Asymmetric Information in Financial Economics:

Asset Pricing, Liquidity Policy and the Resolution of Financial Distress

A thesis presented

by

Spyros Pagratis

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Houghton Street

London WC2A 2AE

England UK

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Abstract

This thesis consists of three self contained essays in financial economics where agents interact under asymmetric information about some latent economic fundamentals.

The chapter on "Asset pricing under noisy rating signals: Does benchmarking on ratings matter?", demonstrates that, in the presence of *noise traders* who benchmark their supply of a traded asset to public signals (ratings), informed traders are induced to rationally overreact to news about fundamentals, leading to excess asset price volatility. The analysis also shows that if market participants use public ratings solely for price discovery purposes then, under no circumstances ratings could weaken price efficiency, even in the presence of *higher order beliefs*.

The chapter on "Prudential liquidity regulation and the insurance aspect of lender of last resort" considers prudential liquidity regulation as *quid pro quo* for emergency liquidity assistance by the central bank. In the presence of bank funding constraints, information-induced bank runs and an objective by the central bank to maintain a balanced budget under its lender of last resort (LOLR) facility, it is shown that prudential liquidity regulation is socially desirable if the banking sector is characterised by sufficient funding constraints, high profit opportunities and a relatively volatile deposit base. Otherwise, liquidity regulation is too costly from a welfare perspective, even after taking into account the social value of LOLR insurance.

Finally, the chapter titled "Co-ordination failure and the signalling role of banks in debt-exchange offers" studies the out-of-court restructuring of debt when a creditor bank makes concessions conditional on other creditors' actions. In line with empirical evidence, the analysis shows that a bank's conditional commitment to restructure injects a degree of *strategic solidity* among other creditors, who then accept restructuring more easily. That leads to resolution of financial distress at lower levels of firm's fundamentals, compared to the situation with no bank in the game, that could imply a lower deadweight cost of inefficient liquidation. It is also discussed how conditional restructuring concessions may lead to a combination of herding incentives and co-ordination problems where, depending on the extent of conditionality, one may dominate the other.

Contents

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Introduction

In economic settings with multiple agents and asymmetric information, there are two equilibrium concepts that one could apply to draw economic predictions. That is, the competitive Rational Expectations Equilibrium (REE) and the Bayesian Nash Equilibrium (BNE). The REE assumes consensus about a deterministic price function that maps agents' information sets into prices and rules out systematic forecast errors by requiring agents' conjectures to coincide with the actual price function that those conjectures generate. The BNE allows for strategic interactions among agents, assuming mutual knowledge of agents' rationality and typically common priors. In a BNE, agents make conjectures about other agents' strategies, which then coincide with agents' best responses to those conjectures.

This thesis contains three self-contained essays in financial economics, where a common theme that emerges is that agents interact under asymmetric information about some latent economic fundamentals. Chapter 1 uses the REE concept to discuss possible asset pricing implications of public announcements regarding the quality of a risky asset. Chapter 2 employs a BNE framework to analyse optimal liquidity holdings by the banking sector in the presence of information induced bank runs and potential emergency liquidity assistance by the central bank. Also within a BNE framework, Chapter 3 discusses the out-of-court restructuring of the contractual obligations of a financially distressed firm, under conditions of asymmetric information among the firm's creditors and in situations where a creditor bank makes concessions conditional on other creditors' actions.

Before undertaking a comprehensive analysis of these issues in further chapters, we briefly summarise and discuss some of the main results in the remaining of this chapter.

1.1 Asset pricing and ratings

Chapter 1 discusses an intertemporal model of asset pricing under asymmetric information, demonstrating how noisy public signals (ratings) about the quality of a risky asset could enhance information efficiency, albeit at a cost of higher asset price volatility. The analysis also draws implications for benchmarking investment decisions on ratings. In particular, it considers a stylised version of benchmarking investment decisions to public information, whereby a residual class of (noise) traders link their net supply of the risky asset to some measure of the probability that the rating next period will fall below a given rating threshold. Thus, benchmarking to ratings can be rationalised as the result of forced sales by a class of regulated investors (e.g. pension funds) that are restricted to hold the risky asset only if its rating is above a prespecified threshold and unload their holdings to the market proportionally to the probability such 'downgrading' will take place.

The main conclusion from the analysis is that price efficiency drops with the extent of benchmarking in the market while volatility increases, suggesting that benchmarking is a possible component of the *excess volatility puzzle*. That is because perceived changes in fundamentals feed into prices not only through changes in perceptions about future income from holding the asset, but also through beliefs about capital gains that depend on the net supply of the asset. Given that benchmarking renders the net supply of the risky asset partly forecastable, informed traders are inclined to trade more aggressively on any item of news that could imply a change in fundamentals in order to exploit perceived mispricings. Thus, they become more prone to misinterpret any item of news as information about fundamentals leading to less informative and more volatile prices.

1.2 Prudential liquidity and the lender of last resort

Chapter 2 discusses a rationale for prudential liquidity standards for banks by considering prudential liquidity as *quid pro quo* for emergency liquidity assistance by the central bank. In the presence of bank funding constraints, information-induced bank runs and an objective by the central bank to maintain a balanced budget under its lender of last resort (LOLR) facility, it is shown that prudential liquidity regulation is socially desirable if the banking sector is characterised by sufficient funding constraints, high profit opportunities and a relatively volatile deposit base. Otherwise, liquidity regulation is too costly from a welfare perspective, even after taking into account the social value of LOLR insurance.

Debt constraints in the model arise from depositors' rational anticipation of bankers' moral hazard problems which, in the spirit of Hellmann, Murdock and Stiglitz (2000), could relate to the extent of *financial liberalisation* of the banking sector. Also, the possibility of information-induced bank runs aims at capturing banks' inherent fragility due to high leverage, short term funding and asymmetric information which, in the presence of adverse economic conditions, could lead to loss of confidence to banking institutions and unanticipated foreclosures of wholesale interbank lines. In the presence of such frictions, the analysis provides a necessary and sufficient condition for prudential liquidity regulation to be socially desirable, showing that the more debt-constrained the banking sector is and the higher its profit opportunities, the more prudential liquidity regulation becomes socially desirable by augmenting the insurance value of LOLR safety net.

1.3 Banks and debt-exchange offers

Chapter 3 discusses the out-of-court restructuring of debt, under conditions of asymmetric information among a firm's creditors and in situations where a creditor bank makes concessions conditional on other creditors' actions. Debt restructuring is modelled as a debt-exchange offer to small creditors and a request for new credit by the bank. Conditionality in the restructuring action is modelled as a minimum tendering rate that needs to be reached before new funds are injected to the firm and old contractual obligations are exchanged with new.

In line with empirical evidence, the analysis suggests that a bank's conditional concession may inject a degree of *strategic solidity* among other creditors, who may then accept to restructure more easily. That could lead to resolution of financial distress at a lower level of firm's fundamentals compared to the situation with no bank in the game.

However, for high minimum tendering rates, herding incentives may dominate coordination problems, as creditors tend to rely more on average opinion – that is reflected in the debt-restructuring outcome – rather than on private signals about the solvency of the firm. In equilibrium, herding incentives are common knowledge and the bank chooses its strategy in a way that accounts for other creditors' incentive to play a low strategy when the bank plays a high one, and vice versa. That leads to multiple equilibria in the restructuring game, where the strategy that is followed by the bank is decreasing in the strategy that is followed by other creditors and vice versa.

Because of the conditionality in creditors' actions and the sequential feature of the game between the bank and other creditors, the analysis becomes involved rather quickly and sometimes unspecified assumptions on endogenous variables are needed to make it whole. For example, we assume a continues mapping from bank's strategy into that of other creditors, although the only thing we know about such a mapping is a set of conditions that are specified by proposition 8. Nevertheless, as the information precision of the bank increases relative to that of other creditors, equilibrium selection no longer matters. However, we intend to revisit this point in future research, considering a more general information structure among creditors.

1.4 Asymmetric information

Asymmetric information typically imposes a requirement on rational agents to use their private signals not only to update their beliefs about some hidden economic fundamentals, but also to infer the beliefs of other agents. In other words, under asymmetric information, rational behaviour requires agents to hold beliefs of higher order, whereby devoting intelligence to anticipate what average opinion is, what average opinion expects average opinion to be, and so on. That is because, a rational agent will realise that her expected utility from undertaking a certain action does not only depend on the actual economic fundamentals, but also on the actions of other agents. In all essays presented in this thesis, a common feature that emerges in the analysis is that tractability in accounting for higher order beliefs, both in a REE and a BNE framework, is preserved without requiring agents to apply extremely sophisticated reasoning.

In chapter 1, rational agents are required to run autoregressive moving average (ARMA) models to forecast observable variables and extract information about fundamentals. In other words, rational agents do not only use past realisations of observable variables to update their beliefs, but also use their forecast errors. Such a modelling technique is consistent with explicit consideration of higher order beliefs and allows us to pin down a unique REE equilibrium in ARMA coefficients, where agents' forecasting models are consistent with the actual law of motion that those models generate. Existence of such an equilibrium is inferred from the results of Marcet and Sargent (1989a,b) on the convergence of least squares learning mechanisms and was firstly applied by Sargent (1991), and later by Hussman (1992), to study rational expectations equilibria with signal extraction from endogenous variables.

In chapters 2 and 3, strategic interactions among agents are modelled using a *global games* methodology. A global game is first defined in Carlsson and van Damme (1993) as a game of incomplete information where the actual pay-off structure is determined by a random draw from a given distribution and where each player receives a noisy signal of the realisation. Carlsson and van Damme (1993) show this result for a two player binary action game. Morris and Shin (1998) and Goldstein and Pauzner (2004) extend the result by Carlsson and van Damme to the case where there is a continuum of agents.

As of Morris and Shin (1998), if a binary action global game satisfies *full (global) strategic complementarities* – i.e. an agent's incentive to take a particular action increases with the proportion of other agents undertaking the same action – and if the upper and lower dominant regions are non empty then, there exists a unique dominant solvable equilibrium strategy. That is a strategy that survives the iterated deletion of strictly dominated strategies. Under such a strategy, all agents undertake a particular action if and only if their signals fall below a critical signal threshold. Goldstein and Pauzner (2004) extend Morris and Shin (1998) by offering a uniqueness result under *one-sided strategic complementarities*. This is when an agent's incentive to undertake an action increases with the proportion of other agents undertaking the same action, provided that that proportion is relatively small.

Global games capture the idea that agents may be induced to undertake certain actions because they believe others will do. Moreover, in a number of economic setups where agents face co-ordination problems, such as those discussed in chapters 2 and 3, global games allow us to pin down a unique equilibrium and to link the probability of economic outcomes to the underlying economic fundamentals. That, in turn, allows us to deliver intuitive comparative static predictions and implications for optimal policy analysis.

Chapter 1
Asset pricing under noisy rating signals: Does benchmarking on ratings matter?

1.1 Introduction

Credit ratings are summary statistics that reflect a rating agency's opinion, as of a specific date, of the creditworthiness and financial robustness of a particular entity. Rating agencies' assessment is mainly based on fundamental analysis and have traditionally measured creditworthiness in the context of a capital, asset quality, management, earnings and liquidity analysis.

Following a series of high-profile credit events (e.g. the Enron bankruptcy in 2001), the Sarbanes-Oxley Act of 2002 has required the US Securities and Exchange Commission (SEC) to conduct a study on rating agencies and their role in securities markets.[1] In the course of that study, market representatives have suggested, among others, that ratings cause undue volatility in securities markets and have called for more transparency regarding the information relied upon by the rating agencies. For example, a market participant[2] has claimed that

> *... one of the first things we wonder is what is it that*

[1] US Securities and Exchange Commission, '*Report on the Role and Function of Credit Rating Agencies in the Operation of the Securities Markets*', January 2003.

[2] Testimony of Cynthia L. Strauss, Director of Taxable Bond Research, Fidelity Investments Money Management Inc., 15 November 2002.

they [the rating agencies] know, and I think that adds unnecessary volatility and uncertainty to the marketplace...

Also the scope and application of credit ratings nowadays stretches beyond the provision of information to market participants. Ratings, for example, have been used to facilitate monitoring the risk of investments by regulated entities and to set capital charges for banks and securities firms. Two notable examples that relate to the use of ratings for capital adequacy purposes are the rules under the New Capital Accord, that have been proposed by the Basel Committee on Banking Supervision and will apply to banks, and the US Net Capital Rule[3] that applies to broker-dealers. Both sets of rules provide for the deduction from capital of a certain percentages of the value of security holdings depending on the credit rating of those securities. Moreover, regulators often restrict certain classes of market participants from investing in securities below a rating threshold, with most notable the dichotomy between *investment* and *subinvestment* grade credits.[4] Rule 2a-7 of the US Investment Company Act, for example, restricts money market funds from investing in commercial paper below a rating threshold. Similar rules apply to insurance companies and pension funds.

From a theoretical perspective, the role of information in asset pricing has been discussed both in a competitive market context and in the presence of strategic interactions among market participants. For example, Kyle (1985) and its extensions[5] consider an

[3] See *Adoption of Alternative Net Capital Requirement for Certain Brokers and Dealers*, Release No. 40 FR 29795 (16 July 1975).

[4] Namely, ratings above or below BBB grade, in Standard & Poor's representation.

[5] See, for example, Michener and Tighe (1991), Holden and Subrahmanyam (1992) and Foster and Viswanathan (1993).

oligopoly of imperfectly informed investors having identical information. They show that, with identical information, there is an intense *pre-emption* phase where informed investors compete very aggressively and, as a result, information is incorporated into prices very quickly. Foster and Viswanathan (1996) introduce heterogeneous information in a Kyle (1985) context showing that trading outcomes depend critically on the initial correlation of private information that traders possess. They show the lower the degree of initial correlation of traders' information – namely the more heterogeneous information becomes – the higher the degree of their monopoly power, with respect to their information advantage, which then gives rise to an *attrition trickle* and an incentive to trade less aggressively. Given that a public signal about fundamentals, such as a public rating, could increase the initial correlation (i.e. reduce heterogeneity) of traders' information, ratings could possibly be viewed as inducing strategic traders to trade more aggressively and prices to incorporate information more quickly.

This paper is in the line of literature initiated by Grossman and Stiglitz (1980) and Hellwig (1980). In the context of a discrete-time asset pricing model of infinite horizon, we consider a competitive asset market where market participants are asymmetrically informed and able to place their orders with a Walrasian auctioneer conditionally on prices.[6] However, in addition to private information that market participant may possess, we introduce a public signal (rating) in every trading round that is produced by a non-trading and non-strategic party (rating agency). Such a public signal is assumed to be produced on the basis of a stylised, time-invariant process (the rating process), which is consistent with

[6] See also Hellwig (1980), Diamond and Verrecchia (1981) and Allen, Morris and Shin (2003).

investors' beliefs.[7] Thus, in this paper we are able to discuss possible asset pricing implications both from the use of ratings for their information content and, in addition, the impact that arises from benchmarking investment decisions and capital requirements on ratings.

Our solution approach involves the calculation of a rational expectations equilibrium (REE) of a securities market with ratings, assuming that the true state-variables of the market are never perfectly revealed neither to investors, nor to the rating agency, but they are observed with some error. Thus, agents need to filter information from the variables that they observe. In particular, the rating agency is assumed to apply a Kalman filter approach to update its ratings on the basis of private information that it observes, while investors are assumed to fit linear econometric models on observable variables. In addition, our modelling approach allows for higher order beliefs to have a material impact on asset prices.[8] That is in line with Bacchetta and van Wincoop (2003) and Allen, Morris and Shin (2003) who argue that under beliefs of higher order asset prices may become biased towards the public information, regardless how sound that information might be. Thus it would be of interest to examine whether a similar result also follows in our set-up and whether a world without ratings would be preferable to a world with, but imprecise, ratings.

However, in discrete-time models with asymmetric information, agents' rationality requires one to address the inferences that agents make from observable variables, know-

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[7] In this paper, we adopt a reduced-form approach to ratings process, by abstracting from the information economics of ratings (e.g. Diamond (1985) and Veldkamp (2003)), from the financial intermediation underpinnings of ratings agencies (e.g. Millon and Thakor (1985)) and the possibility of strategic information revelation (*cheap talk*) by rating agencies.

[8] Higher order beliefs is a basic feature of asset pricing under asymmetric information and it refers to the situation where opinions of other investors' opinions, and higher order than that, may have a material impact on asset prices. That is in line with Keynes' (1936) famous metaphor that the market is similar to a beauty contest, where an agent's subjective payoff from choosing the prettiest face from a list of contestants depends on how close her prediction were to the average opinion of other agents.

ing that others act in a similar fashion. Thus beliefs, beliefs about beliefs and higher order than that, become hidden state variables and the dimension of the state vector, associated with agents' signal extraction problems, becomes unbounded. In order to deal with the problem of infinite regress in expectations, we apply the techniques of Sargent (1991), as applied by Hussman (1992). More specifically, we extend Hussman (1992) by allowing rating announcements to augment investors' information sets and introducing ratings-based frictions, such as ratings-based capital requirements and benchmarking of investment decisions to ratings.

In equilibrium, investors' subjective beliefs have to be consistent with the actual law of motion that those beliefs generate. Thus, equilibrium in our model is calculated as a *fixed point* in the mapping from investors perceived laws of motion to the actual law of motion that investors' perceptions generate. This is by taking as given the econometric techniques that investors apply and assuming that those techniques belong to the same class of linear models. In particular, we focus on the situation where investors fit first-order vector autoregressive moving average (ARMA) models. As of Sargent (1991) and Hussman (1992), the equilibrium in first-order ARMA models is consistent with higher order beliefs and is such that investors have no incentive to increase the order of either the AR or the MA component of their forecasting rules in order to improve their forecasts.

However, for purposes of comparison with the ARMA case, we also describe equilibria (with and without ratings) when investors' forecasting rules are restricted to be first-order vector autoregressive (AR) processes. As we know from Townsend (1983), those first-order autoregressions are always *too short* to give optimal forecasts because of the infinite regress problem. That is, in equilibrium, the prediction errors from first-order vector autoregressions will never be orthogonal to information that lagged two periods or more.[9] We consider the case where investors use vector AR forecasting rules as a proxy for *low market sophistication*, in contrast to *high market sophistication* when investors run ARMA models.

The analysis shows that, when ratings are used for price discovery alone they may increase price volatility, but this is consistent in the model with an increase in price efficiency (i.e. prices become more correlated with fundamentals). Also, the type of forecasting techniques that market participants use to form their beliefs matters for trading outcomes. Moreover, for reasonable levels of rating-based capital requirements, the volatility of prices drops, although at a cost of lower price efficiency.

Yet benchmarking of asset holdings on ratings may cause both a reduction in price efficiency and an increase in volatility. This is despite an optimistic presumption in the model that agents have common knowledge of how the economy works, there are no structural breaks in the economy and investors trust the rating agency in its objective to produce timely, accurate and objective information. In fact, regulatory and other constraints that force a residual class of market players to link their investment decisions to ratings, may generate a sequence of perceived mispricings in the market and drive other investors to overreact to news about fundamentals. That way, benchmarking magnifies the effect of

[9] This is, the prediction errors from first-order vector autoregressions will never be orthogonal to the Hilbert space that is generated by all past history of investors' information.

news on prices in such a way that prices may respond to changes in fundamentals even in excess of the full-information case.[10]

The remainder of the paper is organised as follows: Section 1.2 describes the model and the solution method. Section 1.3 presents the results under no rating-based frictions and discusses persistence implications and comparative statics. Section 1.4 introduces rating-based frictions, such as rating-based capital requirements and benchmarking of asset holdings on ratings, and discusses equilibrium implications. Section 1.5 concludes. Proofs, technical details and figures are included in the appendix.

1.2 The model

We consider a competitive market for a risky asset that pays a risky pay-off D_t that varies over time t. Pay-off D_t consists of two independent factors θ_{1t} and θ_{2t} – hereinafter called *fundamental factors* – that have some persistence over time, as well as of a transitory component u_t

$$D_t = \theta_{1t} + \theta_{2t} + u_t \tag{1.1}$$

We assume that factors where θ_{jt}, $j = 1, 2$, evolve according to the following first-order autoregressive processes

$$\theta_{jt} = \rho\theta_{jt-1} + v_{jt} \quad j = 1, 2 \tag{1.2}$$

with $\{u_t\}$, $\{v_{jt}\}$ be *i.i.d.* white noise innovations with mean zero and variances σ_u^2 and σ_v^2. For simplicity and without loss of generality of our analysis, we assume that the persistence

[10] This is, the hypothetical situation where investors observe perfectly any innovation in fundamentals and they do not need to solve filtering problems.

ρ is the same for the two fundamental factors, but equally one could consider different degrees of persistence, where one of the factors could be thought of as long term and the other as short term.[11] In addition, we assume that the fundamental factors are stationary, i.e. they do not grow explosively for ever, by assuming that $|\rho| < 1$.

The market is populated by N privately informed investors that belong to classes indexed by $j = 1, 2$ depending on the type of private information that they observe. Proportion α belong to class 1 and observe private signals about factor θ_1, while proportion $1 - \alpha$ to class 2, observing signals about θ_2. We consider an overlapping generation of those investors who live for two periods and their preferences over future wealth demonstrate constant absolute risk aversion (CARA) with coefficient $\frac{1}{\phi_j}$. Informed investors are able to trade conditionally on prices – i.e. place limit orders – in the first period and invest their wealth in the risky asset or, alternatively, in a safe asset yielding a return R.

There is also a residual set of traders, called noise traders, who trade both for non-fundamental (liquidity) purposes and for benchmarking reasons, whereby they link their supply of the risky asset to some public information.[12] Non-fundamental trade implies a random supply $\{\varsigma_t\}$ of the asset, which is *i.i.d.* normal with mean zero and variance σ_ς^2, while noise trading for benchmarking reasons is introduced in Section 1.4.2.

Informed investors of class $j = 1, 2$ are assumed to observe private signals s_t^j about the actual realisation of fundamental factor θ_{jt}, which are subject to an element of idiosyn-

[11] However, that would increase the computational intensity of our calculations when we would have to derive an equilibrium of our asset market.

[12] That is discussed in more detail in Section 1.4.

cratic noise η_{jt}

$$s_t^j = \theta_{jt} + \eta_{jt} \qquad j = 1, 2 \tag{1.3}$$

where $\{\eta_{jt}\}$ are $i.i.d.$ white noise innovations, orthogonal to $\{e_{jt}\}$ and $\{v_{jt}\}$, with zero mean and variance σ_η^2. Thus we consider informed investors as having special price discovery skills (e.g. *macro* versus *sector* funds), while such an information structure as given exogenously without modelling explicitly the actual decision of investors to acquire information, as in Grossman and Stiglitz (1980) for example. Instead, we focus exclusively on informed investors' problem of filtering information about fundamentals from observable variables, including their private signals $\{s_t^j\}$.

1.2.1 Ratings

In addition to private signals that informed investors observe and to publicly observed prices and asset pay-offs, we assume that in every trading period a non-trading, independent and non-strategic party (henceforth called the *rating agency*) produces a public signal r_t (henceforth called the *rating*) about the factors that affect the pay-offs of the risky asset.[13] Consistently with real-world features of ratings, we assume that ratings are public signals in the form of *summary statistics*, i.e. they summarise all the information that the rating agency has received over time about the fundamental factors that affect asset pay-offs.[14] In addition, as we discuss below, ratings in this model are updated on the basis of a recur-

13 In this paper, we abstract from the information economics that underpin the existence and functioning of rating agencies, as well as from possible principal-agent problems in the disclosure of information to the ratings agency.

14 In the real world, this may allow the agency to obfuscate the reason behind the rating change when this is based on confidential information.

sive process, which is in analogy to the rating process outlined in *rating policy guidelines* of rating agencies. Finally, we assume that the rating agency uses only its private signals in order to produce its ratings, ignoring any element of public information such as prices,[15] and does not publicly announce the individual elements of its private information.

Thus, the rating r_t is assumed to be an unbiased estimator of the sum of the two fundamental factors conditional on all private signals that the rating agency has observed up to that period. In particular, the rating agency is assumed to be receive noisy private signals s^r_{1t} and s^r_{2t} of the form

$$\begin{aligned} s^r_{1t} &= \theta_{1t} + e_{1t} \\ s^r_{2t} &= \theta_{2t} + e_{2t} \end{aligned} \tag{1.4}$$

where $\{e_{jt}\}$ are *i.i.d.*white noise innovations, orthogonal to $\{u_t\}$, $\{v_{jt}\}$ and $\{\eta_{jt}\}$, with mean zero and variance σ_e^2. By assuming that the rating agency possesses information about both fundamental factors, while individual investors are separated in two groups with each one receiving a different signal, we aim to address possible information advantages of rating agencies relative to individual market participants. That is supported by the adoption of Regulation Fair Disclosure (Regulation FD) by the US Securities and Exchange Commission in October 2000, which prohibits selective disclosure of non-public information by firms, but provides an exception for rating agencies. Having said that, the rating process in this model is given by

$$r_t = E\left[\theta_{1t} + \theta_{2t} \mid s^r_{1s}, s^r_{2s}, s < t\right] \tag{1.5}$$

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5 This is consistent with the Standard & Poor's approach to ratings, as outlined in their *rating policies guidelines*.

Given that the rating r_t in 1.5 depends on all past history of signals s_j^r, we may express it in a recursive form as a function of the previous rating r_{t-1} and the signals observed in period t. That can be achieved by using the following Kalman filter representation.

Lemma 1 *The rating process* $\{r_t\}$*, as defined by* 1.5*, exhibits positive autocorrelation and is generated by the following on-line algorithm.*

$$r_t = \rho\left[1 - \Sigma\left(\Sigma + 1\right)^{-1}\right] r_{t-1} + \rho\Sigma\left(\Sigma + 1\right)^{-1}\left[s_{1t-1}^r + s_{2t-1}^r\right] \tag{1.6}$$

where, parameter Σ *is given by*

$$\Sigma = \frac{1}{2}\left[\frac{\sigma_v^2}{\sigma_e^2} - \left(1 - \rho^2\right) + \sqrt{\left[\frac{\sigma_v^2}{\sigma_e^2} - \left(1 - \rho^2\right)\right]^2 + 4\frac{\sigma_v^2}{\sigma_e^2}}\right] \tag{1.7}$$

Proof *See appendix* ■

From 1.6 and 1.7, the degree of serial correlation in the rating process depends on the relative precision of the rating agency's signal errors, relative to that of fundamental innovations, rather than on the actual levels. Although this is a standard Kalman filter result, in the context of our ratings representation it suggests that the better the access of a rating agency to information the more confident the agency will be to rate more aggressively and to give a rating that may contradict a previous one. Also, the rating process in 1.6 was evaluated on the basis of a steady-state assumption,[16] assuming that the market runs for a long time. That assumption may not fit well in a situation of a regime change (e.g. industry

[16] That is, we use the unconditional variance of the rating forecast error. More details are discussed in the appendix.

liberalisation), or an economy in transition, but in those cases, rational expectations and common knowledge of model parameters would not fit well either.

1.2.2 Definition of rational expectations equilibrium

Informed investors of class $j = 1, 2$ are characterised by the information sets $I_{jt} = \{p_s,\ D_s,\ r_s,\ s_s^j;\ s \leq t\}$, which are a records of data z_{jt} of the form

$$z'_{jt} = \left[\begin{array}{cccc} p_t, & D_t, & r_t, & s_t^j \end{array}\right] \tag{1.8}$$

Let also ζ_j be conditional forecast errors, conditional on investors' information sets and on the type of forecasting techniques that investors use to form their beliefs. Let also ς_t be the net supply of the risky asset in period t, where $\{\varsigma_t\}$ are assumed to be *i.i.d.* white noise innovations. Then, the state vector z_t that describes the market for the risky asset in period t is

$$z'_t = \left[\begin{array}{cccccccccc} p_t & D_t & r_t & s_t^1 & s_t^2 & \theta_{1t} & \theta_{2t} & \varsigma_t & \zeta_{1t} & \zeta_{2t} \end{array}\right] \tag{1.9}$$

State vector z_t includes all variables that are directly and collectively observed by investors, as well as the two latent factors θ_1 and θ_2, the random supply ς of the risky asset and investors' forecast errors ζ_j. Also, the noise of the model at t is specified by a vector ε_t which includes all the white noise innovations

$$\varepsilon'_t = \left[\begin{array}{cccccccc} u_t & \eta_{1t} & \eta_{2t} & v_{1t} & v_{2t} & e_{1t-1} & e_{2t-1} & \varsigma_t \end{array}\right] \tag{1.10}$$

where, the white noise innovations $\{u\}$, $\{v_j\}$, $\{\eta_j\}$ and $\{e_j\}$ are defined by 1.1, 1.2, 1.3, 1.4 and $\{\varsigma\}$ are shocks to the aggregate supply of the risky asset.

The fundamental requirement that a REE must satisfy is that equilibrium prices have to be consistent with the presumption that investors know the actual law of motion of the securities market and choose their demands schedules accordingly. Within a given class of linear forecasting rules (e.g. ARMA), a competitive REE for our securities market is defined by the following steps:

Step 1: *Investors make conjectures about the law of motion of variables that they observe.*[17] *Given their information sets, investors use statistically optimal predictors to derive the perceived law of motion for their observable variables.*

Step 2: *Investors select their demand schedules q_t^j so as to maximise their expected utilities. In order to calculate expected utilities, investors use their perceived laws of motion of the variables they observe.*[18]

Step 3: *Given investors' demand schedules, the price p_t of the risky asset clears the market.*

Step 4: *Investors' perceived laws of motion are correct. That is, there is a fixed point in the correspondence that maps investors' perceived laws of motion to the actual law of motion that those perceptions generate.*

In general, the properties of a REE of our securities market will depend on the type of linear forecasting models that investors are assumed to run. Following Sargent (1991), if an equilibrium is such that investors find it optimal to form their beliefs by fitting more complicated (linear) models on their observable variables, that equilibrium would be de-

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[17] As we discuss below, conjecturing a law of motion about observable variables is equivalent to assume that investors conjecture an actual law of motion for the state vector z_t.

[18] In other words, conditional on their perceived laws of motion, investors form subjective beliefs about the variables they observe and the riskiness of their forecast errors.

fined as a *reduced-order* equilibrium. In contrast, a *full-order equilibrium* would be one where investors have no incentive to increase the order of either the AR or the MA part of their forecasting rules. Given the structure of information that is stipulated in this model, Sargent (1991) and Hussman (1992) have shown that an equilibrium that is calculated on the assumption of investors fitting ARMA(1,1) models on observable variables is of full order and we focus on that type of equilibrium. Thus, conditioning investors' forecasts on an infinite history of data is equivalent to conditioning those forecasts only on first-order lags and the information sets I_{jt} $(j = 1, 2)$ can be restated as: $I_{jt} = \{p_t,\ D_t,\ r_t,\ s_t^j\}$.

1.2.3 Beliefs

Following Sargent (1991) and Hussman (1992), informed investors' perceptions about the law of motion of their observable variables are assumed to be of the general ARMA(1,1) form

$$z_{jt+1} = \mathbf{A}_j z_{jt} + \zeta_{jt+1} + \mathbf{C}_j \zeta_{jt} \qquad j = 1, 2 \tag{1.11}$$

where $z'_{jt} \equiv \begin{bmatrix} p_t, & D_t, & r_t, & s_t^j \end{bmatrix}$, ζ_{jt+1} is the vector of conditional forecast errors and $\mathbf{A}_j$, $\mathbf{C}_j$ are matrices of ARMA coefficients that can be recasted such that 1.11 becomes

$$x_{jt+1} = \mathbf{B}_j x_{jt} + \mathbf{v}_{jt+1} \qquad j = 1, 2 \tag{1.12}$$

with $x_{jt} \equiv \begin{bmatrix} z_{jt} \\ \zeta_{jt} \end{bmatrix}$ be the vector of variables that privately informed investors observe in every period, including their realised forecast errors ζ_{jt}, $\mathbf{v}_{jt+1} = \begin{bmatrix} \zeta_{jt+1} \\ \zeta_{jt+1} \end{bmatrix}$, $\mathbf{B}_j \equiv \begin{bmatrix} \mathbf{A}_j & \mathbf{C}_j \\ 0_4 & 0_4 \end{bmatrix}$ and 0_4 be 4×4 matrices of zeros. Given 1.12, informed investors can forecast x_{jt+1} on the basis of observable x_{jt}

$$E\left[x_{jt+1} \mid x_{jt}\right] = \mathbf{B}_j x_{jt} \tag{1.13}$$

Beliefs 1.13 affect investors' optimal demands for the risky asset and, as a result, prices in equilibrium. In Sections 1.2.4 and 1.2.5 we discuss the solution to investors' optimal portfolio choice problem and we solve for equilibrium prices.

1.2.4 Investor optimisation

We assume that investors of class $j = 1, 2$ demonstrate CARA preferences over future wealth w^j with coefficient of constant absolute risk aversion $\frac{1}{\phi_j}$. We also assume that ratings may influence investment decisions not only through the information they convey to market participants, but also through ratings-based capital requirements. Such capital requirements are assumed to imply an opportunity cost of funds that investors need to set aside as capital, which is proportional to the risky-asset holdings of each individual investor.

Thus, in this model, we examine the possibility that ratings-based capital requirements may have an impact on investment decisions by focusing on the opportunity cost of funds that such requirements would imply for market participants. In particular, we adopt a reduced form approach to ratings-based capital charges whereby investors face an opportunity cost (gain) due to capital requirements at a given period, which is proportional to the extent of deterioration (improvement) in the rating quality of the risky asset over that period. For example, if the rating of the risky asset decreases, then an investor with positive asset holdings would face an opportunity cost of funds due to capital charges, proportional to the quantity of his risky-asset holdings.[19] In particular, we assume that investors of class $j = 1, 2$ choose their optimal demands q_t^j for the risky asset in order to maximise their expected utility over next period's wealth w^j

$$q_t^j = \underset{q_t^*}{Arg\max} E\left[-\exp\left(-w_{t+1}^j/\phi_j\right) \mid I_{jt}\right] \qquad j = 1, 2 \tag{14a}$$

subject to

$$w_{t+1}^j = R\left(w_t^j - q_t^* p_t\right) + q_t^*\left(p_{t+1} + D_{t+1}\right) + kq_t^*\left(r_{t+1} - r_t\right) \tag{14b}$$

where R is the constant gross interest rate on an alternative risk-free investment and, as said before, parameter k is aimed to capture the opportunity cost of funds that investors have to set aside as capital.[20] The above maximisation problem gives the following optimal demands

$$q_t^j = \phi_j \frac{E\left[p_{t+1} + D_{t+1} + kr_{t+1} \mid I_{jt}\right] - Rp_t - kr_t}{Var\left[\zeta_{t+1}^p + \zeta_{t+1}^D + k\zeta_{t+1}^r \mid I_{jt}\right]} \qquad j = 1, 2 \tag{1.15}$$

1.2.5 Market clearing

We assume that investors' optimal demands are aggregated by a central auctioneer who finds, if possible, a market-clearing price.[21] At a rational expectations equilibrium the price

19 In order to preserve the linearity of our model, we are going to assume that a rating increase would imply the release of some capital and, as a result, the investor would face a negative opportunity cost (i.e. a gain).

20 Given that the alternative investment that we consider is the risk-free asset, the opportunity cost k must be inversely related to the level of risk-free interest rates R. Moreover, the opportunity cost parameter k has to take into account the slope in the risk-weights scale that is specified by regulators. For example, the Standardised Approach, under the proposed New Basel Capital Accord, stipulates the following (discrete) scale of risk weights: 0% for assets that are rated between AAA and AA-, 20% for A+ to A-, 50% for BBB+ to BBB-, 100% for BB+ to B- and 150% for assets with a rating below B-.

21 That formulation differs from Kyle (1985) and its extensions, where prices are set by a market-maker on the basis of a *semistrong market efficiency* rule.

p_t must clear the market

$$\alpha N q_t^1 + (1-\alpha) N q_t^2 = \varsigma_t \tag{1.16}$$

where q_t^1 and q_t^2 are agents' optimal demands for the risky asset, as given by 1.15, and $\{\varsigma_t\}$ are $i.i.d.$ white noises with mean zero, variance σ_ς^2 and mutually orthogonal in all lags to any other noise term in the model. From 1.15 and 1.16 the price process p_t becomes

$$p_t = \Lambda^{-1}\left[\alpha\sigma_2^2 N\phi_1 E_1[\cdot] + (1-\alpha)\sigma_1^2 N\phi_2 E_2[\cdot] - Mr_t - \sigma_1^2\sigma_2^2\varsigma_t\right] \tag{1.17}$$

where $E_j[\cdot] \equiv E[p_{t+1} + D_{t+1} + kr_{t+1} \mid I_{jt}]$, $\sigma_j^2 \equiv Var\left[\zeta_{t+1}^p + \zeta_{t+1}^D + k\zeta_{t+1}^r \mid I_{jt}\right]$, for $j = 1, 2$, and parameters Λ, M are given by

$$\Lambda \equiv RN\left[\sigma_2^2\alpha\phi_1 + \sigma_1^2(1-\alpha)\phi_2\right]$$
$$M \equiv kN\left[\sigma_2^2\alpha\phi_1 + \sigma_1^2(1-\alpha)\phi_2\right]$$

Both, subjective beliefs $E_j[\cdot]$ and subjective measures of riskiness σ_j^2 are determined in equilibrium on the basis of investors' perceived laws of motion, as discussed in Section 1.2.3.

1.2.6 Solving for a REE

We assume that investors conjecture that the state vector z_t evolves according to the following law of motion

$$z_t = T(\mathbf{B}) z_{t-1} + V(\mathbf{B})\varepsilon_t \tag{1.18}$$

where $\mathbf{B} \equiv [\mathbf{B}_1\ \mathbf{B}_2]$ and $T(\mathbf{B})$, $V(\mathbf{B})$ are matrices of actual coefficients. If all eigenvalues of $T(\mathbf{B})$ lie inside the unit circle,[22] then equation 1.18 determines a covariance-stationary

[22] It can be easily verified that in our model all eigenvalues of matrix $T(\mathbf{B})$ lie inside the unit circle. This is because of the assumption that the autoregressive parameters are such that $|\rho_j| < 1$ $(j = 1, 2)$.

distribution for the state vector z_t, whose moment matrix $\mathbf{M}_z$ solves

$$\mathbf{M}_z = T(\mathbf{B})\,\mathbf{M}_z T(\mathbf{B})' + V(\mathbf{B})\,\Omega V(\mathbf{B})' \tag{1.19}$$

where Ω is the moment matrix of the vector ε_t of white noise innovations and $\mathbf{B} \equiv [\mathbf{B}_1\ \mathbf{B}_2]$. Given that matrix $V(\mathbf{B})\,\Omega V(\mathbf{B})'$ is symmetric, equation 1.19 defines a discrete-time Lyapunov equation. Then, with all eigenvalues of $T(\mathbf{B})$ less than unity in modulus, there is a unique[23] symmetric matrix M_z that solves equation 1.19. With $\mathbf{M}_z$ in hand we can derive the variance covariance matrices $\mathbf{M}_{x_j}$ of investors' observable variables x_{jt} and the covariance matrix $\mathbf{M}_{zx_j}$ of the state vector z_t with the vector of observable variables x_{jt}, $j = 1, 2$. Using an appropriate selector matrix $\mathbf{u}_j$, matrices $\mathbf{M}_{x_j}$ and $\mathbf{M}_{zx_j}$ are given by

$$\begin{aligned} \mathbf{M}_{x_j} &= \mathbf{u}_j \mathbf{M}_z \mathbf{u}_j' \\ \mathbf{M}_{zx_j} &= \mathbf{M}_z \mathbf{u}_j' \end{aligned} \tag{1.20}$$

Let us now consider the linear projection of vector x_{jt+1}, of investor's $j$observable variables, on its previous realisation x_{jt}

$$E\left[x_{jt+1} \mid x_{jt}\right] = \mathbf{S}_j(\mathbf{B})\,x_{jt} \qquad j = 1, 2 \tag{1.21}$$

Using matrices $\mathbf{M}_{x_j}$ and $\mathbf{M}_{zx_j}$, we are able to evaluate the matrix $\mathbf{S}_j(\mathbf{B})$ of statistically optimal estimators as follows

$$\mathbf{S}_j(\mathbf{B}) = \mathbf{u}_j T(\mathbf{B})\,\mathbf{M}_{zx_j} \mathbf{M}_{x_j}^{-1} \tag{1.22}$$

where $\mathbf{M}_{x_j}$ and $\mathbf{M}_{zx_j}$ are given by 1.20 and $\mathbf{u}_j$ is a matrix that selects the subvector of observable variables x_{jt} from the state-space vector z_t.

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23 From standard theory, there is a unique symmetric matrix $\mathbf{M}_z(\mathbf{B})$ that solves (1.19) *i.f.f.* no eigenvalue of $T(\mathbf{B})$ is the reciprocal of any other eigenvalue of $T(\mathbf{B})$. This is, *i.f.f.* $\mathbf{eig}\,[T(\mathbf{B})]\,\mathbf{eig}\,[T(\mathbf{B})]' - \mathbf{1} \neq \mathbf{0}$. Given that all eigenvalues of $T(\mathbf{B})$ lie inside the unit circle, none of them can be the reciprocal of another eigenvalue of $T(\mathbf{B})$.

Let $\mathbf{S}(\mathbf{B}) \equiv [\mathbf{S}_1(\mathbf{B})\ \mathbf{S}_2(\mathbf{B})]$, then, a rational expectations equilibrium is a fixed point in the correspondence that maps investors' perceptions – as defined by the VAR coefficients $\mathbf{B}$ in 1.13 – into statistically optimal projections $\mathbf{S}(\mathbf{B})$, given the actual law of motion 1.18 that investors' perceptions generate. It is worth emphasising that, in this model, conjectures about the coefficient matrix $\mathbf{B}$ are equivalent to conjectures about the actual law of motion 1.18 of the state vector z_t. Such an equivalence stems from the fact that, for a given coefficient matrix $\mathbf{B}$, equation 1.19 defines a *unique* moment matrix $\mathbf{M}_z$ for the state vector z_t, which in turn, defines matrices $T(\mathbf{B})$ and $V(\mathbf{B})$ of the actual coefficients. In other words, there is a one-to-one relationship between conjectures about coefficient matrix $\mathbf{B}$ and matrices $T(\mathbf{B})$, $V(\mathbf{B})$. That becomes evident in Section 1.A.2 where we outline the fixed-point solution algorithm and how matrices $T(\mathbf{B})$ and $V(\mathbf{B})$ are evaluated.

1.3 Ratings and price discovery

In this section we examine the effect of ratings on price volatility and efficiency under the presumption that they are used solely for price discovery and not for any other purpose, such as to benchmark investment decisions or to set capital requirements. The equilibrium coefficients of investors' forecasting models were calculated using Matlab programs under the following basic parameterisation.

Risk tolerance	$\phi_1=\phi_2=1$
Investor proportions	$a=0.5$
Gross interest rate	$R=1.02$
Constants	$N=1,\ \sigma_u^2=1$
Persistence of fundamentals	$\rho=0.8$
Variance of fundamental innovations	$\sigma_v^2=0.1$
Variance of errors in investors' private signals	$\sigma_\eta^2=1$
Variance of errors in rating agency's signal	$\sigma_e^2=0.1$
Variance of noise in the supply of the risky asset	$\sigma_\varsigma^2=0.01$

The above parameterisation was chosen mainly to illustrate the potential impact of rating announcements on asset prices, but has not been calibrated to match any actual data. Moreover, it allows us to search for a symmetric equilibrium, whereby the coefficients in the forecasting models of each class of investors are equal. In Section 1.3.3, we present a comparative statics analysis where we examine the sensitivity of our results to different levels of risk aversion and precision of rating information.

In order to gauge the impact of ratings on asset prices, we consider two benchmark cases, namely, the case with asymmetric information, but without ratings, and the case of full information.[24] Given the linearity of the model and the assumption that all inno-

[24] Under the full information benchmark, investors are assumed to observe perfectly the realisation of both fundamental factors, but they still remain uncertain about future realisations of these factors. As in Hussman (1992), one can show that for fundamental shocks v_{jt}, $j=1,2$, the price process (p_t) under full information is given by:

$$p_t = \frac{\rho}{(R-\rho)(1-\rho L)}(v_{1t}+v_{2t}) - Constant$$

which implies the following expression for the unconditional variance of prices:

$$Var(p_t) = 2\left(\frac{\rho}{R-\rho}\right)^2\left(\frac{1}{1-\rho^2}\right)\sigma_v^2$$

vations in the model are normally distributed, correlations are considered in terms of the coefficient of linear correlation. Market efficiency is then considered with respect to the informativeness of prices and the extent to which prices correlate with fundamentals.

Section 1.A.3 in the appendix presents the equilibrium coefficients of investors' forecasting techniques and second moments of prices when investors are assumed to run vector ARMA(1,1) models. The ARMA(1,1) case is called *the high-sophistication case*, in the sense that investors cannot improve further their predictions by incorporating more lags in their forecasting models.[25] Also in the appendix we present the equilibrium when investors' forecasting techniques are restricted to a first-order vector AR(1) process. This allows for an examination of the extent to which our results might be sensitive to the assumption of the type of forecasting techniques that investors are using at the REE. The case where investors run simple AR(1) models is called *the low-sophistication case*, in a sense that investors could further improve their forecasts by adding more lags in their time-series models.

Based on the results that we derive under both the high and low-sophistication case, we discuss how the use of ratings for price discovery may impact on market efficiency and price volatility.

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and the covariance of prices with fundamentals θ_{jt}, $j = 1, 2$:

$$Cov\left(p_t, \theta_{jt}\right) = \frac{\rho}{\left(R-\rho\right)\left(1-\rho^2\right)}\sigma_v^2$$

Using the above expressions and the fact that the unconditional variance of fundamentals is $Var\left(\theta_{jt}\right) = \frac{\sigma_v^2}{1-\rho^2}$, we can derive the coefficient of linear correlation of prices with fundamentals under full information.

25 See, for example, Sargent (1991) and Hussman (1992).

1.3.1 Results

Tables I and II below, compare the equilibrium results when there is incomplete information under both the highly sophisticated (ARMA) and less sophisticated (AR) forecasting rules, both with and without ratings. Table I reports the equilibrium variance of asset prices in the different cases, while Table II shows the impact on price efficiency (i.e. how much prices correlate with fundamentals). The benchmark case of full information is also shown in the following tables.

Table I: Price Volatility

Full-information benchmark	
7.3462	
Incomplete information without ratings	
High sophistication	Low sophistication
4.3608	0.3988
Incomplete information with ratings	
High sophistication	Low sophistication
5.0634	4.5017

Table II: Price Efficiency

Full-information benchmark	
0.7071	
Incomplete information without ratings	
High sophistication	Low sophistication
0.4914	0.3927
Incomplete information with ratings	
High sophistication	Low sophistication
0.5429	0.5400

We observe that, in the incomplete information equilibrium, and regardless of the forecasting techniques used, the introduction of ratings increases the volatility of prices but they also enhance price informativeness. In particular, Table II shows that, under both the ARMA and the AR case, the introduction of ratings increases the correlation of prices with fundamentals θ_j $(j = 1, 2)$ albeit at a cost of higher price volatility. The increase in price volatility is much stronger under the low-sophistication case where the introduction of ratings results in an increase in volatility from, approximately, 0.4 to 4.5. However, under the high-sophistication case, the increase is less striking from approximately 4.4 to 5.1.

In the following section we examine how non-fundamental shocks (ς_t) impact on prices under both the ratings and no-ratings case. We also consider the impact of a one-off shock in fundamentals ν_t – i.e. the impact of a single shock in fundamentals that is isolated from the impact of any other shock in the model – as well as the impulse response of prices to pay-off innovations u_t and private signal errors η_t.

1.3.2 Persistence

Under the full-information benchmark, non-fundamental shocks have no persistence on prices because non-fundamental shocks themselves have no persistence. However, when the full information assumption is relaxed, non-fundamental shocks may have a persistent effect on prices. That is because fundamentals are latent variables and investors rely on past values of observable variables to filter information about fundamentals and form their beliefs. Given that prices are affected, through market clearing, by one-off non-fundamental shocks, those shocks may continue to affect prices in future periods through investors' filtering problems. In other words, persistence of non-fundamental shocks on prices is driven by, what Bacchetta and van Wincoop call, persistence of investors' *rational confusion* that eventually dissipates as investors gradually learn about the realisation of fundamentals in previous periods.

Similarly, rational confusion may inhibit investors from responding effectively to fundamental shocks ν_t and it may also drive them to misinterpret non-fundamental noise u_t in asset pay-offs as being fundamental information. The extent to which ratings ameliorate investors' rational confusion and facilitate the incorporation of fundamental information into prices will determine to what extent rating agencies provide a useful service to the market. Finally, errors in investors' private signals may have a different impact on prices under the ratings and the no-ratings case. Private signal errors are expected to affect prices through channels of both subjective beliefs and subjective measures of riskiness. As with all other types of shocks that we consider, the impulse response of prices to private signal errors will be determined by the sign and relative importance of elements in the VAR matrix $T(\mathbf{B})$, as defined by equation 1.18.

The impulse response of prices to various shocks in the model will be determined by the sign and relative importance of elements in the VAR matrix $T(B)$, as defined by equation (1.18). From 1.18, the impulse response of prices to a one standard deviation shock in the i^{th} element of innovations vector ε_t, as defined in 1.10, is given by the following function:

$$f(t) = \left[T(\mathbf{B})^{t-1} V(\mathbf{B})\right]^{(1,i)} \sigma_i \tag{1.23}$$

where $T(\mathbf{B})$ and $V(\mathbf{B})$ are defined by 1.18, σ_i is the standard deviation of the i^{th} element of vector ε_t, superscript $(1, i)$ refers to the i^{th} element in the first row of the matrix in brackets and $t = 1, 2, ...\infty$.

Figure 1.1 illustrates the impulse response of prices to a one standard deviation shock in non-fundamental trade. Under the no-ratings case, the rational confusion that follows the shock induces a price overreaction almost three times larger than the case with ratings. It then takes around 13 trading rounds for most of the rational confusion to unwind, compared to eight trading periods under the ratings case. Similarly, figure 1.2 shows how prices respond to an idiosyncratic shock u in asset pay-offs. We observe that ratings mitigate any undue price impact of a one-off shock in asset pay-offs that is not related to fundamentals.

Figure 1.3 shows the price response to a shock of one standard deviation in fundamentals. Although, initially, the price responds to the shock in the same fashion under both the ratings and the no-ratings case, over the next couple of trading rounds prices tend to move closer towards the full-information benchmark under the ratings case, compared with the no-ratings case. This confirms our earlier finding – by using the equilibrium variance/covariance matrix of our state variables – that ratings improve the informativeness of prices. Finally, in figure 1.4, we report the impact on prices of a one standard deviation shock in private signal errors. The non-monotonicity in the impulse response is due to a particular combination of positive and negative elements in the VAR coefficient matrix $T(\mathbf{B})$, the endogenous nature of prices and forecast errors and the fact that private signals may play a more pronounced role in affecting investors' forecast errors than any other state variable.

1.3.3 Comparative statics

In this section we present a comparative statics of different degrees in risk aversion and of the precision of rating information relative to that of privately informed investors.

Risk aversion

According to financial economics, agents trade securities for two different motives: (i) to share risk when they are endowed with different quantities of the risky asset and (ii), to exploit information when they have access to different information sources and possess different assessments of risky asset pay-offs. The two motives for trading may combine together and affect prices in various ways depending on the model parameters. In particular, as with Hellwig's (1980) static model, as risk aversion increases in the market the risk-sharing motive dominates that of exploiting information. As a result, risk aversion results in less informative prices, which is consistent with the results that our dynamic model produces and are shown in figure 1.5.

As far as price volatility is concerned, it depends both on the degree of price informativeness and serial correlation. On the one hand, we have seen already that price informativeness increases with ratings towards that of the full-information benchmark and this is mainly because prices, by becoming more informative, respond better to fundamental innovations. On the other hand, the higher the serial correlation (in absolute terms) of prices the higher the unconditional variance of the price process. Prices, however, may become serially correlated as a result of serially correlated fundamentals, strong risk-sharing motives, filtering problems, or other externalities that may induce investors to trade with less confidence on private information and place more weight on publicly observed signals, such as prices.

Figure 1.6 reports the impact of risk aversion on price volatility and figure 1.7 the relationship between risk aversion and investors' modelled risk-perceptions. Figures 1.6 and 1.7 illustrate that, *ceteris paribus*, in a market with high risk aversion rational investors are aware that risk-sharing motives dominate those of information exploitation and prices become less informative.[26] As a result, the accuracy of investors' optimal forecasts, which

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[26] This is consistent with Hellwig (1980).

depend among other things on how informative prices are, diminishes. Given that the longrun (unconditional) mean of prices is common knowledge among investors and prices are competitive, less accurate forecasts induce investments to weigh more on the fact that any price deviations from its unconditional mean will be reversed afterwards.[27] Consequently, prices are characterised by strong mean reversion and, as a result, high serial correlation and price volatility.

Rating precision

As far as the effect of the precision of ratings on prices is concerned, figure 1.8 shows that the lower the precision of rating information the less precise investors' optimal forecasts become, but they still remain more precise than in the no-ratings case. Consequently, as the precision of rating information diminishes relative to that of investors' private signals, investors trade less aggressively for information reasons and the informativeness of prices drops towards the no-ratings case benchmark. The relationship between price informativeness and the precision of rating information is illustrated in figure 1.9.

Moreover, as the precision of ratings decreases, relative to that of investors' private information, the market turns out to ignore ratings and the volatility of prices drops towards the level under the no-ratings case. This effect is quite distinct from the impact of risk aversion on prices; while risk aversion induces higher serial correlation in prices and, as a

[27] We could argue that the less accurate investors' forecasts become, the price tends to become a *focal point* around which investors co-ordinate their beliefs. As a result, long-run (unconditional) mean reversion of prices becomes self-fulfilled at earlier trading rounds and it becomes more likely to affect the decisions of currently lived investors. This is consistent with the results of Allen, Morris and Shin (2003) who solve a similar type of equilibrium but with three trading rounds, totally uninformative prices and a public signal about fundamentals that acts as a focal point and *skews* agents beliefs towards it. In our case, however, the focal point is still the price signal itself.

result, higher unconditional volatility of prices, the lower the precision of ratings, relative to that of investors' private signals, the more rational investors tend to ignore ratings and focus more on their private information. This point is illustrated in figure 1.10.

1.4 Ratings and benchmarking

We now turn to examine how prices may be affected by frictions that relate to the use of ratings not only for pure information discovery purposes, but also for rating-based capital requirements and benchmarking of investment decisions on ratings.

1.4.1 Rating-based capital requirements

Regulatory rules often allow regulated entities, such as banks and securities houses, to use credit ratings for capital adequacy purposes. The usual requirement that those entities have to meet is to deduct from capital a certain percentage of the value of their security holdings, depending on the rating that those securities receive from recognised rating agencies. In addition, regulated entities are required by law to maintain a minimum level of capital to withstand potential future losses and, should their capital fall towards that level, they have either to reduce their exposures to risky investments, or to recapitalise.

But setting capital aside for prudential regulation purposes entails an opportunity cost of foregone interest from investing in more profitable risky assets rather than in risk-free securities. This is especially the case when an investor's internal assessment of the fundamental value of traded securities conflicts with that of a rating agency. Consequently, via rating-based capital requirements, ratings could impose a constraint on investment decisions, forcing investors to respond to rating changes in a way that is possibly contrary to their private assessments. That, in turn, could have a material impact on both price efficiency and volatility. What such an impact could be is an open question that we attempt to address through our stylised model in this section.

From Section 1.2.4, parameter k captures the opportunity cost of funds due to rating-based capital charges. So far, k has been set equal to zero, but now turn to examine the case with rating-based capital requirements, that is when $k > 0$, and their impact on the informativeness and volatility of prices. Figure 1.11 shows that the informativeness of prices decreases the higher the parameter k, namely, the higher the incentives that capital adequacy rules offer to investors to forecast next period's rating. At the same time, the volatility of prices drops for an initial range of parameter k and then increases as investors' incentives to forecast the rating process increase further. This is illustrated in figure 1.12.

However, high levels of parameter k would be far from relevant to existing rating-based capital adequacy rules. In particular, the risk-weighting scale of asset holdings under the proposed New Basel Accord, along with the 8% Basel ratio, and low levels of world interest rates would imply a relatively modest level of incentives to forecast ratings for capital adequacy purposes. Thus, any realistic set of rating-based capital rules would be expected to imply a low k, under which both price efficiency and volatility would possibly drop. Moreover, in the real world, the dispersion of information across investors would possibly be higher than in our model, where only two classes of informed investors have been assumed. Higher dispersion of beliefs across investors would lead to greater heterogeneity in asset holdings across portfolios and, as a result, the impact of rating-based capital requirements on the informativeness of prices at an aggregate level would be less pronounced than what our model implies.

1.4.2 Benchmarking noise trades to ratings

An increasing number of policymakers and market participants, including the rating agencies themselves, have pointed to the fact that the use of ratings for reasons other than their information content may impose a negative externality on the efficient functioning of securities markets. In particular, linking investment decisions to ratings, with the most notable example the dichotomy between *investment* and *subinvestment* grade credits, may distort financial markets from pooling information and allocating financial resources in an efficient way.

Such a distortion could arise as a result of both regulatory rules and market practices. In particular, many institutional investors are forced by law, or their own charter, to sell bonds whose credit rating has crossed some critical threshold level. In the United States, for example, regulators place restrictions on the quality of assets pension funds and insurance companies can invest in and those restrictions are explicitly linked to the credit ratings produced by the Nationally Recognised Statistical Rating Organisations (NRSROs). Although these rating-linked constraints may not be necessarily *hard* – in a sense of prescribing *immediate* liquidation of affected assets – they may adversely interfere with investment decisions and drive investors' interest away from assets whose economic value would, otherwise, warrant a better treatment by the market.

In this section we attempt to touch upon the issue of linking investment decisions to ratings and to examine the efficiency implications of such practices. However, the idea of having to liquidate a position in an asset, whose rating has fallen below a certain threshold, implies an optimal investment strategy that allows for the possibility of *downgrade-and-sell* scenarios.[28] In a multi-period context that would require us to track individual investors' asset holdings over time and to incorporate them into the state-space representation of our securities market.

To avoid such a complication, we consider the situation where both fundamental and non-fundamental trade takes place on the basis of a one-period horizon. Then, we relate the non-fundamental trade to ratings by assuming that there is a set of residual market participants who supply the risky asset proportionately to the probability the rating next period will fall below a certain threshold $\overline{r}$. Threshold $\overline{r}$ is assumed common knowledge among investors. For simplicity, we also assume that the residual investors do not learn from prices or asset pay-offs, but they only consider ratings.[29] Thus, the empirical ratings distribution of those investors is conditional only on past rating information. Given the Kalman filter representation of the rating process in lemma 1, the above conditionality can be stated simply in terms of the currently observed rating r_t and not on the basis of the whole history of ratings up to period t.

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28 In that case, evaluating optimal holdings in the risky asset would require techniques similar to those for pricing barrier contracts.

29 This assumption is without loss of generality and is imposed in order to avoid the complication of having to consider non-fundamental investors running econometric models. In a real-world context, one could think of a competitive intra-dealer market and institutional investors with limited price discovery capabilities and restricted access to competitive prices. This would be possibly not far from the realities of corporate bond markets.

Noise traders are assumed to benchmark their supply of the risky asset to some measure of the probability the rating next period will fall below a given threshold $\overline{r}$. Benchmarking, in this way, can be rationalised as the result of forced sales by a class of regulated investors that are restricted to hold the asset only if its rating is above $\overline{r}$ and unload their holdings to the market proportionally to the probability such *downgrading* will take place. For computational convenience and without loss of generality we assume that noise traders consider only ratings for computing such a probability and do not filter information from prices and asset pay-offs. Thus, the total net supply S_t of the risky asset in period t is assumed to be of the form:

$$S_t \simeq A \Pr\left(r_{t+1} \leq \overline{r} \middle| r_t\right) + \varsigma_t \tag{1.24}$$

where A is a constant that captures the extent of benchmarking of noise trades to ratings. Normality is preserved by conditional expectations, thus, by taking the first-order Taylor expansion of the probability term in 1.24 we may express S_t as

$$S_t = A\left(\frac{1}{2} + \frac{1}{\sqrt{2\pi Var\left(r_{t+1}\middle| r_t\right)}}\left[\overline{r} - E\left(r_{t+1}\middle| r_t\right)\right]\right) + \varsigma_t \tag{1.25}$$

where by application of the *Projection Theorem* and from 1.6

$$E\left(r_{t+1}\middle| r_t\right) = \left\{\rho\left[1 - \Sigma\left(\Sigma+1\right)^{-1}\right] + 2\rho\Sigma\left(\Sigma+1\right)^{-1}\frac{Cov\left(r,\theta_j\right)}{Var\left(r\right)}\right\} r_t \tag{1.26}$$

and

$$Var\left(r_{t+1}\middle| r_t\right) = 2\left[\rho\Sigma\left(\Sigma+1\right)^{-1}\right]^2\left[Var\left(\theta_j\right) - \frac{Cov\left(r,\theta_j\right)^2}{Var\left(r\right)}\right] \tag{1.27}$$

where parameter Σ is given by 1.7 and all unconditional second moments in 1.26 and 1.27 can be derived from the equilibrium moment matrix M_z of state vector z_t. By substituting 1.25 into the price equation 1.17 we can derive an expression for the state-space representation of the new price process. We can then compute the price dynamics in the new REE and consider the efficiency implications of benchmarking investment decisions to ratings.

It is worth reiterating that both the benchmarking parameter A and the rating threshold $\overline{r}$ are assumed common knowledge among investors. Thus, the supply of the risky asset that is due to benchmarking of asset holdings on ratings is also common knowledge in every period. That allows us to avoid any further complication of having to consider higher order beliefs about the extent of ratings benchmarking in the market and investors' individual threshold levels. Moreover, regarding the supply of the risky asset, no more noise was added in the model and, as a result, the extent of noise trading ς_t in our securities market remains unaltered. Despite that, however, we will see next that the effect of benchmarking on the second moments of asset prices, and consequently on efficiency, is non trivial.

Assuming a relative precision of 0.9 between private and rating information[30] and by varying the level of benchmarking parameter A, we show that price efficiency drops with the extent of benchmarking (A) in the market while volatility increases, as illustrated in figures 1.13 and 1.14. That occurs despite informed investors being fully rational and no extra source of noise was added in the model. In fact, at the beginning of each trading round, investors observe the realisation of the rating and, by the time investment decisions are made, everyone knows exactly the amount of concurrent residual supply that is due to benchmarking.[31] But, instead of that having a trivial levels-impact on prices, benchmarking on ratings has a material impact on the second moments of prices.

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30 We have repeated the analysis using different levels of relative information precision and the results look qualitatively the same.

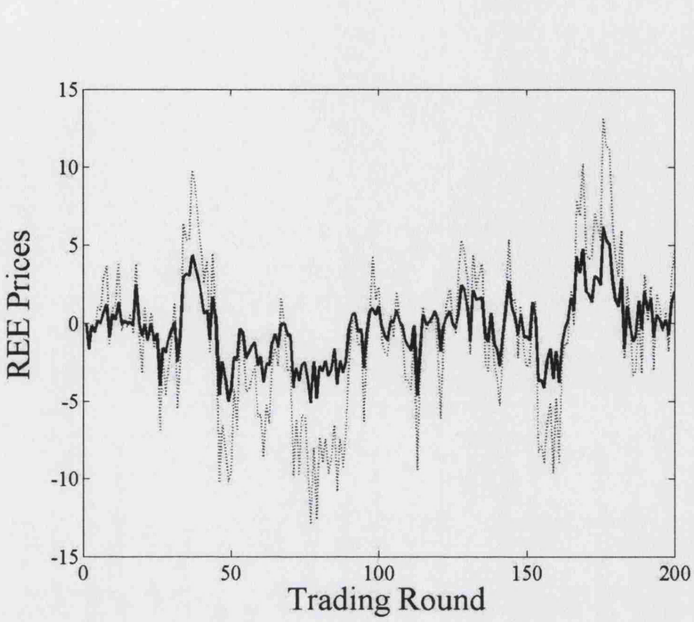
31 In reality, uncertainty about the extent of benchmarking on ratings may further amplify the loss of efficiency.

Such an impact of benchmarking on asset prices can be justified on the grounds that perceived changes in fundamentals feed into prices not only through changes in perceptions about future income from holding the asset, but also through beliefs about capital gains that depend on the net supply of the asset. Given that benchmarking renders the net supply of the risky asset partly forecastable, informed investors are inclined to trade more aggressively on any item of information in order to exploit perceived mispricings and become more prone to misinterpret any item of news as information about fundamentals.

More formally, figures 1.15 to 1.18 report the impulse response of prices to a one standard deviation shock in various noise terms in the model, demonstrating how rational confusion due to asymmetric information could impact on prices. Figures 1.15 to 1.17 show that benchmarking magnifies any undue price response to non-fundamental shocks in pay-offs and errors in private signals and ratings.[32] Figure 1.18 shows that, in the presence of benchmarking of noise trades to ratings, prices overreact to innovations in fundamentals by overshooting even the full information case, which captures the basic accounting identity between prices and asset pay-offs and is represented by a dotted, downward-sloping line in figure 1.18.

The chart below presents a simulation of REE prices with and without benchmarking (solid line), illustrating the magnifying impact of benchmarking on price variations.

32 Notice that, by construction, it is only informed traders who observe pay-offs and private signals. As a result, any price overreaction at least to non-fundamental pay-off shocks and private signal errors is due to trading by informed traders rather than stemming directly from noise trading.



REE prices with benchmarking of noise trades to ratings

Consequently, benchmarking of noise trades to ratings could induce informed traders to trade aggressively on any item of news, sound or not, in order to exploit perceived mispricings in the traded asset. In that case, even relatively unimportant news – i.e. news that is unrelated to fundamentals – could lead to large price swings, resulting in excess asset price volatility and low price efficiency.

1.5 Conclusions and extensions

The role and importance of rating agencies in capital markets has been criticised in recent years because agencies have failed to foresee a number of high-profile credit events, such as the Asian crisis in 1997, the Russian default in 1998 and the Enron bankruptcy in 2001. Agencies have also been criticised for increasing volatility in financial markets, while there have also been voices arguing that ratings are of marginal value to financial markets because the information they provide is *stale* and has already been reflected into share prices.[33]

The model presented in this paper demonstrated that, even if ratings lag the market, they may enhance price efficiency when they are used solely for price discovery by market participants and not for other purposes, such as benchmarking of asset holdings on ratings or rating-based capital requirements. On the other hand, the introduction of ratings could add to asset price volatility, but this was found to be consistent with improved market efficiency. This is under the presumption that investors believe that what the rating agency announces is its *best guess* about fundamentals, and investors, despite having different information, have common knowledge of how the economy works.

We also showed that the quantitative impact resulting from the use of ratings for price discovery purposes may depend on the way that rating information is rationally processed by investors. The lower the sophistication of the forecasting techniques used, the more pronounced the impact of ratings on market outcomes. Qualitatively, however, our results remain robust to the type of forecasting techniques that are used by investors.

Regarding the use of ratings for reasons other than price discovery, we distinguished between two types of ratings-related frictions: (i) rating-based capital requirements that apply to investors on the basis of their individual holdings of a rated asset, (ii) benchmarking of asset holdings to ratings from a residual set of investors (e.g. pension funds, insurance companies) whose sole concern is to sell assets whose rating is likely to fall below a certain threshold.

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33 See, for example, J. DuPratt White Professor of Law at Cornell Law School, testimony in front of the US Senate's Committee on Governmental Affairs, 21 March 2002.

As far as rating-based capital requirements are concerned, our analysis indicated that if investors' incentives to track ratings for capital adequacy purposes are relatively modest then, rating-based capital requirements may reduce price volatility, yet at the cost of lower price efficiency. However, if incentives to track ratings are sufficiently strong then, rating-based capital requirements could, under certain conditions, add to asset price volatility.

In order to analyse the impact of benchmarking asset holdings to ratings, we considered a residual class of (noise) traders that link their net supply of the risky asset to some measure of the probability that the rating next period will fall below a certain threshold. Benchmarking, in this way, was rationalised as the result of forced sales by a class of regulated market participants who face restrictions on the rating quality of assets they hold.

Our results demonstrated that benchmarking of asset holdings to ratings by certain market participants could induce other investors to overreact to any item of news about fundamentals, leading to lower price efficiency and higher asset price volatility. We argued that this is because perceived changes in fundamentals feed into prices not only through changes in perceptions about future income from holding the asset, but also through beliefs about capital gains that depend on the net supply of the asset. Given that benchmarking renders the net supply of the risky asset partly forecastable, informed traders are inclined to trade more aggressively on any item of news that could imply a change in fundamentals, even if they face no restrictions on the rating quality of assets they hold. As a result, informed investors become more prone to misinterpret any item of news as information about fundamentals leading to less informative and more volatile prices.

At this point, it is worth drawing a parallel between our results, in case of benchmarking asset holdings to ratings, and the UK market experience in the second half of 2002. Market commentators at the time attributed the rapid swings in market sentiment partly to a regulatory *resilience test* that applies to life insurance companies. According to that test, firms have to demonstrate solvency in the face of a further 25% decline in their asset holdings. In view of a rapid decline in stock prices that period, the resilience test was suspended for several weeks in order to mitigate forced sales of stocks by major market players.[34]

In a sense, the resilience test that applies to life insurers is a form of benchmarking similar in nature to the rating-based benchmarking that we discussed in this paper. That is, in both cases, a class of market participants benchmarks its investment decisions on a public signal which also conveys information about fundamentals. In our model that public signal was the rating; regarding the resilience-test case, that signal was the price. Similar parallels one could draw with respect to the1987 stock market crash and the role of portfolio insurance, as another form of benchmarking on prices, in exacerbating market turbulence.

Looking forward, the model could be extended to incorporate an explicit objective, by the rating agency, to smooth the rating process (e.g. to avoid rating reversals) and to examine how that might impact on market outcomes. That, of course, would require us to introduce an adjustment cost in the rating process and the rating agency, in the model, to solve a dynamic programming problem rather than running a simple Kalman filter to assign its ratings. Moreover, a different, though still time invariant, rating process could be

[34] See, FSA Guidance Note 4 (2002), '*Resilience test for insurers*'.

adopted that would share more similarities with the actual way that ratings are announced in the marketplace, namely, not in every trading round. A good candidate could be a Markov arrival of rating information under which a rating would be announced in randomly selected periods according to a Markov process. From a modelling perspective, an appealing feature of a Markov formulation would be that, as with the Kalman filter, it has a state-space representation and can be easily incorporated into our framework.

Finally, it would be worth exploring how the results would be affected by an increase in the information dispersion among investors about fundamentals and consider more than two classes of privately informed investors. That would possibly allow us to compare our results with earlier findings on the impact of public information on asset prices, such as in Allen, Morris and Shin (2003).

1.A Appendix

1.A.1 Proof of Lemma 1

Let $\mathbf{s}^{r}_{jt-1}$be the vector of signals that the rating agency receives up to $t-1$ about factor θ_j, $j=1,2$. Given normality of θ_{jt} and signal vector $\mathbf{s}^{r}_{jt-1}$, the conditional distribution of θ_{jt}, conditional on signal vector $\mathbf{s}^{r}_{jt-1}$, is also normal with conditional mean and variance

$$\overline{\theta}_{jt|t-1} \equiv E\left(\theta_{jt} \mid \mathbf{s}^{r}_{jt-1}\right) \tag{1.28}$$

$$\Sigma_{t|t-1} \equiv Var\left(\theta_{jt} \mid \mathbf{s}^{r}_{jt-1}\right) \tag{1.29}$$

Let us suppose that the conditional mean $\overline{\theta}_{jt|t-1}$ and variance $\Sigma_{t|t-1}$ have been calculated and with those in hand we are able to evaluate $\overline{\theta}_{jt+1|t}$ and $\Sigma_{t+1|t}$. From 1.4 we easily derive the conditional expectation of the signals that the rating agency receives in period t, conditional on the agency's signal information up to period $t-1$

$$E\left(s^{r}_{jt} \mid \mathbf{s}^{r}_{jt-1}\right) = E\left(\theta_{jt} \mid \mathbf{s}^{r}_{jt-1}\right) = \overline{\theta}_{jt|t-1} \tag{1.30}$$

Moreover, the forecast error $s^{r}_{jt} - E\left(s^{r}_{jt} \mid \mathbf{s}^{r}_{jt-1}\right)$ is

$$s^{r}_{jt} - E\left(s^{r}_{jt} \mid \mathbf{s}^{r}_{jt-1}\right) = \left(\theta_{jt} - \overline{\theta}_{jt|t-1}\right) + e_{jt} \tag{1.31}$$

Since e_{jt} are independent over time and orthogonal to θ_{jt}, they are also independent of $\overline{\theta}_{jt|t-1}$. This implies that the conditional variance of the forecast error 1.31 is

$$Var\left[\left(s^{r}_{jt} - E\left(s^{r}_{jt} \mid \mathbf{s}^{r}_{jt-1}\right)\right)\right] = \Sigma_{t|t-1} + \sigma^2_e \tag{1.32}$$

where $\sigma_e^2 \equiv Var\left[e_{jt}\right]$. Similarly, the conditional covariance between the forecast errors $s_{jt}^{r} - E\left(s_{jt}^{r} \mid \mathbf{s}_{jt-1}^{r}\right)$ and $\theta_{jt} - E\left(\theta_{jt} \mid \mathbf{s}_{jt-1}^{r}\right)$ is

$$\begin{aligned} & Cov\left[s_{jt}^{r}, \theta_{jt} \mid \mathbf{s}_{jt-1}^{r}\right] \\ = \; & E\left[\left(\theta_{jt} - \overline{\theta}_{jt|t-1} + e_{jt}\right)\left(\theta_{jt} - \overline{\theta}_{jt|t-1}\right)\right] \\ = \; & \Sigma_{t|t-1} \end{aligned} \tag{1.33}$$

From 1.28, 1.29, 1.30, 1.32 and 1.33 we get the conditional joint distribution of signal s_{jt}^{r} and fundamental factor θ_{jt}, conditional on signal information $\mathbf{s}_{jt-1}^{r}$ up to period $t-1$

$$\begin{bmatrix} s_{jt}^{r} \mid \mathbf{s}_{jt-1}^{r} \\ \theta_{jt} \mid \mathbf{s}_{jt-1}^{r} \end{bmatrix} \sim N\left(\begin{bmatrix} \overline{\theta}_{jt|t-1} \\ \overline{\theta}_{jt|t-1} \end{bmatrix}, \begin{bmatrix} \Sigma_{t|t-1} + \sigma_e^2 & \Sigma_{t|t-1} \\ \Sigma_{t|t-1} & \Sigma_{t|t-1} \end{bmatrix}\right) \tag{1.34}$$

Let us now define $\overline{\theta}_{jt|t}$ as the conditional expectation of factor θ_{jt} conditional on signal vector $\mathbf{s}_{jt}^{r}$, namely, all signals s_j^{r} up to period t

$$\begin{aligned} \overline{\theta}_{jt|t} & \equiv E\left(\theta_{jt} \mid \mathbf{s}_{jt}^{r}\right) \\ & = E\left(\theta_{jt} \mid s_{jt}^{r} \middle| \mathbf{s}_{jt-1}^{r}\right) \end{aligned} \tag{1.35}$$

The conditional expectation $\overline{\theta}_{jt|t}$ and the conditional variance $\Sigma_{t|t}$ of the forecast error can be evaluated by applying the *Projection Theorem,* using the join distribution in 1.34

$$\overline{\theta}_{jt|t} = \overline{\theta}_{jt|t-1} + \Sigma_{t|t-1}\left(\Sigma_{t|t-1} + \sigma_e^2\right)^{-1}\left(s_{jt}^{r} - \overline{\theta}_{jt|t-1}\right) \tag{1.36}$$

$$\Sigma_{t|t} = \Sigma_{t|t-1} - \Sigma_{t|t-1}^2\left(\Sigma_{t|t-1} + \sigma_e^2\right)^{-1} \tag{1.37}$$

Moreover, from 1.2 and also the fact that v_{jt} are orthogonal to every element of the signal vector $\mathbf{s}_{jt}^{r}$, we get

$$\begin{aligned}\overline{\theta}_{jt+1|t} &\equiv E\left(\theta_{jt+1} \mid \mathbf{s}_{jt}^{r}\right) \\ &= E\left(\rho\theta_{jt} + v_{jt} \mid \mathbf{s}_{jt}^{r}\right) \\ &= \rho\overline{\theta}_{jt|t}\end{aligned} \tag{1.38}$$

$$\begin{aligned}\Sigma_{t+1|t} &\equiv Var\left(\theta_{jt+1} \mid \mathbf{s}_{jt}^{r}\right) \\ &= Var\left(\rho\theta_{jt} + v_{jt} \mid \mathbf{s}_{jt}^{r}\right) \\ &= \rho^{2}\Sigma_{t|t} + \sigma_{v}^{2}\end{aligned} \tag{1.39}$$

Combining 1.36 with 1.38, and 1.37 with 1.39 we derive the following Kalman filter representation that gives the one-period forecast $\overline{\theta}_{jt+1|t}$ as a function of $\overline{\theta}_{jt|t-1}$

$$\overline{\theta}_{jt+1|t} = \rho\overline{\theta}_{jt|t-1} + \rho\Sigma_{t|t-1}\left(\Sigma_{t|t-1} + \sigma_{e}^{2}\right)^{-1}\left(s_{jt}^{r} - \overline{\theta}_{jt|t-1}\right)$$

or

$$\overline{\theta}_{jt+1|t} = \rho\left[1 - \Sigma_{t|t-1}\left(\Sigma_{t|t-1} + \sigma_{e}^{2}\right)^{-1}\right]\overline{\theta}_{jt|t-1} + \rho\Sigma_{t|t-1}\left(\Sigma_{t|t-1} + \sigma_{e}^{2}\right)^{-1} s_{jt}^{r} \tag{1.40}$$

where $\Sigma_{t+1|t}$ solves

$$\Sigma_{t+1|t} = \rho^{2}\Sigma_{t|t-1} - \rho^{2}\Sigma_{t|t-1}^{2}\left(\Sigma_{t|t-1} + \sigma_{e}^{2}\right)^{-1} + \sigma_{v}^{2} \tag{1.41}$$

Given that $|\rho| < 1$, $\sigma_e^2 > 0$ and $\sigma_v^2 > 0$, the conditional variance $\Sigma_{t|t-1}$ converges to a unique (positive) steady-state constant Σ^* that solves[35]

$$\Sigma^* = \rho^2 \Sigma^* \left[1 - \Sigma^* \left(\Sigma^* + \sigma_e^2\right)^{-1}\right] + \sigma_v^2 \tag{1.42}$$

It is easy to show that the solution to 1.42 is

$$\Sigma^* = \frac{1}{2}\sigma_e^2 \left[\frac{\sigma_v^2}{\sigma_e^2} - \left(1-\rho^2\right) + \sqrt{\left[\frac{\sigma_v^2}{\sigma_e^2} - \left(1-\rho^2\right)\right]^2 + 4\frac{\sigma_v^2}{\sigma_e^2}}\right] \tag{1.43}$$

Independence between θ_1 and θ_2, s_1^r and s_2^r implies that the rating process r_t is given by

$$\begin{aligned} r_t &= E\left[\theta_{1t} + \theta_{2t} \mid s_{1s}^r, s_{2s}^r, s < t\right] \\ &= \overline{\theta}_{1t|t-1} + \overline{\theta}_{2t|t-1} \end{aligned}$$

or, from 1.40

$$r_t = \rho\left[1 - \Sigma\left(\Sigma+1\right)^{-1}\right] r_{t-1} + \rho\Sigma\left(\Sigma+1\right)^{-1}\left[s_{1t-1}^r + s_{2t-1}^r\right] \tag{1.44}$$

where $\Sigma = \frac{1}{2}\left[\frac{\sigma_v^2}{\sigma_e^2} - \left(1-\rho^2\right) + \sqrt{\left[\frac{\sigma_v^2}{\sigma_e^2} - \left(1-\rho^2\right)\right]^2 + 4\frac{\sigma_v^2}{\sigma_e^2}}\right]$.

Q.E.D.

1.A.2 Fixed-point solution algorithm

Following Hussman (1992), we outline here the main steps we need to follow in order to calculate a linear REE equilibrium of our securities market. To derive such an equilibrium we need to evaluate matrices $T\left(\mathbf{B}\right)$ and $V\left(\mathbf{B}\right)$ of the actual law of motion 1.18. We start by choosing arbitrary values for their first row, which corresponds to the price process, and for the conditional variances σ_j^2 and coefficient matrices $\mathbf{B}_j$, $j = 1, 2$. We also define selector

[35] See, for example, Hamilton (1994), Proposition 13.1, page 390.

matrices $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{u}_1$, $\mathbf{u}_2$ that satisfy the following set of equations

$$\begin{aligned} z_{1t} &= \mathbf{e}_1 z_t & x_{1t} &= \mathbf{u}_1 z_t \\ z_{2t} &= \mathbf{e}_2 z_t & x_{2t} &= \mathbf{u}_2 z_t \\ r_t &= \mathbf{e}_r z_t & \varsigma_t &= \mathbf{e}_s \varepsilon_t \end{aligned} \tag{1.45}$$

Let also matrix $\mathbf{c}$ be such that $p_{t+1} + D_{t+1} + kr_{t+1} = \mathbf{c}x_{jt+1}$. Given 1.45, we can easily see that $E\left[p_{t+1} + D_{t+1} + kr_{t+1} \mid I_{jt}\right] = \mathbf{c}\mathbf{B}_j\mathbf{u}_j z_t$ $(j = 1, 2)$ and the equilibrium price 1.17 can be restated as

$$p_t = \Lambda^{-1}\left[\alpha\sigma_2^2 N\phi_1 \mathbf{c}\mathbf{B}_1\mathbf{u}_1 + (1-\alpha)\,\sigma_1^2 N\phi_2\mathbf{c}\mathbf{B}_2\mathbf{u}_2 - M\mathbf{e}_r\right] z_t - \Lambda^{-1}\sigma_1^2\sigma_2^2\mathbf{e}_s\varepsilon_t \tag{1.46}$$

Substituting z_t from 1.18 into the price equation 1.46 we derive the following expression for the price process

$$p_t = \mathbf{d}_p z_{t-1} + \boldsymbol{\epsilon}_p \varepsilon_t$$

where row matrices $\mathbf{d}_p$ and $\boldsymbol{\epsilon}_p$ define the first row of $T(\mathbf{B})$ and $V(\mathbf{B})$, respectively, and they are given by

$$\begin{aligned} \mathbf{d}_p &\equiv \Lambda^{-1}\left[\alpha\sigma_2^2 N\phi_1\mathbf{c}\mathbf{B}_1\mathbf{u}_1 + (1-\alpha)\,\sigma_1^2 N\phi_2\mathbf{c}\mathbf{B}_2\mathbf{u}_2 - M\mathbf{e}_r\right] T(\mathbf{B}) \\ \boldsymbol{\epsilon}_p &\equiv \Lambda^{-1}\left[\alpha\sigma_2^2 N\phi_1\mathbf{c}\mathbf{B}_1\mathbf{u}_1 + (1-\alpha)\,\sigma_1^2 N\phi_2\mathbf{c}\mathbf{B}_2\mathbf{u}_2 - M\mathbf{e}_r\right] V(\mathbf{B}) - \Lambda^{-1}\sigma_1^2\sigma_2^2\mathbf{e}_s \end{aligned}$$

The second row of $T(\mathbf{B})$ and $V(\mathbf{B})$, which corresponds to the pay-off process D_t, is implied by 1.1, while the third row, which corresponds to the rating process, is implied by lemma 1. The fourth and fifth row of $T(\mathbf{B})$ and $V(\mathbf{B})$, which correspond to investors' private signals s_t^j $(j = 1, 2)$ are implied by 1.3, and the sixth and seventh row by 1.2. Row eight of $V(\mathbf{B})$ corresponds to supply of the risky asset and is set equal to

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

With respect to investors' forecast errors ζ_{jt} $(j = 1, 2)$ we define selector matrices $\mathbf{e}_{zj}$ such that

$$\zeta_{jt} = \mathbf{e}_{zj} z_t$$

From the actual law of motion 1.18, from investors' perceptions 1.11 and from selector matrices $\mathbf{e}_{zj}$ and $\mathbf{e}_j$, the forecast errors ζ_{jt} can be written as

$$\begin{aligned} \zeta_{1t} &= \left[\mathbf{e}_1 T\left(\mathbf{B}\right) - \mathbf{A}_1 \mathbf{e}_1 - \mathbf{C}_1 \mathbf{e}_{z1}\right] z_{t-1} + \mathbf{e}_1 V\left(\mathbf{B}\right) \varepsilon_t \\ \zeta_{2t} &= \left[\mathbf{e}_2 T\left(\mathbf{B}\right) - \mathbf{A}_2 \mathbf{e}_2 - \mathbf{C}_2 \mathbf{e}_{z2}\right] z_{t-1} + \mathbf{e}_2 V\left(\mathbf{B}\right) \varepsilon_t \end{aligned} \tag{1.47}$$

Equations in 1.47 define the following matrices $\mathbf{d}_\zeta$ and $\mathbf{e}_\zeta$

$$\begin{aligned} \mathbf{d}_\zeta &\equiv \begin{bmatrix} \mathbf{e}_1 T\left(\mathbf{B}\right) - \mathbf{A}_1 \mathbf{e}_1 - \mathbf{C}_1 \mathbf{e}_{z1} \\ \mathbf{e}_2 T\left(\mathbf{B}\right) - \mathbf{A}_2 \mathbf{e}_2 - \mathbf{C}_b \mathbf{e}_{z2} \end{bmatrix} \\ \mathbf{e}_\zeta &\equiv \begin{bmatrix} \mathbf{e}_1 V\left(\mathbf{B}\right) \\ \mathbf{e}_2 V\left(\mathbf{B}\right) \end{bmatrix} \end{aligned}$$

Matrix $\mathbf{d}_\zeta$ defines rows 9 to 16 of $T\left(\mathbf{B}\right)$, while matrix $\mathbf{e}_\zeta$ defines rows 9 to 16 of $V\left(\mathbf{B}\right)$. It is worth noting that in equations 1.47 selector matrices $\mathbf{e}_1$ and $\mathbf{e}_2$ select elements only from the first five rows of matrices $T\left(\mathbf{B}\right)$ and $V\left(\mathbf{B}\right)$. However, the rows of matrices $T\left(\mathbf{B}\right)$ and $V\left(\mathbf{B}\right)$ that are relevant to ζ_{1t} are rows 9 to 12, while for ζ_{2t} rows 13 to 16. Consequently, $\mathbf{e}_1$ and $\mathbf{e}_2$ do not select any of the coefficients of matrices $T\left(\mathbf{B}\right)$ and $V\left(\mathbf{B}\right)$ that are relevant to the evaluation of forecast errors ζ_{1t} and ζ_{2t}. Thus, there is no need to evaluate a fixed point for the rows of $T\left(\mathbf{B}\right)$ and $V\left(\mathbf{B}\right)$ that correspond to investors' forecast errors.

1.A.3 Equilibrium in the high-sophistication case

Under the benchmark case without ratings, the equilibrium ARMA(1,1) coefficients of the observable variables $\begin{bmatrix} p_t & D_t & s_t^j & \zeta_t^j \end{bmatrix}$ are calculated to be

$$\mathbf{B}_j = \begin{bmatrix} 0.5527 & 0.9359 & -0.0734 & -0.3720 & -0.3491 & 0.2081 \\ -0.0000 & 0.8000 & -0.0000 & 0.0356 & -0.6326 & 0.0793 \\ 0.0000 & -0.0000 & 0.8000 & -0.0120 & 0.0952 & -0.6797 \\ 0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0000 & -0.0000 \\ -0.0000 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0000 \\ -0.0000 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0000 \end{bmatrix}$$

The last three rows of $\mathbf{B}_j$ give the coefficients in the projection of forecast errors ζ_t^j on p_{t-1}, D_{t-1}, s_{t-1}^j and ζ_{t-1}^j. That these coefficients are zero is a necessary condition for ζ_t^j to be conditional vector white noise, conditional on observable information of investors of type $j = 1, 2$. At the REE, from the moment matrix $\mathbf{M}_z$, we derive the following variance-covariance matrix M for the variables $\begin{bmatrix} p_t & D_t & s_t^1 & s_t^2 & \theta_{1t} & \theta_{2t} & \varsigma_t \end{bmatrix}$

$$\mathbf{M} = \begin{bmatrix} 4.3608 & 2.0202 & 0.6740 & 0.6740 & 0.5409 & 0.5409 & -0.0736 \\ 2.0202 & 1.5556 & 0.2778 & 0.2778 & 0.2778 & 0.2778 & -0.0000 \\ 0.6740 & 0.2778 & 1.2778 & 0.0000 & 0.2778 & 0.0000 & 0.0000 \\ 0.6740 & 0.2778 & -0.0000 & 1.2778 & 0.0000 & 0.2778 & -0.0000 \\ 0.5409 & 0.2778 & 0.2778 & 0.0000 & 0.2778 & 0.0000 & 0.0000 \\ 0.5409 & 0.2778 & -0.0000 & 0.2778 & 0.0000 & 0.2778 & -0.0000 \\ -0.0736 & -0.0000 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0100 \end{bmatrix}$$

By introducing ratings in the information sets of investors, the equilibrium ARMA(1,1) coefficients of the observable variables $\begin{bmatrix} p_t & D_t & r_t & s_t^j & \zeta_t^j \end{bmatrix}$ are calculated to be

$$\mathbf{B}_j = \begin{bmatrix} 0.3364 & 1.7162 & -0.0002 & -0.0601 & -0.2136 & -1.2186 & 1.6445 & 0.2319 \\ 0.0000 & 0.8000 & -0.0000 & -0.0000 & 0.0277 & -0.6607 & 0.4579 & 0.0659 \\ 0.0000 & 0.4624 & 0.3376 & -0.0000 & 0.0160 & -0.3819 & 0.2647 & 0.0381 \\ 0.0000 & -0.0000 & 0.0000 & 0.8000 & -0.0138 & 0.0798 & 0.2331 & -0.6844 \\ 0.0000 & -0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0000 & 0.0000 & -0.0000 \\ -0.0000 & 0.0000 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & -0.0000 & 0.0000 \\ -0.0000 & 0.0000 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & -0.0000 & 0.0000 \\ 0.0000 & -0.0000 & 0.0000 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & -0.0000 \end{bmatrix}$$

From the moment matrix $\mathbf{M}_z$ we derive the following variance-covariance matrix $\mathbf{M}$ for the variables $\begin{bmatrix} p_t & D_t & r_t & s_t^1 & s_t^2 & \theta_{1t} & \theta_{2t} & \varsigma_t \end{bmatrix}$

$$\mathbf{M} = \begin{bmatrix} 5.0634 & 2.0202 & 1.0239 & 0.7806 & 0.7806 & 0.6439 & 0.6439 & -0.0668 \\ 2.0202 & 1.5556 & 0.2816 & 0.2778 & 0.2778 & 0.2778 & 0.2778 & 0.0000 \\ 1.0239 & 0.2816 & 0.2816 & 0.1408 & 0.1408 & 0.1408 & 0.1408 & -0.0000 \\ 0.7806 & 0.2778 & 0.1408 & 1.2778 & 0.0000 & 0.2778 & 0.0000 & -0.0000 \\ 0.7806 & 0.2778 & 0.1408 & 1.2778 & 1.2778 & 0.0000 & 0.2778 & 0.0000 \\ 0.6439 & 0.2778 & 0.1408 & 0.2778 & 0.0000 & 0.2778 & 0.0000 & 0.0000 \\ 0.6439 & 0.2778 & 0.1408 & 0.0000 & 0.2778 & -0.0000 & 0.2778 & 0.0000 \\ -0.0668 & -0.0000 & -0.0000 & 0.0000 & 0.0000 & -0.0000 & -0.0000 & 0.0100 \end{bmatrix}$$

We now briefly present the equilibrium forecasting techniques under the *low-sophistication* case where investors are restricted to run vector AR(1) models.[36]

[36] As with the ARMA case, the NREE when the market is using vector AR techniques is calculated using Matlab programs.

1.A.4 Equilibrium in low-sophistication case

Under the benchmark case of asymmetric information without ratings, the equilibrium AR(1) coefficients of the observable variables $\begin{bmatrix} p_t & D_t & s_t^j \end{bmatrix}$ are calculated to be

$$\mathbf{B}_j = \begin{bmatrix} 0.0510 & 0.1044 & 0.0503 \\ 0.1081 & 0.2221 & 0.1068 \\ -0.0235 & 0.1258 & 0.1508 \end{bmatrix}$$

At the REE, from the moment matrix $\mathbf{M}_z$ we derive the following variance-covariance matrix M for the variables $\begin{bmatrix} p_t & D_t & s_t^1 & s_t^2 & \theta_{1t} & \theta_{2t} & \varsigma_t \end{bmatrix}$

$$\mathbf{M} = \begin{bmatrix} 0.3988 & 0.6408 & 0.2220 & 0.2220 & 0.1307 & 0.1307 & -0.0339 \\ 0.6408 & 1.5556 & 0.2778 & 0.2778 & 0.2778 & 0.2778 & -0.0000 \\ 0.2220 & 0.2778 & 1.2778 & 0.0000 & 0.2778 & -0.0000 & -0.0000 \\ 0.2220 & 0.2778 & -0.0000 & 1.2778 & -0.0000 & 0.2778 & 0.0000 \\ 0.1307 & 0.2778 & 0.2778 & -0.0000 & 0.2778 & -0.0000 & -0.0000 \\ 0.1307 & 0.2778 & -0.0000 & 0.2778 & -0.0000 & 0.2778 & -0.0000 \\ -0.0339 & 0.0000 & -0.0000 & 0.0000 & 0.0000 & -0.0000 & 0.0100 \end{bmatrix}$$

We consider now the case with ratings that are used by investors for information discovery purposes only. In this case, the equilibrium AR(1) coefficients of the observable variables $\begin{bmatrix} p_t & D_t & r_t & s_t^j \end{bmatrix}$ are calculated to be

$$\mathbf{B}_j = \begin{bmatrix} 0.0808 & 0.3467 & 2.1872 & 0.1625 \\ 0.0339 & 0.1454 & 0.4974 & 0.0681 \\ 0.0196 & 0.0840 & 0.6251 & 0.0394 \\ -0.0210 & 0.0838 & 0.3266 & 0.1317 \end{bmatrix}$$

From the moment matrix $\mathbf{M}_z$ we derive the following variance-covariance matrix M of the variables $\begin{bmatrix} p_t & D_t & r_t & s_t^1 & s_t^2 & \theta_{1t} & \theta_{2t} & \varsigma_t \end{bmatrix}$

$$\mathbf{M} = \begin{bmatrix} 4.5017 & 1.7513 & 1.0239 & 0.7312 & 0.7312 & 0.6039 & 0.6039 & -0.0572 \\ 1.7513 & 1.5556 & 0.2816 & 0.2778 & 0.2778 & 0.2778 & 0.2778 & 0.0000 \\ 1.0239 & 0.2816 & 0.2816 & 0.1408 & 0.1408 & 0.1408 & 0.1408 & 0.0000 \\ 0.7312 & 0.2778 & 0.1408 & 1.2778 & -0.0000 & 0.2778 & -0.0000 & -0.0000 \\ 0.7312 & 0.2778 & 0.1408 & -0.0000 & 1.2778 & -0.0000 & 0.2778 & -0.0000 \\ 0.6039 & 0.2778 & 0.1408 & 0.2778 & -0.0000 & 0.2778 & -0.0000 & 0.0000 \\ 0.6039 & 0.2778 & 0.1408 & -0.0000 & 0.2778 & -0.0000 & 0.2778 & 0.0000 \\ -0.0572 & 0.0000 & 0.0000 & -0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0100 \end{bmatrix}$$

As expected, the moment matrices of vector $\begin{bmatrix} p_t & D_t & r_t & s_t^1 & s_t^2 & \theta_{1t} & \theta_{2t} & \varsigma_t \end{bmatrix}$ under the ARMA and AR equilibrium differ only with respect to their first row and column that correspond to the second moments of prices. This is because the price process is the only endogenously determined process in the vector, while all other variables are assumed to be exogenous and remain unaffected by the equilibrium allocations of asset holdings among investors.

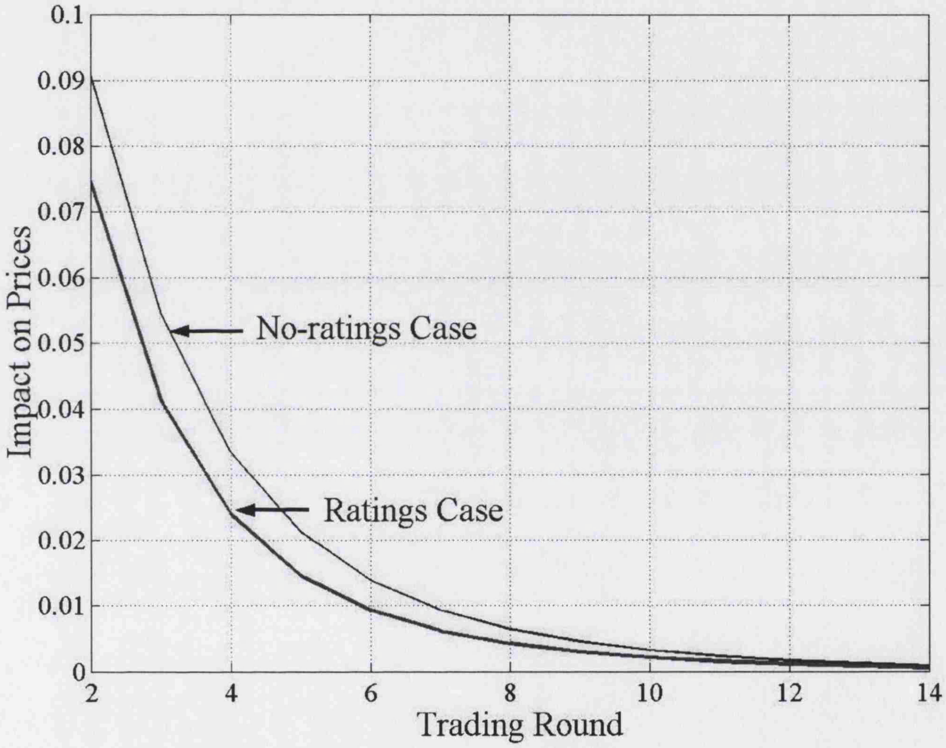

Fig. 1.1. Non-fundamental supply shock (ς)

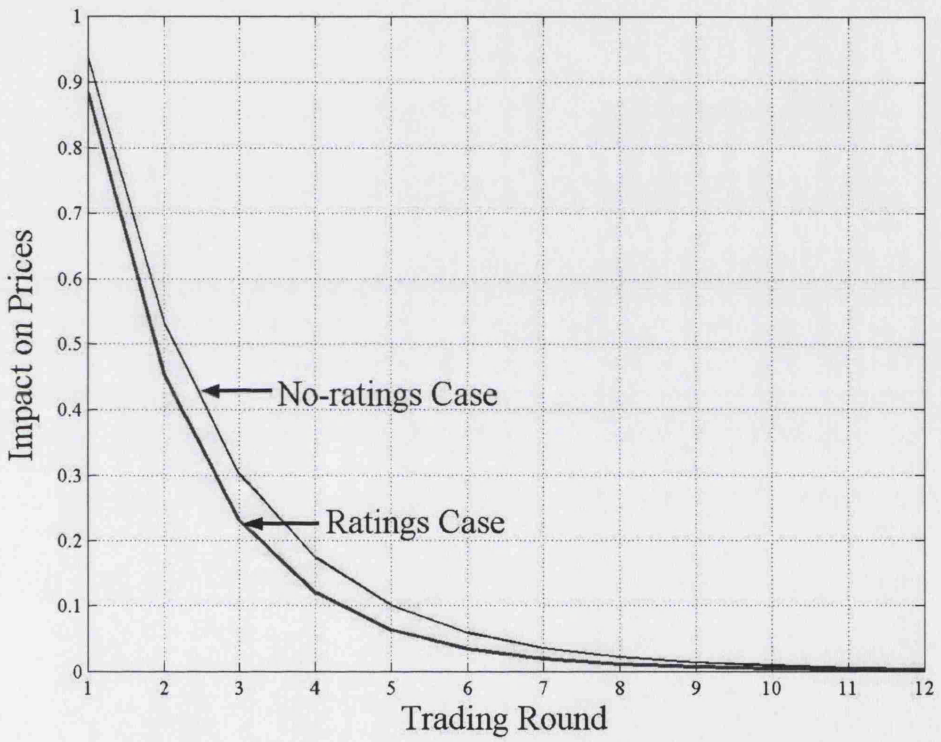

Fig. 1.2. Non-fundamental pay-off shock (u)

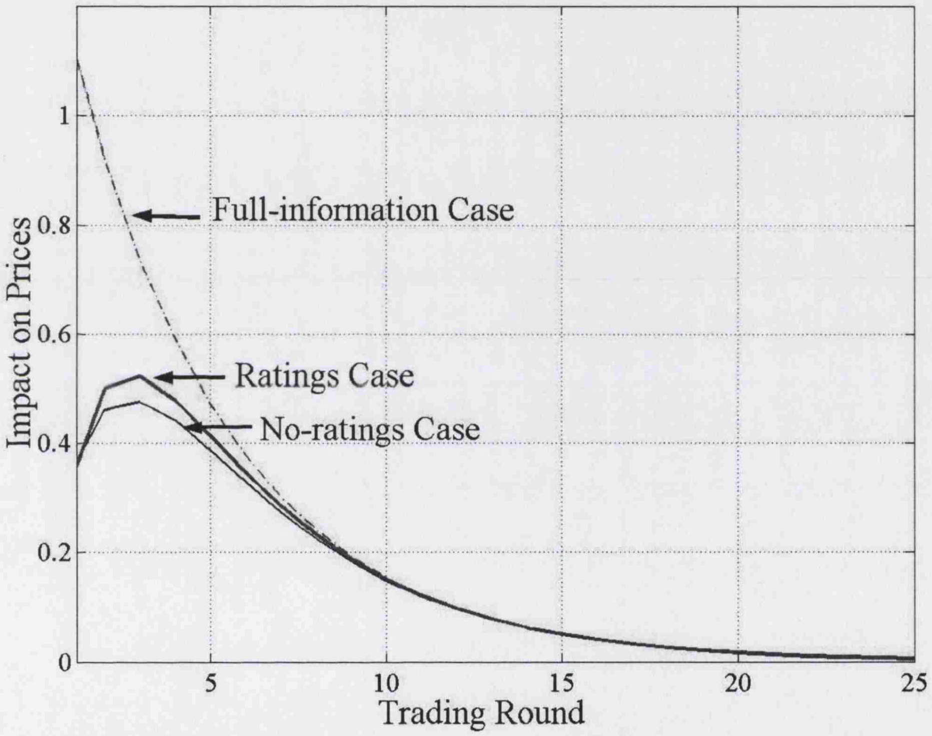

Fig. 1.3. Fundamental shock (v_j)

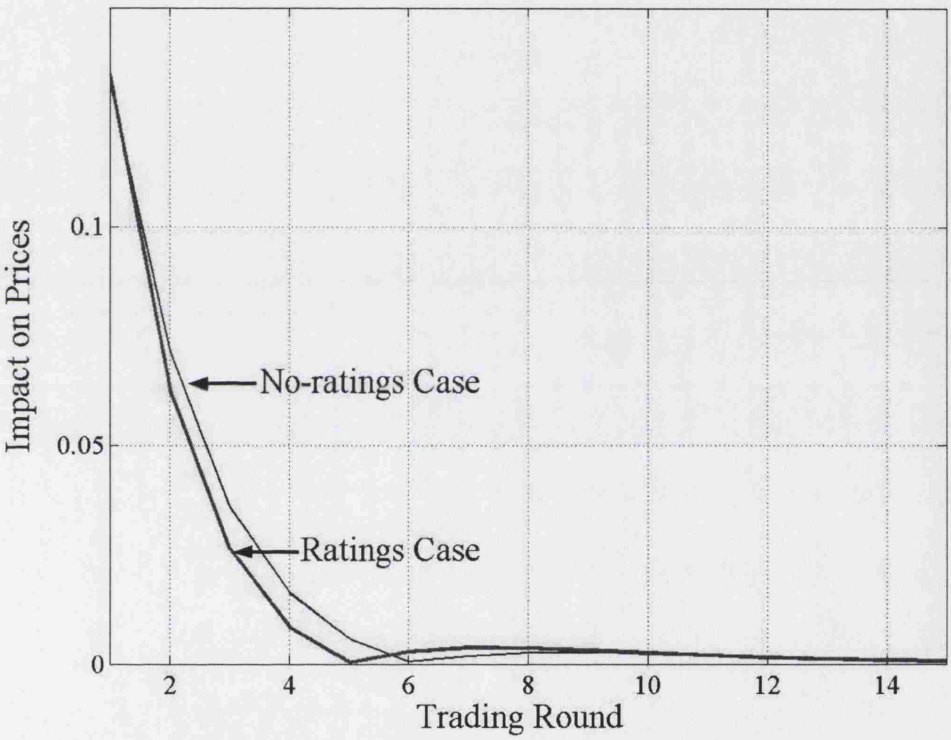

Fig. 1.4. Private signal error (η_j)

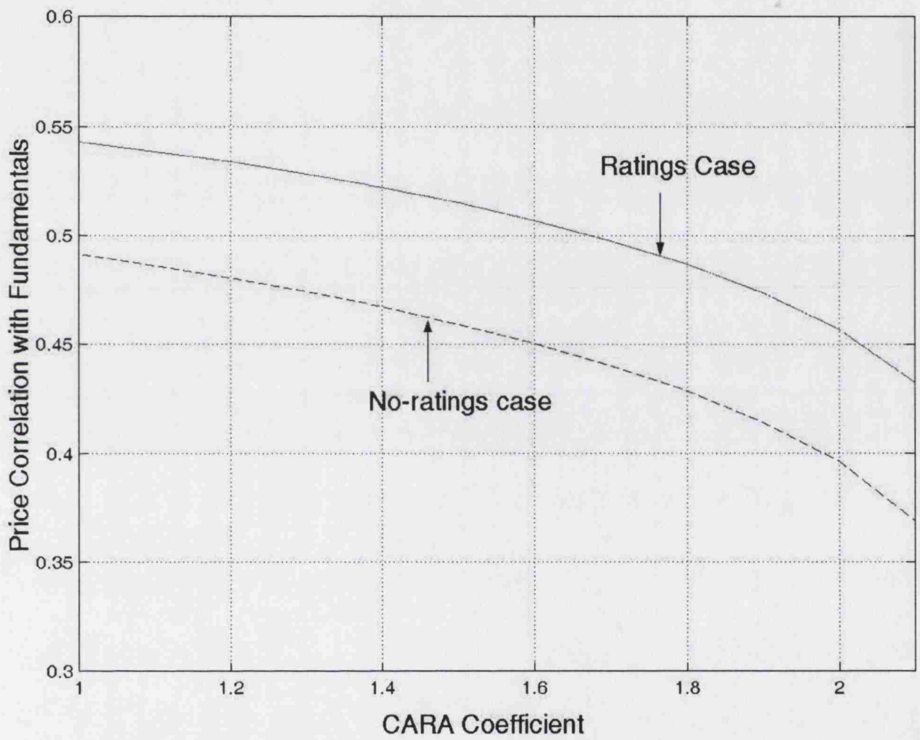

Fig. 1.5. Risk aversion and price efficiency

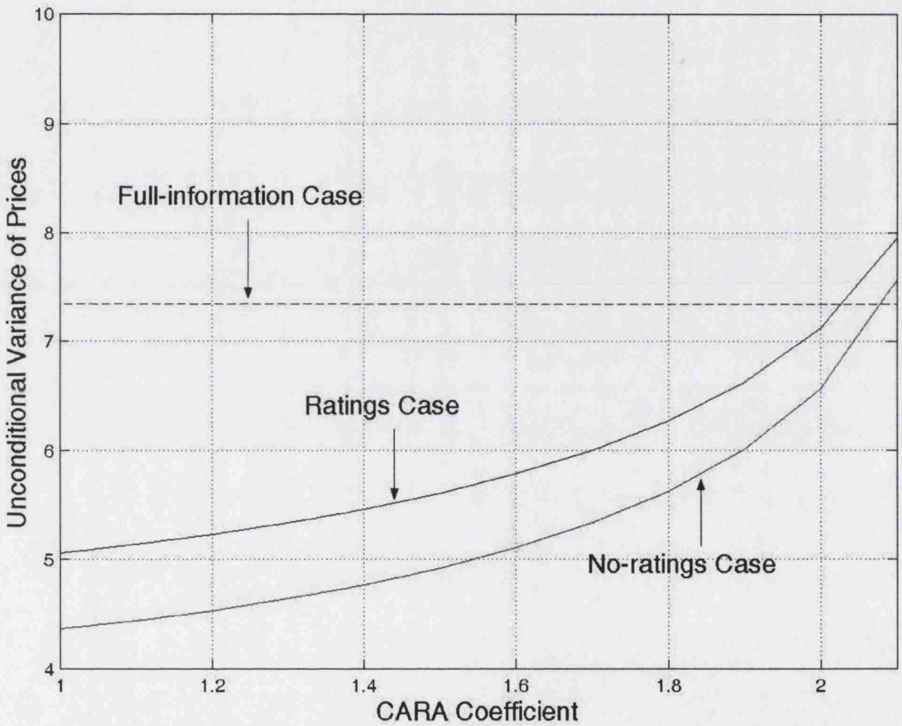

Fig. 1.6. Risk aversion and price volatility

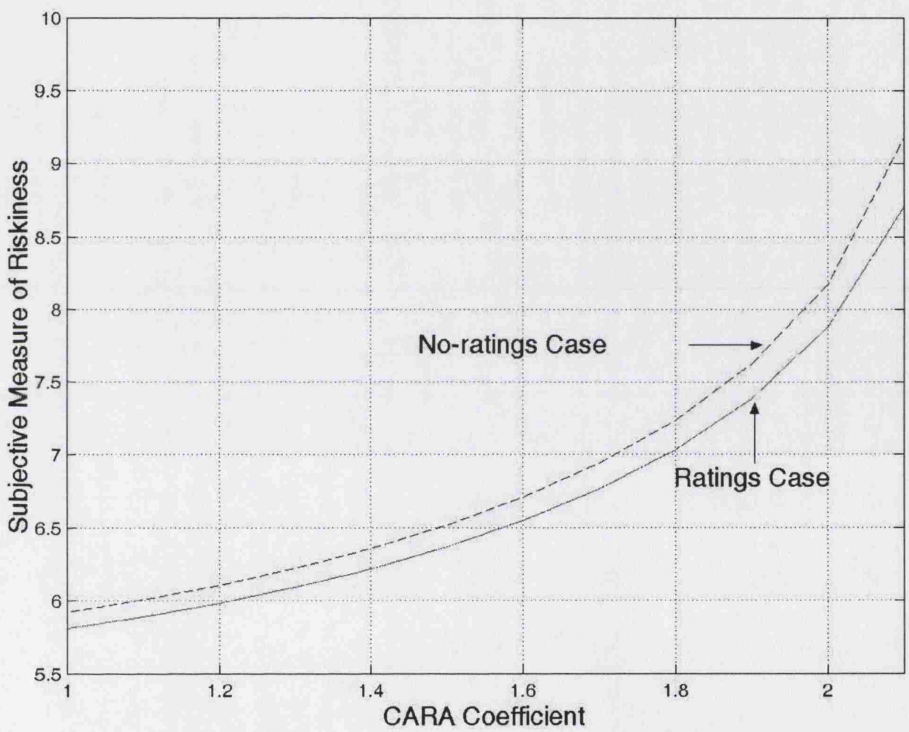

Fig. 1.7. Risk aversion and modelled risk perceptions

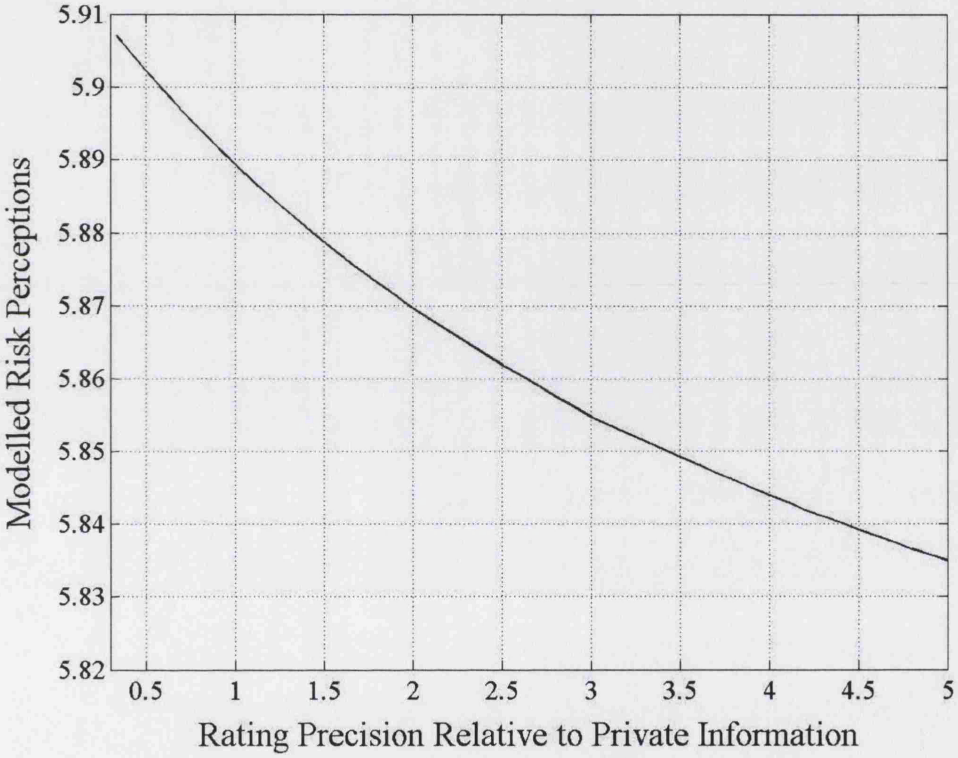

Fig. 1.8. Rating precision and modelled risk perceptions

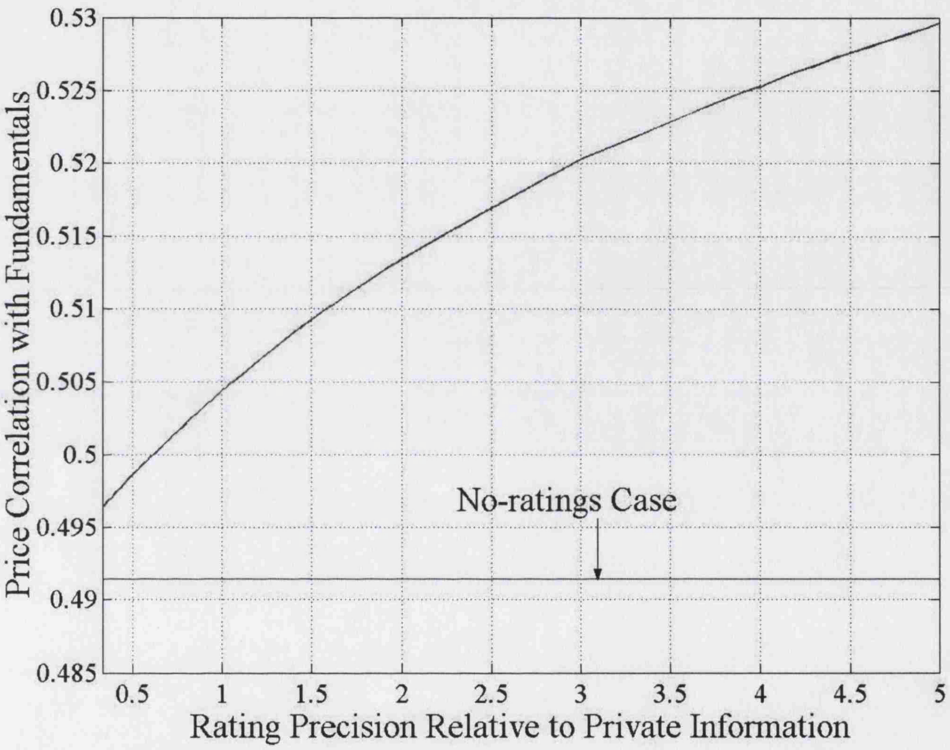

Fig. 1.9. Rating precision and price efficiency

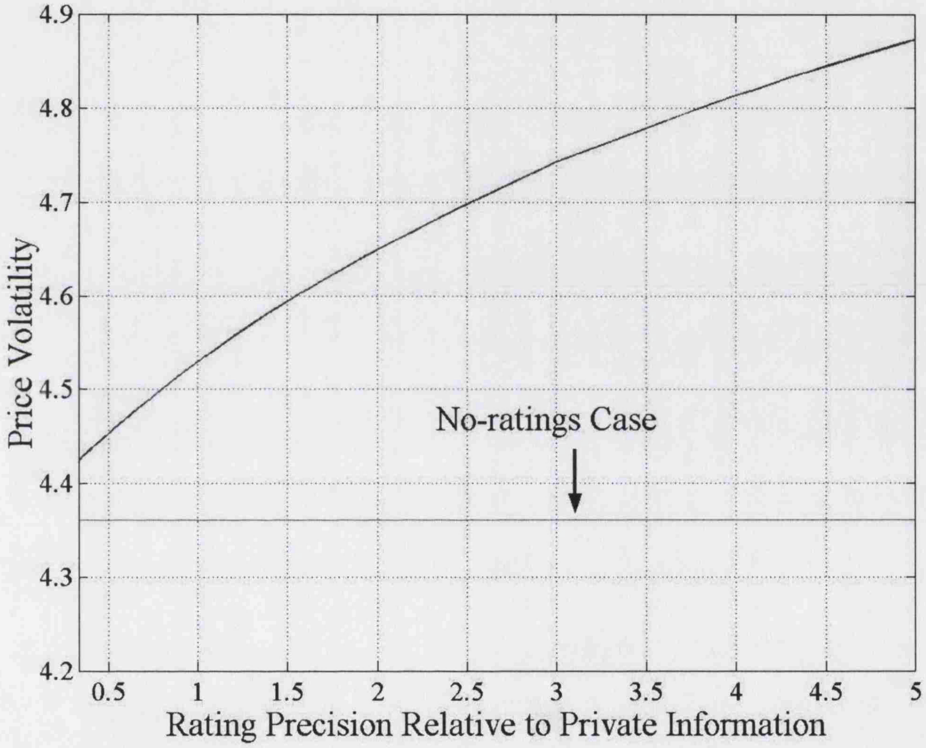

Fig. 1.10. Rating precision and price volatility

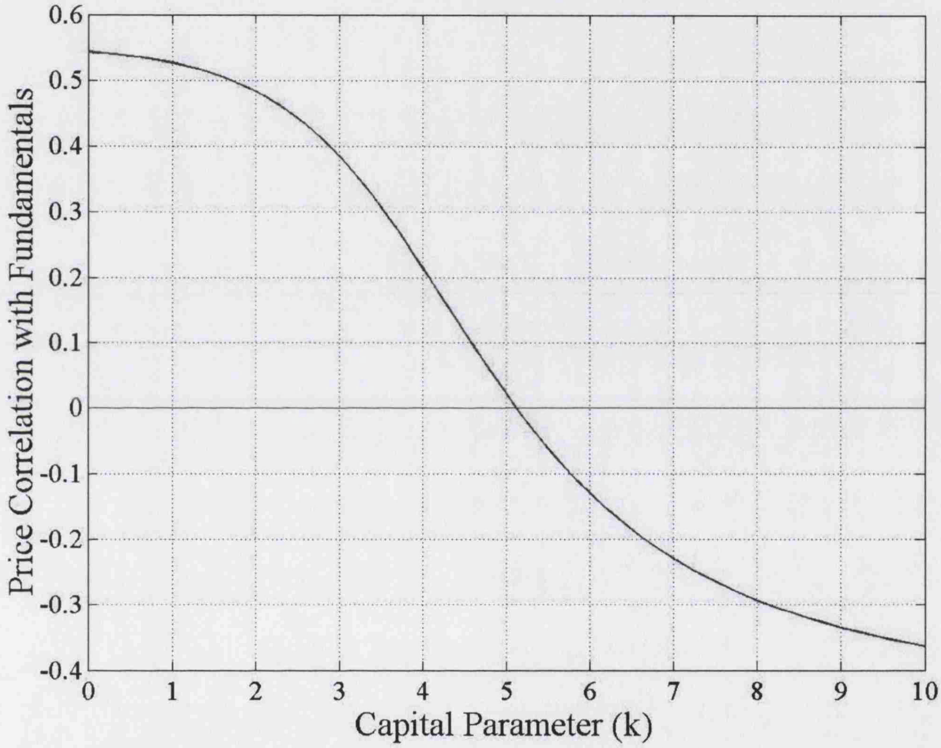

Fig. 1.11. Capital requirements and price efficiency

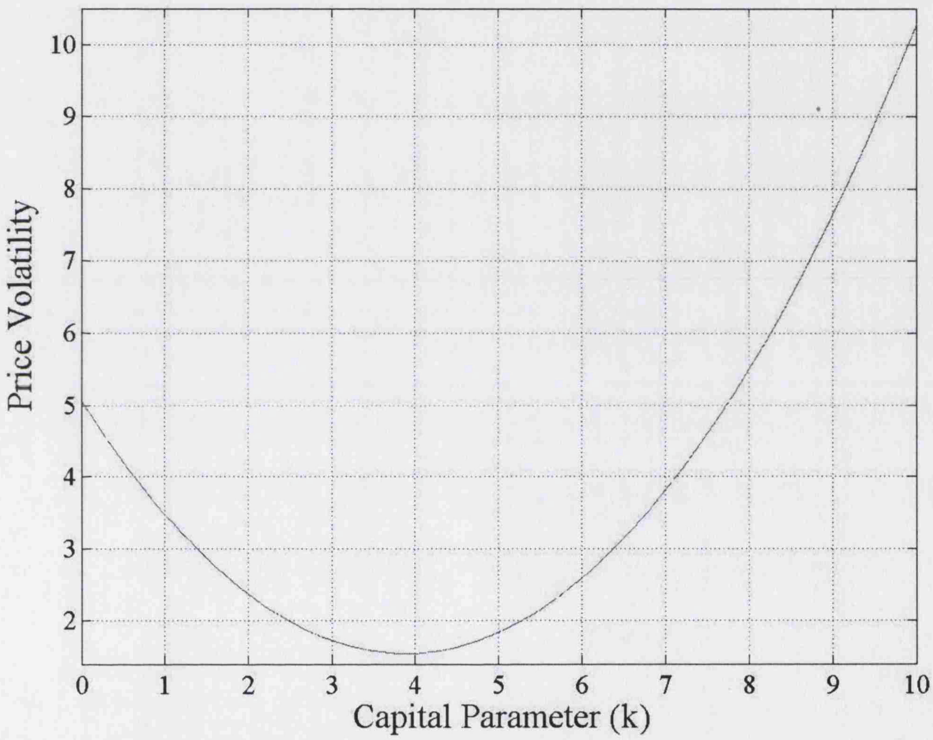

Fig. 1.12. Capital requirements and price volatility

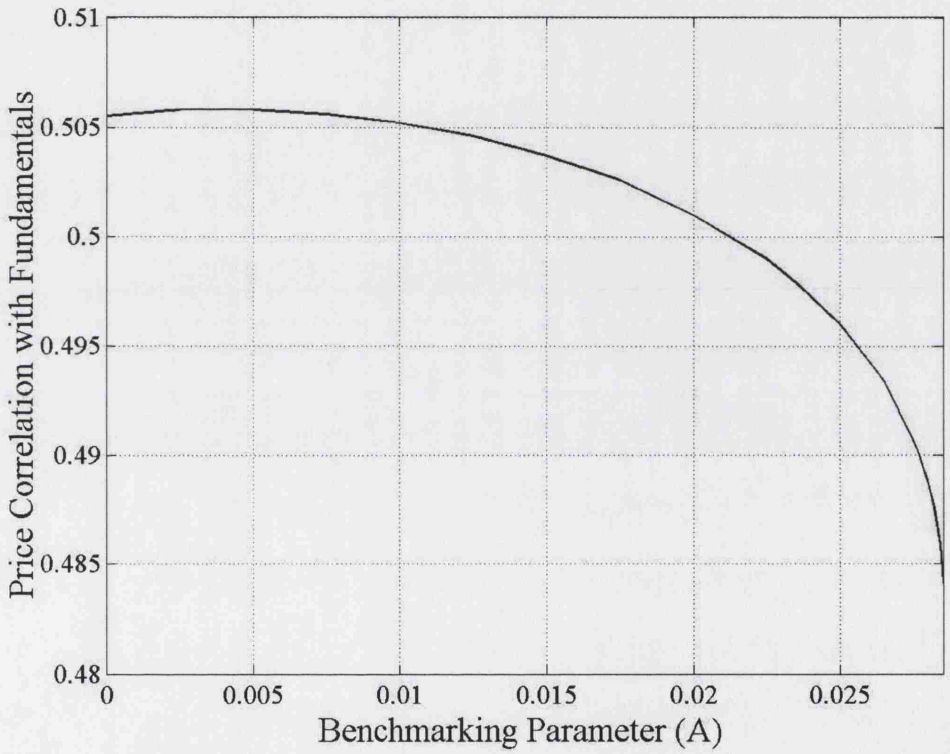

Fig. 1.13. Investment benchmarking (A) and price efficiency

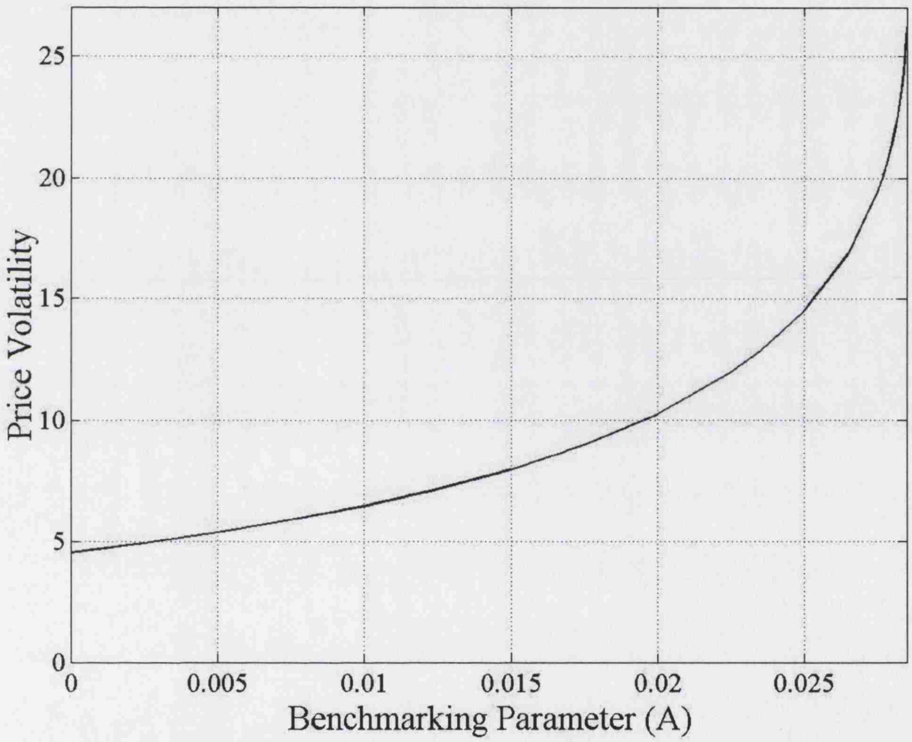

Fig. 1.14. Investment benchmarking (A) and price volatility

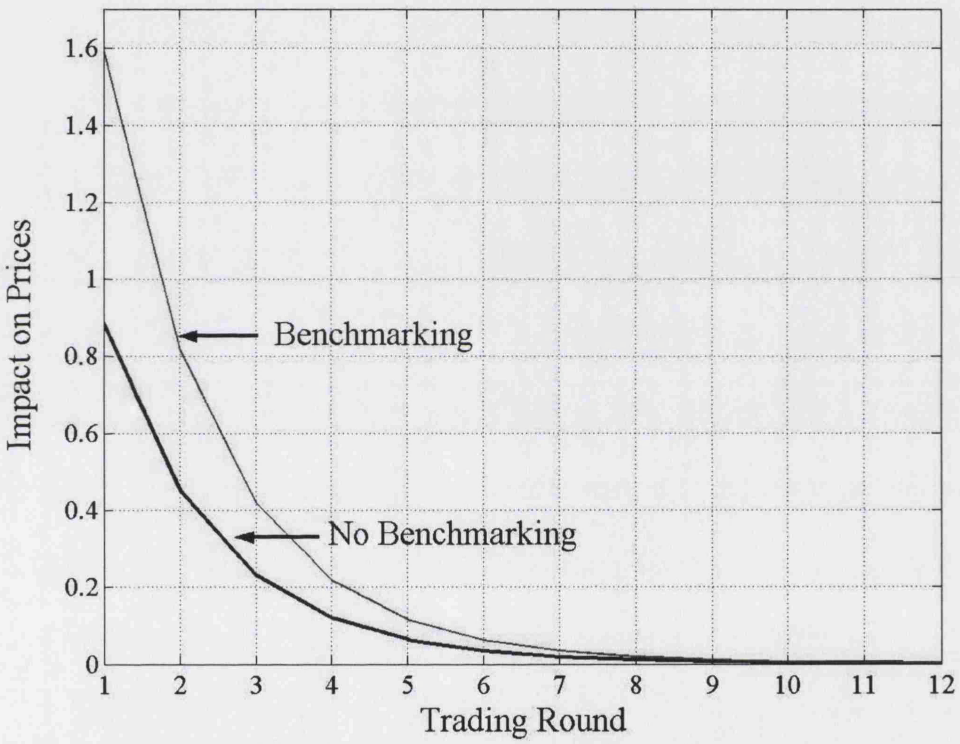

Fig. 1.15. Non-fundamental pay-off shock (u)

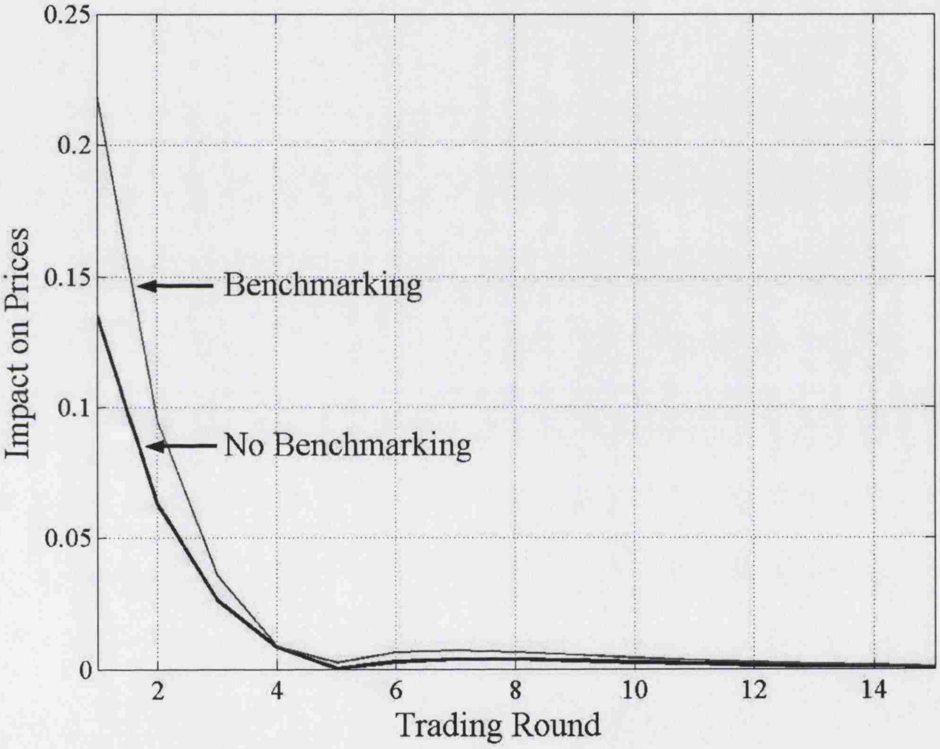

Fig. 1.16. Private signal error (η_j)

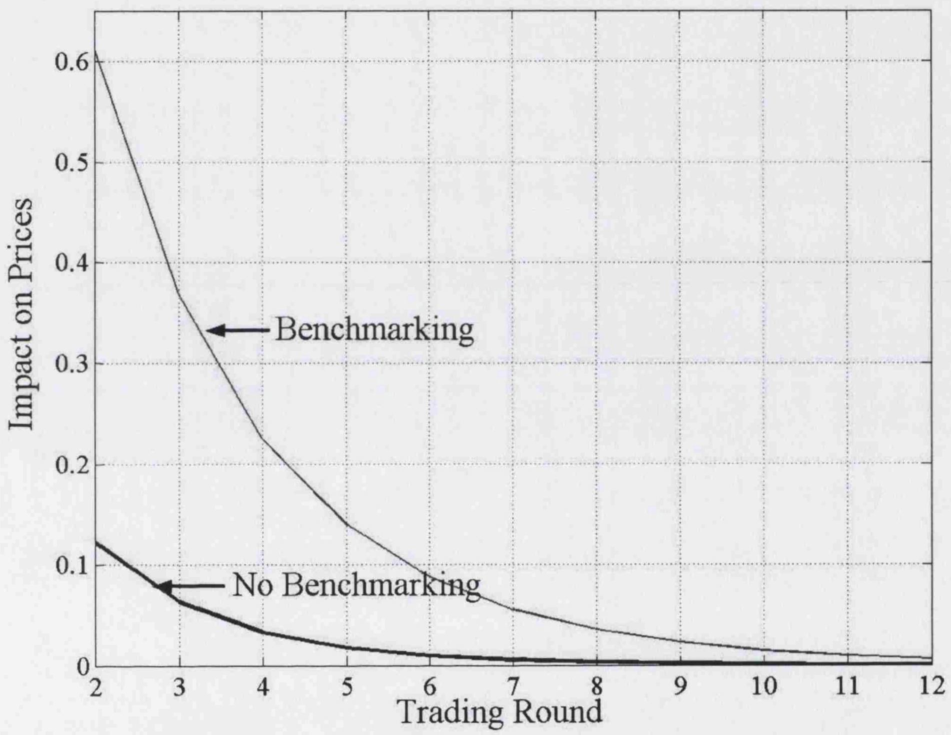

Fig. 1.17. Rating error (e_j)

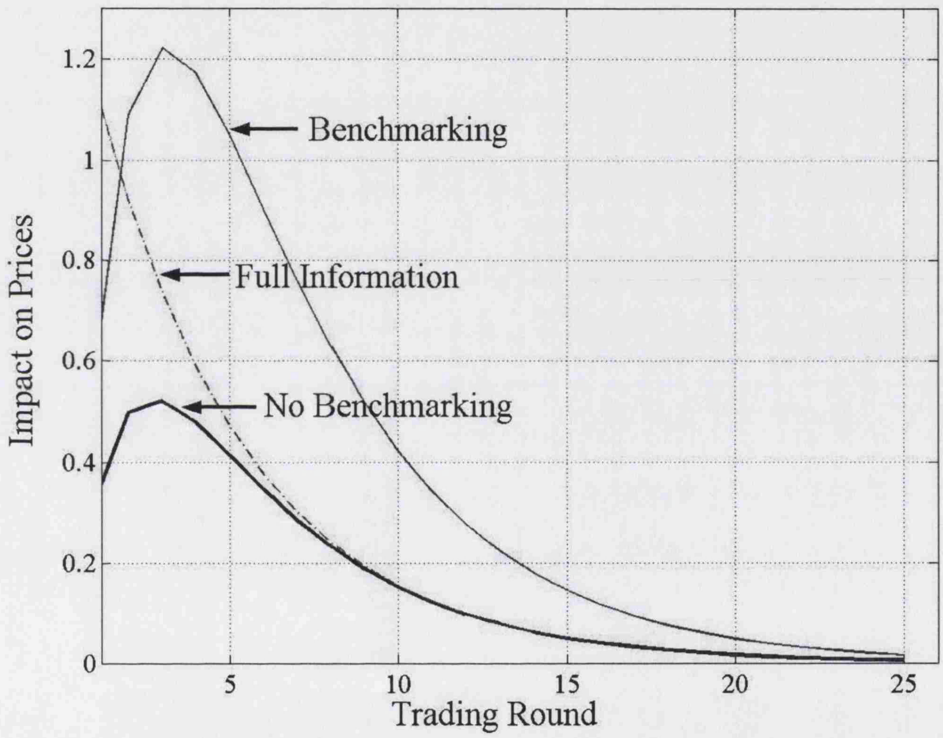

Fig. 1.18. Fundamental shock (v_j)

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Chapter 2
Prudential liquidity regulation and the insurance aspect of lender of last resort

2.1 Introduction

Central banks have been deeply involved in the design of banking regulation even in cases where responsibilities for supervision of banking institutions have been transferred to other authorities. An often cited reason for central banks' involvement in regulatory design is its possible impact on the likelihood of lender of last resort (henceforth LOLR) intervention and the possibility of unintended consequences for central banks' balance sheet.[37] Moreover, the LOLR facility conflicts with the best principles that central banks wish to operate in money markets. That is *predictability* in their actions and lack of *favouritism* in the choice of counterparties.[38] Thus, among central bankers, emergency liquidity assistance has been perceived as literally a last resort policy tool, rather than an alternative to banking regulation that should aim to forestal the need for LOLR intervention.

The focus of this paper is on liquidity regulation, meaning prudential standards for banks specifying an appropriate level of highly liquid assets that banks need to maintain in relation to their liabilities.[39] What we have in mind is a quantitative liquidity require-

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[37] Regulatory design may also impact on financial stability, in which central banks have a vested interest due to possible implications for monetary policy. Bank failures, for example, are often associated with systemic externalities and may have macro implications (e.g. Japan in 1990s), but so does regulatory intervention, depending on design.

[38] I am grateful to Charles Goodhart for pointing this out.

[39] Such a requirement is distinct from reserve balances that banks need to maintain with central banks

ment similar to the Sterling Stock Liquidity Ratio (SSLR) of the UK's Financial Services Authority (FSA).[40] Under FSA's rules, banks need to hold liquid assets to meet outflows over a period of five days, allowing a bank in crisis to continue business for some time and to arrange *more permanent funding solutions*.[41] Conventional wisdom also suggests that the five-day horizon is to cover the worst case scenario where a bank faces a liquidity crisis on a Monday, while authorities are allowed to reach the following weekend and consider the possibility of LOLR intervention without undue pressure from pending market developments.

In the literature, official sector involvement to deal with liquidity crisis in the banking system has received substantial attention both in terms of crisis prevention and crisis management. To mention just a few notable examples, Diamond and Dybvig (1983) argue in favour of a deposit insurance scheme to resolve co-ordination problems among depositors that could lead to bank runs, inefficient liquidation of bank assets and bankruptcy. Bhattacharya and Gale (1987) offer a rationale for official monitoring of liquid asset holdings by banks, suggesting that liquidity shortages may arise as a result of banks' incentives to free-ride on interbank liquidity, rather than holding liquid assets themselves. Dewatripont and Tirole (1994) argue that capital requirements provide an instrument for allocating control

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in certain jurisdictions – including the U.S. – that have typically been used by governments to tax bank profits and to enhance central banks' ability to implement monetary policy. Feinman (1993) offers a detailed discussion of reserve balances with the central bank and their historical evolution.

40 According to a resent survey by the Bank of England on the prudential regulation of banks' liquidity (see *Financial Stability Review*, 15) there is currently no harmonisation of supervisory liquidity requirements at either a G10 or EU level. In February 2000, the Basel Committee on Banking Supervision published "Sound Practices for Managing Liquidity in Banking Organizations", which sets out broad *qualitative* guidelines for how banks should manage liquidity risk.

41 See FSA Interim Prudential sourcebook, pp. 533-534.

rights to the deposit insurance fund if things go badly. Holmström and Tirole (1998, 2000) suggest that the official sector can improve welfare by managing the supply of government debt, given that banks may fail to cross-insure firms if liquidity shocks are correlated. Last but not least, Hellmann, Murdock and Stiglitz (2000) argue in favour of an interest rate ceiling on deposits to complement capital requirements in mitigating bank moral hazard by increasing the franchise value of banks.

Regarding the role of LOLR in dealing with banking crises, Goodfriend and King (1988) argue that solvent banks could perfectly insure against the possibility of a bank run via a sophisticated interbank market, suggesting that central banks should concentrate on maintaining a sufficient amount of liquidity in the banking system, rather than providing the LOLR facility. However, Donaldson (1992) finds evidence supporting the view that liquidity-rich banks may act strategically and abuse the market by charging higher than the competitive rate in case of crisis, which could justify LOLR intervention. Rochet and Vives (2002) argue that the LOLR may prevent inefficient liquidation of bank's assets and improve welfare if the central bank has perfect foresight of bank's fundamentals. Repullo (2003) discusses the effect of LOLR existence on holdings of liquid assets by banks, showing that a bank may end up keeping a lower level of liquid assets, which is consistent with empirical evidence by Gonzalez-Eiras (2003).[42] Repullo (2003) also argues that the existence of the LOLR may lead to more efficient outcomes since holding liquid assets is typically costly. Yet Naqvi (2003) suggests that if the supervisory process of the LOLR is

[42] In December 1996, the central bank of Argentina received access to contingent credit lines from a group of international banks, which enhanced its ability to act as a LOLR. Banks have indicated their reliance on the enhanced ability of the central bank to provide liquidity by reducing their liquid asset holdings by approximately 6.7 %.

subject to noise, then the gains from ex-post efficiency, from holding a lower stock of liquid assets, may be outweighed by ex ante inefficiencies induced by moral hazard which is conducive to lower lending rates in the economy. Goodhart and Huang (2004) favour the existence of an LOLR arguing that the interbank market cannot provide sufficient liquidity when the amount needed to bail out a bank is too large to be accommodated by a single bank and concerted action by a group of banks may be inhibited by co-ordination problems. They also suggest that the interbank market might not be able to provide insurance against liquidity shocks if those shocks happen to be systemic, affecting the whole banking sector.

In this article we consider the implicit costs associated with liquidity regulation as an insurance premium paid by the banking sector,[43] *quid pro quo* for (partial) liquidity insurance by the official sector under the LOLR facility. Then, we investigate how liquidity regulation for banks could be optimally combined with a LOLR policy in order to maximise the expected surplus from financial intermediation and we search for conditions under which a combination of prudential liquidity regulation and LOLR insurance could be welfare improving of a *laisser-faire* regime without prudential liquidity requirements and a LOLR safety net.

The idea of considering regulation, in general, as a form of implicit insurance, and regulatory costs as insurance premia – or *taxes* – against the implicit subsidy that such an insurance would imply, is not new. It dates back to Posner (1971) who explains a number

[43] According to FSA's Interim Prudential sourcebook, p. 495, *banks are reluctant to hold a large stock of immediately available cash or marketable assets, as these generate no return (in the case of cash) or a comparatively low yield (in the case of easily marketable assets, e.g. government bonds).*

of phenomena of regulated industries through the prism of *taxation by regulation*. Buser, Chen and Kane (1981) also consider banking regulations by the Federal Deposit Insurance Corporation (FDIC) as a condition for banks receiving deposit insurance. They interpret the deadweight costs of such regulations as *implicit insurance* premia which develop over and above the explicit fees that are charged by the FDIC.[44] They also argue that the FDIC, through regulatory interference, effectively employs a risk-based – as opposed to a flat – structure of insurance premia. Yet Buser et al. sketch a model of interaction between FDIC regulation and deposit insurance, stopping short of analysing optimal FDIC response. Thus, they offer no insights into welfare implications of FDIC insurance/regulation and whether or not such an official intervention is warranted from a welfare perspective.

Chan, Greenbaum and Thakor (1992) focus on information problems between deposit insurers and banks, showing that the mere notion of a competitive banking industry contradicts the possibility of fairly priced deposit insurance, i.e. when the deposit insurer breaks even on every insured bank. Along the lines of Buser et al. (1981), they show that deposit-linked subsidies – such as underpriced deposit insurance – are necessary in resolving moral hazard problems associated with bankers' risk-shifting motives in the presence of information asymmetries between deposit insurers and banks. However, Chan et al. (1992) abstract from welfare implications of deposit insurance which, from a LOLR perspective, is the main focus of our analysis.

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[44] According to Buser et al. (1981), in order for the FDIC to induce voluntary participation of banks in its regulatory jurisdiction, its sets its explicit insurance premium below market value. As a result, the FDIC insurance fund amounts to only about 0.80 per cent of total deposits in insured banks and FDIC regulations aim at protecting a bank's charter value, serving as a first line of defence against bank losses.

Sleet and Smith (2000) provide a rigorous analysis of the design of a safety net for the banking system in the presence of both full deposit insurance and a LOLR. Using a general equilibrium framework, they argue that the pricing of deposit insurance is irrelevant from a welfare perspective, although the same does not hold as regards the operation of the discount window. Sleet and Smith (2000) illustrate that, in the presence of high discount-window rates, the ex-post costs, associated with banks' perverse incentives to shift risk to the government, offset the ex-ante benefits of inducing banks to invest prudently. But in the presence of multiple equilibria, the LOLR can be used as an *equilibrium selection method.* However, they consider regulation only implicitly via a lump-sum tax that is levied to depositors, allowing the official sector to run a balanced budget. That way, they illustrate that taxing deposits does not affect aggregate welfare, which may be due to the assumption that taxes are levied on deposits rather than on banks' profits.

In this paper we focus on the interaction between LOLR policy and first-best liquidity regulation, how they may affect investment decisions by the banking sector and under what circumstances they becomes socially desirable.[45] The central bank in our model represents the official sector, acting both as an banking regulator and an LOLR. Moreover, we adopt the working hypothesis that the central bank has a welfare maximisation objective, under the constraint to maintain a zero expected cost of potential LOLR intervention.

We also consider a banking sector that faces funding constraints and the possibility of information-induced bank runs. Funding constraints in the model arise from depositors'

[45] Without prejudice to potential information problems between banks and the official sector, those problems are assumed away in our analysis, mainly for analytical tractability. Consequently, the role of prudential liquidity as a possible medium to *buy time* in case of crisis is not analysed in this paper.

rational anticipation of bankers' moral hazard problems, similar to Dewatripont and Tirole (1994) and Holmström and Tirole (1998, 2000). The possibility of information-induced bank runs aims at capturing banks' inherent fragility due to high leverage, short term funding and asymmetric information which, in the presence of adverse economic conditions, may lead to loss of confidence to banking institutions[46] and unanticipated foreclosures of wholesale interbank lines.[47] In the presence of such frictions, the analysis provides a necessary and sufficient condition for prudential liquidity regulation to be socially desirable (proposition 6), showing that the more debt-constrained the banking sector is, the higher its profit opportunities and the less stable its deposit base is, the more prudential liquidity regulation becomes socially desirable.

The remainder of the paper is organized as follows: Section 2.2 discusses the economic environment that we analyse. Section 2.3 presents the benchmark case of no liquidity regulation and Section 2.4 solves for the optimal regulatory contract. Section 2.5 considers welfare implications of prudential liquidity regulation and Section 2.6 concludes. Proofs are included in the appendix.

2.2 Basic environment

We consider a model with three dates ($t = 0, 1, 2$) and three active sectors of risk neutral agents: i) commercial *bankers* that receive uninsured wholesale deposits and extend loans, ii) *fund-managers* who manage depositors' funds that are kept with the bank and iii) a

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46 See, for example, Calomiris and Gorton (1991) and Gorton (1988).

47 See, for example, Rochet and Vives (2002).

central bank that represents the official sector acting both as a banking regulator and a LOLR.

2.2.1 Bankers

As of $t = 0$, the bank can invest in a perfectly diversified portfolio of risky loans with constant returns to scale, where the total amount of investment I is a continuous variable that can be chosen freely. Bank's investment portfolio is financed through capital A and a volume of deposits D. The bank is assumed to invest on behalf of everyone in this economy by choosing an amount of investment that maximises its expected surplus, while it keeps any remaining funds in liquid assets l that pay no interest. That implies the following budget constraint as of $t = 0$:

$$I + l = A + D \tag{2.1}$$

We assume that pay-offs per unit of investment depend on the realisation of a productivity shock $\widetilde{\phi}$, which represents the proportion of loans that *succeed* at $t = 2$ and is uniformly distributed $\widetilde{\phi} \sim U\left(1 - \overline{\phi}, \overline{\phi}\right)$ with $\frac{1}{2} \leq \overline{\phi} \leq 1$. Moreover, loan pay-offs are binary with one unit of loan investment paying a gross return $R > 1$ at $t = 2$, in case of a successful investment, otherwise it pays zero.

As in Diamond (1984), we introduce moral hazard from the bankers' side in the medium to long run, i.e. between periods $t = 1$ and $t = 2$. In particular, bankers in the model act as delegated monitors of loan investments, undertaking an unobservable decision either to manage their loans prudently or to engage in excess risk taking. Excess risk taking yields a private benefit B per unit of investment that is paid out to bankers only if the bank does not fail at $t = 2$, but it also scales down the proportion of successful loans from $\widetilde{\phi}$ to $\frac{\widetilde{\phi}}{\beta}$ with $\beta > 1$. That is consistent with Kane (1989) and Cole, McKenzie and Lawrence (1995) who document that banks opt for a risky investment strategy that pays out a private benefit if the gamble succeeds, but leaves depositors with the losses if the gamble fails.[48]

2.2.2 Fund managers

Deposits in this model are typically repaid at $t = 2$, but can also be withdrawn at $t = 1$. By this is meant that depositors do not face liquidity needs in the interim period. Thus, the emphasis here is shifted from depositors' liquidity insurance to a situation where bank runs may occur because imperfectly informed creditors may refuse to renew their credit lines with the bank.[49] As in Rochet and Vives (2003), we assume that the management of deposits is delegated to a continuum of intermediaries (fund managers) of total measure D, who are able to costlessly monitor the performance of bank loans. Fund managers then decide whether to rollover or withdraw their funds by observing private signals s_i about the proportion of successful loans, of the form:

$$s_i = \widetilde{\phi} + \widetilde{\varepsilon}_i$$

where $\{\widetilde{\varepsilon}_i\}$ are $i.i.d.$ innovations with $\widetilde{\varepsilon}_i \sim U(-\varepsilon, +\varepsilon)$ and $\varepsilon > 0$. By introducing information asymmetries among bank's creditors, the analysis allows for endogenous liquidity

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48 We could also think of private benefit B as the cost of investing in state-of-the-art risk management systems that enhance bankers' ability to improve the credit quality of loan portfolios.

49 We recognise that in a model with only *late* depositors, any problems arising from premature liquidation of deposits could be avoided by prohibiting such an action explicitly in the deposit contract. We could consider *early* depositors in the model by introducing preference shocks á la Diamond and Dybvig (1983), in which case the bank should hold an additional amount of asset as liquid reserves. For simplicity, we do not distinguish between early and late consumers.

shocks to interrelate with the general economic environment, such as the actual economic fundamentals of the banking sector, LOLR policy and prudential liquidity requirements. For example, if a high proportion of creditors observe *bad* signals, then early deposit withdrawals may occur in a large scale and the bank may face a liquidity crisis. But what constitutes a *bad* signal may not only depend on the realisation of $\widetilde{\phi}$, but also on what policies are in place by the official sector, as well as what are creditors' beliefs are about other creditors' actions.[50] We consider the following definition of bank liquidity crisis.

Definition 1 *A bank liquidity crisis occurs if bank's stock of liquid assets at* $t = 1$ *is insufficient to meet fund managers' demand to withdraw their deposits.*

Depending on their actions at $t = 1$, fund managers' pay-offs are as follows:

Fund Managers	**NC&ND**	**C&ND**	**D**
Withdraw	$\pi(1-k)$	$\pi(1-k)$	$\pi(1-k)$
Rollover	π	$\pi(1-k)$	0

NC denotes the events of no liquidity crisis and **ND** that of no bank default. Also, **C** stands for the event of liquidity crisis and **D** for default. Fund managers are assumed to receive a private benefit π if the bank faces no problems (i.e. no liquidity crisis and no default). Otherwise, that pay-off is reduced by a proportion $k \in (0,1)$ in case of premature withdraw of deposits for whatever reason – given that, in the absence of liquidity needs at $t = 1$, deposits are supposed to stay put for two periods – or if they fail to foresee a liquidity crisis and shift their deposits elsewhere. In case of default, fund managers are assumed to lose their total bonus. Such a pay-off structure allows the introduction of information-induced

50 That is in contrast to exogenous liquidity shocks that are assumed, for example, in Holmström and Tirole (1998, 2000).

bank runs in the model using a simple global game argument and preventing the analysis from becoming messier through a more elaborate bank run model with endogenous payoffs à la Diamond and Dybvig.

2.2.3 Central bank

The central bank is assumed to set its LOLR policy in a way that satisfies a two-fold objective: First, to maximise the expected surplus from financial intermediation. Second, to maintain a zero expected *fiscal cost* of LOLR intervention, i.e. to break even in loans extended under its LOLR facility without systematically losing money under its LOLR operations, which would inevitably require it to draw on its fiscal backup. We conjecture a level ϕ^{**} of productivity shock $\widetilde{\phi}$ such that in case of liquidity crisis, the central bank bails out the bank if and only if $\widetilde{\phi} \geq \phi^{**}$. Consequently, given the interpretation of shock $\widetilde{\phi}$, the central bank could be considered as more *forbearing* in accommodating adverse liquidity shocks to the bank, the lower the threshold ϕ^{**} that determines its LOLR policy. In that sense, ambiguity about LOLR intervention is not *constructive*, but simply stems from uncertainty about the actual realisation of shock $\widetilde{\phi}$. Moreover, we assume that any LOLR intervention takes the form of a capital injection, abstracting from the possibility of preferential access to collateral by the central bank.

In relation to its role as banking regulator, the central bank is assumed to prescribe certain holdings of liquid assets by the bank as a proportion to its deposits. As discussed in Section 2.4, given central bank's LOLR policy ϕ^{**}, such a liquidity requirement aims at *implementing* the loan investment by the bank that is socially optimal. In that sense, one may also consider prudential liquidity regulation as an expedient for the central bank to precommit to its LOLR policy. The figure below summarises the sequence of moves.

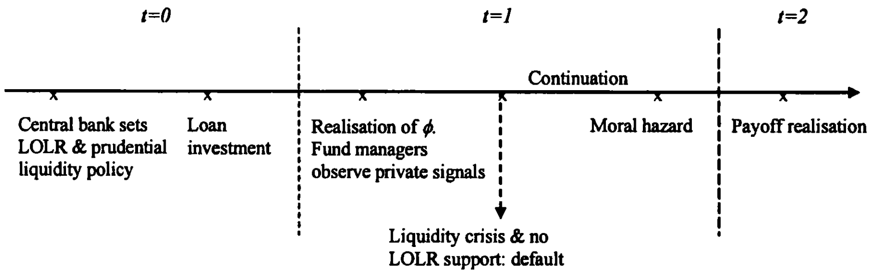

Fig. 2.1. Time line of the model

First, the central bank precommits to a LOLR policy ϕ^{**} requiring the bank to maintain a certain ratio of liquid assets to deposits, while investing all remaining funds in risky loans. As discussed in Section 2.4, such a prudential liquidity ratio turns out to be higher than in the absence of both regulatory restrictions and a LOLR facility and we find a necessary and sufficient condition for that to be socially desirable. Second, a productivity shock ϕ hits the bank's loan portfolio and, on the basis of private signals about ϕ, fund managers decide whether to rollover their deposits with the bank, or to withdraw. If as a result of fund managers' withdrawals the bank faces a liquidity crisis then, if the realised productivity shock ϕ is greater than ϕ^{**}, the central bank intervenes and bails-out the bank, otherwise it lets the bank to fail. If continuation occurs, either because of no liquidity crisis, or due to a bail-out by the central bank, bankers face a moral hazard problem due to risk-taking opportunities for a private benefit B. Finally, investment pay-offs are realised, bank's creditors are repaid and any residual value is assumed by bank's shareholders.

The model time-line in figure 1, aims at capturing the basic presumption that liquidity crises may occur abruptly due to loss of confidence, preceding possible solvency problems that could develop more gradually, for example, due to misconduct of business by bankers (moral hazard). Next, we introduce a number of technical restrictions on the model parameters that allow us to focus on short/medium term prudential liquidity policy, while solvency considerations are left with depositors who, at the deposit raising stage, restrict the debt capacity of the bank to the extent that deposits become incentive compatible.

2.2.4 Basic assumptions

In order for the bank in our model to function as a proper financial intermediary, we impose the following basic assumptions.

Assumption 1: $R > \frac{2(2\overline{\phi}-1)}{\overline{\phi}^2}$.

Assumption 2: $R - \frac{B}{\beta-1} > 0$.

Assumption 3: $R - \frac{B}{\beta-1} < 1$.

Assumption 1 implies that bank's investment has a positive net present value, i.e. the risky investment is socially optimal, and assumption 2 that bank's investment has a higher net present value when bank's managers behave properly rather than when they engage in excess risk taking. Finally, assumption 3 implies that the bank is sufficiently debt constrained that it needs also capital to finance all its investment.[51]

2.2.5 Equilibrium in fund managers' strategies

The liquidity shock that hits the bank at $t = 1$ is characterised here in terms of the equilibrium strategy by fund managers, following realisation of their private signals $\{s_i\}$. In

[51] This point is discussed in more detail in section 2.4.2.

particular, we assume that fund managers follow a trigger strategy s^* that is defined as follows.

Definition 2 *A trigger strategy s^* is a rule of action that maps the realization of a fund manager's signal s_i to one of the following actions: to withdraw if signal s_i is less than s^*, or to rollover if s_i is greater than or equal to s^*.*

As of Morris and Shin (2002) and Goldstein and Pauzner (2004) and given that fund managers' payoffs satisfy global strategic complementarities,[52] if a trigger strategy s^* exists and is unique, then it is the only dominant solvable equilibrium strategy by fund managers.[53] A uniquely determined s^* would then allow us to characterise liquidity shocks to the bank not only in terms of the realised value of productivity shock $\widetilde{\phi}$, but also in terms of the general economic environment, such as the extent of prudential liquidity maintained by the banking sector, central bank's LOLR policy ϕ^{**} and the degree of noise ε in fund managers' signals. Assuming that all model parameters, LOLR policy ϕ^{**} and the prior distribution of $\widetilde{\phi}$ are common knowledge, as well as the realised sample distribution of fund managers is the common distribution of their signals $\{s_i\}$, we prove the following result.

Lemma 2 *The critical level of productivity shock ϕ^* below which a liquidity crisis occurs, is given by:*

$$\phi^* = s^* + \varepsilon \left(1 - 2\frac{l}{D}\right)$$

[52] That is, the incentive of a fund manager to rollover/withdraw increases with the proportion of others undertaking the same action.

[53] In other words, s^* is the only strategy that survives the iterated deletion of strictly dominated strategies. That typically requires non-empty upper and lower dominance regions, namely the existence of a level of fundamentals above (below) which *all* fund managers accept (refuse) rollovering.

Proof *See appendix* ■

We observe that the minimum level of productivity shock ϕ^* that the bank could sustain without incurring a liquidity crisis is decreasing in the proportion of deposits that the bank maintains as liquid assets and increasing in the strategy that is followed by fund managers. Such a strategy is given by the following lemma.

Lemma 3 *Fund managers' equilibrium in trigger strategies s^* is given by:*

$$s^* = \phi^{**} + \varepsilon \left(1 - 2\frac{k}{1-k}\frac{l}{D}\right)$$

where, ϕ^{**} is the level of productivity shock below which the bank is not bailed out and $\frac{l}{D}$ is the ratio of liquid assets to deposits that is maintained by the bank.

Proof *See appendix* ■

The trigger strategy s^* essentially captures fund managers' sensitivity to news about productivity shock $\widetilde{\phi}$. The lower the s^* the less sensitive fund managers' become to private signals about $\widetilde{\phi}$, implying a more stable deposit base, and vice versa. As of lemma 3, s^* increases with the extent of asymmetric information ε among fund managers about fundamentals $\widetilde{\phi}$. In addition, the willingness to rollover deposits with the bank increases in the penalty parameter k, as well as in bank's liquidity ratio $\frac{l}{D}$ and the extent of central bank's readiness to extend the LOLR facility in case of crisis.

Provided that the realisation of $\widetilde{\phi}$ is not too low, such as everyone runs, and not too high, such as no deposits are withdrawn, lemmas 2 and 3 imply that the proportion ρ of deposits withdrawn for a given realisation of the productivity shock is given by

$$\rho(\phi) = \frac{\phi^{**} + 2\varepsilon\left(1 - \frac{k}{1-k}\frac{\lambda}{D}\right) - \phi}{2\varepsilon}$$

As a result, deposit withdrawals at $t = 1$ imply a liquidity shock to the bank that is increasing in the extent of noise ε in fund managers' signals. Moreover, it is increasing in ϕ^{**} meaning that, *ceteris paribus*, the more parial the LOLR insurance the higher the liquidity shock ρ. In addition, ρ decreases in the ratio of liquid assets to deposits, in parameter k and in productivity shock ϕ.

It is also straightforward to show that, for a sufficiently low penalty parameter k, there is a threshold ϕ_U such that if $\widetilde{\phi} \in [\phi_U, \overline{\phi}]$, all fund managers decide to rollover their deposits with the bank. The range $[\phi_U, \overline{\phi}]$ is referred as the *upper dominance region* and corresponds to realisations of fundamentals high enough to prevent any information-induced outflow of funds from the bank. Similarly, there is also a ϕ_L such that if $\widetilde{\phi} \in [1 - \overline{\phi}, \phi_L]$, all fund managers decide to withdraw their funds. The range $[1 - \overline{\phi}, \phi_L]$ is referred as the *lower dominance region*, corresponding to very weak realisations of fundamentals that induce a massive outflow of funds from the bank. Both ϕ_U and ϕ_L, are given by the following result.

Corollary 1 *The upper and lower dominance region for productivity shock $\widetilde{\phi}$ are defined by the following thresholds ϕ_U and ϕ_L, respectively:*

$$\begin{aligned} \phi_U &= \phi^{**} + 2\varepsilon\left(1 - \frac{k}{1-k}\frac{l}{D}\right) \\ \phi_L &= \phi^{**} - 2\varepsilon\frac{k}{1-k}\frac{l}{D} \end{aligned}$$

where ϕ^{**} is given by proposition 3 and defines central bank's LOLR policy, while $\frac{l}{D}$ is bank's ratio of liquid assets to deposits that we solve for in the following section.

Proof *It follows from lemmas 2 and 3 and the fact that $Pr\left(s_i \leq s^* | \, \phi_U\right) = 0$, while $Pr\left(s_i \leq s^* | \, \phi_L\right) = 1$* ∎

Consequently, depending on the realisation of productivity shock ϕ, the event line of the model is as follows.

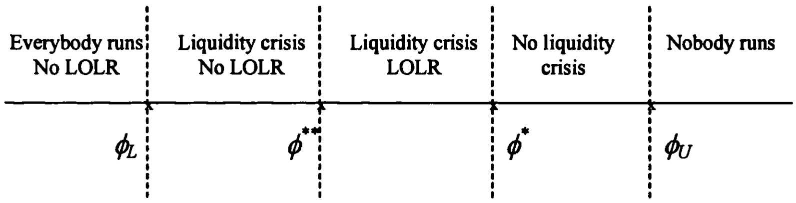

Fig. 2.2. Event line depending on realisation of $\widetilde{\phi}$

Next, we discuss the case without liquidity regulation and LOLR safety net, which is used later as a benchmark to draw welfare implications of official sector involvement both in terms of prudential liquidity regulation and LOLR policy.

2.3 No-regulation benchmark

We consider the simple case where the bank faces no restrictions on its holdings of liquid assets. Moreover, if the bank faces a liquidity shock at $t = 1$, it is possible to raise new funds and avoid liquidation, provided that it has sufficient collateral to pledge. Having assumed a fixed amount of bank capital A, let I_0 be the bank's chosen level of investment, l_0 the amount of voluntary liquid holdings and D_0 the amount of deposits that the bank would be able to raise at $t = 0$. Let also ϕ_0 be a threshold productivity shock below which the bank cannot deal with a liquidity crisis, due to lack of collateral, and is liquidated. Such a threshold satisfies the following break-even condition:

$$\phi_0 R I_0 = D_0 - l_0 \tag{2.2}$$

By substituting $(D_0 - l_0)$ from the budget constraint (2.1) into (2.2) and rearranging, we get

$$\phi_0 = \frac{I_0 - A}{R I_0} \tag{2.3}$$

Let also $m(I_0)$ be the marginal net expected return per unit of investment I_0. Then, the expected surplus of bank's investment is given by $U(I_0) = m(I_0) I_0$.

2.3.1 Voluntary liquidity holdings

In this model, both depositors and the central bank are assumed to make no profits. Thus, the surplus of bank's investment is equal to the bank's net expected utility from investment I_0, which solves the following optimisation problem:

$$\max_{I_0} U(I_0) \tag{2.4a}$$

subject to:

$$\frac{\overline{\phi} + \phi_0}{2}(R I_0 - D_0) \geq \frac{\overline{\phi} + \phi_0}{2\beta}(R I_0 - D_0 + B I_0) \tag{2.4b}$$

$$I_0 + l_0 = A + D_0 \tag{2.4c}$$

Inequality (2.4b) is bank's incentive compatibility condition, (2.4c) is its budget constraint and ϕ_0 is given by (2.3). It is important to realise that the amount of deposits D_0 is set before the bank's choice of its optimal investment and is such that, taking into account the bank's equilibrium investment I_0, bankers have no incentive to take excessive risks for the private benefit B. Thus, the incentive compatibility condition (2.4b) can be written as follows.

$$D_0 \leq \left(R - \frac{B}{\beta - 1}\right) I_0 \tag{4a}$$

Moreover, by assumption 1, bank's investment has positive net present value and also the cost of borrowing from depositors (normalised to zero) is less than the expected net pay-off per unit of investment. Thus, given risk neutrality, the bank has always an incentive to take on more deposits in order to increase its investment. As a result, the incentive compatibility condition (2.4b) binds.

$$D_0 = \left(R - \frac{B}{\beta - 1}\right) I_0 \tag{2.5}$$

We may now show that the proportion of deposits that the bank keeps in liquid assets is given by the following result.

Proposition 1 *In a laisser-faire environment – i.e. under no regulatory requirements and a LOLR safety net – the liquidity ratio $\frac{l_0}{D_0}$ that the bank opts to maintain is given by:*

$$\frac{l_0}{D_0} = 1 - \frac{1}{\left(R - \frac{B}{\beta-1}\right)} + \frac{R\sqrt{\frac{1}{R^2} - C_2^2}}{\left(R - \frac{B}{\beta-1}\right)}$$

where $C_2 \equiv \sqrt{\overline{\phi}^2 - \frac{2(2\overline{\phi}-1)}{R}}$.

Proof *See appendix* ■

In relation to proposition 1, the next result follows.

Corollary 2 *In the absence of prudential liquidity regulation, the bank opts to hold a positive amount of liquid assets if and only if its loans-to-deposits ratio is such that:*

$$\frac{I_0}{D_0} < \frac{1}{1 - \sqrt{1 - R^2 C_2^2}}$$

where $C_2 \equiv \sqrt{\overline{\phi}^2 - \frac{2(2\overline{\phi}-1)}{R}}$.

Proof *It follows immediately from the incentive compatibility condition (2.5) and proposition 1 by setting* $l_0 > 0$ ■

It is worth noting that any voluntary liquidity holdings in this model can be considered as *spare liquidity* in the presence of bank capital A and an optimal amount of loan investment I_0 by the bank. Thus, a stock of liquidity in our benchmark case plays solely a residual role, rather than serving a deeper economic purpose. However, proposition 2 is needed in order to account properly for bank capital and offer a consistent benchmark for comparison with our regulatory case where, as we will see, prudential liquidity enhances the scope of LOLR policy.

2.4 Regulatory contract

The optimal regulatory contract provides for two things: First, an optimal LOLR policy by the central bank. Second, an optimal liquidity ratio that the bank is required to maintain. As discussed in Section 2.2.3, a LOLR policy stipulates an intervention threshold ϕ^{**} such that the expected surplus of bank's investment is maximised under the following constraints: i) The total amount of deposits D is incentive compatible, i.e. bankers are given appropriate incentives not to engage in excess risk-taking. ii) The central bank expects not to incur systematic losses under the LOLR facility. iii) The budget constraint of the bank is satisfied.

As in the no-regulation benchmark, bank's creditors are assumed to make no profits. Thus, the surplus of bank's investment is equal to the bank's expected surplus from loan investment. Let $m(\phi^{**})$ be the marginal net expected return per unit of investment. Then the expected surplus U of bank's investment is $U(\phi^{**}) = m(\phi^{**}) I(\phi^{**})$ and the central bank's optimisation problem can be written as follows:

$$\max_{\phi^{**}} U(\phi^{**}) \tag{2.6a}$$

subject to:

$$\frac{\overline{\phi} + \phi^{**}}{2}(RI - D) \geq \frac{\overline{\phi} + \phi^{**}}{2\beta}(RI - D + BI) \tag{2.6b}$$

$$\frac{\phi^{*} + \phi^{**}}{2} RI \geq D - l \tag{2.6c}$$

$$I + l = A + D \tag{2.6d}$$

where (2.6b) is bank's incentive compatibility condition, (2.6c) is central bank's break-even condition and (2.6d) is bank's budget constraint. Also, l denotes the amount of bank's liquid asset holdings, while A the amount of bank's capital. However, in order to determine central bank's optimal LOLR policy, we first need to establish what is the optimal amount of bank investment $I(\phi^{**})$ for a given LOLR policy ϕ^{**}, that is, for a given level of liquidity insurance extended by the central bank.

2.4.1 Optimal loan investment

As with the no-regulation benchmark, the amount of deposits D is set before the bank's choice of its optimal investment, and is such that bank's managers have no incentive to gamble for the private benefit B. Thus, taking into account bank's equilibrium investment I, the incentive compatibility condition (2.6b) is binding and can be written as follows:

$$D = \left(R - \frac{B}{\beta - 1}\right) I \tag{2.7}$$

We note that the ratio of loans to deposits is fixed and equal to $\left(R - \frac{B}{\beta-1}\right)^{-1}$ regardless of the choice of investment amount I. Given also positive returns to investment and that the central bank makes no profits, the break even condition (2.6c) of the central bank also binds and can be written as follows:

$$\frac{\phi^* + \phi^{**}}{2} RI = (D - l) \tag{2.8}$$

Central bank's break even condition (2.8) implies that, conditional on LOLR intervention, the shareholders of the bank are expected to receive just nothing. Whether they will eventually receive something, i.e. $RI - D - l$, will depend on the actual realisation of productivity shock $\widetilde{\phi}$. Nevertheless, a LOLR policy ϕ^{**} that is consistent with a zero expected cost of the LOLR facility must be such that, conditional on LOLR intervention, shareholders are not expected to receive anything. Having said that, the next result follows.

Proposition 2 *For a given LOLR policy ϕ^{**}, the amount of investment $I(\phi^{**})$ that satisfies simultaneously bank's incentive compatibility condition (2.7), central bank's break* *even condition (2.8) and bank's budget constraint (2.6d), is given by:*

$$I\left(\phi^{**}\right)=\frac{A\left(\frac{1}{R}-\frac{a\varepsilon}{R-\frac{B}{\beta-1}}\right)}{\left(C_1-\phi^{**}\right)} \tag{2.9}$$

where $C_1 \equiv \frac{1}{R}-\varepsilon\frac{a+(1-a)\left(R-\frac{B}{\beta-1}\right)}{\left(R-\frac{B}{\beta-1}\right)}$ *and* $a \equiv \frac{1}{1-k}$.

Proof *See appendix* ■

Having calculated bank's optimal loan investment for given LOLR policy ϕ^{**}, we can now turn to evaluate the optimal LOLR policy ϕ^{**} that maximises the expected surplus from bank's loan investment.

2.4.2 Optimal LOLR policy

Proposition 2 provides the optimal investment $I\left(\phi^{**}\right)$ by the bank for a given LOLR policy ϕ^{**}. With $I\left(\phi^{**}\right)$ in hand, the optimal LOLR policy maximises the expected surplus from bank's investment. We prove the following proposition.

Proposition 3 *Central bank's optimal LOLR policy is to bail out the bank if and only if the level of productivity shock* $\widetilde{\phi}$ *is such that* $\widetilde{\phi} \geq \phi^{**}$, *where:*

$$\phi^{**}=C_1-\sqrt{C_1^2-C_2^2}$$

where $C_1 \equiv \frac{1}{R}-\varepsilon\frac{a+(1-a)\left(R-\frac{B}{\beta-1}\right)}{\left(R-\frac{B}{\beta-1}\right)}$, $C_2 \equiv \sqrt{\overline{\phi}^2-\frac{2\left(2\overline{\phi}-1\right)}{R}}$ *and* $a \equiv \frac{1}{1-k}$.

Proof *See appendix* ■

An optimal LOLR policy ϕ^{**} could also be considered in terms of the induced probability of liquidity crisis at the optimum. In fact, lemmas 2 and 3 imply a one-to-one mapping from central bank's optimal intervention threshold ϕ^{**} to the critical level of productivity shock ϕ^{*} below which a liquidity crisis occurs. Consequently, by choosing an LOLR policy ϕ^{**}, the central bank implicitly induces a certain probability of liquidity crisis in equilibrium. That is consistent with Allen and Gale (1998) and the idea of *optimal financial crisis* that could induce banks to hold efficient portfolios of risky assets. Moreover, the provision of appropriate LOLR insurance, conditional on banks conforming with costly liquidity regulation, is consistent with the principle of *Proportionality* which is widely used in political debate about the extent and intensity of action by the official sector.[54]

2.4.3 Optimal liquidity regulation

Having established central bank's optimal response to a liquidity crisis and the commensurate optimal investment $I(\phi^{**})$ by the bank, that level of investment can be implemented by regulating the level of bank's liquid assets. We prove the following result.

Proposition 4 *Under the optimal regulatory contract, the bank is required to maintain a liquid-assets-to-deposits ratio such that:*

$$\frac{l}{D} = 1 - \frac{1}{\left(R - \frac{B}{\beta-1}\right)} + \frac{R\sqrt{C_1^2 - C_2^2}}{\left(R - \frac{B}{\beta-1} - aR\varepsilon\right)}$$

where $C_1 \equiv \frac{1}{R} - \varepsilon\frac{a+(1-a)\left(R-\frac{B}{\beta-1}\right)}{\left(R-\frac{B}{\beta-1}\right)}$, $C_2 \equiv \sqrt{\overline{\phi}^2 - \frac{2\left(2\overline{\phi}-1\right)}{R}}$ *and* $a \equiv \frac{1}{1-k}$.

Proof *See appendix* ■

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[54] Under the proportionality principle, the content and form of actions by the official sector shall not exceed what is necessary to achieve its policy objective. In our case, such an official sector objective is the maximization of the surplus from financial intermediation, conditional on avoiding systematic losses under the LOLR facility.

From proposition 4, we can easily show that if fund managers' incentives to keep their deposits with the bank are sufficiently high – i.e. the penalty parameter k is not too low – then the prudential liquidity ratio $\frac{l}{D}$ increases with ε, namely with the extent of asymmetric information about bank's fundamentals. That implies that the higher the extent of transparency about bank's fundamentals – i.e. the lower the ε – the lower the prudential liquidity ratio that the bank is required to maintain. Also, from propositions 1 and 4 it is easy to show that, as the noise in fund managers' signals decreases towards zero, the prudential liquidity ratio converges towards the level that the bank would voluntarily hold absent any official sector intervention. Yet for $\varepsilon > 0$, the next result follows.

Proposition 5 *Under the optimal regulatory contract, the ratio of liquid assets to deposits that the bank is required to maintain is higher that in the absence of official sector involvement, i.e. under no liquidity requirements and LOLR insurance.*

Proof *See appendix* ■

Let us now search for conditions under which requiring banks to maintain a higher ratio of liquid assets to deposits than they would hold in a *laisser-faire* regime is, if at all, welfare improving.

2.5 Welfare analysis

From Section 2.4, the expected surplus U of bank's investment under the optimal regulatory contract $\left(\phi^{**}, \frac{l}{D}\right)$ is given by

$$U = \frac{A}{2\left(2\overline{\phi}-1\right)} \frac{\left(C_2^2 - \phi^{**2}\right)}{\left(C_1 - \phi^{**}\right)} \left(1 - \frac{aR}{R - \frac{B}{\beta-1}}\varepsilon\right) \tag{2.10}$$

By substituting the optimal LOLR policy ϕ^{**} from proposition 3 into (2.10) we get

$$U = \frac{A}{2\overline{\phi}-1}\left(C_1 - \sqrt{C_1^2 - C_2^2}\right)\left(1 - \frac{aR}{R - \frac{B}{\beta-1}}\varepsilon\right) \tag{2.11}$$

with $C_1 \equiv \frac{1}{R} - \varepsilon \frac{a+(1-a)\left(R-\frac{B}{\beta-1}\right)}{R-\frac{B}{\beta-1}}$, $C_2 \equiv \sqrt{\overline{\phi}^2 - \frac{2\left(2\overline{\phi}-1\right)}{R}}$ and $a \equiv \frac{1}{1-k}$. Similarly, from Section 2.3, the expected surplus U_0 of bank's investment under no liquidity regulation is given by

$$U_0 = \frac{A}{2\overline{\phi}-1}\left[\frac{1}{R} - \sqrt{\frac{1}{R^2} - C_2^2}\right] \tag{2.12}$$

Clearly, from (2.11) and (2.12) it follows that asymmetric information plays a pivotal role in the analysis of welfare implications of liquidity regulation. We may observe that as the noise in fund managers' signals tends to zero, i.e. $\varepsilon \to 0$, the expected surplus of bank's investment under the optimal regulatory contract becomes equal to the no-regulation benchmark. That is, when asymmetric information among fund managers dissipates, regulating a bank's holdings of liquid assets leads to no welfare improvement/deterioration of the no regulation benchmark.

However, for $\varepsilon > 0$, liquidity regulation may or may not lead to a welfare improvement of the no-regulation case. As we show next, that depends on the extent of bank's funding constraints, as measured by the ratio of optimal loan investment to deposits. For example, for different levels of parameter ε, figure 3 plots the expected surplus of bank's investment under the optimal regulatory contract for a loans-to-deposits ratio equal to 1.4 and 1.95. The horizontal line between the two curves corresponds to the expected surplus from bank's investment under the no-regulation benchmark.

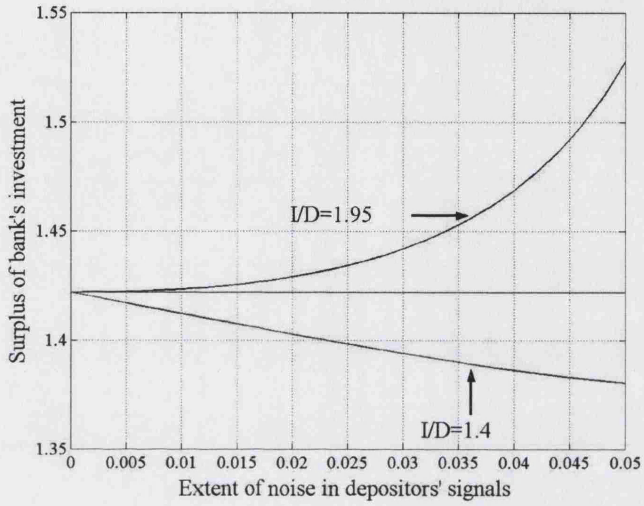

Fig. 2.3. Welfare implications of liquidity regulation

As of figure 2.3, we should expect prudential liquidity regulation, combined with an appropriate LOLR policy, to be socially desirable the more debt-constraint the banking sector is, i.e. the higher the ratio of loans to deposits. Otherwise, prudential liquidity may turn out to be too costly even after taking into account the insurance value of the LOLR. Here we derive a threshold for the loans-to-deposits ratio above which liquidity regulation leads to welfare improvement of the no-regulation case.

Proposition 6 *A necessary and sufficient condition for liquidity regulation to be welfare improving of the no-regulation benchmark is bank's loans-to-deposits ratio to be such* *that:*

$$\frac{I}{D} > \frac{a-1}{a\left(1-\sqrt{1-R^2C_2^2}\right)} \tag{2.13}$$

where $C_2 \equiv \sqrt{\overline{\phi}^2 - \frac{2(2\overline{\phi}-1)}{R}}$ *and* $a \equiv \frac{1}{1-k}$.

Proof *See appendix* ■

Proposition 6 suggests that liquidity regulation becomes socially desirable if and only if the debt capacity of the banking sector is sufficiently low. Essentially, prudential liquidity allows the central bank to extend LOLR insurance while maintaining, on average, a balanced budget under the LOLR facility. Thus, the lower the debt capacity of the banking sector, LOLR insurance becomes increasingly essential for realising the benefits of financial intermediation. That is by raising the expected return per unit of investment and, as a consequence of that, increasing bankers' willingness to employ more funds in productive assets.[55]

However, in this model, the ex ante objective by the central bank to run a balanced budget under the LOLR facility is attained by controlling credit extension through prudential liquidity restrictions. In the absence of such restrictions, bankers would choose to take on excessive risks by investing all available funds in risky loans, free riding, effectively, on central bank's LOLR insurance.[56] But such regulatory restrictions also impose an (opportunity) cost on the banking sector. Thus, if prudential liquidity is to be socially desirable,

55 From (2.24) we may observe that, in the absence of a LOLR, the expected marginal return from investment decreases in the total amount of investment and, as a result, bankers become less willing to employ more funds in productive investments.

56 That is because the marginal expected return from loan investment would not depend anymore on the total loan amount and, thus, bank's expected surplus would be strictly increasing in the total amount lent.

that cost needs to be traded off against the social benefit from LOLR insurance. Not surprisingly, it is when LOLR insurance is mostly needed that its benefits outweigh the costs of liquidity regulation and this is when the banking sector is sufficiently debt constrained.

Proposition 6 also suggests that liquidity regulation is more likely to be welfare improving the higher the potential return R per unit of loan investment. That also is not surprising given that the social value of LOLR insurance is higher when the potential return from keeping a bank alive is relatively high. Finally, we note that the RHS of (2.13) is increasing in parameter k, which determines the penalty that fund managers face for premature withdrawal of their deposits. That implies that the more *stable* the deposit base is – i.e. the higher the penalty parameter k and the lower the incentive to withdraw deposits prematurely – the more difficult it is for inequality (2.13) to be satisfied and, as a result, for prudential liquidity regulation to be welfare improving.

Thus, our model suggests that prudential liquidity regulation, coupled by an appropriate LOLR policy, is expected to be more appropriate for banking systems characterised by high profit opportunities, a less stable deposit base and sufficient funding constraints, which in our model were captured through moral hazard concerns regarding bankers' incentives. In the following section we provide a more formal exposition of the basic trade-off between LOLR insurance benefit and the cost of liquidity regulation that gives rise to proposition 6.

2.5.1 Regulatory cost vs. insurance benefit

From lemmas 2 and 3, the distance between the critical level of productivity shock ϕ^{*}, below which a liquidity crisis arises, and the level of shock ϕ^{**}, below which the central bank does not bailout the bank, is given by

$$(\phi^* - \phi^{**}) = 2\varepsilon\left(1 - \frac{1}{1-k}\frac{l}{D}\right) \tag{2.14}$$

Equation (2.14) implies that, conditional on LOLR intervention, the support of productivity shock $\widetilde{\phi}$ is increasing in ε, i.e. the extent of noise in fund managers' signals, and decreasing in the bank's liquidity ratio $\frac{l}{D}$. Moreover, for a given realisation of $\widetilde{\phi}$ in the interval $[\phi^*, \phi^{**}]$ and investment I by the bank, the expected recovery rate of a LOLR funds can be written as

$$Expected\ recovery\ rate = \min\left[\frac{\frac{I}{D}R\phi}{1-\frac{l}{D}}, 1\right] \tag{2.15}$$

Consequently, the minimum level ϕ_e of productivity shock below which the central bank expects to recover less than the full amount of its LOLR funds is

$$\phi_e = \frac{1-\frac{l}{D}}{\frac{I}{D}R} \tag{2.16}$$

We note that ϕ_e is decreasing in both the bank's liquidity ratio and the ratio of loans to deposits. Not surprisingly, given that the break-even condition (2.6c) of the central bank is binding at the optimum, ϕ_e must be equal to the expected productivity shock conditional on LOLR intervention, i.e. $\phi_e = \frac{\phi^*+\phi^{**}}{2}$. That is shown in figure 4 and can easily be verified by comparing (2.8) with (2.16).

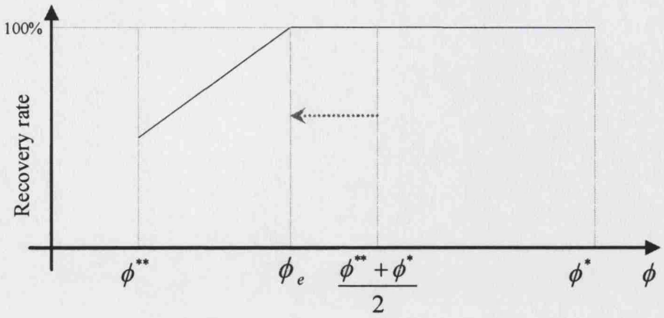

Fig. 2.4. Expected recovery rate of LOLR loan.

Let us now suppose that for a given balance-sheet structure of the bank and LOLR policy ϕ^{**}, ϕ_e was higher than $\frac{\phi^{*}+\phi^{**}}{2}$. Then central bank's break-even condition (2.6c) would be violated and, as of $t = 0$, the central bank would be unable to insure bank's liquidity shocks to a level as low as ϕ^{**}. Given that ϕ_e is decreasing in $\frac{l}{D}$, the central bank would then be able to augment the scope of its liquidity insurance by increasing the reserve requirement to the bank up to the point where ϕ_e becomes equal to the expected productivity shock conditional on LOLR intervention, i.e. $\phi_e = \frac{\phi^{*}+\phi^{**}}{2}$.

Thus, by increasing the liquidity requirement to the bank, the central bank is allowed to provide LOLR insurance against a wider range of productivity shocks that might hit bank's loan portfolio. That, in turn, would result in an increase in bank's expected profits, but on the downside, would also increase the regulatory burden to the bank by requiring it to hold costly liquid assets in excess of what it would opt to maintain in a *laisser-faire* regime. In equilibrium, such a trade-off could result in adverse welfare implications of liquidity regulation, with the regulatory costs from holding liquid assets outweighing the insurance benefits from the LOLR.

However, ϕ_e is not only decreasing in the ratio of liquid assets to deposits, but also in the bank's loans-to-deposits ratio. Thus, *ceteris paribus*, the higher the ratio of loans to deposits the lower the proportion of deposits that should be maintained as liquid assets by the bank, in order for the central bank to satisfy its break even condition. Consequently, there must also be a threshold for the loans-to-deposits ratio above which the insurance benefits brought by the LOLR outweigh the regulatory costs from holding costly liquid assets. Such a threshold was explicitly derived in proposition 6. Finally, for $\varepsilon \longrightarrow 0$ and from (2.14), the analysis collapses to the simple case where $\phi^* = \phi^{**}$ and, in terms of welfare implications, there is no distinction between the case with or without liquidity regulation.

2.6 Concluding remarks

In the spirit of Posner (1971), we regarded prudential liquidity regulation for banks as *quid pro quo* for emergency liquidity assistance by the central bank, where prudential liquidity allows the central bank to run a balanced budget under its LOLR safety net by imposing an implicit insurance premium to the banking sector for (partial) LOLR insurance. In the presence of bank funding constraints, asymmetric information among bank's creditors about the quality of bank's loan portfolio turned out to be key in describing prudential liquidity policy. It was shown that the more diverse creditors' beliefs are about bank's fundamentals, the higher the prudential liquidity ratio that the bank would be required to maintain. However, as asymmetric information about bank's fundamentals dissipates, that ratio converges towards the level that the bank would voluntarily maintain in the absence of intervention by the official sector.

We showed that optimal liquidity regulation implies a ratio of liquid assets to deposits that is higher than what the bank would voluntarily maintain, absent any official sector involvement. However, such a higher liquidity ratio is not to guarantee the existence of a stock of high-quality assets against which central banks can lend in a crisis, but it is rather to enable the bank to meet liquidity shocks from its *own* resources and to a certain level of confidence. In other words, we viewed prudential liquidity regulation serving as *a first line of defence* against bank liquidity problems that allows the central bank to maintain a zero expected cost of LOLR intervention, while counteracting excessive risk-taking due to LOLR safety net.

We also derived a necessary and sufficient condition for liquidity regulation to be welfare improving of the no-regulation case, showing that this is the case if and only if the bank's total loans-to-deposits ratio is above a certain threshold that was explicitly evaluated. Otherwise, liquidity regulation becomes too costly from a welfare perspective, even after taking into account the social value of LOLR insurance. Finally, the more stable a bank's deposit base is – i.e. the lower the *prima facie* incentive of bank's creditors to foreclose their exposures to the bank – the more unlikely it is for prudential liquidity regulation to be welfare improving.

Hellmann et al. (2000) argue that financial liberalisation grants more freedom to banks in determining their lending portfolios, exacerbating moral hazard problems by offering more *gambling* opportunities. Along those lines, and in conjunction to our proposition 6, we may conclude that it is when banks' funding constraints are pronounced due to moral hazard concerns and their profit opportunities are possibly substantial that prudential liquidity regulation is more likely to be welfare improving. But it is in a context of a liberalised financial system that an official safety net could prove to be more valuable and, as a result, its social value could possibly outweigh the cost of prudential liquidity regulation.

2.A Appendix

2.A.1 Proof of Lemma 2

Conditional on productivity shock ϕ, fund managers' signals $\{s_i\}$ are *i.i.d.* uniform with support on $[\phi - \varepsilon, \phi + \varepsilon]$. Given ϕ and fund managers' equilibrium trigger strategies s^*, the proportion of deposits withdrawn is equal to the probability that a given signal is less than or equal to s^*:

$$\Pr\left(s_i \leq s^* | \, \phi\right) = \frac{s^* - \phi + \varepsilon}{2\varepsilon} \tag{2.17}$$

From (2.17), the critical level of shock ϕ^* solves the following:

$$\phi^* = s^* + \varepsilon \left(1 - 2\frac{l}{D}\right) \tag{2.18}$$

Q.E.D.

2.A.2 Proof of Lemma 3

Conditional on observing signal $s_i = s^*$, let P_{00} be the conditional probabilities of no liquidity crisis at $t = 1$ and no default at $t = 2$. Similarly, let P_{10} be the probability of liquidity crisis at $t = 1$ and no default at $t = 2$ conditional on signal $s_i = s^*$. Probabilities P_{00} and P_{10} are calculated as follows:

$$P_{00} = \frac{s^* + \varepsilon - \phi^*}{2\varepsilon} \tag{2.19}$$

By substituting ϕ^* from lemma 2 into (2.19) we get

$$P_{00} = \frac{l}{D} \tag{2.20}$$

where l is the amount of liquid assets held by the bank and D is the total amount of deposits. Similarly, given ϕ^* and ϕ^{**}, P_{10} is given by

$$P_{10} = \frac{\phi^* - \phi^{**}}{2\varepsilon} \tag{2.21}$$

With P_{00} and P_{10} in hand, fund managers' equilibrium in trigger strategies s^* is such that a fund manager who observes signal $s_i = s^*$ is indifferent between withdrawing and rollovering. That is, s^* solves the following:

$$\pi P_{00} + \pi (1 - k) P_{10} = \pi (1 - k) \tag{2.22}$$

where P_{00} and P_{10} are given by equations (2.20) and (2.21). The LHS of (2.22) is the expected pay-off from rollovering, conditional on s^*, while the RHS is the (certain) pay-off from withdrawing. By substituting P_{00} and P_{10} from (2.20), (2.21) into (2.22), we get:

$$\frac{l}{D}\pi + \frac{\phi^* - \phi^{**}}{2\varepsilon}\pi (1 - k) = \pi (1 - k) \tag{2.23}$$

Then, by substituting ϕ^* from lemma 2 into (2.23), s^* is given by:

$$s^* = \phi^{**} + \varepsilon \left(1 - 2\frac{k}{1-k}\frac{l}{D}\right)$$

Q.E.D.

2.A.3 Proof of Proposition 1

Equation (2.3) implies that the net expected return $m(I_0)$ per unit of investment is given by:

$$m(I_0) = \frac{R}{2(2\overline{\phi} - 1)} \left[C_2^2 - \left(\frac{I_0 - A}{RI_0}\right)^2\right] \tag{2.24}$$

where $C_2 \equiv \sqrt{\overline{\phi}^2 - \frac{2(2\overline{\phi}-1)}{R}}$. Then, the bank's expected utility from investment I_0 is:

$$U_0 = m\left(I_0\right) I_0 \tag{2.25}$$

Given (2.24), (2.25) can be written as

$$U_0 = \frac{R}{2\left(2\overline{\phi}-1\right)}\left[C_2^2 - \left(\frac{I_0 - A}{RI_0}\right)^2\right] I_0 \tag{2.26}$$

where the amount of investment I_0 that maximises U_0 in (2.26) is given by:

$$I_0 = \frac{A}{R\sqrt{\frac{1}{R^2} - C_2^2}} \tag{2.27}$$

Thus, from (2.1), (2.4b) and (2.27), the amount of liquid assets l_0 that the bank would hold voluntarily is given by:

$$l_0 = \left[1 - \frac{1}{\left(R - \frac{B}{\beta-1}\right)} + \frac{R\sqrt{\frac{1}{R^2} - C_2^2}}{\left(R - \frac{B}{\beta-1}\right)}\right]\left(R - \frac{B}{\beta - 1}\right) I_0 \tag{2.28}$$

Finally, from (2.4b) and (2.28), the ratio of liquid assets to deposits, under no intervention by the official sector, is given by:

$$\frac{l_0}{D_0} = 1 - \frac{1}{R - \frac{B}{\beta-1}} + \frac{R\sqrt{\frac{1}{R^2} - C_2^2}}{\left(R - \frac{B}{\beta-1}\right)} \tag{2.29}$$

Q.E.D.

2.A.4 Proof of Proposition 2

By substituting (2.6d) into (2.8) we get:

$$\frac{\phi^* + \phi^{**}}{2} RI = I - A \tag{2.30}$$

From lemmas 2 and 3, the term $\frac{\phi^*+\phi^{**}}{2}$ on the LHS of (2.30) can be written as:

$$\frac{\phi^* + \phi^{**}}{2} = \phi^{**} + \varepsilon\left(1 - \frac{1}{1-k}\frac{l}{D}\right) \tag{2.31}$$

Thus, by substituting (2.31) into (2.30) and setting $a \equiv \frac{1}{1-k}$, the central bank's break-even condition becomes:

$$\left[\phi^{**} + \varepsilon\left(1 - a\frac{l}{D}\right)\right] RI = I - A \tag{2.32}$$

Then, from the incentive compatibility condition (2.7) and budget constraint (2.6d), (2.32) becomes:

$$\left(C_1 - \phi^{**}\right) I = A\left[\frac{1}{R} - \frac{a\varepsilon}{R - \frac{B}{\beta-1}}\right] \tag{2.33}$$

where $C_1 \equiv \frac{1}{R} - \varepsilon\frac{a+(1-a)\left(R-\frac{B}{\beta-1}\right)}{\left(R-\frac{B}{\beta-1}\right)}$ and $a \equiv \frac{1}{1-k}$. Then, equation (2.33) implies that, for a given ϕ^{**}, the amount of investment $I\left(\phi^{**}\right)$ that satisfies simultaneously constraints (2.6b), (2.6c) and (2.6d) is given by:

$$I\left(\phi^{**}\right) = \frac{A\left(\frac{1}{R} - \frac{a\varepsilon}{R-\frac{B}{\beta-1}}\right)}{\left(C_1 - \phi^{**}\right)} \tag{2.34}$$

Q.E.D.

2.A.5 Proof of Proposition 3

For a given LOLR policy ϕ^{**}, bank's net expected return per unit of investment is given by:

$$m\left(\phi^{**}\right) = \left[1 - F\left(\phi^{**}\right)\right] \frac{\overline{\phi} + \phi^{**}}{2} R - 1 \tag{2.35}$$

which can be restated as

$$m\left(\phi^{**}\right) = \frac{R}{2\left(2\overline{\phi} - 1\right)}\left(C_2^2 - \phi^{**2}\right) \tag{2.36}$$

where $C_2 \equiv \sqrt{\overline{\phi}^2 - \frac{2(2\overline{\phi}-1)}{R}}$. Given (2.36), central bank's optimisation problem (2.6a) is equivalent to the following unconstrained problem:

$$\max_{\phi^{**}} U(\phi^{**}) \tag{2.37}$$

where $U(\phi^{**}) = m(\phi^{**}) I(\phi^{**})$, $I(\phi^{**})$ is given by proposition 2 and $m(\phi^{**})$ is the net expected return per unit of investment. Given (2.36), (2.37) can be expressed as

$$\max_{\phi^{**}} \frac{A}{2(2\overline{\phi}-1)} \frac{\left(C_2^2 - \phi^{**2}\right)}{\left(C_1 - \phi^{**}\right)} \left(1 - \frac{a\varepsilon R}{R - \frac{B}{\beta-1}}\right) \tag{2.38}$$

where $C_1 \equiv \frac{1}{R} - \varepsilon \frac{a+(1-a)\left(R-\frac{B}{\beta-1}\right)}{\left(R-\frac{B}{\beta-1}\right)}$, $C_2 \equiv \sqrt{\overline{\phi}^2 - \frac{2(2\overline{\phi}-1)}{R}}$ and $a \equiv \frac{1}{1-k}$. However, maximising the expression in (2.38) is equivalent to maximising the following expression of ϕ^{**}:

$$f(\phi^{**}) = C_1 + \phi^{**} - \frac{C_2^2 - C_1^2}{\phi^{**} - C_1} \tag{2.39}$$

The first derivative of $f(\cdot)$ with respect to ϕ^{**} is:

$$\frac{\partial f(\phi^{**})}{\partial \phi^{**}} = 1 + \frac{C_2^2 - C_1^2}{\left(C_1 - \phi^{**}\right)^2} \tag{2.40}$$

while the second derivative is given by:

$$\frac{\partial^2 f(\phi^{**})}{\partial \phi^{**2}} = 2\frac{C_2^2 - C_1^2}{\left(C_1 - \phi^{**}\right)^3} \tag{2.41}$$

We consider two cases: i) $C_1 \leq C_2$ and ii) for $C_1 > C_2$. However, we can easily show that inequality $C_1 \leq C_2$ holds if and only if the bank's loans-to-deposits ratio is such that $\frac{I}{D} \geq \frac{1-(C_2-\varepsilon)R}{2R\varepsilon}$, which for small values of ε implies a high value of $\frac{I}{D}$. Nevertheless, given that the loans-to-deposits ratio under both the regulation and the no-regulation case is equal to $\left(R - \frac{B}{\beta-1}\right)^{-1}$, the case where $C_1 \leq C_2$ can easily be ruled out from corollary 2. Thus, the only relevant case to consider is for $C_1 > C_2$, under which (2.41) becomes negative and the optimal LOLR policy ϕ^{**} is given by:

$$\phi^{**} = C_1 - \sqrt{C_1^2 - C_2^2} \tag{2.42}$$

Q.E.D.

2.A.6 Proof of Proposition 4

From bank's budget constraint (2.1), the liquidity ratio $\frac{l}{D}$ can be expressed as:

$$\frac{l}{D} = \frac{D + A - I}{D} \tag{2.43}$$

By substituting (2.7) and (2.34) into (2.43), we derive the following expression for the ratio of liquid assets to deposits:

$$\frac{l}{D} = 1 - \frac{1}{R - \frac{B}{\beta - 1}} + \frac{R\left(C_1 - \phi^{**}\right)}{\left(R - \frac{B}{\beta-1} - aR\varepsilon\right)} \tag{2.44}$$

where $C_1 \equiv \frac{1}{R} - \varepsilon \frac{a + (1-a)\left(R - \frac{B}{\beta-1}\right)}{\left(R - \frac{B}{\beta-1}\right)}$ and $a \equiv \frac{1}{1-k}.C_1 \equiv \frac{1}{R} - \varepsilon\left(\frac{2}{R - \frac{B}{\beta-1}} - 1\right)$. Then, by substituting ϕ^{**} from proposition 3 into (2.44) and setting $C_2 \equiv \sqrt{\overline{\phi}^2 - \frac{2\left(2\overline{\phi}-1\right)}{R}}$, the optimal liquidity ratio is given by

$$\frac{l}{D} = 1 - \frac{1}{\left(R - \frac{B}{\beta-1}\right)} + \frac{R\sqrt{C_1^2 - C_2^2}}{\left(R - \frac{B}{\beta-1} - aR\varepsilon\right)}$$

Q.E.D.

2.A.7 Proof of Proposition 5

Propositions 1 and 4 imply that $\frac{l_0}{D_0} < \frac{l}{D}$ if and only if:

$$\frac{\sqrt{\frac{1}{R^2} - C_2^2}}{\left(R - \frac{B}{\beta-1}\right)} < \frac{\sqrt{\left[\frac{1}{R} - \varepsilon \frac{a + (1-a)\left(R - \frac{B}{\beta-1}\right)}{\left(R - \frac{B}{\beta-1}\right)}\right]^2 - C_2^2}}{\left(R - \frac{B}{\beta-1} - aR\varepsilon\right)} \tag{2.45}$$

By recasting (2.45), we derive the following inequality:

$$\left(\varepsilon - \frac{K}{aR}\right)^2 < \frac{1}{(1-L)}\left(\frac{a+(1-a)K}{a}\right)^2\left(\varepsilon - \frac{K}{R\left[a+(1-a)K\right]}\right)^2 - \frac{K^2 L}{a^2 R^2 (1-L)} \tag{2.46}$$

where $K \equiv R - \frac{B}{\beta - 1}$, $L \equiv (RC_2)^2$ and $C_2 \equiv \sqrt{\overline{\phi}^2 - \frac{2(2\overline{\phi}-1)}{R}}$. With respect to ε, the geometric locuses defined by the LHS and RHS of (2.46) are parabolas. The parabola defined by the LHS of (2.46) has its vertex at the point $\left[\frac{K}{aR}, 0\right]$ and its focal parameter is $p_{LHS} = \frac{1}{2}$. Similarly, the parabola defined by the RHS of (2.46) has its vertex at the point $\left[\frac{K}{R[a+(1-a)K]}, -\frac{K^2 L}{a^2 R^2 (1-L)}\right]$ and its focal parameter is $p_{RHS} = \frac{(1-L)}{2}\left[\frac{a}{a+(1-a)K}\right]^2$.

From analytical geometry we know that if the focal parameter p of a parabola is positive then the parabola faces upwards. That is definitely the case for the LHS of (2.46), i.e. $\frac{1}{2} > 0$, while for the RHS of (2.46) that is true if and only if $1 - L > 0$, or $C_2 < \frac{1}{R}$. But, given $C_1 > C_2$ and $C_1 < \frac{1}{R}$, it also follows that $C_2 < \frac{1}{R}$. Consequently, the parabola defined by the RHS of (2.46) is also facing upwards.

Finally, we observe that both parabolas intersect at $\left[0, \left(\frac{K}{aR}\right)^2\right]$ and their vertices lie on the right of their intersection point. Thus, for relatively small values of ε, a sufficient condition for (2.46) to hold is $0 > -\frac{K^2 L}{a^2 R^2 (1-L)}$, or $1 - L > 0$. But this has already been shown to be true, implying that the optimal regulatory contract stipulates a higher liquidity ratio than what the bank would voluntarily maintain under no intervention by the official sector.

Q.E.D.

2.A.8 Proof of Proposition 6

Given that $U|_{\varepsilon=0} = U_0$, a necessary and sufficient condition for $U(\varepsilon) > U_0$, for small values of ε, is that the derivative of U with respect to ε, evaluated at $\varepsilon = 0$, is positive, i.e. $\left.\frac{\vartheta U(\varepsilon)}{\vartheta \varepsilon}\right|_{\varepsilon=0} > 0$. With a bit of algebra, we can shown that such a derivative is given by:

$$\left.\frac{\partial U}{\partial \varepsilon}\right|_{\varepsilon=0} = \frac{A}{(2\overline{\phi}-1)}\frac{\left(1-\sqrt{1-L}\right)}{K\sqrt{1-L}}\left[a\left(1-\sqrt{1-L}\right)+(1-a)K\right] \tag{2.47}$$

where $K \equiv R - \frac{B}{\beta-1}$, $L \equiv R^2C_2^2$, $C_2 \equiv \sqrt{\overline{\phi}^2 - \frac{2(2\overline{\phi}-1)}{R}}$ and $a \equiv \frac{1}{1-k}$. It the follows that a necessary and sufficient condition for $\left.\frac{\vartheta U(\varepsilon)}{\vartheta \varepsilon}\right|_{\varepsilon=0} > 0$ is:

$$\frac{1}{K} \equiv \frac{I}{D} > \frac{a-1}{a\left(1-\sqrt{1-R^2C_2^2}\right)}$$

Q.E.D.

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Chapter 3
Co-ordination failure and the signalling role of banks in debt-exchange offers

3.1 Introduction

Empirical evidence suggests that banks play a potentially important role in facilitating the resolution of financial distress. Inspired by Corsetti *et al.* (2001), one possible explanation for this is that the actions of large creditors, such as banks, simply allow small creditors to coordinate more efficiently. In particular, if a bank lender chooses to restructure, this may be taken by smaller, possibly less informed creditors, to imply that the going concern value of the firm, and thus the value of new claims offered, exceeds the liquidation value of the firm. In this way, banks and other large well-informed creditors could facilitate the resolution of financial distress by injecting a degree of *strategic solidity* in credit markets.

In the literature, there has been no theoretical work directed at examining this proposition. Rather, this has tended to concentrate mainly on firms' optimal choices between public and private debt, and the agency and other costs associated with diffused versus concentrated ownership of debt when the firm is *out* of financial distress. This is despite a steady accumulation of empirical work that has examined the role of banks in facilitating public debt exchange offers (out-of-court resolution of financial distress) when creditors face co-ordination problems and banks are assumed to own some proprietary, though not necessarily superior, information about the going concern value of the firm.

Mooradian and Ryan (2004) examine cases of out-of-court restructuring of debt and the resolution of financial distress under Section 3(a)(9) of the US Security Act and compare them to investment-bank-managed exchange offers. They find evidence that when a commercial bank makes a concession – which is almost always conditioned on a successful public debt restructuring – the importance of the exchange offer increases and an investment bank is more likely to be involved. Their results suggest that investment banks play a very important role in certifying the exchange and facilitating debt reduction, but unlike commercial banks, they play little if any role in resolving co-ordination problems.

James (1995, 1996) finds evidence consistent with the hypothesis that bank participation in debt restructuring transactions facilitates public debt exchange offers. That is also supported by evidence provided by Gilson, Kose and Lang (1990) showing that the likelihood of out-of-court debt restructuring is positively related to a firm's reliance on bank debt. In particular, James (1995) finds that forgiveness of principal by banks induces public-debt holders to accept a debt exchange offer more easily and to reduce principal more aggressively.[57] Also the likelihood of achieving minimum tendering rates – which is a typical prerequisite in debt exchange offers – increases.[58] Moreover, he shows that transactions in which banks forgive principal typically involve firms with more severe financial distress (e.g. higher leverage), which suggests that banks make concessions only when their claims are likely to be impaired. James (1996) also reports that, in all cases where

57 In fifteen debt restructuring transactions where banks took no action the average reduction in public debt was 19%, while in 14 cases where the bank reduced principle the average reduction in public debt was 56%.

58 In all cases where banks offer to scale down their loans actual tendering rates are above the minimum specified for success compared to 30% when banks do not make concessions.

banks make concessions, they make their offers contingent upon the successful completion of the public debt exchange offer.

Asquith, Gertner and Scharfstein (1994) analyse how financially distressed firms try to avoid bankruptcy through public/private debt restructuring, asset sales, mergers and capital expenditure reductions. Using a sample of companies with high-yield, *junk* bond issues with financial difficulties, they find evidence that the firm's debt structure affects the way financially distressed firms restructure their claims. In particular, a combination of secured private debt and numerous public debt issues seems to impede out-of-court restructuring and the firm's debt structure affects the way financially distressed firms restructure their claims.

In contrast to James (1996), Asquith *et al.* (1994) find that banks almost never loosen financial constraints by forgiving principal, while loosening financial constraints does not reduce the probability of bankruptcy.[59] They argue, however, that their sample is very specific as it focuses on the high-yield bond market, and the results should not be generalised. Gilson, Kose and Lang (1990) find evidence that the likelihood of out-of-court debt restructuring is positively related to the firm's reliance on bank debt.

In the theoretical front, Bolton and Freixas (2000) discuss a model of corporate finance where both supply and demand influence the availability of finance within an equilibrium set-up with asymmetric information. They argue that banks can help firms in times of distress because they can exploit their superior information/borrower screening skills. In addition, an important feature of their model is banks' ability to securitise senior portions

[59] 59% of firms whose banks loosen financial constraints still went bankrupt vs. 68% of the firms whose banks tighten the constraints, though there are differences in restructuring periods until bankruptcy.

of *rescue* finance they extend to firms in distress (e.g. in a *debtor-in-pocession* situation) and avoid the incentive to liquidate inefficiently a firm in financial distress. In equilibrium, banks choose to increase their supply of loans, provided that they can price effectively for the extra risk and they are not capital constrained. That way bank loans may substitute for other forms of finance and facilitate the resolution of financial distress.

Diamond (1993) argues that, because bank lenders are generally secured, they have little incentive to make concessions. Gertner and Scharfstein (1991) provide a model that illustrates how bank participation in the restructuring transaction can mitigate *holdout* problems among public debt-holders. In doing so, however, they assume common knowledge about the firm's economic fundamentals which allows perfect co-ordination of creditors' actions.

Jaffee and Shleifer (1990) examine how investment banks protect firms from financial distress due to self-fulfilling failure of calls of convertible bonds. They provide an analogy to Diamond and Dybvig's (1983) bank runs model by arguing that, by underwriting the forced conversion of convertible bonds, investment banks essentially provide insurance (a put option) to the firm in the same way that deposit insurance provides protection against bank runs. Yet Jaffee and Shleifer assume that the economic fundamentals of the firm i.e. the value of firm's assets, is common knowledge among creditors and there is no uncertainty about equilibrium behaviour of creditors. This allows perfect co-ordination of creditors' actions and results in multiple Nash equilibria.[60] Moreover, it implies that there is only risk shifting from the firm to the bank and there is no information content in an

60 For a discussion on this issue see Morris and Shin (2000)

investment bank's action to accept the underwriting. It is exactly that information content that is central to our analysis.

In this paper, we consider an out-of-court renegotiation of contractual obligations in a setting that is similar to a debt-exchange offer, where a bank creditor and a continuum of small claimants to a financially distressed firm interact in an environment of asymmetric information about the firm's solvency condition. Solvency is defined with respect to the ability of the firm to repay all its contractual obligations after the completion of a risky project that currently has in place. We investigate the extent to which acceptance by the bank to commit further funds to the firm, facilitates contract revision offers by other creditors.

In our model, the bank is a *large* creditor by virtue of its non-negligible financial mass, compared to all other creditors of the financially distressed firm. However, bank's financial capacity is assumed to be small relative to the balance sheet of the firm and, as a result, insufficient to manufacture a bail-out just by itself. In addition, the financial capacity of each other individual creditor is assumed to be of measure zero and, in that sense, the game is *asymmetric*.

Moreover, the actions by the bank are assumed to be common knowledge among other creditors before they choose their own actions. In that sense, the game is *sequential*. Consequently, in equilibrium, the bank recognises both the information and the money-effect of its own action, while individual small creditors fail to see those effects in isolation and can only consider their impact as a whole.

Throughout this paper, we assume that a bank creditor has an information advantage over other creditors. This is consistent with the literature on the importance of banks' monitoring abilities and how banks might get access to better information compared to other types of creditors.[61] In particular, we focus on the limited case where the relative information precision of small creditors, relative to that of the bank, tends to zero. This is without loss of generality and it is necessary for maintaining tractability in our analysis and deriving a closed form solution for equilibrium strategies.[62]

The basic feature of our analysis is that agents in the model exhibit *one-sided strategic complementarities*. That is, if the proportion of agents who undertake a certain action is relatively small then an agent's incentive to undertake such an action increases with the proportion of other agents undertaking the same action. This allows us to derive Bayesian equilibria of the restructuring game using the results of Goldstein and Pauzner (2004), who extend the *global game* methodology by Carlsson and van Damme (1993) and Morris and Shin (2002). Yet, an innovation of the model that adds to the existing literature on global games is that we consider the action by one class of players – namely, by the bank – as conditional on the actions of other creditors.

The analysis shows that a bank, by making a conditional concession and extending new credit to a firm in financial distress, may inject a degree of *strategic solidity* in credit markets, facilitating the resolution of financial distress. This result is consistent with empir-

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61 That literature includes, among others, Bolton and Freixas (1998, 2000), Holmstrom and Tirole (1997), Houston and James (1996), Rajan (1992), Lummer and McConnell (1989), Fama (1985), to mention but a few.

62 In a similar setting, Corsetti et al (2001) show numerically that the direction of results is robust to any level of relative precision of agents' private information.

ical evidence showing that banks' involvement in debt restructuring transactions facilitates debt-exchange offers by other creditors. In particular, controlling for money-effects, we show that debt-exchange offers become *successful* at lower levels of firm's fundamentals[63] compared to the situation where the bank has no role to play in the debt restructuring.

The intuition that underlies the above proposition is that a bank's involvement in the restructuring can influence the decisions of other creditors to restructure by laying down a marker around which their average opinion coalesce. Also, by conditioning its acceptance of the restructuring on the actions of other creditors, allows the bank to draw on the *collective wisdom* of those creditors, avoiding errors that an unconditional response would possibly entail. That way, a bank lender may influence the beliefs of other creditors in such a way that it could possibly induce a more efficient outcome of the debt renegotiation.

However, for high minimum tendering rates, herding incentives may dominate coordination problems as creditors tend rely more on average opinion, as reflected in the outcome of the debt restructuring, rather than on their private signals about the solvency of the firm. In equilibrium, herding incentives are common knowledge and the bank chooses its strategy in a way that accounts for suppliers' motive to play a low strategy if the bank plays a high one, and vice versa. That leads to multiple equilibria in the restructuring game, where the strategy that is followed by the bank is decreasing in the strategy that is followed by other creditors and vice versa.

In this model, we adopt a fairly generic characterisation of the financially distressed entity and we do not make any specific reference to the ownership structure of that entity,

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63 For our purposes, *fundamentals* of the firm are defined with respect to the cash generating power of the firm's project upon its completion.

the role for equity capital, or potential conflict of interests between shareholders and bondholders in the spirit of Jensen and Meckling (1976). Thus, our analysis by-passes possible conflicts of interest between different classes of security holders and concentrates on the workouts of financial distress, the potential inefficiencies that may arise from creditors' co-ordination problems and how those inefficiencies can be alleviated via the involvement of a bank creditor. Consequently, our analysis does not allow for the simultaneous treatment of both co-ordination problems among creditors, when the firm is *in* financial distress, and potential moral-hazard problems associated with the terms of lending at origination, when the debtor is *out* of financial distress. This, however, is a subject that we consider for future research.

The generic characterisation of the firm's balance sheet allows us to add also some thoughts that stretch beyond the resolution of financial distress in the corporate sector and relate to the resolution of international financial crisis. In particular, we could draw a parallel between the balance sheet of the financially distressed firm in our model, and the capital account of a country during the onset of financial crisis. We could then discuss implications of our model for the *doctrine of catalytic finance* and the scope and rationale for IMF lending to a debtor country when that country faces a situation, or the possibility, of financial distress.

In September 2003, for example, the Brazilian government has authorised the negotiation of a new, one-year deal with the IMF. The government's Treasury secretary, Joaquim Levy, was then quoted as saying[64]

[64] FT, September 12, 2003.

...Obviously, our objective is to walk alone and not depend on the fund. But a one-year renewal could be an important mechanism of information to investors who did not follow Brazil's progress closely.

The above statement is striking given the strong criticism of the IMF by president da Silva for more than twenty years in opposition. But, it also suggests that there is possibly something more than *money* in the involvement of the IMF in the resolution of financial distress. Namely, IMF lending may act as a *mechanism of information* that permits less informed – and possibly *small* – creditors to coordinate more efficiently. This is consistent with the result of our paper that large, informed creditors may act as *gate keepers* to the system and, should debtors' fundamentals justify it, inject a degree of *strategic solidity* among other creditors.

The remainder of the paper is organized as follows: Section 1.2 discusses the model. The solution proceeds in steps in Sections 3.3 and 3.4. Section 3.5 discusses empirical implications and possible extensions and Section 3.6 concludes adding also some thoughts on the role of catalytic finance and collective action clauses in the resolution of international financial crises. Proofs of our results are included in the appendix.

3.2 Basic environment

We consider a two-period setting $\{\tau = 1, 2\}$ where, as of $\tau = 1$, a firm runs an illiquid risky project which generates a random pay-off $\widetilde{R}$ at $\tau = 2$. We also consider two types of risk-neutral claimants to the firm that interact in an environment of asymmetric information about the firm's solvency condition: a continuum of *small*, poorly informed suppliers to the firm of total measure one and a *large* well-informed bank. Both types of claimants and their relationship with the firm are described next.

3.2.1 Suppliers

Every supplier has agreed to deliver at $\tau = 1$ a single unit of homogenous project-specific inputs at a price of one. By project-specific inputs we mean that, if inputs are not delivered to the firm on time, they need to be sold at a discount elsewhere. In other words, the firm is viewed as a monopsonist and full repayment of suppliers is conditional on the firm's ability to pay. Thus, supply contracts may be cancelled (foreclosed) at $\tau = 1$ and suppliers who refuse to deliver their inputs face a certain discount $c \leq 1$, i.e. at a price $(1 - c)$.

That formulation is in line with Berlin, Kose and Saunders (1996) who assume that suppliers cannot be paid up front but only at $\tau = 1$, i.e. when delivery takes place.[65] In that sense, inputs may also relate to factors of production such as capital, with suppliers then representing public debt holders and the firm being a monopsonist as far as redemption of its debt at maturity is concerned.[66]

Moreover, we assume that if a certain proportion of suppliers refuse to deliver their inputs at $\tau = 1$, then the firm's project is interrupted and the firm is liquidated. We consider

[65] Berlin *et al.* (1996) consider a similar situation, yet, with a perfectly co-ordinated set of suppliers, where suppliers may choose to terminate an established supply relationship with a firm should that relationship be severed by the firm.

[66] Should the firm fail to redeem its paper at maturity, holders of public debt would face insolvency costs and a potential discount in their original claims.

the situation where the firm requires a minimum quantity r of inputs for its project to reach its final stage and generate a return R. Otherwise the project must be abandoned and the liquidation value of the firm is then normalised to zero. Consequently, the proportion $(1 - r)$ of suppliers that may not deliver could represent the maximum contraction of firm's operations before it becomes unable to operate as a going concern. Alternatively, and in the context of a debt-exchange offer, the minimum proportion r of inputs could be interpreted as the *minimum tendering rate* in order for a public debt exchange offer to be successful; but we will revisit this point discussing it in more detail later on.

3.2.2 Bank

It extends loans to the firm and is a large creditor by virtue of the face value of its loans relative to individual credit exposures of other creditors (suppliers), which are considered negligible as a proportion of the whole (i.e. of measure zero). Yet the bank is assumed to be of a limited financial capacity, meaning that the total amount of funds it can extend to the firm is small compared to the firm's total input costs.[67] In particular, at $\tau = 1$, the firm has a secured bank debt (loan) with face value B and maturity at $\tau = 2$. Also at $\tau = 1$, the firm is supposed to need an extra amount of funds C in order to cover operating expenses (e.g. labour costs). Failure to meet those obligations would result in severe disruption of firm's operations, abandonment of the project and liquidation, in which case priority rules are assumed to be enforced for secured lenders.

[67] That is more easy to imagine in case of large corporations, or sovereign borrowers, with a diverse set of creditors. Although each of those creditors may represent a small portion of the whole, some of them may be large by virtue of their relative size.

3.2.3 Liquidity crisis

We consider the situation where the firm faces a liquidity shock at $\tau = 1$ and has no resources to pay both for its inputs and for its operating expenses C. Yet, the firm needs at least proportion r of inputs to be delivered at $\tau = 1$ and an amount of funds C in order to reach the final stage of its risky project and avoid costly liquidation. In order to deal with the crisis, the firm requests its existing claimholders to provide extra credit. Henceforth, we will refer to such requests as the *debt restructuring offer*.

In particular, the firm requests the extension of a senior unsecured loan C by the bank that would result in an increase of bank's credit exposure from B to $B + C$. Should the bank agree to provide such a loan, the firm also needs to offer new contracts to suppliers in exchange of old IOU claims and delivery of goods at $\tau = 1$. Such an exchange offer should allow the firm to receive the necessary amount of inputs to carry on with the project.[68]

Considering the firm as resorting to its existing claimholders for credit extension in case of crisis can be justified on grounds of vested interests that existing claimholders have in the resolution of financial crisis and the possibility that may be willing to make concession in order to recover their original exposures. Also, an established working relationships with the firm would imply that existing claimholders may have an information advantage over other potential creditors and, altogether, that may imply more favorable terms in extending distress credit to a solvent, but illiquid, firm. Although the terms of the debt restructuring are taken as given in this paper, focusing instead on how those terms may affect

[68] It is worth reiterating that the financial capacity of the bank is limited and, as a result, the bank is unable to manufacture the bailout of the firm by extending both a new loan C and by repaying the firm's suppliers.

the outcome of the offer, we distinguish between *insolvency* and *illiquidity* in terms of the firm's ability to meet its contractual obligations at $\tau = 2$ from the pay-offs of its risky project. We consider the following definition:

Definition 1 *As of $\tau = 1$ the firm is considered solvent if and only if it is considered capable of meeting all its contractual obligations at $\tau = 2$.*

Finally, before turning to discuss the debt restructuring offer in more detail, it is worth emphasising that this paper assumes away the possibility of a firm prearranging credit lines with its creditors à la Holmström and Tirole (1998, 2000), which would possibly provide some comfort to the firm in case of a liquidity shock. Although we deem that as an interesting extension to our model, here we adopt an ex-post view of liquidity crisis, focusing exclusively on the resolution of financial distress and on ex post decisions of firm's creditors to restructure their claims.

3.2.1 Debt-restructuring offer

Restructuring of supply contracts takes the form of a tender offer (henceforth debt-exchange offer), whereby all suppliers face the same terms of debt-exchange. Under those terms, a supplier is called to delivery a single unit of inputs at $\tau = 1$ in exchange of an unsecured debt claim with face value equal to a_s that is payable at $\tau = 2$. Henceforth, we refer to the face value a_s of the new claims offered as the *debt-exchange ratio* and we assume that it is greater than what suppliers are expected to receive by not delivering to the firm (i.e. $a_s > 1 - c$). Moreover, under the debt-exchange offer, a minimum tendering rate r has to met in order for the offer to be declared as *successful*. Thus, we assume that suppliers' responses to the offer are pooled together and inputs are released in exchange of new contracts only if a minimum tendering rate r is met.

In line with empirical evidence,[69] we assume that acceptance by the bank to make concessions – i.e. to extend new credit to the firm – is made conditional on success of the debt-exchange offer. In other words, a necessary condition for the bank to extend distress funding is that a minimum proportion r of suppliers accept the new contract terms.

3.2.2 Pay-offs of the bank

At date $\tau = 1$ the firm has no collateral to offer but both the old and the new loan to the firm rank first in the firm's capital structure. Yet, if the bank rejects the offer then the firm will be liquidated immediately (e.g. under Chapter 7 proceedings). In that case, the seniority of the old claim to the firm is of no value given that the claim is severely impaired (actually is worthless) due to zero liquidation value of the firm at $\tau = 1$. Given also that bank's agreement to extend new credit to the firm is made conditional upon successful completion of the exchange offer to suppliers, the loss that the bank will incur if default takes place at $\tau = 1$ is limited only to the old loan amount (B). In case of default at $\tau = 2$ the bank has the first claim on what the project has generated up to the total loan amount $(B + C)$. Yet, if there is no default at all, the bank fully recovers both the new and the old loan amount $(B + C)$. The following table summarises the bank's loss function under different

69 See, for example, James (1996).

scenarios:

Bank	*Default at* $\tau = 1$	*Default at* $\tau = 2$	*No Default*
Accept	*-B*	-$LGD \times (B + C)$	0
Reject	*-B*	*-B*	*-B*

where, LGD is the loss-given-default (e.g. internal-systems-based) associated with the situation of default in the final period.[70] For convenience we assume that -$LGD \times (B+C) < -B$, or that $LGD > \frac{B}{B+C}$. This assumption intends to capture the non-trivial nature of bank's commitment to extend new credit at $\tau = 1$. This is, the amount of new credit C is not negligible compared with the original amount B. Moreover, banks usually claim that it is their policy when they extend credit to make sure that the firm is solvent. In other words, the provision of extra security (i.e. enhanced seniority, collateral etc.) other than affecting the terms of lending it is not the driving force behind bank's decision to extend credit or not. As a result, it would be conceptually wrong, on an ex-ante basis, to relate explicitly the bank's pay-off in case of default at $\tau = 2$ to the firm's liquidation value. This would obstruct us from the original objective which is to capture the effect of bank's belief about the *solvency* status of the firm on small claimants' actions. Furthermore, it would computationally burden our analysis making it very specific to distributional assumptions about agents' signals.

70 The use of a fixed LGD is consistent with the *foundations internal-ratings-based* IRB approach that has been proposed by the Basel Committee on Banking Supervision. The IRB approach requires banks to assign a fixed LGD figure to particular classes of credit exposures.

3.2.3 Pay-offs of suppliers

We assume that inputs are project-specific and if suppliers choose not to deliver at date $\tau = 1$ they have to sell the inputs at a discount c elsewhere.[71] Rochet and Vives (2001) use a similar formulation where they interpret a fixed foreclosure cost c as a reputation cost of fund managers due to *bad judgement*. Such an interpretation would be applicable to our model should instead of a continuum of suppliers we would assume a continuum of unsecured creditors (e.g. short term commercial paper investors).

Let us also assume that, as of $\tau = 1$, suppliers expect to receive a small fixed pay-off in case of default at $\tau = 2$, which for notational convenience is set equal to zero.[72] Alternatively, one could consider suppliers' beliefs, regarding their pay-offs in case of default at $\tau = 2$, as endogenously determined on the basis of their signals and conditional on success of the debt-exchange offer at $\tau = 1$. Yet, this would bring undue complication in the model given that what we intend to capture here is suppliers' incentive to avoid the cost of not selling their inputs to the monopsonist firm, or extending credit to an insolvent firm. This assumption is also consistent with empirical evidence. White (1983), for example, observes that unsecured creditors receive little or no pay-off in liquidation.[73]

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[71] We use that assumption in order to avoid the complication of explicitly building the term structure of credit spreads into the model or arbitrarily assume a gross rate of return ($r > 1$) at $\tau = 1$ for every dollar of credit extended by suppliers to the firm at $\tau = 1$.

[72] In a similar setting, Rochet and Vives (2001) use a payoff equal to zero assuming that this is what a fund manager would get for rollovering a credit exposure to an entity that has subsequently defaulted.

[73] In a sample of 178 liquidated firms White (1983) finds that the average payoff rates to unsecured creditors is approximately 2.5%. Nevertheless, for firms reorganising under Chapter 11 proceedings the payoff rates are above 32%. He also argues that some unsecured claims such as trade creditor claims are generally not covered by subordination agreements and rank at the bottom of the seniority ranking.

If there is no default both at $\tau = 1$ and $\tau = 2$, suppliers who restructured are expected to receive an amount a_s per unit of inputs supplied, which is higher than what they would get should they had refused to restructure, i.e. $(1 - c)$. Given the above, suppliers' pay-off function looks as follows:

Suppliers	*Default at* $\tau = 1$	*Default at* $\tau = 2$	*No Default*
Accept Offer	1-c	*0*	a_s
Reject Offer	1-c	1-c	1-c

where $0 < c < 1$ and $1 - c < a_s$.

3.2.4 Information

We assume that the minimum proportion r of required inputs, the level of bank debt B and the aggregate claims by firm's suppliers as well as the level of operating expenses C and the cost c of selling the goods in the outside market are common knowledge among agents. We also assume that prior beliefs about the return $\widetilde{R}$ of the firm's risky project have an improper prior distribution.[74]

At $\tau = 1$, creditors receive private noisy signals about the return of the firm's risky project. Those signals constitute the only information available to creditors about the economic value of firm's investment. Let y be the signal observed by the bank, which is of the following form:

$$y = R + \nu\varepsilon$$

[74] Improper priors allow the analysis to focus exclusively on agents' updated beliefs conditional on their private signals, without taking into account the information contained in the prior distribution. In any case, our results with the improper prior can be seen as the limiting case as the information in the prior density goes to zero. See Hartigan (1983) for a discussion of improper priors, and Morris and Shin (2000) for a discussion of the latter point.

where, $\nu > 0$ is a constant and ε is a normal random variable with zero measure and unit variance, density function $g(\cdot)$ and is independent of R. We denote by $G(\cdot)$ the cumulative distribution function of $g(\cdot)$. Also at $\tau = 1$ each supplier i privately observes the following signal:

$$x_i = R + \sigma\varepsilon_i$$

where, $\sigma > 0$ is a constant and $\{\varepsilon_i\}$ are independent, identically distributed normal random variables, independent of ε, with zero measure, unit variance and density function denoted by $f(\cdot)$. We also denote by $F(\cdot)$ the cumulative distribution function of $f(\cdot)$.

Rational creditors will use their noisy private signals in order to form beliefs about the return $\widetilde{R}$ of the firm's risky project and to infer the beliefs of other creditors. Thus, a creditor will form beliefs not only about the underlying fundamentals of the firm, but also about the beliefs of other creditors, other creditors' beliefs about other creditors' beliefs and so on. This is because rational creditors will realise that their pay-offs do not only depend on the firm's fundamentals, but also on other creditors' whether or not to restructure.

At $\tau = 1$, the bank moves first and decides whether to increase its leverage to the firm. It does so conditional on its private signal y and taking into account the effect of its action on suppliers' behaviour . Suppliers then decide unilaterally whether to extend credit to the firm by delivering their goods at $\tau = 1$ for payment at $\tau = 2$. Their actions are conditional upon their private signals x_i and the commonly observed action by the bank to extend new credit to the firm or not. We consider the following definition:

Definition 2 *A creditor's strategy is a rule of action that maps each realization of its signal to one of two actions: to extend credit to the firm by accepting the offer, or to reject the offer.*

Suppliers strategies are determined at equilibrium by balancing the benefit of a particular strategy against the opportunity cost of that strategy, taking into account strategic complementarities. Given that individually they are unable to influence the solvency of the firm, suppliers fail to account for the effect of their individual decisions on the completion of firm's project. Yet, they are able to account for the effect of their actions as a whole. Thus, they foreclose whenever the expected benefit $(1 - c)$ of doing so is higher than the expected benefit of extending credit to the firm via the new contract. Similarly, the bank accepts to provide new credit to the firm if the total amount it expects to lose from doing so is less than the old loan (B) that it will definitely lose if it will reject the offer.

3.2.5 Equilibrium Concept

Strategic interactions are modeled here by using the methods of Carlsson and van Damme (1993) on *global games* as applied by Morris and Shin (2000, 2001), Rochet and Vives (2001) and Goldstein and Pauzner (2004). With the exception of the latter, the literature on global games typically builds on the assumption of *full strategic complementarities*. This is when an agent's incentive to undertake a certain action increases with the proportion of other agents undertaking the same action. However, this condition is not satisfied in case of debt-exchange offers that are subject to a minimum tendering rate. That is because, a supplier's incentive to tender is highest when the proportion of tendering suppliers reaches the minimum tendering rate – i.e. the tender offer just succeeds – while it drops thereafter given that the firm's liability increases with the proportion of suppliers who tender.

Nevertheless, as in Goldstein and Pauzner (2004), the property of *one-sided strategic complementarities* is still satisfied in our model. Given that a supplier has nothing to lose by accepting an debt-exchange offer that fails – i.e. if the minimum tendering rate is not attained – his incentive to tender increases with the proportion of other tendering suppliers, provided that the proportion of those who tender is relatively small. Thus, given one-sided strategic complementarities, we can borrow the uniqueness result of Goldstein and Pauzner and apply it to our benchmark case with no bank in the game. Then, by introducing a bank in the restructuring and by taking the bank's strategy as given, we can show that suppliers follow a unique Bayesian equilibrium strategy, where the debt-exchange offer succeeds if and only if the actual investment return of the firm is above a certain threshold.

In particular, we suppose that both types of creditors – i.e. suppliers and the bank – follow trigger strategies around critical signal levels x^* and y^* respectively. That is, if the bank observes a signal y that is higher than y^* then, it accepts to extend new credit, otherwise it rejects the offer and the firm is liquidated. Similarly, a supplier delivers his inputs to the firm, in exchange of new claims payable next period, if and only if he observes a signal higher than x^*.

However, we consider a sequential move game where the bank moves first by responding to the debt exchange offer conditionally on other creditors' actions.[75] Conse-

[75] We may justify such a first mover advantage by the bank on the grounds that banks typically have credit policies in place to deal with the situation of financial distress by a debtor. Such policies are less likely to be in place by small creditors whose bargaining power to enforce them would anyway be limited.

quently, an equilibrium here, if any, is a pair (x^*, y^*) where the strategy x^* that is chosen by suppliers takes into account the information that is conveyed in the restructuring decision by the bank, which then internalises suppliers response in its restructuring strategy y^*. In order to derive a closed form solutions for the equilibrium strategies x^* and y^*, we are assuming that the relative precision in the bank's signal $\left(\frac{1}{v}\right)$ relative to that of small creditors $\left(\frac{1}{\sigma}\right)$ is such that $\frac{v}{\sigma} \to 0$. As of Corsetti *et al.* (2001), that assumption is without loss of generality and is imposed solely for analytical tractability.

In order to derive suppliers' equilibrium in trigger strategies, we need first to solve for the critical level R^* of the actual investment return above which proportion of suppliers higher than the minimum tendering rate r accept the debt-exchange offer. We prove the following lemma.

Lemma 4 *Given suppliers' equilibrium in trigger strategies x^*, the critical level of investment return R^* above which the debt-exchange offer succeeds, is given by* $R^* = x^* - \sigma F^{-1}(1-r)$

Proof *See appendix* ∎

In order to assess the role of banks in the resolution of financial distress and how the terms of restructuring impact on creditors' strategies, we consider the benchmark case with no bank in the game where, at the same time, we control for money-effects of bank's involvement in the restructuring. That is, we analyse the situation where suppliers respond to the debt-exchange offer conditionally only on their signals, without observing any restructuring action by the bank.

3.3 Equilibrium with no bank in the game

Let us think of a situation where at $\tau = 1$ the firm does not face a liquidity demand C, but is has outstanding a total amount of debt $B' = B + C$, which is repayable at $\tau = 2$. In addition, if the debt-exchange offer to suppliers succeeds then, the new claims become junior to the debt amount B' and repayable at $\tau = 2$. Assuming that suppliers follow trigger strategies around a critical signal level x^{**}, we can prove the following lemma.

Lemma 5 *Suppose that supplier i observes a signal $x_i = x^{**}$. If there is no liquidation of the firm at $\tau = 1$ then, i supplier's belief about the proportion l^{**} of other suppliers receiving a signal lower than his is given by $l^{**} = \frac{(1-r)}{2}$.*

Proof *See appendix* ■

The critical signal level x^{**}, above which suppliers accept the debt-exchange offer, solves the following equation:

$$\begin{aligned} &(1-c)\Pr\left(R < R^{**} \mid x_i = x^{**}\right) + \\ &+a_s \Pr\left(R > B' + a_s(1-l) \mid x_i = x^{**}\right) = (1-c) \end{aligned} \tag{3.1}$$

where $B' = B + C$, R^{**} is defined as in lemma (4), i.e. $R^{**} = x^{**} - \sigma F^{-1}(1-r)$, and l is the proportion of suppliers who reject the debt-exchange offer.

Proposition 7 *If there is no bank in the game, then under a minimum tendering rate r, suppliers' equilibrium in trigger strategies x^{**} is given by:*

$$x^{**} = (B+C) + a_s\left(\frac{1+r}{2}\right) + \sigma F^{-1}\left[\frac{(1-r)(1-c)}{a_s}\right] \tag{3.2}$$

Proof *See appendix* ∎

Not surprisingly, x^{**} increases in the leverage factors $(B + C)$ and $a_s\left(\frac{1+r}{2}\right)$. Moreover, x^{**} depends on an *asymmetric information premium* – i.e. the third term in (3.2) – which can either be positive or negative depending on the characteristics of the debt-exchange offer, as well as on suppliers' outside option $(1 - c)$ and the extent of noise in suppliers' signals σ.

In particular, for $r \leq 1 - \frac{a_s}{2(1-c)}$, the asymmetric information premium becomes positive as a result of a standard co-ordination problem among suppliers. Otherwise, suppliers are willing to accept a discount in their signals before rejecting the offer, meaning that they become less sensitive to bad news about the firm. That is because, a high minimum tendering rate implies that a debt-exchange offer succeeds only if there is a high level of consensus among suppliers that the firm is solvent. Consequently, by *free riding* on average opinion, a supplier accepts the offer more easily given that he has nothing to lose by accepting an offer that subsequently fails. Similarly, the incentive to free ride on average opinion increases in the debt-exchange ratio a_s. That is because, the higher the a_s the more a supplier's decision to tender is driven more by income considerations – i.e. by what he receives in case of no default – rather than his private information information. Thus, the higher the debt-exchange ratio the lower the asymmetric information premium.

In the following section we consider the case where restructuring decisions by suppliers are sequential to the action by the bank. By taking the form of the game as given and the characteristics of the debt-exchange offer as exogenously determined, we focus on the impact of asymmetric information on creditors' strategies and on how the bank's restructuring strategy relates to the strategy of other creditors. However, we stop short of considering what an optimal debt-exchange offer might be, which is postponed for future research.[76]

3.4 Sequential move with bank in the game

Let us now consider the case where the bank acts as a Stackelberg leader in the restructuring game and its action is publicly observed by the suppliers, affecting their beliefs and, possibly, their willingness to tender. We solve first for suppliers' optimal trigger strategy x^*, taking the bank's strategy y^* as given. By backward induction, we then solve for bank's optimal trigger strategy y^* and, with equilibrium (x^*, y^*) in hand, we discuss implications of bank's involvement in the restructuring for the outcome of the debt-exchange offer.

3.4.1 Suppliers' strategy

We suppose that the bank, having followed a trigger strategy y^*, has accepted the debt restructuring offer. That is, we assume that the bank has observed a signal y greater than y^* and let x^*, in that case, be suppliers' optimal trigger strategy. Given that suppliers are now able to filter information out of bank's restructuring action, their equilibrium strategy is expected to be different from that under the benchmark case with no bank in the game.

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76 In practice, the tender offer may be optimally set by the bank as part of its contingent offer to extend a new loan. It may be also set by bankruptcy law in order to maximise some measure of social welfare. Moreover, in the context of sovereign debt and the issue of bonds under New York Law containing collective action clauses (CACs), it is the debtor country that decides upon the minimum tendering rate. Yet analysing those issues here would require another layer of complexity in our model that would take into account incentives at the initial contracting stage. Given the current level of complexity in our model, we postpone such an analysis for future research.

But before turning to solve for suppliers' strategy x^*, we need to prove first the following lemma.

Lemma 6 *If there is no liquidation of the firm at $\tau = 1$ and a supplier observes a private signal $x_i = x^*$ then, his belief about the proportion l^* of other suppliers receiving a signal lower than x^* is given by*

$$l^* = \frac{\int_{-\infty}^{F^{-1}(1-r)} F\left(\frac{-\sigma t + x^* - y^*}{v}\right) F(t) f(t)\, dt}{(1-r) F\left(\frac{x^*-y^*}{\sqrt{\sigma^2+v^2}}\right)}$$

where, for the limiting case where $\frac{v}{\sigma} \to 0$, it becomes

$$l^* = \begin{cases} \left(\frac{1}{2}\right)^3 \frac{1}{(1-r)F\left(\frac{x^*-y^*}{\sigma}\right)} & \text{if } r < 0.5 \\ \frac{1}{2}\frac{(1-r)}{F\left(\frac{x^*-y^*}{\sigma}\right)} & \text{if } r \geq 0.5 \end{cases}$$

Proof *See appendix* ■

We now solve for suppliers' equilibrium in trigger strategies, focusing on the case where $\frac{v}{\sigma} \to 0$. Although this is without loss of generality, we could apply numerical methods to derive equilibria for arbitrary values of relative precision of $\frac{v}{\sigma}$ of suppliers' signals relative to that of the bank, given that most of our results are also derived generically.

Conditional on bank's positive response to the debt exchange offer, the critical signal level x^* solves the following equation:

$$\begin{gathered}(1-c)\Pr(R < R^* \mid y > y^*) + \\ a_s \Pr(R > B + C + a_s(1-l^*) \mid y > y^*) = (1-c)\end{gathered} \tag{3.3}$$

where the probability terms in (3.3) are taken conditional on signal $x_i = x^*$ and l^* is given by lemma 6. The first term on the LHS of expression (3.3) corresponds to the situation where the debt-exchange offer is not successful, regardless of acceptance by the bank of the debt restructuring offer. If this is the case then, suppliers need to sell their goods elsewhere and face a discount c. The second term refers to the situation where the debt-exchange offer succeeds but there is default at $\tau = 2$ because of insufficient proceeds from the project. Finally, the last term on the LHS corresponds to the situation where the project generates sufficient proceeds at $\tau = 2$ to repay all creditors. The RHS of (3.3) is simply the price that a supplier will get by selling his inputs elsewhere. We prove the following proposition.

Proposition 8 *If suppliers face a debt-exchange offer with minimum tendering rate r and debt-exchange ratio a_s then, conditional on bank's strategy y^*, suppliers' equilibrium in trigger strategies x^* satisfies the following:*

Case 1: If $x^* \geq (B+C) + a_s(1-l^*)$ and $r < 0.5$ then, suppliers optimal strategy x^* solves

$$F\left(\frac{x^* - y^*}{\sigma}\right) = \frac{a_s}{2(1-c)} \tag{3.4}$$

Case 2: If $x^* < (B+C) + a_s(1-l^*)$ and $r < 0.5$ then, suppliers optimal strategy x^* solves

$$a_s F\left(\frac{x^* - (B+C) - a_s(1-l^*)}{\sigma}\right) = (1-c) F\left(\frac{x^* - y^*}{\sigma}\right) \tag{3.5}$$

Case 3: If $r \geq 0.5$ then, suppliers optimal strategy x^* solves

$$a_s F\left(\frac{x^* - (B+C) - a_s(1-l^*)}{\sigma}\right) = (1-c)\left[F\left(\frac{x^* - y^*}{\sigma}\right) - r\right] \tag{3.6}$$

where $l^*(x^*, y^*)$ is given by lemma 6.

Proof *See appendix* ■

Later, when in proposition 10 we solve for equilibria of the form (x^*, y^*), we verify whether the solution for x^* in cases 1 and 2 do in fact satisfy the respective inequalities on x^*. Moreover, by fixing an equilibrium selection of x^* that is continues in bank's strategy y^*, equations (3.4), (3.5) and (3.6) define a unique optimal strategy x^*. We also consider the following corollary to proposition 8.

Corollary 3 *For a minimum tendering rate $r \geq 0.5$, suppliers' equilibrium in trigger strategies x^* satisfies the inequality $x^* < B + C + a_s (1 - l^*)$, where l^* is given by lemma 6.*

Proof *It follows immediately from case 1, in proposition 8, and the fact that $a_s > 1 - c$* ■

Corollary 3 implies that, for a minimum tendering rate $r \geq 0.5$, suppliers are willing to accept the debt-exchange offer even if their private information suggests that the firm is insolvent. This is due to free-riding on average opinion of other creditors and the fact that, if the restructuring offer succeeds then, there is some consensus among other creditors that the firm is solvent.

Let us now solve for bank's equilibrium y^* in trigger strategies where the bank internalises the impact of its action on suppliers' strategies.

3.4.2 Bank's strategy

Conditional on signal y, return R is normally distributed with mean y and standard deviation v. As a result, the bank is able to form beliefs about the proportion of suppliers that will restructure their claims. Assuming that the realised sample distribution of suppliers is always the common distribution of suppliers' signals and focusing on the limiting case where $\frac{v}{\sigma} \to 0$, we consider the following lemma:

Lemma 7 *Conditional on signal $y = y^*$ and on success of a debt-exchange offer with minimum tendering rate $r < 0.5$, bank's belief about the proportion l^b of suppliers who reject the debt-exchange offer is given by: $l^b = F\left(\frac{x^*-y^*}{\sigma}\right)$.*

Proof *See appendix* ∎

Given that rejection by the bank would lead to default by the firm and loss of the original loan amount B, bank's trigger point y^* solves the following equation:

$$-B\Pr\left(R < R^* \mid y = y^*\right) - D\Pr\left(R^* < R < B + C + a_s\left(1 - l^b\right) \mid y = y^*\right) = -B \tag{3.7}$$

where, $D \equiv LGD \times (B + C)$ and, from Section 3.2.2, $LGD > \frac{B}{B+C}$. The first term on the LHS of (3.7) captures the conditional feature of bank's acceptance: Should, the bank agree to provide a new loan C, but proportion of suppliers higher than the critical level $(1 - r)$ reject the offer, there is default at $\tau = 1$ and the bank loses only the original loan amount B. The second term refers to losses that the bank would incur in the situation where default takes place at $\tau = 2$ and both the original and the new loan become impaired. However, in case of no-default at $\tau = 2$ the bank loses nothing and, essentially, faces a *clear exit* from the firm. Should, however, the bank refuse to extend a new loan then, it bears a certain loss of $-B$, which appears on the RHS of equation (3.7). We prove the following proposition.

Proposition 9 *If suppliers follow a trigger strategy x^* then, for the limiting case where $\frac{v}{\sigma} \to 0$, bank's equilibrium in trigger strategies y^* is determined by*

$$y^* = (B + C) + a_s F\left(\frac{y^* - x^*}{\sigma}\right) - vF^{-1}\left(\frac{B}{D}\right) \tag{3.8}$$

where x^* is determined by proposition8, D is the expected loss-given-default on the total credit exposure $(B + C)$ and a_s is the debt-exchange ratio under the debt-exchange offer.

Proof *See appendix* ■

From equation (3.8) and for a given strategy x^* by suppliers, we can easily verify that there is a uniquely determined optimal strategy y^* by the bank. That is illustrated in the following figure.

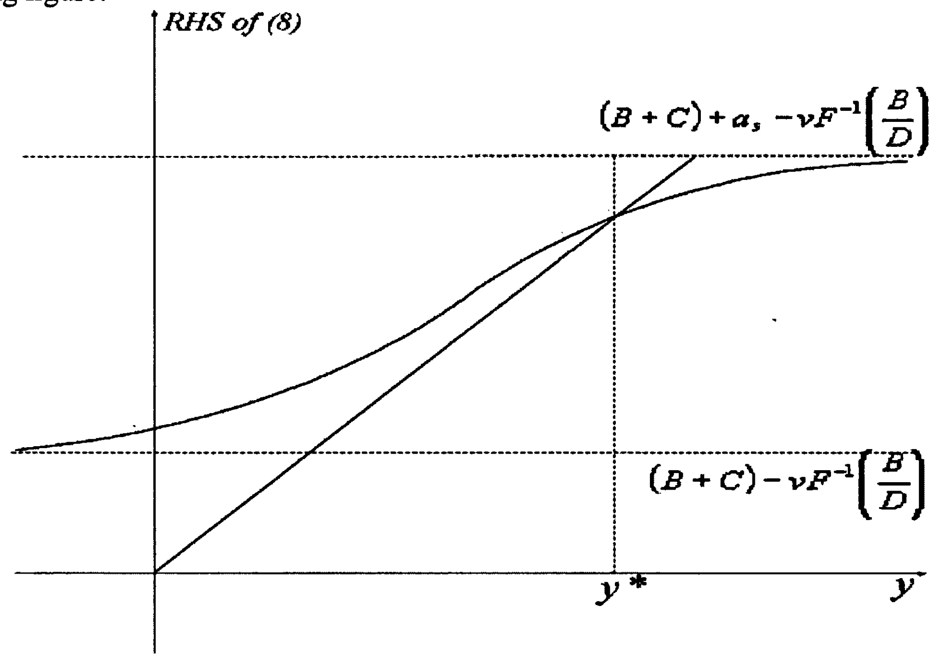

Fig. 3.1. Bank's equilibrium strategy y^* for given x^*.

From equation (3.8) it also follows that the more credit C the bank is asked to provide at $\tau = 1$ and the higher the loss-given-default (LGD) associated with default at $\tau = 2$, the higher the bank's critical signal level y^*. Furthermore, y^* also increases with the debt-exchange ratio a_s of the offer to suppliers, given that the higher it is the more leveraged the firm becomes at $\tau = 2$, if the offer succeeds.

3.4.3 Equilibrium

Equilibria of the form (x^*, y^*) can be calculated by solving simultaneously for x^* and y^* in propositions 8 and 9.

Proposition 10 *If suppliers face a debt-exchange offer with minimum tendering rate r and debt-exchange ratio a_s then, there is a unique equilibrium (x^*, y^*) in suppliers' and bank's strategies if and only if $r < 0.5$ and $\frac{a_s}{2(1-c)} \geq F\left(\frac{2a_s^2(1-r)-(1-c)^2}{4\sigma(1-r)(1-c)}\right)$, where x^* and y^* are given by*

$$x^* = (B + C) + a_s\left(1 - \tfrac{a_s}{2(1-c)}\right) + \sigma F^{-1}\left(\tfrac{a_s}{2(1-c)}\right)$$
$$y^* = (B + C) + a_s\left(1 - \tfrac{a_s}{2(1-c)}\right) - vF^{-1}\left(\tfrac{B}{D}\right)$$

Otherwise, there are multiple equilibria of the form (x^*, y^*), such that

$$y^* = (B + C) + a_s F\left(\frac{y^* - x^*}{\sigma}\right) - vF^{-1}\left(\frac{B}{D}\right)$$

and

$$\begin{cases} y^* > x^* - \sigma F^{-1}\left(\frac{a_s}{2(1-c)}\right) & \text{if } r < 0.5 \\ y^* > x^* - \sigma F^{-1}\left(\frac{a_s}{2(1-c)} + r\right) & \text{if } r \geq 0.5 \end{cases}$$

with l^* given by lemma 6.

Proof *See appendix* ■

For $r < 0.5$, it is easy to verify that condition $\frac{a_s}{2(1-c)} \geq F\left(\frac{2a_s^2(1-r)-(1-c)^2}{4\sigma(1-r)(1-c)}\right)$ is generally satisfied if suppliers' outside option $(1-c)$ is not very close to the debt-exchange ratio a_s and the extent of noise in suppliers' signals is not very low.[77] Consequently, for $r < 0.5$, we consider as standard the unique-equilibrium case, while for completion we also allow for the possibility of multiple equilibria if suppliers information is very precise, or if their outside option is close to the debt-exchange ratio.

However, for $r \geq 0.5$, proposition 10 suggests that there are multiple equilibria of the form (x^*, y^*) where the strategy y^* by the bank is strictly decreasing in suppliers' strategy x^* and vice versa. In other words, if one class of creditors attains a high equilibrium strategy then, acceptance of the offer by that class is perceived as *good news* by the other who then decide to follow a lower strategy.

That effect is illustrated in figure 3.2 where the locus of equilibria of type (x^*, y^*) is shown with a bold line down to the point of intersection with the line $x^* - \sigma F^{-1}\left(\frac{a_s}{2(1-c)}\right)$. Figure 3.2 also illustrates the non-standard, multiple-equilibria case for $r < 0.5$ where, this time, the locus of equilibria is shown with a bold line down to the point of intersection with $x^* - \sigma F^{-1}\left(\frac{a_s}{2(1-c)} + r\right)$.

[77] Both conditions can easily hold here give that $\frac{a_s}{(1-c)} > 1$ and $\frac{v}{\sigma} \to 0$.

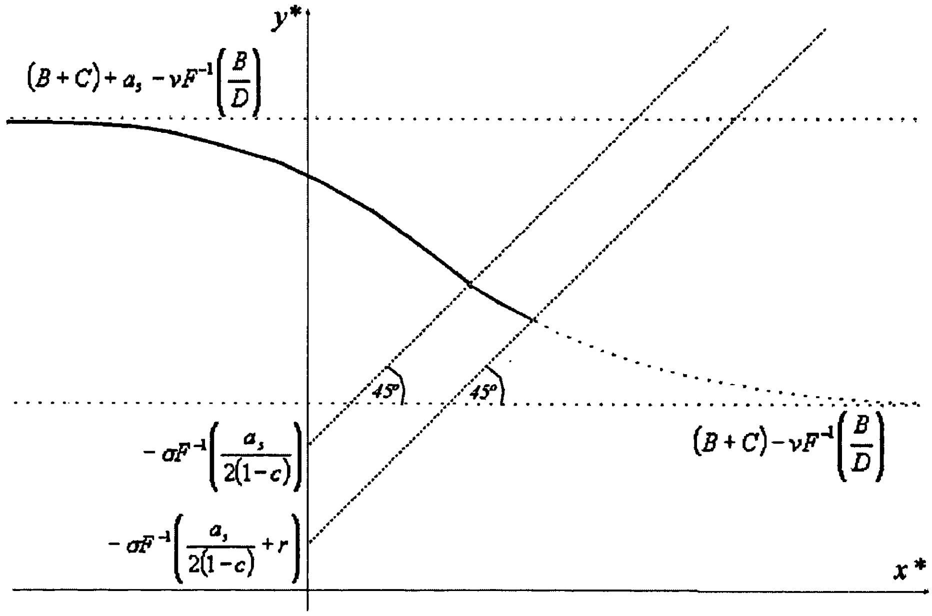

Fig. 3.2. Multiple equilibria (x^*, y^*) (bold line).

Proposition 10 relates, to some extent, to finance literature on *cascading* where a closely related problem is to establish conditions under which parties follow their own signals, or ignore their private information and follow others.[78] In particular, for relatively small minimum tendering rates, i.e. for $r < 0.5$, the information pooling that is attained by a debt-exchange offer is limited because success of the offer requires consensus that the firm is solvent by a small proportion of suppliers. Thus, for a small r, co-ordination problems tend to dominate herding incentives, leading to a unique equilibrium in creditors' strategies, which is common in the global game literature.

However, for a relatively high minimum tendering rates, herding tends to dominate co-ordination problems and creditors rely more on average opinion – as reflected in the out-

78 See, for example, Khanna (1998).

come of the debt-exchange offer and bank's decision to restructure – rather than on their private information about the solvency of the firm. In equilibrium, herding incentives are common knowledge and the bank chooses its strategy in a way that accounts for suppliers' motive to play a low strategy if the bank plays a high equilibrium strategy, and vice versa. That leads to multiple equilibria, where the strategy that is followed by one class of creditors is decreasing in the strategy that is followed by the other, as shown in figure 3.2.

3.5 Empirical implications and extensions

With equilibria of type (x^*, y^*) in hand, we may now assess the impact of bank's conditional concession on the outcome of the debt-exchange offer. We prove the following proposition.

Proposition 11 *If suppliers face a debt-exchange offer with minimum tendering rate* $r \geq 0.5$ *then, bank's involvement in the restructuring induces suppliers to accept the offer more easily compared to the situation with no bank in the game.*

Proof *See appendix* ■

In the real world, minimum tendering rates in debt-exchange offers typically specify qualified majority voting of 75% or above.[79] Consequently, proposition 11 is consistent with empirical evidence by Mooradian and Ryan (2004), James (1995, 1996) and Gilson, Kose and Lang (1990) supporting the proposition that banks' participation in debt

[79] Also, under the G10 model of collective action clauses (CACs), a minimum tendering rate of 75% is specified for altering the financial terms of bonds, while private sector associations typically propose a minimum tendering rate close to 85%.

restructuring transactions facilitates debt-exchange offers and increases the likelihood of out-of-court debt restructuring.[80] Moreover, proposition 11 implies that, for relatively high minimum tendering rates, all equilibria under bank's involvement in the restructuring imply resolution of financial distressed at lower levels of firm's fundamentals, compared to the situation with no bank in the game. Thus, we derive the following corollary.

Corollary 4 *If suppliers face a debt-exchange offer with minimum tendering rate* $r \geq 0.5$ *then, bank's involvement in the restructuring leads to resolution of financial distress at lower levels of firm's fundamentals than with no bank in the game.*

Proof *It follows immediately from lemma 4 and proposition 11* ■

However, one could possibly question whether or not it is desirable for a debt restructuring offer to succeed more easily and, in that respect, whether the impact of a bank's involvement in the restructuring is warranted from a welfare perspective. If herding, for example, induces creditors to become less sensitive to bad news about the firm then, debt restructuring may succeed regardless of the going concern value of the firm being lower than the extra credit it receives to overcome short term liquidity constraints. In that case, all creditors would be better off if the bank had no role to play in the restructuring and the firm had failed. By taking a step backwards, we could then argue that this may have an adverse impact on the firm's cost of capital at the initial contracting stage with counterparties. However, this is an issue that we are not able to address in this model, where we focus exclusively on the restructuring game, and we postpone it for future research.

[80] This result also relates to Shleifer and Vishny (1986) who argue that, in the context of take-over bids, the existence of a large block holder ameliorates hold-out problems among a target firm's shareholders.

Also from an empirical perspective, it would be of interest to analyse the characteristics of successful out-of-court debt restructurings that have subsequently led to an enhancement in credit quality of exposures by those who accepted to restructure. It would also be worth searching for patterns in the characteristics of out-of-court debt restructurings by firms that have subsequently failed, leading to aggravation of credit exposures by original claimholders. Such an analysis would possibly shed some light on factors that might lead to inefficient resolution, if not postponement, of financial distress, such as the extent of conditionality in debt-restructuring actions by certain creditors, or the characteristics of debt-exchange offers. But again, in order to draw predictions and address these issues from a theoretical perspective, we need a richer model where the pre-crisis stage is properly modeled and the structure of the restructuring game is allowed to affect incentives and the cost of capital that a firm faces in an out-of-crisis situation.

Finally, in the real-world, small lenders may "hold the others to ransom" and refuse to negotiate, pressuring instead for a recording of default. This is sometimes done with the intention to force a large creditor – typically a bank – to buy out positions of small creditors at better terms than those creditors would otherwise be able to obtain via a debt-exchange offer. This is also a possibility that our model does not capture. Thus, it would be worth extending the model to analyse liquidity crises where the bargaining power of creditors may not only depend on their size, but also on their voting power and how pivotal their roles are for the resolution of financial distress.

3.6 Conclusions

We developed a model of debt restructuring consistent with institutional characteristics of an out-of-court renegotiation of a firm's contractual obligations. Our results are consistent with empirical evidence, suggesting that banks play a potentially important role in facilitating the resolution of financial distress and catalysing success of debt-exchange offers. In particular, when a bank participates in the debt restructuring, debt-exchange offers succeed at lower levels of the firm's fundamentals compared to the situation where the bank has no role to play in the restructuring. In that sense, involvement by banks in debt restructuring transactions may reduce the possibility of inefficient liquidation due to co-ordination problems among other creditors.

By drawing a parallel between the simple balance sheet of the financially distressed firm in our model, and the capital account of a country during the onset of a financial crisis, we may discuss possible implications of our analysis on the *doctrine of catalytic finance* regarding the resolution of an international financial crisis. That doctrine rests on the premise that "under the right conditions, official assistance and private sector funding are strategic complements".[81] Until before the Argentine crisis in 2001, the doctrine of catalytic finance was the cornerstone of the official community's strategy towards capital account crisis. The main idea was that official assistance to a country that experiences a liquidity crisis could encourage other creditors to act in a way that mitigates the crisis. Since the Argentine default, the doctrine of catalytic finance is less appealing among the

[81] The September 2000 communique of the International Monetary and Finance Committee states that "the combination of catalytic official financing and policy adjustment should allow the country to regain full market access quickly".

G7. In particular, with respect to IMF interventions, there are voices nowadays arguing that the IMF's assistance to a country is exploited by private creditors and, in a sense, the two sources of funding become strategic substitutes during periods of financial crisis, rather than complements. Those voices are further reinforced by a moral-hazard story, according to which, the inability of the IMF to commit not always to intervene exacerbates the moral hazard problem on the part of the debtor country.

As regards the work-outs of financial distress, our analysis supports the presumption that official-sector assistance to a country in financial crisis could encourage other creditors to act in a way that facilitates resolution of the crisis. That is by alleviating co-ordination problems among creditors. However, our analysis also suggests that the extent of conditionality in creditors' actions – captured here through the minimum tendering rate – could affect the balance between co-ordination problems and herding incentives.

In recent years, the issue of conditionality under debt-restructuring transactions has mainly been discussed in relation to *collective action clauses* (CACs) and how they could ameliorate co-ordination and hold-out problems among creditors in the event of crisis.[82] Moreover, there has been some discussion on how sovereign borrowers would choose the level of majority voting under CACs. Thus, the focus has mainly been on how the level of conditionality affects, ex ante, the cost of borrowing given that, ex post, the level of majority voting required to alter the lending terms makes it more or less easy for debtors to force unfavorable credit terms on lenders.[83]

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82 See, for example, Eichengreen and Portes (1995), Buchheit and Gulati (2002), Kletzer (2003) and Haldane *et al.* (2004).

83 See also Eichengreen and Mody (2003).

To our knowledge, the possibility of free riding on average opinion by creditors, as a result of conditionality in restructuring actions, has been ignored. Our analysis suggests that there is a conditionality threshold beyond which co-ordination problems convert into herding problems. That, especially in the corporate sector, could possibly lead to short term resolutions of financial distress, at levels of fundamentals that, under full information, would be difficult to justify concessions by creditors. But, it can also be the case that, under asymmetric information, some degree of conditionality is warranted given that it allows creditors to make better informed decisions by basing their actions on the collective knowledge of other creditors. Thus, we may conclude that conditionality in the provision of financial assistance in case of crisis should be a balancing act.

3.A Appendix

3.A.1 Proof of Lemma 4

Conditional on actual investment return R, signal x_i is normally distributed with mean R and variance σ^2. Given that suppliers' signals $\{x_i\}$ are *i.i.d.*, the critical level of investment return R^*, below which rejection by suppliers generates default, is such that:

$$\Pr\left(x < x^* \mid R = R^*\right) = 1 - r$$

That is,

$$F\left(\frac{x^* - R^*}{\sigma}\right) = 1 - r$$

or

$$R^* = x^* - \sigma F^{-1}\left(1 - r\right)$$

Q.E.D.

3.A.2 Proof of Lemma 5

Given that suppliers' signals are *i.i.d.*, conditional on no default at $\tau = 1$ (i.e. $R > R^{**}$) and on signal x_i, i supplier's belief about the proportion l of other suppliers receiving a signal lower than his is defined as follows:

$$l = \Pr\left(x_j < x_i \mid R > R^{**}\right) \tag{3.9}$$

For $x_i = x^{**}$ equation (3.9) gives the rejection rate l^{**} that one expects to occur when he observes a signal equal to the critical signal level x^{**}:

$$l^{**} = \Pr\left(x_j < x^{**} \mid R > R^{**}\right) =$$

$$= \frac{\Pr\left(x_j < x^{**}, R > R^{**}\right)}{\Pr\left(R > R^{**}\right)} \tag{3.10}$$

Conditional on signal $x_i = x^{**}$, signal x_j is normal with mean x^{**} and variance $2\sigma^2$. Similarly R is also normal with mean x^{**} and variance σ^2. Moreover, conditional on $x_i = x^{**}$, x_j and R are correlated with covariance equal to σ^2. Thus, (x_j, R) is a bivariate normal distribution with mean $\boldsymbol{\mu} = (x^{**}, x^{**})'$ and variance/covariance matrix $\Sigma = \begin{bmatrix} 2\sigma^2 & \sigma^2 \\ \sigma^2 & \sigma^2 \end{bmatrix}$. From the definition of the multivariate normal distribution and given that $\Sigma^{-1} = \frac{1}{\sigma^2}\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ and $|\Sigma| = \sigma^4$, it is easy to show that l^{**} in equation (3.10) is given by the following expression:

$$l^{**} = \frac{\frac{1}{2\pi\sigma^2}\int_{-\infty}^{x^{**}}\int_{x^{**}-\sigma F^{-1}(1-r)}^{+\infty} \exp\left[-\frac{(x_j-R)^2+(x^{**}-R)^2}{2\sigma^2}\right] dR dx_j}{1-r} \tag{3.11}$$

By changing the order of integration in equation (3.11) and by applying the transformation $z = \frac{x_j - R}{\sigma}$, we get the following expression:

$$l^{**} = \frac{\frac{1}{\sqrt{2\pi\sigma^2}}\int_{x^{**}-\sigma F^{-1}(1-r)}^{+\infty}\left\{\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{x^{**}-R}{\sigma}} \exp\left[-\frac{z^2}{2}\right] dz\right\} \exp\left[-\frac{(R-x^{**})^2}{2\sigma^2}\right] dR}{1-r}$$

Let $w = \frac{R-x^{**}}{\sigma}$,

$$l^{**} = \frac{\frac{1}{\sqrt{2\pi}}\int_{-F^{-1}(1-r)}^{+\infty} \exp\left[-\frac{w^2}{2}\right] F\left(-w\right) dw}{1-r}$$

or

$$l^{**} = \frac{\int_{-F^{-1}(1-r)}^{+\infty} F\left(-w\right) f\left(w\right) dw}{1-r}$$

Let $w = -t$ and applying the fact that f is symmetric we finally get:

$$l^{**} = \frac{1}{2}\frac{\int_{-\infty}^{F^{-1}(1-r)} d\left[F\left(t\right)\right]^2}{1-r}$$

or

$$l^{**} = \frac{(1-r)}{2}$$

Q.E.D.

3.A.3 Proof of Proposition 7

By substituting $R^{**} = x^{**} - \sigma F^{-1}(1-r)$ and $l^{**} = \frac{(1-r)}{2}$ in equation (3.1) we get the following equation:

$$(1-c)\int_{-\infty}^{-F^{-1}(1-r)} f(z)\,dz + a_s \int_{\frac{B+C+a_s\frac{1+r}{2}-x^{**}}{\sigma}}^{+\infty} f(z)\,dz = (1-c)$$

or

$$a_s F\left(\frac{x^{**}-(B+C)-a_s\left(\frac{1+r}{2}\right)}{\sigma}\right) = (1-c)(1-r) \tag{3.12}$$

or

$$x^{**} = (B+C) + a_s\left(\frac{1+r}{2}\right) + \sigma F^{-1}\left(\frac{(1-c)(1-r)}{a_s}\right) \tag{3.13}$$

Q.E.D.

3.A.4 Proof of Lemma 6

Similar to the proof of lemma 5, if there is no liquidation of the firm at $\tau = 1$ (i.e. $R > R^*$ and $y > y^*$) then, if a supplier observes a private signal x_i, his belief about the proportion l of other suppliers receiving a signal lower than his is defined as follows:

$$l = \Pr(x_j < x_i \mid R > R^*, y > y^*) \tag{3.14}$$

For $x_i = x^*$ equation (3.14) gives the rejection rate l^* of the debt-exchange offer that one expects to occur when his signal is equal to the critical signal level x^*:

$$l^* = \Pr\left(x_j < x^* \mid R > R^*, y > y^*\right)$$

or

$$l^* = \frac{\Pr\left(x_j < x^*, R > R^*, y > y^*\right)}{\Pr\left(R > R^*\right)\Pr\left(y > y^*\right)} \tag{3.15}$$

From lemma 4, we already know that, conditional on a signal $x_i = x^*$, the probability term $\Pr(R > R^*)$ in the denominator of (3.15) is given by

$$\Pr\left(R > R^*\right) = (1 - r) \tag{3.16}$$

Also conditional on signal $x_i = x^*$, the second probability term in the denominator of (3.15) can be written as

$$\Pr\left(y > y^*\right) = \Pr\left(R - v\varepsilon > y^*\right)$$

or

$$\Pr\left(y > y^*\right) = \Pr\left(x^* - y^* > \sigma\varepsilon_i + v\varepsilon\right)$$

or

$$\Pr\left(y > y^*\right) = F\left(\frac{x^* - y^*}{\sqrt{\sigma^2 + v^2}}\right) \tag{3.17}$$

Regarding the probability term in the numerator of (3.15) then, conditional on signal $x_i = x^*$, signal x_j is normal with mean x^* and variance $2\sigma^2$. Similarly,conditional on $x_i = x^*$, the actual investment return R is normal with mean x^* and variance σ^2, while the signal y that is observed by the bank is normally distributed with mean x^* and variance $(v^2 + \sigma^2)$. Moreover, conditional on $x_i = x^*$, x_j, y and R are mutually correlated with all covariances equal to σ^2. Thus, the vector of random variables (x_j, y, R) follows a multivariate normal distribution with mean vector $\boldsymbol{\mu} = (x^*, x^*, x^*)'$ and variance/covariance matrix $\Sigma = \begin{bmatrix} 2\sigma^2 & \sigma^2 & \sigma^2 \\ \sigma^2 & v^2+\sigma^2 & \sigma^2 \\ \sigma^2 & \sigma^2 & \sigma^2 \end{bmatrix}$. From the definition of the multivariate normal distribution, given that $\Sigma^{-1} = \frac{1}{\sigma^2 v^2} \begin{bmatrix} v^2 & 0 & -v^2 \\ 0 & \sigma^2 & -\sigma^2 \\ -v^2 & -\sigma^2 & \sigma^2+2v^2 \end{bmatrix}$ and $|\Sigma| = \sigma^4 v^2$, it is easy to show that equations (3.15), (3.16) and (3.17) imply the following expression for l^*:

$$l^* = \frac{\frac{1}{\left(\sqrt{2\pi}\right)^3 \sqrt{\sigma^4 v^2}} \int_{-\infty}^{x^*} \left\{ \int_{y^*}^{+\infty} \left[\int_{R^*}^{+\infty} \exp\left(-\frac{v^2(x_j-R)^2+\sigma^2(y-R)^2+v^2(x^*-R)^2}{\sigma^2 v^2}\right) dR \right] dy \right\} dx_j}{(1-r)\, F\left(\frac{x^*-y^*}{\sqrt{\sigma^2+v^2}}\right)} \tag{3.18}$$

By changing the order of integration, (3.18) may easily become

$$l^* = \frac{\frac{1}{\left(\sqrt{2\pi}\right)^3 \sqrt{\sigma^4 v^2}} \int_{R^*}^{+\infty} \left\{ \int_{y^*}^{+\infty} \left[\int_{-\infty}^{x^*} \exp\left(-\frac{(x_j-R)^2}{2\sigma^2}\right) dx_j \right] \exp\left(-\frac{(y-R)^2}{2v^2}\right) dy \right\} \exp\left(-\frac{(R-x^*)^2}{2\sigma^2}\right) dR}{(1-r)\, F\left(\frac{x^*-y^*}{\sqrt{\sigma^2+v^2}}\right)}$$

By applying the transformation $z = \frac{x_j-R}{\sigma}$, l^* becomes

$$l^* = \frac{\frac{1}{\left(\sqrt{2\pi}\right)^2 \sqrt{\sigma^2 v^2}} \int_{R^*}^{+\infty} \left\{ \int_{y^*}^{+\infty} \left[\int_{-\infty}^{\frac{x^*-R}{\sigma}} f(z)\, dz \right] \exp\left(-\frac{(y-R)^2}{2v^2}\right) dy \right\} \exp\left(-\frac{(R-x^*)^2}{2\sigma^2}\right) dR}{(1-r)\, F\left(\frac{x^*-y^*}{\sqrt{\sigma^2+v^2}}\right)}$$

or

$$l^* = \frac{\frac{1}{\left(\sqrt{2\pi}\right)^2 \sqrt{\sigma^2 v^2}} \int_{R^*}^{+\infty} \left[\int_{y^*}^{+\infty} \exp\left(-\frac{(y-R)^2}{2v^2}\right) dy \right] F\left(\frac{x^*-R}{\sigma}\right) \exp\left(-\frac{(R-x^*)^2}{2\sigma^2}\right) dR}{(1-r)\, F\left(\frac{x^*-y^*}{\sqrt{\sigma^2+v^2}}\right)} \tag{3.19}$$

By applying the transformation $u = \frac{y-R}{v}$, l^* becomes

$$l^* = \frac{\frac{1}{\sqrt{2\pi\sigma^2}} \int_{R^*}^{+\infty} \left[\int_{\frac{y^*-R}{v}}^{+\infty} f(u)\, du \right] F\left(\frac{x^*-R}{\sigma}\right) \exp\left(-\frac{(R-x^*)^2}{2\sigma^2}\right) dR}{(1-r)\, F\left(\frac{x^*-y^*}{\sqrt{\sigma^2+v^2}}\right)}$$

or

$$l^* = \frac{\int_{R^*}^{+\infty} F\left(\frac{R-y^*}{v}\right) F\left(\frac{x^*-R}{\sigma}\right) \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(R-x^*)^2}{2\sigma^2}\right) dR}{(1-r)\, F\left(\frac{x^*-y^*}{\sqrt{\sigma^2+v^2}}\right)} \tag{3.20}$$

Let $w = \frac{R-x^*}{\sigma}$,

$$l^* = \frac{\int_{-F^{-1}(1-r)}^{+\infty} F\left(\frac{\sigma w + x^* - y^*}{v}\right) F(-w) f(w)\, dw}{(1-r) F\left(\frac{x^*-y^*}{\sqrt{\sigma^2+v^2}}\right)} \tag{3.21}$$

Applying the transformation $w = -t$ and the fact that f is symmetric – i.e. $f(-t) = f(t)$ – equation (3.21) becomes

$$l^* = \frac{\int_{-\infty}^{F^{-1}(1-r)} F\left(\frac{-\sigma t + x^* - y^*}{v}\right) F(t) f(t)\, dt}{(1-r) F\left(\frac{x^*-y^*}{\sqrt{\sigma^2+v^2}}\right)} \tag{3.22}$$

We consider the following two cases: **1)** $r < 0.5$ and **2)** $r \geq 0.5$.

Case 1: For $r < 0.5$ and considering the limiting case $\frac{v}{\sigma} \to 0$, equation (3.22) becomes

$$l^* = \frac{\int_{-\infty}^{0} F\left(\frac{-\sigma t + x^* - y^*}{v}\right) F(t) f(t)\, dt + \int_{0}^{F^{-1}(1-r)} F\left(\frac{-\sigma t + x^* - y^*}{v}\right) F(t) f(t)\, dt}{(1-r) F\left(\frac{x^*-y^*}{\sqrt{\sigma^2+v^2}}\right)}$$

or

$$l^* = \frac{1}{2} \frac{\int_{-\infty}^{0} d\left[F(t)\right]^2}{(1-r) F\left(\frac{x^*-y^*}{\sigma}\right)}$$

or

$$l^* = \left(\frac{1}{2}\right)^3 \frac{1}{(1-r) F\left(\frac{x^*-y^*}{\sigma}\right)}$$

Case 2: If $r \geq 0.5$ then, for $\frac{v}{\sigma} \to 0$, equation (3.22) becomes

$$l^* = \frac{1}{2} \frac{\int_{-\infty}^{F^{-1}(1-r)} d\left[F(t)\right]^2}{(1-r) F\left(\frac{x^*-y^*}{\sigma}\right)}$$

or

$$l^* = \frac{1}{2} \frac{(1-r)}{F\left(\frac{x^*-y^*}{\sigma}\right)} \tag{3.23}$$

Q.E.D.

3.A.5 Proof of Proposition 8

We first need to solve for the probability that the debt-exchange offer fails, conditional on signal $x_i = x^*$ and acceptance by the bank of the debt restructuring offer. Such a probability can be written as

$$\Pr(R < R^* \mid y > y^*) = \frac{\Pr(R < R^*, y > y^*)}{\Pr(y > y^*)} \tag{3.24}$$

where from the proof of lemma 6, in particular from equation (3.17), we know that conditional on signal $x_i = x^*$ the probability that the bank accepts the offer, i.e. $y > y^*$is given by

$$\Pr(y > y^*) = F\left(\frac{x^* - y^*}{\sqrt{\sigma^2 + v^2}}\right) \tag{3.25}$$

We prove the following lemma.

Lemma 8 *Conditional on signal $x_i = x^*$ and on acceptance by the bank of the debt restructuring offer, the probability that the debt-exchange offer fails is given by*

$$\Pr(R < R^* \mid y > y^*) = \frac{\int_{-\infty}^{-F^{-1}(1-r)} f(w)\, F\left(\frac{x^* - y^* + \sigma w}{v}\right) dw}{F\left(\frac{x^* - y^*}{\sqrt{\sigma^2 + v^2}}\right)}$$

where, for the limiting case where $\frac{v}{\sigma} \to 0$, it becomes

$$\Pr(R < R^* \mid y > y^*) = \begin{cases} 0 & \text{if } r < 0.5 \\ \frac{r}{F\left(\frac{x^* - y^*}{\sigma}\right)} & \text{if } r \geq 0.5 \end{cases}$$

Proof *Conditional on signal $x_i = x^*$, the vector of random variables (R, y) is a bivariate normal distribution with mean vector $\mu = (x^*, x^*)'$ and variance/covariance matrix $\Sigma = \begin{bmatrix} \sigma^2 & \sigma^2 \\ \sigma^2 & v^2 + \sigma^2 \end{bmatrix}$. From the definition of the multivariate normal distribution and the fact* *that* $\Sigma^{-1} = \frac{1}{\sigma^2 v^2}\begin{bmatrix} v^2+\sigma^2 & -\sigma^2 \\ -\sigma^2 & \sigma^2 \end{bmatrix}$ *and* $|\Sigma| = \sigma^2 v^2$, *we can show that*

$$\Pr\left(R < R^*, y > y^*\right) = \frac{1}{\left(\sqrt{2\pi}\right)^2 \sqrt{\sigma^2 v^2}} \int_{-\infty}^{x^*-\sigma F^{-1}(1-r)} \int_{y^*}^{+\infty} \exp\left(-\frac{(R-x^*)^2}{2\sigma^2} - \frac{(R-y)^2}{2v^2}\right) dy \tag{3.26}$$

Considering first the transformation $w = \frac{R-x^*}{\sigma}$, *equation (3.26) becomes*

$$\Pr\left(R < R^*, y > y^*\right) = \int_{-\infty}^{-F^{-1}(1-r)} f(w) \left[\int_{y^*}^{+\infty} \frac{1}{\sqrt{2\pi}v} \exp\left(-\frac{(y-x^*-\sigma w)^2}{2v^2}\right) dy\right] dw$$

or

$$\Pr\left(R < R^*, y > y^*\right) = \int_{-\infty}^{-F^{-1}(1-r)} f(w) \left[\int_{y^*}^{+\infty} \frac{1}{\sqrt{2\pi}v} \exp\left(-\frac{(\sigma w+x^*-y)^2}{2v^2}\right) dy\right] dw \tag{3.27}$$

Let $z = \frac{y-x^*-\sigma w}{v}$. *Then (3.27) becomes*

$$\Pr\left(R < R^*, y > y^*\right) = \int_{-\infty}^{-F^{-1}(1-r)} f(w) \left(\int_{\frac{y^*-x^*-\sigma w}{v}}^{+\infty} f(z)\, dz\right) dw$$

or

$$\Pr\left(R < R^*, y > y^*\right) = \int_{-\infty}^{-F^{-1}(1-r)} f(w)\, F\left(\frac{x^*-y^*+\sigma w}{v}\right) dw \tag{3.28}$$

Consequently, from equations (3.24), (3.25) and 3.28, *the probability that the debt-exchange offer fails, conditional on the bank accepting the debt restructuring offer, is given by*

$$\Pr\left(R < R^* \mid y > y^*\right) = \frac{\int_{-\infty}^{-F^{-1}(1-r)} f(w)\, F\left(\frac{x^*-y^*+\sigma w}{v}\right) dw}{F\left(\frac{x^*-y^*}{\sqrt{\sigma^2+v^2}}\right)} \tag{3.29}$$

For us consider the following cases: 1) $r < 0.5$ *and 2)* $r \geq 0.5$.

Case 1: If $r < 0.5$ *then, for* $\frac{v}{\sigma} \to 0$, *equation (3.28) becomes*

$$\Pr\left(R < R^*, y > y^*\right) = 0$$

Case 2: If $r \geq 0.5$ then, equation (3.28) can be written as

$$\Pr\left(R < R^*, y > y^*\right) = \int_{-\infty}^{0} f(w)\, F\left(\frac{x^* - y^* + \sigma w}{v}\right) dw + \int_{0}^{-F^{-1}(1-r)} f(w)\, F\left(\frac{x^* - y^* + \sigma w}{v}\right)$$

where, for $\frac{v}{\sigma} \longrightarrow 0$, it becomes

$$\Pr\left(R < R^*, y > y^*\right) = \int_{0}^{-F^{-1}(1-r)} f(w)\, dw$$

or

$$\Pr\left(R < R^*, y > y^*\right) = r$$

Summarising, for the limiting case $\frac{v}{\sigma} \longrightarrow 0$, equation (3.29) becomes

$$\Pr\left(R < R^* \mid y > y^*\right) = \begin{cases} 0 & \text{if } r < 0.5 \\ \frac{r}{F\left(\frac{x^* - y^*}{\sigma}\right)} & \text{if } r \geq 0.5 \end{cases} \tag{3.30}$$

■

Let us now solve for the second term in (3.3), namely for the probability that, conditional on signal $x_i = x^*$ and on acceptance by the bank of the restructuring offer, the debt-exchange offer succeeds and the firm is solvent, i.e. it repays all its debt at $\tau = 2$. Such a probability can be written as

$$\Pr\left(R > B + C + a_s\left(1 - l^*\right) \mid y > y^*\right) = \frac{\Pr\left(R > B + C + a_s\left(1 - l^*\right), y > y^*\right)}{\Pr\left(y > y^*\right)} \tag{3.31}$$

We prove the following lemma.

Lemma 9 *Conditional on signal $x_i = x^*$ and on acceptance of the restructuring offer by the bank, the probability of no default at $\tau = 2$ is given by*

$$\Pr\left(R > B + C + a_s\left(1 - l^*\right), y > y^*\right) =$$
$$= \int_{\frac{B + C + a_s(1 - l^*) - x^*}{\sigma}}^{+\infty} f(w)\, F\left(\frac{x^* - y^* + \sigma w}{v}\right) dw$$

where, for the limiting case where $\frac{v}{\sigma} \to 0$, it becomes

$$\Pr\left(R > B + C + a_s\left(1 - l^*\right) \mid y > y^*\right) = \begin{cases} \frac{1}{2F\left(\frac{x^*-y^*}{\sigma}\right)} & \text{if } x^* \geq B + C + a_s\left(1 - l^*\right) \\ \frac{F\left(\frac{x^*-(B+C)-a_s(1-l^*)}{\sigma}\right)}{F\left(\frac{x^*-y^*}{\sigma}\right)} & \text{if } x^* < B + C + a_s\left(1 - l^*\right) \end{cases}$$

with l^* given in lemma 6.

Proof *As in lemma 8, the probability of no default at $\tau = 2$ is given by.*

$$\begin{gathered} \Pr\left(R > B + C + a_s\left(1 - l^*\right), y > y^*\right) = \\ = \frac{1}{\left(\sqrt{2\pi}\right)^2\sqrt{\sigma^2 v^2}} \int_{B+C+a_s(1-l^*)}^{+\infty} \exp\left(-\frac{(R-x^*)^2}{2\sigma^2}\right)\left[\int_{y^*}^{+\infty} \exp\left(-\frac{(y-R)^2}{2v^2}\right) dy\right] dR \end{gathered} \tag{3.32}$$

Considering the transformation $w = \frac{R-x^*}{\sigma}$ equation (3.32) becomes

$$\begin{gathered} \Pr\left(R > B + C + a_s\left(1 - l^*\right), y > y^*\right) = \\ = \int_{\frac{B+C+a_s(1-l^*)-x^*}{\sigma}}^{+\infty} f(w)\left[\int_{y^*}^{+\infty} \frac{1}{\sqrt{2\pi}v} \exp\left(-\frac{(y-x^*-\sigma w)^2}{2v^2}\right) dy\right] dw \end{gathered} \tag{3.33}$$

By setting $z = \frac{y-x^*-\sigma w}{v}$, equation (3.33) becomes

$$\begin{gathered} \Pr\left(R > B + C + a_s\left(1 - l^*\right), y > y^*\right) = \\ = \int_{\frac{B+C+a_s(1-l^*)-x^*}{\sigma}}^{+\infty} f(w)\, F\left(\frac{x^*-y^*+\sigma w}{v}\right) dw \end{gathered} \tag{3.34}$$

Consequently, from equations (3.25), (3.31) and 3.34, the probability of no default at $\tau = 2$, conditional on the bank accepting the debt restructuring offer, is given by

$$\Pr\left(R > B + C + a_s\left(1 - l^*\right) \mid y > y^*\right) = \frac{\int_{\frac{B+C+a_s(1-l^*)-x^*}{\sigma}}^{+\infty} f(w)\, F\left(\frac{x^*-y^*+\sigma w}{v}\right) dw}{F\left(\frac{x^*-y^*}{\sqrt{\sigma^2+v^2}}\right)} \tag{3.35}$$

Let us consider the following two cases: 1) $x^* \geq B + C + a_s\left(1 - l^*\right)$ and 2) $x^* < B + C + a_s\left(1 - l^*\right)$.

Case 1: If $x^* \geq B + C + a_s\left(1 - l^*\right)$ then, for $\frac{v}{\sigma} \to 0$, equation (3.34) becomes

$$\begin{gathered} \Pr\left(R > B + C + a_s\left(1 - l^*\right), y > y^*\right) = \\ = \int_{\frac{B+C+a_s(1-l^*)-x^*}{\sigma}}^{0} f(w)\, F\left(\frac{x^*-y^*+\sigma w}{v}\right) dw + \int_0^{+\infty} f(w)\, F\left(\frac{x^*-y^*+\sigma w}{v}\right) dw \end{gathered}$$

or

$$\Pr\left(R > B + C + a_s\left(1 - l^*\right), y > y^*\right) = \frac{1}{2} \tag{3.36}$$

Case 2: If $x^* < B + C + a_s\left(1 - l^*\right)$ then, for $\frac{v}{\sigma} \to 0$, equation (3.34) becomes

$$\begin{aligned} &\Pr\left(R > B + C + a_s\left(1 - l^*\right), y > y^*\right) = \\ &= \int_{\frac{B+C+a_s(1-l^*)-x^*}{\sigma}}^{+\infty} f\left(w\right) dw \end{aligned}$$

or

$$\begin{aligned} &\Pr\left(R > B + C + a_s\left(1 - l^*\right), y > y^*\right) = \\ &= F\left(\frac{x^* - (B+C) - a_s(1-l^*)}{\sigma}\right) \end{aligned} \tag{3.37}$$

Summarising, for the limiting case $\frac{v}{\sigma} \to 0$, equation (3.35) becomes

$$\Pr\left(R > B + C + a_s\left(1 - l^*\right) \mid y > y^*\right) = \begin{cases} \frac{1}{2F\left(\frac{x^* - y^*}{\sigma}\right)} & \text{if } x^* \geq B + C + a_s\left(1 - l^*\right) \\ \frac{F\left(\frac{x^* - (B+C) - a_s(1-l^*)}{\sigma}\right)}{F\left(\frac{x^* - y^*}{\sigma}\right)} & \text{if } x^* < B + C + a_s\left(1 - l^*\right) \end{cases}$$

■

With lemmas 6, 8 and 9 in hand, we solve for suppliers' equilibrium in trigger strategies x^* for the limiting case where $\frac{v}{\sigma} \to 0$. We consider the following cases: **1)** $x^* \geq B + C + a_s\left(1 - l^*\right)$ and **2)** $x^* < B + C + a_s\left(1 - l^*\right)$, distinguish between $r < 0.5$ and $r \geq 0.5$.

Case 1 $x^* \geq B + C + a_s\left(1 - l^*\right)$.

Subcase 1.1: If $r < 0.5$ then, equation (3.3) becomes

$$F\left(\frac{x^* - y^*}{\sigma}\right) = \frac{a_s}{2\left(1 - c\right)} \tag{3.38}$$

Subcase 1.2: If $r \geq 0.5$ then, equation (3.3) implies that

$$F\left(\frac{x^* - y^*}{\sigma}\right) = r + \frac{a_s}{2\left(1 - c\right)} \tag{3.39}$$

But this case can be rejected because $r + \frac{a_s}{2(1-c)} > 1$.

Case 2 $x^* < B + C + a_s\left(1 - l^*\right)$.

Subcase 2.1: If $r < 0.5$ then, from equation (3.3) becomes

$$a_s F\left(\frac{x^* - (B+C) - a_s(1-l^*)}{\sigma}\right) = (1-c)\, F\left(\frac{x^* - y^*}{\sigma}\right) \tag{3.40}$$

Subcase 2.2: If $r \geq 0.5$ then, equation (3.3) becomes

$$a_s F\left(\frac{x^* - (B+C) - a_s(1-l^*)}{\sigma}\right) = (1-c)\left[F\left(\frac{x^* - y^*}{\sigma}\right) - r\right] \tag{3.41}$$

Q.E.D.

3.A.6 Proof of Lemma 7

Conditional on no default at $\tau = 1$ (i.e. $R > R^*$) and on signal y^*, bank's belief about the proportion l^b of suppliers that receive a signal lower than x^* (i.e. reject firm's offer) is defined as follows:

$$l^b = \Pr\left(x_j < x^* \mid R > R^*, y = y^*\right) =$$

$$= \frac{\Pr\left(x_j < x^*, R > R^*\right)}{\Pr\left(R > R^*\right)} \tag{3.42}$$

Conditional on $y = y^*$ signal x_j is normally distributed with mean y^* and variance $v^2 + \sigma^2$. Similarly, return R is also normal with mean y^* and variance v^2. Moreover, x_j and R are correlated with covariance v^2. Thus, conditional on y^*, (x_j, R) is a bivariate normal distribution with mean $\boldsymbol{\mu} = (y^*, y^*)'$ and variance/covariance matrix $\Sigma = \begin{bmatrix} v^2+\sigma^2 & v^2 \\ v^2 & v^2 \end{bmatrix}$.
From the definition of the multivariate normal distribution and the fact that
$\Sigma^{-1} = \frac{1}{v^2\sigma^2}\begin{bmatrix} v^2 & -v^2 \\ -v^2 & v^2+\sigma^2 \end{bmatrix}$ and $|\Sigma| = v^2\sigma^2$, it is easy to show that l^b in equation (3.42)

is given by the following expression:

$$l^b = \frac{\frac{1}{\left(\sqrt{2\pi}\right)^2\sqrt{\sigma^2 v^2}}\int_{-\infty}^{x^*}\left\{\int_{x^*-\sigma F^{-1}(1-r)}^{+\infty}\exp\left[-\left(\frac{(x_j-R)^2}{2\sigma^2}+\frac{(R-y^*)^2}{2v^2}\right)\right]dR\right\}dx_j}{\Pr\left(y^*-v\varepsilon > x^*-\sigma F^{-1}(1-r)\right)} \tag{3.43}$$

By changing the order of integration in equation (3.43) and by applying the transformation $z = \frac{x_j - R}{\sigma}$, we get the following expression:

$$l^b = \frac{\frac{1}{\sqrt{2\pi v^2}}\int_{x^*-\sigma F^{-1}(1-r)}^{+\infty}\exp\left(-\frac{(R-y^*)^2}{2v^2}\right)\left[\int_{-\infty}^{\frac{x^*-R}{\sigma}} f(z)\,dz\right]dR}{F\left[\frac{y^*-x^*}{v}+\frac{\sigma}{v}F^{-1}(1-r)\right]}$$

or

$$l^b = \frac{\int_{x^*-\sigma F^{-1}(1-r)}^{+\infty} F\left(\frac{x^*-R}{\sigma}\right)\frac{1}{\sqrt{2\pi v^2}}\exp\left[-\frac{(R-y^*)^2}{2v^2}\right]dR}{F\left[\frac{y^*-x^*}{v}+\frac{\sigma}{v}F^{-1}(1-r)\right]}$$

Let the transformation $w = \frac{R-y^*}{v}$

$$l^b = \frac{\int_{\frac{x^*-y^*-\sigma F^{-1}(1-r)}{v}}^{+\infty} F\left(\frac{x^*-y^*-vw}{\sigma}\right) f(w)\,dw}{F\left[\frac{y^*-x^*}{v}+\frac{\sigma}{v}F^{-1}(1-r)\right]} \tag{3.44}$$

For $r < 0.5$ and considering the limiting case where $\frac{v}{\sigma} \rightarrow 0$, equation (3.44) can be expressed as

$$l^b = F\left(\frac{x^*-y^*}{\sigma}\right)$$

Q.E.D.

3.A.7 Proof of Proposition 9

Conditional on signal $y = y^*$, the actual investment return R is normally distributed with mean y^* and variance v^2. Thus, we may express equation (3.7) as

$$-B\int_{-\infty}^{R^*}\frac{1}{\sqrt{2\pi}v}\exp\left(\frac{(R-y^*)^2}{v^2}\right)dR - D\int_{R^*}^{B+C+a_s(1-l^b)}\frac{1}{\sqrt{2\pi}v}\exp\left(\frac{(R-y^*)^2}{v^2}\right)dR = -B \tag{3.45}$$

where, as of lemma 4, for a given strategy x^* by suppliers, R^* is given by $R^*(y^*) = x^*(y^*) - \sigma F^{-1}(1-r)$. Considering the transformation $z = \frac{R-y^*}{v}$ and substituting R^*, equation (3.45) becomes

$$-B\int_{-\infty}^{\frac{x^*-y^*}{v}-\frac{\sigma}{v}F^{-1}(1-r)} f(z)\,dz - D\int_{\frac{x^*-y^*}{v}-\frac{\sigma}{v}F^{-1}(1-r)}^{\frac{B+C+a_s\left(1-l^b\right)-y^*}{v}} f(z)\,dz = -B \tag{3.46}$$

For the limiting case where $\frac{v}{\sigma} \to 0$, we consider two cases: **1)** $r < 0.5$, **2)** $r \geq 0.5$.

Case 1: If $r < 0.5$ then, l^b is given by lemma 7 and equation (3.46) becomes

$$-D\int_{-\infty}^{\frac{B+C+a_s\left(1-l^b\right)-y^*}{v}} f(z)\,dz = -B$$

or

$$y^* = B + C + a_s F\left(\frac{y^*-x^*}{\sigma}\right) - vF^{-1}\left(\frac{B}{D}\right) \tag{3.47}$$

provided that $\lim_{\frac{\sigma}{v}\to\infty}\left[\frac{x^*-y^*}{v} - \frac{\sigma}{v}F^{-1}(1-r)\right] = -\infty$.

Case 2: If $r \geq 0.5$ then, given that l^b is bounder between zero and one, equation (3.46) becomes

$$-B = -B$$

Consequently, for $r > 0.5$, *any* trigger strategy by the bank can be an equilibrium strategy. However, given that the bank in this game acts as a Stackelberg leader, for consistency with the treatment of financial distress in banks' credit policies, we concentrate here on strategies y^* that are characterised by equation (3.47).

Q.E.D.

3.A.8 Proof of Proposition 10

We first consider the case for $r < 0.5$.

Case 1: If $r < 0.5$ then, for $x^* \geq (B+C) + a_s(1-l^*)$, propositions 8 and 9 imply that x^* and y^* solve

$$\left.\begin{array}{r} F\left(\frac{x^*-y^*}{\sigma}\right) = \frac{a_s}{2(1-c)} \\ y^* = (B+C) + a_s - a_s F\left(\frac{x^*-y^*}{\sigma}\right) - vF^{-1}\left(\frac{B}{D}\right) \end{array}\right\} \tag{3.48}$$

By substituting $F\left(\frac{x^*-y^*}{\sigma}\right)$ into y^*, (3.48) becomes

$$\left.\begin{array}{r} F\left(\frac{x^*-y^*}{\sigma}\right) = \frac{a_s}{2(1-c)} \\ y^* = (B+C) + a_s\left(1 - \frac{a_s}{2(1-c)}\right) - vF^{-1}\left(\frac{B}{D}\right) \end{array}\right\} \tag{3.49}$$

Then, by substituting y^* into the first equation and considering the limiting case where $\frac{v}{\sigma} \to 0$, (3.49) becomes

$$\left.\begin{array}{r} x^* = (B+C) + a_s\left(1 - \frac{a_s}{2(1-c)}\right) + \sigma F^{-1}\left(\frac{a_s}{2(1-c)}\right) \\ y^* = (B+C) + a_s\left(1 - \frac{a_s}{2(1-c)}\right) - vF^{-1}\left(\frac{B}{D}\right) \end{array}\right\} \tag{3.50}$$

which implies a unique equilibrium of the form (x^*, y^*).We now need to show that such an equilibrium is consistent with the initial conjecture that $x^* \geq (B+C) + a_s(1-l^*)$. Yet this is the case if and only if the following inequality holds

$$\frac{a_s^2}{2(1-c)} - \sigma F^{-1}\left(\frac{a_s}{2(1-c)}\right) \leq a_s l^* \tag{3.51}$$

By substituting l^* from lemma 6 and $F\left(\frac{x^*-y^*}{\sigma}\right)$ from (3.48), (3.51) becomes

$$\frac{a_s}{2(1-c)} \geq F\left(\frac{2a_s^2(1-r) - (1-c)^2}{4\sigma(1-c)(1-r)}\right) \tag{3.52}$$

However, if (3.52) doesn't hold then, $x^* < (B+C) + a_s(1-l^*)$ and propositions 8 and 9 imply multiple equilibria of the form (x^*, y^*), such that

$$\left.\begin{array}{r} y^* = (B+C) + a_s F\left(\frac{y^*-x^*}{\sigma}\right) - vF^{-1}\left(\frac{B}{D}\right) \\ a_s F\left(\frac{x^*-(B+C)-a_s(1-l^*)}{\sigma}\right) = (1-c)\, F\left(\frac{x^*-y^*}{\sigma}\right) \end{array}\right\}$$

or

$$\left.\begin{array}{c} y^* = (B+C) + a_s F\left(\frac{y^*-x^*}{\sigma}\right) - vF^{-1}\left(\frac{B}{D}\right) \\ a_s F\left(\frac{x^*-y^*}{\sigma} + \frac{y^*-(B+C)-a_s(1-l^*)}{\sigma}\right) = (1-c)\, F\left(\frac{x^*-y^*}{\sigma}\right) \end{array}\right\} \tag{3.53}$$

By partial substitution of y^* into the first equation in (3.53) and for $\frac{v}{\sigma} \to 0$, (3.53) becomes

$$\left.\begin{array}{c} y^* = (B+C) + a_s F\left(\frac{y^*-x^*}{\sigma}\right) - vF^{-1}\left(\frac{B}{D}\right) \\ a_s F\left(\frac{x^*-y^*}{\sigma} + \frac{a_s}{\sigma}\left[l^* - F\left(\frac{x^*-y^*}{\sigma}\right)\right]\right) = (1-c)\, F\left(\frac{x^*-y^*}{\sigma}\right) \end{array}\right\} \tag{3.54}$$

But we know that for (3.53) to hold, x^* must satisfy

$$x^* < (B+C) + a_s(1-l^*) \tag{3.55}$$

where from (3.53) and for $\frac{v}{\sigma} \to 0$, (3.55) is equivalent to

$$\frac{x^*-y^*}{\sigma} + \frac{a_s}{\sigma}\left[l^* - F\left(\frac{x^*-y^*}{\sigma}\right)\right] < 0 \tag{3.56}$$

Combining (3.54) and (3.56) we get multiple equilibria (x^*, y^*), with x^* and y^* satisfying

$$\left.\begin{array}{c} y^* = (B+C) + a_s F\left(\frac{y^*-x^*}{\sigma}\right) - vF^{-1}\left(\frac{B}{D}\right) \\ F\left(\frac{x^*-y^*}{\sigma}\right) < \frac{a_s}{2(1-c)} \end{array}\right\}$$

or

$$\left.\begin{array}{c} y^* = (B+C) + a_s F\left(\frac{y^*-x^*}{\sigma}\right) - vF^{-1}\left(\frac{B}{D}\right) \\ y^* > x^* - \sigma F^{-1}\left(\frac{a_s}{2(1-c)}\right) \end{array}\right\}$$

Let us now consider the case for $r \geq 0.5$.

Case 2: If $r \geq 0.5$ then, propositions 8 and 9 imply multiple equilibria of the form (x^*, y^*), with x^* and y^* solving

$$\left.\begin{array}{c} y^* = (B+C) + a_s F\left(\frac{y^*-x^*}{\sigma}\right) - vF^{-1}\left(\frac{B}{D}\right) \\ a_s F\left(\frac{x^*-(B+C)-a_s(1-l^*)}{\sigma}\right) = (1-c)\left[F\left(\frac{x^*-y^*}{\sigma}\right) - r\right] \end{array}\right\}$$

or

$$\left.\begin{array}{c} y^* = (B+C) + a_s F\left(\frac{y^*-x^*}{\sigma}\right) - vF^{-1}\left(\frac{B}{D}\right) \\ a_s F\left(\frac{x^*-y^*}{\sigma} + \frac{y^*-(B+C)-a_s(1-l^*)}{\sigma}\right) = (1-c)\left[F\left(\frac{x^*-y^*}{\sigma}\right) - r\right] \end{array}\right\} \tag{3.57}$$

By partial substitution of y^* into the second equation and considering the limiting case $\frac{v}{\sigma} \to 0$, (3.57) becomes

$$\left.\begin{array}{r} y^* = (B+C) + a_s F\left(\frac{y^*-x^*}{\sigma}\right) - vF^{-1}\left(\frac{B}{D}\right) \\ a_s F\left(\frac{x^*-y^*}{\sigma} + \frac{a_s}{\sigma}\left[l^* - a_s F\left(\frac{x^*-y^*}{\sigma}\right)\right]\right) = (1-c)\left[F\left(\frac{x^*-y^*}{\sigma}\right) - r\right] \end{array}\right\} \tag{3.58}$$

However, from corollary 3 we know that, for $r \geq 0.5$, suppliers equilibrium strategy x^* satisfies the following inequality:

$$x^* < (B+C) + a_s(1-l^*) \tag{3.59}$$

But given y^* in (3.58), inequality (3.59) can be written as

$$\frac{x^*-y^*}{\sigma} < \frac{a_s}{\sigma}\left[F\left(\frac{x^*-y^*}{\sigma}\right) - l^*\right] + \frac{v}{\sigma}F^{-1}\left(\frac{B}{D}\right)$$

where, for $\frac{v}{\sigma} \to 0$, it becomes

$$\frac{x^*-y^*}{\sigma} + \frac{a_s}{\sigma}\left[l^* - F\left(\frac{x^*-y^*}{\sigma}\right)\right] < 0 \tag{3.60}$$

Thus, combining (3.58) and (3.60) we conclude that, for $r \geq 0.5$, there are multiple equilibria of the form (x^*, y^*) that satisfy

$$\left.\begin{array}{c} y^* = (B+C) + a_s - vF^{-1}\left(\frac{B}{D}\right) - a_s F\left(\frac{x^*-y^*}{\sigma}\right) \\ F\left(\frac{x^*-y^*}{\sigma}\right) < r + \frac{a_s}{2(1-c)} \end{array}\right\}$$

or

$$\left.\begin{array}{c} y^* = (B+C) + a_s - vF^{-1}\left(\frac{B}{D}\right) - a_s F\left(\frac{x^*-y^*}{\sigma}\right) \\ y^* > x^* - \sigma F^{-1}\left(\frac{a_s}{2(1-c)} + r\right) \end{array}\right\}$$

Q.E.D.

3.A.9 Proof of Proposition 11

We need to show that the equilibrium strategy x^* in the presence of a bank is lower than x^{**}, as given by proposition 7, with no bank in the game. For $r \geq 0.5$ and for a given strategy

y^* by the bank, proposition 8 implies that suppliers' equilibrium strategy x^* is given by

$$a_s F\left(\frac{x^* - (B+C) - a_s(1-l^*)}{\sigma}\right) = (1-c)\left[F\left(\frac{x^* - y^*}{\sigma}\right) - r\right] \tag{3.61}$$

where, from lemma 6, l^* is given by $l^* = \frac{1}{2}\frac{(1-r)}{F\left(\frac{x^*-y^*}{\sigma}\right)}$, noticing that $l^* > \frac{(1-r)}{2}$. Also, from proposition 9, bank's equilibrium strategy y^* solves

$$y^* = (B+C) + a_s F\left(\frac{y^* - x^*}{\sigma}\right) - vF^{-1}\left(\frac{B}{D}\right) \tag{3.62}$$

Clearly, the RHS of equation (3.61) is less than $(1-c)(1-r)$, which implies that

$$a_s F\left(\frac{x^* - (B+C) - a_s(1-l^*)}{\sigma}\right) < (1-c)(1-r) \tag{3.63}$$

Given also that $l^* > \frac{(1-r)}{2}$, it follows that

$$a_s F\left(\frac{x^* - (B+C) - a_s\frac{(1+r)}{2}}{\sigma}\right) < a_s F\left(\frac{x^* - (B+C) - a_s(1-l^*)}{\sigma}\right) \tag{3.64}$$

Consequently, from inequalities (3.63), (3.64) and from proposition 7, we derive the following relationship between suppliers equilibrium strategies x^* and x^{**} with and without a bank, respectively.

$$a_s F\left(\frac{x^* - (B+C) - a_s\frac{(1+r)}{2}}{\sigma}\right) < a_s F\left(\frac{x^{**} - (B+C) - a_s\left(\frac{1+r}{2}\right)}{\sigma}\right)$$

where, from the monotonicity of $F(\cdot)$, it follows that $x^* < x^{**}$.

Q.E.D.

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