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ESSAYS IN FINANCIAL ECONOMICS

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Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

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I confirm that Chapter 2 is jointly co-authored with Jing Zeng.

Abstract

In this thesis, I study how information and asset market frictions can affect the investment and funding decisions of financial institutions, and their implications for the efficiency and stability of the financial system as a whole. The first chapter, Self-fulfilling fire sales, shows that while collateralised short-term debt mitigates individual borrowing firms' incentives to take excessive risk, it also exerts pressure on the liquidity of the collateral asset market. When the asset market is not liquid enough, a vicious feedback loop between borrowers' risk-taking incentives and expected asset fire-sale discount can cause instability in this secured funding market. Central bank intervention such as asset purchase is shown to be able to enhance stability and welfare. In the second chapter, Counter-cyclical foreclosure for securitisation, Jing Zeng and I study how information asymmetry in the mortgage securitisation process could distort the foreclosure policy of delinquent mortages. We show that banks would choose to commit to foreclose delinquent mortgages excessively in order to reduce the information friction in the securitisation process. This offers a potential explanation to the large number of foreclosures of delinquent mortgages in the U.S in the aftermath of the Subprime mortgage crisis in 2007-2009. The last chapter, Asset market runs and the collapse of debt maturity, shows that when market-makers have limited risk-absorbing capacity and there is uncertainty in the execution prices of sell orders, a borrower may want to shorten the debt maturity in order to allow his creditor to demand repayment and liquidate the collateral asset ahead of creditors of other firms in the case of default. This strategic shortening of debt maturity in equilibrium amplifies the borrowers risk of failing to roll-over their debt and leads to excessive asset liquidation.

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It goes without saying that any error in the thesis remains mine.

Contents

Chapter 1

Self-fulfilling fire sales: fragility of collateralised short-term debt markets

1.1 Introduction

Financial firms' reliance on collateralised short-term funding such as repurchase agreements (repo) is considered as both a prominent feature and a source of fragility in the modern financial system[1]. These secured loans in this so-called 'shadow banking' system are usually automatically rolled over by creditors in normal times. Yet, the recent crisis has shown that these funding markets can exhibit a 'systemic runs' phenomenon whereby creditors collectively demand tougher borrowing terms or withdraw funding, causing significant distress to the firms and leading to sizeable liquidation of collateral assets at a discount; this phenomenon is commonly known as fire sales[2].

[1] Adrian and Shin (2011) call this a 'Market-based financial system'. See Brunnermeier (2009) and Krishnamurthy (2010b) for detailed reports on the use of repo and asset-backed commercial paper and how these markets collapsed in the Global Financial Crisis of 2007-2009.

[2] Shleifer and Vishny (2011) survey fire sales in finance and macroeconomic literature. Empirically, He et al. (2010) show that 2007Q4 to 2009Q1, hedge funds and broker dealers reduced holdings of securi-

The apparent 'systemic runs' in certain collateralised debt markets however cannot be readily explained by classical bank run models such as Diamond and Dybvig (1983) because the nature of bank debt is different. The first-come-first-served nature of deposit contracts which motivates depositors to front-run each other is absent in repo contracts, for example. As Gorton (2012, p.2) concisely points out:

> '...we know that crises are exits from bank debt... In this form of money (repo), each "depositor" receives a bond as collateral. There is no common pool of assets on which bank debt holders have a claim. So, strategic considerations about coordinating with other agents do not arise. This is a challenge for theory and raises issues concerning notions of liquidity and collateral, and generally of the design of trading securities – private money.'

This paper can be viewed as a response to the above challenge and proposes a new form of coordination failure between *firms* at the ex-ante contracting stage, due to a feedback between the risk-taking incentives of firms and the fire sales of collateral. Under certain conditions, self-fulfilling fire sales and 'systemic runs' can arise.

I present a three-date, competitive equilibrium model of a continuum of firms, each matched with a creditor, and an outside collateral buyer. Each firm is endowed with a divisible asset-in-place which pays a risky dividend at $t = 2$. This asset can be used as collateral to finance an independent, illiquid investment project which becomes successful with some probability and pays a verifiable cash flow at $t = 2$ [3]. Firms are subject to moral hazard problems that at $t = 0$, after borrowing bilaterally from its creditor, each firm privately chooses the success probability of its project by incurring an non-pecuniary effort cost[4]. Pledging collateral to creditors lowers debt yields and thus mitigate firms' incentives

tised assets by $800 billion; these assets were mostly absorbed by commercial banks ($550 bn) and the government ($350 bn). In terms of liabilities, repo finance shrank by $1.5 trillion.

[3]The model can be seen as a competitive equilibrium extension of the borrowers with non-project-related collateral model in Tirole (2006, Section 4.3.5) with multiple risk-taking choices and a market for collateral.

[4]The firms' moral hazard problem can also be modelled as risk-shifting as in Jensen and Meckling (1976)

to shirk, or equivalently to take on excessive project default risk. As the creditor is averse to the systematic risk associated with the collateral dividend, she will seize and liquidate the collateral in a secondary market at $t = 1$ when she knows her firm is insolvent. Finally, the outside collateral buyer is competitive yet capital constrained, hence the market-clearing price of the collateral decreases in the amount of collateral liquidated.

The key novelty of this paper is the feedback between the firms' moral hazard problems and the equilibrium collateral liquidation values which generates a self-fulfilling fire sales phenomenon. When agents expect a lower liquidation value ex-post, creditors require a higher debt yield to break-even. Firms then have to pledge more collateral, or initial margins, in order to maintain incentives; when there is not enough collateral, they engage in more risk-taking. In aggregate, both more pledged collateral and more defaults of firms lead to more collateral being liquidated in the market, resulting in a larger fire-sale discount ex-post. Thus the anticipation of fire sales *causes* fire sales. Figure **??** summarises the phenomenon of *self-fulfilling fire sales* .

The above feedback can be strong enough to produce multiple rational expectation equilibria with different collateral liquidation values. There are two (co-existing) channels through which multiple equilibria can arise. First, as discussed above, there exists a threshold of liquidation value below which there is not sufficient collateral to prevent risk-taking. When the equilibrium liquidation value is just above this threshold, a pessimistic expectation of a liquidation value below this threshold triggers firms' risk-taking, which creates a discrete jump in the amount of collateral liquidated as more firms default ex-post, pushing the market-clearing collateral liquidation value below this threshold. I call this the *risk-taking channel*.

Self-fulfilling fire sales can also arise purely from firms' margin decisions. A lower expected collateral liquidation value requires firms to pledge more collateral; as such, in aggregate more collateral is supplied in the market even when firms' default risks remain unchanged. If the market-clearing price function is sensitive enough in the relevant range,

with an assumption that the cash flow difference between risky and safe project in the case of success is non-verifiable.

multiple equilibria emerge through this *margin channel*.

To the best of my knowledge, this self-fulfilling fragility due to the feedback between endogenous risk-taking and collateral fire sales in the *absence* of aggregate shock has not been documented in the literature previously[5]. This mechanism generates a *systemic run* phenomenon in the collateralised debt market which is different from classic bank run and financial market run models. The source of fragility in this paper stems from a coordination failure between firms with their ex-ante risk-taking and collateral margin decisions, as opposed to depositors' withdrawal decision within a bank or traders' asset liquidation decision in a market at the interim date under a *de facto* sequential service constraint[6]. The coordination failure here operates through the two channels described above: higher default risk or a higher initial margin chosen by an individual firm increases the expected amount of collateral liquidated in the market ex-post. Due to the limited liquidity in the secondary market, this extra supply of collateral marginally lowers the liquidation value, which in turn tightens other firms' ex-ante incentive constraints under rational expectation, requiring them to pledge more collateral or take on excessive risk. As a result, in competitive equilibrium, firms' risk-taking and margin decisions become *strategic complements* due to the joint effect of the firms' incentive constraints and the fire-sale externality in the collateral market.

While the model applies to any situation with multiple borrowing firms and a illiquid collateral market in general, the opaque operations of financial firms such as hedge funds and their reliance on collateralised borrowing make risk-taking concern particularly relevant[7]. In addition, the substantial and contemporaneous increase in debt yields, borrowers'

[5]The margin channel here is similar to the margin spiral in Brunnermeier and Pedersen (2009) while in their model the margins are exogenous and the seed of fragility is an unanticipated, large aggregate shock on asset values.

[6]For instance, in Morris and Shin (2004) a market maker executes sellers' aggregate sell orders sequentially at decreasing prices and a seller's place in the queue for execution is randomly distributed. He and Xiong (2012) provides a recent dynamic bank run type model with coordination failure of roll-over decisions among asset-backed commercial paper holders.

[7]For evidence regarding risk-taking behavior of other financial firms, Becker and Ivashina (2013) and

counter-party risk and collateral spreads during the recent crisis in the wholesale funding markets is consistent with the feedback mechanism between endogenous risk-taking and collateral fire sales in the model[8].

In terms of welfare and policy implications, equilibria with lower collateral liquidation values are less efficient due to firms' inefficient investment decisions, credit rationing, and the inefficient transfer of collateral from firms to creditors. The self-fulfilling nature of the fragility suggests that central banks can *reduce* firms' risk-taking incentives and make the financial system more robust through an ex-ante commitment to intervening in the collateral market, which is opposite to the collective moral hazard concern of bailout and government intervention as noted in Acharya (2009) and Farhi and Tirole (2012). Policies such as asset price guarantee can eliminate the agents' pessimistic expectations and thus the inefficient equilibria. This is in line with the idea that central banks should act as a 'Market Maker of Last Resort' (Buiter and Sibert (2007)) to safeguard the proper functioning of certain key collateral and wholesale funding markets[9].

I conclude the paper with a discussion of the potential unintended consequences of policies to limit post-default fire sales. In the U.S. when firms file for bankruptcy, a provision known as 'automatic stay' prevents creditors from demanding repayments. Repo contracts in practice are usually exempted from automatic stay so that repo lenders can immediately access the collateral. Critics of exemption from automatic stay have argued that it has precipitated the fire sales of collateral during a crisis. While this paper also features potential disorderly fire sales, I find that the ban of stay-exemption may backfire. This is because without stay-exemption, defaulted firms can renegotiate with creditors ex-post to

Kacperczyk and Schnabl (2013) document a 'reach-for-yield' phenomenon in insurance companies and money market mutual funds respectively.

[8]Gorton and Metrick (2012) and Covitz et al. (2013) find significant spikes and volatility in repo rates and ABCP yields in private-label asset-backed-securities markets during the recent crises which correlate positively with proxies for counter-party risks such as the LIBOR-OIS spread.

[9]In a 'longer term' model with endogenous production of collateral, this asset price guarantee policy could encourage the over production of collateral with deteriorating quality. The usual moral hazard concern of government guarantee will hence kick in again.

lower the promised repayment amount of collateral by threatening to file for bankruptcy and delay the transfer of collateral. As the creditors value the immediate access and liquidation of the collateral, they will accept the offer. This renegotiation problem thus reduces the amount of *credibly* pledgeable collateral and worsens the firms' ex-ante moral hazard problem. In short, limiting post-default fire sales can exacerbate the ex-ante risk-taking problem, leading to more pre-default fire sales and dry-ups of some low quality collateral markets.

Related Literature My paper first relates to the recent literature on the fragility of collateralised debt market. Martin et al. (2012) build an infinite-horizon Diamond-Dybvig model with an asset market and characterise liquidity, collateral, and asset liquidation constraints under which banks can ward off an unexpected systemic run by depositors in all banks in the steady-state. Their fragility hence stems from the sequential-service constraint faced by the depositors and an unanticipated aggregate shock to collateral value. In contrast I show the anticipation of fire sales can interact with firms' moral hazard problems and cause fragility.

Models on the use of collateral to mitigate borrowers' moral hazard and adverse selection problems go back to Chan and Thakor (1987) and Besanko and Thakor (1987). See Coco (2000) for a survey. The main difference in my model is that I allow endogenous collateral fire sale discount to study the feedback between firms' moral hazard problems and collateral fire sales. Hombert (2009) also studies a similar feedback but he assumes the solvency of firms are publicly observed so that firms with successful projects can expand and purchase collateral from insolvent firms. In contrast to this paper, he shows that fire sales discourage risk-taking. I assume that solvency of a firm is only observed by its creditor with limited capital thus it is difficult for solvent firms to expand at interim. This adverse selection in the market is arguably more natural for opaque financial firms.

My paper belongs to the self-fulfilling financial crisis literature. Malherbe (2014) shows how liquidity dry-up due to adverse selection can arise from ex-ante self-insurance motives of liquidity hoarding. Diamond and Rajan (2005) shows in a bad aggregate state, systemic failure in the banking system can arise because banks scramble for deposits by raising interest rate which in turns causes more bank failures and further liquidity shortage. In a financial market run context, Morris and Shin (2004) shows how loss-limit constraints on traders' position can trigger coordinated liquidation[10]. My paper contributes to the above literature by highlighting a new type of coordination failure from firms' investment and contracting decisions.

The negative feedback spiral in this paper is similar in spirit to the ones in the literature on asset pricing with constraints. For example, Brunnermeier and Pedersen (2009) and Danielsson et al. (2011) show the existence of an amplifying feedback loop between anticipated and realised asset price volatility when financial institutions operate under a Value-at-Risk constraint. Gromb and Vayanos (2002) and Vayanos (2004) study models with limits to arbitrage due to margin and agency constraint. Building on these insights, Krishnamurthy (2010a) also proposes an asset price guarantee policy to stabilise the asset market. Most of these papers take the constraints as given and focus on the asset pricing and portfolio allocation implications when an exogenous aggregate shock hits. This paper in contrast endogenises the collateralised debt contracts and margin constraints, and the source of risk comes from the endogenous risk-taking of firms.

This paper also relates to the vast literature on the consequences of short-term debt and asset fire-sales. Diamond and Rajan (2011) demonstrate that distressed banks financed with deposit will gamble for resurrection and take the excessive risk of forced liquidation when an aggregate shock hits in the future. Outside collateral buyers who anticipate this fire sales hoard liquidity for asset purchase, leading to a reduction in lending to real sector. Stein (2012) assumes a 'money-like' premium in lenders' preferences for absolutely safe contract and shows that firms tend to create too much safe asset by excessive short-term borrowing and fails to internalise the fire-sale externality when aggregate shocks hit. Eisenbach (2011) shows that the existence of aggregate uncertainty distorts the disciplining effect of short-term debt, and creates inefficiency in both good and bad states. Acharya

[10] For demand-deposit based bank runs models, see Diamond and Dybvig (1983) Rochet and Vives (2004), Goldstein and Pauzner (2005).

et al. (2011) show roll-over risk of short-term debt can cause credit market freeze when bad news hits. My work complements the above literature by showing that the expectation of fire sales can interact with borrowers' risk-taking incentives to generate aggregate risk.

1.2 Model: feedback between risk-taking and fire sales

In this section I first give an overview of the model. Then I analyse the firm-creditor contracting problem at the initial stage $t = 0$ and describe creditors' liquidation decisions and the collateral market at $t = 1$.

1.2.1 Overview of the model

Consider a three-date ($t = 0, 1, 2$) model with a continuum of borrowing firms each matched with a corresponding creditor, and a representative outside collateral buyer. There is a storage technology with returns normalised to zero.

Firms and projects Firms are risk-neutral, identical ex-ante, and each has a unit of common asset-in-place (collateral) with no cash and debt. At $t = 0$ each firm has the opportunity to invest in a project which requires an initial investment of \$1 and will return a verifiable cash flow X in the case of success and X_f otherwise at $t = 2$. Without loss of generality I normalise X_f to 0. Firms are subject to a moral hazard effort-provision problem as in Holmström and Tirole (1997). The success probability of the project depends on the unobservable effort exerted by the firm after financing the project. Effectively the firm can choose the success probability of the project $p_1 > p_2 > p_3$ by incurring a private effort cost $c(p_i) \geq 0$. Shirking here is thus interpreted as risk-taking. Project risk is idiosyncratic, and the realisation of projects is therefore independent across firms.

Collateral assets and financing Aside from the investment opportunity, each firm has one divisible unit of asset (e.g. financial securities) which pays a random, non-negative dividend $\tilde{v}$ with expected value v at $t = 2$. The dividend risk is uncorrelated with the project. The asset is also independent of the operation of the project and can be used as collateral for borrowing. I assume this collateral dividend $\tilde{v}$ to be non-verifiable. As such, the firm can effectively choose $k \in [0, 1]$ fraction of the collateral to pledge to the creditor at the ex-ante contracting stage and keep the remaining $(1 - k)$ fraction beyond the creditor's reach. To fix idea, one can think of a shadow bank who can secretly move assets on and off balance sheet unless the assets are explicitly pledged. While the flexibility to choose k is not crucial to the main result of this paper, this allows me to endogenise the optimal amount of pledged collateral, or initial margin, in the financing contract.

Firms borrow in the form of collateralised short-term debt contract. Specifically, a firm borrows \$1 from its creditor and promises to repay r at $t = 1$ and immediately transfer $k \in [0, 1]$ measure of the collateral to the creditor if repayment is demanded at $t = 1$ and the firm fails to repay. This contract resembles a repurchase agreement (repo) as commonly used in practice, r as the repo rates and k as the initial margin. In Section 1.6, I will discuss the optimality of such a contract and its implementation.

I will make the following assumptions about the net present value and the degree of moral hazard of the project:

Assumption 1. *(NPV and moral hazard intensity of the project) Define* $NPV_i \equiv p_i X - 1 - c(p_i)$, $\Delta p_i \equiv p_i - p_{i+1}$, $\Delta c_i \equiv c(p_i) - c(p_{i+1})$, *and* $A_i \equiv 1 - p_i(X - \frac{\Delta c_i}{\Delta p_i})$ *for* $i = 1, 2$

(i) $NPV_1 \geq NPV_2 > 0 > NPV_3$

(ii) $A_1 > A_2 > 0$ *and*

(iii) $(1 - p_1)A_1 \leq (1 - p_2)A_2$

Assumption 1 is there to preserve the efficiency ranking of actions and at the same time allows risk-taking to arise in equilibrium. Assumption 1(i) implies that prudent investment (p_1) is the efficient action but risk-taking (p_2) is also profitable. Part (ii) and (iii) are about the magnitude of the moral hazard problem, i.e. the absolute and relative size of $\frac{\Delta c_i}{\Delta p_i}$. A_i is the value of collateral required to induce action p_i when the firm and creditor value the collateral symmetrically and (ii) implies that the project cannot be funded without collateral as the firm will choose the negative NPV action (p_3) after financing ($A_i > 0$) and more collateral is needed to induce prudent investment $A_1 > A_2$[11]. Finally the collateral is transferred to the creditor when the project fails with probability $(1 - p_i)$ and (iii) implies that the expected value of collateral lost is weakly lower in the case of prudent investment. Although losing the collateral to the creditor in the case of symmetric valuation is costless, (iii) ensures that p_1 is always the preferred and efficient action even if the creditor values the collateral less because p_1 entails a higher NPV and a smaller expected collateral loss.

Creditors' rollover and collateral liquidation decision At $t = 0$ each firm is matched with a creditor who has cash \$1 to lend. After the financing, at $t = 1$ each creditor receives a private, non-contractible signal about the success or failure of her borrowing firm's project, that is, whether cash flow X or 0 will realise at $t = 2$. I assume the signal is perfect and hence the creditor essentially observes the solvency of the firm she financed. If the project has succeeded, the creditor is willing to roll over her short-term debt to $t = 2$ at the yield r as she knows she will be repaid for sure. When the project fails, the creditor demands repayment and as the insolvent firm cannot repay, the creditor seizes the collateral asset and could potentially sell it on the market[12]. I assume creditors value the collateral less than the firms and the collateral buyer, thus creating a motive for them to sell the collateral at a discount.

Assumption 2. *Creditors' expected utility derived from holding the collateral to $t = 2$ is $\underline{l} \leq v$, i.e. less than the firms' and the collateral buyer's valuation.*

Effectively creditors are averse to the collateral dividend risk and $\underline{l}$ can be understood

[11] To see why the project cannot be funded without external collateral, the minimum repayment to the creditor is $r_i = 1/p_i$ if p_i is chosen. However, the firm would privately choose the negative NPV action p_3 after financing as $p_3(X - r_i) - c_3 > p_i(X - r_i) - c_i$ when $A_i > 0$

[12] Rolling over a failed firm and receiving the collateral risky dividend at $t = 2$ is a weakly dominated strategy for the creditor because seizing the collateral at $t = 1$ gives her the option to sell the collateral in the market.

as their certainty equivalent of the risky dividend. Hence they prefer selling the collateral on the market as long as the market clearing price is above $\underline{l}$. The wedge between the creditors' and the collateral buyer's valuation of the collateral $(v - \underline{l})$ can be motivated by the creditors' lack of expertise in managing the systematic risk associated with the collateral or (indirect) holding cost stemming from tougher regulatory constraints on creditors[13]. As such, from an ex-post perspective, fire sales are an *efficient* transfer of collateral.

I will interpret $\underline{l}$ as the *collateral quality*. For example safe collateral such as U.S. Treasuries will have a high $\underline{l}$ close to v and the creditor can hold such collateral to maturity with minimal cost or limitation. In Section 1.5, I discuss how collateral quality affects fragility and amplifies risk in the financial system.

The assumptions of a perfect signal and ex-post efficient fire sales shut down other sources of inefficiency stemming from the wrongful liquidations of successful project or a coordination failure between creditors trying to front-run each other in the collateral market as in Morris and Shin (2004) and Bernardo and Welch (2004). This allows me to focus on the inefficiency of the coordination failure of firms' ex-ante investment and contracting decisions, which is the main result of this paper.

Collateral buyer and endogenous fire sale discount The final element in the model is illiquidity in the collateral market. At $t = 1$, there is a competitive risk-neutral outside investor who clears the collateral market. However, he has limited capital in the sense that instead of holding cash to purchase the collateral at $t = 1$, he could have invested in a productive technology with decreasing returns to scale which pays off at $t = 2$. I assume the output of this productive technology is non-verifiable and thus creditors cannot directly lend to the collateral buyer. Similar assumptions of a patient investor or outside liquidity provider can be found in Diamond and Rajan (2011), Stein (2012), and Bolton et al. (2011).

As a result the market-clearing price for the collateral offered by the buyer at $t = 1$,

[13] For example, money market mutual funds are typical lenders in the wholesale funding markets and they are subject to the regulation of Rule 2a-7 of the Investment Company Act of 1940 on the amount of holdings of assets with particular rating and maturity. See Kacperczyk and Schnabl (2013).

denoted by $L(\phi;\theta)$, decreases in the amount of collateral sold ϕ and increases in the amount of the buyer's available capital θ. Further discussion on the properties and micro-foundation of the function $L(\phi;\theta)$ will be put forward in Section 1.2.3. The amount of the collateral buyer's capital θ is an exogenous parameter and common knowledge in the model, and is thus *not* a source of aggregate risk.

A time-line summarising the sequence of events is available in the Appendix.

1.2.2 Firms' investment problems: from fire sales to risk-taking

In this section I analyse the ex-ante contracting problem between a firm and its creditor at $t = 0$ while taking the equilibrium collateral liquidation value l as given. Each firm offers a collateralised short-term debt contract to its creditor to raise \$1 for investing in a project. More specifically, the firm promises to repay a r (or gross debt yields r) and should the creditor demand repayment at $t = 1$ (i.e. does not roll over the debt) but the firm fails to repay, the creditor can seize $k \in [0, 1]$ fraction of the collateral asset. This contract is superior to a long-term debt and demanding repayment dominates rolling over a failed firm because a creditor receiving the collateral at $t = 1$ has the option to sell it on the market for l, potentially higher than the utility $\underline{l}$ derived from holding it and getting the risky dividend at $t = 2$. After signing a contract $\{r, k\}$, the firm privately chooses the success probability of the project to maximise its expected net payoff from investing:

$$p(r, k) \equiv \underset{p \in \{p_1, p_2, p_3\}}{\text{argmax}} \quad p(X - r) - (1 - p)kv - c(p) \tag{1.1}$$

which is the expected residual cash flow from the project minus the expected loss of collateral and effort cost. The incentive compatible action $p(r, k)$ for a given contract can be expressed as follows:

$$(IC) \quad p(r,k) = \begin{cases} p_1 & \text{for} \quad r \leq \bar{r}_1(k) \\ p_2 & \text{for} \quad r \in (\bar{r}_1(k), \bar{r}_2(k)] \\ p_3 & \text{otherwise} \end{cases} \tag{1.2}$$

$$\text{where } \bar{r}_i(k) \equiv X - \frac{\Delta c_i}{\Delta p_i} + kv \quad \text{for } i = 1,2 \tag{1.3}$$

Equation (1.2) shows that when the promised repayment r, or debt yields, is higher than certain thresholds $\bar{r}_i(k)$, the firm chooses to take more risk. Pledging more collateral (higher k) increases those thresholds as seen in Equation (1.3) and thus discourages risk-taking because the firm loses more collateral when the project fails. Note that in equilibrium p_3 could not be chosen as investing is a negative NPV action in that case.

The contract offered has to satisfy the creditor's participation constraint. For a given equilibrium collateral liquidation value l, the creditor accepts the contract when

$$(PC) \quad \hat{p}r + (1-\hat{p})kl \geq 1 \tag{1.4}$$

where $\hat{p}$ is the creditor's conjectured project success probability. In the case of failure, the creditor receives measure k of the collateral which is worth $l \in [\underline{l}, v]$ to her in equilibrium.

Knowing the firm's incentive compatibility constraint, the creditor can rationally anticipate the firm's risk-taking decision by looking at the contractual terms $\{r,k\}$. Thus the creditor's conjectured probability $\hat{p}$ is always correct in equilibrium, i.e.

$$(RE) \quad \hat{p} = p(r,k) \tag{1.5}$$

Finally, since pledging collateral to invest risks losing the collateral, the firm would choose to undertake the project only if the expected net payoff of investing is positive. This project-taking (PT) constraint can be written as

$$(PT) \quad U(l) \equiv \max_{\{r,k\}} p(r,k)(X-r) - (1-p(r,k))kv - c(p(r,k)) \geq 0 \tag{1.6}$$

where $U(l)$ is the maximised (indirect) net utility from investing for a given equilibrium collateral liquidation value l when the firm offers the optimal collateralised short-term debt contract $\{r, k\}$.

Formally the firm offers a contract $\{r, k\}$ to the creditor which solves the following optimisation problem:

$$\max_{\{r,k\}} p(X - r) - (1 - p)kv - c(p)$$
$$\text{subject to } (IC), (PC), (RE) \text{ and } (PT)$$

and $k \in [0, 1]$ and $r \geq 0$. In the case of no solution, the firm chooses not to invest in any project.

Before proceeding to the firm's optimal investment decision and financing contract, I will first state some parameter assumptions on the expected value of the collateral v and the NPV of risk-taking, to make the analysis interesting. I will discuss the role of these parameter restrictions after the discussion of Proposition 1. Detailed derivations can be found in the Appendix.

Assumption 3. *(Parameter assumptions on v and the NPV of risk-taking)*

(i) $v \in (A_1, \bar{v})$ *where* $\bar{v} = \dfrac{A_1}{1 - [(1 - p_1)(NPV_2)]/[(1 - p_2)(A_2 + NPV_2)]}$

(ii) $NPV_2 \leq \min\{v - A_2, \dfrac{1 - p_2}{p_2} A_2\}$

Proposition 1. *(Fire sales induce a higher margin or more risk-taking) When Assumptions 1 and 3 hold, there exist two critical values l_{CR}, l_{RT} where $0 \leq l_{CR} < l_{RT} < v$ such that for any given equilibrium collateral liquidation value l, the firm's optimal investment decision $p^*(l)$ and the corresponding contract $\{r(l), k(l)\}$ are as follows:*

1. *for $l \in [l_{RT}, v]$, the firm invests prudently ($p^*(l) = p_1$) and promises debt yield $r_1(l)$ and pledges $k_1(l)$ fraction of the collateral;*

2. *for $l \in (l_{CR}, l_{RT})$, the firm engages in risk-taking and promises debt yield $r_2(l)$ and pledges $k_2(l)$ fraction of the collateral;*

3. *for $l = l_{CR}$, the firm engages in risk-taking with probability $\lambda \in [0,1]$ and forgoes the project with probability $(1-\lambda)$;*

4. *for $l < l_{CR}$, the firm forgoes the investment project ($p^*(l) = \emptyset$) (Credit Rationing)*

The optimal margin and debt yield are

$$k_i(l) = \frac{1 - p_i(X - \Delta c_i/\Delta p_i)}{p_i v + (1-p_i)l} \quad , \quad r_i(l) = \bar{r}_i(k_i(l)) = X - \frac{\Delta c_i}{\Delta p_i} + k_i(l)v \qquad (1.7)$$

and l_{CR} and l_{RT} are implicitly defined in $U(l_{CR}) = 0$ and $k_1(l_{RT}) = 1$.

Proof: *See Appendix.*

Proposition 1 demonstrates the first half of the feedback loop in Figure **??**: anticipation of a lower collateral liquidation value requires the firm to pledge more collateral or take on excessive risk when there is not enough collateral. The key intuition behind this result is that pledging collateral is costly to the firm but good for incentive and there is a finite amount of collateral. The firm in general can repay the creditor in the form of either collateral or future cash generated from the project, but cash is the preferred option because the creditor values the collateral less than the firm in equilibrium ($l \leq v$). As shown in Equation (1.2), the maximum repayment the firm can promise without triggering risk-taking is $r = \bar{r}_1(k)$, which increases with the amount of collateral pledged k. As such, in order to satisfy the creditor's participation constraint under a given liquidation value l, the minimal amount of collateral required to be pledged is $k_1(l)$ which satisfies

$$p_1 \bar{r}_1(k_1(l)) + (1-p_1)k_1(l)l = 1$$

and $k_1(l)$ and $r_1(l) = \bar{r}_1(k_1(l))$ are defined in Equation (1.7). When the liquidation value l decreases, $k_1(l)$ has to increase in order to preserve incentive and satisfy the creditor's participation constraint.

When the liquidation value is high ($l \geq l_{RT}$), the firm can pledge enough collateral $k_1(l) \leq 1$ to induce prudent investment. When l decreases below l_{RT}, implicitly defined in $k_1(l_{RT}) = 1$, even pledging all the collateral cannot simultaneously satisfy the creditor's participation constraint and induce prudent investment, that is, the debt yield required for the creditor to break-even under the prudent investment is too high, i.e.,

$$r = \frac{(1-p_1)l}{p_1} > \bar{r}_1(1)$$

Consequently, for $l < l_{RT}$, risk-taking p_2 is the only feasible action. In this case the firm promises a higher debt yield $r_2(l)$ but still needs to pledge $k_2(l) < 1$ collateral in order to commit to not privately choosing negative NPV action p_3 after financing.

Since risk-taking entails a smaller NPV and a larger expected fire sales cost due to a higher default risk as compared to the prudent investment, the firm would choose to forgo the investment when l is low enough, which I interpret as credit rationing. To see this, the firm's maximised net payoff from investing is

$$U(l) = \underbrace{p^*(l)X - c(p^*(l)) - 1}_{\text{NPV from investment}} - \underbrace{(1-p^*(l))k(l)(v-l)}_{\text{Expected fire-sale cost}} \tag{1.8}$$

which is decreasing in l. Hence there exists a l_{CR} such that the surplus generated from the project equals the expected loss from collateral fire sales, i.e. $U(l_{CR}) = 0$. The firm thus optimally forgoes the investment when $l < l_{CR}$. Finally, the firm is indifferent between no investment and risk-taking at l_{CR} and therefore plays a mixed strategy. The probability of taking on the project is denoted by $\lambda \in [0, 1]$ which will be pinned down in the competitive equilibrium.

Let me briefly discuss the role of Assumption 3. The first part regards the expected value of the collateral $v \in (A_1, \bar{v})$ to allow both prudent investment and risk-taking to arise in equilibrium. When v is low enough, there is insufficient collateral to implement prudent investment whereas with a high enough v, the collateral constraint binds *after* risk-taking becomes unprofitable, i.e. $l_{RT} < l_{CR}$, thus ruling out the possibility of risk-taking. Assumption 3 (ii) ensures risk-taking to be not too profitable otherwise credit rationing will not occur even when the expected fire-sale cost is maximal.

To sum up this subsection, Figure 1.1 graphically summarises the firm's optimal investment decision $p^*(l)$.

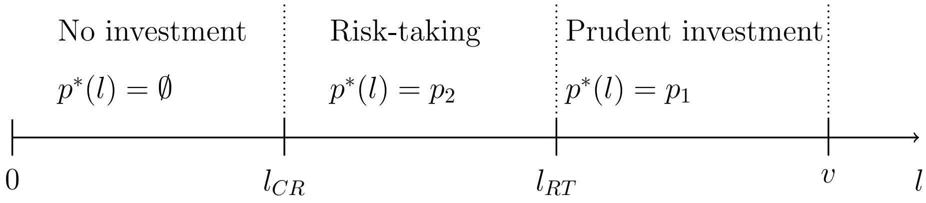


Figure 1.1: The firm's optimal investment decision at different collateral liquidation value l. Mixed strategies are played at the critical thresholds l_{CR}

1.2.3 Collateral market: from risk-taking to fire sales

In this section I will describe the supply and demand of the repossessed collateral asset and the determination of its market-clearing price. There is a competitive collateral buyer with capital $\theta \in [0, +\infty)$ to clear the collateral market at $t = 1$. At $t = 0$ he also has an opportunity to invest in a productive technology with decreasing returns to scale that produces gross return $F(\theta)$ at $t = 2$ where $F(0) = 0$, $F''(\theta) < 0$, $\lim_{\theta \to 0^+} F'(\theta) \to +\infty$ and $F'(\hat{\theta}) = 1$ for some $\hat{\theta} > 0$. Augmented with the storage technology which always returns 1, the investment opportunity gives $F''(\theta) = 0$ and $F'(\theta) = 1$ for $\theta \geq \hat{\theta}$. The output of this technology is assumed to be non-verifiable and he therefore cannot compete with the firms to raise capital from the creditors.

These conditions imply that for the buyer to hoard liquidity I for asset purchase at $t = 1$, he has to forgo some productive investment and thus liquidity carries a premium when $\theta - I < \hat{\theta}$. As the buyer behaves competitively, he takes the collateral liquidation value l as given and optimally hoards liquidity I to maximise his net payoff:

$$\Pi(l) \equiv \max_{I \in [0, \theta]} F(\theta - I) + I\frac{v}{l} - \theta \tag{1.9}$$

and the first order condition is

$$F'(\theta - I^*) \geq \frac{v}{l} \quad \text{with strict equality for } I^* > 0 \tag{1.10}$$

That is, the marginal return of investing in the productive technology has to equal to that of collateral purchase should the buyer decides to participate in the collateral market. For any given amount of liquidated collateral in the collateral market $\phi \in [0, 1]$ at $t = 1$, the market-clearing condition requires $I^* = \phi l$. Thus for $\phi > 0$, $I^* > 0$, and by substituting ϕl into the first order condition, one can re-write the liquidation value l as a function of ϕ and θ, that is, $L(\phi;\theta) \in (0, v]$. The following lemma summarises the properties of this market-clearing collateral liquidation value function.

Lemma 1. *(Market-clearing pricing function for collateral $L(\phi;\theta)$) For a given collateral supply $\phi \in (0,1]$ and the collateral buyer's capital $\theta \in [0, +\infty)$, $L(\phi;\theta)$ satisfies*

(i) $\dfrac{\partial L}{\partial \phi} \leq 0$

(ii) $\dfrac{\partial L}{\partial \theta} \geq 0$

(iii) $\lim_{\theta \to 0} L(\phi;\theta) \to 0$ *and for* $\theta \geq \hat{\theta} + v$, $L(\phi;\theta) = v$.

and $L(0;\theta)$ *is any value* $\in [\dfrac{v}{F'(\theta)}, v]$.

Proof: *direct consequences of total differentiating of the first-order condition Equation 1.10 and application of the definition of $\hat{\theta}$ where $F'(\hat{\theta}) = 1$ for $\theta \geq \hat{\theta}$.* □

Lemma 1 states that the market-clearing price for the collateral is continuous, decreasing in ϕ and increasing in θ. When the collateral buyer's capital is abundant enough, the collateral is always liquidated in fundamental value v whereas with scarce enough capital, he refuses to buy any collateral at any positive price.

Alternatively one could think of the collateral buyer as a competitive, risk-averse market maker with θ being his degree of risk tolerance. This setup is commonly used in the financial market runs literature such as Morris and Shin (2004) and Bernardo and Welch (2004). To keep the analysis as general as possible, I will only impose properties listed in Lemma 1 on any $L(\phi;\theta)$ and place no restrictions on the second-order derivatives, for example. The interpretation of an outside buyer with a productive investment technology is only used again in the welfare analysis section[14].

[14] In the case of a competitive risk-averse market maker, the collateral buyer always breaks even in any equilibrium and his payoff thus does not play a role in the welfare analysis.

Next, I will study how the supply of the collateral asset ϕ is determined. At $t = 0$, the firms and creditors form a conjecture of collateral liquidation value l and all firms adopt their investment strategy as in Proposition 1. Due to the independence of project realisation and the mixed strategy probability λ, the measure of firms with failed projects is deterministic and the measure of collateral repossessed by the creditors is

$$\lambda(l)(1 - p^*(l))k(l)$$

which is a function of measure of firms undertaking investment, the probability of default of their projects, and the amount of collateral pledged to the creditors. As the hold-to-maturity value of the collateral is worth $\underline{l}$ to the creditors, they prefer liquidating the collateral when the liquidation value l is higher than $\underline{l}$. Denote the probability of selling the collateral by $s(l)$, the measure of collateral supplied in the market ϕ is summarised in the following lemma:

Lemma 2. *(Supply of collateral is affected by expected liquidation value via firms' investment) For a given conjectured liquidation value l, the measure of collateral being liquidated at $t = 1$ is given by*

$$\phi(l) = s(l)\lambda(l)(1 - p^*(l))k(l) \tag{1.11}$$

$$\text{where } \lambda(l) = \begin{cases} 0 & \text{for } l < l_{CR} \\ \text{any } \lambda \in [0,1] & \text{for } l = l_{CR} \\ 1 & \text{for } l > l_{CR} \end{cases} ; s(l) = \begin{cases} 0 & \text{for } l < \underline{l} \\ \text{any } s \in [0,1] & \text{for } l = \underline{l} \\ 1 & \text{for } l > \underline{l} \end{cases} \tag{1.12}$$

Proof: *See discussion above.*

Figure 1.2 shows how the supply of collateral depends on the conjectured liquidation value. When the liquidation value is strictly below l_{CR} or $\underline{l}$, there is no collateral liquidated because either no firm undertakes the investment project or creditors prefer to hold the collateral to maturity. At $\max\{l_{CR}, \underline{l}\}$, firms play mixed strategies so that any amount in $[0, (1 - p_2)k_2(\max\{l_{CR}, \underline{l}\})]$ of collateral could be supplied. Beyond this critical value, all firms invest and all creditors choose to sell the asset, and thus the supply of collateral is $(1-p^*(l))k(l)$ which is decreasing and convex in l. Finally, there is a discrete jump at l_{RT} as at this level firms invest prudently and fewer defaults reduce the supply of collateral[15].

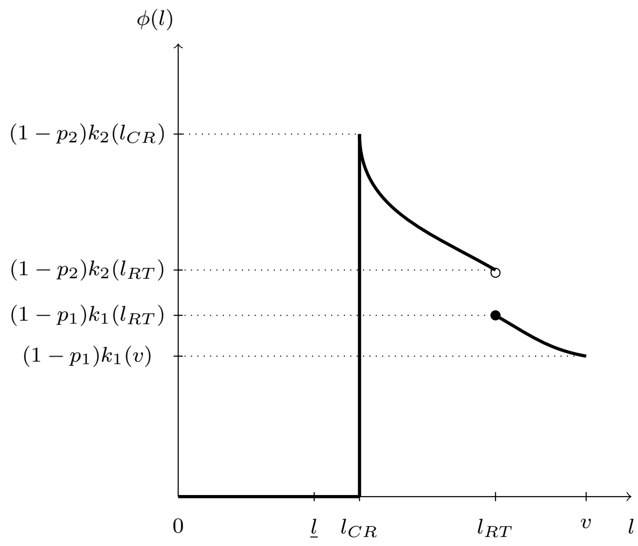


Figure 1.2: Supply of collateral asset ϕ as a function of conjectured liquidation value l

As the market clearing price of the collateral is decreasing in the amount of collateral supplied and more collateral is supplied when firms engage in risk-taking and pledge more collateral, the second half and reverse direction of the feedback loop in Figure **??** is completed: ex-ante firms' risk-taking incentives deepen the fire sale discount in the collateral market. Due to the interdependence nature of moral hazard risk-taking and the equilibrium liquidation value of the collateral, multiple rational expectation equilibria can arise. In the next section I characterise these equilibria and discuss their implications for financial fragility.

[15]The existence of the discrete jump, $(1-p_1)k_1(l_{RT}) < (1-p_2)k_2(l_{RT})$, is a consequence of Assumption 1(iii)

1.3 Competitive equilibrium: self-fulfilling fire sales

This section is devoted to characterising the equilibria and studying their features and implications for fragility.

Definition 1. *For any given amount of collateral buyer's available capital $\theta \in [0, +\infty)$, a symmetric, competitive rational expectation equilibrium consists of an equilibrium liquidation value $\{l^*\}$ and mixed strategy probabilities $\{s^*, \lambda^*\}$such that*

1. *At $t = 0$, agents conjecture the equilibrium liquidation value to be l^*. Firms maximise their expected payoff by implementing the optimal investment strategy $p^*(l^*)$ and offering the optimal contract $\{r(l^*), k(l^*)\}$ as in Proposition 1;*

2. *At $t = 1$, creditors of insolvent firms seize the collateral and supply $\phi(l^*)$ amount of collateral in the market is $\phi(l^*)$ as in Lemma 2;*

3. *The buyer with available capital θ clears the collateral market at the market clearing price $L(\phi(l^*); \theta)$;*

4. *In equilibrium, agents' expectation of collateral liquidation value is correct. That is, $l^* = L(\phi(l^*); \theta)$.*

I will first prove the existence of equilibrium in the next lemma

Lemma 3. *(Existence of equilibria) For any $\theta \in [0, +\infty)$, there exists at least one equilibrium collateral liquidation value l^* that satisfies the equation:*

$$l^* = L(s(l^*)\lambda(l^*)(1 - p(l^*))k(l^*); \theta) \tag{1.13}$$

Proof: *See Appendix.*

While Lemma 3 guarantees that equilibrium exists under any amount of the collateral buyer's capital θ, there can be more than one equilibrium collateral liquidation values l^*

that satisfy Equation (1.13)[16]. The next proposition discusses the main result of this paper: how the parameter θ affects the uniqueness and multiplicity of equilibria.

Proposition 2. *(Fragility and collateral buyer's capital θ) With Assumption 1-3 and for collateral with $\underline{l} < l_{RT}$, there exists two distinct values $\underline{\theta}$, $\bar{\theta} \in (0, +\infty)$ such that*

1. *For $\theta \in [\bar{\theta}, +\infty)$, a unique equilibrium in which all firms invest prudently exists and the equilibrium collateral liquidation value is relatively high, $l^*(\theta) \geq l_{RT}$.*

2. *For $\theta \in [0, \underline{\theta}]$, a unique equilibrium in which firms either engage in risk-taking or forgoes investment exists and $l^*(\theta) < l_{RT}$.*

3. *For $\theta \in (\underline{\theta}, \bar{\theta})$, there exist multiple values of $l^* \in [0, v]$ that satisfy Equation (1.13). As such, multiple rational expectation equilibria exist.*

   (a) *When $l^*(\theta) = l_{CR}$, $(1 - \lambda^*(\theta))$ fraction of the firms are credit rationed where $\lambda^*(\theta) \in [0, 1]$ uniquely satisfies*

   $$L(\lambda^*(\theta)(1 - p_2)k_2(l_{CR}); \theta) = l_{CR} \tag{1.14}$$

   and complete credit rationing occurs for θ such that $L(0; \theta) \leq l_{CR}$

   (b) *When $l^*(\theta) = \underline{l}$, all firms are financed and $(1 - s^*(\theta))$ fraction of the creditors in insolvent firms do not sell the collateral in the market and hold it to maturity where $s^*(\theta) \in [0, 1]$ uniquely satisfies*

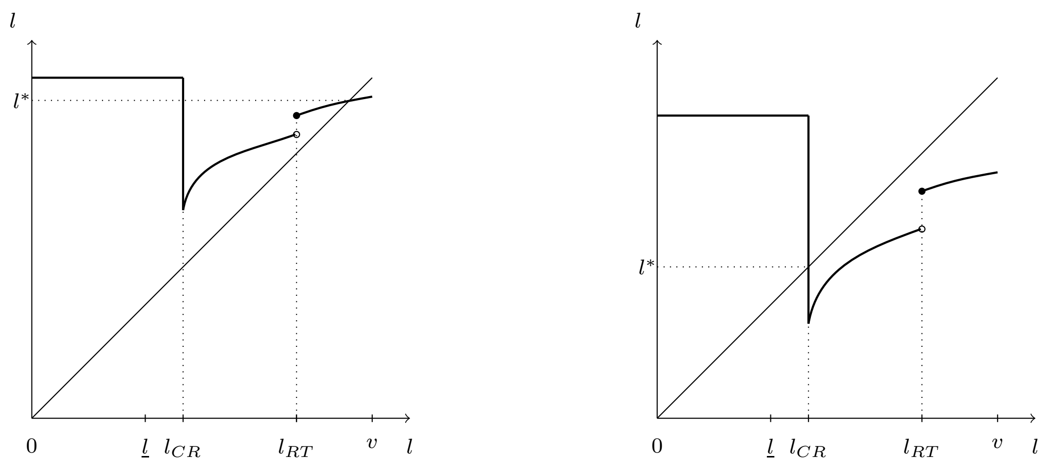
   $$L(s^*(\theta)(1 - p_2)k_2(\underline{l}); \theta) = \underline{l} \tag{1.15}$$

   and no collateral is traded for θ such that $L(0; \theta) \leq \underline{l}$

Proof: *See Appendix*

[16] I disregard the potential continuum of equilibria in which the collateral market clears without any supply or demand of the collateral. These equilibria are exactly the same economically except with a different no-trade price.

Unique equilibrium under extreme θ Figure 1.3 plots the indirect collateral liquidation value function $L(\phi(l))$ and the collateral liquidation value l against l itself. An intersection of the two graphs therefore constitutes an equilibrium (a fixed-point l in Equation (1.13)). Figure 1.3 shows the two cases of unique equilibrium. Intuitively, when θ is large, the competitive collateral buyer's capital is abundant so that he can clear the market at a relatively high price. Consequently, even when all agents in the market are pessimistic that the collateral is going to be liquidated at a low price, as a result firms take on excessive risk and the amount of collateral liquidated is large, this belief will not be vindicated in equilibrium because the collateral buyer has enough capital to clear the market at a price higher than the anticipated one. The same logic applies to the opposite case with $\theta \leq \underline{\theta}$. As a result, there could only be one equilibrium.



(a) when $\theta \geq \bar{\theta}$

(b) when $\theta \leq \underline{\theta}$

Figure 1.3: Cases of unique equilibrium under extreme values of θ

By interpreting the amount of the collateral buyer's capital as a proxy for the aggregate economy, Proposition 2 suggests that the shadow banking system is pro-cyclical, even when the fundamental value of the collateral (v) and firms' investment profitability ($pX - 1 - c$) do not correlate with θ. In a capital-abundant (good) period ($\theta \geq \bar{\theta}$), firms have low default risks, investment returns are high, the amount of credit granted by creditors to firms and by the collateral buyer to the real economy is large, debt yields are low and the collateral liquidation discount is small. In contrast, in a capital-constrained (bad) period firms are stuck in an equilibrium with high default risks, low returns, high borrowing costs, credit being rationed and a large volume of collateral is liquidated at a substantial discount. This pro-cyclicality comes from the fact that the collateral liquidation values are affected by the aggregate capital available for collateral. As a result the moral hazard problem becomes more severe in bad times, creating non-linear amplifications in the system.

Multiple equilibria and fragility When the collateral buyer's capital is between the extreme amount $\underline{\theta}$ and $\bar{\theta}$, the market-clearing price of the collateral becomes more sensitive to the change in the amount of collateral being liquidated. In this case, multiple rational expectation equilibria exist.

The multiple equilibria arises via two channels as shown in Figure 1.4[17]. The first channel is the *risk-taking channel* which is the case for switching equilibrium liquidation value from l_1^* to l_2^* where

$$l_1^* = L((1-p_1)k_1(l_1^*);\theta) \geq l_{RT} > L((1-p_2)k_2(l_2^*);\theta) = l_2^*$$

When the anticipated liquidation value changes from l_1^* to l_2^*, there is not enough collateral to maintain incentives at l_2^*, that is, $k_1(l_2^*) > 1$. Firms thus can only engage in risk-taking, resulting in more defaults and a jump in the amount of collateral being liquidated, $(1-p_2)k_2(l_2^*) < (1-p_1)k_1(l_1^*)$, thus confirming the anticipated lower liquidation value l_2^*. This fragility phenomenon from risk-taking occurs when θ is in the range that produces l^* which is sufficiently close to the risk-taking threshold l_{RT} and the discrete jump in market-clearing price leads to one equilibrium liquidation value above and the other below l_{RT}. As $L(\phi(l);\theta)$ is continuously increasing in θ from 0 to v for any given l, this range of θ always exists, irrespective of the curvature or the elasticity of the market-clearing price function.

Multiple equilibria can also arise from a *margin channel* as in the case from l_2^* to l_3^*, where both are below l_{RT} and thus the firms' default risks are the same. Note also that in

[17] I focus the discussion on stable equilibria only

this case there are some credit rationing in the equilibrium with l_3^*, i.e.,

$$l_2^* = L((1-p_2)k_2(l_2^*);\theta) > L(\lambda^*(\theta)(1-p_2)k_2(l_3^*);\theta) = l_3^*$$

with some $\lambda^*(\theta) \in (0,1)$. When the anticipated liquidation decreases from l_2^* to l_3^*, firms have to pledge more collateral $k_2(l_2^*) > k_2(l_3^*)$ to satisfy their incentive and creditors' break-even constraints. As a result more collateral is liquidated and when the market-clearing price function is sensitive enough in the relevant range, the increase in collateral supply pushes the equilibrium liquidation value to l_3^*.

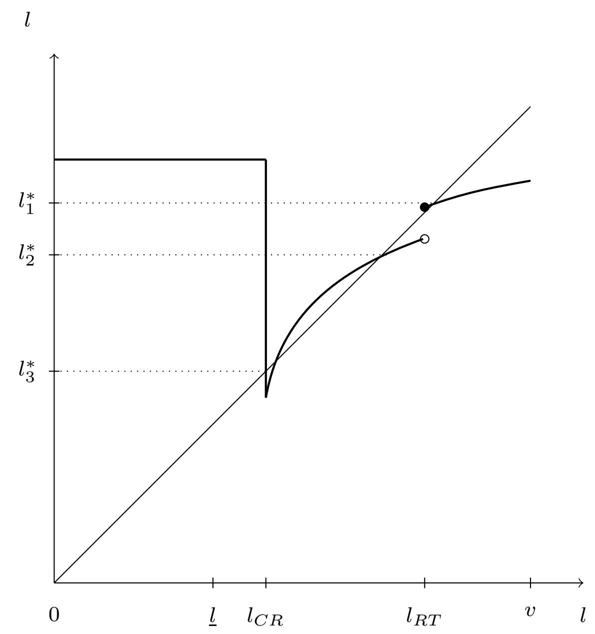


Figure 1.4: Multiple (stable) equilibria via different channels. *Risk-taking channel:* l_1^* to l_2^*; *Margin channel:* l_2^* to l_3^*

Both types of multiple equilibria discussed above are *self-fulfilling* and feature large variations in collateral asset prices, debt yields, the amount of credit rationed, and firms' profitability. There are also some differences in these two channels. Multiple equilibria caused by risk-taking have significant variations in firms' default risk but the change in margins is ambiguous ($k_1(l_1^*) - k_2(l_2^*)$ cannot be signed). Meanwhile, fragility via margin channel causes large changes in initial margins while firms' default risks remain unchanged.

The different effect on margins from the two channels can help to understand the mixed empirical findings on the behaviour of repo haircuts during the Subprime crisis in 2007-2009: Copeland et al. (2011) and Krishnamurthy et al. (2012) found small variations in haircuts in the tri-party repo market while Gorton and Metrick (2012) documented a substantial increase of haircut in the bilateral repo market[18].

To conclude this section, Figure 1.5 summarises how the collateral buyer's capital affects the equilibrium characteristics and fragility in the collateral-based financial system.

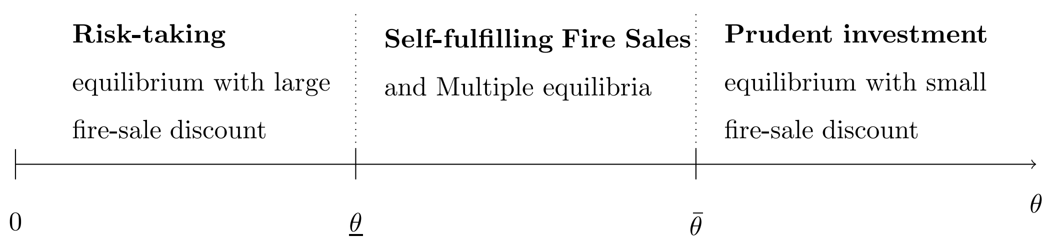


Figure 1.5: Equilibria characterisation under various exogenous amounts of collateral buyer's capital θ.

1.4 Welfare and policy implication: the case for central banks as market-makers of last resort

In this section I will first discuss the welfare implications of the multiple equilibria phenomenon and show that equilibrium with a lower collateral liquidation value is less efficient. Then I argue that this inefficiency creates a role for a social planner, or a central bank in this context, to intervene in and stabilise the collateral market and improve welfare. This role corresponds closely to the idea of *Market-Maker of Last Resort* proposed by various academics and commentators including Willem Buiter and Anne Sibert (see Buiter and Sibert (2007); Buiter (2012)).

[18] Copeland et al. (2011) also finds that lenders in the tri-party repo market are more likely to withdraw funding than to increase haircuts to reduce risk exposure. Credit rationing in this model is analogous to fund withdrawal.

I assume the social planner's objective is to maximise the total net utility of all agents. As the creditors always break even in equilibrium, the social welfare function $W(l^*)$ is defined as the sum of the net payoff of the firm $U(l^*)$ and that of the collateral buyer $\Pi(l^*)$, in equilibrium with collateral liquidation value l^*[19].

$$W(l^*) = U(l^*) + \Pi(l^*) \tag{1.16}$$

where $U(l^*)$ and $\Pi(l^*)$ are defined in Equations (1.8) and (1.9). Consider a collateral asset with quality $\underline{l} < l_{RT}$ in a state θ where multiple equilibria exist. The following proposition shows that the equilibria with lower l^* are associated with lower social welfare.

Proposition 3. *(Inefficiency) When multiple equilibria exist, social welfare $W(l^*)$ is larger in the equilibrium with a higher l^*.*
Proof: *See Appendix.*

Let's compare two equilibria with $l_1^* > l_2^*$. There are four potential sources of welfare loss in the equilibrium with l_2^*: (i) the crowding-out effect on the collateral buyer's investment in productive technology, (ii) the inefficiency from the firms' risk-taking decision when $l_1^* \geq l_{RT} > l_2^*$, (iii) the credit rationing of the firms' positive NPV investment when $l_2^* \leq l_{CR}$, and (iv) the creditors' disutility for holding the collateral to maturity when $l_2^* \leq \underline{l}$.

The self-fulfilling fragility and the inefficiency associated with the lower liquidation value equilibria call for welfare-improving policy intervention. In particular a central bank can coordinate agents into the efficient equilibrium by committing to buy any amount of collateral at a certain price. This kind of asset price guarantee policy can eliminate agents' pessimistic (yet rational) expectation hence ruling out the inefficient equilibria.

Asset Price Guarantee Recall that there are two classes of multiple equilibria that can arise: one involves risk-taking and the other acts through the change in margins.

[19] If the collateral buyer is alternatively modelled as a competitive risk-averse market maker, as suggested in Section 1.2.3, then the equilibria can be Pareto-ranked as both creditors and the collateral buyer always break-even. Only firms have higher payoff in the equilibrium with a higher collateral value.

Consider the risk-taking case with two equilibrium liquidation values $l_1^* \geq l_{RT} > l_2^*$. By committing to buy any amount of collateral at a price $l_{PG} \geq l_{RT}$, the equilibrium with risk-taking l_2^* ceases to exist because when agents know the collateral liquidation value would not fall below l_{RT}, firms can pledge enough collateral to induce prudent investment and thus no risk-taking will happen in the first place.

For the case of multiple equilibria through the margin channel, multiple l^* are both below or above l_{RT}. To pick the equilibrium with the highest l^* the central bank just needs to set the price guarantee l_{PG} strictly higher than the second highest l^* and all equilibria but the one with the highest l^* are eliminated.

Interestingly, as long as the price guarantee is strictly below the highest l^*, the price guarantee facility will never be used because in equilibrium the price offered by the outside buyer is higher than that offered by the central bank. Thus the central bank can stabilise the market and improve welfare by simply promising to intervene. This is similar to the result with deposit insurance in Diamond and Dybvig (1983).

Regarding the funding of this asset purchase programme, the central bank can issue bonds worth l_{PG} to finance the purchase or more accurately give a riskless bond worth l_{PG} to creditors in exchange for collateral. These bonds could be backed by future taxes collected from the payoff of firms' projects. Note that firms cannot individually issue claims backed by the project to finance collateral purchase because of adverse selection, as creditors do not observe other firms' solvency.

The credibility of such a commitment could still be an issue in the off-equilibrium since at $t = 1$ the firms and collateral buyer have made their investment decisions and the fire sales of collateral is simply a zero-sum transfer between the creditors and the buyer[20]. The central bank thus has no interest in tax and redistribution unless he puts an increasingly larger weight on the welfare of creditors than that of the buyer ex-post when the collateral liquidation value decreases.

While my model is very stylised and does not deal with the collective moral hazard

[20]Except the case with $l^* = \underline{l}$ in which creditors have to inefficiently hold some collateral to maturity.

problem as in Farhi and Tirole (2012) and Acharya (2009), it does provide an economic rationale for the central bank to play an active role in stabilising certain important collateral markets in order to prevent systemic runs. I summarise the discussion of this policy discussion in the following proposition:

Proposition 4. *When multiple equilibria exist, asset price guarantee can eliminate the inefficient equilibria at no cost.*

Proof: *See discussion above.*

1.5 Collateral quality and fragility

In this section I will show how collateral quality affects fragility. I interpret creditors' hold-to-maturity utility $\underline{l}$ for a particular class of collateral can be interpreted as collateral quality. The analysis below can be considered as a comparison of equilibria supported by two collaterals with different qualities such as U.S. Treasuries and private-label asset-backed securities, or alternatively, the same class of collateral before and after receiving an exogenous shock on its fundamental risk, like mortgage-backed securities around the breakout of the subprime mortgage crisis in 2007.

Lower quality collateral breeds fragility Collateral quality reflects the creditors' eagerness to liquidate. In the same state, collateral with different qualities can have a different number of equilibria. Figure 1.6 provides an example: for a lower quality collateral $\underline{l}'$, there exist two stable equilibria $l_1^* > l_2^*$ whereas a collateral with higher quality $\underline{l}''$ only supports the equilibrium with the higher liquidation value. This is because creditors' reservation price for the higher quality collateral is higher than the market-clearing price in the low liquidation value equilibrium $\underline{l}' > l_2^*$. The following proposition generalises this argument that low quality collateral breeds fragility, i.e. if multiple equilibria exist in state θ when the collateral quality is $\underline{l}$, they also exist for a lower quality collateral $\underline{l}' < \underline{l}$ in same state.

Proposition 5. *(Low quality collateral breeds fragility) Denote* $\Theta^M(\underline{l})$ *as the set of* $\theta \in [0, +\infty)$ *that permits multiple equlibria to exist when collateral quality is* $\underline{l}$*. Then the set* $\Theta^M(\underline{l})$ *is non-expanding in* $\underline{l}$*.*

Proof: *See Appendix.*

Proposition 5 can explain why the market for high quality collateral like the U.S. Treasuries and agency bonds are rather stable during the crisis while there are substantial variations in repo rates, spreads, and borrowing capacity of lower quality collateral such as private-label ABS and corporate bonds.

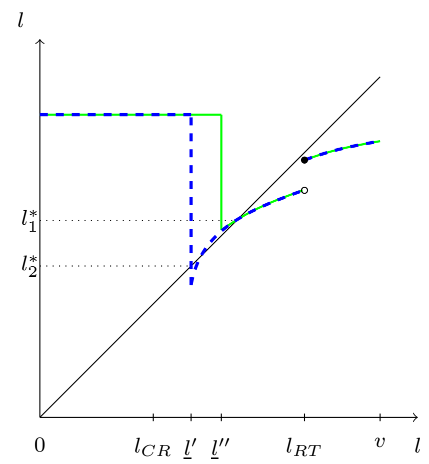
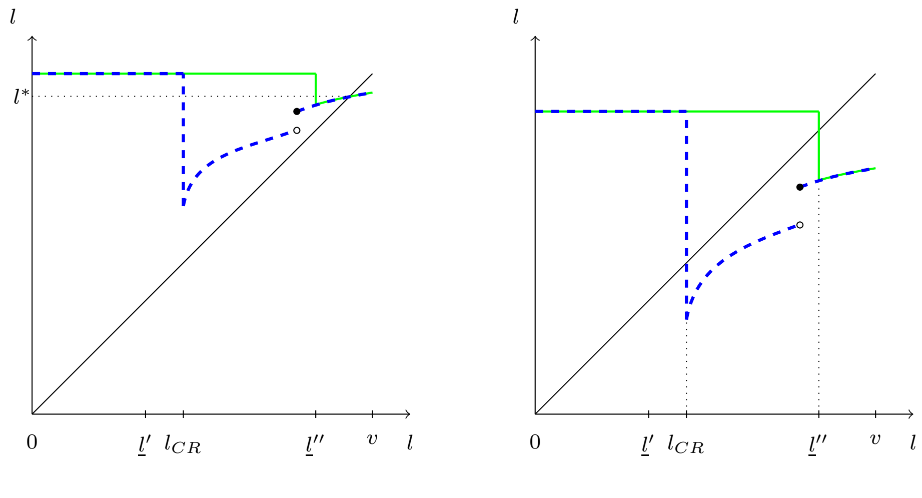


Figure 1.6: Fragility exists for lower quality collateral (blue, dashed) but not for higher quality collateral (red, dotted) in state θ'.

Counter-cyclical credit spread Another well-documented phenomenon during periods of economic distress is that the credit spreads between safe and relatively risky assets increase significantly. Consider again the two collateral assets above with reservation price $\underline{l}'$ and $\underline{l}''$ but in the extreme states with unique equilibria. To make the comparison starker I will take $\underline{l}'' \geq l_{RT}$. In the good state $\theta \geq \bar{\theta}$, there are minimal differences in terms of spreads and margins between the two collaterals because the competitive collateral buying sector has abundant capital to purchase the collateral; as a result, the difference in creditors' reservation prices for the two collateral assets does not appear in equilibrium.

The difference becomes apparent in a state where the collateral buying sector's capital is scarce. Figure 1.7 gives such an example. The differences in quality are amplified due to the moral hazard problem: the lower quality collateral triggers risk-taking in the capital-constrained state, further compounding the problem of scarce capital. This result could explain why there are minimal spread and haircut differences for Treasuries and MBSs in capital-abundant periods while the two markets are markedly different during a crisis. One might also regard the Federal Reserve's Large Scale Asset Purchase programme during the recent crisis as injecting liquidity and pushing the market from the right to the left panel in Figure 1.7. This then suggests a new, moral-hazard based channel to interpret the empirical findings by Krishnamurthy and Vissing-Jorgensen (2013) that the Fed's purchase of MBSs has much a larger reduction in yields than that of Treasuries.



(a) when $\theta \geq \bar{\theta}$ (b) when $\theta \leq \underline{\theta}$

Figure 1.7: Spreads between the lower quality (blue,dashed) and higher quality (red,dotted) collateral assets in good time (a) and bad time (b) respectively.

1.6 Repo as optimal contract and cost of automatic stay

In Section 1.2.2 I restrict attention to collateralised short-term debt contracts with promised repayment r and k measure of collateral being transferred to the lender at $t = 1$, should the firm fail to repay as demanded. In this section I will discuss the optimality and implementability of such a contract. In particular I will show that a repurchase agreement with the exemption of automatic stay can implement the optimal contract. The key friction here is that insolvent firms can threaten to file for bankruptcy protection to delay the transfer of collateral to creditors to $t = 2$, which creates a hold-up problem similar to the one outlined in the incomplete contract literature, as seen in Aghion and Bolton (1992), Hart and Moore (1994),and Diamond and Rajan (2001a)[21]. I conclude this section with a discussion of the potentially negative consequences of forbidding the use of stay-exemption.

A general contract consists of a pair $\{r_s, k_s\}$ and $\{r_f, k_f\}$ which specify cash repayment (r) and the amount of collateral transfer (k) in the case of project success or failure respectively. Timing of the payment is irrelevant for now as the information is fully revealed to both parties at $t = 1$. Recall that project cash flow is X and X_f when the project succeeds or fails respectively. The standard moral hazard result shows that $k_s = 0$ and $r_f = X_f$ are optimal. Intuitively, leaving some returns to the firm in the case of failure and giving collateral to the lender in the case of success worsen the incentive problems. Thus the optimal contract will be a debt contract with promised repayment $r = r_s \geq r_f$ and $k = k_f$ measure of collateral given to the lender only if the project fails.

Furthermore, the firm prefers to commit to transfer the collateral to the lender at $t = 1$ because this allows the lender to liquidate the collateral in the market for price l^* which is greater than $\underline{l}$, the lender's valuation of the collateral at maturity $t = 2$. Improving the lender's payoff in the case of failure allows the firm to promise less repayment, relaxes the

[21] The analysis of the optimal contract here is also similar to that in Acharya and Vishwanathan (2011) with a difference that the hold-up problem there is caused by borrower's ex-post asset-substitution problem; assigning control rights to the lender can thus solve the problem.

incentive constraint, and increases the firm's payoff.

Next is the implementation of the optimal contract. First note that as the creditor's signal about her debtor's solvency is non-contractible, the court cannot enforce payment that is contingent on the signal. As such, the collateralised debt contract has to be demandable at $t = 1$. A general secured short-term debt contract, however, will not be enough if the firm can file for bankruptcy protection at $t = 1$ and delay liquidation to $t = 2$. Specifically, I make the following assumption:

Assumption 4. *(Time-consuming bankruptcy and liquidation procedure) If a firm files bankruptcy protection at $t = 1$, the court needs time to verify its bankruptcy, liquidate the assets and can only execute the repayment to creditors at $t = 2$.*

Assumption 4 is broadly in line with the automatic stay provision in the U.S. that inhibits creditors from collecting debt when a firm files for Chapter 11 bankruptcy protection[22]. In practice the bankruptcy and liquidation of complex securities firms can be time-consuming and costly[23]. In the context of this paper, bankruptcy is costly because the collateral is only worth $\underline{l}$ to creditors $t = 2$, due to their aversion to the collateral dividend risk. Hence at $t = 1$ when the firm fails to repay as requested, it can threaten to file for bankruptcy and make a take-it-or-leave-it offer to the lender with an immediate transfer of $k' \leq k$ units of collateral such that $k'l^* = k\underline{l}$. In other words, the firm cannot credibly commit to transfer k units of collateral to the creditor at $t = 1$ when it is insolvent.

As the source of this renegotiation problem is the delay of the liquidation procedure, a short-term repurchase agreement with the exemption of automatic stay avoids this problem

[22] For instance on the US Federal Courts website, automatic stay is defined as "an injunction that automatically stops lawsuits, foreclosures, garnishments, and all collection activity against the debtor the moment a bankruptcy petition is filed." See: http://www.uscourts.gov/FederalCourts/Bankruptcy/BankruptcyBasics/Glossary.aspx

[23] For example, Lehman Brothers filed for Chapter 11 in September 2008, exited from it in March 2012, and only made the first payment to creditors in April 2012. See "Lehman Exits Bankruptcy, Sets Distribution to Creditors", *Wall Street Journal*, March 06, 2012.

by allowing the repo lender to seize the collateral immediately when the borrower defaults[24]. The following proposition summarises this discussion:

Proposition 6. *(Repo with stay-exemption as optimal contract) The optimal contract is the collateralised short-term debt contract with promised repayment r at $t = 1$ and immediate transfer of k units of collateral to the creditor at $t = 1$ in the case of default. When Assumption 4 holds such that insolvent firms can renegotiate the debt contract by threatening to file for bankruptcy, a short-term repurchase agreement with the exemption of automatic stay avoids this renegotiation problem and implements the optimal contract.*

Proof: *See discussion above.*

Cost of automatic stay Critics of the special stay-exemption status of repo contracts like Roe (2011) have argued that it could cause the disorderly liquidation of collateral assets when some borrowers default, which in turn drives down the price of the collateral and causes systemic risk. They have proposed reform that makes repo lenders also subject to some degree of automatic stay to prevent the above negative spiral. While my model does have the negative spiral fragility, it also suggests that imposing automatic stay may induce *more* fire sale and thus systemic fragility.

Here the key friction caused by automatic stay is that firms can renegotiate the debt contract ex-post. Firms can reduce the promised k units of collateral to $k' = k\frac{l}{l^*}$ ex-post by threatening to enter into bankruptcy protection, which implies the maximum amount of collateral firms can *credibly* pledge is $\frac{l}{l^*} \leq 1$. Thus the collateral constraint becomes easier to bind and firms are more prone to take excessive risk, resulting in more fire sales and fragility in aggregate. Notice that although under automatic stay the lenders cannot seize the collateral and liquidate it in the market at $t = 1$, it is optimal for firms to fire

[24] In principle, an independent sale and repurchase transaction means the collateral rests on the balance sheet of the buyer (repo lender) and thus the automatic stay provision from the default of the seller (repo borrower) should not be applied to the collateral. In practice, nonetheless, repo in the U.S. is treated as *secured loans* and the repo securities are on the balance sheet of the borrower. See Acharya and Öncü (2010) for details and the historical development of the repo market in the U.S.

sale some assets to (partially) repay the lenders. As a result, limiting post-default fire sale worsens firms' risk-taking incentive problem and *increases* pre-default fire sales[25].

1.7 Concluding remarks

This paper shows a novel form of financial fragility stemming from the feedback effect between the risk-taking incentives of borrowing firms and the illiquidity in the collateral asset market. This offers a theory of systemic runs in the modern market-based financial system where traditional strategic considerations of depositors within a financial institution may not arise. When firms collateralise their assets to borrow in the form of short-term debt such as repo, I show that a new kind of coordination failure among firms can arise since firms' risk-taking and margin decisions become *strategic complements* due to the interaction between firms' moral hazard and the fire-sale externality in the collateral market. Fire sales can occur in a self-fulfilling manner and aggregate default risk is endogenously chosen by individual firms.

In terms of policy, this paper provides an economic rationale for central banks to intervene in the collateral market. When the market is moderately illiquid, asset price guarantee can eliminate the rational fear of fire-sales of the market participants at no cost and rule out the inefficient crisis equilibrium. In addition, reform aiming to limit post-default fire sales like banning the special bankruptcy stay-exemption status may actually backfire because this could worsen the incentive problems of the borrowing firms.

[25] In a policy paper, Begalle et al. (2013) makes a similar distinction between pre-default and post-default fire sales and discuss how they can affect each other.

1.8 Appendix

1.8.1 Time-line of events

t=0

- A continuum of firms each needs to borrow \$1 to invest in a project, using an asset-in-place as collateral.
- Each firm offers a collateralised short-term debt to its creditor with promised payment r and k fraction of collateral pledged.
- Each firm privately exerts costly effort to increase the project success probability.
- Collateral buyer with exogenous amount of cash θ optimally hoards cash for collateral purchase at $t = 1$.

t=1

- Projects quality revealed. Each creditor knows whether her firm's project has succeeded or failed.
- Creditors of solvent firms roll over their debt and will receive r at t=2.
- Creditors of insolvent firms seize the k units of collateral and decide whether to sell it in the market.
- Collateral buyer used the hoarded cash to clear the collateral market. The market-clearing price $L(\phi; \theta)$ decreases in the amount collateral sold ϕ and increases in θ.

t=2

- Collateral's dividend realises.
- Succeeded projects' cashflow matures.
- Creditors in solvent firms receive repayment r, and the firms keep the remaining cashflow.

1.8.2 Parametric restrictions in Assumption 3

Parametric restrictions in Assumption 3 are made to ensure $0 \leq l_{CR} < l_{RT} < v$ so that prudent investment, risk-taking and credit rationing can arise in equilibrium. From the implicit definition of l_{RT} and l_{CR}, $k_1(l_{RT}) = 1$ and $U(l_{CR}) = 0$, one can show

$$l_{RT} = \frac{A_1 - p_1 v}{1 - p_1} \text{ and } l_{CR} = v \frac{(1 - p_2)A_2 - p_2 NPV_2}{(1 - p_2)A_2 + (1 - p_2)NPV_2}$$

It is immediate to check that $l_{RT} < v$ and $l_{CR} < l_{RT}$ require $v > A_1$ and $v < \bar{v}$ respectively. To have $U(l_{CR}) = 0$ in equilibrium, one needs $l_{CR} \geq 0$ and $k_2(l_{CR}) \leq 1$ which together give the condition $NPV_2 \leq \min\{v - A_2, \frac{1 - p_2}{p_2} A_2\}$. $\square$

1.8.3 Proofs

Proof of Proposition 1:

First, both (IC) and (PC) are binding at optimal. If (PC) slacks, the firm can decrease r by a small amount to increase profit while (IC) still holds; If (IC) slacks, the firm can reduce k and increase r by a small amount to keep (PC) binding and (IC) still satisfied while a smaller k increases expected payoff due to lower fire-sale cost. To see this, suppose the contrary that $\{r, k\}$ is optimal but (IC) slacks, that is

$$r < X - \frac{\Delta c_i}{\Delta p_i} + kv$$

Plugging the binding (PC) $r = [1 - (1 - p_i)kl]/p_i$ into the above (IC), one can show $k > \frac{A_i}{p_i v + (1 - p_i)l}$. Consider another contract $\{r', k'\}$ such that $k' = k - \epsilon$ and $r' = r + (1 - p_i)\epsilon l/p_i$, (PC) still binds and for a small $\epsilon > 0$ (IC) also holds. However the firm's expected payoff is strictly higher in the case of $\{r', k'\}$, as $NPV_1 - (1 - k')(v - l) > NPV_1 - (1 - k)(v - l)$, contradicting the optimality of $\{r, k\}$.

By binding (PC) and (IC), the optimal contract $\{r(l), k(l)\}$ is described as in Equation 1.7. Note that for a given l, it could be both $\{r_1(l), k_1(l)\}$ and $\{r_2(l), k_2(l)\}$ satisfy the remaining (RE) and (PT) constraints, that is, both prudent investment and risk-taking are feasible choices. Since prudent investment is always superior by Assumption 1, the firm optimally chooses $p_1(l)$ and the contract $\{r_1(l), k_1(l)\}$. Hence the firm chooses prudent investment whenever feasible, that is, when $l \geq l_{RT}$. If not, risk-taking is chosen as long as it is profitable, when $l \geq l_{CR}$. □

Proof of Lemma 3:

For a fixed θ, $L(s(l)\lambda(l)(1 - p(l))k(l); \theta)$ is a mapping from $[0, v] \to [0, v]$. Notice that the function $L(l; \theta)$ is upper semi-continuous from the left and closed from the right. The existence of fixed-point follows from the Lemma in Roberts and Sonnenschein (1976). □

Proof of Proposition 2:

There are three steps in this proof: I first show the existence of extreme regions of θ that only exactly one equilibrium exists. Then I show multiple equilibria must exist under some regions of θ and finally, I characterise the bounds of multiple equilibria regions $\underline{\theta}$, $\bar{\theta}$ for different possible shapes of the market-clearing price function $L(\phi(l);\theta)$.

Step 1: non-empty regions of θ with unique equilibrium

For $\theta \in [\hat{\theta}+v, +\infty)$, $L(\phi(l);\theta) = v$ for all $l \in [0, v]$ according to Lemma 1. Thus there is only prudent investment equilibrium in this region as $l_{RT} < v$. On the other hand, for $l < \max\{\underline{l}, l_{CR}\}$, $\phi(l) = 0$ while the maximum price the collateral buyer willing to pay for the first unit is $\frac{v}{F'(\theta)}$. As $\lim_{\theta \to 0+} F'(\theta) \to +\infty$ and $F''(\theta) < 0$, there exists a $\theta' > 0$ such that $\frac{v}{F'(\theta')} = \max\{\underline{l}, l_{CR}\}$. Then for $\theta \in [0, \theta')$, there is unique equilibrium with complete credit rationing (when $\underline{l} < l_{CR}$) or risk-taking and no collateral traded (when $\underline{l} > l_{CR}$).

Step 2: non-empty set of θ with multiple equilibria

The key of this step is the upward jump of $L(\phi(l);\theta)$ from $l \to l_{RT}$. At $l = l_{RT}$, $\phi(l_{RT}) = (1-p_1)k_1(l_{RT}) = (1-p_1) > (1-p_2)k_2(l_{RT})$, where the strict inequality is implied by Assumption 1(iii). By continuity of $L(;\theta)$, there exists a θ'' such that $L(\phi(l_{RT});\theta'') = l_{RT}$ hence $l^* = l_{RT}$ is an equilibrium with prudent investment at θ''. I am going to show that there also exists at least another equilibrium in the region $l \in [\max\{\underline{l}, l_{CR}\}, l_{RT})$ at this θ''. Due to the discontinuity of $\phi(l)$ at l_{RT}, $L((1-p_2)k_2(l_{RT});\theta'')$ is strictly below l_{RT} and then $L(\phi(l);\theta'')$ must cross the 45-degree line at some $l^* \in [\max\{\underline{l}, l_{CR}\}, l_{RT})$. To reduce notation, I will discuss the case with $l_{CR} > \underline{l}$. If $L((1-p_2)k_2(l_{CR});\theta'') \geq l_{CR}$, then by Intermediate Value Theorem, there exists a $l^* \in [l_{CR}, l_{RT})$ such that $L((1-p_2)k_2(l^*);\theta'') = l^*$ because $L(\phi(l);\theta'')$ is continuous in l and $L((1-p_2)k_2(l_{RT})) < l_{RT}$; If $L((1-p_2)k_2(l_{CR});\theta'') < l_{CR} < L(0;\theta'')$, then there exist a $\lambda^* \in (0,1)$ such that $L(\lambda^*(1-p_2)k_2(l_{CR});\theta'') = l_{CR}$ as at l_{CR}, $L(;\theta'')$ can take any value between $L((1-p_2)k_2(l_{CR});\theta'')$ and $L(0;\theta'')$ due to Lemma 2. In conclusion, there exists multiple equilibria at θ''.

Step 3: Characterise the bounds of $\underline{\theta}$ and $\bar{\theta}$

Let's start with the upper bound $\bar{\theta}$. For $\theta > \theta''$, multiple equilibria can exist because Equation 1.13 has multiple solutions in the region $[l_{RT}, v]$ or at least one solution in $[\max\{\underline{l}, l_{CR}\}, l_{RT})$ or both. Denote θ_1 and θ_2 as the smallest $\theta > \theta''$ that $L((1-p_1)k_1(l); \theta_1) = l$ has exactly one solution in $[l_{RT}, v]$ and $L((1-p_2)k_2(l); \theta_2) = l$ has no solution in $[\max\{\underline{l}, l_{CR}\}, l_{RT})$ respectively. Both θ_1 and θ_2 exist as members in the non-empty set of θ with unique prudent investment equilibrium satisfy these properties. Define $\bar{\theta} = \max\{\theta_1, \theta_2\}$ and as $L(;\theta)$ increases in θ, there is a unique equilibrium with prudent investment for any $\theta \in [\bar{\theta}, +\infty)$. Note that by construction $\theta'' < \bar{\theta}$.
Similarly for $\underline{\theta}$. Denote θ_3 and θ_4 as the largest $\theta < \theta''$ that $L((1-p_2)k_2(l); \theta_3) = l$ has exactly one solution in $[\max\{\underline{l}, l_{CR}\}, l_{RT})$ and $L((1-p_1)k_1(l); \theta_4) = l$ has no solution in $[l_{RT}, v]$ respectively. Define $\underline{\theta} = \min\{\theta_3, \theta_4\}$ and as $L(;\theta)$ increases in θ, there is unique equilibrium with risk-taking (and credit rationing) for any $\theta \in [0, \underline{\theta}]$. Note that by construction, $\underline{\theta} < \theta''$. Finally by the fact that $L(\phi(l); \theta)$ is continuous and strictly increases in θ for $\phi(l) > 0$, any $\theta \in (\underline{\theta}, \bar{\theta})$ contains multiple equilibria and this region is non-empty as $\theta'' \in (\underline{\theta}, \bar{\theta})$. □

Proof of Proposition 3:

By the definition of $U(l^*)$ and $\Pi(l^*)$ in Equation (1.8) and (1.9) and the market-clearing condition $F'(\theta - \phi(l^*)l^*) = v/l^*$, the social welfare function $W(l^*)$ can be expressed as

1. When $l^* > \max\{l_{CR}, \underline{l}\}$, $\phi(l^*) = (1 - p(l^*))k(l^*)$
$$W(l^*) = NPV(l^*) + \int_0^{\theta - \phi(l^*)l^*} [F'(x) - 1]dx \tag{1.17}$$
2. When $l^* = l_{CR}$, $\phi(l_{CR}) = \lambda(1 - p(l_{CR}))k(l_{CR})$
$$W(l_{CR}) = \lambda NPV(l_{CR}) + \int_0^{\theta - \phi(l_{CR})l_{CR}} [F'(x) - 1]dx \tag{1.18}$$
3. When $l^* = \underline{l}$, $\phi(\underline{l}) = s(1 - p(\underline{l}))k(\underline{l})$
$$W(\underline{l}) = NPV(\underline{l}) + \int_0^{\theta - \phi(\underline{l})\underline{l}} [F'(x) - 1]dx - (1 - s)(1 - p(\underline{l}))k(\underline{l})(v - \underline{l}) \tag{1.19}$$

It is then immediate to see that a higher l^* will increase $W(l^*)$ in all cases. $NPV(l^*) = p(l^*)X - 1 - c(p(l^*))$ increases in l^*; $\int_0^{\theta-\phi(l^*)l^*}[F'(x) - 1]dx$ is the net return from collateral buyer's productive investment and increases in l^* as $\phi(l^*)l^*$ decreases in l by market-clearing condition. In case 2, $(1 - \lambda)$ of firms do not invest and in case 3, $(1 - s)$ of creditors could not sell the collateral to the buyer and both entail welfare loss. Therefore equilibria with lower l^* has a lower $W(l^*)$. □

Proof of Proposition 5:

Suppose $\theta \in \Theta^M(\underline{l})$, $L(\phi(l;\underline{l});\theta) = l$ has multiple solutions $\{l^*\}$. What I need to show is that when $\underline{l}$ decreases to any $\underline{l}' < \underline{l}$, there are as least as many solutions. First note that as ϕ is only affected by $\max\{l_{CR}, \underline{l}\}$, changes $\underline{l}$ below l_{CR} will not have any effect in equilibrium. Every member in the set $\{l^*\}$ is at least as large as $\underline{l}$ and when they are strictly larger than $\underline{l}$, they will still be part of the solution for any $\underline{l}' < \underline{l}$. When $\underline{l}$ is one of the solutions and is changed to $\underline{l}'$, there are two cases: If $L((1-p(\underline{l}'))k(\underline{l}');\theta) < \underline{l}'$, the original solution $\underline{l}$ changes to $\underline{l}'$ with $L(s^*(1-p(\underline{l}'))k(\underline{l}');\theta) = \underline{l}'$ for some $s^* \in [0,1]$. This is because at $\underline{l}'$, $L()$ is a correspondence taking any value from $L((1-p(\underline{l}')k(\underline{l}');\theta)$ to $L(0;\theta) \geq \underline{l} > \underline{l}'$. On the other hand, if $L((1-p(\underline{l}'))(\underline{l}');\theta) > \underline{l}'$, the original solution $\underline{l}$ changes to some $\underline{l}'' \in [\underline{l}', \underline{l}]$ where $L((1-p(\underline{l}''))k(\underline{l}'');\theta) = \underline{l}''$. This follows from Intermediate Value Theorem as $L((1-p(l))k(l);\theta)$ is a continuous function in l and $L((1-p(\underline{l}))k(\underline{l});\theta) \leq \underline{l}$. Therefore, decreasing $\underline{l}$ to $\underline{l}'$ does not reduce the number of solutions $\{l^*\}$. □

Chapter 2

Countercyclical foreclosures for securitisation

2.1 Introduction

The epidemic of mortgage foreclosures in the US which started in 2008 has raised concerns from the general public and policy makers.[1] The number of foreclosures started to surge in 2007 and continued to rise into 2010. It has been argued that foreclosures create significant losses for both the lenders and the borrowers, and have major negative externalities to the broader society.[2] In response, the United States government has set up a series of programs in an attempt to reduce mortgage foreclosure, such as the Home Affordable Modification Program. The scale and significant economic implication of foreclosure deserves attention in order to achieve an understanding of its driving force and the underlying mechanism.

Recent studies and reports have suggested that securitisation and the biased incentives of mortgage servicers could have contributed to the wave of foreclosures. For instance, Piskorski et al. (2010) show that, during the recent crisis, mortgages in a securitised pool are more likely to be foreclosed than otherwise similar mortgages on bank portfolios when

[1]For example, the Huffington Post has a designated section for news on the foreclosure crisis.

[2]See for example Pennington-Cross (2006) for a survey on the deadweight loss on foreclosure.

the mortgages become delinquent. In addition, an analysis of the complex compensation structure of the servicers by Thompson (2009) concludes that the servicers' legal and financial incentives bias servicers towards foreclosure instead of modifying delinquent mortgages, even when investors would profit more from modification than foreclosure.

This paper investigates the optimal foreclosure policy of securitisers in a framework of mortgage-backed securitisation under asymmetric information. This framework allows us to answer the following questions. How does information asymmetry in the securitisation process give rise to a foreclosure policy that is *ex post* inefficient? Why does foreclosure appear countercyclical? What is the role of a third-party servicer in the securitisation process? In aggregate, how can a "foreclosure crisis" arise?

We explicitly model the foreclosure decision in the mortgage-backed securitisation and the market for repossessed property. A securitiser has a mortgage pool which returns risky cash flows, and securitisation is motivated by the liquidity needs of the securitiser a là DeMarzo and Duffie (1999). However, some mortgages subsequently become delinquent and the securitiser must decide whether to modify or foreclose the delinquent mortgages. If a mortgage is modified (forbearance), the full repayment is recovered with some probability. If a mortgage is foreclosed, the underlying property is repossessed and sold in a designated market for repossessed properties. Investors in the market for repossessed properties post an aggregate downward sloping demand curve, generating market clearing prices of repossessed properties that are decreasing in the amount of property foreclosed.[3]

The securitiser has private information regarding the probability of recovery on the delinquent mortgages. This may be because the securitiser has access to the specific borrower information. The securitiser then designs and sells a mortgage-backed security to the outside investors. Consistent with existing literature on security design (e.g. Myers and Majluf (1984); Nachman and Noe (1994)), we establish that the securitiser chooses to issue a senior security, or debt, to the outside investors. As in DeMarzo and Duffie (1999), securitisers with high quality mortgage pools signal their type by retaining the residual

[3]This is micro-founded in the model by modelling a mass of investors with heterogeneous renovation costs when investing in repossessed properties.

junior tranche, which entails a liquidity cost.

The main result of the paper is that information asymmetry in the securitisation process leads to countercyclical foreclosure. In the baseline model, we consider the optimal foreclosure policy if the securitiser can choose a set of type-contingent foreclosure rates prior to obtaining any private information on the mortgage pool and commit to the foreclosure policy *ex post*. Compared to the full information benchmark, the optimal foreclosure rates under asymmetric information are more negatively related to the quality of the mortgage pool *ex post*. In other words, signalling concerns lead to more foreclosure when the mortgage pool is of poor quality, and vice versa. This implies that the aggregate foreclosure is negatively related to the overall quality of the mortgage pools in the economy.

The intuition of the above result is as follows. The optimal foreclosure policy under asymmetric information maximises the securitiser's expected payoff by trading off the costs of signalling against the *ex post* inefficiency in the foreclosure decision. In order to reduce the signalling costs, a securitiser designs a policy that discourages the low type from mimicking the high type. The low type's payoff from mimicking comprises the proceeds from selling the debt claim at the high type's price and the value of the retained cash flow. The foreclosure policy of the securitiser has two effects on the incentive for the low type to mimic. Firstly, an inefficiently low foreclosure rate for the high type reduces the incentive to mimic by decreasing the value of the debt security issued by the higher type. Secondly, an excessively high foreclosure rate for the low type discourages mimicking by decreasing the value of the retained junior claim (levered equity), since foreclosure reduces the risk in the overall cash flow from the mortgage pool. This is because, *ex post* under any given market condition, foreclosing a mortgage brings an immediate cash flow equal to the market price of the property, while forbearance entails risk in the potential recovery of the delinquent mortgage. Therefore the equilibrium optimal foreclosure policy is excessively countercyclical.

The ability of the securitisers to commit to the *ex ante* chosen optimal foreclosure policy is crucial in the above mechanism. In an environment where the commitment power is not naturally available, we show that, in equilibrium, third-party servicers play a role in enforcing such commitment. First of all, if the securitiser cannot commit to the *ex ante* chosen optimal foreclosure policy, she would tend to foreclose less *ex post*. Moreover, *ex post* foreclosure policy is positively related to the quality of the mortgage pool. This is because, given that the high quality securitiser retains the junior tranche, which is convex in the cash flows, she benefits from riskier cash flows *ex post*. However, the lack of commitment power hurts the *ex ante* securitisation process, leading to a lower expected payoff in equilibrium for the securitisers.

We then propose a mechanism that resembles the industry practice to enforce such commitment power, which involves mortgage servicers. This is inline with the view of Thompson (2009), who argues that the rise of the servicing industry is a by-product of securitisation.[4] An important function performed by mortgage servicers is the decision of forbearance versus foreclosure. A third-party servicer allows the separation of this decision from the securitiser, potentially enabling the securitiser to commit to a set of *ex ante* chosen foreclosure policies.

In this mechanism, a mortgage originator with a pool of mortgages can choose to (i) securitise the pool himself with the *ex post* servicing done in-house, in which case the foreclosure decision will be made *ex post* as illustrated previously; or (ii) sell the mortgage pool to a securitiser but remain as the servicer of the mortgages. In the latter case, the securitiser offers a compensation contract that includes a payment transfer and fees dependent on the *ex post* cash flow of the mortgage pool. The securitiser then proceeds to issue the optimal mortgage-backed securities, while the servicer makes the *ex post* foreclosure decision according to the incentives given by his compensation.

We show that there exist contracts that implement the optimal foreclosure policy. Because selling the mortgage pool to a securitiser enables commitment and reduces the costs associated with asymmetric information, it is more efficient than in-house servicing. Moreover, for mortgage pools of low quality, the compensation to the servicer is designed to lean

[4] The servicer performs duties including collecting the payments, forwarding the interest and principle to the lenders, and negotiating new terms if the debt is not being paid back, or supervising the foreclosure process.

towards foreclosure. This implements the optimal foreclosure policy which would appear to be excessive *ex post*, namely, the *ex post* foreclosure may result in a loss to the investors. This is evident in the past financial crisis. For example, Levitin (2009) estimates that lenders lose approximately 50% of their investment in a foreclosure situation.

Finally, we extend the model to consider an economy with multiple securitisers who compete in the market for repossessed property when mortgages are foreclosed. The foreclosure policy is still higher for a lower quality mortgage pool, and lower for a high quality one. This leads to countercyclical foreclosure in equilibrium, that is, the overall foreclosure is higher in an economic downturn in which many mortgage pools are of low quality. The prices in the repossessed property market are hence procyclical.

We also examine the two additional sources of friction brought by the competitive environment considered in the market for repossessed property. On the one hand, the fire-sale externality arises with competition, which tends to result in excessive foreclosure in equilibrium. This is because a securitiser does not internalise the negative externality of her decision to foreclose a delinquent mortgage on the other securitisers' payoff due to its price impact. On the other hand, the market power of each securitiser decreases with competition. This tends to increase foreclosure as it reduces the inefficiency associated with oligopoly in terms of insufficient foreclosure. The overall effect suggests that under strong competition, prominent fire-sale externality exacerbates the countercyclical effect of asymmetric information and leads to significant excessive foreclosure in bad economic times, which can be interpreted as a foreclosure crisis.

Related Literature

This paper belongs to the growing body of literature on the incentive problems associated with mortgage securitisation. Various studies argue that securitisation relaxes the *ex ante* lending standards. Keys et al. (2010, 2012), using evidence from securitised subprime loans, show that the ease of securitisation reduces lenders' incentives to carefully screen the mortgage borrowers and that mortgages with higher likelihood to be securitised have higher default rates. Mian and Sufi (2009) find that securitisation of subprime loans is associated with credit expansion and, as a result, counties with a high proportion of subprime mortgages face a larger number of defaults. Elul (2011) also finds securitised prime loans have a higher default rates than otherwise comparable portfolio loans. Our work adds a different dimension to this literature by studying the decision of *ex post* mortgage foreclosure in relation to securitisation.

Our paper also relates to the study of optimal loan modification and foreclosure policy. Wang et al. (2002) show that when a lender (bank) has a high screening cost to ascertain whether a borrower is in distress, it could be optimal for the bank to randomly reject loan workout requests to deter the non-distressed borrower from opportunistically applying for a loan modification. Riddiough and Wyatt (1994) study the case in which the lender's foreclosure cost is private information and the borrowers will infer this cost from past loan foreclosure decisions and consequently decide their default decision and concession request. The lender thus may costly foreclose many loans today to reduce future expected default and loan modification costs. Gertner and Scharfstein (1991) focus on the free-riding problem among multiple creditors and show that when the cost of debt concessions is private but the benefit is shared, a creditor's incentive to grant concessions to a distressed firm is reduced. While the literature typically finds that the frictions lead to excessive foreclosure, this model predicts procyclical foreclosure policy based on the asymmetric information problem which is present in the mortgage-backed securitisation process.

Finally, while this paper is the first to formalise the role played by foreclosure in mortgage-backed securitisation in a model of asymmetric information, several empirical studies identify securitisation as being an important impediment for efficient renegotiation following delinquency, e.g. Agarwal et al. (2011a); Piskorski et al. (2010); Zhang (2011). Particularly related to our model are the empirical findings of Agarwal et al. (2011b). The authors find that the incentives of servicers present an impediment to loss mitigation of delinquent mortgages and attribute this to the holdup problem posed by dispersed investors of the senior tranche when the servicers hold the junior tranche. Our model provides a theoretical argument for distortions in the foreclosure decision of securitised mortgages.

The remainder of this paper is organised as follows. Section 2.2 presents our model of mortgage-backed securitisation with foreclosure policy under asymmetric information. Section 2.3 solves the model and formulates the optimal pre-committed foreclosure policy. In Section 2.4 discusses the role of third-party servicer in the MBS industry. Section 2.5 extends the baseline model to show that fire-sale externality can generate "foreclosure crisis". Finally Section 2.6 summarises the empirical implications produced by the model, and Section 2.7 concludes.

2.2 Model setup

This section sets up the baseline model and comments on the assumptions which are central to the model.

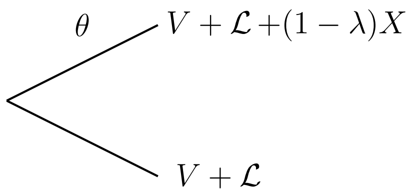
There are four dates: 0, 1, 2 and 3. The baseline model's participants consist of a securitiser and a continuum of outside investors each with one unit of cash. All agents are risk neutral. The securitiser is impatient and has a discount factor $\delta < 1$ between $t = 1$ and $t = 3$. This follows the assumption of DeMarzo and Duffie (1999) and can be interpreted as the securitiser's incentive to raise capital by securitising part of her long term assets as he has access to some positive return investment opportunities. There is no discounting for the outside investors.

Securitiser and mortgage pool

At $t = 0$, the securitiser has a pool of a continuum of identical mortgages that pays off at $t = 3$. We henceforth refer to the securitiser as "she". All mortgages have independent probability to become delinquent at $t = 2$. Therefore a fixed portion of the mortgages becomes delinquent. We normalise the measure of the delinquent mortgages in the pool to 1.The remaining mortgages continue to repay and have an exogenous value of V. The delinquent mortgages can be foreclosed or granted forbearance. In case of foreclosure, the collateral property is repossessed and sold for a liquidation proceed $\mathcal{L}$ to outside investors. In case of forbearance, the fixed mortgage repayment of value X is resumed with probability θ; otherwise the loans are worthless. For simplicity, we assume that the repayments of all delinquent mortgages are perfectly correlated. It can be interpreted to capture the systematic variations in the risk of the mortgages.

Denote λ the fraction of delinquent mortgages foreclosed. The overall cash flow from mortgage pool at $t = 3$ is then $V + \mathcal{L} + (1 - \lambda)X$ with probability θ, and $V + \mathcal{L}$ with probability $(1 - \theta)$, as illustrated in Fig 2.1.

Figure 2.1: Mortgage pool cash flow



At the beginning of $t = 1$, the securitiser receives a private signal regarding the recovering rate of the delinquent mortgages $\theta \in \{\theta_H, \theta_L\}$, where $\theta = \theta_H$ with probability γ. The assumption that the private information only concerns the credit risk of the delinquent mortgages is to simplify analysis and is not central to the model. Nevertheless, one interpretation could be that there is generally less data on delinquent loans, making it more difficult to assess the recovery rate of such borrowers.

After receiving the private information at $t = 1$, the securitiser designs a security that depends on the cash flow of the mortgage pool at $t = 3$, and sells it to outside investors. The securitiser retains the residual cash flow from the mortgage pool after paying off the investors. We will henceforth refer to it as the mortgage-backed securities (MBS). The MBS market is detailed below.

Investors and markets

There are two markets in this model. There is a market of MBS issue at $t = 1$, and a market for distressed property at $t = 2$. The investors are risk neutral, and the discount rate is 0. Since the MBS only pays off at $t = 3$, the investors can participate either in the MBS market, or the distressed property market, or neither.

At $t = 2$ the market for distressed properties opens. Each property is valued at X by the outside investors. However, the investors need to incur a heterogeneous cost $r \geq 0$ per unit capital invested in the distressed properties. This cost r can reflect the significant renovation and repair costs associated with distressed properties, as well as other liens such as unpaid fees and taxes. The heterogeneity in the costs can be driven by the time, skill and experience of the investors to conduct such renovations.

In Section 2.3 and 2.4, we assume that the private cost r of each investor is observable by the securitiser. This allows the monopoly securitiser to implement perfect price discrimination and extract all social surplus. As a baseline model, this setup has the benefit of removing any inefficiency induced by the market structure. This therefore allows a clean representation of the welfare implication of the optimal foreclosure policy under asymmetric information, as presented in Section 2.3.[5] Denote with $I(R)$ the measure of investors with $1 + r \leq R$, with $I(1) = 0$, $I'(R) > 0$ and $I''(R) < 0$.

Given that r is observable to the securitiser, at $t = 2$ the securitiser makes a take it or leave it offer to each investor with a price $b(r)$. The investor then chooses whether or not to accept the offer. An investor is able to accept the offer only if the investor has not invested in the MBS security at $t = 1$. As a tie break convention, we assume that an investor prefers to wait and invest in the distressed property market if the investor expects to be made an offer that will be accepted at $t = 2$, when the investor is indifferent between investing in the MBS security at $t = 1$ and in the distressed property market at $t = 2$. If an investor accepts the offer $b(r)$, the investor purchases a measure $1/b(r)$ of the distressed

[5] The assumption that the private cost r is observable by the securitiser will be relaxed in Section 2.5 to study the effect of competition amongst securitisers in the market for distressed properties.

properties. The cost r is incurred at the end of the periods when the payoffs are realised.

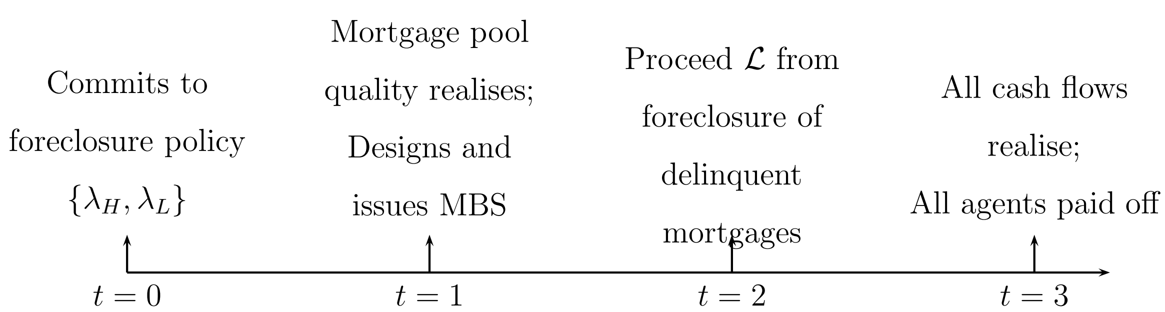
At $t = 1$, the securitiser designs an MBS and issues it to the market. Observing the choice of security on offer, the investors form a belief $\hat{\theta}$ regarding the private information of the issuer and decide whether to subscribe to the issue. The investors strictly prefer to subscribe if the issue is priced below the market valuation, and vice versa. Therefore the market clearing price of the security p is equal to the market value of the security given the investors' belief.

Foreclosure policy

In Section 2.3, we assumed that at $t = 0$, the securitiser commits to a set of foreclosure policies $\{\lambda_H, \lambda_L\}$ contingent on her type when it realises at $t = 1$. We then solve for the optimal foreclosure policy. We relax this assumption in Section 2.4 and provide a mechanism that involves a third party, the mortgage servicer, to implement the optimal foreclosure policy.

The timeline of the model is summarised in Figure 2.2.

Figure 2.2: Baseline model timeline



2.3 Pre-committed foreclosure policy

This section firstly presents the full information (first best) benchmark of the model. We then solve for the optimal foreclosure policy and compare it to the full information benchmark to assess the welfare implications.

2.3.1 First best benchmark

We follow a backward induction process to compute the first best benchmark. First, we solve for the distressed property market pricing equilibrium for a given foreclosure policy. This then allows the characterisation of the securitiser's problem regarding security design and foreclosure policy choice.

Distressed property market

In order to obtain closed-form results, we use in Section 2.3 and 2.4 the following function form for $I(R)$, the measure of investors with renovation cost such that $1 + r \leq R$,

$$I(R) = aX \ln(R), \quad \text{for some } a \in [\underline{a}, \bar{a}]^{6} \tag{2.1}$$

At $t = 2$, a fraction λ_i, $i \in \{H, L\}$ of the delinquent mortgages are foreclosed, and the underlying properties offered to the market. In order to maximise the liquidation proceeds, the securitiser prefers to make offers to investors with the lowest r at the highest prices that will be accepted. An investor will only accept an offer if it allows the investor to at least break even. That is, the payoff to the investor after incurring the renovation cost, $\frac{X}{1+r}$, is (weakly) higher than the price $b(r)$ the investor pays for the property.

$$\frac{X}{1+r} \geq b(r) \tag{2.2}$$

[6] The bounds for the parameter $\bar{a}$ is imposed to guarantee interior solutions in all the relevant sections, where $\underline{a} =$ and $\bar{a} \equiv \frac{\theta_L}{1-\theta_L} \frac{\gamma(1-\delta)\theta_H + (1-\gamma)(\theta_H - \delta\theta_L)}{\gamma(1-\delta)(\theta_H - \delta\theta_H) + (1-\gamma)(\theta_H - \delta\theta_L)}$.

In equilibrium, the securitiser sells to the investors with the highest valuation. In this situation, there exists a threshold $\hat{r}(\lambda_i)$ in equilibrium such that the securitiser makes the following offer $\hat{b}(r)$. We will later solve for the threshold $\hat{r}(\lambda_i)$ by market clearing.

$$\hat{b}(r) = \frac{X}{1+r}, \quad \forall\, r \leq \hat{r}(\lambda_i) \tag{2.3}$$

In equilibrium, the strategies of the investors thus depends on their cost r. All investors with $r \leq \hat{r}(\lambda_i)$ will wait until $t = 2$. They accept the offer $b(r)$ made by the securitiser and purchase measure $\frac{1+r}{X}$ of the distressed properties, if the offer $b(r)$ allows the investor to break even, i.e. satisfies Eq 2.2. All the investors with $r > \hat{r}(\lambda_i)$ compete in the $t = 1$ MBS market.

Finally, the equilibrium threshold $\hat{r}(\lambda_i)$ is given by the clearing condition that the total demand for the properties is equal to the supply,

$$\int_{R=1}^{1+\hat{r}(\lambda_i)} \frac{1}{\hat{b}(r)} dI(R) = \int_{R=1}^{1+\hat{r}(\lambda_i)} \frac{R}{X} dI(R) = \lambda_i \tag{2.4}$$

Using the function form given by Eq. 2.1, the equilibrium threshold $\hat{r}(\lambda_i)$ and liquidation proceed $\mathcal{L}_i(\lambda_i)$ is

$$\hat{r}(\lambda_i) = \frac{\lambda_i}{a} \tag{2.5}$$

$$\mathcal{L}_i(\lambda_i) = aX \ln\left(1 + \frac{\lambda_i}{a}\right) \tag{2.6}$$

The liquidation proceeds are increasing and concave in the foreclosure policy of the mortgage pool. That is,

$$\frac{\partial\, \mathcal{L}_i(\lambda_i)}{\partial \lambda_i} = \frac{a^2 X}{\lambda_i + a} > 0 \tag{2.7}$$

$$\frac{\partial^2\, \mathcal{L}_i(\lambda_i)}{\partial \lambda_i^2} = -\frac{a^2 X}{(\lambda_i + a)^2} < 0 \tag{2.8}$$

First best securitisation and foreclosure

At $t = 1$, a securitiser of type i chooses a security (F_i, f_i) that correspond to the payoffs to outside investors when the delinquent mortgages resume repayments or not respectively.

The securitiser's expected payoff at $t = 1$ is comprised of two parts (Eq. 2.9).

$$p(F_i, f_i) + \delta \left(\theta_i \left[V + \mathcal{L}_i(\lambda_i) + (1 - \lambda_i)X - F_i\right] + (1 - \theta_i) \left[V + \mathcal{L}_i(\lambda_i) - f_i\right]\right) \tag{2.9}$$

The first is the proceeds from issuing an MBS at $t = 1$ backed by the mortgage pool, and the second is the residual cash flow from the mortgage pool at $t = 2$. The proceeds from security issuance is given by the market clearing condition (MC) under full information,

$$(MC) \quad p(F_i, f_i) = \theta_i F_i + (1 - \theta_i) f_i \tag{2.10}$$

We can also rewrite Eq. 2.9 as Eq. 2.11 below. This offers an alternative interpretation comprising of a first part that represents the saving of retention cost due on proceeds $p(F_i, f_i)$ from the security issuance, and a second part that is the intrinsic value of the mortgage pool's cash flows to the securitiser (Eq. 2.11).

$$(1 - \delta)p(F_i, f_i) + \delta \left[V + \mathcal{L}_i(\lambda_i) + (1 - \lambda_i)\theta_i X\right] \tag{2.11}$$

In the first best benchmark, the securitiser chooses the security to maximise its expected payoff subject to the limited liability constraints (LL) on the MBS (Eq. 2.13–2.14) and the market clearing constraint (MC) under full information (Eq. 2.10). From Eq. 2.11 it is clear that the securitisation process does not alter the intrinsic payoff of the mortgage pool, the securitiser simply maximises her proceeds from the MBS issue.

$$(F_i^{FB}, f_i^{FB}) \equiv \arg \max_{(F_i, f_i)} p(F_i, f_i) \tag{2.12}$$
$$s.t. \quad (MC) \quad \text{and}$$
$$(LL) \quad F_i \leq V + \mathcal{L}_i(\lambda_i) + (1 - \lambda_i)X \tag{2.13}$$
$$f_i \leq V + \mathcal{L}_i(\lambda_i) \tag{2.14}$$

Since any security issued would be priced correctly under full information, the securitiser chooses to issue a security backed by the entire cash flow of the mortgage pool to minimise her retention cost. The payoff to a securitiser of type i under full information is $V + \mathcal{L}_i(\lambda_i) + (1 - \lambda_i)\theta_i X$.

We can now formulate the first best foreclosure policy. Anticipating the securitisation process, at $t = 0$ the securitiser chooses a foreclosure policy $(\lambda_H^{FB}, \lambda_L^{FB})$ to commit to, in order to maximise its expected payoff.

$$\begin{aligned}(\lambda_H^{FB}, \lambda_L^{FB}) \equiv \arg \max_{(\lambda_H, \lambda_L)} & \gamma[V + \mathcal{L}_H(\lambda_H) + (1 - \lambda_H)\theta_H X] \\ & +(1-\gamma)[V + \mathcal{L}_L(\lambda_L) + (1 - \lambda_L)\theta_L X] \end{aligned} \tag{2.15}$$

The solutions are characterised by the first order conditions (FOC) because the second order conditions are satisfied. That is, at the first best level of foreclosure, the marginal gain from the property sale in the market is equalised to the expected value of mortgage forbearance (henceforth the forbearance value).

$$(FOC^{FB}): \quad \frac{\partial \mathcal{L}_i(\lambda_i^{FB})}{\partial \lambda_i} - \theta_i X = 0 \qquad \forall\, i \in \{H, L\} \tag{2.16}$$

The following proposition then summarises the first best benchmark results.

Proposition 7. *In the full information equilibrium, the securitiser commits to a foreclosure policy* $(\lambda_H^{FB}, \lambda_L^{FB})$ *at* $t = 0$*, where*

$$\lambda_i^{FB} = \frac{1 - \theta_i}{\theta_i} a \qquad \forall\, i \in \{H, L\} \tag{2.17}$$

She then securitises all of its mortgage pool cash flow at $t = 1$*, which is fairly priced in the market. At* $t = 2$ *the securitiser forecloses fraction* λ_i *if she is of type* i*, and obtains liquidation proceeds* $\mathcal{L}_i^{FB} \equiv \mathcal{L}_i(\lambda_i^{FB}) = aX \ln\left(\frac{1}{\theta_i}\right)$ *from selling the distressed properties in the market.*

In the full information equilibrium, a high type securitiser forecloses a smaller fraction of delinquent mortgages and obtains less liquidation proceed than a low type, $\lambda_H^{FB} < \lambda_L^{FB}$ and $\mathcal{L}_H^{FB} < \mathcal{L}_L^{FB}$. This is because the good type has a higher forbearance value and is therefore less inclined towards foreclosure.

2.3.2 Foreclosure policy under asymmetric information

We now solve for the optimal foreclosure policy given that securitisation occurs under asymmetric information, following a similar backward induction procedure. Notice that given a foreclosure policy, the liquidation proceeds from distressed property sales at $t = 2$ are the same as before. This section therefore focuses on the optimal security to be issued, and presents the equilibrium foreclosure policy.

Securitisation with signalling

At $t = 1$, the securitiser with private information θ_i designs and issues an MBS security backed by the cash flow of the mortgage pool. In this section we restrict our attention to only consider the least cost separating equilibrium.

First, notice that in a separating equilibrium, the low type securitiser always receives the fair price on the security she issues. Therefore she maximises her payoff by selling the entire cash flow from the mortgage pool to outside investors. There is no distortion in the form of inefficient retention for the low type. Given the pre-committed foreclosure policy, denote $U_i(\lambda_i)$ as the equilibrium payoff to a securitiser of type i. Therefore the equilibrium payoff to the low type securitiser is

$$U_L(\lambda_L) = V + \mathcal{L}_L(\lambda_L) + (1 - \lambda_L)\theta_L X \tag{2.18}$$

Next, we solve for the equilibrium security of the high type securitiser in the least cost separating equilibrium. Consider a general security that specifies a set of payoffs $\mathcal{F} \equiv (F_H, f_H, F_L, f_L)$ for each of the two possible cash flow realisations of the mortgage pool respectively for each type of the securitiser. Table 2.1 details the mapping from the realisation of the cash flow to the payoff of the security to the investors.

Specifically, F_i is the payoff of the security if the cash flow of a type i securitiser realises with the delinquent mortgages recovered, and f_i is the payoff of the security if the cash flow of a type i securitiser realises without any recovery. This is because the final cash flow of the mortgage pool will reveal the true type of the securitiser. We restrict our attention to only monotonic security payoffs. That is, a higher realisation of the mortgage pool cash flow should leave both the outside investors and the securitiser a (weakly) higher payoff.[7] In the least-cost separating equilibrium, the optimal security for the high type securitiser

[7] Although this implies some loss of generality, it is not uncommon in the security design literature, e.g.

Table 2.1: Payoffs of the security issued by the high type

Type	Realisation of cash flow	Security payoff $\mathcal{F}$
High	$V + \mathcal{L}_H(\lambda_H) + (1 - \lambda_H)X$	F_H
	$V + \mathcal{L}_H(\lambda_H)$	f_H
low	$V + \mathcal{L}_L(\lambda_L) + (1 - \lambda_L)X$	F_L
	$V + \mathcal{L}_L(\lambda_L)$	f_L

is given by

$$\hat{\mathcal{F}} = \arg \max_{(F_H, f_H, F_L, f_L)} p(\mathcal{F}) \tag{2.19}$$

$$s.t. \quad (MC) \quad p(\mathcal{F}) = \theta_H F_H + (1 - \theta_H) f_H \tag{2.20}$$

$$(LL) \quad \forall\, i \in \{H, L\} \quad \text{and} \tag{2.21}$$

$$\begin{aligned} (IC) \quad U_L(\lambda_L) \geq\ & p(\mathcal{F}) + \delta\theta_L \left[V + \mathcal{L}_L(\lambda_L) + (1 - \lambda_L)X - F_L\right] \\ & + \delta(1 - \theta_L) \left[V + \mathcal{L}_L(\lambda_L) - f_L\right] \end{aligned} \tag{2.22}$$

where Eq. 2.20 is the market clearing condition (MC) when the market believes that the issuer of the security $\mathcal{F}$ is of the high type, and Eq. 2.22 is the incentive compatibility constraint (IC) for the low type to not mimic the security issued by the high type.

Since the monotonicity of the security payoffs depends on the ranking of the cash flow realisations, which depends on the foreclosure decisions of the securitisers. This significantly complicates the analysis as the foreclosure decisions are endogenously determined in equilibrium. For the rest of the paper, we present the results for the relevant case where

Innes (1990) and Nachman and Noe (1994). One potential justification provided by DeMarzo and Duffie (1999) is that, the issuer has the incentive to contribute additional funds to the assets if the security payoff is not increasing in the cash flow. Similarly, the issuers has the incentive to abscond from the mortgage pool if the security leaves the issuer a payoff that is not increasing in the cash flow. If such actions cannot be observed, the monotonicity assumption is without loss of generality.

(λ_H, λ_L) are such that

$$\mathcal{L}_H(\lambda_H) \leq \mathcal{L}_L(\lambda_L) \quad \text{and} \tag{2.23}$$

$$\mathcal{L}_H(\lambda_H) + (1-\lambda_H)X \geq \mathcal{L}_L(\lambda_L) + (1-\lambda_L)X \tag{2.24}$$

It will become clear in Section 2.3.2 that this scenario indeed arises in equilibrium, and we show in Appendix that this is the only equilibrium outcome.

The following proposition summarises the optimal securities in this assuming that Eq. 2.23–2.24 holds.

Proposition 8. *In the least-cost separating equilibrium, the optimal security issued by the low type securitiser is all the equity, whereas that issued by the high type securitiser is a debt with face vaue $\hat{F}(\lambda_H, \lambda_L)$, where*

$$\hat{F}(\lambda_H, \lambda_L) = \begin{cases} V + \frac{(1-\delta\theta_L)\mathcal{L}_L(\lambda_L) + (1-\delta)(1-\lambda_L)\theta_L X - (1-\theta_H)\mathcal{L}_H(\lambda_H)}{\theta_H - \delta\theta_L}, \\ \qquad\qquad \text{if } \frac{1-\theta_H}{\theta_H-\theta_L}[\mathcal{L}_L(\lambda_L) - \mathcal{L}_H(\lambda_H)] \leq (1-\lambda_L)X \\ V + \frac{\mathcal{L}_L(\lambda_L) + (1-\lambda_L)\theta_L X - (1-\theta_H)\mathcal{L}_H(\lambda_H)}{\theta_H}, \qquad \text{otherwise} \end{cases} \tag{2.25}$$

Proof: *See Appendix.*

The result presented in Proposition 8 is two-fold. First, the high type issues a debt security to the outside investors and retains the residual cash flow. The retained cash flow incurs a deadweight loss of $(1-\delta)$. Such costly retention of the mortgage pool cash flow allows the high type securitiser to signal her type and receive a fair market price for the security it issues. This result is in line with DeMarzo and Duffie (1999). Second, in the presence of asymmetric information, the optimal securities issued by the securitiser are debt contracts, with equity being a special case of extremely high face value. This is because debt contract minimises the information sensitivity, a well established intuition in e.g. Myers and Majluf (1984).

The two cases presented in Proposition 8 correspond to when $\hat{F}$ is greater than or smaller than the low type's good realisation of cash flows, $V + \mathcal{L}_L(\lambda_L) + (1-\lambda_L)X$, respectively. That is, the cases correspond to whether the low type would have to default even when the delinquent mortgages resume payments, should she mimic the high type. In the first case, information asymmetry measured by $\frac{1-\theta_H}{\theta_H-\theta_L}$ is large. The high type issues a debt with low face value in order to separate from the low type, because a high face value increases the market price of the security, increasing the incentive for the low type to mimic. This, however, incurs a high retention cost on the high type. In the second case, information asymmetry is less severe, and the high type can separate at a relatively high face value of debt with minimal retention cost. In what follows we assume that information asymmetry is so severe that the first case is true. This allows the asymmetric information to have a material effect and generate interesting implications for the optimal foreclosure policy.

In this case, the high type securitiser enjoys a total payoff of $U_H(\cdot)$ (Eq. 2.26) in equilibrium which is comprised of two parts – the saving of retention cost on the proceeds $p(\hat{F}(\cdot), \hat{f}(\cdot))$ from the security issuance, and the intrinsic value of the mortgage pool's cash flows to the securitiser.

$$U_H(\lambda_H, \lambda_L) = (1-\delta)\hat{p}(\lambda_H, \lambda_L) + \delta\left[V + \mathcal{L}_H(\lambda_H) + (1-\lambda_H)\theta_H X\right] \tag{2.26}$$

$$\begin{aligned}
\text{where} \qquad \hat{p}(\lambda_H, \lambda_L) &\equiv p(\hat{F}_H(\lambda_H, \lambda_L), \hat{f}_H(\lambda_H, \lambda_L)) && (2.27)\\
\hat{F}_H(\lambda_H, \lambda_L) &= \hat{F}(\lambda_H, \lambda_L) && (2.28)\\
\hat{f}_H(\lambda_H, \lambda_L) &= V + \mathcal{L}_H(\lambda_H) && (2.29)
\end{aligned}$$

Optimal ex-ante foreclosure policy

We can now solve for the ex-ante optimal foreclosure policy, given the securitisation game at $t = 1$. The securitiser chooses a foreclosure policy $(\hat{\lambda}_H, \hat{\lambda}_L)$ at $t = 0$ prior to the realisation of her private information, to maximise her expected payoff.

$$(\hat{\lambda}_H, \hat{\lambda}_L) \equiv \arg\max_{(\lambda_H, \lambda_L)} \quad \gamma U_H(\lambda_H, \lambda_L) + (1-\gamma)U_L(\lambda_L) \tag{2.30}$$

The first order conditions that characterise the solutions are

$$(FOC_H): \quad \gamma\left((1-\delta)\frac{\partial \hat{p}(\hat{\lambda}_H, \hat{\lambda}_L)}{\partial \lambda_H} + \delta\left[\frac{\partial \mathcal{L}_H(\hat{\lambda}_H)}{\partial \lambda_H} - \theta_H X\right]\right) = 0 \tag{2.31}$$

$$(FOC_L): \quad \gamma(1-\delta)\frac{\partial \hat{p}(\hat{\lambda}_H, \hat{\lambda}_L)}{\partial \lambda_L} + (1-\gamma)\left[\frac{\partial \mathcal{L}_L(\hat{\lambda}_L)}{\partial \lambda_L} - \theta_L X\right] = 0 \tag{2.32}$$

(FOC_H) is comprised of two components. The first part is the total impact of a change in the foreclosure policy of the high type issuer on her proceeds from security issuance $\frac{\partial \hat{p}(\hat{\lambda}_H, \hat{\lambda}_L)}{\partial \lambda_H}$, and the second is the impact on the total value of the mortgage pool of the high type. (FOC_L) comprises of two components too. The first part is the impact of a change in the foreclosure policy of the low type issuer on the proceeds from security issuance by the high type, $\frac{\partial \hat{p}(\hat{\lambda}_H, \hat{\lambda}_L)}{\partial \lambda_L}$, and the second is the impact on the total value of the mortgage pool of the low type. The first component comes from the fact that, when the foreclosure policy is chosen *ex ante*, the securitiser takes into account the effect of the low type's foreclosure policy on her signalling cost if she is of the high type. The second component encompasses the effect on the low type's proceeds from security issuance since the low type securitises all of her cash flow from the mortgage pool in equilibrium.

Proposition 9. *The optimal foreclosure policy* $(\hat{\lambda}_H, \hat{\lambda}_L)$ *under asymmetric information is more countercyclical than under full information. The equilibrium property prices in equilibrium under asymmetric information are more procyclical.*

$$\hat{\lambda}_H < \lambda_H^{FB} < \lambda_L^{FB} < \hat{\lambda}_L, \tag{2.33}$$

$$\hat{b}(\hat{r}(\hat{\lambda}_H) > \hat{b}(\hat{r}(\lambda_H^{FB}) > \hat{b}(\hat{r}(\hat{\lambda}_L) > \hat{b}(\hat{r}(\lambda_H^{FB}) \tag{2.34}$$

That is, there is insufficient foreclosure if the mortgage pool is of high quality, and excessive foreclosure if the mortgage pool is of low quality.

Proof. We express the high type issuer's proceeds and the total impacts of the foreclosure policy on the high type issuer's proceeds as follows.

$$\hat{p}(\lambda_H, \lambda_L) = \theta_H \hat{F}(\lambda_H, \lambda_L) + (1-\theta_H)\left[V + \mathcal{L}(\lambda_H)\right] \tag{2.35}$$

$$= V + \frac{\theta_H(1-\delta\theta_L)}{\theta_H - \delta\theta_L}\mathcal{L}_L(\lambda_L) + \frac{\theta_H - \delta\theta_H}{\theta_H - \delta\theta_L}(1-\lambda_L)\theta_L X - \frac{\delta\theta_L(1-\theta_H)}{\theta_H - \delta\theta_L}\mathcal{L}_H(\lambda_H) \tag{2.36}$$

$$\frac{\partial \hat{p}(\hat{\lambda}_H, \hat{\lambda}_L)}{\partial \lambda_H} = -\frac{\delta\theta_L(1-\theta_H)}{\theta_H - \delta\theta_L}\frac{\partial \mathcal{L}_H(\hat{\lambda}_H)}{\partial \lambda_H} \tag{2.37}$$

$$\frac{\partial \hat{p}(\hat{\lambda}_H, \hat{\lambda}_L)}{\partial \lambda_L} = \frac{\delta\theta_H(1-\theta_L)}{\theta_H - \delta\theta_L}\frac{\partial \mathcal{L}_L(\hat{\lambda}_L)}{\partial \lambda_L} + \frac{\theta_H(1-\delta)}{\theta_H - \delta\theta_L}\left[\frac{\partial \mathcal{L}_L(\hat{\lambda}_L)}{\partial \lambda_L} - \theta_L X\right] \tag{2.38}$$

It is thus apparent that at the first best level of foreclosure $(\lambda_H^{FB}, \lambda_L^{FB})$, the left hand side of the (FOC_H) (Eq. 2.31) is strictly negative and that of the (FOC_L) (Eq. 2.32) is strictly positive. Therefore the equilibrium is such that there is insufficient foreclosure in the high quality mortgage pool, i.e. the marginal value of foreclosure is greater than the forbearance value of the mortgage $\theta_H X$, and there is excessive foreclosure in the low quality mortgage pool, i.e. the marginal value of foreclosure is lower than the forbearance value of the mortgage $\theta_L X$.

The second part of the proposition regarding property prices follows immediately from the fact that $\hat{b}(\hat{r}(\lambda)) = \frac{aX}{\lambda+1}$ in equilibrium. □

The distortion in the equilibrium foreclosure policy is driven by the signalling concern of the issuer under asymmetric information. Given the equilibrium payoff to the low type issuer, consider the payoff to her is she mimics the high type issuer and issues a debt security. This mimicking payoff is comprised of two parts – the cash proceeds she gets from the security issuance, and the value of the retained cash flow. For a given security issued by the high type, when the high type issuer chooses a less than first best level of foreclosure, the payoff to the debt holders is reduced and hence the value of the security, decreasing the incentive for the low type to mimic. On the other hand, when the low type issuer chooses a higher than first best level of foreclosure, the value of the levered equity she retains decreases, again reducing her incentive to mimic.

The above intuition can be confirmed by the fact that Eq. 2.23 and 2.24 are implied by Proposition 9. This is because the high type is intrinsically riskier than the low type. The results imply that the cash flow from the mortgage pool of the low type is safer in equilibrium than in the first best scenario, and that of the high type is riskier. Therefore in equilibrium the two types become similar, mitigating the asymmetric information problem.

It is also worth noting that the equilibrium foreclosure policy is not time-consistent. Specifically, consider a high type issuer at $t = 2$. Having issued a debt security, the securitiser retains a levered equity stake. This gives her an incentive to prefer the risky cash flow, i.e. that from forbearance, to a safe cash flow, i.e. that from distressed property sales. For a given face value F, the problem of the high type issuer at $t = 2$ is

$$\max_{\lambda_H} \qquad \theta \left[V + \mathcal{L}_H(\lambda_H) + (1 - \lambda_H)X - F \right] \tag{2.39}$$

The solution to the above problem is given by $\frac{\partial \mathcal{L}_H(\hat{\lambda}_H)}{\partial \lambda_H} = X$. Using the functional form of Eq. 2.1, this implies zero foreclosure in the high quality mortgage pool if the foreclosure decision was made *ex post* at $t = 2$. This is even lower that the equilibrium foreclosure policy $\hat{\lambda}_H = \frac{\theta_H - \theta_L}{\theta_H - \delta\theta_L} \frac{1-\theta_H}{\theta_H} a$.

2.4 Servicer and the optimal foreclosure policy

Having established the importance of commitment power to a set of foreclosure policies *ex ante* in the previous section, we now propose a mechanism that resembles the real world and involves a mortgage servicer to implement the desired commitment power.

We now assume that at $t = 0$, the loan originator ("he" henceforth) has the pool of mortgages. The loan originator is risk neutral and is subject to the same liquidity constraint δ as the securitiser. Neither the originator nor the securitiser has information about the quality of the mortgage pool until $t = 1$. At $t = 0$, the loan originator is approached by the securitiser intending to acquire the beneficial rights to the mortgage cash flows. After the sale, the initial originator does not retain any claim to the cash flows, but remains the servicer of the mortgages for a fee to be paid by the securitiser.[8] The servicer performs duties including collecting the payments, forwarding the interest and principle to the lender(s), and negotiating new terms if the debt is not being paid back, or supervising the foreclosure process.

This mechanism rests on the fact that the decision of forbearance versus foreclosure is made by the servicer as opposed to the securitiser. Therefore the institutional distinction between the originator and the securitiser does not play a role. In some cases mortgage servicing is done in-house, meaning that an institution is both the lender and the administrator of the loan. In this section we consider also this case when the originator decides whether or not to sell the mortgage pool to the securitiser. If he does not, he is free to securitiser the loan himself, with the servicing done in-house.

Because the incentives required to implement the optimal foreclosure policy can be different depending on the quality of the mortgage pool, the securitiser offers a menu of compensation contracts to the servicer. For simplicity, assume that all cash flows from the mortgage pool are passed on to the securitiser, and that the securitiser has sufficient funds to pay the fees specified by the contract.

The rest of the game is played in a similar way as before. To summarise, at $t = 0$, the originator is offered a menu of contracts by the securitiser to acquire the mortgage pool. If the originator declines the offer, he is free to continue securitising the mortgage pool at $t = 1$ and becomes an originator-securitiser with in-house servicing. If the originator accepts the offer, he remains as a third-party servicer and the menu of contracts is verifiable. At $t = 1$, the securitiser with private information regarding the quality of the mortgage pool designs a security and sells it to the investors. The originator with private information chooses a contract from the menu and agrees to receive compensation according to the chosen contract. At $t = 2$, the servicer makes the foreclosure decision given the incentives provided by his compensation contract.

[8] Practically, the servicer need not be the originator but they are often the party as the skill set required to perform both functions are similar. There is, however, a secondary market for the transfer of servicing rights through a security called Mortgage Servicing Rights (MSR).

In this section, we first solve for the equilibrium payoff to the originator if he securitises the mortgage pool with in-house servicing. We then solve for the incentive contracts that induce the originator to sell the mortgage pool and implement the optimal foreclosure policy. This finally allows us to comment on the implications of the separation of servicing on securitisation.

2.4.1 Securitisation with in-house servicing

At $t = 2$, the originator-securitiser makes the foreclosure decision to maximise his retained cash flow given his type i and the security issued (F_i, f_i) at $t = 1$,[9]

$$\lambda_i^o \equiv \arg\max_{\lambda_i} \theta_i \left[V + \mathcal{L}_i(\lambda_i) + (1-\lambda_i)X - F_i\right] + (1-\theta_i)\left[V + \mathcal{L}_i(\lambda_i) - f_i\right] \quad (2.40)$$

The foreclosure decision therefore depends on the riskiness of the retained cash flow. If the entire cash flow has been sold to the investors, i.e. $F_i = V + \mathcal{L}_i(\lambda_i) + (1-\lambda_i)X$ and $f_i = V + \mathcal{L}_i(\lambda_i)$, assume that there is no conflict of interest and that the first best foreclosure decision λ_i^{FB} is made. For $F_i < V + \mathcal{L}_i(\lambda_i) + (1-\lambda_i)X$, the originator-securitiser chooses the first best level of foreclosure λ_i^{FB} if he retains some cash flow in the downside, i.e. $f_i < V + \mathcal{L}_i(\lambda_i^{FB})$. Otherwise, he chooses zero foreclosure due to the risk-shifting incentive induced by risky retained cash flow, as shown in the last part of Section 2.3.2.

We now turn to the security design problem at $t = 1$, anticipating the consequential foreclosure decisions. The following proposition characterises the optimal security and the equilibrium foreclosure.

Proposition 10. *With in-house servicing, the security issued by the originator-securitiser in the least-cost separating equilibrium is a standard debt with face value* $F^o \equiv \hat{F}(0, \lambda_L^{FB})$ *if he is of the high type, and an equity contract if he is of the low type. The ex post chosen foreclosure policy is* $(\lambda_H^o, \lambda_L^o) = (0, \lambda_L^{FB})$. *That is, there is excessive forbearance if the mortgage pool is of high quality.*

[9] We maintain the assumption that the security issued must satisfy the monotonicity assumption.

Proof. For the low type issuer, he should optimally securitise the entire cash flow and choose the first best foreclosure policy. This is the first best outcome. For the high type issuer, however, he faces the problem as given by Equation 2.19–2.22.

The proof of the high type's security consists of two parts. First we show that the above securities are indeed the optimal security, if the securities issued are such that the equilibrium foreclosure policy is $(\lambda_H^o, \lambda_L^o) = (0, \lambda_L^{FB})$. Given the equilibrium foreclosure policy, the cash flows of the two types satisfy the conditions in Eq. 2.23 and 2.24. That is,

$$\mathcal{L}_H(0) = 0 \quad < \quad \mathcal{L}_L(\lambda_L^{FB}) = aX \ln\left(\frac{1}{\theta_L}\right) \quad \text{and} \tag{2.41}$$

$$\begin{aligned} \mathcal{L}_H(0) + (1-0)X = X \quad > \quad & \mathcal{L}_L(\lambda_L^{FB}) + (1-\lambda_L^{FB})X \\ & = X - aX\left[\frac{1-\theta_L}{\theta_L} - \ln\left(\frac{1}{\theta_L}\right)\right] \end{aligned} \tag{2.42}$$

Therefore Proposition 8 applies and the equilibrium security is as described above. As the high type issues a risky debt, it indeed chooses zero foreclosure *ex post*. The low type issues equity, it then chooses the first best level of foreclosure.

Secondly we show by contradiction that there does not exist an equilibrium in which another foreclosure policy is chosen. Suppose there is an equilibrium in which the high type originator-securitiser chooses the first best foreclosure level. The resulting cash flows of the mortgage pool still satisfy Eq. 2.23 and 2.24 and the high type would issue a risky debt contract. However, given an outstanding risk debt, the originator-securitiser would not choose the first best level of foreclosure. □

Therefore, if the originator does not sell the mortgage pool, his payoff ω_0 from the subsequent equilibrium is given by

$$\omega_H^o \equiv \gamma U_H(0, \lambda_L^{FB}) \tag{2.43}$$

$$\omega_L^o \equiv (1-\gamma)U_L(\lambda_L^{FB}) \tag{2.44}$$

The expected payoff $\omega^o \equiv \gamma\omega_H^o + (1-\gamma)\omega_L^o$ is lower than the expected payoff obtained if the securitiser can commit to the optimal foreclosure policy with commitment $(\hat{\lambda}_H, \hat{\lambda}_L)$. This therefore creates the incentive to trade between the originator and a securitiser, if the securitiser has the commitment power.

2.4.2 Mortgage servicing contract

We now consider the securitiser's problem at $t = 0$. She would like to acquire the mortgage pool from the originator and provide the originator-servicer with incentive to implement the optimal foreclosure policy $(\hat{\lambda}_H, \hat{\lambda}_L)$ at $t = 2$.

Conjecture an affine contract $(\alpha, \beta, \tau) \in \mathbb{R}^{3+}$ that includes a percentage α of the forbearance cash flow to be paid at $t = 3$, a percentage $\alpha\beta$ of the foreclosure cash flow to be paid at $t = 3$ and a flat transfer τ to be paid at $t = 1$, if the mortgage is of high quality. Similarly for the tie break convention, assume that the originator prefers to accept an offer if he is indifferent between accepting or not. Given a contract, the originator's expected payoff $\omega(\alpha, \beta, \tau)$ given his private information θ is given by

$$\omega_i(\alpha, \beta, \tau) \equiv \max_{\lambda} \tau + \delta\alpha \left[\beta \mathcal{L}_i(\lambda) + (1-\lambda)\theta X\right] \tag{2.45}$$

By construction, the choice of foreclosure policy only depends on β. Specifically, the mortgage servicer chooses λ according to the following first-order condition

$$\beta \frac{\partial \mathcal{L}_i(\lambda)}{\partial \lambda} - \theta_i X = 0 \tag{2.46}$$

A comparison between Eq. 2.46 and (FOC_H) and (FOC_L) (Eq. 2.31 and 2.32) which characterise the optimal foreclosure policy suggests that the securitiser must offer different contracts (β_i) to the servicer depending on the type of the mortgage pool in order to implement $(\hat{\lambda}_H, \hat{\lambda}_L)$ respectively. Specifically, the contract to a servicer with type i mortgage pool must be such

$$\hat{\beta}_H = 1 - (1-\delta)\frac{\theta_L(1-\theta_H)}{\theta_H - \delta\theta_L} \tag{2.47}$$

$$\hat{\beta}_L = \frac{\gamma(1-\delta)\theta_H(1-\delta\theta_L) + (1-\gamma)(\theta_H - \delta\theta_L)}{\gamma(1-\delta)\theta_H(1-\theta_L) + (1-\gamma)(\theta_H - \delta\theta_L)} \tag{2.48}$$

The required incentive contracts are such that $\hat{\beta}_H < 1 < \hat{\beta}_L$. This is because $\beta = 1$ should implement the first best level of foreclosure. Therefore the compensation to the servicer must lean towards forbearance if the mortgage pool is of high quality, and towards foreclosure if the mortgage pool is of low quality, in order to implement the optimal foreclosure policy $(\hat{\lambda}_H, \hat{\lambda}_L)$. Contracts that satisfy Eq. 2.47 and 2.48 implement the optimal foreclosure policy regardless of the specific functional form of $I(R)$.

Since the type of the mortgage pool is not contractible, however, the contracts cannot be type-contingent. Instead, at $t = 0$ when both parties are uninformed, the securitiser can offer a menu of incentive-compatible contracts $\{(\alpha_i, \beta_i, \tau_i)\}_{i\in\{H,L\}}$ to the servicer, who chooses a contract from the menu according to the type of his mortgage pool at $t = 1$. Therefore, the securitiser's problem is to design this menu of contracts to implement a set of foreclosure policies that maximises her expected payoff from the mortgage pool less the fees paid to the servicer. Formally, her maximisation problem is

$$\max_{\{(\alpha_i,\beta_i,\tau_i)\}_{i\in\{H,L\}}} \quad \gamma\left[U_H(\lambda_H, \lambda_L) - \omega_H(\alpha_H, \beta_H, \tau_H)\right] + (1-\gamma)\left[U_L(\lambda_L) - \omega_L(\alpha_L, \beta_L, \tau_L)\right] \tag{2.49}$$

$$\text{s.t.} \quad (PC) \; : \; \gamma\omega_H(\alpha_H, \beta_H, \tau_H) + (1-\gamma)\omega_L(\alpha_L, \beta_L, \tau_L) \geq \omega^o \tag{2.50}$$

$$(IC_H) \; : \; \omega_H(\alpha_H, \beta_H, \tau_H) \geq \omega_H(\alpha_L, \beta_L, \tau_L) \tag{2.51}$$

$$(IC_L) \; : \; \omega_L(\alpha_L, \beta_L, \tau_L) \geq \omega_L(\alpha_H, \beta_H, \tau_H) \tag{2.52}$$

$$(IC_\lambda) \; : \; \lambda_i(\beta_j) \equiv \arg\max_{\lambda} \; [\beta_j \, \mathcal{L}_i(\lambda) + (1-\lambda)\theta_i X] \tag{2.53}$$

where (PC) is the participation constraint for the servicer to prefer loan sales to securitisation with in-house servicing at $t = 0$, (IC_i) are the incentive compatibility constraints for a type i servicer to choose the corresponding contract at $t = 1$, and (IC_λ) is the *ex post* incentive compatibility constraint for a type i servicer to choose the foreclosure policy at $t = 2$, after he has picked the contract j at $t = 1$. Thus his potential deviation at $t = 1$ is considered and the *ex post* foreclosure choice $\lambda_i(\beta_j)$ is implicitly embedded in $\omega_i(\alpha_j, \beta_j, \tau_j)$.

Proposition 11. *At $t = 0$ the securitiser offers an optimal menu of contracts $\{(\hat{\alpha}_i, \hat{\beta}_i, \hat{\tau}_i)\}_{i\in\{H,L\}}$ to the servicer who at $t = 1$ implements the optimal foreclosure policy $(\hat{\lambda}_H, \hat{\lambda}_L)$.*

Proof. We rewrite the servicer's payoff as follows

$$\omega_i(\alpha_i, \beta_i, \tau_i) \;=\; \tau + \delta\alpha_i K(\theta_i, \beta_i) \tag{2.54}$$

$$\text{where} \quad K(\theta_i, \beta_i) \equiv max_\lambda \; [\beta_i \, \mathcal{L}_i(\lambda) + (1-\lambda)\theta_i X] \tag{2.55}$$

The sufficient condition for the securitiser to prefer to implement the optimal foreclosure policy $(\hat{\lambda}_H, \hat{\lambda}_L)$ in equilibrium, which offers her the maximum expected value from the mortgage pool, is for her to pay the minimum fees to the servicer. That is, (PC) binds.

In particular, consider contracts with $\hat{\beta}_i$ and $\hat{\tau}_i$ such that

$$\hat{\tau}_H = \bar{\omega} - \delta\alpha_H K(\theta_H, \hat{\beta}_H) \tag{2.56}$$
$$\hat{\tau}_L = \frac{\omega^o - \gamma\bar{\omega}}{1-\gamma} - \delta\alpha_L K(\theta_L, \hat{\beta}_L) \tag{2.57}$$

for some $\bar{\omega} > \omega^o$. That is, the servicer receives $\bar{\omega}$ if the mortgage pool is of high quality and $\frac{\omega^o - \gamma\bar{\omega}}{1-\gamma} < \bar{\omega}$ otherwise. As shown in (2.47) and (2.48), $\{\hat{\beta}_H, \hat{\beta}_L\}$ implements $(\hat{\lambda}_H, \hat{\lambda}_L)$ while by construction $\{\hat{\tau}_H, \hat{\tau}_L\}$ binds the (PC). Finally we need to choose $\{\alpha_H, \alpha_L\}$ to ensure (IC_H) and (IC_L) satisfied. That is,

$$\begin{aligned}\delta\alpha_H \left[K(\theta_H, \hat{\beta}_H) - K(\theta_L, \hat{\beta}_H)\right] &\geq \frac{\bar{\omega} - \omega^o}{1-\gamma} \\ &\geq \delta\alpha_L \left[K(\theta_H, \hat{\beta}_L) - K(\theta_L, \hat{\beta}_L)\right] \geq 0\end{aligned} \tag{2.58}$$

By the Envelope Theorem, it is straight forward that $K(\theta_i, \beta_i)$ is increasing in θ_i. Therefore there exists $\hat{\alpha}_H$ and $\hat{\alpha}_L$ that satisfy the above inequalities. □

2.4.3 In-house versus third-party mortgage servicing

Third-party mortgage servicers frequently come under public criticism for the foreclosure crisis because of their apparent recklessness in foreclosing mortgages. Levitin and Twomey (2011) asserts that the services' compensation structures create a principal-agent conflict and they do not make the decision whether to foreclose or modify a loan based to maximise the net present value of the loan. Indeed, Credit Suisse reports a loss severity rate of 55% on securitised subprime mortgages in the six months ending in May 2008.[10]

We would like to point out with this model that there need not be an agency conflict in equilibrium. The separation of servicing from securitisation allows the securitiser to

[10]Source: Cordell et al. (2008), page 12. Loss severity measures the total foreclosure costs borne by investors as a proportion of the total unpaid principal on a mortgage.

commit to an ex-ante optimal foreclosure policy, resulting in higher *ex ante* efficiency in securitisation in equilibrium than the in-house servicing case. Through the compensation contract given to the servicer, the optimal foreclosure policy can be implemented.

Nevertheless, the optimal foreclosure policy appears inefficient *ex post*. If the mortgage pool is of low quality, the *ex post* marginal proceeds from foreclosure is lower than the forbearance value as indicated in Proposition 9. In order to implement such foreclosure policy, the compensation given to the servicer is such that $\beta_L > 1$. That is, the servicer receives an incentive tilted towards foreclosure. This is consistent with anecdotal evidence such as that of Goodman (2009).

Moreover, compared to the optimal foreclosure policy with in-house servicing, the foreclosure rate with a third-party servicer is generally higher. Specifically, $\hat{\lambda}_H > \lambda_H^o = 0$ and $\hat{\lambda}_L > \lambda_L^o = \lambda_L^{FB}$. This is an empirically testable implication of our model.

2.5 Fire-sale externality and foreclosure crisis

We have thus far established that the securitiser under asymmetric information implements inefficiently procyclical foreclosure policy. In the baseline model and the model involving a servicer, we have assumed that the securitiser operates as a monopoly with perfect price discrimination in the distressed property market. In this case, the distressed property market is efficient. In this section, we study an extension of the model in which fire-sale externality in the distressed property market exacerbates the countercyclicality to generate a "foreclosure crisis" when the overall quality of the mortgages in the economy is low.

In this extension we make two changes to the baseline model described in Section 2.2. First, we relax the assumption that the investors' private renovation costs r are contractible. Instead, a investor's cost r is only privately known to the investors and is non-verifiable. Second, we consider an economy in which there are $N \geq 1$ securitisers, each endowed with an i.i.d mortgage pool of the size $1/N$.

We solve for the model assuming that all securitisers can commit to a set of foreclosure policies at $t = 0$.[11] We follow the backward induction process to first consider the distressed property market under competition, then characterise the equilibrium foreclosure policy. This finally allows us to study the welfare implications of competition on the securitisers' foreclosure preference.

2.5.1 Distressed property market under competition

In order to obtain closed-form results, we use the following functional form in this section,

$$I(R) = aX\left(1 - \frac{1}{R}\right), \quad \text{for some } a \in [0, \bar{a}] \tag{2.59}$$

At $t = 2$, the total supply of the distressed properties is given by $\Lambda_s \equiv \frac{1}{N}\Sigma_{n=1}^{N}\lambda_i^n$, where $s \in \mathbb{S} \equiv \{H, L\}^N$ is the overall state of the economy given the realisation of all securitisers' types, λ_i^n is the fraction of delinquent mortgages foreclosed by securitiser n given her type i. Because the investor's renovation costs r are private, the market clears at one price b_s such that the measure of investors who are willing to enter the market at this price clears the market, i.e.

$$I(\frac{X}{b_s}) = b_s\Lambda_s = \mathcal{L}_s \tag{2.60}$$

Using the functional form given by Eq. 2.59, the equilibrium property price $b_s(\Lambda_s)$ and total liquidation proceed $\mathcal{L}_s(\Lambda_s)$ is

$$b_s(\Lambda_s) = \frac{aX}{\Lambda_s + a} \tag{2.61}$$

$$\mathcal{L}_s(\Lambda_s) = aX\frac{\Lambda_s}{\Lambda_s + 1} \tag{2.62}$$

For securitiser n, her liquidation proceed is given by

$$\mathcal{L}_i^n(\lambda_i^n, \Lambda^{-n}) \equiv b_s(\frac{1}{N}\lambda_i^n + \Lambda^{-n})\lambda_i^n = aX\frac{\lambda_i^n}{\lambda_i^n + N(\Lambda^{-n} + a)} \tag{2.63}$$

where $\Lambda^{-n} \equiv \Lambda_s - \frac{1}{N}\lambda_i^n$ is the total supply of properties by all the other securitisers.

[11] We have illustrated in the previous section that the exogenous assumption of commitment power is not critical to the implementation of the equilibrium foreclosure policy.

The liquidation proceeds of securitiser n are increasing and concave in her foreclosure policy,

$$\frac{\partial \mathcal{L}_i^n(\lambda_i^n, \Lambda^{-n})}{\partial \lambda_i^n} = aX \frac{N(\Lambda^{-n} + a)}{[\lambda_i^n + N(\Lambda^{-n} + a)]^2} \tag{2.64}$$

$$\frac{\partial^2 \mathcal{L}_i^n(\lambda_i^n, \Lambda^{-n})}{\partial \lambda_i^{n\,2}} = -2aX \frac{N(\Lambda^{-n} + a)a}{[\lambda_i^n + N(\Lambda^{-n} + a)]^3} \tag{2.65}$$

Moreover, there is a fire-sale externality in the equilibrium. That is, the liquidation proceeds of securitiser n is decreasing in the foreclosure policy of her competitor m of type j,

$$\frac{\partial \mathcal{L}_i^n(\lambda_i^n, \Lambda^{-n})}{\partial \lambda_j^m} = -aX \frac{\lambda_i^n}{[\lambda_i^n + N(\Lambda^{-n} + a)]^2} \tag{2.66}$$

This is because an increase in the foreclosure policy of other securitisers leads to a lower market clearing price, which reduces the proceeds obtained by securitiser n for a given foreclosure policy.

2.5.2 Symmetric equilibrium foreclosure policy

At $t = 1$, a securitiser of type i designs and sells an MBS to the investors. Observing all the securities on offer, the investors correctly infer the types of the securitisers in equilibrium. In response, all securitiser are fairly priced as in Section 2.2. Because the securitisers have mortgage pools with independent quality, we restrict our attention to only symmetric equilibria.

Consider for any securitiser n. By the same intuition as in Proposition 8, a low type issuer securitises the entire cash flow from the mortgage pool. For a given set of foreclosure policy $\boldsymbol{\lambda}_i^n \equiv \{\lambda_{i,s}^n\}_{s\in\mathbb{S}}$ for $i \in \{H, L\}$ by securitiser n and a given set of foreclosure policies $\boldsymbol{\Lambda}^{-n} \equiv \{\Lambda_s^{-n}\}_{s\in\mathbb{S}}$ by all other securitisers, the low type securitiser obtains payoff

$$U_L^n(\boldsymbol{\lambda}_L^n; \boldsymbol{\Lambda}^{-n}) = V + \mathbb{E}_L \left[\mathcal{L}_{L,s}^n(\lambda_{L,s}^n, \Lambda_s^{-n}) + (1 - \lambda_{L,s}^n)\theta_L X\right] \tag{2.67}$$

where $\mathbb{E}_i [\cdot]$ is the conditional expectation over all states $s \in \mathbb{S}$ given the securitiser's private information i.

On the other hand, a high type securitiser designs a monotonic security that specifies a set of payoffs $\{(F_{i,s}, f_{i,s})\}_{i\in\{H,L\},s\in\mathbb{S}}$ for the high and low cash flow realisation of the mortgage pool respectively for each type of the securitiser in each state. Since the securitiser does not know the state s, the realisation of the price of the security varies across states. In the least-cost separating equilibrium, the optimal security for the high type securitiser $\{(\hat{F}_{i,s}, \hat{f}_{i,s})\}_{i\in\{H,L\},s\in\mathbb{S}}$ maximises her expected proceed from security issuance subject to the usual limited liability constraints (LL), market clearing constraint (MC) and the incentive compatibility constraint (IC) for the low type not to mimic.

$$\max_{\{(F_{i,s},f_{i,s})\}_{i\in\{H,L\},s\in\mathbb{S}}} \mathbb{E}_H\left[p^n_s(F_{H,s}, f_{H,s})\right] \tag{2.68}$$

$$s.t.\quad (LL)\quad \forall\, i \in \{H, L\}, s \in \mathbb{S} \quad \text{and} \tag{2.69}$$

$$(MC)\quad p^n_s(F_{H,s}, f_{H,s}) = \theta_H F_{H,s} + (1-\theta_H) f_{H,s} \tag{2.70}$$

$$\begin{aligned}(IC)\quad U^n_L(\boldsymbol{\lambda}^n_L) \leq \mathbb{E}_H\left[p^n_s(F_{H,s}, f_{H,s}))\right] + \\ \delta\left(\theta_L\,\mathbb{E}_L\left[V + \mathcal{L}^n_{L,s}(\lambda^n_{L,s}, \Lambda^{-n}_s) + (1-\lambda^n_{L,s})X - F_{L,s}\right]\right. \\ \left.+(1-\theta_L)\,\mathbb{E}_L\left[V + \mathcal{L}^n_{L,s}(\lambda^n_{L,s}, \Lambda^{-n}_s) - f_{L,s}\right]\right)\end{aligned} \tag{2.71}$$

We consider foreclosure policy $(\boldsymbol{\lambda}^n_H, \boldsymbol{\lambda}^n_L)$ such that the resulting cash flows satisfy the following conditions in any pair of states s and s' in which all other securitisers are of the same type, except for securitiser n who is of the high type in state s and the low type in state s'. It will become clear that this is indeed the case in equilibrium.

$$\mathcal{L}^n_{H,s}(\lambda_H) \quad\leq\quad \mathcal{L}^n_{L,s'}(\lambda^n_{L,s'}) \quad \text{and} \tag{2.72}$$

$$\mathcal{L}^n_{H,s}(\lambda^n_{H,s}) + (1-\lambda^n_{H,s})X \quad\geq\quad \mathcal{L}^n_{L,s'}(\lambda^n_{L,s'}) + (1-\lambda^n_{L,s'})X \tag{2.73}$$

Denote the equilibrium face value of the debt $\hat{F}(\boldsymbol{\lambda}^n_H, \boldsymbol{\lambda}^n_L)$. Suppose the face value is such that in all states, a securitiser who issues this security defaults if her cash flow does not recover in case of forbearance of the delinquent mortgages, and never defaults if the cash flow recovers. The conditions for this to be the optimal security is provided in Appendix 2.8.1. This simplifies the analysis and allows direct comparison to the baseline case.

Given the optimal security, the high type securitiser enjoys an expected payoff of $U^n_H(\cdot)$

given by

$$
\begin{aligned}
U_H^n(\boldsymbol{\lambda}_H^n, \boldsymbol{\lambda}_L^n; \boldsymbol{\Lambda}^{-n}) &= (1-\delta)\,\mathbb{E}_H\left[\hat{p}_s^n(\hat{F}(\cdot); \lambda_{H,s}^n, \lambda_{L,s}^n)\right] \\
&\quad + \delta\left(V + \mathbb{E}_H\left[\mathcal{L}_{H,s}^n(\lambda_{H,s}^n, \Lambda_s^{-n}) + (1-\lambda_{H,s}^n)\theta_H X\right]\right) \quad (2.74) \\
\text{where} \quad \hat{p}_s^n(\hat{F}(\cdot); \cdot) &\equiv \theta \hat{F}(\cdot) + (1-\theta)\left[V + \mathcal{L}_{H,s}^n(\lambda_{H,s}^n, \Lambda_s^{-n})\right] \quad (2.75)
\end{aligned}
$$

The equilibrium foreclosure policy in a symmetric equilibrium is therefore chosen by each securitiser to maximise her expected payoff,

$$
(\hat{\boldsymbol{\lambda}}_H^n, \hat{\boldsymbol{\lambda}}_L^n) \equiv \arg \max_{(\boldsymbol{\lambda}_H^n, \boldsymbol{\lambda}_L^n)} \gamma U_H^n(\boldsymbol{\lambda}_H^n, \boldsymbol{\lambda}_L^n; \boldsymbol{\Lambda}^{-n}) + (1-\gamma) U_L^n(\boldsymbol{\lambda}_L^n; \boldsymbol{\Lambda}^{-n}) \quad (2.76)
$$

where $\boldsymbol{\Lambda}^{-n}$ is given by all other securitisers choosing the same foreclosure policy. Denote N_L^s the number of securitisers that are of the low type in state s. Therefore the total amount of foreclosure in equilibrium in state s is given by $\hat{\Lambda}_s(\hat{\lambda}_{H,s}^n, \hat{\lambda}_{L,s}^n) \equiv \frac{N - N_L^s}{N} \hat{\lambda}_{H,s}^n + \frac{N_L^s}{N} \hat{\lambda}_{L,s}^n$.

Proposition 12. *(i) The equilibrium foreclosure policy is higher for a low type issuer and lower for a high type issuer. That is, for any pair of states s and s' in which all other securitisers are of the same type, except for securitiser n who is of the high type in state s and the low type in state s',*

$$
\hat{\lambda}_{H,s}^n < \hat{\lambda}_{L,s'}^n \quad (2.77)
$$

(ii) For $N \geq 2$, the equilibrium foreclosure policy of each securitiser is procyclical. That is, for all states z, z' in which $N_L^z < N_L^{z'}$,

$$
\hat{\lambda}_{i,z}^n > \hat{\lambda}_{i,z'}^n \quad \forall\, i \in \{H, L\} \quad (2.78)
$$

Proof. See Appendix. ☐

Proposition 12 highlights the countercyclicality of the equilibrium under fire-sale externality. This result follows similar intuition to those for the baseline model (Proposition 9). This is because there are more securitisers of low quality in a worse state, which are following a high foreclosure policy than the high quality securitisers. This leads to procyclical distressed property prices. In turn, depressed property prices in a worse state discourages the securitisers from foreclosure, leading to procyclical foreclosure policy individually.

Proposition 13. *There exists a menu of affine contracts* $\{(\alpha_i, \beta_i, \tau_i)\}_{i\in\{H,L\}}$ *such that a securitiser can implement the optimal foreclosure policy in equilibrium through a third-party servicer, where* $\beta_i = \hat{\beta}_i$.

Proof. See Appendix. ☐

Similarly, this equilibrium foreclosure policy can be implemented through a third-party servicer. Because of the independence assumption, there only needs to be two contracts in the menu, one for each type of the mortgage pool. Moreover, the contracts specify the same relative sensitivity towards foreclosure relative to forbearance, $\beta_i = \hat{\beta}_i$, as in the monopoly case, despite a much more complex equilibrium that is being considered in this section. This suggests that it is relatively easy for the securitisers to implement their desired foreclosure policy in practice.

2.5.3 Foreclosure crisis

In this section, we present two benchmarks for comparison in order to understand the equilibrium foreclosure policy characterised above. The equilibrium is affected by three frictions. The first is the fire-sale externality in the distressed property market at $t = 2$, the second is the information asymmetry at $t = 1$, and the last is the market power enjoyed by each securitiser when making the foreclosure policy at $t = 0$.

The first benchmark considered is the full information benchmark (FI) which is absent of the first friction only. Under full information, all securitisers sell their entire cash flows from the mortgage pools to outside investors. The foreclosure policy is thus chosen to maximise the *ex ante* value of each mortgage pool.

$$\begin{aligned}(\boldsymbol{\lambda}_H^{FI}, \boldsymbol{\lambda}_L^{FI}) \;\equiv\; & \arg\max_{(\boldsymbol{\lambda}_H^n, \boldsymbol{\lambda}_L^n)} \gamma\, \mathbb{E}_H \left[\mathcal{L}_{H,s}^n(\lambda_{H,s}^n, \Lambda_s^{-n}) + (1-\lambda_{H,s}^n)\theta_H X\right] \\ & +(1-\gamma)\, \mathbb{E}_L \left[\mathcal{L}_{L,s}^n(\lambda_{L,s}^n, \Lambda_s^{-n}) + (1-\lambda_{L,s}^n)\theta_L X\right] \quad (2.79)\end{aligned}$$

We also present the full information central planner solution – the first best solution (FB). Notice that in the full information benchmark, there are two types of externality ignored by a securitiser. One is the fire-sale externality a securitiser has on other securitisers, and the other is the investor surplus $\frac{X}{b} - (1+r)$ when an investor with private cost r purchases the distressed properties at price b. Therefore in the first best case, a central planner maximises the total surplus in the economy,

$$\begin{aligned}(\boldsymbol{\lambda}_H^{FB}, \boldsymbol{\lambda}_L^{FB}) \;\; \equiv \;\; & \arg \max_{(\boldsymbol{\lambda}_H^n, \boldsymbol{\lambda}_L^n)} \sum_{n=1}^{N} \Big(\gamma\, \mathbb{E}_H \left[\mathcal{L}_{H,s}^n(\lambda_{H,s}^n, \Lambda_s^{-n}) + (1-\lambda_{H,s}^n)\theta_H X\right] \\ & \qquad + (1-\gamma)\, \mathbb{E}_L \left[\mathcal{L}_{L,s}^n(\lambda_{L,s}^n, \Lambda_s^{-n}) + (1-\lambda_{L,s}^n)\theta_L X\right]\Big) \\ & \qquad + \mathbb{E}\left[\int_1^{\frac{X}{b_s(\Lambda_s)}} \left(\frac{X}{b_s(\Lambda_s)} - R\right) dI(R)\right] \qquad (2.80)\end{aligned}$$

where $\boldsymbol{\Lambda}^{-n}$ is given by all other securitisers choosing the same foreclosure policy.

First of all, we highlight the effect of the market power enjoyed by the securitisers, by comparing the two benchmarks. Consider a state s, in which securitiser n is of type i, and in which the number of low type securitisers among the remaining $N-1$ securitisers is N_L^s. The first order conditions that determine a securitiser's foreclosure policy $\lambda_{i,s}^n$ in the full information case and the first best case respectively are given by

$$(FOC^{FI}) \;\; : \;\; \frac{\mathcal{L}_{i,s}^n(\lambda_{i,s}^n, \Lambda_s^{-n})}{\lambda_{i,s}^n} - \theta_i X = 0 \qquad (2.81)$$

$$\begin{aligned}(FOC^{FB}) \;\; : \;\; & \left(\frac{\mathcal{L}_{i,s}^n(\lambda_{i,s}^n, \Lambda_s^{-n})}{\lambda_{i,s}^n} - \theta_i X\right) \\ & + \left(\frac{N-1-N_L^s}{N} \frac{\mathcal{L}_{H,s}^m(\lambda_{i,s}^m, \Lambda_s^{-m})}{\lambda_{i,s}^n} + \frac{N_L^s}{N} \frac{\mathcal{L}_{L,s}^m(\lambda_{i,s}^m, \Lambda_s^{-m})}{\lambda_{i,s}^n}\right) \\ & + aX \left[I\left(\frac{X}{b_s(\Lambda_s)}\right) \frac{\partial}{\partial b_s}\left(\frac{X}{b_s(\Lambda_s)}\right) \frac{\partial b_s(\Lambda_s)}{\partial \Lambda_s} \frac{1}{N}\right] = 0 \qquad (2.82)\end{aligned}$$

It is immediate that the difference between the two first order conditions is in Line 2 and 3 of Eq. 2.82. Specifically, Line 2 of Eq. 2.82 represents the fire-sale externality of securitiser n's foreclosure policy on all other securitisers in a symmetric equilibrium. This effect is negative, suggesting the a securitiser under full information tends to foreclosure excessively relative to the first best case. Line 3 of Eq. 2.82 captures the effect of securitiser n's foreclosure policy on the total investor surplus. This effect is positive, since a higher foreclosure policy tends to lower the market price of the distressed property, benefiting the investors. Failing to account for the investor surplus, a securitiser under full information tends to foreclose insufficiently. The following lemma summarises the important trade-off effects of competition. Two extreme states are of particular interest. Denote $\underline{s}$ the state in which all securitisers have low quality mortgage pools, and $\bar{s}$ the state in which all securitisers have high quality mortgage pools. I will later refer to the former state the "boom" and the latter state the "bust".

Lemma 4. *There exists $\bar{N}, \underline{N}$ such that,*

$$\text{For } N > \bar{N}, \qquad \Lambda_{\bar{s}}^{FI} > \Lambda_{\bar{s}}^{FB} \quad \text{and} \quad \Lambda_{\underline{s}}^{FI} > \Lambda_{\underline{s}}^{FB} \tag{2.83}$$

$$\text{For } N < \underline{N}, \qquad \Lambda_{\bar{s}}^{FI} < \Lambda_{\bar{s}}^{FB} \quad \text{and} \quad \Lambda_{\underline{s}}^{FI} < \Lambda_{\underline{s}}^{FB} \tag{2.84}$$

That is, relative to the first best solution, the full information solution entails excessive foreclosure if the securitisation industry is competitive, but entails insufficient foreclosure if the securitisers enjoy large market power in the extreme states.

Proof. See Appendix. □

This result can be understood by examining the extreme cases. For $N = 1$ when there is only one monopolistic securitiser, there is no fire-sale externality. The market power of the monopoly securitiser therefore leads to insufficient foreclosure in an attempt to maximise her monopoly profit. For $N \to \infty$, the effect of each securitiser's foreclosure policy on the investor surplus diminishes because each securitiser's price impact diminishes. The competitive equilibrium under full information thus lead to excessive foreclosure due to the effect of fire-sale externality.

The following proposition highlights the properties of the equilibrium foreclosure policy, which is affected by asymmetric information as well as the above mentioned trade-off effect of market power.

Proposition 14. *(i) Asymmetric information exacerbates the countercyclicality of the equilibrium foreclosure policy. That is, for any pair of states s, s' in which all other* *securitisers are of the same type in both states except for securitiser n who is of the high type in state s and of the low type in state s′,*

$$\lambda_{H,s}^{\hat{n}} < \lambda_{H,s}^{FI} < \lambda_{L,s'}^{FI} < \hat{\lambda}_{L,s'}^{n} \tag{2.85}$$

In particular,

$$\hat{\Lambda}_{\bar{s}} < \Lambda_{\bar{s}}^{FI} < \Lambda_{\underline{s}}^{FI} < \hat{\Lambda}_{\underline{s}} \tag{2.86}$$

(ii) When competition is strong, the equilibrium entails excessive foreclosure during the bust. That is, for $N > \bar{N}$,

$$\hat{\Lambda}_{\underline{s}} > \Lambda_{\underline{s}}^{FI} > \Lambda_{\underline{s}}^{FB} \tag{2.87}$$

(iii) When competition is weak, the equilibrium entails insufficient foreclosure during the boom. That is, for $N < \underline{N}$,

$$\hat{\Lambda}_{\bar{s}} < \Lambda_{\bar{s}}^{FI} < \Lambda_{\bar{s}}^{FB} \tag{2.88}$$

Proof. See Appendix. □

Part (i) of Proposition 14 follows the same intuitive as the baseline case illustrated in Proposition 9.

Part (ii) can be understood as the "foreclosure crisis" scenario. When competition is strong, fire-sale externality is prominent, leading to excessive foreclosure among low quality mortgage pools. Therefore during the bust, the equilibrium foreclosure is significantly higher than in the first best case. This also leads to depressed property prices. This is consistent with the empirical evidence provided by Piskorski et al. (2010) that the foreclosure rate of delinquent bank-held loans is 3% (13%) to 7% (32%) lower in absolute (relative) terms. The authors also recognise that the primary reason for such findings is whether or not the servicer internalises the costs and benefits from the decision to foreclose a delinquent loan.

Part (iii) is the opposite scenario. If the securitisers have strong market power, they reduce foreclosure to maximise their profits from property sales. During the boom, the securitisers with high quality mortgage pools have the further incentive to avoid foreclosure in order to facilitate their securitisation process under asymmetric information. This results in inefficiently high property prices.

2.6 Empirical implications

This section summarises the empirical implications of our model related to foreclosure policy and characteristics of mortgage servicers' compensation contracts.

1. *In a bad state, securitised mortgages on average have a higher foreclosure rate than comparable bank-held loans.* The main result of our model shows that the asymmetric information friction in the process of mortgage securitisation will exacerbate the countercyclicalilty of foreclosure of delinquent mortgage. In a bad state where most mortgage pools are of low quality, securitised loans on average have a higher foreclosure probability than comparable bank-held loans (no information problem), which is consistent with the empirical finding of Piskorski et al. (2010)

2. *In a bad state, the foreclosure rate of delinquent mortgages in a securitised pool is higher than the ex post efficient level on average.* Specifically we show that when the mortgage pool is of low quality, the proceeds from foreclosing the marginal mortgages are *lower* than their expected recovery value, i.e. its foreclosure will be negative NPV or value-destroying decision from the *ex post* perspective. This is in line with the finding of Levitin (2009).

3. *In a bad state, the third-party servicer's contract on average is biased towards foreclosure.* We show that a securitiser offers an optimal incentive contract to a third-party servicer to implement the optimal foreclosure policy. In a bad state, the incentives for the servicer on average are biased towards foreclosure. This is in line with the anecdotal evidence of Goodman (2009).

4. *Securitised mortgages serviced by third-party servicers are foreclosed more on average than comparable mortgages with in-house servicers.* Our model shows that in-house servicers face a time-inconsistency problem and cannot commit to the ex-ante optimal foreclosure policy. An in-house servicer chooses little foreclosure when he is of the high type because he holds a levered equity claim, but chooses the *ex post* efficient level of foreclosure when he is of the low type. A third-party servicer, on the other hand, implements the optimal foreclosure policy which is insufficient when he is of the high type and excessive when he is of the low type. In either case, the third-party servicer forecloses more than an in-house servicer of the same type. Therefore we expect to observe higher foreclosure rates by third-party servicers on average.

2.7 Conclusion

The recent subprime mortgage crisis has raised concerns regarding the economic and social consequences of mortgage backed securitisation. In particular, the United States experienced a "foreclosure crisis" subsequent to the crisis in 2008 that received much public attention. Recent studies and reports have suggested that securitisation and the biased incentives of mortgage services could have contributed to the foreclosure wave. This paper formally studies the relationship between the foreclosure decision of delinquent loans and the securitisation of mortgages, and examines the role of mortgage servicers in this process.

We investigate the optimal foreclosure decision in a model of mortgage-backed securitisation under asymmetric information. A securitiser with a pool of mortgages has private information regarding the recovery rate of the mortgages that *ex post* become delinquent. The securitiser initially designs and sells a mortgage-backed security, and makes the decision whether to foreclose or modify a mortgage when it becomes delinquent *ex post*.

Relative to the case with full information, we show that the optimal foreclosure policy under asymmetric information involves excessive foreclosure if the mortgage pool is of low quality, and insufficient foreclosure if the mortgage pool is of high quality. This is because the signalling concern at the securitisation stage prompts the securitiser to take procedures at the foreclosure stage to reduce the information sensitivity of the mortgage pool cash flows.

Moreover, we propose a mechanism that involves mortgage servicers that resembles the industry practice, to implement the optimal foreclosure policy. We notice that the optimal foreclosure policy is not time-inconsistent. A securitiser with in-house servicing therefore cannot credibly commit to the *ex ante* optimal policy. In our mechanism, the securitiser designs a contract *ex ante* with a third-party servicer. This rids the securitiser of the commitment problem, and the securitiser can design an incentive contract for the third-party servicer to implement the *ex ante* optimal foreclosure policy.

2.8 Appendix

Proof of Proposition 8:

The optimal security for the low type is to issue all of the equity to outside investors in order to minimise the retention cost. The optimal security for the high type, however, has to satisfy an additional (IC) to prevent the bad type from mimicking. The remainder of the proof characterises the optimal security issued by the high type.

Central to the characterisation is the monotonicity of the security payoffs. The set of permitted securities (F_H, f_H, F_L, f_L) therefore depends on the ranking of the cash flows from each type of securitisers. Notice also that the functional form of the liquidation proceeds $\mathcal{L}_i(\lambda_i)$ implies that, for $\lambda_H < \lambda_L$, $\mathcal{L}_H(\lambda_H) \leq \mathcal{L}_L(\lambda_L)$ and $\mathcal{L}_H(\lambda_H) + (1-\lambda_H)X \geq \mathcal{L}_j(\lambda_L) + (1-\lambda_L)X$.

As will become clear when we solve for the optimal foreclosure policy, this is indeed the case in equilibrium. The solution to this case is a debt contract as presented in Proposition 8 and derived below.

In this case, the liquidation proceeds are less for the high type securitiser than for the low type securitiser. But if the delinquent mortgage recover, the total cash flows are higher for the high type securitiser.

$$\begin{aligned} V + \mathcal{L}_H(\lambda_H) + (1-\lambda_H)X &\geq V + \mathcal{L}_L(\lambda_L) + (1-\lambda_L)X \\ &\geq V + \mathcal{L}_L(\lambda_L) \geq V + \mathcal{L}_H(\lambda_H) \end{aligned} \tag{2.89}$$

The two monotonicity constraints for the insiders and the outsiders, combined with the limited liability constraints (LL), are

$$F_H \geq F_L \geq f_L \geq f_H \geq 0 \tag{2.90}$$

$$\begin{aligned} V + \mathcal{L}_H(\lambda_H) + (1-\lambda_H)X - F_H &\geq V + \mathcal{L}_L(\lambda_L) + (1-\lambda_L)X - F_L \\ &\geq V + \mathcal{L}_L(\lambda_L) - f_L \geq V + \mathcal{L}_H(\lambda_H) - f_H \\ &\geq 0 \end{aligned} \tag{2.91}$$

Firstly, examining the incentive compatibility constraint (IC) Eq. 2.22. The constraint is relaxed by increasing F_L and f_L. That is, decreasing the payoff to the Low type securitiser if she mimics the High type and issues this security. However, this increase will bind either the insider's monotonicity constraint or the outsider's. Depending on which constraints bind, many cases can arise, as discussed below.

(i) Suppose f_L binds the insider's monotonicity constraint, i.e.

$$V + \mathcal{L}_L(\lambda_L) - f_L = V + \mathcal{L}_L(\lambda_L) - f_H \tag{2.92}$$

This implies that $f_L > f_H$. Considering F_L, there are again two scenarios. (a) Suppose F_L binds the insider's monotonicity constraint, i.e.

$$V + \mathcal{L}_L(\lambda_L) + (1 - \lambda_L)X - F_L = V + \mathcal{L}_L(\lambda_L) - f_L \tag{2.93}$$

This implies that $F_L > f_L$ and $F_H \geq V + \mathcal{L}_L(\lambda_L)$. Substituting Eq. 2.92 and 2.93 into the (IC) yields

$$U_L(\lambda_L) \geq \theta_H F_H + (1 - \theta_H) f_H + \delta[V + \mathcal{L}_H(\lambda_H) - f_H] \tag{2.94}$$

It is now clear that the (IC) must bind in order to maximise $\theta_H F_H + (1 - \theta_H) f_H$. Substituting the (IC) into the objective function yields

$$\max_{f_H} \quad [A_1 - (1 - \theta_H) f_H + \delta f_H] + (1 - \theta_H) f_H \tag{2.95}$$

where $A_1 \equiv U_L(\lambda_L) - \delta[V + \mathcal{L}_H(\lambda_H)]$. The solution is therefore to increase f_H until the (LL) binds. The solution in this scenario is given by

$$f_H = V + \mathcal{L}_H(\lambda_H) \tag{2.96}$$

$$f_L = V + \mathcal{L}_L(\lambda_L) \tag{2.97}$$

$$F_L = V + \mathcal{L}_L(\lambda_L) + (1 - \theta_L)X \tag{2.98}$$

$$F_H = V + \frac{\mathcal{L}_L(\lambda_L) + (1 - \lambda_L)\theta_L X - (1 - \theta_H)\,\mathcal{L}_H(\lambda_H)}{\theta_H} \tag{2.99}$$

This is a solution if F_H is indeed such that $F_H \geq V + \mathcal{L}_L(\lambda_L)$, which is equivalent to

$$\frac{1 - \theta_H}{\theta_H - \theta_L}[\mathcal{L}_L(\lambda_L) - \mathcal{L}_H(\lambda_H)] \geq (1 - \lambda_L)X \tag{2.100}$$

I now turn to scenario (b) in which I suppose F_L binds the onsider's monotonicity constraint, i.e.

$$F_L = F_H \tag{2.101}$$

Which implies that $F_H \leq V + \mathcal{L}_L(\lambda_L) + (1-\lambda_L)X$. Following similar reasoning as in the previous scenario, we can substitute Eq. 2.92, 2.101 and the (IC) into the objective function, which again suggests that the solution involves maximising f_H until the (LL) binds. The solution in this scenario is given by

$$f_H = V + \mathcal{L}_H(\lambda_H) \tag{2.102}$$

$$f_L = V + \mathcal{L}_L(\lambda_L) \tag{2.103}$$

$$F_L = F_H = V + \frac{(1-\delta\theta_L)\,\mathcal{L}_L(\lambda_L) + (1-\delta)(1-\lambda_L)\theta_L X - (1-\theta_H)\,\mathcal{L}_H(\lambda_H)}{\theta_H - \delta\theta_L} \tag{2.104}$$

This is a solution if F_L satisfy the (LL), which is equivalent to the violation of the condition given in Eq. 2.100.

(ii) Suppose now that f_L binds the outsider's monotonicity constraint, i.e. $f_L = F_L$. In this case, the only possibility for this to be the case is if $F_L \leq V + \mathcal{L}_L(\lambda_L)$. Under this constraint, we can increase F_L to bind the outsider's monotonicity constraint $F_L = F_H$ because the previous constraint implies that the insider's monotonicity constraint for F_L is not binding. Substituting $f_L = F_L = F_H$ and the (IC) into the objective function implies that the objective function maximised when f_H is minimised to bind the (CC) at f_H. This implies that

$$f_H = 0 \tag{2.105}$$

$$f_L = F_L = F_H = \frac{1-\delta}{\theta_H - \delta}[V + \mathcal{L}_L(\lambda_L) + (1-\lambda_L)\theta_L X] \tag{2.106}$$

However, this is not a solution as this result does not satisfy the (LL) for F_L and f_L.

Therefore for cash flows ranked according to Case 1, the optimal security is a debt contract because the security payoff is always equal to the cash flow apart from the highest payoff, which corresponds to the face value of a debt contract. The optimal face value of the debt is summarised in Proposition 8 corresponding to the two scenarios in part (i) of Case 1.

2.8.1 The optimal securities under competition

Following similar intuition, the low type securitiser issues equity. We now turn to consider the optimal security for the high type when the conditions Eq. 2.72–2.73 for the ranking of the cash flows in each state are satisfied.

Notice first that, in this case, the good cash flows produced by a securitiser in all states are higher than the bad cash flows in all states, i.e.

$$\mathcal{L}^n_{i,s}(\lambda^n_{i,s}, \Lambda^{-n}_s) + (1 - \lambda^n_{i,s})X > \mathcal{L}^n_{i,z}(\lambda^n_{i,z}, \Lambda^{-n}_z) \qquad \forall\, s, z \in S \tag{2.107}$$

This is because the good cash flow is the highest if there is no foreclosure, at $\frac{1}{N}X$, while the bad cash flow is the highest if there is a foreclosure rate of 1, at X. This reflects the intuition that foreclosure reduces the risk in the mortgage pool.

The remainder of the argument proceeds as follows. We first conjecture that the optimal contract a debt contract with face value F such that

$$\max_s\{\mathcal{L}^n_{L,s}(\lambda^n_{L,s}, \Lambda^{-n}_s) \leq F \leq \min_s\{\mathcal{L}^n_{L,s}(\lambda^n_{L,s}, \Lambda^{-n}_s) + (1 - \lambda^n_{L,s})X\}] \tag{2.108}$$

The face value of this debt is relatively low so that the bad securitiser does not have to default should she mimic the good type securitiser and issues this security, but sufficiently high so as to leave zero cash flow to the bad securitisers when their cash flows do not recover. This enables direct comparison of this extension with competition with the baseline case. We then discuss conditions for this security to be the optimal security.

Within the class of debt contracts with face value satisfying the above condition, the optimal face value of the debt $\hat{F}(\boldsymbol{\lambda}^n_H, \boldsymbol{\lambda}^n_L)$ is given by the binding (IC),

$$\begin{aligned}\hat{F}(\boldsymbol{\lambda}^n_H, \boldsymbol{\lambda}^n_L) \;\; &= \;\; V + \frac{1}{\theta_H - \delta\theta_L}\,\mathbb{E}[(1 - \delta\theta_L)\,\mathcal{L}^n_{L,s}(\lambda^n_{L,s}, \lambda^{-n}_s) \\ &\qquad -(1 - \theta_H)\,\mathcal{L}^n_{H,s}(\lambda^n_{H,s}, \lambda^{-n}_s) + (1 - \delta)(1 - \lambda^n_{L,s})\theta_L X]\end{aligned} \tag{2.109}$$

If the face value characterised above indeed satisfies the condition, it is the optimal contract. This can be shown by considering possible deviations. In this case, $f_{H,s}$ and $f_{L,s}$ bind the (LL) for all s. The only possibly deviations are to increase or decrease $F_{i,s}$ for some s. I show in the following that there do not exist any deviations that can increase the good type issuer's payoff while satisfying the monotonicity constraints, the (IC) and the (LL).

(i) Suppose we decrease $F_{L,s}$ by for some s. This violates the (IC), and therefore must be accompanied by a decrease in $F_{H,z}$ for some z, if possible without violating any constraints. This, however, is not optimal as it reduces the payoff to the good type issuer.

(ii) Suppose we increase $F_{L,s}$ by ϵ for some s. This violates the monotonicity constraint on $F_{H,s}$, where also need to increase by at least ϵ. This, however, not violates the (IC) as the payoff to the low type issuer increases by $(\theta_H - \delta\theta_L)\epsilon$. If more monotonicity constraints are violated, we must also increase either $F_{H,z}$ for some z, or both $F_{L,z}$ and $F_{H,z}$ for some z. Similar intuition holds.

Proof of Proposition 12:

This proposition can be proved by examining the relevant first order conditions of the maximisation problem given by Eq. 2.74–2.76.

Consider a pair of states s and s' in which all other securitisers are of the same type, except for securitiser n who is of the high type in state s and the low type in state s'. It follows that the probability of state s conditional on the securitiser n being a high type is equal to the probability of state s' conditional on the securitiser n being a low type. Denote $q^n_{s,s'}$ this conditional probability.

The first order conditions for $\lambda^n_{H,s}(\Lambda^{-n}_s)$ and $\lambda^n_{L,s'}(\Lambda^{-n}_{s'})$, taking the foreclosure policies of other securitisers as given, can be expressed as

$$(FOC_{H,s}) \quad : \quad \gamma\left[(1-\delta)\theta_H\frac{\partial\hat{F}(\cdot)}{\partial\lambda^n_{H,s}} + (1-\delta)(1-\theta_H)\frac{\partial\,\mathcal{L}^n_{H,s}(\cdot)}{\partial\lambda^n_{H,s}}q^n_{s,s'} + \delta\left(\frac{\partial\,\mathcal{L}^n_{H,s}(\cdot)}{\partial\lambda^n_{H,s}} - \lambda^n_{H,s}\theta_H X\right)q^n_{s,s'}\right] = 0 \tag{2.110}$$

$$(FOC_{L,s'}) \quad : \quad \left[\gamma(1-\delta)\theta_H\frac{\partial\hat{F}(\cdot)}{\partial\lambda^n_{L,s'}} + (1-\gamma)\left(\frac{\partial\,\mathcal{L}^n_{L,s'}(\cdot)}{\partial\lambda^n_{L,s'}} - \lambda^n_{L,s'}\theta_L X\right)q^n_{s,s'}\right] = 0 \tag{2.111}$$

where

$$\frac{\partial \hat{F}(\cdot)}{\partial \lambda^n_{H,s}} = -\frac{1-\theta_H}{\theta_H - \delta\theta_L}\frac{\partial \mathcal{L}^n_{H,s}(\cdot)}{\partial \lambda^n_{H,s}} q^n_{s,s'} \tag{2.112}$$

$$\frac{\partial \hat{F}(\cdot)}{\partial \lambda^n_{L,s'}} = \left[\frac{\delta(1-\theta_L)}{\theta_H - \delta\theta_L}\frac{\partial \mathcal{L}^n_{L,s'}(\cdot)}{\partial \lambda^n_{L,s'}} + \frac{1-\delta}{\theta - \delta\theta_L}\left(\frac{\partial \mathcal{L}^n_{L,s'}(\cdot)}{\partial \lambda^n_{L,s'}} - \theta_L X\right)\right] q^n_{s,s'} \tag{2.113}$$

(i) These conditions are very similar to those in the baseline case. Therefore similar reasoning shows that for any foreclosure followed by other securitisers Λ^{-n}, the foreclosure rate of the low type is higher than that of the high type. Therefore this is also true in equilibrium, i.e.

$$\hat{\lambda}^n_{H,s} < \hat{\lambda}^n_{L,s'} \tag{2.114}$$

(ii) In order to prove this part of the proposition, we first need to establish that $\hat{\Lambda}^{-n}_s < \hat{\Lambda}^{-n}_z$ and $\hat{\Lambda}^{-n}_{s'} > \hat{\Lambda}^{-n}_{z'}$. The result then follows.

This part of the proposition concerns equilibrium results. We here impose that the equilibrium is symmetric. The equilibrium foreclosure decision is therefore given by the following fixed point problems.

$$\begin{cases} \hat{\lambda}^n_{H,s} &= \lambda^n_{H,s}(\hat{\Lambda}^{-n}_s), \\ \hat{\lambda}^n_{L,s'} &= \lambda^n_{L,s'}(\hat{\Lambda}^{-n}_{s'}), \end{cases} \tag{2.115}$$

where $\hat{\Lambda}^{-n}_s = \hat{\Lambda}^{-n}_{s'} = \frac{N-N^s_L-1}{N}\hat{\lambda}^n_{H,s} + \frac{N^s_L}{N}\hat{\lambda}^n_{L,s'}$

Notice that the functions $\lambda^n_{H,s}(\cdot)$ and $\lambda^n_{L,s'}(\cdot)$ are decreasing in the argument. This can be easily checked by implicitly differentiating the $(FOC_{H,s})$ and $(FOC_{L,s'})$. The intuition is, an increase the aggregate amount of foreclosure by the other securitisers increases reduces the incentive for the securitiser n to foreclose because of the lower price in the market.

Consider another pair of states z, z' analogous to s, s' such that $N^s_L < N^z_L$, and express the solution to the optimal foreclosure policy in state z as the solution to a system similar to the above. We now prove this part of the proposition by contradiction. Suppose that the equilibrium is such that $\hat{\Lambda}^{-n}_s > \hat{\Lambda}^{-n}_z$. That the functions $\lambda^n_{H,s}(\cdot)$ and $\lambda^n_{L,s'}$ implies that $\hat{\lambda}^n_{H,s} < \hat{\lambda}^n_{H,z}$ and $\hat{\lambda}^n_{L,s'} < \hat{\lambda}^n_{L,z'}$. Combined with the fact that there are fewer low type issuers in state s than in z and the result of part (i), this contradicts with the supposition that the aggregate foreclosure is greater in states s and s' than in states z and z'. Similar arguments show that $\hat{\Lambda}_{s'}^{-n} > \hat{\Lambda}_{z'}^{-n}$ also leads to a contradiction.

Part (ii) of the proposition thus follows immediately. Since $\hat{\Lambda}_{s}^{-n} < \hat{\Lambda}_{z}^{-n}$ and $\hat{\Lambda}_{s'}^{-n} > \hat{\Lambda}_{z'}^{-n}$, it must be that $\hat{\lambda}_{H,s}^{n} > \hat{\lambda}_{H,z}^{n}$ and $\hat{\lambda}_{L,s'}^{n} > \hat{\lambda}_{L,z'}^{n}$.

Proof of Proposition 13:

The proof of this proposition follows closely to that of Proposition 11. First note that the required β_i to implement the optimal foreclosure policy $\lambda_{H,s}^{n}(\Lambda_{s}^{-n})$ and $\lambda_{L,s'}^{n}(\Lambda_{s'}^{-n})$ coincides with the β_i in Proposition 11, i.e. $\beta_i = \hat{\beta}_i$. To see this, given the contract, the servicer's first order conditions are

$$\beta_i \frac{\partial \mathcal{L}_{i,s}^{n}(\cdot)}{\partial \lambda_{i,s}^{n}} - \theta_i X = 0$$

for $i = \{H, L\}$. Thus β_i are chosen to match the coefficients of Eq. (2.110)-(2.111). By direct computation the required β_i is the same as $\hat{\beta}_i$ as in Eq (2.47)-(2.48). The rest of the contractual terms $\{\tau_i, \alpha_i\}$ similarly will also only depend on the servicer's type and do not depend on the aggregate state.

Proof of Lemma 4:

This lemma can be shown by considering the two extremes. For $N = 1$, the LHS of (FOC^{FB}) is larger than the LHS of (FOC^{FI}), because the fire-sale externality term disappears but the investors surplus term in (FOC^{FB}) remains. This leads $\hat{\Lambda}_{s}^{-n,FI} < \hat{\Lambda}_{s}^{-n,FB}$ following the same reasoning as in the previous proof. In a symmetric equilibrium, this also implies that $\hat{\Lambda}_{\bar{s}}^{FI} < \hat{\Lambda}_{\bar{s}}^{FB}$ and $\hat{\Lambda}_{\underline{s}}^{FI} < \hat{\Lambda}_{\underline{s}}^{FB}$.

For $N \to \infty$, the LHS of (FOC^{FB}) is smaller than the LHS of (FOC^{FI}), because when the market is competitive, a securitiser has no price impact and hence does not affect the investors surplus. However, the fire-sale externality is present when there are many securitisers. This lease to the results $\hat{\Lambda}_{\bar{s}}^{FI} > \hat{\Lambda}_{\bar{s}}^{FB}$ and $\hat{\Lambda}_{\underline{s}}^{FI} > \hat{\Lambda}_{\underline{s}}^{FB}$.

Proof of Proposition 14:

(i) We first examine the properties of the (FOC_i) in comparison to (FOC^{FI}). The LHS of $(FOC_{H,s})$ is strictly smaller than the LHS of $(FOC^{FI}_{H,s})$, and the LHS of $(FOC_{L,s'})$ is strictly larger than the LHS of $(FOC^{FI}_{L,s'})$. A symmetric equilibrium can be re-written as the solution to the following system of equations

$$\begin{cases} \hat{\lambda}^n_{H,s} &= \hat{f}_{H,s}(\hat{\lambda}^n_{L,s'}) \\ \hat{\lambda}^n_{L,s'} &= \hat{f}_{L,s'}(\hat{\lambda}^n_{H,s}) \end{cases} \qquad \begin{cases} \lambda^{FI}_{H,s} &= f^{FI}_{H,s}(\lambda^{FI}_{L,s'}) \\ \lambda^{FI}_{L,s'} &= f^{FI}_{L,s'}(\lambda^{FI}_{H,s}) \end{cases} \tag{2.116}$$

for some decreasing functions $\hat{f}_{H,s}(\cdot) < f^{FI}_{H,s}(\cdot)$ and $\hat{f}_{L,s'}(\cdot) < f^{FI}_{L,s'}(\cdot)$.

We now proceed to show this part of the proposition. It is clear that $\lambda^{\hat{n}}_{H,s} < \lambda^{FI}_{H,s} < \lambda^{FI}_{L,s'} < \hat{\lambda}^n_{L,s'}$ is consistent with the above systems of equations. That is, $\hat{\lambda}^n_{H,s} = \hat{f}_{H,s}(\hat{\lambda}^n_{L,s'}) < f^{FI}_{H,s}(\hat{\lambda}^n_{L,s'}) < f^{FI}_{H,s}(\lambda^{FI}_{L,s'}) = \lambda^{FI}_{H,s}$, and similarly for $\hat{\lambda}^n_{L,s'} > \lambda^{FI}_{L,s'}$.

In particular, the reasoning of Proposition 12 applies and implies that $\Lambda^{FI}_{\bar{s}} < \Lambda^{FI}_{\underline{s}}$. This leads to $\hat{\Lambda}_{\bar{s}} < \Lambda^{FI}_{\bar{s}} < \Lambda^{FI}_{\underline{s}} < \hat{\Lambda}_{\underline{s}}$.

Parts (ii) and (iii) follows immediately from Part (i) of this Propositon and Lemma 4.

Chapter 3

Asset market runs and the collapse of debt maturity

3.1 Introduction

Financial institutions' heavy reliance of short-term debt has commonly been seen as one of the important contributing factors for the severity of the Global Financial Crisis 2007-2009. Recent academic research has investigated the roll-over risk and the inefficiency associated with the use of short-term debt[1]. A paper by Gorton et al. (2014) however suggests that there is a further shortening of debt maturity *during* the early stage of the crisis, which endogenously exacerbates the severity of the later shock: the failure of Lehman Brothers. This paper provides a theoretical model to analyse this *endogenous* collapse of debt maturity and hence the build-up of financial fragility in a market distressed period.

I build a competitive equilibrium model of collateralised borrowing in which borrowers have to pledge existing collateral asset to lenders in order to finance some profitable investment projects. Collateral are pledged to enforce repayment and when the borrowers fail

[1] For example, He and Xiong (2012) demonstrate the coordination failure of the roll-over decisions between debt holders maturing at different dates; Eisenbach (2011) shows the roll-over risk of short-term debt can prevent borrower's from taking excessive risk but under aggregate uncertainty this disciplining effect is either too much or too little, resulting in inefficiency.

to repay, the lenders can seize the collateral and sell it in the secondary market. Following the literature on asset market runs such as Bernardo and Welch (2004) and Morris and Shin (2004), I assume two important frictions in the collateral asset market. First, market makers of the collateral asset have limited resources or risk-absorption capacity and thus the equilibrium market clearing price will be decreasing in the amount of collateral being sold; Second, sell orders of the collateral are executed in a random order, i.e. a seller will not know exactly what price she will receive when she submits a sell order. The combination of these two frictions create an incentive for a borrower to adopt short-term debt which gives his lender an advantage to liquidate the collateral asset *ahead* of others and hence at a better price.

More specifically short-term debt provides an *option* for the lenders at the interim date to roll-over their debt or not after they receive some information about the solvency of the borrowers. This option is particularly valuable when the interim signal is accurate and the expected price differential between liquidating earlier and later is large. Therefore, borrowers will prefer short-term debt over long-term debt under these circumstances.

In a competitive equilibrium with many borrower-lender pairs, the expected price differential is endogenous and depends on the aggregate maturity choice of borrowers. The amount of collateral liquidated at the early or the late date depends on how many borrowers use short-term debt and the aggregate amount of collateral liquidated at different dates will affect the price differential. This co-determination of maturity choice and expected price differential gives rise to a debt maturity externality channel and a feedback mechanism which can potentially lead to multiple equilibria with different maturity choices, i.e. a *collapse* of aggregate debt maturity can happen when an all-long-term debt equilibrium switches to an all-short-term debt equilibrium.

Related literature This paper is closely related to the 'maturity rat race' equilibrium in Brunnermeier and Oehmke (2012). Brunnermeier and Oehmke (2012) consider a firm borrows from multiple creditors and show that if the borrower cannot ex-ante commit to a maturity structure, he has the incentives to offer debt with shorter maturity to the new creditor to dilute existing creditors. They show that under some parameter values the unique equilibrium consists of only shortest possible maturity. The maturity collapse mechanism in this paper does not involve diluting existing debt holders and thus borrowers are allowed to ex-ante commit to a maturity structure. The key frictions come from the asset market: offering shorter maturity in this model provides an option for lenders to liquidate the collateral early and at a better price.

This paper builds on the existing models in asset market runs literature. Bernardo and Welch (2004) show the fear of future liquidity shocks can prompt investors to run and sell the asset now rather than later. Morris and Shin (2004) show traders with exogenous loss limits would have incentive to sell before others. Oehmke (2014) studies the optimal collateral liquidation decision of repo lenders after their creditors have defaulted. He shows that when repo lenders are subject to some portfolio risk constraint, the equilibrium price of the collateral asset can overshoot. These papers in essence study a *post-default* asset liquidation problem with an exogenous amount of collateral held by the investors. In contrast, I focus on how borrowing firms' financing problem about the choice of debt maturity can interact with the asset market friction mentioned above to create a aggregate maturity collapse phenomenon.

3.2 Model

Consider a three-date ($t = 0, 1, 2$) model with a continuum of borrowing firms each matched with a corresponding creditor, and a representative outside collateral buyer. There is a storage technology with returns normalised to zero.

Firms and projects Firms are risk-neutral, identical ex-ante, and each has a unit of common asset-in-place (collateral) with no cash and debt. At $t = 0$ each firm has the opportunity to invest in a project which requires an initial investment of \$1 and will return a random non-verifiable cash flow $\tilde{X}$, which is X with probability $p \in (\frac{1}{2}, 1)$ and 0 otherwise at $t = 2$. The project has a positive NPV, i.e. $pX > 1$ and has zero liquidation value at the interim date[2]. The non-verifiable cashflow assumption is common in the incomplete contract literature such as Hart and Moore (1994) and is also used in banking models as in Calomiris and Kahn (1991) and Diamond and Rajan (2001b). The assumption can be motivated by the opaque nature of the balance sheet of the firms. From a modelling perspective, it necessitates the use of external collateral to support the financing because firms can always claim the project realised cashflow to be zero. Finally, the realisations of project cashflow is assumed to be independently distributed across firms, i.e. there is no aggregate uncertainty.

Collateral assets and debt maturity As hinted above, collateral is needed for the financing of projects and here I assume each firm has one divisible unit of asset (e.g. financial securities) which pays an expected cashflow $v > 1$ at $t = 2$. I assume this collateral cashflow to be non-verifiable. As such, the firm can effectively choose $k \in [0, 1]$ fraction of the collateral to pledge to the creditor at the ex-ante contracting stage and keep the remaining $(1-k)$ fraction beyond the creditor's reach. In the shadow banking context, for instance, one can think of the financing acts through an off-balance-sheet investment vehicle and the firm has to ex-ante explicitly transfer asset to this vehicle. Creditors in principles cannot claim payment beyond the balance sheet of the investment vehicle[3].

Firms borrow in the form of collateralised debt contract and can choose to use a short-term or a long-term debt. In the case with the long-term debt, the firm promises to repay r_{LT} to its creditor at $t = 2$ and pledges $k_{LT} \in [0, 1]$ measure of the collateral to the creditor. Should the firm defaults at $t = 2$, the creditor automatically owns the collateral. I assume that due to risk-aversion or regulatory constraint, creditors will always prefer selling the collateral asset in the market to holding them to maturity[4].

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[2]Zero liquidation value for the project at $t = 1$ is for simplicity.

[3]In practice, the Structured Investment Vehicles usually receive (partial) credit guarantee or enhancement from the sponsoring banks. In that case k could be interpreted as the *effective* amount of pledged collateral. For studies regarding SIVs, see Covitz et al. (2013) and Acharya et al. (2013)

[4]Creditors here can be understood as having a liquidity shock with probability one in the setting of Bernardo and Welch (2004), a tight loss limit in Morris and Shin (2004), or a binding portfolio risk

As the project cashflow is non-verifiable, the firm can always threaten to claim default and renegotiate down the repayment. As such, the amount of credible repayment is constrained by the market value of the pledged collateral, i.e.

$$r_{LT} \leq k_{LT} l_2 \tag{3.1}$$

where l_t is the equilibrium expected payoff from liquidating the collateral at date t. For the moment it is convenient to treat l_1 and l_2 as given and assume $v \geq l_1 \geq l_2$. The debt contract above closely resembles a repurchase agreement (repo) as commonly used in the shadow banking sector.

Interim signal and roll over decision If a firm chooses to use the short-term debt, at $t = 0$ it has to pledge k_1 measure of collateral and promises to repay r_1 to the creditor at $t = 1$[5]. Then the debt has to be rolled over by the creditor at $t = 1$, when both the firm and its creditor receive a non-contractible signal about the success or failure of the firm's project, that is, whether cash flow X or 0 will realise at $t = 2$. I assume the following binary signal structure $\tilde{S} = \{S_G, S_B\}$:

$$Prob\{\tilde{S} = S_G | \tilde{X} = X\} = Prob\{\tilde{S} = S_B | \tilde{X} = 0\} = q \tag{3.2}$$

where $q \in (\frac{1}{2}, 1)$ can be interpreted as the quality of the signal. By Bayes rules, the posterior probabilities of success after receiving signal S_i, $i = \{G, B\}$ are

$$p(S_G) := Prob\{\tilde{X} = X | \tilde{S} = S_G\} = \frac{pq}{pq + (1-p)(1-q)} \tag{3.3}$$

and

$$p(S_B) := Prob\{\tilde{X} = X | \tilde{S} = S_B\} = \frac{p(1-q)}{p + q - 2pq} \tag{3.4}$$

constraint in Oehmke (2014).

[5] As shown before, the maximum credible r_1 is $k_1 l_1$

After this interim signal realises, the firm offers a new roll over contract to the creditor. Should the creditor refuses to roll over, the firm is immediately put into liquidation and the creditor can seize and liquidate the collateral asset in the market. Note that the project has no liquidation value and hence the expected cashflow of $p(S_i)X$ is lost if the signal is S_i and the debt is not rolled over.

In principles, the firm can always roll over the short term debt by pledging k_2 measure of collateral such that

$$k_2 l_2 \geq k_1 l_1 \tag{3.5}$$

That is, giving the creditor (weakly) more than she expects to get if she liquidates the firm now. However, after receiving the signal S_i at $t = 1$ the firm will prefer *strategic default* to rolling over the debt after receiving a bad signal if and only if

$$G(l_1, l_2; q) := [1 - p(S_B)]k_2(v - l_2) - k_1(v - l_1) - p(S_B)X \geq 0 \tag{3.6}$$

What equation (3.6) says is if the expected (incremental) cost of rolling over the debt due to fire sale of collateral $(1 - p(S_B))k_2(v - l_2) - k_1(v - l_1)$ is higher than the expected benefit of continuing the project $p(S_B)X$, it is optimal for the firm to default at $t = 1$. Another interpretation is that the firm may voluntarily expose to the roll-over risk of using short-term debt precisely because it allows its creditor to liquidate at an earlier date and a hence better price (recall $l_1 \geq l_2$).

Whether condition (3.6) is satisfied depends crucially on l_1 and l_2 (also the signal quality q). Next, I will discuss the determination of these equilibrium expected payoffs from liquidating the collateral.

Competitive market-makers for collateral and execution uncertainty Following Morris and Shin (2004), I first assume the market-making sector has limited risk-absorption capacity and compete to buy asset by posing limit buy orders for the collateral asset. Market-makers are risk-averse and thus they have to compensated for absorbing risk by posing a downward sloping demand curve for the asset.

Secondly, there is uncertainty in the execution. When a sell order is submitted by a creditor of a defaulted firm, all the orders submitted on the same date are lined up randomly in a queue.

Denote the market-clearing price function as $L(\phi;\gamma)$ which decreases in the amount of collateral already sold and absorbed by the market maker ϕ, decreases in the degree of risk-aversion γ and when $\gamma \to 0$, $L(\phi) \to v$. What it means is that if the total size of the sell orders (each of size 1) are ϕ, the first order will be executed at price $L(0;\gamma)$ and the last one will be at $L(\phi;\gamma)$. For tractibility I use a linear pricing function

$$L(\phi;\gamma) = v - \gamma\phi \tag{3.7}$$

Because of the random queuing nature, the (per unit) expected liquidation proceeds l_t when a creditor submits a sell order at t are:

$$l_1 = v - \frac{1}{2}\gamma\phi_1 \tag{3.8}$$

and

$$l_2 = v - \gamma(\phi_1 + \frac{1}{2}\phi_2) \tag{3.9}$$

where ϕ_1 and ϕ_2 are the aggregate amount of collateral being liquidated at $t = 1$ and $t = 2$ respectively. Importantly as $L(\phi;\gamma)$ decreases in ϕ, $l_1 \geq l_2$. This captures the *advantage of early liquidation* at $t = 1$. At $t = 2$ as the risk-averse market-maker already has ϕ_1 collateral in its inventory, even if $\phi_2 = 0$ the price that one will receive is $l_2 = v - \gamma\phi_1$, essentially 'the end of the queue price' at $t = 1$.

After describing the setup of the model, I will first proceed to discuss the optimal choice between short-term and long-term debt, taking l_1 and l_2 as given. Then I will focus on studying the two extreme pure strategy equilibria with all short-term debt and all long-term debt. Finally I will comment on the co-existence of these two equilibria and compare them in terms of welfare.

3.3 Short-term v.s. Long-term debt: the value of strategic default

As hinted in the previous section, firms might have incentive to default on their short-term debt at $t = 1$ so as to avoid pledging extra costly collateral to roll over the debt. In the section I will show that short-term debt dominates long-term debt exactly when this strategic default condition (3.6) satisfied. And when it is not satisfied, the firm is indifferent between using short-term or long-term debt.

Consider a firm's problem between using short-term and long-term debt. As each firm is of zero-measure, it will take the equilibrium (expected) liquidation payoffs l_1 and l_2 as given since its action will not have any effect on the prices. If the firm choose to use long-term debt, it has to pledge k_{LT} units of collateral such that

$$k_{LT} l_2 \geq 1 \tag{3.10}$$

which is the Participation Constraint of the creditor. In this case the firm' expected net payoff from financing with LT debt is

$$U_{LT}(l_1, l_2) = p(X - k_{LT} l_2 + v) + (1-p)(1-k_2)v - v \tag{3.11}$$

$$= pX - 1 - (1-p)\frac{(v - l_2)}{l_2} \tag{3.12}$$

which is the NPV of the project less the expected fire sale cost $(1-p)k_{LT}(v - l_2)$. The firm will undertake the project only when $U_{LT} \geq 0$ and there is enough collateral to satisfy the credit's PC, i.e. $k_{LT} = \frac{1}{l_2} \leq 1$. I will ignore the potential credit rationing problem by assuming X and v high enough so that the project is always profitable to take and there is always enough collateral for financing.

3.3.1 Short-term debt with strategic default

Now consider a firm issuing short-term debt that will default if a bad signal realises at $t = 1$[6]. For the moment let's assume (and verify later) that the strategic default condition (3.6) is satisfied. Then the firm's expected net utility of financing with defaultable short-term debt is

$$U_{ST}(l_1, l_2) = p[q(X - 1 + v) + (1 - q)(1 - k_1)v] + (1 - p)[q(1 - k_1)v + (1 - q)(1 - k_2)v] - v \tag{3.13}$$

with $k_1 = \dfrac{1}{l_1}$ and $k_2 = \dfrac{k_1 l_1}{l_2} = \dfrac{1}{l_2}$ to satisfy the creditor's participation constraint. Thus the firm will opt for this defautable short-term debt over long-term debt if and only if

$$U_{ST}(l_1, l_2) - U_{LT}(l_1, l_2) \geq 0 \tag{3.14}$$

Simple calculation leads to the following lemma:

Lemma 5. *A firm prefers defaultable short-term debt to long-term debt if and only if it is profitable to default at the interim date when a bad signal arises, i.e.* $U_{ST}(l_1, l_2) - U_{LT}(l_1, l_2) \geq 0$ *if and only if* $G(l_1, l_2, q) \geq 0$.
Proof: *direct comparison of equations (3.6), (3.11) and (3.13).*

Intuitively, a long-term debt is essentially the same as a short-term that is always rolled over. Thus when it is optimal for the firm to default with a bad signal, this extra value of strategic default makes the defaultable short-term debt strictly dominate long-term debt.

Thanks to the above lemma, the function $G(l_1, l_2; q)$ can also tell us when defaultable short-term debt is more likely to be (privately) optimal:

Corollary 1. *Defaultable short-term debt is more likely to be preferred to long-term debt when:*

- *the signal quality is higher*

[6]Obviously no firm will be interested in issuing a short-term debt that will lead to liquidation at $t = 1$ for both good and bad state

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- *the expected collateral liquidation payoff at $t = 1$ is higher*

- *the expected collateral liquidation payoff at $t = 2$ is lower*

Proof: *this is the direct consequence of partial differentiating $G(l_1, l_2; q)$. It is immediate to show that $\frac{\partial G}{\partial q} > 0$, $\frac{\partial G}{\partial l_1} > 0$, and $\frac{\partial G}{\partial l_2} < 0$.*

For a given bad signal, higher signal quality means that the expected loss of the collateral at $t = 2$ and the expected cashflow loss of the project is smaller. The last two bullet points suggest when the advantage to liquidate early is larger, or the adverse price impact of liquidating late is more severe, the value of strategic default increases.

Before we turn to the study of competitive equilibria with endogenous liquidation payoffs, I should highlight that in this paper short-term debt adds value only through the embedded *early exit option*, which is likely to be a zero-sum, private gain because by definition half of the sell orders will be executed later than the other half even if everyone chooses to exit early. In contrast, the loss associated with premature and wrongful liquidation (due to signal inaccuracy) of the project is a real economic loss.

3.4 Competitive Equilibrium: the inefficient maturity collapse

In this section I will endogenise the liquidation payoffs $\{l_1, l_2\}$ and solve for the competitive equilibrium. I will first characterise the two pure strategy equilibria with all firms choosing either long-term or short-term debt. Then I will investigate the possibility of multiple pure strategy equilibria and show that the one with all short-term debt is Pareto-dominated by the one with all long-term debt.

3.4.1 Equilibrium with all long-term debt

Definition 2. *For a given set of parameters, a competitive rational expectation equilibrium with all firms issuing long-term debt is a pair of $\{\hat{l_1}, l^*_{LT}\}$ such that*

- *No firm will deviate to issue short-term debt, i.e.*

$$G(\hat{l}_1, l^*_{LT}; q) \leq 0 \tag{3.15}$$

- *The expected collateral liquidation payoff at* $t = 1$ *is correct,* $\hat{l}_1 = v - \gamma(\frac{\phi_{1,LT}}{2}) = v$, *as* $\phi_{1,LT} = 0$

- *The expected collateral liquidation payoff at* $t = 2$ *is correct, i.e.*

$$l^*_{LT} = v - \gamma(\frac{\phi_{2,LT}}{2}) \tag{3.16}$$

where $\phi_{2,LT} = (1-p)k^*_{LT} = \dfrac{(1-p)}{l^*_{LT}}$ *and* l^*_{LT} *is the largest solution of the equation (3.16).*

The above definition constructs the pure strategy equilibrium with all firms issuing long-term debt[7]. Notice that $\phi_{1,LT} = 0$ because firms do not face roll-over risk and hence there is no liquidation at the interim date. The next proposition links the existence of this equilibrium with the risk-absorption capacity in the market-making sector.

Proposition 15. *Equilibrium with all long-term debt only exists when the market-making sector is not too risk-averse. There exists a* $\gamma_{LT} > 0$ *such that the equilibrium in Definition 2 cannot exist for* $\gamma \in (\gamma_{LT}, \infty)$.

Proof: *See Appendix.*

The intuition of the above proposition is as follows. In equilibrium, no firm can be better off by deviating to issue short-term debt. Since each firm is of measure zero, the collateral liquidation payoff at $t = 1$ if the firm deviates to use short-term debt is $\hat{l}_1 = v$. When γ is larger, that is the market-making sector is more risk-averse and the equilibrium liquidation value l^*_{LT} becomes smaller. Hence the potential deviation, or the early exit option embedded in defaultable short-term debt, becomes more attractive. Proposition

[7]Note that the way I define l^*_{LT} as the largest solution of the equation (3.16) is to rule out multiple equilibria through self-fulfilling fire sale, which is a subject of interest in itself but is not relevant to the study of maturity collapse here.

(15) could be instrumental to understand why equilibrium with long-term debt maturity was hard to find during the recent financial crisis, precisely because the collateral market is in distressed.

Next, similarly I will study the pure strategy equilibrium with all short-term debt.

3.4.2 Equilibrium with all short-term debt

Definition 3. *For a given set of parameters, a competitive rational expectation equilibrium with all firms issuing short-term debt is a pair of* $\{l_1^*, l_2^*\}$ *such that*

- *No firm will deviate to issue long-term debt, i.e.*

$$G(l_1^*, l_2^*; q) \geq 0 \tag{3.17}$$

- *The expected collateral liquidation payoff at* $t = 1$ *is correct, i.e.*

$$l_1^* = v - \gamma(\frac{\phi_{1,ST}^*}{2}) \tag{3.18}$$

where $\phi_{1,ST}^* = (p + q - 2pq)k_1^* = \dfrac{(p + q - 2pq)}{l_1^*}$ *and* l_1^* *is the largest solution of the equation (3.18).*

- *The expected collateral liquidation payoff at* $t = 2$ *is correct,*

$$l_2^* = v - \gamma(\phi_{1,ST}^* + \frac{1}{2}\phi_{2,ST}^*) \tag{3.19}$$

where $\phi_{2,ST}^* = \dfrac{(1-p)(1-q)}{l_2^*}$ *and* l_2^* *is the largest solution of the equation (3.19)*

In a similar manner, the equilibrium with all short-term debt is defined as above. The main difference is that in contrast to the case with all long-term debt, there is liquidation in both dates in equilibrium of all short-term debt because some projects are wrongfully rolled over and default happens at $t = 2$. This makes the characterisation of equilibrium existence harder because both l_1^* and l_2^* are affected in the opposite direction when γ changes. Let's proceed with the non-existence result first.

Proposition 16. *Equilibrium with all short-term debt does not exist if the market-making sector is not risk-averse enough.*

Proof: *if* $\gamma = 0$, $l_1^* = l_2^*$ *and thus* $G(l_1^*, l_2^*; q) < 0$. *By the continuity of the function* $G(l_1^*(\gamma), l_2^*(\gamma); q)$ *in* γ, *there exists an* $\epsilon_\gamma > 0$ *such that for all* $\gamma \in [0, \epsilon_\gamma)$, $G(l_1^*(\gamma), l_2^*(\gamma); q) < 0$.

Proposition 16 shows that there is no point to use short-term debt when the market for collateral is liquid, irrespective of the maturity choices of other firms. As mentioned in the previous section, firms issue short-term debt to try to stay ahead in the queue to avoid facing a severely low price for collateral at $t = 2$. When the market-making sector is not too risk-averse, the price advantage of liquidating early is too small to compensate for the cost associated with a defaultable short-term debt. In other words, there is no rush to exit if the market for collateral is liquid enough.

Unfortunately the complicated dependence of l_1^* and l_2^* on γ, even in the case of linear pricing function, renders the comparative static exercise difficult. For example l_1^* directly depends on γ while changing γ affects l_2^* directly and also through l_1^*, in a system of fixed-point equations. My conjecture is that when the signal quality is accurate enough, which makes the $G(l_1^*, l_2^*; q)$ close to zero, there should exist a non-empty set of Γ_{ST} such that for all $\gamma \in \Gamma_{ST}$, $G(l_1^*, l_2^*; q) > 0$. I will end this subsection with the following conjecture.

Conjecture: *When the signal quality is high enough, there exists a non-empty set* Γ_{ST} *such that for all* $\gamma \in \Gamma_{ST}$, *the equilibrium with all firm issuing short-term debt exists.*

3.4.3 The inefficient maturity collapse

Assuming the conjecture on the existence of the pure strategy short-term debt equilibrium is correct and there exists some γ such that both equilibria exist, then the switching from the all long-term debt equilibrium to the short-term debt equilibrium can be interpreted as *a maturity collapse* in the collateralised lending market.

This multiple equilibria phenomenon arises from the coordination failure of borrowers in the choice of debt maturity. In equilibrium, thanks to the first-come-first-served type of marketing clearing in the collateral market, the more the borrowers opt for short-term debt hence potential early liquidation of collateral, the worse the payoff for borrowers who opt for long term debt because the potential late liquidation of collateral at the 'end of the queue' price becomes more costly. Therefore, the choice of maturity between borrowers becomes strategic complements: a borrowing firm has stronger incentive to choose short-term debt when more firms also choose short-term debt. Interestingly this maturity collapse will only occur when the collateral market is illiquid enough (Proposition 16), which fits quite closely to the notion of 'flight from maturity' during the crisis in Gorton et al. (2014).

Finally in terms of welfare, the all short-term debt equilibrium is *Pareto-dominated* by the long-term debt equilibrium. As discussed earlier, the sole reason for a firm to use short-term debt in the model is to allow its lender to early liquidate the collateral earlier than other lenders and hence to receive a better (expected) price. When all firms opt for short-term debt, however, in expectation all lenders are just liquidating at the same time. All the firms would be better off if they can coordinate to use long-term debt to avoid the wrongful liquidation caused by the inaccuracy of the interim signals. This inefficiency result due to coordination failure suggests a regulation on debt maturity can improve welfare. The following proposition summarises this welfare result.

Proposition 17. *In the case of multiple equilibria, the short-term debt equilibrium is Pareto-dominated by the long-term debt equilibrium.*
Proof: *See Appendix.*

3.5 Conclusion

This paper shows that when market-makers have limited risk-absorbing capacity and there is uncertainty in the execution prices of sell orders, a borrower may want to shorten the debt maturity in order to allow his creditors to demand repayment and liquidate the collateral asset ahead of the others in the case of default. This strategic shortening of debt maturity in equilibrium amplifies the borrowers risk of failing to roll-over their debt and leads to excessive asset liquidation.

A maturity collapse phenomenon can arise when the asset market is illiquid enough. Thanks to the asset market friction, each firm's short maturity choice imposes a negative externality on each other and incentivises other firms to use short-term debt. This reinforcing feedback creates multiple equilibria in terms of aggregate maturity choice, firms' default risk and equilibrium collateral fire-sales discount. The inefficiency in the all short-term debt equilibrium provides an economic rational for social planner to regulation debt maturity.

Appendix

Proof of Proposition 15: For the equilibrium with all long-term debt to exists,

$$G(v, l^*_{LT}(\gamma); q) = (1 - p(S_B))(\frac{v}{l^*_{LT}(\gamma)} - 1) - p(S_B)X \leq 0 \tag{3.20}$$

Note that $l^*_{LT}(\gamma)$ defined in equation (3.16) decreases in γ and $l^*_{LT}(0) = v$. Thus $G(v, l^*_{LT}(\gamma); q) = -p(S_B)X < 0$ and $\frac{dG}{d\gamma} > 0$. Finally there exists a unique $\gamma_{LT} > 0$ that makes $G(v, l^*_{LT}(\gamma_{LT}); q) = 0$. For $\gamma > \gamma_{LT}$, $G(v, l^*_{LT}(\gamma); q) > 0$ and thus the equilibrium does not exist. □

Proof of Proposition 17: Fix a set of parameters with which multiple pure-strategy equilibria arise. Denote the liquidation values of the collateral as $\{\hat{l}_1, l^*_{LT}\}$ in the all long-term debt equilibrium and $\{l^*_1, l^*_2\}$ in the all short-term debt equilibrium. To establish the inefficiency of the short-term debt equilibrium (and *Pareto-dominated* the long-term debt equilibrium), all I need to show is that firms are worse off in the all short-term debt equilibrium (since the competitive marker-maker and the lenders receive no rents in any equilibrium)

$$U_{LT}(\hat{l}_1, l^*_{LT}) > U_{ST}(l^*_1, l^*_2)$$

where U_{LT} and U_{ST} are defined in equations (3.11) and (3.13). I will show the above inequality by proving the following two inequalities in turn:

$$U_{LT}(\hat{l}_1, l^*_{LT}) \geq U_{ST}(\hat{l}_1, l^*_{LT}) > U_{ST}(l^*_1, l^*_2) \tag{3.21}$$

The first inequality directly comes from the definition of the long-term debt equilibrium. If it is not satisfied, a firm will find it optimal to deviate and issue short-term debt given these equilibrium collateral liquidation payoffs.

To prove the second inequality in equation (3.21), I will first show that both $\hat{l}_1 > l^*_1$ and $l^*_{LT} > l^*_2$. With any collateral price function $L(\phi, \gamma)$ that decreases in ϕ, $\hat{l}_1 > l^*_1$ because $\phi_{1,ST} = \frac{(p+q-2pq)}{l^*_1} > \phi_{1,LT} = 0$.

To show $l^*_{LT} > l^*_2$, recall by the no-deviation conditions, $G(\hat{l}_1, l^*_{LT}, q) \leq 0$ and $G(l^*_1, l^*_2, q) \geq 0$. By rearranging the terms one can rewrite

$$G(\hat{l}_1, l^*_{LT}, q) \leq 0 \Leftrightarrow \frac{v}{l^*_{LT}} - 1 \leq \frac{p(S_B)X + (\frac{v}{\hat{l}_1} - 1)}{1 - p(S_B)}$$

and

$$G(l^*_1, l^*_2, q) \geq 0 \Leftrightarrow \frac{v}{l^*_2} - 1 \geq \frac{p(S_B)X + (\frac{v}{l^*_1} - 1)}{1 - p(S_B)}$$

Follow from the earlier result that $\hat{l}_1 > l^*_1$,

$$\frac{v}{l^*_2} - 1 \geq \frac{p(S_B)X + (\frac{v}{l^*_1} - 1)}{1 - p(S_B)} > \frac{p(S_B)X + (\frac{v}{\hat{l}_1} - 1)}{1 - p(S_B)} \geq \frac{v}{l^*_{LT}} - 1$$

Therefore, $l^*_{LT} > l^*_2$. Finally as $U_{ST}(l_1, l_2)$ is strictly increasing in both l_1 and l_2, the second inequality in equation (3.21) follows immediately. □

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