Macroeconomics of Skill Accumulation

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ABSTRACT

This thesis studies three interrelated aspects of skill accumulation in macroeconomics. Each thesis chapter targets different aspects of skill accumulation in a macroeconomic environment: role of trade, role of outside ownership of assets, and role of unemployment. The chapters are thematically united in an explanatory approach which targets market incompleteness which arises from failures of intertemporal commitment: commitment to repay debt, commitment to acquire specific skills and commitment to form future productive matches.

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This thesis is dedicated to my family and friends. Now you know what I have been doing all this time.

I declare that the work submitted is my own and original unless explicit references to other work have been made.

Yong Jin KIM

1 Introduction

This thesis studies three interrelated aspects of skill accumulation in macroeconomics. Each thesis chapter targets different aspects of skill accumulation in a macroeconomic environment: role of trade (Chapter 2), role of outside ownership of assets (Chapter 3), and role of unemployment (Chapter 4). Despite this, the chapters are thematically united in an explanatory approach which targets market incompleteness which arises from failures of intertemporal commitment: commitment to repay debt (Chapter 2), commitment to acquire specific skills (Chapter 3) and commitment to form future productive matches (Chapter 4).

The macroeconomic evironments studied here are key to the individuals incentives to acquire skills. In the framework of chapter 2, the complementarity between old skilled and young unskilled labor in each technology implies that young agents's incentives to enter particular technologies are dependent on past decisions to enter technologies by previous generations. In the framework of chapter 3, the supply of assets across technologies where agents can commit to acquire asset specific skills will determine young agents's incentives to enter each technology. The supply of such assets is dependent on previous generations's decisions to enter technologies. In the framework of chapter 4, agents incentives to acquire skills are dependent on their job matching prospects which are a function of the aggregate vacancy to unemployment ratio.

Another major unifying theme of this thesis is the adoption of the overlapping generations framework. This is an essential methodology for analyzing the repeated incentives to acquire skills by a constant arrival of new labor market entrants in an economy with a stationary population. Chapters 3 and 4 adopt the two period overlapping generations model of Allias (1947), Samuelson (1958) and Diamond (1965), while chapter 4 adopts the perpetual youth model of Blanchard (1985).

Chapter 2

Chapter 2 begins by developing a theory of total factor productivity differences in a framework of technology diffusion. I show how in countries with tighter borrowing constraints, frontier technologies diffuse more slowly, and old outdated technologies continue to be used. I then analyze how countries with different borrowing constraints specialize across new and old technologies through two forms of intra-industry trade. First, I consider international factor mobility and interpret this as a form of (vertically differentiated) intermediate goods trade. Second, I consider trade in (vertically differentiated) final goods. Under both forms of trade, poor countries with tighter borrowing constraints exploit their comparative advantage through specializing in older technologies. However, under intermediate goods trade, poor countries can adopt new technologies faster by gaining access to inputs which complement the use of newer technologies. The patterns of specialization across technologies are dramatically different under each form of trade. Despite this, both forms of trade are consistent with total factor productivity divergence between countries with different borrowing constraints.

At the heart of the economic forces analyzed in Chapter 2 is the inability of multi- period lived agents to borrow against their old period product when young. Countries differ in their ability to solve this borrowing constraint problem which results in their differentiated use of technologies which are in the public domain of the world. The latter occurs in the context of a canonical model of technology diffusion of Chari and Hopenhayn (1991), where agents using newer technologies are associated with a greater demand for borrowing in equilibrium.

Chapter 3

In the third chapter of the thesis I explore in greater detail how economies try to overcome this borrowing problem through the ownership structure of assets. In particular, I show how in a Grossman-Hart-Moore world of incomplete contracts (Grossman and Hart (1986), Hart and Moore (1990)), the borrowing constraint problem of agents who acquire asset specific skills can be overcome when other agents outside the production process own assets (outside ownership).

Chapter 3 shows when agents undertake multi period projects where (i) they acquire asset specific skills over time, and (ii) they are borrowing constrained, outside ownership of assets can improve outcomes. When agents cannot commit to acquire asset specific skills, they are self employed. In the economy described, self employed "entrepreneurs" carry out new projects which in subsequent repetitions of the project implement outside ownership of assets. Outside owners hire "managers" to repeat projects. A necessary condition for entrepreneurs and managers to coexist, is that entrepreneurs are borrowing constrained. My theory of entrepreneurs coincides with their empirical identification as (i) the self employed who are (ii) borrowing constrained.

Another set of necessary conditions for entrepreneurs and managers to coexist is the continuous entry and exit of finite lived projects. After re-labeling assets under outside ownership as "firms", a reduced form characterization of the model coincides exactly with the canonical industry equilibrium model of Hopenhayn (1992). In this sense, my theory provides microfoundations for the interpretation of "firms" in that framework.

In the Chari and Hopenhayn (1991) framework of Chapter 2, new and old technologies coexist because of the following trade-off: new technologies have higher total factor productivity but a scarcity of skilled labor, old technologies have lower total factor productivity but an abundance of skilled labor and skilled and unskilled labor are complements.

Chapter 3 develops an alternative theory of technology diffusion based on the following trade-off: new technologies have higher total factor productivity but agents cannot commit to acquire asset specific skills, old technologies have lower total factor productivity but agents can commit to acquiring asset specific skills. Whether agents can commit to acquire asset specific skills or not determines whether the production process can exploit the gains from outside ownership of assets.

Chapter 4

The final chapter of the thesis explores the incentives to acquire skills in an environment where contractual incompleteness arises naturally. Workers acquire general skills before they are matched with firms, the identity of which is unknown at the time of skill acquisition. The latter means firms cannot be parties to contracts written when skills are acquired. Due to search frictions in matches between workers and firms, matched workers and firms bargain over a bilateral surplus which is increasing in the level of general skills. Since workers must share part of the marginal product of their general skills, the contractual incompleteness is associated with a hold-up problem of inefficient investment.

Chapter 4 analyses the correlation between employment duration, unemployment duration and general skill accumulation in a search equilibrium. I show how the level of general skills and the duration of employment are positively correlated, and both variables are negatively correlated with the duration of unemployment spells. With search frictions, general skill accumulation is associated with a hold up problem since it benefits future job vacancies which workers expect to be matched with. However, if vacancies direct their search to workers of different skill levels, workers indirectly internalize this externality. I derive conditions under which skill accumulation is fully efficient. I also show how Becker's general skill finance rule is modified for a search economy. Extensions analyze indirect complementarities between general and job specific skill accumulation, and the decision of whether to accumulate skills through training or education.

2 Borrowing Constraints and Technology Diffusion: Implications of Intra-industry Trade

An emerging view attributes much of the differences in per capita income levels across countries, to differences in total factor productivity (TFP).[1] This has motivated Prescott (1998) to call for a "theory of TFP differences". If technologies are in the international public domain, an explanation for these persistent TFP differences must be framed in terms of differences in barriers to the adoption of new, high TFP technologies [Parente and Prescott (1994)]. More specifically, a theory of TFP differences needs to address two observations: (i) why is the diffusion of frontier technologies slower in poor countries, and (ii) why do poor countries continue to use and invest in old technologies which have long been discarded from production in rich countries?

This chapter develops a theory of TFP differences in a framework of technology diffusion. The source of my TFP differences are exogenous differences in borrowing possibilities, which result from differences in the quality of institutions which enforce external investor rights. When new technologies are associated with higher "investment" relative to old technologies, tighter borrowing constraints lower output by distorting the allocation of inputs away from high investment activities to low investment activities.

This theory is applied to consider the impact of intra-industry trade on the magnitude of the TFP differences which arise. A running debate in the empirical literature on international technology diffusion is why increased trade and technology transfer between rich and poor countries leads to ambiguous effects on the TFP of poor countries, and ambiguous effects on convergence in TFPs between rich and poor countries [Coe and Helpman (1995), Keller (1998), Rodriguez and Rodrik (1999)]. Among different forms of trade, this literature finds stronger evidence of TFP growth in poor countries looking at intermediate goods trade

[1] Klenow and Rodrigues-Clare (1997), and Hall and Jones (1999) conclude that about 50% of per capita income differences can be traced to TFP differences.

alone [Coe, Helpman and Hoffmaister (1997), Meyer(2001)].[2] I set out to show how despite trade, the TFP differences which arise from differences in borrowing constraints do not subside. Indeed, under some forms of trade, TFP differences diverge.

I address the heterogeneity of TFP outcomes under different forms of trade by considering two forms of intra-industry trade. First, I analyze international mobility of factor inputs and go on to interpret this as a form of (vertically differentiated) intermediate goods trade. Second, I analyze trade in (vertically differentiated) final goods. In this process, I clarify the interaction between two intuitive forces of trade. First, trade leads poor countries to further specialization in old technologies for which they have a comparative advantage (a force for lower incomes). Second, trade leads poor countries to access inputs which complement the use of new technologies (a force for higher incomes). Under intermediate goods trade both forces of trade are present, under final goods trade only the first force is present.

The pattern of specialization across technologies is dramatically different under each form of trade. Intermediate goods trade results in countries with tighter borrowing constraints discarding the very old and very new vintage technologies they use under no trade, and specializing in the intermediate vintage technologies. Intermediate goods trade is consistent with TFP convergence or divergence between countries with different borrowing constraints. Trade in final goods results in countries with tighter borrowing constraints specializing in the very old vintage technologies compared to before trade. Trade in final goods is inconsistent with TFP convergence between countries with different borrowing constraints.

Chari and Hopenhayn (1991) present a canonical analysis of technology diffusion in their "vintage human capital" model. The application of that framework to borrowing constraints and intra-industry trade constitutes the core of this chapter. Among models of technology diffusion the vintage human capital model is unique in predicting continued entry and investment in old technologies. Other

[2] Keller (2001) provides an up to date review of this literature.

models of diffusion which rely on uncertainty of new technologies, strategic issues, or spillover effects seem inappropriate to the question at hand.

The *exogenous* role of enforcement institutions on economic performance is motivated by the literature on financial development (or deepening) and growth. The quality of enforcement institutions are directly linked to the volume of financial trade in an economy. In early seminal work, Goldsmith (1969) established a correlation between economic and financial development by measuring the value of financial intermediary assets to GNP, while country case studies by Cameron (1967) dissected historical relationships between banking development and early industrialization to suggest that the former had an independent and positive growth-inducing role. The modern revival of empirical studies begins with King and Levine (1993) and is reviewed in Levine (1997). Recently, Rousseau (2002) applies standard tools in empirical macroeconomics to link financial revolutions over the past 400 years to surges in real sector activity.

These studies do not unambiguously resolve the issue of causality between enforcement institutions and growth, but recent progress has been made on this front. La Porta, Lopez-de-Silanes, Shleifer and Vishny (1997, 1998) show how various measures of investor rights are systematically linked to the legal origin of enforcement institutions. Since countries typically adopted one of four legal systems (English, French, German, Scandinavian) through occupation or colonization, the implied differences in investor rights are seen as exogenous. In the literature on finance and growth, the La Porta, Lopez-de-Silanes, Shleifer and Vishny indices of investor rights and legal origin are used as instruments to extract the exogenous component of financial development and control for simultaneity bias. In particular, studies find that the main channel through which financial development and investor rights affect growth is through TFP rather than through savings or capital accumulation levels [Beck, Levine and Loayza (2000), Wurgler (2000)]. Instead of highlighting the identity of colonizers, Acemoglu, Johnson and Robinson (2001) argue that the conditions for settlement by colonizers came to determine institutional quality. Using colonialist mortality rates as instruments for institutional quality, they find large effects of this variable on current per capita income levels.[3]

Although the environment studied is inherently dynamic, it is possible to introduce many of the main mechanisms of the theory in terms of a simple static economy (readers may skip this by going to the penultimate paragraph of the introduction). Consider a stationary two period overlapping generations economy, where ex ante identical agents can become entrepreneurs or workers. Workers earn a constant wage in both periods of their lives. Entrepreneurs hire workers from competitive labor markets and maximize profits subject to a production function which is concave in worker inputs. To become an entrepreneur, every agent must acquire skills through learning-by-doing in youth, and in youth such agents receive a fixed income. Equilibrium wages equalize the utility of ex ante identical agents across occupations. Since entrepreneurs earn more than workers, lifetime earnings profiles are steeper for agents planning to become entrepreneurs, and such agents are "investing" in youth in terms of foregone earnings.

Suppose agents have concave utility functions over consumption in each period of their lives. Compare outcomes when subject to budget constraints (i) young agents can borrow as much as they like, and (ii) they can only commit a fixed share of their old period incomes as collateral for loans: borrowing constraints. In the first case, equilibrium wages equalize discounted lifetime earnings across occupations. In the second case, the equilibrium wage is lower, because borrowing constraints have a greater disutility effect on young agents with steeper earnings profiles, who plan to become entrepreneurs. In this second case, although lifetime utilities are lower, discounted lifetime earnings are higher for entrepreneurs. Meanwhile, for workers, discounted lifetime earnings are lower. Since entrepreneurs' input demands are higher, the share of entrepreneurs in the economy must

[3] On the other hand. Rajan and Zingales (2001) show that financial development may not be monotonic over time. and propose an interest group theory of financial development. My argument only relies on stable differences in enforcement institutions, and remains valid as long as institutions are not endogenous to the other variables under analysis.

fall. Borrowing constraints lower aggregate output by distorting the allocation of agents across the two occupations.

Next, suppose that the two cases describe two coexisting economies in the world, and consider outcomes under free international mobility of agents (i.e. factor price equalization). Let every young agent be subject to the borrowing constraints prevailing in her country of origin. Agents who do not face borrowing constraints are sorted into steep earnings profile occupations (entrepreneurs), while agents who face borrowing constraints are sorted into shallow earnings profile occupations (workers). Consider two possible outcomes. If all agents from the borrowing constrained economy become workers, the discounted value of lifetime earnings are equalized across the world. If all agents from the borrowing unconstrained economy become entrepreneurs, agents from this economy have higher discounted lifetime earnings, while *average* discounted lifetime earnings for the economy with borrowing constraints falls. International mobility of agents does not necessarily imply that lifetime earnings converge across economies.

Earnings outcomes under international mobility of agents are identical to outcomes where only entrepreneurs are internationally mobile. Suppose one reinterprets the mobility of entrepreneurs as the mobility of intermediate goods that they produce: i.e. workers are indifferent between working with entrepreneurs or working with intermediate goods that entrepreneurs produce. Then, outcomes under international mobility are interpreted as those under trade in intermediate goods. This analysis already captures two different forces associated with intermediate goods trade. First, agents from economies differentiated by borrowing constraints exploit their comparative advantage by increased specialization across occupations. Second, intermediate goods trade allows agents in the poorer, borrowing constrained economy to participate in a more efficient use of the given technologies.

Now consider final goods trade. Suppose there is no mobility of agents or intermediate goods trade. In each period, a new "fashion" introduces a differentiated good and agents can become fashion specific entrepreneurs through learning-by-doing in youth. It takes one period for a new good to become unfashionable, and all unfashionable goods are perfect substitutes. Although all entrepreneurs and workers produce one period old unfashionable goods, next period's entrepreneurs produce a fixed amount of fashionable goods in the current period. The relative price of fashionable goods is increasing in the scarcity of fashionable goods relative to unfashionable goods. Suppose our two coexisting economies do not engage in any final goods trade. Since there are fewer entrepreneurs in the borrowing constrained economy, in every period the fashionable good is relatively scarce, and the its relative price is higher.

Next, suppose there is trade in fashionable and unfashionable goods. In the borrowing constrained economy, the relative price of fashionable goods will fall. Just like the effect of tighter borrowing constraints, this has a greater disutility effect on young agents who plan to become entrepreneurs, so equilibrium worker wages will fall. In the economy without borrowing constraints the relative price of fashionable goods will rise. This has a greater utility effect on young agents who plan to become entrepreneurs, so equilibrium worker wages will rise. As in the case of intermediate goods trade, the allocation of labor across occupations diverges as a result of trade. This captures the fact that economies exploit their comparative advantage across occupations. However, unlike the case with intermediate goods trade, this is the result of *diverging factor prices* (worker wages). Final goods trade is not associated with agents in the borrowing constrained economy participating in a more efficient use of the given technologies. Overall, the per capita incomes of the two economies must diverge.

In the general framework considered, there is an (exogenously) ever-growing set of technologies which can be ranked according to their TFP. Within each technology, entrepreneurs hire workers to maximize profits. Young agents can become technology specific entrepreneurs through acquiring skills as technology specific workers in youth. Since entrepreneurs in new technologies are more productive, young workers entering relatively older technologies are compensated with higher wages in youth, and this ensures that entrepreneurs in newer technologies earn more than entrepreneurs in older technologies. Earnings profiles are steeper, the newer the technologies that young workers enter into. The number of coexisting technologies (and occupations) is *endogenous* and is determined by the youngest technology such that old, skilled agents in that technology prefer to be workers in another technology rather than utilizing their skills as entrepreneurs. The mechanisms introduced above are considerably strengthened and clarified by endogenizing the number of coexisting occupations which agents choose to participate in. In particular, the effects of borrowing constraints and intra-industry trade on the distribution of the workforce across technology vintages constitute the key findings.

My theory of TFP differences complements existing work which has considered monopoly rights [Parente and Prescott(1999)], vested interests [Krusell and Rios-Rull (1996)], capital labour ratios [Basu and Weil (1998)] and ratios of skilled to unskilled labour [Acemoglu and Zilibotti (2000)]. Jovanovic (1998) presents a model of income inequality which results from different vintages of physical capital which are indivisible. In that paper, he makes a general point that frameworks of inequality can help us understand why despite the mobility of factors of production, international income inequalities may not disappear. My analysis of trade is related to this point. By linking financial development to technology adoption, this paper adds to a literature including Greenwood and Jovanovic (1990), Bencivenga and Smith (1991) and Parente (1994).

Section 1 presents the basic model and conducts comparative statics with borrowing constraints. Section 2 applies the model to trade in intermediate goods and trade in final goods. The last section concludes.

2.1 Model

2.1.1 Preferences

Consider a two period overlapping generations economy, populated by ex ante identical agents who have preferences of the Cobb-Douglas form,

$$u = \sqrt{c_1 c_2} \tag{1}$$

c_1 denotes consumption in youth, and c_2 denotes consumption in old age. A constant population of agents is born every period. Agents can borrow from international capital markets, and face an exogenous world interest factor $R > 1$. Let y_1 and y_2 denote young and old period earnings respectively. The lifetime budget constraint of a worker is,

$$c_1 + \frac{1}{R}c_2 = y_1 + \frac{1}{R}y_2 \tag{2}$$

In credit markets, the imperfect enforcement of investor rights means that young agents can only borrow up to a fraction θ, of their discounted old period earnings for young period consumption,

$$c_1 - y_1 \leq \theta\frac{1}{R}y_2 \tag{3}$$

Borrowing constraints arise because borrowers cannot credibly commit to use more than a fraction θ of their old period earnings as collateral.[4]

2.1.2 Technology

Agents inhabit a "vintage human capital" economy. In every period, a frontier technology is exogenously introduced whose TFP is $\gamma > 1$ times greater than the TFP of last period's frontier technology. Let $\tau \in \{0, 1, 2, ...\}$ index the vintage or

[4] The inalienability of future earnings from agents means that the ex post expropriation problem is particularly severe in this situation (compared to borrowing against physical capital), and the role of third party enforcement of investor rights is important.

age of a technology in period t, *relative* to the newest technology in that period. A vintage τ technology in the current period, becomes the vintage $\tau - 1$ technology in the next period. Date t output in technology τ, is a constant returns to scale function of skilled labour $Z_{\tau,t}$, and unskilled labour $N_{\tau,t}$ inputs,

$$Y_{\tau,t}(Z_{\tau,t}, N_{\tau,t}) = \gamma^{t-\tau} F(Z_{\tau,t}, N_{\tau,t}) \equiv \gamma^{t-\tau} Z_{\tau,t} f(n_{\tau,t}) \tag{4}$$

$n_{\tau,t}$ denotes the number of unskilled agents per skilled agent in vintage τ. The assumption of constant returns implies that $F(0, N)$ is linear in N : define $F(0, 1) \equiv \omega_0 \geq 0$.

Although newer technologies have higher TFP, older technologies will continue to be used if skilled and unskilled labor are complements in production, $F_{ZN} > 0$. This crucial assumption introduces a trade-off between adopting new technologies with higher TFP, but where skilled labor is scarce, versus the continued use of old technologies where skilled labor is abundant but TFP is low.

Every agent acquires technology specific skills through learning-by-doing in youth. All young agents are unskilled. I refer to all unskilled labor as "workers". All old agents are skilled, and may utilize their skills as technology specific "entrepreneurs" who hire workers from competitive labour markets to maximize their earnings. Let $w_{\tau,t}$ denote the period t earnings of a worker in vintage τ. The optimal earnings of an entrepreneur in the corresponding vintage is,[5]

$$\pi_{\tau,t}(w_{\tau,t}) = \max_{n} \gamma^{t-\tau} f(n_{\tau,t}) - n_{\tau,t} w_{\tau,t} \tag{5}$$

A young worker who enters vintage $\tau - 1$ in period $t - 1$, earns $w_{\tau-1,t-1}$ in youth and becomes skilled in vintage τ in period t. Once old, this agent is free be an entrepreneur to earn $\pi_{\tau,t}(w_{\tau,t})$, or exercise the option of being an old worker in any other vintage. Assume that skilled agents choose to be entrepreneurs

[5] Since $F_{ZN} > 0 \Leftrightarrow f''(n) < 0$, the solution to the entrepreneur's maximization problem is unique.

only if they are strictly better off doing so: $\pi_{\tau,t}(w_{\tau,t}) > \max_s w_{s,t}$. Note that young workers in the frontier technology necessarily produce by themselves to earn $w_{0,t} = \gamma^t F(0,1) \equiv \gamma^t \omega_0$. To summarize, the combination of young and old period earnings for a generic agent who is young in period $t-1$ is given by,

$$(y_{1,t-1}, y_{2,t}) = \left(w_{\tau-1,t-1}, \max\left\{\pi_{\tau,t}(w_{\tau,t}), \max_s w_{s,t}\right\}\right) \tag{6}$$

In each period, agents are distributed across technologies of different ages relative to the frontier technology. Let $\mu_{\tau,t}$ denote the share of *old* agents in the population who are skilled in vintage τ, and μ_t the entire distribution. Since period t skilled agents in vintage τ, were period $t-1$ workers in vintage $\tau-1$, we know that $\mu_{\tau-1,t-1}n_{\tau-1,t-1} \geq \mu_{\tau,t}$ for $\forall\tau \geq 1$.

2.1.3 Equilibrium

In a competitive equilibrium: in every period (i) each young worker chooses which technology specific skill to acquire, how much to earn and consume across periods in order to maximize lifetime utility (29) subject to the interest factor R, and the constraints (2), (3) and (6); (ii) old agents maximize their earnings and (iii) the labor markets for technology specific workers clear.

The indirect utility of young workers, $v(y_1, y_2; \theta, R)$ can be rearranged to express utility as the product of the present discounted value of earnings, a constant $\frac{\sqrt{R}}{2}$, and a fraction,

$$\begin{aligned} v(y_1, y_2; \theta, R) &= \frac{\left(y_1 + \frac{1}{R}y_2\right)\frac{\sqrt{R}}{2} if \text{ borrowing constraints do not bind}}{\sqrt{\left(y_1 + \frac{\theta}{R}y_2\right)(1-\theta)y_2} if \text{ borrowing constraints bind}} \\ &= \left(y_1 + \frac{1}{R}y_2\right)\frac{\sqrt{R}}{2}\min\left\{1, 2\left(\frac{\sqrt{(1-\theta)\left(R\frac{y_1}{y_2}+\theta\right)}}{R\frac{y_1}{y_2}+1}\right)\right\} \\ &\equiv \left(y_1 + \frac{1}{R}y_2\right)\frac{\sqrt{R}}{2}\delta\left(R\frac{y_1}{y_2};\theta\right) \end{aligned} \tag{7}$$

This separates out the effects on utility of (i) the level of present discounted earnings and (ii) borrowing constraints and the steepness of the earnings profile. Let $\delta\left(R\frac{y_1}{y_2};\theta\right) \in (0,1]$ denote the "lifetime earnings discount factor". $\delta\left(R\frac{y_1}{y_2};\theta\right)$ is equal to 1 only if borrowing constraints do not bind, is increasing in the level of borrowing constraint θ, $\frac{\partial\delta}{\partial\theta} \geq 0$, and is increasing in the ratio of earnings $\frac{y_1}{y_2}$, $\frac{\partial\delta}{\partial\frac{y_1}{y_2}} \geq 0$. The cross partial, $\frac{\partial^2\delta}{\partial\frac{y_1}{y_2}\partial\theta} \leq 0$ captures the fact that the indifference curves of agents who face different θ will be single crossing in the space of young and old period earnings.

This characterization of indirect utility will assist the analysis of equilibrium outcomes throughout the paper. Using (6), denote the "lifetime earnings discount factor" for the generic agent who is young in period $t-1$ as,

$$\delta_{\tau,t} = \delta_{\tau,t}\left(R\frac{w_{\tau-1,t-1}}{\max\left\{\pi_{\tau,t}(w_{\tau,t}), \max_s w_{s,t}\right\}};\theta\right) \tag{8}$$

Since ex ante identical young workers are utility maximizers, the earnings path for each vintage with positive entry by young agents must yield an indirect utility at least as high as any other vintage,

$$\begin{aligned} &\left(w_{\tau-1,t-1} + \frac{1}{R}\max\left\{\pi_{\tau,t}(w_{\tau,t}), \max_s w_{s,t}\right\}\right)\delta_{\tau,t} \\ \geq\ &\left(w_{\nu-1,t-1} + \frac{1}{R}\max\left\{\pi_{\nu,t}(w_{\nu,t}), \max_s w_{s,t}\right\}\right)\delta_{\nu,t} \end{aligned} \tag{9}$$

for all τ,ν such that $n_{\tau-1} > 0$.

I restrict analysis to that of stationary competitive equilibrium. Such an equilibrium is a collection of earnings profiles $w_{\tau,t}$, $\pi_{\tau,t}(w_{\tau,t})$; input demands $n_{\tau,t}$; distribution functions μ_t such that,

(i) The distribution of agents across vintages is stationary $\mu_{\tau,t} = \mu_\tau$, input demands are stationary $n_{\tau,t} = n_\tau$, and earnings profiles grow at a constant rate $w_{\tau,t} = \gamma^t w_\tau$, $\pi_{\tau,t}(w_{\tau,t}) = \gamma^t\pi_\tau(w_\tau)$.

(ii) Young workers are indifferent among vintages they enter into (9), and old agents maximize their earnings.

(iii) The full employment condition is satisfied: $\sum_{\tau=1}^{\infty} \mu_\tau = 1$.[6]

A stationary equilibrium is characterized by an invariant distribution of agents relative to a constantly changing frontier technology. Although earnings levels are growing at a constant rate, the "lifetime earnings discount factors" across vintages are stationary since they are functions of the ratio of young and old period earnings. The restriction to stationary outcomes means that time indices will be dropped throughout the rest of the paper.

Proposition 1 *A unique stationary competitive equilibrium exists where,*

(i) the terminal vintage is finite $T < \infty$

(ii) skilled agents in vintages 1 *to* $T-1$ *are entrepreneurs,*

so $\mu_\tau = n_{\tau-1}\mu_{\tau-1}$; *young workers who enter vintage* $T-1$ *remain workers in vintage* $T-1$ *when old, so* $\mu_T = \frac{n_{T-1}}{2}\mu_{T-1}$

(iii) worker wages are increasing in vintage $w_{\tau+1} > w_\tau$, *so* $n_{\tau+1} < n_\tau$

(iv) the lifetime earnings discount factor is increasing in vintage $\delta_{\tau+1} \geq \delta_\tau$

Proof in Appendix.

When faced with a given worker wage, entrepreneurs in older vintages are less productive. Young workers are compensated to enter older vintages through higher worker wages. This in turn ensures that entrepreneur earnings are falling in vintage. Eventually, the worker wage which would warrant entry into a very old vintage (vintage T) exceeds the optimized entrepreneur earnings of that vintage such that that vintage and older vintages are not used in production. The slope of the lifetime earnings profile is steeper for those who enter younger vintages. Since δ_τ is falling in the steepness of the lifetime earnings profile, the present discounted value of lifetime earnings is falling in vintage.[7]

[6] In stationary states, this ensures that the full employment constraint of young workers is automatically satisfied.

[7] Thompson (1999) tests whether earnings profiles are steeper in newer technologies using historical data from the Canadian Maritime History Archive. He investigates the variation in wages across three vessel technologies (steam, barques and riggers) and finds strong support for this prediction.

In equilibrium, the indifference condition across coexisting vintages is given by,

$$\begin{aligned}\left(\omega_0 + \frac{\gamma}{R}\pi_1(w_1)\right)\delta_1 &= \left(w_1 + \frac{\gamma}{R}\pi_2(w_2)\right)\delta_2 = \ldots \\ &= \left(w_{T-2} + \frac{\gamma}{R}\pi_{T-1}(w_{T-1})\right)\delta_{T-1} \\ &= \left(w_{T-1} + \frac{\gamma}{R}w_{T-1}\right)\delta_T\end{aligned} \tag{10}$$

Young workers who enter the frontier technology necessarily produce by themselves so earn $w_0 = \omega_0$.

The highest worker wage in the economy is strictly lower than the entrepreneur earnings of the penultimate technology, and weakly higher than the entrepreneur earnings of the terminal technology. Thus, w_{T-1} serves as an index of the number of vintages in use in the economy,

$$\pi_{T-1}(w_{T-1}) > w_{T-1} \geq \pi_T(w_{T-1}) \tag{11}$$

Inequalities (10) and (11) are used to iteratively solve for T and the $T-1$ worker wages. The implied input demands across vintages, (ii) from Proposition 1 and the full employment constraint are combined to solve for μ_1 first, and then the density of old agents across older vintages.

For vintages 1 to $T-2$, Proposition 1 (ii) says that the growth factor of skilled agent density is exactly given by the input demands n_τ, while for vintage $T-1$, this growth factor is given by half the input demand. Since input demands are falling in vintage, there exists a vintage S, such that $n_\tau < 1$ for all $\forall \tau \geq S$. Combining this with (ii) from Proposition 1 implies that the density function of skilled agents across vintages is single peaked at technology S. The vintage with peak density is one way to think about the rate of technology diffusion: the time elapsed between the introduction and peak usage of a technology. I adopt this definition of diffusion throughout.

When a technology is introduced, it is first learned by young workers producing alone. As the technology ages, the supply of entrepreneurs per worker increases $\frac{1}{n_\tau} > \frac{1}{n_{\tau-1}}$, such that workers can exploit the complementarities in skilled and unskilled labor to a greater extent. The process through which as a technology ages, workers can earn progressively higher wages while learning a given set of skills, will define the process of technology "standardization" in this paper.

2.1.4 Differences in borrowing constraints

This section performs comparative statics by varying the degree of borrowing constraint θ. Since labor endowments and the level of skills acquired by workers are held constant, any aggregate output differences resulting from varying the borrowing constraint θ, are TFP differences empirically. A simple example is used to flesh out the main results, then the general result is presented and a discussion of implications follows.

Consider an equilibrium economy where $T = 2$, and borrowing constraints are binding for young workers who enter the frontier technology (i.e. $\delta_1 < 1$). Participation constraints determine the vintage 1 worker wage, given θ and R,

$$\left(\omega_0 + \frac{\gamma}{R}\pi_1(w_1)\right)\delta_1\left(R\frac{\omega_0}{\gamma\pi_1(w_1)};\theta\right) = \left(w_1 + \frac{\gamma}{R}w_1\right)\delta_2\left(R\frac{1}{\gamma};\theta\right) \tag{12}$$

Denote the two earnings paths as "new technology" and "old technology" occupations. Using the results of Proposition 1 and the full employment constraint, the share of skilled agents in technology 2 is equal to half the input demand of skilled agents in technology 1,

$$\mu_2 = 1 - \mu_1 = \mu_1\frac{n_1(w_1)}{2} \tag{13}$$

This solves for the density of skilled agents across the two vintages.

Now suppose that relative to this benchmark economy, young workers in another economy face tighter borrowing constraints $\hat{\theta} < \theta$. All variables relating to this second economy are denoted with hats. Since the new technology occupation has a steeper earnings profile, the marginal disutility of lower θ is greater for this earnings path and $|\triangle\delta_1| > |\triangle\delta_2|$. Intuitively, new technologies require more "investment" in terms of forgone earnings and tighter borrowing constraints raise the "costs" of such investments. Indirect utility across occupations can only be equalized again if the present discounted value of earnings for the new technology occupation increases. The latter can only be achieved through a lower $\hat{w}_1 < w_1$. Ironically, this implies that the earnings profile for the new technology occupation becomes steeper, and leads to a second round of reductions in δ_1, and w_1 and so on. The second and further rounds of effects are necessitated by the fact that the worker wage for the newest technology is anchored at a technologically determined ω_0.

From (13), multiplied reductions in w_1, and the implied increase in input demand $n_1(w_1)$, translate into a shift in the distribution of skilled agents away from the new technology occupation, $\hat{\mu}_1 < \mu_1$. Since vintage 1 worker wages are lower, the terminal vintage conditions for $T = 2$ in (11) may no longer be satisfied (i.e. $\pi_2(\hat{w}_1) > \hat{w}_1$). It may now be worthwhile for technology 2 skilled agents to be entrepreneurs. In this case, the number of coexisting vintages in the economy will expand, $\hat{T} > T$. These insights are generalized in the following Proposition:

Proposition 2 *Compare two economies with different borrowing constraints $\hat{\theta} < \theta$, where $\delta_1 < 1$, then,*

(i) $\hat{w}_\tau < w_\tau$, $n_\tau(\hat{w}_\tau) > n_\tau(w_\tau)$ for all $1 \leq \tau \leq T-1$, and $\hat{\delta}_\tau \leq \delta_\tau$ for all τ where $\mu_\tau > 0$

(ii) the terminal vintage is older in the $\hat{\theta}$ economy $\hat{T} \geq T$

(iii) $\hat{\mu}$ stochastically dominates μ,

$$\sum_{\tau=1}^{S} \hat{\mu}_\tau < \sum_{\tau=1}^{S} \mu_\tau \mathit{for}\ \forall S < \hat{T} \tag{14}$$

Proof in Appendix.

Recall that the density function for skilled agents across vintages peaks at the youngest vintage such that $n_\tau < 1$. Higher input demand within a particular vintage, implies that the density function peaks at an older technology in the $\hat{\theta}$ economy. Thus, (i) technology diffusion is slower and (ii) a larger range of vintages coexist in the economy with tighter borrowing constraints.

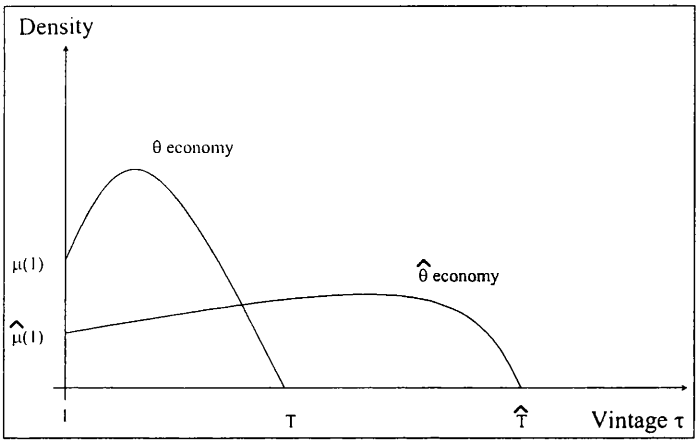

Figure 1: Density of agents across technologies

Consider the youngest vintage $Q \leq \hat{T}$, such that $\hat{\mu}_Q > \mu_Q$. The stochastic dominance result in Proposition 2 (iii) means that such a vintage exists. The higher input demand within vintages and the higher terminal vintage implies that $\hat{\mu}_\tau > \mu_\tau$ for $\forall Q \leq \tau \leq \hat{T}$, and $\hat{\mu}_\tau < \mu_\tau$ for $\forall 1 \leq \tau < Q$. [Figure 1] compares the density functions of skilled agents across vintages in the θ economy and $\hat{\theta}$ economy. Comparing stationary outcomes in these economies, young workers in the $\hat{\theta}$ economy are born into a higher *absolute* stock of skills in older vintages and a lower *absolute* stock of skills in new vintages. In this sense, it is more "appropriate" for young workers in poorer countries to work in older vintages.[8]

[8] This interpretation has implications for the debate between two alternative approaches in analyzing levels differences in per capita income. The factor neutral characteristic of TFP means that technologies are ranked along a single dimension, and there is a unique frontier technology in the world. An alternative approach first formalized by Atkinson and Stiglitz (1969), argues that the menu of technologies exhibits different degrees of efficiency bias toward specific factors of production, so countries with different factor endowments will choose different sets of "appropriate technologies". Caselli and Coleman (2000) find a negative cross country correlation between the efficiency of uneducated labor and educated labor, and present this as evidence in favor of the appropriate technology approach.

My analysis suggests a way to reconcile these two approaches. Differences in the stock of skills across technologies imply that econometric studies will detect higher per worker productivity

A number of further differences in steady state outcomes arise. Lower worker wages within vintages and an older terminal vintage means that the lowest present discounted value of lifetime earnings is lower in the $\hat{\theta}$ economy: $\hat{w}_{\hat{T}-1} + \frac{\gamma}{R}\hat{w}_{\hat{T}-1} < w_{T-1} + \frac{\gamma}{R}w_{T-1}$. The highest discounted value of lifetime earnings in the $\hat{\theta}$ economy is higher: $\omega_0 + \frac{\gamma}{R}\pi_1(\hat{w}_1) > \omega_0 + \frac{\gamma}{R}\pi_1(w_1)$. So more binding borrowing constraints increase the range of discounted lifetime earnings in the economy. Within a particular vintage, the steepness of the earnings profile is higher in the $\hat{\theta}$ economy: $\frac{\gamma\pi_\tau(\hat{w}_\tau)}{\hat{w}_{\tau-1}} > \frac{\gamma\pi_\tau(w_\tau)}{w_{\tau-1}}$. Given $\hat{\theta} < \theta$, the implied fall in the lifetime earnings discount factor, $\hat{\delta}_\tau \leq \delta_\tau$, means the efficiency with which earnings are converted into utility is lower in the $\hat{\theta}$ economy for each vintage.

Since all entrepreneur earnings are higher and the highest worker wage is lower, the skill premium within vintages is higher in the $\hat{\theta}$ economy, $\pi_\tau(\hat{w}_\tau) - \hat{w}_{\hat{T}-1} > \pi_\tau(w_\tau) - w_{T-1}$ for $\forall\tau \leq \hat{T}$. Intuitively, when young workers face borrowing constraints, the "reward" (skill premium) of "investment" (in terms of foregone earnings) becomes higher when the "cost" of investment rises (tighter borrowing constraints). However, the equilibrium effects also lead to higher skill premiums in older vintages where young workers do not face borrowing constraints.

Since input demands are lower within each vintage, $n_\tau(\hat{w}_\tau) < n_\tau(w_\tau)$, it takes longer for a particular ratio of entrepreneur per worker to be realized. This implies that workers in the $\hat{\theta}$ economy have to wait longer to command a given level of wages while learning a set of skills associated with a particular technology. Thus, tighter borrowing constraints slow the rate of technology standardization.

[Figure 2] shows the equilibrium combinations of young and old period earnings of the two economies along their respective indifference curves. Since $\frac{\partial^2\delta}{\partial\frac{y_1}{y_2}\partial\theta} \leq 0$, differences in borrowing constraints imply that these indifference curves are sin-

(for a given measure of workers) in old technologies in poor countries, and higher per worker productivity in new technologies in rich countries. This is the case assuming econometric studies cannot differentiate between skilled and unskilled labor when skills are acquired through learning-by-doing (this is the case in Caselli and Coleman). Suppose educated agents are more likely to work in new technologies than uneducated agents. Then cross country results such as those of Caselli and Coleman would be consistent with the underlying TFP approach of my framework.

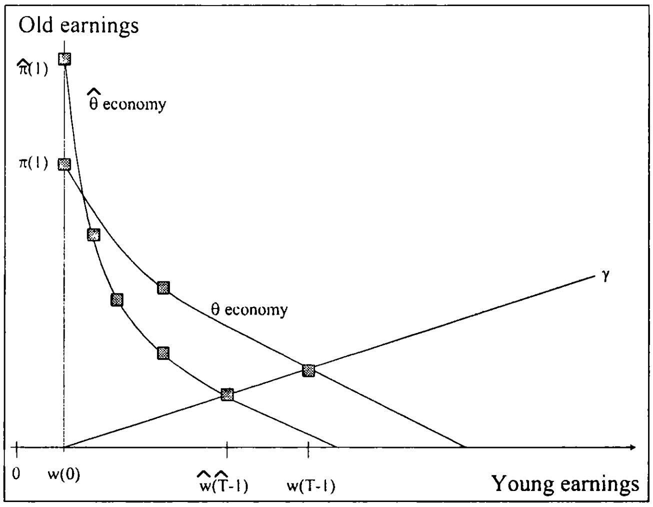

Figure 2: Earnings combinations across technologies

gle crossing. Suppose these two economies coexisted but there is no international worker mobility. Young workers from the $\hat{\theta}$ economy would prefer to migrate to the θ economy to work in older technologies, even when they continue to face borrowing constraint $\hat{\theta}$ after migration. Meanwhile, young workers from the θ economy, would prefer to migrate to the $\hat{\theta}$ economy to work in younger vintages. Workers in the $\hat{\theta}$ economy working in a relatively old vintage may observe higher present discounted earnings and shallower lifetime earnings profiles in *newer* vintages in the θ economy.

2.2 Intra-industry trade

2.2.1 International labor mobility as intermediate goods trade

This section applies the basic model to investigate patterns of specialization across technologies which result from international labor mobility. The mobility of all factors of production must imply there is *factor price equalization.* Since there are only two factors of production in each vintage (entrepreneurs and workers), a sufficient condition for factor price equalization is that only entrepreneurs are internationally mobile.

I interpret outcomes under labor mobility as resulting from trade in intermediate goods in the following way. Consider the earnings of entrepreneurs for each vintage as the price of vintage specific intermediate goods that they produce. Each type of entrepreneur produces one unit of their vintage specific intermediate good. Within each vintage, the entrepreneur intermediate goods are combined with unskilled labor to yield output according to (4). Thus, workers are indifferent between producing with entrepreneurs and producing with intermediate goods produced by entrepreneurs. The analysis is aided by considering outcomes under factor mobility, then interpreting these outcomes as arising from intermediate goods trade.

Consider two coexisting economies of equal size, economy θ and economy $\hat{\theta}$, where $\theta > \hat{\theta}$.[9] The interest rate R is constant. All agents are subject to the borrowing constraints prevailing in their country of origin. Under international mobility of labor, there is a "combined economy" populated by two types of workers who face different borrowing constraints in youth. Denote by primes all variables relating to outcomes in this combined economy. Let μ^W denote the distribution of skilled agents across vintages in the combined population. The density of skilled agents in vintage τ is given by $\mu_\tau^W = \frac{\mu'_\tau + \hat{\mu}'_\tau}{2}$.

A stationary competitive equilibrium with labor mobility is defined as previously, subject to the following modifications. First, young workers in each economy are only indifferent among vintages that their compatriots enter into; i.e. for the θ economy,

$$\begin{aligned} & \left(w'_{\tau-1,t-1} + \frac{1}{R} \max \left\{ \pi_{\tau,t}(w'_{\tau,t}), \max_s w'_{s,t} \right\} \right) \delta_{\tau,t}(\theta) \\ \geq & \left(w'_{\nu-1,t-1} + \frac{1}{R} \max \left\{ \pi_{\nu,t}(w'_{\nu,t}), \max_s w'_{s,t} \right\} \right) \delta_{\nu,t}(\theta) \end{aligned} \tag{15}$$

for all τ, ν such that $n'_{\tau-1} > 0$, and similarly for the $\hat{\theta}$ economy. Second, the full

[9] There is another interesting dimension arising from the relative size of the two economies which is not explored here.

employment constraint is given by: $\sum_{T=1}^{\infty}[\mu'_T + \hat{\mu}'_T] = 2$. Lemma 1 summarizes the pattern of specialization across vintages resulting from labour mobility.

Lemma 1 *Let F, $\hat{F}$ denote the youngest vintage used by entrepreneurs in economy θ and economy $\hat{\theta}$. With international labor mobility, $1 = F' \leq \hat{F}'$, and $T' \leq \hat{T}'$. If borrowing constraints bind for young workers in the $\hat{\theta}$ economy, $\hat{\delta}_{\hat{F}'} < 1$, then $T' \in \{\hat{F}', \hat{F}' - 1\}$.*

Proof in Appendix.

Both the frontier vintage and the terminal vintage are younger in the θ economy. When borrowing constraints are binding for young workers in the $\hat{\theta}$ economy, the technological overlap of skilled agents across the two economies is *at most* one. In equilibrium, the share of agents in the world who migrate to produce with foreign agents is not large. Since younger vintages have higher lifetime earnings, the level of per capita income is higher in the θ economy. The sorting across vintages is the direct consequence of the single crossing property of indifference curves between agents who face different borrowing constraints.

"Technology cycles", a pattern in which new technologies are first learned exclusively in rich countries and are gradually transferred to poor countries, is an equilibrium feature of the analysis. Such cycles are the result of tight borrowing constraint economies delegating the steep earnings profiles activities in the world to loose borrowing constraint economies. Under labor mobility, the former economies can learn new technologies without having to experience steep earnings profiles if they wait until loose constraint economies are in a position to exploit the complementarity between skilled and unskilled labor in production. The latter allows the $\hat{\theta}$ economy to adopt new technologies through a earnings path which is shallow relative to outcomes without labor mobility. As technologies age, they become more standardized as workers can command higher wages while learning a given set of skills. Despite an equal access to the technology frontier, agents from poor countries wait for technologies to become standardized before adopting them from rich countries.

Proposition 3 *With international labor mobility,*

(i) Worker wages within vintages are bounded by the worker wages of the two economies under no mobility $\hat{w}_\tau \leq w'_\tau \leq w_\tau$ *for all* τ *such that* $n'_\tau > 0$.

(ii) The terminal vintage in the world, $\hat{T}'$, *is bounded by the terminal vintage of the two economies under no mobility* $T \leq \hat{T}' \leq \hat{T}$. *The terminal vintage of the* θ *economy can be older or younger* $T \lesseqgtr T'$.

(iii) $\hat{\mu}$ *stochastically dominates* μ^W, μ^W *stochastically dominates* μ,

$$\sum_{\tau=1}^{S} \hat{\mu}_\tau \leq \frac{\sum_{\tau=1}^{S}[\mu'_\tau + \hat{\mu}'_\tau]}{2} \leq \sum_{\tau=1}^{S} \mu_\tau for\ \forall S \leq \hat{T} \tag{16}$$

Proof in Appendix.

Intuitively, the pattern of technology diffusion and the range of coexisting technologies used in the combined economy, is bounded by the pattern of technology use in the two economies without labor mobility. [Figure 3] plots the distribution of skilled agents across vintages in each economy under no mobility, together with the combined distribution of skilled agents with mobility. Compared to the θ economy, the combined economy exhibits slower technology diffusion and older terminal vintage. Compared to the $\hat{\theta}$ economy, the combined economy exhibits faster technology diffusion and younger terminal vintage. Thus, the introduction labor mobility can never increase the age of the terminal vintage in the world.

A clear prediction emerges with respect to the poor $\hat{\theta}$ economy. Before labor mobility, the support of the distribution of skilled agents across vintages is $\{1.\hat{T}\}$. After labor mobility this support is $\{T', \hat{T}'\}$. Labor mobility causes the tight borrowing constraint economy to discard the oldest and newest vintage technologies and specialize in intermediate vintage technologies. For the θ economy, the terminal vintage can increase or decrease following trade, $T \lesseqgtr T'$.

The impact of labor mobility on the relative per capita output between the two economies depends on the difference in their borrowing constraints. Consider

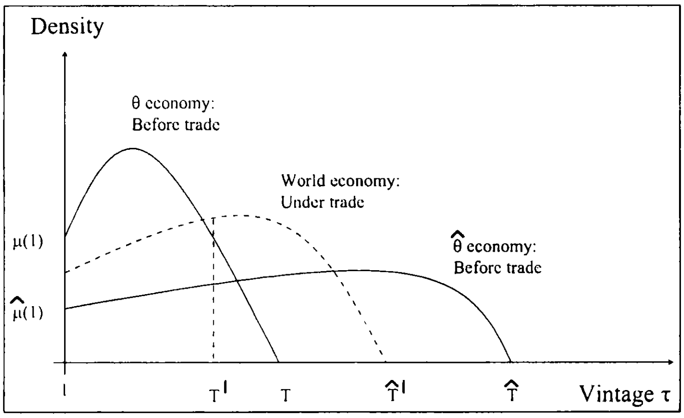

Figure 3: Density of skilled agents under international labor mobility

epsilon differences in borrowing constraints between the two economies: $\hat{\theta} = \theta - \varepsilon$. This corresponds to the case where all three curves in [Figure 3] merge into one curve. Labor mobility is associated with negligible changes in worker wages but large changes in the distribution of agents across technologies. θ economy agents are sorted into high earnings new technologies and $\hat{\theta}$ economy agents are sorted into low earnings old technologies. The ratio of per capita incomes across the two economies, $\frac{y(\theta)}{y(\hat{\theta})}$ must increase. Given much larger differences in borrowing constraints $\hat{\theta} << \theta$, the sorting of agents across technologies is accompanied by large efficiency gains in the level of world output. This introduces the possibility of higher TFP observed in the $\hat{\theta}$ economy. Overall, this may also imply that the ratio of per capita income differences $\frac{y(\theta)}{y(\hat{\theta})}$, is lower with labor mobility: TFP convergence.

[Figure 4] shows the combination of young and old period earnings of agents from both economies under labor mobility. These combinations are mapped along the lower envelope of the indifference curves of agents from the two economies. Agents from both economies must be on higher indifference curves as a result of labor mobility. In the θ economy $w_1' \leq w_1$ ensures lifetime utility is higher for young workers entering the frontier vintage. In the $\hat{\theta}$ economy, $w_{\hat{T}'-1}' \geq \hat{w}_{\hat{T}-1}$ ensures lifetime utility is higher for young workers entering the penultimate vintage. Since participation constraints require lifetime utility to be the same within economies, steady state outcomes with labor mobility increases the lifetime utility of everyone in the combined economy.

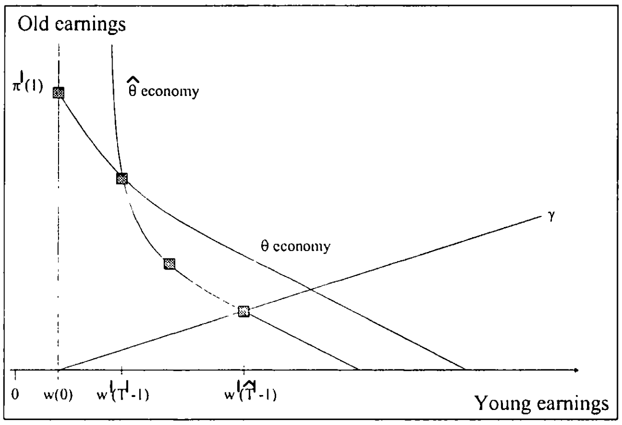

Figure 4: Earnings combinations with international labor mobility

Recall that if workers are indifferent between working with entrepreneurs and working with intermediate goods produced by entrepreneurs, then outcomes under international labor mobility are interpreted as outcomes under trade in intermediate goods. Since a variety of such intermediate goods associated with different technologies, and rankable by their TFP levels, are traded, there is trade in vertically differentiated intermediate goods. Such trade is associated with two forces. First, poor countries specialize in older technologies for which they have a comparative advantage. Second, trade gives workers in poor countries access to a relative abundance of skills in newer technologies which complement the use of new technologies. The combination of these two forces, causes poor countries to specialize in intermediate vintage technologies relative to before trade.

Through trading with richer, loose borrowing constraint countries, workers in poor countries can command higher wages while learning a given set of technology specific skills: that is, they gain access to a faster rate of technology standardization. Although poor countries specialize in the old technologies of the combined economy, the efficiency gains resulting from delegating high TFP activities to rich countries can result in the overall TFP of poor countries rising as a result of intermediate goods trade. Finally, note that the volume of trade in intermediate goods is not large. In particular, from Lemma 1, when borrowing constraints bind for young workers in the $\hat{\theta}$ economy, the number of vintages of intermediate goods traded is at most two.

2.2.2 Final goods trade

Here I do not allow for international labor mobility (or intermediate goods trade), and see what happens when economies differentiated by borrowing constraints trade in vertically differentiated final goods. I begin by showing how the model accommodates vertically differentiated final goods and perform comparative statics with the level of borrowing constraint. Then the implications of trade in vertically differentiated final goods are analyzed.

Technology specific goods are now differentiated by quality which is defined in terms of Lancasterian characteristics. The quality of goods is indexed by $z \geq 0$, and there is a continuum of characteristics indexed by $\xi \geq 0$. Each unit of good z provides one unit of the characteristics $\xi \in [0, z]$. Following Stokey (1991), household preferences are additively separable and symmetric across characteristics. In period t, the utility derived from an allocation of goods, of measure $q_t(z)$ over qualities. is given by,

$$c_t = \int_{\xi=0}^{\infty} v\left(\sum_{z=\xi}^{\infty} q_t(z)\right) d\xi \tag{17}$$

where v is strictly increasing, strictly concave and $v(0) = 0$.[10] I assume preferences are homothetic across goods.

I now interpret new technologies as introducing new goods, where the quality index of the frontier good increases by a factor γ in each period. Let x_τ denote the steady state measure of goods produced using vintage τ technologies, the quality index of which increases by a factor γ each period. The quality $z = 1$ good is the

[10] If v is linear. the analysis collapses to that of the homogenous output economy.

numeraire. Homothetic preferences imply that relative prices are functions of the aggregate supply of goods of different quality, and independent of the distribution of income in the economy. In steady states, the date t relative price of the vintage τ good is given by $\gamma^t p_\tau$ where,

$$\begin{aligned} p_\tau &= p_\tau\left(\frac{x_1}{x_0}, \frac{x_2}{x_0}, ..., \frac{x_{T-1}}{x_0}\right) \qquad (18 \\ &= \frac{v'\left(\sum_{s=0}^{T-1} x_s\right) + (\gamma-1)v'\left(\sum_{s=0}^{T-2} x_s\right) + ... + \gamma^{T-2-\tau}(\gamma-1)v'\left(\sum_{s=0}^{\tau} x_s\right)}{\gamma^{T-1}v'\left(\sum_{s=0}^{T-1} x_s\right)} \text{ if } 0 \leq \tau \leq T - \\ &= \frac{1}{\gamma^\tau} \text{ if } \tau \geq T-1 \end{aligned}$$

From the concavity of $v(\cdot)$, the ratio $\frac{p_{\tau-1}}{p_\tau} \geq \gamma$ is falling in τ and strictly greater than γ for all $\tau \leq T-1$.

A considerable aid to the exposition is to redefine worker wages in terms of the price of final goods they produce, define $p_\tau \nu_\tau \equiv w_\tau$ for all $\tau \geq 1$. This implies that entrepreneur earnings can be rewritten as, $\pi_\tau(p_\tau, w_\tau) = p_\tau \tilde{\pi}(\nu_\tau)$: defining worker wages in this way allows us to separate out the output price component and worker wage component of entrepreneur earnings differences across vintages. Note that the input demand within vintages can now be expressed as, $n_\tau(w_\tau) = \tilde{n}(\nu_\tau)$. Since ω_0 defines the output of vintage $\tau = 0$ goods produced in the frontier technology by workers producing alone, they earn $w_0 = p_0\omega_0$.

A stationary competitive equilibrium is defined as in the case of the economy with homogenous output, subject to some modifications. Now an equilibrium consists of a stationary collection of earning profiles $p_\tau \nu_\tau$, $p_\tau \tilde{\pi}(\nu_\tau)$, input demands $\tilde{n}(\nu_\tau)$, distribution function μ and prices p_τ such that (i) young and old agents maximize their utility, (ii) the full employment constraint is satisfied, (iii) the market for goods of each quality clears: $x_\tau = \mu_\tau f(\tilde{n}(\nu_\tau))$ for all $0 \leq \tau \leq T-1$.

Equilibrium participation constraints are modified as follows,

$$\begin{aligned}\left(p_0\omega_0 + \frac{\gamma}{R}p_1\tilde{\pi}(\nu_1)\right)\delta_1 &= \left(p_1\nu_1 + \frac{\gamma}{R}p_2\tilde{\pi}(\nu_2)\right)\delta_2 = \ldots \\ &= \left(p_{T-2}\nu_{T-2} + \frac{\gamma}{R}\frac{1}{\gamma^{T-1}}\tilde{\pi}(\nu_{T-1})\right)\delta_{T-1} \\ &= \left(\frac{1}{\gamma^{T-1}}\nu_{T-1} + \frac{\gamma}{R}\frac{1}{\gamma^{T-1}}\nu_{T-1}\right)\delta_T\end{aligned} \tag{19}$$

Given the relationship between prices p_τ, note that as before the steepness of the lifetime earnings profile must be falling in vintage. Noting that $p_{T-1} = \frac{1}{\gamma^{T-1}}$ and $p_T = \frac{1}{\gamma^T}$, the conditions on the terminal vintage are modified as follows,

$$\tilde{\pi}(\nu_{T-1}) > \nu_{T-1} \geq \frac{1}{\gamma}\tilde{\pi}(\nu_{T-1}) \tag{20}$$

The vintage for which $\nu_{\tau-1}$ falls within this range determines the age of the terminal vintage T.

Differences in borrowing constraints I use a simple example to flesh out the main mechanisms and then present the general results. Consider an equilibrium economy where $T = 2$ and borrowing constraints are binding in the economy (i.e. $\delta_1 < 1$). Participation constraints determine the vintage 1 worker wage, given θ, R, p_0 and p_1,

$$\left(p_0\omega_0 + \frac{\gamma}{R}p_1\tilde{\pi}(\nu_1)\right)\delta_1\left(R\frac{p_0}{p_1}\frac{\omega_0}{\gamma\tilde{\pi}(\nu_1)};\theta\right) = \left(p_1\nu_1 + \frac{\gamma}{R}p_1\nu_1\right)\delta_2\left(R\frac{1}{\gamma};\theta\right) \tag{21}$$

The share of skilled agents in vintage 2 is equal to half the input demand of skilled agents in vintage 1,

$$\mu_2 = 1 - \mu_1 = \mu_1\frac{\tilde{n}(\nu_1)}{2} \tag{22}$$

Two quality differentiated goods are produced, where their market clearing conditions are given by, $x_0 = \mu_1\omega_0$ and $x_1 = \mu_1 f(\tilde{n}(\nu_1))$. The relative prices of quality differentiated goods (normalized by γ^t) are given by,

$$\begin{aligned} p_0 &= \frac{v'(x_0+x_1)+(\gamma-1)v'(x_0)}{\gamma v'(x_0+x_1)} = 1+\frac{(\gamma-1)[v'(x_0)-v'(x_0+x_1)]}{\gamma v'(x_0+x_1)} \\ p_1 &= \frac{v'(x_0+x_1)}{\gamma v'(x_0+x_1)} = \frac{1}{\gamma} \quad \text{and } p_\tau = \frac{1}{\gamma^\tau} \text{ for all } 1 \leq \tau \end{aligned} \tag{23}$$

Now, relative to this benchmark economy, young workers in another economy face lower borrowing possibilities as a share of their future income, $\hat{\theta} < \theta$. Variables relating to this second economy are denoted with hats. The marginal disutility of tighter borrowing constraints is greater for the steeper, new technology earnings path. Indirect utility across occupations can only be equalized again if the relative discounted value of lifetime earnings for the new technology occupation increases. Given p_0 and p_1, the latter can only be achieved through a lower $\hat{\nu}_1 < \nu_1$.

From (22), the implied increase in input demand $\tilde{n}(\nu_1)$, translates into a shift in the distribution of skilled agents away from the new technology occupation, $\hat{\mu}_1 < \mu_1$. Thus, in the $\hat{\theta}$ economy the ratio of vintage 0 output over vintage 1 output falls, $\frac{\hat{x}_0}{\hat{x}_1} = \frac{\omega_0}{f(\tilde{n}(\hat{\nu}_1))} < \frac{x_0}{x_1} = \frac{\omega_0}{f(\tilde{n}(\nu_1))}$. Since vintage 1 worker wages are lower, the terminal vintage conditions for $T = 2$ in (20) may no longer be satisfied (i.e. $\hat{p}_2\bar{\pi}(\hat{\nu}_1) > \hat{p}_1\hat{\nu}_1$), in which case, the number of coexisting vintages in the economy will expand, $\hat{T} > T$. As a further simplification, assume $\hat{T} = T = 2$. Then, equilibrium relative prices of quality differentiated goods in the $\hat{\theta}$ economy are given by,

$$\begin{aligned} \hat{p}_0 &= 1+\frac{(\gamma-1)[v'(\hat{x}_0)-v'(\hat{x}_0+\hat{x}_1)]}{\gamma v'(\hat{x}_0+\hat{x}_1)} > p_0 \\ \hat{p}_1 &= p_1 = \frac{1}{\gamma} \quad \text{and } p_\tau = \hat{p}_\tau \text{ for all } 1 \leq \tau \end{aligned} \tag{24}$$

While tighter borrowing constraints discourage entry into younger vintages, the resulting reallocation of agents across vintages leads to relative price changes which encourage entry into younger vintages. These relative price changes *dampen* the effect of borrowing constraints on the distribution of agents across vintages. Lemma

2 summarizes the general results which are analogous to those of Proposition 2.

Lemma 2 *Compare two economies with different borrowing constraints $\hat{\theta} < \theta$, where $\delta_1 < 1$, then,*

(i) $\hat{\nu}_\tau < \nu_\tau$, $\tilde{n}(\hat{\nu}_\tau) < \tilde{n}(\nu_\tau)$ *and* $\hat{\delta}_\tau \leq \delta_\tau$ *for all* τ where $\mu_\tau > 0$

(ii) the terminal vintage is older in the $\hat{\theta}$ *economy* $\hat{T} \geq T$

(iii) $\hat{\mu}$ *stochastically dominates* μ,

$$\sum_{\tau=1}^{S} \hat{\mu}_\tau < \sum_{\tau=1}^{S} \mu_\tau for\ \forall S < \hat{T} \tag{25}$$

(iv) *Comparing the* $\hat{\theta}$ *economy and* θ *economy, the ratio of relative prices* $\frac{\hat{p}_\tau}{p_\tau}$ *is strictly falling in vintage until* $\frac{\hat{p}_{\hat{T}-1}}{p_{\hat{T}-1}} = 1$.

Proof in Appendix.

Impact of final goods trade Consider two coexisting economies of equal size, economy θ and economy $\hat{\theta}$, where $\theta > \hat{\theta}$. Denote by primes all variables relating to outcomes with free trade in vertically differentiated final goods. Under free trade, market clearing requires $x'_\tau + \hat{x}'_\tau = \mu'_\tau f(\tilde{n}(\nu'_\tau)) + \hat{\mu}'_\tau f(\tilde{n}(\hat{\nu}'_\tau))$ for all $0 \leq \tau$. The modified full employment constraint is $\sum_{\tau=1}^{\infty}[\mu'_\tau + \hat{\mu}'_\tau] = 2$.

Recall the two example economies from above. Begin by holding the quantity of output across differentiated goods constant in both economies. The ratio of vintage 0 output over vintage 1 output in the world is bounded by the ratio of output of the two economies under autarchy, $\frac{\hat{x}_0}{\hat{x}_1} < \frac{\hat{x}_0+x_0}{\hat{x}_1+x_1} < \frac{x_0}{x_1}$. The ratio of relative prices under trade is also bounded analogously, $\frac{\hat{p}_0}{\hat{p}_1} > \frac{p'_0}{p'_1} > \frac{p_0}{p_1}$. Since $\hat{p}_1 = p'_1 = p_1$, this implies that $\hat{p}_0 > p'_0 > p_0$. Consider the implications of these relative price changes on the participation constraints. In the θ economy, trade encourages further entry into the new technology. In the $\hat{\theta}$ economy, trade discourages entry into the new technology.

To satisfy participation constraints in the θ economy, ν_1 must rise so $\nu'_1 > \nu_1$. The implied decrease in input demand $n(\nu_1)$, translates into a shift in the distribution of agents away from the old technology occupation, $\mu_1' > \mu_1$. In the $\hat{\theta}$ economy, the opposite occurs. To satisfy participation constraints in the $\hat{\theta}$ economy, $\hat{\nu}_1$ must fall so $\hat{\nu}_1' < \hat{\nu}_1$. The implied increase in input demand $n(\hat{\nu}_1)$, translates into a shift in the distribution of agents away from the new technology occupation, $\hat{\mu}_1' < \hat{\mu}_1$. Intuitively, the steady state relative price differences which exist before trade dampen the effect of borrowing constraints on the distribution of agents across technologies. Under trade in final goods, such relative price differences are equalized. Thus, final goods trade *amplifies* the effect of borrowing constraints on the distribution of agents across vintages.

Proposition 4 summarizes the general results of the effects of vertically differentiated final goods trade on the pattern of specialization across vintages.

Proposition 4 *Compare two economies with different binding borrowing constraints $\hat{\theta} < \theta$, before and after trade:*

(i) in the θ economy worker wages in each vintage are higher $p_\tau' \nu_\tau' > p_\tau \nu_\tau$, and the terminal vintage is younger $T' \leq T$ under trade

(ii) *μ stochastically dominates μ',*

$$\sum_{\tau=1}^{S} \mu_\tau < \sum_{\tau=1}^{S} \mu_\tau' \textit{for } \forall S < T \tag{26}$$

(iii) in the $\hat{\theta}$ economy worker wages in each vintage are lower $\hat{p}_\tau' \hat{\nu}_\tau' < \hat{p}_\tau \hat{\nu}_\tau$, and the terminal vintage is older $\hat{T}' \geq \hat{T}$ under trade

(iv) *$\hat{\mu}'$ stochastically dominates $\hat{\mu}$,*

$$\sum_{\tau=1}^{S} \hat{\mu}_\tau' < \sum_{\tau=1}^{S} \hat{\mu}_\tau \textit{for } \forall S < \hat{T}' \tag{27}$$

Proof in Appendix.

Barriers to trade encourage "excessive" entry into high quality production in the $\hat{\theta}$ economy. Under free trade, the distribution of agents across vintages is associated with slower technology diffusion and an older terminal vintage. In the

θ economy, the opposite is true. Barriers to trade encourage excessive entry into relatively low quality production. Under free trade, the distribution of agents across vintages is associated with faster technology diffusion and a younger terminal vintage. [Figure 5] summaries changes in the distribution of skilled agents across vintages caused by free trade in vertically differentiated final goods.

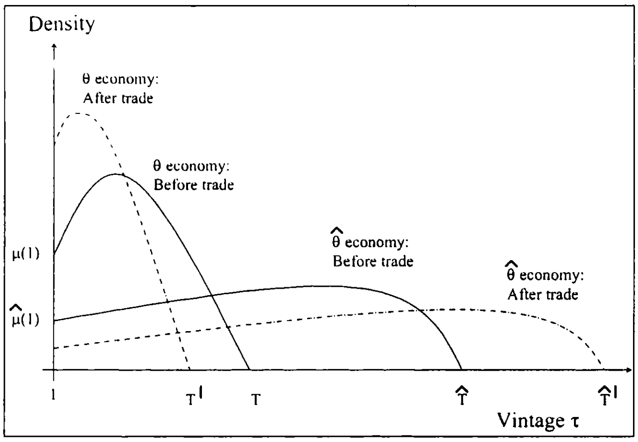

Figure 5: Density of skilled agents under final goods trade

The introduction of trade definitely improves steady state welfare in the θ economy. Since $p'_{T'-1}\nu'_{T'-1} > p_{T-1}\nu_{T-1}$, lifetime welfare has increased for young workers entering the penultimate vintage. From the participation constraints this ensures that welfare has increased throughout the θ economy. In the $\hat{\theta}$ economy, the opposite happens to the worker wage of the penultimate technology, but trade *may* still be associated with higher steady state welfare from the relative price fall of high quality products.

2.3 Conclusion

This paper developed a theory of TFP differences arising from exogenous differences in institutions which enforce external investor rights. In the context of technology diffusion, such differences generated many of the stylized facts about technologies in poorer countries. Although trade and factor mobility may be a force for faster technology adoption and welfare gains, I have shown how some forms of international interaction may not lead to TFP convergence.

Future work should try to verify the links between episodes of institutional reform of external investor rights and accelerated output growth. Institutional reforms constitute a fall in the "barriers to technology adoption" in my framework. Parente and Prescott (1994) define countries which experienced reductions in such barriers as having accelerated growth relative to the U.S. over a sustained period. A further round of such event studies should focus on the impact of trade in generating the predicted pattern of specialization across technologies in the data. In particular, this paper predicts dramatic differences in the use of technologies across industries where trade has predominantly occurred in intermediate goods versus those in which trade has been dominated by final goods.

One potential application of the theory is in understanding residual inequality dynamics between rich and poor nations and their relation to trade. Large increases in residual inequality have been well documented in rich countries since the early 1970s.[11] An independent role for trade in explaining these dynamics has been hindered by three facts: during the period considered (i) job reallocations have occurred within industries as opposed to between industries, (ii) the volume of trade has not increased dramatically and (iii) trade is empirically associated with increased residual inequality in poor countries as well [Zhu and Trefler (2001)]. It is possible to argue that the intermediate goods trade considered in this paper, can lead to increased residual inequality in rich and poor nations, even when the volume of such trade is not large. Recall every agent devotes their youth to learning technologies and acquires skills in old age. An equilibrium feature of workers in newer technologies is their old period component of lifetime earnings is larger. Then, if skill acquisition is stochastic, lifetime inequalities are greater in newer technologies. When trade in intermediate goods causes both rich and poor countries to use newer technologies overall, this would predict greater residual

[11] Acemoglu (2001) provides a review of the related empirical and theoretical literature.

inequality across the world.

2.4 Appendix

Proof of Proposition 1

First show that if $\mu_T = 0$ for some $T \Rightarrow \mu_{T+1} = 0$. Suppose $\mu_T = 0$, $\mu_{T+1} > 0 \Rightarrow w_T = \gamma^{-T}\omega_0, w_{T-1} \geq \gamma^{-T+1}\omega_0 \Rightarrow w_T < w_{T-1}$. This must imply that $\pi_T(w_T) < \pi_{T+1}(w_{T+1})$, but using the definition of $\pi_\tau(w_\tau)$, and noting that $w_{T+1} \geq \gamma^{-T-1}\omega_0$ this leads to a contradiction.

Second show that $T = \infty$ leads to a contradiction. Begin by supposing that $w_\tau \geq w_{\tau+1} \geq \gamma^{-\tau-1}\omega_0 \Rightarrow \pi_\tau \leq \pi_{\tau+1}. \delta_\tau > \delta_{\tau+1}$. Using the definition of $\pi_\tau(w_\tau)$, and noting that $w_\tau \geq \gamma^{-\tau}\omega_0$ this leads to a contradiction. Next suppose $w_\tau < w_{\tau+1} \Rightarrow \pi_\tau \geq \pi_{\tau+1}. \delta_\tau \leq \delta_{\tau+1}$. Since $\pi_\tau(w_\tau)$ is falling in worker wages and vintage, the claim is contradicted.

Next show that $w_\tau < w_{\tau+1}$ for all τ with $\mu_\tau > 0$. Let T be the terminal vintage. Suppose $w_{T-2} \geq w_{T-1} \Rightarrow \pi_T(w_T) \geq \pi_{T-1}(w_{T-1})$. Since T is the terminal vintage, $\max_s w_s > \pi_T(w_T)$ and we have a contradiction. By induction worker wages are increasing in vintage. Since entrepreneur earnings are falling in vintage and worker wages, entrepreneur earnings are falling in vintage and the steepness of the lifetime earnings profile is falling in vintage. Thus, given borrowing constraints are binding $\delta_\tau < \delta_{\tau+1}$.

The existence proof of a unique stationary equilibrium can directly appeal to Proposition 2 and the Theorem in Chari and Hopenhayn (1991).

Proof of Proposition 2

First show that $w_\tau > \hat{w}_\tau$. Suppose $w_1 \leq \hat{w}_1$, from (10) this implies that $w_2 \leq \hat{w}_2$, and by induction $w_\tau \leq \hat{w}_\tau$, all $\tau \leq \hat{T}$. Note from (11) $\hat{T} \leq T$. Since $\frac{\partial \delta}{\partial \theta} > 0$ when $\delta_1 < 1$, revealed preference implies,

$$\begin{aligned} k &= \left[\omega_0 + \frac{\gamma}{R}\pi_1(w_1)\right] \delta\left(R\frac{\omega_0}{\gamma\pi_1(w_1)}, \theta\right) > \left[\omega_0 + \frac{\gamma}{R}\pi_1(w_1)\right] \delta\left(R\frac{\omega_0}{\gamma\pi_1(w_1)}, \hat{\theta}\right) \\ &\geq \left[\omega_0 + \frac{\gamma}{R}\pi_1(\hat{w}_1)\right] \delta\left(R\frac{\omega_0}{\gamma\pi_1(\hat{w}_1)}, \hat{\theta}\right) = \hat{k} \end{aligned}$$

and the terminal vintage conditions (11) imply,

$$\left[w_{\hat{T}-1} + \frac{\gamma}{R} w_{\hat{T}-1}\right] \delta\left(R\frac{1}{\gamma}, \theta\right) \geq \left[w_{T-1} + \frac{\gamma}{R} w_{T-1}\right] \delta\left(R\frac{1}{\gamma}, \theta\right) = k$$

$$[w_{\hat{T}-1}(1 + \frac{\gamma}{R})]\delta\left(\frac{1}{\gamma}, \theta\right) > [w_{\hat{T}-1}(1 + \frac{\gamma}{R})]\delta\left(\frac{1}{\gamma}, \hat{\theta}\right) = \hat{k}$$

Using these inequalities,

$$\begin{aligned} & \left[\omega_0 + \frac{\gamma}{R}\pi_1(w_1)\right]\left(\delta\left(R\frac{\omega_0}{\gamma\pi_1(w_1)}, \theta\right) - \delta\left(R\frac{\omega_0}{\gamma\pi_1(w_1)}, \hat{\theta}\right)\right) \\ \leq \;& k - \hat{k} \leq \left[w_{\hat{T}-1} + \frac{\gamma}{R} w_{\hat{T}-1}\right]\left(\delta\left(R\frac{1}{\gamma}, \theta\right) - \delta\left(R\frac{1}{\gamma}, \hat{\theta}\right)\right) \end{aligned}$$

But $\left[\omega_0 + \frac{\gamma}{R}\pi_1(w_1)\right] > \left[w_{\hat{T}-1} + \frac{\gamma}{R} w_{\hat{T}-1}\right]$, and the fact that $\frac{\partial^2 \delta}{\partial \frac{y_1}{y_2} \partial \theta} \leq 0$ contradicts the inequality. So $w_1 > \hat{w}_1$, and by induction $w_\tau > \hat{w}_\tau$ for all $1 \leq \tau \leq T-1$ and $\delta_\tau \geq \hat{\delta}_\tau$ for all $1 \leq \tau \leq T$, and $\hat{T} \geq T$ from (11).

Using the relationships between successive densities from Proposition 1 (*ii*), the full employment condition can be rearranged to yield the following expression for the density of skilled workers in vintage 1,

$$\frac{1 - \mu_1}{\mu_1} = n_1(w_1) + [n_1(w_1) n_2(w_2)] + \ldots + \left[n_1(w_1) \times \ldots \times \frac{n_{T-1}(w_{T-1})}{2}\right]$$

Since $\hat{T} \geq T$ and $n_\tau(w_\tau) < n_\tau(\hat{w}_\tau)$ for all $\tau \leq T-1$, $\mu_1 > \hat{\mu}_1$. Let Q be the youngest vintage such that $\hat{\mu}_Q > \mu_Q$. Since $\hat{T} \geq T$, $n_\tau(w_\tau) < n_\tau(\hat{w}_\tau)$, and $\mu_1 > \hat{\mu}_1$ stochastic dominance follows for all $S < Q$. It also follows that $\hat{\mu}_S > \mu_S$ for all $\hat{T} \geq S \geq Q$, so $\sum_S^{\hat{T}} \hat{\mu}_\tau > \sum_S^{\hat{T}} \mu_\tau$. This implies that $1 - \sum_{\tau=1}^{S-1} \hat{\mu}_\tau > 1 - \sum_{\tau=1}^{S-1} \mu_\tau$, which establishes the result.

Proof of Lemma 1

By way of contradiction suppose for some vintage S, $\delta'_{S-1} < 1$ and the following two conditions hold,

$$\left[w'_{S-2} + \frac{\gamma}{R}\pi_{S-1}(w'_{S-1})\right] \delta'_{S-1} = \left[w'_{S-1} + \frac{\gamma}{R}\max\{\pi_S(w'_S), w'_{S-1}\}\right] \delta'_S$$

$$\left[w'_{S-2} + \frac{\gamma}{R}\pi_{S-1}(w'_{S-1})\right] \hat{\delta}'_{S-1} \geq \left[w'_{S-1} + \frac{\gamma}{R}\max\{\pi_S(w'_S), w'_{S-1}\}\right] \hat{\delta}'_S$$

This implies that,

$$\begin{aligned} & \left[w'_{S-2} + \frac{\gamma}{R}\pi_{S-1}(w'_{S-1})\right] (\delta'_{S-1} - \hat{\delta}'_{S-1}) \\ \leq \; & \left[w'_{S-1} + \frac{\gamma}{R}\max\{\pi_S(w'_S), w'_{S-1}\}\right] (\delta'_S - \hat{\delta}'_S) \end{aligned}$$

From Proposition 1 $\left[w'_{S-2} + \frac{\gamma}{R}\pi_{S-1}(w'_{S-1})\right] \geq \left[w'_{S-1} + \frac{\gamma}{R}\max\{\pi_S(w'_S), w'_{S-1}\}\right]$, so the inequality violates $\frac{\partial^2 \delta}{\partial \frac{y_1}{y_2} \partial \theta} \leq 0$, when borrowing constraints bind. So $\left[w'_{S-2} + \frac{\gamma}{R}\pi_{S-1}(w'_{S-1})\right] \hat{\delta}'$ $\left[w'_{S-1} + \frac{\gamma}{R}\max\{\pi_S(w'_S), w'_{S-1}\}\right] \hat{\delta}'_S$.

Proof of Proposition 3

Begin with $\hat{w}_\tau \leq w'_\tau$. Suppose $\hat{w}_1 > w'_1$ so under mobility, indirect utility is higher if young workers from economy $\hat{\theta}$ enter the frontier vintage . Participation constraints imply that worker wages are lower in older vintages $\hat{w}_\tau > w'_\tau$. Since workers from the $\hat{\theta}$ economy enter the terminal technology, (11) implies that the indirect utility must be lower for such agents. This is a contradiction. So $\hat{w}_1 \leq w'_1$, and by induction $\hat{w}_\tau \leq w'_\tau$. From (11), higher worker wages implies $\hat{T}' \leq \hat{T}$.

Next consider $w'_\tau \leq w_\tau$. Suppose $w'_1 > w_1$ so under mobility, indirect utility is lower for young workers from economy θ entering the frontier vintage. Participation constraints imply that worker wages are higher in older technologies under mobility $w'_\tau > w_\tau$. From (11), the latter implies that indirect utility would be higher if young workers in the θ economy enter the terminal technology, and this is a contradiction. So $w'_1 \geq w_1$, and by induction $w'_\tau \geq w_\tau$. Result (ii) follows from the argument that the level of the highest worker wages serves as an index of the number of coexisting vintages in (11). The proof of stochastic dominance is similar to that for Proposition 2.

Proof of Lemma 2

Given an equilibrium exists and is unique from the Theorem of Chari and Hopenhayn (1991), I verify such an equilibrium satisfies the conditions of the Lemma. If relative prices between quality differentiated products are held constant, we know from Proposition 2 that when $\hat{\theta} < \theta$, $\tilde{n}(\hat{\nu}_v) > \tilde{n}(\nu_v)$.The task is to prove that this implies $\frac{\hat{p}_{\tau-1}}{p_{\tau-1}} > \frac{\hat{p}_\tau}{p_\tau}$ for all $1 \leq \tau \leq T-1$. From the definition of p_τ and rearranging, we need to show,

$$\frac{v'\left(\sum_{s=0}^{\tau-1}\hat{x}_s\right)}{v'\left(\sum_{s=0}^{\tau-1}x_s\right)}$$

$$> \frac{v'\left(\sum_{s=0}^{T-1}\hat{x}_s\right) + (\gamma-1)v'\left(\sum_{s=0}^{T-2}\hat{x}_s\right) + ... + \gamma^{T-2-\tau}(\gamma-1)v'\left(\sum_{s=0}^{\tau}\hat{x}_s\right)}{v'\left(\sum_{s=0}^{T-1}x_s\right) + (\gamma-1)v'\left(\sum_{s=0}^{T-2}x_s\right) + ... + \gamma^{T-2-\tau}(\gamma-1)v'\left(\sum_{s=0}^{\tau}x_s\right)}$$

This is true if,

$$\frac{v'\left(\sum_{s=0}^{\tau-1}\hat{x}_s\right)}{v'\left(\sum_{s=0}^{\tau-1}x_s\right)} > \frac{v'\left(\sum_{s=0}^{\tau}\hat{x}_s\right)}{v'\left(\sum_{s=0}^{\tau}x_s\right)} > ... > \frac{v'\left(\sum_{s=0}^{T-1}\hat{x}_s\right)}{v'\left(\sum_{s=0}^{T-1}x_s\right)}$$

For any vintage $0 \leq v \leq T-1$,

$$\frac{v'\left(\sum_{s=0}^{v}\hat{x}_s\right)}{v'\left(\sum_{s=0}^{v}x_s\right)} = \frac{v'\left(\hat{\mu}_1\left(\omega_0 + f(\tilde{n}(\hat{\nu}_1)) + \tilde{n}(\hat{\nu}_1)f(\tilde{n}(\hat{\nu}_2)) + ... + \tilde{n}(\hat{\nu}_1)\times ... \times \tilde{n}(\hat{\nu}_{v-1})f(\tilde{n}(\hat{\nu}_v))\right)\right)}{v'\left(\mu_1\left(\omega_0 + f(\tilde{n}(\nu_1)) + \tilde{n}(\nu_1)f(\tilde{n}(\nu_2)) + ... + \tilde{n}(\nu_1)\times ... \times \tilde{n}(\nu_{v-1})f(\tilde{n}(\nu_v))\right)\right)}$$

Given $\tilde{n}(\hat{\nu}_v) > \tilde{n}(\nu_v)$ the result follows. The proof of stochastic dominance is similar to that for Proposition 2.

Proof of Proposition 4

Given an equilibrium exists and is unique from the Theorem of Chari and Hopenhayn (1991), I verify such an equilibrium satisfies the conditions of the Proposition. We need to show, $\frac{\hat{p}_{\tau-1}}{\hat{p}_\tau} > \frac{p'_{\tau-1}}{p'_\tau} > \frac{p_{\tau-1}}{p_\tau}$. Using the proof in Lemma 2, an equilibrium with such relationships between prices exists if, $\tilde{n}(\hat{\nu}_v) > \frac{\tilde{n}(\hat{\nu}'_v)+\tilde{n}(\nu'_v)}{2} > \tilde{n}(\nu_v)$.

Assuming this latter condition holds, the last step is to verify that the implied relationships between prices are consistent this assumed condition. Verifying this is sufficient to confirm the claims of the Proposition. I shall work through the proof for the θ economy. Recall the participation constraints across technologies,

$$\begin{aligned}\left(p_0\omega_0 + \frac{\gamma}{R}p_1\tilde{\pi}(\nu_1)\right)\delta_1 &= \left(p_1\nu_1 + \frac{\gamma}{R}p_2\tilde{\pi}(\nu_2)\right)\delta_2 = \ldots \\ &= \left(p_{T-2}\nu_{T-2} + \frac{\gamma}{R}\frac{1}{\gamma^{T-1}}\tilde{\pi}(\nu_{T-1})\right)\delta_{T-1} \\ &= \left(\frac{1}{\gamma^{T-1}}\nu_{T-1} + \frac{\gamma}{R}\frac{1}{\gamma^{T-1}}\nu_{T-1}\right)\delta_T\end{aligned}$$

Given $\frac{p'_{\tau-1}}{p'_\tau} > \frac{p_{\tau-1}}{p_\tau}$ for all $\tau \leq T-1$, it must be the case that $\nu'_{T-1} > \nu_{T-1}$. Suppose not such that $\nu'_{T-1} \leq \nu_{T-1}$ and the indirect utility of agents is lower under trade. Since $\frac{p'_{T-2}}{p'_{T-1}} > \frac{p_{T-2}}{p_{T-1}}$ this implies $\nu'_{T-2} < \nu_{T-2}$ and by induction $\nu'_\tau < \nu_\tau$ for all $1 \leq \tau \leq T-2$. But since $\frac{p'_0}{p'_1} > \frac{p_0}{p_1}$, this means that indirect utility is higher under trade for agents entering the frontier technology, which is a contradiction. So $\nu'_{T-1} > \nu_{T-1}$ and by induction $\nu'_\tau > \nu_\tau$ for all $1 \leq \tau \leq T-1$ and $\tilde{n}(\nu'_v) < \tilde{n}(\nu_v)$ for all $1 \leq v \leq T-1$. From (20) we also know $T' \leq T$.

In the $\hat{\theta}$ economy a similar logic reveals $\tilde{n}(\hat{\nu}'_v) > \tilde{n}(\hat{\nu}_v)$ for all $1 \leq v \leq \hat{T}-1$, and $\hat{T}' \geq \hat{T}$. The proof is completed by observing that $\tilde{n}(\hat{\nu}'_v) > \tilde{n}(\hat{\nu}_v)$ and $\tilde{n}(\nu'_v) < \tilde{n}(\nu_v)$ is consistent with what was assumed, $\tilde{n}(\hat{\nu}_v) > \frac{\tilde{n}(\hat{\nu}'_v)+\tilde{n}(\nu'_v)}{2} > \tilde{n}(\nu_v)$. The proofs of stochastic dominance are similar to that for Proposition 2.

3 Entrepreneurs versus Managers: Self Employment versus Outside Ownership

This chapter develops a theory of entrepreneurs and managers in the context of self employment versus outside ownership of productive assets. A focal interest is why in capitalist societies, agents who accumulate skills specific to a set of disembodied assets are often hired by outside owners of such assets. It is helpful to outline the structure of the model before discussing motivation.

Two period lived agents in a general equilibrium economy undertake two period projects which combine physical assets with asset specific skills which they accumulate when old through learning by doing when young. When agents productivity streams are steeper than their desired consumption streams and they cannot borrow, outside ownership of physical assets can improve outcomes. Under outside ownership of assets, owners hold up agents with asset specific skills and extract part of their product. Since agents anticipate this ex post hold up by outside owners, when owners compete to attract agents ex ante, they offer up front wages in excess of agents's product. In effect, outside owners implement a stream of transfers to such agents which resembles borrowing. The difference between agents's discounted product and earnings streams constitutes a "dividend" determining the value of scarce productive assets under outside ownership. The arbitrage opportunity for outside ownership arises because of a correlation in periods when agents have asset specific skills, and agents's desire to borrow against their labor product from that period.[12]

Agents who produce with outside owners are called "managers". Another group of agents called "entrepreneurs" undertake new projects where unlike managers, they cannot commit to acquiring skills specific to a set of assets. Then

[12] An important issue is why outside ownership is different from debt. Under standard debt contracts, creditors do not own assets until there is repayment default which happens after production. Unlike outside ownership, there is no ex post hold up. Debt contracts are discussed later.

there are no incentives for outsiders to own assets. Self employed entrepreneurs earn their labor product each period, and are borrowing constrained. However, the subsequent use of entrepreneurs's assets in repetitions of projects is the technology which allows managers to commit to acquiring skills specific to that asset. Thus, by undertaking new projects, entrepreneurs create value to new assets corresponding to the surplus implemented under outside ownership in future repetitions of the project. The key role of entrepreneurs is to supply the economy with assets which outside owners use with hired managers to carry out projects.

An equilibrium where entrepreneurs and managers coexist is constructed by assuming all new projects have higher productivity. In this way, I justify why an endogenous number of productive and less productive projects coexist, and the model can be interpreted as one of gradual technology diffusion. Project lifetimes are finite and there is a continuous entry and exit of projects. The full general equilibrium economy is also populated by workers who produce with managers and entrepreneurs. Workers's lifetime product profiles are shallower than for managers and entrepreneurs, and their savings ensure that at the general equilibrium interest rate, entrepreneurs will be borrowing constrained.

The existing literature models entrepreneurs as providing a different set of scarce resources. Following Schultz (1975), Holmes and Schmitz (1990) assume the ability to exploit arbitrage opportunities is scarce, and is provided by talented entrepreneurs. A related Schumpetarian (1934) view assumes innovation activity is scarce, and entrepreneurs carry out innovations. Following Knight (1921), Kihlstrom and Laffont (1979) assume that risk taking behavior is scarce, and entrepreneurs are relatively risk loving.

In contrast to this theoretical literature, entrepreneurs are empirically identified as (i) the self employed who are (ii) borrowing constrained. Evans and Jovanovic (1989) and Holtz-Eakin, Joulfaian and Rosen (1994) find agents endowed with greater wealth are more likely to become self employed. My theory of entrepreneurs begins with a theory of outside ownership which by default implies particular outcomes under self employment. A necessary condition for outside ownership and self employment to coexist is that the self employed are borrowing constrained. A theory of entrepreneurship where assets which agents can commit to acquire skills specific to are scarce, and new assets are supplied by the self employed coincides well with the empirical identification of entrepreneurs.

Another set of necessary conditions for entrepreneurs and managers to coexist is the continuous entry and exit of finite lived projects. After re-labeling assets under outside ownership as "firms", a reduced form characterization of the model coincides exactly with the canonical industry equilibrium model of Hopenhayn (1992). In this sense, my theory provides microfoundations for the interpretation of "firms" in that framework. By opening up the black box of "firms", and explicitly considering the ownership structure of assets and mode of production, I can provide particular interpretations for the firm entry cost and firm continuation cost assumed in Hopenhayn (1992).

Neher (1999) constructs an agency model of stage financing where successive stages allow entrepreneurs to disembody more of their human capital in physical assets and thereby implement larger loans. My paper considers what happens to such assets after the initial project and how they derive value from their future use. The idea that the ex post hold up problem in bilateral matches can be alleviated through ex ante competition is present in Acemoglu and Shimer (1999) and Kim (1999). In a search unemployment framework, Acemoglu and Shimer (1999) show when the capital investment by firms before they are matched with workers yields product which is extracted by workers through ex post hold up, the ex ante competition between workers to be matched with greater capital will improve firms's chances of being matched with workers. Kim (1999) shows an analogous result when workers invest in general skills before being matched with firms. Both papers show that as long as the search frictions are associated with no externalities (the Hosios condition), there is no investment inefficiency associated with the ex post hold up. Cunat (2002) considers a related mechanism in a model of trade credit where suppliers and customers form bilateral matches, and suppliers anticipating ex post hold up rents from customers, compete for customers by offering ex ante payments. The resulting trade credit coexists with normal bank credit despite its high premium. The current chapter applies the logic of these models to an environment where the combination of ex post hold up and ex ante competition actually improves outcomes, to justify outside ownership arrangements.

Sections 2 and 3 describe the model and equilibrium. Section 4 discusses some of the implications, and section 5 shows how a reduced form version of the model coincides with the Hopenhayn (1992) model. Section 6 concludes.

3.1 Model

Consider a two period overlapping generations economy with a constant population of agents normalized to 2. Ex ante identical agents have preferences over their lifetime given by,

$$u = c_y^{\frac{1}{1+\beta}} c_0^{\frac{\beta}{1+\beta}} \qquad 0 < \beta < 1 \tag{28}$$

Young agents cannot commit to repay loans made against their old period earnings. Thus, there are no credit markets although there may be asset markets. Given their young and old period labor earnings y_1, y_2 and the interest factor of the economy R_t, the indirect utility as a function of earnings for a generation born in period t can be expressed as,

$$\begin{aligned} v(y_1, y_2; R) &= \frac{\left(y_1 + \frac{1}{R_t} y_2\right) (\beta R_t)^{\frac{\beta}{1+\beta}}}{1+\beta} \text{ if borrowing constraints do not bind} \\ &= y_1^{\frac{1}{1+\beta}} y_2^{\frac{\beta}{1+\beta}} \text{ if borrowing constraints bind} \end{aligned} \tag{29}$$

3.1.1 Technology

There is an excess supply of two period projects which are indexed by age, or vintage $\tau \in \{0, 1, ...\}$. A $\tau - 1$ project today becomes a τ project tomorrow. In every period, frontier projects are introduced whose total factor productivity is $\gamma > 1$ times greater than the total factor productivity of last period's frontier projects.

Agents can enter one of three occupations: workers, entrepreneurs or managers. Workers provide unskilled labor in competitive labor markets throughout their lifetimes. Entrepreneurs and managers undertake projects according to a Leontieff technology: one entrepreneur or manager per project. Both entrepreneurs and managers acquire project-specific skills when old, by undertaking projects when young. Period $t-1$ young entrepreneurs or managers have no project specific skills, and yield a constant marginal product, which is assumed to be identical across projects and vintage: $\gamma^{t-1}x_{\tau-1,t-1} = \gamma^{t-1}x \ \forall\tau, t$. Once old in period t, entrepreneurs or managers can combine their project-specific skills with hired workers $n_{\tau,t}$, to yield,

$$y_{\tau,t} = \gamma^{t-\tau} f\left(n_{\tau,t}\right) \qquad \gamma > 1;\ f' > 0, f'' < 0 \tag{30}$$

Old projects are less productive than new projects. There is no uncertainty in this economy.

I assume old, skilled entrepreneurs and managers must work with 1 unit of a project and vintage specific physical asset. Project specific skills are also asset specific skills. Assets live forever, but project and vintage specific assets cannot be reproduced.[13] Let $\tilde{V}_{\tau,t} \geq 0$ denote the value of an asset used by a vintage τ skilled entrepreneur or manager in period t.

What differentiates entrepreneurs from managers is the following. Entrepreneurs work in "virgin" projects where "raw" assets have to be formed in the first period of the project. Raw assets are associated with a commitment problem. Young entrepreneurs cannot commit to acquiring skills specific to a particular set of raw assets. The verified use of an asset in a virgin project, converts raw assets into "seasoned" assets in the second period of a virgin project. Managers work in

[13] This means outside owners must use assets previously used by an entrepreneur. and entrepreneurs can sell assets to appropriate the gains from outside ownership.

"mature" projects where seasoned assets exist in the first period of the project. Seasoned assets are not associated with a commitment problem. Young managers can commit to acquiring skills specific to a particular set of seasoned assets. I simplify the analysis by assuming the raw materials forming assets are in excess supply, so the price of raw assets $\tilde{V}_{0,t} = 0 \ \forall t.$[14]

There can be old, virgin projects, although it will be shown in equilibrium that all virgin projects are new. Assuming the latter, the lifetime of a project is shown in [Figure 6].

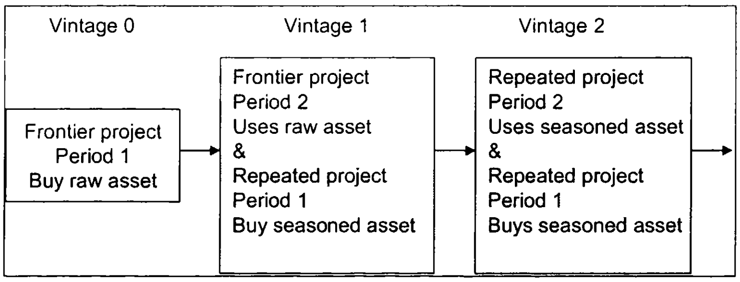

Figure 6: Lifetime of a project

A period $t-1$ new virgin project yields net output $\gamma^{t-1}x - \tilde{V}_{0,t-1} = \gamma^{t-1}x$, plus the skills of the entrepreneur used next period. In period t, the project yields output $\gamma^{t-1}f(n_{1,t}) + \tilde{V}_{1,t}$. A vintage $\tau - 1 \geq 1$ mature project beginning in period $t-1$, yields net output $\gamma^{t-\tau+1}x - \tilde{V}_{\tau-1,t-1}$ plus the skills of the manager used next period. In period t, the project yields output $\gamma^{t-\tau}f(n_{\tau,t}) + \tilde{V}_{\tau,t}$.

Let $\tilde{w}_{\tau,t}$ denote period t worker wages in vintage τ. Since homogenous workers are hired from competitive labor markets $\tilde{w}_{\tau,t} = \tilde{w}_t \ \forall \tau$, workers always earn their marginal product. Similarly, young entrepreneurs and managers have no asset specific skills so always earn *at least* their marginal product. Let $\tilde{\pi}_\tau(\tilde{w}_t)$ denote maximized skilled entrepreneur or manager productivity across projects. Skilled productivities and input demands for unskilled workers are strictly falling in vintage: $\tilde{\pi}_\tau(\tilde{w}_t) > \tilde{\pi}_{\tau+1}(\tilde{w}_t)\,;\ n_\tau(\tilde{w}_t) > n_{\tau+1}(\tilde{w}_t)$. Since old entrepreneurs and

[14] In particular, this assumption allows the economy to avoid outcomes where new and old projects coexist because the raw material costs of old assets have already been sunk.

managers are free to provide unskilled labor, we must have $\tilde{\pi}_{\tau}(\tilde{w}_t) \geq \tilde{w}_t \geq \gamma^t x$. Entrepreneur and manager productivity profiles are steeper than for workers. Outside the project, the best option for an entrepreneur or manager is to become a worker.

Let $\mu_{\tau,t}$ denote the period t measure of old entrepreneurs and managers in vintage τ, and $e_{\tau,t}$ the period t measure of old entrepreneurs only in vintage τ.

3.1.2 Self Employment and Outside Ownership

[Figure 7] shows the timing of events in each period. Agents produce, then conduct asset transactions, and finally consume.

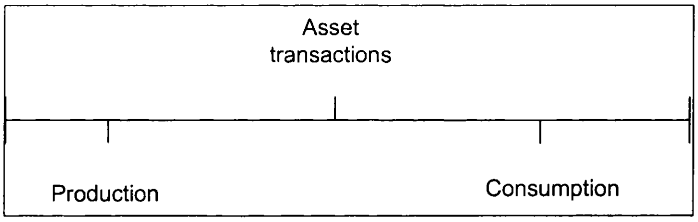

Figure 7: Timeline in period t

I begin with a discussion of managers then consider outcomes for entrepreneurs. If an old manager owns the seasoned asset he produces with, he earns the full product of his asset specific skills plus the resale value of the asset. If outsiders own seasoned assets, the asset owner and manager must bargain over the division of a surplus consisting of the manager's product minus his earnings outside the project: unskilled worker wages. Bargaining takes place before production. I assume that outside owners have full bargaining power such that managers's earnings can be driven down to their outside options when outsiders own assets. This hold up under outside ownership arises because agents cannot contract upon the level of output within projects and the level of asset specific skills of managers. Since managers anticipate second period earnings equal to unskilled wages, under outside ownership, young managers must be offered earnings at least equal to unskilled wages to participate in this occupation. In sum, young managers earnings would exceed their product $w_t \geq x$, while old managers would earn less than their product $\tilde{w}_t \leq \tilde{\pi}_\tau(\tilde{w}_t)$.[15][16]

Let R_{t-1} denote the period $t-1$ (implicit) general equilibrium interest factor in the asset market. Consider an outside owner who buys a seasoned asset for $\tilde{V}_{\tau-1,t-1}$, and hires a young manager in period $t-1$ who realizes output $\gamma^{t-1}x$ for wage $\tilde{w}_{t-1}$. In period t, the owner bargains over the output to receive a surplus $\tilde{\pi}_\tau(\tilde{w}_t) - \tilde{w}_t$ and then sells the asset for $\tilde{V}_{\tau,t}$. Assuming competitive markets for outside ownership, and positive asset values, the net discounted earnings of outside owners must be zero,

$$-\tilde{V}_{\tau-1,t-1} + \gamma^{t-1}x - \tilde{w}_{t-1} + \frac{1}{R_{t-1}}\left(\tilde{\pi}_\tau(\tilde{w}_t) - \tilde{w}_t + \tilde{V}_{\tau,t}\right) = 0 \text{ for } \tau - 1 \geq 1 \quad (31)$$

Rearranging yields an equation for the evolution of seasoned asset values in terms of the difference between the net discounted productivities and earnings of managers. For $\tau - 1 \geq 1$,

$$\tilde{V}_{\tau-1,t-1} = \max\left\{0, \left[\left(\gamma^{t-1}x + \frac{1}{R_{t-1}}\tilde{\pi}_\tau(\tilde{w}_t)\right) - \left(\tilde{w}_{t-1} + \frac{1}{R_{t-1}}\tilde{w}_t\right)\right] + \frac{1}{R_{t-1}}\tilde{V}_{\tau,t}\right\} \quad (32)$$

I verify below in equilibrium that seasoned assets have positive value under outside ownership up to a finite number of vintages.

In multi period projects, the correlation between periods when agents have asset specific skills, and agents wish to borrow against product from that period in borrowing constrained economies, implies there are arbitrage opportunities for other agents who can implement loans for such agents. Since outside owners of

[15] An alternative interpretation coincides with the analysis of firm specific skills in Becker (1964). Outside asset owners appropriate the gains from asset specific skills and "invest" $(w_t - x)$ in young managers.

[16] During their job tenure, managers's product profiles are steeper than their earnings profiles. This analysis is inconsistent with seniority wages: the phenomenon that during job tenure, wage profiles are steeper than product profiles. While empirical studies have shown measured output is consistent with seniority wages, unmeasured components of output (such as the training provided to young by old) may imply that seniority wages does not hold.

seasoned assets carry out cash flows to managers that resemble loans, seasoned assets can have positive value when they are scarce.[17]

Now consider entrepreneurs. Since young entrepreneurs cannot commit to acquiring skills particular to a set of assets, outside owners have no incentive to own raw assets which have value $\tilde{V}_0 = 0$. Thus, entrepreneurs earn their product when young and old. Upon completion of a virgin project, entrepreneurs own the new seasoned assets which have value $\tilde{V}_{1,t} \geq 0$.

3.2 Equilibrium

A competitive equilibrium requires in every period (i) an ownership structure of assets and (ii) agents's choice of occupation, vintage and consumption to maximize lifetime utility subject to the borrowing constraint, earnings across occupations and vintage, the (implicit) interest factor, asset market clearing condition and labor market clearing condition. I restrict attention to steady state growth outcomes where earnings levels, assets prices grow at a constant rate γ, and the distribution of labor across occupations, interest factor and ownership structure of assets are invariant across time: $\tilde{w}_t = \gamma^t w$, $\tilde{\pi}_\tau(\tilde{w}_t) = \gamma^t \pi_\tau(w)$, $\tilde{V}_{\tau,t} = \gamma^t V_\tau$, $\mu_{\tau,t} = \mu_\tau$, $e_{\tau,t} = e_\tau$, $R_t = R$.

Ex ante identical agents become entrepreneurs, managers and workers if their lifetime utility across occupations and vintage is equalized[18],

$$v\left(\gamma^{t-1}x, \gamma^t V_\tau + \gamma^t \pi_\tau(w); R\right) = v\left(\gamma^{t-1}w, \gamma^t w; R\right) \quad \forall \tau \text{ } where \text{ } e_\tau > 0 \tag{33}$$

Note in terms of labor earnings, managers are identical to workers. This comes

[17] Skills may be general with respect to many assets, but merging such assets can make skills de facto specific. Then, although the technology is constant returns to scale in assets, entrepreneurs/managers and workers, there are increasing returns to merging projects where managers's skills are general. This argument unravels if skills are general throughout all projects in the economy. The single merged asset would implement the ex post hold up of managers, but not the ex ante competition to attract managers. Both the ex post hold up and ex ante competition are essential for justifying the arbitrage role of outside ownership.

[18] The use of Cobb Douglas preferences ensures that normalizing earnings by γ^t does not affect the participation constraints.

from the assumption that outside owners have full bargaining power.[19] I guess that entrepreneurs must be borrowing constrained and verify this below in Lemma 2.

Since older projects are less productive, entrepreneurs will only enter frontier projects to become skilled in a $\tau = 1$ project when old. Since managers have to wait for entrepreneurs to supply seasoned assets, the youngest vintage they can enter is $\tau = 1$ to become skilled in $\tau = 2$. So $\mu_1 = e_1$,and $e_\tau = 0 \; \forall \tau > 1$.

Young entrepreneurs entering $\tau = 0$ in period $t-1$ produce $\gamma^{t-1}x$ then form raw assets for free and consume their output. Once old in period t they produce $\gamma^t \pi_1(w)$ and sell seasoned assets for $\gamma^t V_1$ and consume. Although entrepreneurs consume their net labor earnings, young managers and workers may also carry out asset purchases when young which constitutes the savings technology of the economy.

Let $T \geq 1$ denote the oldest or terminal vintage such that,

$$\left(\gamma^{t-1}x + \frac{1}{R}\gamma^t \pi_T(w)\right) - \left(\gamma^{t-1}w + \frac{1}{R}\gamma^t w\right) \geq 0 \tag{34}$$

For younger vintages this inequality holds strictly, and for older vintages the inequality is reversed. Skilled agents coexist in vintages 1 to T. Since older projects are less productive, T is finite and a decreasing function of w and R. The steady state supply of entrepreneurs determines the number of assets and managers in each surviving vintage, so skilled agent densities across coexisting vintages are uniform, $e_1 = \mu_\tau \equiv \mu \; \forall 1 \leq \tau \leq T$.

Solving recursively, the steady state value of a vintage $\tau - 1$ seasoned asset can be expressed as a function of the difference in discounted lifetime productivities of managers and workers given R,

$$\begin{aligned} \gamma^{t-1}V_{\tau-1}(w,R) &= \gamma^{t-1}\sum_{s=\tau}^{T}\frac{\gamma^{s-\tau}}{R^{s-\tau}}\left[\left(x + \frac{1}{R}\gamma\pi_s(w)\right) - \left(w + \frac{1}{R}\gamma w\right)\right] > 0 \quad \mathit{for}\ 1 \leq \tau - 1 \leq T \\ &= 0 \quad \mathit{for}\ \tau > T - 1 \end{aligned}$$

[19]Managers and workers are distinguished by the tenure of employment for asset owners. Two periods for managers. one period for workers.

The difference between discounted lifetime productivities between managers and workers is the dividend earned by the seasoned assets under outside ownership. The right hand side of the equation represents the asset value of the flow of dividends. Asset values are strictly falling in vintage τ, the worker wage $\gamma^t w$, and the interest factor R, and increasing in the age of the terminal vintage T. If managers coexist in the economy, $\gamma^{t-1}V_1 \geq 0 \Rightarrow \gamma^{t-1}x + \frac{1}{R}\gamma^t\pi_1(w) > \gamma^{t-1}w + \frac{1}{R}\gamma^t w$. Given the participation constraint, entrepreneurs can only have higher discounted earnings if they are borrowing constrained.

Given $V_1(w, R)$, the indifference condition between entrepreneurs and workers allows us to solve for the equilibrium worker wage as a function of the interest factor, $w^* = w(R)$.

Lemma 1 *Worker wages are falling in the interest factor* $\frac{dw^*}{dR} \leq 0$.

Proof. The indifference condition is,

$$x^{\frac{1}{1+\beta}}\left(\gamma V_1(w,R) + \gamma\pi_1(w)\right)^{\frac{\beta}{1+\beta}} = \frac{w\left(1+\frac{1}{R}\gamma\right)(\beta R)^{\frac{\beta}{1+\beta}}}{1+\beta}$$

Since V_1 is strictly falling in R, entrepreneurs's utility is (weakly) falling in R. Workers's utility is (weakly) rising in R as long as $\left(\beta - \frac{1}{R}\right) \geq 0$. This inequality must hold under asset market clearing in Lemma 2, so the result follows. ■

The labor market clearing condition for old agents is given by,

$$\frac{\mu}{2}\sum_{\tau=1}^{T} n_\tau(w) = 1 - \mu T \tag{36}$$

Only half of the steady state measure of workers are old. Since T and n_τ are falling in w, μ overall is increasing in w.

Using the steady state measures of μ_τ and e_1, the asset market clearing condition is given by,

$$\gamma^t\mu(T-1)\left[\frac{\sum_{\tau=2}^{T} V_{\tau-1}(w,R)}{T-1} + (w-x)\right] = \gamma^t(1-\mu)\frac{\left(\beta - \frac{1}{R}\gamma\right)w}{1+\beta} \tag{37}$$

This gives us the equilibrium interest factor as a function of the unskilled wage $R^* = R(w)$.[20] The left hand side denotes the demand of aggregate savings consisting of seasoned asset values and the ex ante "loan" to young managers. The right hand side denotes the supply of aggregate savings offered by managers and workers at interest factor R in competitive asset markets. Note when there are no managers, $T = 1$, then the left hand side is zero and the equilibrium interest factor is $\frac{1}{\beta}$.

Lemma 2 *Entrepreneurs must be borrowing constrained given any interest factor clearing the asset market. The interest factor lies between $\frac{\gamma}{\beta} \leq R^* < \frac{1}{\beta}\frac{\gamma\pi_1(w)}{x}$.*

Proof. Suppose not so $R^* \geq \frac{1}{\beta}\frac{\gamma\pi_1(w)}{x}$ and $\left(x + \frac{1}{R}\gamma\pi_1(w)\right) - \left(w + \frac{1}{R}\gamma w\right) = 0 \Rightarrow T = 1$, no assets are traded, and $\mu_2 = 0\ \forall\tau \geq 2$ there are no managers. Since there are no asset market transactions the equilibrium interest factor is $R^* = \frac{\gamma}{\beta}$. This is a contradiction. The bounds on the interest factor follow by inspection. ■

Workers play a crucial role in ensuring that at the general equilibrium interest factor, entrepreneurs and managers may coexist since entrepreneurs are borrowing constrained. The asset market clearing condition can be rewritten in terms of equating the net demand for savings by managers to the supply of savings by workers alone,

$$\gamma^t\mu(T-1)\left[\frac{\sum_{\tau=2}^{T} V_{\tau-1}(w,R)}{T-1} + \frac{\left(1+\frac{\gamma}{R}\right)}{1+\beta}w - x\right] = \gamma^t\left(1-\mu T\right)\frac{\left(\beta - \frac{\gamma}{R}\right)w}{1+\beta}$$

Suppose workers are excluded from buying seasoned assets so the right hand side equals zero. Managers would be buying assets in each others projects, and the equilibrium interest factor would cause managers to consume *at most* x when young. Since entrepreneurs can consume x when young and entrepreneurs have higher discounted lifetime income than managers, entrepreneurs and managers

[20] An alternative formulation of the asset market clearing condition is assets are priced by their discounted future resale price and dividend. $\frac{\mu}{R}\sum_{\tau=2}^{T}\left[V_\tau(w,R) + (\pi_\tau(w) - w)\right]$. The definition of asset values ensures the equivalence of the two formulations.

would not coexist among ex ante identical agents.

Proposition 1 *A steady state equilibrium* $\{w^*, R^*, T^*, \mu^*\}$ exists for the economy.

Proof. Consider the bounds on w^* given the bounds on the interest factor from Lemma 2. The upper bound is given by $\bar{w} \equiv w\left(\frac{\gamma}{\beta}\right)$. The lower bound is given by $\underline{w} \equiv w\left(\beta\frac{x}{\gamma\pi_1(\underline{w})}\right)$, and it is known that entrepreneurs are not borrowing constrained, $V_1\left(w, \beta\frac{x}{\gamma\pi_1(w)}\right) = 0$. So, $w^* \in (\underline{w}, \bar{w}]$.

From Lemma 2, we know $R^*(\underline{w}) \in \left[\frac{\gamma}{\beta}, \beta\frac{x}{\gamma\pi_1(\underline{w})}\right)$ and $R^*(\bar{w}) \in \left[\frac{\gamma}{\beta}, \beta\frac{x}{\gamma\pi_1(\bar{w})}\right)$. Consider the following two cases. Case 1: $T\left(\bar{w}, \frac{\gamma}{\beta}\right) = 1 \Rightarrow R^*(\bar{w}) = \frac{\gamma}{\beta}$. Case 2: $T\left(\bar{w}, \frac{\gamma}{\beta}\right) > 1 \Rightarrow R^*(\bar{w}) > \frac{\gamma}{\beta}$ and $R^*(\underline{w}) > \frac{\gamma}{\beta}$. Under case 1, the economy has a solution where $w^* = \bar{w}$, and $R^* = \frac{\gamma}{\beta}$. Under case 2, the economy has a solution where $w^* \in (\underline{w}, \bar{w})$, and $R^* \in \left(\frac{\gamma}{\beta}, \beta\frac{x}{\gamma\pi_1(\underline{w})}\right)$. ■

3.3 Discussion

3.3.1 Coexistence of Entrepreneurs and Managers

This subsection highlights three necessary assumptions for entrepreneurs and managers to coexist. First, if there are no borrowing constraints and young agents can freely borrow against their old period earnings, only frontier projects would be undertaken and there would be no managers, $T' = 1$. The present discounted value of earnings would be equal for entrepreneurs and workers. In such an economy the credit market clearing condition would be,

$$\mu'(w' - x) = \frac{\left(\beta - \frac{\gamma}{R'}\right) w'}{1 + \beta} \tag{38}$$

Substituting in equilibrium μ' this equation becomes,

$$\frac{(w' - x)}{\frac{n_1(w')}{2} + 1} = \frac{\left(\beta - \frac{\gamma}{R'}\right) w'}{1 + \beta} \tag{39}$$

Similarly, if there were borrowing constraints, but young entrepreneurs could commit to acquire skills specific to raw assets, only frontier projects would be undertaken and the present discounted value of earnings would be equal for managers and workers. In such an economy the asset market clearing condition would be identical to the credit market clearing condition above since the price of raw assets used in virgin projects is zero.

More generally, since positively valued assets substitute for the market failures in the credit and asset markets, asset values would be inversely related to the degree of either of these imperfections.

A third necessary condition is that project productivities increase over time, $\gamma > 1$. Consider outcomes in the limit as $\gamma \to 1$. From the definition of T, $\gamma \to 1 \Rightarrow (i)\ T \to \infty \Rightarrow \mu \to 0$, (ii) asset values converge to a constant independent of $\tau : V_\tau \to V(\hat{w}, \hat{R}) = \frac{\hat{R}}{\hat{R}-1}\left[\left(x + \frac{\gamma}{\hat{R}}\pi(\hat{w})\right) - \left(\hat{w} + \frac{\gamma}{\hat{R}}\hat{w}\right)\right]$, and (iii) input demands for workers are independent of $\tau : n_\tau(\hat{w}) \to n_1(\hat{w})$. The asset market clearing condition at the limit is,

$$\frac{V(\hat{w}, \hat{R}) + (\hat{w} - x)}{\frac{n_1(\hat{w})}{2} + 1} = \frac{\left(\beta - \frac{\gamma}{\hat{R}}\right)\hat{w}}{1 + \beta} \tag{40}$$

Agents in equilibrium are indifferent across occupations, but since assets live forever, nobody becomes an entrepreneur. For entrepreneurs and managers to coexist $\gamma > 1$ must hold, for there to be a continuous entry and exit of finite lived projects.

3.3.2 Debt Contracts

Direct credit markets were shut down in the previous analysis. A substitute savings technology was provided in the form of an asset market. Although managers and workers lend in asset markets, entrepreneurs are borrowing constrained in equilibrium. In reality, entrepreneurs may be able to borrow against the anticipated value of seasoned assets they own and sell after virgin projects are completed. A typical debt contract consisting of a loan and repayment plan which confers ownership rights on all physical assets of entrepreneurs to creditors if entrepreneurs default on repayments could be implemented. Debt repayments take place after production and before asset transactions. Since entrepreneurs can only borrow against a positive asset value $V_1 > 0$, and seasoned assets can only have value under outside ownership, the qualitative results discussed above would not be affected.

Once debt contracts collateralized by seasoned assets are allowed, the participation constraint across occupations becomes,

$$v\left(x + \frac{\gamma}{R}V_1, \gamma\pi_1(w); R\right) = v(w, w; R) \tag{41}$$

Since project productivities fall with age, and managers are at least as well off as entrepreneurs, managers continue to strictly prefer being hired by outside owners, rather than using debt contracts to own assets themselves.

My model predicts that over the life-cycle of a project, external finance would first be provided using debt, and then outside ownership or "equity". Unlike outside owners, debt creditors cannot hold up skilled managers since their ownership of assets occurs after production, conditional on default on repayment. The Modigliani-Miller proposition does not hold here, since seasoned assets can only yield dividends under outside ownership. However, when the gains from outside ownership is not feasible in virgin projects, debt contracts would be written against the anticipated value of assets under outside ownership.[21]

3.3.3 Optimal Terminal Vintage

Is the terminal vintage rule efficient? Efficiency is ensured by a terminal vintage rule which equates the discounted value of a continued project (the manager product) to the discounted opportunity cost (the product of the manager if he was a worker).[22]

[21] Outside owners can borrow up to the resale value of assets as well. In the current model, since all lenders are homogenous, outside owners are indifferent between the mix of debt and equity up to the resale value of the asset.

[22] The two discounted values may not hold with exact equality since the sequence of project depreciations is discrete.

The main analysis assumed that outside owners have full bargaining power such that skilled managers's labor earnings are driven down to their outside option: worker wages $\gamma^t w$. Suppose when bargaining with outside owners, managers can secure a share $0 \leq \theta \leq 1$ of the surplus between his product and outside option. I interpret θ as an exogenous institutional variable which individual agents cannot affect. Let $\gamma^t \varpi_{\tau-1} \geq \gamma^t x$ denote the earnings offered by outside owners to attract young managers who will become skilled in a vintage τ project when old. The new terminal vintage rule sets $T \geq 1$ as the oldest vintage such that,

$$\left(x + \frac{\gamma}{R}\pi_T(w)\right) - \left(\varpi_{T-1} + \frac{\gamma}{R}\left[\theta\left(\pi_T(w) - w\right) + w\right]\right) \geq 0 \tag{42}$$

As long as managers remain borrowing unconstrained, they are willing to work for discounted labor earnings equal to that of workers. If they are borrowing constrained, their discounted labour earnings must be higher for them to remain indifferent across occupations.

$$\begin{aligned} \varpi_{T-1} &= w - \frac{\gamma}{R}\left[\theta\left(\pi_T(w) - w\right)\right] \text{ if borrowing unconstrained} \\ &> w - \frac{\gamma}{R}\left[\theta\left(\pi_T(w) - w\right)\right] \text{ if borrowing constrained} \end{aligned} \tag{43}$$

As long as managers are not borrowing constrained (low θ), the terminal vintage rule is optimal. Once they are borrowing constrained (high θ), the terminal vintage is lower than the optimum. Suppose for instance $\theta = 1$, then $T = 1$, no assets are traded and there are no managers.

An interpretation of this result is that in economies where outside owners's rights are well protected (low θ), no other stakeholder interests need to be protected. In economies where outside owners's interests are poorly protected (high θ), other stakeholder (e.g. managers's) interests need to be considered in the decision to terminate projects.

High θ economies are characterized by a relative scarcity of employment vacancies opened by outside owners. They are also characterized by a relative abundance of self employed entrepreneurs who undertake relatively short lived projects.

3.4 Firms as Assets Under Outside Ownership

This section sets up a canonical model of industry equilibrium and interprets it as a reduced form version of my model. Consider a discrete time economy composed of a continuum of "firms" which produce a homogenous good. The output of an individual firm which is $\tau \geq 1$ periods old in period t is,

$$\gamma^{t-\tau} f(m_{\tau,t} - 1) \qquad \gamma > 1;\ f' > 0, f'' < 0 \tag{44}$$

$m_{\tau,t} \geq 1$ denotes the quantity of labor hired from competitive labor markets at wage w_t. If $m_{\tau,t} \ngeq 1$ output is zero. In each period, a fixed cost $\gamma^t c$ must be incurred to prolong the life of the firm by 1 period. New 0 period old firms are opened at fixed entry cost $\gamma^t F$, and yield no output until they are 1 period old.

In a *steady state growth equilibrium*, the life of a firm is prolonged as long as,

$$-c + \frac{\gamma}{R}\left[\gamma^{-\tau} f(m_\tau - 1) - m_\tau w\right] \geq 0 \tag{45}$$

Let T denote the terminal age of firms, and R the discount factor of the economy. The value of a $1 \leq \tau \leq T$ period old firm is given by,

$$G_\tau \equiv -c \sum_{s=\tau}^{T-1} \frac{\gamma^{s-\tau}}{R^{s-\tau}} + \sum_{s=\tau}^{T} \frac{\gamma^{s-\tau}}{R^{s-\tau}} \left[\gamma^{-s} f(m_s - 1) - m_s w\right] \tag{46}$$

New firms will enter until discounted profits net of the entry cost is zero. In equilibrium,

$$F = -c + \frac{1}{R} G_1 \tag{47}$$

This economy almost exactly describes the industry equilibrium of Hopenhayn (1992). The only differences are that (i) new firms do not yield output until they are 1 period old, (ii) incumbent firms incur fixed costs to keep firms open in the following period as opposed to the current period, and (iii) one extra worker must be hired in production.

Recall the original economy with entrepreneurs, managers and workers. Let (i) $m_\tau = n_\tau + 1$, (ii) $c = w - x$, and (iii) $F = \left(x + \frac{\gamma}{R}\left[\pi_1(w) + V_1\right]\right) - \left(w + \frac{\gamma}{R}w\right)$ the difference in discounted earnings which compensates entrepreneurs for facing borrowing constraints. Assuming managers have no bargaining power $\theta = 0$, they are identical to workers in terms of earnings. Then, a reduced form version of my original economy has been exactly described above when assets under outside ownership are called "firms".

Substituting in for c and m_τ and rearranging, firm values and asset values are equated by,

$$G_\tau = (\pi_\tau(w) - w) + V_\tau \quad \text{for } 1 \leq \tau \leq T \tag{48}$$

The value of a firm coincides exactly with the dividend and resale value of seasoned assets under outside ownership. These substitutions also ensure that the terminal vintage condition is identical for both economies.

In light of my theory, the assumptions underlying the canonical Hopenhayn (1992) model can be interpreted in the following way. "Firms" undergo changes in their ownership structure during their life-cycle from self employment to outside ownership. Under self employment, borrowing constrained owners need to be compensated to open new firms, and this compensation translates into the fixed entry cost F in the Hopenhayn (1992) model. Since the role of the self employed is to supply the economy with seasoned assets, their compensation for facing borrowing constraints can be interpreted as the entry cost for seasoned assets. The "loan" offered by outside owners to attract managers into two period projects translates into the continuation cost c of keeping firms open in the Hopenhayn (1992) model. This "loan" is also the cost of prolonging the use of assets under outside ownership. Unlike in the Hopenhayn (1992) model, both F and c are endogenous to my model.

3.5 Conclusion

This paper developed a theory of entrepreneurs and managers in the context of self employment versus outside ownership of productive assets. A focal interest was why in capitalist societies, agents who accumulate skills specific to a set of disembodied assets are often hired by outside owners of such assets.

Since a fixed level of skills are acquired through learning by doing, the model abstracts away from how there may be underinvestment in skills due to the hold up problem under outside ownership. Such an argument relies on the costs of skills being private to the agent acquiring the skill. Since entrepreneurs unlike managers own the assets they produce with, a richer model could capture the prediction that entrepreneurs accumulate more skills than managers in projects. On the other hand, if investment is costly in output terms, the borrowing constraints facing entrepreneurs could deter skill investment. Different types of skills could be accumulated to different extents depending on whether projects are carried out by entrepreneurs or managers.

Another extension would be to have agents who live for four or more periods, so they can carry out more than one project in their lifetimes. Such a model would predict that entrepreneurs remain entrepreneurs throughout their lifetimes, and always sell assets upon completion of virgin projects. Entrepreneurs would have the largest stock of accumulated wealth, and are in the best position to overcome borrowing constraints which characterize entrepreneurship in the current model.

Finally, this paper sets out a more general research agenda by example. Traditional characterizations of firms in aggregate models have remained a black box. By considering the optimal ownership structure of assets and the mode of production explicitly, stronger microfoundations can be revealed (or not revealed) in the current modelling of firms in aggregate economies.

4 Skill Accumulation in the Search Unemployment Model

This chapter analyses the correlation between employment duration, unemployment duration and general skill accumulation in a search equilibrium. A classic topic in development economics is the correlation of labour force participation in the formal, employment sector with the level of development. When richer countries are characterized by lower unit costs of skills and higher skill levels, I derive implications for unemployment and job tenure durations, and the feedback between such variables and skill levels. The duality of labor markets highlighted by Harris and Todaro (1970), has in the past been explained using a labor turnover model by Stiglitz (1974). The need for an updated analysis along these lines has been recently emphasized by Basu (1997). Mortensen and Pissarides (1994) present a canonical model of endogenous job duration in a search unemployment framework. The extension of that analysis to general skill accumulation constitutes the core of my paper.

With search frictions in matching unemployed workers with job vacancies, the marginal benefit of skills is higher in current matches than in future matches since re-matching takes time. This implies that matches become more resilient towards idiosyncratic shocks, and the expected duration of matches is longer, when skill levels are higher. For the same reason, search frictions imply that the bilateral surplus created between workers and vacancies opened by entrepreneurs, is increasing in general skill levels.

Entrepreneurs who open vacancies receive a fixed share of this surplus. Under free entry of vacancies, a higher surplus translates into a higher vacancy to unemployment ratio, as entrepreneurs "compete" for workers by opening job vacancies. For workers this means that unemployment durations on average are shorter. Since general skills are only productive within matches, longer employment durations and shorter unemployment spells improve the incentives for general skill accumulation. Thus, I show how the level of general skills and the duration of employment are positively correlated, and both variables are negatively correlated with the duration of unemployment spells.

A natural question which arises in analyzing the correlations above is (i) who finances the acquisition of general skills and (ii) is the level of general skill accumulation efficient? With search frictions both entrepreneurs and workers have incentives to pay for general skills, since the bilateral surplus is increasing in skill levels. Under Nash Bargaining of the bilateral surplus, the Becker (1975) rule for general skill finance (workers pay for all of it) is modified in an intuitive way. Employers pay for their bargaining share of total skill cost minus the appreciation in the workers outside option resulting from skill accumulation. Skill accumulation has two effects, increasing the bilateral surplus and improving the worker's outside option and bargaining power. Since the entrepreneur cannot appropriate any share of the improvement in the worker's outside option, the level of the entrepreneur's skill finance is exactly his bargaining share of total skill finance less the unappropriable share.

Workers acquire general skills with the first vacancy they are matched with. I assume skills do not depreciate throughout workers' lifetimes. Since the marginal productivity of workers' general skills is shared with future entrepreneurs who workers expect to be matched with, general skill accumulation is subject to a hold up problem. However, if entrepreneurs open vacancies which are *directed* towards workers of different skill levels, workers will indirectly be able to internalize the externality of skill accumulation, through shorter unemployment spells.

Besides the hold up problem associated with general skill accumulation, the search employment economy is subject to further externalities resulting from (i) job separations causing a negative externality to the pool of existing unemployed workers, and (ii) job creations causing a positive externality on the pool of existing unemployed workers. Hosios (1990) identified a condition under which these latter externalities exactly cancel each other out. I show how given the Hosios condition, with free entry of vacancies, and directed search by vacancies to workers of different skill levels, general skill accumulation is fully efficient. Under these conditions, workers are *exactly* internalizing the externality generated by skill accumulation on future entrepreneurs that workers expect to be matched with. Meanwhile, if vacancies do not direct their search to skilled and unskilled workers, the economy is subject to coordination failure. Workers are unable to internalize the externality of their skill accumulation decisions. I discuss conditions under which skill accumulation is distorted in different directions.

Compared to the Becker analysis, the search framework predicts a larger range of skills are accumulated through training than education. I assume skills acquired through education (before entering the labour market) and training (acquired upon being matched with first job) are perfect substitutes in production. Then for Becker, skills whose unit costs are lower under education are accumulated through education and otherwise through training. In my analysis, a range of skills whose costs are lower under education are accumulated through training, since (i) entrepreneurs participate in the financing of training and (ii) getting the first job takes time so workers discount the benefits of skills acquired through education.

Individual differences in the unit costs of skill accumulation through education give rise to two distinct classes of workers: "white collar" workers who accumulate skills through education and "blue collar" workers who accumulate skills through training. Even small differences in unit costs of skill accumulation can lead to discrete differences in skill levels when one group acquires skills through education and another acquires skills through training. White collar workers who accumulate skills through education have *discretely* longer employment duration and lower unemployment duration.

With search frictions, the duration of matches is increasing in job *specific* skills as well. Ceteris paribus, optimal specific skill accumulation maximizes the bilateral surplus of job matches, and minimizes the unemployment duration of workers. In turn, specific skills respond positively to the expected duration of employment. General and job specific skills complement each other indirectly through the expected duration of employment matches.

In a full employment framework, general skill accumulation decisions under search frictions and exogenous job destruction have been studied by Acemoglu (1997). Non-directed search by vacancies was assumed in that paper. Acemoglu and Shimer (1998) have shown how the externality generated by firms' specific (physical) capital decisions can be indirectly internalized, when workers exercise directed search towards vacancies following wage posting by firms. The paper begins by introducing the skill accumulation process through training. The description of equilibrium is followed by efficiency results and analysis of the economy with non-directed search. The extensions cover education and specific skill accumulation. The last section concludes with suggestions for future research.

4.1 Model

There are two types of agents in the economy: entrepreneurs and workers. Everyone is risk neutral and non-wealth constrained. The share of entrepreneurs in the population is fixed and the size of the labor force who are workers is normalized to 1.[23]

General skills k are embodied in workers and workers can utilize these skills in different matches. Entrepreneurs have a large set of ideas they can implement, and the only shocks in the economy are idea specific ones, and all ideas have an identical and independent productivity distribution at the point of implementation (when idea and worker are first matched).[24]

Productive matches are bilateral (between one worker and one idea), implying the production function is Leontieff. The productivity flow of a particular idea-

[23] This paper concentrates on the worker side of the economy. The analysis of entrepreneurs is trivial in this economy.

[24] The dominace of plant or firm specific idiosyncratic shocks in gross job flows is well documented. See the survey by Davis and Haltiwanger (1998).

worker match is given by,

$$f(k) + x\sigma \qquad \text{where } f' > 0, f'' < 0 \tag{49}$$

The general and idea specific components of productivity are additive.[25] For each idea, $x \in [\underline{x}, \bar{x}]$, and at the point of idea implementation $x = \bar{x}$. The motivation for the last assumption is that only the latest and most productive ideas are adopted at any given time. Bilateral matches are subject to idea specific shocks at Poisson rate λ which give rise to draws of x from a fixed distribution $F(x)$ with $E(x) = 0$ for each idea. When idea specific draws yield a productivity below an optimally determined destruction margin R, bilateral matches are terminated.[26]

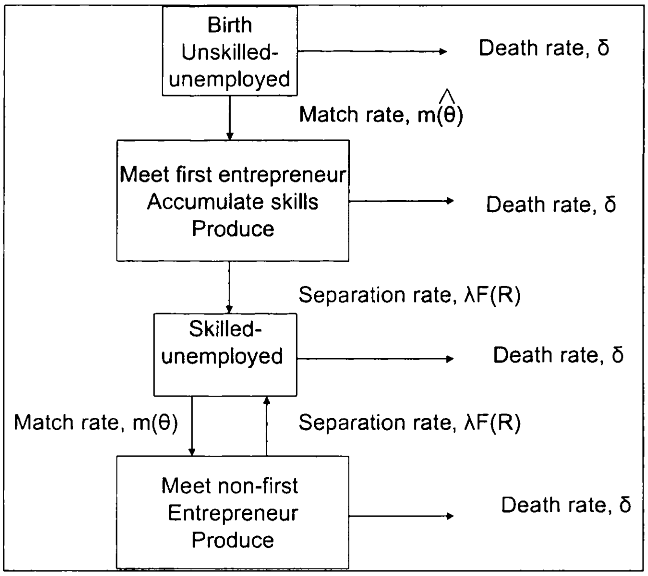

Figure 8: Lifetime of a worker

[Figure 8] documents the possible lifetime paths of workers. All agents are subject to Poisson death shocks at rate δ, which causes their asset values to drop to zero. I assume zero interest rates such that the discount rate of the economy

[25] This formulation is important for the qualitative results. In some past studies, skill levels were multiplicative to the idisyncratic shock component, but it is unclear why general skill levels should increase the variance of idiosyncratic shocks.

[26] Expected employment tenure is then given by $\frac{1}{\lambda F(R)}$.

is equal to the death rate. The assumption of zero interest rates is adopted to facilitate the welfare analysis later.

The skill accumulation decision or workers is made when an unskilled worker is matched with his first entrepreneur. Skill accumulation is instantaneous. Then, all production occurs with workers who have been skilled. I assume that the productivity of matches is not verifiable by third parties. This assumption is used to separate out the output sharing decision from the skill finance sharing decision and will allow me to pin down and analyze a particular optimal skill finance sharing rule.

Entrepreneurs direct vacancies to workers of differentiated skill levels.[27] At any given point in time the distribution of workers with particular skills and the distribution of vacancies searching for particular workers is common knowledge. In equilibrium, only two types of vacancies will be observed, those opened for unskilled workers and those opened for workers skilled at the equilibrium level.

Search frictions in the matching of workers to vacancies are governed by a constant returns to scale match function with the measure of vacancies and unemployed workers as arguments. The flow of skilled worker-idea matches is given by $M \equiv M(u_s, v)$,and the flow of unskilled worker-idea matches is given by $\hat{M} \equiv \hat{M}(1-s, \hat{v})$. u_s is the unemployed share of the labor force who are skilled, v the measure of vacancies opened to skilled workers, s the skilled share of the labor force, and $\hat{v}$ the measure of vacancies opened for unskilled workers.

Let $\theta \equiv \frac{v}{u_s}, \hat{\theta} \equiv \frac{\hat{v}}{(1-s)}$ denote the labour market tightness for skilled and unskilled workers respectively. $m(\theta), m(\hat{\theta})$ are the respective arrival rates of entrepreneurs to skilled and unskilled workers. $\frac{m(\theta)}{\theta} \equiv q(\theta), \frac{m(\hat{\theta})}{\hat{\theta}} \equiv q(\hat{\theta})$ are the respective arrival rates of skilled and unskilled workers to entrepreneurs.[28] The elasticity of the arrival rate of workers $\eta \equiv -\frac{q'(\theta)\theta}{q(\theta)} \equiv -\frac{q'(\hat{\theta})\hat{\theta}}{q(\hat{\theta})} > 0$ is assumed constant.

[27] This necessarily implies general skill levels are verifiable, and skill contingent transfers can be implemented within matches.

[28] The expected unemployment durations for skilled and unskilled workers are given by $\frac{1}{m(\theta)}, \frac{1}{m(\hat{\theta})}$ respectively.

I assume that on-the-job search is less intensive than that off the job, and rule out on-the-job search altogether for simplicity.

4.1.1 Entrepreneurs

Given that entrepreneurs can implement several ideas at the same time, ideas are independent and each entrepreneur can only have at most one idea in each method, each idea can be analyzed in isolation. Since each method can also be analyzed in isolation I drop indices for method for notational convenience.

Entrepreneurial ideas can be in three different states: matched with a skilled worker, searching for a skilled worker (skilled vacancy) and searching for an unskilled worker (unskilled vacancy). The steady state asset value of idea i matched with a skilled worker of skill level k is given by,

$$\delta J(k,x) = f(k) + x\sigma - w(k,x) + \lambda \int [\max\{J(k,\tilde{x}), 0\} - J(k,x)]dF(\tilde{x}) - \delta J(k,x) \tag{50}$$

The flow of expected output to entrepreneurs consists of the product net of wage plus the capital appreciations following productivity shocks minus the capital depreciations following the death of the worker.[29] The outside option of the idea once it has been matched is zero since entrepreneurs weakly prefer to open new ideas as opposed to re-opening old ones. Assume that old ideas are never re-opened.

The asset value of a skilled vacancy searching for a worker of skill level k is given by,

$$\delta V(k) = -c + q(\theta)(J(k,\bar{x}) - V(k)) \tag{51}$$

c is the per period cost of recruitment and its level is assumed the same for all skill levels. During the match process, the identity of skilled workers who will be matched to particular vacancies is unknown so entrepreneurs cannot commit to skill contingent transfers before matches occur. Once matches are realized the incentive for vacancies to offer such contracts disappears.

[29] The other asset equations are straightforward.

The asset value of an unskilled vacancy is given by,

$$\delta \hat{V} = -c + q(\hat{\theta})(J(k, \bar{x}) - \gamma k_e - \hat{V}) \tag{52}$$

$J - \gamma k_e$ is simply the initial asset value of an idea matched with an unskilled worker, where γ is the unit cost of skill accumulation. $k = k_e + k_w$, skill accumulation consists of that financed by entrepreneurs and that financed by workers.

4.1.2 Workers

Workers can find themselves in three different states: skilled and employed, skilled and unemployed and unskilled. The asset value of a skilled worker matched with idea i is given by,

$$\delta W(k, x) = w(k, x) + \lambda \int [\max\{W(k, \tilde{x}), U(k)\} - W(k, x)] dF(\tilde{x}) + \delta(U(k) - W(k, x)) \tag{53}$$

The asset value of a skilled worker searching for a match is given by,

$$\delta U(k) = \max\{\delta \hat{U}, a + m(\theta)(W(k, \bar{x}) - U(k))\} \tag{54}$$

a is the income derived from labour outside the employment sector. Workers skilled in a particular method are unskilled in all other methods so always have an option to search for unskilled worker vacancies. Throughout the analysis, I assume that $a + m(\theta)(W(k, \bar{x}) - U(k) > \delta \hat{U} \quad \forall k > 0$, and later verify this must be true in equilibrium.

The asset value of an unskilled worker searching for a match is given by,

$$\delta \hat{U} = a + m(\hat{\theta})(W(k, \bar{x}) - \gamma k_w - \hat{U}) \tag{55}$$

$W(k, \bar{x}) - \gamma k_w$ is the initial asset value of an unskilled worker matched with his first entrepreneur. Although skills can be acquired outside matches, I assume that it is optimal to delay the skill accumulation decision until the first match is realized. The conditions under which this assumption is valid are verified in the extension on education.

4.1.3 Bargaining

Nash Bargaining is adopted throughout. In a match between an entrepreneur's idea and a skilled worker the Nash Bargaining Rule for Wages is given by,

$$w^*(k,x) = \arg\max(J(k,x))^{1-\beta}(W(k,x) - U(k))^{\beta} \text{ given } x, k \tag{56}$$

where β is the workers' bargaining share. The wage rule determines a renegotiation proof rule for the division of output. In particular this occurs since wage contracts contingent on the level of productivity cannot be enforced. Wages are bargained over given k since the skill investment has already been sunk either in a previous match of the worker or at the beginning of the current match.[30]

Let $S(k,x) \equiv (J(k,x) + W(k,x) - U(k))$ denote the bilateral match surplus between a skilled worker and idea. The first order condition (FOC) of the wage bargaining rule implies that ideas and skilled workers receive their bargaining share of the match surplus,

$$J(k,x) = (1-\beta)S(k,x) \qquad W(k,x) - U(k) = \beta S(k,x) \tag{57}$$

Further bargaining considerations are made when an unskilled worker is matched with an idea for the first time. The Nash Bargaining Rule for Skill Accumulation and Finance is given by,

$$\max_{k,k_e}(J(k,\bar{x}) - \gamma k_e)^{1-\beta}(W(k,\bar{x}) - \hat{U} - \gamma(k-k_e))^{\beta} \tag{58}$$

Let $T(k,\bar{x}) \equiv (J(k,\bar{x}) + W(k,\bar{x}) - \hat{U} - \gamma k)$ denote the bilateral match surplus

[30] Allowing for productivity contingent transfers would mean that this sharing rule relates to one of an infinite set of possible optimal sharing rules.

between an unskilled worker and entrepreneur idea. The FOC for $\hat{k}_e$ implies that ideas and unskilled workers receive their bargaining share of the match surplus,

$$J(k,\bar{x}) - \gamma k_e^* = (1-\beta)T(k,\bar{x}) \qquad W(k,\bar{x}) - \hat{U} - k_w^* = \beta T(k,\bar{x}) \tag{59}$$

Due to non-verifiable productivity, the division of output and the division of skill finance become independent problems. Substituting in the wage bargaining rule (56) for $x = \bar{x}$, gives us the Skill Finance Rule,

$$\gamma k_e^* = (1-\beta)(\gamma k - (U(k) - \hat{U})) \tag{60}$$

Skill accumulation has two effects, increasing the bilateral surplus and improving the workers outside option and bargaining power. Since the entrepreneur cannot appropriate any share of the improvement in the worker's bargaining power, the level of the entrepreneur's skill finance is exactly his bargaining share of total skill finance less the unappropriable share. This modifies the Becker general skill finance rule for the search employment economy.

Since I assume that skill contingent transfers are contractible (between well defined contractual parties, namely a matched worker and entrepreneur), skill levels are set to maximize the asset value of the worker and entrepreneur's idea. However, this optimal skill accumulation rule can also be interpreted as the outcome of Nash Bargaining.[31] The Skill Accumulation Rule is given by,

$$k^* \equiv \arg\max\{J(k,\bar{x}) + W(k,\bar{x}) - \hat{U} - \gamma k\} = \arg\max\{J(k,\bar{x}) + W(k,\bar{x}) - \gamma k\} \tag{61}$$

Skill accumulation is made taking $\hat{U}$ as given since this is a function of skill accumulation outside the current match.

[31] So the verifiablity of skills is not a necessary assumption for the Skill Accumulation Rule.

4.2 Equilibrium

4.2.1 Skilled Worker Sector

The model is solved as follows. First take the skilled worker endogenous variables: the skill accumulation level k, the job destruction margin R, the vacancy to unemployment ratio for skilled workers θ, and the wage rate for workers w_i (note all wage earners are skilled). These are solved using the following four rules.

Job Destruction Rule $S(x = R) \equiv J(R) + W(R) - U(R) \equiv 0$

Free Entry Rule $V(k) = 0$

Nash Bargaining Rule for Wages

$w^*(k, x) = \arg\max(J(k, x))^{1-\beta}(W(k, x) - U(k))^{\beta}$ given x, k

Skill Accumulation Rule $k^* = \arg\max\{J(k, \bar{x}) + W(k, \bar{x}) - \gamma k\}$

A feature of the Skill Accumulation Rule is that the decision internalizes the effect that skill levels have on the labour market tightness θ, faced by the worker. This is a feature of vacancies exercising directed search to workers of different skill levels.

The following equilibrium conditions are derived in Appendix A.

For each idea i the Job Destruction Equation is,

$$[f(k) + R_i^*\sigma] + \frac{\lambda\sigma}{2\delta + \lambda}\int_{R_i^*}^{\bar{x}}(1 - F(\tilde{x}))d\tilde{x} = [a + m(\theta)\beta\frac{(\bar{x} - R_{h\neq i}^*)\sigma}{2\delta + \lambda}] \tag{62}$$

This says the lowest acceptable productivity of a match plus the option value of retaining the existing match in anticipation of productivity improvements equals the opportunity cost of employment.

Differentiating the Job Destruction Equation with respect to k we get,

$$f'(k)+\sigma\frac{\partial R}{\partial k}-\frac{\lambda\sigma(1 - F(R))}{2\delta + \lambda}\frac{\partial R}{\partial k} = -m(\theta)\beta\frac{\sigma}{2\delta + \lambda}\frac{\partial R}{\partial k}+m'(\theta)\beta\frac{\sigma(\bar{x} - R)}{2\delta + \lambda}\frac{\partial \theta}{\partial k} \tag{63}$$

If the opportunity cost of employment is constant, a higher skill level which increases the productivity of matches implies that the destruction margin must fall.

This is for two reasons, (i) the productivity within the current match is higher than elsewhere (second term of LHS), and (ii) the option value of retaining the current match is higher (third term of LHS). The opportunity cost of employment does respond positively to the skill level since as in the current match, skills reduce the destruction margin of future matches and improve the rematch probabilities of workers. Since both these effects are conditional on skills lowering the destruction margin of the current match, overall the destruction margin must respond negatively to skill levels.

From the Job Destruction Rule $S(x) - S(R) = S(x)$, the bilateral surplus created by a match between an idea and a skilled worker can be rewritten as a direct function of the destruction margin only,

$$S(x) = \frac{\sigma(x - R)}{2\delta + \lambda} \tag{64}$$

Note that $-\frac{\sigma}{2\delta+\lambda}\frac{\partial R}{\partial k} = \frac{\partial S(x)}{\partial k} = \frac{\partial S}{\partial k}$.

So from the equation for $S(x)$ we know that $\frac{\partial R}{\partial k} < 0 \Leftrightarrow \frac{\partial S(x)}{\partial k} > 0$. The destruction margin responding negatively with respect to skill levels is equivalent to the statement that the match surplus between an entrepreneurial idea and skilled worker is increasing in skill levels, given the exogenous parameters of our model.

The Job Creation Equation is,

$$\frac{c}{(1-\beta)q(\theta^*)} = \frac{\sigma(\bar{x} - R)}{2\delta + \lambda} = S \tag{65}$$

Entrepreneurs receive fixed shares of the bilateral surplus. Increases in the surplus which occur through reductions in the destruction margin, will invite entry into the competitive vacancy market and increase θ.

Differentiating the Job Creation Equation with respect to k one gets,

$$-\frac{\partial R}{\partial k} = \frac{2\delta + \lambda}{\sigma}\frac{\partial S}{\partial k} = \frac{\eta}{\theta}(\bar{x} - R)\frac{\partial \theta}{\partial k} \tag{66}$$

Since entrepreneurs appropriate a share of the match surplus, higher skill levels will invite entry of vacancies into the market for skilled workers and increase the tightness of the market for skilled workers.

The comparative statics imply $\frac{\partial R}{\partial k} < 0, \frac{\partial \theta}{\partial k} > 0$.

The Skill Accumulation Equation is,

$$\frac{1}{f'(k^*)} = \frac{\delta + m(\theta)\frac{\beta}{\eta}}{\gamma\delta[2\delta + \lambda F(R) + m(\theta)\frac{\beta}{\eta}]} \tag{67}$$

This equation implies $\frac{\partial k}{\partial R} < 0, \frac{\partial k}{\partial \theta} > 0$. The marginal productivity of skills in improving the option value of retaining existing matches is decreasing in the destruction margin. This implies that the marginal productivity of skills in improving the match surplus is decreasing in the destruction margin. The $m(\theta)\frac{\beta}{\eta}$ term in the numerator captures the fact that when unemployment durations are shorter the marginal productivity of general skills is higher during the worker's lifetime. The same term in the denominator captures the fact that shorter unemployment durations reduce the expected duration of any match.

Proposition 1 *Unemployment duration is decreasing and job tenure increasing in the level of general skills. General skills are decreasing in unemployment duration and increasing in job tenure.*

Comparative statics thus reveal a feedback effect between the skill level and labor market variables. Skill levels are positively correlated with the match durations and negatively correlated to unemployment durations of skilled workers. The higher job turnover rate (implied by a higher destruction margin) in countries with lower skill accumulation has been documented by several studies [OECD (1994), Roberts and Tybout (1996)]. The longer duration of unemployment for formal sector skilled and unskilled workers in poorer countries provides an explanation for their lower formal sector labour force participation rates.

The Wage Equation is,

$$w^*(k) = \beta\left(f(k) + x\sigma - \left(a + m(\theta)\beta\frac{\sigma(\bar{x} - R)}{2\delta + \lambda}\right)\right) + a + m(\theta)\beta\frac{\sigma(\bar{x} - R)}{2\delta + \lambda} \quad (68)$$

Wages consist of the opportunity cost of employment plus the workers' share of the flow surplus from the match. Wages are increasing in θ which is increasing in the skill level, reflecting that as skills become less de facto specific, wages approach the marginal product of the match[32].

4.2.2 Unskilled Worker Sector

The remaining endogenous variables, the vacancy to unemployment ratio for unskilled workers $\hat{\theta}$, and the entrepreneur share of general skill finance k_e, are solved by backwards induction using the solutions from the skilled worker sector and the following two rules,

Skill Finance Rule $\gamma k_e^* = (1-\beta)(\gamma k^* - (U(k^*) - \hat{U}))$[33]

No Skill Arbitrage Rule $V(k^*) = \hat{V}$

The second rule says the value of a vacancy searching for workers of the equilibrium skill level, is equivalent to that searching for unskilled workers.

Combining these rules with the Free Entry Rule yields the Job Creation Condition for Unskilled Workers,

$$\frac{c}{q(\hat{\theta}^*)(1-\beta)} = (J(k^*, \bar{x}) + W(k^*, \bar{x}) - \hat{U} - \gamma k^*) = T^* \quad (69)$$

Increases in the vacancy to unemployment ratio for unskilled workers reflect increases in the initial surplus of a match between an idea and an unskilled worker.

Both the solved asset equation for unskilled workers and unemployed skilled workers are linear in their respective labor market tightness measures. The differ-

[32] This Wage Equation for skilled workers defines the implicit labor contract offered by a vacancy to the skilled worker it is matched with.

[33] This rule in conjunction with the Wage Equation for skilled workers defines the contract offered by a vacancy to a unskilled worker it is matched with.

ence in these asset values is given by,

$$U(k) - \hat{U} = \frac{\beta c(\theta - \hat{\theta})}{\delta(1 - \beta)} \tag{70}$$

Substituting this into the Skill Finance Rule yields the equilibrium level of entrepreneur skill finance.

I claimed earlier that $U(k) > \hat{U}\ \forall k > 0$. The Skill Finance Rule and No Skill Arbitrage Rule imply,

$$\frac{c}{q(\theta)} - \frac{c}{q(\hat{\theta})} = (1 - \beta)(\gamma k - (U(k) - \hat{U})) \tag{71}$$

Combining the last two equations, for all $k > 0$ we must have $\theta > \hat{\theta} \Rightarrow U(k) > \hat{U}\ \forall k > 0$. The expected unemployment duration for skilled workers is shorter. Unemployed skilled workers never prefer to behave like an unskilled worker. The entrepreneur share of skill finance is never negative.

4.2.3 Unemployment

The steady state share of skilled workers out of total workers is a direct positive function of the vacancy to unemployment ratio for unskilled workers,

$$s = \frac{m(\hat{\theta})}{m(\hat{\theta}) + \delta} \tag{72}$$

The steady state share of skilled and unemployed workers out of total workers is a positive function of the vacancy to unemployment ratio for skilled workers and a negative function of the job destruction margin,

$$u_s = \frac{\delta + \lambda F(R)}{m(\theta) + 2\delta + \lambda F(R)} s \tag{73}$$

The share of workers who are unemployed is given by one minus the share of workers who are employed,

$$u \equiv 1 - (s - u_s) = 1 - \frac{m(\theta) + \delta}{m(\theta) + 2\delta + \lambda F(R)} \frac{m(\hat{\theta})}{m(\hat{\theta}) + \delta} \tag{74}$$

Higher equilibrium skill levels are associated with lower R, and higher $\{\theta, \hat{\theta}\}$. Skill levels are thus correlated with a larger share of the workforce who are skilled and a larger share of the skilled who are employed.

4.2.4 Efficiency

This section highlights how workers indirectly internalize the positive externality that their skill accumulation decisions have on future entrepreneurs they expect to be matched with. Given θ, workers's skill accumulation generates a positive externality on the share of the match surplus $(1-\beta)S$, enjoyed by future entrepreneurs who workers expect to be matched with. The marginal externality of skill accumulation is given by,

$$(1-\beta)\frac{m(\theta)}{\delta}\frac{\partial S(k)}{\partial k}$$

Where $(1-\beta)\frac{\partial S(k)}{\partial k}$ is the flow of marginal externalities to future entrepreneurs which arrive at rate $m(\theta)$, and this is normalized by the discount rate of the economy to yield a stock measure.

If vacancies undertake directed search to workers of different skill levels, workers internalize the effect that skill accumulation has on their outside option through improved matching prospects $m(\theta)$. The flow of marginal product of skills through improved match prospects is given by $m'(\theta)\frac{\partial\theta}{\partial k}\beta S$. Using (66), and the definition of the elasticity of the arrival rate of workers η, the flow of this marginal product can be rewritten as $\frac{\beta}{\eta}(1-\eta)m(\theta)\frac{\partial S(k)}{\partial k}$. The stock of marginal productivity through improved job prospects is then given by,

$$\frac{\beta}{\eta}(1-\eta)\frac{m(\theta)}{\delta}\frac{\partial S(k)}{\partial k}$$

Under the Hosios condition $\eta = \beta$, the externality of skill accumulation is exactly internalized through improved match prospects. The free entry of vacancies and directed search by vacancies, together with the Hosios condition ensure general skill accumulation is efficient. By inspection, when $\eta > \beta$, there will be under-investment in skills, and when $\eta < \beta$ there will be over-investment in skills.

More generally one can define the social planner's problem and show how under the assumed conditions, the outcomes under a competitive equilibrium coincide exactly with those of the social planner. The social planner's problem is given by,

$$\max_{s,u_s,v,\hat{v},k} Y = \int_0^\infty \left\{ \begin{array}{c} [(1-s)+u_s]a - [v+\hat{v}]c + (s-u_s)f(k) \\ +(s-u_s-b)\bar{x}\sigma + bE(\tilde{x}|\tilde{x} \geq R)\sigma - \hat{M}\gamma k \end{array} \right\} e^{-\delta t}dt \tag{75}$$

subject to the search friction constraints,

$$\frac{ds}{dt} = \hat{M} - \delta s$$

$$\frac{d(s-u_s)}{dt} = \hat{M} + M - (2\delta + \lambda F(R))(s-u_s)$$

$$\frac{db}{dt} = \frac{\lambda(1-F(R))}{2\delta+\lambda}(\hat{M}+M) - (2\delta + \lambda F(R))b \tag{76}$$

The social planner maximizes the discounted flow of net income streams. b denotes the measure of workers who are in matches where idea specific productivity is not at the supremum, $x \neq \bar{x}$.

Proposition 2 *Given the Hosios Condition for no search externalities, the free entry of vacancies and directed search by vacancies, the solution to the social planner's problem is identical to outcomes under the competitive equilibrium.*

Proof in the Appendix B.

Assuming the discount rate is equal to the death rate of the economy simplifies the welfare analysis by allowing us to directly compare steady state solutions of the social planning and real economy rather than having to determine the discounted value of the change in some variable along a convergent path from one solution to the other. Otherwise the assumption that the interest rate is zero is inessential.

Acemoglu (1997) identifies a hold up problem which arises from workers being unable to directly internalize the benefits that their skill accumulation has on future entrepreneurs they expect to be matched with. This is because the identity of the future entrepreneurs or vacancies is unknown and contracts cannot specify parties to the contract. Here, this underinvestment is mitigated since workers are able to indirectly internalize this externality through higher expected match rates given the Free Entry Condition: competition solves the hold up problem. Acemoglu and Shimer (1998) have discussed a similar mechanism for the case of entrepreneurs making match specific ex ante physical investments before opening vacancies.

4.2.5 Equilibrium with Non-directed Search

When the same type of vacancy is opened for workers of every skill level we have non-directed search. By construction then, $\theta \equiv \hat{\theta} \equiv \frac{v}{1-s+u_s}$. The asset equation for vacancies becomes,

$$\delta V = -c + q(\theta)(J(k,x) - \frac{1-s}{1-s+u_s}k_e - V) \tag{77}$$

The expected capital gain through matches is a weighted average of the gains through being matched with a skilled worker and being matched with an unskilled worker. The skill accumulation rule is modified to,

$$k^* \equiv \arg\max\{J(k,\bar{x}) + W(k,\bar{x}) - \gamma k\} \text{ given } \theta \tag{78}$$

Under non-directed search, the tightness of the market becomes an aggregate variable, changes of which are not internalized by individual worker-entrepreneur matches. The new skill accumulation equation,

$$\frac{1}{f'(k^*)} = \frac{\delta + m(\theta)\beta}{\gamma\delta[2\delta + \lambda F(R) + m(\theta)\beta]} \tag{79}$$

reflects the underinvestment which results from this externality. Qualitatively, the feedback between k and $\{R, \theta\}$ is unchanged, but some of the feedback between k and θ is not captured.

The externality that workers cannot internalize consists of the effect of skill levels on future re-match probabilities. This has a positive and negative feedback effect. Higher rematch probability implies workers should invest more (this is represented by the second term on the numerator), but the same effect implies that workers exaggerate the effect of skills on lowering the destruction margin such that workers should invest less (this is represented by the third term in brackets on the denominator). Overall there is under-investment by workers, and the positive feedback implies the existence of coordination failure.

4.3 Extensions

4.3.1 Education

In the classic Becker (1975) analysis, the decision to acquire skills through training or education follows a simple rule. For skills where the marginal cost is lower in training, acquire them on the job, and otherwise acquire them through education.[34]

The education decision is set to solve,

$$h^* = \arg\max\{\hat{U}(h) - \alpha\gamma h\} \text{ s.t. } h \geq 0 \tag{80}$$

Where $\alpha \in (0, 1]$ denotes the relative cheapness of education. For Becker, workers would accumulate all these skills through education.

The education decision is solved by backward induction given the solutions from the unskilled sector, which in turn are solved by backward induction given the solutions from the skilled sector. Define the level of training $t \equiv k - h \geq 0$.

[34] This is the case assuming instantaneous skill accumulation.

The new Job Creation Condition for Unskilled Workers is,

$$\begin{aligned}\frac{c}{q(\hat{\theta}^*)(1-\beta)} &= J(k^*,\bar{x}) + W(k^*,\bar{x}) - \hat{U}(h) - \gamma k^* + \gamma h \quad \text{for } h < k^* \quad (81)\\ &= J(h,\bar{x}) + W(h,\bar{x}) - \hat{U}(h) \quad \text{for } h \geq k^*\end{aligned}$$

When the level of education exceeds the optimizing training level $h \geq k^*$, there is no longer a distinction between skilled and unskilled workers in the search market, i.e. $\hat{\theta} = \theta, \hat{U} = U$. Substituting in for equilibrium $\hat{U}, U$ this condition maps a continuous monotonic increasing relationship between equilibrium $\hat{\theta}$ and h.

The marginal benefit of education is given by,

$$\begin{aligned}\frac{\partial \hat{U}(h)}{\partial h} &= \gamma \frac{m(\hat{\theta})\beta}{\eta\delta + m(\hat{\theta})\beta} > 0 \quad \text{for } h < k^* \quad (82)\\ &= \frac{\partial(J(h,\bar{x}) + W(h,\bar{x}))}{\partial h} \frac{m(\hat{\theta})\beta}{\eta\delta + m(\hat{\theta})\beta} > 0 \quad \text{for } h \geq k^*\end{aligned}$$

Since $\frac{\partial(J(h,\bar{x})+W(h,\bar{x}))}{\partial h} = \gamma$ when $h = k^*$, this marginal benefit function is continuous in h. Since $\frac{\partial(J(h,\bar{x})+W(h,\bar{x}))}{\partial h} \leq \gamma$ when $h \geq k^*$, $\frac{\partial \hat{U}(h)}{\partial h} < \gamma \ for \ \forall \theta < \infty$. So when $\alpha = 1$, workers strictly prefer to accumulate skills through training, and buy no education; $h^* = 0$ the corner solution. Due to discounting and the sharing of skill finance, training is superior to education. Education has some marginal benefit in improving the match probabilities of workers virgin to the labor market but, this benefit is strictly dominated in the case of equal unit costs of training and education.

There exists a critical $\alpha^* < 1$ defined by,

$$\hat{U}(h^*) - \alpha^* h^* = \hat{U}(h = 0) \text{ where } h^* > 0 \quad (83)$$

For a particular method of skills j, if $\alpha_j \geq \alpha^*$ workers prefer to accumulate these skills through training, and otherwise they prefer to accumulate skills through education. So compared to the Becker analysis, the set of skills accumulated through training is larger in a labor market characterized by search frictions.

If $h < k^*$, since $\hat{\theta}$ is monotonically increasing in h, the marginal benefit of education is increasing in the level of education. If $h \geq k^*$, this process slows down and eventually the marginal benefit is falling in education, although it remains positive. Given that the marginal cost of education is constant, these statements imply that $h_j^* > k_h^*$ given $\alpha_j < \alpha^*$. When education is optimally purchased for a particular skill, it is never 'topped-up' by training following the first job match.

Both improvements in the productivity of matches and lower unit cost of skills γ, are channels through which economic development can occur. Both effects raise the marginal productivity of education schedule relative to its marginal costs schedule which implies that the cut off α^* increases. The range of skills accumulated through education increases through the development process.

Overall, the analysis of education in a search economy highlights the 'backwards induction' feature of education incentives. The decision to educate or train is subtle: the incentives to buy education are derived from the employment opportunities available in the job market and the degree to which the unit costs of education are cheaper than that of training.

White versus Blue Collar Workers

The analysis on education suggests sharp predictions about the formation of distinct classes of workers in the economy. Workers with different unit costs of skills may exhibit qualitatively different patterns of skill accumulation. The cut off margin for relative costs of education will be higher for workers with lower unit costs of skill accumulation, $\alpha_L^* > \alpha_H^*$ given $\gamma_L < \gamma_H$. If important skills are characterized by relative costs of education $\hat{\alpha}$, and $\alpha_L^* > \hat{\alpha} \geq \alpha_H^*$, workers with the higher unit costs (blue collar workers) buy no education and become skilled only once matches with entrepreneurs are realized. Blue collar workers train within matches to skill level k^*, whereas workers with low unit costs (white collar workers) accumulate all their skills through education, to a level $h^* > k^*$ (strictly higher skill levels). White collar workers will have longer average tenure, lower unemployment duration and higher specific skill accumulation.

4.3.2 Specific Skill Accumulation

The productivity flow of a particular entrepreneur-worker match, is now a function of idea specific skills z,[35]

$$g(z) + x\sigma \qquad \text{where } g' > 0, g'' < 0 \tag{84}$$

It is also convenient to allow agents to live forever by getting rid of death shocks, so the death rate should now be interpreted as the interest rate. For vacancies, all searching workers are now identical, so the issue of whether search is directed or not is irrelevant. The asset value of a matched idea i, after specific skill investments have been sunk is given by,

$$\delta J(z,x) = g(z) + x\sigma - w(z,x) + \lambda \int [\max\{J(z,\tilde{x}), 0\} - J(z,x)] dF(\tilde{x}) \tag{85}$$

The asset value of an idea searching for a worker is given by,

$$\delta V = -c + q(\theta)(J - \nu z^e - V) \tag{86}$$

The asset value of a worker matched with an idea, after specific skill investment has been sunk is given by,

$$\delta W(z,x) = w(z,x) + \lambda \int [\max\{W(z,\tilde{x}), U\} - W(z,x)] dF(\tilde{x}) \tag{87}$$

The asset value of a worker searching for a job is given by,

$$\delta U = a + m(\theta)(W - \nu z^w - U) \tag{88}$$

Where $z \equiv z^e + z^w$.

[35] Again we assume that specific skill levels do not increase the variance of idiosyncratic shocks.

Again productivity contingent transfers are ruled out and wages are set to maximize the Nash Product for Wages once the specific investment is sunk,

$$w^*(z,x) = \arg\max(J(z,x))^{1-\beta}(W(z,x) - U)^{\beta} \text{ given } x, z \tag{89}$$

Specific Skill Accumulation and Finance are determined by maximizing the Nash Product,

$$\max_{z,z^e}(J(z,\bar{x}) - \nu z^e)^{1-\beta}(W(z,\bar{x}) - \nu(z - z^e) - U)^{\beta} \tag{90}$$

The FOCs yield the Specific Skill Accumulation Rule and Skill Finance Rule,

$$z^* \equiv \arg\max\{J(z,\bar{x}) + W(z,\bar{x}) - \nu z_i\} = \arg\max\{S(z,\bar{x}) - vz\} \tag{91}$$

$$z'^* = (1-\beta)z \tag{92}$$

Unlike general skills, specific skills do not improve the outside option of workers so entrepreneurs simply pay their bargaining share of the costs of specific skill accumulation.

From the FOC for the Skill Accumulation Rule we get the Specific Skill Accumulation Equation,

$$\frac{1}{g'(z^*)} = \frac{1}{\nu(\delta + \lambda F(R))} \tag{93}$$

Intuitively, the incentives for specific skill accumulation are independent of the matching possibilities within the economy and only a function of the expected duration of the current match.

From the Job Destruction Rule the Job Destruction Equation is,

$$[g(z) + R_i^*\sigma] + \frac{\lambda\sigma}{\delta+\lambda}\int_{R_i^*}^{\bar{x}}(1 - F(\tilde{x}))d\tilde{x} = [a + m(\theta)\beta\frac{\sigma(\bar{x} - R_{h\neq i})}{\delta+\lambda}] \tag{94}$$

Unlike general skills, idea specific skills have no effect on the vacancy to unemployment ratio and no effect on the destruction margin of future matches of the worker.

This implies that,

$$
\begin{aligned}
\frac{\partial R}{\partial z} &= -\frac{g'(z)}{\delta + \lambda F(R)} \frac{\delta + \lambda}{\sigma} < 0 && (95)\\
&= -v \frac{\delta + \lambda}{\sigma} < 0 \qquad \text{at } z_h = z_h^* && (96)
\end{aligned}
$$

From the Free Entry Rule the Job Creation Equation is given by,

$$
\frac{c}{q(\theta^*)} = J - \nu z^e = (1 - \beta) \left(\frac{\sigma(\bar{x} - R)}{\delta + \lambda} - \nu z \right) \tag{97}
$$

Taking differentials with respect to z,

$$
\begin{aligned}
\frac{c\eta}{\theta q(\theta)} \frac{\partial \theta}{\partial z} &= (1 - \beta) \left(\frac{g'(z)}{\delta + \lambda F(R)} - \nu \right) > 0 \quad \forall z < z^* \\
&= 0 \qquad \text{at } z = z^*
\end{aligned}
$$

In the economy where specific skill accumulation is undertaken, the equilibrium specific skill level maximizes the tightness of the market.

Combining these results with the analysis for general skill accumulation, general and specific skill are indirect complements. Higher general skills complement specific skills through a lower destruction margin. Optimally determined specific skills complement general skills through a lower destruction margin and higher labor market tightness for skilled workers.

4.4 Conclusion

I have identified the feedback linkages between general skills and specific skills, and between each of these skills with labor market variables. The modified Becker rule for general skill accumulation in a search economy together with the conditions for efficient skill accumulation provide analytical benchmarks for future research in this area. The endogenous determination of skill accumulation through training or education provides insights into the relationship between education and per capita income levels.

The propagation mechanisms developed here are confined to the worker side of the economy. The perfectly elastic supply of vacancies assumed in the paper needs to be modified in an integrated story of worker and entrepreneurial dynamics. The fact that efficient skill accumulation is conditional on free entry of vacancies suggests that the efficiency implications of entrepreneurial dynamics will not be straightforward.

The most natural extension of this work is to apply it to issues of youth unemployment and training, and long term unemployment and training. This could be accommodated by adopting a life-cycle version of the perpetual youth over-lapping generations model, as explored by Gertler (1999). Workers could be in two states young and old where the young face a constant transition probability to become old and the old face a constant probability of death. The model should predict that the longest unemployment durations are suffered by the old unskilled, the shortest by the young skilled, although as a group there is more unemployment among the young then among the old.

A further application of this framework is to consider the importance of the first job for young labor market entrants. Recall the quality of the first job of workers will determine initial levels of general training and thus future job prospects. When ex ante identical young workers within a cohort are matched with first jobs of different quality, it is likely to create substantial within-cohort earnings inequality which persists throughout the careers of the cohort members. Meanwhile, were one to compare between cohorts who entered the labor market at different times, the performance of the aggregate economy at the time of labor market entry is likely to have a persistent effect on the earnings outcomes of different cohort members' careers.

4.5 Appendix A: Skilled Sector Equilibrium

The per period income flow to an entrepreneur from idea i is,

$$(2\delta + \lambda)J(k, x) = f(k) + x\sigma - w(k, x) + \lambda \int \max\{J(k, \tilde{x}), 0\} dF(\tilde{x})$$

The per period surplus income flow to a skilled worker matched with idea i is,

$$(2\delta + \lambda)(W(k, x) - U) = w(k, x) + \lambda \int \max\{W(k, \tilde{x}) - U, 0\} dF(\tilde{x}) - \delta U$$

Summing we get,

$$\begin{aligned}
(2\delta + \lambda)S(k, x) &= f(k) + x\sigma + \lambda \int_{-\infty}^{\bar{x}} \max\{S(k, \tilde{x}), 0\} dF(\tilde{x}) - \delta U \\
&= f(k) + x\sigma + \lambda \int_{R}^{\bar{x}} S(k, \tilde{x}) dF(\tilde{x}) - \delta U \\
& \quad \text{from the definition of } R \\
(2\delta + \lambda)S(k, R) &= 0 = f(k) + R\sigma + \lambda \int_{R}^{\bar{x}} S(k, \tilde{x}) dF(\tilde{x}) - \delta U \\
& \quad \text{the definition of } R \\
\text{so } (2\delta + \lambda)(S(k, x) - S(k, R)) &= (2\delta + \lambda)S(k, x) = (x - R)\sigma \\
\text{so } \int_{R}^{\bar{x}} S(k, \tilde{x}) dF(\tilde{x}) &= \frac{\sigma}{2\delta + \lambda} \int_{R}^{\bar{x}} (\tilde{x} - R) dF(\tilde{x}) \\
&= \frac{\sigma}{2\delta + \lambda} \left\{ \left[(\tilde{x} - R) F(\tilde{x}) \right]_{R}^{\bar{x}} - \int_{R}^{\bar{x}} F(\tilde{x}) d\tilde{x} \right\} \\
& \quad \text{after integration by parts.} \\
&= \frac{\sigma}{2\delta + \lambda} \int_{R}^{\bar{x}} (1 - F(\tilde{x})) d\tilde{x}
\end{aligned}$$

After substituting in, the match surplus can be expressed as,

$$(2\delta + \lambda)S(k, x) = f(k) + x\sigma + \frac{\lambda\sigma}{2\delta + \lambda} \int (1 - F(\tilde{x})) d\tilde{x} - \delta U$$

Setting $x = R$ and $S(R) \equiv 0$ we get the Job Destruction Rule. The Free Entry

Condition implies a positive correlation between the size of the asset value from a newly matched idea and the vacancy to unemployment ratio for skilled workers,

$$J(k,\bar{x}) = \frac{c}{q(\theta)}$$

Combining this with the Surplus Division Rule, we get the Job Creation Equation.

From the function for the bilateral surplus and the wage bargaining rule, the asset value from being unemployed can be rewritten as,

$$\delta U = a + m(\theta)\beta\frac{\sigma(\bar{x}-R)}{2\delta+\lambda}$$

The joint asset value of a new idea matched with an unskilled worker is,

$$J + W - \gamma k = S + U - \gamma k = \frac{a}{\delta} + \left(1 + \frac{m(\theta)\beta}{\delta}\right)\frac{\sigma(\bar{x}-R)}{2\delta+\lambda} - \gamma k$$

The FOC for the Skill Accumulation Rule is initially given by,

$$\frac{m'(\theta)\beta}{\delta}\frac{\sigma(\bar{x}-R)}{2\delta+\lambda}\frac{\partial\theta}{\partial k} - \left(1 + \frac{m(\theta)\beta}{\delta}\right)\frac{\sigma}{2\delta+\lambda}\frac{\partial R}{\partial k} = \gamma$$

The marginal productivity and marginal cost of skills are equated. The first term in the LHS captures the effect that skill levels have on the workers individual rematch probability (conditional on directed search) and the second term captures the effect that skill levels have on improving the size of current and future expected match surpluses. Substituting in the equilibrium equations for $\frac{\partial R}{\partial k}, \frac{\partial\theta}{\partial k}$ we get the Skill Accumulation Equation.

The wage equation is solved by substituting in equilibrium J, W, U into the surplus division rule.

4.6 Appendix B: Efficiency

The current value Hamiltonian of the social planner's problem is given by,

$$
\begin{aligned}
H(s, u_s, v, \hat{v}, k) \quad = \quad & [(1-s)+u_s]a - [v+\hat{v}]c - (s-u_s)f(k) \\
& +(s-u_s-b)\bar{x}\sigma + bE(\tilde{x}|\tilde{x}\ R)\sigma - \hat{M}\gamma k \\
& +\mu(\hat{M} + M - (2\delta+\lambda F(R))(s-u_s)) \\
& +\phi(\hat{M} - \delta s) \\
& +\pi\left(\frac{\lambda(1-F(R))}{2\delta+\lambda}(\hat{M}+M) - (2\delta+\lambda F(R))b\right)
\end{aligned}
$$

The partial FOCs imply,

$$
\frac{\partial H}{\partial u_s} = 0 \Rightarrow \mu = \frac{f(k) + \bar{x}\sigma - a - \pi M_{u_s}\frac{\lambda(1-F(R))}{2\delta+\lambda}}{2\delta + \lambda F(R) + M_{u_s}}
$$

$$
\frac{\partial H}{\partial b} = 0 \Rightarrow \pi = \frac{(E(\tilde{x}|\tilde{x} \geq R) - \bar{x})\sigma}{2\delta + \lambda F(R)}
$$

$$
\frac{\partial H}{\partial v} = 0 \Rightarrow \frac{c}{M_v} = \mu + \pi\frac{\lambda(1-F(R))}{2\delta+\lambda}
$$

The equivalence of this with the Job Creation Equation for skilled workers implies that decentralized entry of vacancies for skilled workers is efficient.

$$
\frac{\partial H}{\partial u_s}|_{\overline{(s-u_s)}} = a + M_{u_s}\left(\mu + \pi\frac{\lambda(1-F(R))}{2\delta+\lambda}\right)
$$

The equivalence of this with the RHS of the Job Destruction Equation implies that decentralized job destruction decisions are efficient.

$$
\frac{\partial H}{\partial s} = 0 \Rightarrow \phi = \frac{f(k) + \bar{x}\sigma - a - \mu(\hat{M}_{(1-s)} + 2\delta + \lambda) - \pi\hat{M}_{(1-s)}\frac{\lambda(1-F(R))}{2\delta+\lambda} + \hat{M}_{(1-s)}\gamma k}{\delta + \hat{M}_{(1-s)}}
$$

$$
\frac{\partial H}{\partial \hat{v}} = 0 \Rightarrow \frac{c}{\hat{M}_{\hat{v}}} = \phi + \mu + \pi\frac{\lambda(1-F(R))}{2\delta+\lambda} - \gamma k
$$

The equivalence of this with the Job Creation Equation for unskilled workers implies that decentralized entry of vacancies for unskilled workers is efficient.

$$\frac{\partial H}{\partial(1-s)}|_{\bar{s}} = a + \hat{M}_{(1-s)}\left(\phi + \mu + \pi\frac{\lambda(1-F(R))}{2\delta+\lambda} - \gamma k\right)$$

The equivalence of this with the flow value of unskilled workers implies that births into the unskilled worker pool are efficient.

$$\frac{\partial H}{\partial k} = 0 \Rightarrow \frac{1}{f'(k)} = \frac{(s-u_s)}{\gamma\hat{M}} = \frac{\delta + m(\theta)}{\gamma\delta[2\delta + \lambda F(R) + m(\theta)]}$$

Immediate inspection shows that skill accumulation is efficient under the Hosios Condition.

Under the Hosios Condition, we have $M_{u_s} = m(\theta)\beta, M_v = q(\theta)(1-\beta), \hat{M}_{(1-s)} = m(\hat{\theta})\beta, \hat{M}_{\hat{v}} = q(\hat{\theta})(1-\beta)$ and $S = \mu + \pi\frac{\lambda(1-F(R))}{2\delta+\lambda}, T = \phi + \mu + \pi\frac{\lambda(1-F(R))}{2\delta+\lambda} - \gamma k$. These imply that the economy is fully efficient subject to search frictions as long as there are no search externalities.

Chapter 1.

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Chapter 2.

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Chapter 3

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