Equilibrium and Optimal Financial and Insurance Contracts under Asymmetric Information

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Thesis submitted for the degree of Doctor of Philosophy (Ph.D) in Economics,
University of London
2003

Abstract

This thesis studies financial and insurance markets under various specifications of asymmetric information.

The opening chapter considers project financing under adverse selection and moral hazard. There are three main contributions. First, the issue of combinations of debt and equity is explained as the outcome of the interaction between adverse selection and moral hazard. Second, it shows that, in the presence of moral hazard, adverse selection may result in the conversion of negative into positive NPV projects leading to an improvement in social welfare. Third, it provides two rationales for the use of warrants. It also shows that, under certain conditions, a debt-warrant combination can implement the optimal contract as a competitive equilibrium.

Chapter 2 examines insurance markets when some clients misperceive risk. Optimism may either increase or decrease precautionary effort and we show that this determines whether optimists or realists are quantity-constrained in equilibrium. Intervention may lead to a strict Pareto improvement on the laissez-faire equilibria. These results provide a more convincing justification for the imposition of minimum coverage requirements than standard models as well as a case for the use of taxes and subsidies in insurance markets.

Chapter 3 focuses on the relationship between coverage and accident rates. In contrast to the prediction of competitive models of asymmetric information that if all agents buy at least some insurance there must be positive correlation between coverage and accident probability, some recent empirical studies find either negative or zero correlation. If optimism discourages precautionary effort there exist separating equilibria that potentially explain the puzzling empirical findings. It is also shown that zero correlation between coverage and risk does not imply the absence of barriers to trade in insurance markets. We conclude with some implications for empirical testing.

Acknowledgements

I am indebted to my supervisors Sudipto Bhattacharya and David de Meza for their guidance, encouragement and insightful comments. I would also like to thank Margaret Bray, Michele Piccione, Jean-Charles Rochet and David Webb for helpful comments on various aspects of this thesis. I am also appreciative of my fellow students, especially Leo Ferraris, for many stimulating discussions.

I am especially grateful to my parents for always supporting my decisions and for the confidence they have put in me. Finally, my greatest debt is to my wife, Maria, for her patience, enduring love and support. I dedicate this thesis to her.

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Introduction

Since the seminal work by Akerlof (1970), an ever-increasing volume of papers has explored the implications of asymmetric information in the functioning of markets. It is now recognised that informational problems abound in many markets and that information asymmetries have significant effects on the equilibrium outcome. Although applications of the asymmetric information (principal-agent) framework can be found in most fields of economics, financial and insurance markets have received particular attention. It has been argued that financiers (insurers) can observe neither the characteristics (adverse selection) nor the actions (moral hazard) of entrepreneurs (insurees). Existing studies have produced interesting insights about the effects of adverse selection and moral hazard and provided explanations to economic phenomena that otherwise would be hard to understand. Nevertheless, puzzles remain. This thesis studies financial and insurance markets under various specifications of asymmetric information and provides possible explanations for some unresolved issues.

Chapter 1 considers project financing under adverse selection and (effort) moral hazard. The key feature of the first part of this chapter is the existence of a pooling equilibrium involving cross subsidisation across types and the issue of both debt and equity. Through the mispricing of equity at individual level, the more prone to shirking type receives the subsidy necessary to induce him to choose the socially efficient high-effort level. This pooling equilibrium has two important implications.

First, in the presence of both adverse selection and moral hazard, in addition to being communication devices, the securities issued are the means of providing the appropriate effort incentives. This double role stems from the interaction between adverse selection and moral hazard and provides an explanation for the issue of combinations of debt and equity even if the issue of equity implies an adverse selection cost. Firms are prepared to incur this cost because it is more than offset by the benefit from relaxing the moral hazard constraint. This result is consistent with the puzzling empirical observation that although equity issue announcements are associated with stock price drops, equity dominates debt as a source of outside financing (Frank and Goyal (2003)).

Second, adverse selection leads to the conversion of negative into positive NPV projects and so to an improvement in social welfare. This result contrasts with those of pure adverse-selection models. In Myers and Majluf (1984) adverse selection leads firms to forego positive NPV projects whereas in de Meza and Webb (1987) it encourages firms to undertake negative NPV projects. Hence, in either case social welfare is lower than under full information about types. The key to this difference is that in the presence of (effort) moral hazard the cross-subsidisation taking place in the pooling equilibrium relaxes this additional constraint and so it can be beneficial. On the contrary, given risk neutrality, in pure adverse selection models there is no channel through which the cross-subsidy can have positive effects but it may have negative ones.

The second part of the chapter analyses the role of warrants and provides two rationales for their use. A considerable fraction of the securities with option features issued by firms are debt-warrant (or equity-warrant) combinations rather than convertible debt. Existing models offer various explanations of why firms issue convertible debt (e.g. Green (1984), Stein (1992)). However, none of them justifies the necessity for the issue of warrants. Under pure adverse selection, warrants can serve as separation devices in cases where other standard securities cannot. In the presence of both adverse selection and moral hazard, warrants allow for the implementation of the socially efficient outcome even if this is not possible when we restrict ourselves to debt, equity and/or convertible debt. Finally, we show that, under certain conditions, a debt-warrant combination can implement the optimal contract as a competitive equilibrium.

In Chapter 2, we explore the implications of optimism in competitive insurance markets when neither the type nor the actions of the insurees are observable. Several recent empirical studies both by psychologists and economists, report that the majority of people tend to be overoptimistic about their ability and the outcome of their actions and underestimate the probability of various risks. For example, Rutter, Quine and Alberry (1998) find that, on average, motorcyclists in Britain both perceive themselves to be less at risk than other motorcyclists and underestimate their absolute accident probability. A large number of papers have investigated the implications of overconfidence and unrealistic optimism in securities markets and firm financing (See De Bondt and Thaler (1995) for a survey). In contrast, research concerning insurance markets has almost entirely been conducted in the context of the standard asymmetric information framework. Insurees know their true accident probability but insurance companies cannot observe the type and/or the actions of the insuree.

In line with the empirical evidence, in this chapter we drop the assumption that all insurees have an accurate estimate of their accident probability. We assume that some agents, the optimists, underestimate it and explore the implications both for the optimists (henceforth Os) themselves and their realistic counterparts (henceforth Rs) in the context of an otherwise standard competitive asymmetric information framework. Except for their misperception of the accident probability, the Os are rational agents who aim at maximising their (perceived) utility and understand the nature and implications of market interactions.

The first part of the chapter is concerned with the positive implications of the interaction between the Os and the Rs. It is shown that if the degree of optimism is sufficiently high there exist separating equilibria where the Os not only take fewer precautions (high-risk type) but also purchase less insurance than the Rs and both types choose the contract they would have chosen if types were observable. That is, because the Os considerably underestimate their accident probability, their presence has no effect on the choices of the Rs. For lower levels of optimism, depending on whether the Os are more or less willing to take precautions, either the Os or the Rs are quantity-constrained. If optimism encourages precautionary effort, the Os themselves are quantity-constrained whereas the Rs make the same choices as under full information about types. If the Os put less effort into reducing their risk exposure, the roles of the two types are reversed.

Moreover, it is shown that, contrary to the conventional wisdom, optimism itself does not necessarily lead to the purchase of less insurance. If optimism encourages precautionary effort, the effect of the lower per unit price may more than offset the effect of the underestimation of the accident probability and result in the Os purchasing more insurance than the Rs.

The second part of the chapter deals with the welfare properties of the laissez-faire equilibria when some clients are optimists. We show that there exist intervention policies that yield strict Pareto gains. If the Rs are quantity-constrained, then a tax on insurance purchase would result in the Os going uninsured, relax their revelation constraint and potentially lead to a strict Pareto gain. If though the Os are quantity-constrained, this logic does not apply. Any attempt to drive out the Rs so as to mitigate the negative externality their presence creates would first drive out the Os. Thus, it would be harmful for the Rs who would pay the tax without gaining anything. However, if the proportion of the Os is sufficiently high, an intervention scheme involving a combination of minimum coverage requirements, taxes and subsidies would lead to a strict Pareto improvement. In the resulting pooling equilibrium the Os subsidise the Rs but purchase more insurance and both types are better off. Because the proportion of the Os is high, the improvement in their true welfare from the higher coverage more than offsets the welfare losses due to the higher per unit premium.

Although the positive results of the imposition of minimum coverage requirements in standard asymmetric information models are similar to ours, the welfare results are quite different. In our model, both types are better off in the pooling equilibrium arising after the intervention whereas in standard models the safe type (the quantity-constrained) is worse off. Therefore, our approach provides a more convincing justification for the imposition of minimum coverage requirements than standard models as well as a case for the use of taxes and subsidies in insurance markets. Finally, intervention schemes involving minimum coverage requirements can be used to create a pure-strategy Nash equilibrium when otherwise none would exist.

Chapter 3 focuses on the relationship between the coverage offered by the insurance contract and the *ex-post* risk of its buyers. Most recent empirical studies find either negative or no correlation. For example, de Meza and Webb (2001) provide casual evidence for a negative relationship in the credit card insurance market. Cawley and Philipson (1999) study of life insurance contracts also shows a negative relationship which, however, is not statistically significant. These findings are at odds with the famous Rothschild-Stiglitz paper (1976) which, along with most other theoretical models of competitive insurance markets under asymmetric information, predicts a positive relationship. This implication is shared by models of pure adverse selection (e.g. Rothschild and Stiglitz (1976)), pure moral hazard (e.g. Arnott and Stiglitz (1988)) as well as models of adverse selection plus moral hazard (e.g. Chassagnon and Chiappori (1997) and Chiappori et.al. (2002)). In fact, Chiappori et.al. (2002) argue that the positive correlation property is extremely general. However, in a recent paper, de Meza and Webb (2001) provide a model where agents are heterogeneous with respect to their risk aversion and face a moral hazard problem. Also, insurance companies pay a fixed administrative cost per claim. In this model, there exist a separating and a partial pooling equilibrium predicting a negative relationship but due to the fixed per claim cost the less risk-averse agents go uninsured.

In this chapter we first show that these (seemingly) contradictory theoretical results can be reconciled. Given that fixed administrative costs are strictly positive, it is shown that the Chiappori et.al. argument holds necessarily true only if, in equilibrium, all agents purchase some insurance. If some agents choose zero coverage, then their assertion is not necessarily true. There can exist separating equilibria that exhibit negative or no correlation between coverage and risk. The presence of these costs results in some agents (the risk tolerant) choosing not to insure. The fact that the administrative costs are now incurred only by the insured agents changes the computation of the premiums which allowed Chiappori et.al (2002) to derive their result.

Therefore, competitive models of insurance markets under asymmetric information can explain the observed negative or no-correlation between coverage and risk in cases where some agents choose zero coverage. However, their prediction is not consistent with negative or no-correlation in insurance markets where all agents opt for strictly positive coverage and there are just two events (loss/no loss), (e.g. the Cawley and Phillipson (1999) findings).

Jullien, Salanie and Salanie (2001) show that if the insurer has monopoly power, negative correlation between risk and coverage is possible even if all agents purchase some insurance and there is just one level of loss. However, insurance markets seem to be fairly competitive and so monopoly is not a good approximation. More importantly, although in Jullien, Salanie and Salanie (2001) the low-risk type is better insured, more coverage is associated with a higher per unit price. Therefore, although they can explain the negative correlation between coverage and risk, the striking observation of Cawley and Phillipson (1999) that insurance premiums exhibit quantity discounts remains a puzzle.

This chapter, by introducing heterogeneity in risk perceptions in a competitive model of asymmetric information, provides an explanation to the puzzling empirical findings. The more optimistic agents (the Os) underestimate their accident probability both in absolute terms and relative to the less optimistic ones (the Rs) and so purchase less insurance. They also tend to be less willing to take precautions. This gives rise to separating equilibria exhibiting negative or no correlation between coverage and risk. Two examples of these equilibria are presented where both the Os and the Rs purchase some insurance.

The first equilibrium predicts both negative correlation between coverage and risk and that per unit premiums fall with the quantity of insurance purchased. The Os not only take fewer precautions (high-risk type) but also purchase less coverage than the Rs. Competition among insurance companies then implies that the Os also pay a higher per unit premium. Because they underestimate their accident probability, the Os purchase low coverage at a high per unit price, although contracts offering more insurance at the same or even lower per unit price are available.

The second equilibrium exhibits no correlation between coverage and risk and involves the Rs being quantity-constrained. In order to reveal their type, the Rs accept lower coverage than they would have chosen under full information about types. Moreover, if we allow for fixed administrative costs, this equilibrium displays a negative relationship between coverage and per unit premiums. Since both types take precautions they have the same accident probability and so are charged the same marginal price. But the fact that the Os purchase less coverage implies that their total per unit premium is higher.

These results have several interesting implications. First, they explain both puzzling empirical findings reported by Cawley and Phillipson (1999): The negative or no correlation between coverage and risk and the fact that insurance premiums display quantity discounts. Second, Cawley and Philipson (1999), Chiappori and Salanie (2000) and Dionne et.al. (2001) argue that the no-correlation empirical findings imply that there is no (risk-related) adverse selection. Thus, there are no information barriers to trade in the life and automobile insurance markets under study. However, as our results suggest, their assertion is not generally true. If insurees differ with respect to their risk perceptions and types are hidden, there exist equilibria involving some agents being quantity-constrained even if the data show zero correlation between coverage and the accident rate. Furthermore, in these cases, there exist intervention policies that yield a strict Pareto improvement on the laissez-faire equilibrium. Third, these equilibria have testable implications that allow us to empirically distinguish our approach from standard asymmetric information models.

Chapter 1

The Roles of Debt, Equity and Warrants Under Asymmetric Information

1.1 Introduction

Following the famous irrelevance proposition of Modigliani and Miller (1958), a vast literature has developed trying to explain the financial choices of firms when they seek outside funds.[1] Despite this research effort, important puzzles remain. Some recent empirical studies find that neither of the two dominant theories of capital structure, the trade-off theory and the pecking-order theory, provides a satisfactory explanation for the observed financing patterns.[2] Firms appear to issue surprisingly large amounts of equity, even after controlling for the various costs (due to financial distress, bankruptcy and agency problems between debtholders and shareholders) associated with debt issues.[3] Moreover, although equity issue announcements are associated with stock price drops (due to adverse selection),[4] equity dominates debt as a source of external financing.[5] Myers and Majluf (1984) show that this adverse-selection problem may lead to underinvestment and so a loss in social welfare.

Furthermore, a considerable fraction of the securities with option features issued by firms are debt-warrant (or equity-warrant) combinations rather than convertible debt.[6] Existing models offer various explanations of why firms issue convertible debt.[7] However, none of them justifies the necessity for the issue of warrants.

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[1] See Harris and Raviv (1991) for a survey.

[2] See, for example, Helwege and Liang (1996), Lemmon and Zender (2001) and Frank and Goyal (2003).

[3] The tax benefits of debt are significant and firms' decisions about financial policies appear to be affected by tax considerations (see, MacKie-Mason (1990) and Graham (1996) and (2000)). Although financial distress and other agency costs are important, they are not large enough to explain these conservative debt policies (see Andrade and Kaplan (1998) and Parrino and Weisbach (1999) and Lemmon and Zender (2001)).

[4] See Lemmon and Zender (2001).

[5] Frank and Goyal (2003) report that net equity issues follow the financing deficit more closely than debt issues.

[6] For example, de Roon and Veld (1998) report that about 30 percent of the convertible securities issued by Dutch companies from 1976 to 1996 were debt-warrant combinations.

[7] Convertible debt is a special case of a debt-warrant combination that obtains when the exercise price of the warrant equals the face value of debt. Green (1984), Constantinides and Grundy (1989), and Stein (1992) provide three different rationales for the use of convertible debt.

This chapter abstracts from taxes, financial distress, bankruptcy and other agency costs and focuses on asymmetric information. We consider a model involving both adverse selection and (effort) moral hazard. There are two types of firms (projects): risky (R), safe (S). Given identical effort levels, the success probability of the safe project is higher but its return in case of success is lower. In the event of failure the return of both types is zero. The entrepreneur can increase the success probability by exerting costly effort. Regardless of the project's type, if the entrepreneur exerts effort the net present value (NPV) of his project exceeds the cost of effort whereas if he shirks the project has negative NPV. That is, exerting effort is socially efficient for both types. Both the project's type and the entrepreneur's action are unobservable.

In this setting, we analyse the roles of debt, equity and warrants and make three contributions. First, we explain the issue of combinations of debt and equity as the outcome of the interaction between adverse selection and moral hazard. Some firms (the risky ones) issue equity even if under pure adverse selection they would have issued just debt. Second, we show that, in the presence of moral hazard, adverse selection may result in the conversion of negative into positive NPV projects and an improvement in social welfare. Third, we provide two rationales for the use of warrants. We also show that, under certain conditions, a debt-warrant combination can implement the optimal contract as a competitive equilibrium.

Two cases are considered: i) pure adverse selection and ii) adverse selection cum moral hazard. In the former case, a combination of securities is only used to convey socially costless information about the type of the project.[8] In the latter case, in addition to transmitting information, the securities issued are the means of providing the appropriate effort incentives. Because of this second role, the introduction of moral hazard into an adverse selection framework has significant effects both on the combinations of securities issued in equilibrium and their pricing.

Regarding the pure adverse selection case, if firms have more information about the quality of their projects than their financiers, then they have an incentive to issue overpriced securities. To the extent that firms cannot credibly signal their type, the resulting adverse-selection problem may lead firms to forego a positive NPV project. Following Myers and Majluf (1984), a great deal of research effort has been devoted to exploring the extent to which this problem can be overcome if firms use different

[8] In this case, we seek methods of financing that result in nondissipative equilibria (Bhattacharya (1980)). That is, equilibria that imply no deadweight losses relative to the full information equilibrium.

combinations of financial instruments to transmit information. It has been shown that debt or equity repurchases in conjunction with the issue of some other security (e.g. equity or convertible debt respectively) may allow for the existence of fully revealing equilibria where the securities issued are correctly priced.[9]

In this chapter, we do not allow for debt or equity repurchases. Firms try to reveal their type by issuing debt-equity or debt-warrant combinations. Equity (warrant) is a convex claim and so its value increases with the variability of returns. In contrast, debt is a concave claim and so its value falls with risk. That is, in relative terms, debt is more valuable for the safe type and equity (warrant) for the risky one. Thus, by issuing more of the less valuable for him security, an entrepreneur can credibly signal his type and reduce the underpricing of his securities. However, the existence of an equilibrium where the securities issued are fairly priced requires that debt is more valuable for the S-type and equity (warrant) for the R-type not only in relative[10] but also in absolute terms. Otherwise, the type whose securities are more valuable can only minimise the underpricing of his securities by issuing just the relatively less valuable for him security.

If the risky projects are mean-increasing or mean-preserving spreads of the safe ones or the risky projects dominate the safe ones by first-order stochastic dominance both conditions are met. In the first case, separation requires the issue of both debt and equity (Heinkel 1982). In the two remaining cases, the adverse-selection problem can be solved (mitigated) by issuing either just equity (mean-preserving spreads) or just debt (first-order stochastic dominance).[11] However, if the risky projects are mean-reducing spreads of the safe ones, both the debt and equity issued by the S-type are more valuable than those issued by the R-type. Thus, the S-type cannot reveal his type and inevitably subsidises the R-type through the mispricing of the relatively less valuable for him security (equity) at individual level.

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[9] See, for example, Brennan and Craus (1987), Constantinides and Grundy (1989) and Heider (2001). However, models that use debt (or equity) repurchases to obtain fully revealing equilibria have a serious shortcoming. They do not explain why firms issued debt (equity) in the past.

[10] Provided the returns of the two types in case of success are different, this (the single-crossing) condition is satisfied regardless of the distributional assumption or the combination of the securities issued. On the contrary, if the safe projects dominate the risky ones by first-order stochastic dominance (the returns of both types in case of success are equal), both conditions are violated. In this case, the equality of returns prevents us from extracting any information about the type of the project. There can exist only pooling equilibria where the S-type provides the R-type with the same amount of subsidy regardless of the securities issued. Notice that if firms have assets in place, the use of collateral could be a solution. However, this solution may not be costless, it may imply deadweight losses (e.g. Besanko and Thakor (1987) and Bester (1987)).

[11] These results are well-known (see de Meza and Webb (1987) and Nachman and Noe (1994)).

The use of warrants, through the appropriate choice of their exercise price,[12] allows for the achievement of full separation even in this case. Since the return of the R-type in case of success is greater, a given increase in the exercise price of the warrant implies that the project's return constitutes a smaller proportion of the total payment to the financier if the warrant is issued by the S-type. That is, as the exercise price rises, the value of the warrant issued by the S-type falls faster. As a result, for a sufficiently high exercise price, the warrant issued by the R-type can be more valuable than that of the S-type even if the S-type equity is more valuable.

This mechanism provides a rationale for the use of warrants. Warrants are issued because they can serve as separation devices when other standard securities (debt, equity and/or convertible debt) cannot.[13]

The introduction of moral hazard into an adverse selection framework has significant effects both on the combinations of the securities issued in equilibrium and their pricing. The distinguishing feature of this part of the chapter is the existence of pooling equilibria involving cross subsidisation across types and the issue of both debt and equity (warrants). These pooling equilibria reflect a trade-off between information revelation and effort incentives. The securities issued by the R- and S-type are priced as a pool. Although, because of perfect competition, debt and equity (warrants) are fairly priced collectively, at individual level they are mispriced. In fact, it is precisely this mispricing that provides the more prone to shirking type with the subsidy necessary to induce him to choose the socially efficient high-effort level.[14]

Consider, for example, the case where the S-type is more prone to shirking and we restrict ourselves to debt and equity. In this case, in the pooling equilibrium the R-type subsidises the S-type through the mispricing of equity. In the absence of moral hazard, the R-type would have issued more debt and less equity. Since, in doing so, he would reduce the subsidy and increase his expected return. However, in the presence of moral hazard, the S-type always mimics the R-type and such a deviation would

[12] It is set such that, in case of success, the option is exercised regardless of the issuer type. Also, the proceeds (from the exercise of the option) are distributed as dividends to the shareholders.

[13] Notice that convertible debt cannot play this role. The mechanism described above works only if the exercise price of the warrant increases while the face value of debt is fixed (in equilibrium, the exercise price of the warrant is strictly greater than the face value of debt). If the two coincide, the increase in the exercise price is exactly offset by the increase in the face value of debt. Hence, the value of convertible debt is strictly greater if it is issued by the S-type regardless of the face value of debt or whether conversion takes place.

[14] If funds are offered at fair terms, the more prone to shirking type chooses the low effort level. Hence, his project NPV is negative and so, if his type is revealed, no rational financier offers funds to him.

destroy his effort incentives. As a result, both the collective and the R-type's net expected return would fall. Since he cannot reveal his type, the R-type accepts to issue just enough equity to induce the S-type to exert effort because the resulting increase in his net expected return (due to the lower interest rate on debt) more than offsets the cost of the incremental subsidy (the adverse selection cost of issuing equity). That is, this pooling equilibrium involves the minimum subsidy consistent with the S-type exerting effort. In any pooling equilibrium involving more than this minimum subsidy, the R-type can still profitably deviate by issuing more debt and less equity.

That is, in the presence of both adverse selection and moral hazard, in addition to being communication devices, debt and equity play a second role. That of incentivising the more prone to shirking type through their mispricing at individual level. This double role stems from the interaction between adverse selection and moral hazard and provides an explanation for the issue of combinations of debt and equity even if the issue of equity implies an adverse selection cost. What is more, in contrast with the pure adverse selection case, the cross-subsidisation is socially beneficial. It converts negative into positive NPV projects and improves social welfare.

However, if firms can only issue debt and equity, it may be the case that, at any given debt level, the proportion of equity issued consistent with exerting effort is strictly lower for the S-type. That is, the pooling equilibrium where both types exert effort may collapse although the R-type would have exerted effort even if a higher proportion of equity was issued (more subsidy was given to the S-type). Because the warrant value falls with the exercise price faster for the S-type, the S-type is willing to increase faster the proportion of equity offered to the financier than the R-type while still exerting effort. As a result, through the appropriate choice of their exercise price, warrants allow for the implementation of the socially efficient outcome even if this is not possible when we restrict ourselves to debt, equity and/or convertible debt. This result provides a second rationale for their use. Finally, we show that, under certain conditions, a debt-warrant combination can implement the optimal contract as a competitive equilibrium.

This chapter is related to three strands in the literature: agency models, pure adverse selection models and models combining adverse selection and moral hazard.

In the celebrated Jensen and Meckling (1976) paper firms issue both debt and equity to minimise the sum of agency costs of these two securities. The agency cost of equity arises from the conflict of interest between management and outside shareholders. The agency cost of debt stems from the conflict of interest between existing shareholders (managers) and would-be debtholders. The issue of debt induces the managers to undertake riskier projects that reduce the value of debt and transfer wealth from debtholders to shareholders (asset substitution problem).[15]

Related to that is the debt overhang problem described by Myers (1977). The existence of risky debt implies that shareholders (managers) may not undertake positive NPV projects because they will incur the total cost of the project but obtain only part of the returns. In fact, if the increase in the value of the outstanding risky debt exceeds the NPV of a project, investment in the project would result in a fall in the shareholders net return.

Therefore, in agency models, the reduction in the agency cost of equity resulting from the issue of debt is offset at the margin by the increase in the agency costs of debt. This trade-off determines the optimal debt-equity ratio (capital structure).

In this context, Green (1984) focuses on the asset substitution problem and develops a rationale for the use of convertible debt (warrants). Convertible debt reverses the convex shape of levered equity over the upper range of the firm's returns (where conversion takes place). As a result, it alters the incentives of the shareholders to take risk and so mitigates the asset substitution problem.

More recently, Biais and Casamatta (1999) consider a model similar to Jensen and Meckling but they completely endogenise the contractual form. Nevertheless, they show that if the risk-shifting problem is more severe, a debt-equity combination (or convertible debt) can implement the optimal contract whereas if the effort problem is more severe stock options are also needed.

Pure adverse selection models emphasise the signalling role of the financing decisions of the firm.[16] If firms have no assets in place and there are no bankruptcy or financial distress costs, by using debt and equity, we can obtain fully revealing

[15] Notice that in our model there is no conflict of interests between shareholders and debtholders (no asset substitution problem) which, given the agency cost of equity, is the driving force of the coexistence of debt and equity in Jensen and Meckling. In our case, moral hazard concerns the choice between different effort levels rather than the choice between a safe and a risky project (the source of the asset substitution problem).

[16] The pure adverse selection part of this chapter belongs to a class of models that seek methods of financing that lead to nondissipative equilibria (Bhattacharya (1980)). Other examples include Heinkel (1982), Brennan and Kraus (1987), and Constantinides and Grundy (1989). Early examples of signalling models in the corporate finance literature are: Leland and Pyle (1977), Ross (1977), Bhattacharya (1979).

equilibria when the risky projects are mean-increasing or mean-preserving spreads of the safe ones or they dominate the safe projects by first-order stochastic dominance.

Moreover, if firms have debt and equity outstanding and debt and equity repurchases are allowed, there potentially exist fully separating equilibria under a wider range of distributional assumptions.[17] Brennan and Kraus (1987) allow only for debt repurchases and consider two cases: first-order stochastic dominance and mean-preserving spreads. They show that fully revealing equilibria can be obtained by issuing equity and repurchasing debt in the first case and by issuing convertible debt in the second. Constantinides and Grundy (1989) allow only equity repurchases and prove that, under first-order stochastic dominance, the issue of convertible debt coupled with equity repurchases leads to full information revelation.

Stein (1992) introduces financial distress costs and provides another justification for the use of convertible debt as well as the issue of debt and equity. In a three-type model, he obtains a fully separating equilibrium where the good type issues debt, the medium type issues convertible debt that is always converted into equity, and the bad type issues equity directly to avoid incurring the distress costs. In this separating equilibrium all firms invest and no distress costs are borne in equilibrium. If convertible debt were not used, this separating equilibrium would not, in general, exist and a situation similar to that described in Myers and Majluf (1984) would arise.

The justifications provided by Green (1984), Brennan and Kraus (1987), and Constantinides and Grundy (1989) for the use of convertible debt rely on the fact that its payoff is concave in the firm's returns for low values of returns and convex for higher values. In Stein (1992), the usefulness of convertible debt stems from the presence of financial distress costs and the inability of a bad firm to force conversion. In our model, the mechanism at work is different. First, it does not depend on financial distress costs. Second, in our case, convertible debt does not improve on a debt-equity combination. Our mechanism relies on the fact that the warrant exercise price can be greater than the face value of debt. By appropriately choosing the exercise price, we can exploit the difference between the returns of the two types of projects and satisfy the revelation or effort incentive constraints under weaker conditions than if warrants were not available.

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[17] If both debt and equity repurchases are allowed, there potentially exist fully separating equilibria under any assumption about the ordering of the distributions of returns as demonstrated by Heider (2001) in a two-type model.

This chapter is most directly linked to models involving both adverse selection and (effort) moral hazard. Darrough and Stoughton (1986) provide such a model where entrepreneurs are risk averse and can issue combinations of debt and equity. However, they only consider separating equilibria where the securities issued are fairly priced. As a result, neither cross-subsidisation across types occurs nor the issue of equity when it implies adverse selection costs can be explained. In contrast, in Vercammen (2002) firms cannot signal their type. Because of his distributional assumption and the fact that firms are restricted to issue only debt a unique pooling equilibrium arises. He shows that the cross-subsidisation that takes place through the mispricing of debt at individual level raises aggregate surplus. Because the low-quality firms are more severely affected by the moral hazard, the cross-subsidisation results in a higher overall effort level and so a lower average failure probability.[18]

In our model, we allow for a wider range of distributional assumptions and firms can use combinations of securities to reveal information about their type. In our case, the pooling equilibrium involves the issue of both debt and equity (warrants) and the minimum subsidy consistent with S-type (the more prone to shirking) exerting effort. In any pooling equilibrium involving more than this minimum subsidy, the R-type (the subsidiser) can profitably deviate by issuing more debt and less equity. However, because the S-type always mimics him, the R-type issues just enough equity to induce the S-type to exert effort because the resulting increase in the R-type's net expected return more than offsets the cost of the incremental subsidy. In other words, if both types exert effort the collective expected return increases so much that both are strictly better off than the case where just debt is issued and the S-type shirks.[19]

Notice that if types were observable, the S-type would not receive financing and so both investment and social welfare would be lower. These results contrast with those of pure adverse-selection models. In Myers and Majluf (1984) adverse selection leads firms to forego positive NPV projects whereas in de Meza and Webb (1987) it encourages firms to undertake negative NPV projects.[20] Hence, in either case social welfare is lower than under full information about types. The key to this difference is that in the presence of (effort) moral hazard the cross-subsidisation taking place in the pooling equilibrium relaxes this additional constraint and so it can be beneficial. On

[18] Although our mechanism is similar, it was independently discovered.
[19] If just debt is issued the S-type does not receive enough subsidy to induce him to exert effort.
[20] De Meza and Webb (1999, 2000) also demonstrate that hidden types may lead to socially excessive lending.

the contrary, in these two pure adverse selection models there is no channel through which the cross-subsidy can have positive effects but it may have negative ones.

However, as de Meza (2002) argues, assuming that agents are risk averse, hidden types may result in an improvement in social welfare even in the absence of moral hazard. If types are observable, the low-quality agents have lower income and so higher expected marginal utility in all states. Therefore, if adverse selection leads to a pooling equilibrium where the high-quality agents subsidise the low-quality ones, the welfare gains of the subsidisees more than offset the welfare losses of the subsidisers and so aggregate welfare rises.

This chapter is organised as follows. Next section describes the basic framework and develops the analytical tools. Section 3 provides some general results about the existence and the type of the equilibria where funds are offered. Section 4 analyses the roles of debt and equity under pure adverse selection and adverse selection cum moral hazard. The roles and the usefulness of warrants are explored in Section 5. In Section 6, we show that, in the adverse selection cum moral hazard case, a debt-warrant combination can implement the optimal contract as a competitive equilibrium. Some brief concluding remarks are provided in Section 7.

1.2 The Model

We consider a simple static (one-period) model of financing involving both adverse selection and effort moral hazard. There are two dates, 0 and 1, and one homogeneous (perishable) good which can be used either for consumption or investment purposes. There are also two groups of agents: entrepreneurs (henceforth Es) and financiers (henceforth Fs). Both the Es and the Fs consume only at date 1.

Each E has an indivisible project but no initial wealth. All projects require the same fixed initial investment I, at date 0. Since the Es have no initial wealth, they need to raise (at least) I from the market.

Each F has a very large amount of initial wealth and can lend at zero interest rate. For simplicity, we assume that there are just two Fs involved in Bertrand competition.

Both the Es and the Fs are risk neutral. The Fs are only interested in the monetary returns of the project. The Es, however, care not only about the pecuniary returns but also about a private benefit B_i. Also, there are no taxes, no bankruptcy or financial distress costs. Finally, there is no conflict of interest between managers and entrepreneurs. In fact, firms are run by entrepreneurs.

Investment takes place at date 0. Returns are realised at date 1 and are observable and verifiable. There are two states of nature: Success, Failure. If a project succeeds it yields X_i. In case of failure, all projects yield 0 regardless of the type of the E.

The probability of success of a project, denoted by $\pi(B_i)$, is related to both the type of the E (project) and the effort level that each E chooses privately. There are two types of Es (projects), R (risky) and S (safe), with respective proportions in the population λ and $1-\lambda$, $0 \le \lambda \le 1$. Given identical effort levels, the success probability of the safe project is higher but its return in case of success is lower: $X_R > X_S$[21] and $\pi_j^S \ge \pi_j^R$. There are two effort levels: Low (shirking), High (working). B_i is a binary variable which denotes the private benefit, in terms of utility, corresponding to each effort level. If the E chooses to shirk, then $B_i = B$ and $\pi(B_i) = \pi_0^i$, if the high effort level is chosen, then $B_i = b$ and $\pi(B_i) = \pi_C^i$ where $B > b \ge 0$ and $1 \ge \pi_C^i > \pi_0^i > 0$. The difference $B - b \equiv C$ can be interpreted as the cost of effort.

If the high effort level is chosen, the NPV of both types of projects exceeds the cost of effort. In contrast, if shirking is chosen, neither project is economically viable (both types of projects have strictly negative NPV). That is,

<u>Assumption 1</u>: $\pi_C^i X_i - I > C > 0 > \pi_0^i X_i - I, \qquad i = R, S$

Assumption 1 also implies $(\pi_C^i - \pi_0^i)X_i > C$, $(i = R, S)$. That is, the choice of the high effort level by either type leads to an increase in the net social surplus and so is socially efficient.

For expositional purposes, we begin by restricting the contract space to debt and outside equity. That is, the Es can borrow by issuing a combination of debt and equity. Debt claims are zero-coupon bonds that are senior to equity.

[21] The remaining case where $X_R = X_S$ and $\pi_j^S > \pi_j^R$ (the safe projects dominate the risky ones by first-order stochastic dominance) is not considered explicitly below. See Footnote 10.

A contract $Z = (\alpha, D)$ provides the E with the required amount of funds, I, in return for a combination of debt of face value D and a proportion of equity of the project α, $0 \le \alpha \le 1$, $D \ge 0$.

Therefore, given risk neutrality and limited liability, the Es seek to maximise:

$$U_i(X_i, \alpha_i, D_i, B_i) = \pi(B_i) Max[(1-\alpha_i)(X_i - D_i), 0] + B_i, \quad i = R, S \qquad (1.1)$$

[22]

where U_i is the expected utility of an E of type i when choosing the contract $Z_i = (\alpha_i, D_i)$. That is, the expected utility of an E consists of two components: i) the expected monetary return and ii) the private benefit, represented by the first and second term respectively in Eq. (1.1).

At date 0, when the contract is signed, the Es know their own type but the Fs cannot observe either the type of each individual E or verify the actions (choice of effort level) of the Es applying for funds. The Fs do, however, know the proportion of each type in the population of Es and the nature of the investment and moral hazard technology. The Fs also wish to maximise their expected profit. The expected profit, P_F, of an F offering a contract (α, D), given limited liability, is given by:

$$P_F = \pi(B_i)\{Max[\alpha(X_i - D), 0] + Min(X_i, D)\} - I, \quad i = R, S \qquad (1.2)$$

1.2.1 Effort Incentive Constraints

Let us first consider the moral hazard problem an E of type i faces. A given contract (α, D) will induce the high effort level if

$$(\pi_C^i - \pi_0^i)(1-\alpha)(X_i - D) \ge C, \quad i = R, S \qquad (1.3)$$

or $(1-\alpha)(X_i - D) \ge c_i$, where $c_i \equiv \dfrac{C}{\pi_C^i - \pi_0^i}$, $i = R, S$ (1.3')

[22] Whenever the Max or Min operators are irrelevant they will be suppresed.

The left-hand side of (1.3) is the increase in the E's net expected return from exerting effort and the right-hand is the cost of effort. The contracts (α, D) satisfying (1.3) or (1.3') are called (effort) incentive compatible. Let IC_i be the set of effort incentive compatible contracts and ICF_i its frontier. The equation of ICF_i is:

$$(1-\alpha)(X_i - D) = c_i, \qquad i = R, S \tag{1.4}$$

The constant c_i tells us how much it costs an E, in utility terms, to increase his success probability by a given amount $(\pi_C^i - \pi_0^i)$. Notice that this cost depends, in general, on the E's type and is inversely related to the "productivity" of effort $(\pi_C^i - \pi_0^i)$. However, the fact that, at any given identical effort level, the safe type's success probability is higher does not imply that it changes more when another effort level is chosen. It may well be true that the risky type is more "productive" in this sense. Thus, c_S can be greater, equal or less than c_R. In combination with X_i, c_i describes the moral hazard "technology". Lemma 1 summarises its key features.

Lemma 1: In the (α, D) space:

a) ICF_i are downward sloping and strictly concave with slope $\left(\frac{d\alpha}{dD}\right)_{ICF_i} = -\frac{1-\alpha}{X_i - D} < 0$. That is, at any (α, D) pair, ICF_R is flatter than ICF_S.

b) ICF_R and ICF_S intersect at some $(1 \geq \alpha \geq 0, D \geq 0)$ if $c_R/X_R \geq c_S/X_S$ and $X_R - c_R \geq X_S - c_S$. Otherwise, either $IC_R \subset IC_S$ or $IC_S \subset IC_R$.

c) Neither IC_R nor IC_S is empty.

Proof: See Appendix 1A.

Figure 1.1 illustrates the case where ICF_R and ICF_S intersect.

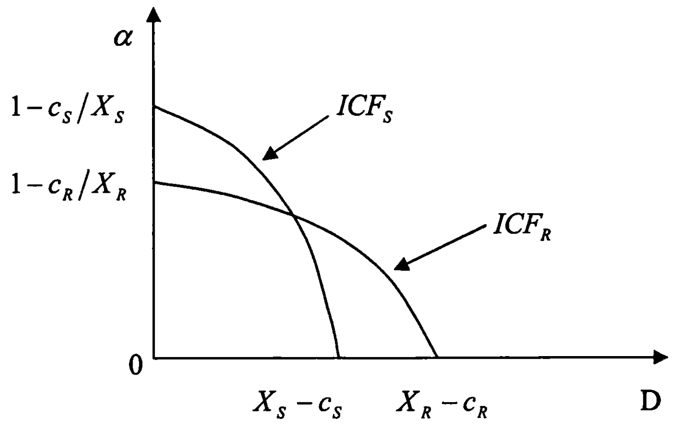


Figure 1.1

1.2.2 Indifference Curves and Revelation Constraints

The family of indifference curves of type i can be derived from Eq. (1.1). It should be noted that the shape of the indifference curves is independent of the probability of success.[23] As a result, no indifference curve of type i crosses ICF_i and therefore the indifference curves do not exhibit kinks in the (α, D) space. For each type, one of the indifference curves coincides with the corresponding ICF.

Lemma 2: Let U_i denote the family of indifference curves of type i, and u_i denote a member of this family. In the (α, D) space, for $0 \leq \alpha < 1$ and $0 \leq D < X_i$

a) u_i are downward sloping and concave with slope $\left(\frac{d\alpha}{dD}\right)_{u_i=\bar{u}} = -\frac{1-\alpha}{X_i - D} < 0$

b) The indifference curves of R and S cross only once.

Proof: See Appendix 1A.

That is, the marginal rate of substitution of debt for equity of the R-type is greater than that of the S-type. Intuitively, regardless of the assumption about the ordering of

[23] This is due to fact that in the event of failure the return is zero.

the distributions of returns, at any given (α, D) pair, equity is more valuable for the R-type and debt for the S-type (even if, in absolute terms, both debt and equity issued by the S-type are more valuable).[24] As a result, the R-type is willing to accept a greater increase in D in exchange for a given reduction in α than the S-type. Technically, the single-crossing condition is satisfied.

Notice also that, due to limited liability, any contract (α, D) above (to the right of) u_S^0 (u_R^0) provides the (R)- S-type with the same level of expected utility as those on u_S^0 (u_R^0). Clearly, the closer to the origin an indifference curve, the higher the expected utility (see Figure 1.2).

For any given pair of contracts $Z_R = (\alpha_R, D_R)$ and $Z_S = (\alpha_S, D_S)$ the revelation constraints are:

$$U_R(Z_R) \geq U_R(Z_S) \tag{1.5}$$

$$U_S(Z_S) \geq U_S(Z_R) \tag{1.6}$$

where U_i, $i = R, S$, is given by Eq. (1.1).

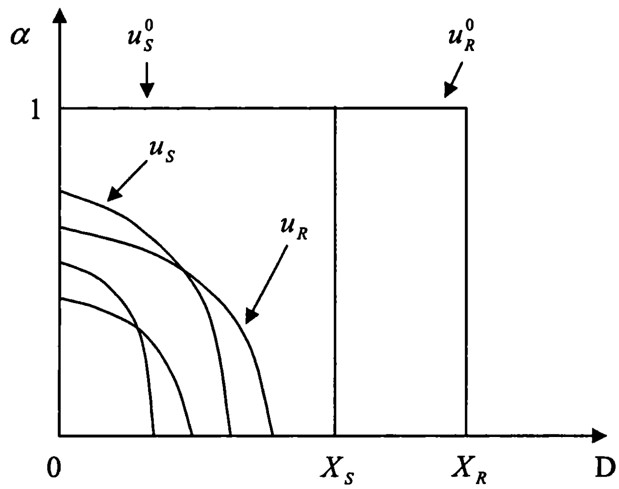


Figure 1.2

[24] Equity is a convex claim and so its value increases, whereas debt is a concave claim and its value falls with the variability of returns, given the expected PV of the project.

1.2.3 Zero-profit Lines

The expected profit of an F offering a contract (α, D) is given by Eq. (1.2). It is clear that the expected profit depends crucially on the effort level chosen (through the success probability of the project). Thus, if a zero-profit line crosses the corresponding effort incentive frontier ICF, it will exhibit a discontinuity because the success probability changes discontinuously when the Es change their effort level. However, given limited liability and the assumption that both types of projects have negative NPV when the low effort level is chosen ($\pi_0^i X_i - I < 0$), the zero-profit lines corresponding to shirking ($\pi^i = \pi_0^i$) do not exist. Any contract (α, D) financing a shirking E is loss-making and no rational F will offer it. Therefore, zero-profit lines can exist only if the high effort level is chosen (by at least one of the two types of Es).

More specifically, the zero-profit line corresponding to the i-type (ZP_i) exists only if the i-type chooses the high effort level (his effort incentive constraint is satisfied) when he receives funds at fair terms.[25] In other words, the existence of a zero-profit line (ZP_i) requires that it belong to the corresponding set of effort incentive compatible contracts (IC_i). Given the investment and moral hazard technology, if both types receive funds at fair terms three different cases may arise: i) the effort incentive constraint is not binding for either type, ii) it is not binding for the one type but is violated for the other, and iii) it is violated for both types. Conditional on the choice of the high effort level there exist three zero-profit lines: that corresponding to the R-type (ZP_R), to the S-type (ZP_S), and the pooling zero-profit line (PZP_H).[26] Lemma 3 summarises the key properties of the zero-profit lines and their relationship with the corresponding indifference curves and effort incentive frontiers. Subsequently, Lemma 4 provides the conditions for the existence of the individual zero-profit lines ZP_R and ZP_S.

[25] By assumption 1, both types of projects have strictly positive NPV when the high effort level is chosen and negative NPV when the Es opt for shirking.

[26] There can also exist another pooling zero-profit line corresponding to the case in which one type opts for the high effort level and the other shirks (PZP_L).

Lemma 3: In the (α, D) space,

a) All ZP_i, PZP_w are downward sloping and strictly concave with slopes:

$$\left(\frac{d\alpha}{dD}\right)_{ZP_i} = -\frac{1-\alpha}{X_i - D} < 0$$

$$\left(\frac{d\alpha}{dD}\right)_{PZP_H} = -\frac{(1-\alpha)\left[\lambda\pi_C^R + (1-\lambda)\pi_C^S\right]}{\lambda\pi_C^R(X_R - D) + (1-\lambda)\pi_C^S(X_S - D)} < 0$$

where $$\left|\left(\frac{d\alpha}{dD}\right)_{ZP_S}\right| \geq \left|\left(\frac{d\alpha}{dD}\right)_{PZP_H}\right| \geq \left|\left(\frac{d\alpha}{dD}\right)_{ZP_R}\right|$$

b) ICF_i, u_i, and ZP_i never cross each other, $i = R, S$.

Proof: See Appendix 1A.

Since all three, zero-profit lines, indifference curves, and effort incentive frontiers corresponding to type i have the same slope, they never cross. One of the indifference curves coincides with the corresponding zero-profit line. However, the location of the zero-profit line relative to the corresponding effort incentive frontier is the key determinant for the existence of the former.

Lemma 4: Suppose both types obtain funds at fair terms, then

a) If $\pi_C^i X_i - I \geq \pi_C^i c_i$, $i = R, S$, then both ZP_S and ZP_R exist.

b) If $\pi_C^R X_R - I \geq \pi_C^R c_R$, $\pi_C^S X_S - I < \pi_C^S c_S$, then only ZP_R exists.

c) If $\pi_C^S X_S - I \geq \pi_C^S c_S$, $\pi_C^R X_R - I < \pi_C^R c_R$, then only ZP_S exists.

d) If $\pi_C^i X_i - I < \pi_C^i c_i$, $i = R, S$, then neither ZP_S nor ZP_R exists.

Proof: If $\pi_C^i X_i - I \geq \pi_C^i c_i$, then the intersection point of ZP_i with the vertical axis, $(I/\pi_C^i X_i)$, lies (weakly) below that of ICF_i, $(1 - c_i/X_i)$. By Lemma 3, ZP_i and ICF_i never intersect (they may coincide). Therefore, ZP_i belongs to IC_i and hence it exists. Conversely, if $\pi_C^i X_i - I < \pi_C^i c_i$, then ZP_i lies outside IC_i and so it does not exist. In the latter case, if the i type obtains funds at fair terms, his effort incentive constraint is violated and so he opts for shirking contradicting the condition ($\pi^i = \pi_C^i$) on which ZP_i is constructed. *Q.E.D.*

Case (a) corresponds to pure adverse selection. Although, moral hazard is present, because for both types the NPV ($\pi_C^i X_i - I$) exceeds the "effective" cost of effort ($\pi_C^i c_i$), it has no bite. If either type obtains funds at fair terms, he exerts effort and so the corresponding zero-profit line exists. In Case (b), financing at fair terms implies that the effort incentive constraint of the R-type is satisfied but that of the S-type is violated (ZP_R belongs to IC_R but ZP_S lies outside IC_S). As a result, the R-type exerts effort and so ZP_R exists whereas the S-type opts for shirking and ZP_S does not exist. In the third case the reverse is true (ZP_S belongs to IC_S but ZP_R lies outside IC_R). In Case (d), the NPV of the project falls short of the "effective" cost of effort for both types. Thus, both types opt for shirking and so no zero-profit line exists. Figure 1.3 provides an illustration for Case (b).

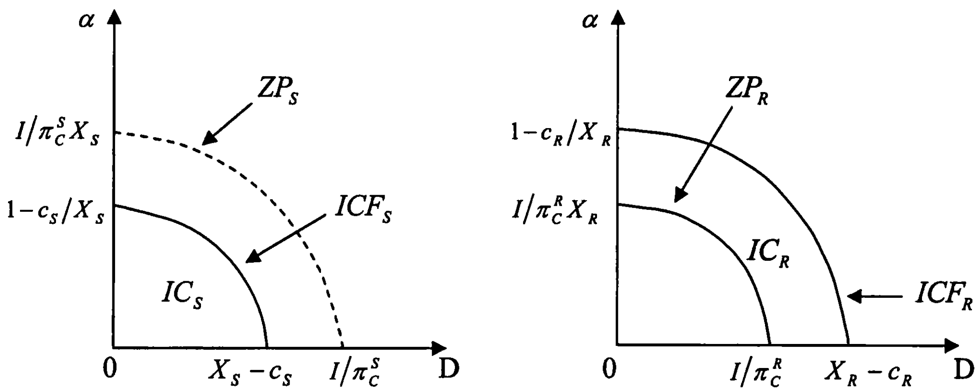


Figure 1.3

1.2.4 Equilibrium

It is well-known that, in most cases, the equilibrium outcome in competitive markets with asymmetric information depends crucially on the game-theoretic specification of the strategic interaction between the informed and uninformed agents. Yet, no agreement has been reached on which game structure is the most appropriate. It is a difficult task to determine the game specification that fits best the case at hand. Here, I assume that the Fs and the Es play the following three-stage game due to Hellwig (1987):

Stage 1: The two Fs simultaneously offer contracts (α, D). Each F may offer any finite number of contracts.
Stage 2: Given the offers made by the Fs, the Es apply for (at most) one contract from one F. If an E's most preferred contract is offered by both Fs, the E chooses each F's offer with probability $1/2$. In the light of the contract chosen, the E decides whether to work or shirk.
Stage 3: After observing the contracts offered by his rival and those chosen by the Es, each F decides which applications will accept or reject. If an application is rejected, the applicant does not receive funds.

This game structure rationalises a Wilson equilibrium (1977) as a perfect Bayesian equilibrium. Unlike the two-stage screening game, it allows for the existence of a (interior) Nash pooling equilibrium when this pooling equilibrium Pareto-dominates any other equilibrium. That is, this equilibrium concept allows agents to exploit all the gains from trade and is a necessary condition for the implementation of the optimal contract as a competitive equilibrium in the adverse selection cum moral hazard case.

We only consider pure-strategy perfect Bayesian equilibria. A pair of contracts (Z_R, Z_S) is an equilibrium if the following conditions are satisfied:[27]

- No contract in the equilibrium pair implies negative (expected) profits for the F. In other words, the Fs' participation or IR constraints are satisfied:

[27] Given limited liability and the strictly positive private benefit, the Es' participation constraints are always satisfied.

$$\pi(B_i)\{Max[\alpha(X_i - D),0] + Min(X_i, D)\} \geq I, \qquad i = R,S \tag{1.7a}$$

- Revelation constraints:

$$U_R(Z_R) \geq U_R(Z_S)$$
$$U_S(Z_S) \geq U_S(Z_R) \tag{1.7b}$$

- Effort incentives constraints:

$$B_i = b \quad \text{if} \quad (1-\alpha)(X_i - D) \geq c_i$$
$$B_i = B \quad \text{if} \quad (1-\alpha)(X_i - D) < c_i \tag{1.7c}$$
$$B_i = 0 \quad \text{if the project is not undertaken.}$$

- Profit maximisation: No other set of contracts, if offered alongside the equilibrium pair at Stage 1, would increase an F's expected profit.

To begin with, because of Bertrand competition, any equilibrium involves zero profits for the Fs. Lemma 5 formalises this argument.

Lemma 5: In any equilibrium whether pooling or separating, both Fs must have zero expected profits.

Proof: Let (α_R, D_R) and (α_S, D_S) be the contracts chosen by the R and S-type respectively (they could be the same contract). Suppose that the two Fs' aggregate expected profits are $P_F > 0$. Then the expected profit of one of the Fs must be no more than $P_F/2$. This F has an incentive to deviate and offer contracts $(\alpha_R - \varepsilon, D_R)$ and $(\alpha_S - \varepsilon, D_S)$, or alternatively $(\alpha_R, D_R - \varepsilon)$ and $(\alpha_S, D_S - \varepsilon)$, for $\varepsilon > 0$. By doing so, he will attract all Es. Since ε can be chosen arbitrarily small, this deviation will yield the deviant F an expected profit arbitrarily close to P_F. Thus, if $P_F > 0$ (at least) one of the Fs has an incentive to deviate and increase his expected profit. This implies that in any equilibrium it must be true that $P_F \leq 0$. However, since Fs have always the option to offer no contracts (or reject all the applications) and make zero profits, in any equilibrium, they cannot make (expected) losses. Therefore, in any equilibrium, both Fs make zero expected profits. *Q.E.D.*

1.3 Types of Equilibria and Provision of Funds: General Results

An important implication of Lemma 5 is that any equilibrium contract must lie on one of the zero-profit lines. This, in turn, implies the following result:

Lemma 6: A separating equilibrium can exist only if both ZP_R and ZP_S exist. If either ZP_S or ZP_R or both do not exist, then no separating equilibrium exists.[28]

Proof: First, given limited liability and the strictly positive private benefit, if funds are offered (whatever the terms they are offered at) both types of Es will always accept them and undertake their project. Thus, there cannot exist a separating equilibrium where only one type invests. Suppose now there is a separating equilibrium in which the R-type chooses contract (α_R, D_R) and the S-type chooses contract (α_S, D_S). By Lemma 5, the contract chosen by the R-type must lie on the R-zero-profit line (ZP_R) and that chosen by the S-type on the S-zero-profit line (ZP_S). Therefore, a separating equilibrium can exist only if both zero-profit lines exist. If one (or both) of the zero-profit lines does not exist, a separating equilibrium cannot exist. *Q.E.D.*

Lemma 6 implies that in cases where one (or both) of the zero-profit lines does not exist, if there exists an equilibrium, it must be pooling. Proposition 1 summarises these results.

[28] Given that the Es' participation constraints are always satisfied, the result in Lemma 6 holds true regardless of the form of the contracts.

Proposition 1: A separating equilibrium can exist only if $\pi_C^i X_i - I \geq \pi_C^i c_i$, $i = R,S$. If $\pi_C^i X_i - I < \pi_C^i c_i$, for either $i = R$, or $i = S$, or $i = R,S$, then the resulting equilibria must be pooling.

The next general result concerns the conditions under which funds are provided.[29]

Proposition 2:

a) If $\pi_C^i X_i - I \geq \pi_C^i c_i$, $i = R,S$, then both types of projects receive financing.

b) If $\pi_C^i X_i - I \geq \pi_C^i c_i$, $\pi_C^k X_k - I < \pi_C^k c_k$, $i = R,S$, $k = R,S$, then funds are offered to both types only if (a part of) either PZP_H or PZP_L exists.

c) If $\pi_C^i X_i - I < \pi_C^i c_i$, $i = R,S$, there exists a unique pooling equilibrium where no E obtains funds (no project is undertaken).

Proof: By Lemma 5, in any equilibrium, funds are offered only along the zero-profit lines. Thus, in any equilibrium, the Fs will offer funds only if (a part of) a zero-profit line exists.

a) By Lemma 4, both ZP_S and ZP_R exist. As a result, PZP_H also exists. Hence, regadless of the type of the equilibrium (separating or pooling) funds are offered.

b) By Lemma 6, in this case, only pooling equilibria can exist. However, the existence of pooling equilibria where funds are offered requires that (a part of) a pooling zero-profit line exist. Thus, (a part of) either PZP_H or PZP_L must exist.[30]

c) By Lemma 4, neither ZP_R nor ZP_S exists. As a result, by Lemma 6, no separating equilibrium exists. Moreover, since neither ZP_R nor ZP_S exists, no pooling zero-profit line exists. If an F offers funds to any E, he will make losses. Therefore, no rational F will do so (the Fs' participation constraints are violated). *Q.E.D.*

---

[29] Under the proposed game structure, in all cases, there exists a pooling equilibrium where funds are not offered.

[30] (A part of) PZP_H exists if it belongs to the intersection of IC_R and IC_S. Contracts offered along it are effort incentive compatible for both types. Thus, both types choose the high effort level and so it actually exists. If PZP_H does not belong to the intersection of IC_R and IC_S, it does not exist. In such a case, (at least) one of the two types shirks contradicting the condition on which PZP_H is drawn. (A part of) PZP_L exists if the following two conditions are satisfied: i) (A part of) it belongs to either IC_S or IC_R and ii) $(1-\lambda)\pi_C^S X_S + \lambda\pi_0^R X_R \geq I$ or $(1-\lambda)\pi_0^S X_S + \lambda\pi_C^R X_R \geq I$.

Notice that even in Case (c), if either type had chosen the high effort level he would have enjoyed a strictly positive expected utility (the sum of two positive components: i) the difference between the NPV of the project and the cost of effort, and ii) the private benefit) instead of zero. However, due to moral hazard, the inducement of this choice is not feasible.

1.4 Types of Equilibria and Methods of Financing: Specific Results

Thus far, no assumption has been made about the ordering of the distributions of returns. However, if an equilibrium exists where funds are provided, then both the type of the equilibrium (pooling or separating) and the method of financing depend, in general, on these assumptions. To proceed further with the analysis, we consider four different assumptions. The risky projects: i) dominate the safe ones by first-order stochastic dominance with respect to returns, ii) are mean-preserving spreads, iii) mean-reducing spreads, and iv) mean-increasing spreads of the safe projects. These distributional assumptions determine the location (intersection) of both the zero-profit lines ZP_R and ZP_S (if they exist) and the effort incentive frontiers ICF_R and ICF_S in the (α, D) space. This, in turn, determines the type of the equilibrium and the method of financing. Lemmas 7 and 8 describe analytically the location of the zero-profit lines and effort incentive frontiers respectively under each assumption.

Lemma 7: If the risky projects

a) dominate the safe projects by first-order stochastic dominance ($\pi_j^R = \pi_j^S$), then ZP_R and ZP_S intersect at $\alpha = 0$. For $\alpha > 0$, ZP_R lies entirely below ZP_S.

b) are mean-preserving spreads of the safe ones ($\pi_j^R X_R = \pi_j^S X_S$), then ZP_R and ZP_S intersect at $D = 0$. For $D > 0$, ZP_S lies entirely below ZP_R.

c) are mean-increasing spreads of the safe ones ($\pi_j^R X_R > \pi_j^S X_S$), then ZP_R and ZP_S intersect at some $(\alpha > 0, D > 0)$.

d) are mean-reducing spreads of the safe projects ($\pi_j^R X_R < \pi_j^S X_S$), then ZP_R and ZP_S do not intersect at any $(1 \geq \alpha \geq 0, D \geq 0)$. ZP_S lies entirely below ZP_R.

Proof: See Appendix 1A.

Intuitively, in Part (a), since both have the same success probability, given its face value, the debt issued by both types is equally valuable. Thus, if both issue only debt, zero profit for Fs requires the issue of the same level of debt. However, if equity is also issued, since the R-type equity is more valuable, an F who just breaks even would ask for a lower proportion of equity if he offered funds to the R-type than to the S-type (given the face value of debt). That is, ZP_R lies below ZP_S at any strictly positive level of equity issued. Under mean-preserving spreads, the equity issued by both types is equally valuable but the debt issued by the S-type is more valuable for the financiers. Thus, the S-type would be asked for the same proportion of equity but a lower face value of debt. Under mean-increasing spreads, equity is more valuable if it is issued by the R-type (its expected return is higher) and debt of given face value if it is issued by the S-type. As a result, a lower proportion of equity and a higher face value of debt is demanded by the R-type. Finally, under mean-reducing spreads, since both debt and equity issued by the S-type are more valuable, a lower proportion of equity and face value of debt is demanded by the S-type.

Lemma 8: If the risky projects

a) dominate the safe projects by first-order stochastic dominance ($\pi_j^R = \pi_j^S$), then ICF_R lies entirely above ICF_S ($IC_S \subset IC_R$).

b) are mean-preserving spreads of the safe ones ($\pi_j^R X_R = \pi_j^S X_S$), then ICF_R and ICF_S intersect at $D = 0$. For $D > 0$, ICF_R lies above ICF_S ($IC_S \subset IC_R$).

c) are mean-increasing ($\pi_j^R X_R > \pi_j^S X_S$) or mean-reducing ($\pi_j^R X_R < \pi_j^S X_S$) spreads of the safe ones, then three cases may arise: i) ICF_R and ICF_S intersect at some $(\alpha > 0, D > 0)$, ii) ICF_S lies above ICF_R ($IC_S \supset IC_R$), and iii) ICF_R lies above ICF_S ($IC_S \subset IC_R$).

Proof: See Appendix 1A.

That is, only the first two assumptions about the ordering of the distributions of returns restrict the location (intersection) of the effort incentive frontiers. These restrictions have the following implications:

Corollary 1: If the risky projects

a) dominate the safe projects by first-order stochastic dominance ($\pi_j^R = \pi_j^S$) and the effort incentive constraint for the R-type is violated, then it is also violated for the S-type (but not necessarily vice versa).

b) are mean-preserving spreads of the safe ones ($\pi_j^R X_R = \pi_j^S X_S$) and one of the effort incentive constraints is violated, then the other one is also violated.

Proof: a) By Lemma 7, ZP_R lies (weakly) below ZP_S. Also, by Lemma 8, ICF_R lies entirely above ICF_S. Therefore, if ZP_R lies above ICF_R (the effort incentive constraint for the R-type is violated), then ZP_S lies necessarily above ICF_S (the effort incentive constraint for the S-type is also violated).

b) By Lemma 7, ZP_R and ZP_S intersect at $D = 0$ and for $D > 0$, ZP_S lies entirely below ZP_R. By Lemma 8, ICF_R and ICF_S intersect at $D = 0$ and for $D > 0$, ICF_R lies above ICF_S. Also, by Lemma 3, ZP_i and ICF_i, $i = R, S$, have the same slope. Thus, if ZP_R (ZP_S) lies above ICF_R (ICF_S), then ZP_S (ZP_R) lies also above ICF_S (ICF_R). That is, if one effort incentive constraint is violated, the other one is also violated. *Q.E.D.*

Now that we have developed the analytical apparatus, we can go on to prove the main results of this chapter. Subsection 1.4.1 examines the pure adverse selection case. In subsection 1.4.2 we consider the case where both adverse selection and moral hazard play a crucial part in the determination and nature of the equilibrium outcome.

1.4.1 The Pure Adverse Selection Case

We first consider the case where the NPV of the project exceeds the "effective" cost of effort for both types (Case (a) of Lemma 4). In this case, as we have seen, no effort incentive constraint is binding if funds are offered at fair terms and so both zero-profit lines ZP_R and ZP_S exist. This, in turn, implies that the pooling zero-profit line PZP_H also exists. Therefore, both separating and pooling equilibria can exist. Moreover, given that the single-crossing condition is satisfied, a "reasonable" pooling equilibrium where both debt and equity are issued can exist only if, in equilibrium, there is no cross-subsidisation across types.

Under pure adverse selection, debt and equity are only used to convey socially costless information about the type of the project. Hence, in any pooling equilibrium where cross-subsidisation takes place, the subsidiser has an incentive to deviate by issuing more of the less valuable for him security. By doing so, he can credibly signal his type, reduce the cross-subsidisation and increase his expected return (utility). As a result, no pooling equilibrium involving cross-subsidisation can sustain. Notice, however, that the breaking of such a pooling equilibrium is possible only if it involves the issue of either both debt and equity or only the more valuable for the deviant security. If only the less valuable for the subsidiser security is issued, such a deviation is not possible and so the equilibrium cannot be broken (corner solution). In such a case, the subsidy is simply minimised (this is the case under mean-reducing spreads).

Therefore, under pure adverse selection, debt and equity can coexist in a pooling equilibrium only if both securities are fairly priced not only collectively but also individually. This, in turn, can occur only if this pooling equilibrium lies at the intersection of the individual zero-profit lines ZP_R and ZP_S (when they intersect at some $(\alpha > 0, D > 0)$). That is, only under mean-increasing spreads. More formally,

Proposition 3: If $\pi_C^i X_i - I \geq \pi_C^i c_i$, $i = R, S$, both types of projects obtain funds but the type of the equilibrium (separating or pooling) and the equilibrium method of financing depend on the ordering of the distributions of returns. In particular,

a) If the risky projects dominate the safe ones by first-order stochastic dominance, there exists a pooling equilibrium where both types issue only debt as well as a continuum of separating equilibria where the risky type issues only debt whereas the safe type issues a combination of debt and equity.
b) If the risky projects are mean-preserving spreads of the safe ones, there exists a pooling equilibrium where both types issue only equity as well as a continuum of

separating equilibria where the safe type issues only equity whereas the risky type issues a combination of debt and equity.

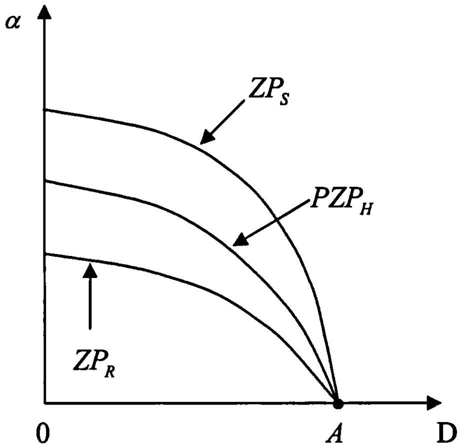
c) If the risky projects are mean-increasing spreads of the safe ones, there exists a continuum of separating equilibria (as well as a pooling equilibrium) where both types issue a combination of debt and equity. The risky type issues (weakly) more debt and less equity.

d) If the risky projects are mean-reducing spreads of the safe ones, there exists a unique pooling equilibrium where both types issue only equity.

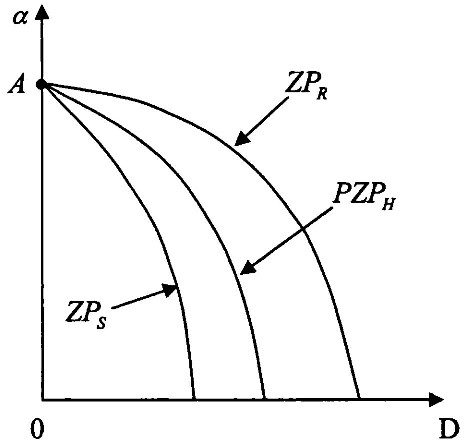
Proof: By Lemma 4, in this case, ICF_i lies above ZP_i, $i = R,S$, and so we can proceed with the analysis ignoring the effort incentive constraints. Let (A_R, A_S) be the equilibrium pair of contracts (in a pooling equilibrium $A_R = A_S = A$). We test whether the pair (A_R, A_S) or A is an equilibrium by considering deviations.

We begin with the case of mean-increasing spreads (Part (c)). First, we have to show that there cannot exist a pooling equilibrium except that at the intersection of ZP_S and ZP_R (point A). By Lemma 5, if there exists a pooling equilibrium it must lie on the pooling zero-profit line (PZP_H). Suppose that the pooling equilibrium contract is contract B that lies on PZP_H to the left of point A (see Figure 1.4c). Consider now the following deviation. An F offers a contract just below B in the area between the indifference curve of the two types through B. Given contract B is still offered, the deviant contract will reasonably attract only an R-type and so is profitable (since it lies above ZP_R). At the same time, contract B becomes loss-making and so any application for it would be rejected at Stage 3. Thus, contract B (any contract on PZP_H to the left of point A) cannot be a pooling equilibrium. By a similar argument, any contract along PZP_H to the right of point A cannot be a pooling equilibrium. However, it is easy to see that there is no profitable deviation from contract A. Therefore, contract A is a pooling equilibrium.

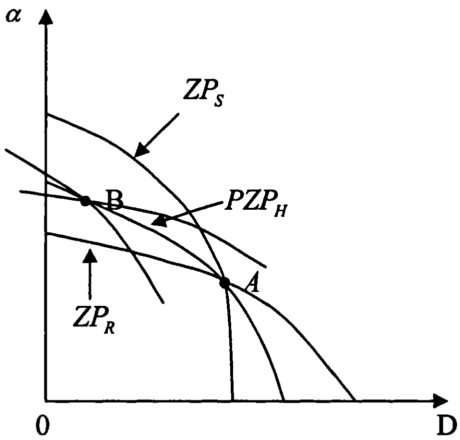
We also have to show that no contract along ZP_R to the left of A and along ZP_S to the right of A can be an equilibrium. All these contracts attract both types and so are loss-making for the financiers (since they lie below PZP_H). Therefore, no rational financier will offer any of them.



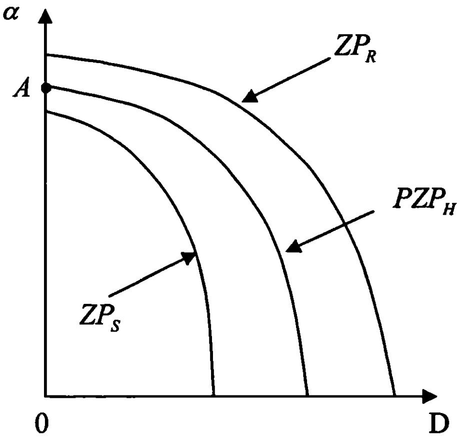
1.4a First-order stochastic dominance



1.4b Mean-preserving spreads



1.4c Mean-increasing spreads



1.4d Mean-reducing spreads

Figure 1.4

Finally, any pair (A_R, A_S), where A_R lies on ZP_R to the right of A and A_S lies on ZP_S to the left of A, is a separating equilibrium. Clearly, all these pairs satisfy the revelation and effort incentive constraints of both types as well as the zero-profit conditions. Furthermore, all separating pairs are equally preferred by both types of Es as well as the Fs and so there is no way to rule any of them out.

In Part (a), clearly, offers below ZP_R are unprofitable. Also, any offer along ZP_R (to the left of point A) would attract both types and so is loss-making. Thus, there cannot exist a separating equilibrium where the R-type issues equity. By an argument similar to that used in Part (c) above, in Part (a) there cannot exist a pooling equilibrium where equity is issued. Consider now an F who deviates by offering a contract in the area between ZP_S and ZP_R. Given contract A is still offered, at Stage 3, the deviant F will reasonably infer that his contract will be chosen by an S-type. As a result, the deviant contract is unprofitable (loss-making) and so any application for it will be rejected at Stage 3. Actually, anticipating the rejection of this application, no S-type would make it at Stage 2. Therefore, contract A which involves both types issuing only debt is a pooling equilibrium (see Figure 1.4a). Finally, any pair (A_R, A_S), where $A_R = A$ and A_S lies on ZP_S to the left of A, is a separating equilibrium. Because debt issued by both types at A is fairly priced, the S-type is indifferent between issuing debt and any debt-equity combination along ZP_S. Also, given contract A, the financiers are equally well off by offering any contract along ZP_S because, given contract A, any such an offer is going to be taken only by the S-type. Thus, none of these separating pairs can be ruled out.

By similar reasoning, we can show that under mean-preserving spreads (Part (b)) there exist a pooling equilibrium where only equity is issued as well as a continuum of separating equilibria where the S-type issues just equity whereas the R-type issues a debt-equity combination along ZP_R (see Figure 1.4b). Similarly, under mean-reducing spreads (Part (d)) there exists a unique pooling equilibrium that involves both types issuing only equity (see Figure 1.4d).[31] *Q.E.D.*

Three remarks should be made here. First, regardless of the distributional assumption, the NPV of all projects (given the high effort level is chosen) exceeds the cost of effort, C, and all projects receive financing. That is, investment is at its optimal level. Second, although under most distributional assumptions a pooling equilibrium exists, with the exception of mean-reducing spreads, the securities issued by both types are fairly priced both collectively (because of perfect competition) and individually. That is, there is no cross-subsidisation across types. On the contrary, under mean-reducing spreads because, in absolute terms, both the debt and equity issued by the S-type are more valuable, in the resulting pooling equilibrium the S-type inevitably subsidises the R-type through the mispricing of equity at individual level (the S-type equity is underpriced and the R-type overpriced). However, because debt is relatively more

[31] These equilibria also obtain in a two-stage signalling or screening game.

valuable than equity for the S-type, in the resulting all-equity pooling equilibrium the cross-subsidisation is minimised. Third, full separation requires that, in absolute terms, debt issued by the S-type and equity issued by the R-type be weakly more valuable for the financiers.

1.4.2 The Adverse Selection cum Moral Hazard Case

In this subsection, we examine the case where the S-type NPV falls short of his "effective" cost of effort (Case (b) of Lemma 4). That is, if the S-type is offered funds at fair terms, his effort incentive constraint is violated and so the corresponding zero-profit line does not exist. Thus, only pooling equilibria can exist. Because the choice of the high effort level is socially efficient, here we focus on pooling equilibria where both types exert effort. These equilibria involve cross-subsidisation across types and Pareto-dominate any other equilibrium. Through the mispricing of equity at individual level, the S-type receives the subsidy necessary to induce him to work.

That is, in the presence of both adverse selection and (effort) moral hazard, in addition to conveying information, debt and equity play a second role. That of incentivising the more prone to shirking type. This double role stems from the interaction between adverse selection and moral hazard and provides an explanation for the issue of combinations of debt and equity even if the issue of equity implies an adverse selection cost. What is more, in contrast with the pure adverse selection case, the cross-subsidisation is socially beneficial. It converts negative into positive NPV projects and improves social welfare.

To illustrate this point, we consider the case where, at any identical effort level, the risky projects dominate the safe ones by first-order stochastic dominance. The remaining cases are analysed in Appendix 1B.

Proposition 4: Suppose the risky projects dominate the safe ones by first-order stochastic dominance ($X_R > X_S$, $\pi_j^R = \pi_j^S = \pi_j$, $j = C,0$) and $\pi_C^R X_R - I > \pi_C^R c_R$, $\pi_C^S X_S - I < \pi_C^S c_S$, $I/\pi_C^R X_R < 1 - c_S/X_S$. Then if $\lambda > \overline{\lambda}_1$ there exists a unique pooling (funding) equilibrium where both types choose the socially efficient high effort level and obtain funds by issuing a combination of debt and equity.

where $\bar{\lambda}_1 \equiv \dfrac{I - \pi_C^S (X_S - c_S)}{(\pi_C^R X_R - \pi_C^S X_S)(1 - c_S/X_S)}$

The equilibrium contract, $A = (\alpha^*, D^*)$, lies at the intersection of ICF_S and PZP_H with α^* and D^* given by:

$$\alpha^* = \frac{I - \left[\lambda \pi_C^R + (1-\lambda)\pi_C^S\right](X_S - c_S)}{\lambda \pi_C^R (X_R - X_S)} \tag{1.8}$$

$$D^* = X_S - c_S/(1 - \alpha^*) \tag{1.9}$$

Proof: We test whether the contract at A is an equilibrium by considering deviations.[32] Offers below ZP_R are clearly loss-making. Any offer in the area between u_R^A (the R-type indifference curve through the equilibrium contract) and ZP_R to the left of ICF_S is going to be taken by both types and so is unprofitable. Thus, we only need to consider the following two deviations: i) Suppose that an F deviates by offering a contract, say A', in the area between u_R^A and ZP_R to the right of ICF_S.

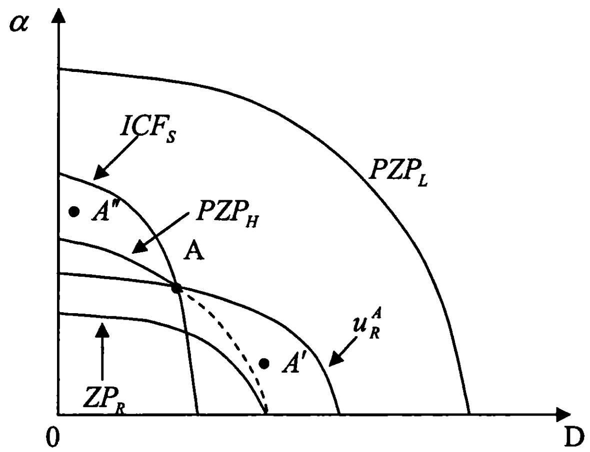


Figure 1.5

[32] In Appendix 1B, we also provide mathematical proofs for our results.

Given that contract A is still offered, the deviant contract, contract A', will reasonably attract only the R-type. This, in turn, implies that contract A is taken only by the S-type and so it becomes loss-making. As a result, at Stage 3, any application for that contract will be rejected. Anticipating that, the S-type will also choose A', at Stage 2, and hence A' becomes also loss-making (since to the right of ICF_S, PZP_H does not exist, and PZP_L lies to the right of u_R^A). Therefore, there is no profitable deviation to the right of A. ii) Consider now an F who deviates by offering a contract, say A'', in the area between ICF_S and u_R^A to the left of (above) A. Given contract A is still offered, contract A'' will reasonably attract only the S-type and so is loss-making. Thus, any application for contract A'' will be rejected at stage 3. Actually, anticipating the rejection of that application at Stage 3, no S-type would make it at Stage 2. Therefore, the pooling equilibrium at A is the unique equilibrium where funds are provided.[33] *Q.E.D.*

The pooling equilibrium at A reflects a trade-off between information revelation and effort incentives. The securities issued by the R- and S-type are priced as a pool. Although, because of perfect competition, debt and equity are fairly priced collectively, at individual level they are mispriced. Not surprisingly, it is precisely this mispricing that provides the more prone to shirking type with the subsidy necessary to induce him to exert effort. More specifically, in the pooling equilibrium of Proposition 4 the R-type subsidises the S-type through the mispricing of the more valuable for him security (equity). Hence, given that the single-crossing condition is satisfied, the R-type has an incentive to deviate by choosing a contract involving more debt and less equity than the equilibrium contract. By doing so, he can credibly signal his type, reduce the cross-subsidisation and increase his expected return.

However, his attempt will be fruitless. If the R-type chooses such a contract, the equilibrium contract becomes loss-making for the financiers and so any application for that will be rejected. As a result, the S-type will always mimic the R-type preventing him from revealing his type and obtaining funds in better terms. What is

[33] It should be noted that uniqueness follows from the application of the "intuitive criterion". All contracts along (the relevant part of) PZP_H correspond to pooling perfect Bayesian equilibria under abritrary out-of-equilibrium beliefs. However, contract A is the only one that survives the "intuitive criterion" (Cho and Kreps, 1987).

more, the deviant contract gives the S-type less subsidy and so destroys his effort incentives. The S-type shirks and the collective expected return falls significantly. A financier who offers a contract involving less equity than the equilibrium contract can break even only if he asks for a considerably greater face value of debt (higher interest rate on debt). But neither type prefers such a deviant contract to the equilibrium contract. Hence, no financier has an incentive to offer a contract involving less equity than the equilibrium contract and so the R-type stays in equilibrium and provides the S-type with just enough subsidy in order to induce him to work.

Loosely speaking, the R-type accepts to issue some equity and induce the S-type to exert effort because the increase in his net expected return (due to the lower interest rate he pays on debt) more than offsets the cost of the incremental subsidy (the adverse selection cost of issuing equity). That is, the R-type is better off in the pooling equilibrium of Proposition 4 where both debt and equity are issued and both types exert effort than in a pooling equilibrium where only debt is issued and so the S-type shirks.[34]

Moreover, the role of debt and equity as communication devices implies that no financier can make a profit by offering a contract involving more equity (subsidy) and less debt than the equilibrium contract. Given that the equilibrium contract is still offered, the deviant contract will not be taken by any E at Stage 2. If an E chooses this contract, the financier will infer that he is an S-type. As a result, the deviant contract is loss-making and any application for that will be rejected at Stage 3. Anticipating that, no E will apply for it at Stage 2.[35]

That is, the existence of the socially efficient pooling equilibrium relies on two factors: i) the endogenous (discrete)[36] choice of the effort level and ii) the three-stage game structure that allows for an (interior) pooling perfect-Bayesian equilibrium even if cross-subsidisation across types takes place and the single-crossing condition is

[34] Notice that in the pooling equilibrium of Proposition 4 the R-type is worse off compared to the case where types are observable and he obtains funds at fair terms. However, social welfare exceeds that under full information about types (see also the discussion in Subsection 1.4.2.2 below).

[35] A similar argument applies if the subsidiser is the S-type. The only difference is that the cross-subsidisation now takes place through the mispricing of the more valuable for the S-type security (debt). That is, in this latter case, the equilibrium contract involves more debt and less equity than the S-type would wish.

[36] We conjecture that, under certain restrictions on the probability and cost functions, this pooling equilibrium exists even if the effort level is a continuous variable.

met. [37] Due to the presence of the third stage agents behave less myopically than in a two-stage screening game and so the non-existence problem is resolved.

If it exists, the pooling equilibrium of Proposition 4 has two interesting implications: First, it provides an explanation for the issue of combinations of debt and equity even if the issue of equity implies an adverse selection cost. Firms issue some equity even if under pure adverse selection they would have issued just debt. Second, in contrast with the pure adverse selection case, the cross-subsidisation is socially beneficial. It converts a negative into a positive NPV project and improves social welfare.

1.4.2.1 Implications for the Issue of Securities

To fix ideas, let us compare the adverse selection cum moral hazard case with the pure adverse selection and pure moral hazard cases. As we have already seen, under pure adverse selection, the securities issued are only used to convey socially costless information about the type of the project. Therefore, firms issue combinations of debt and equity only if both securities are fairly priced not only collectively but also individually. Pooling equilibria involving cross-subsidisation can exist only if the less valuable for the subsidiser security is issued (corner solution). In this case, there is no channel through which the cross-subsidy can have positive effects for the subsidiser. As a result, the subsidiser maximises his return by minimising the subsidy he provides the other type.

In contrast, in the presence of effort moral hazard, if the subsidiser cannot reveal his type, it may be in his interest to incur the adverse selection cost of issuing some of the more valuable for him security. By doing so, he provides the more prone to shirking type with the subsidy necessary to induce him to work and so the collective expected return rises. If the resulting increase in his expected return exceeds this adverse selection cost, the subsidiser's welfare improves. For example, in Proposition 4 the benefit (due to the lower interest rate he pays on debt) for the R-type from

[37] In a two-stage signalling game, such a pooling equilibrium cannot exist. Behaving myopically, the R-type tries to reveal his type by issuing more debt and less equity. However, the S-type always mimics and, more importantly, his effort incentives are destroyed. Therefore, there can exist either pooling equilibria where only debt is issued (corner solution) and the R-type works whereas the S-type shirks or pooling equilibria where both types shirk and so no funds are provided. In either case, the resulting pooling equilibria are Pareto-inferior to that of Proposition 4.

accepting to issue some equity and inducing the S-type to exert effort exceeds the adverse selection cost associated with the equity issue.[38]

In the pure moral hazard case, the Fs observe the type of each individual E. As a result, each type is offered contracts along the corresponding zero-profit line, provided it exists. In the context of our simple model, the mode of financing is irrelevant.[39] All combinations of debt and equity along the existing zero-profit line are offered and are equally preferred by the corresponding type.

1.4.2.2 Implications for Investment and Social Welfare

Under the conditions in Proposition 4, if types were observable only the R-type would receive financing. If the S-type receives funds at fair terms he shirks and so his project NPV is negative. Moreover, financiers have no incentive to transfer resources from the R-type to the S-type to induce the latter to exert effort. Thus, no rational financier will be willing to offer him the required for the investment funds and so the S-type project is not undertaken. That is, under full information about types a potentially positive NPV investment opportunity is forgone. Furthermore, because when the S-type works his project NPV exceeds the cost of effort, the social welfare also worsens.

These results are in sharp contrast with the pure adverse selection case. In Myers and Majluf (1984) adverse selection leads firms to forego positive NPV projects whereas in de Meza and Webb (1987) it encourages firms to undertake negative NPV projects. Hence, in either case social welfare is lower than under full information about types. The key to this difference is that in the presence of (effort) moral hazard the cross-subsidisation taking place in a pooling equilibrium relaxes this additional constraint and so it can be beneficial.[40] On the contrary, given risk neutrality, under

[38] Notice that, although the pooling equilibrium of Proposition 4 involves cross-subsidisation across types of Es, it does not involve cross-subsidisation across debt and equity. Once the equilibrium is determined, the value of these two contracts can be calculated independently and so debt and equity could be traded separately in a secondary market. In fact, the same equilibrium obtains even if instead of one F offering both debt and equity, the Fs specialise in one of the two contracts and debt and equity markets are perfectly competitive (see Appendix 1C for a proof).

[39] This result is due to the assumption that in case of failure the project yields zero regardless of its type. If instead we assume that in case of failure the return is strictly positive then debt becomes the optimal contract (Innes (1990)). All the main results go through under the latter assumption. However, the zero-return assumption simplifies considerably the analysis without losing any insight.

[40] Notice that the pooling equilibrium of Proposition 4 may exist even if the NPV of the S-type project is negative regardless of the effort level. Obviously, in such a case, adverse selection results in overinvestment and a fall in social welfare. Moreover, the pooling equilibrium can exist if there exist

pure adverse selection there is no channel through which the cross-subsidy can have positive effects but it may have negative consequences.

1.5 The Roles of Warrants

So far, the available financial instruments have been debt and equity. The discussion of the previous section illustrated the roles of these two financial contracts as separation devices and means of incentivising the more prone to shirking type. In this section, we introduce financing instruments with option features. More specifically, the Es can borrow the required amount I by issuing a debt-warrant combination.

The warrant gives its holder the right to purchase a prespecified proportion of the firm's equity, η, at an agreed price K (exercise price). The proceeds from the exercise of the option, K, are distributed as dividends to the shareholders. Therefore, a warrant holder will exercise if

$$\eta_i(X_i - D_i + K_i) \geq K_i, \quad K_i \geq 0, \quad i = R,S \tag{1.10}$$

This can be rewritten as

$$K_i \leq \frac{\eta_i}{1-\eta_i}(X_i - D_i), \qquad i = R,S \tag{1.10'}$$

So, given risk neutrality and limited liability, the Es seek to maximise:

$$U_i(X_i, \eta_i, D_i, B_i, K_i) = \pi(B_i) Min\{(1-\eta_i)(X_i - D_i + K_i), Max[(X_i - D_i), 0]\} + B_i \tag{1.11}$$

where U_i is the expected utility of an E of type i when choosing the contract $\Xi_i = (\eta_i, D_i, K_i)$. Similarly, given limited liability, the expected profit of an F offering the contract $\Xi_i = (\eta_i, D_i, K_i)$ is given by:

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more than two types. In this case, it is possible that adverse selection leads to overinvestment but an improvement in social welfare.

$$P_F = \pi(B_i)Max\{\eta_i(X_i - D_i + K_i) - K_i + D_i, Min(X_i, D_i)\} - I \quad (1.12)$$

To make the analysis interesting, we assume that the exercise price is set such that, in case of success, the option is exercised regardless of the type of the project. That is, the exercise price is given by:

$$K_i = \frac{\eta_i}{1-\eta_i}(1-\psi_i)(X_S - D_i) \quad \text{where } \psi_i \in [0,1], \qquad i = R, S \quad (1.13)$$

Eq. (1.13) is a sufficient condition for the warrants issued by both types to be exercised in case of success.[41] Furthermore, without loss of generality, we assume $\psi_R = \psi_S = \psi \in [0,1]$.[42] Basically, this assumption reduces the choice variables (signals) from three (η, D, K) to two (η, D). The choice of η and D completely determines K. By doing so, we considerably simplify the analysis without losing any insight.[43] Using (1.13) and the assumption about ψ, the utility and profit functions simplify respectively to:

$$U_i(X_i, \eta_i, D_i, B_i, K_i) = \pi(B_i)[(1-\eta_i)(X_i - D_i) + \eta_i(1-\psi)(X_S - D_i)] + B_i \quad (1.14)$$

$$P_F = \pi(B_i)\{\eta_i[(X_i - D_i) - (1-\psi)(X_S - D_i)] + D_i\} - I \quad (1.15)$$

1.5.1 Indifference Curves, Effort Incentive and Revelation Constraints

A given contract (η, D) will induce the high effort level if

$$(1-\eta)(X_i - D + K) \geq c_i \quad (1.16)$$

---

[41] This condition is imposed for simplicity. All results go through if instead of X_S in Eq. (1.13) we had X_R or even if we specified a different function for the exercise of the warrant issued by each type. However, these modifications would complicate the analysis without adding any insight.

[42] A combination of debt and equity is a special case of a debt-warrant combination that obtains for $\psi_R = \psi_S = 1$.

[43] No more than two choice variables are necessary for our purposes. Clearly, all the results go through if we increase their number to three by allowing for $\psi_R \neq \psi_S$.

So, the equations of the effort incentive frontiers ICF_S and ICF_R are given respectively by:

$$(1-\psi\eta)(X_S-D)=c_S \tag{1.17}$$

$$(1-\psi\eta)(X_R-D)-\eta(1-\psi)(X_R-X_S)=c_R \tag{1.18}$$

The family of indifference curves of type i can be derived from (1.14). The indifference curves have the same slope as the corresponding effort incentive frontiers. As a result, no indifference curve of type i crosses ICF_i and therefore the indifference curves do not exhibit kinks in the (η, D) space. For each type, one of the indifference curves coincides with the corresponding ICF.

Finally, for any given pair of contracts $\Xi_R=(\eta_R, D_R)$ and $\Xi_S=(\eta_S, D_S)$ the revelation constraints are:

$$U_R(\Xi_R)\geq U_R(\Xi_S) \tag{1.19}$$

$$U_S(\Xi_S)\geq U_S(\Xi_R) \tag{1.20}$$

where U_i, $i=R,S$, is given by Eq. (1.14).

Lemma 9: In the (η, D) space:

a) ICF_R and ICF_S are downward sloping and strictly concave with slopes:

$$\left(\frac{d\eta}{dD}\right)_{ICF_R}=-\frac{1-\psi\eta}{\psi(X_S-D)+(X_R-X_S)}<0$$

$$\left(\frac{d\eta}{dD}\right)_{ICF_S}=-\frac{1-\psi\eta}{\psi(X_S-D)}<0$$

That is, at any (η, D) pair, ICF_R is flatter than ICF_S.

b) If $c_R \geq c_S$ and $X_R - c_R \geq X_S - c_S$, then for any $\psi \in [0, \min[\overline{\psi}, 1]]$, ICF_R and ICF_S intersect at some $(1 \geq \eta \geq 0, D \geq 0)$. Otherwise, either $IC_R \subset IC_S$ or $IC_S \subset IC_R$.

where $\overline{\psi} \equiv \dfrac{(X_R - X_S)(1 - c_S/X_S)}{(X_R - c_R) - (X_S - c_S)}$

c) The indifference curves of the R- and S-type have the same slope as the corresponding effort incentive frontiers.

Proof: See Appendix 1A.

Since $X_R > X_S$, one of the conditions $(c_R \geq c_S)$ for the intersection of ICF_R and ICF_S to occur at some admissible value of the two choice variables (η and D) in this case, is weaker than under a combination of debt and equity $(c_R/X_R \geq c_S/X_S)$.

Intuitively, since $X_R > X_S$, at any given (η, D) pair, a given fall in ψ (increase in the exercise price) implies that the project's return constitutes a smaller proportion of the total payment to the warrantholder if the warrant is issued by the S-type. That is, as the exercise price rises, the warrant value falls faster for the S-type and so the S-type is willing to increase faster the proportion of equity, η, offered to the financier than the R-type while still exerting effort.[44] As a result, for ψ sufficiently low ($\psi \leq \overline{\psi}$), ICF_S and ICF_R intersect at some positive face value of debt, D, even if this is not possible when we restrict ourselves to debt and equity.

As far as the indifference curves are concerned, in the (η, D) space, that of the R-type is flatter. Intuitively, regardless of the assumption about the distribution of returns, at any given (η, D) pair, the warrant is more valuable for the R-type and debt for the S-type (even if, in absolute terms, both debt and the warrant issued by the S-type are more valuable). As a result, the R-type is willing to accept a greater increase

[44] Diagrammatically, in the (η, D) space, as ψ falls ICF_S becomes steeper faster than ICF_R.

in D in exchange for a given reduction in η than the S-type. That is, the single-crossing condition is satisfied.

1.5.2 Zero-profit Lines

The expected profit of an F offering a contract (η, D) is given by (1.15). Given Assumption 1, the zero-profit line corresponding to the i-type (ZP_i) exists only if the i-type exerts effort when he receives funds at fair terms.[45] In other words, the existence of a zero-profit line (ZP_i) requires that it belong to the corresponding set of effort incentive compatible contracts (IC_i). Conditional on the choice of the high effort level there exist three zero-profit lines: that corresponding to the R-type (ZP_R), to the S-type (ZP_S), and the pooling zero-profit line (PZP_H).The equations of the zero-profit lines ZP_S and ZP_R are respectively:

$$\pi_C^S[\eta\psi(X_S - D) + D] = I \tag{1.21}$$

$$\pi_C^R\{\eta[\psi(X_S - D) + (X_R - X_S)] + D\} = I \tag{1.22}$$

Lemma 10 summarises the key properties of the zero-profit lines and their relationship with the corresponding indifference curves and effort incentive frontiers.

Lemma 10: In the (η, D) space,

a) All ZP_R, ZP_S, PZP_H are downward sloping and strictly concave with slopes:

$$\left(\frac{d\eta}{dD}\right)_{ZP_R} = -\frac{1-\psi\eta}{\psi(X_S - D) + (X_R - X_S)} < 0$$

[45] By Assumption 1, the NPV of both types of projects is strictly positive if the high effort level is chosen whereas it is strictly negative if shirking is chosen.

$$\left(\frac{d\eta}{dD}\right)_{ZP_S} = -\frac{1-\psi\eta}{\psi(X_S - D)} < 0$$

$$\left(\frac{d\eta}{dD}\right)_{PZP_H} = -\frac{(1-\psi\eta)\left[\lambda\pi_C^R + (1-\lambda)\pi_C^S\right]}{\lambda\pi_C^R\left[(X_R - X_S) + \psi(X_S - D)\right] + (1-\lambda)\pi_C^S\psi(X_S - D)} < 0$$

where $$\left|\left(\frac{d\alpha}{dD}\right)_{ZP_S}\right| \geq \left|\left(\frac{d\alpha}{dD}\right)_{PZP_H}\right| \geq \left|\left(\frac{d\alpha}{dD}\right)_{ZP_R}\right|$$

b) If $X_R - I/\pi_C^R \geq X_S - I/\pi_C^S$, then for any $\psi \in [0, \min[\hat{\psi}, 1]]$ ZP_R and ZP_S intersect at some $(0 \leq \eta \leq 1, D \geq 0)$.

where $\hat{\psi} \equiv \dfrac{\pi_C^R(X_R - X_S)}{(\pi_C^S - \pi_C^R)X_S}$

c) ICF_i, u_i, and ZP_i never cross each other, $i = R, S$.

Proof: See Appendix 1A.

That is, the intersection of the zero-profit lines ZP_R and ZP_S can occur at some admissible value of the choice variables (η and D) even under mean-reducing spreads. Recall that this is not possible if we restrict the contract space to debt and equity (see Lemma 7). Intuitively, since $X_R > X_S$, at any given (η, D) pair, a given fall in ψ (increase in the exercise price) implies that the project's return constitutes a smaller proportion of the total payment to the warrantholder if the warrant is issued by the S-type. That is, as the exercise price rises, the net payoff of a financier offering funds to the S-type falls faster. As a result, the increase in the proportion of equity, η, required in order for the financier to just break even is greater if the warrant is issued by the S-type.[46] For ψ sufficiently low ($\psi \leq \hat{\psi}$), ZP_S and ZP_R intersect at some positive face value of debt, D, even if this is not possible when we restrict ourselves

[46] Diagrammatically, in the (η, D) space, as ψ falls ZP_S becomes steeper faster than ZP_R.

to debt and equity. In other words, for a sufficiently high exercise price the warrant issued by the R-type becomes more valuable than that of the S-type even if the S-type equity is more valuable.

In general, the intersection of the effort incentive frontiers and the zero-profit lines of the two types at some admissible value for both choice variables requires weaker conditions under debt coupled with a warrant than under a combination of debt and equity. This has important implications for the fair pricing of the securities issued under pure adverse selection and the restrictions on the parameter values required for the existence of the socially efficient pooling equilibrium under adverse selection and (effort) moral hazard. Below we consider each case separately.

1.5.3 The Pure Adverse Selection Case

In this case, as we have seen, no effort incentive constraint is binding and so all three zero-profit lines ZP_R, ZP_S and PZP_H exist.[47] Therefore, both separating and pooling equilibria can exist. Moreover, given that the single-crossing condition is satisfied and, by appropriately choosing ψ, ZP_S and ZP_R can intersect at some admissible value of the choice variables, a "reasonable" pooling equilibrium can exist only if it does not involve cross-subsidisation across types.

Under pure adverse selection, the face value of debt and the proportion of equity (or exercise price) jointly serve as signals conveying socially costless information about the type of the project. Hence, in any pooling equilibrium where cross-subsidisation takes place, the subsidiser has an incentive to deviate by issuing more of the less valuable for him security. By doing so, he can credibly signal his type, reduce the cross-subsidisation and increase his expected return. As a result, no pooling equilibrium involving cross-subsidisation can sustain. That is, in any equilibrium (pooling or separating) the securities issued are fairly priced not only collectively but also individually. Moreover, by choosing $\psi < \min(\hat{\psi},1)$, we can achieve full separation under mean-preserving, mean-increasing and mean-reducing spreads. However, if the risky project dominates the safe one by first-order stochastic dominance, regardless of the value of ψ, there exists a pooling equilibrium where

[47] It can be easily shown that the results of Lemmas 4, 5, and 6 and Propositions 1 and 2 hold true regardless of the form of the contract.

both types issue only debt as well as a continuum of separating equilibria where the R-type issues only debt whereas the S-type issues a debt-warrant combination. More formally,

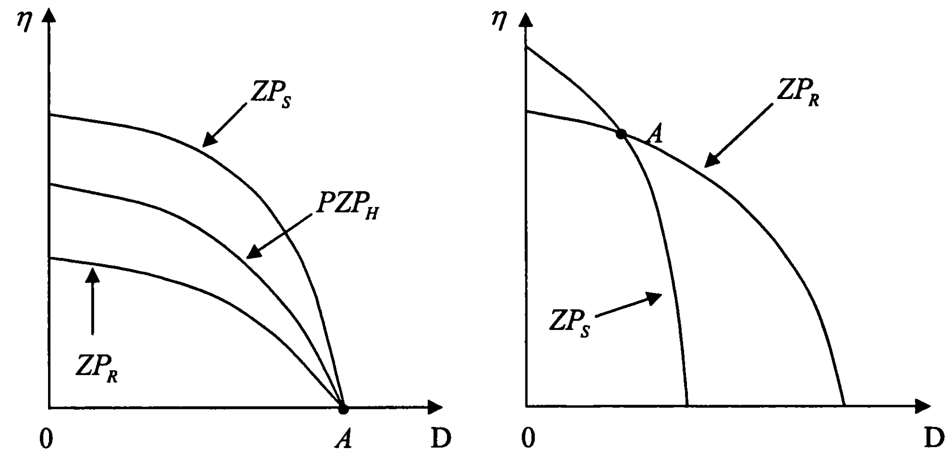
Proposition 5: If $\pi_C^i X_i - I \geq \pi_C^i c_i$, $i = R, S$ and $X_R - I/\pi_C^R > X_S - I/\pi_C^S$, then for any $\psi \in [0, \min[\hat{\psi}, 1]]$ there always exist equilibria (pooling or separating) where both types of projects obtain funds and the securities issued are fairly priced regardless of the distributional assumption. In particular,

a) If the risky projects dominate the safe projects by first-order stochastic dominance, there exists a pooling equilibrium where both types issue only debt as well as a continuum of separating equilibria where the R-type issues only debt whereas the S-type issues a debt-warrant combination.
b) If the risky projects are mean-preserving, mean-increasing, or mean-reducing spreads of the safe ones, there exists a continuum of separating equilibria (as well as a pooling equilibrium) where both types issue a debt-warrant combination with $\eta_R \leq \eta_S$ and $D_R \geq D_S$.

Proof: Similar to Proposition 3 (see Figure 1.6).

Intuitively, full revelation requires that debt be more valuable for the S-type and the warrant for the R-type not only in relative but also in absolute terms.[48] If these two conditions are met the S-type can credibly reveal his type by choosing a contract involving low face value of debt and a warrant with very high exercise price (a high proportion of equity is offered to the financier). The R-type has no incentive to mimic because the cost from the underpricing of such a warrant exceeds the gains from issuing a little overpriced debt. The first condition is satisfied under all four distributional assumptions. By appropriately choosing the warrant exercise price, the second condition can also be satisfied even under mean-reducing spreads.

[48] The first (single-crossing) condition allows agents to send a credible signal while the second rules out pooling equilibria where only the less valuable for the subsidiser security is issued (corner solutions).



1.6a First-order stochastic dominance

1.6b Mean-preserving, mean-increasing or mean-reducing spreads

Figure 1.6

If the risky projects dominate the safe ones by first-order stochastic dominance, debt of given face value issued by both types is equally valuable but the warrant issued by the R-type is more valuable. Hence, in order to avoid subsidising the S-type, in any equilibrium, the R–type issues only debt and debt issued is fairly priced. As a result, the S-type is indifferent between issuing just debt (pooling equilibrium) and any debt-warrant combination along ZP_S (separating equilibria). Therefore, there can exist a pooling equilibrium where only debt is issued as well as a continuum of separating equilibria where the R-type issues just debt whereas the S-type issues a debt-warrant combination along ZP_S.

In summary, under pure adverse selection, a debt-warrant combination allows us to obtain equilibria (pooling or separating) where the securities issued are fairly priced even if it is not possible when we restrict ourselves to debt, equity and/or convertible debt.[49] This result provides a rationale for the use of warrants.

1.5.4 The Adverse Selection cum Moral Hazard Case

In this subsection, we show that a debt-warrant combination allows for the existence of the socially efficient pooling equilibrium under weaker restrictions on parameter values than a debt-equity combination. For expositional simplicity, we only consider

[49] Recall that if we restrict ourselves to debt and equity under mean-reducing spreads there exists a unique pooling equilibrium where only equity is issued and the S-type subsidises the R-type through the mispricing of equity at individual level.

the case where, at any given identical effort level, the risky project dominates the safe one by first-order stochastic dominance (see Appendix 1B for a generalisation of Proposition 6). Also, if both types are offered funds at fair terms, the S-type shirks whereas the R-type exerts effort.

Proposition 6: Suppose the risky projects dominate the safe ones by first-order stochastic dominance ($X_R > X_S$, $\pi_j^R = \pi_j^S = \pi_j$, $j = C,0$) and $\pi_C^R X_R - I > \pi_C^R c_R$, $\pi_C^S X_S - I < \pi_C^S c_S$. Then for any $\psi \leq \overline{\psi}$ and $\lambda \geq \overline{\lambda}_2$, then there exists a unique pooling (funding) equilibrium where both types exert effort and obtain funds by issuing a debt-warrant combination (see Figure 1.7b).

where $$\overline{\lambda}_2 \equiv \frac{I - \pi_C^S(X_S - c_S)}{\pi_C^R(X_R - c_R) - \pi_C^S(X_S - c_S)} < \overline{\lambda}_1$$

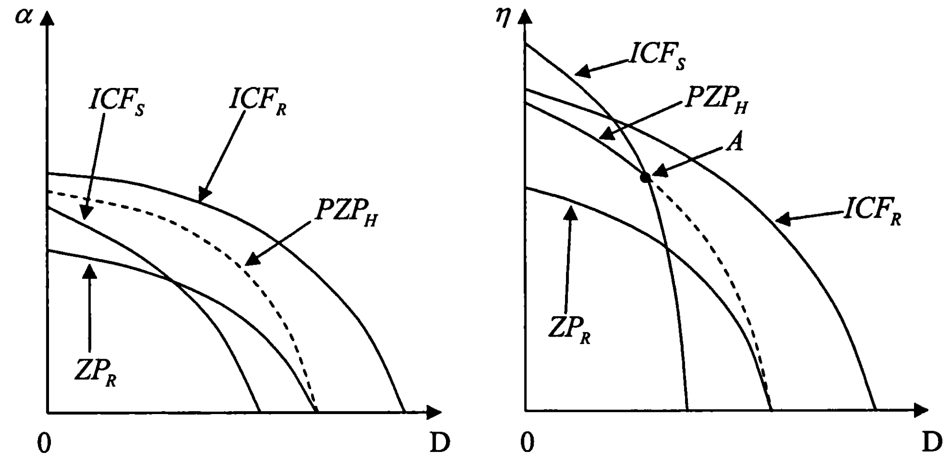
The equilibrium contract, $A = (\eta^*, D^*)$, lies at the intersection of ICF_S and PZP_H with η^* and D^* given by:

$$\eta^* = \frac{I - \left[\lambda\pi_C^R + (1-\lambda)\pi_C^S\right](X_S - c_S)}{\lambda\pi_C^R(X_R - X_S)} \tag{1.23}$$

$$D^* = X_S - c_S/(1 - \psi\eta^*) \tag{1.24}$$

Proof: Similar to Proposition 4 (see Figure 1.7b).

To illustrate the role of warrants, we graphically compare the case where the firms can issue a debt-equity combination with the case they issue a debt-warrant combination (see Figures 1.7a and 1.7b). By Lemma 8, if firms can only issue debt and equity, under this distributional assumption, ICF_S lies entirely below ICF_R. Also, because the R-type equity is more valuable, as his proportion in the population of entrepreneurs, λ, decreases the pooling zero-profit line PZP_H becomes steeper and intersects ICF_S at points corresponding to a higher proportion of equity. A necessary



1.7a Debt-Equity

1.7b Debt-Warrant

Figure 1.7

condition for the existence of the efficient pooling equilibrium is that PZP_H both intersects ICF_S and lies below ICF_R (PZP_H is constructed conditional on both types exerting effort). If λ falls below $\overline{\lambda}_1$, PZP_H lies entirely above ICF_S and so it is not relevant (see Figure 1.7a). As a result, the socially efficient pooling equilibrium collapses although the R-type would exert effort even if a higher proportion of equity was issued.

Because the warrant value falls with the exercise price faster for the S-type, as the warrant exercise price rises both ICF_S and PZP_H become steeper but ICF_S becomes so at a higher rate. As a result, for a sufficiently high exercise price, ICF_S and PZP_H meet again and the existence of the socially efficient pooling equilibrium is restored (see Figure 1.7b). That is, a debt-warrant combination allows for the existence of the efficient pooling equilibrium even if it collapses when firms can issue only debt and equity.

Intuitively, in this case, if firms can only issue debt and equity, at any given debt level, the proportion of equity issued consistent with exerting effort is strictly lower for the S-type. That is, the pooling equilibrium where both types exert effort may collapse although the R-type would have exerted effort even if a higher proportion of equity was issued (more subsidy was given to the S-type). Because the warrant value falls with the exercise price (proportionately) faster for the S-type, the S-type is willing to increase faster the proportion of equity offered to the financier than the R-type while still exerting effort. Thus, because in absolute terms the warrant issued by the S-type is less valuable than his equity, the warrant payoff function between X_R and X_S can be steeper than the equity payoff function without violating the S-type effort incentive constraint. This implies that the difference between the value of the warrants issued by the R- and S-type exceeds the corresponding difference of equity values consistent with both types working. This larger difference allows for the provision of the subsidy necessary to induce the S-type to work when the proportion of the R-type is so low that the socially efficient pooling equilibrium breaks if a debt-equity combination is used.

The mechanism at work here relies on the fact that the warrant exercise price can be chosen independently of (and be greater than) the face value of debt. By choosing a sufficiently high exercise price, we can create a sufficiently convex claim which allows us to exploit the difference between the returns of the two types of projects and satisfy the S-type effort incentive constraint under weaker conditions than if warrants were not available. In other words, through the appropriate choice of their exercise price, warrants allow for the implementation of the socially efficient outcome even if this is not possible when we restrict ourselves to debt, equity and/or convertible debt.[50] [51] This mechanism provides another rationale for their use.

1.6 Optimal Financial Contracts under Adverse Selection and Moral Hazard

If both the type and the actions of the entrepreneurs were observable (and verifiable), both types would exert effort if they were offered funds at fair terms. As a result, the net social surplus (social welfare) would be maximised (first best). However, if the choice of the effort level is not observable and one of the two types (the S-type) shirks if he receives funds at fair terms, the implementation of the socially efficient outcome

[50] In our model, convertible debt does not improve on a debt-equity combination. If convertible debt is used , the exercise price of the option coincides with the face value of debt. This implies that if debt is converted into equity, the payment to the shareholders consists only of the project's return. As a result, the maximum proportion of equity offered to the financiers consistent with the S-type working is exactly the same as under a debt-equity combination. In contrast, if a debt-warrant combination is used, the total payment to the shareholders, if the option is exercised, consists of two components: i) the project's return and ii) the difference between the warrant exercise price and the face value of debt (which can be positive). Hence, the maximum proportion of equity issued consistent with the S-type working can be greater than under a debt-equity combination.

[51] If the roles of the two types reverse and so the required cross-subsidisation takes place through the mispricing of debt, a debt-warrant combination does not improve on a debt-equity combination.

requires cross-subsidisation across types. In this section, we address the following question: Can competitive financial markets implement the socially efficient outcome under the same conditions as a benevolent central authority (social planner) who aims at maximising social welfare?

Competitive financiers have no incentive to transfer resources from the R-type to the S-type to induce the latter to exert effort. Therefore, if types are observable or can be credibly revealed, they offer funds only to the R-type and so competitive markets cannot maximise social welfare. In a competitive environment, the implementation of the first-best solution can be achieved only in a pooling equilibrium where the required cross-subsidisation takes place through the mispricing of the R-type's more valuable security (equity). We begin by characterising the social planner's solution (the optimal contract) under adverse selection and effort moral hazard.

1.6.1 The Social Planner's Solution: The Optimal Contract

The social planner's objective is to induce both types to exert effort whenever feasible. Hence, the social planner will offer the S-type the required subsidy even if he can distinguish the two types, provided the R-type effort incentive constraint is not violated. Since the returns of the two types in case of success are different, observable and verifiable, the social planner can ex post distinguish the two types and promise to offer them funds at fair terms. Moreover, he can commit to making direct lump-sum transfers, τ, from the R-type to the S-type so that the S-type effort incentive constraint and the social planner feasibility constraint are just binding, and the R-type effort incentive constraint is not violated. Mathematically,

$$(X_S - I/\pi_C^S - \tau_S) = c_S \tag{1.25}$$

$$(X_R - I/\pi_C^R - \tau_R) \geq c_R \tag{1.26}$$

$$\lambda\pi_C^R\tau_R + (1-\lambda)\pi_C^S\tau_S = 0 \tag{1.27}$$

Solving (1.25) and (1.26) for τ_S and τ_R respectively and substituting into (1.27), we obtain:

$$\lambda \geq \frac{I - \pi_C^S(X_S - c_S)}{\pi_C^R(X_R - c_R) - \pi_C^S(X_S - c_S)} \equiv \lambda^{SP} = \overline{\lambda}_2 \tag{1.28}$$

Where λ^{SP} is the minimum proportion of the R-type (subsidiser) in the population of entrepreneurs consistent with both types exerting effort. In fact, it is the only restriction on the parameter values the social planner faces in his attempt to implement the socially efficient outcome. That is, the optimal contract involves the resolution of the adverse selection problem and lump-sum transfers.

1.6.2 Implementing the Optimal Contract with Debt, Equity and Warrants

Now that we have characterised the optimal contract, we examine its implementation as a competitive equilibrium using financial instruments observed in the real world. By Proposition 6, we know that, if the risky project dominates the safe one by first-order stochastic dominance, for any $\lambda \geq \overline{\lambda}_2 = \lambda^{SP}$ there exists a pooling equilibrium where both types exert effort and receive funds by issuing a debt-warrant combination. That is, the only restriction on parameter values required for the existence of the socially efficient pooling equilibrium is that the social planner also faces. Therefore, under this distributional assumption, debt coupled with a warrant can implement the optimal contract as a competitive equilibrium.

This really strong result relies on two factors: First, the fact that warrants allow for the intersection of the two effort incentive frontiers at some admissible value of the two choice variables, the proportion of equity, η, and the face value of debt. This, in turn, implies that the socially efficient pooling equilibrium exists until the proportion of the R-type becomes so low that it is impossible to satisfy both effort incentive constraints. This is exactly the constraint the social planner faces. Second, the specific distributional assumption which ensures that the socially efficient pooling equilibrium Pareto-dominates any other equilibrium even if both effort incentive constraints are just binding. In other words, the R-type's benefit from inducing the S-type to exert effort through the mispricing of warrants more than offsets the incremental subsidy (relative to the all-debt equilibrium where the S-type shirks) even if the total subsidy is so high that the R-type effort incentive constraint is just binding.

Under any other distributional assumption and/or a debt-equity combination, the existence of the socially efficient pooling equilibrium requires additional restrictions on the parameter values (see Propositions 1B.1 and 1B.2). If the two effort incentive frontiers, ICF_S and ICF_R, intersect at some admissible value of the choice variables (see Lemmas 1 and 9) $\lambda \geq \overline{\lambda}_2 = \lambda^{SP}$ is still a necessary condition for the existence of the socially efficient pooling equilibrium. However, for some $\lambda \geq \overline{\lambda}_2 = \lambda^{SP}$ this equilibrium collapses because the cost for the R-type of providing the S-type with a higher subsidy exceeds the benefit from inducing him to exert effort. The resulting pooling equilibrium involves the issue of just debt (corner solution) and the S-type shirking.

Notice, however, that because the social planner does not face the latter constraint, whenever ICF_S and ICF_R, intersect at some admissible value of the choice variables, he can implement the optimal contract using a debt-equity or a debt-warrant combination.

1.7 Conclusion

In this chapter, we have analysed and discussed the roles of debt, equity and warrants under adverse selection and (effort) moral hazard. Several interesting results were obtained. First, we explained the issue of combinations of debt and equity as the outcome of the interaction between adverse selection and moral hazard. Firms accept to incur the adverse selection cost of issuing equity because this cost is more than offset by the benefit from relaxing the moral hazard constraint. Second, we showed that, in the presence of moral hazard, adverse selection may result in the conversion of a negative into a positive NPV project and an improvement in social welfare. Third, we provided two rationales for the use of warrants. Under pure adverse selection, warrants can serve as separation devices in cases where other standard securities cannot. Under adverse selection cum moral hazard, warrants allow for the implementation of the socially efficient outcome even if this is not possible when we restrict ourselves to debt, equity and/or convertible debt. Finally, we showed that, under certain conditions, a debt-warrant combination can implement the optimal contract as a competitive equilibrium.

Our focus on a two-type model allowed us to illustrate the effects of the interaction between adverse selection and moral hazard and the mechanism that necessitates the use of warrants in the simplest possible way. Under certain conditions, most results should obtain if we extend the model to allow for more than two types. For example, under pure adverse selection, a debt-warrant combination allows for the existence of equilibria (separating, partial separating or pooling) where the securities issued are fairly priced even if there exist three or more types.

Also, the pooling equilibrium of Proposition 4 (Proposition 6) can obtain if a third type is added. What is more, it may exist even if the NPV of the third type project is negative regardless of whether he exerts effort or not. However, in such a case, the welfare properties of the pooling equilibrium are different. Adverse selection results in overinvestment and possibly in a fall in social welfare. This latter result does not depend on the number of types, it may obtain even with two types if the project of one of them has negative NPV regardless of the effort level.

Another natural extension of the model is to allow for more than two effort levels (possibly a continuum) and check the robustness of the results in the adverse selection cum moral hazard case. We conjecture that under certain distributional assumptions and restrictions on the cost and probability functions, a pooling equilibrium similar to those described in Propositions 4 and 6 should obtain. However, the combination of financial contracts required for its existence as well as the implementation of the optimal contract in this case are interesting open questions.

Appendix 1A: Proofs of Lemmas 1-3 and 7-10

Proof of Lemma 1

a) By totally differentiating (1.4), we obtain:

$$-(X_i - D)d\alpha - (1-\alpha)dD = 0 \quad \Rightarrow \quad \left(\frac{d\alpha}{dD}\right)_{ICF_i} = -\frac{1-\alpha}{X_i - D} < 0$$

Taking into account that ICF_i implicitly defines α as a function of D, we obtain:

$$\left(\frac{d^2\alpha}{dD^2}\right)_{ICF_i} = -\frac{1-\alpha}{(X_i - D)^2} + \frac{\frac{d\alpha}{dD}}{X_i - D} = -\frac{2(1-\alpha)}{(X_i - D)^2} < 0$$

Hence, ICF_i is downward sloping and strictly concave. Also, since $X_R > X_S$, ICF_S is steeper than ICF_R.

b) The effort incentive frontiers of the R- and S-type are respectively:

$$(1-\alpha)(X_R - D) = c_R \tag{1A.1}$$

$$(1-\alpha)(X_S - D) = c_S \tag{1A.2}$$

Using (1A.1) and (1A.2) and solving for α and D we obtain:

$$\overline{\alpha} = \frac{(X_R - c_R) - (X_S - c_S)}{X_R - X_S}, \quad \overline{D} = \frac{X_S c_R - X_R c_S}{c_R - c_S} \tag{1A.3}$$

So, $\quad \overline{\alpha} \geq 0 \;\Leftrightarrow\; X_R - c_R \geq X_S - c_S$

$\quad\quad \overline{\alpha} \leq 1 \;\Leftrightarrow\; c_R \geq c_S$

Also, $\overline{D} \geq 0 \;\Leftrightarrow\; c_R/X_R \geq c_S/X_S \;\Rightarrow\; c_R > c_S$

Therefore, $1 \geq \overline{\alpha} \geq 0$ and $\overline{D} \geq 0 \;\Leftrightarrow\; X_R - c_R \geq X_S - c_S$ and $c_R/X_R \geq c_S/X_S$.

If $X_R - c_R < X_S - c_S$, (the intersection of ICF_S with the horizontal axis lies to the right of that of ICF_R), then because ICF_S is steeper than ICF_R, at any $0 \leq \alpha \leq 1$, ICF_S lies entirely above ICF_R in the (α, D) space. That is, $IC_R \subset IC_S$.
If $c_R/X_R < c_S/X_S$, (the intersection of ICF_S with the vertical axis lies below that of ICF_R), then because ICF_S is steeper than ICF_R, at any $0 \leq \alpha \leq 1$, ICF_S lies entirely below ICF_R in the (α, D) space. That is, $IC_S \subset IC_R$.

c) ICF_i meets the vertical axis at $\alpha_i = 1 - c_i/X_i$ and the horizontal axis at $D_i = X_i - c_i$. By Assumption 1, $X_i > c_i$ and $1 > c_i/X_i$. Also, by Part (a) of this Lemma, ICF_i is downward sloping and strictly concave. Therefore, IC_i cannot be empty (See Figure 1.1). *Q.E.D.*

Proof of Lemma 2

a) For any $0 \leq \alpha \leq 1$, $0 \leq D \leq R_i$, Eq. (1.1) becomes:

$$U_i = \pi(B_i)(1-\alpha)(X_i - D) + B_i, \qquad i = R, S \tag{1A.4}$$

Differentiating (1A.4), we obtain:

$$\left(\frac{d\alpha}{dD}\right)_{u_i = \bar{u}} = -\frac{1-\alpha}{X_i - D} \leq 0$$

$u_i = u$ implicitly defines α as a function of D and so:

$$\left(\frac{d^2\alpha}{dD^2}\right)_{u_i=\bar{u}} = -\frac{2(1-\alpha)}{(X_i - D)^2} \leq 0$$

Hence, the indifference curves of both the R- and the S-type are downward sloping and concave.

b) Since $X_R > X_S$, at any (α, D) pair, u_R is flatter than u_S and hence they cross only once. *Q.E.D.*

Proof of Lemma 3

a) The equations for ZP_i and PZP_H are respectively:

$$\pi_C^i\left[\alpha(X_i - D) + D\right] = I, \qquad i = R, S \tag{1A.5}$$

$$\lambda\pi_C^R\left[\alpha(X_R - D) + D\right] + (1-\lambda)\pi_C^S\left[\alpha(X_S - D) + D\right] = I \tag{1A.6}$$

Differentiating (1A.5) and (1A.6) we obtain the slopes of ZP_i and PZP_H respectively. Since $X_R > X_S$ and $0 \leq \lambda \leq 1$, it is obvious that at any given (α, D) pair,

$$\left|\left(\frac{d\alpha}{dD}\right)_{ZP_S}\right| \geq \left|\left(\frac{d\alpha}{dD}\right)_{PZP_H}\right| \geq \left|\left(\frac{d\alpha}{dD}\right)_{ZP_R}\right|$$

b) By Lemmas 1, 2, and 3

$$\left(\frac{d\alpha}{dD}\right)_{ICF_i} = \left(\frac{d\alpha}{dD}\right)_{u_i=\bar{u}} = \left(\frac{d\alpha}{dD}\right)_{ZP_i} = -\frac{1-\alpha}{X_i - D} < 0, \qquad i = R, S \tag{1A.7}$$

Hence, ICF_i, u_i, ZP_i ($i = R, S$) never intersect. *Q.E.D.*

Proof of Lemma 7

Using (1A.5) and solving for α and D, we obtain the values of α and D where ZP_R and ZP_S intersect in the (α, D) space.

$$\hat{\alpha} = \frac{I(1/\pi_C^R - 1/\pi_C^S)}{X_R - X_S}, \quad \hat{D} = \frac{\pi_C^R X_R - \pi_C^S X_S}{(X_R - X_S)\pi_C^R \pi_C^S / I - (\pi_C^S - \pi_C^R)} \tag{1A.8}$$

Notice that $X_R > X_S$ and $\pi_C^S \geq \pi_C^R$ imply $\hat{\alpha} \geq 0$. Also, $\hat{\alpha} \leq 1 \Leftrightarrow (X_R - X_S)\pi_C^R \pi_C^S / I \geq \pi_C^S - \pi_C^R$. Given $X_R > X_S$, this condition may be violated only under mean-reducing spreads.

That is, in the range of parameters where $\hat{\alpha}$ takes on an admissible value, the denominator of the equation for $\hat{D}$ is positive. Hence, $D \geq 0 \Leftrightarrow \pi_C^R X_R \geq \pi_C^S X_S$. More analytically,

a) If the risky project dominates the safe one by first-order stochastic dominance ($\pi_C^R = \pi_C^S = \pi_C$, $X_R > X_S$), then $\hat{\alpha} = 0$, $\hat{D} = I/\pi_C > 0$. Also, ZP_R is flatter than ZP_S. Hence, for $\alpha > 0$ ZP_R lies below ZP_S in the (α, D) space.
b) If the risky project is a mean-preserving spread of the safe one ($\pi_C^R X_R = \pi_C^S X_S$), then $\hat{\alpha} = I/\pi_C^R X_R$, $\hat{D} = 0$. Hence, since ZP_R is flatter than ZP_S, for $\hat{D} > 0$ ZP_S lies below ZP_R in the (α, D) space.
c) If the risky project is a mean-increasing spread of the safe one ($\pi_C^R X_R > \pi_C^S X_S$), then $\hat{\alpha} > 0$, $\hat{D} > 0$.
d) If the risky project is a mean-reducing spread of the safe one ($\pi_C^R X_R < \pi_C^S X_S$), then $\hat{\alpha} > 0$ but $\hat{D} < 0$. Hence, since ZP_R is flatter than ZP_S, for $D \geq 0$ ZP_S lies below ZP_R in the (α, D) space. That is, ZP_S and ZP_R do not intersect at any admissible value of the two choice variables. *Q.E.D.*

Proof of Lemma 8

a) In this case, $\pi_j^R = \pi_j^S$, $j = C,0$. Hence, $c_R \equiv \dfrac{C}{\pi_C^R - \pi_0^R} = \dfrac{C}{\pi_C^S - \pi_0^S} \equiv c_S$. Using (1A.3) we obtain: $\overline{\alpha} = 1$, $\overline{D} = -\infty$. Also, since ICF_S is steeper than ICF_R, $IC_S \subset IC_R$.

b) Here, $\pi_j^R X_R = \pi_j^S X_S$, $j = C,0$. Hence, $c_S X_R = c_R X_S$. Then, (1A.3) implies $0 < \overline{\alpha} < 1$, $\overline{D} = 0$. Also, since ICF_S is steeper than ICF_R, $IC_S \subset IC_R$. *Q.E.D.*

Proof of Lemma 9

The equations of the effort incentive frontiers ICF_S and ICF_R are given respectively by:

$$(1-\psi\eta)(X_S - D) = c_S \tag{1A.9}$$

$$(1-\psi\eta)(X_R - D) - \eta(1-\psi)(X_R - X_S) = c_R \tag{1A.10}$$

a) By totally differentiating (1A.9) and (1A.10), we obtain the slopes of ICF_S and ICF_R respectively (the equations are provided in the text). Since $X_R > X_S$, at any given (η, D) pair, ICF_S is steeper than ICF_R.

b) Solving (1A.9) and (1A.10) for η and D, we obtain:

$$\overline{\eta} = \frac{(X_R - c_R) - (X_S - c_S)}{X_R - X_S}$$

$$\overline{D} = \frac{(X_S - c_S)[X_R - (1-\psi)X_S] - \psi(X_R - c_R)X_S}{(1-\psi)(X_R - X_S) + \psi(c_R - c_S)} \tag{1A.11}$$

So, $\overline{\eta} \geq 0 \iff X_R - c_R \geq X_S - c_S$

$\overline{\eta} \leq 1 \iff c_R \geq c_S$

Notice that $\overline{\eta}$ is independent of ψ. Also, for $\psi = 1$ the expression for $\overline{D}$ in (1A.11) becomes identical to that in (1A.3). Moreover, for any admissible value of $\overline{\eta}$, the denominator in the expression for $\overline{D}$ is positive. Hence,

$$\overline{D} \geq 0 \quad \Leftrightarrow \quad (X_S - c_S)[X_R - (1-\psi)X_S] \geq \psi(X_R - c_R)X_S \quad \Leftrightarrow$$

$$\psi \leq \frac{(X_R - X_S)(1 - c_S/X_S)}{(X_R - c_R) - (X_S - c_S)} \equiv \overline{\psi} \qquad \text{(1A.12)}$$

Therefore, if $X_R - c_R \geq X_S - c_S$ and $c_R \geq c_S$, then for any $\psi \in [0, \min[\overline{\psi}, 1]]$ $1 \geq \overline{\eta} \geq 0$ and $\overline{D} \geq 0$.

If $X_R - c_R < X_S - c_S$, (the intersection of ICF_S with the horizontal axis lies to the right of that of ICF_R), then because ICF_S is steeper than ICF_R, at any $0 \leq \eta \leq 1$, ICF_S lies entirely above ICF_R in the (η, D) space. That is, $IC_R \subset IC_S$.

If $c_R < c_S$, the intersection of ICF_S and ICF_R occurs at some $\eta > 1$ regardless of the value of ψ. Hence, because ICF_S is steeper than ICF_R, at any $0 \leq \eta \leq 1$, ICF_S lies entirely below ICF_R in the (η, D) space. That is, $IC_S \subset IC_R$.

c) Setting the utility (Eq. (1.14) in the text) of an E of type i equal to a constant and differentiating, we obtain the slopes of the indifference curves which are identical to the corresponding slopes of the effort incentive frontiers. *Q.E.D.*

Proof of Lemma 10

The equations of the zero-profit lines ZP_S and ZP_R are respectively:

$$\pi_C^S[\eta\psi(X_S - D) + D] = I \qquad \text{(1A.13)}$$

$$\pi_C^R\{\eta[\psi(X_S - D) + (X_R - X_S)] + D\} = I \qquad \text{(1A.14)}$$

a) By totally differentiating (1A.13) and (1A.14), we obtain the slopes of ZP_S and ZP_R respectively (the equations are provided in the text). Since $X_R > X_S$, at any given (η, D) pair, ZP_S is steeper than ZP_R. Also, since $0 \le \lambda \le 1$,

$$\left|\left(\frac{d\alpha}{dD}\right)_{ZP_S}\right| \ge \left|\left(\frac{d\alpha}{dD}\right)_{PZP_H}\right| \ge \left|\left(\frac{d\alpha}{dD}\right)_{ZP_R}\right|$$

b) Solving (1A.13) and (1A.14) for η and D, we obtain:

$$\hat{\alpha} = \frac{I(1/\pi_C^R - 1/\pi_C^S)}{X_R - X_S}, \quad \hat{D} = \frac{\pi_C^R(X_R - X_S) - \psi(\pi_C^S - \pi_C^R)X_S}{(X_R - X_S)\pi_C^R\pi_C^S/I - \psi(\pi_C^S - \pi_C^R)} \tag{1A.15}$$

Notice that $X_R > X_S$ and $\pi_C^S \ge \pi_C^R$ imply $\hat{\alpha} \ge 0$. Also, $\hat{\alpha} \le 1 \Leftrightarrow X_R - I/\pi_C^R \ge X_S - I/\pi_C^S$. Given $X_R > X_S$, this condition may be violated only under mean-reducing spreads. Also, in the range of parameters where $\hat{\alpha}$ takes on an admissible value, the denominator of the equation for $\hat{D}$ is positive. Hence,

$$\hat{D} \ge 0 \quad \Leftrightarrow \quad \psi \le \frac{\pi_C^R(X_R - X_S)}{(\pi_C^S - \pi_C^R)X_S} \equiv \hat{\psi} \tag{1A.16}$$

Therefore, if $X_R - I/\pi_C^R \ge X_S - I/\pi_C^S$, for any $\psi \in [0, \min[\hat{\psi}, 1]]$ ZP_S and ZP_R intersect at some admissible value of the two choice variables $(0 \le \eta \le 1, D \ge 0)$ under all four assumptions about the ordering of the distributions of returns.

c) By Lemmas 9 and 10,

$$\left(\frac{d\eta}{dD}\right)_{ICF_i} = \left(\frac{d\eta}{dD}\right)_{u_i = \bar{u}} = \left(\frac{d\eta}{dD}\right)_{ZP_i}, \qquad i = R, S, \tag{1A.17}$$

Hence, ICF_i, u_i, ZP_i ($i = R, S$) never intersect. *Q.E.D.*

Appendix 1B: Generalisation of Propositions 4 and 6

Proposition 1B.1 (Generalisation of Proposition 4): Suppose the following are true $\pi_C^R X_R - I > \pi_C^R c_R$, $\pi_C^S X_S - I < \pi_C^S c_S$, and $I/\pi_C^R X_R < 1 - c_S/X_S$. Then there exists a unique pooling (funding) equilibrium where both types choose the high effort level and obtain funds by issuing both debt and equity if either

a) $IC_S \subset IC_R$, $\lambda > \overline{\lambda}_1$ and $\pi_C^S/\pi_0^S \geq X_R/X_S$ (it is possible if the risky projects dominate the safe ones by first-order stochastic dominance or they are mean-increasing spreads) or

b) $X_R - c_R > X_S - c_S$, $c_R/X_R > c_S/X_S$ (ICF_R and ICF_S intersect), $\lambda > \overline{\lambda}_2$ and $\pi_C^S/\pi_0^S \geq (X_R - c_R)/(X_S - c_S)$ (it's possible only under mean-increasing spreads).

where $\overline{\lambda}_1 \equiv \dfrac{I - \pi_C^S(X_S - c_S)}{(\pi_C^R X_R - \pi_C^S X_S)(1 - c_S/X_S)}$, $\quad \overline{\lambda}_2 \equiv \dfrac{I - \pi_C^S(X_S - c_S)}{\pi_C^R(X_R - c_R) - \pi_C^S(X_S - c_S)}$

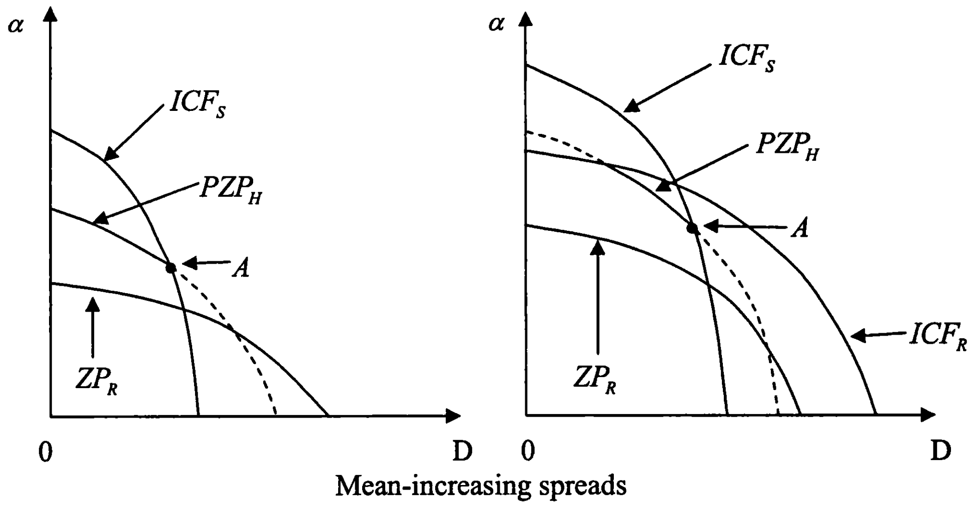
The equilibrium contract, $A = (\alpha^*, D^*)$, lies at the intersection of ICF_S and PZP_H with α^* and D^* given by:

$$\alpha^* = \frac{I - \left[\lambda\pi_C^R + (1-\lambda)\pi_C^S\right]\left(X_S - c_S\right)}{\lambda\pi_C^R(X_R - X_S)} \tag{1B.1}$$

$$D^* = X_S - c_S/(1 - \alpha^*) \tag{1B.2}$$

Proof:

The pooling equilibria described in this proposition exist if the following two conditions are satisfied: i) PZP_H belongs to the intersection of IC_S and IC_R for $\lambda \leq 1$ and ii) the R-type indifference curve through the equilibrium contract, u_R^A, does not intersect PZP_L.



1B.1a $IC_S \subset IC_R$ **1B.1b** ICF_R and ICF_S intersect

Figure 1B.1

a) In this case, since $IC_S \subset IC_R$ the first condition is satisfied if PZP_H crosses ICF_S. Provided ZP_R intersects ICF_S ($I/\pi_C^R X_R < 1 - c_S/X_S$), by Figures 5 and 1B.1, it is clear that PZP_H crosses ICF_S if the intersection point of PZP_H with the vertical axis lies below that of ICF_S. That is, if

$$1 - \frac{c_S}{X_S} > \frac{I}{\lambda \pi_C^R X_R + (1-\lambda)\pi_C^S X_S} \quad \Leftrightarrow \quad \lambda > \frac{I - \pi_C^S (X_S - c_S)}{(\pi_C^R X_R - \pi_C^S X_S)(1 - c_S/X_S)} \equiv \bar{\lambda}_1$$

$I - \pi_C^S (X_S - c_S)$: Minimum subsidy required to induce the S-type to exert effort.

$\pi_C^R X_R - \pi_C^S X_S$: Expected return differential (given the high effort level is chosen).

$1 - c_S/X_S$: Maximum $\alpha \in IC_S$

Regarding the second condition, since $X_R > X_S$ and $0 \le \lambda \le 1$, at any given (α, D) pair, u_R^A is flatter than PZP_L. Therefore, it suffices to show that the intersection point of u_R^A with the horizontal axis lies to the left of that of PZP_L.

The intersection point of PZP_L with the horizontal axis is given by:

$$D = \frac{I}{\lambda \pi_C^R + (1-\lambda)\pi_0^S} \tag{1B.3}$$

Moreover, the expected utility of the R-type in equilibrium is given by:

$$U_R^* = (1-\alpha^*)\pi_C^R (X_R - D^*) + b \tag{1B.4}$$

At $\alpha = 0$, the R-type's expected utility is:

$$(U_R)_{\alpha=0} = \pi_C^R (X_R - D) + b \tag{1B.5}$$

Setting $U_R^* = (U_R)_{\alpha=0}$ and using the expressions for α^* and D^*, we obtain:

$$D = \frac{I - (1-\lambda)\pi_C^S (X_S - c_S)}{\lambda \pi_C^R} \tag{1B.6}$$

Hence, the second condition is satisfied if:

$$\frac{I}{\lambda \pi_C^R + (1-\lambda)\pi_0^S} \geq \frac{I - (1-\lambda)\pi_C^S (X_S - c_S)}{\lambda \pi_C^R} \tag{1B.7}$$

Let $f(\lambda) = \dfrac{I - (1-\lambda)\pi_C^S (X_S - c_S)}{\lambda \pi_C^R}$

and $g(\lambda) = \dfrac{I}{\lambda \pi_C^R + (1-\lambda)\pi_0^S}$

then $f'(\lambda) = -\dfrac{1}{\lambda^2 \pi_C^R}\left[I - \pi_C^S (X_S - c_S)\right] < 0$, $f''(\lambda) = \dfrac{2}{\lambda^3 \pi_C^R}\left[I - \pi_C^S (X_S - c_S)\right] > 0$

Since, by assumption, $I - \pi_C^S(X_S - c_S) > 0$.

Also, $g'(\lambda) = -\frac{(\pi_C^R - \pi_0^S)I}{[\lambda\pi_C^R + (1-\lambda)\pi_0^S]^2} < 0$, $\quad g''(\lambda) = \frac{(\pi_C^R - \pi_0^S)^2 I}{[\lambda\pi_C^R + (1-\lambda)\pi_0^S]^3} > 0$

Assuming $\pi_C^R > \pi_0^S$, both $f(\lambda)$ and $g(\lambda)$ are strictly decreasing and strictly convex.

Furthrermore, $f(\lambda) \le g(\lambda) \;\Rightarrow\; \lambda \le 1$ and $\lambda \ge \frac{I - \pi_C^S(X_S - c_S)}{(\pi_C^R - \pi_0^S)\frac{\pi_C^S}{\pi_0^S}(X_S - c_S)} \equiv \tilde{\lambda}$

Since $0 \le \lambda \le 1$, both $f(\lambda)$ and $g(\lambda)$ are continuous, strictly decreasing and strictly convex, $f(\lambda) \le g(\lambda)$ for $\lambda \ge \tilde{\lambda}$ and $f(\lambda) > g(\lambda)$ for $\lambda < \tilde{\lambda}$, then $f(\lambda) \le g(\lambda)$ for all $\lambda \in [\tilde{\lambda}, 1]$. Therefore, u_R^A does not cut PZP_L for any $\lambda \in [\bar{\lambda}_1, 1]$ if and only if:

$$\bar{\lambda}_1 \ge \tilde{\lambda} \quad \Leftrightarrow \quad \pi_0^S X_R \le \pi_C^S X_S \tag{1B.8}$$

Notice that if the risky projects dominate the safe ones by first-order stochastic dominance $(\pi_j^R = \pi_j^S)$, $j = C, 0$, this condition is automatically satisfied (by Assumption 1). However, under mean-increasing spreads it may be violated. In such a case, $\bar{\lambda}_1 < \tilde{\lambda}$ and hence the socially efficient pooling equilibrium exists only if $\lambda \ge \tilde{\lambda} > \bar{\lambda}_1$.

b) In this case, since PZP_H is flatter than ICF_S and steeper than ICF_R, the first condition is satisfied if

$$\bar{\alpha} \ge \alpha^* \;\Leftrightarrow\; \lambda \ge \frac{I - \pi_C^S(X_S - c_S)}{\pi_C^R(X_R - c_R) - \pi_C^S(X_S - c_S)} \equiv \bar{\lambda}_2 \tag{1B.9}$$

Repeating the steps in Part (a), one can show that, for any $\lambda \in [\bar{\lambda}_2, 1]$, the second condition is satisfied if and only if

$$\frac{\pi_C^S}{\pi_0^S} \geq \frac{X_R - c_R}{X_S - c_S} \tag{1B.10}$$

Q.E.D.

Proposition 1B.2 (Generalisation of Proposition 6): Suppose the following are true $\pi_C^R X_R - I > \pi_C^R c_R$, $\pi_C^S X_S - I < \pi_C^S c_S$ and $X_R - c_R > X_S - c_S$, $c_R \geq c_S$. Then, for any $\psi \in [0, \min[\overline{\psi}, 1]]$, $\lambda \geq \overline{\lambda}_2$ and $\pi_C^S / \pi_0^S \geq (X_R - c_R)/(X_S - c_S)$, then there exists a unique pooling (funding) equilibrium where both types exert effort and obtain funds by issuing a debt-warrant combination (it is possible if the risky projects dominate the safe ones by first-order stochastic dominance or they are mean-increasing spreads).

where $\overline{\lambda}_2 \equiv \dfrac{I - \pi_C^S (X_S - c_S)}{\pi_C^R (X_R - c_R) - \pi_C^S (X_S - c_S)}$

The equilibrium contract, $A = (\eta^*, D^*)$, lies at the intersection of ICF_S and PZP_H with η^* and D^* given by:

$$\eta^* = \frac{I - \left[\lambda \pi_C^R + (1-\lambda)\pi_C^S\right](X_S - c_S)}{\lambda \pi_C^R (X_R - X_S)} \tag{1B.11}$$

$$D^* = X_S - c_S / (1 - \psi \eta^*) \tag{1B.12}$$

Proof: Similar to Proposition 1B.1.

Notice that if the risky projects dominate the safe ones by first-order stochastic dominance we have $X_R > X_S$, $\pi_j^R = \pi_j^S = \pi_j$, $j = C, 0$. This implies:

- $c_R = c_S = c$

- $X_R - c_R > X_S - c_S$
- $\pi_C^S / \pi_0^S \geq (X_R - c_R)/(X_S - c_S) \quad \Leftrightarrow \quad \pi_C^S X_S - \pi_0^R X_R \geq C$ (always true by Assumption 1).

Hence, all the conditions, except for $\lambda \geq \overline{\lambda}_2 = \lambda^{SP}$, required for the existence of the socially efficient pooling equilibrium are automatically satisfied. The remaining condition ($\lambda \geq \overline{\lambda}_2 = \lambda^{SP}$) is identical to that the social planner faces.

Appendix 1C: Separate Bond and Equity (Warrant) Markets

The analysis in the text assumed that the required amount of funds I is provided by the same financier who purchases both debt and equity (warrant). In this appendix, I show that all the results go through even if the buyer of debt and the buyer of equity (warrant) are different (debt and equity (warrant) markets are separate). It suffices to show that the zero-profit lines of an equity-buyer (a warrant-buyer), a bond-buyer and a financier purchasing both debt and equity (warrant) coincide. The following assumptions are made:

i) The project is indivisible.
ii) Es have no storage technology (cannot lend) and the consumption good is perishable.
iii) Bond and equity (warrant) markets are perfectly competitive.

The first assumption implies that the Es borrow at least I. The second implies that no E will borrow more than I. Therefore, Es borrow just I. Given these three assumptions, we have:

$$I_B + I_E = I \tag{1C.1}$$

$$P_{BF} = \left[\lambda \pi_j^R + (1-\lambda)\pi_k^S\right]D - I_B = 0, \quad j = C,0, \quad k = C,0, \tag{1C.2}$$

$$P_{EF} = \alpha\left[\lambda \pi_j^R (X_R - D) + (1-\lambda)\pi_k^S (X_S - D)\right] - I_E = 0 \tag{1C.3}$$

where I_B : Amount the E borrows from the bond-financier

I_E : Amount the E borrows from the equity-financier

P_{BF} : Expected profit of the bond-financier

P_{EF} : Expected profit of the equity-financier

P_F : Expected profit of a financier purchasing both debt and equity

Using (1C.1), (1C.2) and (1C.3) we obtain:

$$\lambda\left[\alpha\pi_j^R(X_R - D) + \pi_j^R D\right] + (1-\lambda)\left[\alpha\pi_k^S(X_S - D) + \pi_k^S D\right] - I$$

$$= P_{EF} = P_{BF} = P_F = 0 \qquad (1C.4)$$

That is, the zero-profit lines of an equity-financier, a bond-financier and a financier purchasing both debt and equity coincide. Therefore, the pooling equilibrium of Proposition 4 obtains regardless of whether the same investor purchases both debt and equity and provides the required amount I or the debt-financier and the equity-financier are different (bond and equity markets are separate).

Similar results can be derived for the individual zero-profit lines. Also, all the results go through if debt and warrants are issued instead of debt and equity.

Separate Bond and Warrant Markets

The expected returns of the warrants issued by the R- and S-type are respectively (using Eq. (1.13) in the text):

$$\eta\pi_j^R\left[\psi(X_S - D) + (X_R - X_S)\right], \qquad j = C,0 \qquad (1C.5)$$

$$\eta\pi_k^S\psi(X_S - D), \qquad k = C,0 \qquad (1C.6)$$

Using Assumptions (i)-(iii) and (1C.5), (1C.6), we have:

$$I_B + I_W = I \tag{1C.7}$$

$$P_{BF} = \left[\lambda\pi_j^R + (1-\lambda)\pi_k^S\right]D - I_B = 0, \quad j = C,0, \quad k = C,0 \tag{1C.8}$$

$$P_{WF} = \eta\left\{\lambda\pi_j^R\left[\psi(X_S - D) + (X_R - X_S)\right] + (1-\lambda)\pi_k^S\psi(X_S - D)\right\} - I_W = 0 \tag{1C.9}$$

where I_B : Amount the E borrows from the bond-financier

I_W : Amount the E borrows from the warrant-financier

P_{BF} : Expected profit of the bond-financier

P_{WF} : Expected profit of the warrant-financier

P_F : Expected profit of a financier purchasing both debt and warrants

Using (1C.7), (1C.8) and (1C.9), we obtain:

$$\lambda\pi_j^R\{\eta[\psi(X_S - D) + (X_R - X_S)] + D\} + (1-\lambda)\pi_k^S[\eta\psi(X_S - D) + D] - I$$

$$= P_F = P_{BF} = P_{WF} = 0 \tag{1C.10}$$

That is, the zero-profit lines of a warrant-financier, a bond-financier and a financier purchasing both debt and warrants coincide. Therefore, the pooling equilibrium of Proposition 6 obtains regardless of whether the same investor purchases both debt and the warrant and provides the required amount I or the debt-financier and the warrant-financier are different (bond and warrant markets are separate).

The above results show that although the pooling equilibria of Propositions 4 and 6 involve cross-subsidisation across types, they do not involve cross-subsidisation across assets (debt and equity or debt and warrants respectively). That is, in equilibrium, the securities issued are fairly priced collectively (because of perfect competition). Therefore, a financier will just break even regardless of whether he holds one of the securities issued or a combination of them.

Chapter 2

Optimism and Insurance under Asymmetric Information: Positive and Welfare Implications

2.1 Introduction

More than two centuries ago in *The Wealth of Nations* Adam Smith argued that "The chance of gain is by every man more or less over-valued and the chance of loss is by most men undervalued" (Smith (1776) Book I, Chapter X). Several recent empirical studies both by psychologists and economists validate his claim. They find that the majority of people tend to be overoptimistic about their ability and the outcome of their actions and underestimate the probability of various risks.[52] For example, Svenson (1981) finds that 90 percent of the automobile drivers in Sweden consider themselves "above average". Similar results are reported by Rutter, Quine and Alberry (1998) for motorcyclists in Britain. On average, motorcyclists both perceive themselves to be less at risk than other motorcyclists and underestimate their absolute accident probability.[53]

A large number of papers have investigated the implications of overconfidence and unrealistic optimism in securities markets and firm financing.[54] In contrast, research concerning insurance markets has almost entirely been conducted in the context of the standard asymmetric information framework. Insurees know their true accident probability but insurance companies cannot observe the type and/or the actions of the insuree.[55]

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[52] However, there is some evidence that people overestimate their accident probability when it is objectively very small (Kahneman and Tversky, (1979)). Also, Viscusi (1990) finds that more individuals overestimate the risk of lung-cancer associated with smoking than underestimate it and on average they greatly overestimate it. However, the analysis for the case of pessimism is similar and is omitted.

[53] See Weinstein and Klein (1996) for a survey.

[54] See De Bondt and Thaler (1995) for a survey.

[55] A notable exception is Villeneuve (2000). He assumes that the insurance company (monopoly) knows better the insuree's accident probability than the insuree himself. The insurer makes a personal

In line with the empirical evidence, this chapter drops the assumption that all insurees have an accurate estimate of their accident probability. It assumes that some agents, the optimists, underestimate it and explores the implications both for the optimists themselves and their realistic counterparts in the context of an otherwise standard competitive asymmetric information framework. More specifically, both the optimists (henceforth Os) and the realists (henceforth Rs) are risk averse and have the same utility function but differ with respect to their perception of the accident probability. All agents can affect their true accident probability by undertaking preventive activities. A higher precautionary effort level implies both a lower accident probability and a higher utility cost (moral hazard). Except for their misperception of the accident probability, the Os are rational agents who aim at maximising their (perceived) utility and understand the nature and implications of market interactions.

The first question we seek to address is under what conditions the presence of the Os affects the choices of the Rs and vice versa? It is shown that if the degree of optimism is sufficiently high there exist separating equilibria where the Os not only take fewer precautions (high-risk type) but also purchase less insurance than the Rs and both types choose the contract they would have chosen if types were observable. That is, because the Os considerably underestimate their accident probability, their presence has no effect on the choices of the Rs. For lower levels of optimism, depending on whether the Os are more or less willing to take precautions,[56] either the Os or the Rs are quantity-constrained.[57] If optimism encourages precautionary effort, the Os themselves are quantity-constrained whereas the Rs make the same choices as under full information about types. If the Os put less effort into reducing their risk exposure, the roles of the two types are reversed.

Contrary to the conventional wisdom, optimism itself does not necessarily lead to the purchase of less insurance. If types are observable and optimism encourages precautionary effort, the effect of the lower per unit price may more than offset the effect of the underestimation of the accident probability and result in the Os purchasing more insurance than the Rs. However, if types are hidden, the presence of

offer conveying information to the insuree. The insuree interprets the signal and can accept or reject the offer.

[56] Although, the Os underestimate their accident probability, they may either overestimate or underestimate its decrease from taking precautions and so be more or less willing to take precautions.

[57] If a type is "quantity-constrained" in equilibrium, it means that he purchases less insurance than he would have purchased under full information about types.

the Rs makes this choice infeasible. The amount of insurance offered at the low per unit price is restricted by the Rs' revelation and effort incentive constraints.

The second part of the chapter deals with the welfare properties of the laissez-faire equilibria described above. Because some of the insurees, the Os, underestimate their accident probability, the definition of the efficiency of the equilibrium is not straightforward. The very presence of the Os raises the question of what is the appropriate efficiency criterion. Should we employ objective probabilities (true expected utility) or subjective probabilities (perceived expected utility)? The answer depends crucially on the origin of the agents' biased estimate. In our environment, the different estimates of the same risk arise because of different perceptions not because of different underlying preferences. Both the Os and the Rs have identical preferences. Hence, the preferences revealed by the insuree's choices coincide with the true underlying preferences. Therefore, the appropriate efficiency criterion seems to be objective rather than subjective probabilities.

Given this criterion, it is possible to find intervention policies that yield strict Pareto gains. If the Rs are quantity-constrained, then a tax on insurance purchase would result in the Os going uninsured, relax their revelation constraint and potentially lead to a strict Pareto gain. In contrast, if the Os are quantity-constrained, this logic does not apply. Any attempt to drive out the Rs so as to mitigate the negative externality their presence creates would first drive out the Os. Thus, it would be harmful for the Rs who would pay the tax without gaining anything.

However, if the proportion of the Os is sufficiently high, an intervention scheme involving a combination of minimum coverage requirements, taxes and subsidies would lead to a strict Pareto improvement. In the resulting pooling equilibrium the Os subsidise the Rs but purchase more insurance and both types are strictly better off. Because the proportion of the Os is high, the improvement in their true welfare from the higher coverage more than offsets the welfare losses due to the higher per unit premium. If neither type is quantity-constrained in the laissez-faire equilibrium, the latter policy can result in the Os purchasing more insurance, at the same per unit price, while the Rs being unaffected. Because the Os were underinsured, they become better off and so a strict Pareto improvement is achieved.

These results provide a justification for the imposition of minimum coverage requirements in insurance markets. However, the imposition of minimum standards only may not achieve the desired outcome. Because the Os underestimate their accident probability, their perceived utility in the pooling equilibrium may be lower than at the allocation without insurance. As a result, they may go uninsured and so be worse off than in the laissez-faire equilibrium. In contrast, a combination of minimum standards, taxes and subsidies would result in the Os purchasing the pooling contract and so being strictly better off. In fact, Finkelstein (2002) finds that the imposition of minimum standards in the US private health insurance market resulted in a decline in the proportion of people with coverage of about 25 percent. Our results suggest that in order for minimum coverage requirements to achieve their objective, they should be accompanied by a mix of taxes and subsidies.

Finally, intervention schemes involving minimum coverage requirements can be used to create a pure-strategy Nash equilibrium when otherwise none would exist. The imposition of the minimum standards renders contracts involving less coverage unattractive and so the equilibrium sustains.

Most obviously, our model is closely related to the standard competitive asymmetric-information models of insurance markets. Following the seminal Rothschild-Stiglitz paper (1976), a huge literature has developed including models of pure moral hazard (e.g. Arnott and Stiglitz (1988)) as well as models of adverse selection cum moral hazard (e.g. Stewart (1994), Chassagnon and Chiappori (1997), de Meza and Webb (2001) and Chiappori et.al. (2002)). Despite the large number of characteristics our model shares with these models, there are important differences in their predictions.

First, if, in equilibrium, all agents purchase strictly positive coverage, standard models predict a positive relationship between coverage and the (average) ex post risk of the buyer of the contract. On the contrary, if optimism discourages precautionary effort, in our framework there exist separating equilibria where the Os not only take fewer precautions and so have a higher accident probability but also purchase less insurance at a higher per unit price. Thus, our approach can simultaneously explain both puzzling empirical findings of Cawley and Philipson (1999): i) that insurance premiums display quantity discounts and ii) the negative correlation between coverage and risk, while standard models cannot.[58]

Second, although the positive results of the imposition of minimum coverage requirements in standard asymmetric information models are similar to ours, the

[58] See Chapter 3 for a extensive treatment of this issue.

welfare results are quite different. In the standard model, social welfare may be higher in the resulting pooling equilibrium but the safe type (the quantity-constrained) is strictly worse off. In contrast, in our model, both types are strictly better off in the pooling equilibrium arising after the intervention. Therefore, our approach provides a more convincing justification for the imposition of minimum coverage requirements than standard models as well as a case for the use of taxes and subsidies in insurance markets.

Many applications of optimism and overconfidence can be found in the burgeoning field of behavioural finance. DeLong et al. (1990, 1991) show that optimistic noise-traders can make higher profits than rational traders. If traders are risk averse, the unpredictability of noise traders' beliefs deters rational traders from betting against them even if prices diverge significantly from fundamental values. De Meza and Southey (1996) and Manove and Padilla (1999) analyse the credit-market problems arising from the presence of overoptimistic entrepreneurs. Barberis, Schleifer, and Vishny (1998) and Daniel, Hirshleifer and Subrahmanyam (1998) explore the implications of investor overconfidence in securities markets.

We proceed as follows: In Section 2, we present the basic framework. Section 3 develops the analytical tools. Section 4 provides some examples of the pooling and separating equilibria that exist in this framework. Section 5 deals with the welfare properties of these equilibria. Section 6 explores the possibility of restoring the existence of equilibrium by intervening in cases where the non-existence problem arises. Finally, section 7 concludes.

2.2 The Model

There are two states of nature: good and bad. In the good state there is no loss whereas in the bad state the individual (insuree) suffers a gross loss of D. Before the realisation of the state of nature all individuals have the same wealth level, W. Also, all individuals are risk averse and have the same utility function but differ with respect to their perception of the probability of suffering the loss. There are two types of individuals, the Rs and the Os. The Rs have an accurate estimate of their true loss probability whereas the Os underestimate it.

Furthermore, all agents can affect the true loss probability by undertaking preventive activities. Given the level of precautionary effort, the true loss probability is the same for both types. We examine the case where agents either take precautions or not (two effort levels). If an individual of type i, ($i = O, R$), takes precautions ($F_i = F$), he incurs a utility cost of F and his true probability of avoiding the loss $p(F_i)$ is p_F. If he takes no precautions ($F_i = 0$), his utility cost is 0 but his true probability of avoiding the loss $p(F_i)$ is p_0, where $p_F > p_0$.

Now, let $p^i = p(F_i, K_i)$ be the (perceived) probability function. Where K_i is the degree of optimism and takes two values: 1 for the Rs $(K_R = 1)$, and $\overline{K} > 1$ for the Os $(K_O = \overline{K} > 1)$. This probability function is assumed to be strictly increasing both in F_i and K_i. As a result, the following relationships are true:

$$p_j^R = p(F_R, K_R) = p(F_R, 1) = p(F_R) = p_j, \qquad j = F, 0 \qquad (2.1)$$

$$p_j^O = p(F_O, K_O) = p(F_O, \overline{K}) > p(F_O) = p_j, \qquad j = F, 0 \qquad (2.2)$$

where p_j is the true probability of avoiding the loss.

In this environment, the (perceived) expected utility of an agent i is given by:

$$EU_i(F_i, K_i, y_i, \lambda_i, W) = p_j^i U(W - y) + (1 - p_j^i) U(W - D + (\lambda - 1)y) - F_i,$$
$$j = F, 0 \quad i = O, R \qquad (2.3)$$

where W: insuree's initial wealth

D: gross loss

y: insurance premium

$(\lambda - 1)y$: net payout in the event of loss, $\lambda > 1$

λy: coverage (gross payout in the event of loss)

Hence, the increase in (perceived) expected utility from taking precautions is:

$$\Delta_i = \left(p_F^i - p_0^i\right)\left[U(W - y) - U(W - D + (\lambda - 1)y)\right] - F, \qquad i = O, R \qquad (2.4)$$

where U is strictly concave and $W-y$, $W-D+(\lambda-1)y$ are the wealth levels in the good and the bad state respectively.

There are two risk neutral insurance companies involved in Bertrand competition. Insurance companies know the true accident probability (given the precautionary effort level) and the perceived accident probabilities of the Os and Rs but they can observe neither the type nor the actions of each insuree. They also know the cost for the insuree corresponding to each precautionary effort level, the utility function of the insurees and the proportion of the Os and Rs in the population.

The insurance contract $(y,\lambda y)$ specifies the premium y and the coverage λy. As a result, since insurance companies have an accurate estimate of the true accident probability, the expected profit of an insurer offering such a contract is:

$$\pi = p_j y-(1-p_j)(\lambda-1)y, \qquad j=F,0 \tag{2.5}$$

Equilibrium

Insurance companies and insurees play the following two-stage screening game:

Stage 1: The two insurance companies simultaneously make offers of sets of contracts $(y,\lambda y)$. Each insurance company may offer any finite number of contracts.

Stage 2: Given the offers made by the insurers, insurees apply for at most one contract from one insurance company. If an insuree's most preferred contract is offered by both insurance companies, he takes each insurer's contract with probability ½. The terms of the contract chosen determine whether the insuree will take unobservable precautions.

We only consider pure-strategy subgame-perfect Nash equilibria (SPNE). Depending on parameter values, three kinds of equilibria can arise: separating, full-pooling and partial-pooling.[59]

A pair of contracts $z_O=(y_O,\lambda_O y_O)$ and $z_R=(y_R,\lambda_R y_R)$ is an equilibrium if the following conditions are satisfied:

[59] In some cases, a non-existence problem similar to that in Rothschild and Stiglitz (1976) arises.

i) The revelation constraints

$$EU_R(z_R) \geq EU_R(z_O)$$
$$EU_O(z_O) \geq EU_O(z_R) \qquad (2.6a)$$

ii) The effort incentive constraints

$$F_i = \begin{cases} F & \text{if } \Delta_i \geq 0, \quad i = O, R \\ 0 & \text{otherwise} \end{cases} \qquad (2.6b)$$

with Δ_i defined in (2.4).

iii) The participation (or IR) constraints of both types:

$$EU_i(z_i) \geq EU_i(z_0), \quad i = O, R \qquad (2.6c)$$

where $z_0 = (y, \lambda y) = (0,0)$

iv) Profit maximisation for insurance companies:

- No contract in the equilibrium pair (z_O, z_R) makes negative expected profits.
- No other set of contracts introduced alongside those already in the market would increase an insurer's expected profits.

2.3 Diagrammatic Analysis

Let $H = W - y$ and $L = W - D + (\lambda - 1)y$ denote the income in the good and bad state respectively of an insuree who has chosen the contract $(y, \lambda y)$. Let also $\overline{H} = W$

and $\overline{L} = W - D$ denote the endowment of an insuree after the realisation of the state of nature.

2.3.1 Effort Incentive Constraints

Let us first consider the moral hazard problem an insuree of type i faces. A given contract $(y, \lambda y)$ will induce an agent of type i to take precautions if

$$(p_F^i - p_0^i)[U(H) - U(L)] \geq F \quad \Leftrightarrow \quad \Delta_i \geq 0, \quad i = O, R \tag{2.7}$$

Let $P_i P_i'$ be the locus of combinations (L, H) such that $\Delta_i = 0$. Since F, $U' > 0$, the $P_i P_i'$ locus lies entirely below the 45^0 line in the (L, H) space. This locus divides the (L, H) space into two regions: On and below the $P_i P_i'$ locus the insurees take precautions (this is the set of effort incentive compatible contracts) and above it they do not. The slope and the curvature of $P_i P_i'$ in the (L, H) space are given respectively by:

$$\left.\frac{dL}{dH}\right|_{P_i P_i'} = \frac{U'(H)}{U'(L)} > 0 \qquad \text{since } U' > 0 \tag{2.8}$$

$$\left.\frac{d^2 L}{dH^2}\right|_{P_i P_i'} = \frac{U'(H)}{U'(L)}\left[A(L)\frac{U'(H)}{U'(L)} - A(H)\right] \tag{2.9}$$

where $A(L) = -\dfrac{U''(L)}{U'(L)}$ is the coefficient of absolute risk aversion.

Since both types have the same utility function, it is clear from the above formulas that the shape of $P_i P_i'$ is independent of the type of the insuree. In addition, $P_i P_i'$ is upward sloping. Also if $U(\cdot)$ exhibits either increasing or constant absolute risk aversion $P_i P_i'$ is strictly concave. If $U(\cdot)$ exhibits decreasing absolute risk aversion, it can be either concave or convex. (See Appendix 2A for a necessary and sufficient condition in order for P_iP_i' to be strictly convex).

However, the position of P_iP_i' does depend upon the insuree's type. Above we assumed that the perceived probability of avoiding the accident is strictly increasing in both the degree of optimism and the precautionary effort level. That is, at any given preventive effort level, the higher the degree of optimism, the greater the underestimation of the true accident probability. Also, given the degree of optimism, the higher the preventive effort level, the lower the perceived accident probability. However, these two restrictions do not imply that a given increase in the preventive effort level will have a greater effect on the perceived accident probability if the degree of optimism is higher. In other words, although the Os underestimate their accident probability at any given precautionary effort level, they may either overestimate or underestimate the decrease in that probability from choosing a higher preventive effort level. If the Os underestimate the decrease in their accident probability from taking precautions, that is, if

$$p_F^R - p_0^R > p_F^O - p_0^O \quad \text{(Case 1)}$$

then, P_RP_R' lies to the left of P_OP_O'. In other words, the Rs' set of effort incentive compatible contracts is strictly greater than that of the Os. If

$$p_F^R - p_0^R < p_F^O - p_0^O \quad \text{(Case 2)}$$

P_RP_R' lies to the right of P_OP_O' and the Rs' set of effort incentive compatible contracts is smaller.[60] Intuitively, given the incremental utility cost of a higher precautionary effort level, the greater the increase in the perceived probability of avoiding the loss from doing so, the more willing one would be to take precautions.

[60] Here, we implicitly assume that the Os either underestimate (case 1) or overestimate (case 2) the decrease in their accident probability from taking precautions regardless of the degree of optimism. However, it is also possible that the Os may underestimate the decrease in their accident probability from taking precautions if the degree of optimism is low and overestimate it at higher levels of optimism or vice versa. The comparative statics with respect to the degree of optimism in Section 2.4 below are conducted under the assumption that the direction of the inequality in cases 1 and 2 does not change as the degree of optimism changes. The analysis for the two cases where the direction of the inequality reverses as the degree of optimism changes is similar and is omitted as it does not produce any new insight.

Some empirical findings suggest that the former case is, in practice, more relevant.[61] However, in principle, both cases are possible and are analysed below.

To make the analysis more interesting, we make the following assumption:

Assumption 1: $(p_F^i - p_0^i)\left[U(\overline{H}) - U(\overline{L})\right] > F$, $\quad i = O, R$

Assumption 1 implies that both $P_R P_R'$ and $P_O P_O'$ pass above the endowment point, and so the effective set of effort incentive compatible contracts is not empty for either type. If Assumption 1 is violated for either type, the corresponding type never takes precautions. Under this assumption, in principle, both types may take precautions in equilibrium, although this is not always the case as we will see below.

2.3.2 Indifference Curves

The indifference curves, labelled I_i, are kinked where they cross the corresponding $P_i P_i'$ locus. Above $P_i P_i'$, insurees of the the i-type do not take precautions, their perceived probability of avoiding the loss is p_0^i, and so the slope of I_i is:

$$\left.\frac{dL}{dH}\right|_{I_i, p=p_0} = -\frac{p_0^i}{1-p_0^i}\frac{U'(H)}{U'(L)} \qquad i = O, R \tag{2.10}$$

On and below $P_i P_i'$ insurees of the i-type do take precautions, their perceived probability of avoiding the loss rises to p_F^i and so the slope of I_i becomes:

$$\left.\frac{dL}{dH}\right|_{I_i, p=p_F} = -\frac{p_F^i}{1-p_F^i}\frac{U'(H)}{U'(L)} \qquad i = O, R \tag{2.11}$$

Hence, just above $P_i P_i'$ the i-type indifference curves become flatter.

Furthermore, because the Os underestimate their accident probability, at any given identical preventive effort level and (L, H) pair, the Os indifference curve is steeper

[61] For example, Viscusi (1990) finds that those who perceive a higher risk are less likely to smoke. The less optimistic agents take more precautions.

in the (L, H) space. Intuitively, the Os are less willing to exchange consumption in the good state for consumption in the bad state because their perceived probability of the bad state occurring is lower than that of the Rs.

2.3.3 Insurers' Zero-profit Lines (Offer Curves)

Using the definitions $H = W - y$ and $L = W - D + (\lambda - 1)y$, and the fact that insurance companies have an accurate estimate of the true accident probabilities, given the precautionary effort level, the insurers' expected profit function becomes:

$$\pi = p_j (W - H) - (1 - p_j)(L - W + D) \tag{2.12}$$

The zero-profit lines are given by:

$$L = \frac{1}{1 - p_j} W - \frac{p_j}{1 - p_j} H - D, \qquad j = F, 0 \tag{2.13}$$

Conditional on the preventive effort level chosen by the two types of insurees, there are three zero-profit lines with slopes:

$$\left. \frac{dL}{dH} \right|_{\pi=0} = -\frac{p_0}{1 - p_0} \qquad \text{(EN' line)} \tag{2.14}$$

$$\left. \frac{dL}{dH} \right|_{\pi=0} = -\frac{p_F}{1 - p_F} \qquad \text{(EJ' line)} \tag{2.15}$$

$$\left. \frac{dL}{dH} \right|_{\pi=0} = -\frac{q}{1 - q} \qquad \text{(EM' line (pooled –line))} \tag{2.16}$$

where $q = \mu p_j + (1 - \mu) p_k$, $j = F, 0$, $k = F, 0$, and μ is the proportion of the Rs in the population of insurees.

Also, at $H = \overline{H} = W$, Eq. (2.13) becomes:

$$L = \overline{L} = W - D \tag{2.17}$$

Eq. (2.17) is independent of the value of p_j. This implies that all three zero-profit lines have the same starting point (the endowment point, E).

2.4 Positive Implications

As we have already mentioned, although the Os underestimate their accident probability, they may either underestimate or overestimate its reduction from taking precautions. We first consider the case where the Os underestimate it. Formally,

Case 1: $p_F^R - p_0^R > p_F^O - p_0^O$

In identifying equilibria, if the degree of optimism is sufficiently high, there exist separating equilibria where the Os go uninsured whereas the Rs take the contract they would have chosen if types were observable. For lower degrees of optimism, depending on parameter values, three kinds of equilibria can arise: separating, full-pooling and partial-pooling where the presence of the Os, in most but not all cases, results in Rs buying less insurance than under full information about types. Below, we present some interesting examples of separating and full-pooling equilibria.[62] We begin with the configuration yielding a separating equilibrium where, although the Os buy some insurance, their presence has no effect on the choice of the Rs.

Proposition 1: If the Os' indifference curve tangent to EN', I_O^*, passes above the intersection of EJ' and $P_R P_R'$ and meets $P_O P_O'$ above EJ', then there exists a unique separating equilibrium (z_R, z_O) where the Rs take precautions whereas the Os do not. Both types choose strictly positive coverage but the Rs purchase more insurance than the Os (see Figure 2.1).[63]

[62] We do not present partial-pooling equilibria because, from our perspective, they do not exhibit any novel feature. However, note that, because of the discreteness of the model, there potentially exist partial-pooling equilibria exhibiting strictly positive profits (see de Meza and Webb (2001)).

[63] This is true if $(p_F - p_0)$ is sufficiently small (precautions do not increase considerably the true probability of avoiding the loss), the degrees of optimism and risk aversion are sufficiently large,

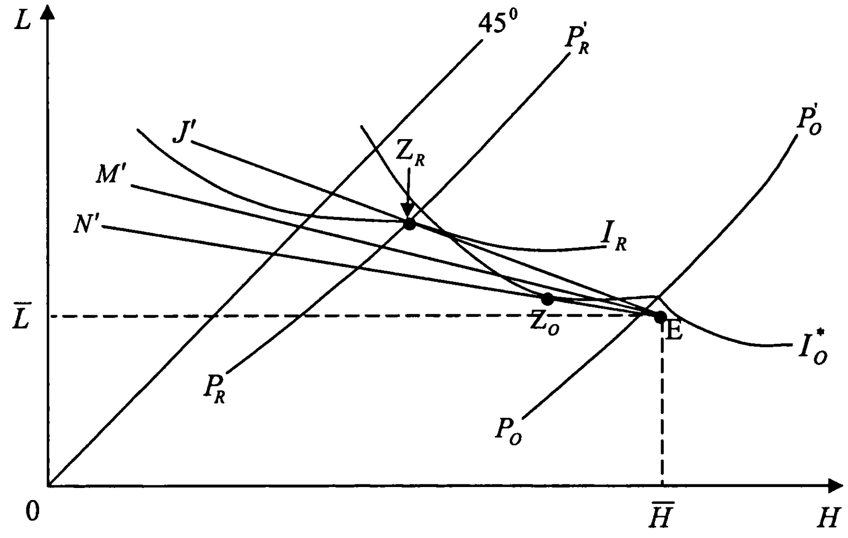

Figure 2.1

Proof: We test whether (z_R, z_O) is an equilibrium by considering deviations. Clearly, the Os strictly prefer z_O to z_R. Offers above EJ' are clearly loss-making. Similarly, offers above P_RP_R' either do not attract any type or, if they do, are unprofitable. Below EJ' and below P_RP_R' there is no offer that attracts the Rs but there are some offers that attract the Os and so are unprofitable (given the equilibrium contract z_R, the Rs are attracted only by contracts that lie above EJ' which are, of course, loss-making). So, there is no profitable deviation and the (z_R, z_O) pair is the unique separating equilibrium. The fact that I_O^* passes above z_R rules out any pooling equilibrium. Therefore, (z_R, z_O) is the unique equilibrium. *Q.E.D.*

Given the contracts offered, because the Os considerably underestimate the reduction in their accident probability from taking precautions, they choose to take no precautions. Also, although insurance is offered at actuarially fair terms, because they underestimate their accident probability, the Os underinsure choosing a contract with low coverage while contracts with higher coverage are available at the same or even lower per unit premium. In other words, the Os not only purchase less coverage and

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$p_F^R - p_0^R$ is sufficiently larger than $p_F^O - p_0^O$ (the distance between P_RP_R' and P_OP_O' is sufficiently large) and P_OP_O' lies sufficiently close to the endowment point, E.

than the Rs but also the take fewer precautions and so their accident probability is higher. Competition among insurers then implies that they also pay a higher per unit premium. Thus, this equilibrium is consistent with both the negative correlation between coverage and risk and the fact that insurance premiums display quantity discounts as reported by Cawley and Phillipson (1999). Standard asymmetric information models cannot simultaneously explain both empirical findings.

Proposition 2: Assume the following are true: i) between $P_R P_R'$ and $P_O P_O'$, I_R is steeper than I_O, ii) the I_O tangent to EN', I_O^*, intersects EJ' above the intersection of EJ' and $P_O P_O'$ and passes below the intersection of EJ' and $P_R P_R'$ and iii) below $P_R P_R'$, EM' does not cut the I_R passing through the intersection of I_O^* and EJ'.[64] Then there exists a unique separating equilibrium (z_R, z_O) where the Rs take precautions whereas the Os do not. Both types choose strictly positive coverage but now the Rs buy less than the Os (see Figure 2.2).[65]

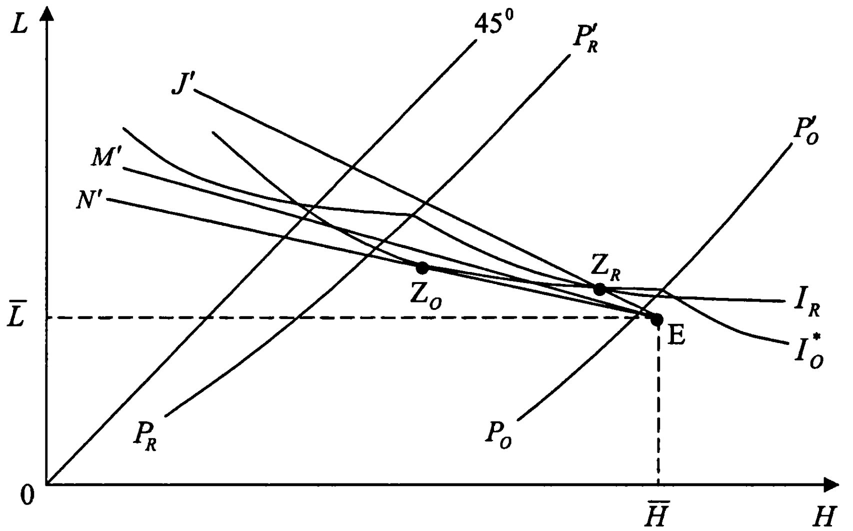

Figure 2.2

[64] Ceteris paribus, if condition (iii) is violated, then there exists no subgame-perfect Nash equilibrium. However, there exists a Wilson pooling equilibrium (Wilson (1977)).
[65] This separating equilibrium obtains under the conditions described in Footnote 59 above but for a lower degree of optimism.

Proof: Offers above EJ' are clearly loss-making. Similarly, offers above $P_R P_R'$ either do not attract any type or, if they do, are unprofitable. Below EJ' and below $P_R P_R'$ there is no offer that attracts only the Rs but there are some offers that attract only the Os and so are unprofitable. Thus, there is no profitable deviation and the (z_R, z_O) pair is the unique separating equilibrium. The fact that below $P_R P_R'$, EM' does not cut the I_R passing through the intersection of I_O^* and EJ' rules out any pooling equilibrium. Therefore, (z_R, z_O) is the unique equilibrium. *Q.E.D.*

Notice that the separating equilibrium in Proposition 2 is qualitatively similar to that in standard adverse-selection models. The high-risk type (the Os) purchase more insurance than the low-risk type (the Rs) and the Rs are quantity-constrained. Compared to Proposition 1, because the Os underestimate less their accident probability (the degree of optimism is lower), they prefer the low-price contract the Rs would have been offered if types were observable to the high-price contract they are offered. Under full information about types, the Rs would have purchased the contract at the intersection of $P_R P_R'$ and EJ', instead of z_R, which involves more insurance. However, in the presence of the Os, this contract is not offered because it violates the Os' revelation and effort incentive constraints and so is loss-making for the insurance companies. In order to reveal their type, the Rs accept lower coverage than they would have chosen if types were observable.

The separating equilibrium in Proposition 1 differs from that in Proposition 2 in two respects: First, in the latter proposition the Rs are quantity-constrained because of the presence of the Os whereas in the former they take the contract they would have chosen under full information about types. Second, the latter equilibrium exhibits positive correlation between the coverage offered by the insurance contract and the accident probability of its buyer whereas the former negative. Both differences are due to the fact that in the latter case the Os' degree of optimism is lower.

If the Os underestimate less the decrease in their accident probability from taking precautions, separating equilibria arise where the Os do take precautions. For the very same reasons as above, the degree of optimism determines whether the Rs are quantity-constrained. If the degree of optimism is sufficiently low, there arises a pooling equilibrium where both types take precautions but the Rs are quantity-constrained (see Appendix 2B).

To summarise, the comparative statics of our model as the degree of optimism changes are as follows. At low levels of optimism there exist pooling equilibria where both types take precautions and buy the contract lying at the intersection of $P_O P_O'$ and EJ'. For higher degrees of optimism, the type of the resulting equilibrium depends crucially on the extent to which the Os underestimate the decrease in their accident probability from taking precautions as well as on the effectiveness of the precautionary effort.

If the configuration yielding the separating equilibria in Propositions 1 and 2 is relevant, then as the degree of optimism rises, separating equilibria similar to that in Proposition 2 arise. For higher levels of optimism, partial-pooling equilibria may arise.[66] As the degree of optimism increases even further, separating equilibria similar to that in Proposition 1 arise. Under the configuration yielding the equilibria in Proposition 2B.1 (Appendix 2B), as the level of optimism rises, separating equilibria similar to that in Proposition 2B.1 arise. In either case, for a sufficiently high degree of optimism, separating equilibria arise where the Os go uninsured but take precautions whereas the Rs take the contract they would have chosen if types could be observed.

That is, if the Os are sufficiently optimistic, their presence has no effect on the choice of the Rs. However, for low and intermediate levels of optimism, if types are not observable, the presence of the Os results in equilibria where the Rs are quantity-constrained. In order to reveal their type, the Rs accept lower coverage than they would have chosen under full information about types.

Let us now consider the case where the Os, although underestimate their accident probability, they overestimate its reduction from taking precautions. Formally,

<u>Case 2:</u> $p_F^R - p_0^R < p_F^O - p_0^O$

Graphically, $P_R P_R'$ lies to the right of $P_O P_O'$ and so the Rs' set of effort incentive compatible contracts is smaller. Intuitively, given the incremental utility cost of a

[66] This occurs if between $P_R P_R'$ and $P_O P_O'$, I_R is flatter than I_O.

higher preventive effort level, the greater the decrease in the perceived accident probability from doing so, the more willing one would be to take precautions.

In this case, there exist only separating and full-pooling equilibria.[67] For low levels of optimism, if an equilibrium exists, it is a pooling one, the equilibrium contract lies at the intersection of $P_R P_R'$ and EJ' and the Os are quantity-constrained. As the degree of optimism rises, separating equilibria arise where, depending on parameter values, the Rs either take precautions and purchase partial insurance or they do not and purchase full insurance.

In the former case, both types take the contracts they would have chosen under full information about types. However, in the latter case, if the degree of optimism is not very high, the resulting separating equilibrium involves the Os, rather than the Rs, being quantity-constrained. Because the Os are more willing to take precautions, their true accident probability is lower and so they are offered insurance at a lower per unit premium. At this lower unit price, because the Os are not very optimistic, they would like to purchase a contract involving a considerable amount of insurance. However, such a contract is not offered because it violates the Rs' revelation and effort incentive constraints and so is loss-making for the insurance companies. In order to reveal their type, the Os accept lower coverage than they would have chosen under full information about types. The following two propositions summarise these results.

Proposition 3: Assume that I_R^P, the Rs' indifference curve through the intersection of $P_R P_R'$ and EJ' (point Z_P), lies above EN'. Then, i) if the degree of optimism is sufficiently low so that the Os' indifference curve through Z_P, I_O^P, is flatter than EJ' and, between $P_R P_R'$ and $P_O P_O'$, EM' does not cut I_O^P, there exists a unique pooling equilibrium at Z_P (see Figure 2.3)[68], ii) if the degree of optimism is sufficiently high so that I_O^P is steeper than EJ', there exists a unique separating equilibrium (z_R, z_O)

[67] The fact that between $P_R P_R'$ and $P_O P_O'$, I_O is steeper than I_R implies that there exist offers involving less coverage which profitably only attracts the Os and rule out any pooling subgame-perfect Nash equilibrium (SPNE) in the area between these two curves. However, there exist Wilson pooling equilibria that Pareto-dominate the (constrained) efficient outcome under realism (the contract at the intersection of $P_R P_R'$ and EJ'). This latter contract is the unique pooling SPNE in this case.

[68] If, between $P_R P_R'$ and $P_O P_O'$, EM' cuts I_O^P, then there exists no SPNE (see also the previous footnote).

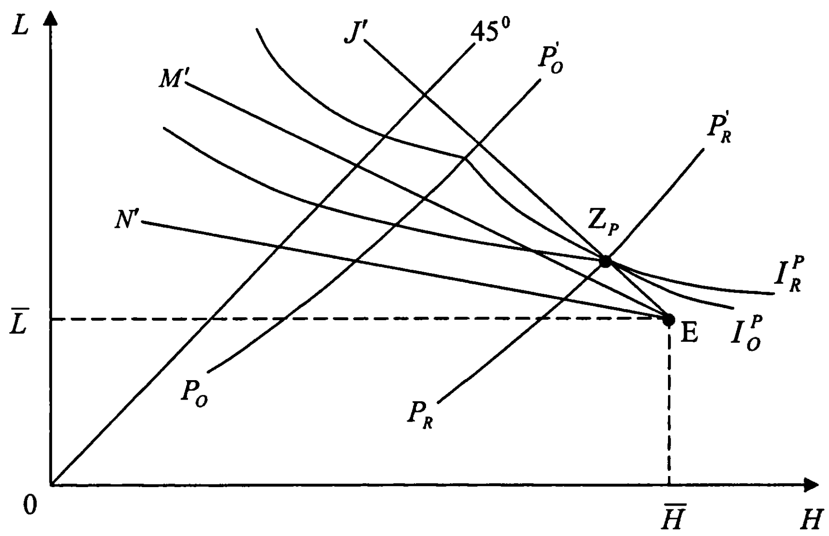

Figure 2.3

where both types take precautions and the Os purchase less insurance than the Rs (see Figure 2.4).[69]

Proof: i) Clearly, offers above EJ' are loss-making. The same is true for offers above $P_O P_O'$. Between $P_R P_R'$ and $P_O P_O'$ and below EJ' there is no offer that attracts the Os and does not attract the Rs, although there are some offers that attract only the Rs. Thus, any offer in this region is unprofitable. Given the equilibrium contract, below EJ' and below $P_R P_R'$ there is no offer that is attractive to either type. Therefore, the pooling contract z_P is the unique equilibrium. ii) Using similar arguments, one can show that the separating pair (z_R, z_O) is the unique equilibrium (see Figure 2.4). *Q.E.D.*

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[69] These results hold true if $(p_F - p_0)$ is sufficiently large (preventive efforts are sufficiently "productive"), and $p_F^R - p_0^R$ is not much smaller than $p_F^O - p_0^O$.

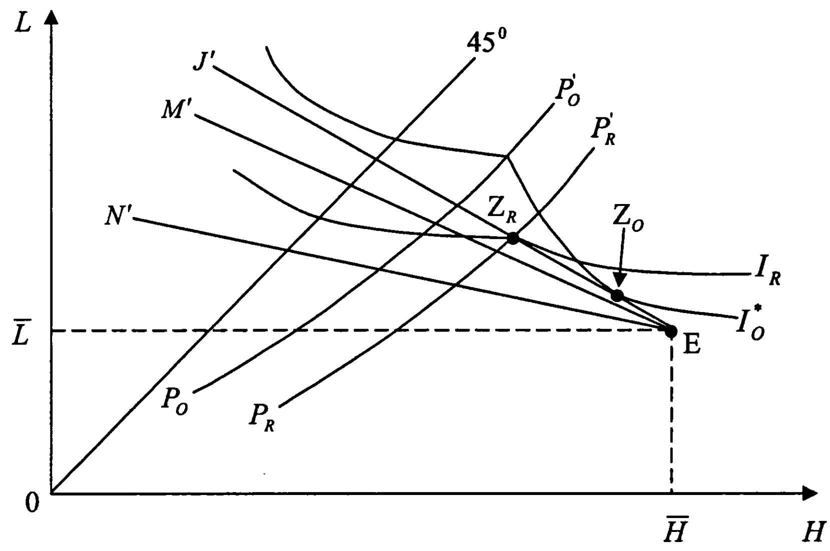

Figure 2.4

Proposition 4: Assume the Rs' indifference curve tangent to EN' at its intersection with the 45-degree line, I_R^*, meets P_RP_R' above EJ'.[70] Then, if the degree of optimism is sufficiently low so that the Os' indifference curve through the intersection of I_R^* and EJ' is flatter than EJ' and EM' does not cut I_O passing through Z_O, there exists a unique separating equilibrium (z_R, z_O) where the Rs take no precautions and buy full insurance whereas the Os take precautions, purchase partial insurance and are quantity-constrained (see Figure 2.5).

Proof: i) Offers above EJ' are clearly loss-making. In the area below EJ' and above P_RP_R' there is no offer that attracts the Os and does not attract the Rs, although there are some offers that attract only the Rs. Thus, any offer in this region is unprofitable. Given the equilibrium contracts, below EJ' and below P_RP_R' there is no offer that is attractive to either type. Hence, the pair (z_R, z_O) is the unique separating equilibrium. Furthermore, the fact that EM' does not cut I_O below (to the right of) P_OP_O' rules out any pooling equilibrium.[71] Therefore, the pair (z_R, z_O) is the unique equilibrium (see Figure 2.5).[72] *Q.E.D.*

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[70] This holds true if $(p_F - p_0)$ is sufficiently small (preventive efforts are not sufficiently effective).

[71] In fact, if between P_RP_R' and P_OP_O', EM' cuts I_O, then there exists no subgame-perfect Nash equilibrium. However, there exist Wilson pooling equilibria.

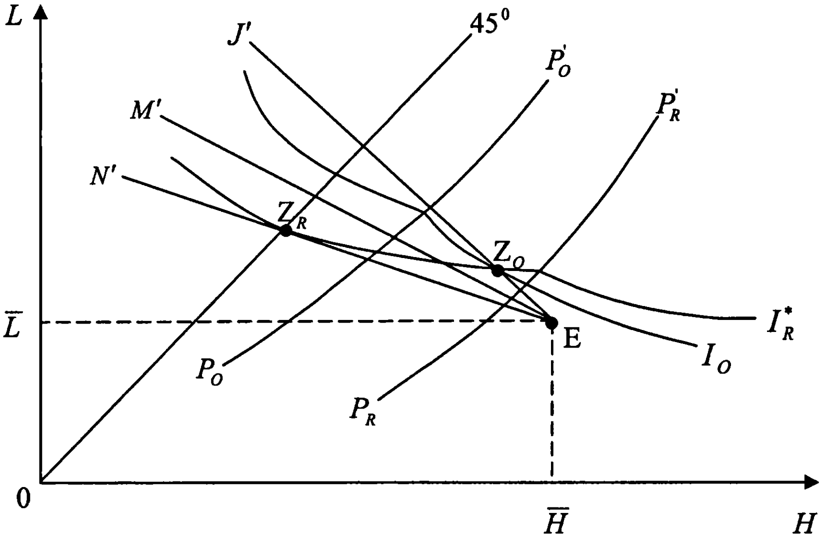

Figure 2.5

Note that, under the configuration yielding the pooling equilibrium illustrated in Figure 2.3, if types were observable but their actions were not, the Os would have purchased more insurance than the Rs. That is, contrary to the conventional wisdom, optimism itself does not necessarily lead to the purchase of less insurance. If the Os overestimate the positive effect of their precautionary efforts on the probability of avoiding the accident, optimism, by relaxing the effort incentive constraint, leads to more precautions and so lower per unit premia. This effect may more than offset the effect of the underestimation of the accident probability and result in the purchase of more insurance. However, under both adverse selection and moral hazard, the presence of the Rs makes this choice infeasible. The amount of insurance offered at the low per unit premium is restricted by the Rs' revelation and effort incentive constraints.

In summary, if neither the type nor the actions of the insurees are observable, the presence of the Os may restrict the choice of the Rs only if the Os not only underestimate their accident probability but also its reduction from taking precautions. If the Os overestimate the decrease in their accident probability from taking precautions, then the presence of the Rs may restrict the choice of the Os but not vice

[72] As the degree of optimism rises, there arises a separating equilibrium where the Rs make the same choices as in Figure 2.5 whereas the Os, whether they purchase insurance or not, they are not quantity-constrained.

versa. More specifically, first, if the degree of optimism is sufficiently high, both types purchase the contract they would have chosen under full information about types. Second, for intermediate and low levels of optimism there exist equilibria where one of the two types of insurees is quantity-constrained. If the Os underestimate the decrease in their accident probability from taking precautions, then the Rs are quantity-constrained whereas if the Os overestimate it, the Os themselves are quantity-constrained.

Finally, two points should be stressed here. First, if all agents purchase some insurance, optimism is a necessary but not a sufficient condition for the existence of a separating equilibrium exhibiting negative correlation between coverage and the accident probability (see Chapter 3 for a more detailed discussion). The negative correlation result also requires that: i) optimism discourage precautionary effort[73], ii) the degree of optimism be sufficiently high (the Os underestimate their accident probability significantly) and iii) preventive effort be not very "productive" (the per unit premium on the contracts offered to the Rs is not significantly lower than that offered to the Os). Second, optimism itself does not necessarily result in the purchase of less insurance. For example, under the configuration yielding the pooling equilibrium in Proposition 3, if types were observable, the Os would have purchased more insurance than the Rs.

2.5 Welfare Implications

In the previous section, we explored the impact of the presence of the Os on the equilibrium outcome when neither the type nor the actions of the insurees are observable. This section deals with the welfare properties of the equilibria described above. In this framework, because some of the insurees, the Os, underestimate their accident probability, the definition of the efficiency of the equilibrium is not straightforward. The very presence of the Os raises the question of what is the appropriate efficiency criterion. Should we employ objective probabilities (true expected utility) or subjective probabilities (perceived expected utility)? The answer depends crucially on the origin of the agents' biased estimate. In our environment, the different estimates of the same risk (accident probability) arise because of different

[73] If optimism encourages precautionary effort, there can only exist equilibria exhibiting either negative or zero correlation (see the analysis of case 2 above).

perceptions not because of different underlying preferences. Both the Os and the Rs have identical preferences. As a result, the preferences revealed by the insuree's choices coincide with the true underlying preferences. Therefore, the appropriate efficiency criterion seems to be objective rather than subjective probabilities.

Given that, it is possible to find intervention policies that yield strict Pareto gains. Below, we employ two different intervention schemes. If the Os or neither type is quantity-constrained in the laissez-faire equilibrium, then only Policy 2 may be effective. On the contrary, if the Rs are quantity-constrained, then only Policy 1 can potentially lead to a strict Pareto improvement.

Policy 1: Imposition of a lump-sum tax, τ, per contract sold (paid by the insurers), with the proceeds returned as a lump-sum subsidy of s to the whole population.

Under Policy 1, the perceived expected utility of an agent i choosing contract $(y, \lambda y)$ is given by:

$$EU_i(F_i, K_i, y_i, \lambda_i, W, s) = p_j^i U(W - y + s) + (1 - p_j^i)U(W - D + (\lambda - 1)y + s) - F_i, \quad j = F, 0, \quad i = O, R \qquad (2.18)$$

Similarly, the actual expected utility is given by:

$$EU_i(F_i, K_i, y_i, \lambda_i, W, s) = p_j U(W - y + s) + (1 - p_j)U(W - D + (\lambda - 1)y + s) - F_i \quad j = F, 0, \quad i = O, R \qquad (2.19)$$

Eq. (2.19) defines the dashed indifference curve I_O^T in Figure 2.9.

Also, the expected profit of an insurance company offering contract $(y, \lambda y)$ is:

$$\pi = p_j y - (1 - p_j)(\lambda - 1)y - \tau, \quad j = F, 0 \qquad (2.20)$$

Using (2.19) and the definitions $H = W - y$ and $L = W - D + (\lambda - 1)y$, we obtain:

$$\pi = p_j(W - H - \tau) - (1 - p_j)(L - W + D - \tau), \quad j = F, 0 \qquad (2.21)$$

So, the zero-profit lines are now given by:

$$L = \frac{1}{1-p_j}(W-\tau) - \frac{p_j}{1-p_j}H - D, \qquad j = F,0 \tag{2.22}$$

The slopes of the zero-profit lines are still given by Eqs. (2.14)-(2.16). However, the tax per contract sold shifts the origin of the zero-profit lines down the 45^o line by the tax amount, τ (point $\hat{J}$ in Figure 2.9).

To make the analysis interesting, we assume that the tax amount is chosen such that at least one type purchases insurance. Then, from the balanced budget condition, the amount of the subsidy is:

$$s = \begin{cases} \tau & \text{if both types purchase insurance} \\ \mu\tau & \text{if only the Rs purchase insurance} \\ (1-\mu)\tau & \text{if only the Os purchase insurance} \end{cases} \tag{2.23}$$

The subsidy shifts both the endowment point, E, up the 45^o line by the amount s (point $\hat{E}$ in Figure 2.9) and the origin of the lines along which the insured can consume (consumption zero-profit (offer) lines) up the 45^o line by the same amount (from $\hat{J}$ to $\tilde{J}$ in Figure 2.9). That is, the consumption zero-profit lines are given by:

$$L = \frac{1}{1-p_j}(W-\tau+s) - \frac{p_j}{1-p_j}H - D \tag{2.24}$$

Policy 2: Imposition of a lump-sum tax, τ, per person paid only by those going uninsured and per contract sold (paid by the insurance companies), with the proceeds distributed to agents (potential insurees) as follows:

i) Those going uninsured receive nothing.
ii) Those buying a contract implying at least a minimum amount of wealth in the bad state, m, receive a subsidy only if the bad state realises. In the good state, they receive no subsidy.

Under Policy 2, the perceived expected utility of an agent i choosing contract $(y, \lambda y)$ is given by:

$$EU_i(F_i, K_i, y_i, \lambda_i, W, s, \tau) =$$

$$\begin{cases} p_j^i U(W-\tau) + (1-p_j^i) U(W-D-\tau) - F_i, & \text{if agent i does not buy insurance} \\ p_j^i U(W-y) + (1-p_j^i) U(W-D+(\lambda-1)y+s) - F_i & \text{if } W-D+(\lambda-1)y \geq m \\ p_j^i U(W-y) + (1-p_j^i) U(W-D+(\lambda-1)y) - F_i & \text{if } W-D+(\lambda-1)y < m \end{cases}$$

$$j = F, 0, \quad i = O, R \tag{2.25}$$

If instead of the perceived probabilities p_j^i we employ the true probabilities p_j, we obtain the true expected utility given by:

$$EU_i(F_i, K_i, y_i, \lambda_i, W, s, \tau) =$$

$$\begin{cases} p_j U(W-\tau) + (1-p_j) U(W-D-\tau) - F_i, & \text{if agent i does not buy insurance} \\ p_j U(W-y) + (1-p_j) U(W-D+(\lambda-1)y+s) - F_i & \text{if } W-D+(\lambda-1)y \geq m \\ p_j U(W-y) + (1-p_j) U(W-D+(\lambda-1)y) - F_i & \text{if } W-D+(\lambda-1)y < m \end{cases}$$

$$j = F, 0, \quad i = O, R \tag{2.26}$$

Eq. (2.26) defines the dashed indifference curve I_O^T in Figures 2.6-2.8 and 2.10, 2.11.

Because under Policy 2 the tax is also paid by those going uninsured, the endowment point shifts down the 45^o line by the tax amount, τ (point $\hat{E}$ in Figures 2.6-2.8 and 2.10, 2.11). Policy 2 has exactly the same effects on the insurers profit function and zero-profit lines as Policy 1. That is, the zero-profit lines are given by Eq. (2.22). Also, because the tax paid by those going uninsured is the same as the tax per contract paid by insurance companies, the origin of the zero-profit lines coincides with the endowment point after the intervention, $\hat{E}$.

To make the analysis interesting, we assume that the tax is sufficiently high so that all agents purchase a contract implying at least a minimum amount of consumption (wealth) m in the bad state. Then, from the balanced budget condition the subsidy is:

$$s = \begin{cases} \tau/(1-p_F) & \text{if precautions are taken} \\ \tau/(1-p_0) & \text{if no precautions are taken}^{74} \\ \tau/(1-q) & \text{if a pooling equilibrium arises} \end{cases} \tag{2.27}$$

where $q = \mu p_j + (1-\mu)p_k$, $j = F,0$, $k = F,0$ and μ is the Rs' proportion in the population of insurees. Because of perfect competition, the insurees pay the tax τ through a higher insurance premium. However, they receive a subsidy of the same expected amount ($s = (1-p_j)\tau/(1-p_j) = \tau$). Therefore, the consumption zero-profit lines are given by:

$$L = \frac{1}{1-p_j}(W-\tau+s) - \frac{p_j}{1-p_j}H - D = \frac{1}{1-p_j}W - \frac{p_j}{1-p_j}H - D \tag{2.28}$$

Eq. (2.28) is identical to (2.13). That is, the consumption zero-profit lines after the implementation of Policy 2 coincide with the zero-profit lines that obtain without any intervention. In other words, the subsidy shifts the origin of the lines along which the insured can consume (consumption zero-profit (offer) lines) up the 45^o line by the same amount as the tax shifts the insurers' zero-profit lines down the 45^o line. As a result, the origin of the consumption zero-profit lines after the intervention coincides with the endowment point before the intervention, E.

For expositional purposes, we begin with the case where there is just one type, the Os. The Os' choice in the laissez-faire equilibrium is driven by their perceived utility. However, as we have argued, the appropriate efficiency criterion is their true utility.

[74] Whether an insuree takes precautions or not can be inferred by the contract he chooses.

Thus, to conclude that the Os are better off after an intervention, we construct a curve along which the Os' true rather than perceived welfare is constant, given their precautionary effort level choice (the I_O^T curve in Figure 2.6). If the I_O^T through the Os' consumption allocation after the intervention passes above that in the laissez-faire equilibrium, the intervention has lead to an improvement in the Os true welfare. Moreover, I_O^T allows us to determine the Os' optimal (second-best) contract. Because the Os underestimate their accident probability, although insurance is offered at actuarially fair terms, in all but one cases they are underinsured in the laissez-faire equilibrium.[75] Application of Policy 2 can always implement the Os' optimal contract.

Proposition 5: i) If the I_O^T tangent to the without-precautions full-insurance contract passes above the intersection of $P_O P_O'$ and EJ', C_O^{PI}, then the optimal contract involves full-insurance (see Figure 2.6).[76] If it passes below C_O^{PI}, then C_O^{PI} is the second-best contract (see Figure 2.7). ii) In either case, application of Policy 2 would result in the Os taking the optimal contract.[77]

Proof: i) Suppose that in the laissez-faire equilibrium the Os take precautions and choose contract Z_O which involves less insurance than the contract offering the maximum amount of insurance consistent with taking precautions, C_O^{PI} (the contract at the intersection of $P_O P_O'$ and EJ'). To determine the optimal contract, we employ I_O^T and compare the without-precautions full-insurance contract, C_O^{FI}, with C_O^{PI}. If the I_O^T tangent to C_O^{FI} passes above C_O^{PI}, then there exist no feasible contract that can increase the Os' true utility. All contracts along EJ' that could improve the Os' true welfare violate their effort incentive constraint and so are loss-making for the insurance companies. On the contrary, if the I_O^T through C_O^{PI} passes above C_O^{FI}, C_O^{PI}

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[75] The only exception is when the Os choose the contract at the intersection of $P_O P_O'$ and EJ' and the I_O^T through this contract lies above the without-precautions full-insurance contract. In this case, the former contract is the Os' optimal (second-best) contract.

[76] This is true if $(p_F - p_0)$ is small (preventive efforts are not very "productive"), and the degree of risk aversion and the perceived cost of precautionary effort are high (the distance between $P_O P_O'$ and the 45-degree line is large).

[77] Notice that mandatory coverage requirements would also work.

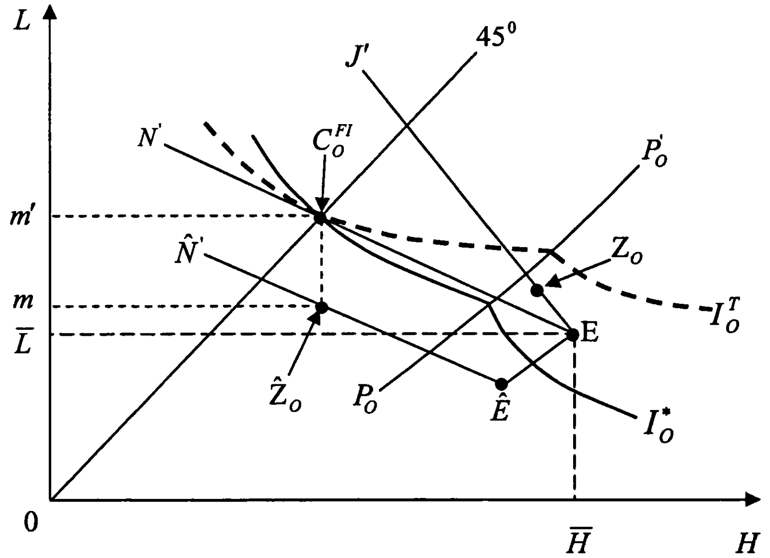

Figure 2.6

is the optimal contract. Since the I_O^T through C_O^{PI} lies strictly above EN', there exists no feasible contract to the left of $P_O P_O'$ that can improve the Os' true welfare. Also, since at C_O^{PI} the Os are partially insured, any contract along EJ' to the right of $P_O P_O'$ implies lower coverage and so lower true utility for the Os.

ii) The tax shifts both the endowment point and the origin of the zero-profit lines down the 45° line to $\hat{\mathrm{E}}$. Also, in order to receive the subsidy, the insurees must purchase a contract implying at least a minimum consumption (wealth) of m in the bad state. Consider, for example, Figure 2.6. After the application of Policy 2, the Os' consumption bundles, C_O^{FI}, consist of two components: the insurance contract, $\hat{Z}_O$, and the subsidy, $\tau/(1-p_0)$. Because the Os underestimate their accident probability, their perceived utility at the laissez-faire allocation Z_O exceeds that at C_O^{FI}. However, since the I_O^T tangent to C_O^{FI} passes above Z_O, their true utility is higher at C_O^{FI}. In either case, since the Os' (perceived) indifference curve through the optimal contract, I_O^*, passes above $\hat{E}$, a combination of minimum coverage requirements, taxes and subsidies can implement the optimal contract. *Q.E.D.*

Notice that the imposition of minimum standards only may have the opposite than the desired outcome. Suppose that the regulator requires that the insurance contracts offered imply at least the minimum amount of consumption m' in the bad state. Consider, for example, Figure 2.7. Since the Os' indifference curve through C_O^{PI} passes below the endowment point E, the Os (the underinsured) would purchase no insurance at all and so they would be worse off than in the laissez-faire equilibrium. However, a combination of minimum coverage requirements, taxes and subsidies can always lead to an increase in the Os' true welfare.[78]

So far, we have identified the Os' optimal contract as well as an intervention scheme that can be employed for its implementation. Needless to say, if types are observable, the Rs always choose their second-best contract. We now proceed to examine how the interaction between the two types of insurees affects the outcome of the intervention when types are hidden. In this case, the question is whether the intervention can lead to a Pareto improvement on the laissez-faire equilibrium. For all equilibria described in the previous section, there exist policies which can be used to

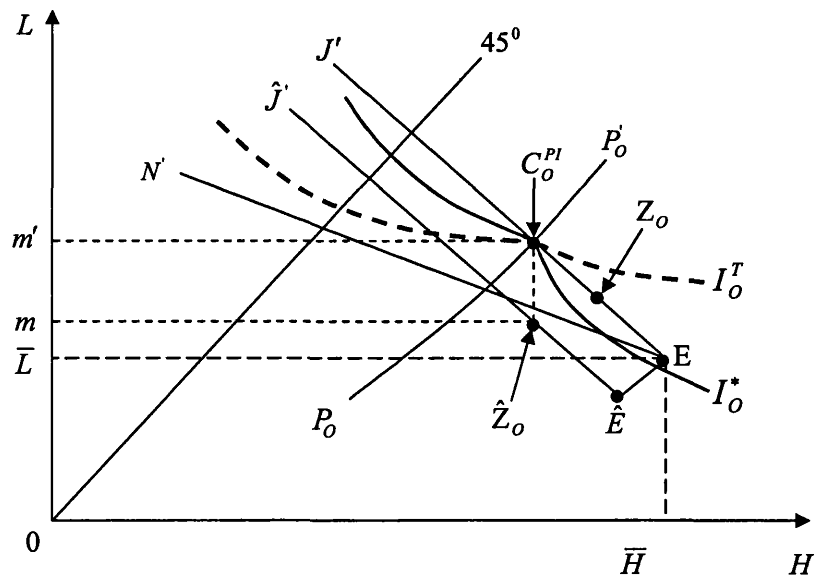

Figure 2.7

[78] However, because more insurance may result in the Os taking no precautions, it is not obvious whether the imposition of a binding upper bound in the amount of insurance the Os can purchase would lead to an increase in their true welfare. In Appendix 2C we show that it is not possible.

achieve this objective. However, the fact that types are not observable prevents the regulator from implementing the second-best contract for both types. Given the requirement that both types be at least as well off as in the laissez-faire equilibrium, the second-best contract can be implemented for at most one of the types. Furthermore, because of the subsidy they receive through the intervention scheme, the type that is less willing to take precautions may be better off in the equilibrium achieved after the intervention than at its second-best allocation.

Proposition 6: In the separating equilibrium of Proposition 1, the application of Policy 2 yields a strict Pareto improvement (see Figure 2.8).

Proof: After the application of Policy 2, the consumption bundles of the Rs and the Os, $\hat{C}_R$ and $\hat{C}_O$ respectively, consist of two components: the insurance contract, $\hat{Z}_R$ and $\hat{Z}_O$ respectively, and the subsidy, $\tau/(1-p_F)$ and $\tau/(1-p_0)$ respectively, which is received only if the accident occurs. Since the Os' indifference curve through $\hat{C}_O$ lies above $\hat{C}_R$, the Os prefer the former allocation. Also, because the Os underestimate their accident probability, their perceived expected utility at the laissez-faire allocation Z_O exceeds that at $\hat{C}_O$.[79] However, their true expected utility is higher at $\hat{C}_O$. Both Z_O and $\hat{C}_O$ involve less than full insurance, but $\hat{C}_O$ involves more coverage at the same, actuarially fair, per unit premium and precautionary effort level. Thus, given risk aversion, $\hat{C}_O$ implies strictly greater actual expected utility. That is, the Os are strictly better off whereas the Rs are as well off. Therefore, a strict Pareto improvement has been achieved.[80] *Q.E.D.*

[79] Notice that after Policy 2 has been applied, Z_O is no longer offered. The tax shifts the origin of the zero-profit lines down the 45^{O} line to $\hat{E}$. Hence, given that the Os take no precautions, in equilibrium, contracts are now offered along $\hat{E}\hat{N}'$ rather than along EN'.

[80] Note that the balanced budget condition is also satisfied. Since both types purchase insurance, the total tax revenue is τ $(=\mu\tau+(1-\mu)\tau)$ which equals the total subsidy given to both types $\mu(1-p_F)\tau/(1-p_F)+(1-\mu)(1-p_0)\tau/(1-p_0)=\tau$.

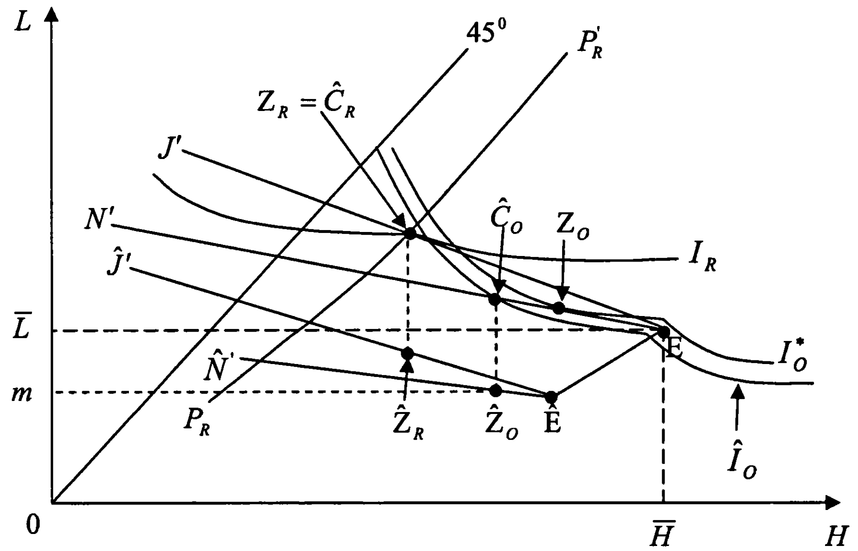

Figure 2.8

In the equilibrium attained after the intervention, both types pay the same per unit premium as in the laissez-faire equilibrium and the Rs still choose their second-best contract, $Z_R = \hat{C}_R$. However, the imposition of the tax and the requirement that the subsidy is received only by those choosing a contract implying at least the minimum amount of consumption m in the bad state results in the purchase of more insurance by the Os which, in turn, leads to an improvement in their true welfare. Moreover, notice that, if a Pareto improvement is to be achieved, the Os' second-best contract (the without-precautions full-insurance contract) cannot be implemented. The Os prefer $\hat{C}_R$ to their second-best allocation. However, because at $\hat{C}_R$ the Os take no precautions, this allocation becomes infeasible if it is chosen by both types. In fact, because optimism discourages precautionary effort, at any feasible allocation chosen by both types, the Rs are strictly worse off than at $\hat{C}_R$. That is, a Pareto improvement requires that the Os be induced to take a contract which they (weakly) prefer to $\hat{C}_R$.

The possibility of a strict Pareto improvement also arises in the separating equilibrium of Proposition 2. However, in this case, Policy 1 rather than Policy 2 is appropriate. Under Policy 1, the subsidy is received regardless of whether or not insurance is purchased whereas the tax is paid only by those buying insurance through a higher per unit premium. The imposition of the tax makes the purchase of insurance less attractive for both types but more so for the Os. As a result, the Os go uninsured, their revelation constraint is relaxed, which allows the Rs to purchase more insurance while still credibly revealing their type.

More specifically, the tax per contract sold shifts the origin of the zero-profit lines down the 45^{O} line to $\hat{J}$. On the contrary, the subsidy shifts the endowment point E up the 45^{O} line to $\hat{\mathrm{E}}$ and the origin of the zero-profit lines along which the insured can consume (consumption zero-profit lines) up the 45^{O} line from $\hat{J}$ to $\widetilde{J}$. In the new equilibrium, the Rs purchase more insurance but subsidise the Os. The question is whether the improvement in their welfare, because of the higher coverage, exceeds the welfare loss due to the subsidy they provide the Os in order to relax their revelation constraint?

Proposition 7: In the separating equilibrium of Proposition 2, applying Policy 1 yields a strict Pareto improvement if the proportion of the Os is sufficiently small. In the new equilibrium, the Os go uninsured (purchase less insurance), the Rs purchase more insurance and both types take precautions (see Figure 2.9).[81]

Proof: After the application of Policy 1, the consumption bundle of the Rs, $\widetilde{C}_R$, consists of the insurance contract $\hat{\mathrm{Z}}_R$ and the subsidy, $s = \mu\tau$, whereas that of the Os', $\hat{E} = \widetilde{C}_O$, consists of their endowment, E, and the subsidy, $s = \mu\tau$. If the proportion of the Os is small (μ is large), $\widetilde{J}\widetilde{J}'$ lies close to EJ' and $\hat{E}$ lies well above E. As a result, the Rs' indifference curve through $\widetilde{C}_R$, $\widetilde{I}_R$, and the Os' indifference curve through $\hat{E} = \widetilde{C}_O$, $\widetilde{I}_O$, lie above the corresponding indifference curves in the laissez-faire equilibrium. That is, in the new equilibrium, the perceived expected utility of both types has increased. Also, since the I_O^T through $\hat{E} = \widetilde{C}_O$ passes above Z_O, the Os' true welfare at $\hat{E} = \widetilde{C}_O$ is strictly greater than at Z_O. That is, both the Rs and the Os are strictly better off.[82] *Q.E.D.*

[81] Given the amount of the tax and the proportion of the Os, the higher the degree of optimism, the more the Os' revelation constraint relaxes and so the greater the improvement in the Rs' welfare.

[82] It may be the case that $\hat{E} = \widetilde{C}_O$ involves higher consumption in both states than Z_O. This clearly implies that the Os are strictly better off after the intervention.

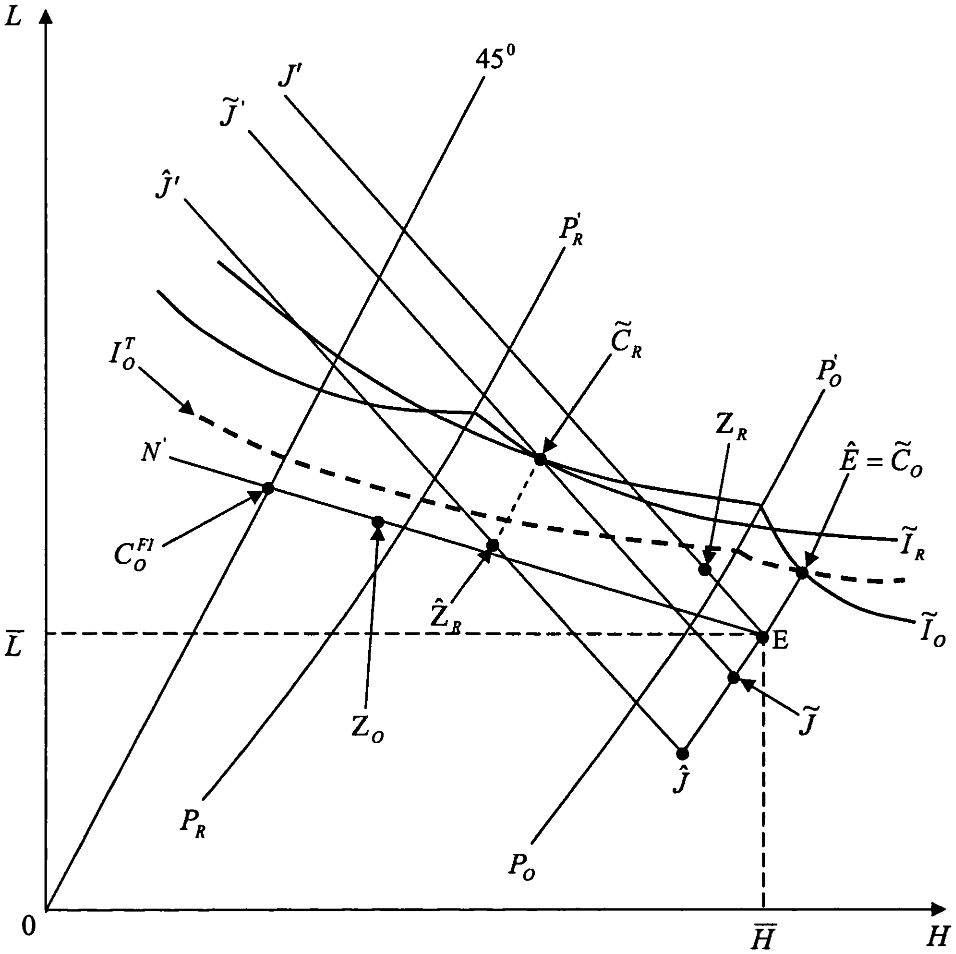

Figure 2.9

Intuitively, if the proportion of the Os is small, the per capita subsidy is high and so its effect both on the Os' utility and revelation constraint is large. This, in turn, allows the Rs to purchase a considerably higher amount of insurance. As a result, the welfare gains of the higher coverage more than offset the welfare loss due to the net tax (tax minus subsidy) the Rs pay.

By similar arguments, applying Policy 1 in the separating and pooling equilibria of Proposition 2B.1 (Appendix 2B) also leads to a strict Pareto improvement. In fact, in the latter case the pooling equilibrium breaks and a separating equilibrium arises.

Surprisingly, although in the separating equilibria of Propositions 2 and 2B.1 the Os are underinsured in the laissez-faire equilibrium, Policy 1, which results in the Os going uninsured, leads to a strict Pareto improvement. On the contrary, Policy 2, which would result in the purchase of more insurance, does not. In both cases the Rs are quantity-constrained in the laissez-faire equilibrium. A policy that would result in the Os purchasing more coverage would tighten rather than relax their revelation constraint. As a result, in order to reveal their type, the Rs would have to purchase even less insurance and so their welfare would worsen. [83]

What is more, the I_O^T through $\hat{E} = \widetilde{C}_O$ lies above C_O^{FI} (the Os' second-best contract). That is, because of the subsidy they receive, the Os' true welfare in the equilibrium arising after the intervention exceeds not only that at the laissez-faire equilibrium but also that at their second-best contract.[84]

This logic does not apply in equilibria where the Os are quantity-constrained.[85] Consider, for example, the pooling equilibrium of Proposition 3 and the separating equilibrium of Proposition 4. In both cases, because the Os underestimate their accident probability, their indifference curve through the contract they choose in equilibrium lies closer to the endowment point, E, than that of the Rs. Thus, Policy 1 would drive out of the market the Os and be harmful for the Rs who would pay the tax without gaining anything. However, if the proportion of the Os is sufficiently high, Policy 2 could lead to a pooling equilibrium on EM' where both the Os and the Rs would be strictly better off. Policy 2 also yields a strict Pareto improvement on the separating equilibrium in Part (ii) of Proposition 3.[86]

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[83] Note that the effectiveness of Policy 1 depends crucially on the fact that if the Os go uninsured, they do take precautions. If the Os never take precautions their indifference curve through $\hat{E} = \widetilde{C}_O$ is flatter than that of the Rs and so they strictly prefer the Rs' consumption allocation, $\widetilde{C}_R$, to $\hat{E} = \widetilde{C}_O$. However, because the Os take no precautions, this allocation is not feasible. In fact, if the Os never take precautions, there exists no intervention policy that yields a Pareto improvement on the equilibrium of Proposition 2.

[84] Notice that the implementation of the Os' second-best allocation is not possible.

[85] It is possible when the Os overestimate the decrease in their accident probability from taking precautions.

[86] The analysis for this last case is similar to that in Proposition 8 and is omitted.

Proposition 8: In the separating equilibrium of Proposition 4, applying Policy 2 yields a strict Pareto improvement if the proportion of the Os is sufficiently high (see Figure 2.10).[87]

Proof: The intervention leads to the breaking of the separating equilibrium. The requirement that the subsidy be received only by those purchasing a contract implying at least the minimum amount of wealth m in the bad state, renders any contract offering less coverage unattractive for the Os. As a result, the pooling equilibrium at $\hat{C}_P$ sustains. Clearly, at $\hat{C}_P$, the Rs are strictly better off. In contrast, because the Os underestimate their accident probability, their perceived utility at the laissez-faire allocation Z_O exceeds that at $\hat{C}_P$. However, since I_O^T passes above Z_O, the Os' true welfare at $\hat{C}_P$ is strictly greater than at Z_O. That is, both the Rs and the Os are strictly better off. Therefore, a strict Pareto improvement has been achieved. *Q.E.D.*

Intuitively, if the proportion of the Os is high, the increase in the per unit premium they are charged is low. As a result, the improvement in their true welfare from the higher coverage more than offsets the welfare losses due to the higher per unit price. Furthermore, the per capita subsidy the Rs receive is high and so the welfare gains arising from the lower per unit premium exceeds the welfare losses due to the lower coverage. Notice, also, that in the laissez-faire equilibrium the Rs choose their second-best contract. Thus, if optimism encourages precautionary effort, it leads to lower per unit premiums and creates the possibility of everyone being strictly better off than in a world where all insurees are realists.[88]

2.5.1 Discussion

The above results provide a justification for the imposition of minimum coverage requirements in insurance markets. Notice, however, that the imposition of minimum standards only may not result in the desired outcome. Consider, for example, Figure 2.10 under the following intervention scheme: the regulator uses no taxes and subsidies but requires that the insurance contracts offered imply at least the minimum

[87] A similar result can be obtained for the pooling equilibrium of Proposition 3.
[88] Obviously, if all insurees were optimists, everyone would be even better off after the intervention.

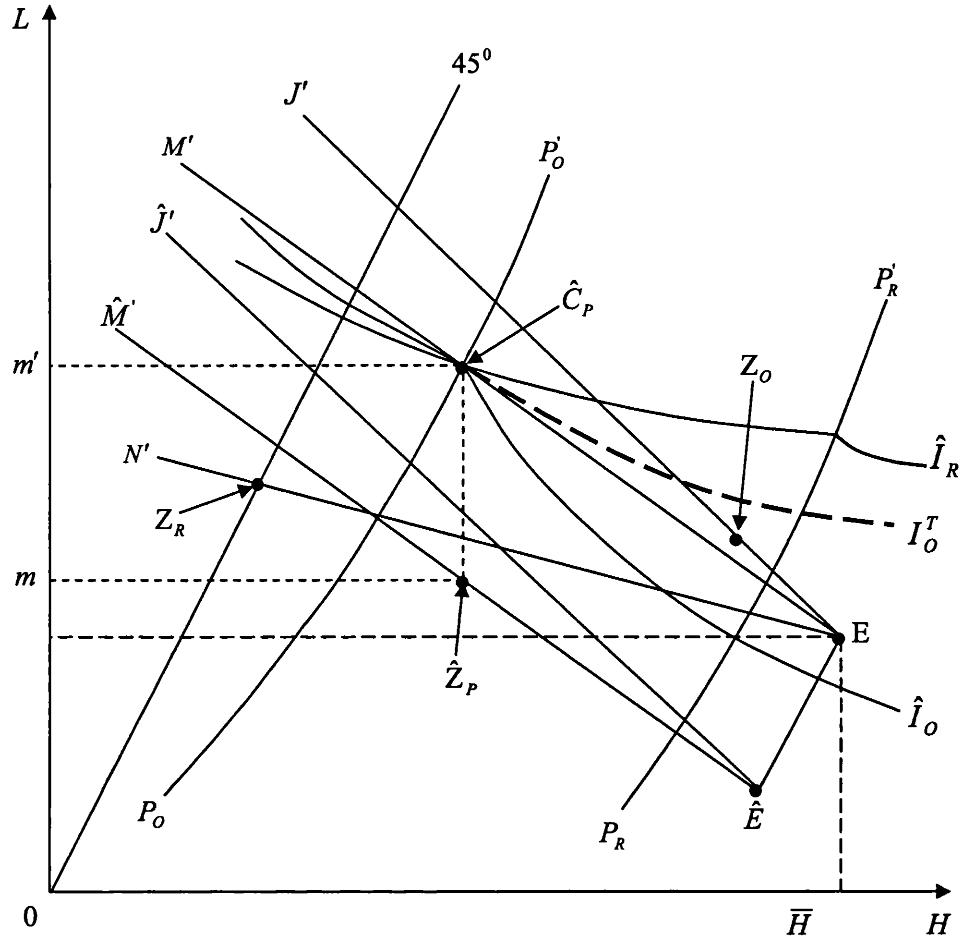

Figure 2.10

amount of wealth m' in the bad state. Since the Os' indifference curve through $\hat{C}_O$ passes below the endowment point E, the Os (the underinsured) will purchase no insurance at all and so they will be worse off than in the laissez-faire equilibrium. In contrast, a combination of minimum standards, taxes and subsidies (Policy 2) will result in the underinsured purchasing more insurance and being strictly better off.

Finkelstein (2002) examines the US market for private health insurance and finds that the imposition of minimum standards has had two effects: First, a decline in the proportion of people with coverage of about 25 percent. Second, a reduction in the amount of insurance purchased by those choosing the most comprehensive policies before their introduction. The results in Proposition 8 are consistent with these empirical findings.[89] Moreover, these results suggest that in order for minimum coverage requirements to achieve their objective, they should be accompanied by a mix of taxes and subsidies.

The positive results of the imposition of minimum coverage requirements in standard asymmetric information models are similar to ours and so are also consistent with these findings. However, the welfare results are quite different. In the standard model, although the social welfare may be higher in the resulting pooling equilibrium, the safe type (the quantity-constrained) is strictly worse off. In contrast, in our model, both types are strictly better off in the pooling equilibrium arising after the intervention. That is, the use of minimum standards is indisputably warranted if, in addition to their type being unobservable, some insurees underestimate their accident probability. Therefore, our model provides a more convincing justification for the imposition of minimum coverage requirements than standard models as well as a case for the use of taxes and subsidies in insurance markets.

2.6 Intervention and Existence of Equilibrium

As we have mentioned, if the proportion of the Rs in the population of insurees is sufficiently high, the equilibria of Propositions 2 and 2B.1 break and a situation arises where no pure-strategy subgame-perfect Nash equilibrium (SPNE) exists.[90] A similar situation arises when the proportion of the Os is sufficiently large in Part (i) of Proposition 3 and Proposition 4. This section explores the possibility of restoring the existence of equilibrium by applying Policy 2 in cases where a laissez-faire SPNE does not exist. For illustration, we consider the configuration yielding the pooling equilibrium of Proposition 3.[91]

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[89] Strictly speaking, in order for the theoretical results to be consistent with the empirical findings a third type should be introduced. This new type should also be optimist but less so than the existing one. In such a case, if only minimum coverage requirements were imposed, the moderately optimist and the realist would choose a pooling contract like $\hat{C}_P$ whereas the most optimistic type would go uninsured. If, however, the minimum standards were accompanied by a mix a taxes and subsidies, there could exist a pooling equilibrium where all three types would have purchased some insurance and would have been strictly better off.

[90] This non-existence situation is similar to that in Rothschild and Stiglitz (1976).

[91] The same logic applies to Propositions 2, 3, and 4 as well as to the Rothschild-Stiglitz model (1976) if the non-existence problem arises.

Proposition 9: In the configuration yielding the pooling equilibrium of Proposition 3, if the proportion of the Os is sufficiently large so that EM' cuts the Os indifference curve through the pooling contract Z_P, then the pooling equilibrium collapses and no SPNE exists.[92] Applying Policy 2, we can restore the existence of the equilibrium. Depending on the size of the tax and the minimum amount of insurance, a unique pooling equilibrium arises on EM' between the tangency point of EM' and the Os indifference curve and the intersection of EM' and $P_O P_O'$.[93] This pooling equilibrium involves the Os taking precautions, the Rs not, and both types purchasing more insurance than in the pooling equilibrium of Proposition 3. Also, the new equilibrium Pareto-dominates the pooling equilibrium of Proposition 3 (see Figure 2.11).

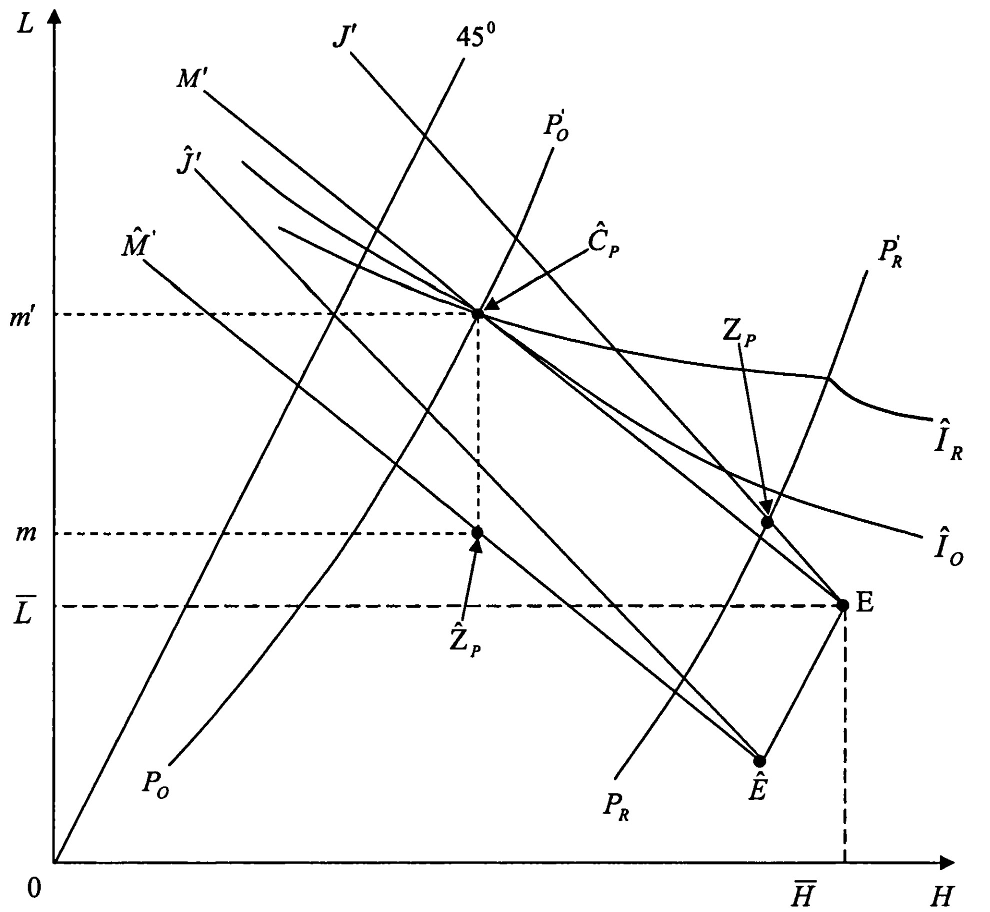

Figure 2.11

[92] However, in all these cases, there exist a Wilson pooling equilibrium (see Wilson (1977)).
[93] If at the intersection of EM' and $P_O P_O'$ the Os indifference curve is flatter than EM', then the pooling equilibrium can only lie at this intersection.

Proof: Since EM' cuts the Os' indifference curve through the pooling contract Z_P, an insurance company can profitably deviate by offering a contract in the area between EM' and the Os' indifference curve through Z_P. This contract attracts both types but since it lies below EM', it implies strictly positive profits for the deviant. As a result, the pooling equilibrium at Z_P breaks. Consider now, for example, the pooling contract $\hat{C}_P$.[94] Without any intervention this contract is not a SPNE. Since the Os indifference curves are steeper than those of the Rs, an insurance company can profitably attract only the Os by offering a contract between $\hat{I}_R$ and $\hat{I}_O$ involving a little less coverage than $\hat{C}_P$.[95] However, after the intervention policy has been applied, the offer of a contract implying less wealth than m is no longer attractive for the Os. The choice of such a contract by an O implies that he receives no subsidy, his consumption bundle lies on the relevant part of $\hat{E}\hat{J}'$ and so he is strictly worse off. Also, any contract offered along $\hat{E}\hat{M}'$ involving a greater amount of insurance attracts only the Rs and so is loss-making. Thus, given the tax and the minimum coverage, $\hat{C}_P$ is the unique pooling equilibrium. Furthermore, the fact that in the area between $P_O P_O'$ and $P_R P_R'$ the Os take precautions but the Rs do not, rules out any separating equilibrium. Therefore, the pooling contract $\hat{C}_P$ is the unique equilibrium. Finally, since the indifference curves of both the Rs and the Os through $\hat{C}_P$, $\hat{I}_R$ and $\hat{I}_O$ respectively, pass above Z_P the perceived utility of both types at $\hat{C}_P$ exceeds that at Z_P. Moreover, since $\hat{C}_P$ involves more coverage than Z_P, the Os underestimate their accident probability and the Os' perceived utility at $\hat{C}_P$ exceeds that at Z_P, their true utility at $\hat{C}_P$ is also greater. Therefore, the pooling equilibrium at $\hat{C}_P$ Pareto-dominates that at Z_P. *Q.E.D.*

If the proportion of the Os is high, the average accident probability does not increase significantly when the Rs take no precautions. Thus, an insurance company can

[94] Notice that $\hat{C}_P$ is the unique Wilson equilibrium without intervention. However, by appropriately choosing the tax and the minimum coverage m, one can support any pooling contract between $\hat{C}_P$ and the intersection of EM' and $P_O P_O'$ along EM' as a SPNE.

[95] The same argument holds true for any pooling contract along the relevant part of EM'.

profitably attract both types by offering a contract involving more insurance than the pooling contract in Proposition 3 and so this pooling equilibrium breaks. Moreover, because the Os not only exhibit a lower true accident probability but also they underestimate it, insurers can always profitably attract them by offering a contract involving a little less insurance. Therefore, there exists no deviant contract which can be supported as a laissez-faire SPNE. However, the imposition of the tax and the requirement that the subsidy is received only by those choosing a contract implying at least the minimum amount of wealth m in the bad state renders unattractive all deviant contracts involving less coverage. As a result, the pooling equilibrium at $\hat{C}_P$ sustains and the existence of the equilibrium is restored. [96]

2.7 Conclusion

We have explored the implications of optimism both on the optimists themselves and their realistic counterparts in a competitive environment where neither the type nor the actions of the insurees are observable. It has been shown that if the degree of optimism is sufficiently high there exist separating equilibria where the Os not only take fewer precautions (high-risk type) but also purchase less insurance than the Rs and both types choose the contract they would have chosen if types were observable. That is, because the Os considerably underestimate their accident probability, their presence has no effect on the choices of the Rs. For lower levels of optimism, depending on whether the Os are more or less willing to take precautions, either the Os or the Rs are quantity-constrained. If optimism encourages precautionary effort, the Os themselves are quantity-constrained whereas the Rs make the same choices as under full information about types. If the Os put less effort into reducing their risk exposure, the roles of the two types are reversed.

Contrary to the conventional wisdom, optimism itself does not necessarily lead to the purchase of less insurance. If types are observable and optimism encourages precautionary effort, the effect of the lower per unit price may more than offset the effect of the underestimation of the accident probability and result in the Os purchasing more insurance than the Rs.

[96] Note that Policy 2 is not unique in this respect. For example, the following intervention scheme could also be used to achieve the same objective: Any insurance contract offered must involve at least the minimum amount of insurance m'.

If the objective is the improvement of the insurees' true welfare, it is possible to find intervention policies that yield strict Pareto gains. If the Rs are quantity-constrained, then a tax on insurance purchase would result in the Os going uninsured, relax their revelation constraint and lead to a strict Pareto improvement. On the contrary, if the Os are quantity-constrained, this logic does not apply. Any attempt to drive out the Rs so as to mitigate the negative externality their presence creates would first drive out the Os. Thus, it would be harmful for the Rs who would pay the tax without gaining anything. However, if the proportion of the Os is sufficiently high, an intervention scheme involving a combination of minimum coverage requirements, taxes and subsidies would lead to a strict Pareto improvement. In the resulting pooling equilibrium the Os subsidise the Rs but purchase more insurance and both types are strictly better off. Because the proportion of the Os is high, the improvement in their true welfare from the higher coverage more than offsets the welfare losses due to the higher per unit premium.

These results provide a justification for the imposition of minimum coverage requirements in insurance markets. However, the imposition of minimum standards only may result in the Os going uninsured and so being worse off than in the laissez-faire equilibrium. In contrast, a combination of minimum standards, taxes and subsidies would lead to the Os purchasing more coverage and so being strictly better off. In fact, Fenkelstein (2002) finds that the imposition of minimum standards in the US private health insurance market resulted in a decline in the proportion of people with coverage of about 25 percent. Our results suggest that in order for minimum coverage requirements to achieve their objective, they should be accompanied by a mix of taxes and subsidies.

Although the positive results of the imposition of minimum coverage requirements in standard asymmetric information models are similar to ours, the welfare results are quite different. In our model, both types are better off in the pooling equilibrium arising after the intervention whereas in standard models the safe type (the quantity-constrained) is strictly worse off. Therefore, our model provides a more convincing justification for the imposition of minimum coverage requirements than standard models as well as a case for the use of taxes and subsidies in insurance markets. Furthermore, intervention schemes involving minimum coverage requirements can be used to create a pure-strategy Nash equilibrium when otherwise none would exist.

Finally, although here we have focused on insurance markets, the introduction of optimism into an asymmetric information framework may have interesting implications for other issues as well. The design of managerial compensation schemes, the choice between self employment and being an employee, the design of securities and other corporate finance issues are only some of them.

Appendix 2A: Curvature of P_iP_i'

The equation of the P_iP_i' locus ($\Delta_i = 0$) is:

$$\Delta_i = (p_F^i - p_0^i)[U(H) - U(L)] - \bar{F} = 0 \qquad \text{(2A.1)}$$

By totally differentiating (2A.1) we obtain:

$$(p_F^i - p_0^i)[U'(H)dH - U'(L)dL] = 0 \;\Rightarrow\; \left.\frac{dL}{dH}\right|_{P_iP_i'} = \frac{U'(H)}{U'(L)} > 0 \qquad \text{(2A.2)}$$

Also P_iP_i' implicitly defines L as a function of H, that is

$$L = g(H) \qquad \text{(2A.3)}$$

Using (2A.2) and taking into account (2A.3) we obtain:

$$\left.\frac{d^2L}{dH^2}\right|_{P_iP_i'} = \frac{U''(H)}{U'(L)} - \frac{U'(H)U''(L)}{[U'(L)]^2}\frac{dg(H)}{dH} = \frac{U''(H)}{U'(L)} - \frac{U'(H)U''(L)}{[U'(L)]^2}\frac{U'(H)}{U'(L)} \Rightarrow$$

$$\left.\frac{d^2L}{dH^2}\right|_{P_iP_i'} = \frac{U'(H)}{U'(L)}\left[\frac{U''(H)}{U'(H)} - \frac{U''(L)}{U'(L)}\frac{U'(H)}{U'(L)}\right] = \frac{U'(H)}{U'(L)}\left[A(L)\frac{U'(H)}{U'(L)} - A(H)\right] \qquad \text{(2A.4)}$$

where $A(\cdot) = -\dfrac{U''(\cdot)}{U'(\cdot)}$ is the coefficient of absolute risk aversion.

P_iP_i' is concave in the (L,H) space iff $\left.\frac{d^2L}{dH^2}\right|_{P_iP_i'} \leq 0$, and using (2A.4) we have:

$$\frac{A(L)}{U'(L)} \leq \frac{A(H)}{U'(H)} \tag{2A.5}$$

Since H>L, increasing or constant absolute risk aversion implies that P_iP_i' is concave in the (L,H) space.

P_iP_i' is strictly convex in the (L,H) space iff $\left.\frac{d^2L}{dH^2}\right|_{P_iP_i'} > 0$ and using (2A.4) we have:

$$\frac{A(L)}{U'(L)} > \frac{A(H)}{U'(H)} \tag{2A.6}$$

Notice that A/U' is the derivative of the inverse of the marginal utility $(1/U')$. This implies that the condition (2A.6) is satisfied iff $(1/U')$ is strictly concave. This condition is stronger than decreasing absolute risk aversion. Therefore, decreasing absolute risk aversion is a necessary but not a sufficient condition for P_iP_i' to be strictly convex in the (L,H) space.

Appendix 2B

Proposition 2B.1: Suppose that, between $P_R P_R'$ and $P_O P_O'$, I_R is steeper than I_O and EM' does not cut I_R through the intersection of EJ' and $P_O P_O'$ (point Z_P). Then, i) if the degree of optimism is sufficiently low so that the Os' indifference curve through Z_P is flatter than EJ' even below $P_O P_O'$ there exists a unique pooling equilibrium where both types take precautions and purchase a strictly positive amount of insurance (see Figure 2B.1). ii) If the degree of optimism is such that the Os' indifference curve is tangent to EJ' between Z_P and E, there exists a unique separating equilibrium where both types purchase strictly positive coverage and take precautions but the Rs buy more insurance than the Os (see Figure 2B.2).[97]

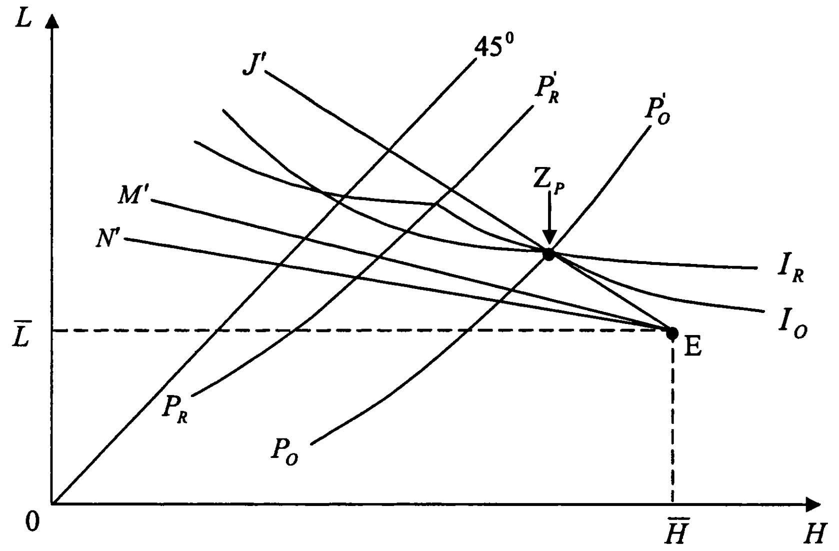

Figure 2B.1

[97] This is true if $(p_F - p_0)$ is sufficiently large (preventive efforts are sufficiently "productive"), the degree of optimism is not very large, and, given the degree of optimism, $p_F^R - p_0^R$ is not much larger than $p_F^O - p_0^O$ (the distance between $P_R P_R'$ and $P_O P_O'$ is not very large).

Proof: i) Clearly, offers above EJ' are loss-making. The same is true for offers above $P_R P_R'$. Between $P_R P_R'$ and $P_O P_O'$ and below EJ' there is no offer that attracts the Rs and does not attract the Os, although there are some offers that attract only the Os. Thus, any offer in this region is unprofitable. Given the equilibrium contract, below EJ' and below $P_O P_O'$ there is no offer that is attractive to either type. Therefore, the pooling contract z_P is the unique equilibrium. ii) Similar arguments can be used to show that the separating pair (z_R, z_O) is the unique equilibrium. *Q.E.D.*

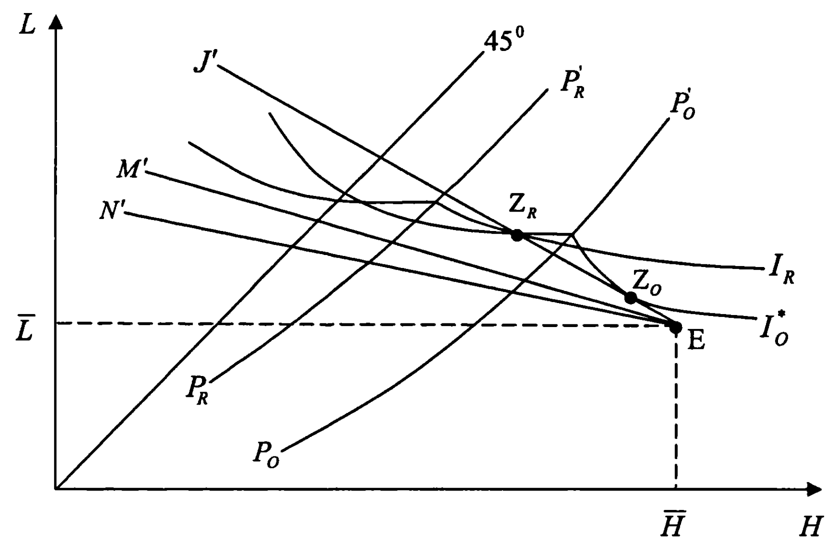

Figure 2B.2

Appendix 2C

Proposition 2C.1: The imposition of a binding maximum coverage requirement can never improve the Os' true welfare.

Proof: It suffices to show that if the Os choose a contract where they do not take precautions, they are strictly better off than at any other contract where they take precautions but they purchase less coverage. Suppose that, given their perceived accident probability, the Os are indifferent between the contract Z_O, where they take precautions and purchase low coverage, and the contract $\hat{Z}_O$, where they do not take precautions but purchase more insurance. Since the I_O^T through $\hat{Z}_O$ passes above Z_O, at $\hat{Z}_O$ the Os' true welfare is strictly greater (see Figure 2C.1). *Q.E.D.*

Intuitively, the Os will choose the contract offering more coverage only if their perceived welfare is greater than at the contract offering lower coverage. Because the Os underestimate their accident probability, their true utility at the high-coverage contract exceeds that at the low-coverage contract even more. For the very same reason, the Os' true welfare at the high-coverage contract may be strictly greater than at the low-coverage contract even if the Os prefer the latter.

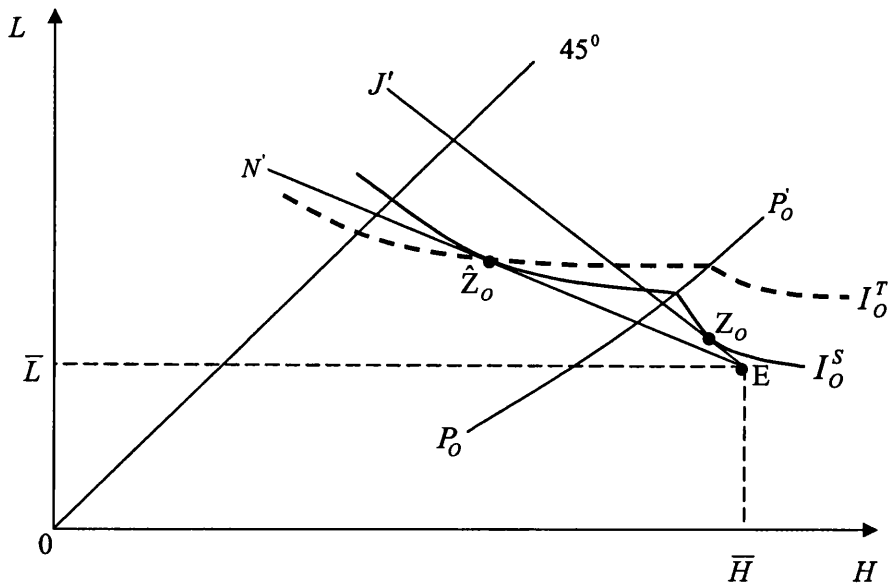

Figure 2C.1

Chapter 3

Asymmetric Information, Heterogeneity in Risk Perceptions and Insurance: An Explanation to a Puzzle

3.1 Introduction

Most recent empirical studies of insurance markets have focused on the relationship between the coverage of the contract and the *ex post* risk (accident rate) of its buyers. The results are mixed. De Meza and Webb (2001) provide casual evidence for a negative relationship in the credit card insurance market.[98] Cawley and Philipson (1999) study of life insurance contracts also shows a negative relationship which, however, is not statistically significant. A similar result is obtained by Chiappori and Salanie (2000)[99] and Dionne, Gourieroux and Vanasse (2001) for the automobile insurance market.[100] In contrast, Finkelstein and Poterba (2000) find a positive relationship in the UK annuities market. Individuals who purchase annuities tend to live longer than those who do not buy.

Starting with the seminal Rothschild-Stiglitz paper (1976), most theoretical models of competitive insurance markets under asymmetric information predict a positive relationship between coverage and the accident probability of the buyer of the contract. This prediction is shared by models of pure adverse selection (e.g. Rothschild and Stiglitz (1976)), pure moral hazard (e.g. Arnott and Stiglitz (1988)) as well as models of adverse selection plus moral hazard (e.g. Chassagnon and Chiappori (1997) and Chiappori et.al. (2002)). In fact, Chiappori et.al. (2002) argue that the positive correlation property is extremely general. However, in a recent paper, de Meza and Webb (2001) provide a model where agents are heterogeneous with respect to their risk aversion and face a moral hazard problem. Also, insurance companies pay

[98] 4.8% of U.K. credit cards are reported lost or stolen each year. The corresponding figure for insured cards is 2.7%.
[99] In the Chiappori and Salanie (2000) study those opting for less coverage purchase the legal minimum of third-party coverage. Dionne et.al (2001) look at contracts with two different levels of deductibles.
[100] All three studies control for observable characteristics known to insurers.

a fixed administrative cost per claim. In this model, there exist a separating and a partial pooling equilibrium predicting a negative relationship but due to the fixed per claim cost the less risk-averse agents go uninsured.

In this chapter, we first show that these (seemingly) contradictory theoretical results can be reconciled. Given that fixed administrative costs are strictly positive, it is shown that the Chiappori et.al. argument holds necessarily true only if, in equilibrium, all agents purchase some insurance. If some agents choose zero coverage, then their assertion is not necessarily true. In this case, there exist separating equilibria that exhibit negative or no correlation between coverage and risk. The presence of these costs results in some agents (the risk tolerant) choosing not to insure. The fact that the administrative costs are now incurred only by the insured agents changes the computation of the premiums which allowed Chiappori et.al (2002) to derive their result.

Therefore, competitive models of insurance markets under asymmetric information can explain the observed negative or no-correlation between coverage and risk in cases where some agents choose zero coverage. For example, the de Meza and Webb (2001) empirical findings are perfectly consistent with the predictions of these models. Furthermore, if the fixed costs per claim are sufficiently high, they can possibly explain similar empirical patterns in cases with more than two events (more than one levels of loss), (e.g. the Chiappori and Salanie (2000) and Dionne et.al (2001) empirical findings).[101] However, their prediction is not consistent with negative or no-correlation in insurance markets where all agents opt for strictly positive coverage and there are just two events (loss/no loss), (e.g. the Cawley and Phillipson (1999) findings).

Jullien, Salanie and Salanie (2001) provide a model where negative correlation between risk and coverage is possible even if all agents purchase some insurance and there is just one level of loss.[102] As far as the insurees are concerned, their model is similar to de Meza-Webb (2001) but in their case the insurer has monopoly power. In order to reveal their type and obtain insurance at a lower per unit price, the less risk-averse insurees accept partial coverage. On the contrary, not only are the more risk-

[101] I would like to thank David de Meza for this point.

[102] Villeneuve (2000) reverses the information structure, he assumes that insurers know better the insuree's accident probability than the insuree himself, and obtains separating equilibria displaying a negative relationship between risk and coverage. In order to convince the high-risk of his type, the monopolistic insurer must offer him a contract that he would not propose to the low-risk type. Profit maximisation then requires that the high-risk type be offered less coverage.

averse agents willing to pay a higher per unit price to purchase more coverage but also take more precautions and so have a lower accident probability. The positive correlation property breaks because the insurer exploits his monopoly power and extracts more surplus from the more risk-averse insurees.

However, insurance markets seem to be fairly competitive and so monopoly is not a good approximation. More importantly, although in Jullien, Salanie and Salanie (2001) the low-risk type is better insured, more coverage is associated with a higher per unit price. Therefore, although they can explain the negative correlation between coverage and risk, the striking observation of Cawley and Phillipson (1999) that insurance premiums exhibit quantity discounts remains a puzzle.

This chapter, by introducing heterogeneity in risk perceptions in a competitive model of asymmetric information, provides an explanation to the puzzling empirical findings. On the one hand, several empirical studies by psychologists indicate that the majority of people tend to be unrealistically optimistic, in the sense that overestimate their ability and the outcome of their actions and underestimate the probability of various risks.[103] [104] On the other hand, Viscusi (1990) finds that more individuals overestimate the risk of lung-cancer associated with smoking than underestimate it and, on average, they greatly overestimate it.[105] Also, those who perceive a higher risk are less likely to smoke. As these studies indicate, regardless of the direction of the bias, people do hold different beliefs about the same or similar risks.[106]

The more optimistic (henceforth Os) agents underestimate their accident probability both in absolute terms and relative to the less optimistic ones (henceforth Rs) and so purchase less insurance. They also tend to be less willing to take precautions. This gives rise to separating equilibria exhibiting negative or no correlation between coverage and risk. Two examples of these equilibria are presented where both the Os and the Rs purchase some insurance.

The first equilibrium predicts both negative correlation between coverage and risk and that per unit premiums fall with the quantity of insurance purchased. The Os not only take fewer precautions (high-risk type) but also purchase less coverage than the

[103] For a survey see Weinstein and Klein (1996). See also de Meza and Southey (1996) and Manove and Padilla (1999) for a discussion of entrepreneurial optimism.

[104] See De Bondt and Thaler (1995) for a survey of the behavioural finance literature.

[105] There is some evidence that people overestimate their accident probability when it is objectively small (e.g. Kahneman and Tversky (1979)).

[106] Given that agents may have different information sets or observe different signals, heterogeneity in risk perceptions is not necessarily inconsistent with rationality (or even rational expectations).

Rs. Competition among insurance companies then implies that the Os also pay a higher per unit premium. Because they underestimate their accident probability, the Os purchase low coverage at a high per unit price, although contracts offering more insurance at the same or even lower per unit price are available.

The second equilibrium exhibits no correlation between coverage and risk and involves the Rs being quantity-constrained. In order to reveal their type, the Rs accept lower coverage than they would have chosen under full information about types. Moreover, if we allow for fixed administrative costs, this equilibrium displays a negative relationship between coverage and per unit premiums. Since both types take precautions they have the same accident probability and so are charged the same marginal price. But the fact that the Os purchase less coverage implies that their total per unit premium is higher.

These results have several interesting implications. First, they explain both puzzling empirical findings reported by Cawley and Phillipson (1999): The negative or no correlation between coverage and risk and the fact that insurance premiums display quantity discounts.

Second, Cawley and Philipson (1999), Chiappori and Salanie (2000) and Dionne et.al. (2001) argue that the no-correlation empirical findings imply that there is no (risk-related) adverse selection. Thus, there are no information barriers to trade in the life and automobile insurance markets under study. Through underwriting, appropriate risk classification and other procedures, insurers can distinguish risks and no additional self-selection mechanism or government intervention is needed. The result in the latter equilibrium suggests that their assertion is not generally true.

Insurance companies may be able to distinguish risks in cases where the accident probability is exogenous. However, in most cases, accident probabilities are endogenous and are affected by the insurees' actions which are unobservable and determined by the insurees' personal characteristics. Although insurers can detect some of these characteristics, it is highly unlikely that they can identify all of them (e.g. degree of risk aversion, risk perceptions). If insurees differ with respect to their risk perceptions and types are hidden, there exist equilibria involving some agents being quantity-constrained even if the data show no correlation between coverage and the accident rate. Furthermore, in these cases, there exist intervention policies that yield a strict Pareto improvement on the laissez-faire equilibrium.

Third, they allow us to empirically distinguish our approach from standard asymmetric information models. To this end, we rely on a very general result derived by Chiappori et.al (2002). If an agent chooses one contract over another offering more coverage, then it must be true that his accident probability under the contract chosen is strictly lower than the per unit premium of the additional coverage offered by the other contract. This is a revealed preference argument. Its validity is independent of the market structure or whether some agents go uninsured. However, because some agents underestimate their accident probability, this prediction fails in both equilibria presented in this chapter.

This chapter is organised as follows. In Section 2 we present a simplified version of the Chiappori et.al. framework and show that if some agents choose zero coverage, the relationship between coverage and risk is not necessarily positive. In Section 3, we present a model where agents differ with respect to their risk perceptions and face a moral hazard problem. Section 4 provides a diagrammatic proof for the existence of the two separating equilibria described above. In Section 5, we consider the welfare properties of the latter equilibrium. Section 6 deals with the empirical implications of our results. Finally, section 7 concludes.

3.2 Reconciliation of Existing Results

To show that the de Meza-Webb (2001) and Chiappori et.al (2002) results are consistent, we employ a simplified version of the latter model. There are two states of nature: good and bad. In the good state the agent incurs no loss whereas in the bad state he incurs a loss of D_θ. The parameter θ represents all the characteristics of the agent (potential insuree) that are his private information (risk, risk aversion, loss, etc). An agent of type θ may privately choose his loss probability $1-p$ in some subset of $[0,1]$. This choice implies a prevention cost that is assumed to be a negative function of the loss probability. In pure adverse selection models this subset is a singleton whereas in moral hazard models where agents choose their preventive effort level, this subset may include two or more points. A contract consists of coverage and premium: $C=(\lambda y, y), \lambda > 1$. The ex post risk of an insuree is a function of the contract he chooses. The average ex post risk of insurees choosing contract C is $1-p(C)$. Also, the following assumptions are made:

Assumption 1: For all contracts offered and all agent types overinsurance is ruled out by assuming $\lambda y \leq D_\theta$.

Assumption 2: Agents are risk averse (in the sense that they are averse to mean-preserving spreads on wealth).

Assumption 3: Insurance companies are risk neutral, and incur a cost per contract $c \geq 0$ and a cost per claim $c' \geq 0$. So, the expected profit of an insurance company offering contract $C = (\lambda y, y)$ to an agent with ex post risk $1-p$ is

$$\pi = y - (1-p)(\lambda y + c') - c$$

Profit Monotonicity (PM) Assumption: If two contracts C_1 and C_2 are chosen in equilibrium and $\lambda_1 y_1 < \lambda_2 y_2$, then $\pi(C_1) \geq \pi(C_2)$.[107]

We now generalize the Chiappori et.al result to cover cases where some agents go uninsured.

Proposition 1: Under Assumptions 1 to 3 and PM if two contracts C_1 and C_2 are chosen in equilibrium and $\lambda_1 y_1 < \lambda_2 y_2$, then $1-p(C_1) < 1-p(C_2)$ is necessarily true if $0 < \lambda_1 y_1 < \lambda_2 y_2$ and $c, c' \geq 0$. If $\lambda_1 y_1 = 0$ and $c > 0$ or $c' > 0$ or $c, c' > 0$, then $1-p(C_1) < 1-p(C_2)$ is not necessarily true, $1-p(C_1) \geq 1-p(C_2)$ is also possible.

Proof: The proof is done through two lemmas.

Lemma 1: Suppose an agent θ chooses the contract $C_1 = (\lambda_1 y_1, y_1) = (0,0)$ over the contract $C_2 = (\lambda_2 y_2, y_2)$ where $\lambda_2 y_2 > 0$. Then it must be true that

$$1 - p(C_1) < \frac{1}{\lambda_2} = \frac{y_2}{\lambda_2 y_2}$$

[107] Assumptions 1-3 and the profit monotonicity (PM) assumption are taken from Chiappori et al.

Proof: See Appendix 3A.

Intuitively, given risk aversion, if the per unit premium under C_2, $1/\lambda_2$, were less than his accident probability under C_1, the agent would be strictly better off taking contract C_2, rather than going uninsured, while keeping $1-p(C_1)$.

Lemma 2: Suppose $C_1=(\lambda_1 y_1, y_1)=(0,0)$ and $C_2=(\lambda_2 y_2, y_2)$ are chosen in equilibrium. If $c>0$ or $c'>0$ or $c,c'>0$, then it may be true that $1-p(C_1)\geq 1-p(C_2)$. If $\lambda_2 y_2>\lambda_1 y_1>0$, then $1-p(C_1)<1-p(C_2)$ is always true.

Proof: By Lemma 1 we have

$$1-p(C_1)<\frac{y_2}{\lambda_2 y_2} \quad \Rightarrow \quad y_2-(1-p(C_1))\lambda_2 y_2>0 \tag{3.1}$$

In this case, $\pi(C_1)$ is (identically) equal to zero. Therefore,

$$\pi(C_1)=0<y_2-(1-p(C_1))\lambda_2 y_2 \tag{3.2}$$

The expected profit for an insurance company offering contract C_2 is

$$\pi(C_2)=y_2-(1-p(C_2))(\lambda_2 y_2+c')-c \tag{3.3}$$

Given (PM), $\pi(C_1)=0$, and the fact that in equilibrium profits cannot be negative, it follows that $\pi(C_1)=\pi(C_2)=0$. Then, using (3.2) and (3.3) we obtain:

$$[1-p(C_2)-(1-p(C_1))](y_2\lambda_2+c')>-[(1-p(C_1))c'+c] \tag{3.4}$$

Given $\lambda_2 y_2>0$ and $c>0$ or $c'>0$ or $c,c'>0$, it is clear from (3.4) that it may well be true that $1-p(C_1)\geq 1-p(C_2)$.

If $\lambda_2 y_2 > \lambda_1 y_1 > 0$, using similar arguments we have:

$$1 - p(C_1) < \frac{y_2 - y_1}{\lambda_2 y_2 - \lambda_1 y_1} \Rightarrow (y_2 - y_1) - (1 - p(C_1))(\lambda_2 y_2 - \lambda_1 y_1) > 0 \qquad (3.5)$$

Using the expected profit functions $\pi(C_i)$, $i = 1,2$, and (3.5) we obtain:

$$\pi(C_1) - \pi(C_2) < [(1 - p(C_2)) - (1 - p(C_1))](\lambda_2 y_2 + c') \qquad (3.6)$$

Given (PM), (3.6) implies $1 - p(C_1) < 1 - p(C_2)$. *Q.E.D.*

Intuitively, if all agents purchase some insurance, they all pay the fixed administrative costs through a higher per unit premium. Also, given risk aversion, if an agent chooses the low-coverage contract, C_1, it must be true that the per unit price of the additional coverage offered by C_2 exceeds his accident probability under C_1. Otherwise, the agent would be strictly better off by choosing C_2 while keeping the same accident probability. Hence, because insurance companies' profit on C_1 is not less than on C_2, the accident probability of an agent choosing C_1 must be strictly lower than an agent choosing C_2.

However, if some agents go uninsured, they do not incur these fixed costs. As a result, although their accident probability is lower than the per unit premium paid by the insured, it is not necessarily lower than the insured's accident probability because the per unit premium paid by the latter covers both their accident probability and the fixed costs. Therefore, the negative correlation equilibria obtained by de Meza and Webb (2001) are perfectly consistent with the predictions of Chiappori et.al. (2002) general framework.

In summary, if some agents choose zero coverage, then both negative and no correlation between coverage and risk can arise. However, if, in equilibrium, all agents choose contracts offering strictly positive coverage, then asymmetric information plus competition among insurance companies imply a strictly positive relationship. Therefore, competitive models of insurance markets under asymmetric information can explain the observed negative or no-correlation between coverage and risk if the comparison is between those who actually purchase some insurance and those choosing not to insure (e.g. the de Meza and Webb (2001) empirical findings). Furthermore, if the fixed costs per claim are sufficiently high, they can possibly explain similar empirical patterns in cases with more than one levels of loss (e.g. the Chiappori and Salanie (2000) and Dionne et.al (2001) empirical findings). However, their prediction is not consistent with negative or no-correlation in insurance markets where all agents opt for strictly positive coverage and there is just one loss level (e.g. the Cawley and Philipson (1999) findings).

Jullien, Salanie and Salanie (2001) provide a model where negative correlation between risk and coverage is possible even if all agents purchase some insurance and there just two events (loss/no loss). As far as the insurees are concerned, their model is similar to de Meza-Webb (2001) but in their case the insurer has monopoly power. In order to reveal their type and obtain insurance at a lower per unit price, the less risk-averse insurees accept partial coverage. In contrast, not only are the more risk-averse agents willing to pay a higher per unit price to purchase more coverage but also take more precautions and so have a lower accident probability. The positive correlation property breaks because the insurer exploits his monopoly power and extracts more surplus from the more risk-averse insurees.

However, insurance markets seem to be fairly competitive and so monopoly is not a good approximation. More importantly, although in Jullien, Salanie and Salanie (2001) the low-risk type is better insured, more coverage is associated with a higher per unit price. Therefore, although they can explain the negative correlation between coverage and risk, the striking observation of Cawley and Phillipson (1999) that insurance premiums exhibit quantity discounts remains a puzzle.

This chapter, by introducing heterogeneity in risk perceptions in a competitive model of asymmetric information, provides an explanation to both puzzling empirical findings. Most standard asymmetric information models of insurance markets (including the Chiappori et.al. (2002) model) implicitly assume that all insurees have an accurate estimate of their accident probability[108] (given the precautionary effort level). Our model retains the assumption of perfect competition among insurance companies but allows agents (insurees) to have different perceptions of the same risk. Except for the misperception of risk, all agents are fully rational. They aim at

[108] Villeneuve (2000) is an exception.

maximising their (perceived) utility and understand the nature and implications of market interactions.

3.3 The Model

There are two states of nature: good and bad. In the good state there is no loss whereas in the bad state the individual (insuree) suffers a gross loss of D. Before the realisation of the state of nature all individuals have the same wealth level, W. Also, all individuals are risk averse and have the same utility function but differ with respect to their perception of the probability of suffering the loss. There are two types of individuals, the Rs and the Os. The Rs have an accurate estimate of their true loss probability whereas the Os underestimate it.[109]

Furthermore, all agents can affect the true loss probability by undertaking preventive activities. Given the level of precautionary effort, the true loss probability is the same for both types. We consider the case where agents either take precautions or not (two effort levels). If an individual takes precautions ($F_i = F$), he incurs a utility cost of F and his true probability of avoiding the loss $p(F_i)$ is p_F. If he takes no precautions ($F_i = 0$), his utility cost is 0 but his true probability of avoiding the loss $p(F_i)$ is p_0, where $p_F > p_0$.

Now, let $p^i = p(F_i, K_i)$ be the (perceived) probability function. Where K_i is the degree of "optimism" and takes two values: 1 for the Rs $(K_R = 1)$, and $\overline{K} > 1$ for the Os $(K_O = \overline{K} > 1)$. This probability function is assumed to be strictly increasing in K_i. As a result, the following relationships are true:

$$p_j^R = p(F_i, K_R) = p(F_i, 1) = p(F_i) = p_j, \qquad i = O, R, \qquad j = F, 0 \tag{3.7}$$

$$p_j^O = p(F_i, K_O) = p(F_i, \overline{K}) > p(F_i) = p_j, \qquad i = O, R, \qquad j = F, 0 \tag{3.8}$$

[109] For expositional simplicity, we assume that the more optimistic are optimists whereas the less optimistic are realists. However, all the results go through if two types are respectively optimists and pessimists.

where p_j is the true probability of avoiding the loss.

In this environment, the (perceived) expected utility of an agent i is given by:

$$EU_i(F_i, K_i, y_i, \lambda_i, W) = p_j^i U(W - y) + (1 - p_j^i) U(W - D + (\lambda - 1)y) - F_i,$$
$$j = F, 0 \quad i = O, R \qquad (3.9)$$

where W: insuree's initial wealth

D: gross loss

y: insurance premium

$(\lambda - 1)y$: net payout in the event of loss, $\lambda > 1$

λy: coverage (gross payout in the event of loss)

Hence, the increase in (perceived) expected utility from taking precautions is:

$$\Delta_i = (p_F^i - p_0^i)[U(W - y) - U(W - D + (\lambda - 1)y)] - F, \qquad i = O, R \qquad (3.10)$$

where U is strictly concave and $W - y$, $W - D + (\lambda - 1)y$ are the wealth levels in the good and the bad state respectively.

There are two risk neutral insurance companies involved in Bertrand competition. Insurance companies know the true accident probability (given the precautionary effort level) and the perceived accident probabilities of the Os and Rs but they can observe neither the type nor the actions of each insuree. They also know the cost for the insuree corresponding to each precautionary effort level, the utility function of the insurees and the proportion of the Os and Rs in the population. In order to make the distinction between the results under different risk perceptions and those of the standard competitive models of asymmetric information clearer, we assume that the costs of processing claims (or underwriting costs) are zero.[110]

The insurance contract $(y, \lambda y)$ specifies the premium y and the coverage λy. As a result, since insurance companies have an accurate estimate of the true accident probability, the expected profit of an insurer offering such a contract is:

$$\pi = p_j y - (1 - p_j)(\lambda - 1)y, \qquad j = F, 0 \qquad (3.11)$$

[110] All results go through if fixed administrative costs are strictly positive but not very large.

Equilibrium

Insurance companies and insurees play the following two-stage screening game:

Stage 1: The two insurance companies simultaneously make offers of sets of contracts $(y, \lambda y)$. Each insurance company may offer any finite number of contracts.

Stage 2: Given the offers made by the insurers, insurees apply for at most one contract from one insurance company. If an insuree's most preferred contract is offered by both insurance companies, he takes each insurer's contract with probability ½. The terms of the contract chosen determine whether the insuree will take unobservable precautions.

We only consider pure-strategy subgame-perfect Nash equilibria (SPNE). Depending on parameter values, three kinds of equilibria can arise: separating, full-pooling and partial-pooling. In this chapter, we only present the two most interesting separating equilibria.

A pair of contracts $z_O = (y_O, \lambda_O y_O)$ and $z_R = (y_R, \lambda_R y_R)$ is an equilibrium if the following conditions are satisfied:

i) The revelation constraints

$$\begin{aligned} &EU_R(z_R) \geq EU_R(z_O) \\ &EU_O(z_O) \geq EU_O(z_R) \end{aligned} \tag{3.12a}$$

ii) The effort incentive constraints

$$F_i = \begin{cases} F & \text{if } \Delta_i \geq 0, \quad i = O, R \\ 0 & \text{otherwise} \end{cases} \tag{3.12b}$$

with Δ_i defined in (3.10).

iii) The participation (or IR) constraints of both types:

$$EU_i(z_i) \geq EU_i(z_0), \quad i = O, R \tag{3.12c}$$

where $z_0 = (y, \lambda y) = (0,0)$

iv) Profit maximisation for insurance companies:

- No contract in the equilibrium pair (z_O, z_R) makes negative expected profits.
- No other set of contracts introduced alongside those already in the market would increase an insurer's expected profits.

3.4 Negative and Zero Correlation Equilibria

Let $H = W - y$ and $L = W - D + (\lambda - 1)y$ denote the income of an insuree who has chosen the contract $(y, \lambda y)$ in the good and bad state respectively. Let also $\overline{H} = W$ and $\overline{L} = W - D$ denote the endowment of an insuree after the realisation of the state of nature.

3.4.1 Effort Incentive Constraints

Let us first consider the moral hazard problem an insuree of type i faces. A given contract $(y, \lambda y)$ will induce an agent of type i to take precautions if

$$(p_F^i - p_0^i)[U(H) - U(L)] \geq F \quad \Leftrightarrow \quad \Delta_i \geq 0, \quad i = O, R \tag{3.13}$$

Let $P_i P_i'$ be the locus of combinations (L, H) such that $\Delta_i = 0$. Since F, $U' > 0$, the $P_i P_i'$ locus lies entirely below the 45^0 line in the (L, H) space. This locus divides the (L, H) space into two regions: On and below the $P_i P_i'$ locus the insurees take precautions (this is the set of effort incentive compatible contracts) and above it they do not. The slope and the curvature of P_iP_i' in the (L, H) space are given respectively by:

$$\left.\frac{dL}{dH}\right|_{P_iP_i'} = \frac{U'(H)}{U'(L)} > 0 \qquad \text{since } U' > 0 \tag{3.14}$$

$$\left.\frac{d^2L}{dH^2}\right|_{P_iP_i'} = \frac{U'(H)}{U'(L)}\left[A(L)\frac{U'(H)}{U'(L)} - A(H)\right] \tag{3.15}$$

where $A(L) = -\frac{U''(L)}{U'(L)}$ is the coefficient of absolute risk aversion.

Since both types have the same utility function, it is clear from the above formulas that the shape of P_iP_i' is independent of the type of the insuree. In addition, P_iP_i' is upward sloping. Also if $U(\cdot)$ exhibits either increasing or constant absolute risk aversion P_iP_i' is strictly concave. If $U(\cdot)$ exhibits decreasing absolute risk aversion, it can be either concave or convex. (See Appendix 2A for a necessary and sufficient condition in order for P_iP_i' to be strictly convex).

However, the position of P_iP_i' does depend upon the insuree's type. Although the Os overestimate their probability of avoiding the loss at any given precautionary effort level, they may either overestimate or underestimate the increase in that probability from choosing a higher preventive effort level. Though both cases are possible, the latter seems to be more reasonable especially if, given that no precautions are taken, the perceived probability of avoiding the accident is high.[111] In this chapter, the analysis is carried out under the assumption that the latter case is relevant. In particular, the following assumption is made:

<u>Assumption 1</u>: $p_F^R - p_0^R > p_F^O - p_0^O$

That is, the Rs' set of effort incentive compatible contracts is strictly greater than that of the Os. It is also assumed that

[111] This assumption is also consistent with Viscusi's (1990) finding that those who perceive a higher risk are less likely to smoke. The more pessimistic agents take more precautions.

Assumption 2: $(p_F^i - p_0^i)\left[U(\overline{H}) - U(\overline{L})\right] > F, \quad i = O, R$

Assumption 2 implies that both $P_R P_R'$ and $P_O P_O'$ pass above the endowment point, and so the effective set of effort incentive compatible contracts is not empty for either type. If Assumption 2 does not hold for either type, the corresponding type never takes precautions.

Two points must be stressed here. First, Assumption 1 is required for but does not necessarily imply a negative relationship between coverage and ex post risk. It may well be the case that Assumption 1 holds and a separating or a partial pooling equilibrium arises exhibiting a positive relationship.[112] Second, although, Assumption 1 is necessary for the negative correlation prediction, Assumption 2 does not need to hold for the Os. In fact, this result obtains more easily if the direction of inequality in Assumption 2 is reversed for the Os. That is, if the Os never take precautions. On the contrary, the no-correlation result requires Assumption 2 but not Assumption 1.[113] It obtains even if the Os overestimate not only their probability of avoiding the accident but also the increase in that probability from taking precautions.

3.4.2 Indifference Curves

The indifference curves, labelled I_i, are kinked where they cross the corresponding $P_i P_i'$ locus. Above $P_i P_i'$, insurees of the the i-type do not take precautions, their perceived probability of avoiding the loss is p_0^i, and so the slope of I_i is:

$$\left.\frac{dL}{dH}\right|_{I_i, p=p_0} = -\frac{p_0^i}{1-p_0^i}\frac{U'(H)}{U'(L)} \qquad i = O, R \tag{3.16}$$

On and below $P_i P_i'$ insurees of the i-type do take precautions, their perceived probability of avoiding the loss rises to p_F^i and so the slope of I_i becomes:

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[112] Chapter 2 provides some examples.

[113] The no-correlation result obtains even if the direction of the inequality in Assumption 2 is reversed. However, this assumption would imply that both types never take precautions and so this case is not very interesting.

$$\left.\frac{dL}{dH}\right|_{I_i, p=p_F} = -\frac{p_F^i}{1-p_F^i}\frac{U'(H)}{U'(L)} \qquad i = O, R \tag{3.17}$$

Hence, just above P_iP_i' the i-type indifference curves become flatter.

Furthermore, because the Os underestimate their accident probability, at any given identical preventive effort level and (L, H) pair, the Os indifference curve is steeper in the (L, H) space. Intuitively, the Os are less willing to exchange consumption in the good state for consumption in the bad state because their perceived probability of the bad state occurring is lower than that of the Rs.

3.4.3 Insurers' Zero-profit Lines (Offer Curves)

Using the definitions $H = W - y$ and $L = W - D + (\lambda - 1)y$, and the fact that insurance companies have an accurate estimate of the true accident probabilities, given the precautionary effort level, the insurers' expected profit function becomes:

$$\pi = p_j(W - H) - (1 - p_j)(L - W + D) \tag{3.18}$$

The zero-profit lines are given by:

$$L = \frac{1}{1-p_j}W - \frac{p_j}{1-p_j}H - D, \qquad j = F, 0 \tag{3.19}$$

Conditional on the preventive effort level chosen by the two types of insurees, there are three zero-profit lines with slopes:

$$\left.\frac{dL}{dH}\right|_{\pi=0} = -\frac{p_0}{1-p_0} \qquad \text{(EN' line)} \tag{3.20}$$

$$\left.\frac{dL}{dH}\right|_{\pi=0} = -\frac{p_F}{1-p_F} \qquad \text{(EJ' line)} \tag{3.21}$$

$$\left.\frac{dL}{dH}\right|_{\pi=0} = -\frac{q}{1-q} \qquad \text{(EM' line (pooled –line))} \qquad (3.22)$$

where $q = \mu p_j + (1-\mu)p_k$, $j = F,0$, $k = F,0$, and μ is the proportion of the Rs in the population of insurees.

Also, at $H = \overline{H} = W$, Eq. (3.19) becomes:

$$L = W - D \qquad (3.23)$$

Eq. (3.23) is independent of the value of p_j. This implies that all three zero-profit lines have the same origin (the endowment point, E).

We can now provide a diagrammatic proof of the existence of the two separating equilibria. The negative correlation result is shown in Proposition 2 whereas Proposition 3 provides an example of a separating equilibrium exhibiting no-correlation between coverage and ex post risk.

Proposition 2: If the Os' indifference curve tangent to EN', I_O^*, passes above the intersection of EJ' and $P_R P_R'$ and meets $P_O P_O'$ above EJ', then there exists a unique separating equilibrium (z_R, z_O) where the Rs take precautions whereas the Os do not. Both types choose strictly positive coverage but the Rs buy more than the Os (see Figure 3.1).[114]

Proof: We test whether (z_R, z_O) is an equilibrium by considering deviations. Clearly, the Os strictly prefer z_O to z_R. Offers above EJ' are clearly loss-making. Similarly, offers above $P_R P_R'$ either do not attract any type or, if they do, are unprofitable. Below EJ' and below $P_R P_R'$ there is no offer that attracts the Rs but

[114] This is true if $(p_F - p_0)$ is sufficiently small and the degrees of optimism and risk aversion are sufficiently large.

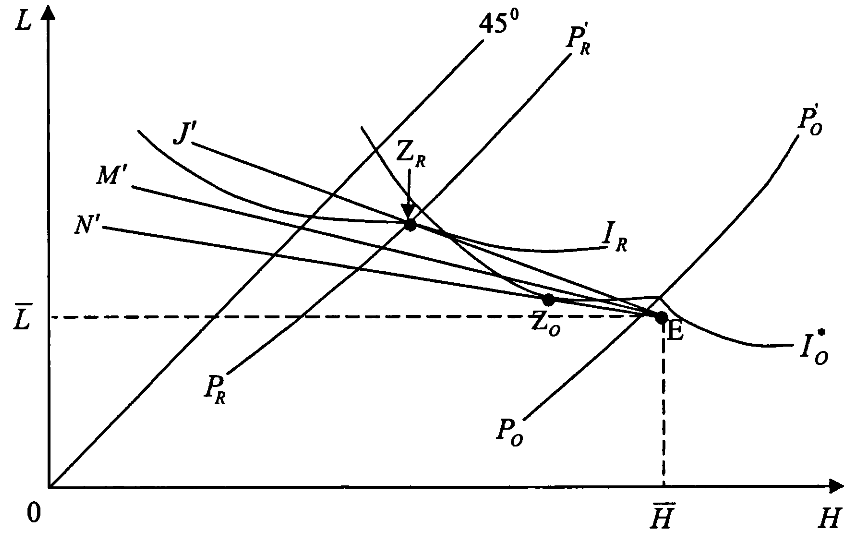

Figure 3.1

there are some offers that attract the Os and so are unprofitable (given the equilibrium contract z_R, the Rs are attracted only by contracts that lie above EJ' which are, of course, loss-making). So, there is no profitable deviation and the (z_R, z_O) pair is the unique separating equilibrium. The fact that I_O^* passes above z_R rules out any pooling equilibrium. Therefore, (z_R, z_O) is the unique equilibrium. *Q.E.D.*

Intuitively, given the contracts offered, because the Os considerably underestimate the reduction in their accident probability from taking precautions, they choose to take no precautions. Also, although insurance is offered at actuarially fair terms, because they underestimate their accident probability, the Os underinsure choosing a contract with low coverage while contracts with higher coverage are available at the same or even lower per unit premium.

That is, this separating equilibrium has two interesting features. The Os not only purchase less coverage than the Rs but also take fewer precautions and so their accident probability is higher. Competition among insurance companies then implies that the Os also pay a higher per unit premium. Therefore, this equilibrium is consistent with both the negative correlation between coverage and risk (point estimate) and the fact that per unit premiums fall with the quantity of insurance purchased as reported by Cawley and Philipson (1999).

Proposition 3: Suppose EM' does not cut I_R through the intersection point of EJ' and I_O^* (the Os' indifference curve tangent to EJ' below P_OP_O' and to the left of E). Then there exists a unique separating equilibrium where both types purchase strictly positive coverage and take precautions but the Rs buy more insurance than the Os. That is, this equilibrium exhibits no correlation between coverage and the accident probability (see Figure 3.2).[115]

Proof: Consider the following deviations. Clearly, offers above EJ' are loss-making. The same is true for offers above P_RP_R'. Between P_RP_R' and P_OP_O' and below EJ' there is no offer that attracts Rs and does not attract the Os, although there are some offers that attract only the Os. Thus, any offer in this region is unprofitable. Given the equilibrium contracts, below EJ' and below P_OP_O' there is no offer that is attractive to either type. Hence, the pair (z_R, z_O) is the unique separating equilibrium. Furthermore, the fact that EM' does not cut I_R below P_RP_R' rules out any pooling equilibrium. Therefore, the pair (z_R, z_O) is the unique equilibrium. *Q.E.D.*

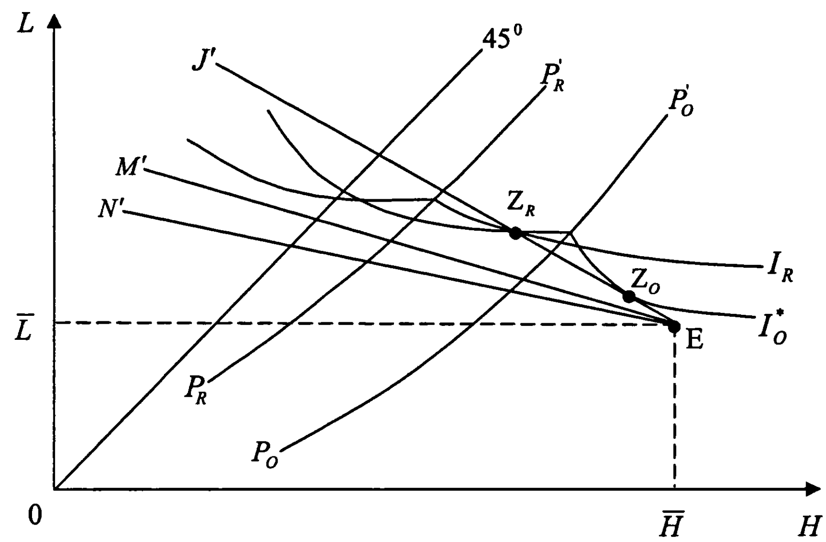

Figure 3.2

[115] This is true if $(p_F - p_0)$ is sufficiently large (preventive efforts are sufficiently "productive"), the degree of optimism is not very large, and, given the degree of optimism, $p_F^R - p_0^R$ is not much larger than $p_F^O - p_0^O$ (the distance between P_RP_R' and P_OP_O' is not very large).

Although they purchase more coverage than the Os, the Rs are quantity-constrained. Under full information about types, the Rs would have purchased the contract at the intersection of $P_R P_R'$ and EJ', instead of z_R, which involves more insurance. However, since types are hidden, this contract is not offered because it violates the Os' revelation and effort incentive constraints and so is loss-making for the insurance companies. In order to reveal their type, the Rs accept lower coverage than they would have chosen if types were observable.

Strictly speaking, the no-correlation prediction is unlikely to be observed in practice. However, if one interprets it as a failure to reject the no-correlation null, then it is consistent with the findings of Cawley and Philipson (1999), Chiappori and Salanie (2000) and Dionne et.al. (2001) about the relationship between coverage and the accident rate. Furthermore, if we allow for administrative and/or underwriting costs, this equilibrium also explains the negative relationship between coverage and per unit premiums (see Figure 3.3). Since both types take precautions they have the same accident probability and so are charged the same marginal price. But the fact that the Os purchase less coverage implies that their total per unit premium is higher. In fact, Cawley and Philipson (1999) find that a fixed production (underwriting) cost and a constant marginal cost explain almost all risk-adjusted variation in prices.

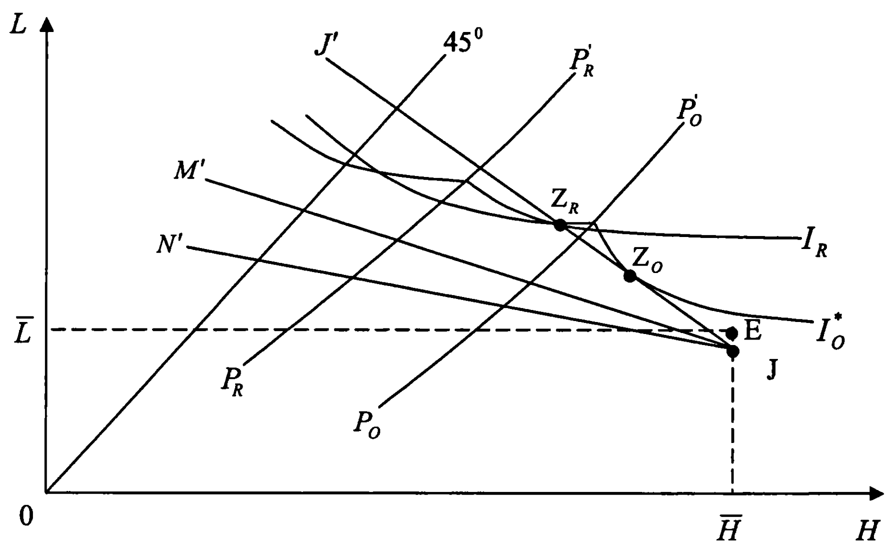

Figure 3.3

3.5 Welfare Implications

Cawley and Philipson (1999), Chiappori and Salanie (2000) and Dionne et.al. (2001) argue that no correlation between coverage and ex post risk implies that there is no (risk-related) adverse selection. As a result, there are no information barriers to trade in the life and automobile insurance markets under study. Through underwriting, appropriate risk classification and other procedures, insurers can distinguish risks and no additional self-selection mechanism or government intervention is needed. However, the separating equilibrium of Proposition 3 suggests that their assertion is not generally true.

Insurance companies may be able to distinguish risks in cases where the accident probability is exogenous. However, in most cases, accident probabilities are endogenous and are affected by the insurees' actions which are unobservable and determined by the insurees' personal characteristics. Although insurers can detect some of these characteristics, it is highly unlikely that they can identify all of them (e.g. degree of risk aversion, risk perceptions). If insurees differ with respect to their risk perceptions and types are hidden, there exist equilibria involving some agents being quantity-constrained even if the data show no correlation between coverage and the accident rate (e.g. equilibrium of Proposition 3).

In this section we explore the welfare properties of this equilibrium. Because some of the insurees, the Os, underestimate their accident probability, the definition of the efficiency of the equilibrium is not straightforward. The very presence of the Os raises the question of what is the appropriate efficiency criterion. Should we employ objective probabilities (true expected utility) or subjective probabilities (perceived expected utility)? The answer to this question depends crucially on the origin of the agents' biased estimate. In our environment, the different estimates of the same risk arise because of different perceptions not because of different underlying preferences. Both the Os and the Rs have identical preferences. As a result, the preferences revealed by the insuree's choices coincide with the true underlying preferences. Therefore, the appropriate efficiency criterion seems to be objective rather than subjective probabilities. However, in this case, regardless of which criterion is used, the suggested intervention policy yields a strict Pareto improvement on the laissez-faire equilibrium.

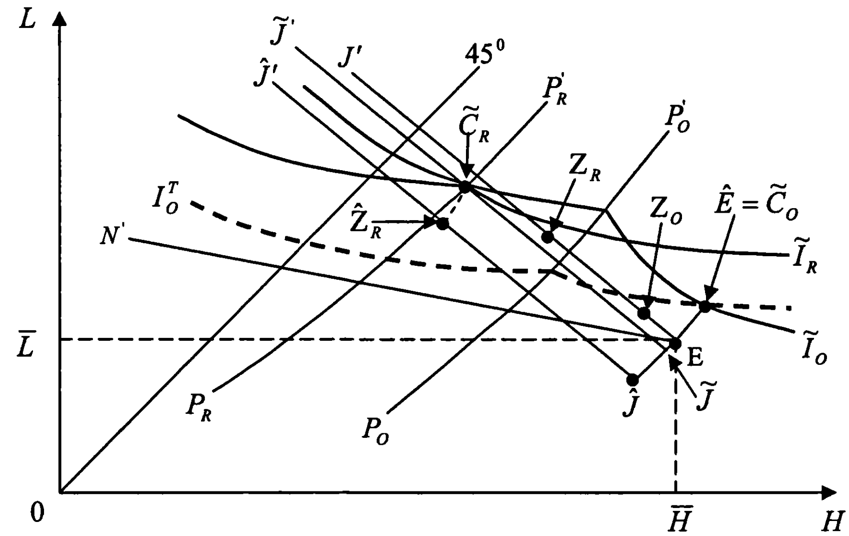

Figure 3.4

Proposition 4: In the separating equilibrium of Proposition 3, introducing a fixed tax per contract sold, with the proceeds returned as a lump-sum subsidy to the whole population yields a strict Pareto improvement if the proportion of the Os is sufficiently small (see Figure 3.4).

Proof: The tax per contract sold shifts the origin of the zero-profit lines down the 45^{O} line to $\hat{J}$. On the contrary, the subsidy shifts the endowment point E up the 45^{O} line to $\hat{\mathrm{E}}$ and the origin of the zero-profit lines along which the insured can consume (consumption zero-profit lines) up the 45^{O} line from $\hat{J}$ to $\tilde{J}$. After the intervention, the consumption bundle of the Rs, $\tilde{C}_R$, consists of the insurance contract $\hat{Z}_R$ and the subsidy whereas that of the Os', $\hat{E} = \tilde{C}_O$, consists of their endowment, E, and the subsidy. If the proportion of the Os is small, $\tilde{J}\tilde{J}'$ lies close to EJ' and $\hat{E}$ lies well above E. As a result, the Rs' indifference curve through $\tilde{C}_R$, $\tilde{I}_R$, and the Os' indifference curve through $\hat{E} = \tilde{C}_O$, $\tilde{I}_O$, lie above the corresponding indifference curves in the laissez-faire equilibrium. That is, in the new equilibrium, the perceived expected utility of both types has increased. To show that the Os' true welfare has also improved, we employ true accident probabilities and construct a curve along which the Os' true welfare is constant, given their precautionary effort level choice (the I_O^T curve in Figure 3.4). Since the I_O^T through $\hat{E} = \widetilde{C}_O$ passes above Z_O, the Os' true welfare at $\hat{E} = \widetilde{C}_O$ is strictly greater than at Z_O. That is, in the new equilibrium both the Rs and the Os are strictly better off.[116] *Q.E.D.*

Intuitively, the imposition of the tax results in the Os going uninsured and mitigates the negative externality their presence creates. In the new equilibrium, the Rs purchase more insurance but subsidise the Os. The question is whether the improvement in their welfare, because of the higher coverage, exceeds the welfare loss due to the subsidy they provide the Os in order to relax their revelation constraint? Because the proportion of the Os is small, the per capita subsidy is high and so its effect both on the Os' utility and revelation constraint is large. This, in turn, allows the Rs to purchase a significantly higher amount of insurance. As a result, the welfare gains of the higher coverage more than offset the welfare loss due to the net tax (tax minus subsidy) the Rs pay.

Surprisingly, although the Os are underinsured in the laissez-faire equilibrium, an intervention scheme which results in the Os going uninsured, leads to a strict Pareto improvement. On the contrary, a policy that would result in the Os purchasing more coverage would tighten rather than relax their revelation constraint. As a result, in order to reveal their type, the Rs would have to purchase even less insurance and so their welfare would worsen.

3.6 Implications for Empirical Testing

The results of Propositions 2 and 3 also allow us to empirically distinguish our approach from standard asymmetric information models. To this end, we rely on a very general result derived by Chiappori et.al (2002) (see Lemma 1 of this chapter). If an agent chooses one contract over another offering more coverage, then it must be true that his accident probability under the contract chosen is strictly lower than the per unit premium of the additional coverage offered by the high-coverage contract. This is a revealed preference argument. Its validity is independent of the market

[116] It may be the case that $\hat{E} = \widetilde{C}_O$ involves higher consumption in both states than Z_O. This clearly implies that the Os are strictly better off after the intervention.

structure or whether some agents go uninsured. For example, it holds in the models of Julien, Salanie and Salanie (2001) and de Meza and Webb (2001) where the positive correlation property breaks.[117] However, in our framework, because some agents (the Os) underestimate their true accident probability, this prediction fails.

Corollary 1: In the separating equilibria of Proposition 2 and 3 it is respectively true that

$$(y_R - y_O)/(\lambda_R y_R - \lambda_O y_O) < 1 - p(Z_O) = 1 - p_0 \tag{3.24}$$

$$(y_R - y_O)/(\lambda_R y_R - \lambda_O y_O) = 1 - p(Z_O) = 1 - p_F \tag{3.25}$$

Proof: See Appendix 3B

In words, in both equilibria the per unit price of the additional insurance offered by the high-coverage contract is not higher than the Os' true accident probability under the low-coverage contract. Nevertheless, due to the underestimation of their accident probability, the Os purchase the low-coverage contract although the high-coverage contract is also available. Therefore, a rejection of this revealed preference argument by the data is consistent with our model but not with standard asymmetric information models.

3.7 Conclusion

Most recent empirical studies on the relationship between coverage and risk find either negative or no correlation. Moreover, Cawley and Philipson (1999) report that, in the US life insurance market, insurance premiums exhibit quantity discounts. Standard asymmetric information models cannot simultaneously explain these empirical findings.

This chapter provides an explanation to this puzzle by introducing heterogeneity in risk perceptions in a competitive model of asymmetric information. The more

[117] It even holds in the Villeneuve (2000) model provided insurance companies do not observe all the insuree's characteristics that affect his accident probability.

optimistic agents (the Os) underestimate their accident probability both in absolute terms and relative to the less optimistic ones (the Rs) and so purchase less insurance. They also tend to be less willing to take precautions. This gives rise to separating equilibria exhibiting negative or no correlation between coverage and risk. Two examples of these equilibria are presented where both the Os and the Rs purchase some insurance.

In the first case, the Os not only take fewer precautions (high-risk type) but also purchase less coverage than the Rs. Competition among insurance companies then implies that the Os also pay a higher per unit premium. Because they underestimate their accident probability, the Os purchase low coverage at a high per unit price, although contracts offering more insurance at the same or even lower per unit price are available. The second equilibrium exhibits no correlation between coverage and risk. Both types take precautions but the Os choose less coverage than the Rs. Nevertheless, the Rs are quantity-constrained. Moreover, if we allow for fixed administrative costs, this equilibrium displays a negative relationship between coverage and per unit premiums.

These results have several interesting implications. First, they explain both puzzling empirical findings: The negative or no correlation between coverage and risk and the fact that insurance premiums display quantity discounts.

Second, Cawley and Philipson (1999), Chiappori and Salanie (2000) and Dionne et.al. (2001) argue that the no-correlation empirical findings imply that there is no (risk-related) adverse selection. Thus, there are no information barriers to trade in the life and automobile insurance markets under study. However, our results suggest that their assertion is not generally true. If insurees differ with respect to their risk perceptions and types are hidden, there exist equilibria involving some agents being quantity-constrained even if the data show no correlation between coverage and the accident rate. Furthermore, in these cases, there exist intervention policies that yield a strict Pareto improvement on the laissez-faire equilibrium.

Third, based on the revealed preference argument of Chiappori et.al (2002), the predictions of the separating equilibria of Propositions 2 and 3 allows us to empirically distinguish our approach from standard asymmetric information models. The rejection of this revealed preference argument by the data is consistent with our model but not with standard asymmetric information models. Clearly, its empirical validity is an interesting topic of future research.

Appendix 3A: Proof of Lemma 1

Consider a contract $C' = (\lambda_2 y_2, y')$ with premium: $y' = (1 - p(C_1))\lambda_2 y_2$.
We will show that the agent prefers C' to C_1. Notice that if the agent still has ex post risk $1-p$ under C' $(1-p(C_1) = 1-p(C'))$, then he faces the following lottery:

$$L' = (-D_\theta + \lambda_2 y_2 - y', 1-p; -y', p)$$

The expectation of this lottery is:

$$(1-p)(-D_\theta + \lambda_2 y_2 - y') - py' = (1-p)(-D_\theta + \lambda_2 y_2) - y' = -(1-p)D_\theta$$

Clearly, it is equal to the expectation of the lottery

$$L_1 = (-D_\theta, 1-p; 0, p)$$

which the agent faces under C_1. Since $0 = \lambda_1 y_1 < \lambda_2 y_2$ and contracts do not overinsure, lottery L_1 is a mean-preserving spread of L'. Thus, given risk aversion, the agent strictly prefers L' to L_1. Furthermore, since under C' he may choose another $1-p' \neq 1-p$ that costs him less than $1-p$, he strictly prefers C' to C_1 and hence to C_2 (by assumption, C_1 is preferred to C_2). However, contracts C' and C_2 offer the same coverage. Therefore, since C' is strictly preferred to C_2, it must be the case that

$$y_2 > y' = (1-p(C_1))\lambda_2 y_2 \quad \Rightarrow \quad 1-p(C_1) < \frac{y_2}{\lambda_2 y_2} = \frac{1}{\lambda_2}$$

Q.E.D.

Appendix 3B: Proof of Corollary 1

Using the zero-profit conditions, we obtain:

$$\pi_i = p(Z_i)y_i - (1-p(Z_i))(\lambda_i - 1)y_i = 0 \quad \Rightarrow \quad \lambda_i = 1/(1-p(Z_i)), \quad i = O,R \quad (3B.1)$$

In the separating equilibrium of Proposition 2 we have:

$$1-p(Z_O) = 1-p_0 > 1-p(Z_R) = 1-p_F \quad \Rightarrow \quad \lambda_R > \lambda_O \quad (3B.2)$$

Therefore,

$$\frac{y_R - y_O}{\lambda_R y_R - \lambda_O y_O} = \frac{y_R - y_O}{\lambda_O(y_R - y_O) + (\lambda_R - \lambda_O)y_R} < \frac{1}{\lambda_O} = 1 - p_0 \quad (3B.3)$$

In the separating equilibrium of Proposition 3 we have:

$$1-p(Z_O) = 1-p(Z_R) = 1-p_F \quad \Rightarrow \quad \lambda_R = \lambda_O \quad (3B.4)$$

Therefore,

$$\frac{y_R - y_O}{\lambda_R y_R - \lambda_O y_O} = \frac{y_R - y_O}{\lambda_R(y_R - y_O)} = \frac{1}{\lambda_R} = 1 - p_F \quad (3B.5)$$

Q.E.D.

Conclusion

In this thesis we studied financial and insurance markets under various specifications of asymmetric information.

In Chapter 1, we analysed and discussed the roles of debt, equity and warrants under adverse selection and (effort) moral hazard. Several interesting results were obtained. First, we explained the issue of combinations of debt and equity as the outcome of the interaction between adverse selection and moral hazard. Firms willingly incur the adverse selection cost of issuing equity because this cost is more than offset by the benefit from relaxing the moral hazard constraint. Second, we showed that, in the presence of moral hazard, adverse selection may result in the conversion of a negative into a positive NPV project and an improvement in social welfare. Third, we provided two rationales for the use of warrants. Under pure adverse selection, warrants can serve as separation devices in cases where other standard securities cannot. Under adverse selection cum moral hazard, warrants allow for the implementation of the socially efficient outcome even if this is not possible when we restrict ourselves to debt, equity and/or convertible debt. We also showed that, under certain conditions, a debt-warrant combination can implement the optimal contract as a competitive equilibrium.

The interaction between adverse selection and moral hazard may also have interesting implications for issues such as internal versus external financial markets and the theory of the firm.

In Chapter 2, we explored the implications of optimism in competitive insurance markets when neither the type nor the actions of the insurees are observable. Optimism may either increase or decrease precautionary effort and we showed that this determines whether optimists or realists are quantity-constrained in equilibrium. There also exist intervention schemes that lead to a strict Pareto improvement on the laissez-faire equilibria. These results provide a more convincing justification for the imposition of minimum coverage requirements than standard models as well as a case for the use of taxes and subsidies in insurance markets.

Chapter 3 focused on the relationship between the coverage offered by the insurance contract and the *ex-post* risk of its buyer. If all agents purchase strictly positive coverage, competitive models of asymmetric information predict a positive relationship between coverage and the accident probability. Yet some recent empirical studies find either negative or zero correlation as well as that per unit premiums fall with quantity. If the more optimistic agents both underestimate their accident probability and are less willing to take precautions, there exist separating equilibria that potentially explain these puzzling empirical findings. It was also shown that zero correlation between coverage and risk does not imply the absence of barriers to trade in insurance markets. We concluded with some testable implications of our results.

Although in this thesis we focused on insurance markets, the introduction of misperception of risk into an asymmetric information framework may have interesting implications for other issues as well. The design of managerial compensation schemes, the choice between self employment and being an employee, the design of securities and other corporate finance issues are only some of them.

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