Monetary Transmission Mechanism: Heterogeneous Information, Inventories, and Credit-Market Imperfections

by

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Abstract

This thesis presents a theoretical investigation into the monetary transmission mechanism. In particular, I focus on heterogeneous information, inventories, and credit-market imperfections as factors that help to propagate monetary disturbances to the economy in a way that can explain plausibly the observed effects of monetary policy.

First, I consider heterogeneous information among price-setters who can only observe the state of the economy through noisy private signals. I construct a model which incorporates the imperfect common knowledge into the Taylor-Calvo staggered price-setting model. The average price chosen in each period depends on the higher-order expectations about not only the current state of the economy but also the future states during the periods the prices will be fixed. The response of inflation to a monetary disturbance is delayed following a sluggish initial adjustment of prices and the response of output is amplified by the imperfection in common knowledge. These results are robust when a noisy public signal in addition to private signals is introduced.

Secondly, I consider inventories by developing simple dynamic general equilibrium models which assume pre-determined prices and incorporate a production-smoothing motive and a sales-facilitating motive for holding inventories. Inventories serve as a source of real rigidities, that is, amplify the persistence of the real effects of monetary policy. Inventories respond procyclically and prices are adjusted gradually to a nominal disturbance only if the sales-facilitating motive is relatively strong enough; otherwise inventories respond countercyclically and prices are adjusted excessively. I also consider the case where production as well as prices is pre-determined.

Lastly, I consider credit-market imperfections by examining a model which incorporates asymmetric information between lenders and borrowers into a standard dynamic New Keynesian model. I calibrate the model using Japanese data and find that the large volatility of Japan's corporate investment can be explained by taking account of the credit-market imperfections. Based on this model, I simulate alternative monetary policy rules and evaluate their performances.

Contents

Introduction

This thesis presents a theoretical investigation into the monetary transmission mechanism. In particular, I focus on heterogeneous information, inventories, and credit-market imperfections as factors that help to propagate monetary disturbances to the economy in a way that can explain plausibly the observed effects of monetary policy.

There are many mechanisms that co-operate in transmitting monetary policy shocks to the economy. The process begins with the open market operations which can affect market interest rates, and then the transmission may proceed through several channels. The interest rate channel is the primary transmission mechanism in both the traditional Keynesian macroeconomic models and the New Keynesian dynamic general equilibrium models. Given some degree of price stickiness, changes in the nominal interest rate translate into those in the real interest rate, which affect investment spending through the traditional "IS curve" or consumption spending through the Euler equation for households' intertemporal allocation. Meanwhile, monetarists emphasize the transmission channels through changes in relative asset prices or relative quantities of assets, which include Tobin's q theory of investment and wealth effects on consumption. Another channel is the credit channel which include the effects of credit-market imperfections through the quantity of bank loans (bank lending channel) and borrowers' net worth (balance sheet channel)[1]. It has been studied extensively since 1990s when many countries experienced large fluctuations in real economic activities accompanied by large swings in asset prices, and I study the case of Japan's economy in Chapter 3. Although the above channels are not mutually exclusive, many empirical studies have tried to assess their relative

[1]Bernanke and Gertler (1995) provides an extensive discussion on the role of the credit channel in monetary transmission.

importance[2].

In theoretical studies, a more fundamental problem has been tackled: *why* rather than *how* does monetary policy have real effects in the short run? Modern macroeconomic theory provides us mainly two explanations: imperfect information about the policy shocks and short-run rigidity in price or wage adjustment. The imperfect information approach was originally developed by Phelps (1970) and Lucas (1972) in the era when the traditional output-inflation relationship collapsed. The costs of acquiring information have been emphasized mainly by monetarists as micro-foundation of several transmission channels[3]. Meanwhile, the recent New Keynesian dynamic general equilibrium models assume short-run rigidity in price or wage adjustment, typically adopting the staggered price-setting a la Taylor (1980) or Calvo (1983), and analyze the effects of interest-rate rules for monetary policy[4].

It is not easy, however, for optimization-based theoretical models to explain the observed effects of monetary policy. While those models have been widely used in monetary policy analyses, several problems have been pointed out. The first is the *persistence* of the policy effects. The Phelps-Lucas imperfect information models imply that the real effects of monetary policy last only insofar as the precise public information about aggregate disturbances is unavailable, which seems contradictory to the observed persistence of business fluctuations despite the availability of macroeconomic data with only short delays. Although some recent imperfect information models including Mankiw and Reis (2002) and Woodford (2003a) can generate persistent real effects of monetary policy, their results rely on assumptions of unrealistically too much unawareness or inattentiveness of economic agents. Meanwhile, the ability of the staggered price-setting models to reproduce quantitatively the observed persistence of business fluctuations are also subject to continued debate. Chari, Kehoe, and McGrattan (2000) argue that implausibly long periods of exogenous rigidity in price adjustment are required to gen-

[2]Other channels include the exchange rate channel, the cost channel, and the expectations channel. For example, Mishkin (1995) and van Els et al. (2001) provide an overview of those transmission mechanisms.

[3]Meltzer (1995) provides a monetarist perspective on the transmission process of monetary shocks as well as a critique of the Keynesian perspectives.

[4]Woodford (2003b) provides a rigorous and extensive framework of New Keynesian, or Neo-Wicksellian as he calls, dynamic general equilibrium models and monetary policy analyses based on it.

erate the realistically persistent output fluctuations. The second problem is the *magnitude.* Many empirical VAR evidences suggest that monetary policy shocks that induce relatively small movements in open market interest rates affect powerfully the real economy. Bernanke and Gertler (1995) use this result to support their argument that the credit channel amplifies the policy effects through the interest rate channel. The third problem is the *co-movement* in output and inflation. The widely used Calvo-type staggered price-setting implies that the price level jumps in the period of disturbance and inflation responds earlier than output, which is contradictory to the stylized fact shown in many empirical studies that monetary policy shocks initially impact on real variables and then have a delayed and gradual effects on inflation.

The following chapters in this thesis are my attempts to develop optimization-based monetary business cycle models that can overcome some of the above problems. The model in each chapter incorporates a key factor that helps to propagate monetary disturbances to the economy in a way that can explain plausibly the observed effects of monetary policy. In **Chapter 1**, I focus on *heterogeneous information* among price-setters who can only observe the state of the economy through noisy private signals. I construct a model which integrates the Woodford (2003a) imperfect common knowledge model with the Taylor-Calvo staggered price-setting model. The average price chosen in each period depends on the higher-order expectations about not only the current state of the economy but also the future states during the periods the prices will be fixed, which makes the initial adjustment of prices to a monetary disturbance more sluggish than that in both the static imperfect common knowledge model and the full-information staggered price-setting model. The response of inflation to a monetary disturbance is delayed following the sluggish initial response and the response of output is amplified by the imperfection in common knowledge. Heterogeneous information which features in this chapter also plays important roles in the following chapters: heterogeneity in the length of the decision lag for price-setting is considered in Chapter 2 and asymmetric information between lenders and borrowers is assumed as the micro-foundation of the financial accelerator effects in Chapter 3.

In **Chapter 2**, I focus on *inventories.* I develop simple dynamic general equilibrium models which assume pre-determined prices and incorporate a production-smoothing motive and a sales-facilitating motive for holding inventories. One of the main challenges in this chapter is to incorporate those inventory-holding motives in a way that is consistent with the stylized facts on inventories. Inventories respond procyclically and prices are adjusted gradually to a nominal disturbance only if the sales-facilitating motive is relatively strong enough; otherwise inventories respond countercyclically and prices are adjusted excessively. When incorporated into the models in an appropriate way, inventories amplify the persistence of the real effects of monetary policy, as Blinder and Fischer (1981) argued in their non-optimization-based model.

In the models in Chapter 1 and 2, monetary policy is defined as just a disturbance driving a stochastic process of aggregate nominal spending and neither the interest rate channel nor any other specific transmission channels are explicitly considered. In **Chapter 3**, I focus on *credit-market imperfections* by examining a model based on Bernanke, Gertler, and Gilchrist (1999) which incorporates asymmetric information between lenders and borrowers into a standard dynamic New Keynesian model, where the credit channel as well as the interest channel is explicitly considered. The model exhibits financial accelerator effects, the mechanism whereby credit-market imperfections help to propagate or amplify various types of shocks including monetary policy shocks to the economy. I calibrate this model using Japanese data and find that the large volatility of Japan's corporate investment can be explained by taking account of this mechanism. Based on this model, I simulate alternative monetary policy rules specified in terms of an interest rate instrument and evaluate their performances.

Although the models in this thesis overcome some problems in the existing optimizing-based monetary business cycle models, they capture only a few aspects of the whole picture of the monetary transmission mechanism. In particular, the models in Chapter 1 and 2 are too simple for practical uses such as policy and empirical research. Meanwhile, the baseline models in those chapters are so tractable and flexible that some extensions can be easily done. In Chapter 1, I extend the baseline model by introducing a noisy public signal as well as private signals, which provides interesting implications for the conduct of monetary policy such as commitments and transparency when the public signal is interpreted as a communication tool of the monetary authority. In Chapter 2, I extend the baseline model by considering the case where production as well as prices is pre-determined. If the decision lag of price-setting is longer than that of production, inventories respond countercyclically at first and then move procyclically, which is consistent with the pattern shown in empirical studies. I also develop a more realistic model for quantitative experiments. I hope those models serve as useful building blocks for future research in various directions.

References

[1] Barnanke, B. S. and Gertler, M. (1995), "Inside the Black Box: The Credit Channel of Monetary Policy Transmission," *Journal of Economic Perspectives*, 9(4): 27-48.

[2] Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999), "The Financial Accelerator in a Quantitative Business Cycle Framework," in Taylor, J. B. and Woodford M. (eds), *Handbook of Macroeconomics*, Vol. 1C, Chap. 21, pp. 1341-1393, North Holland.

[3] Blinder, A. S. and Fischer, S. (1981), "Inventories, Rational Expectations, and the Business Cycle," *Journal of Monetary Economics*, 8: 277-304.

[4] Calvo, G. A. (1983), "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 12: 383-398.

[5] Chari, V. V., Kehoe, P. J. and McGrattan, E. R. (2000), "Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?," *Econometrica*, 68(5): 1151-1179.

[6] Lucas, R. Jr. (1972), "Expectations and the Neutrality of Money," *Journal of Economic Theory*, 4: 103-124.

[7] Mankiw, N. G. and Reis, R. (2002), "Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Philips Curve," *Quarterly Journal of Economics*, 117(4): 1295-1328.

[8] Mishkin F. S. (1995), "Symposium on the Monetary Transmission Mechanism," *Journal of Economic Perspectives*, 9(4): 3-10.

[9] Meltzer A. H. (1995), "Monetary, Credit and (Other) Transmission Processes: A Monetarist Perspective," *Journal of Economic Perspectives*, 9(4): 49-72.

[10] Phelps, E. S. (1970), "Introduction: The New Microeconomics in Employment and Inflation Theory," in Phelps, E., et al. *Microeconomic Foundations of Employment and Inflation Theory*, Norton, New York.

[11] Taylor, J. B.(1980), "Aggregate Dynamics and Staggered Contracts," *Journal of Political Economy*, 88(1): 1-22.

[12] van Els, P., Locarno, A., Morgan, J. and Villetelle, J. P. (2001), "Monetary Policy Transmission in the Euro Area: What Do Aggregate and National Structural Models Tells Us?," European Central Bank, Working Paper No. 94.

[13] Woodford, M. (2003a), "Imperfect Common Knowledge and the Effects of Monetary Policy," in Aghion, P., Frydman, R., Stiglitz, J. and Woodford, M. (eds) *Knowledge, Information and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, Princeton University Press.

[14] Woodford, M. (2003b), *Interest and Prices*, Princeton University Press.

Chapter 1

Imperfect Common Knowledge, Staggered Price-Setting, and the Effects of Monetary Policy

This chapter proposes a model that integrates the Woodford (2003a) imperfect common knowledge model with the Taylor-Calvo staggered price-setting model in order to explain plausibly the observed effects of monetary policy. I drop the Woodford's unrealistic assumption that all price-setters never know the widely available data on the aggregate demand nor even the actual quantity they sold at their own price, and instead assume that the true state of the economy is revealed to all price-setters with a delay of one period. With staggered price-setting, however, the model can generate persistent real effects of monetary policy. The average price chosen in each period depends on the higher-order expectations about not only the current state of the economy but also the future states during the periods the prices will be fixed, which makes the initial adjustment of prices to a monetary disturbance more sluggish than that in both the static imperfect common knowledge model and the full-information staggered price-setting model. I show analytically that the response of inflation is delayed following the sluggish initial response and the response of output continues to be amplified. These results are robust when a noisy public signal in addition to private signals is introduced.

1.1 Introduction

Modern macroeconomic theory provides us mainly two explanations for why monetary policy has real effects in the short run: imperfect information about the policy shocks and short-run rigidity in price or wage adjustment. The imperfect information approach was originally developed by Phelps (1970) and Lucas (1972) in the era when the traditional output-inflation relationship collapsed. However, their arguments were soon criticized for their practical irrelevance: the Phelps-Lucas model implies that the real effects of monetary policy last only insofar as the precise public information about aggregate disturbances is unavailable, which seems contradictory to the observed persistence of business fluctuations despite the availability of macroeconomic data with only short delays[1]. Nowadays many macroeconomic models of business fluctuations assume short-run rigidity in price or wage adjustment, typically adopting the staggered price-setting a la Taylor (1980) or Calvo (1983), in order to analyze the long-lasting real effects of monetary policy.

Meanwhile, some authors have recently reconsidered the imperfect information approach and developed monetary business cycle models that can generate persistent real effects of monetary policy and can also overcome one of the main problems in the Taylor-Calvo staggered price-setting approach, namely the incapability of explaining the observed inflation inertia. Mankiw and Reis (2002) consider sticky information rather than sticky prices, which means parts of the current prices are chosen on the basis of old information. Woodford (2003a) considers imperfect common knowledge about nominal disturbances in an environment among monopolistically competitive suppliers whose optimal pricing strategy depends not only on their own estimates of the aggregate disturbances but also on their expectations of the average estimates by other suppliers. These models can explain in particular the stylized fact shown in many empirical studies[2] that monetary policy shocks initially impact on real variables and then have a delayed and gradual effects

[1]Lucas (1975) develops a monetary business cycle model that can generate persistent real effects of monetary policy by introducing capital accumulation as well as information lags. The persistence can also be generated by introducing inventories into monetary business cycle models based on imperfect information without assuming any nominal rigidity, as I will show in Chapter 2.

[2]For example, Barnanke and Gertler (1995) and Christiano, Eichenbaum, and Evans (2001).

on inflation.

However, those models practically still leave the original problem in the Phelps-Lucas model unsolved. The source of persistence of the real effects of monetary policy in the Mankiw-Reis model is the outdated information that influences on current price-setting. In their model there are always some part of suppliers who set their prices using very old information, as the probability of obtaining new information in each period is constant and identical among all suppliers regardless of how long it has been since their last update. In the Woodford model, all suppliers never know (or pay attention to) the precise information about the aggregate demand nor even the actual quantity they sold at their own price. They choose their prices only on the basis of the history of their subjective observations which contain idiosyncratic perception error. In both models, the persistent real effects of monetary policy would disappear if the true state of the economy were revealed to all suppliers with a delay of only one period. They still fail to explain why price-setters do not use the widely and quickly available macroeconomic data in their model.

In this chapter I propose a model that integrates Woodford's imperfect common knowledge model with the Taylor-Calvo staggered price-setting model in order to overcome the problems in each of them and explain plausibly the observed effects of monetary policy. The model is based on the standard set-up of monopolistic competition developed by Blanchard and Kiyotaki (1987), where each firm's desired price depends on their observation about the overall price index and the output gap. Following the Woodford model, I assume that the price-setters can only observe the state of the economy through noisy private signals that is idiosyncratic to each individual observer so that the overall price is described as the weighted sum of their "higher-order expectations," that is, what others expect about what others expect ... about the aggregate demand[3]. Meanwhile, unlike the Woodford model, the true state of the economy is revealed to all price-setters with a delay of one period. The source of persistence of the real effects of monetary policy in this model is the staggered price-setting. The average prices chosen in each period depends on the higher-order expectations about not only the

[3]Keynes (1936) pointed out the role of higher-order expectations in an asset pricing context by introducing the famous metaphor of financial markets as "beauty contests." Recently, higher-order beliefs have been extensively studied in the theoretical literature on "global games" (Morris and Shin, 2003) and applied to various fields.

current state of the economy but also the future states during the periods the prices will be fixed, which makes the initial adjustment of prices to a monetary disturbance more sluggish than that in both the static imperfect common knowledge model and the full-information staggered price-setting model. Although the higher-order expectations about both current and future states are very complicated, the model can be solved by virtue of the assumption that the true current state will become common knowledge in the next period.

The main results of the model are as follows. The response of inflation to a monetary disturbance is delayed following the sluggish initial adjustment of prices. The response of output is amplified by the imperfection in common knowledge in the period of disturbance and then continues to be larger than that in the corresponding full-information staggered price-setting model, which is nested by my models as a special case, even after the precise information about the initial shock becomes common knowledge. I show the above results analytically first in the baseline model where imperfect common knowledge is incorporated into a simple two-period price-setting model, and then in more general price-setting models which allow for multiple-period staggered price-setting including the one analogized with the Calvo-type price-setting. The Calvo-type price-setting has a problem that the price level jumps in the period of disturbance and inflation responds earlier than output, which is contradictory to the stylized fact I mentioned above. The above results imply that this problem can be overcome by incorporating imperfect common knowledge into the Calvo-analogized model. Moreover, my imperfect common knowledge model can overcome the problem in the Woodford's imperfect common knowledge model that the response of output is relatively weak compared with the Calvo-type price-setting model.

The baseline model is tractable and flexible enough for various extensions. In particular, I extend it by introducing a noisy public signal in addition to the private signals in order to study the consequences of more general information structure following Hellwig (2002) and Amato and Shin (2003). These authors emphasized the separation of information into public and private signals. The baseline model only focuses on the private signals and lack considerations for problems in informational interaction among decision-makers as well as for the availability of macroeconomic data. Whereas Amato and Shin assume price-setters never know the precise information like the Woodford model, I keep the assumption that the true state of the economy is revealed to all price-setters with a delay of one period. I show that the provision of the public signal alleviates the sluggishness in the initial adjustment of prices to some extent but the results in the baseline model such as the delayed response of inflation and the amplified response of output are robust.

The public signal in the extended model may represent noisy information provided by the media, the government, or preliminary data to be revised. When it is interpreted as a communication tool of the monetary authority, the model has interesting implications for the conduct of monetary policy such as commitments and transparency. As Morris and Shin (2002) argued, public information in an economy where decision-makers' information sets are heterogeneous has a disproportionately large effects on their decisions. While the provision of the public signal alleviates the sluggishness in the initial adjustment of prices to monetary disturbances, it exposes firms to additional disturbances, namely informational noise, and could destabilize the economy. Although I do not seek to derive policy or welfare implications from the models in this chapter, the extension to introduce the public signal is important for providing a building block for further extensions in those directions as well as for checking the robustness of the results obtained from the baseline model.

While this chapter is related to the two strands of literature, imperfect common knowledge and staggered price-setting, the attempt to integrate them, as far as I know, has never yet been made. It has been sometimes argued, however, that imperfect information and nominal rigidities are closely related to each other as a plausible explanation for the real effects of monetary policy. Ball and Cecchetti (1988) develop a model in which monopolistically competitive firms gain information by observing the prices set by others and then under certain conditions the staggered price-setting arise endogenously as the equilibrium outcome. Kiley (2000) develops a model with both costs of nominal price adjustment and costs of information acquisition in order to identify empirically the degree of price stickiness. He concludes that the negative relationship between the persistence of detrended output and average inflation in cross-country data is consistent with his endogenous sticky price model where price stickiness acts as a persistence-generating mechanism. Although the staggered price-setting in my models is assumed rather than explained endogenously, my integration approach could be justified by those previous studies as well as by the results I will show in this chapter.

The remainder of the chapter is organized as follows. In Section 1.2, I describe the baseline model and show the main results on the effects of monetary disturbances. In Section 1.3, I extend the baseline model by introducing a noisy public signal in addition to private signals and examine the effects of informational disturbances as well as monetary disturbances. In Section 1.4, I consider more general price-setting environment including the one analogized with the Calvo-type price-setting. Section 1.5 concludes.

1.2 The Baseline Model

In this section I incorporate imperfect common knowledge into a simple two-period staggered price-setting model and then examine analytically the effects of monetary disturbances.

1.2.1 Set-up

Consider an economy where a continuum of monopolistically competitive firms indexed by $i \in [0, 1]$ produce their individual-specific goods and set their prices. Goods are perishable and there are no capital to accumulate as a factor of production. I begin with the following static optimal price-setting condition of firm i[4].

$$p_t^*(i) = E_t^i p_t + \phi E_t^i y_t, \quad 0 < \phi < 1. \tag{1.1}$$

All variables are expressed in logs. $p_t^*(i)$ is i's desired price in period t and would be the actual price if they could set their prices flexibly. p_t is the overall price index and y_t is the output gap. The parameter ϕ is assumed to be less than unity so that their price-setting decisions are strategic complements. When the elasticity of substitution among the differentiated goods is higher or the elasticity of marginal cost with respect to output is lower, this parameter value is smaller and the strategic complimentarity is stronger.

Firms cannot observe precisely the aggregate variables such as p_t and y_t even in the current period t. Moreover, their information sets are heteroge-

[4]This condition can be derived from a standard monopolistic competition model such as Blanchard and Kiyotaki (1987).

neous, which is the main feature of this model. Accordingly the expectation operators conditional on i's information set as of t, E_t^i, are added to p_t and y_t in the above equation. I will describe details of the private information set and the signal extraction problem in the next subsection.

Then I introduce the two-period staggered price-setting a la Taylor (1980). In period t, half of firms in the economy set their prices for the current period t and the next period $t+1$. They must set the same prices for both periods, which means prices are not just pre-determined but fixed. The price chosen by firm i who sets its price in t is given by[5]

$$
\begin{aligned}
x_t(i) &= \frac{1}{2}\left(p_t^*(i) + E_t^i p_{t+1}^*(i)\right) \\
&= \frac{1}{2}\left(E_t^i p_t + \phi\, E_t^i y_t + E_t^i p_{t+1} + \phi\, E_t^i y_{t+1}\right).
\end{aligned}
\tag{1.2}
$$

In period $t+1$ the other half of firms set their prices for $t+1$ and $t+2$, in period $t+2$ the firms who set their prices in t re-set their prices for $t+2$ and $t+3$, and so on. The overall price index is given by

$$
p_t = \frac{1}{2}\left(x_t + x_{t-1}\right) \tag{1.3}
$$

where x_t is the average price chosen by the firms who set their prices in t, that is, $x_t \equiv 2\int_0^{0.5} x_t(i)\, di$ when $t = \cdots, -2, 0, 2, \cdots$, and $x_t \equiv 2\int_{0.5}^{1} x_t(i)\, di$ when $t = \cdots, -1, 1, \cdots$.

Lastly, I specify the demand side of the economy by introducing an exogenous stochastic process for aggregate nominal spending as follows.

$$
m_t - m_{t-1} = \rho\,(m_{t-1} - m_{t-2}) + \sigma\,\epsilon_t, \qquad \epsilon_t \sim N(0,\, 1) \tag{1.4}
$$

where

$$
m_t = p_t + y_t \tag{1.5}
$$

and ϵ_t is Gaussian white noise. One may interpret m_t as "money" that households must hold for their spending. As shown in Christiano, Eichenbaum, and Evans (1998), the above process can be viewed as a plausible stochastic process for the actual money supply (M2) in the U.S. data. Alternatively, m_t can be interpreted more broadly as a generic variable affecting aggregate demand. This simple specification for aggregate demand, however

[5] I assume that the discount rate applied to the firm's profits in the next period is negligible for simplicity.

interpreted it is, allows me to concentrate on examining the consequences of alternative specifications for price-setting behaviours. In my analysis below, I suppose that the disturbance driving the above process for aggregate nominal spending is a monetary policy shock.

1.2.2 Signal Extraction

Here I specify the firms' information set. As in Lucas (1972) and Woodford (2003a), each individual firm estimates the current state of the economy using their private information. In period t, firm i has access to a noisy private signal about the current aggregate demand, m_t, as follows.

$$z_t(i) = m_t + \sigma_u u_t(i), \qquad u_t(i) \sim N(0, 1) \tag{1.6}$$

where $u_t(i)$ is Gaussian white noise distributed independently both of ϵ_t and of $u_t(j)$ for all $j \neq i$. Unlike the Woodford model, this model assumes that the true value of m_t becomes common knowledge among all firms with a delay of only one period in $t + 1$. Therefore the information set of firm i consists of the private signal $z_t(i)$ and the history of realized aggregate nominal spending $\{m_{t-s}\}_{s=1}^{\infty}$. The result of firms' signal extraction for estimating m_t is given by

$$\begin{aligned} E_t^i m_t &\equiv E[m_t \mid z_t(i), m_{t-1}, m_{t-2}, ...] \\ &= b\, z_t(i) + (1 - b)\ \{m_{t-1} + \rho\,(m_{t-1} - m_{t-2})\} \end{aligned} \tag{1.7}$$

where

$$b \equiv \frac{\sigma^2}{\sigma^2 + \sigma_u^2}$$

represents firms' relative reliance on their private signals, which is higher when the precision of the signals is higher (σ_u is smaller) given the variance of aggregate nominal spending.

1.2.3 Higher-Order Expectations

Unlike the Lucas model, this model considers an environment among monopolistically competitive suppliers whose pricing strategy depends on the other suppliers' strategies. The prices chosen by the suppliers depend not only on their own estimates of the current aggregate demand but also on their expectations of the average estimate among the other suppliers, their expectations of the average estimate of that average estimate, and so on.

Averaging (1.7) over i, I have

$$\begin{aligned}\overline{E}_t m_t &= b\, m_t + (1-b)\ \{m_{t-1} + \rho\,(m_{t-1} - m_{t-2})\} \\ &= b\,\sigma\,\epsilon_t + \{m_{t-1} + \rho\,(m_{t-1} - m_{t-2})\},\end{aligned} \tag{1.8}$$

where $\overline{E}_t$ is the average expectations operator. The second line implies that the average estimate is not equal to the true value of m_t defined by (1.4) despite the assumption that the mean of private signals is equal to the true value. The average estimate is more close to the true value when the precision of the private signals is so higher that firms relatively more rely on them. When $\sigma_u = 0$, all firms can access to homogeneous precise signals and the average expectations operator no longer need to be defined.

The average expectations operator, defined in the case of heterogeneous information sets, does not satisfy the law of iterative expectations. The individual firm i's expectation of the average estimate (1.8) can be calculated as

$$\begin{aligned}E^i_t\,[\overline{E}_t\, m_t] &= b\,[\,b\, z_t(i) + (1-b)\ \{m_{t-1} + \rho\,(m_{t-1} - m_{t-2})\}\,] \\ &\quad +(1-b)\ \{m_{t-1} + \rho\,(m_{t-1} - m_{t-2})\}.\end{aligned}$$

Averaging again over i, I have

$$\begin{aligned}\overline{E}_t\,[\overline{E}_t\, m_t] &= b^2\, m_t + (1-b^2)\ \{m_{t-1} + \rho\,(m_{t-1} - m_{t-2})\} \\ &= b^2\,\sigma\,\epsilon_t + \{m_{t-1} + \rho\,(m_{t-1} - m_{t-2})\},\end{aligned}$$

which is different from (1.8). Therefore I need to define the j-th order average expectations as follows.

$$\begin{aligned}\overline{E}^{(0)}_t\, m_t &\equiv m_t \\ \overline{E}^{(j+1)}_t\, m_t &\equiv \overline{E}_t\,[\overline{E}^{(j)}_t\, m_t]\end{aligned}$$

The higher-order average expectations can be calculated as

$$\begin{aligned}E^i_t\,[\overline{E}^{(j)}_t\, m_t] &= b^{j+1}\, z_t(i) + (1-b^{j+1})\ \{m_{t-1} + \rho\,(m_{t-1} - m_{t-2})\} \\ \overline{E}^{(j+1)}_t\, m_t &= b^{j+1}\, m_t + (1-b^{j+1})\ \{m_{t-1} + \rho\,(m_{t-1} - m_{t-2})\} \\ &= b^{j+1}\,\sigma\,\epsilon_t + \{m_{t-1} + \rho\,(m_{t-1} - m_{t-2})\}.\end{aligned} \tag{1.9}$$

Since b is less than 1, the infinite-order average expectation converges to the expectations conditional only on common knowledge about the history of realized aggregate nominal spending.

1.2.4 Solving the model

Now I seek to find a rational expectations equilibrium defined as a set of $\{p_t, y_t\}$ which satisfies the model equations (1.1), (1.2), (1.3), and (1.5) given the exogenous process for aggregate nominal spending (1.4) and the information structure described in the preceding subsections. The key endogenous variable in the model is the reset prices, x_t. Combining equations (1.1) through (1.5), I have

$$\begin{aligned}
x_t(i) &= \frac{1}{2}(E_t^i p_t + \phi E_t^i y_t + E_t^i p_{t+1} + \phi E_t^i y_{t+1}) \\
&= \frac{1}{2}\{\phi E_t^i m_t + (1-\phi) E_t^i p_t + \phi E_t^i m_{t+1} + (1-\phi) E_t^i p_{t+1}\} \\
&= \frac{1}{2}\{\phi E_t^i m_t + \phi E_t^i m_{t+1} + (1-\phi) E_t^i x_t + \frac{1-\phi}{2} E_t^i x_{t+1} + \frac{1-\phi}{2} x_{t-1}\} \\
&= \frac{1}{2}\{\phi (2+\rho) E_t^i m_t - \phi \rho m_{t-1} \\
&\quad + (1-\phi) E_t^i x_t + \frac{1-\phi}{2} E_t^i x_{t+1} + \frac{1-\phi}{2} x_{t-1}\}.
\end{aligned}$$

The price chosen by firm i who sets its price in t depends on its estimate of the current aggregate demand, m_t, of the average price over the firms who set their prices in the same period, x_t, and also of the future average price chosen by the other group of firms, x_{t+1}. It also depends on the past realized aggregate nominal spending, m_{t-1}, and the past average price chosen by the other group of firms, x_{t-1}, which are known in period t so that the expectation operators need not to be added to these terms.

Averaging $x_t(i)$ over the group of firms who set their prices in t, I have

$$\begin{aligned}
x_t &= \frac{1}{2}\{\phi (2+\rho) \overline{E}_t m_t - \phi \rho m_{t-1} + \\
&\quad (1-\phi) \overline{E}_t x_t + \frac{1-\phi}{2} \overline{E}_t x_{t+1} + \frac{1-\phi}{2} x_{t-1}\}
\end{aligned} \tag{1.10}$$

where the average expectations operater is here defined as $\overline{E}_t(\cdot) \equiv 2\int_0^{0.5} E_t^i(\cdot)\, di$ when $t = \cdots, -2, 0, 2, \cdots$, and $\overline{E}_t(\cdot) \equiv 2\int_{0.5}^{1} E_t^i(\cdot)\, di$ when $t = \cdots, -1, 1, \cdots$.

Apart from the average expectations operator, the above equation can be viewed as a second-order difference equation for x_t like the plain two-period staggered price-setting model with full homogenous information sets. I suppose that all firms of both groups believe that the solution form of the difference equation is

$$x_t = \lambda\, x_{t-1} + C_1\, m_{t-1} + C_2\, m_{t-2} + C_3\, \sigma\, \epsilon_t, \tag{1.11}$$

where λ, C_1, C_2, and C_3 are undetermined coefficients, though they cannot precisely observe the realization of aggregate disturbance, ϵ_t, nor x_t in the current period t. The conjecture of this solution form will turn out to be correct by virtue of the assumption that the true value of x_t will be revealed to all firms of both groups in the next period $t+1$. Substituting this solution form into (1.10), I can eliminate the term of x_{t+1}.

$$\begin{aligned}
x_t &= \frac{1}{2}\{\phi\,(2+\rho)\,\overline{E}_t\, m_t - \phi\,\rho\, m_{t-1} + (1-\phi)\,\overline{E}_t\, x_t \\
&\quad + \frac{1-\phi}{2}\,(\lambda\,\overline{E}_t\, x_t + C_1\,\overline{E}_t\, m_t + C_2\, m_{t-1}) + \frac{1-\phi}{2}\, x_{t-1}\} \\
&= \frac{1}{4}\,[\{2\,\phi\,(2+\rho) + (1-\phi)\,C_1\}\,\overline{E}_t\, m_t + \{(1-\phi)\,C_2 - 2\,\phi\,\rho\}\, m_{t-1} \\
&\quad + (2+\lambda)\,(1-\phi)\,\overline{E}_t\, x_t + (1-\phi)\, x_{t-1}\,]
\end{aligned}$$

Note that $E_t^i\, \epsilon_{t+1} (= \overline{E}_t\, \epsilon_{t+1}) = 0$ for all i. Then iterative substitutions for x_t yield higher-order expectations about m_t.

$$\begin{aligned}
x_t &= \frac{2\,\phi\,(2+\rho) + (1-\phi)\,C_1}{4} \sum_{j=1}^{\infty} \left\{\frac{(2+\lambda)\,(1-\phi)}{4}\right\}^{j-1} \overline{E}_t^{(j)}\, m_t \\
&\quad + \frac{(1-\phi)\,C_2 - 2\,\phi\,\rho}{4-(2+\lambda)\,(1-\phi)}\, m_{t-1} + \frac{1-\phi}{4-(2+\lambda)\,(1-\phi)}\, x_{t-1}
\end{aligned} \tag{1.12}$$

This implies that firms consider the weighted sum of higher-order expectations up to the infinite order for choosing their prices. Using (1.9) to substitute for $\overline{E}_t^{(j)}\, m_t$, I obtain

$$\begin{aligned}
x_t &= \frac{b\,\{2\,\phi\,(2+\rho) + (1-\phi)\,C_1\}}{4-(2+\lambda)\,(1-\phi)\,b}\,\sigma\,\epsilon_t \\
&\quad + \frac{2\,\phi\,(2+\rho) + (1-\phi)\,C_1}{4-(2+\lambda)\,(1-\phi)}\,\{m_{t-1} + \rho\,(m_{t-1} - m_{t-2})\} \\
&\quad + \frac{(1-\phi)\,C_2 - 2\,\phi\,\rho}{4-(2+\lambda)\,(1-\phi)}\, m_{t-1} + \frac{1-\phi}{4-(2+\lambda)\,(1-\phi)}\, x_{t-1}
\end{aligned}$$

Matching this with the solution form (1.11), I finally identify the values of undetermined coefficients that provide a stable solution of the difference equation for x_t as follows.

$$\begin{aligned}
\lambda &= \frac{1-\sqrt{\phi}}{1+\sqrt{\phi}} < 1 \\
C_1 &= \frac{2\sqrt{\phi}}{1+\sqrt{\phi}} + \frac{2\rho\sqrt{\phi}(1+\sqrt{\phi}+\rho\sqrt{\phi})}{(1+\sqrt{\phi})\{1+\sqrt{\phi}-\rho(1-\sqrt{\phi})\}} \\
C_2 &= -\frac{2\rho\sqrt{\phi}(1+\sqrt{\phi}+\rho\sqrt{\phi})}{(1+\sqrt{\phi})\{1+\sqrt{\phi}-\rho(1-\sqrt{\phi})\}} \\
C_3 &= \frac{2b\sqrt{\phi}(1+\sqrt{\phi})(1+\sqrt{\phi}+\rho\sqrt{\phi})}{\{4-b(3-2\sqrt{\phi}-\phi)\}\{1+\sqrt{\phi}-\rho(1-\sqrt{\phi})\}}
\end{aligned}$$

The set of equilibrium paths $\{p_t,\, y_t\}$ can be calculated as

$$\begin{aligned}
p_t &= \frac{1}{2}(\lambda x_{t-1} + C_1 m_{t-1} + C_2 m_{t-2} + C_3 \sigma \epsilon_t \\
&\quad + \lambda x_{t-2} + C_1 m_{t-2} + C_2 m_{t-3} + C_3 \sigma \epsilon_{t-1}) \\
&= \lambda p_{t-1} + \frac{1}{2}\{C_1 m_{t-1} + (C_1+C_2) m_{t-2} + C_2 m_{t-3} + C_3 \sigma (\epsilon_t + \epsilon_{t-1})\} \qquad (1.13) \\
y_t &= m_t - \lambda p_{t-1} - \frac{1}{2}\{C_1 m_{t-1} + (C_1+C_2) m_{t-2} + C_2 m_{t-3} + C_3 \sigma (\epsilon_t + \epsilon_{t-1})\} \\
&= \lambda y_{t-1} + \left(1-\lambda+\rho-\frac{C_1}{2}\right) m_{t-1} - \left(\rho + \frac{C_1+C_2}{2}\right) m_{t-2} \\
&\quad - \frac{1}{2}\{C_2 m_{t-3} + C_3 \sigma (\epsilon_t + \epsilon_{t-1})\} \qquad (1.14)
\end{aligned}$$

1.2.5 Impulse Responses

From the solution of the model obtained in the previous subsection, I examine the impulse responses of output and inflation to a monetary disturbance. I compare the responses in my baseline model with those in the full-information two-period staggered price-setting model in order to extract the effects of imperfection in common knowledge. The full-information two-period staggered price-setting model in which all firms can access to homogeneous precise information about the realization of the current aggregate disturbances corresponds to an extreme case, $\sigma_u = 0$ so that $b = 1$, in my baseline model[6].

The impulse responses of price level and output to a unit positive innovation in ϵ_0 are calculated as a set of equilibrium paths where $\epsilon_0 = 1$, $\epsilon_t = 0$ for all $t \neq 0$, $p_{-1} = y_{-1} = 0$, and $\lim_{t\to\infty} y_t = 0$ in (1.13) and

[6]The other extreme case, $b = 0$, means all firms have no information about the current aggregate disturbances, which corresponds to the case of pre-determined prices or information delays studied in Section 3.1 of Chapter 3 in Woodford (2003b).

(1.14). The paths $\{\hat{p}_t, \hat{y}_t\}$ with $0 < b < 1$ are the responses in the baseline model and the paths $\{\hat{P}_t, \hat{Y}_t\}$ with $b = 1$ are those in the full-information staggered price-setting model. The analytical results of the comparison are summarized in the following proposition.

Proposition 1.1. *i) The impulse response of output in the baseline model is persistently larger than that in the full-information staggered price-setting model, i.e.,*

$$\hat{y}_t \geq \hat{Y}_t, \quad t \geq 0.$$

ii) The impulse response of inflation in the baseline model is delayed and then persistently larger than that in the full-information staggered price-setting model, i.e.,

$$\begin{aligned} \hat{p}_t - \hat{p}_{t-1} &< \hat{P}_t - \hat{P}_{t-1}, \quad t = 0, 1. \\ \hat{p}_t - \hat{p}_{t-1} &\geq \hat{P}_t - \hat{P}_{t-1}, \quad t \geq 2. \end{aligned}$$

Proof. i) Taking the difference between $\hat{y}_t$ and $\hat{Y}_t$, I have

$$\begin{aligned} \hat{y}_0 - \hat{Y}_0 &= \frac{4\sqrt{\phi}\,(1+\sqrt{\phi}+\rho\sqrt{\phi})}{(1+\sqrt{\phi})\,\{1+\sqrt{\phi}-\rho\,(1-\sqrt{\phi})\}}\,\frac{(1-b)\,\sigma}{\{4-b\,(3-2\sqrt{\phi}-\phi)\}} > 0 \\ \hat{y}_1 - \hat{Y}_1 &= (1+\lambda)\,(\hat{y}_0 - \hat{Y}_0) \\ \hat{y}_t - \hat{Y}_t &= \lambda\,(\hat{y}_{t-1} - \hat{Y}_{t-1}), \quad t \geq 2. \end{aligned}$$

ii) Taking the difference between $\hat{p}_t - \hat{p}_{t-1}$ and $\hat{P}_t - \hat{P}_{t-1}$, I have

$$\hat{p}_0 - \hat{P}_0 = -\frac{4\sqrt{\phi}\,(1+\sqrt{\phi}+\rho\sqrt{\phi})}{(1+\sqrt{\phi})\,\{1+\sqrt{\phi}-\rho\,(1-\sqrt{\phi})\}}\,\frac{(1-b)\,\sigma}{\{4-b\,(3-2\sqrt{\phi}-\phi)\}}$$

$$\begin{aligned} (\hat{p}_1 - \hat{p}_0) - (\hat{P}_1 - \hat{P}_0) &= \lambda\,(\hat{p}_0 - \hat{P}_0) \\ (\hat{p}_2 - \hat{p}_1) - (\hat{P}_2 - \hat{P}_1) &= -(1-\lambda^2)\,(\hat{p}_0 - \hat{P}_0) \\ (\hat{p}_t - \hat{p}_{t-1}) - (\hat{P}_t - \hat{P}_{t-1}) &= \lambda\,\{(\hat{p}_{t-1} - \hat{p}_{t-2}) - (\hat{P}_{t-1} - \hat{P}_{t-2})\}, \quad t \geq 3. \quad \blacksquare \end{aligned}$$

The differences between my baseline model and the full-information staggered price-setting model originate from the price adjustments in period 0. The initial response of prices in the baseline model is more sluggish due to imperfection in common knowledge; accordingly the price adjustments are delayed and the peak of inflation comes later than that in the full-information staggered price-setting model. The response of output is amplified by the imperfection in common knowledge in period 0, and continue to be larger than that in the full-information staggered price-setting model even after the precise information about the disturbances becomes common knowledge in period 1. The coefficient of the first-order autocorrelation term, λ, as well as the responsiveness to the past aggregate nominal spendings, C_1 and C_2, are not dependent on b and therefore common to the both models. While it takes the same periods for the responses of output and inflation in the both models to die away, the differences between them persist until then.

Sample sets of impulse responses for both models are shown in Figure 1.1. I set the parameter value on the strategic complimentarity, ϕ, to 0.15 following Woodford (2003a) and the AR(1) coefficient on the process for aggregate nominal spending, ρ, to 0.5 following Mankiw and Reis (2002). As for b in the baseline model, I choose the middle value, 0.5, assuming $\sigma = 1$ and $\sigma_u = 1$. With these parameter values, the size of the initial response of prices in the baseline model is about one third of that in the full-information staggered price-setting model. Accordingly the response of output is about 50 percent amplified in period 0, and this rate of amplification is unchanged (even increased) in period 3 when the output gap shrinks to less than 30 percent of its initial response.

The effects of changing parameter values are as follows. Proposition 1.1 implies that the smaller b, the larger response of output and the smaller initial response of prices due to more serious imperfection in common knowledge. A smaller ϕ, that is, a higher degree of strategic complimentarity leads to a larger λ, that is, more persistent responses, and to a smaller C_3, that is, a more sluggish initial response of prices. A smaller ρ, that is, less persistent shock process also leads to a smaller C_3 while it has no effect on λ.

Another interesting comparison is the one between the amplitude of the initial responses in the baseline model and that in a static model of imperfect common knowledge without staggered price-setting. In the static model, averaging (1.1) over i gives the average price of the whole economy as follows

$$\begin{aligned} p_t &= \overline{E}_t\, p_t + \phi\, \overline{E}_t\, y_t \\ &= \phi\, \overline{E}_t\, m_t + (1-\phi)\, \overline{E}_t\, p_t \\ &= \phi \sum_{j=1}^{\infty} (1-\phi)^{j-1}\, \overline{E}_t^{(j)}\, m_t, \end{aligned}$$

where the average expectations operator is now defined as $\overline{E}_t(\cdot) \equiv \int_0^1 E_t^i(\cdot)\, di$. Substituting (1.9) and assuming $m_{t-1} = m_{t-2} = 0$, I have

$$p_t = \frac{b\,\phi}{1 - b\,(1-\phi)}\, \sigma\, \epsilon_t. \tag{1.15}$$

The corresponding result in the baseline model is (1.13) with $p_{t-1} = m_{t-1} = m_{t-2} = m_{t-3} = \epsilon_{t-1} = \rho = 0$, that is,

$$p_t = \frac{b\,\sqrt{\phi}\,(1+\sqrt{\phi})}{4 - b\,(3 - 2\,\sqrt{\phi} - \phi)}\, \sigma\, \epsilon_t, \tag{1.16}$$

which is smaller than (1.15) unless b and ϕ are too small. The difference between (1.15) and (1.16) is the effect of dynamic higher-order expectations about the future states of the economy as well as the current state, which causes a sluggish adjustment of prices.

1.3 Public Information

In this section I introduce a noisy public signal in addition to private signals into the baseline model in order to study the consequences of more general information structure following Hellwig (2002) and Amato and Shin (2003). The public signal in the extended model may represent noisy information provided by the media, the government, or preliminary data to be revised. As Morris and Shin (2002) argued, public information in an economy where decision-makers' information sets are heterogeneous has a disproportionately large effects on their decisions.

1.3.1 Private and Public Signals

First I re-specify the firms' information set. In period t, firm i has access to not only their private signals (1.6) but also a public signal that is not necessarily precise as follows.

$$z_t^P = m_t + \sigma_v\, v_t \qquad v_t \sim N(0,\, 1) \tag{1.17}$$

where v_t is Gaussian white noise distributed independently both of ϵ_t and of $u_t(i)$ for all i. Whereas Amato and Shin (2003) assume that price-setters never know precise information about aggregate disturbances like the Woodford model, I keep the assumption that the true value of m_t will be revealed to all firms with a delay of only one period in $t+1$. Therefore the information set of firm i consists of the private and public signals and the history of realized aggregate nominal spending, of which noisy information, z_t^P, as well as precise information, $\{m_{t-s}\}_{s=1}^{\infty}$, is common knowledge. Following Hellwig (2002), firms' signal extraction for estimating m_t can be calculated as

$$\begin{aligned} E_t^i m_t &\equiv E[m_t \mid z_t(i), z_t^P, m_{t-1}, m_{t-2}, \dots] \\ &= \alpha \Delta z_t(i) + (1-\alpha) \Delta z_t^P \\ &\quad +(1-\Delta) \{m_{t-1} + \rho (m_{t-1} - m_{t-2})\} \end{aligned} \tag{1.18}$$

where

$$\alpha \equiv \frac{\sigma_v^2}{\sigma_v^2 + \sigma_u^2}$$

represents the relative importance of the private signals to the public signal in firms' signal extraction problem, which is higher when the precision of the private signals is higher (σ_u is smaller) given the precision of the public signal, and

$$\Delta \equiv \frac{\sigma^2}{\sigma^2 + \frac{\sigma_u^2 \sigma_v^2}{\sigma_u^2 + \sigma_v^2}} = \frac{\sigma^2}{\sigma^2 + \alpha \sigma_u^2}$$

represents firms' relative reliance on the private and public signals, which is higher when the precision of the composite signal is higher given the variance of aggregate nominal spending.

As in the baseline model, I calculate the higher-order expectations about m_t as follows.

$$\begin{aligned} E_t^i [\overline{E}_t^{(j)} m_t] &= (\alpha \Delta)^{j+1} z_t(i) + \frac{1-(\alpha\Delta)^{j+1}}{1-\alpha\Delta} (1-\alpha) \Delta z_t^P \\ &\quad + \left\{1 - (\alpha\Delta)^{j+1} - \frac{1-(\alpha\Delta)^{j+1}}{1-\alpha\Delta}(1-\alpha)\Delta\right\} \\ &\quad \{m_{t-1} + \rho (m_{t-1} - m_{t-2})\} \end{aligned} \tag{1.19}$$

$$
\begin{aligned}
\overline{E}_t^{(j+1)} m_t &= (\alpha\,\Delta)^{j+1} m_t + \frac{1-(\alpha\,\Delta)^{j+1}}{1-\alpha\,\Delta}(1-\alpha)\,\Delta\, z_t^P \\
&\quad + \left\{1-(\alpha\,\Delta)^{j+1} - \frac{1-(\alpha\,\Delta)^{j+1}}{1-\alpha\,\Delta}(1-\alpha)\,\Delta\right\} \\
&\quad \{m_{t-1} + \rho\,(m_{t-1} - m_{t-2})\} \\
&= \left\{(\alpha\,\Delta)^{j+1} + \frac{1-(\alpha\,\Delta)^{j+1}}{1-\alpha\,\Delta}(1-\alpha)\,\Delta\right\}\sigma\,\epsilon_t \\
&\quad + \frac{1-(\alpha\,\Delta)^{j+1}}{1-\alpha\,\Delta}(1-\alpha)\,\Delta\,\sigma_v\, v_t \\
&\quad + \{m_{t-1} + \rho\,(m_{t-1} - m_{t-2})\} \qquad (1.20)
\end{aligned}
$$

Compared with firm i's own estimate of m_t (1.18), its expectation of the higher-order average expectations (1.19) disproportionately overreact to public information including the public signal and the history of realized aggregate nominal spendings rather than private information. The infinite-order average expectation converges to the expectations conditional only on the common knowledge.

1.3.2 Effects of Monetary Disturbances

Substituting (1.20) into (1.12) in the baseline model, I obtain a stable solution of the difference equation $x_t = \lambda\, x_{t-1} + C_1\, m_{t-1} + C_2\, m_{t-2} + C_3\,\sigma\,\epsilon_t + C_4\,\sigma_v\, v_t$ where λ, C_1, and C_2 are the same as in the baseline model and

$$
\begin{aligned}
C_3 &= \frac{2\sqrt{\phi}\,(1+\sqrt{\phi})\,(1+\sqrt{\phi}+\rho\sqrt{\phi})}{\{1+\sqrt{\phi}-\rho\,(1-\sqrt{\phi})\}} \\
&\quad \left[A\,\frac{\alpha\,\Delta}{4-\alpha\,\Delta\,(3-2\sqrt{\phi}-\phi)} + (1-A)\,\frac{1}{(1+\sqrt{\phi})^2}\right], \\
C_4 &= \frac{8\sqrt{\phi}\,(1+\sqrt{\phi}+\rho\sqrt{\phi})}{(1+\sqrt{\phi})\,\{1+\sqrt{\phi}-\rho\,(1-\sqrt{\phi})\}}\,\frac{(1-\alpha)\,\Delta}{\{4-\alpha\,\Delta\,(3-2\sqrt{\phi}-\phi)\}},
\end{aligned}
$$

and $A \equiv (1-\Delta)\,/\,(1-\alpha\,\Delta)$.

As before, I examine the impulse responses of output and inflation to a monetary disturbance, a unit positive innovation in ϵ_0, and compare them with those in the baseline model as well as the full-information staggered price-setting model. The baseline model without the public signal corresponds to the case of $\sigma_v = \infty$ so that $\alpha = 1$ and $\Delta = b$, and the full-information staggered price-setting model corresponds to that of $\sigma_u = 0$ and $\sigma_v = \infty$ so that $\alpha = 1$ and $\Delta = 1$. The responses of price level and output in this extended model are calculated as a set of equilibrium paths $\{\hat{p}_t^P, \hat{y}_t^P\}$ with $0 < \alpha < 1$ and $0 < \Delta < 1$. The analytical results of the comparison are summarized in the following proposition.

Proposition 1.2. *i) The impulse response of output in the extended model is persistently larger than that in the full-information staggered price-setting model but smaller than that in the baseline model, i.e.,*

$$\hat{y}_t \geq \hat{y}_t^P \geq \hat{Y}_t, \quad t \geq 0.$$

ii) The impulse response of inflation in the extended model is delayed and then persistently larger than that in the full-information staggered price-setting model but smaller than that in the baseline model, i.e.,

$$\begin{aligned} \hat{p}_t - \hat{p}_{t-1} &< \hat{p}_t^P - \hat{p}_{t-1}^P < \hat{P}_t - \hat{P}_{t-1}, \quad t = 0, 1. \\ \hat{p}_t - \hat{p}_{t-1} &\geq \hat{p}_t^P - \hat{p}_{t-1}^P \geq \hat{P}_t - \hat{P}_{t-1}, \quad t \geq 2. \end{aligned}$$

Proof. i) Taking the difference between $\hat{y}_t^P$ and $\hat{Y}_t$, I have

$$\begin{aligned} \hat{y}_0^P - \hat{Y}_0 &= \frac{4\sqrt{\phi}\,(1+\sqrt{\phi}+\rho\sqrt{\phi})}{(1+\sqrt{\phi})\,\{1+\sqrt{\phi}-\rho\,(1-\sqrt{\phi})\}} \frac{(1-\Delta)\,\sigma}{\{4-\alpha\Delta\,(3-2\sqrt{\phi}-\phi)\}} > 0 \\ \hat{y}_1^P - \hat{Y}_1 &= (1+\lambda)\,(\hat{y}_0^P - \hat{Y}_0) \\ \hat{y}_t^P - \hat{Y}_t &= \lambda\,(\hat{y}_{t-1}^P - \hat{Y}_{t-1}), \quad t \geq 2. \end{aligned}$$

These are smaller than $\hat{y}_t - \hat{Y}_t$ in the proof of Proposition 1.1 where $\alpha = 1$ and $\Delta = \beta$.

ii) Taking the difference between $\hat{p}_t^P - \hat{p}_{t-1}^P$ and $\hat{P}_t - \hat{P}_{t-1}$, I have

$$\begin{aligned} \hat{p}_0^P - \hat{P}_0 &= -\frac{4\sqrt{\phi}\,(1+\sqrt{\phi}+\rho\sqrt{\phi})}{(1+\sqrt{\phi})\,\{1+\sqrt{\phi}-\rho\,(1-\sqrt{\phi})\}} \\ &\quad \frac{(1-\Delta)\,\sigma}{\{4-\alpha\Delta\,(3-2\sqrt{\phi}-\phi)\}} \\ (\hat{p}_1^P - \hat{p}_0^P) - (\hat{P}_1 - \hat{P}_0) &= \lambda\,(\hat{p}_0^P - \hat{P}_0) \\ (\hat{p}_2^P - \hat{p}_1^P) - (\hat{P}_2 - \hat{P}_1) &= -(1-\lambda^2)\,(\hat{p}_0^P - \hat{P}_0) \\ (\hat{p}_t^P - \hat{p}_{t-1}^P) - (\hat{P}_t - \hat{P}_{t-1}) &= \lambda\,\{\,(\hat{p}_{t-1}^P - \hat{p}_{t-2}^P) - (\hat{P}_{t-1} - \hat{P}_{t-2})\,\}, \quad t \geq 3. \end{aligned}$$

$(\hat{p}_t^P - \hat{p}_{t-1}^P) - (\hat{P}_t - \hat{P}_{t-1})$ for $t = 0, 1$ are larger and for $t \geq 2$ are smaller than $(\hat{p}_t - \hat{p}_{t-1}) - (\hat{P}_t - \hat{P}_{t-1})$ in the proof of Proposition 1.1 where $\alpha = 1$

and $\Delta = \beta$. ■

The provision of the public signal allows the firms to gain common knowledge so that they can calculate the higher-order average expectations more precisely, which alleviates the sluggishness in their initial adjustment of prices to some extent. Compared with the full-information staggered price-setting model, however, the response of inflation is delayed and that of output is amplified as long as their information sets are heterogeneous.

Sample sets of impulse responses for the extended model are shown in Figure 1.2 as well as those for the two models shown in Figure 1.1. I set σ_v as well as σ_u and σ to 1 so that $\alpha = 0.5$ and $\delta = 2/3$. Other parameter values are the same as in Figure 1.1. With these parameter values, the size of the initial response of prices in the extended model is about 60 percent of that in the full-information staggered price-setting model. Accordingly the response of output is about 30 percent amplified in period 0, and this rate of amplification is unchanged (even increased) in period 3 when the output gap shrinks to less than 30 percent of its initial response. The response of inflation is slightly delayed but peaks at the same period as in the full-information staggered price-setting model and earlier than in the baseline model.

The effects of changing parameter values are the same as in the baseline model. Proposition 1.2 implies that the larger α and the smaller Δ, the larger response of output and the smaller initial response of prices due to the more serious imperfection in common knowledge.

The comparison of the amplitude of the initial responses in the extended model with that in a static model including the public signal as well as private signals can be made as before. The average price in the static model is

$$p_t = \frac{\Delta\,(1-\alpha+\alpha\,\phi)}{1-\alpha\,\Delta\,(1-\phi)}\,\sigma\,\epsilon_t. \tag{1.21}$$

The corresponding result in the extended model is

$$\begin{aligned} p_t \;&=\; \sqrt{\phi}\,(1+\sqrt{\phi}) \\ &\quad \left[A\,\frac{\alpha\,\Delta}{4-\alpha\,\Delta\,(3-2\sqrt{\phi}-\phi)} + (1-A)\,\frac{1}{(1+\sqrt{\phi})^2}\right]\sigma\,\epsilon_t, \end{aligned} \tag{1.22}$$

which is smaller than (1.21) unless Δ and ϕ are too small and α is too close to 1. As before, the difference between (1.21) and (1.22) is the effect of dynamic higher-order expectations about the future states of the economy as well as the current state, which causes a sluggish adjustment of prices.

1.3.3 Effects of Informational Disturbances

While the public signal reduces uncertainty in the firms' higher-order average expectations, it brings another aggregate uncertainty as it contains noise. Under imperfect common knowledge, firms disproportionately overreact to the noisy public signal, which could destabilize the economy. I consider this side effect of public information, studied by Morris and Shin (2002), by examining the impulse responses to a disturbance in the public signal in my model.

The responses of price level and output to a unit positive innovation in v_0 are calculated as a set of equilibrium paths $\{\tilde{p}_t^P, \tilde{y}_t^P\}$ with $0 < \alpha < 1$ and $0 < \Delta < 1$. The responses of output and inflation are given by

$$\begin{aligned}
\tilde{y}_0^P &= -\frac{4\sqrt{\phi}(1+\sqrt{\phi}+\rho\sqrt{\phi})}{(1+\sqrt{\phi})\{1+\sqrt{\phi}-\rho(1-\sqrt{\phi})\}}\frac{(1-\alpha)\Delta\sigma_v}{\{4-\alpha\Delta(3-2\sqrt{\phi}-\phi)\}} \\
\tilde{y}_1^P &= (1+\lambda)\tilde{y}_0^P \\
\tilde{y}_t^P &= \lambda\tilde{y}_{t-1}^P, \quad t \geq 2
\end{aligned}$$

and

$$\begin{aligned}
\tilde{p}_0^P &= \frac{4\sqrt{\phi}(1+\sqrt{\phi}+\rho\sqrt{\phi})}{(1+\sqrt{\phi})\{1+\sqrt{\phi}-\rho(1-\sqrt{\phi})\}}\frac{(1-\alpha)\Delta\sigma_v}{\{4-\alpha\Delta(3-2\sqrt{\phi}-\phi)\}} \\
\tilde{p}_1^P - \tilde{p}_0^P &= \lambda\tilde{p}_0^P \\
\tilde{p}_2^P - \tilde{p}_1^P &= -(1-\lambda^2)\tilde{p}_0^P \\
\tilde{p}_t^P - \tilde{p}_{t-1}^P &= \lambda(\tilde{p}_{t-1}^P - \tilde{p}_{t-2}^P), \quad t \geq 3.
\end{aligned}$$

Firms raise their prices reacting to an upward-biased public signal. Since the exogenous process for aggregate nominal spending is not affected by the informational disturbance, the increase of prices leads to a corresponding decrease of output.

Improving precision of the public signal does not necessarily dampen the amplitude of those responses. While a small σ_v directly leads to a small response of prices, it also leads to firms' large reliance on the public signal, that is, small α and large Δ, therefore indirectly causes high responsiveness to the informational disturbance. If the latter, indirect effect dominates, firms over-react to the public signal so much that improving precision of the public signal over-exposes firms to the noise and amplify the responses.

Sample sets of impulse responses of output and inflation to a *negative* informational disturbance are shown in Figure 1.3.1, where I choose the same parameter values as in Figure 1.2. While firms reduce their prices reacting to the downward-biased public signal, output increases correspondingly. When the output gap starts shrinking, prices starts increasing so that the peak of inflation comes later than that of output. Combining this pattern of responses to a negative informational disturbance with those to a positive monetary disturbance examined in the previous subsection makes the response of inflation more delayed and that of output more amplified, which could offset the effects of providing the public signal that alleviates the sluggishness in the initial adjustment of prices.

In Figure 1.3.2, I plot the initial response of prices, $\tilde{p}_0^P$, as a function of the precision of the public signal, σ_v. While improving precision of the public signal in the range of high precision (small σ_v) dampens the amplitude of the response, it amplifies the response in the range of low precision (large σ_v). This implies that the provision of a public signal could destabilize the economy unless it has sufficiently high precision.

1.4 General Staggered Price-Setting

The models so far are based on a simple two-period staggered price-setting. In this section, I consider more general price-setting environment allowing for multiple-period staggered price-setting. In particular, I am interested in the Calvo-type price-setting which is most widely used in the recent New Keynesian macroeconomic models.

1.4.1 Set-up

Consider an economy where a proportion θ_1 of monopolistically competitive firms $i \in \Theta_1$ set their prices in each period, θ_2 of firms $i \in \Theta_2$ set their prices in every other period, θ_3 of firms $i \in \Theta_3$ in every three periods, and θ_4 of firms $i \in \Theta_4$ in every four periods. Within Θ_j, the periods of price-setting are staggered among equally-sized j groups. I assume the maximum fixed-price length is four periods so that $\sum_{j=1}^{4} \theta_j = 1$.

While the static optimal price-setting condition in the baseline model (1.1) is unchanged, the individual price equation (1.2) is modified as follows.

$$x_t(i) = \begin{cases} p_t^*(i) & \text{for } i \in \Theta_1 \\ \frac{1}{2}\left(p_t^*(i) + E_t^i p_{t+1}^*(i)\right) & \text{for } i \in \Theta_{2,t} \\ \frac{1}{3}\left(p_t^*(i) + E_t^i p_{t+1}^*(i) + E_t^i p_{t+2}^*(i)\right) & \text{for } i \in \Theta_{3,t} \\ \frac{1}{4}\left(p_t^*(i) + E_t^i p_{t+1}^*(i) + E_t^i p_{t+2}^*(i) + E_t^i p_{t+3}^*(i)\right) & \text{for } i \in \Theta_{4,t} \end{cases}$$

where $\Theta_{j,t}$ is $1/j$ of the firms within Θ_j who set their prices in period t. Averaging over those firms $i \in \Theta_1 \cup \Theta_{2,t} \cup \Theta_{3,t} \cup \Theta_{4,t}$, I have

$$x_t = \omega_1 p_t^* + \omega_2 \overline{E}_t p_{t+1}^* + \omega_3 \overline{E}_t p_{t+2}^* + \omega_4 \overline{E}_t p_{t+3}^* \tag{1.23}$$

where

$$p_t^* = \overline{E}_t p_t + \phi \overline{E}_t y_t$$

and

$$\begin{aligned} \omega_1 &= \theta_1 + \frac{1}{2}\theta_2 + \frac{1}{3}\theta_3 + \frac{1}{4}\theta_4 \\ \omega_2 &= \frac{1}{2}\theta_2 + \frac{1}{3}\theta_3 + \frac{1}{4}\theta_4 \\ \omega_3 &= \frac{1}{3}\theta_3 + \frac{1}{4}\theta_4 \\ \omega_4 &= \frac{1}{4}\theta_4. \end{aligned}$$

The average expectations operator $\overline{E}_t$ is now defined as the average of E_t^i over $i \in \Theta_1 \cup \Theta_{2,t} \cup \Theta_{3,t} \cup \Theta_{4,t}$.

The overall price index is given by the weighted sum of prices set in the current and past periods as in the baseline model. (1.3) is now rewritten as

$$p_t = \omega_1 x_t + \omega_2 x_{t-1} + \omega_3 x_{t-2} + \omega_4 x_{t-3}. \tag{1.24}$$

The demand side of the economy, specified by (1.4) and (1.5), is the same as in the baseline model. The information structure is the same as in Section 1.3 where the public as well as private signals is incorporated.

1.4.2 Impulse responses

As before, I examine the impulse responses of output and inflation to a monetary disturbance, and compare them with those in the corresponding full-information staggered price-setting model.

Since the analytical results corresponding to Proposition 1.2 in Section 1.3 are qualitatively little changed, I provide them in Appendix. Here I examine sample sets of impulse responses shown in Figure 1.4, 1.5 and 1.6. In Figure 1.4, in order to analogize the model with the Calvo-type price-setting, I set θ_j to 0.5, 0.25, 0.15, and 0.1 for $j = 1$ to 4 respectively, which implies the constant probability with which each firm gets the opportunity to change its price is approximated by $\omega_1 = 0.7$. Other parameter values are the same as in the extended model in Section 1.3.

As in the two-period staggered price-setting models in the preceding sections, the size of the initial response of prices in the case with only private signals is about one third of that in the corresponding full-information staggered price-setting model, while that in the case with both public and private signals is about 60 percent. Accordingly the initial response of output is more than 80 percent amplified in the case with only private signals, while that is about 50 percent amplified in the case with both public and private signals. Whereas the above rates of amplification are larger than those in the two-period staggered price-setting models in period 0, they monotonically decrease as the output gap shrinks. However, the response of output continues to be amplified, that is, persistently larger than that in the corresponding full-information staggered price-setting model.

One of the main problems in the Calvo-type price-setting model I mentioned in the introduction is that the price level jumps in the period of disturbance and inflation responds earlier than output gap. The above results show that this problem can be overcome by incorporating imperfect common knowledge into the model. It amplifies the initial response of output and delays the response of inflation by making the initial adjustment of prices more sluggish.

Meanwhile, the Woodford model of imperfect common knowledge has a problem that the response of output to a monetary disturbance is relatively weak compared with the Calvo-type price-setting model. The above results show that this problem can also be overcome by integrating imperfect common knowledge and staggered price-setting. The response of output in my integrated model is unambiguously amplified by imperfection in common knowledge.

In Figure 1.5, I set θ_j to 0 for $j = 1$ to 3 and $\theta_4 = 1$, that is, only four-period staggered price-setting with other parameter values unchanged. The size of the initial response of prices in the case with only private signals is about 40 percent of that in the corresponding full-information staggered price-setting model, while that in the case with both public and private signals is more than 60 percent. Following that, the responses of inflation in both cases are slightly delayed but they peak at the same period as in the full-information staggered price-setting model. The initial responses of output are relatively large but the rate of amplification is just about 15 percent in the case with only private signals and about 9 percent in the case with both public and private signals. Those amplification rates are unchanged (even increased) as the output gap shrinks. Since the share of the prices chosen in the period of disturbance under imperfect common knowledge, ω_1, is smaller than that in the previous Calvo-analogized case, the responses are closer to those in the corresponding full-information staggered price-setting model.

Lastly, to complete the discussion, I examine the responses to an informational disturbance. Sample sets of impulse responses of output and inflation to a *negative* informational disturbance for both the Calvo-analogized case and the four-period staggered price-setting case are shown in Figure 1.6. While the amplitude of the responses is larger in the former case, the persistence is larger and the peak comes later in the latter case.

1.5 Concluding Remarks

In this chapter I propose a model that integrates the Woodford (2003a) imperfect common knowledge model with the Taylor-Calvo staggered price-setting model in order to explain plausibly the observed effects of monetary policy. I drop the Woodford's unrealistic assumption that all price-setters never know the widely available data on the aggregate demand nor even the actual quantity they sold at their own price, and instead assume that the true state of the economy is revealed to all price-setters with a delay of one period. With staggered price-setting, however, the model can generate persistent real effects of monetary policy. The average prices chosen in each period depends on the higher-order expectations about not only the current state of the economy but also the future states during the periods the prices will be fixed, which makes the initial adjustment of prices to a monetary disturbance more sluggish than that in both the static imperfect common knowledge model and the full-information staggered price-setting model. The response of inflation is delayed following the sluggish initial response and the response of output continues to be amplified.

The above results are robust in the models in Section 1.3 where a noisy public signal in addition to private signals is introduced and in Section 1.4 where the staggered price-setting environment is generalized. While the provision of the public signal alleviates the sluggishness in the initial adjustment of prices to monetary disturbances, it exposes firms to additional disturbances, namely informational noise, and could destabilize the economy. The Calvo-analogized model in Section 1.4 overcomes the problem in the full-information Calvo model that the price level jumps in the period of disturbance and inflation responds earlier than output, and also overcomes the problem in the Woodford model that the response of output is relatively weak compared with the full-informaiton Calvo model.

Although the models in this chapter are too simple for practical uses such as policy and empirical research, it does not seem too difficult to develop a more realistic and richer dynamic general equilibrium model based on them. Policy implications derived from the extended model can be concerned with the central bank's communication strategy such as commitments and transparency, which corresponds to the precision of the public signal in the model, as well as optimally stabilizing monetary policy rules. As for empirical implications, the effects of information structure such as the relative importance of public information to private information on the characteristics of the monetary policy effects such as the persistence of inflation could be estimated. Although there have already been a few recent attempts to develop richer dynamic general equilibrium models to address those issues[7], most of them assume unrealistically too much unawareness or inattentiveness like the Woodford model. The models in this chapter assume more realistic in-

[7]Amato and Shin (2003) consider a targeting rule expressed in terms of the price level, the output gap, and the natural rate of interest, while Adam (2004) studies optimal monetary policy. Hellwig (2002, 2004) studies the welfare costs of heterogeneous information and the role of monetary policy in the model where precise information will be eventually revealed as in my models. Kawamoto (2004) examines the role of monetary policy in the model with imperfect common knowledge about technology shocks.

formation sets of economic agents, and also have tractability and flexibility. I hope they serve as an alternative building block for future research in various directions.

Appendix

Solution of the General Staggered Price-Setting Model

The key equations of the model are (1.23) and (1.24).

$$\begin{aligned} x_t &= \omega_1\, p_t^* + \omega_2\, \overline{E}_t\, p_{t+1}^* + \omega_3\, \overline{E}_t\, p_{t+2}^* + \omega_4\, \overline{E}_t\, p_{t+3}^* \\ p_t &= \omega_1\, x_t + \omega_2\, x_{t-1} + \omega_3\, x_{t-2} + \omega_4\, x_{t-3} \end{aligned}$$

Combining these, I have

$$\begin{aligned} x_t &= \phi\, (M_1\, \overline{E}_t\, m_t + M_2\, m_{t-1}) \\ &\quad +(1-\phi)\,(W_1\, x_{t-3} + W_2\, x_{t-2} + W_3\, x_{t-1} \\ &\quad +W\, \overline{E}_t\, x_t + W_3\, \overline{E}_t\, x_{t+1} + W_2\, \overline{E}_t\, x_{t+2} + W_1\, \overline{E}_t\, x_{t+3}) \end{aligned}$$

where

$$\begin{aligned} M_1 &\equiv \omega_1 + \omega_2\,(1+\rho) + \omega_3\,(1+\rho+\rho^2) + \omega_4\,(1+\rho+\rho^2+\rho^3) \\ M_2 &\equiv -\{\,\omega_2\,\rho + \omega_3\,(\rho+\rho^2) + \omega_4\,(\rho+\rho^2+\rho^3)\,\} \\ W_1 &\equiv \omega_1\,\omega_4 \\ W_2 &\equiv \omega_1\,\omega_3 + \omega_2\,\omega_4 \\ W_3 &\equiv \omega_1\,\omega_2 + \omega_2\,\omega_3 + \omega_3\,\omega_4 \\ W &\equiv \omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2. \end{aligned}$$

Suppose that all firms believe that the solution form of the above difference equation for x_t is

$$\begin{aligned} x_t &= \lambda_1\, x_{t-1} + \lambda_2\, x_{t-2} + \lambda_3\, x_{t-3} \\ &\quad +C_1\, m_{t-1} + C_2\, m_{t-2} + C_3\, \sigma\, \epsilon_t + C_4\, \sigma_v\, v_t. \end{aligned}$$

As before, I identify the undetermined coefficients as follows.

$$
\begin{aligned}
C_3 &= M^* \left[A \frac{\alpha \Delta}{1 - \alpha \Delta W^*} + (1 - A) \frac{1}{1 - W^*} \right] \\
C_4 &= M^* \frac{(1 - \alpha) \Delta}{(1 - \alpha \Delta W^*)(1 - W^*)}
\end{aligned}
$$

where $A \equiv (1 - \Delta) / (1 - \alpha \Delta)$ and

$$
\begin{aligned}
M^* &\equiv \phi M_1 + (1 - \phi) \{ W_3 C_1 + W_2 C_2 + (W_2 \lambda_1 + W_1 \lambda_2) C_1 + W_1 \lambda_1 C_2 \\
&\quad + W_1 \lambda_1^2 C_1 + (W_2 C_1 + W_1 C_2 + W_1 \lambda_1 C_1 + W_1 C_1)(1 + \rho) + W_1 C_1 \rho \} \\
W^* &\equiv (1 - \phi) \{ W + W_3 \lambda_1 + W_2 \lambda_2 + W_1 \lambda_3 \\
&\quad + (W_2 \lambda_1 + W_1 \lambda_2) \lambda_1 + W_1 \lambda_1 \lambda_2 + W_1 \lambda_1^3 \},
\end{aligned}
$$

while λ_1, λ_2, λ_3, C_1, and C_2 are determined independently of α and Δ by solving the following difference equation

$$
\left(\frac{1}{1 - \phi} - W_1 L^{-3} - W_2 L^{-2} - W_3 L^{-1} - W - W_3 L - W_2 L^2 - W_1 L^3 \right) x_t = \frac{\phi}{1 - \phi} (M_1 + M_2 L) m_t,
$$

where L is the lag operator defined as $L x_t \equiv x_{t-1}$.

Now I compare the responses of price level (or inflation) and output to a unit positive disturbance in ϵ_0, calculated as a set of equilibrium paths $\{ \hat{p}_t^P, \hat{y}_t^P \}$ with $0 < \alpha < 1$ and $0 < \Delta < 1$, with those in the case of only private signals, $\{ \hat{p}_t, \hat{y}_t \}$ with $\alpha = 1$ and $0 < \Delta < 1$, and those in the case of full homogenous information, $\{ \hat{P}_t, \hat{Y}_t \}$ with $\alpha = 1$ and $\Delta = 1$.

The difference in the initial response of prices is given by

$$
\hat{p}_0^P - \hat{P}_0 = - \frac{\omega_1 M^* (1 - \Delta) \sigma}{(1 - \alpha \Delta W^*)(1 - W^*)},
$$

which is negative unless the sign of $(1 - \alpha \Delta W^*)$ is different from that of $(1 - W^*)$. Correspondingly, the difference in the initial response of output is given by

$$
\hat{y}_0^P - \hat{Y}_0 = \frac{\omega_1 M^* (1 - \Delta) \sigma}{(1 - \alpha \Delta W^*)(1 - W^*)},
$$

which is positive on the same condition. The absolute values of the above differences are increasing in α, which implies that $\hat{p}_0^P > \hat{p}_0$ and $\hat{y}_0^P < \hat{y}_0$, that is, the provision of the public signal dampens the initial responses.

The difference in the response of output evolves as follows.

$$\begin{aligned}
\hat{y}_t^P - \hat{Y}_t &= \lambda_1 (\hat{y}_{t-1}^P - \hat{Y}_{t-1}) + \lambda_2 (\hat{y}_{t-2}^P - \hat{Y}_{t-2}) + \lambda_3 (\hat{y}_{t-3}^P - \hat{Y}_{t-3}) \\
&\quad + \frac{\omega_{t+1}}{\omega_1} (\hat{y}_0^P - \hat{Y}_0), \quad 0 \le t \le 3. \\
&= \lambda_1 (\hat{y}_{t-1}^P - \hat{Y}_{t-1}) + \lambda_2 (\hat{y}_{t-2}^P - \hat{Y}_{t-2}) + \lambda_3 (\hat{y}_{t-3}^P - \hat{Y}_{t-3}), \quad t \ge 4,
\end{aligned}$$

which implies that the response of output continues to be amplified as long as positive roots λ_1, λ_2, and λ_3 are chosen.

Meanwhile, the difference in the response of inflation, between $\hat{\pi}_t^P \equiv \hat{p}_t - \hat{p}_{t-1}$ and $\hat{\Pi}_t \equiv \hat{P}_t - \hat{P}_{t-1}$, evolves as follows.

$$\begin{aligned}
\hat{\pi}_t^P - \hat{\Pi}_t &= \lambda_1 (\hat{\pi}_{t-1}^P - \hat{\Pi}_{t-1}) + \lambda_2 (\hat{\pi}_{t-2}^P - \hat{\Pi}_{t-2}) + \lambda_3 (\hat{\pi}_{t-3}^P - \hat{\Pi}_{t-3}) \\
&\quad - \frac{\omega_t - \omega_{t+1}}{\omega_1} (\hat{p}_0^P - \hat{P}_0), \quad 1 \le t \le 3. \\
&= \lambda_1 (\hat{\pi}_3^P - \hat{\Pi}_3) + \lambda_2 (\hat{\pi}_2^P - \hat{\Pi}_2) + \lambda_3 (\hat{\pi}_1^P - \hat{\Pi}_1) \\
&\quad - \frac{\omega_4}{\omega_1} (\hat{p}_0^P - \hat{P}_0), \quad t = 4. \\
&= \lambda_1 (\hat{\pi}_{t-1}^P - \hat{\Pi}_{t-1}) + \lambda_2 (\hat{\pi}_{t-2}^P - \hat{\Pi}_{t-2}) + \lambda_3 (\hat{\pi}_{t-3}^P - \hat{\Pi}_{t-3}), \quad t \ge 5,
\end{aligned}$$

which implies that the sign of the difference in period 0 will be reversed later, that is, the response of inflation is delayed after the initial sluggish adjustment of prices.

References

[1] Adam, K. (2004), "Optimal Monetary Policy with Imperfect Common Knowledge," CEPR Discussion Paper, 4594.

[2] Amato, J. D. and Shin, H. S. (2003), "Public and Private Information in Monetary Policy Models," London School of Economics.

[3] Barnanke, B. S. and Gertler, M. (1995), "Inside the Black Box: The Credit Channel of Monetary Policy Transmission," *Journal of Economic Perspectives*, 9(4): 27-48.

[4] Ball, L. and Cecchetti, S. G. (1988), "Imperfect Information and Staggered Price Setting," *American Economic Review*, 78(5): 999-1018.

[5] Blanchard, O. and Kiyotaki, N. (1987), "Monopolistic competition and the Effects of Aggregate Demand," *American Economic Review*, 77(4): 647-666.

[6] Calvo, G. A. (1983), "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 12: 383-398.

[7] Christiano, L. J., Eichenbaum, M. and Evans, C. L. (1998), "Modeling Money," NBER Working paper, 6371.

[8] Christiano, L. J., Eichenbaum, M. and Evans, C. L. (2001), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," NBER Working paper, 8403.

[9] Hellwig, C. (2002), "Public Announcements, Adjustment Delays and the Business Cycles," University of California at Los Angeles.

[10] Hellwig, C. (2004), "Heterogeneous Information and the Benefits of Transparency," University of California at Los Angeles.

[11] Kawamoto, T. (2004), "Technology Shocks and the Role of Monetary Policy in the Beauty Contest Monetarist Model," Bank of Japan, IMES Discussion Paper, 2004-E-10.

[12] Keynes, J. M. (1936), *The General Theory of Employment, Interest, and Money*, MacMillan, London.

[13] Kiley, M. T. (2000), "Endogenous Price Stickiness and Business Cycle Persistence," *Journal of Money, Credit, and Banking*, 32(1): 28-53.

[14] Lucas, R. Jr. (1972), "Expectations and the Neutrality of Money," *Journal of Economic Theory*, 4: 103-124.

[15] Lucas, R. Jr. (1975), "An Equilibrium Model of the Business Cycle," *Journal of Political Economy*, 83(6): 1113-1236.

[16] Mankiw, N. G. and Reis, R. (2002), "Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Philips Curve," *Quarterly Journal of Economics*, 117(4): 1295-1328.

[17] Morris, S. and Shin, H. S. (2002), "Social Value of Public Information," *American Economic Review*, 92(5): 1521-1534.

[18] Morris, S. and Shin, H. S. (2003), "Global Games: Theory and Applications," in Dewatripont, M., Hansen, L. P. and Turnovsky, S. (eds) *Advances in Economics and Econometrics: Theory and Applications, Eighth World Congress*, Cambridge University Press.

[19] Phelps, E. S. (1970), "Introduction: The New Microeconomics in Employment and Inflation Theory," in Phelps, E., et al. *Microeconomic Foundations of Employment and Inflation Theory*, Norton, New York.

[20] Taylor, J. B.(1980), "Aggregate Dynamics and Staggered Contracts," *Journal of Political Economy*, 88(1): 1-22.

[21] Woodford, M. (2003a), "Imperfect Common Knowledge and the Effects of Monetary Policy," in Aghion, P., Frydman, R., Stiglitz, J. and Woodford, M. (eds) *Knowledge, Information and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, Princeton University Press.

[22] Woodford, M. (2003b), *Interest and Prices*, Princeton University Press.

Figure 1.1: Baseline Model

Figure 1.1.1: Responses of OUTPUT to a positive monetary disturbance

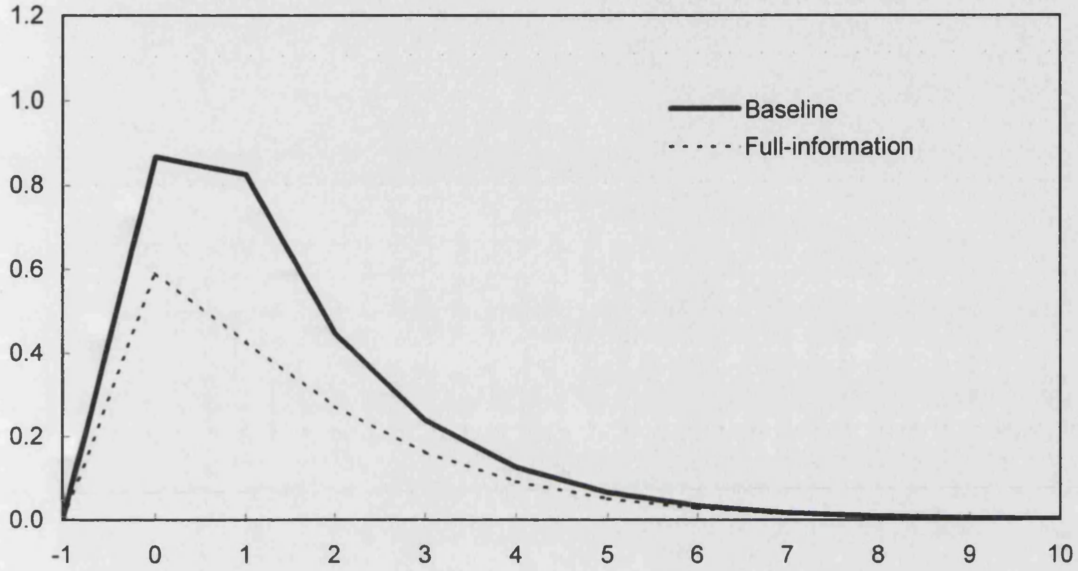


Figure 1.1.2: Responses of INFLATION to a positive monetary disturbance

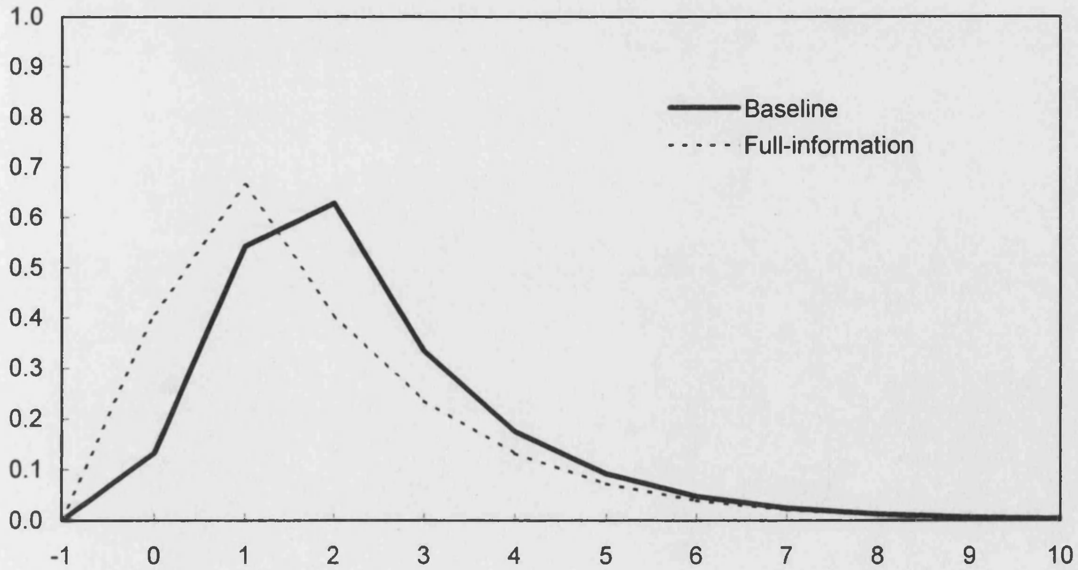


Figure 1.2: Extended Model (Monetary Disturbance)

Figure 1.2.1: Responses of OUTPUT to a positive monetary disturbance

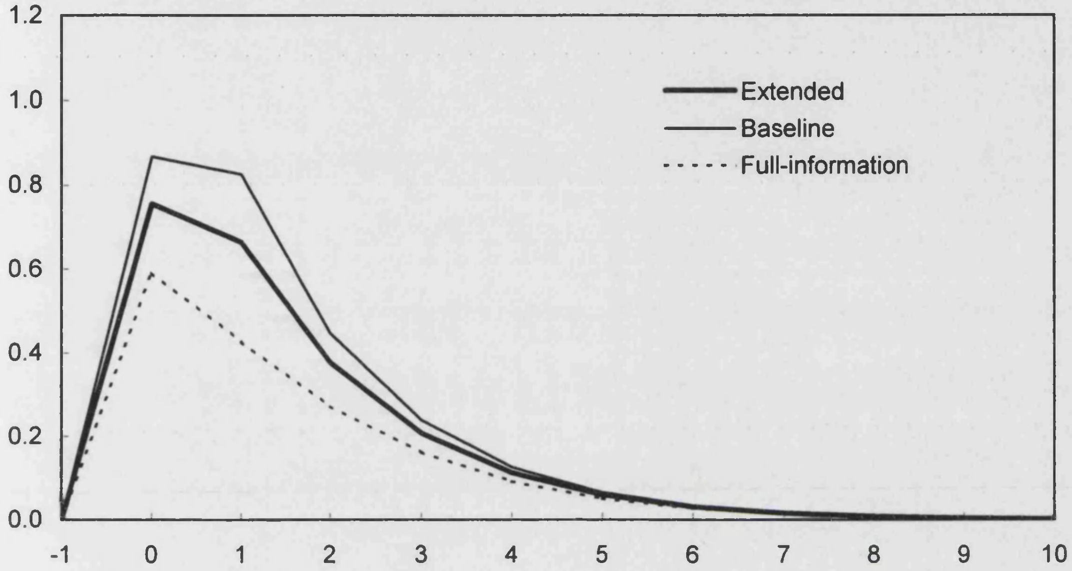


Figure 1.2.2: Responses of INFLATION to a positive monetary disturbance

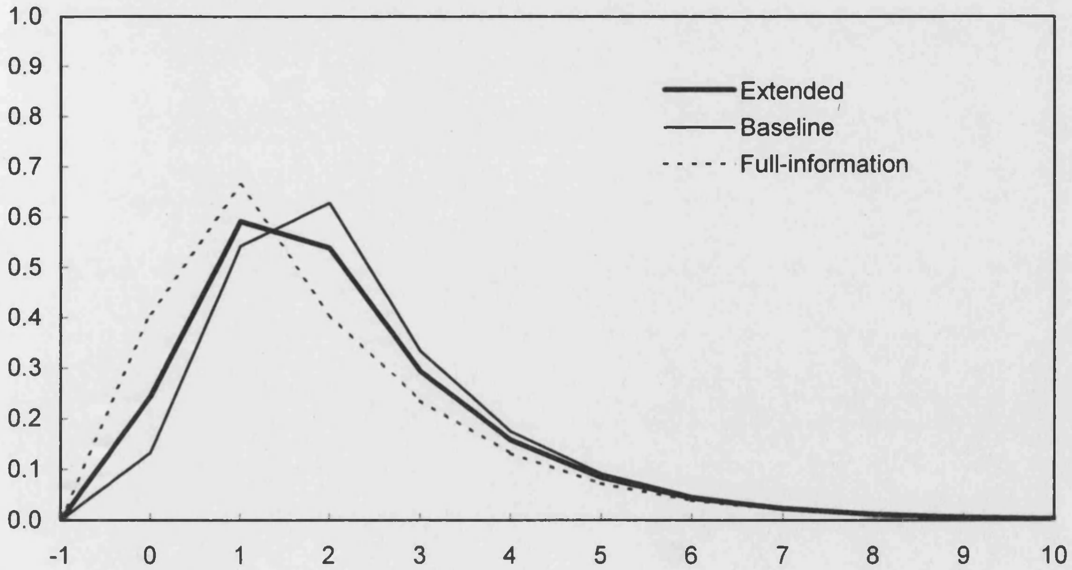


Figure 1.3: Extended Model (Informational Disturbance)

Figure 1.3.1: Responses to a NEGATIVE informational disturbance

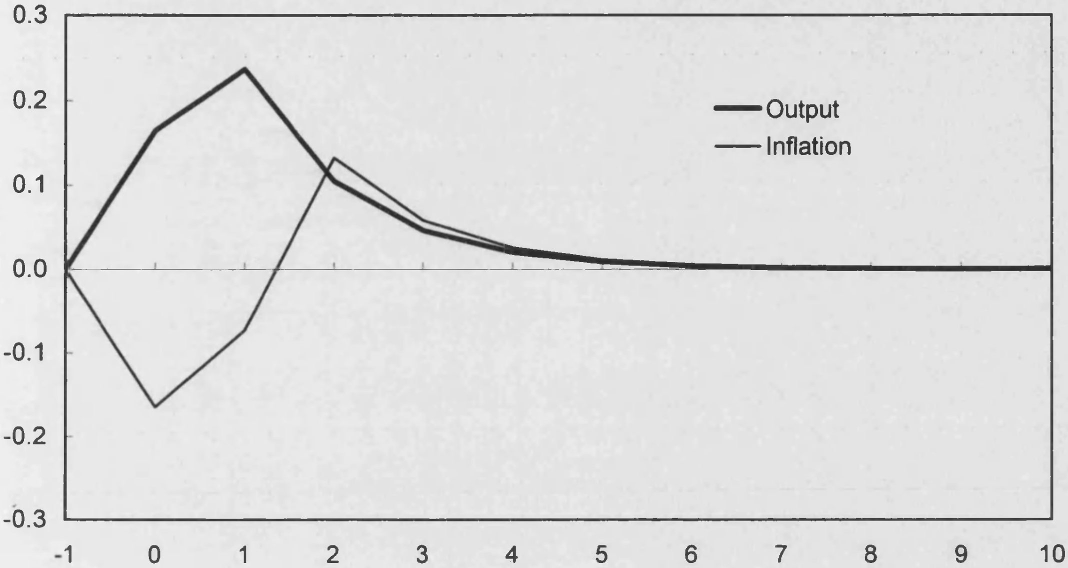


Figure 1.3.2: Initial response of prices v.s. precision of public signal

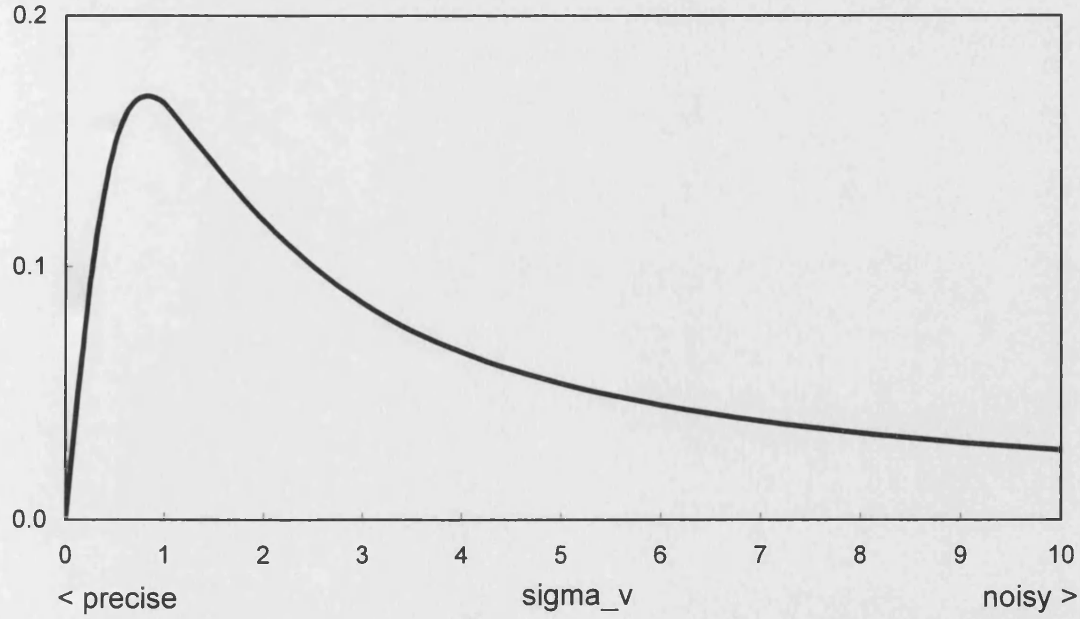


Figure 1.4: Calvo-type price-setting

Figure 1.4.1: Responses of OUTPUT to a positive monetary disturbance

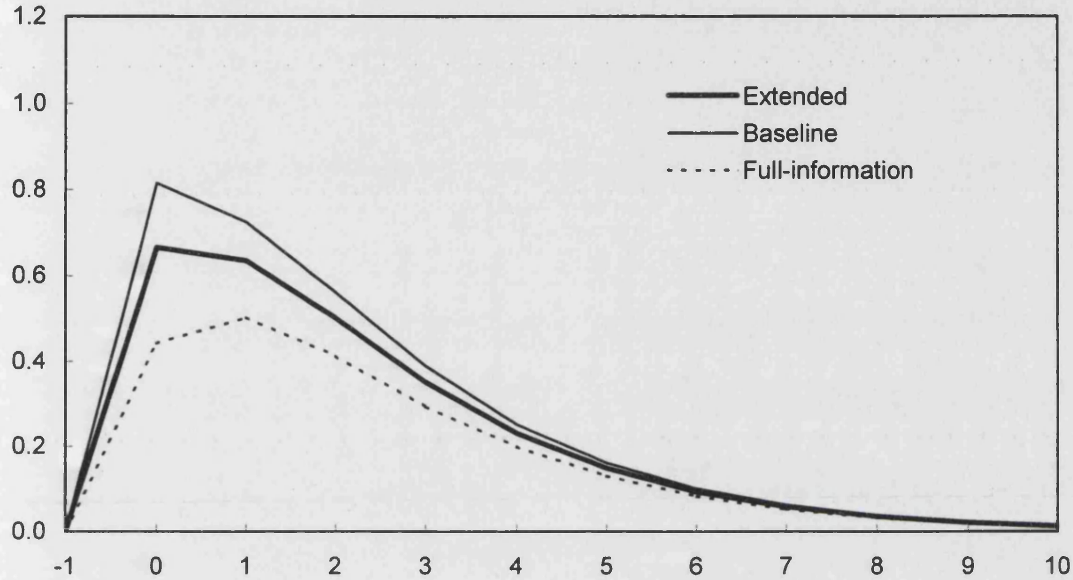


Figure 1.4.2: Responses of INFLATION to a positive monetary disturbance

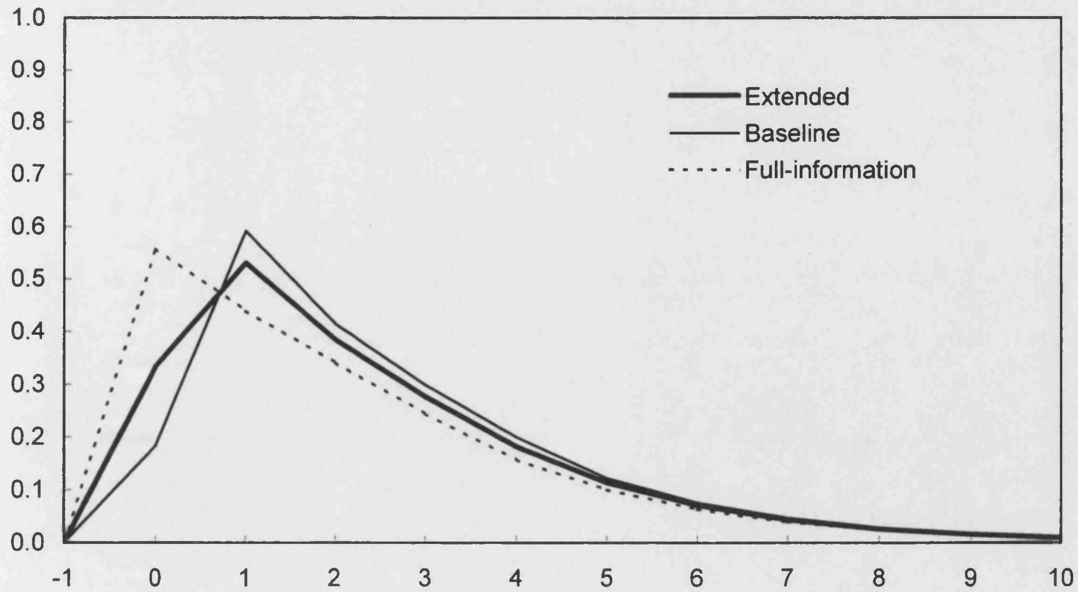


Figure 1.5: Four-period staggered price-setting

Figure 1.5.1: Responses of OUTPUT to a positive monetary disturbance

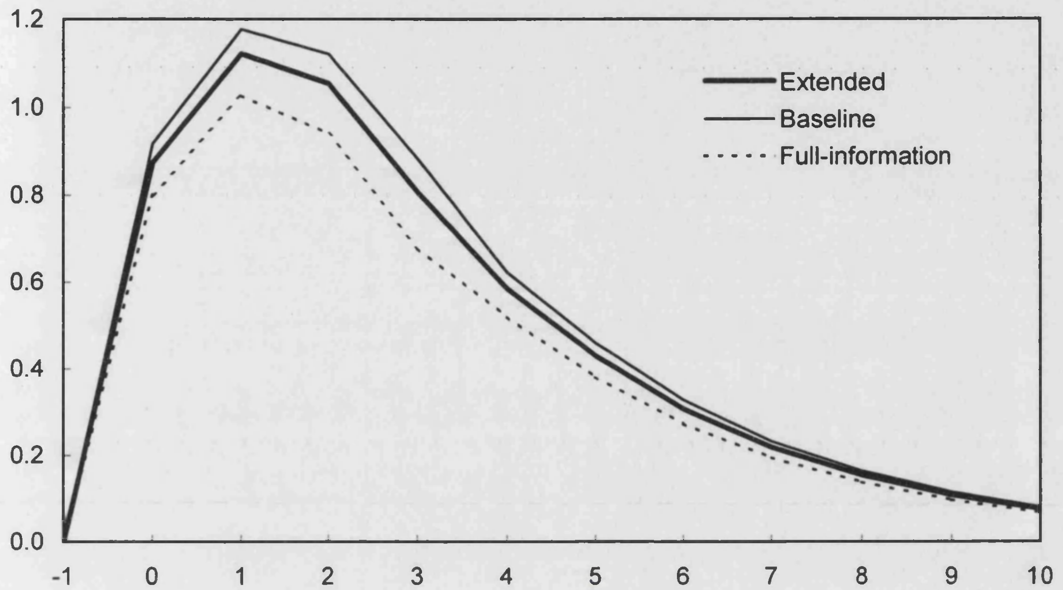


Figure 1.5.2: Responses of INFLATION to a positive monetary disturbance

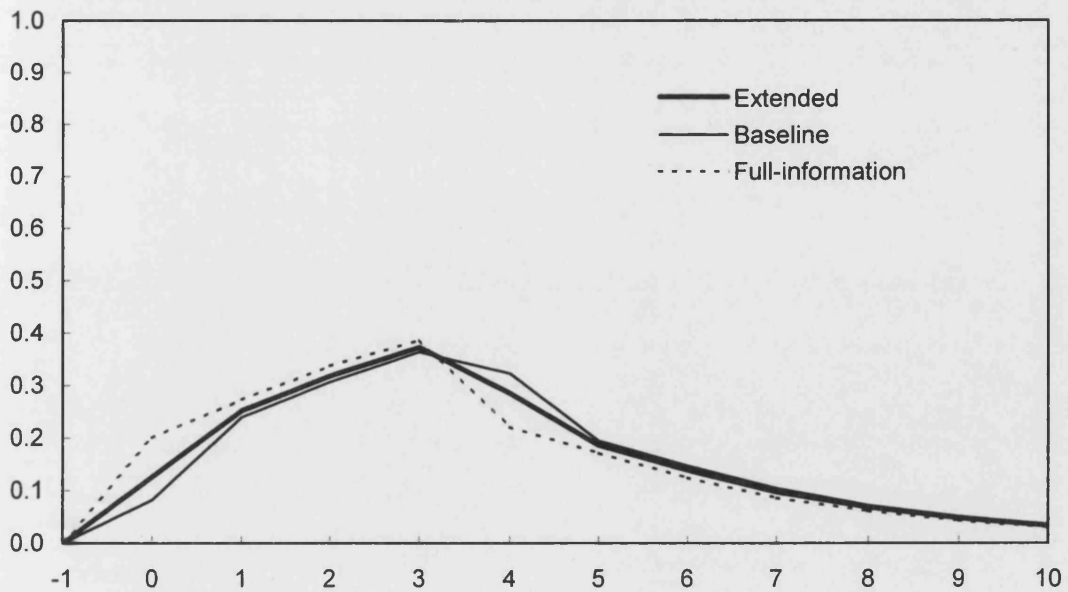


Figure 1.6: General staggered price-setting (Informational Disturbance)

Responses to a NEGATIVE informational disturbance

Figure 1.6.1: Calvo-type price-setting

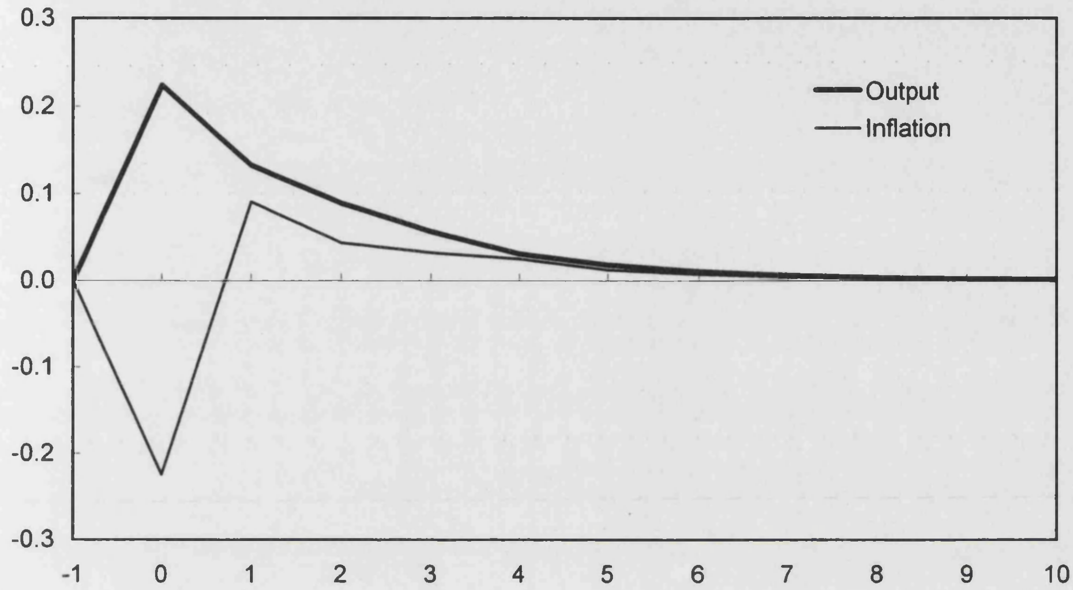
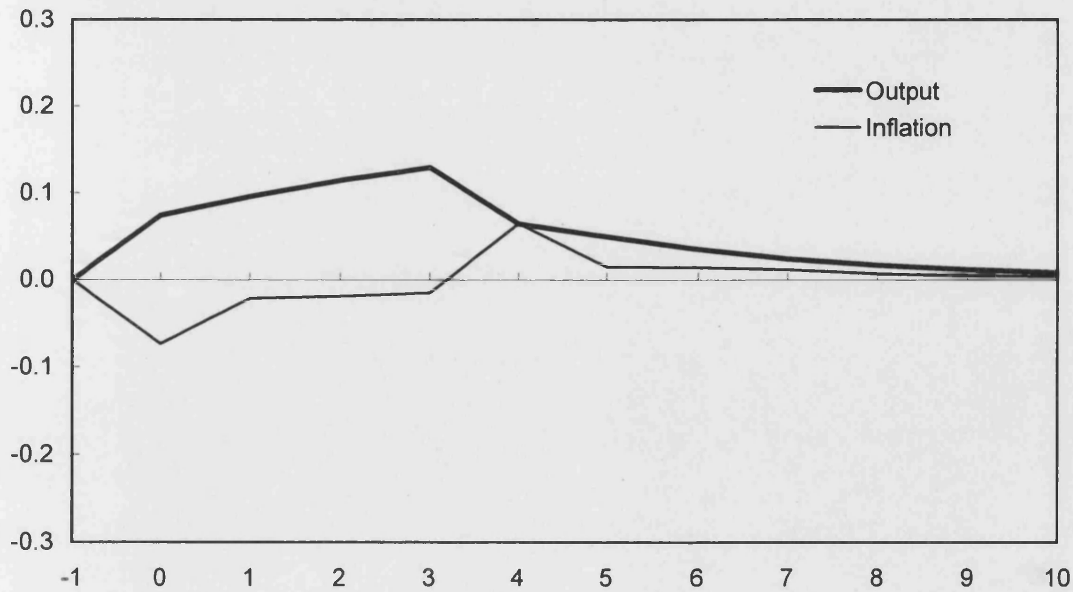


Figure 1.6.2: Four-period staggered price-setting



Chapter 2

Inventories in the Monetary Transmission Mechanism

This chapter studies the role of inventories in the monetary transmission mechanism by developing simple dynamic general equilibrium models which assume pre-determined prices. Inventories serve as a source of real rigidities, that is, amplify the persistence of the real effects of monetary policy. I consider a production-smoothing motive and a sales-facilitating motive for holding inventories. Inventories respond procyclically and prices are adjusted gradually to a nominal disturbance only if the sales-facilitating motive is relatively strong enough; otherwise inventories respond countercyclically and prices are adjusted excessively. I also consider the case where production as well as prices is pre-determined, in which inventories absorb shocks unintendedly as long as production cannot be adjusted. If the decision lag of price-setting is longer than that of production, inventories respond countercyclically at first and then move procyclically, which is consistent with the pattern shown in empirical studies.

2.1 Introduction

There are many mechanisms that co-operate in transmitting monetary shocks to the economy. This paper studies the role of inventories in the monetary transmission mechanism by developing simple dynamic general equilibrium models.

Changes in inventories are key components in business cycles so that economists closely look at inventory data when they assess the current state of business cycles. Although the share of inventory investment in GDP is less than one percent, reduction in inventories arithmetically account for about half of the fall in GDP during post-war U.S. recessions (Ramey and West, 1999). Nonetheless most of existing monetary business cycle models pay no attention to inventories. For example, the standard dynamic New Keynesian models based on the staggered price setting assume that even in the periods monopolistic suppliers cannot set their optimal price they have to produce whatever quantity to meet the demand, that is, the quantity buyers may wish to purchase at their fixed price. If goods are storable, however, those suppliers may wish to hold inventories in order to smooth production or facilitate sales. Gradual adjustments of the stock of inventories could make the monetary policy effects on production or sales more persistent. This idea was proposed by Blinder and Fischer (1981) in their IS-LM framework, but has not been considered in optimization-based monetary business cycle models until very recently.

Probably one of the main reasons for the neglect of inventories is simply that we do not have a conventional dynamic general equilibrium model with inventories that can successfully explain stylized facts. According to Khan and Thomas (2004), a core set of empirical regularities in post-war U.S. data that "any useful model of inventories should seek to address" is as follows: 1) the relative variability of inventory investment is large; 2) the correlation between inventory investment and GDP is positive; 3) the correlation between inventory investment and final sales is positive; 4) the standard deviation of production exceeds that of sales[1]; 5) the correlation between inventory-to-sales ratio and GDP is negative. In addition, Wen (2002) pointed out that inventory investment is procyclical only at relatively low cyclical frequen-

[1]Given the accounting identity that GDP is equal to final sales plus inventory investment, it is sufficient for the fact 3) to imply the fact 4).

cies such as the business-cycle frequencies; it is countercyclical at very high frequencies (2-3 quarters per cycle).

It is not easy for the most popular inventory theory, the production smoothing theory, to explain the stylized fact that inventory investment is positively correlated with sales and accordingly production is more volatile than sales. The production smoothing theory simply claims that firms hold inventories in order to smooth production if their short-term production function is concave. It is proved that this theory can be reconciled with the fact of relatively volatile production if demand shocks are highly serially correlated or cost shocks rather than demand shocks are dominant in the economy. It is not clear, however, whether the production smoothing model can generate volatile production in response to monetary shocks.

Some other theories emphasize sales-related motives for holding inventories[2]. The stock avoidance theory claims that firms hold inventories in order to avoid losses of opportunity for sales when they cannot adjust production to meet positive demand shocks. Bils and Kahn (2000) more emphasizes the relationship between sales and inventories, and assume that sales directly depend on the available inventory stock, i.e., the sum of current production and inventory stock in the beginning of period. They claim that a larger stock of inventories facilitates matching with potential purchasers who arrives with preferences for a specific type of good when the stock is considered as an aggregate of similar goods of different sizes, colours, locations, etc. The relationship between sales and inventories is taken into account by many empirical studies in which a class of the "linear-quadratic models" (Ramey and West, 1999) is typically specified.

One of the main challenges in this chapter is to introduce inventories into a monetary dynamic general equilibrium model in a way that is consistent with the above stylized facts on inventories. Monopolistic suppliers in my models have both the production-smoothing and sales-facilitating motives for holding inventories. The production-smoothing motive is incorporated

[2]Other theories on inventories also include the factor-of-production theory and the (S,s) theory. The former, mainly adopted in the real business cycle models including Kydland and Prescott (1982), claims that inventory stock at each stage of production may facilitate shipment, delivery, distribution, and eventually final production and should be treated as a factor of production. The latter, recently incorporated into a dynamic general equilibrium model by Khan and Thomas (2003), emphasize the role of inventories that saves fixed costs of production or ordering and construct a model that features the S-s type of decision rule for inventory investment.

by assuming that the marginal disutility of production is increasing. The sales-facilitating motive is incorporated by introducing a generic cost of sales that can be saved by holding inventories. This motive induces farmers to keep a close relationship between sales and inventories. If the generic cost function of sales and inventories is constant return to scale, the optimal inventory-to-sales ratio can be explicitly derived. Since the motive for keeping the relationship between sales and inventories causes the fluctuations in sales to amplify those in production, the sales-facilitating motive can be interpreted as a kind of the "accelerator motive" in the literature[3]. In my baseline model which assumes just one-period pre-determined prices, I will show that inventory investment responds procyclically to a nominal disturbance only if the sales-facilitating motive is relatively strong enough. Moreover, in an extended model which assumes production as well as prices is pre-determined, inventories initially absorb the shock unintendedly so that inventory investment responds countercyclically at first and then move procyclically, which is consistent with the cyclical pattern pointed out by Wen (2002) and the estimated VAR evidence by Bernanke and Gertler (1995).

Another challenge is to explain how prices are adjusted to monetary shocks. The models in this chapter assume nominal rigidities such that prices must be determined in advance or based on old information. The price-setting decisions of the monopolistic suppliers are closely related with their inventory-holding decisions. I will show that prices are adjusted gradually to a nominal disturbance, which is consistent with many VAR evidences including Bernanke and Gertler (1995), only if the sales-facilitating motive for holding inventories is relatively strong enough and inventory investment moves procyclically at business-cycle frequencies; otherwise prices are adjusted quickly and even excessively at the early stage of the adjustment.

The intuition behind the above inventory-holding and price-setting behaviour is as follows. Suppose that a positive nominal disturbance occurs in the economy, which means monopolistic suppliers face an unexpected boom in sales because they cannot adjust their prices immediately in the period of disturbance. If the sales-facilitating motive for holding inventories is relatively strong compared with the production-smoothing motive, the suppliers

[3] In more than sixty years ago, Metzler (1941) proposed a mechanism called "inventory accelerator" in which a desired level of inventory stock and its relationship with sales plays a crucial role. He also considered "unintended inventories" I will consider below.

initially produce more than the amount just to meet the demand so that they can hold inventory stock above its normal level to keep the relationship between sales and inventories. Then they start adjusting prices gradually so that sales and inventories can gradually go back to their original levels with keeping their relationship. If the production-smoothing motive is relatively strong, on the other hand, the suppliers meet the initial unexpected demand partially by reducing their existing inventory stock because they strongly wish to avoid changing production so much. Then they raise prices aggressively, even excessively, to dampen sales so that the reduced inventory stock can gradually recover in parallel with production gradually going back to its original level[4].

By incorporating inventories into a monetary business cycle model in an appropriate way, the persistence of monetary policy effects is amplified as Blinder and Fischer (1981) argued. I will show that even my baseline inventory model which assumes that all prices must be determined just one period in advance, or equivalently that true information about disturbances is revealed to all price setters just one period later, can generate the real effects of monetary policy lasting several periods. In an extended model which assumes longer-period pre-determined prices, the real effects persist even after all prices start being adjusted, which implies the model with inventories need not to assume some fraction of prices determined on the basis of unrealistically very old information as the sticky information model of Mankiw and Reis (2002) does for generating persistent real effects. Inventories serve as a source of "real rigidities" (Ball and Romer, 1990), that is, amplify the effects of nominal rigidities.

I will show most of the above results analytically in my sufficiently simplified models where monopolistic "yeoman farmers" produce and directly supply their individual-specific goods and consume all types of those differ-

[4]Another explanation for the relationship between the inventory-holding and price-setting behaviour, interpreting the inventory stock as an asset whose return is decreasing, is as follows. The opportunity cost for monopolistic suppliers to deviate inventory stock upwards from its normal level is equal to the expected deflation rate *of their own products* if the nominal interest rate is fixed. Thus they set higher prices for the next period than the current period (prices must be pre-determined) when they hold inventory stock above its normal level, which implies gradual increases of prices in response to a positive nominal disturbance (the case of strong sales-facilitating motive). On the other hand, they set lower prices for the next period when they hold inventory stock below its normal level, which implies gradual decreases after an excessive increase of prices (the case of weak sales-facilitating motive). See footnote 14 in the baseline model below.

entiated goods. I examine the effects of nominal disturbances by restricting my attention to stationary fluctuations around the steady state and log-linearizing equilibrium conditions. I extend my models step by step from the baseline model which assumes just one-period pre-determined prices to the models which assume both production and prices are pre-determined and the length of the decision lag for price-setting is heterogeneous among farmers. Finally, I develop a more realistic model that includes labour market, capital accumulation, explicit money, and real disturbances and assess quantitatively the cyclical pattern and persistence of aggregate variables. Although not all the results of my simulations quantitatively match with the data, they qualitatively well capture the stylized facts on inventories and support my analytical results.

Recent studies such as Hornstein and Sarte (2001) and Boileau and Letendre (2004) deal with similar problems to this chapter. One of the main differences between their models and mine is the assumption of nominal rigidities. While Hornstein and Sarte assume staggered price-setting a la Taylor (1980)[5] and Boileau and Letendre assume costly price adjustment a la Rotemberg (1982) and Ireland (2001)[6], the models in this chapter assume pre-determined prices a la Fischer (1977) that is a prototype of the sticky information model. The motivations for the assumption are as follows. First, it makes the models so simple that I can easily obtain analytical results. Secondly, the pre-determined prices models cannot generate persistent real effects of monetary policy without inventories, which encourages me to examine how the persistence is generated by introducing inventories into the models. Thirdly, I will consider unintended inventories caused by a decision lag of production. It seems reasonable to assume pre-determined prices together with pre-determined production for examining unintended inventories in response to nominal disturbances. In addition, the pre-determined prices or imperfect information models have the following advantages over the more popular staggered price-setting models. As Mankiw and Reis (2002) ar-

[5]Hornstein and Sarte (2001) consider inventories held for only the production-smoothing motive and show that an aggregation effect intrinsic to price-staggering can make aggregate inventories conform to the stylized facts. Chang, Hornstein and Sarte (2004) use their staggered price-setting model with inventories for studying the response of firms' employment to productivity shocks.

[6]Boileau and Letendre (2004) consider inventories held for alternative motives and compare how those models are consistent with stylized facts and can generate persistent output and inflation through quantitative experiments.

gued, imperfect information models can explain the delay of price adjustment better than staggered price-setting models. Moreover, I will show that the responses of disaggregate inventories to a nominal disturbance in the pre-determined prices models seem more plausible than those in the staggered price-setting models of Hornstein and Sarte (2001) where the level of disaggregate inventory stock oscillates as price-setters change their prices alternately.

The remainder of this chapter is organized as follows. In Section 2.2, I describe the baseline model and show the main results on the effects of nominal disturbances. In Section 2.3, I extend the baseline model by assuming that production as well as prices are pre-determined. I seek to find a condition under which a model assuming pre-determined production can explain more precisely the observed cyclical pattern of inventory-holding behaviour that includes unintended inventories. I also consider the case of heterogeneity in the length of the decision lag for price-setting among farmers. In Section 2.4, I develop a more realistic quantitative model and report the main results of simulations. Section 2.5 concludes.

2.2 The Baseline Model

In this section I develop a basic dynamic general equilibrium model simplified enough to obtain analytical results on the properties of the effects of nominal disturbances on aggregate variables including inventories.

The baseline model in this section assumes a minimum nominal rigidity: all prices must be determined one period in advance due to imperfect information or some other constraints. I will consider the cases of longer-period pre-determined prices, the pre-determination in different periods among price-setters, and pre-determined production as well as prices in the next section. The models in this and next sections neglect some basic and realistic factors in the economy such as labour market, capital accumulation, explicit money, and real disturbances which I will introduce in the quantitative model in Section 2.4.

2.2.1 Set-up

Consider an economy populated by a continuum of infinitely-lived "yeoman farmers" indexed by $i \in [0, 1]$ who produce and directly supply their individual-specific goods and consume all types of those differentiated goods. Farmer i who supplies a good of type i seeks to maximize a discounted sum of utilities of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t^i - \frac{y_t(i)^{1+\eta}}{1+\eta} \right\}, \tag{2.1}$$

where $0 < \beta < 1$ is a discount factor, C_t^i is a constant-elasticity-of-substitution aggregator of farmer i's consumption of each individual good of type j

$$C_t^i \equiv \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{2.2}$$

with $\theta > 1$, and $y_t(i)$ is i's production. I assume η is strictly positive so that the marginal disutility of producing good is increasing and accordingly farmers have incentives to smooth production. The larger η, the stronger production-smoothing motive.

Products are storable in this economy. Sales and production need not match in each period since the gap is adjusted by inventories. The resource constraint for good i is

$$y_t(i) - \Phi(c_t(i), x_t(i)) = c_t(i) + x_t(i) - x_{t-1}(i), \tag{2.3}$$

where $c_t(i) \equiv \int_0^1 c_t^j(i)dj$ is the sales and $x_t(i)$ is the end-of-period inventory stock of good i. $\Phi(c_t(i), x_t(i))$ is a generic function of sales cost that can be saved by holding inventories, which captures the role of inventories to facilitate sales. By virtue of this benefit in addition to the role for smoothing production, farmers hold inventories despite no explicit returns and some physical storage costs which are included in the generic function. I assume that Φ is a non-negative increasing function of $c_t(i)$ and a decreasing function of $x_t(i)$ and the second derivatives satisfy $\Phi_{cc} > 0$, $\Phi_{xx} > 0$ and $\Phi_{cx} < 0$ for relevant region. The absolute value of Φ_{cx} represents the degree of the sales-facilitating motive for holding inventories: the larger $|\Phi_{cx}|$, the stronger sales-facilitating motive. This motive induces farmers to keep a close relationship between sales and inventories.

Financial markets are complete in this economy. Even if the income streams from sales are expected to differ among farmers, they can choose an identical consumption plan under the intertemporal budget constraint for any farmer i from any period t,

$$\sum_{s=t}^{\infty} E_t Q_{t,s} \left[\int_0^1 p_s(j) c_s^i(j) dj \right] \leq W_t + \sum_{s=t}^{\infty} E_t Q_{t,s} [p_s(i) c_s(i)], \tag{2.4}$$

where $Q_{t,s}$ is a stochastic discount factor, W_t is the beginning-of-period wealth, and $p_t(i)$ is the price of good i. Farmers allocate their consumption across differentiated goods at each period so as to maximize the index (2.2), taking as given the level of total expenditure $\int_0^1 p_t(j) c_t^i(j) dj$, which implies

$$c_t^i(j) = \left(\frac{p_t(j)}{P_t} \right)^{-\theta} C_t^i, \tag{2.5}$$

where

$$P_t \equiv \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$$

is the corresponding price index with which i's optimally allocated total expenditure is equal to $P_t C_t^i$. Taking as given this optimal allocation of consumption at each period, farmers then choose optimal path of total consumption. The first order condition at t is given by

$$\frac{1}{C_t} = \beta (1 + i_t) \frac{P_t}{E_t[P_{t+1} C_{t+1}]}, \tag{2.6}$$

where i_t is the riskless nominal interest rate which corresponds to $(1+i_t)^{-1} = E_t Q_{t,t+1}$. Here I drop the superscript of C_t^i and use $C_t \equiv \int_0^1 C_t^i di$ instead since all farmers choose the same consumption plan.

At the same time in each period, farmers make a decision on their production and inventory investment. Since they are monopolistic suppliers who set their prices under demand constraint in sales, their decisions on production are partly combined with those on prices which I will describe in the next paragraph. Meanwhile, farmers can control their inventory investment independently of their price-setting decisions[7]. The first-order condition for optimal inventory-holding is given by

$$y_t(i)^{\eta} \{ 1 + \Phi_x(c_t(i), x_t(i)) \} = \beta E_t y_{t+1}(i)^{\eta}. \tag{2.7}$$

[7]The first-order condition for optimal inventory-holding (2.7) is unchanged even if farmers are price-taker. This does not mean, however, there is no connection between inventory-holding and price-setting behaviours in this model.

They hold inventories so that the marginal cost of increasing inventories in terms of the marginal disutility of production in the current period should be equal to the expected marginal benefit from reducing production in the next period, which includes both the production-smoothing and the sales-facilitating motives. This is a key equation in the model.

Farmers choose their prices one period in advance. Using the information set available only in period $t-1$, farmer i sets its price $p_t(i)$ so as to maximize (2.1) subject to (2.3), (2.4), and the demand constraint $c_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\theta} C_t$ which comes from (2.5) with dropping the superscripts. The first-order condition is given by

$$E_{t-1}\left[\frac{p_t(i)}{P_t}\right] = \frac{\theta}{\theta-1} E_{t-1}\left[C_t\, y_t(i)^{\eta}\left\{1 + \Phi_c(c_t(i),\, x_t(i))\right\}\right]. \tag{2.8}$$

Factors in the expectation operator in the right hand side represent the real marginal cost of supplying good i, which consist of the marginal cost of sales in terms of the marginal disutility of production divided by the marginal utility of consumption.

Lastly, I introduce an exogenous stochastic process for aggregate nominal spending as follows.

$$\ln \mathcal{M}_t = \ln \mathcal{M}_{t-1} + \epsilon_t, \tag{2.9}$$

where $\mathcal{M}_t = P_t C_t$ and ϵ_t is white noise. One may interpret $\mathcal{M}_t$ as "money" that farmers must hold for their spending and the above process may be taken as a monetary policy rule specified by a target path for the money supply. Alternatively, one can image a fiscal-monetary policy rule specified by a target path for the aggregate nominal spending which may be achieved by adjusting the nominal interest rate on the government bond[8]. This simple specification for aggregate demand, however interpreted it is, allows me to concentrate on examining the consequences of alternative specifications for aggregate supply such as inventory-holding and price-setting behaviours. In my analysis in this and next sections, I suppose that the disturbance driving the above process for aggregate nominal spending is a monetary policy shock.

[8] Under the optimal consumption rule of the private sector (2.6), that policy is implemented simply by holding the nominal interest rate on $i \equiv \beta^{-1} - 1$.

2.2.2 Equilibrium

In the symmetric equilibrium, every farmer sets the same price and therefore purchases the same amount of each differentiated good, which implies for any i

$$\begin{aligned} p_t(i) &= P_t \\ c_t(i) &= C_t. \end{aligned}$$

Then equations (2.3), (2.7), and (2.8) are rewritten as

$$Y_t - \Phi(C_t, X_t) = C_t + X_t - X_{t-1} \tag{2.10}$$

$$Y_t^{\eta}\{1 + \Phi_x(C_t, X_t)\} = \beta E_t Y_{t+1}^{\eta} \tag{2.11}$$

$$1 = \frac{\theta}{\theta - 1} E_{t-1}[C_t Y_t^{\eta}\{1 + \Phi_c(C_t, X_t)\}] \tag{2.12}$$

where $Y_t \equiv \int_0^1 y_t(i)di$ and $X_t \equiv \int_0^1 x_t(i)di$.

A rational-expectations equilibrium of the economy is defined as a set of $\{C_t, Y_t, X_t, P_t, i_t\}$ which satisfies the resource constraint (2.10) and the decision rules for consumption (2.6), inventory-holding (2.11), and price-setting (2.12), given the exogenous process for aggregate nominal spending (2.9) where $\mathcal{M}_t = P_t C_t$.

In what follows, I restrict my attention to stationary fluctuations around the steady state. The deterministic steady-state conditions for (2.10), (2.6), (2.11), and (2.12) are as follows.

$$Y - \Phi(C, X) = C \tag{2.13}$$

$$1 = \beta(1 + i) \tag{2.14}$$

$$1 + \Phi_x(C, X) = \beta \tag{2.15}$$

$$1 = \frac{\theta}{\theta - 1} C Y^{\eta}\{1 + \Phi_c(C, X)\} \tag{2.16}$$

Around the steady state, $\Phi(C_t, X_t)$, $\Phi_c(C_t, X_t)$, and $\Phi_x(C_t, X_t)$ may be approximated as follows.

$$\begin{aligned} \Phi(C_t, X_t) &\cong \Phi(C, X) + \Phi_C(C_t - C) + \Phi_X(X_t - X) \\ \Phi_c(C_t, X_t) &\cong \Phi_C + \Phi_{CC}(C_t - C) + \Phi_{CX}(X_t - X) \\ \Phi_x(C_t, X_t) &\cong \Phi_X + \Phi_{CX}(C_t - C) + \Phi_{XX}(X_t - X) \end{aligned}$$

where $\Phi_C \equiv \Phi_c(C, X)$, $\Phi_X \equiv \Phi_x(C, X)$, $\Phi_{CC} \equiv \Phi_{cc}(C, X)$, $\Phi_{CX} \equiv \Phi_{cx}(C, X)$, and $\Phi_{XX} \equiv \Phi_{xx}(C, X)$. Using these and the above steady-state conditions, the whole system consisting of log-linear approximations to the equilibrium conditions can be obtained as follows.

$$\begin{aligned}
\hat{C}_t &= E_t\hat{C}_{t+1} - \{\hat{i}_t - (\hat{P}_{t+1} - \hat{P}_t)\} && (2.17)\\
\eta\hat{Y}_t &= \eta E_t\hat{Y}_{t+1} - \frac{C\,\Phi_{CX}}{\beta}\hat{C}_t - \frac{X\,\Phi_{XX}}{\beta}\hat{X}_t && (2.18)\\
0 &= E_{t-1}\left[\eta\hat{Y}_t + \left(1 + \frac{C\,\Phi_{CC}}{1+\Phi_C}\right)\hat{C}_t + \frac{X\,\Phi_{CX}}{1+\Phi_C}\hat{X}_t\right] && (2.19)\\
\hat{Y}_t &= \frac{C}{Y}(1+\Phi_C)\,\hat{C}_t + \frac{X}{Y}(\beta\hat{X}_t - \hat{X}_{t-1}) && (2.20)\\
\hat{\mathcal{M}}_t &= \hat{P}_t + \hat{C}_t && (2.21)\\
\hat{\mathcal{M}}_t &= \hat{\mathcal{M}}_{t-1} + \epsilon_t && (2.22)
\end{aligned}$$

where $\hat{\mathcal{M}}_t \equiv \ln\mathcal{M}_t$, $\hat{P}_t \equiv \ln P_t$, $\hat{i}_t \equiv \ln(1+i_t)/(1+i)$, and other variables with hat denote log-differences (or rate of deviation) from their steady-state values such as $\hat{X}_t \equiv \ln(X_t/X)$.

If the function $\Phi(C_t, X_t)$ is constant returns to scale, the above system can be simplified. I introduce the function $\phi(Z_t) \equiv \Phi(1, X_t/C_t) = \Phi(C_t, X_t)/C_t$ where $Z_t \equiv X_t/C_t$ is the inventory-to-sales ratio. The assumptions about the function Φ imply $\phi'(Z_t) < 0$ and $\phi''(Z_t) > 0$. Here $\phi''(Z_t)$ rather than $|\Phi_{cx}|$ represents the degree of the sales-facilitating motive. First, the deterministic steady-state conditions (2.13), (2.15), and (2.16) are rewritten as follows.

$$\begin{aligned}
Y &= C\{1+\phi(Z)\} && (2.23)\\
1+\phi'(Z) &= \beta && (2.24)\\
1 &= \frac{\theta}{\theta-1}CY^{\eta}\{1+\phi(Z) - Z\,\phi'(Z)\} && (2.25)
\end{aligned}$$

The optimal steady-state inventory-sales ratio Z is given by (2.24). Around the steady state, the degree of the sales-facilitating motive, $\phi''(Z)$, corresponds to the marginal cost of deviating inventory-to-sales ratio from its optimal level Z. Then the equations (2.18), (2.19), and (2.20) in the above log-linearized system are rewritten as follows.

$$
\begin{aligned}
\eta \hat{Y}_t &= \eta E_t \hat{Y}_{t+1} - \frac{Z \phi''(Z)}{\beta} (\hat{X}_t - \hat{C}_t) & (2.26) \\
0 &= E_{t-1} \left[\eta \hat{Y}_t + \hat{C}_t - \frac{Z^2 \phi''(Z)}{1+\phi(Z) - Z\phi'(Z)} (\hat{X}_t - \hat{C}_t) \right] & (2.27) \\
\hat{Y}_t &= \frac{1+\phi(Z) - Z\phi'(Z)}{1+\phi(Z)} \hat{C}_t + \frac{Z}{1+\phi(Z)} (\beta \hat{X}_t - \hat{X}_{t-1}) & (2.28)
\end{aligned}
$$

In the numerical examples below and the quantitative model in Section 2.4, I assume the function Φ is constant returns to scale for simplicity; otherwise I seek to obtain analytical results without specifying Φ.

2.2.3 Effects of Nominal Disturbances

Using the above log-linearized model, I examine the effects of nominal disturbances on real aggregate variables such as consumption, production, and inventories as well as on prices. The exogenous process for aggregate nominal spending (2.9) implies that a unit positive innovation in ϵ_t raises $E_t \hat{\mathcal{M}}_{t+k}$ by one for all $k \geq 0$. The disturbance I consider below is such an unexpected permanent increase of one unit in the log of aggregate nominal spending.

Suppose that the economy originally stays at its steady state with $\hat{\mathcal{M}}_{t-1} = \hat{P}_{t-1} = 0$, and then in period t the disturbance, $\epsilon_t = 1$, occurs. Since I assume that prices are pre-determined, the initial responses of prices and consumption are

$$
\begin{aligned}
\hat{P}_t &= 0 \\
\hat{C}_t &= 1.
\end{aligned}
$$

The rational-expectations equilibrium I am interested in is one in which deviations of real variables from their steady-state values are stationary, which requires

$$
\begin{aligned}
\lim_{k \to \infty} E_t \hat{P}_{t+k} &= 1 \\
\lim_{k \to \infty} E_t \hat{C}_{t+k} &= 0.
\end{aligned}
$$

Also, $E_t \hat{Y}_{t+k}$ and $E_t \hat{X}_{t+k}$ must converge to 0 as $k \to \infty$. In order to obtain such a unique stationary equilibrium, I assume the following conditions on the parameter and the steady-state values.

$$
\begin{aligned}
&\Phi_C > 0;\ \Phi_X < 0;\ \Phi_{CC} > 0;\ \Phi_{XX} > 0;\ \Phi_{CX} < 0; \\
&\Phi^* \equiv \Phi_{XX}\,\{Y\,(1+\Phi_C + C\,\Phi_{CC}) + \eta\,C\,(1+\Phi_C)^2\} \\
&\qquad + (1-\beta)\,\eta\,C\,(1+\Phi_C)\,\Phi_{CX} - C\,Y\,\Phi_{CX}^2 \ > 0.
\end{aligned} \tag{2.29}
$$

I seek to find a set of sequences $\{\, E_t\hat{C}_{t+k},\ E_t\hat{Y}_{t+k},\ E_t\hat{X}_{t+k},\ E_t\hat{P}_{t+k},\ E_t\hat{i}_{t+k}\,\}$ which satisfies the log-linearized model and is consistent with the above initial and terminal conditions assuming (2.29). The solution is given in the following proposition.

Proposition 2.1. *Suppose that the economy around the steady state is approximated by the model (2.17) through (2.21) and the aggregate nominal spending evolves according to (2.22). All variables with hat are zero in period $t-1$. Prices are determined one period in advance. Then the responses of inventories and consumption to a unit positive innovation in ϵ_t are given by*

$$
\begin{aligned}
E_t\hat{X}_{t+k} &= \lambda\, E_t\hat{X}_{t+k-1} \\
E_t\hat{C}_{t+k} &= \mu\, E_t\hat{X}_{t+k-1}
\end{aligned}
$$

for $k \geq 1$, where $0 < \lambda < 1$ is the smaller of the two real roots of the characteristic polynomial

$$
\beta\,\alpha\,\lambda^2 - \{(1+\beta)\,\alpha - \frac{\Phi^*}{\Phi_{CX}}\}\,\lambda + \alpha = 0
$$

where Φ^* is defined in (2.29) and

$$
\alpha \equiv \left\{ \frac{C\,(1+\Phi_C)}{\beta} - \frac{1+\Phi_C + C\,\Phi_{CC}}{\Phi_{CX}} \right\}\,\eta\,(1+\Phi_C)\ > 0,
$$

and

$$
\mu \equiv \frac{(1-\beta\,\lambda)\,\eta\,X\,(1+\Phi_C) - \lambda\,X\,Y\,\Phi_{CX}}{Y\,(1+\Phi_C + C\,\Phi_{CC}) + \eta\,C\,(1+\Phi_C)^2}\ > 0;
$$

while for $k = 0$

$$
\begin{aligned}
\hat{X}_t &= \frac{C\,(1+\Phi_C)\,\{-\Phi_{CX} - \eta\,\beta\,(1+\Phi_C)/Y\}}{(X\,\Phi_{XX} + \eta\,\beta^2\,X/Y)\,(1+\Phi_C) + \mu\,\beta\,(1+\Phi_C + C\,\Phi_{CC}) + \lambda\,\beta\,X\,\Phi_{CX}} \\
\hat{C}_t &= 1.
\end{aligned}
$$

The responses of production, prices, and nominal interest rate are given by

$$\begin{aligned} E_t\hat{Y}_{t+k} &= \frac{C}{Y}(1+\Phi_C)\,E_t\hat{C}_{t+k} + \frac{X}{Y}(\beta\,E_t\hat{X}_{t+k} - E_t\hat{X}_{t+k-1}) \\ E_t\hat{P}_{t+k} &= 1 - E_t\hat{C}_{t+k} \\ E_t\hat{i}_{t+k} &= 0 \end{aligned}$$

for $k \geq 0$.

The proof is given in Appendix.

Here I assume $\Phi_{XX}+\Phi_{XC} > 0$ for simplicity[9]. Then the main properties of the inventory-holding and price-setting behaviours are summarized by the following corollaries.

Corollary 2.1.1. *i) Inventories respond procyclically, i.e.,* $\hat{X}_t > 0$ *and* $E_t\hat{X}_{t+k} \geq 0$ *for* $k \geq 1$, *if* $-\Phi_{CX} > \eta\,\beta\,(1+\Phi_C)/Y$. *ii) Inventories respond countercyclically, i.e.,* $\hat{X}_t < 0$ *and* $E_t\hat{X}_{t+k} \leq 0$ *for* $k \geq 1$, *if* $-\Phi_{CX} < \eta\,\beta\,(1+\Phi_C)/Y$. *iii) Inventories do not respond, i.e.,* $E_t\hat{X}_{t+k} = 0$ *for* $k \geq 0$, *if* $-\Phi_{CX} = \eta\,\beta\,(1+\Phi_C)/Y$.

Corollary 2.1.2. *i) Prices are expected to be adjusted gradually, i.e.,* $E_t\hat{P}_{t+1} < 1$ *and* $E_t\hat{P}_{t+k} \leq 1$ *for* $k \geq 2$, *if* $-\Phi_{CX} > \eta\,\beta\,(1+\Phi_C)/Y$. *ii) Prices are expected to be adjusted excessively, i.e.,* $E_t\hat{P}_{t+1} > 1$ *and* $E_t\hat{P}_{t+k} \geq 1$ *for* $k \geq 2$, *if* $-\Phi_{CX} < \eta\,\beta\,(1+\Phi_C)/Y$. *iii) Prices are expected to be adjusted instantaneously, i.e.,* $E_t\hat{P}_{t+k} = 1$ *for* $k \geq 1$, *if* $-\Phi_{CX} = \eta\,\beta\,(1+\Phi_C)/Y$.

The deviation from the steady-state level of inventory stock is expected to decay monotonically from its initial response in period t, whether it is positive or negative. The sign of the initial response depends on the difference between $-\Phi_{CX}$ and $\eta\,\beta\,(1+\Phi_C)/Y$, or the coefficient of $\hat{C}_t$ in (2.18), $-C\,\Phi_{CX}/\beta$, and $\eta\,(1+\Phi_C)\,C/Y$, as stated in Corollary 2.1.1[10]. Since $|\Phi_{CX}|$ represents the degree of the sales-facilitating motive for holding inventories and η represents that of the production-smoothing motive, a positive or procyclical response of inventories occurs when the sales-facilitating motive is

[9] This assumption implies $C > X$ if the function Φ is constant returns to scale. Without this assumption, Corollary 2.1.1 and 2.1.2 should be modified as follows: inventories respond procyclically and prices are expected to be adjusted gradually if $-\Phi_{CX} > \eta\,\beta\,(1+\Phi_C)/Y$ but also $(X\,\Phi_{XX} + \eta\,\beta^2\,X/Y)\,(1+\Phi_C) + \mu\,\beta\,(1+\Phi_C+C\,\Phi_{CC}) + \lambda\,\beta\,X\,\Phi_{CX} > 0$.

[10] If the function Φ is constant returns to scale, this condition depends on the difference between $Z\,\phi''(Z)/\beta$ and $\eta\,\{1+\phi(Z) - Z\,\phi'(Z)\}\,C/Y$.

relatively strong while a negative or countercyclical response occurs when the production-smoothing motive is relatively strong[11]. If neither motive dominates, i.e., $-\Phi_{CX} = \eta\beta(1 + \Phi_C)/Y$, inventory stock never deviates from its steady-state level.

The inventory dynamics affect dynamic behaviours of other variables such as consumption, production, and prices. After the adjustment of prices starts in period $t+1$, the deviations from the steady-state values of consumption and production are expected to decay monotonically, in parallel to that of inventories. This means that the real effects of the nominal disturbance persist even after all farmers get precise information about the disturbance and start adjusting their prices. It takes the same periods for the adjustment of prices to be completed as for the real variables including inventory stock to go back to their original levels. If inventories do not respond in the period of disturbance[12], however, prices will be adjusted instantaneously and the real effects will disappear in the next period, as in the standard pre-determined prices or imperfect information models without inventories.

The sign of the initial response of inventories determines how consumption and production are expected to respond and how prices are expected to be adjusted to the disturbance, as stated in Corollary 2.1.2. A sample set of impulse responses of those variables for the case of relatively strong sales-facilitating motive is shown in Figure 2.1.1[13]. Faced with an unexpected boom in sales in period t, farmers produce more than the amount just to meet the demand so that they can hold inventory stock above its normal level for the strong sales-facilitating motive. After period $t+1$, they will adjust their prices gradually so that sales and inventories can gradually go back to their original levels with keeping their close relationship. Meanwhile, production is sharply reduced from t to $t+1$ because they have relatively

[11]In my analytical results, I define the procyclical response of inventories as the deviation of inventory stock upwards from its steady-state level in response to a positive nominal disturbance. In the case of procyclical response of inventories, as I will mention later, inventory *investment* also moves procyclically, that is, its movement is positively correlated with that of production.

[12]This could occur in my model not only when $-\Phi_{CX} = \eta\beta(1 + \Phi_C)/Y$ but also when the cost of deviating inventory stock from its steady-state level is prohibitively high: $\Phi_{XX} = \infty$.

[13]In Figure 2.1.1, I assume the function Φ is constant returns to scale and $\eta = 1.5$, $\beta = 0.99$, $\theta = 10$, $C/Y = 0.99$, $Z \equiv X/C = 2/3$, and $\phi''(Z) = 5$ (which implies $-C\,\Phi_{CX}/\beta = X\,\Phi_{XX}/\beta = Z\,\phi''(Z)/\beta = 3.367$) are chosen for illustrative parameter values. In Figure 2.1.2, $\phi''(Z) = 1$ (which implies $Z\,\phi''(Z)/\beta = 0.6734$) is chosen with other parameter values unchanged.

little concern about smoothing production. This movement of production is correlated with that of *changes* in inventory stock, which means inventory *investment* also moves procyclically. Another sample set for the case of relatively weak sales-facilitating motive is shown in Figure 2.1.2. In this case, since farmers strongly wish to avoid changing production so much, they meet the unexpected demand in period t partially by reducing their existing inventory stock, with relatively little concern about the relationship between sales and inventories. Then in period $t+1$, they raise prices aggressively, even excessively, to dampen sales so that the reduced inventory stock can gradually recover in parallel with production gradually going back to its original level[14].

Comparing these two cases, I find that the case of the strong sales-facilitating motive is more plausible: it is consistent with the stylized fact on inventories that inventory investment is procyclical or production is more volatile than sales, and also consistent with many empirical evidences in the literature that prices are adjusted gradually to nominal disturbances. In other words, assuming a strong sales-facilitating motive is needed for obtaining plausible impulse responses to a nominal disturbance in a simple dynamic general equilibrium model with inventories.

2.3 Unintended Inventories

The baseline model in the previous section assumes that farmers must determine their prices one period in advance while they can control their production without any decision lags. In this section I consider the case in which production as well as prices must be pre-determined. If production as well as prices cannot be adjusted to unexpected demand within a period, inventories are forced to absorb the shock unintendedly, which implies inventories move in the opposite direction to sales at least until production becomes ad-

[14]Equations (2.17), (2.18), and (2.19) imply $E_{t-1}[\hat{\imath}_t-(\hat{P}_{t+1}-\hat{P}_t)] = E_{t-1}[-\frac{C\,\Phi_{CX}}{\beta}\hat{C}-\frac{X\,\Phi_{XX}}{\beta}\hat{X}-\frac{C\,\Phi_{CC}}{1+\Phi_C}(\hat{C}_{t+1}-\hat{C}_t)-\frac{X\,\Phi_{CX}}{1+\Phi_C}(\hat{X}_{t+1}-\hat{X}_t)]$. The left hand side is the marginal opportunity cost of increasing inventories (deviating inventory stock upwards from its steady-state level) and the right hand side is marginal benefit (marginal reduction in the sales cost) from holding inventories both directly through the optimal inventory-holding decision and indirectly through the optimal price-setting decision. Since $\hat{\imath}_t = 0$ for all t, this equation implies that farmers set higher prices for the next period than the current period when they hold inventory stock above its steady-state level, as stated in footnote 4 in the introduction.

justable. Business economists pay much attention to those countercyclical movements in "unintended inventories" when they try to detect a turning point of business cycles. Indeed, as Wen (2002) pointed out, inventory investment in the data moves countercyclically at very high frequencies while it moves procyclically at business cycle frequencies. The estimated VAR evidence of the response of inventory investment to a monetary policy shock in Bernanke and Gertler (1995) also implies those cyclical movements. I seek to find a condition under which a model assuming pre-determined production can explain such a pattern of inventory-holding behaviour.

2.3.1 Identical Lags in Production and Price-setting

First I consider the simplest case in which both production and prices are determined one period in advance, so that the decision lags are identical. In this case, I only have to modify the information set in the inventory-holding decision of the baseline model. The first-order condition (2.7) is replaced with

$$E_{t-1}[\, y_t(i)^\eta \{1 + \Phi_x(c_t(i),\, x_t(i))\}\,] = \beta\, E_{t-1} y_{t+1}(i)^\eta.$$

The corresponding log-linearized equation

$$\eta\, E_{t-1}\hat{Y}_t = \eta\, E_{t-1}\hat{Y}_{t+1} - \frac{C\,\Phi_{CX}}{\beta}\, E_{t-1}\hat{C}_t - \frac{X\,\Phi_{XX}}{\beta}\, E_{t-1}\hat{X}_t \tag{2.30}$$

replaces (2.18).

Then I examine the effects of a nominal disturbance as before. The results are summarized as follows.

Proposition 2.2. *Suppose that the economy around the steady state is approximated by the model (2.17), (2.19), (2.20), (2.21), and (2.30), and the aggregate nominal spending evolves according to (2.22). All variables with hat are zero in period* $t-1$. *Both production and prices are determined one period in advance. Then the responses of inventories and consumption to a unit positive innovation in* ϵ_t *are given by*

$$\begin{aligned} E_t\hat{X}_{t+k} &= \lambda\, E_t\hat{X}_{t+k-1} \\ E_t\hat{C}_{t+k} &= \mu\, E_t\hat{X}_{t+k-1} \end{aligned}$$

for $k \geq 1$, where λ and μ are given in Proposition 2.1. For $k = 0$,

$$\begin{aligned} \hat{X}_t &= -\frac{C\,(1+\Phi_C)}{\beta X} \\ \hat{C}_t &= 1. \end{aligned}$$

The responses of production, prices, and nominal interest rate are given by the same equations in Proposition 2.1.

Corollary 2.2.1. *Inventories respond countercyclically, i.e., $\hat{X}_t < 0$ and $E_t\hat{X}_{t+k} \leq 0$ for $k \geq 1$.*

Corollary 2.2.2. *Prices are expected to be adjusted excessively, i.e., $E_t\hat{P}_{t+1} > 1$ and $E_t\hat{P}_{t+k} \geq 1$ for $k \geq 2$.*

The proof of Proposition 2.2 is given in Appendix.

The above corollaries imply that the sales-facilitating motive, however strong it is, does not work as in the baseline model. Farmers who care about the relationship between sales and inventories are expected to raise their prices aggressively in period $t+1$ in order to dampen sales because inventory stock has reduced unintendedly in period t. Even if farmers have no sales-facilitating motive, they need to dampen sales for the reduced inventory stock to recover gradually without a large adjustment of production. As a result, the response of inventories is countercyclical and the adjustment of prices is excessive for any parameter values within the ranges I assumed.

2.3.2 Different Lags in Production and Price-setting

Since the case of identical decision lags in production and price-setting turned out to be implausible, I consider next the cases of different decision lags. There are two possibilities: a longer decision lag is needed for production than for price-setting, or the opposite. First I consider the former. Leaving the assumption of price-setting in the baseline model still unchanged, I assume production must be determined *two periods* in advance. Then the only log-linearized equation I have to modify is again (2.30), which is replaced with

$$\eta\, E_{t-2}\hat{Y}_t = \eta\, E_{t-2}\hat{Y}_{t+1} - \frac{C\,\Phi_{CX}}{\beta}\, E_{t-2}\hat{C}_t - \frac{X\,\Phi_{XX}}{\beta}\, E_{t-2}\hat{X}_t \qquad (2.31)$$

The effects of a nominal disturbance are summarized as follows.

Proposition 2.3. *Suppose that the economy around the steady state is approximated by the model (2.17), (2.19), (2.20), (2.21), and (2.31), and the aggregate nominal spending evolves according to (2.22). All variables with hat are zero in period $t-1$. Production is determined two periods in advance, while prices are determined one period in advance. Then the responses of inventories and consumption to a unit positive innovation in ϵ_t are given by*

$$\begin{aligned} E_t\hat{X}_{t+k} &= \lambda\, E_t\hat{X}_{t+k-1} \\ E_t\hat{C}_{t+k} &= \mu\, E_t\hat{X}_{t+k-1} \end{aligned}$$

for $k \geq 2$, where λ and μ are given in Proposition 2.1. For $k = 1$,

$$\begin{aligned} E_t\hat{X}_{t+1} &= \frac{C\,(1+\Phi_C)\,(1+\Phi_C+C\,\Phi_{CC})}{\beta X\,\{C\,(1+\Phi_C)\,\Phi_{CX}-\beta\,(1+\Phi_C+C\,\Phi_{CC})\}} < 0 \\ E_t\hat{C}_{t+1} &= -\frac{C\,(1+\Phi_C)\,\Phi_{CX}}{\beta\,\{C\,(1+\Phi_C)\,\Phi_{CX}-\beta\,(1+\Phi_C+C\,\Phi_{CC})\}} < 0\,. \end{aligned}$$

For $k = 0$,

$$\begin{aligned} \hat{X}_t &= -\frac{C\,(1+\Phi_C)}{\beta X} \\ \hat{C}_t &= 1. \end{aligned}$$

The responses of production, prices, and nominal interest rate are given by the same equations in Proposition 2.1.

Corollary 2.3.1. *Inventories respond countercyclically, i.e., $\hat{X}_t < 0$, $E_t\hat{X}_{t+1} < 0$, and $E_t\hat{X}_{t+k} \leq 0$ for $k \geq 2$.*

Corollary 2.3.2. *Prices are expected to be adjusted excessively, i.e., $E_t\hat{P}_{t+1} > 1$ and $E_t\hat{P}_{t+k} \geq 1$ for $k \geq 2$.*

The proof of Proposition 2.3 is given in Appendix.

As in the case of identical decision lags, farmers are expected to raise their prices aggressively in period $t+1$ regardless of their sales-facilitating motive. Again, the response of inventories is countercyclical and the adjustment of prices is excessive for any parameter values within the ranges I assumed.

Therefore, the only remaining possibility of procyclical inventories and gradual price adjustments explained by a model assuming pre-determined production is in the case where a longer decision lag is needed for price-setting than for production. I now assume prices must be determined *two periods* in advance while production must be determined *one period* in advance. (2.19) in the baseline log-linearized model is replaced with

$$0 = E_{t-2}\left[\eta \hat{Y}_t + \left(1 + \frac{C\,\Phi_{CC}}{1+\Phi_C}\right)\hat{C}_t + \frac{X\,\Phi_{CX}}{1+\Phi_C}\hat{X}_t\right] \tag{2.32}$$

while (2.18) is replaced with (2.30) rather than (2.31). The effects of a nominal disturbance are summarized as follows.

Proposition 2.4. *Suppose that the economy around the steady state is approximated by the model (2.17), (2.20), (2.21), (2.30) and (2.32), and the aggregate nominal spending evolves according to (2.22). All variables with hat are zero in period $t-1$. Prices are determined two periods in advance, while production is determined one period in advance. Then the responses of inventories and consumption to a unit positive innovation in ϵ_t are given by*

$$\begin{aligned} E_t\hat{X}_{t+k} &= \lambda\, E_t\hat{X}_{t+k-1} \\ E_t\hat{C}_{t+k} &= \mu\, E_t\hat{X}_{t+k-1} \end{aligned}$$

for $k \geq 2$, where λ and μ are given in Proposition 2.1. For $k = 1$,

$$\begin{aligned} E_t\hat{X}_{t+1} &= \frac{C\,(1+\Phi_C)\{-\Phi_{CX} - \eta\,(1+\beta)\,(1+\Phi_C)/Y\}}{(X\,\Phi_{XX} + \eta\,\beta^2\,X/Y)\,(1+\Phi_C) + \mu\,\beta\,(1+\Phi_C + C\,\Phi_{CC}) + \lambda\,\beta\,X\,\Phi_{CX}} \\ E_t\hat{C}_{t+1} &= 1. \end{aligned}$$

For $k = 0$,

$$\begin{aligned} \hat{X}_t &= -\frac{C\,(1+\Phi_C)}{\beta X} \\ \hat{C}_t &= 1. \end{aligned}$$

The responses of production, prices, and nominal interest rate are given by the same equations in Proposition 2.1.

Corollary 2.4.1. *i) Inventories respond countercyclically at first, i.e.,* $\hat{X}_t < 0$*, and then are expected to move procyclically, i.e.,* $E_t\hat{X}_{t+1} > 0$ *and* $E_t\hat{X}_{t+k} \geq 0$ *for* $k \geq 2$*, if* $-\Phi_{CX} > \eta\,(1+\beta)\,(1+\Phi_C)/Y$*. ii) Inventories respond countercyclically, i.e.,* $\hat{X}_t < 0$*,* $E_t\hat{X}_{t+1} < 0$*, and* $E_t\hat{X}_{t+k} \leq 0$ *for* $k \geq 2$*, if* $-\Phi_{CX} < \eta\,(1+\beta)\,(1+\Phi_C)/Y$*. iii) Inventories respond countercyclically at first, i.e.,* $\hat{X}_t < 0$*, and then are expected to return to their original steady-state level instantaneously, i.e.,* $E_t\hat{X}_{t+k} = 0$ *for* $k \geq 1$*, if* $-\Phi_{CX} = \eta\,(1+\beta)\,(1+\Phi_C)/Y$*.*

Corollary 2.4.2. *i) Prices are expected to be adjusted gradually, i.e.,* $E_t\hat{P}_{t+1} < 1$ *and* $E_t\hat{P}_{t+k} \leq 1$ *for* $k \geq 2$*, if* $-\Phi_{CX} > \eta\,(1+\beta)\,(1+\Phi_C)/Y$*. ii) Prices are expected to be adjusted excessively, i.e.,* $E_t\hat{P}_{t+1} > 1$ *and* $E_t\hat{P}_{t+k} \geq 1$ *for* $k \geq 2$*, if* $-\Phi_{CX} < \eta\,(1+\beta)\,(1+\Phi_C)/Y$*. iii) Prices are expected to be adjusted instantaneously, i.e.,* $E_t\hat{P}_{t+k} = 1$ *for* $k \geq 1$*, if* $-\Phi_{CX} = \eta\,(1+\beta)\,(1+\Phi_C)/Y$*.*

The proof of Proposition 2.4 is given in Appendix.

In this case, prices still cannot be adjusted in period $t+1$ and the boom in sales continues from t to $t+1$. Farmers then try to recover the reduced inventory stock by adjusting production rather than prices, which implies that the growth of production in $t+1$ is fully explained by the change in inventory investment. If the sales-facilitating motive is relatively strong compared with the production-smoothing motive, i.e., $-\Phi_{CX} > \eta\,(1+\beta)\,(1+\Phi_C)/Y$, as shown in Figure 2.2.1[15], farmers will increase production so much that inventory stock will exceed the original level and make it close to the strong sales in period $t+1$. After period $t+2$, they will raise their prices gradually so that sales and inventories can gradually go back to their original levels with keeping their close relationship. If the sales-facilitating motive is relatively weak, i.e., $-\Phi_{CX} < \eta\,(1+\beta)\,(1+\Phi_C)/Y$, as shown in Figure 2.2.2, they will not increase production so much, because of their relatively strong production-smoothing motive, and accordingly the inventory stock in period $t+1$ will be still below its original level. Then in period $t+2$, they need to dampen sales by raising prices aggressively for the inventory stock to recover gradually without a large adjustment of production.

[15] In Figure 2.2.1, $\phi''(Z) = 10$ (which implies $-C\,\Phi_{CX}/\beta = X\,\Phi_{XX}/\beta = Z\,\phi''(Z)/\beta = 6.734$), and in Figure 2.2.2, $\phi''(Z) = 1$ (which implies $\phi''(Z)/\beta = 0.6734$) are chosen for illustrative parameter values with other parameters unchanged from Figure 2.1.1 and 2.1.2.

Here I find that inventories respond procyclically and prices are adjusted gradually to a nominal disturbance in the case of a longer decision lag for price-setting than for production if the sales-facilitating motive is relatively strong enough, although it needs to be stronger than that in the case of no decision lag for production in the baseline model. Whether I consider pre-determined production or not, I need to assume a longer decision lag for price-setting than for production for obtaining the procyclical response of inventories and the gradual adjustment of prices. Moreover, the case considering pre-determined production captures more precisely the observed pattern of inventory-holding behaviour that inventory investment moves countercyclically at very high frequencies and procyclically at business cycle frequencies.

Is it reasonable to assume that the decision lag for price-setting is longer than that for production? One possibility is that there exist some factors that directly cause a substantial decision lag for price-setting such as long-term contracts or commitments. Another possibility is that information processing, or "information stickiness" advocated by Mankiw and Reis (2002), for price-setting is different from that for production. In the case considering pre-determined production, unintended inventories in the period of disturbances have some information about the shocks. When farmers look at their unintended inventories, they may recognize the shocks and use the information in some ways for their decision of production or price-setting in the subsequent periods. The information in unintended inventories might be more quickly processed for the decision of production than that of price-setting, which could cause a longer decision lag for price-setting.

2.3.3 Heterogeneity in price-setting

In the end of this section, I provide a more realistic example for the case of a longer decision lag for price-setting than for production which includes heterogeneity in the length of the decision lag for price-setting among farmers. I assume there are four groups of farmers in the economy. A quarter of farmers have to set their prices one period in advance, another quarter of farmers have to set their prices two periods in advance, another quarter three periods in advance, and the other quarter four periods in advance. Meanwhile, all farmers have to determine their production one period in advance. The equilibrium is no longer symmetric, which means each group of farmers set different prices and accordingly sales, inventories, and production all vary between the groups. The model consists of many equations for each group's decisions, from which I do not seek to derive analytical results as in the case of symmetric equilibrium.

A sample set of impulse responses of aggregate variables to a nominal disturbance is shown in Figure 2.3.1. I choose exactly the same parameter values as in Figure 2.1.1 where the sales-facilitating motive is relatively strong enough. The real effects persist even after all farmers start adjusting their prices, which implies this model need not to assume some fraction of farmers who set their prices on the basis of unrealistically very old information as the Mankiw and Reis (2002) sticky information model does for generating persistent effects. The response of production is hump-shaped, which peaks immediately when it becomes adjustable in period $t+1$. The response of inflation peaks later than that of production as in the sticky information model. On the whole, the results well capture main features of the monetary policy effects reported in, for example, Bernanke and Gertler (1995).

The responses of disaggregate variables are shown in Figure 2.3.2. The four dotted lines in each panel represent the responses of the four groups while the solid line represents the aggregate or average response. The overall price is adjusted gradually as the four groups of farmers start adjusting their prices one after the other from $t+1$ to $t+4$. Sales for goods produced by the farmers who start adjusting their prices earlier decline, while the farmers who cannot adjust their prices face strong demand due to their relatively low prices. Inventories of the farmers who start adjusting their prices earlier respond countercyclically while inventories of those who cannot adjust their prices respond procyclically after production becomes adjustable in $t+1$. Those patterns for disaggregate variables are totally different from those in the staggered price-setting model with inventories which is reported in Hornstein and Sarte (2001). In their model the level of disaggregate inventory stock oscillates as price-setters change their prices alternately, which seems unrealistic.

2.4 Quantitative Experiments

The models so far omit various factors for analytical simplicity. In this section I develop a more realistic model that includes labour market, capital accumulation, explicit money, and real disturbances and assess quantitatively the cyclical pattern and persistence of aggregate variables.

2.4.1 Model

Following Chari, Kehoe, and McGrattern (2000, hereafter CKM) and Boileau and Letendre (2004, hereafter BL), I consider a monetary economy in which a large number of identical and infinity-lived agents consume a homogenous consumption-capital good produced from a continuum of intermediate goods indexed by $i \in [0,\, 1]$. The representative consumer seeks to maximize a discounted sum of utilities of the form

$$E_0 \sum_{t=0}^{\infty} \frac{\beta^t}{1-\sigma} \left[\left(\omega\, C_t^{\frac{\chi-1}{\chi}} + (1-\omega)(M_t/P_t)^{\frac{\chi-1}{\chi}} \right)^{\frac{\chi}{\chi-1}} (1-N_t)^{\psi} \right]^{1-\sigma} .$$

subject to the flow budget constraint

$$P_t\,(C_t + I_t) + M_t + Q_{t,t+1} B_{t+1} = P_t\,(w_t\, N_t + r_t\, K_{t-1}) + M_{t-1} + B_t + T_t + \Pi_t$$

where the capital stock they hold evolves according to

$$I_t - \frac{\nu}{2} \left(\frac{I_t}{K_{t-1}} - \delta \right)^2 K_{t-1} = K_t - (1-\delta) K_{t-1}.$$

M_t is nominal money balances, N_t is hours worked, w_t is the real wage rate, r_t is the rental rate of capital, T_t is nominal net transfers from the government, and Π_t is the aggregate of profits from the producing and retailing firms described below.

Following BL, I assume there are producing and retailing firms owned by consumers. The competitive retailing firms purchase all types of intermediate goods $s_t(i)$ from the producing firms $i \in [0,\, 1]$, aggregate them, and sell them to the consumers. They seek to maximize profits

$$P_t\, S_t - \int p_t(i)\, s_t(i)\, di$$

subject to the aggregation technology

$$S_t = \left[\int_0^1 s_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

where S_t is the aggregate sales to consumers for their consumption and capital investment. The first order condition of their demand for intermediate good i is given by

$$s_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} S_t. \tag{2.33}$$

The monopolistic producing firm i produces differential intermediate good i and sets a price of their own products. Products are storable. They seek to maximize a discounted sum of profits

$$\sum_{s=t}^{\infty} E_t\, Q_{t,s} \left[p_t(i)\, s_t(i) - P_t \left\{ w_t\, n_t(i) + r_t\, k_t(i) \right\} \right]$$

subject to the production technology

$$y_t(i) = A_t\, n_{t-1}(i)^{\alpha}\, k_{t-1}(i)^{1-\alpha},$$

where A_t is the aggregate total factor productivity, the resource constraint for intermediate good i

$$y_t(i) - \Phi(s_t(i),\, x_t(i)) = s_t(i) + x_t(i) - x_{t-1}(i)$$

where $\Phi(\cdot)$ is the same function of sales cost as in the baseline model, and the demand constraint (2.33). The above production technology implies that the producing firms have to determine their inputs for production one period in advance although the productivity shock may change their planned output. Also as in the section 2.3.3, I assume that a quarter of the producing firms have to set their prices one period in advance, another quarter two periods in advance, another quarter three periods in advance, and the other quarter four periods in advance.

Clearing conditions for the final goods, labour, and capital markets are

$$
\begin{aligned}
S_t &= C_t + I_t \\
N_t &= \int n_t(i)\, di \\
K_t &= \int k_t(i)\, di
\end{aligned}
$$

Lastly, the exogenous shock processes for the productivity and money growth are given by

$$
\begin{aligned}
\ln A_t &= (1-\rho^A)\ln A + \rho^A \ln A_{t-1} + \epsilon_t^A \\
\ln(M_t/M_{t-1}) &= (1-\rho^M)\ln(\Delta M) + \rho^M \ln(M_{t-1}/M_{t-2}) + \epsilon_t^M
\end{aligned}
$$

where A is the mean level of the productivity and $\ln(\Delta M)$ is the mean growth rate of money. ϵ_t^A and ϵ_t^M are white noise processes distributed independently of each other and normally with variances $(\sigma^A)^2$ and $(\sigma^M)^2$ respectively. The government is assumed to provide nominal transfers to the consumers in each period so that $T_t = M_t - M_{t-1}$.

2.4.2 Results

I log-linearize the model around the steady state as I did the baseline model, set all parameter and steady-state values, and compute average sample moments such as standard deviations, autocorrelations, and cross correlations over 200 simulations of 200 quarters (50 years).

I choose several parameter values following CKM: $\alpha = 0.67$, $\chi = 0.39$, and $\theta = 10$. ψ is chosen so that the steady-state share of hours worked is 0.333. ν is chosen so that the standard deviation of investment relative to that of production, which is computed from the Hodrick-Prescott-filtered simulated data, is 3.25. The steady-state values of capital stock and investment are chosen so that the annualized capital-output ratio is 2.65 and the investment-output ratio is 0.23 in the steady state.

Some other values are chosen following BL: $\beta = 0.99$, $\delta = 0.025$, and $\sigma = 1.5$. For exogenous shock processes[16], $A = 1$, $\rho^A = 0.979$, $\sigma^A = 0.0072$, $\Delta M = 1$, $\rho^M = 0.69$, and $\sigma^M = 0.006$.

I set $S/Y = 0.99$ which implies the share of inventory investment in GDP is one percent. The quarterly inventory-to-sales ratio is set to $Z \equiv$

[16]BL choose the parameter values on the process of productivity following King and Rebelo (1999) and the process of money growth by estimation using the post-war U.S. quarterly data on M2.

$X/S = 2/3$. These values are roughly consistent with the U.S. historical data. Meanwhile, I assume that ω is so close to 1 that the effects of real money balances on the marginal utility of consumption and labour supply are negligible[17].

As for the parameters on the sales-cost function, I consider both the cases of constant returns to scale (CRS) and the non-CRS cases. For the CRS cases, I consider a case of relatively strong sales-facilitating motive, $\phi''(Z) = 5$ as in Figure 2.1.1, which implies $S^* \equiv -S\,\Phi_{SX}/\beta = 3.367$ and $X^* \equiv X\,\Phi_{XX}/\beta = 3.367$, and a case of relatively weak sales-facilitating motive, $\phi''(Z) = 1$ as in Figure 2.1.2, which implies $S^* = X^* = 0.6734$. For the non-CRS cases, I consider the cases where X^* is 100 times larger than S^* with S^* unchanged from the above CRS cases, which implies $(S^*, X^*) = (3.367, 336.7)$ and $(S^*, X^*) = (0.6734, 67.34)$. These extremely high values of X^* or Φ_{XX} represent prohibitively high costs of deviating inventory stock from its steady-state level. To sum up, I consider four models varying in the values for S^* and X^*: (3.367, 3.367), (0.6734, 0.6734), (3.367, 336.7), and (0.6734, 67.34).

Results are summarized in Table 2.1. First, compared with the models of prohibitively high inventory-deviating costs (3.367, 336.7) and (0.6734, 67.34), the models of reasonable costs (3.367, 3.367) and (0.6734, 0.6734) generate higher autocorrelations of production and inflation, which implies introducing inventories allows us to reproduce persistent business-cycle fluctuations. Secondly, within the models of reasonable inventory-deviating costs, comparing the model of a large sales-facilitating motive (3.367, 3.367) and that of a small sales-facilitating motive (0.6734, 0.6734), we can see that the cross correlation between inventory investment and production, that is, the procyclicality of inventory investment is stronger in the large sales-facilitating model, as predicted by my analytical results in the preceding sections. The relative volatility of sales to production is accordingly smaller in the large sales-facilitating model. As for the first-order autocorrelations, the large sales-facilitating model generates more persistent inflation due to gradual adjustments of prices in response to money-growth shocks, while the small sales-facilitating model generates more persistent production due to relatively large production-smoothing motive. In the small sales-facilitating model, the autocorrelation of inflation is negative mainly due to the exces-

[17]CKM and BL set ω to 0.94.

sive adjustments of prices. Meanwhile, within the models of prohibitively high inventory-deviating costs, there are little differences between the model (3.367, 336.7) and the model (0.6734, 67.34), especially in the procyclicality of inventory investment.

The first row of Table 2.1 shows the corresponding sample moments in the post-war U.S. data reported in BL. The relative volatility of sales to production and the cross correlation between inventory investment and production in the data are between those in the models (3.367, 3.367) and (0.6734, 0.6734). Meanwhile, the autocorrelations of both production and inflation in the simulated data are far smaller than those in the data.

Table 2.1 also shows the results from one of alternative inventory models of BL. The cross correlation between inventory investment and production is barely positive (0.07), which is the largest value obtained from their models. Meanwhile, they succeed in reproducing high autocorrelations of production and inflation.

Although not all the results of my simulation quantitatively match with the data, they qualitatively well capture the stylized facts on inventories[18] and support my analytical results in the preceding sections.

2.5 Concluding Remarks

In this chapter I have studied the role of inventories in the monetary transmission mechanism by developing simple dynamic general equilibrium models. Introducing inventories allows us to generate real effects of monetary policy lasting several periods even in the baseline model which assumes just one-period pre-determined prices, which implies inventories serve as a source of real rigidities. I consider a production-smoothing motive and a sales-facilitating motive for holding inventories. In the baseline model in Section 2.2, I obtain analytical results that inventories respond procyclically and prices are adjusted gradually to a nominal disturbance only if the sales-facilitating motive is relatively strong enough; otherwise inventories respond countercyclically and prices are adjusted excessively. In the ex-

[18]The simulation results also imply that the relative variability of inventory investment is large (0.69 in the model of (3.367, 3.367)) compared with the share of inventory investment in GDP (0.01) and the cross correlation between inventory-to-sales ratio and production is negative (-0.14), which are both consistent with the stylized facts pointed out by Khan and Thomas (2004).

tended models in Section 2.3 which assume production as well as prices is pre-determined, inventories respond countercyclically at first and then move procyclically only if the sales-facilitating motive is relatively strong enough and the decision lag of price-setting is longer than that of production. In the further extended model in Section 2.4 which introduces labour market, capital accumulation, explicit money, and real disturbances, I obtain quantitative results through simulations that support my analytical results in the preceding sections and are consistent with stylized facts on inventories.

Based on the models developed in this chapter, two directions for future research can be pursued. One is policy research. We may have to consider a monetary policy rule that stabilizes both production and sales as well as prices in the models with inventories. Another direction is empirical research. Inventory data have been used for identifying various types of shocks in the economy[19], and could also provide richer evidences on the monetary transmission mechanism. I hope the models in this chapter serve as a useful building block for future research in those directions.

Appendix

Proof of Proposition 2.1

(2.19) in the log-linearized model implies

$$E_t\hat{Y}_{t+k} = -\frac{X\,\Phi_{CX}}{\eta\,(1+\Phi_C)}\,E_t\hat{X}_{t+k} - \frac{(1+\Phi_C + C\,\Phi_{CC})}{\eta\,(1+\Phi_C)}\,E_t\hat{C}_{t+k} \tag{2.34}$$

for $k \geq 1$. Substituting this into (2.18) and (2.20), I have

$$\begin{aligned} E_t\hat{X}_{t+k} \;=\; & -\frac{\beta\,\Phi_{CX}}{(1+\Phi_C)\,\Phi_{XX} - \beta\,\Phi_{CX}}\,E_t\hat{X}_{t+k+1} \\ & + \frac{\beta\,C\,\Phi_{CC} + (\beta - C\,\Phi_{CX})\,(1+\Phi_C)}{(1+\Phi_C)\,\Phi_{XX} - \beta\,\Phi_{CX}}\,E_t\hat{C}_{t+k} \\ & - \frac{\beta\,C\,\Phi_{CC} + \beta\,(1+\Phi_C)}{(1+\Phi_C)\,\Phi_{XX} - \beta\,\Phi_{CX}}\,E_t\hat{C}_{t+k+1} \end{aligned} \tag{2.35}$$

[19] West (1990) estimates the relative importance of cost and demand shocks as a source of fluctuations in GNP using U.S. inventory data.

$$
\begin{aligned}
E_t\hat{C}_{t+k} &= -\frac{\beta\eta X(1+\Phi_C)+XY\Phi_{CX}}{Y(1+\Phi_C+C\Phi_{CC})+\eta C(1+\Phi_C)^2}E_t\hat{X}_{t+k} \\
&\quad+\frac{\eta X(1+\Phi_C)}{Y(1+\Phi_C+C\Phi_{CC})+\eta C(1+\Phi_C)^2}E_t\hat{X}_{t+k-1} \qquad (2.36)
\end{aligned}
$$

for $k \geq 1$. Combining these, I obtain the following second-order difference equation for $\tilde{X}_k \equiv E_t\hat{X}_{t+k}$.

$$A(L)\tilde{X}_{k+1} = 0,$$

where

$$
\begin{aligned}
A(L) &\equiv \alpha L^2 - \{(1+\beta)\alpha - \frac{\Phi^*}{\Phi_{CX}}\}L + \beta\alpha \\
\alpha &\equiv \left\{\frac{C(1+\Phi_C)}{\beta} - \frac{1+\Phi_C+C\Phi_{CC}}{\Phi_{CX}}\right\}\eta(1+\Phi_C) > 0,
\end{aligned}
$$

and L is the lag operator defined as $L\tilde{X}_k \equiv \tilde{X}_{k-1}$. $A(L)$ can be factored as

$$A(L) = (1-\lambda_1 L)(1-\lambda_2 L)$$

where λ_1 and λ_2 are the two roots of the characteristic polynomial

$$\mathcal{A}(\lambda) \equiv \beta\alpha\lambda^2 - \{(1+\beta)\alpha - \frac{\Phi^*}{\Phi_{CX}}\}\lambda + \alpha = 0.$$

On the assumption (2.29), I have two real roots satisfying $0 < \lambda_1 < 1 < \lambda_2$ because $\mathcal{A}(0) > 0$, $\mathcal{A}(1) < 0$, and $\mathcal{A}(\lambda) > 0$ for large enough λ. To be consistent with the terminal condition $\lim_{k\to\infty}\tilde{X}_k = 0$, the smaller root must be taken in the solution form

$$\tilde{X}_k = \lambda\tilde{X}_{k-1}, \qquad (2.37)$$

so that $\lambda \equiv \lambda_1$. Using this, I can rewrite (2.36) as

$$E_t\hat{C}_{t+k} = \mu E_t\hat{X}_{t+k-1} \qquad (2.38)$$

where

$$\mu \equiv \frac{(1-\beta\lambda)\eta X(1+\Phi_C) - \lambda XY\Phi_{CX}}{Y(1+\Phi_C+C\Phi_{CC})+\eta C(1+\Phi_C)^2}.$$

For $k = 0$, (2.20) with the initial conditions $\hat{C}_t = 1$ and $\hat{X}_{t-1} = 0$ implies

$$\hat{Y}_t = \frac{C}{Y}(1 + \Phi_C) + \frac{\beta X}{Y}\hat{X}_t.$$

Substituting this and (2.34) for $k = 1$ into (2.18), I have

$$\begin{aligned}\hat{X}_t &= -\frac{\beta X \Phi_{CX}}{(X\Phi_{XX} + \eta\beta^2 X/Y)(1 + \Phi_C)} E_t\hat{X}_{t+1} \\ &\quad -\frac{\beta(1 + \Phi_C + C\Phi_{CC})}{(X\Phi_{XX} + \eta\beta^2 X/Y)(1 + \Phi_C)} E_t\hat{C}_{t+1} \\ &\quad +\frac{C\{-\Phi_{CX} - \eta\beta(1 + \Phi_C)/Y\}}{(X\Phi_{XX} + \eta\beta^2 X/Y)}.\end{aligned}$$

Substituting (2.37) and (2.38) into this, I obtain

$$\hat{X}_t = \frac{C(1 + \Phi_C)\{-\Phi_{CX} - \eta\beta(1 + \Phi_C)/Y\}}{(X\Phi_{XX} + \eta\beta^2 X/Y)(1 + \Phi_C) + \mu\beta(1 + \Phi_C + C\Phi_{CC}) + \lambda\beta X\Phi_{CX}} \tag{2.39}$$

The full sequence of $E_t\hat{Y}_{t+k}$ for $k \geq 0$ can be obtained by substituting (2.37), (2.38), (2.39) and $\hat{C}_t = 1$ into (2.20).

$$E_t\hat{Y}_{t+k} = \frac{C}{Y}(1 + \Phi_C) E_t\hat{C}_{t+k} + \frac{X}{Y}(\beta E_t\hat{X}_{t+k} - E_t\hat{X}_{t+k-1})$$

Finally, since I consider a disturbance that causes $E_t\hat{\mathcal{M}}_{t+k} = 1$ for all $k \geq 0$, (2.17) and (2.21) implies

$$\begin{aligned} E_t\hat{P}_{t+k} &= 1 - E_t\hat{C}_{t+k} \qquad (2.40)\\ E_t\hat{i}_{t+k} &= 0 \end{aligned}$$

for all $k \geq 0$.

Proof of Proposition 2.2

The solutions for $k \geq 1$ are the same as Proposition 2.1.

For $k = 0$, since production as well as prices are pre-determined, $\hat{Y}_t = 0$, $\hat{P}_t = 0$, and $\hat{C}_t = 1$. Substituting these initial conditions into (2.20), I obtain

$$\hat{X}_t = -\frac{C(1 + \Phi_C)}{\beta X}.$$

Proof of Proposition 2.3

The solutions for $k \geq 2$ and $k = 0$ are the same as Proposition 2.2.

For $k = 1$, production still cannot be adjusted, i.e., $E_t\hat{Y}_{t+1} = 0$, while prices become adjustable in $t+1$. Then (2.19) implies

$$0 = -X\,\Phi_{CX}\,E_t\hat{X}_{t+1} - (1+\Phi_C + C\,\Phi_{CC})\,E_t\hat{C}_{t+1} \tag{2.41}$$

Meanwhile, substituting the solution for $\hat{X}_t$ into (2.20), I have

$$0 = C\,(1+\Phi_C)\,E_t\hat{C}_{t+1} + \beta\,X\,E_t\hat{X}_{t+1} + C\,(1+\Phi_C)/\beta. \tag{2.42}$$

Combining (2.41) and (2.42), I obtain

$$\begin{aligned} E_t\hat{X}_{t+1} &= \frac{C\,(1+\Phi_C)\,(1+\Phi_C + C\,\Phi_{CC})}{\beta\,X\,\{C\,(1+\Phi_C)\,\Phi_{CX} - \beta\,(1+\Phi_C + C\,\Phi_{CC})\}} \\ E_t\hat{C}_{t+1} &= -\frac{C\,(1+\Phi_C)\,\Phi_{CX}}{\beta\,\{C\,(1+\Phi_C)\,\Phi_{CX} - \beta\,(1+\Phi_C + C\,\Phi_{CC})\}}. \end{aligned}$$

Proof of Proposition 2.4

The solutions for $k \geq 2$ and $k = 0$ are the same as Proposition 2.2.

For $k = 1$, prices still cannot be adjusted, therefore, $E_t\hat{C}_{t+1} = 1$, while prices become adjustable in $t+1$. Substituting this condition and the solution for $\hat{X}_t$ into (2.20), I have

$$E_t\hat{Y}_{t+1} = \frac{1+\beta}{\beta}\frac{C}{Y}\,(1+\Phi_C) + \frac{\beta X}{Y}\,E_t\hat{X}_{t+1}.$$

Substituting this and (2.34) for $k = 2$ into (2.30), I have

$$\begin{aligned} E_t\hat{X}_{t+1} &= -\frac{\beta\,X\,\Phi_{CX}}{(X\,\Phi_{XX} + \eta\,\beta^2\,X/Y)\,(1+\Phi_C)}\,E_t\hat{X}_{t+2} \\ &\quad -\frac{\beta\,(1+\Phi_C + C\,\Phi_{CC})}{(X\,\Phi_{XX} + \eta\,\beta^2\,X/Y)\,(1+\Phi_C)}\,E_t\hat{C}_{t+2} \\ &\quad +\frac{C\,\{-\Phi_{CX} - \eta\,(1+\beta)\,(1+\Phi_C)/Y\}}{(X\,\Phi_{XX} + \eta\,\beta^2\,X/Y)}. \end{aligned}$$

Substituting (2.37) and (2.38) into this, I obtain

$$E_t\hat{X}_{t+1} = \frac{C\,(1+\Phi_C)\,\{-\Phi_{CX} - \eta\,(1+\beta)\,(1+\Phi_C)/Y\}}{(X\,\Phi_{XX} + \eta\,\beta^2\,X/Y)\,(1+\Phi_C) + \mu\,\beta\,(1+\Phi_C + C\,\Phi_{CC}) + \lambda\,\beta\,X\,\Phi_{CX}}.$$

References

[1] Barnanke, B. S. and Gertler, M. (1995), "Inside the Black Box: The Credit Channel of Monetary Policy Transmission," *Journal of Eco-*

nomic Perspectives, 9(4): 27-48.

[2] Ball, L. and Romer, D. (1990), "Real Rigidities and the Non-Neutrality of Money," *Review of Economic Studies*, 57: 183-203.

[3] Bils, M. and Kahn, J. A, (2000), "What Inventory Behavior Tells Us About Business Cycles," *American Economic Review*, 90(3): 458-481.

[4] Blinder, A. S. and Fischer, S. (1981), "Inventories, Rational Expectations, and the Business Cycle," *Journal of Monetary Economics*, 8: 277-304.

[5] Boileau, M. and Letendre, M. A. (2004), "Inventories, Sticky Prices, and the Propagation of Nominal Shocks," University of Colorado and McMaster University.

[6] Chang, Y., Hornstein, A. and Sarte, P. D. (2004), "Productivity, Employment, and Inventories," Federal Reserve Bank of Richmond, Working Paper, 04-09.

[7] Chari, V. V., Kehoe, P. J. and McGrattan, E. R. (2000), "Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?," *Econometrica*, 68(5): 1151-1179.

[8] Fischer, S. (1977), "Long-Term Contracts, Rational Expectations, and the Optimal Money Supply Rule," *Journal of Political Economy*, 85: 191-205.

[9] Hornstein, A. and Sarte, P. D. (2001), "Sticky Prices and Inventories: Production Smoothing Reconsidered," Federal Reserve Bank of Richmond, Working Paper, 01-06.

[10] Ireland, P. N. (2001), "Sticky-Price Models of the Business Cycle: Specification and Stability," *Journal of Monetary Economics*, 47: 3-18.

[11] Khan, A. and Thomas, J. K. (2003), "Inventories and the Business Cycle: an Equilibrium Analysis of (S,s) Policies," NBER Working Paper, 10078.

[12] Khan, A. and Thomas, J. K. (2004), "Modeling Inventories over Business Cycle," NBER Working Paper, 10652.

[13] King, R. G. and Rebelo, S. T. (1999), "Resuscitating Real Business Cycles," in Taylor, J. B. and Woodford M. (eds), *Handbook of Macroeconomics*, Vol. 1B, Chap. 14, 927-1007, North Holland.

[14] Kydland, F. and Prescott, E. C. (1982), "Time to Build and Aggregate Fluctuations," *Econometrica*, 50: 1345-1370.

[15] Mankiw, N. G. and Reis, R. (2002), "Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Philips Curve," *Quarterly Journal of Economics*, 117(4): 1295-1328.

[16] Metzler, L. A. (1941), "The Nature and Stability of Inventory Cycles," *The Review of Economic Statistics*, 23: 113-129.

[17] Ramey, V. A. and West, K. D. (1999), "Inventories," in Taylor, J. B. and Woodford, M. (eds), *Handbook of Macroeconomics*, Vol. 1B, Chap. 13, 863-923, North Holland.

[18] Rotemberg, J. J. (1982), "Sticky Prices in the United States," *Journal of Political Economy*, 90: 1187-1211.

[19] Taylor, J. B. (1980), "Aggregate Dynamics and Staggered Contracts," *Journal of Political Economy*, 88(1): 1-22.

[20] Wen, Y. (2002), "Understanding the Inventory Cycle," Cornell University, CAE Working Paper, 02-04.

[21] West, K. D. (1990), "The Sources of Fluctuations in Aggregate Inventories and GNP," *Quarterly Journal of Economics*, 105(4): 939-71.

[22] Woodford, M. (2003), *Interest and Prices*, Princeton University Press.

Figure 2.1: Baseline Model

Figure2.1.1: Strong sales-facilitating motive

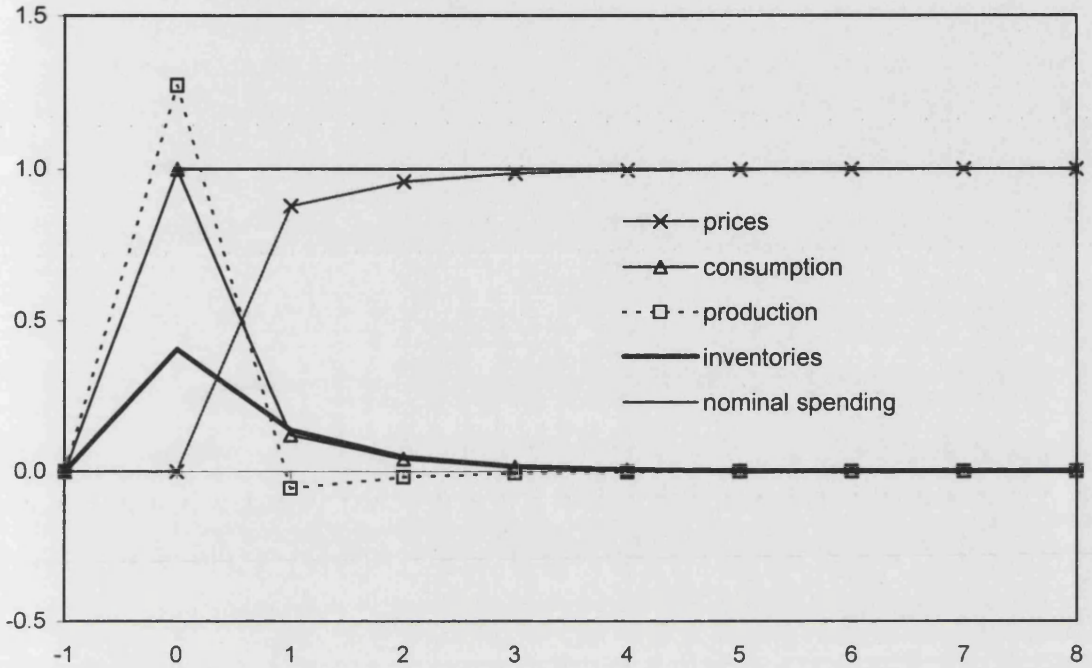


Figure2.1.2: Weak sales-facilitating motive

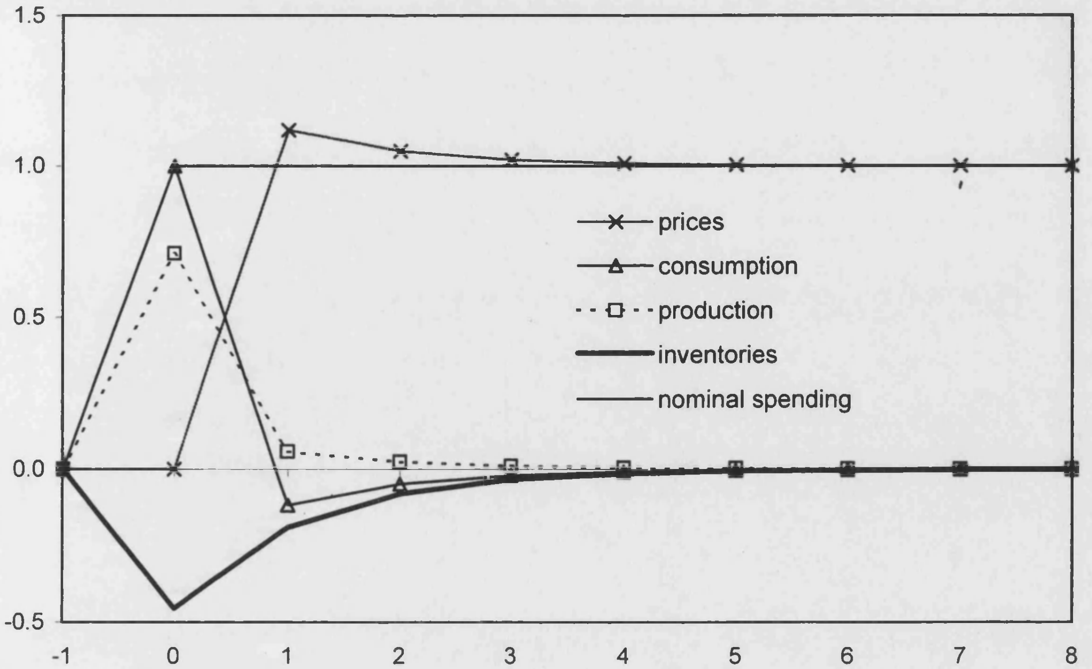


Figure 2.2: Unintended Inventories

Figure2.2.1: Strong sales-facilitating motive

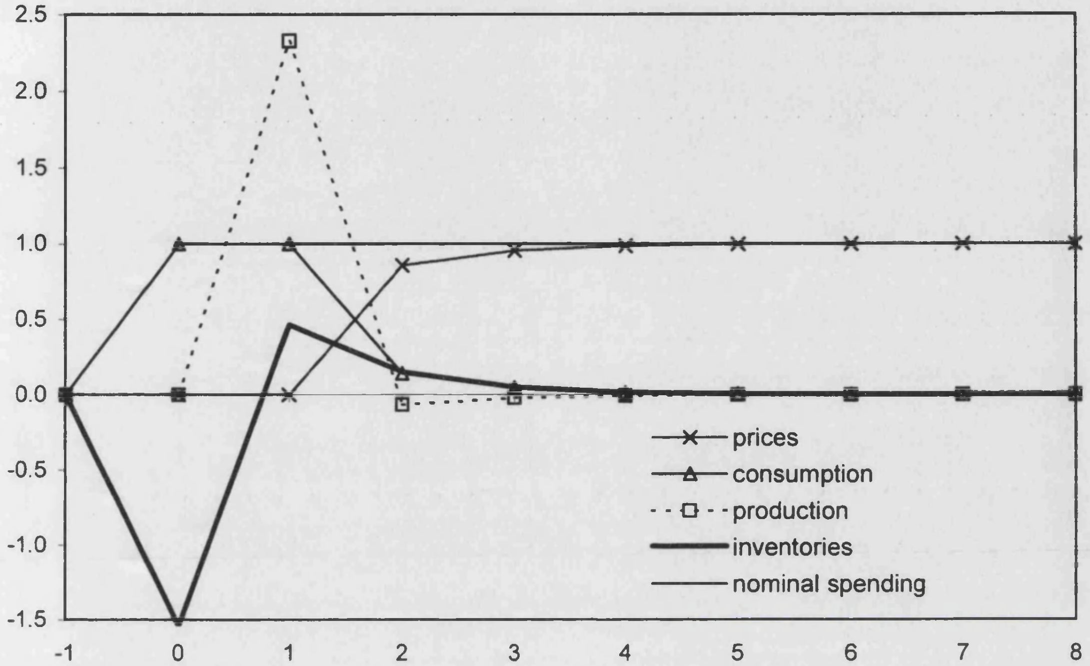


Figure2.2.2: Weak sales-facilitating motive

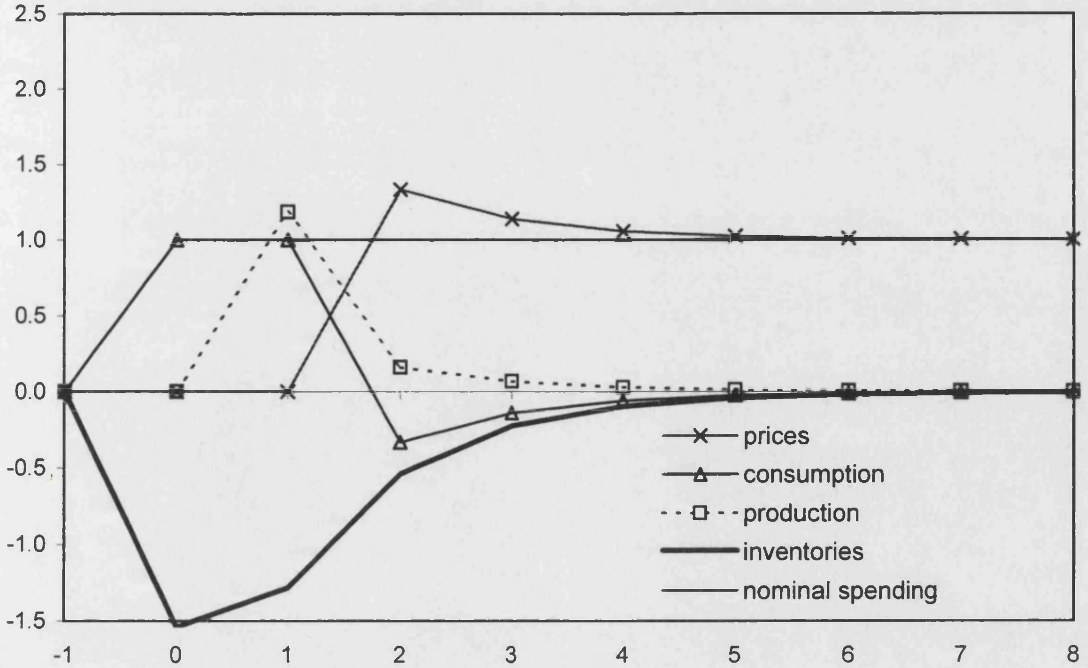


Figure 2.3: Heterogeneity in Price-Setting

Figure2.3.1: Aggregate variables

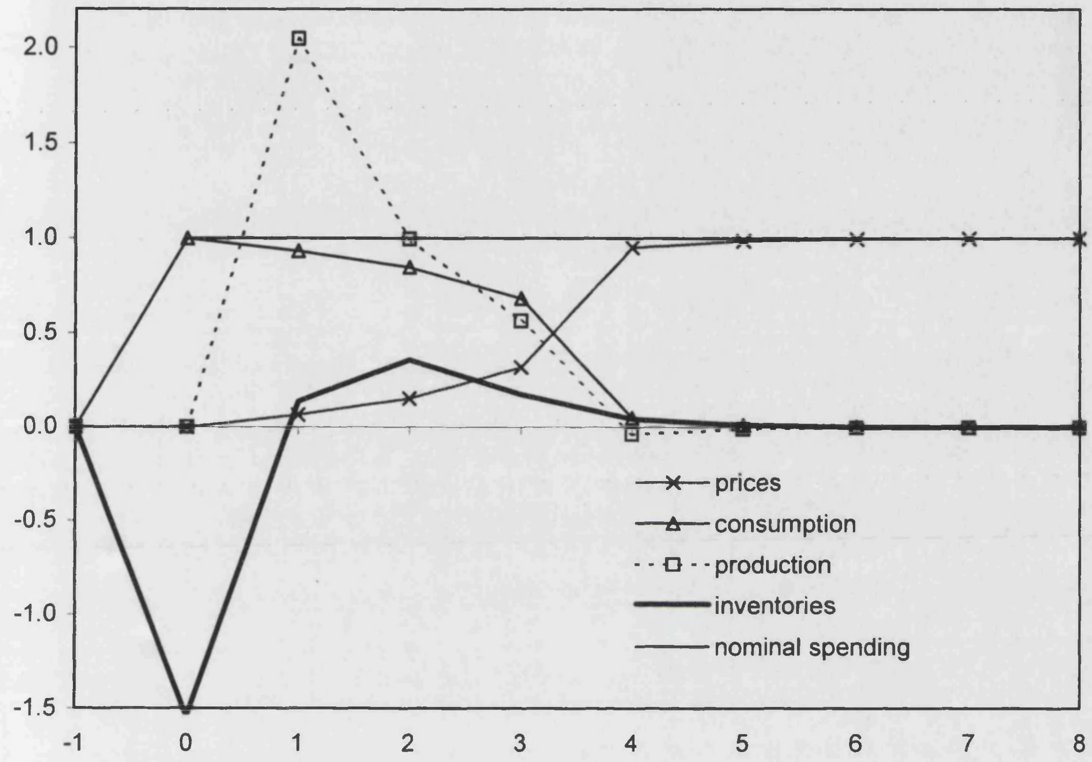


Figure2.3.2: Disaggregate variables

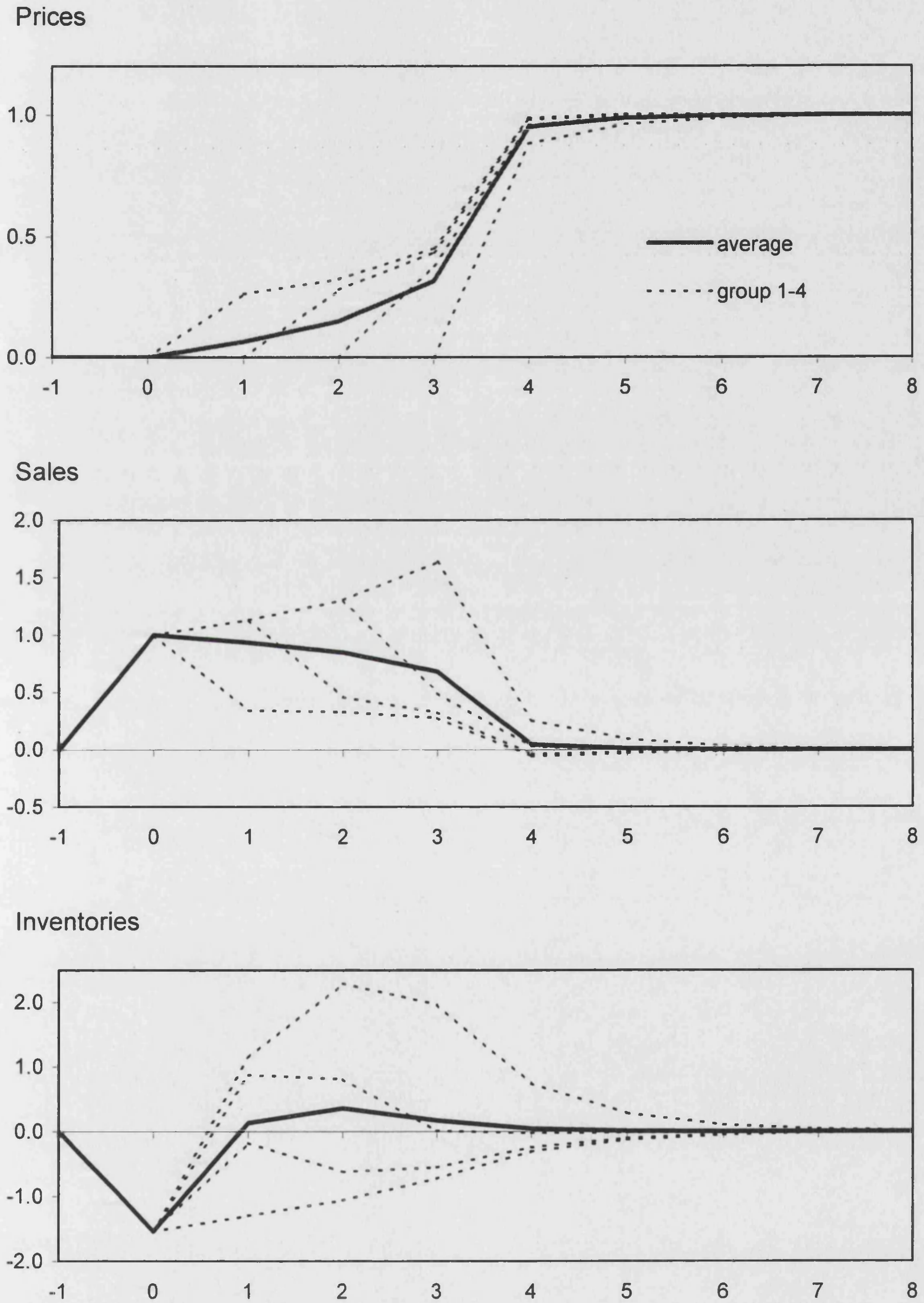


Table 2.1: Quantitative Experiments

	Cross Correlation dX and Y	Relative Volatility S to Y	First-Order Autocorrelation Y	dP
U.S. data	0.50	0.94	0.97	0.78
Boileau=Letendre	0.07	1.37	0.92	0.71
Model				
(S*, X*)				
(3.367, 3.367)	0.69	0.81	0.26	0.16
(0.6734, 0.6734)	0.46	0.98	0.44	-0.01
(3.367, 336.7)	0.73	0.79	0.10	-0.14
(0.6734, 67.34)	0.73	0.79	0.12	-0.21

Notes:

1. Y is production, S is sales, dX is inventory investment, and dP is inflation.
2. U.S. data are those reported in Boileau and Letendre (2004). They calculate the sample moments of quarterly data over 1959:1 to 2000:1 after removing linear-quadratic trends.
3. The results of simulations by Boileau and Letendre (2004) are those from their benchmark shopping-cost model.

Chapter 3

Financial Accelerator Effects in Japan's Business Cycles[1]

This chapter calibrates a dynamic general equilibrium model which incorporates credit-market imperfections using Japanese data. The model exhibits financial accelerator effects, the mechanism whereby credit-market imperfections help to propagate or amplify various types of shocks including monetary policy shocks to the economy. The main result is that the large volatility of Japan's corporate investment can be explained by taking account of this mechanism. I examine the robustness of the results and consider some variations of the model including the adoption of alternative monetary policy rules and the introduction of an asset price bubble.

[1]A revised version of *Bank of Japan Working Paper* released in August 2002.

3.1 Introduction

Japan's economy has experienced large fluctuations especially since the late 1980s. After strong expansion from 1986 to 1991, long-lasting severe recessions have alternated with short-lived moderate expansions. Among several components of real activity to have undergone fluctuations during this period, the key factor has been corporate fixed investment. A large number of studies have investigated the cause of this volatile investment. While the standard neoclassical framework implies that investment is determined by expected future business profitability and the cost of capital, many empirical studies have suggested that financial factors such as balance sheet conditions also influence investment expenditures to some extent, especially those of small and medium sized firms, due to agency costs arising from asymmetric information between firms and financial intermediaries.[2] According to this view, credit market imperfections help to propagate or amplify various types of shocks including monetary policy shocks to the economy. Changes in credit market conditions such as asset prices, corporate debt burdens, and banking capital adequacies are not simply passive reflections of the real economy, but are also in turn major factors that affect real economic activity. These feedback mechanisms, which I will specify later as "financial accelerator" effects, might have become active when asset prices underwent large swings in the late 1980s and early 1990s and when the banking system suffered from broad-scale malfunction in the late 1990s.

The purpose of this chapter is to examine how financial factors due to credit-market imperfections affect economic fluctuations, especially the volatility of corporate fixed investment[3], in a stochastic dynamic general equilibrium setting. Although many academic researchers and practitioners share the view that financial factors may be important, standard macroeconomic frameworks, both real business cycle models and Keynesian models, usually ignore credit-market imperfections.[4] While a large number of previous studies consider financial factors by estimating a single investment

[2]See, for example, Ogawa and Kitasaka (1998), Motonishi and Yoshikawa (1999), and Nagahata and Sekine (2002) for empirical studies which suggest the effects of financial factors on investment in Japan.

[3]Household consumption and investment may also be affected by credit-market imperfections. However, I consider only the effects on corporate investment in this chapter.

[4]In other words, both real business cycle models and Keynesian models inherently subscribe to the Modigliani-Miller theorem (see footnote 14).

function with corresponding explanatory variables, I calibrate a dynamic general equilibrium model which incorporates credit-market imperfections using Japanese data.

I follow the standard approach for calibrating a dynamic general equilibrium model and begin by identifying the facts underlying Japan's business cycles. Here my interest is focused on the cyclical behaviour of variables around their balanced growth paths, as captured by the widely-used Hodrick-Prescott(HP) filter. Chart 3.1 shows standard deviations and cross correlations with output (real GDP) of several detrended series over the period from 1980/Q1 to 2001/Q1. I deal here with GDP, corporate fixed investment, household consumption, total working hours, the inflation rate, and the monetary aggregate.[5] We can see that corporate fixed investment during the sample period is nearly five times more volatile than output, and that this ratio is much larger than that of the U.S. (about three).[6] In previous studies based on real business cycle (RBC) models, the ratio ranges from two to three.[7] The gap between the actual volatility and RBC simulated volatility suggests the existence of some other types of shocks in addition to standard RBC shocks and/or some mechanism that amplifies shocks to the economy. I try to make up for this gap by introducing both nominal shocks (monetary policy shocks) and credit-market imperfections. I first introduce price stickiness into the RBC model, thus setting up the so-called dynamic New Keynesian (DNK) model, and then introduce credit-market imperfections into that DNK model.

The model is based on Bernanke, Gertler and Gilchrist (1999)[8] (BGG hereafter). They embedded a micro financial contracting problem between firms (borrowers) and financial intermediaries (lenders) into a macroeconomic DNK framework in a manner that is both rigorous and yet straight-

[5]GDP, investment, consumption, and working hours are real and per-capita base. Inflation is measured by changes in the GDP deflator. The monetary aggregate is M2+CD (Japan) or M2 (U.S.). All variables except inflation and interest rates are in logarithms and seasonally adjusted.

[6]We can also see that the relative volatility in Japan exceeds four in both subsample periods of 1980s and 1990s.

[7]The relative volatility of investment to output is two in Ohkusa (1991) which calibrated a basic RBC model using Japanese data, and is three in King and Rebelo (1999) which calibrated one using U.S. data.

[8]I slightly change their original model: I introduce a monetary policy rule which reacts to both output and price level rather than inflation, and simplify entrepreneurs' net worth accumulation (omit entrepreneurs' consumption and labour supply).

forward.[9] The model contains households and entrepreneurs (and financial intermediaries) distinctively in order to explicitly motivate lending and borrowing. In addition, there are retailers who set the final (retail) price of output goods, capital producers who transform output goods into capital goods, and a government sector which conducts fiscal and monetary policy. The economy is described as a decentralized rational expectation equilibrium. The only source of economic fluctuation comes from unanticipated shocks: technology shocks, demand shocks, and monetary policy shocks. The model incorporates credit-market imperfections through the assumption that external funds and internal funds are not perfect substitutes: in particular, the difference in the cost of these funds (the external finance premium) depends inversely on the value of entrepreneurs' own wealth (net worth[10]). Procyclical movements in entrepreneurs' net worth caused by unanticipated shocks then lead to countercyclical movements in the external finance premium, and thus enlarge the volatility of investment and amplify economic fluctuations. BGG named this mechanism the "financial accelerator".

I adopt this model to explain the above facts underlying Japan's business cycles. I set parameter values based on actual Japanese data and calculate second moment properties such as standard deviations and cross correlations for both of the price stickiness models: the one with, the other without credit-market imperfections. The main result, which I demonstrate fully in Section 3.4.3, is that introducing credit-market imperfections significantly enlarges the volatility of investment and brings it close to the actual volatility, without loosing the model's fit in other dimensions such as consumption and inflation. This is one piece of evidence supporting the existence of financial accelerator effects in Japan's business cycles: credit-market imperfections help to propagate unanticipated shocks, especially monetary policy shocks, and do indeed amplify economic fluctuations. This result supports the view emphasizing the importance of financial factors.[11]

[9] Other important examples which embed credit-market imperfections into a macroeconomic dynamic general equilibrium framework are Calmstrom and Fuerst (1997) and Kiyotaki and Moore (1997). However, unlike BGG, they do not introduce a nominal rigidity.

[10] The net worth is defined as liquid assets plus the collateral value of illiquid assets less outstanding obligations.

[11] Based on an orthodox growth account approach, Hayashi and Prescott (2002) argue that the problem in 1990s Japan was not a breakdown of the financial system but a low productivity growth rate. Unlike them, however, I focus on cyclical behaviours, so that my argument is not necessarily contradictory to theirs.

I then examine the robustness of the result and consider some variations of the model. First I vary some underlying parameter values. I report the outcomes of varying the values assigned to key parameters governing entrepreneurs' balance sheet conditions, price stickiness, and labour market elasticity. These experiments illustrate various properties of the model and may also be of potential simulative interest in considering possible future states of the economy. I then adopt alternative monetary policy rules and introduce an asset price bubble to the model. In this model, the monetary policy rule has a particularly important role in either stabilizing or destabilizing the economy, and these stabilizing effects are stronger in the case with the financial accelerator than in the case without it. An asset price bubble can be another important source of economic fluctuation in the case with financial accelerator effects, and I find that the large volatility of investment can be better explained by introducing such a bubble as a supplement to the model.

The remainder of this chapter is organized as follows. In Section 3.2, I explain the functioning of the financial accelerator within a minimal setting in which I focus on corporate investment. The rest of the full macroeconomic model is then described in Section 3.3. I calibrate the model using Japanese data and calculate the second moment properties in Section 3.4. I examine how well these properties correspond to those observed in the actual Japanese data, and demonstrate the results in detail. In Section 3.5, I examine the robustness of the results and consider some variations of the model including the adoption of alternative monetary policy rules and the introduction of an asset price bubble as mentioned above. Section 3.6 presents concluding remarks.

3.2 Financial Accelerator Effects on Corporate Investment

In this section, I explain the mechanism whereby credit-market imperfections help to propagate or amplify various types of shocks to the economy, the so-called financial accelerator effects, in a minimal setting in which I focus on corporate investment.[12] The core model described below will be

[12]Other models that explain the financial accelerator more simply in a partial equilibrium setting are given in Bernanke, Gertler, and Gilchrist (1996) and Kasuya and

one part of the full macroeconomic dynamic general equilibrium model I calibrate in this chapter.

3.2.1 The Core Model

I here model the capital-purchasing decisions of "entrepreneurs". There are also external capital producing firms and financial intermediaries providing external funds in this model. Entrepreneurs purchase capital from the capital producers and then produce their output good. In order for the entrepreneurs to purchase capital, they have access to external funds in addition to their own wealth (net worth). Capital producers, on the other hand, purchase the entrepreneurs' output good and transform it into the capital good to sell to entrepreneurs. Although the entrepreneurs' output good will be sold out and finally consumed by some other agents (households), here I do not describe the behaviour of consumers.

Entrepreneurs

Entrepreneurs purchase capital in each period for use in the subsequent period.[13] Capital is used in combination with hired labour to produce the output good as follows

$$Y_t = A_t \left(K_{t-1}\right)^{\alpha} \bar{H}^{1-\alpha}, \tag{3.1}$$

where Y_t, K_t, $\bar{H}$ are output, capital, and the labour input, respectively. A_t is an exogenous technology parameter (total factor productivity). Here the labour input and the payments for it, corresponding to its marginal product, are assumed to be fixed.

Entrepreneurs are risk-neutral. Their demand for capital is determined by comparing the expected marginal return to holding capital with its expected marginal financial cost. Given the production technology (3.1), the

Fukunaga (2003). Here I extract the minimal setting straightforwardly from the dynamic general equilibrium model of BGG.

[13] I here assume that the economy contains a large number of identical entrepreneurs and describe the model below as the representative entrepreneur's problem. (This assumption also holds for the capital producers, households, and retailers I will describe later.) To consider the credit-market imperfections explicitly, I should start from the individual firm subject to idiosyncratic risk on its return to capital, as BGG does. However, some assumptions, such as the independent and identical distributions of the idiosyncratic disturbances and the constant returns to scale in production, allow me to derive aggregate relationships straightforwardly.

ex-post gross return to holding a unit of capital from t to $t+1$, F_{t+1}, is defined as

$$F_{t+1} = \frac{\frac{\alpha Y_{t+1}}{K_t} + Q_{t+1}(1-\delta)}{Q_t}, \tag{3.2}$$

where Q_t is the relative price (in terms of the output good) of a unit of capital which varies depending on the capital production technology (described below) and is taken as given. δ is the depreciation rate of capital. The first term of the numerator, $\frac{\alpha Y_{t+1}}{K_t}$, is the income gain, corresponding to the rent paid to a unit of capital that is assumed to be diminishing returns. The second term is the capital gain, which is all enjoyed by entrepreneurs rather than capital producers who must earn zero profits in competitive equilibrium.

The financial cost condition for the capital-purchasing decision is the main feature of this model. In the standard framework, the expected rate of return is always taken to be equal to the marginal opportunity cost of funds, such as the riskless interest rate, R_{t+1}, which is given independently of entrepreneurs' decision making.

$$\mathrm{E}_t\, F_{t+1} = R_{t+1}.$$

In contrast, here I assume that there exist credit-market imperfections whereby additional costs are imposed on borrowers if they demand external funds (uncollateralized loans) to purchase more of the capital good than they are able to purchase using only internal funds (collateral value).[14] Following BGG, I consider the situation where lenders (financial intermediaries) must pay a cost if they wish to observe borrowers' (entrepreneurs') realized returns. In this situation, the optimal (state-contingent) contract between lenders and borrowers looks like a standard debt contract under which lenders only observe the realized return of the borrowers who could not earn enough to repay at some predetermined rate (contingent on the realized aggregate state); lenders then withdraw all of the observed return from the bankrupt borrowers.[15] In such a situation the auditing cost lenders must pay can be interpreted as the cost of default. Since competitive lenders must receive an

[14]Under perfect credit markets, there are no additional costs of external funds and no distinction between external and internal funds (the Modigliani-Miller theorem).

[15]This type of setting is referred to as "costly state verification (CSV)," a problem analyzed first by Townsend (1979).

expected return to lending (less the auditing cost) equal to the opportunity cost of their funds, the borrowers' expected rate of return ($\mathrm{E}_t F_{t+1}$) must exceed the riskless interest rate (R_{t+1}) to compensate for the default cost. The default risk itself is determined by the extent to which entrepreneurs depend on external funds, and this leads to a relationship between two crucial ratios: the ratio of $\mathrm{E}_t F_{t+1}$ to R_{t+1}, which I call the external finance premium, and the ratio of internal (or alternatively external) funds to the total value of purchased capital, as follows[16]

$$\mathrm{E}_t F_{t+1} = R_{t+1}\, S\left(\frac{N_t}{Q_t K_t}\right) \quad \text{with } N_t \leq Q_t K_t,\ S(1) = 1,\ S'(\cdot) < 0, \tag{3.3}$$

where N_t is entrepreneurs' own wealth (net worth). When the ratio of internal funds is low, the default risk is high, and in this case the external finance premium should be large.[17] Note that the above condition is an *ex-ante* relationship, that is, (3.3) holds only for the expected values conditional on the information available in period t rather than for every realization of F_{t+1}. Note also that entrepreneurs borrow only if they cannot afford their optimal capital stock; their internal funds are always used fully (thus $N_t \leq Q_t K_t$) because the cost of internal funds is always lower than that of external funds for them.[18]

While the conditions of demand for capital (3.2) and financial cost (3.3) mutually determine the static features of the entrepreneurs' capital-purchasing decision, the evolution of their net worth together with the variations in the price of capital play an critical role in the dynamics of this model. The net worth evolves according to

$$N_t = \gamma\ \{ F_t\, Q_{t-1} K_{t-1} - R_t\ S\left(\frac{N_{t-1}}{Q_{t-1} K_{t-1}}\right) (Q_{t-1} K_{t-1} - N_{t-1}) \}, \tag{3.4}$$

where I assume that each entrepreneur has a constant probability γ of surviving to the next period (i.e. there is a probability $1 - \gamma$ that he dies in between periods).[19] The terms in the braces describe the operational firm's

[16]Here I do not describe the optimal contracting problem, which is fully described in Appendix A of BGG.

[17]$S(\cdot)$ is not entrepreneur-specific so that all entrepreneurs choose the same ratio of internal funds.

[18]This is referred to as the "pecking order" assumption.

[19]This assumption preclude the probability either that the net worth diverges or entrepreneurs accumulate enough wealth to be fully self-financed so that credit markets disappear. This is essentially the same situation as that described by Carlstrom and Fuerst (1997) where infinitely lived entrepreneurs discount the future more heavily than

value (equity): the first term is gross return; the second is a repayment term which includes compensation for the potential cost of default (the auditing cost) reflecting the external finance premium. Note that F_t is the *ex-post* rate of return so that the above condition (3.4) holds for every realized return, unlike (3.3). If an unanticipated movement in the returns such as an unpredictable variation in the capital (asset) price, Q_t, occurs within period t, the ex-post return diverges from the financial costs and the net worth is directly affected. Under credit-market imperfections, the net worth responds endogenously to such unanticipated shocks, and this in turn affects economic fluctuations via its interaction with the external finance premium.

Capital Producers

Capital producers purchase the entrepreneurs' output good as a material input, I_t, and combine it with rented capital, K_{t-1}, to produce new capital, $\hat{K}_t$, as follows[20]

$$\hat{K}_t = \Phi\left(\frac{I_t}{K_{t-1}}\right) K_{t-1} \quad \text{with } \Phi(0) = 0,\ \Phi'(\cdot) > 0,\ \Phi''(\cdot) < 0.$$

To capture the empirical fact that corporate investment slightly lags output as shown in Chart 3.1, I assume that the capital producers' purchasing decision has to be made one period in advance. They choose next period expenditure, I_{t+1}, to maximize their expected profits, $\mathrm{E}_t\,[\,Q_{t+1}\,\hat{K}_{t+1} - I_{t+1}\,]$, taking the expected relative price of capital, $\mathrm{E}_t\,Q_{t+1}$, as given. The first-order condition is

$$\mathrm{E}_t\,Q_{t+1} = \mathrm{E}_t\left[\Phi'\left(\frac{I_{t+1}}{K_t}\right)\right]^{-1}. \tag{3.5}$$

Here I normalize the capital production function so that the relative price of capital is unity in the steady state, i.e. $\Phi'(I/K) = 1$ where I and K without time subscripts are the steady-state values. I interpret the capital

do households (in the core model here, their discount rate is higher than the riskless interest rate, R_t). Meanwhile, the birth rate is set to keep the total number of entrepreneurs constant. I do not consider an inheritance problem by assuming implicitly that the bequest which would be left by the dead entrepreneurs perishes immediately on his death, while the new generation of entrepreneurs (and those losing their net worth by default) are given some wealth to begin operations. BGG deal with this problem more explicitly by introducing entrepreneurs' inelastic labour supply and consumption. However, this does not change the essential dynamic properties of the model.

[20]The concavity of the following capital production function ($\Phi''(\cdot) < 0$) implies increasing marginal adjustment costs.

producers' expenditure, I_t, as "investment" for capital. Their decision is linked with the entrepreneurs' capital-purchasing decision described above via the variation in the price of capital.

The aggregate capital stock evolves according to

$$K_t = \Phi\left(\frac{I_t}{K_{t-1}}\right) K_{t-1} + (1-\delta) K_{t-1}, \tag{3.6}$$

where δ is the depreciation rate. Capital is homogeneous so that there is no difference between newly-produced and old capital. Old capital used by entrepreneurs is rented out for the production of new capital, and then returned at the same price as the newly-produced capital.[21]

Equilibrium

A rational expectations equilibrium is defined as a set of endogenous variables $\{Y_t, K_t, N_t, I_t, F_t, Q_t\}$ which satisfies entrepreneurs' decision rules (3.2) and (3.3) (the latter implicitly includes financial intermediaries' decisions), capital producers' decision rule (3.5), and resource constraints (3.1), (3.4), and (3.6), given that the exogenous variables $\{R_t, A_t\}$ follow the stochastic processes defined below:

$$\begin{aligned} r_t &= \rho_r\, r_{t-1} + e^r{}_t && (3.7) \\ a_t &= \rho_a\, a_{t-1} + e^a{}_t, && (3.8) \end{aligned}$$

where r_t and a_t are percentage deviations from the steady states of R_t and A_t, respectively. $e^r{}_t$ and $e^a{}_t$ are random variables distributed normally and uncorrelated both serially and contemporaneously. These are assumed to be stationary autoregressive processes (i.e. they do not contain trend growth).

I can analyze the dynamics of the model by restricting my attention to stationary fluctuations around the steady state. I log-linearize the system as follows.

$$\begin{aligned} y_t &= a_t + \alpha\, k_{t-1} && (3.9) \\ f_t &= (1-\upsilon)(y_t - k_{t-1}) + \upsilon\, q_t - q_{t-1} && (3.10) \\ \mathrm{E}_t\, f_{t+1} &= r_{t+1} - \psi\, [\, n_t - (q_t + k_t)\,] && (3.11) \end{aligned}$$

[21]The rental rate is determined by the zero profit condition for competitive capital producers using constant returns to scale technology. It must be zero in the steady-state.

$$
\begin{aligned}
n_t &= \gamma F \left[\chi f_t - (\chi - 1) r_t \right. \\
&\quad \left. + \psi (\chi - 1)(q_{t-1} + k_{t-1}) + \{ 1 - \psi (\chi - 1) \} n_{t-1} \right] && (3.12) \\
\mathrm{E}_t\, q_{t+1} &= \phi (\mathrm{E}_t\, i_{t+1} - k_t) && (3.13) \\
k_t &= \delta\, i_t + (1 - \delta)\, k_{t-1}. && (3.14)
\end{aligned}
$$

Equations (3.9) through (3.14) correspond to equations (3.1) through (3.6), respectively.[22] Following the convention, all lowercase variables denote percentage deviations from steady state.[23]

3.2.2 Financial Accelerator Effects

I here demonstrate the financial accelerator effects by examining the responses of the six endogenous variables in the core model to one-shot unanticipated shocks to each exogenous variable, consisting of a one percent deviation from the steady state economy.[24] I choose parameter values based on actual data (historical averages), single-equation estimation, or references to preceding studies (shown in Chart 3.2), as detailed in the fuller discussion in Section 3.4.1 below.[25] Each variable converges to the steady-state eventually under those parameter values, which means that the system is stable around the steady-state. Responses to the interest-rate shock are shown in Chart 3.3, and to the technology shock in Chart 3.4.[26]

First I examine the effects of changes in monetary policy which can control the *real* riskless interest rate r_t in Chart 3.3. In the upper panel, ψ is set to zero so that there are no credit-market imperfections (the no financial accelerator case, or the No-FA case). When the interest rate (always equal to the expected rate of return) rises, investment decreases at first (from the subsequent period to the initial shock), but reverts to the steady-state level

[22] I shift the time subscripts in (3.10) backward from (3.2) because it is an ex-post relationship subject to unanticipated shocks within the period.

[23] Uppercase variables without time subscripts denote steady-state values. Greek letters without time subscripts are fixed parameters, e.g. χ is the steady-state value of the reciprocal of the self-financing ratio, $\frac{Q_t K_t}{N_t}$.

[24] When solving the model, I shift the time subscripts of the riskless interest rate appearing in any equation one period backward in order to distinguish the ex-post relationships which are subject to the unanticipated shock from those which are not.

[25] The first-order autoregressive parameter on the shock process for the riskless interest rate, ρ_r, which is endogenized in the whole model later, is here arbitrarily set to 0.5.

[26] The model is solved by the method of undetermined coefficient (the eigen decomposition method). I utilize Harald Uhlig's MATLAB programs which are available at his homepage (http://www.wiwi.hu-berlin.de/wpol/html/toolkit.htm). The detailed methodological exposition is given in Uhlig (1999).

as the shock dies away. In the lower panel depicting the case with credit-market imperfections (the FA case), on the other hand, investment remains lower than the steady-state level even after the shock dies away, and capital and output continue to deviate from their steady-states for an accordingly long time. This persistency comes from the damage to entrepreneurs' net worth caused by the unanticipated fall in the price of capital during the initial period. It takes a long time for the damaged net worth to return to its steady-state level. The damage to the net worth causes the external finance premium to widen in the FA case, which in turn further dampens the demand for capital.

Next, I examine the effects of technology shocks, or changes in a_t, in Chart 3.4. When a positive technology shock boosts output, the demand for capital is stimulated via an upward shift in the marginal product of capital. In the FA case, moreover, the improvement in entrepreneurs' net worth further stimulates the demand for capital via a reduction in the external finance premium. Therefore, the technology shock is also amplified in the FA case.

I will check the impulse responses again in Section 3.4.2 within the full macroeconomic framework. However, the essence of the financial accelerator effects have been demonstrated by the core model.

3.3 The Whole Model

I now develop the core model toward the full macroeconomic dynamic New Keynesian (DNK) framework, including households, retailers, and a government sector, in addition to entrepreneurs, capital producers, and financial intermediaries.

3.3.1 Households

Here I set up a general equilibrium framework within which entrepreneurs' output goods are finally consumed by households. Households live infinitely, and they consume, work, and save. Their intertemporal optimization problem is written as follows

$$\max_{\{C_t\},\{M_t\},\{H_t\}} \quad \mathrm{E}_t \sum_{k=0}^{\infty} \beta^k \, [\, \ln(C_{t+k}) + \zeta \, \ln(\frac{M_{t+k}}{P_{t+k}}) + \xi \, \ln(1 - H_{t+k}) \,]$$

$$s.t. \; C_t + D_{t+1} + \frac{M_t}{P_t} = W_t \, H_t - T_t + V_t + R_t \, D_t + \frac{M_{t-1}}{P_t},$$

where C_t is consumption, $\frac{M_t}{P_t}$ is real money balances, H_t is labour supply, W_t is the real wage, T_t is a lump-sum tax, V_t is dividends received from ownership of retailers (explained below), D_t is deposits held at financial intermediaries, and R_t is the riskless interest rate on these deposits (gross and in real terms). The intermediaries lending funds to many entrepreneurs can perfectly diversify the idiosyncratic risk on their return, which enables households to earn the riskless rate on their deposits.

The first order conditions are

$$\frac{1}{C_t} = \beta \, \frac{1}{\mathrm{E}_t \, C_{t+1}} \, R_{t+1} \tag{3.15}$$

$$\frac{M_t}{P_t} = \zeta \, C_t \, \frac{R^n{}_{t+1}}{R^n{}_{t+1} - 1} \tag{3.16}$$

$$W_t \, \frac{1}{C_t} = \xi \, \frac{1}{1 - H_t}, \tag{3.17}$$

where $R^n{}_{t+1}$ is the (gross) nominal interest rate, defined as $R_{t+1} \, \frac{\mathrm{E}_t \, P_{t+1}}{P_t}$. Now both the real interest rate and labour supply are endogenized.

3.3.2 Retailers

Here I assume that entrepreneurs sell all of their output good to retailers. Retailers then sell final output goods to households, capital producers, and the government sector. Whereas the entrepreneurs' output good is homogeneous, retailers differentiate it slightly at no resource cost and then have the monopolistic power to set the prices of these final output goods. Following Calvo (1983), I assume that the retailers have the opportunity to change their prices in a given period only with probability $1 - \theta$. Then price stickiness, the degree of which is represented by θ, is introduced in keeping with the conventional dynamic New Keynesian framework.

The reason why retailers are incorporated together with entrepreneurs is that the relationship between an individual entrepreneur's demand for capital and his net worth cannot avoid being too complicated to aggregate straightforwardly if entrepreneurs are imperfect competitors. Retailers here are simply a device for introducing price stickiness. Ultimately retailers' monopolistic profits belong to the households who own them, in contrast to entrepreneurs who are independent agents possessing their own wealth.

Retailers who have the opportunity to set their price in a given period t choose their price, P_t^*, that maximizes their expected discounted profits until the period when they are next able to change their price, subject to their individual demand function faced.

$$\max_{P_t^*} \quad \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \frac{\beta\, C_t}{C_{t+k}} \left(\frac{P_t^* - \frac{P_{t+k}}{X_{t+k}}}{P_{t+k}} Y_{t+k}^* \right)$$
$$s.t.\ Y_t^* = \left(\frac{P_t^*}{P_t} \right)^{-\epsilon} Y_t.$$

The discount factor for expected profits consists of two parts: the probability that retailers cannot change their price in the next period, and the households' intertemporal marginal rate of substitution. X_t is the gross mark-up rate of retail (final) price over wholesale price. ${Y^*}_t$ is the demand corresponding to the optimally chosen price P^*. Y_t is aggregate demand. P_t is the aggregate price given by

$$P_t = \{ \theta\, P_{t-1}^{1-\epsilon} + (1-\theta)\, P_t^{*1-\epsilon} \}^{\frac{1}{1-\epsilon}}.$$

The optimally chosen price P^* can be expressed explicitly as follows

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\mathrm{E}_t \sum_{j=0}^{\infty} \theta^j \frac{\beta C_t}{C_{t+j}} \frac{Y_{t+j}^*}{P_{t+j}} \frac{P_{t+j}}{X_{t+j}}}{\mathrm{E}_t \sum_{j=0}^{\infty} \theta^j \frac{\beta C_t}{C_{t+j}} \frac{Y_{t+j}^*}{P_{t+j}}}.$$

I here approximate percentage deviations from the steady state of P and P^* within their local neighbourhood as follows

$$\begin{aligned} p_t &= \theta\, p_{t-1} + (1-\theta)\, p_t^* \\ p_t^* &= (1-\beta\,\theta) \sum_{j=0}^{\infty} (\beta\,\theta)^j\, \mathrm{E}_t\, [\, p_t - x_t\,], \end{aligned}$$

where lowercase variables denote percentage deviations. Then the so-called New Keynesian Phillips curve can be derived as follows

$$\pi_t = -\lambda\, x_t + \beta\, \mathrm{E}_t\, \pi_{t+1}, \tag{3.18}$$

where $\pi_t = p_t - p_{t-1}$ and $\lambda = \theta^{-1}\, (1-\theta)\, (1-\beta\,\theta)$.

By introducing price stickiness, I have to slightly modify the entrepreneurs' behaviour in the core model. Evaluating the rent paid to a unit of capital, $\frac{\alpha Y_{t+1}}{K_t}$, in terms of the retail price of output goods, (3.2) is rewritten as

$$F_{t+1} = \frac{\frac{1}{X_{t+1}} \frac{\alpha Y_{t+1}}{K_t} + Q_{t+1}(1-\delta)}{Q_t}. \tag{3.19}$$

In the same way, evaluating the marginal product of labour in terms of the retail price, I should write the entrepreneurs' labour demand condition as follows

$$W_t = \frac{1}{X_t} \frac{(1-\alpha) Y_t}{H_t}. \tag{3.20}$$

This condition and the households' labour supply condition, (3.17), give the labour market equilibrium.

3.3.3 Government Sector and Monetary Policy Rule

Finally I set the budget constraint and the policy rule of the government sector to close the whole model. The total expenditure of the economy, which is always equal to the aggregate of final output goods, consists of households' consumption, capital producers' investment, and the expenditure of the government sector, G.

$$Y_t = C_t + I_t + G_t. \tag{3.21}$$

Government expenditure is financed by lump-sum taxes and money creation.

$$G_t = \frac{M_t - M_{t-1}}{P_t} + T_t. \tag{3.22}$$

Here I take the nominal interest rate as the instrument of monetary policy. The monetary aggregate is then endogenized in the money demand function, (3.16). Following the spirit of the Taylor rule, I specify the policy rule as follows[27]

$$\begin{aligned} {r^n}_{t+1} &= \rho_n {r^n}_t + (1-\rho_n) {r^{n*}}_{t+1} + {e^n}_t, \qquad {e^n}_t \sim N(0, {\sigma_n}^2) \qquad (3.23) \\ {r^{n*}}_{t+1} &= \nu_y y_t + \nu_p \mathrm{E}_t p_{t+1} \quad \text{with } \nu_y,\ \nu_p > 0, \end{aligned}$$

[27]I do not model the policy objective function explicitly. Within this type of macroeconomic framework, optimizing behaviour should be linked to the underlying structure of economy and would involve imposing a number of additional restrictions throughout the model. Dealing tractably with such restrictions seems beyond the scope of my current treatment.

where $r^{n*}{}_{t+1}$ is the target value of the nominal interest rate and e_t^n is an exogenous random shock to the interest rate which reflects either failure to track the rule or intentional transitory deviations from the rule (pure "policy shocks"). The first equation describes the mechanism of partial adjustment observed in reality, while the second is the policy reaction function which responds to current output and the expected price level in order to stabilize the economy.[28] Note that $r^n{}_{t+1}$, y_t, and p_t are all percentage deviations from steady-state so that they carry the implication of "gaps". According to this rule, the monetary authority (here included in the government sector) raises the nominal interest rate (sets it above its steady-state level) when current output exceeds its steady-state and/or prices are expected to exceed their steady-state level, and vice versa.[29]

The partial adjustment mechanism and the forward-looking reaction (targeting expected future prices rather than current or past prices) follow Clarida, Gali, and Gertler (1998). Unlike Clarida et al., however, I here adopt a price-level reaction rule rather than an inflation reaction rule which would be more popular and nearer to the original Taylor rule.[30] The reason is that using an inflation reaction rule in this model is liable to cause indeterminacy of the steady state equilibrium, as I discuss in Section 3.5.2. I estimate this policy rule in my benchmark parameterization in Section 3.4.1 and try alternative parameterizations in Section 3.5.2.

For fiscal policy, on the other hand, there is no rule. Like the technology parameter, government expenditure, G_t, follows a stationary autoregressive process as follows.

$$g_t = \rho_g \, g_{t-1} + e^g{}_t, \qquad e^g{}_t \sim N(0, \sigma_g{}^2), \tag{3.24}$$

where g_t is the percentage deviation from steady-state and $e^g{}_t$ is a serially uncorrelated random shock.

[28]The above conditions on the coefficients, ν_y, $\nu_p > 0$, are normally imposed to make the policy rule stabilize the economy.

[29]I do not consider the non-negative condition of the nominal interest rate. If the steady-state interest rate is near the zero boundary, the monetary policy shocks may be distributed asymmetrically.

[30]BGG specify a rule reacting only to expected inflation and not to output or the price level.

3.3.4 Equilibrium

I have now described the whole model. As in the core model, I define a rational expectations equilibrium as a set of endogenous variables which satisfies entrepreneurs' decision rules (3.19), (3.3), and (3.20), capital producers' decision rule (3.5), households' decision rule (3.15), (3.16), and (3.17), retailers' decision rule (3.18), and resource constraints (3.1), (3.4), (3.6), and (3.21), given the stochastic processes (3.23), (3.8), and (3.24). Then I log-linearize the system around the steady state as follows.

$$\begin{aligned}
y_t &= \frac{C}{Y}c_t + \frac{I}{Y}i_t + \frac{G}{Y}g_t && (3.25)\\
c_t &= -\,r_{t+1} + c_{t+1} && (3.26)\\
f_t &= (1-\upsilon)(y_t - x_t - k_{t-1}) + \upsilon\, q_t - q_{t-1} && (3.27)\\
\mathrm{E}_t\, f_{t+1} &= r_{t+1} - \psi\,[\, n_t - (q_t + k_t)\,] && (3.28)\\
\mathrm{E}_t\, q_{t+1} &= \phi\,(\mathrm{E}_t\, i_{t+1} - k_t) && (3.29)
\end{aligned}$$

$$\begin{aligned}
y_t &= a_t + \alpha\, k_{t-1} + (1-\alpha)\, h_t && (3.30)\\
h_t &= \frac{\eta_h}{1+\eta_h}(y_t - x_t - c_t) && (3.31)\\
\pi_t &= -\lambda\, x_t + \beta\, \mathrm{E}_t\, \pi_{t+1} && (3.32)\\
k_t &= \delta\, i_t + (1-\delta)\, k_{t-1} && (3.33)\\
n_t &= \gamma F\,[\chi f_t - (\chi-1)\, r_t \\
&\quad + \psi\,(\chi-1)\,(q_{t-1} + k_{t-1}) + \{\,1 - \psi\,(\chi-1)\,\}\, n_{t-1}\,] && (3.34)
\end{aligned}$$

$$\begin{aligned}
r_{t+1} &= {r^n}_{t+1} - \mathrm{E}_t\, \pi_{t+1} && (3.35)\\
\pi_t &= p_t - p_{t-1} && (3.36)\\
m_t - p_t &= c_t - \frac{1}{R}{r^n}_{t+1} && (3.37)\\
{r^n}_{t+1} &= \rho_n\, {r^n}_t + (1-\rho_n)\,(\nu_y\, y_t + \nu_p\, \mathrm{E}_t\, p_{t+1}) \\
&\qquad + {e^n}_t, \quad {e^n}_t \sim N(0,\, {\sigma_n}^2) && (3.38)
\end{aligned}$$

$$\begin{aligned}
a_t &= \rho_a\, a_{t-1} + {e^a}_t, \quad {e^a}_t \sim N(0,\, {\sigma_a}^2) && (3.39)\\
g_t &= \rho_g\, g_{t-1} + {e^g}_t, \quad {e^g}_t \sim N(0,\, {\sigma_g}^2). && (3.40)
\end{aligned}$$

(3.25) is the log-linearized version of the resource constraint, (3.21). Household consumption and corporate investment are described by equations (3.26) through (3.29). (3.26) is the Euler equation of households' consumption derived from (3.15), and corresponds to the IS equation of a Keynesian model. (3.27) through (3.29) are almost the same as equations (3.10), (3.11), and (3.13) in the core model.

(3.30) is the log-linearized version of the production function. (3.31) is the condition for labour market equilibrium derived from (3.17) and (3.20). (3.32) is the New Keynesian Phillips curve (3.18). (3.33) and (3.34), which describe the evolution of the state variables, are the same as (3.14) and (3.12) in the core model.

(3.35) is the Fischer equation defining the relationship between the nominal and real interest rates, (3.36) is the definition of the inflation rate. (3.37) describes households' money demand derived from (3.16), and corresponds to the LM equation of a Keynesian model. (3.38) is the monetary policy rule derived from (3.23), which also prescribes the shock process governing the nominal interest rate.

(3.39) and (3.40) are the shock processes governing the technology parameter (the same as (3.8) in the core model) and government expenditure (the same as (3.24) above), respectively. All of the three random variables in the model, ${e^z}_t$, ${e^a}_t$, and ${e^g}_t$, are uncorrelated both serially and contemporaneously.

3.4 Results

Using the whole log-linearized model, I conduct some quantitative experiments in this section. As I did for the core model above, I again examine the responses of variables to exogenous shocks. In addition, I calculate the second moment properties of the model such as the standard deviations and cross correlations with output (real GDP), and examine how well these properties correspond to those observed in the actual Japanese data.

3.4.1 Parameterization

First of all, I set parameter values to calibrate the model to Japan's economy. As a rule, I choose them based on actual data (historical averages), single-equation estimation, or by reference to preceding studies. A list of the chosen values for the benchmark parameterization is shown in Chart 3.2. Some alternative parameterizations are discussed later in Section 3.5.1.

Steady-state values of the shares of household consumption, corporate fixed investment, and other exogenous expenditures (the expenditure of the government sector in the model) in the total expenditure of the economy are determined as historical averages of their values in the Japanese National Accounts over the period from 1980/Q1 to 2001/Q1, which I take as the main sample period.[31] The capital share, α, and the depreciation rate of capital, δ, are calculated as averages over the sample period, using the Hayashi and Prescott (2002) data, which is constructed from the National Accounts.

Households' discount factor, β $(= 1/R)$, is set to 0.995 following Ohkusa (1991) and Soejima (1997). As is usual with the standard real business cycle (RBC) model, the parameter on leisure preference is determined by the steady-state fraction of time spent working: 0.268 is the historical average in Japan, which implies a labour supply elasticity of 2.737.

The parameter on price stickiness, θ, is set to 0.75, which implies that price adjustments occur on average once every four periods (a year). This value lies within the range of estimation results for the New Keynesian Phillips curve calculated by Fuchi and Watanabe (2002) using Japanese data. The steady-state elasticity of the price of capital (Q) to the ratio of investment to capital stock (I/K) is set to 0.25 following BGG.

I take the steady-state elasticity of the external finance premium (S) with respect to the ratio of capital to net worth (χ), which determine the effectiveness of the financial accelerator, as 0.05 following BGG.[32] Among the principal conditions governing BGG's choice of this value are that χ is 2 and that S is annually 200 basis points. These conditions also offer a close fit to historical averages in Japan as well.[33] I also follow BGG in setting the survival rate of entrepreneurs, γ, to 0.9728.

The parameters relating to the monetary policy rule and exogenous shock

[31] Data before 1980 (partly before 1990) are not available in the new standard (1993 SNA) Japanese National Accounts.

[32] BGG give this value based on a rigorous numerical solution to the optimal contracting problem between entrepreneurs and financial intermediaries which I did not set up precisely in the core model.

[33] According to the previous standard (1968 SNA) National Accounts, the historical average of χ from 1980 to 1998 fiscal year is 1.982. The historical average of the spread between the average contracted lending rate and the official discount rate is annually 241 (long term) or 144 (short term) basis points.

processes are all given by estimation. I estimate the policy reaction function (3.38) using the overnight call rate and the National Accounts data detrended by the Hodrick-Prescott (HP) filter over the sample period.[34] I also estimate the first-order autoregressive parameter on the shock processes governing total factor productivity and exogenous expenditures, ρ_a and ρ_g respectively, using data calculated from the National Accounts.[35,36] I apply the standard errors in these estimations to the standard deviations of the innovations in the corresponding shock processes, σ_n, σ_a and σ_g.

3.4.2 Impulse Responses

I here examine the responses of the whole model to three types of shocks: a monetary policy shock, a technology shock (to total factor productivity) and a demand shock (to exogenous expenditures). Chart 3.5 to 3.7 show the results for each type of shock.

First I examine the effects of monetary policy, here those of a policy shock which works through the above policy rule controlling the nominal interest rate, in Chart 3.5. A tightening policy shock raises the real interest rate as well as the nominal interest rate, which causes the main real variables to respond in almost the same way as those in the core model, where it is the real interest rate that provides the exogenous monetary policy shock. In addition, the variables introduced in the whole model such as money and inflation respond in the right direction theoretically.[37]

The responses to the technology shock in Chart 3.6 are different from

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[34]Following Clarida et al. (1998) and Bernanke and Gertler (1999), I estimate the parameters, ν_y, ν_p, and ρ_n, using the generalized method of moments (GMM). The instrument set includes a constant, plus 1-4 lags of log-differenced real GDP, the log-differenced GDP deflator, and the call rate. For the expected future price level, I take the fourth period (one year) lead of the realized data.

[35]I calculate the total factor productivity, A_t, by subtracting weighted (by α) factor inputs from real GDP in logarithms, and the exogenous expenditures, G_t, by subtracting household consumption and corporate fixed investment from GDP in real terms.

[36]Following much of the RBC literatures, I estimate the first-order autoregressive parameter governing A_t (and G_t) using level data (taking logarithms) together with a constant and a linear trend term. I can alternatively use the HP-filter-detrended data as before, which gives much smaller estimates (around 0.5 and 0.6) than those usually calibrated in the literature.

[37]Inflation responds immediately before the peak of the response of real GDP, which is contradictory to the stylized fact shown in many empirical studies that monetary policy shocks initially impact on real variables and then have a delayed and gradual effects on inflation. As I mentioned in Chapter 1 and 2, it is the Calvo-type price setting which I assume in this chapter to cause this problem.

those of the core model. Under my parameterization, when a positive technology shock hits the economy, entrepreneurs' net worth does not improve but deteriorates. This is partly because the policy reaction function counteracts the positive effects of the shock, and partly because an expansion of the mark up due to price stickiness causes the rent paid to a unit of capital in terms of the retail price to depreciate.[38] The external finance premium thus dampens the demand for capital in the FA case, which counteracts rather than amplifies the positive responses of investment and real GDP. Therefore, the technology shock is less amplified in the FA case, which is the reverse of the outcome in the core model.[39]

A positive demand shock also boosts real GDP and investment, although the rise in the interest rate counteracts this effect, making the increases more modest, as shown in Chart 3.7. Entrepreneurs' net worth here improves and the external finance premium diminishes, which means that the shock is more amplified in the FA case.

3.4.3 Second Moment Properties

As the main quantitative experiment, I calculate second moment properties of the model such as standard deviations and cross correlations with output (real GDP), and examine how well these properties correspond to those observed in the actual Japanese data, illustrated in Chart 3.1. The random variables in this model are the innovations in the three shock process, ${e^n}_t$, ${e^a}_t$, and ${e^g}_t$, and these innovations are assumed to be distributed normally and uncorrelated both serially and contemporaneously.[40] I generate stochastically one hundred simulated sets[41] consisting of a complete dynamic path over the full 85 periods of the sample. For the purpose of comparison with the actual data in Chart 3.1, I calculate the HP-filtered moments from the simulated data. Chart 3.8 shows the results of the simulation-based sample

[38]Meanwhile, households' welfare improves because retailers' monopolistic profits which belong to households increase, accompanied by the expansion of the mark up.

[39]This outcome depends on the parameterization, especially that of the policy reaction and price stickiness terms. In BGG, the policy reaction to output is set to zero (omitted) and hence, like the other shocks, the technology shocks are also more amplified in the FA case.

[40]In the actual data, my parameterization implies that the first order sample autocorrelation of ${e^n}_t$ is 0.313, of ${e^a}_t$ is -0.238, and of ${e^g}_t$ is 0.064, and that the sample cross correlation between ${e^n}_t$ and ${e^a}_t$ is 0.079, between ${e^n}_t$ and ${e^g}_t$ is -0.187, and between ${e^a}_t$ and ${e^g}_t$ is 0.302.

[41]The number of simulated sets, one hundred, follows Soejima (1997).

moments, together with the sample standard errors for each of them.

The main result is that the FA case produces more volatile corporate investment (the standard deviation, shown in Chart 3.8, is 4.36) than the No-FA case (3.36), and corresponds more closely to the actual data (5.32 in Chart 3.1) though still a little bit short. The sample standard errors (0.51 in the FA case and 0.35 in the No-FA case) imply that the difference between the results from the two cases seems significant. We can interpret this result as one piece of evidence supporting the existence of financial accelerator effects in Japan's business cycles.

For other variables, meanwhile, the difference between the two cases does not seem to be significant. Moreover, the volatilities of these variables correspond relatively well to the actual data in both cases. A remarkable exception, however, is total working hours. The volatility the model produces for this variable is, in both cases (2.32 in the FA case and 2.17 in the No-FA case) far larger than that of the actual data (0.62 in Chart 3.1). This point will be discussed later when I vary the parameter value governing the elasticity of the labour supply, η_h, in Section 3.5.1.

For the cross correlations with real GDP, whether the FA or No-FA case, the correspondence of the simulated data to the actual data are poorer than those for the standard deviations. The higher-order autocorrelations of most variables in the simulated data are too small, although this problem is generally shared with many simple RBC models. While the model reproduce the high cross correlations of investment maintained for some lags, it fails to generate the lagged inflation mainly due to the Calvo-type price setting.

3.5 Variations of The Model

I have obtained the result that in the FA case the model produces more volatile corporate investment which corresponds more closely to the actual data than it does in the No-FA case. In this section, I examine the robustness of this result and consider some variations of the model by varying some underlying parameter values, adopting alternative monetary policy rules, and introducing an asset price bubble into the model. These experiments illustrate various properties of the model and may also be of potential simulative interest in considering possible future states of the economy.

3.5.1 Alternative Parameter Values

First I try varying the benchmark parameterization set out in Section 3.4.1. This may be of potential simulative interest in considering possible future states of the economy, as well as providing a robustness check on the sensitivity of the benchmark results. Chart 3.9 shows some alternative results for the standard deviations in the No-FA and FA cases obtained by varying the values for χ, θ, and η_h, which are the three important and sensitive underlying parameters in the model.

First, if the steady state ratio of internal funds is lower (that is, χ is higher) than the benchmark value (the ratio is 0.5, or equivalently χ is 2), the volatility of corporate investment in the FA case increases and comes closer to the actual volatility, although other parameters remain little changed.[42] This is because the effectiveness of the financial accelerator expands as the share of external financing rises.[43] The result is in contrast to the No-FA case, in which changes in χ leave the volatility unchanged.

Secondly, if retailers can change their prices more frequently, for example as a result of deregulation making a more competitive environment, the volatility of the economy as a whole is reduced although that of inflation itself is increased. The range within which I vary the value for θ in the chart is in line with the estimation results of Fuchi and Watanabe (2002). Since the propagation of the monetary policy shock is sensitive to the degree of price stickiness, the volatility caused by the monetary policy shock is sensitive to variation in θ. In the No-FA case, this sensitivity is less than in the FA case.

Thirdly, if households supply their labour less elastically with respect to wages, the volatilities of working hours, investment, and real GDP are reduced while those of consumption and real money balances are increased. The numerical example in the chart suggests that very small elasticity is required in order to reproduce the actual volatility of working hours. In the No-FA case, the sensitivity is once again less than in the FA case.

[42]When χ increases, the elasticity ψ that is fixed here may also increase. Then the volatility of corporate investment may further increase beyond the result shown in Chart 3.9.

[43]However, if the steady state ratio of internal funds is lower than about 20% (that is, χ exceeds 5), the effectiveness of the financial accelerator and the volatility of investment tend to diminish. This is the phase of an "excessive external finance premium" which Kasuya and Fukunaga (2003) pointed out.

3.5.2 Alternative Monetary Policy Rules

In the whole model above, the monetary policy rule has a particularly important role in either stabilizing or destabilizing the economy. Here I compare the performances of alternative policy rules in both the No-FA and FA cases and examine their properties in the context of this model.

First I try varying the estimated benchmark parameter values for the price-level reaction rule, (3.23). The standard deviations for each parameterization in both the No-FA and FA cases are shown in Chart 3.10. Moving from an extreme situation in which monetary policy does not react at all to economic conditions (i.e. both ν_y and ν_p are zero), the benchmark rule stabilizes all variables appearing in the table. As I give larger values to the reaction parameters (for example, $\nu_y = \nu_p = 2$), the economy is further stabilized. We can see that those stabilizing effects are stronger in the FA case than in the No-FA case. Under an active reaction rule such as $\nu_y = \nu_p = 2$, corporate fixed investment in the FA case is strongly stabilized by the policy so that its volatility can be smaller than in the No-FA case and the benchmark result in Section 3.4.3 is overturned.

We can also see that the stabilizing effects of the two terms, ν_y and ν_p, which constitute the policy rule work in different directions. Chart 3.11 shows the stabilizing effects on output (real GDP) and inflation, generally considered to be the two objectives of monetary policy. As I give a larger value to ν_y, inflation becomes volatile while output is stabilized. This means that we are confronted with a tradeoff between the two objectives. Meanwhile, as I increase the value of ν_p above the benchmark value (0.03), both output and inflation are stabilized to some extent. This implies that the benchmark rule leaves room for further stabilization of the economy through more active reaction to the price-level. When ν_p exceeds some threshold, however, inflation continues to be further stabilized but output tends to increase volatility,[44] and we eventually confront the tradeoff.[45]

Next I consider alternative specifications of the policy rule including the

[44]The lower limit on the standard deviation of output is around 0.6, which the rule $\nu_y = \nu_p = 2$ of these examples almost achieves. Any active rule cannot stabilize output over this limit.

[45]With an exogenous nominal shock process to the price setting rule (3.18) instead of the monetary policy rule (the policy rule is then always implemented and completely endogenized), following Clarida, Gali, and Gertler (1999), the tradeoff between the variances of output and inflation would be presented more clearly.

reaction to inflation. I replace the price-level reaction term in the benchmark specification (3.23) with an inflation reaction term as follows.

$$\begin{aligned} {r^n}_{t+1} &= \rho_n \, {r^n}_t + (1-\rho_n)\, {r^{n*}}_{t+1} + {e^n}_t, \qquad {e^n}_t \sim N(0, {\sigma_n}^2) \\ {r^{n*}}_{t+1} &= \nu_y \, y_t + \nu_\pi \, \mathrm{E}_t \, \pi_{t+1} \text{ with } \nu_y, \ \nu_\pi > 0. \end{aligned} \tag{3.41}$$

This specification is nearer to Clarida, Gali and Gertler (1998) and the original Taylor rule.[46] Although this type of specification is popular, it does not work well in this model. When I set ν_π to below unity (say 0.5), both output and inflation are destabilized since the real interest rate moves in the opposite direction to the nominal rate. If I choose a value larger than unity for ν_π, however, prices do not return to the original level after a shock and the steady state equilibrium becomes indeterminate.[47] To avoid this indeterminacy, I should again add a price-level reaction term to the above specification (3.41).

3.5.3 Introducing a Bubble

So far, we regard the asset (capital) price, Q_t, as an endogenous variable which passively reflects the real economy, although in the FA case its unanticipated movements have considerable effects on economic activity through changes in the value of entrepreneurs' net worth. We know, however, from experience in the late 1980s to early 1990s, that if asset markets are influenced by non-fundamental factors, then movements in asset prices may be to some extent considered a further independent source of economic fluctuation, supplementing the three types of shock that I have already considered in the model.

In order to deal with this possibility, I extend the model a little, following Bernanke and Gertler (1999). I introduce a bubble by assuming that the market price of capital, S_t, may differ from capital's fundamental value, Q_t. If a bubble exists at t, it persists and grows with probability p or crashes with probability $1-p$ as follows.

[46]This rule does not target inflation at a certain positive value but at zero. Therefore, it also implicitly targets the steady-state price level.

[47]In Chart 3.10, I show the standard deviations calculated under this parameterization regardless of the indeterminacy.

$$\begin{aligned} S_{t+1} - Q_{t+1} &= \frac{a}{p}(S_t - Q_t) F_{t+1} \quad \text{with probability } p \\ &= 0 \quad\quad\quad\quad\quad\quad\quad \text{with probability } 1-p. \end{aligned}$$

where F_{t+1} is here the fundamental return on capital. Therefore,

$$\mathrm{E}_t [S_{t+1} - Q_{t+1}] = a(S_t - Q_t) \mathrm{E}_t F_{t+1}. \tag{3.42}$$

I assume $0 < p < a < 1$ so that the discount value of the bubble converges to zero over time, while it grows until it bursts. Once the bubble crashes it is not expected to re-emerge, which is an assumption for simplicity. While the relationship between the fundamental value and the fundamental return on capital is prescribed by (3.19), the relationship between the market price and the market return, ${F^S}_{t+1}$, is

$$F^S_{t+1} = \frac{\frac{1}{X_{t+1}} \frac{\alpha Y_{t+1}}{K_t} + S_{t+1}(1-\delta)}{S_t}, \tag{3.43}$$

and the relationship between the fundamental and the market return is

$$F^S_{t+1} = F_{t+1} \left[a(1-\delta) + \{1 - a(1-\delta)\} \frac{Q_t}{S_t} \right]. \tag{3.44}$$

This completes the extension. I now log-linearize the above equations (3.43) and (3.44) around the steady state, shifting the time subscripts as before.

$$f^s{}_t = (1-v)(y_t - x_t - k_{t-1}) + v\, s_t - s_{t-1} \tag{3.45}$$

$$f^s{}_t = f_t - \{1 - a(1-\delta)\}(s_{t-1} - q_{t-1}). \tag{3.46}$$

On condition that the bubble does not burst during t, the realized value in log-linearized terms is written as

$$s_t - q_t = \frac{F}{p(1-\delta)}(s_{t-1} - q_{t-1}). \tag{3.47}$$

I add the above equations (3.45) through (3.47) to the log-linearized model in Section 3.3.4, and replace the fundamental terms, q_t and f_t, by the market terms, s_t and ${f^s}_t$, respectively.[48] I then try generating a bubble that begins with a one percentage point increase in the market price of capital above its fundamental value and bursts four periods (one year) later. Note that agents in the model know only the ex-ante stochastic process governing

[48] I do not replace q_{t+1} in equation (3.28) following Bernanke and Gertler (1999), assuming that capital producers' investment decision is based on the fundamental value rather than the market price.

the bubble. Chart 3.12 shows responses to this exogenous disturbance in both the No-FA and FA cases. We can see that corporate investment in the FA case undergoes a large-scale decline after the collapse of bubble and it takes a long time for it to revert to its original steady state, while in the No-FA case it returns to its steady state almost at once. This drop in investment in the FA case is accompanied by a fall in entrepreneurs' net worth, which is greatly damaged by the collapse of bubble. The financial accelerator effects typically amplify shocks that affect entrepreneurs' net worth. Here I set the value determining the bubble's growth in each period, $\frac{F}{p(1-\delta)}$ in equation (3.47), to 2. Then, as shown in Chart 3.13, the standard deviation of corporate investment in the FA case (5.20) reaches a level that corresponds to the actual data. Thus we see that adding these bubble effects to the benchmark result in Section 3.4.3 achieves a close match to the observed data, although this result is conditional on my assumption that the bubble occurs independently of the other three shocks.[49]

3.6 Concluding Remarks

This chapter calibrates a dynamic general equilibrium model which incorporates credit-market imperfections using Japanese data. The model exhibits financial accelerator effects, the mechanism whereby credit-market imperfections help to propagate or amplify various types of shocks to the economy. The main result is that the large volatility of Japan's corporate investment can be explained by taking account of this mechanism. I examine the robustness of the results and consider some variations of the model by varying some underlying parameter values, adopting alternative monetary policy rules, and introducing an asset price bubble into the model. These experiments illustrate various properties of the model and may also be of potential simulative interest in considering possible future states of the economy.

By varying parameterization and specification, can this model provide a consistent explanation for a variety of different economic situations? In particular, as shown in Chart 3.1, the difference in investment volatility between the Japanese and U.S. economies, or between Japan's economy in

[49]Both the parameterization and the specification of the bubble process here cannot avoid being arbitrary. That is why we did not use this extended model in the benchmark quantitative experiments in Section 3.4.

the 1980s and in the 1990s is remarkably large. There may be some differences in the underlying parameter values governing the effectiveness of the financial accelerator, the performance of monetary policy, or the exogenous shock processes themselves. To investigate the causes of these differences represents another interesting application of this model, and I would like to take this as a subject for future research.[50] Here I can remark that the FA case can generally produce a wider range of results than the No-FA case by varying parameterization and specification.

Finally I should remark that this model is only one of a number of ways of introducing credit-market imperfections into a macroeconomic framework. There are many financial factors I ignore in this model, including banking sector conditions. Therefore other models might give differing quantitative, and possibly qualitative, results. At the same time, the dynamic general equilibrium framework itself still continues to develop, for instance by extension to an open economy setting.[51] I would like to see the results of richer and more diverse studies developing from my approach.

References

[1] Bernanke, B. S. and Gertler, M. (1999), "Monetary Policy and Asset Price Volatility," Federal Reserve Bank of Kansas City, *Economic Review*, Fourth Quarter, pp. 17-59.

[2] Bernanke, B. S., Gertler, M., and Gilchrist, S. (1996), "The Financial Accelerator and the Flight to Quality," *Review of Economics and Statistics*, Vol. 78, pp.1-15.

[3] Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999), "The Financial Accelerator in a Quantitative Business Cycle Framework," in Taylor, J. B. and Woodford M. (eds), *Handbook of Macroeconomics*, Vol. 1C, Chap. 21, pp. 1341-1393, North Holland.

[50] Hall (2001) explains the difference of performance between the U.K. economy in the 1980s and in the 1990s by varying the parameterization on financial conditions in BGG model.

[51] Gertler, Gilchrist and Natalucci (2003) extend the BGG model to an open economy setting and explore the connection between the exchange rate regime and the financial accelerator.

[4] Calvo, G. A. (1983), "Staggered Prices in a Utility-Maximizing framework," *Journal of Monetary Economics*, Vol. 12, pp. 383-398.

[5] Carlstrom, C. T. and Fuerst, T. S. (1997), "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis," *American Economic Review*, Vol. 87, pp. 893-910.

[6] Clarida, R., Gali, J. and Gertler, M. (1998), "Monetary Policy Rules in Practice: Some International Evidence," *European Economic Review*, Vol. 42, No. 6, pp. 1033-1068.

[7] Clarida, R., Gali, J. and Gertler, M. (1999), "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature*, Vol. 37, No.4, pp. 1661-1707.

[8] Fuchi, H. and Watanabe, T. (2002), "Phillips Kyokusen to Kakaku Nenchakusei (Phillips Curve and Price Stickiness)," Institute for Monetary and Economic Studies, Bank of Japan, *Kinyu Kenkyu*, Vol. 21, No. 1, pp. 35-69.

[9] Gertler, M., Gilchrist, S. and Natalucci, F. M. (2003), "External Constraints on Monetary Policy and the Financial Accelerator," NBER Working Paper, 10128.

[10] Hall, S. (2001), "Financial Accelerator Effects in UK Business Cycles," Bank of England, Working Paper, No. 150.

[11] Hayashi, F. and Prescott, E. C. (2002), "The 1990s in Japan: A Lost Decade," *Review of Economic Dynamics*, Vol. 5, No. 1, pp. 206-235.

[12] Kasuya, M. and Fukunaga, I. (2003), "Kinyu Seisaku no Regime Henka: Enkatsu-Suii VAR Model ni yoru Bunseki (Regime-Switching Approach to Monetary Policy Effects: Empirical Studies using a Smooth Transition Vector Autoregressive Model)," Research and Statistics Department, Bank of Japan, Working Paper, 03-7.

[13] King, R. G. and Rebelo, S. T. (1999), "Resuscitating Real Business Cycles," in Taylor, J. B. and Woodford M. (eds), *Handbook of Macroeconomics*, Vol. 1B, Chap. 14, pp. 927-1007, North Holland.

[14] Kiyotaki, N. and Moore, J. (1997), "Credit Cycles," *Journal of Political Economy*, Vol. 105, pp. 211-248.

[15] Motonishi, T. and Yoshikawa, H. (1999), "Causes of the Long Stagnation of Japan during the 1990s," *Journal of the Japanese and International Economics*, Vol. 13, pp. 181-200.

[16] Nagahata, T. and Sekine, T. (2002), "The Effects of Monetary Policy on Firm Investment after the Collapse of the Asset Price Bubble: An Investigation Using Japanese Micro Data," Research and Statistics Department, Bank of Japan, Working Paper, No. 02-3.

[17] Ogawa, K. and Kitasaka, S. (1998), *Shisan Shijou to Keiki Hendou (Asset market and Business Cycles)*, Nihon Keizai Shimbunsha.

[18] Ohkusa, Y. (1991), "Real Business Cycle Riron no Nihon Keizai heno Tekiyou Kanousei (Application of the Real Business Cycle Theory to Japan's Economy)," *Keizai Ronsyu*, Doshisya University.

[19] Soejima, Y. (1997), "Kinyu Gijutsu Kakushin ga Macro Keizai Hendou he Ataeru Eikyou (Effects of Financial Innovation on Macroeconomic Fluctuations)," Institute for Monetary and Economic Studies, Bank of Japan, *Kinyu Kenkyu*, Vol. 16, No. 2, pp. 87-122.

[20] Townsend, R. M. (1979), "Optimal Contracts and Competitive Markets with Costly State Verification," *Journal of Economic Theory*, Vol. 21, pp. 265-293.

[21] Uhlig, H. (1999), "A Toolkit for Analyzing Nonlinear Dynamic Stochastic Models Easily," in Marimon, R. and Scott, A. (eds), *Computational Methods for the Study of Dynamic Economies*, pp. 30-61. Oxford University Press.

Chart 3.1: Actual Data

(1) Standard deviations

	U.S.A.	Japan		
			80/Q1-90/Q4	91/Q1-01/Q1
Real GDP	1.32 <1.00>	1.10 <1.00>	0.94 <1.00>	1.27 <1.00>
Corporate fixed investment	3.88 <2.94>	5.32 <4.82>	4.10 <4.38>	6.44 <5.06>
Private consumption	1.03 <0.78>	0.96 <0.87>	0.82 <0.88>	1.10 <0.86>
Total working hours	1.25 <0.94>	0.62 <0.56>	0.56 <0.60>	0.68 <0.54>
Inflation rate	1.14 <0.86>	1.74 <1.58>	2.08 <2.23>	1.30 <1.02>
Monetary aggregate	1.02 <0.77>	1.61 <1.46>	1.79 <1.92>	1.40 <1.10>

(2) Cross correlations with GDP

1. U.S.A.

	-4	-3	-2	-1	0	1	2	3	4
Real GDP	0.19	0.42	0.63	0.84	1.00	0.84	0.63	0.42	0.19
Corporate fixed investment	-0.15	0.01	0.24	0.48	0.70	0.79	0.77	0.64	0.42
Private consumption	0.44	0.60	0.70	0.80	0.83	0.67	0.50	0.31	0.12
Total working hours	0.06	0.31	0.55	0.78	0.93	0.89	0.75	0.58	0.37
Inflation rate	-0.24	0.00	0.14	0.25	0.33	0.40	0.37	0.43	0.38
Monetary aggregate	0.14	0.11	0.08	0.03	-0.06	-0.14	-0.13	-0.09	-0.07

2. Japan

	-4	-3	-2	-1	0	1	2	3	4
Real GDP	0.35	0.57	0.61	0.73	1.00	0.73	0.61	0.57	0.35
Corporate fixed investment	0.18	0.32	0.49	0.65	0.77	0.79	0.80	0.73	0.59
Private consumption	0.23	0.44	0.29	0.24	0.65	0.25	0.12	0.20	0.07
Total working hours	0.36	0.51	0.59	0.64	0.65	0.65	0.55	0.44	0.25
Inflation rate	0.08	0.14	0.22	0.21	0.11	0.36	0.30	0.26	0.24
Monetary aggregate	0.47	0.55	0.61	0.64	0.62	0.59	0.51	0.42	0.37

< Lead GDP **Lag GDP >**

Notes:

1. Period: 1980/Q1-2001/Q1
2. All variables are in logarithms (except Inflation rate) and have been detrended by the Hodrick-Prescott (HP) filter (λ=1600).
3. GDP, Investment, Consumption, and working hours are real and per-capita base.
4. Inflation rate (rate of change in GDP deflator) is annual rate.
5. Numbers <...> in table (1) represent the relative volatility to GDP.
6. Shaded sells in tables (2) represent the peak.

Sources

Japan: National Accounts, Labor Force Survey, Bank of Japan.

U.S.: National Income and Product Accounts, Monthly Labor Review, Federal Reserve Bank.

Chart 3.2: Notation of Variables and Parameter Values

Y_t	Real GDP, Output	X_t	(Gross) Mark-up Rate
C_t	Private Consumption	A_t	Total Factor Productivity
I_t	Corporate Fixed Investment	H_t	Labor Input Hours
G_t	Government Expenditure	W_t	Real Wage per Hour
R_t	(Gross) Real Interest Rate	R^n_t	(Gross) Nominal Interest Rate
F_t	(Gross) Rate of Returns	P_t	Prices Level
Q_t	Price of Capital Good	$P^{'}_t$	Optimal Price of Retailers
K_t	Capital	Π_t	(Gross) Inflation Rate
N_t	Net Worth	M_t	Money

C/Y	0.538$^+$	...... Steady-State Consumption Share in GDP
I/Y	0.155$^+$	...... Steady-State Investment Share in GDP
G/Y	0.308$^+$	...... Steady-State Exogenous Expenditures Share in GDP
β	0.995*	...... Discout factor of Households
R	-----	...... Steady-State Real Gross Interest Rate ($=1/\beta$)
γ	0.9728*	...... Survival Rate of Enterpreneurs
χ	2$^+$	...... Steady-State Ratio of Capital to Net Worth (=K/N)
S	1.005$^+$	...... Steady-State External Finance Premium (=F/R)
ψ	0.05*	...... Steady-State Elasticity of S to χ ($=S'(\chi)/S(\chi)$)
α	0.358$^+$	...... Capital Share
δ	0.021$^+$	...... Depreciation Rate of Capital
υ	0.8$^+$	...... $=(1-\delta)/\{(1-\delta)+\alpha Y/(XK)\}$
φ	0.25*	...... Steady-State Elasticity of I/K to Q ($=\Phi''(I/K)/\Phi'(I/K)$)
η_h	2.737$^+$	...... Elasticity of Labor Supply to Wage
ξ, ζ	-----	...... Parameter on Household Utility
θ	0.75*	...... Parameter on Price Stickiness
λ	-----	...... $=\theta^{-1}(1-\theta)(1-\theta\beta)$
ε	-----	...... Parameter on Steady-State Markup
ν_y	0.084$^@$	...... Parameter on Monetary Policy Rule for Output Gap
ν_p	0.004$^@$	...... Parameter on Monetary Policy Rule for Price Level
ρ_n	0.920$^@$	...... AR(1) Parameter on Monetary Policy Rule
ρ_a	0.870$^@$	...... AR(1) Parameter on Technology Shock
ρ_g	0.877$^@$	...... AR(1) Parameter on Demand Shock
σ_n	0.104$^@$	...... Standard Deviation of the Monetary Policy Shock
σ_a	0.890$^@$	...... Standard Deviation of the Technology Shock
σ_g	1.275$^@$	...... Standard Deviation of the Demand Shock

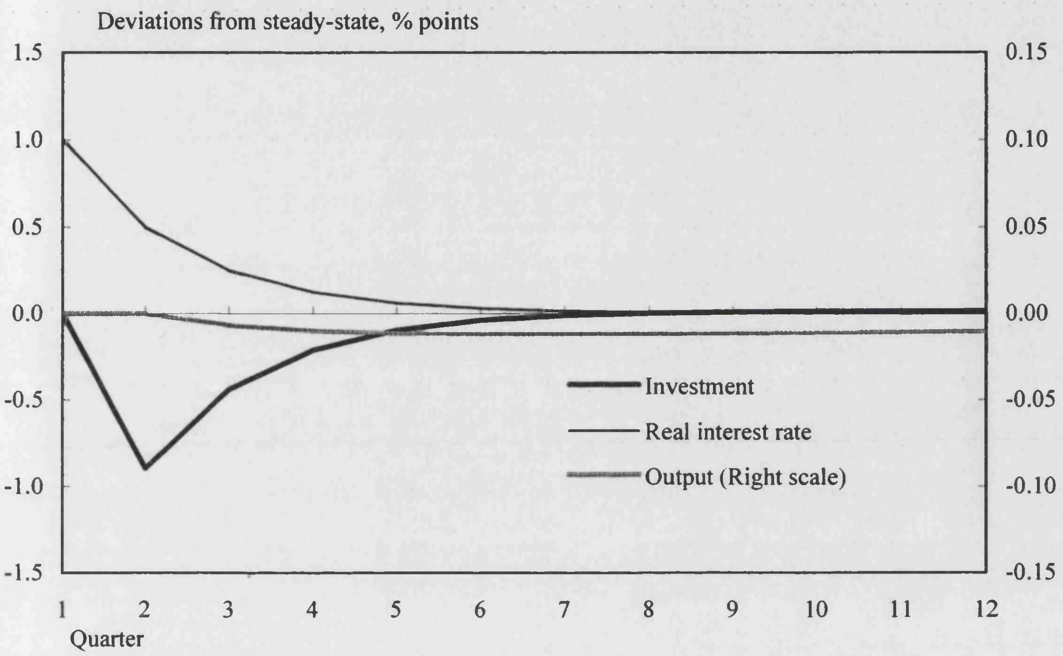
Notes:

1. Uppercase variables denote level, and lowercase variables denote percentage deviations fr the stedy-state. Uppercase variables without time subscripts denote the steady-state value:
2. Values with $^+$ are based on actual data (historical averages), with * on references to preceding studies, and with $^@$ on single-equation estimation.

Chart 3.3: Responses to Monetary Policy Shock (The Core Model)

[Condition] Giving a positive (tightning) monetary policy shock to the real interest rate consisting of annually 1% deviation from the steady-state only in the 1st quarter.

(1) No-FA case



(2) FA case

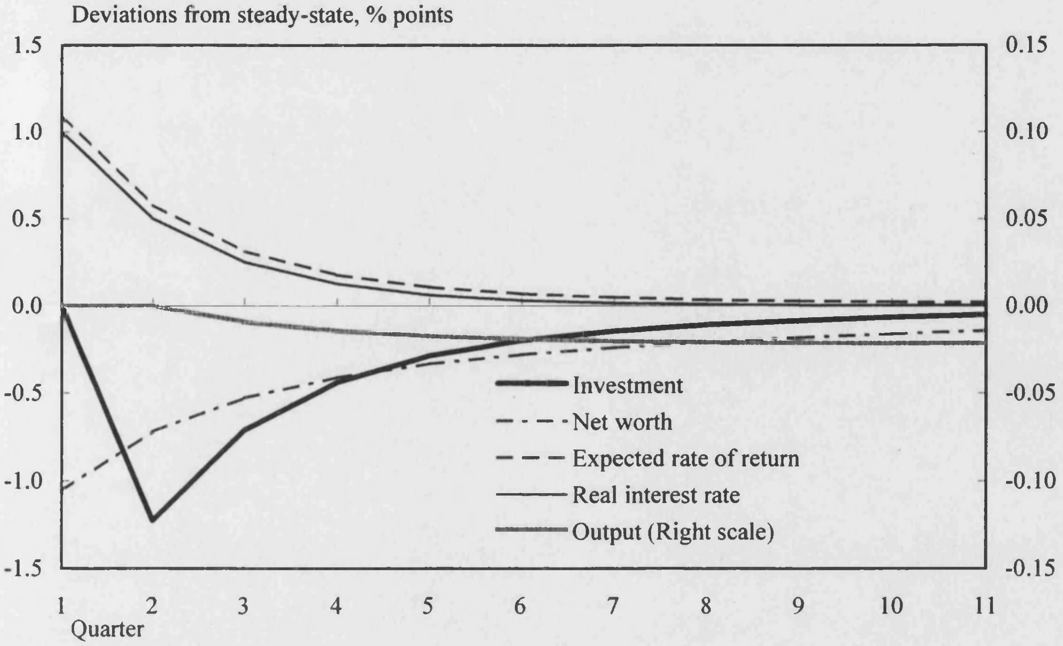
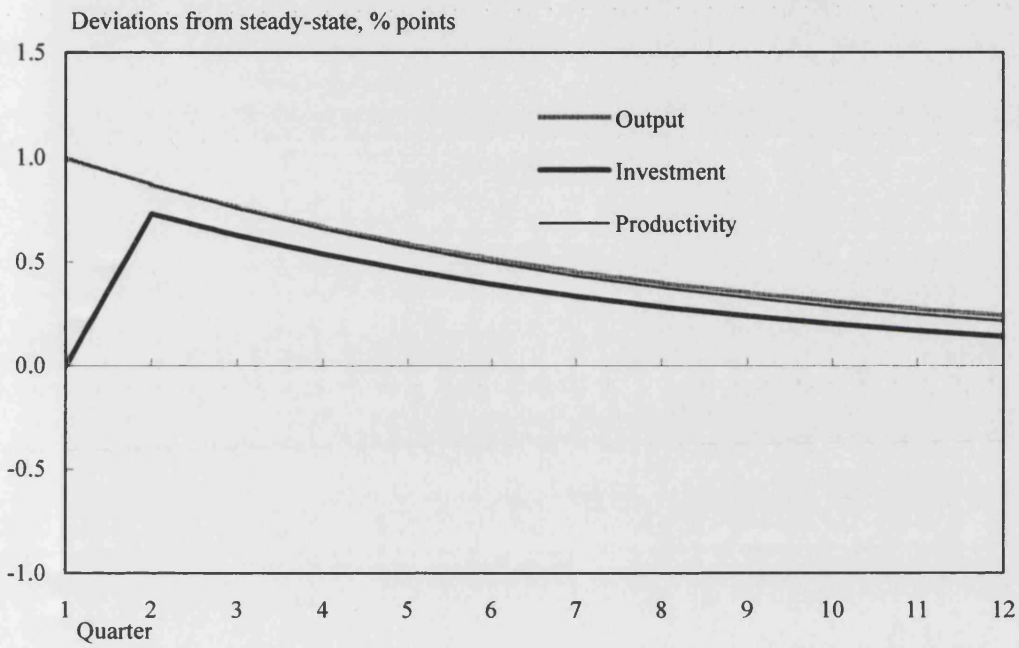


Chart 3.4: Responses to Technology Shock (The Core Model)

[Condition] Giving a positive technology shock consisting of 1% deviation from the steady-state only in the 1st quarter.

(1) No-FA case



(2) FA case

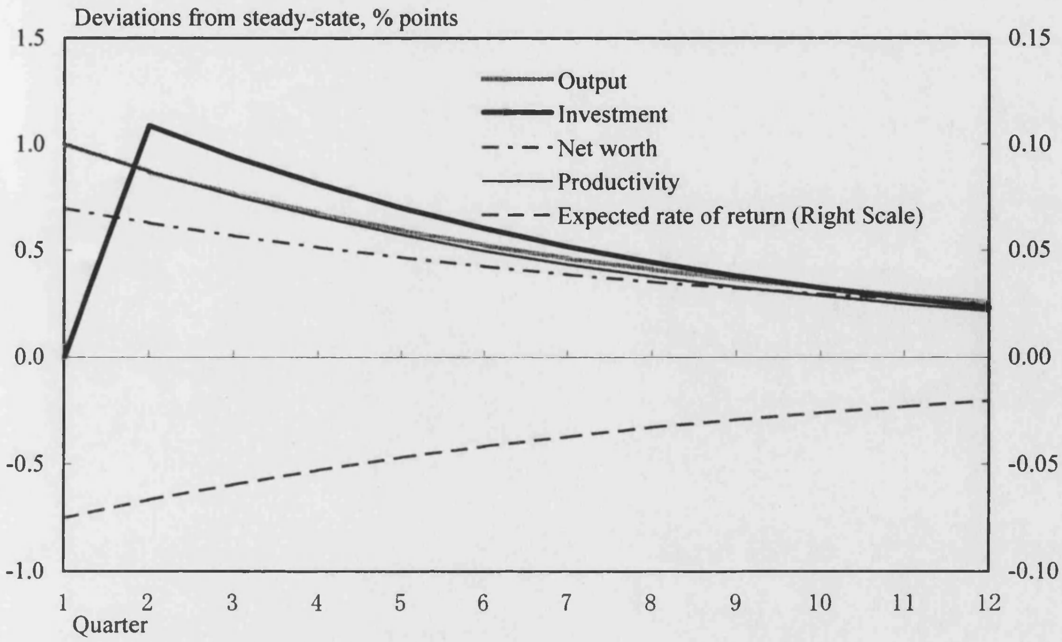
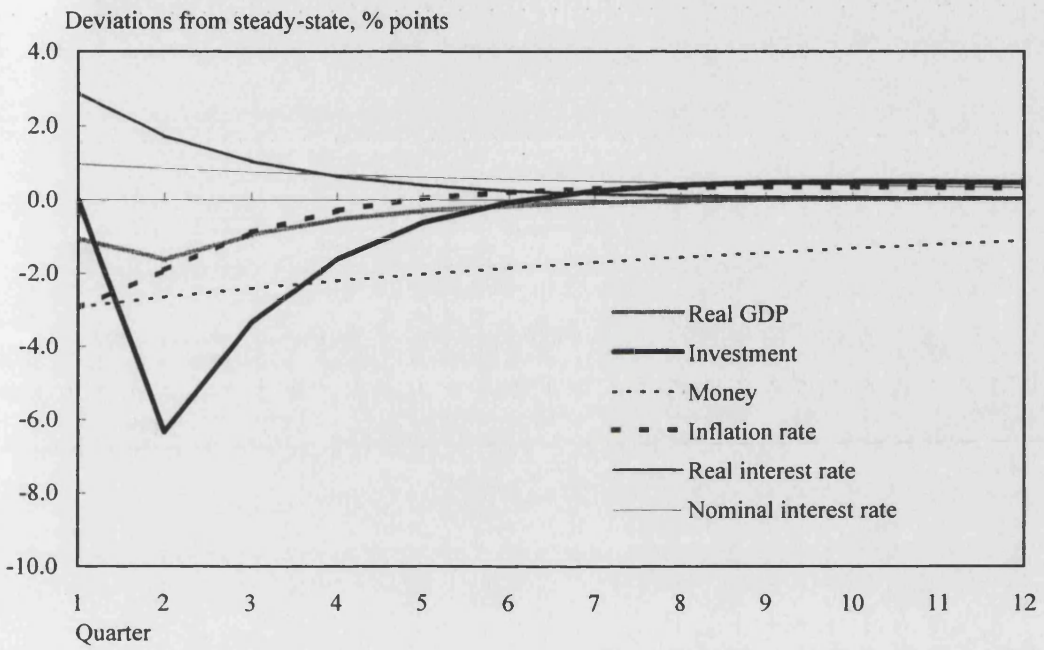


Chart 3.5: Responses to Monetary Policy Shock (The Whole Model)

[Condition] Giving a positive (tightning) monetary policy shock to the nominal interest rate consisting of annually 1% deviation from the steady-state only in the 1st quarter.

(1) No-FA case



(2) FA case

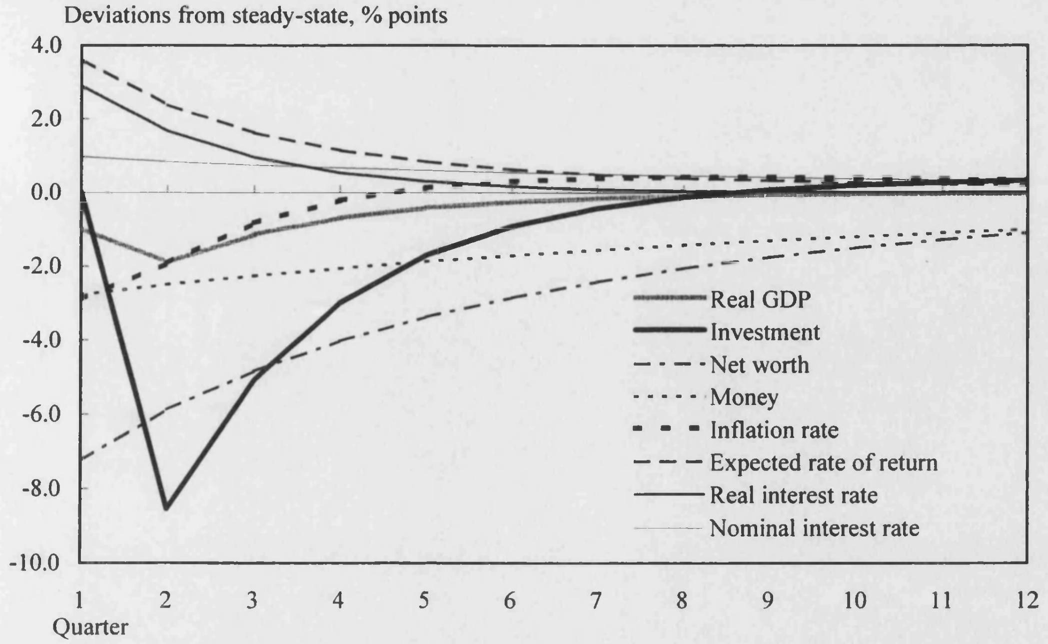
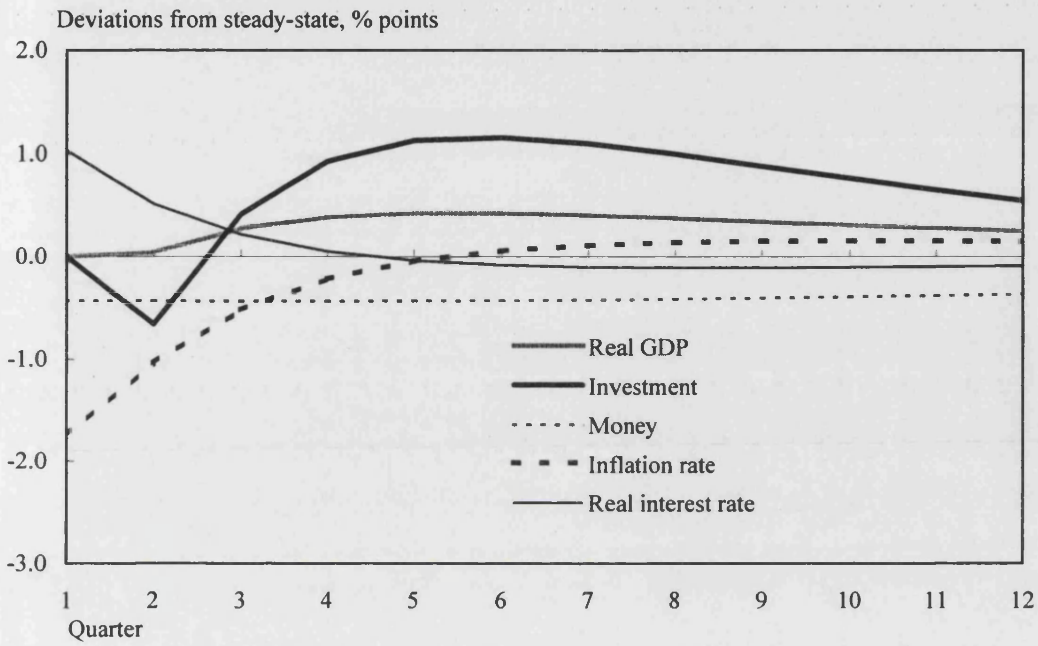


Chart 3.6: Responses to Technology Shock (The Whole Model)

[Condition] Giving a positive technology shock consisting of 1% deviation from the steady-state only in the 1st quarter.

(1) No-FA case



(2) FA case

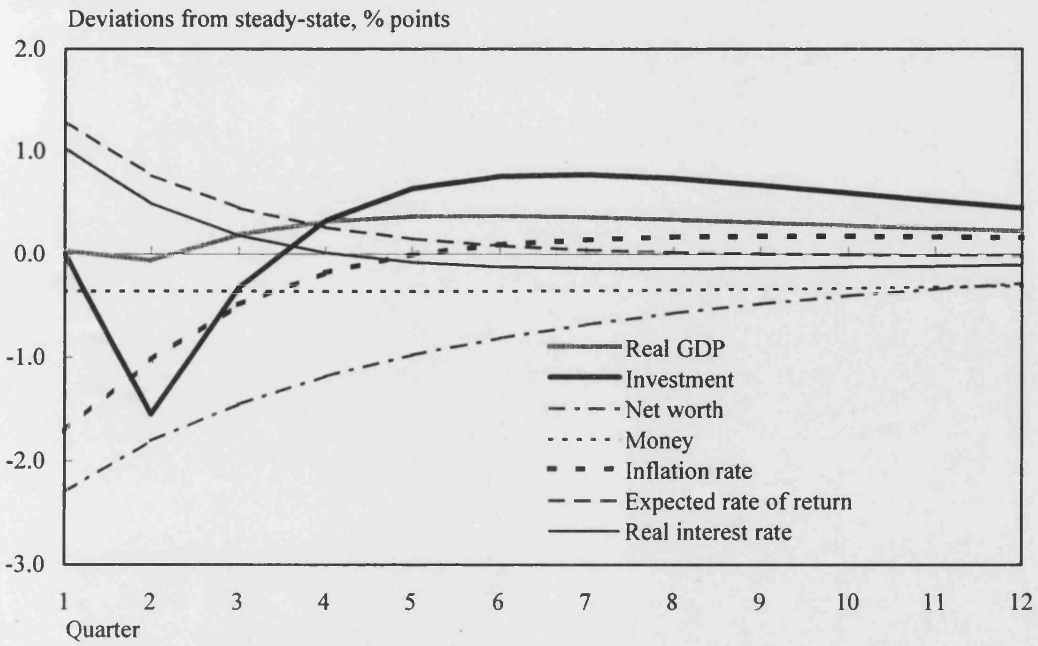
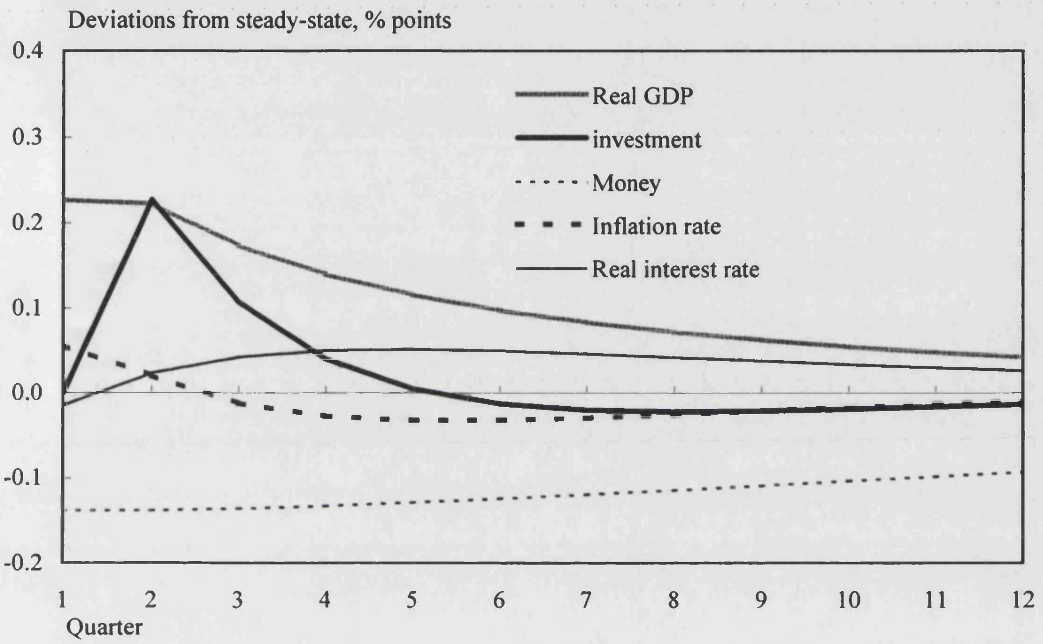


Chart 3.7: Responses to Demand Shock (The Whole Model)

[Condition] Giving a positive demand shock consisting of 1% deviation from the steady-state only in the 1st quarter.

(1) No-FA case



(2) FA case

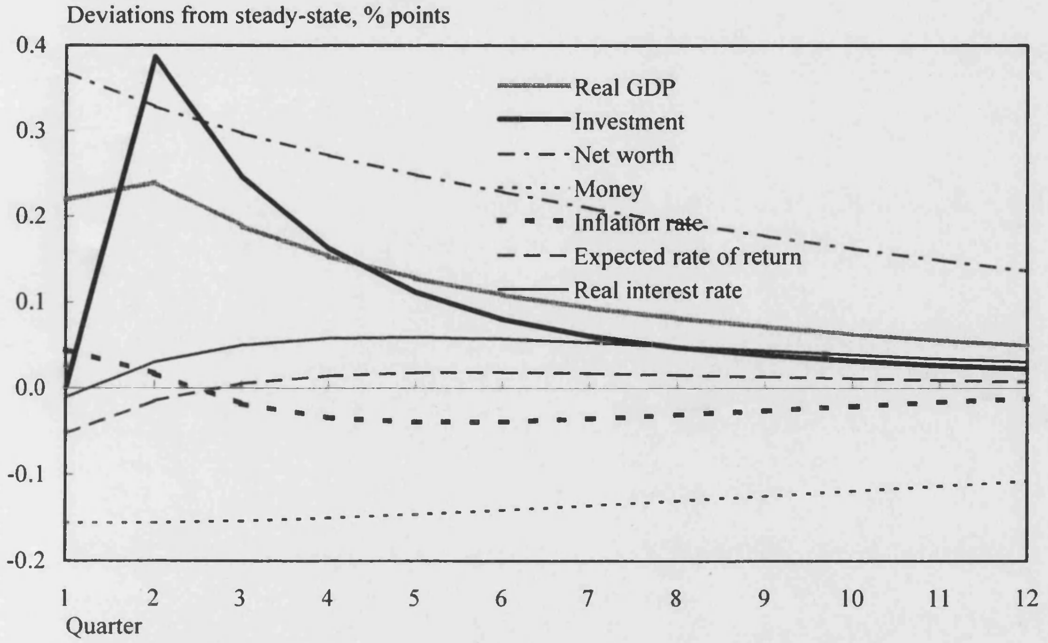


Chart 3.8: Second Moment Properties (Simulated Data)

(1) Standard deviations

	No-FA case	FA case
Real GDP	1.02 (0.13) <1.00>	1.07 (0.15) <1.00>
Corporate fixed investment	3.36 (0.35) <3.31>	4.36 (0.51) <4.07>
Private consumption	1.06 (0.13) <1.04>	1.03 (0.13) <0.96>
Total working hours	2.17 (0.22) <2.13>	2.32 (0.32) <2.17>
Inflation rate	2.28 (0.21) <2.24>	2.26 (0.28) <2.11>
Monetary aggregate	1.65 (0.20) <1.63>	1.56 (0.21) <1.45>

(2) Cross correlations with GDP

1. No-FA case

	-4	-3	-2	-1	0	1	2	3	4
Real GDP	-0.11	0.03	0.28	0.71	1.00	0.71	0.28	0.03	-0.11
	(0.13)	(0.15)	(0.14)	(0.07)	(0.00)	(0.07)	(0.14)	(0.15)	(0.13)
Corporate fixed investment	-0.12	-0.03	0.12	0.38	0.82	0.79	0.32	0.04	-0.11
	(0.13)	(0.14)	(0.15)	(0.13)	(0.04)	(0.05)	(0.13)	(0.15)	(0.12)
Private consumption	-0.06	0.08	0.31	0.69	0.68	0.31	0.09	-0.04	-0.11
	(0.15)	(0.15)	(0.13)	(0.08)	(0.09)	(0.15)	(0.17)	(0.16)	(0.15)
Total working hours	-0.29	-0.24	-0.06	0.36	0.65	0.52	0.26	0.10	0.00
	(0.13)	(0.14)	(0.16)	(0.14)	(0.10)	(0.12)	(0.15)	(0.17)	(0.18)
Inflation rate	-0.23	-0.16	0.04	0.47	0.58	0.33	0.14	0.02	-0.04
	(0.14)	(0.14)	(0.14)	(0.11)	(0.11)	(0.15)	(0.17)	(0.16)	(0.15)
Monetary aggregate	-0.37	-0.34	-0.17	0.27	0.48	0.36	0.26	0.18	0.12
	(0.15)	(0.14)	(0.13)	(0.13)	(0.13)	(0.14)	(0.16)	(0.17)	(0.18)

2. FA case

	-4	-3	-2	-1	0	1	2	3	4
Real GDP	-0.10	0.04	0.29	0.71	1.00	0.71	0.29	0.04	-0.10
	(0.14)	(0.14)	(0.13)	(0.07)	(0.00)	(0.07)	(0.13)	(0.14)	(0.14)
Corporate fixed investment	-0.18	-0.10	0.05	0.34	0.84	0.75	0.34	0.10	-0.04
	(0.14)	(0.13)	(0.13)	(0.11)	(0.04)	(0.06)	(0.13)	(0.14)	(0.13)
Private consumption	0.04	0.18	0.41	0.74	0.59	0.23	0.04	-0.08	-0.14
	(0.14)	(0.15)	(0.14)	(0.10)	(0.11)	(0.14)	(0.14)	(0.15)	(0.16)
Total working hours	-0.29	-0.22	-0.02	0.42	0.67	0.50	0.24	0.08	0.00
	(0.15)	(0.13)	(0.14)	(0.11)	(0.09)	(0.13)	(0.17)	(0.18)	(0.17)
Inflation rate	-0.19	-0.10	0.13	0.57	0.54	0.26	0.07	-0.02	-0.07
	(0.14)	(0.14)	(0.14)	(0.11)	(0.11)	(0.15)	(0.17)	(0.16)	(0.15)
Monetary aggregate	-0.33	-0.27	-0.06	0.41	0.53	0.37	0.25	0.17	0.09
	(0.16)	(0.17)	(0.18)	(0.17)	(0.17)	(0.18)	(0.19)	(0.19)	(0.19)

Notes:

1. All variables are percentage deviations from their steady-state values and have been detrended by the Hodrick-Prescott (HP) filter (λ=1600).
2. Numbers <...> in table (1) represent the relative volatility to GDP.
3. Numbers (...) represent the sample standard error.

Chart 3.9: Alternative Parameter Values

(1) Ratio of internal funds

	No-FA case	(Benchmark)		FA case	(Benchmark)	
	25%	50%	75%	25%	50%	75%
Real GDP	1.02	1.02	1.02	1.18	1.07	1.01
Corporate fixed investment	3.36	3.36	3.36	5.13	4.36	3.77
Private consumption	1.06	1.06	1.06	1.02	1.03	1.04
Total working hours	2.17	2.17	2.17	2.47	2.32	2.21
Inflation rate	2.28	2.28	2.28	2.31	2.26	2.22
Monetary aggregate	1.65	1.65	1.65	1.59	1.56	1.56
	< External funds		**Internal > funds**	**< External funds**		**Internal > funds**

(2) Price stickiness

	No-FA case	(Benchmark)		FA case	(Benchmark)	
	θ=0.9	θ=0.75	θ=0.65	θ=0.9	θ=0.75	θ=0.65
Real GDP	1.47	1.02	0.93	1.70	1.07	0.93
Corporate fixed investment	6.65	3.36	2.60	8.72	4.36	2.92
Private consumption	1.07	1.06	1.04	1.00	1.03	1.04
Total working hours	3.21	2.17	1.70	3.58	2.32	1.77
Inflation rate	0.67	2.28	3.23	0.65	2.26	3.21
Monetary aggregate	1.35	1.65	1.76	1.25	1.56	1.68
	< Sticky		**Flexible >**	**< Sticky**		**Flexible >**

(3) Elasticity of labor supply to wage

	No-FA case	(Benchmark)		FA case	(Benchmark)	
	$\eta_h=\infty$	η_h=2.737	η_h=0.035	$\eta_h=\infty$	η_h=2.737	η_h=0.035
Real GDP	1.05	1.02	0.98	1.11	1.07	0.94
Corporate fixed investment	3.47	3.36	3.12	4.40	4.36	1.98
Private consumption	1.07	1.06	1.26	1.03	1.03	1.93
Total working hours	2.28	2.17	0.75	2.43	2.32	0.61
Inflation rate	1.91	2.28	7.50	1.86	2.26	7.63
Monetary aggregate	1.62	1.65	2.02	1.54	1.56	1.73
	< Elastic		**Inelastic >**	**< Elastic**		**Inelastic >**

Chart 3.10: Alternative Monetary Policy Rules

(1) Varying parameterization (the price-level reaction rule)

1. No-FA case

				Benchmark					
ν_y	**0.000**	**0.084**	**0.000**	**0.084**	**1.000**	**2.000**	**0.084**	**0.084**	**2.000**
ν_p	**0.000**	**0.000**	**0.004**	**0.004**	**0.000**	**0.000**	**1.000**	**2.000**	**2.000**
Real GDP	1.21	1.03	1.19	1.02	0.99	0.90	0.96	0.98	0.67
Corporate fixed investment	3.91	3.46	3.81	3.36	4.45	4.39	2.73	2.83	1.79
Private consumption	1.20	1.07	1.18	1.06	1.22	1.20	1.01	1.04	0.84
Total working hours	2.10	2.23	2.05	2.17	2.97	2.90	1.09	1.00	1.21
Inflation rate	2.11	2.37	2.03	2.28	4.06	4.42	0.60	0.44	1.07
Monetary aggregate	1.81	1.72	1.76	1.65	2.61	2.81	0.98	1.03	0.71

2. FA case

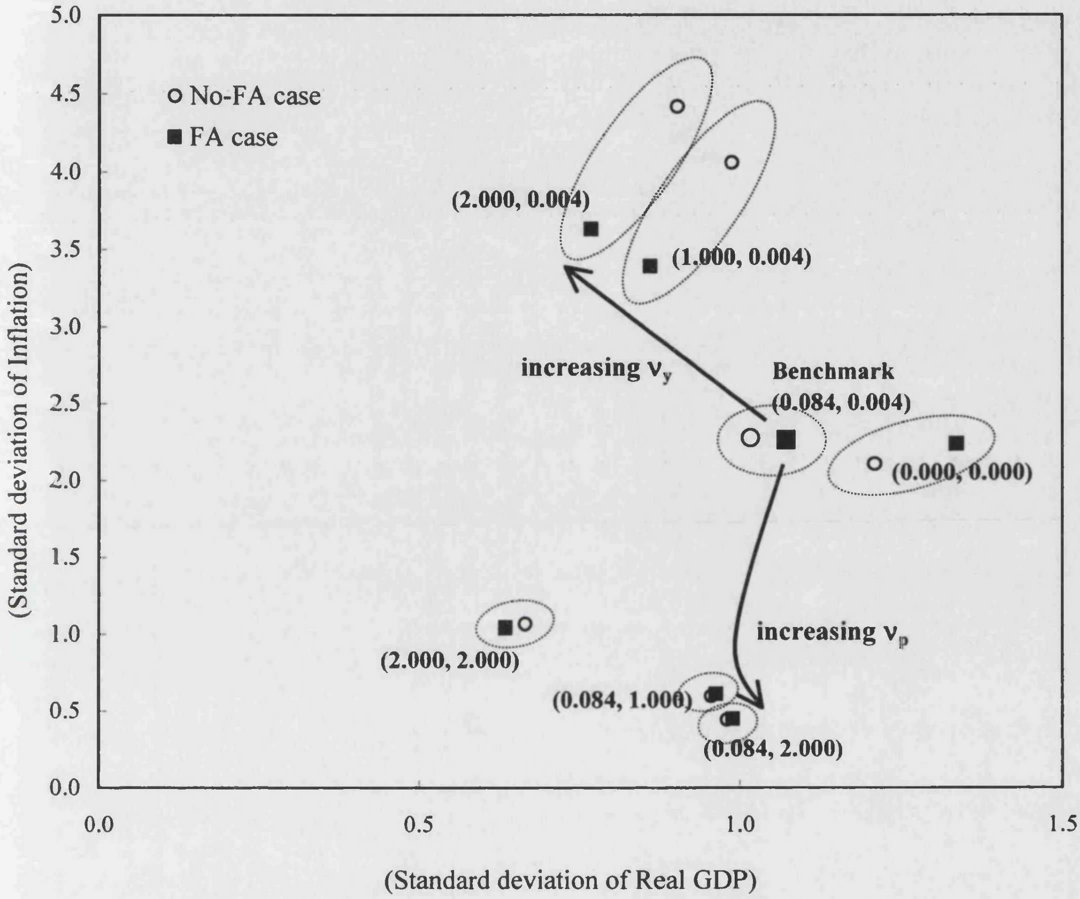
				Benchmark					
ν_y	**0.000**	**0.084**	**0.000**	**0.084**	**1.000**	**2.000**	**0.084**	**0.084**	**2.000**
ν_p	**0.000**	**0.000**	**0.004**	**0.004**	**0.004**	**0.004**	**1.000**	**2.000**	**2.000**
Real GDP	1.34	1.09	1.31	1.07	0.86	0.77	0.96	0.99	0.63
Corporate fixed investment	5.25	4.49	5.10	4.36	5.01	4.91	2.78	2.90	1.49
Private consumption	1.19	1.04	1.17	1.03	1.01	1.00	1.02	1.05	0.88
Total working hours	2.38	2.38	2.32	2.32	2.77	2.69	1.13	1.02	1.21
Inflation rate	2.24	2.34	2.16	2.26	3.39	3.63	0.61	0.45	1.04
Monetary aggregate	1.81	1.60	1.76	1.56	2.00	2.15	0.98	1.03	0.72

(2) Considering the reaction to inflation

	No-FA case				FA case			
ν_y	**0.084**	**0.084**	**0.084**	**0.084**	**0.084**	**0.084**	**0.084**	**0.084**
ν_p	**0.000**	**0.000**	**0.000**	**0.004**	**0.000**	**0.000**	**0.000**	**0.004**
ν_π	**0.000**	**0.500**	**2.000**	**2.000**	**0.000**	**0.500**	**2.000**	**2.000**
Real GDP	1.03	1.12	(0.95)	0.95	1.09	1.19	(1.02)	1.01
Corporate fixed investment	3.46	3.88	(2.90)	2.86	4.49	5.16	(3.85)	3.76
Private consumption	1.07	1.16	(0.99)	0.98	1.04	1.10	(0.99)	0.99
Total working hours	2.23	2.45	(1.84)	1.81	2.38	2.62	(2.04)	2.00
Inflation rate	2.37	2.78	(2.01)	1.95	2.34	2.74	(2.11)	2.05
Monetary aggregate	1.72	1.95	(1.31)	1.28	1.60	1.81	(1.31)	1.29

Notes: (...) in table (2) are calculated regardless of the indeterminacy of the steady state equilibri

Chart 3.11: Alternative Monetary Policy Rules



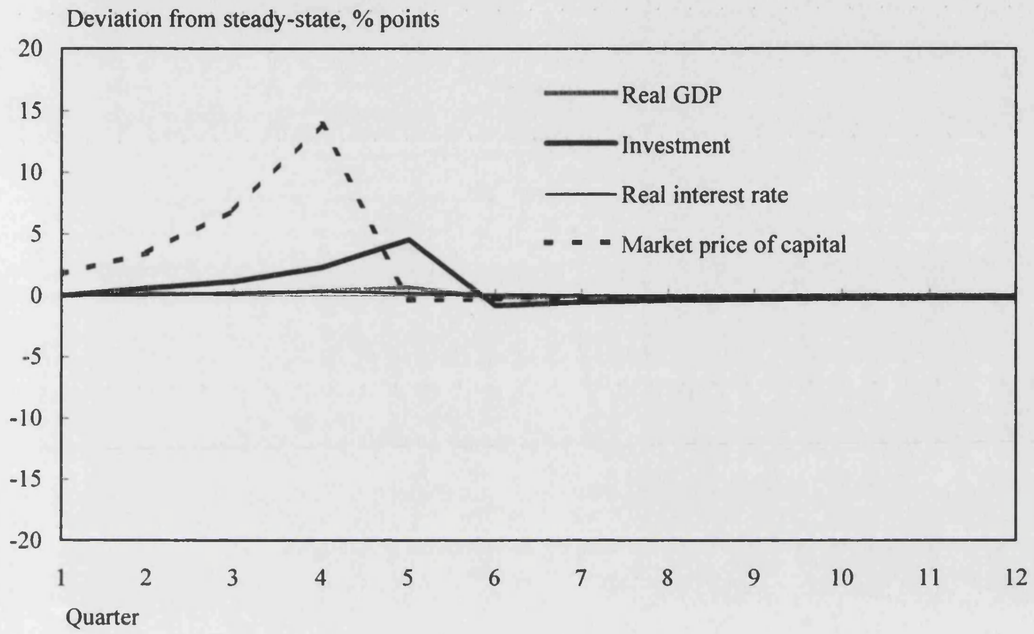
Notes:

1. The standard deviations of GDP and Inflation for each parameterization of our benchmark price-level reaction rule in both No-FA and FA cases (tables (1) in Chart 3.10) are plotted.
2. (.... , ...) represents the set of reaction parameter values (ν_y , ν_p).

Chart 3.12: Responses to Bubble Process

[Condition] Generating a bubble that begins with one percentage point increase above the fundamental value in the 1st quarter and bursts in the 5th quarter.

(1) No-FA case



(2) FA case

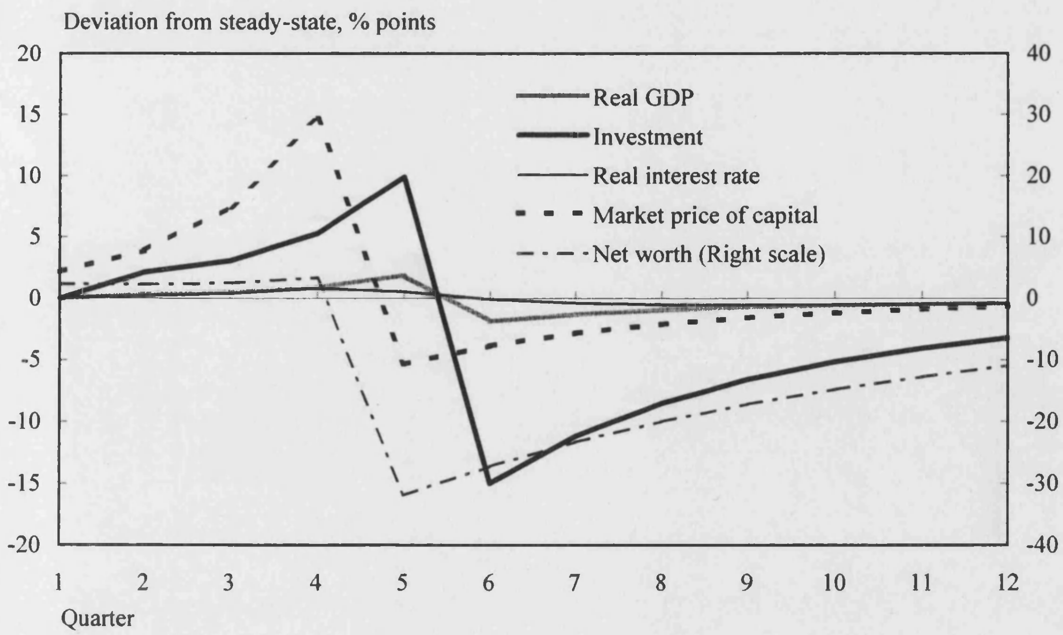


Chart 3.13: Adding Bubble Effects

	No-FA case		FA case	
	Benchmark	Adding Bubble	Benchmark	Adding Bubble
Real GDP	1.13 <1.00>	1.29 <1.00>	1.17 <1.00>	1.35 <1.00>
Corporate Fixed Investment	3.26 <2.88>	4.83 <3.75>	4.05 <3.46>	5.64 <4.19>
Private Consumption	1.35 <1.19>	1.39 <1.08>	1.31 <1.12>	1.35 <1.00>
Total Working Hours	1.60 <1.42>	1.85 <1.43>	1.71 <1.46>	1.98 <1.47>
Price Level	0.67 <0.59>	0.68 <0.53>	0.68 <0.58>	0.69 <0.51>
Monetary Aggregate	1.87 <1.65>	1.90 <1.47>	1.78 <1.52>	1.82 <1.35>

Notes: <...> represents the relative volatility to output.