INFLATION, INTEREST RATES AND TRANSACTIONS COSTS:
THE U.S. EXPERIENCE, 1953-1979

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ABSTRACT

This thesis contains three primary aspects: an analysis of the actual inflation-interest rate relationship in the American economy for the 1953-1979 and 1953-1983 periods in both the short-run and the long-run, a description and analysis of the failure of other real interest rate models to adequately account for the apparent failure of a "full" Fisher effect to operate, and empirical tests of a model (with some variations of it as well) which sheds light on the phenomenon of the underadjustment of nominal rates to inflation even when the longest of runs is taken into account. The essential notion underlying the model is that highly marketable financial assets yield liquidity services in the sense that the holding of them allows an agent to reduce his transactions costs of exchange. Given that the real return on money, which possesses ultimate liquidity, is the negative of the inflation rate then it follows that the real returns on assets which are relatively close to money in terms of liquidity services yielded by them should also fall in the face of increased inflation. The degree to which the real return on any particular asset declines is positively related to its 'moneyness'. Thus the returns on the longer term, relatively illiquid financial assets should be more subject to a full Fisher effect than those on the more marketable assets.

The first chapter reviews some of the theoretical and empirical work on the Fisher effect and offers a detailed explanation of the basic model. An appendix deals with a transactions cost model of underadjustment which has a more explicit role for trading costs. Chapter 2 presents the empirical evidence on underadjustment in the short and long runs and critically reviews some work of others attempting to account for a less than complete Fisher effect. Chapter 3 deals with direct tests of the model's more important implications concerning the degree of adjustment of nominal rates to inflation with respect to an asset's liquidity. In Chapter 4 the hypothesis is entertained and empirically tested that some unexplained 'excess returns' found by researchers of the asset pricing model of security returns represent liquidity premia of the type dealt with in the model. Finally, Chapter 5 summarizes some of the thesis' more important conclusions.
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INTRODUCTION:

This thesis is essentially a careful empirical study of the inflation-interest rate relationship in the American economy for the 1953-1979 era (although the sample period is extended in some sections) and a test of a model which purports to explain the less-than-full adjustment of nominal interest rates to inflation found in various financial asset markets over this period. The model was developed by Fried and Howitt (1983) and is based on the friction associated with transactions costs of exchange. The notion is that markets incorporate in various asset returns an amount which reflects the relative degree of liquidity of the asset in question, liquidity in the sense of transactions costs that are avoided by holding the asset.

Chapter 1 focuses on the theoretical foundations that underlie much of the empirical work on the interest rate-inflation relationship in both the short and long runs and critically discusses some recent work which attempts to explain why researchers have failed to find a full Fisher effect. For example, Blejer and Eden (1979) argue that the measured coefficient on inflation is downward biased due to the exclusion of an inflation variance term in an otherwise standard Fisher effect equation. However evidence is presented that this particular argument has little relevance for our period of interest. It is also demonstrated in the first chapter that the recent focusing of attention on tax-adjusted returns by Darby (1975), Feldstein (1980) et al.,
however theoretically appropriate, fails to account for the stylized fact of less than unit adjustment. Even when allowing for multiple marginal tax rates, the use of plausible magnitudes for these values indicates that we should find an overadjustment, a point that seems to have been overlooked by the writers in this area. It is also argued that real interest rate models such as Mundell-Tobin fail to account for the observed findings.

Fried and Howitt (F-H) developed their model within the context of both continuous time and the long-run, partly because they felt that there were short-term factors that could potentially explain the underadjustment within the business cycle but also because there were no good explanations for the failure of the Fisher effect in the longer run (the failure of superneutrality) other than one that would treat assets as less than perfect substitutes for one another, differing in the liquidity services that they provide. An important contribution of this thesis is to demonstrate that the Fried-Howitt model provides a superior interpretation of the underadjustment findings in the short-run as well as the long-run and that some of the better known potential short-run explanations are either misconceived or do not stand up empirically, at least for the sample period considered in the present work. Chapter 1 also introduces some of the terminology used throughout the present study and provides a discussion of the model that highlights some of its features which are of the greatest relevance to the present work. An appendix to Chapter 1
contains further clarification of the F-H analysis as well as a model which, unlike F-H, explicitly illustrates the role of relative transactions costs in the determination of yield differentials. Additionally, the role of exogenously issued assets in helping to determine interest rate differentials is made more explicit than in F-H.

Chapter 2 is almost exclusively devoted to an analysis of the actual inflation-interest rate relationship in the U.S. economy for the 1953-1979 period although, when possible, the analysis is extended to include the early 1980s, a period of almost unprecedented high real interest rates. One empirical approach used follows that of Mishkin (1981) who invokes a rational expectations analysis to explain the formation of inflation expectations and thus is able to draw inferences about the behavior of the non-observable ex ante real rates from the behavior of the easily measured ex post real rates the two of which are equal under the assumptions made. By extending his list of explanatory variables that could potentially explain real rate movements, confirmation of his findings that variables other than lagged inflation are econometrically insignificant is made. Discussion of a number of other recent analyses of real interest movements (e.g. the supply shock model of Wilcox (1983), the money shock model of Mishkin (1982), the 'stagflation' model of Fama (1982), etc.) are shown to be unable to explain falling real interest rates in the face of increased inflation during our
period. In particular a detailed critique of the Fama (1982) stagflation analysis is provided.

Also, Fama's very well-known work on short-term rates as predictors of inflation is updated to include the inflationary 1970s and early 1980s. Whereas most writers have been content to test the standard Fisher effect equation (or some slightly modified version of it) making some simple prior assumptions about the nature of inflation expectations formation, and then deciding that the data reject the Fama hypothesis (e.g. Nelson and Schwert (1977), Mishkin (1981), et. al.) a unique approach is taken in this chapter. Specifically, the entire Fama analysis which involves the separate testing for constancy of the real rate and market efficiency is applied to the 1953-79 and 1953-83 periods. In other words, in this section, rather than assuming market efficiency at the outset, as many authors have done, and then attributing any deviation in co-movements between nominal yields and inflation to variable ex ante real rates, the assumptions of market efficiency and constant real rates are tested independently, with some fairly surprising results. Although provisional, the results obtained indicate that real rates may have been roughly constant during the relevant periods but that perhaps the U.S. T-bill market was not operating efficiently, even in the 'weakest' sense. These results are stated very tentatively as there are some theoretical pitfalls in this type of work, some of which are discussed in the text.
Another interesting aspect to Chapter 2 is the testing for the longer run co-movements of inflation and interest rates that is undertaken there. Somewhat different results are obtained depending on the particular filtering technique used. However it is argued that the Lucas (1980) approach is preferable to the simpler Summers (1981) one because of the superior underlying theoretical assumption made concerning the expectations generating process of market participants. Interestingly, whereas Lucas inferred from his analysis a close 1-for-1 relationship between inflation and nominal yields, he never tested for it directly, something which is done in an appendix to Chapter 2. There, evidence is provided that while long run or low frequency co-movements between these variables are indeed close, they are significantly less than 1-for-1, specifically nominal rates do not adjust fully to changes in inflation. This result is confirmed in another appendix through the use of more formal cross spectral techniques which, to my knowledge, have not yet been applied to the variables that are of interest to us. In another appendix, it is shown that with proper ARIMA modelling of the time series of inflation and nominal rates, the theoretical presumption of the unidirectional influence from inflation to interest rates is borne out, in contrast to earlier studies which indicated a reverse 'causality'.
Chapter 3 represents the first attempt to test the predictions of the liquidity model directly, the most important being that the greater the 'moneyness' of an asset the more (less) will its real (nominal) yield be affected by a change in inflation. A set of government securities identical in all respects except for maturity length is used in one analysis whereas in another assets that share a common maturity period but differ in other marketability characteristics (and thus trading costs) are used. Also, a careful discussion of the rationale for the particular rankings of assets given in the chapter with respect to the liquidity services they generate, is provided in the text in the appropriate sections. An important point stressed in these sections concerns the necessity of drawing a distinction between liquidity (defined as the absence of transactions costs) and risk factors in the determination of holding period differentials.

In one section of the chapter the argument is put forward, with some supporting empirical evidence, that the apparently anomalous behavior of equity and bond markets in the U.S. during the 1970s, a subject of intense discussion to the present day, is entirely consistent with the predictions of the liquidity model. In this, as well as in other sections, two alternative proxies for expected inflation using Keynesian and rational expectational assumptions are used in the empirical analyses.

Near the end of the chapter, direct OLS estimation of the parameters in an equation mathematically derived from
the liquidity model is used to test the model's implication that an asset's nominal holding period return will be less influenced by a change in inflation the greater is the asset's degree of substitutability with money, with very favorable results.

Chapter 4 deals with the application of the liquidity model to the capital asset pricing model (CAPM) analysis to explain some seemingly inconsistent results obtained by the various CAPM researchers. For instance, a common finding is that the intercept of the empirical security market line (SML) is greater and the slope term is less than predicted by the two-parameter model. If securities are held 'efficiently' (as most studies indicate is indeed the case), an over-estimated intercept term will lead to an under-estimated slope term and vice-versa, a fact that has been overlooked by writers in this area. The primary argument in Chapter 4 is that the apparently anomalous findings of the CAPM researchers has been due to the exclusion of a liquidity factor in the determination of equity returns. In other words, the measured intercept terms for the empirical SML are the proper ones when account is taken of the fact that equities are less liquid than Treasury bills, the latter yielding a return that is taken to represent the 'risk-free' rate. Therefore, the liquidity premia on equities vis a' vis T-bills are the differences between the SML intercepts and the T-bill rates for the corresponding periods. It is easiest to imagine a security market
hyperplane that includes liquidity being measured in the third dimension as opposed to the standard two parameter security market line that shows only the risk-return trade-off. It is also argued that this liquidity factor is identical to that derived from F-H. To empirically test this proposition, the empirical SML intercepts from CAPM tests utilizing five different market portfolio proxies (most analyses use the equally-weighted NYSE common stock index alone) are computed and from them are subtracted the corresponding risk-free rates. These values are taken to be liquidity premia and are regressed on variables which according to the liquidity model should influence them. This is done in both a contemporaneous analysis and in one in which large period-to-period fluctuations in values (a common feature of CAPM analyses which does not violate any of its assumptions, as explained in the text) are attenuated through the application of a smoothing process. Both analyses yield favorable results. Also, a basic methodological discussion and critique of the very influential Roll (1977) work is offered in the chapter.

Finally, Chapter 5 summarizes and draws together in some detail the work presented in the first four chapters. Special attention is paid to highlighting some of the thesis' most important and unique contributions.
CHAPTER 1:
GENERAL DISCUSSION OF THE FISHER EFFECT
AND THE MODEL

I. The Fisher Effect: Discussion and Empirical Work

This chapter has a number of important aims among which are included a discussion of the presumed theoretical relationship between interest rates and inflation in both the long and short run, the presentation of the empirical evidence of others on the foregoing question and a description of a model incorporating the liquidity characteristics of government securities which attempts to explain a result generally found in tests of the relationship between the two variables which is, as we shall see, rejection of the superneutrality hypothesis, i.e., the belief that inflation will not affect real interest rates in the long run.

In most of the relatively more recent analyses, the superneutrality hypothesis is based on the theoretical model of Sidrauski (1967) which incorporates as one of its more important features a given marginal rate of time preference to which is equated the steady state marginal product of capital. As the rate of time preference will be unaffected by changes in inflation, so will be the marginal product of capital. Given the additional assumptions of perfect long-run foresight on the part of the representative household as
well as complete substitutability between capital and bonds, it follows that the real rate of interest is equal to the marginal product of capital and it too will be independent of the inflation rate in the long run. Furthermore, as standard monetary theory predicts that the steady state inflation rate will be equal to the rate of monetary growth the superneutrality of money in a long-run analysis logically follows.

Some writers have argued that an inverse relationship between changes in expected inflation and real rates in the short-run could possibly be explained by invoking the Mundell-Tobin analysis which is the following: an increase in expected inflation raises the opportunity costs of holding money balances which causes people to shift out of money and into interest bearing assets. This causes the equilibrium expected return on these assets to decline. The subsequent reduction in the "cost of capital" will cause firms engaged in a capital expenditures decision-making process to acquire more capital, thus driving down the economy-wide marginal product of capital. This is most clearly seen within the context of the neo-classical growth model. If, as in most analyses, consumption is positively related to wealth then the induced reduction in desired real money holdings brought about by an increase in inflation will cause consumption at each income level to fall and thus savings to rise. However this whole analysis relies greatly on a fairly substantial real balance effect on consumption.
and savings which has not been shown to exist, at least in the U.S. with its low and very stable savings rates. A potential alternative approach, although perhaps one not without its own problems (see Chapter 3), would be to assume that the rate of time preference is positively related to wealth and that money and capital are complementary goods in production. As real money holdings are reduced as a consequence of the increase in inflation, the marginal product of capital will decline thus reducing the desired capital stock. However, the reduction in capital will stop short of the amount necessary to equate capital's new marginal product to the old level because of the decrease in the rate of time preference.

The following section summarizes most of the more important work done in tests of the Fisher effect over approximately the last decade, focusing some attention on some very recent work which seeks to explain its apparent failure in the U.S. economy that the vast majority of researchers in this area have discovered.

In order to empirically test for the presence of a full Fisher effect, most writers have implicitly modified the standard Fisher relationship given in (1)

\[ R_t = \alpha + \pi_t^e + \epsilon_t \]

(where \( R_t \) and \( \alpha \) are the nominal and real interest rates respectively and the expected inflation rate, \( \pi_t^e \), is assumed to be statistically independent of the error term) to become
where $\beta$ is a parameter measuring the extent of the Fisher effect. An approach used by a number of researchers in the late 1960s and early 1970s was to use survey data of inflationary expectations, specifically the data collected semi-annually by Joseph Livingston, a financial columnist, as a proxy for the expected inflation variable. Although the technique of using survey data seemed to fall into disrepute during the early 1970s (for various reasons some of which are discussed below) there has been a recent resurgence of interest in the use of such data, an interest which is related to the present popularity of rational expectations models. The argument for the use of such data by advocates of rational expectations techniques is essentially that agents use certain decision rules in the formation of their expectations, rules which are in some sense embodied in their reported forecasts but which cannot be observed fully by the macroeconometrician. Therefore they argue that survey data are intrinsically more meaningful than such alternative approaches as using, for example, an arbitrarily selected number of lagged values of inflation with equally arbitrary weights attached to them as an inflation expectations proxy. Although this is a very contentious issue, and beyond the scope of the present work, occasional arguments concerning the advantages as well as the disadvantages of using survey data put forth by various writers will be noted as their work is reviewed.
Gibson (1972) and Pyle (1972) come to the conclusion that not only do observed price expectations of the Livingston variety contribute significantly to the determination of nominal interest rates but, in addition, that the coefficient on expected inflation is substantially below unity (roughly .65). However Lahiri (1976) attributes the finding of a low coefficient on the price expectations variable to an "errors in variables" problem, a possibility alluded to by both of the above authors. However, even after applying his corrective techniques among which are included a two-stage least squares estimation of the basic interest rate equation from a reduced form model of interest rate and price level expectations formation, he finds that the coefficient on expected inflation is still significantly below unity, although a bit higher than that found by Gibson and Pyle.

Cargill (1976) and Carlson (1975) point out that Livingston makes adjustments to his survey data before publishing them and Carlson notes that differences between the published forecasts and the actual arithmetic average of the respondents' estimates vary markedly over time. Cargill recomputes the Fisher equation using the average of the respondents' forecasts of inflation for each time period and concludes that the evidence in favor of a close relationship between nominal rates and expected inflation is fairly weak.

In an interesting offshoot of the basic approach of using the Livingston survey data as a proxy for expected inflation, Bomberger and Frazer (1981) use the standard
deviations of the individual period forecasts in a reduced form interest rate equation as a proxy for the degree of inflation uncertainty with the evidence indicating a significantly negative influence of this uncertainty variable. This result is in accordance with the prediction of the model of Blejer and Eden (1979) and could help explain why, in the context of their work, so many authors have found an underadjustment of nominal rates to inflation. The basic argument is as follows: Suppose the proper specification of the Fisher equation is

\[ i = b_0 + b_1 e + b_2 \text{var } \pi \]

where the last value is the expected variance of inflation. In other words, uncertainty is explicitly introduced. That estimate of the parameter on expected inflation is thus \( \hat{b}_1 = b_1 + b_2 c_1 \) where \( c_1 \) is the coefficient on expected inflation when the actual variance of inflation is regressed on it. If the assumption is made that inflation's variance increases directly with its level (evidence that this has indeed been the case during our sample period will be presented below) and if \( b_2 < 0 \), then \( b_1 \) will be downward biased. A similar argument is offered by Kochin (1982). However there is reason to believe that this particular potential explanation may not be appropriate for our period of interest.³ Tests involving the use of alternative proxies for the expected rate of inflation fall into basically two groups, those using some distributed lag of past inflation rates as the proxy and varying proxies based on rational expectational assumptions. The first
approach pre-dates the survey data one compared to which it is usually regarded as inferior given the necessary arbitrariness involved in the selection of the lag length to be used. The most well known work in this area was done by Fisher himself who was unable to confirm the existence of any Fisher effect instead finding that nominal interest rates changed by less than .25% for every one percentage point change in expected inflation.

A very ingenious approach to the study of a key feature implicit in most analyses of the Fisher effect, the constancy of the ex ante real rate of interest (the expected real return from holding an instrument) was undertaken by Mishkin (1981). Essentially his technique involves using the determinable ex post real rates (the nominal interest rates minus actual inflation) to infer behavior about the unobservable ex ante real interest rate. Mishkin illustrates that this is a perfectly acceptable technique under certain assumptions implicit in the rational expectations approach. As it represents probably the most sophisticated rational expectations test (albeit indirect) of the basic validity of the Fisher effect, we will adopt its basic approach, with a different data set, in Chapter 2 when our own tests of the Fisher effect are undertaken. More relevant to our current concern, however, is that Mishkin's results not only tend to confirm the underadjustment hypothesis but also provide evidence for the dominant influence of inflationary expectations on nominal rates, i.e., the lack of empirical support for the belief
that those variables other than inflation which are
generally thought to influence real rates actually do.

Nevertheless, as Fried and Howitt (1983) point out, a
failure to find any evidence for a full Fisher effect using
short-run data does not preclude the validity of any long-
addresses this very issue by utilizing spectral analysis to
filter out short-term co-movements of interest rates and
prices using varying filter lengths and concludes that only
during the post-WWII era does there seem to be any
significant relationship at all between the two variables.
Again, the finding of a less than unit coefficient (for the
1948-79 period) of roughly .6 is the result. Lucas (1980)
uses a two-sided filter to compare the long swings in
interest rates and money growth (not inflation) and finds,
very significantly, a coefficient sufficiently close to
unity. Both of the above works will be dealt with in far
greater detail later when each of the particular filtering
techniques will be used, and comparisons drawn between them,
in a chapter that deals with the present work’s analysis of
the inflation-interest rate relationship.

II. Taxes and the Fisher Effect

A very important related aspect that must be considered
in any analysis of the inflation-interest rate relationship
using modern data is that of taxation of nominal returns.
The implication of incorporating taxes into the analysis is,
of course, that a coefficient of substantially greater than unity would be required for a full Fisher effect to be operative, i.e., for there to be a constant after-tax real interest rate. The basic work on tax-adjusted real returns was done by Darby (1975) who assumed a simple economy-wide marginal tax rate. Gandolfi (1976) extends somewhat the basic Darby analysis by developing a simple model of saving and capital accumulation and illustrates that given the assumption of a relatively stable equilibrium real rate the full Fisher effect will operate only under certain conditions. In his model, full adjustment occurs, in the presence of a positive marginal tax rate, only if savings are completely unresponsive to changes in the real return on savings or if the real interest rate elasticity of investment demand approaches infinity.

Feldstein (1976) further develops the analysis by allowing for the existence of two separate tax rates, one for corporate and one for personal income. His model is further differentiated from Gandolfi's by the feature that even in the absence of any taxes at all in the economy, the full Fisher effect will operate only under the condition of zero interest rate elasticity of the demand for real money balances. Levi and Makin (1978) argue that the usual procedure of considering either the Fisher or Darby hypothesis in isolation is inappropriate and can lead to the incorrect inference that the finding of a unit coefficient on expected inflation implies little or no impact of inflation on real after-tax yields. In their small general
equilibrium model in which the Darby form of the Fisher equation is one reduced form relationship they show that increases in expected inflation will unambiguously decrease after-tax real yields, a result which is consistent with full, under or overadjustment of nominal rates to inflation.

To finish the foregoing literature review on the tax-adjusted Fisher effect (or Darby effect), it should also be noted that Summers (1981) reminds the readers of the difficulties associated with selecting the appropriate marginal tax rate to be used in any empirical analysis of the phenomenon. Partly for this reason and partly for the fact that we are more concerned in the present work with testing for a general underadjustment of nominal rates to inflation than we are with its precise numerical measurement, tax effects will be ignored in Chapter 2 which deals with our own analysis of the inflation-interest rate relationship. Of course, if the underadjustment hypothesis is confirmed with a model which deletes taxes it would be more strongly so in one which incorporates tax effects. For example if we assume, as did Feldstein and Summers (1979) and Summers (1981), a marginal tax rate faced by the holders of interest bearing corporate assets of 33% then it follows that nominal rates would have to rise 1.5 percentage points for every percentage point increase in inflation to maintain a constant real after tax interest rate \( \frac{dr}{d\pi} = \frac{1}{1-t} \) where \( t \) is the appropriate marginal tax rate. However the above analysis would be correct only in the case where there was
little dispersion in the marginal tax rates faced by recipients of interest income. Although the above authors take account of the generally higher marginal rates faced by corporations as opposed to individuals, it is necessary to remember that under the fairly progressive federal income tax system in the U.S., effective marginal tax rates vary considerably among individuals themselves. Thus if there has been some shift in ownership of interest bearing assets among the various income groups in the economy, it would be difficult to incorporate a single marginal tax rate that would provide the required degree of adjustment of nominal rates to inflation (to maintain constant real after tax rates), especially in a longer term analysis. Of course, changes in the tax rates themselves (e.g. the 1981 Reagan tax cuts) would, ceteris paribus, have some influence on the required degree of adjustment.

To further illustrate the complicating factor that the tax system itself is not neutral to inflation, consider the following: suppose an individual were to purchase with borrowed funds a piece of capital, or a claim to capital, for investment purposes in an economy with a given real rate and one in which nominal interest rates adjusted freely to maintain real after tax interest paid at some constant level in the face of inflation. This individual would still be required to pay tax on the increase in the nominal value of his capital brought about by inflation and thus would not be satisfied with an arrangement that would maintain a constant after tax real rate to be received by him. On the other
hand, were he to devote the borrowed funds to obtaining 'psychic' income (e.g. a vacation) that would not be subject to capital gains taxation, such an arrangement would probably be acceptable to him.

A further difficulty arises when allowance is made for the taxation of different forms of income. If, for example, the taxation of equity income is added to the analysis (e.g. as in Summers (1981), Hendershott (1981) and Feldstein and Summers (1979)) it is necessary to modify the simple Darby analysis by replacing unity in the numerator with unity minus the appropriate equity income marginal tax rate. This is true because inflation increases equity values which, ceteris paribus, drives down the required total amount of adjustment. Nevertheless, using plausible values for the respective marginal tax rates, all of the models developed by the authors above predict a more than full adjustment of nominal rates to inflation to maintain constant after tax real rates, with the range of required adjustment across the various models being about 1.15 to 1.30. Although these values are closer to unity than they would be without the inclusion of equity income taxation, they are still far higher than those observed thus indicating that non-neutralities in the tax system cannot be used to account for the apparent underadjustment of nominal rates to inflation over our period.

To summarize the previous sections, most authors have failed to find evidence for the operation of a full Fisher
effect at work using short- or long-run analysis during the post-WWII period in the United States instead finding that nominal rates tend to respond to inflation with a coefficient substantially less than unity. Furthermore, there appears to be no other variable which has been consistently associated with interest rates and inflation in the long run which could possibly account for this underadjustment, at least as far as majority of studies that have considered the effects of alternative potential explanatory variables are concerned. In addition, the inclusion of tax effects in the basic model serves only to make the underadjustment more pronounced. Keeping all of this in mind, we now turn to the next section which attempts to explain this observed phenomenon of a less than unitary relationship within the context of traditional theory's neglect of the differing liquidity services yielded by the various assets that comprise the entire spectrum of financial instruments.

III. The Basic Model and the Nature of Liquidity

The following section contains a somewhat detailed look at a model which seeks to account for the apparent failure of the superneutrality hypothesis to hold in the United States economy, a model developed by Fried and Howitt (1982) hereafter referred to as F-H. What distinguishes this model from others that attempt to explain the empirical absence of the Fisher effect is that "misspecification" in the standard Fisher relationship is attributed to the neglect of a
principle already well established in traditional theory, specifically the friction associated with transactions costs. Standard monetary theory maintains that agents hold money balances as a consequence of their yielding a return in the form of transactions costs that are saved by having them, i.e., a liquidity yield. Indeed, in a perfectly frictionless economy with interest-bearing assets no money would be held. However it is inappropriate in either theory or empirical work to dichotomize the set of all financial assets into one group which includes just the asset which yields only non-pecuniary liquidity services (money) and the other group comprising all others. A much more meaningful approach would involve taking into consideration the varying liquidity yields of the entire spectrum of financial assets. The model to be described below illustrates that with a slight modification of Sidrauski's basic model, a modification consisting merely of the explicit inclusion of the liquidity yields on bonds, the theoretical implication is that the long-run real pecuniary rate of interest will be reduced as a consequence of an increase in the steady state rate of inflation.

At this point it would perhaps be useful to elucidate the terms liquidity and transactions costs. When a particular asset is described as being more liquid than some other asset it is meant that the former instrument can be converted to money (which possesses ultimate liquidity) with lower transactions costs than can the latter instrument.
Although a more detailed discussion of transactions costs must await presentation until Chapter 3 which attempts to empirically test the model's proposition concerning the differential impact of inflation on the yields of assets that possess varying liquidity services, within the context of the type of security analysis that we use in the present study we can think of transactions costs as the amount accruing to a market participant who provides to other investors the opportunity to complete transactions immediately. Oftentimes in empirical work these transactions costs are taken to be approximated by the bid-ask spreads of the various securities. Although, as stated above, a more detailed analysis will be provided in another chapter, we can at this point state that the transactions costs faced by investors dealing in different asset markets will be functionally related to a number of variables including the time rate of transactions in the particular issue and the "thinness" of the market for the issue. Perhaps less important would be the influence of variables such as the absolute size of transactions as well as the amount of insider trading going on in the market. Additionally, it is necessary to keep in mind that there are indeed pure transactions costs that can be clearly distinguished from risk considerations. Costs arising from the physical storing of securities, the writing up in contractual form of the exchange agreements, the enforcement of these contracts, the bookkeeping and administrative services provided, etc. are all unambiguously pure
transactions costs. However it has been argued that price differentials on various instruments faced by a purchaser of, say, securities differing by maturity period alone (e.g. government bonds) only appear to represent differences in transactions costs from the purchaser's perspective but actually arise from risk factors. The argument is that the holders (i.e. dealers) of an inventory of securities will charge purchasers a higher premium for the longer term instruments than for the shorter term ones, ceteris paribus, as the former assets are far more price sensitive thus exposing the dealer to greater capital risk. However from the perspective of the representative purchaser the higher price paid on the longer term asset represents an additional transactions cost even though it ultimately arose from risk considerations. This argument, however, is wrong for several reasons. Not only does it make a very strong implicit assumption about the uncompetitiveness of bond markets (implying that price is exclusively cost-determined) but it ignores the basic fact that dealers do not hold large inventories of the same securities for long periods of time. Although a representative dealer may trade a substantial volume of bonds in the course of a day or week, his net position in bond inventories will typically be very small relative to volume traded. He makes his profits in the 'round-trip' transactions (buying and selling instruments quickly) with these per trade revenues being reflected by the bid-ask spreads. A risk averse dealer does not attempt
to make profits by purchasing large inventories of bonds and then waiting for a price appreciation. An even more fundamental reason for rejecting the particular argument offered above is the following: if a dealer is interested in hedging away the capital risk associated with the holding of an inventory of relatively long term securities why doesn’t he simply arrange to hold a portfolio which he regards as being "optimal" in some sense at each and every moment in time? Clearly the answer must be that it is costly to do so. The dynamics of security markets are such that the average rate at which the purchase orders arrive at the dealer’s seldom match the average rate of arrival of sales order. Thus, any attempt on the part of some representative dealer to hold an optimal portfolio so as to protect himself perfectly from adverse price fluctuations would be very costly, if not impossible, to achieve. Indeed, if the opposite were true i.e. if the purchase orders and sales orders of equal size arrived simultaneously at all times, the bid-ask spreads would be driven to near zero. This would be true, of course, because of the greatly diminished value attached to the liquidity services provided by the dealer in such a world.

The foregoing argument strongly implies that transactions costs will be closely related to the time rate of transactions in the various markets as well as to their thinness. Indeed, this latter factor will serve as supporting evidence for the ranking of the various financial
assets, with respect to their liquidity, in empirical work presented in Chapter 3.

The logical necessity of recognizing the liquidity services yielded by other financial assets besides money has been noted only infrequently in the literature. Fischer (1974) makes an allusion to the question and Patinkin (1965) within the framework of a two-period Fisherian consumption model with interest-bearing bonds notes that if the bonds yield liquidity services (and thus utility) then as the individual sought to solve his inter temporal consumption problem by sliding along the consumption frontier to reach the highest attainable indifference curve, thus changing the amount of bonds held, the entire indifference map would move as well. Nevertheless, aside from these two references, the whole question has been pretty well ignored in the literature.

The fundamental intuitive explanation of the model is provided below;

"Suppose an increase in inflation leaves unaffected the marginal product of capital, because of an invariant rate of time preference. Because it increases the opportunity costs of holding money it will reduce the steady-state demand for real balances. Thus the price level will rise relative to the money supply. The rise in the price level will reduce the values of any outstanding bonds. At the same time, the increase in the opportunity cost of holding money will induce households to substitute bond holdings for money. At the previous equilibrium real rate of interest there will now be an excess demand to hold real bonds, and the rate will be bid down. In the new equilibrium the unchanged marginal product of capital will equal the sum of the real pecuniary yield on bonds, which has fallen, plus the marginal liquidity yield on bonds, which has risen
by an exactly offsetting amount because of the reduction in the outstanding real stocks of financial assets.

The remainder of this chapter will be devoted to an elaboration of the argument above within the context of the F-H liquidity model. Special attention will be paid to the precise nature of the underadjustment of nominal rates to inflation which is a key element of the model. The notation used here is just slightly different from that used by F-H (1983).

Firstly, the model is developed within the framework of a stationary or steady state equilibrium. There are three decision-making units in the economy; the government, households and firms. The government supplies high-powered money, issues bonds on which it pays interest and makes transfer payments to households of a lump sum variety. Governments \( (g) \) and money \( (m) \) grow at the same steady state rate, \( \mu \).

A balanced budget is also assumed with the consequence that the revenues of the government, \( \mu(m + g) \) are equal to its lump sum transfers plus the interest service on outstanding bonds, \( (tr + r_g g) \). The government chooses the growth rate for both assets \( (\mu) \) and thus the proportion of governments to money \( (\gamma) \) which is, of course, a constant.

On the production side, the factors of production are capital \( (K) \) and labor \( (L) \) which are combined to produce output in a constant returns production function \( F(K,L) \). The labor force is comprised of the entire population which
is assumed to be constant. Output per man is denoted as \( y \) where \( y = f\left(\frac{K}{L}\right) = f(k) \). The real rate of interest, \( r - \pi \), is the rental rate on capital (where \( \pi \) is the constant rate of inflation). For profit maximization the necessary condition is that the marginal product of capital \( f'(k) \) equals the real rate of interest, \( r - \pi \). Also, because the production function is homogeneous of degree 1, the Euler condition that the sum of each factor's marginal product times the stock of the particular factor is equal to total output, obtains in this case, i.e.,

\[
y' \cdot L = w' \cdot L + f'(k) \cdot K
\]

The real wage rate, \( w \), is equal, of course, to the marginal product of labor. To further simplify the analysis, \( L \) is set equal to 1.

As the model now stands it is quite standard however, its more unique aspects become apparent in its description of the conditions under which households make decisions. The representative household buys goods at the price \( P \) but incurs certain transactions costs in doing so. The additional feature is added that the household faces costs associated with the storage of goods and bonds. The consequence is that not all of the household's purchases are translated into final consumption. The two types of costs are grouped together under the more general name of trading costs with these trading costs being a positive function of consumer expenditures (\( x \)) as well as real money and bond holdings. Thus, available consumption is equal to expenditures minus trading costs, \( h \), as in the expression below.
\[c = x - h(x; m, g) = l(x; m, g)\]

The partial derivatives \(l_m\) and \(l_g\) represent respectively the marginal liquidity yields of money and Bonds, i.e., the trading costs that are avoided (measured in terms of consumption made available) by holding the marginal units of these assets.

Additional assumptions are invoked including the strict concavity of the liquidity function and the requirement that expenditures are greater than or equal to available consumption which rules out the possibility of negative trading costs. An additional feature, very important for the analysis later, is that these marginal liquidity yields are higher the larger are expenditures.

Thus, the F-H modifications of the basic Sidrauski model is in the inclusion of a trading function similar to that developed by Feige and Parkin (1971). The seminal feature of the Feige-Parkin approach is that money is treated merely as a means to facilitate transactions in the bond and commodity markets as the holding of it allows the representative agent to economize on resources which would otherwise be devoted to exchange. An additional and related aspect of the Feige-Parkin approach is that it allows for costly exchanges between bonds and commodities as well as between money and commodities. The implication of this is that the optimal quantity of money to be held by the representative agent is different from what it would be in
the more traditional Baumol (1952) or Tobin (1958) approaches. For a detailed discussion of the precise nature of the trading function the reader is referred to either Feige and Parkin or F-H (p. 971).

Some additional features of the liquidity model are worth noting; there are assumed to exist claims to capital which yield a nominal rate, $r$, but no return in the form of liquidity services. Utility is a function of consumption alone and marginal utility is positive but strictly decreasing.

Recalling that the marginal rate of time preference is assumed constant and denoting the representative household's total assets by $a$, the following state transition equation obtains.

\[
\dot{a} = w + tr + r(a-m-g) + r_g \cdot g - \pi a - x
\]

In the above expression, $tr$ equals per household transfers and $m$ and $g$ are the real holdings of money and governments respectively.

The representative household seeks to maximize the intertemporal utility function

\[
\int_0^\infty e^{-\delta t} u(l(x;m,g)) \, dt
\]

subject to the constraints imposed by the state transition equation (2). It does so by selecting the optimal time paths for $x$, $m$, $g$ and $a$. The necessary conditions for utility maximization are derived below.
Thus, $\frac{lm}{lx} = r$ and $\frac{lg}{lx} = r - rg$. Selecting the optimal time path of assets is a bit more complex and involves essentially an optimal control problem. First, it is necessary to form a Hamiltonian and impose the condition that the marginal utility of assets ($H_a$) be equal to the marginal utility of consumption.

\[
(3) \quad (d) \quad H = u(\ell(x;m,g))e^{-\delta t} \cdot dt + \left[\dot{\alpha} - (w + tr + r(a-m-g) + rg - \pi a-x)\right]
\]

\[
(3) \quad (e) \quad H_a = -\delta + (r-\pi) = \delta(r-\pi)
\]

but $H_a = \frac{dH}{dt}$ and $H_a = \lambda$, $\frac{dH}{dt} = \dot{\lambda}$

where $\lambda$ tells us the amount by which the objective function is increased with an increase in the constraint, therefore

\[
(3) \quad (f) \quad \lambda = u'1_x \text{ and } \dot{\lambda} = \frac{d}{dt}(u'1_x)
\]

Thus we are left with the following expression:

\[
(4) \quad \frac{d}{dt} [u'(\ell(x;m,g))1_x(x;m,g)] = \delta - (r - \pi)
\]

In a stationary long-run equilibrium, the equilibrium values of $x, a, m,$ and $g$ will be constant and the following equilibrium conditions will be satisfied. First, that the rate of inflation equals the rate of monetary growth i.e.,
\[ \pi = \mu. \] This is a standard result in long-run monetary models and is best visualized by a demand curve for money balances drawn in real money-inflation space, which is invariant due to the fact that expected inflation equals the current rate, and its intersection with a supply function which is stable because of the fixity of real money balances. The result is a stable equilibrium value for inflation, actual and expected, equal to the rate of monetary growth. An additional condition is that total assets are comprised of money, capital and governments \((a=m+k+g)\). The optimal value of \(k\) is chosen to satisfy the profit maximization condition and its marginal product is equal to the fixed rate of time preference, \(f'(k) = \delta\) which is equal to the steady state real rate of interest. Also, expenditures \((x)\) are equal to the output produced by the given capital stock \((k)\) which is equal to income \((y)\).

The equilibrium conditions for holding money balances and governments respectively are thus;

\[ \begin{align*}
1_m(y; m, g) &= (\delta + \mu) l_x(y; m, g) \\
1_g(y; m, g) &= (\delta + \mu - r_g) l_x(y; m, g)
\end{align*} \]

or

\[ \frac{1_m}{l_x} = \delta + \mu \quad \frac{1_g}{l_x} = \delta + \mu - r_g \]

In addition, it should be noted that as the optimal growth rate of both money and governments are selected, so is the ratio of the two assets, \(Y\).
The F-H analysis can now be developed further to illustrate that, with the use of some very reasonable assumptions, the underadjustment of nominal rates to an increase in the steady state rate of inflation is an implication of the model. First, for notational convenience, define $l_m/l_x$ as being equal to $s_m$ and $l_g/l_x$ as $s_g$. Taking the derivative of $s_m$ with respect to $g$ (using the quotient rule) we have the following:

$$(6) \quad s_{mg} = l_m/l_x - (l_m/l_x)^2 l_{xg}$$

If money and bonds are substitutes in producing liquidity ($l_{mg} < 0$), and if the liquidity function is positive as well as strictly concave in both $m$ and $g$, then $s_{mg} < 0$ (of course, this can be generalized to show that $s_{gm} < 0$ as well). Equation 6 ensures also that $s_{mm}$ and $s_{gg}$ are negative as well, indeed even more strongly so in the usual case of less than perfect substitution between the two assets in the sense of producing liquidity.

To illustrate the underadjustment result, it is necessary to rewrite the equilibrium condition with respect to money and bond holdings and take total differentials.

$$(7) \quad l_m - (\delta + \mu) l_x (\tilde{y}; m, \gamma m) = 0$$

$$(8) \quad l_g - (\delta + \mu - r_g) l_x (\tilde{y}; m, \gamma m) = 0$$
\[
(9) \quad l_{mm} \cdot \frac{\partial m}{\partial \mu} + l_{mg} \gamma \frac{\partial m}{\partial \gamma} - l_{x} \frac{\partial \mu}{\partial x} = 0
\]
\[
-(\delta + \mu) l_{xm} \cdot \frac{\partial m}{\partial \mu} = (\delta + \mu) l_{xg} \frac{\partial m}{\partial \gamma} = 0
\]
\[
(10) \quad l_{gm} \cdot \frac{\partial m}{\partial \mu} + l_{gg} \gamma \frac{\partial m}{\partial \gamma} - l_{x} \frac{\partial \mu}{\partial x} + l_{x} \frac{\partial r}{\partial g} - (\delta + \mu - r) l_{xm} \frac{\partial m}{\partial \mu} - (\delta + \mu - r) l_{xg} \gamma \frac{\partial m}{\partial \gamma} = 0
\]

Using Cramer's rule

\[
(11) \quad \begin{bmatrix}
    l_{mm} + l_{mg} \gamma - (\delta + \mu)(l_{xm} + l_{xg} \gamma) & 0 \\
    l_{gm} + l_{gg} \gamma - (\delta + \mu - r)(l_{xm} + l_{xg} \gamma) & l_{x}
\end{bmatrix} \begin{bmatrix}
    \frac{\partial m}{\partial \mu} \\
    \frac{\partial r}{\partial g}
\end{bmatrix} = \begin{bmatrix}
    l_{x} \\
    l_{x}
\end{bmatrix}
\]

\[
(12) \quad \frac{\partial r}{\partial \mu} = 1 - \frac{s_{gm} + \gamma s_{gg}}{s_{mm} + \gamma s_{mg}}
\]

It follows from expression (12) that a given change in inflation will have less than a unitary effect on the nominal pecuniary returns on government securities. It is also obvious that the underadjustment result would still occur if the two assets were not substitutes in providing liquidity services or even if there were a small degree of complementarity between them.

A few more features of the model are worth noting, in particular the signed values of some of the partial derivatives. It follows from the analysis as developed so far that \( \partial m / \partial \mu < 0 \) and \( \partial r / \partial \gamma > 0 \). Even more importantly, the impact of an incremental change in the governments to
money ratio on the liquidity premium is a negative one. Using the equilibrium condition for the holdings of governments, it follows that

\[(13) \frac{\partial (r-r_g)}{\partial \gamma} = \frac{1}{l_1} \frac{\partial l_g}{\partial (\gamma_m)\gamma} - \frac{1}{l_2} \frac{\partial^2 l_1}{\partial \gamma m} < 0\]

IV. The Role of the Government in the Liquidity Model

It should also be noted that it is very important to recognize the restricted range of options open to policy makers in this model. The only fiscal actions potentially available to them would be to change the lump sum taxes or to adjust the governments to money ratio either by a one time increase in the ratio brought about perhaps by a helicopter drop of bonds or, more likely, by periodic adjustments of the ratio in order to take account of changes in the inflation rate. As concerns the latter, F-H (1983) demonstrate (page 973) that it is possible that the government might seek to equate the private gains from government bonds to the constant marginal social costs of maintaining some given stock of bonds and might attempt to do so by following some growth rule for the bond/money ratio based on changes in the inflation rate. In the example of such a rule that they provide, the full adjustment of nominal rates to inflation follows logically as a consequence. However, as they point out, it is necessary to remember that strict adherence to such a rule would imply that policy makers were unconcerned with maintaining socially optimal amounts of real money balances. This
behavior would indeed be "myopic in the extreme" (page 973). Therefore we must content ourselves in this model with a government that merely sets an initial growth rate for both money and bonds, imposes lump sum taxes and makes income transfers of a lump sum variety. However, even these assumptions may not be too restrictive with respect to the results obtained. In one section of the paper they clearly illustrate that in a world without government bonds but with privately issued claims to capital (issued by financial intermediaries whose deposits represent liabilities) the presumption of underadjustment still obtains (pp. 974-5).

The foregoing sections contained many of the model's more important results. Additional ones will be presented, and a more detailed economic interpretation will be provided, in the chapters below that deal with the empirical analysis of the model's implications.
1. See Dornbusch & Frankel (1975) for a useful diagrammatical representation of the Sidrauski model.


3. It is necessary to note, however, that the downward biasedness of the measured bl coefficient relies not only on a positive association between inflation's level and its variance (a positive value of c1) but also on the finding of a negative value for b2, the coefficient on inflation's variance when this variable is added to the standard Fisher equation. However using yearly means of interest rates and inflation and standard deviations within each year over the relevant sample period, OLS estimation indicated that the computed b2 value was not significantly different from zero (see below). This result was confirmed with the use of 6 month testing periods for the 1953-1979 period and also with 6 and 12 month testing periods for the 1953-1983 era. Therefore this particular potential explanation for the apparent underadjustment of nominal rates to inflation in the short-run over the relevant sample period cannot be invoked, at least as far as our sample period is concerned.

1953-1979 1 year testing period

\[
\text{Avg. YMLM} = 7.03 + .64 \text{Avg.}^\pi - .56 \text{Var}^\pi \\
(2.97) \quad (12.55) \quad (-1.01)
\]

D-W= 1.62  \quad R^2 = .88

t-statistics in parentheses

APPENDIX 1

An Alternative Model of Transactions Costs and Underadjustment

In the model of F-H (1983) a representative household seeks to hold optimal inventories of bonds, money and goods with the constraints being that there are lumpy transactions costs associated with the storage of goods (e.g. loss due to spoilage) as well as implicit opportunity costs incurred by holding money balances and bonds. Physical storage costs of bonds and money are excluded from the analysis. These optimal values are provided in equation (12) of their paper (p. 971). Combining (13) with (12) gives rise to (14) which incorporates all of the above-mentioned values in a form which expresses them as determinants of 'available' consumption, in other words expenditures minus total trading costs with this latter value being the sum of transactions and storage costs. Given the additional assumptions that the household receives all of its payments at the beginning of each period, possesses perfect foresight and spends all of its income in a steady stream, it is a simple matter to put into graphical form the time paths of the real holdings of bonds and money as well as commodities. An example with \( n_c = 8 \) and \( n_g = 2 \) is provided in Figure 1 below. The reader should note that the symbol \( g \) represents real governments and \( c \) stands for real consumption units in the present work while \( g \) represents goods and \( b \) bonds in the F-H analysis.
If their equation (13) is modified to a complete cost function by including the opportunity costs of holding bonds (actually negative opportunity costs) as in (1) below, and when this equation is combined with (12) in F-H, some interesting Baumol-like results are obtained.

\[(1) \quad \alpha_c n_c + \alpha_g n_g + \gamma_c - r_g g\]

\[(2) \quad n_c = (\gamma_c X / 2\alpha_c)^{1/2} \quad n_g = (r_g X / 2\alpha_g)^{1/2}\]

\[\bar{m} = (X/2)^{1/2}[(\alpha_g / r_g)^{1/2} - (\alpha_c / \gamma_c)^{1/2}]\]

\[\bar{c} = (X\alpha_c / 2\gamma_c)^{1/2} \quad \bar{b} = (X/2)^{1/2} - (X\alpha_g / 2\gamma_g)^{1/2}\]

A feature of the F-H analysis is the assumption that the extent to which nominal yields will be affected by a change in inflation is entirely dependent on the relative liquidity yields of bonds and money as well as on the proportions in which they are issued exogenously by the government. This holds true also for the influence of a change in the bonds/money (governments/money) ratio on desired real money balances and on nominal bond yields.

An alternative model developed by Orr (1971) incorporates 3 financial assets and is based on a representative household's demand for money in a world in which there are savings in 'shorts' some of which are periodically transferred to 'longs'. The F-H analysis has
actually only two purely financial assets with a third one being physical capital although this latter one can be represented by illiquid claims to capital.

A slightly modified and perhaps simpler alternative to the Orr analysis which has the advantage of explicitly introducing both money/bond ratios and relative transactions costs as determinants of yield differentials is analyzed below. The following differs from F-H in the explicit inclusion in the final equation of not just one but two transactions cost variables (the impact of a single variable transactions cost on yield differentials is only indirectly inferred from their analysis).

It is possible to make the assumption that the pay periods are very short relative to the periods of time between bill purchases. This approach has the advantage in allowing us to circumvent the problem of having to explain the absence of bill to money transactions that would most likely be undertaken by individuals seeking to economize on cash balances in a world where the pay periods were fairly lengthy relative to the periods between bill purchases. Where pay and consumption periods are very short, it is not likely that individuals will actively buy or sell shorts to economize on cash holdings. It should be noted that by regarding the pay period as not only being fairly short but completely exogenous an interesting economic argument concerning the selection of optimal pay periods is ignored. In other words, it seems likely that profit-maximizing
employers and utility-maximizing workers would select some pay period which in some sense would minimize mutually foregone interest. The actual outcome might depend on the relative power of the two groups in a bilateral monopoly context. The main point is that in a more thorough analysis the optimal pay period would tend to vary with changes in interest rates, perhaps even with the endogenously determined short rate.

We will also assume that longer term, relatively illiquid assets (longs) exist and are held by the representative individual. For ease of analysis we will confine our discussion to a period covering the decumulation of longs and their conversion into shorts although it is important to point out that the periodic accumulation of longs is by no means inconsistent with the present analysis.

To begin, the representative individual withdraws $D from longs (illiquid asset sales) $C of which is held as cash to finance near term transactions with the rest being converted into shorts or bills ($B). Average holdings of money ($M) and bills ($B) are defined respectively as

\[
(3) \quad M = \frac{C}{2} \quad \quad B = \frac{D}{2} - M
\]

Assuming a cost of converting bills into money (a) similar to that in the Baumol (1952) analysis as well as a longs to shorts transactions cost (b) and bearing in mind the implicit opportunity costs of holding both bills and money, the total cost function is represented by
Minimizing the cost function leads to the following equations for determining optimal cash withdrawals and illiquid asset sales.

(5) \[ C = \left(\frac{2aT}{r_s}\right)^{1/2} \quad D = \left(\frac{2bT}{r_1}\right)^{1/2} \]

It is now possible to rewrite (3) as

(6) \[ M = \left(\frac{aT}{2r_s}\right)^{1/2} \quad B = \left(\frac{bT}{2} - \frac{r_1 - r_s}{2}\right)^{1/2} - \left(\frac{aT}{2r_s}\right)^{1/2} \]

As per our earlier assumptions, the aggregate level of bills and money are exogenous and we will also assume that the long rate and real transactions (\(T/P\)) are given. The former assumption can be justified by invoking the notion that the long rate is equal to the sum of the steady state marginal product and the steady state inflation rate while the latter can be rationalized by assuming constant or very slowly changing patterns of consumption. The variables are thus the price level, \(P\), and the short rate \(r_s\). The new conditions are thus

(7) \[ M = P \left(\frac{aT}{2r_s}\right)^{1/2} \quad B = P \left(\frac{bT}{2} - \frac{r_1 - r_s}{2}\right)^{1/2} - M \]

(8) \[(a) \quad (B + M)^2 = p^2 \frac{bt}{2(r_1 - r_s)} \]

(b) \[ M^2 = p^2 \left(\frac{at}{2r_s}\right) \]

(c) \[ (B + M)^2 = M^2 \left(\frac{2r_s}{at}\right) + \frac{bt}{2(r_1 - r_s)} \]
Rewriting and solving for the interest rates

\[ \frac{r_s}{(r_1 - r_s)} = \left( \frac{B + M}{M} \right)^2 \cdot \frac{a}{b} \]

or

\[ \frac{r_1}{r_s} = \left( \frac{M}{B + M} \right)^2 \frac{b}{a} + 1 \]

Two important features of the model can be seen in (9). If the steady state inflation rate is equal to the rate of monetary growth (a standard feature of many models in monetary theory as discussed in the text of Chapter 2) and if the inflation rate is raised by increasing the time path of money, ceteris paribus, the real bill rate must fall, i.e. there is a less than full Fisher effect. Also, if the combined transactions cost of simultaneously selling longs and buying bills (b) rises, then, ceteris paribus, the divergence between the yields on these assets rises. This is entirely consistent with the F-H analysis discussed above although the approach used here is somewhat different from theirs. In their model, the only financial assets are money and governments, the relatively liquid and illiquid assets respectively. With only two financial assets, there is only one relevant transactions cost crucial in influencing yield differentials (a non pecuniary yield for money), that associated with exchanges between money and governments. With the introduction of an additional interest-bearing asset in the present analysis an additional transactions cost is created, the cost of long to short transactions. This cost is the relevant one in helping to determine long-short yield differentials. Another result from the present
model which, like the less than full Fisher effect discussed above, coincides with a comparative statics result reached by F-H is that of the negative relationship between increases in the bonds/money ratio and the liquidity premium. Looking at (9), suppose that the government were to increase the amount of bonds either by a one time injection or by raising only the growth rate of bonds. With the long rate tied down, the effect would be to drive up the short rate (decrease the liquidity premium). The effect would probably be even stronger than indicated by (9) if account is taken of the fact that bonds and money are substitutes (albeit imperfect) in producing liquidity services and that the increase in bonds would tend to drive down desired money balances. The most important point to be made in this section is that many of the same results obtained by F-H are obtainable in a model which includes a far more explicit role for transactions costs and, furthermore, does not rely on as many specialized assumptions as does theirs such as the strict concavity of the liquidity functions.

FOOTNOTE

1 The failure of the Fisher effect in this model works in either direction, i.e. the response of nominal rates to decreases in inflation is not complete. This feature of the model may help explain the recent high real interest rates in the face of decreased inflation as experienced in the American economy. This point is discussed in more detail in the next chapter.
CHAPTER 2
THE FISHER EFFECT IN THE SHORT AND LONG RUN

I. General Discussion and Methodology

The purpose of the following chapter is to closely scrutinize the historical record of the relationship between interest rates and inflation during the 1953-1979 period in the United States as the underadjustment of nominal rates to inflation is a key feature of the Fried-Howitt liquidity model. Misspecification in the form of omitted variables in a Fisher type equation is usually cited as the reason for the apparent underadjustment found in the vast majority of the studies in this area. The approach employed in this chapter will be first to delve more deeply into the Fisherian decomposition of nominal interest rates into anticipated inflation rates and ex ante real interest rates both of which involve, of course, agent’s subjective expectations about the future and are thus not directly measurable. A technique used by Mishkin (1981) to analyze some of the characteristics of ex ante real rates will begin the analysis. The basic methodological approach involves the assumption that agent’s expectations are formed rationally. As will be seen, it is then possible to regard the observed ex post real rates as perfect proxies for the non-observable ex ante real rates and then such questions concerning their
variability and their relatedness to specific key economic variables can be addressed.

In the chapter, we will also take a close look at the longer run influence of inflation on interest utilizing filtering techniques which enable us to examine this particular economic relationship at different frequencies. In addition, a particularly interesting approach to the study of the short-term relationship between inflation and nominal rates first undertaken by Eugene Fama (1975) in his work on "efficient markets," with his finding that changes in nominal rates essentially accommodate changes in expected inflation (implying a nearly constant real rate), will be replicated with our data which extend his testing period to include the important inflationary period of the early to late 1970s. Naturally, we are more interested in having a useful framework within which we can examine the relationship between interest rates (real and nominal) and inflation rather than in providing any sort of formal test of the efficient markets hypothesis. Nevertheless, by extending the sample period, interesting results, some of which are quite different from those of Fama, are obtained. The results along with their implications are discussed in a later section of this chapter.

We will begin the empirical analysis of this section with a test of the effect of increases in inflationary expectations on the real rate of interest. Following Mishkin (1981), we will be careful to distinguish the ex ante real rate from the ex post real rate but yet show how
OLS regressions of the latter can be used to infer information about the relationship between the former variable and variables whose values are known when nominal rates (bond prices) are set. The aspect of the following analysis which is of the greatest relevance to the present study is the relationship between ex post real rates and a number of variables which are thought to influence them. We are particularly interested in determining whether inflation is the sole variable that dominates changes in real rates or whether other values as well impinge significantly on real yields.

Again, we invoke the Fisher equation for a one-period bond;

\[ i_t = r_t^A + \pi_t^e \]

where,

- \( i_t \) = the nominal return on a bond held from time period \( t-1 \) to \( t \).
- \( \pi_t^e \) = the inflation rate expected in the market for period \( t-1 \) to \( t \).
- \( r_t^A \) = the real rate of return expected by the market in time period \( t-1 \) for a bond maturing in period \( t \).

Thus, the real rate is simply the difference between the nominal rate set at \( t-1 \) and the expected inflation rate. Because the real rate referred to above is an expected value, it is often termed the ex ante real rate to distinguish it from the ex post real rate which is simply
the nominal rate minus the actual inflation rate, or the actual real return on the one-period bond. The ex post real rate is defined formally as;

\[ (2) \quad r_t^P = i_t - \pi_t = r_t^A - (\pi_t - \pi_t^e) \]

where

- \( r_t^P \) = the ex post real return on a bond held from t-1 to t.
- \( \pi_t \) = the actual inflation rate from period t-1 to t.

One popular approach used by a number of researchers has been to subtract survey measures of inflationary expectations from nominal rates and then to determine the relationship between this real rate proxy and other key economic variables. The survey data most frequently used (e.g., by Carlson (1977), Gibson (1972), et al.) are the Livingston data discussed earlier. Whereas most criticisms of the use of Livingston data focus on their relatively poor predictive power as compared to alternative measures of inflationary expectations, Mishkin's critique lies at the foundations of economic theory.

"One obvious danger with survey data is that there may have been very little incentive for the respondents to answer accurately. A more subtle point that is often unrecognized in the literature is that the behavior of market expectations need not reflect the average expectations of participants in that market. Market expectations are frequently believed to be rational but not because all, most, or even the average market participant is also believed to be rational. Rather, rational expectations are plausible because market expectations can be driven to the rational expectations equilibrium by the
elimination of unexploited profit opportunities. This arbitrage view of expectations formation clearly allows the average expectations of market participants to differ from the market's expectations. On theoretical grounds alone then, we should be skeptical of using survey data to measure inflation expectations in a market.¹

Following Mishkin, we shall approach the problem of determining ex ante real rates via a different route, by assuming rationality in the formation of inflation expectations in the bond market. Rationality implies the following condition:

\[ E(\pi_t - \pi^e_t / \phi_{t-1}) = 0 \]

where \( \phi_{t-1} \) is the set of available information at time \( t-1 \). The expression simply says that any forecast error of inflation is uncorrelated with past information.

Let \( X_{t-1} \) equal a set of variables comprising a subset of \( \phi_{t-1} \) with which \( r_t \) is correlated. Thus, we have;

\[ r_t^A = X_{t-1}\beta + u_t \]

with the error term, \( u_t \), being determined at \( t-1 \). Upon substitution and rewriting the forecast error of inflation as \( e_t \), we get;

\[ r_t^p = X_{t-1} + u_t - e_t \]

Note that equation 5 is capable of being estimated due to the fact that the ex post real rate, \( r_t^p \), is observable. If there can be found a direct relationship between the OLS estimates of (5) and (4) then we can infer a relationship between the ex ante real rate, \( r_t^A \), and variables whose
values are known at $t-1$, even though $r_t^A$ is itself not observable at $t-1$. Put simply, the notion is that the expected OLS estimates of $\beta$ from the two equations are identical for the non-stochastic $X$ case.\(^2\)

II. Empirical Results of the Fisher Equation in the Short-Run

The ex post real rates are calculated by subtracting the percentage changes in the Consumer Price Index over a one month period from the yields on 91 day U.S. Treasury bills with one month left to maturity, with these latter values being established in the secondary market at the beginning of the period.

The price index data were obtained from the Federal Reserve Bulletin while the interest rate series was obtained from "An Analytical Record of Yields and Yield Spreads", a publication of Salomon Brothers, Inc. The CPI is constructed from data collected over the entire month and there is no clear-cut procedure for determining how it should be weighted within a particular month. Consequently, it is difficult to precisely match the interest rate data with the price index data from each month so as to provide for nearly perfect time correspondence of the two series. Although Fama does use monthly series in his testing of the joint hypotheses of market efficiency and the constancy of real rates, nowhere in his writings is there any mention of this potential problem associated with the use of monthly data. Mishkin (1981) recommends the use of non-overlapping
quarterly data to attenuate any potential time consistency problems and uses this approach in his own analysis. Although this potential timing problem was considered it was felt that for purposes of the present study the advantages of having a larger number of degrees of freedom outweighed any of the difficulties associated with the use of a monthly data series.

Along these lines it could be argued that the use of continuously compounded inflation rates by Fama and Mishkin and indeed by most researchers in this area is somewhat less appropriate than the use of discrete values as in this study for the following reason; if the nominal rate as determined in the market is to be regarded as a holding period yield for a representative individual (an implicit assumption in most studies as the overlapping of time periods is not allowed) then the proper measure of inflation is the change in the index of prices from the beginning to the end of the holding period, a discrete value.\(^3\)

As mentioned previously, Fama's tests with ex post real rates indicated that the hypothesis concerning their constancy over the 1953-1971 period could not be rejected. One of his tests involved the measurement of the structure of the autocorrelation function of ex post real rates, a so-called weak form test. The basic notion is that constancy of the real rate implies the null hypothesis that all of the autocorrelations will be equal to zero. This is nothing more than the standard orthogonality property of conditional expectations.\(^4\) The autocorrelations for our period using
one month real rates are presented below in Table 2-1. For purposes of comparative analysis the tests were done using the overall sample period (June 1953-December 1979) as well as what can be regarded as an inflationary sub-period (February 1974-December 1979). The practice of using a special sub-period is repeated throughout the present study.

Also, where possible, the period of analysis is extended in this chapter to the end of 1983, with these results being presented in Appendix 5. One reason for extending the sample period to include the early 1980s is to see if our basic empirical findings hold up when this period of extremely high real interest rates (brought about primarily by a dramatic fall in inflation) is included. Unfortunately, problems concerning data availability precluded the extension of the sample period throughout the present study. Consistency of presentation thus dictated that the 1953-1979 period be used through the text proper although references to these later period results are made in the text whenever appropriate.

It should also be pointed out that the short-run analysis referred to is the testing for the contemporaneous relationship between and among variables or, in special instances, testing done with variables lagged by no more than a few periods. This is to be contrasted with a long-run analysis which attempts to remove business cycle influences in measuring co-movements of variables and which will be dealt with in section III of the present chapter.
TABLE 2-1

Serial Correlation Structure of the Ex Post Real Rate: 1953-1979

<table>
<thead>
<tr>
<th>Lag k</th>
<th>$\rho_1$</th>
<th>$\rho_2$</th>
<th>$\rho_3$</th>
<th>$\rho_4$</th>
<th>$\rho_5$</th>
<th>$\rho_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1953-1979</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>.316</td>
<td>.325</td>
<td>.236</td>
<td>.240</td>
<td>.286</td>
<td>.276</td>
</tr>
<tr>
<td>Approximate Standard error of the autocorrelations:</td>
<td>.124</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Test of $\rho_1 = \rho_2 = \ldots = \rho_6 = 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q(6)=525.7$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1974-1979

<table>
<thead>
<tr>
<th>Lag k</th>
<th>$\rho_1$</th>
<th>$\rho_2$</th>
<th>$\rho_3$</th>
<th>$\rho_4$</th>
<th>$\rho_5$</th>
<th>$\rho_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_1$</td>
<td>.598</td>
<td>.109</td>
<td>.161</td>
<td>.028</td>
<td>-.151</td>
<td>.128</td>
</tr>
<tr>
<td>Approximate standard error of the autocorrelations:</td>
<td>.119</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q(6) = 50.11$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The Q statistic used in the study is the adjusted one suggested by Ljung and Box (1978) and is approximately distributed as $\chi^2(6) = n(n+2)(n-k)^{-1}k$ where $n$ is equal to the number of observations. It constitutes the formal test of the null hypothesis that the values of all the autocorrelations are jointly zero. Its value for the entire sample period of 525.7 is far above the critical values at the .10 and .01 levels of 10.64 and 16.81 respectively.
Furthermore, four of the autocorrelations are more than two standard errors away from zero while the other two are very close to being so. Thus, on the basis of this test alone, we can reject the null hypothesis of constant ex post real rates, at least for the overall period. The results for the inflationary sub-period are mixed. Only the first autoregressive value shows up significantly, however the high value of the Q-statistic tends to support rejection of the null hypothesis of a joint zero value of the autocorrelations. For the 1953-1983 period, the first 3 autocorrelations show up significantly and the Q-statistic is still high although less than it was for 1953-1979 (Appendix 5, table I). Curiously enough, for the extended era's sub-period, 1974-83, none of the autocorrelations is statistically significant yet the Q-value is approximately twice as great as it was for the other sub-period, 1974-79. This finding makes drawing general inferences about the serial correlation structure of ex post real rates from the beginning of the high inflation era even more difficult.

A "semi-strong" form test of the hypothesis of real rate constancy is provided by regressions of the real rate on variables whose values were known when the expected or ex ante real rate was determined i.e., a variable in the publicly available information set $\phi_{t-1}$. A likely candidate to act as an independent variable is the inflation rate lagged by one period, particularly so for our analysis as a fundamental assumption of the model is that increases in inflation will be associated with decreases in real
pecuniary yields. An additional approach would be, following Mishkin, to test for real rate constancy by making use of time trend variables as regressors. In his analysis he tests three models, one which regresses the ex post real rate on a constant term, the one period lagged inflation rate and a series of four time variables (with each successive time variable raised to a higher exponent), a model which omits the lagged inflation value and one with the constant term and lagged inflation value alone. He postulates that real rates have moved with a fourth order polynomial in time as his empirical results indicate that higher order values do not add to the explanatory power of the models. His tests were replicated for our sample period using one month values to maximize the degrees of freedom. The combined results with the lagged inflation and time variables are presented in Table 2-2 below. Again, the ex post real rates are defined as the one month yields on T-bills computed on a discount basis at the beginning of the month minus the proportionate change in the CPI over the corresponding month. The t-values are in parentheses.
TABLE 2-2

Tests of the Constancy of the Real Rate Utilizing
Time Trend Variables

Dependent Variable: Ex post real rates (one month)

<table>
<thead>
<tr>
<th>Coeff. on</th>
<th>Cons. ( \pi_{t-1} )</th>
<th>Time(^1)</th>
<th>Time(^2)</th>
<th>Time(^3)</th>
<th>Time(^4)</th>
<th>R(^2)</th>
<th>D-W</th>
<th>F-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1953-1979</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.21</td>
<td>-.26</td>
<td>.13</td>
<td>2.20</td>
<td>49.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5.93)</td>
<td>(-7.01)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.30</td>
<td>-.12</td>
<td>-4.52</td>
<td>8.61</td>
<td>-4.66</td>
<td>.74</td>
<td>.20</td>
<td>2.07</td>
<td>16.9</td>
</tr>
<tr>
<td>(1.75)</td>
<td>(-2.46)</td>
<td>(-1.42)</td>
<td>(2.15)</td>
<td>(-2.50)</td>
<td>(2.55)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.23</td>
<td></td>
<td>-4.82</td>
<td>8.96</td>
<td>-4.89</td>
<td>.77</td>
<td>.19</td>
<td>1.96</td>
<td>19.3</td>
</tr>
<tr>
<td>(1.65)</td>
<td></td>
<td>(-1.51)</td>
<td>(2.22)</td>
<td>(-2.60)</td>
<td>(2.66)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| 1974-1979 |                |          |          |          |          |       |      |        |
| .92       | -.33           | .17      | 1.78     | 15.7     |          |       |      |        |
| (1.19)    | (-3.96)        |          |          |          |          |       |      |        |
| -1.05     | -.29           | 1.43     | -.723    | 1.61     | -.013    | .24   | 1.79 | 4.0    |
| (-.66)    | (-2.40)        | (.63)    | (-.60)   | (.57)    | (-.54)   |       |      |        |
| -.65      | .83            | -.39     | .08      | -.01     | .12      | 1.33  | 3.3  |        |
| (-.39)    | (.35)          | (-.32)   | (.28)    | (-.24)   |          |       |      |        |

Time trend runs from .01 in 7402 to .71 in 7912 and from .01 to 3.19 from 5306 to 7912

\( \pi_{t-1} \) = inflation rate lagged one month

F-statistic is for the null hypothesis that all the coefficients excluding the constant term are jointly zero. The statistics are distributed as F(3,313) and F(4,313) in the first part and as F(3,66), F(4,66) in the second.

* The superscript represents the value of the exponent on the time variables.
For the overall sample period, the results seem to indicate that ex post real rates have moved with a fourth degree polynomial (quartic function) in time (higher values for the time exponent were not significant). In all three models, the coefficients on the time variables raised to exponents greater than one show up significantly. Additionally, the R squared and F-tests confirm this basic finding. Very importantly for our purposes a significant negative impact of lagged inflation on real yields shows up for the overall sample period with a t-value on the inflation coefficient greater than 7. For the special sub-period, there seems to be no evidence at all for any discernible time trend, however the negative impact of inflation on ex post real rates does appear to be significant, but less so in this instance.

Again, the results obtained by extending the sample period are not markedly different, however it is interesting to note that the contemporaneous inflation-interest rate relationship seems to be less powerful when the early 1980s are included in the analysis. This is most easily seen in the model which regresses the nominal rate on the lagged inflation value alone (Appendix 5, Table II). Both the regression coefficient and the F-value have fallen (from -.26 to -0.14 and 49.2 to 18.4 respectively). This finding is perhaps not too surprising given the earlier observation that the early 1980s was a period characterized by uncommonly high levels of real interest rates, levels surpassing even those of the early stages of the Great
Depression. In fact, the precipitous drop in inflation in the American economy during this time could perhaps be regarded as constituting a structural break that would naturally weaken the relationship between ex post real rates and one period lagged inflation values that we are concerned with in this section. It is also necessary to keep in mind that the rising real yields in the face of the declining inflation of the early 1980s may simply be another aspect of a general underadjustment phenomenon, in this case nominal yields not fully adjusting downwards to declining inflation. This possibility apparently has been ignored given the spate of recent theoretical work attempting to account for the present high levels of real returns. Interestingly, Irving Fisher discussed the less than complete adjustment of nominal yields to declining inflation as well as to rising inflation, a phenomenon which he attributed to a form of money illusion. In other words, Fisher himself did not believe in any Fisher effect.

III. The Influence of Key Variables on Real Rates

Tests of the relationship between real rates and some other variables which have been cited in the literature as being correlated with real rates were performed and the results are provided in Table 2-3 below. These additional potential explanatory variables included the percentage changes in M1 and M3 money supply as well as the employment/population ratios, more precisely the ratio of seasonally adjusted employment to the total U.S. population.
including the armed forces. This contrasts with the standard approach of using unemployment rates as indicators of labor force participation in the economy. In addition, the index of Industrial Production as computed by the Federal Reserve Board served as a proxy for the level of economic activity. The coefficients listed are those on the one month lags of the four additional variables, however the tests were conducted using lags of up to four periods.
TABLE 2-3

Tests of the Correlation Between Real Rates and Other Variables

<table>
<thead>
<tr>
<th>Other explanatory variables</th>
<th>Constant Term of $\pi_{t-1}$ Variable</th>
<th>$R^2$</th>
<th>SER Stat. D-W</th>
</tr>
</thead>
<tbody>
<tr>
<td>1953-1979</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-3.1 %ΔM1</td>
<td>1.39 (-6.46) - .26 (-7.55) - 43.83 (+1.15)</td>
<td>.16</td>
<td>2.33 32.1 2.20</td>
</tr>
<tr>
<td>2-3.2 %ΔM1</td>
<td>0.54 (2.73) - 98.83 (2.44) - 2.70 (-7.91) - .39 (1.53)</td>
<td>.01</td>
<td>2.53 5.9 1.41</td>
</tr>
<tr>
<td>2-3.3 %ΔM3</td>
<td>1.28 (6.84) - 2.70 (-7.91) - .39 (1.53)</td>
<td>.17</td>
<td>2.32 32.8 2.24</td>
</tr>
<tr>
<td>2-3.4 %ΔM3</td>
<td>0.20 (1.45) - .12 (.426) - 12.58 (.478) - .00 (-1.05)</td>
<td>.01</td>
<td>2.55 0.1 1.36</td>
</tr>
<tr>
<td>2-3.5 E/P</td>
<td>1.34 (6.73) - .27 (-7.96) - 8.38 (.30)</td>
<td>.17</td>
<td>2.33 31.9 2.23</td>
</tr>
<tr>
<td>2-3.6 E/P</td>
<td>0.21 (1.39) - 12.58 (.478) - 0.00 (.495)</td>
<td>.01</td>
<td>2.56 0.1 1.34</td>
</tr>
<tr>
<td>2-3.7 IPI</td>
<td>0.02 (.77) - .33 (-3.72) - .00 (-.495)</td>
<td>.29</td>
<td>0.01 5.1 2.09</td>
</tr>
<tr>
<td>2-3.8 IPI</td>
<td>0.06 (1.88) - .00 (-1.05)</td>
<td>.13</td>
<td>0.02 2.4 1.55</td>
</tr>
<tr>
<td>Other explanatory variables</td>
<td>Constant Term of $\pi_{t-1}$</td>
<td>Coeff. Variable</td>
<td>$R^2$</td>
</tr>
<tr>
<td>-----------------------------</td>
<td>-----------------------------</td>
<td>----------------</td>
<td>-------</td>
</tr>
<tr>
<td>1974-1979</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-3.9 %ΔM1</td>
<td>1.37</td>
<td>-.32</td>
<td>-113.90</td>
</tr>
<tr>
<td></td>
<td>(1.79)</td>
<td>(-4.29)</td>
<td>(1.83)</td>
</tr>
<tr>
<td>2-3.10 %ΔM1</td>
<td>-1.85</td>
<td>-.20</td>
<td>-5.77</td>
</tr>
<tr>
<td></td>
<td>(4.23)</td>
<td>(.082)</td>
<td></td>
</tr>
<tr>
<td>2-3.11 %ΔM3</td>
<td>-0.15</td>
<td>-.28</td>
<td>-.28</td>
</tr>
<tr>
<td></td>
<td>(.223)</td>
<td>(.875)</td>
<td></td>
</tr>
<tr>
<td>2-3.12 %ΔM3</td>
<td>-1.85</td>
<td>-.22</td>
<td>-.00</td>
</tr>
<tr>
<td></td>
<td>(6.65)</td>
<td>(.894)</td>
<td></td>
</tr>
<tr>
<td>2-3.13 E/P</td>
<td>0.80</td>
<td>-16.33</td>
<td>.17</td>
</tr>
<tr>
<td></td>
<td>(1.12)</td>
<td>(.478)</td>
<td></td>
</tr>
<tr>
<td>2-3.14 E/P</td>
<td>-1.88</td>
<td>-1.07</td>
<td>.01</td>
</tr>
<tr>
<td></td>
<td>(2.03)</td>
<td>(.028)</td>
<td></td>
</tr>
<tr>
<td>2-3.15 IPI</td>
<td>0.07</td>
<td>-0.00</td>
<td>.39</td>
</tr>
<tr>
<td></td>
<td>(1.87)</td>
<td>(1.41)</td>
<td></td>
</tr>
<tr>
<td>2-3.16 IPI</td>
<td>0.08</td>
<td>-.00</td>
<td>.29</td>
</tr>
<tr>
<td></td>
<td>(2.11)</td>
<td>(-.79)</td>
<td></td>
</tr>
</tbody>
</table>
The results seem to indicate that none of these real variables has any substantial explanatory power for movements of the real interest rate (even with up to four period lagged values of the other independent variables). The only apparent exceptions are provided by the one month models using the percentage changes in M1 for the one period lagged values and for the 3 and 4 period lagged values of the IPI (Not shown) where all of the t-statistics are above 2 (but not exceeding 2.6 in any instance). Nevertheless, the coefficients on the other variables are all insignificant and the decline in the F-values for the equations that omit the lagged inflation variable provide further evidence for the conclusion that these variables have no meaningful impact on real interest rates. However we should be careful about inferring from these results that real factors do not affect the real rate of interest. Rather, the results may indicate nothing more than a lack of sufficient power on the part of our tests to discern co-movements of the real variables and the real rate due to, perhaps, a relatively small amount of cyclical variation in real rates. Nevertheless, the results do seem to indicate that the standard bivariate tests of the Fisher effect are not misspecified, at least as far as the exclusion of the particular key economic variables used above are concerned. However the fact that the time trend variables do show up significantly indicates that there is a variable or variables that are correlated with the trend variables and which could help explain movements of the real interest
rate. We shall argue later that if misspecification is a problem, it is more likely due to the exclusion of a liquidity factor, in other words a factor which incorporates the friction associated with transactions costs of exchange. 6

A preliminary analysis of this very question can be provided by a look at the actual relationship between interest rates and a variable which in the F-H analysis should impinge on them, the ratio of governments to money. It will be recalled that one of the implications of the liquidity model is that nominal pecuniary yields on governments are positively related to this ratio (i.e., \( \frac{d r_g}{d \gamma} > 0 \)). Indeed, with the steady-state nominal return on capital being fixed (in empirical analysis this value can be, as we shall see, approximated by the nominal pecuniary return on long, relatively illiquid bonds), the rising yields on liquid governments is a necessary condition for the inverse relationship between liquidity premia and the governments/money ratio.

To test for this relationship, in one model one month maturity yields were regressed on current monthly inflation values and on the ratio of non-bank private holdings of U.S. Treasury bills with a maturity period of one year or less to M1 money. In another model, only the latter variable served as a regressor. It should be noted that broader measures of money were used as well (old M2 and M3) however the results were too similar to warrant their inclusion below. Both
series were obtained from various issues of the Federal Reserve Bulletin. Table 2-4 below gives the results. Again, t-values are in parentheses.

TABLE 2-4
Tests of the Correlation Between Nominal Rates and the Governments/Money Ratio

<table>
<thead>
<tr>
<th></th>
<th>1953-79</th>
<th>Constant Coeff. on $\pi_{t-1}$</th>
<th>Coeff. on G/M</th>
<th>$R^2$</th>
<th>$F(2,315)$</th>
<th>D-W</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. YM1M*</td>
<td>.746</td>
<td>.375</td>
<td>.497</td>
<td>.58</td>
<td>222.4</td>
<td>1.97</td>
</tr>
<tr>
<td></td>
<td>(1.51)</td>
<td>(18.49)</td>
<td>(3.84)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. YM1M</td>
<td>-.688</td>
<td></td>
<td>1.240</td>
<td>.13</td>
<td>49.3</td>
<td>1.84</td>
</tr>
<tr>
<td></td>
<td>(-.977)</td>
<td></td>
<td>(7.02)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* one month maturity yield

The results indicate that the ratio of liquid governments to money has accounted for some of the variance observed in maturity yields although not as much as did inflation. The important point to note is that the coefficients on the regressors are significant and of the correct sign. However a difficulty arises in that the liquidity model predicts a direct relationship between inflation and the governments/money ratio which, if correct, would introduce a multicollinearity problem in the first model in Table 2-4. To attenuate this problem, partial correlation analysis was undertaken which involves removing on an individual basis the linear influence of inflation on the
dependent variable and on the governments/money ratio and, after subtraction of the fitted values from these regressions from the original series, obtaining a simple correlation value for YM1M and G/M which is then equal to the partial correlation of these two variables. The computed value of .607 confirms our original finding of a substantial positive relationship.

The extent to which nominal rates have adjusted to changes in actual, current inflation over our sample period is perhaps best illustrated by a simple regression of the one month nominal yields on the rates of inflation over the corresponding periods.\(^7\) The results of our regressions are presented below in equations (8) and (9). YM1M represents the one month nominal return on U.S. Treasury bills. Both of our time periods are used in the following analysis.

1953-1979

<table>
<thead>
<tr>
<th>OLS</th>
<th>(8) YM1M = 2.45 + .43(\pi) (1 month)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(23.42) (22.94)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>= .62</td>
</tr>
<tr>
<td>(F(1,316))</td>
<td>= 526.93</td>
</tr>
<tr>
<td>(DW)</td>
<td>= 1.05</td>
</tr>
<tr>
<td>(SER)</td>
<td>= 1.31</td>
</tr>
</tbody>
</table>

1974-1979

<table>
<thead>
<tr>
<th>OLS</th>
<th>(9) YM1M = 3.13 + .40(\pi) (1 month)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(7.82) (9.20)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>= .55</td>
</tr>
<tr>
<td>(F(1,69))</td>
<td>= 84.51</td>
</tr>
<tr>
<td>(DW)</td>
<td>= .90</td>
</tr>
<tr>
<td>(SER)</td>
<td>= 1.22</td>
</tr>
</tbody>
</table>
The results for both periods clearly indicate the tendency of nominal rates during the sample period to rise less than proportionately relative to inflation. Although the high values of the t-statistics on the inflation coefficients indicate clear rejection of the null hypothesis of their being equal to zero, we are more interested in determining whether or not these coefficients are significantly different from 1. Recomputing the t-values for the new null hypothesis, we get:

\[ t(2,319) = 29.68 \quad t(2,68) = 13.52 \]

These values are far above the critical ones at the .01 level of 2.64 and 2.83 respectively. However the low Durbin-Watson statistics indicate the presence of positive serial correlation and a time plot of the residuals seemed to indicate that most of the serial correlation appeared in approximately the last 30 months of observations. However, exclusion of these observations and re-estimation of the equation failed to confirm this piece of casual empiricism. Additionally, the breaking up of the overall sample period into 3 and 4 sub-periods provided evidence that the serial correlation was not substantially greater in any one period.

The use of the generalized Durbin procedure and the Park-Glesjer analysis provided evidence respectively for an AR(1) process and substantial positive heteroscedasticity. Correcting for these problems as well as possible, our fundamental assumption of the underadjustment of nominal rates to inflation still held up for the overall period although the inflation coefficient was not as significant.\(^8\)
This is undoubtedly due to the fact that with the serial correlation correction the model is attempting to explain differences in interest rates and not their levels. Curiously enough, even fairly obvious potential econometric problems such as the above-mentioned heteroscedasticity are rarely corrected for in the various tests of the contemporaneous inflation-interest rate relationship.

For the 1953-1983 and 1974-83 periods, the results are presented in Table III of Appendix 5. Again, OLS estimation produced significant but less than unit coefficients on inflation. However upon correction for positive serial correlation of the first degree using the Cochrane-Orcutt method, as was deemed necessary by the low D-W values obtained, the coefficients drop to fairly low levels. This provides perhaps the most convincing evidence of the diminished relationship between these variables when the early 80s are added to the analysis. Nevertheless, it is necessary to bear in mind that we are concerned with finding a general underadjustment of nominal rates to inflation rather than with its precise numerical value. It should also be noted that in these tests, regressions of the error terms on contemporaneous inflation failed to provide any evidence for any significant positive or negative heteroscedasticity, in contrast to the results obtained with the shorter period lengths.

It was felt that a potentially useful approach to the study of the longer run inflation-interest rate relationship
would be to estimate a series of polynomial distributed lag models utilizing various lag periods as well as different end point restrictions. The interesting aspect of this analysis is that it allows us to look at the relationship between the two variables incorporating the influence of a lagged series of inflation rates, similar to the early Fisher approach but without the basic restriction imposed by the geometric lag estimation technique that he used. In all cases a third degree polynomial was assumed to provide a good approximation to the actual lag structure. The results are given in Table 2-5 below.
<table>
<thead>
<tr>
<th></th>
<th>Mean lag</th>
<th>Sum of lag coefficients</th>
<th>$R^2$</th>
<th>F-statistic</th>
<th>D-W</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>6 lags 1974-9</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>1.66(.46)</td>
<td>.450 (.05)*</td>
<td>.55</td>
<td>38.1</td>
<td>2.08</td>
</tr>
<tr>
<td>B</td>
<td>1.61(.45)</td>
<td>.462 (.05)</td>
<td>.56</td>
<td>26.1</td>
<td>2.12</td>
</tr>
<tr>
<td>C</td>
<td>1.72(.46)</td>
<td>.458 (.05)</td>
<td>.56</td>
<td>19.2</td>
<td>2.13</td>
</tr>
<tr>
<td>D</td>
<td>1.59(.48)</td>
<td>.462 (.05)</td>
<td>.56</td>
<td>19.3</td>
<td>2.11</td>
</tr>
<tr>
<td><strong>6 lags 1953-79</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>2.11(.19)</td>
<td>.496 (.02)</td>
<td>.71</td>
<td>322.0</td>
<td>2.30</td>
</tr>
<tr>
<td>B</td>
<td>2.06(.18)</td>
<td>.500 (.02)</td>
<td>.71</td>
<td>219.1</td>
<td>2.29</td>
</tr>
<tr>
<td>C</td>
<td>2.15(.18)</td>
<td>.500 (.02)</td>
<td>.71</td>
<td>217.6</td>
<td>2.29</td>
</tr>
<tr>
<td>D</td>
<td>2.08(.19)</td>
<td>.501 (.02)</td>
<td>.71</td>
<td>163.8</td>
<td>2.28</td>
</tr>
</tbody>
</table>

*standard errors

A=both constraints
B=far constraint only
C=near constraint only
D=no constraints
TABLE 2-5 (cont.)

<table>
<thead>
<tr>
<th>Mean lag</th>
<th>Sum of lag coefficients</th>
<th>R²</th>
<th>F-statistic</th>
<th>D-W</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 lags</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1974-79</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>3.72 (.63)</td>
<td>.686 (.06)</td>
<td>.72</td>
<td>69.1</td>
</tr>
<tr>
<td>B</td>
<td>3.40 (.52)</td>
<td>.736 (.06)</td>
<td>.79</td>
<td>65.0</td>
</tr>
<tr>
<td>C</td>
<td>4.19 (.61)</td>
<td>.747 (.06)</td>
<td>.75</td>
<td>53.7</td>
</tr>
<tr>
<td>D</td>
<td>3.39 (.65)</td>
<td>.735 (.06)</td>
<td>.79</td>
<td>47.1</td>
</tr>
<tr>
<td>12 lags</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1953-79</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>2.86 (.46)</td>
<td>.505 (.02)</td>
<td>.70</td>
<td>317.1</td>
</tr>
<tr>
<td>B</td>
<td>2.62 (.45)</td>
<td>.511 (.02)</td>
<td>.72</td>
<td>223.9</td>
</tr>
<tr>
<td>C</td>
<td>2.99 (.47)</td>
<td>.507 (.02)</td>
<td>.71</td>
<td>212.6</td>
</tr>
<tr>
<td>D</td>
<td>2.59 (.48)</td>
<td>.511 (.02)</td>
<td>.72</td>
<td>167.5</td>
</tr>
<tr>
<td>16 lags</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1974-79</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>2.46 (1.5)</td>
<td>.462 (.06)</td>
<td>.59</td>
<td>175.1</td>
</tr>
<tr>
<td>B</td>
<td>1.76 (1.6)</td>
<td>.459 (.06)</td>
<td>.60</td>
<td>27.1</td>
</tr>
<tr>
<td>C</td>
<td>1.95 (1.5)</td>
<td>.451 (.07)</td>
<td>.60</td>
<td>27.0</td>
</tr>
<tr>
<td>D</td>
<td>0.01 (1.7)</td>
<td>.436 (.06)</td>
<td>.64</td>
<td>23.3</td>
</tr>
<tr>
<td>16 lags</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1953-79</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>2.31 (.62)</td>
<td>.495 (.02)</td>
<td>.70</td>
<td>314.2</td>
</tr>
<tr>
<td>B</td>
<td>2.07 (.60)</td>
<td>.505 (.02)</td>
<td>.72</td>
<td>226.6</td>
</tr>
<tr>
<td>C</td>
<td>2.36 (.64)</td>
<td>.496 (.02)</td>
<td>.70</td>
<td>208.8</td>
</tr>
<tr>
<td>D</td>
<td>1.85 (.62)</td>
<td>.502 (.02)</td>
<td>.72</td>
<td>170.8</td>
</tr>
</tbody>
</table>

These results provide evidence of the failure of the simple Fisher effect to operate during our period even allowing for fairly lengthy lag effects for the influence of inflation on interest rates. Even after one year, interest rates never adjust to changes in inflation by more than a factor of .75. Although some of the results seem to be confusing (e.g., the sum of the lag coefficients for the 16 lag models are less than those for the 12 lag ones) the above results seem to provide additional evidence in favor
of the hypothesis of the underadjustment of nominal rates to inflation during the 1953-79 and 1974-79 periods.

Once more the results were basically similar for the extended sample period (see Table IV of Appendix 5), however again the evidence indicates the diminishing strength of the relationship between nominal rates and inflation, as illustrated by the declining adjusted R-squared and F-values, when the early 1980s are included in the analysis. This is confirmed by the higher mean lag values which indicate that inflation's influence on nominal yields is being spread out over a longer period and may answer the question as to why we were finding a much weaker contemporaneous relationship between the two values with the extended sample period.
IV. The Long-Run Fisher Effect

Although the above represents an attempt to draw a distinction between the relatively shorter- and longer-term influences of inflation on interest, an alternative, and perhaps more useful, method used by Summers (1981) involving band spectrum regression to filter out high-frequency (short-run) co-movements to more easily test for the existence of a longer run relationship also was employed in this study. A long-run analysis is important for our purposes not only because F-H is essentially a long run, steady state model but, in addition, an underadjustment of nominal rates to inflation (or an over-adjustment for that matter) using short-run analysis is not too surprising a finding if there is reason to believe that the error terms and the expected inflation variable in a Fisher equation are correlated, in other words that the same factors that influence expected inflation impinge on nominal rates as well. This is very plausible, at least in theory, although as we have repeatedly seen it is not clear as to what these outside variables may be. Thus although theory offers different possibilities for the general value of the coefficient on expected inflation in the short-run depending, for instance, on the nature of a particular economic shock being considered, long run theoretical analysis predicts an approximate superneutrality as steady state inflation is determined exclusively by the rate of monetary growth. However it is necessary to be very careful about drawing inferences concerning the breaking down of the
inflation-interest rate relationship even in a short-run analysis. For example, if one were to argue, as some writers have (e.g. Summers (1981)), that a positive or negative money shock would tend to weaken the inflation-interest rate nexus in the short-run, the argument must be based, explicitly or otherwise, on the operation of some sort of liquidity effect. As far as the American experience over the past 30 years or so is concerned, this does not seem to be consistent with the evidence (see Mishkin 1982).

As far as the longer run is concerned, the notion is that cyclical influences such as those discussed above or problems of data alignment might distort the contemporaneous (short-run) relationship between nominal rates and inflation, however by pre-filtering the inflation and interest rate data it is possible to test for low-frequency or long-run co-movements (in other words, co-movements over periods exceeding those of the standard business cycle in the U.S.). Although at least one economist has questioned the use of frequency-domain techniques to empirically test certain long run economic propositions (McCallum (1984)), for the present study it was assumed that such techniques "provide an empirical counterpart for the elusive 'long run' of economic theory" (Geweke (1982) p.1).

Specifically, the Summers method involves the pre-filtering of data using a moving average process to place greater weight on the particular range of frequencies that
the researcher may be interested in. Just as a researcher might be interested in removing data from certain periods from his analysis (e.g. wars, periods of controlled prices, etc.) he might be equally interested in removing the influence of particular frequencies not relevant to his study. Engle (1974) provides the technical discussion that is the most well-known to economists. Engle's approach is to take a p period moving average which will filter out completely data at the frequency of p periods and will almost totally eliminate the power of frequencies up to 2p periods. A slight problem arises however in that the OLS estimates will be inconsistent due to the introduction of substantial serial correlation but, as Engle argues, this particular difficulty can be circumvented by simply raising the OLS standard errors by a factor of p.

Letting $\pi_t$ and $R_t$ denote the inflation rate and some nominal interest rate respectively, Summers' test is of the hypothesis that $B_l$ in an equation of the form given below is equal to 1.

$$R_t = \alpha + B_l \pi_t + u_t$$

where $u_t$ is an unobserved stochastic term. The above relationship is required to hold true only at low frequencies.\(^9\)

Summers initially concluded that inflationary expectations had no impact on interest rates in the short run (over the 1860-1979 sample period) however he employed the above technique in order to determine whether or not his findings could be extended to include longer term effects of
inflation as well. As alluded to above, it is quite plausible that there are positive correlations between injection of money or velocity shocks and interest rates but the initial liquidity effects resulting from such shocks would tend to depress interest rates and thus cause the researcher to believe that no relationship exists between inflation and nominal yields if only high frequency relationships are tested for. Like Summers, we are interested in filtering out these shorter-term relationships. Given that standard monetary analysis supposes that long-run inflation should be neutral in its effects because of the adjustment of nominal rates (tax considerations aside), whatever the short-term influence of inflation on interest, some sort of Fisher relationship should hold in the face of long swings in the rate of inflation, at least partially. An important additional feature of this approach is that by looking at just the low frequency movements, inflation is in principle completely forecastable. Thus actual inflation is an entirely appropriate proxy for expected inflation and the need for the modeling of inflationary expectations is obviated.

The results of our tests are presented below in Table 2-6. For the inflationary sub-period one month interest rates were used with 1, 1.5 and 2 year moving averages and for the entire sample period the tests were conducted using data filtered with 1, 1.5, 2, 3, and 5 year moving averages. Although each moving average will provide us with a
measurement of the longer-run relationship between the two variables by completely filtering out data at the particular period used (and almost completely filtering out data at twice the period used) the 3 and 5 year moving averages are perhaps the only ones that will completely remove any cyclical influences, although the two year filter would come close. This argument is based on assuming the standard reference cycle of approximately 5 years in length as determined by the National Bureau of Economic Research on the basis of the postwar American economic experience (although it should be noted that Zarnowitz (1985) finds an average cycle length of about 46 months in the cycles from the 1850s to the present).

These results provide some evidence for the longer term interest rate-inflation relationship over both of the sample periods (and a less than proportionate one) but only with data pre-filtered with the shorter term moving averages. The finding appears to be that as the length of the filter is increased, the relationship between the two variables becomes increasingly less significant. Therefore, in general terms it is not possible on the basis of these tests alone to reject the null hypothesis of little or no influence of inflation on interest rates when the long run is taken into account, a rather surprising result as most explanations of the anomalous behavior of the two variables rely on short term factors.

Summers finds roughly similar results at least for his 1948-1979 sub-period (using quarterly values of commercial
paper rates), however when he looks at the relationship between inflation and interest rates in earlier periods, he finds virtually no correlation between the two variables with any filter length. Indeed, his measured coefficients on the inflation variable are approximately zero as are the t-values on these coefficients for all of the decadal averages up to WWII. However, by using such early data, his tests could be distorted by a number of factors including the two mentioned earlier, the historical tendency of the Federal Reserve to peg interest rates in order to aid the Treasury in meeting its financing requirements and, perhaps less importantly, the generally poor quality of the consumer price index prior to 1953.

By extending the sample period to include the early 1980s, the results (presented in Table V of Appendix 5) are slightly different showing an even smaller degree of adjustment of nominal rates to inflation over the period marked by the onset of high inflation. However the fact that these results are basically similar is not surprising when one considers the heavy smoothing of the data involved in these tests. In other words, extending the overall sample period by just four years and using data pre-filtered with long term moving averages would not be expected to significantly alter the overall results.
### TABLE 2-6

Moving Average Regressions of One-Month Treasury Bill Rates on Inflation

<table>
<thead>
<tr>
<th></th>
<th>1974-1979</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constant</td>
<td>Coefficient on Inflation *</td>
<td>R^2**</td>
</tr>
<tr>
<td>1 year</td>
<td>1.51</td>
<td>.726</td>
<td>.91</td>
</tr>
<tr>
<td></td>
<td>(.64)</td>
<td>(2.33)</td>
<td></td>
</tr>
<tr>
<td>1.5 years</td>
<td>1.60</td>
<td>.701</td>
<td>.93</td>
</tr>
<tr>
<td></td>
<td>(.54)</td>
<td>(1.72)</td>
<td></td>
</tr>
<tr>
<td>2 years</td>
<td>1.48</td>
<td>.716</td>
<td>.94</td>
</tr>
<tr>
<td></td>
<td>(.43)</td>
<td>(1.45)</td>
<td></td>
</tr>
<tr>
<td>1953-1979</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 year</td>
<td>2.03</td>
<td>.696</td>
<td>.86</td>
</tr>
<tr>
<td></td>
<td>(2.46)</td>
<td>(3.71)</td>
<td></td>
</tr>
<tr>
<td>1.5 years</td>
<td>2.00</td>
<td>.576</td>
<td>.86</td>
</tr>
<tr>
<td></td>
<td>(1.68)</td>
<td>(2.46)</td>
<td></td>
</tr>
<tr>
<td>2 years</td>
<td>1.98</td>
<td>.692</td>
<td>.86</td>
</tr>
<tr>
<td></td>
<td>(1.27)</td>
<td>(1.79)</td>
<td></td>
</tr>
<tr>
<td>3 years</td>
<td>1.87</td>
<td>.559</td>
<td>.86</td>
</tr>
<tr>
<td></td>
<td>(.85)</td>
<td>(1.15)</td>
<td></td>
</tr>
<tr>
<td>5 years</td>
<td>2.10</td>
<td>.577</td>
<td>.85</td>
</tr>
<tr>
<td></td>
<td>(.78)</td>
<td>(.80)</td>
<td></td>
</tr>
</tbody>
</table>

*The t-statistics (in parentheses) are adjusted for the serial correlation problem as discussed in the text.

**As in the Summers (1981) article, only the R-squared values are reported.
TABLE 2-6
Moving Average Regressions of One-Month Treasury Bill Rates on Inflation

<table>
<thead>
<tr>
<th></th>
<th>1974-1979</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constant</td>
<td>Coefficient on Inflation *</td>
<td>R^2**</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>(.43)</td>
<td>(1.45)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

|                  | 1953-1979 |          |          |          |
|                  | Constant  | Coefficient on Inflation * | R^2**    |
| 1 year           | 2.03      | .696     | .86      |
|                  | (2.46)    | (3.71)   |          |
| 1.5 years        | 2.00      | .576     | .86      |
|                  | (1.68)    | (2.46)   |          |
| 2 years          | 1.98      | .692     | .86      |
|                  | (1.27)    | (1.79)   |          |
| 3 years          | 1.87      | .559     | .86      |
|                  | (.85)     | (1.15)   |          |
| 5 years          | 2.10      | .577     | .85      |
|                  | (.78)     | (.80)    |          |

*The t-statistics (in parentheses) are adjusted for the serial correlation problem as discussed in the text

**As in the Summers (1981) article, only the R-squared values are reported.
To summarize our results so far, we find along with Summers that there does not appear to be any long-term relationship between inflation and interest rates with the exception of the 1 and 1.5 year filtered data over the overall period and the one year data over the sub-period. These are, as previously mentioned, fairly surprising results and a somewhat similar technique to filter out short-term influences of inflation on interest was borrowed from Lucas (1980) to see if these results were either confirmed or contradicted. Lucas in his paper was actually testing for the relationship between inflation and M1 growth rates however the technique is perfectly suitable for our analysis as well. Essentially it involves pre-filtering the data with an exponentially-weighted two-sided moving average filter for both series. His theoretical justification for incorporating such a feature is given in footnote 10 of the paper;

"....In general, agents know only the past (arguing for a one-sided backward filter) but they care only about the future, and probably process much more information in forecasting that part of the future relevant to their own decisions than we econometricians can observe (arguing for a one-sided forward filter)."

An additional advantage in utilizing a two-sided moving average is that there is no need to first de-trend the series as was the case earlier.

The tests of the relationship between inflation and nominal yields were redone using the Lucas technique for the 1953-1979 period. Although Lucas inferred from his empirical analysis a close and 1-for-1 co-movement of
inflation and interest rates, he never actually tested for this relationship directly. Instead, he concentrated on M1 growth and inflation and M1 growth and T-bill rates. Following Lucas, various weights were used in the tests and scatter diagrams were examined to test for the relationship between our two variables. The results generally confirmed the existence of a close linear relationship between the two variables, however the value of the slope term shows up consistently at about .8 (as shown by simple OLS regressions) rather than 1, a finding which contradicts the inference drawn from Lucas' work not only by Lucas himself but by F-H (1983). The fit seemed to be better for all filter lengths the closer the weight was to unity, i.e. the closer the filtered observations approached the sample average values of the original series. Appendix 1 to this chapter discusses the Lucas analysis in more detail and presents some sample scatter plots of the two series filtered in accordance with his approach.

Thus we are compelled to conclude that the data support either of the hypotheses concerning the long-run relationship between interest rates and inflation, depending on which filtering process is used. Nevertheless, the superior underlying theoretical approach concerning the activities of agents in processing current and past information in order to make more accurate forecasts of future variable values provides a rationalization for
preferring the Lucas technique to a simpler, one-sided backward filter of the Summers type.

A more mechanical test of the long-run relationship between inflation and interest rates which has the added advantage of enabling the researcher to determine the strength of the association of these variables at numerous different specific frequencies involves the use of cross-spectral techniques. The use of the empirical cross-spectrum to determine the co-movements of inflation and interest appears to be not only unique to the present work but, as shall be seen, is in one sense more informative than either the Lucas or Summers approach. Appendix 3 to this chapter discusses the basics of spectral analysis and presents results for the inflation-interest rate relationship for the 1953-1979 period.

V. The Fama Tests on One Month Bills

This section is devoted to a replication of the tests of the relationship among inflation and both real and nominal rates first undertaken by Eugene Fama (1975), (1976) in his study of the efficiency of the markets for U.S. Treasury Bills. The basic analysis involves testing of the joint hypotheses of constant expected real rates as well as of efficient markets but we will see that it is possible to dichotomize the test to allow for an investigation of these phenomena on an individual basis. The period dealt with is 1953-1979 and thus includes data from the important
inflationary sub-period, data which were excluded in the original Fama analysis.

Following Fama, we define the relevant inflation variable not as the rate of change of some index of prices per se, i.e., an inflation rate, but rather as the change in the purchasing power of money, with this variable at time $t$ being expressed as

$$\rho_t = 1/p_t$$

where $p_t$ represents an index of prices. Thus, the percentage change in the purchasing power of money from period $t-1$ to $t$, $\Delta_t$, is simply

$$\rho_t - \rho_{t-1}/\rho_{t-1} = (p_{t-1}-p_t)/p_t$$

The real rate of interest is thus derived by summing together the one period nominal return, $R_t$, and $\Delta_t$. Note that inflation corresponds to a negative value for $\Delta_t$ while a deflation to a positive value. As the change in the purchasing power of money for the forthcoming period is unknown at $t-1$ so is the real return on a one period bill, unlike the nominal return which is set in the markets at $t-1$. The relationship then becomes (with tildes representing the unknown values)

$$\bar{R}_t = R_t + \bar{\Delta}_t$$

The semi-strong form of market efficiency provides that in an uncertain world the market makes correct use of all relatively costless, relevant information available at $t-1$.
in its assessment of the distribution of $\hat{\Delta}_t$ with the market assessment manifesting itself in the value of the nominal returns that it sets. Put differently, if $\phi_{t-1}^m$ represents the information used by the market at $t-1$ in making its assessment of inflation over the subsequent period then market efficiency implies that $\phi_{t-1}^m$ is equal to $\phi_{t-1}$, the set of all costless, readily available market information relevant to forecasting inflation.

The market's expectation of the change in the purchasing power of money thus becomes

\[
Em(\hat{\Delta}_t/\phi_{t-1}^m, R_t) = Em(\hat{\Delta}_t/\phi_{t-1}, R_t) - R_t
\]

If we make the assumption that the market sets the price of a bill so that it perceives the expected real return to be

\[
Em(\hat{r}_t/\phi_{t-1}^m, R_t) = \alpha_0 + \gamma R_t
\]

then, upon the substitution and re-ordering of terms, we have;

\[
Em(\hat{\Delta}_t/\phi_{t-1}^m) = \alpha_0 + (\gamma-1)R_t = \alpha_0 + \alpha_1 R_t
\]

In equation (15), $\gamma$ can be taken to represent the proportion of a change in nominal rates attributable to changes in the expected real return. If the expected real return is independent of inflation ($\gamma = 0$) then any change in nominal yields reflects exclusively changes in inflationary expectations, i.e., the value of $\alpha_1 = -1$. 

90
Recognizing that $E(\hat{\Delta_t}/\phi_{t-1}, R_t)$ represents the regression function of $\Delta_t$ on $\phi_{t-1}$ and $R_t$, we can easily test for the values of $\alpha_0$ and $\alpha_1$ in (15) through the application of OLS to

$$\Delta_t = \alpha_0 + \alpha_1 R_t + e_t$$

If the expected real return is constant and the efficient markets hypothesis approximates the way financial markets actually do work, then $\alpha_0$ will be equal to $E(r)$ and $\alpha_1$ will be -1.

Unfortunately, as the test of these propositions as stated in (16) now stands, any regression of $\Delta_t$ on $R_t$ will constitute a test of these joint hypotheses. To circumvent this problem, a one-period lagged inflation value can be added to (16) and, if the market makes correct use of the implicit information in $\phi_{t-1}$ relevant to forecasting future inflation, then $E(\alpha_2)$ in 17 is equal to zero.

$$\Delta_t = \alpha_0 + \alpha_1 R_t + \alpha_2 \Delta_{t-1} + e_t$$

This is construed by Fama to serve as a test for the validity of the hypothesis of efficient markets, along with the autocorrelations of the disturbance terms which, if all systematic relationships among the relevant variables are taken into account in the setting of $R_t$, should be jointly equal to zero.

We now turn to the results obtained for our sample period. Of some interest is the fact that in the following analysis the complete Fama tests are replicated, in other
words the constant real rate and efficiency tests are combined, as in Fama, whereas other studies purporting to replicate the Fama tests for longer sample periods only deal with the basic test as presented in (16). This may be a fact of some significance as we shall see later. Also of particular interest is the fact that we are including a period of unprecedentedly high inflation for the American economy whereas Fama cut off his data at July 1971 (to avoid any possible distortions arising from the imposition of the Nixon price controls) thus omitting data from this important inflationary era. Although it should be noted that Mishkin (1981) argues that there is no reasonable justification for the a priori belief that the controls would have a very powerful distortionary influence.

Another aspect of our analysis worth noting is that by including data from the 1970s we will avoid a problem which has served as the basis of some criticism of Fama's work, the selection of a time period (1953-1971) which is seemingly unique in that there appears to have been a complete adjustment of nominal rates to inflation, a result not usually found when other time periods are considered. Our results for the regressions as given in equation (16) are provided in Table 2-7 below. Again, the series used were the one month maturity yields on U.S. T-bills and the monthly percentage changes in the non-seasonally adjusted Consumer Price Index. For purposes of comparison, the
results from the sub-period are included with those from the entire sample period.

TABLE 2-7

Regressions of Changes in Purchasing Power on Nominal Rates

\[ \Delta t = a_0 + a_1 R_1 \]

<table>
<thead>
<tr>
<th>Period</th>
<th>( a_0 )</th>
<th>( a_1 )</th>
<th>( s(a_0) )</th>
<th>( s(a_1) )</th>
<th>( R^2 )</th>
<th>( t(-1)* )</th>
<th>D-W</th>
<th>F-VAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>5309-7912</td>
<td>1.97</td>
<td>-1.40</td>
<td>0.32</td>
<td>0.07</td>
<td>0.57</td>
<td>-5.8</td>
<td>1.79</td>
<td>74.9</td>
</tr>
<tr>
<td>7403-7912</td>
<td>-0.03</td>
<td>-1.26</td>
<td>1.14</td>
<td>0.16</td>
<td>0.45</td>
<td>-1.6</td>
<td>1.81</td>
<td>68.7</td>
</tr>
</tbody>
</table>

* t-statistic based on the hypothesis that \( a_1 = -1 \).

We note that the value of the \( a_1 \) coefficient for the overall period is significantly different from \(-1\) (with a \( t \) value of \(-5.8\)) and thus indicates that changes in purchasing power were associated with less than proportionate changes in nominal interest rates. For the 1974-79 inflation era although the value of the coefficient is less than \(-1\) it is not statistically significantly so thus we cannot reject the Fama hypothesis that changes in nominal rates fully accommodate changes in expected inflation. Interestingly enough, these results seem to be reversed when the sample period is extended to include the early 1980s. In other words, it is not possible to reject the null hypothesis that the interest rate coefficient is significantly different from \(-1\) for the longer period (1953-1983) while rejection is
possible for the shorter period (1974-1983) (see Table IV of Appendix 5). However it is difficult to attach great significance to this finding as the other statistics indicate once more the weakening of the co-movements between nominal rate changes and changes in the purchasing power of money. A comparison of the two longer periods shows that the adjusted R-squared value falls from .57 to .20 while this value falls from .45 to only .02 when the periods are extended by an additional four years. This apparent weakening of the relationship of the two variables is confirmed by an equally dramatic decline in the F-statistics (not shown) from 414.2 to 93.2 and 57.9 to 2.9 respectively. As noted earlier, as this analysis implicitly involves the testing of joint hypotheses, it may be difficult to determine on the basis of these results alone the extent to which this apparent underadjustment during the 1953-79 era represents a falling real rate on the one hand or the failure of markets to operate "efficiently" on the other. Nevertheless, it is possible to devise a test that would isolate the two phenomena, one based on the measurement of the serial correlation of the disturbance terms in (16) and (17). As was noted above, serial correlation of the errors would imply that there is some systematic relationship among the variables that the market is not taking into account when setting bill rates.

A different but obviously related approach, discussed earlier, to test for market efficiency involves the
measurement of the coefficient on a one period lagged value of $\Delta_t$. If all information relevant to the determination of future inflation, some of which information being embodied in past inflation, is used correctly we would expect that the coefficient on $\Delta_{t-1}$ would not be significantly different from zero. However, this hypothesis is meaningful only if past inflation does indeed provide information concerning future price level changes. The autocorrelations of $\Delta_t$ for 12 lags for each period presented below provides evidence that this is the case, at least for the overall period.

| TABLE 2-8 | 
| Autocorrelations of $\Delta_t$ | 

<table>
<thead>
<tr>
<th>$\hat{\rho}_1$</th>
<th>$\hat{\rho}_2$</th>
<th>$\hat{\rho}_3$</th>
<th>$\hat{\rho}_4$</th>
<th>$\hat{\rho}_5$</th>
<th>$\hat{\rho}_6$</th>
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<th>$\hat{\rho}_9$</th>
<th>$\hat{\rho}_{10}$</th>
<th>$\hat{\rho}_{11}$</th>
<th>$\hat{\rho}_{12}$</th>
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</thead>
<tbody>
<tr>
<td>.64</td>
<td>.66</td>
<td>.60</td>
<td>.59</td>
<td>.57</td>
<td>.57</td>
<td>.54</td>
<td>.55</td>
<td>.57</td>
<td>.55</td>
<td>.52</td>
<td>.56</td>
</tr>
</tbody>
</table>

1953-1979

| .67   | .49   | .49   | .38   | .23   | .25   | .24   | .09   | .27   | .25   | .18   | .21   |

1974-1979

$\sigma(\hat{\rho}) = .06(53-79)$  $\sigma(\hat{\rho}) = .13(74-79)$

$Q(302) = 2168.0$  $Q(71) = 308.9$

The values are all positive and in each of the 12 cases are more than two standard errors from zero. Furthermore, the extremely high value of the Q statistic (2168.0) provides additional reason for strongly rejecting the null hypothesis.
of no autocorrelation as the critical value at the .01 level is 26.22. For the sub-period, in only 5 of the 12 cases is the autoregression coefficient more than two standard errors away from zero which would indicate that during this time past inflation did not provide much information useful to the determination of future inflation. This may have something to do with the fact that inflation's variance tends to increase with its absolute level and thus during this period of rapidly rising prices recent past inflation rates became unreliable indicators of future inflation (the empirical evidence for the positive association between inflation's level and its variance is discussed in Appendix 2 to this chapter). However it is necessary to note that the auto-correlation evidence for the sub-period is rather mixed as the Q statistic (308.9) is far above the critical level at the highest listed confidence values. The Q-values drop when the period of analysis is extended to 1983 but are still far above the critical levels. Also, for the 1953-83 period, 11 of the 12 autocorrelations are statistically significant whereas 7 of 12 are the for 1974-83 sub-period (see Table VII of Appendix 5). Thus, the overall results are not substantially different.

The market apparently did not make proper use of the information in past inflation rates in assessing the distribution of the $\Delta_t$'s, at least as far as the Fama test is concerned. Table 2-9 gives the results of the regression
tests which include the one period lagged inflation measure as an explanatory variable.

TABLE 2-9
Regressions of Changes in Purchasing Power on Nominal Rates and Lagged Dependent Variable Values

\[ \Delta_t = a_0 + a_1 R_t + a_2 \Delta_{t-1} + e_t \]

<table>
<thead>
<tr>
<th></th>
<th>( a_0 )</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( s(a_0) )</th>
<th>( s(a_1) )</th>
<th>( s(a_2) )</th>
<th>( R^2 )</th>
<th>D-W</th>
</tr>
</thead>
<tbody>
<tr>
<td>5309-7912</td>
<td>-1.58</td>
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<td>.20</td>
<td>.33</td>
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</tr>
<tr>
<td>7403-7912</td>
<td>.09</td>
<td>-.76</td>
<td>.40</td>
<td>1.04</td>
<td>.20</td>
<td>.10</td>
<td>.53</td>
<td>1.72</td>
</tr>
</tbody>
</table>

For both periods the value of \( a_2 \) is significantly different from zero (with a t-value of about 3.75 in each instance) thus contradicting the results expected in an "efficient" market or at least as so interpreted by Fama. However the corrected R-squared values rise only marginally when this one period lagged variable is added as a regressor indicating, of course, that this variable adds little to the explanatory power of the model. This result was confirmed by comparing the F-values (not shown) for each set of equations.

In summary the results obtained by extending the sample period of Fama's original analysis provide some evidence for the view that not only do nominal interest rates not fully accommodate changes in expected inflation but that the market does not make complete use of the information.
implicit in the past values of inflation in assessing the probability distribution of short-term future inflation rates. This is a rather surprising result as the implication is that the "weakest" form of market efficiency appears to be contradicted. The results for the inflationary sub-period are rather more mixed.

For the 1953-1983 and 1974-1983 periods we find similar results but once again the relationship seems to be a bit weaker. It is interesting to note, however, the robustness of the finding that the market is apparently not taking full account of the informational content of lagged inflation rates in the setting of current nominal rates. One could conceivably argue that financial market participants were subject to inflation illusion throughout most of the 1970s, thus accounting for the results obtained earlier. However that argument would be far too implausible as the period of analysis is extended well into the 1980s.

These results, however, do open up the intriguing (if perhaps not too likely) possibility that the failure to replicate Fama's results by Mishkin et. al. may not be due to large variations in expected real rates but rather to the failure of markets to operate efficiently, or at least the U.S. T-bill market. To my knowledge, this point has been totally neglected in the literature. Recall that in the context of the Fama tests, obtaining estimates of the constant terms and of the coefficients on nominal yields in either model allows testing of the hypothesis that the equilibrium expected real return is constant through time
while obtaining estimates of the coefficient on lagged inflation and of the disturbance terms provides a means to test for market efficiency. This dichotomization of tests is absent in the Mishkin analysis. Rather, what he does is to assume market efficiency at the outset and then proceed to make inferences about the movements of real rates based on this assumption. However, looking at the values of the coefficients on nominal T-bill rates in the model which includes one period lags of the changes in the purchasing power of money as regressors, we see that the values are -1.11 and -1.02 for the 1953-79 and 1953-83 periods respectively (this latter value can be found in Table VIII of Appendix 5) indicating that the null hypothesis of their being equal to -1 cannot be rejected (given a standard error of .10 in each instance) and thus offering evidence in favor of constant real rates within the context of the Fama analysis. Additionally, the coefficients on the lagged inflation values for each period show up significantly negative with t-values of approximately -4 and -3 respectively.

Nevertheless, the combination of variable real rates and efficient markets seems much more likely than one of constant real rates and financial markets that do not operate efficiently. The whole issue centers on the appropriateness of the market efficiency test that Fama used. For instance, Nelson and Schwert (1977) deny the validity of Fama's implicit assumption that most information...
relevant to the determination of future inflation is contained in one-period lagged inflation values, an assumption upon which the autocorrelation tests discussed above are based. Utilizing a more elaborate univariate time series model as a proxy for an 'optimal' predictor of inflation, they find instead that past inflation rates contain almost no information useful in assessing future inflation which implies that the Fama approach does not constitute a very powerful test of his hypothesis of market efficiency. However implicit in their criticism is the notion that higher-order ARMA or ARIMA modeling of inflation leads to better 'predictors' of inflation than a simple first-order autoregressive model, and the evidence on this question is at best mixed as it depends on the time period being considered. Although these questions of market efficiency and the adequacy of univariate time series predictors are very contentious and beyond the scope of the present work, it should be pointed out that it is vital to Fama's analysis of market efficiency that the expected real rate of interest be regarded as a constant. If real yields have exhibited considerable variation then Fama's simple test of the market efficiency notion is inappropriate. However, as discovered earlier, the evidence on the question of real rate constancy during the 1974-79 period is mixed. In all fairness to Fama, however, he has recently re-evaluated his earlier work regarding real rate constancy (in response to overwhelming criticism of his initial tests) in a series of papers incorporating a model which provides for
a negative relationship between expected inflation and the real pecuniary rate of return on financial assets as in Mundell-Tobin. However, the underlying processes generating this negative correlation are quite different from theirs. In a Mundell-Tobin world, increases in expected inflation (which are associated with high nominal interest rates) induce individuals to hold smaller money balances and shift into interest-bearing securities thus driving down the interest rates on these latter assets. The reduction in capital costs generates increased expenditures on capital thereby reducing the marginal returns on these production goods and providing for the negative relationship referred to above.

In Fama's more recent work, however, the processes involved in producing the negative association between expected inflation and real returns on financial assets are substantially different. Rather than having the increase in expected inflation drive down the expected real return more or less directly, the capital expenditure process has a much more fundamental role to play in this latter analysis. More specifically, an increase in the level of economic activity with a given capital stock will result in, of course, a rise in the average output of capital and thus in the expected real return on capital, at least initially. However, if the increased demand for real money balances in response to higher economic growth is accommodated more by a change in prices than in nominal money growth (as his empirical
evidence indicates has been the case in the U.S. from 1953 to the early 1980s) then the partial correlation of economic growth rates and inflation will be negative. Assuming some substitutability between real and financial assets, the negative association between economic activity and inflation (and thus expected inflation) in the economy's monetary sector when combined with the real sector phenomenon of a positive association between real economic activity and the expected return on assets will generate the observed inverse relationship between expected inflation and expected real returns. The foregoing processes explain, according to Fama, the prevalence of the stagflation phenomenon in the U.S. economy throughout most of the 1970s.
VI. A Critique of the Fama Stagflation Model

The question arises as to what are the implications of the Fama analysis for the present work. The basic theoretical foundations and empirical support for his thesis are laid out in Fama (1982). He essentially takes the standard demand function for real money balances expressed in log differences and turns it into a model of inflation by assuming the exogeneity of money, the level of economic activity and the "largely" exogenous nature of nominal interest rates with respect to prices. He then goes on to empirically test for certain values of the parameters in the model (with time subscripts of the independent variables adjusted to conform with the rational expectations approach he takes), the most noteworthy being those on the level of economic activity. The model predicts a negative association between changes in economic activity (and by implication changes in real expected returns on real and financial assets) and movements in the level of prices. This is, of course, the stagflation phenomenon. Assuming almost perfect substitutability between capital and financial assets (an assumption which, it will be remembered, is challenged by the liquidity model), a less than full adjustment of nominal interest rates to inflation can be inferred from his analysis.

There may, however, be reason to question Fama's finding of a negative correlation between these variables. First, the power of his statistical tests is probably very low due to the small number of degrees of freedom in his
analysis. He estimated the various relationships using annual changes in the variables for 1954-77 giving him only about 20 degrees of freedom depending on the particular model being estimated. Re-estimation in OLS form over the comparable sample period and using in this case monthly data for both money growth and either the industrial production index or real personal income, showed that only the regression coefficient on contemporaneous real personal income was both significant and of the correct sign. For the regressions using the index of industrial production proxy, the only significant coefficient was on the one period lagged value and this was positive. Correcting for an AR(1) process, as the low D-W values of 1.6 and 1.3 respectively indicated was necessary, did not alter these basic findings nor did focusing the analysis on the decade of the 1970s, which is generally taken to be the era characterized by the most severe stagflation. While the results with the IPI values clearly contradict the Fama thesis we should be very careful about placing a favorable (to Fama) interpretation upon those results obtained by using real personal income as the economic activity proxy. The rational expectations paradigm that he utilizes posits that the demand for real money balances at time $t$ is functionally related to real transactions expected to occur in the near future implying, of course, that real money demand is forward-looking with respect to real economic activity. Because inflation will depend on actual money
growth rates, when the model is changed from one of real money demand to one of price level changes, the implication is that expected inflation will also be forward-looking with respect to real activity. Thus, in terms of the empirical analysis, it follows that the most significant regression coefficients should be found on the leading values of real activity and not on the current or lagged ones. However, it is precisely on these leading variable values that we find the least significant coefficients. As a typical example, for his 1954-77 period using the Cochrane-Orcutt serial correlation correction technique, the t-value on the one period leading value of real personal income coefficient was only -.159. To determine if this result was due to the selection of an inappropriate lead length, the 2 and 3 month leading values were added to the regression equations. These values showed up equally insignificant (for example in the equation described above, the t-values were -.18, -.18 and -.24 respectively for the 1, 2 and 3 month lead value coefficients). Clearly these results do not support the Fama contention of the negative association between economic activity and price level changes and thus, by implication, the inverse relationship between inflation and the expected real returns on financial assets.

VII. A Critical Discussion of Other Real Rate Models and Conclusion

This chapter has dealt with the actual inflation-interest rate relationship in the American economy during the period of interest in both the short and the long runs,
in other words the contemporaneous relationship and the co-
movements of the variables measured by applying moving
average filters whose adjusted length approximates or
exceeds that of the standard reference cycle. Not only does
the evidence presented indicate the failure of nominal rates
to fully adjust to changes in inflation over the short and
long runs, but there seems to be no evidence for the
influence of variables commonly thought to impinge on real
rates to do so, save for the notable exception of one period
lagged inflation rates. For the short run, these results
for the ex post real rates are presented in Table 2-3 and
are confirmed by Mishkin (1981) (who used a wider range of
variables including real GNP growth, investment to capital
ratios and the GNP gap along with several others) in the
sense that there seems to be no evidence to support the
hypothesis that these other potential explanatory variables
can be used to explain variations in real rates. Even more
relevant for the present study is the fact that Summers
(1983) was unable to account for the underadjustment of
nominal rates to inflation in the long-run by employing the
Engle band spectrum techniques discussed in the text to
variables that proxied for the real returns on capital (both
pre-tax and after-tax) and for risk elements (see page 227
in Summers (1983) for a complete discussion of these proxy
variables).

Of course, it is possible that other variables not
incorporated in either the Mishkin or the present analysis
could conceivably account for variations in real rates, at least in the short run. For instance, the relatively recent emphasis on rational expectations and efficient markets models has led researchers to test for the effects of changes in unanticipated money on real rates. However, Mishkin (1982) concludes that there is no empirical support for the view that unanticipated increases in money are negatively correlated with unanticipated changes in short term interest rates (an earlier paper reached the same conclusion using long term rates), instead finding that the impact was not significantly different from zero. For the pre-October 1979 period (before the Federal Reserve policy shift) the same conclusion is reached by Roley and Walsh (1984). Furthermore, these latter authors were unable to find evidence for a non-zero impact of pre-announced money supply changes on interest rates, another area of research interest for those testing models with rational expectational assumptions. Thus any attempt to explain movements in real rates by invoking the impact of surprises in monetary growth does not seem to be at all consistent with the evidence.

An attempt to account for some of the observed variations in real rates, specifically the decline in real rates to negative levels during the 1970s, based on real factors was done by Wilcox (1983). Basically his argument is that the negative energy supply shocks (and the concommitant rise in energy prices) beginning with the O.P.E.C. oil embargo tended to reduce the demand for the
complementary factors of production, notably real capital. The declining demand for capital would thus, ceteris paribus, put downward pressure on real interest rates. Although his empirical results confirm Fama (1976) and Mishkin (1981) in the finding of the predominant influence of changes in expected inflation on real rate movements, he finds statistical support for the view that supply forces alone pulled down real pre-tax interest rates by a full 1.7 percentage points from 1973 through 1979. Nevertheless, however valid the Wilcox argument may be for his sample period, it does not account for the underadjustment of nominal rates to inflation that occurs long before the oil supply shock of 1973, nor does it explain the underadjustment of nominal rates to inflation after the elimination of the two-tier pricing system for oil by the Carter administration in 1979 to the end of 1983, when neither insufficient energy supplies nor high energy prices (in relative terms) continued to be an important factor in the U.S. economy.

Although it is presently quite fashionable in the U.S. (mostly among non-economists) to posit a strong direct causal relationship from federal budget deficits to real interest rates, there appears to be scant empirical support for this notion (see, for example, Barro (1985)). Nevertheless, there has been some recent work which may help shed some light on the whole question of real rate variability, the empirical results of which match those predicted by F-H. A re-interpretation of the standard
Fisher relationship made by Carmichael and Strebbing (1983) involves regressing the after-tax ex post real interest rate (with variable marginal tax rates) on actual inflation. They begin the analysis by stating that the Fisher hypothesis is one concerning the relationship between the returns on real assets and inflation (which is an arguable point) although financial data are almost always used in the various analyses as data on real capital returns are very difficult to come by. Assuming the unbiasedness of inflationary expectations, their particular "inverted" (sic) form of the Fisher equation simultaneously allows for the incorporation of the Darby analysis while avoiding the errors in variables problem they claim is usually associated with the use of actual inflation as a proxy for expected inflation. They go on to claim that the unique aspect of their approach is that, with the given assumptions, it is possible to dichotomize the impact of a change in inflation into its influence on expected real returns and on the coefficient on inflation (the beta coefficient). Their findings for both the U.S. and Australia from the early 50s to the late 70s indicate that "the impact of inflation has fallen dominantly on real rates of return with little influence on nominal interest rates in either the short-run or the long run" (p. 629) and go on to say "While this evidence provides one explanation of the Fisher paradox, it leaves open the possibility that Fisher's hypothesis may still hold for real assets such as capital" (p. 629). Of course, both statements are entirely consistent with the F-H
analysis. Additionally, on page 625 they discuss the upward trend of a variable in their model, which they interpret as the real risk premium on financial assets, attributing its rise to the growth of implicit interest payments on money during this time, but holding open the possibility of alternative explanations. One possible explanation is that this value is not a risk premium at all but rather a liquidity premium embedded in the real returns on financial assets which has been steadily growing in the face of increased inflation.

To enable us to more clearly understand real rate variability, all within the context of the F-H analysis, direct testing of some of the model's more important implications is undertaken in the chapter to which we now turn.
FOOTNOTES


2. Let the OLS estimate of \( \beta \) for the ex ante model (equation 4) be denoted as \( \hat{\beta} \) and the estimate of \( \beta \) for the ex post form (equation 5) be denoted as \( \beta \). The least squares estimators of from the matrix form of the multiple regression model are the following:

\[
\begin{align*}
(a) \quad E(\hat{\beta}) &= E(X'X)^{-1} X'(X\beta + u) = \beta + E[(X'X)^{-1}X'u] \\
(b) \quad E(\beta) &= E(X'X)^{-1} X'(X\beta + u - e) = \\
&= \beta + E[(X'X)^{-1}X'u] - E[(X'X)^{-1}X'e]
\end{align*}
\]

Because of the rationality of inflationary expectations as assumed in equation (3) (\( E(\hat{e}) = 0 \), \( E(X'e) = E(X)E(e) \)) is equal to zero in equation (b) leaving us with the result that \( E(E(\hat{\beta})) = E(\beta) \). Although the technique of using ex post real rate regressions presents certain technical problems which are discussed fully by Mishkin, for purposes of the present study we will agree with him that this particular method of analysis offers a "dependable way of inferring information about real interest rates" (p. 158). In particular it should be noted that a potential difficulty arises in that making additional assumptions concerning the absence of serial correlation of both the error term in (4) and the joint error term in (5) it can be shown that the variance-covariance matrices will differ by an amount determined by the variance of the forecast errors of inflation. If these forecast errors are large as some researchers believe (e.g., Nelson and Schwert (1977) then the estimates of the \( \beta \)'s from the ex post regressions will be far less precise than they would have been had they been derived from the regression of the equation in the ex ante form as the power of the statistical tests will be so low. For a more formal discussion of this problem the reader is referred to Mishkin (1981) p. 156-7.

3. In practical terms, however, this is probably a moot point as the value of the coefficients along with the relevant statistics are not substantially different using continuous compounding of inflation (i.e. \( \ln(\frac{CPI_{t}}{CPI_{t-1}}) \)) as opposed to the use of discrete values.
4. In formal terms this is true because if the correct assessment of the expected value of \( r_t \) is \( E(\tilde{r}_t^P) \), then for any \( r_{t-k}^P \)
\[
E(\tilde{r}_t^P/r_{t-k}^P) = E(\tilde{r}_t^P)
\]
i.e., there is no way to use a past value of the ex post real rate \( (r_{t-k}^P) \) to formulate an expectation of the value of the real rate at time \( t(\tilde{r}_t^P) \) which is different from \( E(\tilde{r}_t^P) \). An alternative method of stating the same proposition is that the regression function of \( r_t \) on \( r_{t-k}^P \) is the constant \( E(\tilde{r}_t^P) \). If the regression function \( E(\tilde{r}_t^P/r_{t-k}^P) \) is linear in \( r_{t-k}^P \), i.e.
\[
E(r_t/r_{t-k}^P) = \gamma + \alpha_k r_{t-k}^P
\]
then the autoregression coefficient for lag \( k \) is given by \( \alpha_k \) (where \( \gamma \) is a constant). Thus, constancy of the real rate implies that the autocorrelation or autoregression coefficient will be zero for all \( k \).

5. The measurement of the standard error of the autocorrelation is given by
\[
\sigma(\hat{\rho}) = \left( \frac{1}{n-k} \right)^{1/2}
\]

6. It should be noted that Summers (1983) failed to find any strong evidence for the influence of other potential explanatory variables on real interest rates even when allowing for longer-run influences.

7. A note of caution must be injected here. As Sargent (1973) points out, a simple regression of the interest rate on an inflation value to test for the Fisher relationship implicitly places severe macroeconomic restrictions on the fundamental model that Fisher used in his own empirical work. Recall that Fisher posited that expectations of inflation are formed by taking a weight sum of past and current actual rates of inflation, i.e.
\[
\pi^e = v_i (\log P_{t-i} - \log P_{t-i-1})
\]
where the \( v_i \)'s represent the weights used.
One restriction implied in standard tests of the Fisher effect is that these weights sum to unity, an inappropriate assumption if actual inflation is governed by a Markov process, for instance. For example, if actual inflation is determined by the following relationship;

\[ \log P_t - \log P_{t-1} = 0.4(\log P_{t-1} - \log P_{t-2}) + U_t \]

where \( U_t \) is an unpredictable random variable with an expected value of zero and expectations are rational in the sense that people incorporate this observed relationship into their information set to be used when forming expectations of future inflation, then the following would obtain,

\[ \pi_t^e = 0.4(\log P_t - \log P_{t-1}) \]

and the weights would not sum to one. An additional restriction involves the assumption of independence of the expected inflation variable \( \pi_t^e \) and the error term \( (U_t) \) in the basic Fisher equation \( R_t = \alpha + \pi_t^e + U_t \).

A fiscal variable such as government purchases, for example, would tend to be positively related to both the interest rate and the price level (and thus \( \pi_t^e \)) thus contradicting the assumed orthogonality of \( \pi_t^e \) and \( U_t \), implicit in most empirical studies of the Fisher effect. Summers (1983) also addresses this issue.

8. The value of the coefficient on inflation after the corrections is 0.424 with a t-value of 5.20, \( R^2 = 0.123 \), \( F(1, 314) = 45.25 \) and the D-W = 1.62.

9. A critique of this general OLS approach to test for the Fisher effect is offered by McCallum (1983). Suppose inflation and interest rates are generated by the following processes:

\[ R_t = r_t + E_{t}P_{t+1} + v_t \]
\[ p_t = A1 + A2 p_{t-1} + e_t \]

Here \( E_{t+1} \) is the conditional expectation of \( P_{t+1} \) given that all relevant variables in forming expectations at time \( t \) of future inflation are taken into account, i.e., expectations are rational. Thus, the first expression represents the case in which the Fisher effect holds in full and in which the real interest rate fluctuates randomly (as a result of the disturbance term \( v_t \)) around a constant mean value of \( r_t \). Additionally, the rate of inflation is assumed to be exogenous and generated by an AR(1) process; \( e_t \) is white noise and independent of all values of \( v_t \). Thus,
the expected inflation rate is $A_1 + A_2 \ p_t$ so the true relationship between interest rates and inflation is;

$$R_t = (r_t + A_1) + A_2 \ p_t + \nu_t$$

If OLS is used to estimate the above, the slope coefficient corresponding to $\beta$ would take on the value $A_2$ and a researcher using this approach would conclude that the Fisher effect does not hold even though it is explicitly built into the model. His argument however may not be applicable for the sort of long run testing that Summers undertakes.

10. It should also be noted that the true relationship is $r_t = R_t + \Delta r_t + R_t \ \Delta t$, however the last term is far too small to be of any practical significance.

11. See for example Fama and Gibbons (1982).
APPENDIX 1

The Inflation-Interest Rate Relationship
Under the Lucas Approach

Presented below are scatter plots of the filtered values of inflation and nominal T-bill rates for the 1953-1979 period using the Lucas filtering technique discussed in the text above. Our purpose will be to see if the relationship between the two series is a close one and, if so, do the points fall on a 45 degree ray from the origin (which would indicate a 1 for 1 adjustment of nominal rates to inflation) or below the line (which would be evidence of a less than unit relationship in the longer run). Following Lucas (1980) a two-sided exponentially-weighted moving average for each series i is given by:

$$\hat{X}_i(t) = \alpha \sum_{k=-\infty}^{\infty} \beta^{|k|} x_{i,t+k}$$

where $\beta = \text{weight used (} 0 < \beta < 1 \text{)}$ and $\alpha = 1 - \beta / 1 + \beta$. Strictly speaking this expression is appropriate only for use with the infinite record of each variable of both past and future values. Although Lucas utilized a specialized algorithm to circumvent this particular technical problem, it was not used in the present analysis as the straightforward technique of using a finite number of lagged and leading values was thought to provide an approximation sufficient for our purposes, that of illustrating the general tendencies of the two variables to move together with short-
term influences pretty well removed. (See Figures Al-1, Al-2, and Al-3.

The results offer confirmation of our basic assumption of a close and less than one-for-one relationship between inflation and nominal rates with the results being more significant the higher the weight used (i.e., the closer each series approaches its sample average). OLS regressions forcing the line through the origin showed the measured slopes to be .804 and .877 respectively for the .5 and .8 weighted series with the two year values and .881 for the .8 weight with the four-year filter. The numerals in interest rate-inflation space represent the number of observations at that point.
Figure A1-1

LUCAS EXPONENTIALLY-WEIGHTED CENTERED MOVING AVERAGE TECHNIQUE
WEIGHT=.5 FILTER LENGTH=2 YEARS

INTEREST RATE

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<tr>
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<th>5.410</th>
<th>-</th>
<th>5.051</th>
<th>-</th>
<th>4.692</th>
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</tr>
</tbody>
</table>
Figure A1-2

LUCAS EXPONENTIALLY-WEIGHTED CENTERED MOVING AVERAGE TECHNIQUE
WEIGHT=.8  FILTER LENGTH=2 YEARS

INTEREST RATE

INFLATION

-0.975  0.211  1.529  1.645  4.165  5.483  6.801  8.119  9.437  10.755  12.073

0.775  1.66  2.44  3.23  4.02  4.81  5.60  6.39  7.18  7.97  8.76
Figure A1-3

LUCAS EXPONENTIALLY-WEIGHTED CENTERED MOVING AVERAGE TECHNIQUE

WEIGHT = 0.8 FILTER LENGTH = 4 YEARS

INTEREST RATE

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</tbody>
</table>

INFLATION

|-------|--------|-------|-------|-------|-------|-------|-------|-------|---------|--------|--------|
APPENDIX 2

The Level of Inflation and its Variance

The key finding from our empirical analysis of the inflation-interest rate relationship in both the long and short run was the negative association between inflation and real interest rates, a result entirely consistent, as we have seen, with the predictions of the liquidity model. However, this empirical evidence taken alone is not a priori inconsistent with the hypothesis that the market is incorporating an "inflation risk" premium in security returns, i.e., that the inverse relationship between real pecuniary yields and inflation arises from risk as opposed to liquidity considerations. However, if the variance of inflation is an appropriate measure of real return risk and if it is positively related to inflation's level then we should expect to find rising real pecuniary yields in response to increases in inflation. The question of the relationship between inflation's variance and its absolute level is considered in the section to which we now turn.

A straightforward approach to this question involves simply measuring the means and standard deviations during the sample period and then examining a scatter plot of the two series. A yearly average of monthly values combined with a standard deviation within the same year provided one point. The scatter plot is presented below, (Figure A2-1).
Figure A2-1

MEAN AND STANDARD DEVIATION OF INFLATION 1953-1979 (5 PER YEAR)

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<th>Mean of Inflation</th>
<th>Standard Deviation</th>
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<tr>
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</tr>
</tbody>
</table>

Figure A2-2

MEAN AND STANDARD DEVIATION OF INFLATION: QUARTERLY PERCENTAGE CHANGES IN U.S. (Ft. 2 YEAR MEASUREMENT PERIOD 1953-1979)

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<thead>
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<th>Standard Deviation</th>
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<tbody>
<tr>
<td>6.209</td>
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<tr>
<td>9.315</td>
<td>9.315</td>
</tr>
<tr>
<td>10.080</td>
<td>10.080</td>
</tr>
</tbody>
</table>
The plot above provides evidence in favor of a loose positive association between the two variables. The simple correlation coefficient of .58 provided some additional evidence in favor of a positive relationship. Additionally, the relationship appears to be a bit stronger when focusing on the 1960s and 1970s (the correlation coefficient is .69) when the absolute levels of inflation were higher, as easily seen in the scatter plot. An alternative measure of inflation's variability used by Gale (1981) which he calls the "average acceleration" of inflation and which is simply the average of the absolute values of monthly changes in the inflation rate over one year was tried as well with not very meaningful results (the simple correlation coefficient was .11).

Correlation statistics for quarterly values of inflation with biennial measurement periods were obtained with the correlation value for inflation as represented by the percentage change in the consumer price index (.68) being higher than that for the percentage change in the GNP implicit price deflator (.29) over the same period, (Figure A2-2).

The foregoing results based on elementary tests of the variability-level relationship gives some credence to the hypothesis of a positive association, however the relationship is not quite as strong as that found by Taylor (1981) or Gale (1981). Rather the results match more closely those of Klein (1975) for the U.S. in his cross-country study. Furthermore, as in Gale (1981), the
association becomes even less significant with the use of the average acceleration measure of inflation's variability. The important point for our purposes is that the data unambiguously reject the notion of a negative relationship, a finding which tends to discredit any explanation of the underadjustment of nominal interest rates based on the notion the market is incorporating in security returns a premium paid to holders of financial instruments as compensation for bearing greater real return risk in the face of increased inflation. This is not to argue that risk elements are unimportant in the determination of nominal interest rates but rather "their role is secondary to that of transactions costs" (F-H, p. 977).
APPENDIX 3

The Cross Spectral Evidence on the Inflation-Interest Rate Relationship

In the following section the technique of cross-spectral analysis is utilized to more closely determine at which frequency or frequencies (number of cycles per unit time period) is the association between interest rates and inflation the strongest. Recall that both the Summers (1981), (1983) and Lucas (1980) approaches essentially involve the pre-smoothing of the inflation and interest rate data before running OLS regressions (as in the former approach) or eyeballing scatter plots of the filtered series (as in the latter) to look for the presence of any longer run relationship between the variables. Although either technique will pick up any powerful longer run association each lacks precision in the sense of determining the degree of association at many different specified frequencies. For instance, recall that in the Summers approach if one wishes to test for, say, the relationship between two series with a 5-year moving average filter applied to both series, co-movements up to almost 10 years are also eliminated. On the other hand the empirical cross-spectrum provides an estimate of the co-variance occurring between two series at each frequency band. Although formal cross spectral analysis has been used to test for a number of long run economic relationships (e.g. money growth and inflation by Geweke
(1982), as far as I know this is its first application to test for a longer run Fisher effect.

The important values in cross-spectral analysis are the coherence, phase shift and gain. Coherence provides a measure of the linear association between the frequency components of two series and can be regarded as the counterpart in spectral analysis to the coefficient of determination in simple correlation analysis. The plot of coherence against frequency is called the coherence diagram.

The phase (or phase shift, phase angle) statistic provides an estimate of the average lead or lag of one series over another at each frequency band under study. A positive phase value would indicate that the base series has led the crossed series while a negative value the opposite. An important feature concerning the relationship between the coherence and phase spectra is that the meaningfulness of phase statistics varies directly with the level of the coherence values. Thus it is appropriate to infer a lead-lag relationship only at the frequencies with the higher coherence values providing, of course, that the phase values are large in absolute values as well.¹

Finally, gain is a measure of the amplitude differences of the components at each frequency and can be regarded as the regression coefficient obtainable by regressing the crossed series (dependent variable) on the base series (independent variable).

For the analysis, the base series was the one month percentage changes in the non-seasonally adjusted U.S.
consumer price index while the one month maturity yields on U.S. T-bills served as the crossed series. The period covered is from July 1953-December 1979 which comprises 318 months. Thus, the number of observations available far exceeds the minimum required for meaningful spectral results, usually estimated to be about 200. Most economic time series are non-stationary and have most of their power concentrated in the lower frequencies. Non-stationarity arising from powerful upward trends for both series over the sample period was found to be a problem, a not very surprising result. To attenuate this difficulty first differencing was applied to transform the data into a reasonable approximation of stationary series. The approach is identical to that of Sargent (1969) who used first differences in his cross spectral analysis to compare co-movements of different interest rate series throughout the 1950s.2

The statistical computer package used for the analysis was the Bio-Medical Diagnostics Package (BMDP) a feature of which is the automatic selection of 3 different bandwidths, with the smallest bandwidth having the greatest amount of "resolving" power. The undesirable aspect of a small bandwidth is the large variance associated with it. Of course the opposite is true for a large bandwidth. However as the results were very similar across the varying bands only one (the middle one) was chosen for presentation
purposes. The phase, coherence and regression values for the representative bandwidth are provided below.

**TABLE A3-1**

Cross Spectral Analysis of the Inflation-Interest Rate Relationship 1953-1979

<table>
<thead>
<tr>
<th>Period</th>
<th>Phase</th>
<th>Coherence</th>
<th>Regression Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>26.8 years</td>
<td>-.000</td>
<td>.901</td>
<td>.597</td>
</tr>
<tr>
<td>13.4 years</td>
<td>-.001</td>
<td>.884</td>
<td>.597</td>
</tr>
<tr>
<td>6.7 years</td>
<td>-.004</td>
<td>.888</td>
<td>.594</td>
</tr>
<tr>
<td>4.5 years</td>
<td>-.004</td>
<td>.876</td>
<td>.585</td>
</tr>
<tr>
<td>1.5 years</td>
<td>-.083</td>
<td>.309</td>
<td>.335</td>
</tr>
<tr>
<td>1.0 years</td>
<td>.102</td>
<td>.003</td>
<td>.021</td>
</tr>
<tr>
<td>.67 years</td>
<td>.055</td>
<td>.015</td>
<td>.025</td>
</tr>
</tbody>
</table>

* one month maturity yields

The results indicate that the coherence relationship sharply diminishes in strength at periods under about 4 years, peaking at the longest period of 26.8 years and thus providing some evidence in favor of the proposition that the closest association between inflation and interest rates is in the low frequency movements. The regression coefficients are at their highest values when coherence is relatively large and in all cases is just under .6 (this is true for the other frequency bands as well). It should be pointed out that the magnitudes of the regression coefficients are
very close to those found by Summers (1981) for the post
WWII period (1947-1979) and confirms our earlier finding of
a less than 1 for 1 adjustment of nominal rates to inflation
even when the longer run is taken into account.

It was also interesting to discover that no clear
lead-lag relationship was observable at any range of
frequency values. The phase angle statistics at frequencies
corresponding to the highest coherence values were quite
small and the high phase values appeared only sporadically
and then were associated with low coherence values and thus
were not meaningful.

FOOTNOTES

1. Although Hause (1971) cautions the reader against
interpreting phase information in this manner when
economic time series are being discussed.

2. Smith and Marcis (1973) point out however that first
differencing may amplify components with high
frequencies and attenuate those components with low
frequencies.
APPENDIX 4

The Time Series Evidence on Nominal Rates and Inflation

The following section represents an attempt to more closely determine the actual pairwise relationship between price level changes and nominal yields using time series techniques. The basic notion here is that prediction errors from ARIMA models of time series are estimates of the series' "innovations," in other words that part of each actual observation that is not predicted by past observations in the series. The innovations from two different series using both lagged and leading values are then correlated to determine if innovations from series x, for instance, "predict" innovations in series y or vice versa. This analysis is comparatively more sophisticated (and hopefully more meaningful) than the simpler Sims test which attempts to determine causality through OLS regressions of the "independent" variable on a few lagged and leading values of the "dependent" variable.\(^1\) The question of the general lead-lag relationship between interest rates and inflation was indirectly dealt with earlier within the context of Fama's work with his empirical findings that expected monthly inflation rates are fully incorporated in nominal yields (assuming a fixed real rate) set at the beginning of the month with the consequence that short-term interest rates are good predictors of inflation. Other studies (e.g., Nelson and Schwert (1977)) determined
that interest rates "Granger cause" inflation but as Schwert (1979) points out this interpretation may be misleading if efficient assessments of expected inflation are embodied in nominal yields so that interest rates adjust to different values of expected inflation, as Fama (1975) argues is the case. If this latter interpretation is accepted then it is proper to say that "predictable movements of inflation cause movements in the interest rate in the usual sense of the term".2

The following analysis seeks to determine whether inflation has led interest rates or vice versa in the U.S. over the 1953-1979 period. Although the techniques to be used below are interpreted as aspects of causality testing by some authors (see, for example, Haugh (1972) and Pierce (1977)) no such claim is made in the present study.3

For the analysis, the input series was the one month inflation rates as measured by the percentage change in the U.S. consumer price index while the one month yields to maturity computed at the beginning of the month on U.S. Treasury bills served as the interest rate variable.

The first step involved pre-whitening both series as well as possible using the autocorrelation function of either adjusted series as an indicator of the degree to which the modelled series appeared to be generated by a white noise process. The autocorrelation functions for 50 displacement lags are presented below in graphical form.4 INF represents inflation while YMLM refers to the one month
yields to maturity, (Figures A4-1 and A4-2). Additional tests for residual noise in the models included computation of the values of the Akaike Information Criterion or AIC (with the lowest value helping to determine which was the "best" model), deriving Q-statistics for the autocorrelations of the errors and finally a specific application of the more general Kolmogorov-Smirnov test, i.e., deriving cumulative periodograms for both adjusted series, (Figures A4-3 and A4-4).

After the selection of reasonably good models, cross correlation values were computed for various lags and a .05 significance level was used to test the null hypothesis that the cross correlations for the negative as well as for the positive lags were jointly equal to zero. A high significance level on the negative lags along with a low one (under .05) on the positive ones would indicate that the input series (inflation) has "predicted" the output series (interest rates) but not vice versa, an implicit assumption in most tests of the Fisher effect. A test of this sort should provide for an interesting comparison with the work of Cargill and Meyer (1974) who used the Sims analysis to test for the existence of any "feedback" effect from interest rates to inflation over the period of 1950-1970 with their conclusion that feedback did not appear to be a problem.

The model chosen for the interest rate series covering the period from July 1953 - December 1979 was a first-differenced model with a seasonal AR(1) process, (SARIMA
(0,1,0) (1,0,0)) as the initial autocorrelations for the interest rates indicated that this was the process generating this series. In addition, this particular model provided the lowest AIC of a number of alternatives tested (-.05185). The Q-statistic for the residual autocorrelations for an arbitrarily selected 36 lags was Q(36) = 47.37 which is not significant at the .05 level indicating that the adjusted series behaves as if generated by a white noise process.

For the inflation series, the selected model was a first-differenced one with a MA(1) process (ARIMA (0,1,1)) as this provided the lowest AIC value (-.06234) among a number of alternative models. The Q value, Q(36) = 36.05 was insignificant at the .05 level. The cumulative periodogram for the inflation variable provides additional evidence for the existence of a white noise process as the plot lies well within all of the significance boundaries, (Figure A4-3).

Taking these pre-whitened series, cross correlation values were computed with lags of 12, 24, 36 and 48 months. The cross correlation graph for 24 lags is provided below (Figure A4-5). Chi-Square tests for independence as described above were computed and the results are presented below.
TABLE A4-1

Chi-Square Tests for Independence

<table>
<thead>
<tr>
<th>Lags</th>
<th>Negative lags</th>
<th>Positive lags</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Chi-Square</td>
<td>S.L.*</td>
</tr>
<tr>
<td>12 lags</td>
<td>27.13(21.0)**</td>
<td>.007</td>
</tr>
<tr>
<td>24 lags</td>
<td>35.77(36.4)</td>
<td>.057</td>
</tr>
<tr>
<td>36 lags</td>
<td>53.29(51.0)</td>
<td>.031</td>
</tr>
<tr>
<td>48 lags</td>
<td>60.38(65.1)</td>
<td>.108</td>
</tr>
</tbody>
</table>

* significance level
** critical Chi-Square values at the .05 level given in parentheses

The results indicate that as far as the 12 and 36 lag functions are concerned, inflation has "predicted" interest rates in the Granger sense, (i.e. inflation rates seemed to behave as lead indicators of nominal yields) a finding entirely in accordance with the standard a priori belief concerning the relationship between the two variables if not with the empirical results obtained by other authors looking at different periods (e.g. Nelson and Schwert(1979)). The extremely low values of the S.L. for the negative lags compel rejection of the null hypothesis that these cross correlations are jointly equal to zero (e.g. in the case of the 12 period lag we can be 99.3% confident that the negative displacement lag values are not equal to zero taken together) while, conversely, the high S.L. values for the positive lags indicate that we cannot reject the null hypothesis of their being equal to zero. For the 24 and 48...
month lags, we reach the same conclusion with respect to the positive correlations and would be able to reject the null hypothesis for the negative correlations at slightly higher confidence limits.

To determine if these results were attributable to distortions created by the use of monthly values (the time consistency problem discussed in the text), the tests were re-conducted using quarterly values of inflation and interest rates covering the same time period. The results were basically similar across the varying lag periods. For example, for the one year (4 quarters) lag using a first-differenced MA(1) model for inflation (ARIMA (0,1,1)) and a second-differenced MA(1) model for interest rates (ARIMA (0,2,1)), the cross correlation values indicated that the standard assumption of a unidirectional relationship going from inflation to interest rates seemed to be confirmed. The independence tests produced Chi-square values of 8.70 and 9.92 for the negative and positive lags respectively (the critical value being 9.50) while the significance levels were .041 and .069 respectively. Although the negative and positive cross correlation statistics are rather close, at the 95% confidence limit we must reject the null hypothesis for the negative lags and accept it for the positive ones thus confirming our results with the monthly values.

Interestingly enough, when the same models for the monthly series of inflation and interest rates that were used for the 1953-1979 period were applied to the exact same
period (mid 1953-mid 1977) that Nelson and Schwert (1979) were interested in, not only were the A.I.C. values lower with the present study’s univariate time series models but the models themselves served as better predictors of actual inflation and interest rates than did the Nelson and Schwert ones. For example, the A.I.C. values obtained with the present study’s models of inflation and interest rates for the 1953-1977 period were -.06315 and -.07821 respectively whereas the Nelson-Schwert values were -.02175 and -.03814. Thus it seems to follow that their findings of unidirectional influence from interest rates to inflation, a result, as discussed above, in direct conflict with the standard perceptions of the relationship between these variables, were due to inappropriate model selection.
Figure A4-4

Cumulative Periodogram

J16 RESIDUALS: 0, 1, 0x 1, 0, 0 x 12 - YM1 M

Figure A4-5

Cross Correlation

1ST SERIES: J16 RESIDUALS: 0, 1, 0x 1, 0, 0 x 12 - INF
2ND SERIES: J16 RESIDUALS: 0, 1, 0x 1, 0, 0 x 12 - YM1 M
APPENDIX 5

Extended Period Results

I. Serial Correlation Structure of the Ex Post Real Rate:

<table>
<thead>
<tr>
<th>Year</th>
<th>No. of Lags</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1953-83</td>
<td>1</td>
<td>.143</td>
<td>.118</td>
<td>.106</td>
<td>.089</td>
<td>.092</td>
<td>.175</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

std. error of the autocorrelations: .052

<table>
<thead>
<tr>
<th>Year</th>
<th>No. of Lags</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1974-83</td>
<td>1</td>
<td>.172</td>
<td>.108</td>
<td>.121</td>
<td>.096</td>
<td>.094</td>
<td>.199</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

std. error of the autocorrelations: .092

\[ Q(6) \text{1953-83} = 268.67 \]
\[ Q(6) \text{1974-83} = 100.90 \]

II. Tests of the Constancy of the Real Rate Utilizing Time Trend Variables

<table>
<thead>
<tr>
<th>Year</th>
<th>Coeff. Cons.</th>
<th>Coeff. T</th>
<th>Time</th>
<th>Time</th>
<th>Time</th>
<th>Time</th>
<th>R</th>
<th>D-W</th>
<th>F-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>1953-83</td>
<td>(.17) (-3.22)</td>
<td>.11</td>
<td>1.98</td>
<td>11.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.86 - .237</td>
<td>-8.27</td>
<td>12.90</td>
<td>-6.76</td>
<td>1.05</td>
<td>.13</td>
<td>2.03</td>
<td>12.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.61) (-4.96)</td>
<td>(-2.03)</td>
<td>(2.14)</td>
<td>(2.12)</td>
<td>(2.10)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.24</td>
<td>-12.35</td>
<td>17.32</td>
<td>-8.44</td>
<td>1.28</td>
<td>.07</td>
<td>2.50</td>
<td>8.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.78)</td>
<td>(-3.00)</td>
<td>(2.82)</td>
<td>(-2.70)</td>
<td>(2.52)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1974-83</td>
<td>-1.53 -.251</td>
<td>.05</td>
<td>1.85</td>
<td>8.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-1.46) (2.84)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.45 -.310</td>
<td>-1.09</td>
<td>1.11</td>
<td>-.37</td>
<td>.042</td>
<td>.16</td>
<td>2.04</td>
<td>5.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.12) (-3.62)</td>
<td>(-.93)</td>
<td>(.90)</td>
<td>(-.86)</td>
<td>(.84)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.58</td>
<td>-.819</td>
<td>.891</td>
<td>-.288</td>
<td>.032</td>
<td>.06</td>
<td>2.60</td>
<td>3.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.97)</td>
<td>(-.66)</td>
<td>(.64)</td>
<td>(-.63)</td>
<td>(.61)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
III. Contemporaneous Relationship of Nominal Rates and Inflation (corresponding to analysis presented in (8) and (9))

1953-1983

\[ Y_{1M} = 4.04 + 0.232 \pi \text{ (1 month)} \]
\[ R^2 = 0.40 \quad D-W = 0.51 \]
\[ F(1,366)=98.15 \quad S.E.R. = 0.180 \]

1974-1983

\[ Y_{1M} = 7.59 + 0.16 \pi \text{ (1 month)} \]
\[ R^2 = 0.36 \quad D-W = 0.20 \]
\[ F(1,129) = 4.43 \quad S.E.R. = 0.363 \]

AR(1) Correction with Cochrane-Orcutt Technique

1974-1983

\[ Y_{1M} = 8.39 + 0.17 \pi \]
\[ R = 0.89 \quad D-W = 1.72 \]
\[ F(1,116)=947.3 \quad S.E.R. = 1.63 \]

1953-83

\[ Y_{1M} = 5.78 + 0.15 \pi \]
\[ R = 0.96 \quad D-W = 1.81 \]
\[ F(1,362) = 7872.8 \quad S.E.R. = 1.49 \]
### IV. Polynomial Distributed Lag Influence of Inflation on Interest Rates

A = both constraints  
B = far constraint only  
C = near constraint only  
D = no constraints

<table>
<thead>
<tr>
<th>Mean lag</th>
<th>Sum of lag coefficients</th>
<th>$R^2$</th>
<th>F-stat.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>6 lags (1974-1983)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>2.03(.58)</td>
<td>.31(.07)</td>
<td>.10</td>
</tr>
<tr>
<td>B</td>
<td>2.02(.57)</td>
<td>.31(.08)</td>
<td>.10</td>
</tr>
<tr>
<td>C</td>
<td>2.03(.58)</td>
<td>.31(.08)</td>
<td>.09</td>
</tr>
<tr>
<td>D</td>
<td>2.01(.57)</td>
<td>.31(.08)</td>
<td>.09</td>
</tr>
<tr>
<td><strong>6 lags (1953-1983)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>2.37(.17)</td>
<td>.62(.03)</td>
<td>.52</td>
</tr>
<tr>
<td>B</td>
<td>2.36(.17)</td>
<td>.63(.03)</td>
<td>.52</td>
</tr>
<tr>
<td>C</td>
<td>2.38(.17)</td>
<td>.62(.03)</td>
<td>.53</td>
</tr>
<tr>
<td>D</td>
<td>2.36(.17)</td>
<td>.63(.03)</td>
<td>.52</td>
</tr>
<tr>
<td><strong>12 lags (1974-1983)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>5.99(1.28)</td>
<td>.44(.09)</td>
<td>.15</td>
</tr>
<tr>
<td>B</td>
<td>5.68(1.20)</td>
<td>.47(.09)</td>
<td>.17</td>
</tr>
<tr>
<td>C</td>
<td>6.24(1.22)</td>
<td>.47(.09)</td>
<td>.17</td>
</tr>
<tr>
<td>D</td>
<td>5.94(1.21)</td>
<td>.48(.09)</td>
<td>.17</td>
</tr>
<tr>
<td><strong>12 lags (1953-1983)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>5.60(.41)</td>
<td>.70(.03)</td>
<td>.59</td>
</tr>
<tr>
<td>B</td>
<td>5.38(.40)</td>
<td>.71(.03)</td>
<td>.60</td>
</tr>
<tr>
<td>C</td>
<td>5.80(.41)</td>
<td>.71(.03)</td>
<td>.60</td>
</tr>
<tr>
<td>D</td>
<td>5.58(.41)</td>
<td>.72(.03)</td>
<td>.61</td>
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</table>
16 lags  
(1974-83)

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10.04(1.5)</td>
<td>9.39(1.4)</td>
<td>9.57(1.5)</td>
<td>5.99(1.3)</td>
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<tr>
<td></td>
<td>.58(.09)</td>
<td>.61(.09)</td>
<td>.62(.09)</td>
<td>.44(.09)</td>
</tr>
<tr>
<td></td>
<td>.25</td>
<td>.27</td>
<td>.27</td>
<td>.15</td>
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<td></td>
<td>21.0</td>
<td>16.0</td>
<td>12.0</td>
<td>11.3</td>
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16 lags  
(1953-83)

<table>
<thead>
<tr>
<th></th>
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<th>B</th>
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<th>D</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>8.28(.56)</td>
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<td>8.01(.58)</td>
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<td>.75(.03)</td>
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<td></td>
<td>.63</td>
<td>.65</td>
<td>.65</td>
<td>.65</td>
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<tr>
<td></td>
<td>305.5</td>
<td>216.1</td>
<td>162.3</td>
<td>165.6</td>
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V. Moving Average Regressions of One Month T-Bill Rates on Inflation

<table>
<thead>
<tr>
<th></th>
<th>Constant</th>
<th>Coeff. on $\pi$</th>
<th>$R^2$</th>
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</thead>
<tbody>
<tr>
<td>1974-1983</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 year</td>
<td>6.27(.68)</td>
<td>.31(2.11)</td>
<td>.10</td>
</tr>
<tr>
<td>1.5 years</td>
<td>1.98(.39)</td>
<td>.83(1.31)</td>
<td>.73</td>
</tr>
<tr>
<td>2 years</td>
<td>5.55(.08)</td>
<td>.36(.13)</td>
<td>.08</td>
</tr>
</tbody>
</table>

|          |             |                 |       |
| 1953-1983|             |                 |       |
| 1 year   | 2.17(1.06)  | .70(3.01)       | .63   |
| 1.5 years| 4.13(.70)   | .70(2.39)       | .64   |
| 2 years  | 3.67(.48)   | .73(1.11)       | .68   |
| 3 years  | 4.93(.32)   | .76(1.25)       | .73   |
| 5 years  | 6.58(.25)   | .73(.64)        | .78   |
VI. Regressions of Changes in Purchasing Power on Nominal Rates

\[ \Delta t = a_0 + a_1 R_t \]

<table>
<thead>
<tr>
<th></th>
<th>( a_0 )</th>
<th>( a_1 )</th>
<th>( s(a_0) )</th>
<th>( s(a_1) )</th>
<th>( R^2 )</th>
<th>( n )</th>
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</thead>
<tbody>
<tr>
<td>5309-8312</td>
<td>.16</td>
<td>-1.02</td>
<td>.55</td>
<td>.10</td>
<td>.05</td>
<td>.22</td>
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<tr>
<td>7403-8312</td>
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<td>-1.5</td>
<td>2.34</td>
<td>.26</td>
<td>.09</td>
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</table>

VII. Autocorrelations of \( \Delta t \)

<table>
<thead>
<tr>
<th>No. of lags</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>1953-1983</td>
<td>.05</td>
<td>.28</td>
<td>.27</td>
<td>.27</td>
<td>.25</td>
<td>.33</td>
<td>.15</td>
<td>.26</td>
<td>.22</td>
<td>.26</td>
<td>.27</td>
<td>.27</td>
</tr>
<tr>
<td>1974-1983</td>
<td>-.21</td>
<td>.09</td>
<td>.09</td>
<td>.08</td>
<td>.04</td>
<td>.17</td>
<td>-.62</td>
<td>.72</td>
<td>.81</td>
<td>.73</td>
<td>.89</td>
<td>.70</td>
</tr>
</tbody>
</table>

\[ Q(365) = 1077.5 \quad \sigma(\rho) = .05 \quad (1953-83) \]

\[ Q(122) = 149.7 \quad \sigma(\rho) = .09 \quad (1974-83) \]

VIII. Regressions of Changes in Purchasing Power on Nominal Rates and Lagged Dependent Variable Values

\[ \Delta t = a_0 + a_1 R_t + a_2 \Delta t_{-1} + \epsilon_t \]

<table>
<thead>
<tr>
<th></th>
<th>( a_0 )</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( s(a_0) )</th>
<th>( s(a_1) )</th>
<th>( s(a_2) )</th>
<th>( R^2 )</th>
<th>( n )</th>
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</thead>
<tbody>
<tr>
<td>5309-8312</td>
<td>.49</td>
<td>-1.02</td>
<td>-.15</td>
<td>.55</td>
<td>.10</td>
<td>.05</td>
<td>.22</td>
<td>365</td>
</tr>
<tr>
<td>7403-8312</td>
<td>-5.18</td>
<td>-.52</td>
<td>-.23</td>
<td>2.34</td>
<td>.26</td>
<td>.09</td>
<td>.06</td>
<td>119</td>
</tr>
</tbody>
</table>
FOOTNOTES

1. For a discussion of the basic inadequacy of the Sims test particularly with respect to policy questions see Desai (1980), p. 140.


3. For a review of the philosophical causality literature as well as a discussion of the intricate issues of econometric causality see Zellner (1979).

4. This procedure of calculating the cross correlogram functions of the innovations of the two series, comparing one set of innovations to the past, present and future values of the other is one aspect of what is often referred to as the Pierce-Haugh causality test.
CHAPTER 3

THE DIFFERENTIAL IMPACT OF INFLATION ON VARIOUS FINANCIAL INSTRUMENTS

I. Varying Degrees of Adjustment to Inflation in the F-H Model

One of the key implications of the model is that the greater is the "moneyness" of an asset the less will its nominal yield be affected by a change in expected inflation. Although this point will be formally demonstrated later, if we initially assume it to be true we would expect, for instance, that the holding period yields on 1 year T-notes would respond less to changes in inflationary expectations than would the returns on 20 year bonds as the latter are providing a smaller proportion of their total return in the form of liquidity services.

To test this proposition, we could treat the liquidity premium attached to the longer term instrument (vis-a-vis the shorter term security) as the difference between the two respective maturity yields (or, perhaps more interestingly, as the difference in their holding period yields) at a given point in time, provided that the securities were identical with respect to all other relevant characteristics such as default risk. The yield differentials of a series of government securities differing only in term to maturity comes most readily to mind as an appropriate data set for testing purposes.
An alternative and perhaps equally meaningful test would involve yields of assets that differ with respect to other marketability aspects but which are identical with respect to the term to maturity. Some likely candidates for inclusion in such a test are the yields on finance paper, commercial paper, some federal agency securities and Euro-Dollar deposits as well as on the short-term Treasury bills as all of the above assets share a common 3 month maturity period. The notion here is that differences in observed returns among various instruments with a common maturity period reflect differences in their general marketability (liquidity) characteristics. In order to understand more fully the prediction of the model that the closer an asset is to being a substitute for money the less will a given rise in inflation influence its nominal rate of return, it is necessary first to recall the steady state equilibrium conditions with respect to the holding of money balances and governments. This analysis was presented earlier in chapter 1 and some of it is repeated below for the reader's convenience.

\[ l_m(\bar{y}; m, Ym) = (\delta + \mu)l_x(\bar{y}; m, Ym) \]

\[ l_g(\bar{y}; m, Ym) = (\delta + \mu - r_e)l_x(\bar{y}; m, Ym) \]

It will also be recalled that in the steady state the real rate of return will be equal to the exogenously given rate of time preference. Performing the necessary
differentiating and generalizing for a large number of assets, we are left with the following:

\[
(3)\quad \frac{dr_i}{d\mu} = 1 - \frac{s_{im} + \sum_j \Sigma_j s_{ij}}{s_{mm} + \Sigma_j s_{mj}}
\]

where \( s_{im} = l_{im}/l_x \) for notational convenience and \( \gamma = g/m \). Equation (3) is basically identical to Equation (12) in Chapter 1 except that it provides for more assets.

It can be easily deduced from the above expression that the closer assets are to being substitutes for money the less will their yields be affected by a change in inflation (the moneyness of assets would be determined by the extent to which the values of the \( s_{im} \) and the \( s_{ij} \) approach respectively the values of \( s_{mm} \) and the \( s_{mj} \)).

A potentially fruitful approach to test for the existence of this phenomenon involves the direct estimation of the values on the right-hand side of (3) through OLS regressions of various yields on a proxy for the level of economic activity as well as on different measures of money and holdings of government securities differentiated by their maturity periods and then fitting the values obtained into equation (3). The precise technique to be used will be detailed later in this chapter.

II. The Impact of Inflation on Equity and Debt Yields

Before beginning the general empirical analysis concerning the differential impact of inflation on the rates of return on various securities issued both by the private
financial sector and the government, a look at the relationship between inflation and yields on a specific set of assets, equities, may be quite instructive.

Specifically, we shall see that the behavior of equity markets, particularly during the high inflation 1970s, which is regarded as anomalous by a number of important writers, is actually entirely consistent with a theoretical prediction of the liquidity model. That prediction is that the real returns on relatively illiquid equities should be comparatively unresponsive to changes in inflation as opposed to, say, corporate bonds as these former assets represent claims to real capital.

In an earlier chapter we discovered the fairly substantial evidence for the underadjustment of nominal rates to inflation, i.e., a negative association between inflation and real interest rates. A natural counterpart to real interest rates in equity markets is provided by the ratio of after-tax corporate earnings to share values, an earnings/price ratio. Feldstein and Green (1983), however, make the interesting observation that the dividends paid out by the corporate sector as a percentage of real after-tax earnings in the United States has been approximately constant (roughly 45%) for a significantly long period of time, thus it seems appropriate to proxy the real rate of return to equities by the dividend/price ratio for which monthly data were available. Specifically, the overall dividend/price ratios for NYSE common stocks were regressed on two alternative measures of expected inflation, as well
as on actual inflation, to determine whether or not the same negative influence that was found in the relationship between real bill rates and price level changes holds up. Alternative values for expected inflation based on both Keynesian and Rational expectational assumptions were used.

The Keynesian model posits that inflationary expectations at a point in time are equal to a distributed lag of past inflation or

\[ \pi^e_t = \sum w_i \pi_{t-i} \]

From this it is possible to infer the relationship between interest rates and inflation,

\[ R_t = \beta_0 + \beta_1 \sum w_i \pi_{t-i} \]

If the additional restriction of the weights summing to unity is imposed to logically allow for a constant rate of inflation maintained over the \( n \) periods to lead to an identical expected rate of inflation, and the usual assumptions regarding the error terms obtain, the relationship above can be estimated by OLS. The restriction, however, that the weights sum to unity is perhaps unduly severe as Sargent (1973) points out (see footnote 7 of chapter 2). For instance, if inflation follows the stationary stochastic process;

\[ \pi_t = \alpha_1 \pi_{t-1} + e_t \]

the autoregressive predictor of inflation will be \( \alpha_1 \) which needn't necessarily equal unity. However, as we are
assuming that expectations of inflation are formed within the framework of arbitrary rules, rather than on rational forecasts, this point of Sargent's is mentioned merely in passing and is thus neglected in the analysis following.

As far as the rational expectations approach is concerned, the assumption of rationality implies that

\[ \pi_t = \pi_{t-1,t} + u_t \]

where \( \pi_t \) is equal to the actual inflation rate, \( \pi_{t-1,t} \) is equal to the expected inflation rate for period t with the expectation being formed at time \( t-1 \) and \( u_t \) is an error term uncorrelated with any information available at period \( t-1 \). If the error term were correlated with the expected inflation variable this would imply that the expectation was a sub-optimal predictor. The reader may recognize that the above bears great similarity to the Fama efficient markets approach discussed earlier in chapter 2.

Suppose that the actual inflation rate is used as a proxy for the expected rate (which is, of course, the appropriate value for the Fisher effect equation). In this case, we would have the following:

\[ R_t = \beta_0 + \beta_1 \pi_t - u_t. \]

The unbiasedness assumption expressed in equation (7) implies that equation (8) meets the conditions of the classical errors in variables problem. Consistent estimates are obtainable if there exists an instrument (or
instruments) that is (are) positively correlated to the unobservable expected inflation rate but uncorrelated with the expectational errors. However, under the rational expectations approach, any piece of information relevant to the determination of inflation meets these criteria, perhaps particularly the lagged values of inflation (as these would probably have the highest correlation with current inflation). Thus equation (8) is estimated using several lagged values of actual inflation as instruments. These regressions represent tests of the Fisher effect with rational expectational proxies for expected inflation.

Table 3-1 (Part A) below presents estimates of the impact of expected and actual current inflation on the dividend/price ratios on common stocks with both measures of expected inflation. The dividend/price ratio series (DIVPR) was obtained from the CITIBASE computer tapes and covers the 1970-79 period alone as earlier data were not available. However, as the behavior of stock prices and returns seemed to be the most anomalous during the 1970s it is worthwhile to focus on this era and thus a lack of earlier data should not hinder the analysis very much. An arbitrarily selected 9 lagged values were used with the coefficients in the Keynesian case representing the summation of the 9 individual coefficients derived from OLS regressions of the ratios on 9 sequentially lagged monthly inflation values. In the RE case, the 9 lagged values were used as instruments. It should be noted that shorter and longer lag
lengths were also tried with no substantive difference in the results obtained.

Part B of Table 3-1 shows the results of the tests of the influence of inflation on real equity returns while controlling for real economic activity. The inclusion of this additional variable not only differentiates the present analysis from others that focus on inflation and equity returns but serves to make it more consistent with the liquidity model. Controlling for real economic activity becomes even more important a bit later when a yield differential, that between real equity and real corporate bond returns, is taken to be the liquidity premium on bonds and this value is regressed on variables which according to the model should impinge on it. The economic activity proxy used is real personal income although the index of industrial production was also tried with very similar results. The expectations formation processes for real activity were assumed to be identical to the ones for inflation.

**TABLE 3-1**
Regressions of Real Equity Returns on Inflation and Real Activity

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>A.</strong></td>
<td></td>
</tr>
<tr>
<td>Keynesian Expectations</td>
<td>**</td>
</tr>
<tr>
<td>DIVPR = 2.43 - .240 ( \pi )</td>
<td></td>
</tr>
</tbody>
</table>

***(11.82) (0.882)**
R-squared = .37  D-W = .32  F-statistic = 8.75  S.E.R. = .206

Rational Expectations  DIVPR = 2.46 + .229*  
                           (9.04) (6.49)

D-W = .86  S.E.R. = .272

Actual Contemporaneous Values (AR1)  DIVPR = -3.40 + .0032*  
                                           (-1.19) (.287)  7.85

R-squared = .82  D-W = 1.78  F-statistic = 537.7  S.E.R. = .433

B.

Keynesian Expectations  DIVPR = 4.78 + .0098* + 1.05 REALPY  
                         (-4.08) (.037) (.054)

R-squared = .60  D-W = .50  F-statistic = 7.65  S.E.R. = .507

Rational Expectations  DIVPR = -.407 + .199* + .389 REALPY  
                        (-3.01) (5.35) (2.10)

D-W = .82  S.E.R. = .182

Actual Contemporaneous Values (AR1)  DIVPR = 4.25 - .0012* + .007 REALPY  
                                      (7.21) (.097) (.155)

R-squared = .82  D-W = 1.78  F-statistic = 266.5  S.E.R. = 6.08

* Dividend/price ratios on NYSE common stocks

** Indicates the sum of the coefficients in the OLS regressions of the dividend/price ratios on 9 lagged values of inflation
*** t-values

**** Coefficient yielded by the regression of the dividend/price ratio on current inflation where 9 lagged values of inflation are used as instruments. See Summers (1981). Note that with the RE approach, there are no F or R-squared values.

The results are mixed but do strongly contradict the notion that real equity returns fall in response to an increase in inflation either when controlling for real economic activity or not. For both the Keynesian expectations models and equations with actual contemporaneous regressors, the impact of inflation on real returns is insignificant. For the RE model, with and without real personal income as an explanatory variable, the coefficients are positive and significant but also small. In any case, a negative influence could not be found, perhaps a rather surprising result given not only other results from work done in this area but also our earlier findings on the influence of inflation on real financial asset returns. However the results can be reconciled with the predictions of the liquidity model in that equities are relatively illiquid assets representing claims to capital whose real returns should be comparatively uninfluenced by changes in inflation. These results are also consistent with the traditional view of equities as good inflation hedges, at least with respect to dividend yields.

It should be noted that as far as the empirics in the preceding section are concerned, the use of lagged regressors in both expectations formation proxies precluded
a correction for serial correlation. However it is necessary to remember that these proxies used in the present study are fairly rough measures constructed to capture some of the theoretically-predicted features inherent in the two expectations models and are certainly not intended to be exact and definitive values. For the equations with actual inflation this is not the case and an adjustment for first-order serial correlation was made, with the results presented in parts A and B. Not correcting for autoregressive error terms probably accounts for the positive influence of actual inflation on real equity returns (specifically earnings/price ratios) discovered by Summers (1981) in his OLS analysis for the post WWII era. Using OLS estimation with quarterly values of earnings/price ratios and inflation for his 1947-1979 period, I found a significant positive impact of actual inflation on real equity yields, however the coefficient on inflation became insignificantly different from zero when a needed correction for an AR(1) process was made. Even more interesting and relevant for our analysis is the impact of inflation on the spread between equity yields and real corporate yields as measured by an average of corporate bond rates adjusted for inflation. This is important for our purposes because according to the F-H model inflation should impinge more heavily on the real returns on relatively liquid bonds as opposed to the real yields on stocks as the former bear a closer relationship to money. Thus the spread should be positively correlated with inflation. The greater liquidity
of corporate bonds vis à vis stocks is inferred from the fact that round-trip exchanges in the former asset involve lower transactions costs than do exchanges in the latter, and have throughout the period of analysis. Fees charged by brokerage houses for conducting trades in corporate bonds (usually in $1000 units) are based on a flat percentage of the par value of bonds whereas fees for stock trades are based on formulae that take into account the number of shares being traded as well as the dollar value of shares. For average trades (measured in dollar amounts) there is a substantial difference in trading costs between the two assets (between approximately .75 and 1.25 points expressed as a percentage of dollar value). This difference probably has something to do with the fact that only the well known and usually financially-sound corporations issue debt in the form of bonds whereas equity trades often involve shares of companies that are little known and about which it may be costly to obtain trading information. Hence the financial markets tend to place a higher value on the liquidity services offered by brokers in stocks as opposed to dealers in corporate bonds. Indeed, many dealer-customer transactions in long-term bonds are arranged in pairs, where the dealer and customer 'swap' two issues of comparable aggregate value, settling the difference in cash (including some small dealer's fee). These swap arrangements are virtually unknown in equity markets. Much more about the liquidity of various assets, liquidity in the sense of the
absence of transactions costs will be discussed below in sections III and V of this chapter.

The real corporate bond return series was constructed by subtracting the annualized values of monthly inflation (as measured by the percentage changes in the non-seasonally adjusted consumer price index) from annualized monthly values of nominal corporate bond returns (which represent an average of weekly rates determined in the market for highly-rated (AAA) corporate long-term securities). The nominal corporate bond yields were obtained from various issues of the Federal Reserve Bulletin.

For the statistical analysis, the computed real corporate bond rates for each month were subtracted from the dividend/price ratio for the corresponding month and these values were regressed on contemporaneous values of inflation and real personal income as well as on both expectations proxies for these variables.

As referred to earlier, these differences could be regarded as representing differences in liquidity between the two sets of assets. Ideally, within the context of the liquidity model, the ratio of the dollar value of AAA corporates to the dollar value of all the common stocks listed on the NYSE would have been included as a regressor however data on corporate bonds outstanding (as opposed to volume traded) were not available. Nevertheless, as we are primarily concerned in this section with testing for the existence of the wedge that the model predicts inflation will drive between equity and debt yields, the lack of the
additional regressor should not hinder the analysis very much. Indeed, the impact of inflation alone in determining the yield differentials is so great that it is inconceivable that inflation's influence would become econometrically unimportant were the corporates to common shares ratio to be included as an explanatory variable.

The results are presented in Table 3-2 below. Again, the t-values are in parentheses.

<table>
<thead>
<tr>
<th>TABLE 3-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regressions of Equity/Debt Yield Spread on Inflation and Real Income</td>
</tr>
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</table>

A. Keynesian Expectations

\[ \text{DIVPR-RCORP}^* = -8.16 + .954\pi + .591 \text{REALPY} \]

\[ \text{R-squared} = .97 \quad \text{D-W} = 1.56 \]
\[ \text{F-statistic} = 205.43 \quad \text{S.E.R.} = 1.40 \]

B. Rational Expectations

\[ \text{DIVPR-RCORP} = -6.79 + 1.02\pi + .257 \text{REALPY} \]

\[ \text{D-W} = .52 \quad \text{S.E.R.} = .654 \]

C. Actual Contemporaneous Values (OLS)

\[ \text{DIVPR-RCORP} = -6.75 + 1.01\pi + .265 \text{REALPY} \]

\[ \text{R-squared} = .973 \quad \text{D-W} = .93 \]
\[ \text{F-statistic} = 2157.3 \quad \text{S.E.R.} = .609 \]
Actual Contemporaneous Values (AR1)  
\[ \text{DIVPR-RCORP} + -4.93 + .999\pi + .045\ \text{REALPY} \]
\[ (-11.20) \quad (87.25) \quad (.946) \]

\[ \text{R-squared = .995} \quad \text{D-W = 1.73} \]
\[ \text{F-statistic = 5655.0} \quad \text{S.E.R. = .527} \]

* Real corporate bond yield (AAA)

The results indicate clearly that inflation is an extremely important variable in explaining differentials between equity and corporate bond yields, as can be seen by looking at the t-values on the inflation coefficients. Furthermore, regression results with equations that included inflation alone (not shown above) indicate that the high adjusted R-squared values as well as the F-statistics fall little when real personal income is excluded as an explanatory variable. Very interesting is the fact that the coefficient on inflation in all cases is insignificantly different from 1, a finding which when combined with the earlier one of the lack of influence of inflation on real equity returns indicates that real corporate yields are taking roughly the full impact of the adjustment to inflation changes, a finding consistent with our prior notions. The results concerning the influence of real activity on the yield differentials are mixed but in two of the four cases indicate some positive influence, as predicted by the model. Slightly better results (not shown)
were obtained with the IPI serving as the real activity proxy.

Not only do the above findings offer empirical support in favor of the liquidity model but tend to raise doubts about the predictive power of some alternative models that purport to explain falling real rates in the face of inflation. For instance in Chapter 1 a model was discussed which focused on the role of money as a complementary factor of production. In this model, increases in inflation, by reducing the desired level of real money balances, would reduce the marginal physical product of capital and thus the real rate of interest. However this model fails to account for our finding that the real corporate yields decline relative to real equity returns. Or more generally, why, when inflation rises, real rates of return on the relatively liquid assets are drive down more than those on assets that are comparatively illiquid. Of course, the above findings create no problem for the liquidity model with its presumption that assets are imperfect substitutes for one another in the sense of yielding differing liquidity services.

Some interesting evidence for a longer run positive relationship between the equity/debt yield spreads and inflation was obtained by utilizing the Engle band spectrum regression technique described in Chapter 2 in connection with the work of Summers. Using earnings/price ratios (as more remote data were available) and quarterly periods for real corporate bond returns with a three-year filter, the
evidence indicates that even in the longer run, rising inflation is associated with an increase in the divergence between equity and bond returns.

\[
\text{OLS (1947-I - 1979-IV) } \quad \frac{E}{P} - \text{RCORP} = -0.019 + 0.600\pi \\
\quad (-2.4) \quad (7.67)
\]

\[ R^2 = 0.84 \quad \text{F-Stat.} = 677.2 \quad \text{D-W} = 1.80 \]

The selection of a three year filter was not entirely arbitrary. In work of this kind it is necessary to balance the consideration of removing as much of the cyclical influence as possible against the possibility that the selection of too lengthy a filter will entirely eliminate the resolving power of the data. It should be noted, however, that the same basic results were obtained with 2, 4 and 5 year moving average filters.

As mentioned above, Summers finds a similar increase in the spread between equity and debt yields using earning/price ratios as a proxy for equity returns, specifically a rising E/P ratio with a falling real corporate yield. Although Summers believes that some sort of money illusion on the part of financial market participants is responsible for the observed phenomenon, he rejects the money illusion argument put forward by Modigliani and Cohn, specifically that inflation has caused individuals to mis-value the corporate sector. The basis for his rejection of their argument is his finding that the ratio of the total market value of the average corporation relative to the income it generates was roughly constant throughout the 1970s.
The Modigliani and Cohn money illusion argument for the widening of this spread during the late 1960s and throughout most of the 1970s is the following: inflation has caused individuals to commit errors in calculating the value of common stocks (which would, they point out, explain the rising E/P ratio) in two ways. Firstly, agents took inadequate account of the decline in real corporate liabilities associated with a rise in inflation and thus tended to undervalue common stock prices when considering only accounting profits, which fell during the period. They point out that correctly measured profits kept pace with inflation during this time. Secondly, they argue and even provide some casual evidence for the view that agents have incorrectly used nominal rates to capitalize expected equity earnings whereas the correct procedure is, of course, to use real rates of discount in helping to determine the appropriate values of current stock prices.\textsuperscript{3}

It is, however, difficult to understand why presumably rational economic agents would not only commit such major errors but do so consistently. To state the argument in different terms, why should the participants in equity markets be subject to such a large degree of inflation-induced money illusion while money illusion appears to have been absent in other sectors of the economy during the inflationary 1970s? The liquidity argument that the difference in real pecuniary yields is attributable to an increased liquidity premium on corporate bonds which are
relatively more liquid seems to provide a much more reasonable explanation.

Furthermore, an implication of the Summers and Modigliani-Cohn explanations is that there has been some unexploited potential gain to be made by firms by borrowing in the bond markets as opposed to raising capital in the equity markets, but a significant degree of money illusion on their part precluded their taking advantage of this opportunity to reduce their costs of capital. However a simple examination of the time path of the ratio of newly issued corporate debt in the form of bonds to new capital in the form of stock (both common and preferred), taking yearly averages through the 1970s, tends to contradict this implicit argument of these writers. For the first part of the 1970s the ratio fluctuates within the 2-2.5 range (with an average value of 2.19) but rises dramatically to a bit over 5 in 1974, the year that marked the beginning of the relatively high inflation period. Throughout the rest of the 70s the ratio trends downwards but never falls below 3.51, with the average value being 3.87. The obvious inference to be drawn from this is, of course, that firms did indeed borrow more heavily in relative terms in the bond markets as inflation picked up and thus were not completely unaware of any gains to be made with respect to the reduction of capital costs by issuing bonds in preference to stocks. Whether this relative shift away from issuing equities was inflation-induced is not easy to prove empirically, however it is interesting to note that there were no obvious changes in
such economic features as corporate tax policy during the period which could conceivably account for this phenomenon.

III. Tests with Instruments of Varying Marketability

Returning to the main story of this chapter concerning the impact of inflation on assets possessing varying degrees of "moneyness", this next section is devoted to an analysis of the implication of the model that the greater is the substitutability of a particular asset with money, the less will its nominal yield be affected by changes in inflationary expectations. In particular, we will first look at a number of rates of return on assets differing in marketability characteristics but whose maturity periods are the same. Then estimates of the Fisher relationship under alternative expectational assumptions will be provided with the coefficients on the inflation variable serving as the point estimates of the impact of inflation on the one month maturity yields (holding period yields) of the various securities. The group of securities include U.S. Treasury bills, Federal Agency Certificates, Finance Paper and Banker's Acceptances for the overall sample period and all of the above plus certificates of deposit, commercial paper and Eurodollar loans for the inflation sub-period. Both Keynesian and Rational Expectations models were used with the sum of the coefficients on 9 lagged values of inflation representing the value of the overall coefficient in the Keynesian case, as was similarly done in an earlier test.
Also as was done earlier, the lagged values of inflation served as instruments in the test of the Fisher effect within the framework of an RE model.

The securities for the overall period are listed in descending order with respect to the standard perceptions of their relative degree of liquidity. These perceptions are based on the existence of primary and secondary markets for these assets that possess most notably the market characteristics of breadth and depth or the existence in substantial volume of sale and purchase orders for securities very near the current equilibrium values. The breadth of a particular market can be approximated by the money value of the outstanding amount of the security in question or, perhaps even more accurately measured, by the volume of transactions in a particular security over some unit time period. On both of these counts, the ranking of the assets in part 2 of Table 3-3 and in Table 3-4 is correct, as can be seen in Section 1 in Appendix 1 (a description of this section appears below). As far as the dollar value of transactions is concerned, perhaps some economies of scale argument could be invoked to help explain why actively-traded issues such as T-bills tend to have lower transactions costs of exchange than, say, 20 year bonds. A more elaborate argument relating transactions costs to the time rate of transactions is offered below in the section that deals with the empirical analysis with the various maturity government securities.
Section 1 in Appendix 1 provides values for both the amounts outstanding and absolute volume of transactions in the various maturity governments as well as Federal agencies, finance paper and bankers acceptances. The figures are those for licensed dealers and represent averages of daily figures themselves averaged across seven years, 1971-1977. The values were obtained from various issues of the Federal Reserve Bulletin and the Treasury Bulletin. Of course, as far as government securities are concerned, an even quicker method, and perhaps one even more relevant for the present study, to prove the validity of the assertion that transactions costs of exchange tend to rise with the length of time to maturity of the instruments is to note the well-known phenomenon of rising bid-ask spreads on governments as the maturity length rises. A more detailed discussion of bid-ask spreads is offered below; however we can regard them as representing the costs of "round-trip" transactions in securities. It is a very common practice in empirical work to regard these spreads as suitable proxies for pure transactions costs (see, for example, Demsetz (1968)).

For the privately issued assets such as bankers acceptances and large negotiable CD's, bid-ask spreads as such do not exist, however the individual brokerage houses do have "subscription" fees for trading in these assets that can be regarded as close counterparts to spreads. Data on these subscription fees obtained from Merrill Lynch indicate that these fees vary with the different assets in accordance
with their ranking in Table 3-3. For instance, at the two extremes, the transactions costs of dealing in T-bills average out to be about $.15 for every $100 worth of bills whereas the subscription fees involved in dealing with the secondary market for the Euro-dollar deposits average about $1.70 for every $100. For the additional assets included in the 1974-79 analysis, the subscription fees were used of necessity as data on the outstanding amounts and transactions levels of these assets were not available. Table 3-3 reports the results of the tests of the Fisher effect using the various financial instruments.

The results provide some evidence in favor of the proposition that inflation will have relatively less impact on the money yields of those assets that have a higher degree of substitutability with money. Aside from the case of Keynesian expectational assumptions for the sub-period, the coefficients on the inflation variables are all significant and for the overall period rise as we move down the list of securities under both types of expectations models, in accordance with our predictions. Furthermore, while the absolute values of the coefficients under our two expectations generating procedures differ, the relative values seem to remain fairly constant. For example in both the RE and Keynesian cases the point estimates of the impact of inflation on the returns on banker's acceptances lie very close to 16% above the coefficient values for Treasury bills.
Another perhaps meaningful observation is that changes in inflationary expectations seem to have had a more pronounced effect in widening yield differentials during the inflationary sub-period. For example, in the RE case the estimate of inflation's impact on the banker's acceptances rate lies a full 30% above the value for T-Bill yields (.481 vs. .370). This may be a result of market participants being less subject to money illusion during high inflation periods and thus the wedge that the market tends to drive between the more liquid asset and the less liquid one becomes more pronounced.
### TABLE 3-3
Tests of the Fisher Effect with Different Expectational Assumptions and Interest Rates

<table>
<thead>
<tr>
<th></th>
<th>6/1953-12/1979</th>
<th></th>
<th>1974-79</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>d ( r/d\pi^e )</td>
<td>d ( r/d\pi^e )</td>
<td></td>
</tr>
<tr>
<td><strong>Keynesian</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T-Bills</td>
<td>0.587</td>
<td>0.436</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.140)*</td>
<td>(0.019)</td>
<td></td>
</tr>
<tr>
<td>Federal Agencies</td>
<td>0.631</td>
<td>0.473</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.162)</td>
<td>(0.021)</td>
<td></td>
</tr>
<tr>
<td>Finance Paper</td>
<td>0.644</td>
<td>0.480</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.160)</td>
<td>(0.021)</td>
<td></td>
</tr>
<tr>
<td>Bankers</td>
<td>0.679</td>
<td>0.506</td>
<td></td>
</tr>
<tr>
<td>Acceptances</td>
<td>(0.162)</td>
<td>(0.022)</td>
<td></td>
</tr>
<tr>
<td><strong>Rational</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T-Bills</td>
<td>0.598</td>
<td>0.370</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.446)</td>
<td>(0.046)</td>
<td></td>
</tr>
<tr>
<td>Federal Agencies</td>
<td>0.714</td>
<td>0.458</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.596)</td>
<td>(0.059)</td>
<td></td>
</tr>
<tr>
<td>Finance Paper</td>
<td>0.744</td>
<td>0.436</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.540)</td>
<td>(0.057)</td>
<td></td>
</tr>
<tr>
<td>Bankers</td>
<td>0.754</td>
<td>0.481</td>
<td></td>
</tr>
<tr>
<td>Acceptances</td>
<td>(0.532)</td>
<td>(0.062)</td>
<td></td>
</tr>
<tr>
<td>Commercial Paper</td>
<td>0.760</td>
<td>0.491</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.516)</td>
<td>(0.063)</td>
<td></td>
</tr>
<tr>
<td>Certificates of</td>
<td>0.773</td>
<td>0.497</td>
<td></td>
</tr>
<tr>
<td>Deposit</td>
<td>(0.514)</td>
<td>(0.063)</td>
<td></td>
</tr>
<tr>
<td>Euro-Dollar</td>
<td>0.826</td>
<td>0.529</td>
<td></td>
</tr>
<tr>
<td>Deposits</td>
<td>(0.566)</td>
<td>(0.070)</td>
<td></td>
</tr>
</tbody>
</table>

* Standard errors in parentheses
It should be noted that the inclusion of the three additional asset yields for the sub-period presented a problem in that it was difficult to rank a priori the respective securities with respect to the differential degree of liquidity services that they provide. For instance, commercial paper and the large negotiable certificates of deposit issued by commercial banks are regarded in the financial markets as being such close substitutes that it would not be very accurate to say that the two assets have an unambiguous relationship to one another in terms of relative liquidity. Indeed, the closeness of the coefficients for the yields of these two instruments under both types of expectations models would tend to confirm this notion. Another small difficulty arose in the fact that there was a "lumpiness" in the reported movements of the CD rates vis-a-vis the other yields in the sense that the former moved in increments and decrements of no fewer than 5 basis points whereas the other yields were recorded to the nearest one basis point. This is mentioned only in passing as it applied to just one series and thus should not have too adversely affected the overall results.

With regards to this problem of the closeness of the reported coefficients, Section 2 of Appendix 1 to this chapter includes a discussion and the results of a test which provides solid evidence for the statistically significant differences in the measured coefficients on inflation.
IV. Tests with Instruments of Different Maturity Periods

An alternative procedure for determining the validity of the proposition that inflation's influence on the yields of different assets will vary according to the degree of "moneyness" possessed by the particular asset in question involves estimating the parameters in equation (3) through OLS regressions of the liquidity premium on a set of variables including real money balances, real governments and some proxy for economic activity. Recall that in the F-H liquidity model the individual is in equilibrium with respect to bond and money holdings when the following conditions are met:

\[ \frac{1_i}{1_x} = R - R_i \quad \frac{1_m}{1_x} = R \]

where \( R \) and \( R_i \) can be thought of as the nominal rates on a longer term government security and on a shorter term government respectively. The difference between the two values can be taken to represent the liquidity premium attached to the longer term security which reflects the lesser degree of liquidity that it possesses vis-à-vis the short-term security. Rewriting \( \frac{1_i}{1_x} \) as \( s_i \) and assuming that individuals will equate at the margin the total returns (pecuniary and non-pecuniary) of the two securities, we have the following relationship;

\[ R = R_i + s_i \]

Assuming for the sake of simplicity that the long term asset
generates no liquidity service at all, its long-run equilibrium rate of return is simply equal to the sum of the exogenously given marginal rate of time preference plus the rate of inflation which is equal to the rate of monetary expansion in the steady state, i.e.,

(11) \[ R = \delta + \mu \]

The liquidity premium, it will be remembered, is a function of the level of expenditures as well as of the holdings of money balances and government securities, therefore it should be possible in principle to determine the values in equation (3) (the \( s_{im} \), \( s_{mj} \) etc.) by treating the stochastic generalizations of (12) and (13) below as OLS regression equations.

(12) \[ R_i - R = C_i + s_{iy} y + s_{im} m + \sum s_{ij} b_j + e_i \]

(13) \[ -R = C_m + s_{my} y + s_{mm} m + \sum s_{mj} b_j + u_j \]

where \( y \) represents some measure of expenditures or economic activity, \( m \) represents real money balances and the \( b_j \) are the real private holdings of government securities with the \( j \) subscript indicating the maturity period of the various instruments. The liquidity premium is defined as the difference between the holding period (not maturity) yields of the longest term (30 years) instrument and the asset in question, however the term structure of the liquidity premium is assumed to be similar to that in the Hicksian model, one in which the liquidity premium increases with the maturity of the instrument. In Hicks' analysis, this particular
characteristic arises due to an assumed "constitutional weakness" in the bond market created by the combined effects of the preferences of lenders for short bonds in order to reduce capital risk and the preferences for supplying long bonds, in order to minimize capital losses, on the part of borrowers.4

As the maturity period increases, we would expect inflation to exert an ever increasing influence on the yield in question but still below a point-for-point impact as the underadjustment of nominal rates to inflation was the major conclusion of our empirical study in an earlier chapter, as well as of our theoretical analysis (we would expect full adjustment to occur only when the security generates absolutely no liquidity yield at all which is not the case even for the longest maturity financial instruments).

Transactions Costs and Bid-Ask Spreads

The rationale for the assumption that the shorter term securities are more liquid in the sense of having lower transactions costs associated with trading them is based on the well-known inverse relationship between bid-ask spreads and the time rate of transactions in securities. The bid-ask spread can be thought of as the price of the liquidity service provided by the dealer who bridges the time gaps between purchase and sale orders. This gap arises, of course, due to the fact that the orders arrive asynchronously. For an issue which is actively traded the
time gaps tend to be small hence the liquidity services provided by the dealer are, ceteris paribus, less highly valued than those liquidity services provided by the dealer in the relatively non-actively traded or "thin" issues. This inverse relationship between spreads and the time rate of transactions has been verified in every major empirical study of spread determinants (see, for example, Demsetz (1968) and Garbade and Silber (1976)).

Closely related to this, and usually neglected by writers in this area, is the effect of uncertainty on the transactions costs of dealing in particular markets. Specifically, the uncertainty referred to is that about the current equilibrium price of securities. This uncertainty will, of course, tend to be positively related to the thinness of the market in the issue. In other words, the greater the perceived probability distribution of the price of the asset in question (a wide distribution would be associated with an infrequently traded asset) the greater will be the bid-offer spread of the asset, reflecting greater dealer uncertainty about the price. Hence it follows that once again spreads will rise, ceteris paribus, with the thinness of markets in the various issues. This analysis offers implicitly another argument for the predominance of transactions costs considerations over risk considerations in the pricing of securities. It can be argued that the uncertainty referred to above can to some extent be overcome by dealers by devoting resources to the acquisition of knowledge concerning current market
equilibrium prices, i.e. by incurring information or transactions costs. Thus again we can see that transactions costs and not risk considerations are more important in explaining observed differences in holding period yields among securities.

Returning to the empirical analysis, the following covers the inflationary sub-period as data on private holdings alone of government securities does not extend back to 1953. The variables used as regressors were the logs of real, private holdings of bills, other government securities with a maturity period of one year or less, and governments with maturities of 1-5 years, 5-10 years, 10-20, and 20-30 years. The log of the Industrial Production Index proxied for the variable representing the level of economic activity while the log of real M1 money served as the money variable. All of the above series were obtained from the computer tapes of the St. Louis Federal Reserve research department. The rates of return used were the one month holding period yields of the portfolios of securities described above. For the analysis, the holding period returns on 30 year government bonds served as the long-term yields which were assumed to possess virtually no liquidity value and from which were subtracted the holding period yields of the portfolios consisting of the various maturity government securities listed above. Appendix 2 to this chapter describes the precise manner in which the holding period returns were calculated.
To repeat, OLS estimation of the empirical analogs of (12) and (13) was undertaken and the values of the parameters obtained were plugged into the right-hand side of equation (3). The values obtained are intended to provide a means for ranking the assets ordinally with respect to the liquidity services that they provide within the context of the liquidity model and are certainly not meant to be taken as precise numerical measurements of the point impact of inflation on nominal yields. They can be regarded, however, as rough numerical approximations of the influence of inflation on the various yields within the context of F-H. Their precision is limited, however, by the fact that data constraints necessitated using a relatively small, although probably representative, number of portfolios and associated holding period returns.

TABLE 3-4
Tests of Underadjustment (Fried-Howitt Analysis)
(t-values in parentheses)

\[
\frac{d}{d\lambda} \frac{d}{d\pi} = 1 - \left( s_{i m} + \sum s_{i j} s_{j i} \right) / \left( s_{m m} + \sum s_{j m} s_{m j} \right)
\]

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3 month T-Bills</td>
<td>.242 (11.81)</td>
<td></td>
</tr>
<tr>
<td>Within 1 year (Excluding 3 month T-bills)</td>
<td>.362 (10.97)</td>
<td></td>
</tr>
<tr>
<td>1-5 years</td>
<td>.467 (11.11)</td>
<td></td>
</tr>
<tr>
<td>5-10 years</td>
<td>.542 (11.33)</td>
<td></td>
</tr>
<tr>
<td>10-20 years</td>
<td>.608 (10.90)</td>
<td></td>
</tr>
<tr>
<td>20-30 years</td>
<td>.685 (10.98)</td>
<td></td>
</tr>
</tbody>
</table>
The results obtained confirm the hypothesis that the longer-term, relatively less liquid asset yields will tend to follow changes in inflation more closely than will the yields on the short-term assets such as T-Bills. Indeed, the results indicate that for every 100 basis point change in inflation yields on the T-bills change by approximately only 24 basis points while a portfolio consisting of securities of 20-30 year maturities had its yield affected by a full 68 basis points.

It was also interesting to discover that the tests confirmed Cagan (1965) in the procyclical nature of liquidity premia when they are defined as the differences in holding period yields. Furthermore, when the same tests were conducted using the differences in yields to maturity as a measure of the premium, the well known counter-cyclical relationship manifested itself. These determinations were made simply by looking at the signed values of the coefficients on the current period IPI in the OLS regressions of (12) and (13) which, as stated earlier, proxied for the general economic activity variable. These different results, which are dependent on the definition of the liquidity premium used, can be used to support arguments for two alternative theories of short-run fluctuations in the premium. Suppose that an expansion in business activity raises the general level of interest rates, i.e., rates overall are procyclical. In Cagan's "money substitute" theory, the short-term securities are purchased in highly
disproportionate amounts (as agents seek to reduce their real money holdings in response to the increased opportunity costs of holding money) as these assets are very good substitutes for money balances. Hence, their yields are prevented from rising as much as the yields on the longer-term, relatively illiquid securities, i.e., the liquidity premium rises. In other words, when rates are low (and money holdings are high) the marginal value attached to the liquidity provided by short-term assets is also low and individuals are willing to hold the short-term instruments provided that their yields are not too far below those of the long-term security. However as interest rates rise and individuals substitute away from near-monies into higher interest-bearing securities, the marginal valuation of liquidity also rises and the spread between the short and long rate, the liquidity premium increases.

An alternative theory which relies on the Keynesian assumption of a perceived "normal" level for interest rates argues that the liquidity premium will exhibit countercyclical behavior for the following reason: as the overall level of rates are relatively low (below the "normal" level) the substantially greater risk of capital losses of holding long-term bonds during this time will cause agents to hold these securities only if a substantial premium is attached to them, i.e., if their prices are very low. As interest rates overall rise to their perceived "normal" levels, the probability of a significant capital loss diminishes and the liquidity premium embedded in the
long bonds decline thus accounting for the countercyclical nature of the premium.

Although at first glance either measure of returns (holding period yields or yields to maturity) seems equally meaningful, there is good reason for choosing the former measure over the latter. The chief advantage of using holding period yields is that one can look at the total returns on a variety of instruments, differing in maturity lengths, over a specified unit of time without any reference to their respective maturity periods. This is, of course, not true for a maturity yield which is the annualized return to be expected from an instrument only if held over its entire lifetime. Another attractive and related feature of a more technical nature which makes use of the holding period differential definition of the premium preferable to the alternative is the greater number of degrees of freedom obtainable over some given sample period. For example, if we were to define the premium on a one year T-Bill at the beginning of a one year period as the difference between its yield to maturity and the sum of two successive spot yields on 6 month instruments we could quickly run into the problem of having an insufficient number of data points, i.e., too few degrees of freedom. This is true because if we did not allow for the overlapping of time periods, in order to minimize any serial dependence, we would be restricted to using yearly observations. The problem becomes even more
severe, of course, when much longer term securities are considered as in the preceding analysis.

The most important aspect of the preceding discussion for our purposes is that the liquidity premium, properly defined as the difference in holding period yields, is procyclical, a result that would tend to contradict the view that the premium is primarily attributable to risk considerations of the Keynesian variety and confirm the notion that the premium reflects the particular asset's degree of substitutability with money. Cagan's "money substitute" theory is clearly very closely related to liquidity as the absence of transactions costs as in F-H. More recently, Leijonhufvud (1981) attributes what he describes as the "plummeting" of short-term interest rates in the U.S. (from the summer of 1974 to 1981, the period of high inflation in the American economy) to an increased desire on the part of financial market participants for more "flexible" positions. The driving motivation is based on the increasing uncertainty of nominal contracts with the result that long-term commitments tend to be avoided and short assets are actively purchased. However, as F-H (1983) (footnote 8) point out, the implication of the Leijonhufvud argument is that shorter term assets provide greater flexibility (are more liquid) in the sense of having lower transactions costs associated with trading them. Thus his analysis is entirely consistent with theirs.

In summary, this chapter has shown that if we look at the impact of inflation on a variety of instruments
providing varying degrees of liquidity services that inflation has had the least influence on the nominal yields on those securities that possess the most "moneyness". This was found to be true in the instance where securities were differentiated by only their length of time to maturity as well as in the case where the securities were identical with respect to maturity periods but differed in other liquidity characteristics.
1. A potential difficulty arises in the use of a lagged series of some independent variable in a regression equation in that not all of the lagged values will have significant t-ratios, as was our case with the 9 lagged values of inflation used as a proxy for expected inflation. However, this type of technique will still provide unbiased estimates. Furthermore, while the t-statistics on some of the lagged values were insignificant, both the simple correlations and covariances of the dividend/price ratios with each of the 9 lagged values of inflation were so similar across the lagged series as to enable us to reject the notion that some of these lagged values should be excluded.

2. Note that because instrumental variables are used for the rational expectations approach, no R-squared or F-values are reported.

3. This latter notion concerning the role of future expected real earnings in the determination of current equity prices is stated somewhat tenuously as LeRoy and Porter (1979) show that stock prices in the U.S. have been far too volatile to lend support to the view that they represent the discounted value of expected earnings. The notion that there is a F-H type of liquidity return, changing in value over time, incorporated in equity prices may provide an explanation of this seemingly anomalous finding of the above authors. With respect to long term interest rates the identical phenomenon is inferred from the results presented by Shiller (1979) by F-H (1983), footnote 6.

4. Although as Woodward (1983) points out in a restatement of an old argument there is no reason to believe that agents are any more concerned with capital-value risk than with income risk and thus "there is generally no single measure of the riskiness of an asset" p. 348. The implication of this is that the liquidity premium cannot be assumed to be necessarily positive.
APPENDIX 1

Section 1.

Trading Volume and Absolute Levels of Various Maturity Government Securities and Other Assets

A.

U.S. Government Securities *

<table>
<thead>
<tr>
<th>Description</th>
<th>Outstanding Amounts</th>
<th>Transactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bills and other assets within one year</td>
<td>$2651</td>
<td>$5993</td>
</tr>
<tr>
<td>Assets with a maturity period of 1 to 5 years</td>
<td>$487</td>
<td>$2016</td>
</tr>
<tr>
<td>Assets with a maturity period of 5 to 10 years</td>
<td>$231</td>
<td>$845</td>
</tr>
<tr>
<td>Assets with a maturity period of over 10 years</td>
<td>$161</td>
<td>$251</td>
</tr>
</tbody>
</table>

B.

Other Assets *

<table>
<thead>
<tr>
<th>Description</th>
<th>Outstanding Amounts</th>
<th>Transactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Federal Agencies</td>
<td>$788</td>
<td>$1094</td>
</tr>
<tr>
<td>Finance Paper</td>
<td>$514</td>
<td>$861</td>
</tr>
<tr>
<td>Bankers Acceptances</td>
<td>$231</td>
<td>$347</td>
</tr>
</tbody>
</table>

* Averages of daily figures from 1971-77 in millions of dollars: Positions and Sources of Financing and Transactions in by Dealers (Sources: Federal Reserve Bulletin and the Treasury Bulletin)
Section 2.

The following appendix is designed to explain the method used in the tests to determine the statistical significance of the coefficient rankings discussed during the thesis defense. The results provided below supersede those presented in Section 2 of Appendix 1 to Chapter 3 (page 185) of the earlier version of this thesis.

Recall that in both the Keynesian case and the Rational Expectations case, 9 lagged values of actual inflation were used to form proxies for expected inflation under each approach within the context of the Fisher relationship. In the Keynesian case, the sum of the lagged values was taken to be the expectations proxy while in the RE approach, the 9 lagged values served as instruments. With different interest rates as dependent variables, the goal was to determine if the differences in the measured coefficients on the inflation variable were statistically significant. In other words, if $B_1$ is the coefficient on inflation (with yields on Asset 1 as the dependent variable) and $B_2$ is the coefficient on inflation in a Fisher equation with yields on Asset 2 (which differs from Asset 1 with respect to liquidity) serving as the dependent variable, then the test has the null hypothesis that $e'(\hat{B}_1) = e'\hat{B}_2$. To begin, let $X$ represent the matrix of Beta coefficients in mean deviation form. Thus:

$$\text{var}(\hat{B}_1) = s_1^2 (X'X)^{-1} \quad \text{var}(\hat{B}_2) = s_2^2 (X'X)^{-1} \quad \text{cov}(\hat{B}_1, \hat{B}_2) = s_{12} (X'X)^{-1}$$
If RSS is the sum of squares, then

\[ s_1^2 = \frac{RSS_1}{T-10} \quad \text{and} \quad s_2^2 = \frac{RSS_2}{T-10} \]

where \( T \) is equal to the number of observations and \( T-10 \) is equal to the degrees of freedom with 9 lagged values. To calculate \( s_{12} \), divided the covariance by \( T-10 \), i.e.

\[ s_{12} = \frac{\sum \hat{u}_{1t} \hat{u}_{2t}}{T-10} \]

where the \( u \)'s are the residuals. The relevant statistic is a t-statistic defined as

\[ t = \frac{\hat{r} \hat{B}_1 - \hat{r} \hat{B}_2}{\sqrt{V_1 + V_2 - 2C}} \]

where \( V_1 \) is the sum of all elements in \( s_1^2 (X'X)^{-1} \), \( V_2 \) equals the sum of all elements in \( s_2^2 (X'X)^{-1} \) and \( C \) equals the sum of all elements in \( s_{12} (X'X)^{-1} \) which equals \( s_{12}/s_1^2 \) var(\( \hat{B}_1 \)) or \( s_{12}/s_2^2 \) var(\( \hat{B}_2 \)). The computed t-value is then compared to the one relevant for the 95% confidence level. This is done for all of the interest rate series, two at a time, under each of the inflationary expectations approaches. The results obtained are provided below. All of the computed t-values are significant at the .95 level indicating, of course, that the measured coefficients under each of the approaches (Keynesian and RE) are statistically significantly different from one another.
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Section 3.

Originating Equations for Results Presented in Table 3-4

IPI = Log of the monthly Industrial Production Index

LREALM = Log of real money (M1)

LBILLS = Log of real private holdings of Bills (91 day)

LOTH1 = Log of real private holdings of other assets with a maturity period of one year or less (excluding 91 day Bills)

L1Y5Y = Log of real private holdings of notes with a maturity period of 1 to 5 years

L5Y10Y = Log of real private holdings of bonds with a maturity period of 5 to 10 years.

L10Y20Y = Log of real private holdings of bonds with a maturity period of 10 to 20 years

LOV20Y = Log of real private holdings of bonds with a maturity period of over 20 years

RBILLS = One month holding period return on 91-day T-bills

ROTH1 = Holding period return on other assets with a maturity period of one year or less

R1Y5Y = Holding period return on assets with a maturity period of 5 to 10 years

R5Y10Y = Holding period return on assets with a 5 to 10 year maturity period

R10Y20Y = Holding period return on assets with a 10 to 20 year maturity period

ROV20Y = Holding period return on assets with a maturity period of over 20 years

R30Y = Holding period return on 30 year bonds
OLS results estimated in semi-logarithmic form with monthly values for all of the variables.

(1) \[ \text{RBILLS-R30Y} = -6.08 - 0.066 \text{IPI} + 6.41 \text{LREALM} \]
\[ (-1.69) (-4.57) (1.98) \]
\[ + 0.851 \text{LOTHL} + 4.48 \text{L1Y5Y} + 1.57 \text{L5Y10Y} \]
\[ (2.30) (4.82) (2.30) \]
\[ + 2.17 \text{L10Y20Y} + 2.25 \text{LOV20} \]
\[ (3.78) (3.16) \]
\[ R\text{-squared} = .92 \quad F\text{-stat.} = 103.5 \]
\[ \text{S.E.R.} = .387 \quad D-W = 1.79 \]

(2) \[ \text{ROTH1-R30Y} = -9.65 - 0.074 \text{IPI} + 5.59 \text{LREALM} \]
\[ (-2.86) (-5.42) (2.83) \]
\[ + 1.40 \text{LBILLS} + 0.697 \text{LOTHL} + 2.97 \text{L1Y5Y} \]
\[ (1.86) (1.13) (3.37) \]
\[ - 0.78 \text{L5Y10Y} + 1.88 \text{L10Y20Y} + 1.51 \text{LOV20} \]
\[ (-1.39) (3.46) (2.24) \]
\[ R\text{-squared} = .92 \quad F\text{-stat.} = 102.3 \]
\[ \text{S.E.R.} = .366 \quad D-W = 1.81 \]

(3) \[ \text{R1Y5Y-R30Y} = -5.69 - 0.035 \text{IPI} + 5.37 \text{LREALM} \]
\[ (-3.40) (5.18) (2.35) \]
\[ + 0.377 \text{LBILLS} + 0.168 \text{LOTHL} + 0.906 \text{L1Y5Y} \]
\[ (2.01) (1.55) (2.09) \]
\[ + 0.905 \text{L5Y10Y} + 1.09 \text{L10Y20Y} + 0.554 \text{LOV20} \]
\[ (4.06) (4.07) (2.64) \]
\[ R\text{-squared} = .84 \quad F\text{-Stat.} = 49.8 \]
\[ \text{S.E.R.} = .181 \quad D-W = 1.68 \]
(4) $R_{5Y10Y-R30Y} = -6.92 - .051 IPI + 4.04 LREALM$

$( -2.27 ) ( -4.21 ) ( 3.88 )$

$+ .058 LBILLS + 1.08 LOTH1 + 2.28 L1Y5Y$

$( 1.86 ) ( 1.95 ) ( 2.88 )$

$+ 1.64 L5Y10Y + 1.86 L10Y20Y + .818 LOV20$

$( 3.74 ) ( 3.81 ) ( 2.34 )$

$R$-squared $= .83$  
$F$-stat. $= 45.4$

S.E.R. $= .330$  
D-W $= 1.82$

(5) $R_{10Y20Y-R30Y} = -1.10 - .079 IPI + 3.81 LREALM$

$( -2.76 ) ( 2.35 ) ( 3.91 )$

$+ .774 LBILLS + .571 LOTH1 + .551 L1Y5Y$

$( 2.40 ) ( 2.35 ) ( 2.45 )$

$+ .081 L5Y10Y + .464 L10Y20Y + .532 LOV20$

$( 4.38 ) ( 2.98 ) ( 3.83 )$

$R$-squared $= .64$  
$F$-stat. $= 56.35$

S.E.R. $= .157$  
D-W $= 1.80$

(6) $ROV20--R30Y = -1.19 - .051 IPI + 3.39 LREALM$

$( -3.90 ) ( -3.95 ) ( 3.75 )$

$+ .124 LBILLS + .116 LOTH1 + .065 L1Y5Y$

$( 3.42 ) ( 3.48 ) ( 2.41 )$

$+ .200 L5Y10Y + .162 L10Y20Y + .248 LOV20$

$( 2.04 ) ( 3.76 ) ( 1.94 )$

$R$-squared $= .33$  
$F$-stat. $= 31.71$

S.E.R. $= .243$  
D-W $= 1.78$
(7) \[-R30Y = -9.21 - 0.034 \text{ IPI} + 10.36 \text{ LREALM} \]
\[(-4.33) (-3.97) (5.38)\]

\[+ 0.140 \text{ LBILLS} + 0.714 \text{ LOTH1} + 1.55 \text{ L1Y5Y} \]
\[(2.96) (2.83) (2.80)\]

\[+ 1.19 \text{ L5Y10Y} + 1.08 \text{ L10Y20Y} + 0.276 \text{ LOV20} \]
\[(3.88) (3.16) (3.65)\]

R-squared = 0.87  F-stat. = 62.4  S.E.R. = 0.231  D-W = 1.80

Weights used in constructing coefficients in Table 3-4

* GBILLS/M1 = 0.340  G5Y10Y/M1 = 0.078
GOTH1/M1 = 0.120  G10Y20Y/M1 = 0.032
G1Y5Y/M1 = 0.297  GOV20/M1 = 0.029

* all of the ratios are the nominal values of various maturity government securities outstanding (excluding those held by the government and the Federal Reserve) to nominal M1 money

Numerator values as in equation in Table 3-4
With RBILLS - R30Y = 8.39  With R5Y10Y - R30Y = 5.08
With ROTH1 - R30Y = 7.07  With R10Y20Y - R30Y = 4.34
With R1Y5Y - R30Y = 5.90  With ROV20 - R30Y = 3.49

Denominator value (With - R30Y) = 11.08

The results presented above are generally very favorable to the present analysis. Almost without exception the values of the coefficients are of the proper sign, the only exception being the coefficient on L5Y10Y in equation (2) which is not even significant. Additionally, virtually
all of the coefficient values are significant. Although some had t-values below the one appropriate for the 95% confidence level, it was felt that the potential bias problem that would be introduced by the exclusion of these variables outweighed the problem of including econometrically (but not necessarily economically) insignificant regressors. Also, it is necessary to remember that these tests are intended to provide for an empirical verification of the ordinal ranking of the various portfolios with respect to their presumed liquidity and not for precise measurements of the point impact of inflation on portfolio returns.
APPENDIX 2

Method Used to Computate Holding Period Returns

The following section describes the methods by which the holding period yields on assets differing in maturity length were computed. The approach follows that of Cagan (1969). For the short-term (3 month) T-bills which yield no coupon payment but rather whose return comes exclusively in the form of price appreciation, the following formula was utilized.

\[ H_{t,3} = \left( \frac{P_{t+1,2}}{P_{t+3}} \right)^{12} - 1 \]

Here, \( H_{t,3} \) is equal to the one month holding period return on a 3 month bill at time \( t \) and \( P_{t,3}, P_{t+1,2} \) are the prices of the bill at time \( t \) (with three months remaining to maturity) and time \( t+1 \) (with two months left to maturity) respectively. The exponent on the bracketed expression is added for the purpose of annualizing the computed returns.

As the securities are sold at a discount from face value, the prices in (1) can be inferred from the existing maturity yields through the following relationships.

\[ P_{t,3} = \frac{P_{t+3,0}}{1+R_{t,3}}, \quad P_{t+1,2} = \frac{P_{t+3,0}}{(1+R_{t+1,2})} \]

In these expressions, \( P_{t+3,0} \) is equal to the price of the three month asset at time \( t+3 \) with zero months left to maturity (i.e. its face value) and the denominators contain the maturity yields of the three and two month securities.
respectively \((R_{t,3} \& R_{t+1,2})\). Due to such features as the existence of a well-established secondary market for Treasury securities as well as private arbitrage opportunities, the maturity yield of a newly-issued 60 day T-bill can be assumed to be virtually identical to the maturity yield on a secondary T-bill with two months left to maturity. Thus, in this particular case, the one month holding period yield can be determined without any direct data on prices. This can be easily demonstrated by combining (1) and (2) to get

\[
H_{t,3} = \left[ \frac{1 + R_{t,3}}{1 + R_{t+1,2}} \right]
\]

Subtracting this value from the holding period yield of a long-term (30 year) security (obtained from the Ibbotson-Sinquefield data on bond yields) gave the liquidity premium on the long asset used in the empirical analysis of this chapter.

For the longer period assets which yield coupon payments in addition to being subject to capital gains or losses, the holding period returns on Treasury issues grouped into various maturity categories (e.g., 1-5 years, 5-10 years, etc.) were obtained from various issues of a Salomon Brothers publication of economic statistics specifically from a section entitled "Government Rate of Return Indexes." In the Salomon Brothers computation of holding period returns reinvestment is assumed.
CHAPTER 4

THE CAPITAL ASSET PRICING MODEL AND LIQUIDITY PREMIA

The following chapter is devoted to a discussion and empirical analysis of the liquidity premium within the framework of the Sharpe-Lintner two-parameter model of security returns, often referred to as the capital asset pricing model. The sections of this chapter are set out in the following order. First, a somewhat detailed look at the theoretical foundations of the basic capital asset pricing model (hereafter referred to as the CAPM) focusing on particular mathematical features of the model that illustrate its relatedness to standard portfolio choice and utility theory will be offered. The second section will summarize most of the important work done in the empirical testing of the model (e.g., Douglas (1969), Fama and MacBeth (1973)) and will analyze some possible explanations for the unexpected findings of the majority of the tests. For instance a common result emerging from the various studies is that the measured intercept terms for the empirical security market line (a concept to be discussed in more detail later) have been higher than the theoretical values predicted by the CAPM. Another result is that the slope has generally been found to be flatter than would be expected. The notion that the market prices securities in such a way as to include a premium on assets that possess relatively less liquidity value so as to compensate the holders of these instruments will be offered as a possible explanation
for this apparently anomalous behavior of equity returns found by the early researchers in this area. In other words, the underlying notion is that a three-parameter asset pricing model that incorporates a liquidity factor as in F-H serves as the more appropriate model of security returns. The foregoing argument will be tested empirically in the third section of this chapter.

I. Basics of the Capital Asset Pricing Model

The two-parameter model of security returns posits that the market's required rate of return on any asset (or its implicit yield if the asset is a share of stock or a discount bond) will be equal to a "risk-free" rate of return plus a proportion of the difference between the rate of return on the weighted combination of all assets in the portfolio and the risk-free rate. This proportion, commonly called the Beta coefficient, measures the covariance of the particular asset's yield with the yield on the "market portfolio," the value-weighted combination of all assets. The Beta coefficient, as we shall see, incorporates in a sense the particular asset's marginal contribution to the riskiness of the market portfolio and if one of the implications of the two-parameter model is correct, constitutes the only relevant measure of asset risk.

The CAPM in more formal terms is presented below. Let $j$ be a financial asset (say, a share of common stock as these are almost always the assets used in empirical testing of the of the CAPM) then the equilibrium expected rate of
return on \( j \) is described by the following equation:

\[
E(\tilde{R}_j) = R_f + \beta_j (E(\tilde{R}_m) - R_f)
\]

where the tildes denote random values. The variables are defined as follows.

\( \tilde{R}_j \) = the equilibrium expected rate of return on asset \( j \).

\( R_f \) = risk-free rate of return which is faced by both lenders and borrowers.

\( \tilde{R}_m \) = the equilibrium expected rate of return on the so-called market portfolio which is a weighted combination of all assets. The weights are determined by the proportion of the value of each asset in the portfolio to the total value of all assets combined.

\( \beta_j \) = the ratio of the covariance between \( j \)'s rate of return and the variance of the return on the market portfolio, i.e.,

\[
\frac{\text{cov}(R_j, R_m)}{\text{var}(R_m)}
\]

Substituting the last definition into (1) we get;

\[
(2) \quad E(\tilde{R}_j) = R_f + (E(\tilde{R}_m) - R_f)\left(\frac{\text{cov}(\tilde{R}_j, \tilde{R}_m)}{\text{var}(\tilde{R}_m)}\right)
\]

Equation (2) helps to illustrate one of the more important implications of the CAPM namely that an individual asset's return is related to the covariance of its return with the return of all other assets comprising the market portfolio rather than to its own variance. Through judicious portfolio selection all diversifiable risk can be eliminated, at least in principle, yet there will always remain a systematic, non-diversifiable risk which is related to the covariance of the individual asset's and the market
portfolio's returns.

The $\beta$ term is interesting in another respect, alluded to earlier, in that in an amended form it represents the marginal contribution to risk of an asset, i.e., the extent to which the risk of a selected portfolio is increased with a small increment of the asset in question.\(^1\)

In the next section we turn to a discussion of the empirical work done in testing the validity of the two-parameter asset pricing model, focusing attention on some of the unexpected results generally obtained.

II. Tests of the CAPM

The earliest work done with the two-parameter asset pricing model was not so much academic tests of the model per se but rather represented attempts to derive working portfolio evaluation models for use by financial professionals. The types of assets used by the earliest writers in their analyses were the mutual funds as data could be readily obtained on these instruments. A mutual fund is simply an asset which enables a small investor to hold a very diverse portfolio of common or preferred stocks through the purchase of a small share in a large portfolio of stocks held by the issuer of the mutual fund. The purchaser is thus able to circumvent the usual obstacles to diversity in portfolios such as block buying requirements and high brokerage fees.
Jensen (1968) found that there was a positive relationship between the fund returns and the covariance of the returns between the fund portfolios and the market portfolio (as proxied by the Standard & Poor's value-weighted index of selected NYSE common stocks) thus offering evidence in favor of the usefulness of the two-parameter model as a description of the actual process generating the returns on assets. A more direct test using a cross-sectional approach as well as individual equity returns was first undertaken by Douglas (1969). His technique involved testing for the existence of what has become known as the empirical security market line, the presumed linear relationship between the individual asset's average return and its Beta coefficient. More formally, recall the formulation of the basic CAPM as presented in (1) and reproduced below.

\[ E(R_j) = R_f + \beta_j (E(R_m) - R_f) \]

The procedure used was to estimate the cross-sectional regression equation

\[ R_j = A_0 + A_1 \beta_j + e_j \]

where the \( \beta_j \) were obtained from the regressions of individual security returns on an index used as a proxy for the market portfolio, the so-called market model \( (R_j = a_0 + \beta_j R_m + e_j) \). Thus, the tests involve simply comparing the values of \( A_0 \) and \( R_f \) as well as \( A_1 \) and \( R_m - R_f \). If two-parameter model adequately characterizes the way the market
actually prices financial assets it would be expected that $A_0 - R_f$ would not be significantly different from zero while it would be expected to find the $A_1$ coefficient to be statistically significantly positive. The results of the Douglas tests, however, seemed to indicate that not only did the average realized returns on securities not seem to be related to their covariance with the index of returns but were positively related to their own variance over time. Douglas also included in his paper some of the previously unpublished results of Lintner who used a more recent time period (1954-1963) in his cross-sectional analysis of security returns. Lintner’s findings indicate as well that the variance of individual returns is more important in the determination of asset yields than the covariance of returns with the market index (the t-value on the former variance regressor was 6.8). Even more significant, however, was his finding that $A_0$ was much greater than the risk-free proxy rate and that $A_1$ was much less than $R_m - R_f$.

Miller and Scholes (1972) undertook a replication of the Lintner tests for the same time period but on a larger body of data (631 vs. 301 NYSE common stocks). As did Lintner, they took the estimated values of the betas from the first-pass regressions of the annual return for each stock on the average return for all stocks in the sample.
and tested the CAPM in second pass regressions of the individual returns on these estimated betas. As a proxy for the risk-free rate whose value was compared to the computed intercept term \( A_0 \), they used a one-year Treasury note yield to maturity. This particular maturity was selected, of course, so as to match the maturity of the risk-free instrument with the annual data used in the analysis. Also in accordance with the Lintner approach, they used the standard errors of the residuals in the first pass regressions as a measure of non-covariance risk. Their evidence confirmed the Lintner finding of this latter variable's significance in the determination of security returns.

Even more directly relevant for our purposes, however, was the fact that when the regressions were run in risk premium form, in other words, when the proxy risk-free rate was subtracted out from both sides of the equation, the values of the estimated intercepts showed up significantly positive. The t-values on the estimated intercepts were 13.9, 38.3 and 16.2 respectively for the models that included the covariance risk measure alone, the residual (non-Beta) risk measure alone, and both measures of risk as regressors whereas their expected values are zero. Furthermore, the estimated values of \( A_1 \) were significantly less than their theoretical values, in other words the finding of a flatter than expected empirical security market line by Lintner was confirmed.
Miller and Scholes look at a number of potential explanations for these results and reject them one by one, although some tentatively. The most likely candidate for the source of the difficulty would be an errors in variables problem specifically in the individual return covariance measures estimated in the first pass regressions described earlier. The first pass regressions of the market model equation supply only estimates of the actual values and not the "true" beta measures. However, upon further analysis, they reject this particular explanation. From the position of hindsight we can somewhat confidently concur with them in this conclusion as later writers (e.g., Black, Jensen and Scholes (1972)) group the individual securities into various portfolios in order to attenuate any problems associated with individual measurement errors (as well as to reduce computational costs) and still obtain very similar results.

Although the individual securities appeared to plot along the measured security market line very closely, a result in accordance with the CAPM predictions, Miller and Scholes tested more directly for linearity in the basic risk-return relationship to see if some sort of curvature effect could account for the apparent flattening of the line with respect to its expected slope. In other words, they considered the possibility that the true relationship was a curvilinear one. This was done by utilizing a simple quadratic form of the basic equation and re-running the regressions. The results showed that although the additional regressor did show up slightly significantly, its
sign was such that the curvilinear relationship would produce an apparent steepening of the empirical market line rather than a flattening of it. An interesting additional finding was that the \((A_0 - R_f)\) term became even more significantly positive under the quadratic form of the equation. The detrimental effects of other potential biases such as heteroscedasticity, multicollinearity and the selection of an inappropriate proxy for the risk-free rate were also considered but these too seemed to be unable to account for the departures of the empirical results from the theoretical predictions of the model.

Jacob (1971) was the first to use monthly data in her analysis and again the finding of linearity in the risk-return relationship along with a significantly positive value of \((A_0 - R_f)\) and a lower than expected value for \(A_1\) was confirmed.

Black, Jensen and Scholes (1972) were the originators of the approach of grouping securities in order to attenuate any potential errors in variables problem in the measurement of the individual stock betas. Theirs was also the first important direct test of the model to use time series procedures. Estimating the two-factor model in risk premium form, in which the expected value of the intercept term is zero, they found, instead, that the intercept was significantly positive with a t value of over 6.5! In this study, the risk-free rate was proxied by the minimum variance, zero beta portfolio return rather than by the maturity yield of Treasury bills. Once more, the securities
(lumped into 10 portfolios) were remarkably linear in the basic systematic risk-average return relationship with the measured security market line still flatter than would be expected.

Blume and Friend (1973) adopted very similar procedures to Black, Jensen and Scholes for their analysis and discovered virtually the same results except for the last of their three sub-periods (1965-1968) in which the intercept term was much lower and the slope term far higher than expected! However, the linearity of the risk-return trade-off held up. The fact that the usual relationship seemed to be reversed during this time, a period characterized by increased inflationary pressures resulting from an escalation of the Vietnam conflict, may be a fact of some significance to us as we look at the two-factor model test results for the high inflationary 1970s in the third section of this chapter.

No overview of the work done in empirical testing of the asset pricing model would be entirely complete without some discussion of the Fama and MacBeth (1973) paper which is characterized by a great precision and represents still the "state of the art" in CAPM work. In addition, the basic Fama-MacBeth (F-M) approach will be used in our own analysis in the last section of this chapter, hence some degree of early familiarity with their work will be very useful later.
To begin, the F-M approach involves testing the following stochastic generalization (along with several of its variants) of the asset pricing model.

\[
R_{it} = \tilde{C}_{ot} + \tilde{C}_{1t} \beta_{i} + \tilde{C}_{2t} \beta_{i}^2 + \tilde{C}_{3t} \tilde{s}_i + \tilde{e}_{it}
\]

The subscript \( t \) refers to month \( t \) so that \( R_{i,t} \) is the one-period percentage return on security \( i \). The tildes above the coefficients indicate that these values are allowed to vary stochastically over time. The \( \beta_{i}^2 \) term is added to test for linearity (as was similarly done by Miller and Scholes (1972)). This linearity condition is more important than would appear at first glance, as the authors note. If, for example, \( C_2 \) shows up significantly positive, this would imply that high-Beta securities are earning high expected returns, i.e., their prices are too low. By the same token, the low-Beta securities are being priced too high in the market. Thus, the finding of a linear relationship in the risk-return trade-off is essential for acceptance of the hypothesis that prices are determined in financial markets in a manner which is consistent with the desires of investors to hold efficient portfolios.

The \( s_i \) term in equation (4) is meant to represent a measure of risk not deterministically related to covariance risk. Specifically, it denotes the standard errors of the residuals from the so-called market model of security returns referred to earlier and presented below in equation (5).
\[ \tilde{R}_i = \alpha_{im} + \beta_{im} \tilde{R}_m + \tilde{e}_i \]

where \( \alpha_{im} = E(R_i) - \beta_{im} E(R_m) \).

As the errors are assumed to be randomly distributed with an expected value of zero and are independent of the returns on the market portfolio, the following relationship obtains:

\[ \sigma^2(\tilde{R}_i) = \beta^2_{im} \sigma^2(\tilde{R}_m) + \sigma^2(\tilde{e}_i) \]

Equation (6) simply says that the total return variance of asset \( i \) can be split into two component parts, one part which is related to the standard covariance risk measure and the other part which is not. For testing purposes, the residual variances from (16) can be transformed into standard deviations and plugged into regression equation (4) to serve as the measure of the non-Beta risk (the \( s_i \)).

The precise methodology used by F-M in the selection of the individual portfolios must await detailed description until the section of this chapter is reached that concerns the present study's results of the CAPM tests. Suffice it to say at this point, however, that their technique involves the formation of three time periods, the first for the determination and ranking of the individual firm Betas, the second for the selection of the portfolios and the third for the actual testing of the asset pricing model. As noted above, F-M tested three variations of the empirical analog of (4) as well, variations that included the basic two-
factor model as well as those that excluded the $\beta_i^2$ and $s_i$ terms individually.

In the basic form of the model, the "excess returns" described earlier, that is $C_o$ in (4) minus $R_f$, show up significantly with a t-value of 2.55 over the entire 1935-68 sample period and with t-values of 4.56 and 4.84 respectively for the 1951-55 and 1956-60 sub-periods. Over the whole period the measured difference is .0048 or .48 percent per month. In the regressions using the two other variants of (4), this value persists in showing up significantly at least for the 1950s if not for the other sub-periods.

The data are also consistent with the hypothesis of a positive risk-return trade-off. Indeed, the computed value of .0085 for $C_1$ for the overall sample period indicates that bearing risk bore significant rewards during this period; the average incremental returns per unit of covariance risk were .85% per month or over 10% per year.

Also very importantly, the hypotheses that the relationship is a linear one and that covariance risk is the only relevant measure of risk are strongly confirmed by the data. The $C_3$ coefficient lies close to zero for each of the sub-periods in all versions of the tests and the $C_2$ value shows up significantly only for the five year sub-period 1951-55 in the version of the model as presented in (4).

The results of the various tests reviewed in the preceding section are summarized below. First, the evidence is consistent with the hypothesis of a significant positive
linear relationship between realized returns and systematic risk, although the slope of the line is generally less than that predicted by the standard asset pricing model. Second, although attempts to draw a distinction between systematic and non-systematic risk elements in security returns do not always yield definitive results, the majority of tests (the most noteworthy being Fama and MacBeth) indicate that the non-systematic or "diversifiable" risk is statistically meaningless. The implication of this, combined with the other results, is that investors are able and do indeed hold "efficient" portfolios, efficient in the sense of minimizing the variance of returns for the given values of expected returns. The third general finding is that over long time periods, the return on the market portfolio, $R_m$ in (1), is greater than the risk-free rate of interest, in accordance with the prediction of the model. The fourth, almost universal, finding of a measured intercept in the empirical security market line which is significantly greater than the actual risk-free rate for the same periods is the result that is of the greatest concern to us. Fama (1976a) offers a potential explanation of this apparent anomaly in a rare criticism of his own work, a criticism which centers around the inappropriateness of using an equally-weighted index of NYSE stocks as a proxy for the market portfolio. He refers to a study by Fisher (1966) which points out that the standard deviation of the returns on an equally-weighted index of NYSE common stocks is about 1.25 times as high as
the return standard deviation on a value-weighted stock index. Of course, the value-weighted index is the more appropriate measure as it takes into account the varying importance of the shares of different firms by using weights which represent the value of a firm's outstanding shares to the total value of all shares. Thus, the use of an equally-weighted stock measure is not only improper but would tend to overstate the overall degree of riskiness of the market portfolio proxy. In terms of mean-variance analysis the argument is that this particular proxy for the market portfolio lies above and to the right of the true market portfolio on the efficiency frontier. This explains, according to Fama, not only the high measured intercept but the unexpected flatness of the market line found by the great majority of researchers in this area.

The validity of the foregoing argument that the unexpectedly high computed intercepts for the empirical security market line are attributable to the use of an inadequate proxy for the market portfolio can, at least in principle, be put to the test through the use of proxies that stand in closer relationship to the true market portfolio than the equally-weighted index of NYSE equities. Following Fama's implicit suggestion, as well as Roll's (1977) criterion for candidate proxies (to wit that the individual weights should correspond with market value proportions) the CAPM tests will be recomputed utilizing the value-weighted index of portfolio returns as well as indices
consisting of varying proportions of corporate bonds to
stocks as the market portfolio proxies. Our a priori belief
would be that indices containing corporate bonds would be
superior even to a value-weighted stock index in terms of
having an overall level of risk that would more closely
approximate that of the market portfolio.

III. A Discussion of and a Rejoinder to Roll's Critique

Before turning to the empirical sections of this
chapter, a few comments about the Roll (1977) critique of
CAPM testing are perhaps in order, comments including a
discussion of certain methodological problems with his
argument that to my knowledge have not been addressed.
Roll's argument, stated briefly, was that for any
"meaningful" test of the CAPM to be made, complete knowledge
of the true market portfolio's composition must exist which
implies that "every individual asset must be included in a
correct test" (Roll (1977) p. 129). Furthermore, any
"efficient" set of securities serving as the proxy for the
market portfolio and from which a minimum variance, zero-
Beta portfolio can be constructed will generate the linear
relationship predicted in the theoretical CAPM. Because the
market portfolio consists of all assets, financial and real,
any test must utilize a determinable subset of the actual
market portfolio (e.g. the equally-weighted index of
N.Y.S.E. common stocks) and if this subset is ex post
efficient, the asset returns will plot on the empirical
security market line. Thus, the regression tests can prove
only that some selected proxy is efficient, not that the CAPM is valid.

One of the first well-known responses to the Roll attack came from Friend, Westerfield and Granito (1978) who apparently not fully appreciating the fundamental nature of the Roll argument, simply re-computed the CAPM work with a measure of ex ante yields (as in the theoretical model) and seemed to be unable to reject the model's basic hypotheses. However their approach in proxying ex ante returns seems rather ad hoc in that it relies on projections for earnings, dividends and prices based on constant and equal growth rates for these variables.

In a very influential work, Stambaugh (1981) infers from the Roll critique that his major point is that the model's validity may be very sensitive to the specification of the market portfolio. He then delves into this sensitivity question by conducting tests of the CAPM with different compositions (weights) of the market index and with changes in the individual assets comprising the market portfolio. Proponents of the Roll critique might argue that this type of sensitivity analysis as well as other existing evidence on the efficiency or inefficiency of market proxies or even as to a high correlation among the returns on individual proxies is irrelevant to the question of the efficiency of the true market portfolio (M*). As concerns this latter point, whereas it is obvious that there exists no purely exogenous information that would assure a CAPM
researcher that his proxy market portfolio (M) is equal to, or perfectly correlated with, the true portfolio (M*), it may nevertheless be possible to develop a Bayesian interpretation of prior evidence concerning the likelihood that the proxy portfolio returns are perfectly correlated with the true portfolio's returns, as well as to the efficiency of both. In other words, high correlation among proxies (which many researchers have found) may be construed as bearing on the hypothesis that they all lie on or near the security or capital market line and, by implication, have returns that are very highly correlated with the returns on the "true" market portfolio. If these various proxies are also efficient it may also be inferred in a Bayesian sense that the true market portfolio is efficient. An implication of this is that any tests of the CAPM utilizing efficient proxies will be precise enough for econometric purposes. Hence, meaningful analyses of the asset pricing model's validity can be undertaken even if the composition of M* is unknown because the Bayesian priors essentially constitute a type of "exogenous" information, although that which is different from direct knowledge of M*. I believe that this point is of fundamental importance although it seems to have been totally neglected in the literature. Keeping the above argument in mind as well as the fact that the two-parameter asset pricing model is apparently still regarded as an appropriate research paradigm (as evidenced by the substantial number of scholarly articles still being written about it) we turn to the
empirical section of this chapter which seeks to shed some light on some of the apparently anomalous findings of many of the more important CAPM analyses.

IV. Empirical Results of the Short-Run CAPM Tests

The purpose of this section is actually two-fold. We will first be concerned with the recomputation of the two parameter asset pricing model tests using various proxies for the return on the market portfolio. Secondly, we will focus on the "excess returns", if any, from the empirical tests of the asset pricing model and study their behavior with respect to certain key economic variables. In other words, from the measured intercepts of the CAPM test results described above the "risk-free" rates of return will be subtracted and if these differentials are shown to be significantly positive, the hypothesis will be entertained that they individually represent some sort of liquidity premium on common stocks vis-a-vis Treasury bills. Empirical testing of the relationship between these values and variables which according to F-H should influence liquidity premia will then be undertaken.2

An additional concern of ours will be to test the sensitivity of the security market line (SML) to various proxies for the market portfolio. The underlying notion is that with the use of different proxies for the market portfolio it is first necessary to test for the stability of coefficients across the different linear regressions looking
at two at a time. The formal discussion of this will come a bit later in this section.

Keeping Roll's admonition in mind (see footnote 2), it should be noted that none of these proxies may be representative of the true market portfolio and that none of them may be mean-variance efficient. The proxies used in the analysis are formed from combinations of two equity indices and one bond index. The equity indices are the equally-weighted market returns and the value-weighted market returns on NYSE common stocks constructed from data obtained from the computer tapes of the Center for Research in Security Prices at the University of Chicago. The computed monthly equity returns include capital gains and any dividends paid, with appropriate adjustments made for stock splits. The bond returns index was calculated from data taken from the quote sheets of Salomon Brothers and is based on coupon payments along with price changes for roughly 750 corporate bonds rated AA or AAA by Standard and Poor. The index is value-weighted and approximates a bond portfolio with a maturity of 20 years.

The three above indices were combined to form the following market portfolio proxies.

(1) An equally-weighted market index of NYSE securities (common).

(2) A value-weighted market index of NYSE securities.

(3) An index comprising the equally-weighted market index and the bond index, with a 50% weight on each.
(4) An index comprised of the equally-weighted market index and the bond index with a 70% and 30% weight on each respectively.

(5) An index consisting of the equally-weighted market index and the bond index with a 30% and 70% weight on each respectively.

The particular empirical approach follows that of Fama and MacBeth (1973) which is described in great detail in Fama (1976a) and Klemkosky and Vora (1981), from which the CAPM test results used in the present analysis were taken. The specific steps included first the computation of the individual firm betas \((\beta_{im})\) from the market model of security returns (the empirical analog of equation (5)) and then ranking the individual betas in ascending order. The ranked firm betas are then divided into 20 portfolios to reduce the statistical impact of any errors in the measurement of the individual betas, the "errors in variables" problem discussed earlier. To ensure that the different portfolios contained, as nearly as possible, an equal number of securities, the following procedure was used. Let \(n\) be the total number of securities to be apportioned among the 20 portfolios and let \(\text{int}(n/20)\) be the largest integer less than or equal to \(n/20\). All but the first and last portfolios will have \(\text{int}(n/20)\) securities each. If \(n\) is an even number the first and twentieth portfolios will have \(\text{int}(n/20) + 1/2 (n - 20 \text{int}(n/20))\) securities each. If \(n\) is odd, the last portfolio will contain an additional security. This portfolio formation period uses the first three years of monthly data.
Next, the firm betas are recomputed from the market model from the data of the two year portfolio estimation period and the initial portfolio betas are then estimated by averaging the individual betas in each portfolio. The $\beta_{im}$ are updated annually using the estimation period data and adding one succeeding year. The result is that each security will have an additional $\beta_{im}$ calculated.

The next step involves the creation of a monthly time series with $\beta_{pmt}$ (beta value for portfolio p at time t) over the next two year testing period computing $\beta_{pmt}$ as the simple arithmetic average of $\beta_{im}$ in each portfolio thus adjusting $\beta_{pmt}$ on a month-by-month basis to allow for delisting of individual securities.

Finally, to get the independent variables from other periods these steps are repeated and the OLS regressions of

\[
R_{pt} = \gamma_1 + \gamma_2 \beta_{pmt} + e_{pt}
\]

are run to obtain the estimates of the intercept and slope terms for each month of the testing period. Of course, these steps were undertaken five times, once for each proxy for the market portfolio.

To summarize, three years of data are used to compute the individual betas and to allocate them among portfolios. The next two years of data are used to calculate the initial values of the regressors. Finally for the last two years, the testing period, monthly values of the portfolio betas are used to form estimates of the parameters in (7).
For the equally-weighted and value-weighted return indices of NYSE stocks the data used for the analysis begins in June of 1953 and extends to December of 1979 while for the other indices the time period covered is February 1974 to December 1979. Also, it should be noted that not all securities that were available were regarded as being eligible in the sense of having a "sufficient" number of monthly observations previous to the first month of the testing period. To be included in a portfolio, a security had to have data for both years of the estimation period and for at least two years of the formation period. Approximately 1200 securities met these eligibility criteria for the 1974-1979 period. It should additionally be noted that for testing purposes the regressors in (7) were lagged by one period, despite the notation given there. The underlying notion is that adopting such a technique allows for the tests to be "predictive" in nature, in other words we can match the returns for month t with estimates of the risk measures that were available at the beginning of the month.

As mentioned previously, the use of different proxies for the market portfolio immediately raises a question concerning the sensitivity of the computed security market line to the proxy used. The appropriateness of using various market portfolio proxies in the same two-factor model is ultimately a question that can be answered only by an appeal to empiricism, specifically testing for the stability of coefficients across the different linear
regressions. The most likely candidate for a suitable test is the Chow Test under which the null hypothesis is that the set of coefficients in two linear regression equations is stable. The technique involves computing F-ratios of errors and testing to see if the computed values exceed or fall short of the critical F-values. The first result would, of course, imply rejection of the null hypothesis and the second acceptance of it. The following sensitivity test results were obtained from Klemkosky and Vora (1981). The Chow test was conducted on a month-by-month basis where the equation using the equally-weighted stock proxy was compared to the other four proxies, each in turn. Thus, there were four combinations of equations. For only 5 of the 71 months used in the analysis, across all combinations, was the null hypothesis rejected. The highest number of monthly rejections at the .05 level was for the combination which included the 30%/70% equity/bond proxy (four). The other 3 combinations had 2 monthly rejections each. The inference to draw from these results is, of course, that the regression relationships using different proxies for the market portfolio returns are very stable over time, at least for the 1974-1979 period.

The next section is devoted to an empirical analysis of the hypothesis that the "excess returns" from the CAPM found by the majority of the researchers whose work was discussed above, represent liquidity premia of the Fried-Howitt type. We know from an earlier discussion of the model that the
premium is positively related to increases in expected inflation as a key implication of the model is that inflation causes the real pecuniary yield to be bid down as agents seek to substitute bond for money holdings. However, because of the reduction in outstanding real stocks of bonds, at the margin bonds are generating greater non-pecuniary returns in the form of liquidity services. Additionally, given that an assumption of the model is that the greater is the level of expenditures the greater are the marginal liquidity yields of both bonds and money \((l_{xm}, l_{xg} \geq 0)\), we would expect to find that the premium is positively related to the level of economic activity.

Finally, the reader is reminded that the liquidity premium is inversely related to the ratio of the outstanding values of the relatively less liquid asset to the asset which generates a greater portion of its total return in the form of liquidity services.

\[
\vartheta(r-r_g)/\vartheta Y = 1/l_x \vartheta l_g/\vartheta(Ym)Y - l_g/1_x \vartheta l_x/\vartheta Ym < 0
\]

In the F-H analysis using money and bonds, this was due to the fact that if the assets are less than perfect substitutes then an incremental unit of bonds drives down the liquidity yield on bonds vis-a-vis money, i.e., decreases the liquidity premium.

For the empirical analysis the rate of inflation was defined as the monthly percentage change in the non-seasonally-adjusted Consumer Price Index (INF) and real personal income (REALPY) was used as a proxy for the level
of economic activity. Both series were obtained from the computer tapes of CITIBASE. The third independent variable was the ratio of the total value of all outstanding NYSE common shares on a monthly basis to the nominal value of all outstanding 3 month U.S. Treasury Bills held by private investors (C/G). The former series was obtained from Salomon Brothers and the latter from various issues of the Federal Reserve Bulletin. The rationale behind using this particular constructed series is, of course, that in the context of the empirical CAPM tests the returns on individual common stocks proxy for the return on the market portfolio while the T-Bill yield serves as the proxy for the risk-free rate. The dependent variables were obtained by subtracting the one-month yields to maturity from the measured intercepts from the CAPM tests discussed in detail earlier. The results presented immediately below are for the overall period for both the value-weighted and equally-weighted return indexes. Current values of the independent variables were used. T-statistics are in parentheses.
### TABLE 4-1

Results of the CAPM Liquidity Premia Tests

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficients on (1953-1979)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EQW*</td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>27.49 (-1.81)</td>
</tr>
<tr>
<td></td>
<td>(.384) (.98) (.786) (.57)</td>
</tr>
<tr>
<td>AR1</td>
<td>32.96 (-1.72)</td>
</tr>
<tr>
<td></td>
<td>(.202) (.87) (-8.02) (.32)</td>
</tr>
<tr>
<td>VAL**</td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>15.60 (.72)</td>
</tr>
<tr>
<td></td>
<td>(-3.24) (14.28) (-13.62)</td>
</tr>
<tr>
<td>AR1</td>
<td>18.12 (.63)</td>
</tr>
<tr>
<td></td>
<td>(-2.76) (13.55) (-12.92)</td>
</tr>
</tbody>
</table>

EQW* = the measured vertical intercept from the CAPM tests using the equally-weighted measure of equity returns minus the one-month T-bill yield to maturity on three month securities.

VAL** = as above except the value-weighted measure of equity returns is used.

The results provide some evidence that as far as the overall sample period is concerned, the CAPM "excess returns" behave in accordance with liquidity premia of the F-H type. The coefficients on real personal income and on the ratio of common stocks to T-bills are of the proper sign and are significant. Furthermore, the computed F-values are in excess of the critical ones indicating rejection of the null hypothesis of the lack of joint influence of the explanatory variables. The disturbing aspect is that the
coefficients on the inflation values do not show up significantly. To test whether this results was due to some problem of collinearity among the variables Farrar-Glauber analysis was undertaken to more precisely determine which explanatory variables, if any, were linearly dependent. The basic procedure involves plugging the individual diagonal elements from the inverse matrix of simple correlation coefficients of the independent variables into an expression defining an F-value and determining whether the computed F-statistics exceed or fall short of the critical one. At the .01 level, the critical F-value is $F(312,3) = 26.1$. Inflation appeared to be the variable most seriously affected by multicollinearity (with an F-statistic of 105.1) whereas real personal income was only slightly collinear (F-value = 30.5). The ratio of outstanding stocks to T-bills had a computed F-statistic far below the critical value. A number of approaches to correct the collinearity problem were tried including running the regressions with the income variable deleted (as OLS estimation showed that this was the variable with which inflation was most highly correlated) and the coefficient on inflation did show up significantly positive, however in this form the model is, of course, misspecified. In addition the techniques of redefining the variables as their first differences as well as their logarithmic first differences were tried (in order to remove any common trend effect) but the results were not very satisfactory. The technique of principal components as an alternative approach to be used in the face of collinear
series was rejected a priori as the number as well as the nature of the explanatory variables was such as to make the grouping of them into more fundamental values inappropriate. We must therefore content ourselves with the results reported above bearing in mind that the standard errors are inflated as a consequence of multicollinearity (and thus the t as well as the F values will be underestimated).

The results for the inflationary sub-period are not significant. As mentioned earlier, for the 1974-1979 period a larger number of dependent variables was available. In addition to the two used above, three additional values representing the CAPM vertical intercepts using varying proportions of common stock to corporate bond returns as proxies for the market portfolio returns were used.

The AR(1) results with current values of the explanatory variables and with 71 observations are presented below. The selection of the best lag lengths for each regressor did not have any substantive impact on the results so consequently they are not shown. Again, the t-values are in parentheses.
### TABLE 4-2

AR(1) Regressions of CAPM Excess Returns (1974-1979)

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Constant</th>
<th>INF</th>
<th>REALPY</th>
<th>C/G</th>
<th>$R^2$</th>
<th>D-W</th>
<th>$F(3,70)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EQW</td>
<td>301.00 (1.54)</td>
<td>-3.46 (-1.21)</td>
<td>.01 (.09)</td>
<td>-46.69 (-2.15)</td>
<td>2.05</td>
<td>3.30</td>
<td></td>
</tr>
<tr>
<td>VAL</td>
<td>706.31 (1.70)</td>
<td>-3.23 (-.59)</td>
<td>-46.19 (-.97)</td>
<td>-48.97 (-1.14)</td>
<td>2.17</td>
<td>1.38</td>
<td></td>
</tr>
<tr>
<td>EB7030</td>
<td>231.11 (.59)</td>
<td>-3.11 (.82)</td>
<td>-19.39 (-.44)</td>
<td>-3.83 (-.21)</td>
<td>1.92</td>
<td>.37</td>
<td></td>
</tr>
<tr>
<td>EB5050</td>
<td>-1115.21 (-1.17)</td>
<td>-2.86 (-.61)</td>
<td>99.06 (.90)</td>
<td>41.92 (1.58)</td>
<td>2.02</td>
<td>2.11</td>
<td></td>
</tr>
<tr>
<td>EB3070</td>
<td>1205.88 (1.76)</td>
<td>-1.41 (-.25)</td>
<td>-144.70 (-1.87)</td>
<td>-2.81 (-1.87)</td>
<td>2.09</td>
<td>1.22</td>
<td></td>
</tr>
</tbody>
</table>

EB7030 = proxy with 70% weight on equities, 30% on bonds
EB5050 = proxy with 50% weight on equities, 50% on bonds
EB3070 = proxy with 30% weight on equities, 70% on bonds

The results clearly show that the CAPM "excess returns" do not behave like liquidity premia of the F-H type during the inflationary sub-period. Furthermore, the poor results do not seem to be caused by either multicollinearity or heteroscedasticity as indicated by Farrar-Glauber and Park-Glesjer analysis respectively. Although there appeared to be a slight degree of non-linearity in the parameters, the use of non-linear estimation techniques did not substantially affect the results. The insignificant results obtained when focusing on the sub-period are perhaps not too
surprising when we remember that the liquidity model is essentially a long-run one and the sample period dealt with in the immediately preceding analysis is under 6 years. Indeed, it will be remembered that the results over the much longer overall sample period (using approximately 26.5 years of data) were far more significant. Of course, a simple extension of the sample period alone does not provide any information about the true long-run relationships among the variables. However a moving average filter, such as the one discussed earlier in connection with Summers’ work, applied to all of the series would provide for a clearer notion as to how long swings in the "excess returns" have responded to long swings in the explanatory variables. We now turn to the evidence on this question.
V. Low-Frequency CAPM Results

The desirability of smoothing the data on the intercept terms in the type of CAPM work presented below follows from the fact that there is considerable month-to-month variation in both the intercept and slope terms (the interrelationship between the two follows from the fact that an overestimated intercept will lead to an underestimated slope coefficient and vice versa, a fact that, surprisingly, seems to have been totally ignored in discussions of some apparently anomalous empirical CAPM results). In addition, although the intercepts as well as the slopes for both the value-weighted and equally-weighted market indices are on average positive, as predicted by the model, for individual months it is not uncommon to find negative values for either or both. Primarily because of findings like these, some researchers have despaired of the ability of CAPM to yield testable hypotheses and predictions, at least in practice if not in principle (see, for example, Roll (1977)). However I believe that these criticisms are unfounded and can be easily shown to be so by considering the fact that all of the hypotheses derived from the model deal with the relationship between the expected values of risk and return, not with the relationships between actual risk and return which a scarcity of data necessitates the use of in most analyses. It may very well be that for a particular month measured beta risk for the various portfolios may not account for much of the difference among their returns, although over
time we would expect this risk measure to dominate in the
determination of yield differentials. A very important
point is that what matters for tests of the model is not
that these measured values on a period-by-period basis
conform strictly to the predictions of the theoretical CAPM
but rather whether the expected values, which in the
simplest case can be regarded as mathematical averages, have
the proper signs and are of plausible magnitudes. In all of
the important asset pricing model studies this has been
shown to be the case.

Also, the monthly variability of the computed
intercepts and slopes is not at all contrary to the two-
parameter asset pricing model as these values represent, at
least in most empirical analyses including part of the
present one, the monthly returns on a portfolio of NYSE
stocks whose returns have traditionally been quite erratic,
even on very highly diversified portfolios (see, for
example, Fama (1965)).

Stated in this manner, the above arguments implicitly
make the case for applying some sort of averaging process to
the data before searching for the empirical relationships
among the variables involved. Of course, an alternative
technique would be to exclude certain data points (e.g.
those months for which negative intercepts were computed)
and then to undertake the standard OLS estimation. Although
this has been done, it probably introduces a missing
observation bias problem. Therefore, for purposes of the
present study, the approach of pre-smoothing the data was
used. Specifically, an equally-weighted centered moving average filter of various lengths (12, 24 and 36 months) was applied to all series including the intercepts measured in risk-premium form and partial (as opposed to simple) correlations between the dependent and each independent variable in turn were computed for all filter lengths. The centered moving average technique has the advantage of making it unnecessary to first de-trend the series and thus greatly diminishes any spurious correlation between variables. For purposes of comparison, the partial correlations between contemporaneous or current values are included.
### TABLE 4-3
Longer Run Partial Correlation Coefficients of CAPM
Test Variables

<table>
<thead>
<tr>
<th>Current</th>
<th>EQW</th>
<th>VAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>INF</td>
<td>.293</td>
<td>.250</td>
</tr>
<tr>
<td>REALPY</td>
<td>.363</td>
<td>.436</td>
</tr>
<tr>
<td>C/G</td>
<td>-.459</td>
<td>-.507</td>
</tr>
</tbody>
</table>

1 year
- INF .309 .203
- REALPY .515 .614
- C/G -.706 -.813

2 years
- INF .410 .578
- REALPY .573 .707
- C/G -.756 -.907

3 years
- INF .509 .712
- REALPY .588 .729
- C/G -.796 -.708
These values provide striking evidence of a more substantive relationship between the dependent variables and the independent variables, as listed in Tables 4-1 and 4-2, when the influence of erratic movements of, most importantly, the former group is reduced through smoothing. The partial correlations are without exception of the proper sign and generally tend to rise in absolute value with the length of the moving average filter. Perhaps at lower frequencies, for example at frequencies well below the length of the standard reference cycle, the relationships above would be even stronger. However, for the present study, it was felt that the loss of degrees of freedom associated with increasing the filter length did not justify so doing. It is also interesting to note that in most instances the correlations with the value-weighted returns are greater than those with the equally-weighted ones. Our prior would be that the index weighted with market values would be more representative of the true market index and thus it is encouraging that these relationships would show up more strongly.

Also very encouraging from the viewpoint of the present work, these results (along with those presented in Table 4-1) strongly indicate that the incorporation of an F-H type of liquidity premium into the standard CAPM analyses, a premium reflecting the lesser liquidity of the assets comprising the market proxy vis a vis Treasury bills, may account for the seemingly inexplicable CAPM results obtained by the great majority of researchers in this area.
This chapter has dealt with the possibility that the "excess returns" found in early studies of the asset pricing model, that is the differences between the measured intercepts of the security market line and the proxy risk-free rate (when using an index of stock returns as a proxy for the return on the market portfolio) represent a premium incorporated in equity returns reflecting their lower marketability or liquidity characteristics. According to the basic CAPM, the expected value of these differences is zero and thus in empirical analysis should be statistically meaningless. However, in the last section of the foregoing chapter evidence is presented that these differences appear to be systematically related to variables that, according to Fried-Howitt, should impinge on the liquidity premium, at least for the overall sample period if not for the inflationary sub-period. This was shown to be the case using both equally-weighted and value-weighted equity returns as a proxy for the return on the market portfolio. Furthermore, comparing longer swings in the variables by utilizing moving averages and partial correlation coefficients provided further evidence that these excess returns behave as liquidity premia of the F-H variety.
1. To clarify this point, it would perhaps be useful to look at the question within the framework of a constrained optimization problem namely that the individual's goal is, given his preferences, to choose the proportions in which he holds the various assets so as to minimize the variance of the selected portfolio return subject to the joint constraints that the proportions must sum to unity and that the expected portfolio return is simply the weighted sum of the individual asset returns. These constraints are presented in a formal manner below.

(1) \[ \Sigma x_i = 1 \quad (b) \quad E(R_p) = \Sigma x_i E(R_i) \]

Here, \( x_i \) represents the fraction of total invested funds held in asset \( i \) where there are \( n \) assets. The optimization problem can be illustrated in the following Lagrangean expression.

(2) \[ \text{Min } \phi = \left[ \Sigma \Sigma x_i x_j \sigma_{ij} \right] + \lambda [E(R_p) - \Sigma x_i E(R_i)] + \lambda_2 [1 - \Sigma x_i] \]

The first expression on the right hand side is simply the return variance of the portfolio which is expressed as the summation of all the covariances of the yields on assets in the portfolio properly weighted by the \( x_i \)’s. In other words,

(3) \[ \text{var } (R_p) = \Sigma \Sigma x_i x_j \sigma_{ij} \]

or, in standard deviation form

(4) \[ \sigma(R_p) = x_i x_j \sigma_{ij}^{1/2} \]

Rewriting (4);

(5) \[ \sigma(R_p) = \sigma^2(R_p)/\sigma(R_p) \]
\[ \sigma(R_p) = \Sigma x_i x_j \sigma_{ij}/\sigma(R_p) \]
\[ \sigma(R_p) = \Sigma \Sigma x_j (\Sigma x_i \sigma_{ij}/\sigma(R_p)) \]
Differentiating \((R_p)\) with respect to \(x_j\), we have

\[
(6) \quad \frac{\partial \sigma(R_p)}{\partial x_j} = \Sigma x_i \frac{\sigma_{ij}}{\sigma(R_p)} = \Sigma x_i \frac{\text{cov}(R_i, R_j)}{\sigma(R_p)}
\]

Finally, remembering that \(\Sigma x_j = \Sigma x_i = 1\), we get

\[
(7) \quad \frac{\partial \sigma(R_p)}{\partial x_j} = \frac{\text{cov}(R_i, R_j)}{\sigma(R_p)}
\]

where the right hand side of the expression is, of course, a modified form of the Beta coefficient with the standard deviation of the portfolio return rather than its variance in the denominator.

From the optimization problem as stated in (2) and with the explicit inclusion of an asset into the model whose return is risk-free (or at least has zero covariance risk), it is possible to derive the basic CAPM relationship as expressed in (1).

2. I wish to thank Dr. Gautam Vora of Penn State University and Dr. Robert Klemkosky of Indiana University for providing the CAPM intercepts as well as the results of the sensitivity tests discussed later in the text.

3. The rationale for this approach of re-estimating the betas, described in Fama (1976a) pp. 347-8, is to avoid the so-called regression phenomenon. In ranking the betas initially measurement errors are also being ranked in some sense. Although re-estimation of the betas will produce new measurement errors, these will probably be uncorrelated with the original ones.
Appendix 1

Method Used to Compute Partial Correlations

The purpose of the following is to demonstrate how the partial correlation values presented in Table 4-3 were computed and, very importantly, how the positive values on inflation reported there are not inconsistent with the negative values of the OLS inflation coefficients presented in Table 4-1. The partial correlations were obtained from running the SAS statistical package with the PROC REG procedure. Because partial correlation values can be computed automatically only in the OLS procedure in SAS, it was first necessary to correct for the serial correlation problem in PROC AUTOREG and then to manually adjust the data using the Durbin procedure as described in Econometric Models and Economic Forecasts by Pindyck & Rubenfeld, 2nd ed. on page 158. To begin, the smoothed data were run in PROC AUTOREG with both an AR(1) and an AR(2) correction and the AR parameters were obtained. Of course, only the AR parameters that were statistically significant were used in the next step which involved adjusting the data in accordance with the AR parameters obtained. For example, if only the AR(1) parameter is significant and has a value, say, of .6, then the adjusted equation is $Y_t - .6Y_{t-1} = A_0(1-.6) + A_1(X_{1t} - .6X_{1t-1}) + A_2(X_{2t} - .6X_{2t-1}) + A_3(X_{3t} - .6X_{3t-1})$. This equation is then run OLS in PROC REG and the partial correlation values are obtained. This process removes any
serial correlation that exists in either the original contemporaneous or smoothed data and provides for a measure of the correlation of the dependent variable and each independent variable while holding constant the influence of the other two explanatory variables.

To demonstrate how the partial correlation values presented in Table 4-3 of the thesis, particularly those on inflation, are not inconsistent with the negative (actually insignificant) coefficients on inflation in the contemporaneous OLS regressions, consider the following example of how partial correlation values can be computed (and actually were computed for the first version of the thesis). The technique that was employed was borrowed from Statistics by Murray Spiegel (1961) and is presented on page 272 of that text. The technique involves computing partial correlation values from the simple coefficients of correlation and the coefficients of determination obtainable from either the SAS or the TSP statistical package. As an example, assume a generalized equation of the following form:

\[
(1) \quad X_1 = a_1 + a_2 X_2 + a_3 X_3 + a_4 X_4
\]

As in the equation from the thesis, there are three explanatory variables. Let \( r_{12.34} \) equal the partial correlation coefficient between \( X_1 \) and \( X_2 \) keeping constant the effects of \( X_3 \) and \( X_4 \). The formula is:
(2) \[ r_{12.34} = r_{12.3} - r_{14.3} r_{24.3} / \sqrt{(1 - r_{14.3})^2 (1 - r_{24.3})^2} \]

where \( r_{12.3} \), for example, equals the partial correlation coefficient between \( X_1 \) and \( X_2 \) holding the effects of \( X_3 \) constant and is obtainable from the following equation:

(3) \[ r_{12.3} = r_{12} - r_{13} r_{23} / \sqrt{(1 - r_{13})^2 (1 - r_{23})^2} \]

In the above equation, \( r_{12} \) is the simple coefficient of correlation between \( X_1 \) and \( X_2 \) obtainable from the TSP statistical package and \( r_{13}^2 \) is the coefficient of determination (R-squared) between \( X_1 \) and \( X_3 \).

The whole process of computing the partial correlation values then becomes a simple mathematical one. It should be noted that there is no necessary inconsistency between negative simple correlations and positive partial ones. For example, as in the thesis, let the first explanatory variable, \( X_2 \), represent inflation and \( X_1 \) the excess returns as described in the text. If a negative OLS coefficient on inflation is found (as was the case in the thesis) it would tend to be associated with a negative coefficient of correlation, (i.e. \( r_{12} < 0 \)) then it follows from the equations above that \( r_{12.34} \) needn’t necessarily be negative simply because \( r_{12} \) is or, in our case, the negative coefficient on inflation presented in Table 4-1 is not inconsistent with the positive partial correlation presented in Table 4-3.
As concerns the degree of credibility to be offered in the smoothed results presented in Table 4-3, it is necessary to remember why these tests were undertaken in the first place. The contemporaneous OLS results of the CAPM liquidity premia tests presented in Table 4-1 showed that there were some systematic and reasonably strong relationships showing up between the "excess returns" and the explanatory variables, however the presence of multicollinearity (discussed in detail on page 220 of the earlier version of the thesis) made interpretation of the results somewhat problematic. In particular, inflation, seemed to be highly collinear with the other two regressors (which may very well account for the insignificantly negative coefficients reported in 3 of the 4 cases). Therefore it was felt that some test of the general relationship between the dependent variable and each independent variable in turn, a test which simultaneously avoided the collinearity problem while focusing on the longer run (in accordance with the longer run context of the model) would be very useful. Partial correlations of smoothed data were deemed to provide such a test. It should be noted that although smoothing the data did exacerbate somewhat the serial correlation problem discussed in the text, smoothing of the data does not introduce any serial correlation which cannot be corrected for through the use of a commonly accepted method such as the Durbin procedure discussed above. Therefore the results presented in Table 4-3, and discussed in detail above, were
computed correctly and are both conceptually and statistically meaningful.

Finally, it should be noted once more that as far as the reporting of the various statistics is concerned it was not always possible to provide a consistent set of statistics with every equation within the context of some given analysis. For example, in Table 3-2, no R-squared or F-values are provided by the TSP statistical package in the equations with rational expectations assumptions because of the instrumental variables approach used. Under the instrumental variables procedure such statistics would be meaningless.
CHAPTER 5

SUMMARY AND CONCLUSIONS

The purpose of this chapter is to draw together some of the conclusions reached in the first four chapters of the present work as well as to highlight some of the thesis' more important and unique contributions.

The thesis can be regarded as having two primary goals: to provide for a very thorough analysis of the inflation-interest rate relationship in the American economy since the end of the Korean war and to empirically test a model that accounts for the apparent failure during this period of the Fisher effect, i.e. the full adjustment of nominal rates to inflation. An important aspect of the present work with respect to the first stated goal is the updating of the empirical work on the Fisher effect, not only with respect to the time period considered but also in regard to considering recent theoretical models which purport to explain the commonly observed findings of underadjustment. In addition, in certain sections statistical techniques that were either not readily available or widely used by economic researchers until relatively recently are invoked to more carefully test for the actual inflation-interest rate relationship. By extending the sample period to the end of 1983, as is done in several parts of Chapter 2, a period of unusually high real interest rates (the early 1980s) is included in the analysis, with no significant changes in the basic results. Some writers have suggested the need for new
theoretical models to account for recent high real yields. However it is argued in that chapter that high real rates may simply represent another aspect of the general underadjustment phenomenon; in this case nominal yields not fully adjusting downwards to a declining trend in actual and expected inflation.

An important feature of the present work concerns the testing for the influence of certain variables on real interest rates, specifically variables commonly thought to impinge on real rates. For instance, there is no evidence that particular variables that proxy for economic activity (e.g. the industrial production index, employment/population ratios, etc.) have any influence on real yields (see Table 2-3). The only variable that shows up statistically significant in these tests is M1 money growth. However this may be primarily attributable to the high correlation of M1 growth and lagged inflation rather than to the influence of money growth taken by itself. Thus, these tests tend to confirm the findings of Mishkin (1981), Wilcox (1983) et. al. that lagged inflation dominates as a determinant of real rates in the short run. Evidence of the unimportance of various other real variables in the determination of yields in a low-frequency analysis is also discussed.

Some recent theoretical work which purports to account for the underadjustment phenomenon is examined and shown to be either misconceived or inapplicable to the time period of interest. For instance, Fama's 1982 stagflation model
which produces something akin to an inverse expectational
Phillips curve to account for falling real yields in the
face of inflation is critically reviewed. It is
demonstrated that his empirical results are quite sensitive
to the definition of money used and, more importantly, that
the particular coefficient values often show up either
ingsignificantly or with the 'wrong' sign when monthly (as
opposed to his yearly) data are used in order to increase
the degrees of freedom.

Other underadjustment models such as Wilcox's (1983)
oil supply shock model are considered but it is argued that
they are inapplicable to our full sample period. The recent
evidence concerning the unimportance of various money
surprise measures, announcement effects and deficits in
influencing real rates is also discussed in Chapter 2 as is
some recent theoretical work that attempts to account for a
less than full Fisher effect by invoking the variance of
inflation as an explanatory variable. This argument by
Blejer and Eden (1979), which apparently has been somewhat
influential given the number of generally favorable
citations of it in the literature, is shown to be incorrect,
at least for the period of interest.

The Fama work on short-term rates as predictors of
inflation is also replicated with an extended sample period.
Not only does extending the period provide an opportunity to
test the robustness of his findings but avoids the problem
of focusing on an apparently unique period of U.S. economic
history when a full Fisher effect was operating, a problem
with Fama's study. Also, by utilizing the entire Fama analysis in the sense of dichotomizing between real rate constancy and market efficiency tests, a procedure not undertaken by the various critics of Fama's work, some unusual results are obtained. Although preliminary, the results indicate that real rates may have been constant during the study's sample period but that the T-bill markets did not operate efficiently in the sense of incorporating fully the market information implicit in past inflation rates in order to make assessments of future inflation in the setting of nominal yields. These rather surprising results are presented tentatively in the chapter as certain problems do crop up in the interpretation of the Fama tests, some of which are discussed in Chapter 2.

An analysis of the potential effects of a non-neutral tax system on the inflation-interest relationship demonstrates that a tax-adjusted Fisher effect equation would serve only to make the observed underadjustment more pronounced and hence cannot be used to account for falling real yields in the face of inflation. As discussed in the chapter, the use of plausible values for the appropriate marginal tax rates indicates that the coefficient of adjustment should be in the 1.3 to 1.5 range, far higher than the .6 to .8 range usually observed.

The various analyses referred to above show that both the more standard and relatively recent proposed explanations for the failure of the Fisher effect in the short-run are
inappropriate for the American economy for the post 1953 period, thus setting the stage for the more detailed presentation of the liquidity model, as offering a more plausible explanation for underadjustment, a bit later.

An additional feature of the present work is the testing for the longer-term relationship between inflation and nominal yields. This is done by pre-smoothing the data with moving averages which approximate or exceed the length of the standard business cycle. In the analysis three different approaches are used. One borrowed from Summers (1981) involves the use of an equally-weighted one-sided moving average filter and indicates that the inflation-interest rate relationship is close and less than one-for-one with filters of no more than about 2 years in length. It is pointed out in Chapter 2 that the Summers approach while completely eliminating the cyclical influence at the specified filter length also eliminates virtually all of the cyclical influence at lengths of twice the specified one. Thus, assuming the standard reference cycle of about 5 years, a 2 or 2 and 1/2 year moving average of this type would provide a rough measure of the co-movements of these variables at frequencies approximating those of the business cycle.

More favorable results are obtained with the Lucas (1980) approach which involves the use of an exponentially-weighted two-sided moving average filter. As argued in the chapter, it has an advantage over the Summers approach in that it is designed to capture the expectations generating
process of market participants whereas no formal or informal expectations formation process is specified in the Summers analysis. Lucas inferred from his results a close and roughly unitary relationship between inflation and nominal yields although he never tested for it directly. When such a test was undertaken in the present work, it was found that the relationship was indeed close as evidenced by the low S.E.R. and high adjusted R-squared values but certainly less than one-for-one. Specifically, the coefficient on inflation showed up consistently at about .7-.8. Of course, both of these findings are very favorable from the perspective of the present work. These results were confirmed with what is, to the best of my knowledge, the first application of formal spectral techniques to the question of the longer term co-movements of nominal rates and inflation. Evidence presented in an appendix to Chapter 2 indicates that the closest relationship of the two variables (initially first-differenced to remove any non-stationarity) was in the lowest frequency co-movements over 25 years in length. The relationship gradually diminishes in strength as the frequency is increased up to about the length of the standard reference cycle and then drops off sharply. This analysis provides not only the clearest evidence of the close long run tracking of nominal yields to inflation but also of the tendency of nominal rates not to fully adjust to inflation even in the longest of runs (the gain value for
the lower frequency co-movements showed up consistently at about .6). An additional interesting discovery was that no clear lead-lag relationship was discernible in the data when inflation is specified as the base series and interest rates as the crossed series. However this is not too surprising if one's prior is that the T-bill market incorporates changes in expected inflation over the relatively short-term into bill prices with very short lags. Indeed the time series results presented in the Appendix 4 to the same chapter tend to confirm this view.

Turning to the more direct tests of the ability of the liquidity model to account for the underadjustment of nominal yields to inflation, Chapter 3 contains, among other things, a discussion of the seemingly anomalous behavior of equity and debt yields through the inflationary 1970s. This whole as yet unsettled issue centers around the reasons for the divergence of real equity and real debt yields when standard theory predicts that the two should move together rather closely. It is argued that the liquidity model which predicts a relative decline in the real yields on the more liquid asset (corporate bonds) as compared to the less liquid one (shares) in the face of increased inflation provides a superior interpretation of the observed results than the money illusion arguments of Modigliani and Cohn (1979) and Summers (1981) which rely on the existence of 'irrational' market agents. Also, some evidence on relative new debt issuance, i.e. bonds vs. shares, is provided to further illustrate the implausibility of the very
influential money illusion arguments. Another interesting aspect to the tests undertaken in Chapter 3 concerns the use of two alternative proxies for unobservable inflationary expectations based on both Keynesian and rational expectational assumptions.

Also in Chapter 3, empirical evidence of another one of the predictions of the model, specifically that inflation will have the most impact on the real returns on those assets possessing the most 'moneyness', is offered. Assets that share a common maturity period but differ in general marketability characteristics, and thus trading costs of exchange, are ranked in descending order with respect to their liquidity services and evidence is provided that the individual asset's nominal (real) yield is less (more) affected by a given change in inflation the more liquid the asset. The use of the inflationary sub-period for which more data were available allows for the extension of the list of assets to include commercial paper, CD's and Euro-dollar deposit accounts in addition to T-bills, Federal agencies, finance paper and banker's acceptances. An appendix to Chapter 3 provides evidence based on Chow tests that the list of measured coefficients given in Table 3-3 constitutes a statistically significant ranking.

The same arguments and results apply when assets that have common general characteristics (e.g. default risk) but differ in term to maturity are used. Specifically, yields on a set of various maturity government securities with
encashment periods ranging from one month to thirty years are used. As pointed out in the chapter, it is vital in this type of analysis to use holding period returns rather than simple yields to maturity or points on a yield curve. The use of the latter may cause the tests to be distorted because the longer period asset returns may possess significant term premia or contain elements reflecting long term inflationary expectations. The above-mentioned holding period returns were used whenever possible in the present study, another feature which differentiates this work from many other empirical analyses of the inflation-interest relationship. Another interesting aspect to this particular analysis is the use of an equation derived directly from the liquidity model to test its prediction concerning the relationship between an asset's liquidity and the extent to which its nominal return is influenced by inflation. Essentially, OLS analysis of the relationship between holding period yield differentials and variables which according to the liquidity model should impinge on them is used to derive values for the parameters that fit into an equation describing the extent to which a particular asset's nominal yield will change with an increase in inflation.

As referred to above, evidence in favor of the model's important prediction of the differential impact of inflation on various maturity government instruments is offered. For instance, the nominal yields on one month T-bills rise by only about .25 percentage points for every percentage point increase in inflation whereas the returns on the portfolio
of long-term (20-30 year) securities rise by about .7 points for each 1 point rise in inflation. It should be stressed that the particular ranking of these assets in Sections III and IV of Chapter 3 is in no way arbitrary. Nor for that matter is the assumption made earlier that treats corporate bonds as being more liquid in the relevant sense than equities. Empirical evidence based on market 'thinness', the time rate of transactions in the individual securities and actual information on brokerage fees all serve to make the point that the ranking of assets in Chapter 3 with respect to their differential degree of liquidity is indeed correct. Some theoretical arguments based on the time rate of transactions, economies of scale and uncertainty in financial markets are also provided to support the ranking of the assets given in Chapter 3.

Very closely related to all of this is a fairly extensive and unique theoretical discussion dealt with in two chapters concerning the necessity to make a distinction between risk and transaction costs factors in the determination of asset yields. It is demonstrated that it is improper to attribute yield differentials on, say, various maturity government securities to capital risk factors alone. Although capital risk considerations may be important, the inability or unwillingness of dealers to completely hedge away the capital risk associated with the holding of an inventory of securities must ultimately be based on the fact that it is costly to do so. Indeed, the
dominance of transactions cost factors over risk factors in the determination of security yields is argued for in different ways on several occasions throughout the text.

Chapter 4 is concerned with a direct application of the F-H liquidity model to account for some apparently anomalous results obtained by many researchers of the capital asset pricing model's (CAPM) empirical validity. To offer additional confirming evidence of the results obtained concerning the predictive power of the liquidity model in the previous chapter, a related, yet somewhat different, approach is taken here. First, empirical CAPM intercepts using 5 different proxies for the market portfolio are computed and from these intercept values are subtracted the corresponding 'risk-free' rates as proxied by the T-bill yields. The standard CAPM analyses predict that these intercept values will differ from the T-bill rates by a more or less random amount reflecting statistical aberration. However, evidence is provided that far from being meaningless, these differentials are systematically related to variables that according to the liquidity model should impinge on them. The proxy regressors (derived from the model) include the percentage changes in the CPI, real personal income and the ratio of the dollar value of NYSE common stocks to T-bills when the equally-weighted and value-weighted amounts of NYSE stocks were used as proxies for the market portfolio and T-bills were used to represent the 'riskless' asset.

As the CAPM is concerned with the relationship between covariant risk and expected returns (which in the simplest
case equal mathematical averages) it was felt that some sort of moving average process applied to the time series under consideration would be more likely to provide values more representative of the underlying relationships among these variables. For instance, although the various measured SML (security market line) intercepts (minus the risk-free rates) from the CAPM analysis tended to be very volatile on a month-to-month basis, becoming even negative at times, their moving average values were in all cases significantly positive with means that were time independent, indicating, of course, stationarity. Applying the same pre-smoothing processes to the other variables (de-trended to provide stationary series) the computation of partial, as opposed to simple, correlations provided even stronger evidence than the previous tests of these differentials being liquidity yields of the F-H variety. Another related reason for pre-smoothing the data follows from the desirability of eliminating the well-known month-to-month variation in the returns on even very highly diversified portfolios of NYSE common stocks. It should be pointed out that the use of this smoothing technique in this section makes it more conformable to our earlier analyses of the low-frequency inflation-interest rate relationship.

The implication of all this is that it may be possible to account for some of the more confusing results reached by the CAPM researchers by invoking a F-H liquidity return. As discussed in Chapter 4, many of the more important analyses found 'overestimated' intercept terms and 'underestimated'
slope terms for the empirical SMLs. The inclusion of a liquidity premium into the standard two-parameter asset pricing model, a premium reflecting the lesser liquidity of common stocks vis a vis T-bills, may show that mis-specification and not mis-estimation has been the problem. Thus a security market hyperplane with liquidity represented along the z axis provides a better characterization of the underlying processes of security pricing than the simple security market line relating return to risk alone. Neglect of this factor will lead to 'overestimated' intercepts and thus 'underestimated' slopes.

Chapter 4 also contains a discussion of and a response to the very influential Roll (1977) critique. Roll's argument that no 'meaningful' empirical analyses of the CAPM can be undertaken in the absence of any exogenous information on the makeup of the true market portfolio is scrutinized and a rejoinder based on the formation of Bayesian priors by market participants is offered. The argument in the chapter is that while no purely exogenous information on the proper makeup of the market portfolio is likely to exist, the high correlations of average returns and risk among a number of efficiently-held portfolios found by many researchers may constitute a type of indirect information on the efficiency of the 'true' market portfolio.

Finally, some related areas of interest as far as the focus of the present work is concerned are dealt with in the
various appendices. In the appendix to Chapter 1, some amplification and clarification of the liquidity model is undertaken and a model is offered which leads to the same major predictions as F-H but has the advantage over theirs in that the roles of transactions costs and asset ratios in determining yield differentials is made far more explicit. Another distinction is that this latter model, in a step towards more realism, allows for the influence of varying relative transactions costs on interest rate differences whereas the F-H model allows for the influence of only one transactions cost.

The appendices to Chapter 2 include two that deal with the longer-term or low-frequency co-movements of inflation and interest rates using the Lucas filtering approach and more formal spectral techniques. Another provides some empirical evidence in favor of a positive, albeit loose, association between the level of inflation and its variance. This appendix is important not only because the results presented there contrast with those reached by some other writers in this area but also because it does provide some empirical support for the view that, over our sample period, the notion that the observed underadjustment of nominal rates to inflation is attributable to the financial markets' incorporation of inflation premia into security returns is incorrect. Another appendix deals with some time series results that cast doubt on the 'reverse' causality in the inflation-interest rate relationship found by many other writers in this area. With proper ARIMA modeling of the
time series, the standard notion of unidirectional influence from inflation to interest rates is confirmed. The last appendix to Chapter 2 contains some of the results of earlier tests of the relationship between these two variables when the sample period is extended to include the early 1980s.

Chapter 3 has two appendices, one which provides some empirical support for the particular ranking of the assets with respect to their liquidity provided in the chapter and a very short one detailing the method of computing holding period yields used in the chapter.
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