MACROECONOMIC MODELS FOR STUDYING MONETARY POLICY IN ECONOMIES WITH PARTIAL DOLLARISATION

Paul Castillo-Bardález

The London School of Economics and Political Science

A thesis submitted for the Ph.D Degree in Economics
London, July 2007

I declare the work presented in this thesis is my own except for one chapter, which is based on joint work with Diego Winkelried.

My paper with Mr. Winkelried is 'Individual Heterogeneity and Dollarisation Persistence?' Chapter 6 relies on this paper. I was responsible for about 50 per cent of the work involved in that paper.

Paul Castillo-Bardález

I agree with the above statement.

Kosuke Aoki

Gianluca Benigno

ABSTRACT

This thesis studies how monetary policy should be conducted in emerging economies where the domestic currency has been partially replaced by a foreign currency, a phenomenon called 'dollarisation'. The central question is how different forms of dollarisation affect both the transmission mechanism and the goals of the central bank.

A general overview and the motivation of these topics are discussed in the first and second chapters. The third chapter, 'Optimal Monetary Policy and Endogenous Price Dollarisation', shows that having two units of account may be optimal for economies with large sector specific productivity shocks when prices are sticky. In this case, optimal monetary policy implies a certain degree of exchange rate smoothing.

In the fourth chapter, 'Monetary Policy and Currency Substitution' we use a fully micro-founded general equilibrium model where currency substitution is endogenously determined to show how currency substitution can make inflation stabilisation more costly, thus inducing a higher degree of aggregate volatility. We also show that currency substitution does not affect the central banks capability to determine inflation and that in this case exchange rate smoothing is not optimal.

The effect of income distribution on price dollarisation is studied in the fifth chapter. This chapter shows that income inequality can generate an upper boundary for price dollarisation

In the final chapter, 'Dollarisation Persistence and Individual Heterogeneity', we study how the limited capability to process financial information of participants in the dollar deposit market can induce very persistent degrees of financial dollarisation. We further provide empirical evidence supporting our claim.

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ACKNOWLEDGEMENTS

I am grateful, first and foremost, to my supervisors Kosuke Aoki and Gianluca Benigno. Their constant advice and encouragement made possible to bring the experience of the PhD to fruition. I appreciate the time they dedicated to listen to my ideas and to read my drafts. Their insightful suggestions were determinant in guiding my way through the PhD experience.

My gratitude goes also to members of London School of Economics staff who helped me during the years at the PhD: Nobu Kiyotaki who always had time to listen my early ideas, to Evi Pappa, Chris Pissarides, Bernardo Guimaraes, Alex Michaelides, and Andrei Sarychev for their insightful comments and suggestions.

My master degree and the first year of my PhD studies were financed by grants from the Central Bank of Peru. My gratitude to this institution and to a group of very enthusiastic economists who work there: Marco Vega, Carlos Montoro, Diego Winkelried, Vicente Tuesta, Alberto Humala, and Adrian Armas, with whom I had endless discussions about many of the issues of this thesis.

Finally, and most importantly, I thank my parents, Gonzalo and Bertha, for their unconditional support and for teaching me the true value of life.

CHAPTER 1

INTRODUCTION

A fundamental question in monetary theory and one of practical importance for central banks is how monetary policy should be conducted. Recently, a great amount of research has been devoted to providing answers to this question using New Keynesian models[1]. This type of models have proven to be useful tools for understanding how central banks should set interest rates under different types of shocks and for learning about the objectives they should pursue. Most of this literature, however, has studied these issues in single-currency economy models, a relevant case for developed economies but not for a large number of emerging economies that function as dual-currency economies.

This thesis contributes to the literature on monetary policy in emerging market economies on three fronts: first it develops general equilibrium New Keynesian models where endogenously two currencies circulate as medium of payment and unit of account. Second, it shows what type of policies are optimal in this type of environments. Third, it provides insights on how two other key features of emerging market economies,income distribution and the limited ability to process information of participants in the financial market, restrict the use of a foreign currency by domestic agents. Whilst the contribution of this thesis is theoretical, it also provides empirical evidence supporting some of its implications by using as case study Mexico, Peru, Poland and Uruguay.

[1] See for instance, Clarida et al. (1999) and Woodford (2003) for closed economies, and Corsetti and Pesenti (2004), Corsetti and Pesenti (2001), Benigno and Benigno (2003), Benigno (2004) and De Paoli (2004), for open economies.

Dual currency economies are known in the literature and amongst policy makers as *dollarised* economies. This term is used to describe economies where a foreign currency is widely used by its residents alongside or instead of the domestic currency. Distinct forms of dollarisation are recognised in the literature depending on the monetary services provided by the foreign currency and on its legal tender status.

When the domestic currency is the only legal tender in the economy, but the foreign currency is widely accepted, dollarisation is denominated unofficial or *partial*. This type of dollarisation, in turn, can be categorised as *financial dollarisation*(FD), when the foreign currency provides services of reserve of value, *currency substitution* or payments dollarisation(CS), when the domestic economy has been partially replaced in its function of medium of payment, and *price dollarisation*, (PD) when the foreign currency also works as unit of account.

Dollarisation is significant and widespread in many emerging economies, in particular, in those countries with a history of high inflation, such as Bolivia, Egypt, Turkey, Peru, Russia and Uruguay. Yet, it is not exclusive to those economies. Levy-Yeyati (2006) reports that by the end of 2000 more than 44 percent of the banking deposits in emerging market economies where denominated in foreign currency. By the end of 2005 FD, measured by participation of foreign-currency-denominated deposits in the banking system, reached 85 percent in Peru and 55 percent in Bolivia, the two Latin American countries with the longest history of dollarisation. For the same countries, CS, measured by the participation of foreign-currency denominated sight deposits was around 60 percent. These figures are remarkably high even for emerging economies. In transition economies, in particular in Russia, CS is even higher, reaching levels close to 80 percent.

Although not as large as FD or CS, PD is significant. The short-run level of the exchange rate pass-through to domestic prices can be used as an indirect measure of PD[2]. For Peru, Armas et al. (2001), Miller (2003) and Winkelried (2003) estimate values for the short-run degree of pass-through between 15 to 30 percent. More recently, Castillo et al. (2006) estimate values of 40 and 30 percent for CS and PD respectively [3].

1.1 Dollarisation and Monetary Policy

This thesis contains five chapters in addition to the introductory chapter. Chapter 2 draws attention to the importance of dollarisation in emerging economies by providing a set of quantitative measures of its different forms for a wide set of these economies. Chapters 3 and 4 address the issue of how monetary policy should be conducted when agents choose endogenously their unit of account and their medium of payment. Chapter 5 analyses the implications of non-homothetic preferences and income distribution on the degree of PD, whereas Chapter 6 focuses on the implications of the heterogenous ability of the deposit market participants to process information in explaining the degree of persistence of FD.

A fundamental question in monetary theory is how an economy should choose its unit of account. Most of the recent literature in monetary policy that relies on general equilibrium models takes, however, this choice as given [4]. Instead, Chapter 3, explicitly considers the implications of this choice for monetary policy within a general equilibrium small open economy model. Our main result shows

[2] This measure is particularly informative for those goods whose prices are not adjusted frequently. Since, in those cases their prices reflect, at short-run frequencies, mainly exchange rate movements.

[3] This estimation uses Bayesian methods and a stochastic general equilibrium model with Peruvian data.

[4] The only exception to this trend is the literature on endogenous pass-through, which explicitly studies the determinants of exporting firm's invoicing decisions and their implications for real exchange rate volatility, the transmission of foreign shocks and the benefits of international policy coordination See Bacchetta and Wincoop (2005) for a general equilibrium static model of invoicing, and for dynamic frameworks, see Devereux et al. (2004) and Corsetti and Pesenti (2004)

that it is possible to obtain welfare gains from using two units of accounts when: a) sector specific shocks are large enough and b) prices are not too sticky. In these cases having two units of accounts is instrumental in producing less costly adjustment in relative prices across productive sectors. A key implication for monetary policy of this novel result is that zero domestic inflation is not optimal in economies with PD. Instead the central bank faces a trade-off between stabilising domestic inflation, the output gap and the nominal exchange rate. Therefore, this chapter shows that optimal monetary policy implies some degree of exchange rate smoothness. In that sense, the model shows that in economies with PD there is an optimal *fear of floating* [5].

A key parameter in determining the magnitude of the central bank's response to domestic inflation, output gap and of *fear of floating* is the degree of price stickiness. When prices are more sticky the central bank puts a larger weight on domestic inflation and output gap stabilisation, since the welfare costs associated to fluctuations in these two variables are larger in this case. Instead, when prices are not too sticky and sector specific shocks are large, the central banks puts more weight on exchange rate stabilisation since in this case, gains from stabilising relative prices through the exchange rate are larger.

Similarly to Corsetti and Pesenti (2004), Loyo (2001) and Devereux et al. (2004), we also find that individual firm's would choose to invoice in a foreign currency when the volatility of exchange rate is high and firm's marginal costs are positive correlated with the exchange rate. Interestingly then, we show that it is possible to obtain multiple equilibriums in the model, particularly when sector specific shocks are not large enough. In this latter case, firm's optimal invoicing choice would depend on what other firms do. We further provide a full characterisation of these equilibriums.

[5] Terminology of Calvo and Reinhart (2002).

This chapter further shows that optimal monetary policy in economies with endogenous PD implies a positive correlation between the exchange rate and sector specific productivity shocks of firms that invoice in a foreign currency. By inducing this correlation, the central bank can achieve a larger reduction on the relative price distortions generated by sector specific productivity shocks and sticky prices, in comparison with the outcome under a pure inflation target regime. When a subset of firms sets prices in dollars, by altering the parity between the domestic and the foreign currency, the central bank can engineer a change in relative prices in the opposite direction to the one generated by sector specific productivity shocks stabilising relative prices. The model also implies that the larger the degree of price stickiness, the larger the response of the central bank should be to sector specific productivity shocks. This potential behaviour of the central bank, in turn, generates the incentives to some firms to set prices in a foreign currency

Importantly, chapter 4 shows that CS not only affects the economy through different channels than those of PD but more crucially it shows that CS has distinct implications for monetary policy. As this chapter discuss in detail, by optimally choosing the composition of their medium of payments, households open a channel by which the foreign nominal interest rate affects consumption, savings and labour supply decisions, and consequently output gap and inflation dynamics. This new channel introduces a new source of volatility in developing small open economies, in particular in response to foreign interest rate shocks.

Moreover, this chapter shows that the central bank's welfare-based loss function depends on the degree of CS in steady-state. In particular, a central bank operating in this type of environment faces a trade-off between stabilising the output gap, domestic inflation and the nominal interest rate. This trade-off, moreover, depends on the degree of steady-state CS. In particular, when the degree of CS is large, the central puts more weight on domestic inflation and output gap stabilisation than in interest rate smoothing. Consequently, the central bank optimally moves the domestic interest rate more strongly in response to shocks. Also, it turns out to be optimal for the central bank to not smooth exchange rate fluctuations.

In terms of implementable rules, this chapter shows that CS increases the determinacy area of Taylor-type rules that react to domestic inflation. This, though, comes at the cost of increasing the volatility of both the output gap and inflation. Interestingly then, this chapter shows that CS do not limit the central bank's capability to anchor inflation expectations, it only make more costly both domestic inflation and output gap stabilisation. Indeed, the recent experience of highly dollarised Latin American economies, like Bolivia and Peru, is consistent with this result. These economies have successfully anchored inflation expectations and achieved price stability, regardless of their high degrees of both FD and CS.

Developing economies differ from developed ones on several dimensions. Two of them which are particularly relevant are income level differences and agent's access to financial markets. Chapters 5 and 6 we explore the implications of these two key emerging market features. On one hand, chapter 5 focuses on understanding how income distribution affects the choice of unit of account by firms and agent's portfolio decisions. Using a simple overlapping generation monetary model where agents have non-homothetic preferences, this chapter shows that income distribution can limit the extent to which a foreign currency can be used as unit of account. In particular, it shows that even when firms producing necessity goods have marginal costs in foreign currency, they will choose to set prices in domestic currency.

The mechanism that delivers this result is rooted in the link that non-homothetic preferences generate between the demand price elasticity and the income of firm's customers. In this case, the demand price elasticity that firms face is positively correlated with their costumers' income. Hence, when customer's income covaries positively with the exchange rate, which is the case when customers have dollarised their savings, setting prices in foreign currency increases the volatility of a firm's profits, reducing their expected profits. This effect is larger for necessity goods than for luxury goods, and, consequently, firms producing necessity goods are more likely to set prices in domestic currency than firms producing luxury goods. At the aggregate level, the heterogenous pricing behaviour of firms in turn implies that the degree of pass-through of exchange rate to prices would tend to be smaller for low-income countries than for medium-income countries.

On the other hand, Chapter 6 analyses how the limited capability to process information of participants in the deposit market helps to explain the degree of persistence of financial dollarisation. Persistence is the most salient feature of financial dollarisation, and the one that probably causes more concern to policymakers. In this chapter we claim that this persistence is connected to the fact that participants in the dollar deposit market are fairly heterogenous, and so is the way they form their optimal currency portfolio. This chapter develops a simple model where agents differ in their ability to process information, which turns out to be enough to generate persistence upon aggregation. The chapter finds empirical support for this claim with data from three Latin American countries and Poland.

All in all, this thesis provides several novel results on how monetary policy should be conducted in economies with PD and CS. In particular, it shows that in partially dollarised economies the weights of the exchange rate and that of interest rate on the central bank loss function depend on the extent of both PD and CS. PD generates incentives to smooth exchange rate fluctuations, whereas CS reduces the incentives for interest rate smoothing. Importantly, CS also enlarges the area of determinacy for implementable Taylor-type rules, but at the cost of increasing aggregate volatility. Finally, this thesis also provides some novel explanations for two stylised facts on dollarisation: why FD is so persistent and why PD seems to be concentrated in relatively luxury goods. Some empirical evidence supporting the main implications of these two hypotheses is also provided.

CHAPTER 2

QUANTITATIVE MEASURES OF DOLLARISATION

This chapter provides quantitative measures of the three types of dollarisation studied in this thesis. For this purpose, we use data collected from different sources, in particular, the IMF Staff Reports, IMF International Financial Statistics publication and various Central Bank Bulletins. We also use information provided by Baliño et al. (1999), De Nicoló et al. (2005) and Arteta (2003) and Feige (2003).

2.1 Financial Dollarisation

Table 2.1 presents a list of countries where FD is above 10 percent.[1] We measure FD as the average ratio of deposits in foreign currency to total deposits issued by domestic banks for 33 countries for the period from 1991 to 2001.

As Table 2.1 shows, FD is quantitatively important in most of the countries in the sample. On average, it is above 50 percent. In Latin America, Bolivia and Uruguay are the economies with the highest FD measures, with values close to 90 and 80 percent respectively. In the case of Eastern Europe, Armenia and Bulgaria are the economies that exhibit the highest dollarisation ratios, 71 and 51 percent, respectively, whereas in Asia and Africa, Cambodia and Angola exhibit FD levels of 94 and 65 percent respectively.

[1] This table reports only figures of those countries where FD is above 10 percent. We have not considered economies with measures of FD below 10 percent since we are mainly interested in highly dollarised economies. Dollarisation levels below 10 may not be related directly to monetary policy but to other long-term factors such us financial integration.

Table 2.1: Dollarisation Ratios for Selected Countries

	Financial Dollarisation			Inflation		
	Average	*Max.*	*Trend*	*Average*	*Max.*	*Trend*
Angola	65.1	84.0	up	214.0	5000.0	down
Argentina	61.3	74.0	up	-0.3	171.7	down
Armenia	71.4	81.0	up	5.1	4940.8	down
Belarus	51.1	69.0	up	132.0	2221.0	down
Bolivia	90.0	92.0	down	4.1	21.5	down
Bulgaria	50.5	57.0	up	219.4	1058.2	down
Cambodia	93.2	94.0	up	4.1	14.8	down
Costa Rica	41.4	48.0	up	11.4	28.7	down
Croatia	70.4	68.9	up	5.8	1509.4	down
Ecuador	26.9	53.0	up	50.5	96.1	up
Egypt	24.8	32.0	up	3.3	19.8	down
Estonia	19.1	24.0	up	6.4	89.8	down
Georgia	64.6	82.0	up	7.7	162.6	down
Guinea-Bissau	46.0	56.0	up	13.1	69.5	down
Israel	18.9	20.0	up	4.4	19.0	down
Kazakhstan	35.2	60.0	up	10.9	1884.7	down
Kyrgyz Republic	51.9	66.0	up	19.1	35.9	down
Lao People's Dem.Rep	73.0	90.0	up	56.0	128.4	down
Latvia	47.5	52.0	down	4.1	243.0	down
Lithuania	40.6	44.0	up	3.4	410.3	down
Macedonia, FYR	55.4	65.0	up	2.5	126.6	up
Mozambique	48.3	55.0	up	6.7	63.2	up
Nicaragua	67.4	71.0	up	10.5	2944.0	down
Paraguay	55.4	67.0	up	8.3	24.2	up
Peru	65.7	68.0	up	5.0	409.6	down
Poland	21.0	27.0	down	9.9	76.7	down
Romania	37.7	49.0	up	67.9	256.0	down
Russia	34.0	44.0	up	34.1	900.0	up
Turkey	49.0	58.0	up	68.9	106.7	down
Uruguay	80.0	84.0	up	9.1	102.0	down
Vanuatu	67.8	70.0	up	2.8	6.5	up
Vietnam	37.5	43.0	up	2.5	7.3	down
Yemen	49.6	53.0	up	6.7	55.1	up

Note: The average values for dollarisation ratios and inflation correspond to the period 2001-1997, whereas the maximum values to the period 1990-2001. The measure of FD is defined as foreign currency deposits over total deposits.

Sources: IMF Staff Reports, Central Bank bulletins (various issues), Baliño et al. (1999), De Nicoló et al. (2005) and Arteta (2003).

Interestingly, the levels of FD presented in Table 2.1 seem to be more correlated with the history of inflation than the contemporaneous levels of inflation. The correlation coefficient between our measure of FD and the average inflation level for the 33 countries considered in the sample is less than 0.1 percent. This correlation, however, increases up to 20 percent when considering the maximum level of past inflation instead of the average level of inflation. FD, then, seems to persist even many years after an economy has reached low inflation levels. Indeed, this has been the case of economies like Bolivia and Peru, which experimented periods of hyperinflation before the 90's, but still now have levels of FD above 60 percent. The figures presented in Table 2.1, therefore, indicate that financial dollarisation is not only quantitatively important for a large set of emerging economies but also a very persistent phenomenon. Chapter 6 provides a novel explanation of this stylised fact of FD.

2.2 Currency Substitution

Since there is no a direct measure of the amount of foreign currency circulating in dollarised economies, we use indirect measures of CS. For Bolivia and Peru we use as an indirect measure of CS the proportion of dollar-denominated sight deposits in the domestic banking system[2].

For the remaining countries in the sample, we use measures of CS constructed using survey information gathered and organised by the United States of America Customs, on cross border flows of US currency [3]. We complement this information with the figures produced by Feige (2003), who uses survey information on holdings of European currencies in transition economies[4]. The values

[2] Sight deposits are highly correlated with the level of transactions in the economy and hence they contain information on the degree of CS.

[3] Every person that imports or exports currency or monetary instruments for an amount over US 10000, has to fill the report.

[4] These surveys were conducted by the Austrian Central Bank.

Table 2.2: Currency Substitution and Financial Dollarisation Selected Countries 2001

	Currency Substitution	Financial Dollarisation
Bolivia	65	88
Peru	51	66
Poland	27	19
Latvia	79	44
Russia	87	34
Armenia	62	80
Georgia	79	82
Croatia	46	71
Romania	55	49

Note: For Bolivia and Peru measures of CS corresponds to the participation of sight deposits denominated in foreign currency. For the remaining countries the figures correspond to the ratio of foreign currency in circulation over total currency in circulation. Sources: IMF Staff Reports, Central Bank bulletins (various issues), and Feige (2003).

corresponding to this measure of CS are presented in Table 2.2 for a selected group of countries. All figures correspond to 2001. For comparison Table 2.2 also reports the corresponding measures of FD for each country that we included in Table 2.1.

Figures presented in Table 2.2 show that CS is indeed quantitatively large. For some countries it is even larger than FD. The leading examples of this case are Russia and Latvia where CS reaches 81 and 79 percent. This pattern, however, is not uniform across countries. For other economies, like Bolivia and Peru the opposite is observed, FD is higher than CS.

2.3 Price Dollarisation

Although a link may exist between CS and PD in dollarised economies, for a large set of transactions agents pay with a different currency than the one used as unit of account, therefore, these two types of dollarisation may not be quantitatively equivalent.

Table 2.3: Measures of Price Dollarisation Peru 1995-2003

	Average	*Maximum*	*Lima*
Consumer Price Index	6%	16%	3%
Food and Beverages	4%	17%	-4%
Clothes and shoes	15%	36%	23%
Housing Rent fuel and electricity	1%	20%	7%
Furniture and house conservation	23%	39%	25%
Health Care	15%	36%	19%
Education Services	-2%	16%	-6%
Other Services	14%	30%	16%

Note: Measures of PD corresponds to contemporaneous correlation between the exchange rate depreciation and inflation rates for each component of the CPI. For the *average*, CPI's information of 25 Peruvian cities were considered. Source: National Institute of Statistics, INE,

Direct measures of PD are not available for most of the countries included in the sample. However, some indirect measures can be obtained. For instance, the very short-run pass-through of exchange rate movements into domestic prices. This indicator is informative about the degree of PD only when prices are relatively sticky. In this case, within the period that firms do not change their prices, the observed change in the aggregate price index relative to the change in the exchange rate would be proportional to the degree of PD. Using this indirect measures of PD for a series of Latin American countries, Gonzalez-Anaya (2002) finds that FD and PD are not correlated. In particular, economies where the degree of FD is large, like Bolivia and Peru, exhibit relatively low degree of PD.

We further provide evidence that PD is relatively low in economies with large degrees of FD using Peruvian data. In particular, we use monthly data of the Consumer Price index and its components for the period 1995-2003 for 25 Peruvian cities. For each price index we calculate short-run pass-through of exchange rate movements into domestic prices.

Table 2.3 presents the estimated measures of PD for the Peruvian economy. The first column of this table shows the average value of PD for the 25 most important cities in Peru; in the second column, its maximum value and finally in the third column the corresponding value for capital city, Lima. Although the measures presented in Table 2.3 indicate that PD is lower than CS and FD, PD is still significant, particularly for some of the components of the CPI index. The average value of PD is around 6 percent, with a maximum of 16 percent[5]. In terms of components, *furniture and Housing conservation*, *clothes and shoes*,and *health care* are amongst those with the highest degrees of PD, with average values of 23, 15 and 15, respectively. For Lima, the largest city in the country, the same indicators take values above their averages, 25, 19 and 23 respectively.

Interestingly, the pattern of PD across type of goods seems to be related with the average income-level across countries and regions. To illustrate this feature we estimate the correlation between the previously described measures of PD and indirect indicators of the average income-level for the 25 main cities of Peru. In particular, we use four different indicators for the average income level: the per-capita expenditure of families within each city, the number of phones per each 100 people, the number of people at each city that has phone, and the number of public phones per each 100 people. The sample period spans the period from January 1995 to December 2003.

As Table 2.4 shows, there is an statistically significantly positive correlation between the average income level and the degree of PD. The correlation is higher for housing rent and for domestic appliances, and relatively weaker for transport service. This correlation suggest that in poorer cities the degree of PD is much lower than in cities where the average income is higher even for relatively tradable goods as domestic appliances.

[5] Using a time horizon of one and two quarters, Miller (2003) and Winkelried (2003) show that the degree of pass-through ranges between 15 and 30 percent.

Table 2.4: Price Dollarisation and Income Distribution Peru 1995-2003

Selected CPI Components	IND_1	IND_2	IND_3	IND_4
Housing Rent	0.24*	0.25*	0.17	0.19
Domestic Appliances	0.48**	0.58**	0.57**	0.55
Transport Services	0.10	0.13	0.17	0.13

* [**] denotes significance at a 5% [10%] level. IND_1 corresponds to the per-capita expenditure of families within each city, IND_2 to the number of phones per each 100 people, IND_3 number of people at each city that have phone, and IND_4, number of public phones per each 100 people.

Overall, the figures presented in this chapter show that dollarisation is not only quantitatively important for a large number of emerging markets but also a very persistent phenomenon. Consequently, central banks in partially dollarised economies need to take into account explicitly the constraints that dollarisation imposes when designing monetary policy. Chapter 3 and 4 study this issue in detail.

CHAPTER 3

OPTIMAL MONETARY POLICY AND ENDOGENOUS PRICE DOLLARISATION

As chapter 2 has documented there exists a large number of developing economies where two currencies share the basic monetary functions. Amongst these, the function of unit of account is perhaps the one that has been given more attention in the recent new Keynesian literature as a key ingredient for transmission mechanism of monetary policy. This is evident, since in most of the recent models for analysing the design of monetary policy in close and open economies the fundamental, albeit the only function that money plays is the one of unit of account [1]. These models, however, usually assume that domestic firms set prices only in domestic currency, an assumption that is less suitable for developing economies where a large set of domestic prices are denominated in a foreign currency.

The only exception to this trend is the literature on endogenous pass-through, which explicitly studies the determinants of exporting firm's invoicing decisions and their implications for real exchange rate volatility, the transmission of foreign shocks and the benefits of international policy coordination [2]. However, in contrast to develop economies where a foreign currency may be used as unit of account but exclusively for imported goods, in developing economies with PD also a large set of domestically produced goods are invoiced in a foreign currency

[1] See Aoki (2001), Woodford (2003) for close economies and Benigno and Benigno (2003), Benigno (2004), De Paoli (2004) and Galí and Monacelli (2005)for open economies

[2] See Bacchetta and Wincoop (2005) for a general equilibrium static model of invoicing, and for dynamic frameworks, see Devereux et al. (2004) and Corsetti and Pesenti (2004)

for domestic transactions. This distinction generates important differences on how monetary policy should be implemented in develop versus developing economies with PD.

Our goal in this chapter is to twofold: first, to derive a micro-founded welfare-based loss function for the central bank in an economy with PD to account for the distinct implications of PD on monetary policy. Second, to fully characterise the general equilibrium determinants of PD. These are fundamental issues for monetary policy in developing economies for which there are not yet clear-cut answers. For instance, it is usually argued that PD is an endogenous response of firms to bad monetary policy; however, the recent performance of monetary policy in economies with PD like Peru and Bolivia, with annual inflation rates below 3 percent, casts some doubts on the role that bad monetary policy may have as the most important determinant of PD.

Similarly, a usual argument, especially amongst policy makers, is that dollarisation induces central banks to smooth exchange rate fluctuations. Some recent papers as Céspedes et al. (2004) and Chang and Velasco (2001) that study the implications of financial dollarisation on the performance of different exchange rate regimes, however, do not support this view. In those papers, a flexible exchange rate regime still outperforms a fixed exchange rate regime, even when financial frictions are considered. These papers however, do not take into account PD. Instead, this chapter explicitly considers the implications of PD for central bank's exchange rate policy.

The model we use to address these issues belongs to the generation of New Keynesian Small Open Economies models (SOE from now on) developed to study the design of monetary policy[3]. Two particular features, though, distinguish our

[3] Amongst the recent papers that analyse optimal monetary policy in SOE, the closest ones to our model are Sutherland (2000), Galí and Monacelli (2005) and De Paoli (2004).

model from the rest of the literature. First, firms are heterogeneous in their technology -in particular; they face sector-specific productivity shocks- which induce relative price dispersion across firms[4]. Second, as in the endogenous pass-through literature, firms can freely choose either a domestic or a foreign currency for invoicing.

We proceed as follows; first we develop a SOE new Keynesian model, which is derived as the limiting case of a two-country world. We use this model to obtain an aggregated demand and supply relationship conditioned on the degree of PD, which we use later to characterise the optimal firm's invoicing decision and the central bank monetary policy. A novel key implication of PD for the dynamics of the economy is that unexpected fluctuations on the devaluation rate shift the Phillips curve, further complicating inflation stabilisation.

Then, we follow the work of Bacchetta and Wincoop (2005), Devereux et al. (2004), Corsetti and Pesenti (2004) and Loyo (2001) in characterising the firm's invoicing decision using a second-order approximation of the relative expected discounted firm's profits under domestic and foreign currency invoicing. In this dimension our results are similar to those of Devereux et al. (2004) and of Loyo (2001). In particular, we find that firm's are more likely to invoice in a foreign currency when: a) domestic inflation is more volatile than the depreciation rate of the nominal exchange rate, b) the covariance between the domestic price and the nominal exchange rate is high, and c) the nominal exchange rate is positively correlated with a firm's marginal costs.

[4] Several recent papers analyse optimal monetary policy in economies with more than one sector. Aoki (2001) analyses optimal monetary policy for a two-sector close economy model; Benigno (2004) characterises optimal monetary policy in a currency area; Erceg and Levin (2000) consider the case of stickiness in wages and prices, and Huang et al. (2005) analyse the case of price stickiness in the final and intermediate production sectors. Unlike the previous papers, we focus on the optimal choice of unit of account besides optimal monetary policy.

Intuitively, when firms' prices are sticky in both domestic and foreign currency, aggregate and idiosyncratic shocks deviate firms' relative prices from their optimal values generating profit losses. However, firms can mitigate these costs by invoicing in a foreign currency. For instance, when there is a positive correlation between aggregate prices and exchange rates, a firm that invoices in a foreign currency would partially isolate their relative price from domestic inflation, since its relative price would increases in those states of nature where inflation also increase. Similarly, when the nominal exchange rate is positively correlated with the firm's marginal cost, by setting prices in the foreign currency, firms induce a positive correlation between marginal revenues and marginal costs that stabilise firms' profits and increase their expected profits.

Then, we analyse the implications of PD for monetary policy. As we discussed previously PD has first-order effects on domestic inflation through its impact on the Phillips curve. Through this channel, PD generates an endogenous trade-off between stabilising domestic inflation and the output gap making more costly for the central bank to stabilise domestic inflation. Even more importantly, we show that PD also generates second-order welfare effects by contributing to create aggregate relative price dispersion across domestic producers. As Woodford (2003) highlights, the relative price dispersion's determinants are related to the particularities of the price setting structure. In our case, since some firms choose to set prices in a foreign currency, fluctuations in the exchange rate also generate relative price distortions, thereby welfare losses. To properly account for the implications of PD on monetary policy we obtain a welfare-based loss function for the central by taking a second-order approximation of the representative domestic household's utility function around a constrain efficient steady-state as in Woodford (2003) and Benigno and Woodford (2003).

We show that the relevant central bank's loss function in an economy with PD has several distinct features. First, besides domestic inflation and the output gap, the central bank's loss function in an economy with PD depends on the volatility of the nominal exchange rate, its covariance with domestic inflation and and its covariances with the average productivity shock of dollar-price firms. Consequently, in an economy with PD the central bank has incentives to reduce the volatility of the nominal exchange rate and to generate a positive correlation between the domestic exchange rate, domestic prices and dollar-price firm's sector specific productivity shocks. Intuitively, by behaving in this way the central bank minimises the relative price dispersion that aggregate and sector specific shocks generate in an economy with PD when prices are sticky.

Furthermore, we show that there exists a large set of parameter values for which the equilibrium with PD is unique. In particular, for a given degree of price stickiness and labour supply elasticity, equilibrium with PD emerges when sector specific productivity shocks are large enough relative to real exchange rate shocks. Also, for a given size of sector specific productivity shocks, a unique equilibrium with PD exists when the degree of price stickiness is not too large. Intuitively, in general equilibrium two units of account coexist when its benefits, given by the reduction in relative price distortions, are larger than its costs, given by the extra volatility on output gap that the use of the exchange rate by the central bank generates.

Since the benefits of having PD are related to the size of sector-specific productivity shocks, and its costs to the degree of price stickiness, it would be more likely to observe an equilibrium with partial PD when prices are not too sticky (costs are lower) and when domestic sector productivity shocks are larger (benefits are larger). In this sense, our results can be rationalised in terms of Mundell (1961) optimal currency areas theory. As Mundell (1961) proposed, an optimal currency area is a geographical zone where shocks are common. In our model, when sector specific shocks are large enough, indeed there exist two currency areas within the same economy. Thereby, using two units of account become optimal. Furthermore, we also show that it is possible to observe multiple equilibriums, particularly when the degree of price stickiness is too high or when the size of sector specific shocks is two small.

Our results also show that a central bank that aims at anchoring domestic inflation, as for instance in the case of adopting an explicit inflation-targeting regime, generates equilibrium where PD is sub optimally low. On the contrary, a central bank that exhibits an excess of 'fear of floating' would generate a degree of PD that is sub-optimally high. The latter results suggest that there is an optimal degree of 'fear of floating' associated to the optimal degree of PD.

The model used in this chapter is related to Loyo (2001), Devereux et al. (2004) and Corsetti and Pesenti (2004). In those papers, firms also have to decide optimally amongst different units of account. In Loyo (2001) firms have to decide between a real and an imaginary currency to set prices and the central bank can control directly the parity between these two types of currency. In Devereux et al. (2004) and Corsetti and Pesenti (2004), importing and exporting firms have to choose between domestic and foreign currency for invoicing and monetary policy is implemented through money growth rate targes[5]. In contrast to Loyo (2001), in our set up, the central bank does not perfectly control the parity between the domestic and foreign currency but this is determined as an equilibrium outcome in the economy. Also differently from Devereux et al. (2004) and Corsetti and Pesenti (2004) in this chapter we focus on an equilibrium where monetary policy is implemented optimally and where domestic firms can invoice in a foreign for the domestic market.

To set up the model, a number of simplifying assumptions have been made in order to gain clarity. First, although our framework is one of a SOE where terms of trade surely play an important role, we follow Galí and Monacelli (2005) in

[5] A series of other papers study invoicing decisions in different contexts; for instance, Bacchetta and Wincoop (2005), Donnefeld and Zilcha (1991), Giovannini (1988), Johnson and Pick (1997) and Klemperer and Meyer (1986).

choosing a particular type of preferences that eliminate the effects of terms of trade on the economy. This simplification helps to highlight the interplay between the exchange rate and PD[6].

Second, we used a very simple structure of correlations amongst sector specific productivity shocks, which, however, are enough to qualitatively show the implications of the model. Yet, a more complex assumption about this correlation can be made, as in Loyo (2001)[7]. Finally, we choose not to consider any type of financial frictions by using a set up with complete asset markets. This latter choice is made to keep the tractability of the model since it greatly simplifies the characterisation of the micro-founded central bank's loss function [8]. Moreover, adding financial frictions does not alter the invoicing decision of firms in a fundamental way but it highly complicates the determination of the equilibrium[9]

The remainder of this chapter is organised as follows. Section 3.1 describes a simple general equilibrium model of an small open economy where firms face sector specific productivity shocks and price stickiness. Section 3.2 discusses the implications of PD for the dynamic equilibrium of the economy, section 3.3 analyses the relevant loss function for the central bank and the design of optimal monetary policy under PD. Section 3.4 discusses the interplay between the equilibrium level of PD and monetary policy. The final section presents some concluding remarks.

[6] Galí and Monacelli (2005) and De Paoli (2004) show that the effects of terms of trade are eliminated from the economy when both the intertemporal elasticity of substitution and the elasticity of substitution between domestic and foreign goods are equal to 1. These assumptions are chosen in this chapter.

[7] The assumptions of Loyo (2001) on the correlations of shocks, although more complex, are less realistic since they are held under a very particular set of assumptions.

[8] A interesting paper that addresses the issue of optimal monetary policy using a welfare-based central bank's loss function in a two-country world with incomplete markets is Benigno (2001).

[9] It is important to highlight that the complete market assumption is not redundant even when the intertemporal elasticity of substitution and the intertemporal elasticity between domestic and foreign goods are unity. When markets are incomplete the dynamics of the economy depends on the overall stock of foreign debt, which alters the response of the economy to domestic and foreign shocks. Moreover, the central bank's loss function also depends on domestic consumption volatility as Benigno (2001) shows.

3.1 The Model

In this section we outline the model used later in this chapter to study the determinants of PD and its implications for the design of monetary policy. In the model, the world's population is allocated in two economies, a domestic economy of size n and the foreign economy of size $(1-n)$. We focus on the limiting case of a small open economy where domestic shocks do not affect the behaviour of the foreign economy. This case is obtained by making n arbitrary small. In each economy households receive utility from consuming a continuum of differentiated consumption goods and disutility from working. Furthermore, households can smooth consumption using a complete set of stage contingent bonds denominated in domestic currency[10].

On the other hand, monopolistic competitive firms produce consumption goods using labour and a constant return to scale technology. We introduce nominal rigidities in the model by assuming that a fraction of firms, chosen randomly, set prices on period in advance. However, differently from standard SOE models, we further allow firms to use either a foreign or a domestic currency for setting prices[11]. At every period t, there are three types of firms according to their pricing strategies: a) a set of firms with flexible prices, which can respond to contemporaneous shocks, b) firms that set prices one period in advance in pesos, and c) firms that set prices one period in advance but in dollars. The fraction of firms that optimally pre-set prices in the domestic market in foreign currency, our measure of PD, is endogenously determined in equilibrium, whereas the fraction of firms with flexible prices is exogenously given.

We further assume that firms are heterogeneous in their cost structure.

[10] We omit financial frictions in the model for tractability since our main objective is to analyse the implications of nominal frictions for the relationship between PD and monetary policy.

[11] See Bacchetta and Wincoop (2005) for a general equilibrium static model with endogenous invoicing decisions and Devereux et al. (2004) and Corsetti and Pesenti (2004) for dynamic frameworks.

In particular, we assume that firms face sector specific productivity shocks. This assumption together with the one of price stickiness imply that firm's will have different incentives to invoice in a foreign currency depending on the correlation of the exchange rate with the shocks they face. On the other hand, the central bank implement monetary policy optimally by minimizing a welfare-based loss function.

3.1.1 Preferences

We assume the following utility function on consumption and labour

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\log C_t-\frac{N_t^{1+v}}{1+v}\right)\right] \tag{3.1}$$

Where $0<\beta<1$ represents the subjective discount factor, v the inverse of the Frish labour supply elasticity, N_t represents labour hours, and C_t, a consumption basket index. The log preferences on consumption is chosen because it allows to eliminate the effects of terms of trade on SOE models making much easier to understand the interplay between PD and optimal monetary policy[12]. The domestic consumption index is defined by:

$$C_t=\left[(1-\alpha)^{\frac{1}{\eta}}\left(C_{H,t}\right)^{\frac{\eta-1}{\eta}}+\alpha^{\frac{1}{\eta}}\left(C_{F,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{1-\eta}} \tag{3.2}$$

where η represents the elasticity of substitution between domestic and foreign consumption goods, $C_{H,t}$ and $C_{F,t}$ respectively. On the other hand, α is a preference parameter that measures the fraction of consumption allocated in foreign goods. In turn, the domestic consumption basket is as a composite of a continuum of differentiated consumption goods defined by following CES aggregator function:

$$C_{H,t}=\left(\left(\frac{1}{n}\right)^{\frac{1}{\varepsilon}}\int_0^n C_{H,t}(z)^{\frac{\varepsilon-1}{\varepsilon}}d(z)\right)^{\frac{\varepsilon}{\varepsilon-1}}\quad C_{F,t}=\left(\left(\frac{1}{1-n}\right)^{\frac{1}{\varepsilon}}\int_0^n C_{F,t}(z)^{\frac{\varepsilon-1}{\varepsilon}}d(z)\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.3}$$

[12] See Galí and Monacelli (2005) and De Paoli (2004)

where $\varepsilon > 1$ represents the elasticity of substitution between differentiated domestic consumption goods. Associated to this set of preferences there exists a consumption based price index, P_t and corresponding domestic and foreign price indices, $P_{H,t}$ and $P_{F,t}$, respectively, which are defined as follows:

$$P_t = \left((1-\alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right)^{\frac{1}{1-\eta}} \tag{3.4}$$

and

$$P_{H,t} = \left(\frac{1}{n}\int_{\Theta} P_{H,t}^{1-\varepsilon}(z)\,dz + \int_{\Sigma} P_{H,t}^{1-\varepsilon}(z)\,dz + \int_{[0,n]\setminus\Sigma\cup\Theta} P_{H,t}^{1-\varepsilon}(z)\,dz +\right)^{\frac{1}{1-\varepsilon}} \tag{3.5}$$

where, $P_{F,t} = e_t P_{F,t}^{*}$ represents the price index of foreign goods expressed in domestic currency, e_t, the nominal exchange rate, the price of the foreign currency in terms of the domestic currency, and $P_{F,t}^{*}$ the price index of foreign goods in foreign currency, defined by a similar aggregator as equation (3.5). Also, Θ represents the set of firms with flexible prices and Σ the set of firms that pre-set prices in dollars. This latter set is endogenously determined as part of the rational expectations equilibrium. From now on we adopt the convention of denoting with an asterisk $(*)$, foreign variables.

3.1.2 Asset Market Structure

We follow Chari et al. (2002) by assuming that markets are complete domestically and internationally. At each period of time, the world economy faces one of the possible events x_t, drawn from a finitely set ϑ that contains all possible events. History of events up to period t is denoted by ζ_t and the conditional probability of occurrence of state x_{t+1} on the history ζ_t is given by $\Upsilon(x_{t+1} \mid \zeta_t)$. The asset market structure consists of a set of state contingent bonds denominated in domestic currency. Households holdings of these bonds are denote by $B(x_{t+1} \mid x_t)$; and bond's prices in period t and in state x_t are denoted by $\xi(x_{t+1} \mid \zeta_t)$. One unit of each of these bonds pays one unit domestic currency in period $t+1$ if the particular state, x_{t+1} occurs, zero otherwise. Therefore, the household's sequence of budget constraints are given by,

$$P(x_t)\,C_t(x_t) + + \sum_{x_{t+1}\in\vartheta} \xi(x_{t+1} \mid \zeta_t)\,B(x_{t+1} \mid x_t) = W(x_t)\,N(x_t) + B(x_t) + \Xi(x_t) \tag{3.6}$$

where $\Xi(x_t)$ accounts for firm's profits.

3.1.3 Optimal Conditions for Households Decisions

Each household in the domestic economy maximises her utility function given by equation (3.1), subject to their flow budget constraint, equation (3.6). The corresponding household's first order conditions are given by the following set of equations,

$$\xi(x_{t+1} \mid \zeta_t) = \beta\Upsilon(x_{t+1} \mid \zeta_t)\left(\frac{C^{-1}(x_{t+1})}{C^{-1}(x_t)}\frac{P(x_t)}{P(x_{t+1})}\right) \tag{3.7}$$

$$C_t N_t^v = \frac{W_t}{P_t} \tag{3.8}$$

$$\begin{array}{ll} C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t & C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \\ C_{H,t}(z) = \frac{1}{n}\left(\frac{P_{H,t}(z)}{P_{Ht}}\right)^{-\eta} C_{H,t} & C_{F,t}(z) = \frac{1}{1-n}\left(\frac{e_t P^*_{F,t}(z)}{P_{Ft}}\right)^{-\eta} C_{H,t} \end{array} \tag{3.9}$$

Equation (3.7), is the standard Euler condition that defines the optimal path of consumption and savings. When markets are complete, the free risk interest rate is equal to the inverse of the conditional expectation of the state contingent bond prices across all states of nature. Using this equality, the Euler equation can be alternatively written in terms of the free risk nominal interest rate, as follows,

$$\frac{1}{1+i_t} = \beta E_t\left(\frac{C_{t+1}^{-1}}{C_t^{-1}}\frac{P_t}{P_{t+1}}\right) \tag{3.10}$$

Furthermore, by combining the domestic and foreign Euler conditions, we can obtain the following condition,

$$\xi_{t+1} = \beta\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-1}\frac{P^*_t}{P^*_{t+1}}\frac{e_t}{e_{t+1}} = \beta E_t\left(\frac{C_{t+1}^{-1}}{C_t^{-1}}\frac{P_t}{P_{t+1}}\right) \tag{3.11}$$

Denoting by Q_t the real exchange rate, which is defined as the relative price of foreign goods in terms of domestic goods, i.e $Q_t = \frac{P_t^* e_t}{P_t}$ equation (3.11) can be rearranged to obtain the following recursive equation,

$$Q_{t+1} = \left(\frac{C_{t+1}^*}{C_{t+1}}\right)^{-1} \left(\frac{C_t}{C_t^*}\right)^{-1} Q_t \tag{3.12}$$

By iterating backwards equation (3.12) we obtain the following risk sharing condition that relates the real exchange rate to the dynamics of domestic and foreign consumption,

$$Q_t = \varsigma_0 \left(\frac{C_t^*}{C_t}\right)^{-1} \tag{3.13}$$

where ς_0 is a constant defined as follows, $\varsigma_0 = \frac{C_0^{-1}}{(C_0^*)^{-1}} Q_0$. Equation (3.12) shows that under the complete asset market assumption, the real exchange rate satisfies a simple relationship in terms of foreign and domestic consumption.

On the other hand, equation (3.8) determines households' labour supply. This equation shows that households optimally supply labour up to the point where the marginal disutility of working equalises its marginal benefit, given by the real wage expressed in units of utility. The intratemporal allocation of consumption across different good's varieties is determined by equation (3.9). At the optimal, households demand for each variety of consumption good is increasing on total consumption, C_t and decreasing in their corresponding relative prices. In the rest of the world, households solve an identical problem to the one detailed above for domestic households. Therefore, a set of similar optimality conditions describes their behaviour.

3.1.4 Firms

Consumption goods are produced by a continuum of monopolistically competitive firms. Each of them has a constant returns to scale technology to transform labour services $N_t(z)$ into a particular variety of final consumption goods, as follows:

$$A_t(z)Y_{H,t}(z) = N_t(z) \tag{3.14}$$

where $A_t(z)$ represents a negative technology shock, since for higher values of $A_t(z)$, the amount of labour required to produce the same amount output is also higher. We further assume that $A_t(z)$ evolves according the following stochastic process,

$$\ln(A_t(z)) = \epsilon_{z,t} \tag{3.15}$$

with $\epsilon_t \sim N\left(0, \sigma_{\epsilon_z}^2\right)$. and $E\left(\epsilon_{z,t}\epsilon_{j,t}\right) \neq 0$ We further define as A_t the aggregate domestic productivity shock, which is obtained by aggregating $A_t(z)$ over z using the following CES aggregator function, [13]

$$A_t = \left(\frac{1}{n}\int_0^n A_t^{1-\varepsilon}(z)\,dz\right)^{\frac{1}{1-\varepsilon}} \tag{3.16}$$

This definition of aggregate productivity is convenient since allow us to define aggregate output in terms of aggregate employment and productivity in a simple way, as follows,

$$A_tY_{H,t} = N_t \tag{3.17}$$

where,

$$Y_{H,t} = \left(\frac{1}{n}\int_0^n Y_{H,t}(z)\,dz\right) \quad N_t = \left(\frac{1}{n}\int_0^n N_t(z)\,dz\right)$$

Furthermore, each domestic producer faces a downward sloping demand function, which is obtained by aggregating both the domestic and foreign households demand for each good z. After aggregation, the domestic producer's demand function is given by,

$$Y_{H,t}(z) = \int_0^n C_{H,t}^i(z)d(i) + \int_n^1 \left(C_{F,t}^*\right)^i(z)d(i) \tag{3.18}$$

[13] See appendix A.2 for the details of this derivation.

Using equation(3.9) we can write the previous equation as follows:

$$Y_{H,t}(z) = \left(\frac{P_{H,t}(z)}{P_{h,t}}\right)^{-\varepsilon} Y_{H,t} \tag{3.19}$$

Where, $Y_{H,t}$ represents the total demand for domestic goods, which is determined by the following condition:

$$Y_{H,t} = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left((1-\alpha) C_t + \frac{(1-\alpha^*)(1-n)}{n} Q_t^{\eta} C_t^*\right) \tag{3.20}$$

Similarly, foreign firms face a downward demand function given by,

$$Y_{F,t}(z) = \left(\frac{P_{F,t}(z)}{P_{F,t}}\right)^{-\varepsilon} \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} \left(\frac{n}{1-n}\alpha C_t + \alpha^* Q_t^{\eta} C_t^*\right)$$

Where, α^* represents the participation of foreign goods in the consumption basket of foreign households.

Price Setting

A fraction θ of firms in the domestic economy set prices observing the realization of all shocks, whereas the remaining fraction, $(1-\theta)$ set prices one period in advance. Amongst these latter subset of firms a smaller group of them, of mass s, choose to set their prices in foreign currency. Notice that for flexible-price firms the choice of unit of account is irrelevant. These firms can always choose an equivalent price in dollars for the corresponding optimal price in pesos by simply dividing the optimal price in pesos by the current nominal exchange rate. Pricing in foreign currency becomes relevant only when firms face uncertainty about the realization of shocks[14].

A typical firm, z, choose its optimal price $P^{o}_{H,t}(z)$ to maximises the expected value of its profit's flow discounted by the relevant household's discount factor, which is defined by $\Lambda_t = \beta\frac{P_{t-1}C_{t-1}}{P_t C_t}$. Thus, firms' expected profit function under

[14] This irrelevance result is similar to the one discussed by Klemperer and Meyer (1986), who show that the strategy of setting prices or quantities is irrelevant when firms do not face uncertainty

peso invoicing is given by:

$$\Omega(z) = E_{t-1}\left[\left(\left(P_{H,t}(z) - W_t A_t(z)\right) Y_t(z)\Lambda_t\right]\right. \tag{3.21}$$

where the demand for good z is defined as follows,

$$Y_t(z) = \left(\frac{P_{H,t}(z)}{P_{H,t}}\right)^{-\epsilon} Y_{H,t} \tag{3.22}$$

The first order condition that determines the optimal firm's price under peso invoicing is given by

$$E_{t-1}\left[\left(\left(P_{H,t}(z) - \mu W_t A_t(z)\right)\left(\frac{P_{H,t}(z)}{P_{H,t}}\right)^{-\varepsilon} \frac{Y_{H,t}\Lambda_t P_{H,t}}{P_{H,t}}\right] = 0 \tag{3.23}$$

After simplifying the previous expression and defining $\digamma_t = Y_{H,t}\Lambda_t P_{H,t}$, we obtain,

$$P_{H,t}(z) = \mu \frac{E_{t-1}\left(W_t A_t(z)\, P_{H,t}^{\varepsilon-1}\digamma_t\right)}{E_{t-1}\left(P_{H,t}^{\varepsilon-1}\digamma_t\right)} \tag{3.24}$$

As we highlighted before firms choose optimally their unit of account, therefore, if the expected discounted profits under dollar invoicing are larger that under peso invoicing firm's would choose to invoice in dollars. Let's define the price of an individual domestic consumption good in dollars by $d_{H,t}(z)$ and the aggregate domestic price level expressed in dollars by, $d_{H,t}$. Using these two prices, firm's discounted expected profit function under dollar invoicing is given by,

$$\Psi(z) = E_{t-1}\left[\left(\left(d_{H,t}(z) - \frac{W_t}{e_t}A_t(z\right)\right) Y_t(z)\Lambda_t e_t\right] \tag{3.25}$$

and the corresponding firm's demand by the following condition,

$$Y_{H,t}(z) = \left(\frac{d_{H,t}(z)}{d_{H,t}}\right)^{-\varepsilon} Y_{H,t} \tag{3.26}$$

In this case, the optimal price under dollar invoicing is given by the following equation,

$$d_{H,t}(z) = \mu \frac{E_{t-1}\left(\frac{W_t}{e_t}A_t(z) d_{H,t}^{\varepsilon-1}\digamma_t\right)}{E_{t-1}\left(d_{H,t}^{\varepsilon-1}\digamma_t\right)} \tag{3.27}$$

On the other hand, the optimal price of flexible-price firms, those that belong to the set Θ, is given by a mark-up over their corresponding marginal costs,

$$P_{H,t}^{n}(z) = \mu W_t A_t(z)$$

Individual domestic firms pricing decision are therefore summarised as follows,

$$P_{H,t}(z) = \begin{cases} \mu W_t A_t(z) & \text{if } z \in \Theta \\ \mu \frac{e_t E_{t-1} \frac{W_t}{e_t} A_t(z) d_{H,t}^{\epsilon-1} F_t}{E_{t-1}\left(d_{H,t}^{\epsilon-1} F_t\right)} & \text{if } z \in \Sigma \\ \mu \frac{E_{t-1}(W_t A_t(z) P_{H,t}^{\epsilon-1} F_t)}{E_{t-1}\left(P_{H,t}^{\epsilon-1} F_t\right)} & \text{otherwise} \end{cases} \tag{3.28}$$

Firms' Invoicing Decisions

Each firm decides which currency to use for setting prices by comparing their expected profits under dollar and peso invoicing. Firm z will set prices in dollar if and only if the expected profits under this strategy exceeds the corresponding profits under invoicing in pesos. Plugging the optimal pricing rules into their corresponding profit functions, equations (3.21), (3.25) the condition for setting prices in dollars is given by

$$\frac{\Omega(z)}{\Psi(z)} = \frac{\left(E_{t-1}\left(w_t A_t(z) P_{H,t}^{\epsilon}\right)\right)^{1-\epsilon} \left(E_{t-1}\left(p_{H,t}^{\epsilon-1}\right)\right)^{\epsilon}}{\left(E_{t-1}\left(w_t A_t(z) d_{H,t}^{\epsilon}\right)\right)^{1-\epsilon} \left(E_{t-1}\left(d_{H,t}^{\epsilon-1}\right)\right)^{\epsilon}} < 1 \tag{3.29}$$

Therefore, a firm's z optimal invoicing decision, $\sigma_t(z)$ can be defined as follows,

$$\sigma_t(z) = \begin{cases} 1 \text{ if} & \frac{\Omega(z)}{\Psi(z)} < 1 \\ 0 & \text{otherwise} \end{cases}$$

and the set of firm's invoicing in dollars, Σ, as follows:

$$\Sigma = \{z \mid \sigma_t(z) = 1\} \tag{3.30}$$

3.1.5 The Small Open Economy

Following Sutherland (2002) and De Paoli (2004) we parameterise the foreign goods participation in the domestic household's consumption basket, α, in terms of the size of the economy, n and its degree of openness, γ, as follows, $\alpha = (1-n)\gamma$. Similarly, for the case of the foreign economy, $1 - \alpha^* = n\gamma$. With this parameterisation, domestic households consume more imported goods when the economy is more open, this is when γ is larger, or when the size of domestic economy, n, is relatively small.

The SOE is obtained making $n \to 0$. In this case, the foreign-good's participation in the domestic households consumption bundle is determined only by γ, the degree of openness. Furthermore, foreign households consume only foreign produced goods, $\alpha^* = 1$, consequently, changes on domestic aggregated demand have no effects on the foreign economy and its consumer price index coincides with its producer price index $P^* = P^*_F$. In the case of a SOE then, the domestic and foreign demand functions of consumption goods, equations (3.20) and (3.1.4) become:

$$Y_{H,t} = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left(\left(1-\gamma\right)C_t + \gamma Q_t^\eta C_t^*\right) \tag{3.31}$$

$$Y_{F,t}(z) = \left(\frac{P_{F,t}\left(z\right)}{P_{F,t}}\right)^{-\epsilon} \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} \left(Q_t^\eta C_t^*\right) \tag{3.32}$$

In what follows we restrict our analysis to the particular case of $\eta = 1$. With this parameterisation, in the limiting case of $s = 0$, no PD, our SOE is isomorphic to a close economy where domestic inflation, the output gap and the nominal interest rates can be determined independently of foreign shocks. In this way, we focus on the interaction between PD and monetary policy isolating the terms of trade role on monetary policy[15]. Furthermore, this parameterisation simplifies several key equations of the model in a very convenient way. For instance, equation (3.4) becomes,

$$P_t = P_{H,t}^{1-\gamma} P_{F,t}^{\gamma} \tag{3.33}$$

Let's define by Π_t, the gross rate of inflation, $\Pi_t = \frac{P_t}{P_{t-1}}$, and by $\Pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}}$, the gross rate of domestic inflation. Transforming appropriately equation (3.33), the

[15] See De Paoli (2004)and Galí and Monacelli (2005) for a detailed discussion of the role of terms of trade in SOE and the isomorphic close economy representation of open economies

link between Π_t and $\Pi_{H,t}$ can be written as follows,

$$\left(\frac{\Pi_t}{\Pi_{H,t}}\right) = \left(\frac{T_t}{T_{t-1}}\right)^{\gamma} \tag{3.34}$$

where T_t denotes the terms of trade, defined as the foreign goods' price in terms of domestic goods,

$$T_t = \frac{P_{F,t}}{P_{H,t}} \tag{3.35}$$

Also, equation (3.31) become,

$$Y_{H,t} = T_t^{\gamma} C_t \tag{3.36}$$

This latter equation in turn imply that $F_t = 1$. Furthermore, using equations (3.8), (3.17) and (3.36) the real wage in terms of domestic prices is determined by the following condition,

$$w_t = A_t^{v} Y_{H,t}^{1+v} \tag{3.37}$$

Similarly, using equations (3.36) and (3.34) to eliminate consumption and the CPI inflation from the Euler condition, it can be rewritten as follows,

$$\frac{1}{1+i_t} = \beta E_t \left(\frac{Y_{H,t+1}^{-1}}{Y_{H,t}^{-1}} \frac{1}{\Pi_{H,t+1}}\right) \tag{3.38}$$

Finally, the value of the nominal exchange rate can be readily obtained from the definition of the terms of trade, as follows,

$$e_t = \frac{Y_{H,t} P_{H,t}}{Y_t^* P_t^*} \tag{3.39}$$

3.2 The Dynamic Equilibrium

Given a sequence for the nominal interest rate, $\{i_t\}$, foreign productivity shocks, $\{a_t^*\}$ and domestic sector specific productivity shocks, $\{a_t(z)\}$. The dynamic equilibrium of domestic SOE is defined as an allocation for $\{Y_{H,t}\}$, $\{C_t\}$, relative prices, $\{w_t\}$, $\{Q_t\}$, $\{T_t\}$, $\{e_t\}$, $\{\Pi_t\}$ and $\{\Pi_{H,t+1}\}$ and firm's invoicing rules such that conditions (4.42), (4.41), (4.43), (3.36), (3.34), (3.35), (3.28) and (3.30) hold. In what follows, we use lower-case letters to denote variables in deviations respect to their steady-state levels, i.e, $x_t = \log\left(\frac{X_t}{X}\right)$.

3.2.1 The Flexible Price Equilibrium

When prices are flexible, firms can set prices every period observing the realisations of all shocks. In this case, the firm's pricing strategy is irrelevant for the equilibrium allocation. In order to show this point, let's look at the optimal prices of firm z under pesos and dollar invoicing, given by,

$$P^n_{H,t}(z) = \mu W_t A_{z,t} \quad d^n_{H,t}(z) = \mu \tfrac{W_t}{e_t} A_{z,t} \tag{3.40}$$

Since firms can perfectly observe their productivity shocks, $A_{z,t}$ and the nominal exchange rate, the corresponding optimal prices under pesos and dollars invoicing are related through,

$$P^n_{H,t}(z) = d^n_{H,t}(z) e_t \tag{3.41}$$

With these optimal prices, the amount of good z produced is exactly the same under both pricing strategies. Importantly, under the flexible prices allocation, the relative price between two different varieties of goods reaches its optimal value. From equation (3.40), this optimal relative price is given by firms relatively productivity,

$$\frac{P^n_{H,t}(z)}{P^n_{H,t}(z')} = \frac{A_{z,t}}{A_{z',t}} \tag{3.42}$$

This also implies that the usage of labour in the economy reaches its optimal value, given by

$$h^n_t = A_t Y^n_{H,t} \tag{3.43}$$

Also, as appendix A.2 shows, the flexible price equilibrium of this economy, up to a log linear approximation around the steady-state can be nicely characterised by the following three equations that determine the natural interest rate, the potential output and the real exchange rate[16],

$$y^n_{H,t} = -a_t \tag{3.44}$$

$$r^n_t = -E_t\left(a_{t+1} - a_t\right) \tag{3.45}$$

$$q_t = -\left(1-\gamma\right)\left(a_t - a^*_t\right) \tag{3.46}$$

The previous equations show that both the natural level of output and the natural interest rate do not depend on foreign shocks. We obtain this result only because we have chosen a very special type of preferences, ones that exhibit both unitary intertemporal elasticity of substitution and unitary elasticity of substitution between domestic and foreign goods. Under this type of preferences, the substitution and income effect that movements in terms of trade generate cancel out each other eliminating any trade balance deficit[17]. In a more general setup, both domestic output and real interest rate would respond to foreign shocks.

3.2.2 Aggregate Demand

We derive the aggregate demand by taking a log-linear approximation of equation (4.41) around the deterministic steady-state

$$y_{H,t} = E_t y_{H,t+1} - \left(i_t - E_t \pi_{H,t+1}\right) \tag{3.47}$$

As we show in appendix A.2, the aggregate demand equation in terms of output gap consistent with the previous equation is given by,

$$x_t = E_t x_{t+1} - \left(i_t - E_t \pi_{H,t+1} - r^n_t\right) \tag{3.48}$$

[16] We use lower case variables to denote log linear approximation of the original ones with respect to their steady-state.

[17] See Galí and Monacelli (2005) for a detailed discussion of a canonical representation of an small open economy, and De Paoli (2004) for its implications in optimal monetary policy.

The other relevant condition that comes from the aggregate demand side is the one that determines the dynamics of the nominal exchange rate. The nominal exchange rate in this simple economy depends on domestic prices, the output gap and a real exchange rate shock, as follows,

$$e_t = p_{H,t} - p_t^* + x_t - x_t^* + \eta_t \tag{3.49}$$

Where η_t, the composite real exchange rate shock, is defined as follows,

$$\eta_t = a_t^* - a_t \tag{3.50}$$

Equation (3.49) is obtained from the log-linear approximation of equation (4.42), where $a_t = \int_0^n a_t(z)dz$ and a_t^* is the foreign productivity shock, which is assumed to be independent of a_t. Interestingly, under the assumption of complete markets, equation (3.49) is equivalent to the uncover interest parity condition (UIP) . To illustrate this latter point, let's obtain $E_t\Delta e_{t+1}$ using equation (3.49).

$$E_t\Delta e_{t+1} = E_t\pi_{H,t+1} - E_t\pi_{t+1}^* + E_t\Delta x_{t+1} - E_t\Delta x_{t+1}^* + E_t\Delta\eta_{t+1}$$

and then express $E_t\Delta\eta_{t+1}$ in terms of the domestic and foreign natural interest rates, as follows

$$E_t\Delta\eta_{t+1} = E_t\Delta a_{t+1}^* - E_t\Delta a_{t+1} = -r_t^{n*} + r_t^n$$

finally, using the domestic and the foreign Euler equations we obtain the UIP condition,

$$E_t\Delta e_{t+1} = i_t - i_t^*$$

3.2.3 Aggregate Supply

We derive the aggregate supply equation of this economy by aggregating the log linear approximated optimal pricing rules of domestic firms given by equation (3.28).

At each point in time there exist three type of firms differentiated by their pricing strategy. A first type, of mass θ, set prices flexibly observing the realisation of all shocks in the economy. The remaining fraction, $1-\theta$, set prices one period in advance using information up to period, $t-1$. From this second group, a fraction $(1-\theta)\,s$ of firms sets prices in foreign currency, whereas the remaining one, $(1-\theta)\,(1-s)$ does it in pesos. The value of s is endogenously determined as an equilibrium feature in section 3.4.

Firms that set prices in advance are chosen randomly. Up to a first-order approximation the aggregate domestic price index, defined in equation (3.5), is given by,

$$p_{H,t} = \frac{1}{n}\int_0^n p_{H,t}(z)d(z) \tag{3.51}$$

whereas firm's optimal pricing rules by

$$p_{H,t}(z) - p_{H,t} = \begin{cases} w_t + a_t(z) & \text{if } z \in \Theta \\ E_{t-1}\left(w_t + a_t(z)\right) - \left(p_{H,t} - E_{t-1}p_{H,t}\right) + \left(e_t - E_{t-1}e_t\right) & \text{if } z \in \Sigma \\ E_{t-1}\left(w_t + a_t(z)\right) - \left(p_{H,t} - E_{t-1}p_{H,t}\right) & \text{otherwise} \end{cases}$$

where, w_t represents the log-linear approximation of real wages in terms of domestic prices[18]. Aggregating the previous optimal pricing rules across firms, we obtain the following condition for the aggregate supply,

$$\begin{aligned} p_{H,t} - E_{t-1}p_{H,t} &= \frac{\theta}{1-\theta}\left(w_t + a_t\right) + \left(E_{t-1}\left(w_t + a_t\right)\right) \\ &\quad + s\left(E_{t-1}\left(e_t\right) - e_t\right) \end{aligned} \tag{3.52}$$

Taking conditional expectations in period $t-1$ to (3.52) we can easily show that, $E_{t-1}\left(w_t + a_t\right) = 0$, thus, this equation can be written as,

$$p_{H,t} - E_{t-1}p_{H,t} = \frac{\theta}{1-\theta}\left(w_t + a_t\right) + s\left(E_{t-1}\left(e_t\right) - e_t\right) \tag{3.53}$$

[18] A novel feature of the previous pricing rules is that not only unexpected movements in domestic prices generate changes in firm's relative prices but also unexpected movements on the exchange rate. Hence, price stability not only implies stability of marginal costs but also stability of the nominal exchange rate.

Furthermore, from the log linear approximation of equation (4.43) we obtain,

$$w_t = va_t + (1 + v)\, y_{H,t} \tag{3.54}$$

By using equations (3.54) to eliminate real wages and the aggregate productivity shock from equation (3.53), we obtain the following aggregate supply equation in terms of output gap, unexpected changes in prices and in the nominal exchange rate,

$$p_{H,t} = E_{t-1}\left(p_{H,t}\right) + \kappa x_t + s\left(e_t - E_{t-1}\left(e_t\right)\right) \tag{3.55}$$

Where, $x_t = y_t - y_t^n$, $y_t^n = -a_t$ and $\kappa = (1 + \nu)\,\frac{\theta}{(1-\theta)}$, represents the slope of the Phillips curve. Notice that when, $s = 0$, the aggregate supply curve converges to the standard case of small open economy without price dollarisation. Differently, when a positive mass of firms sets prices in dollar, $s \neq 0$, unexpected changes in the nominal exchange rate become a determinant of domestic inflation. Using a simple transformation of equation (3.55), we obtain the Phillips curve in this economy,

$$\pi_{H,t} = E_{t-1}\pi_{H,t} + \kappa x_t + s\left(\Delta e_t - E_{t-1}\left(\Delta e_t\right)\right) \tag{3.56}$$

As equation (3.49) reveals, the output gap and unexpected changes in the depreciation rate are not proportional. Consequently, the central bank cannot induce an equilibrium where simultaneously the domestic inflation, the output gap, and the unexpected change in the exchange rate are all equal to zero. This implies that the central bank will have to accept a higher volatility on output gap in order to stabilise domestic inflation when there exists PD. Consequently, in an economy with PD a monetary policy aiming at $\pi_{H,t} - E_{t-1}\pi_{H,t} = 0$, will be costly. This result differs from the one obtained by Galí and Monacelli (2005), who show that an SOE economy is isomorphic to a close economy under the same type of preferences that we consider in this chapter. Hence, the central bank can stabilise domestic inflation at zero cost. This is true in our model only when $s = 0$.

3.3 Monetary Policy

This section follows Woodford (2003) and Benigno and Woodford (2005) in deriving a central bank welfare-based loss function. Differently from them, however, we focus in an economy with PD. The loss function is obtained from a second-order Taylor expansion of the utility function of the representative household taken around the efficient deterministic steady-state.

By focusing on an efficient steady-state we gain in tractability. In this case a log-linear, instead of a quadratic approximation, of the rational expectations equilibrium is sufficient to obtain an accurate measure of welfare.

In this section we proceed as follows, first we solve for the social planner problem to fully characterise the efficient steady-state allocation. Then, we approximate the household's welfare function around this efficient steady-state. Finally, we derive the optimal central bank's reaction function based on this loss function.

3.3.1 Optimal Allocation

The optimal allocation is obtained by solving the social planner's problem. The social planner chooses an allocation for consumption and labour that maximises the welfare of the representative household given by:

$$U = \left(\ln C - \frac{N^{1+v}}{1+v}\right) \tag{3.57}$$

subject to the technology constraint, a) $N = Y_H A$, b) the link between consumption and output implied by the risk sharing condition, equation (3.13), and c) the aggregate resource constraint, equation (3.36). Notice, however, that these two latter constraints can be combined in just one that relates consumption and output as follows,

$$C = Y_H^{1-\gamma} Y^{*\gamma} \tag{3.58}$$

The efficient allocation then must satisfy,

$$-\frac{U_N}{U_C} = \frac{C}{Y_H}(1-\gamma) \tag{3.59}$$

From the previous condition, it turns out that the efficient output level must be $Y_H = (1-\gamma)^{\frac{1}{1+v}}$ [19]. Importantly, this output level differs from the one obtained in the decentralised equilibrium, which is given by,

$$Y_H = (1-\Phi)^{\frac{1}{1+v}} \tag{3.60}$$

where: $(1-\Phi) = \frac{1-\tau}{\mu}$, represents the distortion associated to monopolistic competition and τ stands for a government subsidy. If this monopolistic distortion is not eliminated it generates an inflationary bias since the central banks finds optimal to target a positive output gap. Besides this distortion, in open economy models, as Benigno and Benigno (2003) show, there exist a deflationary bias given that domestic authorities have incentives to generate a deflationary surprise to appreciate terms of trade. By doing so, the policy makers try to generate a positive welfare effect through a reduction in the disutility of producing goods that more than compensates the reduction in consumption generated by the expenditure switching effect[20]. We set $\tau = 1-(1-\gamma)\mu$. With this subsidy the inflationary bias generated by monopolistic competition and the deflationary bias from terms of trade balance each other, making possible to replicate the efficient output allocation in the decentralised equilibrium.

3.3.2 The Central Bank Loss Function under Price Dollarisation

In order to obtain the central bank loss function we approximate the representative household's utility, equation (3.57), around the efficient steady-state. It turns out

[19] Importantly, this condition characterises optimal monetary policy only under a complete market structure. When markets are incomplete, the risk sharign condition, equation (3.13) does not hold, consequently , equation (3.58) also does not hold. See Benigno (2001) for an analysis of optimal monetary policy with incomplete financial markets in a two country world model

[20] See De Paoli (2004) for a deep discussion of the terms of trade bias in small open economies.

that in an economy with PD the central bank loss function depends on the output gap, $\widehat{x}_t$, unexpected changes in domestic inflation, $\widetilde{\pi}_{H,t}$, unexpected changes in the nominal exchange rate, $\widetilde{e}_t$ the correlation between $\widetilde{\pi}_{H,t}$ and $\widetilde{e}_t$; and the correlation between $\widetilde{e}_t$ and $\widetilde{a}_{s,t}$ [21]. Where this last term represents the average productivity dispersion of firms that have set prices in dollars. The loss function is presented in the next equation,

$$L = -\frac{\Omega}{2}\sum_{t=0}^{t=\infty} \beta^t \left(\Lambda \widehat{x}_t^2 + \widetilde{\pi}_{H,t}^2 + \Lambda_e \Delta \widetilde{e}_t^2 - 2s\widetilde{\pi}_{H,t}\Delta\widetilde{e}_t - 2\Lambda_{ea}\Delta\widetilde{e}_t\widetilde{a}_{s,t}\right) \tag{3.61}$$

where, $\Omega = \overline{u}_c\overline{Y}\left(1-\gamma\right)\varepsilon\frac{(1-\theta)}{\theta}$, $\Lambda_e = s\left(1+\frac{s(1-\theta)}{\theta}\right)$, $\Lambda = \left(1+v\right)\frac{\theta}{(1-\theta)\varepsilon}$, $\Lambda_{ea} = \frac{\theta}{(1-\theta)}$, $\widetilde{a}_{s,t} = \int\limits_{\Sigma}\left(a_t(z)-a_t\right)d(z)$, $\widetilde{\pi}_{H,t} = \left(\pi_{H,t} - E_{t-1}\pi_{H,t}\right)$, and $\widetilde{e}_t = \left(e_t - E_{t-1}e_t\right)$

Several remarks are in order to qualify this loss function. First, since the monopolistic competition distortion is eliminated by the government subsidy, the optimal target for the output gap is zero. Second, fluctuations in terms of trade do not generate welfare losses. This implication comes from the assumptions of unitary elasticity of intertemporal substitution and of unitary elasticity of substitution between domestic and foreign goods under complete markets.

Third, exchange rate volatility generates welfare losses. This is so because when some firms set prices in foreign currency, fluctuations in the nominal exchange rate distort their relative prices increasing the overall relative price dispersion in the economy. Fourth, a positive correlation between the productivity shocks of firms that invoice in foreign currency and the nominal exchange rate increase welfare. In this latter case, this correlation offsets the distortions in relative prices that productivity shocks generate when prices are sticky.

Finally, a positive correlation between the nominal exchange rate and domestic inflation generates welfare gains. This result follows from the fact that for firms with dollar prices, the change in the exchange rate that follows a movement in

[21] The appendix A.5 shows the details of this derivation

domestic inflation reduces the impact of domestic inflation in their relative prices. Importantly, the impact of the exchange rate's second moments on the central bank's loss function crucially depend on the degree of PD, s. When $s = 0$, only domestic inflation and output gap fluctuations generate welfare losses.

PD constraints central bank's objective function mainly because it affects the overall efficiency loss generated by relative price dispersion in the economy. As Woodford (2003) discusses, the determinants of the economy's relative price dispersion depend on the particularities of firm's price setting. In our case, since some firms invoice using a foreign currency, as the above discussion suggest, exchange rate second moments generate welfare losses. In order to illustrate this point let's define the aggregate usage of labour by,

$$N_t = Y_{H,t} A_t \Delta_t \tag{3.62}$$

where Δ_t captures the overall relative price dispersion in the economy and it is defined below

$$\Delta_t = \frac{1}{n} \int_0^n \left(\frac{P_{H,t}(z)}{P_{H,t}} \right)^{-\varepsilon} \frac{A_t(z)}{A_t} d(z) \tag{3.63}$$

Up to a second-order approximation Δ_t is determined by[22]

$$\Delta_t = \frac{\varepsilon}{n} \int_0^n \left[(p_{H,t}(z) - P_{H,t}) - (a_t(z) - a_t) \right]^2 d(z) \tag{3.64}$$

Clearly, to minimise the overall relative price distortions, relative prices need to move in the same direction as relative productivity shocks. Since prices are sticky and a fraction of firms set prices in a foreign currency this cannot be achieved by keeping the domestic price level constant. In this case, the first best allocation is not attainable because the central bank has a limited set of instruments but multiple objectives, keep a continuum of relative prices aligned. However, if some firms set prices in dollars, the central bank can implement a policy superior to zero inflation

[22] See appendix A.1 for the derivation of the economy's overall relative price distortion. Δ_t

by using actively the nominal exchange rate to offset the price misalignments of dollar-price firms.

This is possible because relative prices of dollar-price firms respond to unexpected movements on the exchange rate. Therefore, by inducing a positive correlation between exchange rate movements and domestic inflation the central bank can effectively reduces welfare losses generate by domestic inflation. Furthermore, it can also partially offset the relative price dispersion created by sector specific shocks to dollar-price firms. Importantly then, in an economy with PD the first best allocation is unattainable since the central bank has just one instrument, either inflation, or the nominal exchange rate but more than one objective, $\pi_{H,t} = e_t = \widehat{x}_t = 0$. Therefore, optimal monetary policy can achieve only a second best.

The optimal central bank reaction function that implements this second best is then obtained by minimising the central bank loss function, equation (3.61) subject to the Phillips curve and the dynamics of the nominal exchange rate, equations (3.56) and (3.49) respectively[23]. The first order condition of this problem is given next,

$$(\Lambda_e - s)\,\Delta\widetilde{e}_t - \Lambda_{ea}\widetilde{a}_{s,t} + (1-s)\,\widetilde{\pi}_{H,t} = 0 \tag{3.65}$$

This condition describes the optimal central bank's reaction function. Differently from economies without PD, when there exist PD, domestic inflation is not always optimal. As the previous condition shows, a positive level of inflation is optimal either when an unexpected increase in the nominal exchange rate or a negative productivity shock of dollar-pricing firms hit the economy. In those cases, higher domestic inflation helps to mitigate the effects of these shocks on firm's relative prices. Notice that the optimal increase in the domestic inflation rate in response to these shocks is increasing on the degree of PD, s. This is so because when the degree of PD is higher, exchange rate fluctuations generate larger relative price

[23] The details of the central bank problem under commitment are presented in appendix A.4

distortions.

3.3.3 Equilibrium under Optimal Monetary Policy

In order to analyse the equilibrium responses of domestic inflation, the output gap and the exchange rate, next, we use the central bank's optimal reaction function, equation (3.65), the Phillips curve, equation (3.56) and the exchange rate dynamics, equation (3.49) to fully describe the rational expectations equilibrium under optimal policy. Furthermore, we assume that monetary policy in the foreign economy is also optimal, which implies that $p_t^* = x_t^* = 0$, and then equation (3.49) becomes,

$$e_t = p_{H,t} + x_t + \eta_t \tag{3.66}$$

Likewise, we use variables with $\widetilde{()}$ to denote variable's deviations with respect to its expected value, i.e. $\widetilde{e_t} = e_t - E_{t-1}(e_t)$. Thus, equation (3.66) can be written as follows,

$$\Delta\widetilde{e_t} = \Delta\pi_{H,t} + x_t + \eta_t$$

In equilibrium, the nominal exchange rate, domestic prices and the output gap evolve in terms of two structural shocks, a real exchange shock, $\widetilde{\eta_t}$ and the average dollar-pricing firms' productivity shock, $\widetilde{a}_{s,t}$, according to the following conditions,

$$\Delta\widetilde{e_t} = \varpi_1\widetilde{\eta_t} + \varpi_2\widetilde{a}_{s,t} \tag{3.67}$$

$$\widetilde{\pi}_{h,t} = -\varpi_3\widetilde{\eta_t} + \varpi_4\widetilde{a}_{s,t} \tag{3.68}$$

$$x_t = -\frac{(\varpi_3 + s\varpi_1)}{\kappa}\widetilde{\eta_t} + \frac{(\varpi_4 - s\varpi_2)}{\kappa}\widetilde{a}_{s,t} \tag{3.69}$$

Where the parameters, ϖ_1, ϖ_2,ϖ_3 and ϖ_4 are defined as follows,

$$\begin{array}{ll} \varpi_1 = \frac{\kappa(1-s)}{dd} > 0 & \varpi_2 = \frac{\theta(1+\kappa)}{dd} > 0 \\ \varpi_3 = \frac{\kappa(\Lambda_e - s)}{dd} > 0 & \varpi_4 = \frac{(\kappa+s)\theta}{dd} > 0 \end{array}$$

and,

$$dd = (1-s)\left[\kappa\left(1 - s\left(1-\theta\right)\right) + s\theta\right] > 0$$

Interestingly, as equation (3.67) shows, in equilibrium the nominal exchange rate increases when negative productivity shocks hit firms that invoice in dollars. The impact of these latter shocks on the exchange rate reflects the central bank's optimal reaction function. As we discussed previously, the central bank induces a positive correlation between these shocks and the exchange rate to mitigate their impact on firm's relative prices.

Similarly, the inflation dynamics in an economy with PD exhibits distinct features. As equation (3.68) shows, domestic inflation increases in response to dollar-price firms' negative productivity shocks. By letting domestic inflation to be higher the central bank induces an adjustment on the relative price of firms that invoice in domestic currency, which in turn reduces the gap between dollar and peso goods relative prices.

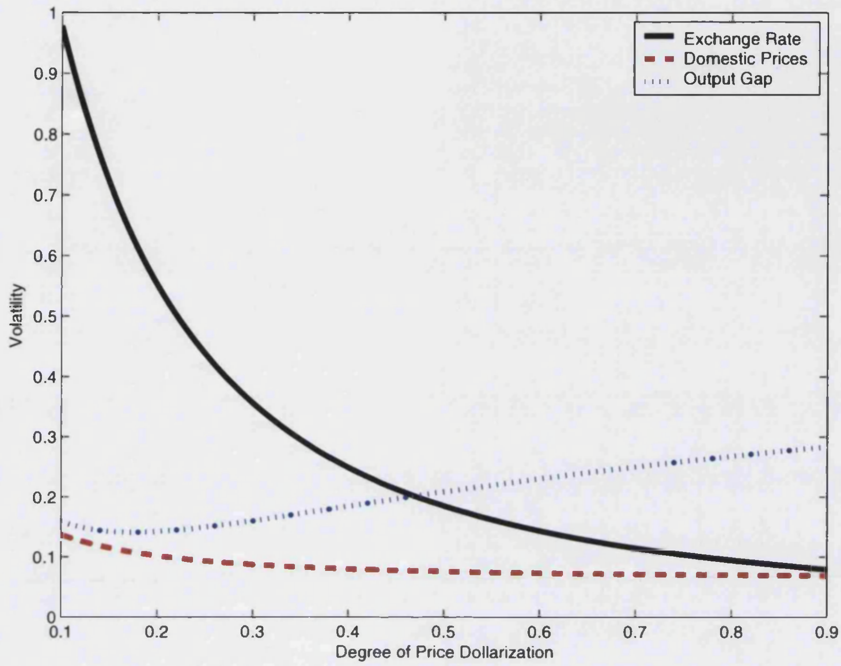
Also, to compensate the weaker response of the nominal exchange to a positive real exchange shock, the output gap falls in response to this shock. On the contrary, when there is no PD, the equilibrium levels of both inflation and output gap are zero and the nominal exchange rate absorbs the impact of real exchange shocks on the economy, in this case we have,

$$\begin{aligned} &\widetilde{\pi}_{h,t} = 0 \quad \Delta\widetilde{e}_t = \widetilde{\eta}_t \\ &\widetilde{x}_t = 0 \end{aligned}$$

To illustrate how the volatility of domestic inflation, the nominal exchange rate and the output gap evolves for different values of PD, we use a calibrated version of the model with the following parameter values $\theta = 0.5$, $v = 1.5$ and $\varepsilon = 10$ and conditions (3.67), (3.68) and (3.69). The results are depicted in figure 3.1.

As figure 3.1 illustrates, when the degree of PD increases, the volatility of both the nominal exchange rate and inflation falls, whereas the corresponding one for the output gap increases. This pattern on volatilities reflects the change

Figure 3.1: Volatility under Optimal Monetary Policy and Price Dollarisation



in the central bank's reaction function in response to different levels of PD. For instance, when the degree of PD is large, fluctuations on the exchange rate become more costly, therefore the central bank has more incentives to smooth exchange rate fluctuations. However, since the central bank has a limited set of policy instruments, a more stable exchange rate come at the cost of a more volatile output gap.

Up to this section we have derived results for optimal monetary policy assuming that the degree of PD is given. However, this variable is not exogenous to monetary policy, on the contrary, it is determined by monetary policy. In the next section we show how firms decide which currency to use for setting its prices and the interaction of this decision with optimal monetary policy.

3.4 Price Dollarisation in General Equilibrium

The general equilibrium level of PD is determined by the dimension of Σ, the set containing all the firms that have optimally chosen to invoice in dollars, :

$$\Sigma = \{z \mid \sigma_t(z) = 1\}$$

where,

$$\sigma_t(z) = \begin{cases} 1 \text{ if} & \frac{\Omega(z)}{\Psi(z)} < 1 \\ 0 & \text{otherwise} \end{cases}$$

To provide more intuition on what factors induce firms to invoicing in dollars we take a second-order approximation to the relative expected profits of invoicing in dollars versus invoicing in pesos. around the deterministic steady state. The details of this derivation are provided in appendix A.5. This approximation shows that a firm's expected profits are larger under dollar invoicing than under peso invoicing if the following condition holds,

$$\left(\frac{1}{2} + s\frac{(1-\theta)}{\theta}\right) E_{t-1}\Delta\widetilde{e}_t^2 - E_{t-1}\left[\Delta\widetilde{e}_t, (\widetilde{a}_t(z) - \widetilde{a}_t)\right] - \frac{1}{\theta}E_{t-1}\left(\widetilde{\pi}_{H,t}\Delta\widetilde{e}_t\right) < 0 \quad (3.70)$$

Condition (3.70) has a very intuitive interpretation. On one hand, it shows that when the expected volatility of the exchange rate is high, firms' expected profits under dollar invoicing are smaller than under peso invoicing. This is so because when firms pre-set prices in dollars, their relative price is more sensitive to exchange rate movements, and therefore a very volatile exchange rate generates large relative price misalignments, reducing firms' expected profits.

The opposite happens when the correlation between domestic prices and the nominal exchange rate is high. In this second case, when firms invoice in dollars their relative price moves in the same direction as domestic inflation, isolating them from unexpected movements on expected inflation, and therefore increasing firms' expected profits. A similar effect has a high correlation between the nominal exchange rate and firms' productivity shock, $a_t(z)$ on the firms' incentives to invoice in dollars.

Interestingly, condition (3.70) is similar to the one obtained by Devereux et al. (2004) and by Loyo (2001) but using different macroeconomic frameworks. Devereux et al. (2004) use a two-country world economy to analyse endogenous pass-through, whereas Loyo (2001) employs a close economy model to analyse the welfare gains of having multiple units of account[24]. Although our macroeconomic framework differs from those of the aforementioned authors in several dimensions, we find similar results on the partial equilibrium determinants of firm's invoicing decisions. As Devereux et al. (2004) highlight, this is so because for small deviations of the exchange rate around the steady state, firms' invoicing decisions are independent on second moment's of aggregate demand, other firm's prices, the household stochastic discount factor and on the financial market structure[25]. However, as we show later, our results are different of the aforementioned authors on two important dimensions: a) the general equilibrium determinants of PD and b) the role of monetary policy.

Using the rational expectations equilibrium solution for the nominal exchange rate and for the domestic price level, given by equations (3.67) and (3.68) we calculate the second moments involved in equation(3.70) in terms of shocks' second moments. Using these second moments, the condition that determines whether a firm set prices in dollars is given by,

$$cov\left[\left(a_t(z) - a_t\right), a_{s,t}\right] > \chi_1 var\left(a_{s,t}\right) + \chi_2 var\left(\eta_t\right) - \chi_3 cov\left(a_{s,t}, \widetilde{\eta}_t\right) \tag{3.71}$$

[24] As appendix A.5 shows, equation (3.70) can be alternative written as,

$$\frac{E_{t-1}\widetilde{e}_t^2}{2} - E_{t-1}\left[\left(\widetilde{W}_t + \widetilde{a}_t(z)\right)\widetilde{e}_t\right] < 0$$

which is similar to the condition in proposition 1 on Devereux et al. (2004).

[25] However, it is important to highlight that our framework differ from those of Devereux et al. (2004) and Loyo (2001) in several dimensions. Different from Devereux et al. (2004) we focus on a SOE and optimal monetary policy, whereas the aforementioned authors focus in a two country model economy where monetary policy is not optimally implemented. Also in contrast with Loyo (2001) who focus in a close economy where the parity between units of account is not constrained by the economic structure. In our model instead, the exchange rate is determined endogenously within the rational expectations equilibrium of the economy.

where the parameters χ_1, χ_2 and χ_3 are defined as follows,

$$\chi_1 = \frac{\frac{\left[(\omega_4-\omega_2)^2-\omega_4^2\right]}{2\theta}+\left(s-\frac{1}{2}\right)\frac{1-\theta}{\theta}\omega_2^2\Big)}{\omega_2} \qquad \chi_2 = \frac{\frac{(\omega_3+\omega_1)^2-\omega_3^2}{2\theta}+\left(s-\frac{1}{2}\right)\frac{1-\theta}{\theta}\omega_1^2}{\omega_2}$$

$$\chi_3 = \frac{\frac{2[(\varpi_4-\varpi_2)(\varpi_3+\varpi_1)+\omega_3\omega_4]}{2\theta}-2\omega_1\omega_2\left(s-\frac{1}{2}\right)\frac{1-\theta}{\theta}}{\omega_2}$$

To better understand the relationship of χ_1, χ_2 and χ_3 with the structural parameters, θ, v, and s, we evaluate their link numerically for a wide range of values. These relationships are depicted in figures 3.5 (a) to (d). As these figures show $\chi_2 > 0$ and $\chi_1 < 0$ for all the parameter values considered. Also, these pictures show that, for a given v, χ_1 is decreasing on θ, and χ_2 is increasing. We use these features of χ_1, χ_2 and χ_3 to characterise the equilibrium PD in the next section. Using the definitions of χ_1, χ_2 and χ_3, the set Σ that contains all firms that optimally choose to invoice in dollars can be defined as follows,

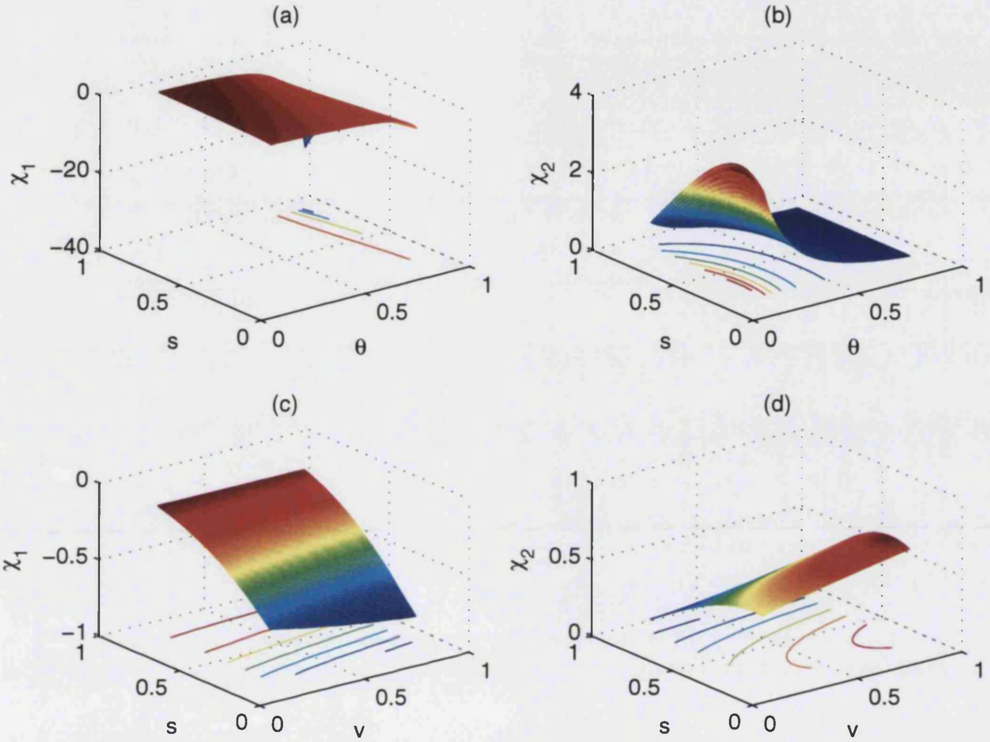
$$\Sigma = \{z : cov\left[\left(a_t(z) - a_t\right), a_{s,t}\right] > \chi_1 var\left(a_{s,t}\right) + \chi_2 var\left(\eta_t\right) - \chi_3 cov\left(a_{s,t}, \eta_t\right)\} \tag{3.72}$$

Notice that condition (3.72) defines a fixed point over the space of sets. Therefore, to evaluate condition (3.72) we need to know the set Σ, and to know the set Σ we need to know which mass of firms satisfy condition (3.72). Thus, we can not tell much about the equilibrium degree of PD of this economy unless we specify some structure for the second moments of $a_t(z)$. Next, we use a very simple case that is enough to obtain qualitatively results about the equilibrium degree of PD and its relationship with optimal monetary policy.

3.4.1 A Simple Case: Two-sector Economy

Let's consider the case of a domestic economy where firms are allocated in two productive sectors 1 and 2. Firms in sector 1 face a productivity shock $a_{1,t}$, whereas

Figure 3.2: Firm's Invoicing Decisions: Key Parameters



Note: In all figures we have considered values for θ, v and s between 0 and 1

firms in sector 2 face the productivity shock $a_{2,t}$. For the sake of simplicity we assume that both $a_{1,t}$ and $a_{2,t}$ have the same mean and variance and also that they are perfectly negatively correlated.

$$E_{t-1}(a_{1,t}) = 0 \qquad E_{t-1}(a_{2,t}) = 0$$

$$E_{t-1}(a_{2,t} - E_{t-1}(a_{2,t}))^2 = \sigma^2 \quad E_{t-1}(a_{1,t} - E_{t-1}(a_{1,t}))^2 = \sigma^2$$

$$E_{t-1}\left(a_{1,t} - E_{t-1}(a_{1,t})\right)\left(a_{2,t} - E_{t-1}(a_{2,t})\right) = -\sigma^2$$

Under these assumptions, aggregate productivity in the domestic economy is not stochastic but constant, hence, $a_t = 0$, $\eta_t = a_t^*$ and $cov\left(a_{s,t}, \eta_t\right) = 0$. Therefore, condition (3.72) simplifies as follows,

$$\Sigma = \left\{ z : cov\left[\left(a_t(z)\right), a_{s,t}\right] > \chi_1 \sigma^2_{a_{s,t}} + \chi_2 \sigma^2_\eta \right\} \tag{3.73}$$

Next we use the previous condition to determine the set of parameter values that guarantee the existence of an equilibrium with PD.

Case 1: Full PD Dollarisation

This case corresponds to $s = (1-\theta)$, an allocation where all firms that pre-set prices choose to invoice in dollars[26]. Furthermore, if $s = (1-\theta)$, then $a_{s,t} = a_t = 0$ and consequently the condition that determines the marginal firm's invoicing decision, condition (3.72) becomes

$$0 > \chi_2 \sigma^2_\eta \tag{3.74}$$

Since, $\sigma^2_\eta > 0$, and $\chi_2(s) > 0$ for all $s \in \langle 0, 1 \rangle$, as we show in figure 3.5, the firms' expected profits under dollar invoicing never would be larger than those under peso invoicing . This is, if all firms that pre-set prices choose to invoice in a foreign currency, a marginal firm would find optimal to deviate from this strategy and to invoice in a domestic currency since by deviating its expected profits would increase. Interestingly then, $s = (1-\theta)$ would not be an equilibrium. To understand why this happens notice that when $a_{s,t} = 0$, the central bank does not respond anymore to dollar-firm's productivity shocks. Consequently, firms' marginal cost would not be correlated with the exchange rate and therefore, firms would not have incentives to invoice in a foreign currency.

[26] In this economy the degree of PD s can not be equal to 1 since only a fraction $(1-\theta)$ of firms pre-set prices. We assume that when indifferent, as in the case of flexible prices, firms set prices in pesos.

Case 2: symmetric case, $n_1 = \frac{1}{2}$

We focus on a symmetric equilibrium where both sectors have equal size: $n_1 = n_2 = \frac{1}{2}$ and where only one sector invoice in a foreign currency. The conditions that characterise this equilibrium are presented next[27],

$$(s - \chi_1) > \chi_2 \tag{3.75}$$

$$-s - \chi_1 - \chi_2 < 0 \tag{3.76}$$

Condition (3.75) guarantees that expected profits under dollar invoicing would be larger than under peso invoicing for type$_1 firms, those belonging to sector 1. On the other hand, c\epsilon$
$2 firms would find optimal to set prices in pesos given that type-1 firms invoice in dollars^{28}. Henc$
θ). However, Since χ_1 and χ_2 depend on s, individual firms invoicing decision would be affected by how other firms choose to invoice and consequently the possibility of multiple equilibrium arise. To analyse all possible equilibriums in this economy it is convenient write conditions (3.75) and (3.76) as follows,

$$\sigma_1 > 0 \quad \sigma_2 < 0 \tag{3.77}$$

In this way, σ_1 and σ_2 can be interpreted as the expected firms' gains of invoicing in dollars relative to invoicing in pesos for type-1 and type-2 firms respectively. These two functions are defined as follows,

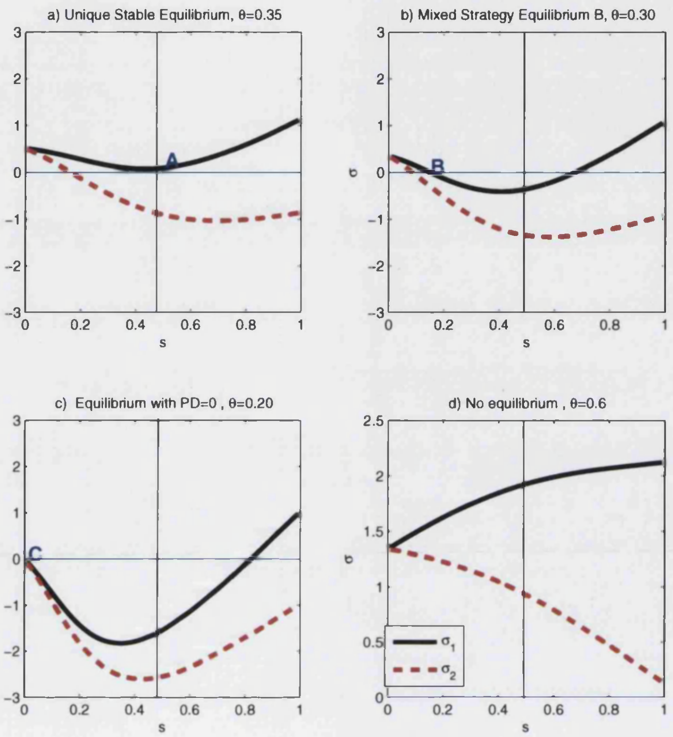
$$\sigma_1 = (s - \chi_1) - \chi_2 \quad \sigma_2 = -(s + \chi_1) - \chi_2$$

To characterise the set of equilibriums let's denote by s_i^* the value of s that solves, $\sigma_i = 0$ and let's plot the relationship between σ_i and s for a given a parameterisation of θ and v. If σ_i is above the zero line for $s \in \langle 0, 1\rangle$, then all type-1 and type-2 firms

[27] These conditions are obtained directly from (3.72) considering that $a_t = 0$, $\eta_t = a_t^*$, $a_{s,t} = sa_{1,t}$ and $cov\,(a_{s,t}, \eta_t) = 0$

[28] For the sake of exposition, we choose the sector 1 as the sector where firms invoice in a foreign currency. However, similar conditions apply for the alternative case where firms in sector 2 are those who choose to invoice in a foreign currency.

Figure 3.3: Equilibrium Price Dollarisation Symmetric Case



Note: The solid line represents the expected relative profits of dollar invoicing versus peso invoicing, for type-1 firms, whereas the dotted line the relative expected profits of type-2 firms. The vertical line is place on $s = 0.5$

would find optimal to invoice in a foreign currency. The opposite would happen if σ_i is below zero for all values of s. If these two curves cut the horizontal axis, on the other hand, the individual firm's decision would depend on what other firms do. Figure 3.3 presents plots of σ_i. In each of these plots, σ_i is measured on the vertical axis and s on the horizontal axis. Figures 3.3 (a) and (b) show the case of a unique equilibrium, whereas figure 3.3 (c) shows the case of an equilibrium without PD and figure 3.3 (d) a case where full PD is not an equilibrium.

The first case, depicted in figure 3.3(a) as point A, corresponds to a pure strategy unique equilibrium. At this point, type-1 firms find optimal to invoice in a foreign currency regardless what other firms do, whereas type-2 firms find optimal to set prices in pesos, given that a mass of $s = n_1(1-\theta)$ type-1 firms invoice in a foreign currency. Notice, however, that firms in sector 2 would like to invoice in dollars if $s \leq s_2^*$ but since all type-1 firms invoice in dollars and their mass is $s = n_1(1-\theta) > s_2^*$ this point is not feasible. Hence, $s = \frac{1}{2}(1-\theta)$ is the unique pure strategy equilibrium in the economy.

The second type of equilibrium is shown at figure 3.3 (b) as point B. In this case, type-1 firms' invoicing decisions depend on what other firms do. In particular, when all type-1 firms choose to invoice in pesos, any one type-1 firm finds optimal to deviate and invoice in dollars. Similarly if all type-1 firms choose to set prices in dollars, any one type-1 firm would find optimal to deviate by invoicing in pesos. Then, an intermediate equilibrium exist at point B, $s = s_1^*$, where some type-1 firms invoice in dollars and some invoice in pesos.

As Devereux et al. (2004) argue, firms can coordinate at this equilibrium by playing mixed strategies. In particular, if every type-1 firm choose to invoice in dollars with probability $\frac{s_1^*}{n_1}$, and to invoice in pesos with probability $\left(1 - \frac{s_1^*}{n_1}\right)$, the unique stable mixed strategy equilibrium would be $s = s_1^*$. On the other hand, figures 3.3 (c) and (d) depict those cases where no equilibrium with PD exists. Figure 3.3 (c) shows the case where $s = 0$. In this case, type-1 and type-2 firms find optimal to set prices in pesos whatever other firms do. Hence, at point C no firm would have incentives to deviated. Notice, however, that if $s > s_1^*$ type-1 firms would have incentives to deviate by invoicing in dollars, but, as in case (b), this allocation is not feasible since $s_1^* > n_1 = \frac{1}{2}$[29]. Finally, figure 3.3(d) depicts a case where both types of firms find optimal to invoice in dollars whatever other firms do. In this case, firm's invoicing decisions are not determined by conditions (3.77),

[29] At this equilibrium type-2 firms choose to invoice in pesos regardless the value of s. The same is true for type-1 firms but for $s < s_1^*$. Since the mass of type-1 firms is $\frac{1}{2} < s_1^*$ it can not be the case that s_1^* firms choose to invoice in dollars.

but instead by conditions (3.74). Thereby, we are back to case 1, where, as we previously shown full PD, $s = \frac{1}{2}(1-\theta)$ is not an equilibrium.

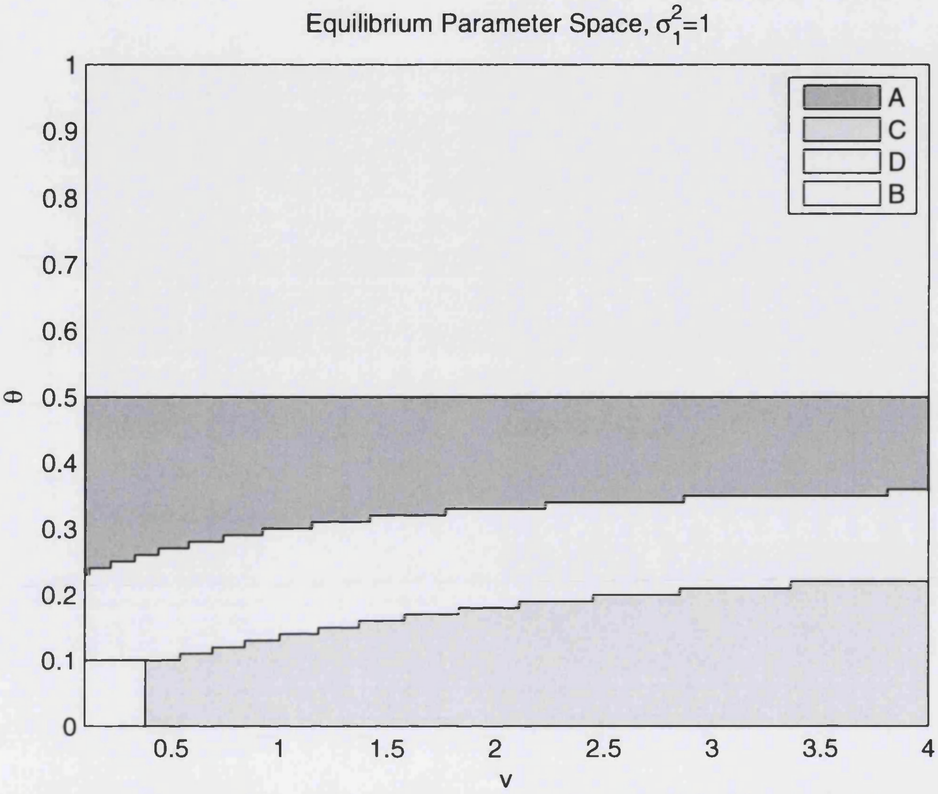
However, to fully characterise the equilibrium with PD it is crucial to determine the set of parameter values that sustain each of the four cases we previously discussed. Unfortunately, since χ_1 and χ_2 are highly non-linear functions of θ, s and v an analytical solution of this problem is not available. Instead we determine the equilibrium solution numerically. To this purpose we calculate σ_1 and σ_2 for a wide range of values for θ and v. In particular, we consider values of θ between 0.1 and 0.99 and for v between 0.1 and 4 . Then we search and store only those parameter values for which conditions (3.77) hold for every s [30]. Figure 3.4 depicts the sub-set of the parameter space that sustain each of the four cases previously discussed. As this figure shows, a unique equilibrium with partial PD, $s = \frac{1}{2}(1-\theta)$ exists when the degree of price stickiness is not too low, zone A. For larger values of θ, both types of firms find optimal to invoice in dollars regardless what other firms do, zone D. This case corresponds to case (d) previously analysed. Whereas for very low values of θ, zone C in the previous figure, the unique equilibrium is $s = 0$.

The previous figures suggest therefore that an equilibrium with partial PD is more likely to hold when the degree of price stickiness is neither too small nor too large, in particular when $\theta \in \langle 0.27, 0.5 \rangle$. Price stickiness affects firms' invoicing decisions through several channels but in this model one that is particularly relevant is the one linked to the central bank's response to the state of the economy. As we discussed in section 3.3.3, the central bank uses the exchange rate to partially eliminate the relative price misalignments generated, when prices are sticky, by aggregate and idiosyncratic shocks.

This strategy, however, is not costless since the output gap volatility increases in response to the central bank policy. Furthermore, this extra cost is larger

[30] In each range of values we consider a grid of 0.01.

Figure 3.4: Unique Price Dollarisation Equilibrium

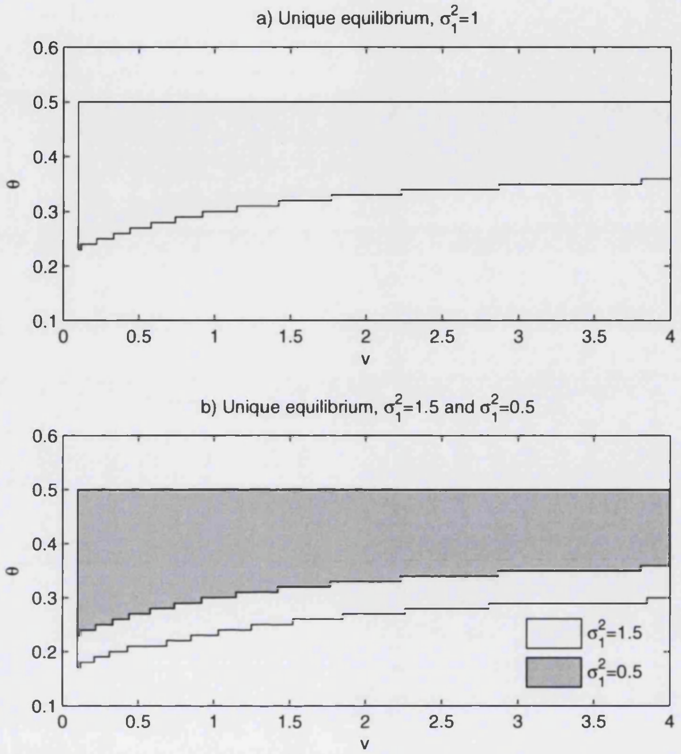


Note: we considered values of θ between 0.1 and 0.99 and for v between 0.1 and 4 and grids points of 0.01

when prices are stickier. Thereby, as equation (3.65) shows in this case the central bank optimally choose to put less weight on $a_{s,t}$, i.e as $\theta \longrightarrow 0$, $\Lambda_{ea} \longrightarrow 0$. This in turn implies that the correlation between firm's marginal costs and the exchange rate is smaller, reducing firm's incentives to invoice in a foreign currency.

Importantly, as figure 3.5 (b) shows, when the size of domestic productivity shocks increase relative to the size of foreign shock, the parameter space that sustains a unique equilibrium, the darker area in the aforementioned figure, is larger. In this case a unique equilibrium exists for values of θ below 0.27, the lower bound

Figure 3.5: Unique Price Dollarisation Equilibrium Large Shocks



Note: we considered values of θ between 0.1 and 0.99 and for v between 0.1 and 4 and grids points of 0.01

of θ when $\frac{\sigma^2}{\sigma_\eta^2} = 1$. Interestingly then, the previous analysis suggest that the larger the domestic shock respect to the foreign one, the larger the likelihood of observing a unique equilibrium with PD[31]. In this latter case, the gains from stabilising relative prices through the use of exchange rate in a dual-currency economy would be larger and consequently, the central bank would be willing to accept a larger cost in terms of output gap volatility. This result can be also interpreted in line with the intuition of Mundell (1961) on optimal currency areas, who defines this concept as

[31] If we consider that each combination of θ and ν correspond to a particular economy. Then, larger the size of the domestic productivity shock, larger it would be the fraction of economies where a unique equilibrium with PD would be observed.

a geographical area that share common real shocks. According to this definition, in our model economy there exist two optimal currency areas when sector specific productivity shocks are large enough. Therefore, as Mundell (1961) suggested in this case it is optimal to have two currencies.

Case 3: Asymmetric Equilibrium, $n_1 > n_2$

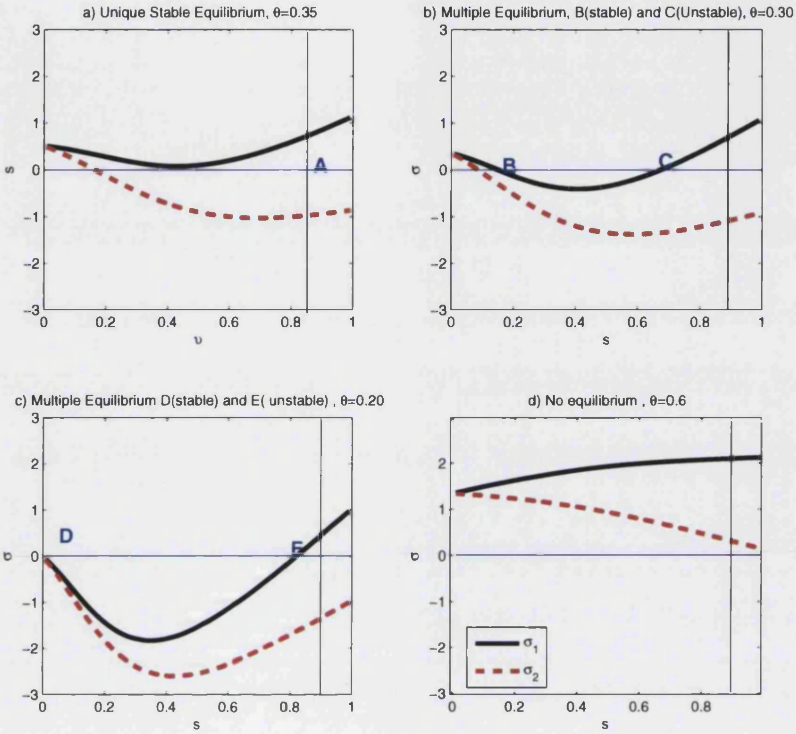
As figure 3.6 (b) to (c) show, in the asymmetric case, there exist more equilibria than in case 2. For instance, point C in figure 3.6 (b) is an equilibrium under mixed strategies but not stable. At this point, type-1 firms choose to invoice in a foreign currency with a probability $\frac{s_1^{*C}}{n_1}$ and choose to invoice in a domestic currency with probability $1 - \frac{s_1^{*C}}{n_1}$.

Type-2 firms, on the other hand, choose to invoice in a domestic currency with probability 1. However, this equilibrium is not stable since type-1 firms have incentives to deviate. For example, if a marginal type-1 firm slightly increases the probability of invoicing in dollars, all type-1 firms would like to follow it to further increase its expected relative gains of invoicing in dollars.. Similarly, if any one type-1 firm deviates by choosing invoicing in pesos with a larger probability, the expected relative gains of invoicing in pesos are larger than the corresponding to invoicing in dollars for all type-1 firms.

Also, in figure 3.6(c), point D is an mixed strategy equilibrium. Similar to the equilibrium at point C, at point D firms also play a mixed strategies and have incentives to deviate. On the other hand, the equilibrium depicted at figure 3.6(a), point A is exactly the same as in case 2, this is $s = \frac{1}{2}(1-\theta)$, and figure 3.6 (d) shows same case as figure 3.3 (d)

Next we perform two simple exercises to analyse the implications of deviations from the optimal policy on the equilibrium PD. In the first one we ask

Figure 3.6: Equilibrium Price Dollarisation Asymmetric Case



Note: The solid line represents the expected relative profits of dollar invoicing versus peso invoicing, for type-1 firms, whereas the dotted line the relative expected profits of type-2 firms. The vertical line is place on $s = 0.7$

whether a central bank that is more adverse to inflation than what is optimal can achieve lower degrees of PD, whereas in the second one we look for the implications of excess of 'fear of floating'.

Price Dollarisation and Inflation Aversion

In order to perform both exercises we parameterise deviations of the central bank from its optimal policy rule. For the first exercise, we use the following alternative central bank reaction function,

$$\left[(\Lambda_e - s)\,\Delta\widetilde{e}_t - \theta\widetilde{a}_{s,t}\right]\varrho + (1-s)\,\widetilde{\pi}_{H,t} = 0$$

where we label by ϱ the index of how much the central bank dislike inflation. When, $\varrho = 0$ we have a central bank that is an inflation nutter, since, it this case, it would implement a policy where,

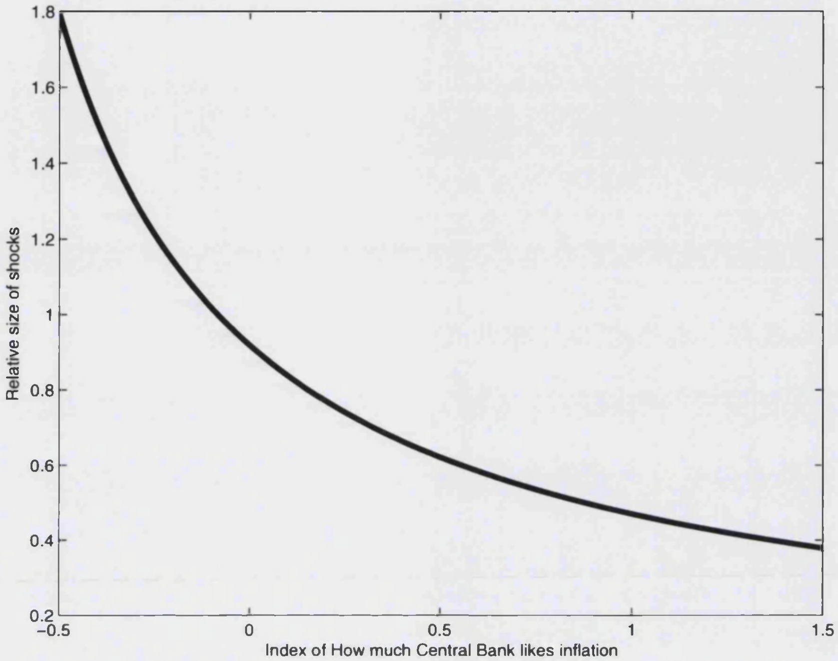
$$\widetilde{\pi}_{H,t} = 0$$

As ϱ increases we have a central bank who tolerates increasingly more inflation. Figure 3.7 shows in the vertical axis the size of relative shocks, $\frac{\sigma_1^2}{\sigma_\eta^2}$ that sustain a unique equilibrium with PD and in the horizontal axis, a measure of how much the central bank likes inflation, $(\varrho - 1)$. We normalised this measure of central bank preferences, so that the optimal monetary policy is reached at zero. As this graph shows, when the central banks tolerates higher inflation levels, the relative size of shocks necessary to sustain an equilibrium with PD falls, making it more likely.

When a central bank tolerates more inflation volatility, firms that set prices in pesos are exposed to higher profit's losses. In this more volatile environment, it turns out optimal for some domestic firms to react by setting prices in dollars . By setting prices in dollars, firms partially isolate their relative prices from inflation. Thus, in our example with just two sectors, when inflation is more volatile PD is sustained as an equilibrium outcome for smaller domestic productivity shocks.

The other interesting insight provided by the previous exercise is that a central bank who implements monetary policy by using an inflation target framework, can effectively reduce PD. As the previous graph shows, an increase in inflation aversion increases the relative size of domestic versus real exchange rate shocks that sustain an equilibrium with PD. This in turn implies, as we previously discussed that the set of parameter values that sustain a unique PD equilibrium increases.

Figure 3.7: Inflation Targeting and Price Dollarisation

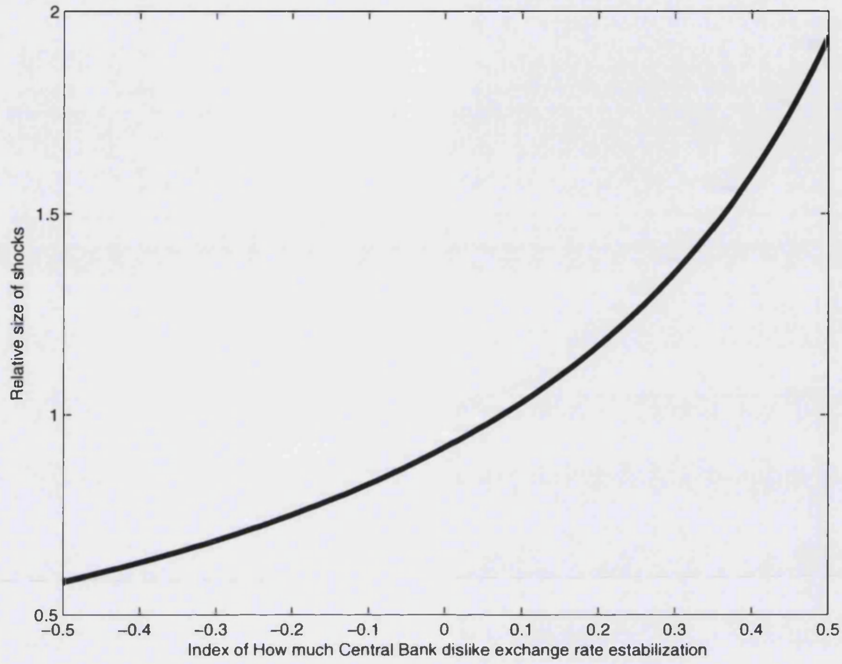


Price Dollarisation and Fear of Floating

In this second exercise, we parameterise "fear of floating" by considering that the central bank deviates from the optimal weights that it puts on exchange rate volatility, Λ_e and on the cross term between $\widetilde{e}_t$ and $a_{s,t}$. More precisely, we shift both parameters by a factor, ϱ_e. When $\varrho_e = 1$, the central bank is using the optimal weights derived in section 3.3. Then, we calculate the critical relative size of domestic sector specific versus real exchange rate shocks, $\frac{\sigma_1^2}{\sigma_\eta^2}$, that uniquely sustain an equilibrium with PD, $s = n_1 (1 - \theta)$.

The results are presented in figure 3.8. As in the previous case, we normalise the degree of fear of floating, by using $1 - \varrho$ on the horizontal axis. This normalisation allows us to locate the equilibrium under optimal monetary policy at point zero on this axis.

Figure 3.8: Excess of Smoothness and Price Dollarisation



As the degree of "fear of floating" increases, an equilibrium with PD can be sustained with relative smaller domestic sector specific shocks, making more likely an equilibrium with $s = n_1(1-\theta)$. Notice that all points on the horizontal axis except zero imply higher welfare losses than when the central bank implements monetary policy optimally. Thus, our model implies that excess of "fear of floating" induces an inefficient high level of PD. As we move from left to right in the previous graph, the size of relative shocks that allow an equilibrium with PD increases, making less likely to observe an equilibrium where, $s = n_1(1-\theta)$.

3.5 Concluding Remarks

In this chapter we have studied how monetary policy should be conducted in an small open economy where firms that faces sector-specific shocks can set prices in two different currencies. The results suggest that in this type of economies optimal monetary policy involves some degree of exchange rate smoothing and an active reaction of the central bank to sector specific productivity shocks. When domestic sector specific productivity shocks are large enough or when price stickiness is not too large, a unique equilibrium with positive degree of price dollarisation is sustainable under optimal monetary policy. As in Mundell (1961), where it is optimal that two countries share a common currency when they face similar real shocks, in this chapter, we show that it might be optimal for a particular economy to have more than one currency when there exist asymmetric productivity shocks within the economy.

The chapter also explores the implications of deviations from optimal policy on the degree of price dollarisation. In particular, we analyse two cases: a)when the central bank is an inflation nutter and b) when it exhibits "excess of fear of floating". A central bank that is more adverse to inflation than society would generate, in equilibrium, a lower level of PD. In that sense, the model predicts that in countries where an explicit inflation targeting is successfully implemented it is less likely to observe price dollarisation. However, in this model, an inflation nutter central bank would induces welfare losses by responding sub-optimally to sector specific productivity shocks.

On the contrary, excessive "fear of floating" leads to an "excessive" degree of price dollarisation, as firms try to take advantage the of benefits that pricing in foreign currency offers in this case. However, excess degree of price dollarisation induces welfare losses for society, since by keeping the nominal exchange rate more stable, the central bank has to tolerate increasingly high levels of volatility in output gap and domestic inflation.

This chapter also shows that PD breaks the isomorphic representation of a small open economy relative to a close economy even when the terms of trade channel is not present. In particular, we show that the dynamics of the economy can not be isolated from exchange rate fluctuations when there exist PD. Furthermore, in this case we show that it is optimal that the central bank smooths exchange rate fluctuations.

Finally, the chapter can be extended in many directions. First, we can use more general assumptions on preferences that allow studying simultaneously the interactions between the channel of terms of trade and sector specific productivity shocks on the design of monetary policy. Second, we can assume a more complex structure on the correlations amongst sector specific productivity shocks, as in Loyo (2001), which it will permit us to generate a continuous mapping between policy and the degree of price dollarisation, finally we could add taxes to analyse the interaction between monetary and fiscal policy when a country faces sector specific productivity shocks.

CHAPTER 4

MONETARY POLICY AND CURRENCY SUBSTITUTION

How should monetary policy be conducted in an economy with CS? Should the central bank put more weight on exchange rate stabilization than on inflation and the output gap stabilization? Is the interest rate channel weaker in this type of economy? Does CS precludes the central bank from controlling inflation? These are highly relevant questions for emerging market economies where CS is still significant, in particular, for those that have recently adopted an inflation-targeting regime like Peru, since this regime relies heavily on structural models for policy analysis and forecasting [1]. This chapter provides some answers to these questions within a micro-founded general equilibrium model of a small open economy with endogenous CS.

We depart from much of the recent literature that uses general equilibrium models for studying the design of monetary policy, such as Benigno and Benigno (2003), Galí and Monacelli (2005), Sutherland (2002) and Woodford (2003) amongst others, by considering that money plays a fundamental role in the economy. The aforementioned papers usually neglect the role of money either by restricting the effects of money to the money market, or by assuming that it does not exist, i.e, cashless economies[2]. The cashless economy assumption is justified for developed economies on empirical grounds, since for those economies there exists some

[1] Peru was the first country with CS to adopt an inflation target regime. However, many other countries with similar features, as Uruguay, Bolivia, and Costa Rica are planning to follow this path. See Armas and Grippa (2005) for a detailed account of the inflation target framework adopted in Peru

[2] A case where the effects of money are restricted to the money market is the one when preferences are separable in money holdings.

empirical evidence showing that the indirect effects of money are relatively small[3]. However, the case for a cashless economy is much harder to make for developing economies where the advantages of using money are larger since a much narrower set of alternative medium of payments is available for transactions than in developed economies[4].

In our setup, two imperfect substitutes medium of payments, a domestic currency, *the peso* and a foreign one, *the dollar* provide liquidity services to households by reducing the transaction costs associated to the consumption of final goods. Households have to use either of these two currencies for transactions. In some transactions households have to pay a higher transaction cost when using pesos, and in others by using dollars. [5].

Under our modelling strategy, households choose optimally the composition of their money holdings by equalizing, at the margin, the sum of the transaction and the opportunity costs of using each alternative currency. This condition completely pins down the degree of CS as an increasing function of the spread between the domestic and the foreign nominal interest rates. Consequently, CS is higher in economies where the domestic nominal interest rates are persistently higher than foreign ones, which is plausible in economies with long-run high inflation levels.

The previously described trading environment is introduced into a very tractable New Keynesian model of a small open economy to evaluate the implications of CS for the design of monetary policy[6]. The model shows that CS generates a channel by which the foreign interest rate distorts consumption, saving and labour

[3] See Ireland (2001) and Woodford (2003) for a detailed discussion on this issue, and Nelson (2002) for a critical view.

[4] In Peru for instance less than 50 percent of the population participates on the financial system, similarly in Bolivia.

[5] Alternative setups to model CS include shopping time and money in utility models For models with non-separable money-in-utility functions, see, Woodford (2003), chapter 3 for a closed economy, and Felices and Tuesta (2005) for an open economy.

[6] Our small open economy model is related to De Paoli (2004), Felices and Tuesta (2005) and Galí and Monacelli (2005).

supply decisions by generating a gap between the marginal utility of consumption and that of income. The relative impact of the foreign interest rate in this gap is increasing on the degree of CS.

Interestingly, the log-linear version of the model economy with CS admits a canonical representation analogous to those without CS, but that differs from the latter in several important dimensions. Firstly, the foreign nominal interest rate appears as an endogenous cost-push shock in the Phillips curve and on the dynamic IS curve. The size of the foreign interest rate impact on the inflation rate and on the output gap depends on the steady-state degree of CS. In particular, in an economy with CS, an increase in the foreign nominal interest rate reduces the output gap and increases inflation[7]. This additional determinant of the inflation dynamics makes it impossible for the central bank to stabilise simultaneously domestic inflation and the output gap.

Second, the welfare based loss function for the central bank in an economy with CS has some new features. Besides output gap and domestic inflation volatility, interest rate volatility generates welfare losses, where the interest rate volatility weight on the central bank loss function depends on the steady-state degree of CS. In particular, as the degree of CS increases it become less costly for the central bank to allow more volatility on the domestic interest rate and the welfare costs of exchange rate smoothing increases. This later result implies that CS does not justify "fear of floating", in the terminology of Calvo and Reinhart (2002).

These new features of an economy with CS have implications for the conduct of monetary policy. First, in economies with a positive level of CS in steady-state, interest rate rules that allow for a flexible exchange rate outperform, in terms of welfare, those that generate some degree of exchange rate smoothing. Second,

[7] This result provides some rationale for the empirical findings of Agénor et al. (2000), Neumeyer and Perri (2005)and Uribe and Yue (2004) who report a negative correlation between the foreign interest rate and output for emerging economies.

interest rate rules with some degree of persistence are desirable, although the gains of interest rate smoothness decrease with the degree of CS.

Also, CS increases the area of determinacy for the rational expectations equilibrium under contemporaneous domestic inflation Taylor rules. In the limit, when CS is full, i.e. when only the foreign currency is used as medium of payment, the area of determinacy coincides with the one of a cashless economy and consequently the Taylor Principle holds. Under all other cases, the set of parameters that allow the determinacy of the rational expectations equilibrium is smaller. In particular, to guarantee determinacy the central bank's response to output should not be too large.

However, it is important to highlight that even though CS increases the area of determinacy, the equilibrium achieved under those rules delivers higher volatility. More precisely, we show that both domestic inflation and output gap volatility monotonically increase with the degree of CS.

This paper is related to some previous work on CS and monetary policy: Felices and Tuesta (2005), use a small open economy model with non-separable money in utility function that depends on both domestic and foreign currency to analyse the effects of dollarisation on monetary policy. Our set up differs from the one in the previous paper in considering a flexible cash-in-advance model to generate endogenous CS. This type of trading friction, by making explicit the trade-off between the transaction and opportunity costs that the agent faces in choosing different types of currency, facilitates the understanding of the main mechanism through which CS affects the economy. Also, Uribe (1997) uses a model with trading frictions but in an economy with flexible prices to analyse the persistency of CS. Gillman (1993) and Den Haan (1990) use models with transaction frictions but in closed economy models and to measure the welfare implications of inflation, and Woodford (2003) studies the implications of transaction frictions for optimal monetary policy in the context of a one-currency closed economy.

The rest of this chapter is organised as follows. In section 4.1, we detail the model economy, although the derivations are presented in appendix B. Section 4.2 discusses the implications of CS for the steady-state and flexible prices equilibrium and it presents the canonical representation of the small open economy under CS. Section 4.3, analyses the implications of CS for monetary policy. Section 4.4 presents some concluding remarks.

4.1 The Model

Following De Paoli (2004) and Sutherland (2002) we derive the small open economy, SOE from now on, as the limiting case of a two country general equilibrium model[8]. Households, consumption goods producers, intermediate goods producers, and the central bank, compose the domestic economy economy. On one hand, domestic households freely choose between dollars and pesos as medium of payment. They also consume a bundle of final consumption goods, supply labour to intermediate goods producers through a competitive labour market, and save using a complete set of state contingent bonds.

Final goods producers, on the other hand, combine domestic and foreign produced intermediate goods as inputs to produce consumption goods. They operate in a perfectly competitive market. On the other hand, intermediate good producers operate in a monopolistic competitive market and use labour as production input. These firms fix prices in advance and face an exogenous probability of changing prices as in Calvo (1983). Monetary policy is implemented by the central bank through an interest rate rule. Only intermediate goods are traded

[8] De Paoli (2004) derives a micro-founded loss function for a central bank in a SOE to study optimal monetary policy, whereas Sutherland (2002), studies the implications of adopting an inflation targeting regime in SOE.

internationally.

4.1.1 Households

Preferences

Households receive utility from the consumption bundle of final goods and disutility from working. Their preferences are described by the following utility function:

$$U_t = E_t \left[\sum_{k=0}^{\infty} \beta^k \left(\frac{C_{t+k}^{1-\sigma}}{1-\sigma} \right) - \frac{1}{1+\varphi} L_{t+k}^{1+\varphi} \right] \tag{4.1}$$

Where E_t represents the expectations operator conditional on information in period t, $\beta \in \langle 0, 1 \rangle$, the household subjective discount factor, $\sigma > 0$, the coefficient of risk aversion and $\varphi > 0$, the inverse of the Frish labour supply elasticity, L_t the number of hours that household work and C_t the composite of a continuum of final consumption goods denoted by $C_t(s)$ and indexed by $s \in [0, 1]$.

$$\ln C_t = \left(\int_0^1 \ln C_t\left(s\right) d\left(s\right) \right) \tag{4.2}$$

Agents in the foreign economy have a similar set of preferences [9]. In what follows we adopt the convention of denoting foreign variables using an asterisks, i.e. C_t^* and L_t^*, represent foreign agent's consumption and working hours in period t.

Transaction Technology

At the domestic and at the foreign economy households are required to use cash for purchasing final consumption goods. Transactions associated to labour services and intermediate domestic and foreign goods do not require cash, these are credit goods

[9] See Appendix B.1 for a description of the foreign economy.

in the terminology of Lucas and Stokey (1987). In contrast to early cash-in-advance models as those of Lucas and Stokey (1987) and Svensson (1985) where households have available only one medium of payment, here, as in Uribe (1997) and Gillman (1993), domestic households are allowed to choose freely between two imperfectly substitute medium of payments. These two competitive medium of payments, which we label as pesos and dollars, are issued by the domestic and foreign central banks at the time the asset market operates [10].

We impose restrictions on the trading environment at the goods market to capture some particular features of this market in economies with CS, for instance, the short supply of foreign notes and coins, which makes more costly the use of foreign currency in small transactions [11]. Also, these restrictions try to capture the costs associated to the exchange rate differentials that households pay when using domestic currency for purchasing goods invoiced in foreign currency. In particular, we assume that households pay a currency-specific real cost at each transaction that depends on the type of good being purchased and the amount of the transaction. For some goods purchasing with pesos is cheaper than with dollars, for others the appositive is true. We index goods by s and denote by $\tau(s)$ and $g(s)$ the proportional costs per good that consumers pay when buying good s with dollars and pesos, respectively. To guarantee a well-defined CS equilibrium level we impose the following restrictions on the transaction cost functions of pesos and dollars,

$$\begin{array}{lll} \tau(s) \succeq 0 & \frac{\partial \tau(s)}{\partial s} > 0 & g(s) \succeq 0 \\ \frac{\partial g(s)}{\partial s} > 0 & g(0) > \tau(0) & g(1) < \tau(1) \\ \frac{\partial \tau(s)}{\partial s} > \frac{\partial g(s)}{\partial s} & & \end{array} \tag{4.3}$$

[10] Uribe (1997) uses a flexible CIA model for domestic and foreign currency to study the determinants of the persistence of currency substitution. On the other hand, Gillman (1993)uses an endogenous cash-credit model to evaluate the welfare effects of inflation.

[11] The sub optimal distribution of foreign notes and coins is natural, since the unitary cost of transporting from abroad notes is decreasing on its denomination.

These assumptions imply that transaction costs are lower in both currencies for small s goods than for large s goods. Also, they imply that it is cheaper to use dollars for purchasing small s goods and to use pesos for large s goods[12]. Furthermore, these assumptions guarantee that there exist a unique threshold good, $\overline{s}_t$ such that goods with index lower than $\overline{s}_t$ are purchased with dollars, whereas, goods with index higher than $\overline{s}_t$ are purchase with pesos. Households choose this threshold level as part of their optimisation problem.

The household's money holdings composition depends, however, not only on the transaction frictions described previously but also on the corresponding opportunity costs of holding each type of currency. The opportunity cost of holding each type of currency in turn crucially depends on the timing of transactions. When all markets operates simultaneously, in particular when the exchange market is open at the time households attend to the goods market, sellers are indifferent between accepting pesos and dollars at the market exchange rate. In this case the only relevant relative cost for the household's currency composition choice is the relative transaction cost between these two alternative means of payment. However, when markets open sequentially the opportunity cost of holding pesos and dollars is not the same, thereby, household's currency composition depends crucially on the relative opportunity cost of pesos versus dollars.

The Timing of Transactions

As in Lucas and Stokey (1987) we assume that at the domestic and foreign economy households belong to a representative family with four members that perform different tasks during each period, but that regroups at the end of each period to pool goods, assets and information. One member, the seller, operates domestic firms,

[12] These assumptions are not too restrictive, we could alternatively assume that, $\tau(s) \succeq 0$, $\frac{\partial \tau(s)}{\partial s} < 0$, and, $g(s) \succeq 0$ and $\frac{\partial g(s)}{\partial s} > 0$. and our results would not change. In both cases, these restrictions guarantee a unique CS equilibrium level.

trades intermediate goods from foreign firms and sells final consumption goods to shoppers. The cash receipts from period t production can be used, however, only in the next period. The second member, the shopper, uses available money holdings to purchase final consumption goods, the third member, the investor, has the task of trading assets: domestic and foreign currency and state contingent bonds. Finally, the fourth member, the worker, supplies labour to intermediate goods producers.

At each period of time markets open sequentially. First opens the asset market, during the morning. Then, after this market has closed, during the afternoon, opens the goods market. At the asset market, domestic and foreign investors meet with the domestic and foreign central banks to trade a set of nominal one-period state contingent bonds denominated in domestic currency. At this market also central banks inject fiat money, pesos and dollars [13].

The central banks set the domestic and the foreign interest rates and the exchange rate is freely determined as an equilibrium outcome at this market. At this time also households observe one of the many states of nature $x_t \in \Psi$ that generate uncertainty. History of events up to period t is denoted by ζ_t and the conditional probability of occurrence of state τ_{t+1} is given by $\Omega(x_{t+1} \mid x_t)$. We denote by $B(x_{t+1} \mid x_t)$ the domestic household's holdings of the state-contingent bonds and by $\xi(x_{t+1} \mid \zeta_t)$ period t bond's prices and state x_t. One unit of each of these bonds pays one unit domestic currency in period $t+1$ if the particular state, τ_{t+1} occurs, otherwise they do not pay. Household enter to the asset market with their stock of wealth carried over from the previous period, $\varpi(x_t)$, plus a domestic currency transfer from the government, $TR_t(x_t)$. They use these resources to purchase a portfolio of state contingent bonds $B(x_{t+1} \mid x_t)$, and to accumulate money holdings of pesos and dollars $M(x_t)$ and $D(x_t)$. We denote by $e(x_t)$ the nominal exchange rate, pesos per dollar. Thereby, the household budget constraint at the asset market,

[13] Note that the timing of transactions in this model is similar to Lucas and Stokey (1987) who assume that the asset market open first. Svensson (1985) instead considers that the goods market open first.

expressed in terms of pesos, is given by:

$$M\left(x_t\right) + D\left(x_t\right) e\left(x_t\right) + \sum_{x_{t+1} \in \Psi} \xi\left(x_{t+1} \mid \zeta_t\right) B\left(x_{t+1} \mid x_t\right) = \varpi\left(x_t\right) + TR\left(x_t\right) \quad (4.4)$$

During the afternoon, at the goods market, shoppers and sellers meet to exchange final consumption goods by currency. Shoppers can pay for each good with any currency, but transactions are costly. Rational households choose their currency's composition to minimise transaction costs and the corresponding opportunity cost of holding both currencies. Importantly, at this market neither households nor final-good producers can change the composition of their money holdings. They have to wait until the next day to exchange domestic and foreign currency at the asset market. This in turn implies that the relevant exchange rate at this market is not the corresponding to the one in the morning but the one expected for the next day. Otherwise final-good producers would not accept dollars in exchange for goods. If this is so, the opportunity cost of holding pesos and dollars would not be the same The next two equations present the two cash-in-advance constraints the affect households consumption possibilities at the goods market.

$$\begin{aligned} M\left(x_t\right) &= \int_{\overline{s_t}}^{1} P\left(s, x_t\right) C\left(s, x_t\right)\left(1 + g(s)d\left(s\right)\right. \\ \sum_{x_{t+1} \in \Psi} e\left(x_{t+1}\right) \Omega\left(x_{t+1} \mid x_t\right) D\left(x_t\right) &= \int_{0}^{\overline{s}_t} P\left(s, x_t\right) C\left(s, x_t\right)\left(1 + \tau(s)\right)d\left(s\right) \end{aligned} \quad (4.5)$$

Under these assumptions, domestic and foreign currency relative opportunity cost would be given by the expected depreciation of the exchange rate. To understand this latter point, let's suppose that the foreign central bank raises its nominal interest rate; rational domestic households would expect a depreciation of the nominal exchange rate for next period. Under these conditions, they would prefer to hold only foreign currency for transactions instead of domestic currency, since by doing so, they will make capital gains. However, holding only dollars is not the optimal choice since pesos and dollars have different transaction costs. By holding only dollars households would have to pay a large transaction cost when purchasing some goods. Thereby, the optimal decision implies to use both pesos and dollars. At the optimal, the relative transaction cost and opportunity costs of using dollars and pesos have to equalise.

Also at the goods market, production of both intermediate and final goods takes places. After the goods market closes, sellers pool their receipts from this market with the wages received by the worker and with the unspent money holdings from the shopper.[14]. Therefore, the households stock of wealth at the beginning of next period is given by wage income, $w(x_t)\,L(x_t)$, the state contingent bond holdings, $B(x_{t+1})$, profits in domestic and foreign currency, $\Xi(x_t)$ and $\Xi^*(x_t)$, and the unspent pesos and dollars and consequently their stock of wealth at the end of period t, is given by:

$$
\begin{aligned}
\varpi(x_{t+1}) &= B(x_{t+1}) + \Xi(x_t) + M(x_t) \\
&\quad - \int_{\overline{s}_t}^{1} P(s, x_t)\, C(s, x_t)\,(1 + g(s)d(s) + w(x_t)\, L(x_t) \\
&\quad + \sum_{x_{t+1}\in\Psi} e(x_{t+1})\,\Omega(x_{t+1} \mid x_t)\, D(x_t) \\
&\quad - \int_{0}^{\overline{s}_t} P(s, x_t)\, C(s, x_t)\,(1 + \tau(s))d(s) \\
&\quad + \sum_{x_{t+1}\in\Psi} e(x_{t+1})\,\Omega(x_{t+1} \mid x_t)\,\Xi^*(x_t)
\end{aligned} \tag{4.6}
$$

[14] This includes the transactions costs which are charged by the sellers during the goods market transactions. This assumption is harmless to our results and it is made only on the sake of simplicity. It does not affect the substitution effects that transaction costs generate in the economy.

We further restrict household decisions to satisfy the following transversality condition.

$$\lim_{n\to\infty} \sum_{x_{t+1}\in\Psi} \xi\left(x_{t+1} \mid \zeta_t\right) \varpi\left(x_{t+1}\right) \geqslant 0$$

Household Optimality Conditions

Each household maximises her utility function given by equation (4.1) subject to the cash-in-advance constraints and the flow budget constraint, equation (4.6). The households' first-order conditions are given by the following set of equations:

Degree of Currency Substitution The degree of CS is determined by the fraction of consumption goods purchased using dollars, $\overline{s}_t$. Optimality implies that for good $\overline{s}_t$ the marginal cost of using dollars and pesos has to be the same. Two costs are associated to money holdings, the standard opportunity costs given by the domestic nominal interest rate, in the case of pesos, and the foreign interest rate, in the case of dollars; and the transaction costs $\tau(s)$ and $g(s)$, respectively. Consequently $\overline{s}_t$ is determined by the following condition,

$$\frac{1+\tau\left(\overline{s}_t\right)}{1+g(\overline{s}_t)} = \frac{1-\frac{1}{(1+i_t)}}{1-\frac{1}{(1+i_t^*)}} \tag{4.7}$$

Condition (4.7) is very similar to the one derived by Baumol (1952), in which the optimal demand for money is obtained when the transaction cost of exchanging bonds by money equalises the nominal interest rate, its opportunity cost. It is also in line with the condition derived by Eichenbaum and Wallace (1985) where the optimal demand for different types of money is determined by equalising their corresponding marginal transaction costs. This condition allows to solve for $\overline{s}_t$, once the transaction cost functions of pesos and dollars are parameterised. To obtain a tractable solution we choose the following exponential cost functions,

$$\tau\left(s_t\right) = \exp(\Psi_o + \Psi_1 s_t) - 1 \quad g\left(s_t\right) = \exp\left(n_o + n_1 s_t\right)) - 1 \tag{4.8}$$

Where by imposing that, $\Psi_1 > n_1$ and $n_o > \Psi_o$, we guarantee that conditions in (4.3) are satisfied. From condition (4.7)the degree of CS is determined by,

$$\overline{s}_t = \frac{\left(n_0 - \Psi_0 + \log\left(\frac{2-\frac{1}{(1+i_t)}}{2-\frac{i_t^*}{(1+i_t^*)}}\right)\right)}{(\Psi_1 - n_1)} \tag{4.9}$$

The previous condition shows that in equilibrium $\overline{s}_t$ is increasing on the level of domestic interest rate, i_t and decreasing on the foreign interest rate, i_t^*. This is a very intuitive result since for instance, when i_t increases, using pesos becomes more costly and consequently the demand for dollars rises, leading to a higher $\overline{s}_t$. Our assumptions also imply that even when both the domestic and the foreign interest rate are zero there exist a positive level of CS. In this case, the degree of CS reaches its lower bound given by,

$$\overline{s}_o = \frac{(n_0 - \Psi_0)}{(\Psi_1 - n_1)} \tag{4.10}$$

This minimum degree of CS, $\overline{s}_o$ captures the fact that even with zero nominal interest rates there exist a set of goods for which is cheaper to use dollars for transactions, $n_o > \Psi_o$. Only in the case where $n_0 = \Psi_0 = 0$ an equilibrium where both nominal interest rates are equal delivers a zero degree of CS. The equilibrium with positive levels of currency substitution and low inflation levels is consistent with the experience of CS in countries like Bolivia and Peru in recent years. The key implication of CS for consumption decisions across goods is, as the following first-order conditions show, that the marginal utility of consumption is larger than the marginal utility of income due to transaction costs:

$$U_{c,t}\frac{\partial c_t}{\partial c_t(s)} = P_t(s)\lambda_t\left(1 + \frac{q_t}{\lambda_t}\right)(1 + g(s)) \text{ for } s \geqslant \overline{s}_t \tag{4.11}$$

$$U_{c,t}\frac{\partial c_t}{\partial c_t(s)} = P_t(s)\lambda_t\left(1 + \frac{n_t}{\lambda_t}\right)(1 + \tau(s)) \text{ for } s < \overline{s}_t \tag{4.12}$$

where, λ_t, q_t and n_t, represent the lagrange multipliers associated to the budget constraint and the two CIA constraints, respectively. The condition that determines the optimal consumption of small s goods, $s < \overline{s}_t$, implies that at the optimum, the good s marginal utility of consumption has to equalise the marginal benefit of savings but adjusted by the cost of using money, which is given by its opportunity cost and its corresponding transaction cost, $\left(1+\frac{n_t}{\lambda_t}\right)(1+\tau(s))$. Similarly, for goods with high index, $s > \overline{s}_t$, optimality implies that the marginal utility of consumption has to equal the marginal utility of income, adjusted by the corresponding transaction cost, $(1+g(s))$, and the opportunity cost of holding domestic currency, $\left(1+\frac{q_t}{\lambda_t}\right)$. From the corresponding first order conditions for holdings of pesos and dollars we obtain:

$$\frac{q_t}{\lambda_t} = 1 - E_t\left(Q_{t,t+1}\right) = 1 - \frac{1}{(1+i_t)} \tag{4.13}$$

$$\frac{n_t}{\lambda_t} = 1 - E_t\left(Q_{t+1}\frac{e_{t+1}}{e_t}\right) = 1 - \frac{1}{(1+i_t^*)} \tag{4.14}$$

In order to analyse the implications of CS in the economy, we aggregate the optimal conditions for the demand of each final good, given by equations (4.11) and (4.12) to obtain the optimality condition for the demand of the consumption bundle. As it is shown in detail in appendix B.3, these conditions imply that the marginal utility of consumption and the marginal utility of income, given by the lagrange multiplier, λ_t, differ by the factor, Υ_t :

$$U_{c,t} = \lambda_t\left(1+\Upsilon_t\right) \tag{4.15}$$

Υ_t measures the distortion associated to the transaction costs in pesos and dollars. In equilibrium, Υ_t depends on both the domestic and the foreign nominal interest rates and the equilibrium degree of CS, $\overline{s}_t$ through function, $\Gamma(\overline{s}_t)$ in the following way:

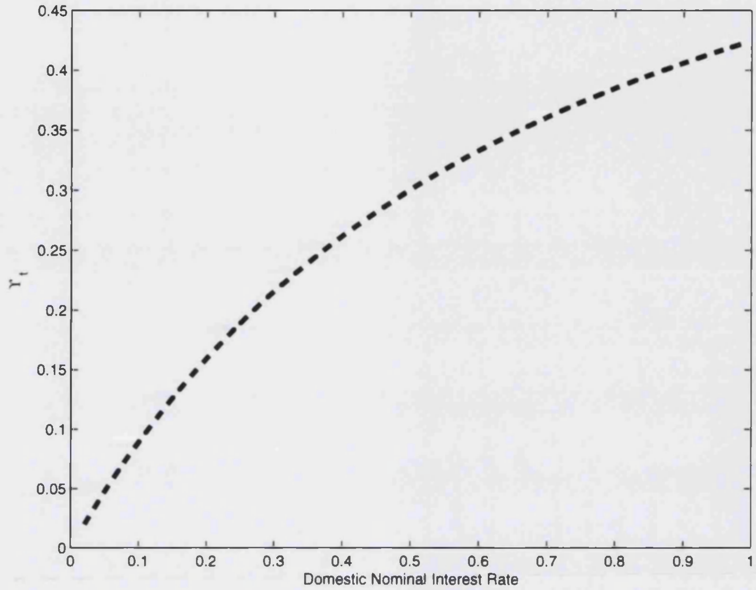
$$\left(1+\Upsilon_t\right) = \left(1+\frac{i_t}{(1+i_t)}\right)\left(1+\Gamma\left(\overline{s}_t\right)\right) \tag{4.16}$$

using the functional forms for the transaction costs in dollars and pesos, defined in equation (4.8), $(1+\Gamma(\overline{s}_t))$ can be written as the following exponential function on $\overline{s}_t$[15] :

$$1+\Gamma\left(\overline{s}_t\right) = \exp\left(\frac{n_1}{2} + n_o - \left(\Psi_1 - n_1\right)\frac{\overline{s}_t^2}{2}\right) \tag{4.17}$$

[15] See appendix, B.3, for details of this derivation

Figure 4.1: The Steady-State Costs of Transaction Frictions



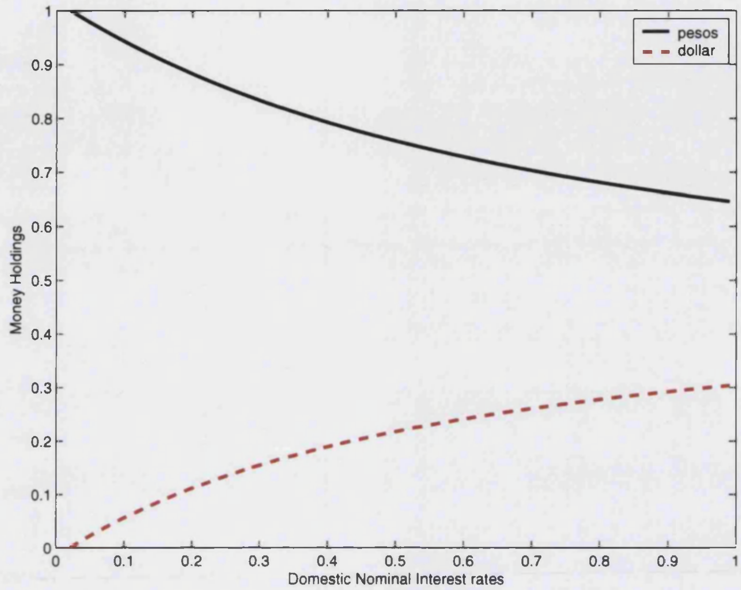
Note: Transaction costs are measured in the vertical axis as fraction of steady-state consumption. The nominal domestic interest rate is measured in the horizontal axis

Observing equation (4.15) it is easy to understand how CS affects the equilibrium of the economy. As this condition shows, transaction costs create a wedge between the marginal utility of consumption and that of income, Υ_t, which distorts the efficient allocation of consumption and labour. This distortion is increasing in both the domestic and the foreign nominal interest rate.

Interestingly, when keeping fixed the foreign interest rates, the marginal effect of i_t on Υ_t is decreasing, since, when CS is allowed, agents can freely substitute domestic currency for foreign currency. Thus, by allowing CS agents can reduce the welfare cost of high nominal interest rates. Figure 4.1 illustrates this latter point by showing that function Υ_t is concave on the nominal interest rate i_t.

Notice that Υ_t is minimised when $i = 0$, although, this condition does not guarantee that transaction frictions are fully eliminated. As we mentioned

Figure 4.2: Money Demand Functions



Note: Money Demand in the vertical axis as ratio of steady-state consumption

previously, only when, $n_0 = \Psi_0 = 0$, a zero nominal interest rate guarantee zero transaction costs.

On the other hand, using the CIA constraints and equation (4.15) we can write the corresponding money demands for domestic and foreign currency as follows,

$$\frac{M_t}{P_t} = C_t \frac{(1-\bar{s}_t)(1+\Upsilon_t)}{2-\frac{1}{1+i_t}} \quad \frac{e_t D_t}{P_t} = C_t \frac{\bar{s}_t(1+\Upsilon_t)}{2-\frac{1}{1+i_t^*}} \tag{4.18}$$

These two money demand functions exhibit standard properties. Both are increasing in the level of domestic consumption, and M_t is decreasing (increasing) on i_t (i_t^*), whereas, D_t is decreasing (increasing) on i_t^* (i_t). Furthermore, taking a log quadratic approximation of the two previous equations around their corresponding steady-states, it is easy to show that M_t is decreasing and a convex function of i_t thus, the model implies that an increase in the volatility of the opportunity cost of holding money would lead to higher money demand. Figure 4.2 plots $\frac{M_t}{P_t}$ and $\frac{e_t D_t}{P_t}$ for different values of the domestic interest rate, holding fixed C_t and i_t^*.

Saving and Portfolio Decisions Savings and household's portfolio decisions are determined by the usual Euler conditions. At the optimum households are indifferent amongst allocating wealth in any period, since the expected present discounted value of the marginal utility of wealth is the same across periods:

$$\frac{1}{(1+i_t)} = E_t\left(\frac{\beta\lambda_{t+1}}{\lambda_t\,(1+\pi_{t+1})}\right) \tag{4.19}$$

Notice that since, $\lambda_t = \frac{U_{c,t}}{(1+\Upsilon_t)}$, the saving decisions of agents will depend, besides the level of the real interest rate, on the degree of CS. Furthermore, since markets are complete, it also holds that:

$$\frac{1}{(1+i_t^*)} = E_t\left(\frac{\beta\lambda_{t+1}e_{t+1}}{e_t\lambda_t\,(1+\pi_{t+1})}\right) \tag{4.20}$$

Combining equations (4.19) and (4.20), we obtain the uncovered interest parity condition (UIP):

$$\frac{(1+i_t)}{(1+i_t^*)} = \frac{E_t\left(\frac{\lambda_{t+1}e_{t+1}}{e_t\lambda_t(1+\pi_{t+1})}\right)}{E_t\left(\frac{\lambda_{t+1}}{\lambda_t(1+\pi_{t+1})}\right)} \tag{4.21}$$

Labour Supply Households supply labour in equilibrium up to the point where the marginal cost of working equalises its marginal benefit:

$$U_{h,t} = \lambda_t\frac{W_t}{P_t} \tag{4.22}$$

The marginal benefit $\lambda_t W_t$ depends, amongst other things, on the level of nominal interest rates and on the degree of currency substitution through λ_t. This is a second channel through which currency substitution affects the economy. Since, real wages affect marginal cost of firms and through the Phillips curve, inflation, the degree of CS, as we discuss in detail in the next sections, will affect inflation dynamics.

Risk Sharing Condition The complete markets assumption implies that the price of the state contingent bond domestically and abroad have to be the same, therefore, we have that the following condition it must hold:

$$\xi_{t+1} = \frac{\beta \lambda_{t+1}^* P_t^*}{\lambda_t^* P_{t+1}^*} = \frac{\beta \lambda_{t+1} P_t e_{t+1}}{\lambda_t e_t P_{t+1}} \tag{4.23}$$

Denoting by Q_t the real exchange rate, the relative price of foreign goods in terms of domestic goods, $Q_t = \frac{P_t^* e_t}{P_t}$ we can transform the previous expression into the following condition:

$$Q_{t+1} = \frac{\lambda_{t+1}^*}{\lambda_{t+1}} \frac{\lambda_t}{\lambda_t^*} Q_t \tag{4.24}$$

Following Chari et al. (2002) we iterate the previous equation backwards to obtain the following risk sharing condition[16]:

$$Q_t = \varsigma_0 \frac{\lambda_t^*}{\lambda_t} \tag{4.25}$$

where the constant term ς_0 is defined as follows:

$$\varsigma_0 = \frac{\lambda_0}{\lambda_0^*} Q_0 \tag{4.26}$$

4.1.2 Firms

Final Good Producers

There is a continuum of final good producers of mass n indexed by j in the domestic economy, which operate under perfect competition, and a mass $1 - n$ of final goods producers in the foreign economy. Domestic final goods producers use home, $Y_{H,t}$, and foreign, $Y_{F,t}$, intermediate goods as inputs into the following production function:

$$Y_t^j = \left[(1-\alpha)^{\frac{1}{\eta}} \left(Y_{H,t}\right)^{\frac{\eta-1}{\eta}} + (\alpha)^{\frac{1}{\eta}} \left(Y_{F,t}\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{1-\eta}} \tag{4.27}$$

[16] Chari et al. (2002) use a model of an open economy with complete markets to analyse the role of price stickiness in explaining the volatility of the real exchange rate.

$$Y_{H,t}^{j}=\left(\left(\frac{1}{n}\right)^{\frac{1}{\epsilon}}\int_{0}^{n}Y_{H,t}\left(z\right)^{\frac{\epsilon-1}{\epsilon}}d\left(z\right)\right)^{\frac{\epsilon}{\epsilon-1}}\quad Y_{F,t}^{j}=\left(\left(\frac{1}{1-n}\right)^{\frac{1}{\epsilon}}\int_{n}^{1}Y_{F,t}\left(z\right)^{\frac{\epsilon-1}{\epsilon}}d\left(z\right)\right)^{\frac{\epsilon}{\epsilon-1}} \tag{4.28}$$

where $\eta > 0$ is the elasticity of substitution between domestic and foreign intermediate goods, whereas, $\epsilon > 1$, is the elasticity of substitution across varieties of intermediate goods. Then the cost minimizing demand functions by firm j of each type of differentiated good is given by the following two conditions:

$$Y_{H,t}^{j}(z)=(1-\alpha)\left(\frac{P_{H,t}(z)}{P_{H,t}}\right)^{-\epsilon}\left(\frac{P_{H,t}}{P_{t}}\right)^{-\eta}Y_{t}^{j} \tag{4.29}$$

$$Y_{F,t}^{j}(z)=\alpha\left(\frac{P_{F,t}(z)}{P_{F,t}}\right)^{-\epsilon}\left(\frac{P_{F,t}}{P_{t}}\right)^{-\eta}Y_{t}^{j} \tag{4.30}$$

The price level charged by final good producers is equal to its marginal cost and it is given by:

$$P_{t}=\left((1-\alpha)P_{H,t}^{1-\eta}+\alpha P_{F,t}^{1-\eta}\right)^{\frac{1}{1-\eta}} \tag{4.31}$$

where:

$$P_{H,t}=\left(\frac{1}{n}\int_{0}^{n}P_{H,t}^{1-\epsilon}\left(z\right)dz\right)^{\frac{1}{1-\epsilon}}\quad P_{F,t}=\left(\frac{1}{n}\int_{0}^{n}P_{F,t}^{1-\epsilon}\left(z\right)dz\right)^{\frac{1}{1-\epsilon}} \tag{4.32}$$

Final goods producers in the foreign economy have a similar technology to that used by domestic intermediate producers, see appendix B.1 for details on the foreign economy.

Intermediate Good Producers

There is a continuum of intermediate good producers of mass n allocated in the domestic economy and of mass $1-n$, in the foreign economy that operate under monopolistic competition. Each of these producers uses a constant returns to scale technology to produce a particular variety of intermediate goods. This technology takes labour as production input as follows :

$$Y_{H,t}(z)=A_{t}L_{t}(z) \tag{4.33}$$

where A_t represents an aggregate productivity shock that follows the following $AR(1)$ process:

$$\ln(A_t) = \chi \ln(A_{t-1}) + \zeta_t \tag{4.34}$$

with $\zeta_t \sim N\left(0, \sigma_\zeta^2\right)$. With this type of technology, the real marginal cost of the representative intermediate good producers is given by

$$mc_t = \frac{W_t}{A_t P_t} \frac{P_t}{P_{H,t}} \tag{4.35}$$

Similarly, the foreign intermediate goods producers uses a constant returns to scale production function given by:

$$Y_{F,t}(z) = A_t^* L_t^*(z) \tag{4.36}$$

where A_t^* represents the foreign productivity shock, which also follows an autoregressive process:

$$\ln(A_t^*) = \chi^* \ln(A_{t-1}^*) + \zeta_t^* \tag{4.37}$$

with $\zeta_t^* \sim N\left(0, \sigma_{\zeta^*}^2\right)$. The aggregate demand for the intermediate good z is obtained by adding up the demand of both the home and foreign final goods producers for this good, as follows:

$$Y_{H,t}(z) = \int_0^n Y_{H,t}^j(z) d(j) + \int_n^1 \left(Y_{H,t}^j\right)(z) d(j) \tag{4.38}$$

In this economy the law of one price holds for a particular good z, therefore we have that: $P_{H,t}(z) = e_t P_{H,t}^*(z)$, and $P_{F,t}(z) = e_t P_{F,t}^*(z)$, consequently, the aggregate demand for home intermediated good z is written as follows:

$$Y_{H,t}(z) = \left(\frac{P_{H,t}(z)}{P_{H,t}}\right)^{-\epsilon} \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left((1-\alpha)\overline{Y}_t + \frac{(1-\alpha^*)(1-n)}{n} Q_t^\eta \overline{Y}_t^*\right) \tag{4.39}$$

Where, $\overline{Y}_t = \int_0^n Y_t^j d(j)$, and $\overline{Y}_t^* = \int_n^1 Y_t^{j*} d(j)$, represent the aggregate production level of final goods at the domestic and foreign economy, respectively. Using a similar derivation for the foreign economy we obtain $Y_{F,t}(z)$, which is given by:

$$Y_{F,t}(z) = \left(\frac{P_{F,t}(z)}{P_{F,t}}\right)^{-\epsilon} \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} \left(\frac{n}{1-n}\alpha\overline{Y}_t + \alpha^* Q^\eta \overline{Y}_t^*\right) \tag{4.40}$$

The Small Open Economy

Following Sutherland (2002), we parameterise the participation of foreign inputs in the production of home and foreign final goods, α , α^*, respectively as follows:

$$\alpha = (1-n)\gamma \quad 1-\alpha^* = n\gamma$$

where n represents the size of the home economy, and γ its degree of openness[17]. This particular parameterisation implies that as the economy becomes more open, the fraction of imported goods used in domestic production increases, whereas as the economy becomes larger, this fraction falls.

The SOE corresponds to the home economy when the size of this economy approaches to zero, $n \longrightarrow 0$. In this case we have that $\alpha \longrightarrow \gamma$ and $\alpha^* \longrightarrow 1$. Consequently, the foreign economy does not use any home produced intermediated good for production of foreign final goods and changes in home aggregate demand have a nil impact on the foreign economy, this is $\frac{\partial Y_{F,t}}{\partial \overline{Y}_t} = 0$. Furthermore, in this limiting case, $P^* = P^*_F$ and:

$$Y_{H,t}(z) = \left(\frac{P_{H,t}(z)}{P_{H,t}}\right)^{-\epsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left((1-\gamma)\overline{Y}_t + \gamma Q_t^{\eta}\overline{Y}_t^*\right) \tag{4.41}$$

$$Y_{F,t}(z) = \left(\frac{P_{F,t}(z)}{P_{F,t}}\right)^{-\epsilon}\left(\frac{P_{F,t}}{P_t}\right)^{-\eta}\left(Q_t^{\eta}\overline{Y}_t^*\right) \tag{4.42}$$

In order to save notation, we define domestic output, $Y_{H,t}$ and foreign output, $Y_{F,t}$, as follows:

$$Y_{H,t} = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left((1-\gamma)\overline{Y}_t + \gamma Q_t^{\eta}\overline{Y}_t^*\right) \tag{4.43}$$

$$Y_{F,t} = \left(\frac{P_{F,t}}{P_t}\right)^{-\eta}\left(Q_t^{\eta}\overline{Y}_t^*\right) \tag{4.44}$$

[17] Sutherland (2002) derives the SOE in a model where the home and foreign economy trade final consumption goods. In this chapter in contrast, the home and the foreign economy trade intermediate goods.

thus, the demand facing individual intermediate goods producing firms can be simply expressed as:

$$Y_{H,t}(z) = \left(\frac{P_{H,t}(z)}{P_{H,t}}\right)^{-\epsilon} Y_{H,t} \quad Y_{F,t}(z) = \left(\frac{P_{F,t}(z)}{P_{F,t}}\right)^{-\epsilon} Y_{F,t} \tag{4.45}$$

Price Setting

At each period t intermediate goods producers face an exogenous probability of changing prices given by $(1-\theta)$. Following Calvo (1983) and Yun (1996), we assume that this probability is independent of the price level chosen by the firm in previous periods and on the date the firm last changed its price. A typical firm choose its price $P^o_{H,t}(z)$ to maximise the present discounted value of its expect flow of profits, given by:

$$E_t\left[\sum_{k=0}^{\infty}(\theta\beta)^k\left(\lambda_{t+k}\left(\frac{P^o_{H,t}(z)}{P_{H,t+k}} - mc_{t+k}\right)\widetilde{Y}_{H,t+k}(z)\right)\right] \tag{4.46}$$

let´s denote by Ψ_{t+k} the inverse of the cumulative domestic inflation level as follows:

$$\Psi_{t+k} = \frac{P_{H,t}}{P_{H,t+k}} \tag{4.47}$$

and by $\widetilde{Y}_{H,t+k}(z)$ the demand of intermediate good z conditioned on that its price has kept fixed at $P^o_{H,t}(z)$:

$$\widetilde{Y}_{H,t+k}(z) = \left(\frac{P^o_{H,t}(z)}{P_{H,t}}\right)^{-\epsilon}\Psi^{-\epsilon}_{t+k}Y_{H,t+k} \tag{4.48}$$

The first order condition that maximises equation (4.46) is given by:

$$E_t\left[\sum_{k=0}^{\infty}(\theta\beta)^k\left(C^{-\sigma}_{t+k}\left(\frac{P^o_{H,t}(z)}{P_{H,t}}\Psi_{t+k} - \frac{\epsilon}{(\epsilon-1)}mc_{t+k}\right)\widetilde{Y}_{H,t+k}(z)\right)\right] = 0 \tag{4.49}$$

As it is shown in appendix B.2, from this first order condition we can derive a non linear recursive representation of the Phillips curve given by the following three equations:

$$N_t = \mu\lambda_t mc_t Y_{H,t} + \theta\beta\pi^{\epsilon}_{H,t+1}N_{t+1} \tag{4.50}$$

$$D_t = \lambda_t Y_{H,t} + \theta\beta\pi_{H,t+1}^{\epsilon-1} D_{t+1} \tag{4.51}$$

$$\theta\left(\pi_{H,t}\right)^{\epsilon-1} = 1 - (1-\theta)\left(\frac{N_t}{D_t}\right)^{1-\epsilon} \tag{4.52}$$

Where, N_t and D_t are auxiliary variables defined in this appendix. A similar set of equations characterises the Phillips curve in the foreign economy.

Real Exchange Rate and Terms of Trade

Next, we define some identities that are helpful in describing the dynamic equilibrium of an open economy. First we define the terms of trades, T_t, as the relative price of foreign goods in terms of domestic goods, as follows:

$$T_t = \frac{P_{F,t}}{P_{H,t}} \tag{4.53}$$

since the domestic economy is small and the law of one price holds, the price of foreign goods is determined by $P_{F,t} = e_t P_t^*$. Using, the previous identity, it is possible to obtain the terms of trade from the following equation,:

$$T_t = \frac{Q_t}{\widetilde{P}_{H,t}} \tag{4.54}$$

where $\widetilde{P}_{H,t} = \frac{P_{H,t}}{P_t}$. Furthermore, using the definition of the consumer price indices for the home and foreign economy and the small open economy assumption, we have that:

$$\left(\frac{Q_t}{T_t}\right)^{\eta-1} = (1-\gamma) + \gamma T_t^{1-\eta} \tag{4.55}$$

Using this last identity, we can define a relationship between CPI inflation and home inflation as follows:

$$\left(\frac{\pi_t}{\pi_{H,t}}\right)^{1-\eta} = \frac{(1-\gamma) + \gamma T_t^{1-\eta}}{(1-\gamma) + \gamma T_{t-1}^{1-\eta}} \tag{4.56}$$

4.1.3 Monetary Policy

The central bank sets monetary policy by choosing the nominal interest rate according to a Taylor rule. We consider the following generic type of Taylor rule,

$$(1+i_t) = \overline{(1+i)}\,(1+i_{t-1})^{\rho_i}\left(\frac{\pi_{i,t}}{\overline{\pi}_t}\right)^{\phi_\pi(1-\rho_i)}\left(\frac{y_t}{\overline{y}_t}\right)^{\phi_x(1-\rho_i)}\left(\frac{e_t}{e_{t-1}}\right)^{\phi_e(1-\rho_i)}$$

where $i = \{H, CPI\}$, $\phi_\pi > 1$, $\phi_x > 0$, $\phi_e > 0$, $\overline{\pi}_t$ represents the inflation target of the domestic central bank, $\overline{y}$, the natural level of output in the domestic economy, H, stands for home prices, and CPI for the consumer price index.

4.1.4 Baseline Parametrisation

The model is calibrated with standard parameter values for small open economies. In particular, we choose, $\sigma = \eta = 1$, to mitigate the effects of terms of trade on the dynamic equilibrium of the economy, as in Galí and Monacelli (2005). The parameter β is set to 0.99, which implies a annual real interest rate of 4 percent. The inverse of the elasticity of labour supply, φ, is set to 3, consistent with micro studies that report low labour supply elasticities. The parameter θ is set to 0.75, which implies that firms keep prices unchanged on average four quarters. The degree of openness of the domestic economy $1-\gamma$ is set to 0.7, whereas, ε is set to 6, which implies a mark up over marginal cost of 20 percent. The persistence of all shocks is set to 0.95 and the variance of their innovations to 0.0071^2. The parameters that characterise the transaction frictions are calibrated to generate a relatively low steady-state level of CS under zero inflation, thus we set $\Psi_0 = 0.01$, $\Psi_1 = 1.1$, $n_0 = \log(2-\beta) + 0.151$ and $\psi_1 = 0.01$, which imply a 15 percent degree of CS.

4.2 Dynamic Equilibrium and Currency Substitution

In order to highlight the effects of CS on the economy we choose a parametrization where the intertemporal elasticity of substitution and the elasticity of substitution between domestic and foreign intermediate goods are equal to 1, i.e. $\sigma = \eta = 1$. In this case, the welfare effects of terms of trade are completely eliminated, since the income and substitution effects that terms of trade generate perfectly cancel out each other. Consequently, domestic and foreign shocks do not affect the current account of the economy. This simplification makes easier to characterise analytically the implications of CS for welfare and optimal monetary policy. We use equations, (4.15), (4.19), (4.25), (4.43), (4.54), (4.55) and (4.56) to obtain domestic consumption in terms of domestic and foreign output and the transaction cost distortion, as the next equation shows,

$$C_t = Y_t^{1-\gamma} (Y_t^*)^\gamma \left(\frac{1}{1+\gamma\Upsilon_t} \right)^{1-\gamma} \frac{1}{(1+\Upsilon_t)^\gamma} \tag{4.57}$$

Using equation (4.57) and the definition of marginal costs, equation (4.35), we eliminate the lagrange multiplier λ_t and the marginal costs, mc_t from equations (4.50) and (4.51) to obtain the following non-linear representation of the Phillips curve,

$$N_t = \mu \left(\frac{Y_{H,t}}{Y_t^*} \right)^\gamma \left(\frac{1+\gamma\Upsilon_t}{1+\Upsilon_t} \right)^{1-\gamma} MC_t + \theta\beta E_t \left(\Pi_{H,t+1}^{\varepsilon} N_{t+1} \right) \tag{4.58}$$

$$D_t = \left(\frac{Y_{H,t}}{Y_t^*} \right)^\gamma \left(\frac{1+\gamma\Upsilon_t}{1+\Upsilon_t} \right)^{1-\gamma} + \theta\beta E_t \left(\Pi_{H,t+1}^{\varepsilon-1} D_{t+1} \right) \tag{4.59}$$

$$\theta\Pi_{H,t+1}^{\varepsilon-1} = 1 - (1-\theta) \left(\frac{N_t}{D_t} \right)^{1-\varepsilon} \tag{4.60}$$

Similarly, we use equations (4.22) and (4.35) to write the real marginal costs in terms of domestic output, $Y_{H,t}$ the transaction cost distortion, Υ_t, productivity, A_t and the relative price distortion generated by domestic inflation, Δ_t, as follows,

$$mc_t = \frac{Y_{H,t}^{1+\varphi}}{A_t^{1+\varphi}} \left(\frac{1+\Upsilon_t}{1+\gamma\Upsilon_t} \right) \Delta_t^\varphi \tag{4.61}$$

Where, $\Delta_t = \int_0^n \left(\frac{P_{H,t}(z)}{P_t}\right)^{-\theta} dz$, can be written recursively as follows,

$$\Delta_t = \theta\Delta_{t-1} + (1-\theta)\left(\frac{1-\theta\Pi_{H,t}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{4.62}$$

Similarly, we use equation (4.57) and equation (4.56) to eliminate consumption from the Euler equation,

$$\frac{1}{(1+i_t)} = \beta E_t\left(\left(\frac{Y_{H,t+1}}{Y_{H,t}}\right)^{-1}\left(\frac{1+\gamma\Upsilon_{t+1}}{1+\Upsilon_{t+1}}\right)\left(\frac{1+\Upsilon_t}{1+\gamma\Upsilon_t}\right)\frac{1}{\Pi_{H,t+1}}\right) \tag{4.63}$$

These transformations allow us to define the rational expectations equilibrium as the solution of the system of Non linear equations given by equations (4.16), (4.17), (4.9), (4.58), (4.59), (4.60), (4.61), (4.62), (4.63) and (4.1.3), for a given a sequence of $\{i_t^*\}$ and $\{y_t^*\}$,.

Equations (4.58) to (4.62) represent the non-linear version of the Phillips curve, whereas equation (4.63) the corresponding aggregate demand. Notice that besides marginal costs, domestic and foreign output and future expected inflation, transaction distortions, Υ_t, which depend on CS, determine inflation. Transaction costs, as we discussed previously, depend on both the domestic and the foreign nominal interest rates as it is established by equations (4.16), (4.16) and (4.9).

It is through this variable that CS affects the economy. When transaction frictions are not present, $\Upsilon_t = 0$, the economy collapses to a standard cashless SOE, as the one analysed by Galí and Monacelli (2005). However, when $\Upsilon_t > 0$ CS plays a role in the transmission mechanism of monetary policy.

In particular, transaction frictions, Υ_t, act as a stochastic tax for holding cash that breaks the equality between the marginal utility of income and consumption. This stochastic tax affects, by making more costly to transform income into consumption, the dynamics of both the aggregate demand and of inflation. The role that CS plays in this mechanism is to determine the weights of both the domestic and the foreign nominal interest rates on Υ_t. In the coming subsections we explain in detail the effects of CS for the steady-state, the flexible and the sticky price equilibrium. In section 4 we address the implications of CS for optimal monetary policy and for the determinacy of the equilibrium of Taylor rules. From now on, we adopt the convention of denoting by capital letters without time subscript, the corresponding steady-state value variables, and by lower case letters their log deviations from their steady-states, i.e. X is the steady-state of X_t and $x_t = \log(\frac{X_t}{X})$.

4.2.1 The Steady State

We analyse a deterministic steady-state where all shocks take their unconditional means, and where both domestic and foreign inflation rates are equal to zero. Since at this steady state, $MC = \frac{1}{\mu}$, from the corresponding steady-state analog of equation (4.61) we obtain the following steady-state level of domestic output,

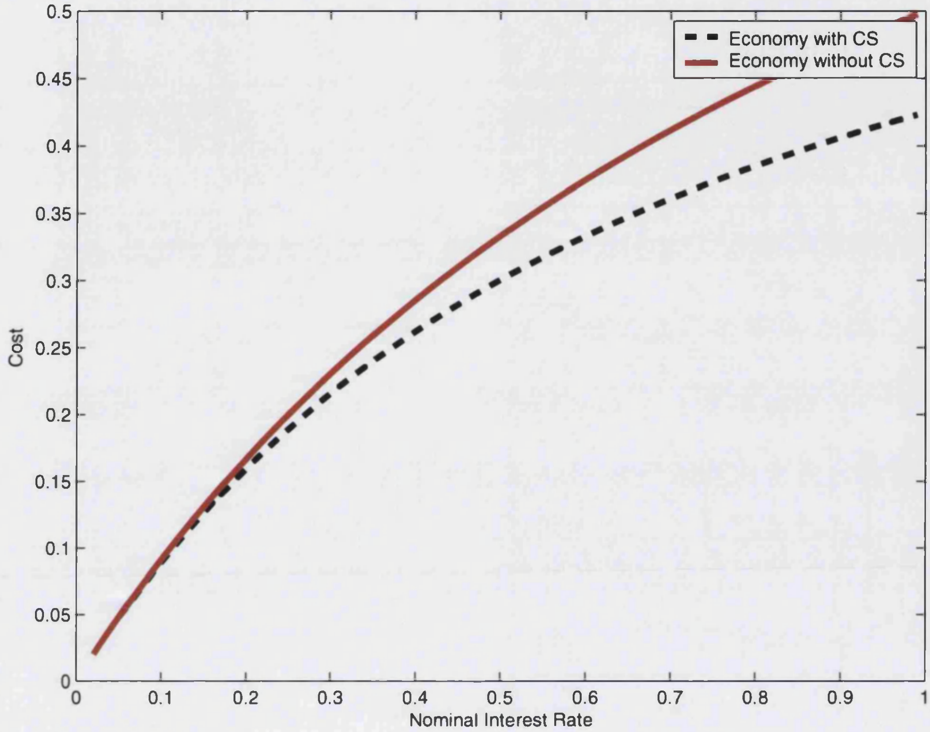
$$Y_H = (1 - \Phi)^{\frac{1}{1+\varphi}} \tag{4.64}$$

where $1 - \Phi = \frac{(1-\tau)(1+\gamma\Upsilon)}{\mu(1+\Upsilon)}$ accounts for the overall distortions in this economy. As equation (4.64) shows, the level of output is below its optimal level of 1. Two factors distort output at the steady-state, the degree of monopolistic competition, measured by the degree of mark-up, μ, that induce firms to produce below its efficient level, and transaction frictions, measured by Υ, that rise the marginal cost of firms above its optimal level.

The size of this second distortion is positively related to both the long-run levels of the domestic and foreign nominal interest rates. In economies where steady-state nominal interest rates are high, the cost associated with transanctions would also be high and therefore consumers would endongenously choose to use more foreign currency as medium of payment.

Figure 4.3 plots the welfare loss associated to the nominal interest rate

Figure 4.3: Transaction Costs and Currency Substitution



Note: On the vertical axis, steady-state costs are measured as proportion of consumption levels. On the horizontal axis the nominal interest rates is measured in units

generated through the distortion on output previously discussed. As this figure shows, this cost is increasing on the nominal interest rate, though it is much lower when CS is allowed. CS reduces the welfare costs of inflation because it permits households to optimally minimise transaction costs by shifting their money demand towards a foreign currency when the cost of using the domestic currency rises.

4.2.2 The Flexible Price Equilibrium

In contrast with the monopolistic distortion that only affects the steady-state of the economy, transaction frictions also distort the dynamic equilibrium of the economy, in particular, they induce an inefficiently dynamic output level. In this economy, the efficient flexible price output level coincides with domestic productivity, yet, as equation (4.65) shows, transaction frictions generate a gap between this efficient level and the output level under flexible prices. Next equation illustrate this latter point,

$$y_t^n = a_t - \frac{1-\gamma}{1+\varphi}\vartheta \upsilon_t \tag{4.65}$$

where, $\upsilon_t = \log(\frac{1+\Upsilon_t}{1+\Upsilon})$ and $\vartheta = \frac{1}{(1+\gamma\Upsilon)}$ [18]. This gap is increasing on both the domestic and the foreign nominal interest rates weighted by the degree of CS as follows,

$$\upsilon_t = \omega\left(\left(1-s\right)i_t + si_t^*\right) \tag{4.66}$$

where, $\omega = \frac{1}{2(1+i)-1}$. Crucially then, in economies where CS is high, the foreign interest rate is the variable that has a larger impact on distorting the dynamic behaviour of output and not the domestic one.

The efficient output level in this economy is achieved when $\upsilon_t = 0$. However, this allocation is not feasible under neither a policy of zero inflation nor a policy of zero domestic nominal interest rates. When inflation is zero, both the nominal interest rate and the degree of CS are positive, therefore, $\upsilon_t \neq 0$. Similarly, when the domestic interest rate is fixed to zero, as equation (4.9) shows, the degree of CS is not necessarily equal to zero, thus, $\upsilon_t \neq 0$.

To achieve the efficient allocation we assume, similarly to Woodford (2003), that the central bank has additional instruments, in particular that it can pay interest on money holdings, i_t^m and that it can tax the holdings of foreign currency, τ_t^m [19]. These two additional instruments can be used to make $\upsilon_t = 0$. Under these assumptions, υ_t and $\overline{s}$ are determined by the following two equations,

$$\upsilon_t = \omega\left(\left(1-s\right)\left(i_t - i_t^m\right) + s\left(i_t^* + \tau_t^m\right)\right) \tag{4.67}$$

[18] Equation (4.65) is obtained by taking a log linear approximation of equation (4.61), details of this derivation are provided in appendix B.4

[19] Woodford (2003) studies models with transaction frictions but with only one currency for close economies and Walsh and Ravenna (2006) studies models with working capital.

$$\overline{s} = \frac{\left(n_0 - \Psi_0 + \log\left(\frac{1+\frac{i-i^m}{(1+i)}}{1+\frac{i^*+\tau^m}{(1+i^*)}}\right)\right)}{(\Psi_1 - n_1)} \tag{4.68}$$

It follows from equations (4.67) and (4.68) that by setting, $i_t = i_t^m$ the central bank can eliminate the distortion generated by the domestic nominal interest rate, and by making $\tau^m = \frac{1}{\beta}\left(2 - \exp\left(n_0 - \Psi_0\right)\right) - 1$, the corresponding distortion generated by the foreign nominal interest rate, thus $\upsilon_t = 0$. Therefore, under this particular set of money holding taxes the central bank can induce an efficient level of domestic output, $y_t^e = a_t$, and equation (4.65) can be written as follows,

$$y_t^n = y_t^e - \frac{1-\gamma}{1+\varphi}\vartheta\upsilon_t \tag{4.69}$$

In what follows, we assume that the economy exhibits some degree of transaction frictions in steady-state, thus, we set $\tau^m = 0$ and $i - i^m$ to be small. Consequently, $\upsilon_t \neq 0$, thereby the flexible price equilibrium and the efficient one do not coincide. This discrepancy affects how the central bank implements monetary policy in a fundamental way. In the next section, we show this issue in detail.

4.2.3 The Equilibrium under Sticky Prices

In order to analyse the effects of CS on the dynamic equilibrium under price stickiness, we take a log-linear approximation of equations from (4.58) to (4.62) around the deterministic steady-state. It turns out that the economy exhibits a canonical representation of three equations: a dynamic aggregate demand, a Phillips curve and an interest rate policy rule. These three equations are presented next:

$$x_t = E_t x_{t+1} - (i_t - E_t\pi_{H,t+1} - r_t^n) + \sigma_i E_t\Delta i_{t+1} + \sigma_{i^*} E_t\Delta i_{t+1}^* \tag{4.70}$$

$$\pi_{H,t} = \beta E_t\pi_{H,t+1} + \kappa x_t + \kappa_i i_t + \kappa_f i_t^* \tag{4.71}$$

$$i_t = \phi_\pi \pi_{H,t} + \phi_x x_t \tag{4.72}$$

where, x_t represents the gap between output under sticky prices and its efficient level, i.e, $x_t = y_t - y_t^e$, and r_t^n the natural interest rate, which is function only of structural shocks. The new set of parameters are defined as follows:

Table 1: Definition of Parameters

$\sigma_i = \omega\vartheta\,(1-\gamma)\,(1-s)$	$\kappa = \lambda\,(1+\varphi)$
$\sigma_{i^*} = \vartheta\omega\,(1-\gamma)\,s$	$\kappa_i = \omega\vartheta\lambda(1-\gamma)\,(1-s)$
$\kappa_f = \vartheta\lambda\omega(1-\gamma)s$	

It is apparent from its canonical representation that the economy with CS exhibits two new features. First, the foreign interest rate shows up in the Phillips curve, equation (4.71), as a cost-push shock, where the magnitude of its impact on inflation depends on the degree of CS. Only when the degree of CS is zero, $s = 0$, the foreign interest rate does not affect the dynamics of inflation. In this case, the economy behaves similarly to the one analysed by Woodford (2003)[20].

The mechanism that generates the channel by which i_t^* appears in the Phillips curve works as follows: transaction costs create a gap between the marginal utility of consumption and that of income, given by v_t. This gap, for a given degree of CS, is increasing in both the domestic and the foreign nominal interest rates,

$$\lambda_t = -c_t - v_t \tag{4.73}$$

Consequently, as interest rates go up, the real value of a given real wage in terms of consumption falls. Since more real resources have to be allocated for transforming wage income into consumption, workers respond by cutting their labour supply. This, in turn, pushes real wages up and accordingly marginal cost rises. The next

[20] Woodford (2003) analyses a model of a close economy where transaction frictions affect the equilibrium of the economy. He finds that in this type of economies, the domestic interest rate affects directly the dynamics of inflation, similarly to the implications of our model.

equation makes explicit this link between transaction and marginal costs,

$$mc_t = (1+\varphi)(y_t - a_t) + \vartheta(1-\gamma)v_t \tag{4.74}$$

The degree of CS determines the relative weight that the domestic and the foreign interest rate have on marginal costs. Equation (4.75) shows how the presence of transaction frictions distorts the proportionality between the real marginal cost and the output gap,

$$mc_t = (1+\varphi)x_t + (1-\gamma)v_t \tag{4.75}$$

Therefore, a central bank that targets $x_t = 0$, can not stabilise the marginal cost of firms, since zero output gap does not imply zero transaction costs, $v_t = 0$[21]. If the central bank does not stabilise marginal costs, can neither stabilise inflation. Consequently, in an economy with CS, it would be impossible for the central bank to simultaneously achieve zero inflation and zero output gap.

The second new feature of this type of economies is a negative effect of i_t^* on aggregate demand. This effect is different to the one based on the intertemporal substitution mechanism. As equation (4.76) shows, its impact on aggregate demand is given by σ_{i^*}, which is increasing on the degree of CS. Also, notice that under CS, the partial response of the output gap to an increase on the domestic nominal interest rate becomes, $-(1+\sigma_i)$. Since as the degree of CS increases, σ_i falls, the output gap become less responsive to changes in the domestic interest rate as CS increases, weakening the interest rate transmission channel of monetary policy:

$$x_t = E_t x_{t+1} - (i_t - E_t\pi_{H,t+1} - r_t^n) - \sigma_i i_t - \sigma_{i^*} i_t^* + \sigma_i E_t i_{t+1} + \sigma_{i^*} E_t i_{t+1}^* \tag{4.76}$$

Under CS, when an agent decides to postpone one unit of income for future consumption, the cost of her decision in period t is given not only by the marginal utility of consumption but also by the transaction cost, v_t. Similarly, the next period benefits of that decision includes, besides the present discounted value of the marginal

[21] Note that transaction costs reach their minimun value only when the domestic nominal interest rate is set close to zero.

utility of consumption, the corresponding expected value of the transaction cost, $E_t v_{t+1}$. Since all shocks in the model are transitory, it holds that $-v_t + E_t v_{t+1} < 0$. Thus, when nominal interest rate increases, the associated transaction cost rises, making more expensive to consume in period t relative to future periods. The interaction of these two effects induces agents to reduce their consumption levels. The effect of transaction costs on savings decisions can be seen more easily by observing the following representation of the Euler equation that results after λ_t is replaced by its equivalent given by equation (4.73),

$$c_t = E_t c_{t+1} - v_t + E_t v_{t+1} - (i_t - E_t \pi_{t+1}) \tag{4.77}$$

In order to illustrate the effects of these new mechanisms on the rational expectations equilibrium, we solve for it, by considering that there exist only one shock in the economy, the foreign nominal interest rate. This assumption help us to obtain simple analytical solutions. Furthermore, we assume that i_t^* follows the following autorregressive process:

$$i_t^* = \rho i_{t-1}^* + \varepsilon_t$$

Under this assumption, the rational expectation equilibrium of equations (4.70), (4.71) and (4.72) is given by the following two equations:

$$x_t = -b i_t^* \tag{4.78}$$

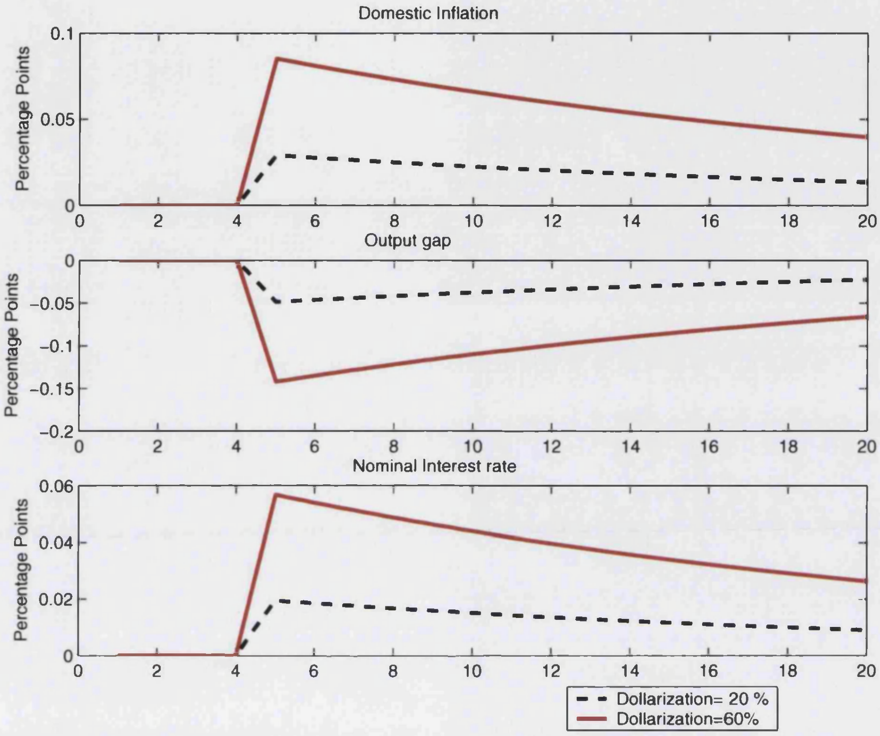
$$\pi_{H,t} = \frac{\kappa_f - b\kappa}{1 - \beta\rho - \kappa_i \phi_\pi} i_t^* \tag{4.79}$$

where,

$$b = \frac{\left(\frac{\phi_\pi - \rho}{(1-\rho)} + \sigma_i \phi_\pi\right) \kappa_f + \sigma_{i^*} \left((1 - \beta\rho - \kappa_i \phi_\pi)\right)}{\left(1 + \left(\frac{1}{(1-\rho)} + \sigma_i\right) \phi_x\right) (1 - \beta\rho - \kappa_i \phi_\pi) + \left(\frac{\phi_\pi - \rho}{(1-\rho)} + \sigma_i \phi_\pi\right) (\kappa + \kappa_i \phi_x)}$$

For most parameterizations, $b > 0$ and $\kappa_f - b\kappa > 0$. Therefore, an increase in foreign interest rate leads to a fall in output gap and to an increase on the domestic inflation rate. However, it is important to highlight that the inflation responses to

Figure 4.4: Response of Output and Inflation to a Foreign Nominal Interest Rate Shock (a)



Note: Dollarisation ratios correspond to steady-state levels

the foreign nominal interest rate is smaller than that of output gap, since the fall in output gap through the standard aggregate demand channel partially offset the direct impact of i_t^* on inflation in the Phillips curve.

These implications are confirmed in figures 4.4 and 4.5 that shows the impulse response functions of domestic inflation, output gap and the nominal interest rate to a positive foreign interest rate shock. These responses were obtained under the benchmark parameterisation for two different levels of the steady-state degree of CS.

As figure 4.4 shows, the response of the three aforementioned variables to the foreign nominal interest rate shock is stronger when the degree of CS in steady-state is higher. It is also important to highlight that as the analytical solution of the equilibrium indicates, the response of inflation is much lower than the one of output gap, since the endogenous response of the domestic nominal interest rates and the output gap partially offsets the initial impact of i_t^* on inflation.

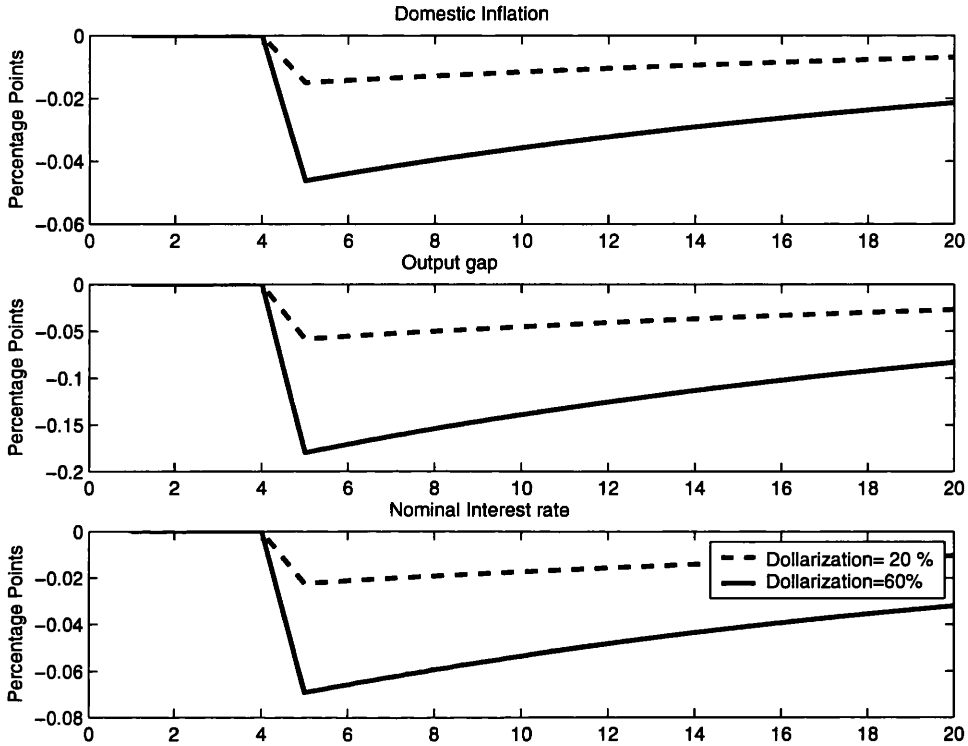
Notice that when the output gap weight on the central bank's reaction function is large enough, $\phi_x \to \infty$, the response of the output gap to i_t^*, measured by b shrinks towards zero. In this case, the effect of i_t^* on domestic inflation reaches its maximum value,

$$\pi_{H,t} = \frac{\kappa_f}{1 - \beta\rho - \kappa_i \phi_\pi} i_t^*$$

On the contrary, when the central bank does not react to the output gap, $\phi_x = 0$, the fall in output more than compensate the direct effect of the foreign nominal interest rate on inflation, $\kappa_f - b\kappa < 0$, thus, both the domestic inflation and the nominal interest rate falls in equilibrium. As in the previous case, here as well, the response of the three variables, output gap, inflation and the domestic nominal interest rate are increasing on the degree of steady-state CS. Figure 4.5 shows the impulse responses of these three variables when $\phi_x = 0$ for two different levels of steady-state CS.

A direct implication of our model is that economies with CS should be more sensitive to foreign nominal interest shocks than economies without CS. This result is in line with the empirical evidence reported by Agénor et al. (2000), Neumeyer and Perri (2005) and Uribe and Yue (2004), who document a negative correlation between domestic output and the foreign nominal interest rate for emerging markets, where CS is more frequent. The next section explores the implications CS for the design of monetary policy; in particular we derive the micro-founded loss function of the central bank, then this loss function is used to evaluate the performance

Figure 4.5: Response of Output and Inflation to a Foreign Nominal Interest Rate Shock (b)



Note: Dollarisation ratios correspond to steady-state levels

of different interest rate rules. Finally, we analyse the implications of CS for the determinacy of the rational expectations equilibrium.

4.3 Monetary Policy under Currency Substitution

In this section we analyse how CS affects monetary policy. In particular, we discuss the implications of CS for the convenience of exchange rate smoothing and for inflation determination. Although, there exist empirical evidence that shows that many central banks in emerging economies, in particular, in economies with dollarisation, tend to actively intervene in the exchange rate market to reduce the volatility of their nominal exchange rates, it is not clear cut whether or not this behaviour is optimal. Authors like Calvo and Reinhart (2002), emphasise the role of financial dollarisation. However, Céspedes et al. (2004), and Gertler et al. (2004), using dynamic stochastic general equilibrium models, find that even with financial dollarisation, a flexible exchange rate outperforms a fixed one. Also, chapter 3 shows that in economies with sector specific productivity shocks it is possible to sustain an equilibrium with price dollarisation, and that in this case, it is optimal for the central bank to allow some degree of exchange rate smoothing.

In order to evaluate the benefits of exchange rate smoothing in economies with CS we derive the micro-founded loss function of the central bank. As in Woodford (2003) and Benigno and Woodford (2005), this loss function comes from a second-order approximation of the utility function of the representative household, around a particular steady-state. We choose a steady-state where the effects of terms of trade are eliminated, but where transaction frictions play a role. In particular, we choose an steady-state of zero inflation, but where, $i - i^m$ is relatively small, thus both domestic and the foreign nominal interest rate distort the dynamics of the economy.

4.3.1 The Central Bank Loss Function

As we show in appendix B.6, the loss function for a central bank in a SOE with CS has the following form,

$$L = \frac{\Omega}{2} \sum_{t=0}^{t=\infty} \beta^t \left[(1-s) \left(\Lambda_i i_t + \Lambda_{ii} i_t^2 + s \Lambda_{ii^*} i_t i_t^* \right) + \Lambda x_t + \pi_{H,t}^2 \right] \tag{4.80}$$

where, Λ_i, Ω, Λ_{ii}, Λ and Λ_{ii^*} are positive parameters. Notice that this loss function differs, in at least two dimensions, from those obtained for economies where transaction frictions are not allowed[22]. First, in an economy with CS, both the domestic and the foreign nominal interest rates generate welfare losses. In particular, the central bank has an incentive to keep domestic interest rates low and stable, but also to induce a negative correlation between the domestic and the foreign interest rates.

To understand why the central bank has this incentive, notice that when CS is positive, $s > 0$, the foreign nominal interest rate also generates transaction costs for households, therefore a central bank, which aims at maximizing households welfare, would have the incentive to move i_t in the opposite direction of i_t^* to compensate the costs generated by fluctuations in the foreign interest rate. Remarkably, this incentive is larger, as the degree of CS increases.

The cross term between domestic and foreign interest on the central bank loss function, also has implications for the desirability of exchange rate smoothing. Since, smoothing exchange rate implies that the central bank has to move the domestic interest rate to mimic the path of foreign domestic, the welfare loss of exchange rate smoothing turns out to be increasing on the degree of CS. Thus, we can argue that CS does not provide a rational for *fear of floating.*

Second, the incentives of the central bank to smooth fluctuations on the domestic nominal interest rate are decreasing on the degree of CS. In the limit, when $s = 1$, the domestic interest rate does not generate welfare losses, thus the central bank has no incentives to smooth domestic nominal interest rate fluctuations. In

[22] For instance Woodford (2003), obtains, for an economy with transaction frictions, a loss function that depends on quadratic terms of inflation, output gap, and the nominal interest rate. He shows that this latter term justifies some degree of interest rate smoothing.

this case, the loss function collapse to the one derived by Woodford (2003)[23]. Interestingly, steady-state CS only affects the interest rate weights on the central bank loss function but not the weights of the output gap and inflation. This is so because CS does not affect production efficiency, only consumption levels.

Next, we use the micro-founded loss function, equation (4.80), to rank different interest rate rules. In particular, we compare the performance of domestic inflation and consumer price inflation interest rate rules under different degrees of interest rate and exchange rate smoothing, the policy rules are parameterised as follows,

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) \left[\phi_\pi \pi_{H,t} + \phi_x x_t + \phi_e E_t \Delta e_t \right] \tag{4.81}$$

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) \left[\phi_\pi \pi_t + \phi_x x_t + \phi_e E_t \Delta e_t \right] \tag{4.82}$$

A policy rule would outperform another if it generates a rational expectations equilibrium that implies a lower expected welfare loss than the alternatives. In order to calculate the expected welfare loss for each interest rate rule we solve up to second-order the rational expectations equilibrium of the economy, defined by equations (4.58) to (4.63), plus the interest rate rule defined previously[24]. The rational expectations equilibrium is calculated for a set of economies, each of one defined for a particular value of ϕ_e and ρ_i.

This strategy allow us to consider the case of economies where the degree of CS is not too small, as it is indeed the case of economies with CS, but at the same time it permit us to keep approximation errors small enough. This in turn allow us to evaluate implementable monetary policy rules in a consistent way.

Figure 4.6 compares the welfare loss generated by implementable interest

[23] Woodford (2003), shows that in a close economy where there exist transaction frictions, the expected loss function of the central bank depends, besides the variance of output gap and inflation, on the variance of the nominal interest rates.

[24] To solve up to second-order the dynamics equilibrium of the economy we use the perturbation method and the code produced by Schmitt-Grohe and Uribe (2004)

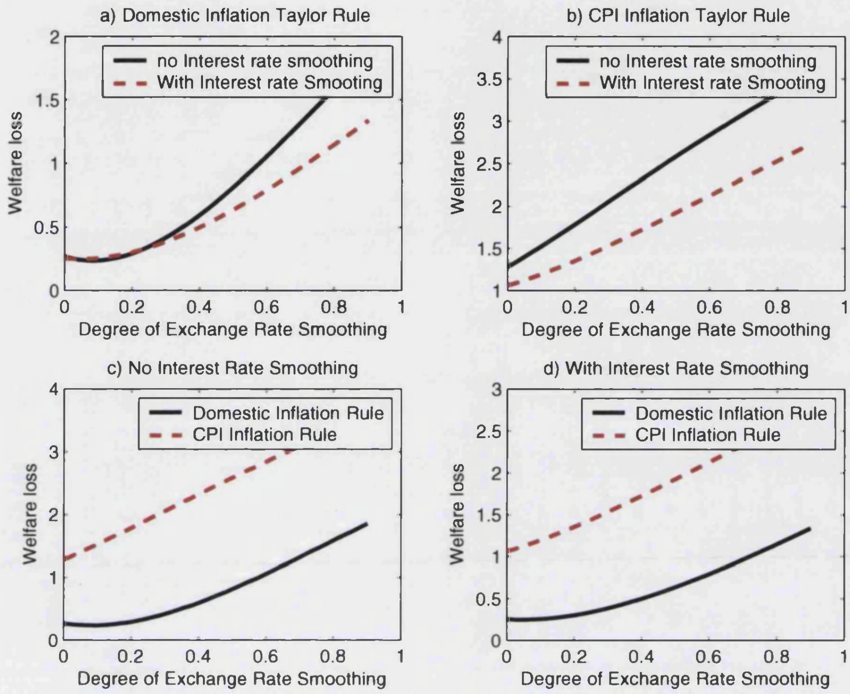
rate rules that react to the CPI and domestic inflation, the output gap, the lag interest rate and the change in the exchange rate, considering different degrees of exchange rate smoothing. In all figures, the horizontal axis measures the degree of exchange rate smoothing, indexed by the value of ϕ_e.

Figures 4.6a and 4.6b, show the welfare losses under domestic inflation rule and CPI inflation, where the solid line represent rules without interest rate smoothing, whereas the dotted line considers those where the central bank reacts also to the one period lagged domestic nominal interest rate. Figure in Panel 4.6c, compares a rule that reacts to domestic inflation to one that reacts to CPI inflation, when both do not consider interest rate smoothing, whereas 4.6d does the same comparison but considering that both rules feature interest rate smoothing.

All three figures show that welfare losses are increasing on the degree of exchange rate smoothing. However, as 4.6c describes, welfare losses are higher for those rules that target the consumer price inflation in comparison with those that target instead domestic inflation. Thus, as in Galí and Monacelli (2005), we also obtain that targeting domestic price inflation allows the central bank to deliver a superior outcome in terms of welfare. This results comes directly from the central bank micro-founded loss function, where the definition of inflation that generates welfare losses is not CPI inflation but domestic inflation. Consequently, a central bank that reacts to CPI inflation generates higher volatility on the output gap and the nominal interest rates which are not compensated by a lower volatility on domestic inflation.

Also, Panel 4.6a and 4.6b, show that rules that consider some degree of interest rate smoothing outperform those that do not smooth the nominal interest rate. Consequently, interest rate smoothing turns to be a desirable objective for the central bank under CS. The intuition of this result is simple, a lower variability on domestic nominal interest rates reduces expected transaction costs, thus expected

Figure 4.6: Welfare Ranking of Interest Rate Rules



Note: In panel (a) and (b) a value of 0.8 for the lag of the nominal interest rate has been consider for rules with interest rate smoothing, those represented with dotted lines. Panel (c) compares both type of rules without interest rate smoothing and Panel (d) does it when a value of 0.8 is used for the parameter of the lagged interest rate. In all cases, a steady-state value of CS of 5 percent has been considered

welfare losses are mitigated.

To sum up, in small open economies with CS the central bank should allow the exchange rate to float and put less weight on interest rate smoothing, in comparison to economies without CS. Also, the steady-state degree of CS does not affect the relative weights the central bank put on output gap and domestic inflation stabilization, it only affects the weight on interest rate smoothing.

4.3.2 Determinacy of Equilibrium

In this subsection we analyse the implications of CS for the determinacy of rational expectations equilibrium. A question that has not been analysed yet in the literature is whether or not CS affects the capability of the central bank to anchor inflation expectations. We address this question in this subsection by showing how the conditions for the determinacy of the rational expectations equilibrium changes with CS for interest rate rules that target domestic inflation. With this purpose and following Woodford (2003), we write the canonical representation of the model economy as follows:

$$E_t z_{t+1} = A z_t + a e_t$$

where: $z_t = \begin{bmatrix} \pi_{H,t} \\ x_t \end{bmatrix}$, and

$$A = \begin{bmatrix} \frac{(1-\kappa_i \phi_\pi)}{\beta} & -\left(\frac{\kappa+\kappa_i\phi_x}{\beta}\right) \\ \frac{(1+\sigma_i)\phi_\pi - (1+\sigma_i\phi_\pi)\frac{(1-\kappa_i\phi_\pi)}{\beta}}{1+\sigma_i\phi_x} & \frac{1+(1+\sigma_i)\phi_x + (1+\sigma_i\phi_\pi)\frac{(\kappa+\kappa_i\phi_x)}{\beta}}{1+\sigma_i\phi_x} \end{bmatrix} \tag{4.83}$$

Since there are two forward looking variables in the model, the rational expectations equilibrium is uniquely determined when both eigenvalues of matrix A are outside the unit circle. As it is detailed in Woodford (2003), the necessary and sufficient conditions for this to hold are:

$$\det A > 1 \tag{4.84}$$

$$\det A + trace(A) > -1 \tag{4.85}$$

$$\det A - trace(A) > -1 \tag{4.86}$$

Writing conditions (4.84),(4.85) and (4.86) in terms of the parameters of the model we obtain:

$$\left(\frac{1}{\beta}-1\right)+\left(\frac{1+\sigma_i(1-\beta)}{\beta}\right)\phi_x+\left(\frac{\kappa+(\beta\sigma_i\kappa-\kappa_i)}{\beta}\right)\phi_\pi>0 \tag{4.87}$$

$$(1-\beta-\kappa_i)\phi_x+\kappa(\phi_\pi-1)>0 \tag{4.88}$$

$$\frac{2(1+\beta)}{\beta}+\left(\frac{(1+\beta)}{\beta}(1+2\sigma_i)+\frac{\kappa_i}{\beta}\right)\phi_x+\left(\frac{\kappa}{\beta}+\frac{2}{\beta}(\sigma_i\kappa-\kappa_i)\right)>0 \tag{4.89}$$

It turns out that conditions (4.87) and (4.89) hold for any pair of positive values of ϕ_x and ϕ_π when the inverse of the elasticity of substitution is large enough. In particular when it satisfies the following lower bound[25].

$$\varphi>\frac{1-\beta}{\beta} \tag{4.90}$$

Therefore, under this parameterisation the only condition that is relevant for determinancy is (4.88). Notice that this condition coincides with the Taylor principle when $\kappa_i = 0$. This occurs when the degree of CS is 1, ($s = 1$), since, $\kappa_i = \lambda(1-\gamma)(1-s)$. Interesting, the model implies that the conditions for determinacy under full CS coincides with those of a cashless economy, panel (a) of figure 4.7.

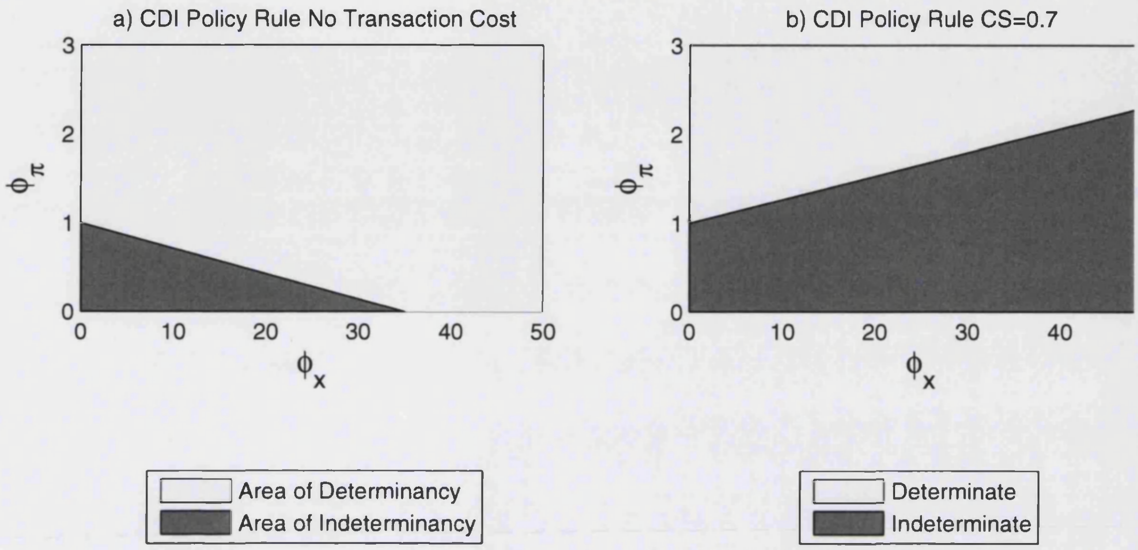
In a cashless economy, the domestic nominal interest rate affects the economy only through its effect on the dynamic IS curve, the same happens when $s = 1$ in an economy with CS. In contrast, when $s \neq 1$, the area of determinacy is much smaller, panel b, figure 4.7. In this case, the domestic interest rate have a direct effect on inflation through the wedge that transaction cost generates between marginal cost and output gap. The effect of the domestic interest rate on inflation is

[25] To understand why this is so, notice that condition (4.87) holds when $(\beta\sigma_i\kappa-\kappa_i)>0$. Since, under our benchmark parameterisation, $\sigma_i=(1-\gamma)(1-s)$, $\kappa_i=\lambda(1-\gamma)(1-s)$ and $\kappa=\lambda(1+\varphi)$. we have that

$$(\beta\sigma_i\kappa-\kappa_i)=(1-\gamma)(1-s)\lambda(\beta(1+\varphi)-1)$$

which is positive for values of φ that satisfy condition (4.90). Similarly, condition (4.89), holds when $(\sigma_i\kappa-\kappa_i)=\lambda(1-\gamma)(1-s)\varphi>0$, which is always true when $\varphi>0$. Therefore, the only condition that determines the set of parameter values for ϕ_x and ϕ_π that render the equilibrium determine is condition (4.88)

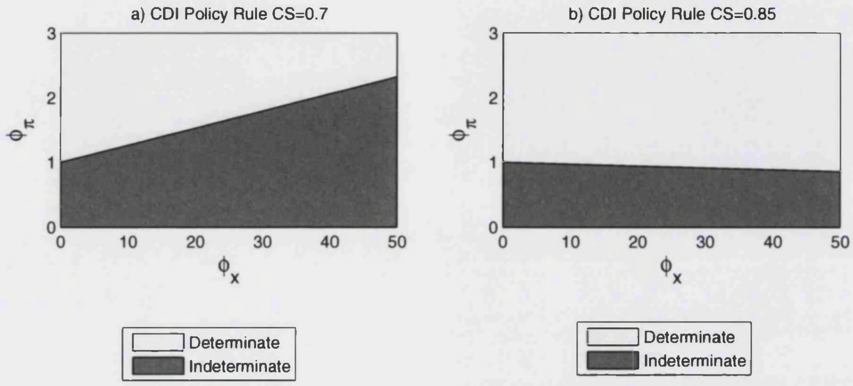
Figure 4.7: Determinacy and Currency Substitution



larger, as the degree of CS decreases, κ_i is smaller. This additional effect of the domestic interest rate on inflation generates the possibility for indeterminacy of the equilibrium.

To see how this mechanism for indeterminacy works, let's suppose that the central bank observes a negative output gap, by the Taylor rule the central bank would reduce the interest rate, however this reduction on nominal interest rates leads to a lower inflation through κ_i. This second round effect generates a further reduction on the nominal interest rate, since the central bank tries to stabilise inflation. When ϕ_x is large enough, the direct effect of nominal interest

Figure 4.8: Determinacy and the Degree of Currency Substitution



rates on inflation is larger than the indirect effect on output gap, therefore, the nominal interest rate and domestic inflation will keep falling and the central bank will not be able to stabilise the economy. This cycle leads to the indeterminacy of the equilibrium. Consequently, in economies with CS as the degree of CS falls the area of determinacy for the rational expectations equilibrium shrinks, this is shown in figure 4.8, panel (a)

On the contrary, when the degree of CS increases, κ_i falls, the area for determinacy of the rational expectations equilibrium increases, figure 4.8, panel (b). This, however does not imply that the CS delivers a more stable economy. As we discussed in the previous section, the volatility of both inflation and output gap increases with the degree of CS. Therefore, even though CS allows the central bank to react more aggressively to stabilise output gap and guarantee the determinacy of the rational expectations equilibrium, at this equilibrium both the output gap and domestic inflation volatility would be larger.

4.4 Concluding Remarks

In this chapter we have developed a very tractable and fully micro-founded model of a small open economy with CS that can be used for monetary policy analysis. The model economy have a canonical representation analogous to their counterparts without CS but differ from the latter ones in two important dimensions: first, the foreign nominal interest rates appears as an endogenous cost push shock in the Phillips curve, where the magnitude of its effect on inflation depends on the degree of CS. Second, the domestic nominal interest rates has a direct effect on inflation, making less effective the use of the nominal interest rate for the control of inflation.

These new features that CS adds to a standard small open economy model have interesting implications for monetary policy. First, the central bank faces a trade-off between stabilizing inflation and the efficient level of output gap, where the magnitude of this trade-off depends on the degree of CS. In particular, as the degree of CS increases, the central bank have to accept a higher volatility of output gap to maintain the volatility of inflation.

Second, CS increases the volatility of inflation, both domestic and CPI, and output gap under a variety typical interest rate target rules. Moreover, rules that smooth the nominal exchange rate perform worse than those that allow more flexibility on the exchange rate. In particular, the volatility of inflation and output gap increases with the degree of smoothness of the exchange rate, similarly to the case of economies without CS. Therefore, CS does not justify "fear of floating", smoothness of the exchange rate.

Finally, CS increases the area of determinacy for the rational expectations equilibrium under contemporaneous domestic inflation Taylor rules. In the limit, when there is full substitution of the domestic currency, the area of determinacy coincides with the one of a cashless economy, therefore the Taylor Principle holds.

In contrast, when there is no CS, but money matters in the dynamic equilibrium, the set of parameters that allow its determinacy shrinks. In particular, for a given reaction of the central bank to inflation deviations, the central bank cannot react too much to output gap to guarantee equilibrium determinacy.

The model can be extended to several directions. For instance, the assumption that transaction costs are rebated to households can be relaxed. Also, the terms of trade distortion can be included to analyse the interaction between this channel and CS. We leave these extensions for future research.

CHAPTER 5

INCOME DISTRIBUTION AND DOLLARISATION

Chapter 2 presented some evidence suggesting that the degree of PD is related to the country's average income-levels. Casual evidence, furthermore, reveals that PD is concentrated mostly on goods whose demand is derived from the consumption of high-income customers; and to a lesser extent on goods associated to the consumption of low-income customers, like necessity goods[1].

Motivated by this evidence, this chapter develops a model where agents' dollarisation decisions and income distribution are related. In particular, the model allows us to study whether differences in income levels matter for the pattern of PD across type of goods and for agents' portfolio decisions regarding foreign-currency denominated assets.

The model is an overlapping generation monetary model with two key assumptions: a) prices are sticky, and b) agents' preferences are non-homothetic[2]. The type of preferences is important in our modelling strategy since it allows differences in income levels to play a role in consumption and portfolio decisions. In particular, under this type of preferences changes in agents' income levels affect the

[1] This pattern of PD is independent of the type of good being considered, whether goods are tradable or not tradable, or whether its purchase implies a large or small payment. For instance in Peru, firms offering education services set prices in different currencies depending on the location of the institution - in rich neighbourhoods prices are in dollars, whilst in poor ones prices are in pesos. Moreover, small transactions like haircuts are charged in dollars in some beauty shops located in rich neighbourhoods, and large transactions, like real estate, are priced in pesos in poor areas.

[2] See Matsuyama (2000) and Matsuyama (2002) for macroeconomic models with non-homothetic preferences

composition of agent's consumption basket. As income rises the participation in the consumption basket of some goods rises, whereas that of the others fall[3]. The former are considered as luxury goods, and the latter, as necessity goods.

We introduce non-homotheticity as in Schmitt-Grohe et al. (2006) by assuming that agents need to consume a fixed minimum amount of each good to receive utility. This amount constitutes a subsistence point. Goods with larger subsistence points can be considered as necessity goods since agents' consumption of these goods fall by less in states of nature where income is low. Whereas goods with low subsistence levels are considered as luxury goods because agents' consumption of these goods increase (fall) by a larger proportion when income also increase (fall).

The key implication of this type of preferences for firms' decisions is that it generates a time-varying demand price elasticity, which depends on the income level of firms' customers. In particular, the demand price elasticity increases when the income of firms customers increases. Firms produce differentiated consumption goods under monopolistic competition using labour and a constant returns-to-scale production function. Also, they can choose between a domestic and a foreign currency for invoicing[4].

The main implications of the model can be summarised as follows. First, income distribution matters for agents' portfolio decisions. More precisely, a low-income agent's portfolio decisions are more sensitive to the volatility of agent's purchasing power, which is crucially determined by the degree of PD, and to a lesser extent to the volatility of the exchange rate. High-income agents's portfolio

[3] This feature is in contrast to the case of homothetic preferences, where only the average level of income in the economy determines the demand for goods, and other moments of income distribution do not play any role.

[4] One of the first works in invoicing decision theory is Klemperer and Meyer (1986), who discuss the decision between Cournot and Bertrand oligopoly competition under uncertainty. Other papers, such as: Giovannini (1988), Donnefeld and Zilcha (1991), Johnson and Pick (1997) and Bacchetta and Wincoop (2005), study the decision of pricing in the exporter's or the importer's currency under international trade. For general equilibrium Devereux et al. (2004), Loyo (2001) and Corsetti and Pesenti (2004).

decisions, instead, respond more strongly to the volatility of exchange rate than the volatility of domestic prices, when forming their portfolios.

At the aggregate level, agents' optimal portfolio decisions imply that FD is positively related with PD. The intuition for the existence of this link is simple. Since PD induces uncertainty in agents' purchasing power, agents optimally choose their portfolio to minimize the effect of this type of uncertainty on their consumption choice.

Second, that firms would optimally choose to invoice depending on the volatility of exchange rate, the correlation of their marginal costs with the exchange rate and on the degree of non-homotheticity of their customers' preferences. In particular, we find that firms would have more incentives to invoice in dollars when the volatility of exchange rate is low, their marginal cost are highly positive correlated with the exchange rate and when the degree of non-homotheticity of their customer's preferences is relatively low.

An interesting result that helps to explain some of the stylised facts on PD discussed above is that optimal firms' invoicing decision implies, under certain conditions, that firms producing necessity goods would not choose to set prices in a foreign currency even when their marginal costs are highly correlated with the exchange rate. This result, couple with the implication of non-homothetic preferences that in low-income countries necessity goods represent a large share of the typical consumption basket, implies that PD does not have a large effect on the degree of exchange rate pass-through. This implication of the model is consistent with what is observed in Bolivia and Peru, where necessity goods, like food, represent more than 45 and 35 percent of their corresponding consumption baskets used to measure the CPI inflation; and the degree of pass-through is below 20 percent, although the levels of FD are above 60 percent.

In those economies then, monetary policy can operate when FD is high, because income distribution limits the effect of dollarisation on the degree of exchange rate pass-through to prices[5].

Why firms that have marginal cost highly positive correlated with the exchange rate do not choose to invoice in dollars?. The answer is related to the effect that exchange rate movements have on firms' profits when their customers have non-homothetic preferences. Under this type of preferences the exchange rate does necessary have the beneficial effects on firms' expected profits that otherwise it would generate, in particular, when firms' marginal costs are highly correlated with the exchange rate.

In this case, invoicing in a foreign currency increases firms' relative prices precisely in those states of nature where the demand-price elasticity is high, thus, increasing firms' profit volatility. Therefore, when firms' customers have non-homothetic preferences, firms' expected profits are reduced under dollar invoicing relative to peso invoicing. For firms producing necessity goods the negative effect of invoicing in dollars dominates its positive effect on expected profits, hence, firms producing these type of goods may choose to invoice in a domestic currency. For firms producing luxury goods, however, the oppositive happens.

Third, we find at the aggregate level it is possible to obtain multiple equilibrium. Importantly, however, we find that the set of multiple equilibrium is reduced when firms' customers have non-homothetic preferences. In particular, in this case, there exist an stable equilibrium where the degree of PD is determined by the size of the sector producing luxury goods. This result is interesting since it rationalises the fact that in economies where income distribution is unequal PD is not very large.

[5] Gonzalez-Anaya (2002) provides evidence showing that low levels of PD coexist with high levels of FD for a large sample of dollarised economies. They measure price dollarisation by the short-run level of pass-through of the exchange rate. Also, see Armas et al. (2001), Miller (2003) and Winkelried (2003) for estimations of degree of exchange rate pass-through to prices for Peru.

Our model is related to previous work on endogenous dollarisation decisions in general equilibrium frameworks. In particular to Sturzenegger (1997) and Ize and Parrado (2002)[6]. In those two papers, endogenous dollarisation decisions are analysed but in different macroeconomic frameworks. Sturzenegger (1997) uses an endogenous cash-in-advance model to analyse the welfare implications of endogenous currency substitution. In his framework, the size of the transaction is the key feature in explaining the pattern of dollarisation. Agents decide the currency in which to trade comparing the fixed cost that implies trading in dollars with the cost of trading in domestic currency, the inflation tax. As the inflation tax is proportional to the value of the transaction, they show that expensive goods are endogenously traded in foreign currency since the benefit of trading with this currency (avoiding the inflation tax) exceeds its cost.

On the contrary, with cheap goods the cost of trading in dollars is higher than the inflation tax, therefore the transaction is made using domestic currency. This approach, however, does not explain why small transactions associated with high-income customers are made in foreign currency. We instead consider that the size of the transaction is not the most important element in determining dollarisation patterns, but the interaction between the firm's customers income-level and the optimal strategies of firms in setting prices.

On the other hand, Ize and Parrado (2002) use a representative agent general equilibrium model to analyse the interaction between PD, FD and monetary policy. In their model, the decisions of FD and PD are endogenous decisions based on minimum variance portfolios. They find that both FD and PD respond to the variance of real exchange rate and inflation, but PD responds as well to monetary policy and to the nature of the shocks. Since this model uses homothetic preferences, it predicts homogenous pricing strategies for firms and consequently it is not able

[6] Recent papers on dollarisation, such as Céspedes et al. (2004) focus more on the effects of liabilities dollarisation on the transmission mechanism of monetary policy and in exchange rates regimes.

to explain the pattern of dollarisation across types of goods.

Also, the model in this chapter is very closely related to the literature on endogenous exchange rate pass-through to prices, in particular to Bacchetta and Wincoop (2005), Devereux et al. (2004), and Corsetti and Pesenti (2004). In all these models exporting and importing firms choose endogenously between a domestic and a foreign currency for invocing by comparing expected profits under each invoicing strategy. Bacchetta and Wincoop (2005) analyse this problem in a general equilibrium static framework, whereas Devereux et al. (2004), and Corsetti and Pesenti (2004) study this problem ina a dynamic general equilibrium setup.

This chapter is organised as follows: in section 5.1 we present the model. In section 5.2 we discuss the optimal financial and price dollarisation decisions of individual agents. Section 5.3 shows determination of the general equilibrium level of PD. Section 5.4 discusses the link between the degree of pass-through and the income level. Finally, section 5.5 presents some concluding remarks. The proofs of the propositions are detailed in the appendix C.

5.1 The Model

The economy is populated by a continuum of two overlapping generations, the young and the old. Each generation lives for two periods and it is composed by a continuum of agents of mass 1 indexed by their labour productivity levels θ_i, which are distributed according to $F(\theta)$ over the support $[\underline{\theta}, \overline{\theta}]$, with $0 < \underline{\theta} < \overline{\theta} < \infty$ and $\int_{\underline{\theta}}^{\overline{\theta}} \theta dF(\theta) = 1$. Agents, when they are young, supply labour inelastically to firms and take portfolio decisions, whereas when they are old only consume a continuum of differentiated consumption goods.

We restrict the set of assets available to young agents to fiat money, consequently their savings are constituted only by their domestic currency money holdings. Young agents have, however, the option of indexing their money holdings to the nominal exchange rate. The nominal exchange rate, e_t is defined as the price of the domestic currency in terms of the foreign one. In addition, it is assumed that the exchange rate depreciation rate, denoted by $\xi_t = \frac{e_{t+1}-e_t}{e_t}$, is normally distributed with a positive mean and constant variance, i.e $\xi_t \sim N(d, \sigma_\xi^2)$. This variable represents the only source of uncertainty in the model. On the other hand, agents' preferences over consumption goods are non-homothetic, which implies that agents' income-level heterogeneity generates heterogeneity in agents' consumption basket composition.

Consumption goods are produced by a continuum of monopolistically competitive firms using all types of labour supplied by young agents and a constant returns to scale technology. One period in advance, these firms choose their prices and the currency for invoicing optimally.

The timing of decisions is as follows, first , before observing the realisation of the exchange rate, young agents choose the degree of indexation of their portfolios that maximises their next period expected utility. Simultaneously firms choose their optimal price and their currency for invoicing. Then the exchange rate is observed, firms produce at the pre-set prices and agents consume.

5.1.1 Preferences

Each agent i lives for two periods, t and $t+1$. When young, during period t, agents supply labour inelastically to firms and take saving decisions. Whereas during period $t+1$, when they are old, agents consume a bundle of differentiated goods, X_{t+1}^i. Preferences over the consumption bundle X_t^i, are defined by the following generic utility function,

$$V_t^i = \beta E_t U\left(X_{t+1}^i\right) \tag{5.1}$$

where U is a continuous and concave function of X_t^i. On the other hand, the consumption bundle is a composite of a continuum of differentiated consumption goods defined by the following Dixit-Stiglitz aggregator,

$$X_t^i = \left[\int_0^1 \left(C_t^i(z) - \gamma_z^i\right)^{\frac{\varepsilon-1}{\varepsilon}} d(z)\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{5.2}$$

where, $\varepsilon > 1$ represents the elasticity of substitution across differentiated goods and γ_z^i a subsistence consumption level for good z. We follow Schmitt-Grohe et al. (2006) in using idiosyncratic subsistence levels to model non-homothetic preferences. As we show in the next sections, non-homotheticity has crucial implications for the portfolio agent's decisions and invoicing firms' decisions. With this type of preferences, the households' marginal valuation of different goods changes with their income levels, how much it changes depends on γ_z .

Goods with larger γ_z represent goods whose consumption is more a necessity. This is so because when agents' income falls their consumption of those goods falls by a smaller proportion in comparison with how much fall agents' consumption of goods with lower γ_z. On the contrary when income rises, the expenditure on goods with lower γ_z increases by a larger proportion that the expenditure on those goods with larger γ_z.

During period t agents receive dividends and wages according to their labour productivity, which is indexed by θ_i. Since agents do not consume during this period they use all their income for savings. The only asset available for savings is money, consequently agents' savings coincide with their money holdings at the end of period t, which are given by,

$$M_t^i = \left(W_t^i + D_t^i\right) \tag{5.3}$$

Where, $W_{i,t}$ represents agent's i wages and $D_{i,t}$ her corresponding dividends. Individual's dividends in turn are defined in terms of agents' labour productivity as follows,

$$D_t^i = \theta_i D_t \tag{5.4}$$

Where D_t represents aggregate dividends. A simple way to rationalise this dividend rule is to assume that workers receive a contingent bonus on the firm's profit level. During period $t+1$, agents use their savings to finance consumption expenditure. They face the following budget constraint,

$$\int_0^1 \frac{P_{t+1}(z)}{P_{t+1}} C_{t+1}^i(z) d(z) \leq \frac{M_{t+1}^i}{P_{t+1}} \tag{5.5}$$

where, $P_{t+1}(z)$ represents the price of good z during period $t+1$, which is defined as follows,

$$P_{t+1}(z) = \begin{cases} P_t^*(z) e_{t+1} \text{ if } z \in \Upsilon \\ P_t(z) \quad \text{otherwise} \end{cases} \tag{5.6}$$

and P_t denotes the aggregate consumer price index given by,

$$P_{t+1} = \left[\int_0^v P_t(z)^{1-\varepsilon} d(z) + \int_v^1 \left(P_t^*(z) e_{t+1}\right)^{1-\varepsilon} d(z) \right]^{\frac{1}{1-\varepsilon}} \tag{5.7}$$

Here, Υ is defined as the set of all firms that set prices in a foreign currency. This set has mass v and measures the degree of price dollarisation in the economy. On the other hand, we denote by η^i the fraction of money holdings of agent i indexed to the nominal exchange rate, her degree of financial dollarisation, and by M_t^i, her money holdings at the end of period t, which is defined by,

$$M_{t+1}^i = M_t^i \left(1 + \eta^i \left[\frac{e_{t+1} - e_t}{e_t}\right]\right) \tag{5.8}$$

Agents Consumption Decisions

The optimal amount of consumption for each type of differentiated good is obtained by minimising the expenditure of consuming the bundle X_t^i. From the first order conditions of this cost minimization problem, we obtain that the consumption of each differentiated good is decreasing on its relative price and increasing in the level of total consumption, as the next equation shows,

$$C_{t+1}^{i}(z) = \left(\frac{P_{t+1}(z)}{P_{t+1}}\right)^{-\varepsilon} X_{t+1}^{i} + \gamma_z^{i} \tag{5.9}$$

Notice that $C_{t+1}^{i}(z)$ is contingent on the realisation of the depreciation of the nominal exchange rate because both, X_{t+1}^{i} and P_{t+1} depend on this variable. On one hand, P_{t+1} is linked to ξ_{t+1} through the degree of PD. In this case, when a large fraction of firms find optimal to invoice in a foreign currency the correlation between P_{t+1} and ξ_{t+1} is positive and high. On the other hand, X_{t+1}^{i} depends on ξ_t through agent's porftolio decisions. If an agent choose to index a large fraction of her money holdings to the exchange rate, then the correlation between X_{t+1}^{i} and ξ_t would be also positive and high. To illustrate this latter point let's link X_{t+1}^{i} and η^{i} using the agent's budget constraint. By aggregating agent i consumption across type of goods we have that,

$$\int_0^1 \frac{P_{t+1}(z)}{P_{t+1}} C_{t+1}^{i}(z) d(z) = X_{t+1}^{i} + \int_0^1 \frac{P_{t+1}(z)}{P_{t+1}} \gamma_z^{i} d(z) \tag{5.10}$$

Let's denote by $C_{t+1}^{i} = \int_0^1 \frac{P_{t+1}(z)}{P_{t+1}} C_{t+1}^{i}(z) d(z)$ the total consumption of agent i, and by $\gamma_{t+1}^{i} = \int_0^1 \frac{P_{t+1}(z)}{P_{t+1}} \gamma_z^{i} d(z)$, her subsistence consumption level. Hence, total consumption level for agent i is defined as follows,

$$C_{t+1}^{i} = X_{t+1}^{i} + \gamma_{t+1}^{i} \tag{5.11}$$

By using equation equations (5.11) and (5.8), equation (5.5), the budget constraint of individual i, can be written as follows,

$$X_{t+1}^{i} = \frac{M_t^{i}}{P_{t+1}} \left(1 + \eta^{i} \xi_{t+1}\right) - \gamma_{t+1}^{i} \tag{5.12}$$

The previous equation shows that for a given price level, when η^{i} is large, the correlation between X_{t+1}^{i} and ξ_{t+1} is also larger.

Portfolio Decisions

The consumption level of agent i, C^i_{t+1} depends not only on her income level but also on the realisation of the exchange rate. This latter variable affects consumption decisions through the portfolio choice of individuals, η^i and also through the the degree of PD. In states of nature where the exchange rate depreciates, those agents with larger η^i would benefit more, since their income levels would increase more than those who have chosen to index a lower proportion of their money holdings to the foreign currency. The opposite would happen in the case of an appreciation, where agents with relatively low values for η^i would face lower reductions in their income levels. Consequently, financial dollarisation become a crucial determinant of both the expected level and the volatility of consumption.

Given the distribution function for the depreciation rate, contingent paths for P_{t+1} and $P_{t+1}(z)$, and a mass υ of firms that set prices in foreign currency, agents choose η^i to maximise their expected discounted utility, equation (5.1), subject to the budget constraint, equation (5.12). The optimal η^i that solves this problem is determined by the following condition,

$$\eta^i = \arg\max \left[\beta E_t U \left(\frac{M^i_t}{P_{t+1}} \left(1 + \eta^i \frac{e_{t+1} - e_t}{e_t}\right) - \gamma^i_{t+1}\right)\right] \tag{5.13}$$

In section 5.2.1 we fully characterise this decision and discuss its main implications.

5.1.2 Firms

Consumption goods are produced by monopolistically competitive firms using labour and a constant return to scale production function. Each firm uses a composite of all types of labour available in the economy. Let's define by $h_t(z)$ the amount of aggregate labour utilised by firm, z, and by θ^i the marginal productivity of labour offered by agent i, then, the production function is defined as follows,

$$Y_t(z) = \left(\int_0^1 \theta^i h_t^i(z) di \right)$$

Under these assumptions, the aggregate nominal wage rate, which for each firm represents their marginal cost, is given by,

$$W_t = \left(\int_0^1 W_t^i di \right)$$

Where, individual wages are determined by the value of agent's marginal productivity of labour,

$$W_t^i = \theta^i P_t \tag{5.14}$$

By aggregating this condition across types of labour we obtain,

$$W_t = \left(\int_0^1 \theta^i P_t di \right) = \overline{\theta} P_t \tag{5.15}$$

Using, the previous equation to eliminate P_t from equation (5.14) we can write the agent's i wages in terms of its productivity and aggregate wages as follows,

$$W_t^i = \theta^i W_t \tag{5.16}$$

Before observing the realisation of the depreciation rate, an important decision that firms take is the currency on which they set prices. They can choose between a domestic and a foreign currency for setting prices one period in advance. Price rigidity is a key building block of the model, since when prices are flexible firms's invoicing decisions do not have any meaningful effect on firms's expected profits and consequently, firms are indifferent between invoicing in domestic or in foreign currency.

Aggregate demand for each type of good is obtained by adding up demand for goods across the distribution of individuals. For a typical firm, its aggregate demand function is defined as follows,

$$Y_t^d(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon} X_t + \gamma_z \tag{5.17}$$

Where the aggregate subsistence level of good z, γ_z is defined as, $\gamma_z = \int_o^1 \gamma_z^i dF\theta^i$. Furthermore, we normalise the subsistence levels of good z, γ_z, as follows $\gamma_z^i = \widetilde{\gamma}_z^i \left(\frac{P(z)}{P}\right)^{-\varepsilon} X$, where, $\widetilde{\gamma}_z^i < 1$ and variables without time subscript denote steady-state variables. This normalisation has the advantage of delivering analytical solutions for the steady-state price level of each type of good.

Notice in equation (5.17) that because preferences are non-homothetic firms face a time varying demand-price-elasticity. As this equation illustrates, the total demand for good z is given by the sum of two types of demands, one which has a constant price elasticity, ε, and another which is completely inelastic. Consequently, with non-homothetic preferences the demand-price-elasticity of good z is a weighted average of 0 and ε where the weights that each of these values receives depend, amongst other factors, on the level of aggregate demand.

When X_t is high, the weight that ε has on the determination of the demand-price-elasticity is also high, increasing the demand-price-elasticity for good z. The opposite happens when, X_t is low. To the extend that X_t is positively correlated with the exchange rate so does the demand-price elasticity.

In particular, in states of the nature where the depreciation of the exchange rate increases(decreases) aggregate demand, the demand-price-elasticity also increases (decreases). Importantly, the effect of exchange rate on the demand-price-elasticity is stronger when γ_z is larger. This novel link that non-homothetic preferences generate between demand-price-elasticity and the exchange rate is a crucial determinant of the firm's invoicing decision, as we show in the next section.

Price Setting

One period in advance firms take two decisions, a) they choose the currency in which set prices and b) they pick up the corresponding optimal price level. At the end of period t, each firm chooses the price level, $P_t(z)$, that maximises their expected profits. When setting prices in domestic currency, the firm's expected profits is given by,

$$E_t\Pi_{t+1}(z) = E_t\left[(P_t(z) - W_{t+1})\left(\left(\frac{P_t(z)}{P_{t+1}}\right)^{-\varepsilon} X_{t+1} + \gamma_z\right)\right] \tag{5.18}$$

and its corresponding first order condition by,

$$\begin{aligned} 0 &= [1-\varepsilon] P_t(z)^{-\varepsilon} E_t\left(P_{t+1}^{\varepsilon} X_{t+1}\right) + \\ &\quad \varepsilon P_t(z)^{-1-\varepsilon} E_t\left(W_{t+1} P_{t+1}^{\varepsilon} X_{t+1}\right) + \gamma_z \end{aligned} \tag{5.19}$$

Whereas, when the firm is setting prices in foreign currency its expected profit function is given by,

$$E_t\Pi_{t+1}^*(z) = E_t\left[(P_t^*(z)e_{t+1} - W_{t+1})\left(\left(\frac{P_t(z)^* e_{t+1}}{P_{t+1}}\right)^{-\varepsilon} X_{t+1} + \gamma_z\right)\right] \tag{5.20}$$

where $P_t^*(z)$ and $\Pi_{t+1}^*(z)$ represent the optimal price in foreign currency and the corresponding expected profit function, respectively. In turn, the first order condition that determines the optimal price in foreign currency is characterised by the following equation,

$$\begin{aligned} 0 &= [1-\varepsilon] P_t^*(z)^{-\varepsilon} E_t\left(P_{t+1}^{\varepsilon} X_{t+1} e_{t+1}^{1-\varepsilon}\right) \\ &\quad + \varepsilon P_t^*(z)^{-1-\varepsilon} E_t\left(e_{t+1}^{-\varepsilon} W_{t+1} P_{t+1}^{\varepsilon} X_{t+1}\right) + \gamma_z E_t e_{t+1} \end{aligned} \tag{5.21}$$

conditions (5.19) and (5.21) determine, for firm z, its optimal prices in pesos and in dollars, respectively. The corresponding second order sufficient conditions are satisfied provided γ_z is not too large. For the case of domestic currency, γ_z has to satisfy the following upper bound to have a well defined maximisation problem.

$$\gamma_z < E_t\left(\frac{W_{t+1}}{P_t(z)}\left(\frac{P_t(z)}{P_{t+1}}\right)^{-\varepsilon} X_{t+1}\right) \tag{5.22}$$

Whereas, for the case of foreign currency price, the corresponding upper bound is given by,

$$\gamma_z < E_t\left(\frac{W_{t+1}}{P_t^*(z)e_{t+1}}\left(\frac{P_t^*(z)e_{t+1}}{P_{t+1}}\right)^{-\varepsilon} X_{t+1}\right) \tag{5.23}$$

Firms Invoicing Decisions

Firms choose the currency for invoicing by comparing the expected value of their profits when using pesos and dollars in setting their prices. Notice that although firms face the same cost structure, they will not choose the same pricing strategy, since they face different demand functions. In particular, firm z will choose to set prices in dollars, if and only if,

$$E_t\Pi^*_{t+1}(z) > E_t\Pi_{t+1}(z) \tag{5.24}$$

Where, $E_t\Pi_{t+1}(z)$ and $E_t\Pi^*_{t+1}(z)$ are defined in equations (5.18) and (5.20), respectively. Firms maximise their profits taking as given, wages, the portfolio decisions of agents and γ_z. Using condition (5.24) we can define the dollarisation decision of firm z and the set Υ that contains all firms which set prices in foreign currency, as follows:

$$dd_t(z) = \begin{cases} 1 \text{ if } E_t\Pi^*_{t+1}(z) > E_t\Pi_{t+1}(z) \\ \quad 0 \text{ otherwise} \end{cases} \tag{5.25}$$

$$\Upsilon = \{z \mid dd_t(z, \eta) = 1\}$$

Consequently, degree of price dollarisation is given by the dimension of Υ,

$$\upsilon = \dim(\Upsilon) \tag{5.27}$$

5.1.3 Market Clearing Conditions and Equilibrium

The equilibrium in the labour market is obtained by aggregating the condition (5.16), which implies that,

$$\frac{W_t}{P_t} = \int_{\underline{\theta}}^{\overline{\theta}} \theta dF(\theta) = 1 \tag{5.28}$$

By further defining the aggregate demand level as:

$$X_t = \int_0^1 X_t^i(z)di \tag{5.29}$$

and by aggregating the condition (5.17) we obtain the following aggregate resources constraint

$$C_t = Y_t = X_t + \gamma \tag{5.30}$$

where, $Y_t = \int_0^1 \frac{P_t(z)Y_t(z)}{P_t}dz$, and $\gamma = \int_0^1 \frac{P_t(z)\gamma_z}{P_t}dz$. On the other hand, the equilibrium degree of financial and price dollarisation is determined by the solving the following fixed point problem,

$$v = \dim\left(\Upsilon(\eta(v)\right) \tag{5.31}$$

Given the income distribution, $F(\theta)$, and the distribution of the depreciation of the exchange rate, a rational expectations Nash equilibrium is defined as a set of allocations $\{C_t^i\}$,$\{C_t\}$,$\{X_t^i\}$, $\{n_t^i\}$, $\{Y_t(z)\}$, $\{dd_t(z)\}$, Υ, v and η; and prices, $\{P_t(z)\}$, $\{P_t^*(z)\}$,W_t and $\{P_t\}$ that satisfy conditions,(5.9), (5.30),(5.31), (5.12), (5.13), (5.17),(5.27), (5.31), (5.25),(5.19), (5.21), (5.28), and (5.7).

Since the model is highly non-linear, analytical solutions are not easy to obtain. For this reason, we use a log-linear approximation of the model around a deterministic steady-state to obtain the equilibrium dynamics and a second-order approximation of the economy around the steady-state to analyse agents' portfolio and firms' invoicing decisions[7]. The log-linear dynamic equilibrium of the economy is provided in appendix C.3, and it is used to calculate the second moments of the endogenous variables, which, in turn, allow us to determine the equilibrium portfolio and invoicing decisions.

[7] In a recent paper Devereux and Sutherland (2006) propose an algorithm to solve open economy macroeconomic models with endogenous portfolio composition, they show that it is enough to fully characterise the portfolio composition to approximate the up to first-order the equations that determine the dynamics of the economy and up to second-order, those that determine the agent's portfolio choice

5.2 Dollarisation Decisions: Partial Equilibrium

5.2.1 Portfolio Decisions

From equation (5.13), the first order condition that characterises the agent's portfolio decision is given by,

$$E_t \left[U_x \left(X_{t+1} \right) \frac{\xi_{t+1}}{P_{t+1}} \right] = 0 \tag{5.32}$$

Following Devereux and Sutherland (2006), we take a second-order approximation of equation (5.32), and a first-order approximation of the agent's budget constraint, equation(5.12), to obtain the agent's optimal portfolio choice. In both cases we approximate these equations around the non-stochastic steady-state.

A critical issue, though, when approximating these equations is how to determine the appropriate steady-state value for the portfolio allocation, since this variable is undetermined at the non-stochastic steady-state. However, as Judd and Guu (2001) show, it is still possible to find an approximation point by locating a bifurcation point in the set of non-stochastic equilibria. Devereux and Sutherland (2006) show that the steady-state gross portfolio derived using their method corresponds to the approximation point derived by Judd and Guu (2001) method. In appendix C.2 we proof that this is indeed the case in our model.

From now on, we use hats to denote deviations of variables with respect to the steady-state values, i.e, $\widehat{x}^i_{t+1} = \frac{X^i_{t+1} - X^i}{X^i}$. The second-order Taylor expansion of the agent's first order optimal portfolio condition evaluated at the steady-state, $X = \overline{X}$, $\Pi = \overline{\Pi}$ and $\xi = 0$, is given by,

$$-\sigma E_t \left(\widehat{x}_{t+1} \xi_{t+1} \right) - E_t \left(\xi_{t+1} \widehat{p}_{t+1} \right) = 0 \tag{5.33}$$

where, $\sigma = \frac{U_{XX} X}{U_X}$. In turn, the corresponding linear approximation of budget constraint is given by,

$$\left(1-\frac{\gamma^i}{C^i}\right)\widehat{x}_t^i = \eta^i\xi_{t+1} - \widehat{p}_{t+1} - \int_0^1 \frac{P(z)\gamma_z^i}{PC^i}\left(\widehat{p}_{t+1}(z) - \widehat{p}_{t+1}\right) d(z) + O\left(\|\xi_t, \sigma_\xi\|^3\right) \tag{5.34}$$

The optimal degree of financial dollarisation of agent i is obtained by solving for the value of η^i that satisfies equation (5.33) after obtaining $E_t\left(x_{t+1}\xi_{t+1}\right)$ and $E_t\left(\xi_{t+1}\pi_{t+1}\right)$ using equation (5.34). The following proposition establishes the conditions that determine the optimal degree of financial dollarisation, η^i.

Proposition 5.2.1. *The optimal degree of financial dollarisation of agent i, η^i, is given by the following condition,*

$$\begin{aligned}\eta^i &= \left(\sigma - \left(1-\frac{\gamma^i}{C^i}\right)\right)\frac{E_t\widehat{p}_{t+1}\xi_{t+1}}{\sigma E_t\xi_{t+1}^2} \\ &\quad + \frac{1}{E_t\xi_{t+1}^2}\left(\int_0^1 \frac{P(z)\gamma_z^i}{PC^i}E_t\left(\left(\widehat{p}_{t+1}(z) - \widehat{p}_{t+1}\right)\xi_{t+1}\right) d(z)\right)\end{aligned}$$

See appendix C.2

Several interesting results follows from the previous proposition: first, when preferences are homothetic, $\gamma_z^i = 0$, all agents have the same degree of financial dollarisation. Consequently, relatively poor and rich agents have exactly the same proportion of their savings indexed to a foreign currency. In this case, the degree of dollarisation of agent's portfolio depends on the degree of PD and on the variance of the nominal exchange rate, as the next equation shows ,

$$\eta^i = (\sigma - 1)\frac{E_t\widehat{p}_{t+1}\xi_{t+1}}{\sigma E_t\xi_{t+1}^2}$$

The previous condition shows that agents choose higher degrees of financial dollarisation when the correlation between the consumer price index and the exchange rate is also higher. By indexing their savings to a foreign currency, agents protect the purchasing power of their money holdings against unexpected changes in domestic inflation[8].

When preferences are non-homothetic, $\gamma^i > 0$, however, additional implications can be obtained from proposition 5.2.1. In this case, agents optimally choose different degrees of financial dollarisation. In particular, for agents with relatively higher income-levels, lower $\frac{\gamma^i}{C^i}$, their degrees of financial dollarisation depend on a lesser extent on the degree of pass-through, measured by the correlation between the exchange rate and the aggregate price level. The opposite happens with low-income agents. In the limit, a very poor agent, whose consumption is very close to her subsistence level, $\frac{\gamma^i}{C^i} \to 1$, the sensitivity of her portfolio choice to the degree of financial dollarisation would reach its maximum value of 1. This latter result follows directly from the fact when preferences are non-homothetic, the level of consumption that generates utility, X^i_{t+1} is more sensitive to the depreciation rate for relatively low-income agents. Therefore, for those agents, the volatility of the exchange rate has a greater negative effect on their expected utility.

5.2.2 Invoicing Decisions

To choose which currency to use as unit of account, firms compare the expected value of their profits under the two alternative pricing strategies, pricing in pesos and pricing in dollars. In particular, firm z will choose to set prices in dollars, if and only if,

$$E_t\Pi^*_{t+1}(z) > E_t\Pi_{t+1}(z) \tag{5.35}$$

In order to gain tractability and ease the interpretation of the determinants of the invoicing firm's decision, as in the case of portfolio decisions, we take a second-order Taylor approximation of both, $\Pi_{t+1}(z)$ and $\Pi^*_{t+1}(z)$ around a deterministic steady-state and then, we use these approximated expected profit functions to evaluate

[8] Ize and Parrado (2002) find a similar result when analysing the dollarisation decision for wage setters. Different from them, in this chapter, this result is a particular case of a more general result where dollarisation decisions differ across agents depending on their income levels

condition (5.35). Also for clarity, in what follows we assume that $E_t\xi = 0$ The following proposition summarises the main result.

Proposition 5.2.2. *A particular firm will choose to set prices in a foreign currency if and only if the following conditions holds,*

$$0 < -\widetilde{\gamma}_z \left[Cov\left(\xi_{t+1}x_{t+1}\right) - \frac{\varepsilon}{2}\left(Var\xi_{t+1}\right)\right]$$
$$-\widetilde{\gamma}_z \left[\varepsilon Cov\left(\xi_{t+1}p_{t+1}\right) + Cov\xi_{t+1}w_{t+1}\right]$$
$$+\left(\varepsilon - 1\right)\left(Cov\xi_{t+1}w_{t+1} - \frac{1}{2}\left(Var\xi_{t+1}\right)\right)$$

Proof. See appendix C.2 □

The condition established in proposition 5.2.2 has two components: the first one, depends crucially on $\widetilde{\gamma}_z$, and it is associated to the time varying price elasticity that non-homothetic preferences generates. The second one, is independent of $\widetilde{\gamma}_z$, and it is determined by the cost structure of the firm. In the particular case of homothetic preferences, i.e $\widetilde{\gamma}_z = 0$,, the demand-price-elasticity is equal to ε, and consequently the firm's invoicing decision is only determined by the cost structure of the firm. In this case, a particular firm would set prices in dollars if the following condition holds

$$E_t\xi_{t+1}w_{t+1} - \frac{1}{2}\left[Var\xi_{t+1}\right] > 0 \tag{5.36}$$

Interestingly, this condition implies that setting prices in dollars would be a dominant strategy for firms when marginal costs are highly correlated with the exchange rate [9]. By setting prices in dollars, firms with this cost structure generate a positive correlation between its revenues and its costs, which stabilise their profits and increase expected profits.

Setting prices in dollars, however, also increases the volatility of firm's revenue, which reduces expected profits. When the former effect is larger than

[9] This condition is similar to condition (3.70) in chapter 3, and to those found by Devereux et al. (2004) and Loyo (2001) but using different macroeconomic frameworks.

the latter one, firms find optimal to invoice in a foreign currency. This is exactly what condition (5.36) establish. Consequently, in this particular case, the model predicts that invoicing in a foreign currency would emerge only in sectors where firm's marginal costs are highly correlated with the exchange rate, as it would be the case of the importing sector.

Let's consider now the case of non-homothetic preferences, $\widetilde{\gamma}_z \neq 0$. In this case, the demand-price-elasticity is not constant, but it depends on aggregate demand, the aggregate price level and the firm's pricing strategy. In particular, this price elasticity covaries positively with the aggregate demand and the aggregate price level.

As we have shown previously, when marginal costs are highly correlated with the exchange rate, setting prices in a foreign currency makes sense, because it allows firms to stabilise expected profits. With non-homothetic preferences, however, the beneficial effect of invoicing in a foreign currency diminish, since, in this case, the demand-price-elasticity is time varying and correlated with the exchange rate. Through this link, when both $Cov\xi_{t+1}p_{t+1}$ and $Cov\left(\xi_{t+1}x_{t+1}\right)$ are positive (negative), setting prices in dollars generate a positive(negative) correlation between the exchange rate and the time varying price elasticity, which in turn increase(reduces) the volatility of profits and, consequently, reduces (increases) expected profits.

This additional effect of the exchange rate on expected profits depends on the value of $\widetilde{\gamma}_z$. Given that $\widetilde{\gamma}_z$ is idiosyncratic, firms choose, different invoicing strategies, even when they have the same cost's structure.

In particular, provided, $Cov\xi_{t+1}w_{t+1} > 0$, $Cov\left(\xi_{t+1}x_{t+1}\right) > 0$, and condition of proposition (5.2.2) holds, only firms with a sufficiently small $\widetilde{\gamma}_z$ are likely to set prices in dollars. On the contrary, firms with a large $\widetilde{\gamma}_z$, large enough to violate the condition established in the previous proposition, find optimal to set prices in domestic currency. This is a remarkable result, since, the latter type of goods can be categorised as of more necessity than those with low values of $\widetilde{\gamma}_z$. Hence, the previous proposition implies, under certain conditions, that only goods which are considered as relatively luxury are invoiced in dollars. In contrast, goods considered as necessity goods, those with larger values for $\widetilde{\gamma}_z$, are invoiced in domestic prices.

This implication of the model fits pretty well the pattern of dollarisation observed in economies like Bolivia and Peru, where, as we discussed previously, firms discriminate prices, by invoicing in dollars those goods consumed by high-income customers and in pesos those goods consumed mainly by low income customers, even when their marginal costs are highly correlated to the exchange rate.

The previous proposition also implies that there exist strategic complementarities amongst firms when choosing their invoicing strategy. In order to visualise this implication, let's rewrite the condition established in the previous proposition as follows,

$$\begin{aligned} 0 &< -\left[(\varepsilon-1)-\widetilde{\gamma}_z\varepsilon\right]\left[\frac{(Var\xi_{t+1})}{2}\right] \\ &\quad -\widetilde{\gamma}_z Cov\left(\xi_{t+1}x_{t+1}\right)+\left[(\varepsilon-1)-\widetilde{\gamma}_z\left(1+\varepsilon\right)\right]Cov\left(\xi_{t+1}p_{t+1}\right) \end{aligned} \tag{5.37}$$

Since the sufficient condition for the optimal price at the steady-state implies that $(\varepsilon-1)-\widetilde{\gamma}_z(1+\varepsilon)>0$, for a given $\widetilde{\gamma}_z$, the previous condition is more likely to hold when the degree of pass-through is higher. This is, when there exist a larger fraction of firms that set prices in dollars. Consequently, larger(smaller) the fraction of firms that set prices in dollars, more (less) likely that at the margin a firm set prices in dollars.

5.3 General Equilibrium

In this section we use conditions (5.27) and (5.25) to solve for the equilibrium level of PD. To fully characterise condition (5.25), it is convenient to rewrite condition (5.37) as follows,

$$\widetilde{\gamma}_z \leq \frac{(\varepsilon-1)\left(Cov\xi_{t+1}w_{t+1} - \frac{(Var\xi_{t+1})}{2}\right)}{\varepsilon\left[Cov\left(w_{t+1}\xi_{t+1}\right) - \frac{1}{2}\left(Var\xi_{t+1}\right)\right] + Cov\left(\xi_{t+1}x_{t+1}\right) + Cov\left(\xi_{t+1}w_{t+1}\right)} \tag{5.38}$$

Interestingly, the degree of FD, η, affects the firms' invoicing decision through its effect on $Cov\left(\xi_{t+1}x_{t+1}\right)$. As it is show in appendix C.3, the degree of FD and the $Cov\left(\xi_{t+1}x_{t+1}\right)$ are linked through the following condition,

$$Cov\left(\xi_{t+1}x_{t+1}\right) = (\eta - \lambda_p v) - \frac{cov(\xi_{t+1}, \varphi_{t+1})}{Var(\xi_{t+1})}$$

where, $\lambda_p = \int_0^1 \frac{1}{1-\frac{\gamma^i}{C^i}} di$ and φ_t is a variable that denotes relative price dispersion, defined in the appendix previously mentioned.The degree of FD, η however, is linked also to PD. By aggregating the individual's portfolio decisions, established by proposition 5.2.1, we obtain the following conditions for η,

$$\eta = \lambda_p v - \frac{v}{\sigma} + \frac{cov(\xi_{t+1}, \varphi_t)}{Var(\xi_{t+1})} \tag{5.39}$$

Clearly, FD is increasing on the degree of PD. Consequently, PD and FD are simultaneously determined. To define this equilibrium, we express equation (5.38) in terms of deep parameters by eliminating the second moments that characterise condition (5.38). We achieve this transformation by using the dynamics equilibrium of the economy, detailed in appendix C.3. After using this equilibrium to write $Cov\left(w_{t+1}\xi_{t+1}\right)$, $Var(\xi_{t+1})$ and the $Cov\left(\xi_{t+1}w_{t+1}\right)$ as function of structural parameters, the firm's condition for invoicing in dollars would be given by as follows,

$$0 < \widetilde{\gamma}^*(z, v) \tag{5.40}$$

where

$$\widetilde{\gamma}^* = (\varepsilon - 1)\left(v - \frac{1}{2}\right) - \widetilde{\gamma}_z\left(\varepsilon\left[v - \frac{1}{2}\right] + v\frac{(\sigma-1)}{\sigma}\right) \tag{5.41}$$

Using equation (5.39)the optimal degree of financial and price dollarisation is obtained as fixed point over the following set,

$$v = \dim(\Upsilon) = \dim\left[\{z \mid \quad 0 \leq \widetilde{\gamma}^*(z, v)\}\right] \tag{5.42}$$

Solving this fixed point is not a trivial task. Nevertheless, it is possible to provide some qualitative implications of the model by analysing the determinants of $\widetilde{\gamma}^*$.

5.3.1 Benchmark case: Homothetic Preferences

In this case, the condition that characterizes firms' optimal invoicing choice, the expected firm's profit of invoicing in dollars relative to pesos, is given by,

$$E_t\Pi_t^*(z) - E_t\Pi_t(z) = \left(v - \frac{1}{2}\right) > 0$$

Notice that since the invoicing decision of a particular firm depends on what other firms do, in particular on the fraction of firms that choose to invoice in a foreign currency, v, there exist multiple equilibriums. According to this condition, a marginal firm would find optimal to invoice in dollars only when $v > 0.5$. Otherwise it would choice to invoice in pesos. Therefore, there exist three possible equilibriums in this case, $v = 0$, $v = 0.5$ and $v = 1$. These are depicted in figure 5.1 as points A, B and C. Two of these equilibriums are stable, points A and C; and one, B, is unstable:

1. Point A, $v = 0$ is an equilibrium point since at this point, given that all firms have chosen to invoice in pesos, an individual marginal firm's expected profits under peso invoicing are larger than under dollar invoicing, thus, firms do not have incentives to deviate from the equilibrium strategy.

2. Point C, $v = 1$ is also an stable equilibrium since, in this case, given that all firms have chosen to invoice in dollars, any individual firm's expected profits

are large under dollar invoicing than under peso invoicing, therefore, it has no incentive to deviate from the equilibrium strategy.

3. Point B, $v = \frac{1}{2}$is also an equilibrium. In this case, firms would be indifferent between choosing invoicing in pesos and in dollars, because its expected profits are exactly the same under these two strategies. Firms can coordinate in this equilibrium by playing mixed strategies by which each firm with probability 0.5 chose to invoice in pesos and with probability 0.5 choose to invoice in dollars. This equilibrium point, however, is not a stable one, since if a small mass of firms choose to deviate slightly from its equilibrium strategy, all firms find optimal to deviate and therefore the equilibrium moves to A or C in figure 5.1

5.3.2 Equilibrium with Non-homothetic Preferences

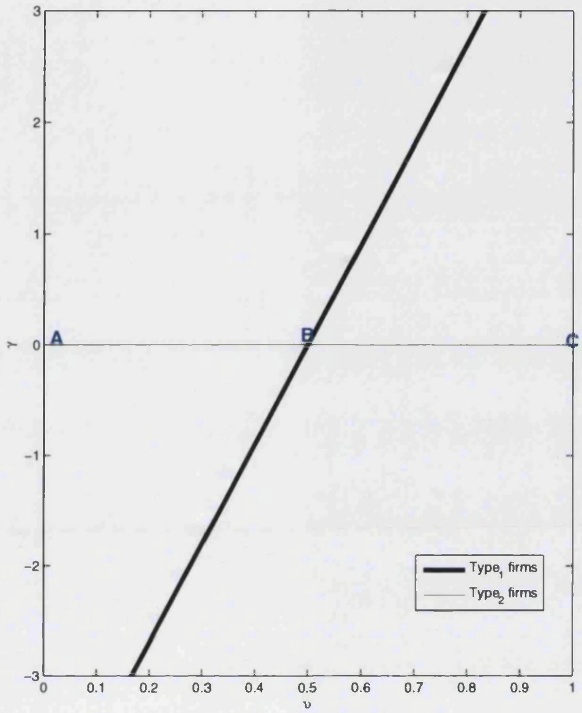
When preferences are non-homothetic the determination of the equilibrium PD is more complex. In particular, the equilibrium depend on proportion of luxury and necessity goods in the economy . To illustrate this point, we focus on a simple 2-good economy case where a mass of n_1 firms produces good 1 and a mass of $1 - n_1$ firms produces good 2. We further assume that good 1 is a necessity and that good 2 is a luxury good. In this case, the equilibrium is determined by two conditions, the optimal invoicing choice of type-2 firms, given by,

$$E_t\Pi_t^*(z) - E_t\Pi_t(z) = \left(v - \frac{1}{2}\right) > 0$$

And the optimal invoice choice of type-1, determined by the following condition

$$E_t\Pi_t^*(z) - E_t\Pi_t(z) = (\epsilon - 1)\left(v - \frac{1}{2}\right) - \gamma\left(\epsilon\left[v - \frac{1}{2}\right] + v\frac{(\sigma - 1)}{\sigma}\right) > 0$$

Figure 5.1: Equilibrium PD Homothetic Preferences



Note: The vertical axis shows the relative expected gains of invoicing in dollars and the horizontal axis the degree of PD

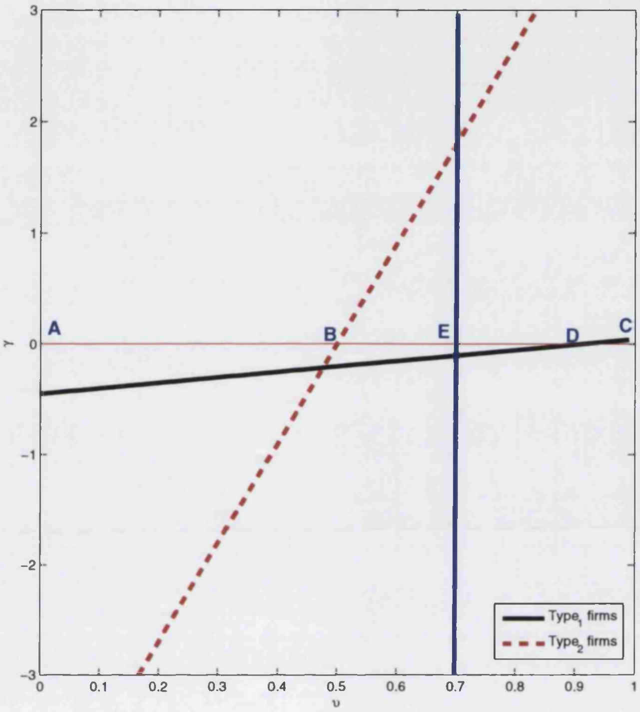
Notice that this latter condition differs form the former one only when $\gamma \neq 0$ and $\sigma > 1$. Otherwise, the equilibrium with non-homothetic preferences would be the same as the one with homothetic preferences. In what follows we focus the analysis on the case $\gamma \neq 0$ and $\sigma > 1$. A first interesting implication of non-homothetic preferences is that as the value of γ increases, type-1 firms would find optimal to invoice in pesos for a larger set of values of υ. To see this point notice that the value of υ that makes type-1 firms indifferent between invoicing in pesos and dollars is increasing on γ. As figure 5.2 shows, for type-1 firms it is optimal to invoice in pesos when $\upsilon \in \langle \upsilon_A, \upsilon_D \rangle$ whereas for type-2 firms this would be the case if $\upsilon \in \langle \upsilon_A, \upsilon_B \rangle$. Effectively then, non-homothetic preferences, by creating a time-varying price elasticity, reduce type-1 firms' incentives to set prices in dollars. The set of equilibriums is depicted in figure 5.2. As this figure shows, besides A, and C, a new stable equilibrium arise in this case, point E, where, $\upsilon = 1 - n_1$. At this equilibrium all type-1 firms choose optimally to invoice in pesos and all type-2 firms choose to invoice in dollars. This equilibrium holds when $n_1 \geq \frac{1}{2}$. Instead, when $n_1 < \frac{1}{2}$, point B, is not anymore equilibrium. In this case, type-1 firms find optimal to invoice in domestic currency regardless what type-2 firms do. Therefore, given that type-1 firms chose to invoice in domestic currency, type-2 firms would also find optimal to do so and the equilibrium would be point A. Similarly, point D would not be an equilibrium either. In this case, the equilibrium would shift to point C.

An interesting implication of the previous results is that in poor economies where necessity goods, type-1 goods, represent a large fraction of consumer's expenditures, $n_1 \to 1$, the degree of PD, $\upsilon = 1 - n_1$ would tend to be small. This key implication of the model is in line with the observed pattern of PD in economies like Bolivia and Peru, low-income countries where the levels of PD are small besides the high levels of FD and CS. Next, we show how non-homothetic preferences have further implications for the degree of pass-through.

5.4 Implications for the Exchange Rate Pass-Through

A key implication of non-homothetic preferences in our model is that the structure of the consumption basket changes as agents' income changes. In particular, the participation of necessity goods on the consumption basket fall as income rises. This implication is consistent with cross-country data on consumption basket composition, which shows that in developing economies the participation of food on

Figure 5.2: Equilibrium PD Non-Homothetic Preferences



Note: The vertical axis shows the relative expected gains of invoicing in dollars and the horizontal axis the degree of PD

the average consumption basket is larger than in developed economies. To illustrate how the average income level affect the degree of pass-through, let's define the following CPI index,

$$P_{CPI,t} = \int_0^1 \alpha(z) P_t(z) dz$$

where, $\alpha(z) = \frac{P(z)C(z)}{PC}$, represents the steady-state participation of good z in the consumption basket of the representative agent. Using this definition of price index the degree of pass-through is defined by $\alpha = \int_0^v \alpha(z) dz$.

Let's further consider that there exist a continuum of economies, which are indexed by their average income level, $\{\overline{\theta}_j\}_{j=1}^{j=N}$. Using the steady-state calculated in appendix C.1, it is easy to calculate the weighs $\alpha(1)$ and $\alpha(2)$ for the two-good economy described in the previous section. The steady-state equilibrium price levels of economy j are given by,

$$P_{j,1} = \frac{\varepsilon}{\varepsilon-1-\widetilde{\gamma}_1} \quad P_{j,2} = \frac{\varepsilon}{\varepsilon-1}$$

and their corresponding consumption levels, by

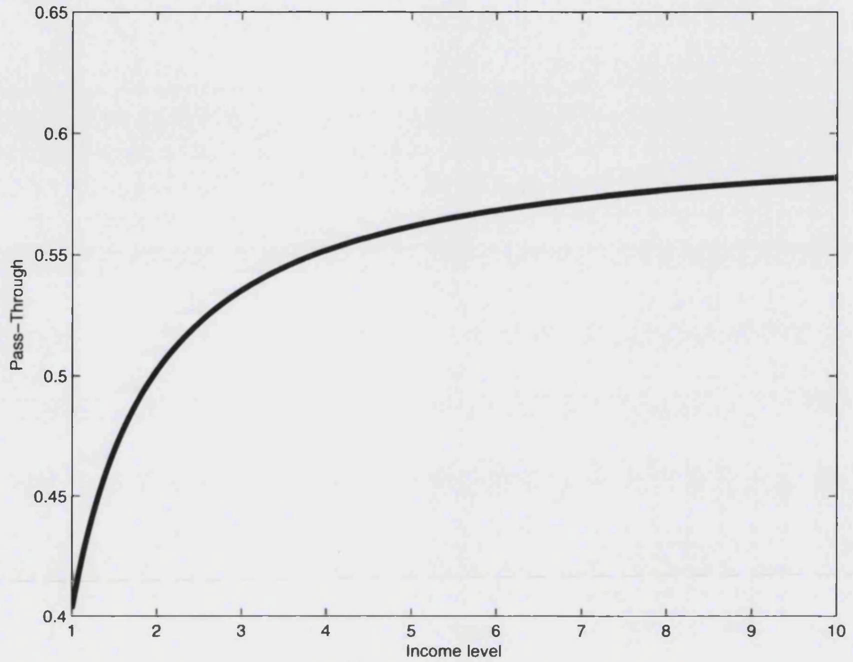
$$C_{j,1} = \left(\frac{P_{j,1}}{P_j}\right)^{-\varepsilon}\left(\overline{\theta}_j - \widetilde{\gamma}_1\frac{P_{j,1}}{P_j}\right) + \widetilde{\gamma}_1\frac{P_{j,1}}{P_j} \quad C_{j,2} = \frac{\varepsilon}{\varepsilon-1}$$

To ease the exposition of the link between income levels and the degree of pass-through, we calculate the degree of pass-through, $\alpha(2)$ for a set of calibrated economies. Since the analytical solution for this parameter value is highly non-linear, to gain in clarity we calibrate its solution using the following values $\varepsilon = 10$, $\widetilde{\gamma}_1 = 0.3$, and $\overline{\theta}_j = [1 : 10]$.

As figure 5.3 illustrates the degree of pass-through is decreasing on the country's income level. For low-income countries, the degree of pass-through is below the degree of PD, whereas the oppositive is true for high-income countries.

This novel result fits very well the pattern of dollarisation in countries like Peru and Bolivia. These are low-income economies, where the degree of PD is relatively large, but the degree of pass-through is much lower. This implication also explains why the degree of pass-through across regions in countries like Peru, as section 2.3 shows, is negatively correlated to their corresponding average-income levels.

Figure 5.3: Pass-Through and Average Income



Note: The vertical axis shows the degree of pass-through and the horizontal axis the country's income level

5.5 Concluding Remarks

This chapter develops a simple overlapping generations model that allows a comprehensive analysis of both agent's portfolio decisions and firm's invoicing problem. The model shows that PD induces FD. A high degree of PD induces agents to save in foreign-exchange indexed assets to avoid the effect of unexpected movements in the exchange rate in their purchasing power. The portfolio decision of agents, though, depends on their income levels. Low-income agent's FD decisions are more sensitive to the degree of PD, whereas those of high-income agents to the expected exchange rate depreciation and its volatility.

Also, the model provides a rationale of why prices of necessity goods tend to be set in domestic currency, even when firms face marginal costs in foreign currency. At the aggregate level, this result implies that the degree of exchange rate pass-through into prices has an upper bound which depends on the average income level.

Two features of the economy turn out to be critical in limiting the scope of a foreign currency in performing the function of unit of account, sticky prices and income distribution. On one hand, the combination of sticky prices and uncertainty about the exchange rate turns the invoicing decision of firms meaningful. Hence, setting prices in a foreign currency may become, under certain conditions, a dominant strategy for firms. On the other hand, differences in customer's income and non-homothetic preferences generate a time varying price elasticity of demand, which reduces the benefits of setting prices in a foreign currency.

Although the model is highly stylised, we believe it captures reasonable well the main stylised facts of PD. However, we would like to explore some extensions to the model, in particular, we would like to provide a numerical solution of the general equilibrium implied by the model. Also, the use of alternative type of non-homothetic preferences are considered in our future research agenda

CHAPTER 6

INDIVIDUAL HETEROGENEITY AND DOLLARISATION PERSISTENCE

In this chapter dollarisation means *deposit dollarisation*[1] which leads eventually to credit dollarisation and to the vulnerability of the financial system of highly dollarised countries. As stressed by Cook (2004) and Céspedes et al. (2004), the efficacy of monetary policy in small open economies with flexible exchange rates is compromised by the negative balance sheet effects generated by dollarisation. In this case, sudden real depreciations can have detrimental consequences on the economic activity by reducing the net worth of firms and generating adverse effects on investment. This situation gives a rationale for a "fear of floating" behavior of central banks, Calvo and Reinhart (2002); Morón and Winkelried (2005).

One of the most salient features of dollarisation, and probably the one that causes more concern to policymakers, is its *persistence*. It is well documented that dollarisation increases sharply during episodes of unduly macroeconomic instability and that it remains stubbornly high even after successful stabilizations.[2] A *prima facie* explanation of the hysteresis is lack of confidence in domestic currency assets as a result of the traumas brought by past inflation, devaluations, banking crises, and so on. This, however, is not very consistent with the strong macroeconomic fundamentals observed in several highly dollarised countries, notably Peru and some

[1] This is also known as asset substitution Reinhart et al. (2003) or financial dollarisation Ize and Levy-Yeyati (2003).

[2] See Guidotti and Rodríguez (1992), Savastano (1996), Quispe (2000) and Kamin and Ericsson (2003).

transition economies in the early 2000s.

An alternative avenue to address this puzzle is to adapt the existing currency substitution literature based on adjustment costs or network externalities. Guidotti and Rodríguez (1992), Sturzenegger (1997) and Uribe (1997) develop models where the cost of using the dollar for transactions depends negatively on the aggregate currency substitution ratio, so once transactions get dollarised, there is no benefit to switch back to using domestic currency if others continue using dollars. An obvious limitation is that this approach refers to the medium-of-exchange and not to the store-of-value function of money. Moreover, these models rely heavily on a knowledge stock that drives the persistence (a "ratchet variable"), so even though they can neatly explain upward trends in the depth of dollarisation, they are not useful in explaining how to de-dollarise, as this may imply an implausible reduction in the knowledge stock.

Ize and Levy-Yeyati (2003) provide a different framework for modelling dollarisation. They derive a minimum variance portfolio (MVP) that depends on the relative volatility of inflation and real depreciation rates. Dollarisation would persist even when inflation is low and stable insofar as the real depreciation volatility is smaller than that of inflation. However, this framework is static whereas persistence is inherently a dynamic phenomenon. In our view, the MVP approach, which is now very popular and has proven successful in explaining cross-sectional variation of dollarisation levels,[3] was not designed to deal with dynamics, since the MVP, the underlying equilibrium level of dollarisation, depends on unconditional moments.[4]

Curiously, the fact that researchers have apparently overlooked is the very

[3] Ize and Levy-Yeyati (2003) provide empirical evidence that the MVP has some explanatory power for the average level of dollarisation across countries. De Nicoló et al. (2005) extends this empirical analysis by considering a broader set of countries.

[4] Dollarisation hysteresis is observed in several countries with high real exchange rate volatility, e.g. Russia. The reason of this apparent contradiction with the portfolio approach may be that it is very difficult to get a sound estimate of the unconditional variances that compose the MVP.

nature of the participants of the dollar deposit market in dollarised economies: depositors are extremely heterogenous, ranging from large entrepreneurs to small firms to non-profit organizations and to individuals (rich and not-so-wealthy).[5] Participation costs in the dollar market are virtually nil due to liberalization, deregulation and, importantly, due to the emergence of informal currency traders – known as *cambistas* in many Latin American countries – who benefit from buying and selling dollars with tighter markups than those in the banking sector.[6] A typical *cambista* would hold a limited amount of money for business (say, between US$2,000 and US$5,000) as she is aiming to meet the dollar demand of individuals or small firms, normally unwilling to pay the higher bank premium to get their savings dollarised.[7] As a result, participation becomes independent of the scale of the transaction and hence widespread.

The aim of this chapter is to draw attention to the fact that heterogeneity of depositors can easily explain the persistence of financial dollarisation. As pointed out by Granger (1980), differences in individual dynamics lead to aggregate persistence. Thus, as it is reasonable to expect that the dynamics of the optimal currency portfolio of a financial expert differs from that of a blacksmith, a persistent aggregate dollarisation ratio arises naturally. There are of course various differences between a financial expert and a blacksmith, but provided that both access the dollar deposit market almost for free, the relevant difference to our analysis centers in their ability to process information and, therefore, to take informed portfolio decisions.[8]

The rest of the chapter is organised as follows. In section 6.1 we briefly

[5] An exception is Sturzenegger (1997) who studies the implications of income inequality on currency substitution, yet with no reference to deposit dollarisation.
[6] Agénor and Haque (1996) provide an account of informal currency markets.
[7] Even large firms may find it profitable to trade with a pool of (well-organised) *cambistas.*
[8] Surely, income differences can also be important if the income gap between the financial expert and the blacksmith is wide. However, we find that in dollarised economies the dollar deposit participation of (many) firms and (a lot of) individuals can be taken roughly as having the same importance.

explore these issues using Peruvian and Polish data.[9] For reasons explained below, these cases illustrate nicely our claim about the interplay between individual heterogeneity and aggregate persistence. Besides, it gives us an idea of how the dollar deposit markets in representative countries are shared amongst various types of depositors.

In section 6.2 we develop a stylised model where agents face noisy information and differ in their ability to forecast when taking portfolio decisions. An important result from this setup is that the dynamics of the individual's optimal portfolio depends on her prediction errors of future dollar returns. It turns out that it is optimal for agents to be cautious when modifying the currency composition of their deposits as there is uncertainty on the quality of the data agents receive. This caution is reflected in portfolios that may adjust in a relatively slow fashion. Finally, we show that upon aggregation of the individual dollarisation decisions it is possible to generate a very persistent economy-wide dollarisation ratio.[10]

In section 6.3 we test the empirical hypotheses of the theoretical model and find supportive evidence from aggregate data of three Latin American countries and Poland. The results suggest that the distributions of "forecasting abilities" behind the aggregate dollarisation ratios are very widely spread. We regard this result as consistent with the idea of financial experts sharing the dollar market with blacksmiths who save in dollars. In section 6.4 we discuss possible extensions to the analysis. Section 6.5 concludes and gives policy recommendations. Derivations and complementary results are shown in the appendix D.

[9] The figures used in section 6.1 come from the Central Bank of Peru and the National Bank of Poland. The facts discussed there are recorded in the annual reports of these institutions.

[10] Our approach is related to other branches of the literature. For instance, Lewbel (1994) uses aggregate information to test heterogeneity on consumption dynamics whereas Michelacci (2004) explains the high degree of persistence of output with the cross-sectional heterogeneity of productive firms.

6.1 Two Illustrative Cases

As documented by Savastano (1996), dollarisation emerges progressively in response to macroeconomic instability, particularly high levels of inflation, showing a well-defined pattern: first agents replace domestic currency as reserve of value, holding usually dollars outside the financial system ("under the mattress"). Then, the dollar is used in some transactions, typically involving real estates and durable goods, and eventually some prices are set in dollars. Most governments later on allow banks to issue deposits in foreign currency to avoid financial disintermediation.[11] The actual experience of various countries shows that within a year an economy can increase its dollarisation ratio enormously, see Figures 6.1(a) and 6.2(a).

On the other side, episodes of *de*dollarisation (i.e., a sustained reduction in the dollarisation ratio) are not very common and thus there is no well-established pattern in the literature. Yet, if ever happened, the dedollarisation process is likely to be slow. The analysis of these events, as opposed to the increase of dollarisation, provide very useful information about the way different depositors decide the currency composition of their savings and on how they respond to news coming from the macroeconomic environment.

6.1.1 Peru in the Early 2000's

Although the Peruvian dollarisation experience shares various of the aforementioned features, it has its own appeal.[12] As shown in Figure 6.1(a), in 1991 (after a four-digit hyperinflation in 1990) the ratio was 60% and has remained fluctuating roughly between 65% and 70% for a decade. Since 2000, it has shown a sustained reduction to about 50% in 2005. Of course, 50% is still a big number, but there are some interesting facts behind this recent drop.

[11] See also Kamin and Ericsson (2003), De Nicoló et al. (2005) and Levy-Yeyati (2006).

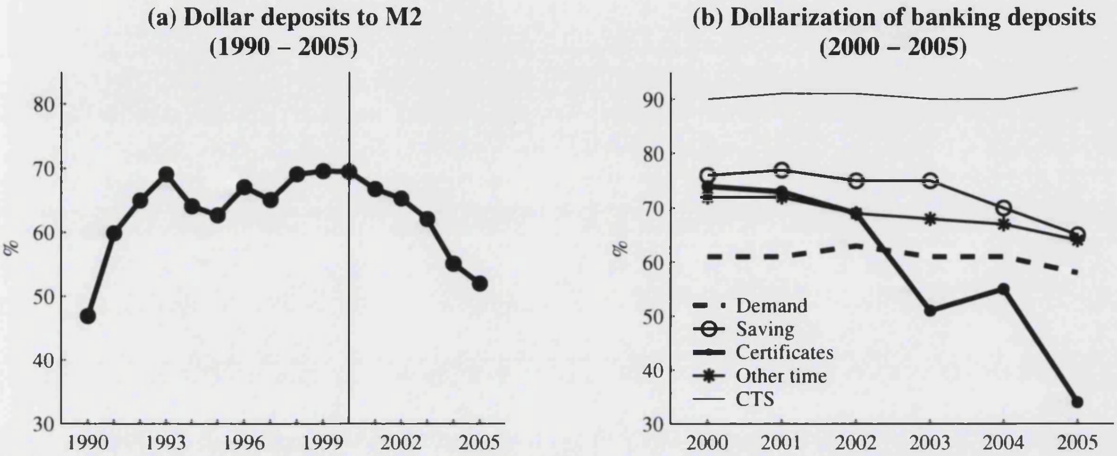
[12] See Quispe (2000) for a careful historical account of the dollarisation experience in Peru.

There are at least two forces driving this decrease. Firstly, after 8 years of announcing inflation targets within a monetary targeting regime (since 1994) and after 5 years of having achieved a one-digit inflation rate, the Central Bank announced the adoption of a fully fledged inflation targeting regime in 2002. This has helped to anchor inflation expectations and has reduced inflation and nominal interest rate volatility. Secondly, between 2001 and 2005, the nominal and real exchange rates have appreciated (6.2% and 5.1%) as a result of a very favorable foreign environment: increasing terms of trade leading to an export boom and very low international interest rates. In a nutshell, the real return to holding deposits dollars vis-à-vis holding deposits in domestic currency has fallen considerably in the early 2000's.

Figure 6.1(b) shows deposit dollarisation by type of deposit: demand, savings and a breakdown of time deposits in certificates, "CTS" and others. A glimpse of the figure reveals that both demand and "CTS" deposits have not reacted to the recent change in the dollar real return trend. Demand deposits accounts for about 20% of total deposits and as the most liquid, almost transactional kind of deposit the flat pattern is justified. On the other side, the CTS is the Peruvian version of an unemployment insurance; by law, it is hold exclusively by individuals and can be claimed only when an individual becomes unemployed. The CTS deposits have reacted even less than the demand deposits, which is puzzling.

The figure also shows a moderate downward trend in the savings and other time deposits. About 80% of the saving and roughly half of the other time deposits are held by individuals. From 2001 to 2005 both ratios have decreased in about 10%. What is remarkable from Figure 6.1(b) is the strong reaction of the certificate of deposits ratio which has fallen in almost 40%, and with no doubts is driving the fall in the aggregate ratio of Figure 6.1(a). The interesting fact is that although the certificate of deposits have similar term than the CTS and the other time deposits, they are mainly held by firms and not individuals.

Figure 6.1: Deposit Dollarisation in Peru



Source: Central Bank of Peru.

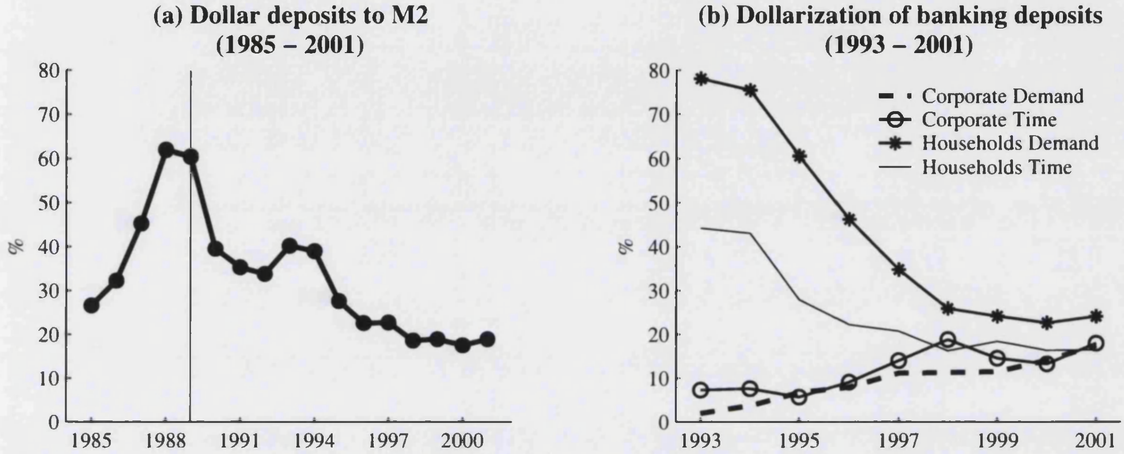
6.1.2 Poland Towards a Market Economy

The Polish experience is regarded as the most successful shift from a planned to a market-oriented economy, and is a thriving example of dedollarisation. By the end 1980's, Poland was on the verge of a profound economic crisis. The huge distortions on relative prices and the cumulative fiscal deficits, inherited from the years of central planning, induced a rapid increase in inflation that reached its historical maximum of 550 percent in 1989. In response to this unstable macroeconomic environment, dollarisation ratios increased rapidly, from levels around 20% in 1985 to a peak of 60% in 1989. This is shown in Figure 6.2(a).

After the introduction of a series of pro-market reforms and of a stabilization program (the so-called "shock-therapy"),[13] dollarisation ratios dropped to averages

[13] A drastic series of institutional and market reforms were put in place in 1990: the government liberalised controls of almost all prices, eliminated most subsidies, abolished administrative allocation of resources in favor of trade, promoted free establishment of private businesses, liberalised the system of international economic relations, and introduced an internal currency convertibility with a currency devaluation of 32%.

Figure 6.2: Deposit Dollarisation in Poland



Source: National Bank of Poland.

of 40% percent by the end of 1993, hand-to-hand with the reduction of inflation (from 500% to 36%). As the macroeconomic conditions kept improving, additional institutional reforms were put in place. Notably, in 1997 the National Bank of Poland was granted independence and a well-defined objective: to guarantee price stability. dollarisation decreased even more reaching by 2001 the level of 18%, comparable with that of developed European economies, as the UK.

A common feature of the Polish experience with the Peruvian one discussed above is the observed heterogeneity of dollarisation dynamics amongst type of deposits. Figure 6.2(b) reveals that by the end of 1993, the difference between the dollarisation ratios of households and firms was of the order of 70% for time deposit and 40% for demand deposits. These differences remained on the range of 20% for more than 4 years.

6.1.3 Moral

The differences between how individuals and firms decide their portfolio composition is obvious. Usually firms have more resources allocated to the management of their funds, whereas individuals often base their decisions on their experience, those of some neighbors and their limited access to information. Moreover, the decision-making even within firms or within individuals is likely to be dissimilar. Our brief inspection of the Peruvian and Polish experiences illustrates our main claim that these differences accounts for much heterogeneity in dollarisation decisions. We next analyze how this translates into persistence.

6.2 A Simple Model

We use a simple framework to show how the combination of imperfect, noisy information on real returns of foreign assets, and specially the heterogeneity amongst market participants can generate a persistent degree of dollarisation.

The model economy is populated by a number of almost identical individuals. They share the same endowment, which is normalised to one, and the same preferences, but they differ in their ability to process information and therefore in their expectations on future outcomes.[14]

Agents choose every period the composition of their portfolio between two assets, one that offers a fixed real return R^P which is denominated in domestic currency (*pesos* from now on) and the other denominated in dollars with real return R_t^D. The real ex-ante excess of return of the dollar over the pesos asset is simply $R_t = R_t^D - R^P$.

[14] Our analysis hold for agents with heterogenous endowments, i.e. wealth/income inequality, as long as they are correlated with the abilities to process information. See appendix D.3 for details.

6.2.1 Portfolio Decision

Depositors are risk adverse. Individual i devotes an amount x_{it} of her savings to the dollar asset and the remaining $1 - x_{it}$ to purchase the asset in pesos. We follow Ize and Levy-Yeyati (2003) in postulating a standard mean-variance utility function. The portfolio decision is ex-ante and based on imperfect information on real returns, so utility for individual i is defined in terms of the conditional expectation for period $t+1$ with information up to period t,[15]

$$\begin{aligned} U_{it} &= E_t\left[x_{it}R^D_{t+1} + (1-x_{it})R^P\right] - 0.5\text{var}_t\left(x_{it}R^D_{t+1} + (1-x_{it})R^P\right) \\ &= E_t\left[x_{it}R_{t+1} + R^P\right] - 0.5\text{var}_t\left(x_{it}R_{t+1}\right) = x_{it}\hat{r}_{it+1} + R^P - 0.5(x_{it})^2 v_{it+1} \end{aligned} \tag{6.1}$$

where $\hat{r}_{it+1}$ and v_{it+1} are the mean and variance of the excess return R_t that individual i expects for period $t+1$, conditional on information up to period t.

The value of x_{it} that maximises (6.1) is

$$x_{it} = \frac{\hat{r}_{it+1}}{v_{it+1}} \tag{6.2}$$

Thus, agents will increase their dollar deposits when they expect a higher real return on this asset for the same expected variance, or when they expect a lower variance given a level of excess of returns.

6.2.2 Forecasting

As equation (6.2) reveals, the only relevant pieces of information for portfolio decisions are the ex-ante excess return and its variance. To make things easier, consider that each agent focuses directly on forecasting R_t, and not on forecasting its components (R^D_t or R^P, which may imply forecasting inflation, depreciation, confiscation risk and so on), and assume that R_t follows a general AR(1) process

$$R_{t+1} = \mu(1-\alpha) + \alpha R_t + w_{t+1} \qquad w_t \sim iid(0, \sigma^2_w) \tag{6.3}$$

[15] We have imposed a value of one to the risk aversion parameter in the utility function. This assumption is harmless to our results.

In period t, the excess return R_t cannot be perfectly observed. What agents observe is an idiosyncratic noise-ridden version of R_t, $S_{it} = R_t + \varepsilon_{it}$ where $\varepsilon_{it} \sim iid(0, \sigma^2_{\epsilon i})$. Our assumption that agents receive different signals can be easily rationalised as a reduced form of a problem where agents face a common signal, but they have different capacity for processing aggregate information. As in Sims (2003), when agents face limited capacity for processing information, they would choose optimally how much effort to allocate in certain activities, as portfolio management. Since individuals face different resources and capacity constraints, when agents have to invest real resources to increase its capacity for processing information on management activities – for instance, to learn how to read and interpret financial news – they can rationally choose to allocate different capacity for processing information on this activity, therefore agents would eventually face different signals.

Each individual has a *forecasting model* of the form

$$\begin{aligned} R_{t+1} &= \mu(1-\alpha) + \alpha\, R_t + w_{t+1} \qquad & w_t \sim iid(0, \sigma^2_w) \\ S_{it} &= R_t + \epsilon_{it} \qquad & \epsilon_{it} \sim iid(0, \sigma^2_{\epsilon i}) \end{aligned} \tag{6.4}$$

Since S_{it} is a noisy indicator, individual i has first to extract R_t from S_{it} (i.e., "nowcasting") and then forecast its mean and variance to implement (6.2). Define $q_i = \sigma^2_w / \sigma^2_{\epsilon i}$ as signal-to-noise ratio which plays a key role in determining how the noisy observations are weighted for signal extraction and prediction. The higher is q_i the more past observations are discounted in forecasting the future. As it can be seen from (6.4), each individual is given a value of q_i to perform her predictions, and this value alone determines the whole forecasting model. This is the only source of (cross-sectional) heterogeneity in this setup. Everything else – α, μ, σ^2_w and the process (6.3) – is of common knowledge across individuals.

That individuals are heterogenous in their ability to extract information from their signal rationalises in a simple manner the fact that those with high q_i (the financial experts) are able to extract more information from the noisy indicator S_{it} than those with low q_i (the blacksmiths). In contrast to the latter, the former might be able to distinguish whether changes in S_{it} reveal underlying movements in R_t or are just due to noise. This in turn implies differences in the speed in which short-run forecasts are adjusted as new information becomes available, and translates directly to portfolio differences amongst market participants. We interpret this heterogeneity as differences in the *ability people have to forecast.*

Define $\hat{r}_{it}$: the optimal predictor of R_t conditional on the information of agent i available at time $t-1$, i.e. conditioned on S_{it} and $v_{it} = E_t\left[(R_t - \hat{r}_{it})^2\right]$ as its mean squared error (MSE). Standard results from the signal extraction literature lead us to the optimal prediction rule[16]

$$\hat{r}_{it+1} = \mu(1-\alpha) + \alpha\,\hat{r}_{it} + k_{it}(S_{it} - \hat{r}_{it}) = \mu(1-\alpha) + (\alpha - k_{it})\,\hat{r}_{it} + k_{it}S_{it} \tag{6.5}$$

where the forecasted value of R_t for next period is the projection of today's forecasted value plus a correction, an updating that is proportional to the latest prediction error incurred $(S_{it} - \hat{r}_{it})$.[17] The value of k_{it}, the *Kalman gain*, is given by the (adjusted) ratio of the MSE of $\hat{r}_{it}$ to the variance of the noisy indicator,

$$k_{it} = \alpha\left(\frac{v_{it}}{v_{it} + \sigma^2_{\epsilon i}}\right) \tag{6.6}$$

The MSE of $\hat{r}_{it}$ evolves according to the following recursion

$$v_{it+1} = \frac{v_{it}(\alpha^2\sigma^2_{\epsilon i} + \sigma^2_w) + \sigma^2_{\epsilon i}\sigma^2_w}{v_{it} + \sigma^2_{\epsilon i}} \tag{6.7}$$

For expositional convenience define $\tilde{v}_{it} = v_{it}\sigma^{-2}_{\epsilon i}$. Then, (6.7) becomes

$$\tilde{v}_{it+1} = \frac{\tilde{v}_{it}(\alpha^2 + q_i) + q_i}{\tilde{v}_{it} + 1} \tag{6.8}$$

[16] The reader that is familiar with state-space modelling will note that the recursions (6.5) and (6.7) above are straightforward applications of the Kalman filter. See Ljungqvist and Sargent (2000, ch. 2 and ch. 21) and Harvey and De Rossi (2006) for further details.

[17] It is important to emphasise that $\hat{r}_{it}$ represents the best forecast of R_t conditional on information up to period $t-1$. Since portfolio decisions are to be taken one period in advance, they do not incorporate the information contained on the signal S_{it}, but this information is taken into account to improve the next period's forecast of R_{t+1}.

It is clear from equation (6.8) that $\tilde{v}_{i\tau+1} = f(\tilde{v}_{i\tau})$. There is a fixed point such that $\tilde{v}_i = f(\tilde{v}_i)$[18] and moreover, since $f'(\tilde{v}_i) < 1$ it is globally stable: regardless of the initial condition $\tilde{v}_{i0}$ we have that $\tilde{v}_{i\tau} \longrightarrow \tilde{v}_i$ and consequently $k_{i\tau} \longrightarrow k_i = \alpha\,\tilde{v}_i(\tilde{v}_i+1)^{-1}$ as $\tau \longrightarrow \infty$. This means that as τ becomes larger, i.e. as each individual has performed the signal extraction exercise a number of times, the updating process defined in (6.5) and (6.7) converges to an equilibrium rule.[19] If it is assumed that this recursive process was initialised long before period t then $\tilde{v}_{it}$ (or v_{it}) and k_{it} can be safely treated as constants that depends on q_i. This fact simplifies the calculations considerably without compromising our conclusions.

To have a better grasp of the way heterogeneity amongst agents affects their forecasts (and portfolios), assume for a moment that $\alpha \longrightarrow 1$ and solve (6.5) recursively to get

$$\hat{r}_{it+1} = k_i \sum_{j=0}^{\infty} (1 - k_i)^j S_{it-j}$$

It is clear from this geometrically distributed lag expression that different draws of q_i (and hence of k_i) are associated with different ways of weighting the available information (the noisy indicators up to period t) in order to produce a forecast.[20]

[18] The fixed point is the positive root of $\tilde{v}_i^2 + [(1-\alpha^2) - q_i]\tilde{v}_i - q_i = 0$.

[19] Convergence is monotonic ($\tilde{v}_{i\tau} \geq \tilde{v}_{i\tau+1} \geq \tilde{v}_i$) because $v_{i\tau+1}$ is based on more information than $v_{i\tau}$.

[20] As noted in Harvey (1989, ch. 4), the forecasting model converges to the popular Exponential Smoothing method (ES) if $\alpha \longrightarrow 1$. However, the scheme explained here is optimal in the sense that it minimises the one step ahead MSE, whereas ES is basically *ad hoc.*

6.2.3 Individual Dynamics

Using the fact that $v_{it} \to v_i$, $k_{it} \to k_i$ and the optimal updating/forecasting rule (6.5), the optimal dollar investment (6.2) boils down to

$$x_{it} = \frac{\hat{r}_{it+1}}{v_{it+1}} = \frac{\hat{r}_{it+1}}{v_i} = \frac{\mu(1-\alpha)}{v_i} + (\alpha - k_i)\left(\frac{\hat{r}_{it}}{v_i}\right) + \left(\frac{k_i}{v_i}\right) S_{it} \tag{6.9}$$

After plugging (6.5) into (6.9), we get

$$x_{it} = a_i x_{it-1} + c_i + b_i S_{it} \tag{6.10}$$

where $a_i = (\alpha - k_i)$, $c_i = \mu(1-\alpha)v_i^{-1}$ and $b_i = (\tilde{v}_i + 1)^{-1}$. The individual's dollarisation ratio follows an autoregressive process and, as such, exhibits some degree of persistence that depends on k_i. It is easy to show that k_i is increasing in q_i, which implies that the individuals with higher q_i (those who gain more information from the signal each period) have less persistent dollarisation ratios. As (6.10) shows, the higher the k_i, the lower the degree of persistence of dollarisation ratios. Furthermore, individuals with low q_i will tend to consider the dollar asset as less risky, since they would attach a higher fraction of the variance of the signal to the noise and not to real excess return.

The dynamics of individual dollarisation decisions shows that with noisy signals of returns, individuals have to rely on past information to optimally forecast them, and have to react with caution to news. To the extent that past portfolio decisions contain past information of returns, it becomes optimal for individuals to make their dollarisation ratios depended on past dollarisation ratios.[21] Thus, our simple model shows that noisy information can render not only persistence but also an higher individual dollarisation ratio.

[21] A similar result but in a different setup can be found in Aoki (2003). In that paper the central bank sets interest rates in an environment with noisy information on output and inflation. The optimal policy rule implies some persistence coming from the cautiousness that the lack of perfect information demands.

6.2.4 Aggregate Dynamics

In a static world the effects of aggregation are well-known: it tends to smooth away individual erratic movements and to fill in discontinuities that may be present at the disaggregate level. Within a dynamic framework, aggregation also increases persistence.[22] To see why consider a group of individuals who hold a small amount of the dollar asset and face an aggregate shock that makes it more attractive (e.g., a strong real depreciation). According to (6.10), these individuals will increase their dollar holdings immediately. But then, they will also revise their expectations about future returns in favor of the dollar asset, thereby perpetuating the impact effect of the shock on aggregate dollarisation. Thus, the moderate persistence in the individual portfolio formation due to the lack of perfect information, summarised in equation (6.10), is exacerbated by aggregation.[23]

Consider that q_i is drawn from a distribution such that the cdf of a_i is $F(a)$. To better understand the workings of aggregation and how aggregate data can help us to draw conclusions about the underlying heterogeneity in dollarisation decisions, it is convenient to focus for a moment on the case where the aggregate signal, S_{it} is an *iid* sequence. We then relax this assumption.

Aggregation When Signals Are *iid*

Appendix D.1 shows that aggregation of (6.10) across the distribution of a renders the following process for the economy-wide dollarisation ratio X_t,

$$X_t = \sum_{j=1}^{\infty} A_j X_{t-j} + \tilde{C} + \tilde{U}_t \tag{6.11}$$

[22] The classic reference for the econometrics of this effect is Granger (1980), which assumes that $F(a)$ (defined below) is a Beta distribution. See also Pesaran (2003) and Zaffaroni (2004) for recent developments.

[23] See Michelacci (2004) for a similar analysis.

where the A_j $(j = 1, 2, \ldots)$ are coefficients, $\tilde{C}$ is a constant and $\tilde{U}_t$ is an aggregate serially uncorrelated disturbance. As suggested before, the remarkable fact is that although at the individual level the dollar share in the portfolio follows an AR(1) process, it becomes AR(∞) at the aggregate – usually known as a process exhibiting *long-memory*.

As stressed by Lewbel (1994), the coefficients in (6.11) are tightly related to the shape of $F(a)$. In appendix D.1 it is also shown that they satisfy the recursion

$$A_s = m_s - \sum_{j=1}^{s-1} m_{s-j} A_j \qquad \text{for } s = 1, 2, \ldots \tag{6.12}$$

where m_s is the s-th *moment* of the distribution of a, $m_s = \int a^s \, \mathrm{d}F(a)$. Hence, it is easy to verify that

$$\begin{aligned} \text{mean}(a) &= m_1 = A_1 \\ \text{variance}(a) &= m_2 - m_1^2 = A_2 \\ \text{skewness}(a) &= (m_3 - 3m_1 m_2 + 2m_1^3)(m_2 - m_1^2)^{-3/2} = (A_3 - A_1 A_2)(A_2)^{-3/2} \end{aligned}$$

These relations allow us to determine how the distribution of forecasting abilities affects persistence at the aggregate level. The higher A_1, the higher the mean which implies that the *average* individual has herself a more persistent behaviour, rendering subsequently a more persistent X_t. On the other side and strikingly, a higher A_2 renders also more persistence: the higher the heterogeneity amongst individuals, the more persistent the aggregate dollarisation ratio. Finally, as pointed out by Zaffaroni (2004), the low frequency behaviour of the aggregate is determined by the shape of the cross sectional distribution as $a \to 1^-$. Hence, a distribution with a heavy left tail ($A_3 < A_1 A_2$), which indicates a higher mass of persistent individuals ($a \approx 1$), would suggest higher aggregate persistence.

It is now clear that this framework can be tested straightforwardly. If the estimates of A_s using aggregate data are inconsistent with the notion of various dynamic processes that have been aggregated into (6.11), then we are to reject the model.[24] The most obvious symptoms of contradiction would be a non-positive estimate of A_2, the variance of a,[25] or a very negative value for A_1, the mean.

Aggregation When Signals are Correlated

Recall now that $S_{it} = R_t + \varepsilon_{it}$, where ε_{it} is an idiosyncratic shock. Then, the aggregation of (6.9) (see appendix D.1) leads to

$$X_t = \sum_{j=1}^{\infty} A_j X_{t-j} + \sum_{r=0}^{\infty} B_j R_{t-j} + \hat{C} + \hat{U}_t \tag{6.13}$$

which as opposed to (6.11) includes a distributed lag of R_t. This difference is clearly a consequence of postulating different assumptions about the nature of S_{it}. Yet, the coefficients A_s ($s = 1, 2, \ldots$) have the same interpretation and implications as before.

6.3 Empirical Evidence

This section tests whether the dynamics of the aggregate dollarisation ratio in selected countries can be regarded as coming from the aggregation of heterogeneous depositors. In other words, we estimate the parameters A_s in equations (6.11) and (6.13) and investigate, from the estimated moments of the underlying distribution $F(a)$, the extent of heterogeneity amongst participants in the dollar deposit market.

It is important to bear in mind that the amount of information about individual behavior that can be inferred from aggregate data, as we attempt to do below, is unquestionably limited. Different assumptions regarding individual decisions can be found to be consistent with a given observed aggregate variable.

[24] Or the assumptions behind the aggregation, see appendix D.1.

[25] Note that $A_2 = 0$ implies a degenerate distribution of a on the point A_1, i.e. a model with a representative agent or identical individuals.

Yet, there are some facts reported below that are supportive to the main hypothesis of this paper and the predictions of the theoretical model.

6.3.1 Baseline Specification

Consider equation (6.11). Three points are worth mentioning before presenting some results. Firstly and unsurprisingly every dollarisation ratio X_t we considered has a unit root[26] and to avoid well-known biases in the estimation of autoregressive coefficients when a unit root is present we estimate (6.11) in first differences,

$$\Delta X_t = \sum_{j=1}^{\infty} A_j \Delta X_{t-j} + U_t^{\dagger} \tag{6.14}$$

Appendix D.1 shows that (6.14) is not only the first-differenced version of (6.11), but is also the result of aggregating (6.10) after first-differentiating. Hence, the coefficients in (6.14) are indeed the same as in (6.11). The disturbance $U_t^{\dagger}$ is autocorrelated and heteroscedastic[27] so robust inference is required.

Secondly, due to data limitations it is not possible to estimate equation (6.14) as it stands. Data are finite, so a truncation in the lags of the $AR(\infty)$ process is unavoidable.

Lastly, if convenient, we consider even richer dynamics than the suggested by our very stylised theoretical model by introducing a MA(1) component in (6.14). In practice, this fact has no other implication for our analysis than to produce better estimates of the A_s. As noted by Lewbel (1994), with a MA component present only a finite number of the moments of $F(a)$ can be recovered as an infinite autoregression in X_t (or in ΔX_t) cannot be separated from the MA parameter, say θ. This is a theoretical rather than empirically substantive concern; as noted earlier, our attempt is not to recover every moment of $F(a)$, but just the first few.

[26] Results of unit root tests are available upon request to the authors. See also appendix D.2.

[27] See Pesaran (2003) for further details.

We gathered information for Peru and Uruguay (two highly dollarised countries), Mexico and Poland. Data are quarterly spanning roughly from the mid-1980's to the mid-2000's. As it is customary in the dollarisation literature, X_t is measured as the ratio of foreign currency deposits from the private sector in the domestic banking system to M2.[28] This information is widely available and our sources are the websites of the various central banks and the International Financial Statistics database, IFS. The regression with the shortest time series (Poland) has $N = 69$ observations; the one with the largest (Peru), $N = 94$.

Results

For each country an ARIMA(2,1,0) was first fitted to equation (6.14). Then, we test for residual autocorrelation and include further lags until the residuals appear serially uncorrelated. In every case, no more than 2 lags is needed, but for Mexico the lag length is 4. For robustness sake we then include a MA component in the best autoregressive specification. Table 6.3.1 reports for each country the best autoregressive model, ARIMA(2,1,1) or ARIMA(4,1,0), and the corresponding ARIMA(2,1,1) or ARIMA(4,1,1) equations. The column labelled θ contains the estimated MA coefficient. For each country we have marked our preferred specification, i.e. the more parsimonious model that describes the data sufficiently well, with a $\star$.

A finding that is robust amongst countries and specifications within the same country, is that the coefficients A_1 and A_2 are significantly positive. Recall that A_1 is the mean of $F(a)$, and A_2 is its variance. Besides, the estimates of the implied third central moment $A_3 - A_1 A_2$ in each country suggest that $F(a)$ is skewed to the left. Provided that $A_1 > 0$, a left-skewed $F(a)$ would be expected

[28] A popular alternative definition of the dollarisation ratio discriminate between residents and non-residents, which includes deposits by residents abroad (Ize and Levy-Yeyati, 2003). We did not include this definition in our empirical work as the corresponding available time series are shorter for the pool of countries analyzed.

Table 6.3.1: ARIMA Models of the Deposit Dollarisation Ratio in Selected Countries

ARIMA model	A_1	A_2	A_3	A_4	θ	$A_3 - A_1A_2$	$\bar{R}^2$
Mexico (1985.Q4 to 2005.Q3, $N = 77$)							
$(4,1,0)$	0.221*	0.199*	−0.192*	0.114**		−0.236*	0.221
	(0.078)	(0.078)	(0.072)	(0.064)		(0.095)	
$(4,1,1)^\star$	0.480*	0.195*	−0.216*	0.251*	−0.097*	−0.310*	0.261
	(0.111)	(0.094)	(0.063)	(0.047)	(0.018)	(0.086)	
Peru (1980.Q1 to 2005.Q3, $N = 94$)							
$(2,1,0)^\star$	0.173*	0.142*				−0.024**	0.200
	(0.063)	(0.058)				(0.013)	
$(2,1,1)$	0.186**	0.139*			−0.058	−0.026	0.173
	(0.094)	(0.065)			(0.143)	(0.016)	
Poland (1985.Q4 to 2002.Q4, $N = 69$)							
$(2,1,0)^\star$	0.474*	0.113*				−0.053*	0.215
	(0.016)	(0.052)				(0.024)	
$(2,1,1)$	0.476*	0.111*			−0.007	−0.053*	0.275
	(0.010)	(0.049)			(0.045)	(0.024)	
Uruguay (1985.Q1 to 2005.Q3, $N = 83$)							
$(2,1,0)$	0.218*	0.290*				−0.063*	0.153
	(0.091)	(0.116)				(0.029)	
$(2,1,1)^\star$	0.265**	0.215*			−0.093*	−0.057**	0.196
	(0.147)	(0.055)			(0.034)	(0.033)	

Maximum likelihood estimates. Figures in parentheses are robust (consistent) standard errors. * [**] denotes significance at a 5% [10%] level. The standard error of the third central moment $A_3 - A_1A_2$ was computed with the delta method. $\bar{R}^2$ is the adjusted R^2. Regressions include a constant and, if necessary, a few dummy variables for outlier removal. In all reported equations, Breusch-Godfrey and Jarque-Bera tests suggested uncorrelated and normally distributed residuals. The preferred specifications are marked with a $\star$.

if the mass of those individuals with relatively persistent portfolios is relatively large respect to the mass of individuals with a close to zero (corresponding to those who change their portfolio quickly). Negative skewness, thus, is consistent with a financial expert sharing the dollar market with a non-expert blacksmith saving in dollars.

A remarkable fact from Table 6.3.1 is that the estimates for Peru are close to those of Uruguay, whereas the Mexican estimates are similar to the Polish. Recall that Peru and Uruguay are heavily dollarised (above 50%), whereas Mexico and Poland, even though have reported sizeable dollarisation ratios by the early or mid-90's, have dollarisation ratios less than 30% by the end of the sample. In Peru and Uruguay the coefficients are of comparable magnitude, $A_2 \approx A_1$, which means that the underlying $F(a)$ is very spread, the a's are fairly heterogeneous[29]. Hence, the highly dollarised economies appear to have a spreader $F(a)$ which is consistent with the idea of decreasing participation costs as dollarisation expands. Furthermore, when parameterise $F(a)$, we found the dollarised countries are more heavily skewed than Mexico and Poland. The estimated of the mass of persistent individuals,$Pr(0 \leq a \leq 1)$, is roughly 0.85 for Peru and Uruguay and about 0.6 for Mexico and Poland.

6.3.2 Augmented Specification

Consider now equation (6.13). In the likely case that signal S_t is not *iid*, then the estimates of Table 6.3.1 may be biased due to the omission of relevant variables. Next, we augment the ARIMA models of Table 6.3.1 to investigate whether this omission changes our main conclusions.

As discussed above, the actual object to be estimated is

$$\Delta X_t = \sum_{j=1}^{p_X} A_j \Delta X_{t-j} + \sum_{j=0}^{p_R} B_j \Delta R_{t-j} + U_t^{\ddagger} \tag{6.15}$$

where p_X and p_R are finite lag lengths. The presence of R_t and its lags in (6.15) follows directly from the fact that the individuals in the theoretical model base their decisions exclusively on this variable. Nonetheless, a richer modelling framework can easily extend (6.15) to

$$\Delta X_t = \sum_{j=1}^{p_X} A_j \Delta X_{t-j} + \sum_{j=0}^{p_R} B_j^D \Delta R_{t-j}^D + \sum_{j=0}^{p_R} B_j^P \Delta R_{t-j}^P + U_t^{\ddagger} \tag{6.16}$$

As $R_t = R_t^D - R_t^P$, equation (6.16) encompasses (6.15) which is a restricted version

[29] These estimates imply a coefficient of variation $\sqrt{A_2}/A_1$ of 2.18 for Peru, 1.75 for Uruguay, 0.91 for Mexico and 0.71 for Poland

with $B_s^D = -B_s^P$ for every s. For this reason, we will focus on (6.16) from now on.[30]

An empirical issue that raises with the introduction of the real returns in the aggregate equations is, precisely, how to measure them. The "true" returns involve expectations of future macroeconomic variables, which historical data are barely available for the countries in our analysis. Call i_t^P and i_t^D the nominal interest rates in domestic currency and US dollars, respectively, δ_t the nominal depreciation (i.e., the percent change of the nominal exchange rate, domestic currency per US dollar) and π_t the CPI inflation. We entertain two measurements of the real returns:

$$R_t^P = \frac{1+i_t^P}{1+\pi_{t+1}} - 1 \qquad\qquad R_t^P = \frac{1+i_t^P}{1+\pi_t} - 1$$

$$R_t^D = \frac{(1+i_t^D)(1+\delta_{t+1})}{1+\pi_{t+1}} - 1 \qquad R_t^D = \frac{(1+i_t^D)(1+\delta_t)}{1+\pi_t} - 1$$

CPI and nominal exchange data are readily available. For i_t^P we use the deposit rate in domestic currency for Peru, Poland and Uruguay and the saving rate in domestic currency for Mexico. For i_t^D, we found data on the interest rate paid to domestic deposits in dollars only in the case of Peru and Uruguay. For Mexico and Poland we approximate i_t^D with the deposit rate in the US.[31] Our sources are still the central banks and the IFS.

Finally, the presence of a contemporaneous return (6.16) may rise the possibility of endogeneity bias. We use a 2SLS procedure to estimate this equation. The instruments are listed in the note to Table 6.3.2. It is worth mentioning that OLS or the exclusion of the contemporaneous returns did not alter the main results

[30] The estimations of (6.15), which are similar to our purposes, are available upon request to the authors.

[31] Unfortunately we could not find time series long enough of country risk to have a better measure of R_t^D in these two countries. The estimation results, though, were robust when we considered the LIBOR rate (in US dollars, at various terms) instead of the US deposit rate.

of this robustness check.[32]

Results

Table 6.3.2 displays the estimation results. To save space we do not report the coefficients of the returns (as they are not of direct interest for our analysis) but do report an F-statistic assessing its overall significance. We set the lag length $p_X = 3$. This is the best choice for Mexico; for the other countries, the optimal is $p_X = 2$, but we still set $p_X = 3$ to ensure that no autoregressive effect is ignored. The choice of p_R, reported in the table, responds to the minimization of the Schwarz criterion.

Recall that by estimating the augmented equations we are assessing whether the results of Table 6.3.1 are robust. So, are they robust? In general they are. A quick comparison of the estimates in Table 6.3.2 with those in Table 6.3.1 reveals that due to the presence of the returns, the fit of the various equations increases, but the estimates of A_1, A_2 and $A_3 - A_1A_2$ do not change much. The notable exception to this pattern is the Mexican case when the returns are measured in the *ex-post* manner, as A_1 losses statistical significance. However, the main claim of the previous sections still holds, qualitatively and almost quantitative: the heterogeneity of decision-makers that underlies the aggregate dollarisation ratios is high, and this fact leads to aggregate dollarisation persistence.

6.4 Caveat: The Role of Learning

An alternative way to rationalise the fact that individuals are heterogeneous in their forecast of R_t is to assume that they cannot perfectly observe the true process that governs the evolution of R_t. For instance, because they do not know the exact

[32] We did not find a significant cointegration relationship between X_t, R_t^P and R_t^D or between X_t and R_t to treat (6.16) as an error correction model. Structural breaks in our 20 year data span may explain this failure. Consistently with this, the *levels* of the returns did not appear to have enough explanatory power in the equations of Table 6.3.2.

Table 6.3.2: Augmented Equations

	A_1	A_2	A_3	$A_3 - A_1A_2$	$H_0 : B = 0$	p_R	$\bar{R}^2$
Mexico (1985.Q4 to 2005.Q3, $N = 77$)							
ex-ante	0.291*	0.202*	−0.273*	−0.331*	11.50*	2	0.554
	(0.096)	(0.066)	(0.092)	(0.113)	[0.000]		
ex-post	0.129	0.287*	−0.240*	−0.278*	22.56*	2	0.565
	(0.108)	(0.089)	(0.080)	(0.091)	[0.000]		
Peru (1980.Q1 to 2005.Q3, $N = 89$)							
ex-ante	0.242*	0.195*	0.003	−0.047*	9.086*	3	0.435
	(0.043)	(0.047)	(0.053)	(0.015)	[0.000]		
ex-post	0.501*	0.138**	−0.027	−0.069**	30.85*	2	0.649
	(0.098)	(0.083)	(0.068)	(0.036)	[0.000]		
Poland (1985.Q4 to 2005.Q3, $N = 68$)							
ex-ante	0.449*	0.132*	−0.002	−0.059*	1.638	3	0.275
	(0.043)	(0.058)	(0.049)	(0.022)	[0.203]		
ex-post	0.586*	0.164*	−0.123	−0.096*	2.402**	4	0.394
	(0.077)	(0.070)	(0.160)	(0.043)	[0.099]		
Uruguay (1985.Q1 to 2005.Q3, $N = 80$)							
ex-ante	0.252*	0.280*	−0.109	−0.070**	3.153*	3	0.124
	(0.104)	(0.114)	(0.140)	(0.038)	[0.049]		
ex-post	0.267*	0.349*	−0.073	−0.093**	2.189	2	0.152
	(0.103)	(0.117)	(0.143)	(0.047)	[0.119]		

2SLS estimates. Instruments for R_t^D and R_t^P (and for the *ex-ante* R_{t-1}^D and R_{t-1}^P) are oil prices changes, US GDP growth and lagged values of these and the R-variables. Figures in parentheses are robust (consistent) standard errors. * [**] denotes significance at a 5% [10%] level. Figures in the $H_0 : B = 0$ column are F-statistics, p-values shown in braces. For Peru, Poland and Uruguay, we set $A_3 = 0$ to compute the third central moment and its standard deviation. Diagnostic tests suggested well-behaved residuals, see notes to Table 6.3.1.

value of α in (6.3). In this case, individuals should form priors on the value of this parameter in order to forecast R_t and to make their portfolio choices. Agents may have different priors on α, but they can update those priors as new information on R_t arrives.[33]

This assumption is plausible in circumstances where the central bank does not have an explicit inflation target or it has one that is not perfectly credible, for instance because it attempts to stabilise simultaneously the exchange rate and the inflation rate. Uncertainty of this type may induce positive expected values for R_t,

[33] For models with learning and heterogenous priors, see Arifovic (1996) and Marimón et al. (2004).

since some agents might expect higher levels of inflation, making more profitable to invest in dollar assets.

Consider a common signal, $S_{it} = S_t = R_t + \varepsilon_t$ where $\varepsilon_t \sim iid(0, \sigma_\varepsilon^2)$ is an aggregate shock. Under this type of uncertainty, the perceived law of motion for R_t of individual i, becomes

$$\hat{r}_{it+1} = \mu\,(1 - \hat{\alpha}_{it}) + \hat{\alpha}_{it}\,\hat{r}_{it} + \omega_t \tag{6.17}$$

Although every agent faces the same signal extraction problem, they portfolio choices differ since they have different priors of α. In this case the optimal portfolio allocation for individual i would be given by

$$x_{it} = \hat{\alpha}_{it}x_{it-1} + \frac{\mu}{v} + \hat{\alpha}_{it}\left(\frac{\sigma_\varepsilon^2}{v_i + \sigma_\varepsilon^2}\right)\xi_{i,t} \tag{6.18}$$

where $\xi_{i,t} = \varepsilon_t + R_t - \hat{r}_{it}$. Notice that the implications for aggregation and heterogeneity are different in this case to those obtained in the baseline model. Here, all agents have the same ability to extract information, but they differ on their priors on α. Since, agents update their beliefs as new information arrives, heterogeneity is not a permanent feature, it only last while agents learn the true value of α.

This fact have remarkable implications, but complicates considerably the empirical implementation of model. Firstly, the degree of aggregate persistence would decrease as agents learn, since the dispersion on the values of $\hat{\alpha}_{it}$ would decrease, therefore, the coefficients of equation (6.11) would be time varying. Although the available sample used in the empirical analysis is relatively short, no strong evidence of time varying parameters was found. Secondly, the speed of the reduction on the degree of persistence would depend on the dispersion of the initial distribution of priors on α: if initial dispersion is high, the reduction on the persistence would be slower. Finally, central banks that adopt a credible inflation target regime for conducting monetary policy, can help not only to reduce the mean value of dollarisation but also its persistence by reducing the dispersion on the priors that individuals have on α.

6.5 Concluding Remarks

In countries with high dollarisation ratios, participation in the dollar deposit market has become massive. Financial deregulation, liberalization, innovation and informal currency markets have allowed a very heterogenous group of agents – from a large firm that uses state-of-art portfolio management techniques to an uninformed individuals who base their portfolio decisions simply on their own experience and limited information – to participate in the same market. This paper shows that such an heterogeneity turns out to be enough to generate persistence in dollarisation ratios upon aggregation. Empirical evidence from three Latin American countries and Poland supports this claim.

The presence of heterogeneity in individual dollarisation decisions has interesting policy implications. Ize and Levy-Yeyati (2003) conclude sensibly that a necessary and sufficient condition for dedollarisation is higher exchange rate flexibility. In our setup this condition is not sufficient (though we reckon it is necessary), as there may exist a mass of individuals that do not respond at all to such a volatility. This makes a case for a more active policy on improving the communication skills of the central bank, in order to better convey its policy of more flexible exchange rates and possibly its commitment to price stability to a broader set of agents, specially to those regarded as uninformed. In this way the policymaker would be contributing to reduce individual heterogeneity and thus aggregate persistence.

This policy implication is particularly relevant for developing economies with an inflation targeting regime or for those evaluating moving towards this regime, as it heavily relies upon transparency and communication strategies. Our analysis suggests that the benefits of the such a policy regime in reducing dollarisation may be condemned to be limited, unless the central bank effectively communicates the implications and benefits of such a regime to the less informed segment of participants in the dollar market.

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A APPENDIXES OF CHAPTER 3

A.1 Dynamic Equilibrium

The equations that determine the dynamic equilibrium of the economy are the following:

1. **Aggregate Demand**

$$\frac{1}{1+i_t} = \beta E_t \left(\left(\frac{Y_{H,t+1}}{Y_{H,t}} \right)^{-1} \frac{1}{\Pi_{H,t+1}} \right) \tag{A.1}$$

$$e_t = \frac{Y_{H,t} P_{H,t}}{Y_t^* P_t^*} \tag{A.2}$$

$$Y_{H,t} = T_t^{-\gamma} C_t \tag{A.3}$$

$$Q_t = \frac{C_t}{Y_t^*} \tag{A.4}$$

$$Q_t = T_t^{1-\gamma} \tag{A.5}$$

2. **Aggregate Supply**

$$\Pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}} \tag{A.6}$$

$$P_{H,t} = \left(\frac{1}{n} \int_{\Theta} P_{H,t}^{1-\varepsilon}(z)\, dz + \int_{\Sigma} P_{H,t}^{1-\varepsilon}(z)\, dz + \int_{[0,n]\setminus\Sigma\cup\Theta} P_{H,t}^{1-\varepsilon}(z)\, dz + \right)^{\frac{1}{1-\varepsilon}} \tag{A.7}$$

$$P_{H,t}(z) = \begin{cases} \mu w_t A_t(z) P_{H,t} & \text{if } z \in \Theta \\ \mu \frac{e_t E_{t-1}\left(w_t A_t(z) d_{H,t}^{\varepsilon}\right)}{E_{t-1}\left(d_{H,t}^{\varepsilon-1}\right)} & \text{if } z \in \Sigma \\ \mu \frac{E_{t-1}(w_t A_t(z) P_{H,t}^{\varepsilon})}{E_{t-1}\left(P_{H,t}^{\varepsilon-1}\right)} & \text{otherwise} \end{cases} \tag{A.8}$$

$$w_t = A_t^{\upsilon} Y_{H,t}^{1+\upsilon} \tag{A.9}$$

$$\sigma_t(z) = \begin{cases} 1 \text{ if } & \frac{\left(E_{t-1}\left(w_t A_t(z) P_{H,t}^{\epsilon}\right)\right)^{1-\epsilon}}{\left(E_{t-1}\left(w_t A_t(z) d_{H,t}^{\epsilon}\right)\right)^{1-\epsilon}} \frac{\left(E_{t-1}\left(p_{H,t}^{\epsilon-1}\right)\right)^{\epsilon}}{\left(E_{t-1}\left(d_{H,t}^{\epsilon-1}\right)\right)^{\epsilon}} < 1 \\ 0 & \text{otherwise} \end{cases} \tag{A.10}$$

$$\Sigma = \{ z \mid \sigma_t(z) < 1 \} \tag{A.11}$$

3. **The foreign Economy**

$$\frac{1}{1+i_t^*} = \beta E_t\left(\left(\frac{Y_{t+1}^*}{Y_t^*}\right)^{-1}\frac{1}{\Pi_{t+1}^*}\right) \tag{A.12}$$

$$\Pi_t^* = \frac{P_t^*}{P_{t-1}^*} \tag{A.13}$$

$$P_t^* = \left(\frac{1}{1-n}\int_{\Theta}(P_t^*(z))^{1-\varepsilon}\,dz + \int_{[n,1]\backslash\Theta}(P_t^*(z))^{1-\varepsilon}\,dz+\right)^{\frac{1}{1-\varepsilon}} \tag{A.14}$$

$$P_{H,t}(z) = \begin{cases} \mu w_t^* A_t^* P_t^* & \text{if } z\in\Theta \\ \mu\frac{E_{t-1}w_t A_t^* P_t^{*\varepsilon})}{E_{t-1}(P_t^{*\varepsilon-1})} & \text{otherwise} \end{cases} \tag{A.15}$$

$$w_t^* = A_t^{*v} Y_t^{*1+v} \tag{A.16}$$

A.2 The Steady-State

We define an steady-state as an equilibrium where both economies have zero inflation, $\Pi = \Pi^* = 1$ and where technology shocks at the home and the foreign economy take their unconditional means, $A = A(z) = A^* = 1$. In this equilibrium, the steady-state analog equations of (A.8) and (A.9) are given by

$$\frac{1}{\mu} = w \tag{A.17}$$

$$w = Y_H^{1+v} \tag{A.18}$$

From the previous two equations it follows immediately that

$$Y_H = (1-\Phi)^{\frac{1}{1+v}} \tag{A.19}$$

similarly for the case of the foreign economy,

$$Y_H^* = (1-\Phi^*)^{\frac{1}{1+v}} \tag{A.20}$$

Where $(1-\Phi) = \frac{1-\tau}{\mu}$. Since the degree of monopolistic competition is the same in both economies, $\Phi = \Phi^*$ and $Y_H = Y_H^*$. Combining the steady-state analogs of equations (A.3),(A.4) and (A.5), we obtain

$$T = \frac{Y_H}{Y^*} = 1 \tag{A.21}$$

finally, from the steady-state analogs of equations (A.4) and (A.5), we have that,

$$\begin{aligned} C &= Y_H = (1-\Phi)^{\frac{1}{\sigma+v}} \\ Q &= 1 \end{aligned} \tag{A.22}$$

The Flexible Price Equilibrium

We analyse the flexible price equilibrium using the log-linear approximation of equations (A.1) to (A.16). The approximation is taken around the deterministic steady-state defined previously. The flexible price equilibrium is an equilibrium where all firms set prices every period observing all shocks in the economy. Under these conditions the log-linear version of equation (A.8), after aggregating across firms, becomes

$$0 = \widehat{w}_t + a_t(z) \tag{A.23}$$

combining equation (A.23) with the log-linear version of equation (A.9) we obtain the output level under flexible prices,

$$y^n_{H,t} = -a_t \tag{A.24}$$

Similarly, from the log-linear approximation of equation (A.1) we obtain the natural interest rate as follows

$$r^n_t = E_t\left(y^n_{H,t+1} - y^n_{H,1}\right) \tag{A.25}$$

On the other hand, the natural real exchange rate level can be obtained by from the log-linear approximation of equation (A.5)

$$q_t = (1-\gamma)\left(y^n_{H,t} - y^*_t\right) = -(1-\gamma)\left(a_t - a^*_t\right)$$

The Aggregate Supply Curve

In order to obtain the domestic aggregate price level, we take the log-linear approximation of equation (A.7), thus we have,

$$p_{h,t} = \frac{1}{n}\int_0^n p_{h,t}(z)d(z) \tag{A.26}$$

Furthermore, from equation (A.8) $p_{h,t}(z)$ if defined as follows,

$$p_{h,t}(z) = \begin{cases} w_t + p_{h,t} + a_t(z) & \text{if } z \in \Theta \\ E_{t-1}\left(w_t + p_{h,t} + a_t(z)\right) & \text{if } z \in \Sigma \\ E_{t-1}\left(w_t + d_{h,t} + a_t(z) + e_t\right) & \text{otherwise} \end{cases} \tag{A.27}$$

The by aggregating $p_{h,t}(z)$ across firms we have,

$$\begin{aligned} p_{h,t} \;=\; & \int_{\Theta} \left[w_t + p_{h,t} + a_t(z)\right] d(z) + \\ & \int_{\Sigma} \left[E_{t-1}\left(w_t + d_{h,t} + a_t(z) + e_t\right)\right] d(z) \\ & + \int_{[0,1]\setminus\Sigma} \left[E_{t-1}\left(w_t + p_{h,t} + a_t(z)\right)\right] d(z) \end{aligned} \tag{A.28}$$

Using the fact that,

$$\int_{\Theta} a_t(z)d(z) = \theta a_t \tag{A.29}$$

$$\int_{[0,n]\backslash\Theta} a_t(z)d(z) = (1-\theta)\, a_t \tag{A.30}$$

We find that,

$$\begin{aligned} p_{h,t} &= \theta\,(w_t + p_{h,t} + a_t) + \\ & (1-\theta)\, s\,(E_{t-1}\,(w_t + p_{h,t} + a_t + e_t) - e_t) \\ & + (1-\theta)\,(1-s)\, E_{t-1}\,(w_t + p_{h,t} + a_t) \end{aligned} \tag{A.31}$$

Rearranging properly we further obtain that,

$$\begin{aligned} p_{h,t} &= \theta\,(w_t + a_t) + (1-\theta)\,(E_{t-1}\,(w_t + a_t)) \\ & + (1-\theta)\, s\,(E_{t-1}\,(e_t) - e_t) + \theta p_{h,t} + (1-\theta)\, E_{t-1} p_{h,t} \end{aligned} \tag{A.32}$$

Using this equation, we can show that,$E_{t-1}\,(w_t + a_t) = 0$, thus, we have,

$$w_t = -a_t + \frac{(1-\theta)}{\theta}\,(p_{h,t} - E_{t-1}\,(p_{h,t})) - \frac{(1-\theta)}{\theta} s\,(e_t - E_{t-1}\,(e_t)) \tag{A.33}$$

Since real wages are given by

$$w_t = (1+v)\, y_{h,t} + v a_{,t} \tag{A.34}$$

the aggregate supply equation, which relates domestic prices with the output gap and inflation expectations errors, and exchange rate expectations errors, would be given by,

$$p_{h,t} = E_{t-1}\,(p_{h,t}) + (1+\nu)\,\frac{\theta}{(1-\theta)} x_t + s\,(e_t - E_{t-1}\,(e_t)) \tag{A.35}$$

Where, $x_t = y_t - y_t^n$ and $y_t^n = a_t$. Using a simple transformation, equation (A.35), we obtain the Phillips curve in this economy,

$$\pi_{h,t} = E_{t-1}\pi_{h,t} + (1+\nu)\,\frac{\theta}{(1-\theta)} x_t + s\,(\Delta e_t - E_{t-1}\,(\Delta e_t)) \tag{A.36}$$

Aggregate Demand

The aggregate demand block is given by the following set of equations,

$$Y_{H,t} = E_t Y_{H,t+1} - (i_t - E_t \pi_{H,t+1}) \tag{A.37}$$

$$y_{H,t} = \gamma t_t + c_t \tag{A.38}$$

$$q_t = c_t - y_t^* \tag{A.39}$$

Combining equations (A.25) and (A.37) we obtain the dynamics of the output gap in this model economy,

$$x_t = E_t x_{t+1} - (i_t - E_t \pi_{H,t+1} - r_t^n)$$

Derivation of Aggregate Productivity

In order to derive equation (3.16) from the main text, lets use the definition of domestic prices, equation (3.51) of the main text, and the definition of optimal price under the flexible price allocation, equation (3.40), as follows,

$$P_{H,t} = \left[\frac{1}{n}\int_0^n P_{H,t}(z)^{1-\varepsilon}dz\right]^{\frac{1}{1-\varepsilon}} = \tag{A.40}$$

$$\left[\frac{1}{n}\int_0^n (\mu W_t A_{z,t})^{1-\varepsilon}\,dz\right]^{\frac{1}{1-\varepsilon}} = \mu W_t\left[\frac{1}{n}\int_0^n A_{z,t}^{1-\varepsilon}dz\right]^{\frac{1}{1-\varepsilon}}$$

thus, by simplifying the previous expression , allow us to write the domestic aggregate price level as a mark-up over nominal marginal,

$$P_{H,t} = \mu W_t A_t \tag{A.41}$$

where, A_t represent the aggregate productivity shock, defined as follows,

$$A_t = \left[\frac{1}{n}\int_0^n A_{z,t}^{1-\varepsilon}dz\right]^{\frac{1}{1-\varepsilon}} \tag{A.42}$$

this last equation corresponds to equation (3.16) in the main text

A.3 The Central Bank Loss Function

In what follows we derive the micro-founded central bank's loss function by using a second-order Taylor approximation of the representative agent's utility function, equation (A.43), around the economy's deterministic steady-state,

$$U = \ln C_t - \frac{h_t^{1+v}}{1+v} \tag{A.43}$$

We use a generic form of the previous utility function to have a general result, thus we approximate,

$$U = U(C_t) - V(h_t)$$

Where total labour depends on output, productivity and the relative price distortion, as follows,

$$h_t = Y_t \Delta_t A_t \tag{A.44}$$

and we know from section 5 that at the first best allocation it must hold that,

$$vY = C(1-\gamma) \tag{A.45}$$

The second order expansion of the utility generated by consumption is given by, :

$$u\left(C_t\right) = \overline{u} + \overline{u}_c \overline{C}\left(\widehat{C}_t + \frac{1}{2}\widehat{C}_t^2\right) + \frac{1}{2}\overline{u}_{cc}\overline{C}^2\widehat{C}_t^2 + o\left(\|\epsilon\|^3\right) \tag{A.46}$$

collecting terms appropriately we have that:

$$u\left(C_t\right) = \overline{u}_c \overline{C}\left(\widehat{C}_t + \frac{1}{2}\left(1-\sigma\right)\widehat{C}_t^2\right) + t.i.p + o\left(\|\epsilon\|^3\right) \tag{A.47}$$

For our particular case where, $\sigma = 1$. equation (A.47) becomes,

$$u\left(C_t\right) = \overline{u}_c \overline{C}\widehat{C}_t + t.i.p + o\left(\|\epsilon\|^3\right)$$

Next we take a second order expansion of $v(h_t)$, we use equation (A.44) to define the aggregate level of labour in terms of output,productivity shocks and price dispersion,thus a second order approximation for the disutility of labor effort is given by,

$$\begin{aligned} v(h_t) &= \overline{v}_h\overline{Y}\left(\widehat{\Delta}_t + \widehat{Y}_t + \frac{1}{2}\left(1 + \frac{\overline{v}_{hh}\overline{Y}}{\overline{v}_h}\right)\widehat{Y}_t^2 + \left(1 + \frac{\overline{v}_{hh}\overline{Y}}{\overline{v}_h}\right)\widehat{Y}_t\widehat{A}_t\right) \\ &\quad + t.i.p + o\left(\|\varepsilon\|^3\right) \end{aligned} \tag{A.48}$$

Notice that since $\widehat{\Delta}_t$ is of order $o\left(\|\varepsilon\|^2\right)$ all terms involving second order terms of $\widehat{\Delta}_t$ are dropped out from equation (A.48) , thus, we have,

$$\begin{aligned} u\left(C_t\right) - v(h_t) &= \overline{u}_c\overline{C}\widehat{C}_t - (1-\gamma)\,\overline{u}_c\overline{C}\left(\widehat{\Delta}_t + \widehat{Y}_t + \frac{1}{2}\left(1+v\right)\widehat{Y}_t^2 + \left(1+v\right)\widehat{Y}_t\widehat{A}_t\right) \\ &\quad + t.i.p + o\left(\|\epsilon\|^3\right) \end{aligned} \tag{A.49}$$

imposing the restriction on the coefficient of risk aversion, $\sigma = 1$, we have:

$$\begin{aligned} &= \overline{u}_c\overline{C}\left(\begin{array}{c} \widehat{C}_t - (1-\gamma)\,\widehat{Y}_t - \frac{1}{2}\left(\left(1-\gamma\right)\left(1+v\right)\right)\widehat{Y}_t^2 \\ -\left(1-\gamma\right)\widehat{\Delta}_t - \left(1-\gamma\right)\left(1+v\right)\widehat{Y}_t\widehat{A}_t \end{array}\right) \\ &\quad + t.i.p + o\left(\|\epsilon\|^3\right) \end{aligned} \tag{A.50}$$

Let's define the following parameters:

$$u_{yy} = -\left(1-\gamma\right)\left(1+v\right) \tag{A.51}$$

$$u_{yA} = \left(1-\gamma\right)\left(1+v\right) \tag{A.52}$$

$$u_{\Delta} = \left(1-\gamma\right) \tag{A.53}$$

Moreover, since:

$$\widehat{C}_t = \left(1-\gamma\right)\widehat{Y}_t + \gamma\widehat{Y}_t^*$$

We can now write the utility function of the representative agent as follows:

$$
\begin{aligned}
u(C_t) - v(h_t) &= \overline{u}_c \overline{Y} \left(-\frac{1}{2} u_{yy} \widehat{Y}_t^2 - u_\Delta \widehat{\Delta}_t - u_{yA} \widehat{Y}_t \widehat{A}_t \right) \\
&\quad + t.i.p + o\left(\|\varepsilon\|^3 \right)
\end{aligned} \tag{A.54}
$$

Rewriting appropriately the quadratic terms we have:

$$
\frac{1}{2} u_{yy} \widehat{Y}_t^2 + u_\Delta \widehat{Y}_t \widehat{A}_t = \frac{1}{2} (1-\gamma) \left((1+v) \left(\widehat{Y}_t^2 - 2\widehat{Y}_t \widehat{A}_t + \widehat{A}_t^2 \right) \right) \tag{A.55}
$$

since we have eliminated all the distortions of the steady-state equilibrium, the quadratic terms of the approximated loss function of the central bank can be written as follows:

$$
\frac{1}{2} u_{yy} \widehat{Y}_t^2 + u_\Delta \widehat{Y}_t \widehat{A}_t = \frac{1}{2} (1-\gamma)(1+v) \widehat{x}_t^2
$$

$$
u(C_t) - v(h_t) = \overline{u}_c \overline{Y} \left(-\frac{1}{2} (1-\gamma)(1+v) \widehat{x}_t^2 - (1-\gamma) \widehat{\Delta}_t \right) + t.i.p + o\left(\|\varepsilon\|^3 \right) \tag{A.56}
$$

Now we have to find the second-order approximation of $\widehat{\Delta}_t$,

$$
\Delta_t = \frac{1}{n} \int_0^n \left(\frac{P_{H,t}(z)}{P_{H,t}} \right)^{-\varepsilon} \frac{A_{z,t}}{A_t} dz \tag{A.57}
$$

In order to simplify notation, let's denote by $R_p(z) = \frac{P_{H,t}(z)}{P_{H,t}}$ and by $R_a(z) = \frac{A_{z,t}}{A_t}$, then Δ_t can be written as follows,

$$
\Delta_t = \frac{1}{n} \int_0^n R_p(z)^{-\varepsilon} R_a(z) dz \tag{A.58}
$$

$$
\begin{aligned}
\Delta_t &= 1 + \frac{1}{n} \int_0^n \left[-\varepsilon r_p(z) + r_a(z) \right] dz \\
&\quad + \frac{1}{2} \frac{1}{n} \int_0^n \left[\varepsilon^2 \left(r_p(z) \right)^2 + \left(r_a(z) \right)^2 \right] dz \\
&\quad - \varepsilon \frac{1}{n} \int_0^n r_p(z) r_a(z) dz
\end{aligned} \tag{A.59}
$$

From the definition of the aggregate domestic price level and aggregate productivity we have,

$$
0 = \tfrac{1}{n} \left[\int_0^n R_p(z)^{1-\varepsilon} \right] dz \quad 0 = \tfrac{1}{n} \left[\int_0^n R_a(z)^{1-\varepsilon} \right] dz \tag{A.60}
$$

A log-quadratic approximation of the previous equations around their corresponding deterministic steady-state give us,

$$0 = \frac{1}{n}\left[\int_0^n r_p(z) + \frac{(1-\varepsilon)}{2} r_p^2(z)\right] dz \quad 0 = \frac{1}{n}\left[\int_0^n r_a(z) + \frac{(1-\varepsilon)}{2} r_a^2(z)\right] dz \tag{A.61}$$

Using the previous equations to eliminate $r_p(z)$ and $r_a(z)$ from equation (A.59) we obtain,

$$\begin{aligned} \Delta_t &= 1 + \frac{1}{2n}\int_0^n \left[(\varepsilon(1-\varepsilon))(r_p(z))^2 - (1-\varepsilon) r_a^2(z)\right] dz \\ &+ \frac{1}{2n}\int_0^n \left[\varepsilon^2 (r_p(z))^2 + (r_a(z))^2\right] dz \\ &- \varepsilon\frac{1}{n}\int_0^n r_p(z) r_a(z) dz \end{aligned} \tag{A.62}$$

Simplifying the previous condition we further obtain,

$$\begin{aligned} \Delta_t &= \frac{1}{2n}\int_0^n \left[\varepsilon (r_p(z))^2 + \varepsilon (r_a(z))^2\right] dz \\ &- \varepsilon\frac{1}{n}\int_0^n r_p(z) r_a(z) dz \end{aligned}$$

Which, can be easily expressed as,

$$\Delta_t = \frac{\varepsilon}{2n}\int_0^n [r_p(z) - r_a(z)]^2 dz$$

Since, $p_{h,t}(z)$ and $p_{h,t}$ have second order effects on Δ_t we only need a first order approximation of $p_{h,t}(z)$, which from the previous section is given by,

$$r_p(z) = \begin{cases} p_{1,t}(z) \\ E_{t-1}p_{1,t}(z) - (p_{h,t} - E_{t-1}p_{h,t}) \\ E_{t-1}p_{1,t}(z) - (p_{h,t} - E_{t-1}p_{h,t}) + (e_t - E_{t-1}(e_t)) \end{cases}$$

where, we denote by $p_{1,t}$ the relative optimal price under flexible prices,

$$p_{1,t}(z) = w_t + a_t + r_a(z)$$

Since, $E_{t-1}(w_t + a_t) = 0$, we have that,

$$E_{t-1}p_{1,t}(z) = E_{t-1}(r_a(z))$$

let's denote by, $\overline{p}_t = w_t + a_t$, thus we have that,

$$r_p(z) = \begin{cases} \overline{p}_t + r_a(z) \\ E_{t-1}\left(r_a(z)\right) - \left(p_{h,t} - E_{t-1}p_{h,t}\right) \\ E_{t-1}\left(r_a(z)\right) - \left(p_{h,t} - E_{t-1}p_{h,t}\right) + \left(e_t - E_{t-1}\left(e_t\right)\right) \end{cases}$$

Therefore, we have that,

$$r_p(z) - r_a(z) = \begin{cases} r_{1,t}(z) = \overline{p}_t \\ r_{2,t}(z) = -\left(r_a(z) - E_{t-1}\left(r_a(z)\right)\right) - \left(p_{h,t} - E_{t-1}p_{h,t}\right) \\ r_{3,t}(z) = -\left(r_a(z) - E_{t-1}\left(r_a(z)\right)\right) - \left(p_{h,t} - E_{t-1}p_{h,t}\right) + \left(e_t - E_{t-1}\left(e_t\right)\right) \end{cases}$$

Furthermore, we have that,

$$\begin{aligned} \int_0^n \left[r_p(z) - r_a(z)\right]^2 dz &= \int_\Theta r_{1,t}(z)^2 dz \\ + \int_{[0,n]\setminus \Sigma U \Theta} r_{2,t}(z)^2 dz & \quad + \int_\Sigma r_{3,t}(z)^2 dz \end{aligned}$$

In order to save notation, lets define by $\widetilde{x}_t = x_t - E_{t-1}\widetilde{x}_t$, then we have

$$\overline{p}_t = \frac{(1-\theta)}{\theta}\widetilde{p}_{h,t} - \frac{(1-\theta)}{\theta}s\widetilde{e}_t$$

therefore,

$$\overline{p}_t^2 = \left(\frac{(1-\theta)}{\theta}\right)^2 + s^2\widetilde{e}_t^2 - 2s\widetilde{p}_{h,t}\widetilde{e}_t$$

Aggregating we have,

$$\int_0^n \left[r_p(z) - r_a(z)\right]^2 dz = M_t + F_t + G_t$$

Where, M_t contains the quadratic terms that come from aggregate variables,

$$M_t = \theta\overline{p}_t^2 \tag{A.63}$$

Next we consider the quadratic terms specific to each group of firms,

$$F_t = (1-\theta)(1-s)\left[\widetilde{r}_a(z)^2 + \widetilde{p}_{h,t}^2\right] + 2\left(\int_\Theta \widetilde{r}_a(z)dz\right)\widetilde{p}_{h,t} \tag{A.64}$$

And finally,

$$G_t = \int_{[0,n]\setminus \Sigma U \Theta} \widetilde{r}_a(z)^2 dz + (1-\theta)s\left(\widetilde{p}_{h,t}^2 + \widetilde{e}_t^2 - 2\widetilde{p}_{h,t}\widetilde{e}_t\right) \tag{A.65}$$

$$+2\left(\widetilde{p}_{h,t} - \widetilde{e}_t\right)\left(\int_{[0,n]\setminus \Sigma U \Theta} \widetilde{r}_a(z)dz\right) \tag{A.66}$$

Adding up, equations (A.63), (A.64) and (A.65) we obtain,

$$\begin{aligned}\int_0^n \left[r_p(z) - r_a(z)\right]^2 dz &= \theta \widetilde{p}_t^2 + (1-\theta)\left(\widetilde{p}_{h,t}^2\right) \\ &\quad + (1-\theta)\, s\widetilde{e}_t^2 + \int_\Sigma \widetilde{r}_a(z)^2 dz \\ &\quad 2\int_\Sigma \widetilde{r}_a(z)\widetilde{p}_{h,t}dz - 2(1-\theta)\, s\left(\widetilde{p}_{h,t}\right)\widetilde{e}_t - 2\int_\Sigma \widetilde{r}_a(z)\widetilde{e}_t dz\end{aligned}$$

Further simplifying, we obtain,

$$\begin{aligned}\int_0^n \left[r_p(z) - r_a(z)\right]^2 dz &= \frac{(1-\theta)^2}{\theta}\left[\widetilde{p}_{h,t}^2 + s^2\widetilde{e}_t^2 - 2s\widetilde{p}_{h,t}\widetilde{e}_t\right] + \\ &\quad (1-\theta)\,\widetilde{p}_{h,t}^2 + (1-\theta)\, s\widetilde{e}_t^2 + \int_{[0,n]\setminus\Theta} \widetilde{r}_a(z)^2 dz \\ + &\quad 2\int_{[0,n]\setminus\Theta} \widetilde{r}_a(z)dz\widetilde{p}_{h,t} - 2(1-\theta)\, s\widetilde{p}_{h,t}\widetilde{e}_t - 2s\int_\Sigma \widetilde{r}_a(z)\widetilde{e}_t dz\end{aligned}$$

Simplifying the previous expression we obtain,

$$\begin{aligned}\frac{1}{2\varepsilon}\Delta_t &= \left(\frac{(1-\theta)}{\theta}\right)\widetilde{p}_{h,t}^2 + \frac{(1-\theta)}{\theta}s\left(1 + \frac{(1-\theta)}{\theta}s\right)\widetilde{e}_t^2 \\ &\quad -2s\left(\frac{(1-\theta)}{\theta}\right)\widetilde{p}_{h,t}\widetilde{e}_t + \left(\int_{[0,n]\setminus\Theta} \widetilde{r}_a(z)^2 dz\right) + \\ &\quad 2\left(\int_{[0,n]\setminus\Theta} \widetilde{r}_a(z)dz\right)\widetilde{p}_{h,t} - 2s\left(\int_\Sigma \widetilde{r}_a(z)d(z)\right)\widetilde{e}_t\end{aligned}$$

Using the following properties of large numbers we have that,

$$\int_\Theta a_t(z)d(z) = \theta a_t$$

$$\int_{[0,n]\setminus\Theta} a_t(z)d(z) = (1-\theta)\, a_t$$

therefore, it follows that,

$$\left(\int_{[0,n]\setminus\Theta} \widetilde{r}_a(z)dz\right) = (1-\theta)\, a_t - (1-\theta)\, a_t = 0$$

Consequently, we obtain the following expression for the distortion generated by the relative price dispersion,

$$\begin{aligned}\frac{1}{2\varepsilon}\Delta_t &= \frac{1}{2}\left(\frac{(1-\theta)}{\theta}\right)\widetilde{p}_{h,t}^2 + \frac{(1-\theta)}{2\theta}s\left(1+\frac{(1-\theta)}{\theta}s\right)\widetilde{e}_t^2 \\ &\quad -s\left(\frac{(1-\theta)}{\theta}\right)\widetilde{p}_{h,t}\widetilde{e}_t + \frac{1}{2}\left(\int_{[0,n]\backslash\Theta}\widetilde{r}_a(z)^2dz\right) \\ &\quad -\left(\int_{\Sigma}\widetilde{r}_a(z)d(z)\right)\widetilde{e}_t\end{aligned}$$

Plugging in the relative price dispersion equation into the welfare function derived previously, we obtain the following quadratic welfare function for this economy with price dollarisation,

$$u(C_t) - v(h_t) = \overline{u}_c\overline{Y}\left(-\frac{1}{2}(1-\gamma)(1+v)\widehat{x}_t^2 - u_\Delta\widehat{\Delta}_t\right) + t.i.p + o\left(\|\varepsilon\|^3\right) \qquad \text{(A.67)}$$

Thus the loss function of the central bank, presented in the main text as equation (3.61) can be written as,

$$W = -\Omega E_0\sum_{t=0}^{\infty}\beta^t L_t$$

$$L_t = \Lambda\frac{1}{2}\widehat{x}_t^2 + \frac{1}{2}\widetilde{p}_{h,t}^2 + \frac{\Lambda_e}{2}\widetilde{e}_t{}^2 - s\widetilde{p}_{h,t}\widetilde{e}_t - \Lambda_{ea}\widetilde{e}_t a_{s,t} + t.i.p + o\left(\|\varepsilon\|^3\right)$$

where,

$$\begin{array}{ll}\Omega = \overline{u}_c\overline{Y}(1-\gamma)\,\varepsilon\frac{(1-\theta)}{\theta} & \Lambda_{ea} = \frac{\theta}{1-\theta} \\ \Lambda = (1+v)\frac{\theta}{(1-\theta)\varepsilon} & a_{s,t} = \left(\int_\Sigma \widetilde{r}_a(z)d(z)\right)\end{array}$$

A.4 Monetary Policy Under Commitment

The central bank choose the path for the domestic inflation, the output gap and the exchange rate to minimize the following lost function

$$\min W = -\Omega E_0 \sum_{t=0}^{\infty} \beta^t L_t$$

$$L_t = \Lambda \frac{1}{2} \widehat{x}_t^2 + \frac{1}{2} \widetilde{p}_{h,t}^2 + \frac{\Lambda_e}{2} \widetilde{e}_t{}^2 - s \widetilde{p}_{h,t} \widetilde{e}_t - \Lambda_{ea} \widetilde{e}_t a_{s,t} + t.i.p + o\left(\|\varepsilon\|^3\right)$$

subject to the constraint of the Phillips curve and the dynamics of the nominal exchange rate, equations (A.68) and (A.69), presented next,

$$\widetilde{p}_{h,t} = \kappa x_t + s \widetilde{e}_t \tag{A.68}$$

$$e_t = p_{h,t} + x_t + \eta_t \tag{A.69}$$

where, $\kappa = (1+\nu) \frac{\theta}{(1-\theta)}$, denotes the slope of the Phillips curve, and η_t represents a shock to the real exchange rate that summarizes the effect of the following structural shocks on the nominal exchange rate,

$$\eta_t = -\left(a_t - a_t^*\right)$$

This shock can be also We solve for the optimal monetary policy under commitment by maximizing the following Lagrangian function, which after applying the law of iterative expectations, can be written as follows,

$$\begin{gathered} E_0 \{ \sum_{t=0}^{\infty} \{ \Lambda \tfrac{1}{2} \beta^t \widehat{x}_t^2 + \tfrac{1}{2} \beta^t \widetilde{\pi}_{h,t}^2 + \beta^t \tfrac{\Lambda_e}{2} \widetilde{e}_t{}^2 - \\ \beta^t s \widetilde{\pi}_{h,t} \widetilde{e}_t - \Lambda_{ea} \beta^t \widetilde{e}_t a_{s,t} \\ + \iota_{1,t} \beta^t \left(\pi_{h,t} - \pi_{h,t} - \kappa x_t - s\left(e_t - e_t\right)\right) + \\ + \iota_{2,t} \beta^t \left(e_t - \pi_{h,t} - x_t - \eta_t\right) \} \} \end{gathered}$$

where, $\iota_{1,t}$ and $\iota_{2,t}$ are the Lagrange multipliers of the Phillips curve and of the equation that constraints the dynamics of the nominal exchange rate. The first order conditions are given by the following three equations,

$$\Lambda \widehat{x}_t - \kappa \iota_{1,t} - \iota_{2,t} = 0 \tag{A.70}$$

$$\widetilde{\pi}_{h,t} - s \widetilde{e}_t - \iota_{2,t} = 0 \tag{A.71}$$

$$\Lambda_e \widetilde{e}_t - s \widetilde{\pi}_{h,t} - \Lambda_{ea} a_{s,t} + \iota_{2,t} = 0 \tag{A.72}$$

These conditions hold at each t, with $t \succeq 1$. They also hold at time 0, given the initial conditions,

$$\iota_{1,-1} = \iota_{2,-1}$$

The optimal plan is a set of policy functions for $\widehat{x}_t$, $\pi_{h,t}$, e_t, $\iota_{1,t}$,and $\iota_{2,t}$, that satisfy conditions,(A.68),(A.69),

(A.70),(A.71),and (A.72), given the initial conditions and the processes for the exogenous variables, $a_{s,t}$, and η_t. To find the allocation under optimal policy we combine equations (A.71) and (A.72) to eliminate, $\iota_{2,t}$ as follows,

$$(\Lambda_e - s)\,\widetilde{e}_t - \Lambda_{ea} a_{s,t} + (1-s)\,\widetilde{\pi}_{h,t} = 0 \tag{A.73}$$

the remaining equations that define the economy are given by,

$$\Lambda \widehat{x}_t - \kappa \iota_{1,t} - \widetilde{\pi}_{h,t} + s\widetilde{e}_t = 0 \tag{A.74}$$

$$e_t = \pi_{h,t} + x_t + \eta_t \tag{A.75}$$

$$\widetilde{\pi}_{h,t} = \kappa x_t + s\widetilde{e}_t \tag{A.76}$$

Since from the Phillips curve we have that, $E_{t-1}x_t = 0$, thus we can write equation (A.75) as follows,

$$\widetilde{e}_t = \widetilde{\pi}_{h,t} + x_t + \widetilde{\eta}_t \tag{A.77}$$

using the previous equation and the Phillips curve we can eliminate the output gap,thus we have, a second condition that relates exchange rate and domestic prices,

$$(\kappa + s)\,\widetilde{e}_t = (1+\kappa)\,\widetilde{\pi}_{h,t} + \kappa\widetilde{\eta}_t \tag{A.78}$$

combining equations (A.73) and (A.78) we solve for the exchange rate and level of domestic prices,

$$\widetilde{e}_t = \varpi_1 \widetilde{\eta}_t + \varpi_2 a_{s,t} \tag{A.79}$$

where,

$$\varpi_1 = \frac{\kappa}{\left[(\kappa+s) + \frac{(1+\kappa)}{(1-s)}(\Lambda_e - s)\right]} \quad \varpi_2 = \frac{\Lambda_{ea}}{\left[\frac{(1-s)}{(1+\kappa)}(\kappa+s) + (\Lambda_e - s)\right]}$$

using, equation (A.73) and equation (A.79) we can find the rational equilibrium of prices,

$$\widetilde{p}_{h,t} = -\varpi_3 \widetilde{\eta}_t + \varpi_4 a_{s,t} \tag{A.80}$$

where the parameters, ϖ_3 and ϖ_4 are defined as follows,

$$\varpi_3 = \frac{\kappa(\Lambda_e + s)}{[(\kappa+s)(1-s) + (1+\kappa)(\Lambda_e - s)]} \quad \varpi_4 = \frac{(\kappa+s)\Lambda_{ea}}{[(1+s)(\kappa-s) + (1+\kappa)(\Lambda_e - s)]}$$

A.5 Endogenous Price Dollarisation

Deriving The Profit Function

In this appendix we derive equations (??) and (3.29) of section 3.4. The firm's expected discounted profit function under peso invoicing is given by,

$$\Omega(z) = E_{t-1}\left[\left(P_{H,t}(z) - W_t A_t(z)\right) P_{H,t}^{-\varepsilon}(z) P_{H,t}^{\varepsilon-1}\right] \tag{A.81}$$

and the corresponding first-order condition for its optimal price is given by

$$E_{t-1}\left[\left(P_{H,t}(z)-\mu W_t A_t(z)\right)P_{H,t}^{\varepsilon-1}\right]=0 \tag{A.82}$$

Using equation (3.24) we eliminate $P_{H,t}(z)$ from equation (A.81), thereby, firm's expected profits can be written as follows,

$$\Omega\left(z\right)=\left(\mu-1\right)\mu^{-\varepsilon}\frac{\left(E_{t-1}\left(W_t A_t(z)\,P_{H,t}^{\varepsilon-1}\right)\right)^{1-\varepsilon}}{\left(E_{t-1}\left(P_{H,t}^{\varepsilon-1}\right)\right)^{-\varepsilon}} \tag{A.83}$$

In terms of real wages, it can be alternative expressed as follows,

$$\Omega\left(z\right)=\left(\mu-1\right)\mu^{-\varepsilon}\frac{\left(E_{t-1}\left(w_t A_t(z)\,P_{H,t}^{\varepsilon}\right)\right)^{1-\varepsilon}}{\left(E_{t-1}\left(P_{H,t}^{\varepsilon-1}\right)\right)^{-\varepsilon}} \tag{A.84}$$

Following similar steps, we obtain the expected discount profit function under dollar invoicing, which is given by the next equation,

$$\Psi\left(z\right)=\left(\mu-1\right)\mu^{-\varepsilon}\frac{\left(E_{t-1}\left(W_t e_t^{-1}A_t(z)\,d_{H,t}^{\varepsilon}\right)\right)^{1-\varepsilon}}{\left(E_{t-1}\left(d_{H,t}^{\varepsilon-1}\right)\right)^{-\varepsilon}} \tag{A.85}$$

To ease the comparison between $\Psi\left(z\right)$ and $\Omega\left(z\right)$, we write $\Psi\left(z\right)$ in terms of $P_{H,t}$ and e_t, as follows,

$$\Psi\left(z\right)=\left(\mu-1\right)\mu^{-\varepsilon}\frac{\left(E_{t-1}\left(W_t A_t(z)\,P_{H,t}^{\varepsilon-1}e_t^{-\varepsilon}\right)\right)^{1-\varepsilon}}{\left(E_{t-1}\left(P_{H,t}^{\varepsilon-1}e_t^{1-\varepsilon}\right)\right)^{-\varepsilon}} \tag{A.86}$$

Equation (A.84) correspond to equation (??) , whereas equation (A.85) to equation (3.29), in the main text.

Equilibrium Condition for PD

After taking a log-quadratic approximation of equation (3.29) around the corresponding expected values of variables that determine $\Omega\left(z\right)$ we obtain ,

$$\begin{aligned}
\log(\Omega\left(z\right)) \simeq\; & -\left(\varepsilon-1\right)\left(\left(\varepsilon-1\right)E_{t-1}\widetilde{p}_{h,t}+\frac{\left(\varepsilon-1\right)^2}{2}E_{t-1}\widetilde{p}_{h,t}^2\right) \\
& -\left(\varepsilon-1\right)\left(E_{t-1}\widetilde{W}_t+E_{t-1}\widetilde{a}_t(z)+\frac{1}{2}E_{t-1}\widetilde{W}_t^2+\frac{1}{2}E_{t-1}\widetilde{a}_t^2(z)\right) \\
& -\left(\varepsilon-1\right)\left(\left(\varepsilon-1\right)E_{t-1}\left[\left(\widetilde{W}_t+\widetilde{a}_t(z)\right)\widetilde{p}_{h,t}\right]\right) \\
& \varepsilon\left(\left(\varepsilon-1\right)E_{t-1}\widetilde{p}_{h,t}+\frac{\left(\varepsilon-1\right)^2}{2}E_{t-1}\widetilde{p}_{h,t}^2\right)+\log(\overline{\Omega\left(z\right)})
\end{aligned}$$

where variables with tilde $\widetilde{x}_t$, represent log deviations from expected values, i.e $\widetilde{x}_t = \log(X_t) - E_{t-1}(\log(X_t))$ Similarly, the log-quadratic approximation of $\Psi(z)$ around the same steady-state is given by

$$\begin{aligned}
\log(\Psi(z)) \simeq\ & \log\overline{\Psi(z)} - (\varepsilon-1)\left((\varepsilon-1)E_{t-1}\widetilde{p}_{h,t} + \frac{(\varepsilon-1)^2}{2}E_{t-1}\widetilde{p}_{h,t}^2\right) \\
& -(\varepsilon-1)\left(-\varepsilon E_{t-1}\widetilde{e}_t + \frac{\varepsilon^2}{2}E_{t-1}\widetilde{e}_t^2\right) \\
& -(\varepsilon-1)\left(E_{t-1}\widetilde{W}_t + E_{t-1}\widetilde{a}_t(z) + \frac{1}{2}E_{t-1}\widetilde{W}_t^2 + \frac{1}{2}E_{t-1}\widetilde{a}_t^2(z)\right) \\
& -(\varepsilon-1)\left((\varepsilon-1)E_{t-1}\left[\left(\widetilde{W}_t + \widetilde{a}_t(z)\right)\widetilde{p}_{h,t}\right]\right) \\
& -(\varepsilon-1)\left(-\varepsilon E_{t-1}\left[\left(\widetilde{W}_t + \widetilde{a}_t(z)\right)\widetilde{e}_t\right] - (\varepsilon-1)\varepsilon E_{t-1}\left[\widetilde{e}_t\widetilde{p}_{h,t}\right]\right) \\
& +\varepsilon\left((\varepsilon-1)E_{t-1}\widetilde{p}_{h,t} + \frac{(\varepsilon-1)^2}{2}E_{t-1}\widetilde{p}_{h,t}^2\right) \\
& +\varepsilon\left((1-\varepsilon)E_{t-1}\widetilde{e}_t + \frac{(1-\varepsilon)^2}{2}E_{t-1}\widetilde{e}_t^2\right) - \varepsilon(1-\varepsilon)^2 E_{t-1}\left[\widetilde{e}_t\widetilde{p}_{h,t}\right]
\end{aligned}$$

Then, $\log\Psi(z) - \log\Omega(z)$ is given by

$$\log\Psi(z) - \log\Omega(z) \simeq E_{t-1}\left(\frac{(1-\varepsilon)\varepsilon}{2}E_{t-1}\widetilde{e}_t^2\right) - (1-\varepsilon)\varepsilon E_{t-1}\left[\left(\widetilde{W}_t + \widetilde{a}_t(z)\right)\widetilde{e}_t\right]$$

Therefore a firm will set prices in dollars when, $\widehat{\Psi(z)} - \widehat{\Omega(z)} < 0$, which holds, when,

$$\frac{E_{t-1}\widetilde{e}_t^2}{2} - E_{t-1}\left[\left(\widetilde{W}_t + \widetilde{a}_t(z)\right)\widetilde{e}_t\right] < 0$$

since we know that,

$$\widehat{W}_t - E_{t-1}\widehat{W}_t = a_t - E_{t-1}a_t + \frac{1}{\theta}\left(p_{h,t} - E_{t-1}(p_{h,t})\right) - \frac{(1-\theta)}{\theta}s\left(e_t - E_{t-1}(e_t)\right)$$

and that,$\widetilde{x}_t = \widehat{x}_t - E_{t-1}\widehat{x}_t$ A firm will set prices in dollars if and only if

$$\left(\frac{1}{2} + s\frac{(1-\theta)}{\theta}\right)E_{t-1}\widetilde{e}_t^2 - E_{t-1}\left[\widetilde{e}_t, \left(\widetilde{a}_t(z) - \widetilde{a}_t\right)\right] - \frac{1}{\theta}E_{t-1}\left(\widetilde{p}_{h,t}\widetilde{e}_t\right) < 0$$

This last equation corresponds to equation (3.70) in the main text,

B APPENDIXES OF CHAPTER 4

B.1 The Foreign Economy

Household's preferences at the foreign economy are given by the following utility function,

$$U_t = E_t \left[\sum_{k=0}^{\infty} \beta^k \left(\frac{C_{t+k}^{*1-\sigma}}{1-\sigma} \right) - \frac{1}{1+\varphi} L_{t+k}^{*1+\varphi} \right] \tag{B.1}$$

Where E_t represents the expectations operator conditional on information up to period t, $\beta \in \langle 0, 1 \rangle$, is the household's subjective discount factor, $\sigma > 0$, the coefficient of risk aversion and $\varphi > 0$, the inverse of the Frish labor supply elasticity, L_t^* stands for the foreign household's working hours and C_t^* for their corresponding consumption level of a final consumption goods, which is defined as follows,

$$\ln C_t^* = \left(\int_0^1 \ln C_t^* (s) \, d(s) \right) \tag{B.2}$$

For simplicity we assume that the foreign economy is a cashless economy. Hence, final good's consumption is not subject to any type of transaction frictions. On the other hand, final good producers's technology in the foreign economy is characterised by the following equations,:

$$Y_t^{j*} = \left((\alpha^*)^{\frac{1}{\eta}} \left(Y_{H,t}^* \right)^{\frac{\eta-1}{\eta}} + (1-\alpha^*)^{\frac{1}{\eta}} \left(Y_{F,t}^* \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{B.3}$$

$$Y_{H,t}^{j*} = \left(\left(\tfrac{1}{n} \right)^{\frac{1}{\epsilon}} \int_0^n Y_{H,t}^* (z)^{\frac{\epsilon-1}{\epsilon}} d(z) \right)^{\frac{\epsilon}{\epsilon-1}} \quad Y_{F,t}^{j*} = \left(\left(\tfrac{1}{1-n} \right)^{\frac{1}{\epsilon}} \int_n^1 Y_{F,t}^* (z)^{\frac{\epsilon-1}{\epsilon}} d(z) \right)^{\frac{\epsilon}{\epsilon-1}} \tag{B.4}$$

and the corresponding demands for domestic and foreign intermediate goods are given by:

$$Y_{H,t}^{j*}(z) = \alpha^* \left(\frac{P_{H,t}^*(z)}{P_{H,t}^*} \right)^{-\varepsilon} \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\eta} Y_t^{j*} \tag{B.5}$$

$$Y_{F,t}^{q*}(z) = (1-\alpha^*) \left(\frac{P_{H,t}^*(z)}{P_{H,t}^*} \right)^{-\varepsilon} \left(\frac{P_{F,t}^*}{P_t^*} \right)^{-\eta} Y_t^{q*} \tag{B.6}$$

Associated to this technology, the corresponding marginal cost of final goods producers, which also represents the price of consumption goods, is given by,

$$P_t^* = \left(\alpha^* \left(P_{H,t}^* \right)^{1-\eta} + (1-\alpha^*) \left(P_{F,t}^* \right)^{1-\eta} \right)^{\frac{1}{1-\eta}} \tag{B.7}$$

where

$$P_{H,t}^{*}=\left(\frac{1}{n}\int_{0}^{n}\left(P_{H,t}^{*}(z)\right)^{1-\epsilon}dz\right)^{\frac{1}{1-\epsilon}}\quad P_{F,t}^{*}=\left(\frac{1}{n}\int_{0}^{n}\left(P_{F,t}^{*}(z)\right)^{1-\varepsilon}dz\right)^{\frac{1}{1-\epsilon}}\tag{B.8}$$

The set of non-linear equations that describe the behaviour of the foreign economy is given by:

Table B.1: Non Linear equations

Phillips Curve	Marginal Cost
$\theta\left(\pi_t^{*}\right)^{\epsilon-1}=1-(1-\theta)\left(\frac{N_t^{*}}{D_t^{*}}\right)^{1-\epsilon}$	$mc_t^{*}=\frac{Y_t^{*\varphi}}{\lambda_t^{*}A_t^{*1+\varphi}}$
$N_t^{*}=\mu\left(Y_t^{*}\right)^{-\sigma}mc_t^{*}Y_t^{*}+\theta\beta\left(\pi_{t+1}^{*}\right)^{\epsilon}N_{t+1}^{*}$	Taylor Rule
$D_t^{*}=\left(Y_t^{*}\right)^{-\sigma}Y_t^{*}+\theta\beta\left(\pi_{t+1}^{*}\right)^{\epsilon-1}D_{t+1}^{*}$	$(1+i_t^{*})=\overline{(1+i_t)}\left(\frac{\pi_t^{*}}{\overline{\pi}^{*}}\right)^{\phi_\pi}\left(\frac{y_t^{*}}{\overline{y}_t^{*}}\right)^{\phi_y}\exp\left(v_t^{*}\right)$
Euler Equation	Marginal Utility
$\frac{1}{1+i_t^{*}}=\beta E_t\left(\frac{\lambda_{t+1}^{*}}{\lambda_t^{*}}\frac{1}{\pi_{t+1}^{*}}\right)$	$Y_t^{*-\sigma}=\lambda_t^{*}$

B.2 The Phillips Curve

Intermediate good producers update their prices randomly. At each updating point they choose the price level, $P_{H,t}^{o}(z)$ that maximises the following profit function:

$$E_t\left[\sum_{k=0}^{\infty}(\theta\beta)^k\left(\lambda_{t+k}\left(\frac{P_{H,t}^{o}(z)}{P_{H,t+k}}-mc_{t+k}\right)\widetilde{Y}_{H,t+k}(z)\right)\right]\tag{B.9}$$

To write the firm's profit function in terms of its contemporaneous relative price, it is useful to denote by Ψ_{t+k} the cumulative domestic inflation,$\Psi_{t+k}=\frac{P_{H,t}}{P_{H,t+k}}$. Using this auxiliary variable, firm's demand, conditioned on the optimal price can be written as follows,

$$\widetilde{Y}_{H,t+k}(z)=\left(\frac{P_{H,t}(z)}{P_{H,t}}\right)^{-\epsilon}\Psi_{t+k}^{-\epsilon}Y_{H,t+k}\tag{B.10}$$

Hence, the firm's first-order condition to determine its optimal relative prices is given by,:

$$E_t\left[\sum_{k=0}^{\infty}(\theta\beta)^k\left(C_{t+k}^{-\sigma}\left(\frac{P_{H,t}^{o}(z)}{P_{H,t}}\Psi_{t+k}-\frac{\epsilon}{(\epsilon-1)}mc_{t+k}\right)\widetilde{Y}_{H,t+k}(z)\right)\right]=0\tag{B.11}$$

From this latter expression, firm's optimal price as defined as follows:

$$\frac{P_{H,t}^{o}(z)}{P_{H,t}} = \frac{\epsilon}{(\epsilon-1)} \frac{E_t\left[\sum\limits_{k=0}^{\infty} (\theta\beta)^k \left(\lambda_{t+k} mc_{t+k} \Psi_{t+k}^{-\epsilon} Y_{H,t+k}\right)\right]}{E_t\left[\sum\limits_{k=0}^{\infty} (\theta\beta)^k \left(\lambda_{t+k} \Psi_{t+k}^{1-\epsilon} Y_{H,t+k}\right)\right]} \tag{B.12}$$

A more tractable representation of the firm's optimal price can be obtained by using two auxiliary variables, N_t and D_t, as in ?. This two auxiliary variables represent the numerator and denominator of the previous equation, as follows

$$N_t = \mu E_t \left[\sum_{k=0}^{\infty} (\theta\beta)^k \left(\lambda_{t+k} mc_{t+k} \Psi_{t+k}^{-\epsilon} Y_{H,t+k}\right)\right] \tag{B.13}$$

and

$$D_t = E_t \left[\sum_{k=0}^{\infty} (\theta\beta)^k \left(\lambda_{t+k} \Psi_{t+k}^{1-\epsilon} Y_{H,t+k}\right)\right] \tag{B.14}$$

Hence, the intermediate good producers' optimal price will be given by:

$$\frac{P_{H,t}^{o}(z)}{P_{H,t}} = \frac{N_t}{D_t} \tag{B.15}$$

where N_t and D_t can be written recursively as follows,

$$N_t = \mu \lambda_t mc_t Y_{H,t} + \theta\beta\pi_{H,t+1}^{\epsilon} N_{t+1} \tag{B.16}$$

$$D_t = \lambda_t Y_{H,t} + \theta\beta\pi_{H,t+1}^{\epsilon-1} D_{t+1} \tag{B.17}$$

On the other hand, since firms have the same cost structure, all of them choose the same optimal price $P_{H,t}^{o}(z)$ at every updating point, whereas the remaining firms maintain fixed their corresponding prices. This imply that upon aggregation, domestic inflation is determined by the following non-linear condition,

$$\theta \left(\pi_{H,t}\right)^{\epsilon-1} = 1 - (1-\theta) \left(\frac{P_{H,t}^{o}(z)}{P_{H,t}}\right)^{1-\epsilon} \tag{B.18}$$

Which, can also be written as follows:

$$\theta \left(\pi_{H,t}\right)^{\epsilon-1} = 1 - (1-\theta) \left(\frac{N_t}{D_t}\right)^{1-\epsilon} \tag{B.19}$$

B.3 Aggregating Consumption Decisions

In this appendix we derive a close form expression for the marginal utility of income, λ_t , which we use later to show the implications of CS for the dynamic equilibrium of the SOE. To achieve this objective, we aggregate the optimal consumption conditions of different varieties of final consumption goods, summarised by the following set of equations

$$U_{c,t}\frac{\partial c_t}{\partial c_t(s)} = P_t(s)\lambda_t\left(1+\frac{q_t}{\lambda_t}\right)(1+g(s)) \text{ for } s \geqslant \overline{s}_t \tag{B.20}$$

$$U_{c,t}\frac{\partial c_t}{\partial c_t(s)} = P_t(s)\lambda_t\left(1+\frac{n_t}{\lambda_t}\right)(1+\tau(s)) \text{ for } s < \overline{s}_t \tag{B.21}$$

Also, from the equilibrium condition for CS we have that,

$$\frac{1+\tau(\overline{s}_t)}{1+g(\overline{s}_t)} = \frac{1+\frac{q_t}{\lambda_t}}{1+\frac{n_t}{\lambda_t}} = \frac{1+i_t}{1+i_t^*} \tag{B.22}$$

Since final good's prices are the same in equilibrium, equations (B.20) and (B.21) can be written as:

$$U_{c,t} = P_t\frac{c_t(s)}{c_t}\lambda_t\left(1+\frac{q_t}{\lambda_t}\right)(1+g(s)) \text{ for } s \geqslant \overline{s}_t \tag{B.23}$$

$$U_{c,t} = P_t\frac{c_t(s)}{c_t}\lambda_t\left(1+\frac{n_t}{\lambda_t}\right)(1+\tau(s)) \text{ for } s < \overline{s}_t \tag{B.24}$$

To facilitate aggregation we take logs to equations (B.23) and (B.24) and then integrate over s . The resulting condition is,

$$\log U_{c,t} = \log(P_t\lambda_t) + (1-\overline{s}_t)\log\left(1+\tfrac{q_t}{\lambda_t}\right) + \int_{\overline{s}_t}^{1}\log(1+g(s))\,ds$$

$$+\overline{s}_t\log\left(1+\tfrac{\eta_t}{\lambda_t}\right) + \int_{0}^{\overline{s}_t}\log(1+\tau(s))\,ds$$

We take logs to equation (B.22) to obtain an expression for $\frac{\eta_t}{\lambda_t}$, which is given by,

$$\log\left(1+\frac{\eta_t}{\lambda_t}\right) = \log\left(1+\frac{q_t}{\lambda_t}\right) - \log(1+\tau(\overline{s}_t)) + \log(1+g(\overline{s}_t)) \tag{B.25}$$

Using equation (B.25) we eliminate $\frac{\eta_t}{\lambda_t}$ from the marginal utility of consumption, obtaining the following expression,

$$\log U_{c,t} = \log(P_t\lambda_t) + \log\left(1+\tfrac{q_t}{\lambda_t}\right) +$$

$$\int_{\overline{s}_t}^{1}\log\tfrac{(1+g(s))}{(1+g(\overline{s}_t))}ds + \int_{0}^{\overline{s}_t}\log\tfrac{(1+\tau(s))}{(1+\tau(\overline{s}_t))}ds + \log(1+g(\overline{s}_t))$$

Hence, by taking antilog to the previous equation, the the marginal utility of aggregate consumption can be written as follows:

$$U_{c,t} = P_t \lambda_t \left(1 + \Upsilon_t\right) \tag{B.26}$$

where, Υ_t represents the aggregate distortion generated by transaction costs. This new variable depends on both the domestic interest rate and the degree of CS, as follows,

$$\left(1 + \Upsilon_t\right) = \left(1 + i\right)\left(1 + \Gamma\left(\overline{s}_t\right)\right) \tag{B.27}$$

where

$$\begin{aligned}\left(1 + \Gamma\left(\overline{s}_t\right)\right) &= \exp(\int_{\overline{s}_t}^{1} \log \frac{\left(1 + g\left(s\right)\right)}{\left(1 + g\left(\overline{s}_t\right)\right)} ds + \\ &\quad \int_{\overline{s}_t}^{1} \log \frac{\left(1 + \tau\left(s\right)\right)}{\left(1 + \tau\left(\overline{s}_t\right)\right)} ds + \log\left(1 + g\left(\overline{s}_t\right)\right))\end{aligned}$$

By using the following functional forms for transaction costs,

$$\tau\left(s_t\right) = \exp(\Psi_o + \Psi_1 s_t) - 1 \tag{B.28}$$

$$g\left(s_t\right) = \exp\left(n_o + n_1 s_t\right)) - 1 \tag{B.29}$$

where, $\Psi_1 > n_1$ $n_o > \Psi_o$, the three components of $\left(1 + \Gamma\left(\overline{s}_t\right)\right)$ are given by,

$$\begin{aligned}\int_{\overline{s}_t}^{1} \log \frac{\left(1 + g\left(s\right)\right)}{\left(1 + g\left(\overline{s}_t\right)\right)} ds &= \int_{\overline{s}_t}^{1} n_1\left(s_t - \overline{s}_t\right) ds = \\ \left[\Psi_1 \left(\frac{s_t^2}{2} - s_t \overline{s}_t\right)\right]_{\overline{s}_t}^{1} &= n_1 \left(\frac{1}{2} - \overline{s}_t + \frac{\overline{s}_t^2}{2}\right)\end{aligned}$$

Similarly, we have,

$$\int_{0}^{\overline{s}_t} \log \frac{\left(1 + \tau\left(s\right)\right)}{\left(1 + \tau\left(\overline{s}_t\right)\right)} ds = \int_{\overline{s}_t}^{1} \Psi_1\left(s_t - \overline{s}_t\right) ds = -\Psi_1 \frac{\overline{s}_t^2}{2} \tag{B.30}$$

$$\log\left(1 + g\left(\overline{s}_t\right)\right) = n_o + n_1 \overline{s}_t \tag{B.31}$$

Therefore, we have that

$$1 + \Gamma\left(\overline{s}_t\right) = \exp\left(\frac{n_1}{2} + n_o - \left(\Psi_1 - n_1\right)\frac{\overline{s}_t^2}{2}\right) \tag{B.32}$$

B.4 The Non Linear Economy

The home economy is fully characterised by the following set of non-linear difference equations:

Non linear equations

Phillips Curve	Terms of Trade
$\theta\left(\Pi_{H,t}\right)^{\epsilon-1}=1-(1-\theta)\left(\frac{N_t}{D_t}\right)^{1-\epsilon}$	$\left(\frac{Q_t}{T_t}\right)^{\eta-1}=(1-\gamma)+\gamma T_t^{1-\eta}$
$N_t=\mu\lambda_t mc_t Y_t+\theta\beta\left(\Pi_{H,t+1}\right)^{\epsilon}N_{t+1}$	CPI inflation
$D_t=\lambda_t Y_t+\theta\beta\left(\Pi_{H,t+1}\right)^{\epsilon-1}D_{t+1}$	$\left(\frac{\Pi_t}{\Pi_{H,t}}\right)^{1-\eta}=\frac{(1-\gamma)+\gamma T_t^{1-\eta}}{(1-\gamma)+\gamma T_{t-1}^{1-\eta}}$
Euler Equation	Taylor Rule
$\frac{1}{1+i_t}=\beta E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\frac{1}{\Pi_{t+1}}\right)$	$(1+i_t)=\bar{i}\left(\frac{\Pi_{H,t}}{\overline{\pi}}\right)^{\phi_\pi}\left(\frac{Y_t}{\overline{Y}_t}\right)^{\phi_x}$
Aggregate Demand	Marginal Utility of Consumption
$Y_t=\left(\frac{Q_t}{T_t}\right)^{-\eta}\left((1-\gamma)C_t+\gamma Q_t^{\eta}Y_t^*\right)$	$C_t^{-\sigma}=\lambda_t\left(1+\Upsilon_t\right)$
Risk Sharing	Transaction cost distortion
$Q_t=\varsigma_0\frac{\lambda_t^*}{\lambda_t}$	$1+\Upsilon_t=\left(1+\frac{i_t}{(1+i_t)}\right)\left(1+\Gamma\left(\overline{s}_t\right)\right)$
Marginal Cost	CS distortion
$mc_t=\frac{T_t Y_t^{\varphi}}{\lambda_t Q_t A_t^{1+\varphi}}$	$1+\Gamma\left(\overline{s}_t\right)=\exp\left(\frac{n_1}{2}+n_o-\left(\Psi_1-n_1\right)\frac{\overline{s}_t^2}{2}\right)$
Demand for money	Dollar transaction cost
$\frac{M_t}{P_t}=\int_0^{\overline{s_t}}C_t(s)\,d(s)$	$\tau\left(s_t\right)=\exp(\Psi_o+\Psi_1 s_t)-1$
Peso transaction costs	CS condition
$g\left(s_t\right)=\exp\left(n_o+n_1 s_t\right))-1$	$\frac{1+\tau(\overline{s}_t)}{1+g(\overline{s}_t)}=\frac{1+i_t}{1+i_t^*}$

Next we reduce the previous system by combining some of its equations. First, we use the marginal utility of consumption to eliminate, λ_t from three equations, the Euler equation, the marginal cost and the risk sharing condition, the resulting conditions are given by the following equations,

$$\frac{1}{1+i_t}=\beta E_t\left(\left(\frac{C_{t+1}}{C_t}\right)^{-1}\frac{1+\Upsilon_t}{1+\Upsilon_{t+1}}\frac{1}{\Pi_{t+1}}\right)\tag{B.33}$$

$$MC_t=\frac{T_t Y_t^{\varphi}C_t\left(1+\Upsilon_t\right)}{Q_t A_t^{1+\varphi}}\tag{B.34}$$

$$Q_t=\frac{C_t\left(1+\Upsilon_t\right)}{Y_t^*}\tag{B.35}$$

Next, we impose $\eta=1$, hence, the terms of trade and CPI inflation equations become,

$$Q_t=T_t^{1-\gamma}\tag{B.36}$$

$$\Pi_t = \Pi_{H,t}\left(\frac{T_t}{T_{t-1}}\right)^{\gamma} \tag{B.37}$$

Domestic aggregate demand can be further simplified by using equation (B.35) as follows,

$$Y_t = \left(\frac{Q_t}{T_t}\right)^{-1}(1+\gamma\Upsilon_t)\,C_t \tag{B.38}$$

From the previous equation we eliminate the real exchange rate by using (B.36), thus domestic output would depend only on terms of trade, domestic consumption and the transaction friction distortion, as follows,

$$Y_t = T_t^{\gamma}(1+\gamma\Upsilon_t)\,C_t \tag{B.39}$$

On th other hand, by plugging in equation (B.39) and (B.37) into equation (B.33), we obtain the dynamic IS , equation (4.63) of the main text,

$$\frac{1}{1+i_t} = \beta E_t\left(\left(\frac{Y_{t+1}}{Y_t}\right)^{-1}\frac{1+\Upsilon_t}{1+\Upsilon_{t+1}}\frac{1+\gamma\Upsilon_{t+1}}{1+\gamma\Upsilon_t}\frac{1}{\Pi_{t+1}}\right) \tag{B.40}$$

Next, we use equation (B.39) and (B.36) to eliminate, C_t and Q_t and T_t from equation (B.34), we obtain the following condition for the marginal costs,

$$MC_t = \frac{Y_t^{1+\varphi}}{A_t^{1+\varphi}}\left(\frac{1+\gamma\Upsilon_t}{1+\Upsilon_t}\right)\Delta_t^{\varphi} \tag{B.41}$$

which corresponds to equation (4.61) of the main text. Notice that, Δ_t measures price dispersion generated by price stickiness,

$$\Delta_t = \int_0^n \left(\frac{P_t(z)}{P_t}\right)^{-\theta} dz$$

from equation (B.34), we can further obtain consumption in terms of the real exchange rate as follows

$$C_t = \frac{Q_t Y_t^*}{(1+\Upsilon_t)}$$

and by plugging in the consumption level obtained in the previous equation into equation (B.39), we obtain terms of trade as function of domestic and foreign output and the transaction distortion.

$$T_t = \frac{Y_t(1+\Upsilon_t)}{Y_t^*(1+\gamma\Upsilon_t)} \tag{B.42}$$

Notice that when, $\Upsilon_t = 0$, terms of trade are determined only by relative levels of output as in standard S.O.E models. We use equations (B.42) and (B.39) to derive an expression that determines the level of consumption in terms of domestic and foreign output and the transaction distortion.

$$C_t = Y_t^{1-\gamma}(Y_t^*)^{\gamma}\left(\frac{1}{1+\gamma\Upsilon_t}\right)^{1-\gamma}\frac{1}{(1+\Upsilon_t)^{\gamma}} \tag{B.43}$$

Finally, using the definition of λ_t we obtain,

$$\lambda_t = \frac{1}{Y_t^{1-\gamma}(Y_t^*)^\gamma}\left(\frac{1+\gamma\Upsilon_t}{(1+\Upsilon_t)}\right)^{1-\gamma} \tag{B.44}$$

We plug in this expression in equations that define the Phillips curve to obtain equations (4.58) and (4.59) of the main text.

B.5 The Log Linear System of Equations

Combining the log linear approximated equations (4.58), (4.59) and (4.60) of section 3, we obtain,

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \lambda mc_t \tag{B.45}$$

where, $\lambda = \frac{(1-\beta\theta)(1-\theta)}{\theta}$. The corresponding log linear approximation of equation (4.61) is given by,

$$mc_t = (1+\phi)(y_t - a_t) + (1-\gamma)\vartheta v_t \tag{B.46}$$

by plugging in combining equation (B.46) into equation (B.45) we obtain the following linear representation of the Phillips curve,

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa(y_t - a_t) + \lambda(1-\gamma)\vartheta v_t \tag{B.47}$$

On the other hand, the dynamic aggregate demand condition in its log linear form is obtained from equation (4.63),

$$i_t = E_t \Delta y_{t+1} + \pi_{H,t+1} + (1-\gamma)\vartheta \Delta v_{t+1} \tag{B.48}$$

The Dynamics of the Flexible Price Allocation

When prices are fully flexible marginal costs are constant, therefore, up to a log linear approximation, $mc_t^n = 0$. Imposing this condition in equation (B.46), we obtain the following definition for the natural level of output,

$$y_t^n = a_t - \frac{(1-\gamma)\vartheta}{1+\phi} v_t \tag{B.49}$$

This equation corresponds to equation (4.65) in section 3.1. Similarly, using equation (B.48) we derive the natural interest rate,

$$r_t^n = E_t \Delta y_{t+1}^n + (1-\gamma)\vartheta \Delta v_{t+1} \tag{B.50}$$

Notice that both the natural level of output and the natural interest rate depend on the distortions generated by transactions frictions. Hence, the flexible price allocation is not efficient. To achieve the efficient allocation we assume similarly to Woodford (2003) that the Central bank has additional policy instruments that it can use to offset the impact of transaction frictions on the economy's flexible price allocation. In particular we assume that the central bank pays interests on domestic money holdings, i_t^m and that it taxes foreign currency holdings, τ_t^m . Under these assumptions, v_t and s will be given by,

$$v_t = \omega \left((1-s) \left(i_t - i_t^m \right) + s \left(i_t^* + \tau_t^m \right) \right) \tag{B.51}$$

$$\overline{s} = \frac{\left(n_0 - \Psi_0 + \log \left(\frac{1+\frac{i-i^m}{(1+i)}}{1+\frac{i^*+\tau^m}{(1+i^*)}} \right) \right)}{(\Psi_1 - n_1)} \tag{B.52}$$

By setting, $i_t = i_t^m$, $i = i^m$ and $\tau^m = \frac{1}{\beta} \left(2 - \exp \left(n_0 - \Psi_0 \right) \right) - 1$, it holds that, $\overline{s} = 0$, and consequently that, v_t. When these conditions hold, transaction frictions do not affect the dynamic equilibrium under flexible prices, thus, the level of output become efficient,

$$y_t^e = a_t$$

This latter equation corresponds to equation (4.69) in section 3.2. By defining the efficient output gap, as $x_t = y_t - a_t$, and by using equations (B.47) and (B.51), we obtain equation (4.71) of section 3.3 . Similarly, by subtracting equations (B.48) and (B.50) we obtain equation (4.70) of section 3.3.

B.6 The Central bank Loss Function

To obtain the welfare based central bank's objective function we approximate up to second-order the domestic household's utility function, U_t

$$U_t = E_t \left[\sum_{k=0}^{\infty} \beta^k \left[U \left(C_{t+k} \right) - V(L_{t+k}) \right] \right] \tag{B.53}$$

This approximation is taken around a deterministic steady-state where there exist a positive but small level of CS. By choosing this particular steady-state we make explicit the effects of transaction costs and CS on household's welfare. Before approximating the utility function, however, it is useful to write it in terms of domestic output. In doing so we use equations (B.54) and (B.55), which link consumption and labour to domestic and foreign output and two additional terms associated to transaction costs and relative price distortions, Υ_t and Δ_t, as follows,

$$C_t = Y_t^{1-\gamma} \left(Y_t^* \right)^{\gamma} G \left(\Upsilon_t \right) \tag{B.54}$$

$$L_t = \frac{Y_t \Delta_t}{A_t} \tag{B.55}$$

where,

$$G(\Upsilon_t) = \left(\frac{1}{1+\gamma\Upsilon_t}\right)^{1-\gamma}\left(\frac{1}{(1+\Upsilon_t)}\right)^{\gamma} \tag{B.56}$$

and Υ_t is defined by equations (4.16), (4.17) and (4.9). Similarly, Δ_t measures the distortions that relative price dispersion generate on labour usage. This latter variable is defined as follows,

$$\Delta_t = \int_0^n \left(\frac{P_{H,t}(z)}{P_t}\right)^{-\theta} dz \tag{B.57}$$

Under the assumption of log utility and using equations (B.54) and (B.56), $U(C_t)$ can be written as follows,

$$U(C_t) = (1-\gamma)\ln(Y_t) + \gamma \ln Y_t^* - (1-\gamma)\ln(1+\gamma\Upsilon_t) - \gamma \ln(1+\Upsilon_t) \tag{B.58}$$

whereas its log-quadratic approximation is given by,

$$U(C_t) = (1-\gamma)y_t + \gamma y_t^* - (1-\gamma)\sigma_\Upsilon\left(\lambda_v v_t + \frac{1-\sigma_\Upsilon}{2}v_t^2\right) + o\left(\|\Upsilon,\epsilon\|^3\right) \tag{B.59}$$

where $v_t = \ln\left(\frac{1+\Upsilon_t}{1+\Upsilon}\right)$, $\sigma_\Upsilon = \frac{\gamma(1+\Upsilon)}{(1+\gamma\Upsilon)}$, and $\lambda_v = \left(\frac{1+\Upsilon+(1-\gamma)}{(1-\gamma)(1+\gamma\Upsilon)}\right)$. The previous equation implies that both the level and the volatility of transaction frictions affect negatively household's welfare. Furthermore, by noticing that $U_c C = 1$, equation (B.59) can be written more compactly as follows,

$$U(C_t) = (1-\gamma)\left(y_t - \frac{\sigma_\Upsilon}{2(1-\sigma_\Upsilon)}(v_t+\overline{v})^2\right) + t.i.p + o\left(\|\Upsilon,\epsilon\|^3\right) \tag{B.60}$$

where, $\overline{v} = \frac{\lambda_v}{1-\sigma_\Upsilon}$. Next, we write v_t in terms of the domestic and the foreign interest rate and the degree of CS, as follows

$$v_t = \ln(1+i_t) + \ln(\frac{1+\Gamma(\overline{s}_t)}{1+\Gamma(\overline{s})}) \tag{B.61}$$

Since only second-order terms of v_t affect welfare it is sufficient to consider a first-order approximation of v_t to obtain an accurate measure of welfare associated to transaction frictions. The log-linear approximation of v_t is given by,

$$v_t = i_t - s(i_t - i_t^*) + o\left(\|\Upsilon,\epsilon\|^2\right) \tag{B.62}$$

simplifying this expression we obtain,

$$v_t = (1-s)i_t + s i_t^* + o\left(\|\Upsilon,\epsilon\|^2\right) \tag{B.63}$$

Alternative, we can write equation (B.59) in terms of the domestic and foreign interest rate by eliminated those terms in equation (B.60) that are independent of policy, as follows

$$\begin{aligned} U(C_t) &= (1-\gamma)\left(y_t - \sigma_\Upsilon(1-s)\lambda_v i_t\right) \\ &\quad -(1-\gamma)(1-s)\sigma_\Upsilon\left[\left(\frac{1-\sigma_\Upsilon}{2}\right)(1-s)i_t^2 + (1-\sigma_\Upsilon)\, s i_t i_t^*\right] \\ &\quad + t.i.p + o\left(\|\Upsilon, \epsilon\|^3\right) \end{aligned}$$

On the other hand, the second-order approximation of $v(L_t)$ is given by,

$$\begin{aligned} v(L_t) &= v(\Delta_t \frac{Y_t}{A_t}) = \overline{v} + \overline{v}_\Delta\left(\Delta_t - 1\right) + \overline{v}_y\left(Y_t - \overline{Y}\right) + \overline{v}_A\left(A_t - 1\right) \\ &\quad + \frac{1}{2}\left[\overline{v}_{yy}\left(Y_t - \overline{Y}\right)^2 + \overline{v}_{\Delta\Delta}\left(\Delta_t - 1\right)^2 + \overline{v}_{AA}\left(A_t - 1\right)^2\right] + \\ &\quad + \overline{v}_{y\Delta}\left(Y_t - \overline{Y}\right)\left(\Delta_t - 1\right) + \overline{v}_{yA}\left(Y_t - \overline{Y}\right)\left(A_t - 1\right) \\ &\quad + \overline{v}_{\Delta A}\left(\Delta_t - 1\right)\left(A_t - 1\right) + o\left(\|\varepsilon\|^3\right) \end{aligned} \tag{B.64}$$

Where $\widehat{\Delta}_t$ can be written in terms of domestic inflation as follows,

$$\widehat{\Delta}_t = \theta\widehat{\Delta}_{t-1}\Pi_{H,t}^{\varepsilon} + (1-\theta)\left(\frac{1-\theta\Pi_{H,t}^{\varepsilon-1}}{1-\theta}\right)^{\frac{-\varepsilon}{1-\varepsilon}} \tag{B.65}$$

and its second-order approximation is given by:

$$\widehat{\Delta}_t = \theta\widehat{\Delta}_{t-1} + \frac{\theta\varepsilon}{1-\theta}\frac{\Pi_{H,t}^2}{2} + o\left(\|\epsilon\|^3\right) \tag{B.66}$$

Under the assumption that the initial relative price distortion is small, this is $\widehat{\Delta}_{-1}$ is of order $o\left(\|\epsilon\|^3\right)$, $\widehat{\Delta}_t$ would be of order $o\left(\|\epsilon\|^2\right)$, thereby cross terms of $\widehat{\Delta}_t$ can be eliminated from equation (B.64). Hence, this latter equation can be written as follows,

$$\begin{aligned} v(L_t) &= \overline{v} + \overline{v}_\Delta\widehat{\Delta}_t + \overline{v}_y\overline{Y}\left(\widehat{Y}_t + \frac{1}{2}\widehat{Y}_t^2\right) + \overline{v}_A\widehat{A}_t + \\ &\quad \frac{1}{2}\left[\overline{v}_{yy}\overline{Y}^2\widehat{Y}_t^2\right] + \overline{v}_{yA}\overline{Y}\widehat{Y}_t\widehat{A}_t + t.i.p + o\left(\|\varepsilon\|^3\right) \end{aligned} \tag{B.67}$$

Furthermore, since at the steady-state it holds that ,

$$(1-\gamma)\,\overline{u}_c C = \overline{v}_y Y \tag{B.68}$$

the total period t utility flow is given by ,

$$\begin{aligned} u\left(C_t\right) - v(h_t) &= -\overline{v}_y\overline{Y}(1-s)\sigma_\Upsilon\left(\lambda_v i_t + \left(\frac{1-\sigma_\Upsilon}{2}\right)(1-s)i_t^2 + (1-\sigma_\Upsilon)\, s i_t i_t^*\right) \\ &\quad -\overline{v}_y\overline{Y}\left(\frac{1}{2}\left((1+\varphi)\right)\widehat{Y}_t^2 + \frac{\overline{v}_\Delta}{\overline{v}_y\overline{Y}}\widehat{\Delta}_t + \frac{\overline{v}_{yA}}{\overline{v}_y}\widehat{Y}_t\widehat{A}_t\right) \end{aligned} \tag{B.69}$$

$$+ t.i.p + o\left(\|\epsilon\|^3\right) \tag{B.70}$$

By defining the following new set of parameters:

$$u_{yy} = (1+\varphi) \tag{B.71}$$

$$u_{yA} = \frac{\overline{v}_{yA}}{\overline{v}_y} \tag{B.72}$$

$$u_{\Delta} = \frac{\overline{v}_{\Delta}}{\overline{v}_y \overline{Y}} \tag{B.73}$$

The utility function of the representative agent up to a second-order can be written as follows:

$$\begin{aligned} u(C_t) - v(h_t) &= -\overline{v}_y \overline{Y} (1-s)\, \sigma_\Upsilon \left(\lambda_v i_t + \left(\frac{1-\sigma_\Upsilon}{2} \right) (1-s) i_t^2 + (1-\sigma_\Upsilon)\, \omega s i_t i_t^* \right) \\ & \quad -\overline{v}_y \overline{Y} \left(\frac{1}{2} u_{yy} \widehat{Y}_t^2 + u_\Delta \widehat{\Delta}_t + u_{yA} \widehat{Y}_t \widehat{A}_t \right) + t.i.p + o\left(\|\varepsilon\|^3 \right) \end{aligned} \tag{B.74}$$

By iterating forward equation (B.65) we obtain:

$$\sum_{t=0}^{t=\infty} \beta^t \widehat{\Delta}_t = \frac{\theta \varepsilon}{(1-\theta)(1-\beta\theta)} \sum_{t=0}^{t=\infty} \beta^t \frac{\pi_{H,t}^2}{2} \tag{B.75}$$

Using this latter equation to eliminate $\widehat{\Delta}_t$, we obtain the following equation,

$$\begin{aligned} & -\overline{v}_Y \overline{Y} \sum_{t=0}^{t=\infty} \beta^t \left(+\frac{1}{2} u_{yy} \widehat{Y}_t^2 + u_\pi \frac{\pi_t^2}{2} + u_\Delta \widehat{Y}_t \widehat{A}_t \right) \\ & -\overline{v}_y \overline{Y} \omega \sigma_\Upsilon (1-s) \sum_{t=0}^{t=\infty} \beta^t \left(\lambda_v i_t + \left(\frac{1-\sigma_\Upsilon}{2} \right) \omega (1-s) i_t^2 + (1-\sigma_\Upsilon)\, \omega s i_t i_t^* \right) \end{aligned} \tag{B.76}$$

Where,

$$u_{yy} = (1+\varphi) \tag{B.77}$$

$$u_\Delta = 1 \tag{B.78}$$

$$u_{yA} = -(1+\varphi) \tag{B.79}$$

we have that:

$$u_\pi = \frac{\theta \varepsilon}{(1-\theta)(1-\beta\theta)} = \frac{\theta \varepsilon}{(1-\theta)(1-\beta\theta)} = \frac{\varepsilon}{\lambda} \tag{B.80}$$

Completing the quadratic form on output and productivity, we have that, :

$$\frac{1}{2} u_{yy} \widehat{Y}_t^2 + u_{yA} \widehat{Y}_t \widehat{A}_t = \frac{1}{2} \left((1+\varphi) \left(\widehat{Y}_t^2 - 2 \widehat{Y}_t \widehat{A}_t + \widehat{A}_t^2 \right) \right) \tag{B.81}$$

by defining the output gap, as: $\widehat{x}_t = \widehat{Y}_t - \widehat{Y}_t^e$, where $\widehat{Y}_t^e$ represent the efficient level of output, the welfare based lost function for a central bank in an economy with currency substitution is given by:

$$-\frac{\varepsilon}{2\lambda}\overline{V}_h\overline{Y}\sum_{t=0}^{t=\infty}\beta^t\left(\frac{(1+\varphi)\lambda}{\varepsilon}\widehat{x}_t^2+\pi_{H,t}^2\right) \tag{B.82}$$

$$-\frac{\lambda}{2\varepsilon}\overline{V}_h\overline{Y}\sigma_\Upsilon\omega(1-s)\sum_{t=0}^{t=\infty}\beta^t\left[2\lambda_v i_t+(1-\sigma_\Upsilon)\omega(1-s)i_t^2+2(1-\sigma_\Upsilon)\omega s i_t i_t^*\right]$$

C APPENDIXES OF CHAPTER 5

C.1 The Steady-State

The steady-state is determined by the following set of equations,

$$(\varepsilon - 1) P(z)^{-\varepsilon} P^{\varepsilon} X \left(\frac{\varepsilon}{\varepsilon - 1} W - P(z) \right) + \gamma_z P(z) = 0 \tag{C.1}$$

$$Y = h \tag{C.2}$$

$$P = \left[\int_0^1 P(z)^{1-\varepsilon} d(z) \right]^{\frac{1}{1-\varepsilon}} \tag{C.3}$$

$$h = \int_0^1 \theta^i h(i) di \tag{C.4}$$

$$\frac{W}{P} = \int_0^1 \frac{W^i}{P} di \tag{C.5}$$

$$\gamma_z = \widetilde{\gamma}_z P(z)^{-\varepsilon} P^{\varepsilon} X \tag{C.6}$$

$$\frac{W^i}{P} = \theta^i \tag{C.7}$$

Using equation (C.6) to simplify equation (C.1) , we obtain the following condition for the optimal price level,

$$\varepsilon W - (\varepsilon - 1) P(z) + \widetilde{\gamma}_z P(z) = 0 \tag{C.8}$$

from this equation we obtain,

$$P^o(z) = \frac{\varepsilon}{\varepsilon - 1 - \widetilde{\gamma}_z} W \tag{C.9}$$

To guarantee that $P^*(z)$ is a maximum, it must hold that $\frac{\partial^2 \Pi(z)}{\partial P(z)^2} (P^*(z)) < 0$. Taking a derivative respect to $P(z)$ of equation (C.1) we obtain the following second order condition for the optimal price,

$$\frac{\partial^2 \Pi(z)}{\partial P(z)^2} = (\varepsilon - 1) \varepsilon P(z)^{-\varepsilon - 1} P^{\varepsilon} X - \varepsilon (1 + \varepsilon) W P(z)^{-\varepsilon - 2} P^{\varepsilon} X < 0$$

simplifying the previous condition, we obtain,

$$\frac{\partial^2 \Pi(z)}{\partial P(z)^2} = (\varepsilon - 1) - (1 + \varepsilon) W P(z)^{-1} < 0$$

after plugging in the optimal price, given by equation (C.9), we obtain the following upper bound for $\widetilde{\gamma}_z$,

$$\frac{\partial^2 \Pi(z)}{\partial P(z)^2} = (\varepsilon - 1) - \frac{(1+\varepsilon)(\varepsilon - 1 - \widetilde{\gamma}_z)}{\varepsilon} < 0$$

By simplifying the previous condition, this upper bound is determined by,

$$\widetilde{\gamma}_z < \frac{(\varepsilon - 1)}{(1+\varepsilon)} < 1$$

Additionally we assume that the government uses a subsidy specific to each industry to eliminate the monopolistic distortion, hence, the equilibrium at the steady-state become efficient.

$$P^o(z) = \frac{\varepsilon \tau_z}{\varepsilon - 1 - \widetilde{\gamma}_z} W = W$$

Therefore,

$$\frac{W}{P} = 1$$

Also, from the aggregate demand condition,

$$X = Y - \gamma$$

We normalize, $Y = C = 1$, then $X = 1 - \gamma$ and $C^i = \theta^i - \gamma^i$

C.2 Proof of Propositions

Proof. **Proposition 5.2.1** The optimal degree of financial dollarisation of agent i, η^i, is given by the following condition,

$$\begin{aligned}\eta^i &= \left(1 - \frac{\gamma^i}{C^i}\right)\frac{(E_t\xi_{t+1})}{\sigma_i E_t \xi_{t+1}^2} + \left(\sigma_i - \left(1 - \frac{\gamma^i}{C^i}\right)\right)\frac{E_t\widehat{p}_{t+1}\xi_{t+1}}{\sigma_i E_t\xi_{t+1}^2} \\ &+ \frac{1}{E_t\xi_{t+1}^2}\left(\int_0^1 \frac{P(z)\gamma_z^i}{PC^i} E_t\left(\left(\widehat{p}_{t+1}(z) - \widehat{p}_{t+1}\right)\xi_{t+1}\right) d(z)\right)\end{aligned}$$

The budget constraint of a typical household is given by,

$$X_{t+1}^i = \frac{M_t^i}{P_{t+1}}\left(1 + \eta^i \xi_{t+1}\right) - \gamma_{t+1}^i$$

where,

$$\gamma_{t+1}^i = \int_0^1 \frac{P_{t+1}(z)}{P_{t+1}} \gamma_z^i d(z)$$

We take a second order approximation of the previous two equations around the deterministic steady-state

$$\begin{aligned}
X_{t+1}^i - X &= \frac{M^i}{P}\eta^i\xi_{t+1} - \frac{M^i}{P}\frac{(P_{t+1}-P)}{P} - \frac{M^i}{P}\eta^i\xi_{t+1}\frac{(P_{t+1}-P)}{P} \\
&\quad \frac{M^i}{P}\left(\frac{P_{t+1}-P}{P}\right)^2 - \int_0^1 \frac{P(z)\gamma_z^i}{P}\left[\frac{P_{t+1}(z)-P(z)}{P(z)}\right]d(z) \\
&\quad + \int_0^1 \frac{P(z)\gamma_z^i}{P}d(z)\left[\frac{P_{t+1}-P}{P}\right] - \int_0^1 \frac{P(z)\gamma_z^i}{P}d(z)\left[\frac{P_{t+1}-P}{P}\right]^2 \\
&\quad + \int_0^1 \frac{P(z)\gamma_z^i}{P}\left[\frac{P_{t+1}(z)-P(z)}{P(z)}\right]\left[\frac{P_{t+1}-P}{P}\right]d(z) + O\left(\|\xi_t,\sigma_\xi\|^3\right)
\end{aligned}$$

We denote by $\widehat{x}_t^i = \frac{X_{t+1}^i - X}{X}$, hence the previous condition can be written as follows,

$$\begin{aligned}
\left(1-\frac{\gamma^i}{C^i}\right)\widehat{x}_{t+1}^i &= \eta^i\xi_{t+1} - \widehat{p}_{t+1} - \eta^i\xi_{t+1}\widehat{p}_{t+1} + \widehat{p}_{t+1}^2 \\
&\quad - \int_0^1 \frac{P(z)\gamma_z^i}{PC^i}\left(\widehat{p}_{t+1}(z) - \widehat{p}_{t+1}\right)d(z) - \frac{\gamma^i}{C^i}\widehat{p}_{t+1}^2 \\
&\quad + \widehat{p}_{t+1}\int_0^1 \frac{P(z)\gamma_z^i}{PC^i}\widehat{p}_{t+1}(z)d(z) + O\left(\|\xi_t,\sigma_\xi\|^3\right)
\end{aligned} \tag{C.10}$$

On the other hand, the utility function up to second order approximation is given by,

$$E_tU\left(C_{t+1}^i - \gamma_{t+1}^i\right) = E_t\left[\Omega^i\left(\widehat{x}_{t+1}^i - \frac{\sigma_i}{2}\widehat{x}_{t+1}^{i2}\right)\right] + O\left(\|\xi_t,\sigma_\xi\|^3\right)$$

Then, the optimal portfolio allocation can be obtained by maximizing the previous utility function respecto to η^i, hence we have the following first order condition,

$$\frac{\partial U\left(X_{t+1}^i\right)}{\partial\eta^i} = E_t\left[\Omega^i\left(\frac{\partial\widehat{x}_{t+1}^i}{\partial\eta^i} - \sigma_i\frac{\partial\widehat{x}_{t+1}^i}{\partial\eta^i}\widehat{x}_{t+1}^i\right)\right] = 0$$

From equation (C.10), we can easily obtain that,

$$\frac{\partial\widehat{x}_{t+1}^i}{\partial\eta^i} = \frac{\left(\xi_{t+1} - \xi_{t+1}\widehat{p}_{t+1}\right)}{\left(1-\frac{\gamma^i}{C^i}\right)}$$

hence,

$$\begin{aligned}
\frac{\partial\widehat{x}_{t+1}^i}{\partial\eta^i}\widehat{x}_{t+1}^i &= \left(\frac{\xi_{t+1}}{\left(1-\frac{\gamma^i}{C^i}\right)}\right) \\
&\quad \frac{\left(\eta^i\xi_{t+1} - \widehat{p}_{t+1} - \int_0^1 \frac{P(z)\gamma_z^i}{PC^i}\left(\widehat{p}_{t+1}(z) - \widehat{p}_{t+1}\right)d(z)\right)}{\left(1-\frac{\gamma^i}{C^i}\right)} \\
&\quad + O\left(\|\xi_t,\sigma_\xi\|^3\right)
\end{aligned}$$

hence, we obtain,

$$\frac{\partial \widehat{x}_{t+1}^i}{\partial \eta^i}\widehat{x}_{t+1}^i = \frac{\left(\eta^i \xi_{t+1}^2 - \widehat{p}_{t+1}\xi_{t+1} - \int_0^1 \frac{P(z)\gamma_z^i}{PC^i}\left(\widehat{p}_{t+1}(z) - \widehat{p}_{t+1}\right)\xi_{t+1}d(z)\right)}{\left(1-\frac{\gamma^i}{C^i}\right)^2} + O\left(\left\|\xi_t, \sigma_\xi\right\|^3\right)$$

$$0 = \frac{\left(\xi_{t+1} - \xi_{t+1}\widehat{p}_{t+1}\right)}{\left(1-\frac{\gamma^i}{C^i}\right)} - \sigma_i \frac{\left(\eta^i \xi_{t+1}^2 - \widehat{p}_{t+1}\xi_{t+1} - \int_0^1 \frac{P(z)\gamma_z^i}{PC^i}\left(\widehat{p}_{t+1}(z) - \widehat{p}_{t+1}\right)\xi_{t+1}d(z)\right)}{\left(1-\frac{\gamma^i}{C^i}\right)^2}$$

Simplifying the previous expression we obtain,

$$0 = \left(1-\frac{\gamma^i}{C^i}\right)\left(E_t\xi_{t+1} - E_t\xi_{t+1}\widehat{p}_{t+1}\right) - \sigma_i\left(\eta^i E_t\xi_{t+1}^2 - E_t\widehat{p}_{t+1}\xi_{t+1} - \int_0^1 \frac{P(z)\gamma_z^i}{PC^i}E_t\left(\left(\widehat{p}_{t+1}(z) - \widehat{p}_{t+1}\right)\xi_{t+1}\right)d(z)\right)$$

hence, the optimal portfolio decision is given by,

$$\eta^i = \left(1-\frac{\gamma^i}{C^i}\right)\frac{\left(E_t\xi_{t+1} - E_t\xi_{t+1}\widehat{p}_{t+1}\right)}{\sigma_i E_t\xi_{t+1}^2} + \frac{\sigma_i}{E_t\xi_{t+1}^2\sigma_i}\left(E_t\widehat{p}_{t+1}\xi_{t+1} + \int_0^1 \frac{P(z)\gamma_z^i}{PC^i}E_t\left(\left(\widehat{p}_{t+1}(z) - \widehat{p}_{t+1}\right)\xi_{t+1}\right)d(z)\right)$$

Alternatively we can write the previous expression as follows,

$$\eta^i = \left(1-\frac{\gamma^i}{C^i}\right)\frac{\left(E_t\xi_{t+1}\right)}{\sigma_i E_t\xi_{t+1}^2} + \left(\sigma_i - \left(1-\frac{\gamma^i}{C^i}\right)\right)\frac{E_t\widehat{p}_{t+1}\xi_{t+1}}{\sigma_i E_t\xi_{t+1}^2} + \frac{1}{E_t\xi_{t+1}^2}\left(\int_0^1 \frac{P(z)\gamma_z^i}{PC^i}E_t\left(\left(\widehat{p}_{t+1}(z) - \widehat{p}_{t+1}\right)\xi_{t+1}\right)d(z)\right)$$

□

Proof. **Proposition 5.2.2**A particular firm will choose to set prices in a foreign currency if and only if the following conditions holds,

$$0 < -\widetilde{\gamma}_z\left[Cov\left(\xi_{t+1}x_{t+1}\right) - \frac{\varepsilon}{2}\left(Var\xi_{t+1} + 2E_t\xi_{t+1}\left(E\xi_{t+1} - E_tp_{t+1}\right)\right)\right] - \widetilde{\gamma}_z\left[\varepsilon Cov\left(\xi_{t+1}p_{t+1}\right) + Cov\xi_{t+1}w_{t+1}\right] + (\varepsilon - 1)\left(Cov\xi_{t+1}w_{t+1} - \frac{\left(Var\xi_{t+1} + 2E_t\xi_{t+1}\left(E\xi_{t+1} - E_tp_{t+1}\right)\right)}{2}\right)$$

Proof. Thus, the profit function of this typical firm can be written as follows,

$$E_t\Pi_{t+1}(z) = E_t\left[(P_t(z) - W_{t+1})\left(\left(\frac{P_t(z)}{P_{t+1}}\right)^{-\varepsilon} X_{t+1} + \gamma_z\right)\right] \tag{C.11}$$

the corresponding second order quadratic Taylor expansion of the previous equation is given by,

$$\begin{aligned}
E_t\Pi_{t+1}(z) \;=\; & (P(z) - W)P(z)^{-\varepsilon}P^{\varepsilon}XE_t\left(\varepsilon\left[p_{t+1} + \frac{\varepsilon}{2}p_{t+1}^2\right] + x_{t+1} + \frac{1}{2}x_{t+1}^2 + \varepsilon x_{t+1}p_{t+1}\right)\\
& -WP(z)^{-\varepsilon}P^{\varepsilon}XE_t\left(\varepsilon p_{t+1}w_{t+1} + w_{t+1}x_{t+1} + w_{t+1} + \frac{1}{2}w_{t+1}^2\right) +\\
& P(z)^{1-\varepsilon}P^{\varepsilon}XE_t\left((1-\varepsilon)\left[P_t(z) + \frac{1-\varepsilon}{2}P_t(z)^2\right]\right)\\
& P(z)^{1-\varepsilon}P^{\varepsilon}XE_t\left((1-\varepsilon)P_t(z)\left(\varepsilon p_{t+1} + x_{t+1}\right)\right)\\
& -WP(z)^{-\varepsilon}P^{\varepsilon}XE_t\left(-\varepsilon\left[P_t(z) + \frac{-\varepsilon}{2}P_t(z)^2\right]\right) +\\
& WP(z)^{-\varepsilon}P^{\varepsilon}XE_t\varepsilon P_t(z)\left(w_{t+1} + \varepsilon p_{t+1} + x_{t+1}\right)\\
& \gamma_z P(z)\left(p_t(z) + \frac{1}{2}P_t(z)^2\right) - \gamma_z WE_t\left(w_{t+1} + \frac{1}{2}w_{t+1}^2\right)
\end{aligned}$$

□

Lets now compare the value of expected profits when the firm sets price in dollars. The steady-state is the same, the only difference is that we replace, $p_t(z)$ by $p_t^*(z)$ and e_t. Thus a second order approximation of the profits function under dollar price setting is given by,

$$
\begin{aligned}
E_t\Pi^*_{t+1}(z) \;=\;& (P(z)-W)(P(z)^{-\varepsilon}P^{\varepsilon}X)\,E_t\left(\varepsilon\left[p_{t+1}+\frac{\varepsilon}{2}p^2_{t+1}\right]+x_{t+1}+\frac{1}{2}x^2_{t+1}+\varepsilon x_{t+1}p_{t+1}\right)\\
& -WP(z)^{-\varepsilon}P^{\varepsilon}XE_t\left(\varepsilon p_{t+1}w_{t+1}+w_{t+1}x_{t+1}+w_{t+1}+\frac{1}{2}w^2_{t+1}\right)+\\
& P(z)^{1-\varepsilon}P^{\varepsilon}XE_t\left((1-\varepsilon)\left[P^*_t(z)+\frac{1-\varepsilon}{2}P^*_t(z)^2\right]\right)\\
& P(z)^{1-\varepsilon}P^{\varepsilon}XE_t\,(1-\varepsilon)\,P^*_t(z)\,(\varepsilon p_{t+1}+x_{t+1})\\
& -WP(z)^{-\varepsilon}P^{\varepsilon}XE_t\left[-\varepsilon\left[P^*_t(z)+\frac{-\varepsilon}{2}P^*_t(z)^2\right]\right]\\
& WP(z)^{-\varepsilon}P^{\varepsilon}XE_t\varepsilon P^*_t(z)\,(w_{t+1}+\varepsilon p_{t+1}+x_{t+1})\\
& P(z)^{1-\varepsilon}P^{\varepsilon}XE_t\left((1-\varepsilon)\left[\xi_{t+1}+\frac{1-\varepsilon}{2}\xi^2_{t+1}\right]\right)\\
& P(z)^{1-\varepsilon}P^{\varepsilon}XE_t\,(1-\varepsilon)\,\xi_{t+1}\,(\varepsilon p_{t+1}+x_{t+1})\\
& -WP(z)^{-\varepsilon}P^{\varepsilon}XE_t\left[-\varepsilon\left[\xi_{t+1}+\frac{-\varepsilon}{2}\xi^2_{t+1}\right]\right]+\\
& WP(z)^{-\varepsilon}P^{\varepsilon}XE_t\varepsilon\xi_{t+1}\,(w_{t+1}+\varepsilon p_{t+1}+x_{t+1})\\
& \gamma_z P(z)\,E_t\left(\xi_{t+1}+\frac{1}{2}\xi^2_{t+1}+P^*_t(z)+\frac{1}{2}P^*_t(z)^2\right)-\gamma_z WE_tw_{t+1}
\end{aligned}
$$

then, we have

$$E_t\Pi^*_{t+1}(z)-E_t\Pi_{t+1}(z)>0$$

The previous condition can be written as follows,

$$
\begin{aligned}
& P(z)^{1-\varepsilon}P^{\varepsilon}XE_t\left((1-\varepsilon)\left[\xi_{t+1}+\frac{1-\varepsilon}{2}\xi^2_{t+1}\right]+(1-\varepsilon)\,\xi_{t+1}\,(\varepsilon p_{t+1}+x_{t+1})\right)\\
& -WP(z)^{-\varepsilon}P^{\varepsilon}XE_t\left[-\varepsilon\left[\xi_{t+1}+\frac{-\varepsilon}{2}\xi^2_{t+1}\right]-\varepsilon\xi_{t+1}\,(w_{t+1}+\varepsilon p_{t+1}+x_{t+1})\right]+\\
& P(z)^{1-\varepsilon}P^{\varepsilon}XE_t\left((1-\varepsilon)\left[P^*_t(z)-P_t(z)+\frac{1-\varepsilon}{2}\left(P^*_t(z)^2-P^2_t(z)\right)\right]\right)\\
& +P(z)^{1-\varepsilon}P^{\varepsilon}XE_t\,(1-\varepsilon)\,(P^*_t(z)-P_t(z))\,(\varepsilon p_{t+1}+x_{t+1})\\
& -WP(z)^{-\varepsilon}P^{\varepsilon}XE_t\left[-\varepsilon\left[P^*_t(z)-P_t(z)+\frac{-\varepsilon}{2}\left(P^*_t(z)^2-P^2_t(z)\right)\right]\right]\\
& +WP(z)^{-\varepsilon}P^{\varepsilon}XE_t\varepsilon\,(P^*_t(z)-P_t(z))\,(w_{t+1}+\varepsilon p_{t+1}+x_{t+1})\\
& \gamma_z P(z)\,E_t\left(\xi_{t+1}+\frac{1}{2}\xi^2_{t+1}+P^*_t(z)-P_t(z)+\frac{1}{2}\left(P^*_t(z)^2-P^2_t(z)\right)\right)>0
\end{aligned}
$$

Rearranging the previous expression in a more convenient way, we have,

$$
\begin{aligned}
&0 < \left[(1-\varepsilon) P(z)^{1-\varepsilon} P^{\varepsilon} X + \varepsilon W P(z)^{-\varepsilon} P^{\varepsilon} X + \gamma_z P(z)\right] E_t \xi_{t+1} + \\
&\left[(1-\varepsilon) P(z)^{1-\varepsilon} P^{\varepsilon} X + \varepsilon W P(z)^{-\varepsilon} P^{\varepsilon} X\right] E_t \left(\xi_{t+1} \left(\varepsilon p_{t+1} + x_{t+1}\right)\right) + \\
&-\frac{\varepsilon}{2} \left[(1-\varepsilon) P(z)^{1-\varepsilon} P^{\varepsilon} X + \varepsilon W P(z)^{-\varepsilon} P^{\varepsilon} X\right] E_t \xi_{t+1}^2 + \\
&\left[(1-\varepsilon) P(z)^{1-\varepsilon} P^{\varepsilon} X + \gamma_z P(z)\right] \frac{E_t \xi_{t+1}^2}{2} + \varepsilon W P(z)^{-\varepsilon} P^{\varepsilon} X E_t \xi_{t+1} w_{t+1} \\
&\left((1-\varepsilon) P(z)^{1-\varepsilon} P^{\varepsilon} X + \varepsilon W P(z)^{-\varepsilon} P^{\varepsilon} X + \gamma_z P(z)\right) \left(P_t^*(z) - P_t(z)\right) \\
&+ \left((1-\varepsilon) P(z)^{1-\varepsilon} P^{\varepsilon} X + \varepsilon W P(z)^{-\varepsilon} P^{\varepsilon} X + \gamma_z P(z)\right) E_t \left(P_t^*(z) - P_t(z)\right) \left(\varepsilon p_{t+1} + x_{t+1}\right) \\
&\varepsilon W P(z)^{-\varepsilon} P^{\varepsilon} X E_t \left(P_t^*(z) - P_t(z)\right) w_{t+1} \\
&-\frac{\varepsilon}{2} \left[(1-\varepsilon) P(z)^{1-\varepsilon} P^{\varepsilon} X + \varepsilon W P(z)^{-\varepsilon} P^{\varepsilon} X\right] \left(P_t^*(z)^2 - P_t^2(z)\right) \\
&\left[(1-\varepsilon) P(z)^{1-\varepsilon} P^{\varepsilon} X + \gamma_z P(z)\right] \frac{\left(P_t^*(z)^2 - P_t^2(z)\right)}{2}
\end{aligned}
$$

From the first order condition of the firm, we have that,

$$(1-\varepsilon) P(z)^{1-\varepsilon} P^{\varepsilon} X + \varepsilon W P(z)^{-\varepsilon} P^{\varepsilon} X = -\gamma_z P(z)$$

and also, we know that, up to log-linear approximation, $P_t^*(z) = P_t(z) - E_t \xi_{t+1}$ thus the previous condition becomes.

$$
\begin{aligned}
&0 < -\gamma_z P(z) E_t \xi_{t+1} \left(\varepsilon p_{t+1} + x_{t+1}\right) \\
&+ \left[(1+\varepsilon) P(z) \gamma_z + (1-\varepsilon) P(z)^{1-\varepsilon} P^{\varepsilon} X\right] \frac{E_t \xi_{t+1}^2}{2} \\
&+ \varepsilon W P(z)^{-\varepsilon} P^{\varepsilon} X E_t \xi_{t+1} w_{t+1} \\
&+ \gamma_z P(z) E_t \xi_{t+1} \left(\varepsilon E_t p_{t+1} + E_t x_{t+1}\right) + \\
&\left[(1+\varepsilon) P(z) \gamma_z + (1-\varepsilon) P(z)^{1-\varepsilon} P^{\varepsilon} X\right] \frac{\left(P_t^*(z)^2 - P_t^2(z)\right)}{2} \\
&- \varepsilon W P(z)^{-\varepsilon} P^{\varepsilon} X E_t \xi_{t+1} E_t w_{t+1}
\end{aligned}
$$

Further simplifying the previous condition we obtain,

$$
\begin{aligned}
&0 < -\gamma_z P(z) \left[\varepsilon Cov \left(\xi_{t+1} p_{t+1}\right) + Cov \left(\xi_{t+1} x_{t+1}\right)\right] \\
&+ \left[(\varepsilon + 1) P(z) \gamma_z + (1-\varepsilon) P(z)^{1-\varepsilon} P^{\varepsilon} X\right] \frac{Var \xi_{t+1}}{2} \\
&+ \varepsilon W P(z)^{-\varepsilon} P^{\varepsilon} X Cov \xi_{t+1} w_{t+1} \\
&+ \left[(\varepsilon + 1) P(z) \gamma_z + (1-\varepsilon) P(z)^{1-\varepsilon} P^{\varepsilon} X\right] E_t \xi_{t+1} \left(E \xi_{t+1} - E_t p_{t+1}\right)
\end{aligned}
$$

Thus, when $\gamma_z = 0$, the firms choose to set prices in dollars only if the correlation between the nominal exchange rate and real wages is high enough,

$$Cov \xi_{t+1} w_{t+1} > \frac{Var \xi_{t+1} + 2 E_t \xi_{t+1} \left(E \xi_{t+1} - E_t p_{t+1}\right)}{2}$$

In the general case we have that,

$$
\begin{aligned}
&0 < \left[-\widetilde{\gamma}_z P(z) P(z)^{-\varepsilon} P^{\varepsilon} X\right] \left[\varepsilon Cov\left(\xi_{t+1} p_{t+1}\right) + Cov\left(\xi_{t+1} x_{t+1}\right)\right] \\
&+\frac{\varepsilon}{2} \left[\widetilde{\gamma}_z P(z) P(z)^{-\varepsilon} P^{\varepsilon} X\right] \left(Var\xi_{t+1} + 2E_t\xi_{t+1}\left(E\xi_{t+1} - E_t p_{t+1}\right)\right) + \\
&(1-\varepsilon)\, P(z)^{1-\varepsilon} P^{\varepsilon} X \frac{\left(Var\xi_{t+1} + 2E_t\xi_{t+1}\left(E\xi_{t+1} - E_t p_{t+1}\right)\right)}{2} \\
&+\varepsilon W P(z)^{-\varepsilon} P^{\varepsilon} X Cov\xi_{t+1} w_{t+1}
\end{aligned}
$$

.Therefore, we can further simplify the previous expression,

$$
\begin{aligned}
&0 < \left[-\widetilde{\gamma}_z P(z)\right] \left[\varepsilon Cov\left(\xi_{t+1} p_{t+1}\right) + Cov\left(\xi_{t+1} x_{t+1}\right)\right] + \\
&\frac{\varepsilon}{2} \widetilde{\gamma}_z P(z) \left(Var\xi_{t+1} + 2E_t\xi_{t+1}\left(E\xi_{t+1} - E_t p_{t+1}\right)\right) \\
&-(\varepsilon - 1)\, P(z) \frac{\left(Var\xi_{t+1} + 2E_t\xi_{t+1}\left(E\xi_{t+1} - E_t p_{t+1}\right)\right)}{2} \\
&+\varepsilon P(z) Cov\xi_{t+1} w_{t+1}
\end{aligned}
$$

since,

$$
P(z) = \left(\varepsilon - 1 - \widetilde{\gamma}_z\right) W
$$

we have that,

$$
\begin{aligned}
&0 < \left[-\widetilde{\gamma}_z P(z)\right] \left[\varepsilon Cov\left(\xi_{t+1} p_{t+1}\right) + Cov\left(\xi_{t+1} x_{t+1}\right)\right] \\
&+\frac{\varepsilon}{2} \left[\widetilde{\gamma}_z P(z)\right] \left(Var\xi_{t+1} + 2E_t\xi_{t+1}\left(E\xi_{t+1} - E_t p_{t+1}\right)\right) \\
&-(\varepsilon - 1)\, P(z) \frac{\left(Var\xi_{t+1} + 2E_t\xi_{t+1}\left(E\xi_{t+1} - E_t p_{t+1}\right)\right)}{2} \\
&+Cov\xi_{t+1} w_{t+1} P(z) \left(\varepsilon - 1 - \widetilde{\gamma}_z\right)
\end{aligned}
$$

We can further eliminate, $P(z)$, hence we obtain,

$$
\begin{aligned}
&0 < \left[-\widetilde{\gamma}_z\right] \left[\varepsilon Cov\left(\xi_{t+1} p_{t+1}\right) + Cov\left(\xi_{t+1} x_{t+1}\right)\right] \\
&+\frac{\varepsilon}{2} \left[\widetilde{\gamma}_z\right] \left(Var\xi_{t+1} + 2E_t\xi_{t+1}\left(E\xi_{t+1} - E_t p_{t+1}\right)\right) \\
&-(\varepsilon - 1) \frac{\left(Var\xi_{t+1} + 2E_t\xi_{t+1}\left(E\xi_{t+1} - E_t p_{t+1}\right)\right)}{2} \\
&+Cov\xi_{t+1} w_{t+1} \left(\varepsilon - 1 - \widetilde{\gamma}_z\right)
\end{aligned}
$$

then we can further simplify

$$
\begin{aligned}
&0 < -\widetilde{\gamma}_z \left[\varepsilon Cov\left(\xi_{t+1} p_{t+1}\right) + Cov\left(\xi_{t+1} x_{t+1}\right)\right] \\
&+\widetilde{\gamma}_z \frac{\varepsilon}{2} \left(Var\xi_{t+1} + 2E_t\xi_{t+1}\left(E\xi_{t+1} - E_t p_{t+1}\right)\right) \\
&-(\varepsilon - 1) \frac{\left(Var\xi_{t+1} + 2E_t\xi_{t+1}\left(E\xi_{t+1} - E_t p_{t+1}\right)\right)}{2} \\
&+Cov\xi_{t+1} w_{t+1} \left(\varepsilon - 1 - \widetilde{\gamma}_z\right)
\end{aligned}
$$

C.3 The Dynamics of the Economy

Aggregate Demand

In order to obtain the aggregate demand of the economy we add the linear approximated budget constraint. across agents This strategy allows to characterize the dynamics of the aggregate demand only up to first order of approximation. However, since we use the dynamics equilibrium of the economy to obtain the second moments of endogenous variables, a first order approximation is sufficient to deliver accurate measures of second moments up to second order approximation. For agent i the first order approximated budget constraint is given by,

$$\begin{aligned}\widehat{x}_{t+1}^{i} &= \frac{\eta^{i}}{\left(1-\frac{\gamma^{i}}{C^{i}}\right)}\xi_{t+1}-\frac{1}{\left(1-\frac{\gamma^{i}}{C^{i}}\right)}\widehat{p}_{t+1} \\ &-\frac{1}{\left(1-\frac{\gamma^{i}}{C^{i}}\right)}\int_{0}^{1}\frac{P(z)\gamma_{z}^{i}}{PC^{i}}\left(\widehat{p}_{t+1}(z)-\widehat{p}_{t+1}\right)d(z)+O\left(\|\xi_{t},\sigma_{\xi}\|^{2}\right) \end{aligned} \quad (C.12)$$

we define aggregate demand by , $\widehat{x}_{t+1}=\int_{0}^{1}\widehat{x}_{t+1}^{i}di$, then, we obtain,

$$\begin{aligned}\widehat{x}_{t+1} &= \int_{0}^{1}\frac{\eta^{i}}{\left(1-\frac{\gamma^{i}}{C^{i}}\right)}di\xi_{t+1}-\int_{0}^{1}\frac{1}{\left(1-\frac{\gamma^{i}}{C^{i}}\right)}di\widehat{p}_{t+1} \\ &-\int_{0}^{1}\frac{1}{\left(1-\frac{\gamma^{i}}{C^{i}}\right)}\int_{0}^{1}\frac{P(z)\gamma_{z}^{i}}{PC^{i}}\left(\widehat{p}_{t+1}(z)-\widehat{p}_{t+1}\right)d(z)di+O\left(\|\xi_{t},\sigma_{\xi}\|^{2}\right) \end{aligned} \quad (C.13)$$

Lets denote by aggregate degree of financial dollarisation by $\eta=\int_{0}^{1}\frac{\eta^{i}}{1-\frac{\gamma^{i}}{C^{i}}}di$ and by $\lambda_{p}=\int_{0}^{1}\frac{1}{1-\frac{\gamma^{i}}{C^{i}}}di$ and by $\varphi_{t+1}=\int_{0}^{1}\frac{1}{1-\frac{\gamma^{i}}{C^{i}}}\int_{0}^{1}\frac{P(z)\gamma_{z}^{i}}{PC^{i}}\left(\widehat{p}_{t+1}(z)-\widehat{p}_{t+1}\right)d(z)di$, therefore, aggregate demand can be written as:

$$\widehat{x}_{t+1}=\eta\xi_{t+1}-\lambda_{p}\widehat{p}_{t+1}-\varphi_{t+1}+O\left(\|\xi_{t},\sigma_{\xi}\|^{2}\right) \quad (C.14)$$

Aggregate Supply

We obtain the aggregate supply by combining the optimal price setting decision of individual firms and the optimality condition at the labour market. From the optimal price setting

$$p_{t+1}(z)=\begin{cases}\kappa_{w}^{z}E_{t}w_{t+1}-\kappa_{x}^{z}\left(\varepsilon E_{t}p_{t+1}+E_{t}x_{t+1}\right), \text{ if } z\in\Upsilon \\ \xi_{t+1}-E_{t}\xi_{t+1}+\kappa_{w}^{z}E_{t}w_{t+1}-\kappa_{x}^{z}\left(\varepsilon E_{t}p_{t+1}+E_{t}x_{t+1}\right) \text{ otherwise}\end{cases} \quad (C.15)$$

where, $\kappa_w^z = \frac{(\varepsilon-1-\widetilde{\gamma}_z)}{(\varepsilon-1-\widetilde{\gamma}_z(1+\varepsilon))}$, $\kappa_x^z = \frac{\widetilde{\gamma}_z}{(\varepsilon-1-\widetilde{\gamma}_z(1+\varepsilon))}$ and Υ represent the set of firms which choose to set prices in domestic currency with mass $1-\upsilon$. Also, from the optimality condition of labor demand we have that,

$$w_{t+1} = p_{t+1} \tag{C.16}$$

By using the previous condition to eliminate real wages from the optimality condition for price setting we obtain,

$$p_{t+1}(z) = \begin{cases} E_t p_{t+1} - \kappa_x^z E_t x_{t+1}, \text{ if } z \in \Upsilon \\ \xi_{t+1} - E_t \xi_{t+1} + E_t p_{t+1} - \kappa_x^z E_t x_{t+1}, \text{ otherwise} \end{cases} \tag{C.17}$$

then the aggregate price level can be determined by aggregating the individual price decisions, as follows,

$$p_{t+1} = \int_0^{\upsilon} (\xi_{t+1} - E_t\xi_{t+1} + E_t p_{t+1} - \kappa_x^z E_t x_{t+1})\, dz + \int_1^{\upsilon} (E_t p_{t+1} - \kappa_x^z E_t x_{t+1})\, dz \tag{C.18}$$

therefore, we obtain,

$$p_{t+1} - E_t p_{t+1} = \upsilon\,(\xi_{t+1} - E_t \xi_{t+1}) - \Gamma E_t x_{t+1} \tag{C.19}$$

where,

$$\Gamma =_0^1 \frac{\widetilde{\gamma}_z}{(\varepsilon - 1 - \widetilde{\gamma}_z\,(1+\varepsilon))} dz$$

Dynamics Equilibrium

The dynamics equilibrium of this economy is given by the path of the endogenous variables $p_{t+1} - E_t p_{t+1}$ and $\widehat{x}_{t+1} - E_t$ in terms of $(\xi_{t+1} - E_t\xi_{t+1})$. To solve this system we start by taking conditional expectations to the aggregate supply equation, then we obtain, $E_t x_{t+1} = 0$, we use this condition into the aggregate demand equation, then we obtain,

$$E_t \widehat{x}_{t+1} = \eta E_t \xi_{t+1} - \lambda_p E_t \widehat{p}_{t+1} - E_t \varphi_t \tag{C.20}$$

which in turn implies that,

$$0 = \eta E_t \xi_{t+1} - \lambda_p E_t \widehat{p}_{t+1} - E_t \varphi_t \tag{C.21}$$

since $E_t\widehat{p}_{t+1}(z) = E_t\widehat{p}_{t+1}$, we have that $E_t\varphi_t = 0$, therefore, we obtain that,

$$E_t \widehat{p}_{t+1} = \frac{\eta}{\lambda_p} E_t \xi_{t+1} \tag{C.22}$$

Using the latter condition, we can write the aggregate demand equation in the following form,

$$\widehat{x}_{t+1} - E_t\widehat{x}_{t+1} = \eta\left(\xi_{t+1} - E_t\xi_{t+1}\right) - \lambda_p\left(\widehat{p}_{t+1} - E_t\widehat{p}_{t+1}\right) - \varphi_{t+1} \tag{C.23}$$

We use the aggregate supply and the previous condition to solve for the dynamics equilibrium of this economy,

$$p_{t+1} - E_t p_{t+1} = v\left(\xi_{t+1} - E_t\xi_{t+1}\right) \tag{C.24}$$

and

$$\widehat{x}_{t+1} - E_t\widehat{x}_{t+1} = \left(\eta - \lambda_p v\right)\left(\xi_{t+1} - E_t\xi_{t+1}\right) - \varphi_t \tag{C.25}$$

Second Moments of Endogenous Variables

Using the previous conditions it is easy to show that,

$$Cov(p_{t+1}, \xi_{t+1}) = vVar(\xi_{t+1}) \tag{C.26}$$

$$Cov(\widehat{x}_{t+1}, \xi_{t+1}) = \left(\eta - \lambda_p v\right)Var(\xi_{t+1}) - cov(\xi_{t+1}, \varphi_{t+1}) \tag{C.27}$$

From the definition of φ_t we have that,

$$cov(\xi_{t+1}, \varphi_t) = -\int_0^1 \frac{1}{\left(1 - \frac{\gamma^i}{C^i}\right)} \int_0^1 \frac{P(z)\gamma_z^i}{PC^i} E\left(\widehat{p}_{t+1}(z) - \widehat{p}_{t+1}\right)\left(\xi_{t+1} - E_t\xi_{t+1}\right) d(z)di \tag{C.28}$$

From the definition of $\widehat{p}_{t+1}(z)$, we are able to simplify the previous expression as follows,

$$\begin{aligned} cov(\xi_{t+1}, \varphi_t) &= -\int_0^1 \frac{1}{\left(1 - \frac{\gamma^i}{C^i}\right)} \int_0^1 \frac{P(z)\gamma_z^i}{PC^i} d(z)diCov(\widehat{p}_{t+1}, \xi_{t+1}) \\ &\quad -\int_0^1 \frac{1}{\left(1 - \frac{\gamma^i}{C^i}\right)} \int_\Upsilon^1 \frac{P(z)\gamma_z^i}{PC^i} d(z)diVar(\xi_{t+1}) \end{aligned} \tag{C.29}$$

Since, at the steady-state, $P(z) = P$,the previous condition can be written as,

$$\begin{aligned} cov(\xi_{t+1}, \varphi_t) &= -\int_0^1 \frac{1}{\left(1 - \frac{\gamma^i}{C^i}\right)C^i} \int_0^1 \gamma_z^i d(z)diCov(\widehat{p}_{t+1}, \xi_{t+1}) \\ &\quad -\int_0^1 \frac{1}{\left(1 - \frac{\gamma^i}{C^i}\right)C^i} \int_\Upsilon^1 \gamma_z^i d(z)diVar(\xi_{t+1}) \end{aligned} \tag{C.30}$$

Lets define by $\lambda_\phi = \int_0^1 \frac{1}{1-\frac{\gamma^i}{C^i}\ C^i} \int_0^1 \gamma_z^i d(z) di$, and by $\lambda_v v \simeq \int_0^1 \frac{1}{1-\frac{\gamma^i}{C^i}\ C^i} \int_\Upsilon^1 \gamma_z^i d(z) di$, then we have,

$$cov(\xi_{t+1}, \varphi_t) = -\lambda_\phi Cov(\widehat{p}_{t+1}, \xi_{t+1}) - \lambda_v v Var(\xi_{t+1}) \tag{C.31}$$

therefore,

$$Cov(\widehat{x}_{t+1}, \xi_{t+1}) = (\eta - (\lambda_p - \lambda_\phi - \lambda_v)\, v)\, Var(\xi_{t+1}) \tag{C.32}$$

we further simplify notation by defining, $\widetilde{\lambda} = (\lambda_p - \lambda_\phi - \lambda_v)$, then we have,

$$Cov(\widehat{x}_{t+1}, \xi_{t+1}) = \left(\eta - \widetilde{\lambda} v\right) Var(\xi_{t+1}) \tag{C.33}$$

D APPENDIXES OF CHAPTER 6

D.1 Aggregation

The derivations herein follow Lewbel (1994) closely. To alleviate the notation we drop the i subscript in this appendix.

Equations (6.11) and (6.12)

Consider equation (6.10),

$$x_t = ax_{t-1} + c + u_t \tag{D.1}$$

where $u_t = bS_t$. Note that c and u_t are individual specific and hence depend on a. Since by assumption S_t is a sequence of serially uncorrelated shocks, so is u_t.

Let E_a be the expectation operator across individuals, $E_a[z] = \int z \, \mathrm{d}F(a)$, such that $X_t = E_a[x_t]$, $C = E_a[c]$ and $U_t = E_a[u_t]$. Aggregation of (D.1) renders

$$X_t = E_a[ax_{t-1}] + C + U_t \tag{D.2}$$

Define a random variable α_s, a scalar $A_s = E_a[\alpha_s]$ and a recursion $\alpha_{s+1} = (\alpha_s - A_s)a$ with initial condition $\alpha_1 = a$. Note that for $s > 1$ the above recursion implies that $\alpha_s = a^s - \sum_{j=1}^{s-1} a^{s-j} A_r$. After taking E_a expectations we get equation (6.12) in the main text, where $m_s = E_a[a^s]$ is the s-th moment of the distribution of a. Note also that

$$\begin{aligned}
E_a[\alpha_s x_{t-s}] &= A_s X_{t-s} + E_a[(\alpha_s - A_s)x_{t-s}] \\
&= A_s X_{t-s} + E_a[(\alpha_s - A_s)ax_{t-(s+1)}] + E_a[(\alpha_s - A_s)c] + E_a[(\alpha_s - A_s)u_{t-s}] \\
&= A_s X_{t-s} + E_a[\alpha_{s+1}x_{t-(s+1)}] + \operatorname{cov}(\alpha_s, c) + \operatorname{cov}(\alpha_s, u_{t-s}) \qquad (D.3)
\end{aligned}$$

where $\operatorname{cov}(\alpha_s, c)$ is the cross-sectional covariance of α_s and c which is time-invariant. On the other side, $\operatorname{cov}(\alpha_s, u_{t-s})$ is the cross-sectional covariance of α_s and u_{t-s} which is time dependent, but as this dependency comes from S_t, it is serially uncorrelated.

Equation (D.3) shows a recursion between $E_a[\alpha_s x_{t-s}]$ and $E_a[\alpha_{s+1}x_{t-(s+1)}]$. After solving it,

$$E_a[ax_{t-1}] = \sum_{j=1}^{\infty} A_j X_{t-j} + \sum_{j=1}^{\infty} \operatorname{cov}(\alpha_j, c) + \sum_{j=1}^{\infty} \operatorname{cov}(\alpha_j, u_{t-j}) \tag{D.4}$$

Let $V_t = \sum_{j=1}^{\infty} \operatorname{cov}(\alpha_j, u_{t-j})$ and $\tilde{V} = E[V_t]$, where E is the expectation operator over time. Define also $\tilde{C} = C + \sum_{j=1}^{\infty} \operatorname{cov}(\alpha_j, c) + \tilde{V}$ and $\tilde{U}_t = U_t + V_t - \tilde{V}$. Then, after plugging (D.4) into (D.2) we get equation (6.11) in the main text, $X_t = \sum_{j=1}^{\infty} A_j X_{t-j} + \tilde{C} + \tilde{U}_t$, where $\tilde{U}_t$ is serially uncorrelated.[34] The underlying assumptions behind the aggregate equation (6.11) are thus, that $\tilde{M}$ and V_t are both finite or the sequences $\{\operatorname{cov}(\alpha_j, c)\}_{j=1}^{\infty}$ and $\{\operatorname{cov}(\alpha_j, u_{t-j})\}_{j=1}^{\infty}$ are absolute summable.

[34] Pesaran (2003) shows that it is heteroscedastic, though.

Equation (6.14)

Consider now equation (D.1) in first differences

$$\Delta x_t = a\Delta x_{t-1} + u_t - u_{t-1} \tag{D.5}$$

so that after aggregation, $\Delta X_t = E_a[a\Delta x_{t-1}] + U_t - U_{t-1}$. Following the same procedure leading to equation (D.4),

$$E_a[a\Delta x_{t-1}] = \sum_{j=1}^{\infty} A_j \Delta X_{t-j} + V_t - V_{t-1} \tag{D.6}$$

so that ΔX_t can be written as

$$\Delta X_t = \sum_{j=1}^{\infty} A_j \Delta X_{t-j} + (U_t + V_t) - (U_{t-1} + V_{t-1}) = \sum_{j=1}^{\infty} A_j \Delta X_{t-r} + U_t^{\dagger} \tag{D.7}$$

which corresponds to the first-difference version or (6.11). The new aggregate error $U_t^{\dagger}$ is serially correlated and the coefficients are the same as those in (6.11).

Equation (6.13)

All the results derived above go through straightforwardly when $S_t = R_t + \varepsilon_t$ where ε_t is *iid*. Coefficients a and b and the noise ε_t are individual specific whereas R_t is an aggregate figure, so $X_t = E_a[ax_{t-1}] + E_a[b]R_t + E_a[b\varepsilon_t]$. Equation (D.4) is now

$$E_a[ax_{t-1}] = \sum_{j=1}^{\infty} A_j X_{t-j} + \sum_{j=1}^{\infty} \text{cov}(\alpha_j, c) + \sum_{j=1}^{\infty} \text{cov}(\alpha_j, b) R_t + \sum_{j=1}^{\infty} \text{cov}(\alpha_j, b\varepsilon_t) \tag{D.8}$$

Call $B_0 = E_a[b]$, $B_j = \text{cov}(\alpha_j, b)$, $\hat{U}_t = \sum_{j=0}^{\infty} W_{t-j}$ where $W_t = \sum_{j=1}^{\infty} \text{cov}(\alpha_j, b\varepsilon_t)$. Further mechanical manipulation leads to (6.13). The aggregate disturbance $\hat{U}_t$ is serially correlated.

D.2 A Brief Note on Fractional Integration

Consider the univariate dynamic model

$$\Phi(L)(1-L)^d X_t = \Theta(L)\eta_t \tag{D.9}$$

where L is the lag operator, $\eta_t \sim iid(0, \sigma_\eta^2)$ and d is the *differencing parameter.* When $d = 0$, X_t is stationary and follows an ARMA process, $\Phi(L)X_t = \Theta(L)\eta_t$. When $d = 1$, X_t has a unit root and hence follows an ARIMA process, $\Phi(L)\Delta X_t = \Theta(L)\eta_t$. More generally, when d takes non-integer values, X_t is said to be a fractionally integrated ARMA (ARFIMA) process. When $d \in (0, 0.5]$, the autocovariance function of X_t declines hyperbolically to zero, making X_t a stationary long-memory process. For $d > 0.5$, X_t is non-stationary (has infinite variance).

Granger (1980) has shown that under particular assumptions about $F(a)$ – the distribution of individual autoregressive coefficients – the aggregation of AR(1) processes like (6.10) leads to (D.9).[35] In our empirical application, we simply imposed $d = 1$ and proceeded. If $d < 1$ truly, then we would have over-differentiated the data, with possible negative effects in our statistical inference.

Table D1 displays estimates of d and tests $H_0 : d = 0$ and $H_0 : d = 1$. We did not find enough evidence to reject $H_0 : d = 1$ whereas $H_0 : d = 0$ is systematically rejected.

Table D1. Estimated Fractional Integration Parameter in Dollarization Ratios

		$H_0 : d = 0$		$H_0 : d = 1$	
	$\hat{d}$	t-stat	p-value	t-stat	p-value
Mexico	0.825	2.376	0.0491	0.505	0.6294
Peru	0.932	3.883	0.0037	0.282	0.7843
Poland	0.955	4.605	0.0025	0.219	0.8333
Uruguay	0.788	2.485	0.0378	0.667	0.5236

The estimation method is that of Geweke and Porter-Hudak (known as GPH). The asymptotic standard error of $\hat{d}$ is $\pi^2/6$ which is used to compute the t-statistics and p-values. Both tests ($H_0 : d = 0$ and $H_0 : d = 1$) are two-tailed. See Baillie (1996) for a review of ARFIMA modelling and for critics to the GPH estimator.

[35] See also Baillie (1996) and Zaffaroni (2004).

D.3 The Distribution of Endowments and Abilities

Our results were derived under the assumption that agents are homogenous in their endowments. In particular, we restricted the analysis to the case where each agent has an endowment of size one. Here, we show that our results hold for a more general case, one in which agents have different size of endowments, but where the distribution of abilities (a) across agents is correlated with that of the endowments. We regard this correlation as plausible in reality.

Consider equation (6.10). For the sake of argument, set $\mu = 0$ so $c_i = 0$, define $\xi_{it} = b_i S_{it}$ and assume that aggregate income is equal to one and that there are two agents in the economy: one with ability a_1 and income n_1 and the other with ability a_2 and income $n_2 = 1 - n_1$. Then,

$$(1 - a_i L) x_{it} = \xi_{it}$$

for $i = 1, 2$(D.10)After generating a common lag polynomial for both process we have that

$$(1 - a_j L)(1 - a_i L) x_{it} = (1 - a_j L) \xi_{it}$$

for $i = 1, 2$ and $i \neq j$(D.11)The aggregate level of dollar deposits, which coincides with the aggregate dollarization ratio, is $X_t = n_1 x_{1t} + n_2 x_{2t}$. Aggregate the equations in (D.3) to get

$$(1 - a_1 L)(1 - a_2 L) X_t = n_1 (1 - a_2 L) \xi_{1t} + n_2 (1 - a_1 L) \xi_{2t} \tag{D.12}$$

Define $\tilde{\xi}_{it} = n_i \xi_{it}$ for $i = 1, 2$. Then, (D.12) boils down to

$$X_t = (a_1 + a_2) X_{t-1} + a_1 a_2 X_{t-2} + \tilde{\xi}_{1t} - a_2 \tilde{\xi}_{1t-1} + \tilde{\xi}_{2t} - a_1 \tilde{\xi}_{2t-1} \tag{D.13}$$

We have that if S_t is an *iid* sequence, the aggregate dollarization ratio follows an ARMA(2,1) process. This simple example can be generalizad to the case of N AR(1) process (hence N ability or endowment levels); in such a case the aggregate dollarization ratio follows an ARMA(N^*, $N^* - 1$) process, where $N^* \leq N$. We can increase the number of agents involved by simply replicating the individual behaviour for a given ability a an arbitrary number of times. Therefore, the aggregation results derived in appendix D.1 go through under the assumption that the distribution of endowments is correlated to that of the abilities to process information. When $N \to \infty$, we get the limiting case exposed in appendix D.2. These derivations apply straightforwardly to the alternative case where S_{it} is not *iid*.