INDUSTRIAL DEVELOPMENT AND INTERNATIONAL TRADE:
TECHNOLOGICAL CAPABILITIES AND COLLUSION

Leopoldo Jose Yanes

Thesis submitted for the degree of Ph.D. in Economics

London School of Economics
September 2005
Abstract

This thesis presents a theoretical analysis of industrialization. Two kinds of models are developed. The first type incorporates the following features:

1) Oligopolistic behaviour and strategic interaction.
2) Endogenous technological capability and market structure.
3) A general equilibrium framework.
4) A dualistic structure (characteristic of many developing countries).
5) Asymmetries in initial conditions.

In part I, chapter 2 develops the benchmark model in autarky. Chapter 3 opens the economy. Under symmetry, trade is welfare improving. Asymmetric initial conditions imply that a backward nation may not benefit from trade with an advanced country, while the advanced nation will always benefit from trade with the backward nation. Subsidizing technological capability allows the backward economy to catch-up with the advanced economy's wage. The subsidy is welfare improving if funded with a lump-sum tax.

In part II we extend the models from part I to variable intensity of competition. Chapter 4 does this for autarky, chapter 5 for the open economy. We encompass intensities of competition ranging from individual to joint profit maximization. In the open economy, intensities of competition lower than individual profit maximization generate a separating surface for the wage rate. Below the surface, trade yields a higher wage rate than autarky (the reverse holds above the surface). The separating surface provides a positive basis for differential trade policy between industries.

The second type of model (chapter 6) is a Big-Push framework with multiple equilibria, and industrialization refers to the transition between these. A Cournot (upstream) industry features endogenous technological capability and vertical linkages to a competitive (downstream) industry. The wage rate depends on whether the economy can fit through a ‘window of opportunity’ whilst on the transition path. One of the central results is that if the wage rate grows too steeply, the window will be missed and the economy may end up in a lower (post transition) level of development.
# Contents

1 Introduction 9

I Introducing Oligopolistic Interactions and Technological Capability into a General Equilibrium Framework 34

2 General Equilibrium with Oligopolistic Interactions: Autarky 35

1 Introduction ......................................................... 35
2 A General Equilibrium Closed Economy Model .................. 37
2.1 Consumers ......................................................... 37
2.2 Industry X ......................................................... 38
2.2.1 Stage 3: Cournot Competition ............................. 38
2.2.2 Stage 2: Competition in Technological Capability ........... 39
2.2.3 Stage 1: The Entry Decision ............................... 40
2.3 The Labour Market and Industry Y ............................ 41
3 Characterization of a Symmetric General Equilibrium .......... 42
An Explanatory Note on Welfare Analysis .......................... 46
4 Analysis of the Symmetric Equilibrium .......................... 46
Notation ...................................................................... 47
4.1 Analysis of Changes in $\beta$ ..................................... 48
4.2 Analysis of Changes in $\sigma$ ..................................... 51
4.3 Analysis of Changes in $\epsilon$ ..................................... 55
4.4 Analysis of Changes in $w_o$ .................................... 57
4.5 Analysis of Changes in $L$ ........................................ 59
5 Remarks Concerning General Features of the Model .......... 64
6 Concluding Remarks ............................................... 66
II Introducing Collusion into a General Equilibrium Framework with Oligopolistic Interactions

4 Collusion in General Equilibrium with Oligopolistic Interactions: Autarky 119

1 Introduction .......................................................... 119
2 Closed Economy General Equilibrium Model with Profit Sharing ..... 120
   Industry X ............................................................... 120
2.1 Stage 3: Quantity Competition with Profit Sharing ................ 121
2.2 Stage 2: Competition in Technological Capability with Profit Sharing 123
2.3 Stage 1: The Entry Decision with Profit Sharing ..................... 125
3 Characterization of a Symmetric General Equilibrium .............. 126
4 Analysis of the Symmetric Equilibrium ............................ 129
4.1 Analysis of Changes in $\gamma$ .................................. 130
4.2 Analysis of Changes in $\sigma$ .................................... 138
5 Remarks Concerning General Features of the Extended Model ..... 144
6 Conclusions .......................................................... 145
Appendix 1: Solving the Final Stage Subgame for Industry X ....... 147
Appendix 2: Second order conditions for the Second Stage Subgame .. 148
Appendix 3: Analysis of Other Variables of Interest .................. 149

5 Collusion in General Equilibrium with Oligopolistic Interactions: Open Economy 154

1 Introduction .......................................................... 154
2 The Open Economy General Equilibrium Model with Profit Sharing .... 156
   Industry X ............................................................... 156
2.1 Stage 3: Quantity Competition with Profit Sharing ................ 157
2.2 Stage 2: Competition in Technological Capability with Profit Sharing 159
2.3 Stage 1: The Entry Decision with Profit Sharing ..................... 160
3 Characterization of a Symmetric General Equilibrium Outcome .... 161
4 Analysis of the Symmetric Equilibrium ............................ 165
4.1 The effect of changing $\beta$ ..................................... 166
4.2 The effect of changing $\gamma$ ..................................... 168
4.3 The effect of changing $\sigma$ ..................................... 170
4.4 The effect of changing $\epsilon$ ..................................... 171
4.5 The effect of changing $u_0$ ....................................... 172
### III Big Push Arguments

#### 6 Endogenous Technological Capability, Trade Policy and the Big Push

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Introduction</td>
<td>189</td>
</tr>
<tr>
<td>2 Model A: Exogenous Technological Capabilities</td>
<td>191</td>
</tr>
<tr>
<td>2.1 Downstream Industry Y</td>
<td>191</td>
</tr>
<tr>
<td>2.2 Upstream Industry X</td>
<td>192</td>
</tr>
<tr>
<td>2.3 The Labour Market</td>
<td>193</td>
</tr>
<tr>
<td>2.4 Equilibrium</td>
<td>195</td>
</tr>
<tr>
<td>2.5 Trade Policy</td>
<td>199</td>
</tr>
<tr>
<td>2.6 Effects of Changing Parameter Values</td>
<td>200</td>
</tr>
<tr>
<td>3 Model B: Endogenous Technological Capabilities</td>
<td>201</td>
</tr>
<tr>
<td>3.1 The Downstream Industry</td>
<td>202</td>
</tr>
<tr>
<td>3.2 The Upstream Industry</td>
<td>203</td>
</tr>
<tr>
<td>3.3 The Labour Market</td>
<td>204</td>
</tr>
<tr>
<td>3.4 Equilibrium</td>
<td>205</td>
</tr>
<tr>
<td>3.5 Trade Policy with Technological Capability</td>
<td>208</td>
</tr>
<tr>
<td>3.6 Effects of Changing Parameter Values</td>
<td>210</td>
</tr>
<tr>
<td>4 Conclusions</td>
<td>211</td>
</tr>
</tbody>
</table>

Appendices:
- Appendix 1: Slope and Convexity of $DD$ and $SS$ | 212 |
- Appendix 2: Proofs | 212 |
- Appendix 3: Stability | 215 |
- Appendix 4: Deriving the solved-out profit function | 216 |
- Appendix 5: A Note on Second Order Conditions | 217 |

#### 7 Conclusions and Extensions for Future Research

Future Research and Limitations | 227 |
This work is dedicated to:

Mariela,
who makes it all worthwhile.

My parents and grandparents,
for believing.

My children,
may you fulfill your promise.
Acknowledgements

Doing economic theory is a process of discovery not unlike a roller coaster ride: Along the track one gets to exert great effort, experience great thrills and enjoy exhilarating bliss. John Sutton inspired me to embark upon this journey, and without his help, guidance, and, indeed, pressure, I would not have set sail, let alone reached port. Moreover, his acute criticism and challenges increased the quality of the research by orders of magnitude. His influence on my career path extends beyond this study. More than a supervisor, he is an inspiration of scientific endeavour.

At the L.S.E., I benefitted greatly from insightful comments and suggestions by Tim Besley, Robin Burgess, Peter Davis, Nobu Kiyotaki, Steve Redding, Alberto Salvo-Farré, Mark Schankerman, and Tony Venables, to whom I am grateful. The Economic Organisation and Public Policy group (where this research began) and (later on) the Economics of Industry group at STICERD were exciting and supportive environments in which to work. I was lucky to have Jian Tong working in a similar research area within the Economics of Industry Group at STICERD. Our many discussions have greatly improved this research, and more importantly, my understanding of the process of development.

While visiting Universitat Pompeu Fabra through the European Doctoral Programme, I benefitted greatly from discussions with Matteo Cervellati and Nicola Pavoni, who, together with Quentin Dupriez, Matthias Messner and Uwe Sunde made my stay in Barcelona a memorable experience. I am grateful to them. I also would like to thank Rodney Beard, John Foster, Stuart McDonald, Fabio Montobbio and Kam Ki Tang for insightful discussions. I am especially grateful to John Quiggin for reading the manuscript and for his many suggestions. I thank seminar participants at the European Doctoral Programme Jamboree at Louvaine Le Neuve (2001), particularly Susana Peralta, at the Australian Economic Theory Workshops (University of Sydney, 2003 and University of Melbourne, 2004), and at the Economics of Industry and Economic Organisation and Public Policy seminars at the L.S.E. for their comments.

Petróleos de Venezuela, S.A. (PDVSA) provided funding for the initial stages of my graduate studies. Their financial support is gratefully acknowledged. I hope the results that follow justify the expense. I was lucky to work with some wonderful people while at PDVSA, we had a great team. I am deeply grateful to Ramón Espinasa and Ronald Pantin, who made it all possible. I thank Osmel Manzano, Bernard Mommer, Klaus Nusser and Javier Peraza for exposing me to the practice of economics and for expanding my horizons.

Finally, my wife, Mariela, offered me unconditional support throughout this endeavour.
Through years of lost week-ends and hardly any holidays, she remained bright and cheerful. Warm thanks to my mother, Aura, for her confidence in me. She, together with my grandmother, 'the two Auras', were an indispensable help towards the end. My father, Leo 'el viejo', is possibly the one who started it all, with his discussions on political economy, imperialism and 'all things left-winged'. The late Oswaldo Ron joined us in many such discussions, adding to the flavour with his unforgettably provocative style. I thank them for the spark.
Chapter 1

Introduction

There is a long standing line of inquiry into how some newly industrializing economies achieved great progress in industrialization, with particular reference to Japan, South Korea and other East Asian economies (usually Taiwan, Hong Kong and Singapore, but Indonesia, Malaysia and Thailand are sometimes included). This industrialization process is often referred to as the 'East Asian Miracle', after the World Bank study (1993). Within this field of research, there are several debates.

First, there is a debate on how to account for these economies' performance, and to what extent productive factors (capital and labour) contributed to this. This debate is about how much of these economies' performance can be accounted for by factor accumulation. The unaccounted for residual (to quote Abramovitz, 1956, p. 11: "the measure of our ignorance") is labelled Total Factor Productivity (TFP), and is usually ascribed to technological progress.

A second debate asks whether government intervention has had anything to do with the economic performance of these economies. This strand of the literature tries to establish whether economic policies introduced by these governments were responsible for their economies' outstanding performance, and which of these policies were in fact determinant to their performance. This is where our main focus of attention lies. Within this debate, there are two aspects which will be our main focus of attention: Firstly, on how latecomers can catch-up with the leading economies. Secondly, on the role of asymmetries in initial conditions and of the intensity of competition in this catching-up process. In this thesis we aim to uncover the theoretical underpinnings of this debate, and to evaluate the arguments analytically. From a positive perspective, we will address the question of whether government intervention can be justified. If such intervention can be justified at all, we then ask whether different industries call for differential treatment in terms of industrial, trade and competition policies, that is, we
ask whether 'industrial targeting' can be justified and what precise form should such policy take? From a normative perspective, we will consider the welfare consequences of such policies.

Several authors have addressed these issues from a historical perspective. Bayly (2004) provides a summary account of this debate in 18th century Europe. During that period mainland Europe was faced with the problem of catching-up with England. On the British side, foreign secretary (and later on, prime minister) Lord Palmerston argued for, and fostered military attacks in the name of, trade liberalization1. Meanwhile, in continental Europe Friedrich List (1885) put forward an early version of the Infant Industry argument, arguing for economies to remain closed during the catching-up phase, and to liberalize trade only after domestic industry could compete against foreign ones on a level ground.

In relation to this debate, Lall (1992) states that: “Such interventions have to be selective, requiring that policy makers identify specific sectors...for promotion...to exploit their growth potential, linkages or externalities...The best combination may be the selective...protection of domestic markets, together with strong incentives for exports activity and domestic competition” (p. 172). However, even assuming that intervention is indeed justifiable (a very strong assumption in itself), with the exception of the study by Dixit and Grossman (1986), there is little in the literature to guide us as to the precise form of economic policy that should be implemented. Reference is often made to promotion of income-elastic goods, whose demand would expand as the world grows wealthier (Amsden, 1989), as well as Lall’s ‘growth potential, linkages or externalities’. Lall (1992) goes on to warn us that “the experience of developing countries is replete with instances of misguided intervention...Consequently, improved methods of intervening are worth striving for” (p. 183). What is required is a precise, testable theory of the form economic policy should take (if such intervention is indeed justifiable in the first instance).

The phenomenal growth of Japan, South Korea, Taiwan, Hong Kong and Singapore has sparked an ongoing flurry of research in economics and in the related discipline of development studies. Murphy, Shleifer and Vishny (1989), state that:

“Virtually every country that experienced rapid growth of productivity and living standards over the last 200 years has done so by industrializing. Countries that have successfully industrialized—turned to the production of manufactures taking advantage of scale economies—are the ones that grew rich, be they eighteenth-

1Milanovic (2003) provides an analysis of the coercive aspects associated to the construction of empires and how this relates to trade liberalization.
century England or twentieth-century Korea and Japan...What is it that allows some but not other countries to industrialize? And can government intervention accelerate the process?" (p. 1003)

In parts I and II of the thesis we develop a standard general equilibrium framework with oligopolistic interactions to address the issues highlighted. In part III some of these questions are brought up again in the context of a Big-Push framework, again with an emphasis on oligopolistic interactions.

Finally, there is a third debate on whether the East Asian growth episode was worthwhile at all. Within the first debate, there is a body of (albeit debatable) evidence which supports the view that after accounting for factor accumulation, there is very little left to explain of East Asian growth. Such a high rate of factor accumulation occurred through sacrifices in current consumption and leisure. This calls into question whether such sacrifices were indeed welfare improving from an intertemporal perspective (Sarell, 1995). Wade (1992) mentions that other indicators of human development in East Asia are way below the level expected for their income. In particular, life expectancy at birth in Sri Lanka is higher than South Korea's (69 years in 1986), whilst the latter's per capita income is six times higher. Pollution has become a major problem in some areas and Seoul has one of the world's highest concentrations of air borne sulfur dioxide. The low level of public health care in Korea has been highlighted in Amsden (1989) and Wade (1992). Dictatorial regimes have carried out widespread repression at great human cost. Such regimes are often hailed as 'benevolent dictatorships', and support for the idea that such regimes are almost necessary to achieve outstanding economic performance is pervasive in the literature, to the extent that some works have been classified as a revival of modernization theory (Amsden, 1989 and Wade, 1990 are cited as examples of the latter by Lie, 1991). There is also a literature which documents the conditions of workers in East Asia (see Choi, 1998; Lie, 1991; Jomo, 1994; Hart-Landsberg, 1992 and Ogle, 1990). The consensus within this literature is that when one looks more closely at the relative sacrifices made by workers versus entrepreneurs, it is the workers (to put it mildly) who have drawn the shortest straw. This view documents appalling work conditions, and claims that the welfare of workers has been the adjustment variable which allowed entrepreneurs to achieve exceptional generation of wealth. Indeed, these accounts resemble a striking resemblance to records of British work conditions during the first industrial revolution (Marx, 1903, v. I, ch. 10 and v. II, ch. 25 and Engels, 1958). We shall have more to say on this issue, especially in chapter 6, where we develop a model of the Big Push. In the context of this model, positive analysis
shows that the wage rate does indeed expand during the industrialization process, but it is essential that it does so at a not-too-fast rate. Otherwise, it runs the risk of thwarting the industrial expansion process. Apart from this issue and the reasonable caveat that not all was the colour of roses in the ‘East Asian Miracle’, we shall not dwell on this debate.

The Sources of Growth Debate

In the economics discipline, the main approach has been to perform growth accounting exercises for these economies, with (albeit to a lesser extent) some focus on theoretical research (for theory, see Lau and Wan, 1993; Lucas, 1993 and Wan, 2004). In the growth accounting literature, Young (1995) carried out a careful growth accounting exercise for Hong Kong, Singapore, Taiwan and South Korea. His findings, summarized in table 1, indicate that total factor productivity (TFP) growth in these economies was not particularly outstanding.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Economy*</td>
<td>2.3</td>
<td>0.2</td>
<td>1.7</td>
<td>2.1</td>
</tr>
<tr>
<td>Manufacturing#</td>
<td>NA</td>
<td>-1.0</td>
<td>3.0</td>
<td>1.7</td>
</tr>
<tr>
<td>Other Industry</td>
<td>NA</td>
<td>NA</td>
<td>1.9</td>
<td>1.4</td>
</tr>
<tr>
<td>Services</td>
<td>NA</td>
<td>NA</td>
<td>1.7</td>
<td>2.6</td>
</tr>
<tr>
<td>Private Sector</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>2.3</td>
</tr>
</tbody>
</table>

NA - Not available. * In the case of Korea and Taiwan, agriculture is excluded. # In the case of Singapore, the years are 1970-1990.

Source: Young (1995), table XII.

Table 1. Average Total Factor Productivity Growth in South East Asia (percent per annum)

To provide a point of reference, in table 2 compares TFP growth rates across some developed and Latin American economies.

<table>
<thead>
<tr>
<th>Country</th>
<th>Period</th>
<th>TFP Growth</th>
<th>Country</th>
<th>Period</th>
<th>TFP Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>1960-1989</td>
<td>0.5</td>
<td>Brazil</td>
<td>1950-1985</td>
<td>1.6</td>
</tr>
<tr>
<td>France</td>
<td>1960-1989</td>
<td>1.5</td>
<td>Chile</td>
<td>1940-1985</td>
<td>0.8</td>
</tr>
<tr>
<td>Germany</td>
<td>1960-1989</td>
<td>1.6</td>
<td>Mexico</td>
<td>1940-1985</td>
<td>1.2</td>
</tr>
<tr>
<td>Italy</td>
<td>1960-1989</td>
<td>2.0</td>
<td>Brazil (M)</td>
<td>1960-1980</td>
<td>1.0</td>
</tr>
<tr>
<td>Japan</td>
<td>1960-1989</td>
<td>2.0</td>
<td>Chile (M)</td>
<td>1960-1980</td>
<td>0.7</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>1960-1989</td>
<td>1.3</td>
<td>Mexico (M)</td>
<td>1940-1970</td>
<td>1.3</td>
</tr>
<tr>
<td>United States</td>
<td>1960-1989</td>
<td>0.4</td>
<td>Venezuela (M)</td>
<td>1950-1970</td>
<td>2.6</td>
</tr>
</tbody>
</table>

M-Manufacturing alone; developed economies are from Dougherty (1991); Latin American economies are from Elias (1990).

Source: Young (1995), table XIV.

Table 2. Comparative Total Factor Productivity Growth in Some Developed and Latin American Economies (percent per annum)

As can be seen from comparing tables 1 and 2, South East Asian TFP growth rates hardly constitute a ‘miracle’. Young’s conclusion is that the spectacular growth rates achieved by these economies were generated by factor accumulation and effective transformation of these factors into output: Lots of perspiration, with a little inspiration. The focus of attention
now becomes how these economies achieved such high rates of factor accumulation, and what mechanisms did they implement to transform these inputs into output efficiently. In a sense, these findings were bad news for growth, since it was hoped that if TFP had contributed substantially to South East Asian performance, there would indeed be hope of maintaining such high growth rates indefinitely. Moreover, rapid factor accumulation was achieved at the expense of consumption and leisure, thus from the perspective of intertemporal welfare, it is not clear whether the sacrifices made yield a positive payoff.

Nonetheless, this is by no means a settled issue. Other studies have found TFP growth rates as high as 4.1% per annum for South Korea (Christensen and Cummings, 1981). Young (1995) reconciles these findings with his own work by carefully delving into the data and methodology used by the other studies and arguing how his estimates are more reliable. In subsequent studies, Young's findings have been criticized and the discussion has re-ignited. Sarel (1995) documents that TFP growth estimates for South East Asia are highly sensitive to the time period chosen and to the relative contributions of labour and capital, and finds TFP growth estimates which are much higher than Young's (1994, 1995). Nelson and Pack (1999) argue that growth accounting is highly sensitive to underlying assumptions on parameter values, and take the view that the methodology is, in general, unreliable. They instead focus on estimating growth rates from cross-country growth regressions, and then comparing these estimates with the observed growth rates for South East Asia. To this end, they use the following growth regression from Levine and Renelt (1992):

\[
GDP_G_i = -0.83 - 0.35RGDP_{60i} - 0.38GPOP_i + 3.17SEC_i + 17.5I_i \quad \text{for } i = 1, \ldots, 101
\]  

(1.1)

where \(GDP_G_i\) is the average growth rate of Gross Domestic Product (GDP) in country \(i\) between 1960 and 1989, \(RGDP_{60i}\) is GDP per capita in purchasing power parity terms in country \(i\) in 1960, \(GPOP_i\) is average population growth in country \(i\) between 1960 and 1989, \(SEC_i\) is secondary school enrollment in country \(i\) in 1960 as a percentage of the appropriate age group, and \(I_i\) is the average investment/GDP ratio in country \(i\) between 1960 and 1989. This equation was estimated for a cross section of 101 countries, all coefficients are significant at the 5% level, and the value of \(R^2\) is 0.46. Using this regression, Nelson and Pack (1999) proceed to predict growth rates for countries which featured high rates of capital accumulation, based on their values of the explanatory variables in equation (1.1), and compare these with the observed outcomes. Their results are shown in table 3, in decreasing order of the values of actual minus predicted growth of GDP per capita.
<table>
<thead>
<tr>
<th>Country</th>
<th>Investment/GDP 1960-1989</th>
<th>Actual minus Predicted growth of GDP per capita</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taiwan</td>
<td>25.0</td>
<td>4.7</td>
</tr>
<tr>
<td>Korea</td>
<td>24.9</td>
<td>3.2</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>27.3</td>
<td>3.1</td>
</tr>
<tr>
<td>Singapore</td>
<td>34.3</td>
<td>1.7</td>
</tr>
<tr>
<td>Ireland</td>
<td>22.2</td>
<td>1.1</td>
</tr>
<tr>
<td>Greece</td>
<td>24.2</td>
<td>0.8</td>
</tr>
<tr>
<td>Panama</td>
<td>24.0</td>
<td>0.2</td>
</tr>
<tr>
<td>Portugal</td>
<td>23.7</td>
<td>-0.2</td>
</tr>
<tr>
<td>Algeria</td>
<td>35.0</td>
<td>-2.6</td>
</tr>
<tr>
<td>Gabon</td>
<td>40.0</td>
<td>-3.0</td>
</tr>
<tr>
<td>Jamaica</td>
<td>25.0</td>
<td>-3.7</td>
</tr>
</tbody>
</table>


Table 3. Investment Ratios and Predicted Minus Actual Growth Rates for Countries with High Rates of Investment (percent per annum)

This methodology indicates that the four South East Asian economies exhibited much higher growth rates than equation (1.1) would predict. Moreover, other high investment economies showed poor performance relative to their predicted values. Why the difference? Nelson and Pack conjecture that:

"a critical element was the...effort of firms...that allowed them to successfully initiate new technologies and absorb new equipment. While other countries with high investment-GDP ratios could purchase machinery that gave them the potential to improve their productivity, this could only be successful when it was combined with domestic effort to absorb the new technology" (1999, p. 431)

Krugman (1994) takes the view that high rates of factor accumulation automatically led to higher output, and that, similarly to Young (1995), there is no ‘miracle’ to explain. Rodrik (2005) argues against the use of cross-country growth regressions: The all encompassing nature of the problem means that endogeneity of explanatory variables (and the associated estimator bias) is almost inescapable. The problem is compounded when one notes that finding appropriate instruments (to be used in instrumental variables techniques which could potentially resolve the endogeneity bias) runs against similar, pervasive, endogeneity problems. Moreover, Rodrik (2005) is particularly skeptic on using policy indicators as explanatory variables, since government policy is highly unlikely to be random at all, let alone exogenous. A recent alternative is the use of panel data techniques to resolve the endogeneity problem (Islam, 1995), but there is still much debate regarding appropriate technique (Lee et al., 1997).

Summing up, whilst significant progress has been made, the magnitude of TFP or unexplained growth in per capita income in South East Asia remains an unresolved question.
Notwithstanding the objections, when analyzed from a macroeconomic perspective, the picture that emerges is that high rates of factor accumulation certainly did account for a lot of South East Asia's growth. However, there remains a considerable fraction of this growth which cannot be accounted for. This fraction is at best 'above standard', and at worst similar to OECD history (with the exception of Singapore, which features much lower TFP growth). That a number of countries which were very poor achieved such high rates of factor accumulation, then managed to transform the latter into output in a relatively efficient manner, and still accomplished TFP growth rates similar or slightly better to those of the OECD is by all standards an outstanding feat. Therein lies the 'miracle'. The emphasis then shifts to asking: What caused the high rates of factor accumulation? How did these economies prevent factor accumulation from going to waste? How did they maintain TFP from collapsing?

**Historical Perspectives**

Views on the underlying causes of the East Asian Miracle vary widely. At one extreme we have the view that such outstanding performance is due to the extent of the free market in these economies, and that government intervention, if anything, hindered what could have been an even more outstanding result. In Robert Wade's view (1990, p. 342) this is highly unlikely, given the paucity of such growth experiences (although the same argument could be reversed by focussing on the historical scarcity of *laissez faire* governments). At the other extreme we find the view that high-quality government interventions were a *sine qua non* condition for such spectacular growth. Ideology plays no minor role in the variance of perspectives, and perhaps this is something we could do without (at least from the perspective of scientific method).

There is a large literature focussing on historical accounts of the East Asian economies, their industries and the behaviour of their governments. The World Bank (1993) carried out an analysis for eight East Asian economies (Japan, South Korea, Taiwan, Singapore, Hong Kong, Indonesia, Malaysia and Thailand) for the latter half of the twentieth century, and concludes by recommending macroeconomic stability, a focus on early education, support of agriculture, building a sound financial system, openness to foreign ideas and technology and letting prices reflect economic scarcities. The study warns that: "promoting specific

---

When considering the magnitude of this 'unexplained' proportion, it is hard to resist the temptation to dismiss it as measurement error. Indeed, the difficulties of accurately measuring capital and labour inputs are paramount. Unexplained growth in the order of 1-2% per year hardly seems sufficient to justify such a strong diversion of attention. Moreover, Young (1995) argues that higher estimates can be explained by reference to methodological flaws in particular TFP studies. The skepticism is compounded when one considers the magnitude of the underground economy. Indeed, with OECD economies estimating up to a sixth of GDP as underground, and backward economies having much higher estimates, it is remarkable that the unexplained fraction has been reduced to so little. Adding to the problem, official statistics in developing economies are also likely to be less reliable.
industries...will generally fail" (p. 367), and then states: "a successful export push, whether it results from an open economy and strong economic fundamentals, or from a combination of strong fundamentals and prudently chosen interventions, offers high economic gains" (ibid.). Although the study does not find support for industrial targeting in general, it does find support for some types of protective trade policies. In particular, policies which promote industries based on their export performance were found to be relatively successful. The study is not without its share of criticism, especially on the issue of the ineffectiveness of industrial targeting (see Rodrik, 1994 and Wade, 1994).

Amsden (1989) studies the South Korean experience in the latter half of the 20th century. In this analysis South Korea is characterized as a late industrializing economy. The notion of a late industrializer refers to economies which are latecomers to the industrialization process. As such they face the disadvantages associated with being a late entrant, as well as the advantages of having a series of inventions available for copying (i.e., learning). Amsden also characterizes the catching-up process of mainland Europe in the 18th century (relative to England) as one of late industrialization. Likewise, countries such as Mexico, Brazil, Turkey, India, Japan and Taiwan are also included in this group (Amsden, 1989, ch. 1). Amsden emphasizes the importance of effective learning in the context of late industrialization. She notes that learning constitutes the critical mechanism by which these economies can expand their technological frontier, and thereby successfully enter into technologically advanced industries. Formal theoretical models of learning in competitive equilibrium in the context of economic growth and international trade have been developed by Stokey (1988, 1991a and 1991b) and Young (1991 and 1993). In such models dynamic gains from learning lend support to the infant industry argument (although static gains from trade do not). Wade (1990) analyses the case of Taiwan, with references to Korea and Japan, focussing on how the government can design policies which improve upon the market mechanism. Both Amsden and Wade take the view that the presence of, and the economic policies of, a developmental state were fundamental to the East Asian Miracle.

Before proceeding, it is convenient to define some key concepts which will be used throughout this study. The first is that of technological capability. This term refers to the knowledge of workers within the firm, which determines the levels of productivity and product quality (Fransman and King, 1984; Lall, 1992 and Sutton, 2004). In this study we will use the term in the narrow sense of 'product quality'. Moreover, the stage-game structure of the models we will be using allows for a more general interpretation of 'product quality'. In this framework, product quality could be re-labelled innovation, R&D or learning. In general, the outcome of sunk investment in stage games admits a similar treatment to the one we will be conferring to
technological capability (product quality). The flexibility of the stage-game approach allow us, then, to treat all of the above as a sunk investment in a particular stage of the firm's decision process.

The second notion is that of initial conditions. Usually, this is taken to reflect the status quo in a country at a given point in time, including sociopolitical, as well as economic, structure. Here we will use this notion in the restricted sense of initial conditions pertaining to technological capability. Nonetheless, initial conditions of technological capability may be interpreted as encompassing all initial conditions which are relevant to the industrial production process.

South Korea

There are some key notions which Amsden identifies as having marked the difference between South Korea and less successful late industrializers. Firstly, she points out the crucial role of large oligopolistic firms, which eventually became internationally competitive and embarked in fierce rivalry: "Korea's economy may be highly concentrated, but its leading firms...engage in intense competition with one another in overseas as well as domestic markets" (Amsden, 1989, p. 129). The extent of concentration in the Korean economy is staggering. In 1974, the combined sales of the top ten chaebol (the Korean diversified business group) was 15.1% of GNP. By 1984, this figure was 67.4%.

Secondly, Amsden claims that the strong emphasis on learning and on shopfloor management within Korean firms proved crucial in attaining technological capability. More specifically, the widespread use of Quality Control Circles, whereby workers set-up teams to improve quality and provide feedback on ways to better the production process, is hailed as one of keys to successful shopfloor management (ibid, p.325).

Thirdly, government use of disciplinary mechanisms is claimed to have enhanced industrial performance. Disciplinary mechanisms are credible incentive schemes whereby the government provides continuing support to enterprises with satisfactory performance, particularly with respect to export targets, productivity and technological goals as opposed to financial indicators. However, there is considerable debate on this point. Kang (1995) questions the autonomy of the Korean State and its ability to discipline the chaebol. He highlights underlying cronyism and how this affected the decisions to bail out business groups.

Fourth, favorable initial conditions allowed firms to bridge the gap between their initial technological capabilities and those of the international market. In particular, the educational level of the workforce was high relative to other countries with similar development levels.
Low levels of initial inequality (whether in terms of income or land distribution) have also been mentioned by Lucas (1993) and Rodrik (1994) as having contributed to higher subsequent growth. Lucas (1993) compares the Philippines with South Korea in the mid-twentieth century, citing rough similarities but crucial differences in the initial distribution of income and in human capital, and suggests that this may help to explain their divergent paths. One might also add the impact of Spanish heritage on the Philippines' institutional framework as opposed to the Japanese colonial influence in Korea.

Fifth, the manipulation of relative prices to increase the relative profitability of industries which did not have a comparative advantage in the short run allowed South Korea to successfully develop its technological capability to a point where industries which were initially unprofitable, became so after a learning phase. Amsden coined the phrase "getting prices wrong" to highlight how the government instated a wide range of subsidies and protection mechanisms to tilt incentives towards entry into industries which would not have been profitable otherwise. This is arguably the most contentious point in the East Asian Miracle. The World Bank (1993) found that, if anything, relative prices in East Asia have been subjected to a lower degree of distortion than elsewhere in the developing world. Although Amsden's evidence on this point is not entirely satisfactory (see the critique in Wade, 1992, p.292), evidence has begun to emerge which points to a maturation process in many Korean industries. A study by Lee (1997) finds that effective rates of protection have gradually decreased for many industries, hinting at an effective process of maturation. She calculates an index of comparative cost for each industry as follows:

\[ C_i(t) = \frac{1 + ERP_i(t)}{1 + ERP_{avg}(t)} \]  

\( C_i(t) \) represents the comparative cost index of industry \( i \) in year \( t \). \( ERP_i(t) \) is the effective rate of protection for industry \( i \) in year \( t \) (see Bhagwati et al., 1998, ch. 15). It is calculated by comparing domestic versus international prices. If domestic prices are higher than international prices, this leads to a positive effective rate of protection. \( ERP_{avg}(t) \) is the average effective rate of protection for all traded goods industries in year \( t \). \( C_i(t) \) then measures how a particular industry's degree of protection weighs up against the average rate of protection in the tradable sector. A \( C_i(t) \) higher than 1 denotes a comparative disadvantage. A \( C_i(t) \) lower than 1 denotes a comparative advantage. Table 4 documents the evolution of an index of comparative cost for several industries in South Korea. An industry which evolves from a high level of comparative cost towards a low level is considered to have matured under protection, in the sense that the difference between its domestic prices and international prices has reduced over time, relative
to other tradeable sectors.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Crops</td>
<td>1.195</td>
<td>1.390</td>
<td>1.524</td>
<td>1.879</td>
<td>3.127</td>
<td>3.336</td>
<td>(-)</td>
<td>(-)</td>
</tr>
<tr>
<td>2 Other Agriculture</td>
<td>0.645</td>
<td>0.696</td>
<td>1.089</td>
<td>1.620</td>
<td>1.414</td>
<td>1.288</td>
<td>1.220</td>
<td>1.264</td>
</tr>
<tr>
<td>3 Forestry</td>
<td>0.842</td>
<td>1.031</td>
<td>1.036</td>
<td>0.480</td>
<td>0.594</td>
<td>0.554</td>
<td>0.700</td>
<td>0.856</td>
</tr>
<tr>
<td>4 Mining</td>
<td>0.846</td>
<td>0.893</td>
<td>0.996</td>
<td>0.742</td>
<td>1.085</td>
<td>1.126</td>
<td>1.141</td>
<td>1.264</td>
</tr>
<tr>
<td>5 Fishery</td>
<td>1.009</td>
<td>0.847</td>
<td>0.656</td>
<td>0.544</td>
<td>0.403</td>
<td>0.413</td>
<td>0.491</td>
<td>0.509</td>
</tr>
<tr>
<td>6 Beverages</td>
<td>0.725</td>
<td>0.755</td>
<td>0.667</td>
<td>0.826</td>
<td>0.793</td>
<td>0.734</td>
<td>0.679</td>
<td>0.730</td>
</tr>
<tr>
<td>7 Tobacco</td>
<td>0.788</td>
<td>0.644</td>
<td>0.379</td>
<td>0.605</td>
<td>0.623</td>
<td>0.466</td>
<td>0.442</td>
<td></td>
</tr>
<tr>
<td>8 Textile</td>
<td>0.823</td>
<td>0.802</td>
<td>0.726</td>
<td>0.829</td>
<td>0.789</td>
<td>0.795</td>
<td>0.830</td>
<td>0.835</td>
</tr>
<tr>
<td>9 Apparel</td>
<td>0.905</td>
<td>0.703</td>
<td>0.716</td>
<td>0.706</td>
<td>0.865</td>
<td>0.831</td>
<td>0.700</td>
<td>0.788</td>
</tr>
<tr>
<td>10 Leather</td>
<td>1.315</td>
<td>1.283</td>
<td>1.107</td>
<td>0.752</td>
<td>0.754</td>
<td>0.511</td>
<td>0.520</td>
<td>0.432</td>
</tr>
<tr>
<td>11 Wood</td>
<td>1.824</td>
<td>1.099</td>
<td>0.816</td>
<td>0.733</td>
<td>0.709</td>
<td>0.748</td>
<td>0.810</td>
<td>0.787</td>
</tr>
<tr>
<td>12 Paper</td>
<td>0.872</td>
<td>0.871</td>
<td>0.764</td>
<td>0.869</td>
<td>0.893</td>
<td>0.833</td>
<td>0.978</td>
<td>0.920</td>
</tr>
<tr>
<td>13 Printing &amp; Publishing</td>
<td>0.635</td>
<td>0.652</td>
<td>1.062</td>
<td>0.729</td>
<td>0.723</td>
<td>0.790</td>
<td>0.864</td>
<td>0.828</td>
</tr>
<tr>
<td>14 Industrial Chemicals</td>
<td>0.875</td>
<td>0.696</td>
<td>0.896</td>
<td>0.920</td>
<td>1.158</td>
<td>1.059</td>
<td>0.844</td>
<td>0.886</td>
</tr>
<tr>
<td>15 Non Industrial Chemicals</td>
<td>1.603</td>
<td>1.473</td>
<td>1.090</td>
<td>0.876</td>
<td>0.796</td>
<td>0.901</td>
<td>1.160</td>
<td>1.003</td>
</tr>
<tr>
<td>16 Petroleum and Coal</td>
<td>0.541</td>
<td>0.532</td>
<td>1.463</td>
<td>(-)</td>
<td>(-)</td>
<td>9.075</td>
<td>1.190</td>
<td>0.548</td>
</tr>
<tr>
<td>17 Rubber</td>
<td>1.156</td>
<td>0.889</td>
<td>0.682</td>
<td>0.773</td>
<td>0.717</td>
<td>0.690</td>
<td>1.024</td>
<td>0.828</td>
</tr>
<tr>
<td>18 Pottery</td>
<td>0.998</td>
<td>0.809</td>
<td>0.714</td>
<td>1.013</td>
<td>0.947</td>
<td>0.950</td>
<td>1.110</td>
<td>0.819</td>
</tr>
<tr>
<td>19 Glass</td>
<td>0.931</td>
<td>0.847</td>
<td>0.673</td>
<td>1.081</td>
<td>0.908</td>
<td>0.842</td>
<td>0.904</td>
<td>0.803</td>
</tr>
<tr>
<td>20 Nonmetallic Mineral</td>
<td>1.032</td>
<td>0.911</td>
<td>0.868</td>
<td>1.030</td>
<td>0.980</td>
<td>1.120</td>
<td>0.888</td>
<td>1.082</td>
</tr>
<tr>
<td>21 Iron and Steel</td>
<td>1.624</td>
<td>1.453</td>
<td>1.028</td>
<td>0.845</td>
<td>0.807</td>
<td>0.803</td>
<td>0.842</td>
<td>0.875</td>
</tr>
<tr>
<td>22 Nonferrous Metal</td>
<td>0.924</td>
<td>0.713</td>
<td>0.999</td>
<td>0.829</td>
<td>0.776</td>
<td>0.619</td>
<td>1.229</td>
<td>1.439</td>
</tr>
<tr>
<td>23 Fabricated Metal</td>
<td>1.155</td>
<td>1.126</td>
<td>0.892</td>
<td>1.126</td>
<td>1.024</td>
<td>1.182</td>
<td>1.092</td>
<td>0.968</td>
</tr>
<tr>
<td>24 General Machinery</td>
<td>1.076</td>
<td>0.820</td>
<td>0.803</td>
<td>1.140</td>
<td>1.028</td>
<td>0.980</td>
<td>1.046</td>
<td>1.187</td>
</tr>
<tr>
<td>25 Electrical Machinery</td>
<td>1.195</td>
<td>1.036</td>
<td>0.824</td>
<td>0.933</td>
<td>0.861</td>
<td>0.869</td>
<td>0.937</td>
<td>0.944</td>
</tr>
<tr>
<td>26 Transportation Equipment</td>
<td>1.698</td>
<td>1.145</td>
<td>1.218</td>
<td>1.439</td>
<td>1.203</td>
<td>1.406</td>
<td>1.315</td>
<td>1.118</td>
</tr>
<tr>
<td>27 Precision Instruments</td>
<td>1.618</td>
<td>1.106</td>
<td>1.158</td>
<td>1.109</td>
<td>1.034</td>
<td>1.041</td>
<td>0.995</td>
<td>1.033</td>
</tr>
<tr>
<td>28 Miscellaneous Products</td>
<td>0.822</td>
<td>0.773</td>
<td>0.918</td>
<td>1.203</td>
<td>1.164</td>
<td>1.054</td>
<td>1.020</td>
<td>1.001</td>
</tr>
<tr>
<td>29 Primary Manufacturing Sector</td>
<td>1.055</td>
<td>1.216</td>
<td>1.393</td>
<td>1.334</td>
<td>1.462</td>
<td>1.575</td>
<td>1.947</td>
<td>2.200</td>
</tr>
</tbody>
</table>

Table 4. Index of Comparative Cost per Industry in South Korea

In table 4, thirteen industries had a comparative cost index above 1 in 1970, of which crops was a senile industry. Of the remaining twelve infant industries, the following matured by crossing (from above) the threshold level of 1: Food (in 1975), leather (in 1980), wood (in 1978), rubber (in 1975), iron and steel (in 1980) and electrical machinery (in 1978). Lee then asks whether any of the remaining industries exhibit a downward trend in their comparative cost indices by fitting a time trend and a constant to the above data. She finds that two additional industries have a significantly negative time trend: Precision instruments (5% level of significance) and non-industrial chemicals (10% level of significance). Thus eight out of twelve infant industries show a process of maturation. Of the remaining industries, nonmetallic minerals, fabricated metal and general machinery show little signs of being what Lee labels 'geriatric infants' (p. 1274), with their comparative cost levels not being too distant from 1. Transportation equipment exhibits an (albeit not significant) downward trend.
The data on comparative cost indices is then related to growth per industry, shown in the following table:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Crops</td>
<td>4.3</td>
<td>2.0</td>
<td>4.7</td>
<td>-0.8</td>
</tr>
<tr>
<td>2 Other Agriculture</td>
<td>4.1</td>
<td>2.5</td>
<td>12.2</td>
<td>1.6</td>
</tr>
<tr>
<td>3 Forestry</td>
<td>-0.3</td>
<td>0.4</td>
<td>0.7</td>
<td>-6.8</td>
</tr>
<tr>
<td>4 Fishery</td>
<td>9.6</td>
<td>1.2</td>
<td>5.1</td>
<td>3.7</td>
</tr>
<tr>
<td>5 Mining</td>
<td>7.5</td>
<td>2.2</td>
<td>2.8</td>
<td>-1.9</td>
</tr>
<tr>
<td>6 Food</td>
<td>3.0</td>
<td>8.0</td>
<td>4.3</td>
<td>4.5</td>
</tr>
<tr>
<td>7 Beverages</td>
<td>13.7</td>
<td>9.3</td>
<td>4.0</td>
<td>13.9</td>
</tr>
<tr>
<td>8 Tobacco</td>
<td>12.5</td>
<td>8.5</td>
<td>4.9</td>
<td>4.7</td>
</tr>
<tr>
<td>9 Textile</td>
<td>23.0</td>
<td>6.6</td>
<td>6.7</td>
<td>4.7</td>
</tr>
<tr>
<td>10 Apparel</td>
<td>25.0</td>
<td>16.7</td>
<td>4.7</td>
<td>4.7</td>
</tr>
<tr>
<td>11 Leather</td>
<td>34.8</td>
<td>8.2</td>
<td>9.4</td>
<td>6.6</td>
</tr>
<tr>
<td>12 Wood</td>
<td>11.4</td>
<td>6.0</td>
<td>3.9</td>
<td>7.5</td>
</tr>
<tr>
<td>13 Paper</td>
<td>14.5</td>
<td>13.4</td>
<td>11.7</td>
<td>16.7</td>
</tr>
<tr>
<td>14 Printing &amp; Publishing</td>
<td>10.6</td>
<td>14.4</td>
<td>15.0</td>
<td>5.7</td>
</tr>
<tr>
<td>15 Industrial Chemicals</td>
<td>26.8</td>
<td>17.6</td>
<td>8.4</td>
<td>13.1</td>
</tr>
<tr>
<td>16 Non Industrial Chemicals</td>
<td>20.8</td>
<td>12.9</td>
<td>13.9</td>
<td>15.3</td>
</tr>
<tr>
<td>17 Petroleum and Coal</td>
<td>8.1</td>
<td>11.9</td>
<td>3.8</td>
<td>8.6</td>
</tr>
<tr>
<td>18 Rubber</td>
<td>23.8</td>
<td>16.7</td>
<td>10.3</td>
<td>7.2</td>
</tr>
<tr>
<td>19 Pottery</td>
<td>8.6</td>
<td>25.5</td>
<td>8.1</td>
<td>1.9</td>
</tr>
<tr>
<td>20 Glass</td>
<td>18.1</td>
<td>13.1</td>
<td>14.5</td>
<td>16.3</td>
</tr>
<tr>
<td>21 Nonmetallic Mineral</td>
<td>14.2</td>
<td>13.0</td>
<td>8.1</td>
<td>9.7</td>
</tr>
<tr>
<td>22 Iron and Steel</td>
<td>33.5</td>
<td>24.6</td>
<td>10.3</td>
<td>9.1</td>
</tr>
<tr>
<td>23 Nonferrous Metal</td>
<td>26.8</td>
<td>21.8</td>
<td>14.9</td>
<td>12.9</td>
</tr>
<tr>
<td>24 Fabricated Metal</td>
<td>21.4</td>
<td>15.5</td>
<td>18.5</td>
<td>15.2</td>
</tr>
<tr>
<td>25 General Machinery</td>
<td>29.7</td>
<td>7.5</td>
<td>21.6</td>
<td>24.7</td>
</tr>
<tr>
<td>26 Electrical Machinery</td>
<td>39.7</td>
<td>24.1</td>
<td>15.3</td>
<td>20.5</td>
</tr>
<tr>
<td>27 Transportation Equipment</td>
<td>21.5</td>
<td>18.3</td>
<td>20.9</td>
<td>19.3</td>
</tr>
<tr>
<td>28 Precision Instruments</td>
<td>37.0</td>
<td>19.3</td>
<td>12.5</td>
<td>19.0</td>
</tr>
<tr>
<td>29 Miscellaneous Products</td>
<td>25.8</td>
<td>17.2</td>
<td>10.8</td>
<td>10.3</td>
</tr>
</tbody>
</table>

Note: Growth rates of output are at 1985 constant prices, percent per annum.
Source: Lee (1997), table 3.

Table 5. Growth of output per industry in South Korea (percent per annum)

For manufacturing industries excluding petroleum and coal, Lee finds that the comparative cost index is positively (and significantly) related to industry growth: Industries which had comparative disadvantage (as measured by a high comparative cost index) are the ones that grew faster. This is interpreted as evidence of successful targeted industrial or trade policy. She also presents growth data for different groupings of industries, as shown in the following table:
<table>
<thead>
<tr>
<th>Industry**</th>
<th>favoured***</th>
<th>Manufacturing</th>
<th>Primary</th>
<th>All Traded</th>
</tr>
</thead>
<tbody>
<tr>
<td>s=5 years</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1970-75</td>
<td>10.8</td>
<td>18.8</td>
<td>28.6</td>
<td>29.1</td>
</tr>
<tr>
<td>1975-80</td>
<td>9.2</td>
<td>17.0</td>
<td>19.7</td>
<td>20.2</td>
</tr>
<tr>
<td>1980-85</td>
<td>5.9</td>
<td>11.9</td>
<td>14.0</td>
<td>15.1</td>
</tr>
<tr>
<td>1985-90</td>
<td>6.4</td>
<td>14.5</td>
<td>16.4</td>
<td>17.1</td>
</tr>
<tr>
<td>s=10 years</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1970-80</td>
<td>10.0</td>
<td>18.0</td>
<td>24.1</td>
<td>24.6</td>
</tr>
<tr>
<td>1975-85</td>
<td>7.5</td>
<td>14.4</td>
<td>16.8</td>
<td>17.6</td>
</tr>
<tr>
<td>1980-90</td>
<td>6.1</td>
<td>13.2</td>
<td>15.2</td>
<td>16.1</td>
</tr>
<tr>
<td>s=15 years</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1970-85</td>
<td>8.6</td>
<td>15.9</td>
<td>20.6</td>
<td>21.3</td>
</tr>
<tr>
<td>1975-90</td>
<td>7.1</td>
<td>14.5</td>
<td>16.7</td>
<td>17.5</td>
</tr>
<tr>
<td>s=20 years</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1970-90</td>
<td>8.0</td>
<td>15.6</td>
<td>19.5</td>
<td>20.3</td>
</tr>
</tbody>
</table>

Note: Growth rates of output are at 1985 constant prices, percent per annum.

*: Heavy and chemical industries are chemicals, petroleum and coal, rubber, nonmetallic mineral, iron and steel, nonferrous metal, fabricated metal, general machinery, electrical machinery, transportation equipment and precision instruments (industries 15-18 and 21-28 from tables 4 or 5), while light industry encompasses all other industries (industries 1-14, 19-20 and 29 from tables 4 or 5).

**: Favoured industries are chemicals, iron and steel, nonferrous metal, general machinery, electrical machinery, transportation equipment and precision instruments (industries 15-16, 22, 23, 25 and 26-28 from tables 4 or 5).

***: Favoured infant industries are all the Favoured Industries excluding industrial chemicals and nonferrous metals (industries 16, 22, 25 and 26-28 from tables 4 or 5).

Source: Lee (1997), table 5.

Table 6. Growth of output for selected groups of industries in South Korea (percent per annum)

In Table 6 it is clear that the group of favoured industries (chemicals, iron and steel, nonferrous metal, general machinery, electrical machinery, transportation equipment and precision instruments) grew at a faster rate. Among the favoured industries, the ones classified as infants (which exclude industrial chemicals and nonferrous metals) grew slightly faster than the two mature ones. Lee does not explain in great detail the procedure followed for defining such groupings, leading to suspicions of selection bias in forming the groupings. However, the interpretation does seem plausible.

Amsden (1989) argues that in South Korea a developmental (and dictatorial) State implemented a wide variety of interventionist policies which directed firms' incentives towards achieving higher technological capability and this led the process of growth. In particular, together with other instruments, the Korean Antitrust Law was actively used as a means of regulating the degree of rivalry within industries. Other instruments included making firms compete for subsidies and other means of support on the basis of performance (particularly export) objectives.

In addition, all commercial banks were government owned and controlled (and this allowed the State to control interest rates and to set up a menu of lending schemes for each industry).
Price controls were regularly used to offset market power in many industries and capital controls were enforced to avoid capital flight and also to maintain local ownership of industries (i.e., capital flows took the form of overseas aid and foreign lending, with foreign ownership or capital outflows being of relatively small significance). The government also implemented an export-oriented import-substitution regime. In essence, a targeted mercantilist trade policy. This consisted of an initial period of import substitution, inward oriented development followed by a subsequent opening of the economy on the exports side, and to a lesser extent on the import side, with the degree of protection differing widely by industry (see table 4, above). Regarding the export push, it is important to highlight the favorable conditions that South Korea enjoyed in gaining access to the USA market.

Amsden (1989) proposes that by enhancing domestic competition among firms, setting export and productivity goals, together with the (credible) threat of not supporting firms with a poor performance, whilst protecting them from foreign competition, the government tilted firm incentives towards increasing performance (as opposed to rent seeking). She asserts that the mix of (targeted) export-oriented import-substitution and disciplinary mechanisms in South Korea was critical for the nation's success.

Historical Perspectives: Critique

In summary, the key questions this literature addresses are: Why did some late industrializers (in particular South Korea, Japan and Taiwan) achieve spectacular success in raising their income levels and their level of technological capability? In other words, What are the fundamental causes of the 'East Asian Miracle'? The means of enquiry are (sometimes quantitative) historical descriptions of the industrialization processes. The answers often come in the form of what Wade has labelled an 'inductive approach to policy and policy-making' (1990, p. 348), whereby the optimal policies are induced from the historical interpretations. In particular, by focusing on the types of policy measures which operated in each period in each economy, the authors aim to identify which of these were crucial to economic performance. This policy choice exercise is often based on expert opinion (usually interviews with policy makers, managers and entrepreneurs). Such a process is likely to be fraught with bias, be it of an ideological nature or due to incomplete/imperfect information or cognition, whether on behalf of the researcher or the expert who provides the opinion. Moreover, in the absence of statistical testing, there is no way of telling whether the influence of a particular policy is indeed (statistically) significant in determining outcomes.

Another problem with this approach is that even in the cases in which we are able to iden-
tify precisely the critical factors (be they policies or other specificities of the economies under study), we are still left in the dark as to (or, at best, with a very vague notion of) the mechanisms through which these policies affect economic performance, and hence the circumstances under which these policies could be used to advantage. The problem is compounded when we consider the variety of policies which were implemented across East Asia, with Thailand and Hong Kong's *laissez faire* versus Japan and Korea's state *dirigisme*; Taiwan's mid-sized public enterprises and small private firms versus Korea's and Japan's large private conglomerates; the impact of Japan's colonial heritage in Korea and Taiwan versus the British heritage in Hong Kong and so forth. The response to this variety has been either to assert that such diversity of interventions supports the hypothesis that they did not make much difference (see for example World Bank, 1993). The alternative response is that most of these interventions were, to a larger or lesser extent, conducive to growth, and that there is a wide range of policy mixes which will work (Wade, 2003, introduction). The problem lies in telling these explanations apart. For this we need a detailed theory of how and why the interventions did (or did not) work.

As we mentioned above in the context of growth regressions, further problems arise when analyzing the direction of causality (i.e., endogeneity of explanatory variables). Hypothetically, economic growth could be caused by responsible fiscal policy and other factors such as good educational systems and infrastructure. However, the reverse is just as likely to be true: It is easier to sustain prudent fiscal policy, good infrastructure, etc. in the context of a prosperous economy. Even in the case of investment, the commonly held notion that it is investment which causes growth is being challenged\(^3\): There is substantial evidence that it is growth which Granger-causes\(^4\) savings, and not the other way round (World Bank, 1993, appendix 5.1 and Carroll and Weil, 1994).

**The Need for Theory**

Nelson and Pack (1999) come to the conclusion that the theories we have so far are lacking in their microfoundations:

"Economic analysis in general, but development economics in particular, needs a better theory of firm behaviour...it needs a realistic theory, that is consistent

---

\(^3\)Although note that such a presumption is not predicted by the quintessential neoclassical growth model: Solow (1956). In Solow's model, accumulation of per-worker capital stock occurs as an endogenous result along the transition path as the economy converges towards its steady state.

\(^4\)Granger causality tests whether a variable is explained by lagged (past) values of another, or vice-versa (Granger, 1969).
with what we have learned empirically, about the processes of firm, industry and national learning, that have been behind The Asian Miracle" (p. 435)

In light of the evidence on the East Asian Miracle, it seems there is a reasonably strong (albeit unproven) case for selective interventions. However, without a clear, positive theory of selective interventions, one is left with little foundation on which to pursue such crucial decisions. What are the precise mechanisms by which these policies worked? Did they work at all or is there some other underlying process?

After a few decades (in Wade's terminology) of 'inductive policy analysis', it is time to apply the lessons we can draw from this literature to build a theory which accounts for some of the most salient features of the process. It is in this spirit that we find a need for formal, testable theories of industrialization. Rodrik (2004, p. 5) puts the problem thus:

"How do we provide guidance to countries besides uttering platitudes ("integrate into the world economy," "maintain sound money and sustainable fiscal balances," etc.)? How do we avoid policy nihilism and an anything-goes kind of approach ("all countries must find their own solutions to their problems")? How do we move forward with a positive agenda for policy reform instead? For those of us working on issues of economic growth, this constitutes the central challenge of our time."

As a starting point for the construction of such theories, in parts I and II we build a framework which takes into consideration some essential features:

1) Oligopolistic behaviour and the associated strategic interaction.
2) An endogenous treatment of technological capability and market structure.
3) A general equilibrium framework.
4) A dualistic structure (characteristic of many developing countries).

Features 1 and 2 emerge from the development studies literature. This emphasizes the importance of large enterprises which successfully embarked upon technologically advanced projects, particularly in the case of South Korea and Japan (Amsden, 1989). Features 3 and 4 are essential to analyze the impact of structural changes as the economy industrializes. In the models we develop later on, we will see that market structure and technological capability interact in an essential manner to affect the wage rate and (dual) structure of the economy. Once we develop models incorporating these features, we will be in a position to modify or extend them by introducing:

5) Asymmetries in initial conditions.
6) Variable intensity of competition.

This will bring forth a coherent analytical framework which will allow us to carry out a positive analysis of the role of economic policy (in particular industrial, trade and competition policies), based on solid micro foundations, and using a standard general equilibrium framework.

We develop two kinds of models. The first type (parts I and II of the thesis), are general equilibrium models characterized by a symmetric equilibrium. The second type of model is presented in part III. This is essentially a Big-Push type model with multiple equilibria, and the question of industrialization is analyzed in the context of a switch between a low wage equilibrium and a high wage equilibrium. In the process we discover a new characterization for the stages of economic growth.

In part I, chapter 2 develops the basic model within a closed economy framework. In chapter 3 this is extended to a two country setting, in which industries in both economies feature strategic interaction and the terms of trade are determined endogenously. We find the usual result that, under symmetry, free trade is welfare improving. The open economy model allows us to address the impact of asymmetries in initial conditions and the question of whether catching-up is feasible and welfare-improving. Allowing for asymmetric initial conditions brings forth the following result: A technologically advanced nation benefits from free trade with all nations of lower or slightly higher technological capability. However, a nation with a significantly lower technological capability will not benefit from a free trade agreement with the advanced country, and is better off in autarky. This sheds light on the negotiation process of bilateral free-trade agreements between advanced and backward economies. Regarding the issue of catching-up, we find that a subsidy to enhance technological capability is an effective instrument which allows the backward economy to match the advanced economy's wage rate. Moreover, if the subsidy is funded via a lump-sum tax, it will be welfare enhancing.

In part II we extend the models developed in part I to allow for a diverse range of intensities of competition. Chapter 4 does this for the closed economy, chapter 5 for the open economy. This will add further oligopolistic content to the models, for we will be able to encompass intensities of competition ranging from individual profit maximization at the firm level to perfect collusion (joint profit maximization) by changing a single parameter. Once we open up the economy, changing the intensity of competition will bring forth one of the central results of the study: For intensities of competition lower than individual profit maximization, there exists a separating surface for the wage rate below which the open economy features a higher wage rate than autarky, whilst above the surface it is the closed economy which features a
higher wage rate. The separating surface occurs in the space of parameters which characterize different types of industries, and thereby provides a positive basis for differential trade policy regarding these industries.

Part III (chapter 6) extends a Big-Push framework borrowed from Venables (1996) to allow for endogenous choice of technological capability in a Cournot oligopolistic (upstream) industry with vertical linkages to a perfectly competitive (downstream) industry. This extension brings forth a novel interpretation of the transition from a low-wage (underdeveloped) equilibrium towards a high-wage (developed) equilibrium. The transition is now characterized by a series of ‘take-off’ points (industrial and technological take-offs), which lead the economy to differing levels of development. The resulting level of development (wage rate) depends crucially on whether the economy can fit through a ‘window of opportunity’ while it is on the transition towards the high-wage equilibrium. If it manages to fit through the window, it will end up with a higher wage rate than would have otherwise been the case (under certain parameter restrictions). The conditions characterizing the ‘window of opportunity’ are formally specified and one of the central results is that if the wage rate grows too quickly, the window will be missed and the economy will end up in lower (yet still high-wage) equilibrium.

We intentionally concentrate exclusively on developing the theoretical aspects of the models in chapters 2-6, and defer the linkage of these findings to historical experience to the conclusions (chapter 7). This has the advantage of allowing us to proceed in a swift and focussed form.

The literature which most closely relates to this study is, for parts I and II, the theory of international trade under imperfect competition. For part II, the literature of interest studies the linkages between the intensity of competition and economic performance. For part III, the relevant literature is that related to Big Push models. We now (briefly) relate our theoretical models to previous work in these specific areas.

The Theory of International Trade Under Imperfect Competition

The first generation of models in this field introduced monopolistic competition (Krugman, 1979, 1980) and strategic interaction (Brander, 1981; Brander and Spencer, 1985; Venables, 1985) into a trade-theoretic structure. These models provided a novel framework which shed light on certain features of international trade previously unaccounted for by the Heckscher-Ohlin model (Bhagwati et al., 1998, chs. 5-6). In particular, the models introduced product differentiation and strategic interaction and this provided an explanation for intra-industry trade. Initially, Krugman did not allow for strategic interaction, whilst Brander, Spencer and Venables did not allow for product differentiation (although Brander, 1981, briefly touches
upon the issue of product variety, p. 9). Among other features of the models, accounting for intra-industry trade constituted an important breakthrough which warranted that the models became part of the mainstream paradigm in economics roughly within a decade (Krugman, 1998, p. 146). Bernhofen (2001) has generalized these models by subsuming product differentiation and strategic interaction into a single framework, using a quadratic utility function with a taste for variety and Cournot oligopoly. This line of work has been continued by Clarke and Collie (2003), who consider the case of Bertrand competition. The literature on trade under imperfect competition is notorious for its variety of models and policy implications. Small changes in assumptions often lead to policy reversals, ranging from the optimality of free trade to intervention via trade policy (tariffs or quotas) or industrial policy (taxes or subsidies). Dixit (1984) comments on the common absence of factor price determination in this literature, and how this proves to be an crucial shortcoming, since inter-industry trade will be determined by such factor prices. Dixit and Grossman (1986) study the case of economies composed of multiple oligopolistic industries, incorporating general equilibrium effects operating via input markets. They provide a formal basis for industrial targeting, i.e., which of the multiple oligopolistic industries should be subsidized/taxed. Their conclusion is that the industries which can most effectively shift rents towards the domestic market are the ones which should be subsidized. However, due to the informational requirements of such a policy, Dixit and Grossman remain skeptical of their findings and conclude:

“our main finding is a negative one, but it has considerable importance for practical policy-making. When several oligopolistic industries are linked together by factor endowment constraints, the optimal rent-extraction policies are generally less beneficial than a partial-equilibrium analysis would suggest, and very demanding of information. The prospects for correct implementation of such policies in practice are not at all good.”

The problem lies, thus, not in the optimality of such industrial targeting (which their analysis proves), but in the implementation of the strategy: How can the worthwhile industries be identified?

Technological Capability in International Trade

The literature on strategic interaction in international trade has been complemented by considering the effects of innovation (i.e., technological capability). Spencer and Brander (1983) analyze the case of R&D rivalry across nations and conclude that subsidies to R&D can be
welfare enhancing from the perspective of an individual nation. Their model also supports
the infant industry argument. However, their argument is based neither on capital market
imperfections (in which credit is not available or is available at an interest rate above the
long run free market shadow value), nor on positive externalities from innovation. The infant
industry argument in Spencer and Brander (1983) relates to the expansion of the national
industry at the expense of the foreign one. Thus a subsidy to local industry can help it achieve
a leadership position.

More recently, Grossman and Helpman (1991) present over a decade’s work on dynamic
models of international trade under monopolistic competition with innovation, including their
famous quality ladder growth model. Murphy and Shleifer (1997) present a model where dif­
fferences in human capital affect product quality. The model assumes a perfectly competitive
setting and a taste for quality (but not for variety). An advanced economy has no incentive
to trade with a backward economy, since consumers in the advanced economy (high in human
capital) have high income levels and prefer the quality good. In the backward economy, con­
sumers have low income levels and cannot afford the quality good. Thus the backward economy
has nothing to offer to the advanced economy, while the rich economy finds it unprofitable to
produce low quality goods to sell to the backward economy.

Motta (1992b) is one of the closest papers to our study. Similarly to us, he introduces a
model of oligopoly with technological capability into an international trade framework. He
uses a model by Sutton (1991, ch. 3) based on perfectly substitutable goods. In our case, we
use a model with imperfect substitutability from Sutton (1998, ch. 2). Motta’s framework
is partial equilibrium, and he allows for differences in qualities between countries. In that
framework trade losses may arise in the short run (i.e., for fixed product quality) for backward
nations, when they engage in free trade with advanced nations. This is due to exit of the
low quality firms. In contrast, in the long run (when product quality can be adjusted), both
nations benefit from trade. This result is similar to what we find in chapter 3, where symmetric
economies with Cournot industries benefit from free trade. Like Motta, our model endogenizes
technological capability in international trade. Unlike Motta, we endogenize market structure,
wage rates, the sectorial distribution of employment and the terms of trade.

New trade theory has a strong interventionist flavour. Brander, Dixit and Spencer, along­
side most authors in this field, warn that such interventionist conclusions should taken cau­
tiously, for two reasons. Firstly, most models assigns a value of unity to the opportunity costs
of industrial policies (subsidies are usually valued in terms of forgone consumption). However,
it may be more accurate to consider alternative government expenditures. If the funds for
subsidies are being taken away from investments, such as health care or human capital, their opportunity cost may be considerably higher than unity. Secondly, a theoretical basis for intervention could be used by special interest groups to justify rent-seeking. Justifiably, this is precisely the type of outcome which has engendered deep mistrust in the economics profession towards interventionist arguments.

The Intensity of Competition

The intensity of rivalry between domestic firms takes a central role in this study. Several authors (Amsden and Singh, 1994; Crafts, 1997, p. 49; Wade, 1990, p. 143) have stressed the importance of this aspect for economic performance. This is probably one of the least developed topics in the industrialization debate (be it on East Asian or elsewhere). A clear terminology has not yet emerged, so we will start by making a few clarifications. Some authors (Aghion et al., 1999), use the term 'competition' to denote product substitutability (or the elasticity of substitution between goods): Increased product substitutability would be associated with decreased mark-ups and hence more intense competition. Others use the term 'competition' to denote the number of firms: A larger number of firms is taken to imply stronger competition (Carlin et al., 2004). The notion we will use for the intensity of competition refers to a structural characteristic of the type of strategic interaction between firms, and is an entirely different concept to product substitutability or to the number of competitors. It was introduced by Sutton (1991, p. 9), and refers to the changing nature of competition as the industry changes from a perfectly competitive setting, to Bertrand competition, to Cournot competition, to joint profit maximization (perfect collusion). The perfectly competitive setting and Bertrand competition would be considered the toughest type of competition, followed by Cournot and then by joint profit maximization (the weakest form of competition, equivalent to a perfectly collusive agreement). The term 'rivalry' can also be used to denote the intensity of competition. The intensity of competition bears considerable similarity to the notion of conjectural variations. However, unlike conjectural variations, it does not rely on notions of (inappropriately dynamic) reaction functions, and is directly measurable (we will see in chapter 4, that it is the extent of cross-ownership within an industry). The intensity of competition can be affected by the anti-trust climate in the economy: Whereas a permissive anti-trust authority may lead to a low intensity of competition, a stringent authority may be associated with a high intensity of competition.

international success derives from the fierce rivalry (intensity of competition) prevalent in the local market. Analyzing the Japanese case, Amsden and Singh find that MITI (the Ministry of International Trade and Industry) intervened in several occasions to curb the intensity of competition between firms, especially during recessions (presumably to avoid exit of otherwise profitable firms). This intervention consisted of encouraging cartels and mergers in industries characterized by economies of scale. On the other hand, there were many instances in which MITI promoted vigorous oligopolistic rivalry. Exposure to international competition was used as a complementary policy to competition policy: “The emphasis on exports and on maintaining oligopolistic rivalry – instead of concentrating resources and subsidies on a single ‘national champion’, which many governments in their industrial policies are prone to do – are the key factors which distinguish Japanese policies from those of other dirigiste countries” (Amsden and Singh, 1994, p. 946). In the Korean case, Amsden and Singh note that there is ample evidence of strong rivalry between big business groups. Aside from anti-trust policy, the Korean government designed a reward scheme in which big business groups would vie for subsidies and other means of support on the basis of technological and export performance. Amsden (1989) describes how the South Korean government went about regulating market structure. Essentially, she argues that the government used industrial licensing and credit allocation to favour specific firms over others. From the following quotation, it is hard not to conclude that the criteria were overtly cronyist (in spite of Amsden’s assertions about reciprocity based on performance):

“In the cement industry, the chaebol belonging to a party elder, the Ssangyong group, was allowed to acquire nearly half of cement-making capacity by the 1980’s, and was then blessed with licenses for capacity expansions despite the existence of a more experienced cement company (the Tongyang corporation) dating to the Japanese colonial period. In the steel industry the small minimills of Japanese colonial heritage were discriminated against in credit allocation in favour of a newly created state-owned integrated enterprise, the Pohang Iron and Steel Company (POSCO). In shipbuilding, seven small experienced shipbuilders were dwarfed and in some cases bankrupted by the government’s assistance to the Hyundai group. In the machinery building sector, all three leading chaebol—the Hyundai, Samsung, and Daewoo groups—were favored over a slew of smaller long-standing firms” (Amsden, 1989, p. 73)

In spite of such close ties the strategy seems to have worked, in the following sense. By
relying on the chaebol, Korea has managed to compete successfully with world leaders and has managed to develop its technological capability. Of course, the normative question of whether such favouritism was welfare improving remains unaddressed.

Competition policy in Japan and South Korea ranged from cartel and merger support to the promotion of fierce rivalry, depending on the industry and on economic conditions prevailing at the time. Amsden and Singh (1994) argue for a non-monotonic relationship between the intensity of competition and economic performance: “If total openness to international competition and maximum domestic competition are not necessarily optimal, what is the appropriate level of openness or degree of domestic competition for an economy?” (p. 943). In their conclusions they state that “during much of the high growth period in Japan, despite all the government restrictions on competition, industrial concentration actually fell” (p. 950). In the models we develop in parts I and II, we will provide a formal treatment of concentration (which will be an endogenous variable), product substitutability and the intensity of competition (both of which will be exogenous parameters). By incorporating the three concepts in an explicit model, the differences and relationships between these notions of ‘competition’ will become clear.

From a theoretical perspective, there are some models which also highlight a similar notion to the intensity of competition. In particular, Eaton and Grossman (1986) build a generalized model which encompasses diverse types of competition by using the conjectural variations approach. Whilst recognizing the shortcomings of conjectural variations, they nonetheless proceed to develop an analysis of industrial and trade policies. After considering a wide variety of settings ranging from Cournot to Bertrand conjectures, strategic (first-mover) governments, and free entry and exit, they find that free trade with no government intervention is optimal only under Stackelberg conjectures when domestic consumption is zero. Dixit (1984) discusses anti-trust policy in the context of a trade model with oligopolistic competition and notes that the disciplining effects of exposure to foreign competition is sometimes used as an argument for the relaxation of anti-trust policy. Dixit shows how it may be desirable to encourage mergers or export cartels of domestic firms in order to strengthen the local industry.

Bernhofen (2001) subsumes product differentiation and strategic interaction into a single framework, using a quadratic utility function with a taste for variety and Cournot oligopoly. In his model ‘competition’ amounts to product substitutability. Aghion et al. (2005) use the elasticity of substitution as a measure of the intensity of competition in an endogenous growth model and find an ‘inverted-U’ relationship between innovation and competition (both theoretically and empirically). In chapters 4 and 5, we shall find a similar relationship between technological capability and the intensity of competition (but only in the case when the
marginal cost of technological capability is high). Carlin et al. (2004) carry out an empirical analysis using surveys for Eastern European transition economies and also find an 'inverted-U' relationship, this time between firm performance (as measured by sales growth) and the number of competitors present in the firm's market.

**Big-Push Perspectives**

Alternative interpretations of the East Asian Miracle are based on the notion that such growth episodes were a transition from a low-income to a high-income equilibrium. The explanations discussed so far highlight the role of a series of institutional causes which impelled high factor accumulation and effectively translated this into high growth rates in the East Asian economies. Rodrik (1995a and 1996) proposes an alternative interpretation of the East Asian Miracle based on the notion of coordination failures. He proposes that these economies had the required resources (initial conditions) to operate at a high level of income, but where unable to do so because they were subject to a coordination failure in that these resources were being used in an inefficient manner. Thus, what the East Asian governments did was to coordinate with economic agents a switch from a low-income equilibrium to a high-income equilibrium, and that it was this transition which sparked growth. In particular, Rodrik argues that the export boom in South Korea and Taiwan was a consequence of the rise in growth and investment: Growth required investment together with imports of machinery. In order to cover the foreign exchange needs of such imports, exports expanded. Presumably (although Rodrik does not clarify this), the mechanism by which exports expand in response to investment is the need of finding larger markets in order to fulfill scale economies of new projects, which the small domestic market could not exploit. So exports are essentially responding to a supply-side expansion. In an comment on Rodrik’s paper, Victor Norman (see the discussion at the end of Rodrik, 1995a) notes that the rise in imports of machinery occurred roughly half a decade after the beginning of the export boom. Norman then states that the hypothesis that it was investment (and the associated machinery imports) which led to exports is hard to sustain, for the latter predated the former. In the models we develop in parts I and II, openness to foreign markets has the effect of disciplining domestic firms, which, when faced with international technological capability, have to choose whether to match their overseas rivals’ technological capability or exit the market. In being exposed to foreign competition, firms have no choice but to increase their technological capability, if they are to achieve positive market share. This entails the notion of an increase in fixed investment as a consequence of the exposure to foreign competition. What we have obtained is essentially a formalization of the argument which
states that exports act as a technology enhancing mechanism: Exports cause investment in technological capability via exposure to foreign competition and to a larger market.

In the same family of Big-Push models we find the paper by Murphy, Shleifer and Vishny (1989), which develops similar arguments for a closed economy. The argument is essentially a formalization of the dual-economy analysis by Lewis (1954): There is a 'traditional' sector with constant returns to scale and a 'modern' sector which features increasing returns to scale. Initially, the economy produces only in the traditional sector, and due to a coordination failure, it is not profitable to enter the modern sector. If this coordination failure can be overcome (by some central coordination mechanism), workers shift to the modern sector, their wages increase and demand for modern goods rises in parallel. Thus a Big-Push of industrialization is obtained. Once the transition is complete, the economy remains in a high-wage industrialized equilibrium. Other Big-Push models include those by Matsuyama (1991, 1992a and 1992b) and Rodriguez-Clare (1996). The key difference between the existing (Big-Push) literature and the model we develop is the introduction of endogenously (and strategically) determined technological capability.
Part I

Introducing Oligopolistic Interactions and Technological Capability into a General Equilibrium Framework
Chapter 2

General Equilibrium with Oligopolistic Interactions: Autarky

1 Introduction

In order to provide an analytical evaluation of the substantive issues in the debate, we develop a model that combines oligopolistic interactions at the industry level with a general equilibrium framework. This chapter constitutes the first step towards developing a general equilibrium model with oligopolistic interactions and international trade. Here we develop the benchmark closed economy model.

Throughout the thesis we will use extensively the notion of 'technological capability'. We will use the term in the narrow sense of 'product quality'. At a deeper level this term encompasses a more fundamental meaning, i.e., the 'knowledge' of workers in the firm, which determines the levels of productivity and product quality (Sutton, 2004). This more profound meaning lies outside the scope of the thesis. Also throughout the thesis we will refer to sunk and fixed costs. It is convenient to clarify that in this research all fixed costs are sunk, and the terms will be used interchangeably.

We present a long run analysis of a general equilibrium, two-sector economy. The analysis is long-run in the sense that market structure and technological capability are determined endogenously and all goods are ultimately consumed (that is, any intertemporal allocation issues have been settled). There is an industrial (modern) sector and a traditional sector, and the only factor of production is labour.

The industrial sector (labelled industry $X$) is characterized by increasing returns to scale (IRTS), which are associated with endogenous sunk costs. In turn, sunk costs determine technological capability. We use a three stage game to model the behavior of firms in the
industrial sector. In the first stage firms decide whether to enter the market. In the second stage, firms invest in technological capabilities, taking the current market structure as given. In the third stage, firms compete in quantities, à la Cournot, taking as given the current market structure and each firm's technological capabilities.

The other industry is a traditional sector (industry Y), characterized by a 1:1 technology (1 unit of labour input produces 1 unit of output), implying constant returns to scale (CRTS). The presence of this sector ensures that the labour market clears. Since the model is general equilibrium, wages are determined endogenously.

There is a representative consumer who consumes goods produced by industries X and Y, subject to a budget constraint. Consumers supply a fixed amount of labour to industries X and Y, for which they perceive the equilibrium wage rate.

Workers are employed by firms in industry X to develop technological capability. The demand for labour from industry X is determined by investment in technological capability and by the number of entrants. For simplicity, we assume there are no variable costs in industry X. Industry Y serves as an outside option to all workers who cannot find employment in industry X.

The following diagram depicts the structure of the model, as explained above.

![Diagram](image)

Figure 1: The Structure of the Autarky Model

The chapter is structured as follows. In Section 2 we develop the model. Section 3 characterizes a symmetric general equilibrium. In Section 4 we analyze how the equilibrium solutions change as we vary the parameters. Section 5 offers some remarks on general features of the model. Section 6 concludes.
2 A General Equilibrium Closed Economy Model

We describe the consumers' problem first, then the production side of the model and finally the labour market. On the production side, the model features two industries (labelled X and Y respectively), without upstream or downstream linkages. For simplicity, the only factor of production in the model is labour. The output of industries X and Y is wholly consumed by workers (there are no savings).

2.1 Consumers

There is a population of L homogeneous consumers, indexed by \( h = 1, \ldots, L \). Each consumer has a perfectly inelastic labour supply, which has been set to one. Consumers choose over two types of good by maximizing a utility function (denoted by \( V \)), subject to a budget constraint. One type of good is labelled Y, and represents the (homogenous) good produced by industry Y (the traditional sector). The other type of product, labelled X, is constituted by a range of vertically differentiated goods produced by industry X. In industry X, each firm produces one good only, labelled \( x_k \). \( x_k \) denotes per-capita consumption of good \( k \). There is a finite number of firms (denoted by \( N+1 \)), and this will define the number of X-type goods, such that \( k = 1, \ldots, N+1 \). Each good in the X industry is associated with a quality level \( (u_k) \), which will represent the producer's technological capability. Thus the representative consumer's problem can be stated as

\[
\begin{align*}
\max_{x_k, y} \quad & V = \sum_{k=1}^{N+1} \left( x_k - \frac{x_k^2}{u_k} \right) - 2\sigma \sum_{k=1}^{N+1} \sum_{l=1}^{N} \frac{x_k}{u_k} \frac{x_l}{u_l} + Y \\
\text{subject to} \quad & \sum_{k=1}^{N+1} p_k x_k + qY = w + \sum_{k=1}^{N+1} s_{hk} \Pi_k
\end{align*}
\]

(2.1)

(2.2)

where \( p_k \) is the price of X-type good \( k \), \( q \) is the price of the Y-type good, \( \sigma \in (0,1) \) is a parameter which measures the substitutability between the X-type goods (alternatively, the degree of horizontal product differentiation), \( w \) is the wage rate, \( s_{hk} \) is the ownership share of (representative) consumer \( h \) in firm \( k \) (such that \( \sum_{h=1}^{L} s_{hk} = 1 \)), and \( \Pi_k \) denotes the net profits of firm \( k \). Free entry implies that net profits are zero, thus firm profits drop out from the budget constraint. The (quadratic) utility function in equation (2.1) is the underlying utility for the

---

1In chapter 6 we develop a model with vertical linkages.

2Subscripts \( k \) and \( l \) will denote firms, while the more commonly used subscripts \( i \) and \( j \) will be reserved to denote countries.

3We denote the number of firms by \( N+1 \) rather than \( N \) since this will make the algebra somewhat more organized in later chapters.
standard linear demand model, with some modifications (see Sutton, 1998, ch. 2). In passing, it is worth noting that to perform welfare analysis we will substitute the equilibrium solutions into \( V \), and we shall label this the welfare indicator, \( W \). From the consumer's problem, we obtain the inverse demand function for good \( k \):

\[
p_k = 1 - 2x_k + \frac{2\sigma}{u_k} \sum_{i \neq k} x_i
\]

The (per-capita) demand function for good \( Y \) is obtained as a residual from the budget constraint (equation 2.2):

\[
Y = \frac{1}{q} \left( w + \sum_{k=1}^{N+1} s_{hk} \Pi_k - \sum_{k=1}^{N+1} p_k x_k \right)
\]

Good \( Y \) will be defined as the numeraire, hence its price \( (q) \) will be set to 1. We seek an equilibrium in which the economy is characterized by symmetric firms, each earning zero net profits. Thus, (per-capita) demand for \( Y \) simplifies to

\[
Y = w - (N + 1)p x
\]

The demand functions specified above are used in obtaining equilibrium solutions for industries \( X \) and \( Y \). We now describe industry \( X \).

2.2 Industry \( X \)

Industry \( X \) features increasing returns to scale, associated with the presence of (endogenous) sunk costs. In this industry, firms play a three stage game, similar to the game featured in Sutton (1998, ch. 2). In the first stage firms decide whether to enter or not. In the second stage, firms invest in building technological capability (by choosing their level of sunk costs). In the final stage, firms compete in quantities, à la Cournot. The equilibrium concept used here is Subgame Perfect Nash Equilibrium (Selten, 1975).

Using backward induction to find a Subgame Perfect Nash Equilibrium, we now move on to the description and solution of the final stage in the firms' decision problem.

2.2.1 Stage 3: Cournot Competition

In this stage, firms choose their optimal quantity, taking as given rivals' quantities, technological capabilities and market structure. Gross profits of firm \( k (\pi_k) \) are equal to revenue \( (p_k x_k) \) minus the cost of producing \( x_k \). For simplicity, when solving the final stage we follow Sutton
(1998) in assuming that variable production costs are zero\(^4\). Hence we write,

\[ \pi_k = p_k x_k \quad \text{for} \quad k = 1, \ldots, N + 1 \quad (2.6) \]

The first order condition for firm \( k \) is given by

\[ p_k + \frac{\partial p_k}{\partial x_k} x_k = 0 \quad \text{for} \quad k = 1, \ldots, N + 1 \quad (2.7) \]

There are \( N + 1 \) such equations in \( x_k \) (one for each firm). In Appendix 1, we solve this system of equations, and find a symmetric Nash equilibrium. We obtain the solution for \( x_k \) in terms of the number of firms, \( N + 1 \), and the vector of firms' technological capabilities, \((u_1, \ldots, u_{N+1})^5\). To simplify exposition, we introduce the following vector notation for firms' technological capabilities: \( U = (u_1, \ldots, u_{N+1})' \). We obtain the following solved-out payoff for firm \( k \) (which will be used to solve the second stage of the game):

\[ \pi_k(U) = \frac{u_k^2}{2(2-\sigma)^2} \left( 1 - \frac{\sigma}{2 + \sigma N} \sum_{i=1}^{N+1} \frac{u_i}{u_k} \right)^2 \quad (2.8) \]

If firms choose a symmetric quality level (denoted by \( u \)), the payoff simplifies to

\[ \pi = \frac{u^2}{2(2 + \sigma N)^2} \quad (2.9) \]

which is the expression derived by Sutton (1998, p. 512, equation 2.2.17'). Note that \( \pi_k(U) \) refers to per-capita gross profit obtained by the firm (the gross profit the firm earns from each consumer). Since population size is given by \( L \), total gross profits are \( L \pi_k(U) \).

2.2.2 Stage 2: Competition in Technological Capability

In the second stage subgame, firms choose how much to invest in order to achieve a certain technological capability, taking as given their rivals' strategies. Their investment is a sunk cost, which is embodied in a fixed outlays function, \( F(.) \), to be defined below. The firm's net profit is

\[ \Pi_k = L \pi_k(U) - F(u_k, w) \quad \text{for} \quad k = 1, \ldots, N + 1 \quad (2.10) \]

\(^4\)The model extends readily to the case of positive variable production costs. However, to reduce the number of parameters, it was decided to set such costs to zero.

\(^5\)Recall that \( u_k \) also represents product \( k \)'s quality level. For our purposes the terms 'product quality' and a firm's 'technological capability' will be synonymous.
where $L$ denotes population size, $\pi_k(U)$ denotes the solved-out gross per-capita profit function (equation 2.8) and $F(u_k, w)$ denotes a fixed outlays function, of the following form: $F(u_k, w) = w f(u_k)$, where $w$ is the wage rate, and $f(u_k) = \varepsilon \left( \frac{u_k}{u_0} \right)^\beta$ is a convex mapping from technological capability ($u_k$) to labour units required to achieve such capability. The mapping $f(u_k)$ is the firm's labour requirement in industry $X$: It measures units of labour required to achieve a certain technological capability $u_k$. We assume $\beta > 2$ (to ensure that the second order conditions hold, see Appendix 2), $\varepsilon > 0$ and $u_k > u_0 \geq 1$. $\beta$ is the elasticity of $f(u_k)$ with respect to $u_k$. $\varepsilon$ is an exogenous set-up cost. $u_0$ represents the initial (inherited) value of technological capability, which is an exogenous parameter. In the open economy case (see chapters 3 and 5), $u_0$ is used as a means of modelling differences in initial conditions across countries.

Firms maximize the objective in equation (2.10) with respect to $u_k$. This leads to the following first order conditions

$$L \frac{\partial \pi_k}{\partial u_k} = \frac{w \varepsilon \beta}{u_k} \left( \frac{u_k}{u_0} \right)^\beta \quad k = 1, \ldots, N + 1 \quad (2.11)$$

Differentiating the solved out payoff (2.8) with respect to $u_k$, we can rewrite the first order conditions as follows:

$$L \frac{2 + \sigma (N - 1)}{(2 - \sigma)^2 (2 + \sigma N)} \left( 1 - \frac{\sigma}{2 + \sigma N} \sum_{i=1}^{N+1} \frac{u_i}{u_k} \right) = \frac{w \varepsilon \beta}{u_k} \left( \frac{u_k}{u_0} \right)^{\beta - 2} \quad k = 1, \ldots, N + 1 \quad (2.12)$$

If firms choose a symmetric quality level (denoted by $u$), the first order conditions simplify to

$$L \frac{2 + \sigma (N - 1)}{(2 - \sigma) (2 + \sigma N)^2} = \frac{w \varepsilon \beta}{u_0} \left( \frac{u}{u_0} \right)^{\beta - 2} \quad (2.13)$$

The first order conditions (together with other equilibrium conditions, to be introduced below) will allow us to solve for the equilibrium level of technological capability (see section 3 for more details). This completes the description of the second stage subgame. Second order conditions for this stage are discussed in Appendix 2.

### 2.2.3 Stage 1: The Entry Decision

In the first stage, firms decide whether to enter or not. We assume there is a sufficiently large pool of potential entrants. Firms enter as long as gross profits are not exhausted by fixed
outlays. This leads to the following free-entry condition

\[ L \pi_k \geq w \varepsilon \left( \frac{u_k}{u_0} \right)^\beta \quad k = 1, \ldots, N + 1 \]  

(2.14)

In equilibrium, ignoring integer effects, entry occurs until condition (2.14) holds with equality. If firms choose a symmetric quality level (denoted by \( u \)), using (2.9), the free entry conditions simplify to

\[ \frac{L}{2(2 + \sigma N)^2} = w \varepsilon \frac{u^{\beta - 2}}{u_0^\beta} \]  

(2.15)

This concludes the description of the Stage 1 subgame, and hence completes the description of the game in industry \( X \).

2.3 The Labour Market and Industry \( Y \)

We first discuss the labour market, and then industry \( Y \). Each consumer has a perfectly inelastic labour supply, which has been set to 1. The consumer decides how much of his labour endowment to allocate between industries \( X \) and \( Y \) (labour is perfectly mobile between industries). Total labour supply is therefore fixed at the size of the population, \( L \). Labour demand stems from sectors \( X \) and \( Y \). The labour requirement of each firm in sector \( X \) is \( f(u_k) = \varepsilon \left( \frac{u_k}{u_0} \right)^\beta \). Employment in industry \( X \) is given by

\[ L_x = \sum_{k=1}^{N+1} f(u_k) \]

In a symmetric equilibrium this simplifies to

\[ L_x = (N + 1) \varepsilon \left( \frac{u}{u_0} \right)^\beta \]  

(2.16)

Any labour not used in industry \( X \) is absorbed by industry \( Y \). We denote employment in industry \( Y \) by \( L_y \). Thus we can write the labour market clearing condition as

\[ L = L_x + L_y \quad \text{where } L_x, L_y \in [0, L] \]  

(2.17)

Industry \( Y \) has a simple 1:1 (constant returns to scale) technology: One unit of labour produces one unit of good \( Y \). The wage rate will adjust to ensure that labour supply matches demand.

We now turn to industry \( Y \). Recall per-capita demand for good \( Y \) from equation (2.4), and its symmetric counterpart (2.5). In a symmetric equilibrium we have that \( x_k = x \) and \( p_k = p \).
Using this, aggregate demand for good \( Y \) can be written as \( \mathcal{T}^D = L Y = L w - (N + 1) L p x \).

We next use the fact that \( L p x = L \pi = w e \left( \frac{u}{u_o} \right)^\beta \), which obtains by the free entry condition in equation (2.15). Thus we can write aggregate demand for good \( Y \) as

\[
\mathcal{T}^D = w \left[ L - (N + 1) \epsilon \left( \frac{u}{u_o} \right)^\beta \right]
\]

(2.18)

Given the 1:1 technology assumed for sector \( Y \), aggregate supply of good \( Y \) is identical to the amount of labour employed in the sector, and can be solved from equation (2.17), to yield

\[
\mathcal{T}^S \equiv L_y = L - (N + 1) \epsilon \left( \frac{u}{u_o} \right)^\beta
\]

(2.19)

If \( L_y > 0 \) then \( w = 1 \), to ensure that \( \mathcal{T}^S = \mathcal{T}^D \). To see this, note that industry \( Y \) uses a (freely available) 1:1 technology, hence the marginal product of labour is given by the price of good \( Y \) (namely, \( q \)), which has been set equal to 1 (since good \( Y \) is the numeraire). However, if demand for labour from sector \( X \) is high enough to make \( L_y = 0 \) (in which case \( L_x = L \)), then industry \( Y \) is non-existent, and industry \( X \) uses all the labour available in the economy. In this case we have that \( w > 1 \) (otherwise workers would switch to production of \( Y \) and earn \( w = 1 \)). Thus we obtain that in general equilibrium \( w \geq 1 \). When \( L_y = 0 \), the equilibrium wage is solved from the labour market clearing condition (equation 2.17).

It is important to note that in this model value added is identical to the wage rate. This can be broken down into the value added generated in each sector, and these are discussed in Appendix 3. This completes the description of the labour market and industry \( Y \). In the following section, we characterize a symmetric general equilibrium.

3 Characterization of a Symmetric General Equilibrium

We have five parameters \((\beta, \sigma, \varepsilon, L \text{ and } u_o)\) and three key variables which determine the rest of the system \((u, N \text{ and } w)\). The general equilibrium solutions to these variables will be functions of the parameters only: \(u(\beta, \sigma, \varepsilon, L, u_o), N(\beta, \sigma, \varepsilon, L, u_o)\) and \(w(\beta, \sigma, \varepsilon, L, u_o)\). We will simplify notation by dropping the arguments, so we write \(u(.)\) for \(u(\beta, \sigma, \varepsilon, L, u_o)\), \(N(.)\) for \(N(\beta, \sigma, \varepsilon, L, u_o)\) and \(w(.)\) for \(w(\beta, \sigma, \varepsilon, L, u_o)\). The solutions will depend on whether \(L_y > 0\) or \(L_y = 0\). The functions \(u(.)\), \(N(.)\) and \(w(.)\) characterize the general equilibrium of the economy, and can be derived explicitly in a symmetric equilibrium. A Symmetric General Equilibrium is characterized by:

a) Market clearing for goods of type \( X \) and \( Y \).
b) Market clearing for labour.

c) A Symmetric Subgame Perfect Equilibrium in Industry $X$, which in turn is characterized by:

i) Firms choosing (Nash) symmetric equilibrium quantities in stage 3 of the game, taking as given technological capabilities and market structure.

ii) Firms choosing (Nash) symmetric equilibrium technological capabilities in stage 2 of the game, taking market structure as given.

iii) Free entry in stage 1 of the game.

We now set out the equilibrium conditions and then use these to seek a symmetric equilibrium.

**Equilibrium Conditions:**

First we have the 'no-profitable deviation' (first order) conditions for choice of technological 
capability from (2.12):

\[
L_\frac{2 + \sigma (N - 1)}{(2 - \sigma)^2 (2 + \sigma N)} \left( 1 - \frac{\sigma}{2 + \sigma N} \sum_{l=1}^{N+1} \frac{u_l}{u_k} \right) = \frac{w \varepsilon \beta u_k^\beta - 2}{u_0^\beta} \quad (2.20)
\]

Second we have the zero profit (free entry) condition, from (2.14):

\[
L \pi_k = w \varepsilon \left( \frac{u_k}{u_0} \right)^\beta \quad (2.21)
\]

Third we have the labour market clearing condition, from (2.17)

\[
L = L_y + L_x \quad (2.22)
\]

Finally, we require market clearing for type $Y$ and type $X$ goods, which will hold by construction. In a symmetric equilibrium, the 'no-profitable deviation' condition for choice of technological capability (2.20) simplifies to

\[
L_\frac{2 + \sigma (N - 1)}{(2 - \sigma)(2 + \sigma N)^2} = \frac{w \varepsilon \beta u_k^\beta - 2}{u_0^\beta} \quad (2.23)
\]

Substituting symmetric gross profits (2.9), the free entry condition (2.21) becomes

\[
\frac{L}{2(2 + \sigma N)^2} = \frac{w \varepsilon u_k^\beta}{u_0^\beta} \quad (2.24)
\]
and using (2.16), the labour market clearing condition (2.22) can be written as

\[ L = L_y + (N + 1) e \left( \frac{u}{u_0} \right)^\beta \]  \hspace{1cm} (2.25)

Conditions (2.23-2.25) allow us to solve for \( u(.) \), \( N(.) \) and \( w(.) \), as follows. From (2.23) we can solve for \( u \) to obtain:

\[ u = \left[ \frac{L u_0^\beta}{w e^\beta (2 - \sigma)(2 + \sigma N)^2} \right]^{\frac{\rho - 3}{2}} \]  \hspace{1cm} (2.26)

To solve for \( N \) we substitute (2.26) into (2.24) and obtain

\[ N + 1 = \frac{(\beta - 2)(2 - \sigma)}{2\sigma} + 1 \]  \hspace{1cm} (2.27)

We now substitute \( N + 1 \) from (2.27) into \( u \) (equation 2.26) to obtain technological capability in terms of exogenous parameters \((\beta, \sigma, e, L, u_0)\) and the wage rate \( w \):

\[ u = \left[ \frac{u_0^\beta L}{w e^\beta (2 - \sigma)(1 - \frac{\beta}{2}) + \sigma} \right]^{\frac{\rho - 3}{2}} \]  \hspace{1cm} (2.28)

Next substitute \( N + 1 \) (from equation 2.27) and \( u \) (equation 2.28) into the labour market clearing condition (2.25) to obtain the equilibrium wage rate. Care needs to be taken in noting that if labour demand from industry \( X \) is insufficient to clear the labour market, industry \( Y \) will absorb any surplus labour. This implies that the wage rate will be equal to the marginal product of labour in industry \( Y \), which is equal to the price of good \( Y \), \( q = 1 \). Thus we have that if \( L_y = 0 \), \( w > 1 \) (otherwise workers would produce using the freely available 1:1 technology), and if \( L_y > 0 \), \( w = 1 \) (in which case they are effectively using such technology).

The equilibrium wage rate is given by

\[ w = \max \left\{ 1, \frac{u_0^\beta}{2} \left[ \frac{L}{e} \right]^\frac{\beta - 2}{2} \left[ \frac{(\beta - 2)(2 - \sigma)}{2\sigma} + 1 \right]^{\frac{\beta - 2}{\beta}} \right\} \]  \hspace{1cm} (2.29)

The solution for the wage rate is substituted into equation (2.28) to obtain the general equilibrium level of technological capability. This depends on whether \( L_y \) is strictly positive or not.

If \( L_y = 0 \) \( (w > 1) \), we have:

\[ u = u_0 \left[ \frac{L}{e} \left( \frac{1}{\frac{(\beta - 2)(2 - \sigma)}{2\sigma} + 1} \right) \right]^\frac{\rho}{\beta} \]  \hspace{1cm} (2.30)

If, on the contrary, \( L_y > 0 \) (such that \( w = 1 \)), then technological capability is obtained simply.
by setting \( w = 1 \) in (2.28):

\[
    u = \left( \frac{\omega_0 L}{2\varepsilon} \frac{1}{\beta (1 - \frac{\delta}{2}) + \sigma^2} \right) \frac{1}{\sigma^2}
\]

(2.31)

The general equilibrium number of firms is simply given by equation (2.27) and no further substitutions are necessary. This completes the characterization of a symmetric general equilibrium in the closed economy. The symmetric general equilibrium outcomes for the closed economy are summarized in the following table:

<table>
<thead>
<tr>
<th>a) If ( L_y = 0 ) (( w &gt; 1 ))</th>
<th>b) If ( L_y &gt; 0 ) (( w = 1 ))</th>
</tr>
</thead>
</table>
| \[
    w = \frac{\omega_0}{2} \left( \frac{L}{\varepsilon} \right)^{3} \frac{1}{(\beta(1-\frac{\delta}{2})+\sigma^2)} + 1
\]
| \[
    u = \omega_0 \left( \frac{L}{\varepsilon} \frac{1}{(\beta(1-\frac{\delta}{2})+\sigma^2)} + 1 \right)^{\frac{1}{3}}
\]
| \[
    N + 1 = \frac{(\beta-2)(2-\sigma)}{2\sigma} + 1
\] |

Table 1: Symmetric General Equilibrium for the Closed Economy.

An interesting feature of the results on market structure is that concentration depends only on \( \beta \) and \( \sigma \). Thus market size \( (L) \), initial conditions \( (u_0) \) and set-up costs \( (\varepsilon) \) do not affect concentration. This is a common feature of vertical product differentiation models, and is related to the 'non-convergence property' discussed in Sutton (1991, 1998) and Cabral (2002), that is, that concentration does not fall to zero as market size increases indefinitely.

**Two Fundamental Effects: Market Structure and Technological Capability**

In this model, there are two fundamental mechanisms which determine the demand for labour and thereby the wage rate. Firstly, we have the technological capability effect, which refers to how changes in technological capability affect the demand for labour at the level of the firm. If technological capability rises, this will increase \( f(u) \) and thus labour demand at the firm level. On the other hand we have the market structure effect, which refers to how industry-level labour demand is affected by market structure. As the number of firms rises, the demand for labour at the industry level will expand—there are more firms with labour requirement \( f(u) \).
The key notion is that the two effects go in opposite directions: As technological capability rises, fixed costs increase, and the market can accommodate fewer firms. Likewise, if technological capability falls, so do fixed costs and a larger number of entrants can survive. One of the central results then is the answer to the question: Which will generate higher labour demand (and higher wage rate), few firms each with high technological capability, or many firms each with low technological capability?

An Explanatory Note on Welfare Analysis

Since net profits are zero and we use a representative consumer, utility is a measure of welfare (denoted by $W$). This is obtained by substituting (symmetric) equilibrium solutions into the utility function (equation 2.1), as follows

$$W = (N + 1) \left[ x - (1 + 2\sigma N) \frac{x^2}{u^2} \right] + Y \quad (2.32)$$

For welfare analysis it is important to note that consumers' income is determined by the wage rate. As we change any given parameter to analyze the comparative statics around a symmetric general equilibrium, we will be interested in the associated welfare change. Welfare is determined by income, and hence by the wage rate. So, in order to obtain the welfare consequences of any given change in parameter values all we need to observe are the changes in the wage rate.

We are now in a position to analyze how the economy reacts to changes in parameter values. This is the task for the next section.

4 Analysis of the Symmetric Equilibrium

We consider how each of the parameters ($\beta, \sigma, \epsilon, L, u_0$) affects outcomes for a symmetric equilibrium. But first, let us summarize the variables to be examined. What follows is a list of the variables which were considered to be the most interesting, for which results are reported.

a) Wage rate (equal to per-capita value added $^6$): $w(.)$ (see table 1).

b) Technological capability: $u(.)$ (see table 1).

c) Number of firms: $N(.) + 1$ (see table 1).

d) Welfare: $W(.)$ (equation 2.32).

$^6$ For more details on the measurement of value added in this model, see Appendix 3.
e) Employment in industry $Y$: $L_Y = L - L_x$ (from equation 2.17).

f) Employment in industry $X$: $L_x (.)$ (equation 2.16).

We analyze how these variables change as parameters are varied. It is worth noting that the analysis we have carried out is an *equilibrium* analysis. As such, all the behavior we discuss refers to changes in the *equilibrium* values of the system as we change one of the exogenous parameters, while holding the other parameters constant (but not the equilibrium values of the variables, these vary according to the equilibrium solutions obtained in section 3).

While the model contains more variables than those presented in this section, these were deemed the most enlightening subset. For the interested reader, the analysis of all remaining variables is presented in Appendix 4.

**Notation**

We will use the notion of 'parameter thresholds' extensively in what follows, so a notational clarification is due. For each of the variables listed above we will analyze slope and concavity with respect to the parameters. However, due to the non-linearity of the system, we often find changes in the slope/concavity of a variable within the interval of change for the parameter at hand. To identify such changes, we have defined 'threshold' values of parameters at which the change occurs.

In the case of a change of slope, the threshold will usually be determined by an extreme point of the variable, and as such will be associated to a vanishing first derivative. For example, if we wish to define a threshold level of $\sigma$ defined by an extreme in $w$ (which will be labelled $\sigma^w$), we would use the condition $\frac{\partial w}{\partial w} \bigg|_{\sigma=\sigma^w} = 0$ to solve for $\sigma^w$.

For a change of concavity, we would use the presence of an inflection point to solve for the threshold value of the parameter. For example, if we wish to pinpoint a change in the concavity of $u$ with respect to $\sigma$, we would define a threshold level of $\sigma$ (call it $\sigma^u$), such that $\frac{\partial^2 u}{\partial \sigma^2} \bigg|_{\sigma=\sigma^u} = 0$, and solve for $\sigma^u$ from this condition.

In order to simplify notation, the notation for thresholds is consistent only within each proposition, not across propositions. If this notational simplification had not been put in place, each parameter threshold would have also required an extra index for the appropriate proposition. Since it will be clear from the context which parameter threshold we are referring to, we have opted for the simpler notation.

---

7 $\frac{\partial}{\partial w} \bigg|_{\sigma=\sigma^w}$ should be read as 'the partial derivative of $w$ with respect to $\sigma$ evaluated at $\sigma^w$.'
Finally, we use the notion of 'piecewise' concavity/convexity of a variable. This notion is useful in the case when a variable exhibits kinks in its trajectory. It refers to functions which are concave/convex within sections that lie between certain thresholds (the kinks), but not across sections. Thus we can have that a function is piecewise concave (when individually considered, all of its sections are concave), but not globally concave (due to the kinks, some sections considered jointly may not be concave, even though each section is concave when considered individually).

4.1 Analysis of Changes in $\beta$

To obtain some intuition of how $\beta$ affects the economy, we proceed to a detailed account of the different forces at work in the model. For this first parameter, we proceed at a leisurely pace to become familiar with the process at hand. $\beta$ is a convenient parameter to start with, since its analysis is relatively simple, and it allows us to introduce the main mechanisms.

Recall that $\beta$ is the elasticity of the firm's labour requirement in industry $X$, $f(u) = \varepsilon \left( \frac{u}{u_0} \right)^\beta$, with respect to technological capability, $u$. Labour requirement is constituted by the physical labour input needed to achieve a certain level of technological capability. $\beta$ is closely related to the marginal cost of technological capability, which is given by $\beta \frac{\varepsilon}{u} \left( \frac{u}{u_0} \right)^\beta$. The marginal cost is monotonically increasing in $\beta$, since $\frac{\varepsilon}{u_0} > 1$.

The following discussion is based on Figure 4.1, shown below. Continuous lines track the equilibrium outcomes of the variables, while dashed lines show the behaviour which would have prevailed had there not been a lower bound of 1 on the wage rate (we shall refer to these as 'shadow' values). The top left graph shows technological capability in terms of $\beta$, the top right graph depicts the wage rate in terms of $\beta$, the bottom left graph shows the number of firms as a function of $\beta$ and the bottom right graph depicts employment in industries $X$ and $Y$ as well as the total number of workers (the economy's labour endowment).
As $\beta$ increases, the marginal cost of technological capability rises. The effects of this are twofold. Firstly, investment in technological capability is reduced (this is the 'technological capability effect'). Secondly, lower investment in technological capability reduces the sunk costs required to enter the industry, leading to an increase in the equilibrium number of firms (the 'market structure effect'). These effects constitute the driving forces for some of the main results of the study. The technological capability effect reduces the demand for labour at the level of the firm ($f(u)$ falls). The market structure effect increases the demand for labour at the level of industry $X$ (there are now more firms demanding workers). The net effect on labour demand will determine the consequences for the wage rate. If the net effect is to increase labour demand (the market structure effect dominates the technological capability effect), then the wage rate will rise. We find that the net effect of increasing $\beta$ is to reduce labour demand, thereby reducing the wage rate (the 'technological capability' effect dominates the 'market structure' effect).

Parallel to the process just described is a process of structural change in the economy. As demand for labour in industry $X$ falls due to the increasing marginal cost of technological capability, any surplus labour must be absorbed by industry $Y$. As soon as the wage rate reaches a level equal to 1, industry $Y$ becomes active. Surplus workers turn to their outside option of using the 1:1 'traditional' technology, to earn their marginal product ($w = q = 1$, 49
since good $Y$ is the numeraire). Thus as the marginal cost of technological capability rises, more workers are made redundant in industry $X$ and employment in this industry contracts. As more workers move to industry $Y$, it expands its share in employment.

The process just described admits several interpretations. Industry $Y$ could be called the 'traditional', 'pre-modern' or 'informal' sector. The key property is that this industry features constant returns to scale (as opposed to industry $X$ which has increasing returns to scale), and offers little scope for expansion of the wage rate. Workers in industry $Y$ could be interpreted as 'covert unemployment', 'informal workers' or 'self employed workers'\(^8\).

The dual structure of the economy helps us consider the effects that different policies have on economic development. In particular, we can envisage two kinds of development configuration:

1) **High-tech**: The economy has few firms each with a high level of technological capability.

2) **Proliferation**: The economy has many firms, each with a low level of technological capability.

We ask the question: Which of these configurations generates a higher level of income (and therefore welfare)? We will see that the answer depends on parameter values, and this provides a testable prediction of the model. In Figure 4.1 we can see that the 'high-tech' configuration is associated with a higher wage rate, relative to the 'proliferation' configuration. However, this is not always the case, as we will see below.

To recap: as $\beta$ rises, $u$ falls at a decreasing rate and $N + 1$ rises linearly. Since the rise in $N + 1$ is insufficient to offset the fall in $u$, labour demand in industry $X$ contracts and $w$ falls, both at decreasing rates. Once the wage rate reaches a level of 1, industry $Y$ becomes active and expands its share in employment (at a decreasing rate), all the while industry $X$ is contracting. It is worth noting that at the level of $\beta$ for which the wage rate reaches the lower bound of 1 (namely, $\beta^{\text{wmax}} = 1$), technological capability begins to fall at a faster rate, thereby generating a kink in the graph of $u$ (and also in that of $w$). This occurs because the increase in marginal cost can no longer be offset by reducing wages: The wage rate had been acting as a buffer to accommodate (albeit partly) the increase in the marginal cost of technological capability. Once $w = 1$, the buffer is no longer available, and the effects of the increase in the marginal cost are borne by technological capability.

The results discussed above and others are stated in a more precise manner in the following proposition (which is continued in Appendix 4)\(^9\).

---

\(^8\) The model could have been simplified by dropping industry $Y$ from the framework and having a wage rate which would adjust to clear the labour market (and would not have a lower bound of 1). However having the dual structure offers interesting insights relating to the process of economic development.

\(^9\) In setting out this proposition and the subsequent ones, care needs to be taken in noting that some of the
Proposition \( \beta \): Effects of \( \beta \)

Recall that \( \beta \in (2, \infty) \).

**a)** Wage rate: Define a threshold level of \( \beta, \beta^{w=1} \), such that \( w = 1 \). The wage rate, \( w \), is strictly decreasing and strictly (piecewise) convex in \( \beta \) for \( \beta < \beta^{w=1} \): \( \frac{\partial w}{\partial \beta} < 0, \frac{\partial^2 w}{\partial \beta^2} > 0 \). For \( \beta \geq \beta^{w=1} \), we have \( w = 1 \).

**b)** Technological capability: \( u \) is strictly decreasing and strictly (piecewise) convex in \( \beta, \frac{\partial u}{\partial \beta} < 0, \frac{\partial^2 u}{\partial \beta^2} > 0 \). When \( \beta \geq \beta^{w=1} \), \( u \) falls at a higher rate, thereby generating a kink in the function.

**c)** Number of firms: \( N+1 \) is strictly increasing and linear in \( \beta, \frac{\partial (N+1)}{\partial \beta} > 0, \frac{\partial^2 (N+1)}{\partial \beta^2} = 0 \).

**d)** Welfare: \( W \) is strictly decreasing and strictly convex in \( \beta, \frac{\partial W}{\partial \beta} < 0, \frac{\partial^2 W}{\partial \beta^2} > 0 \). The behavior of welfare \( (W) \) tracks that of the wage rate, \( w \) (with different values). When \( \beta \geq \beta^{w=1} \), \( W \) falls at a smaller rate, generating a kink in \( W \). The reason why \( W \) becomes flatter is that once the wage rate becomes 1, the fall in welfare is mitigated by the fact that as \( \beta \) increases above \( \beta^{w=1} \) the wage cannot fall any further.

**e)** Employment in industry \( Y \): \( L_Y = 0 \) for \( \beta < \beta^{w=1} \), and it is strictly increasing and strictly concave for \( \beta \geq \beta^{w=1} \): \( \frac{\partial L_Y}{\partial \beta} > 0, \frac{\partial^2 L_Y}{\partial \beta^2} < 0 \).

**f)** Employment in industry \( X \): Recall that \( L_X = L - L_Y \), so its behavior is the opposite of \( L_Y \). Thus for \( \beta < \beta^{w=1} \), \( L_X = L \). For \( \beta \geq \beta^{w=1} \) \( L_X \) is strictly decreasing and strictly convex: \( \frac{\partial L_X}{\partial \beta} < 0, \frac{\partial^2 L_X}{\partial \beta^2} > 0 \).

**Proof:** By inspection of the corresponding variables, their derivatives and associated thresholds.

Following a similar approach to the analysis of \( \beta \), we now proceed to analyze the effects of varying \( \sigma \).

### 4.2 Analysis of Changes in \( \sigma \)

The substitutability of goods in industry \( X \) (horizontal product differentiation) is measured by \( \sigma \). The closer \( \sigma \) is to 1, the closer substitutes the goods are. In the limit, as \( \sigma \to 1 \), type \( X \) goods become perfect substitutes. Likewise, as \( \sigma \to 0 \), the goods become perfectly unsubstitutable.

variables will exhibit kinks. Accordingly, any derivatives will not be defined at the kinks.

To deal with this we have used the notion of piecewise concavity (convexity), introduced above. Thus statements about the (piecewise) concavity of a function refer to the parts of the function which lie (strictly) between the kinks.
We present a similar figure to that used in the previous section. In Figure 4.2, $\sigma$ is plotted on the horizontal axes. Continuous lines represent the symmetric equilibrium outcomes, and dashed lines show the projection of the corresponding outcome, had the wage rate not had a lower bound equal to 1 (the ‘shadow value’). The top left graph in Figure 4.2 depicts technological capability. The top right graph shows the wage rate. The bottom left graph illustrates the number of firms and the bottom right graph depicts employment in industries $X$ and $Y$.

![Figure 4.2: The effect of $\sigma$](image)

Technological capability is increasing in $\sigma$. To study the concavity of technological capability, consider two thresholds: $\sigma_{\text{low}}^{w=1}$ and $\sigma_{\text{high}}^{w=1}$. $\sigma_{\text{low}}^{w=1}$ is the lowest value of $\sigma$ at which the wage rate reaches its lower bound of 1. $\sigma_{\text{high}}^{w=1}$ is the highest value of $\sigma$ at which the wage rate is at its lower bound of 1. For $\sigma < \sigma_{\text{low}}^{w=1}$ and for $\sigma > \sigma_{\text{high}}^{w=1}$ (such that $w > 1$), we have that technological capability is concave in $\sigma$. For $\sigma_{\text{low}}^{w=1} \leq \sigma \leq \sigma_{\text{high}}^{w=1}$ (such that $w = 1$), technological capability is convex in $\sigma$. When $\sigma_{\text{low}}^{w=1} \leq \sigma \leq \sigma_{\text{high}}^{w=1}$ ($w = 1$) we find that technological capability is rising at an increasing rate as $\sigma$ grows, beginning with a flatter slope than was the case when $\sigma < \sigma_{\text{low}}^{w=1}$, (i.e., when $w > 1$). This is due to the wage being fixed at 1: As shown by the ‘shadow’ wage rate, when $\sigma$ increases past $\sigma_{\text{low}}^{w=1}$, had it not been for the lower bound of 1 the wage rate would have continued falling, would have reached a minimum and would have begun growing. The effect of this for technological capability is that when the wage would have continued falling...
(but was fixed at 1), the net marginal benefit of technological capability is rising at a relatively slower rate than its shadow counterpart. Thus technological capability rises at a slower pace than its corresponding shadow value. Once the wage rate reaches a minimum and begins rising, the process is reverted: The shadow wage rate rises faster than the actual wage \((w = 1)\), hence the actual net marginal benefit grows faster than its shadow value, leading to the steeper slope of actual technological capability, relative to its shadow value. Once \(\sigma\) grows past \(\sigma_{high}^{w=1}\), the wage rate becomes strictly greater than 1, and technological capability recovers its underlying (concave and rising) shape.

Meanwhile, the number of firms is decreasing at a decreasing rate in \(\sigma\). The intuition behind the pattern followed by technological capability and by the number of firms is as follows. As \(\sigma\) grows, goods become closer substitutes and the net marginal benefit of escalation\(^{10}\) grows, since any given firm can capture a larger share of the market by unilaterally raising its technological capability. Thus, in equilibrium, each firm finds it optimal to increase its investment in technological capability. In turn, this raises the investment required to survive in the industry, thereby reducing the number of entrants.

We now focus on the determination of the wage rate. So long as employment in industry \(X\) equals labour supply in the whole economy, the wage rate is strictly greater than 1. As soon as labour demand from industry \(X\) falls short of the economy’s labour endowment any surplus labour is employed by industry \(Y\), and this makes the wage rate constant at a value of 1 (which is the constant marginal product of labour in industry \(Y\)). The number of firms combines with technological capability to determine labour demand by industry \(X\), and hence the wage rate. At first we find that the decreasing number of firms is not offset by the increasing technological capability, resulting in a net reduction in the demand for labour by industry \(X\), and a decreasing wage rate. For higher values of \(\sigma\), the decrease in the number of firms is more than offset by the increase in technological capability, resulting in a higher labour demand by industry \(X\) and an increasing wage rate. The wage rate is thus ‘U’-shaped, and may reach its lower bound of 1 for intermediate values of \(\sigma\) \((\sigma_{low}^{w=1} \leq \sigma \leq \sigma_{high}^{w=1})\).

The intuition for the ‘U’-shape of the wage rate is as follows. On the one hand we have that when \(\sigma\) is relatively low, the number of firms is large, with each firm having relatively low technological capability. The effect on labour demand of the falling number of firms is greater than the effect of rising technological capability. Whence the wage rate falls with \(\sigma\) (the ‘market structure’ effect dominates the ‘technological capability’ effect). On the other hand, for

\(^{10}\) The term ‘escalation’ refers to the process whereby a firm increases its technological capability by increasing its investment, in order to outperform its rivals. See Sutton (1998, page 26).
relatively high values of \( \sigma \) the situation is reversed. We now have a small number of firms, each with a relatively high level of technological capability. It is now the increase in technological capability, rather than the fall in the number of firms, that drives the demand for labour from industry \( X \), and hence the wage rate becomes increasing (now it is the 'technological capability' effect that dominates the 'market structure' effect).

Employment exhibits the following pattern. For \( \sigma < \sigma_{low}^{w=1} \) and for \( \sigma > \sigma_{high}^{w=1} \), labour demand from industry \( X \) is sufficient to employ all of the labour force, therefore industry \( Y \) remains inactive (and the wage rate is strictly higher than 1). For \( \sigma_{low}^{w=1} \leq \sigma \leq \sigma_{high}^{w=1} \) labour demand from industry \( X \) is smaller than the economy's labour force, and thus some workers turn to industry \( Y \) for employment (and the wage rate is equal to 1). Within this range, employment in industry \( X \) is decreasing, reaches a minimum and then is increasing. This behaviour reflects the pattern followed by the shadow wage rate. Similarly, employment in industry \( X \) equals the economy's labour endowment minus employment in industry \( Y \), and consequently exhibits complementary behaviour to \( L_Y \): It is increasing, reaches a maximum and becomes decreasing.

It is worth noting (see table 1) that lower values of \( \beta \) or \( \varepsilon \), or higher values of \( L \) or \( u_0 \) will lead to higher wage rate and technological capability (and in the case of \( \beta \), higher concentration). Thus we can also picture a scenario in which the wage rate is overall higher and does not reach its lower bound of 1. It therefore does not have any kinks, exhibiting a pattern like that followed by the shadow wage. Similarly, in this scenario technological capability would be globally concave (following the pattern of the shadow technological capability), also without any kinks in its trajectory. This would imply that industry \( X \) remains the sole employer for any value of \( \sigma \in (0,1) \).

The results discussed above and others are stated more precisely in the following proposition (which is completed in Appendix 4).

**Proposition \( \sigma \): Effects of \( \sigma \)**

Recall that \( \sigma \in (0,1) \).

a) **Wage rate:**

If \( w > 1 \) for all \( \sigma \in (0,1) \) then \( w \) is 'U' shaped: As we increase \( \sigma \), \( w \) is strictly decreasing \( (\frac{\partial w}{\partial \sigma} < 0) \), reaches a global minimum \( (\frac{\partial w}{\partial \sigma} = 0) \) and becomes strictly increasing \( (\frac{\partial w}{\partial \sigma} > 0) \). It follows that \( w \) is strictly convex in \( \sigma \): \( \frac{\partial^2 w}{\partial \sigma^2} > 0 \).

Alternatively, if \( w = 1 \) for some \( \sigma \in (0,1) \), then define two thresholds: \( \sigma_{low}^{w=1} \) and \( \sigma_{high}^{w=1} \), such that for all \( \sigma \in [\sigma_{low}^{w=1}, \sigma_{high}^{w=1}] \), \( w = 1 \) obtains. Provided \( 0 < \sigma_{low}^{w=1} < \sigma_{high}^{w=1} < 1 \), for \( \sigma < \sigma_{low}^{w=1} \) \( w \)
is strictly decreasing and strictly convex in \( \sigma \left( \frac{\partial w}{\partial \sigma} < 0, \frac{\partial^2 w}{\partial \sigma^2} > 0 \right) \). For \( \sigma > \sigma_{\text{high}} \) \( w \) is strictly increasing and strictly convex in \( \sigma \left( \frac{\partial w}{\partial \sigma} > 0, \frac{\partial^2 w}{\partial \sigma^2} > 0 \right) \).

b) Technological capability: \( u \) is strictly increasing in \( \sigma \): \( \frac{\partial u}{\partial \sigma} > 0 \). For \( w > 1 \), we have that \( u \) is strictly concave in \( \sigma \): \( \frac{\partial^2 u}{\partial \sigma^2} < 0 \). When \( w = 1 \), \( u \) is strictly convex in \( \sigma \): \( \frac{\partial^2 u}{\partial \sigma^2} > 0 \). \( u \) is kinked at the points where \( w \) becomes 1.

c) Number of firms: \( N + 1 \) is strictly decreasing and strictly convex in \( \sigma \): \( \frac{\partial (N+1)}{\partial \sigma} < 0, \frac{\partial^2 (N+1)}{\partial \sigma^2} > 0 \).

d) Welfare: \( W \) exhibits similar behavior to the wage rate (with different numerical values). However, when \( w = 1 \), \( W \) becomes flatter, generating a kink in \( W \). This change of slope occurs for the same reason as outlined in part (d) of Proposition 1.

e) Employment in industry \( Y \): \( L_Y = 0 \) for \( w > 1 \). If \( w = 1 \) then \( L_Y > 0 \). In this case, \( L_Y \) is strictly concave: \( \frac{\partial^2 L_Y}{\partial \sigma^2} < 0 \) and \('\gamma'\) shaped: As we increase \( \sigma \), \( L_Y \) is strictly increasing \( \left( \frac{\partial L_Y}{\partial \sigma} > 0 \right) \), reaches a global maximum \( \left( \frac{\partial L_Y}{\partial \sigma} = 0 \right) \) and becomes strictly decreasing \( \left( \frac{\partial L_Y}{\partial \sigma} < 0 \right) \).

f) Employment in industry \( X \): \( L_X \) is equal to \( L - L_Y \). Thus its behavior is the opposite of \( L_Y \). \( L_Y \) will supplement \( L_X \) to use all of the available labour supply (given by \( L \)).

Proof: By inspection of the corresponding variables, their derivatives and associated thresholds.

4.3 Analysis of Changes in \( \varepsilon \)

\( \varepsilon \) can be interpreted as an exogenous set-up cost. The following figure presents the usual variables as a function of \( \varepsilon \). In Figure 4.3, we plot \( \varepsilon \) on the horizontal axes. Continuous lines represent the actual trajectory of the variables, while dashed thin lines show the 'shadow' value of the corresponding variable, had the wage rate not had a lower bound equal to 1. As before, the top left graph shows technological capability, the top right graph shows the wage rate, the bottom left graph shows the number of firms and the bottom right graph shows employment in industries \( X \) and \( Y \).

55
Technological capability is decreasing in $\varepsilon$, at a decreasing rate. When the wage reaches its lower bound of 1, technological capability exhibits a kink and after that it becomes steeper and falls again at a decreasing rate. Intuitively, as set-up costs rise, the marginal cost of technological capability also rises, thereby reducing technological capability. Each of the existing firms (the number of which does not change) chooses a lower level of technological capability, and consequently less work-hours are required. Thus the demand for labour falls in industry $X$, and so does the wage rate. At $\varepsilon^{w=1}$ the demand for labour in industry $X$ becomes smaller than the (fixed) labour supply, and some surplus labour appears. This will be absorbed by industry $Y$, where labour has a marginal product equal to 1. Therefore, the economy’s wage rate reaches its lower bound of 1.

The results discussed above are stated more precisely in the following proposition (continued in Appendix 4).

**Proposition $\varepsilon$: Effects of $\varepsilon$**

Recall that $\varepsilon \in (0, \infty)$.

a) **Wage rate:** Define a threshold $\varepsilon^{w=1}$ as the lowest value of $\varepsilon$ such that $w = 1$. For $\varepsilon < \varepsilon^{w=1}$, $w$ is strictly decreasing and strictly convex in $\varepsilon$: $\frac{\partial w}{\partial \varepsilon} < 0$, $\frac{\partial^2 w}{\partial \varepsilon^2} > 0$. For $\varepsilon \geq \varepsilon^{w=1}$, we have $w = 1$. 

56
b) Technological capability: \( u \) is strictly decreasing and strictly (piecewise) convex in \( \varepsilon : \frac{\partial u}{\partial \varepsilon} < 0, \frac{\partial^2 u}{\partial \varepsilon^2} > 0 \). When \( w \) becomes equal to 1 at \( \varepsilon = \varepsilon^{w=1} \), \( u \) falls at a higher rate, thereby generating a kink.

c) Number of firms: \( N + 1 \) is invariant with respect to \( \varepsilon \).

d) Welfare: \( W \) is strictly decreasing and strictly (piecewise) convex in \( \varepsilon : \frac{\partial W}{\partial \varepsilon} < 0, \frac{\partial^2 W}{\partial \varepsilon^2} > 0 \). \( W \) exhibits a kink at \( \varepsilon = \varepsilon^{w=1} \). This change of slope occurs for the same reason as outlined in part (d) of Proposition \( \beta \).

e) Employment in industry \( Y \): \( L_y = 0 \) for \( \varepsilon < \varepsilon^{w=1} \), otherwise it is strictly increasing and strictly concave: \( \frac{\partial L_y}{\partial \varepsilon} > 0, \frac{\partial^2 L_y}{\partial \varepsilon^2} < 0 \).

f) Employment in industry \( X \): \( L_x = L - L_y \), so its behavior supplements that of \( L_y \).

Proof: By inspection of the corresponding variables, their derivatives and associated thresholds.

4.4 Analysis of Changes in \( u_0 \)

The effect of initial or inherited technological capability (\( u_0 \)) is similar to the reciprocal of the set-up cost \((1/\varepsilon)\). Both parameters affect marginal cost by shifting the fixed outlays function. \( u_0 \) captures the notion of historical disparity in initial conditions. This issue will become central in subsequent chapters, when we introduce international trade. We consider a narrow definition and refer to \( u_0 \) as initial or inherited technological capability, but (similarly to \( u \)), it admits a more general interpretation. Figure 4.4 presents the corresponding graphs. We plot \( u_0 \) on the horizontal axes. Continuous lines represent the actual trajectory of the variables, while dashed thin lines show the 'shadow' value of the corresponding variable, had the wage rate not had a lower bound equal to 1. As before, the top left graph shows technological capability, the top right graph shows the wage rate, the bottom left graph shows the number of firms and the bottom right graph shows employment in industries \( X \) and \( Y \).
For $w = 1$ ($u_0 \leq u_0^{w=1}$) technological capability ($u$) increases at an increasing rate with $u_0$. For $w > 1$ ($u_0 > u_0^{w=1}$), technological capability is still increasing in $u_0$, but at a constant rate. The number of firms is invariant with respect to $u_0$. Thus, to deduce the pattern followed by the wage rate, we need only consider the ‘technological capability’ effect (the ‘market structure effect’ is zero). For sufficiently low values of initial technological capability, the wage is equal to 1, and as $u_0$ grows past $u_0^{w=1}$, the wage begins to rise at an increasing rate. Correspondingly, as technological capability increases, employment in industry $X$ rises, eventually occupying all of the labour force. Meanwhile, employment in industry $Y$ falls, and ultimately industry $Y$ becomes inactive.

The mechanisms at work are the familiar ones. When the wage rate is fixed at 1, technological capability can increase convexly, since the expansion of employment in industry $X$ that this entails is not hindered by rising wages. This can be sustained so long as there is a positive share of the labour force employed in industry $Y$. As soon as there are no more workers left in industry $Y$ on which to draw upon for the expansion of technological capability, wages begin to rise, and it is the wage rate that now rises convexly, while technological capability rises linearly.

The results discussed above and others are stated more precisely in the following proposition (completed in Appendix 4).
Proposition \( u_0 \): Effects of \( u_0 \)

Recall that \( u_0 \in [1, \infty) \).

a) Wage rate: Define a threshold \( u_0^{w=1} \) as the highest value of \( u_0 \) such that \( w = 1 \). For \( u_0 > u_0^{w=1} \), \( w \) is strictly increasing and strictly convex in \( u_0 \): \( \frac{\partial w}{\partial u_0} > 0, \frac{\partial^2 w}{\partial u_0^2} > 0 \). For \( u_0 \leq u_0^{w=1} \), we have \( w = 1 \).

b) Technological capability: For \( u \leq u_0^{w=1} \), \( u \) is strictly increasing and strictly convex in \( u_0 \): \( \frac{\partial u}{\partial u_0} > 0, \frac{\partial^2 u}{\partial u_0^2} > 0 \). For \( u > u_0^{w=1} \), \( u \) is strictly increasing and linear in \( u_0 \): \( \frac{\partial u}{\partial u_0} > 0, \frac{\partial^2 u}{\partial u_0^2} = 0 \).

c) Number of firms: \( N + 1 \) is invariant with respect to \( u_0 \).

d) Welfare: \( W \) is strictly increasing and strictly (piecewise) convex in \( u_0 \): \( \frac{\partial W}{\partial u_0} > 0, \frac{\partial^2 W}{\partial u_0^2} > 0 \). \( W \) exhibits a kink at \( u = u_0^{w=1} \). This change of slope occurs for the same reason as outlined in part (d) of Proposition \( u_0 \).

e) Employment in industry \( Y \): \( L_y = 0 \) for \( u_0 > u_0^{w=1} \), and it is strictly decreasing and strictly concave for \( u_0 \leq u_0^{w=1} \): \( \frac{\partial L_y}{\partial u_0} < 0, \frac{\partial^2 L_y}{\partial u_0^2} < 0 \).

f) Employment in industry \( X \): \( L_x = L - L_y \), so its behavior mirrors that of \( L_y \). Thus for \( u_0 > u_0^{w=1} \), \( L_x = L \) and for \( u_0 \leq u_0^{w=1} \), \( L_x \) is strictly increasing and strictly convex in \( u_0 \): \( \frac{\partial L_x}{\partial u_0} > 0, \frac{\partial^2 L_x}{\partial u_0^2} > 0 \).

Proof: By inspection of the corresponding variables, their derivatives and the associated threshold.

4.5 Analysis of Changes in \( L \)

\( L \) represents the size of the labour force as well as population, and it could also be interpreted as market size. In this section we assume that \( L \) changes continuously, ignoring integer effects.

In analyzing the effects of population change, there are three cases to consider:

(a) \( \beta < 3 \) (the marginal cost of increasing technological capability is low, i.e., \( \beta \) is 'low').

(b) \( \beta > 3 \) (the marginal cost of increasing technological capability is high, i.e., \( \beta \) is 'high').

(c) \( \beta = 3 \).

Figure 4.5a presents the usual graphs for case (a). The top left graph depicts technological capability, the top right graph shows the wage rate, the bottom left graph plots the number of firms, and the bottom right graph exhibits employment in industries \( X \) and \( Y \). As before, the shadow value of a variable, had the wage rate not had a lower bound of 1, is shown as a thin dashed line.
Provided the wage rate is greater than 1 (such that $L > L_{\beta<3}^{w=1}$, where the threshold $L_{\beta<3}^{w=1}$ is defined as the highest value of $L$ at which $w = 1$), increasing the size of the labour force increases technological capability at a decreasing rate. When $L \leq L_{\beta<3}^{w=1}$, the increase in technological capability occurs at an increasing rate and lies beneath its shadow value. This is due to the fact that the wage rate is not rising, which allows technological capability to increase at a faster rate than would be the case had the wage rate been growing.

The number of firms is constant with respect to population size. Thus the ‘market structure’ effect is zero and it is the ‘technological capability’ effect that determines the wage rate. Provided $L > L_{\beta<3}^{w=1}$, the wage rate increases at a decreasing rate as population grows. For such values of $L$ all of the labour force is employed by industry $X$, while industry $Y$ is quiescent. For smaller population size ($L \leq L_{\beta<3}^{w=1}$), the demand for labour from industry $X$ is insufficient to employ all of the work force. Thus, some workers will be absorbed by industry $Y$, where labour has a marginal product equal to 1, and so the wage rate becomes 1.

Regarding the labour market, the increase in technological capability associated with increasing population leads to higher employment in industry $X$, while the ‘market structure’ effect is null. For $L < L_{\beta<3}^{w=1}$, employment in industry $X$ is increasing in population at an increasing rate. When $L \geq L_{\beta<3}^{w=1}$ we have $L_X = L$: Industry $X$ employs all of the labour force (population), and industry $Y$ is no longer active.
Employment in industry $Y$ is obtained as a residual: Any workers not employed by industry $X$ are absorbed by industry $Y$. Thus for $L \geq L^{w=1}_{\beta<3}$, employment in industry $Y$ is zero. For $L < L^{w=1}_{\beta<3}$, employment in industry $Y$ is 'Y'-shaped, increasing in $L$ at first, reaches a maximum, and then is decreasing in $L$ at an increasing rate. To see why this is so, note that since industry $Y$ uses a simple 1:1 technology, its labour employment is equal to aggregate supply of good $Y$ (see equation 2.19). In equilibrium this matches aggregate demand (equation 2.18). In turn, aggregate demand is per-capita demand for good $Y$ (equation 2.5) multiplied by population size. Per-capita demand is the residual income unspent on goods of type $X$, and expenditure on type $X$ goods is equal to profit, which is equal to fixed investment in technological capability (recall that such payments take the form of wages). Thus, per-capita demand for good $Y$ is eventually dependent on technological capability: Higher technological capability increases the demand for goods of type $X$, which reduces the (residual) demand for goods of type $Y$. As we can see from Figure 4.5a, technological capability is increasing in population size. This means that per-capita demand for good $Y$ is decreasing in population.

The change in aggregate demand for good $Y$ is the outcome of two opposing forces, to which we now turn. On the one hand, we have that per-capita demand is falling with population size: As the labour force expands, technological capability expands, while the number of firms is fixed. This increases the expenditure on goods of type $X$, and reduces the (residual) expenditure on good $Y$ (we shall label this the 'industry $X$ crowding-out' effect).

On the other hand, population itself is growing, and thus although per-capita demand of good $Y$ may be falling, the population increase could account for an increase in the aggregate demand of good $Y$ (we shall label this the 'population growth' effect).

From Figure 4.5a, we can see that at first the 'population growth' effect dominates the 'industry $X$ crowding-out' effect, leading to increasing employment in industry $Y$. However, for higher levels of technological capability (and population), the 'industry $X$ crowding-out' effect dominates, thereby leading to a decrease of employment in industry $Y$. Eventually, all of the labour force is employed by industry $X$, and industry $Y$ ceases to be active.

We now turn to case (b), $\beta > 3$ ($\beta$ is high). The corresponding graphs are presented in Figure 4.5b.
In case (b) the marginal cost of technological capability is higher than in case (a). The wage rate exhibits similar behaviour to case (a), although it is overall lower, since this case is associated with a higher $\beta$.

Technological capability is increasing at a decreasing rate. For $L < L_{\beta>3}^{w=1}$, technological capability lies below its shadow value: Since the wage rate is fixed at 1, technological capability is more costly relative to its shadow value. By contrast, when $\beta < 3$ - case (a), $\beta$ is low-, the corresponding population range ($L < L_{\beta<3}^{w=1}$) has technological capability increasing convexly. This difference arises because $\beta > 3$ (high $\beta$) implies a higher marginal cost of technological capability, thus, for a given population increase, technological capability rises at a slower rate.

The number of firms is constant and overall higher than in case (a). Regarding employment, we can see that employment in industry $Y$ is "\cap"-shaped. Employment in industry $X$ is always increasing in population. At first it increases less than proportionately with $L$, then it increases more than proportionally with $L$, eventually catching up with the 45° line. At this point, industry $Y$ becomes inactive and the wage rate becomes strictly greater than 1. The mechanisms behind this process are similar to those described in case (a).

For comparative purposes, Figure 4.5c superimposes Figures 4.5a and 4.5b. Continuous lines represent case (a), $\beta < 3$ (low $\beta$), while dotted lines represent case (b), $\beta > 3$ (high $\beta$).
It is clear from Figure 4.5c that when technological capability and the wage rate are above their lower bounds, they are higher in case (a) — when $\beta$ is low ($\beta < 3$). Correspondingly, the number of firms is higher when $\beta$ is high ($\beta > 3$). Employment in industry $X$ reaches the $45^\circ$ line sooner when $\beta$ is low ($\beta < 3$).

It is interesting to highlight the point where the shadow wage rates cross, $w_{\beta > 3} = w_{\beta < 3}$. At this point, employment levels in industries $X$ and $Y$ also cross their corresponding counterpart when $\beta$ is high/low.

Case (c), $\beta = 3$, features levels of technological capability, number of firms, wage rate and employment which (not surprisingly) lie between those of case (a) and case (b), and follow a similar pattern. The only difference is that when $w = 1$ technological capability is linear in $L$. Since it does not add any new major insights, we have not presented a separate figure for case (c).

The results discussed above and others are summarized in the following proposition (completed in Appendix 4).

**Proposition L: Effects of $L$**

Recall that $L \in (0, \infty)$.

a) **Wage rate:** Define a threshold $L_w = 1$ as the highest value of $L$ such that $w = 1$. For
$L > L^{w=1}$, $w$ is strictly increasing and strictly concave in $L$: $\frac{\partial w}{\partial L} > 0$, $\frac{\partial^2 w}{\partial L^2} < 0$. For $L \leq L^{w=1}$, we have $w = 1$.

b) Technological capability: For $L \geq L^{w=1}$, $u$ is strictly increasing and strictly concave in $L$: $\frac{\partial u}{\partial L} > 0$, $\frac{\partial^2 u}{\partial L^2} < 0$. For $L < L^{w=1}$, we consider three cases:

(a) For $\beta < 3$ $u$ is strictly increasing and strictly convex in $L$: $\frac{\partial u}{\partial L} > 0$, $\frac{\partial^2 u}{\partial L^2} > 0$.

(b) For $\beta > 3$ $u$ is strictly increasing and strictly concave in $L$: $\frac{\partial u}{\partial L} > 0$, $\frac{\partial^2 u}{\partial L^2} < 0$.

(c) For $\beta = 3$ $u$ is strictly increasing and linear in $L$: $\frac{\partial u}{\partial L} > 0$, $\frac{\partial^2 u}{\partial L^2} = 0$.

$c)$ Number of firms: $N + 1$ is constant with respect to $L$, $\frac{\partial N + 1}{\partial L} = 0$.

d) Welfare: $W$ is strictly increasing and strictly (piecewise) concave in $L$: $\frac{\partial W}{\partial L} > 0$, $\frac{\partial^2 W}{\partial L^2} < 0$. $W$ exhibits behavior similar to the wage rate, $w$ (with different numerical values). $W$ exhibits a kink at $L = L^{w=1}$, and is flatter in the region where $w = 1$. This change of slope occurs for the same reason as outlined in part (d) of Proposition $\beta$.

$e)$ Employment in industry $Y$: For $L > L^{w=1}$, $L_y = 0$. For $L \leq L^{w=1}$, $L_y$ is \('U\)-shaped. It is strictly increasing at first ($\frac{\partial L_y}{\partial L} > 0$), reaches a maximum ($\frac{\partial L_y}{\partial L} = 0$) and becomes strictly decreasing afterwards ($\frac{\partial L_y}{\partial L} < 0$). $L_y$ is strictly concave: $\frac{\partial^2 L_y}{\partial L^2} < 0$ for $L \leq L^{w=1}$.

e) Employment in industry $X$: $L_x = L - L_y$, so its behavior supplements that of $L_y$. Thus for $L > L^{w=1}$, $L_x = L$. For $L \leq L^{w=1}$ $L_x$ is strictly increasing and strictly convex: $\frac{\partial L_x}{\partial L} > 0$, $\frac{\partial^2 L_x}{\partial L^2} > 0$.

Proof: By inspection of the corresponding variables, their derivatives and associated thresholds.

This completes the analysis of how the economy changes as we vary each of the parameters. We are now ready to summarize some of the most important results of the model.

5 Remarks Concerning General Features of the Model

One of the most important issues that this framework allows us to address is the following: Under which circumstances (i.e., parameter values) will having few firms each with a high level of technological capability (a 'high-tech' economy) generate a higher wage rate (and hence higher welfare) than having many firms each with a low level of technological capability (a 'proliferation' economy)? If a policy maker could control parameter values in order to change the structure of the economy towards a 'high-tech' configuration or towards a 'proliferation' configuration, which should be chosen?
These are fundamental questions since there is a trade-off for labour demand when choosing whether to opt for few firms with high technological capability or for many firms each with low technological capability. In the first case, labour demand is reduced by having fewer firms, but it is increased by the high level of technological capability that each firm achieves. In the latter case, labour demand is enhanced by the presence of many firms, but it is reduced by the low technological capability that each firm exhibits. It is not clear \textit{a-priori} which leads to the higher level of income and welfare.

The general feature that we have in mind relates to the structure of the demand for (scarce) resources in the economy. As the demand for resources rises, so will their price. Accordingly, the owners of the resources will enjoy a rise in income and welfare. So the problem of development reduces to one of finding the conditions under which the demand for resources is highest. In an oligopoly setting with endogenous technological capability and market structure, an increase in the demand for resources by each firm in the form of fixed costs (increases in technological capability) will lead to reduced entry, and this will curtail the overall increase in the demand for resources. This model, and the extensions that follow in subsequent chapters, provide a framework which allows us to pinpoint exactly the conditions under which each effect dominates the other (the market structure effect versus the technological capability effect). For the closed economy case, these are summarized in the following proposition.

\textbf{Proposition 1: Development Configurations in Autarky}

A 'high-tech' configuration is associated with a higher wage rate (and welfare) than a 'proliferation' configuration, unless $\sigma$ is low.

\textbf{Proof:} By inspection of the equilibrium solution functions for technological capability, the number of firms and the wage rate, shown in table I.

This proposition follows directly from the analysis in section 4: For all parameters, except $\sigma$, it is always the case that parameter values which generate high technological capability and high concentration will be associated with a higher wage rate than parameter values which generate low technological capability and low concentration.

The exception is when $\sigma$ is low. In this case goods are poor substitutes and we observe an economy with many firms each with a low level of technological capability. This proliferation of technological trajectories is associated with a higher wage rate and welfare.

The next set of questions which this framework allows us to address relates to the relationship between the parameters of the model and economic policy. We can envisage a policy maker who designs policies to change some of these parameters. On the one hand, innovation
or industrial policies can be related to changes in $\beta$ (reducing the marginal costs of technological capability, e.g., support to intellectual property, subsidies to R&D), $\epsilon$ (subsidies to set-up costs, e.g., subsidized loans for purchasing capital equipment) and $u_0$ (policies to improve the firm's initial level of technological capability, e.g., public training institutes). On the other hand, demographic and migration policy can address population size ($L$). The extent of horizontal product differentiation (product substitutability), given by $\sigma$, seems harder to change, although Motta and Polo (1998) have endogenized its choice in framework related to ours.

Extension: A Replication Argument

The model presented is constituted by a single oligopolistic industry. This feature is an extreme simplification of a real economy. Nonetheless, the mechanisms at work readily extend to the multiple industry case. Consider an economy composed by a traditional sector like the one we have described, and multiple industries similar to sector $X$ (indexed by $r = 1, ..., R$). Let the products of these industries be non-substitutable with the products of other industries (although each firm's product is substitutable with those of other firms within the same industry). Assume further that there are no supply-side linkages between industries (i.e., there are no intermediate goods). The absence of demand and supply linkages between industries implies that they are only linked via the labour market. Each of these industries will be characterized by a set of parameters ($\beta_r, \sigma_r, \epsilon_r, u_{0r}, L$)$^{11}$, which vary according to the characteristics of each industry. Then we can consider the net effects on the demand for labour of having different mixes or proportions of industries with high or low technological capability (respectively, concentration). The reasoning is straightforward and extends Proposition 1 to the multiple industry case: Having a high proportion of high-tech industries will result in a higher wage rate than a high proportion of proliferation industries, with the exception of those industries in which $\sigma_r$ is low. In the latter case, proliferation will constitute an effective way of expanding the demand for labour (and the wage rate).

6 Concluding Remarks

In this chapter we characterized a general equilibrium model which allows us to assess the impact of the marginal cost of technological capability (related to $\beta$), the substitutability between type $X$ products ($\sigma$), set-up costs ($\epsilon$), population size ($L$) and initial technological capability ($u_0$), on variables such as the wage rate ($w$), technological capability ($u$), market

---

$^{11}$Population size ($L$) is assumed to be an economy-wide variable, and so does not vary between industries.
structure \((N + 1)\), welfare \((W)\), and the structure of the economy (i.e., how employment is distributed between the modern and the traditional industries: \(L_x\) and \(L_y\), respectively). The findings are summarized in Propositions \(\beta-L\) and Proposition 1.

One of the contributions of this chapter is to introduce strategic choice of technological capability (innovation) into a general equilibrium setting. This provides the opportunity of looking at the interaction between the microfoundations of oligopolistic interaction models, as is the case with the innovation/technological capability literature (Brander and Spencer, 1983; Dixit, 1988b and Sutton, 1998); and economy-wide variables, such as the wage rate and the structure of the economy, which have been the usual object of attention in the development economics and international trade literature (Ethier, 1982 and Venables, 1985 and 1996).

The model we have developed will allows us, in the next chapter, to carry out an open-economy analysis with oligopolistic interactions. One of the advantages of this framework is that we can analyze the strategic interaction behind technological capability or innovation using stage games. This will allow us to place strategic interaction in the spotlight, whilst keeping the model tractable.

The mechanism whereby higher technological capability can affect the standard of living in this framework is not via some form of externality (be they across sectors—as in Romer, 1990—or over time—as in Aghion and Howitt, 1992). Rather, the view that emerges is that escalation by any firm (in the sense of this firm investing in higher technological capability) may affect living standards via strategic interaction (oligopolistic competition). This result emerges from the vertical differentiation of the goods being considered: As a firm raises its technological capability, its competitors must match this move or else exit the market. In doing so, firms collectively raise the level of fixed costs associated with surviving in the industry, which are associated with higher concentration. The net effect on the demand for labour will depend on which of two effects dominates. Firstly we have the 'technological capability effect', which raises the demand for labour and hence the wage rate (as well as welfare). Secondly, we have the 'market structure effect', which reduces the demand for labour and hence the wage rate.

In particular, Proposition 1 allows us to address the industrial policy issue of whether development based on small and medium enterprises (SME's) or large enterprises should be preferred. We have found that proliferation (small and medium enterprises) will result in a higher wage rate only if industry \(X\) features a low degree of product substitutability \((\sigma)\). For all other parameter values large enterprises with high technological capability will lead to higher demand for labour. Of course, the idea of having large enterprises is justified subject to the condition that they actually do achieve high technological capability. Otherwise we
would be in the worst possible scenario, where we have the disadvantages of a concentrated market structure together with low technological capability, and the associated low demand for labour, low wage rate and low welfare. However, *ceteris paribus*, having a concentrated market structure with low technological capability is not a Subgame Perfect Nash equilibrium, so this outcome could only arise as a result of exogenously imposed constraints on the system, such as entry restrictions or other types of regulations.
Appendix 1: Solving the Final Stage Subgame for Industry X (Cournot Competition)

In this Appendix, we solve the final stage subgame (Cournot competition), in order to obtain a 'solved-out payoff' function. Recall the first order conditions for this stage of the game (equation 2.7):

\[ p_k + \frac{\partial p_k}{\partial x_k} x_k = 0 \quad \text{for} \quad k = 1, ..., n + 1 \]

Let us begin by substituting the inverse demand function (2.3) and its derivative \( \frac{\partial p_k}{\partial x_k} \) into the first order condition to obtain

\[ 1 - \frac{4x_k}{u_k^2} - \frac{2\sigma}{u_k} \sum_{i \neq k} x_i = 0 \quad (A1.1) \]

Adding and subtracting \( \frac{2\sigma}{u_k} x_k \) and re-organizing equation (A1.1), we arrive at

\[ \frac{x_k}{u_k} = \frac{u_k - \frac{2\sigma}{u_k} \sum_{i=1}^{N+1} x_i}{2(2 - \sigma)} \quad (A1.2) \]

The next step is to sum expression (A1.2) over \( k \), and solve for \( \sum_{i=1}^{N+1} x_i \), to obtain

\[ \sum_{i=1}^{N+1} x_i = \frac{\sum_{i=1}^{N+1} u_i}{2(2 + \sigma N)} \quad (A1.3) \]

Expression (A1.3) is substituted back into equation (A1.2), to yield the following solution for \( x_k \)

\[ x_k = \frac{u_k^2}{2(2 - \sigma)} \left( 1 - \frac{\sigma}{2 + \sigma N} \sum_{i=1}^{N+1} \frac{u_i}{u_k} \right) \quad (A1.4) \]

By imposing symmetry between firms (such that \( u_i = u_k \)), we obtain the following simplification

\[ x = \frac{u^2}{2(2 + \sigma N)} \quad (A1.4') \]

To solve for the price \( (p_k) \), take the inverse demand function (equation 2.3), add and subtract \( 2\sigma x_k^2 \), to obtain

\[ p_k = 1 - 2(1 - \sigma) \frac{x_k}{u_k^2} - \frac{2\sigma}{u_k} \sum_{i=1}^{N+1} \frac{x_i}{u_i} \quad (A1.5) \]
Now substitute $x_k$ from (A1.4) and the expression in (A1.3) into $p_k$ (A1.5). This yields the solution for $p_k$:

$$p_k = \frac{1}{2 - \sigma} \left(1 - \frac{\sigma}{2 + \sigma N} \sum_{i=1}^{N+1} u_i \right)$$  \hspace{1cm} (A1.6)

By imposing symmetry between firms (such that $u_i = u_k$), we obtain the following simplified solution

$$p = \frac{1}{2 + \sigma N}$$ \hspace{1cm} (A1.6')

The solved-out payoff is given by the product of equations (A1.4) and (A1.6). This yields

$$\pi_k = \frac{1}{2(2 - \sigma)^2} \left( u_k - \frac{\sigma}{2 + \sigma N} \sum_{i=1}^{N+1} u_i \right)^2$$ \hspace{1cm} (A1.7)

which is equation (2.8) in the text.

Appendix 2: Second order conditions for the Second Stage Subgame

The second order conditions are obtained by differentiating the first order conditions (2.11) with respect to $u_k$. We obtain the following

$$L \frac{\partial^2 \pi_k}{\partial u_k^2} \leq \omega e^\beta (\beta - 1) \frac{u_k^{\beta-2}}{u_0^\beta}$$ \hspace{1cm} (A2.1)

where

$$\frac{\partial^2 \pi_k}{\partial u_k^2} = \frac{1}{(2 - \sigma)^2} \left[ \frac{2 + \sigma (N - 1)}{2 + \sigma N} \right]^2$$ \hspace{1cm} (A2.2)

Substituting equations (A2.2), (2.28) and (2.27) into the second order condition (A2.1), this simplifies to $\beta \geq 2$. In section 2.2.2 we assume $\beta > 2$ to ensure we attain a maximum.

Appendix 3: Value Added

Since the only production factor in the economy is labour, there are no intermediate goods and profits are zero in all sectors, total value added is identical to the wage rate. This can be divided into value added generated by industry $X$ and by industry $Y$. As usual, there are three ways to measure value added: Using production, income and expenditure. All three approaches lead to the same result for per-capita value added. We shall discuss each in turn.

On the production side of sector $X$, value added is simply the total economy-wide revenue from good $X$, which is given by $L(N + 1)p_x$, since there are no intermediate goods. Upon
dividing by \( L \) we obtain per-capita value added (measured on the production side):

\[
VAP_x = VA_x = (N + 1)px
\]  

(A3.1)

On the income side, value added in sector \( X \) is given exclusively by workers' income \( VAI_x = wL_x \), since labour is the only factor of production. In turn employment in sector \( X \) \( (L_x) \) is equal to \( (N + 1)f(u) \) (where \( f(u) \) was defined in section 2.2.2). The zero profit condition in stage 1 (equation 2.14) can be written as \( L\pi = w f(u) \), so \( f(u) = L\pi/w \). Recall from equation (2.6) that \( \pi = px \), substituting this into \( VAI_x \), we obtain the same result as above: \( VAI_x = VA_x = (N + 1)px \).

On the expenditure side, value added in sector \( X \) is constituted by consumption expenditure of good \( X \), since there is no investment or international trade in this economy. Consumption expenditure of good \( X \) is equal to \( L(N + 1)px \), and dividing by population \( (L) \) we obtain the same result previously found in per-capita terms: \( VAEX = VA_x = (N + 1)px \).

On the production side of sector \( Y \), \( VAP_Y = q(L - L_x) \). Noting that \( q = 1 \), we have that value added in sector \( Y \) is simply the supply of \( Y \) goods, which (given the assumption of 1:1 technology) is the same as employment in sector \( Y \) \( (L_y) \). In per capita terms:

\[
VAP_Y = VA_y = (L - L_x)/L = L_y/L
\]  

(A3.2)

On the income side, value added in \( Y \) is equal to the wage bill in the sector, \( VAI_y = wL_y \). However, recall from section 2.3 that whenever \( L_y > 0 \), the wage rate will be 1. Hence we obtain the same result as with the production side: \( VAI_y = VA_y = L_y/L \).

On the expenditure side, value added is composed by consumption of good \( Y \), which is defined from equation (2.5) as \( L[w - (N + 1)px] \). We now use the zero profit condition \( L\pi = Lpx = w f(u) \) to write value added as \( Lw - (N + 1)w f(u) \). By noting again that if \( Y \) is produced at all, we must have \( w = 1 \), this can be written as \( L - (N + 1)f(u) \). Now \( (N + 1)f(u) \) is none other than \( L_x \), and so we arrive at the same (per-capita) value added as in the production and income side: \( VAEX = VA_y = L_y/L \).

Total (per-capita) value added is the addition of value added for both sectors: \( VA = VA_x + VA_y \).
Appendix 4: Analysis of Other Variables of Interest

For brevity, we decided to discuss only a subset of the economy’s variables in the main body of the chapter. Those variables were deemed the most enlightening in terms of describing the workings of the economy. However, other variables may also be of interest. We provide here a complete analysis of the remaining variables, continuing with the same format as in Section 4.

The remaining variables are:

- **g) Price of good** $X$: $p = \frac{1}{1+\sigma N}$ (equation A1.6', in Appendix 1).
- **h) Per-firm output of good** $X$ **(per-capita)**: $x = \frac{\eta^2}{2(1+\sigma N)}$ (equation A1.4', in Appendix 1).
- **i) Industry output of good** $X$ **(per-capita)**: $X = (N+1)x$.
- **j) Economy output of good** $X$: $X = L X$.
- **k) Demand for Good** $Y$ **(per-capita)**: $Y = w - (N+1)px$ (equation 2.5).
- **l) Aggregate Demand for Good** $Y$: $Y^D = w - (N+1)x \left( \frac{\eta}{u_x} \right)^{\beta}$ (equation 2.18).
- **m) Aggregate Supply of Good** $Y$: $Y^S \equiv L_y = L - (N+1)x \left( \frac{\eta}{u_x} \right)^{\beta}$ (equation 2.19).
- **n) Value added in industry** $X$ **(per-capita)**: $VA_x = (N+1)px$ (equation ??).
- **o) Value added in industry** $Y$ **(per-capita)**: $VA_y = L_y/L$ (equation A3.2). Note that in equilibrium, this is the same as physical per-capita production of good $Y$. To see this note that good $Y$ is the numeraire (its price has been normalized to $q = 1$) and sector $Y$ is assumed to use a 1:1 technology. Thus $VA_y = Y$.

We proceed to complete the corresponding propositions.

**Proposition $\beta$: Effects of $\beta$ (continued)**

- **g) Price of good** $X$: $p$ is strictly decreasing and strictly convex in $\beta$: $\frac{\partial \beta}{\partial \beta} < 0$, $\frac{\partial^2 \beta}{\partial \beta^2} > 0$.
- **h) Per-firm output of good** $X$ **(per-capita)**: $x$ is strictly decreasing and strictly convex in $\beta$: $\frac{\partial \beta}{\partial \beta} < 0$, $\frac{\partial^2 \beta}{\partial \beta^2} > 0$. When $\beta \geq \beta^{w=1}$, $x$ becomes steeper, generating a kink at $\beta = \beta^{w=1}$.
- **i) Industry output of good** $X$ **(per-capita)**: $X$ is strictly decreasing and strictly convex in $\beta$: $\frac{\partial \beta}{\partial \beta} < 0$, $\frac{\partial^2 \beta}{\partial \beta^2} > 0$. $X$ inherits the kink found in $x$, at $\beta = \beta^{w=1}$.
- **j) Economy output of good** $X$: $X$ exhibits behavior identical to $X$, except that it is multiplied by $L$.
- **k) Demand for Good** $Y$ **(per-capita)**: $Y$ has similar properties to $L_y$. It is equal to 0 for $\beta < \beta^{w=1}$, and it is strictly increasing and strictly concave for $\beta \geq \beta^{w=1}$: $\frac{\partial \beta}{\partial \beta} > 0$, $\frac{\partial^2 \beta}{\partial \beta^2} < 0$. Note that, in equilibrium, the upper bound of $Y$ is 1 (in equilibrium, the highest possible
individual demand for $Y$ occurs when all individuals devote all of their time to the production of $Y$, which at most will generate a single unit of good $Y$, per capita).

1) Aggregate Demand for Good $Y$: $T^D$, has the same properties as $Y$, but it is multiplied by $L$ (in equilibrium, $T^D = L_Y$).

m) Aggregate Supply of Good $Y$: $T^S = L_Y$, always matches $T^D$ in equilibrium, and thereby exhibits identical behavior.

n) Value added in industry $X$ (per-capita): $VA_x$, exhibits behavior similar to $w$. The only difference is that when $w = 1$, $VA_x$ does not (like $w$) become flat at a value of 1. Instead it begins to fall at a steeper rate. To see this, note that the wage reduction which occurs as $\beta$ rises acts like a 'buffer' in reducing the shrinkage of sector $X$. Once the wage rate reaches its minimum value of 1, the 'buffer' is exhausted and sector $X$ shrinks at a faster rate.

o) Value added in industry $Y$ (per-capita): $VA_y$, has the same value as per-capita demand for good $Y$ and displays identical behavior.

Proof: By inspection of the corresponding variables, their derivatives and associated thresholds.  

Proposition $\sigma$: Effects of $\sigma$ (continued)

g) Price of good $X$: $p$, is strictly increasing and strictly convex in $\sigma$: $\frac{\partial p}{\partial \sigma} > 0$ and $\frac{\partial^2 p}{\partial \sigma^2} > 0$.

h) Per-firm output of good $X$ (per-capita): $x$ is strictly increasing in $\sigma$: $\frac{\partial x}{\partial \sigma} > 0$. For $w > 1$, we have that $x$ is strictly concave in $\sigma$: $\frac{\partial^2 x}{\partial \sigma^2} < 0$. When $w = 1$, $x$ is strictly convex in $\sigma$: $\frac{\partial^2 x}{\partial \sigma^2} > 0$. $x$ is kinked at the points where $w$ becomes 1.

i) Industry output of good $X$ (per-capita): $X$ is U-shaped in $\sigma$: It is strictly decreasing at first ($\frac{\partial X}{\partial \sigma} < 0$), reaches a minimum ($\frac{\partial X}{\partial \sigma} = 0$), and becomes strictly increasing ($\frac{\partial X}{\partial \sigma} > 0$). $X$ is strictly convex in $\sigma$ ($\frac{\partial^2 X}{\partial \sigma^2} > 0$).

j) Economy output of good $X$: $\chi$ exhibits behavior identical to $X$, except that it is multiplied by $L$.

k) Demand for good $Y$ (per-capita): $Y$ has properties identical to $L_Y$, and we refer the reader to Proposition $\sigma$, part (e).

l) Aggregate Demand for Good $Y$: $T^D$, has the same properties as $Y$, but it is multiplied by $L$ (in equilibrium, $T^D = L_Y$).

m) Aggregate Supply of Good $Y$: $T^S = L_Y$, always matches $T^D$ in equilibrium, and thereby exhibits identical behavior.

n) Value added in industry $X$ (per-capita): $VA_x$, exhibits behavior similar to $w$. The only difference is that when $w = 1$, $VA_x$ does not (like $w$) become flat at a value of 1.
Instead it becomes steeper. To see this, note that the wage variation which occurs as \( \sigma \) changes acts like a 'buffer' in attenuating the adjustment of sector \( X \). Once the wage rate reaches its minimum value of 1, the 'buffer' is exhausted and sector \( X \) adjusts at a faster rate.

**o) Value added in industry \( Y \) (per-capita):** \( VA_y \), has the same value as per-capita demand for good \( Y \) and displays identical behavior.

Proof: By inspection of the corresponding variables, their derivatives and associated thresholds.

Proposition \( \varepsilon \): Effects of \( \varepsilon \) (continued)

g) Price of good \( X \): \( p \) is invariant with respect to \( \varepsilon \): \( \frac{\partial p}{\partial \varepsilon} = 0 \).

h) Per-firm output of good \( X \) (per-capita): \( x \) is strictly decreasing and (piecewise) strictly convex in \( \varepsilon \): \( \frac{\partial x}{\partial \varepsilon} < 0 \), \( \frac{\partial^2 x}{\partial \varepsilon^2} > 0 \). \( x \) exhibits a kink at \( \varepsilon = \varepsilon^{w=1} \).

i) Industry output of good \( X \) (per-capita): \( X \) exhibits behavior similar to \( x \), but is multiplied by \( N+1 \) (which is constant with respect to \( \varepsilon \)).

j) Economy output of good \( X \): \( \chi \), exhibits behavior identical to \( X \), except that it is multiplied by \( L \).

k) Demand for good \( Y \) (per-capita): \( Y \) has properties identical to \( L_y \), and we refer the reader to Proposition \( \varepsilon \), part (e).

l) Aggregate Demand for Good \( Y \): \( T^D \), has the same properties as \( Y \), but it is multiplied by \( L \) (in equilibrium, \( T^D = L_y \)).

m) Aggregate Supply of Good \( Y \): \( T^S \equiv L_y \), always matches \( T^D \) in equilibrium, and thereby exhibits identical behavior.

n) Value added in industry \( X \) (per-capita): \( VA_x \) is strictly decreasing and strictly (piecewise) convex in \( \varepsilon \): \( \frac{\partial VA_x}{\partial \varepsilon} < 0 \), \( \frac{\partial^2 VA_x}{\partial \varepsilon^2} > 0 \), displaying a kink at \( \varepsilon = \varepsilon^{w=1} \).

o) Value added in industry \( Y \) (per-capita): \( VA_y \), has the same value as per-capita demand for good \( Y \) and displays identical behavior.

Proof: By inspection of the corresponding variables, their derivatives and associated thresholds.

Proposition \( u_0 \): Effects of \( u_0 \) (continued)

g) Price of good \( X \): \( p \) is invariant with respect to \( u_0 \): \( \frac{\partial p}{\partial u_0} = 0 \).

h) Per-firm output of good \( X \) (per-capita): \( x \) is strictly increasing and strictly convex in \( u_0 \): \( \frac{\partial x}{\partial u_0} > 0 \), \( \frac{\partial^2 x}{\partial u_0^2} > 0 \), and exhibits a kink at \( u_0 = u_0^{w=1} \).
i) Industry output of good $X$ (per-capita): $X$ exhibits behaviour similar to $x$, but is multiplied by $N + 1$ (which is constant with respect to $u_0$).

j) Economy output of good $X$: $\chi$, exhibits behavior identical to $X$, except that it is multiplied by $L$.

k) Demand for good $Y$ (per-capita): $Y$ has properties identical to $L_Y$, and we refer the reader to Proposition $u_Q$, part (e).

l) Aggregate Demand for Good $Y$: $T^D$, has the same properties as $Y$, but it is multiplied by $L$ (in equilibrium, $T^D = L_Y$).

m) Aggregate Supply of Good $Y$: $T^S \equiv L_Y$, always matches $T^D$ in equilibrium, and thereby exhibits identical behavior.

n) Value added in industry $X$ (per-capita): $VA_x$ is strictly increasing and strictly convex in $u_0$: $\frac{\partial VA_x}{\partial u_0} > 0$, $\frac{\partial^2 VA_x}{\partial u_0^2} > 0$, displaying a kink at $u_0 = u_0^{w=1}$.

o) Value added in industry $Y$ (per-capita): $VA_y$, has the same value as per-capita demand for good $Y$ and displays identical behavior.

Proof: By inspection of the corresponding variables, their derivatives and associated thresholds.

Proposition $L$: Effects of $L$ (continued)

9) Price of good $X$: $p$ is invariant with respect to $L$.

h) Per-firm output of good $X$ (per-capita): For $L > L^{w=1}$, $x$ is strictly increasing and strictly concave in $L$: $\frac{\partial x}{\partial L} > 0$, $\frac{\partial^2 x}{\partial L^2} < 0$. For $L < L^{w=1}$, we consider three cases:

(a) For $\beta < 4$ $x$ is strictly increasing and strictly convex in $L$: $\frac{\partial x}{\partial L} > 0$, $\frac{\partial^2 x}{\partial L^2} > 0$.

(b) For $\beta > 4$ $x$ is strictly increasing and strictly concave in $L$: $\frac{\partial x}{\partial L} > 0$, $\frac{\partial^2 x}{\partial L^2} < 0$.

(c) For $\beta = 4$ $x$ is strictly increasing and linear in $L$: $\frac{\partial x}{\partial L} > 0$, $\frac{\partial^2 x}{\partial L^2} = 0$.

$x$ is kinked at $L = L^{w=1}$.

i) Industry output of good $X$ (per-capita): $X$ is equal to $x$ multiplied by $N + 1$. Thus $X$ is simply a scaled up version of $x$ (with the scaling constant equal to $N + 1$).

j) Economy output of good $X$: $\chi$ is strictly increasing and strictly (piecewise) convex in $L$: $\frac{\partial \chi}{\partial L} > 0$, $\frac{\partial^2 \chi}{\partial L^2} > 0$. $\chi$ exhibits a kink at $L = L^{w=1}$.

k) Demand for good $Y$ (per-capita): For $L > L^{w=1}$ $Y = 0$. For $L < L^{w=1}$ $Y$ is strictly decreasing in $L$: $\frac{\partial Y}{\partial L} < 0$. For $L < L^{w=1}$, we have three cases:

(a) For $\beta > 4$ $Y$ is strictly convex in $L$: $\frac{\partial^2 Y}{\partial L^2} > 0$.

(b) For $\beta < 4$ $Y$ is strictly concave in $L$: $\frac{\partial^2 Y}{\partial L^2} < 0$.

(c) For $\beta = 4$ $Y$ is linear in $L$: $\frac{\partial^2 Y}{\partial L^2} = 0$. 

75
$Y$ is kinked at $L = L^{w=1}$.

1) Aggregate Demand for Good $Y$: $Y^D$, has the same properties as $Y$, but it is multiplied by $L$ (in equilibrium, $Y^D = L Y$, so its properties are identical to those of $L Y$, see part (e) of Proposition $L$).

m) Aggregate Supply of Good $Y$: $Y^S = L Y$, always matches $Y^D$ in equilibrium, and exhibits identical behavior.

n) Value added in industry $X$ (per-capita): $VA_x$ is strictly increasing in $L$: $\frac{\partial VA_x}{\partial L} > 0$.

For $L > L^{w=1}, VA_x$ is strictly concave in $L$: $\frac{\partial^2 VA_x}{\partial L^2} < 0$. For $L \leq L^{w=1}$, we have three cases:

(a) For $p < 4 VA_x$ is strictly convex in $L$: $\frac{\partial^2 VA_x}{\partial L^2} > 0$.

(b) For $p > 4 VA_x$ is strictly concave in $L$: $\frac{\partial^2 VA_x}{\partial L^2} < 0$.

(c) For $p = 4 VA_x$ is linear in $L$: $\frac{\partial^2 VA_x}{\partial L^2} = 0$.

$VA_x$ is kinked at $L = L^{w=1}$.

o) Value added in industry $Y$ (per-capita): $VA_y$, has the same value as per-capita demand for good $Y$ and displays identical behavior.

Proof: By inspection of the corresponding variables, their derivatives and associated thresholds.
Chapter 3

General Equilibrium with Oligopolistic Interactions: Open Economy

1 Introduction

This chapter develops a general equilibrium model of two economies each with two sectors. Each economy is in essence identical to that presented in chapter 2. We assume free trade in the goods market, that is, the economies are joined on the consumption side. Consumers maximize a (quadratic) utility function over two types of goods, labelled type $Y$ and type $X$, subject to a budget constraint. Good $Y$ is homogeneous, while type $X$ goods are vertically differentiated. Consumers have a constant labour supply, equal to their labour endowment, which has been normalized to 1. Labour is assumed immobile between the economies, and each economy features a population of $L_i$ ($i = d, f$) consumers/workers. Labour is used by industry $X$ and any surplus labour above this is absorbed by industry $Y$. Employment in industry $X$ is denoted by $L_{x_i}$ ($i = d, f$), and employment in industry $Y$ is denoted by $L_{y_i}$ ($i = d, f$).

Industry $X$ is characterized by a three stage game. In stage 1 firms decide whether to enter or not, and a zero profit condition emerges. In stage 2, firms compete in technological capabilities (denoted by $u_i$) by investing in sunk costs —labelled $F(u_i)$—, taking market structure as given. Stage 3 portrays competition in quantities ($x_i$), taking market structure and technological capabilities as given. The presence of fixed costs is associated with increasing returns to scale (IRTS).
In principle, industry \( X \) could use labour to generate technological capability (via fixed costs) and to produce type \( X \) goods (via variable costs). We will simplify matters by assuming that variable costs are zero (the analysis can be extended to non-zero variable costs). Industry \( X \) uses labour exclusively to generate technological capability. We can think of this as an appended sector to industry \( X \) (for example, an R&D lab), to highlight the notion that these workers are hired exclusively to further the firm's technological capability. The assumption that all fixed costs are sunk becomes reasonable when we consider that these take the form of wages paid to workers: If the firm exits, it cannot get any of the payments back from its workers.

Industry \( Y \) features a freely available 1:1 technology. The output of industry \( Y \) (denoted by \( Y_t \)) is equal to employment in that industry \( (Y_t = L_{Y_t}) \). This implies a very simple form of constant returns to scale (CRS). Good \( Y \) is treated as the numeraire, so its price is normalized to 1, and the marginal product of labour in industry \( Y \) is 1. We will see that this effectively constitutes the outside option of workers employed in industry \( X \), and it implies that the economy's wage rate is bounded from below at 1.

Labour is the sole input. Consumers' income is constituted by their wage receipts, by profits accruing from shares owned in firms, and by the value of an endowment of type \( Y \) goods. We will see that equilibrium profits are zero, so they drop out from the budget constraint.

We characterize a symmetric general equilibrium, featuring identical values for all variables and parameters in both economies. This chapter provides an analysis of how the equilibrium changes as we manipulate parameter values. The analysis provides the first and second order comparative statics properties of the model. Most parameters of the model can be interpreted as policy variables. Thus the comparative statics exercise provides insight into economic policy. In particular we can analyze different variants of industrial, demographic and trade policies. The characterization of an economy by the values of its parameters admits the following interpretation: We can treat the parameters of the economy as a representation of its institutional or structural characteristics, which lie outside the explanatory scope of the model. We can then analyze the consequences of changing these characteristics by reference to our comparative statics results.

In this chapter we refer to two types of equilibrium. Firstly we have a symmetric equilibrium \textit{within} each economy (but not necessarily \textit{across} economies). In this case all firms within a given country choose identical quantities and technological capability. This type of equilibrium allows countries to have different equilibrium outcomes if their parameter values differ. The other type of equilibrium features symmetry \textit{across} economies. In this equilibrium both countries
feature identical parameter values, and resulting equilibrium outcomes are identical in both economies. In particular, both countries have identical wage rates, welfare levels, numbers of firms and all firms have the same technological capability, sell the same quantity, at an identical price.

We consider the consequences of introducing asymmetries in initial conditions and whether, given some difference in initial conditions, industrial policy aimed at catching-up is feasible and welfare improving. Inter-industry trade may occur when initial conditions differ between the two economies, while intra-industry trade arises in all instances. To facilitate exposition, the structure of the model is summarized in a 'flow-chart' type diagram in Figure 1.

The chapter is structured as follows. In Section 2 we develop the model. In Section 3 we characterize a symmetric general equilibrium. Section 4 analyses the comparative statics properties of this equilibrium, assuming that all parameters are identical in both economies. This section focusses on what happens when both economies face identical changes in their structural/institutional characteristics (parameter values). In Section 5 we carry out an analysis of asymmetric initial conditions and industrial policy for catching-up. This section presents some of the most important results of the thesis. Section 6 concludes.
2 The Open Economy General Equilibrium Model

We present a world economy constituted by two countries, both of which are treated as large economies\(^1\). Both economies have identical structures, although we allow for asymmetry in all parameters except \(\sigma\). Firstly we describe the consumers' problem, then the production side of the model and finally the labour market.

Each of the constituent economies is similar to the closed economy model developed in chapter 2, but now we allow free trade in consumer goods. We label foreign economy variables with subscript ‘\(f\)’ and domestic economy variables with subscript ‘\(d\)’. To avoid duplication of equations, it will be convenient to use subscripts \(i, j = d, f\) with \(i \neq j\) to label expressions which are identical for both economies.

The economies are fully integrated on the consumption side: There is free trade in type \(Y\) and type \(X\) goods. Transport costs are assumed to be insignificant. On the production side, each economy features two industries (labelled \(X\) and \(Y\), as in chapter 2). The only factor of production in the model is labour. Labour is immobile across and perfectly mobile within each of the economies. The output of industries \(X\) and \(Y\) is wholly consumed by workers (saving is not possible), and there are no intermediate inputs.

2.1 Consumers

In each economy there is a population of \(L_i\) homogeneous consumers, indexed by \(h_i = 1, ..., L_i\) for \(i = d, f\). Consumers have identical tastes in both economies, and since the model features free trade in consumption goods and zero transport costs, consumers can purchase goods produced in either country without incurring additional costs. Each consumer has a perfectly inelastic labour supply, which has been normalized to one. Consumers allocate their labour endowment between industries \(X\) and \(Y\). Since there is no labour mobility across countries, consumers earn the (labour market clearing) wage rate prevalent in their country. Consumers maximize the same utility function as in the previous chapter by choosing over two types of good: \(Y\) (a homogenous good) and \(X\) (a vertically differentiated product).

In industry \(X\), each firm produces one good only, labelled \(x_{kd}\) for domestic firms and \(x_{kf}\) for foreign firms, while \(x_k\) refers to firms from either country. \(x_{kd}\) denotes per-capita consumption.

\(^1\) The term 'large economy' is used in the traditional trade theoretic sense: changes in the individual economies affect the world (general) equilibrium. The alternative case would be that of a small economy, where changes in an individual economy do not affect the world (general) equilibrium. In the latter case, terms of trade are usually assumed to be exogenous. Not so in the large economy case, where the terms of trade are determined endogenously.
of good \( k \), produced by the \( k^{th} \) firm in country \( i \). We are using the same index \((k)\) for domestic and foreign firms advisedly, since both will be summed together below. In each economy, there is a finite number of firms, denoted by \( n_i + 1 \) for country \( i \). The worldwide number of firms is denoted by \( N + 1 = n_d + n_f + 2 \). As in chapter 2, we denote the number of firms by \( n_d + 1, n_f + 1 \) and \( N + 1 \), rather than the usual notation of \( n_d, n_f \) and \( N \). This is adopted purely for aesthetic convenience, since it will make the equations somewhat more organized.

We introduce the following vector notation: \( x_d = (x_{1d}, x_{2d}, ..., x_{n_d+1}d) \), \( x_f = (x_{1f}, x_{2f}, ..., x_{n_f+1}f) \) and \( x = (x_d, x_f) \). Since we assume single product firms, the number of firms in each country will determine the number of \( X \)-type goods produced in that country. Each good in the \( X \) industry has a ‘quality level’ associated with it, denoted \( u_{kd} \) in the domestic economy and \( u_{kf} \) in the foreign economy, whilst \( u_k \) denotes technological capability of firms in either country. The quality level represents the producer’s technological capability. In vector notation, we have: \( U_d = (u_{1d}, u_{2d}, ..., u_{n_d+1}d) \), \( U_f = (u_{1f}, u_{2f}, ..., u_{n_f+1}f) \) and \( U = (U_d, U_f) \). The representative consumer’s problem can be stated as

\[
\begin{align*}
\max_{x,Y} & \quad V_i = \sum_{k=1}^{N+1} \left( x_k - \frac{x_k^2}{u_k} \right) - 2\sigma \sum_{k=1}^{N+1} \frac{x_k}{u_k} \sum_{l \neq k}^N \frac{x_l}{u_l} + Y_i \\
{\text{subject to}} & \quad \sum_{k=1}^{N+1} p_k x_k + q_i Y_i = w_i + q_i \bar{Y}_i + \sum_{k=1}^{n_i+1} s_{hki} \Pi_{ki} \quad \text{for } i = d, f
\end{align*}
\]

(3.1)

(3.2)

where \( \sigma \in (0,1) \) measures the substitutability between the \( X \)-type goods (and is identical for both economies), \( p_k \) is the price of the \( k^{th} \) \( X \)-type good (in either country), \( q_i \) is the price of the \( Y \)-type good in economy \( i \), \( w_i \) is the wage rate in economy \( i \), \( Y_i \) is the consumption of type \( Y \) goods in country \( i \), \( \bar{Y}_i \) represents country \( i \)'s (per capita) endowment of type \( Y \) goods, \( s_{hki} \) is the ownership share of consumer \( h \) in firm \( k \) in country \( i \) (with the usual restriction that \( \sum_{h=1}^{n_i} s_{hki} = 1 \)), and \( \Pi_{ki} \) denotes the (net) profits of firm \( k \) in country \( i \). Notice that firm ownership is restricted to a consumer’s economy (i.e., foreign ownership is ruled out). The endowment of type \( Y \) goods, \( \bar{Y}_i \), is a minor variation on the model presented in chapter 2, and it is only included to allow for the possibility that a rich economy may demand more type \( Y \) goods than the poor economy produces (in which case the poor economy depletes its endowment of type \( Y \) goods).

Consumer income comes from wages \((w_i)\), the value of their endowment of type \( Y \) goods \((q_i \bar{Y}_i)\) and dividends generated by the consumer’s shares in industry \( X \) firms \((\sum_{k=1}^{n_i+1} s_{hki} \Pi_{ki})\). In equilibrium we will see that net profits are zero (by free entry), thus the dividend term in the budget constraint will drop out.
To perform welfare analysis we substitute the equilibrium solutions into $V_i$, and we label this welfare indicator ‘$W_i$’ for economy $i = d, f$.

From the consumer’s problem, we obtain the inverse demand facing domestic producers $(p_{kd})$ and foreign producers $(p_{kf})$

$$p_{ki} = 1 - \frac{x_{ki}}{u_{ki}^2} - \frac{2\sigma}{u_{ki}} \left( \sum_{i \neq k} x_{ki} + \sum_{i=1}^{n_i+1} x_{ij} \right) \text{ for } i, j = d, f \text{ and } i \neq j \quad (3.3)$$

Per-capita demand functions for good $Y$ are obtained as a residual from the budget constraint (equation 3.2):

$$Y_i = \frac{1}{q_i} \left( w_i + q_i \overline{Y}_i + \sum_{k=1}^{n_i+1} s_{hki} \Pi_{ki} - \sum_{k=1}^{N+1} p_k \overline{x}_k \right) \text{ for } i = d, f \quad (3.4)$$

Good $Y$ is the numeraire, hence its price is set to 1 in both economies ($q_i = 1$). In a symmetric equilibrium within each country (which is not necessarily symmetric across countries) we have that $x_{ki} = x_i$ for $i = d, f$. In this case, after setting net profits to zero, (per-capita) demand for $Y$ in each country simplifies to

$$Y_i = w_i + \overline{Y}_i - (n_i + 1)p_i x_i - (n_j + 1)p_j x_j \text{ for } i, j = d, f \text{ and } i \neq j \quad (3.5)$$

In a symmetric equilibrium in which both economies have identical outcomes, such that $x_i = x$, demand for $Y$ simplifies further to

$$Y_i = w_i + \overline{Y}_i - (N + 1)p x \text{ for } i = d, f \quad (3.6)$$

Which can be rearranged to yield net demand for $Y$:

$$Y_i - \overline{Y}_i = w_i - (N + 1)p x \text{ for } i = d, f$$

Consumers in economy $i$ will be net consumers (net demanders) of $Y$ when their demand is above their endowment. Otherwise, they are net suppliers of $Y$ (and the economy becomes a net exporter of $Y$). The demand functions specified above are used to obtain the equilibrium solutions for industries $X$ and $Y$. We proceed to describe industry $X$. 

82
2.2 Industry X

Industry X has increasing returns to scale, due to the presence of (endogenous) sunk costs. As in the previous chapter, firms play a three stage game. In the first stage the entry decision is made. In the second stage, sunk investments in technological capability are undertaken. In the third stage, firms face Cournot competition. The equilibrium concept is Subgame Perfect Nash Equilibrium.

Firms in this industry face competition not only from their local rivals, but also from their overseas rivals (since there is free trade in consumer goods), and hire workers exclusively from the local workforce (recall that labour is internationally immobile).

In this chapter and in chapter 5, we will consider two notions of symmetry. Firstly we consider symmetry within each economy, without restricting parameters or solutions to be identical across economies. This will allow us to set out the conditions for a general equilibrium.

Secondly, we consider symmetry both within and across the economies. In this case we impose the restriction that each parameter takes the same value in both economies. We shall refer to the latter case as a symmetric (general) equilibrium.

In keeping with backward induction, we proceed to the description of the final stage in the firms' decision problem.

2.2.1 Stage 3: Cournot Competition

In this stage, firms choose the optimal quantity to produce, taking as given their rivals' strategies, technological capability and market structure. Gross profits of firm \( k \) in country \( i \) are equal to revenue \( (p_{ki}x_{ki}) \) since the cost of producing \( x_{ki} \) has been set to zero, that is, \[ \pi_{ki} = p_{ki}x_{ki} \quad \text{for } i = d, f \] (3.7)

Firms maximize their objective (equation 3.7) by choosing \( x_{ki} \). The first order conditions for the \( k \text{th} \) firm in country \( i \) are given by

\[ p_{ki} + \frac{\partial p_{ki}}{\partial x_{ki}} x_{ki} = 0 \quad \text{for } i = d, f \] (3.8)

For the domestic economy there are \( n_d + 1 \) such equations in \( x \) (one for each firm). For the foreign economy there are \( n_f + 1 \) equations. In Appendix 1, we solve this system of equations, and find a symmetric Nash equilibrium. We obtain the solutions for \( x_d \) and \( x_f \) in terms of the number of domestic and foreign firms \((n_d + 1 \text{ and } n_f + 1)\) and the vectors of firms'
technological capabilities, \( U_d \) and \( U_f \). Using the solutions for \( x_d \) and \( x_f \), after calculations shown in Appendix 1, we obtain a solved-out payoff function, which will be used to solve the second stage of the game. The solved-out payoff is:

\[
\pi_{ki}(U) = \frac{u_{ki}^2}{2(2-\sigma)^2} \left[ 1 - \frac{\sigma}{2 + \sigma(n_i + n_j + 1)} \left( \frac{n_i}{u_{ki}} + \frac{n_j}{u_{ki}} \right) \right]^2 \quad \text{for } i, j = d, f \text{ and } i \neq j
\]  

(3.9)

Note that \( \pi_{ki}(U) \) is per-capita gross profit earned by the firm, that is, \( \pi_{ki}(U) \) is the gross profit the firm earns from each consumer. Since population is \( L_d \) in the domestic economy and \( L_f \) in the foreign economy, and the firm faces a unified world market for consumer goods, total gross profit of the \( k \text{th} \) firm in country \( i \) is given by \((L_d + L_f)\pi_{ki}(U)\).

With within-country symmetry (i.e., firms choose a symmetric quality level within each country, denoted by \( u_d \) and \( u_f \)), \( \pi_{ki} \) simplifies to:

\[
\pi_i = \frac{u_i^2}{2(2-\sigma)^2} \left[ 1 - \frac{\sigma}{2 + \sigma(n_i + n_j + 1)} \left( (n_i + 1) + (n_j + 1) \frac{u_d}{u_i} \right) \right]^2 \quad \text{for } i, j = d, f \text{ and } i \neq j
\]  

(3.10)

With across-country symmetry (setting \( u_d = u_f = u \)) and noting that \( N + 1 = n_d + n_f + 2 \), the above expression simplifies further to

\[
\pi = \frac{u^2}{2(2+\sigma N)^2}
\]

which is the expression we obtained in chapter 2 (equation 2.9). However we must be careful to emphasize that the underlying structure of the economy is substantially different, since we now have two separate countries.

2.2.2 Stage 2: Competition in Technological Capability

In the second stage firms choose their investment in technological capability, taking as given their (local and foreign) rivals' strategies as well as market structure. Sunk investment determines technological capability via a fixed outlays function, \( F(.) \). The firm's net profit is given by

\[
\Pi_{ki} = (L_d + L_f) \pi_{ki}(U) - F(u_{ki}, w_i) \quad \text{for } i = d, f
\]  

(3.11)

where \( L_i \) denotes population size in economy \( i \), \( \pi_{ki}(U) \) denotes the solved-out gross per-capita profit function for country \( i \) (equation 3.9) and \( F(u_{ki}, w_i) \) denotes the fixed outlays function:
F(u_{ki}, u_i) = u_i f(u_{ki}), where \( u_i \) is the wage rate prevailing in country \( i \), and \( f(u_{ki}) = \varepsilon_i \left( \frac{u_{ki}}{u_{o}} \right)^{\beta_i} \) is a convex mapping from technological capability in country \( i \) (\( u_{ki} \)) to labour units required to achieve such capability. The mapping \( f(u_{ki}) \) is the firm's labour requirement in industry \( X \) for country \( i \): It measures units of labour required to achieve a certain technological capability \( u_{ki} \). \( \beta_i > 2 \) is required for the second order conditions to hold. We also assume \( \varepsilon_i > 0 \), and \( u_{ki} > u_o \geq 1 \). \( \beta_i \) is the elasticity of \( f(u_{ki}) \) with respect to \( u_{ki} \). \( \varepsilon_i \) is an exogenous set-up cost. \( u_o \) represents an initial (inherited) value of technological capability, which is an exogenous parameter. In section 5, \( u_o \) is used as a means of modelling differences in initial conditions across countries. It is important to note that we allow each country to have different values for the following parameters: \( \beta_i, L_i, \varepsilon_i, u_o \) \((i = d, f)\). We have constrained the value of \( \sigma \) to be identical in both economies. This appears to be a reasonable assumption for it seems difficult to justify that identical goods should have different degrees of substitutability in different countries.

Firm \( k \) maximizes (3.11) with respect to \( u_{ki} \), taking as given rivals' technological capabilities and market structure. The first order conditions are:

\[
(L_d + L_f) \frac{\partial \pi_{ki}}{\partial u_{ki}} = \frac{w_i \varepsilon_i \beta_i}{u_{ki}} \left( \frac{u_{ki}}{u_o} \right)^{\beta_i} \text{ for } i = d, f \text{ and } k = 1, ..., n_i + 1
\]  

(3.12)

This constitutes a system of \( N+1 \) equations in \( U_d \) and \( U_f \). Taking the derivative of the solved out profit function (equation 3.9) with respect to \( u_{ki} \) and simplifying, we can write (3.12) as follows

\[
\frac{(L_d + L_f) \left[ 2 + \sigma (n_i + n_j) \right]}{(2 - \sigma)^2 \left[ 2 + \sigma (n_i + n_j + 1) \right]} \left[ 1 - \frac{\sigma}{2 + \sigma (n_i + n_j + 1)} \left( \sum_{l=1}^{n_i+1} \frac{u_{i_l}}{u_{ki}} + \sum_{l=1}^{n_j+1} \frac{u_{i_l}}{u_{ki}} \right) \right] = \frac{w_i \varepsilon_i \beta_i \left( \frac{u_{ki}}{u_o} \right)^{\beta_i-2}}{u_{o}^{\beta_i}} \text{ for } i = d, f \text{ and } k = 1, ..., n_i + 1
\]

(3.13)

An equilibrium requires that technological capability be a solution to the system of \( N+1 \) equations in (3.13). In a symmetric equilibrium within each economy, such that \( u_i = u_{ki} = u_i \), the first order conditions simplify to

\[
\frac{(L_d + L_f) \left[ 2 + \sigma (n_i + n_j) \right]}{(2 - \sigma)^2 \left[ 2 + \sigma (n_i + n_j + 1) \right]} \left[ 1 - \frac{\sigma}{2 + \sigma (n_i + n_j + 1)} \left( (n_i + 1) + (n_j + 1) \frac{u_j}{u_i} \right) \right] = \frac{w_i \varepsilon_i \beta_i \left( \frac{u_{ki}}{u_o} \right)^{\beta_i-2}}{u_{o}^{\beta_i}} \text{ for } i, j = d, f \text{ and } i \neq j
\]

(3.14)

This expression embeds the following result:
Proposition 1: On how the technology gap affects technological capability

The net marginal benefit of investment in technological capability is decreasing in the technology gap between countries, i.e., in the ratio of technological capabilities ($u_i/u_j$). This implies that there exists a threshold level of $u_i/u_j$ above which investment in technological capability is not optimal for the laggard economy.

Proof: By inspection of equation (3.14).

The consequence of Proposition 1 is that if the distance between equilibrium levels of technological capability (the technology gap) is sufficiently large, then one of the economies will cease to invest in technological capability: It is simply not optimal to try to catch up with the advanced economy. However, in equilibrium, we find that, in order to have positive market share, firms will effectively catch-up in terms of technological capability. Nonetheless, this does not imply that the economies catch-up in terms of income, since the asymmetry in initial conditions will have implications for market structure and the wage rate. When we analyze the effect of differences in initial conditions or 'history' (denoted by $u_{\omega}$), the result in Proposition 1 will drive some of our conclusions. This completes the description of the second stage subgame. We discuss second order conditions for this stage in Appendix 2.

2.2.3 Stage 1: The Entry Decision

In stage one, firms make their entry decision. Assume there is a sufficiently large pool of potential entrants. Firms will enter as long as net profits are positive. This leads to the following non-negative-profit condition

$$ (L_d + L_f) \pi_{kl} \geq w_i \varepsilon_i \left( \frac{u_{kl}}{u_{ci}} \right)^{\beta_i} \quad \text{for } i = d, f \quad (3.15) $$

Ignoring integer effects, entry occurs until (3.15) holds with equality. Substituting the solved out pay-off $\pi_{kl}$ (equation 3.9) leads to

$$ \frac{L_d + L_f}{2 (2 - \sigma) \varepsilon_i^2} \left[ 1 - \frac{\sigma}{2 + \sigma (n_i + n_f + 1)} \left( \sum_{i=1}^{n_i+1} \frac{u_{kl}}{u_{ki}} + \sum_{i=1}^{n_f+1} \frac{u_{lj}}{u_{kj}} \right) \right]^2 = w_i \varepsilon_i \frac{u_{kl}^{\beta_i - 2}}{u_{ci}^{\beta_i}} \quad (3.16) $$

for $i, j = d, f$ and $i \neq j$ and $k = 1, \ldots, n_i + 1$
This constitutes another requirement for an equilibrium, together with (3.13). Symmetry within each economy allows us to write condition (3.16) as follows

\[
\frac{(L_d + L_f)}{2(2 - \sigma)^2} \left\{ 1 - \frac{\sigma}{2 + \sigma(n_i + n_j + 1)} \left[ (n_i + 1 + (n_j + 1) \frac{u_d}{u_i} \right] \right\}^2 = w_i \epsilon_i \frac{u_i^{\beta_i - 2}}{u_i^{\beta_i}} \quad \text{for } i = d, f
\]

(3.17)

This constitutes a system of two equations in \( n_d \) and \( n_f \). In a symmetric equilibrium, we can find an explicit solution for \( u \) in stage 2, and the equations in (3.17) reduce to a single equation in \( n \), which also has an explicit solution. This completes the description of the first stage of the game played by firms in industry \( X \).

### 2.3 The Labour Market, Industry \( Y \) and the Trade Balance

Firstly we discuss the labour market. This is followed by a description of industry \( Y \) and the trade balance. Each consumer supplies a fixed amount of labour, which has been normalized to 1. Hence, total labour supply is fixed by population size for each country, namely, \( L_d \) and \( L_f \). Labour is internationally immobile (though within each economy it is perfectly mobile between industries). Demand for labour stems from the local industries only (\( X \) and \( Y \)). The labour requirement by each firm in sector \( X \) is \( f(u_{ki}) = \epsilon_i \left( \frac{u_{ki}}{w_i} \right) \beta_i \) for \( i = d, f \). Industry \( X \) employment is given by

\[
L_{x_i} = \sum_{k=1}^{n_i+1} f(u_{ki})
\]

In a symmetric equilibrium, this reduces to

\[
L_{x_i} = (n_i + 1) f(u_i)
\]

(3.18)

Any surplus labour not absorbed by industry \( X \) is employed in industry \( Y \). Employment in industry \( Y \) in country \( i \) is labelled \( L_{yi} \). The labour market clearing condition is written as follows

\[
L_i = L_{x_i} + L_{yi} \quad \text{where } L_{x_i}, L_{yi} \in [0, L_i] \text{ and } i = d, f
\]

(3.19)

The wage rate adjusts to ensure that the labour market clears in each economy, given that labour is internationally immobile and that industry \( Y \) offers a (de-facto) minimum wage rate equal to the marginal product of labour in industry \( Y \).

On the supply side, industry \( Y \) has a simple 1:1 (constant returns to scale) technology: One unit of labour produces one unit of type \( Y \) good. The 1:1 technology prevalent in industry \( Y \)
means that the marginal product of labour in industry $Y$ is equal to the price of type $Y$ good, $q_i$. This good is the numeraire, so we normalize its price to 1. Hence the wage rate is bounded from below at 1.

Aggregate supply of good $Y$ ($Y_{si}$) is composed by production of good $Y$ (which is identical to employment in the sector) and by the economy's endowment of this good ($L_iY_i$). In a symmetric equilibrium, we have

$$Y_{si} = L_d(Y_i) + L_iY_i = L_i(1 + Y_i) - (n_i + 1)f(u_i) \text{ for } i = d, f$$

(3.20)

Per-capita demand for good $Y$ is given in equation (3.4). Aggregate demand for good $Y$ in each country is obtained by multiplying per-capita demand by $L_i$. In a Subgame Perfect Nash Equilibrium featuring symmetry between the firms of each country (but not necessarily between the firms across both countries) we have: $x_{ki} = x_i$ for $i = d, f$. In this case per-capita demand for $Y$ is given in expression (3.5), and aggregate demand can be written as follows

$$T_{Di} = L_i Y_i = L_i \left[u_i + Y_i - (n_i + 1)p_i x_i - (n_j + 1)p_j x_j\right] \text{ for } i, j = d, f \text{ and } i \neq j$$

We next use the fact that in equilibrium $(L_d + L_f)p_ix_i = u_if(u_i)$, which obtains by the free entry condition in equation (3.15). We can then write aggregate demand for good $Y$ as follows

$$T_{Di} = L_i \left[u_i + Y_i - \frac{(n_i + 1)u_if(u_i) - (n_j + 1)w_j f(u_j)}{L_d + L_f}\right] \text{ for } i, j = d, f \text{ and } i \neq j$$

(3.21)

Since industry $Y$ uses a 1:1 technology, the marginal product of labour is given by the price of good $Y$ ($q$), which has been set equal to 1 (good $Y$ is the numeraire). This sector effectively constitutes a worker's outside option: in the worst scenario the worker can always resort to transforming his/her labour endowment with a (freely available) 1:1 technology, and earn an income of $q_i = 1$. Thus we obtain that in (general) equilibrium $u_i \geq 1$. If $L_{yi} > 0$ then $u_i = 1$. However, if demand for labour from sector $X$ is high enough to make $L_{yi} = 0$ (in which case $L_{xi} = L_i$), then industry $Y$ is inactive, and industry $X$ uses all the available labour in the economy. In this case we have that $w_i > 1$ (otherwise workers would shift to industry $Y$ and earn $w_i = 1$).

We now discuss net exports (the trade balance). In sector $Y$ the difference between aggregate supply ($Y_{si}$) and aggregate demand ($T_{Di}$) for type $Y$ good gives the trade balance for good $Y$, to be labelled $TB_{yi}$. Since there are only two countries, we have that $TB_{yd} = -TB_{yf}$.

Exports of good $X$ are given by $L_j(n_i + 1)p_ix_i$, while imports of good $X$ are $L_i(n_j + 1)p_jx_j$. 

---

88
Thus the trade balance for industry $X$ can be written as

$$TB_{Xi} = L_j (n_i + 1) p_i x_i - L_i (n_j + 1) p_j x_j$$

for $i, j = d, f$ and $i \neq j$ \hspace{1cm} (3.22)

The (overall) trade balance for each economy is given by $TB_i = TB_{xi} + TB_{yi}$. In this model the overall trade balance is always zero, hence the trade balance for type $X$ goods must be equal to the negative of the trade balance for type $Y$ good ($TB_{xi} = -TB_{yi}$). Moreover, because there are only two countries in the model, the domestic trade balance for type $X$ goods must be equal to the negative of the foreign trade balance for type $X$ goods ($TB_{xd} = -TB_{xf}$), which in turn is the negative of the foreign trade balance for type $Y$ good ($TB_{xf} = -TB_{yd}$), leading to the conclusion that $TB_{yd} = -TB_{xf}$. Thus we have that $TB_{yd} = -TB_{xf} = TB_{zd} = -TB_{zd}$.

In appendix 3, we provide a discussion of value added and the trade balance in the context of this model, which complements the above discussion. We now proceed to characterize a symmetric general equilibrium for the world economy.

### 3 Characterization of a Symmetric General Equilibrium

We begin by considering a symmetric equilibrium within each economy, such that $u_{ki} = u_{li} = u_i$, $i = d, f$. After setting out the conditions for such an equilibrium, we proceed to obtain explicit solutions for a symmetric equilibrium across both countries, such that all parameters take identical values in both economies ($\beta_i = \beta$, $\epsilon_i = \epsilon$, $L_i = L$, $w_{od} = w_o$ for $i = d, f$ while $\sigma$ is always identical for both economies), as well as technological capability ($u_d = u_f = u$), the number of firms ($n_d = n_f = n$) and the wage rate ($w_d = w_f = w$). We will simply refer to the latter as a ‘symmetric equilibrium’.

In the case of a symmetric equilibrium within each economy, we have nine parameters ($\beta_i$, $\sigma$, $\epsilon_i$, $L_i$, $w_{od}$ for $i = d, f$) and six key variables which determine the rest of the system ($u_i$, $n_i$ and $w_i$). A general equilibrium is characterized by the functions $u_i(\beta_i, \sigma, \epsilon_i, L_i, w_{od})$, $n_i(\beta_i, \sigma, \epsilon_i, L_i, w_{od})$ and $w_i(\beta_i, \sigma, \epsilon_i, L_i, w_{od})$, which can be derived explicitly in a symmetric equilibrium. We will simplify notation by dropping the arguments, so we write $u_i(\cdot)$ for $u_i(\beta_i, \sigma, \epsilon_i, L_i, w_{od})$, $n_i(\cdot)$ for $n_i(\beta_i, \sigma, \epsilon_i, L_i, w_{od})$ and $w_i(\cdot)$ for $w_i(\beta_i, \sigma, \epsilon_i, L_i, w_{od})$. The solutions will depend on whether $L_{yi} > 0$ or $L_{yi} = 0$. A Symmetric General Equilibrium in the open economy is characterized by:

a) World market clearing for goods of type $X$ and $Y$.

b) Market clearing for labour in each economy.

89
c) A Symmetric Subgame Perfect Equilibrium in Industry $X$ in each economy, which is characterized by:

i) Firms choosing (Nash) symmetric equilibrium quantities in stage 3 of the game, taking as given technological capabilities of domestic and foreign rivals and market structure in both economies.

ii) Firms choosing (Nash) symmetric equilibrium technological capabilities in stage 2 of the game, taking market structure in both economies as given.

iii) Free entry in stage 1 of the game for both economies.

Equilibrium Conditions:

The general equilibrium of the world economy is characterized by three conditions for each country (as well as the standard market clearing conditions for goods $X$ and $Y$). We write these for a symmetric equilibrium within each economy.

The first equilibrium condition is a mapping from $u_j$ to $u^*$, such that along this mapping no individual firm wishes to deviate from its strategy. The mapping is defined by the first order conditions for technological capability, $\frac{\partial U_i}{\partial u_i} = 0$ for $i = d, f$ and $k = 1, ..., n_i + 1$ (given by 3.13). Symmetry within each economy allows us to write the mapping as in (3.14):

$$\left(\frac{L_d + L_f}{2(2-\sigma)^2} \frac{[2 + \sigma (n_i + n_j)]}{[2 + \sigma (n_i + n_j + 1)]}\right) \left\{ 1 - \frac{\sigma}{2 + \sigma (n_i + n_j + 1)} \left[ (n_i + 1) + (n_j + 1) \frac{u_j}{u_i} \right] \right\}$$

$$= \frac{w_i e_i \beta_i^i}{u_i^i} \frac{u_i^{i-2}}{u_i^i} \quad \text{for } i, j = d, f \text{ and } i \neq j$$

Secondly we have the free entry condition, as stated in (3.17):

$$\left(\frac{L_d + L_f}{2(2-\sigma)^2} \frac{[2 + \sigma (n_i + n_j)]}{[2 + \sigma (n_i + n_j + 1)]}\right) \left\{ 1 - \frac{\sigma}{2 + \sigma (n_i + n_j + 1)} \left[ (n_i + 1) + (n_j + 1) \frac{u_j}{u_i} \right] \right\}$$

$$= \frac{w_i e_i \beta_i^i}{u_i^i} \frac{u_i^{i-2}}{u_i^i} \quad \text{for } i, j = d, f \text{ and } i \neq j$$

Thirdly we have labour market clearing, (equation 3.19). Upon substituting $L_{yi}$ from (3.18), this becomes:

$$L_i = L_{yi} + L_{xi} = L_{yi} + (n_i + 1) e_i \left( \frac{u_i}{u_{ni}} \right)^{\beta_i}$$

for $i = d, f$

Market clearing for type $Y$ and type $X$ goods in world markets is ensured by construction. These conditions determine the general equilibrium values for technological capability $(u_d, u_f)$, for the number of firms $(n_d + 1, n_f + 1)$ and for the wage rate $(w_d, w_f)$.

In a symmetric equilibrium across both economies, we can find explicit solutions as follows.
Let all parameters take identical values in both economies and let \( u_d = u_f = u \), \( n_d = n_f = n \) and \( w_d = w_f = w \). We obtain an expression for \( u \) from (3.23), given by

\[
u = \left( \frac{Lu_\beta}{we\beta (2 - \sigma)(2 + \sigma (2n + 1))^2} \right)^{\frac{1}{2}}
\]

(3.26)

Noting that \( n = \frac{N-1}{2} \), we can express the above expression in terms of the world number of firms, as follows

\[
u = \left( \frac{2Lu_\beta}{we\beta (2 - \sigma)(2 + \sigma N)^2} \right)^{\frac{1}{2}}
\]

This is the result obtained in equation (2.26), with a doubling of market size (we now have \( 2L \) instead of \( L \)).

The next step is to use condition (3.24) to obtain the number of firms. In a symmetric equilibrium, this simplifies to

\[
\frac{L}{(2 + \sigma (2n + 1))^2} = \frac{u^{\beta - 2}}{u_\sigma}
\]

(3.27)

Substitute \( u \) from equation (3.26) into (3.27) and solve for \( n \). This gives the equilibrium number of firms:

\[
n + 1 = \frac{\beta (2 - \sigma) - 4}{4\sigma} + 1
\]

(3.28)

Noting that \( n = \frac{N-1}{2} \), the number of entrants simplifies to

\[
N + 1 = \frac{(\beta - 2)(2 - \sigma)}{2\sigma} + 1
\]

(3.29)

which is equation (2.27). Note that the number of firms does not change whether we consider a closed economy, or a two country world: Having two countries can be treated as a doubling of market size (population).

Substituting \( n + 1 \) (equation 3.28) into (3.26) we obtain the (symmetric) equilibrium level of technological capability, as a function of parameters and the wage rate:

\[
u = \left( \frac{u_\sigma L}{w e \left[ \frac{1}{\beta (1 - \frac{\beta}{2}) + \sigma} \right]^2} \right)^{\frac{1}{2}}
\]

(3.30)

which is equation (2.28), with twice the population.

This completes the description of the symmetric Subgame Perfect Nash Equilibrium for industry \( X \). To complete the characterization of general equilibrium, we now determine the
equilibrium wage rate.

In a symmetric equilibrium the labour market clearing condition (3.25) simplifies to

$$L = L_y + (n + 1)e \left( \frac{u}{u_o} \right)^\beta$$

(3.31)

The wage rate is obtained by substituting (3.30) and (3.28) into (3.31) and solving for $w$. This yields

$$w = \max \left\{ 1, u_o^2 \left( \frac{L}{e} \right)^\frac{\beta}{\beta - 2} \left[ \frac{\beta (2 - \sigma) - 4}{4\sigma} + 1 \right] \frac{2 + \beta}{\beta (1 - \frac{\beta}{2}) + \sigma}^\beta \right\}$$

(3.32)

If $L_y = 0$ ($w > 1$), the general equilibrium solution for technological capability is obtained by substituting the wage rate in (3.32) into (3.30):

$$u = u_o \left[ \frac{L}{e} \frac{1}{\frac{\beta (2 - \sigma) - 4}{4\sigma} + 1} \right]^\frac{1}{\beta}$$

(3.33)

If $L_y > 0$ ($w = 1$), general equilibrium technological capability is obtained by simply setting $w = 1$ in (3.30):

$$u = \left\{ u_o^2 \frac{L}{e} \frac{1}{\beta (1 - \frac{\beta}{2}) + \sigma}^\beta \right\}^{\frac{1}{\beta - 2}}$$

(3.34)

The general equilibrium number of firms is given by (3.28). This completes the characterization of a symmetric general equilibrium in the open economy. Equilibrium outcomes for all remaining variables are obtained by substituting the solutions obtained above.

To summarize:

1) **Technological Capability:**
   
   If $L_y = 0$ ($w > 1$), then $u(.)$ is given by equation (3.33). On the other hand if $L_y > 0$ ($w = 1$), $u(.)$ is given by equation (3.34).

2) **Number of Firms:**
   
   $n(.)$ is given by equation (3.28).

3) **Wage Rate:**
   
   $w(.)$ is given by equation (3.32).

The symmetric general equilibrium outcomes for the open economy are summarized in the following table (cf. table 1 in chapter 2):
Table 1: Symmetric General Equilibrium Outcomes for the Open Economy.

A natural question to pose relates to the existence of asymmetric equilibria. We can in fact rule out a class of such equilibria, as is shown in the following proposition.

**Proposition 2: Non-existence of a Type of Asymmetric Equilibria**

Consider the following type of equilibrium: Both economies have identical parameter values and firms within each economy set a symmetric level of technological capability, which is not necessarily symmetric across economies. Then there are no equilibria in which firms in one economy set a different level of technological capability to firms in the other economy.

**Proof:** See Appendix 4.

This proposition can be extended to rule out all asymmetric equilibria, provided one is willing to assume that the zero profit condition holds exactly for each firm. The proof is along the same lines to the proof of Proposition 2.

**A Note on Welfare**

The welfare indicator is denoted by $W_i$. Substituting equilibrium solutions into the utility function (equation 3.1) yields the indicator. We find that the utility function contains the term: $2\sigma \sum_{k=1}^{N+1} \frac{\bar{z}_k}{u_k} \sum_{i \neq k} \frac{\bar{z}_i}{u_i}$. This poses the following difficulty: When we impose within-country symmetry ($x_{ki} = x_i$, $u_{ki} = u_i$ for $i = d, f$) the term $\sum_{i \neq k} \frac{\bar{z}_i}{u_i}$ can be written as either $(n_i + 1) \frac{\bar{z}_i}{u_i} + n_j \frac{\bar{z}_i}{u_j}$ or as $n_i \frac{\bar{z}_i}{u_i} + (n_j + 1) \frac{\bar{z}_j}{u_j}$, depending on whether the good we drop is produced by the domestic or the foreign economy. Thus we write the welfare measure as follows

$$W_i = \max (W_{di}, W_{fi})$$

(3.35)
where

\[
W_{ai} = \left[ (n_i + 1) \left( x_i - \frac{x_i^2}{u_i^2} \right) + (n_j + 1) \left( x_j - \frac{x_j^2}{u_j^2} \right) \right]
- 2\sigma \left[ (n_i + 1) \frac{x_i}{u_i} + (n_j + 1) \frac{x_j}{u_j} \right] \left[ (n_i + 1) \frac{x_i}{u_i} + (n_j + 1) \frac{x_j}{u_j} \right] + Y
\]

\[
W_{bi} = \left[ (n_i + 1) \left( x_i - \frac{x_i^2}{u_i^2} \right) + (n_j + 1) \left( x_j - \frac{x_j^2}{u_j^2} \right) \right]
- 2\sigma \left[ (n_i + 1) \frac{x_i}{u_i} + (n_j + 1) \frac{x_j}{u_j} \right] \left[ (n_i + 1) \frac{x_i}{u_i} + (n_j + 1) \frac{x_j}{u_j} \right] + Y
\]

for \( i, j = d, f \) and \( i \neq j \)

In a symmetric equilibrium this simplifies to

\[
W = 2(n+1) \left( x - \frac{x^2}{u^2} \right) - 4\sigma (n+1) (2n+1) \left( \frac{x}{u} \right)^2 + Y
\]

In the next section, we analyze the general equilibrium of the model, and how the economy reacts to changes in parameter values.

4 Analysis of the Symmetric Equilibrium

In this section we present some results and analyze how the key variables in the model change as we vary the parameters of the system. We find the following result:

**Proposition 3: Consequences of Free Trade for the Wage Rate**

In a symmetric equilibrium, free trade results in a higher wage rate.

**Proof:** By comparing the equilibrium wage rates in the open and closed economy models (equations 2.29 and 3.32)\ Hoff.

**Corollary 3:**

In a symmetric equilibrium, free trade results in higher welfare.

**Proof:** By substituting the symmetric general equilibrium outcomes (table 1 in this and the previous chapters) into welfare indicators (equations 2.32 and 3.35) and then comparing these for the open and closed economy\ Hoff.

Proposition 3 and its corollary are familiar results from the literature on trade under imper-
fect competition (see Brander, 1981 and the recent survey by Chui et al., 2002). We proceed to analyze how the key variables of the model change with parameters, for the symmetric equilibrium. In this section we impose the restriction that all parameters are equal in both economies. Thus, we have that $\beta_i = \beta$, $\gamma_i = \gamma$, $\epsilon_i = \epsilon$, $L_i = L$, $u_{oi} = u_o$ ($i = d, f$) and $\sigma$ is always identical for both economies. We change each parameter, holding all other parameters constant.

We will present our analysis with the aid of figures similar to those used in Section 4 of chapter 2. In the figures we will show the equilibrium outcomes for the open economy, and for comparative purposes, we will also depict those for the closed economy. Open economy variables are shown as continuous lines and closed economy variables as dotted lines. In each figure, the top left graphs shows technological capability ($u$), the bottom left graph shows the number of firms in each country ($n + 1$), the top right graph shows the wage rate ($w$) and the bottom right graph shows exports and imports of goods produced by industry $X$. We show the analysis for $w_i > 1$ (the case of $w_i = 1$ is similar to that presented in chapter 2).

### 4.1 Analysis of Changes in $\beta$

We present the main results in Figure 4.1.

---

Equating the wage for the open and closed economies we find that they are equal along the following locus: $\sigma^* = \frac{\beta}{2\lambda - 1}$. $\sigma^*$ defines a boundary or separation in $(\beta, \sigma)$-space. The wage rate in the open economy is lower than under autarky above the $\sigma^*$ locus. Conversely, the wage rate in the open economy is higher than under autarky below the $\sigma^*$ locus.

As $\beta$ goes to infinity, $\sigma^*$ converges to its lower asymptote of 2. However, $\sigma \in (0, 1)$, so the open economy features a higher wage rate than autarky (hence Proposition 3). Nonetheless, the existence of $\sigma^*$ is of interest for chapter 5, where we will find that the region for which the closed economy features a higher wage rate than the open economy does include values of $\sigma \in (0, 1)$.

The wage separation locus emerges because there are two effects generating the difference in wages between the closed and the open economy. On the one hand, opening the economy yields a larger market size (a doubling of population). The increase in market size raises demand for labour via higher technological capability, and thus tends to raise wages, ceteris paribus. On the other hand, in a symmetric equilibrium half of the firms in each economy will exit. This reduces labour demand and tends to lower the wage rate, ceteris paribus. $\sigma^*$ defines the locus where the two effects exactly offset each other. Since welfare is determined by the wage rate, there also exists a separation locus for welfare.

In chapter 5 the wage separation locus becomes relevant, and identifies the conditions under which free trade may or may not be associated with a higher wage rate, relative to autarky.
We can see that technological capability is decreasing convexly, and it is higher in the open economy relative to the closed economy. This can be corroborated by comparing the equilibrium outcomes for technological capability for \( w > 1 \) (equations 2.30 and 3.33), from which we can see that technological capability in the open economy is the same as technological capability in the closed economy, with half the number of firms.

Meanwhile, the number of firms is increasing linearly, with more firms in the closed economy (to be precise, the number of firms in each open economy is half that in the closed economy). This is a reflection of the fact that technological capability is higher in each of the open economies relative to the closed economy, and this implies higher sunk costs.

To understand the consequences of this for the wage rate, we need to consider the demand for labour from industry \( X \). While the number of firms is higher in the closed economy (thereby generating relatively higher demand for labour from industry \( X \)), technological capability is lower (reducing -relative to the open economy- demand for labour from industry \( X \)). The two effects result in a stronger demand for labour in the open economy, leading to the result that the wage rate in the open economy is always higher than in the closed economy.

Finally, we have included in Figure 4.1 a graph depicting imports and exports by industry \( X \). We can observe that imports and exports take the same value, and hence the trade balance...
for industry $X$ is zero, as is the trade balance for industry $Y$ and for the whole economy. This is a result which is generated by the symmetry of the equilibrium under consideration\textsuperscript{3}, and will not extend to the case of asymmetric parameter values (presented in section 5).

The behaviour of exports and imports tracks that of the wage rate (by budget balancedness), and the model exhibits intra-industry trade in equilibrium: In a symmetric equilibrium consumers in both economies choose identical bundles of goods, and since each firm produces a differentiated product, the bundle includes goods from every firm in the world. Thus, since the distribution of firms in the symmetric equilibrium is such that half of the firms are located in each country, half of the output of each country is exported.

4.2 Analysis of Changes in $\sigma$

The following figure summarizes the results.

![Graphs showing technological capability, wage rate, number of firms, and exports-imports, with Open and Closed economies distinguished.](image)

Figure 4.2: The Effect of $\sigma$, open v. closed economy.

We can see that technological capability is increasing concavely, and is higher in the open economy, while the number of firms decreases convexly and is lower (by half) in the open economy.

---

\textsuperscript{3}Under a symmetric equilibrium, the trade balance cannot be different from zero. Otherwise $TB_x = -TB_y$ would not be satisfied: If the trade balance for industry $X$ is zero, the trade balance for industry $Y$ must also be zero.
economy. The wage schedules are 'U'-shaped, and the open economy wage rate lies above the closed economy wage rate. Exports are equal to imports, and they exhibit behaviour similar to the open economy wage rate. The intuition for this pattern of behaviour is similar to that outlined in section 4.2, chapter 2: On the one hand we have that when $\sigma$ is relatively low, the number of firms is large, with each firm having relatively low technological capability. The effect on labour demand of the falling number of firms is greater than the effect of rising technological capability. Whence the wage rate falls with $\sigma$ (the 'market structure' effect dominates the 'technological capability' effect). On the other hand, for relatively high values of $\sigma$ the situation is reversed. We now have a small number of firms, each with a relatively high level of technological capability. It is now the increase in technological capability, rather than the fall in the number of firms, that drives the demand for labour from industry $X$, and hence the wage rate becomes increasing (now it is the 'technological capability' effect that dominates the 'market structure' effect).

4.3 Analysis of Changes in $\varepsilon$

We present the results in the following figure.

![Figure 4.3: The Effect of $\varepsilon$, open v. closed economy.](image)

In Figure 4.3 we see that technological capability, the wage schedule, exports and imports are
decreasing convexly in $\varepsilon$, while the number of firms is invariant in $\varepsilon$. Technological capability is higher in the open economy, while the number of firms is lower. The wage rate is higher in the open economy.

### 4.4 Analysis of Changes in $u_0$

The effect of $u_0$ is shown in the following figure. $u_0$ has an effect similar to $1/\varepsilon$.

![Graph showing the effect of $u_0$, open v. closed economy.](image)

Figure 4.4: The Effect of $u_0$, open v. closed economy.

Figure 4.4 shows that technological capability increases linearly with $u_0$, and it is higher for the open economy. The number of firms does not change with $u_0$, and it is higher in the closed economy. The wage schedule increases convexly with $u_0$, as a consequence of the convexity of the labour requirement, $f(u) = \varepsilon \left( \frac{u}{u_0} \right)^\beta$. We find that the open economy features a higher wage rate. Export and imports of type $X$ goods increase convexly in $u_0$, tracking the wage rate.

### 4.5 Analysis of Changes in $L$

The effect of $L$ is summarized in the following figure.
In Figure 4.5 we can see that technological capability, the wage schedule, exports and imports are increasing concavely in $L$. The number of firms does not change with $L$. Technological capability is higher in the open economy, while the number of firms is lower. The wage rate is higher in the open economy.

This completes the analysis of the symmetric equilibrium with identical parameter values. In the next section, we analyze the case of asymmetries in initial conditions.

5 The Impact of Asymmetric Initial Conditions and the Welfare Consequences of Catching-Up

In section 3 we characterized a symmetric equilibrium, within a model of two identical economies. In this section, we ask the question: What happens when the economies no longer share the same initial conditions? We focus on the consequences of changing initial technological capability for one particular country ($u_{ol}$), while leaving all other parameters fixed. We examine the case of 'large' changes in $u_{ol}$. An analytical characterization is not possible in this case, and the results reported are based on numerical analysis.

After we analyze the impact of asymmetries in initial conditions, we proceed to investigate
the effects of industrial policy to allow the disadvantaged or laggard economy to catch-up with the advanced economy. The policy we will consider takes the form of a subsidy which reduces the marginal cost of technological capability. More specifically, industrial policy will imply that part of the cost of technological capability will be borne by the government. This subsidy will be financed with a lump-sum tax on consumers. We then ask about the welfare properties of such a scheme.

5.1 Asymmetric Initial Conditions

The procedure followed in this section is to start from a symmetric equilibrium, and then let $u_{od}$ diverge from its initial value (which, in a symmetric equilibrium, is identical in both economies). As we do this, we track the general equilibrium solutions of the model by using the equilibrium conditions we presented in section 3, given by expressions (3.23), (3.24) and (3.25). Overall, we have six conditions, three for each economy. These conditions determine the general equilibrium values for technological capability ($u_d, u_f$), for the number of firms ($n_d + 1, n_f + 1$) and for the wage rate ($w_d, w_f$). Once we have solved for these, all other variables in the system can be obtained by substitution. The system of equations in (3.23)-(3.25) yields explicit solutions only in the case of a symmetric equilibrium, in which parameters take the same values in both economies and both countries feature identical technological capabilities, number of firms and wage rates ($u_d = u_f = u$, $n_d + 1 = n_f + 1 = n + 1$ and $w_d = w_f = w$).

In the case that interests us in this section, where we allow for asymmetric initial conditions, the system of equations can only be solved numerically. The procedure is to begin from a symmetric equilibrium, with determined parameter values. We then change $u_{od}$ by a small amount and obtain the numerical solution such that (3.23)-(3.25) hold exactly, taking the initial symmetric equilibrium as a starting point for the equilibrium search algorithm. We then note the new equilibrium values, and again change $u_{od}$ by a small amount. Again we compute the new equilibrium values such that (3.23)-(3.25) hold exactly, taking the previous equilibrium equilibrium values (which are no longer symmetric) as the new starting point for the equilibrium search algorithm. This process was iterated until we had tracked the general equilibrium of the model for a wide range of $u_{od}$.

A graphical representation of the results is offered in Figures 5.1 and 5.2. In these figures, the horizontal axis shows the ratio of domestic initial technological capability (i.e., domestic 'initial conditions') to foreign initial technological capability (i.e., foreign 'initial conditions'): $u_{od}/u_{of}$. Showing the ratio of initial conditions is intuitively appealing, and since the only
parameter we are varying is \( u_{od} \), the ratio of initial conditions \( (u_{od}/u_{of}) \) induces identical behaviour to changes in \( u_{od} \). The vertical axes show the foreign and domestic (non-symmetric) equilibrium values of the key variables in the model.

For the analysis that follows it is convenient to introduce the following notions:

**Horizontal Symmetry**: Two schedules are said to be horizontally symmetric if we can find a point through which we can draw a horizontal benchmark line such that the schedules are symmetric around this line.

**Vertical Symmetry**: Two schedules are said to be vertically symmetric if we can find a point through which we can draw a vertical benchmark line such that the schedules are symmetric around this line.

We will see that some of the schedules presented below exhibit horizontal symmetry, although not vertical symmetry. The reference point around which we draw the benchmark lines will be the symmetric equilibrium.

It is useful to bear in mind three points depicted in Figures 5.1 and 5.2. Firstly we have the threshold \( u_{od}/u_{of} \) \( w_d = 1 \), which is defined as the level of \( u_{od}/u_{of} \) at which \( w_d \) reaches its lower bound of 1. Secondly we have \( u_{od}/u_{of} \) \( w_f = 1 \), which is the level of \( u_{od}/u_{of} \) at which \( w_f \) reaches its lower bound of 1. Finally we have the initial symmetric equilibrium, identified by \( u_{od}/u_{of} = 1 \). This value of \( u_{od}/u_{of} \) is associated with the crossing of the domestic and foreign (open economy) wage schedules, as well as the crossing of schedules for the number of firms, per-capita demand of good \( Y \), exports and imports of type \( X \) goods and the trade balances.

The figures show three regimes, which will be determined by the values taken by the wage rates in both economies. The first regime is characterized by values of \( u_{od}/u_{of} \) such that \( u_{od}/u_{of} \leq u_{od}/u_{of} \) \( w_d = 1 \), where \( u_{od}/u_{of} \) \( w_d = 1 \) is the point at which the domestic wage rate reaches its lower bound of 1. The second regime is characterized by values of \( u_{od}/u_{of} \) such that \( u_{od}/u_{of} \geq u_{od}/u_{of} \) \( w_f = 1 \), where \( u_{od}/u_{of} \) \( w_f = 1 \) is the point at which the foreign wage rate reaches its lower bound of 1. The third regime lies between regimes 1 and 2, and is characterized by values of \( u_{od}/u_{of} \) such that \( u_{od}/u_{of} \) \( w_d = 1 < u_{od}/u_{of} < u_{od}/u_{of} \) \( w_f = 1 \). In this regime, both wage rates lie strictly above their lower bounds: \( w_d > 1 \) and \( w_f > 1 \).

As before, we use the notion of the 'shadow-value' of a variable. This refers to the value the variable would have taken had the wage rate not had a lower bound equal to 1. The counterpart to the shadow value of a variable will be labelled the 'actual' value, or simply referred to by the name of the variable.

Figure 5.1 depicts technological capability (top left graph), the number of firms (bottom left graph), the wage rate (top right graph) and employment in industries \( X \) and \( Y \) (bottom right
Values for the open domestic economy are shown as a continuous thick line. Foreign open economy values are shown as a dashed thick line. Domestic closed economy values are shown as a dotted thin line, and foreign closed economy values are shown as a continuous thin line.

Figure 5.1. The Impact of Changes in Relative Initial Conditions on: Technological Capability, Market Structure, Income and the Distribution of Employment.

Initial conditions \( u_{oi} \) affect the marginal cost of technological capability. A higher level of \( u_{oi} \) reduces the marginal cost of achieving technological capability level \( u_i \) for firms in country \( i \). On the other hand, firms in the other country now face a disadvantage, and must choose their optimal response to the change of technological capability in the advantaged country, in order to maintain a Sub-Game Perfect Nash (General) Equilibrium. In the top left graph, we find that both the domestic and the foreign economies choose identical levels of technological capability (shown by the continuous thick line). While one economy is improving its relative initial conditions (in this case the domestic economy), firms in the other economy must match technological capability in order to maintain a positive market share.

For values of  \( u_{od}/u_{of} \geq u_{od}/u_{of}^{ui} \) = 1 we find that technological capability grows at a slower rate. To see why this occurs, note that for these values of \( u_{od}/u_{of} \) the foreign wage rate is at
its lower bound of 1. Consequently foreign technological capability becomes increasingly more expensive relative to its shadow value. Firms in the domestic economy are content to match foreign firms’ technological capability and thereby spend less on fixed outlays than would have been the case had the foreign wage rate not reached its lower bound.

For $\frac{u_{od}}{u_{of}} \leq u_{od}/u_{of} w_{d}=1$, we find that technological capability is growing at a faster rate in $u_{od}/u_{of}$. For this range, the domestic wage rate is 1 and the shadow wage rate lies below the actual wage rate. As $u_{od}/u_{of}$ grows, the gap between the shadow marginal cost of technological capability and its actual counterpart is shrinking, and thus actual technological capability catches up with shadow technological capability. Correspondingly, firms in the foreign economy match (actual) domestic technological capability.

We also show technological capability for the closed economies. The continuous thin horizontal line shows technological capability for the foreign closed economy (in which nothing changes since $u_{of}$ is held fixed throughout this analysis). The dotted thin line shows technological capability for the closed domestic economy.

The number of firms is shown in the bottom left graph. We can see that as $u_{od}/u_{of}$ grows the number of domestic firms increases, while the number of foreign firms contracts. This reflects the increase in fixed outlays in the foreign economy, which drives out foreign firms, while domestic entrants fill in the gaps left by the foreign firms. This replacement process is accelerated when $u_{od}/u_{of} \geq u_{od}/u_{of} w_{d}=1$. In this case the foreign wage rate is fixed at 1, and the foreign industry $X$ contracts at a faster rate, allowing the domestic industry $X$ to expand. Meanwhile, the world number of firms remains constant (not shown). For $u_{od}/u_{of} \leq u_{od}/u_{of} w_{d}=1$, we find that as $u_{od}/u_{of}$ rises the number of domestic firms rises (and the number of foreign firms falls) at a steeper rate relative to the case when $u_{od}/u_{of} w_{d}=1 < u_{od}/u_{of} < u_{od}/u_{of} w_{d}=1$. This reflects the same mechanism as for $u_{od}/u_{of} \geq u_{od}/u_{of} w_{d}=1$: For $u_{od}/u_{of} \leq u_{od}/u_{of} w_{d}=1$ the domestic wage rate is fixed at 1 and lies above the shadow wage rate for the domestic economy. Moreover, the further to the left we are from $u_{od}/u_{of} w_{d}=1$, the greater the distance between the actual wage rate of 1 and the shadow wage rate. Correspondingly, this implies that as $u_{od}/u_{of}$ grows, the number of domestic firms expands faster as the gap between the domestic shadow wage rate and its actual counterpart is reduced. Meanwhile, in the foreign economy the number of firms falls in order to adjust so that the world number of firms is constant. We can see that the schedules for the number of firms are horizontally symmetric around the symmetric equilibrium.

Turning now to the wage schedules, we have that the increase in foreign labour demand associated to the rise in technological capability is insufficient to offset the fall associated with
the contraction of the number of foreign firms. Accordingly, the foreign wage rate contracts as \( u_{od}/u_{of} \) rises. On the other hand, the domestic industry \( X \) is enjoying an increase in both technological capability and in the number of firms, leading to a rise in the domestic wage rate.

In the wage diagram we have also shown the wage rate for the closed economies: the domestic economy as the dotted thin line and the foreign economy as the continuous thin line. We can see that these lines intersect those of the corresponding open economy. The intersection of the domestic open economy wage with that of the domestic closed economy is labelled \( od\cd\), and we will refer to this point as \( u_{od}/u_{of} \). On the other hand, the intersection of the foreign open economy wage with that of the foreign closed economy is labelled \( of\cf\), and we will refer to this point as \( u_{of}/u_{of} \). The implication is that for values of \( u_{od}/u_{of} \) lower than that associated with point \( od\cd \), the domestic economy achieves a higher wage rate (and welfare) remaining closed, while the foreign economy achieves a higher wage rate under free trade. Similarly, for values of \( u_{of}/u_{of} \) above that associated with point \( of\cf \), we find that it is the foreign economy which achieves a higher wage rate under autarky, while the domestic economy achieves higher wages under free trade.

This raises an important issue: The notion that one economy (the one with relatively better initial conditions) could seek to impose on the other economy a trade regime which is not welfare enhancing for the disadvantaged economy. This may shed some light on the nature of bilateral trade negotiations between advanced and laggard economies. This important result is summarized in the following proposition:

**Proposition 4. Initial Conditions, free trade and the wage rate**

i) For \( u_{od}/u_{of} < u_{od}/u_{of} \cd \), the domestic economy achieves a strictly higher wage rate under autarky, and the foreign economy achieves a strictly higher wage rate under free trade.

ii) For \( u_{od}/u_{of} > u_{od}/u_{of} \cf \), the foreign economy achieves a strictly higher wage rate under autarky, and the domestic economy achieves a strictly higher wage rate under free trade.

iii) For \( u_{od}/u_{of} \cd < u_{od}/u_{of} < u_{od}/u_{of} \cf \), both economies achieve a strictly higher wage rate under free trade.

iv) For \( u_{od}/u_{of} = u_{od}/u_{of} \cd \) the domestic wage rate is equal under autarky or free trade.

v) For \( u_{od}/u_{of} = u_{od}/u_{of} \cf \) the foreign wage rate is equal under autarky or free trade.

This proposition states precisely the conditions under which different trade regimes will generate higher wage rates in different economies. So if the two economies are not too different regarding their initial conditions, free trade will generate higher wage rates in both economies.
However, if initial conditions are too dissimilar across economies, the laggard economy will achieve a higher wage rate by remaining in autarky. On the other hand, the advanced economy will achieve a higher wage rate under free trade. Hence the advanced economy has incentives to negotiate free trade agreements which do not necessarily benefit both parties.

Finally we analyze the bottom right diagram, showing employment in industries $X$ and $Y$. For $u_{od}/u_{of} w_d = 1 < u_{od}/u_{of} w_f^* = 1$, labour demand is sufficiently high to ensure that all of the economies' labour endowment is employed in industry $X$. For $u_{od}/u_{of} \leq u_{od}/u_{of} w_d = 1$ labour demand from the domestic industry $X$ is smaller than the domestic labour endowment. Consequently, as $u_{od}/u_{of}$ falls, employment in the domestic industry $X$ falls below the labour endowment. Parallel to this, employment in the domestic industry $Y$ expands as it absorbs surplus workers. Meanwhile, the foreign economy still features sufficiently high labour demand in industry $X$ to employ all of the labour endowment. For $u_{od}/u_{of} \geq u_{od}/u_{of} w_f^* = 1$ it is the foreign industry $X$ which cannot generate sufficient labour demand to employ all of the foreign labour force, so as $u_{od}/u_{of}$ rises, employment in the foreign industry $X$ contracts. Accordingly, employment in the foreign industry $Y$ rises to take in any redundant workers.

The next figure depicts per-capita demand for type $Y$ goods (left panel), exports and imports of type $X$ goods (centre panel) and the trade balance in type $X$ goods (right panel).

![Figure 5.2](image)

Figure 5.2. The Impact of Changes in Relative Initial Conditions on: Per-Capita Demand of Good $Y$, Exports and Imports of Type $X$ Goods and the Trade Balance.

In the central panel of Figure 5.2 we observe that exports and imports of type $X$ goods closely resemble the pattern displayed by the wage rate. When either of the wage rates reach their lower bound of 1 (that is, when either $u_{od}/u_{of} \leq u_{od}/u_{of} w_d = 1$ or $u_{od}/u_{of} \geq u_{od}/u_{of} w_f^* = 1$), exports of that country fall at a faster rate, reflecting the increased rate of contraction for the country's number of firms. Conversely, the other country's exports expand more rapidly.
as a consequence of the increased rate of growth for that country's number of firms. This is reflected on the trade balance (right panel), which is measured by the distance between a country's exports and imports of type $X$ goods.

Per-capita demand of type $Y$ goods is shown on the left panel. For $u_{od}/u_{of} \ w_d=1 < u_{od}/u_{of} < u_{od}/u_{of} \ w_f=1$, demand for good $Y$ exhibits similar behaviour to the corresponding trade balance. The high wage country is a net consumer of type $Y$ goods, while the low wage country is a net supplier. The low wage country depletes its endowment of type $Y$ goods in order to finance its negative trade balance in type $X$ goods. For $u_{od}/u_{of} \leq u_{od}/u_{of} \ w_d=1$, we find that as the domestic wage rate reaches its lower bound, production of type $Y$ goods expands as $u_{od}/u_{of}$ shrinks. This allows the domestic economy to cover its trade deficit in type $X$ goods, and to eventually become a net consumer of type $Y$ goods too (as evidenced by the crossing of the type $Y$ goods demand and the endowment line, $\overline{Y}$, point 'a' in Figure 5.2). On the other hand, for $u_{od}/u_{of} \geq u_{od}/u_{of} \ w_f=1$, the pattern is similar, with the roles of domestic and foreign reversed.

5.2 Industrial Policy and Catching-Up

Suppose an economy faces a historically given asymmetry in initial conditions, to its disadvantage. In our analysis this corresponds to a location leftward of the symmetric equilibrium in Figures 5.1 and 5.2. We now ask the following questions:

1) Is it possible for this economy to subsidize industry $X$ and catch-up with the advantaged economy (in terms of income and welfare), taking into account the strategic response by foreign rivals?

2) If catching-up is possible, is it welfare improving to do so?

We consider the perspective of the domestic economy (the identity of the disadvantaged economy is immaterial). A lump-sum tax is imposed on consumers, and then redistributed to firms in industry $X$ via a reduction in the marginal cost of technological capability. The reduction in the marginal cost of technological capability can be achieved by reducing $\varepsilon_d$ or $\beta_d$. As we reduce either of these parameters, firms in the domestic economy will increase their fixed outlays and thereby achieve higher technological capability, relative to their optimal level without industrial policy. This generates higher demand for labour, and thus a rise in the wage rate. Concentration does not change in the domestic economy, since fixed outlays paid by firms do not change with the subsidy, hence the only effect of industrial policy occurs via the 'technological capability effect' (see section 4). The difference between fixed outlays
with industrial policy and those obtained without intervention will be covered by the subsidy (financed by the lump-sum tax).

The top panel in Figure 5.3 shows a schedule depicting the level of $\beta_d$ or $\epsilon_d$ required to ensure that the laggard economy catches-up with the advanced economy, for a given level of asymmetry in initial conditions. As the foreign economy’s initial conditions improve relative to the domestic economy’s ($u_{ad}/u_{af}$ falls), the domestic economy must reduce its values of $\beta_d$ or $\epsilon_d$ if it is to maintain a wage rate equal to the foreign economy’s. Levels of $\beta_d$ or $\epsilon_d$ above the schedule imply that the foreign wage rate will be higher than the domestic wage rate. Levels of $\beta_d$ or $\epsilon_d$ below the schedule imply that the foreign wage rate will be lower than the domestic wage rate.

The bottom diagram shows the net welfare gain of catching up with the foreign economy, as measured by the welfare indicator (expression 3.35). The net welfare gain will depend on whether the higher wage rate associated with industrial policy is higher than the wage rate without intervention (net of the lump-sum tax associated to industrial policy). We find that whenever the domestic economy is lagging behind, it is always welfare improving to impose a

---

4 Both parameters generate similar behaviour in the diagrams depicted in Figure 5.3, although their effect on other variables is not always identical.
lump-sum tax in order to finance a subsidy to R&D, in the form of paying for the difference between the non-intervention fixed outlays benchmark and the fixed outlays outcome with industrial policy.

6 Conclusions

In this chapter we have extended the closed economy model presented in chapter 2 to an open economy setting, with strategic interaction and endogenous terms of trade (i.e., we have considered the economies to be large, as opposed to considering the small open economy case). In doing so, the following results have emerged:

We found that the marginal benefit of the vertical differentiation variable is decreasing in the overseas rivals' value of such a variable. In particular, the marginal benefit of technological capability is decreasing in rivals' technological capability. The implication of this is that there exists a threshold for the ratio of technological capabilities (that is, the technology gap), above which the marginal benefit becomes negative. This in turn means that in the absence of industrial policy, catching up will be feasible only if the technological gap is not too wide.

We found that the model exhibits intra-industry trade in all occasions, and free trade is welfare improving whenever both economies are identical. However, when we introduce asymmetric initial conditions, we find that a backward economy (with low technological capability) will benefit from trade with an advanced economy (with high technological capability), provided the gap between technological capabilities is not too large. If the gap is too large, the backward economy achieves a higher wage rate (and welfare) in autarky. On the other hand, the advanced economy always benefits from trading with the backward economy. This clarifies the incentives for bilateral free trade agreements between an advanced and a backward economy.

If we compare the results from chapter 2 (Proposition 1), with the open economy symmetric equilibrium, we find that the development strategy conclusions for the closed economy also follow for the open economy: The 'high-tech' configuration is still associated with a higher wage rate (and welfare) than the 'proliferation' configuration, unless $\sigma$ is low (in which case it is the proliferation configuration which yields the higher wage rate and welfare). So, in considering the symmetric equilibrium, what is good for the closed economy is even better for the open economy (since the latter will feature an even higher wage rate and welfare).

---

5 Recall the definitions given in chapter 2:

1) A 'high-tech' configuration: The economy has few firms, each with a high level of technological capability.

2) A 'proliferation' configuration: The economy has many firms, each with a low level of technological capability.
Once asymmetries in initial conditions are introduced, the question arises of whether catching-up is feasible and welfare enhancing. We analyze this issue by introducing industrial policy in the form of a subsidy to technological capability, funded via a lump-sum tax. The subsidy takes the following shape: The government pays for the extra investment in technological capability required to ensure that the backward economy achieves the same wage rate as the advanced economy. We find that with this policy catching-up is feasible and it is welfare enhancing (after accounting for lump-sum taxes).

These conclusions run the risk of being misinterpreted as supporting government intervention in the form of closing the economy and subsidizing industries in backward economies. It is important to stress that closing the economy is not what is being suggested. Rather, the conclusion is that under Cournot competition trade agreements between economies that are not too dissimilar in their technological capabilities will be beneficial. The focus that emerges is: How can we transform the economy so that it will benefit from free trade? The transformation implies ensuring that the technological capability gap is sufficiently small to achieve welfare gains from free trade, and will invariably lead to higher wages than permanent closure the economy.

This leads us to the second caveat on intervention. Given that we wish to close the technological capability gap, it seems that a subsidy to investment in technological capability is sufficient to achieve this. The developing world is full of examples of failed industrial policies. Such policies are ripe for capture by rent-seekers. Once the subsidies are instituted, the resources devoted to such directly-unproductive rent-seeking activities (Bhagwati, 1982) constitute a heavy burden on the rest of the economy. Moreover, apart from introducing distortions in the allocation of resources, our calculations assume that the opportunity cost of the subsidy is unity (taking the form of forgone consumption). This is a strong assumption, for it is likely that the effectiveness of subsidies in raising technological capability is less than that of private efforts. In particular, if the subsidy loses efficiency as the technological capability gap widens, it is likely that industrial policy will be welfare improving only when the technology gap is not too large.

Such considerations highlight the need to devote resources to studying the precise form of intervention, particularly on how to make such schemes impervious to rent-seeking. The mechanism design literature (see Bolton and Dewatripont, 2005; Salanié, 2005) could provide useful insights in this task. Until such issues have been dealt with, the case for industrial policy remains dubious. Nonetheless, our findings provide a benchmark for such (more detailed) studies.
Appendix 1: Solving the Final Stage Subgame for Industry X

In this Appendix, we solve the final stage subgame (Cournot competition), in order to obtain 'solved-out payoff' functions for domestic and foreign firms. Recall the first order conditions for this stage of the game (equation 3.8):

\[(L_d + L_f) \frac{\partial \pi_{ki}}{\partial u_{ki}} = \frac{w_i \varepsilon_i \beta_i}{u_{ki}} (u_{ki} - \pi_{ci})^{\beta_i} \text{ for } i = d, f \text{ and } k = 1, \ldots, n_i + 1\]

Let us begin by substituting the inverse demand functions (equation 3.3) and its derivative \((\frac{\partial \pi_{ki}}{\partial x_{ki}} \text{ for } i = d, f)\) into the first order conditions. Adding and subtracting \(2\sigma \frac{\pi_{ji}^2}{u_{ki}}\), we obtain

\[1 - 2(2 - \sigma) \frac{x_{ki}}{u_{ki}} - 2\sigma \left( \sum_{i=1}^{n_i+1} \frac{x_{il}}{u_{li}} + \sum_{j=1}^{n_j+1} \frac{x_{lj}}{u_{lj}} \right) = 0 \text{ for } i, j = d, f \text{ and } i \neq j \]  
(A1.1)

We multiply (A1.1) through by \(u_{ki}\) and re-organize the expression as follows

\[\frac{x_{ki}}{u_{ki}} = \frac{u_{ki} - 2\sigma \left( \sum_{i=1}^{n_i+1} \frac{x_{il}}{u_{li}} + \sum_{j=1}^{n_j+1} \frac{x_{lj}}{u_{lj}} \right)}{2(2 - \sigma)} \text{ for } i, j = d, f \text{ and } i \neq j \]  
(A1.2)

The next step is to sum (A1.2) over \(k\) and solve for \(\sum_{j=1}^{n_j+1} \frac{x_{ki}}{u_{ki}}\). This yields

\[\sum_{i=1}^{n_i+1} \frac{x_{il}}{u_{li}} = \frac{\sum_{i=1}^{n_i+1} u_{li} - 2\sigma (n_i + 1) \sum_{i=1}^{n_j+1} \frac{x_{lj}}{u_{lj}}}{2 (2 + \sigma n_i)} \text{ for } i, j = d, f \text{ and } i \neq j \]  
(A1.3)

The equations in (A1.3) constitute a pair of linear equations in \(\sum_{i=1}^{n_i+1} \frac{x_{ki}}{u_{ki}}\) and \(\sum_{i=1}^{n_j+1} \frac{x_{li}}{u_{li}}\). Solving for these we obtain

\[\sum_{i=1}^{n_i+1} \frac{x_{il}}{u_{li}} = \frac{(2 - \sigma) \sum_{i=1}^{n_i+1} x_{ki} - \sigma (n_i + 1) \sum_{j=1}^{n_j+1} x_{lj}}{2(2 - \sigma)(2 + \sigma(n_i + n_j + 1))} \text{ for } i, j = d, f \text{ and } i \neq j \]  
(A1.3')

To obtain the solution for \(x_{ki}\), substitute expression (A1.3') back into (A1.2). This yields the following solution for \(x_{ki}\)

\[x_{ki} = \frac{u_{ki}^2}{2(2 - \sigma)} \left[ 1 - \frac{\sigma}{2 + \sigma(n_i + n_j + 1)} \left( \sum_{i=1}^{n_i+1} \frac{u_{li}}{u_{ki}} + \sum_{j=1}^{n_j+1} \frac{x_{lj}}{u_{ki}} \right) \right] \]  
(A1.4)

for \(i, j = d, f \text{ and } i \neq j\)
Symmetry between firms within each country, such that $u_{ki} = u_{ii} = u_i$ and $u_{kj} = u_{ij} = u_j$, yields the following simplification

$$x_i = \frac{u_i^2}{2(2-\sigma)} \left\{ 1 - \frac{\sigma}{2 + \sigma(n_i + n_j + 1)} \left[ (n_i + 1) + (n_j + 1) \frac{u_j}{u_i} \right] \right\} \quad (A1.4')$$

for $i, j = d, f$ and $i \neq j$

With symmetry across countries, such that $u_d = u_f = u$, and noting that $N = 2n + 1$, this expression becomes

$$x = \frac{u^2}{2(2 + \sigma N)} \quad (A1.4'')$$

which is equation (A1.4').

To solve for prices $(p_{ki})$, add and subtract $2\sigma x_{ki}$ to the inverse demand functions (equation 3.3), to obtain

$$p_{ki} = 1 - 2(1 - \sigma) \frac{x_{ki}}{u_{ki}^2} - \frac{2\sigma}{u_{ki}} \left( \sum_{i=1}^{n+1} \frac{x_{ii}}{u_{ii}} + \sum_{i=1}^{n+1} \frac{x_{ij}}{u_{ij}} \right) \quad \text{for } i, j = d, f \text{ and } i \neq j \quad (A1.5)$$

Next substitute $x_{ki}$ from equation (A1.4) and the expressions in (A1.3') into (A1.5). This yields the solution for $p_{ki}$:

$$p_{ki} = \frac{1}{2 - \sigma} \left[ 1 - \frac{\sigma}{2 + \sigma(n_i + n_j + 1)} \left( \sum_{i=1}^{n+1} \frac{u_{ii}}{u_{ki}} + \sum_{i=1}^{n+1} \frac{u_{ij}}{u_{ki}} \right) \right] \quad \text{for } i, j = d, f \text{ and } i \neq j \quad (A1.6)$$

By imposing symmetry between firms within each economy (such that $u_{ki} = u_{ii} = u_i$ and $u_{kj} = u_{ij} = u_j$), we obtain the following simplified solution

$$p_i = \frac{1}{2 - \sigma} \left\{ 1 - \frac{\sigma}{2 + \sigma(n_i + n_j + 1)} \left[ (n_i + 1) + (n_j + 1) \frac{u_j}{u_i} \right] \right\} \quad (A1.6')$$

for $i, j = d, f$ and $i \neq j$

If there is symmetry across economies, such that $u_d = u_f = u$, and noting that $2n + 1 = N$, this expression becomes

$$p = \frac{1}{2 + \sigma N} \quad (A1.6'')$$

which is equation (A1.6'), but now $N$ represents the worldwide number of firms (minus one).
The solved-out payoff is given by the product of equations (A1.4) and (A1.6). This yields

\[
\pi_{ki}(U) = \frac{u_{ki}^2}{2(2-\sigma)^2} \left[ 1 - \frac{\sigma}{2 + \sigma(n_i + n_j + 1)} \left( \sum_{i=1}^{n_i+1} \frac{u_{ki}}{u_{ki}} + \sum_{i=1}^{n_j+1} \frac{u_{kj}}{u_{kj}} \right) \right]^2 \tag{A1.7}
\]

for \( i, j = d, f \) and \( i \neq j \)

The solved-out payoff is written as equation (3.9) in the main body of this chapter.

**Appendix 2: Second order conditions for the Second Stage Subgame**

The second order conditions are obtained by differentiating the first order conditions (equation 3.12) with respect to \( u_{ki} \) for \( i = d, f \). We obtain the following

\[
(L_d + L_f) \frac{1}{(2-\sigma)^2} \left[ \frac{2 + \sigma(n_i + n_j)}{2 + \sigma(n_i + n_j + 1)} \right]^2 \leq \frac{u_{ki}\beta_i(\beta_i - 1)u_{ki}^{\beta_i-2}}{v_{oi}^{\beta_i}}
\]

for \( i = d, f \) and \( k = 1, ..., n^i + 1 \) \tag{A2.1}

If we substitute out \( u_{ki}, n_i \), and \( w_i \) with their equilibrium values, (A2.1) provides a restriction on \((\sigma, \beta_i)\)-space. This restriction implies that \( \sigma \) cannot be too high and \( \beta_i \) cannot be too low (in any case we require \( \beta_i > 2 \)). If any one of these parameters crosses its bound, the other will need to compensate by moving inward from its bound.

**Appendix 3: Value Added and the Trade Balance**

Value added is identical to the wage rate because labour is the only production factor, profits are zero and there are no international monetary transfers. We can trace value added generated by industry \( X \) and by industry \( Y \). We consider measurement of value added on the production, income and expenditure side. Consider first industry \( X \).

Since there are no intermediate goods, value added in industry \( X \) measured on the production side is equal to world-wide revenue from good \( X \), which is given by \((L_d + L_f)(n_i + 1)p_i x_i\) for \( i = d, f \). Dividing by \( L_i \) yields per-capita value added (as measured by production):

\[
VAP_{xi} = VA_{xi} = \frac{(L_d + L_f)(n_i + 1)p_i x_i}{L_i} \text{ for } i = d, f \tag{A3.1}
\]

As measured by income, value added in industry \( X \) is given by workers' wages \((VAI_{xi} = w_i L_{xi} \text{ for } i = d, f)\). Moreover, employment in industry \( X \) \((L_{xi})\) is given by \((n_i + 1)f_i(w_i)\)
(where \( f_i(u_i) \) was defined in section 2.2.2 as \( f_i(u_i) = \varepsilon_i (\frac{w_i}{\omega_i})^{\beta_i} \)). The free entry condition (2.14) can be written as \((L_d + L_f) \pi_i = w_i f_i(u_i)\), so \( f_i(u_i) = (L_d + L_f) \pi_i / w_i\). From equation (3.7) it follows that \( \pi_i = p_i x_i \), substitution of this into \( VAI_{it} \), yields the same result as above: \( VAI_{it} = VA_{it} = (L_d + L_f) (n_i + 1) p_i x_i \) for \( i = d, f \).

As measured by expenditure, value added in industry \( X \) is made up by consumption expenditure and net exports of good \( X \) (i.e., the trade balance in industry \( X \)), since there are no savings (investment) in this model. Consumption of good \( X \) is \( L_i [(n_i + 1) p_i x_i + (n_j + 1) p_j x_j] \) in country \( i \). Exports of good \( X \) are equal to \( L_j (n_i + 1) p_i x_i \), while imports of good \( X \) are \( L_i (n_j + 1) p_j x_j \) for \( i, j = d, f \) and \( i \neq j \). The trade balance for industry \( X \) can be written as

\[
TB_{xi} = L_j (n_i + 1) p_i x_i - L_i (n_j + 1) p_j x_j \quad \text{for} \ i, j = d, f \quad \text{and} \ i \neq j \tag{A3.2}
\]

By adding consumption expenditure and the trade balance, dividing by population \( (L_i) \) and simplifying we obtain the same result previously found in per-capita terms: \( VAE_{it} = VA_{it} = \frac{(L_d + L_f)}{L_i} (n_i + 1) p_i x_i \).

Value added for the production side in industry \( Y \) is \( VAP_{yi} = q_i (L_i - L_{xi}) \). Noting that \( q_i = 1 \) (good \( Y \) is the numeraire), hence value added in industry \( Y \) is the supply of \( Y \) goods, which (from the 1:1 technology) is the same as employment in industry \( Y \) \( (L_{yi}) \). Per-capita, this is given by:

\[
VAP_{yi} = VA_{yi} = (L_i - L_{xi}) / L_i = L_{yi} / L_i \quad \text{for} \ i = d, f \tag{A3.3}
\]

Measured by income, value added in industry \( Y \) is equal to wages obtained by workers in industry \( Y \), \( VAI_{yi} = w_i L_{yi} \). From section 2.3 we have that whenever \( L_{yi} > 0 \), the wage rate is unity. So we obtain: \( VAI_{yi} = VA_{yi} = L_{yi} / L_i \) (the same result as measured by production).

Measured by expenditure, value added in industry \( Y \) is constituted by consumption and net exports of good \( Y \). Consumption expenditure is defined from equation (3.4) as

\[
L_i [w_i - (n_i + 1) p_i x_i - (n_j + 1) p_j x_j]
\]

Net exports (i.e., the trade balance for industry \( Y \)) are given by the difference between aggre-
gate supply and aggregate demand of good $Y$:

$$TB_{yi} = \frac{L_{yi}}{L_i} - L_i \left[ u_i - (n_i + 1) p_i x_i - (n_j + 1) p_j x_j \right] \text{ for } i, j = d, f \text{ and } i \neq j \quad (A3.4)$$

Aggregate Supply

Aggregate Demand

and we obtain the same (per-capita) value added as measured by production and income:

$$V A E_{yi} = V A_{yi} = L_{yi}/L_i.$$  

Total (per-capita) value added is the sum of value added for both industries: $V A_i = V A_{xi} + V A_{yi}$. The trade balance for each economy is given by $TB_i = TB_{xi} + TB_{yi}$ with the following relationships holding: $TB_i = -TB_j$, $TB_{xi} = -TB_{xj}$ and $TB_{yi} = -TB_{yj}$ for $i, j = d, f$ and $i \neq j$.

Appendix 4: Proof of Proposition 2.

The proof proceeds by obtaining an expression which makes it clear that any asymmetry of the above kind is inconsistent with the general equilibrium conditions of the model. We attain this by working with the equilibrium conditions for each economy: (3.23), (3.24) and (3.25). To simplify these expressions, define:

$$A = 2 + \sigma (n_i + 1)$$

We can then re-write the equilibrium conditions as follows.

The first equilibrium condition is given by expression (3.23), and it is a mapping from $u_j$ to $u_i$, such that along this mapping no individual firm wishes to deviate from its strategy:

$$(L_d + L_f) \frac{(A - \sigma) \sigma}{(2 - \sigma)^2 A^2} \left[ \frac{A}{\sigma} - (n_i + 1) - (n_j + 1) \frac{u_j}{u_i} \right] = w_i \varepsilon \beta \frac{u_i^{\beta - 2}}{u_0^{\beta}} \quad i, j = d, f, i \neq j \quad (A4.1)$$

Secondly we have the zero profit (free entry) condition, from (3.24), which simplifies to

$$(L_d + L_f) \frac{\sigma^2}{2 (2 - \sigma)^2 A^2} \left[ \frac{A}{\sigma} - (n_i + 1) - (n_j + 1) \frac{u_j}{u_i} \right]^2 = w_i \varepsilon \frac{u_i^{\beta - 2}}{u_0^{\beta}} \quad i, j = d, f, i \neq j \quad (A4.2)$$

115
Thirdly we have the labour market clearing conditions

\[ L_i = L_{y_i} + (n_i + 1) \varepsilon \left( \frac{u_i}{u_0} \right)^\beta \quad i = d, f \]  

(A4.3)

If \( L_{y_i} > 0 \), then \( w_i = 1 \). Otherwise \( w_i > 1 \). For the case of \( w_i > 1 \), we can divide the labour market clearing condition for economy \( i \) by that for economy \( j \) to obtain the following expression:

\[ \frac{u_i}{u_j} = \left( \frac{n_j + 1}{n_i + 1} \right)^{1/\beta} \quad i = d, f \]  

(A4.4)

Squaring (A4.1) we obtain:

\[ \left( \frac{L_d + L_f}{(A - \sigma)(2 - \sigma)^2 A^2} \right)^2 \frac{\sigma}{(n_i + 1) - (n_j + 1) \frac{u_j}{u_i}} = \left( \frac{w_i \varepsilon \beta \frac{u_i^{\beta-2}}{u_0^{\beta-2}}}{w_i} \right) \quad i, j = d, f, i \neq j \]  

(A4.5)

Dividing (A4.5) by (A4.2) we obtain the following relationship:

\[ (L_d + L_f) \frac{(A - \sigma)^2}{(2 - \sigma)^2 A^2} = w_i \varepsilon \beta \frac{u_i^{\beta-2}}{u_0^{\beta-2}} \quad i = d, f \]  

(A4.6)

Dividing the expression in (A4.6) for economy \( i \) by the corresponding expression for economy \( j \) we obtain:

\[ \frac{u_i^{\beta-2}}{u_j^{\beta-2}} = \frac{w_j}{w_i} \]  

(A4.7)

We now proceed to divide expression (A4.1) for economy \( i \) by its counterpart in economy \( j \):

\[ \frac{A}{\sigma} - (n_i + 1) - (n_j + 1) \frac{u_j}{u_i} = \frac{w_i \varepsilon \beta \frac{u_i^{\beta-2}}{u_0^{\beta-2}}}{w_j \frac{u_j^{\beta-2}}{u_0^{\beta-2}}} \quad i, j = d, f, i \neq j \]  

(A4.8)

Substituting (A4.7) into (A4.8) we can simplify the latter to:

\[ (n_i + 1) + (n_j + 1) \frac{u_j}{u_i} = (n_j + 1) + (n_i + 1) \frac{u_i}{u_j} \quad i, j = d, f, i \neq j \]  

(A4.9)

We now substitute expression (A4.4) in (A4.9) and we obtain an expression relating \( n_i \) to \( n_j \):

\[ (n_i + 1) \left[ 1 - \left( \frac{n_j + 1}{n_i + 1} \right)^{\beta-1} \right] = (n_j + 1) \left[ 1 - \left( \frac{n_i + 1}{n_j + 1} \right)^{\beta-1} \right] \quad i, j = d, f, i \neq j \]  

(A4.10)

From this expression it is clear that if \( n_i \neq n_j \) the left hand side will be of opposite sign to the

\footnote{The same result obtains if we divide the expressions in (A4.2).}
right hand side, which is a contradiction. It follows that for both economies to have the same number of firms they must also have identical technological capabilities and wage rates.

It remains to consider the case when \( L_{yi} > 0 \). In this case \( u_i = 1 \) and (A4.7) tells us that \( u_i = u_j \), and the result follows through as before.
Part II

Introducing Collusion into a General Equilibrium Framework with Oligopolistic Interactions
Chapter 4

Collusion in General Equilibrium with Oligopolistic Interactions: Autarky

1 Introduction

This chapter extends the closed economy framework developed in chapter 2 by incorporating profit sharing in industry $X$. To achieve this, we introduce a generalized profit function which is a weighted average of a firm's and its rivals' profits. The generalized profit function provides a directly measurable alternative to the (non-measurable) notion of conjectural variations. This function provides the basis for a discussion of competition policy, which is characterized by a parameter in the generalized profit function (denoted by $\gamma$). By varying this parameter, we encompass all intensities of competition ranging from individual profit maximization to perfect collusion (joint profit maximization). This is of interest not only to development economists, but to industrial economists at large, since it provides an alternative to conjectural variations, with the considerable advantage that it is directly measurable.

Aside from the generalized profit function, the structure of the economy is identical to that introduced in chapter 2: We have a two-sector economy, constituted by an industrial sector and a ‘traditional’ sector. The industrial sector (labelled industry $X$) is characterized by increasing returns to scale, which are associated with endogenous sunk costs. In turn, sunk costs determine technological capability. We use the same three stage game as in chapter 2 to model the behavior of firms in the industrial sector. In the first stage firms decide whether to enter the market. In the second stage, firms invest in technological capabilities, taking current
market structure as given. In the third stage, firms compete in quantities, taking as given current market structure and each firm's technological capabilities.

The other industry is a 'traditional' sector (industry Y), characterized by 1:1 constant returns to scale technology (1 unit of labour input produces 1 unit of output). The presence of this sector ensures that the labour market clears. Since the model is general equilibrium, wages are determined endogenously. Labour is the only factor of production.

As before, there is a representative consumer who consumes goods produced by industries X and Y, subject to a budget constraint. Consumers allocate labour between industries X and Y. The demand for labour from industry X stems from investment in technological capability and from the number of firms. Workers are employed by industry X with the sole purpose of developing technological capability. As before, variable costs in industry X are set to zero. Industry Y is the outside option for all workers who cannot find employment in industry X.

The chapter is structured as follows. In Section 2 we develop the model. In section 3 we characterize a symmetric general equilibrium. In Section 4 we analyze how the equilibrium solutions change as we vary parameters. Section 5 discusses some general remarks about the model. We conclude in section 6.

2 Closed Economy General Equilibrium Model with Profit Sharing

The consumers' problem, industry Y and the labour market are identical to those presented in chapter 2. To avoid repetition, we will refer to the equations we derived in chapter 2. Industry X is similar, but now incorporates a generalized profit function.

Industry X

Firms play a three stage game, similar to the game featured in chapters 2 and 3. In the first stage the entry decision is made. In the second stage investment in technological capability is chosen. In the final stage, firms compete in quantities. In all stages firms participate in profit sharing or cross ownership within the industry. The equilibrium concept remains subgame perfection.

One of the central questions of this chapter is the issue of how an economy changes as we vary the 'intensity of competition'. A similar concept (the 'toughness of competition') was introduced by Sutton (1991, ch. 1, p. 9). Both terms refer to the changing nature of competition as the industry changes from a perfectly competitive setting, to Bertrand competition, to
Cournot competition, to joint profit maximization (perfect collusion). The perfectly competitive setting and Bertrand competition would be considered the toughest type of competition, followed by Cournot and then by joint profit maximization (the weakest form of competition, equivalent to a perfectly collusive agreement). It should be noted that this is a different notion to that of 'substitutability', although the two are sometimes used in the literature to refer to 'competition' in a broad sense. Indeed, one of the reasons why we chose the utility function set out in equation (2.1) is that it provides a measure of 'substitutability' ($\sigma$), and this will allow us to disentangle the effects of the 'intensity of competition' and 'substitutability'.

To model the 'intensity of competition', we have allowed firms to maximize a generalized objective function, which will include not only their own profit, but also their rivals' (partners'). This will internalize to some extent the effect that a firm's actions have on its rivals' payoffs, and will allow the 'intensity of competition' to be varied by changing a parameter (labelled $\gamma$).

As required by backward induction, we begin by describing the final stage in the game.

2.1 Stage 3: Quantity Competition with Profit Sharing

Firms choose quantity, taking as given their rivals' strategies, technological capabilities and market structure. We assume that production (variable) costs are zero. Gross profits per consumer for the $k^{th}$ firm are therefore given by

$$\pi_k = p_k x_k$$  \hspace{1cm} (4.1)

and the firm's objective now becomes

$$(1 - \gamma) \pi_k + \gamma \overline{\pi}_{-k}$$  \hspace{1cm} (4.2)

where $\pi_k$ denotes firm $k$'s gross profits per consumer, $\overline{\pi}_{-k}$ is the average of gross (per-consumer) profits of firm $k$'s rivals (given by $\frac{\sum_{h \neq k}^{N} \pi_h}{N}$), and $\gamma \in \left[0, \frac{N}{N+1}\right]$ is the intensity of competition parameter. In Cyert and DeGroot (1973) and Symeonidis (1997), $\gamma$ has been interpreted as a 'coefficient of cooperation'. It measures the extent to which a firm values its rivals' payoffs. It can also be interpreted as the extent of collusion prevalent in the industry. If $\gamma$ is zero, firms take into account only their own profits, and we obtain Cournot competition. As $\gamma$ converges

\footnote{Cyert and DeGroot (1973) and Symeonidis (1997) propose a slightly different formulation. In their studies, the objective function is $\pi_k + \lambda \sum_{h \neq k}^{N} \pi_h$, with $\lambda \in [0, 1]$. For our purposes, it is more convenient to use the formulation set out above.}
to $\frac{N}{N+1}$, firms value their rivals' average profits as much as their own, and we are in the presence of perfect collusion. The upper limit of $\gamma$, namely $\frac{N}{N+1}$, is imposed to avoid the case of a firm valuing its rivals' average profits more than its own.

An alternative interpretation of $\gamma$ is that it is the extent of cross-ownership within the industry, that is, $1 - \gamma$ measures the percentage of firm $k$ owned by its rivals and $\frac{\gamma}{N}$ measures the share that firm $k$ owns in each of its rivals. Under this interpretation, $\gamma$ measures the extent of profit sharing in the industry. The cross-ownership interpretation of $\gamma$ has the advantage of being directly measurable, and although such data is not easily accessible, it does exist. This opens the possibility of future tests for predictions of the model.

Another interpretation for $\gamma$ is that it is a reduced form measure of the sustainability of collusion or cooperation in a repeated game setting (based on some version of the 'Folk Theorem'). A high value for $\gamma$ would imply that it is easy for firms to sustain cooperation in an infinitely repeated game. Reasons which would induce this include high observability, a tough punishment phase, etc. Likewise, low values of $\gamma$ would indicate that collusion is difficult to sustain.

Yet another interpretation of $\gamma$ has been suggested to us. This relates to the presence of 'industrial oligarchies', in which a reduced group of families own a sizeable percentage of firms in an industry. Our parameter $\gamma$ could be related to the extent of within-oligarchy marriages. Marriage can be regarded as a mechanism for the redistribution of property rights within an industry. Thus marriage between families who hold property rights in rival firms can act as a mechanism to reduce the intensity of competition between such firms. It is a well documented fact (Hausmann and Rigobon, 1993; Acemoglu and Robinson, 2001) that in many developing countries (especially in Latin America), a small group of oligarch families hold high percentage of industrial assets. Moreover, it is often the case that the offspring of these families find spouses within the same social stratus. This could be associated to a reduction in the intensity of competition.

There is another implication of the generalized profit function in equation (4.2), which is quite important from the point of view of economic theory. This generalized objective function actually serves as an alternative basis for the characterization of 'conjectural variations', with the advantage that it is directly measurable. So, the model we present for industry $X$ could also be considered as a new way of thinking about 'conjectural variations', without the problems

---

2 We are grateful to Kam Ki Tang for this interpretation.

3 We are grateful to a participant at the Australian Economic Workshop (University of Melbourne, 2004) for this suggestion.
of non-observability that plagued previous models (Dixit, 1986). We shall usually refer to \( \gamma \) as the ‘intensity of competition’.

Firms optimize their objective (equation 4.2) by choosing \( x_k \). The first order condition for the \( k^{th} \) firm is

\[
(1 - \gamma) \left( p_k + \frac{\partial p_k}{\partial x_k} x_k \right) + \gamma \sum_{h \neq k} \frac{\partial p_h}{\partial x_k} x_h = 0 \quad \text{for } k = 1, \ldots, N + 1
\]  

(4.3)

In Appendix 1, we find a symmetric Nash equilibrium and obtain \( x_k \) as a function of the number of firms \( (N + 1) \) and of firms’ technological capabilities, given by the vector \( U = (u_1, \ldots, u_{N+1})' \). Substituting the solutions derived in Appendix 1 into the firm’s objective yields the solved-out payoff that the \( k^{th} \) firm extracts from each consumer (which, following backward induction, is used to solve the second stage of the game):

\[
\pi_k(U) = \frac{1 - \gamma}{2[(2 - \sigma)(1 - \gamma) - \frac{\sigma}{N}]^2} \left[ u_k - \frac{(1 - \gamma + \frac{\gamma}{N})\sigma}{(2 + \sigma N)(1 - \gamma) + \sigma\gamma} \sum_{i=1}^{N+1} u_i \right] 
\]

\[
\left[ (1 - \gamma - \frac{\sigma\gamma}{N}) u_k - \frac{(1 - \gamma)(1 - \gamma - \frac{\gamma}{N})\sigma}{(2 + \sigma N)(1 - \gamma) + \sigma\gamma} \sum_{i=1}^{N+1} u_i \right]
\]  

(4.4)

Total gross profits are given by \( L \pi_k(U) \). If firms choose a symmetric quality level (denoted by \( u \)), profits per consumer simplify to

\[
\pi = \frac{(1 - \gamma)[1 - \gamma(1 - \sigma)]}{2[(2 + \sigma N)(1 - \gamma) + \sigma\gamma]^2} u^2
\]  

(4.5)

Upon setting \( \gamma = 0 \), this expression reduces to

\[
\pi = \frac{u^2}{2(2 + \sigma N)^2}
\]

(4.6)

which is the expression derived in chapter 2 (expression 2.9).

2.2 Stage 2: Competition in Technological Capability with Profit Sharing

In this subgame, firms choose their investment in technological capability, taking as given their rivals' strategies and market structure. The investment is a sunk cost. In this stage the firm's objective is given by

\[
(1 - \gamma)\Pi_k + \gamma\Pi_{-k} \quad \text{for } k = 1, \ldots, N + 1
\]  

(4.7)
where $\Pi_k$ is net profit of the $k^{th}$ firm and $\Pi_{-k}$ is the average net profits of the $k^{th}$ firm's rivals ($\frac{\sum_{h \neq k} \Pi_h}{N}$). Net profit is $\Pi_k = L \pi_k(U) - F(u_k, w)$, where $L$ denotes population size, $\pi_k(U)$ is solved-out gross per-capita profit (equation 4.4) and $F(u_k, w)$ is a fixed outlays function, defined as before: $F(u_k, w) = w f(u_k)$, where $w$ is the wage, and $f(u_k) = e \left( \frac{u_k}{w_0} \right)^\beta$ is the firm's labour requirement in industry $X$. $\beta$ is assumed greater than 2 (to ensure that the second order conditions hold, see Appendix 2), $\epsilon > 0$, and $u_k > u_o > 1$. $\beta$ is the elasticity of $f(u_k)$ with respect to $u_k$. $\epsilon$ is an exogenous set-up cost. $u_o$ represents the initial (inherited) value of technological capability, an exogenous parameter.

Firms maximize their objective (equation 4.7) with respect to $u_k$, with first order conditions:

\[
(1 - \gamma) L \frac{\partial \pi_k}{\partial u_k} + \frac{\gamma}{N} L \sum_{h \neq k} \frac{\partial \pi_h}{\partial u_k} = (1 - \gamma) \frac{w e \beta}{u_k} \left( \frac{u_k}{u_o} \right)^\beta \quad k = 1, \ldots, N + 1
\]  

(4.8)

where the derivatives of the profit function are taken using the solved-out payoff in (4.4):

\[
\frac{\partial \pi_k}{\partial u_k} = \frac{(1 - \gamma)}{2 \left[ (2 - \sigma)(1 - \gamma) - \frac{\sigma^2}{N} \right]^2} \left\{ u_k - \frac{\sigma (1 - \gamma)^\frac{1}{2} \gamma}{(2 + \sigma N)(1 - \gamma) + \sigma} \sum_{i=1}^{N+1} u_i \left[ (1 - \gamma - \frac{\sigma^2}{N}) - \frac{(1 - 1)(1 - \gamma)^\frac{1}{2} \gamma}{(2 + \sigma N)(1 - \gamma) + \sigma} \right] \right. \right.
\left. + \left. (1 - \gamma - \frac{\sigma^2}{N}) u_k - \frac{(1 - \gamma)(1 - \gamma)^\frac{1}{2} \gamma}{(2 + \sigma N)(1 - \gamma) + \sigma} \sum_{i=1}^{N+1} u_i \right[ 1 - \frac{(1 - \gamma)^\frac{1}{2} \gamma}{(2 + \sigma N)(1 - \gamma) + \sigma} \right] \right}\}
\]

(4.9)

\[
\frac{\partial \pi_h}{\partial u_k} = -\frac{(1 - \gamma)}{2 \left[ (2 - \sigma)(1 - \gamma) - \frac{\sigma^2}{N} \right]^2} \left\{ u_k - \frac{\sigma (1 - \gamma)^\frac{1}{2} \gamma}{(2 + \sigma N)(1 - \gamma) + \sigma} \sum_{i=1}^{N+1} u_i \left[ (1 - \gamma - \frac{\sigma^2}{N}) - \frac{(1 - \gamma)(1 - \gamma)^\frac{1}{2} \gamma}{(2 + \sigma N)(1 - \gamma) + \sigma} \right] \right. \right.
\left. + \left. (1 - \gamma - \frac{\sigma^2}{N}) u_k - \frac{(1 - \gamma)(1 - \gamma)^\frac{1}{2} \gamma}{(2 + \sigma N)(1 - \gamma) + \sigma} \sum_{i=1}^{N+1} u_i \right[ 1 - \frac{(1 - \gamma)^\frac{1}{2} \gamma}{(2 + \sigma N)(1 - \gamma) + \sigma} \right] \right\}
\]

(4.10)

The first order conditions (together with other equilibrium conditions, to be introduced below) will allow us to solve for the equilibrium level of technological capability (section 3). If firms choose a symmetric level of technological capability (denoted by $u$), the first order conditions simplify to the following expression:

\[
LT = w e \beta \frac{u^{\beta - 2}}{u_0^\beta} 2^n
\]

(4.11)
where
\[
T = \frac{(1 - \gamma)^2 \{\sigma N [2 - \gamma(2 - \sigma)] + 2 [2 - \sigma - \gamma(2 - \sigma(2 - \sigma))]\}}{\frac{\sigma^2}{2} [2 - \gamma(2 - \gamma)(2 - \sigma)]} \frac{[2 - \sigma)(1 - \gamma) - \frac{\sigma^2}{2}]}{[2 + \sigma N)(1 - \gamma) + \sigma \gamma]^2} \tag{4.12}
\]
Upon setting \( \gamma = 0 \), the first order condition simplifies further to
\[
L \frac{2 + \sigma (N - 1)}{(2 - \sigma)(2 + \sigma N)^2} = \frac{w \varepsilon \beta^\beta}{u_0^\beta} \tag{4.13}
\]
which is expression (2.13).

This completes the description of the second stage subgame (see Appendix 2 for second order conditions).

2.3 Stage 1: The Entry Decision with Profit Sharing

We assume there is a sufficiently large pool of potential entrants. Firms will enter until gross profits equate fixed outlays. This leads to the following free-entry condition
\[
(1 - \gamma)\Pi_k + \gamma \Pi_{-k} \geq 0 \quad k = 1, \ldots, N + 1 \tag{4.13}
\]
Ignoring integer effects, entry occurs until condition (4.13) holds with equality. Since \( \Pi_k = L \pi_k - w \varepsilon \left( \frac{u_k}{u_0} \right)^\beta \), condition (4.13) can be written as follows
\[
(1 - \gamma)L \pi_k + \gamma \frac{\sum_{h \neq k} L \pi_h}{N} \geq (1 - \gamma)w \varepsilon \left( \frac{u_k}{u_0} \right)^\beta + \gamma \frac{\sum_{h \neq k} w \varepsilon \left( \frac{u_k}{u_0} \right)^\beta}{N} \tag{4.14}
\]
where \( \pi_k \) and \( \pi_h \) denote gross profits per consumer (from equation 4.4). The zero profit condition implies that entry by any firm induces a redistribution of property rights to ensure that all firms perceive a fraction \( 1 - \gamma \) of their own net profit and hold claims to a fraction \( \gamma/N \) in each of its rivals' net profits. In a symmetric equilibrium, using (4.5), we can rewrite (4.14) as follows
\[
L \frac{(1 - \gamma)[1 - \gamma(1 - \sigma)]}{2[(2 + \sigma N)(1 - \gamma) + \sigma \gamma]^2} = \frac{w \varepsilon \beta \beta - 2}{u_0^\beta} \tag{4.15}
\]
Upon setting \( \gamma = 0 \), this simplifies further to
\[
\frac{L}{2(2 + \sigma N)^2} = \frac{w \varepsilon \beta \beta - 2}{u_0^\beta} \tag{4.16}
\]
which is expression (2.15).
This completes the description of the game in industry \( X \). In the following section, we characterize a symmetric general equilibrium.

3 Characterization of a Symmetric General Equilibrium

We now have six parameters \((\beta, \gamma, \sigma, \epsilon, L \text{ and } u_0)\). As before, \(u, N\) and \(w\) pin down the general equilibrium. The solutions are functions of the parameters: \(u(\beta, \gamma, \sigma, \epsilon, L, u_0), N(\beta, \gamma, \sigma, \epsilon, L, u_0)\) and \(w(\beta, \gamma, \sigma, \epsilon, L, u_0)\). We drop the arguments for ease of notation, hence we write \(u(.)\) for \(u(\beta, \gamma, \sigma, \epsilon, L, u_0)\), \(N(.)\) for \(N(\beta, \gamma, \sigma, \epsilon, L, u_0)\) and \(w(.)\) for \(w(\beta, \gamma, \sigma, \epsilon, L, u_0)\). Our definition of a symmetric general equilibrium is the same as in chapter 2. We now present the equilibrium conditions. These are used to find a symmetric equilibrium.

Equilibrium Conditions

The first condition corresponds to the first order conditions for technological capability from (4.8):

\[
(1 - \gamma) L \frac{\partial \pi_k}{\partial u_k} + \gamma \frac{1}{N} L \sum_{h \neq k} \frac{\partial \pi_h}{\partial u_k} = (1 - \gamma) \frac{w \epsilon \beta}{u_k} \left( \frac{u_k}{u_0} \right)^\beta \quad k = 1, ..., N + 1
\]

(4.17)

where the derivatives \(\frac{\partial \pi_k}{\partial u_k}\) and \(\frac{\partial \pi_h}{\partial u_k}\) were defined in (4.9) and (4.10), respectively.

The second is the free entry condition, from (4.14):

\[
(1 - \gamma) L \pi_k + \frac{\gamma \sum_{h \neq k} L \pi_h}{N} \geq (1 - \gamma) w \epsilon \left( \frac{u_k}{u_0} \right)^\beta + \gamma \frac{\sum_{h \neq k} w \epsilon \left( \frac{u_h}{u_0} \right)^\beta}{N}
\]

(4.18)

The third equilibrium condition is labour market clearing, from (2.17)

\[
L = L_Y + L_X
\]

(4.19)

Market clearing for type \( Y \) and type \( X \) goods holds by construction. We now consider a symmetric equilibrium, such that all firms set the same technological capability \(u\). In this case, the first order conditions for technological capability (4.17) simplify to

\[
LT = w \epsilon \beta \left( \frac{u^{\beta - 2}}{u_0^\beta} \right) T
\]

(4.20)

where \( T \) was defined in (4.12). After substitution of symmetric gross profits (4.5), the free
entry condition (4.18) simplifies to
\[
L \left( \frac{1}{2} \right) \frac{[1 - \gamma(1 - \sigma)]}{[(2 + \sigma)N]^2} = \frac{we^{b - 2}}{u^b} \tag{4.21}
\]
and use of (2.16), allows the labour market clearing condition (4.19) to be written as follows
\[
L = L_y + (N + 1) \varepsilon \left( \frac{u}{u_o} \right)^\beta \tag{4.22}
\]
We next show that conditions (4.20-4.22) allow us to solve for \(u(.), N(.)\) and \(w(.)\). From (4.20) we solve for \(u\):
\[
u = \left\{ \frac{Lu_o}{2we^\beta} \right\}^{\frac{1}{\beta - 2}} \tag{4.23}
\]
where \(T\) was defined in (4.12). When setting \(\gamma = 0\), the above expression simplifies to
\[
u = \left[ \frac{Lu_o}{we^\beta (2 - \sigma)(2 + \sigma)N^2} \right]^{\frac{1}{\beta - 2}}
\]
which is the result obtained in expression (2.26).

Solving for the number of firms requires substituting (4.23) into (4.21). We obtain a quadratic equation in \(N\):
\[
\sigma(1 - \gamma)^2 [2 - \gamma(2 - \sigma)] N^2 + (1 - \gamma)^2 [2 - \sigma - \gamma [2 - \sigma(2 - \sigma)]) - \beta(2 - \sigma) [1 - \gamma(1 - \sigma)] N - \sigma\gamma [2 - \gamma(2 - \gamma)(2 - \sigma) - \beta(1 - \gamma) [1 - \gamma(1 - \sigma)] = 0
\]
Solving for \(N\) and adding 1, we obtain the equilibrium number of firms:
\[
N + 1 = \frac{Z^c - (1 - \gamma) [2 - \sigma - \gamma [2 - \sigma(2 - \sigma)]) - \beta(2 - \sigma) [1 - \gamma(1 - \sigma)]}{2\sigma(1 - \gamma) [2 - \gamma(2 - \sigma)]} + 1 \tag{4.25}
\]

where
\[
Z^c = \sqrt{\left(1 - \gamma)^2 [2 - \sigma - \gamma [2 - \sigma(2 - \sigma)]) - \beta(2 - \sigma) [1 - \gamma(1 - \sigma)]^2 + 4\sigma^2\gamma [2 - \gamma(2 - \sigma)] [2 - \beta(1 - \gamma) [1 - \gamma(1 - \sigma)] - \gamma(2 - \gamma)(2 - \sigma)]
\]
Note that we have suppressed the negative solution (which features \(-Z^c\) instead of \(Z^c\)), since market forces will ensure that only the positive solution holds. If we set \(\gamma = 0\), the number of
entrants simplifies to
\[ N + 1 = \frac{(\beta - 2)(2 - \sigma)}{2\sigma} + 1 \]
which is expression (2.27). In the case of \( \gamma = 0 \), the negative solution to equation (4.24) becomes zero.

Substituting \( N + 1 \) from (4.25) into \( u \) (equation 4.23), we obtain an expression for technological capability as a function of exogenous parameters \((\beta, \gamma, \sigma, \varepsilon, L, u_o)\) and the wage rate \(w\):
\[
u = \left\{ \frac{2Lu_o^2}{w\varepsilon} \frac{(1 - \gamma)^2 [2 - \gamma(2 - \sigma)]^2 [1 - \gamma(1 - \sigma)]}{[1 - \gamma(1 - \sigma)] [2 [2(1 - \gamma) + \sigma] + \beta(1 - \gamma)(2 - \sigma)] + Z^2} \right\}^{\frac{1}{2}} \quad (4.27)
\]
where \(Z^c\) was defined in equation (4.26). In the case of \( \gamma = 0 \), equation (4.27) simplifies to
\[
u = \left\{ \frac{u_o^2 L}{w 2\varepsilon \left[ \beta \left( 1 - \frac{\sigma}{2} \right) + \sigma \right]} \right\}^{\frac{1}{2}}
\]
which is expression (2.28).

The equilibrium wage rate is obtained by substituting \( u \) (equation 4.27) into the labour market clearing condition (4.22). If labour demand from industry \( X \) is insufficient to clear the labour market at a wage rate higher than 1, workers will shift to industry \( Y \). In this scenario, the wage rate will be equal to the marginal product of labour in industry \( Y \), which is given by \( q = 1 \). Whence, if \( L_Y = 0, w > 1 \) (otherwise workers would shift to the freely available 1:1 technology), and if \( L_Y > 0, w = 1 \) (that is, workers use the 1:1 technology). In a symmetric equilibrium the (general) equilibrium wage rate can be solved to yield
\[
w = \max \left\{ 1, \frac{u_o^2}{2} \left( \frac{L}{\varepsilon} \right)^{\frac{3}{2}} \frac{N + 1}{\beta T} \right\} \quad (4.28)
\]
where \( T \) was defined in (4.12) and \( N \) is given in (4.25). If we set \( \gamma = 0 \), this expression simplifies to:
\[
w = \max \left\{ 1, \frac{u_o^2}{2} \left( \frac{L}{\varepsilon} \right)^{\frac{3}{2}} \left( \frac{(\beta - 2)(2 - \sigma)}{2\sigma} + 1 \right) \right\}
\]
which is equation (2.29).

All that remains to complete the characterization is to obtain the general equilibrium level of technological capability. To obtain this, substitute the solution for the wage rate (4.28) into
equation (4.27). The number of firms is given by equation (4.25). Thus we have characterized a symmetric general equilibrium in autarky. The symmetric general equilibrium solutions in autarky are set out in the table that follows, where $T$ is defined in (4.12) and $Z_c$ in (4.26):

\[
\begin{align*}
\frac{\omega}{\bar{\omega}} &= \max \left\{ \frac{2\nu^2}{\bar{\nu}^2}, \left(\frac{L}{L^*}\right)^\beta \left(\frac{T}{\bar{T}}\right)^{\frac{\beta - 2}{\beta}} \right\} \\
\frac{L^*}{L^*} &= \left\{ \frac{2\nu^2}{\bar{\nu}^2} \frac{(1-\gamma)^2(2-\gamma(2-\sigma))^2(1-\gamma(1-\sigma))}{[1-\gamma(1-\sigma)][2(1-\gamma(2-\sigma))+\beta(1-\gamma(2-\sigma))]} \right\}^{\frac{1}{\beta-2}} \\
N + 1 &= \frac{\gamma^2(2-\sigma)(2-\sigma)(2-\sigma(2-\sigma))}{2\sigma(1-\gamma)(2-\gamma(2-\sigma))} + 1
\end{align*}
\]

Table 1: Symmetric General Equilibrium Outcomes in Autarky with $\gamma > 0$.

As before, concentration does not depend on market size ($L$), initial conditions ($u_0$) or set-up costs ($e$). It depends only on $\beta$, $\gamma$ and $\sigma$. Thus concentration does not fall to zero as market size increases indefinitely (we have ‘non-convergence’).

We are now in a position to analyze how the economy reacts to changes in parameter values. This is the task for the next section.

4 Analysis of the Symmetric Equilibrium

Similarly to chapter 2, we analyze how changing each of the parameters changes outcomes for a symmetric equilibrium. In this section, we report on the same variables as in chapter 2 (section 4), namely:

a) Wage rate (equal to per-capita value added): $W(.)$ (listed in table 1).

b) Technological capability: $u(.)$ (listed in table 1).

c) Number of firms: $N(.) + 1$ (listed in table 1).

d) Welfare: $W(.)$ (equation 2.32).

e) Employment in industry $Y$: $L_y = L - L_x$ (from equation 4.19).

f) Employment in industry $X$: $L_x(\cdot)$ (equation 2.16).

We discuss changes in the equilibrium outcomes of the system as we change each of the parameters, ceteris paribus. The analysis of all remaining variables is presented in Appendix 3. We use the same notation as outlined in chapter 2, section 4. The analyses of $e$, $u_0$ and $L$ do not differ from those in chapter 2 (section 4), and need not be repeated. The analysis of $\beta$ is identical to that presented in section 4.1 in chapter 2, except for the number of firms. When $\gamma > 0$, the number of firms rises convexly with $\beta$. This is a consequence of the presence
of cross-ownership within the industry, which leads to a more than proportional reduction in technological capability (and hence in sunk costs). The reduction of entry costs allows a more than proportional increase in the number of entrants for any given level of $\beta$. We find that the presence of $\gamma$ does affect the analysis of $\sigma$, and there is, of course, the analysis of how $\gamma$ itself affects the system. We begin by presenting the latter.

4.1 Analysis of Changes in $\gamma$

Let us begin with an intuitive explanation of how changes in $\gamma$ affect the equilibrium outcome. As mentioned previously, $\gamma$ is the ‘intensity of competition’ in industry $X$ (alternatively, the extent of cross ownership within the industry). Recall that as $\gamma$ converges to zero, the model collapses to the usual individual firm profit maximization framework, and as $\gamma$ converges to its upper bound ($\gamma^7$), we approach the case of joint profit maximization (perfect collusion). Intermediate values of $\gamma$ generate modes of competition that lie between these extremes.

It is important to note that all of the 3 stages in the game played by firms in industry $X$ are affected by $\gamma$. To see this, consider the objective function in stage 3 (quantity competition):

$$(1 - \gamma)\pi_k + \gamma\bar{\pi}_{-k},$$

where $\pi_k$ is firm $k$'s gross (per capita) profit, and $\bar{\pi}_{-k}$ is the average of gross (per capita) profits of firm $k$'s rivals/partners (given by $\frac{\sum_{i \neq k} \pi_i}{N-1}$). A unilateral increase of quantity by firm $k$ will have two effects. First, there is the marginal benefit associated to the increase in firm $k$'s own profit ($\pi_k$), as it commands a larger market share at the expense of its rivals. The second effect is a marginal cost, which is due to the reduction of its rivals' (partners') profits, and this will reduce firm $k$'s profit via its participation in the other firms' profits (namely $\gamma\frac{\pi}{N}$). There would usually be a third effect, namely, the marginal cost of producing the extra quantity, but we have assumed marginal cost is zero$^4$. Thus the incentive to expand production is curtailed by firm $k$'s consideration of the effects of its actions on its rivals'/partners' profits.

Similarly, in stage 2 (competition in technological capability) $\gamma$ reduces the incentives for firm $k$ to increase its technological capability, since any expansion in sales (due to higher technological capability) will be obtained at the expense of its rivals (partners).

In stage 1 (entry), the effect of higher $\gamma$ is to allow a larger number of entrants to survive. This effect works via a reduction in sunk costs (which are smaller since the industry now features lower technological capability).

$^4$The model can be extended to incorporate non-zero marginal costs. Provided the marginal costs of production are small relative to fixed outlays associated with technological capability, setting these to zero does not change the analysis substantially.
To consider the effects of $\gamma$ on the economy, we present diagrams similar to those in chapter 2. Before proceeding, we need to note that the results that follow (with the exception of the wage rate, as will be explained below) depend on the values taken by $\beta$ and $\sigma$. In particular, we identify two different patterns of behaviour:

Case (a): When $\beta$ and $\sigma$ are ‘sufficiently high’.

Case (b): When $\beta$ and $\sigma$ are not ‘sufficiently high’ (i.e., when either of $\beta$ or $\sigma$ are ‘sufficiently low’).

The wage rate, however, exhibits different behaviour depending on whether $\beta$ is high or low (irrespective of $\sigma$, which only shifts the wage schedule, but does not change its essential properties when plotted against $\gamma$). The meaning of ‘sufficiently high/low’ will be made precise in Proposition $\gamma$ (below). For the following intuitive explanation, we shall use the terms informally.

We start by presenting the case when $\beta$ or $\sigma$ are sufficiently low in Figure 4.1a, where we plot the symmetric equilibrium outcomes for varying values of $\gamma$. The top left graph depicts technological capability, the top right graph shows the wage rate, the bottom left graph plots the number of firms and the bottom right graph shows employment.

Figure 4.1a: The effect of $\gamma$ when $\beta$ or $\sigma$ are sufficiently low

Figure 4.1a shows two scenarios. One in which $\beta$ is low (denoted by $\beta_{lo}$) and another in which $\beta$ is high and $\sigma$ is low (denoted by $\beta_{hi}\sigma_{lo}$). The only variable which changes its overall pattern...
in these two scenarios is the wage rate. Technological capability and the number of firms experience a shift of their levels, but no change in their shape. Employment is not affected, since the wage rate does not reach its lower bound of 1 (for the general case when \( w = 1 \) see Proposition \( \gamma \), below).

When \( \beta \) or \( \sigma \) are sufficiently low, technological capability is decreasing in \( \gamma \) at an increasing rate. As the intensity of competition falls (\( \gamma \) increases), the incentives to invest in technological capability are reduced. The effect of investment in technological capability is to shift outward the firm's demand function, at the expense of its rivals. Given that profit sharing becomes more important as \( \gamma \) rises, it is now less profitable for the individual firm to incur the expenditure required to achieve a given increase in its technological capability. Moreover, the fall in \( u \) occurs at an increasing rate as \( \gamma \) reaches its upper bound. This is an important notion, since it implies that if the intensity of competition is high (\( \gamma \) is low), the effect on the level of technological capability of a reduction in intensity is relatively small. It is when the intensity of competition is already low (\( \gamma \) is high) that further reductions will have a large effect on technological capability.

The pattern seen in the number of firms (or market structure) is the mirror image of the pattern in technological capability. The number of firms increases with profit sharing, at an increasing rate. This reflects the fact that as there is less investment in technological capability, each firm incurs a smaller sunk cost, and this will induce a larger number of entrants. As the intensity of competition reaches its lower bound (profit sharing reaches its upper bound, \( \gamma^* \)), the number of firms becomes increasingly large.

To analyze the wage rate, we consider first the scenario of low \( \beta \). In this case the wage is decreasing in \( \gamma \) at an increasing rate. This means that the 'market structure effect', which increases the demand for labour at the industry level (via a larger number of firms), will not be sufficient to offset the 'technological capability effect', which decreases the demand for labour at the firm level (via a fall in technological capability). Moreover, as with technological capability, it is when \( \gamma \) is already high that a further increase will depress the wage most substantially.

In the remaining scenario of high \( \beta \) and low \( \sigma \), there is a large number of firms each with relatively low technological capability. We find that the wage rate is concave, increasing at first, reaches a global maximum and is decreasing afterwards. Thus, for low values of \( \gamma \) we find that the 'market structure effect' is stronger than the 'technological capability effect', that

---

5A word of caution is due regarding the approach of \( \gamma \) to its upper bound (namely \( \gamma^* = \frac{N}{N+1} \)). We should be careful to note that as \( N \) rises with \( \gamma \), the upper bound (\( \gamma^* \)) will approach the value of 1 from the left. This means that in Figure 4.1a the upper bound will shift to the right as \( \gamma \) increases. Other than this caveat, we need not be concerned about this issue.
is, the increase in the number of firms generates a stronger effect on labour demand than the reduction in technological capability. This situation is reversed for high values of $\gamma$.

Regarding the occupation of the labour force, we find that employment in sector $Y$ is zero throughout, and all workers are employed by industry $X$. The reason behind this is that the parameter values on which Figure 4.1a is based, do not generate a wage rate equal to 1 at any value of $\gamma \in [0, \frac{N}{N+1}]$. In particular, the low levels of $\beta$ generate too high a technological capability to permit $w = 1$.

We proceed to analyze the case when $\beta$ and $\sigma$ are "sufficiently high". We present the corresponding graphs in Figure 4.1b.

Figure 4.1b shows in continuous lines the symmetric equilibrium outcomes. Dashed lines have been used to depict the outcomes that would have prevailed had the wage rate not had a lower bound of 1 (in the two top graphs) and had employment not been restricted to the interval $[0, L]$ (in the bottom right graph), that is, the 'shadow values'.

In this case, technological capability is no longer monotonic, and has a sinusoidal shape.

---

$^6$We could have chosen parameter values which could generate $w = 1$ for some values of $\gamma$, and we shall see this in the case of $\beta$ and $\sigma$ sufficiently high, as well as in Proposition $\gamma$. However it was deemed convenient to keep the analysis simple at this stage.
Aghion et al. (2005) find an 'inverted-U' relationship between innovation and product substitutability (which they identify with 'competition') in the context of an endogenous growth model with quality ladders and a CES production function. As a first approximation, let us concentrate on the outcomes when the wage rate does not have a lower bound. Thus the analysis will refer to actual outcomes for $w > 1$ and to the shadow values (dashed lines) for $w = 1$. In the top left graph we can see that as $\gamma$ increases, $u$ falls, reaches a local minimum, rises to reach a maximum and then falls. Let us discuss the intuition for this pattern.

Disregard the effect of $7$ for the moment, and concentrate on $\beta$ and $\sigma$. When $\beta$ and $\sigma$ are 'sufficiently high', the marginal cost of increasing technological capability (associated to $\beta$) is high, and goods of type $X$ are close substitutes. The industry pattern that emerges from such a parameter configuration is one of intermediate values for technological capability and the number of firms: On the one hand, the high marginal cost of technological capability induces low levels of technological capability and a large number of entrants (via reduced entry costs). On the other hand, the high substitutability of products induces high technological capability and hence a reduced number of firms. This comes about through the market stealing effect: If an individual firm decides to unilaterally increase its technological capability (escalation) it will capture sales from its rivals. The closer substitutes the goods are, the stronger the market stealing effect will be, thereby enhancing the marginal benefit from investment in technological capability. Thus, in equilibrium, higher values of $\sigma$ lead to higher technological capability and fewer firms — due to increased entry costs (a full analysis of the effects of $\sigma$ is provided in section 4.2). The two effects (high $\beta$ and $\sigma$) combine to generate intermediate values of technological capability and of the number of entrants. A similar pattern could also be generated by low $\sigma$ (low substitutability) and low $\beta$ (low marginal cost): The low substitutability induces low technological capability and a large number of firms, whilst the low marginal cost generates high technological capability and a small number of firms. The effect of $\sigma$ works through the marginal benefit of technological capability, while $\beta$ affects the marginal cost of technological capability.

So why do $\beta$ and $\sigma$ 'sufficiently high' generate the pattern in technological capability displayed in Figure 4.1b? For either high or low values of $\gamma$ we see that increases in $\gamma$ lead to lower technological capability. The analysis of this case is similar to the case when $\beta$ is 'sufficiently low' (Figure 4.1a), and need not be repeated. On the other hand, for intermediate values of $\gamma$ we observe an increase in technological capability as $\gamma$ rises (the intensity of competition falls). What is occurring in this range is that the net marginal benefit of technological capability rises with $\gamma$, hence the increment in technological capability. Note that the marginal benefit is en-
hanced by the close substitutability of products. Regarding the marginal costs, we can identify two sources. Firstly, the profit reduction of other firms (in which the firm owns a share \( \frac{1}{N} \)). Secondly, the extra investment required to achieve the new level of technological capability. Thus, marginal costs are not sufficient to offset the enhanced marginal benefit, leading to an increase in technological capability. To see this, consider the case when \( \beta \) is low and \( \sigma \) is high. In this scenario the incentives to increase technological capability are quite strong. Hence the escalation mechanism is enhanced. This would lead to very large profit reductions for rivals/partners, which would be a strong deterrent to the individual firm (since it internalizes, partly albeit, the consequences of its actions on its rivals/partners through ownership of shares in their profits). Thus the individual firm responds by reducing its technological capability as \( \gamma \) rises (as shown in Figure 4.1a). In the case at hand (when \( \beta \) and \( \sigma \) are high) we observe that for an intermediate range of \( \gamma \), the above mechanism reverts. It is because \( \beta \) is high (and hence any escalation is relatively minor), that an individual firm may find it optimal to raise its technological capability as \( \gamma \) rises: The costs arising from the harm it would inflict upon its rivals/partners and the additional required investment are not sufficient to offset the benefit from escalating (which is enhanced by having a high \( \sigma \) and compounded by the relatively low level of \( \gamma \)).

The number of firms rises with \( \gamma \) at first, reaches a local maximum, falls to reach a local minimum (which corresponds exactly with the global maximum of \( u \)) and then rises convexly. The ‘technological capability’ effect and the ‘market structure’ effect combine to determine the wage rate. Initially, the market structure effect dominates the technological capability effect, thus the wage rate is rising and tracks the number of firms. Once technological capability ceases to fall, the balance is reversed, and the technological capability effect takes over the market structure effect. Hence the wage rate tracks technological capability.

Employment in industry \( X \) (\( L_x \)) exhibits properties similar to the wage rate. As \( \gamma \) grows, \( L_x \) rises at an increasing rate at first, then at a decreasing rate, reaching a maximum and then falling at an increasing rate for high values of \( \gamma \). Employment in industry \( Y \) is residual labour not employed in industry \( X \).

Let us now bring into the analysis the lower bound of the wage rate \( (w = 1) \), and link this to employment in both industries. In Figure 4.1b we observe two thresholds for \( \gamma \), \( \gamma_{\text{low}}^{w=1|\beta \geq \beta^w} \) and \( \gamma_{\text{high}}^{w=1|\beta \geq \beta^w} \) (which will be clarified in Proposition \( \gamma \)). When \( \gamma \) is sufficiently low (i.e., \( \gamma \leq \gamma_{\text{low}}^{w=1|\beta \geq \beta^w} \)), the wage becomes 1. Similarly, when \( \gamma \) is sufficiently high (i.e., \( \gamma \geq \gamma_{\text{high}}^{w=1|\beta \geq \beta^w} \)), the wage also becomes 1. This leads to industry \( Y \) becoming active in both cases. For values of \( \gamma \) between the thresholds, all labour is employed in industry \( X \) (leading to zero employment.
in industry $Y$).

For $\gamma \leq \gamma_{\text{low}}^{w=1|\beta|\beta^w}$ actual technological capability is increasing in $\gamma$. This arises because the shadow wage rate lies below the actual wage rate, implying that the actual cost of technological capability is higher than its shadow cost, resulting in an actual technological capability which is below its shadow counterpart. As $\gamma$ grows, the two wages converge. Accordingly actual technological capability converges to its shadow value, generating the observed increase.

Similarly, for $\gamma \geq \gamma_{\text{high}}^{w=1|\beta|\beta^w}$ the wage rate is unity. Hence observed technological capability is below its shadow value. As $\gamma$ grows, the gap between the wage and its shadow value rises, leading to an also rising gap between technological capability and its shadow value.

In addition to the development configurations discussed in chapter 2 (proliferation and high-tech), the inverted-U depicted in the wage graphs in figures 4.1a-b leads to a third option: an intermediate configuration. An intermediate economy is characterized by middling technological capability and concentration, and (so long as $\beta$ is sufficiently high) the highest wage will occur at intermediate levels of $\gamma$ (regardless of $\sigma$).

We summarize the above discussion in the following proposition (which is completed in Appendix 3).

**Proposition $\gamma$: Effects of $\gamma$**

Recall that $\gamma \in [0,1]$.

**a) Wage rate:** Define a threshold level of $\beta$, $\beta^w$, given by the lowest value of $\beta$ such that $\frac{\partial w}{\partial \gamma} |_{\beta=\beta^w} = 0$. For the case when $\beta < \beta^w$, we can find a threshold for $\gamma$, $\gamma_{\text{low}}^{w=1|\beta|\beta^w}$, such that $w = 1$ for all $\gamma \geq \gamma_{\text{low}}^{w=1|\beta|\beta^w}$, provided $\gamma_{\text{low}}^{w=1|\beta|\beta^w} \in \left[0, \frac{N}{N+1}\right]$. When $\beta < \beta^w$ and $\gamma < \gamma_{\text{low}}^{w=1|\beta|\beta^w}$, the wage rate is strictly decreasing and strictly concave in $\gamma$: $\frac{\partial^2 w}{\partial \gamma^2} < 0$.

If $\beta < \beta^w$ and $\gamma \geq \gamma_{\text{low}}^{w=1|\beta|\beta^w}$, the wage rate is equal to 1.

If instead $\beta \geq \beta^w$, we define two thresholds for $\gamma$, $\gamma_{\text{low}}^{w=1|\beta|\beta^w}$ and $\gamma_{\text{high}}^{w=1|\beta|\beta^w}$. These thresholds are ranked as follows: $\gamma_{\text{low}}^{w=1|\beta|\beta^w} < \gamma_{\text{high}}^{w=1|\beta|\beta^w}$ (see Figure 4.1b). If $\gamma \geq \gamma_{\text{high}}^{w=1|\beta|\beta^w}$ then $w = 1$. In the case of $\gamma \leq \gamma_{\text{low}}^{w=1|\beta|\beta^w}$, we face an additional requirement for $w = 1$ within the relevant range of $\gamma$, i.e., for $\gamma_{\text{low}}^{w=1|\beta|\beta^w} \in \left[0, \gamma_{\text{high}}^{w=1|\beta|\beta^w}\right]$. The requirement is that $\sigma$ be sufficiently high: $\sigma > \sigma_{\text{low}}^{w=1}$ (where $\sigma_{\text{low}}^{w=1}$ is such that for all $\sigma > \sigma_{\text{low}}^{w=1}$, $w = 1$ obtains).

When $\beta \geq \beta^w$ and $\gamma \in \left(\max\left(0, \gamma_{\text{low}}^{w=1|\beta|\beta^w}\right), \gamma_{\text{high}}^{w=1|\beta|\beta^w}\right)$, we obtain that as we increase $\gamma$, $w$ is at first strictly increasing ($\frac{\partial w}{\partial \gamma} > 0$), reaches a global maximum ($\frac{\partial^2 w}{\partial \gamma^2} = 0$), and then becomes strictly decreasing ($\frac{\partial^2 w}{\partial \gamma^2} < 0$) in $\gamma$. It also follows that as we increase $\gamma$, $w$ is at first strictly convex ($\frac{\partial^2 w}{\partial \gamma^2} > 0$), reaches an inflection point ($\frac{\partial^2 w}{\partial \gamma^2} = 0$) and finally becomes strictly concave in $\gamma$ ($\frac{\partial^2 w}{\partial \gamma^2} < 0$). If $\gamma \not\in \left(\max\left(0, \gamma_{\text{low}}^{w=1|\beta|\beta^w}\right), \gamma_{\text{high}}^{w=1|\beta|\beta^w}\right)$, then $w = 1$. 

136
b) Technological capability: Define thresholds \((\beta^u, \sigma^u)\) given by the lowest values of \((\beta, \sigma)\) such that \(\frac{\partial u}{\partial \gamma}|_{\beta^u\sigma^u\gamma}=0\). For \(\beta < \beta^u\) or \(\sigma < \sigma^u\), \(u\) is strictly decreasing and strictly concave in \(\gamma\): \(\frac{\partial u}{\partial \gamma} < 0\), \(\frac{\partial^2 u}{\partial \gamma^2} < 0\).

For \(\beta > \beta^u\), \(\sigma > \sigma^u\) we distinguish two cases. Firstly, 
\[\gamma \in \left(\max\left(0, \frac{\gamma_{\text{low}}}{\beta^u}\right), \frac{N}{N+1}\right).\]

Then \(u\) is sinusoidal: it begins being strictly decreasing \(\left(\frac{\partial u}{\partial \gamma} < 0\right)\), reaches a local minimum \(\left(\frac{\partial^2 u}{\partial \gamma^2} < 0\right)\), becomes strictly increasing \(\left(\frac{\partial u}{\partial \gamma} > 0\right)\), reaches a global maximum \(\left(\frac{\partial u}{\partial \gamma} = 0\right)\) and becomes strictly decreasing \(\left(\frac{\partial^2 u}{\partial \gamma^2} < 0\right)\). Correspondingly, \(u\) begins being strictly convex \(\left(\frac{\partial u}{\partial \gamma} > 0\right)\), reaches an inflection point \(\left(\frac{\partial^2 u}{\partial \gamma^2} = 0\right)\) and becomes strictly concave \(\left(\frac{\partial^2 u}{\partial \gamma^2} < 0\right)\) in \(\gamma\). Secondly, if \(\gamma < \gamma_{\text{low}}\), \(\frac{\partial^2 u}{\partial \gamma^2} > 0\).

\(u\) exhibits kinks at \(\gamma = \gamma_{\text{low}}\) and \(\gamma = \gamma_{\text{high}}\).

c) Number of firms: For \(\beta < \beta^u\) or \(\sigma < \sigma^u\) (defined in part (b) of this proposition), \(N + 1\) is strictly increasing and strictly convex in \(\gamma\): \(\frac{\partial (N+1)}{\partial \gamma} > 0\), \(\frac{\partial^2 (N+1)}{\partial \gamma^2} > 0\).

For \(\beta > \beta^u\) and \(\sigma > \sigma^u\), increasing \(\gamma\) induces the following behavior: \(N + 1\) is at first strictly increasing in \(\gamma\) \(\left(\frac{\partial (N+1)}{\partial \gamma} > 0\right)\), reaches a local maximum \(\left(\frac{\partial^2 (N+1)}{\partial \gamma^2} = 0\right)\), becomes strictly decreasing \(\left(\frac{\partial^2 (N+1)}{\partial \gamma^2} < 0\right)\), and then becomes strictly increasing \(\left(\frac{\partial^2 (N+1)}{\partial \gamma^2} > 0\right)\) in \(\gamma\). Correspondingly, \(N + 1\) is initially strictly concave \(\left(\frac{\partial^2 (N+1)}{\partial \gamma^2} < 0\right)\), reaches an inflection point \(\left(\frac{\partial^2 (N+1)}{\partial \gamma^2} = 0\right)\) and finally becomes strictly convex \(\left(\frac{\partial^2 (N+1)}{\partial \gamma^2} > 0\right)\) in \(\gamma\).

d) Welfare: \(W\) exhibits behavior similar to the wage rate (with different numerical values). However, when \(w = 1\), \(W\) becomes flatter, generating a kink in \(W\). This change of slope occurs for the same reason as outlined in part (d) of Proposition \(\beta\), chapter 2: Once the wage rate becomes 1, the fall in welfare is mitigated by the fact that the wage cannot fall any further.

e) Employment in industry \(Y\): \(L_y\), is equal to 0 for \(w > 1\). Industry \(Y\) becomes active when \(w = 1\). Let us begin by introducing a threshold \(\beta^w=1\): the lowest value of \(\beta\) that satisfies \(w = 1\) for all \(\gamma \in \left[0, \frac{N}{N+1}\right]\). When \(\beta \geq \beta^w=1\) we obtain that as we increase \(\gamma\), \(L_y\) is at first strictly decreasing \(\left(\frac{\partial L_y}{\partial \gamma} < 0\right)\), reaches a global minimum \(\left(\frac{\partial^2 L_y}{\partial \gamma^2} = 0\right)\), and then becomes strictly increasing \(\left(\frac{\partial^2 L_y}{\partial \gamma^2} > 0\right)\) in \(\gamma\). It also follows that as we increase \(\gamma\), \(L_y\) is at first strictly concave \(\left(\frac{\partial^2 L_y}{\partial \gamma^2} < 0\right)\), reaches an inflection point \(\left(\frac{\partial^2 L_y}{\partial \gamma^2} = 0\right)\) and finally becomes strictly convex in \(\gamma\) \(\left(\frac{\partial^2 L_y}{\partial \gamma^2} > 0\right)\).

Consider now the case when \(\beta \in (\beta^w=1, \beta^w]\). Recall that threshold level \(\beta^w\) is given by the lowest value of \(\beta\) such that \(\frac{\partial u}{\partial \gamma}|_{\beta=\beta^w}=0\). In this case, we can find two thresholds for \(\gamma\), namely \(\gamma_{\text{low}}\) \(\left(\frac{\partial L_y}{\partial \gamma} = 0\right)\) and \(\gamma_{\text{high}}\) \(\left(\frac{\partial L_y}{\partial \gamma} > 0\right)\), with the following properties:

If \(\gamma \geq \gamma_{\text{high}}\) \(\left(\frac{\partial L_y}{\partial \gamma} > 0\right)\) and \(\gamma_{\text{high}}\) \(\left(\frac{\partial^2 L_y}{\partial \gamma^2} > 0\right)\) in \(\left[0, \frac{N}{N+1}\right]\), then \(w = 1\) and \(L_y\) is strictly increasing and strictly convex in \(\gamma\): \(\frac{\partial L_y}{\partial \gamma} > 0\), \(\frac{\partial^2 L_y}{\partial \gamma^2} > 0\).
If \( \gamma_{\text{low}}^{w=1} > \beta \geq \beta^w \) and \( \gamma_{\text{low}}^{w=1} > \beta \geq \beta^w \in [0, \frac{N}{N+1}] \), then \( w = 1 \) and \( L_y \) is strictly decreasing and strictly concave in \( \gamma; \frac{\partial L_y}{\partial \gamma} < 0, \frac{\partial^2 L_y}{\partial \gamma^2} < 0 \). To ensure that \( \gamma_{\text{low}}^{w=1} > \beta \geq \beta^w \in [0, \frac{N}{N+1}] \) it is sufficient to require \( \sigma > \sigma^{w=1} \) (where \( \sigma^{w=1} \) is such that for all \( \sigma > \sigma^{w=1}, w = 1 \) obtains, as defined in part (a) of this proposition).

Finally, for the case when \( \beta < \beta^w \), we can find a threshold for \( \gamma, \gamma^{w=1} > \beta < \beta^w \), such that \( w = 1 \) for all \( \gamma > \gamma^{w=1} > \beta < \beta^w \). When \( \beta < \beta^w \) and \( \gamma < \gamma^{w=1} > \beta < \beta^w \), \( w = 1 \) and \( L_y = 0 \). If \( \beta < \beta^w \) and \( \gamma > \gamma^{w=1} > \beta < \beta^w \), \( w = 1 \) and \( L_y \) is strictly increasing and strictly convex in \( \gamma; \frac{\partial L_y}{\partial \gamma} > 0, \frac{\partial^2 L_y}{\partial \gamma^2} > 0 \).

As before \( L_y \in [0, L] \) in equilibrium.

f) Employment in industry \( X \): \( L_x \) is equal to \( L - L_y \). Thus its behavior is the reverse of \( L_y \). \( L_y \) supplements \( L_x \) to use all of the available labour supply (given by \( L \)).

Proof: By inspection of the appropriate variables, their derivatives and associated thresholds.

4.2 Analysis of Changes in \( \sigma \)

The effects of \( \sigma \) depend intricately on the values that \( \beta \) and \( \gamma \) take. We distinguish four cases:

(a) When \( \beta \) and \( \gamma \) are very low.
(b) When \( \beta \) and \( \gamma \) are low.
(c) When \( \beta \) is high and \( \gamma \) is high.
(d) When \( \beta \) is high and \( \gamma \) is low.

As previously, the precise meaning of terms such as ‘low’, ‘very low’ or ‘high’, will be clarified in Proposition \( \sigma \), stated below. Notice that the case when \( \beta \) is low and \( \gamma \) is high has not been included. This is because the second order conditions are violated under such a parameter combination (see Appendix 2 for more details on the second order conditions). Nonetheless, to aid exposition, we will refer to this parameter configuration on one occasion.

The analysis proceeds by explaining two diagrams: One for cases (a) and (b) –Figure 4.2a– and the other for cases (c) and (d) –Figure 4.2b–. In Figure 4.2a, \( \sigma \) is plotted on the horizontal axes. Dashed thick lines correspond to case (a), when \( \beta \) and \( \gamma \) are very low (labelled \( \beta^{\text{lo}} \gamma^{\text{lo}} \)), while continuous lines represent case (b), when \( \beta \) and \( \gamma \) are low (labelled \( \beta^{\text{lo}} \gamma^{\text{lo}} \)). Dashed thin lines show the projection of the corresponding function, had the wage rate not had a lower bound equal to 1 (‘shadow values’). The top left graph in Figure 4.2a depicts technological capability. The top right graph shows the wage rate. The bottom left graph illustrates the

1This case corresponds to Figure 4.1a, but it is not depicted. The case shown in the figure is \( \beta < \beta^w \) and \( \gamma < \gamma^{w=1} > \beta < \beta^w \) (for which \( w > 1 \) and \( L_y = 0 \)).
number of firms and the bottom right graph exhibits employment in industries \( X \) and \( Y \).

Figure 4.2a: The effect of \( \sigma \) when: (a) \( \beta \) and \( \gamma \) are very low and (b) \( \beta \) and \( \gamma \) are low.

In case (a), when \( \beta \) and \( \gamma \) are very low (dashed thick lines), technological capability achieves its highest level for all parameter combinations, and is increasing concavely in \( \sigma \). Correspondingly, the number of firms is at its lowest level, and is decreasing convexly in \( \sigma \). The wage rate is at its highest level\(^8\), and is 'U'-shaped. Employment in industry \( X \) equals the economy's labour endowment, and consequently employment in industry \( Y \) is zero (industry \( Y \) is inactive).

The intuition behind this pattern is as follows. When \( \beta \) and \( \gamma \) are very low there are strong incentives to escalate. Firstly, very low values of \( \beta \) imply a very low marginal cost of escalation. Secondly, very low \( \gamma \) implies that the firm cares little for its rivals'/partners' profits, and this also contributes towards a low marginal cost of escalation. The net effect is that as \( \sigma \) grows, the net marginal benefit of escalation grows, and each firm finds it optimal to increase its investment in technological capability. This in turn raises the investment required to survive in the industry, thereby reducing the number of entrants.

The combination of the 'technological capability' effect and the 'market structure' effect will determine the demand for labour in industry \( X \), and hence the wage rate. For low values

\(^8\)That technological capability reaches its highest level, the number of firms its lowest level, and the wage rate its highest level for this parameter configuration follows from the results in sections 4.1 (on the effects of \( \beta \)) and 4.2 (on the effects of \( \sigma \)) in chapter 2.
of \( \sigma \), the effect of increasing technological capability is not sufficient to offset the decreasing number of firms, thus the wage falls (the 'market structure' effect dominates). For high values of \( \sigma \) the balance is reversed, leading to an increasing wage rate. Since demand for labour in industry \( X \) does not fall below the economy's labour endowment, the wage rate does not reach its lower bound of 1, and industry \( Y \) remains inactive throughout.

The only substantive difference between case (b) \( -\beta_{1o} \gamma_{1o} - \) and case (a) \( -\beta_{1o} \gamma_{1o} \) lies in that the wage reaches its lower bound of 1 in case (b) for values of \( \sigma \) between \( \sigma_{\text{low}}^{\text{w} = 1} \) and \( \sigma_{\text{high}}^{w = 1} \). This implies that industry \( Y \) will become active while \( w = 1 \).

In the case of technological capability, we observe that it attains a lower level in its entirety as compared to case (a), while maintaining the same shape so long as \( w > 1 \). When \( w = 1 \) we find that technological capability is rising at an increasing rate as \( \sigma \) grows, beginning with a flatter slope than was the case when \( \sigma < \sigma_{\text{low}}^{w = 1} \), (i.e., when \( w > 1 \)). This is due to the wage being fixed at 1: As shown by the 'shadow' wage rate (dashed thin line), when \( \sigma \) increases past \( \sigma_{\text{low}}^{w = 1} \), had it not been for the lower bound of 1 the wage rate would have continued falling, would have reached a minimum and would have begun growing. The effect of this for technological capability is that when the wage would have continued falling, but was fixed at 1, the net marginal benefit of technological capability is rising at a relatively slower rate than its shadow counterpart. Thus technological capability rises at a slower pace than its corresponding shadow value. Once the wage rate reaches a minimum and begins rising, the process is reverted: The shadow wage rate rises faster than the actual wage \( (w = 1) \), hence the actual net marginal benefit grows faster than its shadow value, leading to the steeper slope of actual technological capability, relative to its shadow value. Once \( \sigma \) passes the value \( \sigma_{\text{high}}^{w = 1} \), the wage rate becomes strictly greater than 1, and technological capability recovers its underlying (concave and rising) shape.

Between \( \sigma_{\text{low}}^{w = 1} \) and \( \sigma_{\text{high}}^{w = 1} \) the wage rate is equal to 1 since demand for labour in industry \( X \) is insufficient to clear the labour market. Thus, surplus labour is absorbed by industry \( Y \). Employment in industry \( Y \) is zero for values of \( \sigma \) below \( \sigma_{\text{low}}^{w = 1} \) and above \( \sigma_{\text{high}}^{w = 1} \). Between these values employment in industry \( Y \) is increasing, reaches a maximum and then is decreasing. This behaviour reflects the pattern followed by the shadow wage rate. Similarly, employment in industry \( X \) \( (L_x) \) equals the economy's labour endowment minus employment in industry \( Y \), and consequently exhibits reciprocal behaviour to employment in industry \( Y \) \( (L_y) \): For values of \( \sigma \) below \( \sigma_{\text{low}}^{w = 1} \) and above \( \sigma_{\text{high}}^{w = 1} \), employment in industry \( X \) is equal to the economy's labour endowment. For \( \sigma \) between \( \sigma_{\text{low}}^{w = 1} \) and \( \sigma_{\text{high}}^{w = 1} \), it is decreasing, reaches a minimum and becomes increasing.
The number of firms is not affected by whether the wage rate reaches its lower bound (see equation ??), and is merely shifted upwards by increasing $\beta$ and $\gamma$.

This completes the analysis of cases (a) and (b). We now present the corresponding graphs for cases (c) —when $\beta$ and $\gamma$ are high— and (d) —when $\beta$ is high and $\gamma$ is low—. These can be seen in Figure 4.2b, below.

Figure 4.2b: The effect of $\sigma$ when: (c) $\beta$ and $\gamma$ are high and (d) $\beta$ is high and $\gamma$ is low.

Figure 4.2b is very similar to Figure 4.2a. Continuous lines represent case (c), when $\beta$ and $\gamma$ are high (labelled $\beta_n\gamma_{hi}$) and dashed thick lines refer to case (d), when $\beta$ is high and $\gamma$ is low (labelled $\beta_n\gamma_{lo}$). As before, dashed thin lines are the projection of the corresponding function, had the wage rate not reached its lower bound of 1: The ‘shadow’ value of the corresponding schedule.

In case (c), when $\beta$ and $\gamma$ are high (continuous lines), technological capability achieves the lowest level for all parameter combinations, and —provided $w > 1$— is increasing concavely in $\sigma$. Once $w = 1$, technological capability becomes decreasing in $\sigma$, at a decreasing rate.

Why does technological capability exhibit such behaviour? On the one hand, when $w > 1$, technological capability rises at a decreasing rate with $\sigma$ because as goods become closer substitutes a firm can capture an increasing market share by escalating its investment in technological capability. However as technological capability continues to rise, the convexity of the fixed outlays function $-F(.)$— implies that escalation becomes increasingly expensive. This
explains the decreasing slope of technological capability.

On the other hand, when \( w = 1 \) and \( \gamma \) is high (regardless of \( \beta \)) we have that technological capability contracts at a decreasing rate. To see what is occurring in this case, note that the shadow wage rate continues decreasing. That the actual wage rate does not fall, but rather remains constant at a value of 1, implies that investment in technological capability becomes increasingly expensive relative to its shadow cost (which is associated with an increasingly lower shadow wage). Furthermore, a high value of \( \gamma \) means that any reduction in profit inflicted on other firms by escalation of a firm is internalized to a great extent. These two effects combine to yield a contraction in technological capability as \( \sigma \) grows.

The analysis in the previous paragraph is valid for any value of \( \beta \). Thus in the case when \( \beta \) is low and \( \gamma \) is high we observe the same pattern as in case (c) --when \( \beta \) and \( \gamma \) are high.

The only difference is that the technological capability schedule is shifted upwards. Moreover, other variables also exhibit similar behaviour in both cases. The case of \( \beta \) low and \( \gamma \) high will not be analyzed explicitly because, as was mentioned previously, the second order conditions may be violated. Moreover, this case does not provide any additional insights to those already presented.

In the bottom left diagram we see the number of firms. In case (c) the number of firms achieves the highest level of our four cases and decreases at a decreasing rate. On the other hand, the wage rate is at its lowest level and also falls at a decreasing rate until it reaches a value of 1, after which it remains constant\(^9\). The wage is falling because the increase in technological capability is not sufficient to offset the exit of firms, leading to a reduction in labour demand.

Employment is shown on the bottom right graph. So long as employment in industry \( X \) equals labour supply in the whole economy, the wage rate is strictly greater than 1. As soon as labour demand from industry \( X \) falls short of the economy's labour endowment, any surplus labour is employed by industry \( Y \). This makes the wage rate constant at a value of 1 (which is the constant marginal product of labour in industry \( Y \)). In this case, employment in industry \( X \) falls at a decreasing rate. Conversely, so long as industry \( X \) generates sufficient employment opportunities, employment in industry \( Y \) is zero. Otherwise it rises at a decreasing rate.

Case (d) --when \( \beta \) is high and \( \gamma \) is low-- is qualitatively similar to case (b) --when \( \beta \) and \( \gamma \) are low-- and is shown in Figure 4.2b for completeness. The only difference lies in that since

\(^9\)That in this case technological capability and the wage rate are at their lowest levels, and that the number of firms is at its highest level for all parameter configurations also follows from the analysis in sections 4.1 and 4.2 in chapter 2.
now $\beta$ is higher, we find that it would take $\sigma > 1$ for the wage rate to return to levels above its lower bound of 1. Thus for admissible values of $\sigma$, technological capability remains below its shadow value while the wage rate lies above its shadow value.

Let us focus on the technological capability graph in Figure 4.2b. What generates the difference in the technological capability schedules once the wage rate reaches unity? As compared to the case when $\gamma$ is high (in which when $w = 1$, technological capability falls), we now find that technological capability continues to increase (albeit below its shadow value). This is a consequence of the incentives that a high intensity of competition generates. When $\gamma$ is low, the firm does not care much for its rivals'/partners' profits. Hence as goods become closer substitutes, firms spend more on technological capability, to capture a greater share of the market.

Note that technological capability in case (d) is strictly greater than in case (c). Similarly, in case (d) the wage rate is (weakly) greater than in case (c). Accordingly, employment in industry $X$ is (weakly) higher in case (d) than in case (c). Consequently, employment in industry $Y$ in case (d) is (weakly) lower than in case (c). The number of firms, on the other hand, is strictly lower in case (d) than in case (c).

So long as $w > 1$, the behaviour induced by increasing $\sigma$ on technological capability and the number of firms is similar in all four cases. This can be summarized as follows: Technological capability is increasing in $\sigma$, at a decreasing rate (regardless of $\beta$ and $\gamma$). The number of firms is decreasing in $\sigma$, at a decreasing rate (regardless of $\beta$ and $\gamma$).

The results discussed above and others are stated more precisely in the following proposition (which is completed in Appendix 3).

**Proposition $\sigma$: Effects of $\sigma$**

Recall that $\sigma \in (0, 1)$.

a) **Wage rate**: Define a threshold $\gamma^w$ to be the highest value of $\gamma$ such that for $\gamma \leq \gamma^w$, $\frac{\partial w}{\partial \sigma} > 0$ holds for some $\sigma \in (0, 1)$.

If $w > 1$ and $\gamma \leq \gamma^w$ then $w$ is 'U' shaped: As we increase $\sigma$, $w$ is strictly decreasing ($\frac{\partial w}{\partial \sigma} < 0$), reaches a global minimum ($\frac{\partial^2 w}{\partial \sigma^2} = 0$) and becomes strictly increasing ($\frac{\partial^2 w}{\partial \sigma^2} > 0$). It follows that $w$ is strictly convex in $\sigma$: $\frac{\partial^2 w}{\partial \sigma^2} > 0$. This corresponds to case (a), above.

Alternatively, if $w = 1$ for some $\sigma \in (0, 1)$, then define two thresholds: $\sigma_{low}^{w=1}\gamma \leq \gamma^w$ and $\sigma_{high}^{w=1}\gamma \leq \gamma^w$, such that for all $\sigma \in [\sigma_{low}^{w=1}\gamma \leq \gamma^w, \sigma_{high}^{w=1}\gamma \leq \gamma^w]$, $w = 1$ obtains. Provided $0$ < $\sigma_{low}^{w=1}\gamma \leq \gamma^w$ < $\sigma_{high}^{w=1}\gamma \leq \gamma^w$ < 1, for $\sigma < \sigma_{low}^{w=1}\gamma \leq \gamma^w$ $w$ is strictly decreasing and strictly convex in $\sigma$ ($\frac{\partial w}{\partial \sigma} < 0, \frac{\partial^2 w}{\partial \sigma^2} > 0$). For $\sigma > \sigma_{high}^{w=1}\gamma \leq \gamma^w$ $w$ is strictly increasing and strictly convex in $\sigma$. 

143
(\frac{\partial w}{\partial \sigma} > 0, \frac{\partial^2 w}{\partial \sigma^2} > 0). This corresponds to case (b).

For \( \gamma > \gamma^w \), define a threshold \( \sigma^{w=1} | \gamma > \gamma^w \) such that for all \( \sigma \geq \sigma^{w=1} | \gamma > \gamma^w \), \( w = 1 \). For \( \sigma < \sigma^{w=1} | \gamma > \gamma^w \), \( w \) is strictly decreasing and strictly convex in \( \sigma (\frac{\partial w}{\partial \sigma} < 0, \frac{\partial^2 w}{\partial \sigma^2} > 0) \). This case corresponds to cases (c) and (d).

b) Technological capability: For \( w > 1 \), we have that \( u \) is strictly increasing and strictly concave in \( \sigma: \frac{\partial u}{\partial \sigma} > 0, \frac{\partial^2 u}{\partial \sigma^2} < 0 \).

When \( w = 1 \), \( u \) is strictly convex in \( \sigma: \frac{\partial^2 u}{\partial \sigma^2} > 0 \). To consider the slope of \( u \), let us define the threshold \( \gamma^u \), such that \( \frac{\partial u}{\partial \sigma} |_{\gamma=\gamma^u} = 0 \). Then for \( \gamma < \gamma^u \), \( u \) is strictly increasing in \( \sigma (\frac{\partial u}{\partial \sigma} > 0) \) and for \( \gamma > \gamma^u \), \( u \) is strictly decreasing in \( \sigma (\frac{\partial u}{\partial \sigma} < 0) \). \( \gamma^u \) defines the cut-off for distinguishing between a low or high \( \gamma \), in cases (b), (c) and (d).

\( u \) is kinked at the point where \( w \) becomes 1.

c) Number of firms: \( N+1 \) is strictly decreasing and strictly convex in \( \sigma: \frac{\partial (N+1)}{\partial \sigma} < 0, \frac{\partial^2 (N+1)}{\partial \sigma^2} > 0 \).

d) Welfare: \( W \) exhibits behavior similar to the wage rate (with different numerical values). However, when \( w = 1 \), \( W \) becomes flatter, generating a kink in \( W \). This change of slope occurs for the same reason as outlined in part (d) of Proposition \( \gamma \).

e) Employment in industry \( Y \): \( L_Y = 0 \) for \( w > 1 \). If \( w = 1 \) then \( L_Y > 0 \). In this case, \( L_Y \) is strictly concave: \( \frac{\partial^2 L_Y}{\partial \sigma^2} < 0 \). To analyze the slope of \( L_Y \), use the threshold \( \gamma^w \) defined in part (a) of this proposition. If \( w = 1 \) and \( \gamma \leq \gamma^w \) then \( L_Y \) is \( \gamma \)-shaped: As we increase \( \sigma \), \( L_Y \) is strictly increasing (\( \frac{\partial L_Y}{\partial \sigma} > 0 \)), reaches a global maximum (\( \frac{\partial L_Y}{\partial \sigma} = 0 \)) and becomes strictly decreasing (\( \frac{\partial L_Y}{\partial \sigma} < 0 \)). This refers to cases (b) and (d).

If \( w = 1 \) but \( \gamma > \gamma^w \) then \( L_Y \) is increasing in \( \sigma (\frac{\partial L_Y}{\partial \sigma} > 0) \) -this is case (c). As before \( L_Y \in [0, L] \) in equilibrium.

f) Employment in industry \( X \): \( L_X \) is equal to \( L - L_Y \). Thus its behavior is the opposite of \( L_Y \). \( L_Y \) supplements \( L_X \) to use all of the available labour supply (given by \( L \)).

Proof: By inspection of the appropriate variables, their derivatives and associated thresholds.

This completes the analysis of how the economy changes as we vary each of the parameters. We now summarize some of the most important results of the model.

5 Remarks Concerning General Features of the Extended Model

In this section we re-take the remarks made in section 5 of chapter 2. In choosing a development configuration, the wage rate is the outcome of the technological capability effect and the market structure effect. In the basic model with Cournot competition (chapter 2), two
development configurations emerged: proliferation and high-tech. In the model with varying degrees of intensity of competition developed in this chapter, another possibility emerges: an intermediate configuration. The following proposition summarizes the changes required to extend Proposition 1 in chapter 2 to the current framework.

**Proposition 1: Development Configurations in Autarky with Adjustable Intensity of Competition**

A 'high-tech' configuration is associated with a higher wage rate (and welfare) than a 'proliferation' configuration, unless:

1. \( \sigma \) is low (for any \( \beta \) and any \( \gamma \)). In this case a 'proliferation' configuration features a higher wage rate (and welfare).

2. \( \beta \) is high and \( \gamma \) takes intermediate values (for any \( \sigma \)). In this case an intermediate configuration with intermediate values for the number of firms and technological capability features a higher wage rate (and welfare).

**Proof**: By inspection of the equilibrium solution functions for technological capability, the number of firms and the wage rate in table 1 and Figures 4.1a-b and 4.2a-b.

This proposition is based on the analysis in Section 4 of this chapter and chapter 2. In case (1), when \( \sigma \) is low, goods are poor substitutes and we observe an economy with many firms each with a low level of technological capability. This proliferation of technological trajectories is associated with a higher wage rate (and welfare). In case (2) we find that for high \( \beta \) and intermediate \( \gamma \) the economy exhibits an intermediate number of firms and technological capability. Since the wage rate is 'U'-shaped, this will be associated with a higher wage rate (and welfare).

### 6 Conclusions

This chapter extends the closed economy model presented in chapter 2 by allowing for varying degrees of intensity of competition (\( \gamma \)). The extension has allowed us to analyze development configurations when competition is less intense than individual profit maximization. We detailed the effects of changes in the intensity of competition in Proposition \( \gamma \). The main results are: If the elasticity of the fixed labour requirement (\( \beta \)) is low, then technological capability and the wage rate decrease concavely in \( \gamma \), while the number of firms increases convexly in \( \gamma \). On the other hand, if \( \beta \) is high, the wage rate achieves a maximum for intermediate values of \( \gamma \). This means that 'more competition is better' is only true when the industry features a low
$\beta$. When $\beta$ is high, there is an intermediate value of $\gamma$ which maximizes the wage rate (and welfare). These insights can serve as a positive and normative basis for competition policy, which is represented by the intensity of competition.

The extended model gives rise to a third development configuration in addition to those introduced in chapter 2 (proliferation and high-tech): An intermediate economy (Proposition 1). This economy is characterized by high $\beta$ and achieves a maximum wage rate for intermediate values of $\gamma$. That is, some degree of collusion may be desirable if the economy is constrained to having a high elasticity of the fixed labour requirement (and the associated high marginal cost of technological capability). This is associated to intermediate values for technological capability and concentration.

A 'replication argument' similar to that introduced in chapter 2 can be made to extend the analysis to multiple industries, provided industries are linked only via the demand for labour inputs (i.e., they have no demand or supply linkages).
Appendix 1: Solving the Final Stage Subgame for Industry X

Let us solve the final stage subgame, in order to obtain a ‘solved-out’ payoff. The first order conditions for this stage are given by (equation 4.3):

\[(1 - \gamma) \left( p_k + \frac{\partial p_k}{\partial x_k} x_k \right) + \gamma \sum_{h \neq k}^{N} \frac{\partial p_h}{\partial x_k} x_h \frac{1}{N} = 0 \quad \text{for} \quad k = 1, \ldots, N + 1 \]

Substituting inverse demand from (2.3) and its derivatives \( \frac{\partial p_h}{\partial x_k} \) into the first order condition yields

\[(1 - \gamma) \left( 1 - \frac{\sigma x_k}{u_k^2} \right) - \left( 1 - \gamma + \frac{\gamma}{N} \right) \frac{2 \sigma}{u_k} \sum_{i \neq k}^{N} \frac{x_i}{u_i} = 0 \quad (A1.1)\]

Add and subtract \( (1 - \gamma + \frac{\gamma}{N}) \frac{2 \sigma x_k}{u_k u_k} \) and re-arrange (A1.1) to obtain

\[x_k = \frac{(1 - \gamma) u_k - \left( 1 - \gamma + \frac{\gamma}{N} \right) \frac{2 \sigma}{u_k} \sum_{i \neq k}^{N} \frac{x_i}{u_i}}{2 \left[ (2 - \sigma)(1 - \gamma) - \frac{\sigma}{N} \right]} \quad (A1.2)\]

Summing (A1.2) over \( k \), and solving for \( \sum_{i=1}^{N+1} \frac{x_i}{u_i} \) yields

\[\sum_{i=1}^{N+1} \frac{x_i}{u_i} = \frac{(1 - \gamma) \sum_{i=1}^{N+1} u_k}{2 \left[ (2 + \sigma N)(1 - \gamma) + \sigma \gamma \right]} \quad (A1.3)\]

Now substitute (A1.3) back into (A1.2). This yields the solution for \( x_k \), as follows

\[x_k = \frac{(1 - \gamma) u_k^2}{2 \left[ (2 - \sigma)(1 - \gamma) - \frac{\sigma}{N} \right]} \left[ 1 - \frac{(1 - \gamma + \frac{\gamma}{N}) \sigma}{(2 + \sigma N)(1 - \gamma) + \sigma \gamma} \sum_{i=1}^{N+1} \frac{u_i}{u_k} \right] \quad (A1.4)\]

Assuming symmetry between firms (such that \( u_i = u_k \)), \( x_k \) simplifies to

\[x = \frac{(1 - \gamma) u_k^2}{2 \left[ (2 + \sigma N)(1 - \gamma) + \sigma \gamma \right]} \quad (A1.4')\]

If we set \( \gamma = 0 \) this expression becomes

\[x = \frac{u_k^2}{2(2 + \sigma N)} \quad (A1.4'')\]

which is expression (A1.4'). Price \( p_k \) can be solved by adding and subtracting \( 2 \sigma \frac{x_k}{u_k^2} \) to the inverse demand function (2.3) to obtain

\[p_k = 1 - 2(1 - \sigma) \frac{x_k}{u_k^2} - \frac{2 \sigma}{u_k} \sum_{i=1}^{N+1} \frac{x_i}{u_i} \quad (A1.5)\]
Next we substitute $x_k$ from (A1.4) and the expression in (A1.3) into $p_k$, to obtain the following solution:

$$p_k = \frac{1}{(2 - \sigma)(1 - \gamma) - \frac{\sigma\gamma}{N}} \left[ \left(1 - \gamma - \frac{\sigma\gamma}{N}\right) - \frac{(1 - \gamma) (1 - \gamma - \frac{\gamma}{N}) \sigma}{(2 + \sigma N)(1 - \gamma) + \sigma\gamma} \sum_{i=1}^{N+1} u_i \right] \quad \text{(A1.6)}$$

Assuming symmetry (that is, $u_i = u_k$), price simplifies to

$$p = \frac{1 - \gamma(1 - \sigma)}{(2 + \sigma N)(1 - \gamma) + \sigma\gamma} \quad \text{(A1.6')}
$$

Upon setting $\gamma = 0$, $p$ reduces further to:

$$p = \frac{1}{(2 + \sigma N)} \quad \text{(A1.6'')}
$$

which is equation (A1.6').

The solved-out payoff is obtained by multiplying (A1.4) and (A1.6), from which

$$\pi_k = \frac{1 - \gamma}{2 [(2 - \sigma)(1 - \gamma) - \frac{\sigma\gamma}{N}]} \left[ u_k - \frac{(1 - \gamma + \frac{\gamma}{N}) \sigma}{(2 + \sigma N)(1 - \gamma) + \sigma\gamma} \sum_{i=1}^{N+1} u_i \right] \quad \text{(A1.7)}$$

this is equation (4.4) in the text.

Appendix 2: Second order conditions for the Second Stage Subgame

Differentiating the first order conditions (4.8) with respect to $u_k$ to obtain the second order conditions yields the following

$$(1 - \gamma)L \frac{\partial^2 \pi_k}{\partial u_k^2} + \frac{\gamma}{N} L \sum_{i \neq k}^N \frac{\partial^2 \pi_i}{\partial u_k^2} \leq (1 - \gamma) \omega \beta (\beta - 1) \frac{u_k^{\beta - 2}}{u_0^\beta} \quad \text{(A2.1)}$$

where

$$\frac{\partial^2 \pi_k}{\partial u_k^2} = \frac{(1 - \gamma)}{[(2 - \sigma)(1 - \gamma) - \frac{\sigma\gamma}{N}]^2} \left[ \left(1 - \gamma - \frac{\sigma\gamma}{N}\right) - \frac{(1 - \gamma) (1 - \gamma - \frac{\gamma}{N}) \sigma}{(2 + \sigma N)(1 - \gamma) + \sigma\gamma} \right]$$

$$\frac{1 - \gamma + \frac{\gamma}{N}) \sigma}{(2 + \sigma N)(1 - \gamma) + \sigma\gamma} \quad \text{(A2.2)}$$

$$\frac{\partial^2 \pi_i}{\partial u_k^2} = \frac{(1 - \gamma)^2 \sigma^2 (1 - \gamma - \frac{\gamma}{N}) (1 - \gamma + \frac{\gamma}{N})}{[(2 - \sigma)(1 - \gamma) - \frac{\sigma\gamma}{N}]^2 [(2 + \sigma N)(1 - \gamma) + \sigma\gamma]^2} \quad \text{(A2.3)}$$
We analyze the second order conditions in the neighbourhood of the symmetric equilibrium. Upon substituting equations (4.29), (4.29), (4.27) and (4.25) into the second order condition (A2.1), we obtain a restriction in \((\beta, \gamma, \sigma)\)-space: The restriction implies that for the second order conditions to hold \(\beta\) cannot be too low (and \(\beta\) must be at least strictly greater than 2 in all cases), \(\gamma\) cannot be too high and \(\sigma\) cannot be too high. This defines bounds on each of these parameters, for given values of the remaining parameters. If a parameter gets close to or crosses its bound, the others may compensate by moving inward and away from their bound, thereby ensuring that the second order conditions continue to hold.

This can be stated more formally by defining the bounds as follows. Let \(\sigma^{SOC}\) be the highest value of \(\sigma\) such that for given values of \((\beta, \gamma)\) expression (A2.1) holds with equality, let \(\gamma^{SOC}\) be the highest value of \(\gamma\) such that for given values of \((\beta, \sigma)\) expression (A2.1) holds with equality and let \(\beta^{SOC}\) be the lowest value of \(\beta\) such that for given values of \((\gamma, \sigma)\) expression (A2.1) holds with equality. We can operationalize the bounds by considering expression (A2.1) with equality as defining implicit functions in \((\beta, \gamma, \sigma)\)-space. From expression (A2.1) with equality we can define (implicit) functions as follows: \(\beta^{SOC} = B(\gamma, \sigma)\) or \(\gamma^{SOC} = \Gamma(\beta, \sigma)\) or \(\sigma^{SOC} = \Sigma(\beta, \gamma)\), with the following properties (which obtain by direct application of the implicit function theorem): \(\frac{\partial \beta^{SOC}}{\partial \gamma} > 0\), \(\frac{\partial \gamma^{SOC}}{\partial \sigma} > 0\), \(\frac{\partial \beta^{SOC}}{\partial \sigma} > 0\), \(\frac{\partial \sigma^{SOC}}{\beta} < 0\), \(\frac{\partial \sigma^{SOC}}{\gamma} < 0\).

**Appendix 3: Analysis of Other Variables of Interest**

This Appendix continues the analysis of the effects of \(\gamma\) and \(\sigma\) for the remaining variables of the model, continuing with the same format as in Appendix 4 in chapter 2.

The remaining variables are:

- **g)** Price of good \(X\): \(p = \frac{1 - \gamma (1 - \sigma)}{(2 + \sigma N)(1 - \gamma) + \sigma}\) (equation A1.6', Appendix 1).
- **h)** Price-quality ratio of good \(X\): \(\xi\)
- **i)** Per-firm output of good \(X\) (per-capita): \(x = \frac{(1 - \gamma)^2}{2(2 + \sigma N)(1 - \gamma) + \sigma}\) (equation A1.4', Appendix 1).
- **j)** Industry output of good \(X\) (per-capita): \(X = (N + 1) x\)
- **k)** Economy output of good \(X\): \(\chi = L X\)
- **l)** Demand for Good \(Y\) (per-capita): \(Y = w - (N + 1) px\) (equation 2.5).
- **m)** Aggregate Demand for Good \(Y\): \(T^D = w \left[ L - (N + 1) \epsilon \left( \frac{w}{u_x} \right)^{\beta} \right]\) (equation 2.18).
- **n)** Aggregate Supply of Good \(Y\): \(T^S = L - (N + 1) \epsilon \left( \frac{w}{u_x} \right)^{\beta}\) (equation 2.19).
o) Value added in industry X (per-capita): $VA_x = (N + 1)p x$ (see Appendix 3, chapter 2).

p) Value added in industry Y (per-capita): $VA_y = L_y/L$ (see Appendix 3, chapter 2). Note that in equilibrium, this is the same as physical per-capita production of good Y. To see this, note that good Y is the numeraire, so its price ($q$) has been set equal to 1, and sector Y is assumed to use a 1:1 technology. Thus $VA_y = Y$.

We proceed to complete the corresponding propositions.

Proposition γ: Effects of γ (continued)

g) Price of good X: $p$ is strictly increasing in γ ($\frac{\partial p}{\partial \gamma} > 0$). To assess the concavity of $p$, let us define threshold levels ($\beta^p, \sigma^p$) such that $\frac{\partial^2 p}{\partial \gamma^2}|_{\beta^p, \sigma^p} = 0$.

For $\sigma < \sigma^p$, $p$ is strictly convex in γ ($\frac{\partial^2 p}{\partial \gamma^2} > 0$), regardless of β. For $\sigma \geq \sigma^p$ and $\beta < \beta^p$, $p$ is strictly concave in γ ($\frac{\partial^2 p}{\partial \gamma^2} < 0$). For $\sigma \geq \sigma^p$ and $\beta \geq \beta^p$, $p$ has an inflection point at $\gamma^p$ ($\frac{\partial^2 p}{\partial \gamma^2}|_{\gamma=\gamma^p} = 0$), is strictly convex for $\gamma < \gamma^p$ ($\frac{\partial^2 p}{\partial \gamma^2} > 0$) and is strictly concave for $\gamma > \gamma^p$ ($\frac{\partial^2 p}{\partial \gamma^2} < 0$).

h) Price-quality ratio of good X: $\frac{x}{u}$ is strictly increasing and strictly convex in γ: $\frac{\partial^2 x}{\partial \gamma^2} > 0$, $\frac{\partial^2 x}{\partial \gamma \partial u} > 0$. Being a composite variable, it inherits its properties from $p$ and $u$.

In particular, when $\sigma \geq \sigma^p$ and $\beta \geq \beta^p$, (i.e., $p$ exhibits an inflection point, see part (g) of this proposition), $\frac{x}{u}$ will become flatter at the inflection point.

Let us increase γ for the case when $\beta \geq \beta^u$ and $\sigma \geq \sigma^u$. In this case $u$ has a sinusoidal shape (see part (b) of this proposition). As γ increases $\frac{x}{u}$ will become flatter as $u$ passes by its local minimum (approaching from the left) and steeper when $u$ passes by its global maximum (also approaching from the left).

When $u$ becomes equal to 1, $\frac{x}{u}$ inherits any kinks from $u$, and it becomes flatter if $u$ becomes steeper (and viceversa).

i) Per-firm output of good X (per-capita): $x$ is strictly decreasing in γ ($\frac{\partial x}{\partial \gamma} < 0$).

To assess the concavity of $x$, consider the threshold levels ($\beta^p, \sigma^p$) (defined in part (g) of this proposition).

For $\sigma < \sigma^p$, $x$ is strictly concave in γ ($\frac{\partial^2 x}{\partial \gamma^2} < 0$), for any $\beta$. For $\sigma \geq \sigma^p$ and $\beta < \beta^p$, $x$ is strictly convex in γ ($\frac{\partial^2 x}{\partial \gamma^2} > 0$). For $\sigma \geq \sigma^p$ and $\beta \geq \beta^p$, $x$ has an inflection point at $\gamma^p$ ($\frac{\partial^2 x}{\partial \gamma^2}|_{\gamma=\gamma^p} = 0$), is strictly concave for $\gamma < \gamma^p$ ($\frac{\partial^2 x}{\partial \gamma^2} < 0$) and is strictly convex for $\gamma > \gamma^p$ ($\frac{\partial^2 x}{\partial \gamma^2} > 0$).

When $u$ becomes 1 (see part (a) of this proposition), $x$ inherits the kinks in $u$, and falls at a faster rate.
j) Industry output of good X (per-capita): $X$ is strictly decreasing ($\frac{\partial X}{\partial \gamma} < 0$) and concave ($\frac{\partial^2 X}{\partial \gamma^2} \leq 0$) in $\gamma$. When $w$ becomes 1, $X$ inherits the kink in $u$, and falls at a faster rate.

k) Economy output of good X: $\chi$, exhibits behavior identical to $X$, except that it is multiplied by $L$.

l) Demand for good Y (per-capita): $Y$ has properties identical to $L_y$, and we refer the reader to Proposition $\gamma$, part (e).

m) Aggregate Demand for Good Y: $T^D$, has the same properties as $Y$, but it is multiplied by $L$ (in equilibrium, $T^D = L_y$).

n) Aggregate Supply of Good Y: $T^S \equiv L_y$, always matches $T^D$ in equilibrium, and thereby exhibits identical behavior.

o) Value added in industry X (per-capita): $VA_x$, exhibits behavior similar to $w$. The only difference is that when $w = 1$, $VA_x$ does not (like $w$) become flat at a value of 1. Instead it becomes steeper. To see this, note that the wage variation which occurs as $\gamma$ changes acts like a 'buffer' in attenuating the adjustment of sector $X$. Once the wage rate reaches its minimum value of 1, the 'buffer' is exhausted and sector $X$ adjusts at a faster rate.

p) Value added in industry Y (per-capita): $VA_y$, has the same value as per-capita demand for good Y and displays identical behavior.

Proof: By inspection of the appropriate variables, their derivatives and associated thresholds.

Proposition $\sigma$: Effects of $\sigma$ (continued)

g) Price of good X: $p$, is strictly increasing in $\sigma$ ($\frac{\partial p}{\partial \sigma} > 0$). To assess the concavity of $p$, let us define a threshold level $\sigma^p$, such that $\frac{\partial^2 p}{\partial \sigma^2}|_{\sigma^p} = 0$. For $\sigma < \sigma^p$, $p$ is strictly convex in $\sigma$ ($\frac{\partial^2 p}{\partial \sigma^2} > 0$). For $\sigma > \sigma^p$, $p$ is strictly concave in $\sigma$ ($\frac{\partial^2 p}{\partial \sigma^2} < 0$). To ensure that $\sigma^p \in (0, 1)$, we need to specify thresholds $\gamma_{low}^p$, $\gamma_{high}^p$, where $\gamma_{low}^p$ is the least value of $\gamma$ such that $\frac{\partial^2 p}{\partial \sigma^2}|_{\gamma = \gamma_{low}^p} = 0$ and $\gamma_{high}^p$ is the highest value of $\gamma$ such that $\frac{\partial^2 p}{\partial \sigma^2}|_{\gamma = \gamma_{high}^p} = 0$. For $\gamma < \gamma_{low}^p$, $\sigma^p \geq 1$, i.e., $p$ is strictly convex in $\sigma$ ($\frac{\partial^2 p}{\partial \sigma^2} > 0$) for $\sigma \in (0, 1)$. For $\gamma > \gamma_{high}^p$, $\sigma^p \leq 0$, i.e., $p$ is strictly concave in $\sigma$ ($\frac{\partial^2 p}{\partial \sigma^2} < 0$) for $\sigma \in (0, 1)$.

h) Price-quality ratio of good X: There exist two thresholds, $\beta p/u$ and $\gamma p/u$, which divide the parameter space into four regions, as follows:

Region 1. For $\beta < \beta p/u$ and $\gamma < \gamma p/u$, $\frac{p}{u}$ is strictly decreasing and strictly convex: $\frac{\partial^2 \frac{p}{u}}{\partial \sigma^2} < 0$, $\frac{\partial^2 \frac{p}{u}}{\partial \gamma^2} > 0$.

Region 2. For $\beta > \beta p/u$ and $\gamma < \gamma p/u$, $\frac{p}{u}$ is 'U'-shaped: As $\sigma$ grows, $\frac{p}{u}$ is strictly decreasing...
\( \frac{\partial X}{\partial \sigma} < 0 \), reaches a minimum and becomes strictly increasing \( \frac{\partial X}{\partial \sigma} > 0 \). \( \frac{\partial^2 X}{\partial \sigma^2} \) is strictly convex \( \frac{\partial^2 X}{\partial \sigma^2} > 0 \).

Region 3. For \( \beta < \beta_p/u \) and \( \gamma > \gamma_p/u \), \( \frac{\partial X}{\partial \sigma} \) is strictly increasing and strictly convex: \( \frac{\partial X}{\partial \sigma} > 0 \), \( \frac{\partial^2 X}{\partial \sigma^2} > 0 \).

Region 4. For \( \beta > \beta_p/u \) and \( \gamma > \gamma_p/u \), \( \frac{\partial X}{\partial \sigma} \) is strictly increasing and strictly concave: \( \frac{\partial X}{\partial \sigma} > 0 \), \( \frac{\partial^2 X}{\partial \sigma^2} < 0 \).

\( \frac{\partial X}{\partial \sigma} \) inherits its properties from \( p \) and \( u \). Thus, it will exhibit a kink at the point where \( u \) has a kink, i.e., when \( w \) reaches its lower bound of 1.

i) Per-firm output of good \( X \) (per-capita): \( x \) is '\( \cap \)'-shaped. As \( \gamma \) rises, \( x \) shifts downward and leftward. As \( \beta \) rises, \( x \) shifts downward and rightward. Accordingly, the section of \( x \) corresponding to \( \sigma \in (0, 1) \) will exhibit non-monotonic behaviour, depending on whether the maximum of \( x \) corresponds to a value of \( \sigma \) within the interval \( (0, 1) \).

Let us define the following thresholds:

i) \( \gamma_{\text{low}}^x \) is such that \( \frac{\partial X}{\partial \sigma} \big|_{\gamma=\gamma_{\text{low}}^x, \sigma=1} = 0 \).

ii) \( \gamma_{\text{high}}^x \) is such that \( \frac{\partial X}{\partial \sigma} \big|_{\gamma=\gamma_{\text{high}}^x, \sigma=0} = 0 \).

iii) \( \beta_{\text{low}}^x \) is such that \( \frac{\partial X}{\partial \sigma} \big|_{\beta=\beta_{\text{low}}^x, \sigma=0} = 0 \).

iv) \( \beta_{\text{high}}^x \) is such that \( \frac{\partial X}{\partial \sigma} \big|_{\beta=\beta_{\text{high}}^x, \sigma=1} = 0 \).

These thresholds partition the parameter space into the following three regions.

Region 1. When \( \gamma < \gamma_{\text{low}}^x \) or \( \beta > \beta_{\text{high}}^x \), \( x \) is strictly increasing in \( \sigma \): \( \frac{\partial X}{\partial \sigma} > 0 \).

Region 2. When \( \gamma_{\text{low}}^x < \gamma < \gamma_{\text{high}}^x \) and \( \beta_{\text{low}}^x < \beta < \beta_{\text{high}}^x \), \( x \) is '\( \cap \)'-shaped in \( \sigma \): At first \( x \) is strictly increasing in \( \sigma \) \( (\frac{\partial X}{\partial \sigma} > 0) \), reaches an interior maximum \( (\frac{\partial X}{\partial \sigma} = 0) \), and becomes strictly decreasing in \( \sigma \) \( (\frac{\partial X}{\partial \sigma} < 0) \).

Region 3: When \( \gamma > \gamma_{\text{high}}^x \) or \( \beta < \beta_{\text{low}}^x \), \( x \) is strictly decreasing in \( \sigma \): \( \frac{\partial X}{\partial \sigma} < 0 \).

\( x \) is concave whenever \( w > 1 \) and convex whenever \( w = 1 \), and exhibits a kink at the point when the wage rate becomes 1.

j) Industry output of good \( X \) (per-capita): \( X \) is strictly convex in \( \sigma \). To assess the slope of \( X \), define thresholds \( (\beta_X, \gamma_X) \) such that \( \frac{\partial X}{\partial \sigma} \big|_{\beta=\beta_X, \gamma=\gamma_X} = 0 \). Then for \( \beta < \beta_X \) and \( \gamma < \gamma_X \), \( X \) is 'U'-shaped in \( \sigma \): It is strictly decreasing at first \( (\frac{\partial X}{\partial \sigma} < 0) \), reaches a minimum \( (\frac{\partial X}{\partial \sigma} = 0) \), and lastly becomes strictly increasing \( (\frac{\partial X}{\partial \sigma} > 0) \). Otherwise, \( X \) is strictly decreasing \( (\frac{\partial X}{\partial \sigma} < 0) \).

k) Economy output of good \( X \): \( \chi \) exhibits behaviour identical to \( X \), except that it is multiplied by \( L \).

l) Demand for good \( Y \) (per-capita): \( Y \) has properties identical to \( L_y \), and we refer the reader to Proposition \( \sigma \), part (e).
m) Aggregate Demand for Good Y: $\mathcal{T}^D$, has the same properties as $Y$, but it is multiplied by $L$ (in equilibrium, $\mathcal{T}^D = L_Y$).

n) Aggregate Supply of Good Y: $\mathcal{T}^S = L_Y$, always matches $\mathcal{T}^D$ in equilibrium, and thereby exhibits identical behavior.

o) Value added in industry X (per-capita): $VA_x$, exhibits behavior similar to $w$. The only difference is that when $w = 1$, $VA_x$ does not (like $w$) become flat at a value of 1. Instead it becomes steeper. To see this, note that the wage variation which occurs as $\sigma$ changes acts like a 'buffer' in attenuating the adjustment of sector $X$. Once the wage rate reaches its minimum value of 1, the 'buffer' is exhausted and sector $X$ adjusts at a faster rate.

p) Value added in industry Y (per-capita): $VA_y$, has the same value as per-capita demand for good Y and displays identical behavior.

Proof: By inspection of the appropriate variables, their derivatives and associated thresholds.\[\square\]
Chapter 5

Collusion in General Equilibrium with Oligopolistic Interactions: Open Economy

1 Introduction

This chapter extends the open economy framework developed in chapter 3 by introducing adjustable intensity of competition (collusion or profit sharing). This is introduced by means of the same generalized profit function used in chapter 4. A firm's objective is now a weighted average of the firm's and its rivals' profits. This allows us to encompass intensities of competition ranging from the benchmark individual profit maximization to perfect collusion (joint profit maximization). Alternatively, this chapter can be regarded as an extension of the closed economy with profit sharing developed in chapter 3, to an open economy setting.

We present a general equilibrium model of two economies, each with two industries. Each economy has the same structure to that presented in chapter 4. There is free trade in the goods market. Consumers choose between a homogeneous type $Y$ good and vertically differentiated type $X$ goods. Each consumer's labour supply is constant, and has been set to 1. Labour is assumed perfectly immobile between the economies, but perfectly mobile within each economy. Each country has a population of $L_i$ ($i = d, f$) consumers/workers. Labour demand stems from industries $X$ (denoted $L_{xi}$ in country $i = d, f$) and $Y$ (denoted $L_{yi}$ in country $i = d, f$). Labour is the only input. Consumers' income is constituted by their wage payments, profits accruing from shares owned in firms, and an endowment of type $Y$ goods. We will see that, in equilibrium, profits are zero, so they drop out from the budget constraint.
In industry $X$, firms play a three stage game. In stage 1 the entry decision is made, and a zero profit condition emerges. In stage 2, firms invest in technological capabilities (denoted $u_t$), taking market structure as given. Investment in technological capability is a sunk cost which implies increasing returns to scale. Industry $X$ uses labour to generate technological capability (in the form of sunk costs). As before, we assume that variable costs are zero. In stage 3 firms choose quantities ($x_t$), taking market structure and technological capabilities as given. In all stages the firm's (generalized) profit function is a weighted average between its own profits and its rivals'.

Industry $Y$ uses a 1:1 technology with constant returns to scale. This technology implies that the output of industry $Y$ is equal to employment in that industry. Good $Y$ is treated as the numeraire, so its price is set to 1. The marginal product of labour in industry $Y$ is, therefore, equal to 1. This constitutes workers' outside option, and establishes a minimum wage of unity.

A symmetric general equilibrium for the world economy is characterized. This features identical values for all variables in both economies, and a symmetric Sub-Game Perfect Nash Equilibrium in industry $X$. The chapter provides an analysis of how the symmetric equilibrium changes as parameters vary. We find a separating surface which divides the parameter space into two areas: One in which trade is welfare improving, and another in which the gains-from-trade-theorem breaks down. The view that emerges is that each trade regime is optimal under certain circumstances. One of the contributions of this research is to provide a framework which allows a clear typification of these circumstances, with a precise set of predictions amenable to testing.

We also performed an exercise where we allow the economies' initial conditions to differ and then to ask whether catching-up is feasible and welfare-enhancing. The procedure is identical to that followed in section 5 of chapter 3: We start from a symmetric equilibrium, and then allow initial conditions to differ between the economies while tracking the new (general) equilibrium solutions (which are no longer symmetric). The asymmetry in initial conditions generates asymmetry between the economies' wage rates, and we end up with an advanced (high-wage) economy and a backward (low-wage) economy. The economy with better initial conditions becomes the leader. In this exercise we find that under certain circumstances, the advanced economy may attain a higher wage rate by engaging in free trade, while the backward economy would achieve higher income by remaining in autarky. This raises the issue of (unfair) trade negotiations, in which a powerful country may coerce other (less powerful) nations into trade regimes that decrease their welfare. Regarding catching-up in wage rates, we found that it is
always feasible and welfare-enhancing. Thus, the results are identical to those in chapter 3, and we refer the reader to section 5 of that chapter for further details.

In considering trade policy, we come across results about the trade balance. On the one hand, we find that there is intra-industry trade in all circumstances. On the other hand, inter-industry trade is only present when there are asymmetric initial conditions, and takes the following form: The high wage (advanced) economy will be a net exporter of type \( X \) goods and a net importer of type \( Y \) goods. The reverse holds for the low wage (backward) economy. This inter-industry trade pattern may require that the backward economy satisfy the high wage economy's demand for type \( Y \) goods by depleting its endowment of this kind of good, since its production of this good will not be sufficient to meet demand by the advanced economy (in parallel with the backward economy's own thirst for imported type \( X \) goods).

The chapter is structured as follows. In Section 2 we develop the model. In Section 3 we characterize a symmetric general equilibrium. Section 4 presents an analysis of the comparative statics properties of the symmetric general equilibrium. Here we analyze how the equilibrium solutions change as we vary parameters (one at a time), assuming that all parameters are identical in both economies. This section focusses on what happens when both economies face identical changes in their structural/institutional characteristics. Section 5 presents results on the existence and properties of the separating surface, and policy implications are addressed. We conclude in Section 6.

2 The Open Economy General Equilibrium Model with Profit Sharing

The treatment of the consumer's problem, the labour market and industry \( Y \) are identical to that presented in chapter 3, and need not be repeated. We will, however, invoke the equations presented in that chapter.

Industry \( X \)

The structure of the game is the same as in previous chapters. To model the 'intensity of competition', firms maximize a generalized objective function, which includes, as well as firm's own profit, their local rivals' (partners'). Thus we have two groups of firms (cartels) which embark upon profit sharing agreements: A domestic cartel and a foreign one.

That all local firms are members of a cartel internalizes the effect that a firm's actions has on its local rivals' payoffs. However, since cartels are constituted only by local firms, the effect of a firm's actions on its overseas rivals is not internalized. As in chapter 4, we will vary
the 'intensity of competition' by changing a parameter (to be labelled 'γ_d' for the domestic economy, and 'γ_f' for the foreign economy).

Recall the two notions of symmetry introduced in chapter 3. First, we have symmetry within each economy, but not necessarily across economies. Second, we have symmetry within and across economies. For symmetry across economies parameters must take the same value in both nations. We label this a symmetric (general) equilibrium.

Backward inducting to find a Subgame Perfect Nash Equilibrium, we proceed to the description and solution of the final stage in the firms' decision problem.

2.1 Stage 3: Quantity Competition with Profit Sharing

In stage three we seek a symmetric Nash Equilibrium in quantities, taking technological capability and market structure as given. Gross (per-consumer) profits of firm k in country i are defined as

\[ \pi_{ki} = p_{ki}x_{ki} \quad \text{for } i = d, f \]  

(5.1)

The firm's objective is now

\[ (1 - \gamma_i)\pi_{ki} + \gamma_i\bar{\pi}_{-ki} \quad \text{for } i = d, f \]  

(5.2)

where:

\( \pi_{ki} \) denotes firm k's gross profits per consumer, located in economy i = d, f.

\( \bar{\pi}_{-ki} \) is the average of gross (per-consumer) profits of firm k's (local) rivals, located in economy i = d, f (\( \bar{\pi}_{-ki} = \frac{\sum_{i \neq k} \pi_{ii}}{n_i} \)).

\( \gamma_i \in [0, \frac{n_i}{n_i+1}] \) is the intensity of the competition parameter in economy i = d, f. An extensive discussion of the meaning of γ was offered in chapter 4, section 2.2.1.

Firms maximize their objective (5.2) by choosing \( x_{ki} \), with the following first order conditions:

\[ (1 - \gamma_i) \left( p_{ki} + \frac{\partial p_{ki}}{\partial x_{ki}} x_{ki} \right) + \gamma_i \sum_{i \neq k} \frac{\partial p_{ki}}{\partial x_{ki}} x_{ki} = 0 \quad \text{for } i = d, f \quad \text{and } k = 1, \ldots, n_i + 1 \]  

(5.3)

In Appendix 1, a symmetric Nash equilibrium is obtained by solving the system in (5.3). We obtain solutions for vectors \( x_d \) and \( x_f \) in terms of market structure \( (n_d + 1 \) and \( n_f + 1 \)) and the vectors of technological capabilities \( (U_d \) and \( U_f \)). Substituting these into the firm's objective
we obtain the solved-out payoff, given by (see Appendix 1 for details)

\[ \pi_{ki}(U) = \frac{1 - \gamma_i}{K_i} \left[ u_{ki} - B_i \sum_{l=1}^{n_i+1} u_{il} - C_i \sum_{l=1}^{n_i+1} u_{ij} \right] \left[ 1 - \gamma_i - \frac{\sigma \gamma_i}{n_i} \right] u_{ki} - D_i \sum_{l=1}^{n_i+1} u_{il} - E_i \sum_{l=1}^{n_i+1} u_{ij} \]

(5.4)

for \( i, j = d, f \) and \( i \neq j \)

where:

\[ K_i = 2 \left[ (1 - \gamma_i)(2 - \sigma) - \frac{\sigma \gamma_i}{n_i} \right]^2 \]

\[ B_i = \frac{\sigma}{A} \left\{ (1 - \gamma_i) [(2 - \sigma) - 2\gamma_i(1 - \sigma)] + \frac{\gamma_i}{n_i} [2 + \sigma n_i - \gamma_i (2 - \sigma + \sigma n_i)] \right\} \]

\[ C_i = \frac{\sigma}{A} (1 - \gamma_j) \left\{ (1 - \gamma_i)(2 - \sigma) - \frac{\sigma \gamma_i}{n_i} \right\} \]

\[ D_i = \frac{\sigma}{A} (1 - \gamma_i) \left\{ (1 - \gamma_i) [(2 - \sigma) - 2\gamma_j(1 - \sigma)] - \frac{\gamma_i}{n_i} [2 + \sigma (n_j (1 - \sigma)) - \gamma_j (1 - \sigma) [2 + \sigma (n_j + 1)]] \right\} \]

\[ E_i = \frac{\sigma}{A} (1 - \gamma_j) \left\{ (1 - \gamma_i)(2 - \sigma) - \frac{\sigma \gamma_i}{n_i} \right\} \]

\[ A = [(1 - \gamma_d)(2 + \sigma n_d) + \gamma_d \sigma] [(1 - \gamma_f)(2 + \sigma n_f) + \gamma_f \sigma] - \sigma^2 (1 - \gamma_d) (n_d + 1) (1 - \gamma_f) (n_f + 1) \]

Gross profit of the \( k^{th} \) firm in country \( i \) is given by \( (L_d + L_f) \pi_{ki}(u) \). Assuming firms choose a symmetric quality level within each country (denoted by \( u_d \) and \( u_f \)), we obtain a simplified expression for the payoffs

\[ \pi_i = \frac{(1 - \gamma_i) u_i^2}{K_i} \left( 1 - B_i (n_i + 1) - C_i (n_j + 1) \frac{u_j}{u_i} \right) \left[ 1 - \gamma_i - \frac{\sigma \gamma_i}{n_i} - D_i (n_i + 1) - E_i (n_j + 1) \frac{u_j}{u_i} \right] \]

(5.6)

Upon setting \( \gamma_d = \gamma_f = 0 \), this expression reduces to

\[ \pi_i = \frac{u_i^2}{2(2 - \sigma)^2} \left\{ 1 - \frac{\sigma}{2 + \sigma (n_i + n_j + 1)} \left[ (n_i + 1) + (n_j + 1) \frac{u_j}{u_i} \right] \right\}^2 \text{ for } i, j = d, f \text{ and } i \neq j \]

(5.7)

which is expression (3.10). Moreover, upon setting \( u_d = u_f = u \) and noting that \( N + 1 = n_d + n_f + 2 \), the above expression simplifies to

\[ \pi = \frac{u^2}{2(2 + \sigma N)^2} \]
which is expression (2.9).

2.2 Stage 2: Competition in Technological Capability with Profit Sharing

In stage two, firms invest in technological capability, taking as given their rivals' technological capabilities (domestic and foreign) and the number of firms (domestic and overseas). The investment is sunk, and materializes through a fixed outlays function, \( F(.) \). Net profit for firm \( k \) in country \( i \) is given by

\[
\Pi_{ki} = (L_d + L_f) \pi_{ki}(U) - F(u_{ki}, w_i) \quad \text{for } i = d, f
\]  

(5.8)

where \( L_i \) stands for population size in economy \( i \), \( \pi_{ki}(U) \) is the solved-out gross per-capita profit function for firm \( k \) in country \( i \) (equation 5.4) and \( F(u_{ki}, w_i) \) is the fixed outlays function:

\[
F(u_{ki}, w_i) = w_i f(u_{ki}), \quad \text{where } w_i \text{ is the wage rate prevailing in country } i, \text{ and } f(u_{ki}) = \beta_i \left( \frac{u_{ki}}{u_{oi}} \right) \text{ is firm } k\text{'s labour requirement to achieve a technological capability of } u_{ki}. \text{ As before, assume } \beta_i > 2 \text{ (required for the second order conditions to hold), } \epsilon_i > 0, \text{ and } u_{ki} > u_{oi} \geq 1.

The firm's objective is given by

\[
(1 - \gamma_i)\Pi_{ki} + \gamma_i \Pi_{-ki} \quad \text{for } i = d, f
\]  

(5.9)

where \( \Pi_{ki} \) is net profit for the \( k^{\text{th}} \) firm and \( \Pi_{-ki} \) is the average net profits of firm \( k\)'s local rivals: \( \frac{\sum_{i=1}^{n} \Pi_{ki}}{n_i} \). Firm \( k \) maximizes the objective in equation (5.9) by choosing \( u_{ki} \). The first order conditions are as follows

\[
(1 - \gamma_i) \left( L_d + L_f \right) \frac{\partial \pi_{ki}}{\partial u_{ki}} + \frac{\gamma_i}{n_i} \left( L_d + L_f \right) \sum_{h \neq k}^{n_i} \frac{\partial \pi_{hi}}{\partial u_{ki}} = (1 - \gamma_i) u_i \epsilon_i \beta_i \left( \frac{u_{ki}}{u_{oi}} \right)^{\beta_i}
\] 

(5.10)

for \( i = d, f \) and \( k = 1, ..., n^i + 1 \)

where \( \frac{\partial \pi_{ki}}{\partial u_{ki}} \) and \( \frac{\partial \pi_{hi}}{\partial u_{ki}} \) are taken using (5.4) to yield the following:

\[
\frac{\partial \pi_{ki}}{\partial u_{ki}} = \frac{1 - \gamma_i}{K_i} \left\{ \left( u_{ki} - B_i \sum_{i=1}^{n_i+1} u_{li} - C_i \sum_{j=1}^{n_j+1} u_{lj} \right) \left( 1 - \gamma_i - \frac{\sigma_i}{n_i} \right) - D_i \right\}
\]

(5.11)

for \( i, j = d, f \) and \( i \neq j \)
\[
\frac{\partial \pi_{hi}}{\partial u_{ki}} = \frac{1 - \gamma_i}{K_i} \left\{ \left( u_{hi} - B_i \sum_{l=1}^{n_i+1} u_{li} - C_i \sum_{l=1}^{n_j+1} u_{lj} \right) D_i + \left( 1 - \gamma_i - \frac{\sigma \gamma_i}{n_i} \right) u_{hi} - D_i \sum_{l=1}^{n_i+1} u_{li} - E_i \sum_{l=1}^{n_j+1} u_{lj} \right\} B_i \right\}
\] (5.12)

for \( i, j = d, f \) and \( i \neq j \)

A Nash equilibrium is constituted by technological capabilities which solve the system in (5.10).

Assuming a symmetric equilibrium within each economy (with \( u_{hi} = u_{ki} = u_i \)) the first order conditions can be written as follows

\[
\left[ \left( 1 - \gamma_i - \frac{\sigma \gamma_i}{n_i} \right) - D_i (n_i + 1) - E_i (n_j + 1) \frac{u_j}{u_i} \right] [(1 - B_i) (1 - \gamma_i) - \gamma_i B_i] +
\] \[1 - B_i (n_i + 1) - C_i (n_j + 1) \frac{u_j}{u_i} \left[ \left( 1 - \gamma_i - \frac{\sigma \gamma_i}{n_i} - D_i \right) (1 - \gamma_i) - \gamma_i D_i \right]
\]

\[
= \frac{K_i}{(L_d + L_f)} w_i \varepsilon_i \beta_i u_{ki}^{-1} \frac{u_{ki}^{-2}}{u_{ci}} \quad \text{for } i = d, f
\] (5.13)

Upon setting \( \gamma_i = 0 \), this expression simplifies to

\[
\frac{(L_d + L_f) [2 + \sigma (n_i + n_j)]}{(2 - \sigma)^2 [2 + \sigma (n_i + n_j + 1)]} \left\{ 1 - \frac{\sigma}{2 + \sigma (n_i + n_j + 1)} \left( (n_i + 1) + (n_j + 1) \frac{u_j}{u_i} \right) \right\}
\]

\[
= \frac{w_i \varepsilon_i \beta_i u_{ki}^{-1}}{u_{ci}} \frac{u_{ki}^{-2}}{u_{ci}} \quad \text{for } i, j = d, f \text{ and } i \neq j
\] (5.14)

which is expression (3.14). It can be shown that Proposition 1 from chapter 3 still holds for \( \gamma_i > 0 \). The net marginal benefit in the first order conditions (5.13) is decreasing in the ratio \( u_i/u_j \). Second order conditions for this stage are discussed in Appendix 2.

2.3 Stage 1: The Entry Decision with Profit Sharing

Assume there is a large number of potential entrants. Firms enter until net profits are zero.

Thus we have the following free-entry condition

\[
(1 - \gamma_i) \Pi_{ki} + \gamma_i \Pi_{-ki} \geq 0 \quad \text{for } i = d, f
\] (5.15)

where \( \Pi_{ki} \) represents net profit, as defined in (5.8), and \( \Pi_{-ki} \) represents average net profits of firm \( k \)'s local rivals: \( \frac{\sum_{k' \neq k} \Pi_{ki}}{n_k} \). In equilibrium, ignoring integer effects, entry occurs until condition (5.15) holds with equality. Moreover, in a symmetric equilibrium condition (5.15) reduces to

\[
\Pi_i \geq 0 \quad \text{for } i = d, f
\] (5.16)
where $\Pi_i$ denotes the symmetric level of net profits for each firm in country $i$. Equation (5.16) holds with equality in equilibrium, thus it can be written as

$$(L_d + L_f) \pi_i = w_i e_i \left( \frac{u_i}{u_{oi}} \right)^{\beta_i} \text{ for } i = d, f$$

(5.17)

Substituting gross profits for a symmetric equilibrium within each economy ($\pi_i$, from equation 5.6), we can rewrite this as follows

$$(1 - \gamma_i) \left( 1 - B_i (n_i + 1) - C_i (n_j + 1) \frac{u_j}{u_i} \right) \left[ 1 - \gamma_i - \frac{\sigma \gamma_i}{n_i} - D_i (n_i + 1) - E_i (n_j + 1) \frac{u_j}{u_i} \right] = \frac{K_i}{(L_d + L_f)} u_i^{\beta_i - 2} \text{ for } i, j = d, f \text{ and } i \neq j$$

(5.18)

If we set $\gamma_i = 0$, condition (5.18) reduces to the condition set out in expression (3.17):

$$\frac{(L_d + L_f)}{2(2 - \sigma)^2} \left\{ 1 - \frac{\sigma}{2 + \sigma (n_i + n_j + 1)} \left[ (n_i + 1) + (n_j + 1) \frac{u_j}{u_i} \right] \right\}^2 = w_i e_i \frac{u_i^{\beta_i - 2}}{u_{oi}^{\beta_i}} \text{ for } i = d, f$$

This completes the description of the first stage of the game played by firms in industry $X$.

We now characterize a symmetric general equilibrium for the world economy.

3 Characterization of a Symmetric General Equilibrium Outcome

Consider a symmetric equilibrium within each economy, with $u_{ki} = u_{ii} = u_i$, $i = d, f$. We first specify the equilibrium conditions, and then obtain solutions for a symmetric equilibrium across both economies, with identical parameters values in both nations ($\beta_i = \beta$, $\gamma_i = \gamma$, $\varepsilon_i = \varepsilon$, $L_i = L$, $u_{oi} = u_o$ for $i = d, f$), and identical technological capability ($u_d = u_f = u$), number of firms ($n_d = n_f = n$) and wage rate ($w_d = w_f = w$).

A general equilibrium is characterized by the functions $u_i(\beta_i, \gamma_i, \varepsilon_i, L_i, u_{oi})$, $n_i(\beta_i, \gamma_i, \varepsilon_i, L_i, u_{oi})$ and $u_i(\beta_i, \gamma_i, \varepsilon_i, L_i, u_{oi})$, for which explicit solutions can be found in a symmetric equilibrium. We write $u_i(.)$ for $u_i(\beta_i, \gamma_i, \varepsilon_i, L_i, u_{oi})$, $n_i(.)$ for $n_i(\beta_i, \gamma_i, \varepsilon_i, L_i, u_{oi})$ and $u_i(.)$ for $u_i(\beta_i, \gamma_i, \varepsilon_i, L_i, u_{oi})$. The definition of a symmetric (general) equilibrium is the same as in chapter 3. We now set out the equilibrium conditions and then use these to seek a symmetric equilibrium.

Equilibrium Conditions:

Six conditions characterize the general equilibrium of the world economy (three for each
country), as well as the standard market clearing conditions for goods X and Y. The conditions are stated for a symmetric equilibrium within each economy.

The first condition is defined by the first order conditions for technological capability (given by 5.10). For a symmetric equilibrium within each economy it can be written as (5.13):

\[
\left[1 - \gamma_i - \frac{\sigma^*}{n_i}\right] - D_i \left(n_i + 1\right) - E_i \left(n_j + 1\right) \frac{u_j}{u_i} \left[\left(1 - B_i\right) \left(1 - \gamma_i\right) - \gamma_i B_i\right] + \\
\left[1 - B_i \left(n_i + 1\right) - C_i \left(n_j + 1\right) \frac{u_j}{u_i}\right] \left[\left(1 - \gamma_i - \frac{\sigma^*}{n_i}\right) - D_i \left(1 - \gamma_i\right) - \gamma_i D_i\right]
\]

\[
= \frac{K_i}{\left(L_d + L_f\right)} w_i \varepsilon_i \beta_i \frac{u_{i\text{t}} - 2}{u_{i\text{t}}} \text{ for } i = d, f
\]

The next condition corresponds to free entry, from (5.18):

\[
\left(1 - \gamma_i\right) \left(1 - B_i \left(n_i + 1\right) - C_i \left(n_j + 1\right) \frac{u_j}{u_i}\right) \left[1 - \gamma_i - \frac{\sigma^*}{n_i} - D_i \left(n_i + 1\right) - E_i \left(n_j + 1\right) \frac{u_j}{u_i}\right]
\]

\[
= \frac{K_i}{\left(L_d + L_f\right)} w_i \varepsilon_i \frac{u_{i\text{t}} - 2}{u_{i\text{t}}} \text{ for } i, j = d, f \text{ and } i \neq j
\]

The third condition is labour market clearing, from (3.19). After substituting \(L_{x}\), from (3.18), we obtain the same condition as in chapter 3:

\[
L_i = L_{yi} + L_{zi} = L_{zi} + \left(n_i + 1\right) \varepsilon_i \left(\frac{u_i}{u_{i\text{t}}}\right) \text{ for } i = d, f
\]

Market clearing for type Y and type X goods in world markets holds by construction. These conditions pin down the general equilibrium outcomes for technological capability (\(u_d, u_f\)), the number of firms (\(n_d + 1, n_f + 1\)) and the wage rate (\(w_d, w_f\)). We set out to find explicit solutions in a symmetric equilibrium across both countries. If all parameters take the same values in both economies and \(u_d = u_f = u, n_d = n_f = n\) and \(w_d = w_f = w\), we can solve for the following expression for \(u\) from (5.19),

\[
u = \left\{\frac{L_{x} \beta}{w e \beta J}\right\}^{\frac{1}{2}}
\]

\[\text{162}\]
where
\[ J = \frac{8 (1 - \gamma)^4 - 2 \sigma^2 (1 - \gamma) \left\{ \frac{\sigma^2}{n} (3 - 2 \gamma) + 2n (1 - \gamma)^2 (1 - 3 \gamma) + \gamma [3 - \gamma (7 - 4 \gamma)] \right\}}{[(2 - \sigma)(1 - \gamma) + \sigma \gamma] \left\{ (2 - \sigma)(1 - \gamma) - \sigma \gamma \right\} [2 + \sigma (2n + 1) - 2 \gamma (\sigma n + 1)]^2} \] (5.23)

Setting \( \gamma = 0 \) and \( n = \frac{N-1}{2} \), expression (5.22) simplifies to

\[ u = \left[ \frac{2 Lu^\sigma}{\omega \beta (2 - \sigma)(2 + \sigma N)^2} \right]^{\frac{1}{3-1}} \]

This is equation (2.26), with twice the size of the market (2L instead of L). Now use condition (5.20) to solve for the number of firms. In a symmetric equilibrium, we have

\[ \frac{L (1 - \gamma) [1 - \gamma (1 - \sigma)]}{[2 + \sigma (2n + 1) - 2 \gamma (\sigma n + 1)]^2} = \omega \frac{u^{\beta - 2}}{u^{\delta}} \] (5.24)

To solve for \( n \), substitute \( u \) from (5.22) into (5.24). This yields the equilibrium number of firms:

\[ n + 1 = \frac{Z + Z^*}{4 \sigma (1 - \gamma) Z^{**} + 1} \] (5.25)

where

\[ Z = \sqrt{Z^* + 8 \sigma^2 Z^{**} \left\{ (1 - \gamma) \left\{ \frac{4 (1 - \gamma)^2 + 2 \gamma \sigma (3 - 2 \gamma)}{-\beta [1 - \gamma (1 - \sigma)] [2 - \sigma - 2 \gamma (1 - \sigma)]} \right\} + \gamma \sigma^2 \right\}} \] (5.26)

\[ Z^* = (1 - \gamma) \left\{ \frac{\beta (2 - \sigma) [1 - \gamma (1 - \sigma)] [2 - \sigma - 2 \gamma (1 - \sigma)]}{-8(1 - \gamma)^2 + 4 \sigma (1 - \gamma) (1 - 3 \gamma) + 2 \gamma \sigma^2 (3 - 4 \gamma)} \right\} - \gamma \sigma^3 (1 - 6 \gamma + 4 \gamma^2) \] (5.27)

\[ Z^{**} = [2 - \gamma (2 - \sigma)] [2 - \sigma - 2 \gamma (1 - \sigma)] \] (5.28)

Setting \( \gamma = 0 \) and noting that \( n = \frac{N-1}{2} \), the number of entrants can be written as follows

\[ N + 1 = \frac{(\beta - 2) (2 - \sigma)}{2 \sigma} + 1 \]

which is equation (2.27), from which we can see that concentration does not change whether we consider a single economy in autarky, or a two country world.

Substituting \( n + 1 \) from (5.25) into (5.22) yields the (symmetric) equilibrium level of tech-
nological capability, as a function of exogenous parameters and the wage:

\[ u = \left\{ \frac{4Lw^2 \gamma (1 - \gamma) [1 - \gamma(1 - \sigma)] (Z^{**})^2}{w \varepsilon \left[ Z^{***} + Z \right]^2} \right\}^{\frac{1}{\beta - 2}} \]  

(5.29)

where \( Z \) and \( Z^{**} \) were defined in (5.26) and (5.28), respectively, and

\[ Z^{***} = (1 - \gamma)^2 \left\{ \frac{\beta (2 - \sigma) [1 - \gamma(1 - \sigma)] [2 - \sigma - 2\gamma (1 - \sigma)]}{+8(1 - \gamma)^2 + 4\sigma(1 - \gamma)(1 + 3\gamma) - 2\sigma^2 (2 - 7\gamma)} \right\} - \gamma \sigma^3 [3 - 2\gamma (5 - 2\gamma)] \]  

(5.30)

If \( \gamma = 0 \), equation (5.29) simplifies to

\[ u = \left\{ \frac{u_o^2 L}{w} \varepsilon \left[ \frac{1}{\beta (1 - \frac{\sigma}{2}) + \sigma} \right] \right\}^{\frac{1}{\beta - 2}} \]

which is equation (3.30).

This completes the characterization of a symmetric Subgame Perfect Nash Equilibrium in industry \( X \). We now determine the equilibrium wage rate, to look for a general equilibrium. For this, use the labour market clearing condition (5.21). In a symmetric equilibrium, this simplifies as follows

\[ L = L_Y + (n + 1)e \left( \frac{u}{u_o} \right)^{\beta} \]  

(5.31)

To solve for the wage rate, substitute (5.29) into (5.31) and solve for \( w \). We obtain the following

\[ w = \max \left\{ 1, \frac{u_o^2}{u_o^2 L} \left( \frac{L}{\varepsilon} \right)^{\frac{2}{\beta}} (n + 1)^{\frac{\beta - 2}{\beta}} J \right\} \]  

(5.32)

where \( J \) was defined in (5.23) and \( n \) in (5.25). To finish the characterization we need to solve for the general equilibrium level of technological capability, which is obtained by substituting the solution for the wage rate (5.32) into equation (5.29). The general equilibrium number of firms is readily given by equation (5.25), for which no further substitutions are necessary. This completes the characterization of an open economy symmetric general equilibrium with profit sharing. The symmetric general equilibrium solutions for the open economy are presented in the following table, where \( J \) is defined in (5.23) and \( Z, Z^*, Z^{**} \) and \( Z^{***} \) in (5.26), (5.27), (5.28), (5.30), respectively:
Table 1: Symmetric General Equilibrium Outcomes for the Open Economy with $\gamma > 0$.

The question of existence of asymmetric equilibria also arises in this framework, and it can be shown that the same non-existence result developed in Proposition 2, chapter 3, goes through in this more general case. Similarly, provided one is willing to accept that the zero profit conditions hold with equality for each firm, this proposition can be extended to rule out all types of asymmetric equilibria. In the next section we analyze how the economy reacts to changes in parameter values.

4 Analysis of the Symmetric Equilibrium

In this section we analyze how the key variables in the model change as we vary the parameters of the system. In this exercise, we start from a situation in which both economies have identical parameter values, and the world economy is located at the symmetric equilibrium characterized in section 3 (table 1). We then vary each parameter, ceteris paribus, such that the change in parameters is identical between the two economies, that is, we impose the restriction that $\beta_i = \beta$, $\gamma_i = \gamma$, $\epsilon_i = \epsilon$, $L_i = L$, $u_{oi} = u_o$ (for $i = d, f$) while $\sigma$ is always identical for both economies.

This is an interesting exercise because it allows us to see what happens when a particular parameter (which has the same value in both economies) is changing at the same rate on a worldwide basis: Starting from identical parameter values, we change the relevant parameter by the same magnitude simultaneously in both economies.

The analysis will focus more incisively on how open economy results compare with those for the closed economy (which was developed in chapter 4). In the symmetric equilibrium with identical parameter values we find that welfare ($W_i$) exhibits qualitatively similar behaviour to the wage rate, thus it will be particularly interesting to study how the wage rate in the open economy fares against that of the closed economy. We concentrate on parameter values for which the wage rate is strictly greater than 1.
The comparison between the closed and open economies allows us to answer one of the fundamental questions of this study: Is it always welfare improving to open the economy? If not, under what conditions will trade lead to higher welfare? After we analyze the different parameter changes, we will be in a position to answer these questions.

We present the analysis with the aid of similar figures to those used in chapter 3, section 4. Open economy variables are shown as continuous lines and closed economy variables as dotted lines. In the horizontal axes we show the parameter in question. The top left graph shows technological capability ($u$). The bottom left graph shows the number of firms ($n+1$). The top right graph depicts the wage rate ($w$) and exports and imports of type $X$ goods are shown in the bottom right diagram.

4.1 The effect of changing $\beta$

We present the main results in Figure 4.1.

In Figure 4.1, technological capability decreases convexly, and the open economy schedule lies above the autarky schedule. Conversely, the number of firms increases convexly, with more firms under autarky. This reflects the fact that in the symmetric equilibrium for the open economy.
economy each country features half of the total number of firms, whereas the closed economy contains the total number of firms. In the limit, as $\gamma$ converges to zero, we find that the worldwide number of firms in the open economy is exactly the same as the total number of firms in the closed economy, thus for $\gamma = 0$ we have $n + 1 = \frac{N+1}{2}$: The number of firms in each country is half the number of firms in the closed economy (this result was initially found in chapter 3). This can only occur if (indeed, it is a reflection of the fact that) technological capability is higher in each of the open economies relative to the closed economy.

The demand for labour from industry $X$ determines the wage rate. Although the number of firms is higher in autarky, technological capability is lower. The two effects result in a stronger demand for labour in the open economy for values of $\beta < \beta^{c-o}$, where $\beta^{c-o}$ is a threshold level of $\beta$ defined by the wage rate in the open economy being equal to the wage rate in the closed economy. For values of $\beta > \beta^{c-o}$ we have the opposite result.

The crossing of the wage schedules is generated by the following behaviour. As $\beta$ falls, technological capability rises at a faster rate in the open economy, while the number of firms falls at a slower rate in the open economy. This combination ensures that the wage rate of the open economy rises at a faster rate as $\beta$ falls, which implies an eventual crossing of the two schedules. This is a result that will also hold for changes in $\gamma$ and $\sigma$, and in it lies the answer to the question: Under what conditions is it welfare improving to open the economy? We will be in a position to propose a full answer to this question once we have completed the analysis of the remaining parameters for the symmetric equilibrium.

Now we ask: Why does technological capability rise at a faster rate as $\beta$ falls in the open economy? The answer to this question lies in the fact that the profit-sharing scheme occurs only between local firms, and not between local and overseas firms. Since firms only internalize the consequences of their escalation on their local rivals, there is an 'overseas market stealing' effect which strengthens the incentive to escalate for the local firms: Local firms do not care about the losses inflicted upon their overseas rivals through their escalation. Of course, this argument also applies to the other economy. Thus in a symmetric equilibrium both countries exhibit technological capability which is not only higher in the open economy relative to the closed economy, but also rises faster.

In the bottom right graph, we can observe that imports and exports take the same value, so trade is balanced for industry $X$. Likewise, trade is also balanced for industry $Y$, implying a zero trade balance overall. This result is specific to the symmetric equilibrium, and breaks down in the case of asymmetric parameters. Budget balancedness implies that exports and imports follow the same pattern as the wage rate, and the model exhibits intra-industry trade
in equilibrium.

4.2 The effect of changing $\gamma$

In considering the effect of $\gamma$, two cases need to be distinguished:

a) When either $\beta$ or $\sigma$ are low.

b) When $\beta$ and $\sigma$ are high.

We present the main results in Figures 4.2a and 4.2b. In Figure 4.2a we present technological capability ($u$), the number of firms ($n+1$), the wage rate ($w$) and exports and imports of goods of type $X$ as functions of $\gamma$. The open economy is represented by continuous lines, while the closed economy is shown as dotted lines.

Figure 4.2a: The effect of $\gamma$ in the symmetric equilibrium, when either $\beta$ or $\sigma$ are low.

Technological capability decreases concavely in $\gamma$, and is higher in the open economy. The number of firms is increasing convexly in $\gamma$, and is lower in the open economy. Regarding the wage rate, we saw (chapter 4, section 4.1) that the autarky wage in case (a) exhibits two patterns of behaviour, which we have reproduced in Figure 4.2a:

(a.1) For $\beta$ low, we find that the closed economy wage is decreasing and concave.

(a.2) For $\beta$ high and $\sigma$ low we find that it is ‘\(\cap\)’-shaped.
On the other hand, the open economy wage exhibits the same pattern in both scenarios: it is decreasing concavely in $\gamma$, and is lower when $\beta$ is high and $\sigma$ low. In both sub-cases (a.1 and a.2) the closed economy wage rate crosses its open economy counterpart, at $\gamma^{o}_{\beta_{io}}$ and at $\gamma^{o}_{\beta_{io}^{*}}$, respectively. Above these thresholds, the closed economy exhibits a higher wage rate. The crossing of the wage schedules responds to mechanisms similar to those discussed when analyzing the effect of changing $\beta$ (see section 4.1).

We have also presented the schedules corresponding to subcases (a.1) and (a.2) for the other variables. It is clear that technological capability, the number of firms and exports are also affected. However these do not change their shape in these two subcases, and all that occurs is a shift of the corresponding schedules. In particular technological capability and exports are lower when $\beta$ is high and $\sigma$ low, while the number of firms is higher.

Finally, exports and imports of type $X$ goods exhibit similar behaviour to the open economy wage rate, and there is intra-industry trade.

As mentioned previously, when $\gamma$ converges to zero the number of firms in any of the open economies converges to half of the number of firms in the closed economy.

We now present the corresponding graph for case (b): When $\beta$ and $\sigma$ are high.

![Graph showing open and closed economy wages and other variables](image)

Figure 4.2b: The effect of $\gamma$ in the symmetric equilibrium, when $\beta$ and $\sigma$ are high

The analysis of Figure 4.2b is similar to that presented in section 4.2, chapter 4. Note that
technological capability is higher in the open economy relative to the closed economy, while the number of firms is correspondingly lower. The wage schedule for the open economy is initially above that for the closed economy, until they cross at $\gamma_{\beta_{ho},\sigma_{ho}}$. We also observe intra-industry trade.

4.3 The effect of changing $\sigma$

The following figure summarizes the results.

We consider two cases:

(a) When $\beta$ and $\gamma$ are very low (labelled $\beta_{vlo},\gamma_{vlo}$).

(b) Other parameter combinations (labelled ‘Others’).

In both cases, technological capability increases concavely, and is higher in the open economy. The number of firms decreases convexly and is lower in the open economy. Exports and imports of type $X$ goods are equal, and they track the open economy wage rate. Technological capability, the number of firms and exports do not change their shape in the two cases and all that occurs is a shift of the corresponding schedules. More specifically, technological capability
and exports (imports) are higher when $\beta$ and $\gamma$ are very low, while the number of firms is lower.

In both cases the open economy wage rate decreases convexly in $\sigma$, and is higher in case (a). The closed economy wage rate decreases convexly for case (b) and is 'U'-shaped when $\beta$ and $\gamma$ are very low. The open and closed economy wage schedules cross at $\sigma_{\beta VIA gamma}^{<}$ in case (a), and at $\sigma_{Close}^{<}$ in case (b). For $\sigma < \sigma_{Close}^{<}$ in case (a), and $\sigma < \sigma_{Close}^{<}$ in case (b), the open economy wage rate is higher, while for values of $\sigma$ above the respective threshold, it is the closed economy which exhibits the higher wage.

4.4 The effect of changing $\varepsilon$

In analyzing the effect of $\varepsilon$ we need to consider the underlying values of $\beta$, $\gamma$ and $\sigma$. These will affect whether the wage rate in the open economy lies above or below that of the closed economy. However, the qualitative pattern followed by technological capability, the number of firms and exports and imports of type $X$ goods with respect to $\varepsilon$ is invariant with respect to $\beta$, $\gamma$ or $\sigma$ (although these schedules will of course shift in accordance with the analysis of sections 4.1-4.3). We present the results in the following figure.

![Figure 4.4: The effect of $\varepsilon$ in the symmetric equilibrium](image)

In Figure 4.4 technological capability, the wage schedule, and exports and imports of type
$X$ goods decrease convexly in $\varepsilon$, while the number of firms is invariant with respect to $\varepsilon$. Technological capability is higher in the open economy, while the number of firms is lower. The wage rate is higher in the open economy for all values of $\varepsilon$ when $\beta, \gamma$ and $\sigma$ are sufficiently low. Conversely, when $\beta, \gamma$ and $\sigma$ are sufficiently high, we find that the wage rate is higher for the closed economy, for all values of $\varepsilon$.

4.5 The effect of changing $u_0$

Figure 4.5 depicts the effect of changing $u_0$ (the effect of which is the same as changing $1/\varepsilon$).

![Graph showing the effect of $u_0$ on technological capability, wage rate, number of firms, and exports/imports of type $X$ goods.]

Figure 4.5: The effect of $u_0$ in the symmetric equilibrium

In figure 4.5 technological capability increases linearly in $u_0$, and it is higher in the open economy. The number of firms is invariant in $u_0$, and it is lower in the open economy (relative to autarky). The wage rate is rising convexly in $u_0$. This results from the convexity of the labour requirement, $f(u) = \varepsilon \left( \frac{u}{u_0} \right)^\beta$. For $\beta, \gamma$ and $\sigma$ sufficiently low, we find that the open economy features a higher wage rate, while for $\beta, \gamma$ and $\sigma$ sufficiently high it is the closed economy wage rate that is higher. Export and imports of type $X$ goods rise convexly in $u_0$, following the wage rate.
4.6 The effect of changing $L$

The effect of changing $L$ is shown in the following figure.

![Diagram showing the effect of $L$]

Figure 4.6: The effect of $L$ in the symmetric equilibrium

Figure 4.6 shows that technological capability, the wage rate, and exports and imports of type $X$ goods rise concavely in $L$. Meanwhile, the number of firms is unchanged. Technological capability is lower in autarky, and the number of firms is higher. For $\beta, \gamma$ and $\sigma$ sufficiently low the wage rate is higher in the open economy. Instead for $\beta, \gamma$ and $\sigma$ sufficiently high, the wage rate is higher in the closed economy.

5 On Free Trade Versus Autarky

In section 4, we saw that the wage schedules for the open and closed economies exhibit a crossing as we vary $\beta, \gamma$ or $\sigma$. Pursuing this notion further, we come across one of the main results of this study, which is summarized in the following proposition. The analysis that follows refers to the case of wage rates strictly above unity.

**Proposition 1:** Existence and properties of a separating surface for the symmetric equilibrium between the open and closed economy wage rates.
There exists a separating surface in \((\beta, \gamma, \sigma)\)-space along which the wage rate for the open economy is equal to the wage rate of the closed economy. In the region of \((\beta, \gamma, \sigma)\)-space lying above the surface, the wage for the open economy is strictly less than the wage for the closed economy. Conversely, in the region lying beneath the surface, the open economy wage is strictly greater than the closed economy wage.

The separating surface can be represented as a function \(\sigma = s(\beta, \gamma)\), with the following properties:

(i) \textbf{Slope with respect to} \(\gamma\): Define a threshold \(\beta^*\) as the largest value of \(\beta\) such that \(\frac{\partial s}{\partial \gamma}_{\beta=\beta^*, \gamma=\gamma} = 0\) for some \(\gamma \in [0, \frac{\pi}{n+1}]\). Then for \(\beta > \beta^*\) we have \(\frac{\partial s}{\partial \gamma} < 0\). For \(\beta \leq \beta^*\), define two thresholds \(\gamma^*\) and \(\gamma^{**}\) with \(\gamma^* < \gamma^{**}\) such that \(\frac{\partial s}{\partial \gamma}_{\beta=\beta^*, \gamma=\gamma^*} = 0\) and \(\frac{\partial s}{\partial \gamma}_{\beta=\beta^*, \gamma=\gamma^{**}} = 0\), which characterize two extreme points. Then for \(\gamma < \gamma^*\) we have \(\frac{\partial s}{\partial \gamma} < 0\) (\(s\) is decreasing), for \(\gamma^* < \gamma < \gamma^{**}\) we have \(\frac{\partial s}{\partial \gamma} > 0\) (\(s\) is increasing) and for \(\gamma > \gamma^{**}\) we have \(\frac{\partial s}{\partial \gamma} < 0\) (\(s\) is decreasing). Hence \(s(\beta, \gamma)\) is \(\gamma^1\)-shaped for \(\beta \leq \beta^*\).

(ii) \textbf{Concavity with respect to} \(\gamma\): Define a threshold \(\gamma^{***}\) such that \(\frac{\partial^2 s}{\partial \gamma^2}_{\gamma=\gamma^{***}} = 0\) (an inflection point). Then for \(\gamma < \gamma^{***}\) we have \(\frac{\partial^2 s}{\partial \gamma^2} > 0\) (\(s\) is strictly convex), and for \(\gamma > \gamma^{***}\) we have \(\frac{\partial^2 s}{\partial \gamma^2} < 0\) (\(s\) is strictly concave).

(iii) \textbf{Slope with respect to} \(\beta\): Define thresholds \(\gamma'\) and \(\gamma''\), with \(\gamma' < \gamma''\), as follows. Let \(\gamma'\) be the smallest value of \(\gamma\) such that \(\frac{\partial s}{\partial \beta}_{\gamma=\gamma'} = 0\), and \(\gamma''\) be the largest value of \(\gamma\) such that \(\frac{\partial s}{\partial \beta}_{\gamma=\gamma''} = 0\). For \(\gamma < \gamma'\) we have \(\frac{\partial s}{\partial \beta} > 0\). For \(\gamma > \gamma''\) we have \(\frac{\partial s}{\partial \beta} < 0\). For \(\gamma' < \gamma < \gamma''\) we have \(\frac{\partial s}{\partial \beta} > 0\) (\(s\) is convex). For \(\beta < \beta'\) we have \(\frac{\partial s}{\partial \beta} < 0\) (\(s\) is decreasing) and for \(\beta > \beta'\) we have \(\frac{\partial s}{\partial \beta} > 0\) (\(s\) is increasing).

(iv) \textbf{Concavity with respect to} \(\beta\): Define a series of thresholds \(\gamma^I < \gamma^{II} < \gamma^{III}\) such that:
- \(\gamma^I\) is the smallest value of \(\gamma\) such that \(\frac{\partial^2 s}{\partial \beta^2}_{\gamma=\gamma^I} = 0\).
- \(\gamma^{II}\) satisfies \(\gamma^I < \gamma^{II} < \gamma^{III}\) and \(\frac{\partial^2 s}{\partial \beta^2}_{\gamma=\gamma^I, \beta=\beta''} = 0\) for some \(\beta'' \in (2, \infty)\), where \(\beta''\) defines an inflection point.
- \(\gamma^{III}\) is the largest value of \(\gamma\) such that \(\frac{\partial^2 s}{\partial \beta^2}_{\gamma=\gamma^{III}} = 0\) for all \(\beta \in (2, \infty)\).

We then have:
- For \(\gamma < \gamma^I\), \(\frac{\partial^2 s}{\partial \beta^2} < 0\) (\(s\) is concave).
- For \(\gamma^I < \gamma < \gamma^{II}\), \(\frac{\partial^2 s}{\partial \beta^2} > 0\) (\(s\) is convex).
- For \(\gamma^{II} < \gamma < \gamma^{III}\) and \(\beta < \beta''\), \(\frac{\partial^2 s}{\partial \beta^2} < 0\) (\(s\) is concave). If \(\gamma^{II} < \gamma < \gamma^{III}\) but \(\beta > \beta''\), then \(\frac{\partial^2 s}{\partial \beta^2} < 0\) (\(s\) is concave).
- For \(\gamma^{III} < \gamma\), \(\frac{\partial^2 s}{\partial \beta^2} > 0\) (\(s\) is convex).
Proof: The proof proceeds by showing how the separating surface is derived and then showing how to analyze its properties.

The separating surface is obtained by equating the wage rate for the closed economy (table 1, chapter 4) and the wage rate for the open economy in the symmetric equilibrium (table 1, this chapter). This generates an equation in $\beta$, $\gamma$, and $\sigma$, from which we can solve (implicitly) for any of these parameters as a function of the others. Thus, any of the functions $\sigma = s(\beta, \gamma)$, $\beta = b(\gamma, \sigma)$ or $\gamma = g(\beta, \sigma)$ are valid (and equivalent) representations of the equation. We chose to present the analysis in terms of $\sigma = s(\beta, \gamma)$.

Using the implicit function theorem we obtain the properties outlined in parts (i)-(iv) of this proposition.\[\]

Corollary 1a: There exists a separating surface for welfare ($W$) with similar properties to the separating surface for the wage rate.

Proof: In a symmetric equilibrium, welfare ($W$) exhibits the same pattern as the wage rate. By equating the equilibrium welfare indicators for the open and closed economies and using the implicit function theorem we obtain the corresponding properties.\[\]

Corollary 1b: There exist separating surfaces for per-firm output of good $X$ ($x$) and labour demand from industries $Y$ ($L_y$) and $X$ ($L_x$) with similar properties to the separating surface for the wage rate.

Proof: By equating the equilibrium values of the corresponding variables for the open and closed economies and using the implicit function theorem.\[\]

Proposition 1 and Corollary 1a state that even in the case when the two economies are in a symmetric equilibrium (i.e., they are identical), there is a region of $(\beta, \gamma, \sigma)$-space for which free trade is not optimal for either of the economies: Both countries are better off in autarky.

Since our framework features economies with an increasing returns to scale industry, the result that there may not be gains from trade is not unexpected. Indeed, it is a well documented fact that when the economy features some industries with increasing returns to scale, gains from trade may not accrue (see Markusen and Melvin, 1989). The novelty of Proposition 1 lies in the characterization of a new set of mechanisms that generate the result. In particular, the separating surface is originated by the trade-off between the 'technological capability' effect and the 'market structure' effect.

We plot the separating surface in the following figure.
Proposition 1, Corollary 1a and Figure 5.1 provide a framework to inform decisions about the optimal trade regime. The key issue lies in identifying the industry’s values of $\beta, \gamma$ and $\sigma$. With this information it becomes clear which trade regime will lead to higher wages. The parameters $\beta$ and $\sigma$ can be estimated econometrically and $\gamma$ is directly measurable (it is the extent of cross-ownership within the industry).

Let us illustrate the mechanisms behind the separating surface as shown in Figure 5.1, by reference to two extreme parameter configurations. Firstly, consider the configuration where $\beta, \gamma$ and $\sigma$ are high, in which autarky yields a higher wage rate than free trade. This case features a high marginal cost of technological capability, a low intensity of competition and goods which are close substitutes. High values of $\beta$ and $\gamma$ reduce the level of technological capability, and induce entry by a large number of firms. Recall that the number of firms is always larger in the closed economy relative to the open economy, and that technological capability is always lower in the closed economy, relative to the open economy. As the economy is opened, firms exit and technological capability of the survivors rises. The net outcome for the wage rate depends on the net of these effects on labour demand. In this case, the exit effect of opening the economy is not offset by the increase in technological capability. Moreover, a high $\sigma$ implies that market-stealing is strong, and competition by foreign rivals is substantially...
harmful. This, together with the fact that there is a lot of (local) profit-sharing going on (high \( \gamma \)) leads to the conclusion that autarky welfare-dominates free trade in this scenario.

Secondly, consider the case when \( \beta, \gamma \) and \( \sigma \) are sufficiently low (i.e., the economy is located close to the origin in Figure 5.1). In this scenario free trade results in higher wages relative to autarky. This case features a low marginal cost of technological capability (low \( \beta \)), high intensity of competition (low \( \gamma \)) and low substitutability between type \( X \) goods (low \( \sigma \)). In this case, the economy is characterized by few firms, each with high technological capability. When the economy is opened, some of these firms exit, but the resulting increase in technological capability more than compensates for this. A low \( \sigma \) implies that foreign competition is not as harmful as when \( \sigma \) is high.

These configurations are but two examples of possible parameter combinations. A more complete analysis can be constructed by providing a taxonomy of industry types. Sutton (1991 and 1998) provides the elements for constructing such a taxonomy of industries based on the level of marginal cost of technological capability (related to \( \beta \)) and product substitutability or horizontal product differentiation (\( \sigma \)). To illustrate, we present some benchmark cases in Figure 5.2, below. The vertical and horizontal axes measure \( \sigma \) and \( \beta \), respectively.

![Figure 5.2: A taxonomy of industries based upon their product substitutability (\( \sigma \)) and the marginal cost of technological capability (related to \( \beta \)).](image)

In case A, a high degree of horizontal product differentiation (high \( \sigma \)) together with a low marginal cost of technological capability (low \( \beta \)) results in a highly concentrated industry with high technological capability (top left box in Figure 5.2). Examples of this include the aircraft industry (Sutton 1998, ch. 16), digital switches, colour film (ibid., ch. 5) and liquid...
Reducing product substitutability we move downwards, to case B. Provided the marginal cost of technological capability remains low, the industry is now characterized by lower concentration and higher technological capability. An example of this type of industry are flowmeters (ibid., ch. 6).

As we move rightward horizontally (holding $a$ constant) the marginal cost of technological capability rises with $\beta$. Accordingly, technological capability is lower relative to the left side of the diagram. If products from one firm are not easily substitutable with those of other firms (including overseas rivals), then the industry will have the least concentrated structure and also the least technological capability of all the possibilities in the diagram (case C). The cement industry could be included in this category (ibid., ch. 12), since high transport costs make it uneconomic to ship the product over long distances, thus rendering a structure similar to Hotelling's horizontal product differentiation model (Hotelling, 1929).

On the other hand, if products are easily substitutable (case D), but the marginal cost of technological capability remains high, the industry will feature lower concentration and higher technological capability than in case C. An example of this is the flour industry (Sutton 1991, ch. 7).

At this point a caveat must be made. The characterization of industries in terms of a limited set of parameters (in this case $\beta$ and $\sigma$) necessarily implies simplification, and even those industries offered as examples will have many other determinants which are not being considered. That is the trade-off between encompassing a wide range of industries and tailoring the model to suit a particular industry: Generality versus specificity (Sutton, 1996). These examples are offered as an illustration of the mechanisms at work in the models we have developed.

We continue with the analysis by projecting the contours of the separating surface $s(\beta, \gamma)$ for given values of $\gamma$ onto $(\sigma, \beta)$-space. The procedure is to take a given value of $\gamma = \gamma^o$, and plot the function $s(\beta, \gamma^o)$ in $(\sigma, \beta)$-space. This is repeated for different values of $\gamma \in [0, \frac{1}{N+1}]$. The result is a family of separating curves along which the open economy wage rate is equal to that of the closed economy. The curves divide the space into a region where the closed economy wage rate is higher than its open economy counterpart (lying above the curve), and a region when the reverse holds (beneath the curves). Figure 5.3 presents the family of separating curves.
Each curve in Figure 5.3 represents a locus where the open and closed economy wage rates are equal, for a given value of \( \gamma \). The curves divide the parameter space into two regions. The region above each curve features a higher wage rate for the closed economy, while the region below features a higher wage rate for the open economy. As the intensity of competition decreases (\( \gamma \) increases), the curves shift down, implying that for higher \( \gamma \) a larger region of \((\sigma, \beta)\) parameter space will feature a higher wage rate in autarky relative to the open economy. Note that in the top left corner (high \( \sigma \) and low \( \beta \)) the closed economy will feature a higher wage rate, for any value of \( \gamma \) different from zero. Similarly, the bottom left corner (low \( \sigma \) and \( \beta \)) always features a higher wage rate for the open economy, regardless of the value of \( \gamma \).

Superimposing on this diagram the taxonomy we developed in Figure 5.2, we can draw some conclusions for trade policy. In an economy characterized by an industry featuring low product substitutability (low \( \sigma \)) and low marginal cost of technological capability (low \( \beta \)), free trade will generate a higher wage rate relative to autarky, for any intensity of competition (e.g., flowmeters). On the other hand, if the economy features an industry with high product substitutability and low marginal cost of technological capability, free trade will generate a lower wage rate relative to autarky, for any intensity of competition but the harshest (e.g., aircraft). For other industries, the optimal trade regime depends intricately on the intensity of competition. We can see that reducing the intensity of competition implies a 'crowding-out' of the industries for which free-trade is optimal.

In the limit, as the intensity of competition reaches its minimum level (\( \gamma \) converges to \( \frac{N}{N+1} \)),

Figure 5.3: Contours of the separating surface for the wage rate for different values of \( \gamma \).
the only industries for which free trade will be optimal are those characterized by low product substitutability (since this lessens the harmful effect of competition from overseas rivals). This is shown in Figure 5.3 by the lowest separating curve, labelled $\gamma_{\text{high}}$. The negative slope of the curve implies that lower marginal costs of technological capability (lower $\beta$) serve as a trade-off for higher levels of product substitutability.

Similarly, as the intensity of competition becomes high, the only industries for which it will be optimal to have a closed economy are those which have a low marginal cost of technological capability and products that are highly substitutable. In this case we find 'crowding-out' of industries for which autarky is optimal.

We summarize these results in the following proposition.

**Proposition 2: Optimal Trade Regimes for Different Types of Industry**

i) If the intensity of competition is high ($\gamma$ is low, but different from zero), the economy will achieve higher wages in autarky only if it is characterized by an industry with sufficiently high product substitutability (high $\sigma$) and a sufficiently low marginal cost of technological capability (low $\beta$).

ii) If the intensity of competition is low ($\gamma$ is high), the economy will achieve higher wages under free-trade only if it is characterized by an industry with sufficiently low product substitutability (low $\sigma$) and sufficiently low marginal cost of technological capability (low $\beta$).

iii) For intermediate values of the intensity of competition, the optimal trade regime is obtained by checking whether the values of $\beta$ and $\sigma$ for the economy lie above or below the separating curve given by $\sigma = s(\beta, \gamma^o)$, where $\gamma^o$ denotes the economy's intensity of competition.

**Proof:**
Follows from Figure 5.3■.

**A Replication Argument Revisited**
In chapter 2, section 5, we discussed how the model can be extended to a multiple industry setting and the likely consequences of this. This analysis also applies to the models we have developed in chapters 3-5. In moving from our simple model to a more realistic setting, we can envisage each economy as being composed by multiple industries (suppose there are $R_i$ industries in each economy), each with its own characteristic parameter values ($\beta_{r_i}$, $\gamma_{r_i}$, $\sigma_{r_i}$, $\varepsilon_{r_i}$, $w_{or}$, and $L_i$ for $r_i = 1, ..., R_i$ and $i = d, f$). If we introduce multiple industries to the model in this chapter, the analysis of the separating surface applies to each industry. Thus, having
a larger proportion of industries with the appropriate type of trade policy\textsuperscript{1} would generate higher demand for labour and consequently higher wages.

Closely related to these notions is the connection between trade and growth. The empirical evidence is mildly supportive of a positive correlation between free-trade and economic growth. However, this is a highly debated claim, with many studies finding a very weak or even non-existent link (Rodriguez and Rodrik, 1999). How, then, can this be reconciled with our findings?

The view that emerges is that by having a generalized opening of the economy, labour demand from some industries will contract, whilst labour demand from others will expand. The overall effect on the wage rate (i.e., income) depends on what types of industries generate most of the economy's labour demand. If they are mostly high-$\sigma$ and low-$\beta$ industries and the economy exhibits low intensity of competition, we can expect an overall reduction in income levels. If on the other hand, most industries have low values of $\sigma$ and high values of $\beta$, together with a high intensity of competition, we can expect free-trade will be associated to an increase in income levels.

This analysis raises the question: How to go about changing these parameters? From section 4 it is clear that there is potential for gains-from-trade. How, then, can we prepare the economy to make the most of free-trade? The intensity of competition (the extent of profit-sharing: $\gamma$) is an anti-trust issue. The substitutability of products ($\sigma$) seems to be an intrinsic characteristic of goods, and this hints towards promotion of industries with low product substitutability (which could be associated with goods that exhibit a high degree of horizontal product differentiation). The marginal cost of technological capability (associated to $\beta$) points towards development of institutions that support the creation of technological capability, innovation, product quality, etc. To recapitulate, the focus is now on changing the institutions of the economy in order to make it suitable for opening. This reflects a shift of emphasis towards modifying the institutional framework of the economy, which in this model is captured by structural parameters.

We conjecture that some aspects of Proposition 1 are more general than the current model implies. In particular, consider an $n$-industry (e.g., $n$-country or $n$-region) model with limited factor supply, and in which factor demand derives from investment in technological capability and from the number of firms. In such a setting we expect to find a (welfare) separating surface in the space of parameters which affect both the number of firms and technological capability.

\textsuperscript{1}By 'appropriate' we mean the type of trade policy (autarky versus free-trade) which generates the highest demand for labour for that particular industry.
The existence of this surface is warranted by the trade-off in factor demand between a larger number of firms versus higher technological capability. The economic policy consequences of this are similar to the ones we have discussed above. This extension lies outside the scope of this study and will be the topic of future research.

This completes the analysis of the symmetric equilibrium under identical parameter values. As mentioned in the introduction, we found that the inclusion of \( \gamma \) does not affect our previous results regarding asymmetric initial conditions or catching-up (chapter 3, section 5). The only difference will be a relocation or shift of the initial symmetric equilibrium, in the manner described in section 4 of this chapter. Thus it was not deemed necessary to recreate the analysis here.

6 Conclusions

In this chapter we extended the closed economy model presented in chapter 4 to an open economy setting, with strategic interaction and endogenous terms of trade. This chapter can also be regarded as a generalization of the open economy model developed in chapter 3, to allow for varying degrees of intensity of competition.

If we compare the results from chapter 4 (Proposition 1), with the open economy symmetric equilibrium, we find that the conclusions for development configurations need to be modified slightly for the open economy. From the analysis in section 4 we conclude that a 'high-tech' configuration\(^2\) is still associated with a higher wage rate (and welfare) than other configurations, with two exceptions:

1) \( \sigma \) is low, in which case a 'proliferation' configuration yields higher wages.

2) \( \beta \) and \( \sigma \) are high and \( \gamma \) is intermediate, in which case an 'intermediate' configuration yields higher wages.

Notice that for the intermediate economy to yield a higher wage now requires not only high-\( \beta \) and intermediate \( \gamma \), but also high-\( \sigma \) (whereas in chapter 4 it did not depend on \( \sigma \)). This can be gleaned from the analysis of Figures 4.2a-b. With one exception, the cases when a particular development configuration achieves a higher wage rate are reinforced in the open economy. In particular, the wage rates achieved under each configuration will be higher under free-trade. The exception is the case of high-\( \beta \) and low-\( \sigma \). In this case, the closed economy achieves a

\(^2\)Recall the definitions from chapters 2 and 4:

1) **High-tech configuration**: Industry \( X \) has few firms each with a high level of technological capability.

2) **Proliferation configuration**: Industry \( X \) has many firms, each with a low level of technological capability.

3) **Intermediate configuration**: Industry \( X \) has an intermediate number of firms, each with an intermediate level of technological capability.
higher wage rate with an intermediate level of \( \gamma \) (leading to an intermediate configuration). In the open economy with high-\( \beta \) and low-\( \sigma \), a higher wage rate is achieved with low-\( \gamma \) (associated with a high-tech configuration).

We found that in the symmetric equilibrium, there is a separating surface in \((\beta, \gamma, \sigma)\)-space, above which autarky is associated with a higher wage rate. Thus the gains from trade theorem breaks down. This is not surprising, since industry \( X \) features increasing returns to scale. The result allows us to analyze alternative development strategies. In general the opening of the economy entails a reduction in the number of firms and an increase in technological capability. The optimal trade regime will depend on the characteristics of each industry. This leads to the surprising conclusion that different industries need different trade regimes. Moreover, this conclusion highlights the need of transforming the economy before opening it to trade, so that it is characterized by industries which effectively will ensure that free-trade is welfare-enhancing. The transformation takes the form of modifications in the economy's institutional framework (i.e., changes in parameter values). We find that if parameters are changed to maximize the wage rate, free-trade will always be optimal (see Figures 4.1-4.6). Thus, free-trade and development always go hand in hand.

We also found that the results on asymmetric initial conditions and catching-up from chapter 3 (section 5) follow through for the extended model with adjustable intensity of competition.
Appendix 1: Solving the Final Stage Subgame for Industry X

Let us solve the final stage subgame (quantity competition) to obtain 'solved-out payoff' for domestic and foreign firms. We begin by substituting the inverse demand functions (equation 3.3) and its derivatives \( \frac{\partial p_i}{\partial x_{ki}} \) and \( \frac{\partial p_i}{\partial x_{kj}} \) for \( i = d, f \) into the first order conditions (equation 5.3). From this, we obtain

\[
(1 - \gamma_i) \left[ 1 - 2(2 - \sigma) \frac{x_{ki}}{u_{ki}} - \frac{2\sigma}{u_{ki}} \left( \sum_{l=1}^{n_i+1} x_{li} + \sum_{l=1}^{n_j+1} x_{lj} \right) \right] - \frac{\gamma_i}{\gamma_j} \frac{2\sigma}{u_{ki}} \sum_{l \neq k} x_{li} u_{li} = 0
\]

(A1.1)

for \( i, j = d, f \) and \( i \neq j \)

We proceed to add and subtract \( \frac{\gamma_i}{\gamma_j} \frac{2\sigma}{u_{ki}} \frac{x_{ki}}{u_{ki}} \) in each of the two equations in (A1.1). After re-organizing we arrive at

\[
x_{ki} = \frac{(1 - \gamma_i) u_{ki} - (1 - \gamma_i + \frac{\gamma_i}{n_i}) 2\sigma \sum_{l=1}^{n_i+1} \frac{x_{li}}{u_{li}} - (1 - \gamma_j) 2\sigma \sum_{l=1}^{n_j+1} \frac{x_{lj}}{u_{lj}}}{2 \left[ (2 - \sigma) (1 - \gamma_i) - \frac{\sigma \gamma_i}{n_i} \right]}
\]

(A1.2)

for \( i, j = d, f \) and \( i \neq j \)

The next step is to sum each of the two expressions in (A1.2) over \( k \). This yields

\[
\sum_{l=1}^{n_i+1} \frac{x_{li}}{u_{li}} = \frac{(1 - \gamma_i) \left[ \sum_{l=1}^{n_i+1} u_{ki} - 2\sigma(n_i + 1) \sum_{l=1}^{n_j+1} \frac{x_{lj}}{u_{lj}} \right]}{2 \left[ (1 - \gamma_i) (2 + \sigma n_i) + \gamma_i \sigma \right]}
\]

for \( i, j = d, f \) and \( i \neq j \) (A1.3)

The equations in (A1.3) are linear in \( \sum_{l=1}^{n_i+1} \frac{x_{li}}{u_{li}} \) and \( \sum_{l=1}^{n_j+1} \frac{x_{lj}}{u_{lj}} \). Solving for these yields

\[
\sum_{l=1}^{n_i+1} \frac{x_{li}}{u_{li}} = \frac{(1 - \gamma_i) \left\{ \left[ (1 - \gamma_j) (2 + \sigma n_j) + \gamma_j \sigma \right] \sum_{l=1}^{n_i+1} u_{ki} - \sigma (1 - \gamma_j) (n_i + 1) \sum_{l=1}^{n_i+1} x_{li} \right\}}{2A}
\]

(A1.3')

for \( i, j = d, f \) and \( i \neq j \)

where \( A \) is defined in the main body of the chapter (see equation 5.5), and is reproduced here for convenience

\[
A = [(1 - \gamma_d) (2 + \sigma n_d) + \gamma_d \sigma] [(1 - \gamma_f) (2 + \sigma n_f) + \gamma_f \sigma] - \sigma (1 - \gamma_d) (n_d + 1) (1 - \gamma_f) (n_f + 1)
\]
The solution for \( x_{ki} \) is obtained by substituting (A1.3') into (A1.2). This gives the following result

\[
x_{ki} = \frac{(1 - \gamma_i)u_{ki}^2}{2 \left[ (2 - \sigma)(1 - \gamma_i) - \frac{\sigma \gamma_i}{n_i} \right]} \left( 1 - B_i \sum_{l=1}^{n_i+1} \frac{u_{li}}{u_{ki}} - C_i \sum_{l=1}^{n_j+1} \frac{u_{lj}}{u_{kj}} \right)
\]

for \( i, j = d, f \) and \( i \neq j \)  

(A1.4)

where \( B_i \) and \( C_i \) are defined in equation (5.5) as follows

\[
B_i = \frac{\sigma}{A} \left\{ (1 - \gamma_i) \left[ (2 - \sigma) - 2\gamma_i(1 - \sigma) \right] + \frac{\gamma_i}{n_i} \left[ 2 + \sigma n_i - \gamma_i \left( 2 - \sigma + \sigma n_i \right) \right] \right\}
\]

\[
C_i = \frac{\sigma}{A} (1 - \gamma_f) \left[ (1 - \gamma_i)(2 - \sigma) - \frac{\sigma \gamma_i}{n_i} \right]
\]

for \( i, j = d, f \) and \( i \neq j \)

By imposing symmetry between firms (but not across countries), such that \( u_{ki} = u_{lk} = u_i \) and \( u_{kj} = u_{jk} = u_j \), we obtain the following simplified expression

\[
x_i = \frac{(1 - \gamma_i)u_i^2}{2 \left[ (2 - \sigma)(1 - \gamma_i) - \frac{\sigma \gamma_i}{n_i} \right]} \left[ 1 - B_i (n_i + 1) - C_i (n_j + 1) \frac{u_j}{u_i} \right]
\]

(A1.4')

for \( i, j = d, f \) and \( i \neq j \)

Upon setting \( \gamma_d = \gamma_f = 0 \) and imposing symmetry across countries, such that \( u_d = u_f = u \), this expression becomes

\[
x = \frac{u^2}{2 \left[ 2 + \sigma N \right]}
\]

(A1.4'')

which is equation (A1.4'').

Prices \((p_{ki})\) are obtained by adding and subtracting \( 2\sigma u_{ki}^2 \) to the inverse demand functions (equation 3.3). This yields

\[
p_{ki} = 1 - 2(1 - \sigma) \frac{x_{ki}}{u_{ki}^2} - \frac{2\sigma}{u_{ki}} \left( \sum_{l=1}^{n_i+1} \frac{x_{li}}{u_{li}} + \sum_{l=1}^{n_j+1} \frac{x_{lj}}{u_{lj}} \right)
\]

for \( i, j = d, f \) and \( i \neq j \)  

(A1.5)

Substituting \( x_{ki} \) from (A1.4) and the expressions in (A1.3') into (A1.5). Thus we obtain the following solution for \( p_{ki} \):

\[
p_{ki} = \frac{1}{(2 - \sigma)(1 - \gamma_i) - \frac{\sigma \gamma_i}{n_i}} \left[ \left( 1 - \gamma_i - \frac{\sigma \gamma_i}{n_i} \right) - D_i \sum_{l=1}^{n_i+1} \frac{u_{li}}{u_{ki}} - E_i \sum_{l=1}^{n_j+1} \frac{u_{lj}}{u_{kj}} \right]
\]

(A1.6)

for \( i, j = d, f \) and \( i \neq j \)
where

\[
D_i = \frac{\sigma}{A} (1 - \gamma_i) \left\{ \frac{(1 - \gamma_i) [(2 - \sigma) - 2\gamma_f(1 - \sigma)]}{\frac{\gamma_i}{\bar{n}_i} \{2 + \sigma [n_f (1 - \sigma) - \gamma_f (1 - \sigma)] [2 + \sigma (n_f + 1)]\}} \right\}
\]

\[
E_i = \frac{\sigma}{A} (1 - \gamma_f) \left\{ (1 - \gamma_i) (2 - \sigma) - \frac{\sigma \gamma_i}{\bar{n}_i} \right\}
\]

as defined in equation (5.5). Assuming symmetry between firms (such that \(u_{ki} = u_{ii} = u_i\) and \(u_{kj} = u_{ij} = u_j\)), we obtain the following simplification

\[
p_i = \frac{1}{(2 - \sigma)(1 - \gamma_i) - \frac{\sigma \gamma_i}{\bar{n}_i}} \left[ (1 - \gamma_i - \frac{\sigma \gamma_i}{\bar{n}_i}) - D_i (n_i + 1) - E_i (n_j + 1) \frac{u_i}{u_i} \right]
\]

for \(i, j = d, f\) and \(i \neq j\) (A1.6')

Upon setting \(\gamma_d = \gamma_f = 0\) and imposing symmetry across countries, such that \(u_d = u_f = u\), this expression becomes

\[
p = \frac{1}{2 - \sigma N}
\]

which is equation (A1.6'').

The solved-out payoff is the product of (A1.4) and (A1.6):

\[
\pi_{ki}(u) = \frac{1 - \gamma_i}{K_i} \left( u_{ki} - B_i \sum_{l=1}^{n_i+1} u_{li} - C_i \sum_{l=1}^{n_j+1} u_{lj} \right) \left[ (1 - \frac{\sigma \gamma_i}{\bar{n}_i}) u_{ki} - D_i \sum_{l=1}^{n_i+1} u_{li} - E_i \sum_{l=1}^{n_j+1} u_{lj} \right]
\]

for \(i, j = d, f\) and \(i \neq j\)

where

\[
K_i = 2 \left[ (1 - \gamma_i) (2 - \sigma) - \frac{\sigma \gamma_i}{\bar{n}_i} \right]^2
\]

as defined in equation (5.5). The solved-out payoff is equation (5.4) in the main body of the chapter.
Appendix 2: Second order conditions for the Second Stage Subgame

To obtain the second order conditions, differentiate the first order conditions (5.10) with respect to $u_{kt}$ for $i = d, f$. This yields

$$(1 - \gamma_i) \frac{\partial^2 \pi_{ki}}{\partial u_{ki}^2} + \frac{\gamma_i}{n_i} \sum_{j \neq k} \frac{\partial^2 \pi_{ki}}{\partial u_{kj}^2} \leq \frac{(1 - \gamma_i) w_i \epsilon_i \epsilon_i (\beta_i - 1) u_{ki}^{\beta_i - 2}}{(L_d + L_f) u_{oi}^{\beta_i}}$$

for $i = d, f$ and $k = 1, ..., n_i + 1$ (A2.1)

Calculating the derivatives gives

$$\frac{\partial^2 \pi_{ki}}{\partial u_{ki}^2} = \frac{2(1 - \gamma_i)(1 - B_i)}{K_i} \left( 1 - \gamma_i - \sigma \gamma_i - D_i \right)$$

$$\frac{\partial^2 \pi_{ki}}{\partial u_{kj}^2} = \frac{2(1 - \gamma_i) D_i B_i}{K_i}$$

Upon substituting (5.33) and (5.33) into (A2.1), we obtain

$$(1 - \gamma_i) (1 - B_i) \left( 1 - \gamma_i - \frac{\sigma \gamma_i}{n_i} - D_i \right) + \gamma_i B_i D_i \leq \frac{K_i w_i \epsilon_i \epsilon_i (\beta_i - 1) u_{ki}^{\beta_i - 2}}{2 (L_d + L_f) u_{oi}^{\beta_i}}$$

(A2.1')

If we further substitute out $u_{kt}$, $n_i$ and $w_i$ with their equilibrium values, (A2.1') provides a restriction on the parameter space $(\sigma, \beta_i, \gamma_i)$. This restriction exhibits similar behaviour to the one derived in Appendix 2, chapter 4, in that it implies that $\sigma$ cannot be too high, $\beta_i$ cannot be too low (and in particular $\beta_i$ cannot be less than or equal to 2 for any value of $\sigma$ and $\gamma_i$) and $\gamma_i$ cannot be too high. If any one of these parameters crosses its bound, the others will need to compensate by moving inward from their bounds.
Part III

Big Push Arguments
Chapter 6

Endogenous Technological Capability, Trade Policy and the Big Push

1 Introduction

This chapter introduces a model of the 'Big Push'. The point of departure is similar to Venables (1996), and the main innovation with respect to that paper is that the framework is extended by allowing firms to choose technological capability. The argument of this chapter is that not all industrialization processes are similar. By analyzing the structural characteristics facing firms in different industrial environments, we develop some insights into why the process differs from one setting to another. The process of economic development is characterized by a sequence of 'take-offs'. There is an industrial take-off and a technological take-off. The industrial take-off leads to an initial Big-Push of industrialization. Once this process is underway, the economy may (or may not) achieve a second take-off point: Technological take-off. If it does, it will reach a higher income level than would have been the case otherwise, under certain conditions.

The modelling of technology in this chapter follows the endogenous sunk costs literature (Sutton 1991, 1998). In this literature, fixed outlays raise consumers' willingness-to-pay for a good, in the form of an increase in a shift parameter for the firm's demand schedule (alternatively, they increase the quality of a product). As before we associate a firm's technological capability to the quality of its products. The framework can easily be extended to deal with process innovation, learning-by-doing within the firm and network effects (see Sutton 1998, chs. 14 and 15).
The model, then, consists of a three sector economy. There are demand and cost linkages between two sectors, labelled $X$ and $Y$. In addition to sectors $X$ and $Y$, there is a residual ('rest of the economy') sector, which is used to close the model. Sector $X$ is the upstream industry. This industry has increasing returns to scale and is imperfectly competitive. Industry $X$ supplies intermediate goods of type $X$ to the (perfectly competitive) downstream industry $Y$. Industry $Y$ uses intermediate type $X$ goods from the upstream sector and labour to produce $Y$ (final output). The demand and cost linkages constitute a pecuniary externality: On the one hand, an increase in downstream industry output benefits upstream firms by raising intermediate goods demand (demand linkage). On the other hand, an expansion in upstream industry leads to lower price/quality ratios through either reduced concentration or enhanced technological capability. In turn, this reduces costs for downstream firms (cost linkage). The externalities allow the existence of multiple equilibria.

The existence of multiple equilibria is an essential feature of the results that follow. There are three equilibria (two stable and one unstable). One of the stable equilibria corresponds with high output in both sectors, a low price/quality ratio for type $X$ goods and high wages. The other stable equilibrium features a high price/quality ratio for intermediate type $X$ goods, low output in both sectors and a low wage rate. All equilibria are equally feasible, in the sense that the economy's resources (i.e., its labour endowment) do not change in moving from one equilibrium to another. All that may change is the distribution of resources between sectors, technological capability or market structure in the upstream industry and the efficiency with which resources are used. An essential assumption is that firms cannot commit to the high output equilibrium (i.e., there is a coordination failure).

We begin from an initial situation in which the presence of high tariffs results in an equilibrium featuring low output with a high price/quality ratio for the upstream industry ($X$), and therefore a low output level for the downstream industry ($Y$) and a low wage rate. We analyze the effect of a reduction in tariff levels: When tariffs are reduced sufficiently, the low output equilibrium is no longer feasible and the economy takes-off and converges to the high output equilibrium. Thus it experiences a 'Big Push' of industrialization.

We analyze two variants of this model. In the first variant, the technological capability of firms in the upstream industry $X$ is fixed exogenously, as in Venables (1996, Model I). Firms play a two-stage game: In the first stage the entry decision is taken (a zero profit, or free entry, condition is introduced). In the second stage, firms compete in quantities, à la Cournot.

In the second variant of the model, firms choose a fixed investment in order to achieve a certain technological capability, which is now endogenously determined. The outcome of
these outlays takes, as noted above, the form of an increase in a shift parameter in the firm’s demand schedule (the shift parameter is labelled technological capability). The game is now a 3-stage game. In the new (intermediate) stage firms choose how much to invest building up their technological capability. The mapping from fixed outlays \((F)\) to technological capability is a convex function (i.e., the marginal cost of increasing technological capability is increasing). The remaining stages of the game are as in the first variant of the model: In the first stage of the game (entry), we determine the number of firms that prevail at equilibrium. In the final stage (Cournot competition), firms with higher technological capability (‘quality’) enjoy a greater level of demand for a given price.

Both upstream \((X)\) and downstream \((Y)\) sectors produce tradable goods. In principle, tariffs can be imposed on either of the sectors (or on both). Since we are modelling a small open economy, the rest of the world is exogenous.

To complete the description of the economy’s equilibrium, we introduce a labour market. For simplicity, labour supply is assumed to be perfectly inelastic. Labour demand derives both from the \(X\) and \(Y\) sectors, and a residual ‘Rest of the Economy’ sector. Labour productivity in this residual sector is diminishing: As demand for labour rises from the \(X\) and \(Y\) sectors, less labour is used in the ‘Rest of the Economy’, so that its marginal productivity rises, thereby increasing wages in the whole economy.

2 Model A: Exogenous Technological Capabilities

This model is essentially that presented in Venables (1996, Model I), with a number of technical extensions. Since Model B builds upon and extends this model, it is convenient to analyze the conditions which underlie the results of Model A. We begin by analyzing the downstream industry.

2.1 Downstream Industry \(Y\)

Downstream firms minimize costs, \(wL_y + pX\), subject to the production function \(Y = AL_y^\alpha X^{1-\alpha}\), where \(A = \alpha^{-\alpha}(1 - \alpha)^{\alpha-1}\) is a constant, \(w\) is the wage rate, \(L_y\) is labour demand by sector \(Y\), \(p\) is the price of intermediate output, \(X\) is aggregate intermediate output demand, \(Y\) is downstream industry output and \(\alpha\) is the share of labour in costs \((0 \leq \alpha \leq 1)\). Solving for the firm’s cost function, we obtain

\[
C(w, p, Y) = wp^{1-\alpha}Y
\]
Since we have constant returns to scale, perfect competition and hence zero profits, at equilibrium average and marginal costs coincide with price. Setting $Y = 1$, the price of output $Y$ (denoted by $q$) can be expressed as

$$q = w^a p^{1-a}$$  \hspace{1cm} (6.1)

This is simply the zero profit condition for sector $Y$. Throughout the analysis $q$ will be exogenously given, and $q > 1$ is assumed. Equation (6.1) can be used to express conditional factor demands in terms of industry revenue, obtaining the following

$$L_Y = \alpha \frac{q^Y}{w}$$  \hspace{1cm} (6.2)

$$X = (1 - \alpha) \frac{q^Y}{p}$$  \hspace{1cm} (6.3)

2.2 Upstream Industry $X$

Upstream firms play a two-stage game. The equilibrium concept is Subgame Perfect Nash Equilibrium. The first stage involves the entry decision by firms. It is convenient in simplifying the expressions that follow, to denote the number of firms as $N + 1$ (rather than $N$). In the second stage, firms compete à la Cournot, viz, firm $i$ chooses the output level $x_i$ which maximizes profit: $\pi_i = (p - wc)x_i$, where $c$ denotes the constant level of marginal cost, $p$ is the price of upstream goods and $w$ is the wage rate (as defined in section 2.1). The inverse demand schedule for $X$ can be obtained from the conditional factor demand for aggregate intermediate output, using equation (6.3) above. In a symmetric equilibrium, the first order condition for this problem simplifies to

$$p \left( 1 - \frac{1}{N} \right) = wc$$

Using (6.3), we define upstream industry revenue as follows: $S = pX = (1 - \alpha) qY$. Using the first order condition and the definition of industry revenue, the symmetric solutions for price, quantity and profits can be expressed as

$$p = \frac{wc}{N}$$  \hspace{1cm} (6.4)

$$x = \frac{S}{wc (N + 1)^2}$$  \hspace{1cm} (6.5)

$$\pi = \frac{S}{(N + 1)^2}$$  \hspace{1cm} (6.6)

The entry condition requires that final stage (gross) profits, as defined in (6.6), should just cover set-up costs. Setting the expression for gross profits in (6.6) equal to the setup cost $\epsilon$,
we can solve for the equilibrium number of upstream firms (denoted by \( N + 1 \)),

\[
N + 1 = \sqrt{\frac{S}{w\epsilon}} \tag{6.7}
\]

For simplicity, the number of firms is treated throughout as a continuous variable. The case of a discrete number of entrants is discussed in Venables (1996). It is convenient to assume \( \epsilon \geq 1 \). As \( \epsilon \) falls, the number of firms increases, and price falls. In the limit, as \( \frac{S}{\epsilon} \rightarrow \infty \), \( (N + 1) \rightarrow \infty \), and price converges to marginal cost \((wc)\). This is the convergence property: Market structure converges to the competitive solution as entry costs become small and/or industry revenue becomes large. On the other hand as \( \epsilon \rightarrow S/w \), we converge to the monopoly solution. In this limit, we need to impose a ceiling on price (otherwise \( x \rightarrow 0 \) and \( p \rightarrow \infty \)). This ceiling will be the import price of intermediate goods.

The aggregate supply of intermediate output at equilibrium, is given by (recall that the number of firms is denoted by \( N + 1 \))

\[
X = (N + 1)x \tag{6.8}
\]

and labour demand by sector \( X \) equals

\[
L_x = (N + 1)(xc + \epsilon) \tag{6.9}
\]

### 2.3 The Labour Market

Labour supply is perfectly inelastic at \( L_e \). Labour demand comes from industries \( X \) and \( Y \) and a 'rest of the economy' (residual) sector. This residual sector exhibits diminishing marginal productivity of labour: As demand for labour rises in sectors \( X \) and \( Y \), less labour is available to the rest of the economy, its marginal productivity rises, thus pushing up the wage rate for the whole economy. This behaviour implies the following wage function

\[
\frac{w}{q} = MPL(L_e - L_y - L_x) \quad MPL' < 0, MPL'' < 0 \tag{6.10}
\]

where \( MPL(.) \) denotes the marginal product of labour in the residual sector, \( MPL' \) and \( MPL'' \) denote (respectively) the first and second derivatives, \( q \) is the price of the final good, and \( w \) is the nominal wage rate. Whence, the real wage rate is a decreasing and concave function of the amount of labour used in the residual sector. To see why concavity is required, note
that a rising wage imposes an external diseconomy on sectors $X$ and $Y$. In order to achieve the ‘Big Push’, this effect needs to be curtailed: It is necessary to assume that the marginal productivity of labour ($MPL$) does not fall too quickly, so that the wage rate does not rise too steeply as the upstream and downstream sectors expand output (thereby reducing employment in the residual sector).

It is convenient, before turning to a full analysis of the model, however, to remark on a basic feature of equilibrium; and to use this to motivate an assumption regarding a functional form we wish to impose, as follows: It is easy to see that there is a negative, monotonic relationship between employment in the residual sector and the level of output in the downstream industry. We can therefore define a function $\omega(Y)$ as follows$^1$,

$$MPL(L_e - L_y - L_x) \equiv \omega(Y) \quad \omega' > 0, \omega'' < 0 \quad (6.11)$$

where $\omega'$ and $\omega''$ denote the first and second derivatives. Rather than choose a specific functional form for $MPL(L_e - L_y - L_x)$, it is analytically convenient to instead impose a suitable functional form on $\omega(Y)$ as follows

$$\omega(Y) = Y^{1/\theta}, \text{ with } \theta > 1 \quad (6.12)$$

This reduced form equation appropriately captures the behaviour of the system. Upon substituting the number of firms (equation 6.7) and firm level intermediate output (equation 6.5) into labour demand by sector $X$ (equation 6.9), labour market clearing can be stated as

$$L_e = \frac{qY}{w} + L_r \quad (6.13)$$

where $L_r$ denotes the amount of labour employed in the residual sector. This completes the description of the model$^2$.

$^1$Note that there is no need to include intermediate output ($X$) because this depends positively and monotonically on $Y$. To see this, substitute the number of firms (equation 6.7) and firm level intermediate output (equation 6.5) into aggregate intermediate output supply (equation 6.8).

$^2$Note that consumers have not been modelled explicitly. An alternative model with consumers is available upon request. This model yields similar results, and for simplicity the model is closed using the above reduced form wage equation (as in Venables, 1996).
2.4 Equilibrium

Equilibrium is characterized by means of two conditions, the intersections of which define the equilibria for the economy. The first condition is obtained by solving for \( p \) from equation (6.1):

\[
p = \left( \frac{q}{w^{\alpha}} \right)^{\frac{1}{1-\alpha}} \quad \text{DD}
\]

This schedule is labelled \( DD \). It is convenient to denote the value of \( p \) satisfying \( DD \) as \( p_y \) (we shall refer to this as 'demand price'). \( DD \) implies the downstream industry (\( Y \)) is operating with zero profits. Note that \( DD \) is downward sloping in \( w \): For a given price of final output \( q \), a higher wage rate \( w \) allows a smaller demand price \( p_y \) to be paid for intermediate output, in order to break even. It is easy to see that \( DD \) is convex with respect to \( w \) (see appendix 1).

To obtain the second condition, substitute (6.12) and (6.7) into (6.4). This yields

\[
p = \frac{wc}{1 - \sqrt{\frac{\varepsilon}{1-\alpha} \left( \frac{q}{w} \right)^{\theta-1}}} \quad \text{SS}
\]

This schedule is labelled \( SS \). It is convenient to denote the value of \( p \) satisfying \( SS \) by \( p_x \) (we shall refer to this as the 'supply price'). \( SS \) implies zero profits in the upstream industry (\( X \)) and labour market clearing. To avoid changes in the concavity/convexity of the \( SS \) schedule, we assume:

\[
\text{A1. } w > w^* = q \left( \frac{\varepsilon}{1-\alpha} \right)^{\frac{1}{\theta-1}}. \text{ Note that } w^* > 1.
\]

There are two effects at work in the \( SS \) schedule. The first is that as \( w \) increases, the marginal cost of intermediate output rises linearly, thereby increasing supply price, this will make \( SS \) upward sloping at high values of the wage rate. The effect can be seen in the numerator of \( SS \). The second effect works through the number of firms: The wage rate increase reflects higher demand for labour by sectors \( X \) and \( Y \), which means higher production levels in both sectors. As sales in the upstream industry (\( X \)) rise, the number of entrants increases, thereby reducing supply price (\( p_x \)). This effect can be observed in the denominator of \( SS \), and is prevalent at low values of the wage rate, making \( SS \) downward sloping.

Equating the \( DD \) and \( SS \) schedules yields

\[
\varepsilon = \frac{q}{w^{\alpha}} - \sqrt{\frac{\varepsilon}{1-\alpha} \left( \frac{q}{w} \right)^{\theta+\frac{1+\alpha}{\theta-\alpha}}} \quad (6.13)
\]

195
The values of \( w \) which solve (6.13), denoted by \( w_e \), are the equilibrium wage rates.

We now provide an account of how the model works. Before doing so, it is useful to begin by solving for aggregate intermediate output \( (X) \) in terms of the wage rate. This helps towards the construction of a figure with \( p_y, p_x \) on the vertical axis, and \( X \) on the horizontal axis. This figure will provide a useful illustration. Combining (6.7), (6.12), (6.5) and (6.8), it follows that

\[
X = \frac{(1 - \alpha)^{\frac{1}{2}}}{c} \left( \frac{w}{q} \right)^{\frac{\epsilon - 1}{2}} \left[ (1 - \alpha)^{\frac{1}{2}} \left( \frac{w}{q} \right)^{\frac{\epsilon - 1}{2}} - \epsilon \right]
\]  

where \( X \) represents upstream output. Equations (DD), (SS) and (6.14) allow us to write \( p_x \) and \( p_y \) as functions of \( X \). Figure 1 exhibits the functions \( p_x(X) \) and \( p_y(X) \), which are labelled SS and DD, respectively, in the figure. When interpreting the figure, it is important to note that, under A1, there is a monotonic relationship between \( w \) and \( X \), so we may interpret a rise in \( w \) as a rightward movement along the horizontal axis of the figure (i.e., the horizontal axis can be relabelled \( w \) instead of \( X \)).

**EXOGENOUS TECHNOLOGICAL CAPABILITIES**

Figure 1: SS shows the function \( p_x(X) \). DD shows the function \( p_y(X) \). Equilibria E1 and E3 are stable, while E2 is unstable.

We can explore the workings of the model by using Figure 1 to examine what happens when the exogenously fixed price of imports falls. The vertical axis measures the price of the intermediate good, the horizontal axis shows aggregate output of the intermediate good. Three schedules are
shown in Figure 1: The DD curve \((p_y)\), the SS curve \((p_x)\), and the price of imports after tariffs \((p_m)\). \(p_m\) can be controlled by tariff policy. To the left of point A, the import price becomes binding and domestic upstream firms cannot set \(p_x\) above \(p_m\) without losing the domestic market to imports. Since the economy is small, supply of imports is assumed infinitely elastic.

**DD** is a zero profit schedule. Pairs \((p, X)\) lying below **DD** imply positive profits for downstream firms, while pairs lying above imply negative profits. The downstream industry will be in equilibrium only along the **DD** locus. The opposite holds for **SS**: For pairs \((p, X)\) lying below **SS**, upstream firms have negative profits (given market structure, i.e., \(N + 1\)) and exit ensues, pushing price up. For pairs \((p, X)\) above **SS** profits are positive (given \(N + 1\)) and entry ensues, driving price down. Hence, with flexible prices, the **SS** locus is attracting. Note, however, that **SS** is not quite a zero profit schedule. Rather, it is a schedule along which the labour market clears and the upstream industry has zero profits, given the number of firms.

All equilibria must lie on the **DD** locus, since that is the only way the downstream industry will be in equilibrium. If the price level is fixed (as is the case to the left of point A, where import price, \(p_m\), is below supply price, \(p_x\)), the upstream industry can, however, be in equilibrium out of the **SS** locus, since the number of firms will adjust to make profits zero, but the subsequent price adjustment will not take place. Clearly, in this case the number of firms will differ from that determined when prices can fluctuate freely (as is the case when the economy lies on the **SS** curve).

In equilibrium, supply price will be given by \(p_s = \min(p_x, p_m)\), so that if upstream firms set a price higher than the price of imports, they will not be able to sell. We begin from the case when \(p_m\) is very high, so that \(p_s = p_x\). Under A1, there are two equilibria, denoted by \(E_2\) and \(E_3\) in the diagram. \(E_2\) is an unstable equilibrium, so we identify \(E_3\) with the equilibrium outcome (see Appendix 3 for further discussion of stability).

Now let \(p_m\) fall, so that \(p_s = p_m\) (this is the case shown in the diagram). Here there are three equilibria, two stable equilibria at \(E_1\) and \(E_3\), and an unstable equilibrium at \(E_2\). For a high protection level (\(p_m\) is high), let the upstream industry be initially located at \(E_1\). This equilibrium features a high upstream price, low upstream and downstream output, a large 'rest of the economy' sector and therefore a low wage rate. The high price is due to the small number of firms in the upstream sector, and high concentration is supported by small industry sales. Upstream firms sell at a price marginally below \(p_m\). The number of upstream firms operating at \(E_1\) is smaller than the number associated with **SS** at production level \(X_{E1}\). As discussed above, this is because \((p, X)\) pairs lying below **SS** imply negative profits for upstream firms, and exit ensues until profits with price \(p_m\) are zero.
Equilibrium $E_1$ is stable. To the left of $E_1$ demand price is above supply price (equal to the price of imports), hence it is profitable for upstream firms to enter and aggregate intermediate output expands to $E_1$ (note that price is fixed at $p_m$). Meanwhile, other effects are at work: As upstream and downstream output rises, less labour is used in the residual sector, and wages rise. To the right of $E_1$, demand price is below supply price ($p_m$), thus firms exit driving price back up to $E_1$, output falls in both sectors, the residual sector expands and wages fall.

The second equilibrium is $E_3$, characterized by a low upstream price, high output in both industries, low output in the rest of the economy and a high wage. Low concentration in the upstream industry generates a price lower than the price of imports. The low price supports a high downstream output level, which in turn supports high upstream output.

This equilibrium is again stable: To the left, demand price is above supply price and upstream firms earn positive profits, entry ensues, price falls, downstream and upstream outputs increase, shifting the economy back to $E_3$. As before, along the transition the rest of the economy shrinks and wages rise. To the right of $E_3$, demand price is below supply price, upstream firms earn negative profits, exit ensues and the process takes the economy back to $E_3$.

The third equilibrium is $E_2$, which is unstable. To the left of $E_2$ firms exit, and to the right they enter, converging to $E_1$ and $E_3$, respectively. $E_2$ is the 'industrial take-off' point, in the sense that if the economy is shifted past it, it will experience a large industrial expansion.

The different configurations of equilibria that may arise in the current framework are set out in Proposition 1. To this end, it is convenient to specify some conditions, which allow us to formulate the subsequent proposition and impose some restrictions on parameters.

**Conditions:**

- **C1.** $\epsilon \leq \left\{ \frac{1-\alpha}{\alpha} \frac{4}{[\alpha+(1-\alpha)(\beta-1)]^2} \right\} \left[ \frac{(\beta-1)(1-\alpha)}{2(1-\alpha)(\beta-1)} \right]$.  
- **C2.** $\lim_{w \to -\infty} \frac{\beta}{\beta} < 1$.  
- **C3.** $\lim_{w^* \to w^*} \frac{\beta}{\beta} < 1$.  
- **C4.** $p_x$ and $p_y$ have no changes in concavity/convexity for $w \in (w^*, \infty)$.

We also introduce a further assumption, which confines the analysis to values of $X$ lying above $X^*(w^*)$ in Figure 1.

- **A2.** $p_m < q \left( \frac{1-\alpha}{\alpha} \right)^{\beta-1}(1-\alpha)$.
Proposition 1: Equilibria

I) If C1 holds with strict inequality, then under A1, A2, C2, C3 and C4 there are three equilibria: Two intersections of $DD$ and $SS$ ($E_2$ and $E_3$) and one intersection of $DD$ and $p_m$ ($E_1$).

II) If C1 holds with equality, then under A1, A2, C2, C3 and C4 there are two equilibria: One tangency point between $DD$ and $SS$ ($E'_2$) and one intersection of $DD$ and $p_m$ ($E_1$).

III) If C1 does not hold, then there is a unique equilibrium given by the intersection of $DD$ and $p_m$ ($E_1$).

Proof: see Appendix 2.

The analysis has concentrated on the more interesting case shown in Figure 1 and stated in Proposition 1, part I (Parts II and III are not illustrated). We proceed to analyze the effect of tariffs.

2.5 Trade Policy

We will interpret a change in tariff levels in what follows as a shift in the value of $p_m$. The effects of tariffs on intermediate goods are discussed in the following proposition.

Proposition 2: Tariffs on Intermediate Goods

If the economy is at $E_1$, a tariff reduction (increase) for intermediate output raises (lowers) output in both sectors. If tariffs fall sufficiently, a large output expansion (‘Big Push’) is induced. If the fall in the price of imports is sufficiently large, it will eliminate the upstream industry and shift the downstream industry to a high production level.

Proof:

The effect of increasing tariffs is to raise $p_m$. Since $p_x$ and $p_y$ are downward sloping, $E_1$ shifts to the left, reducing aggregate upstream and downstream output and the wage rate (see Figure 1). Protection of the upstream industry generates a contraction of both industries. Reducing tariffs lowers $p_m$ and moves $E_1$ to the right, increasing output in both sectors.

If the reduction is enough to shift $p_m$ below $p(w_{E2})$, by the instability of $E_2$ (see Appendix 3), the economy will start an automatic expansion process and converge to $E_3$. At $E_3$, $p_m$ need not be binding.

Let $w_{\text{min}} = q \left[ \frac{e}{1-\alpha} \frac{(1+\theta^2)}{4} \right]^{\frac{1}{4\alpha}}$. This specifies the minimum of $p_x$ for $w \in (w^*, \infty)$. If $p_m$ falls below $p_x(w_{\text{min}})$, the price of imports after tariffs is too low for the domestic upstream industry to operate and the new equilibrium is given by the intersection of $p_y$ and $p_m$.\[199\]
Proposition 2 has some interesting implications. It says that non-protectionist trade policy can help domestic industry to develop. If import price falls sufficiently, the upstream industry could be eliminated, leaving the country only with the downstream industry. This will be associated with a higher wage rate than if the upstream industry had survived, since the non-competitive nature of this industry imposes a negative (pecuniary) externality on downstream firms.

To analyze protection of the downstream industry consider the following proposition.

**Proposition 3: Tariffs on Downstream Goods**

Protection in the downstream industry raises output for both sectors. If the economy is initially located at $E_1$ and protection is sufficient, an industrial expansion from equilibrium $E_1$ to $E_2$ can be triggered.

**Proof:**

It is useful to relabel the horizontal axis in Figure 1. Note that under A1, $X$ (equation 6.14) is a monotonically increasing function of $\frac{1}{q}$. By similar reasoning as before, Figure 1 can be interpreted as having $\frac{1}{q}$ on the horizontal axis. Then changes in $q$ will shift $p_y$, while reflecting movements along $p_x$ (as well as $p_y$). To see this, note that $q$ enters $p_x (SS)$ only through $\frac{1}{q}$ in the denominator. $p_y$ can be written as $p_y = q \left( \frac{a}{b} \right)^{\frac{1}{1-a}}$ from which it is clear that changes in $q$ not only generate movements along $p_y$ but shifts as well.

Protection of the downstream industry is equivalent to increasing $q$. This will shift $p_y$ up. Equilibria $E_1$ and $E_3$ are shifted to the right, whereas $E_2$ moves leftward and upward. If the economy is at $E_3$ or $E_1$, increases in production for both sectors follow. However if $p_y$ shifts past the crossing between $p_m$ and $p_x$ (point $A$ in Figure 1), then an expansion to $E_3$ is triggered.

Propositions 2 and 3 imply that a combination of protection for downstream firms and tariff reduction for upstream firms has the potential to generate a 'Big Push' of industrialization.

### 2.6 Effects of Changing Parameter Values

We consider each parameter in turn.

$\alpha$: Under A1, the effect of increasing $\alpha$ is to shift $p_x$ up and $p_y$ down (see equations $DD$ and $SS$). Thus there is a value of $\alpha$ above which the only equilibrium is $E_1$. This is defined by C1, taking as given other parameters. It is also the case that by reducing $\alpha$ an expansion
from $E_1$ to $E_3$ can be triggered. This defines a value of $\alpha$ below which the only equilibrium is $E_3$ (the reasoning is similar to Proposition 3).

$\theta$ : Increasing $\theta$ does not affect $p_y$, and shifts $p_x$ down (see equations $DD$ and $SS$). To see this, bear in mind that $\frac{\alpha}{\nu} < 1$, thus $(\frac{\alpha}{\nu})^{\theta-1}$ is decreasing in $\theta$. The value of $\theta$ below which only $E_1$ exists, is given by $C_1$ (other parameters being given). There is also a value of $\theta$ above which a ‘Big Push’ of industrialization is triggered.

c : Increasing $c$ only affects $p_x$ by shifting it up (see equations $DD$ and $SS$). Thus, there exists a value of $c$ (defined by $C_1$) above which only $E_1$ exists. There is also a value of $c$ below which the economy ends up at $E_3$.

e : The effect is similar to $c$. The value of $\varepsilon$ above which only $E_1$ exists is also defined by $C_1$, taking as given the other parameters. Values of $\varepsilon$ below a certain threshold generate a shift towards $E_3$.

$q$ : see Proposition 3.

The model discussed so far features exogenous technological capabilities. In the next model, we go beyond the framework of Venables (1996), in allowing upstream firms to choose their level of technological capability. We examine in this setting when an economy can generate an industrial expansion (Big Push), and we further explore how this affects the quality of its products. Here, an industrial expansion may involve firms acquiring higher technological capabilities in order to produce a higher quality product.

3 Model B: Endogenous Technological Capabilities

In this model upstream firms can invest to achieve higher technological capabilities. In this case the intermediate good may no longer be homogeneous. Investment in technological capability is also referred to as ‘escalation’ (Sutton, 1998). Escalation leads to higher quality products (i.e., higher technological capability). The introduction of quality as an attribute of intermediate goods implies that good $i$ will have an associated price ($p_i$) and quality level ($u_i$), which are inseparable features. The technological capability of each upstream firm will be embedded in the (single) good it produces.

The model will retain the general structure of the previous section: It will have a downstream industry, an upstream industry and a residual sector.
3.1 The Downstream Industry

As in section 2.1, the downstream industry has constant returns to scale and is perfectly competitive. Taking the wage rate \( w \) and the price-quality ratio \( (p_i/u_i) \) as given, downstream firms choose \( L_y \) and \( x_i \) in order to minimize costs: \( wL_y + \sum_{i=1}^{N+1} (p_i/u_i)^\alpha x_i \), subject to \( Y = AL_y^\alpha \left( \sum_{i=1}^{N+1} x_i \right)^{1-\alpha} \), where \( p_i \) is the price of intermediate output \( i \) (produced exclusively by upstream firm \( i \)), \( u_i \) is the quality level of good \( i \) (alternatively, technological capability of upstream firm \( i \)), \( 0 < \varphi < 1 \) is the extent to which quality reduces costs (below it will be seen that it also represents an externality). Other variables are as defined in section 2.1. A good’s quality is relevant because it reduces the production cost of downstream firms. This can be justified by considering how low quality inputs hinder the production process. To mention but two examples, by making the production process more prone to mechanical failures or by generating more losses via unsaleable products\(^3\).

When downstream firms choose which upstream input to buy, they will choose the one with the lowest price/quality ratio and make all their planned purchases from the firm offering the chosen variety. Since upstream goods are perfect substitutes, upstream firms must have identical price/quality ratios: \( \frac{P_i}{u_i} = \lambda \) for all \( i \). In a symmetric equilibrium all upstream firms have identical prices, quantities and technological capabilities. In this case we write the solution for the downstream firms’ problem in terms of \( \frac{P}{u} \) (as opposed to \( \frac{P_i}{u_i} \)), and use \( X \) instead of \( \sum_{i=1}^{N+1} x_i \) (where \( X \) is aggregate upstream output). Solving the downstream firms’ problem yields the following cost function:

\[
C(w, \frac{P}{u}, Y) = w^{1-\alpha} \left( \frac{P}{u^\alpha} \right)^\alpha Y
\]

As in section 2.1, setting \( Y = 1 \) and \( C(w, \frac{P}{u}, 1) = q \), we obtain the zero profit condition for the downstream industry:

\[
q = w^{1-\alpha} \left( \frac{P}{u^\alpha} \right)^\alpha
\]  

(6.15)

Using (6.15), we express conditional factor demands in terms of downstream industry revenue:

\[
L_y = \alpha \frac{qY}{w} \quad \text{(6.16)}
\]

\[
X = (1 - \alpha) \frac{qY}{P/u^\alpha} \quad \text{(6.17)}
\]

\(^3\)There are other ways of introducing \( u_i \) into the problem solved by downstream firms. For example, it could be introduced as a multiplicative factor in the production function, yielding \( Y = AL_y^\alpha \left( \sum_{i=1}^{N+1} u_i x_i \right)^{1-\alpha} \). However, the chosen alternative gives simpler results.
3.2 The Upstream Industry

To allow endogenous choice of technological capability, the two stage game in section 2.2 is modified by introducing an intermediate stage in which firms choose investment in technological capability (i.e., product quality). As before, the entry decision is made in the first stage. In the intermediate stage firms choose fixed outlays (investment in technological capability). Finally, in the third stage firms compete à la Cournot, by choosing quantities. The equilibrium concept is Subgame Perfect Nash Equilibrium, and the game is solved by backward induction. We seek a symmetric Nash Equilibrium in each stage.

In the third stage, upstream firms choose \( x_i \) in order to maximize gross profits \( \pi_i = (p_i - wc)x_i \), taking other firms' quantities, technological capability and market structure as given. Marginal cost is a constant, denoted by \( c \). Upstream firms offer a unique price/quality ratio, defined by \( \frac{p_i}{x_i} = \lambda \) for all \( i \). This means that upstream industry revenue can be written as \( S = \sum_{j}^{N+1} p_j x_j = \lambda \sum_{j}^{N+1} u_j x_j \). From which

\[
\lambda = \frac{S}{\sum_{j}^{N+1} u_j x_j} \tag{6.18}
\]

The final stage profit function for upstream firms can be written as \( \pi_i = (\lambda u_i - wc) x_i \). Differentiating with respect to \( x_i \), we obtain the first order condition:

\[
\lambda u_i + \frac{\partial \lambda}{\partial x_i} u_i x_i = wc \tag{6.19}
\]

After calculations shown in Appendix 4, the following solved-out profit function is obtained

\[
\pi_i = S \left(1 - \frac{N}{\sum_{j}^{N+1} u_j} \right)^2 \tag{6.20}
\]

The profit function no longer depends on \( x_i \), it only depends on \( u_i \) and \( N + 1 \). If firms choose symmetric technological capabilities, setting \( u_i = u_j \) yields \( \pi = \frac{S}{(N+1)^2} \), which is the profit function obtained in (6.6). Under symmetry, \( p \) and \( x \) are also identical to those in equations (6.4) and (6.5), respectively.

In the second stage, firms choose \( u_i \) to maximize net profit: \( \pi_i = F(u_i) \), where \( \pi_i \) is given in (6.20) and \( F(u_i) \) denotes the fixed outlays function, \( F(u_i) = \varepsilon u_i^\beta \). This function is convex in \( u_i \) (\( \beta > 1 \)), i.e., the marginal cost of increasing technological capability is increasing. A firm's technological capability (resp., the quality of a product) is assumed to be bounded below by 1 (\( u_i \geq 1 \)). Zero investment in technological capability gives \( F = \varepsilon \) and the model collapses to
the exogenous technological capability model presented in section 2.

Before solving for the optimal technological capability level, let us solve for industry revenue from equation (6.17). This yields $S = (1 - \alpha) q Y u^\beta$. In solving for the optimal $u_i$, we have assumed that upstream firms take $S$ as given. Thus the (symmetric) quality level entering $S$ constitutes an externality. By increasing their own technological capability and by the symmetric response of rivals, firms de facto increase overall industry sales. This market expansion effect is not taken into account when firms optimally choose their own investment. The first order condition for the intermediate stage is

$$\frac{\partial \pi_i}{\partial u_i} = \frac{\partial F(u_i)}{\partial u_i}$$

from which we solve for the symmetric (Nash) equilibrium level of technological capability:

$$u = \left[ \frac{2(1 - \alpha) Y}{\varepsilon \beta} \left( \frac{q}{w} \right) \frac{N^2}{(N + 1)^3} \right]^{\frac{1}{\beta - \varepsilon}}$$

(6.21)

In the entry stage we set profits (6.20) equal to fixed outlays ($F(u_i)$). Substituting (6.21) into the zero profit condition, we solve for the number of entrants,

$$N + 1 = \frac{\beta}{4} \left( 1 + \sqrt{1 + \frac{8}{\beta}} \right) + 1$$

(6.22)

The number of firms is increasing in $\beta$: As the marginal cost of achieving a given technological capability rises, quality falls, permitting a greater number of entrants. Also note that the number of firms depends only on $\beta$, and is independent of industry revenue (cf. equation (6.7) in section 2.2). This is the non-convergence property discussed in Sutton (1991, 1998) and Cabral (2002).

3.3 The Labour Market

The labour market is exactly as in section 2.3, except for labour demand from the upstream sector. Now labour demand from sector $X$ is given by

$$L_x = (N + 1)(x \varepsilon + su^\beta)$$

(6.23)

Upon substituting $u$ from (6.21) and following a similar procedure to section 2.3, the labour market clearing condition has labour demand from sectors $X$ and $Y$ and (residual) labour used in the rest of the economy adding up to the total labour endowment.
**A Window of Opportunity**

Since technological capability is bounded from below \((u \geq 1)\), there will be a "technological take-off" wage rate (denoted by \(w_s\)), above which firms start investing in technological capability. Below the "technological take-off" point, firms do not invest and the workings of the model are identical to Model A (exogenous technological capability). The switch point is defined by setting \(u = 1\) in (6.21). Substituting \(Y\) out using (6.12), and solving for \(w_s\) yields

\[
w_s = q \left[ \frac{\varepsilon \beta}{2(1 - \alpha) N^2} \right]^{\frac{1}{1-\alpha}} \tag{6.24}
\]

The "technological take-off" wage \((w_s)\) is increasing in \(q, \varepsilon, \alpha, \beta\) and decreasing in \(\theta\). The key insight is that for an endogenous increase in technological capability to take place, the technological take-off wage rate must lie (recall Figure 1) between the unstable equilibrium wage and the high output wage \((w_{E2} \leq w_s \leq w_{E3})\). The formal condition for this is derived below (condition C6 in section 3.5). This condition effectively constitutes a *window of opportunity* for the economy to begin developing its technological capability. We will see below that if the economy manages to fit through this "window of opportunity", under certain conditions it will achieve a higher wage rate than would have otherwise been the case.

3.4 Equilibrium

As in section 2.4, we characterize equilibrium by two conditions. To obtain the first condition, solve for \(p/u\) from equation (6.15). This condition is labelled \(D'D'\), and it holds when \(w \geq w_s\) (recall that \(w_s\) is the "technological take-off" wage rate, and it defines the wage level at which firms start investing). When \(w < w_s\) the model collapses to Model A (exogenous technological capabilities). In this case, the condition is simply \(DD\) (which is the same as in section 2.4).

\[
p = \left( \frac{q}{u^\alpha} \right)^{\frac{1}{1-\alpha}} \quad \text{if } w < w_s \quad \text{DD}
\]

\[
\frac{p}{u} = \frac{1}{u^{1-\varphi}} \left( \frac{q}{u^\alpha} \right)^{\frac{1}{1-\alpha}} \quad \text{if } w \geq w_s \quad \text{D'D'}
\]

Similarly to section 2.4, the value of \(p\) satisfying \(DD\) or \(D'D'\) is denoted by \(p_y\) ("demand price").

The price quality ratio for sector \(X\) is obtained by using equation (6.4) and dividing by \(u\). This condition is stated below as \(S'S'\), and it holds for \(w \geq w_s\). For \(w < w_s\), the relevant
condition is \( SS \), from section 2.4.

\[
P = \begin{cases} 
\frac{wc}{1 - \sqrt{1 - \alpha \left( \frac{\varepsilon}{\phi} \right)^{\theta - 1}}} & \text{if } w < w_s \\
\frac{wc N + 1}{u} & \text{if } w \geq w_s
\end{cases}
\]

The value of \( p \) satisfying \( SS \) or \( S'S' \) is denoted by \( p_x \) ("supply price").

Combining (6.12), (6.21), \((D'D')\) and \((S'S')\) yields the explicit solution for the equilibrium wage rate (for \( w \geq w_s \)):

\[
\hat{w} = q \left\{ c^{(1 - \alpha)(\theta - 1)} \left( \frac{2(1 - \alpha)}{\varepsilon \beta} \right)^\theta N^{\beta + \varphi} \frac{N^{\beta + \varphi}}{(N + 1)^{\beta + 2\varphi}} \right\}^{\beta - \frac{1}{\varphi - [\theta - \alpha](\theta - 1)]}
\]

(6.25)

We can see that \( \hat{w} \) is increasing in \( q \). Provided \( \beta > \varphi [\theta - \alpha (\theta - 1)] \), \( \hat{w} \) is increasing in \( \theta \) and \( \varphi \), and decreasing in \( c, \varepsilon \) and \( \alpha \). For \( \beta < \varphi [\theta - \alpha (\theta - 1)] \) the comparative statics properties are reversed. \( \hat{w} \) is non-monotonic in \( \beta \), at first decreasing and later increasing. A more detailed analysis is provided in section 3.6, on the effects of changing parameter values.

The equilibrium value of technological capability is obtained by plugging \( \hat{w} \) from (6.25) and \( Y \) from (6.12) into \( u \) (equation 6.21). This yields:

\[
u = \left[ \left( \frac{2(1 - \alpha)}{\varepsilon \beta} \right)^\theta N^{1 + \alpha + \theta (1 - \alpha)} \right]^{\beta - \frac{1}{\varphi - [\theta - \alpha](\theta - 1)]}
\]

(6.26)

A diagram similar to Figure 1 will be helpful in showing how the model with endogenous technological capability works. As previously, aggregate intermediate output production \( X \) is plotted in the horizontal axis. To see that \( X \) is indeed monotonically increasing in \( w \), we express \( X \) as a function of \( w \). This is achieved by combining equations (6.12), (6.21), (6.5) and (6.8), to obtain

\[
X = \frac{1}{c} \left\{ \left( \frac{2}{\varepsilon \beta} \right)^\varphi \left( 1 - \alpha \right) \left( \frac{w}{q} \right)^{\theta - 1} \right\}^\theta N^{\beta + \varphi} \frac{N^{\beta + \varphi}}{(N + 1)^{\beta + 2\varphi}} \}
\]

(6.27)

To construct Figure 2, we follow a similar procedure to Figure 1: We use equations \((DD), (D'D'), (SS), (S'S')\) and (6.27) to write \( p_x, \frac{p_x}{u}, p_y \) and \( \frac{p_x}{u} \) as functions of \( X \). Whence we plot \( p_x(X), \frac{p_x}{u}(X), p_y(X) \) and \( \frac{p_x}{u}(X) \). \( p_x(X) \) is labelled \( SS \), \( \frac{p_x}{u}(X) \) is labelled \( S'S' \), \( p_y(X) \) is labelled \( DD \) and \( \frac{p_x}{u}(X) \) is labelled \( D'D' \). Thick lines represent the equilibrium outcomes of the model, while dotted (continuation) lines represent the outcomes under exogenous technological
ENDOGENOUS TECHNOLOGICAL CAPABILITIES

Figure 2: SS shows the function $p_u(X)$. $S'S'$ shows the function $p_u^2(X)$. DD shows the function $p_y(X)$. D'D' shows the function $p_y^2(X)$.

The configuration presented in Figure 2 and the effects of changing parameter values (section 3.6) rely on the following restriction on parameter values:

$A3. \theta - (1 - \varphi) > \beta > \varphi [\theta - \alpha(\theta - 1)]$

The analysis of Figure 2 is similar to that of Figure 1. Note the structural change occurring at $X_a$. $X_a$ is the 'technological take-off' point (cf. $X_{E2}$, the 'industrial take-off' point). At $X_a$, $u = 1$ and it will be optimal to start investing in technological capability. This can be noted by the change of slope in $DD$ and $SS$. As quality rises with wages, the price/quality ratios fall at a faster rate.

We now discuss the slopes of $D'D'$ and $S'S'$. By substituting (6.12) and (6.21) into $D'D'$, it can be seen that $D'D'$ is always downward sloping in $w$. After similar substitutions it can be shown that $S'S'$ can be either downward or upward sloping in $w$, depending on whether $\theta - 1 \geq \beta - \varphi$. If $\theta - 1 > (\varphi)\beta - \varphi$, $S'S'$ has a negative (positive) slope. If $\theta - 1 = \beta - \varphi$, the price/quality ratio is independent of the wage, hence it is also independent of $X$, thereby leading to a horizontal $S'S'$ schedule. For the case depicted in Figure 2, we have that $\theta - 1 > \beta - \varphi$ (which is encompassed by the left inequality in $A3$). Below we present some conditions (C5-C7)
which ensure that $\theta - 1 > \beta - \varphi$.

The mechanism behind the slope of $S'S'$ is of a fundamentally different nature than that behind $SS$. In $SS$, as wages rise, firms enter, thereby inducing a lower price. As wages rise further, marginal cost keeps increasing. This effect eventually dominates the slope of $SS$, making it positive. In $S'S'$, however, the number of firms is fixed by equation (6.22), and the value of the slope depends on two effects. Firstly, the increasing quality level of intermediate goods makes $S'S'$ downward sloping (ceteris paribus). Secondly, the increasing wage rate (and hence increasing marginal cost) makes $S'S'$ upward sloping (ceteris paribus).

To summarize, up to $w_s$ price has been falling for a while due to entry of firms in the upstream industry, and ceteris paribus, it would eventually begin to rise (due to increasing marginal costs). However, if $\theta - 1 > \beta - \varphi$, the endogenous investment in technological capability gives a 'second breath' to the process, and though firms no longer enter, competition in technological capability (escalation) leads to a new phase of further reductions in the upstream price/quality ratio.

As in section 2, there are three equilibria, and tariff policy can move the economy from $E_1$ to $E_2$, as before. This requires that $w_s \in [w_{E2}, w_{E3}]$. In this case, the shift between $E_1$ and $E_2$ will be accompanied by an increase in technological capability. Moreover, under certain conditions, $w_{E2} < \bar{w}$ (the precise conditions are specified below).

If $\theta - 1 > \beta - \varphi$ (which is implied by A3), the net effect of the increase in product quality dominates (otherwise, the increasing marginal cost effect dominates). This parameter restriction, namely $\theta - 1 > \beta - \varphi$, has an intuitive interpretation, to which we now turn. It says that $\theta$ is large relative to $\beta - \varphi$. This means that the wage function (equation 6.12) has wages rising relatively slowly. On the other hand a small value of $\beta - \varphi$ implies that the marginal cost of building technological capability is relatively low and/or the externality (market expansion) effect is relatively strong. Thus in this scenario, the transition from equilibrium $E_1$ towards equilibrium $E_2$ (the 'Big Push') is accompanied by a relatively slow-rising wage rate, and a relatively fast-rising technological capability.

### 3.5 Trade Policy with Technological Capability

Let us introduce some conditions which will be helpful in presenting an important result:

- **C5.** $c \leq \left[ \frac{2(1-\alpha)}{\epsilon} \frac{N^{1+\theta - \alpha(\theta - 1)}}{(N+1)^{1+2+\alpha(1+\theta - 1)}} \right]^{1/2}$

- **C6.** $c \leq \left[ 1 - \frac{\epsilon}{\sqrt{2(1-\alpha)}} \frac{N^{3/2}}{N} \right] \left[ \frac{2(1-\alpha)}{\epsilon} \frac{N^2}{(N+1)^{1+\theta - \alpha(\theta - 1)}} \right]^{1/2}$

208
The result is stated in the following proposition (recall Figure 2).

**Proposition 4: Tariffs on Intermediate Goods with Technological Capability**

Let the economy be at $E_1$ and let C1-C6 hold, then:

I) A sufficiently large reduction in intermediate output tariffs generates an industrial expansion which will shift the economy to $E_q$, together with an increase in upstream firms' technological capabilities.

II) The equilibrium with endogenous technological capabilities ($E_\xi$) will feature a higher wage than the equilibrium where technological capability is at a minimum ($E_3$, with $u = 1$).

III) If tariffs are lowered sufficiently, the upstream industry may cease to exist.

Additionally,

i) $w_s \leq \bar{w} \iff C7$

ii) $w_{E_2} \leq w_s \leq w_{E_3} \iff C6$

iii) $w_{E_3} \leq \bar{w} \iff C5 \land C6 \land C7$

**Proof:** See Appendix 2.

Proposition 4 specifies the consequences of reducing $p_m/u$. In Figure 2 (as in Model A) once $p_m/u$ falls below $E_2$, an industrial expansion follows, converging to either $E_3$ (if $w_s \notin [w_{E_2}, w_{E_3}]$) or to $E_\xi$ (if $w_s \in [w_{E_2}, w_{E_3}]$). In the latter case, the transition triggers a rise in technological capability as the economy crosses the technological threshold $w_s$. The wage rate associated with $E_\xi$ will be higher than that associated with $E_3$ under the conditions set out above. If $p_m/u$ falls below $E_\xi$ then the upstream industry cannot compete with imports and ceases to exist. Equilibrium will now lie at the intersection of $p_m/u$ and $D'D'$, and (as in Model A) the economy will feature a higher wage rate.

Having set out the effects of tariffs for the upstream industry, we now discuss the effect of tariffs for the downstream industry.

**Proposition 5: Tariffs on Downstream Goods with Technological Capability**

Protection of the downstream industry will expand output in both sectors, and if sufficient, it can trigger effects similar to those described in Proposition 4.
Proof:

Note the similarity to Proposition 3. Recall that protection of the downstream industry can be modelled as an increase in \( q \). Note also that \( X \) in (6.27) is monotonically increasing in \( \frac{w}{q} \), and the horizontal axis in Figure 2 can be relabelled \( \frac{w}{q} \). In this representation, \( S'S' \) does not shift with changes in \( q \), whilst \( D'D' \) does. The rest of the analysis proceeds in accordance to Propositions 3 and 4.

Regarding the ‘technological take-off’ point \((w_s)\), note that increasing \( q \) shifts \( w_s \) and \( \bar{w} \) proportionately. This, together with the upward shift in \( D'D' \), guarantees that if \( w_s < w_{K3} \) held initially, it will continue to hold at the new level of \( q \). Therefore, if the economy featured the possibility of ‘technological take-off’ at the initial \( q \), this will still hold at the new \( qM \).

3.6 Effects of Changing Parameter Values

For \( w < w_s \), all effects of changing parameter values remain as in Model A. The properties discussed here are specifically for \( w \geq w_s \). The issue of interest is how \( \bar{w}, w_s, D'D' \) and \( S'S' \) change with parameters.

\( \alpha \) : Increases in \( \alpha \) shift \( D'D' \) and \( S'S' \) up. If \( \beta > \varphi [\theta - \alpha(\theta - 1)] \) (A3), \( \bar{w} \) is decreasing in \( \alpha \), hence the upward shift in \( S'S' \) is greater than the shift in \( D'D' \). If \( \beta < \varphi [\theta - \alpha(\theta - 1)] \), \( \bar{w} \) can be (but not necessarily is) increasing in \( \alpha \) and the shift in \( S'S' \) would be smaller than the shift in \( D'D' \). \( w_s \) is increasing in \( \alpha \) (see 6.24). Thus for \( \beta > \varphi [\theta - \alpha(\theta - 1)] \), there exists a value of \( \alpha \) above which an increase in technological capabilities does not occur.

\( \theta \) : Increases in \( \theta \) shift down both \( D'D' \) and \( S'S' \). For \( \beta > \varphi [\theta - \alpha(\theta - 1)] \), \( \bar{w} \) is increasing in \( \theta \) and \( S'S' \) shifts by more than \( D'D' \). The opposite holds if \( \beta < \varphi [\theta - \alpha(\theta - 1)] \). \( w_s \) is decreasing in \( \theta \). If \( \beta > \varphi [\theta - \alpha(\theta - 1)] \), there is a value of \( \theta \) below which \( w_s > \bar{w} \) and it will not be optimal to invest in technological capability.

\( c \) : Changes in \( c \) do not affect \( D'D' \). As in section 2.7, increasing \( c \) shifts \( S'S' \) up. This is reflected in \( \bar{w} \), which is decreasing in \( c \), as long as \( \beta > \varphi [\theta - \alpha(\theta - 1)] \). \( w_s \) does not depend on \( c \). Consequently, there exists a value of \( c \) above which no escalation is optimal.

\( \varepsilon \) : If \( \beta > \varphi [\theta - \alpha(\theta - 1)] \), \( \bar{w} \) is decreasing in \( \varepsilon \). Rising \( \varepsilon \) is associated with upward shifts in \( D'D' \) and \( S'S' \), with the shift in \( S'S' \) being greater. \( w_s \) is increasing in \( \varepsilon \). This means that there is a value of \( \varepsilon \) above which investment in technological capability does not take place.

\( \varphi \) : Under A3, \( \bar{w} \) increases with \( \varphi \), whereas \( w_s \) is not affected. Rising \( \varphi \) shifts \( S'S' \) down and \( D'D' \) up.

\( \beta \) : \( w_s \) is increasing in \( \beta \). However, \( \bar{w} \) is non-monotonic in \( \beta \). If A3 holds, \( \bar{w} \) is at first
decreasing and later increasing in $\beta$. Thus there exists a value of $\beta$ for which $\frac{\partial \varepsilon}{\partial \beta} = 0$.

4 Conclusions

In this chapter we have extended the Venables (1996) model of the 'Big-Push' to a richer setting in which firms engage in development of their technological capabilities. The main features of the analysis have been shown to carry over to this setting. Moreover, this has allowed us to examine the effects of the 'Big-Push' mechanism on the technological capability of firms (product quality).

The main contribution is the following. In the endogenous technological capability setting, a tariff reduction for the upstream sector (alternatively, tariff increase for the downstream sector), can induce an industrial expansion, together with an increase in upstream firms' technological capabilities. The investment in technological capability will take place if the 'technological take-off' point (denoted by the wage rate $w_4$) is associated to a sufficiently low wage (see Figure 2 and Proposition 4). The technological take-off can be bypassed if the technological take-off wage rate is too high. In this case the economy misses the window of opportunity, and ends up with a thwarted process of industrial development in which technological capability does not rise. Thus the economy may industrialize, but the industries into which it successfully enters will be technologically backward and (under certain conditions) the economy will achieve a lower wage rate than if the technological take-off had been achieved. The key notion is that the wage rate cannot rise too steeply along the transition path towards the high-wage equilibrium. Otherwise, it runs the risk of impeding entry into technologically advanced industries, which (under certain conditions) are associated to a higher wage rate at the high-wage equilibrium.

The model also sheds light on some possible reasons why many developing countries managed to (partially) industrialize, whilst very few countries managed to enter successfully into high-technology industries: The conditions required for achieving technological take-off are indeed stringent.
Appendix 1: Slope and Convexity of $DD$ and $SS$

For $DD$, the relevant derivatives are given by:

$$\frac{\partial p_y}{\partial w} = -\frac{\alpha}{1 - \alpha} \frac{p_y}{w} < 0$$

$$\frac{\partial^2 p_y}{\partial w^2} = \frac{\alpha}{1 - \alpha} \left( \frac{\alpha}{1 - \alpha} + 1 \right) \frac{p_y}{w^2} > 0$$

For $SS$, similar calculations yield:

$$\frac{\partial p_x}{\partial w} = \frac{p_z}{w} \left[ 1 - \frac{(\theta - 1) \sqrt{\frac{\epsilon}{1 - \alpha} \left( \frac{\bar{q}}{w} \right)^{\theta - 1}}}{1 - \sqrt{\frac{\epsilon}{1 - \alpha} \left( \frac{\bar{q}}{w} \right)^{\theta - 1}}} \right]$$

$$\frac{\partial^2 p_x}{\partial w^2} = \frac{\epsilon}{w} \left( \frac{\theta - 1}{2} \right) \sqrt{\frac{\epsilon}{1 - \alpha} \left( \frac{\bar{q}}{w} \right)^{\theta - 1}} \left\{ \frac{(\theta - 1) \sqrt{\frac{\epsilon}{1 - \alpha} \left( \frac{\bar{q}}{w} \right)^{\theta - 1}}}{1 - \sqrt{\frac{\epsilon}{1 - \alpha} \left( \frac{\bar{q}}{w} \right)^{\theta - 1}}} + \left( \frac{\theta - 3}{2} \right) \right\}$$

Appendix 2: Proofs

Proof of Proposition 1:

The proof proceeds by construction and exclusion of cases not specified in parts I-III of the proposition. We examine the basic properties that are required of $DD$, $SS$ and $p_m$, and then show how these properties are met. Finally, it is shown that the cases of zero or strictly more than three equilibria can be excluded.

It will be useful to keep Figure 1 in mind. Also recall that since $X$ is a monotonically increasing function of $w$ (see equation 6.14), the figure with $w$ on the horizontal axis has the same properties as Figure 1.

To see why C1-C4 are necessary (and taken together, sufficient), consider C1-C3. To obtain two crossings between $p_x$ and $p_y$, there must be three ranges for $w$. Firstly, for $w \in (w^*, w_{E1})$, $p_x > p_y$ (C3). Secondly for $w \in [w_{E2}, w_{E3}]$, $p_x < p_y$ (C1). Finally, for $w \in (w_{E3}, \infty)$, $p_x > p_y$ (C2). This guarantees at least two crossings (at least one tangency point if C1 holds with equality). Including C4 guarantees exactly two crossings (exactly one tangency point if C1 holds with equality).

Let us analyze C1 first, while C2-C4 hold. To guarantee that $p_x$ and $p_y$ cross or are at least tangential, a range where $p_x \leq p_y$ is a necessary condition. Indeed this range is defined by two equilibria, as follows: $w \in [w_{E2}, w_{E3}]$. C1 is necessary and sufficient for $p_x \leq p_y$. To see this, consider the case when $p_x$ is tangent to $p_y$. In this case $\frac{p_y}{p_x}$ has a maximum at $p_x = p_y$, defined
by \[ \frac{\partial (p_x)}{\partial w} \bigg|_{w_p} = 0 \]. This yields \( w_p = q \left\{ \frac{(2 + (1-\alpha)(\theta-1))^2 - \frac{e}{1-\alpha}}{1-\alpha} \right\}^{\frac{1}{\theta+1}} \). Substituting \( w_p \) into \( p_x \) and imposing the condition \( \frac{p_x}{p_y} \geq 1 \) we obtain C1.

Consider C2: As \( w \to \infty, \frac{p_x}{p_y} \to 0 \) and C2 holds.

To check that C3 holds, let \( w \to w^* \) from above. Then \( p_x \to \infty \) and \( p_y \to p_y(w^*) \), which is finite. This yields \( p_x > p_y \).

To check C4, calculate \( \frac{\partial^2 p_x}{\partial w^2} \) and \( \frac{\partial^2 p_y}{\partial w^2} \) (see Appendix 1). The analysis focuses on positive \( w \). \[ \frac{\partial^2 p_x}{\partial w^2} = 0 \iff w \to \infty \left( p_y \text{ becomes linear} \right), \text{ otherwise } \frac{\partial^2 p_x}{\partial w^2} > 0. \] To analyze \( p_x \) note that \( \frac{\partial^2 p_x}{\partial w^2} \geq 0 \) for \( w > w^* \), and \( \frac{\partial^2 p_x}{\partial w^2} < 0 \) for \( 0 < w < w^* \). Hence to the left of \( w^* \), \( p_x \) is concave, where as to the right of \( w^* \), \( p_x \) is convex. For \( w > w^* \) there are no changes of concavity/convexity in \( p_x \). Note that \( \frac{\partial^2 p_x}{\partial w^2} \to 0 \) as \( w \to q \left[ \frac{\varepsilon}{1-\alpha} \left( \frac{1+\varepsilon}{\varepsilon} \right)^2 \right]^{\frac{1}{\theta+1}}, \) but this does not define a change in concavity/convexity. Rather, it reflects the fact that \( p_x \) becomes linear as the wage grows.

In order to have at least one firm in the upstream industry, assumption A1 implies an upper bound on \( p_m \), which is defined by \( p_m < p_y(w^*) \). Performing these calculations yields A2. For \( w > w^* \) there is at least one firm in the upstream industry. For \( p_m \) higher than the upper bound, the domestic upstream industry is non-existent.

It remains to show that zero and more than three equilibria cannot exist.

Consider the zero equilibrium case. If there is no equilibrium, \( p_m \) and \( p_y \) do not cross \( (E_1 \) does not exist \( ) \) and C1 does not hold. \( p_m \) is simply a horizontal line, while \( p_y \) is a hyperbola, hence they will always cross - unless \( p_m \) is exactly zero (an unfeasible price). Therefore, \( E_1 \) always exists. This is true regardless of whether A1 and A2 hold.

To exclude strictly more than three equilibria, note that \( E_1 \) always exists, so what is required is that there are more than two crossings of \( SS \) and \( DD \). By C2 and C3, the number of equilibria will be even. To see this, note that \( p_x > p_y \) as \( w \to w^* \) and as \( w \to \infty \), hence an odd number of crossings is not possible. To exclude an even number of crossings higher than two, note that this would require changes in the concavity/convexity of \( p_x \) and/or \( p_y \), but this cannot be by C4.\]

**Proof of Proposition 4:**

The configuration shown in Figure 2 is in line with Proposition 4. C1 (strictly)-C4 hold to guarantee that \( DD \) and \( SS \) cross exactly twice (see Proposition 1).

Part I follows similar reasoning to Proposition 2: If tariffs for the intermediate goods are reduced sufficiently \( (p_m/u \text{ falls below } E_2) \), equilibrium \( E_1 \) ceases to exist and the economy converges to \( E_5 \).
To see how i, ii and iii relate to parts I, II, and III, consider each of the former:

i) For an endogenous increase in technological capabilities to take place, the technological take-off wage rate \( w_s \) must lie below the equilibrium wage rate \( \tilde{w} \). This will hold if and only if \( \frac{\tilde{w}}{w_s} \geq 1 \). Substituting \( \tilde{w} \) and \( w_s \) from (6.25) and (6.24), respectively, yields C5. 'i' is a necessary condition for part I.

ii) Industrialization will be characterized by an increase in technological capabilities if and only if \( w_a \leq \left[ w_e^2, w_{E3} \right] \). Substituting \( w_s \) into the equilibrium condition (6.13), condition C6 is obtained. 'ii' is another necessary condition for part I.

iii) In order to have \( \tilde{w} \geq w_{E3} \) (part II), it is useful to plot the equilibrium condition (6.13). This can be seen in Figure 3, where the left hand side (LHS, equal to \( c \)) has been plotted against the right hand side (RHS, equal to \( \left( \frac{q}{w} \right)^{1-\alpha} - \sqrt{\frac{\epsilon}{1-\alpha}} \left( \frac{q}{w} \right)^{\frac{\alpha+1}{1-\alpha}} \)).

In Figure 3 it can be seen that for \( \tilde{w} \geq w_{E3} \) to hold, we require LHS \( \geq \) RHS. This is also the condition required for \( \tilde{w} \geq w_{E3} \). But this also holds for \( \tilde{w} \leq w_{E2} \). In order to rule out this case, both C5 (or 'i') and C6 (or 'ii') are necessary. Substituting \( \tilde{w} \) into the equilibrium condition (6.13), setting LHS \( \geq \) RHS and simplifying yields C7. Note that since \( 0 \leq c \leq 1 \) (from C1) and the exponents of \( c \) in C7 are positive, C7 yields a restriction of the form \( c \leq c^* \), where \( c^* \) solves C7 with equality. C5, C6 and C7 are each necessary for 'iii', and together they are sufficient. Part II and 'iii' are equivalent.

For part III, note that if the tariff is reduced such that \( p_m < p_x(\tilde{w}) \), the upstream industry will not be able to attain a positive market share, even with high technological capability. As in Model A, the economy achieves a higher wage rate in this case.
Appendix 3: Stability

Let \( w_E \) be the wage rate associated with equilibrium ‘\( E \)’. Recall that upstream firms cannot sell at a price higher than the price of imports. Hence, supply price is given by \( p_s = \min(p_x, p_m) \).

Let us introduce the following definition of stability:

**Definition: Stability**

I) An equilibrium is **left-stable** if and only if for \( w < w_E, p_y > p_s \). Otherwise it is **left-unstable**.

II) An equilibrium is **right-stable** if and only if for \( w > w_E, p_y < p_s \). Otherwise it is **right-unstable**.

An equilibrium is **stable** if and only if it is left and right stable. An equilibrium is **unstable** if and only if it is left and right unstable.

To see how this definition works, we now apply it to the equilibria considered in Proposition 1. Consider first the case when \( p_m \) is not binding.

**Left-stability:** Recall \( E_3 \) in Figure 1. If \( p_y > p_x \), the market price is \( p_y \). Upstream firms make positive profits and entry into the upstream industry follows. This increases aggregate supply of intermediate output, lowering supply price by moving along \( p_x \). In turn, higher aggregate supply of intermediate output reduces equilibrium price, and this allows a higher level of final output. Thus, the gap between \( p_x \) and \( p_y \) is reduced. Higher output in downstream and upstream sectors leaves less labour available for the rest of the economy, increasing wages. The process continues until \( p_y = p_x \) at \( w_E \) (convergence).

**Left-instability:** Recall \( E_2 \) in Figure 1. If \( p_y < p_x \), exit follows, thereby decreasing aggregate intermediate output, \( p_x \) rises, further increasing the gap between \( p_x \) and \( p_y \). The reduction of aggregate intermediate and downstream output increases labour used in the rest of the economy, thereby reducing wages. This process takes the economy to successively lower wages and increases the gap between \( p_x \) and \( p_y \) (divergence). Similarly, \( E_2' \) is left-unstable.

**Right-stability:** Recall \( E_3 \) in Figure 1. If \( p_y < p_s \), arguments similar to the left-instability case indicate that the economy converges to a smaller wage rate where \( p_y = p_x \). Similar reasoning shows that \( E_2' \) is also right-stable.

**Right-instability:** Recall \( E_2 \) in Figure 1. If \( p_y > p_x \), similar reasoning to the left-stability case shows the economy diverging to a larger wage rate where the gap between \( p_y \) and \( p_x \) grows (locally).

If \( p_m \) is binding (as at \( E_1 \)), the above reasoning goes through with a slight change. The
gap between \( p_s \) and \( p_y \) is not reduced (augmented) by upstream output price changes. Rather, the mechanism at work are changes in the aggregate supply of upstream goods, for a fixed price. These changes are an optimal reply by upstream firms to unfulfilled demand by downstream firms, which occur by either entry or exit (as appropriate). Changes in \( X \) will lead to movements along \( p_y \) (while \( p_s \) is fixed at \( p_m \)). These movements along \( p_y \) will either reduce or expand the gap between \( p_m \) and \( p_y \), depending on whether the stable or unstable case (respectively) is being considered.

It follows that \( E_1 \) and \( E_3 \) are stable equilibria, \( E_2 \) is unstable and \( E_2' \) is right-stable and left-unstable.

**Appendix 4: Deriving the solved-out profit function**

By perfect substitutability, upstream firms set \( \frac{E_i}{u_i} = \lambda \). Equation (6.18) is reproduced here for convenience:

\[
\lambda = \frac{S}{\sum_{j}^{N+1} u_j x_j} \tag{A3.1}
\]

Firms maximize \( \pi_i = (p_i - wc) x_i = (\lambda u_i - wc) x_i \). The first order condition is

\[
\lambda u_i - \frac{\lambda^2 u_i}{S} u_i x_i = wc \tag{A3.2}
\]

From (A3.2) solve for \( u_i x_i \) and sum this over all firms to get

\[
\sum_{j}^{N+1} u_j x_j = S \left( \frac{N + 1}{\lambda} - \frac{wc}{\lambda^2} \sum_{j}^{N+1} \frac{1}{u_j} \right) \tag{A3.3}
\]

Substitute \( \sum_{j}^{N+1} u_j x_j \) from (A3.1) into (A3.3) and solve for \( \lambda \) to obtain:

\[
\lambda = \frac{wc}{N} \sum_{j}^{N+1} \frac{1}{u_j} \tag{A3.4}
\]
This can be substituted into (A3.2) to give the following solutions for $p_i$ and $x_i$ and, using these, $\pi_i$:

\[
x_i = \frac{S}{w} \frac{N}{\sum_{j=1}^{N+1} u_j} \left( 1 - \frac{N}{\sum_{j=1}^{N+1} u_j} \right)
\]

\[
p_i = \lambda u_i = \frac{w c}{N} \sum_{j=1}^{N+1} \frac{u_i}{u_j}
\]

\[
\pi_i = S \left( 1 - \frac{N}{\sum_{j=1}^{N+1} u_j} \right)^2
\]

$\pi_i$ is equation (6.20) in the text.

Appendix 5: A Note on Second Order Conditions

Since the downstream firms' problem is strictly convex, it follows that the second order conditions hold. The upstream sector's second order conditions hold for any $N + 1 > 1$, which is guaranteed by $w > w^*$ (A1).
Chapter 7

Conclusions and Extensions for Future Research

This study was motivated by reference to a variety of industrialization experiences, particularly those of some East Asian economies (especially South Korea, Japan and Taiwan). We now summarize our theoretical findings, and relate them to the historical experience.

After a series of disappointing results from structural reforms based on prescriptions of the 'Washington consensus' (particularly in Latin America and Sub-Saharan Africa) and the huge literature detailing how East Asian economies diverged from such prescriptions to achieve outstanding performance, the economics profession is looking for a better understanding of how such heterodox policies could have worked. Rodrik (2004, p.1) puts the problem succinctly:

"Perhaps the disappointments in Africa are due to special circumstances: the ravages of civil war and crises in public health. But how can one explain the Latin American story, which is one of poor growth and productivity performance in the 1990s—much worse than in the 1950-1980 period? Fiscal discipline, privatization and openness to trade have produced an economic performance that does not even begin to match the performance under import substitution. And that is a puzzle of major proportions."

This thesis contributes by providing detailed theoretical insight into the design of industrial and trade policy, using models which capture some essential features of the East Asian experience, within a framework which is standard in economic analysis.

A theoretical model is by necessity a simplification of reality, and as such is not expected to explain all aspects of the latter. Rather, it is a starting point which uncovers a set of
mechanisms so that we may begin to shed light into particular processes or systems. The models we have developed here address the issue of industrialization. The view that emerges from them is essentially that of development as the transformation of a dual economy into an industrial economy (Lewis, 1954). While the transition can be caused by different mechanisms, the study as a whole points towards an understanding of this transformation process. The models in parts I and II provide an understanding of industrialization as a process in which an economy changes its underlying institutional structure, and it is these changes that lead to its transformation. The model in part III provides an alternative perspective. In that case, the transformation is viewed as a process triggered by an initial change (the crossing of a threshold). Then, if a series of conditions are fulfilled, the economy may cross a sequence of take-offs which will determine at which level of income it ends up.

The models we construct feature some aspects of the East Asian industrialization process that have been identified as salient by a number of authors (Amsden, 1989 and Wade, 1990, among others). As outlined in chapter 1 (introduction), these features are:

1) Oligopolistic behaviour and the associated strategic interaction.
2) An endogenous treatment of technological capability and market structure.
3) A general equilibrium framework.
4) A dualistic structure (characteristic of many backward economies).

Chapter 2 presents the benchmark closed economy model. Strategic interaction takes the form of Cournot competition. The model highlights the trade-off between the market structure effect and the technological capability effect. These effects are the fundamental driving force for many subsequent results. They constitute the underlying determinants of the demand for labour from industry $X$: Labour demand stems from the sunk investment in technological capability that each firm carries out. Accordingly, the number of firms present in the industry also affects labour demand. The key notion is that, so long as actual and shadow values of outcomes do not diverge, technological capability and the number of firms always move in

---

1This caveat refers to a minor exception discussed in chapter 4 (section 4.1), relating to Figure 4.1b: For $\beta$ and $\sigma$ high and $\gamma$ low, both technological capability and the number of firms rise together. This occurs for the following reasons:

On the one hand, for these parameter values the wage rate is unity, hence the shadow value of the wage rate lies below the actual outcome. Accordingly, actual technological capability lies below its shadow value (since its actual marginal cost is higher than its shadow marginal cost). As $\gamma$ increases, the wage rate rises above unity and actual outcomes converge to their shadow values. Since actual technological capability was below its shadow counterpart, we observe that actual technological capability rises to meet its shadow value (meanwhile shadow technological capability is decreasing in $\gamma$).

On the other hand, for low $\gamma$, the number of firms is increasing in $\gamma$. Thus, we find that actual outcomes of technological capability and the number of firms are moving in the same direction. However, the underlying mechanism whereby one variable moves in the opposite direction of the other still applies (the shadow value of technological capability is falling). It has only been concealed by the divergence of shadow and actual values.
opposite directions: Higher sunk costs increase concentration. Thus, a rise in technological capability implies a fall in the number of firms and vice versa. We envisaged two development configurations:

**High-tech:** The economy is characterized by few firms, each with high technological capability.

**Proliferation:** The economy is characterized by many firms, each with low technological capability.

The key question is then, which configuration will deliver the highest demand for labour and hence the highest wage rate? Proposition 1 summarizes the result: Unless $\sigma$ is low, a high-tech development configuration is associated with a higher wage rate (and welfare). If $\sigma$ is low, then a proliferation configuration is associated with a higher wage (and welfare). Essentially this means that the technological capability effect always dominates the market structure effect, except when goods are poor substitutes (i.e., a high degree of horizontal differentiation). This proposition is easily extended to encompass an economy composed by multiple industries, which are only linked via the labour market. This means that there are neither inter-industry demand linkages (goods are not substitutes across industries), nor supply linkages (there are no intermediate products). In this simple (but enlightening) case, if a planner chooses policies such that all industries except those with low within-industry product substitutability achieve maximum technological capability, maximum demand for labour will ensue (thus the wage rate and welfare will be maximized). Meanwhile, if a planner chooses policies so that concentration is minimized in industries with highly horizontally differentiated products, labour demand from these industries will also be maximized.

In chapter 3 we open the autarky model to trade. The international economy features two countries, each identical to the autarky model. Firms are Cournot competitors and there is strategic interaction between each firm and its rivals (domestic and foreign). We find a symmetric general equilibrium, such that each country's industry $X$ features subgame perfection. With identical parameter values in both nations, we can rule out asymmetric equilibria in which firms in one nation have a different technological capability to the other nation's firms (Proposition 2). Moreover, if profits are exactly zero for each firm, the result immediately extends to all types of asymmetric equilibria. Thus multiplicity of equilibria is not an issue, and we can safely focus on the symmetric equilibrium without need to worry about equilibrium switching phenomena.

We find that under Cournot competition free trade always results in a higher wage rate, and hence trade is welfare improving (Proposition 3). Moreover the conclusions about development
configurations are reinforced for the open economy (high-tech beats proliferation unless \( \sigma \) is low, in which case the reserve holds).

A fundamental result is that a firm's marginal benefit of technological capability is decreasing in the technological gap, i.e., in the ratio of the firm's technological capability to its overseas rivals' (Proposition 1). The consequence of this is that if the gap is too wide, firms in the backward economy do not find it optimal to invest in technological capability (the net marginal benefit is negative).

The opening of the economy to foreign competition forces domestic firms to either match their foreign rivals' technological capability or exit. Thus, in being exposed to a larger market size, firms in both nations raise their technological capability, and this leads to the associated increase in sunk investments, and a rise in concentration. This mechanism sheds light on how the export boom preceded the investment boom in South Korea and Taiwan (Rodrik, 1995a, see the comment by Victor Norman).

When we allow for asymmetries in initial conditions, the symmetry of the equilibrium is broken in the sense that the initially symmetric equilibrium outcomes will shift to new values which are no longer symmetric (although they still correspond to the same equilibrium, i.e., multiplicity of equilibria does not arise). The procedure to track equilibrium outcomes towards the new outcomes is as follows: We begin from a symmetric equilibrium with identical parameter values. Then we introduce a small change in the ratio of initial conditions \( \left( \frac{\text{Net}}{w_t} \right) \) and calculate the new equilibrium outcome so that equilibrium conditions hold exactly at the new outcome. There are three equilibrium conditions for each country:

1) A condition for the optimal choice of technological capability in industry \( X \).

2) A free-entry condition in industry \( X \).

3) A labour market clearing condition.

The new outcome (which is no longer symmetric) then becomes the starting point for the next round of iteration. The procedure was repeated until we encompassed a wide range of relative initial conditions. We found that the gains-from-trade-theorem needs to be qualified in the following manner: Provided the asymmetries are not too large, both nations gain from free trade. However, if initial conditions are too different (the technological gap is too wide), the backward nation may find that autarky leads to a higher wage rate. Meanwhile the advanced nation still benefits from free trade. This incompatibility of interests gives rise to issues of unfair trade negotiations, in which an advanced country pressures a backward nation into a trade agreement which is not beneficial to it. The lesson that can be taken from this result is that before exposing domestic industry to foreign competition, it is important to compare
the relative technological capabilities of both industries. If the local industry is lagging too far behind, then rather than expose it to trade straight away, it may be more convenient to enhance its technological capability first to ensure the technological gap is reduced sufficiently and the domestic economy benefits from trade.

Bhagwati (1988) has discussed at length the trend in the USA during the mid-eighties to spous a kind of 'strategic' trade policy. The policies spoused by this trend amount to little less than coercing small economies into trade diversion towards the USA. The support of such policies often calls upon some variant of the infant industry argument, whereby temporary interventions may generate a permanent advantage to the small nation. The small nation would then presumably overtake the USA in a particular industry. In order to prevent this overtaking, interest groups in the USA support removing interventions (industrial policies) in foreign economies, which could give rivals an 'unfair' advantage. The problem with this prescription, as embodied in the 1988 Trade Act, is that the USA would itself decide whether foreign rivals enjoy such 'unfair' advantages. Given the bargaining power of the USA, it is likely that politically weaker rivals can be coerced into trade-distorting measures, such as voluntary import expansions, where they divert trade from other nations to the USA. Proposition 4 in chapter 3 states the circumstances under which these incentives arise, and pins down parameter values that identify the cases under which it will be optimal for the laggard nation to open up to free trade with the advanced nation, and when it will not be in its interest. The basic thrust of the proposition is that free trade is always welfare enhancing, provided the technological capabilities of the two nations are not too dissimilar. It is when technological capabilities become very different that the laggard nation could be better off remaining closed (although free trade will still be welfare improving to the advanced nation). This proposition provides positive and normative criteria for the formation of trading blocks, under conditions of dual economies with oligopolistic industries. Basically, nations should only set up free trade agreements with nations whose technological capabilities are not too dissimilar to their own. When technological capabilities of the laggard economies expand (presumably as a consequence of free trade within the 'backward trading block') and the technology gap with the advanced nation is reduced, it may be optimal to engage in free trade with the advanced nation. The last decade has seen the USA embark upon a series of bilateral free trade agreements with small nations (particularly within Latin America). The theoretical findings we have developed here lead us to predict that these small nations will not benefit from such agreements.

The next questions addressed in chapter 3 relate to catching-up and industrial policy: Is a subsidy to investment in technological capability (financed with a lump-sum tax) an
effective mechanism for catching-up? If so, is it welfare improving? We find affirmatively for both questions. Given some level of asymmetry in initial conditions, assume the government subsidizes investment in technological capability to ensure that the wage rate of the laggard economy matches that of the advanced economy, and this subsidy is financed with a lump-sum tax. This subsidy will effectively result in the laggard nation catching-up with the advanced nation in terms of their wage rates, even after accounting for any strategic response by foreign oligopolists. The social costs of the subsidy do not offset its social benefits, and there is a net welfare gain in the laggard economy.

In chapter 4 we introduce adjustable intensity of competition into the benchmark autarky model from chapter 2. This is done by means of a generalized profit function for firms in industry $X$. This device allows us to vary the intensity of competition from individual profit maximization to perfect collusion (joint profit maximization), by changing a single parameter ($\gamma$). Additionally, the way we have modelled the intensity of competition can be interpreted as conjectural variations, with the advantage that $\gamma$ is directly measurable (it is the extent of cross-ownership within the industry).

The effects of increasing the intensity of competition depend on the values of $\beta$ (related to the marginal cost of technological capability) and $\sigma$ (product substitutability or horizontal product differentiation). If either $\beta$ or $\sigma$ are low, then stronger intensity of competition increases technological capability and concentration. If $\beta$ is low (for any $\sigma$) then it also increases the wage rate. If on the other hand $\beta$ and $\sigma$ are high, technological capability and concentration are non-monotonic in $\gamma$. If $\beta$ is high, the wage rate is shaped like an inverted-U in $\gamma$. This means that for industries in which the marginal cost of technological capability is high, there is an intermediate level of collusion (intensity of competition) which maximizes the wage rate. This leads to a revision of the results on development configurations from previous chapters.

A high-tech configuration leads to higher wages, with two exceptions:

1) When $\sigma$ is low (as before). In this case a proliferation configuration attains higher wages.

2) When $\beta$ is high and $\gamma$ takes intermediate values. In this case we find that an intermediate configuration yields higher wages. An intermediate economy is characterized by intermediate levels of technological capability and concentration.

In chapter 5 we consider the open economy version of the autarky model in chapter 4. The opening of the economy is performed essentially in the same manner as in chapter 3. Regarding development configurations, the results from autarky require a slight modification. Now the high-tech configuration leads to higher wages except when:

1) $\sigma$ is low (as before), in which case proliferation yields higher wages.
2) When both $\beta$ and $\sigma$ are high and $\gamma$ takes intermediate values, in which case the intermediate configuration yields higher wages.

In chapter 4 (autarky with adjustable intensity of competition) we found that when technological capability features a high marginal cost (high $\beta$), there is an optimal level for the intensity of competition (alternatively, an optimal level of collusion) which maximizes the wage rate. In the open economy with adjustable intensity of competition, we find a similar result, but now we not only require high $\beta$, but also a high degree of product substitutability (high $\sigma$). This can help to explain the behaviour of Japan's MITI, which sometimes intervened to curb competition, by encouraging cartelization and other means of reducing the intensity of competition (Amsden and Singh, 1994). We would also expect that as the economy reduces its marginal cost of technological capability ($\beta$ falls), it would benefit more from subsequent reductions in the intensity of competition. That is, as an economy matures from backward to advanced, the optimal competition policy changes from medium levels of collusion (intermediate intensity of competition), to tough competition.

Other results from the open economy with adjustable intensity of competition relate to trade policy. We found that there exists a separating surface in the space of $(\beta, \gamma, \sigma)$ which equates the wage of the open economy with adjustable intensity of competition to that of its autarky counterpart (Proposition 1, chapter 5). For combinations of $(\beta, \gamma, \sigma)$ lying above the surface, autarky results in a higher wage. Conversely, for combinations below the surface, free-trade yields higher wages. The separation arises from the trade-off between the technological capability effect and the market structure effect. When the economy is opened, two things happen. First, technological capability rises as a result of the increase in market size. Second, some firms exit. The two effects exactly offset each other at the separating surface. The implication is that different industries require different trade regimes, and these are characterized in Proposition 2 (chapter 5). A precise prediction as to which types of industry should be opened and which left in autarky is provided. In particular, as the intensity of competition rises, free-trade becomes optimal for more and more industries. In the limit, as the intensity of competition reaches its maximum ($\gamma = 0$, i.e., individual profit maximization), the model collapses to that of chapter 3 and free-trade is always welfare improving. Proposition 2 and Figure 5.3 (chapter 5) provide a typification of the optimal trade regime depending on the industry type. Industries which feature low substitutability are quite resilient to free-trade, and will generally benefit from it, especially those with low marginal costs of technological capability (low $\beta$). On the other hand, industries with high substitutability tend to be better off in autarky, particularly those with low marginal costs of technological capability (low $\beta$).
substitutability makes foreign competition potentially very harmful (a strong market structure effect), and if this is compounded with strong incentives for escalation (low $\beta$), industries are likely to contract their labour demand and thus depress the economy's wage rate.

We must be careful not to let these results be misinterpreted. The conclusion is not that we should close this or that industry, full stop. That would be throwing the baby out with the bath water. Rather, we need to step back and ask the more fundamental question: What can we do to increase the wage rate? Most of the parameters are amenable to being controlled by a policy maker. $\beta$ (relating to the marginal cost of technological capability) and $\epsilon$ (set-up costs) can be influenced by industrial policy. $\gamma$ relates to competition (antitrust) policy. $L$ (population) can be controlled to some extent by migration and demographic policy. Initial conditions (or rather future initial conditions) can be modified by the actions taken in the present, for example investment in human capital. Product substitutability ($\sigma$) seems harder to modify. It may be that firms can modify this by investing in horizontal product differentiation (see Motta and Polo, 1998). However, in the absence of a formal theoretical treatment, it seems difficult to suggest how a policy maker could control it.

Such modification of parameter values points towards changes in the institutional framework of the economy. With these notions in mind we can then focus on what the optimal parameter values are in order to achieve the highest demand for labour, wage rate and welfare. From Figures 4.1-4.6 we learn that higher wages will follow from lower $\beta$, lower $\gamma$ (if either of $\beta$ or $\sigma$ are low), intermediate $\gamma$ (if both $\beta$ and $\sigma$ are high), low $\sigma$, low $\epsilon$, high $u_0$ and high $L$. If a policy maker can control these parameters, then the economy can be shifted towards a higher wage rate. Changes in parameter values leading to higher wages also imply that free-trade is optimal. The conclusion is, then, to ask firstly, where in $(\beta, \gamma, \sigma)$-space is the economy located? Then, to ask what trade regime is optimal at the current location and whether it is feasible (and welfare improving) to modify the institutional framework of the economy (parameter values) in order to increase the wage rate and allow the economy to make the most out of free-trade.

This brings us to the question of industrial policy, regarding asymmetries in initial conditions and catching-up. We find the same results as in chapter 3: Catching-up is feasible for all parameter values, and industrial policy to this end is welfare improving. Thus industrial policy directed at controlling $\beta$ or $\epsilon$ in order to shift the economy towards a high-wage state (catching-up) is, in principle, feasible.

Expanding this reasoning to a multiple industry scenario leads to similar conclusions. Different industries will be characterized by different parameter values. Thus the policy maker's problem is to modify parameter values so that labour demand from each industry is maximized.
To recap, it seems there is a strong case for tough competition policy, except in the case of industries in which it is hard to raise technological capability (high $\beta$) and goods are easily substitutable (high $\sigma$). In this case, an intermediate intensity of competition may be desirable. Regarding trade policy, if competition is tough ($\gamma = 0$), then free trade is optimal. For lower intensities of competition, the optimal trade regime may differ between industries. The general picture is that if parameters can be controlled, shifting these to achieve higher wages always guarantees that free-trade is optimal. On the other hand, if $\sigma$ cannot be controlled, we find grounds for maintaining high $\sigma$ industries in autarky.

In chapter 6 we present a model of the 'Big Push' with endogenous technological capabilities and multiple equilibria. This chapter presents a vision of the process of development based a sequence of take-offs. The transition from a low wage to a high wage equilibrium can be triggered by trade policy (industrial take-off). Along the transition path, if the wage rate does not rise too steeply, the economy may cross a technological take-off, and this will trigger investment in technological capability. There is a window of opportunity through which the economy must fit in order to cross the technological take-off, and the conditions for this are indeed stringent. Under certain conditions, this will lead to a high-wage equilibrium which features a higher wage rate than would have been the case without investment in technological capability. If the economy does not manage to fit through the window of opportunity, it will end up with a thwarted industrialization process in which it has achieved a higher wage rate, but has not increased its technological capability. Thus, the economy industrializes by expanding industry along a low-tech path.

The view that emerges from Amsden (1989) and Wade (1990) is that the Japanese, South Korean and Taiwanese economies successfully used industrial, competition and trade policies in order to increase their income levels. If we look at the structure of these economies at the beginning of their transformation, and compare that picture with the current status, what we see are economies which began being mostly agrarian societies, and are now characterized by a series of high-tech industries. Comparing this with less successful industrializers, a crucial difference seems to be the extent to which the North East Asian economies managed to develop their technological capabilities (their entry into high-tech industries). They first went through an import substitution phase, and only later did they open their economies (even then, the opening was most generous on the exports side, with import liberalization being less forthcoming). During the import substitution phase they began the transformation of the economy with policies aimed at increasing technological capability. Later on, when their economies were opened, their products were sufficiently competitive to successfully enter world markets. How
did they successfully raise their technological capability during the import substitution phase? Appropriate use of competition and industrial policies is supported by our theoretical findings as a key ingredient to their performance. It remains to be seen whether the specifics of North East Asian industrial and competition policies were in line with our theoretical predictions. Studies like that by Lee (1997) for trade policy, preferably at a more disaggregated level, constitute a useful starting point to elucidate the effects of industrial and competition policies on industrial output growth.

Wade (2003, p. xlviii) asserts the need for multilateral agencies to become involved in the design of development strategies that take into account the lessons from the East Asian experience, namely the strategic use of industrial and trade policies. However, the problem is that the formal theoretical basis for the design of such policies has been lacking (with the exception of the work by Dixit and Grossman, 1986, which only deals with industrial policy, i.e., subsidies). Wade (ibid.) proposes the use of input-output tables to guide the promotion criteria for such industrial policies. The criteria he supports is that the planners should aim to achieve a 'dense' input-output matrix, i.e., an economy with lots of internal linkages. Amsden (1989) proposes supporting industries which feature high income elasticities. Lall (1992) mentions growth potential, linkages and externalities. None of these authors provide a positive or normative formal analysis to found such important claims. This thesis provides a formal, testable theoretical framework which allows a positive and normative analysis of industrial, competition and trade policies. More work is needed to make the theories more realistic and general, not to mention thorough testing.

Future Research and Limitations

Possible extensions for future research include using Bertrand competition instead of Cournot as the underlying structure of the final stage subgame. This offers the possible advantage of encompassing a complete coverage of intensities of competition, all the way from perfect collusion (joint profit maximization) to Bertrand (considered to be the most intense of all forms of competition).

Other extensions include relaxing the 1:1 structure of industry Y to allow a more general structure, possibly with diminishing marginal returns. Alternatively, we could drop industry Y altogether. However, this would entail losing the dual structure of the economy, which was deemed an attractive feature from the development economics perspective.

A strategic analysis for government policy (in the spirit of the strategic trade policy liter-
ature, e.g. Brander and Spencer, 1985) lies outside the scope of the thesis, and incorporating such decisions into the current framework is a promising extension. Moreover, reference has been made to how a policy maker could control several of the parameters of the model. This raises the question of how such control would be achieved, the costs associated with it and whether such modifications would indeed pay off in terms of welfare. Some progress has been made in this direction by considering industrial policy for catching-up in the presence of asymmetries in initial conditions. Another possibility is to consider the symmetric equilibrium and ask whether shifting the economy from some initial parameter configuration towards a new configuration (presumably associated with higher wage rates) would be feasible and welfare improving. Such extensions will be addressed in the future.

The Big-Push model in chapter 6 lends itself to the incorporation of exports. This is a worthwhile extension, and we have already made some progress in this direction. Also of interest is the reversal of the upstream and downstream sectors: We can imagine a situation in which it is the downstream sector where the imperfectly competitive behaviour lies, while the upstream sector is perfectly competitive.

The use of the generalized profit function introduced in chapter 4 invites an analysis of how firms choose their intensity of competition (γ). Future work on this could shed light on the optimal distribution of property rights within an industry. This addresses the question of: What game (Bertrand, Cournot, perfect collusion, etc.) would firms like to play? Again, this framework has the advantage of dealing with directly measurable conjectural variations.

An extension of the models in chapters 2-5 to multiple time periods is a necessary step towards a full assessment and reconsideration of the infant industry argument, especially regarding the role of expectations and intertemporal strategic interaction (first mover advantages, preemption, entry deterrence, etc.). Another seemingly promising avenue is to translate this framework into a dynamic general equilibrium setting with dynamic oligopoly. This is an extension on which we have begun to work. However, the difficulties of modelling dynamic oligopoly are not to be underestimated. In general, the models developed in this thesis would benefit from dynamic analysis, and it is in this direction that our efforts are being directed. Nonetheless, the framework as it stands can be interpreted as a long run model of general equilibrium. The model is long run since (long run) variables such as market structure and technological capability are determined endogenously, all production is ultimately consumed and any external trade imbalances must be paid for. In this sense the analysis may be interpreted as relating to the final resting point of the system after ‘all is said and done’ and any issues of intertemporal allocation of resources have been resolved. Dynamic analysis is
expected to bring forth issues relating to time (in)consistency of economic policy, descriptions of transitional paths, intertemporal resource allocation and intertemporal strategic interaction.

Finally, a word of warning is necessary. In this study we have uncovered various justifications for government interventionism, be it in the form of preferential trade agreements, subsidies for catching-up (chapter 3) or targeted trade policy (chapters 5 and 6). Although the literature on trade under imperfect competition is no longer 'new', it is still in its infancy with regards to the design of optimal policies for interventions, let alone the testing of such policies. Thus, to quote Dixit (1984, p. 4): “Any attempts to justify protection [or more generally intervention] on the basis of these results would be, at a minimum, premature”. In particular, policy design exercises need to take into account the literature on mechanism design (see Bolton and Dewatripont, 2005). This lies outside the scope of this study. Hence the usual caveats must be emphasized:

Firstly, the opportunity cost of subsidies has been assumed to be forgone consumption, so it remains to be shown whether this is a plausible assumption (i.e., whether the opportunity cost of subsidies is unity). In particular, if subsidies to industries are to occur at the expense of social investments, such as public health or human capital, we expect the case of interventions to be substantially weakened.

Secondly, industrial targeting (whether in the form of subsidies or protectionist trade policy) is a rent-seeker's dream. Such results are more than likely to serve as ammunition for vested interests in implementing DUP (directly unproductive) activities to capture rents (Bhagwati, 1982). The efficiency losses from such efforts can reach a considerable magnitude and should not be understated (Murphy, Shleifer and Vishny, 1993).
Bibliography


Christensen, L. R. and D. Cummings (1981). "Real Product, Real Factor Input


From It”. IMF Working Paper Number 95/98.


