Economic Analysis of the Post-1990 Stagnation in Japan

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Abstract

This dissertation analyzes the mechanism of the stagnation in Japan after 1990, focusing on the relationship between the credit market and the productivity slowdown. The key mechanism is that the credit market has a function to reallocate the production resources from the less-productive to the more-productive producers, and that if this function is hampered, then the average productivity level of the economy falls and the productivity slowdown occurs. This dissertation consists of three chapters. Chapter 1 estimates the productivity growth rate in Japan, and confirms the productivity slowdown in the 1990's. Chapter 2 provides a heterogeneous agents model with different productivity levels of the agents, and analyzes how the restriction on collateral liquidation affects the productivity slowdown. This analysis links the feature of the Japanese credit market with the productivity slowdown after 1990. Chapter 3 analyzes the capital- and the investment-output ratios under a credit crunch, and shows that the credit market shock is consistent with the observed feature of these ratios in Japan after 1990.

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Acknowledgement

I have been struggling to complete this dissertation in the past 6 years. I am indebted to Nobuhiro Kiyotaki and Charles Goodhart for their continuous and patient support as the co-advisers of my PhD research. My conversation with Professor Kiyotaki started from literally asking what is economic analysis, and he has guided me to learn what it is. Professor Goodhart has been giving me tough questions to point out the gap between my analysis and practice, and helping me to ask and answer the questions relevant to the empirical observation. While it is left for the future research to answer his fundamental question what is the role of banking in the Japanese stagnation, the experience of working with him will help me to clarify this important question.

I thank Kosuke Aoki for his kind support as the adviser in the final year of my PhD study, and also Francesco Caselli, Satoshi Kawanishi, Ryuzo Miyao, Kalin Nikolov, Kozo Ueda, Noriyuki Yanagawa, the seminar participants at Bank of Japan, Kobe U, LSE, Shiga U and Sophia U, and especially Alexander Michaelides for their comments. I am grateful for Wouter Den Haan's lecture on the numerical methods in economics, without which I would not be able to complete this dissertation.

Last but not least, I appreciate all the support from my wife Reiko and my parents Shigeo and Takako during the stagnation in my research for the past years.

Introduction

The recent stagnation in Japan after 1990 has set a big challenge to macroeconomics. The question is why the economic growth rate had been low for more than a decade, despite the strong growth until 1990 and the large fiscal and monetary expansions after 1990. To explain this phenomenon, Fumio Hayashi and Edward Prescott have published the important work in the Review of Economic Dynamics in 2002. They estimate the decline of the productivity growth rate in Japan, and show that this empirical finding explains the dynamics of the Japanese economy after 1990 in the standard exogenous growth model. After their analysis, researchers started investigating the mechanism of the productivity slowdown, and empirically found that it was accompanied by biased credit and resource allocations across the firms. This PhD dissertation adds to this effort by clarifying the theoretical mechanism of the interaction between the biased credit and resource allocations and the productivity slowdown.

The theoretical analyses in this dissertation extend the path-breaking work of Nobuhiro Kiyotaki and John Moore, which was published in the Journal of Political Economy in 1997. The analyses in this dissertation clarifies that the insight of their work synthesizes the classical debate on the source of the economic fluctuation between Edward Prescott and Lawrence Summers in the Fall 1986 issue of the Federal Reserve Bank of Minneapolis Quarterly Review. In this debate, Prescott advocates that productivity shocks are the driving force of the economic fluctuation, while Summers argues that breakdown of exchange in the economy is more likely to explain the economic fluctuation.

The insight of Kiyotaki and Moore is that the productivity levels differ across the producers in the economy, and that the credit market reallocates the production resources from the less-productive to the more-productive. As shown in this dissertation, a corollary of their insight is that if the intertemporal exchange in the credit market is broken down, then it reduces the resource allocation to the more-productive, and lowers the average productivity level of the economy. Hence, the insight of Kiyotaki and Moore implies that Prescott and Summers look at the same mechanism of the economic fluctuation from different perspectives. This dissertation shows that the synthesis of the Prescott-Summers debate is important to understand the mechanism of the productivity slowdown in Japan after 1990.

This dissertation consists of three chapters. Chapter 1 estimates the productivity growth rate in Japan, and confirms the productivity slowdown in the 1990's. Chapter 2 provides a heterogeneous agents model with different productivity levels of the agents, and analyzes how the restriction on collateral liquidation affects the productivity slowdown. This analysis links the feature of the Japanese credit market with the productivity slowdown after 1990. Chapter 3 analyzes the capital- and the investment-output ratios under a credit crunch, and shows that the credit market shock is consistent with the observed feature of these ratios in Japan after 1990.

Chapter 1

Productivity Growth in Japan for 1974-1998

Abstract

This chapter estimates the productivity growth rate in Japan for 1974-1998, controlling for unobserved capacity utilization and non-constant returns to scale in production. We adopt the total factor productivity (TFP) as the productivity measure. We compare the results between the two sets of the proxies for unobserved capacity utilization: the working-hours of labor and the energy inputs. We observe a productivity slowdown in the 1990's by both of the proxies.

1.1 Introduction

The Japanese economy has experienced a long stagnation after 1990. The average growth rate of real GDP per working-age population dropped from 3.26% in the 1981-1990 period to 0.95% in the 1990-2003 period. There has been proposed a number of hypotheses to explain this phenomenon. One of the main hypotheses is a productivity slowdown, as observed by Hayashi and Prescott (2002) in the 1990s. They use the Solow residual (the growth accounting), and estimate that the aggregate productivity growth rate dropped from 3.7% in the 1983-1990 period to 0.3% in the 1990-2000 period. They argue that this is the main cause of the stagnation in Japan after 1990.

A potential problem of their estimation is that the Solow residual contains some other components than the productivity growth. The Solow residual is the residual of the output growth net of the growth of the observed inputs, such as capital and labor, and thus measures the growth of the unobserved components in production. The unobserved components include productivity of the inputs and unobserved capacity utilization. The productivity of the inputs consists of the productivity level independent of the amounts of the inputs, so-called total factor productivity (TFP),[1] and the effect of non-constant returns to scale, which varies with the amounts of the inputs. Over all, the Solow residual contains the growth and change of the three effects; the unobserved capacity utilization, TFP, and the effect of non-constant returns to scale.

Hayashi and Prescott estimate the growth rate of TFP by the Solow residual. Thus, they implicitly assume that the effects of unobserved capacity utilization and non-constant returns to scale are negligible without confirming this in the data. But after Hayashi and Prescott's work, several works estimate the TFP growth rates by controlling for the unobserved capacity utilization. Most of the works confirm the TFP slowdown in Japan in the 1990's. Fukao and Kwon (2004) contain a survey.

However, Kawamoto (2004) obtains a different result that there was no TFP slowdown in the 1990s, and concludes that the unobserved capacity utilization and non-constant returns to scale in production caused the observed decline of the Solow residual in the 1990s. His result is important, because his estimation controls for both of the unobserved capacity utilization and non-constant returns to scale in production in a unified framework induced from the firm's cost minimization, while the preceding works use the measures of unobserved capacity utilization estimated independently of their estimation methods. Kawamoto follows the literature to control for the unobserved capacity utilization and non-constant returns to scale in production by regression, and "purify" the Solow residual, such as Hall (1989), Burnside, Eichenbaum and Rebelo (1993 and 1995), Basu (1996) and Basu and Kimball (1997). He especially applies the method of Basu and Kimball to the Japanese data.

The Basu-Kimball method controls for the unobserved capacity utilization

[1] Basu, Fernald, and Kimball (2004) calls the Solow residual as "TFP" and what we call as TFP here as "technology." But in this chapter, we choose to follow the terminology in the macroeconomic theory literature.

by the working-hours per worker as the proxy. The assumption behind this proxy is that the firms optimize all the margins of the capacity utilization, and endogenously correlate the working-hours per worker (the observed margin) with the unobserved margins. But there is a concern to directly apply this method to the Japanese data in the 1990s. As will be described in the main section, the statutory workweek of labor kept declining over the 1990's since the 1988 revision of the Labor Standards Law.[2] Such a regulatory restriction would change the environment of the firm's cost minimization, and then the correlation between the working-hours per worker and the unobserved capacity utilization. As will be shown later, the effect of the regulatory change on the working-hours per worker was especially significant for the 1988-1993 period. The large exogenous decline of the working-hours per worker in the early 1990's could lead to over-estimating the decline of the unobserved capacity utilization, and underestimating the decline of TFP growth rates in the 1990's.

In this chapter, we separate the effect of the reduction of the statutory workweek of labor from the working-hours of labor, and then apply the Basu-Kimball method to estimate the TFP growth rates in Japan. For the separation, we exploit the observation that the reduction of the working-days took the main part in the reduction of the statutory workweek of labor. Under the firm's cost minimization problem robust to the exogenous reduction of the working-days by laws, we use the change of the average working-hours of a worker per day as the proxy of the unobserved capacity utilization. We observe a TFP slowdown in the 1990's by this method.

Also, to be immune from the effect of the reduction of the statutory workweek of labor, we use the energy-inputs as the proxy for the unobserved capacity utilization, and provide another estimate of the TFP growth rates. This method is similar to Burnside, Eichenbaum and Rebelo (1995) and Basu (1996). We show that a TFP slowdown is observed in the 1990's by this method as well.

The rest of the chapter is organized as follows: Section 1.2 describes the regulatory change of the working-hours per worker. Section 1.3 describes the model. Section 1.4 shows the estimation results by the Basu-Kimball method. Section

[2] In fact, Kawamoto is concerned with the secular down-ward trend in the working-hours, and uses the over-time to control for the unobserved capacity utilization. But we will argue below that the over-time is also affected by the reduction of the statutory workweek of labor.

1.5 shows the estimation results by the energy inputs. Section 1.6 concludes.

1.2 Statutory workweek of labor in the 1990's

In this section, we describe the reduction of the statutory workweek of labor in the 1990's. By the late 1980's, the Japanese economy had achieved the successful economic growth and the large current account surplus. In 1988, the government set a long-term policy to reduce the working-hours of Japanese workers to increase their leisure time and consumption.[3] The objective of this policy was to correct the trade imbalance and to enhance the workers' welfare. In the same year, the Labor Standards Law was revised to reduce the statutory workweek of labor. This revision gradually reduced the statutory workweek from 48 hours to 46 hours by March 1991, and to 44 hours by March 1994. From April 1994, the workweek was reduced to 40 hours. But the medium- and small-sized establishments with not more than 300 employees were allowed to continue having 44 hours until March 1997.[4] Even after 1997, the labor laws set a "guidance" period for the government to induce the medium- and small- establishments to observe the 40-hour workweek by 1999. A policy description issued by the Ministry of Labor in February 1998 noted that 20% of these establishments were yet to observe the statutory workweek.[5] Thus, the implementation of the 40-hour statutory workweek was only gradual over the 1990's.

The reduction of the statutory workweek was mainly implemented through the reduction of the working days. Figure 1.2 shows the average working-days per year in the economy, and Figure 1.3 shows the average working-hours of a worker per day. Figure 1.2 shows that there was a significant decline between 1988 and 1993 and a slight downward trend between 1993 and 1999. Figure 1.3 shows that the average working-hours of a worker per day was stable despite the

[3]The cabinet set this policy in a policy statement named as "Japan cooperating with the rest of the world: the 5-year economic management plan."

[4]There were a few exceptions. The allowance was not applied to the financial and the communication industries, and the civil servants. All establishments in the mining, the transportation and courier, the cleaning, and the butchery industries received this special allowance. In the theater industry, only the establishment with not more than 100 employees received the special allowance.

[5]Labor Standards Bureau, "On management of the labor standards administration", February 1998.

reduction of the statutory workweek. Later, we will exploit this observation to exclude the effect of the regulatory change from the estimation.

The decline of the average working-hours of a worker per day after 1996 in Figure 1.3 was not an economy-wide phenomenon, but driven by the transportation and communication and the wholesale and retail industries. In these industries, the shares of the part-time workers significantly increased, which contributed to the decline of the average working-hours of a worker per day. Figure 1.4 shows that the average over-time of a worker per day was also fluctuating around the constant level, so that the reduction of the working-days drove the permanent reduction of the working-hours in all kinds.

1.3 Model

In this chapter, we consider that the production function takes the following generalized Cobb-Douglas form:

$$Y_{i,t} = A_{i,t} \left\{ \mu_i \left[(F_{i,t} H_{i,t} N_{i,t})^{\alpha_{i,N}} (U_{i,t} K_{i,t-1})^{\alpha_{i,K}} \right]^{\rho_i} + (1 - \mu_i)(M_{i,t})^{\rho_i} \right\}^{\gamma_i / \rho_i}, \quad \rho_i < 1. \tag{1.1}$$

The subscript t indicates year t. The subscript i is the index for the representative firm in industry i. μ_i, $\alpha_{i,j}$, ρ_i and γ_i are constants for $j = N, K$. $\alpha_{i,N}$ and $\alpha_{i,K}$ are the factor share of labor and capital, respectively, in the value-added $(F_{i,t} H_{i,t} N_{i,t})^{\alpha_{i,N}} (U_{i,t} K_{i,t-1})^{\alpha_{i,K}}$. $1/(1 - \rho_i)$ is the elasticity of substitution between the value-added and the intermediate inputs. γ_i is the returns to scale in production. $Y_{i,t}$ is the gross output. $A_{i,t}$ is the TFP level. $F_{i,t}$ is the unobserved labor effort-intensity per worker-hour. $H_{i,t}$ is the average working-hours of a worker per day. We discuss the reason for this definition of $H_{i,t}$ in the next section. $N_{i,t}$ is the number of worker-days, so that $H_{i,t} N_{i,t}$ is worker-hours. $U_{i,t}$ is the running-time of capital. $K_{i,t-1}$ is the amount of capital. and $M_{i,t}$ is the amount of intermediate inputs.

We consider existence of the adjustment costs of capital and labor, so that these two inputs are quasi-fixed. Then, because the firms cannot completely adjust the amounts of capital and labor when they are hit by productivity and demand (price) shocks, they absorb the shocks by adjusting the three margins of the capacity utilization, $F_{i,t}$, $H_{i,t}$, and $U_{i,t}$. We will describe these assumptions more precisely later in the specification of the firm's cost minimization problem.

We estimate the growth rate of $A_{i,t}$ by a stationary linear regression. We take a log-linear approximation of the function (1.1) around its steady state,[6] and its difference such that

$$\begin{aligned}\Delta\ln Y_{i,t} =& \Delta\ln A_{i,t} + \eta_{i,N}(\Delta\ln F_{i,t} + \Delta\ln H_{i,t} + \Delta\ln N_{i,t}) \\ &+ \eta_{i,K}(\Delta\ln U_{i,t} + \Delta\ln K_{i,t-1}) + \eta_{i,M}\Delta\ln M_{i,t}, \end{aligned} \tag{1.2}$$

where

$$\eta_{i,N} \equiv \left(\frac{\gamma\mu\alpha_N\left[(FHN)^{\alpha_N}(UK)^{\alpha_K}\right]^{\rho}}{\mu\left[(FHN)^{\alpha_N}(UK)^{\alpha_K}\right]^{\rho} + (1-\mu)M^{\rho}}\right)_{i,SS} \tag{1.3}$$

$$\eta_{i,K} \equiv \left(\frac{\gamma\mu\alpha_K\left[(FHN)^{\alpha_N}(UK)^{\alpha_K}\right]^{\rho}}{\mu\left[(FHN)^{\alpha_N}(UK)^{\alpha_K}\right]^{\rho} + (1-\mu)M^{\rho}}\right)_{i,SS} \tag{1.4}$$

$$\eta_{i,M} \equiv \left(\frac{\gamma(1-\mu)M^{\rho}}{\mu\left[(FHN)^{\alpha_N}(UK)^{\alpha_K}\right]^{\rho} + (1-\mu)M^{\rho}}\right)_{i,SS}. \tag{1.5}$$

$(\cdot)_{i,SS}$ indicates the steady-state value of the variable inside the bracket for the firm i. We omit the firm index i and the time index t for the variables inside the bracket. Thus, $\eta_{i,j}$ is the elasticity of production to the input at the steady state for $j = N, K, M$.

In the literature, it is popular to calibrate the elasticities of production to the inputs by the cost-shares of the inputs. But this calibration requires that the marginal returns of the inputs equal their marginal costs at the steady state.[7] This assumption does not hold, if the firms are credit-constrained, because the firms cannot invest into capital as much as they like, and the marginal return to capital becomes larger than its marginal cost. Also, it is necessary to assume that the adjustment costs are negligible around the steady state. We avoid to make these assumptions, and estimate the elasticities of production to the inputs by regression.

By decomposing the TFP growth $\Delta\ln A_{i,t}$ into the trend growth and the

[6] i.e., the trend growth path.

[7] See Hall (1989) for more detail of the calibration.

deviation from it, we obtain the following regression form:

$$
\begin{aligned}
\Delta \ln Y_{i,t} = & \overline{\Delta \ln A_i} + \eta_{i,N}(\Delta \ln F_{i,t} + \Delta \ln H_{i,t} + \Delta \ln N_{i,t}) \\
& + \eta_{i,K}(\Delta \ln U_{i,t} + \Delta \ln K_{i,t-1}) + \eta_{i,M} \Delta \ln M_{i,t} \\
& + error, \qquad (1.6)
\end{aligned}
$$

where $\overline{\Delta \ln A_i}$ is log of the trend TFP growth rate, and the error term is the deviation in year t, $\Delta \ln A_{i,t} - \overline{\Delta \ln A_i}$. Thus, we estimate the TFP growth rates by the sum of the constant term and the error term. The problem in estimating (1.6) is that the regressors include the two margins of the unobserved capacity utilization, $\Delta \ln F_{i,t}$ and $\Delta \ln U_{i,t}$. We adopt two methods to approximate them in the following.

1.4 Proxy of the unobserved capacity utilization: Average working-hours of a worker per day

In the first method, we approximate the unobserved capacity utilization by the average working-hours of a worker per day. We consider the following cost minimization problem of the firm:

$$
\begin{aligned}
\min_{\{F_t, H_t, N_t, U_t, K_t, M_t\}_{t=0}^{\infty}} \quad & \mathrm{E}_t \left\{ \sum_{t=0}^{\infty} \frac{(1+r_0)}{\prod_{j=0}^{t}(1+r_j)} \left[w_t G(F_{i,t}, H_{i,t}) V(U_{i,t}) N_{i,t} + P_{M,i,t} M_{i,t} \right.\right. \\
& \left.\left. + \, \Xi \left(N_{i,t}, N_{i,t-1}, K_{i,t}, K_{i,t-1} \right) \right] \right\} \qquad (1.7)
\end{aligned}
$$

$$
\text{s.t. } Y_{i,t} \geq \overline{Y}
$$

$w_s G(F_{i,t}, H_{i,t}) V(U_{i,t})$ is the total compensation for the workers. The level coefficient w_t is taken as given by the firm. $G(\cdot, \cdot)$ is the premium for the labor effort-intensity and the working-hours of each worker per day. $V(\cdot)$ is the shift premium to compensate the shift-work at undesirable time, such as the night. It is assumed that the shift premium is a function of the running-time of capital. The intuition for this assumption is that the firm needs to have the workers work longer at undesirable time to run the capital stock longer. G and V are multiplicative with each other in the total compensation by assumption.

$\Xi(\cdot,\cdot,\cdot,\cdot)$ is the adjustment-cost function for labor and capital. The adjustment cost makes these inputs quasi-fixed, and causes the fluctuation of the capacity utilization. We do not use any further properties of the adjustment-cost functions in this chapter. r_s is the nominal interest rate. $P_{M,i,t}$ is the price of the intermediate inputs for the firm i at each year s. These prices are all nominal. Note that the nominal cost minimization problem is equivalent to the real problem. If we divide the minimization problem by the current output price, we can transform it into the cost minimization problem with real variables after a few steps of calculation. The constraint is the quantity constraint for the cost minimization, given an arbitrary amount of production $\overline{Y}$.

Note that the cost-minimization problem is robust to imperfect competition in the output market and existence of the demand shocks, as we can take the value of $\overline{Y}$ as given. In appendix, we can show that $F_{i,t}$ and $U_{i,t}$ are determined by the following functions:

$$F_{i,t} = F(H_{i,t}) \tag{1.8}$$

$$U_{i,t} = U\left(H_{i,t}, \frac{\alpha_{i,K}}{\alpha_{i,N}}\right). \tag{1.9}$$

Hence, both $F_{i,t}$ and $U_{i,t}$ depend on $H_{i,t}$. The intuition is that in the cost minimization, the firm optimizes all the margins of the capacity utilization. This behavior creates the endogenous correlation between the average working-hours of a worker per day, i.e. the observed margin, with the unobserved margins of the capacity utilization.

We take log-linear approximations of these functions around the steady state and their differences, and substitute them into (1.6). Then, we obtain

$$\begin{aligned}\Delta \ln Y_{i,t} =& \overline{\Delta \ln A_i} + \eta_{i,N}(\Delta \ln H_{i,t} + \Delta \ln N_{i,t}) + \eta_{i,K}\Delta \ln K_{i,t-1} \\ &+ \eta_{i,M}\Delta \ln M_{i,t} + (\eta_{i,N} f_{i,H} + \eta_{i,K} u_{i,H})\,\Delta \ln H_{i,t} + error,\end{aligned} \tag{1.10}$$

where

$$f_{i,H} = \left(\frac{F' \cdot H}{F}\right)_{i,SS} \tag{1.11}$$

$$u_{i,H} = \left(\frac{U_1' \cdot H}{U}\right)_{i,SS}. \tag{1.12}$$

U_j is the derivative of U by its jth argument. Note that $\alpha_{i,K}/\alpha_{i,N}$ is constant and disappears when we take its difference.

Basu and Kimball (1997) propose this method of approximating the unobserved capacity utilization by the working-hours of labor. Their original estimation uses the working-hours per worker as the proxy for the unobserved capacity utilization, rather than the average working-hours of a worker per day. However, as described in the last section, there was a continuous structural change in the working-hours per worker in Japan over the 1990's. If we define $H_{i,t}$ by the total working-hours per worker, then we need to introduce a time-varying constraint on the choice of $H_{i,t}$ in the cost minimization problem (1.7). This violates the stationary relationships between $H_{i,t}$, and $F_{i,t}$ and $U_{i,t}$ shown in (1.8) and (1.9). To avoid this problem, we define $H_{i,t}$ by the average working-hours of a worker per day, exploiting the observation that the reduction of the working-hours per worker was mainly implemented through the reduction of the working-days, and that the average working-hours of a worker per day did not show mean change in the 1990's. A straightforward interpretation of our cost minimization problem is that the firms take the working-days per worker as given under the Labor Standards Laws, and compensate the workers if they work longer in each day. Also note that our cost minimization problem is robust to both endogenous and exogenous changes of the working-days per worker, given our specification of the total compensation function to the workers is correct.

1.4.1 Data and the estimation method

We estimate (1.10) by the industry-level data in Japan between 1974 and 1998. Thus, we estimate the behavior of the representative firm for each industry. The data are taken from the Japan Industry Productivity (JIP) database, constructed by Fukao, Miyagawa, Kawai, and Inui, et al (2003). This database contains annual price and quantity data of the gross output, the capital stock, the worker-hours, and the intermediate inputs for 1973-1998. Annual working-hour data over the sample period are taken from *Maitsuki Kinro Tokei*, an establishment survey conducted by Ministry of Health, Labor, and Welfare.

We use the beginning-of-the-year value of the capital stock as the operating capital stock for each year, as the model assumes one period lag between investment into capital and production. The database provides two series of the worker-hours; the quality-unadjusted worker-hours and the quality-adjusted worker-hours. The quality-unadjusted worker-hours are the plain sum of worker-hours across the different types of workers. The quality-adjusted worker-hours are Divisia indexes, using the wage share of each type of the workers in each year as the time-varying weight. We will report the estimation results under both types of the worker-hours.

For estimation, we use three-stage least squares (3SLS) to exploit the covariance of the productivity shocks across the industries. We estimate the variance-covariance matrix of the error terms by the Newey-West estimator with the bandwidth of 2 to take into account the serial correlation of the productivity shocks. The instrumental variables are 1) log difference of the real oil prices in yen at the end of the year, deflated by CPI, 2) log difference of Nominal US federal fund rates, 3) log difference of national corporate tax rates,[8] and 4) a dummy variable on monetary contraction, which takes one in 1973, 1980, and 1990, and zero otherwise. The last variable is provided by Kawamoto (2004). All the instrumental variables are lagged from 2 years to 5 years.

The sample period of the estimation is only 25 years. Given that the instruments are the time-series variables, the number of the industries has to be less than the sample years, since the estimated variance-covariance matrix of the error terms would be singular otherwise. We need to take the inverse of the estimated matrix to obtain the optimal weighting matrix for 3SLS. To deal with this problem, we classify the industries into the heavy-manufacturing sector, the light-manufacturing sector, and the non-manufacturing sector, and apply 3SLS for each sector. We assume that the regression-coefficients are identical across the industries within each sector to mitigate the small sample problem. Table 1.1 shows the industry classification. See appendix for further discussion on the data detail.

[8]The rates in 2) and 3) are gross.

1.4.2 Estimation results

We allow the estimated trend growth rate of TFP $\overline{\Delta \ln A_i}$ to change its value in the 1990's, and introduce the time-dummies in the constant term. Thus, we estimate the following regression:

$$\begin{aligned}\Delta \ln Y_{i,t} =& \overline{\Delta \ln A_{i,-90}} + \overline{\Delta \ln A_{i,91-}} + \eta_{j,N}(\Delta \ln H_{i,t} + \Delta \ln N_{i,t}) + \eta_{j,K}\Delta \ln K_{i,t-1} \\ &+ \eta_{j,M}\Delta \ln M_{i,t} + (\eta_{j,N} f_{j,H} + \eta_{j,K} u_{j,H})\,\Delta \ln H_{i,t} + error, \qquad (1.13)\end{aligned}$$

The subscript j is the index for each sector, as we assume the identical coefficients within the sector. $\overline{\Delta \ln A_{i,-90}}$ is the trend growth rate of TFP until 1990, and $\overline{\Delta \ln A_{i,91-}}$ is the one after that. We allow the values of these dummy variables to vary across the industries to estimate the industry-specific trend growth rates of TFP. Despite the possible trend change in the 1990's, we still estimate the constant elasticities of the output to the inputs $\eta_{j,h}$ for $h = N, K, M$ over the sample period, assuming that the change of the elasticities was not significant even under the possible trend change in the 1990's. We make this assumption to mitigate the problem of the small sample size in the 1990's (8 years.)

Tables 1.2 and 1.3 show the regression result of (1.13). In Table 1.2, we use quality-unadjusted worker-hours for $H_{i,t}N_{i,t}$. The estimates for the light-manufacturing sector and the non-manufacturing sector show that the capital growth does not significantly affect the output growth in these sectors. Burnside, Eichenbaum and Rebelo (1995) note a similar observation in the US data. This result implies that the amount of capital is not a good measure for the capital service, even after controlling for the unobserved capacity utilization by the average working-hours of a worker per day. J_1 is the J-statistic for the over-identifying restriction test. The J-statistics imply that the over-identifying restriction test is rejected at 1% level for all the industries. We find that it is difficult to satisfy the test even with other sets of instrumental variables. This may be due to the small sample size, which allows non-zero sample correlation between the error term of the regression and the instrumental variables even if their asymptotic correlation is zero.[9] J_2 is the p-value for the Wald-test for constant

[9]In the sectoral comparison, the manufacturing sectors show far better fit than the non-manufacturing sector, given that the instrument variables are correctly chosen. This suggests that the functional form of the production function differs between the manufacturing and the non-manufacturing sectors.

returns to scale. Note that the sum of the coefficients to $\Delta\ln H_{i,t-1} + \Delta\ln N_{i,t-1}$, $\Delta\ln K_{i,t-1}$, and $\Delta\ln M_{i,t-1}$ equals the returns to scale in production γ_i. Hence, $\gamma_i = 1$ is tested. The p-values imply that the heavy-manufacturing and the non-manufacturing sectors show constant returns to scale.

In Table 1.3, we use quality-adjusted worker-hours for $H_{i,t}N_{i,t}$. The feature of the result for the manufacturing sectors is similar to Table 1.2. The result for the non-manufacturing sector differs. In this sector, the absolute sizes of the coefficients are smaller in Table 1.3 than in Table 1.2, except the coefficient to the capital growth. The coefficient for the capital growth has a significantly minus sign in Table 1.3.

Table 1.5 shows the average TFP growth rates under the quality-unadjusted worker-hours and the quality-adjusted worker-hours, respectively. We calculate the aggregate TFP growth rates in the table by taking the weighted average of the estimated industrial TFP growth rates. We use

$$\frac{\omega_{i,t}}{1 - s_{M,i,t}} \tag{1.14}$$

as the weight for the industry i in year t, where $\omega_{i,t}$ is the nominal value-added share of the industry, and $s_{M,i,t}$ is the ratio of the intermediate-inputs expenditure to the revenue in the industry. See appendix for more detail of this weight. The common feature of the estimates is a TFP slowdown in the 1990's, especially in the manufacturing sectors. Thus, our estimates have different implication from Kawamoto (2004)'s observation that there was no TFP slowdown. The reason for the different result is that our estimation method differs from Kawamoto's. The main difference is that we use the average working-hours of a worker per day for $H_{i,t}$ rather than the working-hours per worker, and do not pre-calibrate the values of $\eta_{i,H}$, $\eta_{i,K}$, and $\eta_{i,M}$ by the cost-shares in each industry. Also, the classification of the industries into the sectors is different.

1.5 Proxy of the unobserved capacity utilization: Energy inputs

In this section, we consider the energy inputs as an alternative proxy for the unobserved capacity utilization. The motivation for this alternative is to avoid the influence of the amendment of the labor standards on the estimation.

We assume that the running time of capital is complementary with electricity, gas, and heat-supply, and follows the function

$$U_{i,t} = U\left(\frac{Z_{i,t}J_{i,t}}{K_{i,t-1}}\right), \tag{1.15}$$

where $J_{i,t}$ is the sum of the real values of electricity, gas, and heat-supply, and $Z_{i,t}$ is the energy-efficiency of capital. Electricity and gas are used as the energy to run the facilities. The heat-supply supplies cold and hot water, which is used for air-conditioning of the facilities. We call these intermediate inputs as the energy inputs. Accordingly, we redefine $M_{i,t}$ as the intermediate inputs net of $J_{i,t}$.

We modify the cost-minimization problem (1.7) as

$$\begin{aligned}\min_{\{F_t,H_t,N_t,U_t,K_t,M_t\}_{t=0}^{\infty}} \mathrm{E}_t \Bigg\{ \sum_{t=0}^{\infty} \frac{(1+r_0)}{\prod_{j=0}^{t}(1+r_t)} \big[\, & w_t G(F_{i,t}, H_{i,t}) N_{i,t} + P_{M,i,t}M_{i,t} + P_{J,i,t}J_{i,t} \\ & + \Xi\left(N_{i,t}, N_{i,t-1}, K_{i,t}, K_{i,t-1}\right)\big]\Bigg\}\end{aligned} \tag{1.16}$$

$$\begin{aligned}\text{s.t. } & Y_{i,t} \geq \overline{Y} \\ & U_{i,t} = U\left(\frac{Z_{i,t}J_{i,t}}{K_{i,t-1}}\right).\end{aligned}$$

$P_{J,i,t}$ is the price of the energy inputs. We add the cost of $J_{i,t}$ and the new function of the running-time of capital (1.15), and take out the shift premium $V(U_{i,t})$ from the total compensation function to the workers. The last modification implies that the running-time of capital is complementary with the energy inputs in the current method, while it is complementary with labor in the previous method. The intra-temporal first-order conditions for $J_{i,t}$ and $M_{i,t}$ imply

$$\frac{P_{J,i,t}J_{i,t}}{P_{M,i,t}M_{i,t}} = \frac{\mu\alpha_{i,K}\left[(F_{i,t}H_{i,t}N_{i,t})^{\alpha_{i,N}}(U_{i,t}K_{i,t})^{\alpha_{i,K}}\right]^{\rho} u_{i,J}}{(1-\mu)(M_{i,t})^{\rho}}, \tag{1.17}$$

where

$$u_{i,J} = \frac{U'(Z_{i,t}J_{i,t}/K_{i,t-1})}{U_{i,t}} \frac{Z_{i,t}J_{i,t}}{K_{i,t-1}}. \tag{1.18}$$

Thus, $u_{i,J}$ is the elasticity of the running-time of capital to the ratio of the effective energy-inputs $Z_{i,t}J_{i,t}$ to the capital stock $K_{i,t-1}$. We assume this elasticity is constant for each firm over the sample period. Taking log and difference of (1.17), we obtain

$$\begin{aligned}\Delta\ln U_{i,t} = \frac{1}{\rho_i \alpha_{i,K}} \{ & \Delta\ln(P_{J,i,t}J_{i,t}) - \Delta\ln(P_{M,i,t}M_{i,t}) \\ & + \rho_i \left[\Delta\ln M_{i,t} - \alpha_{i,N}(\Delta\ln F_{i,t} + \Delta\ln H_{i,t} + \Delta\ln N_{i,t}) \right. \\ & \left. - \alpha_{i,K}\Delta\ln K_{i,t-1}\right]\}. \end{aligned} \tag{1.19}$$

Substituting (1.19) and the definitions of η's (1.3)-(1.5) into (1.6), we obtain

$$\Delta\ln Y_{i,t} = \overline{\Delta\ln A_i} + \gamma_i \Delta\ln M_{i,t} + \frac{\eta_{i,K}}{\rho_i \alpha_{i,K}} [\Delta\ln(P_{J,i,t}J_{i,t}) - \Delta\ln(P_{M,i,t}M_{i,t})] + error. \tag{1.20}$$

Note that this equation is robust to existence of the growth of energy-efficiency of capital $Z_{i,t}$ under the assumption of constant $u_{i,J}$ for each industry. Also, labor and capital do not appear in the regressors. (1.17) provides the intuition behind this result such that the expenditure ratio of the energy inputs to the intermediate inputs $P_{J,i,t}J_{i,t}/(P_{M,i,t}M_{i,t})$ controls for the service from both labor and capital in the production relative to the service from the intermediate inputs, including the unobserved capacity utilization.

(1.20) is similar to the regression form derived by Basu (1996). The difference is that Basu uses the log-difference of the price ratio between the value-added and the intermediate inputs in the regressors, rather than the expenditure ratio between the energy inputs and the intermediate inputs, $\Delta\ln(P_{J,i,t}J_{i,t}) - \Delta\ln(P_{M,i,t}M_{i,t})$. Even though (1.17) implies that we can replace the expenditure ratio in (1.20) with the price ratio between the energy inputs and the intermediate inputs by changing the associated coefficient in the regression, we use the expenditure ratio, since doing so is robust to the different prices set by the firms for the same type of goods to different customers in reality. We can use the energy inputs rather than the value-added in the expenditure ratio because of the assumption (1.15). The advantage to use the energy inputs is that we only need to consider the intra-temporal cost minimization by the firm to obtain the regression form (1.20). If we use the value-added in the expenditure ratio, then we need to consider the inter-temporal cost minimization problem, because labor and capital are not immediately adjustable due to the adjustment costs to the sizes of the employment and the capital stock.

1.5.1 Estimation results

We use 3SLS and the same data as in the previous section 1.4. We introduce the industry-specific time-dummies into the constant terms, and assume that the other coefficients are constant within the sector as in the previous section. Table 1.4 shows the estimation result. It shows the minus sign of the coefficient to the log-difference of the expenditure ratio between the energy inputs and the intermediate inputs for all the sectors. This implies that the sign of ρ_i is minus, so that the value-added consisting of labor and capital is more complementary with the intermediate inputs than the Cobb-Douglas case, in which $\rho_i = 0$.

The p-values in J_2 imply that the value of γ is significantly lower than 1 for all the sectors. Also, in the light-manufacturing sector and the non-manufacturing sector, we have insignificant coefficients to the log-difference of the expenditure ratio between the energy inputs and the intermediate inputs. These observations suggest that the expenditure ratio does not fully control for the services from labor and capital, which might result in the significant decreasing returns to scale in our estimates. This result implies that the number of the average working-hours of a worker per day may be a better proxy than the energy inputs to control for the unobserved capacity utilization for the manufacturing sectors. But the result for the non-manufacturing sector is mixed, as the J-statistic in J_1 implies that (1.20) has a better fit than (1.13) for the non-manufacturing sector.

Table 1.5 shows the average TFP growth rates estimated by the regression (1.20). We use the same weight (1.14) as the previous section to aggregate the industrial TFP growth rates. As in the previous section, we observe a TFP slowdown in the 1990's, especially in the manufacturing sectors. Hence, we confirm that this observation is robust to the different proxies of the unobserved capacity utilization considered in this chapter.

1.6 Conclusion

In this chapter, we estimate the TFP growth rates in Japan for 1974-1998, controlling for the unobserved capacity utilization and the returns to scale in production. We use the two proxies to control for the unobserved capacity utilization; the average working-hours of a worker per day and the expenditure ratio between the energy inputs and the intermediate inputs. For both of the methods, we observe that there was a TFP slowdown in the 1990's, especially in the manufacturing sectors.

We observe constant returns to scale in the heavy-manufacturing sector and the non-manufacturing sector, when we use the average working-hours of a worker per day as the proxy to control for the unobserved capacity utilization. However, we observe significant decreasing returns to scale for all the sectors, when we use the expenditure ratio between the energy inputs and the intermediate inputs. The reason for the latter result is likely that the expenditure-ratio does not fully control for the service from the value-added consisting of labor and capital. However, we also observe that the estimation with the energy inputs has a better fit for the non-manufacturing sector than the one with the average working-hours of a worker per day. This suggests that the manufacturing and the non-manufacturing sectors might have different types of the cost of the unobserved capacity utilization.

Appendix

1.A.1 The firm's cost minimization problem to derive (1.8) and (1.9)

We consider the intra-temporal cost-minimization of (1.7) by $F_{i,t}$, $H_{i,t}$, and $U_{i,t}$, taking the other variables as given. The first-order conditions are

$$w_s G_1(F_{i,t}, H_{i,t}) V(U_{i,t}) N_{i,t} = \lambda_{i,t} \eta_{i,N} \frac{Y_{i,t}}{F_{i,t}}$$
$$w_s G_2(F_{i,t}, H_{i,t}) V(U_{i,t}) N_{i,t} = \lambda_{i,t} \eta_{i,N} \frac{Y_{i,t}}{H_{i,t}}$$
$$w_s G(F_{i,t}, H_{i,t}) V'(U_{i,t}) N_{i,t} = \lambda_{i,t} \eta_{i,K} \frac{Y_{i,t}}{U_{i,t}}$$

G_j is the derivative of G by its jth term. $\lambda_{i,t}$ is the Lagrange multiplier for the quantity constraint of the cost minimization. These equations imply

$$F_{i,t}G_1(F_{i,t}, H_{i,t}) = H_{i,t}G_2(F_{i,t}, H_{i,t})$$
$$\frac{V'(U_{i,t})}{V(U_{i,t})} = \frac{\alpha_{i,K}}{\alpha_{i,N}} \frac{H_{i,t}G_2(F_{i,t}, H_{i,t})}{U_{i,t}G(F_{i,t}, H_{i,t})}.$$

We obtain the second equation by substituting the definitions of $\eta_{i,N}$ and $\eta_{i,K}$ given by (1.3) and (1.4). We assume that there exist implicit functions for $F_{i,t}$ and $U_{i,t}$. Then, we obtain the following functions:

$$F_{i,t} = F(H_{i,t})$$
$$U_{i,t} = U\left(\frac{\alpha_{i,K}}{\alpha_{i,N}}, H_{i,t}, F_{i,t}\right).$$

The first equation is (1.8). We obtain (1.9) by substituting the first equation into the second.

1.A.2 Further discussion on the data detail

Iron & steel (33) and Other iron & steel (34) show that the amounts of the value-added, i.e. the revenue minus the intermediate-inputs cost, significantly declined in 1976 and in 1977, respectively. The industry code of the JIP database is in parenthesis. The firms in these industries often produce both types of goods. We merge the two industries into Iron and steel to eliminate their anomalous observations.

We correct worker-hours data in 1985. The both quality-adjusted and quality-unadjusted worker-hours data show sharp temporary drops in 1985 for 6 service industries: Other transportation service (63), Other social service (64), Equipment rental for industrial use(71), Other business services (72), Laundry, Barber, and Public Bath (77), and Other Personal Services (78). They are sufficiently large to make the drop of worker-hours in the aggregate service industries. We do not find such movement in worker-hours data of the aggregate service industries from the National Accounts or *Maitsuki Kinro Tokei*. We regard them as typos in the JIP database, and replace the both types of worker-hours in that year by the average of the data in 1984 and 1986.

1.A.3 Derivation of (1.14)

We describe the derivation of the weight to aggregate the industrial TFP growth rates given by (1.14). First, consider the Divisia index of the real GDP growth rate. We follow Nakamura, et al. (1992), pp.110-111, to obtain the index. In continuous time, the nominal GDP growth rate can be written as

$$\frac{d}{dt}\left\{\sum_i (P_{i,t}Y_{i,t} - P_{M,i,t}M_{i,t})\right\} = \sum_i \left(\frac{dP_{i,t}}{dt}Y_{i,t} - \frac{dP_{M,i,t}}{dt}M_{i,t}\right) + \sum_i \left(P_{i,t}\frac{dY_{i,t}}{dt} - P_{M,i,t}\frac{dM_{i,t}}{dt}\right).$$

By dividing both sides by $\sum_i (P_{i,t}Y_{i,t} - P_{M,i,t}M_{i,t})$,

$$\begin{aligned}
&\frac{\frac{d}{dt}\{\sum_i (P_{i,t}Y_{i,t} - P_{M,i,t}M_{i,t})\}}{\sum_i (P_{i,t}Y_{i,t} - P_{M,i,t}M_{i,t})} \\
=&\frac{d}{dt}\left\{\ln\left(\sum_i (P_{i,t}Y_{i,t} - P_{M,i,t}M_{i,t})\right)\right\} \\
=&\sum_i \frac{P_{i,t}Y_{i,t} - P_{M,i,t}M_{i,t}}{\sum_i (P_{i,t}Y_{i,t} - P_{M,i,t}M_{i,t})} \cdot \frac{P_{i,t}Y_{i,t}\frac{dP_{i,t}/dt}{P_{i,t}} - P_{M,i,t}M_{i,t}\frac{dP_{M,i,t}/dt}{P_{M,i,t}}}{P_{i,t}Y_{i,t} - P_{M,i,t}M_{i,t}} \\
&+\sum_i \frac{P_{i,t}Y_{i,t} - P_{M,i,t}M_{i,t}}{\sum_i (P_{i,t}Y_{i,t} - P_{M,i,t}M_{i,t})} \frac{P_{i,t}Y_{i,t}\frac{dY_{i,t}/dt}{Y_{i,t}} - P_{M,i,t}M_{i,t}\frac{dM_{i,t}/dt}{M_{i,t}}}{P_{i,t}Y_{i,t} - P_{M,i,t}M_{i,t}} \\
=&\sum_i \omega_{i,t}\frac{d(\ln P_{i,t})/dt - s_{M,i,t}d(\ln P_{M,i,t})/dt}{1 - s_{M,i,t}} + \sum_i \omega_{i,t}\frac{d(\ln Y_{i,t})/dt - s_{M,i,t}d(\ln M_{i,t})/dt}{1 - s_{M,i,t}},
\end{aligned}$$

where

$$\omega_{i,t} = \frac{P_{i,t}Y_{i,t} - P_{M,i,t}M_{i,t}}{\sum_i (P_{i,t}Y_{i,t} - P_{M,i,t}M_{i,t})}$$
$$s_{M,i,t} = \frac{P_{M,i,t}M_{i,t}}{P_{i,t}Y_{i,t}}.$$

We define the real GDP growth rate by the second term of the right-hand side of the equation. In discrete-time, we approximate dx/dt by $x_t - x_{t-1}$, and obtain

$$\sum_i \omega_{i,t}\frac{\Delta \ln Y_{i,t} - s_{M,i,t}\Delta \ln M_{i,t}}{1 - s_{M,i,t}} \tag{1.21}$$

as the real GDP growth rate. If we substitute (1.13) into (1.21), the real GDP growth rate becomes

$$\sum_i \omega_{i,t}\frac{\Delta \ln A_{i,t} + \eta_{i,N}\Delta \ln(F_{i,t}H_{i,t}N_{i,t}) + \eta_{i,K}\Delta \ln(U_{i,t}K_{i,t-1}) + (\eta_{i,M} - s_{M,i,t})\Delta \ln M_{i,t}}{1 - s_{M,i,t}}. \tag{1.22}$$

It can be shown that if there is no mark-up, then $\eta_{i,M} = s_{M,i,t}$ at the steady state, so that the real GDP growth rate is decomposed into the TFP growth rate and the input growth rate, including the unobserved capacity utilization. Hence, we use $\omega_{i,t}/1 - s_{M,i,t}$ as the weight attached to the industry i's TFP growth rate in year t.

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Table 1.1: Industry classification

JIP code	Heavy-manufacturing	JIP code	Light-manufacturing
27	Basic chemicals	11	Livestock products
28	Chemical fibers	12	Processed fishery products
29	Other chemicals	14	Other foods
30	Petroleum products	15	Beverages
31	Coal products	16	Tobacco
33-34*	Iron & steel	17	Natural textile
35	Nonferrous metal	19	Other textile products
36	Metal products	20	Apparel
37	General machinery	21	Lumber& Wood
38	Electrical machinery for industrial use	22	Furniture & glass
		23	Paper
39	Electrical machinery for household use	24	Publishing & printing
		25	Leather
40	Other electrical machinery	26	Rubber products
41	Motor vehicles	32	Stone, clay & glass
42	Ships	45	Miscellaneous manufacturing
43	Other transportation equipment		
44	Precision instruments		

JIP code	Non-manufacturing
46	Construction
47	Civil engineering
48	Electric utilities
49	Gas utilities
53	Wholesale trade
54	Retail trade
55	Finance
56	Insurance
57	Real estate
59	Railroad transportation
60	Trucking
61	Water transportation
62	Air transportation
63	Other transportation services
64	Telephone & telegraph
71	Equipment rental for industrial use
72	Other business services
73	Amusement & recreation services
74	Radio & television
75	Restaurants
76	Hotels & other lodging places
77	Laundry, barber, & public bath

Note: 33 – 34* implies a merged industry. See appendix for the data detail.

Table 1.1 continued.

JIP code	Excluded from the analysis (15.8%)
1	Rice & Wheat
2	Other Farms
3	Stockbreeding & Sericulture
4	Veterinary & Agricultural services
5	Forestry
6	Fishing
7	Coal mining
8	Metal mining
9	Oil & Gas extraction
10	Other mining
13	Rice-polishing & flour-milling
18	Chemical textile
50	Waterworks
51	Water services for industrial use
52	Waste disposal
58	Housing (including imputed services)
65	Postal services
66	Private educational services
67	Research
68	Private medical services
69	Other social services
70	Advertising
78	Other personal services
79	Educational services (Government)
80	Medical services (Government)
81	Other services (Government)
82	Medical services (Non-profit)
83	Other services (Non-profit)
84	Unclassified

Table 1.2: Regression coefficients of (1.13): Quality-unadjusted worker-hours

	Heavy -manufacturing	Light -manufacturing	Non -manufacturing
η_N	0.1542 (0.1215)	-0.0757 (0.1163)	0.3387 (0.0394)
η_K	0.1760 (0.1162)	-0.0977 (0.1566)	0.0042 (0.0307)
η_M	0.7289 (0.0636)	0.5648 (0.0997)	0.6118 (0.0221)
$\eta_N f_H + \eta_K u_H$	0.9544 (0.4059)	0.5339 (0.6049)	1.0848 (0.2355)
J_1	790	1065	12100
J_2	0.7078	0.0027	0.2742

Note: The standard-error is in the parenthesis. The regression (1.13) is

$$\Delta\ln Y_{i,t} = \overline{\Delta\ln A_{i,-90}} + \overline{\Delta\ln A_{i,91-}} + \eta_{j,N}(\Delta\ln H_{i,t} + \Delta\ln N_{i,t}) + \eta_{j,K}\Delta\ln K_{i,t-1} + \eta_{j,M}\Delta\ln M_{i,t} + (\eta_{j,N} f_{j,H} + \eta_{j,K} u_{j,H})\,\Delta\ln H_{i,t} + error,$$

The industry-specific time-dummies, $\overline{\Delta\ln A_{i,-90}}$ and $\overline{\Delta\ln A_{i,91-}}$, are omitted in the table. J_1 is the J-statistic for the over-identifying restriction test. J_2 is the p-value for the Wald-test of the constant returns to scale in production, i.e. $\eta_N + \eta_K + \eta_M = 1$. The firm, sector, time indexes i, j, t are omitted in the table.

Table 1.3: Regression coefficients of (1.13): Quality-adjusted worker-hours

	Heavy -manufacturing	Light -manufacturing	Non -manufacturing
η_N	0.1751 (0.1263)	-0.0487 (0.1238)	0.1979 (0.0557)
η_K	0.1561 (0.1101)	-0.0995 (0.1639)	-0.1475 (0.0392)
η_M	0.715 (0.062)	0.5492 (0.101)	0.4181 (0.0255)
$\eta_N f_H + \eta_K u_H$	0.9512 (0.4036)	0.5965 (0.6168)	-0.0382 (0.3586)
J_1	757	1022	12766
J_2	0.7620	0.0039	0.0000

Note: See the note for Table 1.2.

Table 1.4: Regression coefficients of (1.20)

	Heavy -manufacturing	Light -manufacturing	Non -manufacturing
γ	0.8244	0.4653	0.4788
	(0.0412)	(0.0742)	(0.0421)
$\eta_K/(\rho\alpha_K)$	-0.1308	-0.0467	-0.0167
	(0.0338)	(0.0449)	(0.0150)
J_1	1150.8	1006.3	2631.4
J_2	0.0000	0.0000	0.0000

Note: The standard-error is in the parenthesis. The regression (1.20) is

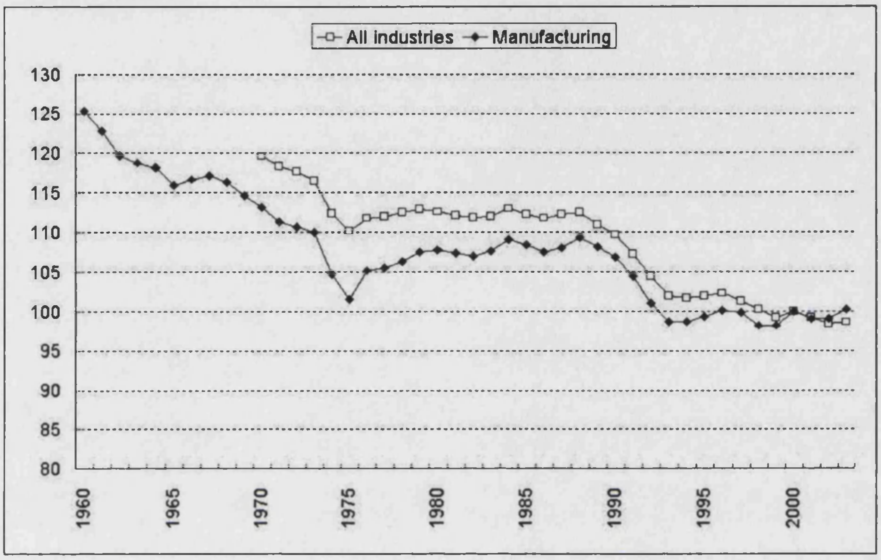
$$\Delta\ln Y_{i,t} = \overline{\Delta\ln A_{i,-90}} + \overline{\Delta\ln A_{i,91-}} + \gamma_j \Delta\ln M_{i,t} + \frac{\eta_{j,K}}{\rho_j \alpha_{j,K}} [\Delta\ln(P_{J,i,t}J_{i,t}) - \Delta\ln(P_{M,i,t}M_{i,t})] + error.$$

The industry-specific time-dummies, $\overline{\Delta\ln A_{i,-90}}$ and $\overline{\Delta\ln A_{i,91-}}$, are omitted in the table. J_1 is the J-statistic for the over-identifying restriction test. J_2 is the p-value for the Wald-test of the constant returns to scale in production, i.e. $\gamma = 1$. The firm, sector, time indexes i, j, t are omitted in the table.

Table 1.5: Average TFP growth rates (%)

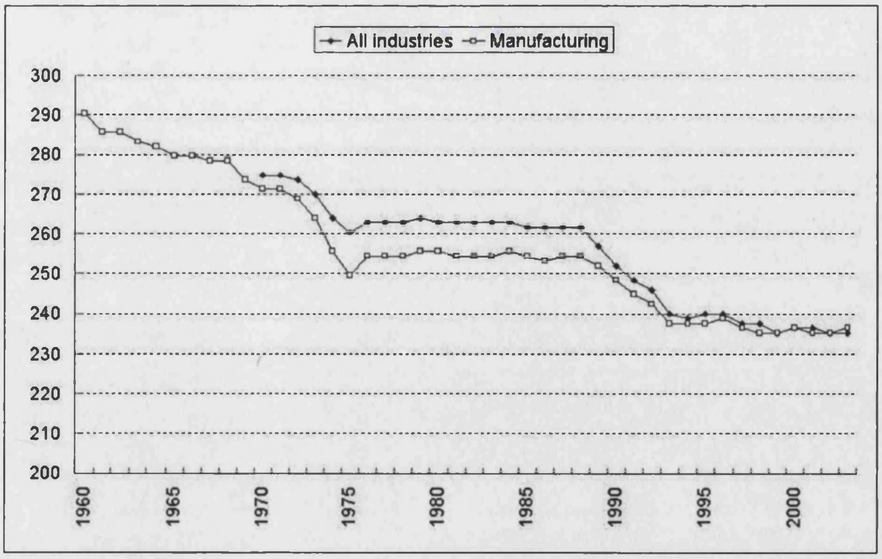
		All the sectors	Manufacturing	Non -manufacturing
Regression (1.13) (Quality-unadjusted worker-hours)	1980-1990	1.32	3.45	0.19
	1990-1998	0.54	- 0.03	0.94
Regression (1.13) (Quality-adjusted worker-hours)	1980-1990	3.89	3.65	4.02
	1990-1998	1.23	- 0.02	1.95
Regression (1.20)	1980-1990	3.85	5.22	3.06
	1990-1998	0.69	-0.06	1.37

Figure 1.1: Total working-hours per worker, (2000=100.)



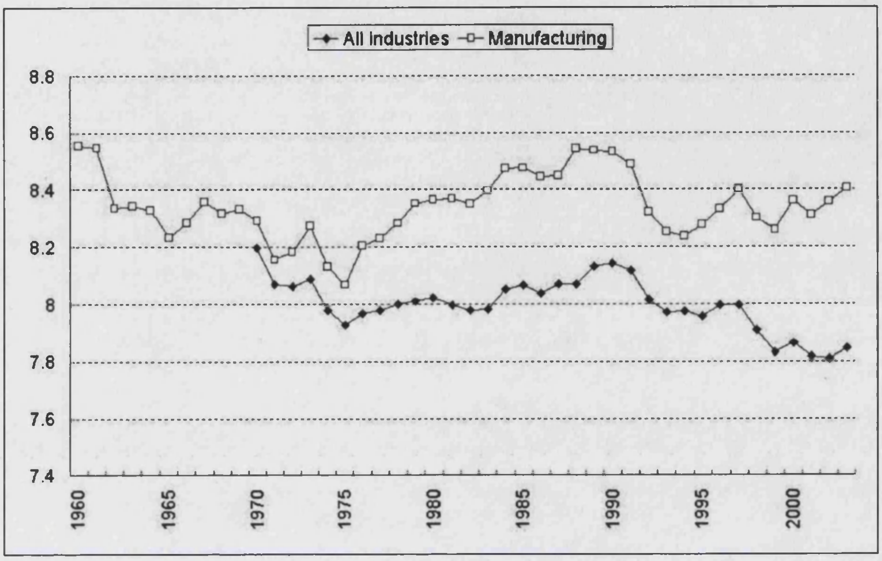
Note: The data for all the industries are only available from 1970.

Figure 1.2: Working-days per year (days.)



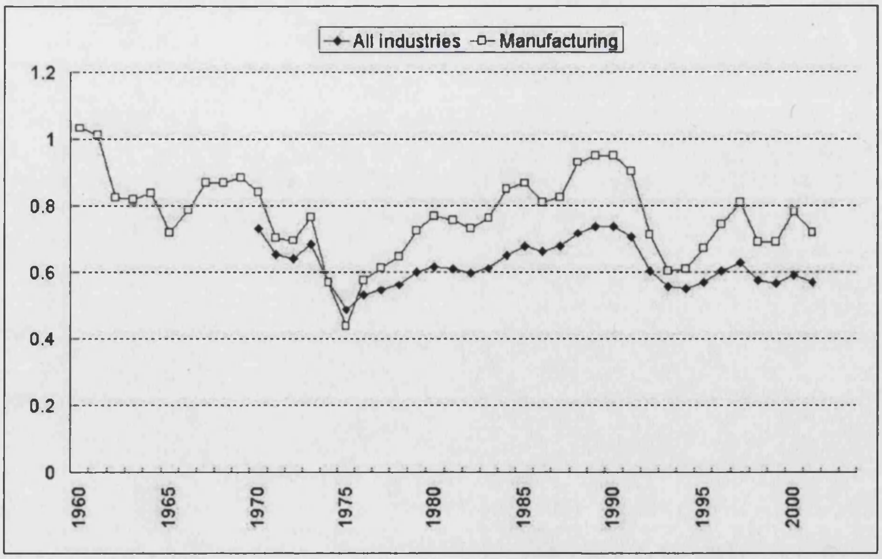
Note: The data for all the industries are only available from 1970.

Figure 1.3: Average working-hours of a worker per day (hours.)



Note: The data for all the industries are only available from 1970.

Figure 1.4: Average over-time of a worker per day (hours.)



Note: The data for all the industries are only available from 1970.

Chapter 2

Firm Dynamics, Bankruptcy Laws and Total Factor Productivity

Abstract

This chapter analyzes how legal restriction on liquidating collateral assets interacts with a productivity slowdown in an economy with credit market frictions and heterogeneous productivity levels across the producers. We find that the legal restriction mitigates the downward fluctuation of the TFP level at the onset of the productivity slowdown, but also that a shock to the legal restriction endogenously deepens the productivity slowdown. We analyze entry and exit of the producers, and show that the low-productive producers start remaining in production under the productivity slowdown. We discuss implication of the model for the long stagnation in Japan after 1990.

2.1 Introduction

As estimated in the previous chapter, the productivity slowdown was one of the features of the long stagnation in Japan after 1990. Hayashi and Prescott (2002) estimate that the total factor productivity (TFP) growth rate dropped from 3.7% in the 1983-1990 period to 0.3% in the 1990-2000 period. While they do not control for fluctuation of unobserved capacity utilization and non-constant returns to scale in production, the previous chapter estimates that the average TFP growth rate dropped from 1.32% in the 1980-1990 period to 0.54% in the 1990-1998 period, controlling for these two factors. Figures 2.1 and 2.2 show the level and the growth rate of TFP measured by the Solow residual.

The observed productivity slowdown was accompanied with the prolonged non-performing loans problem in the banking sector. Figure 2.3 shows that the loan write-offs by the banks became large only from the mid 1990's, and remained persistently high after that, despite the fact that the non-performing loans problem had originated in real-estate related lending made in the late 1980's.[1] One of the reasons behind the continuing loan losses was the declining collateral values due to the large fall of the land price.[2] This implies that the banks did not liquidate loans and collateral assets as quickly as the decline of the land price. Figure 2.4 shows the declining real land price index in Japan in the 1990's.

One of the reasons for the slow liquidation of the collateral assets was the mortgage laws.[3] Until recently, there had been strong protection of the borrowers' collateral assets against foreclosure by the lenders. They were regarded as an impediment in the economy[4] and led to the revision of the laws in 2003. In fact, the slow liquidation of the collateral assets also matches with the analysis of the Japanese economy before 1990. The analysis highlighted the soft approach of the Japanese banks toward financially distressed firms, which let the firms continue their operation.[5] The restriction on liquidating collateral assets would induce the banks to take such a soft approach, because the restriction would prevent the banks from swiftly terminating the operation of the financially distressed firms.

A natural question that arises is how this feature of the credit market would

[1]See Hoshi (2001) for an empirical analysis of the origin of the non-performing loans problem.

[2]See Bank of Japan (2002) for an analysis on the causes of the new loan losses of the banks.

[3]As another explanation, recent works argue that the banks extended loans to inefficient defaulting firms in the non-performing loans problem, and that the efficiency of the economy was impaired. See Hosono and Sakuragawa (2003), Peek and Rosengren (2003) and Caballero, Hoshi, and Kashyap (2004). This issue will be also analyzed in this paper .

[4]Ministry of Justice issued a report on this problem in 2002; "A supplementary note for the interim proposal for the Law to Revise a Part of the Civil Laws for a Reform of the Mortgage and Civil Execution System." See appendix for more detail of the mortgage laws.

[5]Corbett (1987), Sheard (1994) and Hoshi and Kashyap (2001, Ch.5) contain empirical surveys and case studies of the Japanese main-bank system.

interact with the productivity slowdown. In this paper, we answer this question by constructing a general-equilibrium model with the legal restriction on liquidating collateral assets. To analyze the transactions in the credit market, we need to consider a heterogeneous agents model. This is because homogeneous agents do not have any need for exchange with each other, including credit transactions. Hence, our analysis of the productivity slowdown departs from Hayashi and Prescott (2002), which analyze the effect of an exogenous aggregate productivity slowdown in the representative agent model.[6] In the class of the heterogeneous agents models, we focus on heterogeneity of the productivity levels across the agents. This choice is motivated by the considerable evidence that there occurred distortions in the resource allocation across the firms behind the productivity slowdown in the 1990's.[7] We model the autoregressive productivity transition process of each agent similar to the empirical firm dynamics, and analyze the resource allocation across the agents with different productivity levels.

To analyze the effect of the legal restriction on liquidating collateral assets, we model the collateralized debt contracts in the credit market assuming that the agents can only pledge to repay their debts up to the collateral values of their assets in the model. This assumption follows Kiyotaki and Moore (1997). Focusing on the collateralized debt contracts is important for the analysis of the Japanese economy, because collateralized loans are one of the features of the Japanese credit market, and caused the loan losses to the banking sector in the 1990's through the fall of the collateral value of the assets.

Using this model, we find two types of the effects of the legal restriction on liquidating collateral assets. One is the ex-post effect. When a negative productivity shock hits the economy, then the asset price falls, which reduces the asset value owned by the producers. This causes debt-overhang of the producers, and induces the lenders to liquidate their assets. This is a resource shift from the more-productive agents to the less-productive in the economy, because those

[6]Barseghyan (2002) and Dekle and Kletzer (2003) also provide general equilibrium analysis of the Japanese stagnation. Both consider the effect of the government forbearance in the non-performing loans problem in the banking sector. Barseghyan introduces the accumulation of non-performing loans by the insolvent banks in the standard overlapping generation model, and Dekle and Kletzer do so in the infinite-horizon representative agent model.

[7]Saita and Sekine (2001), Miyagawa (2003) and Nakakuki, Otani and Shiratsuka (2004).

who borrow and engage in production are the more-productive. The restriction on liquidating collateral assets mitigates this resource shift after the negative productivity shock, and prevents a sharp fall of the TFP level of the economy due to the biased resource allocation across the producers. On the flip side of the coin, hampering the liquidation of the producers' collateral assets increases the loan losses that the lenders suffer after the shock. This result matches with the observation in Japan such that the decline of the TFP level was not sharp in the early 1990's, while the loan losses accounted by the banks became large in the 1990's.

The other effect of the legal restriction on liquidating collateral assets is the ex-ante effect. The restriction reduces the collateral value of the assets and the borrowing limit of the agents. This prevents the more-productive agents in the economy from accumulating the production resources at their hands through borrowing. Then, the TFP level of the economy falls. This effect is important to understand the effect of the judicial precedents of the supreme court cases in 1989 and 1991 that strengthened the legal restriction on liquidating collateral assets.[8] Such a legal shock to the credit market causes the level-down effect on the TFP trend, and an endogenous productivity slowdown in the transition to the new trend. Also, by this level-down effect on TFP, the asset price falls after the shock, which induces default of the producers. Hence, the legal shock causes the endogenous productivity slowdown, the fall of the asset price, and the loan losses to the lenders. This result implies that the legal shock to the credit market helps to explain the feature of the Japanese economy in the 1990's.

We also find that the ex-ante effect of the legal restriction on liquidating collateral assets is important to understand the effect of an increase in the productivity heterogeneity in the economy. Fukao and Kwon (2004) find the increases of the persistence of each firm's productivity level and the productivity gap across the firms over the 1990's in the panel data of the Japanese manufacturing firms. Our model implies that such a shock to the productivity dynamics lets the more-productive agents have more time to accumulate net-worth by keeping investing into their high-productive production. This shifts the resource alloca-

[8]Yamazaki, Seshimo, Ohta and Sugihara (2005) point out this fact. See appendix for more detail.

tion toward them, and raises the TFP level of the economy. However, the legal restriction on liquidating collateral assets reduces the borrowing limit and the leverage the agents can take. This curbs the positive effects of the shock on the net-worth accumulation of the more-productive producers and on the TFP level. This result implies that the legal restriction prevented the increase in the productivity heterogeneity across the firms from mitigating the productivity slowdown in Japan after 1990. Over all, our model implies that the legal restriction mitigated the downward fluctuation of the TFP level at the onset of the productivity slowdown, but endogenously deepened the productivity slowdown over time in the long stagnation in Japan.

In the analysis of the productivity slowdown described above, we investigate the effect of entry and exit of the agents in production. This is important, because some researchers find that low-productive "zombie firms" were financed by the lenders and remained in production, which worsened the productivity slowdown in the 1990's.[9] We show that the low-productive producers start remaining in production under the productivity slowdown, because the productivity slowdown lowers the input prices more than the productivity levels under the credit market frictions,[10] and then raises the rate of return to investment for each producer. Hence, it becomes viable for the low-productive producers to remain in production.[11]

Besides the results described so far, we also investigate the cause of the decline of the firms' borrowing in the 1990's. One possible reason for the decline of the borrowing is that the low rates of return to investment induced the firms to avoid taking risk of debt-overhang and reduce their borrowing.[12] Another

[9]Kobayashi, Saita, and Sekine (2002), Hosono and Sakuragawa (2003), Peek and Rosengren (2003), and Caballero, Hoshi, and Kashyap (2004).

[10]Kiyotaki and Moore (1997) find this propagation mechanism.

[11]The effect of the restriction on liquidating collateral assets on the entry and exit of the producers is that under the exogenous aggregate productivity slowdown, the restriction mitigates the downward fluctuation of the TFP level and the fall of the input prices after the shock. Then, it prevents the low-productive producers from remaining in production as well. The intensified restriction endogenously causes the productivity slowdown and the stay of the low-productive producers in production.

[12]Motonishi and Yoshikawa (1999) explain the decline of the firms' investment and borrowing by the decline of their demand for the capital stock. However, Hayashi and Prescott (2002) report that the firms continued financing investment by their internal reserves in the 1990's despite cutting back the amount of borrowing. The risk of debt-overhang can explain this observation such that the low rates of return to investment induced the firms to avoid taking risk of the ex-post debt-overhang and refrain from borrowing.

explanation for the decline of the firms' borrowing is that the fall of the collateral value of the assets reduced the borrowing limit of the firms, and constrained their borrowing. To address this question, we consider possible debt-overhang by delay of production to analyze the risk-taking behavior of the agents, and investigate the change of their credit demand under the productivity slowdown. We find that because the productivity slowdown reduces the input prices more proportionally than the productivity level of each agent, each agent has the higher rate of return to investment, and becomes more inclined to borrow up to the limit. Consequently, the potential credit demand of the agents increases. However, the productivity slowdown reduces the asset price and the collateral value of the assets. This reduces the agents' borrowing. Therefore, our analysis suggests that the fall of the collateral value of the assets caused the decline of the firms' borrowing in the 1990's, rather than the risk-averse behavior of the firms.

Related to this result, we find that the more-productive producers become more fragile to the unexpected fall of the asset price than the less-productive producers, because the more-productive producers borrow more. This analysis indicates the mechanism behind the observation that the more-productive firms exit from production, while the less-productive firms stay in production in the 1990's.[13]

Theoretically, this chapter is related to the literature that analyzes the effect of policies and institutions on the TFP level of the economy. In this literature, Caselli and Gennaioli (2002) is the most closely related work to this chapter. They analyze the effect of credit market frictions on the TFP level of the economy through letting inefficient family members inherit the firms. Our contribution is to analyze the TFP effect of the credit market frictions under various shocks to the deep parameters of the economy.

This chapter also adds to the recent literature on the general equilibrium analysis of bankruptcy laws. Bolton and Rosenthal (2002) and Biais and Mariotti (2003) are among the preceding works. They analyze the effect of the bankruptcy laws on the resource allocation across the agents, considering liq-

[13]Nishimura, Kiyota and Nakajima (2003), and Fukao and Kwon (2004) analyze the case in Japan in the 1990's. Barlevy (2003) analyzes a similar observation over the business cycles in United States.

uidity shocks. Our contribution is to analyze this effect in the heterogeneous agents model with different productivity levels, and to specify its effect on the level and the growth of TFP in the economy.

The rest of the chapter is organized as follows: Section 2.2 describes the model. Section 2.3 shows the dynamic responses to structural changes in the model. Section 2.4 analyzes the mechanism of the decline of the firm's borrowing under the productivity slowdown. Section 2.5 discusses implication of the analysis for the long stagnation in Japan. Section 2.6 concludes.

2.2 Model

Consider a discrete-time economy with homogeneous and perishable goods and a continuum of agents. Each agent is risk-neutral, and maximizes the following utility function:

$$E_t\left(\sum_{s=t}^{\infty}\beta^{s-t}\,c_{i,s}\right), \tag{2.1}$$

where i is the index for each agent, and $c_{i,s}$ is consumption at date s. $\beta \in (0,1)$ is the discount factor for future utility, and E_t denotes expectation formed at date t.

The factor of production in the economy is land. If an agent i invests $l_{i,t}$ units of land into production at date t, then she harvests the amount $A_{i,t}l_{i,t}$ of goods at date $t+1$. $A_{i,t}$ is the productivity level which the agent knows when she invests the land into production at date t. After the harvest, the agent can start new production.

$A_{i,t}$ changes over time for each agent i. After the production, she receives a stochastic shock to determine her productivity level for the next production. The productivity transition follows an autoregressive process such that

$$\ln(A_{i,t}) = gt + \rho\ln(A_{i,t-1}) + \epsilon_{i,t}, \quad \rho \in (0,1), \quad \epsilon_{i,t} \sim \text{i.i.d.}N(0,\sigma^2). \tag{2.2}$$

g is the exogenous productivity growth rate. ρ is the autoregressive coefficient. g and ρ are constant and common to all the agents. $\rho \in (0,1)$ implies that the productivity level is persistent. $\epsilon_{i,t}$ is the idiosyncratic productivity shock to each agent. Hereafter, we omit the agent index i to simplify the notation.

The production technology is specific to each agent. Once an agent has invested land into production, only she has the necessary skill to obtain the full returns from the production at the next date. This assumption implies that in the credit contract, the lending agent (the lender) can seize only the land of the borrowing agent (the borrower-producer), if the borrower-producer does not cooperate and defaults. We assume that the agents cannot collectively punish the defaulting borrower-producer by excluding her from the credit market, and that the defaulting borrower-producer can borrow from a new lender without any default record and start new production. We also assume that the borrower-producer has a strong bargaining power against the lender, and can reduce the amount of the repayment of the borrowing down to the collateral value of the land for the lender by renegotiating the contract between the timings of the land-investment and the harvest. Anticipating this, the lender lends to the borrower-producer only up to the collateral value of the land. Thus, the pledgeable repayment of the borrowing by the borrower-producer is constrained by

$$b_{t+1} \leq E_t v_{t+1} l_t. \tag{2.3}$$

b_{t+1} is the repayment of the borrowing by the borrower-producer at date $t+1$, and v_{t+1} is the collateral value per unit of land then.

Note that it is the expected value of v_{t+1} that determines the borrowing limit of the borrower-producer, rather than the realized value. To obtain (2.3), we assume that the borrower-producer must work using her specific skill before knowing the shock to the economy, and can renegotiate the credit contract only before then. This assumption implies the credit contract becomes the debt contract. This will matter when we analyze the effect of unexpected shocks in the next section.[14]

To specify the collateral value of land, we assume that the laws in this economy allow the lender to liquidate only a fraction θ ($\in [0,1]$) of the defaulting

[14]The reason of our focus on collateralized debt contracts is that they are a prominent feature of the Japanese credit market, which had led to the non-performing loans problem by the large decline of the collateral value of land after 1990. Modeling collateral lending, we will analyze how the legal restriction on liquidating collateral assets would affect the aggregate production later. The motivation for this analysis is that the Japanese mortgage laws have been restrictive against foreclosure of collateral assets, as described in appendix.

borrower's land every date. Implicitly, we assume that the laws and the court system are inefficient, and do not allow swift foreclosure of the collateral land by the lender. The unliquidated fraction of the collateral land remains at disposal of the borrower until the next date. Then, the lender can liquidate the fraction θ of the remaining collateral land again.[15] Under this assumption, the collateral value of a unit of the collateral land for the lender is[16]

$$v_t = \theta q_t + E_t\left[\frac{(1-\theta)\theta q_{t+1}}{1+r_t} + ...\right] = \theta q_t + (1-\theta)\frac{E_t v_{t+1}}{1+r_t}. \tag{2.4}$$

q_t is the land price at date t, and r_t is the interest rate for the borrowing made at date t. We assume that the interest rate is competitively determined in the credit market, and taken as given by the agents.[17]

2.2.1 Agent's behavior

Under these assumptions, the agent's optimization problem at date t is defined as

$$\begin{aligned} \max_{\{c_s, l_s, b_{s+1}\}_{s=t}^{\infty}} \quad & E_t\left(\sum_{s=t}^{\infty} \beta^{s-t} c_s\right) && (2.5) \\ \text{s.t.} \quad & c_s + q_s(l_s - l_{s-1}) = A_{s-1} l_{s-1} - b_s + \frac{b_{s+1}}{1+r_s} \\ & b_{s+1} \leq E_s v_{s+1} l_s \\ & c_s, l_s \geq 0. \end{aligned}$$

c_s is consumption, l_s is the land invested into the next production, and b_s is the amount of debt-repayment at date s. If b_s is negative, it is the return to the lending of the agent. r_s is the interest rate, q_s is the land price, and v_s is

[15]This assumption about the partial liquidation reflects a difference between the mortgage laws and the bankruptcy laws, such that the mortgage laws themselves do not reduce the borrower's liabilities, and ultimately allow full-liquidation of collateral assets by the lenders after possible delays. The result of the model would be applicable to the effect of bankruptcy laws, as long as they reduce the liabilities of the borrowers and restrict ex-post liquidation of collateral assets by the lenders.

[16]v_t would be the market price of the mortgage right to liquidate a unit of the borrower-producer's collateral land, if the lender sold it to the other agents. Also in each date, the timing of the events is as follows: the harvest of the production, transactions in the land and the credit markets including liquidation of land, and investment of land into production. Thus, the liquidated land is sold to the other agents and invested into production.

[17]Note that the borrower-producers gain the strong bargaining power against the lenders only after making investment. The interest rate is competitively determined in the credit market before they borrow from the lenders and invest into production.

the collateral value of a unit of land at date s. The agent takes the current and expected future values of q_s, r_s and v_s as given. A_{s-1} is the productivity level of the investment made at the previous date. The first constraint is the flow-of-funds constraint. The second constraint is the borrowing constraint. The third constraint is the non-negativity constraint for consumption and the land-investment.

The expectation is taken for the future values of q_s, r_s and v_s, and the productivity levels in the future, given the productivity level for the current investment, A_t. We assume that the agents form rational expectation. As the productivity transition shocks to the agents are idiosyncratic, there is no aggregate uncertainty, and we can replace the expected future values of q_s, r_s and v_s with the realized values in the economy.

We solve the optimization problem (2.5) by calculating the shadow value of the net-worth by the Lagrange multiplier. The agent chooses the behavior that provides the highest shadow value of the net-worth, given the predetermined amount of the net-worth. See appendix for more detail of the optimization.

We can show that the agent's consumption, borrowing (lending), and investment depend on the level of A_t such that[18]

$$(c_t,\ l_t,\ b_{t+1}) = \begin{cases} (\ 0,\ +,\ E_t v_{t+1} l_t\ ) & \text{if } A_t \in (\underline{A}_t^P, \infty). \quad \text{(Borrower-producers.)} \\ (\ 0,\ 0,\ -\ ) & \text{if } A_t \in (\overline{A}_t^C, \underline{A}_t^P). \quad \text{(Lender-savers.)} \\ (\ +,\ 0,\ 0\ ) & \text{if } A_t \in (0, \overline{A}_t^C). \quad \text{(Consumers.)} \end{cases} \tag{2.6}$$

$\underline{A}_t^P > \overline{A}_t^C$. $\underline{A}_t^P$ is the lower-bound of the productivity level to engage in production. $\overline{A}_t^C$ is the upper-bound to consume their net-worth. The marginal agents with $A_t = \underline{A}_t^P$ are indifferent between investment and lending, and those with $A_t = \overline{A}_t^C$ are indifferent between lending and consumption.[19]

[18]We numerically show this under the parameter values calibrated for the Japanese economy which we will specify later.

[19]Their behavior can be written as

$$(c_t,\ l_t,\ b_{t+1}) = \begin{cases} (\ 0,\ +,\ \leq E_t v_{t+1} l_t\ ) & \text{if } A_t = \underline{A}_t^P. \\ (\ \geq 0,\ 0,\ (-\infty, \infty)\ ) & \text{if } A_t = \overline{A}_t^C. \end{cases}$$

The ranges in the brackets imply that the agent is indifferent to the corresponding variable in the specified range.

(2.6) only specifies the signs of c_t, l_t and b_{t+1} for some cases. The values of such variables are determined so as to satisfy the flow-of-funds constraint such that

$$l_s = \frac{A_{s-1}l_{s-1} + q_s l_{s-1} - b_s}{q_t - \frac{E_t v_{t+1}}{1+r_t}} \qquad \text{if } A_t \in (\underline{A}_t^P, \infty). \tag{2.7}$$

$$b_{s+1} = -(1+r_s)(A_{s-1}l_{s-1} + q_s l_{s-1} - b_s) \qquad \text{if } A_t \in (\overline{A}_t^C, \underline{A}_t^P). \tag{2.8}$$

$$c_s = A_{s-1}l_{s-1} + q_s l_{s-1} - b_s \qquad \text{if } A_t \in (0, \overline{A}_t^C). \tag{2.9}$$

The intuition for the agents' behavior in (2.6) is that when an agent has a sufficiently high productivity level ($\geq \underline{A}_t^P$), she finds the production cost implied by the interest rate and the land price relatively cheap, compared to the high rate of return to her investment. Then, she engages in production, and borrows up to the limit to invest as much as possible. If an agent has a medium productivity level ($\in (\overline{A}_t^C, \underline{A}_t^P)$), then she does not find it profitable to invest into production, and exit from or does not enter production. But she expects to become high-productive in the near future, and saves her net-worth to invest into production then. If an agent has a low productivity level ($< \overline{A}_t^C$), then she expects to remain low-productive for the future, because the productivity level is persistent as assumed by the productivity transition process (2.2). Because she discounts future consumption, she finds it optimal to consume all the net-worth at her hand immediately, rather than waiting for becoming high-productive.[20]

Over all, the agents determine their behavior by comparing the rates of return to investment and lending, and the marginal utility of consumption. Because the rates of return to investment and lending depend on the current and future interest rates and land prices, the levels of $\underline{A}_t^P$ and $\overline{A}_t^C$ depend on these prices. We can show that $\underline{A}_t^P$ satisfies

$$\underline{A}_t^P = (1+r_t)q_t - q_{t+1}. \tag{2.10}$$

[20](2.6) implies that if the agents consume, the agents make no investment or lending, so that their net-worth at the next date is zero. Given the borrowing constraint, the flow-of-funds constraint implies that such agents cannot do any economic activity without net-worth from the next-date. Hence, consumption is endogenous exit from the economy. To keep the population of the economically-active agents in the economy positive, we assume there are new-entrants into the economy every date with arbitrarily small amount of net-worth. Hence, the equilibrium obtained above is the limit case when we take the new-entrants' net-worth to the limit at zero in the model. We consider the limit case to simplify the analysis, as generally the net-worth of the new firms is not large in the data, compared to the incumbents.

Hence, the marginal producer's productivity level is equal to the cost of holding land for a period. See appendix for the implicit function to determine $\overline{A}_t^C$.

In (2.6), there appear the two state variables of the agent; A_t and the net-worth. A_t affects the type of the agent's behavior, and the net-worth affects the amount of the investment, the lending, or the consumption through the flow-of-funds constraint of the agent as shown in (2.7)-(2.9). This is because the agent decides the levels of her investment, lending and consumption by comparing the expected rates of return to investment and lending with the marginal utility of consumption. The linear utility function (2.1) makes the marginal utility unity, independent of the consumption level. Also, because the production function is linear to the land-investment, and the interest rate is taken as given, the rates of return to investment and lending are independent of the levels of investment and lending. Therefore, the choice of the levels of investment, lending and consumption becomes discrete such as spending all the net-worth on one of the three types of the actions and nothing on the others, depending on the productivity level of the agent.

2.2.2 Equilibrium conditions

Now, we consider the market clearing conditions of the land and the credit markets. Given the agent's behavior specified above, we can obtain the land market clearing condition as

$$\int_{\underline{\alpha}_t^P}^{\infty} L_t(\alpha_t)d\alpha_t = 1, \tag{2.11}$$

where

$$\alpha_t \equiv \ln(A_t). \tag{2.12}$$

The definitions of $\underline{\alpha}_t^P$ and $\overline{\alpha}_t^C$, respectively, are $\ln(\underline{A}_t^P)$ and $\ln(\overline{A}_t^C)$. $L_t(\alpha_t)$ is the land-investment distribution function at date t. The left-hand side of the equation is the aggregate land demand, and the right-hand side is the aggregate supply. We assume that the supply of land in the economy is fixed and normalized to be 1.

The credit market clearing condition is

$$\int_{\overline{\alpha}_t^C}^{\underline{\alpha}_t^P} S_t(\alpha_t)d\alpha_t = \frac{E_t v_{t+1}}{1+r_t}\int_{\underline{\alpha}_t^P}^{\infty} L_t(\alpha_t)d\alpha_t. \tag{2.13}$$

$S_t(\alpha_t)$ is the lending distribution function at date t. The left-hand side of the equation is the aggregate lending, and the right-hand side is the aggregate borrowing.

To specify the aggregate land demand and the aggregate lending, we need to obtain the land-investment and the lending distribution functions $L_t(\alpha_t)$ and $S_t(\alpha_t)$. To do so, we first aggregate the flow-of-funds constraint of the optimization problem (2.5) to obtain the aggregate net-worth such that

$$\begin{aligned} W_t(\alpha_t) = &\int_{\underline{\alpha}_{t-1}^P}^{\infty} (\exp(\alpha_{t-1}) + q_t - E_{t-1}v_t)L_{t-1}(\alpha_{t-1})f(\alpha_t|\alpha_{t-1})d\alpha_{t-1} \\ &+ \int_{\overline{\alpha}_{t-1}^C}^{\underline{\alpha}_{t-1}^P} (1+r_{t-1})S_{t-1}(\alpha_{t-1})f(\alpha_t|\alpha_{t-1})d\alpha_{t-1}, \end{aligned} \tag{2.14}$$

where

$$f(\alpha_t|\alpha_{t-1}) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{[\alpha_t - gt - \rho\alpha_{t-1}]^2}{2\sigma^2}\right). \tag{2.15}$$

$W_t(\alpha_t)$ is the aggregate net-worth distribution function at date t. $f(\alpha_t|\alpha_{t-1})$ is the conditional probability to have α_t given α_{t-1}, which is defined by the productivity transition process (2.2).

From the aggregate net-worth, we can obtain the land-investment and the lending distribution functions as[21]

$$L_t(\alpha_t) = \frac{W_t(\alpha_t)}{q_t - E_t v_{t+1}/(1+r_t)} \quad \text{for } \alpha_t \in [\underline{\alpha}_t^P,\ \infty), \tag{2.16}$$

$$S_t(\alpha_t) = W_t(\alpha_t) \quad \text{for } \alpha_t \in [\overline{\alpha}_t^C, \underline{\alpha}_t^P). \tag{2.17}$$

Given the parameters of the model (β, g, ρ, σ, θ), and the initial values and functions of $\underline{\alpha}_{-1}^P$, $\overline{\alpha}_{-1}^C$, $E_{-1}v_0$, $L_{-1}(\alpha_{-1})$ and $(1+r_{-1})S_{-1}(\alpha_{-1})$, we define equilibrium as

- $\{c_s, l_s, b_{s+1}\}_{s=0}^{\infty}$ solve the optimization problem (2.5), given the current and future prices $\{q_s, r_s, v_s\}_{s=0}^{\infty}$; Consequently, $\{\underline{\alpha}_s^P,\ \overline{\alpha}_s^C\}_{s=0}^{\infty}$ are derived;

[21]In (2.16) and (2.17), we assume the marginal producers with $\alpha_{t-1} = \underline{\alpha}_{t-1}^P$ borrow up to the limit. This is valid, as this is weakly optimal for these agents, and their size in the economy is zero. The similar argument holds for the marginal lenders with $\alpha_{t-1} = \overline{\alpha}_{t-1}^C$.

- $\{L_s(\alpha_s), S_s(\alpha_s)\}_{s=0}^{\infty}$ are recursively determined by (2.14)-(2.17), given $\{q_s, r_s, E_s v_{s+1}, \underline{\alpha}_s^P, \overline{\alpha}_s^C\}_{s=0}^{\infty}$, and the initial values and functions;
- $\{q_s, r_s, v_s\}_{s=0}^{\infty}$ are determined in order to satisfy the market clearing conditions (2.11) and (2.13), given the definition of v_s in (2.4);
- the agents form rational expectation about the future prices $\{q_s, r_s, v_s\}_{s=t+1}^{\infty}$ at every date t;
- there is no bubble in the land price, so that at every date t,

$$\lim_{s\to\infty} E_t \left[\frac{q_s}{\prod_{h=t}^{s-1}(1+r_h)} \right] = 0. \tag{2.18}$$

The goods market clears in equilibrium by Walras' Law.

As described above, the productivity transition shocks to the agents are idiosyncratic, and there is no aggregate uncertainty in the economy. By the rational expectation, we can replace the expected values of the future prices with the realized values in equilibrium.

2.3 Dynamic analysis

2.3.1 Definition of TFP and the renegotiation of the debts after unexpected shocks

Using this model, we analyze how changes of the parameter values affect the land allocation and the average productivity in the economy measured by total factor productivity (TFP). TFP is the average productivity of the aggregate factors of production. To define TFP in our model, we first derive the aggregate output at date t as

$$Y_t = \int_{\underline{\alpha}_{t-1}^P}^{\infty} \exp(\alpha_{t-1}) L_{t-1}(\alpha_{t-1}) d\alpha_{t-1}. \tag{2.19}$$

As the land is the sole factor of production in our model, we define TFP by dividing the aggregate output by the supply of land in the economy. Because the supply of land is fixed to be 1, TFP coincides with the aggregate output;

$$TFP_t = Y_t. \tag{2.20}$$

(2.19) and (2.20) imply that TFP is the weighted average of the individual productivity levels of the producers.

In the following analysis, we consider the dynamic response of the economy to unexpected shocks. In some cases, the land price declines so much that the output and the asset value owned by the borrower-producer become smaller than the value of her debt after the shock. In this case, the lender can take all the borrower-producer's output, and liquidate her land up to the fraction θ. This is implied by the assumption above such that the borrower-producer has worked for production with her specific skill before knowing the shock to the economy. After the shock, the lender does not need cooperation from the borrower-producer to produce the output, and can fully enforce the debt repayment up to the legal restriction on liquidating the collateral assets. Over the unpaid debt with the unliquidated fraction of the land, the lender and the borrower-producer renegotiate. Without cooperation of the borrower-producer, the lender would be able to only gradually liquidate the land from the next date under the legal restriction on liquidating collateral assets. In this case, the payoff to the lender at the next date $t+1$ would be $E_t v_{t+1}$ per unit of the unliquidated fraction of the land. By the strong bargaining power of the borrower-producer, we assume that the borrower-producer can reduce the repayment for the unpaid debt at the next date down to this value in the renegotiation. This assumption implies that if the size of the unliquidated fraction of the land is $(1-\theta)l_{t-1}$ at date t, then the lender rolls over the amount $E_t[v_{t+1}/(1+r_t)](1-\theta)l_{t-1}$ of the debt, and writes off the rest of the unpaid debt. We assume that all the lenders in the economy suffer from the loan losses at the same rate to their lending after the unexpected shock.[22]

2.3.2 Effect of restricting collateral liquidation under an aggregate productivity slowdown

In this subsection, we analyze how different levels of the legal restriction on liquidating collateral assets affect the dynamics of the economy under an exoge-

[22]Implicitly, we can consider that the banks intermediate between the borrower-producers and the lenders without any cost, and they equally apply the loan losses to the deposits of the lenders.

nous aggregate productivity slowdown. This analysis is related to Hayashi and Prescott (2002), which analyze the effect of the aggregate productivity slowdown in the standard exogenous growth model for the long stagnation in Japan after 1990. We extend their analysis by taking into account the slow liquidation of collateral assets in the Japanese credit market. We especially highlight the Japanese mortgage laws, which had been restrictive against foreclosure of the collateral land and ended up with the reform in 2003. We investigate how such legal restriction on liquidating collateral assets would interact with the productivity slowdown.

We consider an unexpected permanent decline of g at date 0, and numerically compute the dynamics of the model after the shock. g is the exogenous productivity growth rate common to all the agents. We adopt the base-line parameter values in Table 2.1, and the change of g in Table 2.2. We choose the base-line parameter values by matching the steady-state values in the model with the Japanese macro and micro data before the productivity slowdown after 1990. For existence of the steady state, we can show that there exists a balanced-growth path with the growth rate $\exp(g/(1-\rho))$.

Table 2.1: The base-line parameter values for numerical calculation.

β	g	ρ	σ	θ
0.9510	0.0076	0.614	0.0797	0.1

Table 2.2: The experiment 1

An unexpected permanent shock to g at date 0	$0.0076 \rightarrow 0.0058$

In Table 2.1, the values of ρ and σ are taken from Fukao and Kwon (2004)'s estimation of the productivity transition process (2.2) for the Japanese manufacturing firms in the 1990's. We use the estimates of ρ and σ for their earliest subsample period in the 1990's.[23] Then, we choose the value of g to match the long-run TFP growth rate in the model, $\exp(g/[1-\rho])-1$, with 1.98%, which was the average TFP growth rate for 1975-1990.[24] The benchmark values of β

[23]It is 1994-1997. Ideally, we should use the estimates for the 1980's, but their panel data only start from 1994.

[24]This is measured by the Solow residual. The estimate is different from Hayashi and

and θ are chosen to match the interest rate and the collateralizable fraction of the current land value[25] with 5% and 77% at the steady state, given the other base-line parameter values. 5% is the average real interest rate in the early 1980's. We choose this period because of the oil shocks in the 1970's and the asset-price bubble in the late 1980's. For the collateralizable fraction of the current land value, several banks issue their policies on this fraction in their annual reports. In many cases, their fractions are 60%. The Finacial Service Agency also announces the guideline for their inspectation of the banks' balance sheets, and set 70% as the healthy collateralizable fraction of the current land value. I choose $\theta = 0.1$ to match with the conservative estimate of the collateralizable fraction, 77%. In Table 2.2, we choose the change of g to make the long-run TFP growth rate decline from 1.98% to 0.95%. 0.95% is the average TFP growth rate in Japan for 1990-1998.[26]

We assume that the economy is at the steady state before the shock at date 0. When we numerically compute the dynamics of the model after the shock, we calculate the equilibrium sequence of $\{q_s, r_s, \underline{\alpha}_s^P, \overline{\alpha}_s^C\}_{s=0}^{\infty}$ which converges to the new steady state under the new value of g. In the iteration to find this sequence, we compute the distribution functions $L_t(\alpha_t)$ and $S_t(\alpha_t)$ by approximating the integral in (2.14) by Gaussian-quadrature. See appendix for more detail of the computation.

Figure 2.5 shows the dynamics of TFP_t, q_t and r_t. To show the dynamics of the economy under different levels of θ, we choose high, middle and low values of θ, which are 1, 0.5 and 0.1, respectively.[27] First, the land price immediately drops after the shock, as the agents in the economy form rational expectation

Prescott (2002). This is because they use GNP as the output and include the capital stock oversea owned by Japanese. Here, we use GDP and the domestic capital stock in the calculation.

[25]For example, the collateralizable fraction of the current land price is 50%, if you can borrow \$5000 against the collateral land worth \$10000 at the current market value. In our model, this fraction is given by $[v_{t+1}/(1+r_t)]/q_t$. We can show that this is $\theta/\{(1+r_t)/\exp(g/[1-\rho]) - (1-\theta)\}$ at the steady state.

[26]This figure is measured by the Solow residual. We use the data of the 1968 Standard of the National Account. The sample period ends in 1998, because the government changed the standard of the data in this year. Some data necessary to calculate the Solow residual is not available for the period before 1990 under the new standard. We choose to use the old standard to keep the consistency of the estimates of the TFP growth rates between the periods before and after 1990.

[27]The collateralizable fraction of the current land value under $\theta = 1, 0.5$ and 0.1, respectively, are around 97%, 95%, and 77% at the steady state.

and take into account that the productivity slowdown reduces the future land productivity. This reduces the asset value owned by the borrower-producers, and leads to their default. The enforcement of their debt repayments shifts the net-worth allocation to the lenders. Because the borrower-producers are the more-productive agents in the economy, this implies that the net-worth allocation is biased toward the less-productive agents. This shift of the net-worth allocation is accompanied with the shift of the land allocation to the less-productive agents. This is because the borrowing constraint prevents the borrower-producers from financing all the cost of purchasing land by borrowing, and requires them to make down-payments from their own net-worth.

The loss of the net-worth of the more-productive agents prevents them from buying as much land as before, and propagates the reduction of the land price. By this propagation, the cost of investment falls more proportionally than the decline of the TFP level.[28] This increases the rate of return to investment for each agent, which induces the less-productive agents to start engaging in production. Consequently, the threshold productivity level to engage in production, $\underline{A}_t^P$, is lowered, as shown in Figure 2.6. This change implies that even if an incumbent producer becomes low-productive by the productivity transition process (2.2), she does not exit from production. Similarly, the low-productive lenders start entering production.

The shift of the land allocation toward the less-productive producers causes an endogenous propagation of the exogenous productivity shock, and the temporary drop of the TFP level from the new long-run trend. Note that the borrowing constraint plays the important role in this endogenous decline of the TFP level as described above. If we considered a perfectly competitive market model like Hayashi and Prescott (2002), then the TFP trend would immediately kink after the shock without any temporary drop.

Over time, the more-productive agents recover the net-worth by the high rates of return to their investment, and accumulate land for their production. Then, the TFP level gradually returns to the new long-run trend.[29] The real in-

[28]Kiyotaki and Moore (1997) find this mechanism. Our contribution is to find that this mechanism explains why the low-productive producers start remaining in production under the productivity slowdown.

[29]By this recovery, the TFP growth rate overshoots the new steady-state level after the

terest rate falls because the decline of the future land price reduces the collateral value of land, and then the borrowing limit of the agents.

In the comparison across the different levels of θ, the lower the level of θ, the smaller the temporary drop of the TFP level from the new trend. This is because the lower value of θ prevents the enforcement of the debt repayments after the decline of the land price. This lets the more-productive producers retain more net-worth, and mitigates the shift of the net-worth allocation to the less-productive agents. Figure 2.7 shows the net-worth distribution across the agents, and that the loss of the net-worth of the more-productive producers becomes less as θ is lower. On the flip side of the coin, the lenders suffer from the larger loan losses as θ is lower, as shown in Table 2.3.[30]

Table 2.3: The loan-loss rate at date 0 after the shock to g.

θ	1	0.5	0.1
Loan-loss rate	21.82%	22.41%	25.69%

2.3.3 Effect of intensified restriction on collateral liquidation

In this subsection, we analyze the dynamics of the economy after the legal restriction on liquidating collateral assets is intensified. The motivation for this analysis is the supreme court cases in 1989 and 1991, which strengthened the legal restriction on liquidating collateral assets. These cases reinforced the protection of the borrowers' lease contracts on the collateral land and buildings, which would delay foreclosure of the collateral assets by the lenders.[31] Using

sharp drop at date 0.

[30]The loan losses at date 0 are caused by the unexpected decline of the land price. From date 0 onward, the agents rationally expect the future land prices, so that the lenders lend only up to the amounts under which they can avoid the borrowers' default.

[31]Under the mortgage laws, foreclosure of collateral properties rescinds the lease contracts on the collateral properties, if the lender has set the mortgage right before the lease contracts. But until the reform in 2003, the laws had protected the preceded lease contracts against foreclosure for 3-5 years. One way to prevent the borrower from abusing this regulation was to have the lease contract between the borrower and the lender. However, the supreme court in 1989 denied the validity of such a contract unless there was actual use of the property by the lender. See appendix for more detail of the supreme court cases in 1989 and 1991.

Table 2.4: The experiment 2

An unexpected permanent shock to θ at date 0	$0.1 \rightarrow 0.031$

our general-equilibrium model, we analyze how the economy responds to such a legal shock, focusing on the level and the growth of TFP.

We consider an unexpected permanent decline of θ at date 0 from 0.1 to 0.031. We use the base-line parameter values in Table 2.1 for the other parameters in the numerical calculation. This shock replicates that the average TFP growth rate dropped from 1.98% to 1.11% for 1990-1995 in Japan. Here, we consider the decline of the average TFP growth rate for 5 years after the shock rather than a decade, provided that the shock occurred in 1990. This is because the productivity slowdown under the shock to θ is not permanent, as will be shown later. Instead, this exercise shows that the legal shock to the credit market endogenously causes a medium-term productivity slowdown. Even though $\theta = 0.031$ might seem too low, we can calculate that the corresponding collateralizable fraction of the land value is 51% at the steady state, given the other base-line parameter values in Table 2.1. This figure is not immediately unrealistic.

We assume that the economy is at the steady state before the shock at date 0, and calculate the equilibrium dynamics of the economy converging to the new steady state under the new value of θ. Figure 2.8 shows the dynamics of TFP_t, q_t and r_t. The TFP level endogenously drops after the shock, and its trend also levels down. This is because the decline of θ reduces the collateral value of land and the borrowing limit of the agents. This in turn limits the leverage the agents can take, and decreases the rates of return to unit net-worth invested by the borrower-producers. Therefore, their net-worth accumulation slows down. As they are the more-productive agents in the economy, consequently the land allocation shifts to the less-productive agents, which causes the level-down effect on the TFP level of the economy.

The land price responds to this level-down of the future land productivity, and immediately drops after the shock. This lowers the cost of investment. Then, it becomes viable for the less-productive agents to remain in and enter production after the shock, and the level of $\underline{A}_t^P$ declines, as shown in Figure 2.9. This entry and exit effect contributes to the endogenous level-down effect on the TFP level in the economy.

The real interest rate falls after the shock because of the reduced borrowing limit of the borrower-producers. This directly reduces the borrower-producers' borrowing. Then, the real interest rate decreases to clear the credit market.

2.3.4 Comparison between the exogenous aggregate productivity slowdown and the endogenous slowdown under the intensified restriction on liquidating collateral assets

The main feature of the dynamics after the decline of θ is similar to the case of the shock to g. Both of the shocks cause the declines of the TFP growth rate, the land price and the real interest rate. But there are differences as well. One of the differences between the two shocks is that the TFP level only gradually shifts to the new trend after the shock to θ, while it undershoots the new trend after the shock to g. This is because the decline of θ only gradually shifts the land allocation to the less-productive agents by limiting the leverage the more-productive agents can take. Also, the decline of θ lets the borrower-producers renegotiate the debt repayments at date 0 under the intensified restriction on liquidating collateral assets.[32] This shifts the net-worth allocation to the more-productive agents at the impact of the shock to θ, as shown in Figure 2.10, and contributes to the slow shift of the land allocation to the less-productive agents after the shock. In contrast, under the shock to g, the unexpected decline of the land price reduces the asset value owned by the borrower-producers, and the enforcement of their debt repayments immediately shifts the net-worth allocation to the less-productive agents at the impact of the shock. This in turn causes the immediate shift of the land allocation to the less-productive agents after the shock.

The gradual decline of the TFP level after the shock to θ is transformed into the continuous stagnation of the TFP growth rate. In contrast, the fluctuation

[32]The loan-loss rate is 32.44%.

of the TFP growth rate is larger under the decline of g. The TFP growth rate sharply declines after the shock to g, but quickly jumps up and overshoots the new long-run TFP growth rate. The overshooting of the TFP growth rate occurs, as the TFP level recovers from the initial drop to the new trend after the shock.

Note that the shock to g directly reduces the long-run TFP growth rate $\exp(g/[1-\rho])$, which is implied by the productivity transition process (2.2), while the shock to θ causes the level-down of the TFP trend without affecting its long-run growth rate. Hence, the shock to g reduces the TFP level much larger than the shock to θ in the long run. This is transformed into the larger decline of the land price after the shock to g than θ, for the given level of the decline in the TFP growth rate at the impact of the shock. This feature again indicates that the fluctuation of the economy after the shock is larger under the shock to g than θ.

2.3.5 Effect of an increase in the heterogeneity of the agents' productivity levels

In this subsection, we analyze the dynamics of the economy after an increase in the heterogeneity of the agents' productivity levels. This is motivated by the empirical analysis of Fukao and Kwon (2004). They estimate the productivity transition process (2.2) for the Japanese manufacturing firms by the panel data in the 1990's, and find that the level of ρ increased during the 1990's, and that the productivity gap across the firms also increased. Given this observation, we analyze the effect of such a structural change of the firm dynamics and how the level of the legal restriction on liquidating collateral assets affects the effect.

We consider an unexpected permanent increase of ρ at date 0 from 0.614 to 0.765. These values are estimated by Fukao and Kwon for 1994-1997 and 1998-2001, respectively. The immediate effect of this change is an increase of the long-run growth rate of the economy $\exp(g/[1-\rho])$. To analyze the level effect of the increase of ρ, we set $g = 0$. We use the base-line parameter values in Table 2.1 for the other parameters. We assume that the economy is at the steady state before the shock at date 0, and calculate the equilibrium dynamics

Table 2.5: The experiment 3

An unexpected permanent shock to ρ at date 0 under $g = 0$	$0.614 \rightarrow 0.765$

of the economy converging to the new steady state under the new value of ρ.

Figure 2.11 shows the dynamics of TFP_t, q_t and r_t. The TFP level rises after the shock. This is because the more-productive agents become more likely to remain in production after the shock, and increase their net-worth accumulation through reinvesting their net-worth into their high-productive production. This shifts the land allocation toward the more-productive agents and raises the TFP level. The rise of the TFP level also raises the land price.

The real interest rate declines because the size of the lenders increases in the economy. The rise of the land price raises the cost of investment and then $\underline{A}_t^P$, as shown in Figure 2.12. Also, $\overline{A}_t^C$ decreases, because the more net-worth accumulation by the more-productive agents enhances the payoff for the less-productive agents to wait and lend their net-worth until they become high-productive and engage in production.

However, the comparison across the different levels of θ shows that the lower the level of θ, the smaller the positive effect of an increase in ρ on the TFP level. This is shown by the TFP growth rate in Figure 2.11. The reason is that the lower level of θ reduces the collateral value of land and limits the leverage the agents can take. This weakens the effect of an increase in ρ to facilitate the net-worth accumulation of the more-productive producers.

2.4 The risk-taking of the producers under the productivity slowdown

2.4.1 The motivation of the analysis

In this section, we introduce idiosyncratic delay of each agent's production to analyze how the risk-taking behavior of the producers changes under the productivity slowdown. The motivation of this analysis is the decline of the firms' borrowing observed in Japan in the 1990's.

One possible reason for the decline of the borrowing is that the low rates of return to investment induced the firms to avoid taking risk of ex-post debt-overhang and refrain from borrowing. To this point, readers may argue that the productivity slowdown under the decreasing returns to scale in production could be the important factor for the decline of the firms' credit demand, rather than the risk-concern of the producers. But as Hayashi and Prescott (2002) note, the firms kept financing their investment by the internal reserves in the 1990's. The decreasing returns to scale in production do not explain why the firms spent their own funds on investment while cutting back their borrowing. Hence, we need to consider the risk associated with borrowing in this line of explanation.

Another explanation for the decline of the firms' borrowing is that the fall of the collateral value of the assets reduced the borrowing limit for the firms, and constrained their borrowing. By using our model, we will clarify which cause is consistent with the productivity slowdown.

2.4.2 Extension of the model

The factor of production in the economy is land as before. If an agent invests l_t units of land into production at date t, then she harvests the amount $A_t l_t$ of goods at date $t+1$ with probability μ, but the harvest may be delayed until date $t+2$ with probability $1-\mu$. If the delay occurs, the agent needs to reinvest land into production. If she reinvests l_{t+1} of land at date $t+1$, then she will harvest the amount $A_{t+1}l_{t+1}$ of goods at date $t+2$. Note that the delay only postpones the timing of the production. Even though the agent has failed to produce the output, she learns from the failure and can harvest the output at the next date. Since the harvest occurs with probability 1 after the delay, the reinvestment is ex-post more efficient than the initial investment into new production under the same productivity level. Thus, the shock of delay is a type of the liquidity shocks.

We assume that the learning from the delay of production is only applied up to the size of the previous investment, and that the reinvestment l_{t+1} cannot exceed the initial investment level l_t.[33] We assume that the shock of delay is

[33]This assumption implies that the agents with delayed production cannot start new production by any means. In this sense, they become infertile by the negative productivity shock.

idiosyncratic.

If the agent does not have delayed production, then her productivity level changes subject to the productivity transition process (2.2). If the production is delayed at date $t+1$, the agent can continue the production she initiated, and does not receive the stochastic shock. Her productivity level grows by[34]

$$\ln(A_{t+1}) = \frac{g}{1-\rho} + \ln(A_t). \tag{2.21}$$

(2.2) implies that $g/(1-\rho)$ is the long-run growth rate of the average productivity level in the economy. By (2.21), we assume that the productivity level of the agent with delayed production keeps up with the exogenous productivity growth in the rest of the economy. Table 2.6 summarizes the production function.

Table 2.6: Production function

date t	date $t+1$		date $t+2$
l_t of land	$\nearrow$ (w.p. μ)	$A_t l_t$ of goods	
(A_t is known.)	$\searrow$ (w.p. $1-\mu$)	No goods l_{t+1} of land ($\leq l_t$)	$\rightarrow$ $\exp(\frac{g}{1-\rho})A_t l_{t+1}$ of goods

We assume that it is too costly to write contingent contracts, and only consider the debt contracts in the credit market. The other assumptions of the model do not change. See appendix for the agent's optimization problem and the equilibrium conditions. We can show that agents without delayed production

They can only continue the production by using their experience from the failure.

[34] Note that if the lenders do not invest into production at the current date, then they can start new production without delayed production at the next date. Thus, the assumption above implies that these lenders will have the productivity transitions defined by (2.2). If the lenders invest into their own production while lending, and have delay at the next date, then their productivity transitions will be subject to (2.21).

behave such that

$$(c_t,\ l_t,\ b_{t+1}) =$$

$$\begin{cases} (\ 0, & +, & E_t v_{t+1} l_t &) & \text{if } A_t \in (\underline{A}_t^{SPH}, \infty). & \text{(Constrained producers.)} \\ \left(\ 0, \right. & +, & E_t\left[\dfrac{v_{t+2}}{1+r_{t+1}}\right] l_t & \left. \right) & \text{if } A_t \in (\underline{A}_t^{SPL}, \underline{A}_t^{SPH}). & \text{(Unconstrained producers.)} \\ (\ 0, & 0, & - &) & \text{if } A_t \in (\overline{A}_t^{SC}, \underline{A}_t^{SPL}). & \text{(Lender-savers.)} \\ (\ +, & 0, & 0 &) & \text{if } A_t \in (0, \overline{A}_t^{SC}). & \text{(Consumers.)} \end{cases} \tag{2.22}$$

$\underline{A}_t^{SPH} > \underline{A}_t^{SPL} > \overline{A}_t^{SC}$.[35] $\underline{A}_t^{SPH}$ is the lower-bound of the productivity level to borrow up to the limit. $\underline{A}_t^{SPL}$ is the lower-bound to engage in production. $\overline{A}_t^{SC}$ is the upper-bound to consume their net-worth. The values of $c_t,\ l_t,\ b_{t+1}$ are

$$l_s = \begin{cases} \dfrac{y_s + q_s l_{s-1} - b_s}{q_t - \frac{E_t v_{t+1}}{1+r_t}} & \text{if } A_t \in (\underline{A}_t^{SPH}, \infty), \\ \dfrac{y_s + q_s l_{s-1} - b_s}{q_t - E_t\left[\frac{v_{t+2}}{(1+r_t)(1+r_{t+1})}\right]} & \text{if } A_t \in (\underline{A}_t^{SPL}, \underline{A}_t^{SPH}), \end{cases} \tag{2.23}$$

$$b_{s+1} = -(1+r_s)(y_s + q_s l_{s-1} - b_s) \qquad \text{if } A_t \in (\overline{A}_t^{SC} \underline{A}_t^{SPL}), \tag{2.24}$$

$$c_s = y_s + q_s l_{s-1} - b_s \qquad \text{if } A_t \in (0, \overline{A}_t^{SC}), \tag{2.25}$$

where

$$y_s = \begin{cases} A_{s-1} l_{s-1}, & \text{if the production succeeds.} \\ 0, & \text{if the production is delayed.} \end{cases}$$

The difference of (2.22) from the previous case (2.6) is that the more-productive producers borrow up to the limit as before, but the less-productive producers do

[35]As before, the levels of $\underline{A}_t^{SPH}$, $\underline{A}_t^{SPL}$ and $\overline{A}_t^{SC}$ depend on the current and future interest rates and land prices. See appendix for the implicit functions to determine these levels. The marginal agents with $A_t = \underline{A}_t^{SPH}$ and $\underline{A}_t^{SPL}$ are indifferent between investment and lending, and those with $A_t = \overline{A}_t^{SC}$ are indifferent between lending and consumption. These marginal agents' behavior can be written as

$$(c_t,\ l_t,\ b_{t+1}) = \begin{cases} \left(\ 0, \right. & +, & \left[E_t\left[\frac{v_{t+2}}{1+r_{t+1}} l_t\right], E_t v_{t+1} l_t\right] & \left. \right) & \text{if } A_t = \underline{A}_t^{SPH}. \\ \left(\ 0, \right. & \geq 0, & \leq E_t\left[\frac{v_{t+2}}{1+r_{t+1}} l_t\right] & \left. \right) & \text{if } A_t = \underline{A}_t^{SPL}. \\ (\ \geq 0, & 0, & (-\infty, \infty) &) & \text{if } A_t = \overline{A}_t^{SC}. \end{cases}$$

The ranges in the brackets imply that the agent is indifferent to the corresponding variable in the specified range.

not do so. This is because if they borrow much at the current date and get hit by the delay of production at the next date, then they will have debt-overhang and be forced to sell the land to repay the debts. More specifically, the flow-of-funds constraint and the borrowing constraint of the delayed producer at date $t+1$ imply

$$\begin{aligned} l_{t+1} &\leq \frac{q_{t+1}l_t - b_{t+1}}{q_{t+1} - E_{t+1}v_{t+2}/(1+r_{t+1})} \\ &= (1-\theta)l_t, \qquad \text{if } b_{t+1} = E_t v_{t+1} l_t. \end{aligned} \tag{2.26}$$

The right-hand side of the first line is the amount of the net-worth divided by the minimum down-payment to buy a unit of land. The last equality is obtained by substituting $b_{t+1} = E_t v_{t+1} l_t$ and (2.4).[36] (2.26) implies that $l_{t+1} < l_t$, given $\theta > 0$. Thus, the producers have to liquidate their land if they borrow up to the limit and get hit by the delay of production. This liquidation of land is costly, as the delayed producer would be able to harvest the production at the following date if they reinvested the liquidated land into delayed production. The less-productive producers do not take the risk of this debt-overhang, because they do not have sufficiently large return from the immediate success of the initial investment. Therefore, the less-productive producers refrain from borrowing up to the limit.

This argument immediately implies that all the producers with delayed production borrow up to the limit, because they do not have the delay of production again in the next date. Also, we can show that all the agents with delayed production continue to engage in production under the parameter values we consider. Thus, their behavior is similar to (2.6). See appendix for the specification of their behavior.

2.4.3 Reason for the different risk-taking across the producers

In this subsection, we more analytically explain the reason that the less-productive producers find it optimal to borrow less than the limit, while the more-productive

[36]As described above, the agents form rational expectation, and the values of the expected prices are replaced with the realized values. Then, we can obtain the last equality by substituting (2.4).

producers borrow up to the limit. For this purpose, we compare the rates of return to investment and lending. Note that the agent chooses the action accompanied with the highest rate of return, given the predetermined net-worth.

We focus on the steady state to analytically compare the rates of return to investment and lending.[37] For the producers without delayed production at the steady state, we define $\eta_{1,t}$, $\eta_{2,t}$ and $\eta_{3,t}$, respectively, as the rates of return for borrowing up to limit, borrowing just up to the level where the agent can fully reinvest her land when she has the delay of production, and lending. We can calculate them such that

$$\eta_{1,t} = \frac{E_t \left[\mu(A_t + q_{t+1} - v_{t+1}) + (1-\mu)(1-\theta)\beta \left(\exp\left(\frac{g}{1-\rho}\right) A_t + q_{t+2} - v_{t+2} \right) \right]}{q_t - \frac{E_t v_{t+1}}{1+r_t}}, \tag{2.27}$$

$$\eta_{2,t} = \frac{E_t \left[\mu \left(A_t + q_{t+1} - \frac{v_{t+2}}{1+r_{t+1}} \right) + (1-\mu)\beta \left(\exp\left(\frac{g}{1-\rho}\right) A_t + q_{t+2} - v_{t+2} \right) \right]}{q_t - E_t \left[\frac{v_{t+2}}{(1+r_t)(1+r_{t+1})} \right]}, \tag{2.28}$$

$$\eta_{3,t} = 1 + r_t. \tag{2.29}$$

For each of (2.27) and (2.28), the first term of the numerator is the expected rate of return to investment from the immediate success. The second term is the one from the delay of production. This term is multiplied by $(1-\theta)$ in (2.27), because the fraction θ of land will be liquidated if production is delayed. The denominator is the required down-payment for unit land-investment.

Figure 2.13 draws (2.27)-(2.29) as functions of A_t, given the current and future interest rates and land prices. First, we can show that the slope of (2.27) is larger than (2.28), if $\mu > 0.5$.[38] This implies that the more-productive producers prefer to borrow up to the limit, and that only the less-productive producers may find it optimal to borrow less than the limit.

[37] See appendix for the expression of the rates of return to investment and lending in the general dynamics, which are described as the shadow value of the net-worth. They depend on the expected future shadow-values of the net-worth at different dates, which we need to numerically calculate. However, at the steady state, the expected future shadow-values of the net-worth stay constant for each productivity level, so that we can normalize the rates of return to investment and lending by the constant value in the comparison across them, given the current productivity level.

[38] At the steady state, the slope of (2.27) is larger than (2.28) if and only if $\mu > \beta(\theta+r)/[1+\beta(\theta+r)]$. As we can show $\beta(1+r) < 1$ in any equilibrium, $\mu > 0.5$ is sufficient to satisfy this inequality for all $\theta \in [0,1]$.

Second, the intercept of (2.28) is higher than (2.27). This is because less borrowing and investment raise the rate of return to investment, when the productivity level is low and investment is inefficient. Then, we can show that (2.28) surpasses $1+r_t$ at the lower value of A_t than (2.27). Denote such threshold levels of A_t for (2.27) and (2.28) by A_t' and A_t'', respectively. To analytically confirm $A_t' > A_t''$, we can obtain from (2.27)-(2.29) that

$$\left[\mu + (1-\mu)\beta \exp\left(\frac{g}{1-\rho}\right)\right](A_t' - A_t'')$$
$$= (1-\mu)\theta\left[\beta\left(\exp\left(\frac{g}{1-\rho}\right)A_t' + q_{t+2} - v_{t+2}\right) - \left(q_{t+1} - \frac{v_{t+2}}{1+r_{t+1}}\right)\right]. \quad (2.30)$$

The left-hand side is the gap between the expected productivities of investment under A_t' and A_t''. The right-hand side is the expected return to reinvesting the liquidated fraction θ of land when the agent with A_t' is hit by the delay of production.[39] (2.30) implies that for borrowing up to the limit to be viable, the productivity level must be sufficiently high to compensate the expected loss from losing the profitable opportunity to reinvest into delayed production. This is consistent with the intuitive explanation for (2.22) provided in the previous subsection.

Note that considering delay of production under credit market frictions lets us analyze the producers' risk-concern without adding the producers' concave utility function to the model. This is a strength of our model, as we can keep focusing on the producers' profit maximization to induce their behavior.

2.4.4 Dynamic analysis of the producers' risk-taking behavior

In this subsection, we describe the dynamic responses of the economy after the unexpected shocks to g and θ. We use the base-line parameter values in Table 2.1 and consider the shocks to g and θ in Tables 2.2 and 2.4. We choose to use $\mu = 0.9$. The qualitative result of the analysis does not change by different levels of μ.[40]

[39] We can show that the right-hand side of (2.30) is positive at the steady state, if $\mu < \beta(1+r_t)$. $\beta(1+r_t)$ is very close to 1 in equilibrium, under the base-line parameter values shown in Table 2.1.

[40] Given the base-line parameter values, this parameter value implies that the net-worth share of the unconstrained producers among all the producers is around 12% at the steady

The responses of the economy are similar to the ones depicted in Figures 2.5-2.10. Here, we focus on the size of the unconstrained producers, who do not borrow up to the limit in the economy. Figure 2.14 shows the aggregate borrowing-output ratio, the net-worth share of the unconstrained producers among all the producers, and the ratio between $\underline{A}_t^{SPH}$ and $\underline{A}_t^{SPL}$ under the shock to g. The figure shows that the size of the unconstrained producers in the economy decreases, so that the decline of the borrowing-output ratio is not due to the risk-averse behavior of the producers, but the reduction of the collateral value of land and the borrowing limit of the producers. The mechanism of this result is that the borrowing constraint propagates the negative effect of the aggregate productivity slowdown on the land price. Then, the cost of investment decreases more than the decline of the TFP level. This increases the rate of return to investment for each producer. The higher rates of return to investment induce more of the producers to take the maximum leverage.

Figure 2.15 shows the similar figures to Figure 2.14 under the shock to θ. The figure shows that the reduction of the collateral value of land by the decline of θ decreases the borrowing limit of the producers and then the borrowing-output ratio. Also, the size of the unconstrained producers decreases. This is because the decline of θ allows less liquidation of the collateral assets in case of the delay of production, and constrains the risk-taking of more producers. This direct effect increases the size of the constrained producers in the economy.

2.5 Implication for the long stagnation in Japan after 1990

2.5.1 Endogenous productivity slowdown

Here, we discuss implication of the dynamic analysis for the long stagnation in Japan after 1990. Hayashi and Prescott (2002) show the importance of the productivity slowdown to account for the long stagnation. In our analysis of

state. To the author's knowledge, there is no estimate for the net-worth or asset share of the unconstrained firms in the Japanese economy. A close estimate is Ogawa and Suzuki (2000), but they only reports the estimated percentage of the unconstrained firms among the listed Japanese firms.

the shock to g, we show that the exogenous aggregate productivity slowdown is endogenously propagated by the shift of the net-worth to the low-productive producers.

Also in the analysis of the shock to θ, we show that the intensified legal restriction on liquidating collateral assets limits the leverage that the agents can take, and hampers the more-productive agents from accumulating land. This endogenously reduces the TFP level, and causes the decline of the TFP growth rate in transition to the new TFP trend. Hence, our model implies that the legal shock to the restriction on collateral liquidation such as the supreme court cases in 1989 and 1991 endogenously contributed to the productivity slowdown. This result indicates that by considering the role of the credit market in the economy, we need to less rely on the exogenous productivity shock to account for the productivity slowdown in Japan in the 1990's.

2.5.2 Remaining low-productive "zombie" firms under the productivity slowdown

We show that the low-productive producers start remaining in production under the productivity slowdown, whether the slowdown is caused by the shock to g or θ. This negative exit effect further propagates the decline of the TFP level in the productivity slowdown. This result is in line with the empirical analysis of "zombie firms", such that the low-productive firms remained in production being financed by the banks, and reduced the productivity level of the economy in the 1990's.[41] In our model, this phenomenon occurs because the productivity slowdown causes the falls of the land price and the interest rate, and reduces the cost of investment for the producers. This makes it viable for the less-productive agents to engage in production.[42]

This mechanism in our model contrasts with the analysis of Caballero, Hoshi and Kashyap (2004), which argue that the insolvent banks kept financing the

[41] Kobayashi, Saita, and Sekine (2002), Hosono and Sakuragawa (2003), Peek and Rosengren (2003), and Caballero, Hoshi, and Kashyap (2004).

[42] Barseghyan (2002) shows a similar mechanism for entry of producers by the fall of the manager wage. However, he does not consider any shock to the agents' productivity levels. In this chapter, we show that the less-productive producers remain in production even under the exogenous productivity slowdown.

insolvent unproductive firms to hide their loan losses, and letting them stay in production in the 1990's. In their model, this phenomenon would raise the input demand and the input prices. Thus, they need to consider other shocks to explain the observed declines of the interest rate and the land price in the 1990's, as shown in Figures 2.4 and 2.16. Our work suggests that the productivity shock and the credit market shock are among such shocks consistent with the finding of "zombie firms" remaining in production.[43]

2.5.3 Lack of large fluctuation at the onset of the productivity slowdown

Our analysis of the shock to g shows that under the exogenous aggregate productivity slowdown, the downward fluctuation of the TFP level and the decline of the land price are large at the onset of the shock, if there is no restriction of liquidating collateral assets ($\theta = 1$). This feature of the exogenous aggregate productivity slowdown does not match with the observation in Japan in the 1990's. Figure 2.1 shows that there was not much temporary decline of the TFP level from the new trend in the early 1990's.

We show that considering the restriction on liquidating collateral assets mitigates the downward fluctuation of the TFP level, and makes the dynamics under the exogenous aggregate productivity slowdown closer to the observation. Also, this feature of the credit market in our model explains the large loan losses to the lenders after the exogenous slowdown, which is in line with the non-performing loans problem in the 1990's.[44] Note that if we consider the exogenous aggregate productivity slowdown in the perfectly competitive market model such as Hayashi and Prescott (2002), we can only explain no temporary downward fluctuation of the TFP level, and cannot explain the combination of this phenomenon with the large loan-losses associated with the decline of the collateral value of the assets.

[43]The model of Caballero, Hoshi and Kashyap implies that the increase of the input price deters healthy productive firms from entering the production, and further reduces the average productivity level in the economy. Our model does not explain this deterrence effect of "zombie firms".

[44]The data issued by the Financial Service Agency shows that the ratio of the cumulative loan losses for the banks during 1992-1997 to the loan outstanding of the banks on March 1992 is 9.8%.

In the analysis of the shock to θ, we show that the intensified restriction of liquidating collateral assets causes the gradual level-down effect on the TFP trend, and the continuous stagnation of the TFP growth rate. The decline of the land price at the onset of the shock is also much smaller than the case of the exogenous aggregate productivity slowdown. This feature of the shock to θ is in line with the observation in the early 1990's such that the productivity slowdown occurred without a very large fall of the land price, as shown in Figure 2.4.

The remained question is why the land price only gradually declined in the 1990's, while the TFP growth rate persistently stagnated. In our model, the exogenous aggregate productivity slowdown causes the persistent decline of the TFP growth rate and the large decline of the land price at the onset of the shock, while the intensified legal restriction on liquidating collateral assets causes the temporary decline of the TFP growth rate and the small decline of the land price. Our analysis suggests that we need to consider more aspects of the economy to account for this phenomenon.

2.5.4 The cause of the decline of the firms' borrowing

Under both of the shocks to g and θ, the productivity slowdown is accompanied with the fall of the borrowing-output ratio of the firms. This matches with the observation in Japan, as shown in Figure 2.17. In the discussion by academics and professionals to explain this observation, there have appeared two possible mechanisms. One is that the firms did not take risk due to low rates of return to their investment, and reduced their borrowing. The other is that the borrowing of the firms was constrained by the fall of the collateral value of land. Our model implies that the size of the unconstrained borrower-producers in the economy decreases under the productivity slowdown, as shown in Figures 2.14 and 2.15, so that the cause of the stagnant firms' borrowing in Japan was due to the fall of the collateral value of land, rather than the risk-averse behavior of the firms.

2.5.5 The exit of the more-productive firms despite the stay of the less-productive

The model extended in Section 2.4 has implication to the observation that the more-productive firms exited from production, while the less-productive stayed in the 1990's. Nishimura, Kiyota and Nakajima (2003) and Fukao and Kwon (2004) observe this phenomenon in the Japanese firm data in the 1990's. As shown by (2.22), the more-productive producers take more leverage than the less-productive producers. This is because the less-productive producers are afraid of having the debt-overhang in the delay of production. This makes the more-productive producers more fragile to the unexpected decline of the land price. More specifically, the conditions for the producers to avoid liquidating land to repay their debts are[45]

$$\frac{E_t v_{t+1}}{1+r_t} \geq E_{t-1} v_t - A_{t-1} \text{ for the constrained producers.} \tag{2.31}$$

$$\frac{E_t v_{t+1}}{1+r_t} \geq E_{t-1}\left[\frac{v_{t+1}}{1+r_t}\right] - A_{t-1} \text{ for the unconstrained producers.} \tag{2.32}$$

(2.4) implies that $E_{t-1}v_t > E_{t-1}[v_{t+1}/(1+r_t)]$, and that the lowest level of $E_t v_{t+1}$ to satisfy the condition is higher for the constrained producers than the unconstrained around the threshold productivity level to take the maximum leverage $\underline{A}_{t-1}^{SPH}$.[46] This is because their productivity levels are close, but the amounts of borrowing are different.

2.5.6 The effect of the increased productivity gap across the firms

Fukao and Kwon (2004) estimate the productivity transition process (2.2) for the Japanese manufacturing firms over the 1990's. They find increases of ρ and the productivity gap across the firms. The latter empirical finding matches with the discussion in the media such that the firms were polarized between "winner firms" and "loser firms" in the 1990's. Our analysis on an increase of ρ clarifies

[45] From the flow-of-funds constraint, the net-worth per unit of land for the producers without delayed production is $A_{t-1} + q_t - b_t$. To avoid liquidating land, this has to be greater than the minimum down-payment required to buy a unit of land $q_t - E_t v_{t+1}/(1+r_t)$.

[46] Here, we distinguish the productivity level of the producers by A_{t-1}, rather than A_t. This is because in the empirical analysis, the firms are classified by their productivity level for the current production, rather than the expected productivity level for the next production.

the general equilibrium effect of this structural change in the firm dynamics. We show that it has a positive effect on the TFP level, but that this effect becomes smaller as θ is lower. Hence, the legal restriction on liquidating collateral assets prevented the positive effect of the observed increase in ρ from mitigating the long stagnation in Japan.

2.6 Conclusion

This chapter has analyzed the effect of legal restriction on liquidating collateral assets on the TFP level of the economy by constructing a heterogeneous agents model with credit market frictions. Using this model, we have analyzed the dynamic responses of the economy to structural changes, and discussed the implication of the model for the long stagnation in Japan after 1990.

We have first considered an exogenous aggregate productivity slowdown. We find that the legal restriction on liquidating collateral assets mitigates the downward fluctuation of the TFP level of the economy after the shock, while raising the loan-loss rate to the lenders. Second, we have investigated the effect of the intensified legal restriction on liquidating collateral assets, and show that such a legal shock endogenously reduces the TFP growth rate, and also causes loan losses to the lenders by unexpectedly reducing the land price. Third, we have found that an increase of persistence of the agent's productivity level raises the TFP level of the economy, but that the legal restriction on liquidating collateral assets reduces this positive effect. In relation to the long stagnation in Japan, our model implies that the legal restriction mitigated the downward fluctuation of the TFP level at the onset of the productivity slowdown, but endogenously deepened the productivity slowdown over time.

In the analysis, we have found that the low-productive producers remain in production under the productivity slowdown. This is because the land price is lowered during the slowdown, which in turn reduces the cost of investment. The low cost of investment makes it viable for the low-productive producers to engage in production. Also, our model implies that the more-productive producers take the maximum leverage by borrowing up to the limit, while the less-productive producers do not do so, fearing the debt-overhang in case that they are hit by delay of production. Hence, the more-productive producers are more fragile to the unexpected fall of the land price than the less-productive, if their productivity levels and income from production are close. These features match with the observed firm dynamics in the long stagnation in Japan.

A remarkable point of our result is that we have shown that the credit market shock can endogenously cause the productivity slowdown, so that we need to less rely on the exogenous productivity slowdown to account for the long stagnation in Japan after 1990. Although Hayashi and Prescott (2002) plays down the role of the credit market in the long stagnation in Japan after 1990, our result implies that the credit market shock is important to understand the mechanism of the long stagnation, as it is one of the distinct features of the Japanese economy in the 1990's. Related to this point, Hayashi and Prescott (2002) highlight the increase of the aggregate capital-output ratio observed in the 1990's as an evidence of the importance of the productivity shock in the long stagnation, and the stable aggregate investment-output ratio as an evidence for irrelevance of the credit market shock. However, the next chapter shows that a credit market shock can increase the aggregate capital-output and investment-output ratios in a model of credit market frictions with sunk cost of investment, so that their evidence does not necessarily indicate irrelevance of the credit market shock.

In our model, the exogenous aggregate productivity slowdown causes the persistent decline of the TFP growth rate and the large decline of the land price at the onset of the shock, while the intensified legal restriction on liquidating collateral assets causes the temporary decline of the TFP growth rate and the small decline of the land price. Even though the latter result matches with the small decline of the land price under the decline of the TFP growth rate in the early 1990's, either type of the shocks does not account for the lasting gradual decline of the land price under the persistent stagnation of the TFP growth rate over the 1990's and even after that. Our analysis suggests that we need to consider more aspects of the economy to account for this combination of the observations in Japan after 1990. This is left for the future research.

Appendix

2.A.1. Mortgage laws in Japan

In this part, we describe restriction on liquidating collateral assets under the mortgage laws in Japan, and the supreme court cases in 1989 and 1991 which intensified the restriction. The restriction was taken as an impediment against swift liquidation of collateral assets by the lenders during the 1990's, which led to the revision of the mortgage laws in 2003. Our description in this section refers to the report issued by the Ministry of Justice in 2002; "A supplementary note for the interim proposal for the Law to Revise a Part of the Civil Laws for a Reform of the Mortgage and Civil Execution System." We also refer to Yamazaki, Seshimo, Ohta and Sugihara (2005) for the supreme court cases in 1989 and 1991.

First, we describe the mortgage laws. We use one paragraph for each restriction on liquidating collateral asset.

(Protection of short-term lease contracts.) Execution of the mortgage by the lenders rescinds the lease contract of the mortgaged land property and the building on it, if the mortgage agreement precedes the lease contract. But the Civil Law protected the preceded lease contract, if its duration was within a certain length (5 years for the land lease, and 3 years for the building lease,) unless the lenders suffered loss from the protection. Even though existence of the lease contract reduced the collateral values of the land properties and the buildings, the court did not necessarily recognize them as the loss to the lenders. The borrowers abused this article of the law to lower the collateral values of the properties and the buildings, and demanded compensations from the lenders to cancel the lease contracts in some cases. By the revision of the laws in 2003, the maximum length of the lease contracts for the protection is now 6 months, unless the lenders have agreed a longer lease contracts beforehand.

(Protection of buildings on mortgaged land properties.) When a lender foreclosed on the land property, she could auction the building on it as well, if the mortgage agreement preceded the construction of the building. But the lender could only auction the land property, if the owner of the building was a third party who had constructed it even after the mortgage agreement. Even though the buyer of the foreclosed property could obtain a court order to destroy the third party's building, it was costly to go through the necessary procedure. Thus, this protection could lower the collateral values of the land properties. By the revision of the laws in 2003, this protection of the third party's building was abolished.

(Civil Execution Law.) The court order to remove the occupant from the foreclosed property had to correctly identify the occupant. This requirement let malicious borrowers to deter foreclosures of the land properties by keeping changing the occupants to dodge the court. By the revision of the laws in 2003, this requirement was relaxed.

(Foreclosure auction system.) The lender has to use the public auction managed by the court to foreclose on the mortgaged land property. The court sets the minimum price for the bids at every auction. Idee and Taguchi (2002) examine the foreclosure auction data from the court district of the second largest city, Osaka, for 1997-2000, and observe that improvement of the auction procedure in 1998, including a relaxation of the minimum price rule, had a positive effect on the success rate of foreclosures.

(Compulsory foreclosure.) If the borrower sold or lease the mortgaged land property to a third party, then the third party could offer to cancel the mortgage by paying the lender a certain value of money. To counter the offer, the lender had to foreclose on the land property through the public auction. If they could not sell it for more than 110% of the offered value, then she had to buy the land property by herself for 110% of the offered value. Since the execution of the mortgage would rescind the sales contract of the mortgaged land property to the third party, the payment from the lender would be made to the borrower, and offset by the lender's secured claim. By the revision of the laws in 2003, the lender does not have to buy the property even if the foreclosure fails. It was costly for the lender to go through the procedure to use the public auctions without choosing the favorable timing of foreclosure. The lender had to accept too low offer from the third party to cancel the mortgage in some cases. By the revision of the laws in 2003, the lender is now given more time to foreclose on the mortgaged property after the third party's offer to cancel the mortgage.

(Administration of the mortgaged property.) The lenders were not entitled to administer the properties before execution of the mortgage, and could only seize the payments from the tenants to the borrower, if the borrower defaulted. But the payments included the administration fees of the property, so that the seizure could hamper the administration of the property and lower its value, if there were multiple lenders. By the revision of the laws in 2003, it is now possible for the lenders to request the court to appoint an administrator to the mortgaged property, if the borrower defaults.

Second, we describe the two supreme-court cases in 1989 and 1991. The case in 1989 was about the protection of short-term lease contracts described above. To prevent this protection from being abused, the lender could make a lease contract of the property with the borrower by her own. But the supreme court judged that such a lease contract by the lender is not accompanied with actual use of the property, and invalid. This judicial precedents implies that, for abuse of short-term lease contracts, the lender can rescind them by the Civil laws only after the borrower has made an abused short-term lease contract. The supreme-court case in 1991 was about whether the lender could move out the occupant from the mortgaged property after rescinding an abused short-term lease contract. The court judged that the mortgage right did not include the right to do so. This judicial precedent had been effective until the supreme court allowed the lender to move out the occupant in another case in 1999.

2.A.2 Solving the agent's optimization problem

Here, we describe the agent's optimization problem in the model extended in Section 2.4. This model nests the basic model described in Section 2.2 by setting $\mu = 1$. The recursive form of the agent's optimization problem is

$$
\begin{aligned}
&V_t(\hat{y}_t + \hat{q}_t l_{t-1} - \hat{b}_t,\ \hat{A}_t,\ \hat{y}_t) \\
&= \max_{\{\hat{c}_t, l_t, \hat{b}_{t+1}\}} \hat{c}_t + \hat{\beta} E_t V_{t+1}(\hat{y}_{t+1} + \hat{q}_{t+1} l_t - \hat{b}_{t+1},\ \hat{A}_{t+1},\ \hat{y}_{t+1}) \qquad (2.33) \\
&\quad \text{s.t. } \hat{c}_t + \hat{q}_t l_t = \hat{y}_t + \hat{q}_t l_{t-1} - \hat{b}_t + \frac{\hat{b}_{t+1}}{1+\hat{r}_t} \\
&\quad \hat{b}_{t+1} \le \hat{v}_{t+1} l_t \\
&\quad \hat{y}_t = \begin{cases} \hat{A}_{t-1} l_{t-1}, & \text{if the production succeeds.} \\ 0, & \text{if the production is delayed.} \end{cases} \\
&\quad \hat{c}_t, l_t \ge 0 \\
&\quad l_t \le l_{t-1} \quad \text{if the production is delayed.}
\end{aligned}
$$

where

$$
\begin{aligned}
\hat{z}_t &= \frac{z_t}{\exp(gt/[1-\rho])}, \quad \text{for } z_t = c_t,\ b_t,\ v_t,\ y_t \text{ and } q_t. \\
\hat{A}_{t-1} &= \frac{A_{t-1}}{\exp(gt/[1-\rho])} \\
\hat{\beta} &= \beta \exp\left(\frac{g}{1-\rho}\right) \\
1 + \hat{r}_t &= \frac{1+r_t}{\exp(g/[1-\rho])}
\end{aligned}
$$

We normalize the problem to make it stationary. y_s is the amount of the output at date s. The first constraint is the flow-of-funds constraint. The second constraint is the borrowing constraint. The third constraint is the production function. The forth constraint is the non-negativity constraint for consumption and the land-investment. The last constraint is the maximum reinvestment into delayed production. The expectation is taken for the success and delay of the harvest of production, the values of $\hat{q}_s$, $\hat{r}_s$ and $\hat{v}_s$, and the productivity levels in the future, given the productivity level for the current investment A_t and whether the harvest is delayed or not at the current date.

For the state variables of the problem, $\hat{y}_t$ turns out independently of the net-worth in the argument of the value function V_t. This is because the success and delay of production change the success probability of the next production. The functional form of V_t changes over time, according to the series of the future interest rates and land prices.

We start from solving the agent's optimization problem at the steady state, where $\hat{q}_t$ and $\hat{r}_t$ are constant. V_t takes an identical functional form over time. We recursively apply the Lagrange method and the envelope theorem to calculate the optimum conditions. In so doing, note that the function $E_t V_t$ is not differentiable at $l_t = 0$, because the agent with $l_t = 0$ will receive the productivity transition shock ϵ_{t+1} in (2.2) for sure at the next date, but the agent with $l_t > 0$ has the probability of the delay of production, and will not receive the productivity transition shock in such a case. We split the choice set of the agent into the two sub-domains between $l_t \geq 0$ with having the probability of the delay, and $l_t = 0$ without having the probability of the delay. The function $E_t V$ is differentiable in each sub-domain. We solve the agent's optimum behavior for each sub-domain, and then identify the agent's optimum behavior for the whole choice set by the one giving the higher payoff between the two. We can show that the action under $l_t = 0$ with having the probability of the delay is always dominated the one under $l_t = 0$ without having the probability of the delay, so that adding $l_t = 0$ with having the probability of the delay into the choice set of the agent does not change the optimum behavior of the agent, but only makes each of the two sub-domains closed.

After obtaining the optimum conditions at the steady state, we consider the dynamics converging to the steady state. By recursively applying the Lagrange method and the envelope theorem, we obtain the shadow values of the net-worth for the agents without delayed production such that

$$\lambda^S_{1,t}(\hat{\alpha}_t) = \frac{E_t\left[\mu\hat{\beta}(\hat{\alpha}_t + \hat{q}_{t+1} - \hat{v}_{t+1})\lambda^S_{t+1}(\hat{\alpha}_{t+1})\right]}{\hat{q}_t - \frac{E_t\hat{v}_{t+1}}{1+\hat{r}_t}} + \frac{\left[(1-\mu)(1-\theta)\hat{\beta}^2\left(\hat{\alpha}_t + \hat{q}_{t+2} - \hat{v}_{t+2}\right)\lambda^S_{t+2}(\hat{\alpha}_{t+2})\right]}{\hat{q}_t - \frac{E_t\hat{v}_{t+1}}{1+\hat{r}_t}} \tag{2.34}$$

$$\lambda^S_{2,t}(\hat{\alpha}_t) = \frac{E_t\left[\mu\hat{\beta}\left(\hat{\alpha}_t + \hat{q}_{t+1} - \frac{\hat{v}_{t+2}}{1+\hat{r}_{t+1}}\right)\lambda^S_{t+1}(\hat{\alpha}_{t+1})\right]}{\hat{q}_t - E_t\left[\frac{\hat{v}_{t+2}}{(1+\hat{r}_t)(1+\hat{r}_{t+1})}\right]} + \frac{E_t\left[(1-\mu)\hat{\beta}^2\left(\hat{\alpha}_t + \hat{q}_{t+2} - \hat{v}_{t+2}\right)\lambda^S_{t+2}(\hat{\alpha}_{t+2})\right]}{\hat{q}_t - E_t\left[\frac{\hat{v}_{t+2}}{(1+\hat{r}_t)(1+\hat{r}_{t+1})}\right]} \tag{2.35}$$

$$\lambda^S_{3,t}(\hat{\alpha}_t) = \hat{\beta}(1+\hat{r}_t)E_t[\lambda^S_{t+1}(\hat{\alpha}_{t+1})] \tag{2.36}$$

$$\lambda^S_{4,t}(\hat{\alpha}_t) = 1 \tag{2.37}$$

$$\lambda^S_t(\hat{\alpha}_t) = \max\left\{\lambda^S_{1,t}(\hat{\alpha}_t),\ \lambda^S_{2,t}(\hat{\alpha}_t),\ \lambda^S_{3,t}(\hat{\alpha}_t),\ \lambda^S_{4,t}(\hat{\alpha}_t)\right\} \tag{2.38}$$

$\hat{\alpha}_t = \ln(\hat{A}_t)$. $\lambda^S_{j,t}$ for $j = 1,2,3,4$ is the Lagrange multipliers for the flow-of-funds constraint of the agent without delayed production. The subscripts 1, 2, 3 and 4, respectively, correspond to the cases under investing and borrowing up to the limit, investing and borrowing less than the limit, lending and consuming. $\lambda^S_t(\hat{\alpha}_t)$ is the value of the Lagrange multiplier when the agent takes the optimal behavior, given the productivity level.

We can obtain $\underline{A}^{SPH}_t$, $\underline{A}^{SPL}_t$ and $\overline{A}^{SC}_t$ by

$$\lambda^S_{1,t}(\hat{\alpha}^{SPH}_t) = \lambda^S_{2,t}(\hat{\alpha}^{SPH}_t) \tag{2.39}$$

$$\lambda^S_{2,t}(\hat{\alpha}^{SPL}_t) = \lambda^S_{3,t}(\hat{\alpha}^{SPL}_t) \tag{2.40}$$

$$\lambda^S_{3,t}(\hat{\alpha}^{SC}_t) = \lambda^S_{4,t}(\hat{\alpha}^{SC}_t) \tag{2.41}$$

where

$$\hat{\alpha}^{SPH}_t \equiv \ln(\underline{A}^{SPH}_t) - g(t+1)/(1-\rho) \tag{2.42}$$

$$\hat{\alpha}^{SPL}_t \equiv \ln(\underline{A}^{SPL}_t) - g(t+1)/(1-\rho) \tag{2.43}$$

$$\hat{\alpha}^{SC}_t \equiv \ln(\overline{A}^{SC}_t) - g(t+1)/(1-\rho) \tag{2.44}$$

We can numerically show that $\underline{A}^{SPH}_t > \underline{A}^{SPL}_t > \overline{A}^{SC}_t$ in the dynamics under the parameter values we consider. Thus, we can obtain (2.22).

To obtain the Lagrange multiplier for the agent with delayed production, we only need to insert $\mu = 1$ into (2.34) for the borrower-producers. For the lenders and the consumers, the values of the Lagrange multiplier are the same as (2.36) and (2.37). Denote the values of the Lagrange multiplier corresponding to (2.34), (2.36) and (2.37) by $\lambda^D_{1,t}$, $\lambda^D_{3,t}$ and $\lambda^D_{4,t}$, respectively. We define $\hat{\alpha}^{DPH}_t$ and $\hat{\alpha}^{DC}_t$ as

$$\lambda^D_{1,t}(\hat{\alpha}^{DPH}_t) = \max\{\lambda^D_{3,t}(\hat{\alpha}^{DPH}_t), \lambda^D_{4,t}(\hat{\alpha}^{DPH}_t)\} \tag{2.45}$$

$$\max\{\lambda^D_{1,t}(\hat{\alpha}^{DC}_t), \lambda^D_{3,t}(\hat{\alpha}^{DC}_t)\} = \lambda^D_{4,t}(\hat{\alpha}^{DC}_t) \tag{2.46}$$

We can show that $\hat{\alpha}^{DPH}_t \geq \hat{\alpha}^{DC}_t$. If $\lambda^S_{3,t}(\hat{\alpha}_t)) < \max\{\lambda^S_{1,t}(\hat{\alpha}_t)), \lambda^S_{4,t}(\hat{\alpha}_t))\}$ for all $\hat{\alpha}_t$, then $\hat{\alpha}^{DPH}_t = \hat{\alpha}^{DC}_t$. Thus, we obtain the optimum behavior for the agents with delayed production as

$$(\hat{c}_t,\ l_t,\ \hat{b}_{t+1}) = \begin{cases} (\ 0,\ \ +,\ \ E_t\hat{v}_{t+1}l_t\ ) & \text{if } \hat{\alpha}_t \in (\hat{\alpha}^{DPH}_t, \infty). \\ (\ 0,\ \ 0,\ \ -\ ) & \text{if } \hat{\alpha}_t \in (\hat{\alpha}^{DC}_t, \hat{\alpha}^{DPH}_t). \\ (\ +,\ \ 0,\ \ 0\ ) & \text{if } \hat{\alpha}_t \in (-\infty, \hat{\alpha}^{DC}_t). \end{cases} \tag{2.47}$$

The marginal agents with $\hat{\alpha}_t = \hat{\alpha}^{DPH}_t$ are indifferent between investment and lending, and those with $\hat{\alpha}_t = \hat{\alpha}^{DC}_t$ are indifferent between lending and consumption. (2.47) only shows the signs of some variables. The values of these variables take similar forms as the first equation in (2.7), (2.8) and (2.9).

2.A.3 The equilibrium conditions

The delayed producers should have invested into production at the previous date, and must have the productivity level equal to or more than $\exp(\frac{g}{1-\rho})\underline{A}^{SPL}_{t-1}$. To simplify the notation, we denote the lowest productivity level of the delayed producers who continue their production by $\hat{\alpha}^{DP}_t$ such that

$$\hat{\alpha}^{DP}_t \equiv \max\left\{\hat{\alpha}^{SPL}_{t-1},\ \hat{\alpha}^{DPH}_t\right\} \tag{2.48}$$

From the previous section in the appendix, we obtain the land market clearing condition as

$$\int_{\hat{\alpha}^{SPL}_t}^{\infty} L^S_t(\hat{\alpha}_t)d\hat{\alpha}_t + \int_{\hat{\alpha}^{DP}_t}^{\infty} L^D_t(\hat{\alpha}_t)d\hat{\alpha}_t = 1 \tag{2.49}$$

$L^j_t(\hat{\alpha}_t)$ is the land-investment distribution function at date t, for $j = S, D$. The superscripts S and D, respectively, imply the distribution for the agents without and with delayed production.

The credit market clearing condition is

$$\int_{\hat{\alpha}_t^{SC}}^{\hat{\alpha}_t^{SPL}} \hat{S}_t^S(\hat{\alpha}_t)d\hat{\alpha}_t + \int_{\hat{\alpha}_t^{DC}}^{\hat{\alpha}_t^{DP}} \hat{S}_t^D(\hat{\alpha}_t)d\hat{\alpha}_t = \frac{E_t\hat{v}_{t+1}}{1+\hat{r}_t}\int_{\hat{\alpha}_t^{SPH}}^{\infty} L_t^S(\hat{\alpha}_t)d\hat{\alpha}_t + E_t\left[\frac{\hat{v}_{t+2}}{(1+\hat{r}_t)(1+\hat{r}_{t+1})}\right]\int_{\hat{\alpha}_t^{SPL}}^{\hat{\alpha}_t^{SPH}} L_t^S(\hat{\alpha}_t)d\hat{\alpha}_t + \frac{E_t\hat{v}_{t+1}}{1+\hat{r}_t}\int_{\hat{\alpha}_t^{DP}}^{\infty} L_t^D(\hat{\alpha}_t)d\hat{\alpha}_t \quad (2.50)$$

$\hat{S}_t^j(\hat{\alpha}_t)$ is the normalized lending distribution function at date t, for $j = S, D$. The superscript S is for the agents without delayed production, and D for those with delayed production. The left-hand side of the equation is the aggregate lending, and the right-hand side is the aggregate borrowing. The first term in the right-hand side is the borrowing of the constrained borrower-producers without delayed production. The second term is the borrowing of the unconstrained borrower-producers without delayed production. The third term is the borrowing of the borrower-producers with delayed production, who borrow up to the limit.[47]

To specify the aggregate land demand and the aggregate lending, we need to obtain the land-investment and the lending distribution functions $L_t^j(\hat{\alpha}_t)$ and $\hat{S}_t^j(\hat{\alpha}_t)$ for $j = S, D$. To do so, we first aggregate the flow-of-funds constraint of the optimization problem (2.33) to obtain the aggregate net-worth. For the agents without delayed production represented by the superscript S,

$$\begin{aligned}
\hat{W}_t^S(\hat{\alpha}_t) = & \int_{\hat{\alpha}_{t-1}^{SPH}}^{\infty} (\exp(\hat{\alpha}_{t-1}) + \hat{q}_t - E_{t-1}\hat{v}_t)\mu L_{t-1}^S(\hat{\alpha}_{t-1})f(\hat{\alpha}_t|\hat{\alpha}_{t-1})d\hat{\alpha}_{t-1} \\
& + \int_{\hat{\alpha}_{t-1}^{SPL}}^{\hat{\alpha}_{t-1}^{SPH}} \left(\exp(\hat{\alpha}_{t-1}) + \hat{q}_t - E_{t-1}\left[\frac{\hat{v}_{t+1}}{1+\hat{r}_t}\right]\right)\mu L_{t-1}^S(\hat{\alpha}_{t-1})f(\hat{\alpha}_t|\hat{\alpha}_{t-1})d\hat{\alpha}_{t-1} \\
& + \int_{\hat{\alpha}_{t-1}^{SC}}^{\hat{\alpha}_{t-1}^{SPL}} (1+\hat{r}_{t-1})\hat{S}_{t-1}^S(\hat{\alpha}_{t-1})f(\hat{\alpha}_t|\hat{\alpha}_{t-1})d\hat{\alpha}_{t-1} \\
& + \int_{\hat{\alpha}_{t-1}^{DP}}^{\infty} (\exp(\hat{\alpha}_{t-1}) + \hat{q}_t - E_{t-1}\hat{v}_t)L_{t-1}^D(\hat{\alpha}_{t-1})f(\hat{\alpha}_t|\hat{\alpha}_{t-1})d\hat{\alpha}_{t-1} \\
& + \int_{\hat{\alpha}_{t-1}^{DC}}^{\hat{\alpha}_{t-1}^{DP}} (1+\hat{r}_{t-1})\hat{S}_{t-1}^D(\hat{\alpha}_{t-1})f(\hat{\alpha}_t|\hat{\alpha}_{t-1})d\hat{\alpha}_{t-1} \quad (2.51)
\end{aligned}$$

where

$$f(\hat{\alpha}_t|\hat{\alpha}_{t-1}) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{[\hat{\alpha}_t + \frac{g}{1-\rho} - \rho\hat{\alpha}_{t-1}]^2}{2\sigma^2}\right) \quad (2.52)$$

$\hat{W}_t^S(\hat{\alpha}_t)$ is the normalized aggregate net-worth distribution function for the agents who do not have delayed production and have the normalized productivity level $\hat{\alpha}_t$ at

[47] If $\hat{\alpha}_t^{DC} < \hat{\alpha}_{t-1}^{SPL}$ and $\hat{\alpha}_t^{DP} = \hat{\alpha}_{t-1}^{SPL}$, then there is no delayed producer who terminates her production or lends her net-worth. $\hat{S}_t^D(\hat{\alpha}_t)$ is zero for all $\hat{\alpha}_t$ in such a case.

date t. If we add $gt/(1-\rho)$, and then take the exponential, then we can obtain the level of the aggregate net-worth distribution. $f(\hat{\alpha}_t|\hat{\alpha}_{t-1})$ is the conditional probability to have $\hat{\alpha}_t$ given $\hat{\alpha}_{t-1}$, which is defined by the productivity transition (2.2).

For the agents with delayed production represented by the superscript D,[48]

$$\hat{W}_t^D(\hat{\alpha}_{t-1}) = \begin{cases} (\hat{q}_t - E_{t-1}\hat{v}_t)(1-\mu)L_{t-1}^S(\hat{\alpha}_{t-1}), & \text{for } \alpha_{t-1} \in [\hat{\alpha}_{t-1}^{SPH}\infty). \\ \left(\hat{q}_t - E_{t-1}\left[\frac{\hat{v}_{t+1}}{1+\hat{r}_t}\right]\right)(1-\mu)L_{t-1}^S(\hat{\alpha}_{t-1}), & \text{for } \hat{\alpha}_{t-1} \in [\hat{\alpha}_{t-1}^{SPL}, \hat{\alpha}_{t-1}^{SPH}). \end{cases} \tag{2.53}$$

$\hat{W}_t^D(\hat{\alpha}_t)$ is the normalized aggregate net-worth distribution function for the agents with delayed production. From the aggregate net-worth, we can obtain the land-investment and the lending distribution functions as

$$L_t^S(\hat{\alpha}_t) = \frac{\hat{W}_t^S(\hat{\alpha}_t)}{\hat{q}_t - E_t\hat{v}_{t+1}/(1\hat{r}_t)} \quad \text{for } \hat{\alpha}_t \in [\hat{\alpha}_t^{SPH},\ \infty) \tag{2.54}$$

$$L_t^S(\hat{\alpha}_t) = \frac{\hat{W}_t^S(\hat{\alpha}_t)}{\hat{q}_t - E_t\left[\frac{\hat{v}_{t+2}}{(1+\hat{r}_t)(1+\hat{r}_{t+1})}\right]} \quad \text{for } \hat{\alpha}_t \in [\hat{\alpha}_t^{SPL},\ \hat{\alpha}_t^{SPH}) \tag{2.55}$$

$$\hat{S}_t^S(\hat{\alpha}_t) = \hat{W}_t^S(\hat{\alpha}_t) \quad \text{for } \hat{\alpha} \in [\hat{\alpha}_t^{SC}, \hat{\alpha}_t^{SPL}) \tag{2.56}$$

$$L_t^D(\hat{\alpha}_t) = \frac{\hat{W}_t^D(\hat{\alpha}_t)}{\hat{q}_t - E_t\hat{v}_{t+1}/(1+\hat{r}_t)} \quad \text{for } \hat{\alpha}_t \in [\hat{\alpha}_t^{DP},\ \infty) \tag{2.57}$$

$$\hat{S}_t^D(\hat{\alpha}_t) = \hat{W}_t^D(\hat{\alpha}_t) \quad \text{for } \hat{\alpha}_t \in [\hat{\alpha}_t^{DC},\ \hat{\alpha}_t^{DP}) \tag{2.58}$$

Given the parameters of the model (β, μ, g, ρ, σ, θ), and the initial values and functions of $\hat{\alpha}_{-1}^{SPH}$, $\hat{\alpha}_{-1}^{SPL}$, $\hat{\alpha}_{-1}^{DP}$, $\overline{\alpha}_{-1}^{jC}$, $E_{-1}v_0$, $E_{-1}[v_1/(1+r_0)]$, $L_{-1}^j(\alpha_{-1})$ and $(1+\hat{r}_{-1})\hat{S}_{-1}^j(\hat{\alpha}_{-1})$ for $j=S,D$, we define equilibrium as

- $\{\hat{c}_s, l_s, \hat{b}_{s+1}\}_{s=0}^{\infty}$ solve the optimization problem (2.33), given the current and future prices $\{\hat{q}_s, \hat{r}_s, \hat{v}_s\}_{s=0}^{\infty}$; Consequently, $\{\hat{\alpha}_s^{SPH},\ \hat{\alpha}_s^{SPL},\ \hat{\alpha}_s^{DP},\ \hat{\alpha}_s^{jC}\}_{s=0}^{\infty}$ are derived for $j=S,D$;

- $\{L_s^j(\hat{\alpha}_s), \hat{S}_s^j(\hat{\alpha}_s)\}_{s=0}^{\infty}$ are recursively determined by (2.51)-(2.58), given $\{\hat{q}_s$, $\hat{r}_s$, $E_s\hat{v}_{s+1}$, $E_s[\hat{v}_{s+2}/(1+\hat{r}_{s+1})]$, $\hat{\alpha}_s^{SPH}$, $\hat{\alpha}_s^{SPL}$, $\hat{\alpha}_s^{DP}$, $\hat{\alpha}_s^{jC}\}_{s=0}^{\infty}$ for $j=S,D$, and the initial values and functions;

- $\{\hat{q}_s, \hat{r}_s, \hat{v}_s\}_{s=0}^{\infty}$ are determined in order to satisfy the market clearing conditions (2.49) and (2.50), and the definition of v_s in (2.4);

- the agents form rational expectation about the future prices $\{\hat{q}_s, \hat{r}_s, \hat{v}_s\}_{s=t+1}^{\infty}$ at every date t;

[48] In (2.53), we assume the marginal producers with $\hat{\alpha}_{t-1} = \hat{\alpha}_{t-1}^{SPL}$ and $\hat{\alpha}_{t-1}^{SPH}$, respectively, borrow $E_{t-1}\hat{v}_t l_{t-1}$ and $E_{t-1}[\hat{v}_{t+1}/(1+\hat{r}_t)]l_{t-1}$. This is valid, as these amounts of borrowings are weakly optimal, and their size in the economy is zero.

- there is no bubble in the land price, so that at every date t.

The goods market clears in equilibrium by Walras' Law.

2.A.4 Calculating the equilibrium dynamics.

We describe the calculation method for the transitory dynamics between the steady states. To simplify the notation, we denote $\hat{q}_t - \hat{v}_{t+1}/(1+\hat{r}_t)$ by $\hat{u}_t$. We conduct the following iteration:

1. Calculate the steady states under the parameter values before and after the shock. The steady state before the shock provides the initial condition of the economy. Use the new steady-state values for $\{\hat{v}_s\}_{s=t+1}^{t+2}$ and $\{\hat{\alpha}_s^{SPH}, \hat{\alpha}_s^{SPL}, \hat{\alpha}_s^{SC}, \hat{\alpha}_s^{DP}\}_{s=t-1}^{t}$ at each date t in the next step.

2. From date 0 onward, we calculate $\hat{u}_t$, $\hat{r}_t$ and $\hat{W}_t^j(\hat{\alpha}_t)$ for $j = S, D$ by the market clearing conditions and the integral equation for the net-worth distribution (2.11)-(2.58) until they converge to the new steady-state levels, given $\hat{q}_0$, $\{\hat{v}_s\}_{s=t+1}^{t+2}$ and $\{\hat{\alpha}_s^{SPH}, \hat{\alpha}_s^{SPL}, \hat{\alpha}_s^{SC}, \hat{\alpha}_s^{DP}\}_{s=t-1}^{t}$ obtained in the step 3, and $\hat{W}_{t-1}^j(\hat{\alpha}_{t-1})$ for $j = S, D$ and $\hat{r}_{t-1}$ at each date t. $\hat{W}_{t-1}^j(\hat{\alpha}_{t-1})$ for $j = S, D$ and $\hat{r}_{t-1}$ are updated forward.

3. From the date of convergence toward date 0, we calculate $\hat{u}_t$, $\hat{r}_t$, $\hat{\alpha}_t^{SPH}$, $\hat{\alpha}_t^{SPL}$, $\hat{\alpha}_t^{SC}$, and $\hat{\alpha}_t^{DP}$ by all the equilibrium conditions, given $\hat{W}_{t-1}^j(\hat{\alpha}_{t-1})$ for $j = S, D$ and $\hat{r}_{t-1}$ obtained in the step 2, $\hat{\alpha}_{t-1}^{SPH}$, $\hat{\alpha}_{t-1}^{SPL}$, $\hat{\alpha}_{t-1}^{SC}$ and $\hat{\alpha}_{t-1}^{DP}$ obtained in the previous step 3, and $\{\hat{v}_s, \lambda_s^S(\hat{\alpha}_s)\}_{s=t+1}^{t+2}$ at each date t. $\{\hat{v}_s, \lambda_s^S(\hat{\alpha}_s)\}_{s=t+1}^{t+2}$ are updated backward, and $\hat{q}_0$ is also calculated.

4. Check the convergence of the series of $\{\hat{u}_s, \hat{r}_s\}_{s=t}^{200}$ at the steps 2 and 3 with the series at the step 3 in the previous iteration by supnorm. All the series converge to the new steady state by date 200 in the cases we consider. The convergence criterion is $1e-5$ for the ratios between the two series. If they do not converge, return to the step 2.

In the step 3, we need to solve the integral equations for $\lambda_t^S(\hat{\alpha}_t)$ and $\hat{W}_t^S(\hat{\alpha}_t)$. To approximate the values of the integrals, we apply the Legendre nodes and weights for the domains $(-\infty, \hat{\alpha}_t^{SC})$, $[\hat{\alpha}_t^{SC}, \hat{\alpha}_t^{SPL})$, $[\hat{\alpha}_t^{SPL}, \hat{\alpha}_t^{SPH})$, $[\hat{\alpha}_t^{SPH}, \infty)$ and $[\hat{\alpha}_t^{DP}, \infty)$ at each date t by replacing $\pm\infty$ with a high and a low values, as in the steady-state calculation.

In the cases we consider, the delayed producers always continue their production, so that we do not need to consider the domains $(-\infty, \hat{\alpha}_t^{DC})$ and $[\hat{\alpha}_t^{DC}, \hat{\alpha}_t^{DP})$.

By (2.34)-(2.38), we can show that the value of $\lambda_t^S(\hat{\alpha}_t)$ only depends on $\{\hat{u}_s, \hat{r}_s\}_{s=t}^{t+1}$ and $\{\hat{v}_s,\ \lambda_s^S(\hat{\alpha}_s)\}_{s=t+1}^{t+2}$. As $\hat{v}_t$ is determined by $\{\hat{u}_t,\ \hat{r}_s\}_{s=t}^{\infty}$, $(\hat{\alpha}_t^{SPH},\ \hat{\alpha}_t^{SPL},\ \hat{\alpha}_t^{SC},\ \hat{\alpha}_t^{DP})$ can be uniquely specified by the series of $\{\hat{u}_s, \hat{r}_s\}_{s=t}^{\infty}$. Hence, we only need to check the convergence of $\{\hat{u}_s, \hat{r}_s\}_{s=0}^{\infty}$ in the step 4.

The intuition of this iteration process is that we obtain the equilibrium series backward from the new steady state in the step 3, because the agent's behavior and the land price are forward-looking. However, the net-worth distribution is history-dependent, so we update the time-path of the net-worth distribution from the old steady state to the new steady state in the step 2 in each iteration.

2.A.5 Estimating the TFP growth rate by the Solow residual.

We approximate the aggregate production function of the Japanese economy by the Cobb-Douglas function. The capital share is 0.361, taken from Hayashi and Prescott. The output is GDP. The capital stock is the sum of the inventory, the net fixed assets, and the intangible non-financial assets, net of the amounts held by the government. The National Accounts provide the nominal value of the capital stock evaluated at the replacement cost. The labor force is the number of the employed workers times the average hours worked. The output and the capital stock are deflated by the GDP deflator. Hayashi and Prescott (2002) calculate the real value of the capital stock in the same way. We take the output, the capital stock and the number of the employed workers from the 1968 SNA for 1970-1998 and from the 1993 SNA for 1990-2002. The 1993 SNA is only available for this period. We take the average hours worked from "Maitsuki Kinro Tokei Chosa," an establishment survey conducted by the Ministry of Welfare and Labor.

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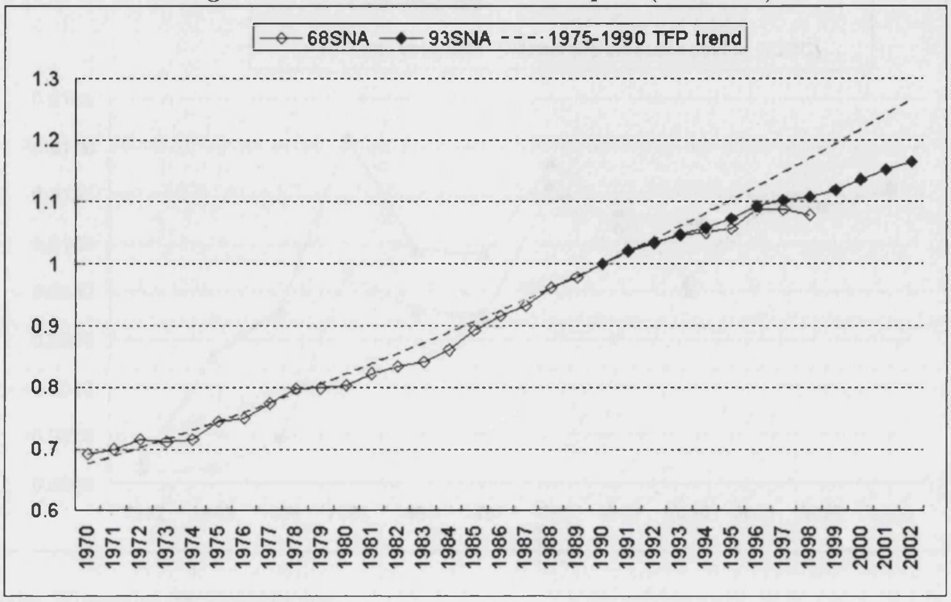
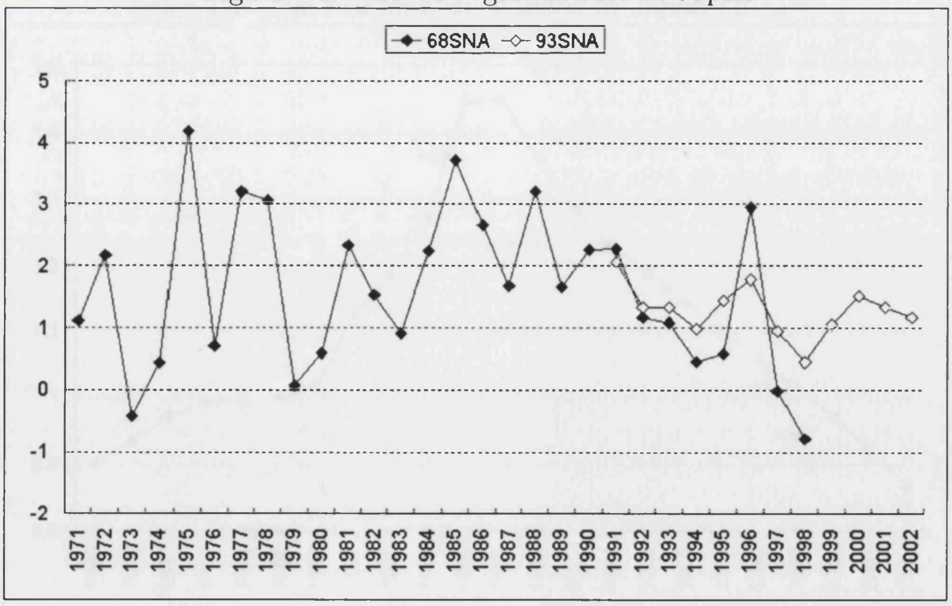
Figure 2.1: The TFP level in Japan (1990=1.)



Source: National Accounts.

Note: We calculate the TFP growth rates by the Solow residual. "63SNA" is calculated by the data of the 1963 Standard of the National Account. This data is only available for 1970-1998. "93SNA" corresponds to the 1993 standard. This data is only available for 1990-2002. "1975-1990 TFP trend" grows at the constant rate 1.98%. 1.98% is the average TFP growth rate for 1975-1990 calculated by the 1963 SNA. See appendix for more detail of the calculation.

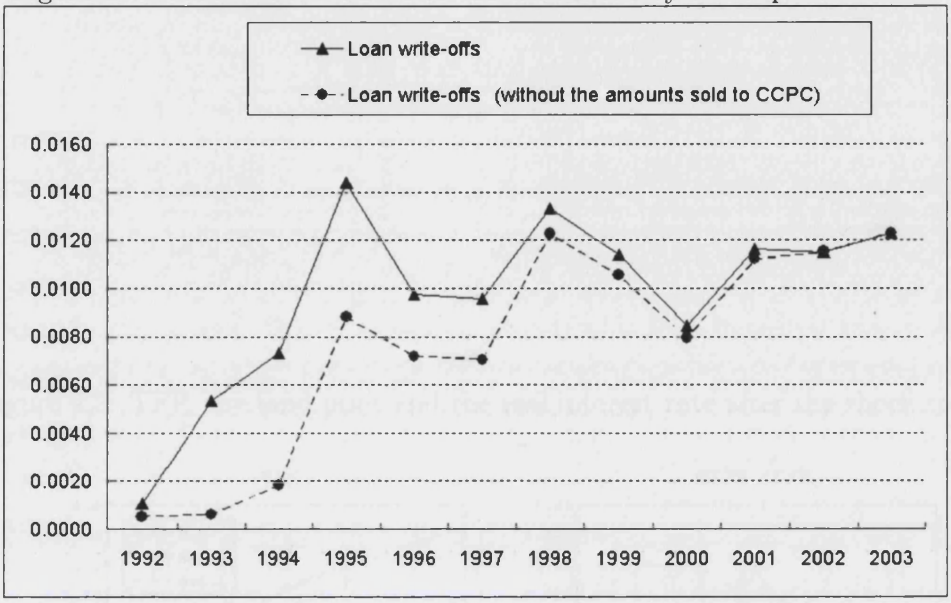
Figure 2.2: The TFP growth rate in Japan



Source: National Accounts.

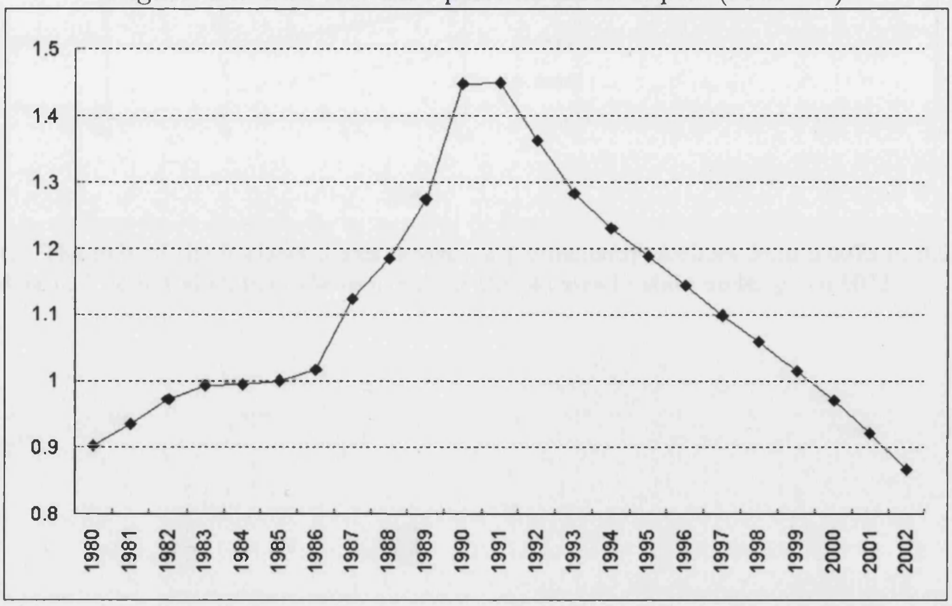
Note: We calculate the TFP growth rates by the Solow residual. See the note of Figure 2.1.

Figure 2.3: Flow of the loan write-offs accounted by the Japanese banks



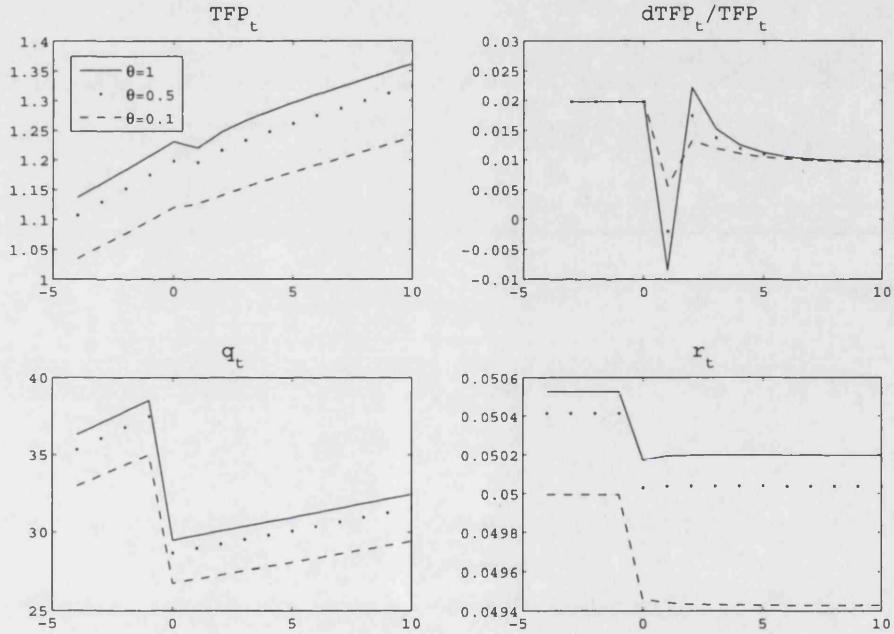
Source: Financial Service Agency.
Note: The figure is normalized by the aggregate bank lending each year. "The loan write-offs without the amounts sold to CCPC" excludes the amounts of the write-offs by loan sales to Cooperative Credit Purchasing Co., Ltd (CCPC). This is because CCPC was set up by the banks only to intermediate liquidation of collateral assets. Liquidation was not swift. As of March 1998, CCPC had only collected 24% of the purchased values of the mortgages by then. Thus, a large part of the accounted loan write-offs by the loan sales to CCPC was not liquidated in fact.

Figure 2.4: The real land price index in Japan (1985=1.)



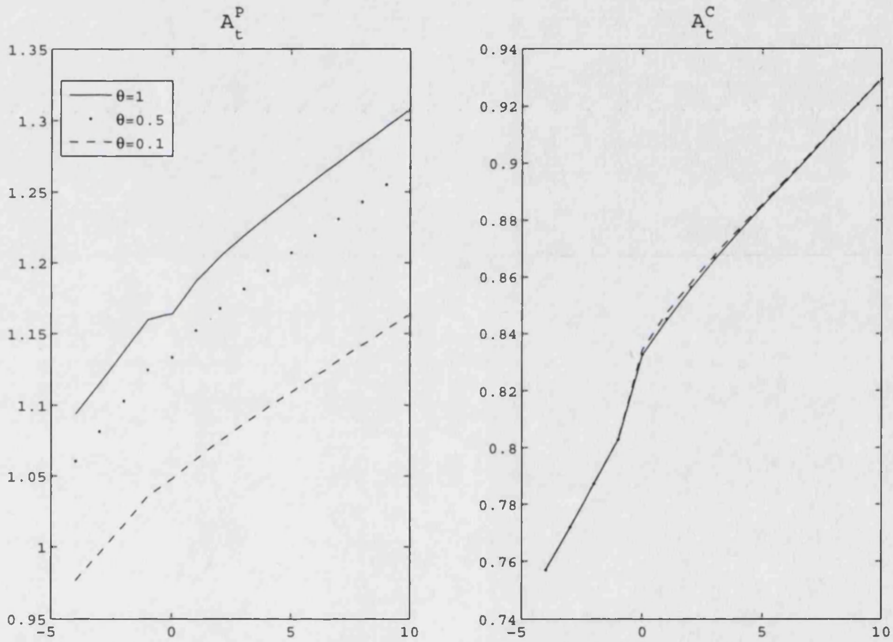
Source: Japan Real Estate Institute, and National Accounts.
Note: The real land price is the Nationwide City Land Price Index divided by the GDP deflator.

Figure 2.5: TFP, the land price and the real interest rate after the shock to g



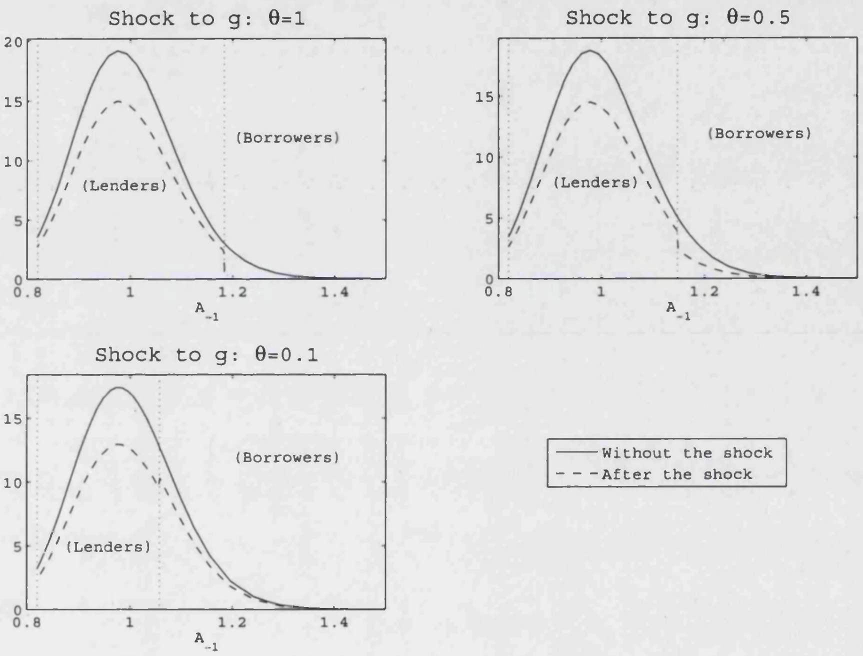
Note: The unit of the horizontal axis is year. g permanently declines from 0.0073 to 0.0058 at date 0. The initial state of the dynamics is the old steady state under $g = 0.0073$.

Figure 2.6: The entry and exit threshold for the productivity level after the shock to g



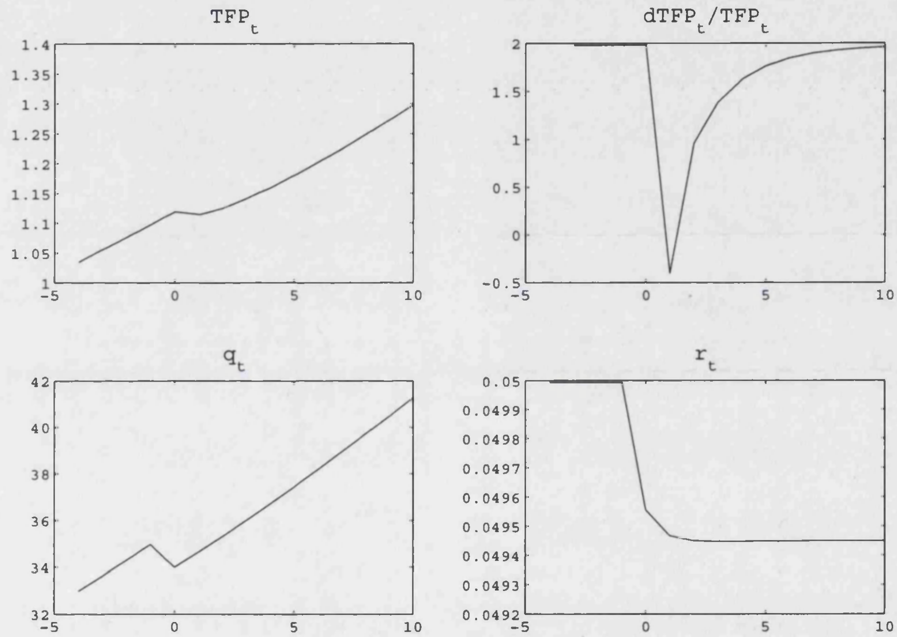
Note: The unit of the horizontal axis is year. g permanently declines from 0.0073 to 0.0058 at date 0. The initial state of the dynamics is the old steady state under $g = 0.0073$. $\underline{A}_t^P$ is the threshold of the productivity level to engage in production. $\overline{A}_t^C$ is the threshold of the productivity level to consume all the net-worth.

Figure 2.7: The net-worth distribution at date 0 after the shock to g



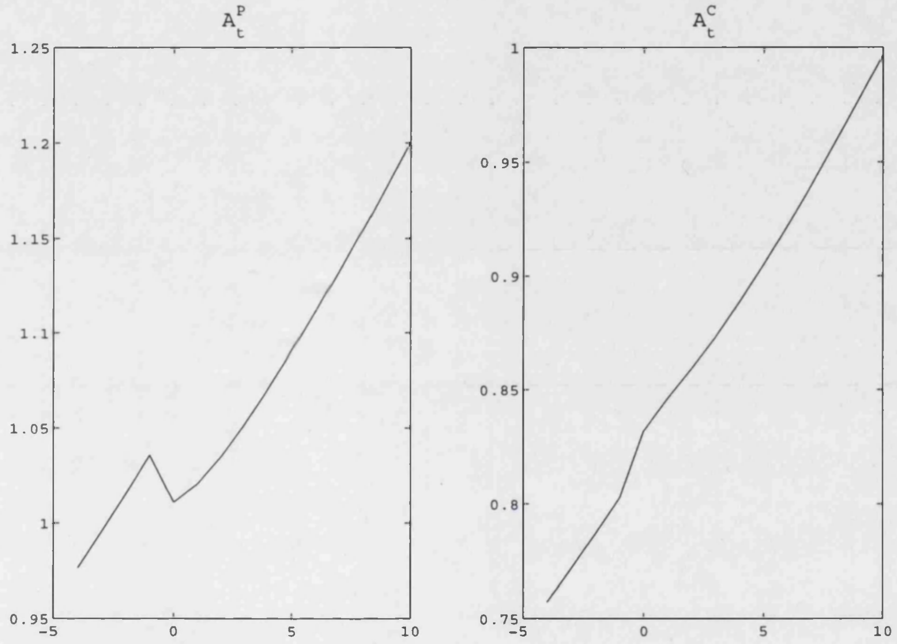
Note: g permanently declines from 0.0073 to 0.0058 at date 0. The initial state of the dynamics is the old steady state under $g = 0.0073$. The figure shows the net-worth distributions of the agents at date 0 over the productivity levels A_{-1} before the productivity transitions. There are three vertical dotted lines in each graph. For the agents without delayed production, the left vertical line is the border between the consumers and the lenders. The right line is the one between the lenders and the borrowers. The figure includes the net-worth of all the borrower-producers with and without delayed production. The distribution named as "Without the shock" is the distribution without the shock to g, and the one named as "After the shock" is the actual distribution after the shock.

Figure 2.8: TFP, the land price and the real interest rate after the shock to θ



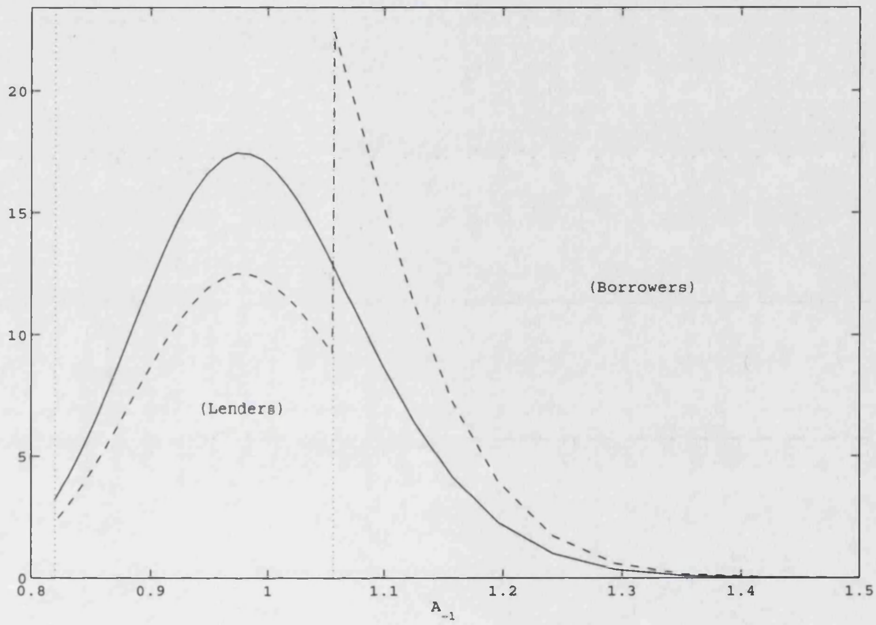
Note: The unit of the horizontal axis is year. θ permanently declines from 0.1 to 0.031 at date 0. The initial state of the dynamics is the old steady state under $\theta = 0.1$.

Figure 2.9: The entry and exit threshold for the productivity level after the shock to θ



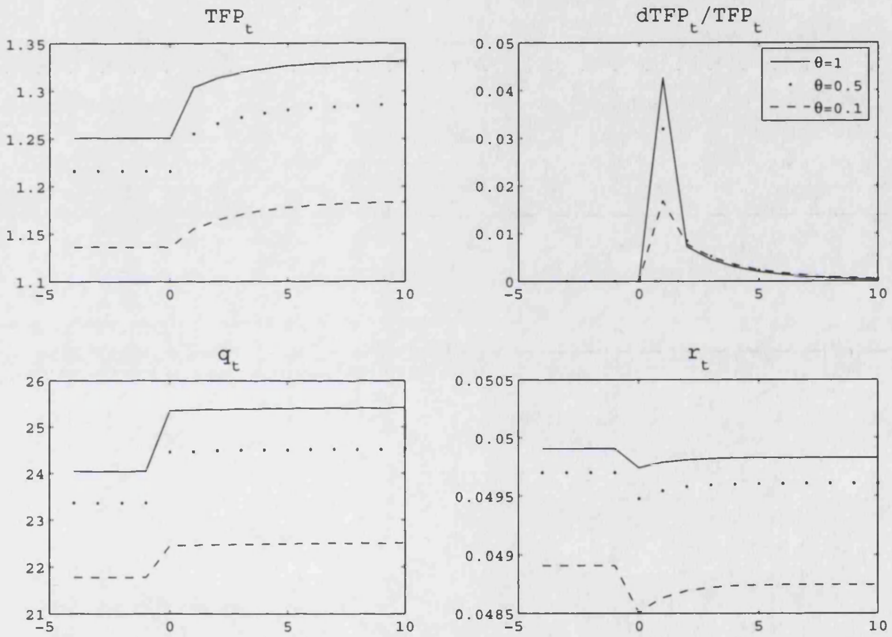
Note: The unit of the horizontal axis is year. θ permanently declines from 0.1 to 0.031 at date 0. The initial state of the dynamics is the old steady state under $\theta = 0.1$. $\underline{A}_t^P$ is the threshold of the productivity level to engage in production. $\overline{A}_t^C$ is the threshold of the productivity level to consume all the net-worth.

Figure 2.10: The net-worth distribution at date 0 after the shock to θ



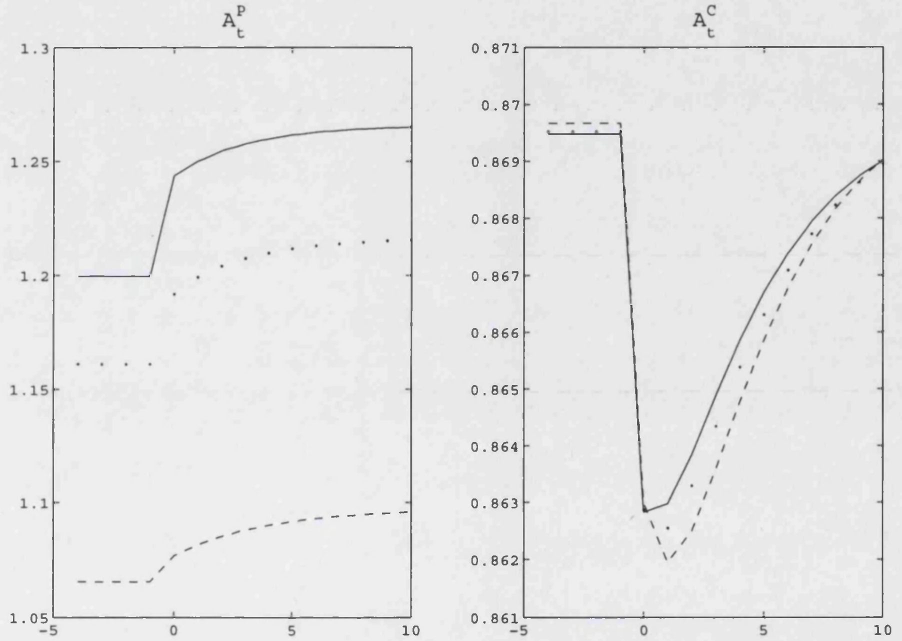
Note: θ permanently declines from 0.1 to 0.031 at date 0. The initial state of the dynamics is the old steady state under $\theta = 0.1$. The figure shows the net-worth distributions of the agents at date 0 over the productivity levels A_{-1} before the productivity transitions. There are three vertical dotted lines in each graph. For the agents without delayed production, the left vertical line is the border between the consumers and the lenders. The right line is the one between the lenders and the borrowers. The figure includes the net-worth of all the borrower-producers with and without delayed production. The distribution named as "Without the shock" is the distribution without the shock to θ, and the one named as "After the shock" is the actual distribution after the shock.

Figure 2.11: TFP, the land price and the real interest rate after the shock to ρ under $g = 0$



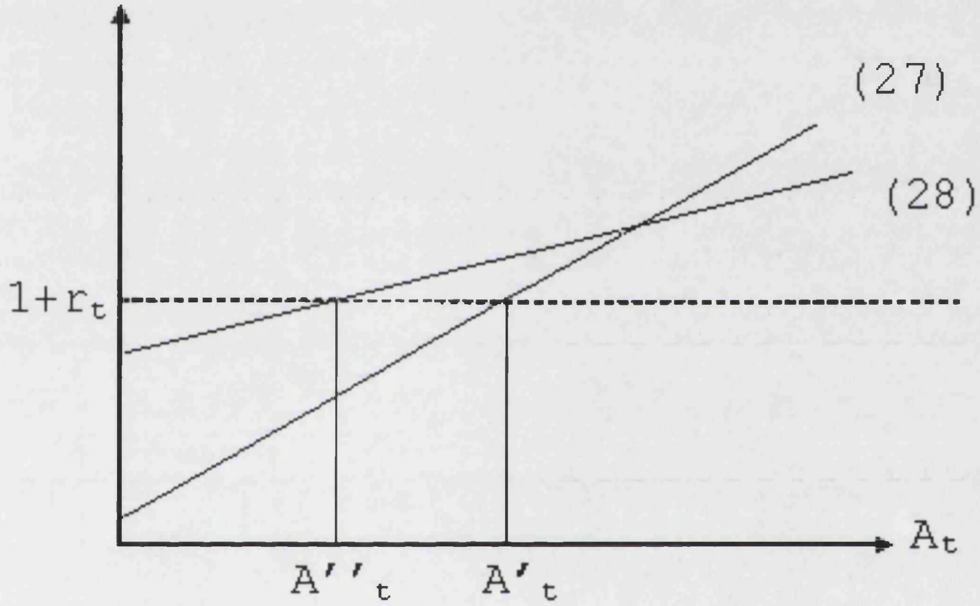
Note: The unit of the horizontal axis is year. $g = 0$. ρ permanently increases from 0.614 to 0.765 at date 0. The initial state of the dynamics is the old steady state under $\rho = 0.614$.

Figure 2.12: The entry and exit threshold for the productivity level after the shock to ρ under $g = 0$



Note: The unit of the horizontal axis is year. $g = 0$. ρ permanently increases from 0.614 to 0.765 at date 0. The initial state of the dynamics is the old steady state under $\rho = 0.614$. $\underline{A}_t^P$ is the threshold of the productivity level to engage in production. $\overline{A}_t^C$ is the threshold of the productivity level to consume all the net-worth.

Figure 2.13: The rate of return to unit net-worth invested or lent under the possible delay of production



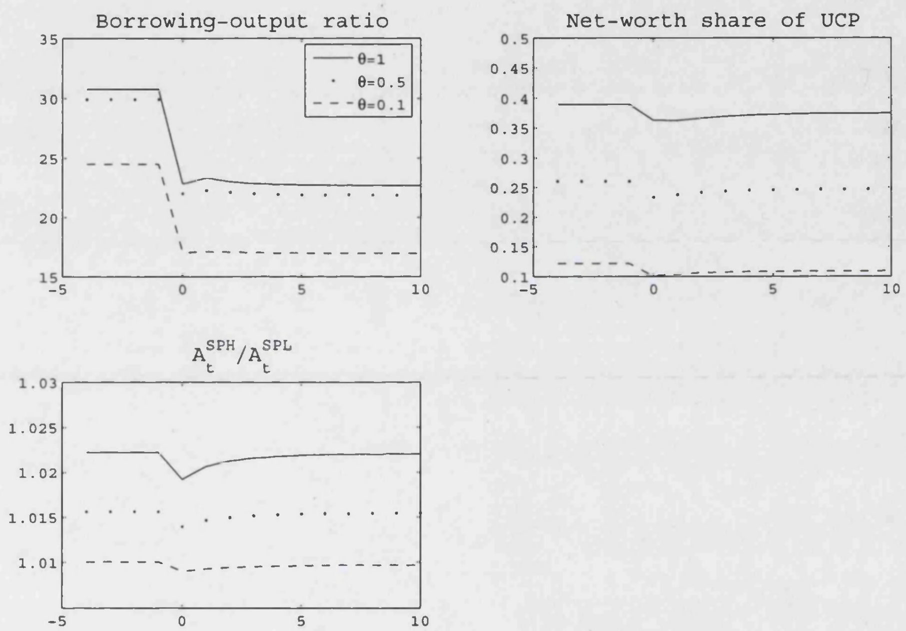
Note: The curves denoted as (2.27) and (2.28), respectively, are the rates of return to unit net-worth for borrowing up to the limit and less than the limit under each productivity level A_t at the steady state. They correspond to the equations (2.27) and (2.28) such that

$$(2.27) = \frac{E_t\left[\mu(A_t + q_{t+1} - v_{t+1}) + (1-\mu)(1-\theta)\beta\left(\exp\left(\frac{g}{1-\rho}\right)A_t + q_{t+2} - v_{t+2}\right)\right]}{q_t - \frac{E_t v_{t+1}}{1+r_t}}$$

$$(2.28) = \frac{E_t\left[\mu\left(A_t + q_{t+1} - \frac{v_{t+2}}{1+r_{t+1}}\right) + (1-\mu)\beta\left(\exp\left(\frac{g}{1-\rho}\right)A_t + q_{t+2} - v_{t+2}\right)\right]}{q_t - E_t\left[\frac{v_{t+2}}{(1+r_t)(1+r_{t+1})}\right]}$$

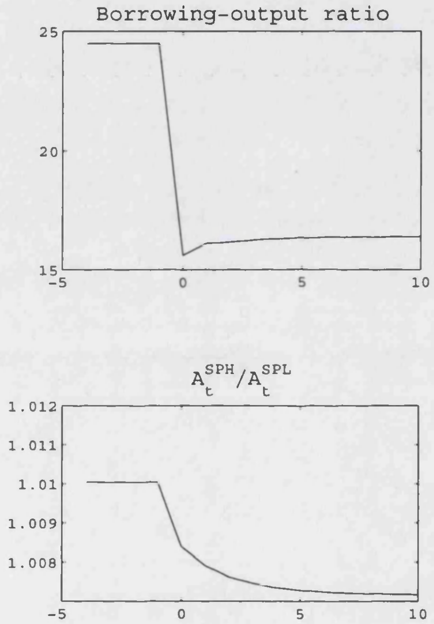
In drawing the curves, we take the current and future interest rates and land prices as given. The dotted horizontal line is the rate of return to unit net-worth for lending.

Figure 2.14: The borrowing-output ratio, the net-worth share of the unconstrained producers and $\underline{A}_t^{SPH}/\underline{A}_t^{SPL}$ after the shock to g



Note: The unit of the horizontal axis is year. g permanently declines from 0.0073 to 0.0058 at date 0. The initial state of the dynamics is the old steady state under $g = 0.0073$. "The net-worth share of UCP" is the net-worth share of the unconstrained producers among all the producers.

Figure 2.15: The borrowing-output ratio, the net-worth share of the unconstrained producers and $\underline{A}_t^{SPH}/\underline{A}_t^{SPL}$ after the shock to θ

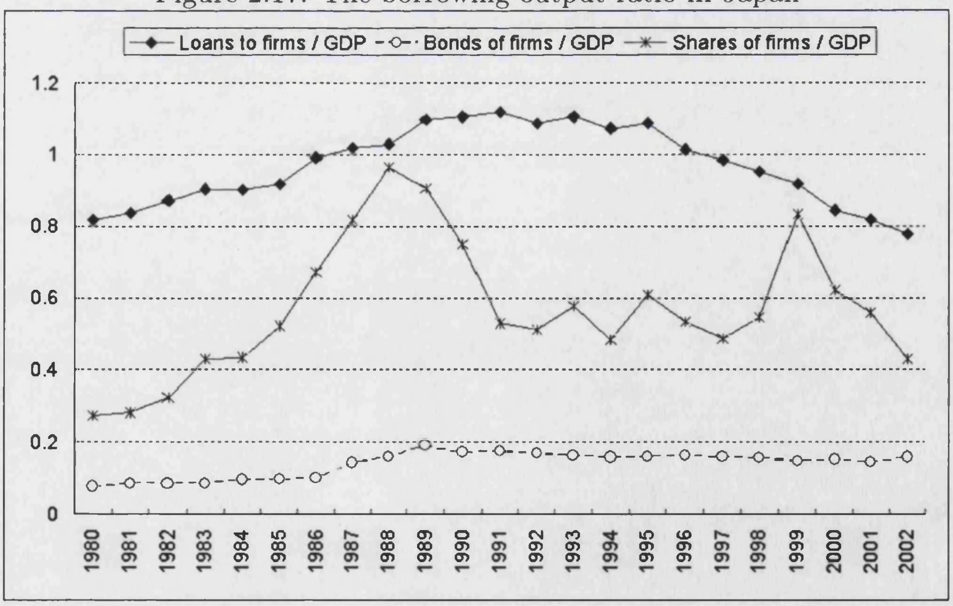


Note: The unit of the horizontal axis is year. θ permanently declines from 0.1 to 0.031 at date 0. The initial state of the dynamics is the old steady state under $\theta = 0.1$. "The net-worth share of UCP" is the net-worth share of the unconstrained producers among all the producers.

Figure 2.16: The real interest rate in Japan (%)

Source: Bank of Japan.
Notes: The figure shows the ex-post annual real interest rates. The real interest rates are calculated by the long-term nominal interest rates minus the realized inflation rates. The inflation rates are calculated by GDP deflators.

Figure 2.17: The borrowing-output ratio in Japan



Source: National Accounts and Flow of Funds Statistics.
Notes: "Loans to firms", "Bonds of firms" and "Shares of firms" are taken from the aggregate liabilities of the private firms in Flow of Funds Statistics. Shares include private shares. The output is GDP.

Chapter 3

Sunk Cost of Investment and Credit Crunch

Abstract

This chapter investigates the effect of a credit crunch on macroeconomic indicators in a model of credit market frictions. We show that a persistent credit crunch increases the capital- and the investment-output ratios in the economy, if there exists a sufficiently large sunk cost of investment. In the dynamic analysis of a temporary credit crunch, we show that the credit crunch reduces borrowing of the producers and their debt-repayments ex-post, and subsequently increases their net-worth. This makes the capital- and the investment-output ratios overshoot the steady state level in the recovery from the shock. We discuss implication of the model to the long stagnation in Japan after 1990.

3.1 Introduction

One of the features of the long stagnation in Japan after 1990 was the non-performing loans problem in the banking sector. Several researchers discuss whether there is a link between this problem and the stagnation of the investment growth in Japan in the 1990's. Estimating the investment function, Motonishi and Yoshikawa (1999) find that the decline of demand for the capital stock mostly explained the investment of the firms in the industry-level data, while Nagahata and Sekine (2005) and Ogawa (2003) find that the firms' investment was affected by the supply conditions of the bank loans, such as the bank capital, in the firm-level data.

Hayashi and Prescott (2002) join this discussion from a different perspective by using a general equilibrium analysis. Using the standard exogenous growth model, they show that a productivity slowdown well explains the observed decline of the detrended value-added, increase of the capital-output ratio (Figure 3.1), and decrease of the after-tax return on capital in Japan after 1990. The credit crunch does not play any role in their model. The reason for them to use the model with the complete credit market is that the investment-output ratio did not decline in the 1990's despite the fall of the bank loans to the firms (Figure 3.2). They argue that this observation indicates that there was no credit crunch constraining the firms' investment.

In this chapter, we investigate how a credit crunch would affect the capital- and the investment-output ratios in a model of credit market frictions. We find three effects of a credit crunch. The first effect is to hamper the investment into the capital stock. This effect makes the capital stock more scarce in production relative to the fixed-supplied factor of production, such as land and labor, and works toward increasing the marginal productivity of capital and reducing the capital-output ratio.

The second effect of a credit crunch is obtained from the insight of Kiyotaki and Moore (1997) such that the credit market facilitates the resource reallocation from the less-productive producers to the more-productive through borrowing. As a corollary, a trouble in the financial intermediation hampers the resource allocation to the more-productive producers, and reduces the average productivity level in the economy. This works toward reducing the marginal productivity of capital and increasing the capital-output ratio in the economy. As this second effect works in the opposite direction to the first effect above, the total effect of a credit crunch on the capital-output ratio is ambiguous.

In the steady-state analysis, we find that the level of sunk cost of investment is important to determine the direction of the total effect of a persistent credit crunch. In the model, we assume that a producer has to incur sunk cost to expand the size of production by investment. The cost is sunk because it is not transferable to the other producers as a part of the capital stock. Examples of such cost are the adjustment costs to find new facilities and location of the new production, and to find the channel to sell the new products. Also, some of the capital stock is producer-specific and not resaleable. Our model implies that if the level of the sunk cost is small, then the first effect of a credit crunch dominates the second, and causes a decline of the capital-output ratio by the reduction of the investment. But if the sunk cost is sufficiently large, then a credit crunch increases the capital-output ratio by the endogenous decline of the productivity level.

To understand this result, note that if a factor of production is transferred between the producers, then the buyer of the factor has to incur the sunk cost of investment to enhance her production size. A credit crunch reduces the aggregate sunk cost of investment in the economy by restricting the resource reallocation from the less-productive producers to the more-productive in the credit market. Hence, in aggregate, the producers can invest more in the capital stock instead of the sunk cost. This effect mitigates the shortage of the capital stock under a credit crunch, and makes it more likely that a credit crunch increases the capital-output ratio by the endogenous decline of the productivity level. The investment-output ratio follows the same result as the capital-output ratio by this mechanism.

Note that the productivity gap across the producers is the key component of the endogenous decline of the average productivity level by a credit crunch. We show that the sunk cost of investment contributes to explain the significant productivity gap in the economy. This is because the producers who enhance the production sizes by investment are the more-productive, as they earn more revenues from the production to reinvest into the next production. The sunk cost of investment makes their investment more expensive, and reduces their input demand. This in turn decreases the input price, and lets the less-productive producers stay in production.

Besides the effect of a persistent credit crunch, we also discuss the dynamic response of the economy to a temporary credit crunch, and compare it with the case of an aggregate negative productivity shock. We find that a credit crunch and an negative productivity shock cause similar dynamic responses of the output, the average productivity of the economy measured by total factor productivity, the capital stock, and the factor price. The difference is their effects on the net-worth of the producers, which in turn creates the different response of the capital-output ratio. A credit crunch limits the borrowing of the producers and reduces their debt-repayments ex-post. Subsequently, the reduced debt-repayments increase the net-worth. This lets the capital stock recover quickly. However, the recovery of the aggregate output becomes slower than the capital stock, as the credit crunch shifts the production resources to the low-productive producers, and reduces the average productivity of the economy. Then, the capital-output ratio overshoots the steady-state level in the recovery under the credit crunch. In contrast, a negative productivity shock reduces the output and then the net-worth of the producers. The reduction of the net-worth constrains the investment of the producers, and the accumulation of the capital stock. Hence, the capital-output ratio remains lower than the steady-state level in the recovery period after the negative productivity shock.

A related literature is the general equilibrium analysis of credit market frictions such as Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). Our model is built upon the Kiyotaki and Moore model. Casselli and Genaiolli (2002) consider the effect of credit market frictions on the average productivity level of the economy, and argue that more credit market frictions allow the inefficient family transfer of the production resources, and reduces the average productivity level. Our model includes the effect of credit market frictions on the average productivity level, too, but also analyzes its implication to the capital- and the investment-output ratios, which Hayashi and Prescott (2002) take as important indicators to identify the cause of the stagnation in Japan after 1990.

Holmstrom and Tirole (1998) and Chen (2001) analyze the macroeconomic effect of a credit crunch by formally modeling the role of the bank capital in the financial intermediation. Especially, Chen provides a dynamic analysis. The difference of our analysis from his analysis is that our analysis considers the productivity gap across the producers and the endogenous decline of the average productivity level in the economy.

The rest of the chapter is organized as follows: Section 3.2 describes the model. Section 3.3 shows the steady-state equilibrium and the effect of a credit crunch on the total-factor-productivity level. Section 3.4 compares the dynamic effects of a credit crunch and a negative productivity shock. Section 3.5 concludes.

3.2 Model

We consider a discrete-time economy with homogeneous goods and a continuum of agents. There are producers and lenders in the economy. The producers are risk-neutral, and each of them maximizes the following utility function:

$$E_t\left(\sum_{s=t}^{\infty}\beta^{s-t}\,c_s\right), \tag{3.1}$$

where c_s is consumption at date s, $\beta \in (0,1)$ is the discount factor for future utility, and E_t is expectation formed at date t. Each producer exits from the economy with probability $1-\gamma$ at the end of each date. When she exits, she consumes all of her wealth. We follow Bernanke, Gertler, and Gilchrist (1999) to set this assumption. This assumption of the producer's behavior lets us simply model the regular exit of the firms, and preclude the possibility that the producers will ultimately accumulate enough net-worth to self-finance their investment.

Each producer has the following production technology at each date t:

$$y_{t+1} = A_{t+1}k_t^{\alpha}l_t^{1-\alpha}, \tag{3.2}$$

where k_t is the capital stock invested in production at date t, l_t is the size of land invested at date t, y_{t+1} is the output of goods at date $t+1$, and A_{t+1} is the productivity level at date $t+1$. Hence, there is a period lag between investment and production. $\alpha \in (0,1)$ is the constant factor share of capital.

A_{t+1} takes either of the two values A^H or A^L at each date $t+1$, determined by the idiosyncratic stochastic shock to each producer. $A^H > A^L$. Each producer knows the value of A_{t+1} only after making investment at date t. μ^H ($\in (0,1)$) denotes the probability that $A_{t+1} = A^H$ at the next date, when a producer has $A_t = A^H$ at the current date. Similarly, μ^L denotes the probability of having A^H at the next date, when $A_t = A^L$. We assume

$$\mu^H > \mu^L, \tag{3.3}$$

Table 3.1: Transition matrix for the producers

		date $t+1$		
		H	L	Exit
date t	High productivity (H)	$\mu^H\gamma$	$(1-\mu^H)\gamma$	$(1-\gamma)$
	Low productivity (L)	$\mu^L\gamma$	$(1-\mu^L)\gamma$	$(1-\gamma)$
	Exit from the economy (Exit)	0	0	1

so that the high-productivity level today is followed by the higher expected productivity level tomorrow than the low-productivity level today. In summary, the transition matrix for the producers is given by Table 3.1. We call the producers with μ^H as high-productive, and those with μ^L as low-productive.

The producers rent the capital stock and the land from the lenders. To finance the rental cost of capital and land, there is a competitive one-period debt market, in which one unit of goods at date t is exchanged for a claim to $(1+r)$ units of goods at date $t+1$. We assume that only debt contracts are feasible in the credit market because of the prohibitively high cost to write and enforce contingent contracts. We assume that the economy is the small-open economy, and that the interest rate r is exogenous. Thus, the lenders have deep pockets.[1] To simplify the analysis, we assume

$$\beta(1+r) = 1. \tag{3.4}$$

The production technology is specific to each producer. Once a producer has invested into the capital stock and the land at date t, only she has the necessary skill to obtain the full return from the production at date $t+1$. Without the skill of the producer who initiated the investment, the other producers and lenders can obtain only a fraction θ of the full returns, besides the capital stock and the land invested in production. There is no record keeping of the credit history. Each producer is free to walk away from the production and from any debt obligation between the dates of investment and harvest, and can start new production without any default record. We assume that the debtor-producer

[1] Given the exogenous interest rate in the small-open economy, the exact form of the household's utility maximization to determine the households' savings is irrelevant to the model.

has strong bargaining power, and can reduce her debt-repayment down to the fraction θ of the expected output by renegotiation with the lender between the dates of investment and harvest. Anticipating the possibility of the renegotiation, the lender limits the amount of credit at date t, so that the debt-repayment of the debtor-producer at the next date, b_{t+1}, will not exceed the fraction θ of the expected output. This implies that

$$b_{t+1} \leq \theta E_t \left[y_{t+1} \right], \tag{3.5}$$

where b_{t+1} is the amount of the debt-repayment at date $t+1$.[2]

Note that the borrowing is constrained by the expected output at the next date in (3.5). We assume that the producer needs to input their skill before knowing the realization of the productivity level at date $t+1$, so that the producer can renegotiate the debt contract only before then. After the output realizes, the debtor-producer loses the strong bargaining power to renegotiate the debt contract. We also assume

$$A^L > \theta E[A \,|H]. \tag{3.6}$$

In (3.6), $E[A \,|i] = \mu^i A^H + (1 - \mu^i) A^L$, which is the conditional expectation of A_{t+1} at date t, given $A_t = A^i$ for $i = H, L$. Thus, the face value of the debt b_{t+1} is always repaid at date $t+1$. We set this assumption to prevent the low-productive producers from losing all the net-worth by default and being unable to continue their production due to the borrowing constraint (3.5).[3] Then, we can analyze the resource allocation between the high- and the low-productive producers.

θ is the indicator of the efficiency of the credit market. We analyze the effect of a credit crunch by lowering the level of θ. This is a short-cut assumption to simply model the effect of the credit-supply condition of the financial intermediaries. Implicitly, we assume that the capital of the financial intermediaries (the banks) affects the pledgeable fraction of the output by the borrower to the lenders, and that a crunch of the bank capital reduces θ. Holmstrom and Tirole

[2]The values of the capital stock and the land do not enter the right-hand side of (3.5), as the producers rent these inputs and incur debts to finance the rental costs for them.

[3]The borrowing constraint prevents the producers from finance all the production cost by borrowing, and requires them to make down-payments to buy the production resources from their net-worth.

(1998) and Chen (2001) analyze such a mechanism that a decline of the bank capital implies less supply of the informed capital, and raises the returns paid to the informed capital. Therefore, the borrowers have less pledgeable repayments from their production to the uninformed lenders (the depositors).

We consider the bank-capital specific shock to the economy, and treat the shock to θ independently of the productivity shock. An example of such a shock is the analysis provided by Hoshi and Kashyap (1999) such that the Japanese banks lost their traditional borrowers, the large firms, to the corporate bond market after the financial liberalization in the 1980's, and lent to inefficient small-sized firms due to the lack of experience with this new kind of borrowers. These loans turned out to be the non-performing loans problem in the 1990's. Such a mistake made by the banks would reduce the bank capital independently of the productivity level of the economy. But it is also true that the bank capital is endogenous and would be affected by the productivity shock. From this view, our analysis should be interpreted as extracting the indirect effect of the productivity shock through a credit crunch and highlighting its characteristic separately from the direct effect of the productivity shock.

We further assume that the producers have to incur sunk cost to enhance the size of production by investment. The amount of the sunk cost of investment is assumed to be

$$z \cdot \max\{k_t^{\alpha} l_t^{1-\alpha} - k_{t-1}^{\alpha} l_{t-1}^{1-\alpha},\, 0\}, \tag{3.7}$$

where z is a positive constant. We call $k_t^{\alpha} l_t^{1-\alpha}$ as production units. (3.7) reflects the adjustment cost for new production units such as costs to find new land, facilities, location of production, and channels to sell their new products. In (3.7), we assume that this installation of production units is producer-specific, and not transferable across the producers. Hence, its cost is sunk. The producers need to pay the sunk cost only once for the existing production units, so that the amount of the sunk cost is only proportional to the size of the new production units, rather than the whole production units.

Given (3.1)-(3.5) and the stochastic exit of the producers, the producer's optimization problem is defined as

$$\max_{\{c_s,k_s,l_s,b_{s+1}\}_{s=t}^{\infty}} \quad E_t\left(\sum_{s=t}^{\infty}\beta^{s-t}\,c_s\right) \tag{3.8}$$

$$\begin{aligned}
\text{s.t.}\quad & c_s+\left(1-\frac{1-\delta}{1+r}\right)k_s+u_s l_s+z\cdot\max\{k_s^{\alpha}l_s^{1-\alpha}-k_{s-1}^{\alpha}l_{s-1}^{1-\alpha},\,0\}\\
& =A_s k_{s-1}^{\alpha}l_{s-1}^{1-\alpha}-b_s+\frac{b_{s+1}}{1+r}\\
& b_{s+1}\le\theta E_s A_{s+1}k_s^{\alpha}l_s^{1-\alpha}\\
& c_s,k_s,l_s\ge 0\\
& c_s=A_s k_{s-1}^{\alpha}l_{s-1}^{1-\alpha}-b_s \text{ if the producer exits.}
\end{aligned}$$

c_s, k_s, l_s and b_s, respectively, are consumption, the capital stock, the land investment, and the amount of debt at date s. δ is the depreciation rate of the capital stock. r is the exogenous interest rate. $1-(1-\delta)/(1+r)$ is the rental cost of capital, and u_s is the rental cost of land at date s.[4] A_s is the productivity level of the producer. The first constraint is the flow-of-funds constraint, the second the borrowing constraint, the third the non-negativity constraint for consumption, the capital stock and the land-investment, and the forth the consumption when the producer exits from the economy.

3.2.1 Equilibrium

In this chapter, we focus on the case where (a) the borrowing constraint (3.5) is binding for all the producers;[5] (b) the high-productive producers increase the sizes of their production units from the previous levels and incur the sunk cost of investment; (c) the low-productive producers decrease their production units from the previous levels by short of net-worth, and do not incur the sunk cost of investment; and (d) consumption is 0 for each producer until she exits from the economy. Hereafter, we solve the model under this conjecture, and then choose parameter values such that this conjecture is verified. The appendix contains the derivation of the necessary and sufficient conditions for such parameter values from the optimization conditions of (3.8). In the numerical simulation below, we check that these conditions are satisfied.

[4] $u_t = q_s - E_s q_{s+1}/(1+r)$, where q_s is the land price at date s.

[5] More precisely, the producers exiting from the economy do not borrow, so that they are not constrained by the borrowing constraint.

It is immediately shown from the cost minimization for a given amount of the production units $k_s^\alpha l_s^{1-\alpha}$ that

$$k_s = \frac{\alpha u_s}{[1-(1-\delta)/(1+r)]\,(1-\alpha)} l_s \tag{3.9}$$

at the optimum of (3.8). (3.9) implies that the capital stock is proportional to the land, given the rental cost of land u_s. We can aggregate the both sides of (3.9) and obtain from (3.2) and (3.5) with equality that

$$K_t^i = \frac{\alpha u_t}{[1-(1-\delta)/(1+r)]\,(1-\alpha)} L_t^i \tag{3.10}$$

$$Y_t^H = A^H \left(\frac{\alpha u_{t-1}}{[1-(1-\delta)/(1+r)]\,(1-\alpha)}\right)^\alpha (\mu^H L_{t-1}^H + \mu^L L_{t-1}^L) \tag{3.11}$$

$$Y_t^L = A^L \left(\frac{\alpha u_{t-1}}{[1-(1-\delta)/(1+r)]\,(1-\alpha)}\right)^\alpha [(1-\mu^H) L_{t-1}^H + (1-\mu^L) L_{t-1}^L] \tag{3.12}$$

$$B_{t+1}^i = \theta E[A\,|i] \left(\frac{\alpha u_t}{[1-(1-\delta)/(1+r)]\,(1-\alpha)}\right)^\alpha L_t^i \tag{3.13}$$

for $i = H, L$. K_t^i, L_t^i, Y_t^i, and B_{t+1}^i, respectively, are the aggregate capital stock, the aggregate land-investment, and the aggregate output at date t, and the aggregate value of the producers' debt-repayments at date $t+1$, for the high-productive ($i = H$) and the low-productive ($i = L$) producers.

Aggregating the flow-of-funds constraints and substituting (3.10) and (3.13) into them, we obtain

$$\left[\Omega_t + z - \frac{\theta E[A\,|H]}{1+r}\right] \phi_t^\alpha L_t^H = \gamma \left[V_t^H + z\phi_{t-1}^\alpha(\mu^H L_{t-1}^H + \mu^L L_{t-1}^L)\right] \tag{3.14}$$

$$\left[\Omega_t - \frac{\theta E[A\,|L]}{1+r}\right] \phi_t^\alpha L_t^L = \gamma V_t^L, \tag{3.15}$$

where

$$\Omega_t \equiv \left(\frac{1-(1-\delta)/(1+r)}{\alpha}\right)^\alpha \left(\frac{u_t}{1-\alpha}\right)^{1-\alpha} \tag{3.16}$$

$$\phi_t \equiv \frac{\alpha u_t}{[1-(1-\delta)/(1+r)]\,(1-\alpha)} \tag{3.17}$$

$$V_t^H = \left[(A^H - \theta E[A\,|H])\mu^H L_{t-1}^H + (A^H - \theta E[A\,|L])\mu^L L_{t-1}^L\right] \phi_{t-1}^\alpha \tag{3.18}$$

$$V_t^L = \left[(A^L - \theta E[A\,|H])(1-\mu^H) L_{t-1}^H + (A^L - \theta E[A\,|L])(1-\mu^L) L_{t-1}^L\right] \phi_{t-1}^\alpha. \tag{3.19}$$

Ω_t is the marginal cost of production except the sunk cost of investment. ϕ_t is the capital-land ratio common to all the producers, which is derived from

(3.9). V_t^H and V_t^L are the aggregate net-worth of the high-productive and the low-productive producers at date t, respectively.

(3.14) is the aggregate flow-of-funds constraint for the high-productive producers, who incur the sunk cost of investment. z appears in the right-hand side of (3.14), because the producers do not have to incur the sunk cost for the existing production units. (3.15) is the aggregate flow-of-funds constraint for the low-productive producers.

We normalize the economy by setting the fixed supply of land equal to 1. The market clearing condition for land is

$$L_t^H + L_t^L = 1. \tag{3.20}$$

We define equilibrium such that each producer chooses $\{c_s, k_s, l_s, b_{s+1}\}_{s=t}^{\infty}$ to maximize (3.8), and $\{u_s\}_{s=t}^{\infty}$ clears the land market every date. Under the conjecture (a)-(d) described at the beginning of this subsection, (3.14), (3.15) and (3.20) specify the equilibrium values of $\{u_s, L_s^H, L_s^L\}$ for $s \geq 0$, given the predetermined aggregate net-worth in the right-hand sides of (3.14) and (3.15) at the initial date. Then, (3.10)-(3.13) determine the aggregate investment, the aggregate output, the aggregate value of debts, which recursively determines the aggregate net-worth of the producers at the next date by (3.18) and (3.19). Note that we have assumed that the productivity transition is idiosyncratic for each producer. This assumption implies that there is no aggregate uncertainty, except unexpected shocks. Hence, we assume that the agents have perfect foresight under the rational expectation.

Remember that each producer is exiting from the economy in some date by assumption. To keep the total population of the producers always positive in the model, we assume that there is always new entry of the producers into the economy. To obtain (3.14) and (3.15), we assume that new producers enter the economy with an arbitrarily small net-worth.[6] The net-worth of the new entrants does not play any other significant role in the model than to keep the population of the producers always positive. In (3.14) and (3.15), we take the new entrants' net-worth to the limit at zero.

[6]Under the binding borrowing constraint, a positive net-worth is necessary to pay the down-payment to start the production.

3.3 Steady-state equilibrium

3.3.1 The long-run effect of a persistent credit crunch

In this section, we consider the steady-state equilibrium, and analyze the long-run effect of a persistent credit crunch by comparative statics with respect to θ. The steady-state equilibrium is defined such that all the variables in (3.10)-(3.20) are constant over time. Substituting (3.18), (3.19) and (3.20) into (3.14) and (3.15), we can specify the steady-state equilibrium by the following two equations:

$$\frac{L_{SS}^H}{1-L_{SS}^H} = \frac{\Omega_{SS} - \theta E[A\,|L]/(1+r) - \gamma(1-\mu^L)(A^L - \theta E[A\,|L])}{\gamma(1-\mu^H)(A^L - \theta E[A\,|H])} \tag{3.21}$$

$$\begin{aligned}\Omega_{SS} =& \{\gamma(E[A\,|H] - E[A\,|L]) - [1-\gamma(\mu^H - \mu^L)]z\}L_{SS}^H + \gamma(E[A\,|L] + \mu^L z) \\ &+ \theta\left(\frac{1}{1+r} - \gamma\right)\{(E[A\,|H] - E[A\,|L])L_{SS}^H + E[A\,|L]\}.\end{aligned} \tag{3.22}$$

The subscript SS implies the steady-state value of the variable. We obtain (3.21) from (3.15), and (3.22) by adding (3.14) and (3.15). We can interpret these equations such that (3.21) is the response of the aggregate land-investment by the high-productive producers, L_{SS}^H, to the marginal production cost, Ω_{SS}, through the producers' individual behavior, and that (3.22) is the response of Ω_{SS} to L_{SS}^H through the market-clearing price determination.

We can find the steady-state equilibrium by drawing (3.21) and (3.22) in the $L_{SS}^H - \Omega_{SS}$ plane, and show that there exists unique steady-state equilibrium under the conjecture (a)-(d) described in the section 3.2.1.[7] Figures 3.3 and 3.4 contain the diagrams. We can show that (3.6) implies that (3.21) is upward-sloping in Ω_{SS}.[8] There are two cases for the slope of (3.22), either non-negative (Figure 3.3) or negative (Figure 3.4). The slope of (3.22) with respect to L_{SS}^H is

$$\left[\frac{\theta}{1+r} + (1-\theta)\gamma\right](E[A\,|H] - E[A\,|L]) - [1-\gamma(\mu^H - \mu^L)]z.$$

Thus, the sign of the slope depends on the levels of z and $(E[A|H] - E[A|L])$.

Now we analyze the comparative statics with respect to θ. We assume

$$\frac{1}{1+r} > \gamma. \tag{3.23}$$

[7] $A^H > \theta E[A\,|L]$ implies that the intercept of (3.22) is larger than (3.21). Note that Ω_{SS} goes to ∞ as L_{SS}^H goes to 1 in (3.21). This proves existence of unique steady-state equilibrium.

[8] (3.21) is the aggregate flow-of-funds constraint for the low-productive. Their land-investment declines with Ω_{SS}. Then, L_{SS}^H increases with Ω_{SS} in this equation.

(3.23) implies that the survival rate of the producers is not too high. Under this assumption, $E[A|H] > E[A|L]$ and the conjecture (a)-(d) described in the previous section, we can show that the right-hand sides of (3.21) and (3.22) are increasing in θ.[9] This implies that a decline of θ reduces L_{SS}^H given Ω_{SS} in (3.21), and Ω_{SS} given L_{SS}^H in (3.22). In Figures 3.3 and 3.4, a decline of θ shifts the curve (3.21) leftward, and the curve (3.22) downward.

The intuition behind the reduction of L_{SS}^H in (3.21) is that a decline of θ reduces the borrowing of the producers by tightening their borrowing constraints. Given the value of Ω_{SS}, it limits the leverage taken by the producers. As the high-productive producers have the higher rate of return to investment than the low-productive, the high-productive suffer from the more loss of the rate of return to investment by the limited leverage. Hence, the gap of the rate of return between the high- and the low-productive becomes narrower, and the relative share of the net-worth shifts to the low-productive in the long run. Then, the land allocation also shifts to the low-productive. The intuition behind the reduction of Ω_{SS} in (3.22) is that the reduction of the producers' borrowing by a decline of θ reduces the aggregate expenditure on the inputs in production. This in turn decreases the rental cost of land u_{SS} and the marginal cost of production Ω_{SS}.

The shifts of (3.21) and (3.22) cause an unambiguous decline in L_{SS}^H as shown in Figures 3.3 and 3.4. Ω_{SS} unambiguously decreases in Figure 3.3, but its direction of change is ambiguous in Figure 3.4. The reason for this ambiguous result on Ω_{SS} in Figure 3.4 is the negative slope of (3.22). In this case, z is relatively large compared to the degree of the heterogeneity of the productivity levels across the producers ($E[A|H]-E[A|L]$). Note that if L_{SS}^H is lowered, then it is accompanied with less aggregate expenditure on the sunk cost of investment arising from the resource reallocation from the low-productive producers

[9] The derivative of the right-hand side of (3.21) with respect to θ is proportional to

$$E[A|H][\Omega_{SS} - \gamma(1-\mu^L)A^L] - E[A|L]A^L\left[\frac{1}{1+r} - \gamma(1-\mu^L)\right].$$

The necessary condition for the case that the low-productive producers cut back their production sizes is

$$\Omega_{SS} - \frac{\theta E[A|L]}{1+r} > A^L - \theta E[A|L].$$

By substituting this inequality into Ω_{SS}, we can show that the sign of the derivative is positive.

to the high-productive. Then, the saved expenditure on the sunk cost is instead spent on the inputs. Because now the sunk cost is large, saving the sunk cost contributes to increase the land price and the cost of production in (3.22). Therefore, in Figure 3.4, the reduction of the borrowing by a decline of θ causes the two contradicting effects on Ω_{SS}; the effect of limiting the borrowing of the producers and reducing the aggregate expenditure on the inputs, and the effect of saving the aggregate sunk cost by restricting the resource reallocation to the high-productive producers. These contradicting effects make the change of Ω_{SS} ambiguous.

Result 1 summarizes these results.

Result 1: There exists unique steady state. L_{SS}^{H} decreases as θ declines. Ω_{SS} decreases as θ declines, when z is small relative to $(E[A\,|H] - E[A\,|L])$. The direction of the change in Ω_{SS} is ambiguous, when z is large relative to $(E[A|H]-E[A\,|L])$.

We measure the average productivity level in the economy by total factor productivity (TFP), and define the TFP level by

$$\begin{aligned}\mathrm{TFP}_t &\equiv \frac{Y_t^H + Y_t^L}{(K_{t-1}^H + K_{t-1}^L)^{\alpha}} \\ &= E[A\,|H]L_{t-1}^H + E[A\,|L](1 - L_{t-1}^H). \qquad (3.24)\end{aligned}$$

The second equality is obtained from (3.10)-(3.12).[10] (3.24) implies that the aggregate TFP level is determined by the land allocation between the high-productive and the low-productive producers. Since (3.24) implies that TFP_{SS} is increasing in L_{SS}^{H}, we immediately obtain Result 2 from Result 1.

Result 2: TFP_{SS} is increasing in θ.[11]

Now we consider the change of the capital- and the investment-output ratios.

[10]In (3.24), we use the gross output, rather than the value-added net of the sunk cost of investment.

[11]Caselli and Genaiolli (2003) also show that credit market frictions reduce the TFP level of the economy.

Table 3.2: Parameter values for simulation

α	0.86	γ	0.9
r	0.025	δ	0.1
μ^H	0.84	μ^L	0.28
A^H	1.225	A^L	1
θ	0.75	z	0.2

From (3.10)-(3.12), we can obtain the capital-output ratio at the steady state as

$$\frac{K_{SS}^H + K_{SS}^L}{Y_{SS}^H + Y_{SS}^L} = \frac{\phi_{SS}^{1-\alpha}}{TFP_{SS}} = \frac{\alpha \Omega_{SS}}{[1-(1-\delta)/(1+r)]TFP_{SS}}. \tag{3.25}$$

The second equality is obtained from (3.17), the definition of ϕ_t. Thus, the capital-output ratio at the steady state is decreasing in TFP_{SS}, and increasing in Ω_{SS}. Result 1 implies that a decline of θ tends to decrease the capital-output ratio through Ω_{SS}, if the level of the proportional sunk cost z is small. But this effect is weak if the level of z is large and the decline of Ω_{SS} is small. Result 2 implies that a decline of θ contributes to increase the capital-output ratio through TFP_{SS} in all the cases.

To clarify this result, we numerically calculate the derivative of the capital-output ratio with respect to θ at the steady-state equilibrium under the different sets of z and A^H/A^L. A^H/A^L represents the degree of the heterogeneity of the productivity levels across the producers. We define the parameters of the model at annual frequency, and adopt the base-line parameter values shown in Table 3.2 for the other parameters than z and A^H. These parameters are chosen to match the steady-state of the model with the Japanese macro and micro-level data. See appendix for the detail of the calibration.

Figure 3.5 shows the sign of the derivative of the capital-output ratio with respect to θ over the $z - A^H/A^L$ plane. The figure implies that the derivative is negative when z is sufficiently large. The area of the negative derivative is not very responsive to A^H/A^L.[12] Thus, the level of z is more important to determine the sign of the derivative.

[12]The area of the negative derivative is expanding as A^H/A^L increases when A^H/A^L is

We consider the implication of these results for the investment-output ratio. The aggregate investment into the capital stock is defined as

$$I_t \equiv K_t^H + K_t^L - (1-\delta)(K_{t-1}^H + K_{t-1}^L). \tag{3.26}$$

As K_t^H and K_t^L are constant at the steady state, I_{SS} equals $\delta(K_{SS}^H + K_{SS}^L)$. Hence, the investment-output ratio follows the capital-output ratio in the comparative statics in this section. We obtain the following result:

Result 3: Lowering the level of θ increases the steady-state values of the capital- and the investment-output ratios, if z is sufficiently large.

3.3.2 The role of sunk cost of investment to explain the heterogeneity of the producers

In the previous subsection, we have shown that the heterogeneity of the productivity levels across the producers plays the crucial role in the endogenous decline of the average productivity level by a credit crunch. Note that the heterogeneity would not exist in the perfectly competitive market, because given the constant-returns-to-scale production technology, all the production resources would go to the most productive producers. As clarified by Kiyotaki and Moore (1997), credit market frictions are important to explain the existence of the heterogeneity. Credit market frictions limit the borrowing of the most productive producers, and their purchase of the production resources. Hence, the most productive producers cannot accumulate all the production resources in the economy at their hands, and the less productive producers can also engage in production.

In this subsection, we argue that it is also important to consider the sunk cost of investment to explain the existence of the heterogeneity of the productivity levels across the producers. For this purpose, we consider the economy

relatively low. This is because the high heterogeneity of the producers increases the endogenous decline of the productivity level by a credit crunch. But the area starts slightly shrinking as A^H/A^L increases when A^H/A^L is relatively large. This is because the high productivity gap allows the high-productive producers accumulate a large part of the capital stock and the land for their production. In such a case, a marginal reduction of the resource allocation to them by a decline of θ does not much affect the average productivity level of the economy. Hence, the capital-output ratio becomes more likely to be increasing in θ through the capital stock accumulation.

without the sunk cost of investment, and numerically calculate the model to find how much heterogeneity can occur at the steady state without the sunk cost of investment.

Set $z = 0$, and consider the marginal case where the low-productive producers engage in production, but do not borrow up to the borrowing limit. Then, the marginal productivity of their investment equals the marginal cost of production:

$$E[A|L] = (1+r)\Omega_{SS}. \tag{3.27}$$

Note that Ω_{SS} is the marginal cost to purchase a production unit $k_t^{\alpha} l_t^{1-\alpha}$, and $E[A|L]$ is the expected marginal productivity of the production unit for the low-productive producers. If this condition holds, then the marginal productivity of the investment for the high-productive producers, $E[A|H]$, exceeds the marginal cost, and that the high-productive producers borrow up to the borrowing limit. Then, we can obtain the amount of the high-productive's investment into the capital stock and the land from their aggregate flow-of-funds constraint as follows:

$$\left(\Omega_{SS} - \frac{\theta E[A|H]}{1+r}\right)\phi_{SS}^{\alpha} L_{SS}^{H} = \gamma V_{SS}^{H} + V^{N}, \tag{3.28}$$

$$V_{SS}^{H} = \mu^{H}(A^{H} - \theta E[A|H])\phi_{SS}^{\alpha} L_{SS}^{H} + \mu^{L}(1+r)\gamma V_{SS}^{L}, \tag{3.29}$$

$$V_{SS}^{L} = (1-\mu^{H})(A^{L} - \theta E[A|H])\phi_{SS}^{\alpha} L_{SS}^{H} + (1-\mu^{L})(1+r)\gamma V_{SS}^{L}. \tag{3.30}$$

(3.28) is the aggregate flow-of-funds constraint of the high-productive producers. V^N in the right-hand side of the equation is the net-worth of the new entrants into the economy. In this section, we explicitly put an arbitrarily small value of V^N in the equation for presentation purpose. Note that (3.27) implies that the rate of return to investment for the low-productive producers is equal to the market rate of return (1+r), so that their net-worth increases by this rate as shown in the last two equations (3.29) and (3.30).

Solving (3.27)-(3.30) for L_{SS}^{H}, we obtain

$$\begin{aligned} L_{SS}^{H} = & \phi_{SS}^{-\alpha} V^{N} \left\{ \frac{E[A|L]}{1+r} - \frac{\theta E[A|H]}{1+r} - \gamma \left[(A^{H} - \theta E[A|H])\mu^{H} \right.\right. \\ & \left.\left. + \frac{(1+r)\gamma\mu^{L}(1-\mu^{H})(A^{L} - \theta E[A|H])}{1-(1+r)\gamma(1-\mu^{L})} \right] \right\}^{-1}. \end{aligned} \tag{3.31}$$

If $L_{SS}^{H} \in [0,1)$, then the high-productive producers do not use all the land in the economy, so that the low-productive producers stay in production at the steady state. Otherwise, the low-productive do not engage in production. Among such cases, $L^H_{SS} > 1$ does not occur in our model, because V^N is arbitrarily small.[13] If the denominator of (3.31) is negative, then $L^H_{SS} < 0$, given $\phi_{SS} > 0$. This implies that the high-productive producers accumulate more net-worth than the cost of investment. The high-productive's borrowing constraints do not bind, and they use all the production resources in the economy. We investigate under which parameter values this case occurs.

For this analysis, we compare the different levels of the heterogeneity across the producers. We measure the heterogeneity across the producers by the relative sizes of μ^H and μ^L. Respectively, they are the transition probabilities from each productivity level, high and low, at the current date to the high productivity level at the next date. Thus, the ex-ante productivity levels of the producers become more heterogeneous as the gap between μ^H and μ^L gets larger. We numerically calculate the model to find the set of (μ^H, μ^L) under which L^H_{SS} is in $[0, 1)$. Besides μ^H, μ^L and z, we use the values in Table 3.2 for the other parameter values.

Figure 3.6 shows the set of (μ^H, μ^L) under which the low-productive producers stay in production. The figure implies that the low-productive producers are more likely to stay in production when the values of μ^H and μ^L are close, i.e. the heterogeneity across the producers is low. The set of such μ^H and μ^L does not contain $(\mu^H, \mu^L) = (0.84, 0.28)$, which is calibrated from the Fukao and Kwon (2004)'s micro data analysis of the Japanese firms. Hence, we need the sunk cost of investment to explain these calibrated values. Figure 3.5 is drawn under $(\mu^H, \mu^L, z) = (0.84, 0.28, 0.2)$, and shows that both of the high- and the low-productive producers stay in production even under such a high level of the heterogeneity implied by μ^H and μ^L.

The intuition for this result is that the high-productive producers receive high incomes from their production, and reinvest them to expand their production sizes. The sunk cost of investment makes it expensive to expand the production sizes, and restricts the accumulation of the production resources by the high-productive producers together with credit market frictions. Hence, the low-

[13] We can numerically show this. If $L^H_{SS} > 1$, then there is an excess demand for land from the high-productive producers, and the rental cost of land u_{SS} goes up. This will raise Ω_{SS}, and makes the rate of return to investment by the low-productive less than its marginal cost.

productive producers can stay in production under the less demand for the production resources from the high-productive and the less input prices.

3.4 Dynamic effect of a temporary credit crunch

In this section, we analyze the dynamics of the model to a temporary credit crunch. The dynamic equilibrium is recursively given by (3.14), (3.15) and (3.20). We numerically calculate the dynamic equilibrium under the parameter values given in Table 3.2.

3.4.1 Credit crunch

We consider an unexpected decline of θ from 0.75 by 1% at date 0, and its gradual recovery following

$$\theta_t = 0.75 + 0.5(0.75 - \theta_{t-1}),$$

where θ_t is the level of θ associated with the borrowing at date t and the debt repayment at date $t+1$. 0.75 is the steady state value of θ. 0.5 is the autoregressive coefficient to the mean deviation. The gradual recovery of θ makes the duration of the credit crunch temporary. We assume that the decline of θ_0 does not affect the debt-repayments of the producers at date 0, but only affects the borrowing limit. To investigate the effect of the sunk cost of investment, we consider the dynamic responses of the economy for $z = 0$ and $z = 0.2$.

Figure 3.7 and 3.8 show the dynamic paths of θ_t and the macroeconomic indicators Y_t, TFP_t, K_t, u_t, B_{t+1}, I_t and L_t^H. Y_t is the aggregate output $Y_t^H + Y_t^L$, and K_t is the aggregate capital stock equal to $K_t^H + K_t^L$. B_{t+1} is the aggregate debts repaid at the next date equal to $B_{t+1}^H + B_{t+1}^L$. I_t is the aggregate investment net of the sunk cost as defined by (3.26). The values in Figure 3.7-3.12 are % deviations from their steady-state levels, except for L_t^H. L_t^H is the aggregate land share for the more-productive producers.

The dynamics under $z = 0.2$ shows that the decline of θ reduces the investment into the capital stock by tightening the borrowing constraints, and also causes a resource shift to the low-productive producers and reduces the total factor productivity level. The mechanism behind the resource shift is the same as the one described in the previous section, such that a decline of θ limits the leverage taken by the producers, and that the high-productive producers suffer from the more loss of the rate of return to investment than the low-productive.[14] Both declines of investment and TFP reduce the aggregate output. The rental cost of land declines with the fall of the borrowing of the producers and the aggregate expenditure on land.

The feature of the dynamics under $z = 0$ is the same as $z = 0.2$, except TFP_t and L_t^H. We can numerically show that under $z = 0$, the low-productive producers stop engaging in production, and only the more-productive producers use the production resources. Thus, the total factor productivity is constant to be A^H. Hence, as described in the section 3.3.2, this numerical example highlights that the sunk cost of investment contributes to explain the productivity gap across the producers and the endogenous fluctuation of the total factor productivity level by the credit market shock.

Figure 3.9 shows the dynamic paths of V_t^H, V_t^L, K_{t-1}/Y_t, and I_t/Y_t. A notable feature is that the net-worth of the producers increases after the credit crunch. This is because the reduction of the borrowing reduces the debt-repayments ex-post, and increases the net-worth of the producers.[15] The reduced input price u_t under the credit crunch contributes to this phenomenon, as it makes the reduction of the capital stock and the revenue from the production less than the reduction of the borrowing. Then, the net-worth still increases despite the reduced output.[16]

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[14]The reason for the slight increase of TFP at date 1 is that the decline of θ_0 does not change the predetermined net-worth at date 0, but only increases the required down-payments for the producers to buy a production unit. Because of the existence of the sunk cost of investment, the required down-payments for the low-productive producers, $\Omega_t - \theta E[A\,|L]/(1+r)$, more proportionally increases than the one for the high-productive producers, $\Omega_t + z - \theta E[A|H]/(1+r)$. Hence, the resource allocation shifts to the high-productive producers at date 0, and increases the TFP level at date 1. After date 1, the mechanism described in the main part works through the net-worth of the producers.

[15]Readers might ask why the producers do not individually refrain from the borrowing to maximize their expected net-worth, given the producer's utility function assumed by (3.1). This is because the producers take the rental cost of land as given in the competitive land market. credit market frictions and the binding borrowing constraints limit the amount of the borrowing, which reduces the expenditure on the inputs and hence the input price. The low input price creates a positive gap between the marginal return to investment and its marginal cost. Then, the producers individually find it profitable to borrow up to their borrowing limits. This mechanism is found by Biais and Mariotti (2004).

[16]The comparison between $z = 0$ and $z = 0.2$ shows that the sunk cost of investment reduces this effect for the more-productive producers, and increases it for the less-productive producers. This is because under $z = 0.2$, the land share of the more-productive producers

The capital-output ratio initially declines because the net-worth is predetermined at the impact of the initial shock to θ, and the recovery of the net-worth by the reduction of the debt-repayments has not yet taken place. The decline of θ just reduces the capital stock and the capital-output ratio by limiting the borrowing of the producers. But the subsequent increase of the net-worth contributes to a quick recovery of the capital stock. Given the endogenous decline of the productivity level, the output stays low, and the capital-output ratio overshoots the original steady-state level in the recovery period. The dynamics of the investment and the investment-output ratio follow this mechanism as well.

3.4.2 Negative productivity shock

For comparison, we derive the dynamic response to an unexpected negative productivity shock. It is a short-cut to assume an unexpected shock, but the simulation below still highlights the difference between a productivity shock and a credit crunch. We keep the productivity gap across the producers A_t^H/A_t^L constant, where A_t^j is the level of A^j at date t for $j = H, L$. We assume that A_t^L unexpectedly declines by 1% at date 0, and thereafter follows

$$A_t^L = 1 + 0.5(1 - A_{t-1}^L),$$

1 is the steady-state level of A_t^L. 0.5 is the autoregressive coefficient. As A_t^H/A_t^L is kept constant for all t, all the producers are hit by the same productivity shock proportional to their productivity level.

Figures 3.10-3.12 show the same sets of variables as Figures 3.7-3.9, except the productivity shock instead of the shock to θ. The qualitative feature of Figures 3.10-3.11 is the same as Figures 3.7-3.8.[17] Despite this similarity, Figure 3.12 shows that the net-worth and the capital-output ratio behave differently from the case of the shock to θ. The decline of the output by the productivity

declines after the shock to θ, and then their aggregate net-worth. On the flip side of the coin, the aggregate net-worth of the less-productive producers more increases.

[17]Only L_t^H shows the different feature from the case of the shock to θ. L_t^H increases after the productivity shock, rather than decreases. In our model, the low-productive producers are credit-constrained, and suffer from shortage of the net-worth. Then, the negative productivity shock reduces the low-productive's net-worth more proportionally than the high-productive's, and the production resource shifts toward the high-productive. But the direct effect of the negative productivity shock dominates the effect of the resource shift, and reduces the total factor productivity of the economy.

shock first increases the capital-output ratio, since the capital stock is predetermined at the impact of the initial shock. But it also reduces the net-worth of the producers.[18] This reduction of the net-worth then decreases the investment into the capital stock and the capital-output ratio. This implies that the effect of the reduction of the net-worth is strong enough to create shortage of the capital stock in production and to raise the marginal productivity of capital despite the decline of the productivity level.

3.5 Conclusion and the implication to the long stagnation in Japan after 1990

In this chapter, we have analyzed the effect of a credit crunch in the model of credit market frictions. We have shown that the capital- and the investment-output ratios do not necessarily decline in a credit crunch. When the sunk cost of investment is sufficiently large, a persistent credit crunch increases these ratios by restricting the resource reallocation across the producers in the credit market and causing the endogenous decline of the average productivity level.

In the dynamic analysis of a temporary credit crunch, we have shown that the credit crunch leads to the subsequent increase of the net-worth of the producers by restricting their borrowing and reducing their debt-repayments. This increase of the net-worth of the producers contributes to a quick recovery of the capital stock, while the credit crunch suppresses the aggregate output by the endogenous decline of the average productivity level. Then, the capital- and the investment-output ratios overshoot the original steady state level in the recovery period after the shock.

While this chapter analyzes the model where all the producers are credit-constrained, readers might wonder what if the less-productive producers are not credit-constrained, as Kiyotaki and Moore (1997) formulate. This assumption would strengthen our result, because a credit crunch would still cause the resource shift to the less-productive and the endogenous decline of the average productivity level, but it would not constrain the aggregate investment into the

[18]The effect of the sunk cost of investment on the dynamics of the net-worth is similar to the shock to θ, which is described in the footnote 16.

capital stock, as the less-productive producers invest into the capital stock without the borrowing constraint. Then, a credit crunch would increase the capital- and the investment-output ratios. Instead of this assumption, we consider that all the producers are credit-constrained. This is to include the effect in the analysis that a credit crunch restrains the aggregate investment, which appears in the discussion over the credit crunch described in the introduction. Then, we have analyzed the changes of the capital- and the investment-output ratios under the credit crunch, taking into account this effect.

Our result that the capital- and investment-output ratios do not necessarily decrease under the credit crunch provides a counter example to Hayashi and Prescott (2002)'s conjecture to preclude a credit market shock as one of the causes for the long stagnation in Japan after 1990. Hence, the evidence raised by Hayashi and Prescott does not necessarily indicate irrelevance of the credit market shock in the 1990's. The reason for obtaining this result is that our model considers the productivity gap across the producers, and endogenizes the average productivity (total factor productivity) level of the economy, while Hayashi and Prescott take it exogenous.

Note that our result does not deny the main result of Hayashi and Prescott that the standard exogenous growth model with a productivity slowdown well predicts the observed increase of the capital-output ratio. Instead, we have shown that qualitatively similar macroeconomic observations can occur under the productivity shock and the credit market shock, but that the dynamics of the producers' net-worth are different between these shocks. We suggest that investigation into the firm-level data is necessary to identify the source of the problem. From this view, the micro-data analysis of the Japanese firms such as Nagahata and Sekine (2005) and Ogawa (2003) would be important to supplement the macroeconomic analysis.

Appendix

3.A.1 Derivation of the optimization condition of (3.8)

Substituting (3.9) into k_s, we can rewrite the optimization problem (3.8) as

$$\max_{\{c_t, c_t', k_t, l_t', l_t'', b_{t+1}\}} \quad c_0 + E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \gamma^{t-1} \left[\gamma c_t + (1-\gamma) c_t' \right] \right\}$$

$$\begin{aligned}
\text{s.t.} \quad & c_t + \frac{u_t}{1-\alpha}(l_t' + l_t'') + z \left(\frac{\alpha u_t}{1-\alpha} \right)^{\alpha} l_t' = A_t \left(\frac{\alpha u_{t-1}}{1-\alpha} \right)^{\alpha} (l_{t-1}' + l_{t-1}'') - b_t + \frac{b_{t+1}}{1+r} \\
& b_{t+1} \leq \theta E_t A_{t+1} \left(\frac{\alpha u_t}{1-\alpha} \right)^{\alpha} (l_t' + l_t'') \\
& \left(\frac{\alpha u_t}{1-\alpha} \right)^{\alpha} l_t'' \leq \left(\frac{\alpha u_{t-1}}{1-\alpha} \right)^{\alpha} (l_{t-1}' + l_{t-1}'') \\
& l_t' \geq 0 \\
& c_t \geq 0 \\
& c_t' = A_t k_{t-1}^{\alpha} l_{t-1}^{1-\alpha} - b_t.
\end{aligned}$$

c_t' is the consumption when the producer exits from the economy. l_t' is the land investment more than the previous land investment, and l_t'' is the one up to the previous level. In terms of notation in (3.8), $l_t = l_t' + l_t''$. Thus, the max-operator in the flow-of-funds constraint in (3.8) is replaced by the third and the forth constraints.

We substitute the first and last constraints into c_s and c_s', and form the Lagrange function as follows:

$$\begin{aligned}
L = & \sum_{t=0}^{\infty} (\beta\gamma)^t E_0 \Bigg\{ (1 + \eta_{4,t}) \left[A_t \left(\frac{\alpha u_{t-1}}{1-\alpha} \right)^{\alpha} (l_{t-1}' + l_{t-1}'') - b_t + \frac{b_{t+1}}{1+r} \right. \\
& \left. - \frac{u_t}{1-\alpha}(l_t' + l_t'') - z \left(\frac{\alpha u_t}{1-\alpha} \right)^{\alpha} l_t' \right] \\
& + \eta_{1,t} \left[\theta E_t A_{t+1} \left(\frac{\alpha u_t}{1-\alpha} \right)^{\alpha} (l_t' + l_t'') - b_{t+1} \right] \\
& + \eta_{2,t} \left[\left(\frac{\alpha u_{t-1}}{1-\alpha} \right)^{\alpha} (l_{t-1}' + l_{t-1}'') - \left(\frac{\alpha u_t}{1-\alpha} \right)^{\alpha} l_t'' \right] \\
& + \eta_{3,t} l_t' \Bigg\} \\
& + \sum_{t=0}^{\infty} \beta^t \gamma^{t-1} (1-\gamma) E_0 \left[A_t \left(\frac{\alpha u_{t-1}}{1-\alpha} \right)^{\alpha} l_{t-1}^{1-\alpha} - b_t \right].
\end{aligned}$$

$\eta_{j,t}$ is the Lagrange multiplier for the $j+1$ th constraint in the optimization problem above for the high-productive and the low-productive producers, respectively, for $j = 1, 2, 3, 4$.

First of all, the borrowing limit must be less than the cost of production to have the borrowing constraints binding, and requires the producers to spend some down-payment to make investment. The sufficient condition for this case is

$$\Omega_t - \frac{\theta E[A|H]}{1+r} > 0.$$

The Euler equations are obtained as

$$b_{t+1}: \quad 1 + \eta_{4,t} = (1+r)[\eta_{1,t} + \beta + \beta\gamma E_t \eta_{4,t+1}] \tag{3.32}$$

$$l'_t: \quad \eta_{2,t} + \hat{\eta}_{3,t} = (1 + \eta_{4,t})z \tag{3.33}$$

$$\begin{aligned} l''_t: \quad & (1+\eta_{4,t})\left[\Omega_t - \frac{\theta E_t A_{t+1}}{1+r}\right] + \eta_{2,t} \\ & = \beta\gamma E_t\left[(1+\eta_{4,t+1})(A_{t+1} - \theta E_t A_{t+1}) + \eta_{2,t+1}\right] + \beta(1-\gamma)(1-\theta)E_t A_{t+1} \end{aligned} \tag{3.34}$$

where $\hat{\eta}_{3,t} \equiv \eta_{3,t}\{(1-\alpha)[1-(1-\delta)/(1+r)]/(\alpha u_t)\}^{\alpha}$.

In the main section, we conjecture the binding borrowing constraints, the expansion of the production sizes by the high-productive producers, the reduction of the production sizes by the low-productive producers, and zero consumption until exit. This case corresponds to

$$\begin{gathered} \eta^H_{1,t} > 0,\ \eta^H_{2,t} > 0,\ \eta^H_{3,t} = 0,\ \eta^H_{4,t} > 0, \text{ and} \\ \eta^L_{1,t} > 0,\ \eta^L_{2,t} = 0,\ \eta^L_{3,t} > 0,\ \eta^L_{4,t} > 0, \text{ for all } t \geq 0. \end{gathered} \tag{*}$$

The subscripts H and L denote the Lagrange multipliers for the high-productive and the low-productive producers, respectively. Thus, the value of the Lagrange multiplier only depends on the current productivity level of the producer. In the steady state, these Lagrange multipliers take constant values. Given the convergence to the steady state, we can numerically calculate the Lagrange multipliers, and check that the conditions above are satisfied.

Besides the optimization conditions above, the high-productive producers must have enough net-worth to expand their production sizes. Also, the low-productive producers must be short of the net-worth to expand their production sizes. These conditions are

$$\begin{aligned} \Omega_t - \frac{\theta E[A|H]}{1+r} &< A^H - \theta E[A|H] \\ \Omega_t - \frac{\theta E[A|L]}{1+r} &> A^L - \theta E[A|L]. \end{aligned}$$

3.A.2 Parameter values for the simulation in Table 3.2

We define the parameters of the model at annual frequency. The parameters are chosen to match the steady-state of the model with the Japanese macro and micro-level data. α is chosen to match with the average ratio of the land value to the value of the capital stock held by the non-financial companies. This ratio had been stable for 1970-1985 before the asset-price bubble in the late 1980's, and its average was 0.814. γ is chosen to replicate the ratio of the capital-income to the value of the capital stock equal to 6.6. This number is equivalent to have the capital-output ratio equal to 2.2 in the industrial data complied by Fukao et al (2003), given that 0.33 is the capital share in the aggregate Cobb-Douglas production function of capital and labor. r is taken from Hayashi and Prescott (2002)'s calibration of the steady-state time-discount rate. δ is the conventional value for the depreciation rate for the capital stock, which is adopted by Kiyotaki and West (2004). μ^H, μ^L and A^H are calibrated to match with the firm-level data analysis of the Japanese manufacturing firms, conducted by Fukao and Kwon (2004). A^L is normalized to be 1. θ is chosen to match the average ratio of the retention from the capital income of the non-financial companies, which was stable around 0.25 over the 1980's. We choose $z = 0.2$, under which both of the high- and low-productive producers engage in production with binding borrowing constraints. This parameter value roughly implies that 20% of the expenditure for investment is sunk.

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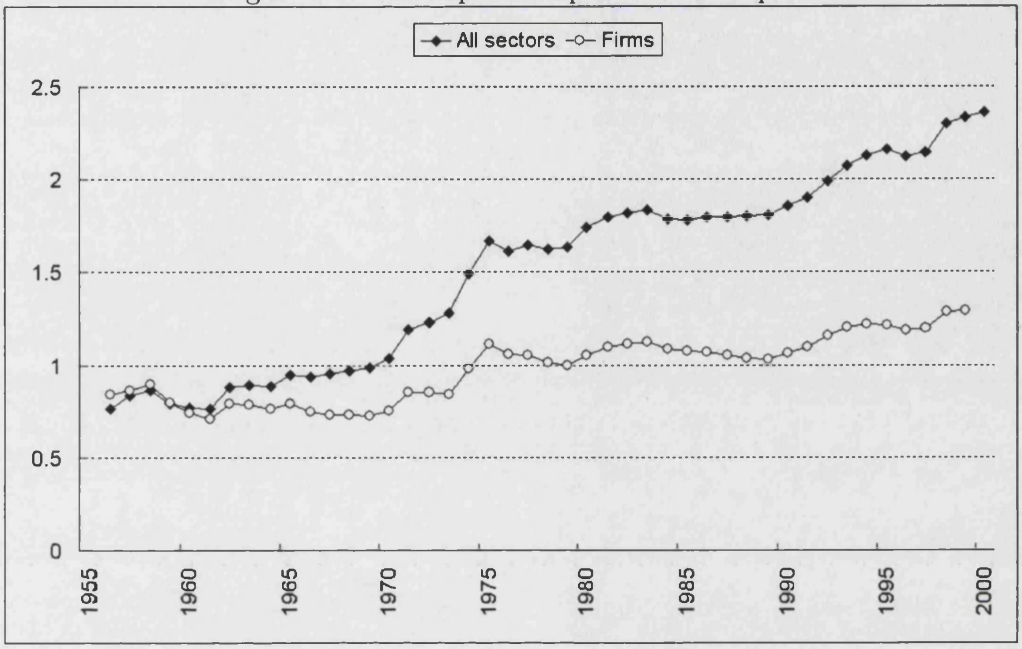
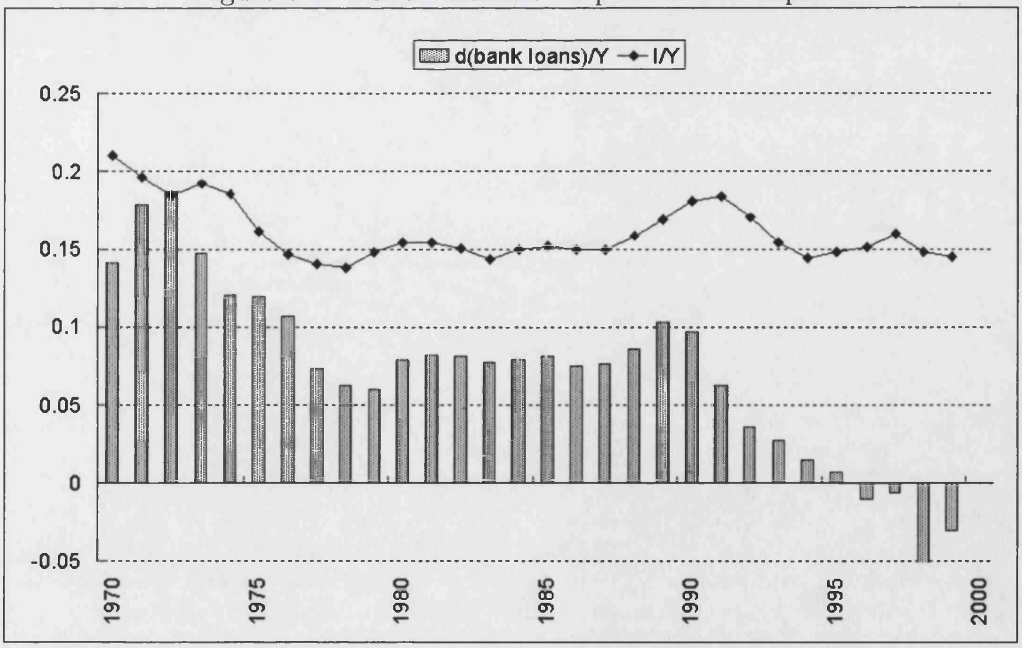
Figure 3.1: The capital-output ratio in Japan



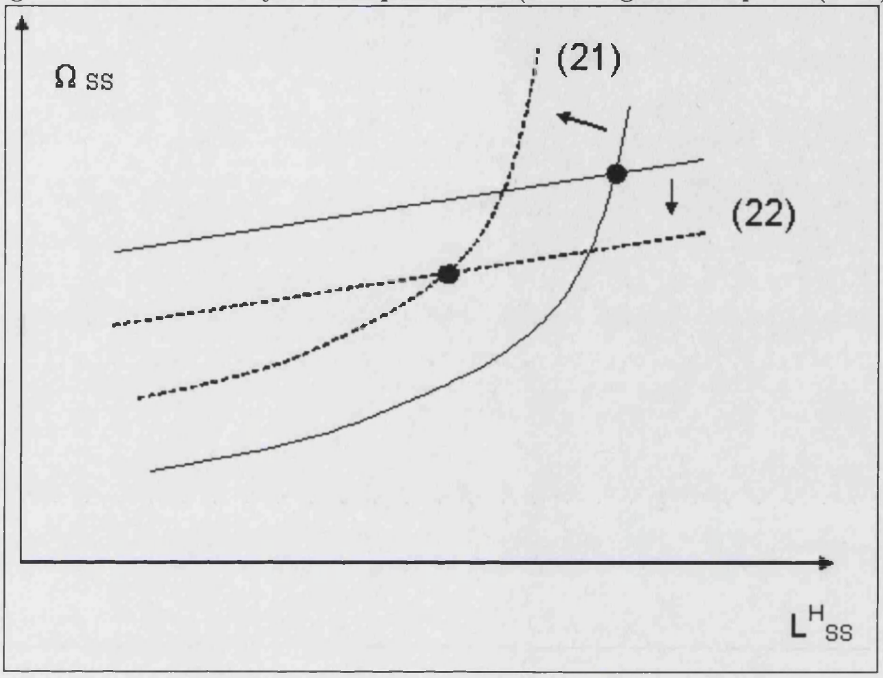
Source: Data appendix of Hayashi and Prescott (2002).
Note: "All sectors" is quoted from Hayashi and Prescott. The capital stock includes the residential capital and the capital in the foreign countries, and the output is GNP. "Firms" is the capital stock of the firms over GNP.

Figure 3.2: The investment-output ratio in Japan



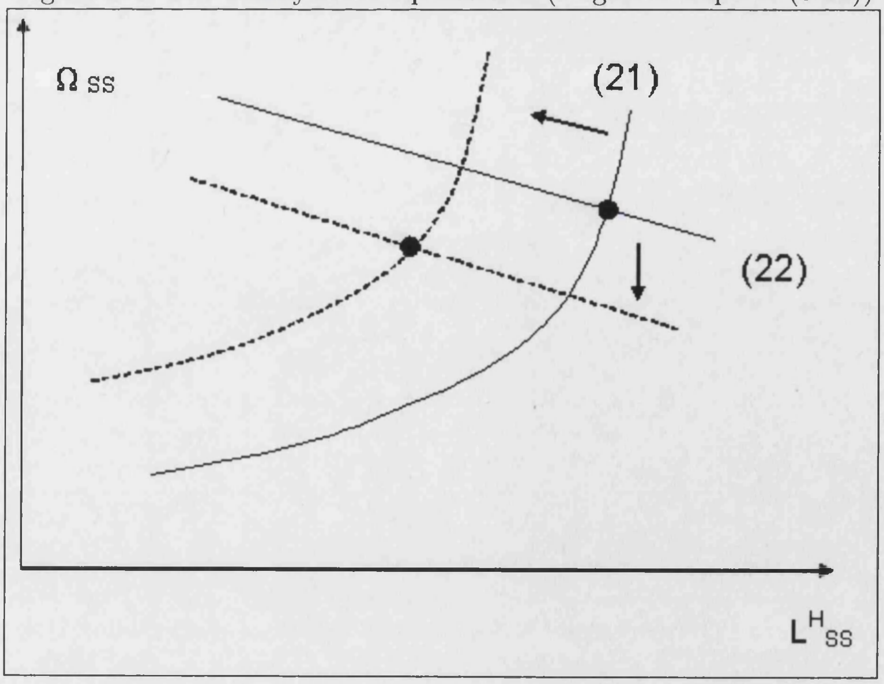
Source: Data appendix of Hayashi and Prescott (2002).
Note: "d(bankloans)/Y" is the annual change of the bank loans to the firms over GNP. "I/Y" is the investment by the firms over GNP.

Figure 3.3: The steady-state equilibrium (Non-negative slope of (3.22))



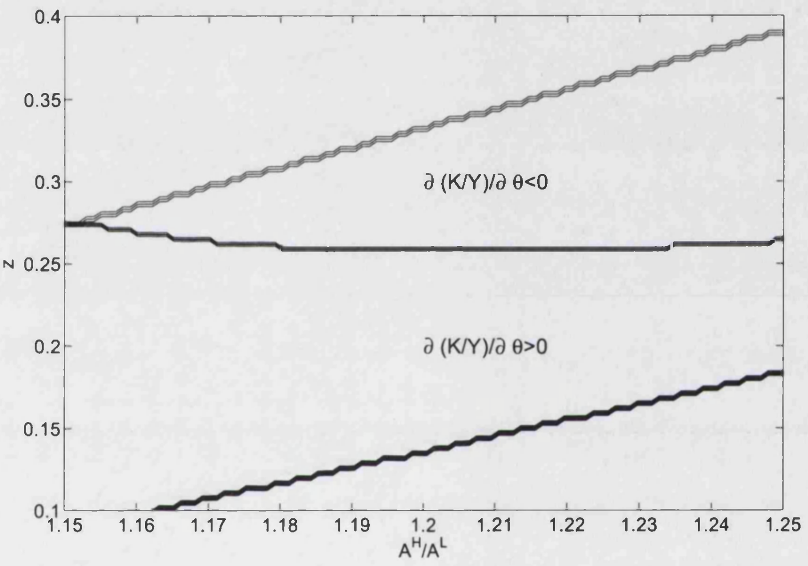
Note: The shifts of the curves are made by a reduction of θ.

Figure 3.4: The steady-state equilibrium (Negative slope of (3.22))



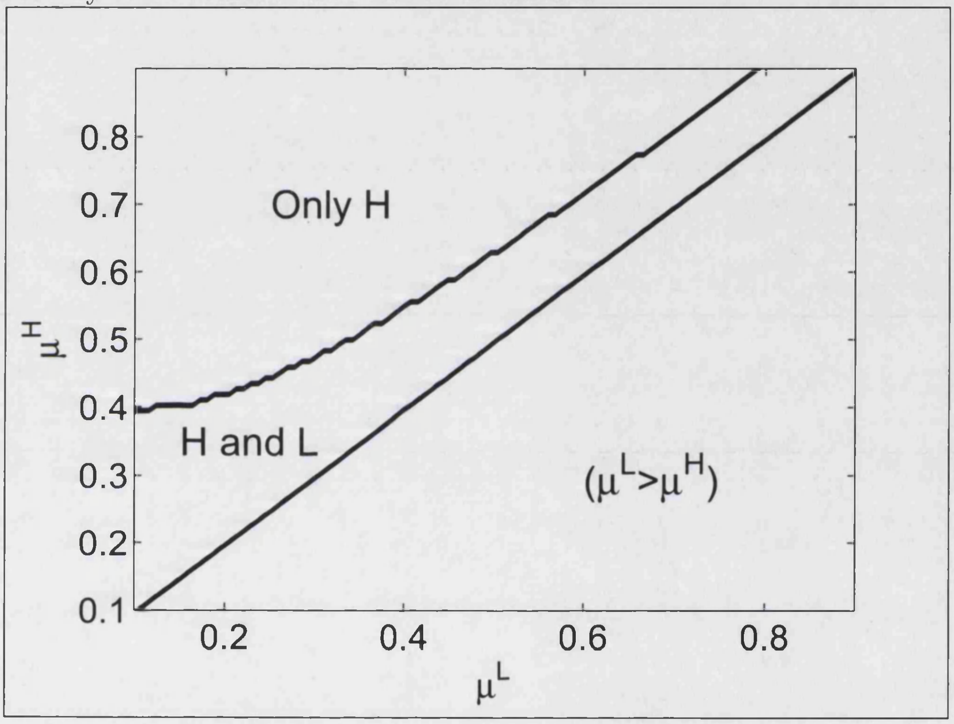
Note: The shifts of the curves are made by a reduction of θ.

Figure 3.5: $\partial(K/Y)/\partial\theta$ around $\theta = 0.75$ at the steady state



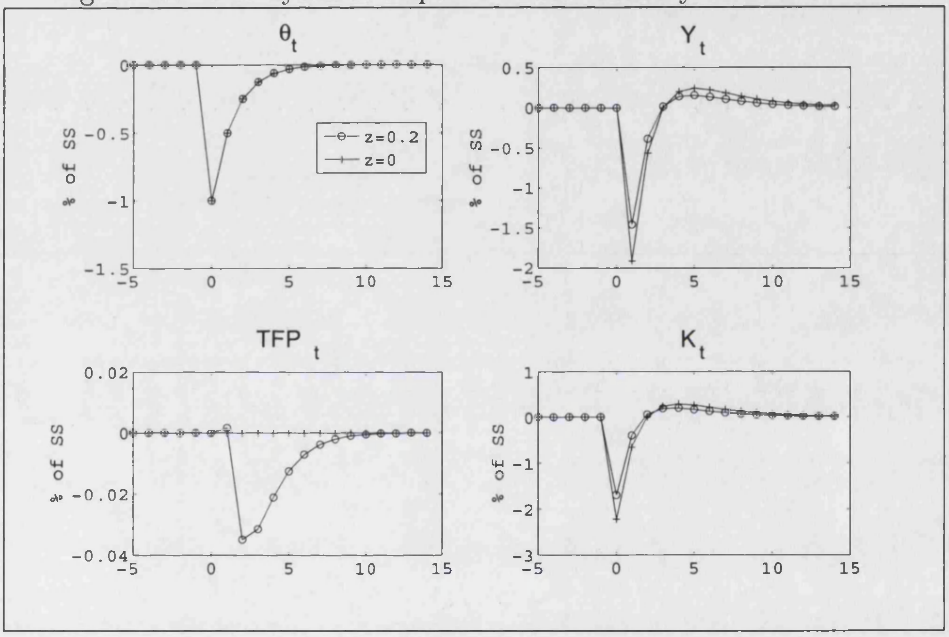
Note: In the area "$\partial(K/Y)/\partial\theta < 0$", the derivative is negative around $\theta = 0.75$ at the steady state under the values of z and A^H/A^L and the other parameter values specified in Table 3.2. In the area "$\partial(K/Y)/\partial\theta > 0$", the derivative is positive. In the blank area at the lower-right corner, the low-productive producers do not borrow up to their borrowing limits because their relative productivity level is too low. In the blank area at the upper-left corner, the high-productive producers do not borrow up to the capacities because the sunk cost of investment to expand their production sizes is too high.

Figure 3.6: The heterogeneity of the productivity levels across the producers at the steady state without sunk cost of investment



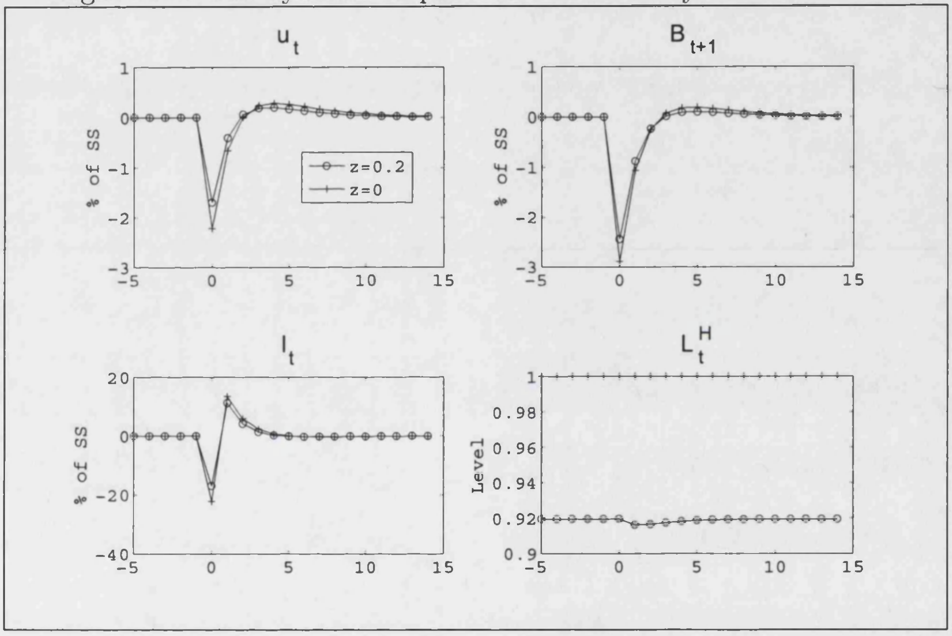
Note: The model is simulated under $z = 0$. In the area "Only H", only the high-productive producers engage in production at the steady state. In the area "H and L", the low-productive producers also engage in production, either having binding borrowing constraints or not. The lower triangle area denoted by "$(\mu^L > \mu^H)$" is excluded from the analysis, as $\mu^H > \mu^L$ by assumption.

Figure 3.7: The dynamic response of the economy to a shock to θ



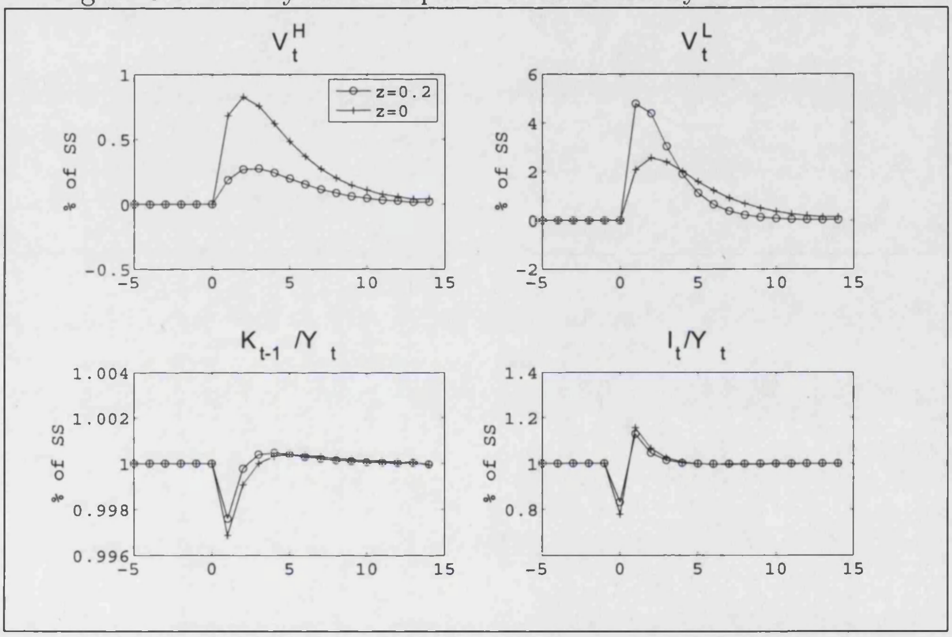
Note: The parameter values are described in Table 3.2. "% of SS" is the % deviation from the steady state level of each variable. The time frequency is a year.

Figure 3.8: The dynamic response of the economy to a shock to θ



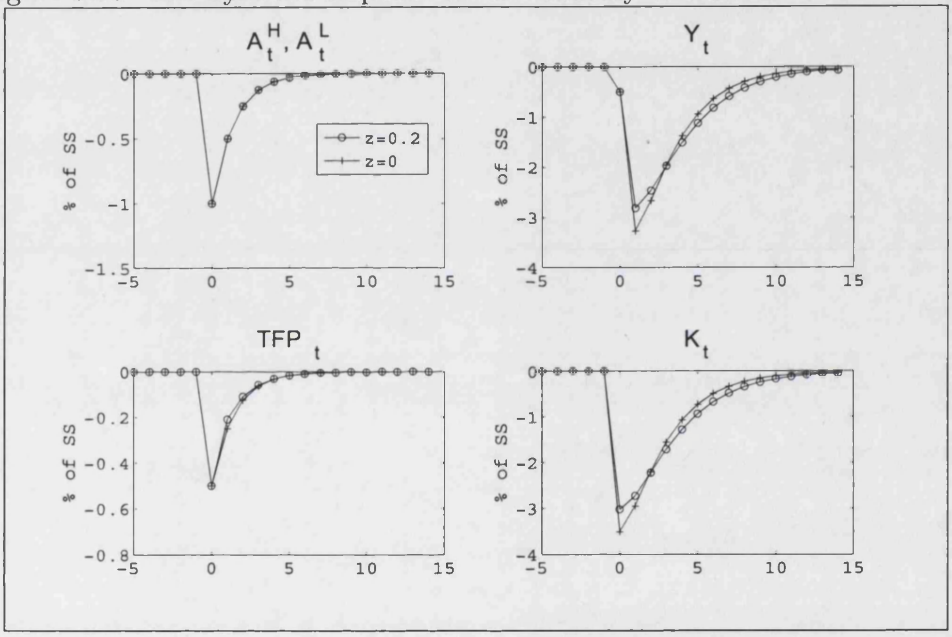
Note: The parameter values are described in Table 3.2. "% of SS" is the % deviation from the steady state level of each variable. The time frequency is a year.

Figure 3.9: The dynamic response of the economy to a shock to θ



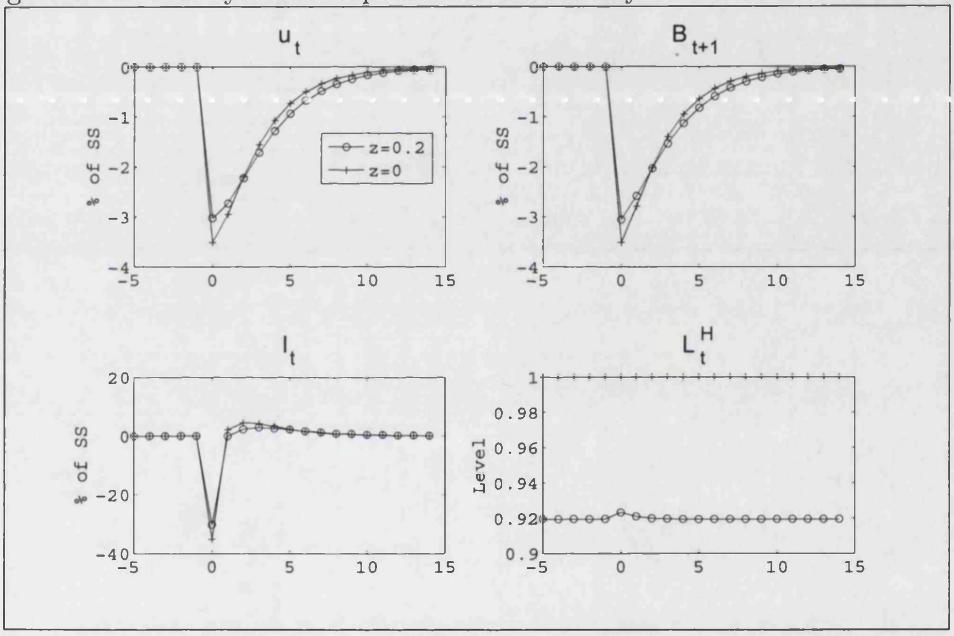
Note: The parameter values are described in Table 3.2. "% of SS" is the % deviation from the steady state level of each variable. The time frequency is a year.

Figure 3.10: The dynamic response of the economy to a shock to A^H and A^L



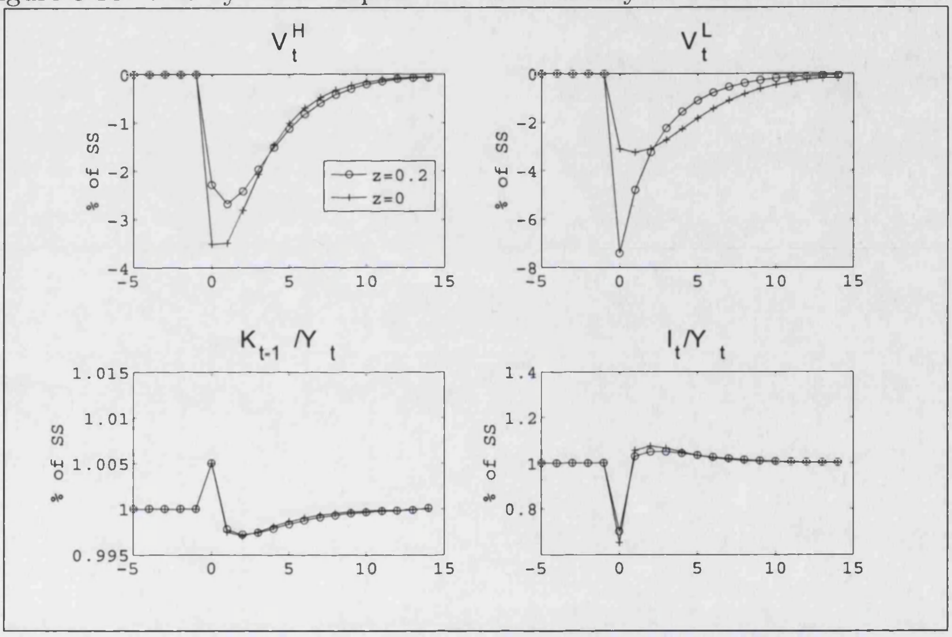
Note: The parameter values are described in Table 3.2. "% of SS" is the % deviation from the steady state level of each variable. The time frequency is a year.

Figure 3.11: The dynamic response of the economy to a shock to A^H and A^L



Note: The parameter values are described in Table 3.2. "% of SS" is the % deviation from the steady state level of each variable. The time frequency is a year.

Figure 3.12: The dynamic response of the economy to a shock to A^H and A^L



Note: The parameter values are described in Table 3.2. "% of SS" is the % deviation from the steady state level of each variable. The time frequency is a year.

Concluding remarks

In this dissertation, the chapter 1 empirically confirms the productivity slowdown in Japan after 1990 by controlling for the unobserved capacity utilization and non-constant returns to scale in production. The chapter 2 shows that productivity slowdowns can be caused by shocks to the credit market and the firm dynamics, and clarifies the mechanism of the remaining low-productive firms in production and the decline of the firms' borrowing under the productivity slowdown. The chapter 3 shows that the credit market shock can cause the features of the capital- and the investment-output ratios in Japan after 1990, and that these features do not necessarily indicate irrelevance of the credit market shock in the long stagnation in Japan after 1990, as opposed to the conclusion of Fumio Hayashi and Edward Prescott in their work published in the Review of Economic Dynamics in 2002.

Over all, this dissertation clarifies that the credit market shock hampers the resource allocation across the firms, and causes an endogenous productivity slowdown. This result is important, as the disturbance in the credit market is one of the distinct features of the Japanese economy after 1990. Also, this result clarifies that the insight of the work of Nobuhiro Kiyotaki and John Moore published in the Journal of Political Economy in 1997 synthesizes the classical Prescott-Summers debate on the source of the economic fluctuation.

Even though the role of banking is only implicitly analyzed in the chapter 3, the result of the dissertation implies that further research on banking in the general equilibrium framework is promising to further understanding of the long stagnation in Japan. This dissertation also clarifies that the gradual decline of the land price under the sustaining productivity slowdown is a puzzling feature of the Japanese economy after 1990. As mentioned at the end of the chapter 2, it is necessary to investigate how the supply of effective land was increasing under the large public investment in the 1990's, and also how people updated their expectation over the future land prices under the unexpected and sustaining productivity slowdown in the 1990's. It is for the future research to analyze these aspects of the economy.