Section 1, we are concerned in this appendix with the effect that a possible structural break may have had on the above results, in particular one due to World War II. Table 3.12 corresponds to Table 3.3, i.e., reporting results of the tests for white noise u_t, based only on post-war data. There are numerous non-rejections in Table 3.12 (i) for the lower d-values, and some lack of monotonicity of $\hat{r}$ in d. In Tables 3.12 (ii) and (iii) we again find significant $\hat{r}$, even for d = 2.25 in case of the GNP deflator and consumer prices, while in few of the series the null is not rejected when d = 0.5 in one or both these tables. However, the greater amount of non-rejections could be largely due to the smaller sample size, and, qualitatively, we see that as in Table 3.3 industrial production and unemployment rate are the least nonstationary series, whereas consumer prices, GNP deflator, wages and money stock are the most nonstationary ones, and in nine series in both Tables 3.3 (iii) and 3.10 (iii) (albeit not entirely the same ones) the unit root null is not rejected. Very similar results were obtained when we used the time domain version of the tests, with the non-rejections occurring at practically the same d-values as in Table 3.12.

Looking at Figure 3.1 we observe that the series might have a different growth rate after World War II. In Table 3.13 we give results of the tests for white noise u_t but including dummy variables for the changing slope in the trend function of the series in 1946. Thus, instead of (15), we consider

$$y_t = \beta_1 + \beta_{21} t + (\beta_{22} - \beta_{21}) dt + x_t, \qquad t = 1,2,..., \tag{21}$$

where $dt = t - t^*$ if $t > t^*$ and 0 otherwise, and t^* refers to the period of time corresponding to 1945. Monotonicity is now always achieved and the unit root null hypothesis is rejected in favour of more nonstationary alternatives in the same five series as in Table 3.3 (iii). In fact, all non-rejections values of d in Table 3.13 are exactly the same as in Table 3.3 (iii) except for velocity and consumer prices, where the null is rejected for d = 0.75 in Table 3.3 (iii) but not in Table 3.13. In view of these results we can conclude by saying that there is no significant improvement when including dummy variables for the changing growth at least for white noise u_t.

Allowing AR u_t with the dummies for the changing trend, results were similar to those when we included a simple linear time trend in the model, with unemployment and industrial production as the less nonstationary series, and money, consumer prices and GNP deflator as the most nonstationary ones. In Table 3.14, we resume these results choosing for each series a particular order of the autoregression, using the same criterion as in Table 3.9. Comparing results here with those in Table 3.9 (iii) we see that in nine of the fourteen series k is the same, and in five of them the non-rejections occur at exactly the same values of d.

In view of all these results, we could conclude by saying that in the provided model, the presence of a possible structural break on the data does not greatly affect the main conclusions obtained in the chapter.

FIGURE 3.1: Extended version of Nelson and Plosser's (1982) data.

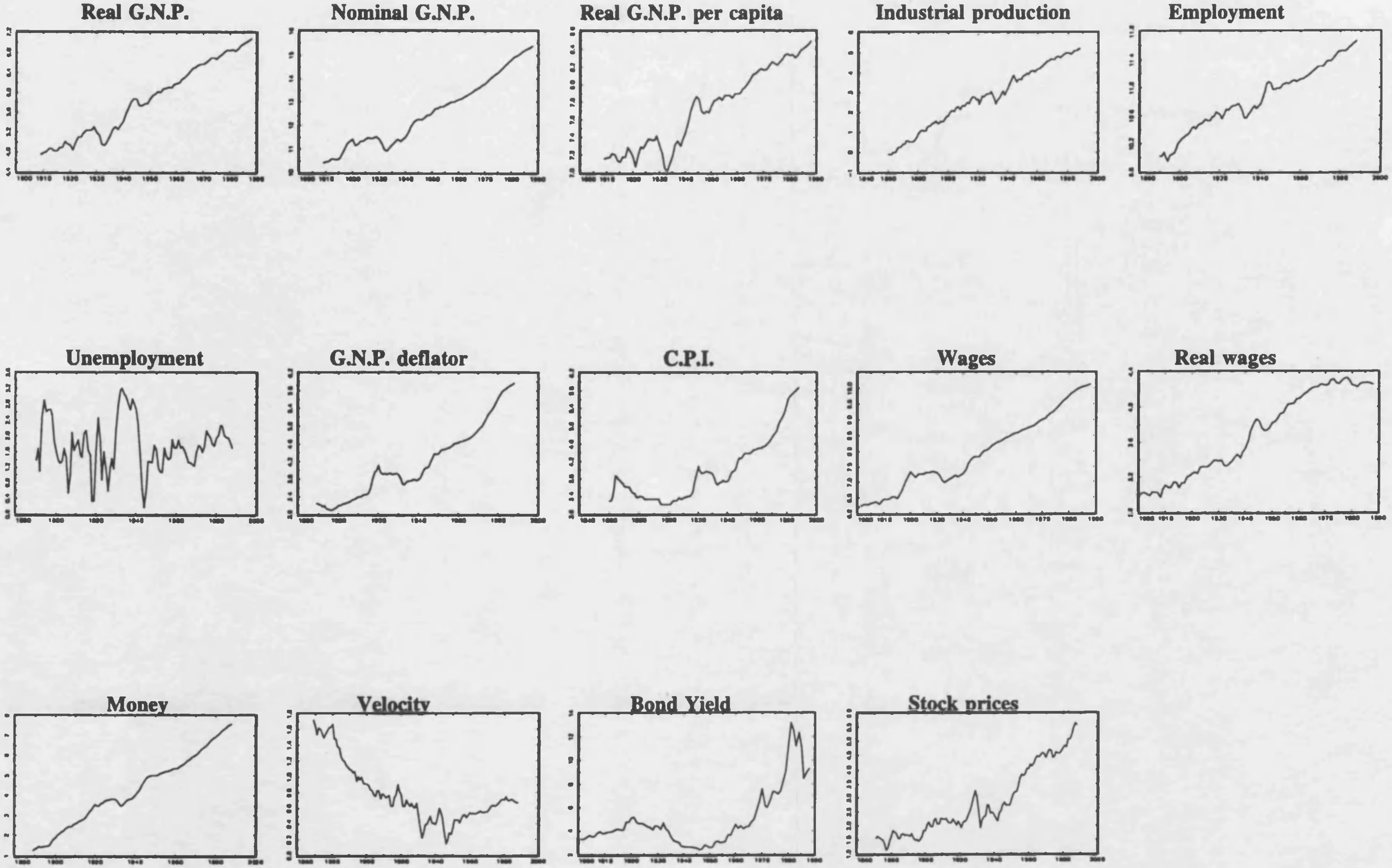

FIGURE 3.2: Autocorrelation functions of the extended version of Nelson and Plosser's (1982) data.

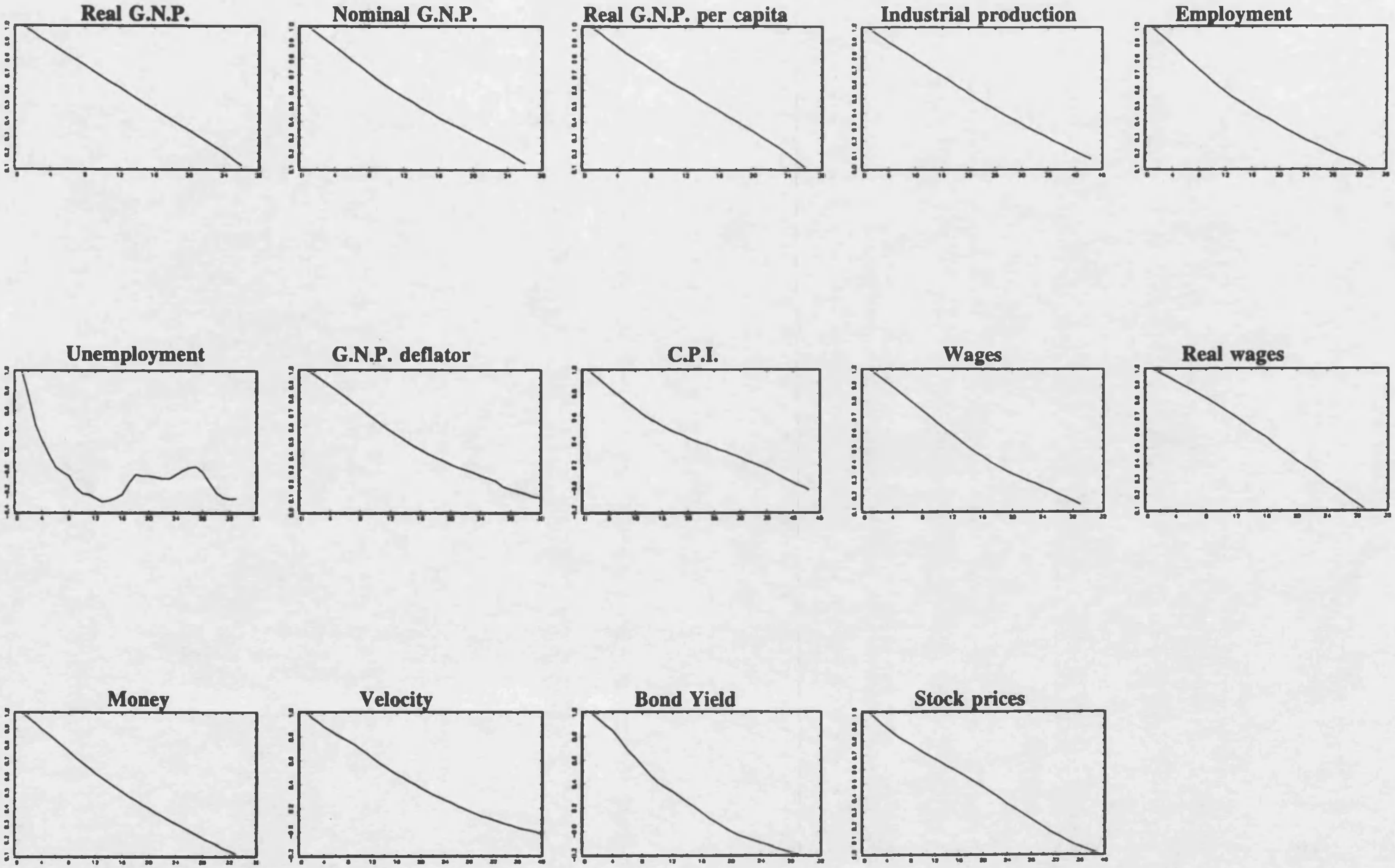

FIGURE 3.3: Various estimates of the spectral density of the extended version of Nelson and Plosser's (1982) data.

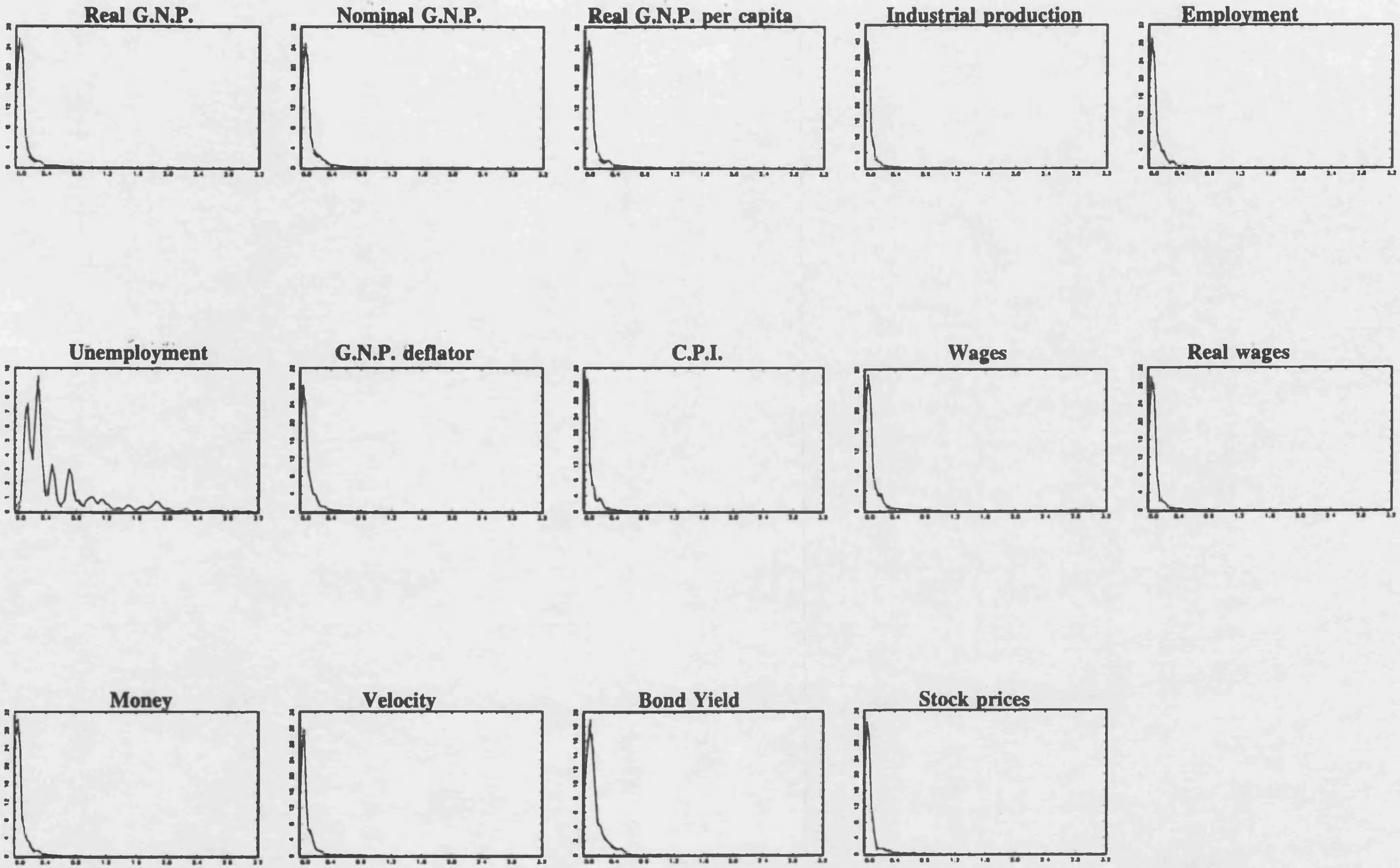

TABLE 3.1

Sample autocorrelations of the raw extended Nelson and Plosser data.

Series	n	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}	r_{11}	r_{12}	r_{13}	r_{14}
Real GNP	80	.96	.92	.89	.85	.82	.78	.75	.71	.68	.64	.61	.58	.54	.51
Nominal GNP	80	.96	.91	.87	.83	.79	.75	.70	.66	.62	.59	.55	.52	.49	.45
Real p.cap GNP	80	.96	.92	.87	.84	.80	.77	.73	.69	.66	.63	.59	.56	.53	.50
Ind.production	129	.97	.94	.92	.89	.87	.84	.82	.80	.77	.75	.72	.70	.68	.66
Employment	99	.96	.92	.88	.84	.80	.76	.73	.69	.65	.62	.58	.55	.53	.50
Unemployment	99	.75	.46	.31	.17	.04	.00	-.04	-.15	-.21	-.22	-.26	-.29	-.28	-.25
GNP deflator	100	.96	.93	.89	.85	.81	.77	.73	.69	.65	.61	.57	.54	.50	.47
Cons. prices	129	.96	.92	.88	.85	.81	.78	.74	.71	.67	.64	.61	.58	.55	.53
Wages	89	.96	.92	.89	.85	.81	.77	.74	.70	.66	.62	.58	.55	.51	.48
Real wages	89	.97	.95	.92	.89	.86	.84	.81	.78	.75	.72	.69	.66	.62	.59
Money stock	100	.96	.93	.89	.86	.82	.79	.76	.72	.69	.65	.62	.59	.56	.53
Velocity	120	.95	.91	.88	.84	.81	.79	.76	.73	.69	.66	.62	.58	.55	.51
Bond yield	89	.95	.89	.84	.77	.68	.62	.55	.47	.41	.37	.33	.29	.25	.20
C.stock prices	118	.96	.92	.88	.85	.82	.80	.77	.74	.72	.69	.67	.64	.62	.59

The natural logs of all the data are used except for the bond yield, n is the sample size and r_i is the ith order sample autocorrelation. The large sample standard error under the null hypothesis of no autocorrelation is $1/\sqrt{T}$ or roughly 0.10 for series of length considered here. Real p.cap GNP is real per capita GNP; Unemployment r. is unemployment rate; Cons. prices is consumer prices index; and C.stock prices is common stock prices.

TABLE 3.2

Sample autocorrelations of the first differences of the extended Nelson and Plosser data.

Series	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}	r_{11}	r_{12}	r_{13}	r_{14}	s(r)
Real GNP	.33	.02	-.18	-.22	-.17	.01	.07	-.05	-.21	-.20	-.00	-.03	.03	.10	.11
Nominal GNP*	.44	.10	-.08	-.20	-.04	.16	.15	.07	-.06	-.10	-.02	-.16	-.22	-.17	.11
Real p.cap GNP	.32	.01	-.17	-.20	-.16	.01	.08	-.05	-.20	-.19	-.00	-.05	.01	.08	.11
Ind.production	.03	-.12	-.01	-.09	-.25	.04	.13	-.01	-.18	-.01	.10	-.10	.10	.11	.08
Employment	.31	-.06	-.08	-.16	-.19	.00	.10	.01	-.16	-.14	-.02	-.10	-.03	.09	.10
Unemployment r.	.09	-.29	.01	-.02	-.17	.00	.14	-.11	-.10	.04	-.00	-.09	-.02	-.00	.10
GNP deflator*	.49	.28	.15	.04	.11	.09	.05	.07	.02	.03	-.01	-.13	-.16	-.20	.10
Cons. prices*	.62	.24	.11	.10	.14	.12	.08	.09	.08	.06	-.02	-.10	-.08	-.13	.08
Wages*	.47	.13	.01	-.05	-.05	.09	.16	.02	-.11	-.12	-.12	-.35	-.24	-.20	.10
Real wages	.22	-.03	-.06	-.06	-.07	-.06	.06	.10	-.03	-.11	.00	-.05	.10	.13	.10
Money stock*	.62	.31	.15	.03	-.02	-.00	-.02	-.07	-.12	-.14	-.19	-.33	-.40	-.30	.10
Velocity	.12	-.02	-.14	-.13	-.09	.11	.07	.06	-.05	-.02	.07	-.12	.15	.05	.09
Bond yield	.17	-.14	.13	.02	-.24	-.10	-.00	-.01	-.08	.06	.20	.05	-.05	.11	.10
C.stock prices	.19	-.13	-.06	-.10	-.21	-.01	.12	.05	.01	.13	.03	-.11	-.17	-.00	.09

r_i is the ith order autocorrelation coefficient.
*: Time series where the unit root hypothesis is rejected in Table 3.3 (ii) and (iii) below. s(r) is the large sample standard error under white noise, namely 1/T.

TABLE 3.3

$\hat{r}$ in (2.9) with white noise u_t

	d							
	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
(i): with no regressors.								
Real GNP	1.87'	1.94'	-0.43'	-2.20	-3.19	-3.78	-4.15	-4.41
Nominal GNP	2.12	2.08	-0.42'	-2.23	-3.21	-3.79	-4.16	-4.41
Real per capita GNP	2.67	2.18	-0.45'	-2.23	-3.21	-3.79	-4.16	-4.42
Industrial production	2.45	0.87'	-0.80'	-2.36	-3.46	-4.17	-4.67	-5.02
Employment	3.56	2.80	-0.41'	-2.43	-3.50	-4.13	-4.53	-4.81
Unemployment rate	3.62	1.26'	-0.78'	-2.07	-2.88	-3.41	-3.78	-4.05
GNP deflator	2.19	2.30	-0.47'	-2.44	-3.51	-4.13	-4.53	-4.81
Consumer prices	4.87	4.71	0.39'	-2.46	-3.88	-4.63	-5.08	-5.38
Wages	1.98	2.13	-0.44'	-2.32	-3.34	-3.94	-4.33	-4.60
Real wages	1.89'	1.93'	-0.53'	-2.32	-3.32	-3.92	-4.31	-4.58
Money stock	1.01'	1.35'	0.66'	-1.55'	-3.01	-3.83	-4.33	-4.66
Velocity	9.55	3.46	-0.73'	-2.79	-3.84	-4.45	-4.85	-5.14
Bond yield	6.14	3.46	0.19'	-1.60'	-2.61	-3.28	-3.78	-4.17
Common stock prices	3.56	2.88	0.02'	-2.09	-3.31	-4.03	-4.49	-4.81
(ii): with an intercept.								
Real GNP	7.33	2.62	1.10'	-0.20'	-1.37'	-2.30	-3.00	-3.54
Nominal GNP	7.27	3.24	2.12	0.66'	-0.77'	-1.85'	-2.64	-3.24
Real per capita GNP	7.72	3.25	1.23'	-0.28'	-1.46'	-2.37	-3.05	-3.57
Industrial production	7.93	1.11'	-0.83'	-2.39	-3.46	-4.17	-4.66	-5.01
Employment	6.10	1.87'	1.11'	-0.32'	-1.64'	-2.58	-3.25	-3.74
Unemployment rate	2.94	0.71'	-0.93'	-2.08	-2.86	-3.40	-3.78	-4.07
GNP deflator	10.46	6.66	4.48	1,46'	-0.89'	-2.39	-3.34	-3.97
Consumer prices	15.81	1.50	7.41	3.21	0.54'	-1.05'	-2.12	-2.89
Wages	7.99	3.65	2.62	1.04'	-0.63'	-1.84'	-2.69	-3.29
Real wages	9.14	3.62	1.12'	-0.93'	-2.23	-3.05	-3.60	-4.00
Money stock	7.25	2.49	2.82	2.78	0.88'	-0.89'	-2.13	-2.98
Velocity	8.61	3.75	0.33'	-1.83'	-3.04	-3.83	-4.40	-4.82
Bond yield	10.38	4.65	0.71'	-1.36'	-2.51	-3.20	-3.65	-3.97
Common stock prices	10.49	3.96	0.35'	-1.52'	-2.62	-3.36	-3.90	-4.30
(iii): with a linear time trend.								
Real GNP	5.95	3.46	1.39'	-0.18'	-1.39'	-2,31	-3.01	-3.54
Nominal GNP	10.74	6.69	3.23	0.81'	-0.78'	-1.87'	-2.65	-3.24
Real per capita GNP	5.84	3.42	1.33'	-0.26'	-1.46'	-2.37	-3.05	-3.57
Industrial production	5.33	1.42'	-1.00'	-2.51	-3.50	-4.18	-4.66	-5.01
Employment	6.93	3.84	1.37'	-0.40'	-1.67'	-2.58	-3.25	-3.74
Unemployment rate	2.95	0.71'	-0.93'	-2.08	-2.86	-3.39	-3.77	-4.05
GNP deflator	14.32	10.37	5.75	1.77'	-0.86'	-2.40	-3.35	-3.97
Consumer prices	10.72	15.13	8.42	3.43	0.56'	-1.11'	-2.12	-2.83
Wages	11.13	7.43	3.95	1.26'	-0.59'	-1.83'	-2.69	-3.30
Real wages	9.11	4.86	1.26'	-0.95'	-2.23	-3.02	-3.56	-3.98
Money stock	12.03	9.36	6.30	3.34	0.91'	-0.84'	-2.07	-2.94
Velocity	13.85	5.41	0.35'	-1.90'	-3.09	-3.85	-4.41	-4.84
Bond yield	10.37	4.52	0.71'	-1.35'	-2.51	-3.20	-3.65	-3.97
Common stock prices	9.52	3.79	0.37'	-1.48'	-2.60	-3.34	-3.87	-4.27

': Non-rejection values for the null hypothesis at 95% significance level.

TABLE 3.4

$\hat{r}$ in (17) with white noise u_t

	d							
	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
(i): with no regressors.								
Real GNP	1.43'	1.83'	-0.10'	-1.73'	-2.66	-3.16	-3.47	-3.69
Nominal GNP	2.76	2.91	-0.09'	-1.79'	-2.67	-3.16	-3.47	-3.69
Real per capita GNP	3.22	3.00	-0.11'	-1.80'	-2.68	-3.17	-3.48	-3.48
Industrial production	2.12	0.21'	-0.82'	-2.11	-3.03	-3.66	-4.09	-4.40
Employment	3.13	2.69	-0.08'	-2.02	-2.98	-3.53	-3.88	-4.11
Unemployment rate	4.01	1.36'	-0.52'	-1.70'	-2.43	-2.91	-3.23	-3.46
GNP deflator	2.75	2.83	-0.25'	-2.03	-2.98	-3.53	-3.88	-4.12
Consumer prices	5.09	4.97	0.37'	-2.13	-3.39	-4.07	-4.47	-4.73
Wages	2.90	3.03	-0.09'	-1.87'	-2.80	-3.33	-3.66	-3.89
Real wages	1.09'	1.15'	-0.04'	-1.84'	-2.77	-3.30	-3.64	-3.87
Money stock	1.01'	1.70'	0.85'	-1.19'	-2.47	-3.21	-3.67	-3.97
Velocity	9.20	3.38	-0.51'	-2.38	-3.33	-3.88	-4.24	-4.49
Bond yield	6.92	3.58	0.34'	-1.38'	-2.34	-2.91	-3.28	-3.54
Common stock prices	3.38	2.89	-0.03'	-1.66'	-2.64	-3.28	-3.72	-4.05
(ii): with an intercept.								
Real GNP	8.93	2.29	1.18'	-0.17'	-1.15'	-1.92'	-2.51	-2.96
Nominal GNP	7.63	3.26	2.77	0.60'	-0.64'	-1.54'	-2.21	-2.71
Real per capita GNP	7.43	3.20	1.15'	-0.23'	-1.22'	-1.98	-2.55	-2.98
Industrial production	7.31	1.63'	-0.86'	-2.12	-3.04	-3.67	-4.10	-4.40
Employment	5.60	2.17	1.19'	-0.28'	-1.40'	-2.20	-2.78	-3.20
Unemployment rate	2.50	0.61'	-0.78'	-1.77'	-2.44	-2.90	-3.24	-3.48
GNP deflator	10.81	6.59	5.44	1.34'	-0.78'	-2.05	-2.86	-3.40
Consumer prices	15.15	11.73	7.46	2.91	0.50'	-0.92'	-1.86'	-2.54
Wages	7.07	4.38	3.41	0.96'	-0.50'	-1.54'	-2.27	-2.78
Real wages	9.06	3.37	1.05'	-0.74'	-1.85'	-2.56	-3.04	-3.38
Money stock	7.64	2.56	2.41	2.88	0.85'	-0.70'	-1.79'	-2.53
Velocity	8.32	3.96	0.42'	-1.58'	-2.66	-3.34	-3.84	-4.20
Bond yield	10.52	4.29	0.61'	-1.15'	-2.12	-2.71	-3.09	-3.36
Common stock prices	10.70	4.18	0.44'	-1.32'	-2.28	-2.92	-3.38	-3.74
(iii): with a linear time trend.								
Real GNP	4.99	2.92	1.17'	-0.15'	-1.16'	-1.93'	-2.51	-2.96
Nominal GNP	8.88	5.70	2.76	0.69'	-0.65'	-1.56'	-2.22	-2.71
Real per capita GNP	4.89	2.90	1.14'	-0.20'	-1.22'	-1.98	-2.55	-2.98
Industrial production	4.64	1.27'	-0.87'	-2.20	-3.07	-3.67	-4.10	-4.40
Employment	5.99	3.30	1.18'	-0.34'	-1.43'	-2.21	-2.78	-3.20
Unemployment rate	2.51	0.62'	-0.78'	-1.77'	-2.44	-2.90	-3.23	-3.47
GNP deflator	12.30	9.66	5.35	1.58'	-0.76'	-2.06	-2.87	-3.40
Consumer prices	17.88	13.34	7.49	3.08	0.51'	-0.97'	-1.86'	-2.49
Wages	9.42	6.45	3.40	1.06'	-0.50'	-1.55'	-2.28	-2.79
Real wages	7.70	4.04	1.06'	-0.78'	-1.88'	-2.56	-3.02	-3.37
Money stock	10.29	8.09	5.45	2.88	0.78'	-0.73'	-1.78'	-2.53
Velocity	12.12	5.15	0.44'	-1.65'	-2.70	-3.35	-3.85	-4.22
Bond yield	8.77	3.88	0.61'	-1.14'	-2.12	-2.71	-3.09	-3.36
Common stock prices	8.15	3.53	0.43'	-1.27'	-2.27	-2.91	-3.38	-3.73

': Non-rejection values for the null hypothesis at 95% significance level.

TABLE 3.5

Sample autocorrelations of the $y_t - \beta_1 - \beta_2 t$, where β_1 and β_2 are OLS estimates from the d-th differenced (16), using the "best" choice for d in each series from Table 3.3 (iii).

Series	d_0	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}	r_{11}	r_{12}	r_{13}	r_{14}
Real GNP	1.25	.90	.75	.59	.47	.38	.33	.29	.22	.16	.13	.15	.16	.19	.21
Nominal GNP	1.50	.94	.85	.75	.65	.58	.51	.43	.33	.23	.13	.05	-.02	-.08	-.12
Real p.cap GNP	1.25	.90	.74	.59	.46	.38	.33	.28	.21	.14	.11	.11	.12	.15	.17
Ind.production	1.00	.84	.66	.53	.40	.30	.28	.25	.18	.12	.12	.12	.09	.10	.07
Employment	1.25	.91	.76	.63	.51	.42	.37	.31	.23	.15	.10	.08	.06	.06	.06
Unemployment r	0.75	.75	.47	.33	.18	.05	.01	-.03	-.14	-.20	-.21	-.25	-.28	-.27	-.24
GNP deflator	1.50	.96	.92	.88	.83	.79	.74	.69	.65	.60	.56	.52	.48	.44	.40
Cons.prices	1.50	.98	.95	.93	.89	.85	.81	.78	.74	.70	.66	.63	.59	.56	.52
Wages	1.50	.94	.86	.77	.67	.58	.49	.40	.30	.20	.11	.03	-.03	-.07	-.09
Real Wages	1.25	.96	.92	.87	.83	.80	.77	.74	.70	.66	.62	.60	.57	.54	.51
Money Stock	1.75	.97	.92	.87	.81	.76	.70	.65	.61	.56	.51	.47	.43	.40	.37
Velocity	1.00	.94	.88	.82	.78	.74	.71	.67	.63	.58	.53	.48	.43	.40	.36
Bond Yield	1.00	.94	.87	.82	.74	.65	.60	.54	.48	.44	.41	.37	.32	.27	.22
C.stock prices	1.00	.91	.79	.69	.62	.56	.54	.53	.50	.47	.42	.36	.29	.24	.22

r_i is the ith order autocorrelation coefficient.

TABLE 3.6

Sample autocorrelations of d-th differences of the $y_t - \beta_1 - \beta_2 t$ in Table 3.5

Series	d_0	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}	r_{11}	r_{12}	r_{13}	r_{14}
Real GNP	1.25	.15	-.05	-.21	-.20	-.15	.06	.12	-.01	-.18	-.18	.04	-.03	.03	.10
Nominal GNP	1.50	.08	-.11	-.17	-.30	-.07	.18	.11	.07	-.09	-.09	.11	-.08	-.13	-.01
Real p.cap GNP	1.25	.14	-.06	-.20	-.19	-.15	.06	.12	-.01	-.18	-.16	.06	-.05	.01	.09
Ind.production	1.00	.03	-.12	-.01	-.09	-.25	.04	.13	-.01	-.18	-.01	.10	-.10	.10	.11
Employment	1.25	.14	-.17	-.09	-.14	-.19	.03	.14	.03	-.15	-.11	.03	-.09	-.02	.13
Unemployment r	0.75	.24	-.16	.00	-.04	-.16	-.02	.08	-.12	-.12	-.00	-.04	-.12	-.06	-.05
GNP deflator	1.50	.00	-.05	-.05	-.19	.03	.00	-.03	.04	-.02	.06	.04	-.10	-.05	-.04
Cons.prices	1.50	.25	-.21	-.18	-.09	.03	.01	-.03	.01	.03	.06	-.03	-.12	-.00	-.09
Wages	1.50	.10	-.16	-.11	-.13	-.14	.08	.20	.02	-.11	-.02	.08	-.31	-.04	-.00
Real Wages	1.25	.03	-.13	-.10	-.06	-.07	-.08	.06	.10	-.03	-.12	.02	-.08	.09	.12
Money Stock	1.75	.07	-.17	-.06	-.10	-.11	.03	.02	-.00	-.03	.02	.09	-.14	-.23	-.04
Velocity	1.00	.12	-.02	-.14	-.13	-.09	.11	.07	.06	-.05	-.02	.07	-.12	.15	.05
Bond Yield	1.00	.17	-.14	.13	.02	-.24	-.10	-.00	-.01	-.08	.06	.20	.05	-.05	.11
C.stock prices	1.00	.19	-.13	-.06	-.10	-.21	-.01	.12	.05	.01	.13	.03	-.11	-.17	-.00

r_i is the ith order autocorrelation coefficient.

TABLE 3.7

$\hat{r}$ in (2.9) and $\dot{r}$ in (17) for logged bond yield with white noise u_t

	d							
	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
FREQUENCY DOMAIN ($\hat{r}$)								
with no regressors:	6.38	3.17	-0.11'	-2.15	-3.18	-3.80	-4.24	-4.56
with an intercept:	10.59	4.96	1.24'	-1.55'	-2.87	-3.57	-4.01	-4.32
with a time trend:	13.59	6.62	1.23'	-1.54'	-2.87	-3.57	-4.00	-4.31
TIME DOMAIN ($\dot{r}$)								
with no regressors:	6.43	3.92	0.18'	-1.79'	-2.78	-3.31	-3.64	-3.86
with an intercept:	12.61	6.06	1.06'	-1.31'	-2.42	-3.02	-3.39	-3.65
with a time trend:	11.41	5.65	1.06'	-1.30'	-2.42	-3.02	-3.39	-3.65

': Non-rejection values for the null hypothesis at 95% significance level.

TABLE 3.8

Joint test of (18) against (19) in model (15) with white noise u_t.

	d							
	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
Real GNP	123.47	56.27	17.86	2.87'	2.35'	5.43'	9.12	12.59
Nominal GNP	124.15	68.39	31.86	6.47	1.64'	3.74'	7.20	10.68
Real p. cap. GNP	114.77	37.92	7.70	0.92'	2.23'	5.63'	9.34	12.77
Ind. production	180.50	82.66	21.30	8.18	12.28	17.47	21.78	25.18
Employment	123.29	62.94	19.65	3.02'	3.09'	6.73	10.59	14.01
Unemployment rate	8.68	0.52'	0.88'	4.35'	8.24	11.66	14.46	16.77
GNP deflator	89.28	102.58	43.13	5.59'	0.86'	5.77'	11.27	15.84
Consumer prices	319.17	175.01	69.35	13.46	1.24'	1.39'	4.51'	8.62
Wages	143.97	78.63	37.49	7.46	1.47'	3.80'	7.54	11.10
Real wages	158.31	61.83	14.09	2.64'	5.45'	9.75	13.58	16.73
Money stock	147.86	92.02	63.05	24.79	4.55'	2.46'	6.03	10.42
Velocity	119.75	29.69	2.40'	3.85'	9.65	15.20	19.88	23.68
Bond yield	127.76	28.46	1.65'	1.98'	6.33	10.26	13.35	15.82
C.stock prices	181.33	45.69	5.58'	2.89'	6.94	11.36	15.29	18.67

': Non-rejection values for the null hypothesis at 95% significance level.

TABLE 3.9

$\hat{r}$ in (2.9) with AR(k) u_t for a particular choice of k.

	k	d 0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
(i): with no regressors.									
Real GNP:	1	-2.29	-0.47'	-0.87'	-1.77'	-2.52	-3.06	-3.46	-3.75
Nominal GNP	1	-4.19	-1.73'	-1.25'	-1.66'	-2.20	-2.66	-3.03	-3.32
Real p.cap.GNP	1	-3.01	-0.43'	-0.86'	-1.78'	-2.53	-3.07	-3.46	-3.76
Indust. prod.	2	-3.71	-1.20'	-1.99	-2.23	-2.60	-2.98	-3.33	-3.69
Employment:	1	-2.57	0.02'	-0.76'	-1.89'	-2.73	-3.31	-3.72	-4.04
Unemployment:	2	0.32'	-0.21'	-0.51'	-1.01'	-1.56'	-2.10	-2.58	-3.01
GNP deflator:	3	0.16'	-0.84'	-1.27'	-1.86'	-2.36	-2.72	-3.01	-3.27
Consumer prices:	2	0.33'	-0.75'	-0.65'	-1.14'	-1.75'	-2.31	-2.82	-3.28
Wages:	1	-1.27'	-0.23'	-0.91'	-1.90'	-2.66	-3.19	-3.58	-3.88
Real wages:	1	-3.78	-1.07'	-1.21'	-2.01	-2.70	-3.21	-3.59	-3.88
Money stock:	3	12.63	-0.04'	-1.85'	-2.47	-2.87	-3.10	-3.23	-3.33
Velocity:	1	-0.10'	0.79'	-0.99'	-2.39	-3.26	-3.83	-4.23	-4.54
Bond yield:	1	1.07'	0.29'	-1.37'	-2.17	-2.66	-3.07	-3.45	-3.80
C. stock prices:	1	4.29	-0.02'	-1.97	-3.02	-3.66	-4.05	-4.31	-4.50
(ii): with an intercept.									
Real GNP:	3	-7.65	-0.98'	-1.94'	-2.46	-2.75	-2.99	-3.17	-3.28
Nominal GNP:	4	-4.38	-6.71	0.36'	-0.19'	-1.05'	-1.98	-2.79	-3.36
Real p.ca	3	-12.15	-1.04'	-1.94'	-2.41	-2.72	-2.96	-3.14	-3.25
Industrial prod.:	4	-4.43	-1.71'	-2.67	-2.80	-2.99	-3.10	-3.14	-3.17
Employment:	2	-7.64	-2.04	-1.69'	-1.92'	-2.28	-2.61	-2.88	-3.11
Unemployment:	2	-0.77'	-0.88'	-1.16'	-1.59'	-2.05	-2.50	-2.90	-3.25
GNP deflator:	4	-5.70	-7.34	-0.82'	-0.40'	-0.77'	-1.42'	-2.12	-2.70
Consumer prices:	5	8.74	-0.18'	-1.42'	-1.65'	-1.75'	-2.23	-2.72	-2.94
Wages:	2	-3.10	-3.80	-1.00'	-1.66'	-2.14	-2.53	-2.83	-3.05
Real wages:	4	-0.56'	-2.65	-0.47'	-1.75'	-2.49	-2.89	-3.06	-3.13
Money stock:	1	-4.38	-3.47	-1.23'	-1.99	-2.26	-2.65	-3.04	-3.38
Velocity:	4	1.63'	2.06	0.15'	-1.70'	-2.87	-3.52	-3.90	-4.14
Bond yield:	1	-1.50'	0.24'	-1.31'	-2.41	-3.16	-3.68	-4.03	-4.31
C. stock prices:	4	-5.53	-0.12'	-1.65'	-2.84	-3.46	-3.76	-3.84	-3.78
(iii): with a linear time trend.									
Real GNP	3	-0.92'	-1.41'	-1.97	-2.43	-2.76	-3.00	-3.17	-3.27
Nominal GNP:	2	0.03'	-0.80'	-1.46'	-2.18	-2.69	-2.95	-3.05	-3.06
Real per capita	3	-0.96'	-1.43'	-1.94'	-2.38	-2.72	-2.97	-3.13	-3.24
Indust. product.	4	-0.66'	-1.70'	-2.66	-3.33	-3.73	-3.93	-4.06	-4.14
Employment:	4	-0.99'	-1.12'	-1.65'	-2.25	-2.78	-3.22	-3.55	-3.77
Unemployment:	2	-0.77'	-0.87'	-1.16'	-1.58'	-2.04	-2.48	-2.90	-3.28
GNP deflator:	1	1.11'	-1.02'	-1.21'	-1.07'	-1.75'	-2.35	-2.86	-3.30
Consumer prices	5	10.44	-0.96'	-1.33'	-1.37'	-1.70'	-2.19	-2.71	-2.92
Wages:	1	1.41'	-0.14'	-1.62'	-2.06	-2.49	-2.86	-3.17	-3.44
Real wages:	1	-0.11'	-0.22'	-1.26'	-2.16	-2.73	-3.06	-3.31	-3.57
Money stock:	1	3.47	1.32'	-1.07'	-2.06	-2.22	-2.53	-2.91	-3.29
Velocity:	3	1.96	1.59'	-0.73'	-2.54	-3.38	-3.59	-3.68	-3.76
Bond yield:	1	0.06'	0.05'	-1.31'	-2.39	-3.16	-3.67	-4.02	-4.29
C. stock prices:	2	1.37'	-0.21'	-1.41'	-2.17	-2.61	-2.85	-3.00	-3.17

': Non-rejection values for the null hypothesis at 95% significance level.

TABLE 3.10

$\hat{r}$ in (2.9) with a time trend and Bloomfield exponential u_t and the same value of k as in Table 3.9 (iii).

	k	d 0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
Real GNP:	3	3.20	-0.14'	-2.33	-3.72	-4.70	-5.52	-6.34	-7.24
Nominal GNP:	2	8.88	2.65	-0.38'	-2.06	-3.14	-3.92	-4.56	-5.16
Real per c. GNP	3	3.07	-0.16'	-2.27	-3.65	-4.66	-5.51	-6.36	-7.30
Ind. production:	4	1.14'	-2.18	-4.01	-5.24	-6.17	-6.95	-7.86	-9.12
Employment:	4	2.79	-0.38'	-2.68	-4.28	-5.56	-6.73	-8.00	-9.36
Unemployment:	2	0.56'	-0.70'	-1.68'	-2.71	-3.87	-5.09	-6.40	-7.67
GNP deflator:	1	10.79	4.05	0.86'	-0.58'	-1.72'	-2.70	-3.61	-4.45
Consumer prices	5	57.09	18.96	5.13	0.03'	-2.08	-3.56	-5.53	-7.24
Wages:	1	8.03	2.94	0.32'	-1.14'	-2.17	-2.99	-3.71	-4.35
Real wages:	1	3.73	1.02'	-0.77'	-2.09	-3.06	-3.76	-4.36	-4.97
Money stock:	1	11.77	5.58	1.80'	-0.20'	-1.41'	-2.32	-3.12	-3.89
Velocity:	3	11.15	3.01	-0.95'	-3.92	-5.88	-6.80	-7.62	-8.76
Bond yield:	1	4.95	1.18'	-0.91'	-2.41	-3.58	-4.47	-5.17	-5.77
C. stock prices:	2	5.85	0.87'	-1.57'	-3.08	-4.17	-4.95	-5.50	-6.03

':Non-rejection values for the null hypothesis at 95% significance level.

TABLE 3.11

$\hat{r}$ in (2.9) with a time trend and Bloomfield exponential u_t for a particular choice of k

	k	d 0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
Real GNP:	1	2.01	-0.14'	-1.33'	-2.03	-2.55	-3.05	-3.57	-4.12
Nominal GNP:	3	11.71	4.04	0.01'	-2.46	-4.18	-5.42	-6.39	-7.24
Real per c. GNP	4	3.94	0.10'	-2.38	-4.12	-5.51	-6.73	-7.97	-9.30
Ind. production:	5	3.52	0.40'	-1.64'	-3.61	-5.36	-6.18	-6.66	-7.67
Employment:	3	3.50	0.15'	-1.91'	-3.25	-4.29	-5.25	-6.27	-7.37
Unemployment:	1	-0.13'	-1.47'	-2.56	-3.52	-4.35	-5.06	-5.65	-6.15
GNP deflator:	4	24.72	9.83	2.62	-0.23'	-1.68'	-2.50	-3.61	-5.41
Consumer prices	5	57.09	18.96	5.13	0.03'	-2.08	-3.56	-5.53	-7.24
Wages:	1	8.03	2.94	0.32'	-1.14'	-2.17	-2.99	-3.71	-4.35
Real wages:	4	7.75	2.41	-0.56'	-2.54	-4.26	-5.30	-5.55	-5.97
Money stock:	1	11.77	5.58	1.80'	-0.20'	-1.41'	-2.32	-3.12	-3.89
Velocity:	4	13.30	3.73	0.29'	-3.32	-6.71	-8.17	-9.10	-10.60
Bond yield:	1	4.95	1.18'	-0.91'	-2.41	-3.58	-4.47	-5.17	-5.77
C. stock prices:	1	4.21	0.29'	-1.70'	-2.94	-3.83	-4.52	-5.08	-5.61

':Non-rejection values for the null hypothesis at 95% significance level.

TABLE 3.12

$\hat{r}$ in (2.9) with white noise u_t for the extended Nelson and Plosser data starting in 1946.

	d							
	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
(i): with no regressors.								
Real GNP	1.20'	1.00'	-0.59'	-1.86'	-2.61	-3.06	-3.36	-3.56
Nominal GNP	1.20'	1.02'	-0.56'	-1.83'	-2.59	-3.05	-3.34	-3.55
Real per capita GNP	1.43'	1.07'	-0.60'	-1.87'	-2.61	-3.06	-3.36	-3.56
Industrial production	0.91'	0.99'	-0.45'	-1.74'	-2.51	-2.99	-3.30	-3.51
Employment	1.52'	1.10'	-0.58'	-1.86'	-2.60	-3.05	-3.35	-3.55
Unemployment rate	0.40'	-0.33'	-1.47'	-2.24	-2.71	-3.03	-3.25	-3.42
GNP deflator	1.07'	1.01'	-0.49'	-1.78'	-2.55	-3.01	-3.32	-3.53
Consumer prices	1.23'	1.27'	-0.52'	-1.99	-2.82	-3.32	-3.65	-3.87
Wages	1.15'	1.00'	-0.56'	-1.83'	-2.59	-3.04	-3.34	-3.55
Real wages	1.28'	0.97'	-0.68'	-1.93'	-2.65	-3.09	-3.37	-3.57
Money stock	0.85'	0.80'	-0.63'	-1.86'	-2.60	-3.05	-3.34	-3.54
Velocity	0.27'	0.89'	0.01'	-1.60'	-2.60	-3.12	-3.43	-3.64
Bond yield	1.38'	0.66'	-0.32'	-1.21'	-1.89'	-2.40	-2.80	-3.11
Common stock prices	0.52'	0.61'	-0.89'	-2.24	-2.96	-3.33	-3.54	-3.69
(ii): with an intercept.								
Real GNP	3.33	0.54'	-0.29'	-1.13'	-2.07	-2.71	-3.12	-3.40
Nominal GNP	4.04	1.22'	0.20'	-0.68'	-1.81'	-2.59	-3.03	-3.30
Real per capita GNP	3.43	0.60'	-0.63'	-1.60'	-2.32	-2.82	-3.17	-3.42
Industrial production	2.77	-0.17'	-1.21'	-2.01	-2.57	-2.92	-3.16	-3.33
Employment	4.21	1.34'	-0.20'	-1.44'	-2.14	-2.56	-2.85	-3.09
Unemployment rate	0.99'	-0.60'	-1.49'	-2.09	-2.53	-2.84	-3.09	-3.28
GNP deflator	4.90	2.49	1.88'	1.37'	0.34'	-0.58'	-1.20'	-1.61'
Consumer prices	5.58	2.92	2.42	1.67'	0.33'	-0.70'	-1.36'	-1.78'
Wages	3.86	1.16'	0.53'	0.43'	-0.44'	-1.57'	-2.30	-2.74
Real wages	4.55	2.91	1.94'	0.01'	-1.35'	-2.10	-2.54	-2.84
Money stock	5.33	2.79	1.71'	0.68'	-0.72'	-1.70'	-2.20	-2.50
Velocity	1.68'	0.40'	-0.29'	-1.62'	-2.51	-3.00	-3.28	-3.46
Bond yield	4.92	1.83'	0.08'	-1.07'	-1.86'	-2.37	-2.72	-2.98
Common stock prices	2.60	0.75'	-0.32'	-1.56'	-2.30	-2.69	-2.92	-3.10
(iii): with a linear time trend.								
Real GNP	3.13	0.80'	-0.77'	-1.76'	-2.38	-2.78	-3.09	-3.35
Nominal GNP	9.15	5.56	1.55'	-1.08'	-2.36	-2.96	-3.30	-3.51
Real per capita GNP	1,63'	0.14'	-0.99'	-1.82'	-2.40	-2.79	-3.10	-3.36
Industrial production	1.58'	-0.48'	-1.68'	-2.35	-2.75	-3.00	-3.19	-3.36
Employment	4.17	1.41'	-0.50'	-1.59'	-2.20	-2.58	-2.85	-3.08
Unemployment rate	0.32'	-0.69'	-1.49'	-2.09	-2.53	-2.84	-3.09	-3.28
GNP deflator	11.26	9.51	6.54	3.39	1.27'	0.24'	-0.52'	-1.28'
Consumer prices	11.06	8.69	5.57	2.58	0.50'	-0.68'	-1.37'	-1.80'
Wages	9.08	6.66	3.75	1.08'	-0.79'	-1.91'	-2.64	-3.13
Real wages	9.24	5.91	2.19	-0.25'	-1.44'	-2.03	-2.44	-2.78
Money stock	11.06	10.43	8.28	4.20	0.54'	-1.16'	-1.83'	-2.23
Velocity	4.05	1.99	-0.28'	-1.88'	-2.75	-3.16	-3.38	-3.51
Bond yield	2.76	1.45'	0.06'	-1.08'	-1.87'	-2.38	-2.73	-2.98
Common stock prices	4.75	1.68'	-0.53'	-1.70'	-2.31	-2.70	-2.99	-3.19

': Non-rejection values for the null hypothesis at 95% significance level. All the series are transformed to natural logarithms except bond yield.

TABLE 3.13

r̂ in (2.9)) including dummy variables for the changing slope in 1945, and white noise u_t.

	d							
	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
Real GNP	5.57	3.33	1.38'	-0.13'	-1.21'	-2.06	-2.83	-3.46
Nominal GNP	7.61	5.23	2.80	0.76'	-0.71'	-1.74'	-2.57	-3.22
Real per capita GNP	5.58	3.45	1.34'	-0.21'	-1.30'	-2.14	-2.89	-3.50
Industrial production	5.07	1.32'	-1.03'	-2.45	-3.29	-3.96	-4.57	-5.01
Employment	6.94	3.83	1.37'	-0.36'	-1.64'	-2.55	-3.22	-3.71
Unemployment rate	2.90	0.69'	-0.95'	-2.06	-2.77	-3.27	-3.70	-4.03
GNP deflator	10.62	7.87	4.60	1.50'	-0.88'	-2.41	-3.35	-3.98
Consumer prices	14.05	10.10	6.25	2.93	0.50'	-1.08'	-2.08	-2.82
Wages	8.54	6.11	3.50	1.19'	-0.50'	-1.67'	-2.58	-3.27
Real wages	8.75	4.67	1.16'	-0.98'	-2.20	-2.95	-3.52	-3.97
Money stock	11.50	9.10	6.32	3.52	0.88'	-1.09'	-2.23	-2.99
Velocity	4.83	1.54'	-0.62'	-1.96	-2.88	-3.80	-4.52	-4.98
Bond yield	5.39	2.32	0.07'	-1.50'	-2.54	-3.21	-3.65	-3.98
Common stock prices	4.35	1.70'	-0.26'	-1.61'	-2.52	-3.25	-3.84	-4.27

': Non-rejection values for the null hypothesis at 95% significance level. All the series are transformed to natural logarithms except bond yield.

TABLE 3.14

r̂ in (2.9) with dummy variables for the changing growth and AR(k) u_t for a particular choice of k.

		d							
	k	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
Real GNP:	3	1.18'	-1.58'	-2.03	-2.39	-2.51	-2.54	-2.73	-2.98
Nominal GNP:	2	-0.04'	-1.88'	-1.98	-2.36	-2.69	-2.83	-2.91	-2.95
Real per	1	0.12'	-2.36	-2.56	-2.48	-2.37	-2.32	-2.48	-2.75
Indust. product.	4	-0.76'	-1.75'	-2.69	-3.14	-2.82	-2.69	-3.08	-3.48
Employment:	1	0.31'	-1.84'	-2.43	-2.71	-2.79	-2.86	-3.17	-3.58
Unemployment:	2	-0.78'	-0.89'	-1.18'	-1.54'	-1.75'	-1.99	-2.46	-2.98
GNP deflator:	1	0.44'	-1.92'	-1.79'	-1.26'	-1.69'	-2.33	-2.86	-3.30
Consumer prices	5	10.12	-0.06'	-0.15'	-1.73'	-1.82'	-2.40	-2.90	-3.16
Wages:	1	1.53'	-0.68'	-2.06	-2.20	-2.44	-2.64	-2.90	-3.22
Real wages:	2	-0.94'	-0.06'	-0.95'	-1.91'	-2.43	-2.61	-2.77	-3.04
Money stock:	1	3.42	1.34'	-1.07'	-1.78'	-1.91'	-2.52	-2.97	-3.32
Velocity:	4	-0.15'	0.56'	-1.44'	-1.68'	-2.27	-2.73	-2.82	-3.60
Bond yield:	1	-0.10'	-1.76'	-1.87'	-2.60	-3.20	-3.68	-4.02	-4.29
C. stock prices:	4	-1.24'	-1.84'	-2.58	-3.06	-3.01	-2.98	-3.11	-3.13

': Non-rejection values for the null hypothesis at 95% significance level.

CHAPTER 4

SEASONAL FRACTIONAL INTEGRATION IN MACROECONOMIC TIME SERIES

In this chapter the tests of Robinson (1994c) are applied to quarterly U.K. and Japanese consumption and income series that were analyzed in Hylleberg, Engle, Granger and Yoo (HEGY, 1990) and Hylleberg, Engle, Granger and Lee (HEGL, 1993) respectively. We show that seasonal fractional integration, even with different amplitudes at different frequencies might be an alternative plausible way of modelling these series.

4.1 INTRODUCTION AND SUMMARY

Many economic time series contain important seasonal components and it is a common belief that modellers need to pay specific attention to the nature of seasonality rather than essentially to ignore it. The concept of seasonality is seldom defined rigorously. It seems clear that any definition of seasonality must include something like a 'systematic intra-year movement', though the relevant question should be how systematic such movement is. In order to resolve that question we need to consider the causes of what we call a seasonal movement as done by Thomas and Wallis (1971), Granger (1978) and Hylleberg (1986). Following Hylleberg (1992) these causes can be grouped into three classes: a) weather (i.e. temperature, precipitation, etc.); b) calendar events (i.e. the timing of religious or secular festivals, etc.); and c) timing decisions (i.e. school vacation, industry vacation, etc.). Some of these causes may be unchanging over long periods (Christmas), while others may change at discrete intervals (vacations), and still others are continuously varying but predictable (Easter), while other varying causes are unpredictable (the weather). The following definition of seasonality in Economics is found in Hylleberg (1992): "Seasonality is the systematic, although not necessarily regular, intra-year movement caused by the changes of the weather, the calendar, and timing decisions, directly or indirectly through the production and consumption decisions made by the agents of the economy. The decisions are influenced by endowments, the expectations and preferences of the agents, and the production techniques available in the economy". Seasonality and its appropriate modelling have been the focus of interest in recent years, however, there is little consensus on how seasonality should be treated in empirical applications on aggregate data. Since the statistical properties of different seasonal models are distinct, the imposition of one kind when another is present can result in serious bias or loss of information, and it is therefore useful to establish what kind of seasonality is present in the data.

Traditionally, seasonal fluctuations have been considered a nuisance that obscure the more important components of the series (presumably the growth and cyclical components, eg. Burns and Mitchell, (1946)), and seasonal adjustment procedures have been implemented to eliminate seasonality. Of the large number of seasonal adjustment procedures, the most widely used was the Census X-11 method, described in Shiskin et al. (1967). This method uses a set of moving averages to produce seasonally adjusted data. The X-11 program and other methods that have been empirically developed later (such as the ARIMA X-11) tend to produce what their developers feel are desirable seasonal adjustments, but their statistical properties are difficult to assess from a theoretical view-point. These methods dominated applied time series econometrics until quite recently, and a survey and a discussion of some of the major issues involved in the seasonal adjustment of time series data can be found in Bell and Hillmer (1984). In the past few years, a new viewpoint has emerged, in which seasonal fluctuations are not taken as nuisance but as integral part of the economic data. Contributors to this view include Ghysels (1988), Barsky and Miron (1989), Braun and Evans (1995), Chatterjee and Ravikumar (1992) and Hansen and Sargent (1993) among others. The first two articles point out that seasonal adjustment might lead to mistaken inferences about economic relationships between time series data. In fact, seasonal fluctuations have been found to be economically significant and an important source of variation in economic time series. Thus, seasonal adjustment might cause a significant loss of valuable information about the time series behaviour of economic variables.

Three classes of time series models commonly used to model seasonality can be called:

a) Purely deterministic seasonal processes

b) Stationary stochastic seasonal ARMA processes

c) Integrated (and fractionally integrated) seasonal processes.

A purely deterministic seasonal process is a process generated by seasonal dummies such as:

$$y_t = m_0 + \sum_{i=1}^{s-1} m_i \, S_{it} \tag{1}$$

where s is the number of time periods in a year and the m_i are the coefficients corresponding to the seasonal dummies S_{it}. The seasonal dummy definition simply allows for the mean of the series to vary by season, so the presence of seasonal dummy seasonality raises no interesting statistical issues per se. The reason for using models like (1) is that the factor that might produce the seasonal variation can be readily identifiable (eg. school calendars, the timing of tax collection, etc.). This means that there may be situations in which we have identifying restrictions available for the seasonal variation. For example, a December boom in output can reasonably be attributed to a demand shift (Christmas) as opposed to an improvement in the technology. Therefore, identifying restrictions provided by considering the sources of seasonal dummy variation can be exploited in evaluating competing economic hypotheses.

A stationary stochastic seasonal ARMA process can be expressed as

$$\Phi_P(L^s)\, y_t = \Theta_Q(L^s)\, \epsilon_t \qquad \epsilon_t \sim iid \tag{2}$$

where $\Phi_P(L^s)$ and $\Theta_Q(L^s)$ are polynomials in L^s (the seasonal lag operator, $L^s x_t = x_{t-s}$) of orders P and Q respectively, with the roots of $\Phi_P(L^s)$ outside the unit circle and the roots of $\Theta_Q(L^s)$ outside or on the unit circle. If the roots of $\Theta_Q(L^s)$ are strictly outside the unit circle, the process is invertible, and (2) can be written as an infinite autoregression of form

$$\rho(L^s)\, y_t = \epsilon_t \qquad \epsilon_t \sim iid \tag{3}$$

with all the roots of $\rho(L^s)=0$ lying outside the unit circle and where some of them are complex pairs with seasonal periodicities. More precisely, the spectrum of a process like this will be given by

$$f(\lambda) = \frac{\sigma^2}{2\pi\,|\rho(e^{i\lambda s})|^2}, \tag{4}$$

where $\sigma^2 = \mathrm{Var}(\varepsilon_t)$ and $f(\lambda)$ will have peaks at some of the seasonal frequencies λ_s. An example of this type of process would be

$$y_t = \rho\, y_{t-s} + \epsilon_t \tag{5}$$

with $|\rho| < 1$, and s=4, for example, for quarterly data; in this case, the spectrum would have a peak at the seasonal frequencies $\pi/2$ (and $3\pi/2$) and π as well as at zero frequency. A crucial fact about series displaying stationary stochastic seasonality of this form is that they are not qualitatively different from series displaying no-seasonal stationary stochastic ARMA behaviour. Consider, for example, the case of s=1 in (5). The spectrum of this process differs from that with s=4 in that most of its power is located now at the so-called business cycle frequencies. For both processes (s=1 and s=4) however, the spectrum has power at all frequencies, including both the seasonal frequencies and the business cycle frequencies. The relative amount of power at the two sets of frequencies differs, but there is no logical way to say how much of the power at particular frequencies is due to particular lags in the AR representation.

While it is common practice to model a seasonal component as having a deterministic or stationary path of forms a) and b), there may be cases where it is appropriate to allow the model of the seasonal component to drift substantially over time. This possibility is implicit in the practice of seasonal differencing (see eg. Box and Jenkins (1970)) whereby a process observed s times per year would be transformed to its s-period difference, on the assumption that the process contains an integrated seasonal component. If the lag polynomial in (3) is given by $(1-L^s)$ corresponding to a seasonal unit root, then it can be factorized as $(1-L^s) = (1-L)(1+L+L^2+...+L^{s-1}) = (1-L)S(L)$. That is, the seasonal difference operator can be broken down into the product of the first difference operator and the moving-average seasonal filter S(L) containing further roots of modulus unity. Engle et al. (1989) define a variable y_t to be seasonally integrated of orders d and D (denoted SI(d,D)), if $(1-L)^d S(L)^D y_t$ is stationary. Thus for quarterly data, in the terminology established above, if $(1-L^4)^d y_t$ is stationary, then $y_t = SI(d,d)$ with $S(L) = (1+L+L^2+L^3)$. Further, noting that

$$(1-L^4) = (1-L)(1+L+L^2+L^3) = (1-L)(1+L)(1+L^2), \qquad (6)$$

the quarterly seasonal process above has four roots of modulus unity: one at zero frequency, one at the two cycles per year corresponding to frequency π, and two complex pairs at one cycle per year corresponding to frequencies $\pi/2$ and $3\pi/2$ (of a cycle 2π), all of them with the same integration order d; however, in view of the decomposition in (6) we see that a seasonal process might also present different integration orders for each of these frequencies. Then, if the innovations of such a process are an I(0) series[1], the process will be stationary if all the integration orders are in the interval (-1/2, 1/2), and we say that y_t has seasonal long memory at a given frequency if the integration order at that frequency is greater than zero. A seasonal series might also display only a single root at a particular frequency. For example, an integrated process with a single root at two cycles per year is

$$(1+L)^d y_t = \epsilon_t, \tag{7}$$

and at one cycle per year it is

$$(1+L^2)^d y_t = \epsilon_t. \tag{8}$$

Thus, if ε_t is an I(0) series and if $0 < d < 1/2$, y_t will be in both cases covariance stationary with the spectral density unbounded at frequency π in (7), and at frequency $\pi/2$ in (8).

Combining these last two approaches (b and c) in the classical Box-Jenkins (1970) framework, the modelling of time series with seasonal components takes place by applying the seasonally fractional differences in addition to the first fractional differences in the stationary producing phase of the process. The final model can be written as

$$\phi(L)\ \Phi(L^s)\ (1-L)^d\ (1-L^s)^D y_t = \theta(L)\ \Theta(L^s)\ \epsilon_t \tag{9}$$

where the seasonal and non-seasonal autoregressive (AR) operators, $\Phi(L^s)=(1-\Phi_1L^s-\ldots-\Phi_PL^{sP})$ and $\phi(L)=(1-\phi_1L-\ldots-\phi_pL^p)$ have zeros outside the unit circle; the seasonal and non-seasonal moving average (MA) operators, $\Theta(L^s)=(1+\Theta_1L^s+\ldots+\Theta_QL^{sQ})$ and $\theta(L)=(1+\theta_1+\ldots+\theta_qL^q)$ have zeros outside or on the unit circle, and the ε_t's are independently distributed with zero mean and variance σ^2. Then we say that y_t follows an ARIMA (p,d,q) x (P,D,Q) model. ARIMA models of the form of (9) have been widely used in the literature to model seasonality and they have been found to be flexible enough to describe the behaviour of many actual nonstationary and seasonal time series.

Finally, time varying coefficient models of the forms a), b) and c) have also arisen in the seasonal literature and they can be called periodically (deterministic, autoregressive or integrated) seasonal processes. Possible economic motivations for

[1] We define now ahead an I(0) process u_t, $t=0,\pm1,\ldots$, as a covariance stationary process with spectral density bounded and bounded away from zero at any frequency.

time-varying parameter models are that economic agents may have seasonally varying utility functions (Osborn (1988)), seasonally varying expectations (Franses (1992)), and/or periodic adjustment costs. A good survey of time varying models of seasonality can be found in Hylleberg (1986, Chapter 6).

We attempt now to describe a brief and partial summary of the main findings on modelling seasonality in economic time series. Starting with deterministic patterns, Barsky and Miron (1989) considered a model that included both deterministic and stochastic seasonal components and investigated the seasonal fluctuations in a wide selection of post-war quarterly U.S. macroeconomic variables. Their empirical results suggested that deterministic seasonals played a very important role in explaining the variation of the data, while models of stationary indeterministic seasonalities played a secondary role. Deterministic seasonal models have also been proposed in Clare et al. (1995) for describing the monthly U.K. returns on the FT-A share index, and in McDougall (1995) for some New Zealand macroeconomic series. Also deterministic models, but allowing for time variation in the magnitude of the seasonal dummy coefficients have been analyzed by Stephenson and Farr (1972) and Hylleberg (1986), while Canova (1992) analyzed seasonality as a sum of a deterministic process and a stationary stochastic process. In this model, deterministic seasonals are captured by seasonal dummies, and the stochastic seasonals are accounted for by a set of uncertain linear restrictions on the AR coefficients of the model. He uses a Bayesian AR approach, and its method is applied to ten quarterly U.S. macroeconomic series, concluding that seasonality can be well modelled as the sum of deterministic seasonals and a stationary AR process.

Nelson and Plosser (1982) and subsequent work have indicated that a unit root model provides a better approximation to the trend in many economic time series when compared to a deterministic-trend model. As in that work, seasonal unit roots have emerged in this literature, motivated essentially by the possible changing nature of the seasonal component in economic time series. Hylleberg, Engle, Granger and Yoo (1990) (henceforth HEGY) found evidence for seasonal unit roots in the quarterly U.K. nondurable consumption and disposable income, using a general procedure that allows tests for unit roots at some seasonal frequencies without maintaining that unit roots are present at all of them. This procedure (that will be explained briefly in the next section) allows the model to include a constant, seasonal dummies or a time trend. A good deal of empirical work has been done following this general approach: Otto and Wirjanto (1990), after applying the HEGY procedure to fourteen quarterly Canadian economic time series found evidence in favour of seasonal unit roots. Similar evidence was found in Lee and Siklos (1991, 1994), and also in Linden (1994) for the Finnish economy. Beaulieu and Miron (1993) extended the HEGY procedure for monthly data and examined twelve U.S. macroeconomic series in monthly and quarterly data. In contrast with most of these previous studies, they concluded that evidence in favour of a seasonal unit root was weak. In the same line, Osborn (1990) using the Osborn et al. (1988) and the HEGY tests, found little support for seasonal unit roots in a survey of thirty quarterly U.K. macroeconomic variables. The findings of Beaulieu and Miron (1993) have been seriously questioned by Hylleberg et al. (1993). Their conclusions are that seasonality in many cases is variable, not fixed. Abeysinghe (1994) examined the consequences of using seasonal dummies in regression when seasonality is generated by seasonal unit roots. It was shown in that paper that subtracting fixed seasonal means from a seasonally integrated series changes the covariance structure of the series, and often the mean subtracted series may take the appearance of a stationary series in small samples, suggesting that spurious regressions can arise in practice. Mills and Mills (1992) proposed a model with both deterministic and stochastic factors, these factors being estimated by signal extraction. Using standard F-statistics, they analyzed a set of quarterly U.K. macroeconomic series and concluded that both forms of seasonality are present in the data, with the majority of the series containing both seasonal and non-seasonal unit roots. Finally, Hylleberg, Engle, Granger and Lee (1993) (henceforth HEGL) performed the HEGY test on the quarterly series of the Japanese real consumption and real disposable income. Their results showed that the income series was integrated of order 1 at all frequencies, 0, $\pi/2$ (and $3\pi/2$), and π, while the consumption series was integrated of order 1 at frequencies 0 and π, but the tests had some difficulty in separating unit roots at frequency $\pi/2$ from a deterministic seasonal pattern. Osborn (1993) in the discussion following that paper suggests that a nonstationary periodically AR(1) or a periodically integrated I(1) process could be a better modelization. Theoretical analyses of periodic models have been developed by Tiao and Grupe (1980) and their application to economic data appears in Osborn and Smith (1989), Franses and

Romijn (1993), Franses (1994), and Franses and Paap (1994) among others.

In relation to fractional models, few empirical studies have been carried out. The notion of a fractional Gaussian noise model with seasonality was suggested by Jonas (1981) and extended in a Bayesian framework by Carlin et al. (1985) and Carlin and Dempster (1989). In Porter-Hudak (1990) a seasonal fractionally integrated model was applied to some quarterly U.S. monetary aggregates with the conclusions that a fractional ARMA model could be more appropriate than the usual ARIMA models for these aggregate data. Advantages of seasonal fractionally differencing models for forecasting monthly data are illustrated in Sutcliffe (1994) and another empirical application is found in Ray (1993).

In the next section we briefly describe some of the common tests for seasonal integration, and compare them with Robinson's (1994c) tests for nonstationary hypotheses which permit us to test seasonal fractional integration of any stationary or nonstationary degree in raw time series. In Section 3, the tests of Robinson (1994c) are applied to some macroeconomic data of Japan and United Kingdom that were used in HEGL (1993) and HEGY (1990) respectively, and finally Section 4 contains some concluding remarks of these empirical applications.

4.2 TESTS FOR SEASONAL INTEGRATION

In this section we present some of the most commonly used tests for seasonal integration and compare them with Robinson's (1994c) tests for nonstationary hypotheses described in Chapter 2. As previously mentioned, the latter tests are very general and they will permit us to test seasonal fractional and non-fractional integration at some or all seasonal frequencies.

We first consider the Dickey, Hasza and Fuller (DHF, 1984) test which is basically an extension of the test of Dickey and Fuller (1979) to processes such as

$$(1-\rho_s L^s)\, y_t = \epsilon_t \qquad \epsilon_t \sim iid\,(0,\sigma^2) \tag{10}$$

where $\rho_s = 1$. The test is based on the auxiliary regression of form

$$(1-L^s)\, y_t = \pi\, y_{t-s} + \epsilon_t \tag{11}$$

and the test statistic is the 't-value' corresponding to π in (11). Due to the nonstandard asymptotic distributional properties of the 't-values' under the null hypothesis H_o: $\pi = 0$, DHF (1984) provide the fractiles of simulated distributions which give us the critical values to be applied when testing the null against the alternatives: H_1: $\pi < 0$. In order to whiten the errors in (11), the auxiliary regression may be augmented by lagged values of $(1-L^s)y_t$, and with deterministic parts as an intercept, seasonal dummies or a trend, but unfortunately this changes the distribution of the test statistic. (An extension of these tests, accommodating deterministic seasonal trends is developed in Cho et al. (1995)). In addition, the use of the correct augmentation is of great importance for the size and power of the tests in finite samples. Another limitation of DHF's (1984) test is that it is a joint test for roots at long run and seasonal frequencies, and therefore it does not allow for unit roots at some but not all the seasonal frequencies. Also the alternative is a specific s^{th} order autoregression. Similar problems arise in the tests proposed by Bhargava (1985). As an example, if we take s=4 in (11), extending the decomposition in (6), $(1-L^4) = (1-L)(1+L)(1-iL)(1+iL)$, with roots $L = +1,-1,+i,-i$, all of length 1 and corresponding to the zero frequency ($L = 1$), the semiannual π, ($L = -1$), and the annual frequencies $\pi/2$ and $3\pi/2$, ($L = \pm i$), if the data are quarterly. Athola and Tiao (1987) proposed tests for the case of complex roots in the quarterly case, and all these previous ideas are the basis for the extension of the DHF's (1984) tests by HEGY (1990) who propose a test for the quarterly case that, unlike the DHF's (1984) tests, looks at unit roots at some frequencies without maintaining that unit roots are present at all of them. The test is based on the auxiliary regression

$$(1-L^4)\,y_t = \pi_1 y_{1\,t-1} + \pi_2 y_{2\,t-1} + \pi_3 y_{3\,t-2} + \pi_4 y_{3\,t-1} + \epsilon_t \qquad (12)$$

where $y_{1t} = (1+L+L^2+L^3)y_t$ removes the seasonal unit roots and leaves in the zero frequency unit root; $y_{2t} = -(1-L+L^2-L^3)y_t$, leaves the root at frequency π; and $y_{3t} = -(1-L^2)y_t$ leaves the unit roots at frequencies $\pi/2$ and $3\pi/2$. The existence of unit roots at 0, π, $\pi/2$ (and $3\pi/2$) implies that $\pi_1=0$, $\pi_2=0$, and $\pi_3=\pi_4=0$ respectively. The t-values on π_1 and π_2 are shown to have the known Dickey-Fuller distribution (see Fuller (1976)) under the null of $\pi_1=0$ and $\pi_2=0$ respectively, while the t-value on π_3 has a DHF distribution with s=2 conditional on $\pi_4=0$. Also a joint test of $\pi_3=\pi_4=0$ is proposed based on the F-value, and the critical values of the distribution are tabulated. To show how these limiting distributions relate to the classical unit root tests, a test of $\pi_1 = 0$ in (12) will have the familiar Dickey-Fuller distribution if $\pi_2 = \pi_3 = \pi_4 = 0$ since the model can be written in the form

$$y_{1t} = (1+\pi_1)\,y_{1t-1} + \epsilon_t,$$

and similarly, if $\pi_1 = \pi_3 = \pi_4 = 0$, the model becomes

$$y_{2t} = -(1+\pi_2)\, y_{2t-1} + \epsilon_t,$$

and testing $\pi_2 = 0$ above is a test for a root of -1 which was shown by Dickey and Fuller (1979) to be the mirror of the Dickey-Fuller distribution. A test of $\pi_3 = 0$ can be written as

$$y_{3t} = -(1+\pi_3)\, y_{3t-2} + \epsilon_t,$$

assuming that $\pi_4 = 0$ which is therefore the mirror of the DHF distribution for biannual seasonality, and the inclusion of y_{3t-1} in (12) recognizes potential phase shifts in the annual component. Since the null is that $\pi_3 = \pi_4 = 0$, the assumption that $\pi_4 = 0$ may merely reduce the power of the test against some alternatives.

A crucial fact in these tests is that the same limiting distributions are obtained when it is not known a priori that some of the π's are zero: if the π's other than the one to be tested are truly nonzero, then the process does not have unit roots at these frequencies and the corresponding y's are stationary. The regression is therefore equivalent to a standard augmented unit root test. If however some of the other π's are zero, there are other unit roots in the regression, but the corresponding y's are now asymptotically uncorrelated, and the distribution of the test statistics will not be affected by the inclusion of a variable with a zero coefficient which is orthogonal to the included variables. To see this, note that the homogeneous solutions to equations

$$(1-L)\, y_t = \epsilon_t; \quad (1+L)\, y_t = \epsilon_t; \quad \text{and} \quad (1+L^2)\, y_t = \epsilon_t,$$

are given, respectively, by

$$S_{1t} = \sum_{j=0}^{t-1} \epsilon_{t-j}; \quad S_{2t} = \sum_{j=0}^{t-1} (-1)^j \epsilon_{t-j}; \quad \text{and} \quad S_{3t} = \sum_{j=0}^{int[(t-1)/2]} (-1)^j \epsilon_{t-2j}.$$

The variances of these series are given by

$$V(S_{1t}) = V(S_{2t}) = t\,\sigma^2, \quad V(S_{3t}) = \left(int\left[\frac{t-1}{2}\right]+1\right)\sigma^2,$$

where σ^2 is the variance of ε_t, and if these series are excited by the same $\{\varepsilon_t\}$ and t is divisible by four, the covariances are all zero, and at other values of t, the covariances are at most σ^2, so the series are asymptotically uncorrelated as well as being uncorrelated in finite samples for complete years of data. Thus, for example, when testing $\pi_1 = 0$ in (12), if we suppose that $\pi_2 = 0$ but y_2 is still included in the regression, y_1 and y_2 will be asymptotically uncorrelated since they have unit roots at different frequencies and both will be asymptotically uncorrelated with lags of y_4 which is stationary. Therefore, the test for $\pi_1 = 0$ will have the same limiting distribution regardless of whether y_2 is included in the regression, and similar arguments can be used for the other cases. As in DHF (1984) test, the auxiliary regression has to be augmented with lagged values of the dependent variable in order to whiten the errors, and deterministic components can be introduced in the auxiliary regression (12), however, again the distribution changes. An extension of this procedure for monthly data can be found in Beaulieu and Miron (1993), and another extension to allow joint HEGY-type tests for the presence of unit roots at zero and all seasonal frequencies, and only for seasonal frequencies is given in Ghysels et al. (1994). It is shown in this article that the test statistics will have the same limiting distribution as the sum of the corresponding squared t-statistics for π_i (i=1,2,3,4) in the former, and π_i (i=2,3,4) in the latter test.

Other seasonal unit roots tests presented in the literature are Hasza and Fuller (1982) which discuss using an F-type statistic in the model

$$(1-L)(1-L^4)y_t = \beta_1 z_{4\,t-1} + \beta_2 z_{5\,t-4} + \epsilon_t \tag{13}$$

with $z_{4t} = (1-L)y_t$ and $z_{5t} = (1-L^4)y_t$, and again possible augmented autoregressions in the left hand side in (13). The null hypothesis will correspond to a I(1,1) process (using the Box-Jenkins' (1970) terminology) and the alternatives are I(0,0), or I(1,0) or I(0,1). Dickey and Pantula (1987) pointed out the inappropriateness of such tests since they are two sided in nature whereas the alternative of stationarity is one-sided. Using the same model (13), Osborn et al. (1988) proposed t-ratios on β_1 and β_2 for the same null and alternatives hypotheses as in Hasza and Fuller (1982) and a score test was proposed by Ahn and Cho (1993) as an another alternative test for seasonal AR unit root.

All these procedures referred to so far are tests for a unit root in the seasonal AR operator and have stochastic nonstationary as the null hypothesis. Canova and Hansen (1995) extended the test of Kwiatkowski et al. (1992) to the seasonal case, and proposed a LM test, (the CH test), based on the residuals from a regression extracting the deterministic seasonal components and other deterministic components, for testing the null of stationarity around a deterministic seasonal pattern. In Hylleberg (1995), small sample properties of HEGY (1990) and CH (1995) tests for seasonal unit roots in quarterly time series are evaluated and compared. He concludes that both tests complement each other. Finally, Tam and Reinsel (1996) propose in a recent article a test for a unit root in the seasonal MA operator, testing the null of deterministic seasonality against the alternative of stochastic nonstationary. They consider the (integrated) SMA(1) model,

$$y_t = \mu_t + \epsilon_t \qquad t = 1-s, \ldots 0 \tag{14}$$

$$(1-L^s)\, y_t = (1-\alpha L^s)\, \epsilon_t \qquad t = 1,2\ldots, \tag{15}$$

where μ_t is a deterministic seasonal mean function, so that $\mu_t - \mu_{t-s} = 0$, and ε_t is, initially, a white noise process. Thus, a test of $\alpha = 1$ in (15) can be interpreted as a test of deterministic seasonality against the alternative, $\alpha < 1$ of stochastic integrated seasonality. The test can be extended to a more general case with ε_t in (15) replaced by u_t, where u_t is a stationary and invertible ARMA process $\phi(L)\, u_t = \theta(L)\, \varepsilon_t$, and also to allow for a deterministic linear trend in the series y_t, leading to a different nonstandard null limit distribution.

All tests presented so far particularized the case of a unit root at some or all seasonal frequencies. Robinson's (1994c) univariate tests described in Chapter 2 are more general in the sense that they allow us to test any integer or fractional root of any order, and therefore do not concentrate merely on the unit root situation. Similarly to HEGY's (1990) tests, they improve tests of DHF (1984) allowing for roots at all seasonal frequencies, but unlike these tests, the model will allow us to test different amplitudes and different frequencies not only under the alternative but also under the null hypothesis. In HEGY (1990) they test the presence of a unit root at each frequency and several null hypotheses will be tested for each case of interest. Extending (12) to allow augmentations of the dependent variable and deterministic paths, the auxiliary regression in HEGY (1990) is

$$\phi(L)\,(1-L^4)\, y_t = \pi_1 y_{1t-1} + \pi_2 y_{2t-1} + \pi_3 y_{3t-2} + \pi_4 y_{3t-1} + \eta_t + \epsilon_t \tag{16}$$

where $\phi(L)$ is a stationary lag polynomial in the fourth difference of y_t and η_t is a deterministic process that might include an intercept, a time trend and/or seasonal dummies. If we cannot reject the null hypothesis $\pi_1 = 0$ against the alternative $\pi_1 < 0$ in (16), the process will have a unit root at zero frequency whether or not other (seasonal) roots are present in the model. In Robinson's (1994c) tests, the null will include a particular model that might include the unit root at zero frequency as the only root in the process but it can also be specified in a way that allows roots at various seasonal frequencies. Thus in his model, substituting (1.9) by

$$y_t = \eta_t + x_t \qquad t = 1,2,.... \qquad (17)$$

and specifying (1.10) as

$$(1 - L)^{1+\theta} x_t = u_t \qquad t = 1,2,...., \qquad (18)$$

and (2.2), the null hypothesis (1.12) will imply that the process has a single unit root at this zero frequency and no other roots will be present given the requirements on u_t in (18) which, in this Robinson's (1994c) setting, are that u_t must be a covariance stationary process with at most weak parametric autocorrelation. Thus, we rule out here the possibility of testing, as in HEGY (1990), for a unit root whether or not other roots are present in the process, since the spectrum of u_t must be bounded and bounded away from zero at any frequency. In fact, the test statistics will be functions of the hypothesized differenced series which have short memory under the null and thus, if we suppose the process has some seasonal roots, we must specify at what particular frequencies these roots are located and which are their integration orders, in order to take appropriate differences to satisfy the conditions on u_t. Thus, we could have instead of (18)

$$(1 - L^2)^{d+\theta} x_t = u_t \qquad t = 1,2,.... \qquad (19)$$

or alternatively

$$(1 - L + L^2 - L^3)^{d+\theta} x_t = u_t \qquad t = 1,2,.... \qquad (20)$$

or

$$(1 - L^4)^{d+\theta} x_t = u_t \qquad t = 1,2,..... \qquad (21)$$

In all these situations, with d = 1, under the null (1.12), the process will display a unit root at zero frequency, with another one at frequency π in (19); with two complex ones corresponding to $\pi/2$ and $3\pi/2$ in (20), or with all them as in (21). Similarly, using HEGY's (1990) tests once more, the non-rejection of the null $\pi_2 = 0$ in (16) will imply that the process displays a unit root at frequency π independently of other possible roots in the process, and this hypothesis can be consistent with (2.2) and (17) jointly with (19) or (21) among other possibilities covered by Robinson's (1994c) tests. Furthermore, testing sequentially, (or jointly as in Ghysels et al. (1994)), the different null hypotheses in (16), if we cannot reject that $\pi_i = 0$ for i=1,2,3 and 4, the overall null hypothesized model in HEGY (1990) becomes

$$\phi(L)\,(1-L^4)\,y_t \;=\; \eta_t + \epsilon_t \qquad t = 1,2,\ldots, \qquad (22)$$

and we can compare it with the set-up in Robinson (1994c) using (2.2), (17), and (21) with

$$\phi(L)\,u_t \;=\; \epsilon_t \qquad t = 1,2,\ldots \qquad (23)$$

which, with d = 1, under the null (1.12) becomes

$$\phi(L)\,(1-L^4)\,y_t = \phi(L)\,(1-L^4)\,\eta_t + \epsilon_t \qquad t = 1,2,\ldots \qquad (24)$$

Clearly, if we do not include any explanatory variable in (16) and (17), (i.e. $\eta_t \equiv 0$), (24) becomes (22), and if $\eta_t \neq 0$ the difference between the two models will only come through the deterministic components of the process. Similarly, if we cannot reject that π_1 and π_2 are equal to zero but we reject a joint test of $\pi_3 = \pi_4 = 0$ in (16), a plausible model in HEGY (1990) would be

$$\phi(L)\,(1-L^2)\,y_t \;=\; \eta_t + \epsilon_t, \qquad t = 1,2,\ldots \qquad (25)$$

and the corresponding setting in Robinson's (1994c) tests would be (2.2), (17), and (23) jointly with

$$(1-L^2)^{1+\theta}x_t \;=\; u_t, \qquad t = 1,2,\ldots$$

Robinson's (1994c) tests will also allow us to test different integration orders for each of the frequencies. Thus, instead of (21) we could consider for instance,

$$(1-L)^{d_1+\theta_1}\,(1+L)^{d_2+\theta_2}\,(1+L^2)^{d_3+\theta_3}x_t \;=\; u_t, \qquad t = 1,2,\ldots$$

and test the null hypothesis: $\theta = (\theta_1,\theta_2,\theta_3)' = 0$, for different values of d_1, d_2 and d_3. This possibility is also ruled out in HEGY (1990) and the other tests presented above, which just concentrate on the unit root situations.

Finally, we can also compare the tests of Robinson (1994c) with those proposed in Tam and Reinsel (1996). For a general ARMA case, they considered the model

$$y_t \;=\; \mu_t \;+\; u_t \qquad t \;=\; 1-s,\ldots 0 \qquad (26)$$

$$(1-L^s)\;y_t = (1-\alpha L^s)\;u_t \qquad t \;=\; 1,2\ldots, \qquad (27)$$

where μ_t is as in (14), (i.e., $\mu_t - \mu_{t-s} = 0$), and u_t is a stationary and invertible ARMA(p,q) process. They tested the null hypothesis

$$H_o\!: \alpha = 1 \qquad (28)$$

in (27) against the alternative H_a: $\alpha < 1$. The non-rejection of the null (28) in (26) and (27) would imply that y_t follows a deterministic seasonal pattern plus a stationary stochastic process, (i.e., like (26) with t = 1,2,...), while its rejection would give us evidence in favour of seasonal integration. We can now take fractional alternatives instead of the MA alternatives in the right hand side in (27), and also allow for fractional integration instead of the unit root case in its left hand side. Thus, instead of (27) we could consider

$$(1-L^s)^d \, y_t = (1-L^s)^\gamma \, u_t \qquad t = 1,2... \tag{29}$$

with d > 0, and given the common factors appearing in both sides in (29), calling $\delta = \gamma - d$, the model can be rewritten as (26) with

$$(1-L^s)^\delta \, y_t = u_t \qquad t = 1,2.. \tag{30}$$

and we can test here the null hypothesis H_o: $\delta = 0$, against the alternative: H_a: $\delta >$ 0. The non-rejection of H_o in (26) and (30) would imply that y_t behaves like (26) with t = 1,2,..., (i.e. a deterministic seasonal plus a stationary process), while its rejection would imply that y_t follows a seasonal fractionally integrated process. Note that the same models under the null and alternative hypotheses can be obtained using Robinson's (1994c) setting in (2.2), (17) and (18), with η_t in (17) replaced by μ_t, and $\theta = \delta - 1$ in (18).

Finally we stress again that the standard null limit χ^2 distribution in Robinson's (1994c) tests is constant across different specifications of $\rho(L;\theta)$ and regressors, and thus does not require any nonstandard distribution unlike the tests presented previously. In the next section we apply different versions of Robinson's (1994c) tests to macroeconomic data of Japan and United Kingdom that were analyzed in HEGL (1993) and HEGY (1990) respectively.

4.3 EMPIRICAL RESULTS

The relationship between consumption and income is arguably one of the most important in macroeconomics. The most influential and perhaps most widely tested view of the relationship between these two variables is the permanent income hypothesis (PIH), (see Hall (1989) for a literature review). Thus, it is not surprising that a considerable amount of effort and empirical evidence have been generated to determine the nature of the time series behaviour of these two variables separately as well as, of course, their joint statistical relationship.

The finding that many macroeconomic time series, including consumption and income, might be represented as integrated processes (Nelson and Plosser (1982) and subsequent work) raises the possibility that unit roots may be common, leading to the concept of cointegration suggested by Granger (1981). Indeed this notion has found a natural application in the PIH since Hall's (1978) influential article showed that U.S. consumption behaved like a random walk. Other papers that test various versions of the PIH in the context of cointegration analysis are King et al. (1991), Han and Ogaki (1991) and Corbae et al. (1994). Relatively less attention has been paid to the case of quarterly data. Examples in the literature that study the case of seasonal integration and cointegration for consumption and income with quarterly data are HEGY (1990), Wirjanto (1991), HEGL (1993) and Lee and Siklos (1994).

In this section we concentrate on the univariate treatment of these two variables, and apply different versions of Robinson's (1994c) tests to some seasonally unadjusted, quarterly data for Japan and United Kingdom, using the same data sets as in HEGL (1993) and HEGY (1990) respectively.

For both countries we follow the same procedure. We test H_0 (1.12), in a model given by (1.10); (2.2) and

$$y_t = \alpha + \beta_0 t + \beta_1 S_{1t} + \beta_2 S_{2t} + \beta_3 S_{3t} + x_t \qquad t = 1, 2, \ldots T \qquad (31)$$

where θ in (1.10) is a (p x 1) vector, S_{1t}, S_{2t} and S_{3t} in (31) are the seasonal dummies, u_t in (1.10) is a I(0) process (i.e., with spectral density positive and finite at any frequency), and $\rho(L;\theta)$ in (1.10) is a known function of L and θ adopting different forms for the different cases considered. We test in a sequential way. Since the data are quarterly, we start by assuming that the process x_t in (31) has four roots and take

$$\rho(L;\theta) \;=\; (1 - L^4)^{d+\theta}, \qquad (32)$$

and given that θ is in this case a scalar (p=1), we test H_o (1.12) against the one-sided alternatives

$$H_{a1}: \theta < 0, \quad \text{or} \quad H_{a2}: \theta > 0. \qquad (33)$$

Thus, under (1.12) the series will follow an I(d) process with one root at zero frequency; one at frequency π; and two complex ones corresponding to frequencies $\pi/2$ and $3\pi/2$, all them with the same integration order d.

In order to allow different integration orders at different frequencies we also consider

$$\rho(L;\theta) = (1-L^2)^{d_1+\theta_1}(1+L^2)^{d_2+\theta_2}, \tag{34}$$

and more generally,

$$\rho(L;\theta) = (1-L)^{d_1+\theta_1}(1+L)^{d_2+\theta_2}(1+L^2)^{d_3+\theta_3}. \tag{35}$$

Therefore, $\theta = (\theta_1,\theta_2)'$ under (34) and $(\theta_1,\theta_2,\theta_3)'$ under (35), and we test H_o (1.12) against the two-sided alternative

$$H_a: \theta \neq 0. \tag{36}$$

Clearly, when departures are actually of the specialized form (32), a test of (1.12) directed against (36) will have greater power than ones directed against (34) or (35), but these tests have power against a wider range of alternatives.

Following this sequential way of testing we next assume x_t displays only three roots: two of them complex, corresponding to frequencies $\pi/2$ and $3\pi/2$, and one real that might be either at zero frequency or at frequency π. Thus, we perform the tests when

$$\rho(L;\theta) = (1-L+L^2-L^3)^{d+\theta} \tag{37}$$

and

$$\rho(L;\theta) = (1+L+L^2+L^3)^{d+\theta}, \tag{38}$$

and extending now the tests to allow different integration orders at the complex and at the real roots, we also consider two-sided tests where

$$\rho(L;\theta) = (1-L)^{d_1+\theta_1}(1+L^2)^{d_2+\theta_2} \tag{39}$$

and

$$\rho(L;\theta) = (1+L)^{d_1+\theta_1}(1+L^2)^{d_2+\theta_2}. \tag{40}$$

In a further group of tests, we assume that the hypothesized model contains only two roots, one at zero frequency and the other at frequency π. Again we start looking at one-sided tests against

$$\rho(L;\theta) = (1-L^2)^{d+\theta} \tag{41}$$

and then at two-sided tests against

$$\rho(L;\theta) = (1-L)^{d_1+\theta_1}(1+L)^{d_2+\theta_2}. \tag{42}$$

Finally we consider the possibility that the process might be well modelled with a single root (or perhaps two complex ones), and therefore we look at

$$\rho(L;\theta) = (1-L)^{d+\theta}, \tag{43}$$

$$\rho(L;\theta) = (1 + L)^{d+\theta}, \qquad (44)$$

and finally,

$$\rho(L;\theta) = (1 + L^2)^{d+\theta}. \qquad (45)$$

Note that all these specifications of $\rho(L;\theta)$ above can be viewed as particular cases of the general form specified in (1.11) and thus, satisfying the conditions required in Theorems 1-4 in Chapter 2. In all these cases the tests will be performed for different model specifications in (31). First we assume that $\alpha = \beta_0 = \beta_1 = \beta_2 = \beta_3 = 0$ a priori, i.e., we will include no regressors in (31). Next we take α as unknown and $\beta_i = 0$ for i=0,...3 a priori, i.e., introducing only an intercept. The other three cases are: a time trend (α and β_0 unknown, and $\beta_i = 0$ for i=1,...3 a priori); an intercept and the dummy variables ($\beta_1 = 0$ a priori, and the rest of the parameters unknown); and the general case of an intercept, a time trend and the dummies (i.e., with all parameters in (31) unknown). In all cases we consider a wide range of null hypothesized values for d (and for the d_i's when $p > 1$), going from 0.50 to 2.25 with 0.25 increments, and white noise u_t, though in some cases of interest, when testing a single parameter (i.e., $p = 1$), we extend the results to I(0) parametric autocorrelation in u_t, allowing seasonal and/or non-seasonal AR. We first present the results for Japanese data.

4.3.a The Japanese case

We analyze here the log of total real consumption (c_t), the log of real disposable income (y_t), and the difference between them (c_t-y_t) in Japan from 1961.1 through 1987.4 in 1980 prices. In Figure 4.1 we plot the original time series, their sample autocorrelations and estimates of the spectral density function[2], observing much clearer seasonal components in y_t and c_t-y_t than in c_t. This can be better viewed in Figure 4.2 which shows the four quarters of each of the individual series after subtracting the average value of the year. Graphs like these were advocated by Franses (1994) and in case of a constant seasonal pattern, the series should be parallel. In these figures we observe that both series seem to present a changing

[2] As in Chapter 3, they are estimates of the standardized spectral density function, using Barlett, Tukey and Parzen lag windows of size T-1.

seasonal pattern, more marked for y_t than for c_t.

These series have been analyzed in HEGL (1993) to test the presence of seasonal integration and cointegration. In this work (and in an earlier version (HEGL (1991)), they apply the HEGY (1990) tests to these data and their conclusions can be summarized as follows: for c_t, integration is obtained at frequencies 0, $\pi/2$, π and $3\pi/2$ (of a cycle 2π) if there are no regressors in the model or if only a time trend is included; however, if dummies are also included, only two unit roots are observed: one at zero frequency and other at frequency π. Therefore, a plausible model for this series would be (22) when $\eta_t \equiv 0$ or $\eta_t = (1,t)'$, but (25) if η_t includes the seasonal dummies. For y_t, unit roots are not rejected at any of these frequencies when there are no regressors or a time trend and/or dummies are introduced, but if only an intercept is included the unit root at zero frequency is rejected. Finally, for c_t-y_t, the unit root nulls are not rejected at these frequencies, independently of the regressors used, so (22) again seems to be an appropriate way of modelling this series. We now set out the main results obtained across this work.

The first thing we do is to plot the individual series after applying the filter $(1 - L^4)$ and thus, removing all possible unit roots at zero and at seasonal frequencies. This is done in Figure 4.3, which also shows the autocorrelation functions and estimates of the spectral density function. In this figure we observe that both individual series still present a nonstationary appearance, with a possible different mean before and after 1973. This may be related to the different slope observed in the trend of both series after the oil crisis in that year. (See again Figure 4.1)[3]. In the same figure we still observe significant autocorrelations in both series, with a slow decay and/or oscillation, while estimates of the spectral density have a large value around zero frequency, both of which could be indicative of fractional integration greater than a unit root, especially at this zero frequency. For c_t-y_t, plots seem to accommodate better to a possible stationary situation.

Tables 4.1a and 4.1b report the one-sided test statistic $\hat{r}$ in (2.9) when $\rho(L;\theta)$ in (1.10) adopts the form in (32). Therefore we assume x_t has four roots: one at

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[3] In order to deal with the problem of a possible structural break in the slope of the series, we calculate in Appendix 4.1 some of the tests performed below for the two subsamples: 1961.1-1973.4 and 1974.1-1987.4. In Appendix 4.2 we perform the tests including dummy variables for the changing growth of the series.

zero frequency, one at frequency π, and two complex at frequencies $\pi/2$ and $3\pi/2$. In Table 4.1a we take u_t as white noise. We observe here that if we take all parameters in (31) to be zero a priori, we cannot reject (1.12) for d = 0.75 and d = 1 in either individual series (c_t and y_t), while in c_t-y_t, these two cases are also not rejected as well as the case of d = 0.50. When regressors such as an intercept, a trend or seasonal dummies are included in (31), the unit root hypothesis is rejected in both series in favour of more nonstationary alternatives (d > 1), but in some cases we observe a lack of monotonic decrease in the test statistic with respect to d. As we explained in Chapter 3, such monotonicity is something that we should expect of any reasonable statistic, given correct specification and adequate sample size because they are one-sided test statistics. In particular, we observe that monotonicity is not captured when we include an intercept, and an intercept and the dummies for c_t, and an intercept and the dummies for y_t. Looking at c_t-y_t, monotonicity is always achieved and the nulls of d = 1 and of d = 0.75 are never rejected. We could conclude from this table that if x_t is an I(d) process of form $(1-L^4)^d x_t = u_t$, and u_t is in fact white noise, the two individual series are clearly nonstationary with d greater than 1 in most cases; however their difference seems less integrated (with $d \leq 1$), suggesting that some fractional cointegration could exist between both series, for the cointegrating vector (1,-1), using a simplistic version of the permanent income hypothesis theory as discussed by Davidson et al. (1978), for instance.

The fact that d = 1 is not rejected for c_t and y_t when there are no regressors, and for c_t-y_t independently of the regressors used in (31), is consistent with the results in HEGL (1993) though they allow augmentations incorporating significant lagged values at the fourth difference of the series. Thus, in Table 4.1b we suppose that u_t follows an AR(k) process, with k=1,2,3 and 4. Monotonicity is now observed in many cases, especially when we consider the series c_t-y_t, and the order of the autoregression is low. The range of non-rejection values of d goes from 0.50 through 1 for c_t and c_t-y_t, and from 0.50 through 1.25 for y_t. When d > 1.25, the null is rejected in all cases where monotonicity is achieved. The lower integration orders observed in this table compared with Table 4.1a can be in large part due to the fact that the AR estimates are Yule-Walker, entailing roots that are automatically less than one in absolute value but which can be arbitrarily close to one, and therefore, they pick up at least part of the nonstationary component of the series.

If we concentrate on the AR(1), we see that the unit root is not rejected for y_t but is for c_t when the dummies are included in the model, again in line with HEGL (1993).

We also calculated the test statistics when u_t was a seasonal AR process with one and two parameters. However, we do not report the results here since, though we observed many non-rejections, in any single case we obtained monotonic decrease in the test statistic with respect to the integration order d. This can suggest that the use of seasonal AR for modelling u_t when x_t is given by $(1-L^4)^d x_t = u_t$, is not a correct way of specifying it, given that seasonality can be picked up either by seasonal integration above or by deterministic patterns (seasonal dummies) in (31). Similarly, supposing u_t follows a mixed seasonal and non-seasonal AR process, the monotonic decrease in $\hat{r}$ with respect to d was again unlikely achieved, probably because of the same reasons as above.

In all cases considered so far we have assumed that the four roots in x_t must have the same integration order. In the following table we allow this integration order to differ between complex roots and real ones. Table 4.1c corresponds to two-sided tests, reporting $\hat{R}$ in (2.9) when $\rho(L;\theta)$ adopts the form in (34) and $\alpha = (\beta_i)_{i=0,3} = 0$ <u>a priori</u>; α unknown and $(\beta_i)_{i=0,3} = 0$ <u>a priori</u>; and finally α and β_0 unknown and $(\beta_i)_{i=1,3} = 0$ <u>a priori</u>, i.e., we consider the cases of no regressors, an intercept and a time trend respectively. We present results for values of d_1 and d_2 ranging between 0.50 and 1.50. When there are no regressors, the null hypothesis is rejected in all cases for both individual series with the lowest value of the test statistic achieved when $d_1 = 1$ and $d_2 = 0.50$, suggesting that in these two series perhaps the complex roots are not required and a model with only two roots (one at zero frequency and the other at frequency π) might be more plausible. A test of this hypothesis will be conducted later. Looking at c_t-y_t, we observe some non-rejection cases: if $d_1 = d_2$, the null is not rejected when this integration order is 0.50, 0.75 and 1. These three possibilities were also non-rejected in Table 4.1a when we considered one-sided tests, however, the lowest value is now achieved when $d_1 = 0.75$ and $d_2 = 0.50$. Including an intercept or a time trend, we observe now some non-rejections for c_t and y_t. Starting with c_t, the null hypothesis is not rejected when $d_1 = 1.25$ or 1.50 and $d_2 = 0.50$, 0.75 or 1, observing therefore a greater degree of integration at the $0/\pi$ frequencies than at the complex ones. Similarly, for y_t, all non-rejections occur when d_1 is slightly greater than d_2, and for c_t-y_t, the lowest test statistic is obtained at $d_1 = d_2 = 0.75$. The null hypothesis of a unit root at all frequencies ($d_1 = d_2 = 1$) is either non-rejected in this series which is again consistent with Table 4.1a and with results given in HEGL (1993).

In Table 4.1d we are slightly more general in the specification of $\rho(L;\theta)$, and a different integration order is allowed at each frequency. Therefore $\rho(L;\theta)$ takes the form in (35) and again in this table, we present results of $\hat{R}$ for cases of no regressors, an intercept, and a time trend, with white noise u_t. As in Table 4.1c, when there are no regressors the null is always rejected for the individual series with the lowest value obtained at $d_1 = 1.50$ and $d_2 = d_3 = 0.50$, indicating therefore the importance of the root at zero frequency as was pointed out before in view of Figure 4.3. For c_t-y_t there are non-rejections at some alternatives with the lowest value obtained at $d_1 = 1.50$, $d_2 = 0.50$ and $d_3 = 1$, but the case of $d_1 = d_2 = d_3 = 1$ is rejected in this series. Finally, including an intercept or a time trend, results are similar in both cases. For c_t, the lowest statistic is obtained when $d_1 = 1.50$, $d_2 = 1.00$ and $d_3 = 0.50$; for y_t, when $d_1 = 1.50$, and $d_2 = d_3 = 1.00$; and for c_t-y_t, when $d_1 = 1.00$, $d_2 = 0.50$ and $d_3 = 1.00$. All these results corroborate the importance of the root at zero frequency over the others for the three time series.

In Tables 4.2a and 4.2b we suppose x_t contains only three roots: two complex corresponding to the annual frequencies $\pi/2$ and $3\pi/2$, and a real one at zero frequency. In both tables we take u_t as white noise. Table 4.2a gives results of $\hat{r}$ when $\rho(L;\theta)$ adopts the form (37). The unit root is rejected for all series and all specifications in (31) and we observe that the only non-rejection cases correspond to both individual series when $d = 0.75$ if there are no regressors or if the seasonal dummies are included. The great amount of rejections observed in this table is in line with results for two-sided tests in Table 4.2b, where we allow a different integration order at zero and at complex frequencies. In this table $\rho(L;\theta)$ takes the form in (39) and we observe that the null hypothesis is always rejected. When there are no regressors, the lowest statistic is obtained at $d_1 = 1$ and $d_2 = 0.50$ for the individual series, indicating again the importance of the root at zero frequency. In all the other cases and in the three series, the lowest statistic corresponds to $d_1 = d_2 = 0.50$, which is consistent with the results given in Table 4.2a where the lowest test statistic corresponded to $d = 0.50$. Rejections of the unit root in this table might be in large part due to no inclusion of the root at frequency π, and this is corroborated by looking at Figure 4.4 which shows plots of the series, their sample autocorrelations and estimates of the spectral density after applying the filter $(1-L+L^2-L^3)$ on them. In this figure we see that the series are clearly nonstationary, with significant autocorrelations and with the estimates of the spectrum showing a large value around the frequency π and therefore suggesting the need to include the root at such frequency. Excluding the root at zero frequency and taking $\rho(L;\theta)$ as given by (38) or (40) resulted in rejecting the null for all cases in the three series. This was not surprising given the random walk character observed in the series in Figure 4.1 and the importance of the root at this zero frequency previously mentioned.

Following this sequential way of performing the tests, we assume x_t has only two roots, one at zero frequency and the other at frequency π. As in previous cases, we start plotting the differenced series, their sample autocorrelations and estimates of the spectral density for the unit root case. This is done in Figure 4.5, and we observe that even removing the unit roots at these two frequencies, the series are still nonstationary, suggesting in view of the correlograms and the estimates of the spectral density that the root at complex frequencies might also be important. First we take $\rho(L;\theta)$ as in (41) so θ consists of a single parameter. Tables 4.3a and 4.3b give results for one-sided tests with white noise and seasonal AR u_t respectively. Results with non-seasonal and mixed seasonal and non-seasonal AR u_t were not very conclusive with monotonicity only obtained at a few specifications in (31). In Table 4.3a we observe that though monotonicity is always achieved, results are quite variable across the different specifications in (31). Starting with c_t, if there are no regressors, the non-rejection values of d range between 0.75 and 1.25; when a time trend is considered, the only non-rejection case occurs at d = 0.50, and including dummies the values of d where the null is not rejected are 1 and 1.25. For y_t, if there are no regressors, the null is not rejected when d = 0.75 and 1; including an intercept, the only non-rejection value occurs at d = 0.5, and with seasonal dummies, the only non-rejection value of d is 0.75. For c_t-y_t, the null is rejected in favour of stationary alternatives for the first three cases, however, including dummies, the null is not rejected when d = 0.50. For the unit root null, our results are consistent with those of HEGL (1993). In fact, the unit root is not rejected for c_t when the dummies are included, but is nearly always rejected for y_t and c_t-y_t, due perhaps to exclusion of the unit root at the complex frequencies $\pi/2$ and $3\pi/2$, as was suggested by these authors.

Allowing seasonal AR u_t, we see in Table 4.3b that the monotonic decrease in $\hat{r}$ with respect to d is always achieved. In this table we observe that for c_t, the values of d range between 0.5 and 1.25, and the unit root is now never rejected. However, looking at y_t, the unit root null is rejected in favour of less nonstationary alternatives in all cases except when there are no regressors where the unit root is not rejected. Since this null hypothesis is not rejected for c_t, but it is for y_t and c_t-y_t, again results in this case with seasonal AR u_t support the evidence found in HEGL (1993) that only two unit roots (at zero and π frequencies) were present in consumption. For c_t-y_t, only when there are no regressors and $d = 0.50$ is the null non-rejected, and in all other cases, stationary alternatives seem more plausible, so again here, we could say that a certain degree of fractional cointegration seems to exist at these two frequencies, according to the permanent income hypothesis.

Extending now the tests to allow different integration orders at these two frequencies, results are given in Table 4.3c. We observe across this table just a single case where the null is not rejected and it corresponds to c_t when there are no regressors and $d_1 = 1.25$ and $d_2 = 0.50$. Results here are consistent with those given in Table 4.3a when we tested a scalar θ, especially for cases of an intercept and a time trend: with an intercept, we saw in Table 4.3a that the only non-rejection case was for y_t with $d = 0.50$. In Table 4.3c this hypothesis is rejected but it corresponds to the lowest value obtained across the table. Similarly for the case of a time trend, the only non-rejection value in Table 4.3a corresponded to c_t with $d = 0.50$ and again this hypothesis is the one which produces the lowest statistic in Table 4.3c.

Finally, we examine the case of x_t containing a single root, and first we concentrate on the case of this root located at zero frequency. Thus $\rho(L;\theta)$ will adopt the form (43). This can be motivated by looking again at Figure 4.1 where we see that both individual series may have a random walk character (though this is less likely for c_t-y_t). In Figure 4.6 we plot first differences of the series, their sample autocorrelations and estimates of the spectral density, observing in the latter peaks at both frequencies $\pi/2$ and π, especially pronounced at π. Looking at the results for white noise u_t (in Table 4.4a), we observe that the unit root null is not rejected for c_t and y_t when there are no regressors, but is strongly rejected for c_t-y_t in favour of stationary alternatives. There are few non-rejections in this table (only 5 of the 120 cases presented), and apart from the two cases of a unit root, the other three non-rejection cases correspond to d = 0.5 with a time trend for c_t, and d = 0.75 with seasonal dummies for y_t. In case of c_t-y_t, the null is rejected in favour of stationary alternatives for the whole variety of specifications in (31), suggesting that at this zero frequency, a degree of fractional cointegration might also occur and referring again to the permanent income hypothesis.

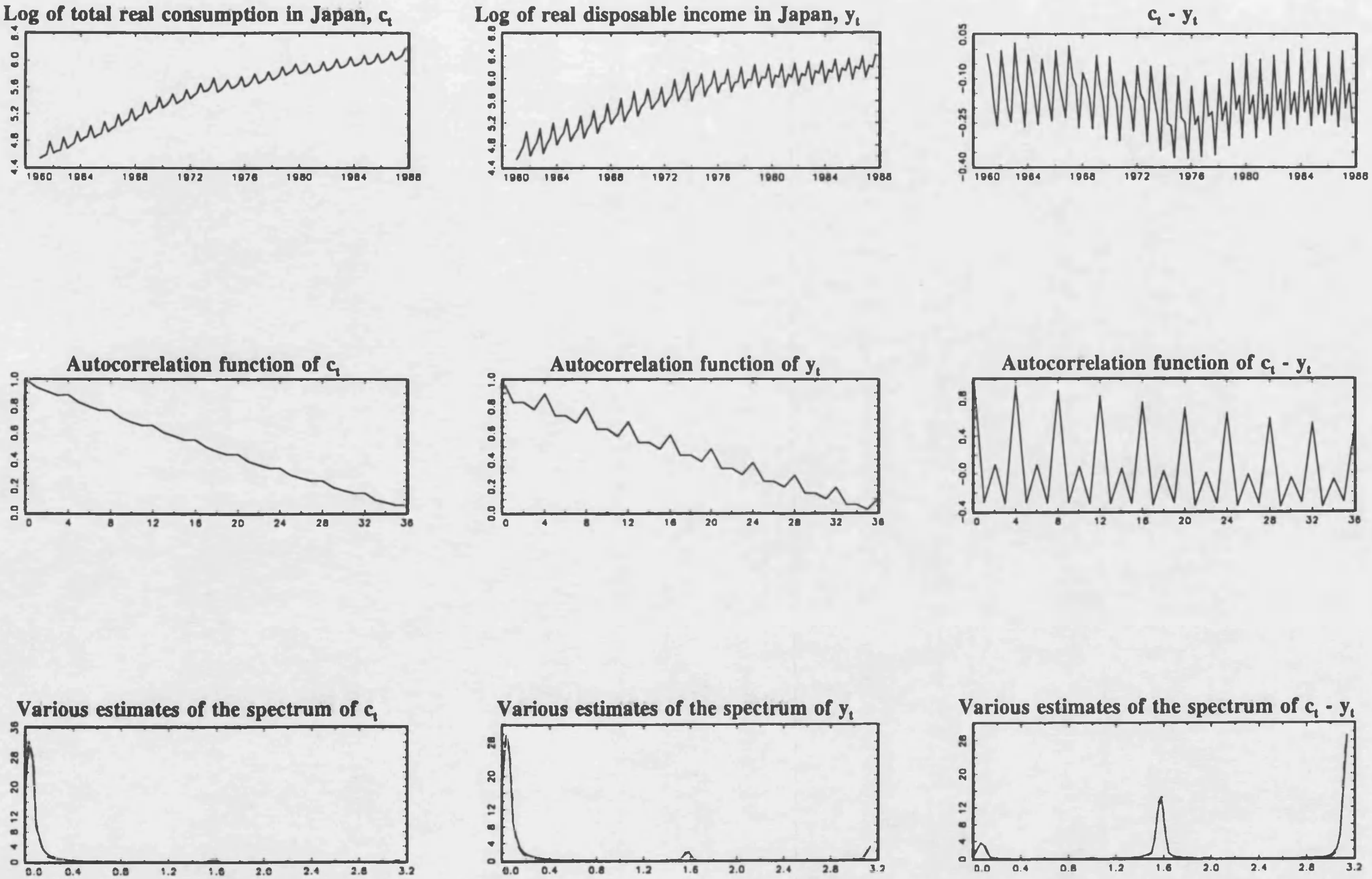
We do not report here the results for non-seasonal AR u_t, mainly because we observed very few cases where monotonicity was achieved across the different values of d. This might be explained because seasonality is not captured now by first differences, and deterministic components do not seem to be sufficient to pick up this effect. However, we report in Table 4.4b results for seasonal AR u_t, observing that monotonicity is now achieved in practically all cases, with results very similar when we take only one or two parameters. If there are no regressors, the unit root null is not rejected for c_t and y_t. In the latter, the null d = 0.75 is also non-rejected; however, for c_t-y_t, the only non-rejection case occurs at d = 0.50. Including some regressors, the case of d = 0.75 is not rejected for c_t in any specification in (31); the values of d range between 0.50 and 0.75 for y_t and is always rejected in favour of stationary alternatives for c_t-y_t, so here again we found evidence in favour of the permanent income hypothesis. The great proportion of rejections of the unit root null observed across this table is in line with HEGL (1993) who suggest that other unit roots apart from the one at zero frequency might be required when modelling these series. In Table 4.4c we present results for mixed seasonal and non-seasonal AR u_t. Monotonicity is obtained now in some cases: when we include an intercept or a time trend for c_t; a time trend for y_t, and a time trend and the dummies for c_t-y_t. For c_t and y_t, the non-rejection values of d range between 0.75 and 1.75, while for c_t-y_t are slightly smaller going from 0.50 through 1.50.

Finally, we also performed the tests for white noise u_t, assuming that x_t has a single root at frequency π, (i.e., $\rho(L;\theta)$ adopting the form in (44)), and taking x_t as an I(d) process with two complex roots corresponding to frequencies $\pi/2$ and $3\pi/2$, (i.e., $\rho(L;\theta)$ as in (45)). However, we do not present the results here, since H_o

was always rejected for all series and all specifications in (31). This may be due to the fact that we do not allow for presence of other roots and thus, modelling the series with a single root at these seasonal frequencies does not seem appropriate.

As a conclusion we can summarize the main results obtained for the Japanese case by saying that if x_t is I(d) with four seasonal roots of the same order d, and u_t is white noise, the values of d where the null hypothesis is not rejected are greater than or equal to one for c_t and y_t, and less than or equal to one for c_t-y_t. If u_t is AR, d ranges in most cases between 0.50 and 1 for the three series, and allowing different integration orders for the different frequencies, the most noticeable fact is the relative importance of the root at the zero frequency over the others. Excluding one of the real roots (either at zero or at frequency π), the null hypothesis is rejected in practically all situations, indicating the importance of these roots. Taking x_t as I(d) with two roots, at zero and at frequency π, if u_t is white noise, the null hypothesis is not rejected for c_t when d ranges between 0.75 and 1.25 while for y_t and c_t-y_t the non-rejection cases correspond to $d < 1$. Modelling here u_t as seasonal AR, the unit root is not rejected for c_t but is for the other two series, and if we permit different integration orders at these two frequencies the only non-rejection case occurs for c_t with the integration order at zero frequency slightly greater than at frequency π. Finally, if we assume that the process has a single root at zero or at frequency π (or two complex ones corresponding to frequencies $\pi/2$ and $3\pi/2$), the unit root will be rejected in practically all cases in favour of less nonstationary alternatives. Results presented in this section are consistent with those given in HEGL (1993) for the case of unit roots. Thus, y_t and c_t-y_t can be well modelled as I(1) processes with four unit roots, while c_t might also be described as an I(1) process with only two unit roots, at zero and at frequency π. As a final comment, given that the values of d where the null is not rejected are in practically all situations smaller for c_t-y_t than for the individual series, we could conclude by saying that there might exist a certain degree of fractional cointegration between these two variables at zero and at seasonal frequencies for a given cointegration vector (1,-1), according to the permanent income hypothesis.

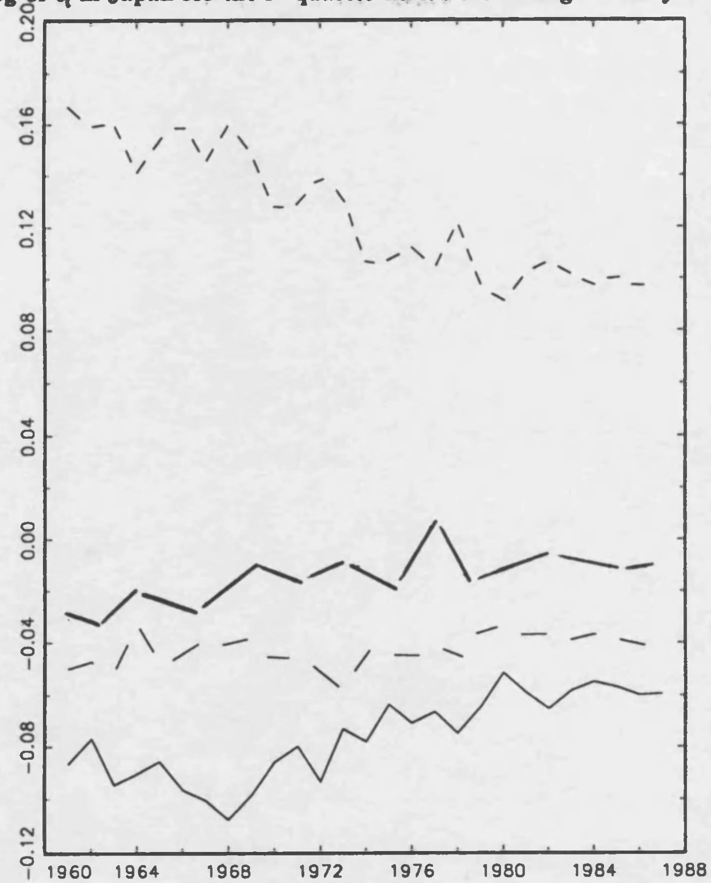
FIGURE 4.1



Log of total real consumption in Japan, c_t
Log of real disposable income in Japan, y_t
c_t - y_t
Autocorrelation function of c_t
Autocorrelation function of y_t
Autocorrelation function of c_t - y_t
Various estimates of the spectrum of c_t
Various estimates of the spectrum of y_t
Various estimates of the spectrum of c_t - y_t

FIGURE 4.2

Log of c_t in Japan for the i[th] quarter minus the average of the year



Log of yt in Japan for the i[th] quarter minus the average of the year

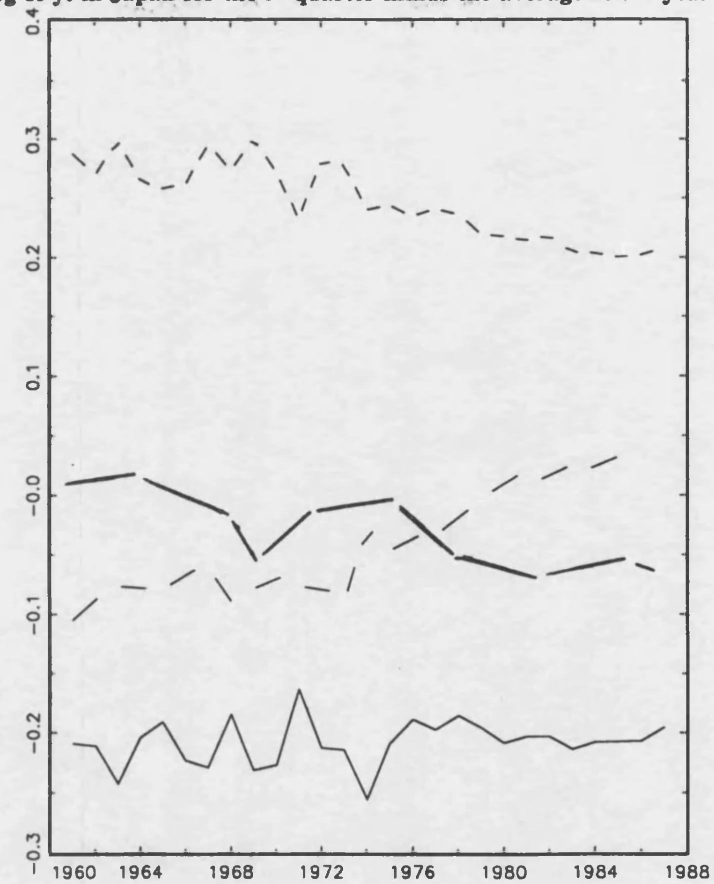
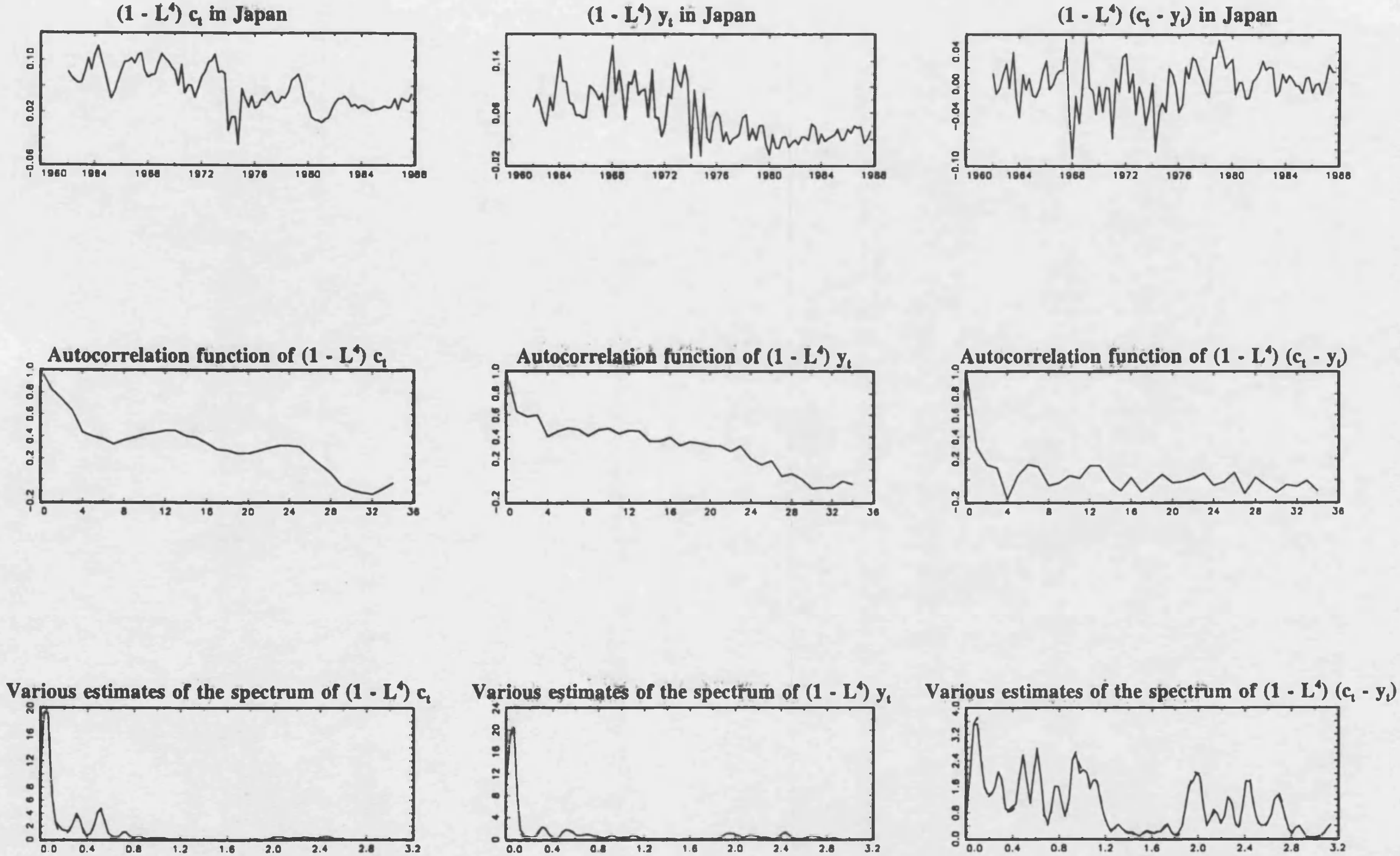


FIGURE 4.3



(1 - L4) ct in Japan
(1 - L4) yt in Japan
(1 - L4) (ct - yt) in Japan
Autocorrelation function of (1 - L4) ct
Autocorrelation function of (1 - L4) yt
Autocorrelation function of (1 - L4) (ct - yt)
Various estimates of the spectrum of (1 - L4) ct
Various estimates of the spectrum of (1 - L4) yt
Various estimates of the spectrum of (1 - L4) (ct - yt)

FIGURE 4.4

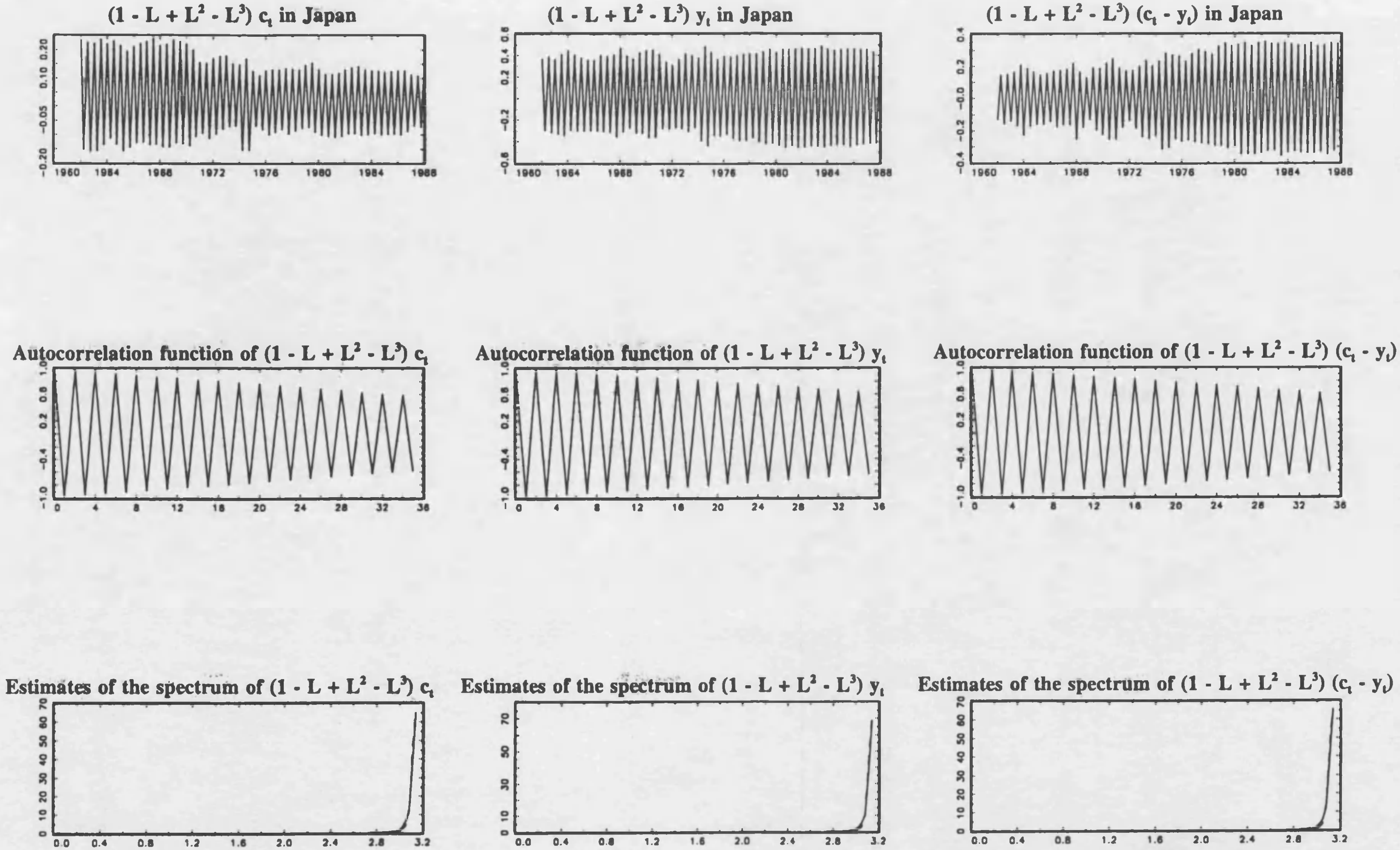

(1 - L + L² - L³) c_t in Japan
(1 - L + L² - L³) y_t in Japan
(1 - L + L² - L³) (c_t - y_t) in Japan
Autocorrelation function of (1 - L + L² - L³) c_t
Autocorrelation function of (1 - L + L² - L³) y_t
Autocorrelation function of (1 - L + L² - L³) (c_t - y_t)
Estimates of the spectrum of (1 - L + L² - L³) c_t
Estimates of the spectrum of (1 - L + L² - L³) y_t
Estimates of the spectrum of (1 - L + L² - L³) (c_t - y_t)

FIGURE 4.5

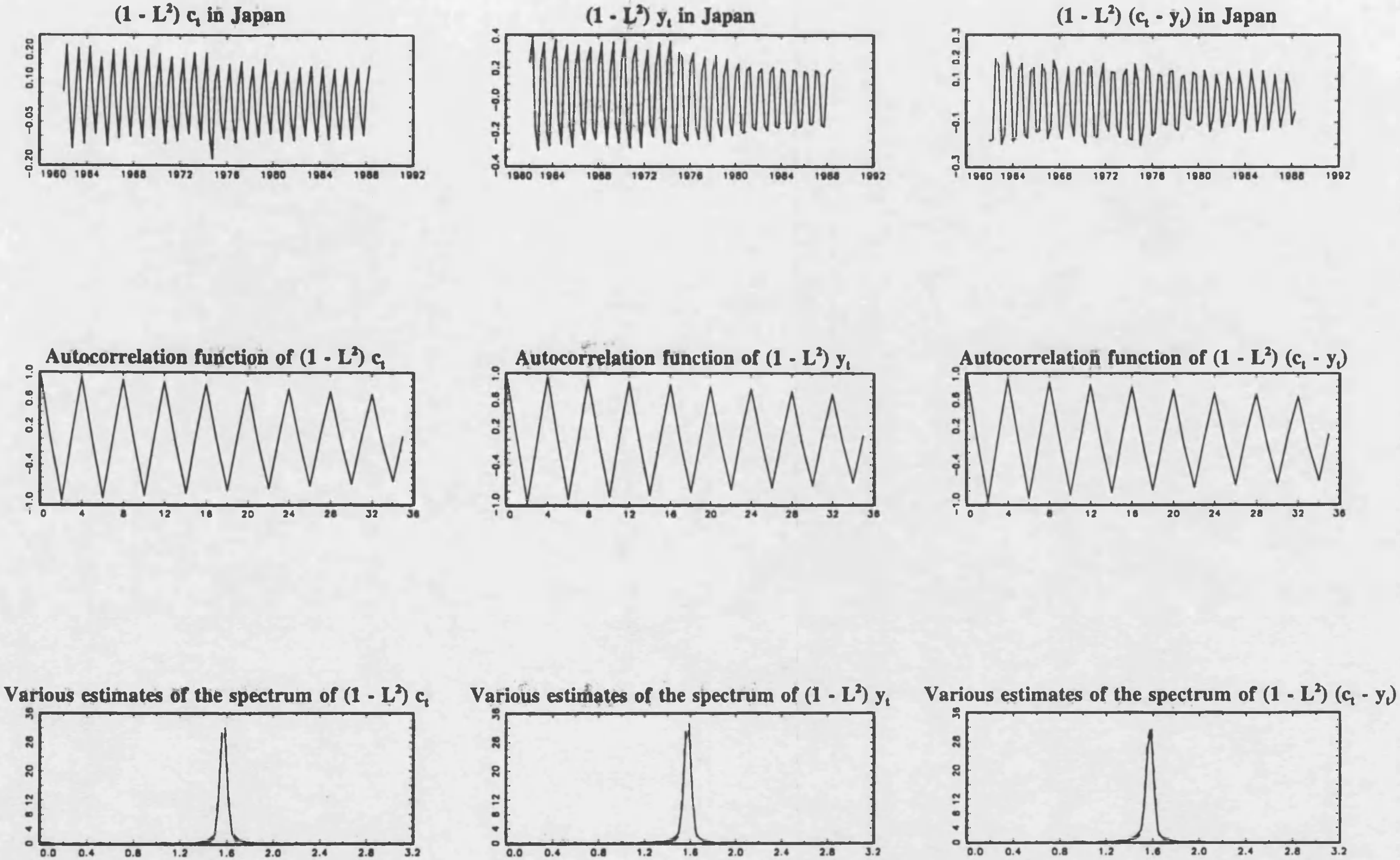
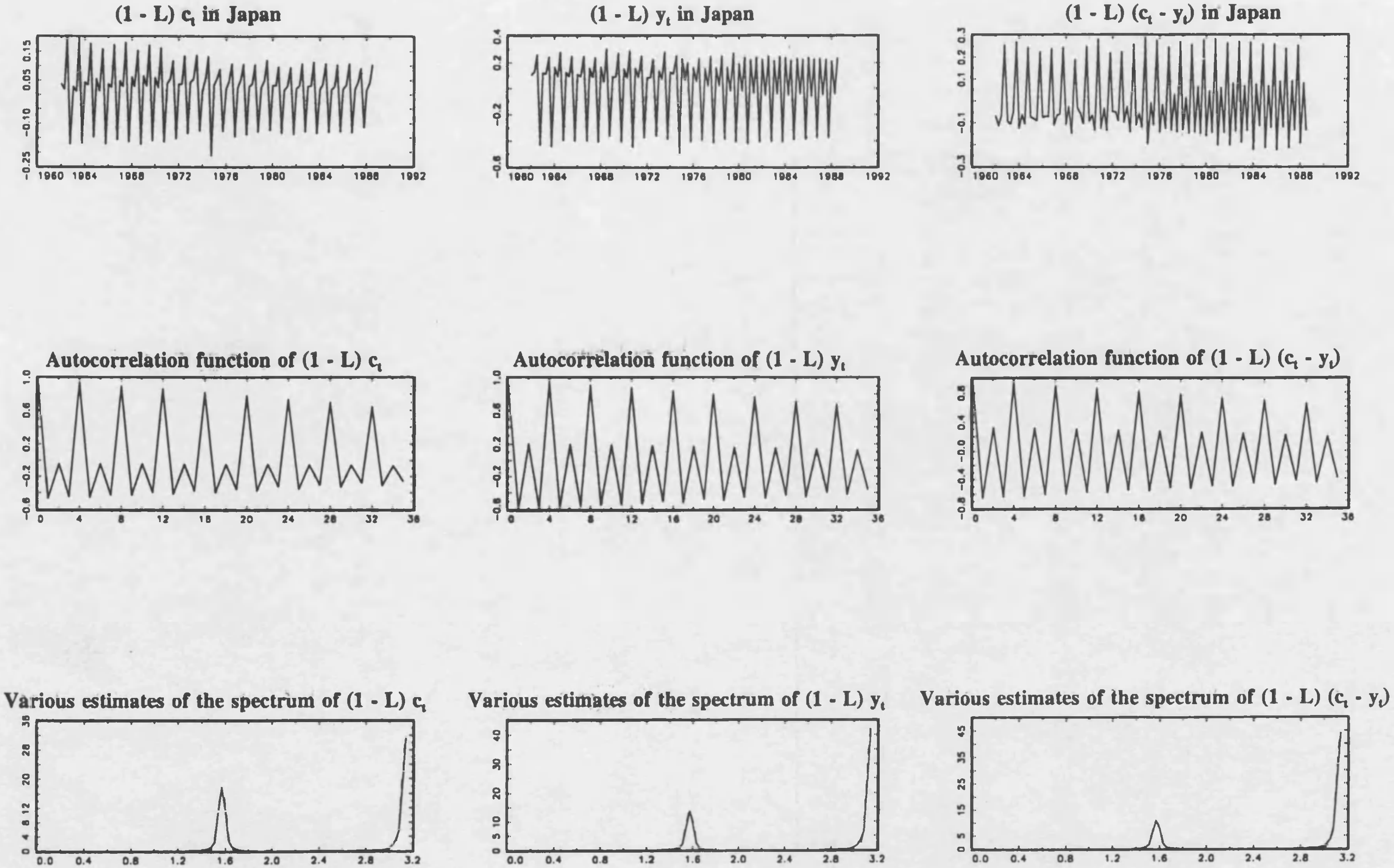

(1 - L²) c_t in Japan
(1 - L²) y_t in Japan
(1 - L²) (c_t - y_t) in Japan
Autocorrelation function of (1 - L²) c_t
Autocorrelation function of (1 - L²) y_t
Autocorrelation function of (1 - L²) (c_t - y_t)
Various estimates of the spectrum of (1 - L²) c_t
Various estimates of the spectrum of (1 - L²) y_t
Various estimates of the spectrum of (1 - L²) (c_t - y_t)

FIGURE 4.6



(1 - L) c_t in Japan
(1 - L) y_t in Japan
(1 - L) (c_t - y_t) in Japan
Autocorrelation function of (1 - L) c_t
Autocorrelation function of (1 - L) y_t
Autocorrelation function of (1 - L) (c_t - y_t)
Various estimates of the spectrum of (1 - L) c_t
Various estimates of the spectrum of (1 - L) y_t
Various estimates of the spectrum of (1 - L) (c_t - y_t)

TABLE 4.1a

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 - L^4)^{d+\theta}$ and white noise u_t

(Japanese case)

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	2.61	0.77'	-1.02'	-2.36	-3.22	-3.76	-4.12	-4.37
I	4.36	2.64	3.05	1.36'	-0.89'	-2.54	-3.50	-4.04
I,T	9.12	7.28	3.83	0.00'	-2.72	-3.76	-4.01	-4.17
I,D	4.41	2.80	4.39	2.95	0.34'	-1.78'	-3.06	-3.76
I,T,D	10.02	8.34	5.14	1.04'	-2.11	-3.51	-3.99	-4.24

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	2.54	0.72'	-1.05'	-2.38	-3.23	-3.77	-4.13	-4.38
I	4.70	3.34	2.21	-0.08'	-2.10	-3.37	-4.06	-4.44
I,T	7.80	6.04	2.54	-0.91'	-3.11	-3.76	-3.77	-3.86
I,D	4.95	4.12	4.78	2.33	-0.57'	-2.63	-3.72	-4.25
I,T,D	10.28	8.48	5.10	0.84'	-2.30	-3.69	-4.19	-4.44

c_t-y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	1.53'	-0.08'	-1.77'	-2.93	-3.63	-4.05	-4.33	-4.52
I	2.41	0.46'	-1.54'	-2.84	-3.60	-4.05	-4.34	-4.54
I,T	2.34	0.45'	-1.54'	-2.86	-3.58	-3.82	-3.89	-4.02
I,D	3.42	0.35'	-1.79'	-3.06	-3.76	-4.15	-4.39	-4.55
I,T,D	3.31	0.34'	-1.79'	-3.06	-3.76	-4.15	-4.39	-4.55

': Non-rejection values of the null hypothesis (1.12) at 95% significance level; Letters in bold correspond to the cases where monotonicity with respect to d is achieved.

c_t: Log of total consumption in Japan, 1961.1 to 1987.4
y_t: Log of disposable income in Japan, 1961.1 to 1987.4

--: No intercept, no time trend and no seasonal dummies.
I : Intercept.
I,T: Intercept and time trend.
I,D: Intercept and seasonal dummies.
I,T,D: Intercept, time trend and seasonal dummies.

TABLE 4.1b

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 - L^4)^{d+\theta}$ and AR(k) u_t

(Japanese case)

k = 1

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-3.13	-3.50	-3.83	-4.13	-4.38	-4.57	-4.71	-4.83
I	-1.59'	-0.67'	-0.51'	-1.79'	-2.78	-3.50	-3.99	-4.30
I,T	2.57	1.01'	-0.65'	-2.01	-3.19	-3.82	-4.09	-4.27
I,D	-2.87	-3.21	-3.31	-3.51	-3.73	-4.05	-4.35	-4.56
I,T,D	-1.05'	-2.67	-3.30	-3.63	-4.12	-4.48	-4.64	-4.74

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-3.01	-3.47	-3.82	-4.12	-4.37	-4.57	-4.71	-4.83
I	-0.03'	0.87'	0.23'	-1.38'	-2.67	-3.52	-4.03	-4.34
I,T	3.09	2.07	0.24'	-1.64'	-3.09	-3.67	-3.80	-3.96
I,D	-2.51	-2.37	-1.71'	-1.88'	-2.50	-3.34	-3.99	-4.36
I,T,D	0.29'	-1.41'	-1.61'	-1.98	-3.08	-3.91	-4.28	-4.49

c_t-y_t\d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	0.87'	-0.84'	-2.29	-3.21	-3.77	-4.13	-4.37	-4.54
I	1.94'	-0.01'	-1.78'	-2.91	-3.59	-4.01	-4.28	-4.48
I,T	1.89'	-0.02'	-1.78'	-2.93	-3.58	-3.86	-4.00	-4.16
I,D	1.34'	-1.29'	-2.66	-3.46	-3.95	-4.25	-4.44	-4.58
I,T,D	1.29'	-1.29'	-2.66	-3.46	-3.95	-4.25	-4.45	-4.58

k = 2

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-3.19	-3.53	-3.81	-4.09	-4.34	-4.54	-4.70	-4.82
I	-1.53'	-0.51'	-0.85'	-2.14	-2.96	-3.56	-4.01	-4.36
I,T	1.77'	0.16'	-1.35'	-2.37	-3.30	-3.88	-4.15	-4.34
I,D	-2.90	-3.26	-3.56	-3.82	-3.99	-4.24	-4.48	-4.66
I,T,D	-1.23'	-2.84	-3.60	-3.92	-4.29	-4.60	-4.74	-4.83

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-3.08	-3.50	-3.80	-4.09	-4.34	-4.54	-4.70	-4.82
I	-0.29'	0.75'	0.20'	-1.31'	-2.54	-3.41	-3.96	-4.30
I,T	2.69	1.61'	0.04'	-1.55'	-3.04	-3.66	-3.77	-3.93
I,D	-2.54	-2.57	-2.53	-2.78	-3.05	-3.57	-4.07	-4.39
I,T,D	0.11'	-1.99	-2.59	-2.72	-3.33	-3.97	-4.31	-4.51

c_t-y_t\d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	0.80'	-0.88'	-2.27	-3.18	-3.75	-4.11	-4.36	-4.53
I	1.85'	0.03'	-1.72'	-2.89	-3.60	-4.02	-4.30	-4.49
I,T	1.81'	-0.01'	-1.72'	-2.91	-3.59	-3.85	-3.97	-4.12
I,D	0.45'	-1.67'	-2.77	-3.47	-3.94	-4.24	-4.44	-4.58
I,T,D	0.40'	-1.68'	-2.77	-3.47	-3.94	-4.24	-4.44	-4.58

cont...

k = 3

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-3.25	-3.57	-3.80	-4.06	-4.33	-4.56	-4.75	-4.91
I	-1.53'	-0.28'	-0.61'	-2.03	-2.85	-3.49	-4.01	-4.38
I,T	1.74'	0.13'	-1.37'	-2.31	-3.29	-3.97	-4.26	-4.47
I,D	-2.95	-3.30	-3.69	-3.97	-4.16	-4.41	-4.67	-4.88
I,T,D	-1.34'	-2.89	-3.71	-4.07	-4.47	-4.82	-4.99	-5.08

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-3.13	-3.53	-3.80	-4.07	-4.34	-4.57	-4.76	-4.92
I	-0.23'	0.79'	0.19'	-1.20'	-2.34	-3.28	-3.93	-4.34
I,T	2.30	1.17'	-0.16'	-1.42'	-2.92	-3.65	-3.77	-3.95
I,D	-2.66	-2.82	-3.05	-3.38	-3.55	-3.85	-4.21	-4.47
I,T,D	0.08'	-2.21	-3.13	-3.34	-3.67	-4.13	-4.40	-4.57

c_t-y_t\d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	0.78'	-1.07'	-2.44	-3.29	-3.85	-4.22	-4.49	-4.68
I	2.00	-0.04'	-1.78'	-2.89	-3.59	-4.03	-4.33	-4.54
I,T	1.95'	-0.06'	-1.79'	-2.91	-3.59	-3.84	-3.96	-4.12
I,D	0.25'	-2.00	-2.98	-3.56	-3.97	-4.26	-4.46	-4.60
I,T,D	0.20'	-2.01	-2.98	-3.56	-3.97	-4.26	-4.46	-4.60

k = 4

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-3.29	-3.52	-3.60	-3.71	-3.89	-4.10	-4.29	-4.45
I	-1.93'	-1.24'	-1.54'	-2.05	-2.21	-2.59	-3.14	-3.62
I,T	0.11'	-1.21'	-1.75'	-1.72'	-2.42	-3.43	-3.83	-4.07
I,D	-3.03	-3.19	-2.74	-2.60	-2.36	-2.51	-3.06	-3.62
I,T,D	-1.35'	-2.41	-2.63	-2.23	-2.60	-3.58	-4.09	-4.35

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-3.21	-3.50	-3.59	-3.71	-3.90	-4.10	-4.30	-4.46
I	-1.16'	-0.89'	-1.34'	-1.36'	-1.54'	-2.22	-2.98	-3.53
I,T	-1.89'	-2.34	-1.77'	-1.06'	-1.94'	-2.90	-2.95	-3.15
I,D	-2.69	-2.76	-3.14	-3.28	-2.89	-2.92	-3.45	-3.96
I,T,D	-0.05'	-2.53	-3.36	-2.91	-2.74	-3.47	-4.00	-4.29

c_t-y_t\d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-4.68	-2.68	-2.58	-3.00	-3.44	-3.79	-4.06	-4.25
I	-4.61	-2.21	-1.76'	-2.34	-2.99	-3.49	-3.84	-4.09
I,T	-4.72	-2.24	-1.77'	-2.36	-2.99	-3.11	-3.07	-3.22
I,D	-2.11	-2.72	-2.86	-3.19	-3.63	-3.99	-4.24	-4.41
I,T,D	-2.22	-2.74	-2.86	-3.19	-3.63	-3.99	-4.25	-4.41

': Non-rejection values of the null hypothesis (1.12) at 95% significance level; Letters in bold correspond to the cases where monotonicity in the value of the tests with respect to d is achieved.

TABLE 4.1c

$\hat{R}$ in (2.9) with $\rho(L;\theta) = (1 - L^2)^{d1+\theta 1} (1 + L^2)^{d2+\theta 2}$ and white noise u_t (Japanese case)

		No intercept and no trend			Intercept			Intercept and time trend		
d_1	d_2	c_t	y_t	c_t-y_t	c_t	y_t	c_t-y_t	c_t	y_t	c_t-y_t
0.50	0.50	41.03	39.76	5.25'	64.79	63.91	6.83	167.85	107.69	6.49
0.50	0.75	47.92	46.58	12.86	72.81	75.60	15.02	192.19	150.17	14.47
0.50	1.00	53.35	51.97	19.48	79.24	83.19	23.76	201.74	168.45	23.03
0.50	1.25	57.72	56.32	24.32	84.92	89.31	30.32	207.12	178.07	29.41
0.50	1.50	61.28	59.88	28.05	90.11	94.67	35.22	210.65	183.90	34.14
0.75	0.50	17.12	16.72	0.42'	22.81	13.95	4.30'	77.49	29.81	4.23'
0.75	0.75	22.42	22.01	2.95'	34.46	30.38	0.50'	117.38	68.97	0.52'
0.75	1.00	27.06	26.61	8.97	42.28	43.89	5.08'	137.50	100.85	5.16'
0.75	1.25	31.19	30.72	14.18	48.55	53.78	11.17	150.13	123.02	11.27
0.75	1.50	34.94	34.45	18.52	54.07	61.56	16.86	159.27	138.97	16.96
1.00	0.50	7.76	7.64	3.58'	8.74	8.21	10.28	11.04	8.56	10.27
1.00	0.75	10.73	10.62	1.45'	22.43	5.76'	2.66'	29.89	6.90	2.67'
1.00	1.00	13.33	13.22	4.71'	35.55	14.50	2.39'	48.00	18.11	2.41'
1.00	1.25	15.72	15.59	7.98	45.91	26.86	4.86'	62.37	33.81	4.89'
1.00	1.50	17.98	17.84	10.53	54.34	38.69	7.33	74.08	49.01	7.35
1.25	0.50	8.07	7.98	8.32	1.82'	11.98	15.19	1.96'	14.05	15.31
1.25	0.75	9.93	9.91	4.61'	3.85'	2.95'	7.92	0.36'	5.22'	8.04
1.25	1.00	11.30	11.30	6.64	11.73	0.30'	6.31	5.01'	0.43'	6.41
1.25	1.25	12.40	12.40	9.30	20.03	4.29'	8.09	10.88	2.47'	8.20
1.25	1.50	13.37	13.37	11.08	27.56	9.91	9.77	16.30	6.18	9.88
1.50	0.50	10.37	10.25	12.16	3.37'	16.22	18.62	6.01	19.15	19.08
1.50	0.75	12.16	12.13	7.72	0.37'	9.18	11.92	3.78'	14.25	12.22
1.50	1.00	13.30	13.31	8.85	2.37'	3.32'	9.31	5.14'	7.65	9.29
1.50	1.25	13.99	14.02	11.58	6.04	3.81'	11.01	7.96	8.00	10.92
1.50	1.50	14.45	14.48	13.53	9.44	5.56'	13.00	9.94	9.71	12.92

': Non-rejection values for the null hypothesis (1.12) at 95% significance level.

TABLE 4.1d

$\hat{R}$ in (2.9) with $\rho(L;\theta) = (1 - L)^{d1+\theta1} (1 + L)^{d2+\theta2} (1 + L^2)^{d3+\theta3}$ and white noise u_t (Japanese case)

			No intercept and no trend			Intercept			Intercept and time trend		
d_1	d_2	d_3	c_t	y_t	c_t-y_t	c_t	y_t	c_t-y_t	c_t	y_t	c_t-y_t
0.50	0.50	0.50	103.66	101.27	21.28	141.71	136.00	18.03	281.38	181.08	17.54
0.50	0.50	1.00	125.45	122.99	49.44	166.31	169.76	49.62	334.06	276.40	48.41
0.50	0.50	1.50	138.97	136.53	63.60	183.92	188.44	66.73	346.47	298.25	64.88
0.50	1.00	0.50	117.27	114.99	43.61	154.99	157.32	44.85	320.99	259.76	43.82
0.50	1.00	1.00	136.62	134.40	94.74	177.59	188.17	129.43	366.44	370.88	127.32
0.50	1.00	1.50	148.39	146.29	120.92	194.11	205.87	176.67	377.28	395.97	173.92
0.50	1.50	0.50	123.50	121.33	57.72	164.44	169.23	63.62	335.74	292.35	62.24
0.50	1.50	1.00	140.31	138.24	107.66	185.10	196.22	152.38	371.11	383.19	150.03
0.50	1.50	1.50	150.64	148.71	131.71	200.60	212.57	196.08	379.89	403.28	193.24
1.00	0.50	0.50	18.90	18.50	2.03'	9.87	3.73'	4.01'	10.73	3.66'	4.01'
1.00	0.50	1.00	29.47	28.91	2.04'	32.10	4.74'	0.53'	36.42	4.94'	0.54'
1.00	0.50	1.50	38.39	37.60	3.03'	45.26	8.71	1.04'	50.98	8.85	1.04'
1.00	1.00	0.50	31.34	30.89	6.50'	24.98	12.03	11.13	28.27	12.33	11.12
1.00	1.00	1.00	45.88	45.45	16.30	81.47	39.02	7.86	100.22	44.08	7.87
1.00	1.00	1.50	57.62	57.14	29.12	113.61	79.39	17.82	142.03	92.80	17.82
1.00	1.50	0.50	39.66	39.20	8.21	40.61	16.16	11.31	47.66	17.03	11.30
1.00	1.50	1.00	54.65	54.24	26.23	106.91	65.41	15.61	135.75	77.02	15.62
1.00	1.50	1.50	66.40	65.97	43.02	138.79	115.71	32.58	179.06	142.14	32.60
1.50	0.50	0.50	10.33	10.11	2.94'	9.57	3.89'	3.94'	11.15	4.33'	3.99'
1.50	0.50	1.00	13.25	13.06	1.78'	31.88	3.95'	1.19'	35.86	4.40'	1.20'
1.50	0.50	1.50	14.41	14.16	2.00'	44.10	5.65'	1.34'	48.52	6.02'	1.35'
1.50	1.00	0.50	14.23	14.01	11.25	3.21'	14.26	16.43	4.79'	16.22	16.73
1.50	1.00	1.00	19.69	19.56	7.84	3.62'	1.58'	7.17'	3.57'	3.72'	7.13'
1.50	1.00	1.50	23.28	23.11	11.94	11.54	5.77'	10.30	9.18	8.36	10.26
1.50	1.50	0.50	19.04	18.81	12.79	5.20'	16.52	18.79	6.49'	19.23	19.22
1.50	1.50	1.00	27.05	26.95	12.38	14.48	6.84'	10.62	9.75	8.65	10.47
1.50	1.50	1.50	32.83	32.72	20.29	30.45	13.79	16.00	18.91	12.28	15.69

': Non-rejection values for the null hypothesis (1.12) at 95% significance level.

TABLE 4.2a

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 - L + L^2 - L^3)^{d+\theta}$ and white noise u_t

(Japanese case)

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	5.30	1.41'	-2.75	-5.93	-7.91	-9.04	-9.68	-10.04
I	2.33	-9.68	-10.83	-11.00	-11.08	-11.11	-11.13	-11.14
I,T	-4.90	-9.68	-10.81	-11.04	-11.10	-11.12	-11.13	-11.13
I,D	8.20	1.06'	-6.85	-9.38	-10.32	-10.62	-10.73	-10.79
I,T,D	10.90	1.31'	-6.96	-9.79	-10.49	-10.66	-10.73	-10.78

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	4.13	-0.12'	-4.39	-7.27	-9.87	-9.72	-10.18	-10.44
I	-6.23	-10.90	-11.07	-11.10	-11.11	-11.12	-11.13	-11.13
I,T	-9.47	-10.82	-11.06	-11.11	-11.12	-11.13	-11.13	-11.14
I,D	8.55	-0.05'	-7.66	-9.76	-10.45	-10.69	-10.80	-10.86
I,T,D	9.94	-0.52'	-7.78	-9.96	-10.51	-10.70	-10.80	-10.86

c_t-y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-10.28	-10.85	-11.03	-11.09	-11.11	-11.12	-11.12	-11.12
I	-10.60	-11.01	-11.09	-11.11	-11.11	-11.11	-11.12	-11.12
I,T	-10.58	-11.00	-11.09	-11.11	-11.11	-11.12	-11.12	-11.12
I,D	-6.85	-9.60	-10.42	-10.70	-10.81	-10.86	-10.90	-10.92
I,T,D	-6.90	-9.60	-10.42	-10.70	-10.81	-10.85	-10.88	-10.91

': Non-rejection values of the null hypothesis (1.12) at 95% significance level. Letters in bold correspond to the cases where monotonicity with respect to d is achieved.

c_t: Log of total consumption in Japan, 1961.1 to 1987.4
y_t: Log of disposable income in Japan, 1961.1 to 1987.4

--: No intercept, no time trend and no seasonal dummies.
I: Intercept.
I,T: Intercept and time trend.
I,D: Intercept and seasonal dummies.
I,T,D: Intercept, time trend and seasonal dummies.

TABLE 4.2b

$\hat{R}$ in (2.9) with $\rho(L;\theta) = (1 - L)^{d1+\theta 1} (1 + L^2)^{d2+\theta 2}$ and white noise u_t. (Japanese case)

			No intercept and no trend				Intercept				Intercept and a time trend		
d_1	d_2		c_t	y_t	c_t-y_t		c_t	y_t	c_t-y_t		c_t	y_t	c_t-y_t
0.50	0.50	131.72	106.24	107.21		85.25	52.79	113.57		55.44	93.01	137.12	
0.50	0.75		149.78	119.39	112.08		95.94	54.71	118.15		50.50	97.28	117.80
0.50	1.00		163.55	128.88	113.78		103.93	54.96	119.45		51.57	98.47	119.11
0.50	1.25		174.36	135.96	114.58		110.64	54.78	119.89		52.00	98.88	119.56
0.50	1.50		183.05	141.40	115.05		116.53	54.48	120.05		52.24	99.07	119.73
0.75	0.50		42.42	32.73	114.68		92.65	116.10	118.47		91.85	114.23	118.27
0.75	0.75		53.76	40.51	118.93		96.77	120.15	122.41		97.13	118.40	122.19
0.75	1.00		63.05	46.29	120.34		97.39	121.09	123.47		98.74	119.46	123.24
0.75	1.25		70.86	50.73	120.95		97.11	121.26	123.80		99.39	119.77	123.56
0.75	1.50		77.58	54.24	121.28		96.54	121.21	123.90		99.72	119.85	123.65
1.00	0.50		15.33	21.28	118.04		111.70	119.31	120.03		111.45	119.12	120.00
1.00	0.75		21.27	27.52	121.65		117.11	122.92	123.30		116.72	122.68	123.26
1.00	1.00		25.75	32.00	122.78		118.76	123.83	124.12		118.22	123.54	124.08
1.00	1.25		29.19	35.31	123.24		119.45	124.08	124.34		118.78	123.75	124.29
1.00	1.50		31.92	37.84	123.47		119.81	124.16	124.38		119.03	123.79	124.33
1.25	0.50		24.31	36.49	119.77		115.82	120.47	120.95		116.62	120.64	120.97
1.25	0.75		31.37	45.21	122.81		120.45	123.47	123.67		121.21	123.62	123.69
1.25	1.00		36.65	51.72	123.74		121.79	124.18	124.33		122.51	124.32	124.34
1.25	1.25		40.64	56.70	124.09		122.32	124.37	124.48		122.99	124.48	124.49
1.25	1.50		43.73	60.64	124.25		122.59	124.41	124.50		123.22	124.51	124.50
1.50	0.50		38.10	51.35	120.87		118.10	121.33	121.63		118.63	121.52	121.69
1.50	0.75		47.40	61.80	123.44		122.04	123.85	123.92		122.53	124.01	123.97
1.50	1.00		54.49	69.56	124.19		123.19	124.45	124.46		123.63	124.58	124.50
1.50	1.25		59.99	75.49	124.45		123.65	124.60	124.58		124.05	124.72	124.61
1.50	1.50		64.36	80.17	124.56		123.90	124.64	124.59		124.27	124.74	124.61

': Non-rejection values of the null hypothesis (1.12) at 95% significance level.

TABLE 4.3a

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 - L^2)^{d+\theta}$ and white noise u_t

(Japanese case)

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	4.42	1.78'	-0.47'	-1.91'	-2.75	-3.25	-3.58	-3.80
I	2.75	-4.04	-4.61	-4.75	-4.84	-4.88	-4.91	-4.92
I,T	-0.96'	-3.71	-4.58	-4.82	-4.89	-4.92	-4.94	-4.95
I,D	6.87	3.85	1.94'	-0.84	-2.80	-3.73	-4.14	-4.34
I,T,D	12.14	7.99	2.04	-1.93'	-3.49	-3.99	-4.21	-4.35

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	4.13	1.55'	-0.66'	-2.06	-2.87	-3.35	-3.67	-3.89
I	-1.23'	-4.72	-4.83	-4.87	-4.90	-4.92	-4.93	-4.94
I,T	-3.38	-4.51	-4.81	-4.89	-4.92	-4.94	-4.95	-4.96
I,D	6.57	0.44'	-2.84	-4.05	-4.55	-4.73	-4.79	-4.81
I,T,D	7.78	1.25'	-2.86	-4.28	-4.66	-4.72	-4.72	-4.74

c_t-y_t\d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-4.25	-4.63	-4.80	-4.87	-4.91	-4.93	-4.94	-4.95
I	-4.55	-4.81	-4.87	-4.89	-4.91	-4.92	-4.92	-4.93
I,T	-4.51	-4.79	-4.86	-4.89	-4.91	-4.92	-4.93	-4.94
I,D	-1.11'	-3.40	-4.20	-4.50	-4.63	-4.69	-4.73	-4.76
I,T,D	-1.14'	-3.39	-4.20	-4.50	-4.62	-4.66	-4.67	-4.69

': Non-rejection values of the null hypothesis (1.12) at 95% significance level. Letters in bold correspond to the cases where monotonicity in the value of the tests with respect to d is achieved.

c_t: Log of total consumption in Japan, 1961.1 to 1987.4
y_t: Log of disposable income in Japan, 1961.1 to 1987.4

--: No intercept, no trend and no seasonal dummies.
I: Intercept.
I,T: Intercept and trend.
I,D: Intercept and seasonal dummies.
I,T,D: Intercept, trend and seasonal dummies.

TABLE 4.3b

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 - L^2)^{d+\theta}$ and seasonal AR(K) u_t
(Japanese case)

K = 4

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	1.67'	0.96'	-0.52'	-2.00	-2.95	-3.50	-3.81	-4.01
I	0.50'	-1.25'	-1.74'	-2.10	-2.42	-2.67	-2.89	-3.09
I,T	0.95'	-0.84'	-1.73'	-2.18	-2.50	-2.76	-3.00	-3.21
I,D	1.91'	1.42'	-0.31'	-1.68'	-2.85	-3.48	-3.78	-3.95
I,T,D	3.15	1.32'	-0.29'	-2.23	-3.24	-3.61	-3.79	-3.93

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	1.41'	0.63'	-0.85'	-2.21	-3.05	-3.52	-3.80	-3.98
I	-0.18'	-2.44	-2.46	-2.58	-2.76	-2.94	-3.11	-3.26
I,T	-0.91'	-2.00	-2.39	-2.62	-2.83	-3.03	-3.22	-3.41
I,D	1.53'	-1.56'	-3.19	-3.66	-3.94	-4.11	-4.22	-4.30
I,T,D	0.09'	-2.01	-3.20	-3.76	-4.00	-4.10	-4.13	-4.14

c_t-y_t\d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-1.86'	-2.34	-2.69	-2.94	-3.13	-3.29	-3.42	-3.53
I	-2.56	-2.79	-2.90	-3.01	-3.13	-3.25	-3.37	-3.48
I,T	-2.42	-2.72	-2.88	-3.02	-3.16	-3.29	-3.41	-3.55
I,D	-2.28	-3.26	-3.70	-3.94	-4.10	-4.20	-4.28	-4.34
I,T,D	-2.28	-3.25	-3.70	-3.94	-4.10	-4.18	-4.22	-4.23

K = 8

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	1.69'	0.88'	-0.53'	-2.02	-2.99	-3.53	-3.84	-4.03
I	1.23'	-0.64'	-1.65'	-2.14	-2.57	-2.90	-3.16	-3.38
I,T	1.84'	-0.38'	-1.62'	-2.27	-2.70	-3.01	-3.28	-3.51
I,D	2.11	1.31'	-0.52'	-1.79'	-2.85	-3.47	-3.81	-4.02
I,T,D	3.64	1.45'	-0.51'	-2.26	-3.19	-3.58	-3.81	-3.98

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	1.49'	0.58'	-0.87'	-2.21	-3.05	-3.52	-3.81	-4.01
I	0.80'	-2.57	-2.65	-2.79	-3.02	-3.22	-3.41	-3.58
I,T	-0.65'	-2.06	-2.56	-2.85	-3.10	-3.31	-3.51	-3.71
I,D	1.78'	-1.53'	-3.37	-3.85	-4.13	-4.29	-4.40	-4.46
I,T,D	0.33'	-2.11	-3.39	-3.94	-4.18	-4.27	-4.29	-4.30

c_t-y_t\d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-1.71'	-2.39	-2.89	-3.23	-3.45	-3.61	-3.74	-3.85
I	-2.68	-3.01	-3.16	-3.31	-3.45	-3.58	-3.71	-3.82
I,T	-2.51	-2.93	-3.15	-3.32	-3.48	-3.61	-3.74	-3.89
I,D	-2.33	-3.33	-3.81	-4.06	-4.22	-4.33	-4.40	-4.45
I,T,D	-2.33	-3.33	-3.81	-4.06	-4.22	-4.30	-4.33	-4.34

': Non-rejection values of the null hypothesis (1.12) at 95% significance level. Letters in bold correspond to monotonocity with respect to d.

TABLE 4.3c

$\hat{R}$ in (2.9) with $\rho(L;\theta) = (1 - L)^{d1+\theta1} (1 + L)^{d2+\theta2}$ and white noise u_t. (Japanese case)

		No intercept and no time trend			Intercept			Intercept and time trend		
d_1	d_2	c_t	y_t	c_t-y_t	c_t	y_t	c_t-y_t	c_t	y_t	c_t-y_t
0.50	0.50	63.67	58.38	18.26	34.93	7.74	20.83	7.15	12.41	20.43
0.50	0.75	72.47	67.13	17.99	44.65	9.18	20.85	8.78	11.44	20.46
0.50	1.00	79.47	74.24	17.20	54.82	11.70	20.29	11.30	10.22	19.93
0.50	1.25	85.18	80.15	16.25	65.42	15.58	19.54	15.21	9.05	19.23
0.50	1.50	89.94	85.14	15.22	76.26	20.99	18.66	20.81	8.10	18.40
0.75	0.50	21.45	19.22	21.17	17.19	22.08	22.54	14.80	20.34	22.36
0.75	0.75	27.81	25.23	21.55	16.67	22.32	23.18	14.41	20.46	22.98
0.75	1.00	33.37	30.59	21.26	15.33	21.87	23.15	13.36	20.00	22.94
0.75	1.25	38.30	35.44	20.80	13.70	21.14	23.00	12.12	19.35	22.78
0.75	1.50	42.74	39.86	20.26	11.99	20.19	22.77	10.85	18.57	22.56
1.00	0.50	6.44	6.18	22.31	20.65	22.58	22.75	20.50	22.47	22.73
1.00	0.75	9.51	8.92	23.11	21.44	23.32	23.62	21.24	23.17	23.60
1.00	1.00	12.28	11.41	23.09	21.33	23.38	23.72	21.09	23.20	23.70
1.00	1.25	14.85	13.76	22.92	20.94	23.29	23.71	20.67	23.08	23.68
1.00	1.50	17.28	16.02	22.70	20.41	23.14	23.67	20.12	22.91	23.64
1.25	0.50	5.75'	6.22	22.60	21.39	22.70	22.72	21.80	22.84	22.73
1.25	0.75	7.42	7.73	23.69	22.60	23.65	23.79	23.08	23.81	23.81
1.25	1.00	8.73	8.86	23.82	22.76	23.79	23.95	23.31	23.98	23.97
1.25	1.25	9.84	9.80	23.77	22.65	23.77	23.96	23.28	23.99	23.99
1.25	1.50	10.86	10.64	23.68	22.45	23.72	23.95	23.16	23.96	23.98
1.50	0.50	7.58	8.19	22.55	21.63	22.67	22.56	21.92	22.80	22.58
1.50	0.75	9.01	9.59	23.91	23.20	23.84	23.86	23.55	24.00	23.89
1.50	1.00	9.97	10.48	24.13	23.53	24.06	24.08	23.93	24.24	24.11
1.50	1.25	10.63	11.03	24.15	23.55	24.08	24.11	24.00	24.29	24.16
1.50	1.50	11.11	11.39	24.11	23.49	24.05	24.11	23.99	24.28	24.16

': Non-rejection values for the null hypothesis (1.12) at 95% significance level.

TABLE 4.4a

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 - L)^{d+\theta}$ and white noise u_t

(Japanese case)

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	8.47	3.43	-0.37'	-2.49	-3.61	-4.27	-4.70	-4.99
I	3.17	-4.31	-4.61	-4.83	-5.02	-5.18	-5.33	-5.46
I,T	-1.51'	-3.93	-4.59	-4.85	-5.04	-5.19	-5.33	-5.46
I,D	12.74	3.01	-2.47	-4.54	-5.37	-5.68	-5.83	-5.93
I,T,D	16.98	5.30	-2.52	-4.86	-5.47	-5.69	-5.82	-5.91

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	7.35	2.47	-1.07'	-2.98	-3.98	-4.57	-4.95	-5.21
I	-2.71	-4.98	-5.11	-5.27	-5.42	-5.55	-5.67	-5.78
I,T	-4.03	-4.82	-5.10	-5.28	-5.43	-5.56	-5.68	-5.78
I,D	11.76	-0.13'	-3.38	-4.26	-4.62	-4.81	-4.96	-5.08
I,T,D	10.31	0.31'	-3.42	-4.35	-4.64	-4.79	-4.90	-5.00

c_t-y_t\d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-4.74	-5.09	-5.31	-5.47	-5.60	-5.72	-5.82	-5.91
I	-4.95	-5.16	-5.32	-5.47	-5.60	-5.71	-5.82	-5.91
I,T	-4.89	-5.14	-5.32	-5.47	-5.60	-5.72	-5.83	-5.91
I,D	-2.88	-4.56	-5.10	-5.35	-5.51	-5.63	-5.74	-5.82
I,T,D	-2.91	-4.56	-5.10	-5.35	-5.50	-5.60	-5.67	-5.73

': Non-rejection values of the null hypothesis (1.12) at 95% significance level. Letters in bold correspond to the cases of monotonicity with respect to d.

c_t: Log of total consumption in Japan, 1961.1 to 1987.4
y_t: Log of disposable income in Japan, 1961.1 to 1987.4

--: No intercept, no trend and no seasonal dummies.
I: Intercept.
I,T: Intercept and trend.
I,D: Intercept and seasonal dummies.
I,T,D: Intercept, trend and seasonal dummies.

TABLE 4.4b

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 - L)^{d+\theta}$ and seasonal AR(K) u_t
(Japanese case)

K = 4

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	5.44	2.82	-0.39'	-2.51	-3.66	-4.32	-4.75	-5.04
I	3.82	-1.27'	-2.71	-3.36	-3.97	-4.44	-4.82	-5.11
I,T	2.44	-1.16'	-2.62	-3.49	-4.13	-4.62	-4.97	-5.22
I,D	6.23	1.08'	-2.71	-4.05	-4.77	-5.18	-5.43	-5.60
I,T,D	5.09	0.33'	-2.81	-4.21	-4.84	-5.19	-5.42	-5.57

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	4.46	1.69'	-1.14'	-2.87	-3.84	-4.46	-4.88	-5.18
I	3.17	-3.51	-2.91	-3.41	-3.94	-4.38	-4.74	-5.03
I,T	-1.13'	-2.17	-2.85	-3.48	-4.06	-4.56	-4.92	-5.17
I,D	5.43	-1.67'	-3.90	-4.46	-4.78	-4.99	-5.14	-5.25
I,T,D	0.80'	-2.56	-3.98	-4.53	-4.80	-4.95	-5.02	-5.06

c_t-y_t\d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-1.60'	-2.64	-3.40	-3.96	-4.40	-4.74	-5.01	-5.23
I	-3.07	-3.17	-3.50	-3.95	-4.37	-4.72	-5.00	-5.22
I,T	-2.63	-3.04	-3.49	-3.96	-4.43	-4.83	-5.13	-5.33
I,D	-3.05	-4.08	-4.61	-4.94	-5.17	-5.33	-5.45	-5.53
I,T,D	-3.06	-4.08	-4.61	-4.94	-5.15	-5.26	-5.28	-5.27

K = 8

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	5.37	2.74	-0.40'	-2.52	-3.66	-4.32	-4.75	-5.05
I	5.27	-1.17'	-2.74	-3.44	-4.06	-4.54	-4.91	-5.20
I,T	2.96	-1.01'	-2.65	-3.57	-4.23	-4.72	-5.07	-5.32
I,D	6.53	1.35'	-2.69	-4.08	-4.82	-5.23	-5.48	-5.63
I,T,D	5.75	0.48'	-2.81	-4.23	-4.87	-5.23	-5.46	-5.61

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	4.61	1.74'	-1.04'	-2.78	-3.78	-4.42	-4.84	-5.14
I	4.30	-3.57	-2.96	-3.47	-4.01	-4.46	-4.82	-5.11
I,T	-1.15'	-2.20	-2.89	-3.54	-4.13	-4.64	-5.00	-5.26
I,D	5.78	-1.65'	-3.94	-4.48	-4.77	-4.97	-5.11	-5.22
I,T,D	0.99'	-2.63	-4.02	-4.54	-4.80	-4.94	-5.00	-5.04

c_t-y_t\d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-1.54'	-2.68	-3.48	-4.05	-4.49	-4.83	-5.11	-5.32
I	-3.12	-3.23	-3.58	-4.04	-4.46	-4.81	-5.10	-5.32
I,T	-2.66	-3.10	-3.56	-4.05	-4.52	-4.92	-5.22	-5.42
I,D	-3.09	-4.15	-4.68	-5.00	-5.22	-5.38	-5.49	-5.57
I,T,D	-3.11	-4.15	-4.67	-4.99	-5.20	-5.31	-5.34	-5.34

': Non-rejection values of the null hypothesis (1.12) at 95% significance level, and the letters in bold correspond to monotonicity with respect to d.

TABLE 4.4c

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 - L)^{d+\theta}$ and seasonal and non-seasonal AR(k,K) u_t. (Japanese case)

k = 1 & K = 4

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-7.17	-0.35'	-0.41'	-1.75'	-2.76	-3.47	-3.99	-4.39
I	5.64	2.51	0.44'	-0.49'	-1.42'	-2.23	-2.96	-3.60
I,T	7.42	2.66	0.54'	-0.74'	-1.79'	-2.70	-3.42	-4.00
I,D	-9.46	2.88	1.29'	-0.54'	-2.06	-3.15	-3.89	-4.41
I,T,D	-14.04	1.26'	1.21'	-0.87'	-2.24	-3.15	-3.83	-4.32

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-3.40	0.47'	-0.07'	-1.24'	-2.18	-2.92	-3.55	-4.07
I	7.14	-0.90'	0.35'	-0.30'	-1.07'	-1.81'	-2.50	-3.11
I,T	3.23	1.39'	0.44'	-0.42'	-1.34'	-2.26	-3.03	-3.60
I,D	-9.16	-1.56'	-3.73	-4.44	-4.86	-5.12	-5.27	-5.33
I,T,D	-7.65	-2.95	-3.84	-4.57	-4.93	-5.02	-4.90	-4.71

c_t-y_t\d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	2.21	0.89'	-0.13'	-0.95'	-1.66'	-2.29	-2.85	-3.34
I	-0.00'	0.07'	-0.29'	-0.93'	-1.60'	-2.25	-2.83	-3.35
I,T	0.76'	0.28'	-0.27'	-0.96'	-1.74'	-2.56	-3.25	-3.76
I,D	-0.69'	-2.04	-2.91	-3.52	-3.96	-4.28	-4.49	-4.59
I,T,D	-0.70'	-2.04	-2.91	-3.51	-3.91	-3.98	-3.75	-3.44

k = 2 & K = 4

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-8.45	-1.91'	-0.60'	-1.36'	-2.13	-2.71	-3.18	-3.57
I	8.21	5.94	3.59	2.26	0.84'	-0.59'	-2.05	-3.47
I,T	11.68	6.17	3.65	1.95'	0.33'	-1.31'	-2.82	-4.19
I,D	-3.80	-2.02	-2.29	-1.35'	-1.84'	-2.75	-3.62	-4.33
I,T,D	-2.90	-9.36	-2.48	-0.93'	-1.64'	-2.54	-3.38	-4.04

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-4.81	-1.68'	-0.80'	-1.26'	-1.80'	-2.33	-2.83	-3.31
I	11.30	1.65'	2.81	1.68'	0.34'	-1.01'	-2.36	-3.62
I,T	7.39	4.40	2.91	1.51'	-0.03'	-1.72'	-3.25	-4.49
I,D	-11.28	0.29'	-1.71'	-2.88	-3.79	-4.46	-4.92	-5.21
I,T,D	-12.83	-1.98	-1.87'	-3.07	-3.90	-4.28	-4.23	-4.05

c_t-y_t\d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	4.97	3.15	1.62'	0.26'	-1.00'	-2.20	-3.32	-4.33
I	2.24	2.14	1.41'	0.29'	-0.91'	-2.13	-3.29	-4.33
I,T	3.26	2.41	1.44'	0.24'	-1.14'	-2.66	-4.02	-5.07
I,D	-0.83'	-1.46'	-2.47	-3.38	-4.15	-4.75	-5.16	-5.38
I,T,D	-0.89'	-1.47'	-2.47	-3.37	-4.06	-4.25	-3.93	-3.51

cont...

k = 1 & K = 8

$c_t \setminus d$	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-7.55	-0.51'	-0.42'	-1.75'	-2.77	-3.48	-4.00	-4.40
I	8.73	2.91	0.54'	-0.51'	-1.53'	-2.41	-3.18	-3.85
I,T	8.55	3.15	0.66'	-0.80'	-1.94'	-2.91	-3.67	-4.26
I,D	-9.30	3.39	1.47'	-0.47'	-2.07	-3.14	-3.82	-4.27
I,T,D	-13.11	1.57'	1.34'	-0.80'	-2.20	-3.09	-3.71	-4.14

$y_t \setminus d$	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-2.90	0.62'	0.08'	-1.10'	-2.10	-2.91	-3.55	-4.06
I	9.00	-0.97'	0.35'	-0.34'	-1.16'	-1.95'	-2.66	-3.30
I,T	3.35	1.46'	0.46'	-0.47'	-1.44'	-2.41	-3.22	-3.81
I,D	-8.68	-1.52'	-3.88	-4.56	-4.95	-5.18	-5.30	-5.35
I,T,D	-7.51	-3.09	-3.99	-4.67	-5.00	-5.09	-4.97	-4.79

$c_t - y_t \setminus d$	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	2.56	1.00'	-0.15'	-1.04'	-1.79'	-2.44	-3.02	-3.51
I	0.03'	0.07'	-0.32'	-1.01'	-1.73'	-2.40	-3.00	-3.51
I,T	0.88'	0.31'	-0.30'	-1.05'	-1.88'	-2.73	-3.44	-3.94
I,D	-0.73'	-2.23	-3.15	-3.74	-4.15	-4.43	-4.60	-4.67
I,T,D	-0.75'	-2.23	-3.15	-3.73	-4.10	-4.15	-3.91	-3.62

k = 2 & K = 8

$c_t \setminus d$	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-8.85	-1.95	-0.62'	-1.34'	-2.00	-2.52	-2.95	-3.32
I	12.08	6.14	3.71	2.32	0.84'	-0.58'	-1.94'	-3.20
I,T	12.45	6.60	3.80	1.97	0.31'	-1.27'	-2.65	-3.83
I,D	-3.45	-0.58'	-1.42'	-1.21'	-1.73'	-2.51	-3.20	-3.71
I,T,D	-2.92	-5.68	-1.64'	-0.81'	-1.50'	-2.29	-2.95	-3.45

$y_t \setminus d$	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-3.74	-1.19'	-0.54'	-1.01'	-1.59'	-2.14	-2.65	-3.11
I	13.98	1.60'	2.50	1.54'	0.31'	-0.87'	-1.97	-2.97
I,T	6.91	4.05	2.70	1.37'	-0.03'	-1.46'	-2.71	-3.68
I,D	-11.73	0.48'	-1.81'	-2.98	-3.86	-4.46	-4.84	-5.04
I,T,D	-12.07	-2.01	-1.98	-3.16	-3.95	-4.30	-4.22	-4.02

$c_t - y_t \setminus d$	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	5.16	3.14	1.53'	0.20'	-0.95'	-1.98	-2.90	-3.68
I	2.23	2.04	1.31'	0.23'	-0.88'	-1.95'	-2.91	-3.73
I,T	3.26	2.33	1.34'	0.18'	-1.09'	-2.41	-3.54	-4.34
I,D	-0.82'	-1.54'	-2.50	-3.89	-3.88	-4.30	-4.54	-4.62
I,T,D	-0.89'	-1.54'	-2.50	-3.28	-3.82	-3.90	-3.57	-3.19

': Non-rejection values of the null hypothesis (1.12) at 95% significance level. Letters in bold correspond to the cases where monotonicity in the value of the tests with respect to d is achieved.

4.3.b The U.K. case

We analyze here the quarterly United Kingdom data set used in HEGY (1990). c_t is now the log of consumption expenditure on non-durables and y_t is logged personal disposable income for the time period 1955.1 through 1984.4. The data are shown in Figure 4.7 and we see that both individual series may have a random-walk character, implying that we would expect to find a unit root at zero frequency. We also observe in this figure that slopes are in both series slightly flatter after oil-crisis in 1973, suggesting the possibility of a structural break in that year[1]. In relation to seasonal pattern, it seems clear that c_t contains a much stronger and less changing seasonal pattern than y_t, although even the seasonal consumption pattern changes over the sample period. This can be better viewed in Figure 4.8 showing the four quarters for each time series. The conclusions obtained in HEGY (1990) were that c_t could be I(1) at each of the frequencies 0, $\pi/2$, π and $3\pi/2$ of a cycle (2π), so a plausible model for c_t would be

$$\phi(L)\,(1-L^4)\,c_t = \eta_t + \epsilon_t \qquad t=1,2,\ldots, \tag{46}$$

where ε_t is an iid process, η_t can be zero, but also any kind of deterministic process (as an intercept, a time trend or seasonal dummies), and $\phi(L)$ is the possible augmentation of the fourth difference of c_t. For y_t, their results suggested that income contained only two roots, one at zero frequency and other at frequency π, so the model would become

$$\phi(L)\,(1-L^2)\,y_t = \eta_t + \epsilon_t \qquad t=1,2,\ldots, \tag{47}$$

again for different specifications in η_t. Finally for c_t-y_t, they found evidence of four unit roots if the dummies were not introduced in the model, but two unit roots of form as in (47) if they were included.

Following the same line as in the Japanese case, we will present results of Robinson's (1994c) tests assuming first that the model specified in (1.10); (2.2) and (31) has four roots on the unit circle. In Figure 4.9 we plot the series, their sample autocorrelations and estimates of the spectral density after removing the unit root at

[1] Similarly to Japanese data, we will perform at the end of this chapter some of the tests for the two subsamples: 1955.1-1973.4 and 1974.1-1984.4 (in Appendix 4.1), and including dummy variables for the changing trend (in Appendix 4.2).

zero and at seasonal frequencies. We observe in this figure that the series might have a stationary appearance, though still observe significant autocorrelations at some lags in c_t and y_t. Tables 4.5a and 4.5b are analogous to Tables 4.1a and 4.1b above, showing results of one-sided statistic $\hat{r}$ in (2.9) when $\rho(L;\theta)$ takes the form given in (32). Table 4.5a gives results for white noise u_t, and we observe in this table that the monotonic decrease in $\hat{r}$ with respect to d is always achieved for all specifications in (31) and for the three series. For c_t and y_t, the null hypothesis is never rejected when d = 0.75 and d = 1, and also the case of d = 1.25 is not rejected when we include as regressors an intercept and dummies. For c_t-y_t, the values of d where H_o is not rejected are slightly smaller (d = 0.50 and d = 0.75), and in this series we see that the unit root null is clearly rejected in all cases in favour of less nonstationary alternatives, suggesting that if the two individual series were in fact I(1), a degree of fractional cointegration may exist for a given cointegration vector (1,-1). The fact that the unit root null is never rejected for c_t is consistent with HEGY (1990), however we observe that this hypothesis is either non-rejected for y_t, while HEGY (1990) found evidence of only two unit roots (at zero and at frequency π) in this series. Various tests of this hypothesis will be performed later in a further group of tests.

In Table 4.5b we take AR(k) u_t with k=1,2,3 and 4. Monotonicity is achieved in practically all cases and the unit root null is rejected in all situations across this table. The non-rejection values correspond to d = 0.50 and d = 0.75, and in those cases where the former is rejected, always is in favour of stationary alternatives. As we explained before for the Japanese case, this smaller degree of nonstationary (compared with Table 4.5a), could be in large part due to competition between integration orders and AR parameters in describing the nonstationary component. Allowing u_t to be seasonal or a mixed seasonal and non-seasonal AR, we observed few cases where monotonicity was achieved, suggesting that they were not a correct specification in this case since seasonality might be better explained either by quarterly integration or by seasonal dummies.

Table 4.5c gives results of the two-sided test statistic $\hat{R}$ in (2.9) when θ is (2x1). $\rho(L;\theta)$ is now given in (34) and therefore we allow different integration orders for the real and complex roots. If there are no regressors in the model, H_o is rejected in all cases for the individual series and the lowest statistic is achieved when

$d_1 = 1$ and $d_2 = 0.5$, indicating perhaps the importance of real roots over complex ones. For c_t-y_t, we observe in this table that all non-rejections correspond to values of d_2 (i.e. the integration order of the complex roots) smaller than d_1 (i.e. the integration order for the two real roots), and the lowest statistic is now obtained at $d_1 = 0.75$ and $d_2 = 0.50$. Including a constant or a time trend gives similar results in both cases: for c_t, all non-rejections occur when $d_1 = 1.00$, 1.25 or 1.50 and when $d_2 = 0.50$ and 0.75, with the lowest value obtained at $d_1 = 1$ and $d_2 = 0.5$. For y_t, we observe only three non-rejection cases corresponding to $d_1 = 1.00$, 1.25 and 1.50, with $d_2 = 0.50$, which might indicate that complex roots are not required when modelling this series, as was pointed out in HEGY (1990); for c_t-y_t, there are some more non-rejections, with the lowest statistic obtained at $d_1 = 0.75$ and $d_2 = 0.5$. Thus, we observe in all cases a greater degree of integration for real roots than for complex ones, and also smaller integration orders for c_t-y_t than for c_t and y_t.

Finally in this group of tables, we extend these tests to allow different integration orders at zero and at frequency π. In this case $\rho(L;\theta)$ takes the form given in (35) and results appear in Table 4.5d. They are consistent with the previous ones: in fact, when there are no regressors, the null is always rejected for c_t and y_t, while for c_t-y_t there are some non-rejections, with the lowest value achieved at $d_1 =1$ and $d_2 = d_3 = 0.50$, (i.e. the same alternative as in Table 4.5c). Including a constant or a time trend, the lowest statistic occurs when $d_1 = 1$ and $d_2 = d_3 = 0.50$ for c_t and c_t-y_t, and when $d_1 = 1.50$, $d_2 = 1.00$ and $d_3 = 0.50$ for y_t. All these results seem to emphasize the importance of the root at zero frequency over the others, given its greater integration order.

In the following group of tables we suppose x_t can be well modelled as an I(d) process with three roots and first, in Tables 4.6a and 4.6b, we show results for white noise u_t, excluding the root at frequency π. In Table 4.6a, $\rho(L;\theta)$ adopts the form in (37), and we observe few cases where the null is not rejected, corresponding to c_t and y_t when $d = 0.50$ or 0.75; however, looking at c_t-y_t, we see that the null is always rejected for all specifications in (31) in favour of stationary alternatives. The unit root null is rejected in all series for all cases considered, which is in line with HEGY (1990), who suggested the need of the unit root at frequency π for the three series. This can also be viewed through Figure 4.10 which shows plots of the series, their sample autocorrelations and estimates of the spectral density after removing the unit root at zero and at frequencies $\pi/2$ and $3\pi/2$, showing, especially at the latter, the importance of the root at frequency π when modelling in this way.

Results for two-sided tests when $\rho(L;\theta)$ is of form of (39) are given in Table 4.6b. We see that the null hypothesis is always rejected across this table. If there are no regressors, the lowest statistics are obtained when $d_1 = 1$ and $d_2 = 0.50$ for c_t and y_t, and when $d_1 = d_2 = 0.50$ for c_t-y_t, indicating once more the importance of the root at zero frequency for the individual series when modelling in this way, and including an intercept or a time trend, the lowest values appear when d_1 and d_2 are 0.50 or 0.75. Performing the tests when excluding the root at zero frequency resulted in rejecting the null in all cases. This was observed when using both the one-sided tests with $\rho(L;\theta)$ as in (38), and the two-sided ones with $\rho(L;\theta)$ as in (40). Thus, we could conclude by saying that real roots are, as in the Japanese case, both important when modelling these series.

In the next group of tables we suppose that x_t has only two roots: one at zero frequency and the other one corresponding to frequency π. Plots for the unit root case are given in Figure 4.11, and we observe that sample autocorrelations are still significant but smaller for y_t than for the other two series, and the estimates of the spectral density have a peak at frequency $\pi/2$ in all them, with larger values for c_t and c_t-y_t than for y_t. First we take $\rho(L;\theta)$ as in (41) so the same integration order is assumed at both frequencies. This way of specifying the model is interesting in view of results in HEGY (1990), who suggested that only two unit roots at these frequencies were present in y_t and in some cases for c_t-y_t. Results for white noise u_t are given in Table 4.7a. Monotonicity is now always achieved and the non-rejection values occur when $d = 0.75$ and 1 for c_t and y_t, and $d = 0.50$ for c_t-y_t, suggesting again the possibility of a fractional cointegration relationship at these two frequencies for the cointegrating vector (1,-1). The hypothesis of two unit roots ($d = 1$) is always rejected for c_t if we include regressors. These rejections are in line with HEGY (1990), who indicated that complex unit roots should be included. For y_t we observe that $d = 1$ is not rejected in 3 of the 5 possible specifications in (31) which is also consistent with HEGY (1990).

Allowing non-seasonal AR u_t, results varied widely depending on the order of the autoregression, and though we do not report results here, again the values of d where the null was not rejected were smaller for c_t-y_t than for c_t and y_t. Results for seasonal AR u_t are given in Table 4.7b. Monotonicity is always achieved and the non-rejections occur for values of d ranging between 0.50 and 1 for the individual series, but only the case of d = 0.50 is not rejected for c_t-y_t. We observe in this table more non-rejections for y_t than for the other two series when testing the unit root hypothesis which is once more consistent with HEGY (1990). Performing the tests when u_t was mixed seasonal and non-seasonal AR, we observed less cases where monotonicity was achieved, though the same conclusions as in the previous tables hold with non-rejection values for d smaller for c_t-y_t than for the individual series.

In Table 4.7c we allow integration orders to differ at zero and at frequency π, and thus, we take now $\rho(L;\theta)$ as in (42). If there are no regressors, the null is always rejected and the lowest statistic is obtained at d_1 = 1.25 and d_2 = 0.50 for the individual series, and at d_1 = 0.50 and d_2 = 1.50 for c_t-y_t, so if there are no regressors and x_t displays two real roots, the root at zero frequency appears more important than the seasonal one for the individual series, but the one at frequency π is the most important when modelling c_t-y_t. Including a constant or a time trend, results are consistent with those given in Table 4.7a. In that table we saw that the only non-rejection case for a model with an intercept or a time trend corresponded to y_t with d = 0.75. In Table 4.7c, this alternative is narrowly rejected but is not the case of d_1 = 0.75 and d_2 = 0.50, and in all the other situations, the null hypothesis is always rejected as was in Table 4.7a.

In the last group of tables we assume x_t has a single root located at zero frequency (in Tables 4.8a and 4.8b), at frequency π (in Table 4.8c), and finally we suppose x_t contains two complex roots corresponding to frequencies $\pi/2$ and $3\pi/2$ (in Table 4.8d). Thus, $\rho(L;\theta)$ takes the form in (43) in the first two tables. Plots of first differences of the series, their sample autocorrelations and estimates of the spectral density function are given in Figure 4.12. In this figure we observe that the seasonal component still remains in all of them, and though the unit root at zero frequency has been removed, the estimates of the spectral density still present large peaks at frequencies $\pi/2$ and π, more pronounced for c_t-y_t and c_t than for y_t. Starting with the case of white noise u_t (in Table 4.8a), as with the Japanese case, we observe that if there are no regressors, the unit root null is not rejected for c_t and y_t, but is strongly rejected for c_t-y_t in favour of stationary alternatives. There are few non-rejections in this table and they correspond to values of d ranging between 0.50 and 1 for the individual series; for c_t-y_t, the only two non-rejection cases occur at d = 0.50 if the dummies are included, but for the remaining specifications, this null is strongly rejected in favour of stationary alternatives. The fact that the unit root is rejected in this table for all series when some regressors are included in (31) can be consistent with HEGY (1990), who suggest the need of at least one seasonal unit root.

Allowing u_t to be non-seasonal AR, we observed in many cases a lack of monotonic decrease in the value of the test statistic with respect to d. This can be explained since seasonality is not described now by first differences, and seasonal dummies on their own do not seem to be sufficient to pick up this effect. To corroborate this, if u_t follows a seasonal AR process (in Table 4.8b), we see that monotonicity is always achieved. In this table, H_o is not rejected in some cases for c_t and y_t when d = 0.75 and d = 1. For c_t-y_t, only if the regressors include an intercept and the dummies, d = 0.50 is not rejected but in all other cases, this hypothesis is always strongly rejected in favour of stationary alternatives, suggesting again that at zero frequency, fractional cointegration might occur claiming the simplistic version of the PIH. Performing the tests with mixed seasonal and non-seasonal AR, monotonicity was achieved in some cases, with the non-rejections occurring in practically all cases when $d \geq 1$ for c_t and y_t, but when $d < 1$ for c_t-y_t.

Finally, Tables 4.8c and 4.8d give results for white noise u_t and $\rho(L;\theta)$ of forms given in (44) and (45) respectively. We observe in both tables that including regressors, the null is always rejected in all series due perhaps to exclusion of the root at zero frequency. The only non-rejection cases observed across these two tables correspond to c_t and y_t when there are no regressors, but is now difficult to distinguish here a proper integration order for the seasonal roots since the values of d where the null is not rejected vary widely in both series, from 0.50 through 1.50 in Table 4.8c, and from 0.50 through 1.25 in Table 4.8d.

To summarize the main results obtained in the U.K. case, we can say that if x_t is I(d) with four roots of the same order and u_t is white noise, the values of d where the null is not rejected range between 0.75 and 1 for the individual series and are slightly smaller for the difference c_t-y_t. If u_t follows an AR process, d ranges between 0.50 and 0.75 for the three series considered. Allowing different integration orders at each frequency, we observe that the root at zero frequency seems more important than the seasonal ones, and the seasonal root at frequency π appears also more important than the two complex ones corresponding to frequencies $\pi/2$ and $3\pi/2$. Modelling x_t as I(d) with three roots, results strongly reject the null when the excluded root corresponds to frequency zero. If the excluded root is the real seasonal π, results also reject the null in practically all cases, suggesting the importance of these two roots when modelling these series. If we take x_t as an I(d) process with two real roots, the model seems more appropriate for y_t than for c_t or c_t-y_t, which is in line with results in HEGY (1990). Finally, modelling x_t as fractionally integrated with a single root at zero frequency, the range of d where H_o is not rejected goes from 0.50 to 1 for the individual series but close to stationarity for c_t-y_t, but using a single seasonal root at frequency π or a pair of complex ones at frequencies $\pi/2$ and $3\pi/2$ seems inappropriate in view of the great proportion of rejections.

FIGURE 4.7

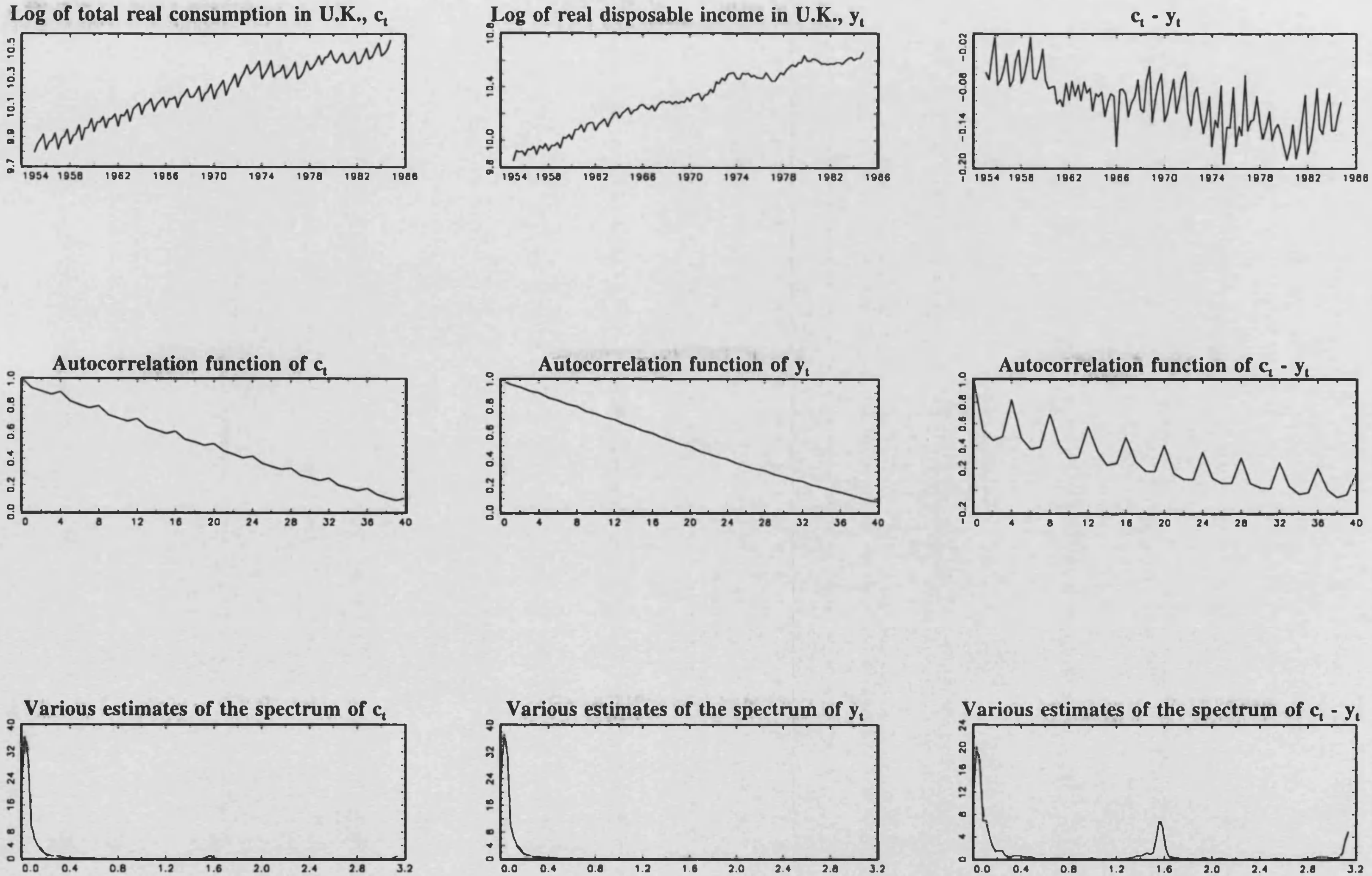
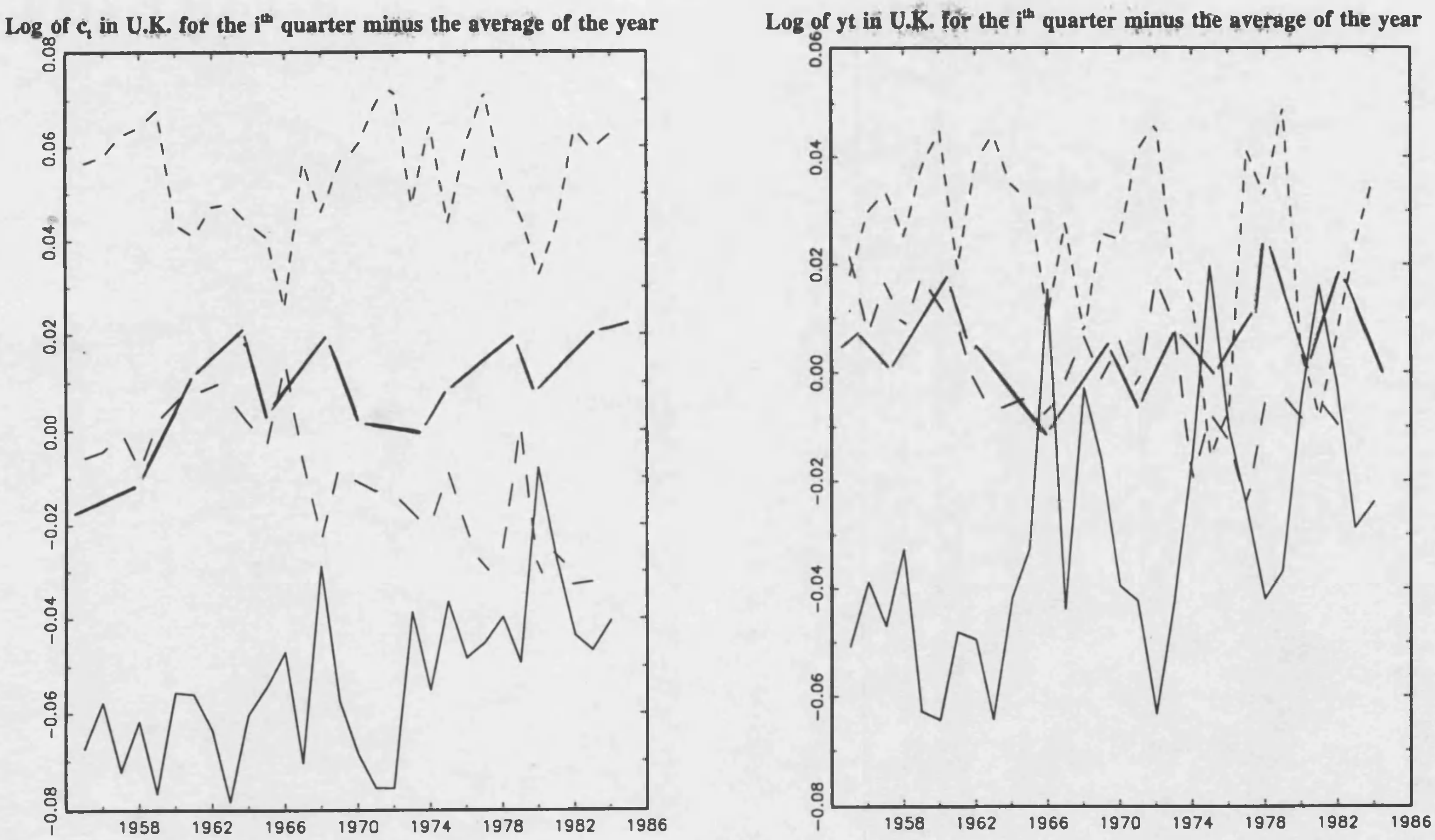
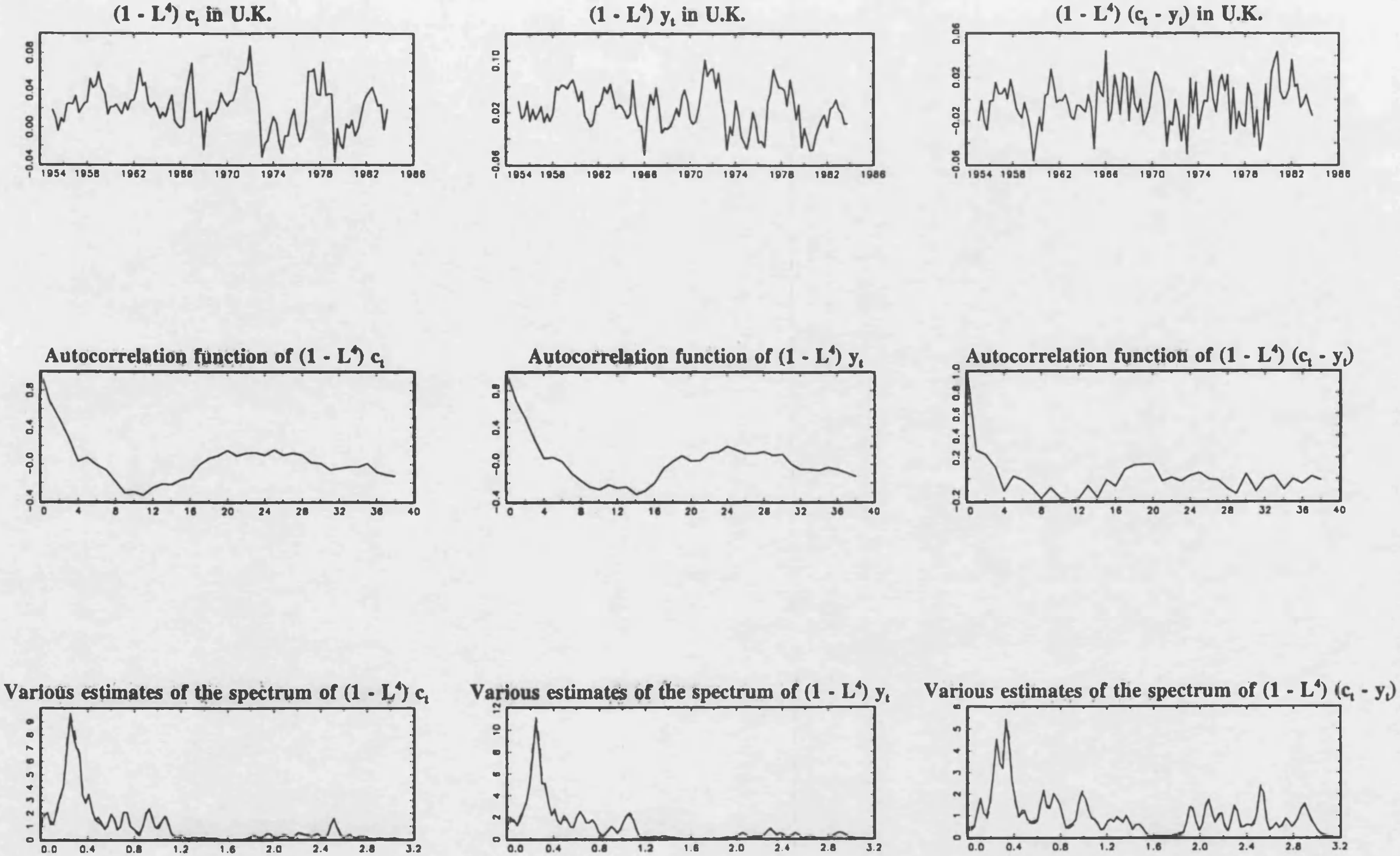

Log of total real consumption in U.K., c_t
Log of real disposable income in U.K., y_t
c_t - y_t
Autocorrelation function of c_t
Autocorrelation function of y_t
Autocorrelation function of c_t - y_t
Various estimates of the spectrum of c_t
Various estimates of the spectrum of y_t
Various estimates of the spectrum of c_t - y_t

FIGURE 4.8



Log of c_t in U.K. for the i^{th} quarter minus the average of the year
0.08
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
-0.08
1958 1962 1966 1970 1974 1978 1982 1986
Log of yt in U.K. for the i^{th} quarter minus the average of the year
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
-0.08
1958 1962 1966 1970 1974 1978 1982 1986

FIGURE 4.9



(1 - L⁴) c_t in U.K.
(1 - L⁴) y_t in U.K.
(1 - L⁴) (c_t - y_t) in U.K.
Autocorrelation function of (1 - L⁴) c_t
Autocorrelation function of (1 - L⁴) y_t
Autocorrelation function of (1 - L⁴) (c_t - y_t)
Various estimates of the spectrum of (1 - L⁴) c_t
Various estimates of the spectrum of (1 - L⁴) y_t
Various estimates of the spectrum of (1 - L⁴) (c_t - y_t)

FIGURE 4.10

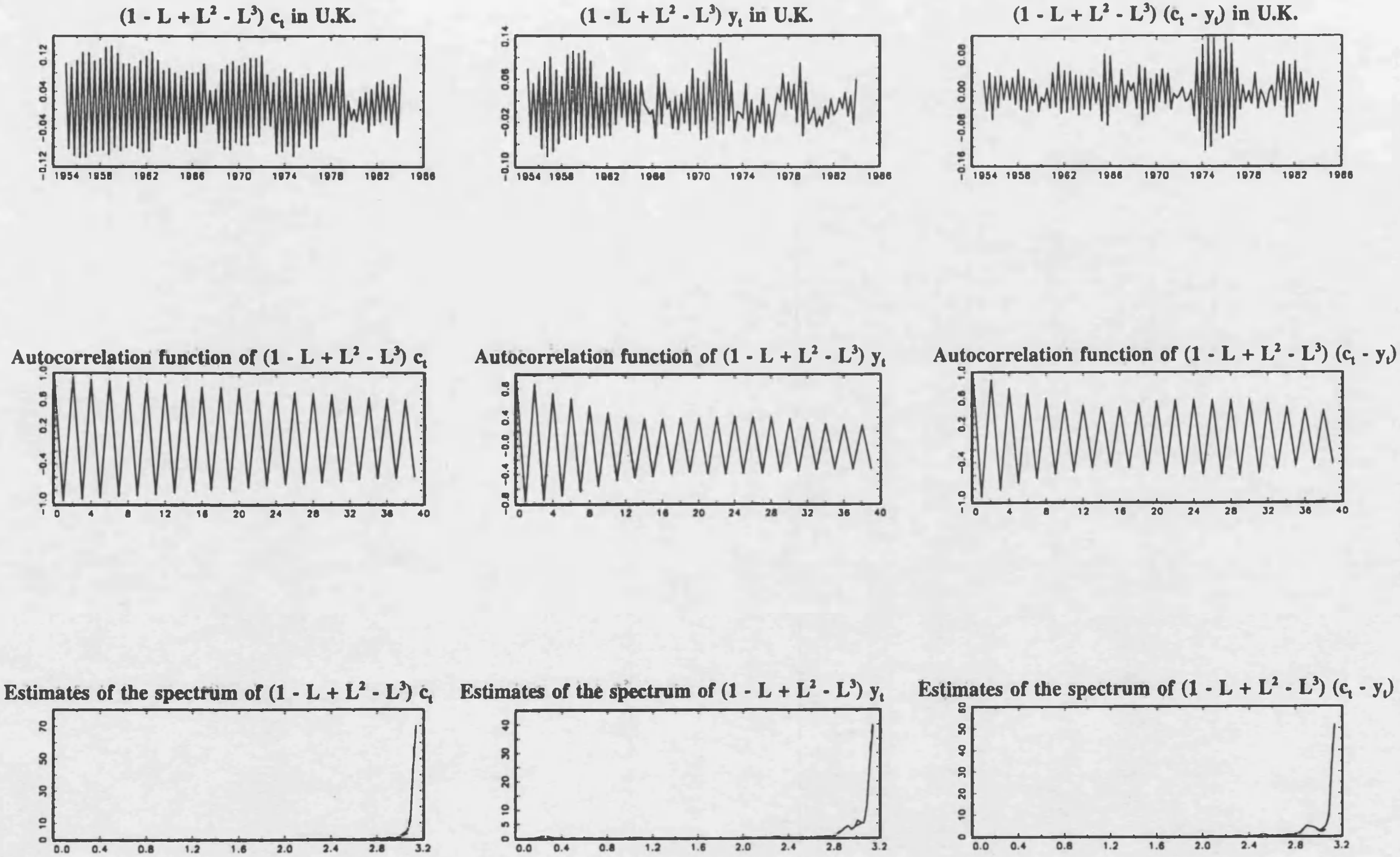

$(1 - L + L^2 - L^3)\ c_t$ in U.K.
$(1 - L + L^2 - L^3)\ y_t$ in U.K.
$(1 - L + L^2 - L^3)\ (c_t - y_t)$ in U.K.
Autocorrelation function of $(1 - L + L^2 - L^3)\ c_t$
Autocorrelation function of $(1 - L + L^2 - L^3)\ y_t$
Autocorrelation function of $(1 - L + L^2 - L^3)\ (c_t - y_t)$
Estimates of the spectrum of $(1 - L + L^2 - L^3)\ c_t$
Estimates of the spectrum of $(1 - L + L^2 - L^3)\ y_t$
Estimates of the spectrum of $(1 - L + L^2 - L^3)\ (c_t - y_t)$

FIGURE 4.11

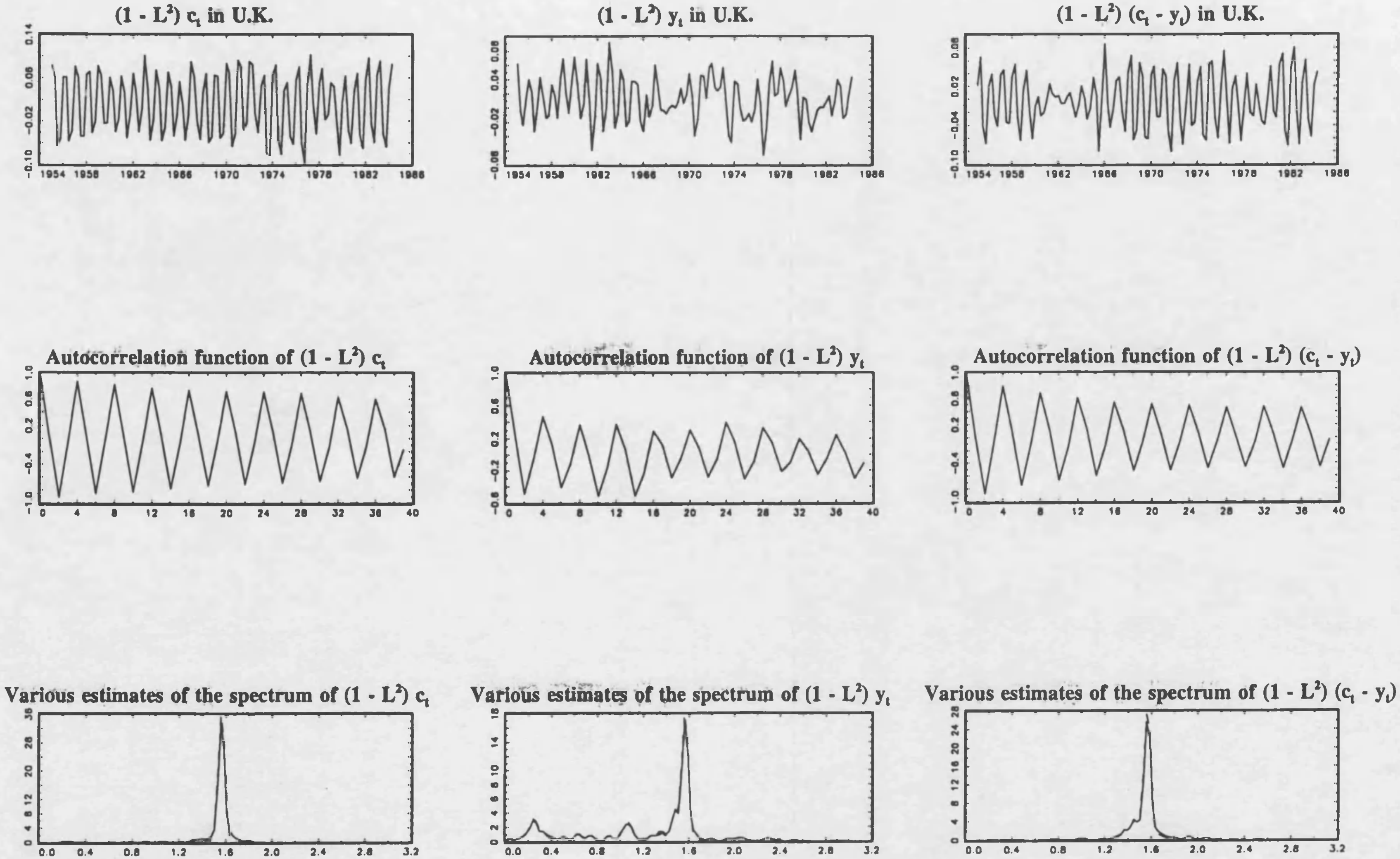

(1 - L²) c_t in U.K.
(1 - L²) y_t in U.K.
(1 - L²) (c_t - y_t) in U.K.
Autocorrelation function of (1 - L²) c_t
Autocorrelation function of (1 - L²) y_t
Autocorrelation function of (1 - L²) (c_t - y_t)
Various estimates of the spectrum of (1 - L²) c_t
Various estimates of the spectrum of (1 - L²) y_t
Various estimates of the spectrum of (1 - L²) (c_t - y_t)

FIGURE 4.12

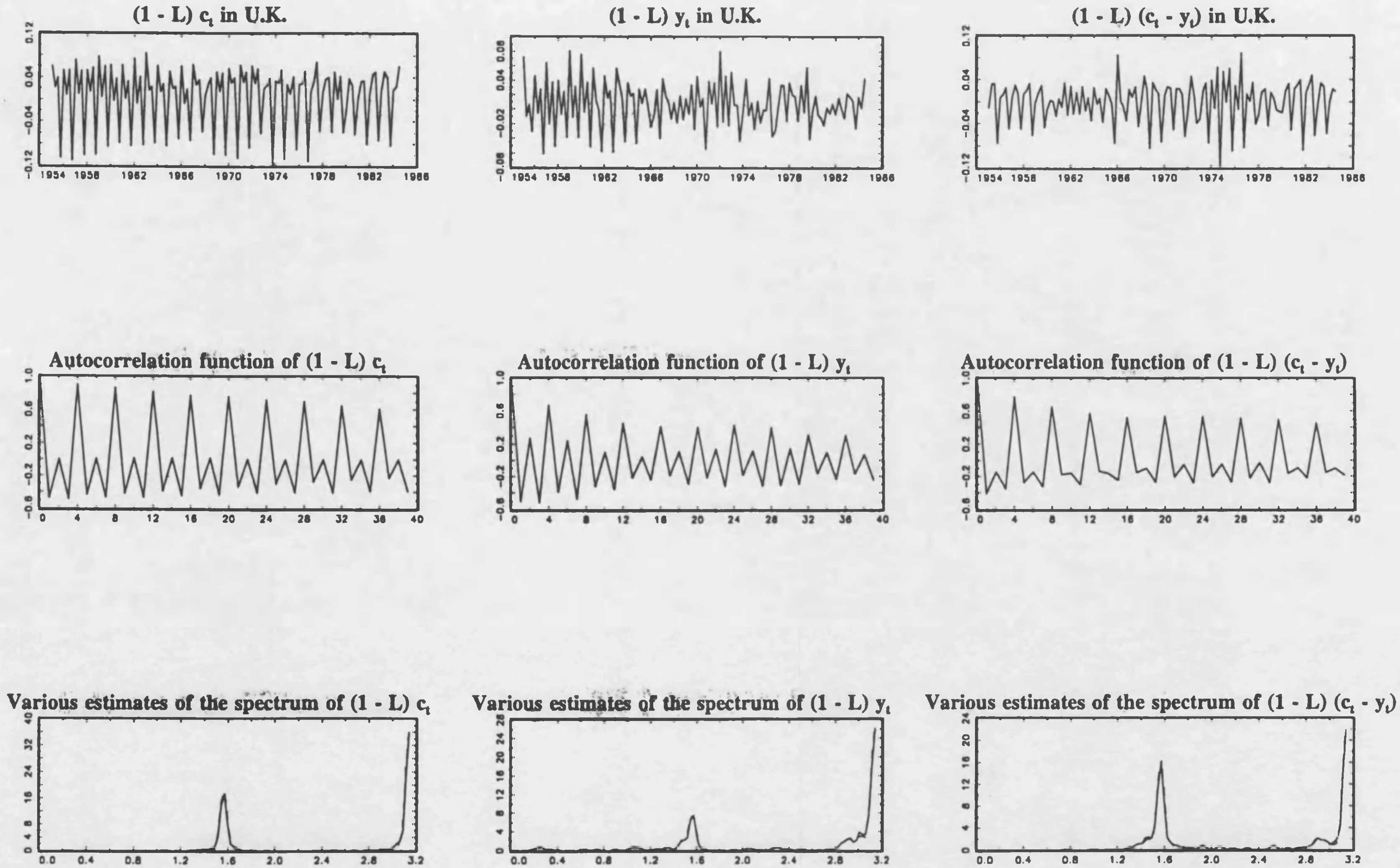

(1 - L) c_t in U.K.
(1 - L) y_t in U.K.
(1 - L) (c_t - y_t) in U.K.
Autocorrelation function of (1 - L) c_t
Autocorrelation function of (1 - L) y_t
Autocorrelation function of (1 - L) (c_t - y_t)
Various estimates of the spectrum of (1 - L) c_t
Various estimates of the spectrum of (1 - L) y_t
Various estimates of the spectrum of (1 - L) (c_t - y_t)

TABLE 4.5a

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 - L^4)^{d+\theta}$ and white noise u_t

(U.K. case)

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	3.31	1.02'	-1.00'	-2.43	-3.32	-3.88	-4.25	-4.51
I	5.09	1.31'	-1.11'	-2.00	-2.79	-3.42	-3.86	-4.18
I,T	2.65	0.41'	-1.26'	-2.33	-3.02	-3.46	-3.75	-3.99
I,D	5.17	1.32'	-1.09'	-1.87'	-2.62	-3.24	-3.70	-4.04
I,T,D	2.70	0.31'	-1.25'	-2.23	-2.87	-3.34	-3.72	-4.04

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	3.29	1.01'	-1.00'	-2.42	-3.31	-3.87	-4.24	-4.50
I	5.16	1.31'	-1.11'	-2.00	-2.79	-3.42	-3.86	-4.18
I,T	2.50	0.45'	-1.06'	-2.11	-2.84	-3.37	-3.76	-4.07
I,D	5.16	1.21'	-0.97'	-1.76'	-2.53	-3.16	-3.64	-4.00
I,T,D	2.41	0.39'	-1.06'	-2.06	-2.76	-3.28	-3.69	-4.02

c_t-y_t\d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-0.66'	-1.48'	-2.21	-2.84	-3.32	-3.69	-3.99	-4.24
I	1.09'	-1.37'	-2.39	-3.05	-3.53	-3.88	-4.15	-4.37
I,T	-0.20'	-1.44'	-2.39	-3.06	-3.53	-3.86	-4.11	-4.32
I,D	1.34'	-1.19'	-2.21	-2.89	-3.41	-3.79	-4.08	-4.32
I,T,D	-0.01'	-1.26'	-2.21	-2.92	-3.43	-3.82	-4.11	-4.35

': Non-rejection values of the null hypothesis (1.12) at 95% significance level, and the letters in bold correspond to the cases where monotonicity with respect to d is achieved.

c_t: Log of total consumption in U.K., from 1955.1 to 1984.4
y_t: Log of disposable income in U.K., from 1955.1 to 1984.4

--: No intercept, no time trend and no seasonal dummies.
I: Intercept.
I,T: Intercept and time trend.
I,D: Intercept and seasonal dummies.
I,T,D: Intercept, time trend and seasonal dummies.

TABLE 4.5b

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 - L^4)^{d+\theta}$ and AR(k) u_t. (U.K. case)

k = 1

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-3.26	-3.62	-3.96	-4.27	-4.52	-4.72	-4.87	-4.98
I	-0.84'	-0.78'	-2.10	-3.13	-3.76	-4.17	-4.44	-4.63
I,T	1.07'	-0.82'	-2.32	-3.25	-3.81	-4.16	-4.39	-4.55
I,D	-2.27	-2.65	-3.34	-3.75	-4.05	-4.29	-4.49	-4.65
I,T,D	-1.08'	-2.64	-3.38	-3.81	-4.10	-4.32	-4.50	-4.65

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-3.26	-3.62	-3.96	-4.27	-4.52	-4.71	-4.86	-4.98
I	-1.81'	-1.77'	-2.59	-3.32	-3.85	-4.23	-4.49	-4.69
I,T	-0.24'	-1.69'	-2.69	-3.40	-3.90	-4.25	-4.50	-4.68
I,D	-2.43	-2.52	-3.01	-3.47	-3.87	-4.18	-4.43	-4.62
I,T,D	-1.23'	-2.32	-2.99	-3.51	-3.90	-4.21	-4.44	-4.63

c_t-y_t\d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-0.86'	-1.85'	-2.60	-3.17	-3.59	-3.91	-4.17	-4.38
I	-0.30'	-1.79'	-2.66	-3.25	-3.69	-4.01	-4.25	-4.45
I,T	-0.62'	-1.80'	-2.66	-3.26	-3.69	-3.99	-4.22	-4.41
I,D	-0.29'	-1.67'	-2.52	-3.13	-3.58	-3.93	-4.20	-4.41
I,T,D	-0.57'	-1.69'	-2.52	-3.14	-3.60	-3.94	-4.21	-4.43

k = 2

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-3.30	-3.62	-3.91	-4.21	-4.48	-4.69	-4.85	-4.98
I	-1.11'	-1.10'	-2.25	-3.18	-3.77	-4.16	-4.42	-4.61
I,T	0.45'	-1.17'	-2.47	-3.32	-3.85	-4.18	-4.39	-4.54
I,D	-2.35	-2.80	-3.49	-3.88	-4.15	-4.36	-4.54	-4.68
I,T,D	-1.29'	-2.81	-3.53	-3.93	-4.20	-4.39	-4.55	-4.68

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-3.29	-3.61	-3.91	-4.21	-4.47	-4.68	-4.85	-4.98
I	-2.13	-2.27	-2.89	-3.47	-3.92	-4.26	-4.51	-4.69
I,T	-1.10'	-2.19	-2.96	-3.54	-3.97	-4.29	-4.51	-4.69
I,D	-2.62	-2.81	-3.20	-3.59	-3.92	-4.20	-4.43	-4.61
I,T,D	-1.79'	-2.64	-3.18	-3.61	-3.95	-4.23	-4.45	-4.62

c_t-y_t\d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-0.90'	-2.02	-2.79	-3.31	-3.69	-3.97	-4.20	-4.40
I	-0.68'	-1.99	-2.83	-3.39	-3.78	-4.07	-4.29	-4.47
I,T	-0.71'	-1.96	-2.82	-3.39	-3.78	-4.06	-4.27	-4.44
I,D	-0.69'	-1.90'	-2.72	-3.29	-3.70	-4.02	-4.26	-4.46
I,T,D	-0.67'	-1.88'	-2.71	-3.29	-3.71	-4.03	-4.28	-4.47

cont...

k = 3

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-3.35	-3.64	-3.88	-4.16	-4.44	-4.69	-4.88	-5.04
I	-1.13'	-1.10'	-2.30	-3.26	-3.85	-4.24	-4.50	-4.69
I,T	0.51'	-1.14'	-2.54	-3.43	-3.97	-4.29	-4.49	-4.63
I,D	-2.40	-2.88	-3.59	-3.97	-4.23	-4.45	-4.63	-4.78
I,T,D	-1.26'	-2.85	-3.62	-4.05	-4.31	-4.49	-4.64	-4.78

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-3.35	-3.63	-3.87	-4.16	-4.44	-4.68	-4.88	-5.04
I	-2.20	-2.40	-3.11	-3.70	-4.15	-4.49	-4.74	-4.92
I,T	-1.28'	-2.42	-3.22	-3.80	-4.23	-4.54	-4.75	-4.91
I,D	-2.70	-2.94	-3.38	-3.77	-4.11	-4.39	-4.63	-4.81
I,T,D	-1.97	-2.82	-3.37	-3.81	-4.17	-4.44	-4.66	-4.83

c_t-y_t\d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-0.94'	-2.06	-2.84	-3.37	-3.74	-4.02	-4.25	-4.44
I	-0.80'	-2.04	-2.87	-3.43	-3.82	-4.10	-4.32	-4.50
I,T	-0.77'	-2.01	-2.86	-3.43	-3.82	-4.09	-4.30	-4.47
I,D	-0.76'	-1.93'	-2.75	-3.33	-3.74	-4.06	-4.30	-4.48
I,T,D	-0.71'	-1.92'	-2.75	-3.33	-3.75	-4.07	-4.31	-4.50

k = 4

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-3.39	-3.59	-3.67	-3.80	-4.01	-4.23	-4.43	-4.59
I	-1.42'	-1.49'	-2.54	-3.37	-3.89	-4.22	-4.41	-4.53
I,T	-0.95'	-1.61'	-2.71	-3.56	-4.09	-4.37	-4.46	-4.47
I,D	-2.39	-2.80	-3.63	-4.03	-4.28	-4.46	-4.59	-4.67
I,T,D	-1.18'	-2.81	-3.67	-4.16	-4.43	-4.55	-4.62	-4.67

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-3.39	-3.60	-3.68	-3.81	-4.02	-4.23	-4.43	-4.59
I	-2.16	-2.19	-3.01	-3.54	-3.88	-4.12	-4.29	-4.41
I,T	-1.68'	-2.59	-3.22	-3.69	-4.03	-4.25	-4.38	-4.44
I,D	-2.71	-2.93	-3.45	-3.75	-3.98	-4.16	-4.30	-4.40
I,T,D	-2.29	-3.06	-3.50	-3.83	-4.08	-4.25	-4.36	-4.43

c_t-y_t\d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-3.19	-3.27	-3.60	-3.78	-3.88	-3.94	-4.00	-4.07
I	-2.67	-2.91	-3.37	-3.63	-3.79	-3.90	-3.98	-4.06
I,T	-3.01	-3.06	-3.40	-3.65	-3.79	-3.87	-3.93	-4.00
I,D	-1.72'	-2.43	-3.00	-3.34	-3.58	-3.77	-3.92	-4.04
I,T,D	-1.93'	-2.58	-3.03	-3.35	-3.60	-3.80	-3.96	-4.09

': Non-rejection values of the null hypothesis (1.12) at 95% significance level, and the letters in bold correspond to monotonicity across the values of d.

TABLE 4.5c

$\hat{R}$ in (2.9) with $\rho(L;\theta) = (1 - L^2)^{d1+\theta1}\,(1 + L^2)^{d2+\theta2}$ and white noise u_t. (U.K. case)

		No intercept and no trend			Intercept			Intercept and time trend		
d_1	d_2	c_t	y_t	c_t-y_t	c_t	y_t	c_t-y_t	c_t	y_t	c_t-y_t
0.50	0.50	52.45	52.15	3.42'	79.34	83.17	11.36	33.55	40.66	3.65'
0.50	0.75	60.69	60.37	9.92	88.99	91.84	22.06	46.54	48.22	10.31
0.50	1.00	67.35	66.99	14.87	96.04	99.10	31.11	54.20	53.60	15.80
0.50	1.25	72.87	72.47	18.35	102.02	105.50	38.64	59.62	57.75	19.94
0.50	1.50	77.53	77.09	20.95	107.41	111.28	45.04	63.87	61.09	23.15
0.75	0.50	19.80	19.76	1.05'	12.96	18.85	0.86'	7.51	14.80	0.86'
0.75	0.75	25.89	25.85	5.65'	23.48	26.73	4.90'	16.69	21.37	4.82'
0.75	1.00	31.25	31.19	10.25	31.01	33.24	9.40	23.30	26.30	9.24
0.75	1.25	36.06	35.98	13.73	36.87	38.92	13.17	28.11	30.26	12.80
0.75	1.50	40.45	40.34	16.43	41.85	44.05	16.36	31.94	33.59	15.69
1.00	0.50	8.31	8.29	2.03'	0.86'	5.43'	2.76'	1.03'	5.61'	2.75'
1.00	0.75	11.56	11.57	4.20'	6.07	10.23	4.48'	6.47	10.40	4.46'
1.00	1.00	14.42	14.44	7.73	11.13	14.03	7.61	11.48	14.06	7.59
1.00	1.25	17.08	17.10	10.62	14.86	17.17	10.23	15.03	17.03	10.22
1.00	1.50	19.61	19.62	12.90	17.78	19.92	12.30	17.74	19.60	12.30
1.25	0.50	8.60	8.55	4.99'	0.98'	3.89'	5.88'	1.36'	4.47'	5.91'
1.25	0.75	10.58	10.56	5.34'	4.14'	7.44	6.20	4.78'	7.98	6.26
1.25	1.00	12.05	12.04	7.84	8.23	10.23	8.52	8.93	10.61	8.58
1.25	1.25	13.24	13.24	10.04	11.18	12.46	10.57	11.71	12.59	10.63
1.25	1.50	14.30	14.31	11.73	13.34	14.42	12.07	13.60	14.27	12.13
1.50	0.50	11.09	11.01	8.22	2.96'	5.40'	8.93	3.22'	6.04	8.89
1.50	0.75	12.97	12.92	7.49	5.14'	8.19	8.41	5.57'	8.93	8.37
1.50	1.00	14.16	14.12	9.30	8.68	10.28	10.22	9.35	11.04	10.20
1.50	1.25	14.90	14.87	11.08	11.10	11.67	12.03	11.89	12.38	12.04
1.50	1.50	15.39	15.36	12.34	12.54	12.76	13.28	13.35	13.35	13.31

': Non-rejection values for the null hypothesis (1.12) at 95% significance level.

TABLE 4.5d

$\hat{R}$ in (2.9) with $\rho(L;\theta) = (1 - L)^{d1+\theta 1} (1 + L)^{d2+\theta 2} (1 + L^2)^{d3+\theta 3}$ and white noise u_t. (U.K. case)

			No intercept and no trend			Intercept			Intercept and time trend		
d_1	d_2	d_3	c_t	y_t	c_t-y_t	c_t	y_t	c_t-y_t	c_t	y_t	c_t-y_t
0.50	0.50	0.50	127.05	126.62	10.53	164.90	171.34	28.14	76.44	95.29	11.08
0.50	0.50	1.00	152.82	152.31	26.92	193.94	198.38	59.74	112.61	117.81	28.76
0.50	0.50	1.50	169.81	169.18	35.71	212.63	218.33	81.57	127.96	130.52	39.38
0.50	1.00	0.50	142.22	141.65	26.75	184.11	191.31	59.23	104.44	118.39	29.54
0.50	1.00	1.00	165.31	164.67	53.77	209.65	215.12	105.01	142.48	139.65	59.23
0.50	1.00	1.50	180.43	179.68	67.65	226.66	232.99	133.04	158.39	151.31	75.56
0.50	1.50	0.50	150.03	149.37	37.56	196.00	203.51	80.41	117.98	128.19	42.68
0.50	1.50	1.00	170.47	169.75	65.60	218.48	224.71	126.01	150.78	146.37	73.77
0.50	1.50	1.50	184.05	183.24	78.90	234.23	241.05	151.84	164.90	156.38	89.06
1.00	0.50	0.50	21.14	21.23	2.00'	2.11'	7.68'	3.10'	2.15'	7.91	3.05'
1.00	0.50	1.00	32.90	33.08	11.08	13.72	18.10	12.88	13.78	18.22	12.76
1.00	0.50	1.50	42.95	43.14	17.44	21.12	25.66	19.76	20.99	25.47	19.62
1.00	1.00	0.50	34.51	34.56	4.70'	11.11	23.34	4.20'	11.61	24.20	4.21'
1.00	1.00	1.00	50.50	50.61	14.55	35.02	42.05	11.58	35.77	42.70	11.60
1.00	1.00	1.50	63.55	63.64	23.00	49.17	55.45	18.64	49.29	55.41	18.68
1.00	1.50	0.50	43.38	43.39	9.64	19.96	35.22	8.32	20.30	35.77	8.33
1.00	1.50	1.00	59.88	59.92	27.72	49.19	56.68	23.42	49.65	56.71	23.42
1.00	1.50	1.50	72.94	72.96	41.97	64.53	70.88	37.01	64.43	70.11	36.92
1.50	0.50	0.50	11.07	10.99	9.41	8.22	12.24	10.38	8.67	12.65	10.37
1.50	0.50	1.00	14.13	14.11	26.61	28.72	28.64	27.95	29.62	29.17	27.95
1.50	0.50	1.50	15.38	15.37	38.31	41.79	42.13	39.74	42.71	42.74	39.74
1.50	1.00	0.50	15.57	15.54	6.04'	2.54'	6.03'	6.53'	2.62'	6.32'	6.50'
1.50	1.00	1.00	21.47	21.53	9.41	8.79	11.69	10.53	8.87	11.84	10.52
1.50	1.00	1.50	25.43	25.52	13.63	13.54	15.15	15.15	13.47	14.94	15.17
1.50	1.50	0.50	20.77	20.74	8.93	6.09'	12.07	9.28	6.03'	12.23	9.24
1.50	1.50	1.00	29.37	29.42	11.63	19.63	23.26	11.43	19.50	23.13	11.42
1.50	1.50	1.50	35.65	35.72	16.43	29.30	31.77	15.58	28.84	31.08	15.60

': Non-rejection values of the null hypothesis (1.12) at 95% significance level.

TABLE 4.6a

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 - L + L^2 - L^3)^{d+\theta}$ and white noise u_t.

(U.K. case)

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	6.41	1.90'	-2.64	-6.04	-8.15	-9.36	-10.05	-10.44
I	0.35'	-10.39	-11.20	-11.38	-11.48	-11.53	-11.57	-11.59
I,T	-8.32	-10.51	-11.16	-11.38	-11.48	-11.54	-11.57	-11.59
I,D	8.42	-2.38	-8.04	-9.80	-10.63	-11.00	-11.18	-11.26
I,T,D	1.58'	-4.47	-8.11	-9.86	-10.65	-11.01	-11.17	-11.26

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	6.39	1.91'	-2.64	-6.04	-8.16	-9.37	-10.06	-10.45
I	5.36	-7.41	-9.87	-10.65	-11.08	-11.30	-11.41	-11.47
I,T	-2.70	-7.59	-9.78	-10.68	-11.09	-11.30	-11.41	-11.47
I,D	7.67	-3.58	-8.37	-10.00	-10.77	-11.12	-11.28	-11.37
I,T,D	0.77'	-4.94	-8.40	-10.05	-10.79	-11.13	-11.29	-11.37

c_t-y_t\d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-8.09	-9.94	-10.83	-11.21	-11.38	-11.46	-11.50	-11.52
I	-7.06	-10.01	-10.86	-11.21	-11.36	-11.44	-11.48	-11.50
I,T	-7.96	-10.03	-10.86	-11.21	-11,36	-11.44	-11.48	-11.50
I,D	-3.04	-7.68	-9.46	-10.32	-10.75	-10.97	-11.09	-11.16
I,T,D	-4.34	-7.67	-9.46	-10.32	-10.75	-10.97	-11.09	-11.15

': Non-rejection values of the null hypothesis (1.12) at 95% significance level, and the letters in bold correspond to the cases where monotonicity with respect to d is achieved.

c_t: Log of total consumption in U.K., from 1955.1 to 1984.4
y_t: Log of disposable income in U.K., from 1955.1 to 1984.4

--: No intercept, no time trend and no seasonal dummies.
I: Intercept.
I,T: Intercept and time trend.
I,D: Intercept and seasonal dummies.
I,T,D: Intercept, time trend and seasonal dummies.

TABLE 4.6b

$\hat{R}$ in (2.9) with $\rho(L;\theta) = (1 - L)^{d1+\theta 1} (1 + L^2)^{d2+\theta 2}$ and white noise u_t. (U.K. case)

		No intercept and no trend			Intercept			Intercept and a time trend		
d_1	d_2	c_t	y_t	c_t-y_t	c_t	y_t	c_t-y_t	c_t	y_t	c_t-y_t
0.50	0.50	170.02	169.51	67.67	65.47	155.32	54.54	65.47	155.32	54.54
0.50	0.75	192.35	191.80	80.49	73.99	173.08	64.15	73.99	173.08	64.15
0.50	1.00	209.87	209.22	86.55	79.74	187.54	68.41	79.74	187.54	68.41
0.50	1.25	224.08	223.33	89.88	84.32	199.93	70.54	84.32	199.93	70.54
0.50	1.50	235.89	235.04	91.97	88.24	210.84	71.71	88.24	210.84	71.71
0.75	0.50	51.35	51.42	86.72	104.81	60.14	88.31	104.81	60.14	88.31
0.75	0.75	64.85	64.97	100.30	110.45	64.14	101.52	110.45	64.14	101.52
0.75	1.00	76.12	76.22	106.66	112.02	65.63	107.61	112.02	65.63	107.61
0.75	1.25	85.76	85.82	110.15	112.51	66.17	110.89	112.51	66.17	110.89
0.75	1.50	94.19	94.21	112.34	112.64	66.28	112.90	112.64	66.28	112.90
1.00	0.50	16.48	16.50	99.53	118.83	89.13	100.58	118.83	89.13	100.58
1.00	0.75	22.94	23.03	112.41	124.78	96.27	113.20	124.78	96.27	113.20
1.00	1.00	27.85	27.96	118.28	126.79	100.15	118.95	126.79	100.15	118.95
1.00	1.25	31.68	31.78	121.45	127.76	102.69	122.06	127.76	102.69	122.06
1.00	1.50	34.75	34.85	123.40	128.38	104.56	123.98	128.38	104.56	123.98
1.25	0.50	25.48	25.45	107.22	123.13	101.91	107.50	123.13	101.91	107.50
1.25	0.75	32.89	32.95	118.75	128.18	108.90	118.75	128.18	108.90	118.75
1.25	1.00	38.39	38.48	123.88	129.83	112.69	123.78	129.83	112.69	123.78
1.25	1.25	42.53	42.62	126.61	130.63	115.16	126.48	130.63	115.16	126.48
1.25	1.50	45.72	45.81	128.27	131.12	116.94	128.14	131.12	116.94	128.14
1.50	0.50	40.53	40.55	112.17	125.92	109.96	112.07	125.92	109.96	112.07
1.50	0.75	50.42	50.54	122.24	130.23	116.46	121.88	130.23	116.46	121.88
1.50	1.00	57.92	58.08	126.64	131.67	119.99	126.21	131.67	119.99	126.21
1.50	1.25	63.72	63.89	128.96	132.38	122.28	128.53	132.38	122.28	128.53
1.50	1.50	68.32	68.49	130.37	132.85	123.90	129.95	132.85	123.95	129.95

': Non-rejection values of the null hypothesis (1.12) at 95% level.

TABLE 4.7a

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 - L^2)^{d+\theta}$ and white noise u_t

(U.K. case)

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	5.23	2.04	-0.47'	-2.00	-2.87	-3.38	-3.72	-3.95
I	2.06	-4.26	-4.74	-4.86	-4.95	-5.01	-5.04	-5.06
I,T	-3.21	-4.30	-4.71	-4.89	-4.98	-5.03	-5.06	-5.09
I,D	7.14	0.17'	-2.49	-3.40	-3.98	-4.33	-4.53	-4.66
I,T,D	2.60	-0.66'	-2.50	-3.48	-4.03	-4.34	-4.54	-4.66

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	5.18	2.00	-0.51'	-2.03	-2.89	-3.40	-3.74	-3.97
I	6.47	-0.69'	-2.81	-3.64	-4.16	-4.47	-4.65	-4.76
I,T	1.99	-1.05'	-2.80	-3.72	-4.23	-4.49	-4.65	-4.76
I,D	7.52	1.52'	-1.16'	-2.38	-3.23	-3.75	-4.07	-4.28
I,T,D	4.09	0.96'	-1.18'	-2.50	-3.29	-3.78	-4.08	-4.28

c_t-y_t\d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-3.97	-4.47	-4.77	-4.93	-5.01	-5.05	-5.07	-5.08
I	-3.11	-4.35	-4.70	-4.86	-4.94	-4.98	-5.01	-5.03
I,T	-3.76	-4.40	-4.70	-4.86	-4.94	-4.99	-5.02	-5.04
I,D	-0.54'	-3.03	-3.84	-4.27	-4.51	-4.66	-4.75	-4.82
I,T,D	-1.64'	-3.06	-3.85	-4.27	-4.51	-4.66	-4.75	-4.81

': Non-rejection values of the null hypothesis (1.12) at 95% significance level, and the letters in bold correspond to the cases where monotonicity in the value of the tests with respectto d is achieved.

c_t: Log of total consumption in U.K., from 1955.1 to 1984.4
y_t: Log of disposable income in U.K., from 1955.1 to 1984.4

--: No intercept, no time trend and no seasonal dummies.
I: Intercept.
I,T: Intercept and time trend.
I,D: Intercept and seasonal dummies.
I,T,D: Intercept, time trend and seasonal dummies.

TABLE 4.7b

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 - L^2)^{d+\theta}$ and seasonal AR(K) u_t. (U.K. case)

K = 4

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	1.93’	1.17’	-0.49’	-2.09	-3.10	-3.66	-3.99	-4.19
I	0.59’	-1.39’	-2.26	-2.65	-2.92	-3.11	-3.27	-3.40
I,T	-0.77’	-1.68’	-2.28	-2.67	-2.94	-3.15	-3.34	-3.51
I,D	1.87’	-0.54’	-2.40	-3.13	-3.59	-3.87	-4.05	-4.17
I,T,D	0.80’	-1.12’	-2.41	-3.17	-3.60	-3.88	-4.05	-4.18

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	1.91’	1.13’	-0.54’	-2.12	-3.12	-3.69	-4.01	-4.20
I	1.55’	-0.99’	-2.27	-2.87	-3.26	-3.51	-3.68	-3.80
I,T	0.56’	-1.26’	-2.44	-3.13	-3.53	-3.77	-3.94	-4.06
I,D	2.05	0.55’	-1.46’	-2.59	-3.39	-3.88	-4.16	-4.33
I,T,D	1.95’	0.17’	-1.49’	-2.69	-3.45	-3.90	-4.17	-4.34

c_t-y_t\d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-2.14	-2.76	-3.25	-3.58	-3.80	-3.93	-4.02	-4.08
I	-1.97	-2.71	-3.09	-3.34	-3.51	-3.63	-3.74	-3.83
I,T	-2.19	-2.81	-3.19	-3.42	-3.58	-3.71	-3.81	-3.91
I,D	-1.35’	-2.92	-3.54	-3.91	-4.14	-4.29	-4.40	-4.48
I,T,D	-1.86’	-2.92	-3.54	-3.91	-4.14	-4.29	-4.40	-4.48

K = 8

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	1.94’	1.08’	-0.49’	-2.11	-3.14	-3.71	-4.03	-4.22
I	1.46’	-0.87’	-2.01	-2.65	-3.09	-3.39	-3.60	-3.75
I,T	-0.19’	-1.15’	-2.00	-2.66	-3.13	-3.46	-3.71	-3.92
I,D	2.08	-0.45’	-2.14	-2.90	-3.47	-3.85	-4.09	-4.26
I,T,D	0.74’	-0.99’	-2.15	-2.93	-3.47	-3.84	-4.09	-4.26

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	1.92’	1.05’	-0.54’	-2.15	-3.17	-3.73	-4.04	-4.23
I	2.64	-1.07’	-2.76	-3.57	-4.15	-4.52	-4.74	-4.88
I,T	0.54’	-0.91’	-1.95’	-2.72	-3.28	-3.67	-3.93	-4.12
I,D	2.24	0.54’	-1.50’	-2.59	-3.38	-3.87	-4.16	-4.34
I,T,D	1.86’	0.10’	-1.53’	-2.70	-3.43	-3.89	-4.17	-4.35

c_t-y_t\d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-1.91’	-2.65	-3.30	-3.75	-4.02	-4.19	-4.29	-4.34
I	-2.04	-3.03	-3.56	-3.89	-4.09	-4.22	-4.32	-4.40
I,T	-1.95’	-2.74	-3.28	-3.62	-3.84	-3.98	-4.10	-4.19
I,D	-1.34’	-2.91	-3.54	-3.92	-4.16	-4.31	-4.42	-4.49
I,T,D	-1.92’	-2.91	-3.54	-3.92	-4.16	-4.31	-4.41	-4.48

’: Non-rejection values of the null hypothesis (1.12) at 95% significance level. Letters in bold correspond to the cases where monotonocity in the value of the tests with respect to d is achieved.

TABLE 4.7c

$\hat{R}$ in (2.9) with $\rho(L;\theta) = (1 - L)^{d1+\theta 1} (1 + L)^{d2+\theta 2}$ and white noise u_t. (U.K. case)

		No intercept and no trend			Intercept			Intercept and a time trend		
d_1	d_2	c_t	y_t	c_t-y_t	c_t	y_t	c_t-y_t	c_t	y_t	c_t-y_t
0.50	0.50	81.29	80.38	16.06	26.81	101.13	10.85	11.79	25.48	14.58
0.50	0.75	91.34	90.37	16.37	36.50	115.60	10.93	11.41	34.07	14.88
0.50	1.00	99.46	98.47	15.89	47.24	128.02	10.49	10.60	41.85	14.42
0.50	1.25	106.20	105.24	15.08	59.01	139.06	10.00	9.86	49.40	13.68
0.50	1.50	111.95	111.01	14.15	71.59	149.02	9.68	9.40	56.85	12.84
0.75	0.50	25.29	24.99	19.09	18.45	4.73'	18.05	18.45	5.34'	18.37
0.75	0.75	32.42	32.03	20.13	18.45	8.54	19.12	18.89	8.81	19.50
0.75	1.00	38.66	38.21	20.12	17.44	11.96	19.18	18.42	11.25	19.61
0.75	1.25	44.23	43.76	19.63	16.05	15.71	18.78	17.62	13.57	19.26
0.75	1.50	49.27	48.81	18.88	14.48	20.06	18.17	16.65	16.15	18.68
1.00	0.50	7.24	7.25	20.56	21.21	6.40	19.78	21.08	6.44	19.80
1.00	0.75	10.61	10.54	22.31	22.44	9.50	21.59	22.28	9.60	21.61
1.00	1.00	13.63	13.50	22.84	22.56	10.58	22.20	22.37	10.70	22.22
1.00	1.25	16.43	16.26	22.80	22.32	10.84	22.27	22.13	10.98	22.30
1.00	1.50	19.09	18.90	22.46	21.90	10.88	22.08	21.71	11.03	22.11
1.25	0.50	6.36	6.50	20.82	21.75	8.43	20.02	21.94	8.80	20.02
1.25	0.75	8.21	8.30	23.13	23.41	12.62	22.36	23.62	13.09	22.36
1.25	1.00	9.65	9.70	24.05	23.77	14.25	23.32	23.99	14.76	23.32
1.25	1.25	10.86	10.87	24.35	23.76	14.59	23.67	24.01	15.12	23.67
1.25	1.50	11.97	11.94	24.33	23.62	14.39	23.73	23.88	14.92	23.73
1.50	0.50	8.26	8.43	20.47	21.94	9.54	19.68	22.14	9.82	19.69
1.50	0.75	9.86	10.02	23.22	24.01	14.73	22.45	24.23	15.11	22.46
1.50	1.00	10.93	11.06	24.43	24.53	17.12	23.70	24.77	17.55	23.72
1.50	1.25	11.67	11.77	24.95	24.64	17.94	24.24	24.89	18.40	24.26
1.50	1.50	12.21	12.28	25.13	24.62	18.04	24.45	24.88	18.51	24.47

': Non-rejection values for the null hypothesis (1.12) at 95% significance level.

TABLE 4.8a

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 - L)^{d+\theta}$ and white noise u_t

(U.K. case)

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	9.89	3.91	-0.30'	-2.55	-3.73	-4.43	-4.87	-5.18
I	1.57'	-4.49	-4.76	-5.01	-5.23	-5.42	-5.59	-5.74
I,T	-3.32	-4.31	-4.74	-5.02	-5.25	-5.44	-5.61	-5.76
I,D	11.91	-0.91'	-3.37	-4.28	-4.83	-5.18	-5.42	-5.61
I,T,D	3.84	-1.13'	-3.34	-4.34	-4.87	-5.21	-5.45	-5.64

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	9.83	3.87	-0.31'	-2.55	-3.73	-4.42	-4.86	-5.17
I	8.65	-3.00	-4.31	-4.95	-5.37	-5.65	-5.85	-6.00
I,T	1.13'	-2.69	-4.27	-4.99	-5.41	-5.67	-5.87	-6.02
I,D	11.76	-0.86'	-3.49	-4.60	-5.24	-5.61	-5.85	-6.02
I,T,D	4.76	-0.77'	-3.44	-4.66	-5.28	-5.64	-5.87	-6.04

c_t-y_t\d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-3.66	-4.26	-4.63	-4.87	-5.06	-5.22	-5.38	-5.52
I	-3.00	-4.20	-4.61	-4.87	-5.07	-5.24	-5.40	-5.54
I,T	-3.50	-4.23	-4.61	-4.87	-5.07	-5.24	-5.39	-5.54
I,D	-1.09'	-3.67	-4.42	-4.85	-5.13	-5.34	-5.51	-5.65
I,T,D	-1.95'	-3.63	-4.42	-4.85	-5.13	-5.34	-5.50	-5.65

': Non-rejection values of the null hypothesis (1.12) at 95% significance level, and the letters in bold correspond to the cases of monotonicity with respect to d.

c_t: Log of total consumption in U.K, from 1955.1 to 1984.4
y_t: Log of disposable income in U.K, from 1955.1 to 1984.4

--:	No intercept, no time trend and no seasonal dummies.
I:	Intercept.
I,T:	Intercept and time trend.
I,D:	Intercept and seasonal dummies.
I,T,D:	Intercept, time trend and seasonal dummies.

TABLE 4.8b

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 - L)^{d+\theta}$ and seasonal AR(K) u_t. (U.K. case)

K = 4

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	6.13	3.29	-0.29'	-2.58	-3.79	-4.48	-4.93	-5.24
I	3.04	-0.71'	-1.43'	-2.31	-3.13	-3.83	-4.42	-4.89
I,T	1.52'	-0.18'	-1.38'	-2.35	-3.22	-3.98	-4.60	-5.07
I,D	5.11	-1.00'	-2.80	-3.75	-4.41	-4.88	-5.22	-5.48
I,T,D	2.34	-0.91'	-2.74	-3.78	-4.45	-4.93	-5.28	-5.54

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	6.11	3.25	-0.31'	-2.59	-3.79	-4.48	-4.92	-5.23
I	3.89	-1.44'	-2.44	-3.30	-3.98	-4.52	-4.95	-5.29
I,T	1.67'	-0.87'	-2.35	-3.32	-4.02	-4.56	-4.99	-5.32
I,D	5.05	-1.03'	-2.90	-3.92	-4.59	-5.05	-5.38	-5.63
I,T,D	2.94	-0.74'	-2.83	-3.95	-4.62	-5.08	-5.41	-5.66

c_t-y_t\d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-2.46	-3.73	-4.54	-5.02	-5.32	-5.53	-5.68	-5.81
I	-2.28	-3.71	-4.45	-4.92	-5.25	-5.49	-5.67	-5.80
I,T	-2.59	-3.75	-4.45	-4.92	-5.25	-5.48	-5.65	-5.77
I,D	-1.80'	-3.76	-4.53	-5.00	-5.31	-5.53	-5.69	-5.82
I,T,D	-2.23	-3.72	-4.53	-5.01	-5.31	-5.52	-5.68	-5.81

K = 8

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	6.03	3.20	-0.28'	-2.58	-3.79	-4.49	-4.94	-5.25
I	4.52	-0.15'	-1.36'	-2.43	-3.30	-4.00	-4.56	-5.01
I,T	2.39	0.28'	-1.30'	-2.47	-3.39	-4.15	-4.74	-5.18
I,D	5.58	-0.41'	-2.47	-3.64	-4.40	-4.88	-5.23	-5.48
I,T,D	2.61	-0.42'	-2.42	-3.66	-4.43	-4.93	-5.28	-5.53

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	6.02	3.16	-0.31'	-2.59	-3.79	-4.48	-4.93	-5.23
I	4.60	-0.85'	-2.25	-3.34	-4.12	-4.67	-5.09	-5.41
I,T	2.10	-0.39'	-2.17	-3.36	-4.16	-4.71	-5.12	-5.44
I,D	5.33	-0.87'	-2.79	-3.90	-4.61	-5.08	-5.42	-5.66
I,T,D	2.84	-0.64'	-2.73	-3.92	-4.64	-5.11	-5.44	-5.69

c_t-y_t\d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-2.32	-3.71	-4.59	-5.09	-5.39	-5.60	-5.76	-5.88
I	-2.17	-3.75	-4.53	-5.01	-5.33	-5.57	-5.74	-5.87
I,T	-2.48	-3.78	-4.53	-5.01	-5.33	-5.56	-5.72	-5.84
I,D	-1.78'	-3.76	-4.52	-4.97	-5.28	-5.49	-5.65	-5.78
I,T,D	-2.29	-3.73	-4.52	-4.98	-5.27	-5.48	-5.63	-5.76

': Non-rejection values of the null hypothesis (1.12) at 95% significance level, and the letters in bold correspond to the cases where monotonocity in the value of the tests with respect to d is obtained.

TABLE 4.8c

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 + L)^{d+\theta}$ and white noise u_t

(U.K. case)

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-0.34'	-0.51'	-0.92'	-1.34'	-1.71'	-2.04	-2.32	-2.55
I	-4.65	-4.67	-4.69	-4.71	-4.72	-4.73	-4.74	-4.74
I,T	-3.84	-3.97	-4.07	-4.16	-4.23	-4.30	-4.36	-4.41
I,D	-4.71	-4.73	-4.75	-4.77	-4.77	-4.77	-4.77	-4.77
I,T,D	-4.59	-4.61	-4.63	-4.64	-4.66	-4.67	-4.68	-4.69

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-0.71'	-0.76'	-1.09'	-1.46'	-1.80'	-2.11	-2.37	-2.60
I	-4.69	-4.71	-4.73	-4.74	-4.74	-4.75	-4.75	-4.76
I,T	-4.54	-4.60	-4.63	-4.65	-4.66	-4.67	-4.68	-4.69
I,D	-4.71	-4.74	-4.75	-4.77	-4.77	-4.77	-4.77	-4.77
I,T,D	-4.63	-4.63	-4.64	-4.65	-4.66	-4.67	-4.68	-4.69

c_t-y_t\d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-4.13	-4.24	-4.30	-4.35	-4.39	-4.42	-4.45	-4.46
I	-4.18	-4.30	-4.38	-4.44	-4.49	-4.53	-4.57	-4.60
I,T	-3.18	-3.37	-3.50	-3.61	-3.71	-3.81	-3.90	-3.99
I,D	-4.48	-4.56	-4.62	-4.65	-4.68	-4.69	-4.71	-4.72
I,T,D	-3.78	-4.01	-4.14	-4.24	-4.31	-4.37	-4.41	-4.45

': Non-rejection values of the null hypothesis (1.12) at 95% significance level, and the letters in bold correspond to the cases of monotonicity with respect to d.

c_t: Log of total consumption in U.K, from 1955.1 to 1984.4
y_t: Log of disposable income in U.K, from 1955.1 to 1984.4

--: No intercept, no time trend and no seasonal dummies.
I: Intercept.
I,T: Intercept and time trend.
I,D: Intercept and seasonal dummies.
I,T,D: Intercept, time trend and seasonal dummies.

TABLE 4.8d

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 + L^2)^{d+\theta}$ and white noise u_t

(U.K. case)

c_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-0.16'	-0.41'	-1.19'	-1.91'	-2.53	-3.05	-3.50	-3.87
I	-7.06	-7.11	-7.15	-7.17	-7.19	-7.21	-7.22	-7.23
I,T	-7.02	-7.16	-7.19	-7.20	-7.21	-7.21	-7.21	-7.21
I,D	-7.10	-7.11	-7.22	-7.25	-7.26	-7.26	-7.26	-7.25
I,T,D	-6.92	-6.97	-7.00	-7.03	-7.05	-7.08	-7.09	-7.11

y_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-0.25'	-0.66'	-1.35'	-2.02	-2.61	-3.12	-3.55	-3.91
I	-7.09	-7.13	-7.17	-7.19	-7.21	-7.22	-7.23	-7.24
I,T	-6.98	-7.05	-7.07	-7.08	-7.08	-7.09	-7.09	-7.09
I,D	-7.11	-7.17	-7.22	-7.25	-7.26	-7.26	-7.26	-7.26
I,T,D	-6.92	-6.92	-6.92	-6.94	-6.96	-6.98	-7.00	-7.01

c_t-y_t\d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
--	-6.76	-6.86	-6.87	-6.87	-6.86	-6.86	-6.86	-6.86
I	-6.85	-7.03	-7.12	-7.17	-7.20	-7.22	-7.23	-7.25
I,T	-5.90	-6.44	-6.68	-6.81	-6.88	-6.93	-6.97	-6.99
I,D	-6.82	-6.98	-7.07	-7.12	-7.16	-7.18	-7.20	-7.21
I,T,D	-5.67	-6.23	-6.48	-6.60	-6.68	-6.74	-5.77	-6.80

': Non-rejection values of the null hypothesis (1.12) at 95% significance level, and the letters in bold correspond to the cases of monotonicity with respect to d.

c_t: Log of total consumption in U.K, from 1955.1 to 1984.4
y_t: Log of disposable income in U.K, from 1955.1 to 1984.4

--: No intercept, no time trend and no seasonal dummies.
I: Intercept.
I,T: Intercept and time trend.
I,D: Intercept and seasonal dummies.
I,T,D: Intercept, time trend and seasonal dummies.

4.4 CONCLUDING REMARKS

We have presented a variety of model specifications for quarterly consumption and income data in Japan and the U.K.. Given the number of possibilities covered by Robinson's (1994c) tests, it is difficult to draw clear conclusions about which might be the best way of modelling these series. In fact, using these tests, the null hypothesized model will permit different deterministic paths; different lagged functions $\rho(L)$, allowing roots at some or all seasonal frequencies (as well as at zero frequency), each of them with a possible different integration order; and will also allow different ways of modelling the I(0) disturbances u_t. Looking at the results presented above as a whole, some common features are observed for all series in both countries and they can be summarized as follows:

First, modelling x_t as a quarterly I(d) process (i.e., of the form: $(1-L^4)^d x_t = u_t$, t=1,2,...) seems to be appropriate when u_t is white noise or a non-seasonal AR, however, if u_t is seasonal AR, results are worse in both countries, in the sense that monotonicity in the test statistic with respect to d is not achieved in most cases. This can be explained because seasonality can be captured in this case either by quarterly integration above or by seasonal dummy variables in (31). We also observe that the integration order seems slightly smaller if u_t is AR rather than white noise, due perhaps to AR picking up part of the nonstationary component of the series. The results emphasize the importance of real roots over complex ones, given the greater integration order observed in the former roots, and this is even clearer when we allow different integration orders for each frequency. Excluding one real root results in rejecting the null in practically all situations. If $\rho(L,\theta)$ is given by (41), we observe some non-rejections if u_t is white noise, and allowing I(0) autocorrelation, results are now better for seasonal AR than for non-seasonal AR processes. This can be explained because the lagged function $\rho(L)$ does not seem to capture now seasonality at all and therefore, the seasonal AR component may play an important role in this situation. Separating here the roots at zero and at frequency π, results emphasize the importance of the root at the long run frequency, but modelling the series as a simple I(d) process with a single root does not seem appropriate in most cases.

Another common feature observed across all these tables is the fact that the integration order for the individual series seems to range between 0.50 (or 0.75) and 1.25, independently of the lagged function used when modelling x_t and the inclusion or not of deterministic parts in (31), indicating clearly the nonstationary nature of these series; however, c_t-y_t seems less integrated in practically all situations. Therefore, if we consider that the series are well modelled by a given $\rho(L)$, a certain degree of fractional cointegration would exist between consumption and income for a given cointegration vector (1,-1), using a very simplistic version of the permanent income hypothesis.

We can compare these results with those obtained in HEGL (1993) and HEGY (1990) for unit root situations. Results in HEGL (1993) for Japanese data indicated the presence of unit roots at all frequencies for y_t and c_t-y_t, and the same conclusions hold for c_t if the dummies were not included in the model, but only the two real unit roots would be present if these dummy variables were included. If we look now at our tables, we observe that the unit root null is not rejected for y_t in any specification in (31) when $\rho(L,\theta)$ adopts the form in (32) with AR u_t. Similarly for c_t-y_t, we cannot reject the unit root null for the same $\rho(L;\theta)$ and white noise u_t. For c_t, the null of four unit roots is not rejected when there are no dummies, but if they are included non-rejections will occur when $\rho(L,\theta)$ takes the form of (41) with white noise or seasonal AR u_t. For the U.K. case, results in HEGY (1990) suggested that four unit roots could be present for c_t, and for c_t-y_t if the dummies were not included, and two real unit roots for y_t, and for c_t-y_t if these were included. Our results again show a certain consistency with theirs given that the unit root null is not rejected for c_t if $\rho(L,\theta)$ adopts the form in (32) with white noise u_t, and for y_t this hypothesis is not rejected if $\rho(L,\theta)$ takes the form of (41) and u_t is white noise or seasonal AR.

Finally, in the appendices of this chapter, (and set out below), we have considered the possibility of a structural change occurring in the slope of the trend function of the series, and due to oil price shock in 1973. First, in Appendix 4.1, we performed some of the tests in Section 3 on two subsamples, splitting the data in that year. In Appendix 4.2, we modelled the shock as exogenous, including dummies in the regression model to correct the changing growth in the series. In both cases results were similar to those in Section 3, finding therefore little evidence of structural change in these data. Though the results presented in this chapter can lead to ambiguous conclusions, we find them interesting in the sense that they suggest an alternative way of modelling seasonality, allowing fractional roots at some or all seasonal frequencies as well as at zero frequency, and allowing also different integration orders in each of these frequencies.

APPENDICES TO CHAPTER 4

As mentioned in Chapter 2, Perron (1989, 1993) found that several stylized facts, such as the 1929 crash and the 1973 oil price shock, might be a cause of non-rejection of the unit root at zero frequency in macroeconomic data, and that when these were taken into account, deterministic trend models might be preferable. Following this work, we are concerned by the effect that a possible structural break in the long run component of the series may have had on results in Section 3, in particular, one due to oil crisis in 1973. Tables in Appendix 4.1 correspond to some of the tests performed in Section 3, splitting the sample period in two subsamples basing on pre and post-oil crisis data. Tables in Appendix 4.2 correspond to similar tests, but treating the price oil shock as exogenous and modelling the change in the slope of the trend function with dummy variables.

APPENDIX 4.1

a) The Japanese case

The sample periods are now 1961.1 - 1973.4 and 1974.1 - 1987.4. Thus, we have 52 observations in the first subsample and 56 in the second one. Tables 4.9a and 4.9b report results of $\hat{r}$ in (2.9) in both subsamples respectively, when $\rho(L;\theta)$ adopts the form given in (32) and u_t is white noise or non-seasonal AR. We focus on these types of disturbances since these were the cases where monotonicity was most likely achieved across the different models for u_t. Results are similar in both tables and non-rejections occur in practically all cases when d ranges between 0.50 and 1. We observe that if u_t is white noise, the non-rejection d's are slightly smaller than those in Table 4.1a, but allowing AR u_t, results are in line with those in Table 4.1b.

Excluding the root at frequency π gave us similar results to those in Table 4.2a, with the only non-rejection cases corresponding to c_t and y_t with $d = 0.75$ and no regressors. Excluding the root at zero frequency, the null was rejected in practically all cases in favour of stationary alternatives as in Section 3.

In Tables 4.10a and 4.10b we examine the case of two real roots in x_t, with white noise and seasonal and non-seasonal AR u_t. Again results are similar in both samples, and we see that the non-rejection values of d range between 0.50 and 1.25 for c_t and y_t with white noise or seasonal AR and also for c_t-y_t with seasonal AR, but these hypotheses are practically always rejected if u_t follows a non-seasonal AR. Again these results are in line with those in Section 3 when we looked at the whole period of time.

Tables 4.11a and 4.11b perform the tests when x_t contains a single root located at zero frequency and u_t is white noise or seasonal AR. Comparing these results with those in Tables 4.4a and 4.4b we observe now some more non-rejections, which might be related with the smaller sample size, but in general, we see that the values of d where the null was not rejected in Tables 4.4a and 4.4b are non-rejected when we split the sample size in Tables 4.11a and 4.11b, with values of d ranging now between 0.50 and 1.25 in the first subsample, and between 0.50 and 1.50 in the second one.

b) The U.K. case

The sample periods are now 1955.1 - 1973.4 and 1974.1 - 1984.4, so that we have 76 observations in the first subsample and 44 in the second one. Tables 4.12a and 4.12b report the one-sided test statistic $\hat{r}$ in (2.9) when x_t contains four roots on the unit circle, and u_t is white noise and a non-seasonal AR. Results are similar in both samples with the non-rejections occurring at the same values of d in both tables and ranging in most cases between 0.50 and 1. These non-rejection d's also coincide with those given in Tables 4.5a and 4.5b when we considered the whole period of time.

Excluding the root at frequency π we obtained few non-rejection cases, corresponding to c_t and y_t with d = 0.50 and d = 0.75. The same conclusions were obtained when we considered the whole sample period in Table 4.6a. Excluding the root at zero frequency, the null was rejected in all cases and in the three series, as it also was in Section 3.

Tables 4.13a and 4.13b present results when x_t contains only two real roots, and again results are similar in both subsamples, with the non-rejections occurring for c_t and y_t when d ranges between 0.50 and 1.25 for white noise and seasonal AR u_t. Looking at c_t-y_t, the null is practically always rejected in the first subsample though in the other subsample is not rejected the case of seasonal AR(1) u_t, with d ranging again between 0.50 and 1. Comparing these results with those for the whole sample period, we observe that they are rather similar, with non-rejections also ranging between 0.50 and 1 in most cases for individual series, and rejecting the null in favour of stationary alternatives for c_t-y_t.

Tables 4.14a and 4.14b show results when x_t contains a single root located at zero frequency. These tables correspond to Tables 4.8a and 4.8b in Section 3. Starting with white noise u_t, we saw in Table 4.8a that the only non-rejection cases for c_t corresponded to d = 1 with no regressors and to d = 0.50 with an intercept. For y_t, the non-rejections were d = 1 with no regressors and d = 0.50 with a time trend. In Tables 4.14a and 4.14b we see that these cases are among the few where the null is not rejected. Allowing seasonal AR u_t, the null hypothesis was not rejected in Table 4.8b for c_t when d was 0.75 or 1, and for y_t when d was 0.75. Splitting the sample we see that these cases are either non-rejected as well as the case of d = 0.50.

As a conclusion of all these tables we see that results do not differ much when we split the sample period in two subsamples based on pre and post-oil crisis data from those obtained when we considered the whole period of time. In fact, apart from a somewhat greater proportion of non-rejections, due to smaller sample sizes, the values of d where the null are not rejected are practically the same in all series across both countries.

APPENDIX 4.2

Another way of dealing with the problem of a structural change in the long run component of the series might be to include some dummy variables in the regression model in order to take into account of a possible change in the slope of the trend function. The model becomes (1.10) and (2.2) with

$$y_t = \alpha + \beta_{01} t + (\beta_{02} - \beta_{01}) dt + \sum_{i=1}^{3} \beta_i S_{it} + x_t \quad t = 1, 2, \ldots T \qquad (48)$$

where dt = t - t^* if t > t^*, and 0 otherwise, and t^* refers to the period of time at which the change in the slope occurs, (in our case, the fourth quarter of 1973).

Across Tables 4.15a-4.17b, we perform Robinson's (1994c) tests in (1.10); (2.2) and (48), testing the null (1.12) against the one-sided alternatives (33), for some specialized forms $\rho(L;\theta)$ used in Section 3, and for a range of values of d from 0.00 through 2.00 with 0.25 increments, treating separately the cases α, β_{01}, β_{02} unknown and $\beta_1 = \beta_2 = \beta_3 = 0$ a priori, and (α, β_{01}, β_{02}, β_1, β_2, β_3) unknown. Note that non-rejections of H_o when d = 0 would imply that the series are I(0) stationary around deterministic functions, with the slowdown in growth after 1973 modelled as exogenous as was advocated by Perron (1989) and others.

Tables 4.15a and 4.15b give results of $\hat{r}$ in (2.9) for Japanese and U.K. data respectively, with $\rho(L;\theta)$ as in (32) and white noise and non-seasonal AR u_t. Looking at the Japanese data, we see that if we do not include seasonal dummies, there is a lack of monotonic decrease in $\hat{r}$ with respect to d when d is in the stationary region. Including these dummies, monotonicity is always achieved and the null is not rejected for c_t when d ranges between 0.25 and 0.75; for y_t when d = 0.50, and for c_t-y_t when d is 0.50 or 0.75. Note that the null is always rejected when d = 0, in favour of alternatives with d > 0, rejecting therefore that the series follow a deterministic trend model with I(0) u_t.

Results for the U.K. data are given in Table 4.15b and we see that monotonicity is achieved even for the case of $(\beta_i)_{i=1,2,3} = 0$ a priori, in y_t and c_t-y_t. In this case we see that the null is not rejected when d = 0, however, we observe that lower statistics are obtained when d is slightly greater. Including seasonal dummies, the non-rejection values of d range in most of the cases between 0.25 and 0.75. Comparing results in these two tables with those given in Section 3 when we considered a simple linear time trend, (in Tables 4.1a, 4.1b, 4.5a and 4.5b), we observe that there is now a somewhat larger proportion of non-rejections at smaller values of d, though most non-rejections in those tables are non-rejected now when we consider the structural break.

Excluding one of the real roots, either at frequency π or at zero frequency resulted in rejecting the null in practically all cases in all series, as was the case in the Section 3 tests.

In Tables 4.16a and 4.16b we take $\rho(L;\theta)$ as in (41), and x_t contains only two real roots. We observe in both tables that the non-rejection d's tend to be slightly smaller by about 0.25 than those in Section 3. Including seasonal dummies, the null is not rejected when d = 0 for y_t with white noise, AR(1) and seasonal AR(1) u_t, and for c_t-y_t with seasonal AR using Japanese data (in Table 4.16a), and with seasonal and non-seasonal AR(1) u_t using U.K. data, (in Table 4.16b), though in most of cases, lower statistics are obtained if d is greater than 0. Stationary alternatives of this type were also plausible in tests of Section 3 in view of Tables 4.3b and 4.7b. We also observe that the unit root null is not rejected for Japanese consumption and U.K. income as in the tests of Section 3.

Finally, in Tables 4.17a and 4.17b, we suppose x_t contains a single root located at zero frequency. Monotonicity is obtained in some cases for Japanese data and in all of them for U.K. data. The non-rejection values of d are 0.50 and 0.75 for c_t in Japan, and for c_t and y_t in the U.K., and range between 0.00 and 0.50 for Japanese y_t and for c_t-y_t in both countries. Again results here are in complete analogy with those in Section 3 when we included a simple linear trend.

Results in this second appendix show that even correcting the model of a possible structural change in the slope of the trend function, does not significantly affect the results in Section 3. Though we observe in these tables slightly smaller non-rejection d's than those in Section 3, these non-rejections occur in many cases at the same values of d, suggesting that the structural break in the trend function is not relevant at all when modelling this series. Finally, since the null of d = 0 is rejected in most of cases where monotonicity is achieved, these results suggest that deterministic trend models of the form advocated by Perron (1989) are inappropriate in these series.

APPENDIX 4.3

The Fortran program used to obtain Robinson's (1994c) univariate tests is described in this appendix. If the null hypothesized model is

$$y_t = \beta' \mu_t + x_t \qquad t = 1,2,\ldots T$$

$$\rho(L)\, x_t = u_t \qquad t = 1,2,\ldots T$$

$$x_t = 0 \qquad t \leq 0$$

$$u_t \sim I(0),$$

the test statistic is given by: **TEST(I,L,K,IQ),**

where I = 1,2,...7, and

I = 1 means that $\mu_t \equiv 0$ (i.e. (--) in Tables 4.1a - 4.17b)

I = 2 means that $\mu_t \equiv 1$ (i.e. (I))

I = 3 means that $\eta_t = (1,t)'$ (i.e. (I,T))

I = 4 means that $\eta_t = (1,S_{1t},S_{2t},S_{3t})'$ (i.e. (I,D))

I = 5 means that $\eta_t = (1,t,S_{1t},S_{2t},S_{3t})'$ (i.e. (I,T,D))

I = 6 means that $\eta_t = (1,t_1,t_2)'$ (i.e. (I,T^*) and

I = 7 means that $\eta_t = (1,t_1,t_2,S_{1t},S_{2t},S_{3t})'$ (i.e. (I,T^*,D),

where t_1 and t_2 are dummy variables for the changing growth in the trend function.

L = 1,2,...7, and

L = 1 means that $\rho(L) = (1 + L^2)^d$

L = 2 means that $\rho(L) = (1 - L + L^2 - L^3)^d$

L = 3 means that $\rho(L) = (1 - L)^d$

L = 4 means that $\rho(L) = (1 + L + L^2 + L^3)^d$

L = 5 means that $\rho(L) = (1 + L)^d$

L = 6 means that $\rho(L) = (1 - L^2)^d$

L = 7 means that $\rho(L) = (1 - L^4)^d$.

K = 1,2,...ND, where 'ND' can be any integer number, and it corresponds to the value of d above, using the relation: $d = K/4 + 0.25$. Thus,

K = 1 means that d = 0.50

K = 2 means that d = 0.75

K = 3 means that d = 1.00 and so on.

Finally, IQ = 1,2,...11, where

IQ = 1 means that u_t is an AR(1) process.

IQ = 2 means that u_t is an AR(2) process.

IQ = 3 means that u_t is an AR(3) process.

IQ = 4 means that u_t is an AR(4) process.

IQ = 5 means that u_t is a white noise process.

IQ = 6 means that u_t is a seasonal AR(1) process.

IQ = 7 means that u_t is a seasonal AR(2) process.

IQ = 8 means that u_t is a seasonal and non-seasonal AR(1,1)

IQ = 9 means that u_t is a seasonal and non-seasonal AR(1,2)

IQ = 10 means that u_t is a seasonal and non-seasonal AR(2,1)

IQ = 11 means that u_t is a seasonal and non-seasonal AR(2,2).

```
      PROGRAM APPENDIX 4.3
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      PARAMETER(N=*,N2=(N/2)-1,N1=N/4,N3=3*N/4,ND=4,NS=**,NK=N-1,NWA=5*N)
      DIMENSION XL(N),Y(8,N),R(7,N),V(N,2),W(N,2),S(7,N),X(8,7,N),A1(2,1),
     +  A2(3,2),A3(4,3),A4(5,4),A5(6,5),A6(7,6),O2(2,2),O3(3,3),O4(4,4),B(7),
     +  O5(5,5),O6(6,6),CF2(2),CF3(3),CF4(4),CF5(5),CF6(6),AL(8,8),U(7,N),
     +  P(N),G(N),EX(N,8),RE(N),SX(8,8),YE(8),XP(8,8),P1(1),P2(2),P3(3),P4(4),
     +  P5(5),p6(6),CV(N),PAR(4),MM(7),SX2(3,2),CR(N),XP2(2,2),T2(2),SX3(4,3),
     +  XP3(3,3),T3(3),WA(NWA),SX4(5,4),XP4(4,4),T4(4),MMM(7),AE(2,2),PARA(2),
     +  WWA(NWA),NM(7),AA(4,2,2),AO(2,2,2),AU(2,2,2),XB(4),XGU(2,2,N),
     +  GG(2,2,N),VRR(2,2),EXX(4,2,2,N),XSA(2,2),EX11(4,4,4),XA10(2,2),
     +  EX1(4,4,4),WW1(3,2),FE(4,4),WW2(4,3),ZZ2(3,3),ZZ3(3,3),WW2(3,2),
     +  WW4(5,4),ZZ4(4,4),ZZ1(2,2),XGO(2,2,N),XXB(4),TEST(7,7,ND,11)
      XN=N
      PI=3.141592654
      OPEN(100,FILE='****.DAT',STATUS='OLD')
      DO 1 I=1,N
        XI=I
        XL(I)=2.*PI*XI/XN
        READ(100,101) Y(1,I)
        Y(2,I)=1.
        Y(3,I)=I
        Y(4,I)=0.
        Y(5,I)=0.
        Y(6,I)=0.
1     CONTINUE
101   FORMAT(F9.7)
      Y(4,1)=1.
      Y(5,2)=1.
      Y(6,3)=1.
      DO 2 I=1,N1-1
        J1=4.*I+1
        J2=4.*I+2
        J3=4.*I+3
        Y(4,J1)=1.
        Y(5,J2)=1.
        Y(6,J3)=1.
2     CONTINUE
      DO 3 I=1,NS
        Y(7,I)=I
        Y(8,I)=0.
3     CONTINUE
      DO 4 I=NS+1,N
        Y(7,I)=NS
        Y(8,I)=I-NS
4     CONTINUE
      DO 5 F=1,N-1
        R(1,F)=LOG(ABS(2.*COS(XL(F))))
        R(3,F)=LOG(ABS(2.*SIN(XL(F)/2.)))
        R(2,F)=R(1,F)+R(3,F)
5     CONTINUE
      R(1,N1)=0.
      R(1,N3)=0.
      R(2,N1)=0.
      R(2,N3)=0.
      DO 6 F=1,N2
        R(5,F)=LOG(2.*COS(XL(F)/2.))
        R(4,F)=R(5,F)+R(1,F)
        R(6,F)=R(5,F)+R(3,F)
```

```
          R(7,F)=R(1,F)+R(5,F)+R(3,F)
6       CONTINUE
        R(4,N1)=0.
        R(7,N1)=0.
        DO 1000 K=1,ND
          XK=K
          D=XK/4.+0.25
          V(1,1)=0.
          V(2,1)=D
          V(1,2)=D
            DO 7 J=2,N
              JM=2.*J
              V(JM,1)=((J-D-1)/J)*V(JM-2,1)
7           CONTINUE
            DO 8 J=2,N
              V(J,2)=((J-D-1)/J)*V(J-1,2)
8           CONTINUE
            DO 9 J=1,N
              W(J,1)=(-1)*V(J,1)
              W(J,2)=(-1)*V(J,2)
9           CONTINUE
            DO 10 I=1,8
              DO 11 J=2,N
                DO 12 L=1,7
                  S(L,J)=0.
12              CONTINUE
                IS=INT((J-1)/2)
                DO 13 M=1,IS
                  IM=2*M
                  S(1,J)=S(1,J)+((-1)**M)*W(IM,1)*Y(I,J-IM)
13              CONTINUE
                DO 14 M=1,J-1
                  S(3,J)=S(3,J)+W(M,2)*Y(I,J-M)
                  S(5,J)=S(5,J)+((-1)**M)*W(M,2)*Y(I,J-M)
14              CONTINUE
11            CONTINUE
              DO 15 J=1,2
                X(I,1,J)=Y(I,J)
15            CONTINUE
              X(I,3,1)=Y(I,1)
              X(I,5,1)=Y(I,1)
              DO 16 J=3,N
                X(I,1,J)=S(1,J)+Y(I,J)
16            CONTINUE
            DO 10 J=2,N
              X(I,2,1)=X(I,1,1)
              X(I,3,J)=S(3,J)+Y(I,J)
              X(I,4,1)=X(I,1,1)
              X(I,5,J)=S(5,J)+Y(I,J)
              X(I,6,1)=X(I,2,1)
              X(I,7,1)=X(I,4,1)
                DO 17 M=1,J-1
                  S(2,J)=S(2,J)+W(M,2)*X(I,1,J-M)
                  S(4,J)=S(4,J)+((-1)**M)*W(M,2)*X(I,1,J-M)
                  S(6,J)=S(6,J)+W(M,2)*X(I,5,J-M)
17              CONTINUE
              X(I,2,J)=S(2,J)+X(I,1,J)
              X(I,4,J)=S(4,J)+X(I,1,J)
              X(I,6,J)=S(6,J)+X(I,5,J)
```

```
            DO 18 M=1,J-1
              S(7,J)=S(7,J)+W(M,2)*X(I,4,J-M)
18          CONTINUE
          X(I,7,J)=S(7,J)+X(I,4,J)
10      CONTINUE
      DO 1000 L=1,7
        IF (L.LE.3) THEN
          NNN=N
        ELSE
          NNN=N2+1
        END IF
        DO 19 I1=1,6
        DO 19 I2=1,6
          A1(I1,I2)=0.
          A2(I1,I2)=0.
          A3(I1,I2)=0.
          A4(I1,I2)=0.
          A5(I1,I2)=0.
          A6(I1,I2)=0.
19      CONTINUE
        DO 20 IS=1,7
          B(IS)=0.
20      CONTINUE
        DO 21 J=1,N
          A1(1,1)=A1(1,1)+X(2,L,J)**2.
          A3(1,1)=A1(1,1)
          A6(1,1)=A1(1,1)
          A4(1,1)=A1(1,1)
            DO 22 I1=1,2
              A3(1,I1+1)=A3(1,I1+1)+X(2,L,J)*X(I1+6,L,J)
              A6(1,I1+1)=A6(1,I1+1)+X(2,L,J)*X(I1+6,L,J)
              A3(I1+1,1)=A3(1,I1+1)
              A6(I1+1,1)=A6(1,I1+1)
                DO 23 I2=1,2
                  A2(I1,I2)=A2(I1,I2)+X(I1+1,L,J)*X(I2+1,L,J)
                  A3(I1+1,I2+1)=A3(I1+1,I2+1)+X(I1+6,L,J)*X(I2+6,L,J)
                  A6(I1+1,I2+1)=A6(I1+1,I2+1)+X(I1+6,L,J)*X(I2+6,L,J)
23              CONTINUE
22          CONTINUE
              DO 24 I1=2,4
                A4(1,I1)=A4(1,I1)+X(2,L,J)*X(I1+2,L,J)
                A6(1,I1+2)=A6(1,I1+2)+X(2,L,J)*X(I1+2,L,J)
                A4(I1,1)=A4(1,I1)
                A6(I1+2,1)=A6(1,I1+2)
                A6(2,I1+2)=A6(2,I1+2)+X(7,L,J)*X(I1+2,L,J)
                A6(3,I1+2)=A6(3,I1+2)+X(8,L,J)*X(I1+2,L,J)
                A6(I1+2,2)=A6(2,I1+2)
                A6(I1+2,3)=A6(3,I1+2)
                  DO 25 I2=2,4
                    A4(I1,I2)=A4(I1,I2)+X(I1+2,L,J)*X(I2+2,L,J)
                    A6(I1+2,I2+2)=A6(I1+2,I2+2)+X(I1+2,L,J)*X(I2+2,L,J)
25                CONTINUE
24            CONTINUE
              DO 26 I1=1,5
              DO 26 I2=1,5
                A5(I1,I2)=A5(I1,I2)+X(I1+1,L,J)*X(I2+1,L,J)
26            CONTINUE
              DO 27 IS=1,7
                B(IS)=B(IS)+X(1,L,J)*X(IS+1,L,J)
```

```
27          CONTINUE
21       CONTINUE
         CALL F01ABF(A1,2,1,O1,1,P1,IFAIL)
         CALL F01ABF(A2,3,2,O2,2,P2,IFAIL)
         CALL F01ABF(A3,4,3,O3,3,P3,IFAIL)
         CALL F01ABF(A4,5,4,O4,4,P4,IFAIL)
         CALL F01ABF(A5,6,5,O5,5,P5,IFAIL)
         CALL F01ABF(A6,7,6,O6,6,P6,IFAIL)
         O2(1,2)=O2(2,1)
         O3(1,2)=O3(2,1)
         O3(1,3)=O3(3,1)
         O3(2,3)=O3(3,2)
         DO 28 I=1,3
         DO 28 J=I+1,4
           O4(I,J)=O4(J,I)
28       CONTINUE
         DO 29 I=1,4
         DO 29 J=1+I,5
           O5(I,J)=O5(J,I)
29       CONTINUE
         DO 30 I=1,5
         DO 30 J=1+I,6
           O6(I,J)=O6(J,I)
30       CONTINUE
         CF1=O1*B(1)
         DO 31 I=1,2
           CF2(I)=0.
         DO 31 J=1,2
           CF2(I)=CF2(I)+O2(I,J)*B(J)
31       CONTINUE
         DO 32 IS=1,3
           CF3(IS)=O3(IS,1)*B(1)+O3(IS,2)*B(6)+O3(IS,3)*B(7)
32       CONTINUE
         DO 33 IS=1,4
           CF4(IS)=O4(IS,1)*B(1)+O4(IS,2)*B(3)+O4(IS,3)*B(4)+O4(IS,4)*B(5)
33       CONTINUE
         DO 34 I=1,5
           CF5(I)=0.
         DO 34 J=1,5
           CF5(I)=CF5(I)+O5(I,J)*B(J)
34       CONTINUE
         DO 35 IS=1,6
           CF6(IS)=O6(IS,1)*B(1)+O6(IS,2)*B(6)+O6(IS,3)*B(7)+O6(IS,4)*B(3)+
   +       O6(IS,5)*B(4)+O6(IS,6)*B(5)
35       CONTINUE
         DO 36 J=1,N
           U(1,J)=X(1,L,J)
           U(2,J)=X(1,L,J)-CF1*X(2,L,J)
           U(3,J)=X(1,L,J)-CF2(1)*X(2,L,J)-CF2(2)*X(3,L,J)
           U(4,J)=X(1,L,J)-CF4(1)*X(2,L,J)-CF4(2)*X(4,L,J)-
   +       CF4(3)*X(5,L,J)-CF4(4)*X(6,L,J)
           U(5,J)=X(1,L,J)-CF5(1)*X(2,L,J)-CF5(2)*X(3,L,J)-
   +       CF5(3)*X(4,L,J)-CF5(4)*X(5,L,J)-CF5(5)*X(6,L,J)
           U(6,J)=X(1,L,J)-CF3(1)*X(2,L,J)-CF3(2)*X(7,L,J)-CF3(3)*X(8,L,J)
           U(7,J)=X(1,L,J)-CF6(1)*X(2,L,J)-CF6(2)*X(7,L,J)-CF6(3)*X(8,L,J)-
   +       CF6(4)*X(4,L,J)-CF6(5)*X(5,L,J)-CF6(6)*X(6,L,J)
36       CONTINUE
       DO 1000 I=1,7
         UME=0.
```

```
      DO 37 J=1,N
        UME=UME+(1./XN)*U(I,J)
37    CONTINUE
      SVAR=0.
      DO 38 J=1,N
        SVAR=SVAR+(U(I,J)-UME)**2.
38    CONTINUE
      VAR=SVAR/XN
      DO 39 LL=1,N-1
        CV(LL)=0.
          DO 40 J=1,N-LL
            CV(LL)=CV(LL)+(U(I,J)-UME)*(U(I,J+LL)-UME)
40        CONTINUE
        CR(LL)=CV(LL)/SVAR
39    CONTINUE
      DO 41 F=1,N-1
        XC=0.
        XS=0.
          DO 42 J=1,N
            XJ=J
            XC=XC+U(I,J)*COS(XL(F)*XJ)
            XS=XS+U(I,J)*SIN(XL(F)*XJ)
42        CONTINUE
        P(F)=(XC**2.+XS**2.)/(2.*PI*XN)
        RE(F)=R(L,F)
41    CONTINUE
     DO 1000 IQ=1,11
      IF (IQ.LE.5) THEN
        MM(1)=IQ
        NPAR=IQ
        CALL G13ADF(MM,CR,NK,VAR,NPAR,WWA,NWA,PAR,RV,ISF,IFAIL)
          DO 43 IPAR=1,IQ
            AL(IPAR,IQ)=PAR(IPAR)
43      CONTINUE
        IF (IQ.EQ.5) THEN
          DO 44 IPAR=1,IQ
            AL(IPAR,IQ)=0.
44        CONTINUE
        END IF
      DO 45 F=1,N-1
        S1=0.
        S2=0.
          DO 46 IO=1,IQ
            S1=S1+AL(IO,IQ)*SIN(XL(F)*IO)
            S2=S2+AL(IO,IQ)*COS(XL(F)*IO)
46        CONTINUE
        G(F)=1./((1.-S2)**2.+S1**2.)
45    CONTINUE
      VR=0.
      DO 47 F=1,N-1
        VR=VR+(2.*PI/XN)*P(F)/G(F)
47    CONTINUE
      DO 48 F=1,N-1
      DO 48 IP=1,IQ
        EXE=0.
          DO 49 IO=1,IQ
            EXE=EXE+AL(IO,IQ)*COS((IP-IO)*XL(F))
49        CONTINUE
        EX(F,IP)=2.*(COS(IP*XL(F))-EXE)*G(F)
```

```
48      CONTINUE
        DO 50 I1=1,IQ
        DO 50 I2=1,IQ
          XA=0.
          XAA=0.
          SX(I1,I2)=0.
          YE(I1)=0.
            DO 51 F=1,NNN-1
              XA=XA+((-1)*2.*PI/XN)*RE(F)*P(F)/G(F)
              XAA=XAA+(2./XN)*RE(F)**2.
              SX(I1,I2)=SX(I1,I2)+EX(F,I1)*EX(F,I2)
              YE(I1)=YE(I1)+RE(F)*EX(F,I1)
51          CONTINUE
50      CONTINUE
          IF (IQ.EQ.1) THEN
            XP(1,1)=1./SX(1,1)
            YEA=(2./XN)*YE(1)*XP(1,1)*YE(1)
          ELSE IF (IQ.EQ.2) THEN
            DO 52 I1=1,2
            DO 52 I2=1,2
              SX2(I1,I2)=SX(I1,I2)
52          CONTINUE
            CALL F01ABF(SX2,3,2,XP2,2,T2,IFAIL)
            XP2(1,2)=XP2(2,1)
            YEA=0.
              DO 53 M1=1,2
              DO 53 M2=1,2
                YEA=YEA+(2./XN)*YE(M1)*XP2(M1,M2)*YE(M2)
53            CONTINUE
          ELSE IF (IQ.EQ.3) THEN
            DO 54 I1=1,3
            DO 54 I2=1,3
              SX3(I1,I2)=SX(I1,I2)
54          CONTINUE
            CALL F01ABF(SX3,4,3,XP3,3,T3,IFAIL)
            DO 55 I1=1,2
            DO 55 I2=1+I1,3
              XP3(I1,I2)=XP3(I2,I1)
55          CONTINUE
            YEA=0.
            DO 56 M1=1,3
            DO 56 M2=1,3
              YEA=YEA+(2./XN)*YE(M1)*XP3(M1,M2)*YE(M2)
56          CONTINUE
          ELSE IF (IQ.EQ.4) THEN
            DO 57 M1=1,4
            DO 57 M2=1,4
              X4(M1,M2)=SX(M1,M2)
57          CONTINUE
            CALL F01ABF(SX4,5,4,XP4,4,T4,IFAIL)
            DO 58 I1=1,3
            DO 58 I2=1+I1,4
              XP4(I1,I2)=XP4(I2,I1)
58          CONTINUE
            YEA=0.
            DO 59 M1=1,4
            DO 59 M2=1,4
              YEA=YEA+(2./XN)*YE(M1)*XP4(M1,M2)*YE(M2)
59          CONTINUE
```

```
        ELSE IF (IQ.EQ.5) THEN
          YEA=0.
        END IF
      YA=XAA-YEA
      TEST(I,L,K,IQ)=((XN/YA)**0.5)*XA/VR
      ELSE IF (IQ.GT.5.AND.IQ.LT.8) THEN
        IOO=IQ-5
        MMM(4)=IOO
        MMM(7)=4
        NPAR=IOO
        CALL G13ADF(MMM,CR,NK,VAR,NPAR,WA,NWA,PARA,RV,ISF,IFAIL)
        DO 60 IPAR=1,IOO
          AE(IPAR,IOO)=PARA(IPAR)
60      CONTINUE
        DO 61 F=1,N-1
          S1=0.
          S2=0.
            DO 62 IO=1,IOO
              S1=S1+AE(IO,IOO)*SIN(XL(F)*4.*IO)
              S2=S2+AE(IO,IOO)*COS(XL(F)*4.*IO)
62          CONTINUE
          G(F)=1./((1.-S2)**2.+S1**2.)
61      CONTINUE
        VR=0.
        DO 63 F=1,N-1
          VR=VR+(2.*PI/XN)*P(F)/G(F)
63      CONTINUE
        DO 64 F=1,N-1
        DO 64 IP=1,IOO
          EXE=0.
            DO 65 IO=1,IOO
              EXE=EXE+AE(IO,IOO)*COS(4.*(IP-IO)*XL(F))
65          CONTINUE
          EX(F,IP)=2.*(COS(4.*IP*XL(F))-EXE)*G(F)
64      CONTINUE
        DO 66 I1=1,IOO
        DO 66 I2=1,IOO
          XA=0.
          XAA=0.
          SX(I1,I2)=0.
          YE(I1)=0.
        DO 66 F=1,NNN-1
          XA=XA+((-1)*2.*PI/XN)*RE(F)*P(F)/G(F)
          XAA=XAA+(2./XN)*RE(F)**2.
          SX(I1,I2)=SX(I1,I2)+EX(F,I1)*EX(F,I2)
          YE(I1)=YE(I1)+RE(F)*EX(F,I1)
66      CONTINUE
        IF (IQ.EQ.6) THEN
          XP(1,1)=1./SX(1,1)
          YEA=(2./XN)*YE(1)*XP(1,1)*YE(1)
        ELSE IF (IQ.EQ.7) THEN
          DO 67 I1=1,2
          DO 67 I2=1,2
            SX2(I1,I2)=SX(I1,I2)
67        CONTINUE
          CALL F01ABF(SX2,3,2,XP2,2,T2,IFAIL)
          XP2(1,2)=XP2(2,1)
          YEA=0.
          DO 68 M1=1,2
```

```
          DO 68 M2=1,2
            YEA=YEA+(2./XN)*YE(M1)*XP2(M1,M2)*YE(M2)
68        CONTINUE
        END IF
        YA=XAA-YEA
        TEST(I,L,K,IQ)=((XN/YA)**0.5)*XA/VR
      ELSE IF (IQ.GT.7.AND.IQ.LT.12) THEN
        IF (IQ.EQ.8) THEN
          IZ1=IQ-7
          IZ2=IQ-7
          NM(1)=IZ1
          NM(4)=IZ2
          NM(7)=4
          NPAR=IZ1+IZ2
          CALL G13ADF(NM,CR,NK,VAR,NPAR,WA,NWA,PARA,RV,ISF,IFAIL)
          DO 69 IPAR=1,NPAR
            AA(IPAR,IZ1,IZ2)=PARA(IPAR)
69        CONTINUE
        ELSE IF (IQ.EQ.9) THEN
          IZ1=IQ-7
          IZ2=IQ-8
          NM(1)=IZ1
          NM(4)=IZ2
          NM(7)=4
          NPAR=IZ1+IZ2
          CALL G13ADF(NM,CR,NK,VAR,NPAR,WA,NWA,PARA,RV,ISF,IFAIL)
          DO 70 IPAR=1,NPAR
            AA(IPAR,IZ1,IZ2)=PARA(IPAR)
70        CONTINUE
        ELSE IF (IQ.EQ.10) THEN
          IZ1=IQ-9
          IZ2=IQ-8
          NM(1)=IZ1
          NM(4)=IZ2
          NM(7)=4
          NPAR=IZ1+IZ2
          CALL G13ADF(NM,CR,NK,VAR,NPAR,WA,NWA,PARA,RV,ISF,IFAIL)
          DO 71 IPAR=1,NPAR
            AA(IPAR,IZ1,IZ2)=PARA(IPAR)
71        CONTINUE
        ELSE IF(IQ.EQ.11) THEN
          IZ1=IQ-9
          IZ2=IQ-9
          NM(1)=IZ1
          NM(4)=IZ2
          NM(7)=4
          NPAR=IZ1+IZ2
          CALL G13ADF(NM,CR,NK,VAR,NPAR,WA,NWA,PARA,RV,ISF,IFAIL)
          DO 72 IPAR=1,NPAR
            AA(IPAR,IZ1,IZ2)=PARA(IPAR)
72        CONTINUE
        END IF
      AO(1,1,1)=AA(1,1,1)
      AO(1,1,2)=AA(1,1,2)
      AO(1,2,1)=AA(1,2,1)
      AO(2,2,1)=AA(2,2,1)
      AO(1,2,2)=AA(1,2,2)
      AO(2,2,2)=AA(2,2,2)
      AU(1,1,1)=AA(2,1,1)
```

```
      AU(1,2,1)=AA(3,2,1)
      AU(1,1,2)=AA(2,1,2)
      AU(2,1,2)=AA(3,1,2)
      AU(1,2,2)=AA(3,2,2)
      AU(2,2,2)=AA(4,2,2)
      DO 73 F=1,N-1
      IZ1=2
      IZ2=2
      DO 73 I1=1,IZ1
      DO 73 I2=1,IZ2
        S1=0.
        S2=0.
        S3=0.
        S4=0.
          DO 74 IO=1,I1
            S1=S1+AO(IO,I1,I2)*SIN(XL(F)*IO)
            S2=S2+AO(IO,I1,I2)*COS(XL(F)*IO)
74        CONTINUE
          DO 75 IO=1,I2
            S3=S3+AU(IO,I1,I2)*SIN(XL(F)*4.*IO)
            S4=S4+AU(IO,I1,I2)*COS(XL(F)*4.*IO)
75        CONTINUE
        GO=((1.-S2)**2.+S1**2.)
        GU=((1.-S4)**2.+S3**2.)
        XGO(I1,I2,F)=1./GO
        XGU(I1,I2,F)=1./GU
        XG=GO*GU
        GG(I1,I2,F)=1./XG
73    CONTINUE
      DO 76 I1=1,IZ1
      DO 76 I2=1,IZ2
        VRR(I1,I2)=0.
      DO 76 F=1,N-1
        VRR(I1,I2)=VRR(I1,I2)+(2.*PI/XN)*P(F)/GG(I1,I2,F)
76    CONTINUE
      DO 77 F=1,NNN-1
        EXX(1,1,1,F)=2.*(COS(XL(F))-AO(1,1,1))*XGO(1,1,F)
        EXX(2,1,1,F)=2.*(COS(4.*XL(F))-AU(1,1,1))*XGU(1,1,F)
        EXX(1,2,1,F)=2.*(COS(XL(F))-AO(1,2,1)-AO(2,2,1)*COS((-1)*XL(F)))*
     +  XGO(2,1,F)
        EXX(2,2,1,F)=2.*(COS(2.*XL(F))-AO(1,2,1)*COS(XL(F))-AO(2,2,1))*
     +  XGO(2,1,F)
        EXX(3,2,1,F)=2.*(COS(4.*XL(F))-AU(1,2,1))*XGU(2,1,F)
        EXX(1,1,2,F)=2.*(COS(XL(F))-AO(1,1,2))*XGO(1,2,F)
        EXX(2,1,2,F)=2.*(COS(4.*XL(F))-AU(1,1,2)-AU(2,1,2)*COS((-4.)*XL(F)))
     +  *XGU(1,2,F)
        EXX(3,1,2,F)=2.*(COS(8.*XL(F))-AU(1,1,2)*COS(4.*XL(F))-AU(2,1,2))*
     +  XGU(1,2,F)
        EXX(1,2,2,F)=2.*(COS(XL(F))-AO(1,2,2)-AO(2,2,2)*COS((-1)*XL(F)))*
     +  XGO(2,2,F)
        EXX(2,2,2,F)=2.*(COS(2.*XL(F))-AO(1,2,2)*COS(XL(F))+AO(2,2,2))*
     +  XGO(2,2,f)
        EXX(3,2,2,F)=2.*(COS(4.*XL(F))-AU(1,2,2)-AU(2,2,2)*COS((-4.)*XL(F)))
     +  *XGU(2,2,F)
        EXX(4,2,2,F)=2.*(COS(8.*XL(F))-AU(1,2,2)*COS(4.*XL(F))+AU(2,2,2))*
     +  XGU(2,2,F)
77    CONTINUE
      DO 78 I1=1,2
      DO 78 I2=1,2
```

```
              XSA(I1,I2)=0.
                DO 79 F=1,NNN-1
                  XSA(I1,I2)=XSA(I1,I2)+RE(F)*P(F)/GG(I1,I2,F)
79              CONTINUE
               XA10(I1,I2)=(-2.*PI/XN)*XSA(I1,I2)
78          CONTINUE
            XAA=0.
            DO 80 F=1,NNN-1
             XAA=XAA+(2./XN)*RE(F)**2.
80          CONTINUE
            DO 81 I1=1,4
            DO 81 I2=1,4
              EX1(1,I1,I2)=0.
              EX1(2,I1,I2)=0.
              EX1(3,I1,I2)=0.
              EX1(4,I1,I2)=0.
            DO 81 F=1,NNN-1
              EX1(1,I1,I2)=EX1(1,I1,I2)+(1./XN)*(EXX(I1,1,1,F)*EXX(I2,1,1,F))
              EX1(2,I1,I2)=EX1(2,I1,I2)+(1./XN)*(EXX(I1,2,1,F)*EXX(I2,2,1,F))
              EX1(3,I1,I2)=EX1(3,I1,I2)+(1./XN)*(EXX(I1,1,2,F)*EXX(I2,1,2,F))
              EX1(4,I1,I2)=EX1(4,I1,I2)+(1./XN)*(EXX(I1,2,2,F)*EXX(I2,2,2,F))
81          CONTINUE
            DO 82 I1=1,2
            DO 82 I2=1,2
              WW1(I1,I2)=EX1(1,I1,I2)
82          CONTINUE
            CALL F01ABF(WW1,3,2,ZZ1,2,QQ1,IFAIL)
            ZZ1(1,2)=ZZ1(2,1)
            DO 83 I1=1,2
            DO 83 I2=1,2
              EX11(I1,I2,1)=ZZ1(I1,I2)
83          CONTINUE
            DO 84 I1=1,3
            DO 84 I2=1,3
              WW2(I1,I2)=EX1(2,I1,I2)
84          CONTINUE
            CALL F01ABF(WW2,4,3,ZZ2,3,QQ2,IFAIL)
            ZZ2(1,2)=ZZ2(2,1)
            ZZ2(1,3)=ZZ2(3,1)
            ZZ2(2,3)=ZZ2(3,2)
            DO 85 I1=1,3
            DO 85 I2=1,3
              EX11(I1,I2,2)=ZZ2(I1,I2)
85          CONTINUE
            DO 86 I1=1,3
            DO 86 I2=1,3
              WW3(I1,I2)=EX1(3,I1,I2)
86          CONTINUE
            CALL F01ABF(WW3,4,3,ZZ3,3,QQ3,IFAIL)
            ZZ3(1,2)=ZZ3(2,1)
            ZZ3(1,3)=ZZ3(3,1)
            ZZ3(2,3)=ZZ3(3,2)
            DO 87 I1=1,3
            DO 87 I2=1,3
              EX11(I1,I2,3)=ZZ3(I1,I2)
87          CONTINUE
            DO 88 I1=1,4
            DO 88 I2=1,4
              WW4(I1,I2)=EX1(4,I1,I2)
```

```
88      CONTINUE
        CALL F01ABF(WW4,5,4,ZZ4,4,QQ4,IFAIL)
        DO 89 I1=1,3
        DO 89 I2=1+I1,4
          ZZ4(I1,I2)=ZZ4(I2,I1)
89      CONTINUE
        DO 90 I1=1,4
        DO 90 I2=1,4
          EX11(I1,I2,4)=ZZ4(I1,I2)
90      CONTINUE
        DO 91 IM=1,4
        DO 91 IN=1,4
          FE(IM,IN)=0.
91      CONTINUE
        DO 92 F=1,NNN-1
        DO 92 IK=1,4
          FE(1,IK)=FE(1,IK)+(1./XN)*RE(F)*EXX(IK,1,1,F)
          FE(2,IK)=FE(2,IK)+(1./XN)*RE(F)*EXX(IK,2,1,F)
          FE(3,IK)=FE(3,IK)+(1./XN)*RE(F)*EXX(IK,1,2,F)
          FE(4,IK)=FE(4,IK)+(1./XN)*RE(F)*EXX(IK,2,2,F)
92      CONTINUE
        SXS1=0.
        SXS2=0.
        SXS3=0.
        SXS4=0.
        DO 93 I1=1,4
        DO 93 I2=1,4
          SXS1=SXS1+EX11(I1,I2,1)*FE(1,I1)*FE(1,I2)
          SXS2=SXS2+EX11(I1,I2,2)*FE(2,I1)*FE(2,I2)
          SXS3=SXS3+EX11(I1,I2,3)*FE(3,I1)*FE(3,I2)
          SXS4=SXS4+EX11(I1,I2,4)*FE(4,I1)*FE(4,I2)
93      CONTINUE
        XB(1)=XAA-2.*SXS1
        XB(2)=XAA-2.*SXS2
        XB(3)=XAA-2.*SXS3
        XB(4)=XAA-2.*SXS4
        DO 94 IV=1,4
          IF(XB(IV).LT.0) THEN
            XXB(IV)=0.0000000001
          ELSE
            XXB(IV)=XB(IV)
          END IF
94      CONTINUE
          TEST(I,L,K,8)=((XN/XXB(1))**(0.5))*XA10(1,1)/VRR(1,1)
          TEST(I,L,K,9)=((XN/XXB(2))**(0.5))*XA10(2,1)/VRR(2,1)
          TEST(I,L,K,10)=((XN/XXB(3))**(0.5))*XA10(1,2)/VRR(1,2)
          TEST(I,L,K,11)=((XN/XXB(4))**(0.5))*XA10(2,2)/VRR(2,2)
        END IF
1000  CONTINUE
      END
```

TABLE 4.9a

$\hat{t}$ in (2.9) with $\rho(L;\theta) = (1 - L^4)^{d+\theta}$ for Japanese data from 1961.1 to 1973.4.

Series: c_t \ d	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
White Noise u_t:								
–	0.16'	-0.64'	-1.49'	-2.23	-2.80	-3.20	-3.49	-3.70
I	1.08'	-0.02'	-1.74'	-2.27	-2.48	-2.89	-3.30	-3.63
I,T	-0.36'	-1.25'	-2.15	-2.81	-3.24	-3.46	-3.52	-3.56
AR(1) u_t:								
–	-2.82	-3.10	-3.33	-3.54	-3.72	-3.87	-3.99	-4.08
I	-1.60'	-1.18'	-1.31'	-2.10	-2.64	-3.09	-3.42	-3.66
I,T	-0.41'	-1.06'	-1.84'	-2.50	-2.97	-3.25	-3.40	-3.51
AR(2) u_t:								
–	-2.89	-3.15	-3.35	-3.53	-3.70	-3.85	-3.98	-4.08
I	-1.65'	-1.17'	-1.22'	-2.03	-2.58	-3.02	-3.35	-3.59
I,T	-0.43'	-1.06'	-1.85'	-2.50	-2.97	-3.25	-3.41	-3.52
Series: y_t \ d	**0.50**	**0.75**	**1.00**	**1.25**	**1.50**	**1.75**	**2.00**	**2.25**
White Noise u_t:								
–	0.16'	-0.64'	-1.48'	-2.22	-2.79	-3.19	-3.48	-3.69
I	1.00'	-0.12'	-1.65'	-2.42	-2.85	-3.24	-3.57	-3.82
I,T	-0.14'	-0.84'	-1.76'	-2.52	-3.04	-3.29	-3.37	-3.44
AR(1) u_t:								
–	-2.74	-3.08	-3.32	-3.53	-3.71	-3.86	-3.98	-4.08
I	-0.55'	-0.31'	-1.11'	-1.99	-2.57	-3.05	-3.41	-3.67
I,T	-0.22'	-0.70'	-1.43'	-2.13	-2.68	-3.03	-3.23	-3.38
AR(2) u_t:								
–	-2.80	-3.12	-3.34	-3.53	-3.70	-3.85	-3.90	-4.08
I	-0.77'	-0.37'	-1.02'	-1.90'	-2.44	-2.90	-3.28	-3.55
I,T	-0.46'	-0.83'	-1.48'	-2.15	-2.68	-3.01	-3.19	-3.35
Series: c_t-y_t\d	**0.50**	**0.75**	**1.00**	**1.25**	**1.50**	**1.75**	**2.00**	**2.25**
White Noise u_t:								
–	-0.04'	-0.75	-1.55	-2.22	-2.74	-3.12	-3.41	-3.63
I	-0.16'	-0.98	-1.87	-2.56	-3.05	-3.39	-3.64	-3.82
I,T	-0.27'	-1.03	-1.88	-2.54	-2.99	-3.26	-3.40	-3.53
AR(1) u_t:								
–	-0.11'	-0.88'	-1.73'	-2.40	-2.88	-3.24	-3.50	-3.70
I	-0.14'	-0.86'	-1.69'	-2.37	-2.86	-3.23	-3.50	-3.69
I,T	-0.27'	-0.93'	-1.71'	-2.36	-2.84	-3.14	-3.34	-3.50
AR(2) u_t:								
–	-0.10'	-0.87'	-1.72'	-2.39	-2.89	-3.23	-3.49	-3.69
I	-0.29'	-1.14'	-1.97	-2.58	-3.02	-3.33	-3.56	-3.73
I,T	-0.62'	-1.27'	-1.99	-2.57	-2.98	-3.23	-3.37	-3.51

': Non-rejection values of the null hypothesis (1.12) at 95% significance level. The letters in bold correspond to those cases where monotonicity with respect to d is achieved.

TABLE 4.9b

$\hat{t}$ in (2/9) with $\rho(L;\theta) = (1 - L^4)^{d+\theta}$ for Japanese data from 1974.1 to 1987.4.

Series: c_t \ d	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
White Noise u_t:								
–	0.48'	-0.46'	-1.42'	-2.22	-2.82	-3.24	-3.53	-3.74
I	0.79'	-0.62'	-1.45'	-1.65'	-2.13	-2.67	-3.11	-3.45
I,T	0.66'	-0.04'	-1.09'	-2.02	-2.69	-2.99	-3.00	-3.04
AR(1) u_t:								
–	-2.89	-3.15	-3.38	-3.59	-3.77	-3.92	-4.05	-4.14
I	-1.12'	-0.87'	-0.93'	-1.49'	-2.20	-2.78	-3.20	-3.52
I,T	0.55'	0.18'	-0.63'	-1.51'	-2.18	-2.61	-2.89	-3.13
AR(2) u_t:								
–	-2.94	-3.19	-3.39	-3.57	-3.75	-3.90	-4.03	-4.14
I	-1.17'	-0.72'	-0.48'	-0.79'	-1.38'	-1.93'	-2.40	-2.78
I,T	-0.64'	-0.36'	-0.24'	-0.98'	-1.65'	-2.13	-2.47	-2.77
Series: y_t \ d	**0.50**	**0.75**	**1.00**	**1.25**	**1.50**	**1.75**	**2.00**	**2.25**
White Noise u_t:								
–	0.52'	-0.44'	-1.41'	-2.22	-2.82	-3.24	-3.54	-3.75
I	0.13'	-0.89'	-1.73'	-2.38	-2.90	-3.29	-3.57	-3.77
I,T	0.11'	-0.62'	-1.62'	-2.44	-2.96	-2.99	-2.74	-2.70
AR(1) u_t:								
–	-2.84	-3.14	-3.39	-3.60	-3.79	-3.94	-4.06	-4.15
I	0.06'	-0.63'	-1.35'	-2.02	-2.62	-3.06	-3.39	-3.62
I,T	0.01'	-0.50'	-1.27'	-2.07	-2.65	-2.87	-2.89	-2.98
AR(2) u_t:								
–	-2.89	-3.17	-3.38	-3.58	-3.70	-3.91	-4.04	-4.15
I	-0.06'	-0.63'	-1.32'	-1.90'	-2.48	-2.94	-3.29	-3.55
I,T	-0.00'	-0.51'	-1.21'	-1.98	-2.60	-2.79	-2.73	-2.82
Series: c_t-y_t\d	**0.50**	**0.75**	**1.00**	**1.25**	**1.50**	**1.75**	**2.00**	**2.25**
White Noise u_t:								
–	0.74'	-0.30'	-1.41'	-2.30	-2.94	-3.37	-3.66	-3.86
I	0.09'	-1.06'	-2.09	-2.76	-3.18	-3.48	-3.69	-3.84
I,T	-0.27'	-1.17'	-2.10	-2.75	-3.12	-3.14	-3.02	-3.08
AR(1) u_t:								
–	-0.77'	-1.25'	-2.33	-3.06	-3.51	-3.80	-4.00	-4.13
I	0.09'	-0.94'	-1.86'	-2.57	-3.05	-3.38	-3.61	-3.78
I,T	-0.45'	-1.08'	-1.87'	-2.57	-3.02	-3.17	-3.18	-3.28
AR(2) u_t:								
–	-1.00'	-1.52'	-2.10	-2.74	-3.25	-3.61	-3.87	-4.05
I	-0.06'	-0.98'	-1.84'	-2.53	-3.03	-3.37	-3.60	-3.77
I,T	-0.38'	-1.08'	-1.85'	-2.53	-2.99	-3.12	-3.11	-3.22

': Non-rejection values of the null hypothesis (1.12) at 95% significance level, and the letters in bold correspond to those cases where monotonicity with respect to d is achieved.

TABLE 4.10a

$\hat{t}$ in (2.9) with $\rho(L;\theta) = (1 - L^2)^{d+\theta}$ for Japanese data from 1961.1 to 1973.4

Series: c_t \ d	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
White Noise u_t:								
–	-1.86'	0.56'	-0.66'	-1.58'	-2.17	-2.56	-2.82	-3.01
I	0.52'	-3.31	-3.86	-3.80	-3.81	-3.83	-3.85	-3.87
I,T	-3.67	-3.77	-3.81	-3.84	-3.86	-3.88	-3.89	-3.90
AR(1) u_t:								
–	-1.99	-2.13	-2.31	-2.53	-2.76	-2.94	-3.08	-3.19
I	-2.66	-3.41	-3.85	-3.81	-3.81	-3.83	-3.85	-3.87
I,T	-3.67	-3.77	-3.81	-3.84	-3.86	-3.88	-3.89	-3.90
SAR(1) u_t:								
–	0.80'	0.22'	-0.67'	-1.61'	-2.30	-2.72	-2.97	-3.12
I	0.15'	-0.62'	-1.51'	-1.72'	-1.92'	-2.10	-2.27	-2.41
I,T	-0.83'	-1.21'	-1.52'	-1.78'	-2.00	-2.19	-2.37	-2.53
Series: y_t \ d	**0.50**	**0.75**	**1.00**	**1.25**	**1.50**	**1.75**	**2.00**	**2.25**
White Noise u_t:								
–	1.88'	0.59'	-0.61'	-1.52'	-2.12	-2.52	-2.79	-2.98
I	-2.10	-3.85	-3.85	-3.81	-3.82	-3.83	-3.85	-3.86
I,T	-3.75	-3.79	-3.81	-3.82	-3.85	-3.87	-3.89	-3.90
AR(1) u_t:								
–	-1.85'	-2.02	-2.21	-2.46	-2.69	-2.89	-3.04	-3.16
I	-2.85	-3.85	-3.86	-3.82	-3.83	-3.84	-3.85	-3.87
I,T	-3.75	-3.80	-3.81	-3.83	-3.85	-3.87	-3.89	-3.91
SAR(1) u_t:								
–	0.78'	0.21'	-0.65'	-1.56'	-2.23	-2.65	-2.90	-3.06
I	-0.05'	-1.03'	-1.46'	-1.69'	-1.92'	-2.12	-2.29	-2.45
I,T	-0.75'	-1.10'	-1.43'	-1.72'	-1.97	-2.19	-2.41	-2.60
Series: c_t-y_t\ d	**0.50**	**0.75**	**1.00**	**1.25**	**1.50**	**1.75**	**2.00**	**2.25**
White Noise u_t:								
–	-3.46	-3.59	-3.69	-3.76	-3.81	-3.84	-3.86	-3.88
I	-3.81	-3.82	-3.79	-3.80	-3.81	-3.83	-3.84	-3.85
I,T	-3.72	-3.76	-3.78	-3.80	-3.82	-3.85	-3.87	-3.89
AR(1) u_t:								
–	-3.49	-3.62	-3.71	-3.77	-3.82	-3.85	-3.87	-3.89
I	-3.81	-3.83	-3.80	-3.80	-3.82	-3.83	-3.85	-3.86
I,T	-3.73	-3.77	-3.79	-3.81	-3.83	-3.85	-3.87	-3.89
SAR(1) u_t:								
–	-1.09'	-1.37'	-1.65'	-1.90'	-2.10	-2.28	-2.43	-2.56
I	-1.93'	-1.77'	-1.77'	-1.93'	-2.10	-2.27	-2.43	-2.57
I,T	-1.51'	-1.60'	-1.74'	-1.93'	-2.13	-2.33	-2.52	-2.70

': Non-rejection values of the null hypothesis (1.12) at 95% significance level and the letters in bold correspond to those cases where monotonicity with respect to d is achieved.

TABLE 4.10b

$\hat{t}$ in (2.9) with $\rho(L;\theta) = (1 - L^2)^{d+\theta}$ for Japanese data from 1974.1 to 1987.4

Series: c_t \ d	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
White Noise u_t:								
–	2.22	0.72'	-0.62'	-1.60'	-2.22	-2.62	-2.80	-3.07
I	-1.97	-3.93	-3.90	-3.88	-3.90	-3.92	-3.94	-3.96
I,T	-3.67	-3.80	-3.86	-3.90	-3.93	-3.96	-3.98	-4.00
AR(1) u_t:								
–	-2.09	-2.20	-2.37	-2.61	-2.84	-3.02	-3.16	-3.27
I	-2.90	-3.93	-3.91	-3.88	-3.90	-3.93	-3.94	-3.96
I,T	-3.69	-3.81	-3.87	-3.90	-3.94	-3.96	-3.98	-4.00
SAR(1) u_t:								
–	0.88'	0.31'	-0.63'	-1.64'	-2.35	-2.79	-3.04	-3.20
I	-0.24'	-0.72'	-1.08'	-1.50'	-1.83'	-2.10	-2.32	-2.50
I,T	0.27'	-0.49'	-1.10'	-1.55'	-1.89'	-2.17	-2.42	-2.64
Series: y_t \ d	**0.50**	**0.75**	**1.00**	**1.25**	**1.50**	**1.75**	**2.00**	**2.25**
White Noise u_t:								
–	2.30	0.80'	-0.55'	-1.53'	-2.16	-2.57	-2.84	-3.03
I	-3.63	-3.66	-3.88	-3.88	-3.90	-3.91	-3.93	-3.94
I,T	-3.71	-3.81	-3.85	-3.89	-3.92	-3.94	-3.96	-3.98
AR(1) u_t:								
–	-2.01	-2.15	-2.33	-2.57	-2.80	-2.99	-3.13	-3.24
I	-3.64	-3.96	-3.89	-3.89	-3.91	-3.92	-3.94	-3.95
I,T	-3.71	-3.82	-3.87	-3.90	-3.93	-3.95	-3.96	-3.98
SAR(1) u_t:								
–	0.90'	0.37'	-0.55'	-1.55'	-2.28	-2.74	-3.00	-3.17
I	-1.15'	-2.06	-1.83'	-1.97	-2.14	-2.30	-2.45	-2.59
I,T	-1.47'	-1.59'	-1.78'	-1.99	-2.18	-2.33	-2.47	-2.64
Series: c_t-y_t\d	**0.50**	**0.75**	**1.00**	**1.25**	**1.50**	**1.75**	**2.00**	**2.25**
White Noise u_t:								
–	-1.73'	-2.74	-3.30	-3.59	-3.73	-3.81	-3.86	-3.90
I	-3.41	-3.76	-3.85	-3.88	-3.90	-3.92	-3.93	-3.94
I,T	-3.58	-3.77	-3.84	-3.89	-3.92	-3.92	-3.93	-3.95
AR(1) u_t:								
–	-2.31	-3.04	-3.44	-3.65	-3.77	-3.84	-3.88	-3.91
I	-3.41	-3.77	-3.86	-3.90	-3.92	-3.93	-3.94	-3.95
I,T	-3.58	-3.78	-3.86	-3.90	-3.93	-3.93	-3.94	-3.95
SAR(1) u_t:								
–	-0.81'	-1.43'	-1.83'	-2.08	-2.26	-2.41	-2.53	-2.65
I	-1.55'	-1.95'	-2.12	-2.23	-2.34	-2.44	-2.55	-2.65
I,T	-1.82'	-1.98	-2.11	-2.24	-2.37	-2.42	-2.46	-2.56

': Non-rejection values of the null hypothesis (1.12) at 95% significance level and the letters in bold correspond to those cases where monotonicity with respect to d is achieved.

TABLE 4.11a

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 - L)^{d+\theta}$ for Japanese data from 1961.1 to 1973.4.

Series: c_t \ d	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
White Noise u_t:								
–	4.42	1.77'	-0.45'	-1.88'	-2.72	-3.23	-3.56	-3.78
I	0.64'	-3.48	-3.69	-3.78	-3.91	-4.03	-4.15	-4.25
I,T	-3.39	-3.52	-3.66	-3.80	-3.93	-4.05	-4.16	-4.26
SAR(1) u_t:								
–	3.38	1.57'	-0.44'	-1.89'	-2.74	-3.25	-3.58	-3.80
I	3.39	0.51'	-1.80'	-2.19	-2.67	-3.10	-3.46	-3.75
I,T	0.03'	-0.92'	-1.69'	-2.32	-2.85	-3.28	-3.61	-3.87
SAR(2) u_t:								
–	3.40	1.57'	-0.44'	-1.89'	-2.74	-3.25	-3.58	-3.80
I	4.66	0.44'	-1.82'	-2.23	-2.71	-3.15	-3.51	-3.81
I,T	0.07'	-0.94'	-1.72'	-2.36	-2.89	-3.32	-3.66	-3.92
Series: y_t \ d	**0.50**	**0.75**	**1.00**	**1.25**	**1.50**	**1.75**	**2.00**	**2.25**
White Noise u_t:								
–	4.33	1.72'	-0.48'	-1.90'	-2.75	-3.27	-3.60	-3.83
I	-2.19	-3.48	-3.53	-3.66	-3.81	-3.94	-4.07	-4.17
I,T	-3.25	-3.36	-3.51	-3.67	-3.82	-3.96	-4.08	-4.18
SAR(1) u_t:								
–	3.22	-1.43'	-0.51'	-1.91'	-2.75	-3.27	-3.61	-3.85
I	-3.65	-1.15'	-1.47'	-2.09	-2.67	-3.16	-3.54	-3.83
I,T	0.32'	-0.59'	-1.42'	-2.16	-2.78	-3.30	-3.68	-3.94
SAR(2) u_t:								
–	3.26	1.44'	-0.51'	-1.91'	-2.75	-3.27	-3.61	-3.85
I	4.44	-1.27'	-1.48'	-2.11	-2.71	-3.20	-3.59	-3.88
I,T	0.41'	-0.54'	-1.41'	-2.18	-2.82	-3.34	-3.73	-3.99
Series: c_t-y_t \ d	**0.50**	**0.75**	**1.00**	**1.25**	**1.50**	**1.75**	**2.00**	**2.25**
White Noise u_t:								
–	-2.85	-3.08	-3.31	-3.50	-3.66	-3.80	-3.93	-4.05
I	-3.18	-3.22	-3.33	-3.49	-3.65	-3.80	-3.93	-4.05
I,T	-3.03	-3.16	-3.32	-3.50	-3.66	-3.81	-3.94	-4.06
SAR(1) u_t:								
–	-0.19'	-1.01'	-1.79'	-2.45	-2.99	-3.41	-3.74	-3.98
I	-1.50'	-1.42'	-1.85'	-2.44	-2.97	-3.40	-3.73	-3.97
I,T	-0.70'	-1.19'	-1.83'	-2.46	-3.02	-3.48	-3.81	-4.04
SAR(2) u_t:								
–	0.11'	-0.82'	-1.71'	-2.44	-3.01	-3.45	-3.78	-4.03
I	-1.43'	-1.35'	-1.79'	-2.43	-2.99	-3.44	-3.78	-4.02
I,T	-0.50'	-1.06'	-1.76'	-2.45	-3.04	-3.52	-3.86	-4.03

': Non-rejection values of the null hypothesis (1.12) at 95% significance level and the letters in bold correspond to those cases where monotonicity with respect to d is achieved.

TABLE 4.11b

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 - L)^{d+\theta}$ for Japanese data from 1974.1 to 1987.4.

Series: c_t \ d	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
White Noise u_t:								
–	5.02	2.02	-0.39'	-1.91'	-2.78	-3.30	-3.64	-3.88
I	-1.83'	-3.37	-3.36	-3.49	-3.65	-3.80	-3.95	-4.07
I,T	-2.98	-3.16	-3.34	-3.51	-3.68	-3.83	-3.90	-4.08
SAR(1) u_t:								
–	3.67	1.78'	-0.38'	-1.92'	-2.80	-3.33	-3.66	-3.90
I	2.64	2.22	0.37'	-0.69'	-1.55'	-2.27	-2.86	-3.33
I,T	3.83	1.74'	0.27'	-0.82'	-1.74'	-2.51	-3.09	-3.51
SAR(2) u_t:								
–	3.70	1.77'	-0.38'	-1.92'	-2.80	-3.33	-3.67	-3.90
I	4.14	2.05	0.37'	-0.68'	-1.54'	-2.25	-2.85	-3.34
I,T	3.76	1.71'	0.28'	-0.81'	-1.71'	-2.48	-3.08	-3.52
Series: y_t \ d	**0.50**	**0.75**	**1.00**	**1.25**	**1.50**	**1.75**	**2.00**	**2.25**
White Noise u_t:								
–	3.05	2.02	-0.41'	-1.93'	-2.81	-3.35	-3.69	-3.93
I	-4.09	-4.13	-4.17	-4.29	-4.42	-4.52	-4.61	-4.68
I,T	-3.93	-4.02	-4.16	-4.30	-4.43	-4.54	-4.63	-4.70
SAR(1) u_t:								
–	3.59	1.70'	-0.43'	-1.94'	-2.82	-3.35	-3.70	-3.94
I	-0.68'	0.64'	-0.20'	-1.13'	-1.92'	-2.58	-3.12	-3.53
I,T	1.98	0.90'	-0.22'	-1.20'	-2.07	-2.81	-3.39	-3.81
SAR(2) u_t:								
–	3.64	1.69'	-0.43'	-1.93'	-2.81	-3.34	-3.70	-3.94
I	-0.71'	0.63'	-0.91'	-1.15'	-1.97	-2.64	-3.17	-3.59
I,T	1.92'	0.91'	-0.21'	-1.22'	-2.11	-2.86	-3.44	-3.87
Series: c_t-y_t \ d	**0.50**	**0.75**	**1.00**	**1.25**	**1.50**	**1.75**	**2.00**	**2.25**
White Noise u_t:								
–	-3.42	-4.03	-4.34	-4.51	-4.62	-4.70	-4.76	-4.81
I	-4.19	-4.31	-4.41	-4.52	-4.62	-4.69	-4.75	-4.80
I,T	-4.20	-4.29	-4.41	-4.53	-4.63	-4.71	-4.77	-4.82
SAR(1) u_t:								
–	1.81'	0.51'	-0.54'	-1.40'	-2.12	-2.71	-3.19	-3.56
I	1.21'	0.40'	-0.51'	-1.38'	-2.13	-2.75	-3.24	-3.62
I,T	1.13'	0.44'	-0.51'	-1.42'	-2.24	-2.92	-3.45	-3.83
SAR(2) u_t:								
–	2.59	0.88'	-0.45'	-1.43'	-2.18	-2.77	-3.24	-3.62
I	1.33'	0.47'	-0.48'	-1.42'	-2.20	-2.82	-3.31	-3.69
I,T	1.24'	0.54'	-0.48'	-1.46'	-2.32	-3.00	-3.52	-3.92

': Non-rejection values of the null hypothesis (1.12) at 95% significance level, and the letters in bold correspond to those cases where monotonicity with respect to d is achieved.

TABLE 4.12a

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 - L^4)^{d+\theta}$ for U.K. data from 1955.1 to 1973.4

Series: c_t \ d	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
White Noise u_t:								
--	1.51'	0.11'	-1.22'	-2.66	-2.98	-3.45	-3.78	-4.01
I	1.92'	-0.34'	-2.19	-2.69	-3.15	-3.60	-3.95	-4.20
I,T	-0.06'	-1.04'	-1.94'	-2.65	-3.18	-3.55	-3.76	-3.89
AR(1) u_t:								
--	-3.01	-3.30	-3.57	-3.82	-4.03	-4.20	-4.33	-4.43
I	-1.39'	-1.52'	-2.38	-3.13	-3.59	-3.89	-4.10	-4.24
I,T	-0.02'	-1.20'	-2.38	-3.14	-3.58	-3.83	-3.98	-4.08
AR(2) u_t:								
--	-3.06	-3.32	-3.55	-3.78	-4.00	-4.17	-4.32	-4.43
I	-1.67'	-1.85'	-2.49	-3.10	-3.52	-3.80	-4.01	-4.17
I,T	-0.34'	-1.45'	-2.49	-3.17	-3.57	-3.80	-3.94	-4.04
Series: y_t \ d	**0.50**	**0.75**	**1.00**	**1.25**	**1.50**	**1.75**	**2.00**	**2.25**
White Noise u_t:								
--	1.52'	0.12'	-1.21'	-2.25	-2.97	-3.45	-3.77	-4.00
I	1.54'	-0.87'	-2.00	-2.31	-2.91	-3.50	-3.96	-4.25
I,T	0.41'	-0.58'	-1.53'	-2.36	-3.05	-3.59	-3.96	-4.19
AR(1) u_t:								
--	-3.01	-3.30	-3.57	-3.82	-4.03	-4.20	-4.33	-4.43
I	-2.16	-2.47	-3.06	-3.53	-3.87	-4.13	-4.31	-4.44
I,T	-0.70'	-2.08	-3.03	-3.59	-3.92	-4.14	-4.30	-4.41
AR(2) u_t:								
--	-3.06	-3.32	-3.55	-3.78	-3.99	-4.17	-4.32	-4.43
I	-2.42	-2.77	-3.22	-3.64	-3.94	-4.16	-4.33	-4.45
I,T	-1.08'	-2.28	-3.17	-3.69	-3.99	-4.19	-4.33	-4.42
Series: c_t-y_t \ d	**0.50**	**0.75**	**1.00**	**1.25**	**1.50**	**1.75**	**2.00**	**2.25**
White Noise u_t:								
--	-0.16'	-0.71'	-1.47'	-2.21	-2.82	-3.28	-3.63	-3.88
I	-0.67'	-1.80'	-2.34	-2.81	-3.22	-3.54	-3.79	-3.98
I,T	-0.38'	-1.46'	-2.26	-2.83	-3.23	-3.53	-3.76	-3.94
AR(1) u_t:								
--	-0.54'	-1.51'	-2.30	-2.87	-3.29	-3.60	-3.83	-4.01
I	-0.99'	-2.00	-2.61	-3.08	-3.43	-3.69	-3.89	-4.04
I,T	-0.65'	-1.77'	-2.56	-3.09	-3.44	-3.68	-3.87	-4.02
AR(2) u_t:								
--	-0.60'	-1.53'	-2.31	-2.88	-3.29	-3.59	-3.82	-3.99
I	-0.99'	-2.01	-2.62	-3.08	-3.43	-3.69	-3.89	-4.04
I,T	-0.70'	-1.78'	-2.57	-3.09	-3.44	-3.68	-3.87	-4.02

': Non-rejection values of the null hypothesis (1.12) at 95% significance level and the letters in bold correspond to those cases where monotonicity with respect to d is achieved.

TABLE 4.12b

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 - L^4)^{d+\theta}$ for U.K. data from 1974.1 to 1984.4

Series: c_t \ d	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
White Noise u_t:								
--	-0.22'	-0.91'	-1.62'	-2.25	-2.75	-3.11	-3.38	-3.57
I	0.31'	-0.83'	-1.83'	-2.39	-2.74	-3.00	-3.21	-3.38
I,T	-1.10'	-1.57'	-1.99	-2.34	-2.63	-2.81	-2.89	-2.95
AR(1) u_t:								
--	-2.87	-3.08	-3.27	-3.46	-3.62	-3.75	-3.86	-3.96
I	-0.38'	-1.05'	-1.97	-2.66	-3.10	-3.38	-3.57	-3.70
I,T	-0.73'	-1.41'	-2.13	-2.68	-3.06	-3.30	-3.43	-3.51
AR(2) u_t:								
--	-2.93	-3.13	-3.30	-3.45	-3.60	-3.74	-3.85	-3.95
I	-0.51'	-1.11'	-1.97	-2.62	-3.05	-3.33	-3.52	-3.66
I,T	-0.73'	-1.45'	-2.15	-2.68	-3.05	-3.28	-3.41	-3.49
Series: y_t \ d	**0.50**	**0.75**	**1.00**	**1.25**	**1.50**	**1.75**	**2.00**	**2.25**
White Noise u_t:								
--	-0.22'	-0.91'	-1.62'	-2.25	-2.74	-3.11	-3.37	-3.57
I	0.34'	-0.66'	-1.35'	-1.76'	-2.10	-2.42	-2.71	-2.97
I,T	-0.80'	-1.08'	-1.40'	-1.74'	-2.06	-2.35	-2.59	-2.77
AR(1) u_t:								
--	-2.86	-3.07	-3.27	-3.45	-3.62	-3.75	-3.86	-3.95
I	-2.07	-2.26	-2.56	-2.87	-3.13	-3.34	-3.50	-3.63
I,T	-1.85'	-2.24	-2.57	-2.86	-3.11	-3.31	-3.45	-3.55
AR(2) u_t:								
--	-2.92	-3.12	-3.29	-3.45	-3.60	-3.75	-3.85	-3.95
I	-2.20	-2.40	-2.68	-2.95	-3.18	-3.35	-3.49	-3.60
I,T	-2.03	-2.41	-2.71	-2.97	-3.18	-3.34	-3.46	-3.55
Series: c_t-y_t \ d	**0.50**	**0.75**	**1.00**	**1.25**	**1.50**	**1.75**	**2.00**	**2.25**
White Noise u_t:								
--	-0.52'	-0.98'	-1.50'	-1.98	-2.40	-2.74	-3.03	-3.27
I	-0.60'	-1.26'	-1.82'	-2.28	-2.66	-2.97	-3.23	-3.45
I,T	-0.60'	-1.26'	-1.83'	-2.28	-2.64	-2.93	-3.17	-3.39
AR(1) u_t:								
--	-0.72'	-1.54'	-2.17	-2.62	-2.96	-3.22	-3.44	-3.61
I	-0.62'	-1.30'	-1.89'	-2.35	-2.72	-3.02	-3.27	-3.49
I,T	-0.62'	-1.31'	-1.89'	-2.35	-2.71	-2.99	-3.22	-3.43
AR(2) u_t:								
--	-1.19'	-1.89'	-2.39	-2.74	-3.02	-3.25	-3.45	-3.61
I	-0.83'	-1.55'	-2.11	-2.52	-2.85	-3.12	-3.35	-3.54
I,T	-0.83'	-1.56'	-2.12	-2.52	-2.84	-3.09	-3.31	-3.50

': Non-rejection values of the null hypothesis (1.12) at 95% significance level and the letters in bold correspond to those cases where monotonicity with respect to d is achieved.

TABLE 4.13a

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 - L^2)^{d+\theta}$ for U.K. data from 1955.1 to 1973.4.

Series: c_t \ d	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
White Noise u_t:								
--	3.25	1.17'	-0.59'	-1.76'	-2.47	-2.91	-3.19	-3.39
I	0.56'	-3.67	-4.16	-4.22	-4.27	-4.31	-4.33	-4.34
I,T	-3.41	-3.88	-4.11	-4.23	-4.29	-4.32	-4.34	-4.36
AR(1) u_t:								
--	-2.25	-2.36	-2.59	-2.89	-3.15	-3.35	-3.50	-3.61
I	-2.69	-3.79	-4.18	-4.24	-4.28	-4.32	-4.33	-4.35
I,T	-3.52	-3.93	-4.14	-4.24	-4.30	-4.32	-4.34	-4.36
SAR(1) u_t:								
--	1.25'	0.61'	-0.61'	-1.83'	-2.65	-3.12	-3.40	-3.56
I	0.41'	-0.81'	-2.17	-2.50	-2.67	-2.80	-2.91	-3.01
I,T	-0.96'	-1.70'	-2.18	-2.47	-2.66	-2.80	-2.93	-3.07

Series: y_t \d	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
White Noise u_t:								
--	3.23	1.15'	-0.61'	-1.79'	-2.49	-2.93	-3.21	-3.41
I	2.92	-1.57'	-3.13	-3.59	-3.88	-4.04	-4.14	-4.20
I,T	-0.53'	-2.07	-3.04	-3.59	-3.80	-4.01	-4.07	-4.13
AR(1) u_t:								
--	-2.29	-2.39	-2.61	-2.91	-3.17	-3.37	-3.52	-3.63
I	-2.60	-2.82	-3.43	-3.75	-3.95	-4.00	-4.16	-4.21
I,T	-2.22	-2.89	-3.38	-3.73	-3.94	-4.05	-4.11	-4.16
SAR(1) u_t:								
--	-1.25'	0.59'	-0.64'	-1.86'	-2.67	-3.14	-3.41	-3.57
I	0.33'	-1.15'	-2.30	-2.73	-2.94	-3.07	-3.17	-3.24
I,T	-0.63'	-1.62'	-2.29	-2.68	-2.90	-3.01	-3.08	-3.15

Series: c_t-y_t \d	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
White Noise u_t:								
--	-3.48	-3.82	-4.06	-4.20	-4.27	-4.31	-4.33	-4.34
I	-3.22	-3.86	-4.05	-4.14	-4.19	-4.22	-4.24	-4.26
I,T	-3.52	-3.89	-4.06	-4.14	-4.19	-4.23	-4.26	-4.28
AR(1) u_t:								
--	-3.61	-3.88	-4.09	-4.21	-4.28	-4.31	-4.33	-4.34
I	-3.39	-3.90	-4.07	-4.15	-4.19	-4.22	-4.24	-4.26
I,T	-3.62	-3.93	-4.07	-4.15	-4.20	-4.23	-4.26	-4.28
SAR(1) u_t:								
--	-1.97	-2.37	-2.70	-2.94	-3.11	-3.23	-3.31	-3.35
I	-1.91'	-2.38	-2.64	-2.80	-2.92	-3.03	-3.12	-3.20
I,T	-2.08	-2.42	-2.64	-2.80	-2.93	-3.04	-3.15	-3.25

': Non-rejection values of the null hypothesis (1.12) at 95% significance level, and the letters in bold correspond to those cases where monotonicity with respect to d is achieved.

TABLE 4.13b

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 - L^2)^{d+\theta}$ for U.K. data from 1974.1 to 1984.4.

Series: c_t \ d	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
White Noise u_t:								
--	1.48'	0.30'	-0.78'	-1.59'	-2.14	-2.49	-2.74	-2.91
I	-2.19	-3.24	-3.42	-3.48	-3.53	-3.57	-3.60	-3.62
I,T	-2.90	-3.21	-3.38	-3.48	-3.54	-3.59	-3.63	-3.66
AR(1) u_t:								
--	-2.11	-2.20	-2.33	-2.52	-2.71	-2.87	-3.00	-3.10
I	-2.71	-3.31	-3.45	-3.51	-3.55	-3.59	-3.61	-3.63
I,T	-3.04	-3.29	-3.43	-3.51	-3.57	-3.61	-3.64	-3.67
SAR(1) u_t:								
--	-0.55'	0.02'	-0.79'	-1.64'	-2.25	-2.63	-2.85	-2.99
I	0.17'	-0.89'	-1.44'	-1.72'	-1.92'	-2.08	-2.22	-2.34
I,T	-0.03'	-1.08'	-1.44'	-1.71'	-1.93'	-2.11	-2.28	-2.44

Series: y_t \d	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
White Noise u_t:								
--	1.46'	0.28'	-0.79'	-1.61'	-2.16	-2.51	-2.75	-2.92
I	1.20'	-0.58'	-1.51'	-2.12	-2.56	-2.87	-3.00	-3.21
I,T	0.47'	-0.56'	-1.44'	-2.10	-2.57	-2.90	-3.12	-3.27
AR(1) u_t:								
--	-2.14	-2.23	-2.35	-2.54	-2.73	-2.89	-3.01	-3.11
I	-2.11	-2.26	-2.51	-2.75	-2.95	-3.12	-3.25	-3.34
I,T	-1.71'	-2.16	-2.48	-2.74	-2.96	-3.14	-3.27	-3.37
SAR(1) u_t:								
--	0.54'	0.02'	-0.82'	-1.66'	-2.27	-2.64	-2.86	-3.00
I	0.41'	-0.57'	-1.34'	-1.86'	-2.23	-2.45	-2.59	-2.67
I,T	0.38'	-0.51'	-1.28'	-1.84'	-2.22	-2.46	-2.62	-2.72

Series: c_t-y_t \d	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
White Noise u_t:								
--	-1.69'	-2.43	-3.01	-3.36	-3.54	-3.63	-3.68	-3.70
I	-2.75	-3.19	-3.41	-3.52	-3.58	-3.61	-3.63	-3.64
I,T	-2.71	-3.17	-3.40	-3.52	-3.59	-3.62	-3.65	-3.67
AR(1) u_t:								
--	-2.42	-2.85	-3.20	-3.44	-3.58	-3.65	-3.69	-3.72
I	-2.77	-3.20	-3.42	-3.53	-3.59	-3.62	-3.64	-3.65
I,T	-2.74	-3.19	-3.41	-3.53	-3.59	-3.63	-3.66	-3.68
SAR(1) u_t:								
--	-0.46'	-1.12'	-1.67'	-2.07	-2.34	-2.52	-2.65	-2.73
I	-1.43'	-1.93'	-2.23	-2.40	-2.52	-2.62	-2.70	-2.77
I,T	-1.48'	-1.94'	-2.23	-2.41	-2.53	-2.63	-2.72	-2.81

': Non-rejection values of the null hypothesis (1.12) at 95% significance level, and the letters in bold correspond to those cases where monotonicity with respect to d is achieved.

TABLE 4.14a

$\hat{t}$ in (2.9) with $\rho(L;\theta) = (1 - L)^{d+\theta}$ for U.K. data from 1955.1 to 1973.4

Series: c_t \ d	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
White Noise u_t:								
-	6.66	2.63	-0.40'	-2.17	-3.14	-3.71	-4.08	-4.33
I	0.05'	-4.00	-4.26	-4.44	-4.61	-4.76	-4.88	-4.98
I,T	-3.49	-3.95	-4.24	-4.45	-4.62	-4.76	-4.89	-4.99
SAR(1) u_t:								
-	4.53	2.27	-0.39'	-2.19	-3.17	-3.74	-4.11	-4.37
I	3.54	-0.54'	-1.64'	-2.30	-2.94	-3.50	-3.96	-4.32
I,T	0.69'	-0.66'	-1.57'	-2.34	-3.03	-3.63	-4.09	-4.44
SAR(2) u_t:								
-	4.53	2.24	-0.39'	-2.19	-3.17	-3.75	-4.11	-4.37
I	5.03	-0.36'	-1.76'	-2.46	-3.08	-3.61	-4.05	-4.40
I,T	1.10'	-0.65'	-1.71'	-2.49	-3.16	-3.73	-4.18	-4.52

Series: y_t \ d	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
White Noise u_t:								
-	6.63	2.61	-0.42'	-2.18	-3.14	-3.71	-4.07	-4.33
I	-4.14	-3.13	-4.14	-4.53	-4.77	-4.93	-5.05	-5.14
I,T	-1.49'	-3.25	-4.10	-4.54	-4.77	-4.90	-5.01	-5.11
SAR(1) u_t:								
-	4.51	-2.23	-0.42'	-2.20	-3.18	-3.74	-4.11	-4.36
I	3.25	-0.56'	-2.35	-3.11	-3.65	-4.05	-4.34	-4.54
I,T	0.73'	-0.11'	-2.31	-3.10	-3.64	-3.95	-4.11	-4.28
SAR(2) u_t:								
-	4.52	2.20	-0.42'	-2.21	-3.18	-3.74	-4.11	-4.36
I	5.09	1.28'	-1.85'	-3.02	-3.66	-4.07	-4.35	-4.55
I,T	2.40	-0.01'	-1.86'	-2.97	-3.60	-3.89	-4.08	-4.29

Series: c_t-y_t \d	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
White Noise u_t:								
-	-2.72	-3.23	-3.55	-3.74	-3.89	-4.02	-4.16	-4.29
I	-2.41	-3.18	-3.48	-3.68	-3.85	-4.01	-4.16	-4.31
I,T	-2.72	-3.21	-3.48	-3.68	-3.85	-4.01	-4.15	-4.29
SAR(1) u_t:								
-	-1.69'	-2.79	-3.46	-3.83	-4.05	-4.21	-4.34	-4.47
I	-0.99'	-2.29	-2.97	-3.41	-3.75	-4.02	-4.24	-4.42
I,T	-1.49'	-2.38	-2.98	-3.41	-3.75	-4.00	-4.18	-4.33
SAR(2) u_t:								
-	-1.46'	-2.70	-3.46	-3.86	-4.09	-4.25	-4.39	-4.52
I	-0.81'	-2.28	-3.01	-3.47	-3.80	-4.07	-4.29	-4.47
I,T	-1.36'	-2.37	-3.02	-3.47	-3.81	-4.06	-4.24	-4.40

': Non-rejection values of the null hypothesis (1.12) at 95% significance level, and the letters in bold correspond to those cases where monotonicity with respect to d is achieved.

TABLE 4.14b

$\hat{t}$ in (2.9) with $\rho(L;\theta) = (1 - L)^{d+\theta}$ for U.K. data from 1974.1 to 1984.4

Series: c_t \ d	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
White Noise u_t:								
-	3.88	1.46'	-0.53'	-1.83'	-2.59	-3.06	-3.36	-3.57
I	-1.69'	-2.56	-2.71	-2.90	-3.09	-3.26	-3.41	-3.55
I,T	-2.04	-2.41	-2.68	-2.90	-3.11	-3.29	-3.44	-3.57
SAR(1) u_t:								
-	3.00	1.30'	-0.53'	-1.84'	-2.61	-3.07	-3.37	-3.58
I	1.70'	-0.06'	-0.79'	-1.48'	-2.08	-2.59	-3.02	-3.35
I,T	1.23'	0.11'	-0.77'	-1.52'	-2.17	-2.73	-3.17	-3.49
SAR(2) u_t:								
-	-3.04	1.31'	-0.53'	-1.84'	-2.61	-3.08	-3.38	-3.59
I	2.82	0.72'	-0.38'	-1.30'	-2.03	-2.59	-3.03	-3.36
I,T	2.23	0.87'	-0.35'	-1.34'	-2.12	-2.74	-3.19	-3.50

Series: y_t \ d	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
White Noise u_t:								
-	3.84	1.42'	-0.57'	-1.85'	-2.61	-3.07	-3.37	-3.58
I	2.61	-0.47'	-1.83'	-2.61	-3.11	-3.43	-3.65	-3.81
I,T	1.69'	-0.39'	-1.78'	-2.62	-3.13	-3.44	-3.63	-3.76
SAR(1) u_t:								
-	2.96	1.26'	-0.57'	-1.86'	-2.62	-3.08	-3.38	-3.59
I	1.56'	-0.56'	-1.72'	-2.48	-2.99	-3.35	-3.60	-3.79
I,T	1.58'	-0.34'	-1.66'	-2.50	-3.03	-3.37	-3.57	-3.72
SAR(2) u_t:								
-	3.01	1.27'	-0.57'	-1.87'	-2.63	-3.08	-3.38	-3.39
I	1.28'	-0.72'	-1.73'	-2.51	-3.03	-3.37	-3.60	-3.77
I,T	0.59'	-0.55'	-1.67'	-2.52	-3.06	-3.39	-3.57	-3.70

Series: c_t-y_t \d	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
White Noise u_t:								
-	-1.67'	-2.60	-3.17	-3.47	-3.64	-3.77	-3.88	-3.97
I	-2.79	-3.11	-3.32	-3.50	-3.65	-3.78	-3.89	-3.99
I,T	-2.71	-3.08	-3.32	-3.50	-3.66	-3.78	-3.89	-3.99
SAR(1) u_t:								
-	0.30'	-1.19'	-2.38	-3.12	-3.54	-3.78	-3.94	-4.05
I	-2.10	-2.61	-2.92	-3.21	-3.47	-3.68	-3.84	-3.97
I,T	-2.00	-2.54	-2.91	-3.22	-3.48	-3.69	-3.85	-3.97
SAR(2) u_t:								
-	0.47'	-1.09'	-2.40	-3.19	-3.61	-3.85	-4.01	-4.12
I	-2.11	-2.67	-3.00	-3.30	-3.55	-3.76	-3.92	-4.03
I,T	-2.00	-2.60	-2.99	-3.30	-3.57	-3.78	-3.94	-4.06

': Non-rejection values of the null hypothesis (1.12) at 95% significance level, and the letters in bold correspond to those cases where monotonicity with respect to d is achieved.

TABLE 4.15a

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 - L^4)^{d+\theta}$ and dummy variables for the changing trend in Japanese data.

Series: c_t \ d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
White Noise u_t:									
I,T*	0.40'	1.11'	0.88'	-0.60'	-2.00	-2.94	-3.52	-3.85	-4.06
I,T*,D	3.63	1.54'	-0.30'	-1.65'	-2.62	-3.30	-3.76	-4.05	-4.26
AR(1) u_t:									
I,T*	0.47'	1.10'	1.14'	-0.17'	-1.72'	-2.78	-3.43	-3.83	-4.09
I,T*,D	5.31	2.11	-1.20'	-3.05	-3.85	-4.25	-4.48	-4.63	-4.73
AR(2) u_t:									
I,T*	0.91'	1.23'	1.05'	-0.32'	-1.84'	-2.86	-3.48	-3.87	-4.14
I,T*,D	3.85	0.70'	-1.84'	-3.30	-4.00	-4.37	-4.59	-4.72	-4.81

Series: y_t \ d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
White Noise u_t:									
I,T*	0.22'	0.73'	1.01'	-0.25'	-1.84'	-2.91	-3.48	-3.67	-3.72
I,T*,D	5.49	2.52	-0.37'	-2.13	-3.06	-3.62	-3.99	-4.24	-4.43
AR(1) u_t:									
I,T*	0.29'	0.90'	0.94'	-0.17'	-1.61'	-2.67	-3.30	-3.61	-3.77
I,T*,D	5.55	2.59	-0.47'	-2.32	-3.21	-3.71	-4.04	-4.28	-4.45
AR(2) u_t:									
I,T*	0.29'	0.61'	0.51'	-0.44'	-1.74'	-2.73	-3.32	-3.58	-3.73
I,T*,D	5.10	2.45	-0.47'	-2.36	-3.24	-3.73	-4.05	-4.28	-4.45

Series: c_t-y_t\d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
White Noise u_t:									
I,T*	0.75'	1.11'	0.87'	-0.57'	-2.04	-3.01	-3.55	-3.78	-3.87
I,T*,D	6.09	3.55	0.80'	-1.21'	-2.51	-3.33	-3.85	-4.18	-4.40
AR(1) u_t:									
I,T*	0.96'	1.45'	0.85'	-0.59'	-2.04	-3.00	-3.56	-3.84	-4.00
I,T*,D	6.23	3.68	0.39'	-1.89'	-3.00	-3.61	-4.00	-4.27	-4.45
AR(2) u_t:									
I,T*	1.16'	1.45'	0.84'	-0.64'	-2.08	-3.03	-3.57	-3.82	-3.96
I,T*,D	6.09	3.55	0.80'	-1.21'	-2.51	-3.33	-3.85	-4.18	-4.40

': Non-rejection values of the null hypothesis (1.12) at 95% level. The letters in bold correspond to those cases where monotonicity is achieved. T^* means that we include dummy variables for the change in the slope in the trend.

TABLE 4.15b

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 - L^4)^{d+\theta}$ and dummy variables for the changing trend in U.K. data.

Series: c_t \ d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
White Noise u_t:									
I,T*	0.63'	0.54'	-0.30'	-1.30'	-2.10	-2.71	-3.23	-3.67	-3.96
I,T*,S	2.03	0.40'	-0.78'	-1.58'	-2.18	-2.69	-3.20	-3.66	-4.00
AR(1) u_t:									
I,T*	0.64'	0.85'	0.44'	-1.06'	-2.42	-3.29	-3.85	-4.23	-4.46
I,T*,S	4.37	1.25'	-1.41'	-2.84	-3.50	-3.89	-4.18	-4.41	-4.59
AR(2) u_t:									
I,T*	0.66'	0.73'	-0.13'	-1.28'	-2.52	-3.33	-3.86	-4.21	-4.44
I,T*,S	4.45	1.11'	-1.55'	-2.97	-3.62	-3.99	-4.25	-4.46	-4.62

Series: y_t \ d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
White Noise u_t:									
I,T*	1.66'	0.89'	-0.11'	-1.01'	-1.77'	-2.43	-3.02	-3.57	-4.00
I,T*,S	2.00	0.75'	-0.28'	-1.10'	-1.78'	-2.40	-2.98	-3.53	-3.96
AR(1) u_t:									
I,T*	1.61'	1.06'	-0.45'	-1.80'	-2.76	-3.43	-3.93	-4.31	-4.58
I,T*,S	3.02	0.42'	-1.41'	-2.42	-3.05	-3.54	-3.94	-4.28	-4.53
AR(2) u_t:									
I,T*	0.76'	0.03'	-1.15'	-2.19	-2.97	-3.54	-3.97	-4.31	-4.57
I,T*,S	1.45'	-0.49'	-1.89'	-2.69	-3.20	-3.61	-3.96	-4.26	-4.51

Series: c_t-y_t\d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
White Noise u_t:									
I,T*	1.45'	0.81'	-0.37'	-1.54'	-2.43	-3.08	-3.54	-3.88	-4.14
I,T*,S	2.98	1.26'	-0.20'	-1.36'	-2.24	-2.90	-3.40	-3.79	-4.11
AR(1) u_t:									
I,T*	1.38'	0.55'	-0.70'	-1.85'	-2.68	-3.27	-3.69	-4.00	-4.24
I,T*,S	2.83	0.87'	-0.64'	-1.72'	-2.52	-3.12	-3.57	-3.92	-4.28
AR(2) u_t:									
I,T*	1.44'	0.55'	-0.78'	-2.00	-2.84	-3.40	-3.79	-4.06	-4.28
I,T*,S	3.23	1.03'	-0.71'	-1.90'	-2.71	-3.28	-3.69	-4.01	-4.27

': Non-rejection values of the null hypothesis (1.12) at 95% level and the letters in bold correspond to those cases where monotonicity is achieved. T^* means that we include dummy variables for the change in the slope in the trend.

TABLE 4.16a

$\hat{t}$ in (2.9) with $\rho(L;\theta) = (1 - L^2)^{d+\theta}$ and dummy variables for the changing trend in Japanese data.

Series: c_t \ d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
White Noise u_t:									
I,T*	-2.46	-3.99	-4.55	-4.75	-4.83	-4.86	-4.87	-4.86	-4.86
I,T*,D	5.99	2.90	0.20'	-1.62'	-2.73	-3.38	-3.77	-4.02	-4.19
AR(1) u_t:									
I,T*	-2.59	-3.99	-4.56	-4.75	-4.83	-4.86	-4.87	-4.86	-4.86
I,T*,D	6.69	1.22'	-1.68'	-2.68	-3.21	-3.60	-3.86	-4.06	-4.21
SAR(1) u_t:									
I,T*	0.88'	-0.13'	-0.86'	-1.40'	-1.81'	-2.14	-2.36	-2.47	-2.55
I,T*,D	2.63	1.32'	-0.31'	-1.69'	-2.57	-3.08	-3.39	-3.59	-3.73

Series: y_t \ d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
White Noise u_t:									
I,T*	-2.68	-4.46	-4.82	-4.88	-4.90	-4.90	-4.89	-4.89	-4.89
I,T*,D	0.32'	-2.51	-3.78	-4.28	-4.51	-4.62	-4.68	-4.70	-4.71
AR(1) u_t:									
I,T*	-4.06	-4.51	-4.82	-4.88	-4.90	-4.90	-4.89	-4.89	-4.89
I,T*,D	0.34'	-2.56	-3.80	-4.29	-4.51	-4.62	-4.68	-4.70	-4.71
SAR(1) u_t:									
I,T*	-1.23'	-1.89'	-2.09	-2.26	-2.42	-2.59	-2.73	-2.83	-2.92
I,T*,D	-1.07'	-2.39	-3.08	-3.44	-3.66	-3.83	-3.95	-4.02	-4.04

Series: c_t-y_t\d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
White Noise u_t:									
I,T*	-1.56'	-4.01	-4.69	-4.83	-4.87	-4.89	-4.88	-4.88	-4.89
I,T*,D	5.10	0.27'	-2.66	-3.84	-4.30	-4.51	-4.61	-4.65	-4.67
AR(1) u_t:									
I,T*	-2.47	-4.03	-4.69	-4.84	-4.87	-4.89	-4.89	-4.89	-4.90
I,T*,D	5.20	0.11'	-2.83	-3.91	-4.33	-4.52	-4.62	-4.65	-4.67
SAR(1) u_t:									
I,T*	-0.35'	-1.86'	-2.43	-2.70	-2.86	-2.99	-3.10	-3.19	-3.28
I,T*,D	0.45'	-1.37'	-2.73	-3.38	-3.70	-3.90	-4.03	-4.12	-4.15

': Non-rejection values of the null hypothesis (1.12) at 95% level and the letters in bold correspond to those cases where monotonicity is achieved. T* means that we include dummy variables for the change in the slope in the trend.

TABLE 4.16b

$\hat{t}$ in (2.9) with $\rho(L;\theta) = (1 - L^2)^{d+\theta}$ and dummy variables for the changing trend in U.K. data.

Series: c_t \ d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
White Noise u_t:									
I,T*	-1.26'	-3.00	-4.01	-4.51	-4.76	-4.89	-4.96	-4.99	-5.03
I,T*,S	3.77	1.63'	-0.28'	-1.80'	-2.89	-3.62	-4.09	-4.37	-4.55
AR(1) u_t:									
I,T*	-1.20'	-3.22	-4.15	-4.57	-4.79	-4.91	-4.97	-5.01	-5.07
I,T*,S	2.79	-0.88'	-2.38	-3.05	-3.53	-3.94	-4.25	-4.46	-4.60
SAR(1) u_t:									
I,T*	1.21'	0.02'	-0.93'	-1.67'	-2.22	-2.61	-2.88	-3.05	-3.17
I,T*,S	2.52	1.00'	-0.49'	-1.74'	-2.64	-3.25	-3.64	-3.89	-4.04

Series: y_t \ d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
White Noise u_t:									
I,T*	4.14	2.07	-0.16'	-1.91'	-3.09	-3.81	-4.23	-4.49	-4.65
I,T*,S	5.59	3.57	1.55'	-0.23'	-1.66'	-2.69	-3.39	-3.85	-4.12
AR(1) u_t:									
I,T*	2.40	-0.11'	-1.95'	-2.98	-3.65	-4.10	-4.40	-4.60	-4.73
I,T*,S	5.64	1.28'	-1.12'	-2.24	-2.92	-3.43	-3.82	-4.11	-4.30
SAR(1) u_t:									
I,T*	2.65	1.21'	-0.38'	-1.70'	-2.60	-3.18	-3.53	-3.75	-3.89
I,T*,S	3.37	2.16	0.81'	-0.59'	-1.87'	-2.87	-3.55	-3.96	-4.20

Series: c_t-y_t\d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
White Noise u_t:									
I,T*	-0.61'	-2.70	-3.86	-4.40	-4.72	-4.86	-4.93	-4.96	-4.99
I,T*,S	2.90	0.23'	-1.84'	-3.13	-3.87	-4.28	-4.52	-4.66	-4.75
AR(1) u_t:									
I,T*	-0.77'	-2.92	-3.95	-4.47	-4.73	-4.87	-4.93	-4.97	-4.99
I,T*,S	1.56'	-0.65'	-2.26	-3.31	-3.95	-4.32	-4.55	-4.68	-4.76
SAR(1) u_t:									
I,T*	0.16'	-1.24'	-2.18	-2.77	-3.12	-3.34	-3.49	-3.60	-3.69
I,T*,S	1.29'	-0.47'	-1.97	-2.97	-3.56	-3.91	-4.14	-4.28	-4.39

': Non-rejection values of the null hypothesis (1.12) at 95% level and the letters in bold correspond to those cases where monotonicity is achieved. T* means that we include dummy variables for the change in the slope in the trend.

TABLE 4.17a

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 - L)^{d+\theta}$ and dummy variables for the changing trend in Japanese data.

Series: c_t \ d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
White Noise u_t:									
I,T*	-2.78	-3.84	-4.25	-4.49	-4.68	-4.82	-4.91	-5.02	-5.20
I,T*,D	9.24	3.27	-1.20'	-3.58	-4.67	-5.17	-5.41	-5.58	-5.75
SAR(1) u_t:									
I,T*	5.08	2.28	-0.04'	-1.58'	-2.62	-3.29	-3.25	-2.61	-2.83
I,T*,D	6.58	2.82	-0.27'	-2.24	-3.46	-4.23	-4.66	-4.86	-5.02
SAR(2) u_t:									
I,T*	5.98	2.92	0.17'	-1.56'	-2.69	-3.38	-3.32	-2.68	-2.97
I,T*,D	6.42	3.18	0.15'	-2.06	-3.48	-4.33	-4.79	-5.02	-5.22

Series: y_t \ d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
White Noise u_t:									
I,T*	-4.45	-4.68	-4.83	-4.97	-5.12	-5.25	-5.33	-5.43	-5.57
I,T*,D	0.34'	-2.30	-3.45	-3.98	-4.28	-4.48	-4.63	-4.73	-4.82
SAR(1) u_t:									
I,T*	0.27'	-1.35'	-1.88'	-2.24	-2.80	-3.32	-3.39	-3.31	-3.23
I,T*,D	0.11'	-1.98	-3.17	-3.85	-4.29	-4.59	-4.78	-4.78	-4.78
SAR(2) u_t:									
I,T*	0.58'	-1.29'	-1.88'	-2.26	-2.85	-3.38	-3.42	-2.99	-3.26
I,T*,D	0.33'	-1.95'	-3.23	-3.91	-4.32	-4.60	-4.77	-4.78	-4.79

Series: c_t-y_t\d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
White Noise u_t:									
I,T*	-3.81	-4.65	-4.97	-5.16	-5.31	-5.44	-5.53	-5.62	-5.74
I,T*,D	3.43	-1.67'	-3.88	-4.74	-5.13	-5.35	-5.51	-5.60	-5.67
SAR(1) u_t:									
I,T*	0.98'	-1.41'	-2.49	-2.97	-3.43	-3.87	-4.06	-3.91	-4.05
I,T*,D	1.27'	-1.72'	-3.29	-4.10	-4.59	-4.92	-5.13	-5.21	-5.21
SAR(2) u_t:									
I,T*	1.55'	-1.23'	-2.50	-3.05	-3.54	-3.98	-4.13	-3.96	-4.14
I,T*,D	1.53'	-1.57'	-3.32	-4.19	-4.68	-4.99	-5.20	-5.28	-5.29

': Non-rejection values of the null hypothesis (1.12) at 95% level and the letters in bold correspond to those cases where monotonicity is achieved. T* means that we include dummy variables for the change in the slope in the trend.

TABLE 4.17b

$\hat{r}$ in (2.9) with $\rho(L;\theta) = (1 - L)^{d+\theta}$ and dummy variables for the changing trend in U.K. data.

Series: c_t \ d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
White Noise u_t:									
I,T*	-0.50'	-2.79	-3.86	-4.41	-4.75	-4.98	-5.14	-5.31	-5.51
I,T*,S	8.75	4.46	0.59'	-2.04	-3.55	-4.36	-4.86	-5.18	-5.43
SAR(1) u_t:									
I,T*	6.09	3.66	1.45'	-0.14'	-1.31'	-2.24	-2.84	-3.17	-3.68
I,T*,S	7.61	4.23	1.03'	-1.31'	-2.83	-3.79	-4.41	-4.84	-5.18
SAR(2) u_t:									
I,T*	5.88	4.63	2.46	0.35'	-1.26'	-2.41	-3.06	-3.38	-3.89
I,T*,S	5.22	3.34	1.38'	-0.71'	-2.48	-3.68	-4.42	-4.88	-5.21

Series: y_t \ d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
White Noise u_t:									
I,T*	8.53	3.15	-0.92'	-3.21	-4.37	-5.00	-5.37	-5.64	-5.86
I,T*,S	11.32	6.56	1.73'	-1.72'	-3.69	-4.72	-5.30	-5.64	-5.87
SAR(1) u_t:									
I,T*	7.68	4.01	0.96'	-1.06'	-2.39	-3.30	-3.97	-4.78	-4.90
I,T*,S	8.74	5.27	1.56'	-1.25'	-2.97	-3.98	-4.62	-5.07	-5.41
SAR(2) u_t:									
I,T*	5.43	3.45	1.44'	-0.60'	-2.24	-3.39	-4.14	-4.66	-5.06
I,T*,S	6.63	4.12	1.27'	-1.15'	-2.86	-3.96	-4.66	-5.12	-5.44

Series: c_t-y_t\d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
White Noise u_t:									
I,T*	0.87'	-2.19	-3.60	-4.25	-4.61	-4.84	-4.96	-5.09	-5.27
I,T*,S	6.31	1.05'	-2.13	-3.68	-4.44	-4.85	-5.12	-5.30	-5.46
SAR(1) u_t:									
I,T*	2.48	-0.66'	-2.62	-3.75	-4.45	-4.89	-5.10	-5.12	-5.16
I,T*,S	4.05	0.23'	-2.34	-3.76	-4.54	-5.01	-5.30	-5.48	-5.62
SAR(2) u_t:									
I,T*	2.51	-0.33'	-2.51	-3.79	-4.54	-4.99	-5.18	-5.18	-5.24
I,T*,S	3.41	0.01'	-2.40	-3.77	-4.53	-4.98	-5.26	-5.43	-5.57

': Non-rejection values of the null hypothesis (1.12) at 95% level and the letters in bold correspond to those cases where monotonicity is achieved. T* means that we include dummy variables for the change in the slope in the trend.

CHAPTER 5

MULTIVARIATE TESTS OF FRACTIONALLY INTEGRATED HYPOTHESES

5.1 INTRODUCTION

In this chapter we extend the tests of Robinson (1994c) described in Chapter 2 to a general multivariate context, testing the presence of unit roots and other nonstationarities on the residuals in a multiple time series system. The multivariate case is relevant in order to analyze the interrelationships between different variables, and it can provide a more detailed insight into properties and stochastic behaviour than the univariate work. In particular, we will initially take the underlying I(0) sequence to be contemporaneously correlated but uncorrelated in time, then go on to extend the treatment to a general case of I(0) parametric autocorrelation.

Multivariate tests for unit roots have been widely analyzed in the literature, and they are commonly related to the problem of cointegration, testing the number of common unit roots in a system of equations, (e.g., Johansen (1988)). The test statistics presented in this chapter go beyond that in the sense that they will allow us to test not only unit roots, but also fractional roots of any order for each one of the time series analyzed, and some tests for cointegration, or more generally, fractional cointegration will be developed in the next chapter.

We consider a multivariate regression model of form

$$Y_t = Z_t(\boldsymbol{\delta}) + X_t \qquad t = 1,2,... \tag{1}$$

with

$$X_t = 0, \qquad t \leq 0 \tag{2}$$

where the column vectors Y_t and X_t each have N components, and by δ we mean a (K x 1) vector of real parameters, and $Z_t(\delta)$ is a (N x 1) vector of possibly non-linear functions of δ and, in general a number of predetermined variables. We will assume that under the null hypothesis to be tested and described below, X_t in (1) and (2) satisfies

$$\boldsymbol{\Phi}(L)\, X_t = U_t, \qquad t = 1,2,... \tag{3}$$

where $\Phi(L)$ is a (N x N) diagonal matrix function of the backshift operator L, and

U_t is a (Nx1) I(0) vector process[1] with mean zero and weak parametric correlation. We consider a given matrix function $\Phi(z;\theta)$ of the complex variate z and the p-dimensional vector θ of real-valued parameters, where $\Phi(z;\theta) = \Phi(z)$ for all z such that $|z| = 1$ if and only if the null hypothesis defined by

$$H_o: \theta = 0 \qquad (4)$$

holds, where there is no loss of generality in using the vector of zeros instead of an arbitrary given vector. In doing so, we can cast (3) in terms of a nested composite parametric null hypothesis, within the class of alternatives

$$\Phi(L;\theta)\, X_t = U_t \qquad t = 1,2,.... \qquad (5)$$

We take $\Phi(z)$ to have u^{th} diagonal element of form

$$\rho_u(z) = (1-z)^{\gamma_1^u}(1+z)^{\gamma_2^u}\prod_{j=3}^{h^u}(1-2\cos w_j^u z+z^2)^{\gamma_j^u}$$

for a given h^u, given distinct real numbers w_j^u, j=3,4,..h^u on the interval $(0,\pi)$ and given real numbers γ_j^u for j=1,...h^u. Thus, a model like (3) will include a wide range of possibilities to be tested for each time series, such as I(d) processes with a single root at zero frequency, if $\rho_u(z) = (1-z)^d$; quarterly I(d) processes with four roots if $\rho_u(z) = (1-z^4)^d$; 1/f noise processes if $\rho_u(z) = (1-z)^{1/2}$, etc.

We specify now $\Phi(z;\theta)$ in a way such that we take each diagonal element of $\Phi(z;\theta)$, $\rho_u(z;\theta)$, to depend on θ but not necessarily involving all elements of θ. To do that, we take

$$\rho_u(z;\theta) = (1-z)^{\gamma_1^u+\theta_{i_1}^u}(1+z)^{\gamma_2^u+\theta_{i_2}^u}\prod_{j=3}^{h^u}(1-2\cos w_j^u z+z^2)^{\gamma_j^u+\theta_{i_j}^u} \qquad (6)$$

where for each combination (u,j), $\theta_{ij}^u = \theta_l$ for some l; and for each l, there is at least one combination (u,j) such that $\theta_{ij}^u = \theta_l$, where θ_l corresponds to the l^{th} element of θ. This is a fairly general specification in the sense that we allow for duplications not only within equations but also across equations. Furthermore, this way of specifying $\Phi(z;\theta)$ permits us to specifically consider situations where θ is the same across all equations, and also the case when θ is taken as strictly different for each equation. This will be illustrated with some examples in Section 4.

We adopt the normalization $\rho_u(0;\theta) = 1$ for all θ and u = 1,2,...N, and we assume that $\rho_u(z;\theta)$ is differentiable in θ on a neighbourhood of $\theta = 0$ for all $|z|$

[1] We define an I(0) vector process U_t, t = 0,±1,..., as a covariance stationary vector process with spectral density matrix $f(\lambda)$ that is finite and positive definite.

= 1. Also we assume that for any u,v = 1,2,...,N

$\det(E_{uv}) < \infty$ (7)

$$\textit{where} \quad E_{uv} = \frac{1}{4\pi}\int_{-\pi}^{\pi}\left(\epsilon_{(u)}(\lambda)\,\overline{\epsilon}_{(v)}(\lambda)' + \epsilon_{(v)}(\lambda)\,\overline{\epsilon}_{(u)}(\lambda)'\right)d\lambda$$

$$\textit{and} \quad \epsilon_{(u)}(\lambda) = \frac{\partial \log \rho_u(e^{i\lambda};\theta)}{\partial\theta} \qquad (8)$$

for real λ, and $\bar{\varepsilon}_{(u)}(\lambda)$ as the conjugate vector of $\varepsilon_{(u)}(\lambda)$. Note that the (p x 1) vector $\varepsilon_{(u)}(\lambda)$ is independent of θ given the linearity of log $\rho_u(e^{i\lambda};\theta)$ with respect to θ in (6). In particular, its real part takes the form

$$\delta_{1l}^{u}\log\left|2\sin\frac{\lambda}{2}\right| + \delta_{2l}^{u}\log\left(2\cos\frac{\lambda}{2}\right) + \sum_{j=3}^{h^u}\delta_{jl}^{u}\log\left|2\left(\cos\lambda - \cos w_j^u\right)\right|,$$

for l=1,...,p and $|\lambda| < \pi$, where $\delta_{jl}^{u} = 1$ if $\theta_{ij}^{u} = \theta_l$ and 0 otherwise. Condition (7) is not satisfied if we include AR alternatives of form: $\rho_u(z;\theta) = (1 - (1+\theta)z)$, but it is satisfied by fractional alternatives of form: $\rho_u(z;\theta) = (1-z)^{d+\theta}$, for example.

It should also be noted that under the null hypothesis, defined in (4), the model will be completely specified by (1)-(3), and it can be redefined as

$$\Phi(L)\,Y_t = W_t(\delta) + U_t \qquad (9)$$

where $W_t(\delta) = (W_{1t}(\delta);W_{2t}(\delta);...;W_{Nt}(\delta))'$, with $W_{ut}(\delta) = \rho_u(L)Z_{ut}(\delta)$. (9) is a very general form of a regression model which includes multivariate linear and non-linear models and simultaneous equation systems, and its possible non-linear nature is motivated given that in economics and the physical sciences, multivariate regression models that are essentially of a non-linear nature have frequently been proposed to describe phenomena that may be of a continuous nature but are sampled at equal intervals of time. (See e.g. Robinson (1972), (1977)).

The initial discussion of the tests will assume that U_t in (3) is a white noise vector process, so the only nuisance parameters will be the elements of $Z_t(\delta)$ in (1) and those of the variance-covariance matrix of U_t. Then, we will extend the tests to a quite general form of I(0) autocorrelation in U_t, which will include as specific examples, the type of multiple autoregressive-moving average (ARMA) models.

We will start by presenting the functional forms of the test statistics based on the three general principles when deriving nested parametric hypotheses, namely, the score, Wald and likelihood-ratio principles, and we will do so for the two situations mentioned above, that is, white noise and weak parametric autocorrelation in U_t. As usual, it should be possible to show that the tests based on these three principles will have the same null limit distribution (a χ_p^2 distribution where p is the number of restrictions tested). However, we do not present rigorous proofs of the asymptotic properties, but rather informal statements. It will undoubtedly be possible to extend the asymptotic null and local distribution theory of Robinson (1994c) for the scalar case, to our multivariate situation under natural generalizations of his conditions. Once we have obtained the functional forms of the tests, we will rewrite them for two cases of particular interest: First, when θ in (5) is the same across all diagonal elements in $\Phi(z;\theta)$ and then, we will consider the case when θ is strictly different for each element in $\Phi(z;\theta)$. In the final part of this chapter, some simulations based on Monte Carlo experiments will be carried out in order to study the finite-sample behaviour of versions of the tests. Appendices 5.1 and 5.2 show the derivations of the test statistics of Sections 2 and 3, and Appendix 5.3 describes the Fortran program used in obtaining the score test statistics.

5.2 SCORE TEST FOR WHITE NOISE U_t

In this section we describe a score test for the null hypothesis (4) in a model given by (1), (2) and (5), under the presumption that U_t in (5) is a vector sequence of zero mean uncorrelated in time random variables, with unknown variance-covariance matrix K. One definition for the score test is as follows. Let $L(\eta)$ be an objective function (such as the negative of the log-likelihood) and take $\eta = (\theta',\upsilon')'$, where $\tilde{\eta} = (0',\tilde{v}')'$ are the values that minimizes $L(\eta)$ under the null hypothesis. A score test (see Rao (1973), page 418) is then given by

$$\frac{\partial L(\eta)}{\partial \eta'}\left[E_o\left(\frac{\partial L(\eta)}{\partial \eta}\frac{\partial L(\eta)}{\partial \eta'}\right)\right]^{-1}\frac{\partial L(\eta)}{\partial \eta}\Big|_{\substack{\theta=0\\ v=\tilde{v}}} \qquad (10)$$

where the expectation is taken under the null hypothesis prior to substitution of $\tilde{v}$. However, the same asymptotic behaviour will be expected if we replace the inverted matrix appearing in (10) by alternative forms such as the sample average or the Hessian. For convenience in the derivation below, we will make use of the expected information matrix, so the score test will take the form

$$\frac{\partial L(\eta)}{\partial \eta'}\left[E_o\frac{\partial L^2(\eta)}{\partial \eta\,\partial \eta'}\right]^{-1}\frac{\partial L(\eta)}{\partial \eta}\Big|_{\substack{\theta=0\\ v=\tilde{v}}}. \qquad (11)$$

We now describe the test statistic. We take L in (11), with $\eta = (\theta',\delta',\alpha')'$ and $\alpha = v(K)$, to be the negative of the log-likelihood based on Gaussian U_t. In Appendix 5.1 it is shown that (11) takes the form

$$\hat{S}^t = T\,\hat{a}^{t\prime}\,(\hat{A}^t)^{-1}\,\hat{a}^t \qquad (12)$$

where $\hat{a}^t$ is a (p x 1) vector of form

$$\hat{a}^t = -\sum_{u=1}^{N}\sum_{v=1}^{N}\hat{\sigma}^{uv}\sum_{s=1}^{T-1}\psi_s^{(u)}\,C_{uv}(s;\hat{\delta}), \qquad (13)$$

and $\psi_s^{(u)}$ is obtained by expanding

$$\psi_{(u)}(\lambda) = Re[\epsilon_{(u)}(\lambda)] \quad as \quad \sum_{s=1}^{\infty}\psi_s^{(u)}\cos\lambda s.$$

$$\hat{A}^t = \sum_{u=1}^{N}\sum_{v=1}^{N}\hat{\sigma}^{uv}\hat{\sigma}_{uv}\sum_{s=1}^{T-1}\left(1-\frac{s}{T}\right)\psi_s^{(u)}\,\psi_s^{(v)\prime}, \qquad (14)$$

$\hat{\sigma}^{uv}$ is the $(u,v)^{th}$ element of $\hat{K}^{-1}$; $\hat{\sigma}_{uv}$ is the $(u,v)^{th}$ element of $\hat{K}$; and $C_{uv}(s;\hat{\delta})$ is the $(u,v)^{th}$ element of $C_{\hat{U}}(s)$, where

$$\hat{K} = \frac{1}{T}\sum_{t=1}^{T}\hat{U}_t(\hat{\delta})\,\hat{U}_t(\hat{\delta})'; \qquad C_{\hat{U}}(s) = \frac{1}{T}\sum_{t=1}^{T-s}\hat{U}_t(\hat{\delta})\,\hat{U}_{t+s}(\hat{\delta})';$$

$\hat{U}_t(\delta) = \Phi(L)Y_t - W_t(\delta)$, and $\hat{\delta}$ must be at least a $T^{1/2}$- consistent estimate of the true value δ.

Clearly, as in the univariate tests of Chapter 2, concise formulas for $\psi_s^{(u)}$ are available in some simple cases; for example, $\psi_s^{(u)} = -s^{-1}$, when $\rho_u(L;\theta) = (1 - L)^{d+\theta}$. However, we can also express the test statistic in the frequency domain and, under certain suitable conditions[2], approximate this to obtain an alternative, asymptotically equivalent, form. $\hat{a}^t$ in (13) can be written as

$$-\frac{1}{2}\sum_{u=1}^{N}\sum_{v=1}^{N}\hat{\sigma}^{uv}\int_{-\pi}^{\pi}(\epsilon_{(u)}(\lambda)+\bar{\epsilon}_{(v)}(\lambda))\,I_{uv}(\lambda;\hat{\delta})\,d\lambda,$$

where $\varepsilon_{(u)}(\lambda)$ is as in (8) and $\bar{\varepsilon}_{(v)}(\lambda)$ is the conjugate vector of $\varepsilon_{(v)}(\lambda)$; $I_{uv}(\lambda;\hat{\delta})$ is the $(u,v)^{th}$ element in the cross-periodogram of $\hat{U}_t(\delta) = (\hat{U}_{1t}(\delta);...;\hat{U}_{Nt}(\delta))'$:

$$I_{uv}(\lambda,\hat{\delta}) = W_u(\lambda;\hat{\delta})\,\overline{W_v(\lambda;\hat{\delta})}, \quad W_u(\lambda;\hat{\delta}) = \frac{1}{\sqrt{2\pi T}}\sum_{t=1}^{T}\hat{U}_{ut}(\hat{\delta})\,e^{i\lambda t},$$

where the line over $W_v(\lambda;\hat{\delta})$ denotes complex conjugate. To see the previous result note that $\hat{a}^t$ in (13) can be decomposed into

$$-\left(\sum_{u=1}^{N}\hat{\sigma}^{uu}\sum_{s=1}^{T-1}\psi_s^{(u)}C_{uu}(s;\hat{\delta})+\frac{1}{2}\sum_{u=1}^{N}\sum_{\substack{v=1\\v\neq u}}^{N}\hat{\sigma}^{uv}\sum_{s=1}^{T-1}(\psi_s^{(u)}C_{uv}(s;\hat{\delta})+\psi_s^{(v)}C_{vu}(s;\hat{\delta}))\right),$$

and it can be shown that

---

[2] These conditions are basically a generalization of those of Robinson (1994c), requiring technical assumptions on ρ_u (and thus on $\varepsilon_{(u)}(\lambda)$) to justify approximating integrals by sums.

$$\sum_{s=1}^{T-1} \psi_s^{(u)} C_{uu}(s;\hat{\delta}) = \frac{1}{2}\int_{-\pi}^{\pi} (\epsilon_{(u)}(\lambda) + \overline{\epsilon}_{(u)}(\lambda))\, I_{uu}(\lambda;\hat{\delta})\, d\lambda,$$

$$\text{and} \quad \sum_{s=1}^{T-1} (\psi_s^{(u)} C_{uv}(s;\hat{\delta}) + \psi_s^{(v)} C_{vu}(s;\hat{\delta})) =$$

$$\frac{1}{2}\int_{-\pi}^{\pi} (\epsilon_{(u)}(\lambda) + \overline{\epsilon}_{(v)}(\lambda))\, I_{uv}(\lambda;\hat{\delta})\, d\lambda + \frac{1}{2}\int_{-\pi}^{\pi} (\epsilon_{(v)}(\lambda) + \overline{\epsilon}_{(u)}(\lambda))\, I_{vu}(\lambda;\hat{\delta})\, d\lambda .$$

Also, under suitable conditions, keeping $\hat{\sigma}^{uv}$ and $\hat{\sigma}_{uv}$ fixed, $\hat{A}^t$ in (14) becomes asymptotically

$$\sum_{u=1}^{N}\sum_{v=1}^{N} \hat{\sigma}^{uv}\hat{\sigma}_{uv}\sum_{s=1}^{\infty} \psi_s^{(u)}\psi_s^{(v)\prime}, \tag{15}$$

and using Parseval's relationship, this quantity can be expressed as

$$\sum_{u,v=1}^{N} \hat{\sigma}^{uv}\hat{\sigma}_{uv}\frac{1}{4\pi}\int_{-\pi}^{\pi}\left(\epsilon_{(u)}(\lambda)\overline{\epsilon}_{(v)}(\lambda)' + \epsilon_{(v)}(\lambda)\overline{\epsilon}_{(u)}(\lambda)'\right)d\lambda = \sum_{u,v=1}^{N} \hat{\sigma}^{uv}\hat{\sigma}_{uv}E_{uv},$$

since (15) can also be decomposed into

$$\sum_{u=1}^{N} \hat{\sigma}^{uu}\hat{\sigma}_{uu}\sum_{s=1}^{\infty} \psi_s^{(u)}\psi_s^{(u)\prime} = \sum_{u=1}^{N} \hat{\sigma}^{uu}\hat{\sigma}_{uu}\frac{1}{2\pi}\int_{-\pi}^{\pi} (\epsilon_{(u)}(\lambda)\overline{\epsilon}_{(u)}(\lambda)')\, d\lambda,$$

$$\text{and} \quad \frac{1}{2}\sum_{u=1}^{N}\sum_{\substack{v=1\\ v\neq u}}^{N} \hat{\sigma}^{uv}\hat{\sigma}_{uv}\sum_{s=1}^{\infty} (\psi_s^{(u)}\psi_s^{(v)\prime} + \psi_s^{(v)}\psi_s^{(u)\prime}) =$$

$$\sum_{u=1}^{N}\sum_{\substack{v\neq u\\ v=1}}^{N} \hat{\sigma}^{uv}\hat{\sigma}_{uv}\frac{1}{4\pi}\int_{-\pi}^{\pi} \epsilon_u(\lambda)\,\overline{\epsilon}_v(\lambda)' + \epsilon_v(\lambda)\,\overline{\epsilon}_u(\lambda)'\, d\lambda$$

Therefore, the score statistic in (12) can be approximated in the frequency domain by the expression

$$\hat{S}^f = T\, \hat{a}^{f\prime}\, (\hat{A}^f)^{-1}\, \hat{a}^f \tag{16}$$

where

$$\hat{a}^f = \frac{-\pi}{T}\sum_{u=1}^{N}\sum_{v=1}^{N} \hat{\sigma}^{uv}\sum_{r}^{*} (\epsilon_{(u)}(\lambda_r) + \overline{\epsilon}_{(v)}(\lambda_r))\, I_{uv}(\lambda_r;\hat{\delta}), \tag{17}$$

and

$$\hat{A}^f = \frac{1}{2T}\sum_{u=1}^{N}\sum_{v=1}^{N} \hat{\sigma}^{uv}\hat{\sigma}_{uv}\sum_{r}^{*} (\epsilon_{(u)}(\lambda_r)\overline{\epsilon}_{(v)}(\lambda_r)' + \epsilon_{(v)}(\lambda_r)\overline{\epsilon}_{(u)}(\lambda_r)'), \tag{18}$$

$\lambda_r = 2\pi r/T$, and the sums on the asterisk are over λ_r in M where $M = \{\lambda;\ -\pi < \lambda \leq \pi;\ \lambda \notin (\rho_l-\lambda;\rho_l+\lambda),\ l=1,2,...,s\}$, such that ρ_l, $l=1,2,...s$ are the distinct poles on $\varepsilon_{(u)}(\lambda)$ on $(\pi,\pi]$ for $u=1,2,...,N$. Note that if, for example, $\rho_u(L;\theta)$ is given by $(1-L)^{d+\theta}$, we calculate $\varepsilon_{(u)}(\lambda_r)$ as

$$Re[\epsilon_{(u)}(\lambda_r)] = \psi_{(u)}(\lambda_r) = \log\left|2\sin\frac{\lambda_r}{2}\right|, \quad \text{and} \quad Im[\epsilon_{(u)}(\lambda_r)] = \frac{\lambda_r - \pi}{2}$$

with r=1,2,...,T-1.

We should expect that under some regularity conditions, (basically a natural generalization of those in Robinson (1994c)), the test described below will have the same optimal asymptotic properties as Robinson's (1994c) univariate tests. These conditions impose a martingale difference assumption on the white noise vector U_t;[3] also W as defined in Appendix 5.1 must be a positive definite matrix; and $\rho_u(z;\theta)$, u=1,2...,N must belong to Class H as defined in Chapter 2, with $\varepsilon_{(u)}(\lambda)$ satisfying the same conditions as $\psi(\lambda)$ in that chapter. We believe that under these conditions, (12) and (16) will have a null limit χ_p^2 distribution, and under local alternatives of form (2.6), a $\chi_p^2(\nu)$ distribution with a non-centrality parameter ν, which is optimal under Gaussianity of U_t.

Thus, a large-sample 100α%-level test for rejecting H_o (4) against the alternative: H_1: $\theta \neq 0$, will be given by the rule: "Reject H_o if $\hat{S}^t$ (or $\hat{S}^f$) > $\chi^2_{p,\alpha}$", where $P(\chi_p^2 > \chi^2_{p,\alpha}) = \alpha$.

5.3 SCORE TEST FOR WEAKLY PARAMETRICALLY CORRELATED U_t

The test statistics presented in Section 2 can be robustified to allow weakly parametrically autocorrelated U_t. We can consider the model in (1), (2), and (5), with U_t in (5) as a vector process with N components generated by a parametric model of form

$$U_t = \sum_{j=0}^{\infty} A(j;\tau)\,\epsilon_{t-j} \qquad t = 1,2,\ldots, \tag{19}$$

where ε_t is a vector white noise process, and K is now the unknown variance-covariance matrix of ε_t. In relation with (19), the corresponding spectral density matrix is

$$f(\lambda;\tau) = \frac{1}{2\pi} k(\lambda;\tau)\; K\; k(\lambda;\tau)^*, \tag{20}$$

where $k(\lambda;\tau) = \sum_{j=0}^{\infty} A(j;\tau)\,e^{i\lambda j}$, and k^* means the complex conjugate transpose of k.

A number of conditions are required on A and f in Appendix 5.2 when deriving the test statistic; their practical implications being that though U_t is capable of exhibiting a much stronger degree of autocorrelation than multiple autoregressive

[3] That is, $E(U_t \mid B_{t-1}) = 0$ and $E(U_t U_t' \mid B_{t-1}) = K$, where B_t is the σ-field of events generated by U_s, $s \leq t$.

moving average ARMA processes, its spectral density matrix must be finite, with eigenvalues bounded and bounded away from zero. Thus, it cannot include fractional processes with positive or negative differencing parameters.

By extending the argument in Section 2 and Appendix 5.1, we show in Appendix 5.2 that, under Gaussianity of U_t, an approximate score statistic for testing (4) in (1), (2), (5) and (19) is

$$\tilde{S} = T \tilde{b}' \tilde{B}^{-1} \tilde{b} \tag{21}$$

and $\tilde{B}$ is $\tilde{C} - \tilde{D}' \tilde{E}^{-1} \tilde{D}$, where

$$\tilde{b} = -\frac{1}{2T} \sum_r^* \sum_{u=1}^N \sum_{v=1}^N \left(\epsilon_{(u)}(\lambda_r) + \bar{\epsilon}_{(v)}(\lambda_r)\right) I_{uv}(\lambda_r;\hat{\delta})\, \hat{f}^{vu}(\lambda_r;\tilde{\tau}),$$

$$\tilde{C} = \frac{1}{2T} \sum_r^* \sum_{u,v=1}^N \left(\epsilon_{(u)}(\lambda_r)\bar{\epsilon}_{(v)}(\lambda_r)' + \bar{\epsilon}_{(v)}(\lambda_r)\epsilon_{(u)}(\lambda_r)'\right) \hat{f}_{uv}(\lambda_r;\tilde{\tau})\, \hat{f}^{vu}(\lambda_r;\tilde{\tau}),$$

$$\tilde{D}' = -\frac{1}{2T} \sum_r^* \sum_{u,v=1}^N \left(\epsilon_{(u)}(\lambda_r) + \bar{\epsilon}_{(v)}(\lambda_r)'\right) \hat{f}^{vu}(\lambda_r;\tilde{\tau}) \frac{\partial \hat{f}_{uv}(\lambda_r;\tilde{\tau})}{\partial \tau'},$$

and

$$(\tilde{E})_{uv} = \frac{1}{2T} \sum_r^* \operatorname{tr}\left(\hat{f}^{-1}(\lambda_r;\tilde{\tau}) \frac{\partial \hat{f}(\lambda_r;\tilde{\tau})}{\partial \tau_u} \hat{f}^{-1}(\lambda_r;\tilde{\tau}) \frac{\partial \hat{f}(\lambda_r;\tilde{\tau})}{\partial \tau_v} \right).$$

$I_{uv}(\lambda;\hat{\delta})$ is the $(u,v)^{th}$ element of the periodogram of $\hat{U}_t$, $I_U(\lambda;\hat{\delta})$, as was given in Section 2; $\hat{f}_{uv}(\lambda_r;\tilde{\tau})$ and $\hat{f}^{uv}(\lambda_r;\tilde{\tau})$ correspond to the $(u,v)^{th}$ elements of $\hat{f}(\lambda_r;\tilde{\tau})$ and $\hat{f}^{-1}(\lambda_r;\tilde{\tau})$ respectively, with

$$\hat{f}(\lambda;\tilde{\tau}) = \frac{1}{2\pi} k(\lambda;\tilde{\tau})\ \hat{K}\ \ k(\lambda;\tilde{\tau})^*$$

and

$$\tilde{\tau} = \operatorname{argmin}_{\tau \in T^*} \left(\frac{T}{2} \log\det \hat{f}(\lambda_r;\tau) + \frac{1}{2} \sum_r^* \operatorname{tr}[\hat{f}^{-1}(\lambda_r;\tau) I_U(\lambda_r;\hat{\delta})] \right), \tag{22}$$

where T^* is a compact subset of q-dimensional Euclidean space.

We can see that the test statistic obtained in (21) becomes (16) when we consider the case of white noise U_t. In such situation, $\hat{f}_{uv}(\lambda_r;\tau) = \hat{\sigma}_{uv}/2\pi$, and $\hat{f}^{uv}(\lambda_r;\tau) = 2\pi\,\hat{\sigma}^{uv}$. Then,

$$\tilde{b} = \frac{-\pi}{T} \sum_r^* \sum_{u=1}^N \sum_{v=1}^N \left(\epsilon_{(u)}(\lambda_r) + \bar{\epsilon}_{(v)}(\lambda_r)\right) I_{uv}(\lambda_r;\hat{\delta})\, \hat{\sigma}^{vu} =$$

$$= \frac{-\pi}{T} \sum_{u=1}^N \sum_{v=1}^N \hat{\sigma}^{uv} \sum_r^* \left(\epsilon_{(u)}(\lambda_r) + \bar{\epsilon}_{(v)}(\lambda_r)\right) I_{uv}(\lambda_r;\hat{\delta}) = \hat{a}^f \text{ in } (17).$$

Similarly,

$$\tilde{C} = \frac{1}{2T}\sum_{r}^{*}\sum_{u=1}^{N}\sum_{v=1}^{N}\left(\epsilon_{(u)}(\lambda_r)\overline{\epsilon}_{(v)}(\lambda_r)' + \epsilon_{(v)}(\lambda_r)\overline{\epsilon}_{(u)}(\lambda_r)'\right)\hat{\sigma}_{uv}\hat{\sigma}^{vu} = \hat{A}^{f}$$

in (18) and finally, $\tilde{D}$ and $\tilde{E}$ are now zero matrices, so we have that $\tilde{S}$ in (21) takes the same form as $\hat{S}^f$ in (16).

Extending the conditions in Robinson (1994c) to this multivariate context, we should expect that, allowing a martingale difference assumption on ε_t in (19), with $\sum_{j=1}^{\infty} j^{1/2}\|A(j;\tau)\| < \infty$, where $\|A\|$ means any norm for the matrix A, for example the square root of the maximum eigenvalue of A^*A; with W as a positive definite matrix; ρ_u, u=1,...,N, satisfying the same conditions as in Section 2; and $f_{uv}(\lambda;\tau)$, and $\partial f_{uv}(\lambda;\tau)/\partial\tau$ satisfying a Lipschitz condition in λ of order $\eta > 1/2$, for all u,v=1,...,N, then, under H_o (4): $\tilde{S} \to_d \chi_p^2$ as $T \to \infty$, and $\tilde{S}$ should also satisfy the same asymptotic efficiency properties as $\hat{S}^t$ and $\hat{S}^f$ in Section 2.

5.4 PARTICULAR CASES OF THE SCORE TESTS

In the preceding sections we have presented three different versions of the score test statistic: (12), which corresponds to the time domain representation of the test for white noise U_t; (16), which approximates (12) in the frequency domain; and (21) which is the frequency domain version of the test statistic for weakly parametrically autocorrelated U_t. In this section we consider two particular cases of interest for each version of these tests. The first case corresponds to the test statistic when we take θ in (5) as a (p x 1) vector containing exactly the same elements across all diagonal elements in $\Phi(z;\theta)$, while the second case takes this vector θ as strictly different for each diagonal element in $\Phi(z;\theta)$.

We illustrate this with two simple examples in a bivariate model: First we test if one of the series is an $I(d_1)$ process and if the other is $I(d_2)$. Thus, we consider that both series have a root at the same zero frequency. In the second example, we consider that the series might differ in the number of roots in its bivariate representation. Thus, we test the same hypothesis, $(I(d_1))$, for the first series and a quarterly $I(d_2)$ process in the second one. Therefore, the model will be specified, under the null hypothesis, in the first of these examples as

$$\begin{pmatrix}(1-L)^{d_1} & 0\\ 0 & (1-L)^{d_2}\end{pmatrix}\begin{pmatrix}X_{1t}\\ X_{2t}\end{pmatrix} = \begin{pmatrix}U_{1t}\\ U_{2t}\end{pmatrix} \qquad t = 1,2,...., \qquad (E1)$$

and in the second as

$$\begin{pmatrix} (1-L)^{d_1} & 0 \\ 0 & (1-L^4)^{d_2} \end{pmatrix} \begin{pmatrix} X_{1t} \\ X_{2t} \end{pmatrix} = \begin{pmatrix} U_{1t} \\ U_{2t} \end{pmatrix} \qquad t = 1,2,...., \qquad (E2)$$

where $X_t = (X_{1t}, X_{2t})' = 0$ for $t \leq 0$, and $U_t = (U_{1t}, U_{2t})'$ follows an I(0) process.

5.4.a Same θ across the equations

We consider the model in (1), (2), and (5), but now we take $\Phi(z;\theta)$ to be of form such that its u^{th} diagonal element is

$$\rho_u(z;\theta) = (1-z)^{\gamma_1^u + \theta_{i_1}} (1+z)^{\gamma_2^u + \theta_{i_2}} \prod_{j=3}^{h^u} (1 - 2\cos w_j z + z^2)^{\gamma_j^u + \theta_{i_j}},$$

and for each j, $\theta_{ij} = \theta_l$ for some l, and for each l, there is at least one j such that $\theta_{ij} = \theta_l$. Therefore we take the parameter vector θ to be exactly the same across all equations in (5), and the difference between one equation and another comes now through the coefficients γ_i^u for i=1,2,...,h^u and u=1,2,...,N. Thus, in the first example, the model will be specified as

$$\begin{pmatrix} (1-L)^{d_1+\theta} & 0 \\ 0 & (1-L)^{d_2+\theta} \end{pmatrix} \begin{pmatrix} X_{1t} \\ X_{2t} \end{pmatrix} = \begin{pmatrix} U_{1t} \\ U_{2t} \end{pmatrix} \qquad t = 1,2,....,$$

and we will test here the null hypothesis, H_o: $\theta = 0$, against the alternative, H_a: $\theta \neq 0$. Given that in this case θ is a scalar, we can also consider one-sided tests for the same null hypothesis against the alternatives: H_{a1}: $\theta < 0$ or H_{a2}: $\theta > 0$.

In the second example, the model will take the form

$$\begin{pmatrix} (1-L)^{d_1+\theta_1}(1+L)^{\theta_2}(1+L^2)^{\theta_3} & 0 \\ 0 & (1-L)^{d_2+\theta_1}(1+L)^{d_2+\theta_2}(1+L^2)^{d_2+\theta_3} \end{pmatrix} \begin{pmatrix} X_{1t} \\ X_{2t} \end{pmatrix} = \begin{pmatrix} U_{1t} \\ U_{2t} \end{pmatrix},$$

which, under the null hypothesis, H_o: $\theta = (\theta_1, \theta_2, \theta_3)' = 0$, becomes (E2), implying that X_{2t} behaves as a quarterly I(d_2) process, and therefore, with all roots with the same integration order d_2. Clearly we could also have tested the model, allowing different integration orders at zero and at seasonal frequencies.

This specification is a particular case of the general model presented in Section 1 where now

$$\epsilon_{(u)}(\lambda) = \frac{\partial \log \rho_u(e^{i\lambda};\theta)}{\partial \theta} = \epsilon(\lambda) \qquad \textit{for all} \quad u=1,2,...,N. \qquad (23)$$

(23) implies that $\psi_s^{(u)} = \psi_s$ for all u=1,2,...,N, and then, we can immediately describe the functional forms of the three test statistics. Starting with white noise U_t, substituting (23) in (12) - (14), the time domain version of the test statistic is

$$\hat{S}^{t^1} = T\,\hat{a}^{t^{1\prime}}\,(\hat{A}^{t^1})^{-1}\,\hat{a}^{t^1} \tag{24}$$

where

$$\hat{a}^{t^1} = -\sum_{s=1}^{T-1}\psi_s\sum_{u=1}^{N}\sum_{v=1}^{N}\hat{\sigma}^{uv}C_{uv}(s;\hat{\delta}) = -\sum_{s=1}^{T-1}\psi_s\,tr\left[\hat{K}^{-1}\,C_{\hat{U}}(s)\right],$$

and

$$\hat{A}^{t^1} = \sum_{s=1}^{T-1}\left(1-\frac{s}{T}\right)\psi_s\psi_s'\sum_{u=1}^{N}\sum_{v=1}^{N}\hat{\sigma}^{uv}\hat{\sigma}_{uv} = N\sum_{s=1}^{T-1}\left(1-\frac{s}{T}\right)\psi_s\,\psi_s'.$$

Expressing now the test statistic in terms of its frequency domain representation

$$\hat{S}^{f^1} = T\,\hat{a}^{f^{1\prime}}\,(\hat{A}^{f^1})^{-1}\,\hat{a}^{f^1} \tag{25}$$

$$where\quad \hat{a}^{f^1} = \frac{-\pi}{T}\sum_{u=1}^{N}\sum_{v=1}^{N}\hat{\sigma}^{uv}\sum_{r}^{*}\left(\epsilon(\lambda_r)+\bar{\epsilon}(\lambda_r)\right)I_{uv}(\lambda_r;\hat{\delta})$$

$$\hat{A}^{f^1} = \frac{1}{2T}\sum_{r}^{*}\left(\epsilon(\lambda_r)\bar{\epsilon}(\lambda_r)'+\epsilon(\lambda_r)\bar{\epsilon}(\lambda_r)'\right)\sum_{\substack{u=1\\v=1}}^{N}\hat{\sigma}^{uv}\hat{\sigma}_{uv} = \frac{1}{T}\sum_{r}^{*}\epsilon(\lambda_r)\bar{\epsilon}(\lambda_r)'\sum_{\substack{u=1\\v=1}}^{N}\hat{\sigma}^{uv}\hat{\sigma}_{uv}$$

$$= \frac{N}{T}\sum_{r}^{*}\epsilon(\lambda_r)\;\bar{\epsilon}(\lambda_r)' = \frac{2N}{T}\sum_{r}^{*}\psi(\lambda_r)\;\psi(\lambda_r)'.$$

Finally, allowing weak parametric autocorrelation in U_t, substituting (23) in (21), we obtain that the test statistic is

$$\tilde{S}^1 = T\,\tilde{b}^{1\prime}\left(\tilde{C}^1 - \tilde{D}^{1\prime}\,\tilde{E}^{-1}\,\tilde{D}^1\right)^{-1}\tilde{b}^1 \tag{26}$$

where

$$\tilde{b}^1 = -\frac{1}{2T}\sum_{r}^{*}\left(\epsilon(\lambda_r)+\bar{\epsilon}(\lambda_r)\right)\sum_{u=1}^{N}\sum_{v=1}^{N}I_{uv}(\lambda_r;\hat{\delta})\hat{f}^{vu}(\lambda_r;\tilde{\tau}) =$$

$$\frac{-1}{T}\sum_{r}^{*}\psi(\lambda_r)\sum_{u=1}^{N}\sum_{v=1}^{N}I_{uv}(\lambda_r;\hat{\delta})\hat{f}^{vu}(\lambda_r;\tilde{\tau}) = \frac{-1}{T}\sum_{r}^{*}\psi(\lambda_r)\,tr\left[I_U(\lambda_r;\hat{\delta})\hat{f}(\lambda_r;\tilde{\tau})^{-1}\right],$$

$$\tilde{C}^1 = \frac{1}{2T}\sum_{r}^{*}\left(\epsilon(\lambda_r)\bar{\epsilon}(\lambda_r)'+\epsilon(\lambda_r)\bar{\epsilon}(\lambda_r)'\right)\sum_{u=1}^{N}\sum_{v=1}^{N}\hat{f}_{uv}(\lambda_r;\tilde{\tau})\,\hat{f}^{vu}(\lambda_r;\tilde{\tau})$$

$$= \frac{N}{T}\sum_{r}^{*}\epsilon(\lambda_r)\;\bar{\epsilon}(\lambda_r)' = \frac{2N}{T}\sum_{r}^{*}\psi(\lambda_r)\;\psi(\lambda_r)', \tag{27}$$

$$\tilde{D}^{1'} = -\frac{1}{2T}\sum_r^* \left(\epsilon(\lambda_r) + \bar{\epsilon}(\lambda_r)\right)\sum_{u=1}^N \sum_{v=1}^N \hat{f}^{vu}(\lambda_r;\tilde{\tau})\frac{\partial \hat{f}_{uv}(\lambda_r;\tilde{\tau})}{\partial \tau'} =$$

$$\frac{-1}{T}\sum_r^* \psi(\lambda_r)\left[tr\left(\hat{f}(\lambda_r;\tilde{\tau})^{-1}\frac{\partial \hat{f}(\lambda_r;\tilde{\tau})}{\partial \tau_1}\right);\ldots;tr\left(\hat{f}(\lambda_r;\tilde{\tau})^{-1}\frac{\partial \hat{f}(\lambda_r;\tilde{\tau})}{\partial \tau_q}\right)\right], \tag{28}$$

and

$$\tilde{E}_{uv} = \frac{1}{2T}\sum_r^* tr\left[\hat{f}(\lambda_r;\tilde{\tau})^{-1}\frac{\partial \hat{f}(\lambda_r;\tilde{\tau})}{\partial \tau_u}\hat{f}(\lambda_r;\tilde{\tau})^{-1}\frac{\partial \hat{f}(\lambda_r;\tilde{\tau})}{\partial \tau_v}\right]. \tag{29}$$

5.4.b Different θ's across equations

A second case of interest might be when we take the (p x 1) vector θ appearing in (5) to be equal to $(\theta^{1\prime};\theta^{2\prime};\ldots;\theta^{N\prime})'$, where θ^u is a $(p_u$ x 1) vector affecting only the uth equation. That is, the vector of parameters involving θ will be strictly different for each equation. We can now write down the uth diagonal element in $\Phi(z;\theta)$ as

$$\rho_u(z;\theta^u) = (1-z)^{\gamma_1^u + \theta_{i_1}^u}(1+z)^{\gamma_2^u + \theta_{i_2}^u}\prod_{j=3}^{h^u}(1-2\cos w_j z + z^2)^{\gamma_j^u + \theta_{i_j}^u} \tag{30}$$

where for each j, $\theta_{i_j}^u = \theta_l^u$ for some l, and for each l, there is at least one j such that $\theta_{i_j}^u = \theta_l^u$. Thus, in the first of the examples mentioned above, the model will be of form

$$\begin{pmatrix} (1-L)^{d_1+\theta_1^1} & 0 \\ 0 & (1-L)^{d_2+\theta_1^2} \end{pmatrix}\begin{pmatrix} X_{1t} \\ X_{2t} \end{pmatrix} = \begin{pmatrix} U_{1t} \\ U_{2t} \end{pmatrix} \qquad t = 1,2,\ldots,$$

with $\theta = (\theta^1;\ \theta^2)' = (\theta_1^1;\ \theta_1^2)'$, and in the second example

$$\begin{pmatrix} (1-L)^{d_1+\theta_1^1}(1+L)^{\theta_2^1}(1+L^2)^{\theta_3^1} & 0 \\ 0 & (1-L)^{d_2+\theta_1^2}(1+L)^{d_2+\theta_2^2}(1+L^2)^{d_2+\theta_3^2} \end{pmatrix}\begin{pmatrix} X_{1t} \\ X_{2t} \end{pmatrix} = \begin{pmatrix} U_{1t} \\ U_{2t} \end{pmatrix}$$

with $\theta = (\theta^{1\prime};\ \theta^{2\prime})' = (\theta_1^1, \theta_2^1, \theta_3^1;\ \theta_1^2, \theta_2^2, \theta_3^2)'$.

Again this way of specifying the model is a particular case of the general model presented in Section 1. We need to define the $(p_u$ x 1) vectors

$$e_{(u)}(\lambda) = \frac{\partial \log \rho_u(e^{i\lambda};\theta^u)}{\partial \theta^u}; \qquad f_{(u)}(\lambda) = Re[e_{(u)}(\lambda)],$$

for all u=1,2,...,N, sharing the same properties as $\varepsilon_{(u)}(\lambda)$ and $\psi_{(u)}(\lambda)$ in Sections 1-3.

To show that this is a particular case of the general specification given before, we just need to note that

$$\epsilon_{(u)}(\lambda) = P_u \, e_{(u)}(\lambda) \tag{31}$$

where P_u is a $(p \times p_u)$ matrix of 1's and 0's of form

$$P_u = \begin{pmatrix} 0 \\ --- \\ I_{p_u} \\ --- \\ 0 \end{pmatrix},$$

and substituting (31) in (12), (16) and (21) we can easily obtain the functional forms for the three test statistics. Starting again with white noise U_t, in the time domain representation, and noting that $\psi_s^{(u)} = P_u f_s^{(u)}$ where $f_s^{(u)}$ comes from expanding $f_{(u)}(\lambda)$ as $\sum_{s=1}^{\infty} f_s^{(u)} \cos \lambda s$, the test statistic takes the form

$$\hat{S}^{t^2} = T \, \hat{a}^{t^2\prime} \, (\hat{A}^{t^2})^{-1} \, \hat{a}^{t^2} \tag{32}$$

$$\text{where } \hat{a}^{t^2} = \begin{pmatrix} \hat{a}_1^{t^2} \\ \hat{a}_2^{t^2} \\ \cdot \\ \cdot \\ \hat{a}_N^{t^2} \end{pmatrix}, \quad \text{with } \hat{a}_u^{t^2} = -\sum_{v=1}^{N} \hat{\sigma}^{uv} \sum_{s=1}^{T-1} C_{uv}(s;\hat{\delta}) f_s^{(u)};$$

$$\text{and } \hat{A}^{t^2} = \begin{pmatrix} \hat{a}_{11}^{t} & \dots & \hat{a}_{1N}^{t} \\ \cdot & \dots & \cdot \\ \hat{a}_{N1}^{t} & \dots & \hat{a}_{NN}^{t} \end{pmatrix}, \quad \text{with } \hat{a}_{uv}^{t} = \hat{\sigma}^{uv} \hat{\sigma}_{uv} \sum_{s=1}^{T-1} \left(1 - \frac{s}{T}\right) f_s^{(u)} f_s^{(v)\prime}. \tag{33}$$

The corresponding test statistic in the frequency domain representation is

$$\hat{S}^{f^2} = T \, \hat{a}^{f^{2\prime}} \, (\hat{A}^{f^2})^{-1} \, \hat{a}^{f^2} \tag{34}$$

$$\text{where } \hat{a}^{f^2} = \begin{pmatrix} \hat{a}_1^{f^2} \\ \hat{a}_2^{f^2} \\ \cdot \\ \cdot \\ \hat{a}_N^{f^2} \end{pmatrix}, \quad \text{with } \hat{a}_u^{f^2} = \frac{-2\pi}{T} \sum_{v=1}^{N} \hat{\sigma}^{uv} \sum_{r}^{*} e_{(u)}(\lambda_r) I_{uv}(\lambda_r;\hat{\delta}),$$

and

$$\hat{A}^{f2} = \begin{pmatrix} \hat{a}^f_{11} & \dots & \hat{a}^f_{1N} \\ . & \dots & . \\ \hat{a}^f_{N1} & \dots & \hat{a}^f_{NN} \end{pmatrix}, \tag{35}$$

with

$$\hat{a}^f_{uv} = \frac{1}{T}\hat{\sigma}^{uv}\hat{\sigma}_{uv}\sum_r^* e_{(u)}(\lambda_r)\bar{e}_{(v)}(\lambda_r)' = \frac{2}{T}\hat{\sigma}_{uv}\hat{\sigma}^{uv}\sum_r^* f_{(u)}(\lambda_r)f_{(v)}(\lambda_r)'. \tag{36}$$

Finally, the test statistic in the frequency domain for weakly parametrically autocorrelated U_t takes the form

$$\tilde{S}^2 = T\,\tilde{b}^{2\prime}\left(\tilde{C}^2 - \tilde{D}^{2\prime}(\tilde{E})^{-1}\tilde{D}^2\right)^{-1}\tilde{b}^2 \tag{37}$$

$$\tilde{b}^2 = \begin{pmatrix} \tilde{b}^2_1 \\ \tilde{b}^2_2 \\ . \\ \tilde{b}^2_N \end{pmatrix} \quad with \quad \tilde{b}^2_u = -\frac{1}{T}Re\left(\sum_r^* e_{(u)}(\lambda_r)\sum_{v=1}^N I_{uv}(\lambda_r;\hat{\delta})\hat{f}^{vu}(\lambda_r;\tilde{\tau})\right),$$

$$\tilde{C}^2 = \begin{pmatrix} \tilde{c}_{11} & \dots & \tilde{c}_{1N} \\ . & \dots & . \\ \tilde{c}_{N1} & \dots & \tilde{c}_{NN} \end{pmatrix}$$

with

$$\tilde{c}_{uv} = \frac{1}{T}Re\left(\sum_r^* e_{(u)}(\lambda_r)\bar{e}_{(v)}(\lambda_r)'\hat{f}_{uv}(\lambda_r;\tilde{\tau})\hat{f}^{vu}(\lambda_r;\tilde{\tau})\right), \tag{38}$$

$$\tilde{D}^{2\prime} = \begin{pmatrix} \tilde{D}'_1 \\ \tilde{D}'_2 \\ \dots \\ \tilde{D}'_N \end{pmatrix} \tag{39}$$

with

$$\tilde{D}'_u = -\frac{1}{T}Re\left(\sum_r^* e_{(u)}(\lambda_r)\sum_{v=1}^N \hat{f}^{uv}(\lambda_r;\tilde{\tau})\frac{\partial\hat{f}_{vu}(\lambda_r;\tilde{\tau})}{\partial\tau'}\right), \tag{40}$$

and $\tilde{E}_{uv}$ remains unchanged, i.e. as in (29).

5.5 WALD TESTS

Once we have obtained the functional forms of the score test statistics, we can use and extend the derivations of previous sections to obtain representations of the tests based on the Wald and likelihood-ratio principles. In this section we concentrate on Wald tests, and present functional forms of the three cases studied before, i.e., the time domain and the frequency domain versions of the tests for white noise U_t, and the frequency domain representation when U_t is weakly parametrically autocorrelated.

5.5.a Wald test for white noise U_t

Here we describe a Wald test for the null hypothesis (4) in the model (1), (2), and (5) under the presumption that U_t in (5) is a vector sequence of zero mean uncorrelated random variables, with unknown variance-covariance matrix K. Recalling from Section 2, $\eta = (\theta', \delta', \alpha')'$, $L(\eta)$ is the negative of the log-likelihood based on Gaussian U_t, (with a minimum at $\eta = \bar{\eta}$), and given the asymptotic block diagonality of the second derivative matrix of $L(\eta)$, (see (A13) in Appendix 5.1), a general form of the Wald test can be written as

$$\bar{\theta}' \, E\left(\frac{\partial^2 L(\bar{\eta})}{\partial\theta \, \partial\theta'} \right) \bar{\theta}, \tag{41}$$

though any other $T^{1/2}$-consistent estimate of η, under (4) could also be adopted in (41).

We start specifying the test statistic in its time domain representation. Denoting $\dot{\eta}$ any admissible value of η, the negative of the log-likelihood, apart from a constant, can be expressed as

$$L^t(\dot{\eta}) = \frac{T}{2} \log\det(K(\dot{\alpha})) + \frac{1}{2} \sum_{t=1}^{T} U_t(\dot{\theta},\dot{\delta})' K(\dot{\alpha})^{-1} U_t(\dot{\theta},\dot{\delta}) \tag{42}$$

where $U_t(\dot{\theta},\dot{\delta}) = \Phi(L;\dot{\theta}) Y_t - W_t(\dot{\delta})$, and the supscript 't' on $L(\dot{\eta})$ indicates the time domain form of the log-likelihood. By the same arguments as those given in Appendix 5.1 it can be shown that

$$\frac{\partial^2 L^t(\dot{\eta})}{\partial\dot{\theta}\,\partial\dot{\theta}'} = \sum_{u=1}^{N} \sum_{s=1}^{T-1} \psi_s^{(u)} \sum_{t=1}^{T-s} \left(\sum_{m=1}^{\infty} \psi_m^{(u)\prime} U_{u,t-m}(\dot{\theta},\dot{\delta}) \right) \sum_{v=1}^{N} \dot{\sigma}^{uv} U_{v,t+s}(\dot{\theta},\dot{\delta}) \; +$$

$$\sum_{u=1}^{N} \sum_{s=1}^{T-1} \psi_s^{(u)} \sum_{t=1}^{T-s} U_{u,t}(\dot{\theta},\dot{\delta}) \sum_{v=1}^{N} \dot{\sigma}^{uv} \sum_{m=1}^{\infty} \psi_m^{(v)\prime} U_{v,t+s-m}(\dot{\theta},\dot{\delta}),$$

where $\dot{\sigma}^{uv} = \{K(\dot{\alpha})^{-1}\}_{uv}$ and $U_{ut}(\dot{\theta},\dot{\delta})$ is the u^{th} element of $U_t(\dot{\theta},\dot{\delta})$. Taking now the expectation in this last expression, evaluated under the null (4) and at $\dot{\delta} = \bar{\delta}$, it is zero for the first summand, and for the second term becomes

$$(T-s)\sum_{u=1}^{N}\sum_{v=1}^{N}\sum_{s=1}^{T-1}\psi_s^{(u)}\psi_s^{(v)\prime}\;\hat{\sigma}^{uv}\;E(\hat{U}_{u,t}(\delta)\,\hat{U}_{v,t}(\delta))\;=$$

$$=\;T\sum_{u=1}^{N}\sum_{v=1}^{N}\;\hat{\sigma}^{uv}\hat{\sigma}_{uv}\sum_{s=1}^{T-1}\left(1-\frac{s}{T}\right)\psi_s^{(u)}\,\psi_s^{(v)\prime},\qquad(43)$$

given the uncorrelatedness in U_t.

Substituting now (43) in (41), we obtain that a Wald test statistic in the time domain context takes the form

$$\hat{W}^t\;=\;T\,\hat{\theta}^{t\prime}\,\hat{A}^t\,\hat{\theta}^t\qquad(44)$$

where $\hat{\theta}^t$ is obtained throughout the minimization of $L^t(\eta)$ in (42), using $T^{1/2}$ - consistent estimates $\hat{\delta}$ and $\hat{\alpha}$, under the null hypothesis (4), and

$$\hat{A}^t\;=\;\sum_{s=1}^{T-1}\left(1-\frac{s}{T}\right)\sum_{u=1}^{N}\sum_{v=1}^{N}\;\hat{\sigma}^{uv}\hat{\sigma}_{uv}\psi_s^{(u)}\,\psi_s^{(v)\prime},$$

that is, adopting the same form as in (14).

For the frequency domain version of the test statistic, we can approximate the negative of the log-likelihood function as

$$L^f(\dot{\eta})\;=\;\frac{T}{2}\log\det\left(\frac{1}{2\pi}K(\dot{\alpha})\right)\;+\;\pi\sum_r^{*}tr\left[K(\dot{\alpha})^{-1}I_U(\lambda_r;\dot{\theta};\dot{\delta})\right]\qquad(45)$$

where $I_U(\lambda_r,\dot{\theta},\dot{\delta})$ is now the cross-periodogram of $U_t(\dot{\theta},\dot{\delta})$ evaluated at λ_r=2πr/T.

Starting with the derivation with respect to $\dot{\theta}$,

$$\frac{\partial L^f(\dot{\eta})}{\partial\dot{\theta}}\;=\;\frac{\partial}{\partial\dot{\theta}}\left(\pi\sum_r^{*}\;tr\left[K(\dot{\alpha})^{-1}I_U(\lambda_r;\dot{\theta};\dot{\delta})\right]\right)\;=$$

$$\pi\sum_r^{*}\left(\frac{\partial}{\partial\dot{\theta}}vec'\left(I_U(\lambda_r;\dot{\theta};\dot{\delta})\right)\right)vec\left(K(\dot{\alpha})^{-1}\right)\;=\;\pi\sum_r^{*}\sum_{u=1}^{N}\sum_{v=1}^{N}\frac{\partial I_{uv}(\lambda_r;\dot{\theta};\dot{\delta})}{\partial\dot{\theta}}\dot{\sigma}^{vu}$$

and using the same arguments as in Appendix 5.2, under suitable conditions, this last expression becomes asymptotically

$$\pi\sum_r^{*}\sum_{u=1}^{N}\sum_{v=1}^{N}\left(\epsilon_{(u)}(\lambda_r)+\bar{\epsilon}_{(v)}(\lambda_r)\right)I_{uv}(\lambda_r;\dot{\theta};\dot{\delta})\,\dot{\sigma}^{vu},$$

and thus, $\partial^2L^f(\eta)/\partial\dot{\theta}\,\partial\dot{\theta}'$ evaluated at $\dot{\theta}=0$ and at $\dot{\delta}=\hat{\delta}$ becomes asymptotically

$$\pi\sum_r^{*}\sum_{u=1}^{N}\sum_{v=1}^{N}\left(\epsilon_{(u)}(\lambda_r)+\bar{\epsilon}_{(v)}(\lambda_r)\right)\left(\epsilon_{(u)}(\lambda_r)+\bar{\epsilon}_{(v)}(\lambda_r)\right)'I_{uv}(\lambda_r;\hat{\delta})\,\hat{\sigma}^{vu},$$

whose expectation for large T will be given by

$$\frac{1}{2}\sum_r^{*}\sum_{u=1}^{N}\sum_{v=1}^{N}\left(\epsilon_{(u)}(\lambda_r)+\bar{\epsilon}_{(v)}(\lambda_r)\right)\left(\epsilon_{(u)}(\lambda_r)+\bar{\epsilon}_{(v)}(\lambda_r)\right)'\;\hat{\sigma}_{uv}\,\hat{\sigma}^{vu}.$$

Therefore, a Wald test statistic in this context will adopt the form

$$\hat{W}^f = T\,\hat{\theta}^{f\prime}\,\hat{A}^f\,\hat{\theta}^f \qquad (46)$$

where $\hat{\theta}^f$ is obtained now throughout the mininimization of $L^f(\eta)$ in (45) with $T^{1/2}$ - consistent estimates $\hat{\delta}$ and $\hat{\alpha}$ under the null, and

$$\hat{A}^f = \frac{1}{2T}\sum_r^* \sum_{u=1}^N \sum_{v=1}^N \left(\epsilon_{(u)}(\lambda_r) + \bar{\epsilon}_{(v)}(\lambda_r)\right)\left(\epsilon_{(u)}(\lambda_r) + \bar{\epsilon}_{(v)}(\lambda_r)\right)' \,\hat{\sigma}^{uv}\,\hat{\sigma}_{uv}$$

$$= \frac{1}{2T}\sum_r^* \sum_{u=1}^N \sum_{v=1}^N \left(\epsilon_{(u)}(\lambda_r)\,\bar{\epsilon}_{(v)}(\lambda_r)' + \bar{\epsilon}_{(v)}(\lambda_r)\,\epsilon_{(u)}(\lambda_r)'\right) \hat{\sigma}^{uv}\,\hat{\sigma}_{uv},$$

by the same arguments as those given in previous sections.

5.5.b Wald test for weakly parametrically correlated U_t

Analogously to what we did for the score test, we can now robustify the test statistic in (46), to allow for weak parametric autocorrelation in U_t. We take U_t as in (19) and again here, the same conditions as those given in Section 3 and Appendix 5.2 will be required on U_t to obtain the test statistic. Recalling η from Section 3, the Wald test in this context will take the form

$$\theta'\,F^{\theta\theta}\,\theta\;\big|_{\eta=\bar{\eta}}$$

where $\bar{\eta}$ is the value that minimizes $L(\eta)$ in Appendix 5.2, though again any other $T^{1/2}$-consistent estimate can be adopted, and

$$F^{\theta\theta} = F_{\theta\theta} - F_{\theta\tau}\,F_{\tau\tau}^{-1}\,F_{\tau\theta}$$

$$\textit{where}\;\; F = \begin{pmatrix} F_{\theta\theta} & F_{\theta\tau} \\ F_{\tau\theta} & F_{\tau\tau} \end{pmatrix}$$

is the expected information matrix. Now, given the derivations carried out in Appendix 5.2, a Wald test statistic will adopt the form

$$\tilde{W} = T\,\tilde{\theta}'\,(\tilde{C} - \tilde{D}'\,(\tilde{E})^{-1}\,\tilde{D}\,)\,\tilde{\theta} \qquad (47)$$

with $\tilde{C}$,$\tilde{D}$ and $\tilde{E}$ as in (21), $\tilde{\tau}$ as in (22) and $\tilde{\theta}$ obtained by minimizing $L(\eta)$ in (B4) in Appendix 5.2 with $\tau = \tilde{\tau}$.

5.5.c Particular cases

We can stress the two cases of interest mentioned in Section 4. First, we consider θ is exactly the same parameter vector across all equations in (5). The test statistic for white noise U_t in the time domain representation takes the form

$$\hat{W}^{t^1} = T\,\hat{\theta}^{t^1\prime}\,(\hat{A}^{t^1})\,\hat{\theta}^{t^1} \tag{48}$$

$$\text{with } \hat{A}^{t^1} = N\sum_{s=1}^{T-1}\left(1-\frac{s}{T}\right)\psi_s\,\psi_s',$$

and $\hat{\theta}^{t1}$ as in (44) but minimizing $L^t(\eta)$ with $\Phi(L;\theta)$ as defined in Section 5.4.a.

The frequency domain version is

$$\hat{W}^{f^1} = T\,\hat{\theta}^{f^1\prime}\,(\hat{A}^{f^1})\,\hat{\theta}^{f^1} \tag{49}$$

$$\text{with } \hat{\theta}^{f^1} \text{ as in (46), and } \hat{A}^{f^1} = \frac{2N}{T}\sum_{r}^{*}\psi(\lambda_r)\,\psi(\lambda_r)',$$

and if U_t is weakly parametrically autocorrelated, the test statistic becomes

$$\tilde{W}^1 = T\,\tilde{\theta}^{1\prime}\,(\tilde{C}^1 - \tilde{D}^{1\prime}\,(\tilde{E})^{-1}\,\tilde{D}^1)\,\tilde{\theta}^1 \tag{50}$$

with $\tilde{\theta}^1$ as in (47), and $\tilde{C}^1$, $\tilde{D}^1$ and $\tilde{E}$ as in (27), (28) and (29) respectively.

Finally, we consider the different versions of the test statistics when we take the parameter vector θ to be strictly different for each equation in (5). The time domain representation for white noise U_t is

$$\hat{W}^{t^2} = T\,\hat{\theta}^{t^2\prime}\,(\hat{A}^{t^2})\,\hat{\theta}^{t^2} \tag{51}$$

with $\hat{\theta}^{t2}$ as in (44), i.e. minimizing (42) under the null hypothesis (4) and using now the new matrix $\Phi(L;\theta)$ specified in (30) and $\hat{A}^{t2}$ as in (33). The frequency domain version of the test statistic is

$$\hat{W}^{f^2} = T\,\hat{\theta}^{f^2\prime}\,(\hat{A}^{f^2})\,\hat{\theta}^{f^2} \tag{52}$$

with $\hat{\theta}^{f2}$ as in (46) and $\hat{A}^{f2}$ as in (35) and (36); and finally, if U_t is weakly parametrically autocorrelated, the test statistic becomes

$$\tilde{W}^2 = T\,\tilde{\theta}^{2\prime}\,(\tilde{C}^2 - \tilde{D}^{2\prime}\,(\tilde{E})^{-1}\,\tilde{D}^2)\,\tilde{\theta}^2 \tag{53}$$

with $\tilde{\theta}^2$ as in (47) and $\tilde{C}^2$, $\tilde{D}^2$ and $\tilde{E}$ as in (38-40).

5.6 LIKELIHOOD RATIO TESTS

Finally, we can also compute pseudo likelihood ratio test statistics under the same situations as in previous sections. Starting with the case of white noise U_t, a pseudo log-likelihood ratio test will adopt the form

$$LR = 2\,(L(\hat{\eta}) - L(\bar{\eta}))$$

where $L(\eta)$ is the negative of the log-likelihood; $\hat{\eta} = (0'; \hat{\delta}'; \hat{\alpha}')'$ as in Section 2, and $\bar{\eta} = (\hat{\theta}'; \hat{\delta}; \bar{\alpha}')'$, where $\hat{\theta}$ minimizes $L(\theta'; \hat{\delta}'; \hat{\alpha}')$ and $\bar{\alpha}$ is obtained using $\hat{\theta}$ and

$\hat{\delta}$. First we concentrate on the time domain version of the test. From previous sections, we can write

$$L^t(\hat{\eta}) = \frac{T}{2}\log\det K(\hat{\alpha}) + \frac{1}{2}\sum_{t=1}^{T} \hat{U}_t(\delta)'K(\hat{\alpha})^{-1}\hat{U}_t(\delta)$$

$$= \frac{T}{2}\log\det K(\hat{\alpha}) + \frac{T}{2}tr\left[K(\hat{\alpha})^{-1}\frac{1}{T}\sum_{t=1}^{T} \hat{U}_t(\delta)\hat{U}_t(\delta)'\right]$$

$$= \frac{T}{2}\log\det K(\hat{\alpha}) + \frac{NT}{2}, \qquad (54)$$

and similarly,

$$L^t(\bar{\eta}) = \frac{T}{2}\log\det K(\bar{\alpha}) + \frac{1}{2}\sum_{t=1}^{T} U_t(\hat{\theta}^t,\hat{\delta})'K(\bar{\alpha})^{-1}U_t(\hat{\theta}^t,\hat{\delta})$$

$$= \frac{T}{2}\log\det K(\bar{\alpha}) + \frac{T}{2}tr\left[K(\bar{\alpha})^{-1}\frac{1}{T}\sum_{t=1}^{T} U_t(\hat{\theta}^t,\hat{\delta})U_t(\hat{\theta}^t,\hat{\delta})'\right]$$

$$= \frac{T}{2}\log\det K(\bar{\alpha}) + \frac{NT}{2}. \qquad (55)$$

Using (54) and (55), we can write a pseudo log-likelihood ratio test statistic as

$$LR^t = T\log\frac{\det K(\hat{\alpha})}{\det K(\bar{\alpha})} \qquad (56)$$

where

$$K(\hat{\alpha}) = \frac{1}{T}\sum_{t=1}^{T} \hat{U}_t(\delta)\hat{U}_t(\delta)',$$

$\hat{U}_t(\delta) = U_t(\hat{\delta})$, and $\hat{\delta}$ is as given in Section 2 (i.e., a $T^{1/2}$-consistent estimate of δ under the null hypothesis) , and

$$K(\bar{\alpha}) = \frac{1}{T}\sum_{t=1}^{T} U_t(\hat{\theta}^t,\hat{\delta})U_t(\hat{\theta}^t,\hat{\delta})'$$

and $\hat{\theta}^t$ obtained throughout the minimization of $L^t(\eta)$ based on $\hat{\delta}$ and $\hat{\alpha}$.

Similarly, we can derive the test statistic in its frequency domain representation. Again from previous sections we have that

$$L^f(\hat{\eta}) = \frac{T}{2}\log\det\left(\frac{1}{2\pi}K(\hat{\alpha})\right) + \pi\sum_{r}^{*} tr[K(\hat{\alpha})^{-1}I_U(\lambda_r;\hat{\delta})]$$

$$= \frac{-NT}{2}\log 2\pi + \frac{T}{2}\log\det K(\hat{\alpha}) + \pi\, tr\left[K(\hat{\alpha})^{-1}\sum_{r}^{*} I_U(\lambda_r;\hat{\delta})\right]$$

$$= C + \frac{T}{2}\log\det(K(\hat{\alpha}), \quad where \quad C = \frac{NT}{2}(1 - \log 2\pi)$$

and similarly,

$$L^f(\bar{\eta}) = C + \frac{T}{2}\log\det K(\bar{\alpha})$$

so, a pseudo LR test statistic in this context can be approximated by

$$LR^f = T\log\frac{\det K(\hat{\alpha})}{\det K(\bar{\alpha})} \tag{57}$$

where now

$$K(\hat{\alpha}) = \frac{2\pi}{T}\sum_r^* I_U(\lambda_r;\hat{\delta})$$

and

$$K(\bar{\alpha}) = \frac{2\pi}{T}\sum_r^* I_U(\lambda_r;\hat{\theta}^f;\hat{\delta}),$$

and $\hat{\theta}^f$ minimizes the frequency domain version of the log-likelihood based on $\hat{\delta}$ and $\hat{\alpha}$.

Extending the tests for weak parametric autocorrelation in U_t, the test statistic takes the form

$$\overline{LR} = T\log\frac{\det K(\hat{\alpha})}{\det K(\bar{\alpha})} \tag{58}$$

where

$$K(\hat{\alpha}) = \frac{2\pi}{T}\sum_r^* tr[\hat{f}(\lambda_r;\tilde{\tau})^{-1}\, I_U(\lambda_r;\hat{\delta})]$$

and

$$K(\bar{\alpha}) = \frac{2\pi}{T}\sum_r^* tr[\hat{f}(\lambda_r;\tilde{\tau})^{-1}\, I_U(\lambda_r;\tilde{\theta}^f;\hat{\delta})],$$

with $\hat{f}$, $\tilde{\tau}$, $\tilde{\theta}^f$ and $\hat{\delta}$ as they were given in all previous pages.

Finally, for the two particular cases considered in Section 4, the test statistics will take the same form as in (56), (57) and (58) with the only difference in the specification of the matrix $\Phi(L;\theta)$ appearing in (5).

5.7 FINITE SAMPLE PERFORMANCE

In this final section we examine the finite sample behaviour of some of the test statistics presented in previous sections, by means of Monte Carlo simulations. All calculations were carried out using Fortran and the NAG's library random number generator, on LSE's VAX computer. Given the variety of tests and the number of possibilities covered by them, we concentrate on a bivariate model where the null hypothesis will be two time series following a random walk. We will consider a model of form

$$\begin{pmatrix} (1-L)^{1+\theta_1} & 0 \\ 0 & (1-L)^{1+\theta_2} \end{pmatrix} \begin{pmatrix} X_{1t} \\ X_{2t} \end{pmatrix} = \begin{pmatrix} U_{1t} \\ U_{2t} \end{pmatrix} \qquad t = 1,2,...,T \qquad (59)$$

$$X_t = (X_{1t}, X_{2t})' = 0 \quad \text{for all } t \le 0, \qquad (60)$$

where under the null hypothesis given by:

$$H_o\text{: } \theta = (\theta_1, \theta_2)' = 0, \qquad (61)$$

$U_t = (U_{1t}, U_{2t})'$ will be initially, a white noise vector process with mean zero and variance-covariance matrix Σ. First, and without loss of generality, we assume that $\Sigma = I_2$, but we also present results, for a given positive definite matrix Σ, in order to check if the test statistics are robust for a different specification of Σ. We look first at rejection frequencies of the score test statistic given in (32), for fractional alternatives, where $(\theta_i)_{i=1,2}$ in (59) takes values: -0.8; -0.6; -0.4; -0.2; 0; 0.2; 0.4; 0.6 and 0.8. Then, we generate Gaussian series for different sample sizes (50, 100 and 200 observations) taking 5000 replications of each case, and present results for four different nominal sizes: 10%, 5%, 2.5% and 1%. The reason for focusing on the test statistic given in (32), (i.e., the time domain version), rather than in its frequency domain representation (i.e., (34)), is that the latter form of the test statistic is much more expensive computationally in terms of CPU time. We know that in finite samples, the results of the two test statistics can vary substantially, though asymptotically the difference will be negligible. Furthermore, in view of the empirical results presented in the following chapter, we see that even when the sample size is not very large, results in both cases are very similar, rejecting the null hypothesis for the same type of situations.

In Table 5.1 we present rejection frequencies of the test statistic $\hat{S}^{t2}$ in (32) when $\Sigma = I_2$, for three different sample sizes (T = 50, 100 and 200) and a nominal size of 10%. Tables 5.2-5.4 are similar to Table 5.1 but with nominal sizes of 5%, 2.5% and 1%, respectively. Looking across these tables, we see that the sizes of the tests are too small in all cases, however they tend to improve as we increase the number of observations. For example, we observe in Table 5.1 (α = 10%), that when the sample size is 50, the size is 3.3%, but increases to 5.3% when T = 100, and to 7.2% when T = 200. Similarly in Table 5.2 ($\alpha = 5\%$), the sizes are 1.2% for T = 50, 2.0% for T = 100, and 3.2% for T = 200. The same behaviour is observed in Tables 5.3 and 5.4, with all sizes smaller than nominal ones but increasing with the number of observations. If we concentrate now on small departures from the null (61), we observe that these rejection frequencies increase strongly, especially when the sample size is large (e.g. T = 200). This increase is more marked when θ_1 and θ_2 take the same value, though it is also noticeable when θ_1 and θ_2 are different. In Table 5.1c (T = 200, $\alpha = 10\%$) we see that the lowest rejection probability, apart from that of the true model ($\theta_1 = \theta_2 = 0$), is 0.827 which is obtained when $\theta_1 = 0$ and $\theta_2 = -0.2$, and becomes 0.993 when $\theta_1 = \theta_2 = -0.2$. Similarly in Table 5.2c, (when T = 200 and $\alpha = 5\%$), the values for the same alternatives are 0.671 and 0.997; in Table 5.3c ($\alpha = 2.5\%$) are 0.495 and 0.941, and in Table 5.4c ($\alpha = 1\%$) 0.279 and 0.848.

Another remarkable feature of these results is that when the sample size is small (e.g. T = 50), it seems that there is a bias toward positive values of θ_1 and θ_2. This bias is especially clear when the nominal size is also small. We can see through Tables 5.2a, 5.3a and 5.4a that if θ_1 and θ_2 are both greater than or equal to 0, rejection frequencies are always greater than those obtained when the values of θ_1 and θ_2 were less than or equal to 0. Taking nominal sizes of 2.5% and 1%, this bias also appears for a sample size of 100 observations (Tables 5.3b and 5.4b); however, increasing the sample size to 200 observations, the bias tends to disappear. A particularly poor result is obtained in Table 5.4a (T =50; $\alpha = 1\%$), when θ_1 (or θ_2) is equal to 0 and θ_2 (or θ_1) is negative. In such situations, the rejection probabilities never exceed 0.100. Again these results improve considerably when we increase the sample size to 100 or 200 observations (Tables 5.4b and 5.4c). Finally we observe that in all cases, rejection frequencies increase with absolute value of θ and with sample size T, and when T = 200, the rejection probability of 1 is obtained in most of cases when $|\theta_i|_{i=1,2} \geq 0.4$ for $\alpha = 10\%$ and 5%, and when $|\theta_i|_{i=1,2} \geq 0.6$ for $\alpha = 2.5\%$ and 1%.

Tables 5.5-5.8 report rejection frequencies of the same statistic as above, but now we take Σ as a positive definite matrix of form: $\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$. In doing so, we can see if the test statistic is robust to a different specification of the variance-covariance matrix of the differenced residuals. Table 5.5 is the counterpart of Table 5.1 for the new variance-covariance matrix Σ. Similarly, Tables 5.6-5.8 corresponds to Tables 5.2-5.4 above. We observe now that sizes are slightly greater than before, but again too small with respect to nominal ones though increasing with the sample size T. In Table 5.5 ($\alpha = 10\%$), we see that sizes are now 3.9% for T = 50; 6.1% for T = 100; and 7.5% for T = 200. Across Tables 5.6-5.8 we see that in five cases (Tables 5.6c, 5.7b, 5.7c, 5.8a and 5.8c), sizes are the same as when $\Sigma = I_2$, while in the other four cases (Tables 5.6a, 5.6b, 5.7a and 5.8b) they are slightly greater, but not exceeding in 0.02% those results obtained across Tables 5.2-5.4. A bias for positive values of θ_1 and θ_2 is again observed when nominal sizes and sample sizes are small; however, the pathological cases observed in Table 5.4a have now disappeared (Table 5.8a). All rejection frequencies increase with sample size T, but in a few cases, we now observe a lack of monotonicity of these rejections with respect to $(\theta_i)_{i=1,2}$, when the sample size is small and $(\theta_i)_{i=1,2}$ takes low values. Comparing these results in Tables 5.5-5.8 with those obtained in Tables 5.1-5.4, we see that in most of the cases, rejection frequencies are now slightly greater, but in general, results are similar across all tables, suggesting that the test statistic is not affected much by the different specifications of the variance-covariance matrix Σ.

In Tables 5.9 and 5.10 we present empirical sizes of the test in the frequency domain representation. Table 5.9 reports sizes of the test statistic $\hat{S}^{f2}$ in (34), assuming first, in Table 5.9a, that $\Sigma = I_2$, while in Table 5.9b we take $\Sigma = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$. As in all previous tables, we see that sizes are very small when T = 50, however they improve considerably when we increase the sample size. Comparing empirical sizes in Table 5.9a with those in Tables 5.1-5.4, we see that they are very similar. When T = 50 the sizes are now slightly smaller than in the time domain versions of the tests, but when T = 100 or 200, they are slightly greater. We should mention here that results obtained in Table 5.9 (and also in Table 5.10) have been obtained using 1000 replications of each case, (unlike the 5000 replications used in Tables 5.1-5.8). Therefore the difference may be largely due to the different number of replications used. When $\Sigma \neq I_2$ (Table 5.9b) the same conclusions hold, with empirical sizes smaller than nominal ones but increasing with T, and observing few differences with respect to empirical sizes obtained in the time domain representation of the tests across Tables 5.5-5.8. Comparing results in Table

5.9b with those obtained in Table 5.9a, we again observe few differences, with the highest one occurring when T = 50 and α = 10%; in this case, the empirical sizes are 2.8% in Table 5.9a and 3.6% in Table 5.9b, while in the remaining cases, the differences are not greater than 0.03% between both tables.

Finally, Table 5.10 reports sizes for the test statistic $\tilde{S}^2$ in (37), i.e., the frequency domain representation of the test when U_t is weakly parametrically autocorrelated. In Table 5.10a we assume that U_t follows a VAR(1) representation, and we choose the parameterization

$$\begin{pmatrix} U_{1t} \\ U_{2t} \end{pmatrix} = \begin{pmatrix} 0.5 & 0.2 \\ 0.3 & 0.5 \end{pmatrix} \begin{pmatrix} U_{1t-1} \\ U_{2t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix}, \tag{62}$$

where ε_t is normally distributed with mean zero and variance-covariance matrix I_2. In Table 5.10b we consider a VMA(1) structure on U_t using the same parameters as in the VAR(1) case. That is,

$$\begin{pmatrix} U_{1t} \\ U_{2t} \end{pmatrix} = \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix} + \begin{pmatrix} 0.5 & 0.2 \\ 0.3 & 0.5 \end{pmatrix} \begin{pmatrix} \epsilon_{1t-1} \\ \epsilon_{2t-1} \end{pmatrix}, \tag{63}$$

and again ε_t normally distributed with mean 0 and variance I_2.

In both tables we see that sizes are now too large for all nominal sizes, especially when T = 50; however, as we increase the number of observations, these empirical sizes reduce and then tend to approximate to nominal ones. Thus, for the VAR(1) case (Table 5.10a), we see that if the number of observations is 200, the sizes are 10.4% for α = 10%; 6.0% for α = 5%; 3.1% for α = 2.5%; and 1.2% for α = 1%. When the VMA(1) structure is considered (Table 5.10b), empirical sizes are now slightly greater than in the VAR(1) case, but again we observe a considerable improvement when we increase the number of observations. Similar results were obtained when we used different parameters in (62) and (63) and a different variance-covariance matrix for the residuals ε_t.

As a conclusion, we can summarize the results obtained across these tables by saying that the score test statistics obtained in this chapter seem to be adequate to test the null hypothesis of a random walk in this bivariate context. Though sizes are smaller than nominal ones in most of cases, the performance of these tests seems quite good even for small departures of the null hypothesis (61), especially as we increase the number of observations.

APPENDIX 5.1: DERIVATION OF THE SCORE STATISTIC $\hat{S}^t$

The negative of the log-likelihood under (1), (2), (5), and Gaussianity of U_t, can be expressed, apart from a constant as

$$L(\theta,\dot{\delta},\dot{\alpha}) = \frac{T}{2}\log\det K(\dot{\alpha}) + \frac{1}{2}\sum_{t=1}^{T} U_t(\theta,\dot{\delta})' K(\dot{\alpha})^{-1} U_t(\theta,\dot{\delta})$$

$$= \frac{T}{2}\log\det K(\dot{\alpha}) + \frac{1}{2}\sum_{t=1}^{T} X_t(\dot{\delta})'\Phi(L;\theta) K(\dot{\alpha})^{-1}\Phi(L;\theta) X_t(\dot{\delta}), \qquad (A1)$$

for any admissible $\dot{\alpha}$ and $\dot{\delta}$, where $U_t(\theta,\dot{\delta}) = \Phi(L;\theta)X_t(\dot{\delta})$ and $X_t(\dot{\delta}) = Y_t - Z_t(\dot{\delta})$.

Starting with the first derivatives in (11),

$$\frac{\partial L(\theta,\dot{\delta},\dot{\alpha})}{\partial\theta} = \frac{\partial}{\partial\theta}\left[\frac{1}{2}\sum_{t=1}^{T} X_t(\dot{\delta})'\Phi(L;\theta) K(\dot{\alpha})^{-1}\Phi(L;\theta) X_t(\dot{\delta})\right]$$

$$= \sum_{t=1}^{T}\left[\frac{\partial\rho_1(L;\theta)}{\partial\theta}X_{1t}(\dot{\delta});\ldots;\frac{\partial\rho_N(L;\theta)}{\partial\theta}X_{Nt}(\dot{\delta})\right]K(\dot{\alpha})^{-1}U_t(\theta,\dot{\delta}) =$$

$$\sum_{t=1}^{T}\left[\frac{\partial\log\rho_1(L;\theta)}{\partial\theta}U_{1t}(\theta,\dot{\delta});\ldots;\frac{\partial\log\rho_N(L;\theta)}{\partial\theta}U_{Nt}(\theta,\dot{\delta})\right]K(\dot{\alpha})^{-1}U_t(\theta,\dot{\delta})$$

where $U_t(\theta,\dot{\delta}) = (U_{1t}(\theta,\dot{\delta});..;U_{Nt}(\theta,\dot{\delta}))'$ and $X_t(\dot{\delta}) = (X_{1t}(\dot{\delta});..;X_{Nt}(\dot{\delta}))'$, and evaluating now this last expression at $\theta = 0$ we obtain

$$\sum_{t=1}^{T}\left[\epsilon_{(1)}(L)\,U_{1t}(\dot{\delta});\ldots;\epsilon_{(N)}(L)\,U_{Nt}(\dot{\delta})\right]K(\dot{\alpha})^{-1}U_t(\dot{\delta}) \qquad (A2)$$

where $\epsilon_{(u)}(L) = \dfrac{\partial\log\rho_u(L;\theta)}{\partial\theta}$ *can be expanded as* $\sum_{s=1}^{\infty}\psi_s^{(u)}L^s$,

in view of (8) and below, and the expression in (A2) becomes

$$\sum_{t=1}^{T}\left[\left(\sum_{s=1}^{\infty}\psi_s^{(1)}U_{1,t-s}(\dot{\delta})\right);\ldots;\left(\sum_{s=1}^{\infty}\psi_s^{(N)}U_{N,t-s}(\dot{\delta})\right)\right]K(\dot{\alpha})^{-1}U_t(\dot{\delta}) =$$

$$\sum_{t=1}^{T}\left[\left(\sum_{s=1}^{\infty}\psi_s^{(1)}U_{1,t-s}(\dot{\delta})\right);\ldots;\left(\sum_{s=1}^{\infty}\psi_s^{(N)}U_{N,t-s}(\dot{\delta})\right)\right]\begin{bmatrix}\sum_{v=1}^{N}\dot{\sigma}^{1v}U_{vt}(\dot{\delta})\\ --\\ \sum_{v=1}^{N}\dot{\sigma}^{Nv}U_{vt}(\dot{\delta})\end{bmatrix} \qquad (A3)$$

$$\sum_{u=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{\infty}\psi_s^{(u)}U_{u,t-s}(\dot{\delta})\sum_{v=1}^{N}\dot{\sigma}^{uv}U_{v,t}(\dot{\delta}) =$$

$$= \sum_{u=1}^{N} \sum_{s=1}^{T-1} \psi_s^{(u)} \sum_{t=1}^{T-s} U_{ut}(\dot{\delta}) \sum_{v=1}^{N} \dot{\sigma}^{uv} U_{v,t+s}(\dot{\delta})$$

$$= T\sum_{u=1}^{N} \sum_{v=1}^{N} \dot{\sigma}^{uv} \sum_{s=1}^{T-1} \psi_s^{(u)} C_{uv}(s,\dot{\delta}) , \qquad (A4)$$

where $\dot{\sigma}^{uv}$ is the $(u,v)^{th}$ element of $K(\dot{\alpha})^{-1}$ and

$$C_{uv}(s,\dot{\delta}) = \frac{1}{T}\sum_{t=1}^{T-s} U_{u,t}(\dot{\delta})\, U_{v,t+s}(\dot{\delta}) .$$

Calling $L_o = L(\dot{\eta})_{\theta=0}$, the first derivative with respect to $\dot{\delta}$ is

$$\frac{\partial L_o}{\partial \dot{\delta}} = \frac{\partial}{\partial \dot{\delta}}\left[\frac{1}{2} \sum_{t=1}^{T} \left(Y_t - Z_t(\dot{\delta})\right)' \Phi(L)\, K(\dot{\alpha})^{-1} \Phi(L) \left(Y_t - Z_t(\dot{\delta})\right)\right]$$

$$= \frac{\partial}{\partial \dot{\delta}} [\frac{1}{2}\sum_{t=1}^{T} W_t(\dot{\delta})' K(\dot{\alpha})^{-1} W_t(\dot{\delta}) - \frac{1}{2} \sum_{t=1}^{T} W_t(\dot{\delta})'\, K(\dot{\alpha})^{-1} \Phi(L)\, Y_t -$$

$$\frac{1}{2} \sum_{t=1}^{T} Y_t' \,\Phi(L)\, K(\dot{\alpha})^{-1}\, W_t(\dot{\delta})]$$

$$= \frac{\partial}{\partial \dot{\delta}}\left[\frac{1}{2}\sum_{t=1}^{T} W_t(\dot{\delta})' K(\dot{\alpha})^{-1} W_t(\dot{\delta}) - \sum_{t=1}^{T} W_t(\dot{\delta})' K(\dot{\alpha})^{-1} \Phi(L)\, Y_t\right]$$

$$= \sum_{t=1}^{T} \frac{\partial W_t(\dot{\delta})}{\partial \dot{\delta}} K(\dot{\alpha})^{-1} W_t(\dot{\delta}) - \sum_{t=1}^{T} \frac{\partial W_t(\dot{\delta})}{\partial \dot{\delta}} K(\dot{\alpha})^{-1} \Phi(L)\, Y_t$$

$$= - \sum_{t=1}^{T} \frac{\partial W_t(\dot{\delta})}{\partial \dot{\delta}} K(\dot{\alpha})^{-1} U_t(\dot{\delta}) . \qquad (A5)$$

From (A1) we have that L_o can be expressed as

$$\frac{T}{2} \log\det K(\dot{\alpha}) + \frac{1}{2} tr[K(\dot{\alpha})^{-1} S(\dot{\delta})],$$

where $S(\dot{\delta}) = \sum_{t=1}^{T} U_t(\dot{\delta})\, U_t(\dot{\delta})'$, and differentiating L_o with respect to $\dot{\alpha}$ leads to

$$\frac{T}{2} tr\,[K(\dot{\alpha})^{-1} (dK(\dot{\alpha}))] - \frac{1}{2} tr\,[K(\dot{\alpha})^{-1} (dK(\dot{\alpha}))\, K(\dot{\alpha})^{-1} S(\dot{\delta})] \qquad (A6)$$

$$= -\frac{1}{2} tr\,[(dK(\dot{\alpha})\, K(\dot{\alpha})^{-1} (S(\dot{\delta}) - TK(\dot{\alpha}))\, K(\dot{\alpha})^{-1}]$$

$$= -\frac{1}{2}(vec(dK(\dot{\alpha})))'(K(\dot{\alpha})^{-1} \otimes K(\dot{\alpha})^{-1})\, vec(S(\dot{\delta}) - TK(\dot{\alpha}))$$

$$= -\frac{1}{2} dv(K(\dot{\alpha}))' D_m' (K(\dot{\alpha})^{-1} \otimes K(\dot{\alpha})^{-1})\, vec(S(\dot{\delta}) - TK(\dot{\alpha})), \quad (A7)$$

where D_m is the duplication matrix, and using the well known result that tr[ABCD] = (vec A)'(D' $\otimes$ B)(vec C). Then, from (A7) we easily observe that

$$\frac{\partial L_o}{\partial \dot{\alpha}} = -\frac{1}{2} D_m'(K(\dot{\alpha})^{-1} \otimes K(\dot{\alpha})^{-1})\, vec\,(S(\dot{\delta}) - TK(\dot{\alpha})). \qquad (A8)$$

Next we look at the second derivative matrices appearing in (11), and first concentrate on the (pxp) matrix $\partial^2 L_o/\partial\theta\partial\theta'$. From the equality above (A4)

$$\frac{\partial L_o}{\partial \theta} = \sum_{u=1}^{N}\sum_{s=1}^{T-1} \psi_s^{(u)} \sum_{t=1}^{T-s} U_{ut}(\dot{\delta}) \sum_{v=1}^{N} \dot{\sigma}^{uv} U_{v,t+s}(\dot{\delta}),$$

and then we have that

$$\frac{\partial^2 L_o}{\partial\theta\partial\theta'} = \sum_{u=1}^{N}\sum_{s=1}^{T-1} \psi_s^{(u)} \sum_{t=1}^{T-s} \frac{\partial \log \rho_u(L;0)}{\partial\theta'} U_{ut}(\dot{\delta}) \sum_{v=1}^{N} \dot{\sigma}^{uv} U_{v,t+s}(\dot{\delta}) \; +$$

$$\sum_{u=1}^{N}\sum_{s=1}^{T-1} \psi_s^{(u)} \sum_{t=1}^{T-s} U_{ut}(\dot{\delta}) \sum_{v=1}^{N} \dot{\sigma}^{uv} \frac{\partial \log \rho_v(L;0)}{\partial\theta'} U_{v,t+s}(\dot{\delta})$$

$$= \sum_{u=1}^{N}\sum_{s=1}^{T-1} \psi_s^{(u)} \sum_{t=1}^{T-s} \left(\sum_{m=1}^{\infty} \psi_m^{(u)\prime} U_{u,t-m}(\dot{\delta}) \right) \sum_{v=1}^{N} \dot{\sigma}^{uv} U_{v,t+s}(\dot{\delta}) \; +$$

$$\sum_{u=1}^{N}\sum_{s=1}^{T-1} \psi_s^{(u)} \sum_{t=1}^{T-s} U_{ut}(\dot{\delta}) \sum_{v=1}^{N} \dot{\sigma}^{uv} \left(\sum_{m=1}^{\infty} \psi_m^{(v)\prime} U_{v,t+s-m}(\dot{\delta}) \right).$$

In order to form (11), we need to take the expectation of this last expression. (Note that it is evaluated at $\theta = 0$, i.e. under H_o (4)). It is zero for the first summand given the uncorrelatedness in U_t and since it involves terms of the form $U_{u,t-m}$ and $U_{v,t+s}$, for m,s > 0. The expectation of the second summand is

$$\sum_{u=1}^{N}\sum_{s=1}^{T-1} \psi_s^{(u)} \sum_{v=1}^{N} \dot{\sigma}^{uv} \psi_s^{(v)\prime} \sum_{t=1}^{T-s} E\left(U_{ut}(\dot{\delta})\, U_{vt}(\dot{\delta})\right) =$$

$$\sum_{u=1}^{N}\sum_{v=1}^{N}\sum_{s=1}^{T-1} (T-s)\, \psi_s^{(u)} \psi_s^{(v)\prime} \dot{\sigma}^{uv} \dot{\sigma}_{uv} = T \sum_{s=1}^{T-1} \left(1 - \frac{s}{T}\right) \sum_{u=1}^{N}\sum_{v=1}^{N} \dot{\sigma}^{uv} \dot{\sigma}_{uv} \psi_s^{(u)} \psi_s^{(v)\prime}.$$

Again from the equality above (A4), we have that $\partial L_o/\partial\theta$ can be rewritten as

$$\sum_{u=1}^{N}\sum_{s=1}^{T-1}\psi_s^{(u)}\sum_{t=1}^{T-s}\left(\rho_u(L)\,Y_{ut}-W_{ut}(\hat{\delta})\right)\sum_{v=1}^{N}\dot{\sigma}^{uv}\left(\rho_v(L)\,Y_{v,t+s}-W_{v,t+s}(\hat{\delta})\right),$$

and from this expression, we observe that

$$\frac{\partial^2 L_o}{\partial\theta\partial\hat{\delta}'}=\frac{\partial}{\partial\hat{\delta}'}\Big[\sum_{u=1}^{N}\sum_{s=1}^{T-1}\psi_s^{(u)}\sum_{t=1}^{T-s}\Big(W_{ut}(\hat{\delta})\sum_{v=1}^{N}\dot{\sigma}^{uv}W_{v,t+s}(\hat{\delta})-\rho_u(L)\,Y_{ut}\ \times$$

$$\times\sum_{v=1}^{N}\dot{\sigma}^{uv}\,W_{v,t+s}(\hat{\delta})-W_{ut}(\hat{\delta})\sum_{v=1}^{N}\dot{\sigma}^{uv}\,\rho_v(L)\,Y_{v,t+s}\Big)\Big]=$$

$$\sum_{u=1}^{N}\sum_{s=1}^{T-1}\psi_s^{(u)}\sum_{t=1}^{T-s}\Big(\frac{\partial W_{ut}(\hat{\delta})}{\partial\hat{\delta}'}\sum_{v=1}^{N}\dot{\sigma}^{uv}W_{v,t+s}(\hat{\delta})+W_{ut}(\hat{\delta})\sum_{v=1}^{N}\dot{\sigma}^{uv}\frac{\partial W_{v,t+s}(\hat{\delta})}{\partial\hat{\delta}'}$$

$$-\rho_u(L)\,Y_{ut}\sum_{v=1}^{N}\dot{\sigma}^{uv}\frac{\partial W_{v,t+s}(\hat{\delta})}{\partial\hat{\delta}'}-\frac{\partial W_{ut}(\hat{\delta})}{\partial\hat{\delta}'}\sum_{v=1}^{N}\dot{\sigma}^{uv}\,\rho_v(L)\,Y_{v,t+s}\Big)$$

$$=-\left(\sum_{u=1}^{N}\sum_{s=1}^{T-1}\psi_s^{(u)}\sum_{t=1}^{T-s}\left[\frac{\partial W_{ut}(\hat{\delta})}{\partial\hat{\delta}'}\sum_{v=1}^{N}\dot{\sigma}^{uv}U_{v,t+s}(\hat{\delta})+U_{ut}(\hat{\delta})\sum_{v=1}^{N}\dot{\sigma}^{uv}\frac{\partial W_{v,t+s}(\hat{\delta})}{\partial\hat{\delta}'}\right]\right)$$

$$=-\sum_{u=1}^{N}\sum_{s=1}^{T-1}\psi_s^{(u)}\sum_{v=1}^{N}\dot{\sigma}^{uv}\sum_{t=1}^{T-s}\left[\frac{\partial W_{ut}(\hat{\delta})}{\partial\hat{\delta}'}U_{v,t+s}(\hat{\delta})+U_{ut}(\hat{\delta})\frac{\partial W_{v,t+s}(\hat{\delta})}{\partial\hat{\delta}'}\right].\qquad (A9)$$

For the derivation of $\partial^2 L_o/\partial\theta\,\partial\dot{\alpha}'$, we have that calling $P_t(\hat{\delta})=[P_{1t}(\hat{\delta});\ldots;P_{Nt}(\hat{\delta})]$ the (p x N) matrix appearing in (A3), then

$$\frac{\partial L_o}{\partial\theta}=\sum_{t=1}^{T}P_t(\hat{\delta})\,K(\dot{\alpha})^{-1}U_t(\hat{\delta}),$$

and differentiating this expression with respect to $\dot{\alpha}$, we obtain

$$-\sum_{t=1}^{T}P_t(\hat{\delta})\,\dot{K}^{-1}(d\dot{K})\,\dot{K}^{-1}U_t(\hat{\delta})=-\sum_{t=1}^{T}\left(U_t(\hat{\delta})\otimes P_t(\hat{\delta})\right)vec(\dot{K}^{-1}(d\dot{K})\,\dot{K}^{-1})$$

$$=-\sum_{t=1}^{T}\left(U_t(\hat{\delta})\otimes P_t(\hat{\delta})\right)(\dot{K}^{-1}\otimes\dot{K}^{-1})\,D_m dv(\dot{K})$$

where $\dot{K}=K(\dot{\alpha})$, and therefore,

$$\frac{\partial^2 L_o}{\partial\theta\partial\dot{\alpha}'}=-\sum_{t=1}^{T}\left(U_t(\hat{\delta})\otimes P_t(\hat{\delta})\right)\left(K(\dot{\alpha})^{-1}\otimes K(\dot{\alpha})^{-1}\right)D_m.\qquad (A10)$$

Finally in order to complete the Hessian in (11) we still have to calculate some second derivatives with respect to $\dot{\delta}$ and $\dot{\alpha}$. From (A5)

$$\frac{\partial^2 L_o}{\partial\dot{\delta}\,\partial\dot{\delta}'} = \frac{\partial}{\partial\dot{\delta}'}\left(-\sum_{t=1}^{T}\frac{\partial W_t(\dot{\delta})}{\partial\dot{\delta}}\,K(\dot{\alpha})^{-1}\,U_t(\dot{\delta})\right)$$

$$= \frac{\partial}{\partial\dot{\delta}'}\left(-\sum_{t=1}^{T}\frac{\partial W_t(\dot{\delta})}{\partial\dot{\delta}}\,K(\dot{\alpha})^{-1}\,(\Phi(L)\;Y_t - W_t(\dot{\delta}))\right) =$$

$$\sum_{t=1}^{T}\left(\frac{\partial W_t(\dot{\delta})}{\partial\dot{\delta}}K(\dot{\alpha})^{-1}\frac{\partial W_t(\dot{\delta})}{\partial\dot{\delta}'}-\left(U_t(\dot{\delta})'K(\dot{\alpha})^{-1}\otimes I_k\right)\frac{\partial vec}{\partial\dot{\delta}'}\left(\frac{\partial W_t(\dot{\delta})}{\partial\dot{\delta}}\right)\right). \quad (A11)$$

Next we consider $\partial^2 L_o/\partial\dot{\delta}\,\partial\dot{\alpha}'$, and since

$$\frac{\partial L_o}{\partial\dot{\delta}} = -\sum_{t=1}^{T}\frac{\partial W_t(\dot{\delta})}{\partial\dot{\delta}}K(\dot{\alpha})^{-1}U_t(\dot{\delta})\,,$$

differentiating this expression with respect to $\dot{\alpha}$,

$$\sum_{t=1}^{T}\frac{\partial W_t(\dot{\delta})}{\partial\dot{\delta}}K(\dot{\alpha})^{-1}(dK(\dot{\alpha}))\,K(\dot{\alpha})^{-1}U_t(\dot{\delta})$$

$$= \sum_{t=1}^{T}\left(U_t(\dot{\delta})'\otimes\frac{\partial W_t(\dot{\delta})}{\partial\dot{\delta}}\right)vec\,[K(\dot{\alpha})^{-1}(dK(\dot{\alpha}))\,K(\dot{\alpha})^{-1}]$$

$$= \sum_{t=1}^{T}\left(U_t(\dot{\delta})'\otimes\frac{\partial W_t(\dot{\delta})}{\partial\dot{\delta}}\right)(K(\dot{\alpha})^{-1}\otimes K(\dot{\alpha})^{-1})\,D_m\;dv(K(\dot{\alpha}))\,,$$

and therefore,

$$\frac{\partial^2 L_o}{\partial\dot{\delta}\,\partial\dot{\alpha}'} = \sum_{t=1}^{T}\left(U_t(\dot{\delta})'\otimes\frac{\partial W_t(\dot{\delta})}{\partial\dot{\delta}}\right)(K(\dot{\alpha})^{-1}\otimes K(\dot{\alpha})^{-1})\,D_m. \quad (A12)$$

The final term in (11) that we should look at is $\partial^2 L_o/\partial\dot{\alpha}\,\partial\dot{\alpha}'$. Differentiating (A6) with respect to $\dot{\alpha}$, and recalling again $\dot{K} = K(\dot{\alpha})$, we have

$$-\frac{T}{2}tr[\dot{K}^{-1}(d\dot{K})\,\dot{K}^{-1}(d\dot{K})] + \frac{1}{2}tr[\dot{K}^{-1}(d\dot{K})\,\dot{K}^{-1}(d\dot{K})\,\dot{K}^{-1}S(\dot{\delta})]$$

$$+\;\frac{1}{2}tr\,[\dot{K}^{-1}(d\dot{K})\,\dot{K}^{-1}(d\dot{K})\,\dot{K}^{-1}S(\dot{\delta})]$$

$$= -\frac{T}{2}tr\,[(d\dot{K})\,\dot{K}^{-1}(d\dot{K})\,\dot{K}^{-1}] + tr\,[(d\dot{K})\,\dot{K}^{-1}(d\dot{K})\,\dot{K}^{-1}S(\dot{\delta})\,\dot{K}^{-1}]$$

$$=-\frac{T}{2}vec(d\dot{K})'(\dot{K}^{-1}\otimes\dot{K}^{-1})\,vec(d\dot{K}) + vec(d\dot{K})'(\dot{K}^{-1}S(\dot{\delta})\,\dot{K}^{-1}\otimes\dot{K}^{-1})\,vec(d\dot{K})$$

$$= -\frac{T}{2} dv(\dot{K})' D_m' (\dot{K}^{-1} \otimes \dot{K}^{-1}) D_m dv(\dot{K}) + dv(\dot{K})' D_m' (\dot{K}^{-1} S(\hat{\delta}) \dot{K}^{-1} \otimes \dot{K}^{-1}) D_m dv(\dot{K}),$$

obtaining as a final expression for $\partial^2 L_o / \partial \dot{\alpha} \partial \dot{\alpha}'$

$$-\frac{T}{2} D_m' (K(\dot{\alpha})^{-1} \otimes K(\dot{\alpha})^{-1}) D_m + D_m' (K(\dot{\alpha})^{-1} S(\hat{\delta}) K(\dot{\alpha})^{-1} \otimes K(\dot{\alpha})^{-1}) D_m .$$

We can get now consistent and efficient estimates of δ and α by equating (A5) and (A8) to zero; however, for practical purposes and in order to simplify the computations, we can take any $T^{1/2}$-consistent estimates of δ and α. We will assume that $\hat{\delta}$ is a consistent estimate of δ and we will take $\hat{K} = K(\hat{\alpha}) = T^{-1} S(\hat{\delta})$. It follows then from previous pages that

$$\frac{\partial L(0, \hat{\delta}, \hat{\alpha})}{\partial \theta} = \sum_{u=1}^{N} \sum_{s=1}^{T-1} \psi_s^{(u)} \sum_{t=1}^{T-s} \hat{U}_{ut}(\hat{\delta}) \sum_{v=1}^{N} \hat{\sigma}^{uv} \hat{U}_{v,t+s}(\hat{\delta}) =$$

$$T \sum_{u=1}^{N} \sum_{s=1}^{T-1} \sum_{v=1}^{N} \psi_s^{(u)} \hat{\sigma}^{uv} \frac{1}{T} \sum_{t=1}^{T-s} \hat{U}_{ut}(\hat{\delta}) \hat{U}_{v,t+s}(\hat{\delta}) =$$

$$T \sum_{u=1}^{N} \sum_{s=1}^{T-1} \sum_{v=1}^{N} \psi_s^{(u)} \hat{\sigma}^{uv} C_{uv}(s; \hat{\delta}) .$$

$$\text{Also, } E\left(\frac{\partial^2 L(0, \hat{\delta}, \hat{\alpha})}{\partial \theta \partial \theta'} \right) = T \sum_{u=1}^{N} \sum_{v=1}^{N} \hat{\sigma}^{uv} \hat{\sigma}_{uv} \sum_{s=1}^{T-1} \left(1 - \frac{s}{T}\right) \psi_s^{(u)} \psi_s^{(v)\prime} = T \hat{A}^t,$$

and the asymptotic expectation matrix in (11) multiplied by 1/T will take the form

$$\begin{pmatrix} \bar{A} & 0 & 0 \\ 0 & W & 0 \\ 0 & 0 & \frac{1}{2} D_m' (K^{-1} \otimes K^{-1}) D_m \end{pmatrix} \qquad (A13)$$

where $\bar{A} = \sum_{u=1}^{N} \sum_{v=1}^{N} \sigma^{uv} \sigma_{uv} \sum_{s=1}^{\infty} \psi_s^{(u)} \psi_s^{(v)\prime}$, and

$$W = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \left(\frac{\partial W_t(\delta)}{\partial \delta} K^{-1} \frac{\partial W_t(\delta)}{\partial \delta'} \right)$$

is a positive definite matrix by assumption. (Note that the block diagonality in (A13) follows from expressions (A9), (A10) and (A12), given that $\hat{\sigma}^{uv}$ consistently estimates σ^{uv} and $\hat{U}_t(\delta)$ has zero expectation).

APPENDIX 5.2: DERIVATION OF THE SCORE STATISTIC $\tilde{S}$

For the derivation of the score test statistic in this context of weak parametric autocorrelation in U_t, we assume that k and K in (20) are parameterized separately, so τ is taken to specify k and α to specify K. Thus, the spectral density matrix of $U_t(\theta;\delta)$ for any admissible δ and τ is

$$f(\lambda;\dot{\alpha};\dot{\tau}) = \frac{1}{2\pi} k(\lambda;\dot{\tau})\, K(\dot{\alpha})\, k(\lambda;\dot{\tau})^* \qquad (B1)$$

$$\text{where} \quad k(\lambda;\dot{\tau}) = \sum_{j=0}^{\infty} A(j;\dot{\tau}) e^{i\lambda j}.$$

It is also assumed that $A(0;\tau) = I_N$ (the N-rowed identity matrix) for any τ in Euclidean space R^q, and that $f(\lambda;\alpha;\tau)$ is a finite, positive matrix, with eigenvalues bounded and bounded away from zero at any frequency on a neighborhood N^* of τ and M^* of α. Also, we assume that each element of $\hat{f}(\lambda;\tau)$, $\hat{f}_{uv}(\lambda;\tau)$, as defined below (B4), must be continuous in (λ,τ) for $\tau \in N^*$ and have first and second derivatives with respect to τ continuous in (λ,τ) for $\tau \in N^*$.

Taking now $\eta = (\theta';\alpha';\delta';\tau')'$, the negative of the log-likelihood based on Gaussianity of U_t can be expressed as

$$l(\dot{\eta}) = \frac{1}{2}\log\det J(\dot{\alpha};\dot{\tau}) + \frac{1}{2} U(\theta,\dot{\delta})' J^{-1}(\dot{\alpha};\dot{\tau})\, U(\theta,\dot{\delta}), \qquad (B2)$$

where $U(\theta,\delta) = (U_1(\theta,\delta); U_2(\theta,\delta);\ldots;U_T(\theta,\delta))'$, and $J(\alpha,\tau)$ is a (NT x NT) matrix with $J_{s-t}(\alpha,\tau) = \int_{-\pi}^{\pi} e^{i(s-t)\lambda} f(\lambda;\dot{\alpha};\dot{\tau})d\lambda$ in the (t,s) block of N^2 elements, for any admissible α, δ and τ. However, given the computational difficulty of this expression, especially when N and T are large, under suitable conditions, (B2) can be approximated by

$$L(\theta;\dot{\alpha};\dot{\delta};\dot{\tau}) = \frac{T}{2}\log\det f(\lambda_r;\dot{\alpha};\dot{\tau}) + \frac{1}{2}\sum_r^{*} \mathrm{tr}[f^{-1}(\lambda_r;\dot{\alpha};\dot{\tau}) I_U(\lambda_r;\theta;\dot{\delta})], \qquad (B3)$$

where $I_U(\lambda_r;\theta;\delta)$ is the periodogram of $U_t(\theta;\delta)$ evaluated at frequencies $\lambda_r = 2\pi r/T$ and the sum on * is as described in page 182.

Calling now $\hat{\delta}$ any $T^{1/2}$-consistent estimate of δ and $\hat{\alpha}$ as defined in Appendix 5.1, we can concentrate both out and consider

$$\hat{L}(\theta;\dot{\tau}) = L(\theta;\hat{\alpha};\hat{\delta};\dot{\tau}) = \frac{T}{2}\log\det \hat{f}(\lambda_r;\dot{\tau}) + \frac{1}{2}\sum_r^{*} \mathrm{tr}[\hat{f}^{-1}(\lambda_r;\dot{\tau})\hat{I}_U(\lambda_r;\theta)], \qquad (B4)$$

where

$$\hat{f}(\lambda_r;\dot{\tau}) = \frac{1}{2\pi} k(\lambda_r;\dot{\tau}) K(\hat{\alpha}) k(\lambda_r;\dot{\tau})^*,$$

and

$$\hat{I}_U(\lambda_r;\theta) = \hat{W}(\lambda_r;\theta)\overline{\hat{W}(\lambda_r;\theta)}, \quad with \quad \hat{W}(\lambda_r;\theta) = \frac{1}{\sqrt{2\pi T}}\sum_{t=1}^{T} U_t(\theta;\hat{\delta})e^{i\lambda_r t}.$$

Then we can express a score test statistic as:

$$\frac{\partial\hat{L}(\theta;\dot{\tau})}{\partial\theta'}\left[E\left(\frac{\partial^2\hat{L}(\theta;\dot{\tau})}{\partial\theta\partial\theta'}\right)-E\left(\frac{\partial^2\hat{L}(\theta;\dot{\tau})}{\partial\theta\partial\dot{\tau}'}\right)\left(E\left(\frac{\partial^2\hat{L}(\theta;\dot{\tau})}{\partial\dot{\tau}\partial\dot{\tau}'}\right)\right)^{-1}E\left(\frac{\partial^2\hat{L}(\theta;\dot{\tau})}{\partial\dot{\tau}\partial\theta'}\right)\right]^{-1}\frac{\partial\hat{L}(\theta;\dot{\tau})}{\partial\theta}\Big|_{\theta=0,\dot{\tau}=\tilde{\tau}}, \quad (B5)$$

where the expectation is taken under the null hypothesis (4) prior to substitution of $\tilde{\tau}$, where $\tilde{\tau}$ can be any consistent estimate of τ under (4).

We start with $\partial\hat{L}(\theta;\dot{\tau})/\partial\theta$, and from (B4), we see that it is

$$\frac{\partial}{\partial\theta}\left(\frac{1}{2}\sum_r^{*} tr[\hat{f}^{-1}(\lambda_r;\dot{\tau})\hat{I}_U(\lambda_r;\theta)]\right)$$

$$= \frac{1}{2}\sum_r^{*}\left(\frac{\partial}{\partial\theta}vec'(\hat{I}_U(\lambda_r;\theta))\right)vec(\hat{f}^{-1}(\lambda_r;\dot{\tau})')$$

$$= \frac{1}{2}\sum_r^{*}\sum_{u=1}^{N}\sum_{v=1}^{N}\frac{\partial\hat{I}_{uv}(\lambda_r;\theta)}{\partial\theta}\hat{f}^{vu}(\lambda_r;\dot{\tau}), \quad (B6)$$

where $\hat{I}_{uv}(\lambda_r;\theta)$ is the $(u,v)^{th}$ element of $\hat{I}_U(\lambda_r;\theta)$, and $\hat{f}^{uv}(\lambda_r;\tau)$ is the $(u,v)^{th}$ element of $\hat{f}^{-1}(\lambda_r;\tau)$. We first concentrate on

$$\frac{\partial\hat{I}_{uv}(\lambda_r;\theta)}{\partial\theta}\Big|_{\theta=0} = \frac{\partial}{\partial\theta}\left(\frac{1}{2\pi T}\sum_{t=1}^{T}\sum_{s=1}^{T}U_{u,t}(\theta;\hat{\delta})U_{v,s}(\theta;\hat{\delta})e^{i(t-s)\lambda_r}\right)\Big|_{\theta=0}$$

$$= \frac{1}{2\pi T}\sum_{t=1}^{T}\sum_{s=1}^{T}\left(\frac{\partial\log\rho_u(L;\theta)}{\partial\theta}U_{u,t}(\theta;\hat{\delta})\right)U_{v,s}(\theta;\hat{\delta})e^{i(t-s)\lambda_r}\Big|_{\theta=0} +$$

$$\frac{1}{2\pi T}\sum_{t=1}^{T}\sum_{s=1}^{T}U_{u,t}(\theta;\hat{\delta})\left(\frac{\partial\log\rho_v(L;\theta)}{\partial\theta}U_{v,s}(\theta;\hat{\delta})\right)e^{i(t-s)\lambda_r}\Big|_{\theta=0}$$

$$= \frac{1}{2\pi T}\sum_{t=1}^{T}\sum_{s=1}^{T}\sum_{m=1}^{\infty}\psi_m^{(u)}U_{u,t-m}(\hat{\delta})U_{v,s}(\hat{\delta})e^{i(t-s)\lambda_r} + \frac{1}{2\pi T}\sum_{t=1}^{T}\sum_{s=1}^{T}U_{u,t}(\hat{\delta})\sum_{m=1}^{\infty}\psi_m^{(v)}U_{v,s-m}(\hat{\delta})e^{i(t-s)\lambda_r}$$

$$=\sum_{m=1}^{T-1} \psi_m^{(u)} e^{i\lambda_r m} \frac{1}{2\pi T} \sum_{t=1}^{T-m} \sum_{s=1}^{T} U_{ut}(\hat{\delta}) U_{vs}(\hat{\delta}) e^{i\lambda_r(t-s)} + \sum_{m=1}^{T-1} \psi_m^{(v)} e^{-i\lambda_r m} \frac{1}{2\pi T} \sum_{t=1}^{T} \sum_{s=1}^{T-m} U_{ut}(\hat{\delta}) U_{vs}(\hat{\delta}) e^{i\lambda_r(t-s)}$$

and, under suitable conditions, (with m=1,2,...,M < T-1, for sufficiently large M), this expression becomes asymptotically

$$\left(\epsilon_{(u)}(\lambda_r) + \bar{\epsilon}_{(v)}(\lambda_r)\right) I_{uv}(\lambda_r;\hat{\delta}). \qquad (B7)$$

Substituting now (B7) in (B6) we obtain that $\partial \hat{L}(\theta;\dot{\tau}) / \partial\theta \,|_{\theta=0}$ is asymptotically

$$\frac{1}{2}\sum_{r}^{*} \sum_{u=1}^{N} \sum_{v=1}^{N} \left(\epsilon_{(u)}(\lambda_r) + \bar{\epsilon}_{(v)}(\lambda_r)\right) I_{uv}(\lambda_r;\hat{\delta}) \hat{f}^{vu}(\lambda_r;\dot{\tau}). \qquad (B8)$$

We next examine the second derivative matrices appearing in (B5), and we start with $\partial^2 \hat{L}(\theta;\dot{\tau}) / \partial\theta\,\partial\theta'$.

$$\frac{\partial^2 \hat{L}(\theta;\dot{\tau})}{\partial\theta\partial\theta'} = \frac{1}{2}\sum_{r}^{*} \sum_{u=1}^{N} \sum_{v=1}^{N} \left(\epsilon_{(u)}(\lambda_r) + \bar{\epsilon}_{(v)}(\lambda_r)\right) \frac{\partial \hat{I}_{uv}(\lambda_r;\theta)}{\partial\theta'} \hat{f}^{vu}(\lambda_r;\dot{\tau})$$

and using again (B7), this last expression evaluated at $\theta = 0$, becomes for large T

$$\frac{1}{2}\sum_{r}^{*} \sum_{u=1}^{N} \sum_{v=1}^{N} \left(\epsilon_{(u)}(\lambda_r) + \bar{\epsilon}_{(v)}(\lambda_r)\right) \left(\epsilon_{(u)}(\lambda_r)' + \bar{\epsilon}_{(v)}(\lambda_r)'\right) I_{uv}(\lambda_r;\hat{\delta}) \hat{f}^{vu}(\lambda_r;\dot{\tau}),$$

whose asymptotic expectation is

$$\frac{1}{2}\sum_{r}^{*} \sum_{u=1}^{N} \sum_{v=1}^{N} \left(\epsilon_{(u)}(\lambda_r) + \bar{\epsilon}_{(v)}(\lambda_r)\right) \left(\epsilon_{(u)}(\lambda_r)' + \bar{\epsilon}_{(v)}(\lambda_r)'\right) \hat{f}_{uv}(\lambda_r;\dot{\tau}) \hat{f}^{vu}(\lambda_r;\dot{\tau}),$$

given that, heuristically, if $f(\lambda;\tau)$ is continuous in λ, $E(I_{uv}(\lambda)) \rightarrow_{T\rightarrow\infty} f_{uv}(\lambda;\tau)$, for fixed λ. (See Brillinger (1981)). We can write this last expression as

$$\frac{1}{2}\sum_{r}^{*} \sum_{u=1}^{N} \sum_{v=1}^{N} \left(\epsilon_{(u)}(\lambda_r)\epsilon_{(u)}(\lambda_r)' + \epsilon_{(u)}(\lambda_r)\bar{\epsilon}_{(v)}(\lambda_r)' + \bar{\epsilon}_{(v)}(\lambda_r)\epsilon_{(u)}(\lambda_r)' + \bar{\epsilon}_{(v)}(\lambda_r)\bar{\epsilon}_{(v)}(\lambda_r)'\right) \, x$$

$$x \; \hat{f}_{uv}(\lambda_r;\dot{\tau}) \hat{f}^{vu}(\lambda_r;\dot{\tau}) = \frac{1}{2}\sum_{r}^{*} \sum_{u=1}^{N} \epsilon_{(u)}(\lambda_r)\epsilon_{(u)}(\lambda_r)' \sum_{v=1}^{N} \hat{f}_{uv}(\lambda_r;\dot{\tau}) \hat{f}^{vu}(\lambda_r;\dot{\tau})$$

$$+ \frac{1}{2}\sum_{r}^{*} \sum_{v=1}^{N} \bar{\epsilon}_{(v)}(\lambda_r)\, \bar{\epsilon}_{(v)}(\lambda_r)' \sum_{u=1}^{N} \hat{f}_{uv}(\lambda_r;\dot{\tau}) \hat{f}^{vu}(\lambda_r;\dot{\tau})$$

$$+ \frac{1}{2}\sum_{r}^{*} \sum_{u=1}^{N} \sum_{v=1}^{N} \left(\epsilon_{(u)}(\lambda_r)\bar{\epsilon}_{(v)}(\lambda_r)' + \bar{\epsilon}_{(v)}(\lambda_r)\epsilon_{(u)}(\lambda_r)'\right) \hat{f}_{uv}(\lambda_r;\dot{\tau}) \hat{f}^{vu}(\lambda_r;\dot{\tau}), \qquad (B9)$$

which first two summands will be approximately zero noting that

$$\sum_{v=1}^{N} \hat{f}_{uv}(\lambda_r;\dot{\tau})\hat{f}^{vu}(\lambda_r;\dot{\tau}) = \sum_{u=1}^{N} \hat{f}_{uv}(\lambda_r;\dot{\tau})\hat{f}^{vu}(\lambda_r;\dot{\tau}) = 1 \quad and$$

$$\sum_{r}^{*} \epsilon_{(u)}(\lambda_r)\epsilon_{(u)}(\lambda_r)' = \sum_{r}^{*} \bar{\epsilon}_{(v)}(\lambda_r)\,\bar{\epsilon}_{(v)}(\lambda_r)' \simeq 0, \qquad (B10)$$

for all u,v = 1,2,..,N. To see this last result, note that approximating the sum by an integral

$$\int_{-\pi}^{\pi} \epsilon_{(u)}(\lambda)\epsilon_{(u)}(\lambda)'d\lambda = \int_{-\pi}^{\pi}\sum_{s=1}^{\infty} \psi_s^{(u)} e^{i\lambda s}\sum_{m=1}^{\infty} \psi_m^{(u)'} e^{i\lambda m}d\lambda$$

$$= \int_{-\pi}^{\pi}\sum_{s=1}^{\infty} \psi_s^{(u)}(\cos\lambda s + i\sin\lambda s)\sum_{m=1}^{\infty} \psi_m^{(u)'}(\cos\lambda m + i\sin\lambda m)d\lambda$$

$$= \sum_{s=1}^{\infty}\sum_{m=1}^{\infty} \psi_s^{(u)}\psi_m^{(u)'}\left(\int_{-\pi}^{\pi}\cos\lambda s\cos\lambda m\,d\lambda - \int_{-\pi}^{\pi}\sin\lambda s\sin\lambda m\,d\lambda\right) = 0,$$

and identically for the second term in (B10).

Now we look at the (p x q) matrix $\partial^2\hat{L}(\theta;\tau)/\partial\theta\partial\tau'$ in (B5) which, evaluated at $\theta = 0$, is

$$\frac{\partial}{\partial\dot{\tau}'}\left(\frac{1}{2}\sum_{r}^{*}\sum_{u=1}^{N}\sum_{v=1}^{N}(\epsilon_{(u)}(\lambda_r) + \bar{\epsilon}_{(v)}(\lambda_r))I_{uv}(\lambda_r;\hat{\delta})\hat{f}^{vu}(\lambda_r;\dot{\tau})\right)$$

$$= \frac{1}{2}\sum_{r}^{*}\sum_{u=1}^{N}\sum_{v=1}^{N}(\epsilon_{(u)}(\lambda_r) + \bar{\epsilon}_{(v)}(\lambda_r))I_{uv}(\lambda_r;\hat{\delta})\frac{\partial\hat{f}^{vu}(\lambda_r;\dot{\tau})}{\partial\dot{\tau}'},$$

and whose expectation for large T is

$$\frac{1}{2}\sum_{r}^{*}\sum_{u=1}^{N}\sum_{v=1}^{N}(\epsilon_{(u)}(\lambda_r) + \bar{\epsilon}_{(v)}(\lambda_r))\hat{f}_{uv}(\lambda_r;\dot{\tau})\frac{\partial\hat{f}^{vu}(\lambda_r;\dot{\tau})}{\partial\dot{\tau}'}. \qquad (B11)$$

This last expression can also be shown in terms of the derivatives of $\hat{f}$ with respect to τ (instead of the derivatives of its inverse, $\hat{f}^{-1}$). (B11) can be expressed as

$$\frac{1}{2}\sum_{r}^{*}\sum_{u=1}^{N} \epsilon_{(u)}(\lambda_r) \sum_{v=1}^{N} \hat{f}_{uv}(\lambda_r;\dot{\tau}) \frac{\partial \hat{f}^{vu}(\lambda_r;\dot{\tau})}{\partial \dot{\tau}'} \tag{B12}$$

$$+ \frac{1}{2}\sum_{r}^{*}\sum_{v=1}^{N} \bar{\epsilon}_{(v)}(\lambda_r)\sum_{u=1}^{N} \hat{f}_{uv}(\lambda_r;\dot{\tau}) \frac{\partial \hat{f}^{vu}(\lambda_r;\dot{\tau})}{\partial \dot{\tau}'}, \tag{B13}$$

where $\dfrac{\partial \hat{f}^{vu}(\lambda_r;\dot{\tau})}{\partial \dot{\tau}'} = \left(\dfrac{\partial \hat{f}^{vu}(\lambda_r;\dot{\tau})}{\partial \dot{\tau}_1};\ldots;\dfrac{\partial \hat{f}^{vu}(\lambda_r;\dot{\tau})}{\partial \dot{\tau}_q}\right).$

Now using the relationship

$$\frac{\partial \hat{f}^{-1}(\lambda_r;\dot{\tau})}{\partial \dot{\tau}_i} = -\hat{f}^{-1}(\lambda_r;\dot{\tau}) \frac{\partial \hat{f}(\lambda_r;\dot{\tau})}{\partial \dot{\tau}_i} \hat{f}^{-1}(\lambda_r;\dot{\tau}),$$

we have that

$$\hat{f}(\lambda_r;\dot{\tau}) \frac{\partial \hat{f}^{-1}(\lambda_r;\dot{\tau})}{\partial \dot{\tau}_i} = -\frac{\partial \hat{f}(\lambda_r;\dot{\tau})}{\partial \dot{\tau}_i} \hat{f}^{-1}(\lambda_r;\dot{\tau})$$

and

$$\frac{\partial \hat{f}^{-1}(\lambda_r;\dot{\tau})}{\partial \dot{\tau}_i} \hat{f}(\lambda_r;\dot{\tau}) = -\hat{f}^{-1}(\lambda_r;\dot{\tau}) \frac{\partial \hat{f}(\lambda_r;\dot{\tau})}{\partial \dot{\tau}_i},$$

implying this two equalities that

$$\sum_{v=1}^{N} \hat{f}_{uv}(\lambda_r;\dot{\tau}) \frac{\partial \hat{f}^{vu}(\lambda_r;\dot{\tau})}{\partial \dot{\tau}_i} = -\sum_{v=1}^{N} \frac{\partial \hat{f}_{uv}(\lambda_r;\dot{\tau})}{\partial \dot{\tau}_i} \hat{f}^{vu}(\lambda_r;\dot{\tau}) \tag{B14}$$

and

$$\sum_{u=1}^{N} \frac{\partial \hat{f}^{vu}(\lambda_r;\dot{\tau})}{\partial \dot{\tau}_i} \hat{f}_{uv}(\lambda_r;\dot{\tau}) = -\sum_{u=1}^{N} \hat{f}^{vu}(\lambda_r;\dot{\tau}) \frac{\partial \hat{f}_{uv}(\lambda_r;\dot{\tau})}{\partial \dot{\tau}_i} \tag{B15}$$

respectively. Substituting now (B14) in (B12) and (B15) in (B13), (B11) becomes

$$-\frac{1}{2}\sum_{r}^{*}\sum_{u=1}^{N} \epsilon_{(u)}(\lambda_r) \sum_{v=1}^{N} \hat{f}^{vu}(\lambda_r;\dot{\tau}) \frac{\partial \hat{f}_{uv}(\lambda_r;\dot{\tau})}{\partial \dot{\tau}'}$$

$$-\frac{1}{2}\sum_{r}^{*}\sum_{v=1}^{N} \bar{\epsilon}_{(v)}(\lambda_r)\sum_{u=1}^{N} \hat{f}^{vu}(\lambda_r;\dot{\tau}) \frac{\partial \hat{f}_{uv}(\lambda_r;\dot{\tau})}{\partial \dot{\tau}'}$$

$$= -\frac{1}{2}\sum_{r}^{*}\sum_{u=1}^{N}\sum_{v=1}^{N} \left(\epsilon_{(u)}(\lambda_r) + \bar{\epsilon}_{(v)}(\lambda_r)\right)\hat{f}^{vu}(\lambda_r;\dot{\tau}) \frac{\partial \hat{f}_{uv}(\lambda_r;\dot{\tau})}{\partial \dot{\tau}'}. \tag{B16}$$

Finally, we look at the (q x q) matrix $\partial^2\hat{L}(\theta;\dot{\tau})/\partial\dot{\tau}\partial\dot{\tau}'$. The u[th] element of $\partial\hat{L}(\theta;\dot{\tau})/\partial\dot{\tau}$ is

$$\frac{\partial\hat{L}(\theta;\dot{\tau})}{\partial\dot{\tau}_u} = \frac{1}{2}\sum_r^* tr\left(\hat{f}^{-1}(\lambda_r;\dot{\tau})\frac{\partial\hat{f}(\lambda_r;\dot{\tau})}{\partial\dot{\tau}_u}\right) + \frac{1}{2}\sum_r^* tr\left(\frac{\partial\hat{f}^{-1}(\lambda_r;\dot{\tau})}{\partial\dot{\tau}_u}\hat{I}_U(\lambda_r;\theta)\right) =$$

$$= \frac{1}{2}\sum_r^* tr\left(\hat{f}^{-1}(\lambda_r;\dot{\tau})\frac{\partial\hat{f}(\lambda_r;\dot{\tau})}{\partial\dot{\tau}_u} + \frac{\partial\hat{f}^{-1}(\lambda_r;\dot{\tau})}{\partial\dot{\tau}_u}\hat{I}_U(\lambda_r;\theta)\right)$$

$$= \frac{1}{2}\sum_r^* tr\left(\hat{f}^{-1}(\lambda_r;\dot{\tau})\frac{\partial\hat{f}(\lambda_r;\dot{\tau})}{\partial\dot{\tau}_u} - \hat{f}(\lambda_r;\dot{\tau})^{-1}\frac{\partial\hat{f}(\lambda_r;\dot{\tau})}{\partial\dot{\tau}_u}\hat{f}^{-1}(\lambda_r;\dot{\tau})\hat{I}_U(\lambda_r;\theta)\right).$$

Then, $\dfrac{\partial\hat{L}(\theta;\dot{\tau})}{\partial\dot{\tau}_u\partial\dot{\tau}_v}$, *evaluated at* $\theta = 0$, *becomes:*

$$\frac{1}{2}\sum_r^* tr\Big(\frac{\partial\hat{f}^{-1}(\lambda_r;\dot{\tau})}{\partial\dot{\tau}_v}\frac{\partial\hat{f}(\lambda_r;\dot{\tau})}{\partial\dot{\tau}_u} + \hat{f}^{-1}(\lambda_r;\dot{\tau})\frac{\partial^2\hat{f}(\lambda_r;\dot{\tau})}{\partial\dot{\tau}_u\partial\dot{\tau}_v} - \frac{\partial\hat{f}^{-1}(\lambda_r;\dot{\tau})}{\partial\dot{\tau}_v}\frac{\partial\hat{f}(\lambda_r;\dot{\tau})}{\partial\dot{\tau}_u}\hat{f}^{-1}(\lambda_r;\dot{\tau})I_U(\lambda_r;\hat{\delta})$$

$$- \hat{f}^{-1}(\lambda_r;\dot{\tau})\frac{\partial^2\hat{f}(\lambda_r;\dot{\tau})}{\partial\dot{\tau}_u\partial\dot{\tau}_v}\hat{f}^{-1}(\lambda_r;\dot{\tau})I_U(\lambda_r;\hat{\delta}) - \hat{f}^{-1}(\lambda_r;\dot{\tau})\frac{\partial\hat{f}(\lambda_r;\dot{\tau})}{\partial\dot{\tau}_u}\frac{\partial\hat{f}^{-1}(\lambda_r;\dot{\tau})}{\partial\dot{\tau}_v}I_U(\lambda_r;\hat{\delta})\Big) =$$

$$\frac{1}{2}\sum_r^* tr\Big(-\hat{f}^{-1}(\lambda_r;\dot{\tau})\frac{\partial\hat{f}(\lambda_r;\dot{\tau})}{\partial\dot{\tau}_v}\hat{f}^{-1}(\lambda_r;\dot{\tau})\frac{\partial\hat{f}(\lambda_r;\dot{\tau})}{\partial\dot{\tau}_u} + \hat{f}^{-1}(\lambda_r;\dot{\tau})\frac{\partial^2\hat{f}(\lambda_r;\dot{\tau})}{\partial\dot{\tau}_u\partial\dot{\tau}_v} + \hat{f}^{-1}(\lambda_r;\dot{\tau})\ x$$

$$x\ \frac{\partial\hat{f}(\lambda_r;\dot{\tau})}{\partial\dot{\tau}_v}\frac{\partial\hat{f}(\lambda_r;\dot{\tau})}{\partial\dot{\tau}_v}\hat{f}^{-1}(\lambda_r;\dot{\tau})I_U(\lambda_r;\hat{\delta}) - \hat{f}^{-1}(\lambda_r;\dot{\tau})\frac{\partial^2\hat{f}(\lambda_r;\dot{\tau})}{\partial\dot{\tau}_u\partial\dot{\tau}_v}\hat{f}^{-1}(\lambda_r;\dot{\tau})\ I_U(\lambda_r;\hat{\delta}) +$$

$$\hat{f}^{-1}(\lambda_r;\dot{\tau})\frac{\partial\hat{f}(\lambda_r;\dot{\tau})}{\partial\dot{\tau}_u}\hat{f}^{-1}(\lambda_r;\dot{\tau})\frac{\partial\hat{f}(\lambda_r;\dot{\tau})}{\partial\dot{\tau}_v}\hat{f}^{-1}(\lambda_r;\dot{\tau})I_U(\lambda_r;\hat{\delta})\ \Big),$$

and whose asymptotic expectation is

$$\frac{1}{2}\sum_r^* tr\left(\hat{f}^{-1}(\lambda_r;\dot{\tau})\frac{\partial\hat{f}(\lambda_r;\dot{\tau})}{\partial\dot{\tau}_u}\hat{f}^{-1}(\lambda_r;\dot{\tau})\frac{\partial\hat{f}(\lambda_r;\dot{\tau})}{\partial\dot{\tau}_v}\right). \qquad (B17)$$

Substituting now (B8), (B9), (B11) and (B17) in (B5), evaluated at $\dot{\tau} = \tilde{\tau}$, we form (21).

APPENDIX 5.3

In this appendix we describe the Fortran program used to calculate the multivariate score statistics given in (32), (34) and (37). This program was used in the simulations carried out in Section 7 above, and will also be used in Chapter 6 where a number of empirical applications will be performed. If the null hypothesized model is

$$\begin{pmatrix} Y_{1t} \\ Y_{2t} \end{pmatrix} = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \begin{pmatrix} 1 \\ t \end{pmatrix} + \begin{pmatrix} X_{1t} \\ X_{2t} \end{pmatrix} \qquad t = 1,2,\ldots,T$$

$$\begin{pmatrix} (1-L)^{d_1} & 0 \\ 0 & (1-L)^{d_2} \end{pmatrix} \begin{pmatrix} X_{1t} \\ X_{2t} \end{pmatrix} = \begin{pmatrix} U_{1t} \\ U_{2t} \end{pmatrix} \qquad t = 1,2,\ldots,T$$

$$X_t = (X_{1t}, X_{2t})' = 0 \qquad t \leq 0$$

$$\begin{pmatrix} U_{1t} \\ U_{2t} \end{pmatrix} \sim I(0),$$

the test statistic will be given by: **TST(I,K1,K2,J)**, where

I = 1,2,3 and

I = 1 means that $B_{ij} = 0$ for i,j = 1,2,

I = 2 means that $B_{i2} = 0$ for i = 1,2, and

I = 3 means that B_{ij} is unknown for i = 1,2.

K1 and K2 = 1,...,ND, where ND can be any integer number, and they correspond to d_1 and d_2 above according to the relationship: d_i = Ki/10, i=1,2, and finally,

J = 1,2,3,4 where

J = 1 means that U_t is a VAR(1) process,

J = 2 means that U_t is a VMA(1) process,

J = 3 means that U_t is a white noise process, (and the test statistic is calculated in the frequency domain), and

J = 4 means that U_t is a white noise process, (and the test statistic is calculated in the time domain).

PROGRAM APPENDIX 5.3

```
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      PARAMETER(NV=2,NV1=NV+1,N=***,N1=N-1,ND=2,NPAR=10,ICM=NPAR,
     + LWK=3000,LIW=100,NQ=4,NQ1=NQ+1,NIRE=4)
      DIMENSION XL(0:N1),XRE(N1),XIM(N1),CO(0:N1),SI(0:N1),Y(4,N),VV(ND,N), XME(3,ND,N),
     + W(ND,N),UD(NV1,N),S(ND,N),X(4,ND,N),U(3,NV,ND,N),PRE(NV,NV,N1),PIM(NV,NV,N1),
     + PAR(NPAR),QQ(NV1,NV),V(NV1,N),G(NPAR),CM(ICM,NPAR),IW(LIW),WK(LWK),PARM(NPAR),
     + QQM(NV1,NV), VM(NV1,N),GM(NPAR),CMM(ICM,NPAR),IWM(LIW),WKM(LWK),QQ1(NV,NV),
     + P7(NV),FRED(NV,NV,N1,NQ),FIMD(NV,NV,N1,NQ),FRE(NV,NV,N1),BA(NV),FIM(NV,NV,N1),
     + FRE1(NV,NV,N1),FIM1(NV,NV,N1,E(NQ,NQ,N1),EXE(NQ1,NQ),EXE1(NQ,NQ),P3(NQ),XC(NV,NV),
     + DK(NV,NQ), XXB(NV1,NV),XXB1(NV,NV),Z1(NV,NV,N1),Z2(NV,NV,N1),P4(NV),UM(NV),
     + XKV(N1,NV),XKV1(NV,NV),XAF(NV),XAA(NV1,NV),Q(NV),XAA1(NV,NV),XAT(NV),
     + XXC(NV,NV,N1),TST(3,ND,ND,NIRE)

      LOGICAL MEAN,PLD(NPAR),EXACT
      XN=N
      PI=3.141592654
      PI2=2.*PI
      PIC2=PI**2.
      ZPI=1./PI2
      OPEN(100,FILE='DATA1.DAT',STATUS='OLD')
      OPEN(101,FILE='DATA2.DAT',STATUS='OLD')
      XL(0)=0.
      DO 1 I=1,N
        XI=I
        XL(I)=2.*PI*XI/XN
        READ(100,102) Y(1,I)
        READ(101,103) Y(2,I)
        Y(3,I)=1.
        Y(4,I)=I
1     CONTINUE
102   FORMAT(F**.**)
103   FORMAT(F**.**)
      DO 2 I=1,N1
        XRE(I)=LOG(ABS(2.*SIN(XL(I)/2.)))
        XIM(I)=(XL(I)-PI)/2.
2     CONTINUE
      DO 3 J=0,N1
        CO(J)=COS(XL(J))
        SI(J)=SIN(XL(J))
3     CONTINUE
      DO 4 K1=1,ND
        XK1=K1
        D1=XK1/10.
        VV(K1,1)=D1
          DO 5 J=2,N
            VV(K1,J)=((J-D1-1)/J)*VV(K1,J-1)
5         CONTINUE
          DO 6 J=1,N
            W(K1,J)=(-1)*VV(K1,J)
6         CONTINUE
          DO 7 I=1,4
            DO 8 J=2,N
              S(K1,J)=0.
                DO 9 M=1,J-1
                S(K1,J)=S(K1,J)+W(K1,M)*Y(I,J-M)
9               CONTINUE
8             CONTINUE
          X(I,K1,1)=Y(I,1)
        DO 7 J=2,N
          X(I,K1,J)=S(K1,J)+Y(I,J)
7       CONTINUE
        DO 10 M=1,3
          XUM=0.
            DO 11 J=1,N
              XUM=XUM+(1./XN)*X(M,K1,J)
11          CONTINUE
          DO 12 J=1,N
            XME(M,K1,J)=X(M,K1,J)-XUM
12        CONTINUE
10      CONTINUE
        A1=0.
        A2=0.
        A3=0.
        A4=0.
        A5=0.
        A6=0.
        A7=0.
        XNU1=0.
        XNU2=0.
        XDE=0.
        DO 13 J=1,N
          A1=A1+X(3,K1,J)**2.
          A2=A2+X(3,K1,J)*X(4,K1,J)
          A3=A3+X(4,K1,J)**2.
          A4=A4+X(3,K1,J)*X(1,K1,J)
          A5=A5+X(4,K1,J)*X(1,K1,J)
          A6=A6+X(3,K1,J)*X(2,K1,J)
          A7=A7+X(4,K1,J)*X(2,K1,J)
          XNU1=XNU1+XME(3,K1,J)*XME(1,K1,J)
          XDE=XDE+XME(3,K1,J)**2.
          XNU2=XNU2+XME(3,K1,J)*XME(2,K1,J)
13      CONTINUE
        B1=(A3*A4-A2*A5)/(A1*A3-A2**2.)
        B2=(A1*A5-A2*A4)/(A1*A3-A2**2.)
        B3=(A3*A6-A2*A7)/(A1*A3-A2**2.)
        B4=(A1*A7-A2*A6)/(A1*A3-A2**2.)
        C1=XNU1/XDE
        C2=XNU2/XDE
        DO 14 J=1,N
          U(1,1,K1,J)=X(1,K1,J)
          U(1,2,k1,J)=X(2,K1,J)
          U(2,1,K1,J)=X(1,K1,J)-C1*X(3,K1,J)
          U(2,2,K1,J)=X(2,K1,J)-C2*X(3,K1,J)
          U(3,1,K1,J)=X(1,K1,J)-B1*X(3,K1,J)-B2*X(4,K1,J)
          U(3,2,K1,J)=X(2,K1,J)-B3*X(3,K1,J)-B4*X(4,K1,J)
14      CONTINUE
4     CONTINUE
      DO 1000 I=1,3
      DO 1000 K1=1,ND
      DO 1000 K2=1,ND
```

```
      DO 15 J=1,N
        UD(1,J)=U(I,1,K1,J)
        UD(2,J)=U(I,2,K2,J)
15     CONTINUE
      DO 16 M1=1,NV
      DO 16 M2=1,NV
      DO 16 J=1,N1
        AA1=0.
        BB1=0.
          DO 17 IT=1,N
          DO 17 IS=1,N
            SIGN=1.
            IF((IT-IS).LT.0) THEN
            SIGN=-1.
            ENDIF
            LL=ABS((IT-IS)*J)-INT(ABS((IT-IS)*J/XN))*XN
            AA1=AA1+UD(M1,IT)*UD(M2,IS)*CO(LL)
            BB1=BB1+UD(M1,IT)*UD(M2,IS)*SIGN*SI(LL)
17         CONTINUE
        PRE(M1,M2,J)=AA1/(PI2*XN)
        PIM(M1,M2,J)=BB1/(PI2*XN)
        PIM(1,1,J)=0.
        PIM(2,2,J)=0.
16     CONTINUE
      DO 18 I1=1,NPAR
        PAR(I1)=0.
        PLD(I1)=.FALSE.
        PARM(I1)=0.
18     CONTINUE
      DO 19 I1=1,NV1
      DO 19 I2=1,NV
        QQ(I1,I2)=0.
        QQM(I1,I2)=0.
19     CONTINUE
      DO 20 I7=1,NV
        UM(I7)=0.
          DO 21 J=1,N
            UM(I7)=UM(I7)+(1./XN)*UD(I7,J)
21         CONTINUE
20     CONTINUE
      DO 22 I2=1,NV
      DO 22 J=1,NV
      DO 22 L=1,N-1
        XXC(I2,J,L)=0.
          DO 23 K=1,N-L
          XXC(I2,J,L)=XXC(I2,J,L)+(1./XN)*(UD(I2,K)-UM(I2))*(UD(J,K+L)-UM(J))
23         CONTINUE
22     CONTINUE
      DO 24 I7=1,NV
      DO 24 J=1,NV
        XKK=0.
          DO 25 K=1,N
            XKK=XKK+(UD(I7,K)-UM(I7))*(UD(J,K)-UM(J))
25         CONTINUE
        XKV(I7,J)=XKK/XN
24     CONTINUE
```

```
      IFAIL=0.
      CALL F01ABF(XKV,N1,NV,XKV1,NV,Q,IFAIL)
      DO 26 I7=1,NV-1
      DO 26 J=I7+1,NV
        XKV(J,I7)=XKV(I7,J)
        XKV1(I7,J)=XKV1(J,I7)
26     CONTINUE
      DO 27 I8=1,NV
      DO 27 J=1,NV
        XAA(I8,J)=(PIC2/6.)*XKV(I8,J)*XKV1(I8,J)
27     CONTINUE
      CALL F01ABF(XAA,NV1,NV,XAA1,NV,P,IFAIL)
      DO 28 i8=1,NV-1
      DO 28 j=i8+1,NV
        XAA(J,I8)=XAA(I8,J)
        XAA1(I8,J)=XAA1(J,I8)
28     CONTINUE
      DO 29 I2=1,NV
        XAT(I2)=0.
        XAF(I2)=0.
          DO 30 L=1,N1
          DO 30 k=1,NV
            ZZL=L
            XAT(I2)=XAT(I2)+(1./ZZL)*XKV1(I2,K)*XXC(I2,K,L)
            XAF(I2)=XAF(I2)+((-1)*PI2/XN)*XKV1(I2,K)*(XRE(L)*PRE(I2,K,L)-XIM(L)*PIM(I2,K,L))
30         CONTINUE
29     CONTINUE
      STIME=0.
      SFREQ=0.
      DO 31 I2=1,NV
      DO 31 J=1,NV
        STIME=STIME+XAT(I2)*XAA1(I2,J)*XAT(J)
        SFREQ=SFREQ+XAF(I2)*XAA1(I2,J)*XAF(J)
31     CONTINUE
     DO 1000 IRE=1,4
      EXACT=.TRUE.
      MEAN=.TRUE.
      IPRINT=-1
      CTOL=0.0001
      MCAL=5000
      ISHOW=0
      IFAIL=-1
      IF (IRE.EQ.1) THEN
        call g13dcf(nv,n,1,iq,mean,par,npar,qq,nv1,ud,pld,exact,iprint,
     +   ctol,mcal,ishow,niter,rlogl,v,g,cm,icm,wk,lwk,iw,liw,ifail)
        r1=par(1)
        r2=par(2)
        r3=par(3)
        r4=par(4)
        ifail=0
        call f01abf(qq,nv1,nv,qq1,nv,p7,ifail)
        qq1(1,2)=qq1(2,1)
        do 32 j=1,n1
          fa=pi2*(qq1(1,1)*(1.+r1**2-2.*r1*co(j))+qq1(2,2)*(r3**2.)+2.*qq1(1,2)*r3*(r1-co(j)))
          fb=pi2*(qq1(1,1)*r2*(r1-co(j))+qq1(2,2)*r3*(r4-co(j))+qq1(1,2)*(1.+r1*r4+r2*r3-(r1+r4)*co(j)))
          fc=pi2*(qq1(1,1)*r2-qq!(2,2)*r3+qq1(1.2)*(r4-r1))*si(j)
```

```
      fd=pi2*(qq1(2,2)*(1.+r4**2-2.*r4*co(j))+qq1(1,1)*(r2**2.)+2.*qq1(1,2)*r2*(r4-co(j)))
      detf=fa*fd-(fb**2.)-(fc**2.)
      fe=fd/detf
      ff=(-1)*fb/detf
      fg=(-1)*fc/detf
      fh=fa/detf
      fj=2.*(qq1(1,1)*(r1-co(j))+qq1(1,2)*r3)
      fk=qq1(1,1)*r2+qq1(1,2)*(r4-co(j))
      fl=(-1)*qq1(1,2)*si(j)
      fm=qq1(1,1)*(r1-co(j))+qq1(1,2)*r3
      fn=qq1(1,1)*si(j)
      fo=2.*(qq1(1,2)*(r4-co(j))+qq1(1,1)*r2)
      fp=2.*(qq1(1,2)*(r1-co(j))+qq1(2,2)*r3)
      fq=qq1(2,2)*(r4-co(j))+qq1(1,2)*r2
      fr=(-1)*qq1(2,2)*si(j)
      fs=qq1(1,2)*(r1-co(j))+qq1(2,2)*r3
      ft=qq1(1,2)*si(j)
      fu=2.*(qq1(2,2)*(r4-co(j))+qq1(1,2)*r2)
      f1=fe*fj+ff*fk+fg*fl
      f2=fg*fk-ff*fl
      f3=fe*fk
      f4=fe*fl
      f5=ff*fj+fh*fk
      f6=(-1)*(fg*fj+fh*fl)
      f7=ff*fk+fg*fl
      f8=ff*fl-fg*fk
      f9=ff*fm+ff*fn
      f10=fg*fm-ff*fn
      f11=fe*fm+ff*fo
      f12=fe*fn+fg*fo
      f13=fh*fm
      f14=(-1)*fh*fn
      f15=fh*fo+ff*fm+fg*fn
      f16=ff*fn-fg*fm
      f17=fe*fp+ff*fq+fg*fr
      f18=fg*fq-ff*fr
      f19=fe*fq
      f20=fe*fr
      f21=ff*fp+fh*fq
      f22=(-1)*(fp*fg+fh*fr)
      f23=ff*fq+fg*fr
      f24=ff*fr-fg*fq
      f25=ff*fs+fg*ft
      f26=fg*fs-ff*ft
      f27=fe*fs+ff*fu
      f28=fe*ft+fu*fg
      f29=fh*fs
      f30=(-1)*(fh*ft)
      f31=ff*fs+fg*ft+fu*fh
      f32=ff*ft-fg*fs
      e(1,1,j)=f1**2.-f2**2.+f7**2.-f8**2.+2.*(f3*f5-f4*f6)
      e(1,2,j)=f1*f9+f3*f13+f5*f11+f7*f15-f2*f10-f4*f14-f6*f12-f8*f16
      e(1,3,j)=f1*f17+f3*f21+f5*f19+f7*f23-f2*f18-f4*f22-f6*f20-f8*f24
      e(1,4,j)=f1*f25+f3*f29+f5*f27+f7*f31-f2*f26-f4*f30-f6*f28-f8*f32
      e(2,2,j)=f9**2.-f10**2.+f15**2.-f16**2.+2.*(f11*f13-f12*f14)
      e(2,3,j)=f9*f17+f11*f21+f13*f19+f15*f23-f10*f18-f12*f22-f14*f20-f16*f24
      e(2,4,j)=f9*f25+f11*f29+f13*f27+f15*f31-f10*f26-f12*f30-f14*f28-f16*f32
      e(3,3,j)=f17**2.-f18**2.+f23**2.-f24**2.+2.*(f19*f21-f20*f22)
      e(3,4,j)=f17*f25+f19*f29+f21*f27+f23*f31-f18*f26-f20*f30-f22*f28-f24*f32
      e(4,4,j)=f25**2.-f26**2.+f31**2.-f32**2.+2.*(f27*f29-f28*f30)
      fre1(1,1,j)=fa
      fim1(1,1,j)=0.
      fre1(1,2,j)=fb
      fim1(1,2,j)=fc
      fre1(2,1,j)=fb
      fim1(2,1,j)=(-1)*fc
      fre1(2,2,j)=fd
      fim1(2,2,j)=0.
      fre(1,1,j)=fe
      fim(1,1,j)=0.
      fre(1,2,j)=ff
      fim(1,2,j)=fg
      fre(2,1,j)=ff
      fim(2,1,j)=(-1)*fg
      fre(2,2,j)=fh
      fim(2,2,j)=0.
      fred(1,1,j,1)=fj
      fred(1,2,j,1)=fk
      fred(2,1,j,1)=fk
      fred(2,2,j,1)=0.
      fimd(1,1,j,1)=0.
      fimd(1,2,j,1)=fl
      fimd(2,1,j,1)=(-1)*fl
      fimd(2,2,j,1)=0.
      fred(1,1,j,2)=0.
      fred(1,2,j,2)=fm
      fred(2,1,j,2)=fm
      fred(2,2,j,2)=fo
      fimd(1,1,j,2)=0.
      fimd(1,2,j,2)=fn
      fimd(2,1,j,2)=(-1)*fn
      fimd(2,2,j,2)=0.
      fred(1,1,j,3)=fp
      fred(1,2,j,3)=fq
      fred(2,1,j,3)=fq
      fred(2,2,j,3)=0.
      fimd(1,1,j,3)=0.
      fimd(1,2,j,3)=fr
      fimd(2,1,j,3)=(-1)*fr
      fimd(2,2,j,3)=0.
      fred(1,1,j,4)=0.
      fred(1,2,j,4)=fs
      fred(2,1,j,4)=fs
      fred(2,2,j,4)=fu
      fimd(1,1,j,4)=0.
      fimd(1,2,j,4)=ft
      fimd(2,1,j,4)=(-1)*ft
      fimd(2,2,j,4)=0.
32    continue
      ELSE IF(IRE.EQ.2) THEN
        call g13dcf(nv,n,ip,1,mean,parm,npar,qqm,nv1,ud,pld,exact,iprint,
     +  ctol,mcal,ishow,niterm,rloglm,vm,gm,cmm,icm,wkm,lwk,iwm,liw,ifail)
```

```
t1=parm(1)*(-1)
t2=parm(2)*(-1)
t3=parm(3)*(-1)
t4=parm(4)*(-1)
do 33 j=1,n1
  fa=zpi*(qqm(1,1)*(1.+t1**2+2.*t1*co(j))+qqm(2,2)*t2**2.+2.*qqm(1,2)*t2*(t1+co(j)))
  fb=zpi*(qqm(1,1)*t3*(t1+co(j))+qqm(2,2)*t2*(t4+co(j))+qqm(1,2)*(1.+t2*t3+t1*t4+(t1+t4)*co(j)))
  fc=zpi*(qqm(2,2)*t2-qqm(1,1)*t3+qqm(1,2)*(t1-t4))*si(j)
  fd=zpi*(qqm(2,2)*(1.+t4**2+2.*t4*co(j))+qqm(1,1)*t3**2.+2.*qqm(1,2)*t3*(t4+co(j)))
  detf=fa*fd-(fb**2.)-(fc**2.)
  fe=fd/detf
  ff=(-1)*fb/detf
  fg=(-1)*fc/detf
  fh=fa/detf
  fj=2.*(t1*qqm(1,1)+qqm(1,1)*co(j)+qqm(1,2)*t2)
  fk=qqm(1,1)*t3+qqm(1,2)*t4+qqm(1,2)*co(j)
  fl=(qqm(1,2)-qqm(1,1))*si(j)
  fm=2.*(qqm(1,2)*t1+qqm(1,2)*co(j)+qqm(2,2)*t2)
  fn=qqm(1,2)*t3+qqm(2,2)*t4+qqm(2,2)*co(j)
  fo=qqm(2,2)*si(j)
  fp=qqm(1,2)*t2+qqm(1,1)*t1+qqm(1,1)*co(j)
  fq=(-1)*qqm(1,1)*si(j)
  fr=2.*(t4*qqm(1,2)+qqm(1,2)*co(j)+qqm(1,1)*t3)
  fs=qqm(2,2)*t2+qqm(1,2)*t1+qqm(1,2)*co(j)
  ft=(-1)*qqm(1,2)*si(j)
  fu=2.*(t4*qqm(2,2)+qqm(2,2)*co(j)+qqm(1,2)*t3)
  f1=fe*fj+ff*fk+fg*fl
  f2=fg*fk-ff*fl
  f3=fe*fk
  f4=fe*fl
  f5=ff*fj+fh*fk
  f6=(-1)*(fg*fj+fh*fl)
  f7=ff*fk+fg*fl
  f8=ff*fl-fg*fk
  f9=fe*fm+ff*fn+fg*fo
  f10=fg*fn-ff*fo
  f11=fe*fn
  f12=fe*fo
  f13=ff*fm+fh*fn
  f14=(-1)*(fg*fm+fh*fo)
  f15=ff*fn+fg*fo
  f16=ff*fo-fg*fn
  f17=ff*fp+fg*fq
  f18=fg*fp-ff*fq
  f19=fe*fp+fr*ff
  f20=fe*fq+fr*fg
  f21=fh*fp
  f22=(-1)*(fh*fq)
  f23=ff*fp+fg*fq+fr*fh
  f24=ff*fq-fg*fp
  f25=ff*fs+fg*ft
  f26=fg*fs-ff*ft
  f27=fe*fs+ff*fu
  f28=fe*ft+fu*fg
  f29=fh*fs
  f30=(-1)*(fh*ft)
  f31=ff*fs+fg*ft+fu*fh
  f32=ff*ft-fg*fs
  e(1,1,j)=f1**2.-f2**2.+f7**2.-f8**2.+2.*(f3*f5-f4*f6)
  e(1,2,j)=f1*f9+f3*f13+f5*f11+f7*f15-f2*f10-f4*f14-f6*f12-f8*f16
  e(1,3,j)=f1*f17+f3*f21+f5*f19+f7*f23-f2*f18-f4*f22-f6*f20-f8*f24
  e(1,4,j)=f1*f25+f3*f29+f5*f27+f7*f31-f2*f26-f4*f30-f6*f28-f8*f32
  e(2,2,j)=f9**2.-f10**2.+f15**2.-f16**2.+2.*(f11*f13-f12*f14)
  e(2,3,j)=f9*f17+f11*f21+f13*f19+f15*f23-f10*f18-f12*f22-f14*f20-f16*f24
  e(2,4,j)=f9*f25+f11*f29+f13*f27+f15*f31-f10*f26-f12*f30-f14*f28-f16*f32
  e(3,3,j)=f17**2.-f18**2.+f23**2.-f24**2.+2.*(f19*f21-f20*f22)
  e(3,4,j)=f17*f25+f19*f29+f21*f27+f23*f31-f18*f26-f20*f30-f22*f28-f24*f32
  e(4,4,j)=f25**2.-f26**2.+f31**2.-f32**2.+2.*(f27*f29-f28*f30)
  fre(1,1,j)=fa
  fim(1,1,j)=0.
  fre(1,2,j)=fb
  fim(1,2,j)=fc
  fre(2,1,j)=fb
  fim(2,1,j)=(-1)*fc
  fre(2,2,j)=fd
  fim(2,2,j)=0.
  fre1(1,1,j)=fe
  fim1(1,1,j)=0.
  fre1(1,2,j)=ff
  fim1(1,2,j)=fg
  fre1(2,1,j)=ff
  fim1(2,1,j)=(-1)*fg
  fre1(2,2,j)=fh
  fim1(2,2,j)=0.
  fred(1,1,j,1)=fj
  fred(1,2,j,1)=fk
  fred(2,1,j,1)=fk
  fred(2,2,j,1)=0.
  fimd(1,1,j,1)=0.
  fimd(1,2,j,1)=fl
  fimd(2,1,j,1)=(-1)*fl
  fimd(2,2,j,1)=0.
  fred(1,1,j,2)=fm
  fred(1,2,j,2)=fn
  fred(2,1,j,2)=fn
  fred(2,2,j,2)=0.
  fimd(1,1,j,2)=0.
  fimd(1,2,j,2)=fo
  fimd(2,1,j,2)=(-1)*fo
  fimd(2,2,j,2)=0.
  fred(1,1,j,3)=0.
  fred(1,2,j,3)=fp
  fred(2,1,j,3)=fp
  fred(2,2,j,3)=fr
  fimd(1,1,j,3)=0.
  fimd(1,2,j,3)=fq
  fimd(2,1,j,3)=(-1)*fq
  fimd(2,2,j,3)=0.
  fred(1,1,j,4)=0.
  fred(1,2,j,4)=fs
  fred(2,1,j,4)=fs
  fred(2,2,j,4)=fu
```

```
      fimd(1,1,j,4)=0.
      fimd(1,2,j,4)=ft
      fimd(2,1,j,4)=(-1)*ft
      fimd(2,2,j,4)=0.
33    continue
      END IF
      DO 34 I1=2,NQ
      DO 34 J=1,I1-1
      DO 34 K=1,N1
       E(I1,J,K)=E(J,I1,K)
34    CONTINUE
      DO 35 I1=1,NQ
      DO 35 J=1,NQ
       EX=0.
        DO 36 K=1,N1
         EX=EX+E(I1,J,K)
36      CONTINUE
        EXE(I1,J)=(1/(2.*XN))*EX
35    CONTINUE
      IFAIL=0.
      CALL f01AAF(EXE,NQ1,NQ,EXE1,NQ,P3,IFAIL)
      DO 37 I1=1,NQ-1
      DO 37 J=I1+1,NQ
       EXE1(I1,J)=EXE1(J,I1)
37    CONTINUE
      DO 38 I5=1,NV
       BX=0.
        DO 39 J=1,N1
        DO 39 K=1,NV
         BX=BX+((XRE(J)*PRE(I5,K,J)-XIM(J)*PIM(I5,K,J))*FRE1(K,I5,J)-
     +      (XIM(J)*PRE(I5,K,J)+XRE(J)*PIM(I5,K,J))*FIM1(K,I5,J))
39      CONTINUE
       BA(I5)= ((-1)/XN)*BX
38    CONTINUE
      DO 40 I5=1,NV
      DO 40 J=1,NV
       XCX=0.
        DO 41 K=1,N1
         XCX=XCX+(XRE(K)**2.+XIM(K)**2.)*(FRE(I5,J,K)*FRE1(J,I5,K)-FIM(I5,J,K)*FIM1(J,I5,K))
41      CONTINUE
       XC(I5,J)=(1./XN)*XCX
40    CONTINUE
      IF (IRE.EQ.1) THEN
       do 42 i5=1,nv
       do 42 j=1,nv
       do 42 k=1,n1
        z1(i5,j,k)=xre(k)*fre(i5,j,k)-xim(k)*fim(i5,j,k)
        z2(i5,j,k)=xre(k)*fim(i5,j,k)+xim(k)*fre(i5,j,k)
42     continue
       do 43 i5=1,nv
       do 43 k=1,nq
        dkd=0.
         do 44 j=1,nv
         do 44 l=1,n1
          dkd=dkd+(z1(i5,j,l)*fred(j,i5,l,k)-z2(i5,j,l)*fimd(j,i5,l,k))
44        continue
        dk(i5,k)=(1./xn)*dkd
43     continue
       do 45 i5=1,nv
       do 45 j=1,nv
        sus=0.
         do 46 m=1,nq
         do 46 nh=1,nq
          sus=sus+dk(i5,m)*exe1(m,nh)*dk(j,nh)
46       continue
        xxb(i5,j)=xc(i5,j)-sus
45     continue
       ifail=0
       call f01aaf(xxb,nv1,nv,xxb1,nv,p4,ifail)
       xxb1(1,2)=xxb1(2,1)
       tst(i,k1,k2,1)=0.
       do 47 i5=1,nv
       do 47 j=1,nv
        tst(i,k1,k2,1)=tst(i,k1,k2,1)+xn*ba(i5)*xxb1(i5,j)*ba(j)
47     continue
      ELSE IF(IRE.EQ.2) THEN
       do 48 i5=1,nv
       do 48 k=1,nq
        dkd=0.
         do 49 j=1,nv
         do 49 l=1,n1
        dkd=dkd+(xre(l)*fre1(i5,j,l)-xim(l)*fim1(i5,j,l))*fred(j,i5,l,k)
     +    -(xim(l)*fre1(i5,j,l)+xre(l)*fim1(i5,j,l))*fimd(j,i5,l,k)
49        continue
        dk(i5,k)=(-1/xn)*dkd
48     continue
       do 50 i5=1,nv
       do 50 j=1,nv
        sus=0.
         do 51 m=1,nq
         do 51 nh=1,nq
          sus=sus+dk(i5,m)*exe1(m,nh)*dk(j,nh)
51        continue
        xxb(i5,j)=xc(i5,j)-sus
50     continue
       ifail=0.
       call f01aaf(xxb,nv1,nv,xxb1,nv,p4,ifail)
       xxb1(1,2)=xxb1(2,1)
       tst(i,k1,k2,2)=0.
       do 52 i5=1,nv
       do 52 j=1,nv
        tst(i,k1,k2,2)=tst(i,k1,k2,2)+xn*ba(i5)*xxb1(i5,j)*ba(j)
52     continue
      ELSE IF(IRE.EQ.3) THEN
       tst(i,k1,k2,3)=xn*sfreq
      ELSE IF(IRE.EQ.4) THEN
       tst(i,k1,k2,4)=xn*stime
      END IF
1000  CONTINUE

      END
```

TABLE 5.1

Rejection frequencies of $\hat{S}^{t2}$ in (32) with $\Sigma = I_2$

True model: $\theta_1 = \theta_2 = 0$. **No. of replications: 5000**

α = 10 %

Table 5.1a): T = 50

θ_1 / θ_2	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
-0.8	1.000	.999	.998	.977	.939	.952	.988	.998	1.000
-0.6	.999	.998	.984	.904	.772	.829	.953	.994	.999
-0.4	.996	.982	.910	.677	.428	.576	.871	.982	.997
-0.2	.979	.902	.674	.322	.128	.316	.751	.957	.996
0	.933	.768	.430	.126	**.033**	.206	.660	.939	.992
0.2	.954	.823	.563	.308	.203	.336	.725	.944	.994
0.4	.985	.952	.863	.746	.661	.724	.894	.980	.997
0.6	.999	.994	.978	.953	.930	.944	.979	.995	.999
0.8	1.000	.999	.999	.995	.992	.992	.998	.999	1.000

Table 5.1b): T = 100

θ_1 / θ_2	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
-0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.6	1.000	1.000	1.000	1.000	.999	.999	1.000	1.000	1.000
-0.4	1.000	1.000	1.000	.989	.933	.979	.999	1.000	1.000
-0.2	1.000	1.000	.989	.805	.417	.751	.987	1.000	1.000
0	1.000	.998	.935	.411	**.053**	.516	.964	.999	1.000
0.2	1.000	.999	.971	.756	.518	.767	.985	.999	1.000
0.4	1.000	1.000	.999	.987	.967	.984	.998	1.000	1.000
0.6	1.000	1.000	1.000	.999	.999	1.000	1.000	1.000	1.000
0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 5.1c): T = 200

θ_1 / θ_2	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
-0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.2	1.000	1.000	1.000	.993	.834	.984	1.000	1.000	1.000
0	1.000	1.000	1.000	.827	**.072**	.849	.999	1.000	1.000
0.2	1.000	1.000	1.000	.988	.858	.984	1.000	1.000	1.000
0.4	1.000	1.000	1.000	1.000	.999	.999	1.000	1.000	1.000
0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE 5.2

Rejection frequencies of $\hat{S}^{t2}$ in (32) with $\Sigma = I_2$

True model: $\theta_1 = \theta_2 = 0$. **No. of replications: 5000**

α = 5 %

Table 5.2a): T = 50

θ_1 / θ_2	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
-0.8	.999	.998	.981	.889	.744	.811	.947	.993	.999
-0.6	.997	.986	.918	.708	.462	.601	.879	.984	.998
-0.4	.980	.919	.726	.400	.181	.358	.778	.962	.996
-0.2	.888	.700	.394	.132	.039	.209	.660	.935	.990
0	.741	.457	.170	.037	**.012**	.147	.585	.910	.987
0.2	.805	.591	.350	.205	.137	.256	.646	.923	.989
0.4	.944	.873	.769	.659	.582	.648	.846	.968	.996
0.6	.993	.977	.959	.927	.904	.920	.967	.992	.998
0.8	1.000	.999	.996	.991	.987	.988	.997	.999	1.000

Table 5.2b): T = 100

θ_1 / θ_2	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
-0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.6	1.000	1.000	1.000	.998	.989	.996	1.000	1.000	1.000
-0.4	1.000	1.000	.998	.958	.803	.925	.998	1.000	1.000
-0.2	1.000	.998	.958	.618	.221	.632	.976	1.000	1.000
0	1.000	.987	.798	.206	**.020**	.426	.945	.999	1.000
0.2	1.000	.996	.922	.637	.425	.693	.976	.999	1.000
0.4	1.000	1.000	.997	.975	.949	.975	.997	1.000	1.000
0.6	1.000	1.000	1.000	.999	.999	1.000	1.000	1.000	1.000
0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 5.2c): T = 200

θ_1 / θ_2	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
-0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.4	1.000	1.000	1.000	1.000	.999	1.000	1.000	1.000	1.000
-0.2	1.000	1.000	1.000	.997	.680	.966	1.000	1.000	1.000
0	1.000	1.000	.999	.671	**.032**	.793	.999	1.000	1.000
0.2	1.000	1.000	.999	.969	.808	.972	1.000	1.000	1.000
0.4	1.000	1.000	1.000	.999	.999	.999	1.000	1.000	1.000
0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE 5.3

Rejection frequencies of $\hat{S}^{t2}$ in (32) with $\Sigma = I_2$

True model: $\theta_1 = \theta_2 = 0$. **No. of replications: 5000**

α = 2.5 %

Table 5.3a): T = 50

θ_1 / θ_2	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
-0.8	.997	.982	.905	.672	.416	.563	.869	.982	.998
-0.6	.985	.926	.750	.428	.187	.371	.784	.965	.995
-0.4	.905	.745	.471	.170	.054	.232	.682	.940	.992
-0.2	.666	.412	.174	.044	.010	.143	.585	.906	.986
0	.404	.182	.053	.011	**.004**	.105	.521	.880	.981
0.2	.548	.358	.228	.137	.100	.198	.575	.895	.984
0.4	.863	.776	.681	.578	.516	.581	.798	.951	.993
0.6	.976	.960	.935	.900	.872	.891	.956	.988	.998
0.8	.999	.997	.993	.986	.979	.984	.994	.999	1.000

Table 5.3b): T = 100

θ_1 / θ_2	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
-0.8	1.000	1.000	1.000	.999	.998	.999	1.000	1.000	1.000
-0.6	1.000	1.000	1.000	.995	.955	.983	.999	1.000	1.000
-0.4	1.000	1.000	.992	.880	.602	.834	.993	1.000	1.000
-0.2	1.000	.992	.880	.413	.092	.526	.963	.999	1.000
0	.997	.953	.591	.089	**.010**	.354	.924	.998	1.000
0.2	.999	.980	.836	.527	.349	.621	.963	.999	1.000
0.4	1.000	.999	.992	.961	.928	.963	.996	1.000	1.000
0.6	1.000	1.000	1.000	.999	.999	.999	1.000	1.000	1.000
0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 5.3c): T = 200

θ_1 / θ_2	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
-0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.4	1.000	1.000	1.000	1.000	.996	.999	1.000	1.000	1.000
-0.2	1.000	1.000	1.000	.941	.495	.934	1.000	1.000	1.000
0	1.000	1.000	.996	.495	**.014**	.742	.999	1.000	1.000
0.2	1.000	1.000	.999	.941	.754	.957	1.000	1.000	1.000
0.4	1.000	1.000	1.000	.999	.999	.999	1.000	1.000	1.000
0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE 5.4

Rejection frequencies of $\hat{S}^{t2}$ in (32) with $\Sigma = I_2$

True model: $\theta_1 = \theta_2 = 0$. **No. of replications: 5000**

$\alpha = 1\%$

Table 5.4a): T = 50

θ_1/θ_2	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
-0.8	.962	.874	.651	.289	.096	.293	.740	.956	.994
-0.6	.875	.704	.406	.134	.035	.204	.661	.933	.990
-0.4	.646	.403	.164	.039	.010	.141	.583	.905	.986
-0.2	.288	.136	.040	.006	.001	.096	.500	.869	.979
0	.098	.034	.010	.003	**.001**	.070	.438	.832	.973
0.2	.281	.205	.136	.091	.066	.141	.500	.853	.976
0.4	.734	.663	.573	.494	.442	.500	.739	.932	.989
0.6	.950	.928	.898	.862	.830	.850	.931	.983	.997
0.8	.995	.991	.985	.977	.970	.974	.988	.997	.999

Table 5.4b): T = 100

θ_1/θ_2	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
-0.8	1.000	1.000	1.000	.995	.979	.991	1.000	1.000	1.000
-0.6	1.000	1.000	.997	.956	.805	.925	.998	1.000	1.000
-0.4	1.000	.998	.962	.688	.307	.685	.982	1.000	1.000
-0.2	.997	.958	.692	.199	.026	.416	.941	.998	1.000
0	.976	.801	.295	.025	**.004**	.281	.892	.997	1.000
0.2	.990	.923	.685	.405	.279	.530	.944	.999	1.000
0.4	.999	.997	.980	.937	.897	.945	.994	.999	1.000
0.6	1.000	1.000	1.000	.999	.998	.999	1.000	1.000	1.000
0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 5.4c): T = 200

θ_1/θ_2	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
-0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.4	1.000	1.000	1.000	.999	.975	.997	1.000	1.000	1.000
-0.2	1.000	1.000	.999	.848	.275	.886	.999	1.000	1.000
0	1.000	1.000	.971	.279	**.005**	.673	.999	1.000	1.000
0.2	1.000	1.000	.998	.892	.686	.932	1.000	1.000	1.000
0.4	1.000	1.000	1.000	.999	.998	.999	1.000	1.000	1.000
0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE 5.5

Rejection frequencies of $\hat{S}^{t2}$ in (32) with $\Sigma = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$

True model: $\theta_1 = \theta_2 = 0$. **No. of replications: 5000**

α = 10 %

Table 5.5a): T = 50

θ_1 / θ_2	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
-0.8	1.000	.999	.998	.997	.998	.999	.999	1.000	1.000
-0.6	1.000	.998	.989	.969	.982	.997	.999	1.000	1.000
-0.4	.998	.987	.912	.770	.841	.975	.998	1.000	1.000
-0.2	.998	.968	.768	.346	.319	.816	.983	.999	1.000
0	.998	.985	.834	.323	**.039**	.431	.921	.994	.999
0.2	.999	.997	.974	.813	.442	.340	.824	.987	.998
0.4	1.000	.999	.998	.981	.917	.824	.895	.985	.999
0.6	1.000	1.000	1.000	.999	.995	.987	.987	.996	.999
0.8	1.000	1.000	1.000	.999	.999	.999	.999	.999	1.000

Table 5.5b): T = 100

θ_1 / θ_2	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
-0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.6	1.000	1.000	1.000	1.000	1 000	1.000	1.000	1.000	1.000
-0.4	1.000	1.000	.999	.996	.998	1.000	1.000	1.000	1.000
-0.2	1.000	1.000	.996	.813	.764	.997	1.000	1.000	1.000
0	1.000	1.000	.998	.768	**.061**	.821	.998	1.000	1.000
0.2	1.000	1.000	1.000	.995	.825	.765	.992	1.000	1.000
0.4	1.000	1.000	1.000	1.000	.999	.994	.998	1.000	1.000
0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 5.5c): T = 200

θ_1 / θ_2	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
-0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.2	1.000	1.000	1.000	.993	.991	1.000	1.000	1.000	1.000
0	1.000	1.000	1.000	.990	**.075**	.989	1.000	1.000	1.000
0.2	1.000	1.000	1.000	.988	.989	.983	1.000	1.000	1.000
0.4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE 5.6

Rejection frequencies of $\hat{S}^{t2}$ in (32) with $\Sigma = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$

True model: $\theta_1 = \theta_2 = 0$. **No. of replications: 5000**

$\alpha = 5$ %

Table 5.6a): T = 50

θ_1 / θ_2	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
-0.8	.999	.998	.990	.979	.986	.997	.999	1.000	1.000
-0.6	.997	.987	.932	.866	.921	.986	.998	.999	1.000
-0.4	.988	.930	.743	.503	.625	.920	.992	.999	1.000
-0.2	.975	.861	.500	.152	.144	.688	.964	.996	.999
0	.986	.913	.618	.149	**.014**	.327	.882	.987	.999
0.2	.996	.984	.916	.685	.340	.256	.766	.978	.997
0.4	.999	.998	.992	.960	.879	.764	.846	.976	.997
0.6	1.000	1.000	.999	.998	.991	.979	.977	.992	.999
0.8	1.000	1.000	.999	.999	.999	.999	.999	.999	1.000

Table 5.6b): T = 100

θ_1 / θ_2	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
-0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.4	1.000	1.000	.999	.982	.993	1.000	1.000	1.000	1.000
-0.2	1.000	.999	.983	.630	.571	.989	1.000	1.000	1.000
0	1.000	1.000	.992	.594	**.021**	.750	.998	1.000	1.000
0.2	1.000	1.000	1.000	.985	.754	.689	.988	1.000	1.000
0.4	1.000	1.000	1.000	1.000	.999	.989	.997	1.000	1.000
0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 5.6c): T = 200

θ_1 / θ_2	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
-0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.2	1.000	1.000	1.000	.979	.970	1.000	1.000	1.000	1.000
0	1.000	1.000	1.000	.968	**.032**	.979	1.000	1.000	1.000
0.2	1.000	1.000	1.000	1.000	.980	.972	1.000	1.000	1.000
0.4	1.000	1.000	1.000	.999	1.000	.999	1.000	1.000	1.000
0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE 5.7

Rejection frequencies of $\hat{S}^{t2}$ in (32) with $\Sigma = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$

True model: $\theta_1 = \theta_2 = 0$. **No. of replications: 5000**

$\alpha = 2.5$ %

Table 5.7a): T = 50

θ_1 / θ_2	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
-0.8	.997	.987	.942	.889	.921	.981	.996	.999	.999
-0.6	.984	.928	.786	.634	.753	.939	.992	.999	.999
-0.4	.937	.786	.487	.243	.366	.817	.977	.996	.999
-0.2	.876	.627	.247	.051	.005	.553	.934	.991	.999
0	.918	.737	.374	.061	**.005**	.251	.835	.982	.998
0.2	.980	.937	.814	.556	.261	.196	.709	.968	.995
0.4	.996	.992	.973	.927	.826	.702	.798	.963	.996
0.6	.999	.999	.998	.993	.983	.969	.968	.988	.997
0.8	.999	.999	.999	.999	.999	.998	.997	.998	1.000

Table 5.7b): T = 100

θ_1 / θ_2	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
-0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.6	1.000	1.000	1.000	.999	1.000	1.000	1.000	1.000	1.000
-0.4	1.000	1.000	.993	.939	.975	1.000	1.000	1.000	1.000
-0.2	1.000	.999	.938	.431	.380	.972	1.000	1.000	1.000
0	1.000	1.000	.974	.392	**.010**	.668	.996	1.000	1.000
0.2	1.000	1.000	.999	.971	.689	.618	.983	1.000	1.000
0.4	1.000	1.000	1.000	1.000	.997	.985	.997	1.000	1.000
0.6	1.000	1.000	1.000	1.000	1.000	.999	1.000	1.000	1.000
0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 5.7c): T = 200

θ_1 / θ_2	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
-0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.2	1.000	1.000	1.000	.942	.922	1.000	1.000	1.000	1.000
0	1.000	1.000	1.000	.920	**.014**	.966	1.000	1.000	1.000
0.2	1.000	1.000	1.000	1.000	.968	.957	1.000	1.000	1.000
0.4	1.000	1.000	1.000	1.000	1.000	.999	1.000	1.000	1.000
0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE 5.8

Rejection frequencies of $\hat{S}^{t2}$ in (32) with $\Sigma = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$

True model: $\theta_1 = \theta_2 = 0$. **No. of replications: 5000**

$\alpha = 1$ %

Table 5.8a): T = 50

θ_1 / θ_2	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
-0.8	.966	.890	.722	.552	.651	.883	.975	.993	.998
-0.6	.885	.720	.452	.263	.402	.797	.965	.992	.998
-0.4	.716	.446	.184	.006	.137	.643	.938	.991	.998
-0.2	.553	.260	.062	.007	.016	.404	.880	.984	.998
0	.638	.400	.140	.021	**.001**	.182	.769	.973	.995
0.2	.879	.791	.634	.415	.190	.140	.632	.947	.994
0.4	.971	.955	.930	.870	.763	.640	.735	.947	.994
0.6	.993	.993	.989	.982	.971	.949	.952	.982	.996
0.8	.999	.999	.998	.999	.997	.996	.996	.998	.999

Table 5.8b): T = 100

θ_1 / θ_2	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
-0.8	1.000	1.000	1.000	.999	1.000	1.000	1.000	1.000	1.000
-0.6	1.000	1.000	.999	.994	.997	1.000	1.000	1.000	1.000
-0.4	1.000	.999	.963	.792	.907	.998	1.000	1.000	1.000
-0.2	1.000	.995	.798	.213	.205	.938	.999	1.000	1.000
0	1.000	.998	.898	.198	**.003**	.579	.992	1.000	1.000
0.2	1.000	1.000	.997	.934	.600	.533	.976	1.000	1.000
0.4	1.000	1.000	1.000	1.000	.993	.974	.994	1.000	1.000
0.6	1.000	1.000	1.000	1.000	1.000	.999	1.000	1.000	1.000
0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 5.8c): T = 200

θ_1 / θ_2	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
-0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.2	1.000	1.000	1.000	.853	.810	1.000	1.000	1.000	1.000
0	1.000	1.000	1.000	.819	**.005**	.942	1.000	1.000	1.000
0.2	1.000	1.000	1.000	1.000	.948	.929	1.000	1.000	1.000
0.4	1.000	1.000	1.000	1.000	1.000	.999	1.000	1.000	1.000
0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE 5.9

Table 5.9a: Empirical sizes of $\hat{S}^{\Omega}$ in (34) with $\Sigma = I_2$

True model: $\theta_1 = \theta_2 = 0$. **No. of replications: 1000**

$T \setminus \alpha$	10%	5%	2.5%	1%
50	0.028	0.012	0.001	0.000
100	0.058	0.019	0.010	0.006
200	0.074	0.038	0.020	0.008

Table 5.9b: Empirical sizes of $\hat{S}^{\Omega}$ in (34) with $\Sigma = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$

True model: $\theta_1 = \theta_2 = 0$. **No. of replications: 1000**

$T \setminus \alpha$	10%	5%	2.5%	1%
50	0.036	0.012	0.002	0.000
100	0.057	0.021	0.008	0.005
200	0.076	0.035	0.017	0.006

TABLE 5.10

Table 5.10a: Empirical sizes of $\tilde{S}^2$ in (37) with a VAR(1) structure on U_t

True model: $\theta_1 = \theta_2 = 0$. **No. of replications: 1000**

$T \setminus \alpha$	10%	5%	2.5%	1%
50	0.134	0.074	0.040	0.017
100	0.123	0.069	0.035	0.014
200	0.104	0.060	0.031	0.012

Table 5.10b: Empirical sizes of $\tilde{S}^2$ in (37) with a VMA(1) structure on U_t

True model: $\theta_1 = \theta_2 = 0$. **No. of replications: 1000**

$T \setminus \alpha$	10%	5%	2.5%	1%
50	0.207	0.154	0.127	0.097
100	0.137	0.090	0.054	0.045
200	0.131	0.062	0.038	0.023

CHAPTER 6

EMPIRICAL APPLICATIONS OF THE MULTIVARIATE TESTS AND FRACTIONAL COINTEGRATION

6.1 INTRODUCTION

In this chapter we will consider several pairs of variables that have been widely analyzed in the literature mainly in order to detect the presence of cointegrating relationships between them. In particular we will analyze the common behaviour between consumption and income, wages and prices, and nominal G.N.P. and money, using the same data as in Engle and Granger (1987), and the relationship between stock prices and dividends, using the data in Campbell and Shiller (1987). All these pairs of variables have been studied by many authors. Thus, the relationship between consumption and income has been analyzed by Davidson, Hendry, Srba and Yeo (henceforth DHSY (1978)), and also in Hall (1978), Campbell and Mankiw (1990), Qin (1991) and Ermisch and Westaway (1994) among others. The relation between wages and prices appears in Hall (1988), Mehra (1977, 1991), Ashenfelter and Card (1982), Stein (1979, 1984) and Darrat (1994). An article relating stock prices and dividends is Campbell and Shiller (1987), and those relating nominal G.N.P. and money include McElhattan (1976), Hafer (1984) and Sims (1994). All these groups of variables were also analyzed from a Bayesian viewpoint in DeJong (1992).

For each data set the analysis will be conducted as follows: first, we will calculate Robinson's (1994c) univariate tests for each series in order to detect what might be the proper integration order of each individual series, and will compare these results with those obtained using classical Dickey-Fuller (DF) and augmented Dickey-Fuller (ADF) tests for unit roots. We will also present results of the multivariate score tests of Chapter 5, investigating how plausible a fractionally integrated bivariate representation of the two series together might be, and finally, we will study the possibility of fractional cointegration for each pair of these variables.

The components of a (Nx1) vector X_t are said to be fractionally cointegrated of order d,b, ($X_t \sim CI(d,b)$), if a): all components of X_t are integrated of order d

($X_{it} \sim I(d)$), and b): there exists a vector r ($r \neq 0$) such that $N_t = r'X_t$ is integrated of order d-b ($N_t \sim I(d-b)$) with $b > 0$.[1] The vector r is called the cointegrating vector and $r'X_t$ will represent an equilibrium constraint operating in the long run component of X_t. If X_t has more than two components, then there may be more than one cointegrating vectors r, though in what follows, we will assume that X_t has only two components, so that $X_t = (y_t, z_t)'$ where y_t and z_t correspond to the pairs of variables that will be analyzed later. In this bivariate context, a necessary condition for cointegration is that both individual series must be integrated of the same order and thus, a plausible way of testing CI(d,d-b) might be to consider a joint test of the null hypothesis

$$H_o: \theta_1 = \theta_2 = 0 \quad \text{and} \quad \delta = 0 \tag{1}$$

against the alternative H_a: $\theta_1 \neq \theta_2$ or $\delta \neq 0$ in a model given by

$$y_t = \alpha z_t + x_t \qquad t = 1,2,.... \tag{2}$$

$$(1-L)^{d+\theta_1} y_t = v_{1t} \qquad t = 1,2,.... \tag{3}$$

$$(1-L)^{d+\theta_2} z_t = v_{2t} \qquad t = 1,2,.... \tag{4}$$

$$(1-L)^{b+\delta} x_t = v_{3t} \qquad t = 1,2,.... \tag{5}$$

with $z_t = x_t = 0$ for $t \leq 0$, where α is a scalar, and with possibly correlated white noise disturbances v_{1t}, v_{2t} and v_{3t}. However, we observe in this set-up that, if we take $b < d$ in (5), the null hypothesis (1) would imply that y_t and z_t are fractionally cointegrated of order d,d-b, but rejections of the null do not guarantee no-cointegration since $\theta_1 = \theta_2$ with δ greater than or smaller than zero might still imply that both series are fractionally cointegrated. Similarly, if we take $b = d$ in (5), the null hypothesis (1) would imply no cointegration, but the alternative would not guarantee cointegration at all, given that θ_1 might be different from θ_2, and also δ

[1] A more general definition of fractional cointegration allowing different integration orders for each series is found in Marinucci and Robinson (1997). They define $X_t \sim CI(d_1,...d_N,b)$ if $X_{it} \sim I(d_i)$ for all i, and there exists a (Nx1) vector $r \neq 0$ such that $N_t = r'X_t \sim I(d)$, where $d = \max_{1\leq i\leq N} (d_i - b)$. Note that this property is possible and meaningful if and only if $b > \max_{1\leq i\leq N} d_i - \min_{1\leq i\leq N} d_i$.

might be greater than zero.

An alternative procedure might be to test initially that both individual series, y_t and z_t, are integrated of order d, using either Robinson's (1994c) univariate tests described in Chapter 2 or the multivariate version in Chapter 5, and test the null hypothesis

$$H_o: \delta = 0 \tag{6}$$

against the one-sided alternative H_a: $\delta < 0$, in a model given by (2) and (5) with b = d, but in this case the parameter vector α will not be identified under the null hypothesis (6). We can illustrate this with a simple example. Suppose now that the two series y_t and z_t are jointly generated as a function of possibly correlated white noise disturbances ε_{1t} and ε_{2t} according to the model

$$y_t = \beta\, z_t + u_{1t}, \qquad (1-L)^d\, u_{1t} = \epsilon_{1t} \tag{7}$$

$$y_t = \alpha\, z_t + u_{2t}, \quad (1-L)^{d+\delta}\, u_{2t} = \epsilon_{2t} \quad \text{with } \delta < 0. \tag{8}$$

Clearly the parameters α and β are unidentified in the usual sense as there are no exogenous variables and the errors are contemporaneously correlated. The reduced form of the system will take the form

$$y_t = \frac{-\alpha}{\beta-\alpha}\, u_{1t} + \frac{\beta}{\beta-\alpha}\, u_{2t}$$

$$z_t = \frac{1}{\beta-\alpha}\, u_{1t} - \frac{1}{\beta-\alpha}\, u_{2t},$$

and since y_t and z_t are linear combinations of u_{1t} and u_{2t}, both variables will be integrated of order d. Equation (8) describes a particular linear combination of the random variables which is integrated of order smaller than d, and thus, y_t and z_t are fractionally cointegrated. Since the null hypothesis is taken to be no cointegration, or $\delta = 0$, if α were known, a test for the null hypothesis (6) could be constructed in (2) and (5) with d = b, taking x_t as the series to be integrated of order d under the null. However, if α is unknown, it must be estimated from the data, but if the null is true, α is not identified. Thus, only if the series are cointegrated can α be estimated by the cointegrating regression.

We will present here a testing procedure that follows a similar methodology to the one proposed in Engle and Granger (1987). First, we test that both series are integrated of the same order d. This can be done using either Robinson's (1994c) univariate tests, or the multivariate version described in Chapter 5. We could consider the model

$$\begin{pmatrix} (1-L)^{d+\theta_1} & 0 \\ 0 & (1-L)^{d+\theta_2} \end{pmatrix} \begin{pmatrix} y_t \\ z_t \end{pmatrix} = \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix} \qquad t = 1,2,\ldots,$$

$$(y_t,\ z_t)' = 0 \qquad t \leq 0$$

and test the null hypothesis

$$H_o\colon \theta_1 = \theta_2 = 0 \tag{9}$$

against the alternative H_a: $\theta = (\theta_1, \theta_2)' \neq 0$, for a prescribed value $d > 1/2$, and white noise or weakly autocorrelated ε_{1t} and ε_{2t}. Thus, the non-rejection of the null hypothesis (9) will imply that both series are nonstationary with the same integration order d.

Once we have checked this, we can estimate the cointegrating parameters from the cointegrating regression. Since all the linear combinations of y_t and z_t except the one defined in (8) will be integrated of order d, the least squares estimate from the regression of y_t on z_t, under cointegration, will produce a good estimate of α. In standard cointegration analysis (in which cointegration of order 1,1 is considered), Stock (1987) showed that the least squares estimate of the cointegrating parameter was consistent and converged in probability at the rate $T^{1-\delta}$ for any $\delta > 0$, rather than the usual rate $T^{1/2}$. Cheung and Lai (1993) and others extended the analysis to the case of fractional cointegration, and showed that the least squares estimate was also consistent though with possible different convergence rates, according to the cointegration order. In particular, they showed that under the general hypothesis of cointegration of order d,b with $d > 1/2$ and $b > 0$, the least squares estimate was consistent and converged at the rate $T^{b-\delta}$ and thus, included the Stock's (1987) convergence result as a special case with $b = 1$. Given the consistency of the least squares estimate of α in (8), we can now use Robinson's (1994c) univariate tests for testing the integration order in the equilibrium errors $e_t = y_t - \hat{\alpha} z_t$, with $\hat{\alpha}$ as the least squares estimate of α, and the test statistic will still remain with the same standard limit distribution. Thus, we could consider the model

$$(1 - L)^{b+\theta}\, e_t = u_t \qquad t = 1,2,... \tag{10}$$

$$e_t = 0, \qquad t \leq 0,$$

with I(0) u_t, and test the null hypothesis:

$$H_o: \theta = 0, \tag{11}$$

for different values of b, using Robinson's (1994c) univariate tests. We could take b = d in (10), and test H_o (11) against the alternative

$$H_a: \theta < 0, \tag{12}$$

and the test statistic will have an asymptotic null N(0,1) distribution. Rejections of (11) against (12) will imply that y_t and z_t are fractionally cointegrated, given that the equilibrium errors e_t present a smaller integration order than the individual series y_t and z_t. However, given that the equilibrium errors are not actually observed but obtained from minimizing the residual variance of the cointegrating regression, in finite samples the residual series might be biased toward stationarity, and thus, we would expect the null hypothesis to be rejected more often than suggested by the nominal size of Robinson's (1994c) tests. A similar problem arises in Engle and Granger (1987) and Cheung and Lai (1993) when testing cointegration. In order to cope with this problem, the empirical size of Robinson's (1994c) tests for cointegration in finite samples is obtained using a simulation approach.

In Table 6.1 we report the empirical size of Robinson's (1994c) tests for cointegration corresponding to different sample sizes (T = 50, 100, 200 and 300). We use the Monte Carlo method in 50,000 replications, assuming that the true system is of two I(d) processes with Gaussian independent white noise disturbances, that are not cointegrated, (i.e., b = d in (10)), and take values of d ranging from 0.6 through 1.5 with 0.1 increments. For simplicity, we assume that u_t in (10) is white noise, so we use the test statistic $\tilde{r}$ given in (2.3), though we could also have extended the analysis to cover the case of weak parametric autocorrelation in u_t. We observe in this table that the empirical distributions are similar across different values of d. They have a negative mean and the critical values are smaller than those given by the Normal distribution, which is consistent with the earlier discussion that, when testing H_o (11) against (12), the use of standard critical values will result in the cointegration tests rejecting the null hypothesis of no cointegration too often. On the other hand, when testing (11) against alternatives of form H_a: $\theta > 0$, using the Normal distribution, we should expect not to reject the null so often as when using the finite sample critical values. We also see in this table that the empirical distributions are positively skewed with kurtosis greater than 3, though increasing the sample size, the three statistics (mean, skewness and kurtosis) approximate to the values corresponding to the Normal distribution. The Fortran code used in this experiment is given in Appendix 6.1.

We next examine the power property of Robinson's (1994c) tests for cointegration relative to the ADF and Geweke and Porter-Hudak (GPH,1983) tests for cointegration. We consider a bivariate I(1) system, claimed to be non-cointegrated under the null hypothesis. The ADF unit root test recommended by Engle and Granger (1987) is given by the usual t-statistic for b_0 in

$$(1-L)\,e_t = b_0 e_{t-1} + b_1 (1-L)\,e_{t-1} + \ldots + b_p (1-L)\,e_{t-p} + \epsilon_t,$$

where e_t are the equilibrium errors and the lag parameter p can be selected using some model-selection procedures such as the Akaike and the Schwarz information criterions. The GPH test for cointegration proposed by Cheung and Lai (1993) is based on the estimation of the fractional differencing parameter d, in the linear regression

$$\ln(I(\lambda_j)) = \beta_0 + \beta_1 \ln(4\sin^2(\lambda_j/2)) + u_t,$$

where $\lambda_j = 2\pi j/T$ and $I(\lambda_j)$ is the periodogram of e_t at the ordinate j. Given that the least squares estimate of β_1 provides a consistent estimate of 1-d (see Robinson (1995a)), hypothesis testing concerning the value of d is based on the t-statistic of the regression coefficient.

In Table 6.2 we perform the power function of the three tests (ADF, GPH and Robinson) for cointegration against fractional and AR alternatives. Results for ADF and GPH tests have been taken from Cheung and Lai (1993) and the Monte Carlo experiment conducted is described in Appendix 6.2. The power of a test is measured as the percentage of the time the test can reject a false null hypothesis of no cointegration. We have performed Robinson's (1994c) test statistics $\tilde{r}$ in (2.3) and $\hat{r}$ in (2.9), for four different possibilities, assuming that the differenced series are white noise and AR processes of orders 1, 2 and 3, for 5% and 10% significance levels. We use the asymptotic critical values given by the Normal distribution, mainly because of the different models used for the disturbances and of the optimal asymptotic properties of Robinson's (1994c) tests stressed in Chapter 2. Furthermore, in the empirical applications carried out below, we use the standard N(0,1) distribution, given that the test statistics will be performed not only for different models for the disturbances, but also for different values of d, and also including deterministic paths, as an intercept and a linear time trend.

When testing against fractional alternatives, Robinson's (1994c) tests perform better than the ADF and GPH tests, and this is observed for white noise disturbances but also if they follow AR processes. The highest rejection frequencies are obtained with white noise disturbances if the integration order ranges between 0.05 and 0.75, but when this parameter approximates to 1, better results are obtained for weakly parametrically autocorrelated disturbances.

When testing against AR alternatives, again better statistical power properties are observed in Robinson's (1994c) tests relative to ADF and GPH tests, with higher rejection frequencies obtained at all values of the AR parameter ϕ. If this parameter ranges between 0.05 and 0.55, results are better when the disturbances are white noise, but if it ranges between 0.55 and 0.95, the tests behave better for weakly parametrically autocorrelated disturbances. The relative pronounced difference in power between Robinson's (1994c) tests and the ADF and GPH tests for cointegration is not surprising given that the ADF test assumes a strict I(0) and I(1) distinction and the GPH test requires estimation of the differencing parameter, whereas Robinson's (1994c) tests do allow fractional differencing and do not require estimation of the fractional differencing parameter. The performance of Robinson's (1994c) tests in this context of cointegration is examined at the end of each of the examples considered. We start now describing the first of these relationships.

TABLE 6.1

Empirical sizes of Robinson's (1994c) tests for cointegration.*

T = 50

d:	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
Perc.										
0.1%	-2.94	-2.94	-2.95	-2.93	-2.93	-2.93	-2.93	-2.92	-2.93	-2.92
0.5%	-2.65	-2.66	-2.66	-2.67	-2.66	-2.66	-2.66	-2.66	-2.65	-2.65
1.0%	-2.51	-2.52	-2.53	-2.52	-2.52	-2.52	-2.52	-2.51	-2.50	-2.50
2.5%	-2.29	-2.30	-2.31	-2.30	-2.30	-2.30	-2.29	-2.29	-2.28	-2.27
5.0%	-2.09	-2.10	-2.11	-2.11	-2.10	-2.09	-2.08	-2.08	-2.07	-2.07
10.0%	-1.84	-1.85	-1.85	-1.84	-1.84	-1.84	-1.83	-1.82	-1.82	-1.81
20.0%	-1.50	-1.51	-1.51	-1.51	-1.50	-1.50	-1.49	-1.49	-1.48	-1.48
30.0%	-1.25	-1.26	-1.26	-1.26	-1.25	-1.25	-1.24	-1.23	-1.23	-1.22
40.0%	-1.02	-1.03	-1.03	-1.03	-1.02	-1.01	-1.01	-1.00	-0.99	-0.99
50.0%	-0.79	-0.81	-0.81	-0.80	-0.80	-0.79	-0.78	-0.78	-0.77	-0.76
60.0%	-0.55	-0.57	-0.57	-0.57	-0.56	-0.55	-0.54	-0.54	-0.53	-0.53
70.0%	-0.29	-0.30	-0.31	-0.30	-0.30	-0.29	-0.28	-0.27	-0.26	-0.26
80.0%	0.04	0.02	0.01	0.01	0.02	0.03	0.03	0.05	0.06	0.06
90.0%	0.52	0.50	0.50	0.50	0.51	0.52	0.53	0.54	0.54	0.55
95.0%	0.96	0.94	0.93	0.93	0.94	0.95	0.96	0.97	0.97	0.97
97.5%	1.35	1.34	1.34	1.36	1.37	1.38	1.38	1.38	1.39	1.40
99.0%	1.87	1.85	1.86	1.86	1.87	1.87	1.87	1.88	1.88	1.88
99.5%	2.30	2.27	2.24	2.24	2.25	2.24	2.22	2.23	2.25	2.23
99.9%	3.06	3.08	3.08	3.07	3.04	3.03	3.05	3.04	3.06	3.01
Mean:	-0.70	-0.72	-0.72	-0.72	-0.71	-0.70	-0.70	-0.69	-0.68	-0.68
Skewness	0.59	0.59	0.59	0.59	0.58	0.57	0.56	0.56	0.55	0.54
Kurtosis	3.67	3.68	3.69	3.70	3.68	3.64	3.60	3.59	3.53	3.50

T = 100

d:	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
Perc.										
0.1%	-2.96	-2.95	-2.95	-2.97	-2.96	-2.94	-2.95	-2.96	-2.96	-2.96
0.5%	-2.64	-2.65	-2.64	-2.63	-2.63	-2.62	-2.62	-2.61	-2.60	-2.60
1.0%	-2.48	-2.49	-2.48	-2.48	-2.47	-2.47	-2.46	-2.45	-2.45	-2.44
2.5%	-2.23	-2.24	-2.24	-2.23	-2.23	-2.22	-2.21	-2.21	-2.20	-2.20
5.0%	-2.01	-2.00	-2.00	-2.01	-2.00	-2.00	-1.99	-1.99	-1.99	-1.98
10.0%	-1.74	-1.75	-1.75	-1.74	-1.74	-1.72	-1.71	-1.71	-1.71	-1.70
20.0%	-1.38	-1.39	-1.39	-1.38	-1.38	-1.37	-1.36	-1.36	-1.35	-1.35
30.0%	-1.11	-1.12	-1.12	-1.12	-1.11	-1.10	-1.09	-1.09	-1.08	-1.08
40.0%	-0.87	-0.88	-0.88	-0.88	-0.87	-0.86	-0.85	-0.84	-0.84	-0.84
50.0%	-0.63	-0.65	-0.65	-0.64	-0.64	-0.63	-0.62	-0.61	-0.61	-0.60
60.0%	-0.39	-0.40	-0.40	-0.40	-0.39	-0.38	-0.38	-0.37	-0.36	-0.36
70.0%	-0.11	-0.13	-0.13	-0.13	-0.12	-0.12	-0.11	-0.10	-0.09	-0.09
80.0%	0.21	0.19	0.19	0.19	0.20	0.20	0.21	0.22	0.22	0.22
90.0%	0.69	0.68	0.67	0.67	0.67	0.68	0.69	0.70	0.71	0.72
95.0%	1.11	1.10	1.10	1.10	1.11	1.11	1.12	1.13	1.13	1.14
97.5%	1.50	1.49	1.48	1.49	1.49	1.50	1.49	1.50	1.51	1.52
99.0%	1.99	1.98	1.96	1.96	1.97	1.97	1.98	1.97	1.98	1.98
99.5%	2.36	2.31	2.29	2.29	2.29	2.31	2.33	2.36	2.34	2.34
99.9%	3.15	3.13	3.11	3.11	3.09	3.08	3.08	3.10	3.13	3.12
Mean:	-0.56	-0.57	-0.58	-0.57	-0.56	-0.56	-0.55	-0.54	-0.54	-0.53
Skewness	0.46	0.46	0.46	0.45	0.45	0.45	0.45	0.44	0.44	0.45
Kurtosis	3.41	3.40	3.39	3.39	3.40	3.40	3.38	3.35	3.34	3.34

cont...

T = 200

d:	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
Perc.										
0.1%	-3.04	-3.08	-3.07	-3.14	-3.19	-3.20	-3.12	-3.12	-3.10	-3.06
0.5%	-2.71	-2.73	-2.70	-2.70	-2.66	-2.64	-2.62	-2.61	-2.63	-2.64
1.0%	-2.50	-2.48	-2.47	-2.46	-2.45	-2.46	-2.45	-2.44	-2.44	-2.43
2.5%	-2.21	-2.20	-2.20	-2.21	-2.20	-2.20	-2.20	-2.19	-2.18	-2.18
5.0%	-1.95	-1.97	-1.97	-1.97	-1.97	-1.96	-1.94	-1.94	-1.93	-1.93
10.0%	-1.64	-1.65	-1.67	-1.66	-1.66	-1.65	-1.63	-1.62	-1.62	-1.61
20.0%	-1.26	-1.28	-1.28	-1.29	-1.28	-1.27	-1.26	-1.25	-1.25	-1.25
30.0%	-0.89	-1.00	-1.01	-1.00	-1.00	-0.99	-0.99	-0.98	-0.98	-0.97
40.0%	-0.74	-0.75	-0.75	-0.75	-0.74	-0.73	-0.73	-0.73	-0.72	-0.72
50.0%	-0.49	-0.50	-0.52	-0.51	-0.51	-0.50	-0.49	-0.48	-0.48	-0.48
60.0%	-0.24	-0.26	-0.27	-0.27	-0.26	-0.25	-0.24	-0.23	-0.23	-0.23
70.0%	0.02	0.01	0.01	0.01	0.01	0.02	0.03	0.03	0.03	0.04
80.0%	0.36	0.33	0.32	0.31	0.33	0.33	0.34	0.35	0.36	0.36
90.0%	0.83	0.81	0.79	0.79	0.79	0.79	0.81	0.81	0.81	0.81
95.0%	1.22	1.20	1.19	1.19	1.21	1.22	1.23	1.23	1.24	1.27
97.5%	1.59	1.55	1.52	1.54	1.55	1.58	1.59	1.60	1.63	1.64
99.0%	2.07	2.02	2.00	1.99	2.01	2.02	2.06	2.07	2.08	2.11
99.5%	2.36	2.34	2.34	2.37	2.33	2.38	2.37	2.48	2.49	2.49
99.9%	2.85	2.80	2.79	2.84	2.98	3.03	3.10	3.03	3.05	3.07
Mean:	-0.44	-0.46	-0.46	-0.46	-0.46	-0.45	-0.44	-0.43	-0.43	-0.42
Skewness	0.31	0.30	0.30	0.31	0.32	0.32	0.33	0.34	0.35	0.36
Kurtosis	3.18	3.17	3.18	3.23	3.26	3.27	3.28	3.28	3.30	3.31

T = 300

d:	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
Perc.										
0.1%	-2.96	-2.96	-3.04	-3.12	-3.19	-3.22	-3.19	-3.17	-3.17	-3.15
0.5%	-2.52	-2.56	-2.63	-2.61	-2.60	-2.59	-2.60	-2.59	-2.61	-2.61
1.0%	-2.41	-2.42	-2.44	-2.45	-2.44	-2.44	-2.44	-2.44	-2.44	-2.44
2.5%	-2.17	-2.18	-2.19	-2.20	-2.18	-2.17	-2.16	-2.14	-2.13	-2.13
5.0%	-1.90	-1.91	-1.92	-1.92	-1.91	-1.90	-1.89	-1.88	-1.87	-1.87
10.0%	-1.59	-1.60	-1.60	-1.61	-1.60	-1.60	-1.60	-1.59	-1.58	-1.58
20.0%	-1.20	-1.20	-1.22	-1.22	-1.21	-1.21	-1.21	-1.21	-1.20	-1.19
30.0%	-0.92	-0.93	-0.93	-0.93	-0.92	-0.91	-0.91	-0.90	-0.90	-0.89
40.0%	-0.67	-0.68	-0.68	-0.68	-0.67	-0.67	-0.67	-0.66	-0.65	-0.65
50.0%	-0.43	-0.44	-0.44	-0.43	-0.43	-0.43	-0.43	-0.42	-0.42	-0.42
60.0%	-0.18	-0.19	-0.19	-0.19	-0.19	-0.19	-0.18	-0.18	-0.18	-0.18
70.0%	0.09	0.07	0.07	0.07	0.07	0.08	0.09	0.09	0.09	0.09
80.0%	0.41	0.40	0.38	0.38	0.40	0.41	0.41	0.42	0.42	0.42
90.0%	0.91	0.89	0.87	0.87	0.87	0.87	0.88	0.89	0.89	0.89
95.0%	1.33	1.31	1.30	1.29	1.29	1.30	1.31	1.31	1.31	1.34
97.5%	1.68	1.65	1.67	1.66	1.69	1.71	1.73	1.74	1.74	1.75
99.0%	2.10	2.08	2.07	2.10	2.12	2.12	2.12	2.15	2.15	2.15
99.5%	2.40	2.32	2.33	2.35	2.41	2.41	2.44	2.41	2.42	2.41
99.9%	2.92	2.90	2.88	2.87	2.91	2.94	2.85	2.84	2.84	2.84
Mean:	-0.37	-0.39	-0.39	-0.39	-0.39	-0.38	-0.38	-0.37	-0.37	-0.36
Skewness	0.31	0.29	0.28	0.28	0.29	0.29	0.30	0.30	0.30	0.30
Kurtosis	3.07	3.08	3.10	3.13	3.15	3.15	3.15	3.14	3.13	3.13

*: The empirical size is obtained based on 50,000 replications in simulation, assuming that the true system is two non-cointegrated I(d) processes. The test statistic is $\tilde{r}$ in (2.3), and the Fortran code used in this experiment is given in Appendix 6.1.

TABLE 6.2

Power of the ADF, GPH and Robinson tests for cointegration against fractional alternatives: $(1 - L)^d u_{2t} = \varepsilon_{2t}$.*

		d									
Size	Test	.95	.85	.75	.65	.55	.45	.35	.25	.15	.05
5%	ADF (p = 4)	.06	.07	.10	.14	.19	.26	.36	.50	.61	.73
	GPH (μ = .55)	.06	.09	.15	.21	.30	.37	.47	.56	.61	.64
	GPH (μ = .575)	.06	.10	.16	.24	.33	.42	.53	.62	.67	.71
	GPH (μ = .60)	.06	.11	.18	.28	.40	.52	.63	.73	.78	.81
	ROBINSON (W.N.)	**.07**	**.22**	**.50**	**.78**	**.94**	**.99**	**.99**	**1.00**	**1.00**	**1.00**
	ROBINSON (AR(1))	**.15**	**.22**	**.35**	**.52**	**.71**	**.85**	**.94**	**.97**	**.99**	**.99**
	ROBINSON (AR(2))	**.22**	**.26**	**.31**	**.41**	**.54**	**.67**	**.78**	**.86**	**.92**	**.95**
	ROBINSON (AR(3))	**.30**	**.32**	**.35**	**.41**	**.50**	**.59**	**.68**	**.76**	**.82**	**.85**
10%	ADF (p = 4)	.11	.13	.18	.24	.32	.41	.53	.67	.78	.87
	GPH (μ = .55)	.12	.17	.26	.35	.46	.56	.65	.72	.76	.78
	GPH (μ = .575)	.12	.18	.27	.38	.50	.60	.71	.77	.81	.83
	GPH (μ = .60)	.12	.19	.30	.43	.57	.68	.79	.85	.88	.90
	ROBINSON (W.N.)	**.16**	**.37**	**.66**	**.88**	**.97**	**.99**	**1.00**	**1.00**	**1.00**	**1.00**
	ROBINSON (AR(1))	**.26**	**.36**	**.51**	**.69**	**.84**	**.94**	**.98**	**.99**	**.99**	**.99**
	ROBINSON (AR(2))	**.32**	**.37**	**.45**	**.57**	**.69**	**.81**	**.89**	**.94**	**.97**	**.98**
	ROBINSON (AR(3))	**.40**	**.43**	**.47**	**.55**	**.64**	**.73**	**.81**	**.87**	**.91**	**.94**

Power of the ADF, GPH and Robinson tests for cointegration against autoregressive alternatives: $(1 - \phi L)u_{2t} = \varepsilon_{2t}$.*

		φ									
Size	Test	.95	.85	.75	.65	.55	.45	.35	.25	.15	.05
5%	ADF (p = 4)	.07	.16	.29	.42	.53	.61	.66	.73	.75	.77
	GPH (μ = .55)	.07	.17	.33	.49	.59	.64	.67	.69	.68	.66
	GPH (μ = .575)	.07	.17	.35	.52	.63	.69	.73	.75	.74	.72
	GPH (μ = .60)	.07	.18	.37	.56	.70	.76	.81	.84	.83	.83
	ROBINSON (W.N.)	**.07**	**.21**	**.46**	**.72**	**.90**	**.98**	**.99**	**1.00**	**1.00**	**1.00**
	ROBINSON (AR(1))	**.18**	**.36**	**.59**	**.76**	**.88**	**.94**	**.97**	**.98**	**.99**	**.99**
	ROBINSON (AR(2))	**.27**	**.42**	**.58**	**.70**	**.80**	**.86**	**.90**	**.93**	**.95**	**.96**
	ROBINSON (AR(3))	**.37**	**.49**	**.60**	**.69**	**.75**	**.80**	**.83**	**.86**	**.87**	**.88**
10%	ADF (p = 4)	.14	.28	.46	.60	.71	.78	.82	.86	.88	.89
	GPH (μ = .55)	.14	.29	.50	.66	.75	.78	.81	.82	.81	.79
	GPH (μ = .575)	.14	.30	.52	.69	.78	.82	.85	.86	.85	.84
	GPH (μ = .60)	.14	.30	.54	.72	.82	.87	.90	.91	.92	.91
	ROBINSON (W.N.)	**.16**	**.38**	**.65**	**.87**	**.97**	**.99**	**.99**	**1.00**	**1.00**	**1.00**
	ROBINSON (AR(1))	**.30**	**.54**	**.76**	**.89**	**.95**	**.98**	**.99**	**.99**	**.99**	**.99**
	ROBINSON (AR(2))	**.39**	**.58**	**.74**	**.84**	**.90**	**.94**	**.96**	**.97**	**.98**	**.98**
	ROBINSON (AR(3))	**.47**	**.63**	**.74**	**.82**	**.87**	**.90**	**.92**	**.93**	**.94**	**.95**

*: ADF is augmented Dickey-Fuller test statistic and p is the lag parameter selected using AIC and SIC criterions. GPH is Geweke-Porter-Hudak test statistic and μ is the value used in the sample-size function $n=T^{\mu}$. Results for ADF and GPH have been taken from Cheung and Lai (1993), (pages 108 and 109). Robinson's tests are $\bar{r}$ in (2.3) and $\hat{r}$ in (2.9). The power of each test is based on 10,000 replications and the Monte Carlo experiment with the Fortran code is described in Appendix 6.2.

6.a CONSUMPTION AND INCOME

The data are U.S. quarterly real per capita consumption on non-durables and real per capita disposable income from 1947.I to 1981.II and plots of the series are given in the upper part of Figure 6.A1. In these plots we observe that both series seem to present similar nonstationary behaviour, increasing slowly during the 50's, growing at a higher rate since 1960 with a sharp decay after the crisis in 1973, and increasing strongly afterwards. The nonstationary character of these two series can be better viewed through the other plots in this figure, which show the sample autocorrelations and estimates of the spectral density function[1]: we observe here a very slow and persistent decay in the autocorrelations, and a peak on the estimated spectrums at zero frequency, which might indicate that both series require some kind of differencing in order to get stationarity.

These two series were analyzed from an error correction point of view in Davidson, Hendry, Srba and Yeo (DHSY, 1978) and from a time series viewpoint in Hall (1978) and others. In the first of these studies, evidence was presented for the error correction model of consumption behaviour from both theoretical and empirical points of view: consumers make plans which may be frustrated; they adjust next period's plans to recoup a portion of the error between consumption and income. Hall (1978) found evidence that U.S. consumption was a random walk and that past values of income had no explanatory power which implied that income and consumption were not cointegrated. Neither of these studies modelled income itself and it was taken as exogenous in DHSY (1978). Engle and Granger (1987) performed first the DF and ADF tests to check if both individual series were I(1). Then they performed several cointegration tests in order to check if both variables were in fact cointegrated, concluding that they were, though income may be exogenous in view of the error correction representation. Using the same data set, DeJong (1992) used a Bayesian approach to analyze the cointegration inference in these variables and he concluded that when trend-stationary was given zero prior probability the cointegration inference was often supported. When this prior was relaxed, however, the data supported the trend stationary representations.

[1] As in previous chapters, they are estimates of the standardized spectral density function, using Barlett, Tukey and Parzen lag windows of size T-1.

As we have just mentioned above, Engle and Granger (1987) started by testing if the two series were individually integrated of order 1. They regressed the change in consumption on its past level and two past changes obtaining a t-test of 0.77, and therefore suggesting that the series was not stationary in level. Running the same model with second differences on lagged first differences and two lags of second differences, the t-test was -5.26 indicating that the first difference was stationary. For income, four past lags were used and the two test statistics were -0.01 and -6.27 respectively, again establishing that income was I(1).

In Table 6.A1 we perform Robinson's (1994c) univariate tests in order to investigate more deeply what the appropriate integration order for each of the individual series might be. Therefore, we consider the same model as that used in Chapter 3 for Nelson and Plosser's data. In both series we calculate the one-sided test statistic $\hat{r}$ in (2.9), for different specifications in the regression model (i.e., including no regressors, an intercept, and an intercept and a time trend), and for different modelizations for the disturbances (as white noise, and seasonal and non-seasonal AR processes). In this table we observe that the unit root null hypothesis is never rejected in either of the series, and though there are some other cases where the null is also non-rejected, (as when d takes the value 0.9 or 1.1), the lowest statistics across different values of d are obtained in practically all situations when d = 1. If the disturbances follow a non-seasonal AR, we observe in some cases, a lack of monotonic decrease in $\hat{r}$ with respect to d. This may be due to the fact that the data are quarterly, and though they are deseasonalized, certain seasonal structure may remain, especially for consumption. In fact, if the disturbances follow a seasonal AR process, monotonicity is always achieved, with the non-rejection values ranging again from 0.9 through 1.1, and with the lowest statistics corresponding to the unit root case.

In view of this table we can say that the two series are I(1) though slight variations in the integration order might also be plausible, which is not at all surprising given the smoothness in the behaviour of the fractional processes apart from the case of the boundary situation between stationary and non-stationary processes, i.e., when d = 0.5. In this case, the null was always decisively rejected in favour of more nonstationary alternatives, indicating strong evidence against the trend-stationary representations. We also observed that the results are not greatly affected by the different regressors in the model, and they seem to corroborate the findings of Engle and Granger (1987) and others, that both series are integrated of order 1. Next, in the following four tables, we present results of the multivariate tests.

We start by specifying the model in its more general form which, in this bivariate set-up will take the form

$$\begin{pmatrix} y_t \\ z_t \end{pmatrix} = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}\begin{pmatrix} 1 \\ t \end{pmatrix} + \begin{pmatrix} X_{1t} \\ X_{2t} \end{pmatrix} \qquad t = 1,2,...T \tag{13}$$

and

$$\begin{pmatrix} (1-L)^{d_1+\theta_1} & 0 \\ 0 & (1-L)^{d_2+\theta_2} \end{pmatrix}\begin{pmatrix} X_{1t} \\ X_{2t} \end{pmatrix} = \begin{pmatrix} U_{1t} \\ U_{2t} \end{pmatrix} \qquad t=1,2,...T \tag{14}$$

with

$$X_{it} = 0 \qquad \text{for } i = 1,2 \qquad \text{and} \qquad t \leq 0, \tag{15}$$

where y_t and z_t are the original time series, (in this case, consumption and income); $U_t = (U_{1t}, U_{2t})'$ is a stationary I(0) vector process, and the null hypothesis is given by:

$$H_o\text{: } \theta_1 = \theta_2 = 0. \tag{16}$$

We present results of the score test statistics in (5.32) - (5.37), depending basically on the choice for the disturbance vector U_t in (14), and the inclusion or not of restrictions in the elements of the matrix B in (13), and give results for values of d_1 and d_2, ranging from 0.60 through 1.40 with 0.10 increments.

In Tables 6.A2 and 6.A3 we report results of the score test statistics in (5.32) and (5.34), respectively, i.e., the time and the frequency domain versions of the tests for white noise U_t. In both tables we start presenting results imposing B = 0 a priori, i.e., including no regressors in (13); then, we take $B_{12} = B_{22} = 0$ a priori, i.e., including only an intercept, and finally we consider the model in its more general form, i.e., imposing no restrictions on the regression model (13). Clearly, if $d_1 = d_2 = 1$, the model behaves, for t > 1, as a random walk vector process if $B_{12} = B_{22} = 0$, and as a random walk with an intercept if $B \neq 0$. The first thing we observe in these two tables is that results are very similar in both domains, with all non-rejection cases occurring for the same values of d_1 and d_2 in both tables; thus, the difference between the time and the frequency domain versions of the tests seems small even though the sample size is not very large (T = 138). In the upper part of these two tables we observe that when there are no regressors, the only non-rejection case occurs when $d_1 = d_2 = 0.9$, and any departure from this case increases strongly the value of the test statistic. Thus, we see in Table 6.A2 that if there are no regressors, the lowest statistic is 3.68 corresponding to $d_1 = d_2 = 0.9$, and the closest departures are $d_1 = d_2 = 1$ and $d_1 = d_2 = 0.8$ with $\hat{S}^{t2} = 7.00$ and 9.52 respectively. Similarly, in the upper part of Table 6.A3, the only non-rejection case is $d_1 = d_2 = 0.9$ with a test statistic of 3.84, and again the closest departures are $d_1 = d_2 = 0.8$ and $d_1 = d_2 = 1$ with $\hat{S}^{f2} = 7.02$ and 7.64 respectively. Including an intercept or a time trend there are more non-rejections and all of them occur for values of d_1 and d_2 around 1, with the lowest statistics again obtained at $d_1 = d_2 = 0.9$ in both cases and in both tables. The fact that the null hypothesis is not rejected in these tables for the case of two unit roots but is rejected for any value of d_1 and d_2 smaller than 0.9 or greater than 1.1 corroborates the results of Table 6.A1 that both series might be integrated of order 1.

In the next two tables, a richer structure is allowed in U_t. First, in Table 6.A4, we assume U_t is VAR(1) and we observe in the upper part of this table that if there are no regressors, the null is always decisively rejected. Thus, modelling these series with no regressors appears not to be a correct way of specifying the model. Including an intercept or a time trend, we observe some non-rejections, always for values of d_1 and d_2 smaller than 1.2 and in most of cases for values of d_1 (the integration order of consumption) equal or slightly smaller than d_2 (the integration order of income). The lowest statistics are now obtained at $d_1 = 0.6$ and $d_2 = 0.7$ when including an intercept, and at $d_1 = 0.7$ and $d_2 = 0.8$ when a time trend is considered. Results in this table appear less nonstationary than in the previous ones, (related with white noise U_t), but this can be explained by the fact that the parameters in the VAR representation have been obtained using the method of maximum likelihood throughout a quasi-Newton algorithm, and in some cases these parameters can be close to nonstationary. In fact, looking at the lower part of the table, the lowest statistic when including a time trend is obtained at $d_1 = 0.6$ and $d_2 = 0.7$ taking a value of 0.28, and with an estimated structure on U_t as follows

$$\begin{pmatrix} U_{1t} \\ U_{2t} \end{pmatrix} = \begin{pmatrix} 0.36 & -0.08 \\ 1.03 & 0.13 \end{pmatrix} \begin{pmatrix} U_{1t-1} \\ U_{2t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix}.$$

Therefore, the determinant of the AR polynomial evaluated at $z = 1$ takes the value of 0.639. However, though it is not shown in the table, other non-rejection cases are also obtained for values of d_1 and d_2 smaller than 0.6, and as we approximate to stationary, the determinant in these situations will be approximately zero. For example, when $d_1 = 0.5$ and $d_2 = 0.6$, the value of the test statistic is 0.79 and the determinant is 0.309, and if $d_1 = 0.4$ and $d_2 = 0.5$, the value of the test is 1.57 and the corresponding value of the determinant is 0.185. Thus, we could say that competition between the VAR structure on U_t and the orders of differencing may exist, for picking up the nonstationary component of the series, and as these parameters in the VAR representation approximate to the nonstationary case, integration orders seem to be smaller in both series. We also see here that the null hypothesis of two unit roots ($d_1 = d_2 = 1$) is not rejected in this case, and a model like this, for $t > 1$, behaves like the one performed in Engle and Granger (1987) though they allowed higher order autoregressions. They used this unrestricted VAR representation to establish that the joint distribution of consumption and income was an error correction model, through a way of eliminating those parameters that were not significant in the VAR representation. Our results show that though this case of $d_1 = d_2 = 1$ may be possible, other fractional possibilities for d_1 and d_2 might be even more plausible, in view of the lower statistics obtained in some cases.

Finally in these multivariate tests, we present results when U_t is VMA(1). Results are given in Table 6.A5, and as in Table 6.A4, if we do not include regressors, the null is always rejected, suggesting that a model with no regressors is not the correct way of specifying it. We also see here that the lowest statistic occurs in this case at $d_1 = d_2 = 1$, with $\tilde{S}^2 = 7.63$. Including a constant or a time trend there are a lot of non-rejections and all of them correspond to values of d_1 and d_2 greater than or equal to 0.7. This is not surprising given that the VMA(1) structure on U_t is always stationary and therefore, the nonstationarity character of the series must be picked up mainly through the differencing orders. We also observe in this table that the lowest statistics with an intercept and with a time trend are achieved in both cases at $d_1 = d_2 = 0.9$, that is, for the same values as in Tables 6.A2 and 6.A3 when

U_t was white noise. As a conclusion of these multivariate tests it seems clear that both series are nonstationary with integration orders fluctuating around 1 in most of the cases, but also smaller if U_t follows a VAR representation and greater if it follows a VMA process.

In the final part of this section we try to find if a cointegrating relationship might possibly exist between these two variables. In order to examine this problem, we run the regression of consumption (c_t) on income (y_t) and a constant, and its reverse, as was done in Engle and Granger (1987), and the resulting equations were

$$c_t = \underset{(85)}{0.52} + \underset{(123)}{0.23}\, y_t \qquad \text{(t-values)} \tag{17}$$

and

$$y_t = \underset{(-50)}{-0.22} + \underset{(123)}{4.30}\, c_t. \qquad \text{(t-values)} \tag{18}$$

In Table 6.A6 we have performed Robinson's (1994c) univariate tests on these estimated residuals, and the structure in this table is similar to Table 6.A1, showing results of $\hat{r}$ in (2.9) for the different cases of no regressors, an intercept, and an intercept and a time trend. The most noticeable thing observed here is that the unit root null is rejected in practically all cases (except when the disturbances follow an AR(1) process, but in this case there is a wide range of values of d where the null is not rejected, and furthermore, monotonicity is not achieved for values of d smaller than 0.6). Apart from this situation, all the other non-rejections always take place for values of d smaller than or equal to 0.8, which is in sharp contrast to results presented in Table 6.A1 for the original time series, where the null was not rejected when d was greater than or equal to 0.9. Therefore these results suggest that both series are fractionally cointegrated given that the estimated residuals display a lower integration order than the individual series. We also observe in this table that the lowest statistics across different values of d are always obtained at d = 0.7, independently of the inclusion or not of an intercept and/or a time trend in the model, indicating therefore that the estimated residuals from the cointegrating regressions are still nonstationary but with a mean-reversion property.

Engle and Granger (1987) studied the cointegration relationship between these two variables, testing the null of nonstationarity in the estimated residuals in (17) and (18) and therefore, testing the null hypothesis of no cointegration between them. Using the cointegrating regression Durbin-Watson test (CRDW), the null was rejected at 5% significance level but hardly at 1%, and using the DF and ADF tests, was rejected for the latter but hardly for the former even at 5% significance level. A problem with these testing procedures is that they only concentrate on the case of I(0) residuals and do not consider other fractionally integrated possibilities.

In Table 6.A7 we again use Robinson's (1994c) univariate tests in order to check if these estimated residuals might be stationary. Therefore, we perform the same test statistic as in Table 6.A6, but now choosing values of d ranging from 0.00 through 0.50. We see in this table that the null is always rejected, and even at the boundary case of d = 0.50, the null is decisively rejected in favour of more nonstationary alternatives and thus, finding conclusive evidence against stationary residuals.

Therefore, we have found certain evidence of fractional cointegration for consumption and income, with the deviations from an equilibrium following a nonstationary fractional process with the integration order smaller than one. The distinction between I(d) processes with d = 1 and d < 1 is important from an economic point of view: if x_t is an I(d) process with $d \in [0.5, 1)$, the process will be covariance nonstationary but mean-reverting since an innovation will have no permanent effect on the value of x_t. This is in contrast to an I(1) process which will be both covariance nonstationary and not mean-reverting, and the effect of an innovation will persist forever. Results presented above give evidence that the equilibrium errors display mean reversion and the effect of a shock to the system will eventually die out so that an equilibrium relationship between consumption and income will prevail in the long run.

As a conclusion, we can summarize the main results obtained in this section by saying that consumption and income both seem nonstationary with the integration order fluctuating around 1, independently of the inclusion or not of an intercept or a time trend in the model. This unit root behaviour observed in the series is obtained when we use the univariate representation of the tests but also when the multivariate tests are performed, though here, if U_t follows a VAR(1) process, integration orders can be smaller due to competition with VAR parameters in modelling the nonstationarity.

Finally, both series seem fractionally cointegrated with the integration order of the residuals in the cointegrating regressions fluctuating around 0.70, and therefore with the equilibrium errors displaying slow mean reversion, unlike the individual series where shocks seem to persist forever. These results are interesting in that they seem to connect the opposing findings between Hall (1978) and others, who came to the conclusion that both variables were not cointegrated, and Engle and Granger (1987) and others, who found cointegration between consumption and income. Our results suggest that both variables might be fractionally cointegrated, with nonstationary equilibrium errors but with a mean reverting behaviour.

FIGURE 6.A1

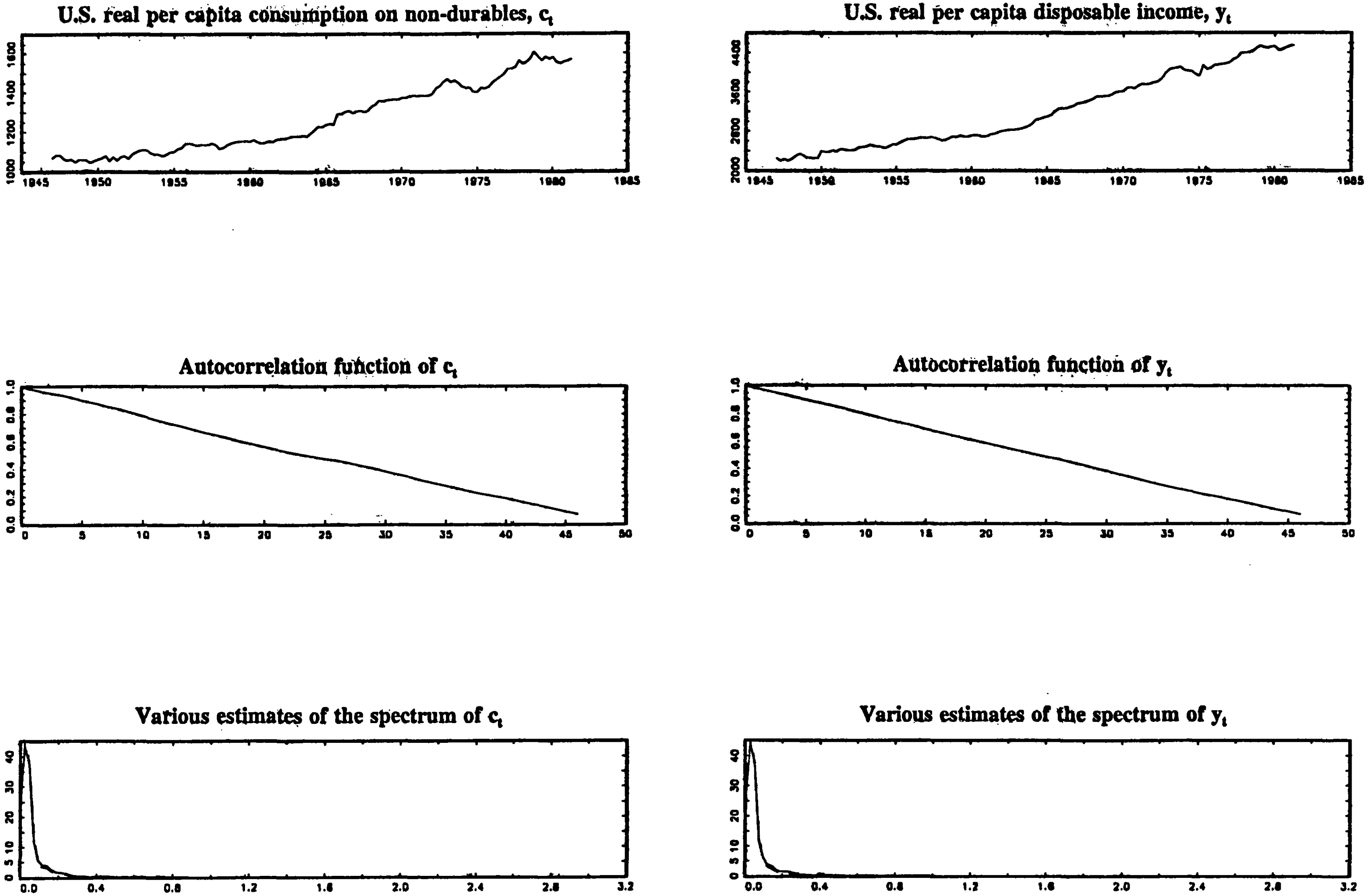

U.S. real per capita consumption on non-durables, c_t
1000 1200 1400 1600
1945 1950 1955 1960 1965 1970 1975 1980 1985
U.S. real per capita disposable income, y_t
2000 2800 3600 4400
1945 1950 1955 1960 1965 1970 1975 1980 1985
Autocorrelation function of c_t
0.0 0.2 0.4 0.6 0.8 1.0
0 5 10 15 20 25 30 35 40 45 50
Autocorrelation function of y_t
0.0 0.2 0.4 0.6 0.8 1.0
0 5 10 15 20 25 30 35 40 45 50
Various estimates of the spectrum of c_t
0 5 10 20 30 40
0.0 0.4 0.8 1.2 1.6 2.0 2.4 2.8 3.2
Various estimates of the spectrum of y_t
0 5 10 20 30 40
0.0 0.4 0.8 1.2 1.6 2.0 2.4 2.8 3.2

TABLE 6.A1

$\hat{r}$ in (2.9) for U.S. Consumption and Income

Consumption

d	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	7.13	4.81	2.74	1.00’	-0.40’	-1.49’	-2.33	-2.98	-3.50	-3.91
AR(1)	-0.02	0.24	0.06	-0.37	-0.88	-1.41	-1.89	-2.32	-2.70	-3.03
AR(2)	-1.40	-0.94	-0.81	-0.92	-1.18	-1.49	-1.81	-2.12	-2.41	-2.67
SAR(1)	5.52	4.09	2.51	0.96’	-0.39’	-1.50’	-2.36	-3.03	-3.55	-3.97
SAR(2)	5.39	4.02	2.49	0.96’	-0.39’	-1.50’	-2.37	-3.04	-3.56	-3.98
b) Intercept.										
W.N.	16.57	11.00	5.78	2.23	0.05’	-1.35’	-2.32	-3.04	-3.59	-4.03
AR(1)	-3.77	2.32	2.54	1.24	0.10	-0.73	-1.38	-1.93	-2.42	-2.85
AR(2)	-3.82	-2.51	0.18	0.13	-0.32	-0.68	-0.93	-1.16	-1.40	-1.65
SAR(1)	6.94	6.47	4.55	2.10	0.11’	-1.31’	-2.31	-3.06	-3.63	-4.08
SAR(2)	6.87	6.56	5.30	2.91	0.60’	-1.06’	-2.19	-2.97	-3.54	-3.98
c) Intercept and a time trend.										
W.N.	9.40	6.50	3.89	1.73’	0.04’	-1.25’	-2.24	-2.99	-3.57	-4.01
AR(1)	1.13	1.48	1.25	0.71	0.06	-0.62	-1.26	-1.85	-2.36	-2.82
AR(2)	-0.51	-0.37	-0.24	-0.25	-0.37	-0.57	-0.81	-1.07	-1.35	-1.62
SAR(1)	7.10	5.43	3.55	1.71’	0.11’	-1.20’	-2.22	-3.00	-3.60	-4.06
SAR(2)	7.62	6.14	4.32	2.37	0.57’	-0.91’	-2.04	-2.87	-3.49	-3.94

Income

d	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	6.74	4.40	2.36	0.65’	-0.73’	-1.80’	-2.63	-3.27	-3.77	-4.16
AR(1)	1.05’	0.80’	0.41’	-0.12’	-0.71’	-1.30’	-1.84’	-2.31	-2.72	-3.08
AR(2)	-0.54’	-0.57’	-0.63’	-0.81’	-1.09’	-1.44’	-1.79’	-2.14	-2.46	-2.75
SAR(1)	5.22	3.68	2.09	0.58’	-0.75’	-1.82’	-2.67	-3.32	-3.82	-4.22
SAR(2)	5.15	3.67	2.11	0.60’	-0.74’	-1.84’	-2.69	-3.34	-3.84	-4.24
b) Intercept.										
W.N.	18.35	13.52	7.24	2.55	-0.09’	-1.55’	-2.44	-3.08	-3.56	-3.95
AR(1)	-3.32	1.83	3.37	1.41	-0.30	-1.35	-2.01	-2.47	-2.85	-3.18
AR(2)	-3.87	-1.37	2.11	1.20	-0.12	-0.98	-1.46	-1.75	-1.96	-2.14
SAR(1)	6.87	7.35	6.02	2.85	0.17’	-1.49’	-2.51	-3.18	-3.68	-4.07
SAR(2)	7.53	6.12	4.82	2.60	0.19’	-1.48’	-2.52	-3.21	-3.70	-4.09
c) Intercept and a time trend.										
W.N.	11.10	7.28	4.06	1.61’	-0.17’	-1.43’	-2.35	-3.03	-3.55	-3.95
AR(1)	3.59	3.09	1.96	0.75’	-0.31’	-1.17’	-1.85’	-2.40	-2.83	-3.18
AR(2)	1.58	1.86	1.31	0.57	-0.16	-0.78	-1.27	-1.66	-1.94	-2.15
SAR(1)	7.73	6.22	4.12	1.94’	0.07’	-1.35’	-2.39	-3.13	-3.67	-4.07
SAR(2)	6.70	5.53	3.86	1.92’	0.11’	-1.33’	-2.39	-3.15	-3.69	-4.09

’: Non-rejection values of the null hypothesis (1.12) with $\rho(L;\theta) = (1-L)^{d+\theta}$ at 95% significance level when monotonicity in the test statistic with respect to d is observed.

TABLE 6.A2

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with no regressors and white noise U_t. d_1 and d_2 are the differencing orders for consumption and income respectively.

d_1\d_2	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	158.76	214.49	259.27	265.21	254.36	237.55	219.35	201.78	185.71
0.7	113.59	45.57	135.21	187.45	197.06	189.79	176.82	162.57	148.85
0.8	195.62	67.13	9.52	83.65	129.55	140.33	136.53	127.68	117.58
0.9	233.77	157.57	45.19	3.68'	54.21	88.82	98.92	97.78	92.45
1.0	236.76	187.60	116.69	29.20	7.00	39.79	63.65	72.10	72.57
1.1	225.45	186.93	140.02	80.07	19.67	12.65	34.16	49.93	56.52
1.2	209.33	175.59	139.38	99.20	53.30	16.55	18.51	33.07	43.43
1.3	192.43	161.33	130.62	100.35	69.17	37.27	17.93	23.92	34.09
1.4	176.46	147.11	119.77	95.00	72.17	50.03	29.84	21.64	28.72

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with an intercept and white noise U_t. d_1 and d_2 are the differencing orders for consumption and income respectively.

d_1\d_2	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	184.65	112.00	136.87	179.24	205.61	217.40	221.66	222.83	222.94
0.7	232.62	83.11	44.39	62.31	83.46	96.33	103.07	106.55	108.44
0.8	281.84	105.17	20.48	10.54	20.06	29.06	35.21	39.33	42.21
0.9	311.28	132.30	27.67	1.14'	2.21'	7.26	11.67	15.18	18.01
1.0	323.68	149.17	38.83	4.77'	1.65'	4.59'	7.86	10.74	13.24
1.1	326.94	157.27	46.89	10.01	4.99'	7.02	9.79	12.34	14.63
1.2	326.68	160.62	51.95	14.44	8.62	10.28	12.86	15.31	17.51
1.3	325.59	161.85	55.15	17.92	11.85	13.36	15.90	18.34	20.54
1.4	324.62	162.26	57.25	20.64	14.59	16.06	18.62	21.09	23.33

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with a time trend and white noise U_t. d_1 and d_2 are the differencing orders for consumption and income respectively.

d_1\d_2	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	73.46	42.55	47.62	62.90	76.47	85.58	90.88	93.68	95.04
0.7	84.23	29.59	16.54	22.60	32.86	41.63	47.82	51.82	54.31
0.8	99.27	33.50	7.25	4.16'	9.46	16.03	21.55	25.68	28.61
0.9	111.15	42.72	9.56	0.30'	1.55'	5.80'	10.16	13.81	16.64
1.0	118.44	51.25	15.50	2.92'	1.65'	4.29'	7.70	10.82	13.41
1.1	122.12	57.37	21.34	7.33	4.72'	6.46	9.29	12.08	14.47
1.2	123.58	61.26	25.95	11.59	8.40	9.68	12.25	14.88	17.20
1.3	123.93	63.60	29.31	15.12	11.76	12.87	15.34	17.95	20.28
1.4	123.84	64.99	31.71	17.89	14.59	15.67	18.13	20.77	23.15

': Non-rejection values of the null hypothesis (5.4) at 95% significance level.

TABLE 6.A3

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with no regressors and white noise U_t. d_1 and d_2 are the differencing orders for consumption and income respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	122.55	172.91	215.06	221.57	212.69	198.34	182.67	167.51	153.64
0.7	104.72	33.79	111.52	157.70	166.76	160.97	150.09	138.00	126.29
0.8	184.68	61.22	7.02	72.15	112.35	122.14	119.24	111.84	103.25
0.9	217.32	143.00	40.55	3.84'	49.58	80.36	89.56	88.83	84.32
1.0	217.96	168.65	104.29	26.61	7.64	38.33	60.21	68.05	68.64
1.1	206.37	167.29	124.77	72.11	18.89	13.15	33.84	48.84	55.11
1.2	190.92	156.84	124.29	89.42	49.42	16.82	18.77	32.97	43.16
1.3	175.06	143.98	116.71	90.87	64.12	36.09	18.60	23.98	33.90
1.4	160.21	131.23	107.28	86.50	67.29	48.12	30.04	22.35	28.64

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with an intercept and white noise U_t. d_1 and d_2 are the differencing orders for consumption and income respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	157.37	95.36	120.74	157.40	179.09	188.52	191.90	192.86	193.02
0.7	203.96	68.94	37.29	53.74	72.31	83.64	89.71	92.96	94.82
0.8	247.10	88.92	16.75	8.94	17.74	26.10	31.92	35.89	38.69
0.9	270.05	112.12	23.24	0.85'	2.11'	6.99	11.33	14.81	17.60
1.0	278.38	126.19	33.18	4.30'	1.70'	4.59'	7.85	10.75	13.25
1.1	279.82	132.92	40.49	9.32	4.98'	6.96	9.71	12.29	14.59
1.2	279.00	135.81	45.22	13.64	8.60	10.19	12.75	15.20	17.42
1.3	277.89	137.00	48.29	17.05	11.81	13.27	15.77	18.22	20.43
1.4	277.11	137.53	50.38	19.73	14.53	15.97	18.48	20.96	23.21

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with a time trend and white noise U_t. d_1 and d_2 are the differencing orders for consumption and income respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	69.22	39.25	44.85	59.61	72.38	80.83	85.72	88.27	89.49
0.7	80.97	27.07	14.93	20.92	30.72	39.06	44.96	48.79	51.18
0.8	95.91	31.25	6.44	3.75'	8.92	15.27	20.64	24.67	27.54
0.9	106.91	40.06	8.77	0.23'	1.55'	5.72'	10.01	13.62	16.44
1.0	113.30	48.03	14.46	2.80'	1.70'	4.32'	7.69	10.80	13.39
1.1	116.36	53.75	20.08	7.11	4.73'	6.45	9.26	12.03	14.43
1.2	117.48	57.43	24.56	11.30	8.37	9.65	12.18	14.80	17.12
1.3	117.67	59.67	27.85	14.79	11.72	12.83	15.26	17.85	20.17
1.4	117.54	61.04	30.21	17.55	14.54	15.62	18.04	20.66	23.04

': Non-rejection values of the null hypothesis (5.4) at 95% significance level.

TABLE 6.A4

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with no regressors and a VAR(1) structure on U_t. d_1 and d_2 are the differencing orders for consumption and income respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	204.84	141.87	120.55	108.22	83.98	198.70	252.71	504.53	4512.0
0.7	411.69	253.45	144.94	116.66	91.14	141.66	177.43	334.94	775.54
0.8	96.71	599.55	220.34	95.25	53.30	104.83	138.94	263.32	514.89
0.9	96.07	102.14	106.35	177.36	115.98	109.78	162.26	341.17	623.47
1.0	172.19	129.77	103.78	75.04	23.51	376.02	440.21	585.36	743.66
1.1	254.57	200.31	199.41	575.05	113.04	26.59	133.87	318.95	550.06
1.2	8543.7	1800.7	980.56	131.15	175.73	98.58	23.72	117.00	353.22
1.3	119.10	102.25	13.51	170.54	219.82	124.11	62.39	19.96	191.08
1.4	655.37	551.04	127.18	246.93	361.62	310.23	165.95	57.48	17.51

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with an intercept and a VAR(1) structure on U_t. d_1 and d_2 are the differencing orders for consumption and income respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	6.39	0.98'	6.24	12.11	14.76	15.08	14.45	13.72	13.32
0.7	18.13	6.03	0.44'	7.82	16.59	22.40	25.32	26.28	26.09
0.8	23.47	22.59	3.65'	1.08'	7.95	15.51	21.22	24.93	27.00
0.9	22.15	36.20	15.79	1.92'	2.35'	7.08	12.23	16.71	20.20
1.0	18.50	42.28	27.99	8.30	3.09'	4.40'	7.50	11.09	14.58
1.1	14.94	42.91	35.81	15.60	7.04	5.83'	7.04	9.27	11.99
1.2	12.44	40.60	39.31	21.50	11.89	9.21	9.07	10.07	11.81
1.3	11.27	37.07	39.82	25.50	16.54	13.37	12.46	12.56	13.40
1.4	11.38	33.39	38.59	27.85	20.49	17.63	16.50	16.08	16.25

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with a time trend and a VAR(1) structure on U_t. d_1 and d_2 are the differencing orders for consumption and income respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	7.20	0.28'	3.64'	10.65	16.95	20.95	22.52	22.24	20.90
0.7	20.33	4.70'	0.34'	4.72'	11.86	18.34	22.82	25.08	25.47
0.8	32.32	15.05	2.56'	1.04'	5.63'	11.94	17.66	21.80	24.13
0.9	39.05	26.09	9.53	2.00'	2.39'	6.51	11.60	16.21	19.64
1.0	40.64	34.09	17.93	6.67	3.09'	4.34'	7.71	11.60	15.11
1.1	38.56	38.07	25.02	12.84	6.67	5.42'	6.86	9.46	12.30
1.2	34.40	38.57	29.62	18.64	11.55	8.69	8.55	9.84	11.74
1.3	29.57	36.68	31.70	23.07	16.45	12.98	11.86	12.14	13.14
1.4	35.21	33.63	31.90	25.96	20.65	17.41	15.95	15.63	15.96

': Non-rejection values of the null hypothesis (5.4) at 95% significance level.

TABLE 6.A5

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with no regressors and a VMA(1) structure on U_t. d_1 and d_2 are the differencing orders for consumption and income respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	999347	89912	75.37	80.31	83.55	91.00	109.73	166.80	521.03
0.7	------	------	34.38	46.92	54.66	62.72	75.02	97.33	160.94
0.8	54.31	398.45	2790.6	15.66	27.77	38.76	50.00	63.10	80.82
0.9	651.20	929.68	114.53	302.14	7.63	18.72	31.71	41.72	49.63
1.0	3311.8	5175.0	13017	3543.0	11.32	7.73	34.83	28.38	32.09
1.1	6793.2	5472.7	5703.4	6937.4	8866.2	29731	38.07	24.26	22.05
1.2	8109.1	5077.3	3902.3	3447.5	3343.2	2739.0	1873.9	73.13	18.91
1.3	8727.7	5379.6	3471.9	2461.0	1908.3	1553.8	1115.9	519.46	50.21
1.4	8285.1	6023.2	3600.7	2100.6	1338.8	933.86	679.56	607.46	138.35

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with an intercept and a VMA(1) structure on U_t. d_1 and d_2 are the differencing orders for consumption and income respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	67.90	37.18	53.88	75.16	86.76	90.29	88.61	83.61	77.60
0.7	94.77	30.46	13.55	26.53	40.02	46.94	48.38	45.81	40.97
0.8	111.08	44.92	7.19	3.34'	11.73	18.92	22.31	22.16	19.41
0.9	114.94	55.93	14.29	0.53'	1.59'	5.84'	8.96	9.92	8.85
1.0	112.26	59.87	20.57	3.98'	1.10'	1.88'	3.16'	3.98'	3.86'
1.1	107.04	59.29	23.45	7.05	2.99'	2.20'	1.45'	1.35'	1.49'
1.2	101.15	56.28	23.56	8.44	4.37'	3.43'	1.98'	0.89'	0.65'
1.3	95.45	52.06	21.72	8.18	4.67'	4.03'	2.91'	1.58'	0.78'
1.4	90.98	47.78	18.76	6.63	3.86'	3.66'	3.14'	2.22'	1.25'

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with a time trend and a VMA(1) structure on U_t. d_1 and d_2 are the differencing orders for consumption and income respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	24.52	10.82	16.81	27.99	36.90	41.46	41.49	37.91	34.04
0.7	34.56	8.89	3.40'	9.34	17.46	23.46	25.81	24.44	21.19
0.8	43.16	14.45	1.78'	1.07'	5.66'	10.79	14.01	14.47	12.74
0.9	47.51	20.36	5.16'	0.28'	1.08'	3.65'	6.02	7.42	7.24
1.0	48.11	24.05	9.10	2.56'	1.10'	1.25'	1.58'	2.81'	3.61'
1.1	46.05	25.14	11.69	5.06'	2.76'	1.85'	0.55'	0.51'	1.32'
1.2	42.24	23.99	12.42	6.55	4.26'	3.21'	1.62'	0.39'	0.41'
1.3	37.64	21.20	11.36	6.60	4.81'	4.06'	2.88'	1.54'	0.57'
1.4	33.56	17.72	9.04	5.32'	4.23'	3.94'	3.36'	2.54'	1.29'

': Non-rejection values of the null hypothesis (5.4) at 95% level and "---" means that the test statistic is greater than 99999.

TABLE 6.A6

$\hat{r}$ in (2.9) for the estimated residuals

c_t - 0.52 - 0.23 y_t

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	0.98'	-0.24'	-1.27'	-2.12	-2.83	-3.40	-3.87	-4.26	-4.58	-4.85
AR(1)	0.50'	-0.63'	-0.87'	-1.17'	-1.52'	-1.87'	-2.23	-2.58	-2.91	-3.23
AR(2)	-2.12	-2.01	-1.94	-1.90	-1.90	-1.93	-1.99	-2.06	-2.16	-2.28
SAR(1)	1.42'	0.06'	-1.12'	-2.10	-2.89	-3.52	-4.02	-4.42	-4.74	-5.01
SAR(2)	1.42'	0.05'	-1.14'	-2.12	-2.91	-3.54	-4.04	-4.43	-4.75	-5.01
b) Intercept.										
W.N.	0.99'	-0.32'	-1.44'	-2.36	-3.09	-3.66	-4.11	-4.47	-4.76	-4.99
AR(1)	0.07'	-0.05'	-0.40'	-0.87'	-1.38'	-1.85'	-2.29	-2.68	-3.04	-3.35
AR(2)	-1.69	-1.46	-1.38	-1.43	-1.55	-1.70	-1.85	-2.00	-2.15	-2.30
SAR(1)	1.47'	0.06'	-1.20'	-2.26	-3.10	-3.74	-4.23	-4.61	-4.90	-5.14
SAR(2)	1.51'	0.11'	-1.17'	-2.24	-3.09	-3.74	-4.24	-4.61	-4.91	-5.14
c) Intercept and a time trend.										
W.N.	1.17'	-0.24'	-1.42'	-2.36	-3.09	-3.66	-4.11	-4.47	-4.75	-4.98
AR(1)	0.35'	0.10'	-0.34'	-0.86'	-1.38'	-1.86'	-2.29	-2.68	-3.02	-3.31
AR(2)	-1.47	-1.33	-1.32	-1.41	-1.55	-1.70	-1.86	-2.00	-2.13	-2.23
SAR(1)	1.67'	0.17'	-1.16'	-2.25	-3.10	-3.74	-4.23	-4.60	-4.89	-5.12
SAR(2)	1.73'	0.22'	-1.12'	-2.23	-3.09	-3.75	-4.24	-4.61	-4.90	-5.12

y_t + 0.22 - 4.30 c_t.

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	0.95'	-0.26'	-1.27'	-2.12	-2.81	-3.39	-3.86	-4.25	-4.57	-4.84
AR(1)	-0.57'	-0.69'	-0.91'	-1.21'	-1.54'	-1.89'	-2.24	-2.58	-2.91	-3.22
AR(2)	-2.18	-2.06	-1.98	-1.94	-1.93	-1.95	-2.00	-2.08	-2.17	-2.28
SAR(1)	1.38'	0.04'	-1.13'	-2.10	-2.89	-3.51	-4.01	-4.41	-4.73	-4.99
SAR(2)	1.38'	0.03'	-1.15'	-2.12	-2.90	-3.53	-4.03	-4.42	-4.74	-5.00
b) Intercept.										
W.N.	0.90'	-0.37'	-1.48'	-2.39	-3.11	-3.67	-4.12	-4.47	-4.76	-5.00
AR(1)	-0.01'	-0.10'	-0.43'	-0.90'	-1.40'	-1.87'	-2.30	-2.69	-3.04	-3.36
AR(2)	-1.75	-1.49	-1.41	-1.45	-1.57	-1.71	-1.86	-2.02	-2.16	-2.31
SAR(1)	1.38'	0.00'	-1.24'	-2.29	-3.12	-3.75	-4.24	-4.62	-4.91	-5.14
SAR(2)	1.42'	0.04'	-1.21'	-2.27	-3.11	-3.76	-4.25	-4.62	-4.92	-5.15
c) Intercept and a time trend.										
W.N.	1.12'	-0.28'	-1.45'	-2.38	-3.11	-3.68	-4.12	-4.47	-4.76	-4.98
AR(1)	0.33'	0.08'	-0.36'	-0.88'	-1.39'	-1.87'	-2.31	-2.69	-3.02	-3.31
AR(2)	-1.47	-1.33	-1.33	-1.42	-1.56	-1.72	-1.87	-2.01	-2.14	-2.24
SAR(1)	1.62'	0.12'	-1.20'	-2.28	-3.12	-3.76	-4.24	-4.61	-4.90	-5.13
SAR(2)	1.68'	0.18'	-1.16'	-2.26	-3.11	-3.76	-4.25	-4.62	-4.91	-5.13

': Non-rejection values of the null hypothesis (1.12) with $\rho(L;\theta) = (1-L)^{d+\theta}$ at 95% significance level when monotonicity in the value of the tests with respect to d is observed.

TABLE 6.A7

$\hat{r}$ in (2.9) for the estimated residuals with d ≤ 0.50

c_t - 0.52 - 0.23 y_t

a) No intercept and no time trend.

Residuals \ d	0.00	0.10	0.20	0.30	0.40	0.50
White Noise:	11.45	9.60	7.69	5.80	4.02	2.40
Seas. AR (1):	9.00	8.14	7.09	5.84	4.42	2.91
Seas. AR (2):	8.84	8.07	7.08	5.85	4.44	2.92

b) Intercept.

Residuals \ d	0.00	0.10	0.20	0.30	0.40	0.50
White Noise:	11.45	9.60	7.69	5.82	4.05	2.44
Seas. AR (1):	9.00	8.13	7.09	5.84	4.42	2.94
Seas. AR (2):	8.84	8.07	7.07	5.85	4.45	2.98

c) Intercept and a time trend.

Residuals \ d	0.00	0.10	0.20	0.30	0.40	0.50
White Noise:	11.57	9.78	7.96	6.17	4.43	2.75
Seas. AR (1):	9.06	8.22	7.22	6.05	4.71	3.23
Seas. AR (2):	8.90	8.16	7.22	6.07	4.75	3.28

y_t + 0.22 - 4.30 c_t.

a) No intercept and no time trend.

Residuals \ d	0.00	0.10	0.20	0.30	0.40	0.50
White Noise:	11.27	9.42	7.54	5.69	3.94	2.35
Seas. AR (1):	8.87	8.03	7.00	5.76	4.35	2.86
Seas. AR (2):	8.72	7.97	6.98	5.77	4.36	2.86

b) Intercept.

Residuals \ d	0.00	0.10	0.20	0.30	0.40	0.50
White Noise:	11.27	9.42	7.52	5.66	3.91	2.33
Seas. AR(1):	8.87	8.03	6.99	5.73	4.31	2.83
Seas. AR(2):	8.72	7.97	6.97	5.74	4.33	2.87

c) Intercept and a time trend.

Residuals \ d	0.00	0.10	0.20	0.30	0.40	0.50
White Noise:	11.49	9.70	7.89	6.10	4.36	2.69
Seas. AR (1):	9.01	8.18	7.18	6.00	4.66	3.18
Seas. AR (2):	8.86	8.12	7.18	6.03	4.70	3.22

6.b PRICES AND WAGES

The nature of the relationship between prices and wages has long been the subject of ongoing debate. The expectations augmented Phillips-curve theory contends that the two variables are mutually causal. However, the original wage type Phillips-curve model argues it is inflation that cause wage growth rather than vice versa. The price mark-up scheme holds an opposing view and asserts that wage growth plays an independent causal role in the inflationary process, and other theories, (eg. the monetarist) deny the presence of any reliable linkage between wages and prices.

Researchers have also expended enormous effort attempting to investigate empirically the relationship between these two variables but with mixed results. For example, Mehra (1977) and Ashenfelter and Card (1982) report results suggesting a bidirectional causality; Barth and Bennett (1975) and Stein (1984) find causality running from prices to wages without feedback, while Shannon and Wallace (1986) report results showing causality only in the reverse situation. Gordon (1977), Bazdarich (1978) and Batten (1981) find no causal linkage between the two variables. Such remarkably mixed evidence is unfortunate in light of the implications for economic and public policy. Mehra (1991) employed the technique of cointegration modelling U.S. quarterly data. His model encompassed three basic variables, namely, prices, wages and an output-gap proxy. He tested each of the three variables (in logs) for the presence of unit roots, finding evidence of two unit roots in prices and wages, but a single unit root in the output-gap variable. Applying cointegration techniques, he concluded that first differences (but not levels) of prices and wages were cointegrated. Darrat (1994) used an error correction representation which also included other relevant variables (such as money supply, exchange and interest rates) and he concluded that wages and prices were not cointegrated and therefore did not exhibit a reliable long run relationship. His results were consistent with Gordon (1988), supporting the view that wages and prices are irrelevant to each other.

In order to examine these two variables, we use the same data set as in Engle and Granger (1987). They analyzed logs of C.P.I. and production worker wage in manufacturing throughout the 1950's, 60's and 70's with monthly data and found no evidence of cointegration either for the individual decades or for the whole sample.

Over the whole period of time, the Durbin-Watson cointegration test (CRDW) from the cointegrating regression in either direction was 0.0054, suggesting that it was insignificantly different from zero. The ADF test of the regression of prices on wages was -0.6 and for the reverse regression 0.2. Adding a twelfth lag, the test statistics were 0.88 and 1.55 respectively. None of these values approached the critical value of 3.2 and therefore, their evidence accepted the null hypothesis of no cointegration between wages and prices. For individual decades, again none of the ADF tests were significant at even 10% level and the largest statistic was obtained in the 1950's when regressing prices on wages, but still below the critical value. Thus, they found conclusive evidence that prices and wages were not cointegrated.

Figures 6.B1 and 6.B2 contain plots of the original series for C.P.I. and wages, their sample autocorrelations and estimates of the spectral density function, considering the whole sample size and individual decades as well. We observe in the up-left hand side of these figures that both series increases over the whole sample period, with a possible changing growth around 1973 due perhaps to the oil crisis in that year. The slow decay of sample autocorrelations and the peak in the estimated spectrums at zero frequency suggest the nonstationary component of the series.

In Tables 6.B1-6.B4 we present results of Robinson's (1994c) univariate tests, reporting $\hat{r}$ in (2.9), when testing (1.12) in (1.9) and (1.10) with $\rho(L;\theta) = (1 - L)^{d+\theta}$, using the whole sample size in Table 6.B1, and each individual decades in Tables 6.B2-6.B4. In Table 6.B1 we observe that the unit root null hypothesis is never rejected for C.P.I. when we do not include regressors. However, including an intercept or an intercept and a time trend, this hypothesis is always strongly rejected in favour of more nonstationary alternatives, observing also in these cases, a lack of monotonic decrease in $\hat{r}$ with respect to d for most specifications of the disturbances. We see that the only non-rejection value of d among those cases where monotonicity is achieved occurs at d = 1.4 when including a time trend with white noise and seasonal AR(2) u_t. For wages, we see in the lower part of this table, that the non-rejection values of d always range between 1 and 1.2 if we do not include regressors, with monotonicity achieved in all cases, and with the lowest statistics occurring at d = 1 for white noise and seasonal AR u_t, and at d = 1.1 for non-seasonal AR u_t. However, including an intercept and an intercept and a time trend, monotonicity is again unlikely to be achieved, and in those cases where it is, the non-rejection values of d are 1 and 1.1.

Looking at individual decades, in Tables 6.B2-6.B4, results are very similar to those in Table 6.B1. Starting with C.P.I., we observe in the upper part of these tables that the non-rejection values of d range between 0.9 and 1.1 if we do not include regressors, with monotonicity achieved for cases of white noise and seasonal AR disturbances, and with the lowest statistics obtained in all decades when d = 1. Including an intercept, monotonicity is never achieved in the 50's, (Table 6.B2), though this property is captured for seasonal AR u_t in the 60's, (Table 6.B3), and for white noise and seasonal AR(2) u_t in the 70's, (Table 6.B4). In all these cases the non-rejection values of d are always greater than 1, with the lowest statistics occurring at d = 1.2 in the 60's and at d = 1.4 in the 70's. Including an intercept and a time trend, monotonicity is again only achieved for white noise and seasonal AR disturbances, with non-rejection d's ranging between 1.3 and 1.5 in the 50's and 70's, and between 1.1 and 1.3 during the 60's. For wages, we see in the lower part of these tables that the non-rejection values of d range in most cases between 0.9 and 1.1 in the three decades. This is observed independently of the regressors used in the model and the ways of modelling the disturbances, and the lowest statistics appear in practically all cases when d takes values 0.9 and 1.

As a conclusion of these univariate tests, we see that C.P.I. and wages might be both individually integrated or order 1 if we do not include regressors. However, including an intercept or an intercept and a time trend, wages seems to be also I(1) though C.P.I. appears as more nonstationary, (i.e., with d > 1), especially when we consider the whole sample period, and the decades of the 50's and 70's.

In the next group of tables, we calculate the multivariate score tests first in Tables 6.B5-6.B8, considering the whole sample period, and then studying the different decades separately. Tables 6.B5 and 6.B6 give results of the multivariate score tests of Chapter 5 in the time and the frequency domain respectively, when the disturbances follow a white noise vector process. Thus, we report the statistic $\hat{S}^{t2}$ in (5.32) in Table 6.B5, and $\hat{S}^{f2}$ as given in (5.34) in Table 6.B6. In both tables results are very similar, with few non-rejection cases, and occurring at the same values of d_1 and d_2: if there are no regressors, the only non-rejection value corresponds to the case of two unit roots (i.e. $d_1 = d_2 = 1$), but including an intercept or a time trend, this case is rejected, and the only non-rejection occurs now when d_1 (the integration order of log of C.P.I.) is 1.4, and d_2 (the integration order of log of wages) is 1. Therefore, these results corroborate the findings of the univariate tests above, that both series are I(1) if we do not include regressors but C.P.I. might be of a higher integration order if an intercept or a time trend is included.

In Table 6.B7 we allow U_t to follow a VAR(1) process: if we do not include regressors, the two unit roots null is strongly rejected, and the only non-rejection case occurs at $d_1 = d_2 = 1.4$, with $\tilde{S}^2 = 3.36$. Including an intercept or an intercept and a time trend, there are some isolated cases where the null is not rejected but results do not show much consistency, suggesting perhaps that this is not a correct way of specifying the model. This might be related to the lack of monotonic decrease observed in $\hat{r}$ with respect to d in Table 6.B1 when u_t was AR and the model included an intercept and/or a time trend. If we take U_t as a VMA(1) process, in Table 6.B8, the null is always rejected if there are no regressors, and including an intercept and an intercept and a time trend, the only cases where the null hypothesis is not rejected are $d_1 = 1.4$ and d_2 ranging between 1.1 and 1.3, once more showing a higher integration order of C.P.I. over wages. The lowest statistics occur in both cases when $d_1 = 1.4$ and $d_2 = 1.3$, with $\tilde{S}^2 = 2.83$ when including an intercept, and with $\tilde{S}^2 = 2.30$ when including an intercept and a time trend. Thus, we can summarize the results of the multivariate tests on the thirty year period by saying that both series might be I(1) when we do not include regressors, but integration orders higher than one might be required, especially for prices, when including an intercept and/or a time trend.

The next group of tables follow the same structure as the previous ones, but we concentrate now on individual decades. Starting with the 1950's, we observe in Tables 6.B9 and 6.B10 that if U_t is a white noise vector process, the non-rejection cases coincide in both tables and correspond to $d_1 = d_2 = 0.9$, 1 and 1.1 when there are no regressors, and to $d_1 = 1.3$ and 1.4 and d_2 ranging from 0.8 to 1.1 when an intercept or a time trend is included. The lowest statistics in these two tables are obtained in both domains when we include a time trend and $d_1 = 1.4$ and $d_2 = 0.9$, with the test statistics $\hat{S}^{t2} = 0.19$ in Table 6.B9, and $\hat{S}^{f2} = 0.09$ in Table 6.B10. Allowing weak parametric autocorrelation in U_t, we do not report results here, however, the main conclusions obtained were: with VAR(1) U_t and no regressors, the only non-rejection case corresponded to $d_1 = d_2 = 1.4$ (that is, for the same values as in Table 6.B7 when we considered the thirty year period), and including regressors, the null was almost never rejected, with the lowest statistics appearing when d_1 ranged between 1 and 1.3 and $d_2 = 1.1$; with VMA(1) U_t, the only non-rejection case with no regressors was $d_1 = 0.9$ and $d_2 = 0.8$, and including regressors, the non-rejections occurred in practically all cases when d_1 and d_2 were greater than 1, and $d_1 > d_2$.

Looking at the 1960's, in Tables 6.B11-6.B13, results are more definite. Thus, if we do not include regressors, the null of $d_1 = d_2 = 1$ is the only non-rejection case for white noise U_t. (See the upper part of Tables 6.B11 and 6.B12). This hypothesis was also non-rejected for the VAR(1) case, and though it is rejected for VMA(1) disturbances, (in Table 6.B13), it does correspond to the lowest statistic across all possibilities presented in that table. Thus, these results are in complete analogy with the univariate ones presented in Table 6.B3 where the unit root case was the most plausible alternative for both series when modelling with no regressors. Including an intercept or a time trend, the non-rejection cases correspond to d_1 between 1.2 and 1.4 and d_2 between 0.9 and 1.1 for white noise and VMA(1) disturbances, corroborating again the results in Table 6.B3 that C.P.I. is of a higher integration order than wages when including an intercept and/or a time trend. In fact, the lowest statistics appear for white noise U_t when $d_1 = 1.2$ and $d_2 = 1$ with an intercept, and when $d_1 = 1.3$ and $d_2 = 1$ with an intercept and a time trend. For VMA disturbances, the lowest statistics occur at $d_1 = 1.4$ and $d_2 = 1.1$ for both cases of an intercept and an intercept and a time trend. Allowing VAR(1) U_t, we do not report results since they did not show much coherence, suggesting that the VAR representation, when including an intercept or an intercept and a time trend, was not a correct way of specifying the model in this decade.

Finally, in Tables 6.B14-6.B16, we concentrate on the 1970's. Results in these tables are similar to those given previously. Starting with white noise U_t, we see in Tables 6.B14 and 6.B15 that if there are no regressors, the only non-rejection cases occur when $d_1 = d_2 = 1$ and 1.1, using the time domain version of the test, but also when $d_1 = d_2 = 0.9$ using its frequency domain representation. In both cases the lowest statistics correspond to the two unit roots null, with $\hat{S}^{t2} = 2.48$ in Table 6.B14 and $\hat{S}^{f2} = 2.89$ in Table 6.B15. However, as in previous tables, if we include an intercept or a time trend, these hypotheses are strongly rejected and the non-rejection values are d_1 = 1.3 and 1.4, and d_2 between 0.8 and 1.1. If U_t follows a VAR(1) process, (in Table 6.B16), the null hypothesis of two unit roots is the only non-rejection case if there are no regressors, and any departure from this case strongly increases the value of the test statistic. This hypothesis is also not rejected if we include regressors though lower statistics are obtained for smaller values of d_1 and d_2. In fact, the lowest statistics are obtained in these cases when $d_1 = 1$ and $d_2 = 0.7$ when including an intercept, and when $d_1 = 1.1$ and $d_2 = 0.7$ with a time trend. Allowing VMA(1) U_t we do not report results, since the null was always rejected when modelling with no regressors, and few non-rejections appeared with an intercept and a time trend when d_1 = 1.4 and d_2 ranged from 0.9 to 1.4.

Results of univariate and multivariate tests above suggest that both series might be integrated of order 1 if we do not include regressors in the model. This is observed for the whole sample size but also when individual decades are considered. In view of these results we next examine if both variables might be fractionally cointegrated and, as we did in the previous section, we run the cointegrating regressions of one of the variables against the other.

The next tables report results of Robinson's (1994c) univariate tests on the estimated residuals from the cointegrating regressions. When we consider the whole sample period, these regressions are

$$\log CPI_t = 3.91 + 0.70 \log W_t$$
$$(429.3) \quad (101.1) \quad (\text{t-values})$$

and

$$\log W_t = -5.31 + 13.6 \log CPI_t.$$
$$(-84.1) \quad (101.1) \quad (\text{t-values})$$

Table 6.B17 reports $\hat{r}$ in (2.9) on the estimated residuals above. We see in this table that if the disturbances are modelled with non-seasonal AR processes, monotonicity in $\hat{r}$ with respect to d is never achieved, indicating that the model might be misspecified in these cases. If the disturbances are white noise or seasonal AR, monotonicity is always achieved and the unit root null hypothesis is always rejected in favour of more nonstationary alternatives. This is observed in both series

of residuals and for the three cases of no regressors, an intercept and an intercept and a time trend. In fact, the only cases where the null hypothesis is not rejected occur when d = 1.1, therefore providing conclusive evidence against the hypothesis of cointegration between both variables at least when we consider the whole sample period. Thus, results in this table indicate that the estimated residuals from the cointegrating regressions are nonstationary and non-mean reverting, supporting the view of Gordon (1988) and others that prices and wages move separately without any reliable long run relationship.

If we concentrate on individual decades, starting with the 1950's, we see in Table 6.B18 that the non-rejections always occur for values of d greater than 1 if the disturbances are modelled as white noise or seasonal AR. If they are non-seasonal AR, there is a wide range of values of d where the null is not rejected, some of which are smaller than 1. However, the bulk of the results in this table reject the null when d = 1 in favour of alternatives with d > 1, with the lowest statistics appearing in most cases when d takes values 1.2 and 1.3, suggesting therefore that both series of residuals are nonstationary and are not mean-reverting in this decade.

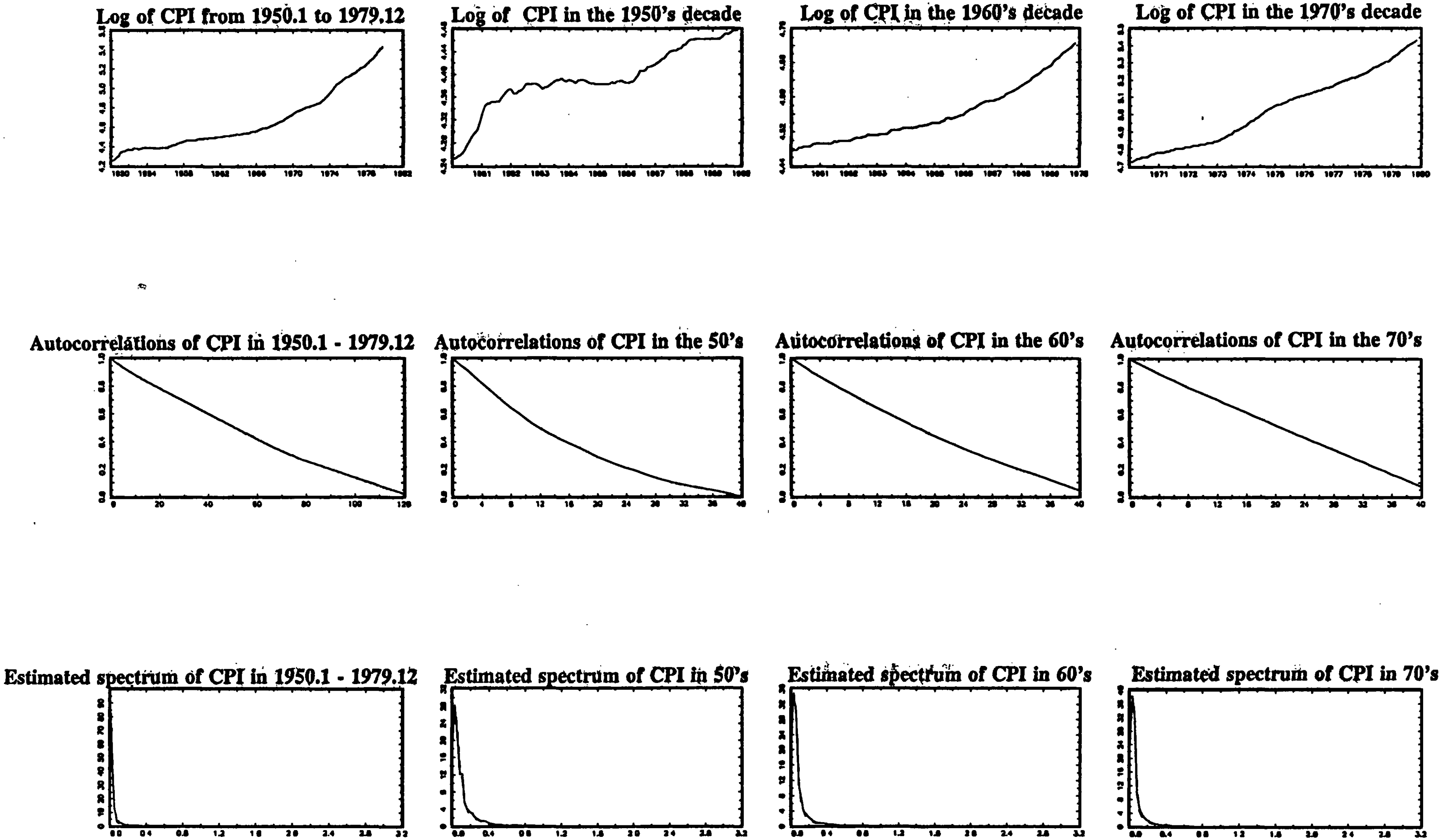
Results for the 1960's are given in Table 6.B19. Monotonicity is achieved across all specifications for the disturbances. If they follow a non-seasonal AR(1) process, there is a wide range of values of d where the null is not rejected, which makes it difficult to distinguish an appropriate integration order in this case. The lowest statistics across the different values of d are obtained in these cases at d = 0.9 when including no regressors, and at d = 0.8 if we include an intercept or a time trend. Results for white noise or seasonal AR are more definite, with non-rejection values of d ranging between 0.8 and 1.1, and with the lowest statistics obtained in all cases when d = 0.9. Thus, we conclude the analysis of this decade by saying that C.P.I. and wages might be slightly fractionally cointegrated, with the estimated residuals from the cointegrating regressions showing a small component of mean reversion.

Results for the 1970's, in Table 6.B20, are very similar to those given in the previous table. If the disturbances follow an AR(1) process, the non-rejections occur when d ranges between 0.6 and 1.1, with the lowest statistic obtained in all cases at d = 0.9. If they are white noise or seasonal AR, the band of non-rejection d's narrows, going from 0.8 through 1.1, with the lowest values obtained again at d =

0.9. Thus, we also observe in this decade nonstationary residuals but with a small component of mean reversion.

Summarizing the main results obtained in this section, we see that prices and wages are both individually integrated of order 1, though prices might display a higher integration order when an intercept or an intercept and a time trend is included in the model. The multivariate tests support this view, finding two unit roots when there are no regressors, but rejecting this hypothesis in favour of more nonstationarities for prices when including an intercept or a time trend. Looking at the possibility of fractional cointegration, prices and wages seem to move apart when we consider the whole sample size, though during the 1960's and 1970's, a small degree of fractional cointegration might occur between both variables, with the estimated residuals from the cointegrating regressions being nonstationary, but showing a small component of mean reversion.

FIGURE 6.B1



Log of CPI from 1950.1 to 1979.12
Log of CPI in the 1950's decade
Log of CPI in the 1960's decade
Log of CPI in the 1970's decade
Autocorrelations of CPI in 1950.1 - 1979.12
Autocorrelations of CPI in the 50's
Autocorrelations of CPI in the 60's
Autocorrelations of CPI in the 70's
Estimated spectrum of CPI in 1950.1 - 1979.12
Estimated spectrum of CPI in 50's
Estimated spectrum of CPI in 60's
Estimated spectrum of CPI in 70's

FIGURE 6.B2

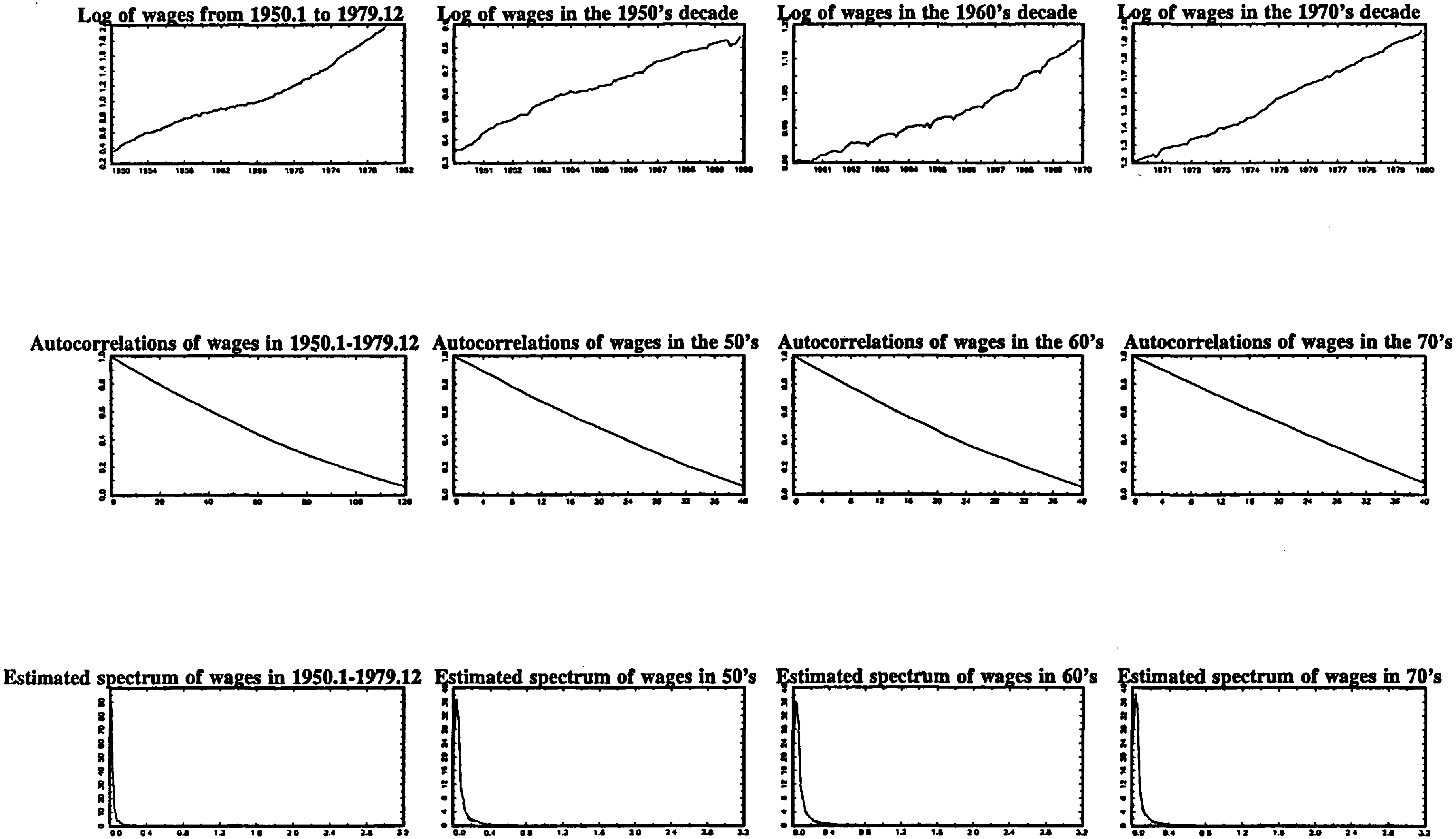

Log of wages from 1950.1 to 1979.12
Log of wages in the 1950's decade
Log of wages in the 1960's decade
Log of wages in the 1970's decade
Autocorrelations of wages in 1950.1-1979.12
Autocorrelations of wages in the 50's
Autocorrelations of wages in the 60's
Autocorrelations of wages in the 70's
Estimated spectrum of wages in 1950.1-1979.12
Estimated spectrum of wages in 50's
Estimated spectrum of wages in 60's
Estimated spectrum of wages in 70's

TABLE 6.B1

$\hat{r}$ in (2.9) for log of U.S. C.P.I. and wages

C.P.I. (1950.1 - 1979.12)

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	17.44	11.40	6.41	2.60	-0.17'	-2.16	-3.60	-4.66	-5.47	-6.10
AR(1)	4.02	3.93	2.71	1.20'	-0.26'	-1.50'	-2.52	-3.35	-4.03	-4.59
AR(2)	1.34	1.94	1.51	0.64	-0.35	-1.28	-2.10	-2.79	-3.38	-3.87
SAR(1)	11.04	8.56	5.47	2.41	-0.18'	-2.17	-3.65	-4.74	-5.57	-6.21
SAR(2)	10.13	7.96	5.22	2.35	-0.18'	-2.17	-3.66	-4.76	-5.59	-6.23
b) Intercept.										
W.N.	43.39	45.74	45.58	40.32	30.67	19.76	10.69	4.40	0.38	-2.19
AR(1)	-7.00	-6.74	-6.70	-5.66	3.81	9.50	7.66	4.67	2.01	-0.03
AR(2)	-4.36	-4.31	-4.11	-4.00	-1.95	3.49	4.74	3.14	1.24	-0.39
SAR(1)	8.56	8.50	8.84	9.94	11.54	11.05	7.85	3.79	0.31	-2.20
SAR(2)	11.20	11.24	10.88	9.44	8.54	7.87	5.99	3.06	0.11	-2.24
c) Intercept and a time trend.										
W.N.	56.07	52.88	47.95	40.63	31.00	20.54	11.37	4.68	0.33'	-2.35
AR(1)	-2.92	-4.76	-5.82	-4.21	4.76	9.80	8.05	4.88	1.93	-0.29
AR(2)	1.19	-1.25	-2.78	-3.28	-1.57	3.33	4.92	3.32	1.18	-0.64
SAR(1)	11.69	9.87	9.45	10.41	11.61	11.12	8.16	4.00	0.29	-2.36
SAR(2)	15.70	13.34	11.04	9.37	8.49	7.75	6.03	3.15	0.06'	-2.40

Wages (1959.1 - 1979.12)

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	27.84	16.41	8.91	3.93	0.57'	-1.73'	-3.33	-4.47	-5.32	-5.97
AR(1)	18.06	12.05	7.26	3.85	1.36'	-0.49'	-1.86'	-2.89	-3.67	-4.29
AR(2)	13.91	11.01	7.05	4.12	1.92'	0.24'	-1.04'	-2.02	-2.78	-3.39
SAR(1)	15.42	11.92	7.55	3.65	0.55'	-1.74'	-3.38	-4.56	-5.42	-6.08
SAR(2)	11.80	9.97	6.73	3.36	0.46'	-1.77'	-3.40	-4.57	-5.43	-6.09
b) Intercept.										
W.N.	34.79	31.88	22.60	10.38	2.40	-1.69'	-3.85	-5.13	-5.98	-6.60
AR(1)	-8.66	-5.99	9.41	10.06	5.20	1.31	-1.06	-2.53	-3.51	-4.23
AR(2)	-5.93	-6.36	-0.65	7.23	5.09	1.97	-0.20	-1.55	-2.42	-3.02
SAR(1)	8.93	8.73	10.06	7.72	2.41	-1.54	-3.84	-5.21	-6.10	-6.74
SAR(2)	9.85	7.89	6.66	5.53	1.90'	-1.61'	-3.86	-5.23	-6.12	-6.75
c) Intercept and a time trend.										
W.N.	43.40	32.26	19.88	9.47	2.49	-1.59'	-3.90	-5.25	-6.10	-6.69
AR(1)	-2.25	12.29	14.45	10.39	5.42	1.44	-1.19	-2.82	-3.83	-4.51
AR(2)	-4.38	-0.21	8.33	8.61	5.43	2.11	-0.38	-1.98	-2.94	-3.49
SAR(1)	9.59	11.74	11.38	7.52	2.49	-1.44	-3.89	-5.35	-6.24	-6.84
SAR(2)	6.93	7.00	7.16	5.47	1.95	-1.52	-3.91	-5.37	-6.26	-6.86

': Non-rejection values of the null hypothesis (1.12) with $\rho(L;\theta) = (1-L)^{d+\theta}$ at 95% significance level when monotonicity in the test statistic with respect to d is observed.

TABLE 6.B2 (1950's decade)

$\hat{r}$ in (2.9) for log of U.S. C.P.I. and wages.

C.P.I. (1950.1 - 1959.12)

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	7.46	5.00	2.82	1.02'	-0.39'	-1.48'	-2.30	-2.92	-3.41	-3.79
AR(1)	-0.51	0.31	0.29	-0.13	-0.68	-1.25	-1.78	-2.23	-2.62	-2.95
AR(2)	-2.77	-1.37	-0.81	-0.76	-0.97	-1.29	-1.64	-1.98	-2.29	-2.57
SAR(1)	5.20	3.94	2.44	0.94'	-0.40'	-1.48'	-2.32	-2.96	-3.46	-3.85
SAR(2)	5.02	3.80	2.38	0.92'	-0.40'	-1.48'	-2.33	-2.97	-3.46	-3.85
b) Intercept.										
W.N.	9.66	10.87	11.54	10.37	8.24	5.91	3.75	1.90	0.40	-0.78
AR(1)	-2.64	-0.82	-0.37	-0.27	0.44	0.74	0.54	0.09	-0.45	-0.99
AR(2)	-3.48	-0.85	0.02	-0.22	-0.20	-0.16	-0.32	-0.63	-1.03	-1.45
SAR(1)	7.25	8.01	8.22	7.55	6.42	4.98	3.37	1.78	0.37	-0.79
SAR(2)	7.25	8.05	8.16	7.27	5.96	4.50	2.99	1.53	0.21	-0.88
c) Intercept and a time trend.										
W.N.	18.12	16.55	14.15	11.25	8.26	5.52	3.21	1.38'	-0.01'	-1.05'
AR(1)	-0.19	-1.02	-1.48	-0.78	0.45	0.64	0.26	-0.28	-0.83	-1.29
AR(2)	0.05	-0.89	-1.25	-0.95	-0.35	-0.24	-0.50	-0.92	-1.36	-1.73
SAR(1)	9.68	9.10	8.45	7.61	6.38	4.77	3.00	1.35'	-0.02'	-1.05'
SAR(2)	10.23	9.56	8.53	7.28	5.86	4.30	2.68	1.16'	-0.12'	-1.12'

Wages (1950.1 - 1959.12)

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	5.82	4.23	2.59	1.07'	-0.22'	-1.26'	-2.08	-2.72	-3.23	-3.64
AR(1)	1.10	1.17	0.91	0.44	-0.12	-0.71	-1.25	-1.75	-2.18	-2.55
AR(2)	0.32	0.38	0.37	0.21	-0.09	-0.46	-0.86	-1.25	-1.62	-1.97
SAR(1)	4.75	3.64	2.35	1.01'	-0.22'	-1.27'	-2.11	-2.76	-3.28	-3.69
SAR(2)	4.58	3.51	2.27	0.98'	-0.24'	-1.27'	-2.11	-2.76	-3.28	-3.69
b) Intercept.										
W.N.	9.95	5.01	0.99'	-0.53'	-1.26'	-1.95'	-2.61	-3.20	-3.70	-4.11
AR(1)	-7.57	-2.58	-0.82	-0.21	0.14	0.11	-0.19	-0.60	-1.05	-1.49
AR(2)	-7.74	-8.83	-5.01	-2.81	-1.62	-1.04	-0.82	-0.81	-0.91	-1.07
SAR(1)	5.16	3.57	1.03'	-0.43'	-1.21'	-1.94'	-2.63	-3.24	-3.75	-4.16
SAR(2)	5.23	3.57	1.08'	-0.42'	-1.22'	-1.94'	-2.63	-3.23	-3.73	-4.13
c) Intercept and a time trend.										
W.N.	7.96	4.87	2.26	0.26'	-1.19'	-2.23	-2.98	-3.54	-3.96	-4.28
AR(1)	1.84	2.28	1.87	1.12	0.33	-0.38	-0.97	-1.43	-1.78	-2.06
AR(2)	-1.93	-1.27	-0.89	-0.95	-1.20	-1.48	-1.72	-1.89	-1.98	-1.99
SAR(1)	5.46	3.87	2.04	0.29'	-1.15'	-2.24	-3.03	-3.60	-4.01	-4.34
SAR(2)	5.34	3.76	1.99	0.27'	-1.16'	-2.24	-3.02	-3.58	-3.99	-4.30

': Non-rejection values of the null hypothesis (1.12) with $\rho(L;\theta) = (1-L)^{d+\theta}$ at 95% significance level when monotonicity in the test statistic with respect to d is observed.

TABLE 6.B3 (1960’s decade)

$\hat{r}$ in (2.9) for log of U.S. C.P.I. and wages.

C.P.I. (1960.1 -1969.12)

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	7.40	4.97	2.82	1.03’	-0.38’	-1.46’	-2.28	-2.91	-3.40	-3.78
AR(1)	-0.38	0.36	0.30	-0.12	-0.67	-1.24	-1.76	-2.22	-2.61	-2.94
AR(2)	-2.54	-1.25	-0.75	-0.72	-0.94	-1.26	-1.61	-1.95	-2.26	-2.54
SAR(1)	5.23	3.95	2.45	0.95’	-0.38’	-1.46’	-2.30	-2.95	-3.44	-3.83
SAR(2)	5.06	3.82	2.40	0.94’	-0.38’	-1.46’	-2.31	-2.95	-3.45	-3.84
b) Intercept.										
W.N.	17.44	17.55	16.65	13.54	8.73	4.01	0.59	-1.49	-2.69	-3.41
AR(1)	-5.26	-6.00	-7.08	-3.30	4.11	4.00	2.05	0.14	-1.21	-2.06
AR(2)	-2.78	-2.95	-3.60	-4.60	-1.97	2.40	1.88	0.34	-1.06	-2.03
SAR(1)	4.89	4.48	3.99	3.96	3.90	2.49	0.34’	-1.49’	-2.70	-3.45
SAR(2)	7.46	7.16	5.97	3.93	2.55	1.45’	-0.08’	-1.61’	-2.73	-3.46
c) Intercept and a time trend.										
W.N.	23.06	20.81	17.64	13.59	9.09	4.85	1.47’	-0.90’	-2.41	-3.33
AR(1)	-3.64	-4.53	-4.09	1.86	4.99	4.45	2.74	0.81	-0.82	-1.95
AR(2)	-1.53	-2.67	-3.53	-3.70	-0.73	2.41	2.20	0.89	-0.62	-1.88
SAR(1)	6.39	5.13	4.68	4.64	4.18	2.83	0.89’	-1.00’	-2.43	-3.36
SAR(2)	10.13	7.72	5.39	3.74	2.68	1.65’	0.29’	-1.21’	-2.48	-3.37

Wages (1960.1 - 1969.12)

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	7.12	4.83	2.77	1.04’	-0.35’	-1.42’	-2.25	-2.88	-3.38	-3.77
AR(1)	0.60	0.80	0.54	0.04	-0.55	-1.14	-1.67	-2.14	-2.53	-2.87
AR(2)	-1.04	-0.50	-0.36	-0.50	-0.80	-1.17	-1.55	-1.90	-2.22	-2.51
SAR(1)	5.36	4.00	2.48	0.98’	-0.35’	-1.43’	-2.27	-2.92	-3.42	-3.82
SAR(2)	5.17	3.88	2.43	0.97’	-0.35’	-1.43’	-2.28	-2.92	-3.43	-3.82
b) Intercept.										
W.N.	14.41	11.02	6.08	1.69’	-1.03’	-2.51	-3.35	-3.89	-4.27	-4.57
AR(1)	-7.60	-4.52	2.52	2.45	0.89	-0.40	-1.23	-1.77	-2.19	-2.54
AR(2)	-3.49	-4.42	-4.51	-1.06	-0.84	-1.35	-1.71	-1.87	-1.93	-1.97
SAR(1)	4.65	4.58	4.14	1.80’	-0.84’	-2.54	-3.49	-4.05	-4.43	-4.71
SAR(2)	5.65	4.39	3.32	1.58’	-0.81’	-2.57	-3.54	-4.10	-4.46	-4.73
c) Intercept and a time trend.										
W.N.	11.00	7.01	3.56	0.90’	-0.99’	-2.29	-3.17	-3.79	-4.24	-4.57
AR(1)	4.50	4.44	3.50	2.25	1.02’	-0.05’	-0.91’	-1.58’	-2.12	-2.54
AR(2)	-0.78	-0.32	0.08	-0.15	-0.62	-1.09	-1.47	-1.73	-1.91	-2.02
SAR(1)	7.51	5.91	3.63	1.23’	-0.80’	-2.27	-3.26	-3.93	-4.39	-4.71
SAR(2)	5.55	4.62	3.12	1.15’	-0.77’	-2.27	-3.30	-3.97	-4.42	-4.73

’: Non-rejection values of the null hypothesis (1.12) with $\rho(L;\theta) = (1-L)^{d+\theta}$ at 95% significance level when monotonicity in the test statistic with respect to d is observed.

TABLE 6.B4 (1970's decade)

$\hat{r}$ in (2.9) for log of U.S. C.P.I. and wages.

C.P.I. (1970.1 - 1979.12)

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	7.26	4.90	2.78	1.02'	-0.38'	-1.45'	-2.28	-2.91	-3.40	-3.78
AR(1)	-0.31	0.36	0.29	-0.12	-0.67	-1.24	-1.76	-2.21	-2.60	-2.94
AR(2)	-2.39	-1.21	-0.75	-0.73	-0.94	-1.26	-1.61	-1.95	-2.26	-2.55
SAR(1)	5.20	3.92	2.43	0.94'	-0.38'	-1.46'	-2.30	-2.95	-3.44	-3.83
SAR(2)	5.04	3.80	2.38	0.93'	-0.38'	-1.46'	-2.30	-2.95	-3.45	-3.84
b) Intercept.										
W.N.	16.23	15.98	15.79	14.22	10.90	7.09	3.88	1.46'	-0.34'	-1.68'
AR(1)	-5.74	-6.30	-6.79	-7.47	-1.75	2.88	3.30	2.65	1.68	0.66
AR(2)	-3.39	-3.38	-3.15	-3.41	-4.16	-3.12	-0.11	0.80	0.75	0.35
SAR(1)	5.16	4.93	4.96	5.05	5.06	4.45	3.08	1.42	-0.18	-1.53
SAR(2)	6.63	6.44	6.35	5.78	4.89	4.09	3.04	1.67'	0.17'	-1.22'
c) Intercept and a time trend.										
W.N.	20.41	19.02	17.11	14.61	11.59	8.26	4.99	2.14	-0.10'	-1.71'
AR(1)	-2.09	-3.57	-4.82	-5.06	-0.71	2.91	3.62	3.09	1.99	0.74
AR(2)	1.73	0.02	-1.32	-2.43	-3.33	-3.44	-1.20	0.50	0.79	0.44
SAR(1)	9.83	8.23	7.00	6.17	5.53	4.70	3.43	1.74'	-0.03'	-1.57'
SAR(2)	10.21	9.13	7.90	6.59	5.36	4.27	3.14	1.79'	0.24'	-1.28'

Wages (1970.1 - 1979.12)

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	6.58	4.51	2.60	0.95'	-0.38'	-1.44'	-2.25	-2.88	-3.37	-3.76
AR(1)	0.29	0.54	0.37	-0.06	-0.60	-1.15	-1.67	-2.13	-2.53	-2.86
AR(2)	-1.36	-0.78	-0.55	-0.60	-0.83	-1.14	-1.49	-1.83	-2.14	-2.43
SAR(1)	5.04	3.76	2.32	0.89'	-0.39'	-1.44'	-2.27	-2.92	-3.42	-3.81
SAR(2)	4.92	3.67	2.29	0.88'	-0.39'	-1.45'	-2.28	-2.92	-3.42	-3.81
b) Intercept.										
W.N.	15.14	12.46	7.11	1.42'	-1.35'	-2.33	-2.78	-3.10	-3.40	-3.67
AR(1)	-6.45	-6.81	-0.24	-0.23	-2.05	-2.79	-3.02	-3.18	-3.36	-3.56
AR(2)	-5.99	-9.30	-2.30	0.76	-0.53	-1.27	-1.39	-1.41	-1.50	-1.66
SAR(1)	5.05	4.43	3.64	1.10'	-1.33'	-2.35	-2.81	-3.14	-3.43	-3.70
SAR(2)	5.86	4.51	3.01	0.98'	-1.29'	-2.35	-2.82	-3.14	-3.43	-3.70
c) Intercept and a time trend.										
W.N.	6.04	3.46	1.37'	-0.20'	-1.34'	-2.16	-2.75	-3.20	-3.54	-3.81
AR(1)	2.39	1.18'	0.00'	-1.04'	-1.88'	-2.53	-3.02	-3.40	-3.67	-3.88
AR(2)	2.51	1.94'	1.22'	0.45'	-0.28'	-0.90'	-1.41'	-1.81'	-2.10	-2.29
SAR(1)	4.84	3.07	1.32'	-0.17'	-1.32'	-2.17	-2.79	-3.24	-3.59	-3.85
SAR(2)	4.49	2.92	1.31'	-0.14'	-1.30'	-2.16	-2.79	-3.25	-3.60	-3.86

': Non-rejection values of the null hypothesis (1.12) with $\rho(L;\theta) = (1-L)^{d+\theta}$ at 95% significance level when monotonicity in the test statistic with respect to d is observed.

TABLE 6.B5 (1950.1 - 1979.12)

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with no regressors and white noise U_t. d_1 and d_2 are the differencing orders for log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	1818.1	1635.2	1227.0	914.79	860.39	804.72	747.77	693.58	644.53
0.7	1674.7	1253.6	767.76	658.72	649.68	627.87	591.05	548.66	506.62
0.8	1589.8	987.83	412.54	267.03	330.75	375.03	380.84	365.88	342.69
0.9	1560.4	879.34	345.75	649.54	87.87	17.14	218.64	233.33	230.42
1.0	1526.4	827.74	411.02	105.15	0.99'	44.04	103.13	139.32	154.53
1.1	1471.6	774.45	445.48	197.88	40.87	8.15	40.44	77.73	103.15
1.2	1403.2	715.06	439.25	246.03	105.38	31.61	25.86	47.73	71.34
1.3	1330.9	655.89	413.42	256.91	144.93	70.60	39.75	43.08	57.83
1.4	1261.2	601.23	381.86	249.63	151.65	99.34	63.45	52.68	58.19

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with an intercept and white noise U_t. d_1 and d_2 are the differencing orders for log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	2215.7	2040.1	2160.2	2503.4	2492.8	2953.6	3023.2	3054.8	3071.2
0.7	2157.8	1920.5	1994.0	2324.5	2274.4	2772.9	2843.1	2875.1	2891.7
0.8	2108.3	1753.3	1672.0	1920.5	1896.8	2324.2	2391.1	2422.4	2438.9
0.9	2054.6	1575.6	1250.4	1325.5	1341.9	1617.2	1673.1	1700.9	1716.3
1.0	1773.1	1270.0	804.65	684.95	740.46	862.97	901.03	922.22	935.17
1.1	2111.6	1445.6	672.47	342.53	312.37	335.99	352.73	363.28	370.62
1.2	2221.2	1495.9	605.73	171.27	90.31	97.57	109.88	118.75	125.44
1.3	2313.6	1551.4	601.37	119.62	18.71	22.85	34.01	42.49	49.05
1.4	2372.0	1589.9	613.16	113.03	5.62'	10.14	21.17	29.63	36.19

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with a time trend and white noise U_t. d_1 and d_2 are the differencing orders for log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	2226.6	1895.5	2001.1	2254.0	2557.1	2601.7	2668.5	2700.9	2717.4
0.7	2332.2	1834.6	1824.9	2035.7	2238.5	2378.3	2448.9	2483.9	2501.9
0.8	2309.7	1702.2	1558.7	1693.0	1812.9	1992.8	2060.8	2095.8	2114.2
0.9	2155.5	1487.0	1207.6	1235.5	1283.7	1450.7	1508.0	1539.2	1556.4
1.0	1957.3	1214.7	792.36	696.84	740.46	856.11	896.65	920.70	935.07
1.1	1744.1	1035.2	553.53	373.42	357.08	380.67	399.14	410.93	418.75
1.2	1639.0	927.85	401.22	167.97	115.18	122.13	135.44	145.05	151.99
1.3	1603.3	893.43	348.32	92.38	24.52	27.88	39.58	48.59	55.33
1.4	1601.1	892.90	341.89	78.33	5.71'	9.44	20.89	29.85	36.60

': Non-rejection values of the null hypothesis (5.4) at 95% significance level.

TABLE 6.B6 (1950.1 - 1979.12)

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with no regressors and white noise U_t. d_1 and d_2 are the differencing orders for log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	2195.7	1322.8	1011.1	1072.2	990.35	919.59	852.48	790.50	735.08
0.7	1926.0	1038.3	615.45	557.15	571.13	560.78	531.46	494.47	457.84
0.8	1672.0	926.84	347.63	220.52	294.42	342.85	351.96	340.02	319.66
0.9	1500.2	941.15	360.09	55.96	76.61	16.20	208.44	223.62	221.83
1.0	1376.9	947.92	467.33	116.44	0.73'	44.85	103.07	138.17	153.22
1.1	1274.2	914.21	515.51	222.19	43.61	8.20	42.85	80.43	105.53
1.2	1182.8	857.63	510.35	275.33	113.01	30.78	25.73	50.36	74.98
1.3	1101.4	794.65	481.20	286.43	155.44	71.78	37.72	42.87	60.45
1.4	1029.7	733.89	444.88	277.41	160.28	102.35	62.00	50.17	57.95

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with an intercept and white noise U_t. d_1 and d_2 are the differencing orders for log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	2066.2	1928.2	1995.4	2251.51	2797.6	2633.0	2701.5	2735.0	2753.0
0.7	1925.3	1738.3	1782.0	2033.68	2616.6	2414.6	2483.2	2516.9	2534.9
0.8	1863.2	1565.7	1491.8	1682.53	2179.5	2026.3	2091.3	2123.8	2141.5
0.9	1808.0	1398.8	1135.7	1192.81	1504.0	1443.6	1497.6	1526.1	1542.4
1.0	2043.5	1459.4	879.55	731.92	796.83	800.69	837.59	859.08	872.51
1.1	1803.8	1225.0	599.36	329.30	303.96	326.74	343.33	354.04	361.53
1.2	1884.9	1249.6	521.67	161.71	91.70	98.87	111.06	119.99	126.75
1.3	1964.9	1293.0	509.97	107.55	19.43	23.68	34.70	43.19	49.79
1.4	2020.2	1328.0	519.12	99.45	5.21'	9.94	20.81	29.26	35.87

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with a time trend and white noise U_t. d_1 and d_2 are the differencing orders for log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	2278.7	1953.7	2073.1	2336.6	2470.5	2687.8	2752.8	2783.4	2798.5
0.7	2324.4	1814.9	1809.2	2026.9	2244.6	2371.6	2440.5	2473.8	2490.6
0.8	2289.3	1652.3	1500.3	1635.4	1869.9	1934.6	2000.9	2034.3	2051.7
0.9	2151.4	1442.1	1143.7	1166.5	1354.3	1378.5	1434.3	1464.2	1480.5
1.0	1934.2	1236.6	838.36	750.69	796.83	798.29	837.71	860.92	874.67
1.1	1790.2	1037.1	531.70	343.67	325.41	349.43	367.61	379.18	386.83
1.2	1696.2	943.74	395.77	156.22	102.60	110.41	123.68	133.23	140.09
1.3	1662.2	913.73	349.72	89.03	21.15	25.36	37.14	46.13	52.82
1.4	1658.1	913.01	344.76	77.44	5.23'	9.73	21.28	30.24	36.93

': Non-rejection values of the null hypothesis (5.4) at 95% significance level.

TABLE 6.B7 (1950.1 - 1979.12)

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with no regressors and a VAR(1) structure on U_t. d_1 and d_2 are the differencing orders for log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	304.56	318.83	341.97	339.79	334.83	324.35	327.78	176.68	574.47
0.7	314.06	228.84	260.87	280.64	290.36	293.07	308.76	306.05	111.79
0.8	210.15	157.28	215.17	264.16	288.45	300.68	316.48	352.22	388.97
0.9	164.16	120.38	98.36	331.68	343.30	344.86	358.71	403.73	477.21
1.0	275.78	27.31	83.46	70.29	2919.0	854.12	30.34	247.70	596.84
1.1	96.90	89.40	193.08	281.39	223.15	1402.9	374.73	164.54	393.21
1.2	53.02	100.21	100.77	27.94	14.47	18.90	85.59	399.88	606.75
1.3	47.34	77.12	128.88	126.49	80.79	41.44	17.36	83.05	218.00
1.4	96.15	123.54	198.48	322.34	375.18	268.36	131.06	31.17	3.36'

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with an intercept and a VAR(1) structure on U_t. d_1 and d_2 are the differencing orders for log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	2.62'	1.27'	2.23'	4.63'	6.51	7.55	9.37	12.57	16.67
0.7	12.59	7.60	10.75	14.77	17.05	17.88	19.14	21.78	25.47
0.8	31.74	19.01	23.41	29.79	32.87	33.50	33.96	35.69	38.68
0.9	39.54	16.94	21.38	26.80	26.67	24.35	23.02	23.84	26.39
1.0	35.43	2.45'	16.85	15.23	3.22'	0.33'	5.65'	13.72	21.35
1.1	9.58	3.64'	47.16	45.17	24.56	30.07	45.82	59.96	70.32
1.2	47.78	8.92	82.66	78.64	39.66	34.07	42.04	50.93	58.24
1.3	50.90	6.30	106.64	93.15	34.90	17.71	19.36	24.94	30.63
1.4	48.49	1.55'	123.68	98.69	28.58	5.77'	5.05'	9.63	14.94

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with a time trend and a VAR(1) structure on U_t. d_1 and d_2 are the differencing orders for log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	28.80	16.50	11.40	11.57	13.24	25.73	29.83	34.00	38.01
0.7	45.00	36.99	28.84	27.01	27.89	40.38	44.12	47.99	51.76
0.8	53.65	54.17	43.32	37.19	35.90	45.87	49.12	52.59	56.07
0.9	39.30	58.27	47.95	31.61	23.50	25.54	28.23	31.50	34.91
1.0	9.67	55.78	56.20	23.33	3.22'	7.52	19.71	30.54	38.60
1.1	5.28'	71.60	94.39	51.20	23.78	39.38	59.77	74.89	84.76
1.2	9.50	83.11	129.07	80.63	40.72	38.79	52.48	63.39	71.02
1.3	6.87	80.04	141.51	88.11	35.30	16.84	25.63	34.15	40.74
1.4	2.82'	75.18	144.24	87.57	27.80	3.18'	10.60	18.68	25.19

': Non-rejection values of the null hypothesis (5.4) at 95% significance level.

TABLE 6.B8 (1950.1 - 1979.12)

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with no regressors and a VMA(1) structure on U_t. d_1 and d_2 are the differencing orders for log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	------	------	------	436.51	489.73	545.22	696.44	1133.9	1818.0
0.7	------	------	------	80.21	285.62	357.50	511.95	950.71	1851.9
0.8	88.58	47317	------	------	119.74	224.71	421.00	880.94	1782.6
0.9	248.96	232.36	14158	------	75.37	157.87	491.88	1024.3	1778.6
1.0	221929	6483.7	8745.1	------	------	603.67	1187.9	1743.6	2114.3
1.1	------	------	95874	69882	43503	335548	27129	7316.8	4247.7
1.2	10343	11845	12148	11328	9190.1	6693.7	7107.9	6865.0	4752.9
1.3	2807.2	3741.6	4371.4	4279.6	3674.8	2986.7	2239.3	2867.1	2294.5
1.4	1282.6	1792.8	2270.8	2348.9	1956.0	1681.6	1196.6	1093.2	1645.9

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with an intercept and a VMA(1) structure on U_t. d_1 and d_2 are the differencing orders for log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	1592.3	1585.0	1613.3	1706.8	1813.1	1897.0	1953.5	1989.2	2010.5
0.7	1343.2	1321.9	1353.9	1450.6	1556.1	1633.8	1681.6	1709.0	1723.0
0.8	1151.5	1025.7	990.95	1049.6	1135.8	1204.4	1247.6	1272.2	1284.1
0.9	1040.9	798.44	659.57	647.18	692.54	740.88	774.75	794.79	804.46
1.0	1006.4	676.49	442.86	356.93	354.68	376.13	395.93	408.60	414.10
1.1	1023.8	638.95	339.97	200.31	161.66	153.26	157.88	162.04	181.21
1.2	1062.2	644.87	305.80	133.37	73.45	55.26	55.40	56.53	80.37
1.3	1101.1	663.61	301.13	110.03	39.05	16.84	15.76	15.84	40.20
1.4	1128.0	681.51	305.80	104.28	27.75	3.97'	2.90'	2.83'	21.49

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with a time trend and a VMA(1) structure on U_t. d_1 and d_2 are the differencing orders for log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	1754.7	1636.9	1710.4	1846.5	1967.6	2048.8	2095.2	2118.4	2125.7
0.7	1575.8	1337.7	1322.4	1412.3	1518.2	1599.9	1651.3	1679.4	1690.1
0.8	1379.9	1057.0	956.65	985.28	1057.8	1126.0	1174.1	1202.9	1215.7
0.9	1200.2	825.02	655.47	622.96	653.50	698.09	735.01	759.09	770.28
1.0	1061.1	660.85	444.48	363.13	354.68	373.42	395.26	411.03	417.48
1.1	969.82	562.67	320.93	208.33	172.10	162.74	167.58	173.53	202.04
1.2	918.79	513.00	260.33	131.40	79.54	61.03	61.25	62.70	97.28
1.3	894.99	492.29	235.75	99.51	40.41	18.09	16.97	16.79	53.14
1.4	886.81	486.50	228.71	89.38	27.34	3.70'	2.61'	2.30'	29.47

': Non-rejection values of the null hypothesis (5.4) at 95% significance level, ant "---" means that the test statistic exceeds 999999.

TABLE 6.B9 (1950's decade)

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with no regressors and white noise U_t. d_1 and d_2 are the differencing orders for log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	146.56	224.79	264.76	271.49	262.44	246.98	229.60	212.45	196.58
0.7	41.68	45.70	136.52	186.71	197.60	191.50	179.16	165.17	151.50
0.8	156.59	57.83	10.41	78.69	125.47	137.61	134.42	125.74	115.61
0.9	211.85	154.73	55.32	1.19'	45.76	82.66	94.09	93.31	87.98
1.0	217.69	182.39	122.76	40.42	0.87'	29.49	55.96	65.74	66.57
1.1	206.56	178.77	139.01	86.53	26.55	4.21'	22.90	41.04	49.00
1.2	190.62	165.83	134.39	98.59	57.84	19.06	9.11	21.41	33.69
1.3	174.28	151.05	123.83	96.05	68.18	39.96	17.52	14.40	22.46
1.4	159.15	136.96	112.40	89.15	68.19	48.98	31.31	19.54	19.44

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with an intercept and white noise U_t. d_1 and d_2 are the differencing orders for log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	220.62	195.98	191.70	193.62	197.29	201.26	204.92	208.06	210.67
0.7	187.40	162.98	156.90	157.82	161.59	166.22	170.62	174.33	177.28
0.8	150.39	127.63	120.81	120.59	123.75	128.32	132.97	137.00	140.23
0.9	111.56	90.86	84.45	83.54	85.85	89.87	94.31	98.34	101.67
1.0	76.25	57.21	51.85	50.80	52.39	55.69	59.64	63.45	66.71
1.1	49.32	31.23	26.97	26.13	27.27	29.93	33.37	36.84	39.98
1.2	32.37	14.61	11.18	10.63	11.55	13.79	16.83	20.07	23.01
1.3	23.92	6.10	3.14'	2.84'	3.67'	5.66'	8.44	11.48	14.33
1.4	21.15	3.13'	0.38'	0.20'	1.01'	2.86'	5.51'	8.44	11.24

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with a time trend and white noise U_t. d_1 and d_2 are the differencing orders for log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	226.47	218.11	228.38	242.93	255.00	263.24	268.45	271.74	274.03
0.7	205.47	187.09	191.05	202.98	214.51	223.05	228.73	232.44	235.01
0.8	170.43	143.62	139.98	147.28	156.59	164.32	169.88	173.71	176.45
0.9	131.31	99.29	89.19	91.63	98.07	104.38	109.37	113.06	115.83
1.0	98.09	64.00	49.75	48.46	52.39	57.27	61.55	64.96	67.65
1.1	75.19	41.00	24.76	21.23	23.46	27.27	30.96	34.09	36.67
1.2	62.11	28.61	11.70	7.09	8.38	11.55	14.86	17.79	20.28
1.3	56.20	23.40	6.42	1.38'	2.23'	5.08'	8.18	10.99	13.43
1.4	54.60	22.25	5.37'	0.19'	0.86'	3.55'	6.56	9.32	11.74

': Non-rejection values of the null hypothesis (5.4) at 95% significance level.

TABLE 6.B10 (1950's decade)

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with no regressors and white noise U_t. d_1 and d_2 are the differencing orders for log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	111.58	174.91	206.29	210.78	202.77	189.80	175.45	161.42	148.49
0.7	30.70	34.06	108.49	148.89	157.54	152.65	142.78	131.58	120.60
0.8	125.51	45.28	7.29	65.05	103.85	114.16	111.99	105.26	97.21
0.9	170.28	124.62	44.96	0.83'	40.40	72.14	82.29	82.18	78.12
1.0	175.22	147.87	101.75	34.72	1.40'	28.54	52.25	61.19	62.34
1.1	166.47	145.81	116.47	75.44	24.46	4.85'	24.21	41.16	48.52
1.2	153.73	135.97	113.75	87.05	53.74	18.38	9.56	23.82	35.80
1.3	140.57	124.38	105.77	85.85	64.03	39.11	16.67	14.59	25.30
1.4	128.31	113.16	96.75	80.56	64.71	48.33	31.00	17.93	19.42

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with an intercept and white noise U_t. d_1 and d_2 are the differencing orders for log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	218.56	196.19	191.91	193.78	197.47	201.40	204.95	207.96	210.49
0.7	183.06	160.80	154.98	156.09	160.00	164.61	168.87	172.41	175.25
0.8	144.32	123.51	117.01	117.07	120.43	124.99	129.49	133.33	136.42
0.9	105.22	86.31	80.20	79.57	82.09	86.11	90.40	94.24	97.42
1.0	70.84	53.54	48.41	47.60	49.39	52.70	56.52	60.15	63.28
1.1	45.22	28.88	24.80	24.14	25.47	28.16	31.48	34.82	37.81
1.2	29.34	13.37	10.08	9.68	10.76	13.03	15.97	19.04	21.90
1.3	21.52	5.56'	2.72'	2.52'	3.50'	5.52'	8.22	11.11	13.87
1.4	19.05	2.93'	0.28'	0.20'	1.14'	3.03'	5.58'	8.38	11.08

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with a time trend and white noise U_t. d_1 and d_2 are the differencing orders for log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	218.02	208.99	219.15	233.09	244.51	252.37	257.44	260.74	263.06
0.7	195.02	175.35	179.13	190.58	201.57	209.77	215.32	219.02	221.60
0.8	161.43	133.45	129.94	137.05	146.03	153.53	159.00	162.82	165.55
0.9	124.88	92.23	82.72	85.33	91.67	97.88	102.83	106.53	109.29
1.0	93.61	59.43	46.21	45.39	49.39	54.27	58.57	62.01	64.71
1.1	71.63	37.78	22.87	20.00	22.41	26.28	30.02	33.20	35.80
1.2	58.81	25.90	10.51	6.67	8.20	11.45	14.83	17.81	20.32
1.3	52.84	20.78	5.44'	1.23'	2.34'	5.27'	8.44	11.31	13.77
1.4	51.12	19.59	4.40'	0.09'	1.02'	3.80'	6.87	9.69	12.12

': Non-rejection values of the null hypothesis (5.4) at 95% significance level.

TABLE 6.B11 (1960's decade)

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with no regressors and white noise U_t. d_1 and d_2 are the differencing orders for log of CPI and wages.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	331.23	411.10	373.07	334.27	299.61	269.15	242.70	219.95	200.56
0.7	228.78	157.52	308.70	284.52	252.81	223.92	198.78	177.27	159.03
0.8	296.92	200.43	49.47	218.39	207.24	183.67	161.58	142.46	126.29
0.9	288.72	240.21	159.15	8.81	147.00	146.18	130.50	115.05	101.63
1.0	266.70	226.38	183.17	117.88	2.09'	96.87	102.51	93.32	83.39
1.1	242.59	204.99	169.58	134.44	84.43	5.83'	65.57	74.05	69.46
1.2	219.75	183.75	151.74	123.86	97.57	61.09	11.50	48.10	56.96
1.3	199.28	164.54	134.87	110.58	90.66	72.34	46.73	16.95	39.45
1.4	181.42	147.81	120.04	98.31	81.70	68.61	56.49	38.92	21.80

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with an intercept and white noise U_t. d_1 and d_2 are the differencing orders for log of CPI and wages.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	341.78	360.59	392.15	432.12	464.37	483.65	493.96	499.71	503.32
0.7	303.38	320.79	353.56	394.49	426.99	446.18	456.35	461.98	465.51
0.8	251.19	253.24	274.73	308.26	336.36	353.25	362.32	367.44	370.74
0.9	200.65	173.38	168.20	183.90	202.33	214.67	221.78	226.07	229.02
1.0	179.28	118.68	80.02	73.47	80.70	88.23	93.41	96.94	99.56
1.1	192.38	105.84	40.74	17.68	17.62	22.40	26.59	29.73	32.19
1.2	218.65	116.16	35.58	4.07'	1.03'	4.97'	8.91	11.96	14.37
1.3	240.53	129.37	40.96	5.76'	1.78'	5.59'	9.53	12.58	15.00
1.4	254.50	138.89	46.70	9.88	5.60'	9.42	13.38	16.46	18.89

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with a time trend and white noise U_t. d_1 and d_2 are the differencing orders for log of CPI and wages.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	301.10	325.78	350.89	372.05	387.89	398.89	406.33	411.45	415.11
0.7	263.92	281.15	304.45	325.88	342.40	353.96	361.76	367.09	370.85
0.8	212.68	216.84	232.49	249.94	264.51	275.14	282.53	287.67	291.35
0.9	155.57	143.56	147.23	157.26	167.69	176.18	182.51	187.15	190.58
1.0	109.80	82.42	73.34	75.02	80.70	86.72	91.82	95.86	99.01
1.1	86.50	47.87	28.90	24.04	26.08	30.25	34.46	38.07	41.00
1.2	82.63	37.42	12.53	3.88'	3.93'	7.17	10.95	14.36	17.19
1.3	88.14	39.64	11.77	1.35'	0.58'	3.50'	7.15	10.51	13.32
1.4	95.37	45.34	16.10	4.91'	3.86'	6.71	10.36	13.73	16.55

': Non-rejection values of the null hypothesis (5.4) at 95% significance level.

TABLE 6.B12 (1960's decade)

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with no regressors and white noise U_t. d_1 and d_2 are the differencing orders of log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	251.43	315.77	290.51	261.28	234.08	209.71	188.32	169.84	154.04
0.7	204.88	114.84	238.73	225.37	202.30	180.08	160.24	142.99	128.20
0.8	254.81	178.99	34.48	173.07	169.25	152.27	135.24	120.03	106.90
0.9	245.06	207.82	143.44	6.03	121.46	124.50	113.12	100.96	90.02
1.0	225.40	194.69	161.49	108.28	2.57'	84.61	91.68	84.87	76.79
1.1	204.52	176.20	149.33	121.81	79.52	6.63	60.83	69.46	65.98
1.2	184.87	158.06	134.02	112.51	91.23	58.99	12.05	47.00	55.51
1.3	167.26	141.65	119.57	101.02	85.16	69.67	45.93	17.26	39.84
1.4	151.89	127.29	106.79	90.33	77.24	66.38	55.59	38.52	21.96

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with an intercept and white noise U_t. d_1 and d_2 are the differencing orders of log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	303.02	328.04	354.40	381.50	401.98	414.35	421.45	425.86	428.93
0.7	246.26	266.98	295.51	324.95	346.62	359.38	366.56	370.94	373.97
0.8	196.02	197.82	216.73	241.66	261.08	272.71	279.30	283.39	286.26
0.9	157.73	130.53	127.48	139.93	153.30	162.24	167.68	171.25	173.88
1.0	143.79	87.25	57.83	54.52	60.67	66.69	70.99	74.09	76.52
1.1	154.77	76.63	26.86	11.89	13.34	17.74	21.48	24.37	26.71
1.2	175.40	84.23	22.61	1.66'	1.25'	5.21'	8.85	11.71	14.02
1.3	193.04	94.74	27.06	3.47'	2.49'	6.43	10.09	12.97	15.29
1.4	204.79	102.73	32.05	7.27	6.11	10.08	13.79	16.68	19.02

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with a time trend and white noise U_t. d_1 and d_2 are the differencing orders of log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	306.35	339.12	367.27	386.95	399.49	407.29	412.33	415.85	418.50
0.7	242.14	263.14	288.33	308.37	321.98	330.68	336.32	340.18	343.00
0.8	181.99	185.08	201.12	217.32	229.58	237.94	243.56	247.49	250.38
0.9	130.56	114.04	117.23	126.54	135.55	142.52	147.61	151.36	154.19
1.0	96.15	62.89	53.49	55.41	60.67	65.94	70.31	73.77	76.50
1.1	81.87	37.47	18.97	15.36	17.83	21.82	25.66	28.91	31.56
1.2	81.99	31.68	8.11	1.46'	2.47'	5.85'	9.45	12.61	15.23
1.3	87.93	35.02	9.11	1.13'	1.57'	4.77'	8.32	11.47	14.09
1.4	94.23	40.30	13.41	4.92'	5.20'	8.38	11.94	15.12	17.76

': Non-rejection values of the null hypothesis (5.4) at 95% significance level.

TABLE 6.B13 (1960's decade)

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with no regressors and a VMA(1) structure on U_t. d_1 and d_2 are the differencing orders for log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	------	223.70	119.79	70.42	33.95	43.51	709.51	12778	------
0.7	54.25	------	164.04	241.17	79.28	77.15	177.59	1551.3	------
0.8	22.23	38.38	------	91.44	114.23	114.53	109.38	284.41	1137.7
0.9	10.23	66.70	50.12	------	122.51	148.93	144.47	165.32	229.69
1.0	1237.6	42.95	212.94	117.43	10.07	611.56	181.51	934.05	122.71
1.1	923625	137.90	330.74	462.54	300.35	25489	736.78	162.68	90.47
1.2	------	------	------	531.73	503.82	428.84	2132.2	755.45	167.07
1.3	811793	701643	2349.4	590.39	446.51	332.75	291.95	485.85	146.41
1.4	------	427061	105494	938.07	346.69	227.70	188.95	161.35	149.31

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with an intercept and a VMA(1) structure on U_t. d_1 and d_2 are the differencing orders for log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	191.75	212.84	225.22	233.36	240.12	246.47	252.53	258.06	262.13
0.7	134.87	151.57	166.21	177.64	186.64	193.85	199.68	204.14	205.90
0.8	93.39	96.64	105.87	115.88	124.74	131.92	137.43	141.17	141.37
0.9	71.55	60.08	59.29	63.89	70.11	75.97	80.67	83.67	82.88
1.0	65.26	42.71	31.89	29.88	32.38	36.26	39.88	42.23	41.37
1.1	67.97	38.38	20.09	12.62	11.88	13.99	16.70	18.71	18.36
1.2	74.15	40.39	17.31	6.21	3.27'	4.15'	6.21	8.02	8.25
1.3	79.97	43.89	18.13	5.01'	0.80'	0.89'	2.43'	4.00'	4.53'
1.4	83.82	46.27	19.25	5.32'	0.56'	0.17'	1.28'	2.57'	3.15'

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with a time trend and a VMA(1) structure on U_t. d_1 and d_2 are the differencing orders for log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	175.91	195.91	211.22	220.85	226.68	230.63	234.03	237.61	240.97
0.7	122.43	134.45	147.73	157.99	165.17	170.35	174.54	178.20	180.37
0.8	84.40	86.09	93.97	102.04	108.64	113.83	118.08	121.53	122.73
0.9	60.98	53.19	54.57	58.93	63.62	67.86	71.58	74.59	75.19
1.0	48.95	33.98	29.51	30.01	32.38	35.29	38.22	40.73	41.17
1.1	44.51	24.82	16.01	13.37	13.74	15.43	17.62	19.74	20.32
1.2	44.26	21.82	10.25	5.50'	4.46'	5.26'	6.93	8.79	9.64
1.3	45.65	21.79	8.74	2.81'	0.95'	1.22'	2.55'	4.25'	5.33'
1.4	46.83	22.42	8.80	2.42'	0.22'	0.22'	1.34'	2.92'	4.13'

': Non-rejection values of the null hypothesis (5.4) at 95% significance level and "---" means that the tests statistic exceeds 999999.

TABLE 6.B14 (1970's decade)

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with no regressors and white noise U_t. d_1 and d_2 are the differencing orders of the log of CPI and wages respectively.

d_1\d_2	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	376.13	403.36	360.48	323.14	290.45	261.62	236.36	214.46	195.65
0.7	210.93	165.83	302.64	276.22	246.05	218.71	194.73	174.03	156.34
0.8	285.14	207.31	42.62	215.48	202.73	180.19	159.07	140.65	124.94
0.9	278.06	240.23	174.39	6.46	147.58	144.63	129.24	114.21	101.10
1.0	257.09	224.40	188.27	133.86	2.48'	100.16	102.94	93.38	83.45
1.1	233.90	202.55	171.62	140.84	97.70	5.44'	70.21	75.54	70.17
1.2	211.80	181.27	152.64	127.09	103.26	70.56	9.55	52.86	58.84
1.3	191.91	162.17	135.26	112.49	93.59	76.56	52.70	13.74	43.53
1.4	174.51	145.55	120.17	99.55	83.43	70.72	59.29	42.27	17.72

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with an intercept and white noise U_t. d_1 and d_2 are the differencing orders of the log of CPI and wages respectively.

d_1\d_2	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	362.76	352.82	386.54	434.95	462.79	472.48	475.86	477.79	479.37
0.7	373.15	330.80	356.39	409.53	442.05	453.83	457.87	459.96	461.56
0.8	380.08	303.33	297.57	339.55	370.35	382.39	386.69	388.83	390.39
0.9	368.75	266.75	215.69	229.28	249.70	258.93	262.51	264.40	265.83
1.0	348.29	229.78	137.51	119.50	126.86	132.01	134.40	135.91	137.20
1.1	332.45	203.42	85.74	48.82	49.03	52.09	53.92	55.32	56.62
1.2	323.90	188.54	59.45	16.04	14.64	17.46	19.31	20.80	22.22
1.3	320.67	182.01	48.88	4.29'	3.04'	5.99	7.95	9.53	11.04
1.4	321.12	181.07	46.82	2.28'	1.20'	4.17'	6.13	7.73	9.29

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with a time trend and white noise U_t. d_1 and d_2 are the differencing orders of the log of CPI and wages respectively.

d_1\d_2	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	274.35	283.15	295.59	306.50	314.59	320.29	324.32	327.28	329.55
0.7	261.87	264.83	274.80	285.09	293.30	299.26	303.54	306.67	309.06
0.8	234.89	230.74	236.11	243.93	250.95	256.37	260.40	263.42	265.75
0.9	193.37	182.61	182.68	187.03	192.01	196.28	199.66	202.32	204.42
1.0	142.96	127.39	122.93	123.99	126.86	129.87	132.52	134.74	136.59
1.1	93.88	75.79	68.49	67.34	68.76	70.90	73.05	75.00	76.70
1.2	56.60	37.81	29.37	27.28	28.11	29.95	31.97	33.89	35.61
1.3	35.90	17.20	8.62	6.42	7.24	9.14	11.26	13.28	15.11
1.4	29.19	10.64	2.19'	0.13'	1.10'	3.14'	5.37'	7.49	9.39

': Non-rejection values of the null hypothesis (4.4) at 95% significance level.

TABLE 6.B15 (1970's decade)

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with no regressors and white noise U_t. d_1 and d_2 are the differencing orders of log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	291.50	316.70	284.83	255.48	229.24	205.83	185.20	167.25	151.82
0.7	180.61	126.62	240.83	222.32	199.09	177.47	158.23	141.45	127.00
0.8	237.77	175.69	32.31	176.19	168.12	150.82	134.08	119.21	106.35
0.9	230.45	200.95	149.05	5.22'	125.44	124.73	112.81	100.70	89.90
1.0	212.56	187.71	160.34	117.15	2.89'	89.18	92.79	85.27	77.05
1.1	193.04	169.76	146.93	123.62	88.56	5.95'	65.60	71.10	66.78
1.2	174.44	152.20	131.46	112.57	94.15	66.54	9.92	51.57	57.40
1.3	157.67	136.32	117.11	100.53	86.25	72.58	51.36	13.95	43.89
1.4	142.95	122.40	104.50	89.66	77.64	67.65	57.96	41.84	17.80

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with an intercept and white noise U_t. d_1 and d_2 are the differencing orders of log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	314.14	314.89	338.55	366.90	382.22	387.97	390.68	392.57	394.12
0.7	302.00	269.29	288.78	323.74	344.11	351.92	355.26	357.32	358.89
0.8	306.90	237.54	230.90	259.35	280.05	288.70	292.39	294.51	296.07
0.9	303.32	210.46	167.87	176.81	191.53	198.77	202.06	203.99	205.43
1.0	288.61	183.26	110.19	97.30	103.64	108.26	110.67	112.25	113.55
1.1	272.69	160.21	68.85	42.69	44.14	47.23	49.11	50.52	51.81
1.2	262.49	145.42	45.49	14.49	14.71	17.50	19.33	20.77	22.13
1.3	258.65	138.87	35.47	3.32'	3.52'	6.35	8.20	9.71	11.15
1.4	259.19	138.15	33.54	1.20'	1.41'	4.19'	6.02	7.54	9.02

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with a time trend and white noise U_t. d_1 and d_2 are the differencing orders of log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	272.76	284.02	296.71	306.66	313.60	318.35	321.71	324.23	326.19
0.7	240.61	244.75	254.56	263.86	270.98	276.07	279.74	282.47	284.58
0.8	204.94	200.99	205.97	212.94	219.07	223.80	227.37	230.08	232.20
0.9	164.13	153.11	152.91	156.75	161.19	165.05	168.17	170.66	172.66
1.0	120.19	104.33	99.92	100.91	103.64	106.54	109.13	111.32	113.15
1.1	79.35	61.13	54.23	53.38	54.96	57.21	59.44	61.44	63.17
1.2	48.96	30.20	22.39	20.83	21.98	24.01	26.16	28.14	29.88
1.3	32.43	13.88	6.04	4.44'	5.61'	7.71	9.93	11.99	13.80
1.4	27.56	9.18	1.50'	0.03'	1.34'	3.54'	5.85'	7.97	9.83

': Non-rejection values of the null hypothesis (5.4) at 95% significance level.

TABLE 6.B16 (1970's decade)

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with no regressors and a VAR(1) structure on U_t. d_1 and d_2 are the differencing orders of log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	117.96	214.71	264.33	270.33	246.36	2204.2	1227.5	904.01	6076.6
0.7	77.93	142.21	171.16	209.92	227.56	240.45	466.10	1311.0	7120.8
0.8	42.07	68.57	112.16	152.05	198.54	249.87	28950	1463.3	6863.1
0.9	33.80	41.94	63.61	114.90	154.33	240.39	499.99	1256.9	5603.7
1.0	832.06	647.84	480.58	220.72	4.51'	376.55	1192.8	2485.8	5098.7
1.1	170247	673495	100878	14366	1961.5	215.20	363.83	2581.4	4838.0
1.2	735696	674218	423953	58132	10531	1279.5	134.14	956.36	3221.4
1.3	53448	295376	610815	134237	21935	6046.0	1058.1	11.16	1264.0
1.4	51775	181795	576768	274222	38219	11392	4737.3	1307.7	9.54

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with an intercept and a VAR(1) structure on U_t. d_1 and d_2 are the differencing orders of log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	1.34'	2.47'	4.16'	5.58'	6.88	7.76	8.42	9.16	10.08
0.7	4.08'	1.82'	3.62'	6.08	7.67	8.50	9.15	9.92	10.91
0.8	7.92	5.27'	7.01	10.78	13.06	13.83	14.30	15.02	16.09
0.9	6.88	5.52'	7.35	11.54	14.31	14.95	15.18	15.83	16.95
1.0	2.78'	0.49'	0.99'	1.58'	4.43'	6.60	7.81	8.82	9.91
1.1	5.96'	1.95'	3.75'	2.83'	7.57	11.65	13.53	14.56	15.42
1.2	11.62	5.37'	9.19	7.01	11.33	15.21	16.82	17.66	18.41
1.3	11.99	4.92'	10.02	5.96'	9.61	13.35	14.91	15.78	16.63
1.4	8.70	2.99'	8.67	2.80'	5.95'	9.73	11.43	12.47	13.52

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with a time trend and a VAR(1) structure on U_t. d_1 and d_2 are the differencing orders of log of CPI and wages respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	5.33'	5.05'	5.97'	7.45	9.15	10.89	12.58	14.14	15.49
0.7	10.92	10.16	10.76	12.14	13.82	15.58	17.29	18.89	20.30
0.8	14.56	13.38	13.57	14.70	16.25	17.93	19.60	21.19	22.62
0.9	10.90	9.57	9.38	10.23	11.61	13.19	14.80	16.33	17.73
1.0	2.67'	1.53'	1.42'	2.57'	4.43'	6.51	8.51	10.30	11.83
1.1	2.30'	1.27'	1.54'	3.39'	6.07	8.89	11.45	13.60	15.32
1.2	7.02	5.91'	6.05	7.84	10.54	13.41	16.03	18.23	19.99
1.3	8.07	6.96	6.74	8.08	10.43	13.07	15.58	17.77	19.57
1.4	5.17'	4.17'	3.71'	4.74'	6.84	9.37	11.87	14.13	16.05

': Non-rejection values of the null hypothesis (5.4) at 95% significance level.

TABLE 6.B17

$\hat{r}$ in (2.9) for the estimated residuals.

log CPI_t - 3.91 - 0.70 log W_t

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	35.07	24.95	15.55	8.10	2.83	-0.68'	-2.97	-4.49	-5.52	-6.26
AR(1)	8.95	13.04	10.43	6.81	3.46	0.74	-1.29	-2.75	-3.79	-4.53
AR(2)	2.41	7.93	8.23	5.83	3.14	0.76	-1.13	-2.54	-3.54	-4.23
SAR(1)	15.27	14.51	11.52	7.19	2.91	-0.50'	-2.91	-4.53	-5.62	-6.38
SAR(2)	11.41	10.88	9.25	6.26	2.70	-0.49'	-2.87	-4.51	-5.63	-6.40
b) Intercept.										
W.N.	37.90	27.78	17.78	9.61	3.84	0.12'	-2.23	-3.76	-4.81	-5.58
AR(1)	-0.39	7.12	8.42	5.88	2.91	0.45'	-1.31	-2.52	-3.35	-3.96
AR(2)	-1.75	2.15	5.47	4.45	2.24	0.16	-1.43	-2.51	-3.23	-3.70
SAR(1)	13.99	13.69	12.14	8.30	3.88	0.26'	-2.23	-3.86	-4.95	-5.72
SAR(2)	9.71	9.39	8.73	6.56	3.26	0.08'	-2.28	-3.87	-4.95	-5.73
c) Intercept and a time trend.										
W.N.	38.07	27.85	17.76	9.57	3.85	0.14'	-2.21	-3.74	-4.80	-5.58
AR(1)	-0.94	7.00	8.42	5.87	2.91	0.48	-1.28	-2.49	-3.34	-3.96
AR(2)	-1.98	2.05	5.48	4.44	2.24	0.19	-1.39	-2.48	-3.21	-3.70
SAR(1)	13.73	13.65	12.14	8.29	3.88	0.29'	-2.20	-3.84	-4.94	-5.72
SAR(2)	9.64	9.37	8.74	6.56	3.26	0.10'	-2.26	-3.86	-4.95	-5.73

log W_t + 5.31 - 13.6 log CPI_t

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	32.40	22.63	13.86	7.04	2.22	-1.02'	-3.16	-4.60	-5.60	-6.31
AR(1)	10.42	12.26	9.42	6.00	2.89	0.37	-1.52	-2.90	-3.89	-4.61
AR(2)	3.85	8.19	7.54	5.15	2.64	0.43	-1.33	-2.65	-3.60	-4.27
SAR(1)	15.42	14.02	10.69	6.39	2.32	-0.86'	-3.12	-4.65	-5.69	-6.43
SAR(2)	11.91	10.99	8.94	5.74	2.21	-0.82'	-3.08	-4.64	-5.70	-6.45
b) Intercept.										
W.N.	36.33	26.17	16.47	8.73	3.33	-0.17'	-2.40	-3.87	-4.88	-5.63
AR(1)	1.41	8.00	8.08	5.41	2.53	0.20	-1.47	-2.61	-3.41	-4.01
AR(2)	-0.97	3.20	5.48	4.10	1.91	-0.09	-1.58	-2.61	-3.28	-3.73
SAR(1)	14.65	13.85	11.80	7.78	3.43	-0.04'	-2.41	-3.97	-5.03	-5.78
SAR(2)	9.89	9.56	8.67	6.26	2.91	-0.19'	-2.46	-3.99	-5.03	-5.78
c) Intercept and a time trend.										
W.N.	36.82	26.51	16.59	8.75	3.33	-0.16'	-2.39	-3.86	-4.88	-5.63
AR(1)	-0.05	7.59	8.10	5.41	2.53	0.21	-1.46	-2.60	-3.41	-4.01
AR(2)	-1.61	2.76	5.46	4.11	1.91	-0.07	-1.57	-2.59	-3.27	-3.73
SAR(1)	14.04	13.71	11.82	7.78	3.43	-0.03'	-2.40	-3.97	-5.02	-5.78
SAR(2)	9.65	9.44	8.65	6.26	2.91	-0.18'	-2.45	-3.98	-5.03	-5.78

': Non-rejection values of the null hypothesis (1.12) at 95% significance level when monotonicity in the test statistic with respect to d is observed.

TABLE 6.B18 (1950's decade)

$\hat{r}$ in (2.9) for the estimated residuals.

log CPI(50)$_t$ - 4.16 - 0.36 log W(50)$_t$

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	9.35	7.32	5.44	3.75	2.30	1.07'	0.03'	-0.84'	-1.57'	-2.19
AR(1)	2.02	2.06	1.61	1.09	0.58	0.07	-0.41	-0.86	-1.28	-1.67
AR(2)	3.00	2.27	1.71'	1.23'	0.81'	0.40'	0.02'	-0.35'	-0.70'	-1.04'
SAR(1)	7.53	6.46	5.15	3.73	2.34	1.08'	0.01'	-0.89'	-1.63'	-2.24
SAR(2)	7.51	6.42	5.12	3.72	2.35	1.10'	0.03'	-0.86'	-1.61'	-2.23
b) Intercept.										
W.N.	13.24	11.69	9.34	6.69	4.25	2.24	0.68'	-0.49'	-1.38'	-2.06
AR(1)	1.32'	0.82'	0.56'	0.40'	0.03'	-0.47'	-0.99'	-1.44'	-1.82'	-2.12
AR(2)	3.64	1.59'	0.42'	-0.07'	-0.46'	-0.91'	-1.36'	-1.76'	-2.08	-2.32
SAR(1)	8.78	8.25	7.31	5.92	4.18	2.38	0.79'	-0.48'	-1.44'	-2.15
SAR(2)	8.62	7.69	6.44	5.05	3.56	2.04	0.63'	-0.55'	-1.46'	-2.15
c) Intercept and a time trend.										
W.N.	14.88	12.56	9.71	6.80	4.24	2.18	0.62'	-0.55'	-1.42'	-2.08
AR(1)	0.19	-0.14	0.18	0.33	0.01	-0.50	-1.03	-1.48	-1.84	-2.12
AR(2)	1.28'	0.14'	-0.13'	-0.22'	-0.51'	-0.94'	-1.39'	-1.78'	-2.08	-2.30
SAR(1)	8.81	8.24	7.33	5.94	4.17	2.33	0.72'	-0.55'	-1.48'	-2.17
SAR(2)	8.71	7.64	6.39	5.03	3.54	2.00	0.57'	-0.61'	-1.51'	-2.18

log W(50)$_t$ + 10.29 - 2.48 log CPI(50)$_t$

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	10.80	8.40	6.20	4.29	2.69	1.36'	0.26'	-0.64'	-1.39'	-2.03
AR(1)	2.66	2.84	2.14	1.39	0.73	0.15	-0.36	-0.82	-1.23	-1.60
AR(2)	4.46	3.21	2.27	1.55'	0.95'	0.44'	-0.01'	-0.41'	-0.76'	-1.09'
SAR(1)	8.15	7.13	5.78	4.26	2.75	1.39'	0.24'	-0.70'	-1.47'	-2.10
SAR(2)	8.03	6.94	5.63	4.17	2.71	1.39'	0.25'	-0.68'	-1.45'	-2.09
b) Intercept.										
W.N.	13.64	11.48	8.82	6.09	3.68	1.76'	0.30'	-0.79'	-1.62'	-2.26
AR(1)	0.76	0.54	0.57	0.41	-0.02	-0.56	-1.08	-1.52	-1.87	-2.15
AR(2)	2.46	1.00'	0.24'	-0.16'	-0.57'	-1.04'	-1.49'	-1.88'	-2.17	-2.38
SAR(1)	8.69	8.06	7.04	5.54	3.73	1.93'	0.39'	-0.81'	-1.69'	-2.35
SAR(2)	8.43	7.36	6.09	4.68	3.17	1.64'	0.27'	-0.85'	-1.71'	-2.35
c) Intercept and a time trend.										
W.N.	14.29	11.88	9.00	6.15	3.68	1.73'	0.26'	-0.83'	-1.64'	-2.27
AR(1)	0.21	0.08	0.42	0.38	-0.03	-0.58	-1.10	-1.54	-1.88	-2.15
AR(2)	1.35'	0.32'	-0.01'	-0.23'	-0.60'	-1.06'	-1.51'	-1.89'	-2.16	-2.36
SAR(1)	8.67	8.05	7.05	5.55	3.72	1.90'	0.34'	-0.85'	-1.72'	-2.37
SAR(2)	8.43	7.33	6.06	4.67	3.16	1.62'	0.23'	-0.89'	-1.74'	-2.37

': Non-rejection values of the null hypothesis (1.12) at 95% significance level when monotonicity in the value of the tests with respect to d is observed.

TABLE 6.B19 (1960's decade)

$\hat{r}$ in (2.9) for the estimated residuals.

log CPI(60)$_t$ - 3.88 - 0.68 log W(60)$_t$

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	6.21	3.93	2.03	0.50'	-0.71'	-1.67'	-2.42	-3.02	-3.51	-3.91
AR(1)	1.04'	0.75'	0.32'	-0.13'	-0.56'	-0.95'	-1.31'	-1.64'	-1.96	-2.24
AR(2)	0.32'	-0.45'	-1.11'	-1.62'	-1.97	-2.21	-2.36	-2.46	-2.52	-2.57
SAR(1)	6.23	4.19	2.24	0.58'	-0.74'	-1.75'	-2.52	-3.11	-3.59	-3.98
SAR(2)	6.14	4.19	2.25	0.58'	-0.72'	-1.70'	-2.44	-3.01	-3.47	-3.86
b) Intercept.										
W.N.	5.51	3.38	1.64'	0.25'	-0.86'	-1.75'	-2.46	-3.04	-3.51	-3.91
AR(1)	0.18'	0.08'	-0.16'	-0.46'	-0.76'	-1.06'	-1.34'	-1.62'	-1.99	-2.15
AR(2)	-0.59'	-1.23'	-1.73'	-2.10	-2.36	-2.52	-2.61	-2.65	-2.68	-2.70
SAR(1)	5.65	3.63	1.79'	0.26'	-0.94'	-1.86'	-2.57	-3.13	-3.58	-3.96
SAR(2)	5.55	3.61	1.79'	0.28'	-0.88'	-1.76'	-2.43	-2.97	-3.41	-3.79
c) Intercept and a time trend.										
W.N.	5.51	3.37	1.63'	0.25'	-0.86'	-1.74'	-2.45	-3.03	-3.51	-3.91
AR(1)	0.21'	0.10'	-0.16'	-0.46'	-0.76'	-1.05'	-1.33'	-1.61'	-1.98	-2.15
AR(2)	-0.56'	-1.20'	-1.72'	-2.10	-2.35	-2.51	-2.59	-2.63	-2.66	-2.68
SAR(1)	5.66	3.63	1.78'	0.26'	-0.94'	-1.86'	-2.56	-3.13	-3.58	-3.96
SAR(2)	5.57	3.62	1.78'	0.28'	-0.88'	-1.75'	-2.42	-2.96	-3.41	-3.79

log W(60)$_t$ + 5.60 - 1.44 log CPI(60)$_t$

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	6.11	3.87	1.99	0.48'	-0.73'	-1.68'	-2.43	-3.04	-3.52	-3.92
AR(1)	1.09'	0.77'	0.33'	-0.12'	-0.56'	-0.96'	-1.32'	-1.66'	-1.98	-2.28
AR(2)	0.36'	-0.44'	-1.09'	-1.58'	-1.93'	-2.16	-2.31	-2.41	-2.48	-2.53
SAR(1)	6.17	4.14	2.21	0.57'	-0.75'	-1.76'	-2.53	-3.13	-3.60	-3.99
SAR(2)	6.14	4.17	2.23	0.57'	-0.74'	-1.72'	-2.46	-3.04	-3.51	-3.89
b) Intercept.										
W.N.	5.27	3.19	1.52'	0.17'	-0.91'	-1.78'	-2.48	-3.06	-3.53	-3.92
AR(1)	0.14'	0.00'	-0.24'	-0.52'	-0.80'	-1.08'	-1.36'	-1.62'	-1.89'	-2.16
AR(2)	-0.64'	-1.32'	-1.82'	-2.17	-2.40	-2.54	-2.62	-2.65	-2.67	-2.69
SAR(1)	5.48	3.46	1.65'	0.17'	-1.00'	-1.90'	-2.60	-3.15	-3.60	-3.97
SAR(2)	5.45	3.46	1.65'	0.18'	-0.94'	-1.80'	-2.46	-2.99	-3.43	-3.81
c) Intercept and a time trend.										
W.N.	5.26	3.19	1.51'	0.17'	-0.91'	-1.78'	-2.48	-3.05	-3.52	-3.92
AR(1)	0.10'	0.00'	-0.24'	-0.52'	-0.80'	-1.08'	-1.35'	-1.62'	-1.88'	-2.15
AR(2)	-0.69'	-1.32'	-1.81'	-2.17	-2.40	-2.53	-2.61	-2.64	-2.66	-2.67
SAR(1)	5.47	3.46	1.65'	0.16'	-1.00'	-1.90'	-2.59	-3.15	-3.60	-3.97
SAR(2)	5.42	3.46	1.65'	0.18'	-0.94'	-1.79'	-2.45	-2.98	-3.43	-3.81

': Non-rejection values of the null hypothesis (1.12) at 95% significance level when monotonicity in the value of the tests with respect to d is observed.

TABLE 6.B20 (1970's decade)

$\hat{r}$ in (2.9) for the estimated residuals.

log CPI(70)$_t$ - 3.60 + 0.91 log W(70)$_t$

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	7.37	4.78	2.48	0.57'	-0.91'	-2.01	-2.82	-3.41	-3.84	-4.17
AR(1)	1.99	1.87'	1.25'	0.40'	-0.50'	-1.34'	-2.05	-2.61	-3.05	-3.39
AR(2)	0.08	0.68	0.65	0.28	-0.27	-0.87	-1.45	-1.93	-2.32	-2.62
SAR(1)	5.58	4.00	2.27	0.60'	-0.84'	-1.97	-2.82	-3.43	-3.88	-4.21
SAR(2)	5.57	4.14	2.52	0.86'	-0.62'	-1.83'	-2.73	-3.39	-3.86	-4.21
b) Intercept.										
W.N.	6.88	4.36	2.17	0.39'	-0.98'	-2.00	-2.75	-3.30	-3.72	-4.04
AR(1)	1.42'	1.40'	0.90'	0.16'	-0.63'	-1.36'	-1.98	-2.48	-2.88	-3.20
AR(2)	-0.48	0.24	0.35	0.11	-0.32	-0.81	-1.29	-1.70	-2.04	-2.32
SAR(1)	5.18	3.62	1.94'	0.36'	-0.97'	-2.00	-2.78	-3.35	-3.77	-4.09
SAR(2)	5.16	3.73	2.13	0.55'	-0.84'	-1.96	-2.77	-3.36	-3.80	-4.13
c) Intercept and a time trend.										
W.N.	6.88	4.34	2.15	0.38'	-0.98'	-1.99	-2.74	-3.30	-3.71	-4.04
AR(1)	1.43'	1.41'	0.89'	0.16'	-0.63'	-1.35'	-1.97	-2.47	-2.87	-3.19
AR(2)	-0.47	0.25	0.35	0.11	-0.31	-0.80	-1.27	-1.69	-2.03	-2.31
SAR(1)	5.19	3.61	1.93'	0.36'	-0.97'	-2.00	-2.77	-3.34	-3.77	-4.09
SAR(2)	5.17	3.74	2.13	0.54'	-0.84'	-1.94'	-2.76	-3.36	-3.80	-4.12

log W(70)$_t$ + 3.92 - 1.08 log CPI(70)$_t$

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	7.36	4.78	2.47	0.57'	-0.91'	-2.02	-2.83	-3.42	-3.85	-4.17
AR(1)	2.04	1.88'	1.24'	0.39'	-0.52'	-1.36'	-2.06	-2.63	-3.07	-3.41
AR(2)	0.17	0.71	0.65	0.26	-0.29	-0.90	-1.48	-1.97	-2.36	-2.66
SAR(1)	5.62	4.02	2.28	0.61'	-0.83'	-1.97	-2.82	-3.44	-3.89	-4.22
SAR(2)	5.61	4.17	2.54	0.88'	-0.61'	-1.82'	-2.73	-3.39	-3.86	-4.21
b) Intercept.										
W.N.	6.81	4.29	2.12	0.36'	-0.99'	-2.00	-2.75	-3.30	-3.71	-4.04
AR(1)	1.43'	1.38'	0.86'	0.13'	-0.65'	-1.38'	-1.99	-2.49	-2.89	-3.20
AR(2)	-0.40	0.25	0.34	0.09	-0.33	-0.82	-1.30	-1.71	-2.05	-2.32
SAR(1)	5.17	3.59	1.91'	0.34'	-0.98'	-2.01	-2.78	-3.34	-3.77	-4.09
SAR(2)	5.17	3.72	2.11	0.52'	-0.86'	-1.96	-2.77	-3.36	-3.80	-4.12
c) Intercept and a time trend.										
W.N.	6.82	4.29	2.11	0.36'	-0.99'	-2.00	-2.74	-3.30	-3.71	-4.03
AR(1)	1.41'	1.38'	0.86'	0.13'	-0.65'	-1.37'	-1.99	-2.48	-2.88	-3.20
AR(2)	-0.44	0.25	0.34	0.09	-0.33	-0.82	-1.28	-1.70	-2.04	-2.31
SAR(1)	5.17	3.59	1.90'	0.34'	-0.98'	-2.01	-2.77	-3.34	-3.76	-4.09
SAR(2)	5.16	3.72	2.11	0.52'	-0.86'	-1.95'	-2.77	-3.36	-3.79	-4.12

': Non-rejection values of the null hypothesis (1.12) at 95% significance level when monotonicity in the value of the tests with respect to d is observed.

6.c GROSS NATIONAL PRODUCT AND MONEY

In this section we examine the relationship between nominal G.N.P. and nominal money. This is based upon the quantity theory equation: M V = P Y, and most empirical applications stem from the assumption that velocity is constant or at least stationary. Under this general condition, log M, log P and log Y should be cointegrated with known unit parameters, and similarly, nominal G.N.P. and nominal money should also be cointegrated.

A test of this hypothesis was conducted in Engle and Granger (1987) and DeJong (1992). In the first of these articles, four different measures of money were used: M1, M2, M3 and the total liquid assets L, and in each case, the sample period was 1959.1 through 1981.2 quarterly. After applying the ADF tests on the estimated residuals of the regression of log G.N.P. on log of each of the monetary aggregates (and also the reverse situations), only in one of the cases, results of the tests were significant at the 5% significance level and it corresponded to the regression of log of M2 on log of G.N.P. Therefore, they concluded that the only stable relationship was between M2 and nominal G.N.P. but for the other aggregates, the tests rejected the hypothesis of cointegration and stationarity of velocity.

DeJong (1992) examined the relationship between nominal G.N.P. and M2 for the same period of time. First, the integration inference was investigated for the individual series using the DeJong and Whiteman's (1991) Bayesian approach: Using zero trend priors, results strongly supported the inference of integration, however, when non-zero trends were considered the evidence was in favour of trend-stationary alternatives. Similarly, when testing the hypothesis of cointegration, if zero-trend priors were given, nominal G.N.P. and money seemed cointegrated but for a more general prior, this result was relatively implausible. The relationship between these two variables has also been studied in Moazzani and Gupta (1995) who considered a dynamic regression model for estimating the long run relationship between nominal G.N.P. and money. Their results supported the neutrality proposition implied by the quantity theory of money. Among other studies also analysing the relation between output and money are Stein (1982), Lothian (1985), Geweke (1986), Dwyer and Hafer (1988) and Hayakawa (1988).

We consider here the same data set as in Engle and Granger (1987) and DeJong (1992), i.e. logs of G.N.P. and the four monetary aggregates, M1, M2, M3 and L in the United States, from 1959.1 through 1981.2 quarterly. Plots of the raw time series, their sample autocorrelations and estimates of the spectral density function are given in Figure 6.C1 and we observe that all series exhibit a nonstationary character with a smooth trend over time. The slow decay observed in the sample autocorrelations and the peaks around zero frequency in the estimates of the spectral density function suggest that all series might display a unit root component.

We start by examining the nonstationary nature of each individual series. In Engle and Granger (1987) all of them were assumed to be I(1) while in DeJong (1992), the integration inference was supported only if zero trend priors were considered. In Tables 6.C1 and 6.C2 we have calculated Robinson's (1994c) univariate tests for logs of G.N.P. and money respectively, performing $\hat{r}$ in (2.9), for different specifications in (1.9), with $\rho(L;\theta) = (1 - L)^{d+\theta}$ in (1.10), and modelling the disturbances as white noise, seasonal and non-seasonal AR processes. The first thing we observe in these two tables is that if the disturbances follow a non-seasonal AR process, monotonicity in $\hat{r}$ with respect to d is never achieved, implying perhaps that a seasonal component still remains on the series even though they have been deseasonalized previously.

Starting with log of G.N.P., in Table 6.C1, we observe that if we do not include regressors, all non-rejections occur when d ranges between 0.8 and 1.1, and in all cases the lowest statistics across different values of d are obtained when d = 1; however, including an intercept or an intercept and a time trend, we see that the unit root null is always rejected in favour of more nonstationary alternatives, with d ranging now between 1.1 and 1.3, and with the lowest statistics occurring at d = 1.1 when including an intercept, and at d = 1.2 with an intercept and a time trend.

Similar evidence is found when we analyze the log of the monetary aggregates in Table 6.C2: if there are no regressors, the unit root null is never rejected, and though other possibilities with d slightly greater than or smaller than 1 seem also plausible, the lowest statistics across different values of d are always obtained when d = 1; however, including an intercept and an intercept and a time trend, this hypothesis is always decisively rejected in favour of other more nonstationary alternatives, with d ranging now between 1.1 and 1.4 for the log of M1, but greater than 1.3 for the other measures of money, and thus, strongly rejecting the trend-stationary representations advocated by some authors. Therefore, we can conclude the analysis of the individual series by saying that all of them seem to be I(1) when modelling with no regressors, but integration orders greater than one should be required when we include an intercept and/or a time trend in the model.

In the following group of tables we calculate the multivariate score tests of Chapter 5 for each pair of variables, i.e., the log of G.N.P. and logs of each monetary aggregate M1, M2, M3 and L. In Tables 6.C3 and 6.C4 we analyze the first of these relationships (i.e., using M1 as the monetary aggregate), reporting results of the time and the frequency domain versions of the test statistics with white noise U_t. We observe that if we do not include regressors, the only non-rejection cases occur when $d_1 = d_2 = 1$ and 1.1. In both tables the lowest values appear for the case of two unit roots, obtaining the test statistics $\hat{S}^{t2} = 0.09$ in Table 6.C3, and $\hat{S}^{f2} = 0.07$ in Table 6.C4. Including an intercept or an intercept and a time trend, results are similar in both cases, and the null hypothesis of two unit roots is always decisively rejected in favour of other more nonstationary alternatives with d_1 and d_2 greater than or equal to 1.1. In fact, the lowest statistics are obtained in both domains at $d_1 = 1.1$ and $d_2 = 1.2$ when including an intercept, and at $d_1 = d_2 = 1.2$ with an intercept and a time trend. Allowing U_t to be weakly autocorrelated, we do not report results, but the most interesting cases were obtained here when U_t followed a VAR(1) process and we did not include regressors, in which case the only non-rejection case occurred again when $d_1 = d_2 = 1$, and when U_t followed a VMA(1) process and we included regressors, with the non-rejection cases occurring then when d_1 and d_2 were greater than or equal to 1. Thus, results of the multivariate tests, when using M1 as the measure of money, support the evidence found in the univariate tests in Tables 6.C1 and 6.C2 that both series might be I(1) when modelling with no regressors but greater integration orders might be required when including an intercept or an intercept and a time trend.

Very similar results are obtained when we analyze the same relationship but using now the other monetary aggregates. Tables 6.C5-6.C7 show results using M2 instead of M1. Starting with white noise U_t, (in Tables 6.C5 and 6.C6), we see that the non-rejections occur at the same values of d_1 and d_2 in both domains: if there are no regressors, the null hypothesis is not rejected if $d_1 = d_2 = 1$, 1.1 and 1.2, and the lowest statistics are obtained at $d_1 = d_2 = 1.1$ in the time domain, with $\hat{S}^{t2} = 0.89$, (in Table 6.C5), and at $d_1 = d_2 = 1$ in the frequency domain with $\hat{S}^{f2} = 0.88$ (in Table 6.C6). Including an intercept, these hypotheses are strongly rejected and the non-rejections appear now when d_1 ranges between 1.1 and 1.3 and $d_2 = 1.4$. Finally, including an intercept and a time trend, the two unit roots null is again rejected and the non-rejection values range between 1.1 and 1.4 for d_1, and are 1.3 and 1.4 for d_2. Therefore, we again observe here greater integration orders in both variables when including an intercept or an intercept and a time trend in the model. Allowing U_t to be VAR(1), the most interesting case appeared here when we did not include regressors, where the null $d_1 = d_2 = 1$ was the only non-rejection case. Including an intercept and an intercept and a time trend, there were more non-rejections but they did not show much coherence, suggesting that the model might be misspecified in these cases. This might be related to the lack of monotonic decrease observed in $\hat{r}$ with respect to d in the univariate results in Tables 6.C1 and 6.C2 with AR disturbances. If U_t is VMA(1), we observe in Table 6.C7 that the two unit roots hypothesis is always decisively rejected in favour of alternatives with d_1 and d_2 greater than 1. The lowest statistics appear in this case when $d_1 = 1.3$ and $d_2 = 1.4$ (with no regressors), 1.2 (with an intercept), and 1.3 (with an intercept and a time trend). Thus, results of the multivariate tests, when using M2 as the measure of money, are again in line with those obtained with the univariate tests in Tables 6.C1 and 6.C2, suggesting that both series might be I(1) when modelling with no regressors, but rejecting this hypothesis in favour of more nonstationarities when an intercept and/or a time trend is included.

In Tables 6.C8-6.C10 we consider M3 as the monetary aggregate. Across Tables 6.C8 and 6.C9 (related with white noise U_t), we observe only five cases where the null is not rejected: when $d_1 = d_2 = 1$, 1.1 and 1.2 with no regressors, with the lowest statistics at $d_1 = d_2 = 1.1$ in the time domain (Table 6.C8) and at $d_1 = d_2 = 1$ in the frequency domain (Table 6.C9), and when $d_1 = 1.3$ and 1.4 and $d_2 = 1.4$ with an intercept and a time trend. Thus, results here are once more in complete analogy with those obtained in Tables 6.C1 and 6.C2 for the univariate tests, failing to reject the unit root null when we do not include regressors, but rejecting this hypothesis in favour of more nonstationarities if we include an intercept and/or a time trend in the model. If U_t is VAR(1), in Table 6.C10, the two unit roots null is the only non-rejection case if we do not include regressors, with a test statistic $\tilde{S}^2 = 0.53$, and including an intercept and an intercept and a time trend, there are much more non-rejections with the lowest statistics corresponding to $d_1 = 1$ and $d_2 = 0.8$ with an intercept, and to $d_1 = 1.1$ and $d_2 = 0.8$ with an intercept and a time trend. In these cases we also observe non-rejections for values of d_1 and d_2 smaller than one. As we explained in previous sections, this may be due to the fact that the VAR parameters have been obtained throughout a quasi-Newton algorithm which can give us results arbitrarily close to stationarity and thus, it is the competition between the VAR parameters and the differencing orders in describing the nonstationary, which causes this smaller degree of integration observed in some cases. Though we do not report results, we also computed the test statistic when U_t was VMA(1): if there were no regressors, the only two non-rejection cases occurred when $d_1 = 1.4$ and $d_2 = 1.2$, and when $d_1 = 1.3$ and $d_2 = 1.4$; however, including an intercept and an intercept and a time trend, results were similar in both cases, with all non-rejections occurring when d_1 and d_2 were equal to or greater than 1. This was not surprising given that the VMA representation is always stationary, and the nonstationary component of the series must be mainly described throughout the differencing parameters.

Finally in Tables 6.C11-6.C13 we use the total liquid assets L as the measure of money, and the non-rejections occur practically at the same values as in previous tables. Thus, if U_t is white noise (in Tables 6.C11 and 6.C12), $d_1 = d_2 = 1$ and 1.1 are the only two non-rejection cases when we do not include regressors, but these hypotheses are decisively rejected in favour of more nonstationary alternatives when including an intercept and an intercept and a time trend. In fact, the lowest statistics are obtained in these cases when $d_1 = 1.1$ and $d_2 = 1.4$ with an intercept, and when $d_1 = 1.2$ and $d_2 = 1.4$ with an intercept and a time trend. If U_t is VAR (1), (in Table 6.C13), the two unit roots null is the only non-rejection case with no regressors (as was in Table 6.C10 when using M3), but including an intercept or an intercept and a time trend, there are more non-rejection values, with the lowest statistics in both cases achieved at $d_1 = 0.9$ and $d_2 = 0.8$. Here we also observe that the null is not rejected when $d_1 = d_2 = 0.6$, but as we explained above, this smaller degree of integration might be due to competition between the VAR parameters and the differencing orders in describing the nonstationary. We do not report the results for VMA(1) U_t, but the main conclusions here were that all non-rejections occurred when d_1 and d_2 were greater than 1, with the lowest statistics obtained at $d_1 = 1.1$ and $d_2 = 1.2$ when including an intercept, and at $d_1 = 1.2$ and $d_2 = 1.3$ with an intercept and a time trend.

As a conclusion of the multivariate tests presented above we see that results here are in line with those obtained previously with the univariate tests. Thus, the two unit roots null hypothesis is not rejected when we do not include regressors in the model, but this hypothesis is decisively rejected in favour of more nonstationary alternatives, with integration orders greater than one in both variables, when including an intercept and/or a time trend.

In the final part of this section we look at the possibility of fractional cointegration between both variables. Across Tables 6.C14-6.C17 we present results of Robinson's (1994c) univariate tests on the estimated residuals, after regressing log of G.N.P. on log of each of the monetary aggregates, and their reverses, performing $\hat{r}$ as given in (2.9), for different specifications in (1.9) and different types of disturbances. First in Table 6.C14 we take M1 as the measure of money. The most noticeable thing observed here is that the unit root null is rejected in most cases when we do not include regressors, and this hypothesis is always rejected in favour of less nonstationary alternatives. In fact, apart from the case of AR(2) disturbances, all non-rejections take place when d ranges between 0.6 and 0.9, with the lowest statistics occurring at d = 0.7 with AR(1) disturbances, and at d = 0.8 for the remaining cases; including an intercept or an intercept and a time trend, the non-rejections appear in most of cases when d ranges between 0.6 and 1.1 and the lowest statistics are obtained when d takes values 0.8 or 0.9. Therefore we could infer from this table that a certain degree of fractional cointegration might exist between G.N.P. and M1, with the estimated residuals from the cointegrating regressions being nonstationary but with a small component of mean reversion.

In Table 6.C15 we perform the same statistics as above but using M2 as the measure of money. In this table we observe that the monotonic decrease in $\hat{r}$ with respect to d is only achieved for white noise and seasonal AR disturbances. In these cases, the non-rejection d's always range between 1 and 1.3 with the lowest statistics obtained in all cases when d = 1.1. This is observed in both series of residuals and for the three cases of no regressors, an intercept, and an intercept and a time trend. Therefore, these results show that the estimated residuals from the cointegrating regressions, when using M2 as the monetary aggregate, are nonstationary and non-mean reverting, suggesting that no reliable long-run relationship exists between these two variables.

Similar results are obtained in Table 6.C16, when using M3 as the measure of money. We again observe a lack of monotonic decrease in the value of the test statistic with respect to d when the disturbances are non-seasonal AR. For the remaining specifications, the non-rejection values of d always range between 1 and 1.3, with the lowest statistics occurring in all cases when d = 1.1. Thus, given that the null hypothesis is practically always rejected when d is smaller than one, these results suggest that M3 is non-cointegrated with G.N.P.

Finally, in Table 6.C17, we take L as the measure of money. Results in this table are very similar to those given in Table 6.C14 when using M1, with monotonicity achieved in all cases, and the non-rejection values of d ranging between 0.7 and 1.1 for white noise and seasonal AR disturbances, and between 0.6 and 1.2 for non-seasonal AR. In this table we observe that the unit root null is almost never rejected, however, the lowest statistics appear in all cases when d is smaller than 1. Thus, results in this table suggest that L might be fractionally cointegrated with G.N.P., with the estimated residuals from the cointegrating regressions showing a certain component of mean reversion.

Engle and Granger (1987) used the ADF tests to check if the estimated residuals were stationary and in particular, if they followed an I(0) process. Their results rejected the hypothesis of stationarity in all cases except when regressing log of M2 on log of G.N.P., in which case the test statistic was significant at the 5% level. Our results show that all the estimated residuals appear as nonstationary, and regressing log of M2 on log of G.N.P., the estimated regression was

$$\log M2 = \underset{(168.9)}{6.48} + \underset{(180.4)}{0.99} \log GNP, \qquad \text{(t-values)}$$

and we see in the lower part of Table 6.C15 that testing here the null hypothesis for different values for d, as we approximate to the stationary case, the values of the test statistic increase strongly, implying the rejection of the null in favour of more nonstationary alternatives. Only if the disturbances follow a non-seasonal AR process, we observe some non-rejections even for d = 0.6, but in these cases we observe a lack of monotonic decrease in $\hat{r}$ with respect to d which might indicate that the model is misspecified.

We can summarize now the main results obtained in this section by saying that nominal G.N.P. and the different measures of money are all individually integrated of order 1 when we do not include regressors in the model; however, including an intercept or an intercept and a time trend, greater integration orders must be required. Similar results were obtained when using the multivariate tests, finding two unit roots in all cases when modelling with no regressors, but rejecting this hypothesis in favour of more nonstationarities when including an intercept and/or a time trend. Testing the possibility of a cointegrating relationship between G.N.P. and money, only when using M1 or L as the monetary aggregates, we found a certain degree of fractional cointegration, with the estimated residuals from the cointegrating regressions being nonstationary but mean reverting. Using M2 or M3 as the measures of money, the equilibrium errors were nonstationary and not mean reverting, with the integration order of the estimated residuals equal to or greater than 1 in practically all cases.

FIGURE 6.C1

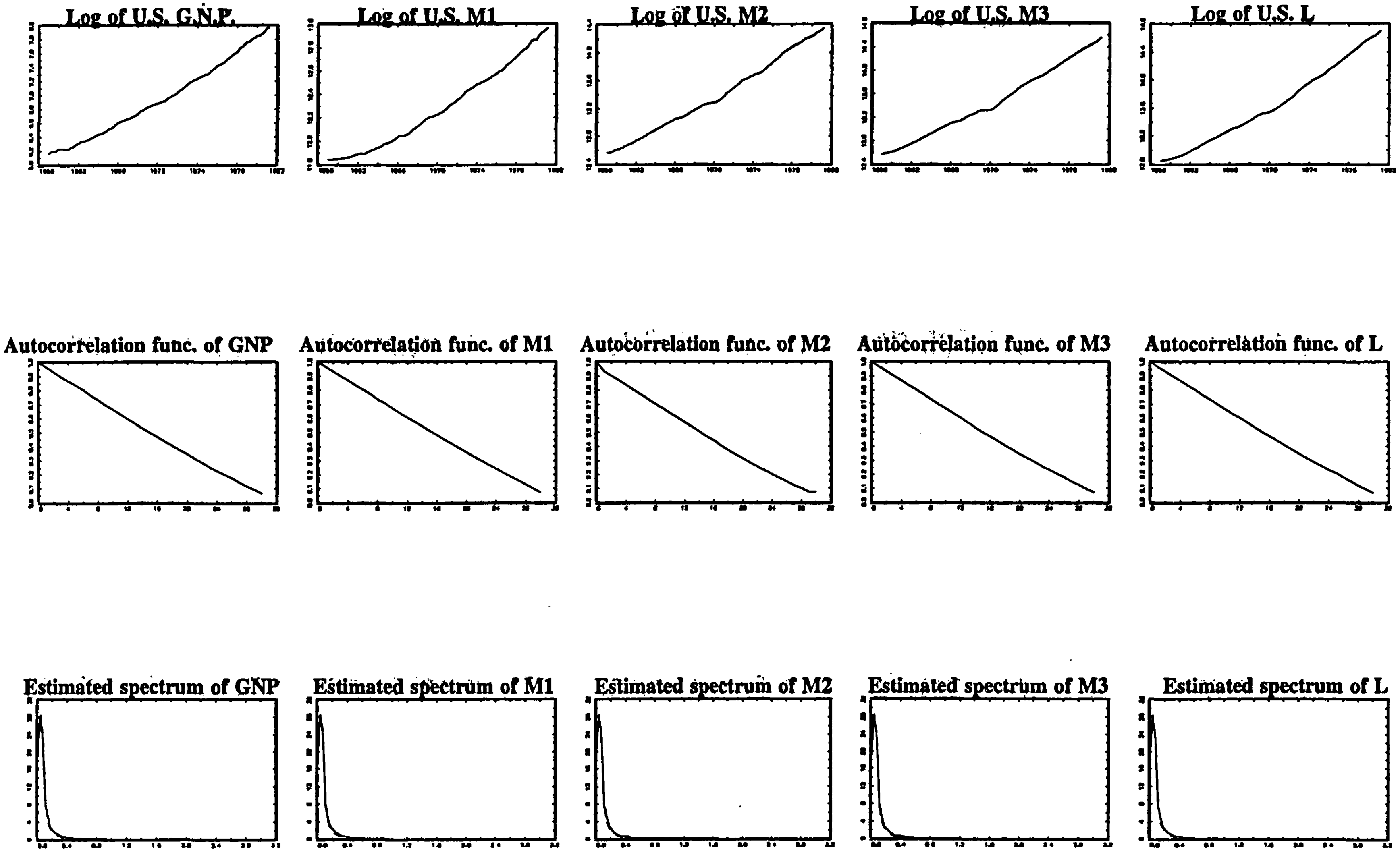

Log of U.S. G.N.P.
Log of U.S. M1
Log of U.S. M2
Log of U.S. M3
Log of U.S. L
Autocorrelation func. of GNP
Autocorrelation func. of M1
Autocorrelation func. of M2
Autocorrelation func. of M3
Autocorrelation func. of L
Estimated spectrum of GNP
Estimated spectrum of M1
Estimated spectrum of M2
Estimated spectrum of M3
Estimated spectrum of L

TABLE 6.C1

$\hat{r}$ in (2.9) for log of G.N.P.

Log of nominal G.N.P.

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	5.48	3.72	2.10	0.70'	-0.44'	-1.35'	-2.05	-2.60	-3.03	-3.38
AR(1)	-0.88	-0.26	-0.19	-0.44	-0.82	-1.26	-1.67	-2.05	-2.39	-2.67
AR(2)	-2.84	-1.73	-1.20	-1.05	-1.13	-1.34	-1.59	-1.85	-2.11	-2.35
SAR(1)	4.18	3.09	1.87'	0.65'	-0.44'	-1.35'	-2.07	-2.63	-3.07	-3.41
SAR(2)	4.12	3.04	1.84'	0.64'	-0.44'	-1.35'	-2.07	-2.63	-3.07	-3.42
b) Intercept.										
W.N.	13.01	12.58	11.53	8.26	3.49	0.09'	-1.48'	-2.18	-2.60	-2.94
AR(1)	-4.94	-5.14	-4.32	0.93	1.38	-0.64	-1.92	-2.45	-2.69	-2.88
AR(2)	-2.29	-2.11	-2.14	-2.77	0.04	-0.69	-1.80	-2.28	-2.46	-2.58
SAR(1)	4.46	4.08	3.84	3.78	2.60	0.11'	-1.51'	-2.24	-2.65	-2.98
SAR(2)	6.05	5.61	4.85	3.66	2.41	0.24'	-1.57'	-2.36	-2.75	-3.05
c) Intercept and a time trend.										
W.N.	14.28	11.96	9.18	6.28	3.64	1.50'	-0.11'	-1.26'	-2.05	-2.60
AR(1)	-0.79	-0.76	0.64	1.36	0.97	0.22	-0.61	-1.34	-1.91	-2.31
AR(2)	-1.42	-1.81	-0.77	0.34	0.49	0.13	-0.43	-1.02	-1.52	-1.87
SAR(1)	6.90	6.03	5.19	4.15	2.77	1.24'	-0.16'	-1.27'	-2.07	-2.62
SAR(2)	7.06	5.96	4.93	3.93	2.77	1.39'	-0.02'	-1.22'	-2.10	-2.68

': Non-rejection values of the null hypothesis (1.12) with $\rho(L;\theta) = (1-L)^{d+\theta}$ at 95% significance level when monotonicity in the value of the tests with respect to d is observed.

TABLE 6.C2

$\hat{r}$ in (2.9) for the log of money.

log of M1

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	5.65	3.80	2.13	0.70'	-0.46'	-1.37'	-2.07	-2.62	-3.05	-3.39
AR(1)	-1.17	-0.33	-0.20	-0.43	-0.82	-1.25	-1.67	-2.05	-2.38	-2.67
AR(2)	-3.44	-1.99	-1.30	-1.10	-1.16	-1.36	-1.61	-1.87	-2.12	-2.35
SAR(1)	4.19	3.11	1.87'	0.64'	-0.46'	-1.37'	-2.09	-2.65	-3.08	-3.42
SAR(2)	4.12	3.05	1.85'	0.64'	-0.46'	-1.37'	-2.09	-2.65	-3.08	-3.43
b) Intercept.										
W.N.	14.56	14.58	13.86	11.00	6.30	2.05	-0.47	-1.70	-2.32	-2.70
AR(1)	-3.44	-2.95	-2.20	-0.23	2.33	0.73	-1.06	-2.17	-2.74	-3.06
AR(2)	-1.91	-1.82	-2.19	-2.43	2.19	2.24	0.96	-0.22	-0.92	-1.32
SAR(1)	5.18	5.22	5.15	4.64	3.61	1.58	-0.43	-1.65	-2.30	-2.69
SAR(2)	6.83	6.75	6.27	5.00	3.58	1.83'	-0.36'	-1.87'	-2.62	-3.02
c) Intercept and a time trend.										
W.N.	15.64	13.99	11.72	8.94	6.02	3.35	1.17'	-0.45'	-1.59'	-2.35
AR(1)	-1.82	-2.06	-1.20	1.15	1.92	1.41	0.42	-0.68	-1.69	-2.48
AR(2)	-1.68	-2.77	-3.46	-0.45	1.77	2.36	2.19	1.53	0.62	-0.32
SAR(1)	6.99	6.00	5.18	4.44	3.54	2.33	0.92'	-0.42'	-1.53'	-2.31
SAR(2)	7.77	6.59	5.44	4.45	3.55	2.51	1.20'	-0.25'	-1.55'	-2.50

log of M2

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	5.62	3.79	2.12	0.70'	-0.45'	-1.36'	-2.06	-2.61	-3.04	-3.38
AR(1)	-1.12	-0.31	-0.19	-0.42	-0.80	-1.24	-1.66	-2.04	-2.37	-2.66
AR(2)	-3.36	-1.96	-1.29	-1.09	-1.15	-1.34	-1.59	-1.85	-2.10	-2.34
SAR(1)	4.18	3.10	1.87'	0.65'	-0.45'	-1.36'	-2.08	-2.64	-3.07	-3.42
SAR(2)	4.11	3.04	1.85'	0.64'	-0.45'	-1.36'	-2.08	-2.64	-3.08	-3.42
b) Intercept.										
W.N.	12.71	12.38	12.06	10.49	7.50	5.09	3.87	3.13	2.40	1.61'
AR(1)	-5.02	-5.30	-4.91	-4.07	-3.89	-3.97	-3.13	-2.29	-1.66	-1.29'
AR(2)	-3.63	-4.14	-5.25	-9.77	-7.21	-4.14	-3.33	-2.78	-2.34	-2.00
SAR(1)	4.91	4.86	5.31	6.27	6.41	5.19	4.14	3.42	2.71	1.93'
SAR(2)	5.94	5.68	5.72	6.22	6.37	4.68	3.19	2.58	2.05	1.37'
c) Intercept and a time trend.										
W.N.	14.18	12.73	11.11	9.39	7.67	6.03	4.52	3.15	1.96	0.96'
AR(1)	1.69	1.15	0.41	-0.56	-1.84	-2.95	-2.96	-2.52	-2.17	-1.94'
AR(2)	5.31	4.11	2.02	-0.80	-2.74	-3.31	-3.32	-3.18	-3.00	-2.79
SAR(1)	9.39	8.97	8.44	7.75	6.86	5.81	4.63	3.40	2.21	1.16'
SAR(2)	9.49	9.11	8.62	7.94	6.99	5.76	4.29	2.75	1.32'	0.18'

cont...

log of M3

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	5.61	3.79	2.13	0.71'	-0.44'	-1.35'	-2.06	-2.61	-3.03	-3.38
AR(1)	-1.09	-0.30	-0.19	-0.42	-0.80	-1.24	-1.66	-2.04	-2.37	-2.66
AR(2)	-3.28	-1.91	-1.27	-1.07	-1.14	-1.33	-1.58	-1.84	-2.10	-2.33
SAR(1)	4.19	3.11	1.88'	0.65'	-0.45'	-1.36'	-2.08	-2.64	-3.07	-3.42
SAR(2)	4.11	3.05	1.85'	0.65'	-0.45'	-1.36'	-2.08	-2.64	-3.07	-3.42
b) Intercept.										
W.N.	12.67	12.36	12.18	11.03	8.46	6.07	4.75	3.97	3.25	2.47
AR(1)	-5.22	-5.57	-5.18	-3.92	-2.49	-2.31	-2.71	-2.62	-2.29	-2.00
AR(2)	-3.21	-3.27	-3.07	-3.50	-6.21	-3.49	-2.87	-2.61	-2.42	-2.31
SAR(1)	4.86	4.79	5.24	6.28	6.90	6.18	5.21	4.48	3.77	2.95
SAR(2)	5.92	5.68	5.81	6.43	6.92	6.30	5.28	4.58	3.90	3.09
c) Intercept and a time trend.										
W.N.	15.35	14.02	12.47	10.78	9.06	7.39	5.85	4.45	3.22	2.17
AR(1)	3.68	3.14	2.40	1.60	0.75	-0.33	-1.71	-2.57	-2.63	-2.45
AR(2)	15.73	16.47	2.85	-1.10	-2.07	-2.33	-2.47	-2.59	-2.70	-2.77
SAR(1)	9.39	9.09	8.79	8.40	7.84	7.07	6.06	4.89	3.65	2.47
SAR(2)	9.47	9.13	8.79	8.41	7.92	7.22	6.27	5.08	3.74	2.45

log of L

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	5.61	3.78	2.12	0.70'	-0.45'	-1.36'	-2.06	-2.61	-3.04	-3.38
AR(1)	-1.09	-0.30	-0.19	-0.42	-0.81	-1.24	-1.66	-2.04	-2.37	-2.66
AR(2)	-3.29	-1.92	-1.27	-1.08	-1.15	-1.34	-1.59	-1.85	-2.11	-2.34
SAR(1)	4.18	3.10	1.87'	0.64'	-0.46'	-1.37'	-2.08	-2.64	-3.08	-3.42
SAR(2)	4.11	3.04	1.85'	0.64'	-0.46'	-1.37'	-2.09	-2.64	-3.08	-3.42
b) Intercept.										
W.N.	13.47	13.61	14.08	13.83	11.71	8.03	4.94	3.25	2.28	1.46
AR(1)	-4.63	-4.63	-3.90	-2.56	-1.27	-0.96	-1.75	-1.93	-1.75	-1.56
AR(2)	-2.78	-2.67	-2.26	-1.91	-1.69	-1.38	-1.78	-2.09	-2.03	-1.86
SAR(1)	5.03	5.06	5.51	6.26	6.88	6.65	5.11	3.62	2.62	1.77
SAR(2)	6.40	6.39	6.63	6.92	7.03	6.66	5.17	3.52	2.51	1.69
c) Intercept and a time trend.										
W.N.	17.83	16.95	15.74	14.14	12.19	9.99	7.70	5.47	3.46	1.82'
AR(1)	1.12'	1.05'	0.84'	0.44'	-0.03'	-0.49'	-0.96'	-1.15'	-1.22'	-1.38'
AR(2)	4.27	3.98	3.07	1.79'	0.53'	-0.60'	-1.08'	-1.23'	-1.44'	-1.66'
SAR(1)	8.55	8.21	7.93	7.68	7.44	7.07	6.40	5.28	3.75	2.12
SAR(2)	9.29	8.79	8.30	7.88	7.50	7.07	6.44	5.41	3.88	2.09

': Non-rejection values of the null hypothesis (1.12) with $\rho(L;\theta) = (1-L)^{d+\theta}$ at 95% significance level when monotonicity in the test statistic with respect to d is observed.

TABLE 6.C3 (log GNP - log M1)

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with no regressors and white noise U_t. d_1 and d_2 are the differencing orders for log of GNP and M1 respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	327.75	220.70	222.86	207.30	189.24	171.62	155.42	140.99	128.39
0.7	286.78	211.35	189.96	182.08	165.21	148.16	132.52	118.69	106.70
0.8	251.21	229.92	89.34	153.30	142.01	126.55	112.09	99.36	88.42
0.9	224.75	199.35	176.88	16.61	118.30	107.29	94.68	83.46	73.87
1.0	202.01	175.82	152.11	131.47	0.09'	89.11	80.17	70.80	62.73
1.1	181.94	155.83	132.63	112.83	95.94	3.12'	67.23	60.82	54.37
1.2	164.30	138.60	116.36	97.91	82.97	70.44	7.79	52.23	48.07
1.3	148.93	123.79	102.64	85.63	72.36	62.10	53.50	11.46	42.74
1.4	135.67	111.14	91.07	75.46	63.72	55.04	48.55	42.95	14.43

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with an intercept and white noise U_t. d_1 and d_2 are the differencing orders for log of GNP and M1 respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	181.44	205.91	228.20	239.65	256.30	276.14	287.17	289.73	289.52
0.7	187.89	159.76	170.06	192.79	222.47	250.75	265.55	269.02	268.60
0.8	200.38	155.82	125.67	124.10	148.48	177.84	194.11	198.34	198.05
0.9	215.12	178.36	123.82	74.02	60.97	74.32	86.36	90.54	90.95
1.0	244.82	218.60	159.30	77.90	23.06	12.09	16.21	19.30	20.43
1.1	274.02	254.75	196.81	103.79	27.04	1.43'	1.03'	3.56'	5.04'
1.2	288.81	272.85	216.61	120.61	35.74	3.95'	1.89'	4.42'	6.12
1.3	294.09	279.36	224.13	127.83	40.46	6.24	3.45'	6.04	7.87
1.4	296.13	281.79	227.05	130.96	42.93	7.74	4.59'	7.19	9.10

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with a time trend and white noise U_t. d_1 and d_2 are the differencing orders for log of GNP and M1 respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	139.79	152.58	162.65	165.11	164.35	163.18	162.03	160.94	160.32
0.7	130.34	115.68	112.51	114.18	118.35	123.21	126.83	128.52	128.85
0.8	141.42	109.48	85.03	72.38	70.75	75.16	80.47	84.01	85.53
0.9	157.49	121.45	81.75	51.46	37.33	35.66	39.20	43.01	45.35
1.0	173.28	139.85	93.21	49.74	23.06	13.47	13.46	16.37	18.97
1.1	186.48	157.53	109.22	58.20	22.33	5.94'	2.44'	4.15'	6.69
1.2	196.03	171.22	123.63	69.08	27.53	6.35	0.20'	0.92'	3.34'
1.3	202.12	180.40	134.23	78.45	33.78	9.56	1.60'	1.66'	4.00'
1.4	205.68	185.94	141.12	85.21	39.01	13.00	3.88'	3.51'	5.83'

': Non-rejection values of the null hypothesis (5.4) at 95% significance level.

TABLE 6.C4 (log GNP - log M1)

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with no regressors and white noise U_t. d_1 and d_2 are the differencing orders for log of GNP and M1 respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	247.09	179.02	178.03	164.74	149.81	135.32	121.99	110.11	99.73
0.7	220.00	153.52	154.94	147.19	133.51	119.82	107.21	95.98	86.19
0.8	193.73	177.77	62.09	127.12	117.49	105.20	93.66	83.41	74.49
0.9	173.22	156.10	139.79	10.43	100.75	91.75	81.71	72.67	64.82
1.0	155.21	138.39	122.47	107.62	0.07'	78.57	71.33	63.74	57.08
1.1	139.16	122.95	107.92	94.46	82.16	3.58'	61.60	56.35	50.97
1.2	124.97	109.44	95.39	83.16	72.75	63.36	8.07	49.60	46.11
1.3	112.57	97.69	84.60	73.55	64.47	57.04	50.31	11.69	41.72
1.4	101.87	87.57	75.37	65.40	57.49	51.29	46.36	41.71	14.72

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with an intercept and white noise U_t. d_1 and d_2 are the differencing orders for log of GNP and M1 respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	146.23	183.61	217.13	227.01	230.20	232.49	231.93	230.01	228.86
0.7	155.66	118.40	137.15	163.35	183.40	196.02	199.86	199.00	197.66
0.8	176.01	115.24	86.67	92.22	113.25	131.29	138.76	139.43	138.41
0.9	187.88	136.17	83.46	47.99	42.82	52.84	59.99	61.98	61.97
1.0	203.57	163.85	108.73	49.08	13.90	8.09	11.23	13.45	14.42
1.1	218.77	186.51	134.21	66.99	16.51	1.64'	0.95'	3.11'	4.55'
1.2	227.12	198.15	147.92	79.13	23.10	2.87'	1.99'	4.19'	5.83'
1.3	230.83	203.01	153.73	84.83	27.05	5.07'	3.64'	5.87'	7.62
1.4	232.86	205.40	156.49	87.71	29.41	6.73	5.01'	7.24	9.04

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with a time trend and white noise U_t. d_1 and d_2 are the differencing orders for log of GNP and M1 respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	126.82	141.94	160.70	168.86	168.02	162.43	155.57	149.89	146.75
0.7	116.53	92.28	93.59	102.03	108.46	110.81	109.90	107.52	105.43
0.8	133.33	85.16	60.71	54.76	57.52	61.75	64.25	64.77	64.36
0.9	152.25	98.03	56.52	33.61	26.18	26.74	29.30	31.23	32.20
1.0	166.48	114.76	66.25	31.49	13.90	8.76	9.35	11.31	13.01
1.1	175.58	128.67	79.01	38.06	13.28	3.13'	1.37'	2.76'	4.69'
1.2	180.67	138.20	89.79	46.40	17.42	3.75'	0.09'	0.85'	2.71'
1.3	183.19	143.99	97.38	53.47	22.36	6.53	1.55'	1.82'	3.72'
1.4	184.41	147.27	102.23	58.59	26.57	9.51	3.68'	3.61'	5.49'

': Non-rejection values of the null hypothesis (5.4) at 95% significance level.

TABLE 6.C5 (log GNP - log M2)

Multivariate score test in the time domain ($\hat{S}^{t2}$ in (5.32)) with no regressors and white noise U_t. d_1 and d_2 are the differencing orders of log of GNP and M2.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	315.95	219.22	221.80	206.63	188.85	171.41	155.32	140.96	128.40
0.7	286.77	208.60	188.07	181.14	164.70	147.88	132.38	118.64	106.69
0.8	250.94	230.99	100.05	151.62	141.28	126.19	111.91	99.29	88.40
0.9	224.40	199.77	178.47	27.51	117.03	106.79	94.46	83.36	73.84
1.0	201.62	176.01	152.83	133.10	2.08'	88.29	79.88	70.68	62.69
1.1	181.54	155.91	133.06	113.55	97.27	0.89'	66.79	60.68	54.33
1.2	163.88	138.62	116.65	98.34	83.53	71.33	5.32'	52.08	48.04
1.3	148.51	123.78	102.84	85.92	72.68	62.45	54.00	9.62	42.75
1.4	135.26	111.11	91.22	75.67	63.93	55.22	48.72	43.17	13.08

Multivariate score test in the time domain ($\hat{S}^{t2}$ in (5.32)) with an intercept and white noise U_t. d_1 and d_2 are the differencing orders of log of GNP and M2.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	254.48	304.75	323.89	323.63	317.35	306.46	294.69	286.94	283.47
0.7	197.52	229.61	269.65	289.66	294.47	286.47	273.32	263.52	258.76
0.8	175.66	165.22	182.72	207.06	220.30	216.35	203.74	192.80	186.47
0.9	184.74	155.61	125.41	109.32	108.91	106.52	98.37	89.85	83.53
1.0	220.01	192.02	137.87	75.35	41.04	31.13	26.80	22.74	18.73
1.1	256.16	233.76	174.89	90.88	32.70	13.48	8.81	6.60	4.28'
1.2	275.74	257.40	199.32	108.61	39.64	14.78	8.65	6.38	4.44'
1.3	283.51	267.23	210.58	118.51	45.32	17.55	10.32	7.65	5.62'
1.4	286.67	271.34	215.71	123.77	49.09	19.86	11.94	8.92	6.71

Multivariate score test in the time domain ($\hat{S}^{t2}$ in (5.32)) with a time trend and white noise U_t. d_1 and d_2 are the differencing orders of log of GNP and M2.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	204.58	212.63	216.03	211.95	202.21	189.57	176.59	165.34	157.29
0.7	164.61	165.19	168.82	168.52	163.28	154.50	144.19	134.23	126.18
0.8	132.77	121.07	117.86	116.17	112.63	106.69	99.13	91.20	84.17
0.9	118.45	95.82	81.93	73.68	67.80	62.28	56.46	50.55	45.15
1.0	120.42	91.76	68.53	52.02	41.04	33.48	27.72	22.95	19.01
1.1	130.75	100.39	71.50	48.23	31.83	21.06	14.05	9.37	6.21
1.2	142.39	112.72	81.34	53.79	33.13	19.24	10.50	5.22'	2.19'
1.3	151.85	123.73	91.78	62.01	38.55	22.22	11.84	5.71'	2.42'
1.4	158.33	131.80	100.20	69.52	44.45	26.46	14.77	7.82	4.15'

': Non-rejection values of the null hypothesis (5.4) at 95% significance level.

TABLE 6.C6 (log GNP - log M2)

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with no regressors and white noise U_t. d_1 and d_2 are the differencing orders for log of GNP and M2 respectively.

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	237.57	178.16	177.36	164.31	149.56	135.19	121.94	110.11	99.76
0.7	219.58	150.83	153.81	146.61	133.18	119.64	107.12	95.96	86.20
0.8	193.32	178.06	69.27	126.09	117.03	104.96	93.55	83.36	74.49
0.9	172.82	156.13	140.49	17.67	99.93	91.42	81.56	72.61	64.81
1.0	154.83	138.33	122.75	108.47	0.88'	78.01	71.12	63.66	57.06
1.1	138.80	122.86	108.06	94.82	82.98	1.37'	61.28	56.25	50.96
1.2	124.62	109.33	95.46	83.37	73.09	64.01	5.79'	49.50	46.11
1.3	112.23	97.58	84.64	73.69	64.67	57.30	50.74	10.03	41.80
1.4	101.54	87.46	75.39	65.49	57.62	51.44	46.52	41.94	13.57

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with an intercept and white noise U_t. d_1 and d_2 are the differencing orders for log of GNP and M2 respectively.

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	217.20	279.92	298.85	287.65	266.59	247.05	234.59	228.84	226.92
0.7	155.42	184.23	226.16	239.74	232.17	216.46	202.94	195.19	192.00
0.8	147.19	124.00	139.68	161.45	166.43	156.88	144.40	135.43	130.57
0.9	159.26	119.25	91.15	81.96	81.70	77.10	69.15	61.90	56.68
1.0	183.06	146.35	99.72	55.60	32.47	24.65	20.52	16.77	13.20
1.1	205.72	175.29	126.50	67.18	26.93	12.56	8.39	6.14	3.91'
1.2	218.51	192.07	144.71	80.98	32.87	14.04	8.60	6.26	4.34'
1.3	224.33	199.76	153.75	89.19	37.98	16.78	10.32	7.58	5.57'
1.4	227.30	203.55	158.38	93.97	41.64	19.25	12.13	9.03	6.83

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with a time trend and white noise U_t. d_1 and d_2 are the differencing orders for log of GNP and M2 respectively.

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	194.16	205.53	209.88	204.31	191.93	176.67	161.74	149.48	141.40
0.7	144.82	144.86	149.17	148.61	142.12	131.79	120.17	109.47	101.31
0.8	118.12	101.71	97.54	95.79	91.94	85.42	77.33	69.16	62.24
0.9	112.07	82.83	67.06	58.95	53.47	48.14	42.41	36.66	31.58
1.0	118.40	82.92	58.00	42.25	32.47	25.81	20.63	16.31	12.78
1.1	128.84	92.06	62.08	40.40	26.02	16.79	10.79	6.76	4.07'
1.2	138.48	102.84	70.98	45.75	27.79	15.98	8.58	4.12'	1.60'
1.3	145.59	111.84	79.87	52.95	32.71	18.85	10.06	4.89'	2.17'
1.4	150.24	118.28	86.94	59.46	37.97	22.73	12.82	6.93	3.86'

': Non-rejection values of the null hypothesis (5.4) at 95% significance level.

TABLE 6.C7 (log GNP - log M2)

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with no regressors and a VMA(1) structure on U_t. d_1 and d_2 are the differencing orders of log of GNP and M2 respectively.

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	87.10	147.19	18107	70959	444225	-------	-------	-------	-------
0.7	93.96	841.89	434.86	17950	96958	-------	-------	-------	-------
0.8	5067.4	137.81	417.98	322.76	13765	92644	-------	-------	-------
0.9	339893	14503	132.27	20.74	183.44	5979.4	29475	88172	380063
1.0	164959	57884	11215	194.69	11.53	125.84	1860.4	2541.0	7930.7
1.1	391644	191334	41179	418.07	244.56	6451.4	22.68	278.86	216.24
1.2	-------	858214	119252	18131	63.21	85.52	582.00	4.84’	16.03
1.3	195901	-------	403564	33732	3706.1	385.40	23.55	442.25	0.95’
1.4	1392.2	201563	-------	420984	3719.4	211.68	15.06	6.63	135.84

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with an intercept and a VMA(1) structure on U_t. d_1 and d_2 are the differencing orders of log of GNP and M2 respectively.

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	-----	113.11	138.36	127.58	109.29	99.23	95.54	95.17	96.03
0.7	-----	31.19	97.68	103.67	97.06	90.32	86.17	84.68	84.87
0.8	-----	6.87	52.38	64.58	61.35	56.55	52.36	49.98	49.25
0.9	-----	103.97	31.44	33.55	26.41	21.58	18.07	15.82	14.62
1.0	-----	454.53	34.08	22.04	9.66	5.00’	2.95’	1.88’	1.22’
1.1	-----	1114.7	46.27	23.06	5.28’	1.07’	0.31’	0.29’	0.40’
1.2	-----	-------	57.58	29.97	5.79’	0.61’	0.19’	0.46’	0.87’
1.3	-----	10.03	63.96	36.59	8.08	0.74’	0.05’	0.15’	0.40’
1.4	-----	18.39	66.34	40.71	10.48	6.27	18.50	0.36’	0.11’

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with a time trend and a VMA(1) structure on U_t. d_1 and d_2 are the differencing orders of log of GNP and M2 respectively.

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	65.93	82.51	83.36	77.50	69.78	62.44	56.51	52.40	50.31
0.7	37.32	52.84	55.43	52.07	46.76	41.30	36.51	32.83	30.48
0.8	22.22	34.20	35.40	32.31	28.14	23.96	20.27	17.30	15.17
0.9	17.65	26.42	24.53	20.17	16.11	12.69	9.92	7.71	6.11
1.0	19.44	26.25	20.67	14.18	9.66	6.62	4.52’	3.02’	2.02’
1.1	24.22	30.35	21.39	12.13	6.65	3.75’	2.15’	1.20’	0.65’
1.2	29.49	36.12	25.18	12.89	5.66’	2.47’	1.20’	0.67’	0.45’
1.3	33.66	41.64	30.66	15.96	6.30	2.08’	0.78’	0.54’	0.58’
1.4	35.88	45.44	36.04	20.47	8.43	2.62’	0.81’	0.60’	0.77’

’: Non-rejection values of the null hypothesis (5.4) at 95% significance level and "---" means that the value of the test statistics exceeds 999999.

TABLE 6.C8 (log GNP - log M3)

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with no regressors and white noise U_t. d_1 and d_2 are the differencing orders of log of GNP and M3 respectively.

$d_1\backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	306.41	223.07	223.01	207.20	189.17	171.62	155.47	141.08	128.49
0.7	283.38	195.82	190.48	181.96	165.10	148.13	132.55	118.76	106.79
0.8	248.93	228.46	89.17	153.18	141.85	126.50	112.10	99.42	88.50
0.9	222.97	198.37	176.66	22.95	118.05	107.20	94.68	83.50	73.94
1.0	200.49	175.05	151.90	131.89	1.54'	88.95	80.15	70.84	62.79
1.1	180.60	155.18	132.45	112.97	96.52	1.05'	67.19	60.85	54.42
1.2	163.06	138.04	116.21	97.99	83.21	70.93	5.18'	52.29	48.13
1.3	147.78	123.29	102.51	85.69	72.50	62.29	53.83	9.21	42.83
1.4	134.58	110.68	90.96	75.50	63.82	55.15	48.67	43.16	12.54

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with an intercept and white noise U_t. d_1 and d_2 are the differencing orders of log of GNP and M3 respectively.

$d_1\backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	257.81	307.59	327.51	326.04	319.91	312.77	303.72	295.79	290.48
0.7	207.15	229.99	268.39	288.44	295.16	292.85	283.78	274.41	267.91
0.8	190.59	175.16	184.23	204.14	219.34	222.27	214.78	205.09	197.61
0.9	200.34	175.22	143.09	117.06	110.52	111.10	107.06	100.26	93.77
1.0	233.73	213.68	166.90	100.27	52.38	36.37	31.88	28.45	24.55
1.1	267.34	253.52	205.94	124.28	52.04	21.24	12.84	9.72	7.01
1.2	284.98	274.93	229.19	144.15	62.67	24.51	13.18	9.20	6.43
1.3	291.62	283.27	239.07	154.11	69.84	28.49	15.54	10.83	7.75
1.4	294.19	286.51	243.21	158.96	74.18	31.54	17.74	12.56	9.17

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with a time trend and white noise U_t. d_1 and d_2 are the differencing orders of log of GNP and M3 respectively.

$d_1\backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	207.44	213.75	219.14	218.21	211.58	201.31	189.46	177.83	167.98
0.7	174.79	168.29	169.71	170.62	168.16	162.37	154.28	145.27	136.74
0.8	154.24	134.00	123.95	119.19	115.76	111.62	106.17	99.75	93.16
0.9	148.84	120.66	98.51	83.27	73.51	66.90	61.55	56.54	51.69
1.0	154.84	124.89	95.16	70.30	52.38	40.57	32.86	27.48	23.40
1.1	165.40	137.11	104.48	73.62	49.08	31.90	20.79	13.83	9.47
1.2	175.38	149.82	117.07	83.46	54.81	33.58	19.36	10.49	5.27'
1.3	182.66	159.65	127.96	93.61	62.89	39.11	22.60	12.07	5.87'
1.4	187.16	166.10	135.69	101.57	70.08	44.94	26.95	15.19	8.15

': Non-rejection values of the null hypothesis (5.4) at 95% significance level.

TABLE 6.C9 (log GNP - log M3)

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with no regressors and white noise U_t. d_1 and d_2 are the differencing orders of log of GNP and M3 respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	230.53	180.78	178.17	164.69	149.78	135.34	122.05	110.19	99.83
0.7	217.86	141.97	155.34	147.13	133.46	119.82	107.25	96.05	86.28
0.8	192.46	176.95	62.09	127.04	117.39	105.17	93.69	83.46	74.56
0.9	172.32	155.61	139.79	14.89	100.55	91.68	81.71	72.71	64.89
1.0	154.52	138.05	122.44	108.04	0.69'	78.42	71.30	63.77	57.14
1.1	138.59	122.69	107.90	94.65	82.72	1.51'	61.54	56.37	51.03
1.2	124.48	109.23	95.38	83.30	73.00	63.87	5.61'	49.64	46.17
1.3	112.13	97.52	84.61	73.66	64.64	57.27	50.70	9.58	41.84
1.4	101.47	87.43	75.38	65.49	57.62	51.44	46.52	41.97	12.95

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with an intercept and white noise U_t. d_1 and d_2 are the differencing orders of log of GNP and M3 respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	217.96	276.66	298.35	289.63	271.40	254.84	243.35	236.54	232.52
0.7	169.76	182.07	221.08	237.73	234.71	223.85	212.65	204.55	199.49
0.8	166.56	134.24	138.59	157.63	166.87	163.32	154.22	145.72	139.62
0.9	176.32	137.33	104.37	86.63	83.54	82.01	76.91	70.62	64.98
1.0	196.26	164.31	121.10	73.07	41.27	29.78	25.51	22.00	18.25
1.1	215.65	190.41	148.23	90.23	41.44	19.54	12.61	9.46	6.66
1.2	226.43	204.82	164.84	105.02	49.91	22.57	13.26	9.36	6.51
1.3	231.24	211.17	172.57	113.01	56.00	26.29	15.60	11.01	7.84
1.4	233.73	214.25	176.39	117.38	60.04	29.37	17.94	12.88	9.39

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with a time trend and white noise U_t. d_1 and d_2 are the differencing orders of log of GNP and M3 respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	196.67	204.49	211.93	211.37	203.57	191.41	177.69	164.70	154.18
0.7	159.02	147.41	148.61	150.26	147.82	141.26	132.11	122.19	113.13
0.8	145.88	115.68	102.38	97.60	94.71	90.72	85.06	78.35	71.61
0.9	148.10	108.49	81.87	66.42	57.78	52.16	47.39	42.65	38.00
1.0	156.68	115.49	81.55	57.12	41.27	31.43	25.06	20.49	16.89
1.1	165.69	126.74	90.57	61.04	39.59	25.33	16.30	10.63	7.01
1.2	172.50	136.75	101.09	69.57	44.85	27.30	15.76	8.59	4.32'
1.3	176.72	143.84	109.65	77.92	51.73	32.20	18.81	10.26	5.19'
1.4	179.00	148.23	115.57	84.39	57.80	37.27	22.72	13.16	7.37

': Non-rejection values of the null hypothesis (5.4) at 95% significance level.

TABLE 6.C10 (log GNP - log M3)

Multivariate score tests ($\tilde{S}^2$ in (5.37)) <u>with no regressors</u> and a VAR(1) structure on U_t. d_1 and d_2 are the differencing orders for log of GNP and M3 respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	127.14	228.18	251.89	212.29	3614.9	------	------	------	------
0.7	158.17	122.35	226.26	215.35	2396.9	------	------	------	------
0.8	252.33	95.01	102.35	248.81	1757.1	522665	------	------	------
0.9	185.48	160.72	61.04	82.88	1357.4	29792	244167	------	------
1.0	270.18	185.29	135.21	76.38	0.53’	11670	11692	14106	23538
1.1	9766.0	15690	5545.6	3350.7	1271.6	8.95	7545.8	6391.3	21604
1.2	56356	297999	556356	6873.0	2206.4	5981.3	6.58	5244.4	16478
1.3	174476	566107	------	------	5002.6	4397.8	4525.3	7.50	4990.9
1.4	963196	-----	------	------	13636	27720	18486	4332.8	14.40

Multivariate score tests ($\tilde{S}^2$ in (5.37)) <u>with an intercept</u> and a VAR(1) structure on U_t. d_1 and d_2 are the differencing orders for log of GNP and M3 respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	12.64	68.23	95.67	9.54	0.96’	0.41’	1.87’	2.98’	3.42’
0.7	0.37’	13.47	24.66	10.48	2.03’	0.46’	1.17’	2.04’	2.55’
0.8	4.39’	1.34’	14.76	14.51	5.70’	1.42’	0.63’	0.91’	1.28’
0.9	5.54’	0.20’	4.28’	12.46	11.68	6.99	3.57’	1.99’	1.39’
1.0	4.65’	1.32’	0.16’	3.76’	7.62	7.54	5.28’	3.31’	2.12’
1.1	4.34’	1.95’	0.34’	0.34’	2.82’	4.28’	3.69’	2.65’	1.88’
1.2	4.70’	2.31’	1.27’	0.43’	2.05’	4.58’	5.24’	4.78’	4.16’
1.3	5.36’	2.75’	2.16’	1.30’	2.31’	5.22’	6.90	7.04	6.60
1.4	6.18	3.41’	3.11’	2.41’	2.96’	5.67’	7.80	8.42	8.24

Multivariate score tests ($\tilde{S}^2$ in (5.37)) <u>with a time trend</u> and a VAR(1) structure on U_t. d_1 and d_2 are the differencing orders for log of GNP and M3 respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	18.78	24.39	19.31	11.66	6.03	2.65’	0.84’	0.18’	0.38’
0.7	8.25	16.85	19.79	16.58	11.50	7.00	3.66’	1.48’	0.33’
0.8	3.55’	8.58	14.46	16.61	15.20	12.02	8.43	5.20’	2.75’
0.9	3.04’	3.01’	6.73	10.85	13.01	12.81	10.85	8.03	5.21’
1.0	4.80’	1.44’	1.53’	4.34’	7.62	9.61	9.70	8.19	5.90’
1.1	7.25	2.71’	0.19’	0.79’	3.37’	6.01	7.39	7.11	5.62’
1.2	9.47	5.15’	1.43’	0.30’	1.73’	4.16’	6.05	6.56	5.76’
1.3	11.09	7.63	3.70’	1.54’	1.97’	3.95’	6.00	6.99	6.72
1.4	12.11	9.69	6.05	3.39’	3.05’	4.55’	6.55	7.90	8.08

’: Non-rejection values of the null hypothesis (5.4) at 95% significance level, and "---" means that the value of the test statistics exceeds 99999.

TABLE 6.C11 (log GNP - log L)

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with no regressors and white noise U_t. d_1 and d_2 are the differencing orders of log of GNP and L respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	310.01	224.20	223.39	207.35	189.21	171.60	155.42	141.02	128.43
0.7	283.18	192.54	191.37	182.21	165.18	148.13	132.51	118.71	106.73
0.8	248.77	228.06	79.26	153.82	142.01	126.52	112.08	99.37	88.45
0.9	222.85	198.09	176.22	15.40	118.47	107.27	94.67	83.47	73.89
1.0	200.40	174.84	151.60	131.51	0.15'	89.20	80.17	70.81	62.75
1.1	180.52	155.01	132.24	112.74	96.27	2.68'	67.32	60.85	54.40
1.2	163.00	137.89	116.03	97.81	83.05	70.80	7.38	52.36	48.12
1.3	147.72	123.16	102.36	85.55	72.38	62.20	53.78	11.22	42.87
1.4	134.53	110.56	90.83	75.39	63.73	55.09	48.63	43.16	14.28

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with an intercept and white noise U_t. d_1 and d_2 are the differencing orders of log of GNP and L respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	201.40	273.78	323.14	330.75	321.47	312.96	305.92	296.84	289.19
0.7	193.65	184.69	226.96	264.63	280.96	287.90	287.52	279.52	270.81
0.8	206.04	178.15	165.07	170.24	188.27	207.88	216.60	212.79	205.18
0.9	224.73	208.57	181.01	138.26	99.77	92.90	100.95	102.96	99.83
1.0	258.42	254.81	231.89	176.06	94.64	38.27	24.41	23.82	23.25
1.1	289.62	293.75	276.09	219.95	125.25	43.54	12.19	5.23'	3.26'
1.2	304.82	312.33	297.45	242.69	145.60	55.79	17.28	7.21	3.96'
1.3	309.90	318.40	304.51	250.73	154.08	62.60	21.61	10.35	6.57
1.4	311.67	320.32	306.63	253.25	157.29	66.00	24.37	12.67	8.69

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with a time trend and white noise U_t. d_1 and d_2 are the differencing orders of log of GNP and L respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	172.97	194.79	221.92	234.35	232.64	221.95	206.70	190.41	176.12
0.7	175.14	158.78	160.77	167.12	169.92	167.75	161.66	153.29	144.49
0.8	194.79	165.84	141.92	124.64	114.05	108.14	104.36	100.92	97.17
0.9	214.94	189.94	156.75	120.51	89.62	68.92	57.79	52.89	50.93
1.0	232.25	215.49	183.35	139.55	94.64	58.68	35.79	24.32	20.18
1.1	245.40	236.20	208.33	163.78	112.64	66.85	33.97	15.09	6.88
1.2	253.88	250.26	226.66	183.94	131.22	80.66	41.50	16.94	4.90'
1.3	258.43	258.31	237.96	197.50	145.29	93.10	50.70	22.56	7.75
1.4	260.40	262.12	243.83	205.25	154.17	101.90	58.21	28.16	11.61

': Non-rejection values of the null hypothesis (5.4) at 95% significance level.

TABLE 6.C12 (log GNP - log L)

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with no regressors and white noise U_t. d_1 and d_2 are the differencing orders for log of GNP and L respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	233.14	181.47	178.40	164.78	149.79	135.31	122.00	110.14	99.77
0.7	218.00	139.16	155.89	147.28	133.50	119.80	107.21	96.00	86.23
0.8	192.53	176.90	54.56	127.45	117.48	105.18	93.66	83.42	74.52
0.9	172.38	155.55	139.63	9.47	100.84	91.73	81.70	72.68	64.85
1.0	154.57	138.00	122.32	107.85	0.05'	78.61	71.33	63.75	57.11
1.1	138.64	122.66	107.81	94.52	82.56	3.24'	61.66	56.37	51.01
1.2	124.53	109.20	95.31	83.20	72.89	63.76	7.80	49.71	46.16
1.3	112.18	97.49	84.54	73.58	64.56	57.19	50.64	11.63	41.87
1.4	101.51	87.40	75.32	65.42	57.56	51.38	46.48	41.95	14.78

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with an intercept and white noise U_t. d_1 and d_2 are the differencing orders for log of GNP and L respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	164.88	252.50	314.09	320.22	298.89	272.12	250.64	236.62	229.05
0.7	164.33	139.26	187.06	232.00	243.21	234.81	220.86	208.03	199.40
0.8	184.49	135.89	115.74	127.88	150.36	161.44	159.26	151.06	143.32
0.9	197.72	162.42	126.74	92.89	71.96	70.34	73.30	71.70	67.77
1.0	215.49	193.40	163.21	118.83	65.26	29.35	19.09	17.25	15.81
1.1	231.87	217.48	193.09	149.89	87.53	33.99	11.28	5.02'	2.72'
1.2	240.47	229.27	207.72	166.34	102.93	44.06	16.14	7.32	3.95'
1.3	244.12	233.85	213.24	172.79	109.90	50.04	20.40	10.56	6.64
1.4	246.05	235.97	215.56	175.44	113.05	53.39	23.34	13.14	8.98

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with a time trend and white noise U_t. d_1 and d_2 are the differencing orders for log of GNP and L respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	159.37	178.68	213.93	234.36	236.64	225.75	207.36	186.63	168.11
0.7	165.86	130.90	131.56	145.48	155.77	157.18	50.64	139.24	126.62
0.8	193.35	140.05	106.72	92.91	90.56	91.25	90.18	86.11	79.99
0.9	215.47	165.60	120.29	85.26	62.88	51.10	45.62	42.63	40.07
1.0	229.38	188.31	143.83	100.90	65.26	40.28	25.58	18.30	15.22
1.1	236.72	203.73	163.95	120.66	79.67	46.49	23.80	10.88	5.01'
1.2	239.52	212.39	177.34	136.34	94.50	57.72	30.14	12.78	3.94'
1.3	239.77	216.25	184.75	146.40	105.54	67.83	37.93	17.84	6.78
1.4	239.08	217.43	188.16	151.89	112.42	75.03	44.36	22.90	10.48

': Non-rejection values of the null hypothesis (5.4) at 95% significance level.

TABLE 6.C13 (log GNP - log L)

Multivariate score tests ($\tilde{S}^2$ in (5.37)) <u>with no regressors</u> and a VAR(1) structure on U_t. d_1 and d_2 are the differencing orders for log of GNP and L respectively.

d_1\d_2	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	132.62	223.58	243.89	209.06	2803.7	------	------	------	------
0.7	181.56	125.09	219.89	208.22	1925.9	------	------	------	------
0.8	271.18	108.86	99.10	240.86	1515.9	568672	------	------	------
0.9	195.76	214.33	73.18	82.56	1365.3	27025	183711	------	------
1.0	273.63	189.77	141.86	82.37	0.55’	9200.2	10787	13189	22436
1.1	10897	172755	5652.2	3135.5	1043.6	7.24	7153.4	6180.7	21671
1.2	662272	337983	475161	92183	1747.2	5635.0	7.07	5135.3	16846
1.3	191153	636489	------	798930	4680.8	5033.1	4464.3	13.55	5659.1
1.4	------	------	------	------	13736	31204	21053	4574.4	22.39

Multivariate score tests ($\tilde{S}^2$ in (5.37)) <u>with an intercept</u> and a VAR(1) structure on U_t. d_1 and d_2 are the differencing orders for log of GNP and L respectively.

d_1\d_2	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	4.86’	73.37	45.21	242.72	11.09	2.31’	0.60’	1.32’	2.12’
0.7	6.93	3.85’	29.68	27.68	11.47	4.37’	1.70’	1.43’	1.79’
0.8	16.10	2.61’	2.96’	14.67	14.74	8.77	3.44’	1.69’	1.58’
0.9	11.62	6.34	0.90’	1.57’	7.64	11.07	8.74	4.80’	2.72’
1.0	7.27	5.90’	3.44’	1.20’	1.71’	4.20’	6.03	5.63’	4.38’
1.1	5.97’	4.87’	4.33’	3.51’	3.08’	3.36’	3.29’	2.83’	2.37’
1.2	6.54	4.54’	4.45’	4.84’	5.00’	5.42’	5.76’	5.24’	4.43’
1.3	7.60	4.86’	4.63’	5.49’	6.17	6.88	7.66	7.81	7.17
1.4	8.67	5.53’	5.09’	6.04	6.97	7.97	8.93	9.48	9.14

Multivariate score tests ($\tilde{S}^2$ in (5.37)) <u>with a time trend</u> and a VAR(1) structure on U_t. d_1 and d_2 are the differencing orders for log of GNP and L respectively.

d_1\d_2	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	5.45’	26.21	37.24	29.14	17.16	8.93	4.26’	1.66’	0.31’
0.7	1.78’	3.05’	15.83	24.63	24.15	18.78	12.35	6.95	3.16’
0.8	7.98	1.04’	1.76’	9.68	17.39	20.93	19.89	15.57	10.25
0.9	11.26	5.82’	0.98’	1.20’	6.20	12.35	16.54	17.16	14.44
1.0	11.49	9.44	5.07’	1.52’	1.71’	4.94’	8.99	11.75	12.00
1.1	10.02	10.78	8.76	5.02’	2.80’	3.09’	4.86’	6.73	7.62
1.2	7.93	10.47	10.75	8.33	5.69’	4.53’	4.64’	5.13’	5.36’
1.3	6.02	9.31	11.21	10.40	8.37	6.86	6.22	5.96’	5.62’
1.4	4.77’	8.01	10.73	11.23	10.15	8.92	8.13	7.63	7.13

’: Non-rejection values of the null hypothesis (5.4) at 95% significance level and "---" means that the value of the test statistics exceeds 999999.

TABLE 6.C14 (log GNP & log M1)

$\hat{r}$ in (2.9) for the estimated residuals.

log GNP + 12.08 - 1.54 log M1

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	3.15	1.26'	-0.22'	-1.32'	-2.13	-2.71	-3.13	-3.44	-3.67	-3.86
AR(1)	0.93'	0.14'	-0.71'	-1.50'	-2.16	-2.68	-3.06	-3.33	-3.52	-3.65
AR(2)	0.65'	0.33'	-0.25'	-0.94'	-1.63'	-2.25	-2.74	-3.12	-3.38	-3.56
SAR(1)	2.40	0.96'	-0.32'	-1.35'	-2.14	-2.73	-3.15	-3.47	-3.70	-3.89
SAR(2)	2.60	1.22'	-0.07'	-1.16'	-2.00	-2.63	-3.09	-3.43	-3.68	-3.87
b) Intercept.										
W.N.	4.16	2.49	1.05'	-0.14'	-1.07'	-1.77'	-2.29	-2.68	-2.99	-3.23
AR(1)	1.61'	0.88'	0.03'	-0.82'	-1.57'	-2.20	-2.68	-3.06	-3.36	-3.60
AR(2)	2.00	2.04	1.75	1.24	0.60	-0.06	-0.66	-1.18	-1.63	-2.02
SAR(1)	2.69	1.58'	0.55'	-0.39'	-1.19'	-1.83'	-2.33	-2.71	-3.01	-3.26
SAR(2)	3.04	2.13	1.19'	0.17'	-0.80'	-1.62'	-2.25	-2.73	-3.08	-3.36
c) Intercept and a time trend.										
W.N.	5.14	3.07	1.33'	-0.04'	-1.05'	-1.79'	-2.33	-2.74	-3.06	-3.30
AR(1)	2.09	1.22'	0.23'	-0.73'	-1.56'	-2.23	-2.76	-3.17	-3.50	-3.74
AR(2)	2.32	2.30	1.94'	1.34'	0.62'	-0.10'	-0.77'	-1.38'	-1.91'	-2.36
SAR(1)	3.08	1.91'	0.74'	-0.31'	-1.18'	-1.85'	-2.36	-2.76	-3.08	-3.32
SAR(2)	3.30	2.40	1.38'	0.27'	-0.78'	-1.65'	-2.32	-2.82	-3.19	-3.47

log M1 - 7.81 - 0.64 log GNP

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	3.24	1.31'	-0.19'	-1.31'	-2.12	-2.70	-3.12	-3.43	-3.67	-3.85
AR(1)	1.02'	0.20'	-0.67'	-1.48'	-2.16	-2.68	-3.06	-3.34	-3.53	-3.66
AR(2)	0.78'	0.44'	-0.17'	-0.88'	-1.59'	-2.22	-2.73	-3.11	-3.38	-3.56
SAR(1)	2.44	0.99'	-0.30'	-1.34'	-2.14	-2.72	-3.15	-3.46	-3.70	-3.89
SAR(2)	2.65	1.26'	-0.04'	-1.14'	-1.99	-2.63	-3.09	-3.42	-3.67	-3.87
b) Intercept.										
W.N.	4.30	2.58	1.10'	-0.11'	-1.06'	-1.76'	-2.28	-2.68	-2.99	-3.23
AR(1)	1.69'	0.94'	0.07'	-0.79'	-1.56'	-2.19	-2.68	-3.06	-3.36	-3.60
AR(2)	2.06	2.08	1.79'	1.27'	0.62'	-0.04'	-0.65'	-1.18'	-1.63'	-2.02
SAR(1)	2.75	1.63'	0.58'	-0.38'	-1.19'	-1.83'	-2.32	-2.71	-3.01	-3.26
SAR(2)	3.08	2.17	1.22'	0.20'	-0.79'	-1.61'	-2.25	-2.73	-3.08	-3.36
c) Intercept and a time trend.										
W.N.	5.18	3.11	1.35'	-0.02'	-1.04'	-1.78'	-2.33	-2.74	-3.06	-3.30
AR(1)	2.11	1.25'	0.25'	-0.71'	-1.55'	-2.22	-2.75	-3.17	-3.49	-3.74
AR(2)	2.32	2.32	1.96	1.36'	0.64'	-0.09'	-0.76'	-1.37'	-1.91'	-2.35
SAR(1)	3.10	1.92'	0.75'	-0.30'	-1.17'	-1.84'	-2.36	-2.76	-3.08	-3.32
SAR(2)	3.31	2.41	1.39'	0.28'	-0.77'	-1.64'	-2.31	-2.81	-3.19	-3.47

': Non-rejection values of the null hypothesis (1.12) at 95% significance level when monotonicity in the test statistic with respect to d is observed.

TABLE 6.C15 (log GNP & log M2)

$\hat{r}$ in (2.9) for the estimated residuals.

log GNP + 6.49 - 1.00 log M2

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	7.82	6.06	4.35	2.78	1.40'	0.23'	-0.72'	-1.48'	-2.08	-2.55
AR(1)	-1.61	-1.47	-1.58	-1.77	-2.03	-2.35	-2.67	-2.99	-3.27	-3.51
AR(2)	-0.82	-0.67	-0.65	-0.74	-0.90	-1.12	-1.37	-1.64	-1.90	-2.14
SAR(1)	6.31	5.07	3.75	2.43	1.20'	0.12'	-0.79'	-1.52'	-2.11	-2.58
SAR(2)	6.45	5.12	3.68	2.25	0.95'	-0.16'	-1.06'	-1.76'	-2.30	-2.73
b) Intercept.										
W.N.	7.65	5.87	4.15	2.58	1.22'	0.11'	-0.79'	-1.51'	-2.07	-2.51
AR(1)	-1.52	-1.07	-0.70	-0.72	-0.98	-1.35	-1.75	-2.14	-2.48	-2.78
AR(2)	-1.55	-0.97	-0.60	-0.46	-0.49	-0.64	-0.86	-1.11	-1.36	-1.60
SAR(1)	5.94	4.75	3.50	2.25	1.07'	0.03'	-0.84'	-1.54'	-2.10	-2.55
SAR(2)	5.92	4.70	3.41	2.11	0.87'	-0.19'	-1.06'	-1.73'	-2.26	-2.66
c) Intercept and a time trend.										
W.N.	7.69	5.88	4.14	2.57	1.23'	0.11'	-0.78'	-1.49'	-2.04	-2.44
AR(1)	-1.54	-1.07	-0.71	-0.74	-0.99	-1.35	-1.74	-2.11	-2.43	-2.68
AR(2)	-1.63	-0.98	-0.60	-0.46	-0.49	-0.64	-0.85	-1.08	-1.30	-1.46
SAR(1)	5.93	4.75	3.50	2.25	1.07'	0.04'	-0.83'	-1.53'	-2.07	-2.48
SAR(2)	5.90	4.70	3.41	2.10	0.87'	-0.19'	-1.05'	-1.72'	-2.22	-2.60

log M2 - 6.48 - 0.99 log GNP

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend										
W.N.	7.79	6.04	4.34	2.77	1.39'	0.23'	-0.72'	-1.48'	-2.08	-2.56
AR(1)	-1.63	-1.52	-1.63	-1.81	-2.08	-2.39	-2.71	-3.02	-3.30	-3.54
AR(2)	-0.82	-0.69	-0.68	-0.77	-0.93	-1.15	-1.40	-1.67	-1.93	-2.17
SAR(1)	6.31	5.07	3.75	2.43	1.20'	0.12'	-0.79'	-1.53'	-2.12	-2.59
SAR(2)	6.46	5.12	3.68	2.25	0.94'	-0.16'	-1.06'	-1.76'	-2.31	-2.73
b) Intercept.										
W.N.	7.61	5.85	4.14	2.57	1.23'	0.11'	-0.79'	-1.50'	-2.06	-2.51
AR(1)	-1.52	-1.09	-0.72	-0.73	-0.99	-1.35	-1.75	-2.13	-2.48	-2.78
AR(2)	-1.50	-0.97	-0.61	-0.47	-0.50	-0.64	-0.86	-1.10	-1.36	-1.60
SAR(1)	5.95	4.75	3.50	2.25	1.07'	0.03'	-0.83'	-1.54'	-2.10	-2.54
SAR(2)	5.93	4.69	3.41	2.10	0.87'	-0.19'	-1.05'	-1.73'	-2.25	-2.66
c) Intercept and a time trend.										
W.N.	7.67	5.87	4.14	2.57	1.23'	0.12'	-0.78'	-1.48'	-2.03	-2.44
AR(1)	-1.55	-1.08	-0.72	-0.74	-0.99	-1.35	-1.74	-2.11	-2.43	-2.67
AR(2)	-1.64	-0.99	-0.61	-0.47	-0.50	-0.64	-0.85	-1.08	-1.30	-1.46
SAR(1)	5.93	4.75	3.50	2.25	1.07'	0.04'	-0.83'	-1.52'	-2.07	-2.48
SAR(2)	5.89	4.69	3.41	2.10	0.87'	-0.18'	-1.04'	-1.71'	-2.22	-2.60

': Non-rejection values of the null hypothesis (1.12) at 95% significance level when monotonicity in the test statistic with respect to d is observed.

TABLE 6.C16 (log GNP & log M3)

$\hat{r}$ in (2.9) for the estimated residuals.

log GNP + 5.26 - 0.90 log M3

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	7.28	5.48	3.77	2.25	0.94'	-0.13'	-0.99'	-1.67'	-2.21	-2.63
AR(1)	-1.59	-1.51	-1.55	-1.74	-2.02	-2.34	-2.65	-2.94	-3.19	-3.41
AR(2)	-1.09	-0.88	-0.85	-0.96	-1.16	-1.42	-1.70	-1.99	-2.25	-2.49
SAR(1)	5.95	4.69	3.35	2.05	0.87'	-0.13'	-0.94'	-1.59'	-2.11	-2.52
SAR(2)	6.13	4.88	3.50	2.12	0.87'	-0.18'	-1.02'	-1.67'	-2.18	-2.59
b) Intercept.										
W.N.	7.33	5.54	3.82	2.29	1.00'	-0.06'	-0.89'	-1.55'	-2.07	-2.48
AR(1)	-1.17	-0.82	-0.70	-0.88	-1.21	-1.60	-1.97	-2.31	-2.61	-2.85
AR(2)	-0.99	-0.53	-0.35	-0.39	-0.58	-0.85	-1.14	-1.43	-1.70	-1.94
SAR(1)	5.83	4.67	3.41	2.13	0.95'	-0.07'	-0.89'	-1.54'	-2.05	-2.46
SAR(2)	5.93	4.80	3.55	2.26	1.03'	-0.04'	-0.91'	-1.57'	-2.09	-2.49
c) Intercept and a time trend.										
W.N.	7.40	5.56	3.83	2.29	1.00'	-0.05'	-0.89'	-1.55'	-2.06	-2.46
AR(1)	-1.14	-0.80	-0.70	-0.88	-1.21	-1.60	-1.97	-2.31	-2.59	-2.81
AR(2)	-1.00	-0.52	-0.34	-0.39	-0.58	-0.84	-1.14	-1.43	-1.68	-1.88
SAR(1)	5.83	4.68	3.41	2.13	0.95'	-0.07'	-0.89'	-1.54'	-2.04	-2.43
SAR(2)	5.92	4.80	3.56	2.26	1.03'	-0.04'	-0.90'	-1.57'	-2.08	-2.47

log M3 - 5.82 - 1.09 log GNP

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	7.25	5.45	3.75	2.23	0.94'	-0.13'	-1.00'	-1.68'	-2.22	-2.64
AR(1)	-1.56'	-1.57'	-1.60'	-1.78'	-2.05	-2.36	-2.67	-2.96	-3.21	-3.43
AR(2)	-1.14	-0.92	-0.89	-1.00	-1.20	-1.45	-1.73	-2.01	-2.28	-2.51
SAR(1)	5.94	4.68	3.34	2.04	0.86'	-0.13'	-0.95'	-1.60'	-2.11	-2.53
SAR(2)	6.12	4.86	3.48	2.10	0.86'	-0.19'	-1.03'	-1.68'	-2.19	-2.60
b) Intercept.										
W.N.	7.29	5.51	3.81	2.29	1.00'	-0.05'	-0.88'	-1.54'	-2.06	-2.48
AR(1)	-1.19	-0.85	-0.72	-0.89	-1.22	-1.60	-1.97	-2.31	-2.60	-2.85
AR(2)	-1.00	-0.55	-0.36	-0.40	-0.58	-0.85	-1.14	-1.43	-1.70	-1.94
SAR(1)	5.83	4.66	3.41	2.13	0.95'	-0.06'	-0.89'	-1.54'	-2.05	-2.45
SAR(2)	5.93	4.80	3.55	2.26	1.03'	-0.04'	-0.90'	-1.57'	-2.08	-2.49
c) Intercept and a time trend.										
W.N.	7.39	5.56	3.83	2.29	1.00'	-0.05'	-0.88'	-1.54'	-2.06	-2.45
AR(1)	-1.15	-0.81	-0.70	-0.89	-1.22	-1.60	-1.97	-2.31	-2.59	-2.81
AR(2)	-1.01	-0.53	-0.35	-0.39	-0.58	-0.85	-1.14	-1.43	-1.68	-1.88
SAR(1)	5.83	4.68	3.42	2.13	0.95'	-0.06'	-0.89'	-1.53'	-2.04	-2.43
SAR(2)	5.92	4.80	3.56	2.26	1.03'	-0.04'	-0.90'	-1.57'	-2.08	-2.46

': Non-rejection values of the null hypothesis (1.12) at 95% significance level when monotonicity in the value of the tests with respect to d is observed.

TABLE 6.C17 (log GNP & log L)

$\hat{r}$ in (2.9) for the estimated residuals.

log GNP + 6.17 - 0.96 log L

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	3.04	1.64'	0.47'	-0.49'	-1.25'	-1.86'	-2.34	-2.73	-3.04	-3.29
AR(1)	-0.39'	-0.71'	-1.13'	-1.57'	-1.98	-2.35	-2.66	-2.92	-3.13	-3.31
AR(2)	0.04'	-0.10'	-0.39'	-0.76'	-1.16'	-1.55'	-1.90'	-2.20	-2.45	-2.65
SAR(1)	2.83	1.61'	0.48'	-0.48'	-1.26'	-1.88'	-2.37	-2.75	-3.06	-3.31
SAR(2)	2.88	1.70'	0.58'	-0.40'	-1.21'	-1.85'	-2.35	-2.74	-3.05	-3.31
b) Intercept.										
W.N.	3.03	1.67'	0.54'	-0.38'	-1.13'	-1.74'	-2.23	-2.63	-2.96	-3.23
AR(1)	-0.39'	-0.71'	-1.13'	-1.57'	-2.00	-2.39	-2.73	-3.02	-3.27	-3.47
AR(2)	0.03'	-0.08'	-0.34'	-0.68'	-1.06'	-1.43'	-1.79'	-2.12	-2.41	-2.66
SAR(1)	2.82	1.63'	0.55'	-0.38'	-1.14'	-1.75'	-2.24	-2.64	-2.97	-3.24
SAR(2)	2.87	1.73'	0.65'	-0.30'	-1.09'	-1.73'	-2.24	-2.65	-2.98	-3.25
c) Intercept and a time trend.										
W.N.	3.06	1.70'	0.56'	-0.37'	-1.12'	-1.73'	-2.23	-2.62	-2.94	-3.18
AR(1)	-0.38'	-0.71'	-1.13'	-1.57'	-2.00	-2.38	-2.72	-3.01	-3.23	-3.37
AR(2)	0.06'	-0.06'	-0.33'	-0.67'	-1.05'	-1.43'	-1.78'	-2.10	-2.34	-2.49
SAR(1)	2.85	1.65'	0.56'	-0.37'	-1.13'	-1.75'	-2.24	-2.63	-2.95	-3.19
SAR(2)	2.90	1.75'	0.67'	-0.29'	-1.09'	-1.73'	-2.23	-2.64	-2.96	-3.20

log L - 6.43 - 1.03 log GNP

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	3.06	1.66'	0.49'	-0.47'	-1.24'	-1.85'	-2.34	-2.72	-3.03	-3.29
AR(1)	-0.38'	-0.70'	-1.13'	-1.57'	-1.98	-2.35	-2.66	-2.93	-3.14	-3.31
AR(2)	0.06'	-0.08'	-0.37'	-0.75'	-1.15'	-1.54'	-1.89'	-2.19	-2.45	-2.65
SAR(1)	2.85	1.63'	0.50'	-0.47'	-1.25'	-1.87'	-2.36	-2.74	-3.05	-3.31
SAR(2)	2.90	1.72'	0.60'	-0.38'	-1.19'	-1.84'	-2.34	-2.73	-3.05	-3.30
b) Intercept.										
W.N.	3.04	1.68'	0.55'	-0.38'	-1.13'	-1.74'	-2.23	-2.63	-2.96	-3.23
AR(1)	-0.38'	-0.70'	-1.13'	-1.57'	-2.00	-2.39	-2.73	-3.02	-3.27	-3.47
AR(2)	0.04'	-0.07'	-0.34'	-0.68'	-1.05'	-1.43'	-1.79'	-2.12	-2.41	-2.66
SAR(1)	2.83	1.64'	0.55'	-0.38'	-1.14'	-1.75'	-2.24	-2.64	-2.97	-3.23
SAR(2)	2.88	1.73'	0.65'	-0.30'	-1.09'	-1.73'	-2.24	-2.65	-2.98	-3.25
c) Intercept and a time trend.										
W.N.	3.06	1.70'	0.56'	-0.37'	-1.12'	-1.73'	-2.23	-2.62	-2.94	-3.18
AR(1)	-0.38'	-0.71'	-1.13'	-1.57'	-2.00	-2.38	-2.72	-3.01	-3.23	-3.37
AR(2)	0.06'	-0.06'	-0.33'	-0.67'	-1.05'	-1.43'	-1.78'	-2.10	-2.34	-2.49
SAR(1)	2.86	1.66'	0.56'	-0.37'	-1.13'	-1.75'	-2.24	-2.63	-2.95	-3.19
SAR(2)	2.91	1.75'	0.67'	-0.29'	-1.08'	-1.73'	-2.23	-2.64	-2.96	-3.20

': Non-rejection values of the null hypothesis (1.12) at 95% significance level when monotonicity in the test statistic with respect to d is observed.

6.d STOCK PRICES AND DIVIDENDS

In this section we study the relationship between stock prices and dividends. If the present value model were true, a linear combination of both variables (which must be integrated of order 1) should be stationary and thus, prices and dividends would be cointegrated. Though much literature exists on this topic, little consensus exists about what might be the correct model specification for these two variables. Thus, using the DF and ADF tests with a time trend, Shiller (1981) tested and rejected the hypothesis of integration for Standard and Poor's (S&P) dividends over the years 1872-1978; however, Kleidon (1986), using the same tests with an intercept on a shorter S&P's data set (1926-1979) argued that dividends and prices were both integrated. Perron (1988) used tests of Phillips (1987) and Phillips and Perron (1988) for testing unit roots on the S&P's 1871-1984 data set and found evidence of a unit root on prices but stationary around a deterministic trend on dividends.

Campbell and Shiller (1987) and DeJong (1992) tested a present value model in the stock market using time-series data for real U.S. annual prices and dividends, on a broad stock index from 1871 to 1986. In the first of these articles, they applied the ADF tests, with and without a time trend, on both individual series, and their results suggested that both series were integrated of order 1. Using the DF and ADF tests on the residuals from the cointegrating regressions, their results were mixed: the former test rejected the null hypothesis of no cointegration at the 5 per cent level while the latter narrowly failed to reject it at the 10 per cent level. DeJong (1992) used a Bayesian approach to investigate the integration inference for these two variables and his evidence was in favour of trend-stationary representations. Similarly, Koop (1991b), using a different data set, came to the same conclusions that both variables were stationary with a time trend, and even assuming unit roots, he found little evidence of cointegration. Finally Fama and French (1988), Lo and MacKinlay (1988) and Poterba and Summers (1988) developed "variance ratio" tests, and suggested that stock prices exhibited mean reverting behaviour. In contrast with these previous studies, DeJong and Whiteman (1992) found no mean reversion in S&P prices and dividends.

In this section we use the same data set as in Campbell and Shiller (1987) and DeJong (1992), studying the relationship between real U.S. annual dividends and stock prices from 1871 to 1986. Figure 6.D1 contains plots of both series, their sample autocorrelations and estimates of the spectral density function. In this figure we observe that both series seem to present a very similar nonstationary behaviour, with some peaks and troughs, especially pronounced during the crisis in 1929 and 1973. The nonstationary character of the series may be better viewed through sample autocorrelations, which decay very slowly and persistently, and from estimates of the spectral density function, which show a very large value around zero frequency and thus, suggesting the unboundedness of the spectrum at such frequency.

As in previous sections, we start calculating Robinson's (1994c) univariate tests in order to investigate more deeply what might be the appropriate integration order for each series. Clearly, under the trend stationary representations suggested in Perron (1988), Koop (1991b), DeJong (1992) and others, we should expect not to reject the null hypothesis when this integration order is zero.

Table 6.D1 reports results of $\hat{r}$ in (2.9), when testing (1.12) in a model given by (1.9) and (1.10) with $\rho(L;\theta) = (1 - L)^{d+\theta}$, for cases of no regressors, an intercept and an intercept and a time trend, and with white noise, non-seasonal and seasonal AR disturbances. Starting with stock prices, we see in the upper part of this table that the monotonic decrease in $\hat{r}$ with respect to d is only achieved for white noise and seasonal AR disturbances. In these cases, the non-rejection values of d always range between 0.8 and 1.1, with the lowest statistics occurring when d takes values 0.9 and 1. This is observed independently of the inclusion or not of an intercept and/or a time trend in the model, and of the different ways of modelling the disturbances. Looking at dividends, (in the lower part of this table), we observe that if there are no regressors and the disturbances follow a non-seasonal AR process, there is a wide range of non-rejection values of d, but apart from this case, the values of d where the null is not rejected always range between 0.9 and 1.4, with the lowest statistic occurring in most of cases when d = 1.1. Including regressors, such as an intercept or an intercept and a time trend, results are similar in both cases, with the non-rejection d's ranging from 0.9 through 1.3, and with the lowest statistics occurring again at d = 1.1.

To conclude with respect to this table, we see that both series might be well-modelled as I(1) processes, independently of the inclusion or not of an intercept and/or a time trend in the model, and of the way of modelling the disturbances. We also observe that the integration order seems to be slightly greater for dividends than for prices: in fact, for dividends, integration orders of 1 or 1.1 might be most preferable, while for prices, the most plausible ones are 0.9 and 1. Given that the null hypothesis is not rejected for prices when d = 0.9 (and in some cases when d = 0.8), these results might indicate the presence of a small component of mean reversion in this series and thus, they would be consistent with those obtained in Fama and French (1988), Lo and McKinlay (1988) and Poterba and Summers (1988); however, since the null is practically always rejected in both series when d = 0.6 in favour of more nonstationary alternatives, and the bulk of these results fail to reject the unit root null, we could conclude by rejecting the trend-stationary representations proposed in DeJong (1992) and others, and supporting the view in Campbell and Shiller (1987) that both series are integrated of order 1. Next we look at the results in a multivariate context.

Tables 6.D2-6.D5 contain results of the multivariate score tests of Chapter 5. In Tables 6.D2 and 6.D3 we report the test statistics in the time and the frequency domain respectively, assuming that U_t is a white noise vector process. Results in both tables are similar and we observe that the subset of values of d_1 and d_2 where the null is not rejected remains the same for the different cases of no regressors, an intercept, and an intercept and a time trend. These values range between 0.8 and 1.1 for d_1 (the integration order of prices), and between 0.8 and 1.3 for d_2 (the integration order of dividends). Therefore, we again observe here a greater integration order for dividends than for prices. The two unit root null hypothesis (i.e. $d_1 = d_2 = 1$) is never rejected in these tables, though lower statistics are obtained in some cases when d_1 and/or d_2 are smaller than 1. In fact, looking at the results in the time domain, we see in the upper part of Table 6.D2, that if there are no regressors, the lowest statistic is obtained when $d_1 = 0.9$ and $d_2 = 1$, with $\hat{S}^{t2} = 0.19$. Including an intercept and an intercept and a time trend, the lowest statistics appear in both cases at $d_1 = d_2 = 0.9$, with $\hat{S}^{t2} = 0.21$ when including an intercept, and with $\hat{S}^{t2} = 0.20$ when including an intercept and a time trend. Similarly, looking at the results in the frequency domain, in Table 6.D3, the lowest statistics occur at $d_1 = 0.9$ and $d_2 = 1$ when modelling with no regressors, and at $d_1 = d_2 = 0.9$ when including an intercept and an intercept and a time trend.

In Table 6.D4 a VAR(1) structure is assumed for U_t. Here again results seem quite robust to different regressors. We observe that the two unit roots null hypothesis is always rejected in favour of less nonstationary alternatives, with d_1 and d_2 ranging between 0.6 and 0.9. The lowest statistics are obtained in this table when $d_1 = d_2 = 0.6$, which is close to the stationary region, and this is observed for the three cases of no regressors, an intercept and an intercept and a time trend. As we explained in previous sections, this smaller degree of nonstationarity observed in this table compared with Tables 6.D2 and 6.D3 (referred to white noise U_t), might be due to the fact that the VAR parameters have been obtained using the method of maximum likelihood throughout a quasi-Newton algorithm, which can give us parameters arbitrarily close to nonstationary. Thus, competition may exist between the VAR parameters and the differencing orders in describing the nonstationary component of the series, and as we approximate to stationary, the value of the determinant in the VAR representation will be approximately zero.

In Table 6.D5 U_t is assumed to be VMA(1). We observe here a greater proportion of non-rejections compared with Table 6.D4, with the lowest statistics obtained in the three cases when $d_1 = d_2 = 0.6$, that is, for the same values as in Table 5.D4. However, the two unit roots null is not rejected in this table, and other possibilities with d_1 and d_2 greater than 1 are either non-rejected. This can be explained because U_t is now always stationary and thus, the nonstationary component of the series might be mainly described throughout the differencing parameters.

As a conclusion of the univariate and multivariate tests presented above, we see that both series might be integrated of order 1, though slight variations in the integration order, higher for dividends than for prices, also seem plausible. This unit root behaviour observed in the series is obtained independently of the regressors used in the model. If U_t is VAR(1), the integration orders are smaller in both series, perhaps due to the competition with the VAR parameters, but these orders vary widely when U_t is VMA(1).

In the final part of this section, we calculated the cointegrating regressions of one of the variables against the other, and its reverse, and the resulting regressions were:

$$p_t = -0.12 + 30.99\, d_t,$$
$$(-6.27) \quad (24.97) \qquad \text{(t-values)}$$

and

$$d_t = 0.005 + 0.027\, p_t,$$
$$(13.17) \quad (24.97) \qquad \text{(t-values)}$$

where p_t corresponds to stock prices and d_t to dividends. Campbell and Shiller (1987) performed the DF and ADF tests for no cointegration in the estimated residuals above, finding mixed results: the test statistics rejected the null at the 5% level but narrowly failed to reject it at the 10% level.

Table 6.D6 contains results of Robinson's (1994c) univariate tests on these estimated residuals. We see that the non-rejections always occur at the same values of d for the different cases of no regressors, an intercept, and an intercept and a time trend. We also observe in this table that the unit root null hypothesis is rejected in all cases except if the disturbances follow an AR(2) process, but in this case there is a wide range of values of d where the null is not rejected and, though it is not shown in the table, there is a lack of monotonic decrease in the value of the test statistic with respect to d for values of d smaller than 0.6. All other non-rejection values always take place when d ranges between 0.6 and 0.9, with the lowest statistics across different values of d occurring in most of cases when d = 0.7. Thus, given that the unit root null is practically always rejected in favour of less nonstationary alternatives, we may conclude that both series are fractionally cointegrated, with the estimated residuals from the cointegrating regression showing a mean reverting behaviour.

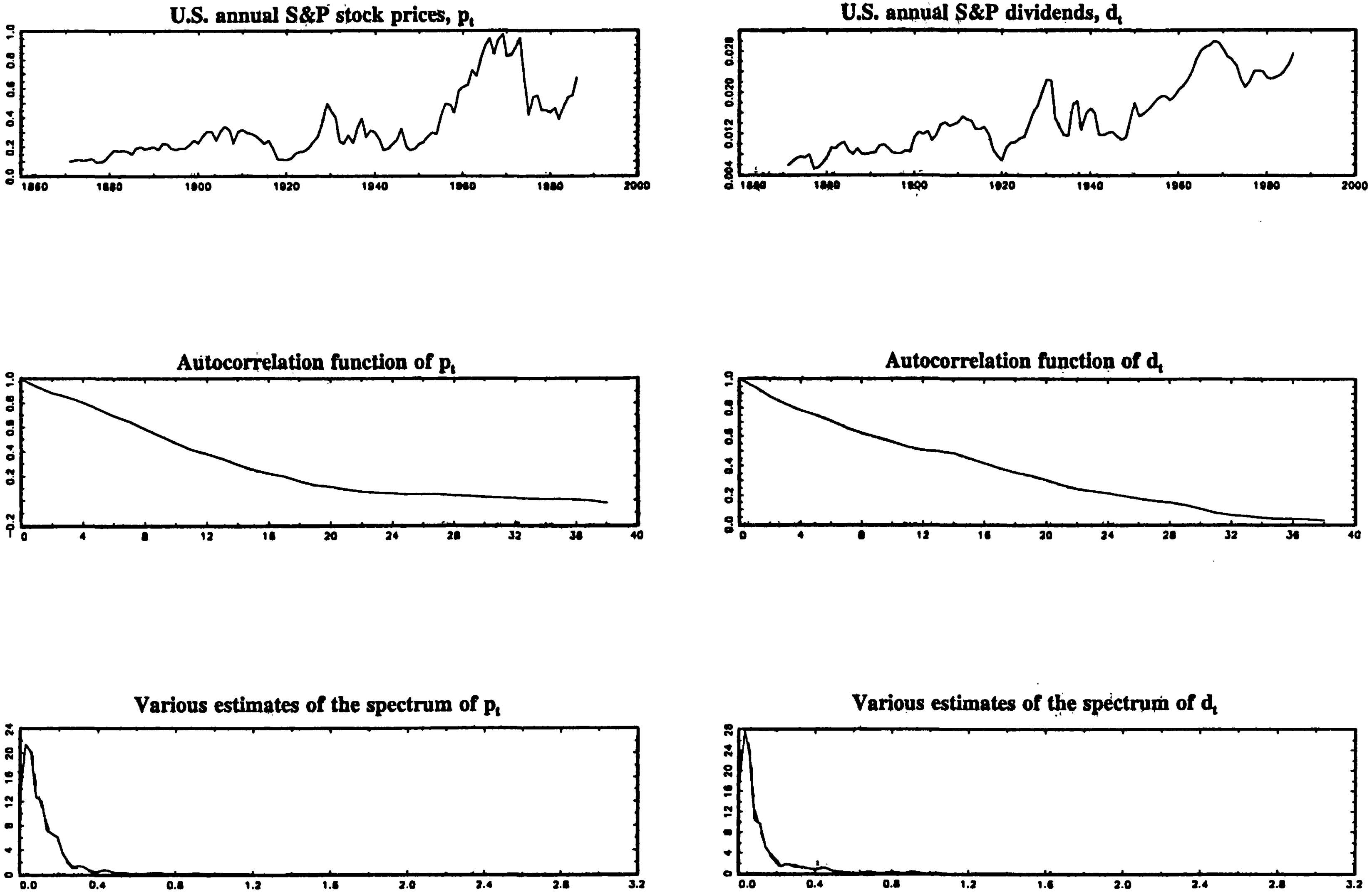
We also see in the same table that the null hypothesis is not rejected when d = 0.6. Thus, it might also be of interest to test if the estimated residuals are stationary. In Table 6.D7 we calculate the same tests as in Table 6.D6, with d ranging now from 0.0 through 0.5. We observe here that the null hypothesis is always rejected for all values of d, and even at the boundary case of d = 0.5, it is decisively rejected in favour of more nonstationary alternatives with d > 0.5. Therefore we might conclude by saying that the estimated residuals are clearly nonstationary, with integration orders fluctuating around 0.7 and thus, showing mean reverting behaviour.

All these results alleviate the mixed evidence found in Campbell and Shiller (1987) when testing the null of no cointegration with the classical DF and ADF tests.

As mentioned in previous sections, a problem in these testing procedures is that they only concentrate on I(0) stationary and I(1) nonstationary residuals, and thus, do not consider other fractionally integrated possibilities. Our results suggest that dividends and prices might be fractionally cointegrated, with the equilibrium errors from the cointegrating regressions, though nonstationary, displaying mean reverting behaviour. Thus, a shock to the system will eventually die out, implying a reliable long run relationship between prices and dividends in the stock market.

Summarizing the main results obtained in this section, we see that prices and dividends appear both individually integrated or order 1 independently of the inclusion or not of an intercept or a time trend in the model, though slight variations in this integration order (smaller than one for prices, but greater than one for dividends) might also be plausible. The multivariate tests corroborate these findings if the disturbances are white noise but integration orders smaller than one might be more appropriate if the disturbances are weakly autocorrelated. Results of Robinson's (1994c) univariate tests on the estimated residuals from the cointegrating regressions indicate that both variables are in fact fractionally cointegrated with the integration orders of the cointegrating residuals fluctuating around 0.7 and thus implying mean reversion.

FIGURE 5.D1



U.S. annual S&P stock prices, p_t
U.S. annual S&P dividends, d_t
Autocorrelation function of p_t
Autocorrelation function of d_t
Various estimates of the spectrum of p_t
Various estimates of the spectrum of d_t

TABLE 6.D1

$\hat{r}$ in (2.9) in the stock market.

S&P's stock prices (1871 - 1986)

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	6.02	4.03	2.35	0.95'	-0.19'	-1.10'	-2.03	-2.40	-2.86	-3.23
AR(1)	0.18	0.24	-0.07	-0.56	-1.14	-1.72	-2.28	-2.78	-3.21	-3.58
AR(2)	0.72	1.36	1.64	1.67	1.47	1.11	0.63	0.09	-0.49	-1.05
SAR(1)	4.37	3.01	1.72'	0.57'	-0.43'	-1.26'	-1.97	-2.50	-2.95	-3.31
SAR(2)	4.19	2.90	1.66'	0.53'	-0.45'	-1.27'	-1.97	-2.50	-2.94	-3.31
b) Intercept.										
W.N.	6.06	3.89	2.14	0.73'	-0.40'	-1.30'	-2.00	-2.55	-2.99	-3.33
AR(1)	-1.02	-0.48	-0.61	-1.02	-1.54	-2.09	-2.60	-3.06	-3.45	-3.79
AR(2)	-0.55	0.57	1.03	1.11	0.95	0.60	0.15	-0.37	-0.90	-1.42
SAR(1)	4.05	2.72	1.45'	0.31'	-0.67'	-1.47'	-2.13	-2.66	-3.09	-3.43
SAR(2)	3.94	2.63	1.39'	0.28'	-0.68'	-1.48'	-2.13	-2.66	-3.08	-3.42
c) Intercept and a time trend.										
W.N.	5.76	3.85	2.16	0.75'	-0.39'	-1.30'	-2.01	-2.56	-3.00	-3.34
AR(1)	-0.45	-0.27	-0.51	-0.97	-1.52	-2.09	-2.61	-3.08	-3.48	-3.81
AR(2)	0.14	0.84	1.16	1.18	0.97	0.60	0.12	-0.41	-0.95	-1.47
SAR(1)	4.14	2.78	1.49'	0.34'	-0.65'	-1.47'	-2.13	-2.66	-3.09	-3.44
SAR(2)	3.92	2.67	1.43'	0.30'	-0.67'	-1.48'	-2.13	-2.66	-3.09	-3.43

S&P's dividends (1871 - 1986)

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	5.41	4.03	2.84	1.79'	0.86'	0.04'	-0.68'	-1.30'	-1.84'	-2.30
AR(1)	-0.32'	-0.78'	-1.09'	-1.37'	-1.64'	-1.92'	-2.20	-2.48	-2.75	-3.01
AR(2)	0.08'	-0.07'	-0.22'	-0.38'	-0.57'	-0.79'	-1.03'	-1.30'	-1.57'	-1.85'
SAR(1)	5.40	4.15	2.99	1.91'	0.93'	0.07'	-0.68'	-1.32'	-1.87'	-2.34
SAR(2)	5.44	4.16	2.97	1.88'	0.90'	0.03'	-0.71'	-1.35'	-1.90'	-2.36
b) Intercept.										
W.N.	5.36	3.62	2.37	1.36'	0.50'	-0.25'	-0.89'	-1.45'	-1.94'	-2.36
AR(1)	-2.82	-2.44	-2.35	-2.39	-2.50	-2.66	-2.83	-3.02	-3.21	-3.39
AR(2)	-1.78	-1.38	-1.27	-1.28	-1.36	-1.48	-1.64	-1.83'	-2.03	-2.23
SAR(1)	5.08	3.69	2.48	1.44'	0.54'	-0.25'	-0.92'	-1.49'	-1.98	-2.41
SAR(2)	5.17	3.72	2.45	1.38'	0.47'	-0.30'	-0.97'	-1.53'	-2.02	-2.43
c) Intercept and a time trend.										
W.N.	4.78	3.52	2.38	1.38'	0.51'	-0.24'	-0.90'	-1.46'	-1.97	-2.37
AR(1)	-1.98	-2.12	-2.21	-2.33	-2.48	-2.65	-2.84	-3.04	-3.23	-3.42
AR(2)	-1.16	-1.12	-1.14	-1.21	-1.33	-1.48	-1.66	-1.86	-2.06	-2.28
SAR(1)	4.80	3.63	2.51	1.47'	0.55'	-0.24'	-0.92'	-1.50'	-2.00	-2.42
SAR(2)	4.78	3.60	2.45	1.41'	0.49'	-0.30'	-0.97'	-1.55'	-2.03	-2.45

': Non-rejection values of the null hypothesis (1.12) with $\rho(L;\theta) = (1-L)^{d+\theta}$ at 95% significance level when monotonicity in the test statistic with respect to d is observed.

TABLE 6.D2

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with no regressors and white noise U_t. d_1 and d_2 are the differencing orders for stock prices and dividends respectively.

d_1\d_2	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	21.15	22.56	28.13	34.29	39.48	43.20	45.51	46.69	47.09
0.7	14.93	8.27	8.51	11.99	16.47	20.68	24.06	26.51	28.14
0.8	18.74	6.75	2.08'	1.97'	4.32'	7.63	10.97	13.86	16.17
0.9	25.61	11.24	3.42'	0.19'	0.30'	2.08'	4.62'	7.28	9.72
1.0	31.98	17.22	7.96	2.97'	1.16'	1.48'	3.01'	5.08'	7.26
1.1	36.58	22.47	12.95	7.09	4.19'	3.45'	4.10'	5.54'	7.34
1.2	39.36	26.29	17.14	11.10	7.69	6.32	6.37	7.31	8.72
1.3	40.70	28.72	20.20	14.38	10.86	9.19	8.89	9.48	10.61
1.4	41.09	30.06	22.24	16.81	13.41	11.68	11.20	11.60	12.54

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with an intercept and white noise U_t. d_1 and d_2 are the differencing orders for stock prices and dividends respectively.

d_1\d_2	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	21.16	21.89	28.33	35.36	40.94	44.65	46.75	47.66	47.81
0.7	16.13	7.91	8.31	12.30	17.05	21.25	24.47	26.71	28.15
0.8	20.65	6.73	1.81'	1.96'	4.47'	7.75	10.92	13.63	15.77
0.9	27.76	11.39	3.16'	0.21'	0.36'	2.11'	4.50'	6.98	9.25
1.0	34.04	17.34	7.63	2.82'	1.19'	1.52'	2.93'	4.83'	6.85
1.1	38.46	22.47	12.49	6.83	4.15'	3.47'	4.05'	5.36'	7.00
1.2	41.06	26.17	16.56	10.72	7.57	6.31	6.32	7.15	8.43
1.3	42.29	28.52	19.53	13.90	10.66	9.13	8.82	9.33	10.34
1.4	42.62	29.82	21.52	16.26	13.15	11.57	11.12	11.44	12.28

Multivariate score tests in the time domain ($\hat{S}^{t2}$ in (5.32)) with a time trend and white noise U_t. d_1 and d_2 are the differencing orders for stock prices and dividends respectively.

d_1\d_2	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	18.42	20.44	26.46	32.99	38.32	41.96	44.06	45.00	45.17
0.7	12.57	7.15	7.97	11.81	16.42	20.55	23.75	25.98	27.41
0.8	16.22	5.69'	1.66'	1.91'	4.39'	7.64	10.81	13.51	15.65
0.9	22.95	10.10	2.94'	0.20'	0.36'	2.10'	4.50'	6.98	9.24
1.0	29.18	15.94	7.37	2.80'	1.19'	1.51'	2.92'	4.83'	6.85
1.1	33.64	21.04	12.21	6.81	4.15'	3.46'	4.04'	5.35'	6.99
1.2	36.30	24.72	16.26	10.70	7.58	6.30	6.31	7.13	8.42
1.3	37.57	27.05	19.21	13.87	10.66	9.12	8.81	9.31	10.33
1.4	37.93	28.34	21.18	16.22	13.14	11.56	11.10	11.43	12.26

': Non-rejection values of the null hypothesis (5.4) at 95% significance level.

TABLE 6.D3

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with no regressors and white noise U_t. d_1 and d_2 are the differencing orders for stock prices and dividends respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	22.79	22.91	27.64	33.33	38.31	42.00	44.42	45.79	46.42
0.7	17.17	9.39	8.73	11.60	15.73	19.80	23.21	25.79	27.61
0.8	21.33	8.37	2.80'	1.98'	3.85'	6.90	10.17	13.13	15.59
0.9	28.45	13.23	4.56'	0.67'	0.09'	1.50'	3.86'	6.51	9.03
1.0	34.98	19.50	9.45	3.69'	1.23'	1.07'	2.32'	4.29'	6.49
1.1	39.69	24.95	14.71	8.12	4.54'	3.26'	3.55'	4.80'	6.54
1.2	42.53	28.90	19.09	12.38	8.30	6.36	6.00	6.67	7.97
1.3	43.91	31.40	22.28	15.83	11.67	9.44	8.68	8.98	9.96
1.4	44.29	32.77	24.38	18.37	14.37	12.09	11.16	11.24	12.00

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with an intercept and white noise U_t. d_1 and d_2 are the differencing orders for stock prices and dividends respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	20.99	21.89	28.39	35.39	40.96	44.56	46.62	47.52	47.67
0.7	15.91	7.84	8.28	12.25	16.94	21.09	24.27	26.49	27.92
0.8	20.45	6.66	1.79'	1.92'	4.38'	7.60	10.73	13.41	15.55
0.9	27.55	11.33	3.17'	0.23'	0.33'	2.03'	4.37'	6.82	9.07
1.0	33.81	17.26	7.65	2.86'	1.21'	1.50'	2.87'	4.74'	6.74
1.1	38.19	22.36	12.49	6.88	4.20'	3.49'	4.04'	5.31'	6.93
1.2	40.77	26.03	16.53	10.76	7.63	6.35	6.33	7.13	8.39
1.3	41.98	28.36	19.49	13.93	10.71	9.18	8.85	9.33	10.32
1.4	42.31	29.66	21.47	16.29	13.20	11.63	11.15	11.46	12.27

Multivariate score tests in the frequency domain ($\hat{S}^{f2}$ in (5.34)) with a time trend and white noise U_t. d_1 and d_2 are the differencing orders for stock prices and dividends respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	18.50	20.49	26.49	33.01	38.32	41.95	44.05	45.00	45.19
0.7	12.66	7.18	7.97	11.79	16.37	20.49	23.68	25.93	27.38
0.8	16.35	5.75'	1.68'	1.89'	4.34'	7.56	10.72	13.44	15.59
0.9	23.14	10.20	3.00'	0.20'	0.34'	2.05'	4.43'	6.91	9.19
1.0	29.40	16.09	7.48	2.86'	1.21'	1.50'	2.89'	4.79'	6.82
1.1	33.89	21.22	12.36	6.92	4.22'	3.50'	4.05'	5.34'	6.98
1.2	36.57	24.93	16.43	10.85	7.68	6.37	6.35	7.16	8.43
1.3	37.85	27.27	19.41	14.04	10.80	9.23	8.88	9.37	10.37
1.4	38.21	28.56	21.38	16.40	13.30	11.68	11.20	11.50	12.32

': Non-rejection values of the null hypothesis (5.4) at 95% significance level.

TABLE 6.D4

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with no regressors and a VAR(1) structure on U_t. d_1 and d_2 are the differencing orders for stock prices and dividends respectively.

d_1\d_2	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	1.12'	5.02'	11.60	18.83	25.11	29.58	31.99	32.57	31.79
0.7	2.83'	2.12'	5.38'	11.31	18.30	24.83	29.87	32.98	34.26
0.8	9.17	4.44'	3.46'	5.87'	10.86	17.20	23.52	28.78	32.47
0.9	18.00	10.98	6.66	5.27'	6.83	10.87	16.41	22.28	27.55
1.0	26.92	19.47	13.39	9.23	7.51	8.42	11.67	16.46	21.85
1.1	34.14	27.80	21.45	15.92	11.92	11.54	13.11	16.31	20.54
1.2	38.73	34.46	29.08	23.43	18.34	15.03	14.53	15.84	18.64
1.3	40.43	38.71	35.11	30.33	25.20	19.98	17.95	17.55	18.75
1.4	39.84	40.49	38.99	35.74	31.39	25.27	22.40	20.72	20.48

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with an intercept and a VAR(1) structure on U_t. d_1 and d_2 are the differencing orders for stock prices and dividends respectively.

d_1\d_2	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	1.76'	7.55	15.70	23.74	30.12	34.10	35.66	35.28	33.67
0.7	2.51'	2.89'	7.78	14.87	22.27	28.57	32.95	35.22	35.70
0.8	8.70	4.04'	4.36'	8.16	14.02	20.53	26.47	31.04	33.95
0.9	17.60	9.93	6.33	6.30	9.11	13.84	19.44	24.91	29.51
1.0	26.40	18.04	12.12	9.06	8.73	10.79	14.63	19.45	24.45
1.1	33.23	26.02	19.52	14.72	12.02	11.54	13.11	16.31	20.54
1.2	37.21	32.30	26.62	21.42	17.40	15.03	14.53	15.84	18.64
1.3	38.36	36.08	32.21	27.69	23.38	19.98	17.95	17.55	18.75
1.4	37.38	37.48	35.72	32.63	28.90	25.27	22.40	20.72	20.48

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with a time trend and a VAR(1) structure on U_t. d_1 and d_2 are the differencing orders for stock prices and dividends respectively.

d_1\d_2	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	2.34'	7.87	15.71	23.59	29.92	33.89	35.44	35.04	33.43
0.7	2.70'	3.21'	7.93	14.93	22.34	28.69	33.09	35.37	35.84
0.8	8.53	4.26'	4.49'	8.23	14.14	20.75	26.77	31.39	34.32
0.9	17.32	10.12	6.47	6.33	9.14	13.96	19.67	25.22	29.88
1.0	26.17	18.29	12.36	9.14	8.73	10.82	14.73	19.64	24.71
1.1	33.11	26.33	19.88	14.92	12.09	11.56	13.15	16.42	20.71
1.2	37.02	32.61	27.07	21.73	17.56	15.10	14.59	15.92	18.76
1.3	38.07	36.34	32.65	28.07	23.62	20.12	18.05	17.65	18.87
1.4	36.95	37.63	36.12	33.02	29.18	25.46	22.53	20.84	20.60

': Non-rejection values of the null hypothesis (5.4) at 95% significance level.

TABLE 6.D5

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with no regressors and a VMA(1) structure on U_t. d_1 and d_2 are the differencing orders for stock prices and dividends respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	0.05’	1.01’	3.46’	5.94’	7.84	8.91	8.98	7.73	4.57’
0.7	1.92’	0.74’	2.05’	4.09’	6.00	7.33	7.87	7.25	4.80’
0.8	4.79’	1.92’	2.14’	3.67’	5.43’	6.88	7.70	7.55	5.72’
0.9	6.70	2.77’	2.06’	3.11’	4.80’	6.39	7.49	7.72	6.45
1.0	7.42	3.27’	1.87’	2.30’	3.72’	5.38’	6.72	7.31	6.54
1.1	7.08	3.35’	1.78’	1.72’	2.72’	4.22’	5.67’	6.54	6.15
1.2	5.96’	2.89’	1.53’	1.28’	1.89’	3.12’	4.54’	5.61’	5.60’
1.3	4.63’	2.08’	1.04’	0.85’	1.24’	2.11’	3.33’	4.48’	4.85’
1.4	3.63’	1.32’	0.48’	0.39’	0.71’	1.32’	2.16’	3.11’	3.70’

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with an intercept and a VMA(1) structure on U_t. d_1 and d_2 are the differencing orders for stock prices and dividends respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	0.00’	1.74’	4.86’	7.61	9.55	10.65	10.93	10.28	8.26
0.7	1.83’	0.98’	2.95’	5.40’	7.48	8.94	9.70	9.64	8.33
0.8	4.75’	1.88’	2.68’	4.74’	6.84	8.54	9.65	10.03	9.27
0.9	6.59	2.48’	2.22’	3.87’	6.04	8.02	9.50	10.28	9.99
1.0	7.16	2.78’	1.71’	2.69’	4.65’	6.77	8.55	9.72	9.87
1.1	6.72	2.75’	1.40’	1.78’	3.29’	5.30’	7.22	8.65	9.16
1.2	5.60’	2.29’	1.07’	1.13’	2.13’	3.83’	5.76’	7.41	8.25
1.3	4.32’	1.59’	0.63’	0.62’	1.24’	2.45’	4.13’	5.88’	7.08
1.4	3.36’	1.01’	0.23’	0.22’	0.62’	1.37’	2.48’	3.92’	5.28’

Multivariate score tests ($\tilde{S}^2$ in (5.37)) with a time trend and a VMA(1) structure on U_t. d_1 and d_2 are the differencing orders for stock prices and dividends respectively.

$d_1 \backslash d_2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.6	0.07’	1.86’	4.73’	7.37	9.33	10.48	10.80	10.17	8.14
0.7	1.46’	1.22’	3.08’	5.46’	7.56	9.08	9.89	9.87	8.57
0.8	3.85’	1.95’	2.80’	4.80’	6.91	8.66	9.84	10.27	9.56
0.9	5.33’	2.40’	2.30’	3.90’	6.06	8.07	9.61	10.45	10.23
1.0	5.82’	2.64’	3.11’	2.73’	4.65’	6.79	8.62	9.84	10.06
1.1	5.42’	2.63’	1.52’	1.85’	3.30’	5.30’	7.26	8.74	9.31
1.2	4.38’	2.18’	1.21’	1.24’	2.18’	3.84’	5.78’	7.46	8.37
1.3	3.13’	1.47’	0.77’	0.76’	1.34’	1.57’	4.16’	5.92’	7.16
1.4	2.21’	0.80’	0.30’	0.34’	0.74’	1.46’	2.54’	3.45’	3.02’

’: Non-rejection values of the null hypothesis (5.4) at 95% significance level.

TABLE 6.D6

$\hat{r}$ in (2.9) for the estimated residuals.

p_t + 0.12 - 30.99 d_t

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	1.48'	0.21'	-0.79'	-1.58'	-2.20'	-2.68	-3.06	-3.37	-3.62	-3.83
AR(1)	-0.83'	-1.38'	-1.97	-2.49	-2.95	-3.35	-3.68	-3.95	-4.18	-4.37
AR(2)	1.06'	0.82'	0.44'	-0.03'	-0.54'	-1.04'	-1.52'	-1.97	-2.38	-2.74
SAR(1)	1.15'	0.05'	-0.87'	-1.61'	-2.22	-2.70	-3.10	-3.42	-3.70	-3.93
SAR(2)	1.15'	0.05'	-0.87'	-1.61'	-2.21	-2.70	-3.10	-3.42	-3.70	-3.93
b) Intercept.										
W.N.	1.46'	0.18'	-0.83'	-1.63'	-2.24	-2.73	-3.11	-3.41	-3.66	-3.86
AR(1)	-0.86'	-1.45'	-2.03	-2.57	-3.04	-3.44	-3.76	-4.03	-4.24	-4.42
AR(2)	1.02'	0.75'	0.32'	-0.18'	-0.71'	-1.22'	-1.70'	-2.13	-2.52	-2.87
SAR(1)	1.12'	0.01'	-0.92'	-1.66'	-2.27	-2.75	-3.15	-3.47	-3.74	-3.97
SAR(2)	1.12'	0.01'	-0.91'	-1.66'	-2.26	-2.75	-3.15	-3.47	-3.74	-3.97
c) Intercept and a time trend.										
W.N.	1.46'	0.18'	-0.83'	-1.63'	-2.24	-2.73	-3.11	-3.41	-3.66	-3.86
AR(1)	-0.87'	-1.45'	-2.04	-2.58	-3.05	-3.44	-3.76	-4.02	-4.24	-4.42
AR(2)	1.03'	0.73'	0.31'	-0.19	-0.71'	-1.22'	-1.69'	-2.12	-2.51	-2.85
SAR(1)	1.11'	0.00'	-0.92'	-1.67'	-2.27	-2.75	-3.15	-3.47	-3.74	-3.97
SAR(2)	1.11'	0.00'	-0.92'	-1.66'	-2.26	-2.75	-3.15	-3.47	-3.74	-3.97

d_t - 0.005 - 0.027 p_t

d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
a) No intercept and no time trend.										
W.N.	1.10'	-0.02'	-0.90'	-1.60'	-2.15	-2.60	-2.97	-3.27	-3.52	-3.74
AR(1)	-1.40'	-1.85'	-2.29	-2.70	-3.07	-3.38	-3.66	-3.89	-4.10	-4.27
AR(2)	0.31'	0.07'	-0.26'	-0.63'	-1.02'	-1.40'	-1.78'	-2.14	-2.47	-2.78
SAR(1)	1.00'	-0.03'	-0.87'	-1.56'	-2.11	-2.57	-2.95	-3.28	-3.55	-3.79
SAR(2)	0.99'	-0.03'	-0.88'	-1.56'	-2.11	-2.57	-2.95	-3.28	-3.55	-3.79
b) Intercept.										
W.N.	0.95'	-0.13'	-0.99'	-1.68'	-2.22	-2.67	-3.02	-3.32	-3.57	-3.78
AR(1)	-1.53'	-2.00	-2.45	-2.86	-3.22	-3.53	-3.79	-4.01	-4.20	-4.36
AR(2)	0.21'	-0.09'	-0.46'	-0.87'	-1.27'	-1.66'	-2.02	-2.35	-2.66	-2.94
SAR(1)	0.86'	-0.15'	-0.97'	-1.64'	-2.19	-2.64	-3.02	-3.34	-3.61	-3.85
SAR(2)	0.86'	-0.15'	-0.97'	-1.64'	-2.19	-2.64	-3.02	-3.34	-3.62	-3.85
c) Intercept and a time trend.										
W.N.	0.89'	-0.15'	-1.00'	-1.68'	-2.22	-2.66	-3.02	-3.32	-3.57	-3.77
AR(1)	-1.57'	-2.03	-2.47	-2.87	-3.22	-3.53	-3.79	-4.01	-4.19	-4.35
AR(2)	0.18'	-0.13'	-0.49'	-0.88'	-1.28'	-1.65'	-2.01	-2.34	-2.65	-2.92
SAR(1)	0.81'	-0.17'	-0.98'	-1.65'	-2.19	-2.64	-3.02	-3.34	-3.61	-3.85
SAR(2)	0.81'	-0.17'	-0.98'	-1.64'	-2.19	-2.64	-3.02	-3.34	-3.61	-3.85

*: p_t corresponds to stock prices and d_t to dividends. ': Non-rejection values of the null hypothesis (1.12) at 95% significance level when monotonicity in the test statistic with respect to d is observed.

TABLE 6.D7

$\hat{r}$ in (2.9) for the estimated residuals with d ≤ 0.50*

p_t + 0.12 - 30.99 d_t

a) No intercept and no time trend.

Residuals \ d	0.00	0.10	0.20	0.30	0.40	0.50
White Noise:	12.84	10.90	8.85	6.78	4.80	3.01
Seas. AR (1):	8.54	7.56	6.43	5.15	3.78	2.42
Seas. AR (2):	8.11	7.32	6.31	5.09	3.75	2.40

b) Intercept.

Residuals \ d	0.00	0.10	0.20	0.30	0.40	0.50
White Noise:	12.84	10.90	8.84	6.77	4.79	2.99
Seas. AR (1):	8.54	7.56	6.43	5.15	3.77	2.39
Seas. AR (2):	8.11	7.32	6.30	5.09	3.74	2.38

c) Intercept and a time trend.

Residuals \ d	0.00	0.10	0.20	0.30	0.40	0.50
White Noise:	13.02	11.05	8.97	6.85	4.83	3.01
Seas. AR (1):	8.60	7.60	6.45	5.16	3.77	2.39
Seas. AR (2):	8.24	7.42	6.36	5.12	3.75	2.38

d_t - 0.005 - 0.027 p_t

a) No intercept and no time trend.

Residuals \ d	0.00	0.10	0.20	0.30	0.40	0.50
White Noise:	12.74	10.51	8.26	6.10	4.15	2.47
Seas. AR (1):	8.09	7.31	6.26	4.98	3.58	2.22
Seas. AR (2):	8.10	7.32	6.26	4.97	3.57	2.21

b) Intercept.

Residuals \ d	0.00	0.10	0.20	0.30	0.40	0.50
White Noose:	12.74	10.51	8.23	6.02	4.01	2.31
Seas. AR (1):	8.09	7.31	6.25	4.94	3.49	2.09
Seas. AR (2):	8.10	7.32	6.25	4.93	3.48	2.07

c) Intercept and a time trend.

Residuals \ d	0.00	0.10	0.20	0.30	0.40	0.50
White Noise:	11.04	9.14	7.22	5.37	3.66	2.16
Seas. AR (1):	8.12	7.11	5.92	4.61	3.25	1.97
Seas. AR (2):	8.03	7.07	5.91	4.60	3.24	1.97

*: p_t corresponds to stock prices and d_t to dividends.

6.2 FINAL COMMENTS

In this chapter we have analyzed several pairs of variables claimed by many authors to be cointegrated. In particular, we have examined the relationship between consumption and income, wages and prices, G.N.P. and money, and stock prices and dividends in United States, using the same data sets as in Engle and Granger (1987) and Campbell and Shiller (1987). All these pairs of variables were also analyzed from a Bayesian viewpoint in DeJong (1992).

Using Robinson's (1994c) univariate tests, we started modelling the individual series, finding that all of them might be integrated of order one, though slight variations in this integration order might also be plausible. This unit root behaviour observed in the series was obtained independently of the way of modelling the disturbances and of the inclusion or not of deterministic regressors in the model; however, in some of these series, (in particular, nominal G.N.P., prices and money), integration orders greater than one were required when we included an intercept and an intercept and a time trend, and showing therefore, strong evidence against the trend-stationary representations advocated by some authors.

The multivariate versions of the tests corroborated the findings of the univariate tests when modelling together each pair of variables. Thus, two unit roots were found in most cases if we did not include regressors in the model and the disturbances were white noise or VMA(1) processes, though greater integration orders were more plausible in some series if we included an intercept and an intercept and a time trend. If the disturbances were VAR(1), the integration orders were slightly smaller and this might be explained by competition between the differencing orders and the VAR parameters in describing the nonstationary component of the series.

Finally, applying the tests of Robinson (1994c) on the estimated residuals from the regression of one of the variables against the other, results indicated that consumption and income, and stock prices and dividends were fractionally cointegrated, with the integration orders of the estimated residuals fluctuating around 0.7 in both cases, and therefore, with equilibrium errors displaying mean reversion. Nominal G.N.P. was not cointegrated with nominal money when using M2 or M3 as monetary aggregates, however, using M1 or L, results suggested the presence of a small component of fractional cointegration, with the integration orders of the residuals from the cointegrating regression fluctuating around 0.8 in most of cases. Finally, prices and wages were clearly non-cointegrated when we considered the thirty year period (i.e. 1950-79) and when we looked at the 1950's, though a small degree of mean reversion appeared during the 1960's and 1970's.

Results in this chapter alleviate the mixed conclusions obtained in previous works when classical cointegration was considered. In classical cointegration the individual series are assumed to be I(1) but the estimated residuals must be strictly I(0) processes. In this chapter we have shown that though the individual series might be I(1), the estimated residuals from the cointegrating regressions might be fractionally integrated, with the integration orders smaller than 1 (but greater than 0.5) in some cases and thus, being nonstationary but with a mean reverting behaviour.

APPENDIX 6.1

In this appendix we describe the Fortran code used to calculate the empirical size of Robinson's (1994c) tests for cointegration in Table 6.1.

```
C       "EMPIRICAL SIZE OF ROBINSON'S (1994c) TESTS FOR COINTEGRATION"
        parameter (n=**,nnd=10,nore=50000)
        implicit double precision (a-h,o-z)
        dimension ds(nnd),xcv(nnd,nore),xl(n-1),psi(n-1),save(nnd),add(n),
+         u1(n),u2(n),x1(n),x2(n),z(n),yw(n),zd(n),p(n-1),w(nnd,nore)
        dimension sm(nnd),s3(nnd),s4(nnd),skur(nnd),ske(nnd),var(nnd)
        xnore=nore
        pai=3.141592654
        ds(1)=0.50
        do 1 i=2,nnd
          ds(i)=ds(i-1)+0.10
1       continue
        xn=n
        xn1=n-1
        do 2 i=1,n-1
          xi=i
          xl(i)=(pai*2.*xi)/xn
          psi(i)=log(abs(2.*sin(xl(i)/2.)))
2       continue
        b=0
        do 3 i=1,n-1
          b=b+(1./xn1)*psi(i)**2.
3       continue
        xb=(2.*b)**(-0.5)
        do 9999 nd=1,nnd
          d=ds(nd)
          add(1)=d
            do 4 j=2,n
              xj=j
              add(j)=((xj-d-1.)/xj)*add(j-1)
4       continue
```

```
      do 9998 ijk=1,nore
        call g05cbf(ijk)
            do 5 i=1,n
              u1(i)=g05ddf(.0,1.0)
              u2(i)=g05ddf(.0,1.0)
5           continue
            x1(1)=u1(1)
            x2(1)=u2(1)
            do 6 i=2,n
              x1(i)=0.
              x2(i)=0.
                do 7 j=1,i-1
                  x1(i)=x1(i)+add(j)*x1(i-j)
                  x2(i)=x2(i)+add(j)*x2(i-j)
7               continue
              x1(i)=x1(i)+u1(i)
              x2(i)=x2(i)+u2(i)
6           continue
            xalfa1=0.
            xalfa2=0.
            do 8 i=1,n
              xalfa1=xalfa1+x1(i)*x2(i)
              xalfa2=xalfa2+x2(i)**2.
8           continue
            do 9 i=1,n
              z(i)=x1(i)-(xalfa1/xalfa2)*x2(i)
9           continue
            do 10 i=2,n
              yw(i)=0.
                do 11 j=1,i-1
                  yw(i)=yw(i)+((-1)*add(j))*z(i-j)
11          continue
              zd(1)=z(1)
              zd(i)=yw(i)+z(i)
10          continue
           do 12 j=1,n-1
              ct=0.
              st=0.
                do 13 i=1,n
                  xi=i
                  ct=ct+zd(i)*cos(xi*xl(j))
                  st=st+zd(i)*sin(xi*xl(j))
13              continue
              p(j)=(ct**2.+st**2.)/(pai*2.*xn)
12          continue
            a=0.
            do 14 j=1,n-1
              a=a+(pai*2./(-xn))*psi(j)*p(j)
14          continue
            vm0=0.
            do 15 i=1,n
              vm0=vm0+(1./xn)*zd(i)
15          continue
            c0=0.
            do 16 i=1,n
              c0=c0+(1./xn)*(zd(i)-vm0)**2.
16          continue
            stat=(xn**0.5)*xb*a/c0
            w(nd,ijk)=stat
9998  continue
```

```
9999    continue
        do 17 k=1,nnd
          lim=nore-1.
            do 18 i=1,lim
              start=i+1
            do 18 j=start,nore
              if (w(k,i)-w(k,j)) 24,24,25
18            save(k)=w(k,i)
                  w(k,i)=w(k,j)
                  w(k,j)=save(k)
18          continue
17      continue
        do 19 i=1,nnd
          sm(i)=0.
            do 20 j=1,nore
              sm(i)=sm(i)+(1./xnore)*w(i,j)
20          continue
19      continue
        do 21 i=1,nnd
          var(i)=0.
          ske(i)=0.
          skur(i)=0.
            do 22 j=1,nore
              var(i)=var(i)+(1./xnore)*(w(i,j)-sm(i))**2.
              ske(i)=ske(i)+(1./xnore)*(w(i,j)-sm(i))**3
              skur(i)=skur(i)+(1./xnore)*(w(i,j)-sm(i))**4
22          continue
          s3(i)=ske(i)/(var(i)**1.5)
          s4(i)=skur(i)/(var(i)**2.)
21      continue
        end
```

APPENDIX 6.2

To illustrate the potential difference in power between the tests of Robinson (1994c) and the GPH and ADF tests for cointegration, a Monte Carlo experiment, similar to that in Engle and Granger (1987) and Cheung and Lai (1993) is conducted. We consider a bivariate system where y_t and z_t are given by

$$y_t + z_t = u_{1t} \quad \text{(C1)}$$

$$y_t + 2\, z_t = u_{2t} \quad \text{(C2)}$$

where $(1 - L)u_{1t} = \varepsilon_{1t}$, and u_{2t} is generated, alternatively, as an autoregressive process

$$(1 - \rho L)u_{2t} = \varepsilon_{2t}, \quad \text{(C3)}$$

or as a fractional white noise process

$$(1 - L)^d\, u_{2t} = \varepsilon_{2t}, \quad \text{(C4)}$$

where the innovations ε_{1t} and ε_{2t} are generated as independent standard normal variates. Thus, if $\rho = 1$ in (C3) or $d = 1$ in (C4), the two series are I(1) and non-cointegrated; if u_{2t} is generated by (C3) and $|\rho| < 1$, y_t and z_t are cointegrated, and (C2) is their cointegrating relationship; alternatively, if u_{2t} is generated by (C4) and d < 1, y_t and z_t are fractionally cointegrated. As in Engle and Granger (1987) and Cheung and Lai (1993), we used samples of size T = 76, and sample series of y_t and z_t were generated setting the initial values of u_1 and u_2 equal to zero, creating 126 observations, of which the first 50 were discarded to reduce the effect of the initial conditions. We report the rejection frequencies at 5% and 10% significance level, based on 10,000 replications.

```
C       "POWER FUNCTION OF ROBINSON'S (1994c) TESTS FOR COINTEGRATION
        AGAINST FRACTIONAL AND AUTOREGRESSIVE ALTERNATIVES".

        parameter (n=126,nn=76,nm=50,nnd=10,nore=10,nwa=5*nn,nk=nn-1)
        implicit double precision (a-h,o-z)
        dimension ds(nnd),xl(nn-1),psi(nn-1),cosl(nn,nn),sinl(nn,nn),
+         add(n),u1(n),u2(n),u3(n),x1(nn),x2(nn),z(nn),p(nn-1),e1(n),e2(n),
+         cv(2),xx1(nm+1:n),xx2(nm+1:n),uu2(n,2),zz(nn),cr(nn-1)
+         stat(4,nnd),nrej(4,2,nnd,2),xnrej(4,2,nnd,2),mm(7),
+         wwa(nwa),par(3),al(3,3),g(nn-1),ex(nn-1,3),sx(3,3),ye(3),
+         xp(1,1),sx2(3,2),xp2(2,2),sx3(4,3),xp3(3,3),t2(2),t3(3)

        do 1 i=1,4
        do 1 j=1,2
        do 1 k=1,nnd
        do 1 l=1,2
          nrej(i,j,k,l)=0
1       continue
        cv(1)=-1.64
        cv(2)=-1.28
        pai=3.141592654
        ds(1)=0.05
        do 2 i=2,nnd
          ds(i)=ds(i-1)+0.10
2       continue
        xn=nn
        xn1=nn-1
        do 3 i=1,NN-1
          xi=i
          xl(i)=(pai*2.*xi)/xn
          psi(i)=log(abs(2.*sin(xl(i)/2.)))
3       continue
        b=0
        do 4 i=1,nn-1
          b=b+(1./xn1)*psi(i)**2.
4       continue
        xb=(2.*b)**(-0.5)
        do 5 i=1,nn
        do 5 j=1,NN
          xi=i
          cosl(i,j)=cos(xi*xl(j))
          sinl(i,j)=sin(xi*xl(j))
5       continue
        do 9000 nd=1,nnd
          d=ds(nd)
          add(1)=d
          do 6 j=2,n
            xj=j
```

```
       add(j)=((xj-d-1.)/xj)*add(j-1)
6     continue
      do 9001 ijk=1,nore
        call g05cbf(ijk)
        do 7 i=1,n
          e1(i)=g05ddf(.0,1.0)
          e2(i)=g05ddf(.0,1.0)
7       continue
        u1(1)=e1(1)
        uu2(1,1)=e2(1)
        do 8 i=2,n
          uu2(i,1)=0.
          do 9 j=1,i-1
              uu2(i,1)=uu2(i,1)+add(j)*uu2(i-j,1)
9         continue
          uu2(i,1)=uu2(i,1)+e2(i)
8       continue
        do 10 i=1,n
          uu2(i,2)=0.
          do 11 j=0,i-1
            uu2(i,2)=uu2(i,2)+(d**j)*e2(i-j)
11        continue
10      continue
        do 12 i=2,n
          u1(i)=0.
          do 13 k=0,i-1
            u1(i)=u1(i)+e1(i-k)
13        continue
12      continue
        do 9002 l=1,2
          do 14 i=1,n
            u2(i)=uu2(i,l)
14        continue
          do 15 i=nm+1,n
            xx1(i)=2.*u1(i)-u2(i)
            xx2(i)=u2(i)-u1(i)
15        continue
          do 16 i=1,nn
            x1(i)=xx1(i+50)
            x2(i)=xx2(i+50)
16        continue
          xalfa1=0.
          xalfa2=0.
          do 17 i=1,nn
            xalfa1=xalfa1+x1(i)*x2(i)
            xalfa2=xalfa2+x2(i)**2.
17        continue
          do 18 i=1,nn
            zz(i)=x1(i)-(xalfa1/xalfa2)*x2(i)
18        continue
          z(1)=zz(1)
          do 19 i=2,nn
            z(i)=zz(i)-zz(i-1)
19        continue
          ume=0.
          do 20 i=1,nn
            ume=ume+(1./xn)*z(i)
20        continue
          svar=0.
          do 21 i=1,nn
```

```
              svar=svar+(z(i)-ume)**2.
21         continue
           var=svar/xn
           do 22 j=1,nn-1
             cvk=0.
             do 23 i=1,nn-j
               cvk=cvk+(z(i)-ume)*(z(i+j)-ume)
23           continue
             cr(j)=cvk/svar
22         continue
           do 24 j=1,nn-1
             ct=0.
             st=0.
             do 25 i=1,nn
               ct=ct+z(i)*cosl(i,j)
               st=st+z(i)*sinl(i,j)
25           continue
             p(j)=(ct**2.+st**2.)/(pai*2.*xn)
24         continue
           ta=0.
           ta2=0.
           do 26 j=1,nn-1
             ta=ta+psi(j)*p(j)
             ta2=ta2+p(j)
26         continue
           a=(pai*2.*ta)/(-xn)
           va=(pai*2.*ta2)/xn
           stat(4,nd)=(xn**0.5)*xb*a/va
           do 9003 iq=1,3
             mm(1)=iq
             npar=iq
             fail=0
             call g13adf(mm,cr,nk,var,npar,wwa,nwa,par,rv,isf,ifail)
             do 27 ipar=1,iq
               al(ipar,iq)=par(ipar)
27           continue
             do 28 if=1,nn-1
               s1=0.
               s2=0.
               do 29 io=1,iq
                 s1=s1+al(io,iq)*sin(xl(if)*io)
                 s2=s2+al(io,iq)*cos(xl(if)*io)
29             continue
               g(if)=1./((1.-s2)**2.+s1**2.)
28           continue
             vr=0.
             do 30 if=1,nn-1
               vr=vr+(2.*pai/xn)*p(if)/g(if)
30           continue
             do 31 if=1,nn-1
             do 31 ip=1,iq
               exe=0.
               do 32 io=1,iq
                 exe=exe+al(io,iq)*cos((ip-io)*xl(if))
32             continue
               ex(if,ip)=2.*(cos(ip*xl(if))-exe)*g(if)
31           continue
             do 33 i1=1,iq
             do 33 i2=1,iq
               xr=0.
```

```
                xrr=0.
                sx(i1,i2)=0.
                ye(i1)=0.
                do 34 if=1,nn-1
                  xr=xr+((-1)*2.*pai/xn)*psi(if)*p(if)/g(if)
                  xrr=xrr+(2./xn)*psi(if)**2.
                  sx(i1,i2)=sx(i1,i2)+ex(if,i1)*ex(if,i2)
                  ye(i1)=ye(i1)+psi(if)*ex(if,i1)
34              continue
33            continue
              if(iq.eq.1) then
                xp(1,1)=1./sx(1,1)
                yer=(2./xn)*ye(1)*xp(1,1)*ye(1)
              else if(iq.eq.2) then
                do 35 i1=1,2
                do 35 i2=1,2
                  sx2(i1,i2)=sx(i1,i2)
35              continue
                call f01abf(sx2,3,2,xp2,2,t2,ifail)
                xp2(1,2)=xp2(2,1)
                yer=0.
                do 36 m1=1,2
                do 36 m2=1,2
                  yer=yer+(2./xn)*ye(m1)*xp2(m1,m2)*ye(m2)
36              continue
              else if(iq.eq.3) then
                do 37 i1=1,3
                do 37 i2=1,3
                  sx3(i1,i2)=sx(i1,i2)
37              continue
                call f01abf(sx3,4,3,xp3,3,t3,ifail)
                do 38 i1=1,2
                do 38 i2=1+i1,3
                  xp3(i1,i2)=xp3(i2,i1)
38              continue
                yer=0.
                do 39 m1=1,3
                do 39 m2=1,3
                  yer=yer+(2./xn)*ye(m1)*xp3(m1,m2)*ye(m2)
39              continue
              end if
              ya=xrr-yer
              stat(iq,nd)=((xn/ya)**0.5)*xr/vr
9003        continue
            do 40 iq=1,4
            do 40 i=1,2
              if(stat(iq,nd).lt.cv(i)) then
                nrej(iq,i,nd,l)=nrej(iq,i,nd,l)+1
              endif
40          continue
9002      continue
9001    continue
        xnore=nore
        do 41 iq=1,4
        do 41 i=1,2
        do 41 l=1,2
          xnrej(iq,i,nd,l)=nrej(iq,i,nd,l)/xnore
41      continue
9000  continue
      end
```

CHAPTER 7

CONCLUSIONS

Fractional integration has recently emerged in the literature as an alternative viable way of modelling economic time series. In this thesis we have concentrated on testing fractional and seasonal fractional integration and cointegration in macroeconomic time series. We used a testing procedure suggested by Robinson (1994c), which is very general in the sense that it allows us to test one or more integer or fractional roots of arbitrary order anywhere in the unit circle on the complex plane. These tests are described in Chapter 2; they are derived via the score principle, are efficient when directed against appropriate alternatives, and have standard null and local limit distributions. The empirical distribution of the tests in finite samples is also computed in Chapter 2, and some simulations, comparing the finite sample behaviour of Robinson's (1994c) tests, using both, the size-corrected and the asymptotic critical values, with some other existing unit roots tests, are also carried out at the end of the chapter.

In Chapter 3 we concentrate on cases where the singularity in the spectrum occurs at zero frequency. We use a version of Robinson's (1994c) tests for testing unit roots and other nonstationarities on an extended version of Nelson and Plosser's (1982) data. These are fourteen U.S. macroeconomic variables in historical annual data. We model each series for different cases of no regressors, an intercept, and an intercept and a time trend, and for different types of disturbances, which, also unusually, include the Bloomfield exponential spectral model. The conclusions vary substantially across the fourteen series and across different models for the disturbances. When they are white noise, the unit root hypothesis is rejected in five series, in each of which a somewhat greater degree of nonstationarity is indicated, though even when the unit root is not rejected there is also evidence of possible fractional integration. With AR disturbances there are fewer rejections and evidence of smaller degree of nonstationarity, due perhaps to competition between the differencing parameter and the autoregression in describing the nonstationarity. Using the Bloomfield model, we find strong evidence in favour of single values for the differencing parameter, most of which are 0.75 and 1. Overall, the consumer price index and money stock seem the most nonstationary, followed by the G.N.P.

deflator and wages, whereas industrial production and unemployment rate seem the closest to stationary.

In Chapter 4 we concentrate on seasonality and thus, we allow the singularities to occur not only at zero frequency but also at the seasonal frequencies. Robinson's (1994c) tests are now applied to quarterly U.K. and Japanese consumption and income series, using the same data sets as in HEGY (1990) and HEGL (1993) respectively. We present a variety of model specifications for both series. However, given the number of possibilities covered by Robinson's (1994c) tests, it is difficult to draw clear conclusions about which might be the best way of modelling them. In fact, the null hypothesized model includes different deterministic paths; different lagged functions, allowing roots at some or all seasonal frequencies (as well as at zero frequency), each of them with a possible different integration order; and different ways of modelling the I(0) disturbances. Looking at the bulk of the results, some common features are observed for all series in both countries. Thus, modelling the series as quarterly I(d) processes, (i.e., with $\rho(L) = (1 - L^4)^d$) seems appropriate when the disturbances are white noise or non-seasonal AR. Allowing different integration orders at real and at complex roots, the results emphasize the importance of the real roots over the complex ones, given the greater integration order observed in the former roots, and this is even clearer when we allow different integration orders at each frequency. Excluding one real root results in rejecting the null in practically all situations. When modelling with $\rho(L) = (1 - L^2)^d$, results are now better for seasonal AR than for the other cases, and separating here the roots at zero and at frequency π, results emphasize the importance of the long run frequency; however, modelling the series with a simple I(d) process with $\rho(L) = (1 - L)^d$ seems inappropriate in most of the cases. Looking at individual series, integration orders range between 0.50 and 1.25 in both countries and for both series, indicating clearly the nonstationary nature of these series; however its difference seems less integrated, suggesting that a certain degree of fractional cointegration exists for a given cointegrating vector (1,-1), using a simplistic version of the permanent income hypothesis. These results are consistent with those obtained in HEGY (1990) and HEGL (1993) for the unit root case, though we show that seasonal fractional integration, even allowing different integration orders at different frequencies, might be an alternative plausible way of modelling these series.

In Chapter 5 we extend the tests of Robinson (1994c) to the multivariate case, testing the presence of unit roots and other nonstationary hypotheses on the residuals in a multiple time series system. The multivariate case provides a more detailed insight into properties and stochastic behaviour than the univariate work. We describe the functional forms of the test statistics based on the three general principles when deriving nested parametric hypotheses, namely, the score, Wald and likelihood principles. Some particular cases of the tests, leading to neat forms of the test statistics are also presented, and finally, some simulations based on Monte Carlo experiments are carried out in order to study its finite sample behaviour. We show that results based on the score test seem to be adequate to test the null of a random walk in a bivariate context.

The multivariate tests of Chapter 5 are applied in Chapter 6 to some pairs of variables that might be cointegrated. In particular, we use the same data sets as in Engle and Granger (1987) and Campbell and Shiller (1987), studying the relationships between consumption and income, prices and wages, nominal G.N.P. and money, and stock prices and dividends. First, using Robinson's (1994c) univariate tests, we found that all individual series might be I(1) when modelling with no regressors, though in some of them, (in particular, prices, nominal G.N.P. and money) higher integration orders should be required when including an intercept and/or a time trend. The multivariate tests support this view, finding two unit roots when modelling without regressors, but rejecting this hypothesis in favour of more nonstationarities in some of them when including deterministic paths.

Finally we also presented a testing procedure for testing the hypothesis of fractional cointegration of given orders d, d-b, in the bivariate case. This procedure follows a similar methodology to the one proposed in Engle and Granger (1987). In the first step we test that both individual series are integrated with the same integration order d. This can be done using either Robinson's (1994c) univariate tests or the multivariate version described in Chapter 5. Once we have checked that, we can again use Robinson's (1994c) univariate tests, testing if the estimated residuals from the cointegrating regressions are fractionally integrated of order b, with $b < d$, and the test statistic will still remain with the same standard limit distribution. The empirical sizes of the tests on finite sample is obtained and the power properties of these tests relative to ADF and GPH tests for cointegration are also evaluated and compared. Robinson's (1994c) tests behave better than the ADF and the GPH tests for cointegration when testing against both fractional and AR alternatives. This is not surprising if we take into account that the ADF test assumes a strict I(0) and I(1) distinction and the GPH test requires estimation of the fractional differencing parameter, whereas Robinson's (1994c) tests do allow fractional integration and do not require estimation of d.

Performing the tests on the estimated residuals from the cointegrating regressions for each pair of variables, results suggest that consumption and income, and stock prices and dividends are fractionally cointegrated, with the equilibrium errors from the cointegrating regressions fluctuating around 0.7 in most cases, and thus being nonstationary but mean-reverting. Nominal G.N.P. seems non-cointegrated with money when using M2 and M3 as measures of money, though a small component of mean reversion appears on the estimated residuals when using M1 or L. Finally, prices and wages are clearly non-cointegrated when looking at the thirty year period, though a small degree of mean reversion appears in the 60's and 70's.

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