



Labour Market Policy and Individual Saving Behaviour in  
Markets with Search Frictions

by

Vasileios Gkionakis

London School of Economics and Political Science

submitted for the  
PhD in Economics

UMI Number: U615897

All rights reserved

INFORMATION TO ALL USERS

The quality of this reproduction is dependent upon the quality of the copy submitted.

In the unlikely event that the author did not send a complete manuscript and there are missing pages, these will be noted. Also, if material had to be removed, a note will indicate the deletion.



UMI U615897

Published by ProQuest LLC 2014. Copyright in the Dissertation held by the Author.  
Microform Edition © ProQuest LLC.

All rights reserved. This work is protected against  
unauthorized copying under Title 17, United States Code.



ProQuest LLC  
789 East Eisenhower Parkway  
P.O. Box 1346  
Ann Arbor, MI 48106-1346

THESES

F

8802

BIB 1134223

## Abstract

The present dissertation evaluates specific labour market policies and investigates individual saving behaviour in economies characterized by search and matching frictions in the labour market.

The first chapter investigates the optimality of state provided unemployment insurance in a search theoretic framework with saving and borrowing constraints. The model is solved numerically, since an analytic solution is not possible, and then calibrated using features of the US economy. The results demonstrate that when individuals have access to saving, the importance of unemployment benefits provision diminishes significantly. Ex post heterogeneity among agents, matters however. Individuals that were unlucky not to accumulate enough assets to buffer the unemployment risk, would still prefer to receive non-trivial amounts of state provided benefits during their unemployment spell.

The second chapter of the thesis is concerned with the interaction between saving, consumption and search. It starts by documenting that the excess sensitivity of consumption growth to lagged labor income growth conceals a negative sensitivity of consumption growth to lagged unemployment growth. To understand this empirical regularity, we embed search frictions in a heterogeneous agent, precautionary savings model and study the implications for unemployment and consumption dynamics both at the microeconomic and macroeconomic level.

The third and final chapter employs a standard search and matching model with no saving, in order to study the effects of firing taxes on the job destruction

rate, when probation period – or temporary contract - policies are implemented. It is shown that, contrary to conventional wisdom, firing taxes can amplify the job turnover rate by providing incentives to destroy surviving matches at the end of the probation period. Moreover, low skill workers are shown to be more severely affected while wage inequality across different productivity groups may increase.

## Contents

List of Tables	6
List of Figures	8
Introduction	9
Chapter 1. Optimal unemployment insurance with search, saving and liquidity constraints	12
1.1. Introduction	12
1.2. The Model	18
1.3. Model Results	37
1.4. Risk aversion	44
1.5. Heterogeneity, wealth and unemployment insurance	46
1.6. Conclusion	49
Appendix 1.A	52
Appendix 1.B	52
Chapter 2. Chapter Two: Precautionary Saving, Search and Incomplete Information <sup>1</sup>	68

---

<sup>1</sup>Co-authored with Alex Michaelides, LSE.



2.1. Introduction	68
2.2. The Consumption-Unemployment Relationship in the Data	73
2.3. The Complete Information Model	78
2.4. Incomplete Information Model	98
2.5. Conclusion	106
Appendix 2.A	107
Appendix 2.B	108
Chapter 3. Chapter Three: Short Job Tenures and Firing Taxes in the Search Theory of Unemployment	126
3.1. Introduction	126
3.2. The model	132
3.3. Equilibrium	142
3.4. Productivity Differences	148
3.5. Computational experiments	152
3.6. Conclusions	155
Appendix 3.A	157
Appendix 3.B	159
References	173

## List of Tables

Table 1.1	56
Table 1.2	57
Table 1.3	59
Table 1.4	61
Table 1.5	62
Table 1.6	64
Table 1.7	66
Table 2.1	112
Table 2.2	113
Table 2.3	114
Table 2.4	116
Table 2.5	117
Table 2.6	119
Table 2.7	121
Table 2.8	123

Table 3.1	164
Table 3.2	165
Table 3.3	166
Table 3.4	168

## List of Figures

Figure 1.1	67
Figure 1.2	67
Figure 2.1	125
Figure 2.2	125
Figure 2.3	125
Figure 2.4	125
Figure 3.1.a	170
Figure 3.1.b	170
Figure 3.2.a	170
Figure 3.2.b	170
Figure 3.3	171
Figure 3.4	172
Figure 3.5	172

## Introduction

The present thesis is built upon a particular approach to the theory of unemployment. More specifically, it employs the search theory of unemployment in an attempt to examine certain features of labour market policy on one hand and saving behaviour at the individual as well as the aggregate level on the other.

The approach of search frictions in the labour market has been greatly supported in the past years by the emergence of new data on job and worker flows and has also proved popular in examining equilibrium unemployment and unemployment dynamics. More importantly, because the notion of risk is manifested not only through the danger of exiting the employment state but also through the risk of remaining unemployed for certain periods of time, this approach constitutes a solid theoretical foundation for the study of policy associated with the welfare state (like the provision of unemployment insurance for example) as well as the analysis of instruments that guard against the risk of income loss (like individual saving).

The first chapter of the thesis is concerned with determining the optimal level of unemployment insurance provision in a model where consumers exit the unemployment pool at a predetermined rate while their chances of becoming re-employed at some subsequent period are affected by their individual choice of search effort on one hand and the aggregate-wide matching frictions on the other. At the same time, these consumers are allowed to accumulate assets in order to buffer the

risk associated with income and thus consumption loss in some future state (precautionary saving). The latter is an important feature of this economy because it immediately raises the issue of the substitutability between private and public provision. Similar scenarios have been studied by Fredriksson and Holmlund (2001), Costain (1999) and Lentz (2005), but the present work takes a step further by introducing the notion of endogenous search decision, an endogenous aggregate matching rate and a wage determination process that depends explicitly on the workers' outside option. In addition, heterogeneity among agents in the current thesis is generated not only through different employment/unemployment histories but also through labour income fluctuations which - despite their simplistic modelling - may amplify the individual wealth differences of ex-ante identical consumers and can have an impact on the design of optimal unemployment insurance provision. The results of the analysis suggest that individual saving is a powerful weapon that can be used very effectively against the unemployment risk and therefore the need for additional state provided assistance diminishes significantly when the welfare metric is aggregate welfare. However, it is shown that heterogeneity does matter because consumers who have been unlucky and unable to accumulate assets would prefer to receive non-trivial levels of unemployment insurance and enjoy considerable gains to their individual consumption profile.

The second chapter documents a stylized fact between lagged unemployment growth rate and current non-durables aggregate consumption growth, namely that the latter is negatively related to the former over and above the excess sensitivity of

consumption growth to income growth. To explain this correlation - and in general to understand the individual and aggregate implications - this chapter utilizes the framework of the previous one, i.e. a model that embeds search frictions in the precautionary saving literature. This investigation suggests that when households can distinguish aggregate from idiosyncratic job destruction shocks (complete information), the model cannot replicate the macroeconomic stylized facts, even though it does better than the model without any search. However, the introduction of incomplete information improves these predictions moving the model's results closer in line with the empirical evidence, mainly because the signal extraction problem does not allow the complete smoothing of unemployment shocks.

The last chapter of the thesis abstracts from saving and search decisions and takes a much simpler approach to the relationship between probation period policies and firing taxes. More specifically, the model argues that firing tax regulations are not always beneficial for job destruction when accompanied by probation (or temporary contract) periods. This is because these tax-free periods provide incentives to firms to dispose off employees before the firing tax policy is initiated, thus amplifying the destruction rate and increasing unemployment. It is also shown, that lower productivity workers may suffer more when compared with more productive peers, as a result of the implementation of such policies.

## CHAPTER 1

# Optimal unemployment insurance with search, saving and liquidity constraints

### 1.1. Introduction

The present chapter examines the optimality of unemployment insurance (UI henceforth) provision in a model where impatient but prudent and borrowing constrained individuals have access to savings, choose their search intensity during the unemployment spell and receive labour income when employed. In this framework, the aggregate matching rate<sup>1</sup> is determined endogenously while the wage formation is structured in a way that it depends explicitly on the workers' wealth accumulation and their outside opportunity. The results of this research suggest that the presence of savings diminishes the importance UI provision and that the welfare gains of eliminating any state provided assistance can be non trivial. Although this may be true from an aggregate perspective, heterogeneity between individuals does matter. Agents who were unlucky enough to accumulate very limited amounts of savings are still better off when receiving UI at historically observed

---

<sup>1</sup>In the standard search and matching framework, the aggregate matching rate is determined by the availability of vacancies and unemployed individuals. It is assumed to affect positively the individual transition probabilities from the unemployment to the employment state.



levels. As a result, both the optimal net replacement rate ( $NRR^2$ ) and the accompanied welfare benefits are a decreasing function of the buffer stock of savings of individuals.

As a starting point, the adverse effect of UI on search decisions and the aggregate matching rate is an important ingredient. Unemployment insurance reduces search incentives (the moral hazard effect) and consequently increases unemployment, something which decreases the aggregate matching rate. The latter reduces further the individual job finding probability and demonstrates that the optimality of UI provision cannot be fully evaluated in partial equilibrium models with a fixed matching rate coefficient. Access to savings together with the wage determination process are the other crucial components. When agents cannot save, equilibrium labour income is increasing in UI because of the improvement in the "threat" point of individuals at higher unemployment benefits. Because the equilibrium wage increases as the state assistance becomes more generous, some of the adverse effects of UI are partly offset, which in turn entails that some insurance may be welfare improving. However, when saving is an option and the wage agreement depends explicitly on the agents' stock of wealth, two new things emerge: Firstly, the equilibrium wage is higher than before, and secondly, it remains relatively flat across all levels of unemployment insurance. The first result is due to the fact that workers and searchers have - on average - improved their outside option value by

---

<sup>2</sup>The net replacement rate is the ratio of the after tax unemployment benefit over the after tax labour income received when employed expressed in *percentage* terms. Note that in this chapter the terms "ratio" and "rate" will be used interchangeably.

having accumulated assets. To understand the second feature of the model one has to recall that impatient consumers will reduce their savings whenever faced with lower risk. Although a more generous UI provision tends to improve individuals' threat point, the reduction in accumulated assets, that follows the increase in benefits, acts towards the opposite direction. It turns out that the combined effect is negligible and hence the wage is very inelastic to unemployment insurance provision. Overall, agents enjoy a higher wage income than before (even at zero UI) and on average they have enough savings to buffer the unemployment risk. At the same time, a zero net replacement rate implies that the government does not distort search incentives at all (i.e. there is no moral hazard) and therefore the unemployment rate is kept to a minimum. However, a zero UI may not be the optimal policy for individuals with very few assets, a result that raises the issue of a non-uniform UI provision, even for an economy that is populated by *ex ante* identical consumers.

All in all, the model incorporates the basic ingredients of the precautionary savings literature pioneered by the seminal works of Zeldes (1989), Deaton (1991) and Carroll (1992, 1997) to the search theoretical framework of Mortensen (1978) and Pissarides (1984, 1985 and 2000) that has been widely used to explain equilibrium unemployment and unemployment dynamics. Indeed, such a unified framework seems appropriate for the study of UI mainly because of the interaction between savings and search effort. In the standard buffer stock saving literature, households accumulate assets in order to help buffer negative shocks to income [Deaton (1991)

and Carroll (1992, 1997)]. On the other hand, models that embed variable search effort explore endogenous search decisions by associating the search intensity with the probability of exiting the unemployment state [a few examples include Hopenhayn and Nicolini (1997), Costain (1999) and Lentz (2005)]. It becomes therefore evident that savings and search are two instruments that guard against the unemployment risk and because they are both costly for an impatient agent there can be a potential interaction between them. More specifically, the extent to which an agent has access to savings as an instrument of self insurance will affect the extent to which she can finance her consumption during the unemployment spell. The latter will be reflected in the intensity of search and the demand for additional insurance in the form of state provided benefits.

Research on this area was first pioneered by the work of Hansen and Imrohoroglu (1992). They studied the optimality of UI in a model where agents have access to a storage technology (with zero interest rate) and can either accept or reject exogenous employment opportunities. Hence, their approach abstracts from search decisions but moral hazard is modelled by assigning positive probabilities of receiving UI benefit to those agents that reject employment opportunities. In general, they find that positive levels of the unemployment benefit are optimal, as long as the degree of moral hazard is not extreme. Along these lines, Joseph and Weitzenblum (2003) employ a partial equilibrium model with search and savings to examine the UI provision and its beneficial role for low paid workers in France. They find that once agents are allowed to accumulate assets then the replacement

rate should be set close to 30 *percent*. Somewhat similar results are obtained by Wang and Williamson (1996). Finally, Lentz (2005) uses a partial equilibrium job search model with savings for the Danish economy and estimates that the optimal *NRR* lies in the range of 43 to 82 *percent* depending on the relationship between the discount rate and the interest rate. Costain (1999) takes a step further by using a general equilibrium framework. However, because his wage is a weighted average of the marginal product of labour and the disutility from work, labour income determination is independent of the outside opportunities of a worker [a point made also by Fredriksson and Holmlund (2001)]. Since his calibration is based on a quarterly frequency and benefits are assumed to be paid for six months, the moral hazard effect is not strong. Thus, Costain finds that the optimal replacement rate for patient agents is close to 50 *percent* and quite higher for impatient individuals.

The present model is similar to the models analysed by Costain (1999) and Lentz (2005) in that savings are considered a costly means of self insurance against the unemployment risk. My divergence from Lentz however, is that the aggregate matching rate is not taken as given and that labour income is determined endogenously. In contrast with Costain on the other hand, the wage determination that I employ is made explicitly dependant on the outside opportunities of workers and

searchers. In addition, my calibration is made over a monthly frequency while I allow for a fixed level of unemployment benefit to be paid indefinitely<sup>3</sup>, assumptions that may impact on the degree of moral hazard manifested in the model.

More specifically, I embed variable search intensity in the precautionary savings model with borrowing constraints. Like in Hopenhayn and Nicolini (1997), Davidson and Woodbury (1998), Fredriksson and Holmlund (2001), Costain (1999) and Lentz (2005) the effort exerted to find a job during the unemployment spell<sup>4</sup> is unobservable by any third party and thus is a source of moral hazard. In addition, I model explicitly the dependence of labour income on the workers' resources by resolving to wage posting similar in spirit to Moen (1997) and Pissarides (2001). Impatient and prudent consumers face idiosyncratic labour income uncertainty and the exogenous risk of becoming unemployed in which case they will receive a state provided unemployment assistance. As a result, those individuals that can afford to, choose to save in order to finance consumption in bad periods. At the same time, firms post vacancies at a given cost and wait for the arrival of workers.

The model is solved numerically since an analytical solution is not possible and then calibrated to match certain relevant features of monthly US data. For the benchmark scenario, access to savings dramatically diminishes the role of UI provision when aggregate welfare is the welfare metric. Because US unemployment

---

<sup>3</sup>It is true that the current US system provides unemployment benefits during the first 6 months of the unemployment spell. However, there are other forms of assistance (e.g. housing benefits) that can be extended for a period of 60 months.

<sup>4</sup>In this model, on the job search is discarded.

duration is usually short, individuals can buffer it effectively by accumulating relatively few amounts of assets. Very poor individuals however, still prefer to receive non-trivial levels of state provided assistance in order to finance consumption during the unemployment spell.

In contrast with most of the existing literature, I show that a higher degree of risk aversion is not necessarily associated with stronger demand for unemployment insurance. The result follows because more risk averse individuals accumulate on average more assets and are thus more able to buffer the unemployment risk. On top of that, the endogeneity of the matching rate is important since a declining rate - at higher UI levels - has a more negative impact the welfare of more risk averse individuals.

This chapter is organized as follows. The next section outlines the model. It begins with the household problem and then proceeds to discuss the firm side of the economy, the transition rates, and the description of the search equilibrium. Section 3 presents the benchmark results while section 4 is devoted to risk aversion. Section 5 discusses the implications of *ex post* heterogeneity and section 6 concludes the chapter.

## 1.2. The Model

### 1.2.1. Households

I consider the problem of an infinitely lived household  $j$  that maximizes expected intertemporal utility and faces two types of risk: The first one takes the form

of idiosyncratic labour income fluctuations and the second one is driven by the positive probability of job loss in some subsequent period. The former is implemented in order to generate a higher degree of heterogeneity among agents. When employed, the individual receives a wage and when unemployed she receives an unemployment benefit which is assumed to be strictly less than her labour income. Therefore, the individual needs to decide how much to consume and save in periods of employment and how much to consume, save and search in periods of unemployment.

In the present work, normative aspects of unemployment insurance such as duration and eligibility are not considered. In addition, I do not investigate the possibility of quitters. In effect, I am ignoring the case in which benefits are not being paid to those who do not search at all but, as it turns out in the simulations, no agent chooses to optimally set her search effort equal to zero.

Formally:

$$(1.1) \quad \max_{\{c_{jt}, s_{jt}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t [U(c_{jt}) - \eta_{jt} G(s_{jt})]$$

subject to

$$(1.2) \quad c_{jt} + b_{jt} \leq x_{jt}$$

$$(1.3) \quad x_{jt+1} = (1+r)b_{jt} + (1-\eta_{jt+1})(1-\tau_{t+1})w_{jt+1}\epsilon_{jt+1} + \eta_{jt+1}(1-\tau_{t+1})z$$

$$(1.4) \quad c_{jt} \geq 0$$

$$(1.5) \quad b_{jt} \geq 0$$

$$(1.6) \quad s_{jt} \geq 0$$

$$(1.7) \quad \Pr(\eta_{jt} = 1 | \eta_{jt-1} = 0) = \delta$$

$$(1.8) \quad \Pr(\eta_{jt} = 0 | \eta_{jt-1} = 0) = 1 - \delta$$

$$(1.9) \quad \Pr(\eta_{jt} = 0 | \eta_{jt-1} = 1) = \mu(\lambda_t s_{jt})$$

$$(1.10) \quad \Pr(\eta_{jt} = 1 | \eta_{jt-1} = 1) = 1 - \mu(\lambda_t s_{jt})$$

All variables are in real terms.  $b_{jt}$  is the real amount of the riskless asset (bonds) held between the beginning of period  $t$  and the beginning of period  $t + 1$  while  $\beta$  is the discount factor that satisfies  $0 < \beta < 1$ .  $U(c_{jt})$  is a function that describes the felicity derived from consumption ( $c_{jt}$ ) and is assumed to be strictly increasing and concave.  $G(s_{jt})$  stands for the disutility of search ( $s_{jt}$ ) in period  $t$ , with  $G(\cdot)$  being increasing and convex. As such, preferences apart from being additively



separable over time, they are also separable over consumption and search. This implies that search costs are independent of consumption and thus wealth [for an elaborate discussion on the issue of non-separability and its consequences for the search profile see Lentz and Tranæs (2002)].  $x_{jt}$  denotes cash on hand at the beginning of period  $t$  [(1.2) will hold with strict equality given local non-satiation],  $r$  is the net riskless rate on savings which is assumed time-invariant and exogenous,  $w_{jt}$  is wage income received at the beginning of period  $t$  while  $\tau_t$  is the marginal tax rate, common to all individuals.  $z$  is the state provided unemployment benefit which is assumed to be strictly less than wage income.  $\eta_{jt} \in \{0, 1\}$  denotes the state of employment at time  $t$ , where  $\eta = 1$  stands for unemployed and  $\eta = 0$  for employed.  $\epsilon_{jt+1}$  is an i.i.d. shock with mean 1 and *s.d.*  $\sigma_\epsilon$ <sup>5</sup>. The latter, together with the probability of job loss are the two sources of uncertainty. Finally,  $\delta$  is the exogenous probability of becoming unemployed while  $\mu(\lambda_t s_{jt})$  is the endogenous probability of finding a job in the next period with  $\lambda_t$  denoting the aggregate matching rate.

I assume that the period-by-period felicity function is of the constant relative risk aversion (*CRRA*) form:

$$U(c_{jt}) = \frac{c_{jt}^{1-\rho} - 1}{1-\rho}, \quad \rho \neq 1, \quad \rho > 0$$

$$(1.11) \quad U(c_{jt}) = \ln(c_{jt}), \quad \rho = 1$$

---

<sup>5</sup>The way this error component is introduced resembles an expenditure shock i.e. the individual encounters a shock after the wage has been agreed and paid. I do it in this way because it does not affect the firm's problem and hence simplifies the solution considerably. As mentioned previously, this shock is mainly used in order to amplify agent heterogeneity.

while the disutility of search will be given by:

$$(1.12) \quad G(s_{jt}) = s_{jt}^\gamma, \quad \gamma > 1$$

When an individual is unemployed, she will have to decide how much to search in order to improve her chances of finding a job in the subsequent period. In this, I follow the standard practice in the literature [see, for example, Costain (1999) and Lentz (2005)] and assume that the probability of finding a job in period  $t + 1$  is given by:

$$(1.13) \quad \mu(\lambda_t s_{jt}) = 1 - \exp(-\lambda_t s_{jt})$$

The latter is strictly increasing and concave in  $s_{jt}$  and satisfies the properties that

$$\lim_{s_{jt} \rightarrow 0} \mu(\lambda_t s_{jt}) \rightarrow 0$$

and

$$\lim_{s_{jt} \rightarrow \infty} \mu(\lambda_t s_{jt}) \rightarrow 1$$

Given the above specification and by defining as  $V_{jt}^e$  and  $V_{jt}^u$  the value functions of the employment and unemployment states respectively, the recursive formulation of the household problem can be written as:

$$(1.14) \quad V_{jt}^e(x_{jt}) = \max_{b_{jt+1}} \{U(x_{jt} - b_{jt}) + \beta[(1 - \delta)E_t V_{jt+1}^e(x_{jt+1}) + \delta E_t V_{jt+1}^u(x_{jt+1})]\}$$

$$\begin{aligned}
V_{jt}^u(x_{jt}) &= \max_{b_{jt+1}, s_{jt}} \{U(x_{jt} - b_{jt}) - G(s_{jt}) + \\
(1.15) \quad &+ \beta[\mu(\lambda_t s_{jt}) E_t V_{jt+1}^e(x_{jt+1}) + (1 - \mu(\lambda_t s_{jt})) E_t V_{jt+1}^u(x_{jt+1})]\}
\end{aligned}$$

The solution to this problem consists of three policy functions, namely consumption when employed, consumption when unemployed and the choice of search intensity. I will discuss later the appropriate solution method since the present model is analytically intractable when  $b_{jt} \geq 0$ .

### 1.2.2. Firms

Regarding the firm side of the economy, I closely follow the standard search and matching framework pioneered by Mortensen (1978) and Pissarides (1984, 1985 and 2000). For simplicity, I abstract from endogenous capital decisions and endogenous job destruction. I will assume that each job employs one worker while I keep the productivity of a job ( $p$ ) constant in time. As a result, the value function  $J$  for an active job  $i$  (i.e. a job that is already occupied by a worker) satisfies the following equation:

$$(1.16) \quad J_{it} = p - k - w_{it} + \frac{1 - \delta}{1 + r} J_{it+1}$$

where  $k$  is the total capital cost, assumed to be exogenous and fixed.

At the beginning of each period a number of unemployed agents (queue) apply to each job vacancy  $i$  based on the wage contract  $w_{it}$  that is posted by the individual firm  $i$ . The choice of the wage contract is such that the firm maximizes profits

from a vacant job subject to providing a minimum  $V_t^u$  to its job applicants and subject to the constraint imposed by (1.16).

Based on the posted wage contract, the search intensity, the number of vacancies and the number of unemployed individuals, some searchers will become matched and some vacant jobs will become filled. The rest will continue to search until they meet their partner in a subsequent period.

Before proceeding to the next section and in order to facilitate the notation used, one should note that the infinite structure of the problem implies a stationary equilibrium at which  $J$ , vacancies, unemployment and wages are all constant. Thus, all time subscripts that refer to aggregate and firm variables can, and will be dropped.

### 1.2.3. Transition probabilities

As discussed earlier, the individual transition probability from the unemployment to the employment pool is an increasing function of search intensity and the matching coefficient. The latter is common to all agents and is implemented in order to capture the idea of search externalities. That said, it is harder for an individual to find a job the more unemployed agents there are in the economy and easier the higher the number of vacancies available. More specifically, I assume that the functional form of the matching coefficient is given by:

$$(1.17) \quad \lambda = \Theta \frac{u^\alpha v^{1-\alpha}}{u}$$

where  $u$  is the unemployment rate,  $v$  is the vacancy rate (expressed in terms of total population which is assumed to be equal to the constant size of the labour force  $L$ ) and  $\alpha$  is assumed to be constant and take values in the open interval  $(0, 1)$ .  $\Theta$  is a scaling factor. It is straightforward to show that  $\lambda$  is increasing in  $v/u$ , a variable which is commonly referred to in the literature as market tightness  $(\theta)$ . The average transition probability is then defined by:

$$(1.18) \quad \bar{\mu}[\theta, h(s_j)] = \int_j [1 - \exp(-\lambda(\theta)s_j)] dj, \quad \bar{\mu}_\theta > 0$$

for some function  $h$  of all individual search intensities. One might equally think of  $h$  as a non-linear function of average search effort. For the sake of simplicity, I will denote this average probability as  $\bar{\mu}(\theta, h)$ <sup>6</sup>. One has to remember however, that this average transition rate is influenced by market tightness and individual search intensities.

Regarding transitions from vacancies to active jobs, I will concentrate only at the symmetric equilibrium i.e. the case at which each firm posts the same wage contract (see below) and hence market tightness is the same in all firm queues. By definition then, the probability of finding a worker is given by:

$$(1.19) \quad q = \frac{\text{matches}}{vL} = \frac{\text{matches } u}{uL} \frac{1}{v} = \bar{\mu}(\theta, h) \frac{1}{\theta} \equiv q(\theta, h)$$

---

<sup>6</sup>Note that in the case of a linear probability function  $\mu(s_{jt}) = \lambda_t s_{jt}$ , the average transition rate would simply be  $\bar{\mu}(\theta, \bar{s}) = \lambda \bar{s}$ , where  $h = \bar{s}$ .

since  $\bar{\mu}$  is by definition equal to  $matches/uL$ . As a result, one can write:

$$(1.20) \quad \bar{\mu}(\theta, h) = \theta q(\theta, h)$$

#### 1.2.4. Search equilibrium

I have so far specified the returns to workers, unemployed agents and active jobs for given transitional rates and wages. In this section I close the model by analysing job creation that consists of the description of a wage rule and market tightness.

The important thing to realize is that the agents' asset accumulation poses a considerable difficulty in deriving wage rules since individuals are now heterogeneous with respect to their wealth. If one wanted to tackle the problem by resolving to Nash bargaining in individual meetings (a common approach in search and matching models) or any other rule that is based on a one-to-one negotiation, she would be immediately confronted with the problem of keeping track of an infinite-dimensional object. Since the agent's surplus (i.e. the difference between the value of the employment and unemployment state) is specific to each individual, it follows that the division of the joint surplus and thus the wage are also agent-specific. On the other hand, assuming that *individual* agreements are based on market averages does not provide a convincing story. To overcome this difficulty, I base my approach on wage posting similar in spirit to Moen (1997) and Pissarides (2001).

Suppose that firm  $i$  has a vacancy to fill and therefore it posts a wage contract, conditional on the arrival of the destructive shock. Assume that unemployed agents allocate themselves to each queue in such a way so that no one can be made better off by changing queue. Thus, each one enjoys the return of the unemployment state which is agent-specific and faces her own prospect of becoming employed in the next period.

The expected return of an unemployed agent  $j$  that has joined queue  $i$  is  $V_{i,jt}^u$  and is given by (1.15)<sup>7</sup>. However, in order to make each firm post a single wage contract to all of its job applicants, I will assume that every firm has knowledge only about the average values of wealth, search and transition probability in the economy. This is the only assumption I make regarding the firm's information set. On one hand, it can be considered as sensible since it can be nearly impossible to observe the wealth stock of each and every individual and on the other hand it makes the problem tractable as it implies a single wage offer. Note that conceptually this assumption is not the same as conjecturing that the outcome of the Nash bargaining is based on market averages because wage posting does not involve firm-agent meetings that are agent-specific in nature.

Denote as  $\bar{V}_i^u$  and  $\bar{V}_i^e$  the average values of unemployment and employment states respectively in queue  $i$  with:

$$(1.21) \quad \bar{V}_i^u = \bar{U}^u - \bar{G} + \beta[\theta_i q_i \bar{V}_i^e + (1 - \theta_i q_i) \bar{V}_i^u]$$

---

<sup>7</sup>This is the same expression as that of equation (1.15) but with a subscript  $i$  included to denote that the individual has joined queue  $i$ .

$$(1.22) \quad \bar{V}_i^e = \bar{U}^e + \beta[(1 - \delta)\bar{V}_i^e + \delta\bar{V}_i^u]$$

where  $\bar{U}^u = U(\bar{c}^u)$ ,  $\bar{U}^e = U(\bar{c}^e)$  and  $\bar{G} = G(\bar{s})$  denote utility when unemployed, employed and disutility of search evaluated at the average values of consumption and search respectively. Note that the structure of our setup implies that all mean values are constant and thus no time subscripts and no expectation operators are required. Let also  $\bar{V}^u$  be the returns from joining the queue with the most attractive posted contract. Then the constraint facing firm  $i$  is:

$$(1.23) \quad \bar{V}_i^u \geq \bar{V}^u$$

The important implication of the above is that it explicitly takes into account the agents' outside option value i.e. the benefits received when unemployed and the buffer stock of wealth and thus it creates a feedback channel between the UI benefit, the asset holdings and the equilibrium labour income.

The firm is assumed to select a wage by maximizing the present discounted value of a vacant job subject to (1.23). Vacant jobs can enter the market at any time to participate in the matching process. Let  $\Pi_i$  be the expected profits from joining queue  $i$  with a vacant job. Assume now that there are no job creation costs but maintaining a vacant job open costs  $pc$  per period, for some  $c > 0$ . The value of the vacancy then, satisfies:

$$(1.24) \quad \Pi_i = -pc + \frac{1}{1+r} [q_i J_i + (1 - q_i) \Pi_i]$$



Given the flat wage profile, the maximization problem is:

$$(1.25) \quad \max_{w_i, \theta_i} \Pi_i = \frac{1+r}{r+q_i} \left( -pc + \frac{q_i}{1+r} J_i \right)$$

subject to

$$(1.26) \quad J_i = \frac{1+r}{r+\delta} (p - k - w_i)$$

$$(1.27) \quad \bar{V}_i^u = \frac{1}{1 - \beta[1 - \theta_i q_i]} (\bar{U}^u - \bar{G} + \beta \theta_i q_i \bar{V}_i^e) \geq \bar{V}^u$$

$$(1.28) \quad \bar{V}_i^e = \frac{1}{1 - \beta(1 - \delta)} (\bar{U}^e + \beta \delta \bar{V}^u)$$

Maximization with respect to  $w_i$  and  $\theta_i$  gives:

$$(1.29) \quad \zeta \frac{1}{1 - \beta[1 - \theta_i q_i]} \left[ \frac{1+r}{r+q_i} \frac{q_i}{r+\delta} - \frac{dq_i}{dw_i} \frac{1}{r+q_i} (J_i - \Pi_i) \right]^{-1} =$$

$$\left( \bar{U}_{w_i}^u - \bar{G}_{w_i} + \beta \frac{d[\theta_i q_i]}{dw_i} (\bar{V}_i^e - \bar{V}_i^u) + \beta \frac{\theta_i q_i}{1 - \beta(1 - \delta)} \bar{U}_{w_i}^e \right)$$

$$(1.30) \quad \frac{dq_i}{d\theta_i} \frac{1}{r+q_i} (J_i - \Pi_i) =$$

$$-\zeta \frac{1}{1 - \beta[1 - \theta_i q_i]} \left[ \bar{U}_{\theta_i}^u - \bar{G}_{\theta_i} + \beta \frac{d[\theta_i q_i]}{d\theta_i} (\bar{V}_i^e - \bar{V}_i^u) + \beta \frac{\theta_i q_i}{1 - \beta(1 - \delta)} \bar{U}_{\theta_i}^e \right]$$

where  $\zeta$  is the Lagrange multiplier. Any changes to the wage and market tightness will affect individual search intensity and thus the average transition probabilities,

an effect that needs to be taken into account by the firm when making its decision.

The wage rule is then derived by substituting (1.29) into (1.30):

$$(1.31) \quad \varepsilon(J_i - \Pi_i) \left[ \frac{\bar{U}_{w_i}^u - \bar{G}_{w_i} + \beta \frac{d[\theta_i q_i]}{dw_i} (\bar{V}_i^e - \bar{V}_i^u)}{\theta_i q_i} + \beta \frac{\bar{U}_{w_i}^e}{1 - \beta(1 - \delta)} \right] =$$

$$\left[ \frac{1 + r}{r + \delta} - \frac{\frac{dq_i}{dw_i} (J_i - \Pi_i)}{q_i} \right] \left[ \frac{\bar{U}_{\theta_i}^u - \bar{G}_{\theta_i}}{q_i} + \beta(1 - \varepsilon)(\bar{V}_i^e - \bar{V}_i^u) + \beta \frac{\theta_i \bar{U}_{\theta_i}^e}{1 - \beta(1 - \delta)} \right]$$

where  $1 - \varepsilon$  is the elasticity of  $\theta_i q_i$  - the average job finding probability - with respect to market tightness (see Appendix 1.A). Note that with linear probability and Cobb Douglas matching rate,  $\varepsilon = \alpha$ . In this case however, with a non-linear probability function,  $\alpha$  is a parameter that controls the relevant elasticity but is not exactly equal to it.

Given market tightness, (1.31) is solved to determine the wage contract that maximizes  $\Pi_i$ . In the absence of savings and variable search intensity, when utility is linear and  $\beta = \frac{1}{1+r}$ , (1.31) reduces to:

$$(1.32) \quad (V_i^e - V_i^u) = \frac{\varepsilon}{1 - \varepsilon} U_c^e (J_i - \Pi_i)$$

which is equation (41) in Pissarides (2001). Although (1.32) is a simplification of (1.31), it provides the basic intuition. Both of these equations entail that the agreed wage rate will depend on workers' outside option. As a result, when unemployment benefits decrease there will be pressure for a decrease in wages since individuals' position deteriorates. But a reduction in benefits makes prudent agents save more,

something which tends to improve their "bargaining strength". The net effect will clearly impact on the equilibrium wage, the unemployment and vacancy rates and thus on aggregate welfare. It is worth noting that monotonicity of both sides of (1.31) cannot in general be established but the simulations yielded always a single wage solution.

In order to derive the value of market tightness, I focus on the symmetric equilibrium i.e.  $\theta_i = \theta$  for all  $i$  and I impose the usual zero-profit condition on new job creation by assuming that firms will create vacant jobs up to the point where all rents from job creation are exhausted. This implies that  $\Pi_i = 0$  for all  $i$ . Hence, at the symmetric equilibrium:

$$(1.33) \quad \frac{pc}{q} = \frac{(p - k - w)}{r + \delta}$$

The left hand side is the expected cost of maintaining a job vacant (since  $\frac{1}{q}$  is the expected duration of a vacancy) while the right hand side is the expected profit from an active job. In other words, (1.33) is something like a labour demand curve that equates revenue to cost.

### 1.2.5. Solution Method

I generalize the Deaton (1991) solution to allow for search during the unemployment state by deriving the three Euler equations associated with the three control variables (see Appendix 1.B):

$$(1.34) \quad U_c(c_{jt}^e) = \max\{U_c(x_{jt}^e), \beta(1+r)[(1-\delta)E_t U_c(c_{jt+1}^e) + \delta E_t U_c(c_{jt+1}^u)]\}$$

$$(1.35) \quad \begin{aligned} U_c(c_{jt}^u) = & \max\{U_c(x_{jt}^u), \beta(1+r)[\mu(\lambda s_{jt})E_t U_c(c_{jt+1}^e) \\ & + (1-\mu(\lambda s_{jt}))E_t U_c(c_{jt+1}^u)]\} \end{aligned}$$

and

$$(1.36) \quad G_s(s_{jt}) = \beta \mu_s(\lambda s_{jt})(E_t V_{jt+1}^e - E_t V_{jt+1}^u)$$

where the superscripts  $e$  and  $u$  refer to the employment and unemployment states respectively. The first two Euler equations are a straightforward generalization of Deaton (1991). If the individual is employed and liquidity constrained in period  $t$ , consumption cannot be higher than her cash on hand  $x_{jt}^e$  and as a result the marginal utility of consumption cannot be lower than  $U_c(x_{jt}^e)$ . If on the other hand, the borrowing limit is not binding, then optimality dictates that current marginal utility should be equated to the expected discounted future one (Hall, 1978). The latter takes into account the possibility that the individual may become unemployed in the next period, hence it is weighted by the (exogenous) probabilities  $\delta$  and  $1 - \delta$ . The same logic applies to an unemployed individual in the current period. Now, however, expected marginal utility of next period consumption has to be weighted by the (endogenous) probabilities of finding a job or not.

The third Euler equation derives the decision for search. It states that at the optimum an unemployed agent will choose search effort to equate the marginal cost of search to the discounted future benefits of it. The discounted expected benefits are the product of the marginal increase in the probability of finding employment from an additional unit of search ( $\mu_s(\lambda s_{jt})$ ) and the discounted expected benefit from employment relative to unemployment. The latter benefit is quantified by the term  $\beta(E_t V_{jt+1}^e - E_t V_{jt+1}^u)$  which computes the discounted expected difference between employment and unemployment in  $t + 1$ .

Finally, one should note that the state variable  $x$  evolves differently according to whether the individual is unemployed or not in the current period and the next. The four different possibilities are:

$$(1.37) \quad x_{jt+1}^{ee} = (1 + r)b_{jt}^e + (1 - \tau)w\epsilon_{jt+1}$$

$$(1.38) \quad x_{jt+1}^{eu} = (1 + r)b_{jt}^e + (1 - \tau)z$$

$$(1.39) \quad x_{jt+1}^{uu} = (1 + r)b_{jt}^u + (1 - \tau)z$$

$$(1.40) \quad x_{jt+1}^{ue} = (1 + r)b_{jt}^u + (1 - \tau)w\epsilon_{jt+1}$$

### 1.2.6. Calibration of Parameters

The model is calibrated to match features of US data. I consider the unit of analysis to be one month because typically unemployment duration is short and a quarterly calibration will throw away too much information.

I set  $\beta$  equal to 0.996 (which corresponds to a monthly discount rate of 0.0042 and an annual one of 0.05) and the constant real interest rate,  $r$ , equal to 0.0016 (which corresponds to an annual interest rate of 0.02). Such a relationship between the two aforementioned parameters implies consumer impatience. Borrowing constraints are not relevant for patient agents as these individuals are inherently inclined to accumulate positive amounts of assets and hence there would be no upper bound to wealth.

Regarding the coefficient of risk aversion, some of the literature on unemployment insurance assumes values close to 1<sup>8</sup>. Others, like Acemoglou and Shimer (1999) and Joseph and Weitzenblum (2003) use a value of 3, while Hansen and Imrohoroglu (1992) set it equal to 2.5. Finally, Lentz (2005) estimates its value

---

<sup>8</sup>See for instance, Hopenhayn and Nicolini (1997), Costain (1999), Davidson and Woodbury (1998), Fredriksson and Holmlund (2001) and Wang and Williamson (2002).

to be around 2.2 for the Danish economy. I choose to be somewhere in the middle and set the value of  $\rho$  equal to 1.2 but I also report results for a different parameterization.

I set  $\sigma_\epsilon$  equal to 0.05 which results in an 8 percent standard deviation, very close to the estimated figure of 10 *percent* (Carroll, 1992). In my case however, this is not exactly a labour income shock but it suffices to amplify the heterogeneity among individuals. I normalize productivity  $p$  by 1 and set  $\delta$  equal to 0.03 based on the estimates of Abowd and Zellner (1985) and the Job Openings and Labour Turnover Survey for the period 2000 - 2004. Finally  $k$  is set equal to 0.35, which implies that capital has a share of 35 *percent* of total productivity.

Regarding the baseline *NRR*, Gruber (1997) documents that the average replacement ratio in 1987 in his sample (the *Panel Study of Income Dynamics, PSID*) is around 0.426 while Meyer (1990) reports an average *NRR* of around 0.7 (using Continuous Wage and History Data, 1985a and 1985b). In addition, Hopenhayn and Nicolini (1997) report that with some variation across states "*...the current system provides workers with a replacement rate of approximately 60 percent...*". The OECD in Benefits and Wages (2002) calculates an average *NRR* of around 0.35. The latter however is the total benefits paid to unemployed agents for a total duration of 60 months. On the other hand, the average *NRR* (including some housing benefits) for the first month of the unemployment spell ranges from 0.57 to 0.6 (depending on the marital status of the agent) in the same study. Since

in my simulation no agent is unemployed for more than 4 consecutive months, I consider it sensible to aim for a benchmark  $NRR$  close to 0.5.

Regarding the value of  $\alpha$ , I choose to follow Blanchard and Diamond (1989) who estimate it to be in the range of 0.45 to 0.55, failing to reject constant returns to scale. For this reason I set its baseline value equal to 0.5.

I set the value of  $\Theta$  equal to 1.3 in order to generate an unemployment rate in the neighborhood of 5.5 to 6 *percent*. This target value for the unemployment rate is based on the monthly series that is readily available from the Bureau of Labour Statistics and spans the last thirty years. At the same time, I set  $\gamma$  equal to 2 (which implies a quadratic utility for search) in order to match an average consumption drop in the range of 10 to 14 *percent*. This last targeted value is based on two empirical studies. The first one is due to Gruber (1997) who documents - by using data from the *PSID* - that transitions from the employment to the unemployment state are on average associated with a 7 *percent* drop in food consumption. His sample however, contains information about food consumption only. In a related study Browning and Crossley (2001) by using a Canadian data set, find that mean total consumption falls by around 14 *percent* with unemployment.

Finally, I choose to calibrate  $c$  in order to achieve a market tightness in the range 0.5 to 1 [like Shimer (2005)] and not exceed a cost of maintaining a vacancy of 2 monthly wages [like Holmlund and Fredriksson (2001)]. For this reason I set  $c$  equal to 0.2 which resulted in an expected cost of maintaining a vacancy



approximately equal to around 0.5 of monthly wage and an equilibrium market tightness equal to 0.82.

Table 1.1 summarizes these benchmark parameter values.

### 1.3. Model Results

#### 1.3.1. Policy functions

The consumption policy functions are plotted in figure 1 and the search intensity (expressed as the probability of finding a job when unemployed) in figure 2. Consumption is equal to cash on hand when the borrowing constraint is binding and saving becomes positive beyond a certain point generating a concave consumption function. As it is evident, an unemployed agent starts saving earlier than an employed one because of the increased uncertainty to finance her consumption in the next period. Notice also that the difference between the two policy functions diminishes as wealth increases because accumulated assets become more and more sufficient to finance consumption despite of the fact that an unemployed agent receives lower income than that of a worker. On the other hand, search intensity is flat for low values of cash on hand and decreases beyond a certain level. The profile is flat because for poor and borrowing constrained individuals an extra unit of cash on hand will be consumed entirely, leaving thus the incentives for search unchanged. These incentives however, decrease with wealth for richer agents since people who are able to save can afford longer unemployment spells. It should be emphasized that this negative wealth effect on search decisions is consistent

with the findings of Cremieux et al. (1995), Lentz and Tranæs (2002) and Algan, Cheron, Hairault and Langot (2003) who report that wealth has a positive and statistically significant effect on unemployment duration.

### **1.3.2. Benchmark model results**

This part of the chapter is concerned with determining the optimal level of unemployment insurance from an aggregate perspective. More specifically, I vary the benefit provision and discuss the implications for aggregate welfare. I begin my analysis by considering the model where individuals do not have access to any means of saving. In order to make this version of the model comparable with the benchmark scenario (the model with savings), I elect to re-calibrate some of the parameters to achieve approximately the same targeted values that were set for the baseline case. For this reason, I set  $c$  equal to 0.45 and  $\Theta$  equal to 1.225 so that the simulated unemployment and vacancy rates as well as the transition probabilities between the two models closely resemble one another when the UI benefit is set equal to its benchmark value. This makes the comparison of consumer behaviour fair and straightforward.

The simulated results for this model are reported in Table 1.2. In general, mean individual consumption of the employed falls with unemployment insurance following the increase in taxation, a measure necessary to finance the generosity of the system. However, average consumption of the unemployed rises because the unemployment benefit increases. As a result, unemployment insurance is having

a significant effect in smoothing consumption drops from the employment to the unemployment state. Still however, these drops are quite pronounced because of the absence of any instrument of self-insurance.

The equilibrium wage rate increases with the level of unemployment insurance, a result which is quite standard in all search models of unemployment that feature an endogenous wage process but lack the concept of savings. The reason is that higher benefits imply that the value of the unemployment state increases relative to that of employment. As a result, agents need to be compensated more in order to retain incentives to search or work, hence the wage rises with UI.

Turning to the rest of the simulated results, one observes that average search intensity declines with UI provision, evidence that individuals reduce the costly activity of search when compensated more during the unemployment spell (the moral hazard effect). At the same time, the unemployment rate increases while vacancies decline following the increase in the cost of labour. The aggregate matching rate decreases sharply and consequently the average job finding probability is lower at higher levels of UI.

I now proceed to examine the welfare implications of UI provision. The last row of Table 1.2 calculates the aggregate welfare gain obtained when moving from each different level of the unemployment benefit to the optimal one. More specifically, it measures the percentage increase in the certainty equivalence of consumption  $c_z$

at benefit  $z$ , where the latter is defined as the solution to the following equation:

$$(1.41) \quad W_z = \sum_{t=0}^{\infty} \left( \frac{c_z^{1-\rho} - 1}{1-\rho} \right)$$

where

$$(1.42) \quad W_z = \int_j [(1-u)V_j^e + uV_j^u] dj, \text{ at benefit } z$$

As can be readily seen from the calculations in Table 1.2, the optimal level of unemployment insurance is equal to 0.3 where the implied replacement rate is 47.78 percent. In other words, in the absence of any means of self-insurance the baseline scenario appears to be the optimal one. As benefits rise, there is an initial improvement in aggregate welfare because unemployed individuals are able to consume more. Although taxation increases, some the adverse effects of higher UI are partly offset by the increases in the equilibrium wage rate which acts so as to improve the welfare of the already employed (at least compared with a case of a constant wage). As a result, some UI provision is beneficial in such an environment. As unemployment insurance keeps rising further however, the disincentive effect together with the reduction in vacancies grow too large and thus the unemployment rate increases quickly, something which deteriorates aggregate welfare. Overall, the welfare gains from unemployment insurance provision can be very large. If, for example, the initial provision is set to zero, then the certainty

equivalence of consumption would rise by an impressive 10.58 *percent*, should the policy makers decide to increase benefits to the 0.3 level.

I now proceed by solving and simulating the model with the endogenous wage process in the presence of savings, using the baseline parameterization discussed in the previous section. Results are reported in Table 1.3. Mean consumption of both the employed and the unemployed falls with unemployment insurance, although on average it is higher than that reported in the previous case. The most important thing to document, is that mean individual savings decrease sharply with unemployment insurance. As the social assistance becomes progressively more generous, impatient agents optimally decide to reduce asset accumulation since the unemployment state becomes less costly to endure and the threat of facing the liquidity constraint diminishes. The latter illustrates the substitution between public and private insurance. More specifically, when the implied net replacement rate is approximately equal to 78 *percent*, savings of the employed are around 8 times lower than those associated with a 0 unemployment benefit. On the other hand, mean individual consumption drop seems to vary little at all levels of unemployment insurance, evidence that individual savings are quite sufficient to smooth the transition from the employment to the unemployment state. At a replacement rate of 78 *percent* however, this consumption drop is reduced to 12.10 *percent*.

Turning now to the aggregate implications of UI, one immediately observes two things that are in contrast with the previous results. The first one indicates that

the equilibrium wage rate is on average around 2.6 *percent* higher than the wage negotiated in the no-savings scenario. This follows the improved threat point of agents who are now in a position to "negotiate" a higher labour income. The other thing is that the equilibrium wage rate has a relatively flat profile across different levels of unemployment insurance. The reason is that the negotiation process is subject to opposing forces. On one hand, the increase in benefits tends to put upward pressure on wages because it makes the unemployment state less costly and thus firms need to make contracts more generous to attract job applicants. On the other hand, as benefits rise impatient individuals respond by lowering their stock of assets thereby worsening their "bargaining strength". It turns out that for the given parameterization, the two effects almost cancel each other and therefore, the equilibrium wage remains relatively unchanged at all levels of unemployment insurance. At a 0 *percent* replacement rate, equilibrium labour income is around 4.2 *percent* higher than the one obtained previously.

The moral hazard effect is clearly present since average search effort declines with UI provision and is almost halved as the replacement rate increases from 0 to 78 *percent*. Naturally, unemployment increases and the matching rate together with the average job finding probability decrease sharply while taxation rises in order to finance the generosity of the system.

Turning now to the welfare implications, one notices that the gains of abolishing the unemployment insurance scheme are non trivial. Based on the results of the simulated model, the optimal replacement rate is found to be equal to 0 *percent*

while the certainty equivalence of consumption would rise by 0.76 *percent* if the policy maker decided to reduce the UI from its baseline level to the optimal one. At the same time the unemployment rate would decline by almost 17 *percent* following the increase in the intensity of search effort.

Results are different from before because agents have access to a means of self insurance that enables them to protect themselves against the unemployment risk. At the same time, the wage negotiation process can be important. With a sufficient stock of savings and no further improvement in labour income (like before), the adverse effects of UI on aggregate welfare through a declining job finding probability due to moral hazard are quite large, indicating that in such an economy the importance of UI provision diminishes significantly.

Naturally, one may wonder how important the wage negotiation process is. Costain (1999) correctly observes that subtle changes to the wage rule may impact considerably on the final result. As it was mentioned previously, the labour income determination can be extremely complicated in the presence of wealth heterogeneity. Nevertheless, it is true that changes to the wage negotiation process and its sensitivity to different UI provision levels can be very important. The present work argues that the aforementioned wage rule strikes a reasonable compromise between modelling tractability and economic intuition. At the same time, it advocates that it is the combination of both - savings and the specific wage determination - that leads to the results reported. Table 1.4 illustrates exactly this point; I maintain the assumption that agents are precautionary savers but this time a vary exogenously

the wage to show that the UI optimality and the associated welfare benefits do not depend *only* on the agents' buffer stock of assets. My benchmark scenario remains unchanged but now the wage increases exogenously as UI benefits rise. Initially (Scenario 1 in Table 1.4), I increase the wage by a constant 5% with every 0.1 increase in the unemployment benefit. This enables employed individuals to increase their consumption and as a result the certainty equivalence of consumption rises monotonically with UI. In the second case (Scenario 2), I allow for a concave increase in the wage profile, something that initially boosts consumption of employed considerably. This time however, the gains from higher wages diminish and are offset by the losses due to higher unemployment. As a result, the benchmark case is the optimal one. Together, these results outline the fact that it is the joint contribution of savings and the wage process that matters for the determination of the optimal unemployment insurance program.

#### 1.4. Risk aversion

In this section, I check the robustness of the previous findings by raising the coefficient of risk aversion to 2.4. I re-calibrate each model in order to attain approximately the same targeted values as before, so I set  $c$  equal to 0.65 and  $\Theta$  equal to 0.7 for the no-savings scenario while the same parameters are assigned the values of 0.18 and 0.92 respectively for the model with savings.

Results for the first case are reported in Table 1.5. Given that agents are now more risk averse, they are willing to negotiate a lower wage at each level



of unemployment insurance and hence their average consumption is reduced. The path of equilibrium labour income is still increasing however, because benefits affect the relative values of the two states. Without any means of self-insurance, a higher degree of risk aversion induces agents to want more state provided assistance than before. Hence, the optimal unemployment benefit is found to be equal to 0.4 where the implied net replacement rate is 64.66 *percent* (an increase of around 35 *percent* compared with the previous case). Understandably, unemployment insurance is more welfare improving than before. Changing benefits from the baseline system to the optimal one generates an increase in the certainty equivalence of consumption equal to 0.52 *percent*. In the absence of savings, these numbers reveal how sensitive our conclusions can be to the choice of the degree of risk aversion. These findings are in line with previous relevant studies that establish a positive relationship between risk aversion and demand for state provided insurance [see, for instance, Davidson and Woodbury (1998)].

I now turn to the case with savings, in an attempt to see if these conclusions carry over to the richer model. Table 1.6 reports individual and aggregate results when the degree of risk aversion is equal to 2.4. The most apparent feature of the simulated data is that individual savings increase considerably. In the current framework, an increase of risk aversion is equivalent to an increase in prudence. Hence, more prudent agents decide optimally to save more. Apart from the extreme case of  $z = 0.5$ , assets are on average around 55 *percent* higher than in the baseline model and as a result consumption drops are dramatically smaller at all levels of

UI. Following the increase in asset accumulation, the equilibrium wage income turns out to be approximately the same as the one negotiated with a lower degree of risk aversion. All though *per se* risk averse individuals would be willing to accept a lower wage, the larger buffer stock of assets improves their position in the bargain. Hence, the equilibrium wage falls on average by a very small 0.05 *percent* compared to a 2.65 *percent* in the no-savings scenario.

As before, the last row of Table 1.6, reports the welfare gains of UI. Despite the increase in risk aversion, the optimal unemployment benefit is found to be equal to 0 and the aggregate welfare benefits of moving from the baseline replacement rate to the optimal one are still non-trivial (0.99 *percent*). This contrasts with the previous findings and with most of the existing theoretical literature. This divergence can be attributed to two reasons: the increase in the amount of self-insurance and the endogenous matching rate. Since people accumulate more assets they become better insured against the unemployment hazard. At the same time, more risk averse agents are more adversely affected by the reduction in the aggregate matching rate. Consequently, unemployment insurance provision is still not a desirable feature, even for more risk averse individuals, at least on the aggregate.

### **1.5. Heterogeneity, wealth and unemployment insurance**

Our previous discussion suggests that savings work as an instrument that substitutes public insurance provision. The latter, embodied in a framework with an endogenous wage process resulted in a zero optimal UI, from an aggregate welfare

perspective. However, the present model is a model of heterogeneous agents in which each individual holds a different amount of assets. Some individuals may have been unlucky enough and were not able to build any stock of savings while others may have accumulated very few assets. One is tempted to wonder therefore, if the previous results vary among different wealth groups. If they do, then it would be interesting to see the implications for the optimal  $NRR$  and its associated welfare gains for each different group as well as determining the minimum amount of savings necessary to buffer completely the unemployment hazard. These issues are both intriguing and important because they are related to the uniform provision of the UI system and means testing.

In order to explore this area, I divide individuals to groups based on their wealth holdings (expressed as percentages of average savings). Following my previous practice, I vary the UI provision and calculate the mean of the certainty equivalence of consumption in each group. This enables me to identify the optimal replacement rate and the accompanied welfare gains of UI provision for each wealth group.

Table 1.7 summarizes results for six different savings groups, namely individuals who hold assets between 0%–10%, 10%–20%, 20%–30%, 30%–40%, 40%–50% and more than 50% of average wealth. For simplicity, I will refer to these groups as groups 1, 2, 3, 4, 5 and 6. The first row of Table 1.6 reports the optimal UI, the second the implied optimal  $NRR$  and the last one the average welfare gains obtained for each individual group when the unemployment benefit changes from

0 (which is the optimal policy based on aggregate welfare) to each group's optimal level.

The pattern is unambiguous. Both, the optimal UI provision and its welfare benefits fall with savings. Individuals who could not build a significant stock of assets require more insurance than richer individuals. More specifically, very poor agents (those that belong to group 1) would prefer a 46.88 replacement rate while those of group 2 would opt for a lower *NRR* equal to 31.25. Groups 3, 4 and 5 would see their welfare maximized if the replacement rate was equal to 15.63 but richer individuals would benefit if UI was set equal to zero. It is worth noting that the welfare gains of UI do not simply vary between groups that desire different replacement rates but vary between groups with the same optimal *NRR*. In terms of the certainty equivalence of consumption, group 3 benefits five times more than group 5 despite the fact that all these individuals would choose the same replacement rate.

Table 1.7 provides also information about the amount of savings necessary to buffer fully the unemployment risk. When an individual is able to accumulate assets greater than or equal to 50 *percent* of average wealth, then she is fully covered against the hazard of unemployment. When  $z = 0$  and the implied *NRR* = 0%, mean savings are equal to 1.6655. Therefore, assets equal to around 1.3 (or more) of the equilibrium monthly wage would be sufficient to guard against the unemployment risk without the need of additional state provided assistance. This result should be expected to vary however, with different unemployment durations.

Overall, this section outlines that heterogeneity (and thus the wealth distribution) does matter for the optimality of UI. The results are interesting because, after all, this is an economy populated by *ex ante* identical consumers who enjoy the same level of wage. Due to idiosyncratic labour income fluctuations and different employment histories however, not all individuals prefer the same amount of state provided unemployment insurance. Poor agents require non-trivial replacement rates while richer consumers would be better off with no additional insurance.

## 1.6. Conclusion

This paper has investigated the optimality and the accompanied welfare implications of unemployment insurance provision. Such a task was undertaken by incorporating the basic ingredients of the buffer stock saving model into the search theoretic approach to the labour market. The equilibrium wage was made explicitly dependant to the outside options of workers/searchers while the aggregate matching rate was endogenously determined. The resulting model was not analytically tractable so a numerical solution method had to be employed. Calibration was done on a monthly frequency since average US unemployment duration is short and any lower frequency would throw away too much information.

Savings together with the moral hazard effect and the wage profile across different UI benefits, seem to diminish greatly the importance of unemployment insurance provision. Based on an aggregate welfare metric, the optimal net replacement rate in the benchmark scenario was found to be equal to 0 *percent* with non-trivial

gains for consumption. A higher degree of risk aversion did not seem to have an effect on this finding, because more prudent individuals accumulate more assets and thus are well covered against the unemployment risk.

*Ex post* heterogeneity is important however. A common wage income is not enough to suggest a uniform UI system. Different employment histories and idiosyncratic income fluctuations lead individuals to accumulate different amounts of assets. This affects not only the desirability of UI provision but the associated welfare gains among different wealth groups.

Since savings are so important, modelling idiosyncratic labour income fluctuations more realistically - i.e. using an  $AR(1)$  process - might have implications for the optimality of unemployment insurance. It is not clear however, at which direction the results would point. Persistent shocks will negatively affect the ability of unlucky individuals to accumulate enough assets to buffer the unemployment risk, something which could indicate that some *UI* provision is beneficial. However, agents that recognize the persistent nature of the shocks might be inclined to save even more in good times, thereby increasing their stock of assets. The final outcome would depend on the net effect between these two opposing forces.

Future work could examine the case of multiple benefits being offered at the same time to different individuals based on their stocks of assets. Transitional dynamics might also be taken into account, similar in spirit to Joseph and Weitzblum (2003) and Lentz (2006). Finally, endogenizing the interest rate may be an

additional dimension worth exploring as this would affect the cost of savings and thus the distribution of wealth holdings.

## Appendix 1.A: The elasticity of the job finding rate

Define as  $e_q$  the elasticity of  $q$  with respect to market tightness, i.e.:

$$(1.A.1) \quad e_q = q \frac{\theta}{q}, e_q < 0$$

The elasticity of the average job finding rate ( $\bar{\mu} = \theta q$ ) is then given by:

$$(1.A.2) \quad e_\mu = [q + \theta q \theta] \frac{\theta}{\theta q} = 1 + e_q, e_\mu > 0$$

By defining:

$$(1.A.3) \quad \varepsilon = -e_q, \varepsilon > 0$$

it follows that the elasticity of the average job finding rate is given by:

$$(1.A.4) \quad e_\mu = 1 - \varepsilon$$

## Appendix 1.B: Solution algorithm

As discussed in the paper, there are two value functions depending on employment status ( $V^e, V^u$ ). These are reproduced here to facilitate the analysis that



follows:

$$(1.B.1) \quad V_{jt}^e(x_{jt}) = \max_{b_{jt+1}^e} \{U(x_{jt} - b_{jt}^e) + \beta[(1 - \delta)E_t V_{jt+1}^e(x_{jt+1}) + \delta E_t V_{jt+1}^u(x_{jt+1})]\}$$

$$V_{jt}^u(x_{jt}) = \max_{b_{jt+1}^u, s_{jt}} \{U(x_{jt} - b_{jt}^u) - G(s_{jt}) + \beta[\mu(s_{jt})E_t V_{jt+1}^e(x_{jt+1}) + (1 - \mu(s_{jt}))E_t V_{jt+1}^u(x_{jt+1})]\} \quad (1.B.2)$$

Combining the first order necessary condition with respect to savings, the two envelope conditions  $\{\frac{\partial V_{jt}^e}{\partial x_{jt}} = U_c(c_{jt}^e)$  and  $\frac{\partial V_{jt}^u}{\partial x_{jt}} = U_c(c_{jt}^u)\}$  and the possibility of a binding liquidity constraint, we can derive the first two Euler equations given by (1.34) and (1.35). The third Euler equation can be derived by differentiating (1.B.2) with respect to search intensity. One should note that because of the endogenous probabilities of finding employment in the next period, the value functions cannot be shown to be strictly concave [see Lentz and Tranæs (2004)] but the numerical solution (see below) yielded functions that were always globally concave.

Because of the max operators in the households' policy functions [equations (1.34) and (1.35)], the problem cannot be solved analytically. For this reason I proceed with numeric evaluation. The single state variable (cash on hand,  $x_t$ ) is discretized into a certain number of grid points (say 100) with more points at lower values of cash on hand where the value function is more curved and the policy functions will have a kink. With this discretization at hand the solution algorithm is as follows:

- 1.: Make an initial guess about  $u$ ,  $v$  (and hence  $\lambda$ ),  $\bar{\mu}$ ,  $\tau$  and  $\varepsilon$
- 2.: Make an initial guess about the wage rate  $w$
- 3.: Make an initial guess about the three policy and the two value functions
- 4.: Given the values in 1 and 2 as well as the conditions in 3, solve for savings when employed and unemployed and search using (1.34), (1.35) and (1.36) and the value functions using (1.14) and (1.15). Update at 3 until policy and value functions converge
- 5.: Using the converged policy and value functions and the values in 1 and 2, simulate over all households to obtain  $\bar{V}_i^e$  and  $\bar{V}_i^u$  and find the wage that solves (1.31).
- 6.: With the policy functions from 4 and the wage rate from 5, simulate again to obtain an update for  $u$ ,  $v$ ,  $\lambda$ ,  $\bar{\mu}$ ,  $\tau$  and  $\varepsilon$
- 7.: Check initial with updated values. If their difference is sufficiently close to zero stop. Otherwise go back to 1 and update the values. Keep iterating until all seven variables and all policy and value functions have converged.

Interpolations along the single continuous state variable are performed using cubic spline interpolation and the upper bound of cash on hand is found by a trial and error method that ensures simulated liquid assets never exceed the chosen upper bound for cash on hand. Simulations are carried over 2000 individuals, which is a sufficient number for the i.i.d. shock to wash out. The time periods in each simulation are set so that the economy reaches a stationary equilibrium. Finally, iterations are terminated when initial and updated variables have converged up

to the fourth digit. This makes comparative statics reliable for policy and welfare conclusions.

Table 1.1  
Benchmark parameter values

<i>Monthly frequency</i>			
Discount factor $\beta$	0.996	Scaling factor $\Theta$	1.3
Interest rate $r$	0.0016	Unempl. benefit ( $z$ )	0.3
Risk aversion $\rho$	1.2	Job destruction rate $\delta$	0.03
Disutility of search $\gamma$	2	Productivity $p$	1
<i>s.d.</i> of income uncertainty $\sigma_\epsilon$	0.05	Maintenance cost $c$	0.2
Elast. of the matching rate coef. $\alpha$	0.5	Capital cost $k$	0.35

Notes to Table 1.1: The benchmark value of  $z$  was set equal to 0.3 because in the simulations it yielded a *NRR* close to our targeted value of 50 percent.

Table 1.2  
Benchmark results, no savings

*Monthly frequency*

	Unemployment benefit					
	0.0	0.1	0.2	0.3	0.4	0.5
<b>Individual results</b>						
Mean indiv. consumption (empl.)	0.6195	0.6199	0.6164	<i>0.6102</i>	0.5995	0.5748
Mean indiv. consumption (unempl.)	0.0000	0.0993	0.1968	<i>0.2916</i>	0.3811	0.4562
Mean indiv. consumption drop (%)	100.00	83.98	68.07	<i>52.22</i>	36.42	20.64
<b>Aggregate results</b>						
Wage	0.6207	0.6242	0.6264	<i>0.6279</i>	0.6292	0.6301
Implied net repl. rate (%)	0.00	16.02	31.93	<i>47.78</i>	63.57	79.35
Average search	1.0370	0.8840	0.7549	<i>0.6469</i>	0.5367	0.4007
Unemployment rate (%)	3.62	4.10	4.82	<i>5.70</i>	7.22	10.80
Vacancy rate (%)	5.98	5.24	4.82	<i>4.46</i>	4.13	3.81
Aggregate matching rate	1.5757	1.3848	1.2245	<i>1.0832</i>	0.9265	0.7284
Average job finding rate (%)	80.47	70.59	60.33	<i>50.42</i>	39.19	25.32
Tax rate (%)	0.00	0.69	1.59	<i>2.81</i>	4.71	8.77
Welfare gain (%)	10.58	3.67	0.87	<i>0.00</i>	0.37	3.44

Notes to Table 1.2: The unemployment benefit is increased by the discrete amount of 0.1 each time. The case of zero UI is actually approximated by a benefit of 0.03 in order

to ensure that the code is successfully solved numerically (since the possibility of a zero consumption would imply an infinite marginal utility). This practice is implemented across all models and all simulations. Welfare gains measure the percentage change in the certainty equivalence of consumption by moving from each level of unemployment insurance to the optimal one.

Table 1.3  
Benchmark results, savings

*Monthly frequency*

	Unemployment benefit					
	0.0	0.1	0.2	0.3	0.4	0.5
<b>Individual results</b>						
Mean indiv. consumption (empl.)	<i>0.6179</i>	0.6163	0.6138	0.6104	0.6043	0.5912
Mean indiv. consumption (unempl.)	<i>0.5265</i>	0.5241	0.5214	0.5194	0.5174	0.5196
Mean indiv. savings (empl.)	<i>1.8106</i>	1.5548	1.1520	0.7955	0.4722	0.2138
Mean indiv. savings (unempl.)	<i>1.1663</i>	0.9728	0.6745	0.4256	0.2190	0.0951
Mean indiv. consumption drop (%)	<i>14.78</i>	14.95	15.06	14.90	14.38	12.10
<b>Aggregate results</b>						
Wage	<i>0.6399</i>	0.6400	0.6400	0.6399	0.6397	0.6392
Implied net repl. rate (%)	<i>0.00</i>	15.62	31.25	46.88	62.48	78.16
Average search	<i>0.7501</i>	0.7105	0.6578	0.5928	0.5117	0.3995
Unemployment rate (%)	<i>4.63</i>	4.87	5.21	5.70	6.62	8.57
Vacancy rate (%)	<i>4.54</i>	4.52	4.53	4.55	4.64	4.71
Aggregate matching rate	<i>1.2980</i>	1.2624	1.2231	1.1698	1.0965	0.9708
Average job finding rate (%)	<i>62.08</i>	59.16	55.21	50.00	42.91	32.15
Tax rate (%)	<i>0.00</i>	0.08	1.69	2.76	4.24	6.84
Welfare gain (%)	<i>0.00</i>	0.12	0.37	0.76	1.55	3.50

Notes to Table 1.3: The unemployment benefit is increased by the discrete amount of 0.1 each time. The case of zero UI is actually approximated by a benefit of 0.03 in order to ensure that the code is successfully solved numerically (since the possibility of a zero consumption would imply an infinite marginal utility). This practice is implemented across all models and all simulations. Welfare gains measure the percentage change in the certainty equivalence of consumption by moving from each level of unemployment insurance to the optimal one.



Table 1.4  
 Optimal UI, savings and exogenous wage

<i>Monthly frequency</i>						
Unemployment benefit						
	<b>0.0</b>	<b>0.1</b>	<b>0.2</b>	<b>0.3</b>	<b>0.4</b>	<b>0.5</b>
Scenario 1						
Wage	0.5527	0.5804	0.6094	0.6399	0.6719	<i>0.7055</i>
Welfare gain (%)	16.67	11.50	6.82	4.83	2.74	<i>0.00</i>
Scenario 2						
Wage	0.5689	0.5974	0.6213	<i>0.6399</i>	0.6527	0.6592
Welfare gain (%)	8.25	3.55	0.39	<i>0.00</i>	1.02	3.34

Notes to Table 1.4: The unemployment benefit is increased by the discrete amount of 0.1 each time. The case of zero UI is actually approximated by a benefit of 0.03 in order to ensure that the code is successfully solved numerically (since the possibility of a zero consumption would imply an infinite marginal utility). This practice is implemented across all models and all simulations. Welfare gains measure the percentage change in the certainty equivalence of consumption by moving from each level of unemployment insurance to the optimal one.

Table 1.5

 $\rho = 2.4$ , no savings*Monthly frequency*

	Unemployment benefit					
	0.0	0.1	0.2	0.3	0.4	0.5
<b>Individual results</b>						
Mean indiv. consumption (empl.)	0.5819	0.5996	0.6029	0.5985	<i>0.5863</i>	0.5520
Mean indiv. consumption (unempl.)	0.0000	0.0994	0.1970	0.2914	<i>0.3789</i>	0.4441
Mean indiv. consumption drop (%)	100.00	83.42	67.32	51.31	<i>35.36</i>	19.54
<b>Aggregate results</b>						
Wage	0.5829	0.6032	0.6120	0.6162	<i>0.6189</i>	0.6215
Implied net repl. rate (%)	0.00	16.58	32.68	48.68	<i>64.66</i>	80.45
Average search	2.5877	1.8596	1.4010	1.1016	<i>0.8506</i>	0.5961
Unemployment rate (%)	3.05	3.48	4.39	5.70	<i>7.91</i>	13.51
Vacancy rate (%)	9.54	6.59	5.35	4.72	<i>4.21</i>	3.64
Aggregate matching rate	1.2381	0.9631	0.7732	0.6366	<i>0.5104</i>	0.3633
Average job finding rate (%)	95.93	83.32	66.15	50.41	<i>35.22</i>	19.47
Tax rate (%)	0.00	0.60	1.48	2.86	<i>5.26</i>	11.17
Welfare gain (%)	107.26	20.36	4.62	0.52	0.00	4.32

Notes to Table 1.5: The unemployment benefit is increased by the discrete amount of 0.1 each time. The case of zero UI is actually approximated by a benefit of 0.03 in order

to ensure that the code is successfully solved numerically (since the possibility of a zero consumption would imply an infinite marginal utility). This practice is implemented across all models and all simulations. Welfare gains measure the percentage change in the certainty equivalence of consumption by moving from each level of unemployment insurance to the optimal one.

Table 1.6

 $\rho = 2.4$ , savings*Monthly frequency*

	Unemployment benefit					
	0.0	0.1	0.2	0.3	0.4	0.5
<b>Individual results</b>						
Mean indiv. consumption (empl.)	0.6159	0.6144	0.6127	0.6092	0.6034	0.5897
Mean indiv. consumption (unempl.)	0.5634	0.5605	0.5596	0.5553	0.5493	0.5440
Mean indiv. savings (empl.)	2.7064	2.4242	1.6880	1.1899	0.7466	0.4091
Mean indiv. savings (unempl.)	2.0528	1.8219	1.1922	0.7924	0.4523	0.2370
Mean indiv. consumption drop (%)	8.53	8.78	8.66	8.86	8.97	7.75
<b>Aggregate results</b>						
Wage	0.6397	0.6401	0.6406	0.6406	0.6406	0.6404
Implied net repl. rate (%)	0.00	15.62	31.22	46.83	62.44	0.7808
Average search	0.9723	0.9305	0.9141	0.8310	0.7236	0.5684
Unemployment rate (%)	4.75	5.05	5.24	5.70	6.61	8.70
Vacancy rate (%)	5.22	4.98	4.70	4.69	4.70	4.61
Aggregate matching rate	0.9645	0.9136	0.8708	0.8336	0.7758	0.6701
Average job finding rate (%)	60.59	57.13	54.73	49.89	42.93	31.66
Tax rate (%)	0.00	0.08	1.70	2.76	4.23	6.93
Welfare gain (%)	0.00	0.20	0.60	0.99	1.76	3.80

Notes to Table 1.6: The unemployment benefit is increased by the discrete amount of 0.1 each time. The case of zero UI is actually approximated by a benefit of 0.03 in order to ensure that the code is successfully solved numerically (since the possibility of a zero consumption would imply an infinite marginal utility). This practice is implemented across all models and all simulations. Welfare gains measure the percentage change in the certainty equivalence of consumption by moving from each level of unemployment insurance to the optimal one.

Table 1.7

## Optimal UI and savings

*Monthly frequency*

	<b>Grp 1</b>	<b>Grp 2</b>	<b>Grp 3</b>	<b>Grp 4</b>	<b>Grp 5</b>	<b>Grp 6</b>
Optimal UI	0.3	0.2	0.1	0.1	0.1	0.0
Implied net repl. rate (%)	46.88	31.25	15.63	15.63	15.63	0.0
Welfare gain (%)	0.67	0.19	0.05	0.02	0.01	0.0

Notes to Table 1.7: Welfare gains measure the percentage change in the certainty equivalence of consumption by moving from a 0 replacement rate (i.e. the optimal *NRR* in the benchmark model based on aggregate welfare) to the optimal one for each different wealth group.

Groups are arranged as follows:

Grp 1: individuals with savings less than 10 percent of mean savings

Grp 2: individuals with savings between 10 and 20 percent of mean savings

Grp 3: individuals with savings between 20 and 30 percent of mean savings

Grp 4: individuals with savings between 30 and 40 percent of mean savings

Grp 5: individuals with savings between 40 and 50 percent of mean savings

Grp 6: individuals with savings more than 50 percent of mean savings

Figure 1.1: Consumption policy function

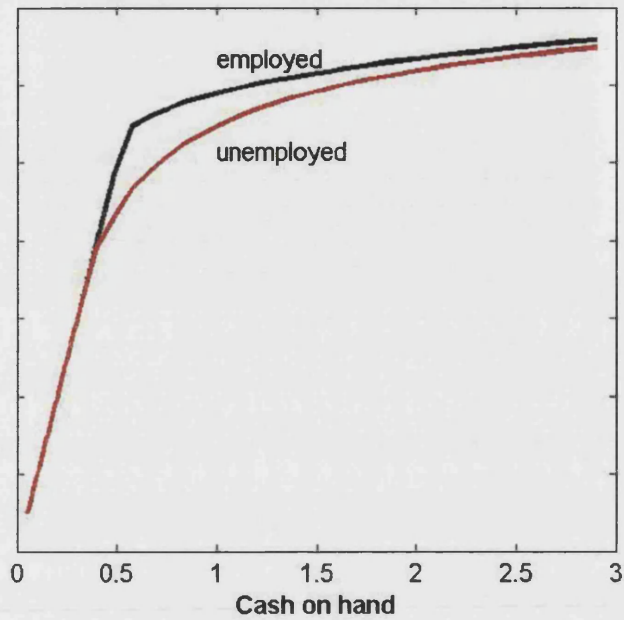
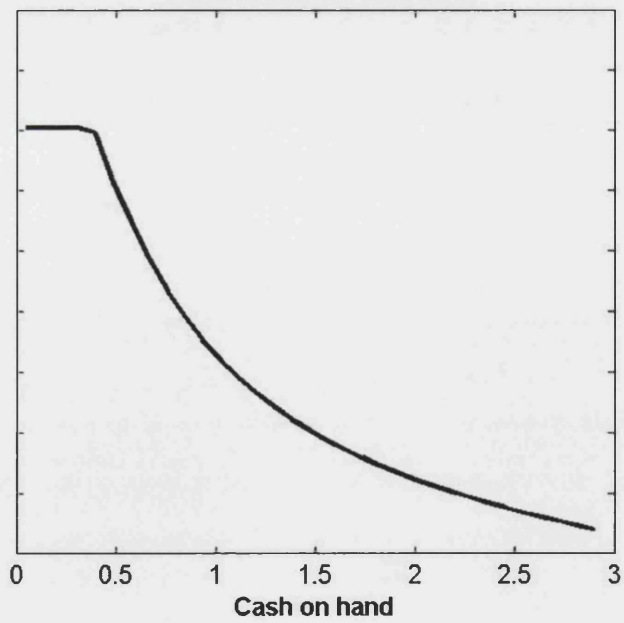


Figure 1.2: Probability of finding employment policy function



## CHAPTER 2

# Chapter Two: Precautionary Saving, Search and Incomplete Information<sup>1</sup>

### 2.1. Introduction

Following our previous discussion, the present chapter attempts to identify the combined dynamics of consumption and unemployment in the presence of search and saving. Based on the seminal work by Zeldes (1989), Deaton (1991) and Carroll (1992, 1997), precautionary savings models that feature liquidity constraints, undiversifiable labour income risk and some notion of impatience have been widely used to explain consumption dynamics<sup>2</sup>. At the same time, models featuring search frictions have been used to explain both the existence of equilibrium unemployment and unemployment dynamics. Although problems remain in explaining some business cycle phenomena [for example, the large variability in the vacancy-unemployment ratio, Shimer (2005)], the Mortensen-Pissarides (1994) framework has been extensively used to understand unemployment fluctuations [for instance, Cole and Rogerson, (1999)]. Nevertheless, few studies have combined

---

<sup>1</sup>Co-authored with Alex Michaelides, LSE.

<sup>2</sup>Hubbard, Skinner and Zeldes (1995), Attanasio, Banks, Meghir and Weber (1999), Gourinchas and Parker (2002) and Cagetti (2003) offer supporting evidence from microeconomic data for the life-cycle model.



the basic insights from the two approaches to jointly examine the implications for unemployment and consumption dynamics<sup>3</sup>.

We think that this provides a potentially important gap in the literature because it is hard to argue a priori that unemployment and consumption should be studied separately, especially when both can be determined (fully or partially) by endogenously selected instruments like saving and search.

In order to confirm this intuition, we document a stylized fact from aggregate data, namely that non-durables consumption growth is negatively related to unemployment growth over and above the excess sensitivity of consumption growth to labour income growth. Carroll and Dunn (1997) is an earlier study that is consistent with this finding but they focus on unemployment expectations from survey data rather than actual unemployment figures. This finding seems consistent with microeconomic evidence from, for instance, Gruber (1997), who finds that food consumption drops by around 7% during the unemployment spell.

Given these empirical results, we see the purpose of this chapter as twofold: firstly we are interested in building, simulating and aggregating a model that features two jointly determined instruments that guard against the income risk - namely saving and search - and then trying to understand any potentially interesting results from their interaction on the aggregate level. Secondly, we want to

---

<sup>3</sup>Hansen and Imrohoroglu (1992), Wang and Williamson (1996), Costain (1999) and Alvarez and Veracierto (2001) are some examples that do so but these papers do not explicitly incorporate undiversifiable, idiosyncratic labor income risk.

see whether our current framework is capable or not of explaining the empirical regularities that we observe in the data.

For these reasons, we embed search frictions in the precautionary savings model, thereby endogenously modeling consumption and unemployment. Recently, a number of papers has taken similar routes. Gomes, Greenwood and Rebelo (2001) investigate the joint comovement of unemployment and consumption in a general equilibrium setting but they do not endogenize the search intensity decision during the unemployment state and do not focus on the aggregate relationships documented in this chapter. Benitez-Silva (2000) solves a model similar to the microeconomic model we study and finds that search intensity is decreasing over the life-cycle (a prediction consistent with microeconomic evidence) but does not investigate the aggregate implications of the model. More recently, Lentz (2005) and Algan et. al. (2003) offer empirical evidence suggesting that wealthier households search less than poorer ones for a new job and therefore exhibit higher unemployment durations. Nevertheless, our understanding of how these non-linear microeconomic models aggregate and whether they can replicate the observed comovements between consumption and unemployment remains at a nascent stage. Our study takes a step in that direction by investigating how a precautionary savings and search model - with ex post heterogeneity - aggregates.

To this end and apart from attempting to explain the empirical regularities, we view our setup as a step towards enhancing our understanding from combining

precautionary savings and search models for aggregate fluctuations. We therefore abstract at this stage from general equilibrium considerations and take both wages and interest rates as exogenous. We make this choice because even the partial equilibrium model is quite complex and needs to be solved numerically and substantial comparative statics need to be performed in trying to understand the economic intuition and implications of the model. The theoretical setup extends the saving and liquidity constraints model proposed by Deaton (1991) and Carroll (1992, 1997) to accommodate search frictions and study the interaction between unemployment and consumption dynamics. We build a microeconomic model with heterogeneous agents to understand the model's comparative statics and then aggregate the model explicitly (avoiding the fallacy of composition<sup>4</sup> i.e. that what is true at the individual is also true at the composite level) to study the interaction between aggregate endogenous variables.

As far as the empirical phenomena are concerned, we are interested in seeing whether the richer framework that embeds search and savings can account for the excess smoothness of consumption growth (i.e. the fact that consumption growth is half as volatile as income growth), the excess sensitivity of consumption growth to lagged income and unemployment growth and the contemporaneous negative correlations between unemployment growth and income and consumption growth.

---

<sup>4</sup>Attanasio and Weber (1993) emphasize the importance of aggregation bias when testing economic theory. Their work can be viewed as emphasizing the potential problems raised by the fallacy of composition and the importance of aggregation.

Overall, we find that when households can distinguish aggregate from idiosyncratic job destruction shocks (i.e. when there is complete information), the model goes some way in explaining the observed patterns but is still poor in replicating the actual magnitudes. It nevertheless, does better than the model without any search. Introducing incomplete information improves our results. Optimal consumption and search intensity choices maintain the same shape as a function of wealth, yet the signal extraction problem does not allow the complete smoothing of unemployment shocks. As a result, the contemporaneous correlations between unemployment and both consumption and earnings growth become more negative than in the complete information model, moving the model's predictions closer in line with the empirical evidence. At the same time, consumption growth becomes smoother than earnings growth, and is closer to empirical magnitudes than the complete information model. Moreover, excess sensitivity of consumption growth to lagged labour income growth continues to persist at the macroeconomic level - in line with the empirical findings. However, the sensitivity of consumption growth to lagged unemployment growth cannot be robustly replicated in terms of statistical significance, even though the model does seem to approach this prediction more closely than the complete information specification. We view these simulation results as encouraging and suggest that future extensions might offer a better understanding of the joint comovement between consumption and unemployment over the business cycle.

This chapter is organized as follows. Section 2 uncovers a robust negative relationship between unemployment changes and consumption growth. Section 3 describes the economic environment and discusses the numerical solution method and the results for the complete information model. Section 4 analyzes the incomplete information model results and section 5 concludes.

## 2.2. The Consumption-Unemployment Relationship in the Data

In this section we demonstrate a robust negative correlation between consumption growth and unemployment growth<sup>5</sup>. So far the existing literature has established that consumption growth exhibits an excess sensitivity to lagged (or predictable) income growth, *in contrast* with what the permanent income hypothesis predicts. More specifically, consumption should not be expected to respond to predictable income changes because permanent income consumers should have revised their consumption by the time they knew that their income will change in the future. Following similar reasoning, the existence of unemployment risk should have made consumers build the appropriate (or at least a sufficient) stock of assets so that income fluctuations, due to unemployment spells, should not impact on consumption expenditure.

---

<sup>5</sup>The results and conclusions do not change qualitatively when trends are removed by an HP filter. This holds both for excess sensitivity regressions run with detrended variables and the relative smoothness ratios between consumption, unemployment and labor income. We chose not to report these results both due to space considerations and because we wanted to relate our empirical results with the “excess sensitivity” literature that performs the analysis in growth rates.

In our work, we use U.S. quarterly data<sup>6</sup> from 1959:01 to 2002:04. Aggregate real per capita consumption  $C_t$ , is measured as the sum of consumption of non-durables (excluding shoes and clothing) and services deflated by the chain-type price index of personal consumption expenditures. Real per capita after tax income is denoted as  $Y_t$  (see Appendix 2.A for the construction of the series) which is also deflated by the same price index. The construction of both series is in line with what is proposed by Blinder and Deaton (1985) and Pischke (1995), for instance. Table 2.1 summarizes the basic properties of the aggregate data. Specifically, consumption growth is less than half as volatile as labour income growth (with the relative standard deviation of consumption to labour income growth being around 0.4). The latter is another stylized fact that cannot be explained by the permanent income hypothesis and is referred to in the literature as the "*excess smoothness of consumption*" (in parallel with the term "*excess sensitivity*"). Finally, the unemployment rate growth exhibits higher volatility than labour income growth.

---

<sup>6</sup>Annual aggregate consumption data do not suffer from many of the problems that plague the construction of quarterly data (Wilcox, (1992)). Nevertheless, the relationship between unemployment and consumption is probably strongest even at the monthly frequency given the mean duration of unemployment. In section 3 we discuss in detail the choice of frequency for the model, which is related with the choice in the data.

### **2.2.1. OLS results**

Table 2.2 replicates the robust excess sensitivity of consumption growth to lagged labour income changes (Panel A), that has been widely documented in the empirical literature as violating the permanent income hypothesis. The point estimate from the regression is statistically significant at better than 5 percent level and equal to .109. Adding lagged unemployment growth as an additional determinant of consumption growth generates the results in Panel B. Both estimates are statistically significant at better than 5 percent with the unemployment coefficient being negative, implying that when the unemployment rate rises, future consumption growth is revised downwards. Moreover, the earnings excess sensitivity coefficient estimate is adjusted downwards indicating that part of the excess sensitivity to labour income could be due to omitting unemployment from the first regression. Overall, the *OLS* regressions indicate that consumption is sensitive to both labour income growth and unemployment changes.

### **2.2.2. Instrumental variables (IV)**

**2.2.2.1. Two-stage least squares (2SLS).** We next check the robustness of these correlations by estimating the response of consumption growth to expected labour income growth and expected unemployment growth, proxying expected values with actual contemporaneous growth rates and using instrumental variables techniques to estimate the relevant coefficients. This is essentially reproducing the

Campbell-Mankiw (1989) results, while extending the analysis to investigate the potential empirical impact of unemployment growth on consumption fluctuations.

Table 2.3 uses three different sets of instruments to first reproduce the Campbell-Mankiw (1989) results and then investigate the potential effect of expected unemployment on consumption growth. All instruments are lagged for two periods to avoid the potential effects of measurement error following an MA(1), a common route to tackle this problem in the literature. The estimates are highly significant and the signs of all coefficients are consistent with the *OLS* results. Specifically, omitting unemployment growth from the specification generates a coefficient on expected labour income growth equal to 0.44, while including unemployment growth reduces this coefficient to 0.236 and generates a coefficient on expected unemployment growth equal to  $-0.04$ , with both coefficients statistically significant at the 5% level. These conclusions are robust to alternative instrument specifications and are reported in Table 2.3. Moreover, the overidentifying restrictions test (last column of Table 2.3) indicates that the model cannot be rejected at better than 5 percent level of significance.

**2.2.2.2. Robustness to Weak IVs.** Two requirements must be satisfied for IV estimation to be unbiased and produce reliable inference: instrument exogeneity and instrument relevance. Exogeneity refers to the instruments being orthogonal to the error term while relevance dictates that instruments must be “significantly” related with the endogenous regressors. Instruments that are weakly correlated with what they are instrumenting can produce biased estimates and the first order



asymptotics may be a poor guide for their actual distribution. As a result, standard statistical inference can be unreliable.

We first test all three sets of instruments (with the null being that the instruments are weak) using the minimum eigenvalue of the matrix analog of the  $F$ -statistic (Stock and Yogo, 2003). Both in terms of  $2SLS$  bias and  $2SLS$  size distortion, there seems to be no evidence to reject the null of weak IVs<sup>7</sup>. For this reason we compute three fully robust Gaussian tests when instruments are weak. These statistics can test for the joint significance of the estimates in the presence of weak instruments. This is due to the fact that all statistics have well defined asymptotics which do not depend on instrument relevance. In what follows, we abstract from explicitly reviewing the tests and refer the reader to the relevant papers below.

We start by employing the *Anderson – Rubin (AR)* statistic, due to Anderson and Rubin (1949) and Moreira (2001). Under the general conditions of weak instruments  $AR \xrightarrow{d} \chi_K^2/K$  (where  $K$  is the number of  $IV$ 's) irrespective of instrument relevance. Our null hypothesis is that both endogenous variables are statistically insignificant. Table 1.4 summarizes the results of the  $AR$  test. With four instruments the test suggests that income and unemployment rate growth are jointly significant at the 5 percent level and adding more instruments does not alter this conclusion. We next report results from the Kleibergen statistic (Kleibergen, 2001) which rejects the null for all instrument sets at the 5% level. Moreover, the

---

<sup>7</sup>Results not reported for space considerations.

$LR$  statistic provides results along these lines rejecting the null at the 5 percent level of significance for all instrument sets.

Taken together, these robust inference tests suggest that the estimates are jointly significant despite the presence of weak instrumental variables, in contrast with the predictions of the permanent income hypothesis.

### 2.3. The Complete Information Model

We now proceed by building our basic model. Typically, heterogeneous agent consumption models with undiversifiable labour income risk and occasionally binding liquidity constraints are solved at an annual frequency. Although there is no strong a priori reason for this choice, a number of potential explanations do exist. One possibility arises from the fact that most household level data (like the Panel Study of Income Dynamics - *PSID*) collect information on household labour income at an annual frequency. Given that in theoretical models the labour income process usually needs to be specified in advance to derive the consumption implications, researchers write down and solve models at an annual frequency for which labour income dynamics can be based on observed outcomes. This will not work in our case for two reasons: firstly because we want to study high frequency interactions between consumption and unemployment and secondly - and most importantly - the typical duration of an unemployment spell in the U.S. is roughly a quarter. On the other end, even though good unemployment data at the monthly frequency exist and are readily available from the BLS, the same does not hold for

aggregate consumption data. In fact, we know from Wilcox (1992) that aggregate consumption data suffer from serious problems even at the quarterly frequency. We think that solving a monthly model and time aggregating to a quarterly frequency strikes a reasonable compromise between the measurement error inherent in aggregate consumption data and the rich time series dynamics exhibited empirically by unemployment (and the theoretical search model) at the monthly frequency. Equivalently, we think that studying the implications of a search model at an annual frequency will neither be intuitively appealing nor empirically plausible since most households find a new job after an unemployment spell of around one quarter. We next detail the specific assumptions we make.

### 2.3.1. Labour Income

Labour income risk is undiversifiable because of moral hazard and adverse selection considerations, and it cannot be ignored by households. When employed, we assume that labour income of household  $i$  follows:

$$(2.1) \quad Y_{it} = P_{it}U_{it}U_t$$

where

$$(2.2) \quad P_{it} = G_t P_{it-1} N_{it}$$

This process is decomposed into a permanent,  $P_{it}$ , and a transitory idiosyncratic component,  $U_{it}$ , and a transitory aggregate component,  $U_t$ . We assume that  $\ln U_{it}$  and  $\ln N_{it}$  are each independent and identically (normally) distributed with zero means and variances  $\sigma_u^2$  and  $\sigma_n^2$ , respectively. The log of  $P_{it}$  evolves as a random walk with a stochastic drift assumed to be common to all individuals (the aggregate shock). This stochastic drift,  $\ln G_t$  is assumed to be normally distributed with mean  $\mu_g$  and variance  $\sigma_g$ . In addition to the permanent and transitory idiosyncratic components ( $P_{it}$  and  $U_{it}$ ), we also assume that there is a transitory aggregate component ( $U_t$ ) in the labour income process that is log-normally distributed with mean  $-0.5\sigma_{ug}^2$  and variance  $\sigma_{ug}^2$ . We make this assumption because it is empirically defensible for a small variance of  $\sigma_{ug}^2$  (as we assume) and it will be shown to generate more smoothness in aggregate consumption through its transitory nature. Given these assumptions, the growth in individual labour income follows

$$(2.3) \quad \Delta \ln Y_{it} = \ln G_t + \ln N_{it} + \ln U_{it} - \ln U_{it-1} + \ln U_t - \ln U_{t-1}$$

where the unconditional mean growth for individual earnings is  $\mu_g$ , and the unconditional variance equals  $(\sigma_g^2 + \sigma_n^2 + 2\sigma_u^2 + 2\sigma_{ug}^2)$ . Individual earnings growth in (2.3) has a single Wold representation that is equivalent to the MA(1) process for individual earnings growth estimated using household level data (Abowd and Card [1989], MaCurdy (1981) and Pischke [1995]). Moreover, the individual earnings growth is negatively serially correlated while aggregate shocks have a small

(statistically insignificant) negative serial correlation with the exact value of this correlation depending on  $\sigma_{ug}^2$ . We calibrate these values to match the properties of the constructed aggregate labour income process<sup>8</sup>. Furthermore, aggregate shocks are a small component of the total volatility in individual wage growth, consistent with the small magnitude of time effects in earnings regressions<sup>9</sup>.

### 2.3.2. Job Destruction

There is an exogenous probability that the individual will become unemployed in a particular period. We denote this probability by  $\delta_{it}$  (the job destruction rate) and as can be directly observed by the subscripts used it has an idiosyncratic nature. During unemployment, the individual has access to unemployment insurance which is a constant fraction ( $\omega$ ) of labour income:  $ub_{it} = \omega Y_{it}$  and the duration of this benefit is assumed to last as long as the agent is unemployed. Given the lower mean wage received in this state, individuals will want to start working so that unemployment duration does not persist for long periods in equilibrium.

---

<sup>8</sup>In our quarterly data aggregate earnings growth has a first order autocorrelation equal to  $-0.06$  and the standard deviation equals  $0.022$  (at an annual frequency). In our baseline specification, we use  $\sigma_{ug} = 0.005$  which implies that the first order autocorrelation in the simulated aggregate series at a quarterly frequency will be  $\frac{-\sigma_{ug}^2}{\sigma_g^2/4 + 2\sigma_{ug}^2} = \frac{-0.005^2}{0.02^2/4 + 2*0.005^2} = -0.06$  while the total standard deviation of the aggregate shock will be  $\sqrt{0.02^2 + 2 * 0.005^2} = .021$ .

<sup>9</sup>More recently, the variance of earnings shocks has received attention in microeconomic work. Specifically, Meghir and Pistaferri (2004) argue that the variance of earnings shocks is serially correlated while Storesletten, Telmer and Yaron (2004) argue that the variance of earnings shocks rises in downturns. We abstract from these more complicated specifications in this paper. In earlier work, Quah (1990) has shown that decompositions of aggregate labor income shocks exist that can explain the observed smoothness of consumption. This might also be true for the unemployment process but we do not explore the implications of this assumption in this paper.

There is an aggregate and an idiosyncratic component to job destruction rates. The aggregate component is exogenously assumed to have the same properties as the monthly, recently compiled, process from JOLTS (Job Openings and labour Turnover Survey) compiled by the Bureau of labour Statistics<sup>10</sup>. Empirically, the job destruction rate is a very persistent process [with an AR(1) coefficient of 0.75] and we therefore use this specification as an exogenous input in the model. Moreover, job destruction rates are known to be countercyclical in the data (see, for example, Hall (2004) and Shimer (2005)). We therefore introduce a negative correlation between the aggregate innovation in the job destruction rate and the aggregate earnings shock but we perform robustness checks by varying these parameters. The idiosyncratic component will simply reflect the very high volatility of job destruction observed at the individual, relative to the aggregate level (Davis, Haltiwanger and Schuh (1996)).

To closely follow these restrictions, we model the log of the exogenous job destruction probability as the sum of an aggregate AR(1) component given by

$$\ln \delta_t = \varkappa + \phi(\ln \delta_{t-1} - \mu) + \ln \epsilon_t$$

---

<sup>10</sup>Details can be found at <http://www.bls.gov/jlt/home.htm#overview>. We use the total non-farm separations rate for calibrating the aggregate component of the job destruction rate.

and an i.i.d idiosyncratic component given by a log-normally distributed variable  $\zeta_{it}$  with mean  $-0.5\sigma_\zeta^2$  and variance  $\sigma_\zeta^2$ . The probability of exogenous job destruction is then given by<sup>11</sup>

$$\delta_{it} = \exp(\ln \delta_t + \ln \zeta_{it})$$

The negative correlation between  $\ln \epsilon_t$  and  $\ln G_t$  captures the countercyclicality of the aggregate component of the job destruction rate and is denoted by  $\rho_{eg}$  while the total standard deviation of  $\ln \delta_t$  is denoted by  $\sigma_\delta$ .

### 2.3.3. Search Effort

When an individual is unemployed, she will have to decide how much to search in order to find a job. In this we follow Lentz (2005) and assume that the probability of finding a job in period  $t + 1$  is a positive function of the search effort ( $s_{it}$ ):

$$(2.4) \quad \mu(\lambda s_{it}) = 1 - \exp(-\lambda s_{it})$$

The latter is strictly increasing and concave in  $s_{it}$  and  $\lambda$  is now an exogenous parameter that controls the offer arrival rate. In a general equilibrium framework  $\lambda$  would be endogenously determined and would reflect market tightness (as it was discussed in the previous chapter).

---

<sup>11</sup>The probability is always between zero and one without any further transformation given the empirically-based choices for the parameters describing job destruction.

### 2.3.4. Model

We consider the problem of a household that maximizes expected intertemporal utility

$$(2.5) \quad \text{MAX}_{\{B_t, s_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t) v(\eta_t s_t)$$

subject to

$$(2.6) \quad C_t + B_t \leq X_t$$

$$(2.7) \quad X_{t+1} = (1 + r)B_t + (1 - \eta_{t+1})Y_{t+1} + \eta_{t+1}\omega Y_{t+1}$$

$$(2.8) \quad C_t \geq 0$$

$$(2.9) \quad B_t \geq 0$$

$$(2.10) \quad s_t \geq 0$$

As previously, all variables are in real terms.  $B_t$  is the real amount of the riskless asset (bonds) which is held between the beginning of period  $t$  and the



beginning of period  $t + 1$ .  $E_t$  is the mathematical expectation operator and  $\beta$  is the discount factor that satisfies  $0 < \beta < 1$ .  $U(C_t)$  is the utility function at time  $t$ ,  $X_t$  is cash on hand at the beginning of period  $t$ ,  $r$  is the riskless rate which is assumed time-invariant, and  $Y_t$  is labour income received at the beginning of period  $t$ .  $\eta_t \in \{0, 1\}$  denotes the state of employment at time  $t$ , where  $\eta = 1$  stands for unemployed and  $\eta = 0$  for employed. Finally  $\omega$  is the replacement ratio.

The budget constraint (2.6) will hold with equality, given the assumption of non-satiation. We assume that the utility function is given by:

$$(2.11) \quad U(C_t, s_t) = \frac{C_t^{1-\rho}}{1-\rho} \exp[(1-\rho)(-s_t)], \quad \rho \neq 1, \quad \rho > 1$$

The utility specification differs from that of the previous chapter and recent empirical studies of search intensity [Lentz (2005), Algan et. al. (2003)] but has been used by other authors [for instance, Meghir, Low and Pistaferri (2004)]. We work with this specification because it allows the introduction of steady state growth in the economy, a feature of preferences that is of pivotal importance in the study of business cycle dynamics since developed economies in general (and the U.S. in particular) exhibit steady state growth (even though the growth rate might differ across countries and for the same country over time)<sup>12</sup>. Given that further extensions of the model to study business cycles need to admit balanced growth paths, we take the view that the utility specification needs to be carefully chosen to

---

<sup>12</sup>King, Plosser and Rebelo (1988) emphasize this point in the context of real business cycle models. This specification of search in the utility function also ensures that the standard properties of the utility function are respected:  $U_C > 0$ ,  $U_{CC} < 0$ ,  $U_s < 0$  and  $U_{ss} > 0$ .

both admit balanced growth paths and preserve the precautionary savings motive and we feel this utility function is a reasonable choice. In addition, it makes our analysis tractable as it allows for the normalization of the problem (see below).

### 2.3.5. Calibration of Parameters

We consider the unit of analysis to be one month and set  $\beta$  equal to  $\frac{1}{1+0.05/12}$ , and the constant real interest rate,  $r$ , equal to  $0.02/12$ . Carroll (1992) estimates the variances of the idiosyncratic shocks using data from the *PSID*, and our benchmark simulations use values close to those: 10 percent per quarter for  $\sigma_u$  and  $8/\sqrt{12}$  percent for  $\sigma_n$ . We set  $\sigma_g$  equal to  $0.02/\sqrt{12}$ , and  $\mu_g$  equal to  $0.02/12$ . Based on the monthly JOLTS data, the standard deviation of the logarithm of aggregate job destruction is set at 0.06 and the first order autoregressive coefficient ( $\phi$ ) equal to 0.75 with mean at  $-3.45$ . This generates a mean job destruction rate close to 3%, consistent with Shimer (2005). There is no clean way of picking the correlation between the aggregate earnings shock and the job destruction rate because the aggregate earnings data are at a quarterly frequency and we need this parameter at a monthly frequency. We therefore experiment and report results for different values of this parameter, bearing in mind the range of parameters reported by Hall (2004) and Shimer (2005); Hall (2004) uses a strongly countercyclical process while Shimer (2005) uses a slightly acyclical one. We therefore use a benchmark correlation equal to  $-0.7$  and also report results for a correlation equal to zero.

Finally, we set the replacement rate during unemployment equal to 60% (but we also report results for the 50% case).

We pick two parameters to generate two predictions from the model. Specifically, we pick the benchmark coefficient of relative risk aversion so that consumption during unemployment is on average 7 – 10 percent lower than during employment (consistent with Gruber (1997)). Moreover, because the model is supposed to capture the behavior of people facing unemployment shocks (that is, relatively poorer people in the economy), we do not want high asset accumulation. A low coefficient of relative risk aversion achieves both goals, and we set  $\rho = 1.2$  to achieve these aims. Moreover, we adjust the parameter through which search intensity affects the probability of finding a job when unemployed ( $\lambda$ ) to generate in simulated data an unemployment rate close to 6 percent and an average probability of finding employment after one month of around 50 percent [consistent with data from the BLS, as reported by Shimer (2005)]. This results in a value for  $\lambda$  of 1.0.

### 2.3.6. Solution Method

We generalize the Deaton (1991) solution to allow for search during the unemployment state by deriving three Euler equations associated with the three control variables. Letting  $U_C$  denote the marginal utility of consumption and  $V^e, V^u$  the

value of being employed and unemployed respectively, the three Euler equations are given by (see Appendix 2.B for further details)<sup>13</sup>:

(2.12)

$$U_C(C_{it}^e) = \text{MAX}\{U_C(X_{it}), \beta(1+r)E_t[(1-\delta_{it+1})U_C(C_{it+1}^e) + \delta_{it+1}U_C(C_{it+1}^u)]\}$$

$$(2.13) \quad U_C(C_{it}^u) = \text{MAX}\{U_C(X_{it}), \beta(1+r)[\mu(\lambda s_{it})E_t U_C(C_{it+1}^e) + (1-\mu(\lambda s_{it}))E_t U_C(C_{it+1}^u)]\}$$

and

$$(2.14) \quad U_s(C_{it}^u) = \beta\mu'(\lambda s_{it})E_t[V_{it+1}^e - V_{it+1}^u]$$

The three Euler equations were discussed extensively in the previous chapter, hence no further comments will be added here.

Given the non-stationary process followed by labour income, we normalize asset holdings and cash on hand by the permanent component of earnings  $P_{it}$ , denoting the normalized variables by lower case letters (Carroll, 1992)<sup>14</sup>. Defining

<sup>13</sup>We let  $E_t$  denote the expectation conditional on information at time  $t$  but we exclude the uncertainty about the next period employment status in this notation to illustrate how the Euler equations generalize explicitly from the case where the agent remains always employed.

<sup>14</sup>The ability to normalize this specification of utility and derive aggregate implications is vital if the business cycle properties of the model are to be further studied in richer settings.

$Z_{t+1} = \frac{P_{t+1}}{P_t}$ , taking advantage of the homogeneity of degree  $(-\rho)$  of marginal utility implied by CRRA preferences, and using the identity  $c_{it+1}^e = x_{it+1} - b_{it+1}^e$  (see Appendix 2.B for the proposed numerical algorithm), we have

$$(2.15) \quad U_c(x_{it} - b_{it}^e) = \text{MAX}\{U_c(x_{it}), \beta(1+r)E_t Z_{t+1}^{-\rho} [(1 - \delta_{t+1})U_c(c_{it+1}^e) + \delta_{t+1}U_c(c_{it+1}^u)]\}$$

$$(2.16) \quad \begin{aligned} U_s(x_{it} - b_{it}^u) = & \text{MAX}\{U_c(x_{it}), \beta(1+r)E_t Z_{t+1}^{-\rho} [\mu(\lambda s_{it})U_c(c_{it+1}^e) \\ & + (1 - \mu(\lambda s_{it}))U_c(c_{it+1}^u)]\} \end{aligned}$$

and

$$U_s(x_{it} - b_{it}^u) = \beta\mu'(\lambda s_{it})E_t[Z_{t+1}^{1-\rho}[V_{it+1}^e - V_{it+1}^u]]$$

The normalized state variable  $x$  evolves differently according to whether the individual is unemployed or not in the current period and the next. The four different possibilities are:

$$(2.17) \quad x_{it+1}^{e,e} = b_{it}^e(1+r)Z_{t+1}^{-1} + U_{t+1}U_{it+1}$$

$$(2.18) \quad x_{it+1}^{e,u} = b_{it}^e(1+r)Z_{t+1}^{-1} + \omega U_{t+1}U_{it+1}$$

$$(2.19) \quad x_{it+1}^{u,u} = b_{it}^u(1+r)Z_{t+1}^{-1} + \omega U_{t+1}U_{it+1}$$

$$(2.20) \quad x_{it+1}^{u,e} = b_{it}^u(1+r)Z_{t+1}^{-1} + U_{t+1}U_{it+1}$$

### 2.3.7. Benchmark Model Results

The consumption policy functions are plotted in figure 2.1 and the search intensity (expressed as the probability of finding a job when unemployed) in figure 2.2. Saving is zero when the constraints are binding and increases with cash on hand beyond a certain point generating a concave consumption function. The relevant range of the consumption functions for the unemployed is around mean earnings during employment (one) and in that region either the liquidity constraint is binding or consumption is lower than consumption during employment (figure 2.1). This is corroborated from the simulation results reported later.

Search intensity is increasing in wealth for low values of cash on hand and decreases beyond a certain level of cash on hand (figure 2.2). As wealth increases, the expected value of being employed relative to being unemployed shrinks (the difference in the expected value functions on the right hand side in the third Euler equation, (2.14)). Thus, search incentives are decreasing in wealth beyond a certain level and richer agents can afford longer unemployment spells. For the

levels of wealth that the liquidity constraint is binding, on the other hand, the marginal utility of consumption is very high and the only dimension/margin of precautionary saving activity remains the intensity of search. As a result, search intensity increases over low wealth levels (specifically, over wealth levels that the liquidity constraint remains binding during the unemployment state) and decreases beyond that level of cash on hand that generates positive saving. Moreover, in the presence of persistent job destruction shocks, search intensity depends on the current realization of job destruction. A high job destruction realization that is expected to persist for a number of periods reduces the attractiveness of finding a job and generates a lower search intensity for a given level of cash on hand (figure 2.2).

Figure 2.3 illustrates what happens when the unemployment insurance system becomes more generous by increasing the replacement ratio from 0.6 to 0.8. Search intensity decreases for a given level of cash on hand, while maintaining the same non-monotonic shape as a function of wealth.

### **2.3.8. Time Series Analysis**

We simulate the model at a monthly frequency and report both individual and aggregate statistics. Individual statistics are reported at a monthly frequency to understand the predictions of the microeconomic model for consumer behavior.

Aggregate statistics, on the other hand, will be compared to the quarterly frequency stylized facts that were reported earlier. We therefore time-aggregate individual statistics to a quarterly frequency by either adding up monthly values (for consumption and earnings, for instance) or picking the third value in the monthly simulations to correspond to the quarter (in the case of unemployment for example). The procedure is done over the number of monthly frequency observations in the sample, with a certain number of initial simulations (100) discarded from the statistics to allow the economy to reach its steady state (this typically takes place quickly, after around 20 – 30 months). Simulations are performed over 2000 individuals and are averaged over 100 Monte Carlo draws reducing substantially simulation bias in the reported statistics.

**2.3.8.1. Individual Statistics.** Table 2.5 produces average time series statistics from tracking different individual histories over time. The first column reproduces results for the case without any unemployment risk (the Deaton (1991) model).<sup>15</sup> Impatient households consume on average their mean labour income and save around 23% of their mean earnings. The standard deviation of individual consumption growth is substantially lower than the standard deviation of labour income growth (0.037 and 0.144 respectively) as households can buffer very well their labour income shocks with a small amount of savings. This shows that even

---

<sup>15</sup>The results are very similar to the Carroll (1997) model in this column but we impose the constraint explicitly which might lead to some differences on the level of the consumption function depending on expected benefits during unemployment.



the model without search and unemployment risk can generate excess consumption smoothness *at the microeconomic level*.

Comparing our benchmark specification to these results, we note that saving rises substantially in the presence of unemployment risk but the mean level of consumption is only 2.4% lower relative to the no-unemployment case. Consumption drops when moving from employment to unemployment and the drop is around 8%: the choice of risk aversion coefficient was guided to generate such a drop, consistent with the empirical results in Gruber (1997). Unconditional labour income growth becomes more volatile reflecting the lower benefits (wages) received during unemployment spells and the standard deviation of consumption growth also rises as a result but is still substantially smoother than labour income growth.

This substantial consumption smoothing is achieved by high accumulation of bonds (around a month's mean earnings during the employment state) on one hand and search effort on the other. The latter can ensure that the unemployment spell will not last for long periods of time and, therefore allows the individual to use more of her savings to smooth consumption effectively (of course, saving is higher when employed relative to when unemployed, reflecting the desire to smooth consumption across states). In effect, the individual is able to control *partially* how long she will stay unemployed and therefore can control more effectively the consumption-transition from the employment to the unemployment state.

Eliminating transitory aggregate shock uncertainty (setting  $\sigma_{ug} = 0$ ) leaves almost unaltered the microeconomic implications of the model since aggregate shocks

make up a small component of the total uncertainty faced by the household, and thus have a negligible impact on results. As a result, the microeconomic implications of the model remain almost unchanged by this. Moreover, the same conclusion for the same reason arises when the correlation between aggregate shocks and job destruction shocks is set equal to zero. Reducing the unemployment benefit, on the other hand, generates higher unconditional saving (from 0.946 to 1.24), while also increasing the search intensity during unemployment from 0.7 to 0.79 implying an increase in the probability of exiting unemployment within a month from 50.4% to 54.7%. Higher saving and search intensity imply that the higher labour income uncertainty does not translate to higher consumption uncertainty, generating a very smooth consumption growth series.

**2.3.8.2. Aggregate Statistics.** Table 2.6 reports the statistics from averaging individual consumption decisions in the cross section and then computing the time series statistics implied by the model at a quarterly frequency to be compared to the quarterly data. Here, we are interested in two things: firstly, we want to see the workings of the model on the aggregate level and secondly we are interested in identifying whether the search-saving framework can account for the excess smoothness and excess sensitivity of consumption growth at the macro level.

The first column reports the results from aggregating the Deaton (1991) model. Individual consumption smoothing does not survive the aggregation procedure since what is important for aggregate statistics is the reaction to the aggregate shock, not the smoothing of individual transitory shocks. The relative smoothness

ratio (defined as the ratio of the standard deviation of consumption to earnings growth) is 0.86, implying that some consumption smoothing does survive the aggregation procedure, but not to the extent that it can replicate the magnitudes observed in the data<sup>16</sup>. More specifically, permanent shocks are not smoothed and since aggregate shocks are mostly permanent this translates into a volatile consumption growth series at the macro level.

Almost, all the search models generate a smoother aggregate consumption growth series, a direct result of the higher saving taking place to buffer unemployment shocks in addition to labour income fluctuations and the search effort. Specifically, the relative smoothness ratio (s.d. of consumption growth relative to s.d. of earnings growth) varies from 0.72 to 0.87, with the more plausible parameterizations generating a figure around 0.72. Consumption growth is therefore smoother than the standard buffer stock saving model but not as smooth as in the data, indicating that incorporating unemployment risk goes - at least - some way in explaining the observed patterns.

The excess sensitivity coefficient on labour income is statistically significant and of a magnitude comparable to the data. The main reasons for this positive correlation are firstly related to the fact that we have assumed a small transitory

---

<sup>16</sup>In the annual frequency model, the relative smoothness ratio is closer to one. In the monthly model this is not the case because the ratio of transitory to permanent shock variances is much higher than in the annual model implying higher individual saving rates and higher individual consumption smoothing, some of which survives the aggregation procedure.

component to aggregate labour income shocks, that makes consumption of impatient and liquidity constrained individuals to respond to lagged income growth and secondly, the presence of unemployment which in effect makes income shocks due to unemployment to be transitory. When individuals are employed however, this positive correlation is re-enforced by the presence of search effort. The reason is that impatient employed consumers are more likely to consume out of current temporary income increases, despite the possibility of future unemployment risk, because they can protect more effectively from the latter by raising their search effort during the unemployment spell. On the other hand however, the model as it stands, seems incapable of replicating the sensitivity to lagged unemployment growth as the latter is shown to be statistically insignificant in the simulations. Finally, the correlation between unemployment growth and both consumption and earnings growth is negative and statistically significant but not as high in absolute value as that observed in the data.

An interesting observation arises from the comparative statics involving the aggregate quantities: setting the transitory aggregate shock standard deviation equal to zero ( $\sigma_{ug} = 0$ ) and the correlation between job destruction and the aggregate earnings shock equal to zero ( $\rho_{\epsilon g} = 0$ ). The microeconomic implications of either of these changes were almost identical to the benchmark simulations in table 2.5. Now, however, the absence of a transitory aggregate shock reduces the consumption smoothing present in the model as aggregate shocks are more permanent

(relative smoothness rises from 0.72 to 0.87). Perhaps equivalently, the contemporaneous correlation between consumption and labour income growth rises from 0.80 to 0.94, as consumption reacts more to the presence of a higher degree of permanent shocks. In the absence of negative correlation between job destruction and earnings shocks, on the other hand, the correlation between consumption growth and unemployment growth is statistically insignificant from zero (contrary to the empirical evidence) illustrating the importance of this correlation in accounting for the empirical evidence.

A more general point can also be made from these comparative statics. Even though the magnitude of these aggregate shocks is small relative to the total uncertainty faced by the household, understanding aggregate fluctuations does require experimenting with, and including, aggregate shocks in the microeconomic model: microeconomic implications alone may not be informative about the aggregate implications of a particular model.

Overall, we conclude that incorporating unemployment risk does go some way in explaining the smoothness of consumption growth that we observe in the data but still the smoothness ratio is higher than the empirical one. At the same time, the assumption of transitory aggregate income shocks together with the presence of search effort can improve the model's predictions as far as the excess sensitivity of consumption to lagged income growth is concerned. However, the model does not seem capable of accounting for the excess sensitivity with respect to lagged unemployment growth while the correlations between the latter and

current consumption and income growth are quite lower in absolute value than the ones we find in the data.

#### **2.4. Incomplete Information Model**

So far, the arrival of a job destruction shock was due to aggregate and idiosyncratic reasons and households were able to distinguish between them. If, for example, there was an aggregate income shock that was automatically translated into an aggregate job destruction shock - due to the negative correlation we have assumed - the individual was able to observe the true source of job destruction variation and correctly attribute it to the aggregate income shock. We now investigate the implications of being unable to distinguish between aggregate and idiosyncratic job destruction shocks following the same methodology as in the previous section. The original idea comes from the classic 1973 Lucas signal extraction model with producers unable to distinguish movements in aggregate prices and wages from individual ones. The closest precursors of this assumption in the context of consumption models can be found in various forms in Deaton (1991), Pischke (1995) and Ludvigson and Michaelides (2001), in the context of earnings shocks. The idea is simple. Given that economy-wide shocks account for a very small fraction of the variance in individual earnings growth [Pischke (1995), for instance], households may have little incentive to distinguish aggregate from idiosyncratic shocks to their earnings. Pischke (1995) shows that informational assumptions can have important effects on aggregate consumption when individual households

behave according to the permanent income hypothesis while Deaton (1991) and Ludvigson and Michaelides (2001) illustrate the importance of this assumption in the context of variants of buffer stock saving models. Nevertheless, experimenting with this assumption in the context of the current model reveals a substantial impact on saving behavior due to the presence of the unemployment state: failure to distinguish aggregate from idiosyncratic shocks actually generates a substantial increase in saving to smooth consumption. We therefore introduce incomplete information in the ability of the household to understand whether the unemployment state has arrived due to an aggregate or an idiosyncratic job destruction shock. We later show that this assumption does not affect the microeconomic predictions of the model but can have substantially different implications for aggregate dynamics from the complete information model.

There are some other similar (but different) assumptions about information. Goodfriend (1992) assumes that information about the macroeconomy arrives with a lag, while Gabaix and Laibson (2001) and Reis (2003) assume that individuals respond to the aggregate shocks infrequently, and this may happen when aggregate shocks have cumulated to an amount that cannot be optimally ignored. In our setup, the individual optimally chooses to not spend time and effort to separate the aggregate from the idiosyncratic job destruction shocks because aggregate job destruction makes up a very small component of total idiosyncratic job destruction. This assumption can also be motivated based on recent work by Sims (2003) as

coming from “limited information-processing capacity” on the part of rational individuals (Sims (2003), p. 666).

### 2.4.1. Job Destruction

If aggregate and idiosyncratic shocks cannot be separated by the individual, the assumptions about the job destruction process we have made so far imply that the individual will base decisions on an AR(1) that has the same autocovariance generating function as the true process. Specifically, the true process is

$$(2.21) \quad \ln \delta_t + \ln \zeta_{it} = \varkappa + \phi(\ln \delta_{t-1} - \mu) + \ln \epsilon_t + \ln \zeta_{it}$$

but the household will be observing

$$(2.22) \quad \ln q_{it} = \mu + \psi \ln q_{it-1} + \ln p_{it}$$

where  $\varkappa$  is a constant and  $\ln p_{it}$  a zero-mean white noise process with variance  $\sigma_p^2$ .

How are the three parameters  $\{\mu, \psi, \sigma_p^2\}$  pinned down? They are determined by matching the mean, variance and first order autocovariance implied by the actual earnings process (2.21) with the one perceived by the individual given by (2.22).

The variance of the actual job destruction process is  $\sigma_\delta^2 + \sigma_\zeta^2$  and this has to equal



the variance of the perceived job destruction process,  $\sigma_q^2$ . To infer  $\psi$ , we match the first order autocovariance between the actual and the perceived process to get that

$$\psi\sigma_q^2 = \phi\sigma_\delta^2$$

so that

$$\psi = \frac{\phi\sigma_\delta^2}{\sigma_q^2} = \frac{\phi\sigma_\delta^2}{\sigma_\delta^2 + \sigma_\zeta^2}$$

Given that we have set values for the two variances  $\sigma_\delta^2$  and  $\sigma_\zeta^2$  and for the autoregressive parameter  $\phi$  already, we can infer both  $\psi$  and  $\sigma_q^2$ . It is useful to note that incomplete information as  $\sigma_\zeta^2$  increases, the idiosyncratic component in the job destruction process becomes more important and therefore incomplete information is less severe for individuals. In the limit, the coefficient  $\psi$  will tend to zero as  $\sigma_\zeta^2$  tends to infinity<sup>17</sup>.

---

<sup>17</sup>In the benchmark calibration the correlation between the job destruction rate and aggregate labor income growth is non-zero. According to the true earnings process, we know that

$$\text{cov}(\delta_t, \Delta \ln Y_{it}) = \sigma_{\delta g} = \rho_{\epsilon g} \sigma_\delta \sigma_g$$

Matching this with the covariance between the perceived job destruction rate and the actual labor income process implies that the perceived correlation between the job destruction rate and the innovation in aggregate labor income is given by

$$\frac{\rho_{\epsilon g} \sigma_\epsilon}{\sqrt{\sigma_\epsilon^2 + \sigma_\zeta^2}}$$

which again is distorted relative to the truth by the amount of idiosyncratic uncertainty in the job destruction process.

### 2.4.2. Benchmark Model Results

The model is solved with a similar set of Euler equations as in the complete information case. The consumption policy functions are similar in shape and not very different in level as the ones plotted in figure 2.1 (the complete information case). Consumption equals cash on hand when the liquidity constraint is binding and increases with cash on hand beyond a certain point generating a concave consumption function.

Search intensity (expressed as the probability of finding a job when unemployed) is plotted in figure 2.4. Search intensity is increasing in wealth for low values of cash on hand and decreases beyond a certain level of cash on hand, having the same shape as in the complete information model. The key difference from figure 2.2 (the complete information case) is that the differences across job destruction states are not visible any more. The signal extraction problem is responsible for this result: when the job destruction process is correctly attributed to the true AR(1) process, the search profile will crucially depend on it. However, when agents believe that the process is governed by idiosyncratic shocks then the autoregressive parameter of the perceived job destruction process is very close to zero and this makes the policy function very similar to what would result from a simple i.i.d. shock. This is intuitive: given the larger variance of the idiosyncratic relative to the aggregate destruction shock, the perceived process inherits mostly

the process associated with the idiosyncratic i.i.d. shock. Nevertheless, the level of search, on average, is very similar across the two different information cases.

### 2.4.3. Time Series Analysis

**2.4.3.1. Individual Statistics.** Table 2.7 produces average time series statistics from tracking different individual histories over time. Comparing the benchmark complete information specification to the benchmark incomplete information results, we note the striking similarity in implications for the microeconomic problem between the two radically different information structures. Almost all entries are exactly identical and the same is also true for the no-transitory aggregate variance case ( $\sigma_{ug} = 0$ ), the lower unemployment insurance model ( $\omega = 0.5$ ) and the case where the correlation between job destruction and aggregate earnings shocks is zero. The reason for these results comes from the fact that the aggregate job destruction shock is a very small component of total idiosyncratic job destruction uncertainty, implying that carefully basing decisions on the aggregate component does not substantially affect behavior *at the microeconomic level*.

The only (small) differences appear when comparing the low idiosyncratic job destruction variance case ( $\sigma_{\zeta} = 0.1$ ). This is so because the signal extraction problem is actually larger when the aggregate component of job destruction is more prominent - in relative terms - in the total job destruction variance. As a result, there are stronger differences between the complete and incomplete information cases here but still the differences are very small in magnitude.

**2.4.3.2. Aggregate Statistics.** So far we established that there are extremely small differences at the microeconomic level driven by the fact that the aggregate component of the job destruction shock is usually small. We now turn to the aggregate implications of our informational assumptions. Table 2.8 reports the incomplete information results. Comparing the benchmark results across complete (Table 2.6) and incomplete information (Table 2.8) models, we note the following;

First, there is a greater degree of consumption smoothing taking place in the incomplete information model with relative smoothness falling from 0.72 to 0.60 bringing the model's prediction closer in line with the empirical data. As already stated, what matters for aggregation are the aggregate shocks. The aggregate component of the job destruction shock however, is mistakenly attributed to idiosyncratic nature. The latter is attempted to be smoothed by agents (unwittingly) through savings and therefore does not wash out on the aggregate. Once again, the presence of search effort facilitates the smoothing of transition; when faced with a (perceived) idiosyncratic negative shock and becomes unemployed, an agent would like to use her savings - if available - to smooth what she believes is a temporary change. The fact that she can search and affect the probability of re-employment implies that she can use more of her savings and thus maintain a smoother consumption profile.

Second, the correlation between consumption and earnings growth is unchanged but there is a substantial increase - in absolute value - in the correlation between both consumption and unemployment growth (from  $-0.16$  to  $-0.36$ ) and

between earnings and unemployment growth (from  $-0.28$  to  $-0.62$ ). The reason that consumption growth is now more negatively related to unemployment growth is straightforward: Agents try to adjust their savings (and thus consumption) due to the idiosyncratic shock that they think has hit them. Along the same lines, search effort responds more and as a result there is a greater negative correlation between unemployment and income growth.

Third, the excess sensitivity coefficient on lagged labour income is slightly higher (from  $0.12$  to  $0.18$ ) while the lagged unemployment growth coefficient is negative and approaches the 10% level of statistical significance. As already mentioned, savings and search effort respond by more to the mistakenly perceived idiosyncratic job destruction shock and as a result, next period's consumption growth becomes more sensitive to unemployment changes. Still however, this link is not strong enough to generate a very robust statistical relationship.

These conclusions are robust to different perturbations of the structural parameters as table 2.8 illustrates. We conclude that the incomplete information, infinite horizon, partial equilibrium model goes some way towards explaining some of the observed time series regularities but further work is needed to understand the implications of the model for unemployment dynamics over the business cycle and its joint determination with consumption.

## 2.5. Conclusion

This chapter documents that the excess sensitivity of consumption growth to lagged (expected) labour income growth conceals a robust negative sensitivity of consumption growth to lagged (expected) unemployment growth. Incorporating search frictions in a heterogeneous agent, precautionary savings model and aggregating individual life histories cannot fully replicate the aggregate stylized facts. Nevertheless, introducing incomplete information (defined as the inability to distinguish between aggregate and idiosyncratic shocks) delivers time series predictions that are relatively consistent with the macroeconomic empirical evidence but further work is needed to understand the joint determination of consumption and unemployment over the business cycle. Future work can, for instance, extend the model in general equilibrium using variants of the methodology suggested by Krusell and Smith (1998) and/or incorporate preferences that do not allow for the perfect planning assumed by exponential discounting (Laibson (1997) or O'Donoghue and Rabin (1999)), so that the unemployment shock does generate a larger negative impact on consumption.

## Appendix 2.A: Data construction

This appendix gives data details for our aggregate variables.

$Y$  denotes after tax per capita labour income deflated by the consumption of nondurables and services deflator.

$C$  denotes per capita consumption of nondurables and services (excluding shoes and clothing) deflated in the same way.

To construct  $Y$  we use the following series from the Bureau of Economic Analysis (BEA):

Add wage income ( $WGSAL$ ) and other labour income ( $OTHLAB$ ) and subtract personal contributions for social insurance ( $CONTRIB$ ). We do not add in transfer payments because, throughout the sample, these data are heavily influenced by retroactive payments and massive one-time increments, as well as seasonal adjustment problems. To determine the taxes paid on this labour income, we construct  $TAOLAB = \frac{WGSAL+OTHLAB}{PERSINC}$ , where  $PERSINC$  is the total disposable personal income (that is, wages, rent, interest and dividends).  $TAOLAB$  is thus the share of labour income in total disposable income. After tax labour income is then

$$ATLABINC = WGSAL + OTHLAB - CONTRIB - TAOLAB * TAXPAY,$$

where  $TAXPAY$  is defined as personal tax and nontax payments from the national income and product accounts. We multiply proprietors' income by  $TAOLAB$

(new variable called  $PROPINC$ ) and construct the share of proprietors' labour income in total disposable income as  $TAOPROP = \frac{PROPINC}{PERSINC}$ , so that after tax proprietors' income is

$$ATPROPINC = PROPINC - TAOPROP * TAXPAY$$

and the real after tax per capita series used in the paper follows as

$$Y = \frac{(ATLABINC + ATPROPINC)}{(POP * JC)}$$

where  $POP$  is the population series and  $JC$  is the personal consumption chain type deflator.

To obtain the unemployment series  $u_t$ , we use monthly data from the Bureau of labour Statistics and pick each third value to build the quarterly series. Real interest rates are constructed as nominal 3-month Treasury Bill rates minus actual inflation based on the Consumer Price Index.

## Appendix 2.B: Euler Equations and Numerical Solution Method

There are two value functions associated with the utility cost model depending on employment status ( $V^e, V^u$ ). They are determined recursively according to

(2.B.1)

$$V_t^e(X_{it}) = \text{MAX}_{B_{it}^e} U(X_{it} - B_{it}^e) + \beta E_t[(1 - \delta_{t+1})V_{t+1}^e(X_{it+1}) + \delta_{t+1}V_{t+1}^u(X_{it+1})]$$



and

$$\begin{aligned}
V_t^u(X_t) &= \text{MAX}_{B_{it}^u, s_{it}} U(X_{it} - B_{it}^u) v(s_{it}) \\
&\quad + \beta E_t [\mu(s_{it}) V_{t+1}^e(X_{it+1}) + (1 - \mu(s_{it})) V_{t+1}^u(X_{it+1})]
\end{aligned} \tag{2.B.2}$$

Combining the first order necessary condition with respect to  $B_{it}^u$ , the two envelope conditions  $\{\frac{\partial V_t^e}{\partial X_{it}} = U_c(C_{it}^e)$  and  $\frac{\partial V_t^u}{\partial X_{it}} = U_C(C_{it}^u)\}$  and the possibility of a binding liquidity constraint, we can derive the first two Euler equations given by (2.15) and (2.16). The third Euler equation can be derived by differentiating (2.B.2) and imposing the possibility of a binding constraint on search intensity. The normalization by the growing components is done to make the model stationary and allow balanced growth paths. This utilizes the fact that the value functions are homogeneous of degree  $(1 - \rho)$ , a property that they inherit from the CRRA utility function.

Two sufficient conditions for the individual Euler equations to define a contraction mapping are the conditions in Theorem 1 of Deaton and Laroque (1992) for a mathematically identical model of commodity prices ( $\beta = \frac{1}{1+d}$ ):

$$(2.B.3) \quad \frac{1+r}{1+d} E_t Z_{t+1}^{-\rho} < 1$$

If this condition holds, there will exist a unique set of optimum policies satisfying the three Euler equations. We next simplify these conditions to gain an intuitive understanding of the economics of the problem. Given that  $Z_{t+1} = G_{t+1}N_{t+1}$ , with  $\{N\}$  being log normally distributed, we have  $E_t(G_{t+1}N_{t+1})^{-\rho} = \exp(-\rho\mu_g + \frac{\rho^2\sigma_g^2}{2}) * \exp(-\rho\mu_n + \frac{\rho^2\sigma_n^2}{2})$ . Then

$$\begin{aligned} E_t \frac{1+r}{1+d} Z_{t+1}^{-\rho} &= E_t \frac{1+r}{1+d} E_t Z_{t+1}^{-\rho} & (2.B.4) \\ &= \frac{1+r}{1+d} * \exp(-\rho\mu_g + \frac{\rho^2\sigma_g^2}{2}) * \exp(-\rho\mu_n + \frac{\rho^2\sigma_n^2}{2}) \end{aligned}$$

Taking logs of the two conditions and using the approximation  $\log(1+x) \approx x$  for small  $x$ , (2.B.3) becomes

$$(2.B.5) \quad \frac{r-d}{\rho} + \frac{\rho}{2}(\sigma_n^2 + \sigma_g^2) < \mu_g$$

which is the condition derived by Deaton (1991) with  $\mu_n = 0$  and is the same condition as in Carroll (1997) where  $\mu_n$  is non-zero.

The condition can be satisfied for high  $\mu_g$  or  $d$ . First, a high expected earnings growth profile (as measured by  $\mu_g$ ) guarantees that the individual will not want to accumulate an infinite amount bonds but would rather borrow now, expecting earnings to increase in the future. Second, if the rate of time preference exceeds the expected stock return, more risk averse (higher  $\rho$ ) individuals will not satisfy the convergence conditions.

The single state variable (normalized cash on hand,  $x_t = \frac{X_t}{P_t}$ ) is discretized into say (100) grid points with more points at lower values of cash on hand where the value function is more curved and the policy functions have a kink. Given that the value functions converge, we can solve simultaneously the three functional equations for the three policy functions. Note that the value functions also need to be computed and updated until convergence when computing the search intensity function. Interpolations along the single continuous state variable are performed using cubic spline interpolation and the upper bound of cash on hand is found by a trial and error method that ensures simulated liquid assets never exceed the chosen upper bound for cash on hand.

The incomplete information model is solved in a similar manner.

**Table 2.1***Properties of Aggregate Time Series U.S. Data**Quarterly Frequency*

1959:01 - 2002:04

Time Series	s.d.	First Autoc.
$\Delta C_t$	.44	<b>.336</b>
$\Delta Y_t$	1.1	-.082
$\Delta u_t$	5.98	<b>.393</b>

<i>Contemporaneous Correlations</i>		
$Corr(\Delta C_t, \Delta Y_t)$		<b>.489</b>
$Corr(\Delta C_t, \Delta u_t)$		<b>-.422</b>
$Corr(\Delta Y_t, \Delta u_t)$		<b>-.432</b>

Notes to Table 2.1:  $Y$  denotes real, after tax, per capita labour income,  $C$  denotes real, per capita consumption of nondurables and services (excluding shoes and clothing) and  $u$  denotes the unemployment rate.  $\Delta$  is used to denote the growth rate in a variable. Details for the construction of these variables can be found in Appendix 2.A. Bold variables denote statistical significance at the 5% level.

**Table 2.2**

*Excess Sensitivity of Consumption Growth to  
labour Income Growth and Unemployment Growth*

1959:01 - 2002:04

*OLS estimates*

Panel A			
Dependent variable	Regressor	<i>Adj.R</i> <sup>2</sup>	
$\Delta C_t$	$\Delta Y_{t-1}$		
	.109*	.077	
	(.036)		
Panel B			
Dependent variable	Regressors		<i>Adj.R</i> <sup>2</sup>
$\Delta C_t$	$\Delta Y_{t-1}$	$\Delta u_{t-1}$	
	.077*	-.014*	.102
	(.031)	(.004)	

Notes to Table 2.2:  $Y$  denotes real, after tax, per capita labour income,  $C$  denotes real, per capita consumption of nondurables and services (excluding shoes and clothing) and  $u$  denotes the unemployment rate.  $\Delta$  is used to denote the growth rate in a variable. Details for the construction of these variables can be found in Appendix 2.A.\* denotes statistical significance at the 5% level. Standard errors for the OLS estimates are given in parentheses.

**Table 2.3** *IV Estimates*

Panel A				
Dependent Variable	<i>IVs</i>	Regressors		<i>Jstat.</i>
$\Delta C_t$	1st set	$\Delta Y_t$	$\Delta u_t$	
		.441*	–	.002
		(.137)	–	
		.236*	–.041*	.008
		(.110)	(.018)	
Panel B				
Dependent Variable	<i>IVs</i>	Regressors		<i>Jstat.</i>
$\Delta C_t$	2nd set	$\Delta Y_t$	$\Delta u_t$	
		.420*	–	.020
		(.121)	–	
		.231*	–.039*	.028
		(.098)	(.016)	
Panel C				
Dependent Variable	<i>IVs</i>	Regressors		<i>Jstat.</i>
$\Delta C_t$	3rd set	$\Delta Y_t$	$\Delta u_t$	
		.323*	–	.048
		(.070)	–	
		.185*	–.032*	.030
		(.082)	(.015)	

Notes to Table 2.3:  $Y$  denotes real, after tax, per capita labor income,  $C$  denotes real, per capita consumption of nondurables and services (excluding shoes and clothing),  $u$  denotes the unemployment rate and  $r$  is the real short term interest rate defined as the difference between the nominal three month U.S. Treasury Bill rate and inflation (constructed from the CPI). Data range: 1959:01 - 2002:04, quarterly.  $\Delta$  is used to denote the growth rate in a variable. Details for the construction of these variables can be found in Appendix 2.A. \* denotes statistical significance at the 5% level and standard errors are given in parentheses. The first set of instruments is:  $\Delta Y_{t-2}$ ,  $\Delta Y_{t-3}$ ,  $\Delta u_{t-2}$ ,  $\Delta u_{t-3}$ . The second set of instruments is the first set plus  $\ln(C_{t-2}/Y_{t-2})$ ,  $\ln(C_{t-3}/Y_{t-3})$  and the third set of instruments is the second set plus  $r_{t-2}$ ,  $r_{t-3}$ . For the regressions without  $\Delta u_t$  the respective instrument sets do not include  $\Delta u_{t-2}$ ,  $\Delta u_{t-3}$ .

**Table 2.4***Fully Robust Inference with Weak Instruments*

Test	1st set of <i>IVs</i>	2nd set of <i>IVs</i>	3rd set of <i>IVs</i>
A-R	8.02**	7.03**	6.97**
Critical Values (10%,5%)	(1.94, 2.37)	(1.77, 2.09)	(1.67, 1.94)
Kleibergen	16.1**	12.4*	25.6**
Critical Values (10%,5%)	(7.78, 9.49)	(10.64, 12.59)	(13.36, 15.51)
Moreira (LR)	23.5**	23.8**	39.7**
Critical Values (5%)	(4.20)	(4.73)	(4.73)

Notes to Table 2.4: Different tests of the null that the *IVs* are weak in the regressions of Table 2.3. \* (\*\*) denotes that  $H_0$  is rejected at 10% (5%). Numbers in parentheses indicate the critical values of the respective statistics at the 10% and 5% level of statistical significance respectively. The first set of instruments is:  $\Delta Y_{t-2}$ ,  $\Delta Y_{t-3}$ ,  $\Delta u_{t-2}$ ,  $\Delta u_{t-3}$ . The second set of instruments is the first set plus  $\ln(C_{t-2}/Y_{t-2})$ ,  $\ln(C_{t-3}/Y_{t-3})$  and the third set of instruments is the second set plus  $r_{t-2}$ ,  $r_{t-3}$ . The critical values for the *LR* test are reproduced from Moreira (2003) and refer only to 5% significance. Moreira does not report any values for the case of six and eight exogenous variables but since the statistic is increasing in the number of instruments we report the threshold for 10 *IVs*.



**Table 2.5***Complete Information (Individual Statistics) - monthly frequency*

Variable	DC	Bench	$\sigma_{ug} = 0$	$\sigma_{\zeta} = 0.1$	$\omega = .5$	$\rho_{\epsilon g} = 0$
Mean $c$	1.00	.976	.976	.976	.973	.976
Mean $c^e$		.982	.982	.982	.977	.982
Mean $c^u$		.902	.902	.902	.904	.902
Mean $b$	.226	.946	.946	.946	1.24	.946
Median $b$	.224	.957	.958	.957	1.27	.957
Mean $b^e$		.980	.981	.980	1.28	.980
Mean $b^u$		.453	.453	.453	.623	.453
Mean $s$		.70	.699	.70	.79	.70
$\sigma(\Delta C_{it})$	.037	.043	.043	.043	.046	.043
$\sigma(\Delta Y_{it})$	.144	.191	.191	.191	.223	.191
Prob Exit Un (%)		50.4	50.4	50.4	54.7	50.4
Drop in $c$ (%)		8.15	8.14	8.15	7.45	8.16
Drop in $b$ (%)		53.8	53.8	53.8	51.3	53.8

Notes to Table 2.5: All simulations are performed at a monthly frequency to capture high frequency unemployment dynamics. DC refers to the model without unemployment risk which is identical to Deaton (1991) and similar to Carroll (1997), except for the frequency in decision making. For the benchmark specification,  $\beta = 1 - \frac{.05}{12}$ ,  $r = .02/12$ ,  $\omega = 0.6$ ,  $\mu_g = .02/12$ ,  $\sigma_u = 0.1$ ,  $\sigma_g = .02/\sqrt{12}$ ,  $\sigma_{ug} = 0.005$ ,  $\sigma_N = .08/\sqrt{12}$ ,  $\phi = 0.75$ ,  $\sigma_{\zeta} = 0.2$ ,  $\rho_{\epsilon g} = -0.7$ ,  $\sigma_{\delta} = 0.06$  and  $\kappa = -3.45$ . Lower case variables

are normalized by the permanent component of individual labour income.  $\lambda$  is chosen so that the mean probability of finding employment after one month is around 50%. This generates a value for  $\lambda = 1$ . The risk aversion coefficient is chosen to generate a drop in consumption of around 8% in mean consumption from the employment to the unemployment state generating  $\rho = 1.2$ . The last two rows refer to the drop in mean normalized consumption and savings from an employment to an unemployment state. The statistics are computed over 172\*3 periods over 2000 individuals and averaged over 100 simulation draws.

**Table 2.6***Complete Information Model (Aggregate Statistics) - quarterly frequency*

Variable	DC	Bench	$\sigma_{ug} = 0$	$\sigma_{\zeta} = 0.1$	$\omega = .5$	$\rho_{\epsilon g} = 0$
Mean $\Delta C_t$	.005	.005	.005	.005	.005	.005
$\sigma(\Delta C_t)$	.008	.008	.008	.008	.008	.008
Mean $\Delta Y_t$	.005	.005	.005	.005	.005	.005
$\sigma(\Delta Y_t)$	.01	.016	.01	.01	.011	.01
$\sigma(\Delta C_t)/\sigma(\Delta Y_t)$	.862	.718	.87	.80	.77	.80
<i>Corr</i> ( $\Delta C_t, \Delta Y_t$ )	<b>.91</b>	<b>.80</b>	<b>.94</b>	<b>.87</b>	<b>.86</b>	<b>.86</b>
<i>Corr</i> ( $\Delta C_t, \Delta U_t$ )		<b>-.16</b>	<b>-.16</b>	<b>-.16</b>	<b>-.16</b>	<b>-.01</b>
<i>Corr</i> ( $\Delta Y_t, \Delta U_t$ )		<b>-.28</b>	<b>-.34</b>	<b>-.32</b>	<b>-.34</b>	<b>-.18</b>
Sensitivity ( $\Delta Y_{t-1}$ )	<b>.16</b>	<b>.12</b>	<b>.17</b>	<b>.14</b>	<b>.14</b>	<b>.15</b>
S.E. ( $\Delta Y_{t-1}$ )	.06	.06	.07	.06	.06	.06
Sensitivity ( $\Delta U_{t-1}$ )		-.002	.001	-.001	-.001	-.001
S.E. ( $\Delta U_{t-1}$ )		.004	.002	.004	.004	.004
Mean Un (%)		6.04	6.04	6.04	5.56	6.04

Notes to Table 2.6: See notes to Table 2.5. For this table, cross sectional averages are first taken and then the aggregate statistics of interest are computed over the 100 simulation draws. The sensitivity rows report the coefficients from a regression using the simulated data of consumption growth on lagged labour income growth and lagged unemployment growth, respectively. S.E. are the standard errors from these regressions. Mean un is the mean rate of unemployment. Numbers in bold are different from zero at

the 5% statistical significance level. The statistics are computed over 172 periods over 2000 individuals.

**Table 2.7***Incomplete Information Model (Individual Statistics) - monthly frequency*

Variable	Bench	$\sigma_{ug} = 0$	$\sigma_{\zeta} = 0.1$	$\omega = .5$	$\rho_{\epsilon g} = 0$
Mean $c$	.976	.978	.977	.975	.978
Mean $c^e$	.981	.983	.982	.979	.983
Mean $c^u$	.903	.903	.900	.907	.903
Mean $b$	.949	.949	.934	1.24	.949
Median $b$	.957	.963	.944	1.27	.960
Mean $b^e$	.980	.981	.967	1.28	.981
Mean $b^u$	.453	.453	.444	.623	.451
Mean $s$	.701	.701	.701	.791	.701
$\sigma(\Delta C_{it})$	.043	.042	.043	.045	.042
$\sigma(\Delta Y_{it})$	.191	.188	.190	.218	.188
Prob Exit Un (%)	50.4	50.4	50.4	54.7	50.4
Drop in $c$ (%)	8.1	8.1	8.3	7.39	8.14
Drop in $b$ (%)	53.8	53.9	54.1	51.3	54.0

Notes to Table 2.7: All simulations are performed at a monthly frequency to capture high frequency unemployment dynamics. DC refers to the model without unemployment risk which is identical to Deaton (1991) and similar to Carroll (1997), except for the frequency in decision making. For the benchmark specification,  $\beta = 1 - \frac{.05}{12}$ ,  $r = .02/12$ ,  $\omega = 0.6$ ,  $\mu_g = .02/12$ ,  $\sigma_u = 0.1$ ,  $\sigma_g = .02/\sqrt{12}$ ,  $\sigma_{ug} = 0.005$ ,  $\sigma_N = .08/\sqrt{12}$ ,  $\phi = 0.75$ ,  $\sigma_{\zeta} = 0.2$ ,  $\rho_{\epsilon g} = -0.7$ ,  $\sigma_{\delta} = 0.06$  and  $\kappa = -3.45$ . Lower case variables

are normalized by the permanent component of individual labour income.  $\lambda$  is chosen so that the mean probability of finding employment after one month is around 50%. This generates a value for  $\lambda = 1$ . The risk aversion coefficient is chosen to generate a drop in consumption of around 8% in mean consumption from the employment to the unemployment state generating  $\rho = 1.2$ . The last two rows refer to the drop in mean normalized consumption and savings from an employment to an unemployment state. The statistics are computed over  $172 \times 3$  periods over 2000 individuals and averaged over 100 simulation draws..

**Table 2.8***Incomplete Information Model (Aggregate Statistics) - quarterly frequency*

Variable	Bench	$\sigma_{ug} = 0$	$\sigma_{\zeta} = 0.1$	$\omega = .5$	$\rho_{\epsilon g} = 0$
Mean $\Delta C_t$	.005	.005	.005	.005	.005
$\sigma(\Delta C_t)$	.009	.009	.008	.008	.008
Mean $\Delta Y_t$	.005	.005	.005	.005	.005
$\sigma(\Delta Y_t)$	.014	.012	.011	.014	.012
$\sigma(\Delta C_t)/\sigma(\Delta Y_t)$	.604	.681	.722	.595	.707
<i>Corr</i> ( $\Delta C_t, \Delta Y_t$ )	<b>.795</b>	<b>.879</b>	<b>.871</b>	<b>.811</b>	<b>.777</b>
<i>Corr</i> ( $\Delta C_t, \Delta U_t$ )	<b>-.36</b>	<b>-.36</b>	<b>-.32</b>	<b>-.33</b>	-.006
<i>Corr</i> ( $\Delta Y_t, \Delta U_t$ )	<b>-.55</b>	<b>-.62</b>	<b>-.50</b>	<b>-.60</b>	<b>-.385</b>
Sensitivity ( $\Delta Y_{t-1}$ )	<b>.131</b>	<b>.181</b>	<b>.154</b>	<b>.152</b>	<b>.151</b>
S.E. ( $\Delta Y_{t-1}$ )	.053	.063	.062	.05	.06
Sensitivity ( $\Delta U_{t-1}$ )	-.002	-.001	-.002	-.002	-.002
S.E. ( $\Delta U_{t-1}$ )	.002	.002	.003	.002	.002
Mean Un (%)	5.6	5.56	5.88	5.12	5.55

Notes to Table 2.8: See notes to Table 2.7. For this table, cross sectional averages are first taken and then the aggregate statistics of interest are computed over the 100 simulation draws. The sensitivity rows report the coefficients from a regression using the simulated data of consumption growth on lagged labour income growth and lagged unemployment growth, respectively. S.E. are the standard errors from these regressions. Mean un is the mean rate of unemployment. Numbers in bold are different from zero at

the 5% statistical significance level. The statistics are computed over 172 periods over 2000 individuals.



Figure 2.1: Consumption policy functions

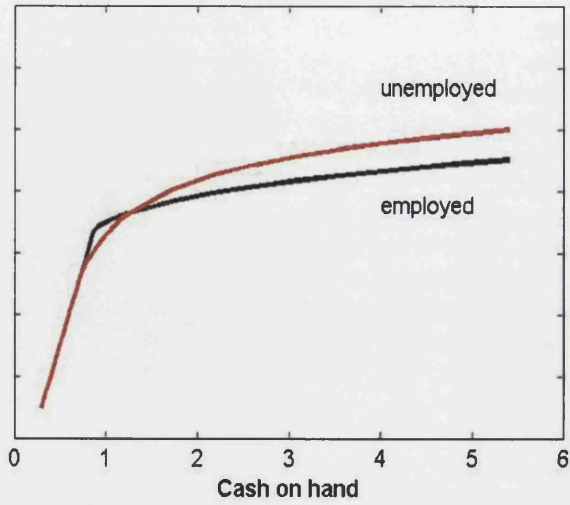


Figure 2.2: Probability of finding employment

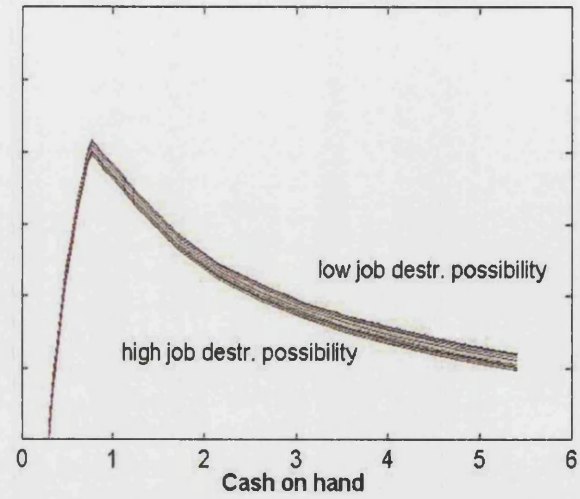


Figure 2.3: Change in the replacement ratio

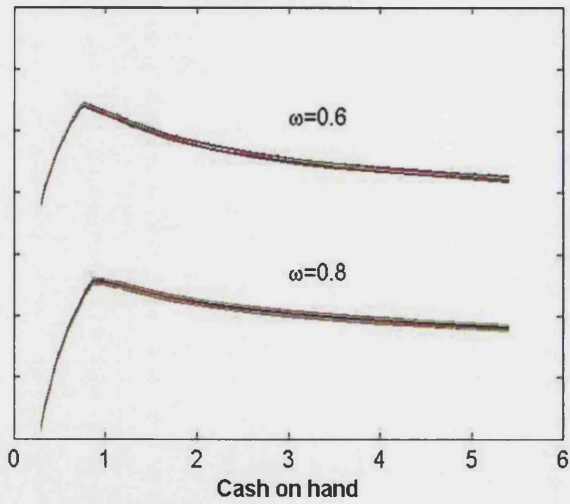
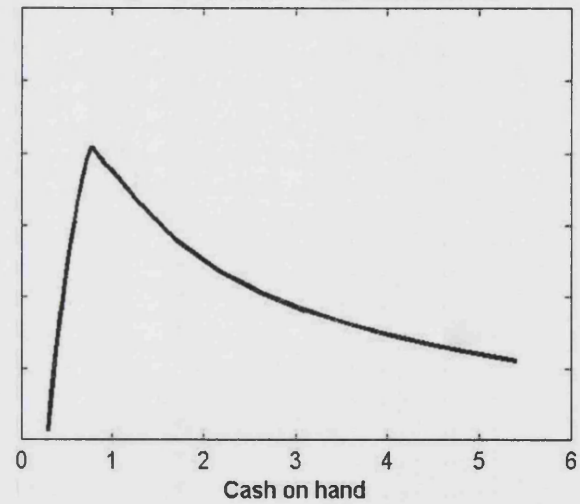


Figure 2.4: Probability of finding employment, incomplete information



## CHAPTER 3

# Chapter Three: Short Job Tenures and Firing Taxes in the Search Theory of Unemployment

### 3.1. Introduction

It has long been argued that differences in unemployment rates and labour market performance across countries can be considered as structural and should be sought in the institutional arrangements and the different policy regulations employed by individual governments. Emphasis in the theoretical literature has been placed upon Employment Protection Legislation (*EPL*) and its consequences for leading labour market indicators such as job creation and job destruction. *EPL* consists of a set of rules that affect the process of dismissals. Hence, it is an additional cost that has to be incurred by the firm when laying off employees. Distinction in these restrictions falls in two categories: severance compensation and firing taxes. The latter is a penalty imposed by the government outside the firm-worker pair while the former is a pure transfer from the employer to the fired employee. Contract theory has established that in the presence of full wage flexibility, the two parties can write contracts in such a way so as to render the effects of these transfers neutral (see Lazear, 1988 and 1990). As a result, the vast

majority of the literature has focused on the concept of firing taxes<sup>1</sup>. This chapter will move along these lines<sup>2</sup>.

Firing taxes include administrative, procedural, legal and any other financial penalties that a ruling judge may impose on the firm when a separation is initiated. From a theoretical point of view, these costs are shown to suppress firing and hiring making thus the recruitment and dismissal processes smoother over the business cycle (see Mortensen and Pissarides, 2001; Bertola et al., 2000). Hence, sharp employment reductions are expected to occur less frequently in economies with stringent *EPL*. Empirical results are however and to some extent inconclusive.

Table 3.1 presents cross country comparisons for the *EPL* ranking of selected economies during the late 1980s and 1990s. Individual scores are first decomposed into two different contractual arrangements (temporary and regular contracts) and then an overall rating is reported. By using the job turnover rate (*JTR*) as a proxy for labour market flexibility, Figure 3.1.a illustrates the relationship between *EPL* and *JTR* for 17 OECD economies in the late 1980s. It appears that a negative relationship does tend to manifest itself in the data, although it is quite surprising to observe that heavily regulated countries such as Italy, France and Sweden have the same or even higher *JTR* with that of the US, an economy with relaxed *EPL* policies (a point made also by Bertola et al., 2000). However, if *JTR* of continuing establishments only is taken into account, the direction of association becomes

---

<sup>1</sup>See Garibaldi and Violante (2004) for a discussion on wage rigidity and severance compensation.

<sup>2</sup>At the time this chapter was written (2001-2002), a paper with similar results was published in Labour Economics [Cahuc and Postel-Vinay, Labour Economics 9 (2002) 63-91].

more ambiguous (see Figure 3.1.b). On top of this, the legislation governing the Difficulty of Dismissal on permanent contracts<sup>3</sup> - which is a major component of *EPL* (with a correlation coefficient of around 0.87) and is the closest in meaning to what is considered as firing taxes since it excludes legislation regarding severance compensation - is shown to be unrelated if not somewhat positively related to job turnover (see Figures 3.2.a and 3.2.b). It seems that the protective policies affect the dynamics of the labour market in more ways than the theoretically established ones and probably in opposite directions<sup>4</sup>.

One possible explanation for the overall pattern is that restrictions do not usually cover workers in short tenures. Thus, stringent regulations may enlarge the pool of short term jobs - as firms attempt to circumvent termination laws - and as a result aggravate *JTR*. In practice, there are two sources of labour market legislation that can give rise to such short tenures: probation periods which offer an initial adjustment period for both the employer and the employee and temporary or fixed term contracts (*TC*). The former usually provide a firing tax-free period of a few months but can be extended for up to two years, as in the case of the UK (see Table 3.2, Panel A). Temporary contracts are likely to be more relevant for the purpose of this study since they involve fixed term employer-employee

---

<sup>3</sup>The terms “regular” and “permanent” contract will be used interchangeably throughout this paper.

<sup>4</sup>It should be noted that one needs to be a little cautious about these conclusions because no controls have been used (controlling for firm size could turn out to be important but unfortunately this is not possible in the data) and because the sample size is relatively small, excluding economies like Greece, Portugal and Spain which have notoriously stringent policies.

relations that vary from two to three years. It is important to emphasize that in these cases, notice periods and severance compensation are prohibited and workers cannot initiate procedures for unfair dismissal. That said, temporary contracts are in effect dismissal cost-free periods. Concerning duration, agreements can usually be renewed but in general they cannot exceed a certain predetermined cumulative period<sup>5</sup> (see Table 3.2, Panel B). As a result, when the contract expires the pair is either left with the option to separate or continue by writing a regular contract in which case layoff costs become operational<sup>6</sup>.

During the mid-1980s Germany and Spain relaxed restrictions on *TCs*. Büchtemann (1991, 1993) and Milner et al. (1995) note that despite similar reforms in the regulation of fixed term contracts the impact was sharply different in the two countries and has taken many years to unfold. The OECD, in the Employment Outlook 1999, suggests that one candidate explanation could be identified on the basis that "*...the potential future firing costs due to EPL that were associated with hiring a worker on a permanent contract remained larger in Spain than in Germany*" (OECD, 1999, ch. 2 p.71). Under such an assumption - and besides adversely affecting *JTR* - such policies could influence the composition of employment regarding permanency on the job.

---

<sup>5</sup>If such a condition does arise, "*...courts can be called upon to examine the validity of the reason given and may declare the fixed term unjustified, judging that its main purpose is to circumvent termination laws*" (OECD, 1999, ch.2 p.59).

<sup>6</sup>It is true that in the past, temporary contracts were limited to so-called specific projects or seasonal work. However, the majority of the countries have by now either lifted or significantly relaxed these requirements. Hence, continuation on regular contract terms is straightforward if beneficial to both parties.

Keeping in mind the well known limitations dictated by the nature and availability of information of *EPL* indicators<sup>7</sup>, Table 3.3 summarizes the estimation results of regressing the share of temporary employment on the measures of employment protection of 24 OECD countries for the late 1990s. The first two columns refer to a simple univariate regression while the last two include additional variables to control for country specific effects. The number of additional variables however, ought to be kept to a minimum given the limited number of observations available. In the estimations both indicators of *EPL* in regular contracts were used. The dependent variable (*TE*) is the average of temporary employment shares between 1998 and 2002 while the policy indicators refer to the late 1990s<sup>8</sup>. This was done to avoid any potential endogeneity problems and because changes in policy may require time until their effects become evident. Not surprisingly, the sign of the estimated coefficients in the univariate regressions is within the lines of the previous discussion, their magnitude is quite considerable and they are statistically significant at conventional levels. Regarding the multivariate regressions, the unemployment rate (*UR*) is included because higher unemployment rate might make agents more willing to commit themselves to temporary contractual agreements<sup>9</sup> and the labour productivity growth (*LPGR*) is added on the basis that higher rate of growth may induce more investment in human capital and thus encourage

---

<sup>7</sup>See Bertola et al. (2000) for an extended discussion on the issue.

<sup>8</sup>Estimations were also performed by averaging the variables over 1996-2002 but with no noticeable changes in the results.

<sup>9</sup>There may be an endogeneity issue here, but this will not affect estimates for employment protection.

longer employer-employee relationships. Finally, *GDP* growth (*GDPGR*) is included as a standard control variable. The estimation documents that legislation is a significant determinant of temporary employment. The estimated coefficients are marginally lower than before and they are still statistically significant at better than 5 percent <sup>10</sup> (this result is consistent with the findings of Grubb and Wells, 1993). Overall, the regulation governing dismissals in permanent contracts seems to matter for the composition of employment. For this reason, one needs a theoretical grounding of how this mechanism depends on policy design and how it affects the equilibrium. To do so, such policies must be explicitly modelled<sup>11</sup>.

To incorporate regulations of this kind in a search model of unemployment, one does not need necessarily to resort to a model of endogenous job destruction. Indeed, I will use the simplest framework possible to raise the issue by building on a model of exogenous job separation. Of course, under such an assumption, the link between termination taxes and employment protection is broken<sup>12</sup>. This needs not be of serious concern however, as the purpose of this paper is to identify the conditions under which firing restrictions may amplify the separation rate. All in all, I make no attempt to justify why dismissal taxes are in place. Yet, I use such

---

<sup>10</sup>Additional variables were also included in the regression but without any changes in the results. Examples include union density, subsidies to regular employment as well as the *EPL* indicators for temporary employment.

<sup>11</sup>Although not exactly the same in reality, I will use the terms “probation period” and “temporary contract (*TC*)” interchangeably for the needs of this paper.

<sup>12</sup>Although and as it will become clear later in the paper, such policies can be justified on the grounds of reducing wage inequality.

an assumption as given and study its consequences for wages, unemployment rate and job destruction.

Overall, the study raises the issue of the complexity of such policies and makes an effort to fill in a potential gap in the literature. Moreover, it advocates that some of the observed phenomena concerning turnover rates as well as the composition of employment regarding permanency on the job could, at least partially and in principle be addressed within the discussed mechanism.

The next section describes the structure of the model: it starts with an overview of the notation and assumptions and then discusses the two alternative environments upon which, the model will be built. Section 3 explains the possible equilibria outcomes and studies the relationship between firing taxes, job destruction and unemployment rate. Section 4 illustrates the effects of such policies on different productivity groups and wage differentials while section 5 proceeds with some computational experiments. Section 6 concludes this chapter.

## **3.2. The model**

### **3.2.1. Notation and assumptions**

The model put forward is a standard search and matching model of exogenous job destruction pioneered by Mortensen (1978) and Pissarides (1984, 1985 and 2000). One usual assumption is that each firm employs one worker. Firms without workers post vacancies at a cost of  $pc$ , where  $p$  is the job's productivity. The vacancy cost is proportional to productivity under the assumption that it is more



costly to acquire more productive workers. It is also sunk and thus the investment undertaken by the firm is irreversible. In equilibrium, job creation is governed by profit maximization by taking into account expected revenue and cost of a newly created match. The matching function  $m(v, u)$  which directs employer-employee meetings is assumed to be increasing in both arguments, concave and homogeneous of degree one in vacancies ( $v$ ) and unemployment ( $u$ )<sup>13</sup>.

All realized job matches yield a pure economic rent. If the worker and the firm separate, each party will have to go through a costly search process in order to meet its next partner. It is this rent that has to be shared in the wage contract. Here, I assume that this surplus is divided in fixed proportions between the firm and the worker in a bargaining process where  $\beta$  represents labour's share.

Negative shocks arrive to existing matches at the Poisson rate  $\delta$ . When this happens the productivity of the job is reduced to zero and hence the match dissolves. However, the latter may not be the only source of job destruction. After the worker and the firm match, the job enters a probation time period  $T$  whose length is determined by policy. During this time, employers can costlessly fire existing workers. If the match survives and this period elapses, the firm becomes liable to a firing tax  $pF$ , again imposed by policy. Therefore, the implementation of a firing restriction that becomes operational after a predetermined period of time introduces a process of endogenous separation decisions since firms may find

---

<sup>13</sup>Empirically, studies establish the existence of an aggregate matching function with constant returns to scale. See Divine and Kiefer (1991) and Petrongolo and Pissarides (2001) for a survey of the issue.

it optimal to destroy the match instead of becoming liable to the termination tax in the future.

To simplify what follows, and without loss of generality, assume that the date of job creation is always denoted as  $\tau = 0$ . The remaining of the notation that will be used is:  $V$  the value of a vacant job (irrespective of time),  $J$  the value of an active job at date of creation,  $J(t)$  the value of an active job at  $t$  (created at  $\tau = 0$ ),  $W$  the value of employment at date of creation,  $W(t)$  the value of employment at  $t$  (created at  $\tau = 0$ ),  $U$  the value of unemployment (irrespective of time),  $z$  the unemployment income and  $r$  the interest rate.

### 3.2.2. Regime 1: Firms dismiss workers at $T$

I begin by examining the equilibrium where firms find it beneficial to dispose off the match at the end of  $T$  (this will be referred to as regime 1 and be denoted with the script 1). Given a matching function  $m(v, u)$  the probability that a worker will arrive to a vacant job is equal to  $m(v, u)/v$  while the probability that an unemployed agent will find an unoccupied working opportunity is  $m(v, u)/u$ . Using the homogeneity of the matching function, one can rewrite the latter two as functions of the ratio of vacancies to unemployment. Let  $\theta$  be equal to  $v/u$  and refer to it as the market tightness. Therefore, the rate at which vacant jobs become filled is denoted by  $q(\theta) \equiv m(\theta, 1)$  and the rate at which unemployed agents move into employment is  $\theta q(\theta) \equiv \theta m(\theta, 1)$ .

The asset pricing equation for the return of a vacant job is given by:

$$(3.1) \quad rV^1 = -pc + q(\theta) [J^1 - V^1]$$

A vacant job costs  $pc$  per unit of time and changes state at a rate  $q(\theta)$ . When job creation takes place, it yields a net return of  $J^1 - V^1$ . Free entry implies that  $V^1 = 0$  and thus:

$$(3.2) \quad J^1 = \frac{pc}{q(\theta)}$$

(3.2) will be referred to as the job creation condition. It states that in equilibrium firms will create vacant jobs to the point where the value of a newly created match equals the expected cost of maintaining a vacancy.

Following similar reasoning:

$$(3.3) \quad rJ^1(t) = p - w_1(t) - \delta J^1(t) + \dot{J}^1(t)$$

where  $w_1(t)$  is the wage rate in regime 1,  $t \in [0, T]$  and  $\lim_{t \rightarrow T} \dot{J}^1(t) = 0$ . Note that in regime 1, firms never pay the firing tax since they always dismiss employees at the end of  $T$ . Finally, the job loses value as time progresses because it approaches the end of its life-cycle.

The values of employment and unemployment satisfy the following asset pricing equations respectively:

$$(3.4) \quad rW^1(t) = w_1(t) - \delta [W^1(t) - U^1] + \dot{W}^1(t)$$

$$(3.5) \quad rU^1 = z + \theta q(\theta) [W^1 - U^1]$$

The capital value of employment is equal to its net return  $w_1(t)$  minus the risk of changing state to unemployment and the capital value of unemployment is equal to unemployment income adjusted for the possibility that the agent will become employed.  $z$  may refer to a state provided benefit or imputed income from leisure or a combination of the two. I will discuss later the implications of such assumptions when I study productivity differences.

**3.2.2.1. Wage determination.** When the worker and the firm meet they share the surplus match value in fixed proportions. Thus the wage is set to maximize the Nash product  $(J^1 - V^1)^{1-\beta} (W^1 - U^1)^\beta$ . By assuming that the same bargaining holds for all future renegotiations, one obtains:

$$(3.6) \quad w_1^* = (1 - \beta)z + \beta p(1 + c\theta)$$

(see Appendix 3.A for an explicit derivation of  $w_1$ ).

The wage is constant in  $t$  since both, employer and employee, make capital losses as  $t \rightarrow T$ . These losses must be shared according to the Nash bargaining rule.

**3.2.2.2. Job value.** Given the wage and an exogenously probation period set by policy, the optimal value of a job in regime 1 at any  $t \in [0, T]$  is given by:

$$(3.7) \quad J^1(t) = \int_t^T [p - w_1^*] e^{-(r+\delta)(s-t)} ds$$

By setting  $t = 0$  one obtains:

$$(3.8) \quad J^1 = \int_0^T [p - w_1^*] e^{-(r+\delta)s} ds$$

or, because both  $p$  and  $w_1^*$  are independent of time and  $w_1^*$  is given by (3.6):

$$(3.9) \quad (r + \delta) J^1 = [p - (1 - \beta)z - \beta p(1 + c\theta)][1 - e^{-(r+\delta)T}]$$

Clearly,  $J^1$  converges to 0 as  $T \rightarrow 0$  and to  $[p - (1 - \beta)z - \beta p(1 + c\theta)]/(r + \delta)$  as  $T \rightarrow +\infty$ .

**3.2.2.3. Equilibrium in regime 1.** A solution to the model of regime 1 consists of a job value and a market tightness pair  $(J^{1*}, \theta^*)$  that solves (3.2) and (3.9). Since the former is an upward sloping curve in the  $(J, \theta)$  space while the latter is a downward sloping one, existence of unique equilibrium is guaranteed. Equilibrium

market tightness is then determined by:

$$(3.10) \quad [p - (1 - \beta)z - \beta p(1 + c\theta^*)] [1 - e^{-(r+\delta)T}] - (r + \delta) \frac{pc}{q(\theta^*)} = 0$$

Inspection of (3.10) easily establishes that the l.h.s. is decreasing in  $\theta^*$  and increasing in  $T$  implying that equilibrium market tightness increases monotonically in the probation period. The intuition for this is simple: an increase in  $T$  makes the expected life of a job longer. As a result the job value curve shifts upwards determining higher market tightness, higher job value and higher wage. Since both, the equilibrium job value and equilibrium market tightness are monotonically increasing in  $T$  it immediately follows that  $J^{1*}$ ,  $\theta^*$  and  $w_1^*$  are always less than their respective values in a policy-free equilibrium.

### 3.2.3. Regime 2: Firms do not destroy surviving matches at $T$

In regime 2 all firms chose not to dismiss workers in surviving matches at the end of the fixed horizon. Alternatively, a  $TC$  is transformed to a permanent contract with probability one, conditional on that the match has survived to  $T$ .

As before, the asset pricing equation for a new vacancy is given by:

$$(3.11) \quad rV^2 = -pc + q(\theta) [J^2 - V^2]$$

where again profit maximization implies:

$$(3.12) \quad J^2 = \frac{pc}{q(\theta)}$$

Consider now the job value for any  $t \in [0, T]$ :

$$(3.13) \quad J^2(t) = \int_t^T [p - w_2] e^{-(r+\delta)(s-t)} ds + \int_T^\infty [p - w_2 - \delta pF] e^{-(r+\delta)(s-t)} ds$$

and by setting  $t = 0$

$$(3.14) \quad J^2 = \int_0^T [p - w_2] e^{-(r+\delta)s} ds + \int_T^\infty [p - w_2 - \delta pF] e^{-(r+\delta)s} ds$$

The value of a job equals its net return  $p - w_2$ . The job also runs a risk of being destroyed which will result in the loss of  $J^2$ . If the match survives to  $T$ , the firm is locked in and will incur a firing tax  $pF$  with probability  $\delta$  at each  $t > T$ .

The dismissal payment is assumed to be proportional to productivity on the grounds that it is more costly to get rid off a more productive worker than a less productive one. Of course, after  $TC$  is terminated, the asset pricing equation becomes:

$$(3.15) \quad (r + \delta)J^{2T} = p - w_2 - \delta pF$$

Finally, the equations for the values of employment and unemployment respectively, are:

$$(3.16) \quad W^2(t) = \int_t^T [w_2 + \delta U^2] e^{-(r+\delta)(s-t)} ds + \int_T^\infty [w_2 + \delta U^2] e^{-(r+\delta)(s-t)} ds$$

$$(3.17) \quad rU^2 = z + \theta q(\theta) [W^2 - U^2]$$

**3.2.3.1. Wage determination.** As before, the wage is set to split the surplus match value in fixed proportions. Consider now a firm at the date of job creation (or at any  $t \in [0, T]$ ). If it separates from the employee its loss will be  $J^2(t)$ . On the other hand, if destruction takes place after  $T$ , its loss will be  $J^{2T} + pF$ . This difference suggests the presence of a two-tier wage: an initial wage until  $T$  and a second one from  $T$  onwards.

The initial wage is given by:

$$(3.18) \quad w_2^* = (1 - \beta)z + \beta p(1 + c\theta)$$

while the second one is:

$$(3.19) \quad w_{2T}^* = (1 - \beta)z + \beta p(1 + c\theta) + \beta r p F$$

(see Appendix 3.A for a formal derivation).



$w_{2T}$  - the “insider” wage - is shown to be higher than the “outsider” one ( $w_2$ ). This is because of the fact that after  $T$  has elapsed the firm is locked in and as a result the “continuation” wage increases in the firing tax.

Some authors reject the plausibility of a two-tier wage (see Lindbeck and Snower, 1988) by arguing that the worker has no credible threat to force renegotiation. In any case, this issue has to be challenged on empirical grounds although its implications would affect the predictions of the model quantitatively but not qualitatively. It has to be noted however that Friesen (1996) who studied the wages of workers subject to different regulations from different Canadian provinces, found that after controlling for education, firm size, occupation and industry, incumbent workers protected from legislation appeared to extract higher wages than workers not protected by law and that starting wages tended to fall to offset subsequent increases.

**3.2.3.2. Equilibrium in regime 2.** A solution to the model of regime 2 consists of a job value and a market tightness pair ( $J^{2*}, \theta^{**}$ ) that solves (3.12) and (3.14) by first substituting out the wages in (3.14). For the same reason as before, existence of unique equilibrium is guaranteed. Equilibrium market tightness is now determined by:

$$(3.20) \quad [p - (1 - \beta)z - \beta p(1 + c\theta^{**})] - e^{-(r+\delta)T} pF(\beta r + \delta) - (r + \delta) \frac{pc}{q(\theta^{**})} = 0$$

Clearly, equilibrium  $\theta$  is again shown to be increasing in  $T$ .

### 3.3. Equilibrium

Given the two alternatives, the equilibrium can be identified by establishing the conditions under which the strategy of an individual firm  $i$  is optimal given the strategy chosen by the rest of the firms in the economy<sup>14</sup>. In other words, one seeks to find a range of  $F$  for which given that all firms, except firm  $i$ , dismiss workers in surviving matches at  $T$ , dismissal at  $T$  corresponds to the optimal response of firm  $i$  as well and a range of  $F$  for which given that all firms, except firm  $i$ , do not dismiss workers in surviving matches at  $T$ , not dismissal at  $T$  corresponds to the optimal response of firm  $i$  as well.

**Proposition 1.** *Let  $\theta^*$  be the equilibrium market tightness in regime 1. Then for any finite  $T$ , there exists an  $F^*$  where:*

$$(3.21) \quad F^* = \frac{[p - (1 - \beta)z - \beta p(1 + c\theta^*)]}{p(\beta r + \delta)}$$

*such that:*

- (a) *For  $F > F^*$  there exists a unique Nash equilibrium in which all firms dismiss workers in surviving matches at  $T$ .*
- (b) *For  $F = F^*$  some dismissal may take place at  $T$ .*

**Proof.** See Appendix 3.B □

---

<sup>14</sup>Implicit in this formulation is the assumption that firm  $i$  is too small to affect market tightness.

**Proposition 2.** *Let  $\theta^{**}$  be the equilibrium market tightness in regime 2. Then for any finite  $T$ , there exists an  $F^{**}$  where:*

$$(3.22) \quad F^{**} = \frac{[p - (1 - \beta)z - \beta p(1 + c\theta^{**})]}{p(\beta r + \delta)}$$

*such that:*

- (a) *For  $F < F^{**}$  there exists a unique Nash equilibrium in which all firms do not dismiss workers in surviving matches at  $T$ .*
- (b) *For  $F = F^{**}$  some dismissal may take place at  $T$ .*

**Proof.** See Appendix 3.B □

**Proposition 3.** *Let  $\theta^*$  and  $\theta^{**}$  be the equilibrium values of market tightness in regimes 1 and 2 respectively. Then for  $F = F^{**}$ :*

$$(3.23) \quad F^* = F^{**} \text{ always}$$

**Proof.** See Appendix 3.B □

Given Propositions 1, 2 and 3, Corollary 1 follows:

**Proposition 4.** *For any increasing, concave and homogeneous of degree one matching function and for any finite  $T$ , there exists an  $\tilde{F}$  such that:*

$$(3.24) \quad \tilde{F} = \frac{[p - (1 - \beta)z - \beta p(1 + c\theta^*)]}{p(\beta r + \delta)}$$

*and:*

**Corollary 5.** (1)

- (a) For all  $F > \tilde{F}$  all firms chose to destroy surviving matches at  $T$
- (b) For all  $F < \tilde{F}$  all firms chose not to destroy surviving matches at  $T$
- (c) For  $F = \tilde{F}$  some dismissal may take place at  $T$ .

Proposition 4 suggests that implementing a probation period regulation (or facilitating the use of *TCs*) may result in jobs being destroyed not only due to the arrival of an adverse shock but also as the outcome of endogenous separation decisions at  $T$ . As a result, such a policy may adversely affect the job turnover rate because of the more frequent firing taking place.

### 3.3.1. Equilibrium unemployment

In this section I discuss the implications for equilibrium unemployment. There are two cases to consider: when  $T = 0$  in which case the model collapses to the Pissarides version (2000, ch. 9) and when  $T$  is non-zero and finite<sup>15</sup>.

When  $T = 0$  there is no equilibrium in regime 1 because in any meaningful equilibrium the job value must be strictly positive. Therefore, by equating job creation to job destruction, steady state equilibrium unemployment is given by:

$$(3.25) \quad u^{**0} = \frac{\delta}{\delta + \theta^{**0} q(\theta^{**0})}$$

---

<sup>15</sup>Clearly, I abstract from the possibility of  $T \rightarrow +\infty$  since trivially this implies that there are in effect no firing restrictions.

(where the superscript 0 indicates that  $T = 0$ ). Since  $\theta^{**}$  is increasing in the probation period and  $u$  is decreasing in  $\theta$ <sup>16</sup>,  $u^{**0}$  is unambiguously higher than its respective value in a policy-free environment.

If  $T$  is non-zero and finite one needs to consider whether  $F < \tilde{F}$ ,  $F > \tilde{F}$  or  $F = \tilde{F}$ .

Under the assumption that  $F < \tilde{F}$ , equilibrium unemployment rate is again given by (3.25) where now the equilibrium value of market tightness is higher, implying a lower value for unemployment.

When  $F > \tilde{F}$  and with an exogenous arrival rate of the adverse shock drawn from a Poisson distribution, the fraction of newly created jobs that survive to  $T$  is given by  $e^{-\delta T}$  and therefore job destruction is now determined by:

$$(3.26) \quad JD^T = \delta(1 - u^T) + JC^T e^{-\delta T}$$

(where again the superscript  $T$  indicates the finite probation horizon). Hence, equilibrium unemployment rate is given by:

$$(3.27) \quad u^{*T} = \frac{\delta}{\delta + \theta^{*T} q(\theta^{*T}) (1 - e^{-\delta T})}$$

---

<sup>16</sup>Implicit to this, is the assumption that the direct effect of  $\theta$  on  $u$  more than offsets the indirect effect through  $q(\theta)$ .

Comparison with (3.25) reveals that it is not necessary for unemployment to be higher when  $T = 0$ . More specifically, if:

$$(3.28) \quad \theta^{*T} q(\theta^{*T}) (1 - e^{-\delta T}) < \theta^{**0} q(\theta^{**0})$$

then  $u^{*T}$  will be higher than  $u^{**0}$ . In other words, the introduction of a trial period (or  $TC$ ) which seems to “alleviate” the policy restrictions, may actually amplify the unemployment rate by enabling employers to destroy surviving matches at the end of  $T$ . The latter is more likely to be true when  $T$  is relatively small and as a result the more frequent job destruction will cause unemployment to rise.

Finally when  $F = \tilde{F}$  some dismissal may take place. The equilibrium cannot be predicted ex ante but the possible outcomes can be Pareto ranked, at least in terms of unemployment. To see this consider what happens when  $F = \tilde{F}$ : equilibrium market tightness is the same in both regimes. Therefore,  $u^T(\theta^{*T})$  is unambiguously higher than  $u^T(\theta^{**T})$  because of the dismissal occurring at  $T$ . All other cases are clearly worse off than  $u^T(\theta^{**T})$  since some destruction takes place at  $T$ , but better off than  $u^T(\theta^{*T})$  because some workers are being kept at jobs when the  $TC$  terminates.

### 3.3.2. Firing taxes, probation period and the job destruction rate

Examination of (3.24) establishes that there is a close link between the two policy instruments. Namely,  $\partial \tilde{F} / \partial T < 0$  since  $\partial \theta^* / \partial T > 0$  always. As  $T$  expands,  $J^{1*}$

increases (the job value curve shifts upwards). This is also true for regime 2, but this positive effect is partly offset by the more heavily discounted profits after  $T$ . As a result,  $J^{1*}$  increases faster than  $J^{2*}$  in  $T$  and hence a lower  $F$  is required to make firms willing to switch/stay to regime 1.

As the firing tax increases and approaches  $\tilde{F}$  from below, equilibrium unemployment rate rises, makes a jump upwards when  $F = \tilde{F}$  and remains fixed thereafter (while equilibrium market tightness decreases until  $\tilde{F}$  and then stays the same irrespective of  $F$ ). This situation is depicted in Figure 3.

All the above have implications for the steady state job destruction rate i.e. the inflow of workers into the unemployment pool. When firms chose optimally not to dismiss employees at  $T$ , the job destruction rate is equal to  $\delta$ . As the firing restriction becomes more harsh and exceeds the critical value  $\tilde{F}$ , the job destruction rate makes an upward jump and remains constant for all  $F > \tilde{F}$ . More specifically, its value is given by:

$$(3.29) \quad JDR = \delta \frac{1}{1 - e^{-\delta T}}$$

$JDR$  rises because at each  $t$  a fraction  $\delta$  of matches dissolves and an additional proportion of jobs is destroyed as it reaches the end of the fixed period. The only thing that matters now is the length of  $T$ . Indeed, (3.29) reveals that expansions of  $T$  cause  $JDR$  to decrease as a result of a less frequent dismissal.

Overall, heavy layoff costs are likely to induce employers to destroy matches at  $T$ . When this happens, the resulting job separation rate will increase and it will be higher the shorter the length of the  $TC$  is.

By taking the assumption of firing taxes as given, the analysis establishes a theoretical approach within which such regulations can be examined explicitly. In this respect, it demonstrates the complexity of dismissal policies and offers a potential explanation for the observed patterns. Firstly, it suggests that temporary tenures are likely to be observed mostly when firing restrictions on regular contracts are more severe, something consistent with the data. Secondly, it argues that outflows of jobs and workers may be inversely affected by such ruling procedures, something which may be hidden behind the ambiguous empirical relationship between  $EPL$  and the job turnover rates.

### 3.4. Productivity Differences

This section discusses productivity differences in segmented markets and the effects of policy on worker groups of diverse skills.

Before proceeding, one must stress the importance of the assumptions governing the unemployment income. As the analysis suggests, this income is independent of worker skill. Clearly, this makes sense if the latter is defined to be imputed income from leisure activities. Of course, it can include other forms of income as well such as state provided unemployment benefit but these would have to be made proportional to  $p$  (or  $w$ ). What turns out to be crucial for the results that follow is



that at least some portion of the income that has to be given up when transition from unemployment to employment takes place, is independent of productivity. This assumption reassures that market tightness and wages increase with skill and that unemployment rates fall. A second important issue is that skill markets are segmented. That is, each firm and worker participate solely in one market with the same level of productivity. Finally, the length of the probation period (or  $TC$ ) must be irrespective of skill. This is actually not a bad claim. The vast majority of real world policy schemes coincides with this assertion. In practice, some distinctions are made for blue and white collar workers but no other differentiation is made for the productivity differences within each group.

To facilitate the analysis, I will consider the simplest case possible: two different levels of productivity which will be referred to as "high" and "low" skill. Given the discussion of the previous section one can formally derive the following Proposition:

**Proposition 6.** *Let  $p^h$  and  $p^l$  denote the productivity levels of high and low skilled workers respectively, with:*

$$(3.30) \quad p^h = ap^l, \text{ and } a > 1$$

*Then for any finite and common among skill groups  $T$ , it is true that:*

$$(3.31) \quad \tilde{F}^h > \tilde{F}^l \text{ always}$$

**Proof.** See Appendix 3.B

□

Proposition 4 establishes that there is a range of firing taxes such that for any  $F \in (\tilde{F}^l, \tilde{F}^h)$ , employers in high skilled jobs keep their workers while firms in low productivity matches dismiss employees at the end of  $T$ . Workers in these two segmented markets enjoy different wages, face different unemployment and job turnover rates not only because their different productivity levels determine different equilibrium market tightness but also because the initiation of such a policy causes different firm responses.

The argument can be generalized. Hence, Proposition 5 follows:

**Proposition 7.** *Let there be a distribution of productivities in the economy with CDF  $P(p)$ . Then, there is a distribution of critical tax levels across different productivity groups, so that  $G(F)$  is the proportion of firms that chose to destroy surviving matches at the end of  $T$  (where  $G(\cdot)$  is the CDF of the firing restriction).*

**Proof.** See Appendix 3.B □

Based on Proposition 5,  $JDR$  is determined by:

$$(3.32) \quad JDR = G(F)\delta \frac{1}{1 - e^{-\delta T}} + [1 - G(F)]\delta$$

Clearly, a rise in the firing tax unambiguously amplifies the job destruction rate as it augments the proportion of jobs that are being destroyed at  $T$ .

### 3.4.1. Wage inequality

Given different tax thresholds among different productivity clusters, wage inequality is one of the first things that come immediately to mind. This is particularly relevant since the implementation of policies of this sort is usually justified on the grounds of protecting lower-skilled groups that are more vulnerable to the unemployment risk and the reduction in real wages. For this reason, I now raise the question of whether wage inequality rises or falls when a policy is initiated.

**Proposition 8.** *Let  $ap^l$  and  $p^l$  denote the productivity levels of high and low skilled workers respectively,  $\forall a > 1$ . Then, for any increasing, concave and homogeneous of degree one matching function, wage inequality is highest when no policy is implemented and lowest when  $T = 0$ . For any finite and non-zero  $T$ , the wage differential increases with  $T$  and is higher when  $F \in (\tilde{F}^l, \tilde{F}^h)$ .*

**Proof.** See Appendix 3.B □

The reason that wage inequality decreases with policy is based on the proportionality of the firing tax to the productivity level. As a consequence, the downward shift of the job value curve is less smooth for the high-skilled group resulting in a reduction in the difference of the two equilibrium tightness values which in turn causes a decrease in the wage differential. Since temporary predetermined tenures introduce a tax-free period, it follows that inequality rises with  $T$ . Moreover, inequality is magnified whenever  $F \in (\tilde{F}^l, \tilde{F}^h)$  because insiders'

wage increases further relative to that of the other group and it is only high-skilled workers that become insiders. Overall, this is the most interesting result. Such policy designs may not only intensify job destruction margins but aggravate wage differentials by endogenously creating a dichotomy between insiders and outsiders i.e. employees who enjoy the benefits of protection and those who do not.

### 3.5. Computational experiments

In this section I provide computed solutions that can proliferate the model's implications for policy design and put to the test its ability to replicate real world examples. For the most part I will use the parameter values that are provided in Mortensen (1994). The numerical values for the fixed period and the level of firing taxes are deduced from OECD data, Guell (2003) and Garibaldi and Violante (2004).

The functional form of the aggregate matching function is assumed to be log linear, so that:

$$(3.33) \quad q(\theta) = A\theta^{-\eta}, \text{ for some } A > 0$$

I normalize the time period to be one quarter and without loss of generality set the average productivity  $p$  equal to 1 while concentrating on the implications for the high productivity group ( $p = 1.5$ ) and the low productivity one ( $p = 0.5$ ) - which I assume to be of equal proportions. The interest rate is set to 0.01. The recruiting cost, the value of leisure and labour's share are fixed to 0.3, 0.349 and

0.5 respectively as in Mortensen (1994). In what follows, I will abstract from the possibility of search externalities<sup>17</sup> and concentrate only on efficient equilibrium outcomes by setting the elasticity of matches with respect to unemployment ( $\eta$ ) equal to 0.5.

By trial and error, I adjust the values of  $A$  and  $\delta$  so that on average the model reproduces an equilibrium unemployment rate around 10 percent and an expected duration of unemployment of four to five months. This gives me  $A = 0.65$  and  $\delta = 0.08$ .

As far as the length of  $T$  is concerned, I use the information provided by the OECD regarding the maximum cumulated duration allowed for fixed term contracts (see Table 3.2, Panel B). For this reason I set  $T$  equal to 8, 10, 12 and 14<sup>18</sup>. Information for the firing taxes is not easily identified. Garibaldi and Violante (2004), using information from Guell (2003) estimate that the total firing costs in Italy endured by firms upon separation, amount to around seven monthly wages. The latter is adjusted for the relevant probability of a worker's appeal for unfair dismissal being granted. However, this estimate includes severance compensation as well and needs to be decomposed into its two different components. Garibaldi and Violante find that on average firing taxes are equal to around 24 to 34 percent of the entire firing charge incurred. Based on these findings, I assume that the policy maker decides for the level of the firing restriction according to the wage of

---

<sup>17</sup>See Hosios (1990).

<sup>18</sup>I set  $T$  equal to 14 as well, under the assumption that sometimes the temporary contract may be extended for a short period of time.

the average productivity group ( $p = 1$ ) that would have been agreed, should the temporary contract become permanent (since it is the stringency governing regular employment that is of main concern). The wage however is endogenously derived and as a result a numerical value cannot be explicitly computed. Therefore, I start with an initial guess for  $F$  and change it accordingly until the latter and the one implied by the equilibrium in regime 2 converge.

Simulation results are illustrated in Table 1.4. When the temporary contract lasts for 2 years there is no equilibrium in regime 1 (not reported). When the period is extended to 10 quarters, a firing tax residing at the higher band will induce lower productivity firms to destroy surviving matches at  $T$ . This suggest that the model has the ability to offer theoretically one candidate explanation as to why temporary contracts grew only modestly in countries like Ireland and Italy i.e. economies with quite short maximum cumulated period allowed.

As the period increases to 3 and 4 years, only modest firing charges are required to make employers of low productivity jobs willing to terminate the contract at  $T$ . This might replicate the cases of Portugal and Spain which allow quite prolonged  $TCs$  and have seen a remarkable expansion of temporary employment during the 1990's and early 2000's.

The average job destruction rates are clearly amplified by the presence of heavy dismissal costs. The percentage increase in average  $JDR$  ranges from 40 percent (when  $T = 10$  and  $F = 0.732$ ) to 24 percent (when  $T = 14$  and  $F = 0.631$ ).

Figures 4 and 5 deliver a clear illustration of the disadvantage of low productivity workers when common firing restrictions are in place. The wage differential increases with  $T$  (Figure 5) and jumps upwards when it is optimal for low-skilled firms to dismiss workers at the end of  $T$ . The area within the two curves in Figure 4 corresponds to low skilled workers being fired when the contract expires but high productivity matches, that have survived to  $T$ , being retained. In an endogenous job destruction framework, tightening termination laws would reduce the separation rate in those matches that are preserved but aggravate it in the rest. The overall outcome would depend on the severity of the reform, the preexisting composition of employment in terms of contractual agreements and on the distribution of productivity. It is this dualism of firing policies that may be responsible for the ambiguous overall patterns that we observe in the data.

### 3.6. Conclusions

Empirical results are to some extent inconclusive about the effects of dismissal policies on job and labour flows. Some countries have relatively elevated turnover rates despite their strict layoff policies. At the same time, temporary contractual agreements have seen a notable expansion in economies where heavy firing restrictions on regular employment have remained largely in place.

This paper has suggested one reason that could partially explain the observed phenomena. Probation periods (or temporary contracts) that enable firms to dissolve matches costlessly, may provide incentives to dispose off employees when

this time period elapses, thereby increasing flows into unemployment. In addition, the model predicts that the number of temporary job tenures will increase when termination costs on regular employment are high and probation periods long. Hence, it gives a clear warning to policy makers when planning the institutional arrangements. As the results suggest, the two policy instruments are closely linked and one should be cautious for the individual practices about to be exercised as they may influence the composition of employment, create groups of insiders and outsiders and affect the wage differentials among different productivity clusters.

On the purely empirical side, this work provides an additional motivation for collecting data regarding the transition rates of different skill groups. In that way, not only one could test the predictions of the model more accurately but provide more insight for institutional design.

In any case, the structural differences in employment performance indicators call for theoretical improvements in the areas of friction and labour market flexibility modelling. That could be one way to deepen our understanding of the dynamics and try to reconcile some apparent inconsistencies.



## Appendix 3.A: Wage determination

To derive the wage in regime 1 we use the first order condition  $\beta [J^1(t) - V^1] = (1 - \beta) [W^1(t) - U^1]$ . Given (3.3) and (3.4) we have:

$$(A1) \quad \beta J^1(t) = \int_t^T \beta [p - w_1] e^{-(r+\delta)(s-t)} ds$$

and

$$(A2) \quad (1 - \beta) [W^1(t) - U^1] = \int_t^T (1 - \beta) [w_1 - rU^1] e^{-(r+\delta)(s-t)} ds$$

Subtracting the two equations and by taking into account that  $V^1 = 0$ :

$$(A3) \quad 0 = \int_t^T [\beta p - w_1 + (1 - \beta)rU^1] e^{-(r+\delta)(s-t)} ds$$

Differentiating (A3) wrt  $t$ , we obtain:

$$(A4) \quad w_1 = \beta p + (1 - \beta)rU^1$$

Consider now that  $W^1 - U^1 = \frac{\beta}{1 - \beta} J^1 = \frac{\beta}{1 - \beta} \frac{pc}{q(\theta)}$  and substitute it into (3.5) to get:

$$(A5) \quad rU^1 = z + \theta pc \frac{\beta}{1 - \beta}$$

Hence, (A5) and (A4) imply that:

$$(A6) \quad w_1^* = (1 - \beta)z + \beta p(1 + c\theta)$$

The wage profile is constant in  $t$  since both, employer and employee make capital losses as  $t \rightarrow T$ . These losses must be shared according to the Nash bargaining procedure. Implicit to this of course is the fact that the wage is renegotiated continuously.

To derive the wage in (3.19), we first note that:

$$(A7) \quad \beta J^2(T) = \beta \frac{p - w_{2T} - \delta p F}{r + \delta}$$

and that:

$$(A8) \quad (1 - \beta)[W^{2T} - U^2] = (1 - \beta) \frac{w_{2T} - rU^2}{r + \delta}$$

Using the sharing rule  $\beta [J^{2T} + pF - V^2] = (1 - \beta) [W^{2T} - U^2]$ , and (A7) and (A8) we obtain:

$$(A9) \quad w_{2T} = \beta p + (1 - \beta)rU^2 + \beta r p F$$

and by substituting  $rU^2$  out of (A9):

$$(A10) \quad w_{2T}^* = (1 - \beta)z + \beta p(1 + c\theta) + \beta r p F$$

The initial wage ( $w_2^*$ ) is then determined by taking the sharing rule  $\beta [J^{2T} + pF - V^2] = (1 - \beta) [W^{2T} - U^2]$  as given.

$w_2^*$  is irrespective of  $F$  because the worker has still no credible threat to negotiate an increase since she knows that if she is fired at any  $t$  before  $T$ , the firm will

not have to incur any firing cost. It is this asymmetry between the two parties that causes the wage to be lower in all negotiations for  $t \in [0, T]$ .

## Appendix 3.B: Proofs of propositions

### *Proof of Proposition 1*

The optimality of “dismissal at  $T$ ” implies that:

$$(B1) \quad J^1(\theta^*) \geq J^2(\theta^*)$$

Note that this rule is always time consistent in the sense that it reassures the optimality of the response at  $T$  as well. The intuition for this is simple: dismissal at  $T$  will be optimal if expected profits from continuation are non-positive. Substituting  $\theta^*$  into  $J^2$  results in  $J^2(\theta^*) = J^1(\theta^*) + J^{2T}(\theta^*)$ . By implication of (B1), the optimality rule is reduced to:

$$(B2) \quad J^{2T}(\theta^*) \leq 0$$

which defines the following “critical level” of the firing tax:

$$(B3) \quad F^* = \frac{[p - (1 - \beta)z - \beta p(1 + c\theta^*)]}{p(\beta r + \delta)}$$

### *Proof of Proposition 2*

The proof of Proposition 2 is analogous to that of Proposition 1 with the optimally rule now being:

$$(B4) \quad J^1(\theta^{**}) \leq J^2(\theta^{**})$$

which implies:

$$(B5) \quad J^{2T}(\theta^{**}) \geq 0$$

This gives rise to the following firing tax:

$$(B6) \quad F^{**} = \frac{[p - (1 - \beta)z - \beta p(1 + c\theta^{**})]}{p(\beta r + \delta)}$$

*Proof*

*Proof of Proposition 3*

The result follows directly by setting  $F = F^{**}$  and substituting into (3.20).

Doing so produces:

$$(B7) \quad [p - (1 - \beta)z - \beta p(1 + c\theta^{**})] (1 - e^{-(r+\delta)T}) - (r + \delta) \frac{pc}{q(\theta^{**})} = 0$$

Comparing (3.10) and (B7), it immediately follows that  $\theta^* = \theta^{**}$  and by (3.21) and (3.22) we have that  $F^* = F^{**}$ . Apparently, one derives the same result by setting  $F = F^*$  and substituting into (3.10).

To understand the importance and relevance of proposition 3, it is useful to think of  $F^{**}$  as a function of  $F$  (since  $\theta^{**}$  is - after all - a function of  $F$ ). Inspection

of (3.20) and (3.22) establishes that  $F^{**}(F)$  is monotonically increasing and that  $F^{**} > 0$  when  $F = 0$ . This implies that  $F^{**}(F)$  lies above the 45 degree line when  $F < F^*$  and below it when  $F > F^*$ . In other words, when  $F < F^*$  it is also true that  $F < F^{**}$ .

*Proof of Proposition 5*

The first thing we need to establish is that  $\theta^{*h}$  is higher than  $\theta^{*l}$ . This is merely a result driven by the fact that high-skill workers enjoy lower relative returns from leisure activities. The two equilibrium values are determined by:

$$(B8) \quad (r + \delta) \frac{p^l c}{q(\theta^{*h})} + \beta p^l c \theta^{*h} [1 - e^{-(r+\delta)T}] = [p^l - \frac{(1-\beta)z}{a} - \beta p^l] [1 - e^{-(r+\delta)T}]$$

for high skilled when  $F = \tilde{F}^h$ , and

$$(B9) \quad (r + \delta) \frac{p^l c}{q(\theta^{*l})} + \beta p^l c \theta^{*l} [1 - e^{-(r+\delta)T}] = [p^l - (1-\beta)z - \beta p^l] [1 - e^{-(r+\delta)T}]$$

for low skilled when  $F = \tilde{F}^l$ . Since the l.h.s.'s in both equations are increasing in market tightness, it immediately follows that for any  $a > 1$ ,  $\theta^{*h} > \theta^{*l}$ . Given the derivation of the critical tax in (3.24), the difference between  $\tilde{F}^h$  and  $\tilde{F}^l$  is given by:

$$(B10) \quad \tilde{F}^h - \tilde{F}^l = \frac{(1-\beta)z}{p^l(\beta r + \delta)} \left( \frac{a-1}{a} \right) - \frac{\beta c}{\beta r + \delta} (\theta^{*h} - \theta^{*l})$$

Therefore, the relevant question is whether  $\theta^{*h} - \theta^{*l} \leq \frac{(1-\beta)z}{p^l c \beta} \left( \frac{a-1}{a} \right)$ .

Let

$$(B11) \quad \theta^{*h} - \theta^{*l} = \frac{(1-\beta)z}{p^l c \beta} \left( \frac{a-1}{a} \right)$$

Then subtracting (B9) from (B8) and substituting  $\theta^{*h} - \theta^{*l}$  out from (B11), we find that  $(r + \delta)c \left( \frac{1}{q(\theta^{*h})} - \frac{1}{q(\theta^{*l})} \right) = 0$  which cannot be true since  $\theta^{*h} > \theta^{*l}$  implies that  $\frac{1}{q(\theta^{*h})} > \frac{1}{q(\theta^{*l})}$ .

Assume now that:

$$(B12) \quad \theta^{*h} - \theta^{*l} > \frac{(1-\beta)z}{p^l c \beta} \left( \frac{a-1}{a} \right)$$

say  $\theta^{*h} - \theta^{*l} = \Delta \frac{(1-\beta)z}{p^l c \beta} \left( \frac{a-1}{a} \right)$  for some  $\Delta > 1$ . Then (B8), (B9) and (B12) imply that  $(r + \delta)c \left( \frac{1}{q(\theta^{*h})} - \frac{1}{q(\theta^{*l})} \right) < 0$  which can never be true. Hence it must be that  $\theta^{*h} - \theta^{*l} < \frac{(1-\beta)z}{p^l c \beta} \left( \frac{a-1}{a} \right)$  and thus:

$$(B13) \quad \tilde{F}^h - \tilde{F}^l > 0$$

### *Proof of Proposition 6*

Let  $P(p)$  be the *CDF* of productivities across segmented markets. Since  $\tilde{F}$  is a continuous function of  $p$  then it follows that there is a distribution of  $\tilde{F}$  in the economy with e.g.  $G(\tilde{F})$  as the *CDF*. Thus, for any actual firing restriction  $F$ ,  $G(F)$  represents the percentage of firms that chose to get rid off the match at the end of the probation period.

*Proof of Proposition 7*

To establish the result in Proposition 7 we make use of (3.10) and subtract the two equilibrium conditions (i.e. when both productivity groups destroy at  $T$ ) to obtain:

$$(B14) \quad [1 - e^{-(r+\delta)T}] \left[ (1 - \beta)z \frac{a-1}{a} - \beta p_l c(\theta^h - \theta^l) \right] - \\ + (r + \delta) p_l c \left[ \frac{1}{q(\theta^h)} - \frac{1}{q(\theta^l)} \right] = 0$$

Now note that when  $T$  tends to infinity, (B14) refers to the “no tax” case. As  $T$  increases from 0 to  $\infty$ , the l.h.s. of (B14) increases as well. Since the l.h.s. is decreasing in  $(\theta^h - \theta^l)$  it is true that  $(\theta^h - \theta^l)$  increases as  $T$  rises, for any decreasing choice of  $q(\cdot)$ . Since the wage differential  $(\Delta w)$  is increasing in  $(\theta^h - \theta^l)$  it follows that  $\Delta w$  rises as  $T$  rises. Given that the wage of high productivity insiders will have a firing tax component while the low productivity outsiders will not be protected by such regulations,  $\Delta w$  is amplified when  $T$  is finite and  $F \in (\tilde{F}^l, \tilde{F}^h)$ .

Table 3.1 (*Source is OECD Employment Outlook 1999*)*EPL rankings*

	Regular employment		Temporary employment		Overall <i>EPL</i> strictness	
	Late 1980s	Late 1990s	Late 1980s	Late 1990s	Late 1980s	Late 1990s
Austria	2.6	2.6	1.8	1.8	2.2	2.2
Belgium	1.5	1.5	4.6	2.8	3.1	2.1
France	2.3	2.3	3.1	3.6	2.7	3.0
Germany	2.7	2.8	3.8	2.3	3.2	2.5
Ireland	1.6	1.6	0.3	0.3	0.9	0.9
Netherlands	3.1	3.1	2.4	1.2	2.7	2.1
Switzerland	1.2	1.2	0.9	0.9	1.0	1.0
UK	0.8	0.8	0.3	0.3	0.5	0.5
Greece	2.5	2.4	4.8	4.8	3.6	3.6
Italy	2.8	2.8	5.4	3.8	4.1	3.3
Portugal	4.8	4.3	3.4	3.0	4.1	3.7
Spain	3.9	2.6	3.5	3.5	3.7	3.1
Turkey	—	2.6	—	4.9	—	3.8
Denmark	1.6	1.6	2.6	0.9	2.1	1.2
Finland	2.7	2.1	1.9	1.9	2.3	2.0
Norway	2.4	2.4	3.5	2.8	3.0	2.6
Sweden	2.8	2.8	4.1	1.6	3.5	2.2
Czech Rep.	—	2.8	—	0.5	—	1.7
Hungary	—	2.1	—	0.6	—	1.4
Poland	—	2.2	—	1.0	—	1.6
Canada	0.9	0.9	0.3	0.3	0.6	0.6
Mexico	—	2.3	—	—	—	—
US	0.2	0.2	0.3	0.3	0.2	0.2
Australia	1.0	1.0	0.9	0.9	0.9	0.9
Japan	2.7	2.7	—	2.1	—	2.4



Table 3.2 (*Source is OECD Employment Outlook 1999*)

<i>Panel A: Trial period (in months)</i>			<i>Panel B: Regulation of Fixed-term contracts</i>		
	Late 1980s	Late 1990s	Maximum cumulated period allowed (in months)		
			Late 1980s	Late 1990s	
Austria	1.0	1.0			
Belgium	3.3	3.3			
France	1.6	1.6	Belgium	24.0	30.0
Germany	6.0	6.0	France	24.0	18.0
Ireland	12.0	12.0	Germany	18.0	24.0
Netherlands	2.0	2.0	Ireland	12.0	12.0
Switzerland	2.0	2.0	Italy	9.0	15.0
UK	24.0	24.0	Portugal	30.0	30.0
Greece	2.0	3.0	Spain	36.0	36.0
Italy	0.8	0.8			
Portugal	1.0	2.0			
Spain	1.7	2.5			
Denmark	1.5	—			
Finland	4.0	4.0			
Norway	1.0	1.0			
Sweden	6.0	6.0			
Canada	3.0	3.0			
US	—	—			

Table 3.3

*Estimation results*

Dependent variable	<i>TE</i>	<i>TE</i>	<i>TE</i>	<i>TE</i>
Adjusted $R^2$	0.17	0.14	0.37	0.40
Constant	5.19*	5.93**	4.93	3.65
	(3.15)	(3.18)	(4.12)	(4.27)
<i>EPL_RE</i>	3.24**		2.67**	
	(1.34)		(1.21)	
<i>DD</i>		2.28**		2.21**
		(1.05)		(0.91)
<i>UR</i>			0.53*	0.61**
			(0.31)	(0.30)
<i>LPGR</i>			-3.04**	-3.08**
			(1.00)	(0.97)
<i>GDPGR</i>			1.11*	1.23*
			(0.78)	(0.77)

Notes to Table 3.3: *Sources are* OECD Statistical Compendium 2003 and OECD Employment Outlook 1999.\*(\*\*) denotes stat. significance at 10% (5%) level. *TE* is temporary employment as % of total dependent employment (average 1998-2002), *EPL\_RE* is

employment protection legislation for regular employment (late 1990s), *DD* denotes difficulty of dismissal (late 1990s), *UR* is unemployment rate (average 1998-2002), *LPGR* denotes labour productivity % change (average 1998-2002) and *GDPGR* is real *GDP* growth rate (average 1998-2002). The sample includes 24 countries: Austria, Belgium, Canada, Czech Republic Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Japan, Mexico, Netherlands, Norway, Poland, Portugal, Spain, Sweden, Switzerland, Turkey, UK and US. For the multivariate regression, 23 observations were used since there were no data for *LPGR* for Turkey.

Table 3.4

*Simulation results*

	High productivity ( $p = 1.5$ )			Low productivity ( $p = 0.5$ )		
$T = 10$ (2.5 years)						
	$F = 0.520$	$F = 0.626$	$F = 0.732$	$F = 0.520$	$F = 0.626$	$F = 0.732$
$\tilde{F}$		1.138			0.656	
$u$	7.92%	7.96%	8.01%	13.14%	13.32%	<b>21.90%</b>
$JDR$	8.00%	8.00%	8.00%	8.00%	8.00%	<b>14.52%</b>
$T = 12$ (3 years)						
	$F = 0.522$	$F = 0.629$	$F = 0.735$	$F = 0.522$	$F = 0.629$	$F = 0.735$
$\tilde{F}$		1.03			0.60	
$u$	7.89%	7.93%	7.96%	13.00%	<b>19.65%</b>	<b>19.65%</b>
$JDR$	8.00%	8.00%	8.00%	8.00%	<b>12.96%</b>	<b>12.96%</b>
$T = 14$ (3.5 years)						
	$F = 0.523$	$F = 0.631$	$F = 0.738$	$F = 0.523$	$F = 0.631$	$F = 0.738$
$\tilde{F}$		0.967			0.565	
$u$	7.87%	7.89%	7.92%	12.88%	<b>18.06%</b>	<b>18.06%</b>
$JDR$	8.00%	8.00%	8.00%	8.00%	<b>11.87%</b>	<b>11.87%</b>

Notes to Table 3.4:  $T$  is measured in quarters. The values for  $F$  are based on 24, 29 and 34 percent of seven months' wages of average productivity ( $p = 1$ ) that is agreed, when the temporary contract becomes permanent. The rest of the parameter values are:  $A = 0.65$ ,  $\delta = 0.08$ ,  $r = 0.01$ ,  $c = 0.3$ ,  $b = 0.349$ ,  $\beta = 0.5$  and  $\eta = 0.5$ . Numbers in bold indicate equilibrium in regime 1

Figure 3.1.a: Employment Protection Legislation and Job Turnover  
Late 1980s

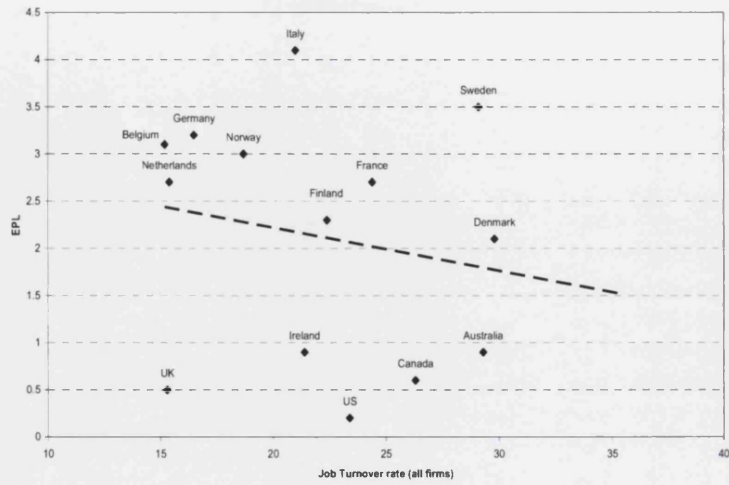
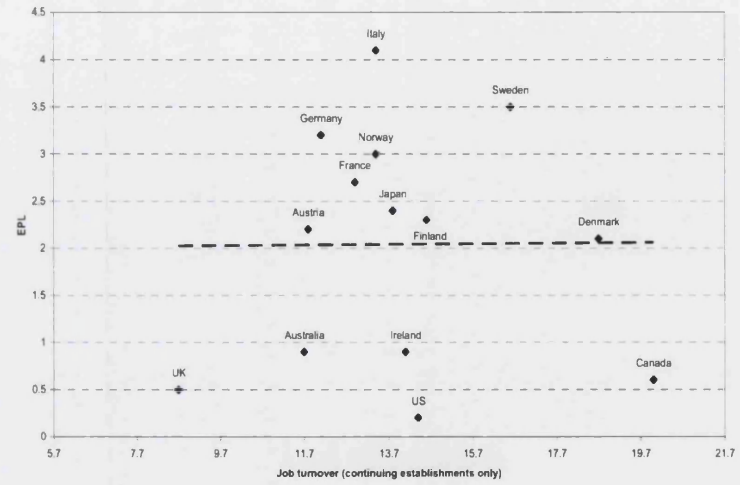


Figure 3.1.b: Employment Protection Legislation and Job Turnover  
Late 1980s



170

Figure 3.2.a: Difficulty of Dismissal and Job Turnover  
Late 1980s

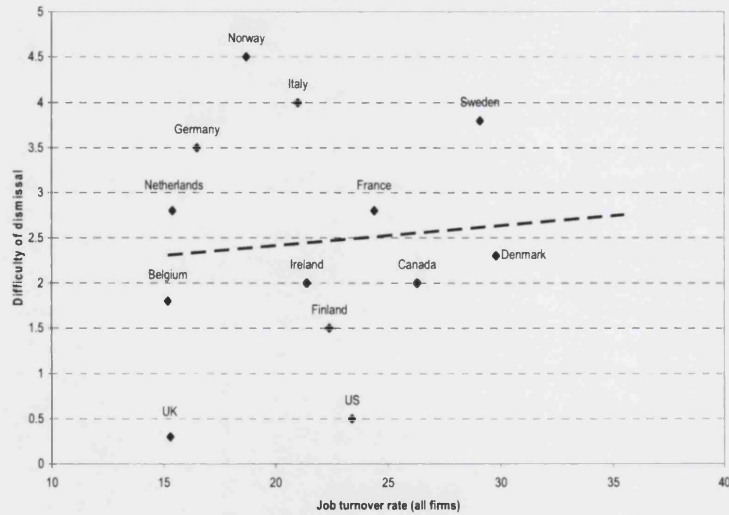
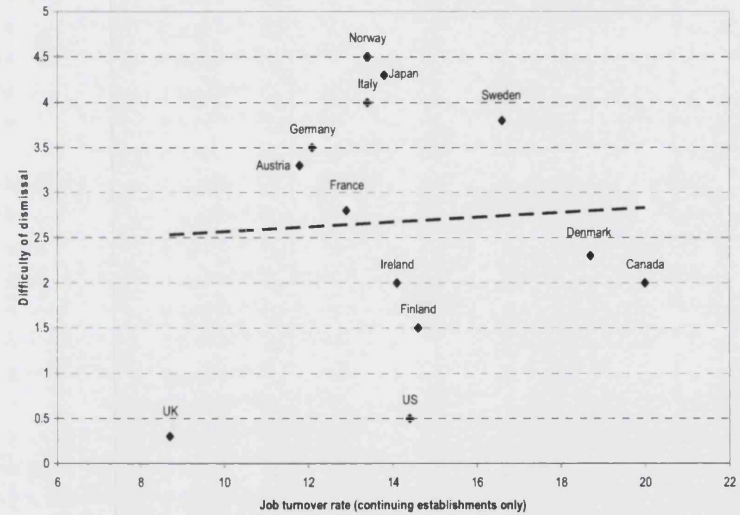
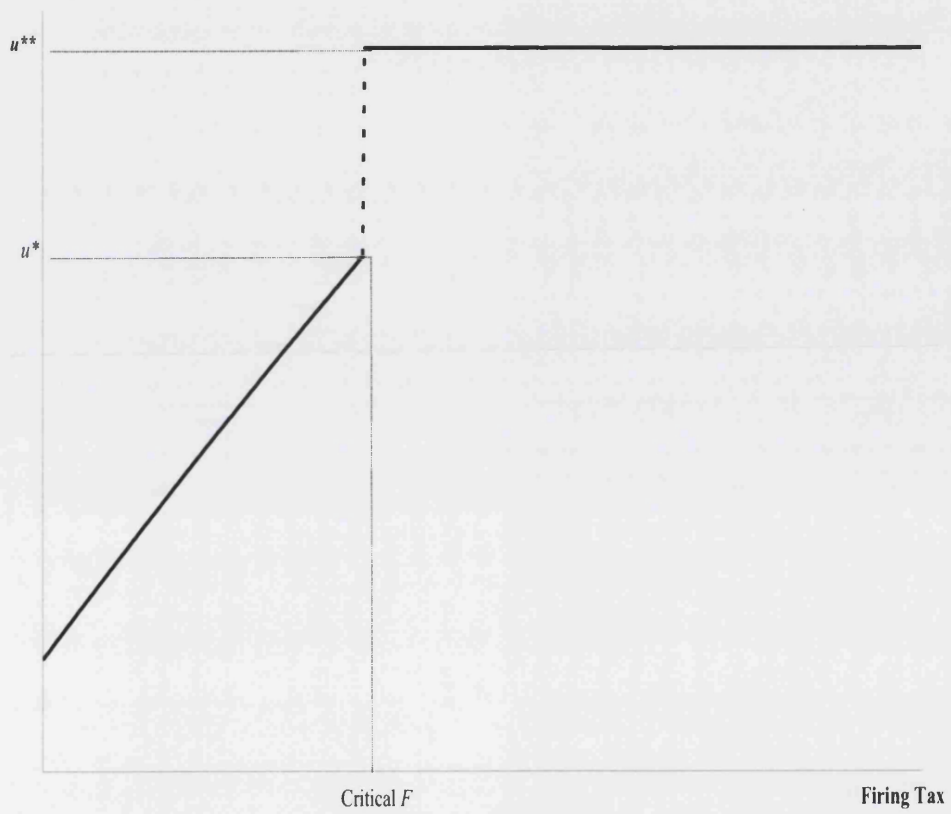


Figure 3.2.b: Difficulty of Dismissal and Job Turnover  
Late 1980s

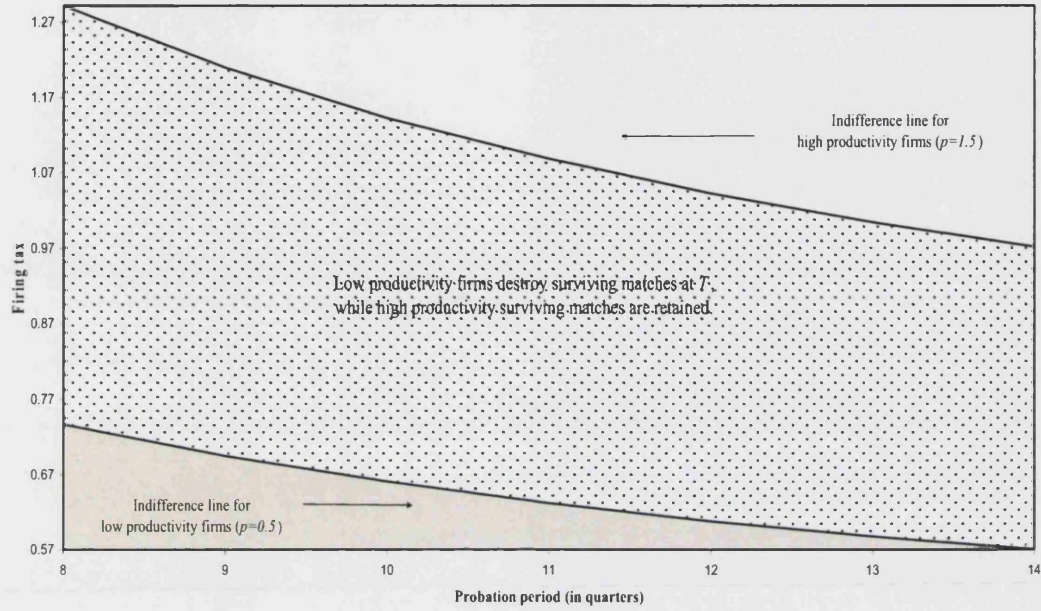


Equil. Unempl.  
Rate

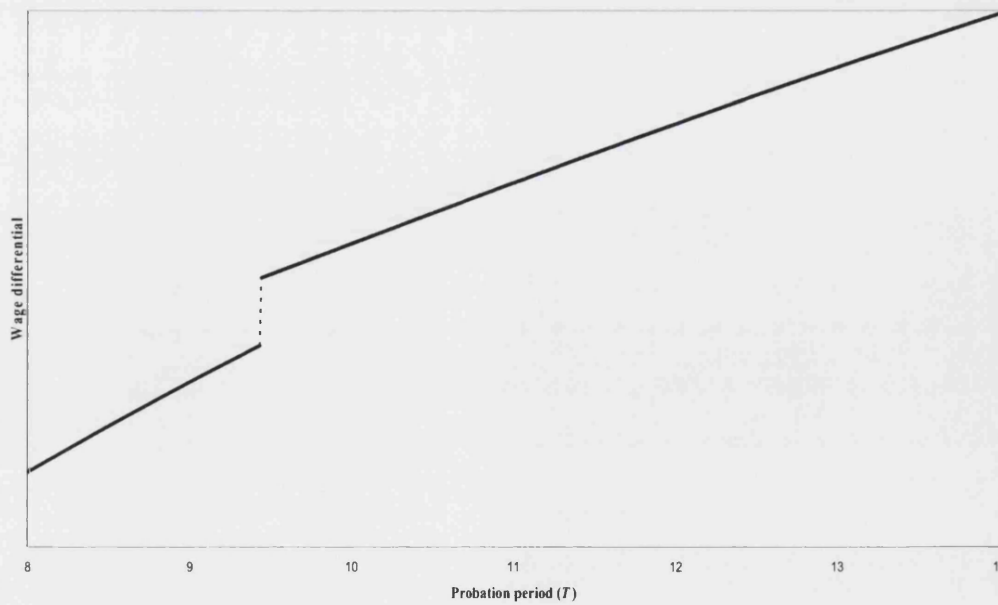
Figure 3.3: Equilibrium unemployment rate and firing tax



**Figure 3.4: Simulations**  
**Regions for  $T$  and  $F$  for low and high productivity matches**



**Figure 3.5: Simulations**  
**Wage inequality and  $T$**





## References

- [1] Abowd, J. and A. Zellner, 1985, Estimating Gross labour-Force, *Journal of Business and Economic Statistics* 3(3), 254-283.
- [2] Abowd, J. and D. Card, 1989, On the Covariance Structure of Earnings and Hours Changes, *Econometrica*, 57, 411-445.
- [3] Acemoglu, D. and R. Shimer, 1999, Efficient Unemployment Insurance, *The Journal of Political Economy*, 107(5), 893-928.
- [4] Alvarez, F. and M. Veracierto, 2001, Severance Payments in an Economy with Frictions, *Journal of Monetary Economics*, 47(3), 477-98.
- [5] Anderson, T. W. and H. Rubin, 1949, Estimation of the Parameters of a Single Equation in a Complete System of Stochastic Equations, *Annals of Mathematical Statistics*, 20, 46-63.
- [6] Attanasio, O. and G. Weber, 1993, Consumption Growth, the Interest Rate and Aggregation, *Review of Economic Studies*, 60, 631-49.
- [7] Attanasio, O., J. Banks, C. Meghir, and G. Weber, 1999, Humps and Bumps in Lifetime Consumption, *Journal of Business and Economic Statistics*, 17, 22-35.
- [8] Algan, Y., A. Cheron, J. Hairault and F. Langot, 2003, Wealth Effect on labour Market Transitions, *Review of Economic Dynamics*, 6, 156-78.
- [9] Benitez-Silva, H., 2000, A Dynamic Model of Job Search Behavior over the Life Cycle with Empirical Applications, SUNY Stonybrook working paper.

- [10] Bertola, G., T. Boer and S. Cazes, 2000, Employment protection in industrialized countries: The case for new indicators, *International Labour Review* 139(1), 57-72.
- [11] Binmore, K., A. Rubinstein and A. Wolinsky, 1986, The Nash bargaining solution in economic modelling, *Rand Journal of Economics* 17, 176-188.
- [12] Blanchard, O. and P. Diamond, 1989, The Beveridge Curve, *Brookings Papers in Economic Activity*, 0(1), 1-60.
- [13] Blinder, A. and A. Deaton, 1985, The time series consumption function revisited, *Brooking Paper on Economic Activity*, 465-521.
- [14] Browning, M. and T. Crossley, 2001, Unemployment Insurance Benefit Levels and Consumption Changes, *Journal of Public Economics*, 80(1), 1-23.
- [15] Büchtemann, C., 1991, Does (de-) regulation matter? Employment protection in West Germany, in: E. Matzner and W. Streeck, eds., *Beyond keynesianism: The socio-economic of production and full employment*, (Edward Elger, Cambridge, UK) 111-136.
- [16] Büchtemann, C., 1993, Employment security and labour markets, in: C. Buchtemann, ed., *Employment security and labour market behavior: Interdisciplinary approaches and international evidence*, (ILR Press, Ithaca, New York) 3-68.
- [17] Cagetti, M., 2003, Wealth Accumulation over the life cycle and Precautionary Savings, *Journal of Business and Economic Statistics*, 21, 339-353.
- [18] Cahuc, P. and F. Postel-Vinay, 2002, Temporary jobs, employment protection and labor market performance, *Labour Economics*, (9), 63-91.
- [19] Campbell, J. and G. Mankiw, 1989, Consumption, income and interest rates: Reinterpreting the time series evidence, *NBER Macroeconomics Annual*, 185-216.
- [20] Carroll, C., 1992, The Buffer-Stock Theory of Saving: Some Macroeconomic Evidence, *Brookings Papers on Economic Activity*, 2, 61-156.

- [21] Carroll, C., 1997, Buffer Stock Saving and the Life Cycle / Permanent Income Hypothesis, *Quarterly Journal of Economics*, CXII, 3-55.
- [22] Carroll, C. and W. Dunn, 1997, Unemployment Expectations, Jumping (S,s) Triggers and Household Balance Sheets, *NBER Macroeconomics Annual*, ed. by Benjamin S. Bernanke, and Julio Rotemberg, 165-229, MIT Press.
- [23] Cole, H. and R. Rogerson, 1999, Can the Mortensen-Pissarides matching model match the business-cycle facts?, *International Economic Review*, 40 (4), 933-959.
- [24] Costain, J., 1999, Unemployment Insurance with Endogenous Search Intensity and Precautionary Saving, UPF working paper.
- [25] Davis, S., J. C. Haltiwanger and S. Schuh, 1996, *Job Creation and Job Destruction*, Cambridge, Mass: MIT Press.
- [26] Deaton, A., 1991, Saving and Liquidity Constraints, *Econometrica*, 59, 1221-1248.
- [27] Deaton, A. and G. Laroque, 1992, On the Behavior of Commodity Prices, *Review of Economic Studies*, 59, 1-23.
- [28] Delacroix, A., 2003, Transitions into unemployment and the nature of firing costs, *Review of Economic Dynamics* 6, 651-671.
- [29] Davidson, C., and S. Woodbury, 1998, The Optimal Dole with Risk Aversion and Job Destruction, Michigan State University working paper.
- [30] Devine, T. and N. Kiefer, 1991, *Empirical labour economics: The search approach* (Oxford University press, Oxford).
- [31] Fredriksson, P. and B. Holmlund, 2001, Optimal Unemployment Insurance in Search Equilibrium, *Journal of Labour Economics*, 19(2), 370-399.
- [32] Friesen, J., 1996, The response of wages to protective labour legislation: Evidence from Canada, *Industrial and labour Relations Review* 49(2), 243-255.
- [33] Gabaix, X. and D. Laibson, 2001, The 6D Bias and the Equity-Premium Puzzle, *NBER Macroeconomics Annual*, 257-312.

- [34] Garibaldi, P. and G. Violante, 2004, The employment effects of severance payments with wage rigidities, Working paper.
- [35] Gomes, J., J. Greenwood and S. Rebelo, 2001, Equilibrium Unemployment, *Journal of Monetary Economics*, 48, 109–152.
- [36] Goodfriend, M., 1992, Information aggregation bias, *American Economic Review*, 82, 508-19.
- [37] Gourinchas P. and J. Parker, 2002, Consumption over the Life Cycle, *Econometrica*, 70, 47-89.
- [38] Grubb, D. and W. Wells, 1993, Employment regulation and patterns of work in EC countries, *OECD Economic Studies* 21, 7-58.
- [39] Gruber, J., 1997, The Consumption Smoothing Benefits of Unemployment Insurance, *American Economic Review*, 87, 192-205.
- [40] Guell, M., 2003, Fixed-term contracts and unemployment: An efficiency wage analysis, Industrial Relations Section, Princeton University, Working paper no. 433.
- [41] Hall, R., 1978, Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence, *Journal of Political Economy*, 86(6), 971-87.
- [42] Hall, R., 2004, Employment Fluctuations with Equilibrium Wage Stickiness, Stanford University mimeo.
- [43] Hansen, G. and A. Imrohoroglu, 1992, The Role of Unemployment Insurance in an Economy with Liquidity Constraints and Moral Hazard, *Journal of Political Economy*, 100, 118-142.
- [44] Hopenhayn, H. and J. Nicolini, 1997, Optimal Unemployment Insurance, *The Journal of Political Economy*, 105(2), 412-438.
- [45] Hubbard, G., J. Skinner and S. Zeldes, 1995, Precautionary Saving and Social Insurance, *Journal of Political Economy*, 103, 360-399.
- [46] Job Openings and Labour Turnover Survey, Bureau of Labour Statistics.

- [47] Joseph, G. and T. Weitzblum, 2003, Optimal Unemployment Insurance: Transitional Dynamics vs. Steady State, *Review of Economic Dynamics*, 6, 869-884.
- [48] King, R., C. I. Plosser and S. Rebelo, 1988, Production, Growth and Business Cycles, *Journal of Monetary Economics*, 21, 195-232.
- [49] Kleibergen, F., 2001, Pivotal Statistics for Testing Structural Parameters in Instrumental Variables Regression, *Econometrica*, 70, 1781-1804.
- [50] Krusell, P. and A. Smith, 1998, Income and Wealth Heterogeneity in the Macroeconomy, *Journal of Political Economy*, 106, 867-896.
- [51] Lazear, E., 1988, Employment at will, job security, and work incentives, in: R. Hart, ed., *Employment, Unemployment and labour Utilization*, (Unwin Hyman, Boston).
- [52] Laibson, D., 1997, Golden Eggs and Hyperbolic Discounting, *Quarterly Journal of Economics*, 112(2), 443-77.
- [53] Lazear, E., 1990, Job security provisions and employment, *Quarterly Journal of Economics* 105, 699-726.
- [54] Lentz, R., 2005, Optimal Unemployment Insurance in an Estimated Job Search Model with Savings, University of Wisconsin Madison working paper.
- [55] Lentz, R. and T. Tranæs, 2004, Job Search and Savings: Wealth Effects and Duration Dependence, University of Wisconsin Madison working paper.
- [56] Lindbeck, A. and D. Snower, 1998, *The insider-outsider theory of employment and unemployment* (MIT Press, Cambridge, MA).
- [57] Low H., C. Meghir and L. Pistaferri, 2004, Wage Risk, Employment Risk and Precautionary Saving, UCL mimeo.
- [58] Lucas, R., 1973, Some International Evidence on Output-Inflation Tradeoffs, *American Economic Review*, 63 (3), 326-334.

- [59] Ludvigson S. and A. Michaelides, 2001, Does Buffer Stock Saving Explain the Smoothness and Excess Sensitivity of Consumption?, *American Economic Review*, 91, 631-647.
- [60] MaCurdy, T. E., 1981, The Use of Time Series Processes to Model the Error Structure of Earnings in Longitudinal Data Analysis, *Journal of Econometrics*, 18, 83-114.
- [61] Meghir, C. and L. Pistaferri, 2004, Income Variance Dynamics and Heterogeneity, *Econometrica*, 72 (1), 1-32.
- [62] Metcalf D., S. Milner and G. Nombela, 1995, Employment protection legislation and labour market outcomes in Spain, Centre for Economic Performance, Discussion paper no. 244, 1-62.
- [63] Meyer, B. 1990, Unemployment Insurance and Unemployment Spells, *Econometrica*, 58(4), 757-782.
- [64] Moen, E., 1997, Competitive search equilibrium, *The Journal of Political Economy*, 105, 385-411.
- [65] Moreira, M. J., 2001, Tests with Correct Size When Instruments Can Be Arbitrarily Weak, unpublished manuscript, University of California, Berkeley.
- [66] Moreira, M. J., 2003, A Conditional Likelihood Ratio Test for Structural Models, *Econometrica*, 71, 1027-1048.
- [67] Mortensen, D., 1978, Specific capital and labour turnover, *Bell Journal of Economics*, 9, 572-86.
- [68] Mortensen, D., 1994, Reducing supply side disincentives to job creation, in: *Reducing unemployment: Current issues and policy options*, (Federal Reserve Bank of Kansas City, Kansas City).
- [69] Mortensen, D. and C. Pissarides, 1994, Job Creation and Job Destruction in the Theory of Unemployment, *The Review of Economic Studies* 61, 397-415.
- [70] Mortensen, D. and C. Pissarides, 2003, Taxes, subsidies and equilibrium labour market outcomes, in: E. Phelps, ed., *Designing inclusion: Tools to*

raise low-end pay and employment in private enterprise, (Cambridge University Press, Cambridge, UK).

- [71] O'Donoghue T. and M. Rabin, 1999, Doing it Now or Later, *American Economic Review*, 89 (1), 103-24.
- [72] OECD, 1996, *Employment Outlook* (Paris).
- [73] OECD, 1999, *Employment Outlook* (Paris).
- [74] OECD, 2002, *Benefits and Wages*, OECD Indicators, Paris.
- [75] Petrongolo, B. and C. Pissarides, 2001, Looking into the black box: A survey of the matching function, *Journal of Economic Literature* 39, 390-431.
- [76] Pischke J.S., 1995, Individual Income, Incomplete Information and Aggregate Consumption, *Econometrica*, 63, 805-840.
- [77] Pissarides, C., 1984, Efficient job rejection, *The Economic Journal*, 94, 97-108.
- [78] Pissarides, C., 1985, Short -run equilibrium dynamics of unemployment, vacancies and real wages, *American Economic Review*, 75, 676-690.
- [79] Pissarides, C., 2000, *Equilibrium unemployment theory*, MIT Press, Cambridge, MA.
- [80] Pissarides, C., 2001, Employment Protection, *Labour Economics*, 8, 131-159.
- [81] Quah, D., 1990, Permanent and Transitory Movements in labour Income: An Explanation For 'Excess Smoothness' in Consumption, *Journal of Political Economy*, 98 (3), 449-475.
- [82] Reis, R., 2003, *Inattentive Consumers*, mimeo, Harvard University.
- [83] Sims, C., 2003, Implications of Rational Inattention, *Journal of Monetary Economics*, 20, 665-690.
- [84] Shimer, R., 2005, The Cyclical Behaviour of Equilibrium Unemployment and Vacancies: Evidence and Theory, *American Economic Review*, 95(1), 25-49.

- [85] Stock, J. and M. Yogo, 2003, Testing for Weak Instruments in Linear IV Regression, mimeo, Harvard University.
- [86] Stock, J., J. Wright and M. Yogo, 2002, A Survey of Weak Instruments and Weak Identification in Generalized Method of Moments, *Journal of Business and Economic Statistics*, 20, 518-529.
- [87] Storesletten, K., C. Telmer and A. Yaron, 2004, Cyclical Dynamics in Idiosyncratic labour Market Risk, *Journal of Political Economy*, 112, 3, 695-717
- [88] Wang, C. and S. Williamson, 1996, Unemployment Insurance with Moral Hazard in a Dynamic Economy, *Carnegie Rochester Conference Series on Public Policy*, 44, 1-41.
- [89] Wilcox, D., 1992, The Construction of U.S. Consumption Data: Some Facts and Their Implications for Empirical Work, *American Economic Review* 82, 4, 922-41
- [90] Zeldes, S., 1989, Optimal Consumption with Stochastic Income: Deviations from Certainty Equivalence, *Quarterly Journal of Economics*, 104(2), 275-98.