London School of Economics and Political Science

# Essays on Microeconomics

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# Declaration

I certify that the thesis I have presented for examination for the MPhil/PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).



I warrant that this authorization does not, to the best of my belief, infringe the rights of any third party.

I declare that my thesis consists of approximately 30,000 words.

## Statement of conjoint work

I confirm that Chapter 3 was jointly co-authored with Thiemo Fetzer and Amar Shanghavi. Mr. Shanghavi and myself conceived the original idea. I formulated and solved the theoretical model based on discussions with my co-authors. I took part in the regression specification and the discussion and interpretation of the empirical results. In total, I believe I contributed with a third of the work.

*"To think is to forget a difference"*

Jorge Luis Borges

# Abstract

This thesis consists of three chapters. Using economic theory, they analyze the effect of certain changes in the environment on some variables of economic interest.

The first chapter studies the effect of securitization on asset pricing when agents have heterogeneous beliefs about future dividends, prices and interest rates. The securities are constrained to belong to tranches of different payment priority, mimicking collateralized debt obligations (CDO). Securitization weakly increases the gap between the price of an underlying asset and any perceived present value of its dividends. The necessary and sufficient conditions for this increase to be strict are identified. In cases where there is a type of agent more sophisticated than all others, securitization can decrease the rate of return some agents receive without increasing the rate of return of none.

The second chapter checks the robustness of a surprising result in Dekel et al. (2007). The result states that strict Nash equilibria might cease to be evolutionary stable when agents are able to observe the opponent's preferences with a very low probability. The chapter shows that the result is driven by the assumption that there is no risk for the observed preferences to be mistaken. In particular, when a player may observe a signal correlated with the opponent's preferences, but the signal is noisy enough, it is shown that all strict Nash equilibria are evolutionary stable.

The third chapter studies one dimension of the social cost of bad public infrastructure in developing countries. It uses an extensive period of power rationing in Colombia throughout 1992 as a natural experiment and exploit exogenous spatial variation in the intensity of power rationing as an instrumental variable. It is estimated that power rationing increased the probability that a mother had a baby nine months later by five percent. Women who were exposed to the shock and had an additional child tend to be in worse socio-economic conditions more than a decade later.

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# Preface

This thesis is composed of three independent chapters. The first chapter is inspired by the events related to the 2007-2008 financial crisis. In a step towards understanding the crisis, the chapter presents an asset pricing model in an environment where risk-neutral agents only differ in their beliefs. The effect of the introduction of asset backed securities, known as securitization, is studied. The asset backed securities are modeled to replicate the instruments that nowadays are notorious for their role in the crisis, like mortgage backed securities (MBS) and collateralized debt obligations (CDO). The paper identifies the necessary conditions for the introduction of these instruments to increase the gap between the underlying asset price and its fundamental value. Surprisingly, securitization could decrease the rates of return some agents receive without increasing the rates of return of none.

The model suggests that regulators should consider restricting the issuing or trading of new financial instruments until parallel financial innovations allow agents to take short positions on those instruments. In this way, unsustainable asset price increases could be avoided. In fact, the original motivation for the chapter was to model financial innovations as shocks that explain both the boom and the bust of the business cycle. The optimal regulatory policy is still an open research field.

The second chapter is motivated by the insight that individuals are not necessarily motivated by selfish interests. But how can we explain that non-selfish preferences are not wiped out by natural selection? A possibility is that preferences are observable so agents reciprocate good behavior. In real life, however, preferences are never truly observed. What might be observed though are signals correlated with preferences, like the social, cultural and ethnic background. The chapter shows that if the frequency and precision of those signals are high enough, then natural selection favors preferences which lead to efficient outcomes. On the other hand, no matter what the frequency is, when the signal is noisy enough, natural selection favors preferences which lead to Nash equilibrium outcomes.

The third chapter studies a more applied question. In 1992, Colombia faced a continued shortage of electricity for almost a year. How did this affect socioeconomic outcomes in the long run? We focus on the effect on fertility and education. We formalize the intuitive idea that blackouts increase unintended pregnancy by decreasing the opportunity cost of sex. In turn, having children makes it more difficult not to drop out of school or college. We find statistical evidence that this was the case in Colombia. We additionally find that having a blackout baby does not imply that women fully adjust their fertility behavior in the long-run. In other words, the number of babies they have across their life-cycle was on average higher. Overall, the chapter suggest that energy infrastructure investment has to account for the social returns it gets from reduced fertility and higher educational attainment.

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# Chapter 1

# Asset value and securitization under heterogeneous beliefs


## Abstract

*This paper studies the effect of securitization on asset pricing when agents have heterogeneous beliefs about the stochastic process governing dividends, prices and interest rates. For this purpose, the asset pricing model of Harrison and Kreps (1978) is modified to account for the possibility for agents to issue asset-backed securities. The securities are constrained to belong to tranches of different payment priority, mimicking collateralized debt obligations (CDO). The introduction of asset-backed securities weakly increases the gap between the price of an underlying asset and any perceived present value of its dividends. A necessary condition for this increase to be strict is the absence of beliefs regarding the next-period price of the underlying asset which first-order stochastically dominate all other beliefs. In states of the world where investors with divergent beliefs buy securities from different tranches, the underlying asset is traded at a price higher than what anyone thinks it is worth. Since securities with a return below the market interest rate may be traded across agents, securitization has mixed effects on portfolio returns. It is shown that when there is an agent who is more sophisticated than all others, securitization can decrease the portfolio return of some agents without increasing the portfolio return of none.*

## 1.1 Introduction

The introduction of financial instruments that allowed banks to cash in the loans they issued facilitated the boom in house prices that preceded the 2007 financial crisis (Brunnermeier (2009)). Subprime mortgages were pooled together and sold in the form of mortgage-backed securities (MBS), meaning that mortgage repayments went to MBS investors rather than the mortgage originators. Furthermore, MBS were pooled together and sold in the form of collateralized debt obligations (CDO), implying that the MBS payouts went to CDO investors. The CDO backed by the same pool of MBS were issued with different degrees of payment priority, allowing the ones with the highest priority to be highly rated by rating agencies. The process was iterated even further to create squared CDO (Figure 1.1). Instruments like MBS, CDO and squared CDO are known as asset-backed securities (ABS), which are securities whose payout is derived from and collateralized by some underlying assets. The possibility of creating high-rated securities backed by subprime mortgages eased the credit towards subprime borrowers, fueling an increase in house prices.[1]

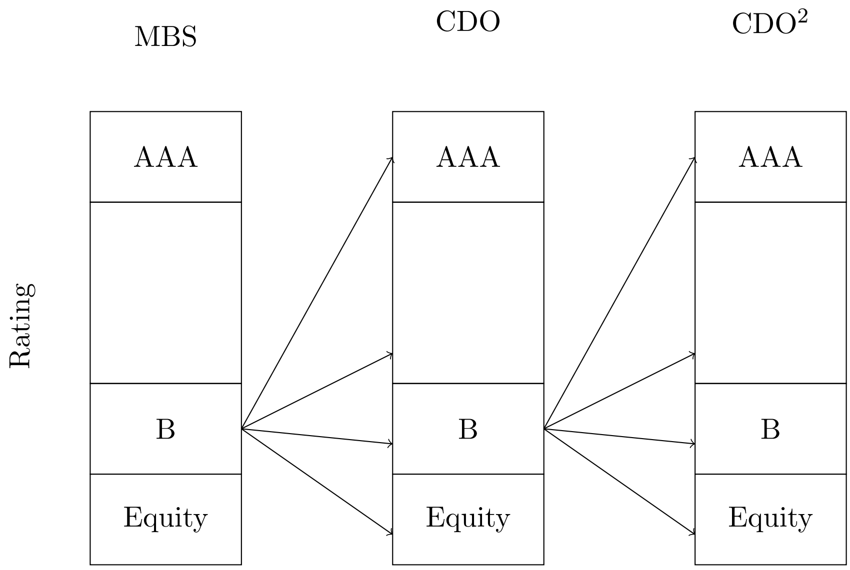


FIGURE 1.1: Multiple rounds of ABS

This paper studies the effect of the introduction of ABS on the price of the underlying assets. It does it in an environment where investors may not take into account all information available, potentially leading to heterogeneous beliefs about the future payouts of the underlying assets. The paper compares a benchmark scenario, where ABS are absent, with scenarios that include an increasing variety of ABS. The underlying hypothesis is that, due to belief disagreement, the gradual introduction of ABS inflated the price of the underlying assets above their fundamental value.

[1]For a historical account of security innovation, see Matthews (1994).

In order to give an intuition of its results, the paper begins with a motivating example. In the benchmark of this example, two risk-neutral agents trade a short-term bond and a long-term asset. The benchmark is a particular case of the Harrison and Kreps (1978) model, in which ABS are absent. Next, the paper presents a scenario where the two agents can also trade securities backed by the long-term asset. These asset-backed securities resemble real-life CDOs in their payoff structure. By comparing this scenario with the benchmark, the example illustrates how a specific form of belief disagreement allows this class of ABS to translate into an increase in the price of the long-term asset. Intuitively, the asset-backed securities allow the payoff of the long-term asset to be "sliced" in two, allowing both agents to participate in the cash flow generated by the underlying asset. As a result, the demand of the long-term asset increases.

The paper extends the motivating example to a more general case. In this general model, a finite number of risk-neutral agents trade three types of assets in each of many infinitely countable periods. These assets are: i) an infinitely-supplied, risk-free, one-period bond, ii) a finitely-supplied, long-term asset that pays an uncertain dividend every period, and iii) securities backed by the latter.

The way in which the securities backed by the long-term asset are modeled is as follows. By issuing asset-backed securities, the owner of an underlying asset commits to transferring the cash flow generated by the underlying asset to the investors who buy the ABS. The ABS could simply be pass-through securities in which the cash flow goes to investors in proportion to the securities bought. However, more sophisticated ABS could be created by "slicing" or *tranching* the cash flow into tiers or *tranches*. The ABS associated with the most senior tranche promise a fixed amount of cash unless the total cash flow from the underlying asset is below this amount. The next tranche entitles a fixed amount unless the total cash flow, net of the senior tranche payoff, is below this amount, and so on. This waterfall payment structure is also used by Malamud et al. (2010) and Grodecka (2013). For simplicity, the ABS are assumed to have a maturity of one period. The process of issuing ABS is defined as *securitization*. The process of issuing ABS with a waterfall payment structure is defined as *tranching securitization* or *tranching* for short.

The equilibrium conditions follow from the following assumptions. Firstly, it is assumed that no naked positions are allowed, which means that the ownership of the long-term asset is necessary in order to issue the ABS. Secondly, it is assumed that there is an exogenous limit on the short-selling of the ABS. Since the long-term asset is in finite supply, these assumptions imply that the supply of the ABS is finite as well. The fact that one-period bonds are in infinite supply implies that any agent can borrow as much as she wants at the market interest rate. Since there are no credit constraints, the equilibrium price of each ABS is given by the willingness to pay of whoever is the most optimistic about the ABS' next payout. The paper shows that this equilibrium exists and is unique. If no short-selling constraints are assumed, the absence of credit constraints will lead to an infinite supply of ABS and no equilibrium will exist.

The paper proceeds to check how an increasing number of tranches affects the equilibrium. Compared to the benchmark, tranching securitization can lead to an increase in the underlying asset price only if, in the benchmark scenario, no agent has a perceived distribution regarding the next-period price of the underlying asset that first-order stochastically dominates the perceived distributions of all other agents. If this is the case, tranching creates securities that are tailor-made for investors with different beliefs. Since ownership of the asset is necessary to issue ABS, the demand for the underlying asset increases. As a result, the gap between the asset price and any perceived present value of its dividends widens. Furthermore, since different tranches could be bought by investors with different beliefs, each investor will think that the others are overpaying for theirs. Hence, the whole asset will be traded at a price higher than what anyone thinks it is worth.

Although securitization is modeled as a refinement of the existing tranches, it alternatively can be seen as the process of issuing securities backed by ABS. This paper shows that the two representations are equivalent. Therefore, the model implicitly captures what Geanakoplos (1996) denominates as *pyramiding arrangements*: using the long-term asset as collateral, an agent may borrow from another agent, who in turn uses the issued debt as collateral to borrow from a third agent. In the original representation, this implies that mezzanine tranches are implicitly borrowing from senior tranches and lending to junior tranches.

Finally, the paper studies the effect of the introduction of ABS on the rates of return that agents receive on their portfolio. For this purpose, it uses the behavioral framework of Eyster and Piccione (2013), who take the model of Harrison and Kreps (1978) but provide more structure on agents' beliefs. In their model, agents are characterized by a partition of the state space. This formalizes the idea that people have a coarse understanding of their environment. The finer the partition, the more sophisticated they are. Eyster and Piccione (2013) show that an agent might obtain an average portfolio return lower than the one obtained by a less sophisticated agent. This is because a heterogeneous and coarse understanding of the environment implies a form of winner's curse: buying the long-term asset means that all other agents are pessimistic about its payoff in the next period. This paper shows that a consequence of this is that the introduction of new ABS could decrease the portfolio return some agents receive without increasing the return of none.

This paper is organized as follows. The rest of this introduction presents a literature review. Section 1.2 presents the motivating example that illustrates the setup and gives a hint of the main results. Section 1.3 describes the general model and defines the equilibrium. Section 1.4 proves the existence and uniqueness of the equilibrium. Section 1.5 studies the effect of tranching securitization on the price of the underlying asset. Section 1.6 shows that multiple rounds of tranching are equivalent to a refinement of the initial tranching. Section 1.7 studies the effect of tranching on actual portfolio returns. Section 1.8 comments on relaxing the waterfall payment constraint. Finally section 1.9 presents some conclusions and suggestions for further research.

### 1.1.1 Literature Review

The paper belongs to the branch of asset pricing literature under belief disagreement, of which Scheinkman and Xiong (2004) offer a survey. Milgrom and Stokey (1982) show that, if rational agents differ in information only but coincide on priors, there would be no trade. Because of this, numerous studies depart from the assumption of common priors. Miller (1977) and Harrison and Kreps (1978) were pioneers in this regard. A non-exhaustive list of other studies on asset pricing with heterogeneous beliefs includes Harris and Raviv (1993), Zapatero (1998), Chiarella and He (2001), Scheinkman and Xiong (2003), Cao and Ou-Yang (2009), Xiong and Yan (2010), Geanakoplos (2010), He and Xiong (2010), Cao (2011) and Hanson and Sunderam (2013). Empirically, Hong and Stein (2007) advocate the use of heterogeneous beliefs to explain observed patterns in financial markets.

The paper also intersects with the literature on financial innovation and security design, of which Duffie and Rahi (1995) offer a survey. Recent examples include Brock et al. (2009), Che and Sethi (2010), Fostel and Geanakoplos (2012), Kubler and Schmedders (2012) and Simsek (2013a,b), which share with this paper the aim of studying the potentially destabilizing effects of financial innovation. In Brock et al. (2009), the expectations with a higher average profit are favorably selected by the market. The learning process might overshoot, though. Under incomplete markets, this implies that the more Arrow-Debreu securities are introduced, the more likely the markets are to become unstable. Kubler and Schmedders (2012) also study the effect of financial innovation under heterogeneous beliefs, but in the context of an overlapping generations model. Their focus is on the effect of completing the markets on asset price volatility. Once markets become complete, wealth starts to shift across generations. As a consequence of the different propensities to consume between generations, asset price volatility increases. Simsek (2013b) shows that, under heterogeneous beliefs, the introduction of new assets could increase rather than decrease the average portfolio variance, as measured by the variance of agents' net worth.

The works of Fostel and Geanakoplos (2012) and Simsek (2013a), in particular, are closely related to this paper. Both works build upon on a two-period general equilibrium model where agents diverge in their beliefs regarding the dividend of a physical asset. In Fostel and Geanakoplos (2012), there are only two states and a continuum of beliefs. In Simsek (2013a), there is a continuum of states but only two beliefs. Both papers work with the assumption that agents cannot commit to pay what they promise. Therefore, creditors demand collateral in the form of either cash or the physical asset. This paper shares their prediction regarding the effect of tranching on asset prices, but in a context of partial equilibrium, infinite periods, any finite number of states and any finite number of beliefs. Furthermore, the aforementioned representation equivalence implies that the paper includes the case where promises themselves can be used as collateral. The paper also goes beyond the aim of these two works by studying the effect of securitization on the return each type of agent receives in the long run.

Another closely related paper to this is Garmaise (2001). His paper studies the problem of a firm designing a security to be auctioned among two investors. Investors do not have rational expectations: rather they use history to achieve statistical consistency, in a way similar to Eyster and Piccione (2013). Apart from this, the problem is essentially static. Furthermore, the firm is constrained to issue only one security. The optimal security maximizes differences in opinion, which is the same motivation found in this paper for issuers to tranche as much as possible any asset they own.

## 1.2 Motivating example

Consider an environment where trade happens in each of many infinite countable periods. Each period there is an infinitely-supplied bond used as a numeraire that entitles a payoff $R > 1$ next period. The bond can be traded for a finitely-supplied asset that yields a random dividend $d$ every period. There are three possible states of the world: $l$, $m$ and $h$. The realizations of the dividend are such that $d(l) < d(m) < d(h)$. Securities backed by the long-term asset can be issued and traded.

There are two types of infinitely-lived agents, $\mathcal{A}$ and $\mathcal{B}$. Both are risk-neutral. In every state, agent $\mathcal{A}$ thinks that the probability of transiting to any other state next period is $1/3$. Meanwhile, agent $\mathcal{B}$ thinks that the probability of staying in the current state is $1 - \epsilon$ and uniform otherwise. The disagreement may emerge as a consequence of agents using different forecasting tools or not using all information available. There are many motivation for heterogeneous beliefs, but all certainly require at least some agents to depart from rational expectations: if information is used efficiently, the realizations of the stochastic process will feed back into agents' beliefs. Implicitly, the model assumes that the learning process has stalled.

### 1.2.1 Benchmark: A world without asset-backed securities

In the benchmark scenario, there are no asset-backed securities. Because of limited short-selling, the market clears when the long-term asset's highest expected return matches the return of the short-term bond. Let $q : \{l, m, h\} \to \mathbb{R}_+$ be the function that maps states of the world into asset prices. Notation is simplified by assuming that the asset price is *cum-dividend* and that the declared dividend is to be paid next period.

Consider the state $x \in \{l, m, h\}$. When considering buying the long-term asset, both agents are willing to pay $d(x)/R$ plus the discounted asset price they expect for next period. For agent $\mathcal{A}$, the expected price of the long-term asset is $\frac{1}{3}(q(l) + q(m) + q(h))$. For agent $\mathcal{B}$, the same expectation is close to $q(x)$. If, as happens in equilibrium, $q(l) < q(m) < q(h)$, agent $\mathcal{A}$ buys the long-term asset in state $l$ whereas agent $\mathcal{B}$ buys it in state $h$. Ex-ante, it is not clear who will buy the asset in state $m$. Still, the solution to the price function $q$ when $\epsilon$ goes to zero is given by

$$q(l) = \frac{d(l) + \frac{1}{3}(q(l) + q(m) + q(h))}{R} \tag{1.1}$$

$$q(m) = \frac{d(m) + \max\left\{q(m), \frac{1}{3}(q(l) + q(m) + q(h))\right\}}{R} \tag{1.2}$$

$$q(h) = \frac{d(h) + q(h)}{R} \tag{1.3}$$

For instance, if $R = 2$, $d(l) = 0$, $d(m) = 2$ and $d(h) = 3$, then $q(l) = 1$, $q(m) = 2$ and $q(h) = 3$.

To illustrate the conditions under which the introduction of ABS increases the long-term asset price, consider the perceived distributions regarding the asset price for the next period. Let $G_i(t|x)$ be the probability that the next-period asset price is less than or equal to $t \in \mathbb{R}_+$, as perceived by agent $i \in \{\mathcal{A}, \mathcal{B}\}$ conditional on being in state $x \in \{l, m, h\}$. Figure 1.2 illustrates these conditional distributions for each of the three states. In states $l$ and $h$, the distributions can be ranked according to first-order stochastic dominance, as one distribution always takes values no lower than the other. The agent whose beliefs first-order stochastically dominate that of the other will buy

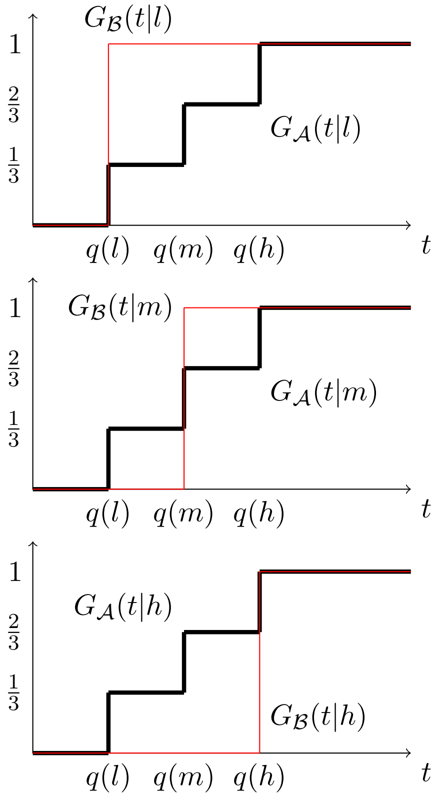


FIGURE 1.2: Perceived distributions on the asset price

the asset. In state $m$, there is no stochastic dominance between $G_{\mathcal{A}}(\cdot|m)$ and $G_{\mathcal{B}}(\cdot|m)$. Still, whoever has highest expectation regarding the asset price will buy the asset. In this state, however, agent $\mathcal{B}$ is more optimistic regarding the price not being below any value below $q(m)$, whereas agent $\mathcal{A}$ is more optimistic regarding the price not being below any value above $q(m)$. This divergence is exploited by "slicing" the asset into two securities: one with a payout attached to realizations below $q(m)$ and the other with a payout attached to realizations above $q(m)$. This is illustrated in the next subsection.

### 1.2.2 A world with asset-backed securities

Assume now the any owner of the long-term asset can issue two kinds of ABS, each of which have a maturity of one-period. The claims of the ABS add up to the realization of the payout of the long-term asset. Each ABS belongs to one of two tranches: the *senior* tranche and the *junior* tranche. Denote by $q' : \{l, m, h\} \to \mathbb{R}_+$ the asset price function in the scenario with these two kinds of ABS. The ABS in the senior tranche guarantees a payoff $q'(m)$ for next period unless the state becomes $l$, in which case it pays $q'(l)$. The payoff from the junior tranche is $q'(h) - q'(m)$ if the realized state is $h$ and 0 otherwise. In short, if $\phi_s$ and $\phi_j$ denote the payoffs from senior and junior tranches as a function of the realization $q'(x)$, then

$$\phi_s(q'(x)) = q'(x) - (q'(x) - q'(m))^+ \quad (1.4)$$
$$\phi_j(q'(x)) = (q'(x) - q'(m))^+ \quad (1.5)$$

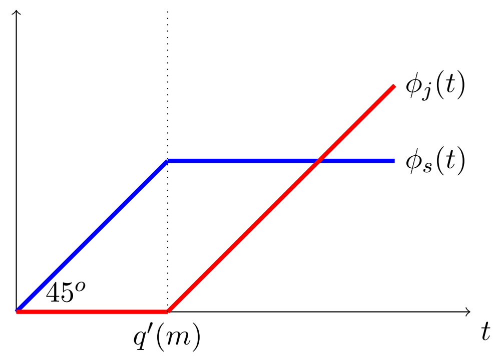


FIGURE 1.3: Payoff function from senior and junior tranches

Figure 1.3 plots the payoff from each tranche as a function of the cum-dividend price of the underlying asset. Note that i) owning the junior tranche is the same as having a one-period purchased call option on the underlying asset with strike price $q'(m)$ ii) owning the senior tranche is equivalent to owning the underlying asset entangled with a one-period written put option with strike price $q'(m)$. In this particular example, the size of each tranche is given by $q'(m)$. Such a threshold is known as an *attachment point*.

On an alternative interpretation, the buyer of the junior tranche becomes the owner of the asset, but has a debt liability to the buyer of the senior tranche and is using the underlying asset as collateral. In this example, the junior buyer promises to pay $q'(m)$ to the senior buyer in the next period. However, if the value of the collateral is $q'(l)$, the junior buyer defaults and the senior buyer seizes the collateral.

Given the assumption about the existence of an exogenous constraint on short-selling, the price of each ABS is given by the willingness to pay from whoever is most optimistic about the payout of the ABS next period, discounted by the interest rate. To start with, assume $q'(l) < q'(m) < q'(h)$. At state $m$, agent $\mathcal{B}$ expects a payoff close to $q'(m)$ from the senior tranche, while agent $\mathcal{A}$ expects $\frac{2}{3}q'(m) + \frac{1}{3}q'(l)$. Hence, the senior tranche will be bought by agent $\mathcal{B}$. Meanwhile, agent $\mathcal{B}$ expects a payoff close to zero from the junior tranche, while agent $\mathcal{A}$ expects $q'(h) - q'(m)$ with one-third probability. Hence agent $\mathcal{A}$ will buy the junior tranche. If $p(s, x)$ and $p(j, x)$ denote the prices of senior and junior tranches in state $x \in \{l, m, h\}$, then, as $\epsilon$ tends to zero,

$$\begin{aligned} p(s,m) &= \frac{q'(m)}{R} \\ p(j,m) &= \frac{\frac{1}{3}\left(q'(h) - q'(m)\right)}{R} \end{aligned}$$

Since different agents might have claims on the same asset, the introduction of ABS can break the equivalence between any single expectation regarding the value of the underlying asset and the total value of the securities. To show this, note that at state $m$, the expectation on the asset price for next period is close to $q'(m)$ for agent $\mathcal{B}$ and $\frac{1}{3}(q'(l) + q'(m) + q'(h))$ for agent $\mathcal{A}$. Then

$$\begin{aligned} p(s,m) + p(j,m) &> \frac{q'(m)}{R} \\ p(s,m) + p(j,m) &> \frac{\frac{1}{3}\left(q'(l) + q'(m) + q'(h)\right)}{R} \end{aligned}$$

which means that the payoff from selling the securities surpasses any expectation regarding the discounted price of the underlying asset.[2]

The equilibrium price for the long-term asset is given by a non-arbitrage condition. Any agent could buy the long-term asset and sell the asset-backed securities derived from it. In equilibrium, the long-term asset cum-dividend price has to equal the discounted declared dividend plus the market value of the asset-backed securities:

$$q'(x) = \frac{d(x)}{R} + p(s,x) + p(j,x) \tag{1.6}$$

If the left-hand side of (1.6) were greater than the right hand side, there would be no demand for the long-term asset. On the other hand, if the left-hand side were lesser than the right-hand side, the demand would be infinite.

For states $l$ and $h$, the equilibrium conditions can be derived by identifying who buys the tranches at these states. At state $l$, the expectations of $\mathcal{A}$ on the payoff from any tranche are greater than those of $\mathcal{B}$. The opposite is true at state $h$. Hence both tranches are bought by $\mathcal{A}$ at state $l$ and both tranches are bought by $\mathcal{B}$ at state $h$. Adding up the tranche prices yields the discounted long-term asset price expected by whoever buys both tranches. Hence the price equations for $q'(l)$ and $q'(h)$ resemble equations (1.1) and (1.3). The same is not true for $q'(m)$, however, since at state $m$ each tranche is

[2]See Fostel and Geanakoplos (2012) for a similar result.

bought by different agents. In summary, the equilibrium price function $q'$ is given by

$$q'(l) = \frac{d(l) + \frac{1}{3}\left(q'(l) + q'(m) + q'(h)\right)}{R} \tag{1.7}$$

$$q'(m) = \frac{d(m) + q'(m) + \frac{1}{3}(q'(h) - q'(m))}{R} \tag{1.8}$$

$$q'(h) = \frac{d(h) + q'(h)}{R} \tag{1.9}$$

By comparing equation (1.8) with equation (1.2), it can be seen that the introduction of ABS has increased the asset price at state $m$. This in turns increases the price at state $l$, since $\mathcal{A}$'s expectation at state $l$ is higher. Going back to the numerical example where $R = 2$, $d(l) = 0$, $d(m) = 2$ and $d(h) = 3$, it can be show that the asset price increases by $\frac{1}{20}$ at state $l$ and by $\frac{1}{4}$ at state $m$. Thanks to the reselling opportunity, the introduction of ABS increases the asset price on states in which ABS appear to be innocuous.

## 1.3 The Model

In this section, the motivating example is extended to any finite number of states, any finite number of types of agents and any number of ABS with a waterfall payment structure. All agents perceive the stochastic process governing dividends and interest rates as an irreducible Markov chain, but they may disagree on the transition probabilities.

It is assumed that the short-selling of the ABS or the long-term asset is not possible. Alternatively, it can be assumed that there is an exogenous constraint on the short-selling of the ABS and of the long-term asset. Under this assumption, the price of any ABS will be given by the willingness to pay from whoever is most optimistic about the ABS future payout. Without any limit on the amount of ABS that can be short-sold, agents would be willing to take positions that would imply infinite bets on the future state of the world. If that were the case, there will be no equilibrium.

### 1.3.1 Dividends and interest rates

The background structure of the model is given by an exogenous stochastic process governing a) the interest rate on the infinitely-supplied bond and b) the dividend generated by the finitely-supplied long-term asset. The state-space, denoted by $X$, is assumed to be finite. Buying a bond when the state is $x \in X$ entitles the owner to a payoff $R(x) > 1$ next period. Buying the long-term asset when the state is $x \in X$ entitles the owner to a dividend $d(x) \geq 0$ next period and the right to keep, sell or securitize the asset.

The state of the world is publicly observed and evolves according to a Markov process. The transition probability from state $x \in X$ to state $y \in X$ is given by $P(x, y)$, which is assumed to be strictly positive for every pair of states. This assumption is sufficient for the existence of a function $\mu : X \to (0, 1)$ such that

$$\mu(y) = \sum_{x \in X} P(x, y)\mu(x)$$

for every $y \in X$. In the long-run, the world spends a fraction $\mu(x)$ of the time at state $x \in X$, regardless of the initial state.

### 1.3.2 Beliefs

Each agent has a theory which represents their beliefs regarding the behavior of dividends, interest rates and prices. The set of theories in the market is represented by $\mathbf{C}$ and is assumed to be finite. Denote by $Q_{\mathcal{F}}(x, y)$ the transition probability from state $x \in X$ to state $y \in X$ perceived by theory $\mathcal{F} \in \mathbf{C}$. An agent is said to have rational expectations if $Q_{\mathcal{F}} = P$.

Consider a random variable $g : X \to \mathbb{R}$ which could be a price, dividend, interest rate or any function of them. The expectation of agent $\mathcal{F}$ of $g$ one period forward, conditional on $x \in X$, is given by

$$E_{\mathcal{F}}(g)(x) := \sum_{y \in X} g(y) Q_{\mathcal{F}}(x, y)$$

Most of the results in this paper are agnostic about the source of heterogeneity of beliefs. Henceforth, the true stochastic process $P$ has no role within these results. However, sometimes the performance of each theory is compared with the truth, which should be interpreted as the long-term outcome. In this case, beliefs satisfy the constraints of Eyster and Piccione (2013). In their framework, each theory $\mathcal{F} \in \mathbf{C}$ is equivalent to a partition of $X$. An element of $\mathcal{F}$ is known as a block of theory $\mathcal{F}$. If $x$ and $z$ are in the same block, then the transition probabilities perceived by theory $\mathcal{F}$ in states $x$ and $z$ are the same. In particular, for $x, y \in X$,

$$Q_{\mathcal{F}}(x, y) := \frac{\sum_{z \in \mathcal{F}(x)} P(z, y)\mu(z)}{\sum_{z \in \mathcal{F}(x)} \mu(z)} \tag{1.10}$$

where $\mathcal{F}(x)$ is the block in $\mathcal{F}$ that contains $x$. Note that if every block in $\mathcal{F}$ is a singleton, then an agent with theory $\mathcal{F}$ has rational expectations. On the other hand, if $\mathcal{F} = \{X\}$, then $Q_{\mathcal{F}}(x, y) = \mu(y)$ for all $x \in X$. In the Eyster and Piccione (2013) framework, the long-run average of any random variable $g : X \to \mathbb{R}$ can be obtained by taking the expectation for $g$ next period by any $\mathcal{F} \in \mathbf{C}$ and averaging across states:

$$\sum_{x \in X} \left(E_{\mathcal{F}}\left(g\right)\left(x\right)\right)\mu(x) = \sum_{y \in X} g(y)\mu(y)$$

In this case, all agents achieve some degree of statistical consistency despite not necessarily having rational expectations.

Consider the motivating example of section 1.2. The set of beliefs was given by $\mathbf{C} = \{\mathcal{A}, \mathcal{B}\}$. The transition probabilities were given by $Q_{\mathcal{A}}(x,y) = \frac{1}{3}$ and $Q_{\mathcal{B}}(x,x) = 1 - \epsilon$ for $x, y \in X := \{l, m, h\}$. The case studied corresponded to the limiting case where $\epsilon \to 0$. The example was agnostic regarding the true Markov process $P : X^2 \to (0,1)$. However, if $P(x,x) = 1 - \epsilon$ and $P(x,y) = \epsilon/2$ for $x, y \in X$ and $x \neq y$, the beliefs for $\mathcal{A}$ and $\mathcal{B}$ can be represented as the partitions $\{\{l, m, h\}\}$ and $\{\{l\}, \{m\}, \{h\}\}$, respectively. Agent $\mathcal{A}$'s expectations, despite not being rational, achieve the aforementioned statistical consistency.

### 1.3.3 Asset-backed securities

The ownership of the long-term asset allows the possibility of obtaining liquidity (bonds) not only by reselling the asset but also through issuing and selling ABS. It is assumed that the ABS have a maturity of one-period. Each ABS is associated with a particular interval or *tranche*. The security is a contract in which the owner of the long-term asset (the issuer) commits to transferring to the holder of the security (the investor) a (non-negative) amount of bonds next period. This amount depends on the realization of the asset price relative to the tranche when payment is due. The securities are backed by the long term asset in the sense that the net transfers from the issuer to all investors are always equal to the payout of the underlying asset.

Formally, let $\mathcal{T}$ be a collection of non-overlapping intervals that partition the set $[0, \infty)$, which is the set of all conceivable realizations of the asset price. The partition $\mathcal{T}$ is known as a *tranching* and an interval $\tau \in \mathcal{T}$ is known as a *tranche*. Define $\underline{\tau} := \inf(\tau)$ and $\overline{\tau} := \sup(\tau)$. The payoff function $\phi_\tau : \mathbb{R}_+ \to \mathbb{R}_+$ from a security associated with tranche $\tau$ is given by

$$\phi_\tau(t) := (t - \underline{\tau})^+ - (t - \overline{\tau})^+ \tag{1.11}$$

where $t \in \mathbb{R}_+$ is a realization for the underlying asset price.

The point $\underline{\tau}$ is known as the *attachment point* of tranche $\tau$. This is because the tranche only starts to pay off when the realization for the asset price is above $\underline{\tau}$. Above $\overline{\tau}$, the payoff from tranche $\tau$ stalls at $\overline{\tau}-\underline{\tau}$ and the payoff from the next junior tranche (if any) detaches from 0. Figure 1.4 illustrates the payoff function for a generic tranche.

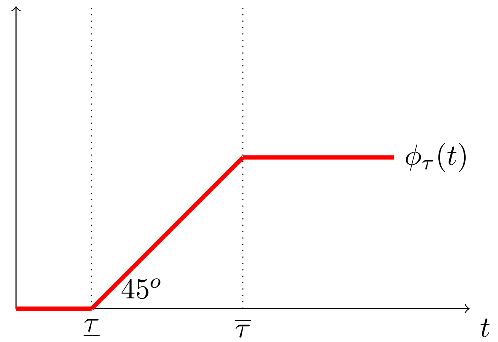


FIGURE 1.4: Payoff function of a generic tranche $\tau$

The securities are backed up by the long-term asset in the sense that they are *budget balanced*: if $t$ is the realization of the asset price at the expiration date, the tranches payoffs add up to $t$:

$$\sum_{\tau\in\mathcal{T}} \phi_\tau(t) = t$$

In addition, there is *limited liability* for the issuer: $\phi_\tau(t) \geq 0$ for all $\tau \in \mathcal{T}$ and $t \in \mathbb{R}_+$.

In the example of subsection 1.2.1, there was no securitization so the tranching was given by $\mathcal{T} = \{[0, \infty)\}$. In subsection 1.2.2, there were only two tranches and the attachment point was calibrated so that the tranching was $\mathcal{S} = \{[0, q'(m)), [q'(m), \infty)\}$. The senior tranche was $s = [0, q'(m))$ while the junior tranche was $j = [q'(m), \infty)$. Hence, the payoff functions in equations (1.4) and (1.5) could be equivalently defined by using the expression in (1.11).

### 1.3.4 Equilibrium

In equilibrium, the demand for the long-term asset and the demand for each security have to match their respective supply. The exogenous limit on short-selling guarantees that this supply is finite. For $x \in X$, let $p(\tau, x)$ be the price of the security associated with tranche $\tau \in \mathcal{T}$ and $q(x)$ be the price of the long-term asset.

**Definition 1.1.** *For a given tranching $\mathcal{T}$, a waterfall equilibrium is a pair of functions $q : X \to \mathbb{R}_+$ and $p : \mathcal{T} \times X \to \mathbb{R}_+$ such that*

$$q(x) = \frac{d(x)}{R(x)} + \sum_{\tau\in\mathcal{T}} p(\tau, x) \tag{1.12}$$

*and*

$$p(\tau, x) = \frac{\max_{\mathcal{F}\in\mathbf{C}} E_{\mathcal{F}}\left(\phi_\tau(q)\right)(x)}{R(x)} \tag{1.13}$$

*for $x \in X$ and $\tau \in \mathcal{T}$.*

Equation (1.12) states that, in equilibrium, zero profits are made by buying the long-term asset and selling the securities derived from it. Equation (1.13) states that the price of each security is given by the highest discounted expectation regarding the payoff from the corresponding tranche.

## 1.4 Existence and uniqueness

The existence and uniqueness of a waterfall equilibrium is proved by constructing the following object: For any tranching $\mathcal{T}$, define a mapping $\Psi_{\mathcal{T}}$ from the set of price functions $\mathbb{R}_+^X$ to itself. For $q \in \mathbb{R}_+^X$ and $x \in X$, the mapping is given by

$$\Psi_{\mathcal{T}}(q)(x) = \frac{d(x) + \sum_{\tau \in \mathcal{T}} \max_{\mathcal{F} \in \mathbf{C}} E_{\mathcal{F}} \left( (q - \underline{\tau})^+ - (q - \overline{\tau})^+ \right)(x)}{R\,(x)} \tag{1.14}$$

By substituting equation (1.13) into equation (1.12), it follows that a long-term asset price function $q$ must be a fixed point of $\Psi_{\mathcal{T}}$ in order to be part of an equilibrium.

**Proposition 1.1.** *For every tranching $\mathcal{T}$, there is a unique waterfall equilibrium.*

*Proof.* The mapping $\Psi_{\mathcal{T}}$ is a contraction mapping and therefore has a unique fixed point since it satisfies Blackwell's sufficient conditions:

1. (monotonicity) For all $q, q' \in \mathbb{R}_+^X$ such that $q(x) \leq q'(x)$ for all $x \in X$,

$$\Psi_{\mathcal{T}}(q)(x) \leq \Psi_{\mathcal{T}}(q')(x)$$

   for all $x \in X$.

2. (discounting) For any $q \in \mathbb{R}_+^X$ and $c \geq 0$,

$$\Psi_{\mathcal{T}}(q + c)(x) \leq \Psi_{\mathcal{T}}(q)(x) + \left( \min_{y \in X} R\,(y) \right)^{-1} c$$

   for all $x \in X$.

□

## 1.5 The effects of tranching

This section studies the effect of tranching securitization, understood as the introduction of new ABS with a waterfall payment constraint, on the equilibrium price of the long-term asset. Formally, tranching securitization means a refinement of the existing tranching:

**Definition 1.2.** *An economy engages in tranching securitization if it switches from a tranching $\mathcal{T}$ to a tranching $\mathcal{S}$, where $\mathcal{S}$ is a refinement of $\mathcal{T}$.*

This means that agents who buy the long-term asset are able to sell a greater variety of tranches. It is shown in section 1.6 that this definition is equivalent to being able to issue new ABS backed by the existing ABS.

### 1.5.1 Effect on asset prices

Denote by $q_{\mathcal{T}}$ the waterfall equilibrium price for the long-term asset for tranching $\mathcal{T}$. Further securitization cannot decrease the price of the underlying asset:

**Lemma 1.1.** *If $\mathcal{S}$ is a refinement of $\mathcal{T}$, then*

$$q_{\mathcal{T}}(x) \leq q_{\mathcal{S}}(x)$$

*for all $x \in X$*

Proof: See appendix.

Intuitively, there is no harm to the issuer from slicing the cash flow of the underlying asset into more tranches: if the issuer is lucky, investors who were outbid when the big tranches were issued will push harder when bidding for smaller tranches. At worst, investors will be indifferent.[3]

It is worth comparing the benchmark scenario, where there is no tranching, with any scenario where there is some degree of tranching:

**Corollary 1.1.** *Define $\underline{q} := q_{\{[0,\infty)\}}$ as the price of the long-term asset in the absence of any tranching. Then, for any tranching $\mathcal{T}$,*

$$\underline{q}(x) \leq q_{\mathcal{T}}(x) \tag{1.15}$$

*for every $x \in X$.*

[3]It should be noted, though, that this result will hold even if the waterfall constraint is absent - see section 1.8.

*Proof.* Since any tranching $\mathcal{T}$ is a refinement of $\{[0,\infty)\}$, inequality (1.15) follows from Lemma 1.1 □

The function $\underline{q}$ is a lower bound on asset prices in the sense that any tranching $\mathcal{T}$ cannot decrease the asset price below $\underline{q}(x)$ at any $x \in X$. Intuitively, the price of the long-term asset without any tranching, $\underline{q}(x)$, is given by the highest expectation on the cash flow coming from the whole asset. In contrast, the price of the long-term asset when there is some degree of tranching, $q_{\mathcal{T}}(x)$, is obtained by adding up the highest expectations on the cash flow coming from each tranche. The highest expectation on the cash flow from the whole asset cannot be higher than the sum of the highest expectations on the cash flow from each tranche.

Still, a satisfactory explanation for the issuing of ABS will have to show that it *strictly* increases the asset price in at least one state. If this is the case, then it will be natural to assume that anyone who buys the long-term asset will issue ABS until the tranching stops increasing the asset value. A tranching is said to be *issuer-optimal* if no other tranching can increase the asset price in any state:

**Definition 1.3.** *A tranching $\mathcal{T}$ is issuer-optimal if*

$$q_{\mathcal{S}}(x) \leq q_{\mathcal{T}}(x)$$

*for any tranching $\mathcal{S}$ and all $x \in X$.*

In order to characterize the necessary and sufficient conditions for a tranching to be issuer-optimal, the following object is introduced: Let $I_A$ be the indicator function for event $A$. For $t \in \mathbb{R}_+$, define $G^{\mathcal{T}}_{\mathcal{F}}(t|x)$ as the probability at state $x$ that the asset price next period is no higher than $t$, as perceived by theory $\mathcal{F}$ when the tranching is $\mathcal{T}$:

$$G^{\mathcal{T}}_{\mathcal{F}}(t|x) := E_{\mathcal{F}}\left(I_{\{q_{\mathcal{T}} \leq t\}}\right)(x)$$

**Proposition 1.2.** *A tranching $\mathcal{T}$ is issuer-optimal if and only if, for each $\tau \in \mathcal{T}$ and $x \in X$, there is a theory $\mathcal{G} \in \mathbf{C}$ such that*

$$G^{\mathcal{T}}_{\mathcal{G}}(t|x) \leq G^{\mathcal{T}}_{\mathcal{F}}(t|x)$$

*for all $t \in \tau$ and all $\mathcal{F} \in \mathbf{C}$.*

PROOF: See appendix.

Proposition 1.2 states that a tranching is issuer-optimal if and only if for every state and every tranche, there is someone who is more optimistic than everyone else regarding the asset price not being below any realization in the tranche. If this is not the case, then there is a tranching that will increase the asset price in every state. This is because at some state there will be someone willing to outbid whoever is buying the current tranche whenever a smaller tranche is issued.

A natural question is whether the benchmark with no tranching is issuer-optimal. Denote by $\underline{G}_{\mathcal{F}}(\cdot|x)$ the cumulative distribution function of the asset price for theory $\mathcal{F}$ when there is no tranching, i.e.,

$$\underline{G}_{\mathcal{F}}\left(t|x\right) := E_{\mathcal{F}}\left(I_{\{\underline{q} \leq t\}}\right)(x)$$

for $t \in \mathbb{R}_{+}$.

**Corollary 1.2.** *The tranching* $\{[0, \infty)\}$ *is issuer-optimal if and only if, for each* $x \in X$*, there is a theory* $\mathcal{G} \in \mathbf{C}$ *such that* $\underline{G}_{\mathcal{G}}\left(\cdot|x\right)$ *first-order stochastically dominates* $\underline{G}_{\mathcal{F}}\left(\cdot|x\right)$ *for all* $\mathcal{F} \in \mathbf{C}$*.*

*Proof.* Follows directly from proposition 1.2 □

If there is only one theory in the market, the first-order stochastic dominance condition trivially holds. As expected, the benchmark without tranching is issuer-optimal if there is no belief disagreement.

As an example of a tranching that is not issuer-optimal, consider $\{[0, \infty)\}$ in subsection 1.2.1: there is no first-order stochastic dominance between beliefs at state $m$, as illustrated by Figure 1.2. On the other hand, the tranching $\{[0, q'(m)), [q'(m), \infty)\}$ in subsection 1.2.2 is issuer-optimal: for each tranche and each state there is always an agent whose conditional cumulative distribution of the asset price caps the other one along the realizations in the tranche.

Finally, it is worth remarking that whenever the introduction of ABS increases the asset price, the asset will be traded at a price higher than anyone thinks it is worth:

**Proposition 1.3.** *If* $\{[0, \infty)\}$ *is not issuer-optimal, then there is a tranching* $\mathcal{T}$ *in which the waterfall equilibrium* $q_{\mathcal{T}}$ *is such that*

$$q_{\mathcal{T}}(x) > \frac{d(x) + \max_{\mathcal{F} \in \mathbf{C}} E_{\mathcal{F}}\left(q_{\mathcal{T}}\right)(x)}{R(x)}$$

*in at least one state* $x \in X$*.*

*Proof.* By Definition 1.3, there is a tranching $\mathcal{T}$ such that

$$\underline{q}(x) \leq q_{\mathcal{T}}(x)$$

with strict inequality for at least one $x \in X$.

Suppose that

$$q_{\mathcal{T}}(x) \leq \frac{d(x) + \max_{\mathcal{F} \in \mathbf{C}} E_{\mathcal{F}} \left(q_{\mathcal{T}}\right)(x)}{R(x)}$$

for every $x \in X$. Since

$$\frac{d(x) + \max_{\mathcal{F} \in \mathbf{C}} E_{\mathcal{F}} \left(q_{\mathcal{T}}\right)(x)}{R(x)} \leq \Psi_{\mathcal{T}} \left(q_{\mathcal{T}}\right)$$

then

$$q_{\mathcal{T}}(x) = \frac{d(x) + \max_{\mathcal{F} \in \mathbf{C}} E_{\mathcal{F}} \left(q_{\mathcal{T}}\right)(x)}{R(x)}$$

for every $x \in X$. But if that is the case, then both $q_{\mathcal{T}}$ and $\underline{q}$ are fixed points of $\Psi_{\{[0,\infty)\}}$, which is a contradiction to the uniqueness established in Proposition 1.1. □

Intuitively, in an equilibrium different from the no-securitization benchmark, there has to be at least one state in which the tranching allows for an investor who otherwise would not have bought the whole asset to buy a piece of it. Since investors with different theories will be buying different tranches of the same asset, each of them will think that the others are overpaying for theirs.

### 1.5.2 Relation to Fundamentals

Consider the relationship between the asset price and the perceived fundamental values, namely the present value of dividends perceived by each theory in the market. Let $v_{\mathcal{F}}(x)$ be the present value of dividends perceived by $\mathcal{F} \in \mathbf{C}$ at sate $x \in X$. This value function is given by the recursive solution to

$$v_{\mathcal{F}}(x) = \frac{d(x) + E_{\mathcal{F}} \left(v_{\mathcal{F}}\right)(x)}{R(x)} \tag{1.16}$$

for $x \in X$.

**Proposition 1.4.** *For any tranching $\mathcal{T}$ and any collection of theories $\mathbf{C}$,*

$$v_{\mathcal{F}}(x) \leq q_{\mathcal{T}}(x)$$

*for all* $x \in X$ *and all* $\mathcal{F} \in \mathbf{C}$*. Furthermore, if* $\{[0, \infty)\}$ *is not issuer-optimal, then there is a tranching* $\mathcal{T}$ *such that*

$$v_{\mathcal{F}}(x) < q_{\mathcal{T}}(x)$$

*for all* $x \in X$ *and* $\mathcal{F} \in \mathbf{C}$*.*

*Proof.* The definition of waterfall equilibrium for the benchmark case

$$\underline{q}(x) = \frac{d(x) + \max_{\mathcal{F} \in \mathbf{C}} E_{\mathcal{F}}\left(\underline{q}\right)(x)}{R(x)}$$

and equation (1.16) imply

$$v_{\mathcal{F}}(x) \leq \underline{q}(x) \tag{1.17}$$

for all $x \in X$ and $\mathcal{F} \in \mathbf{C}$. The first claim follows from inequality (1.17) and inequality (1.15). The second claim follows from inequality (1.17) and Lemma 1.4 in the appendix. □

Proposition 1.4 states that if the benchmark equilibrium without securitization is not issuer-optimal, then further securitization will eventually increase the gap between the asset price and any perceived present value of dividends. Because of reselling opportunities, securitization increases this gap at every state.

## 1.6 Iterated securitization

In section 1.5, increased securitization has been modeled as a refinement of the existing tranching, which is equivalent to creating more tranches for the cash flow coming from the long-term asset. Securitization, however, is also seen as issuing securities backed by asset-backed securities, as when CDO are created based on existing MBS or squared CDO are created based on existing CDO. This section shows the equivalence between these two interpretations. Remarkably, any tranching is equivalent to issuing debt using debt securities as collateral and iterating this process a finite number of times.

### 1.6.1 Example

Consider the case where $X = \{a, b, c, d\}$ and the probability of a state going back to itself is close to one and uniform otherwise. Hence, the system spends a quarter of the time in each state. The interest rate is assumed to be constant. The collection of theories in

Table 1.1: Security payoffs for the issuer-optimal tranching

| States | $\phi_e$ | $\phi_j$ | $\phi_m$ | $\phi_s$ |
|---|---|---|---|---|
| $a$ | $q(a)-q(b)$ | $q(b)-q(c)$ | $q(c)-q(d)$ | $q(d)$ |
| $b$ | 0 | $q(b)-q(c)$ | $q(c)-q(d)$ | $q(d)$ |
| $c$ | 0 | 0 | $q(c)-q(d)$ | $q(d)$ |
| $d$ | 0 | 0 | 0 | $q(d)$ |

the market is given by $\mathbf{C} = \{\mathcal{A}, \mathcal{B}, \mathcal{C}\}$, where

$$\begin{aligned} \mathcal{A} &= \{\{a,d\},\{b,c\}\} \\ \mathcal{B} &= \{\{a,b,d\},\{c\}\} \\ \mathcal{C} &= \{\{a,b,c,d\}\} \end{aligned}$$

All theories in the market achieve statistical consistency: the perceived transition probability for agent $\mathcal{A}$ from states $a$ or $d$ to states $a$ or $d$ tends to one half, the perceived transition probability for agent $\mathcal{B}$ from states $a$, $b$ or $d$ to states $a$, $b$ or $d$ tends to one third and the perceived transition probability for agent $\mathcal{C}$ from any state to any other tends to one fourth.

For this example, it can be shown that a tranching that is issuer-optimal is one where there are four tranches and the attachment points coincide with the realizations of the asset price. Let $q$ be the waterfall equilibrium price function for the issuer-optimal tranching. Assume that the dividend function is such that $q(a) > q(b) > q(c) > q(d)$. In order of seniority, the tranches are labeled $s$ (senior), $m$ (mezzanine), $j$ (junior) and $e$ (equity). Table 1.1 presents the payoff function for each tranche.

In state $d$, theory $\mathcal{A}$ buys the tranche $e$, theory $\mathcal{B}$ buys the tranche $j$ and theory $\mathcal{C}$ buys the tranche $m$. All agents have the same willingness to pay regarding tranche $s$. In state $d$, three different types of agents have claims on the underlying asset.

Instead of a unique round of securitization with four tranches, consider three rounds of securitization. In the first round, the underlying asset is securitized into debt and equity tranches. In the second round, the debt tranche of the first round is securitized into other debt and equity tranches. In the third round, the debt tranche of the second round is securitized into more debt and equity tranches (Figure 1.5). Let $\phi_e^i$ and $\phi_f^i$ be the payoffs from the equity and the debt tranches in the $i$-th round of securitization, where $i \in \{1,2,3\}$. Table 1.2 shows the assumed payoff for each tranche. By construction, the payoff from the debt and the equity tranches add up to the payoff from the debt tranche from the previous round.

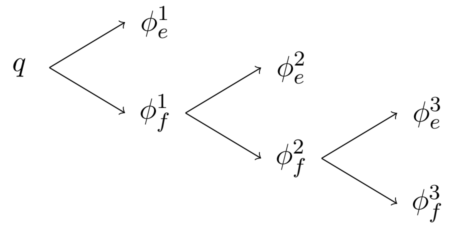


FIGURE 1.5: Iterated securitization

TABLE 1.2: Security payoffs for the iterated case

| States | $\phi_e^1$ | $\phi_f^1$ | $\phi_e^2$ | $\phi_f^2$ | $\phi_e^3$ | $\phi_f^3$ |
|---|---|---|---|---|---|---|
| $a$ | $q(a)-q(b)$ | $q(b)$ | $q(b)-q(c)$ | $q(c)$ | $q(c)-q(d)$ | $q(d)$ |
| $b$ | 0 | $q(b)$ | $q(b)-q(c)$ | $q(c)$ | $q(c)-q(d)$ | $q(d)$ |
| $c$ | 0 | $q(c)$ | 0 | $q(c)$ | $q(c)-q(d)$ | $q(d)$ |
| $d$ | 0 | $q(d)$ | 0 | $q(d)$ | 0 | $q(d)$ |

Consider the equilibrium in state $d$. The equity tranche of the 1st round will be bought by theory $\mathcal{A}$, the equity tranche of the 2nd round will be bought by theory $\mathcal{B}$, and the equity tranche of the 3rd round will be bought by theory $\mathcal{C}$. The value of the debt tranche on the 3rd round will be $q(d)$, discounted by the interest rate. Given this and the willingness to pay by agent $\mathcal{C}$ on the equity tranche of the 3rd round, the value of the debt tranche in the 2nd round will be $q(d)$ plus three quarters of $q(c)-q(d)$, discounted by the interest rate. Given this and the willingness to pay by agent $\mathcal{B}$ on the equity tranche of the 2nd round, the value of the debt tranche in the 1st round will be $q(d)$ plus three quarters of $q(c)-q(d)$ plus two thirds of $q(b)-q(c)$, discounted by the interest rate. Given this and the willingness to pay by agent $\mathcal{A}$ on the equity tranche of the 1st round, the value of the asset at state $d$ will be the dividend at $d$ plus $q(d)$ plus three quarters of $q(c)-q(d)$ plus two thirds of $q(b)-q(c)$ plus half of $q(a)-q(c)$, discounted by the interest rate. This is the same asset price equation for the aforementioned case of one round of securitization with four tranches.

A way of reading the equilibrium transactions at state $d$ is that agent $\mathcal{A}$ is borrowing from agent $\mathcal{B}$ using the underlying asset as collateral. In turn, agent $\mathcal{B}$ is borrowing from agent $\mathcal{C}$ using the debt issued by $\mathcal{A}$ as collateral. The equilibrium features a *pyramiding arrangement*, as defined by Geanakoplos (1996).

### 1.6.2 General Case

In order to define what will be labeled an *iterated securitization equilibrium*, consider the following structure. Let there be a tree where $\mathbb{N}$ is its finite set of nodes, $r \in \mathbb{N}$ is its root node and $\mathbb{Z} \subset \mathbb{N}$ is its set of terminal nodes. Let $a : \mathbb{N}/\{r\} \to \mathbb{N}/\mathbb{Z}$ be a function where $a(n)$ denotes the predecessor of $n$. The root node $r$ represents the long-term asset.

Each node $n \in \mathbb{N}/\{r\}$ represents a security backed by asset $a(n)$. Nodes in $\mathbb{Z}$ represent securities that are not further securitized.

For $m \in \mathbb{N}/\mathbb{Z}$, denote by $S(m)$ the set of $m$'s successors:

$$S(m) = \{n \in \mathbb{N}/\{r\} : a(n) = m\}$$

Let $(\tau_n)_{n\in\mathbb{N}}$ be a collection of intervals such that

1. $\tau_r = [0, \infty)$ and
2. for $m \in \mathbb{N}/\mathbb{Z}$, the collection $(\tau_n)_{n\in S(m)}$ partitions the interval

$$[0, \sup(\tau_m) - \inf(\tau_m)]$$

Each security in $\mathbb{N}/\{r\}$ is a straddle with a maturity of one period. Its payoff is a function of the cash flow from its predecessor. Specifically, if $t$ is the cash flow from asset $a(n)$, then the cash flow for security $n \in \mathbb{N}/\{r\}$ would be given by:

$$\zeta_n(t) = (t - \inf(\tau_n))^+ - (t - \sup(\tau_n))^+ \tag{1.18}$$

Hence, for $m \in \mathbb{N}/\mathbb{Z}$:

$$\zeta_m(t) = \sum_{n\in S(m)} \zeta_n(t)$$

This confirms that the cash flow from securities in $S(m)$ add up to the cash flow from asset $m$.

Let $\gamma_n : X \to \mathbb{R}_+$ be the cash flow for security $n \in \mathbb{N}$ as a function of the state of the world. If $q : X \to \mathbb{R}_+$ is the price function for the long-term asset, then $\gamma_r = q$ and

$$\gamma_n(x) = \zeta_n(\gamma_{a(n)}(x)) \tag{1.19}$$

for $n \in \mathbb{N}/\{r\}$ and $x \in X$.

A tuple $\Sigma = (\mathbb{N}, r, \mathbb{Z}, a, (\tau_n)_{n\in\mathbb{N}})$ is labeled an *iterated securitization structure*.

**Definition 1.4.** *For a given iterated securitization structure $\Sigma$, an iterated securitization equilibrium is a function $p_\Sigma : \mathbb{N} \times X \to \mathbb{R}_+$ such that, for $x \in X$,*

*1.*

$$p_\Sigma(r, x) = \frac{d(x)}{R(x)} + \sum_{n\in S(r)} p_\Sigma(n, x) \tag{1.20}$$

2. *For* $m \in \mathbb{N}/(\mathbb{Z} \cup \{r\})$,

$$p_\Sigma(m,x) = \sum_{n \in S(m)} p_\Sigma(n,x) \tag{1.21}$$

3. *For* $z \in \mathbb{Z}$,

$$p_\Sigma(z,x) = \frac{\max_{\mathcal{F} \in \mathbf{C}} E_{\mathcal{F}}(\gamma_z)(x)}{R(x)} \tag{1.22}$$

*where* $\gamma_r = p_\Sigma(r,\cdot)$ *and, for* $n \in \mathbb{N}/\{r\}$, $\gamma_n$ *is given by equation (1.19).*

The function $p_\Sigma(r,\cdot)$ is the price function for the long-term asset. Equation (1.20) states that it is given by the discounted declared dividend plus the added prices of the securities derived from it. Equation (1.21) states that for the securities that are not terminal nodes, their price is given by the added prices of the securities derived from them. Finally, equation (1.22) gives the equilibrium for the terminal nodes, i.e. the securities that are not further securitized. Their price is given by the highest expectation regarding the cash flow coming from them. Ultimately, the cash flow from the securities at the terminal nodes will depend on the price of the long-term asset in the next period.

**Proposition 1.5.** *For any iterated securitization structure* $\Sigma$, *there is a tranching* $\mathcal{T}$ *such that*

$$p_\Sigma(r,x) = q_{\mathcal{T}}(x)$$

*for* $x \in X$

Proof: See appendix.

Proposition 1.5 states the equivalence between the equilibrium concepts in Definitions 1.2 and 1.4. For any iterated securitization equilibrium, the price of the long-term asset is the same as in a waterfall equilibrium for a suitable tranching $\mathcal{T}$. Intuitively, multiple rounds of securitization are redundant since the same outcome can be obtained by tranching the long-term asset into $|\mathbb{Z}|$ securities in a single round.

## 1.7 Effect on rates of return

This section studies the effect of the introduction of ABS on the excess return obtained by each theory in the market, defined as the actual payoff of forgoing one bond to buy ABS instead whenever this is perceived as worthwhile.

The effect of the introduction of new ABS on returns is subtle. Agents who are not optimistic about the overall expected payoff from an underlying asset could be the most optimistic regarding the payoff being above a certain threshold. Without ABS, they will not buy the asset. By introducing the appropriate tranching, they will buy a security with an attachment point at the aforementioned threshold. However, this means they will be buying the leftovers from whoever would have bought the overall asset without any ABS. On the other hand, by being outbid by less sophisticated agents, they may be being stopped from buying tranches that do not have a promising payoff. Except for particular cases, the effect of the introduction of ABS on returns is ambiguous.

Formally, consider the strategy of holding one bond and exchanging it for asset-backed securities only when $\mathcal{F} \in \mathbf{C}$ perceives this to be worthwhile. Denote by $\Delta_{\mathcal{F}}$ the return of this strategy in excess of always holding a bond. Let $\psi_\tau := \phi_\tau(q_{\mathcal{T}})$ be the function that maps states of the world into equilibrium payoffs for tranche $\tau \in \mathcal{T}$. Denote by $\mathbf{B}(\tau, x)$ the set of theories that buy tranche $\tau \in \mathcal{T}$ at state $x \in X$:

$$\mathbf{B}(\tau, x) = \{\mathcal{F} \in \mathbf{C} : E_{\mathcal{F}}(\psi_\tau)(x) \geq E_{\mathcal{G}}(\psi_\tau)(x) \text{ for all } \mathcal{G} \in \mathbf{C}\}$$

The *actual* as opposed to the *perceived* payoff from buying the security in tranche $\tau \in \mathcal{T}$ at state $x \in X$ is given by:

$$T(\psi_\tau)(x) := \sum_{y \in X} \psi_\tau(y) P(x, y)$$

Finally, let $p(\tau, \cdot)$ be the equilibrium price function for tranche $\tau \in \mathcal{T}$. The excess return from buying asset-backed securities whenever $\mathcal{F} \in \mathbf{C}$ perceives this as worthwhile is:

$$\Delta_{\mathcal{F}} = \sum_{x \in X} \left( \sum_{\tau \in \mathcal{T}} \left(T(\psi_\tau)(x) - R(x) p(\tau, x)\right) I_{\{\mathcal{F} \in \mathbf{B}(\tau, x)\}} \right) \mu(x) \tag{1.23}$$

It is worth remembering that, in equilibrium, every agent *perceives* their excess return to be zero. If it were positive, their demand for the long-term asset would exceed the supply. If it were negative, the agent would be buying the short-term bond instead of buying the long-term asset. Still, the *actual* average excess return $\Delta_{\mathcal{F}}$ is not necessarily zero.[4]

Equation (1.23) could be rewritten as the average gap between actual and perceived payoffs from each tranche. Since the equilibrium price for tranche $\tau$ is given by

$$p(\tau, x) = \frac{\max_{\mathcal{F} \in \mathbf{C}} E_{\mathcal{F}}(\psi_\tau)(x)}{R(x)}$$

[4]In fact, the actual excess returns could even be positive: see Example 5 in Eyster and Piccione (2013).

it follows that

$$\Delta_{\mathcal{F}} = \sum_{x \in X} \left( \sum_{\tau \in \mathcal{T}} \left( T(\psi_\tau)(x) - E_{\mathcal{F}}(\psi_\tau)(x) \right) I_{\{\mathcal{F} \in \mathbf{B}(\tau,x)\}} \right) \mu(x)$$

meaning that the excess return is an average of the gap between actual and expected payoffs whenever the tranches are bought.

An immediate result obtained from the statistical consistency of beliefs is that if $\mathbf{C} = \{\mathcal{F}\}$, then $\Delta_{\mathcal{F}} = 0$, i.e. an agent able to match the long-run frequency of the payoff from the asset-backed securities will get a return equal to that from holding the short-term bond as long as there are no competing theories.

When there are competing theories, the return received could be above or below the market interest rate. With no securitization, Eyster and Piccione (2013) show that, for the case where there is a theory that refines any other, no agent can earn a return above the interest rate. What is true for the whole underlying asset is still true when it is sliced into asset-backed securities. To show this, assume that $\mathcal{G} \in \mathbf{C}$ refines every $\mathcal{F} \in \mathbf{C}$. Define

$$\delta_{\mathcal{F}}(\tau) := \sum_{x \in X} \left( T(\psi_\tau)(x) - E_{\mathcal{F}}(\psi_\tau)(x) \right) \left( I_{\{\mathcal{F} \in \mathbf{B}(\tau,x)\}} \right) \mu(x)$$

as the average excess return from buying tranche $\tau \in \mathcal{T}$ in the states in which $\mathcal{F} \in \mathbf{C}$ perceives this as worthwhile. Proposition 3 in Eyster and Piccione (2013) implies that $\delta_{\mathcal{G}}(\tau) = 0$ and $\delta_{\mathcal{F}}(\tau) \leq 0$ for every $\tau \in \mathcal{T}$. Then trivially $\Delta_{\mathcal{G}} = 0$ and $\Delta_{\mathcal{F}} \leq 0$. Whenever there exists a theory that refines every other, the excess return is invariably zero for the finest theory and non-positive for the rest, no matter the tranching.

In light of this result, it is no surprise that the introduction of ABS can weakly decrease the return received by every agent, as occurs in section 1.2 if $\mathcal{A} = \{\{l, m, h\}\}$ and $\mathcal{B} = \{\{l\}, \{m\}, \{h\}\}$. In states where, without securitization, the most sophisticated agent would have bought the whole asset, the tranches not bought by this agent will yield a return that, on average, is below the market interest rate.

Nevertheless, the introduction of ABS *can* increase the return of some agents as long as they are not the most sophisticated ones. As an example, consider a state space $X = \{l, m, h\}$ where the probability of a state going back to itself is close to one and uniform otherwise. The collection of theories is $\mathbf{C} = \{\mathcal{A}, \mathcal{B}, \mathcal{C}\}$, where:

$$\begin{aligned} \mathcal{A} &= \{\{l\}, \{m, h\}\}, \\ \mathcal{B} &= \{\{l, h\}, \{m\}\}, \\ \mathcal{C} &= \{\{l, m, h\}\}. \end{aligned}$$

The interest rate is constant and the dividend is such that, in the benchmark equilibrium, $q(l) < q(m) < q(h)$.

TABLE 1.3: Expected payoffs from the long-term asset in the benchmark

| States | $T(\psi)$ | $E_{\mathcal{A}}(\psi)$ | $E_{\mathcal{B}}(\psi)$ | $E_{\mathcal{C}}(\psi)$ | $\mathbf{B}$ |
|---|---|---|---|---|---|
| $l$ | $q(l)$ | $q(l)$ | $\frac{1}{2}(q(l)+q(h))$ | $\frac{1}{3}(q(l)+q(m)+q(h))$ | $\{\mathcal{B}\}$ |
| $m$ | $q(m)$ | $\frac{1}{2}(q(m)+q(h))$ | $q(m)$ | $\frac{1}{3}(q(l)+q(m)+q(h))$ | $\{\mathcal{A}\}$ |
| $h$ | $q(h)$ | $\frac{1}{2}(q(m)+q(h))$ | $\frac{1}{2}(q(l)+q(h))$ | $\frac{1}{3}(q(l)+q(m)+q(h))$ | $\{\mathcal{A}\}$ |

Consider the case without any ABS. Table 1.3 presents i) the true expected price $T(\psi)$, ii) the expected price by each agent and, given these, iii) the set $\mathbf{B}$ of agents who will buy the long-term asset. All these objects are functions of the state of the world. If the dividend at state $h$ is high enough, the asset is bought by $\mathcal{B}$ at state $l$ and by $\mathcal{A}$ at states $m$ and $h$. Agent $\mathcal{C}$, who is less sophisticated than the other two, has a "loser's blessing" that allows him to have a non-negative excess return:

$$\underline{\Delta}_{\mathcal{A}} = -\frac{1}{6}(q(h)-q(m)) + \frac{1}{6}(q(h)-q(m)) = 0$$

$$\underline{\Delta}_{\mathcal{B}} = -\frac{1}{6}(q(h)-q(l)) < 0$$

$$\underline{\Delta}_{\mathcal{C}} = 0$$

Consider now the case where ABS are introduced. A collection of three tranches with attachment points 0, $q(l)$ and $q(m)$ is an issuer-optimal tranching if $q$ is its waterfall equilibrium and $q(l) < q(m) < q(h)$. The most senior tranche pays $q(l)$ at every state so there is no excess return from it. The excess returns for the mezzanine and the junior tranches can be obtained from Tables 1.4 and 1.5, respectively. These tables present i) the actual expected payoffs $T(\psi_\tau)$, ii) the expected payoff for each agent and, given these, iii) the set $\mathbf{B}(\tau,\cdot)$ of buyers for each of these tranches.

TABLE 1.4: Expected payoffs from the mezzanine tranche

| States | $T(\psi_m)$ | $E_{\mathcal{A}}(\psi_m)$ | $E_{\mathcal{B}}(\psi_m)$ | $E_{\mathcal{C}}(\psi_m)$ | $\mathbf{B}(m,\cdot)$ |
|---|---|---|---|---|---|
| $l$ | 0 | 0 | $\frac{1}{2}(q(m)-q(l))$ | $\frac{2}{3}(q(m)-q(l))$ | $\{\mathcal{C}\}$ |
| $m$ | $q(m)-q(l)$ | $q(m)-q(l)$ | $q(m)-q(l)$ | $\frac{2}{3}(q(m)-q(l))$ | $\{\mathcal{A}\}$ |
| $h$ | $q(m)-q(l)$ | $q(m)-q(l)$ | $\frac{1}{2}(q(m)-q(l))$ | $\frac{2}{3}(q(m)-q(l))$ | $\{\mathcal{A}\}$ |

TABLE 1.5: Expected payoffs from the junior tranche

| States | $T(\psi_j)$ | $E_{\mathcal{A}}(\psi_j)$ | $E_{\mathcal{B}}(\psi_j)$ | $E_{\mathcal{C}}(\psi_j)$ | $\mathbf{B}(j,\cdot)$ |
|---|---|---|---|---|---|
| $l$ | 0 | 0 | $\frac{1}{2}(q(h)-q(m))$ | $\frac{1}{3}(q(h)-q(m))$ | $\{\mathcal{B}\}$ |
| $m$ | 0 | $\frac{1}{2}(q(h)-q(m))$ | 0 | $\frac{1}{3}(q(h)-q(m))$ | $\{\mathcal{A}\}$ |
| $h$ | $q(h)-q(m)$ | $\frac{1}{2}(q(h)-q(m))$ | $\frac{1}{2}(q(h)-q(m))$ | $\frac{1}{3}(q(h)-q(m))$ | $\{\mathcal{A}\}$ |

Thanks to the introduction of ABS, $\mathcal{C}$ is lured into buying the mezzanine tranche at state $l$. As a consequence, $\mathcal{C}$'s return with securitization is lower than without. Meanwhile, at the expense of agent $\mathcal{C}$, the excess return for $\mathcal{B}$ is higher than what it would be without securitization:

$$\Delta_{\mathcal{A}} = 0 = \underline{\Delta}_{\mathcal{A}}$$

$$\Delta_{\mathcal{B}} = -\frac{1}{6}\left(q(h) - q(m)\right) > \underline{\Delta}_{\mathcal{B}}$$

$$\Delta_{\mathcal{C}} = -\frac{2}{9}\left(q(m) - q(l)\right) < \underline{\Delta}_{\mathcal{C}}$$

Thanks to the introduction of ABS, some agents manage to get rid of toxic assets and increase their portfolio return.

## 1.8 Other types of contracts

Apart from having their real-life counterparts, the only appealing features of securities with a waterfall structure are: i) their payoff is monotone in the cash flow from the underlying asset, ii) as shown in section 1.6, they implicitly represent any case where only debt and equity can be issued and debt itself can be used as collateral. However, no argument has been given in this paper for agents to choose or be constrained by this class of contracts.

Without being constrained to issue ABS with a waterfall payment structure, agents could exploit the divergence of beliefs even further by creating contracts which do not satisfy this constraint. For instance, consider the equilibrium of subsection 1.2.2. The senior tranche pays $q(m)$ if the state in the next period is $m$ or $h$ and $q(l)$ otherwise, At state $m$, this tranche is bought by agent $\mathcal{B}$. This agent could create a new security, backed by the senior tranche, which pays $q(m)$ at state $h$ and 0 otherwise. Such a security does not satisfy the waterfall constraint since its payoff does not exclusively depend on the cash flow from the underlying tranche. This new security is (almost) worthless for $\mathcal{B}$, given that a realization $h$ next period looks very unlikely. However, agent $\mathcal{A}$ would be willing to pay $\frac{1}{3}\frac{q(m)}{R}$ for it.

As an alternative to the waterfall equilibrium, consider the following asset-backed securities: for $y \in X$, the $y$-security pays $q(y)$ in the next period if the realized state is $y$ and 0 otherwise. The equilibrium price function for the long-term asset, denoted by $\bar{q}$, will be given by

$$\bar{q}(x) = \frac{d(x) + \sum_{y \in X} \bar{q}(y) \max_{\mathcal{F} \in \mathbf{C}} Q_{\mathcal{F}}(x, y)}{R(x)} \tag{1.24}$$

for $x \in X$. It is easy to show that there exists such an equilibrium and that $q_{\mathcal{T}}(x) \leq \bar{q}(x)$ for all $x \in X$ and any tranching $\mathcal{T}$.[5] It follows that the lack of a belief that stochastically dominates all others in at least one state is a necessary condition for tranching to increase the asset price only because of the waterfall constraint. Instead, this new kind of securitization structure will increase the price as long as there is any disagreement regarding the transition probabilities.

## 1.9 Conclusion

This paper has shown the effect of the introduction of asset-backed securities under heterogeneous beliefs and short-selling constraints. The heterogeneity is motivated by assuming that agents are not capable of processing all information available. Hence, different agents may ignore different information, leading to divergent expectations about the assets' payouts. This heterogeneity can be exploited by financial institutions through the issuing of asset-backed securities to be traded among those agents with divergent beliefs.

Since the issued securities have to be backed by an underlying asset, the introduction of asset-backed securities weakly increases the demand of the underlying asset. When the asset-backed securities have a waterfall payment structure, the necessary condition for this increase to be strict is the absence of beliefs, represented by perceived distributions on the price of the underlying asset, that first-order stochastically dominate all other beliefs. If this condition holds, the introduction of asset-backed securities may lead to an increase in the price of the underlying asset.

The paper has also studied the effect of securitization on agents' average portfolio returns. Whoever buys an asset may by subject to a winner's curse: the return they expect on the asset, which is equal to the return on safe bonds, may be above the return the asset on average generates. As a result, the introduction of asset-backed securities may be a blessing in disguise for some agents: in states of the world where an agent is buying an underperforming asset, securitization may lead her to sell away a fraction of the asset payout, allowing her to inadvertently transfer some of the losses the asset generates to other agents. However, the paper has also shown that the introduction of asset-backed securities may decrease the portfolio return some agents receive without increasing the portfolio return of none.

The setup of the model implies that agents have no objective reason to trade. Specifically, if agents have rational expectations, they will not trade any ABS, since they are all

[5] Allen and Gale (1988) also show that, without short selling, the market value of an asset is maximized when the securities are such that each of them gets all the payoff in a single state.

assumed to be risk-neutral. In this context, restrictions on the trading of ABS will have no effect on the objective well-being of agents, but will help to close the gap between the price of the underlying asset and its fundamental value. Nevertheless, the optimal policy for the case where there are different degrees of risk aversion is still an open question.

In a full theoretical account of the 2007 crisis, both the boom and the bust of asset prices should be explained. This paper explains how the introduction of asset-backed securities allows belief disagreement to translate into an increase in asset prices. However, it does not explain how asset prices may eventually collapse. One possible option to explain this is to allow for agents to update their beliefs based on the return they get on the asset-backed securities. Once they realize that the actual return is below their expectations, the demand for the assets collapses. Another possible option is the introduction of liquidity constraints. In this framework, the agents who buy asset-backed securities with a return below the short-term bond will get into a debt spiral and eventually will hit a liquidity constraint. Since they will no longer be able to rollover their debts in order to buy asset-backed securities, the demand of the underlying assets will fall. A formalization of these narratives is left for future research.

## 1.10 Appendix

Proof of Lemma 1.1: Since $\mathcal{S}$ is a refinement of $\mathcal{T}$, for any function $g : X \to \mathbb{R}_+$ and any $x \in X$, the following inequality holds:

$$\sum_{\tau\in\mathcal{T}} \max_{\mathcal{F}\in\mathbf{C}} E_{\mathcal{F}}\left(\phi_\tau\left(g\right)\right)(x) \leq \sum_{\upsilon\in\mathcal{S}} \max_{\mathcal{F}\in\mathbf{C}} E_{\mathcal{F}}\left(\phi_\upsilon\left(g\right)\right)(x)$$

Hence $\Psi_{\mathcal{T}}(g)(x) \leq \Psi_{\mathcal{S}}(g)(x)$ and so $q_{\mathcal{T}}(x) \leq \Psi_{\mathcal{S}}(q_{\mathcal{T}})(x)$.

Define $\Psi^1_{\mathcal{S}} := \Psi_{\mathcal{S}}$ and

$$\Psi^n_{\mathcal{S}} := \Psi_{\mathcal{S}}(\Psi^{n-1}_{\mathcal{S}})$$

for $n = 2, 3, \ldots$ . Since $\Psi_{\mathcal{S}}$ is monotone (see proof of Proposition 1.1), $q_{\mathcal{T}}(x) \leq \Psi_{\mathcal{S}}(g)(x)$ for all $g$ such that $q_{\mathcal{T}}(x) \leq g(x)$ for $x \in X$. Hence if $q_{\mathcal{T}}(x) \leq \Psi^{n-1}_{\mathcal{S}}(q_{\mathcal{T}})(x)$, then $q_{\mathcal{T}}(x) \leq \Psi^n_{\mathcal{S}}(q_{\mathcal{T}})(x)$. Since $\Psi_{\mathcal{S}}$ is a contraction and $q_{\mathcal{S}}$ is its fixed point, this implies $q_{\mathcal{T}}(x) \leq q_{\mathcal{S}}(x)$ for all $x \in X$.

□

Definition 1.5 and Lemmas 1.2, 1.3 and 1.4 will be used to prove Proposition 1.2.

**Definition 1.5.** *A tranching $\mathcal{T}$ satisfies the dominance condition if, for each $\tau \in \mathcal{T}$ and $x \in X$, there is a theory $\mathcal{G} \in \mathbf{C}$ such that*

$$G_{\mathcal{G}}^{\mathcal{T}}(t|x) \leq G_{\mathcal{F}}^{\mathcal{T}}(t|x)$$

*for all $t \in \tau$ and all $\mathcal{F} \in \mathbf{C}$.*

**Lemma 1.2.** *If $\mathcal{T}$ satisfies the dominance condition and $\mathcal{S}$ is refinement of $\mathcal{T}$, then $q_{\mathcal{S}} = q_{\mathcal{T}}$*

*Proof.* For any price function $q$, the payoff from a bounded tranche $\tau$ expected for next period by theory $\mathcal{F}$ on state $x$ can be written as

$$\begin{aligned} E_{\mathcal{F}}(\phi_\tau(q))(x) &= \int_{\inf(\tau)}^{\sup(\tau)} \phi_\tau(t)\, dG_{\mathcal{F}}^{\mathcal{T}}(t|\, x) + \int_{\sup(\tau)}^{\infty} \phi_\tau(t)\, dG_{\mathcal{F}}^{\mathcal{T}}(t|\, x) \quad (1.25) \\ &= \sup(\tau) - \inf(\tau) - \int_{\inf(\tau)}^{\sup(\tau)} G_{\mathcal{F}}^{\mathcal{T}}(t|\, x) d\phi_\tau(t) \quad (1.26) \end{aligned}$$

Therefore, if $\mathcal{T}$ satisfies the dominance condition and $\mathcal{S}$ is a refinement of $\mathcal{T}$, then

$$\begin{aligned} \sum_{\upsilon \in \mathcal{S}: \upsilon \subset \tau} \max_{\mathcal{F} \in \mathbf{C}} E_{\mathcal{F}}(\phi_\upsilon(q_{\mathcal{T}}))(x) &= \max_{\mathcal{F} \in \mathbf{C}} E_{\mathcal{F}} \left( \sum_{\upsilon \in \mathcal{S}: \upsilon \subset \tau} \phi_\upsilon(q_{\mathcal{T}}) \right)(x) \\ &= \max_{\mathcal{F} \in \mathbf{C}} E_{\mathcal{F}}(\phi_\tau(q_{\mathcal{T}}))(x) \end{aligned}$$

for any $\tau \in \mathcal{T}$ and $x \in X$. Using equations (1.11) and (1.14), it follows that

$$\begin{aligned} \Psi_{\mathcal{S}}(q_{\mathcal{T}}) &= \Psi_{\mathcal{T}}(q_{\mathcal{T}}) \\ &= q_{\mathcal{T}} \end{aligned}$$

□

**Lemma 1.3.** *If $\mathcal{T}$ satisfies the dominance condition, then*

$$q_{\mathcal{S}}(x) \leq q_{\mathcal{T}}(x)$$

*for all $x \in X$ and any tranching $\mathcal{S}$.*

*Proof.* Let $\mathcal{S} \vee \mathcal{T}$ the coarsest common refinement between $\mathcal{T}$ and $\mathcal{S}$. By Lemma 1.1,

$$q_{\mathcal{S}}(x) \leq q_{\mathcal{S} \vee \mathcal{T}}(x)$$

for $x \in X$. By Lemma 1.2,

$$q_{\mathcal{S} \vee \mathcal{T}}(x) = q_{\mathcal{T}}(x)$$

for $x \in X$. □

**Lemma 1.4.** *If $\mathcal{T}$ does not satisfy the dominance condition, then there is a tranching $\mathcal{S}$ such that*

$$q_{\mathcal{T}}(x) < q_{\mathcal{S}}(x)$$

*for all $x \in X$.*

*Proof.* Since $\mathcal{T}$ does not satisfy the dominance condition, there is a state $x \in X$ and a tranche $\tau \in \mathcal{T}$ such that

$$\begin{aligned}\min_{\mathcal{F}\in\mathbf{C}} \int_{\inf(\tau)}^{\sup(\tau)} G_{\mathcal{F}}^{\mathcal{T}}(t|\, x) d\phi_{\tau}(t) \;\; > \;\; & \min_{\mathcal{F}\in\mathbf{C}} \int_{\inf(\tau)}^{\inf(\upsilon)} G_{\mathcal{F}}^{\mathcal{T}}(t|\, x) d\phi_{\tau}(t) \\ & + \min_{\mathcal{F}\in\mathbf{C}} \int_{\inf(\upsilon)}^{\sup(\upsilon)} G_{\mathcal{F}}^{\mathcal{T}}(t|\, x) d\phi_{\tau}(t) \\ & + \min_{\mathcal{F}\in\mathbf{C}} \int_{\sup(\upsilon)}^{\sup(\tau)} G_{\mathcal{F}}^{\mathcal{T}}(t|\, x) d\phi_{\tau}(t)\end{aligned}$$

for some $\upsilon \subset \tau$. Let $\mathcal{S}$ be a refinement of $\mathcal{T}$ such that $\upsilon \in \mathcal{S}$. From equations (1.14) and (1.26), it then follows that

$$\Psi_{\mathcal{T}}\left(q_{\mathcal{T}}\right)(x) < \Psi_{\mathcal{S}}\left(q_{\mathcal{T}}\right)(x)$$

Hence $q_{\mathcal{T}}$ is not a waterfall equilibrium for tranching $\mathcal{S}$. But since $\mathcal{S}$ is a refinement of $\mathcal{T}$, Lemma 1.1 implies

$$q_{\mathcal{T}}(y) \leq q_{\mathcal{S}}(y) \tag{1.27}$$

for all $y \in X$ with strict inequality for $y = x$. Since for every theory the perceived transition probability from any state to any other is always positive, the inequality is strict for every $y \in X$. □

Proof of Proposition 1.2: Follows directly from Definition 1.5 and Lemmas 1.3 and 1.4.

□

Proof of Proposition 1.5: From equations (1.20), (1.21) and (1.22), it follows that

$$p_{\Sigma}(r, x) = \frac{d(x)}{R(x)} + \sum_{z\in\mathbb{Z}} \frac{\max_{\mathcal{F}\in\mathbf{C}} E_{\mathcal{F}}\left(\gamma_z\right)(x)}{R(x)} \tag{1.28}$$

for $x \in X$.

Let $b(z)$ be the branch that connects $r$ with $z \in \mathbb{Z}$. Since $\gamma_r = p_\Sigma(r, \cdot)$, equations (1.18) and (1.19) imply

$$\begin{aligned} \gamma_z(x) &= \left(p_\Sigma(r,x) - \sum_{n \in b(z)} \inf(\tau_n) - \inf(\tau_z)\right)^+ \\ &- \left(p_\Sigma(r,x) - \sum_{n \in b(z)} \inf(\tau_n) - \sup(\tau_z)\right)^+ \end{aligned} \tag{1.29}$$

for $x \in X$.

By replacing (1.29) into (1.28) and using the equilibrium concept in Definition 1.2, it follows that $p_\Sigma(r, \cdot) = q_{\mathcal{T}}$ if the attachment points for tranching $\mathcal{T}$ are given by

$$\left(\sum_{n \in b(z)} \inf(\tau_n) - \inf(\tau_z)\right)_{z \in \mathbb{Z}}$$

□

# Chapter 2

# A Note on *The Evolution of Preferences*


## Abstract

*This note checks the robustness of a surprising result in Dekel et al. (2007). The result states that strict Nash equilibria might cease to be evolutionary stable when agents are able to observe the opponent's preferences with a very low probability. This note shows that the result is driven by the assumption that there is no risk for the observed preferences to be mistaken. In particular, when a player may observe a signal correlated with the opponent's preferences, but the signal is noisy enough, all strict Nash equilibria are evolutionary stable.*


## 2.1 Introduction

This note expands the evolutionary game theory model of Dekel et al. (2007) in order to check the robustness of their results. In their model, each player has a probability $p \in [0, 1]$ of observing the preferences of the opponent they are matched with. With complementary probability, they observe nothing. Their model yields the following results. Firstly, any efficient strict Nash equilibrium is evolutionary stable. Secondly, when $p$ is close enough to 1, a pure strategy profile has to be efficient in order to be stable.[1] Thirdly, when $p = 0$, any strict Nash equilibrium is stable. This last result, however, is not continuous: there are strict Nash equilibria which are not stable for

[1]This result is reminiscent of the "secret handshake" result of Robson (1990) and the "information leaks lead to cooperation" result of Matsui (1989).

very low levels of $p$.[2] This note checks the robustness of these results when there is a probability that the observed preferences do not correspond to the opponents' actual preferences. It shows that the lack of stability of strict Nash equilibria for very low levels of $p$ is driven by the assumption that the signals that players receive on the opponents' preferences are fully accurate. When the signals are noisy enough, all strict equilibria are stable.

The setup of the extended model is as follows. A large but finite number of players are randomly and uniformly matched with each other. In each match, each player observes a signal from her opponent. With a probability $p$, the player observes the opponent's preferences. With probability $q$, the player observes some preferences randomly drawn from the population. With probability $1 - p - q$, the player observes no signal. Players in a match play a Bayesian Nash equilibrium (BNE) of an incomplete information game, where the type of each player is given by her preferences and the signal she observes. The outcome of the match induces some payoff in terms of fitness. However, this fitness function does not necessarily represent players' preferences. Generally speaking, an outcome is stable if no entrants -players with preferences different from those of incumbents- induce a BNE far away from the BNE previously played among incumbents and no entrants outperform incumbents in terms of the average fitness obtained across matches.

The results of the extended model are as follows. Just as in Dekel et al. (2007), efficient strict Nash equilibria are always stable (Proposition 2.1). Additionally, efficiency is a necessary condition for stability when the frequency $(p+q)$ and the precision $(p/q)$ of the signal are high enough (Proposition 2.2). Furthermore, when the precision of the signal is low enough, (a) a pure-strategy profile has to be a Nash equilibrium to be stable, and (b) any strict Nash equilibrium is stable (Proposition 2.3). This last result implies that, as long as signals are noisy enough, the stability of strict Nash equilibria is robust to the introduction of low frequency signals.

The rest of this note is organized as follows. Section 2.2 presents the setup of the extended model. Section 2.3 defines the stability concept. Section 2.4 presents the results regarding the necessary and sufficient conditions for the stability of pure-strategy profiles. Section 2.5 presents some final comments. The proofs of all propositions are presented in the appendix.

[2]In the context of strict coordination games, these results lead the authors to side in favor of the selection of payoff-dominant equilibria (Harsanyi and Selten (1988)) as opposed to risk-dominant equilibria (Carlsson and Van Damme (1993), Ellison (1993), Kandori et al. (1993)).

## 2.2 Model

The underlying structure of the model is given by a symmetric game $G = (A, \pi)$ where $A$ is a finite action set, $\triangle$ is the simplex on $A$ and $\pi \in \mathbb{R}^{\triangle^2}$ is an **objective fitness** function. In particular, $\pi(\alpha, \beta)$ is the fitness players get when their strategy is $\alpha \in \triangle$ and their opponent's is $\beta \in \triangle$. Throughout the note, Latin letters are used instead of Greek letters when referring to pure strategies.

### 2.2.1 Preferences

Preferences over outcomes are not necessarily represented by a positive affine transformation of $\pi$. Still, preferences will be subject to evolutionary pressures that will define their chances of survival. The question is which kind of preferences will prevail.[3] The preferences are represented by von Neumann-Morgenstern (vNM) **utility** functions. Let $U \subset \mathbb{R}^{\triangle^2}$ be the set of vNM functions. For players with a utility function $u \in U$, the utility they get when their strategy is $\alpha$ and their opponent's is $\beta$ is $u(\alpha, \beta)$. Since $u$ does not have to be a positive affine transformation of $\pi$, preferences do not necessarily rank outcomes the same way as fitness does. The fitness function $\pi$ will reappear in the analysis when assessing which preferences are able to survive evolutionary pressures.

### 2.2.2 Signals

The environment is populated by a large but finite number of players. Players are randomly matched with each other an infinite number of times. In each match, players play a game whose action set is given by $A$. For every match, players may receive a signal that helps them to update their beliefs regarding their opponent's preferences. This signal is private information. Consider a given distribution $\mu$ over utility functions in the existing population. Denote by $C_\mu$ the support of $\mu$. The signal a player receives comes from the set:

$$X_\mu := C_\mu \cup \emptyset$$

The following events may arise when a player is matched with another: (a) with probability $p$, the player observes her opponent's preferences, (b) with probability $q$, the player observes preferences that are drawn according to $\mu$, and (c) with probability $1 - p - q$, the player receives no signal. Throughout the note, it is assumed that $p$ and $q$ are both

[3] This is known as the *indirect approach* of evolutionary game theory, pioneered by Güth and Yaari (1992) and Güth (1995). In this approach, individuals are endowed with preferences, rather than with behavioral strategies that they blindly follow. Natural selection operates on preferences, not on strategies. The strategic outcome induced by a distribution of preferences is be part of the definition of stability.

positive. This means that, unless the population is monomorphic, players can never be sure about the preferences of their opponents.[4] The model of Dekel et al. (2007) corresponds to the limiting case where $q = 0$.

### 2.2.3 Beliefs

In order to choose the best strategy according to their preferences, players have to form expectations about the signal their opponent receives. Let $g_\mu(x|u)$ be the probability that the opponent receives a signal $x \in X_\mu$ when the player's utility function is $u \in C_\mu$. We have:

$$g_\mu(x|u) = \begin{cases} p + q\mu(x) & \text{if } x = u \\ q\mu(x) & \text{if } x \in C_\mu/\{u\} \\ 1 - p - q & \text{if } x = \emptyset \end{cases}$$

Simultaneously, players try to infer their opponent's preferences from the signal they receive. It is assumed that players know the distribution of preferences in the population and that this is common knowledge. Let $f_\mu(u|x)$ be the probability that an opponent has preferences represented by $u \in C_\mu$, conditional on a signal $x \in X_\mu$. Bayes rule implies

$$f_\mu(u|x) = \begin{cases} \mu(u) & \text{for } x = \emptyset \\ \frac{p+q\mu(u)}{p+q} & \text{for } x \in C_\mu \end{cases}$$

### 2.2.4 Equilibrium play

A players private information in each match is given by $(u, y) \in C_\mu \times X_\mu$, where $u$ represents their preferences and $y$ the signal they receive from their opponent. Let $b : C_\mu \times X_\mu \to \triangle$ be a behavioral function such that $b(u, y)$ denotes the strategy played by agents with utility $u \in C_\mu$ when they receive a signal $y \in X_\mu$.

It is assumed that natural selection operates long after individuals have learned the preferences supported by $\mu$, their frequency and the strategies played by each type in $C_\mu \times X_\mu$. Rationality implies that players will play a Bayesian Nash Equilibrium (BNE)[5]:

**Definition 2.1.** A function $b$ is a Bayesian Nash Equilibrium for a given a distribution over preferences $\mu$ if

$$b(u, y) \in \arg\max_{\alpha \in \triangle} \sum_{v \in C_\mu} \sum_{x \in X_\mu} u(\alpha, b(v, x)) f_\mu(v|y) g_\mu(x|u)$$

[4]A population is monomorphic if $C_\mu$ is a singleton.

[5]For an environment with no observability where people play self-confirming equilibria, see Gamba (2011)

for $(u, y) \in C_\mu \times X_\mu$.

The set of BNE for a distribution of preferences $\mu$ is denoted by $\mathbb{B}(\mu)$. The tuple $(\mu, b)$ constitutes a *configuration* if $b \in \mathbb{B}(\mu)$. A configuration describes, not only the distribution of players preferences, but also the strategies players use. The distribution over strategies induced by a configuration is known as its *outcome*.

## 2.3 Stability

The evolutionary feasibility of a distribution of preferences depends on the performance of the incumbent preferences relative to the mutant preferences that may enter the population. The performance of preferences is measured in terms of fitness. In particular, it is measured by the *average fitness* obtained after infinitely many rounds of matching. Denote by $\Pi_{(\mu,b)}(u)$ the average fitness players with preferences $u \in C_\mu$ get under a configuration $(\mu, b)$. When receiving a signal $y \in X_\mu$, a player with utility function $u$ plays $b(u, y)$. The expected fitness from this play is

$$\sum_{v \in C_\mu} \sum_{x \in X_\mu} \pi\left(b(u,y), b(v,x)\right) g_\mu\left(x|u\right) \mu(v)$$

The average fitness for players with utility function $u$ is obtained by averaging their expected fitness across the signals they receive:

$$\Pi_{(\mu,b)}(u) = \sum_{v \in C_\mu} \sum_{y \in X_\mu} \sum_{x \in X_\mu} \pi\left(b(u,y), b(v,x)\right) g_\mu\left(x|u\right) g_\mu\left(y|v\right) \mu(v)$$

The appearance of mutant preferences can disturb the equilibrium being played by incumbents. It is assumed that the equilibrium played after mutants' entrance is such that incumbents keep playing the same action when observing a signal corresponding to incumbents. Formally, consider a distribution $\tilde{\mu}$ whose support may or may not include preferences in the support of $\mu$. An equilibrium $\tilde{b}$ is *focal* relative to the configuration $(\mu, b)$ if incumbents (those with utility function in $C_\mu$) keep playing the same strategy when facing any signal in $X_\mu$. The subset of $\mathbb{B}(\tilde{\mu})$ which is focal relative to $(\mu, b)$ is denoted by

$$\mathbb{F}_{(\mu,b)}(\tilde{\mu}) := \left\{\tilde{b} \in \mathbb{B}(\tilde{\mu}) : \ \tilde{b}(u,y) = b(u,y) \text{ for all } (u,y) \in C_\mu \times X_\mu\right\}$$

In order to define stability, some additional notation has to be previously introduced. Consider a configuration $(\mu, b)$. Assume that a fraction $\epsilon \in (0, 1)$ of the population mutates and switches to preferences represented by a vNM function $\tilde{u}$. After mutation, the distribution of preferences is given by

$$\tilde{\mu}(u) = \begin{cases} \epsilon & \text{if } u = \tilde{u} \\ (1-\epsilon)\mu(u) & \text{otherwise} \end{cases} \tag{2.1}$$

for $u \in U$. For a given $\tilde{u}$ and a given pre-entry distribution $\mu$, the set $\mathcal{M}_{\bar{\epsilon}}(\mu, \tilde{u})$ denotes the set of post-entry distributions in which the fraction of entrants is no larger than $\bar{\epsilon} \in (0,1)$. In other words, the distribution $\tilde{\mu}$ is in $\mathcal{M}_{\bar{\epsilon}}(\mu, \tilde{u})$ if there is $\epsilon \in [0, \bar{\epsilon}]$ such that equation (2.1) holds.

A formal definition can be given now with the help of the notation introduced so far:

**Definition 2.2.** A configuration $(\mu, b)$ is stable if there is $\bar{\epsilon} \in (0,1)$ such that, for every $\tilde{u} \in U$,

$$\tilde{\mu} \in \mathcal{M}_{\bar{\epsilon}}(\mu, \tilde{u})$$

implies

1. $\mathbb{F}_{(\mu,b)}(\tilde{\mu}) \neq \emptyset$
2. $\Pi_{(\tilde{\mu},\tilde{b})}(\tilde{u}) \leq \Pi_{(\tilde{\mu},\tilde{b})}(u)$ for $\tilde{b} \in \mathbb{F}_{(\mu,b)}(\tilde{\mu})$ and $u \in C_\mu$

A strategy in $\triangle$ is said to be stable if it is the outcome of a stable configuration. A distribution of preferences $\mu$ is stable if there is $b \in \mathbb{B}(\mu)$ such that the configuration $(\mu, b)$ is stable.

Generally speaking, stability means that, at least when the fraction of mutants is small, the following conditions hold: 1) there is at least one post-entry equilibrium in which mutants do not disturb the behavior of incumbents and 2) entrants do not outperform incumbents in any of these equilibria.[6]

## 2.4 Results

Define $p+q$ as the *frequency* of the signal and define $p/q$ as the *precision* of the signal. In general, when the frequency and the precision of signals are high, stability favors the efficient outcomes of $G$. When the precision is low, stability favors the Nash equilibrium outcomes of $G$.

[6]The stability concept in definition 2.2 is slightly more demanding than the one provided by Dekel et al. (2007). In Definition 2.2, if the set of focal equilibria is empty, the configuration is not stable. In Dekel et al. (2007), if the set of focal equilibria is empty, stability demands that entrants do not outperform incumbents in any BNE "close" to $b$. The results of this note can be extended to the latter and more relaxed definition of stability.

**Definition 2.3.** A strategy profile $(\alpha, \alpha) \in \triangle^2$ is an efficient outcome of $G$ if

$$\pi(\alpha, \alpha) \geq \pi(\beta, \beta)$$

for all $(\beta, \beta) \in \triangle^2$.

### 2.4.1 Stability of efficient strict Nash equilibria

The first result shows the robustness of efficient strict Nash equilibria to any kind of noise: no matter what values $p$ and $q$ take, an outcome which is an efficient strict Nash equilibrium of $G$ is a stable outcome:

**Proposition 2.1.** *If $(a, a)$ is both a strict Nash equilibrium and an efficient outcome of $G$, then it is stable.*

Proof: See Appendix.

The stability of a pure-strategy profile $(a, a)$ which is both efficient and a strict Nash equilibrium can be supported by a monomorphic population for whom playing $a$ is a dominant strategy. Entrants who are not playing $a$ for all signals they receive will not outperform incumbents, either because they will not be best-responding to incumbents, or because they will not be able to coordinate among themselves on an outcome more efficient than $(a, a)$.

### 2.4.2 Stability under high frequency and high precision

The following result shows that efficiency is a necessary condition for stability when *both* the precision and the frequency of the signal are high enough. To show this, define $\mathcal{N}(\bar{\delta})$ as the set of probability vectors whose difference from the situation of full observability is no higher than $\bar{\delta} \in (0, 1)$:

$$\mathcal{N}(\bar{\delta}) := \left\{(q, p) \in (0, 1)^2 \ : \ p + q \leq 1 \text{ and } |(q, p) - (0, 1)| \leq \bar{\delta}\right\}$$

**Proposition 2.2.** *If $(a, a)$ is not an efficient outcome of $G$, then there is a $\bar{\delta} \in (0, 1)$ such that $a$ is not a stable outcome for any $(q, p) \in \mathcal{N}(\bar{\delta})$.*

Proof: See Appendix.

Intuitively, consider a configuration that induces a non-efficient play $(a, a)$. When both the signal frequency and the signal precision are high enough, there are entrants who will usually play $a$ against incumbents and will usually play the efficient strategy against entrants. The proof relies on showing that there is a focal equilibrium in which the incumbents keep playing $a$ regardless of the signal. If this is the case, the entrants will achieve a higher average fitness than the incumbents.

### 2.4.3 Stability under low precision

The following proposition states that, when the signal is very noisy, (a) a pure-strategy profile has to be a Nash equilibrium to be stable, and (b) any strict Nash equilibrium is stable. This result is *independent* of the frequency of the signal. Formally, define $\sigma := p/q$ as the precision of the signal:

**Proposition 2.3.**

*(a) If $(a, a)$ is not a Nash equilibrium of $G$, then there is a $\bar{\sigma} \in (0, 1)$ such that $a$ is not stable for any $\sigma \in [0, \bar{\sigma})$.*

*(b) If $(a, a)$ is a strict Nash equilibrium of $G$, then there is a $\bar{\sigma} \in (0, 1)$ such that $a$ is stable for any $\sigma \in [0, \bar{\sigma})$.*

PROOF: See Appendix.

The intuition for the necessity result in (a) is as follows: If $(a, a)$ is not a Nash equilibrium, then there is a strategy $\tilde{a}$ that offers a higher fitness than $a$ when the opponent plays $a$. Consider an entrant for whom playing $\tilde{a}$ is a dominant strategy. In any focal equilibria, incumbents play $a$ when receiving a signal corresponding to an incumbent. Since the signal is almost uninformative, this implies that entrants will rarely get a payoff different than $\pi(\tilde{a}, a)$. Meanwhile, since the fraction of entrants is small, incumbents will usually get $\pi(a, a)$. Since $\pi(\tilde{a}, a) > \pi(a, a)$, the entrants will take over the population.

In order to give an intuition for the sufficiency result in (b), consider a population for whom playing $a$ is a dominant strategy. In order to have any chance of outperforming the incumbents, entrants will have to play $a$ when receiving no signal and to play $a$ when receiving the signal that their opponent is an incumbent. Suppose that an entrant receives a signal that her opponent is a fellow entrant. Since the signals is very noisy and the fraction of entrants is small, she is most likely to be facing an incumbent who plays $a$. Therefore, if she responds be playing something other than $a$, she will achieve a lower fitness than incumbents, since $(a, a)$ is a strict Nash equilibrium of $G$.

In Dekel et al. (2007), the authors give an example of a strict coordination game in which there is a (risk-dominant) strict Nash equilibrium that is not stable even for negligibly but positive levels of observability. In contrast, propositions 2.1 and 2.3 imply:

**Corollary 2.1.** *For strict coordination games, (a) the payoff-dominant equilibrium is always stable, and (b) the risk-dominant equilibrium is stable if the signal precision is sufficiently low.*

In other words, the stability of a risk-dominant equilibrium that is not pay-off dominant is actually robust to the introduction of noisy signals. The significance of this result is that the equilibrium selection in favor of payoff-dominant equilibria is not as appealing as it seems when players can make inferences about opponents' preferences. Once it is acknowledged that the information regarding opponents' preferences could be very noisy, the selection against payoff-dominated equilibria ceases to be straightforward.

## 2.5 Conclusion

It is clearly unrealistic to assume that preferences can be perfectly observed. On the other hand, it is also unrealistic to assume that no information regarding the opponents' preferences exists. This note has studied the intermediate case where there is some imprecise information regarding opponents' preferences. No matter whether the signals on the opponent's preferences are noisy or not, there are always stable preferences that support efficient strict Nash equilibria. Whenever the signals are accurate and frequent enough, a pure strategy profile has to be efficient to be stable. Finally, the note has checked the robustness of the most puzzling result of Dekel et al. (2007), which is that strict Nash equilibria might cease to be stable when a low-frequency signal is introduced. Once information distortions are acknowledged, the "folk" result is reinstated: if the signal is noisy enough, all strict Nash equilibria remain stable.

An assumption made throughout this note is that the probability of observing no signal is independent of types. It seems plausible, however, that observing a signal becomes more likely as the divergence of preferences decreases. In such a setup, it is conjectured that there will be stable configurations where players: i) play an efficient strategy if the opponent is perceived as having similar preferences, and ii) play a Nash strategy in any other case.

## 2.6 Appendix

Proposition 2.1 *If $(a, a)$ is both a strict Nash equilibrium and an efficient outcome of $G$, then it is stable.*

*Proof.* Consider a configuration $(\mu, b)$ where $C_\mu = \{u\}$ and playing $a$ is a strictly dominant strategy for $u$. Denote by $\tilde{u}$ the utility function for an entrant. Consider a focal equilibrium $\tilde{b} \in \mathbb{F}_{(\mu,b)}(\tilde{\mu})$ such that $\tilde{\mu}$ is such that (2.1) holds for some $\epsilon \in (0,1)$. Since $a$ is strictly dominant for $u$, $\mathbb{F}_{(\mu,b)}(\tilde{\mu})$ is not empty and $\tilde{b}(u,x) = a$ for $x \in \{u, \tilde{u}, \emptyset\}$.

To shorten notation, denote the strategies an entrant follows when receiving signals $u$, $\tilde{u}$ and $\emptyset$ by

$$\begin{aligned}\gamma_u &:= \tilde{b}(\tilde{u}, u)\\ \gamma_{\tilde{u}} &:= \tilde{b}(\tilde{u}, \tilde{u})\\ \gamma_\emptyset &:= \tilde{b}(\tilde{u}, \emptyset)\end{aligned}$$

The fitness for incumbents will be

$$\Pi_{(\tilde{\mu},\tilde{b})}(u) = (1-\epsilon)\pi(a,a) + \epsilon \begin{pmatrix} p + q(1-\epsilon) \\ q\epsilon \\ 1-p-q \end{pmatrix}' \begin{pmatrix} \pi(a,\gamma_u) \\ \pi(a,\gamma_{\tilde{u}}) \\ \pi(a,\gamma_\emptyset) \end{pmatrix} \tag{2.2}$$

whereas the fitness for the entrant will be

$$\begin{aligned}\Pi_{(\tilde{\mu},\tilde{b})}(\tilde{u}) = (1-\epsilon) &\begin{pmatrix} p + q(1-\epsilon) \\ q\epsilon \\ 1-p-q \end{pmatrix}' \begin{pmatrix} \pi(\gamma_u, a) \\ \pi(\gamma_{\tilde{u}}, a) \\ \pi(\gamma_\emptyset, a) \end{pmatrix} \\ + \epsilon &\begin{pmatrix} p + q\epsilon \\ q\,(1-\epsilon) \\ 1-p-q \end{pmatrix}' \begin{pmatrix} \pi(\gamma_{\tilde{u}},\gamma_{\tilde{u}}) & \pi(\gamma_{\tilde{u}},\gamma_u) & \pi(\gamma_{\tilde{u}},\gamma_\emptyset) \\ \pi(\gamma_u,\gamma_{\tilde{u}}) & \pi(\gamma_u,\gamma_u) & \pi(\gamma_u,\gamma_\emptyset) \\ \pi(\gamma_\emptyset,\gamma_{\tilde{u}}) & \pi(\gamma_\emptyset,\gamma_u) & \pi(\gamma_\emptyset,\gamma_\emptyset) \end{pmatrix} \begin{pmatrix} p + q\epsilon \\ q\,(1-\epsilon) \\ 1-p-q \end{pmatrix}\end{aligned} \tag{2.3}$$

Consider the case where either $\gamma_u$ or $\gamma_\emptyset$ are not equal to $a$. Subtracting (2.3) from (2.2) yields

$$\Pi_{(\tilde{\mu},\tilde{b})}(u) - \Pi_{(\tilde{\mu},\tilde{b})}(\tilde{u}) = \pi(a,a) - (p+q)\pi(\gamma_u, a) - (1-p-q)\pi(\gamma_\emptyset, a) + o(\epsilon) \tag{2.4}$$

Since $(a,a)$ is a strict Nash equilibrium, there is $\epsilon$ small enough such that (2.4) is positive.

Consider the case where $\gamma_u$ and $\gamma_\emptyset$ are both equal to $a$. Subtracting (2.3) from (2.2) yields

$$\begin{aligned} \Pi_{(\tilde{\mu},\tilde{b})}(u) - \Pi_{(\tilde{\mu},\tilde{b})}(\tilde{u}) &= \epsilon q \left(\pi(a,a) - \pi(\gamma_{\tilde{u}}, a)\right) \\ &+ \epsilon p^2 \left(\pi(a,a) - \pi(\gamma_{\tilde{u}}, \gamma_{\tilde{u}})\right) \\ &+ \epsilon p 2 \left(1-p\right) \left(\pi(a,a) - \frac{1}{2}\pi(a,\gamma_{\tilde{u}}) - \frac{1}{2}\pi(\gamma_{\tilde{u}}, a)\right) + o(\epsilon^2) \end{aligned} \tag{2.5}$$

The first term on the right hand side of (2.4) is positive because $(a, a)$ is a strict Nash equilibrium. The second and third terms of (2.4) are positive because $(a, a)$ is an efficient outcome. Hence, there is an $\epsilon$ small enough such that (2.5) is positive.

It has been shown that, for every entrant $\tilde{u} \in U$, there is $\epsilon$ small enough such that they do not outperform incumbents. Still, it has to be shown that there is a uniform barrier $\bar{\epsilon}$ for which the fitness of any entrant cannot be higher than the fitness of incumbents when the fraction of entrants is no higher than $\bar{\epsilon}$. To show this, rewrite the strategy for the entrants in the following way. For $x \in \{u, \tilde{u}, \emptyset\}$, find $t_x \in [0, 1]$ and $\beta_x \in \triangle$ such that

$$\gamma_x = t_x a + (1 - t_x)\beta_x$$

where the support of $\beta_x$ does not include $a$.

The fitness for the incumbents under configuration $(\tilde{\mu}, \tilde{b})$ is given by

$$\begin{aligned} \Pi_{(\tilde{\mu},\tilde{b})}(u) &= (1-\epsilon)\pi(a,a) \\ &+ \epsilon \begin{pmatrix} p + q(1-\epsilon) \\ q\epsilon \\ 1-p-q \end{pmatrix}' \begin{pmatrix} t_u \\ t_{\tilde{u}} \\ t_\emptyset \end{pmatrix} \pi(a,a) \\ &+ \epsilon \begin{pmatrix} p + q(1-\epsilon) \\ q\epsilon \\ 1-p-q \end{pmatrix}' \begin{pmatrix} 1-t_u & 0 & 0 \\ 0 & 1-t_{\tilde{u}} & 0 \\ 0 & 0 & 1-t_\emptyset \end{pmatrix} \begin{pmatrix} \pi(a,\beta_u) \\ \pi(a,\beta_{\tilde{u}}) \\ \pi(a,\beta_\emptyset) \end{pmatrix} \end{aligned}$$

whereas the fitness for the entrants is given by

$$\begin{aligned}
\Pi_{(\tilde{\mu},\tilde{b})}(\tilde{u}) &= (1-\epsilon)\begin{pmatrix} p+q(1-\epsilon) \\ q\epsilon \\ 1-p-q \end{pmatrix}^{'}\begin{pmatrix} t_u \\ t_{\tilde{u}} \\ t_{\emptyset} \end{pmatrix}\pi(a,a) \\
&+ (1-\epsilon)\begin{pmatrix} p+q(1-\epsilon) \\ q\epsilon \\ 1-p-q \end{pmatrix}^{'}\begin{pmatrix} 1-t_u & 0 & 0 \\ 0 & 1-t_{\tilde{u}} & 0 \\ 0 & 0 & 1-t_{\emptyset} \end{pmatrix}\begin{pmatrix} \pi(\beta_u,a) \\ \pi(\beta_{\tilde{u}},a) \\ \pi(\beta_{\emptyset},a) \end{pmatrix} \\
&+ \epsilon\begin{pmatrix} p+q\epsilon \\ q(1-\epsilon) \\ 1-p-q \end{pmatrix}^{'}\begin{pmatrix} \pi(\gamma_{\tilde{u}},\gamma_{\tilde{u}}) & \pi(\gamma_{\tilde{u}},\gamma_u) & \pi(\gamma_{\tilde{u}},\gamma_{\emptyset}) \\ \pi(\gamma_u,\gamma_{\tilde{u}}) & \pi(\gamma_u,\gamma_u) & \pi(\gamma_u,\gamma_{\emptyset}) \\ \pi(\gamma_{\emptyset},\gamma_{\tilde{u}}) & \pi(\gamma_{\emptyset},\gamma_u) & \pi(\gamma_{\emptyset},\gamma_{\emptyset}) \end{pmatrix}\begin{pmatrix} p+q\epsilon \\ q(1-\epsilon) \\ 1-p-q \end{pmatrix}
\end{aligned}$$

The difference between fitness payoffs is then given by:

$$\begin{aligned}
&\Pi_{(\tilde{\mu},\tilde{b})}(u) - \Pi_{(\tilde{\mu},\tilde{b})}(\tilde{u}) = \\
&\begin{pmatrix} p+q(1-\epsilon) \\ q\epsilon \\ 1-p-q \end{pmatrix}^{'}\begin{pmatrix} 1-t_u & 0 & 0 \\ 0 & 1-t_{\tilde{u}} & 0 \\ 0 & 0 & 1-t_{\emptyset} \end{pmatrix}\begin{pmatrix} \pi(a,a)-\pi(\beta_u,a) \\ \pi(a,a)-\pi(\beta_{\tilde{u}},a) \\ \pi(a,a)-\pi(\beta_{\emptyset},a) \end{pmatrix} \\
&-\epsilon\begin{pmatrix} p+q(1-\epsilon) \\ q\epsilon \\ 1-p-q \end{pmatrix}^{'}\begin{pmatrix} 1-t_u & 0 & 0 \\ 0 & 1-t_{\tilde{u}} & 0 \\ 0 & 0 & 1-t_{\emptyset} \end{pmatrix}\begin{pmatrix} \pi(a,a)-\pi(\beta_u,a)-\pi(a,\beta_u) \\ \pi(a,a)-\pi(\beta_{\tilde{u}},a)-\pi(a,\beta_{\tilde{u}}) \\ \pi(a,a)-\pi(\beta_{\emptyset},a)-\pi(a,\beta_{\emptyset}) \end{pmatrix} \\
&+\epsilon\begin{pmatrix} p+q(1-\epsilon) \\ q\epsilon \\ 1-p-q \end{pmatrix}^{'}\begin{pmatrix} t_u \\ t_{\tilde{u}} \\ t_{\emptyset} \end{pmatrix}\pi(a,a) \\
&-\epsilon\begin{pmatrix} p+q\epsilon \\ q(1-\epsilon) \\ 1-p-q \end{pmatrix}^{'}\begin{pmatrix} \pi(\gamma_{\tilde{u}},\gamma_{\tilde{u}}) & \pi(\gamma_{\tilde{u}},\gamma_u) & \pi(\gamma_{\tilde{u}},\gamma_{\emptyset}) \\ \pi(\gamma_u,\gamma_{\tilde{u}}) & \pi(\gamma_u,\gamma_u) & \pi(\gamma_u,\gamma_{\emptyset}) \\ \pi(\gamma_{\emptyset},\gamma_{\tilde{u}}) & \pi(\gamma_{\emptyset},\gamma_u) & \pi(\gamma_{\emptyset},\gamma_{\emptyset}) \end{pmatrix}\begin{pmatrix} p+q\epsilon \\ q(1-\epsilon) \\ 1-p-q \end{pmatrix}
\end{aligned}$$

Efficiency implies that

$$\begin{pmatrix} p+q\epsilon \\ q(1-\epsilon) \\ 1-p-q \end{pmatrix}^{'}\begin{pmatrix} \pi(\gamma_{\tilde{u}},\gamma_{\tilde{u}}) & \pi(\gamma_{\tilde{u}},\gamma_u) & \pi(\gamma_{\tilde{u}},\gamma_{\emptyset}) \\ \pi(\gamma_u,\gamma_{\tilde{u}}) & \pi(\gamma_u,\gamma_u) & \pi(\gamma_u,\gamma_{\emptyset}) \\ \pi(\gamma_{\emptyset},\gamma_{\tilde{u}}) & \pi(\gamma_{\emptyset},\gamma_u) & \pi(\gamma_{\emptyset},\gamma_{\emptyset}) \end{pmatrix}\begin{pmatrix} p+q\epsilon \\ q(1-\epsilon) \\ 1-p-q \end{pmatrix} \leq \pi(a,a)$$

Therefore

$$\Pi_{(\tilde{\mu},\tilde{b})}(u) - \Pi_{(\tilde{\mu},\tilde{b})}(\tilde{u}) \geq$$
$$\begin{pmatrix} p+q(1-\epsilon) \\ q\epsilon \\ 1-p-q \end{pmatrix}' \begin{pmatrix} 1-t_u & 0 & 0 \\ 0 & 1-t_{\tilde{u}} & 0 \\ 0 & 0 & 1-t_\emptyset \end{pmatrix} \begin{pmatrix} \pi(a,a)-\pi(\beta_u,a) \\ \pi(a,a)-\pi(\beta_{\tilde{u}},a) \\ \pi(a,a)-\pi(\beta_\emptyset,a) \end{pmatrix} -$$
$$\epsilon \begin{pmatrix} p+q(1-\epsilon) \\ q\epsilon \\ 1-p-q \end{pmatrix}' \begin{pmatrix} 1-t_u & 0 & 0 \\ 0 & 1-t_{\tilde{u}} & 0 \\ 0 & 0 & 1-t_\emptyset \end{pmatrix} \begin{pmatrix} 2\pi(a,a)-\pi(\beta_u,a)-\pi(a,\beta_u) \\ 2\pi(a,a)-\pi(\beta_{\tilde{u}},a)-\pi(a,\beta_{\tilde{u}}) \\ 2\pi(a,a)-\pi(\beta_\emptyset,a)-\pi(a,\beta_\emptyset) \end{pmatrix}$$

Since $(a,a)$ is a strict Nash equilibrium, there is an $\bar{\epsilon}$ such that the right hand side of this inequality is non-negative for all $\epsilon \in (0,\bar{\epsilon})$ and any $(t_x,\beta_x)_{x\in\{u,\tilde{u},\emptyset\}}$.

□

Proposition 2.2 *If $(a,a)$ is not an efficient outcome of $G$, then there is a $\bar{\delta} \in (0,1)$ such that $a$ is not a stable outcome for any $(q,p) \in \mathcal{N}(\bar{\delta})$.*

*Proof.* Since $a$ is not efficient, there is a $\beta \in \triangle$ such that there is a $\bar{\delta}$ such that

$$\pi(a,a) < q\left(\pi(\beta,a)-\pi(a,a)\right) + \begin{pmatrix} p \\ 1-p \end{pmatrix}' \begin{pmatrix} \pi(\beta,\beta) & \pi(\beta,a) \\ \pi(a,\beta) & \pi(a,a) \end{pmatrix} \begin{pmatrix} p \\ 1-p \end{pmatrix} \tag{2.6}$$

for all $(q,p) \in \mathcal{N}(\bar{\delta})$.

The task is to find for every $\epsilon$ an entrant $\tilde{u}$ for which there is a focal equilibrium such that the entrant outperforms the incumbents. Consider a post-entry focal equilibrium in which the incumbents play $a$ no matter the signal and entrants i) play $a$ when perceiving an incumbent or receiving no signal and ii) play $\beta$ when perceiving and entrant. Such a focal equilibrium exists when $\epsilon$ is small enough and when $\tilde{u}$ is chosen suitably.

Suppose $a$ is a stable outcome for configuration $(\mu,b)$, where $b(u,x)=a$ for all $(u,x) \in C_\mu \times X_\mu$. The post-entry payoff for incumbents in this post-entry focal equilibrium is:

$$\Pi_{(\tilde{\mu},\tilde{b})}(u) = \pi(a,a) + \epsilon^2 q\left(\pi\left(a,\beta\right) - \pi\left(a,a\right)\right)$$

for $u \in C_\mu$. The only possibility for an incumbent to receive a payoff not equal to $\pi(a,a)$ is to encounter an entrant and being misperceived as an entrant, which is an event that occurs with probability proportional to $\epsilon^2$.

Meanwhile, the payoff for entrants is given by:

$$\begin{aligned}\Pi_{(\tilde{\mu},\tilde{b})}(\tilde{u}) &= (1-\epsilon)\left((1-q\epsilon)\,\pi(a,a)+q\epsilon\pi(\beta,a)\right)\\ &+ \epsilon\begin{pmatrix} p+q\epsilon \\ 1-p-q\epsilon\end{pmatrix}'\begin{pmatrix}\pi(\beta,\beta) & \pi(\beta,a)\\ \pi(a,\beta) & \pi(a,a)\end{pmatrix}\begin{pmatrix} p+q\epsilon \\ 1-p-q\epsilon\end{pmatrix},\end{aligned}$$

This can be rewritten as

$$\begin{aligned}\Pi_{(\tilde{\mu},\tilde{b})}(\tilde{u}) &= (1-\epsilon)\pi(a,a)+\epsilon q\left(\pi(\beta,a)-\pi(a,a)\right)\\ &+ \epsilon\begin{pmatrix} p \\ 1-p\end{pmatrix}'\begin{pmatrix}\pi(\beta,\beta) & \pi(\beta,a)\\ \pi(a,\beta) & \pi(a,a)\end{pmatrix}\begin{pmatrix} p \\ 1-p\end{pmatrix}+o(\epsilon^2).\end{aligned}$$

Stability implies:

$$\Pi_{(\tilde{\mu},\tilde{b})}(u) \geq \Pi_{(\tilde{\mu},\tilde{b})}(\tilde{u})$$

Therefore:

$$\begin{aligned}\epsilon\pi(a,a) &\geq \epsilon q\left(\pi(\beta,a)-\pi(a,a)\right)\\ &+ \epsilon\begin{pmatrix} p \\ 1-p\end{pmatrix}'\begin{pmatrix}\pi(\beta,\beta) & \pi(\beta,a)\\ \pi(a,\beta) & \pi(a,a)\end{pmatrix}\begin{pmatrix} p \\ 1-p\end{pmatrix}+o(\epsilon^2)\end{aligned}$$

This contradicts inequality (2.6) when $(q,p)\in\mathcal{N}(\bar{\delta})$

□

Proposition 2.3

*(a) If $(a,a)$ is not a Nash equilibrium of $G$, then there is a $\bar{\sigma}\in(0,1)$ such that $a$ is not stable for any $\sigma\in[0,\bar{\sigma})$.*

*(b) If $(a,a)$ is a strict Nash equilibrium of $G$, then there is a $\bar{\sigma}\in(0,1)$ such that $a$ is stable for any $\sigma\in[0,\bar{\sigma})$.*

*Proof.* *(a)* Since $(a,a)$ is not a Nash equilibrium, there is a degenerate distribution $\tilde{a}\in\triangle$ such that

$$\pi(a,a)<\pi(\tilde{a},a)$$

Therefore, there is $\bar{\sigma}\in(0,1)$ such that, for any $\sigma\in[0,\bar{\sigma})$,

$$\pi(a,a)-\left(\sigma q\pi\left(\tilde{a},\underline{\beta}\right)+(1-\sigma q)\,\pi(\tilde{a},a)\right)<0 \tag{2.7}$$

where

$$\underline{\beta} \in \arg\min_{\beta \in \triangle} \pi\left(\tilde{a}, \beta\right)$$

Suppose there is a stable configuration $(\mu, b)$ with outcome $a$. Consider an entrant $\tilde{u}$ for whom playing $\tilde{a}$ is a strictly dominant strategy. Since, $(\mu, b)$ is stable, there is a distribution $\tilde{\mu}$ as in (2.1) such that $\mathbb{F}_{(\mu,b)}(\tilde{\mu})$ is not empty for $\epsilon$ small enough.

For $\tilde{b} \in \mathbb{F}_{(\mu,b)}(\tilde{\mu})$, the average payoff for incumbents is given by

$$\begin{aligned}
\sum_{u \in C_\mu} \Pi_{(\tilde{\mu},\tilde{b})}(u)\mu(u) &= (1-\epsilon)\begin{pmatrix} 1-q\epsilon \\ q\epsilon \end{pmatrix}' \begin{pmatrix} \pi\left(a,a\right) & X \\ Y & Z \end{pmatrix} \begin{pmatrix} 1-q\epsilon \\ q\epsilon \end{pmatrix} \pi\left(a,a\right) \\
&+ \epsilon\left(\sigma+\epsilon\right)q \sum_{v \in C_\mu} \pi\left(\tilde{b}(v,\tilde{u}), \tilde{a}\right)\mu(v) \\
&+ \epsilon\left(1-\left(\sigma+\epsilon\right)q\right)\pi\left(a,\tilde{a}\right)
\end{aligned}$$

where

$$\begin{aligned}
X &:= \sum_{v \in C_\mu} \pi\left(\tilde{b}(v,\tilde{u}), a\right)\mu(v) \\
Y &:= \sum_{w \in C_\mu} \pi\left(a, \tilde{b}(w,\tilde{u})\right)\mu(w) \\
Z &:= \sum_{v \in C_\mu}\sum_{w \in C_\mu} \pi\left(\tilde{b}(v,\tilde{u}), \tilde{b}(w,\tilde{u})\right)\mu(v)\mu(w)
\end{aligned}$$

Hence

$$\sum_{u \in C_\mu} \Pi_{(\tilde{\mu},\tilde{b})}(u)\mu(u) = \pi(a,a) + o(\epsilon)$$

Meanwhile, the payoff for an entrant is given by

$$\begin{aligned}
\Pi_{(\tilde{\mu},\tilde{b})}(\tilde{u}) &= (1-\epsilon)(\sigma+\epsilon)q \sum_{u \in C_\mu} \pi\left(\tilde{a}, \tilde{b}(u,\tilde{u})\right)\mu(u) \\
&+ (1-\epsilon)\left(1-\left(\sigma+\epsilon\right)q\right)\pi\left(\tilde{a},a\right) \\
&+ \epsilon\pi(\tilde{a},\tilde{a})
\end{aligned}$$

Therefore:

$$\Pi_{(\tilde{\mu},\tilde{b})}(\tilde{u}) \geq \left(\sigma q \pi\left(\tilde{a},\underline{\beta}\right) + \left(1-\sigma q\right)\pi\left(\tilde{a},a\right)\right) + o(\epsilon)$$

Stability implies that, for all $\epsilon$ below certain threshold:

$$\sum_{u \in C_\mu} \Pi_{(\tilde{\mu},\tilde{b})}(u)\mu(u) - \Pi_{(\tilde{\mu},\tilde{b})}(\tilde{u}) \geq 0$$

This implies

$$\pi(a,a) - \left(\sigma q \pi\left(\tilde{a}, \underline{\beta}\right) + (1-\sigma q)\,\pi\left(\tilde{a}, a\right)\right) + o(\epsilon) \geq 0$$

For $\sigma \in [0, \bar{\sigma})$, this contradicts inequality (2.7).

*(b)* Consider a configuration $(\mu, b)$ where $C_\mu = \{u\}$ and playing $a$ is a strictly dominant strategy for $u$. Denote by $\tilde{u}$ the utility function for an entrant. Consider a focal equilibrium $\tilde{b} \in \mathbb{F}_{(\mu,b)}(\tilde{\mu})$ such that $\tilde{\mu}$ is such that (2.1) holds for some $\epsilon \in (0,1)$. Since $a$ is strictly dominant for $u$, $\mathbb{F}_{(\mu,b)}(\tilde{\mu})$ is not empty and $\tilde{b}(u,x) = a$ for $x \in \{u, \tilde{u}, \emptyset\}$.

To shorten the notation, denote the strategies an entrant follows when receiving signals $u$, $\tilde{u}$ and $\emptyset$ by

$$\gamma_u := \tilde{b}(\tilde{u}, u)$$
$$\gamma_{\tilde{u}} := \tilde{b}(\tilde{u}, \tilde{u})$$
$$\gamma_\emptyset := \tilde{b}(\tilde{u}, \emptyset)$$

Consider the case where $\gamma_u$ or $\gamma_\emptyset$ are not equal to $a$. Since $(a,a)$ is a strict Nash equilibrium of $G$, it follows that

$$(p+q)\,\pi\left(\gamma_u, a\right) + (1-(p+q))\,\pi\left(\gamma_\emptyset, a\right) < \pi\left(a,a\right)$$

If this is the case, an entrant who is not best responding with $a$ when receiving signals $u$ or $\emptyset$ will not be able to outperform the incumbents whenever the fraction of entrants is sufficiently small.

Consider the case where $\gamma_u$ and $\gamma_\emptyset$ are equal to $a$. Decompose the entrant's strategy when receiving signal $\tilde{u}$ in the following way: find $t_{\tilde{u}} \in [0,1)$ and $\beta_{\tilde{u}} \in \triangle$ such that:

$$\gamma_{\tilde{u}} = t_{\tilde{u}} a + (1 - t_{\tilde{u}})\beta_{\tilde{u}}$$

where the support of $\beta_{\tilde{u}}$ does not include $a$.

The fitness for the incumbent in the focal equilibrium is given by

$$\Pi_{(\tilde{\mu},\tilde{b})}(u) = (1-\epsilon)\pi\left(a,a\right) + \epsilon\left(\pi\left(a,a\right) + (1-t_{\tilde{u}})\,q\epsilon\left(\pi\left(a,\beta_{\tilde{u}}\right) - \pi\left(a,a\right)\right)\right)$$

meaning that not receiving a payoff $\pi(a,a)$ is an event of order $\epsilon^2$: it requires being matched with an entrant and being misperceived as an entrant.

The fitness for the entrant is given by:

$$\begin{aligned}\Pi_{(\tilde{\mu},\tilde{b})}(\tilde{u}) &= (1-\epsilon)\left(\pi\left(a,a\right)+\left(1-t_{\tilde{u}}\right)q\epsilon\left(\pi\left(\beta_{\tilde{u}},a\right)-\pi\left(a,a\right)\right)\right) \\ &+ \epsilon\begin{pmatrix} p+q\epsilon \\ 1-p-q\epsilon\end{pmatrix}^{'}\begin{pmatrix}\pi(\gamma_{\tilde{u}},\gamma_{\tilde{u}}) & \pi(a,\gamma_{\tilde{u}}) \\ \pi(\gamma_{\tilde{u}},a) & \pi(a,a)\end{pmatrix}\begin{pmatrix} p+q\epsilon \\ 1-p-q\epsilon\end{pmatrix}\end{aligned}$$

Using the fact that $p=\sigma q$, the fitness for the entrant can be rewritten as:

$$\begin{aligned}\Pi_{(\tilde{\mu},\tilde{b})}(\tilde{u}) &= (1-\epsilon)\pi\left(a,a\right)+\left(1-t_{\tilde{u}}\right)q\epsilon\left(\pi\left(\beta_{\tilde{u}},a\right)-\pi\left(a,a\right)\right) \\ &+ \epsilon\begin{pmatrix} (\sigma+\epsilon)q \\ 1-(\sigma+\epsilon)q\end{pmatrix}^{'}\begin{pmatrix}\pi(\gamma_{\tilde{u}},\gamma_{\tilde{u}}) & \pi(a,\gamma_{\tilde{u}}) \\ \pi(\gamma_{\tilde{u}},a) & \pi(a,a)\end{pmatrix}\begin{pmatrix} (\sigma+\epsilon)q \\ 1-(\sigma+\epsilon)q\end{pmatrix}\end{aligned}$$

Therefore, the difference between payoffs will be given by:

$$\begin{aligned}&\Pi_{(\tilde{\mu},\tilde{b})}(u)-\Pi_{(\tilde{\mu},\tilde{b})}(\tilde{u}) = \epsilon\left(1-t_{\tilde{u}}\right)q\left(\pi\left(a,a\right)-\pi\left(\beta_{\tilde{u}},a\right)\right)+ \\ &\epsilon\left(\pi\left(a,a\right)-\begin{pmatrix} (\sigma+\epsilon)q \\ 1-(\sigma+\epsilon)q\end{pmatrix}^{'}\begin{pmatrix}\pi(\gamma_{\tilde{u}},\gamma_{\tilde{u}}) & \pi(a,\gamma_{\tilde{u}}) \\ \pi(\gamma_{\tilde{u}},a) & \pi(a,a)\end{pmatrix}\begin{pmatrix} (\sigma+\epsilon)q \\ 1-(\sigma+\epsilon)q\end{pmatrix}\right)\end{aligned}$$

The expression can be rewritten as:

$$\begin{aligned}&\Pi_{(\tilde{\mu},\tilde{b})}(u)-\Pi_{(\tilde{\mu},\tilde{b})}(\tilde{u}) = \left(1-t_{\tilde{u}}\right)\epsilon q\left(\pi\left(a,a\right)-\pi\left(\beta_{\tilde{u}},a\right)\right)+ \\ &\epsilon(\sigma+\epsilon)q(\sigma+\epsilon)q\left[\pi\left(a,a\right)-\begin{pmatrix} t_{\tilde{u}} \\ 1-t_{\tilde{u}}\end{pmatrix}^{'}\begin{pmatrix}\pi\left(a,a\right) & \pi(a,\beta_{\tilde{u}}) \\ \pi(\beta_{\tilde{u}},a) & \pi(\beta_{\tilde{u}},\beta_{\tilde{u}})\end{pmatrix}\begin{pmatrix} t_{\tilde{u}} \\ 1-t_{\tilde{u}}\end{pmatrix}\right]+ \\ &\epsilon(\sigma+\epsilon)q(1-(\sigma+\epsilon)q)\left[\pi\left(a,a\right)-\begin{pmatrix} 1 \\ 0\end{pmatrix}^{'}\begin{pmatrix}\pi\left(a,a\right) & \pi(a,\beta_{\tilde{u}}) \\ \pi(\beta_{\tilde{u}},a) & \pi(\beta_{\tilde{u}},\beta_{\tilde{u}})\end{pmatrix}\begin{pmatrix} t_{\tilde{u}} \\ 1-t_{\tilde{u}}\end{pmatrix}\right]+ \\ &\epsilon(\sigma+\epsilon)q(1-(\sigma+\epsilon)q)\left[\pi\left(a,a\right)-\begin{pmatrix} t_{\tilde{u}} \\ 1-t_{\tilde{u}}\end{pmatrix}^{'}\begin{pmatrix}\pi\left(a,a\right) & \pi(a,\beta_{\tilde{u}}) \\ \pi(\beta_{\tilde{u}},a) & \pi(\beta_{\tilde{u}},\beta_{\tilde{u}})\end{pmatrix}\begin{pmatrix} 1 \\ 0\end{pmatrix}\right]\end{aligned} \tag{2.8}$$

Since $(a,a)$ is a strict Nash equilibrium, it follows that:

$$\begin{aligned}&\pi\left(a,a\right)-\begin{pmatrix} t_{\tilde{u}} \\ 1-t_{\tilde{u}}\end{pmatrix}^{'}\begin{pmatrix}\pi\left(a,a\right) & \pi(a,\beta_{\tilde{u}}) \\ \pi(\beta_{\tilde{u}},a) & \pi(\beta_{\tilde{u}},\beta_{\tilde{u}})\end{pmatrix}\begin{pmatrix} t_{\tilde{u}} \\ 1-t_{\tilde{u}}\end{pmatrix} \\ &> \\ &\pi\left(a,a\right)-\begin{pmatrix} t_{\tilde{u}} \\ 1-t_{\tilde{u}}\end{pmatrix}^{'}\begin{pmatrix}\pi\left(a,a\right) & \pi(\bar{\beta},\beta_{\tilde{u}}) \\ \pi\left(a,a\right) & \pi(\bar{\beta},\beta_{\tilde{u}})\end{pmatrix}\begin{pmatrix} t_{\tilde{u}} \\ 1-t_{\tilde{u}}\end{pmatrix}\end{aligned}$$

where

$$\bar{\beta} := \arg \max_{\zeta \in \{a, \beta_{\tilde{u}}\}} \pi(\zeta, \beta_{\tilde{u}})$$

Substituting this inequality into (2.8) yields:

$$\begin{aligned} \Pi_{(\tilde{\mu},\tilde{b})}(u) - \Pi_{(\tilde{\mu},\tilde{b})}(\tilde{u}) &> (1 - t_{\tilde{u}})\, \epsilon q \left(\pi\left(a, a\right) - \pi\left(\beta_{\tilde{u}}, a\right)\right) \\ &+ (1 - t_{\tilde{u}})\, \epsilon(\sigma + \epsilon)^2 q^2 \left(\pi(a, a) - \pi(\bar{\beta}, \beta_{\tilde{u}})\right) \\ &+ (1 - t_{\tilde{u}})\, \epsilon(\sigma + \epsilon) q(1 - (\sigma + \epsilon) q) \left(\pi\left(a, a\right) - \pi(a, \beta_{\tilde{u}})\right) \\ &+ (1 - t_{\tilde{u}})\, \epsilon(\sigma + \epsilon) q(1 - (\sigma + \epsilon) q) \left(\pi\left(a, a\right) - \pi(\beta_{\tilde{u}}, a)\right) \end{aligned} \tag{2.9}$$

Therefore, we can find a suitable threshold for $\sigma$ such that the right-hand side of (2.9) is non-negative for any $t_{\tilde{u}} \in [0, 1]$ and $\epsilon$ small enough.

□

# Chapter 3

# An urban legend? The long-term effects of fertility shocks[1]


## Abstract

*This paper studies one dimension of the social cost of bad public infrastructure in developing countries. We use an extensive period of power rationing in Colombia throughout 1992 as a natural experiment and exploit exogenous spatial variation in the intensity of power rationing as an instrumental variable. We show that power rationing induced a "mini baby boom" nine months later. In particular, it increased the probability that a mother had a baby by five percent. We estimate that every tenth power outage baby was not adjusted for 12 years later, resulting in an overall increase in total fertility. This increase has indirect social costs, as women who were exposed to the shock and had an additional child find themselves in worse socio-economic conditions more than a decade later.*


## 3.1 Introduction

The idea of baby booms following a blackout has been a subject of contention for a long time. It first came into prominence in popular culture after the great New York blackout of 1965, which left over 30 million people without electricity for 13 hours. However, the seminal work by Udry (1970) concluded that there was no significant impact of the great New York blackout on fertility 9 months later. Since then, the theory has been termed as an "urban legend" by the President of the Population Association of America.

[1] Coauthored with Thiemo Fetzer and Amar Shanghavi.

Unlike the New York blackout or indeed most power outages in the developed world, which are limited in time and space, developing countries have been experiencing great power uncertainty. Many states in Africa experience rolling blackouts, which last weeks if not months and for several hours a day.[2] Thus, though the evidence may not be supporting the existence of baby booms after a blackout in developed countries, the frequency of blackouts over a longer period of time make it more plausible to causally link blackouts in developing countries to baby booms 9 months later.

If periods of unexpected power rationing entail baby booms, there may also be large *hidden social costs*. Women may not continue their studies because of unexpected pregnancies. In turn, unexpected babies may experience worse health and educational outcomes than the expected ones. Fertility shocks have implications for investment in human capital and asset accumulation in general, affecting the labor market outcomes and the life path of parents and their children. In short, extensive periods of power rationing may entail changes in fertility behavior and these changes may lead to large social costs.

In order to test these hypotheses, we carry out three empirical exercises, all based on an episode of power rationing in Colombia that lasted from February 1992 to March 1993. In the first one, we provide evidence that there is a causal effect of power outages on short-run fertility. In order to identify this effect, we construct two novel datasets. First, we use the IPUMS micro-sample for the 2005 population census of Colombia to construct a retrospective mother-level birth history by linking mothers to children within the household. Next, we combine this dataset with municipality level variation in night lights as measured by satellite images for the period 1992/1993 and construct a variable of treatment intensity for the power crisis. By using the retrospective mother data and the intensity of the blackout on the municipalities where they live, we estimate that a full blackout leads to an increase of 0.005 percentage points in the probability of having a child the year after. When evaluated at the mean probability of having a child in any given year, this results in an increase in probability of having a birth by 5 percent. Using the mean intensity of the power outage across Colombia, we are able to calculate a back-of-the-envelope estimate of approximately 10,000 additional births in 1993 due to the blackout.

Our second empirical exercise shows that this increase in short-term fertility is not dynamically adjusted through fewer children in the future, implying an overall increase in the total number of children 12 years later. In order to identify this long-run effect, we apply a difference-in-difference approach using our mothers' data. We select two groups

[2] Reliable electricity provision has been identified as a key driver for growth (see Dinkelman, 2011). In developing countries, electricity supply to firms and households is extremely erratic, constraining development (see Abeberese, 2012; Fisher-Vanden et al., 2012; Foster and Steinbuks, 2009; Reinikka and Svensson, 2002).

of women: the ones who were already pregnant when the blackout started and the ones who got pregnant during the blackout. For each group, we check how their long-run fertility varies with the blackout intensity. Since the two groups are otherwise similar, any difference in the variation between the two groups can only be attributed to the blackout. In this way, we estimate that a tenth of the short-run increase in fertility was not adjusted for 12 years after the blackout. Additional placebo exercises confirm the long-lasting effect of the blackout on the total number of children. Further, consistent with our priors, younger mothers (aged 30 or less in 1992) respond to the treatment in 1993. However, they are also better able to adjust their lifetime fertility in the long run. In contrast, though the older cohort has a weaker response to the blackout in 1993, they are less likely to be able to adjust their lifetime fertility due to an unexpected baby, thus driving most of our long-run effect. We conclude that 1 out of 10 blackout babies were not fully adjusted for 12 years later. Given the small size of the short and long run effects, we are not surprised that most demographers have failed to find any significant effects of power outages on fertility.

In our third empirical exercise, we study the effect of unintended pregnancy on the long-term socio-economic outcomes of mothers. This is a question of key interest to labor economists (see e.g. Angrist and Evans, 1998; Ashcraft et al., 2013), but has mainly been studied in developed country contexts. In order to identify the effect of childbearing on mother outcomes, we use our previous exercise to predict the number of children a woman has because of the blackout. Then, we use these predicted values as an explanatory variable for the mothers' socio-economic outcomes, in order to estimate the causal effect of having additional children. The biggest challenge to this exercise is to ensure that the blackout did not affect the socio-economic outcomes through any other channel different than the number of children 12 years later. This concern is addressed in the first stage of our estimation, since both treatment and control were exposed to the power outage, but only the treatment group could physically get pregnant. From this exercise, we are able to paint a consistent picture of the negative consequences of unplanned motherhood following the blackout. We provide evidence showing that women who had an unplanned baby due to the blackout end up with worse educational attainment and are more likely to be single mothers living in poorer housing conditions. These findings suggest that there may have been significant welfare consequences through long-term persistence of the fertility shock. To our knowledge, this is the first paper to link long-run socioeconomic outcomes of power rationing with unplanned motherhood and more generally the long-run consequences of unintended pregnancies in a developing country context.

This paper contributes to several strands of literature. Firstly, it contributes to a limited but growing empirical literature on examining the impact of electricity infrastructure in developing countries (see Dinkelman, 2011; Rud, 2012). In particular, it provides evidence of the influence of infrastructure on fertility behavior. On this strand, the work most closely related to ours is Burlando (2014a), who looks at the impact of a month-long power outage in Zanzibar on village-level fertility outcomes. He finds a mini baby boom 9 months after the blackout, with an increase in village level births by 20%.[3]

Second, this paper also contributes to a better understanding of fertility responses to other aggregate shocks. For example, Evans et al. (2008) and Pörtner (2008) study the effect of natural disasters and hurricanes in particular. They find a negative relationship between hurricane advisories and baby booms 9 months after the event. As the type of advisory goes from least severe to most severe, the fertility effect of the specific advisory type decreases monotonically from positive to negative. Our paper also relates to the literature understanding the role of culture, media and leisure on fertility (Ferrara et al., 2012; Jensen and Oster, 2009; Kearney and Levine, 2014). These studies have found a link between television programming and fertility behavior, including smaller family sizes.

Finally, we also provide evidence on the long-run consequences of unwanted pregnancies and the effect of children on their mother's labor market and socio-economic outcomes (Angrist and Evans, 1998; Jacobsen et al., 1999). In the Colombian context, this is particularly worrying as unwanted pregnancies and early childbearing are common in Latin America and the Caribbean. According to Koontz and Conly (1994), women who begin childbearing as teenagers are estimated to have two to three more children than women who delay their first birth until their twenties or later (Gutmacher Institute, 1997). Not only does early motherhood lead to more children in the future, but high adolescent fertility rates are linked to low educational attainment and poverty (Koontz and Conly, 1994). In developed countries the findings are similar, where early motherhood leads to lower educational outcomes, worse housing conditions and worse labor market outcomes (Levine and Painter, 2003; Ashcraft et al., 2013; Kaplan et al., 2004).

The rest of the paper proceeds as follows. In Section 3.2, we discuss a theoretical background that leads to the predictions that we test. Section 3.3 provides a brief background on the context of the 1992 black out in Colombia. In section 3.4, we describe the data and how we constructed our main dependent variables. Section 3.5 provides the empirical strategy and the key results. The conclusion follows in Section 3.6.

[3] In related work, Burlando (2014a) finds that in-utero exposure of the mother to the power outage resulted in lower birth weight.

## 3.2 Theoretical background

An unexpected, long-lasting and intense blackout -like the one experienced by Colombia in 1992- might have an impact on fertility behavior through different channels. Many people, if not losing their jobs, may be forced to get back home earlier from work, given the scheduled blackout times. Leisure activities outside home were less attractive if not more dangerous because of the lack of night light. Inside the household, leisure activities involving electricity became prohibitively expensive at the running blackout times. All these channels suggest that the opportunity cost of sex decreased as a consequence of a long-lasting blackout.

The dominance of these substitution effects is not straightforward, though. If childbearing is costly, a decline in current income decreases the present value of wealth, which decreases the affordability of children in the future and therefore might decrease sex activity in the present.[4] Additionally, disposition towards sex activity might decrease as income falls (e.g. stress). These wealth and income effects might dominate the substitution effects. Therefore, the effect of the blackout on fertility is theoretically ambiguous.

Having children at an earlier stage might affect the life-cycle asset-accumulation of both parents and children. Parenthood might force early school retirement for teenage parents when children demand time and money. Unexpected children might have less access to nutrition and education than those who were planned.[5] Since the available data is for only 12 years after the blackout, we are not ready yet to fully assess the impact on children. Hence, both the theoretical and empirical exercises we carry out focus on the effect of fertility shocks on parents' human, physical and social capital.

A theoretical model, presented in Appendix 3.8.1, highlights the channels through which unexpected blackouts may affect a mother's lifetime fertility and education. It formalizes the following two ideas: Firstly, if the substitution effect dominates, the blackout decreases the opportunity cost of sex and therefore it leads to more pregnancies. Secondly, childcare competes with education for time and other resources, leading to a lower educational attainment for mothers. This comparative statics parallel the empirical exercises carry out in Section 3.5.

[4] As per the 1990 Colombia Demographic Health Survey, condom use was only prevailing among 1.6% of the respondents, while 35% of respondents were using no contraceptive method at all.

[5] Burlando (2014b) finds evidence suggesting that in utero exposure to a power outage induced income effect resulted in lower birth weight.

## 3.3 Historical background

We study the Colombian context where power rationing was in place from February 1992 to March 1993.

In 1992 Colombia derived roughly 80 % of its electricity consumption from hydroelectric sources. About 40% of this energy is produced in fourteen hydro-electric power plants that are located mainly in the Caldas and Antioquia departments of central Colombia. These are located to the north and east of Medellin, where the Mountain ranges of the Andes typically provide ample rainfall runoff water that may be used for hydroelectric power generation.

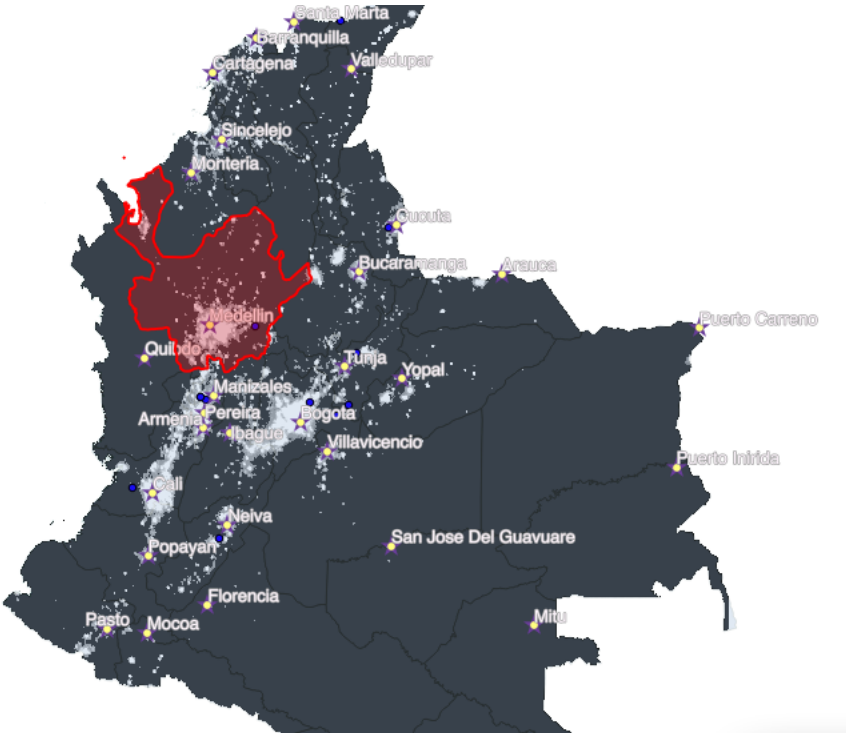


FIGURE 3.1: Colombia Administrative Regions, night lights emissions in 1992 and Provincial Capital Cities. Antioquia departmento is highlighted.

Especially Antioquia departmento, highlighted in Figure 3.1, is a major location for hydro-electric power generation. It has several power plants that are sited between two rivers, being supplied with water from one and emptying the water into another after it passes through the turbines. This makes these power plants particularly vulnerable to reductions in water flows.[6]

In 1992, El Nino droughts led to a dramatic depletion of water reservoirs that feed most major power plants. Some power stations needed to cut back power production

[6]Department of Energy, An Energy Overview of Colombia, `http://goo.gl/nnhWBN`.

dramatically as there was simply too little water to produce electricity with. One of the biggest energy firms estimated that throughout the year there was a shortfall equivalent to roughly 20% of the annual production of 1991.[7]

The resulting power rationing was felt by Colombians across the country, which is why the period from 1992 to 1993 is referred to by Colombians simply as the "Black Out". There are no country-wide figures available, as even for some departments, no power production or consumption data is available for that period. For our purposes, it is important to note that the short-fall in production was not evenly spread across the country. In the north-east, historically thermal power generation from coal has been available and some parts of the South had not been connected to the national electricity grid. Further, electricity losses along the transmission lines generate a natural gradient. All these contribute to creating spatial variation in the intensity of the power rationing.[8]

Since 1993, Colombia has had a very stable electricity supply. In response to the power outages and rolling blackouts of 1993, the government heavily invested in infrastructure and de-regulation of the energy sector.[9] The previous period of power rationing caused by low hydro inflows due to a climate phenomenon was in 1983 (McRae, 2010). Based on various newspaper articles from the period, the timing of the previous drought-driven blackout and the strong response of the government to the shock, we claim that the 1992/93 episode was not anticipated. This will help us to identify the effect of blackouts on fertility and the effect of fertility on long-run socio-economic outcomes.

## 3.4 Data

### 3.4.1 Detecting Power Outages from Remote Sensing

Satellite-derived night lights data has been used by economist to map economic activity (see Doll, 2008), economic growth (Henderson et al., 2012) or the evolution of agglomeration clusters over time (see Storeygard, 2012; Fetzer and Shanghavi, 2014). This data has the advantage of being available where reliable GDP statistics do not exist. Furthermore, they allow the study of the geography of urbanization. Political economists have been using this data to map the role of ethnic origin of a leader and the provision

[7]See `http://www.tebsa.com.co/history.htm`, accessed on 20.06.2013.

[8]This is akin to Costa (2013), who studies the long run effects of power rationing in the South of Brazil due to power production shortages following droughts in 2001. The lack of integration in the power network created distinct spatial variation in the extent of power rationing in his context.

[9]In particular, a lot of investment has also been made to increase access to on-demand thermal power plants capacity. According to `https://www.cia.gov/library/publications/the-world-factbook/geos/co.html`, the share of hydroelectric power generation capacity is now only 67%.

of public goods (see Hodler and Raschky, 2014), since in some contexts data on the provision of public goods is not available. However, to the best of our knowledge, the night lights data has not been exploited to study abnormal variation in night light intensity, which may be caused by power outages.

We exploit luminosity data to map the geographic heterogeneity of the extent to which there was indeed a blackout. Figure 3.2 highlights our approach to measure this heterogeneity indirectly, using night lights luminosity data available from US run Defense Metrological Satellite Program (DMSP).

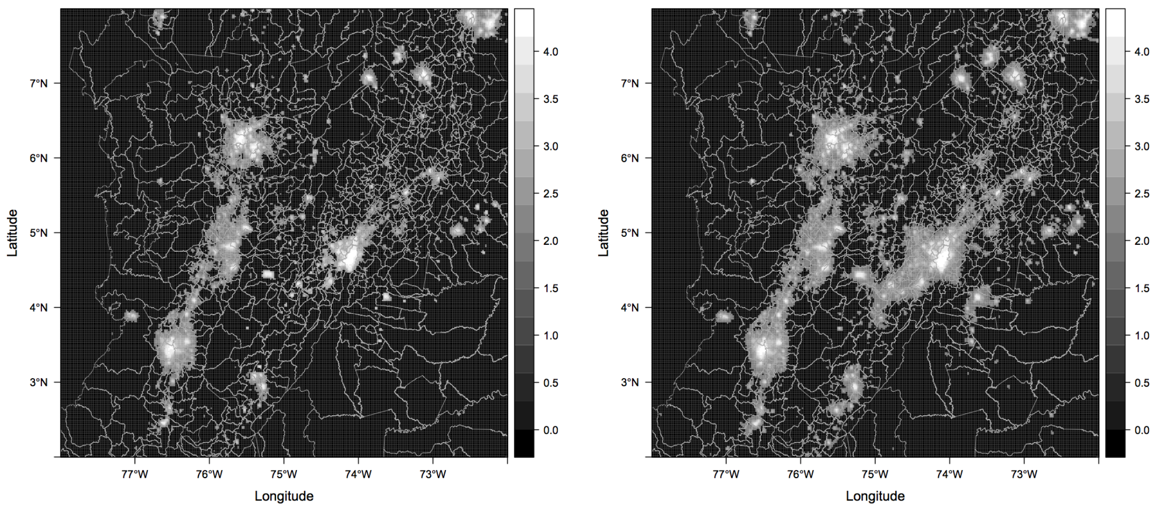


Figure 3.2: Light intensity in Colombia, 1992 (left) and 1993 (right) on identical log-scales.

The figures depict the luminosity variable around three main urban centers in Colombia in 1992 (left) and 1993 (right). The northern area is the Medellin metropolitan area, while in the south is Colombia's second biggest city Cali. The light-blob to the right is the metropolitan area of Bogota. The differences in the pictures are dramatic. Especially around Bogota, the broader geographic area appeared to be completely dark in 1992, while it is lit in 1993.

Note that the night light data series is only available from 1992 onward. Hence, we cannot compare the light intensity in the year 1992 (the year of the outage) with preceding years, as this data simply does not exist. However, we may be able to compare the 1992 lighting intensity with the intensity in 1993 or 1994. This measure will, of course, be subject to measurement error since the 1993 luminosity is an outcome variable in itself. However, as we are studying micro-level data, it is hard to believe that the micro mother-level variation we exploit has a direct effect on 1993 luminosity. In fact, in appendix 3.8.3 we show that it is difficult to argue that the year-on-year variation between

1992 and 1993 is capturing something other than the power-outage, since the pictures for 1993 and 1994 look almost identical.

We construct the power outage intensity variable at municipality level as being essentially the ratio of the average population-weighted municipality luminosity in 1992 over that measure for 1993, i.e. it is constructed as:

$$O_m = 100 \times \left(1 - \frac{Lights_{1992}}{Lights_{1993}}\right)$$

The weighting by population becomes necessary as the IPUMS data merges several municipalities that have population sizes less than 20,000 to ensure that users of the data are not able to reverse-engineer who the individuals in the sample were. This makes the treatment intensity construction more tedious and less clean, but does not represent a significant issue. In total we are left with 515 municipalities that have population above 20,000.[10] The average measure suggests an outage extent of around 30%. Total luminosity was 30% lower in 1992 compared to 1993. This maps well into the estimates from Tebsa, a big power generator at the time, who estimate that the shortfall was around 20% relative to the annual production in 1991.[11]

Before turning to the empirical specification and the main results, we discuss the census data that is used throughout the paper.

### 3.4.2 Census Data

An analysis that studies fertility effects at the aggregate level may fail to discover any statistical effect due to low power conjoint with small effect sizes. In order to address this, we construct individual level birth histories using the 2005 micro sample census of Colombia provided by IPUMS. The micro-data sample covers 10% of the population and was hailed to be the most successfully conducted population census.[12] For the short run outcomes, we construct a retrospective panel of mothers using the matched mother to children data for the period 1990 to 1995. We restrict our analysis to mothers aged between 18 and 45 in 1993. This ensures that no woman was younger than 15 years old in 1990, the earliest year of analysis in our sample. The panel structure of the data allows us to identify the effect of the blackout by exploiting within-mother variation in the timing of birth of babies, instead of cross-region or within-region variation. This

[10] Our results are robust to weighting by the geographic size of municipalities. Note that the measure may actually take on negative values, indicating that there are places that were more luminous in 1992 than in 1993; however, the bulk of the municipalities exhibit a positive measure.

[11] See `http://www.tebsa.com.co/history.htm`, accessed on 20.06.2013.

[12] See `http://unstats.un.org/unsd/censuskb20/KnowledgebaseArticle10236.aspx`, accessed 20.06.2013.

gives us a total number of 624,667 mothers with 369,140 children born in this period. We restrict the short run analysis to a tight 6 year window, given the dynamic nature of changes in fertility both over time and space. Due the exogenous timing of the shock, focusing on the years immediately before and after the blackout allows us to mitigate some of the concerns related to parallel trends across the different municipalities, which would be needed to study fertility dynamics over a longer horizon.

For the long run analysis, we exploit the exogenous timing of the blackout caused by El-Nino rainfall shortages and take advantage of the timing of births, giving us a quasi-random experiment where mothers were not treated by the blackout due to biological constraints. We restrict our analysis to comparing women who gave birth in 1993 to women who gave birth in 1992. Further, we only consider women who have fewer than 8 children. The reason to focus on this group of women is driven by the fact that a non-trivial number of mothers report having more than 7 children. This is approximately 10% of the sample of women who gave birth in 1992/1993. It is very likely that a woman that has 12 children is very different than a woman that has total number of children closer to the sample mean of 4. Thus we restrict the sample to this sub-population as it allows us to identify the impact of an unintended pregnancy on an average woman (especially with declining fertility rates over time). This leaves us with 56,656 and 61,282 women in the sample for 1992 and 1993, respectively. The summary statistics for the key variables in the short and long-run are presented in Appendix 3.8.2.

We now proceed to detail the empirical strategy and present the main results.

## 3.5 Empirical Strategy

We separate the empirical analysis into three steps. Firstly, we look at the short run implications of the power outages on mother-level fertility behavior. Secondly, we show that these effects persisted - i.e. that the power outage is associated with a life-time increase in fertility. Thirdly, we ask how long-run fertility correlates with economic outcomes for the mothers, thus highlighting the possibility of there being welfare consequences.

### 3.5.1 Short Run Fertility Effects

Our dependent variable is a dummy variable $B_{imt} = 1$, if mother $i$ from municipality $m$ gave birth in year $t$. We estimate the following linear probability model specification:

$$B_{imt} = a_i + b_t + \gamma \times O_m \times T_t + \epsilon_{imt} \tag{3.1}$$

where we include mother fixed effects $a_i$ and time fixed-effects $b_t$. We add the sub-index $m$ for municipality, since the treatment intensity is fixed at municipality level. The treatment assignment is $T_t = 1$ for $t = 1993$, i.e. the year in which babies conceived in 1992 are being born.[13] Note that $T_t$ is perfectly collinear with the time-fixed effects $b_t$ and that the power outage measure $O_m$ is invariant at municipality level, thus perfectly collinear with the mother fixed effects $a_i$.

The coefficient of interest is $\gamma$, which measures the average difference in the probability of giving birth for a mother. The interaction term exploits variation across municipalities in the degree of power-outage intensity measured by $O_m$. The coefficient $\gamma$ represents the causal effect of power-outage intensity on the probability of giving birth under the following assumption. After controlling for mother fixed effects and exogenous covariates, the changes in probability of birth for mothers living in municipalities which experienced lower power-outage treatment provide a counterfactual for mothers living in municipalities which were hit by higher power-outage treatment.

Municipality unobserved characteristics could be a source of violation of the identifying assumption represented by model 3.1. However, unobservable time-invariant characteristics of municipalities such as geography, history and culture are not because the set of mother fixed effects in each municipality captures such municipality fixed effects. On the other hand, due to the short time frame we consider for the analysis (1990-1995), we are not very concerned about time-varying unobserved factors which may be correlated with the intensity of power-outage and probability of birth.[14]

### 3.5.2 Long-run Fertility Outcomes

A short-run fertility shock may be fully compensated by a reduction in fertility in the future. In particular, the total number of children in the lifetime of a women may be not affected by the power outage, as increased fertility behavior during the blackout is compensated with less fertility in later years.[15]

We study whether women who experienced a fertility shock were adjusting their fertility behavior 12 years after the blackout. The key difficulty for this exercise is to find an adequate treatment and control-group for which a difference-in-difference methodology

[13] It is clear that this is an Intention to Treat design, as we do not actually observe fertility and sexual behavior around the time, i.e. we cannot rule out that some women who were assigned treatment did actually receive treatment.

[14] Another obvious concern is migration. We address this in the robustness checks and show that our results are robust to studying women who most likely did not migrate.

[15] Nevertheless, one may still find an effect of unexpected children on the mother's socio-economic outcomes. Since in 2005, most of these children were only 12 years old, we are not yet able to study consequences on the "power outage babies" themselves. This is left to the next census round.

can be applied on the cross-sectional data on the number of children per women that comes from the 2005 census. This is not straightforward as cohorts in different age-groups may be differing in many ways. Hence, it is difficult to verify a common trends assumption.

Selecting a good counterfactual cohort may help us address issues regarding the *underlying mechanisms*. In particular, it is possible that the power outage had a direct effect on incomes and through that affected long-run outcomes at the mother-level. This would lead to a violation of the exclusion restriction for our later instrumental variables exercise. A second concern is that the power-outage may have led women to become mothers at an earlier age. Since these women are sexually active for a longer period of time, we would expect them to have, in total, more children. It has been shown that early motherhood has been found to adversely affect the mother (see e.g. Ashcraft et al., 2013), which would lead to yet another violation of the exclusion restriction for our exercise.

We choose our treatment and control to capture the unplanned motherhood mechanism. In particular, we choose our control group such that both treatment and control were exposed to the power outage (and thus, a possible income shock), but only the treatment group could *physically receive the treatment*. To address the channel of early motherhood affecting socio-economic outcomes, we present results for a constrained sample of mothers both in the control and treatment group who had given birth at least once before 1992 and 1993, respectively.

With this in mind, we chose the control group as women who gave birth during 1992, the period in which the blackout occurred. These women were, if anything, only partially treated. Even if they were no longer pregnant at the end of the blackout, these women are biologically less likely to be responsive to treatment in form of changing fertility behavior. Post-delivery, the likelihood of having another child immediately is very low as post natal care takes up a large chunk of the mothers' time. As per the sample, only 3.4 per cent of the women gave birth in both 1992 and 1993.[16] In general, women who gave birth in 1992 were exposed to the treatment in many ways but were physically constrained to be responsive to any changes in sexual behavior during the blackout. Thus, they can constitute a control group for women who gave birth in 1993.[17]

The assignment of treatment and control group constitutes a very good counterfactual, as their age-profiles and hence their physiological fertility profiles are very similar since

[16]Since the treatment was for a year, while pregnancy lasts for only 9 months, it is possible for a mother to have given birth twice in two years. Our first stage is robust to the exclusion of mothers who gave birth in both years. A balance check is presented in Table 3.8. Not conditioning for the control variables we include in the regression, the treatment and control groups compare quite well.

[17]In order to give further support to our argument, we use a comparison between woman who gave birth in 1992 and women who have birth in 1991 as a placebo test.

they reproduced around the same period of time. Furthermore, the choice of treatment and control group helps us rule out alternative mechanisms that could violate our instrumental variables identification strategy for the effects of unplanned motherhood on the socio-economic outcomes of the mother.

The specification we estimate is a difference in difference specification. This allows us to test the impact of the blackout on total number of children 12 years onwards, comparing women who gave birth in 1993 to women who gave birth in 1992.

In particular, we estimate:

$$tch_{ami} = b_{ma} + \beta_1 M_i + \beta_2 O_m + \beta_3 M_i \times O_m + \mathbf{X_{im}}'\pi + \epsilon_{ami} \tag{3.2}$$

where $tch_{ami}$ is the total number of children born to mother $i$ in municipality $m$ in a ten year age cohort $a$. The variable $b_{ma}$ is a set of municipality-age-fixed effects. It controls for common shocks to women of the same age in a municipality. These fixed-effects are very demanding, but take out a lot of age specific heterogeneity that could be due to age or time-specific events at municipality level. Note that in this setup, we cannot control for mother-fixed effects, as there is only cross-sectional variation in the dependent variable. The variable $O_m$ measures, as before, the intensity of the power outage in 1992. The treatment $M_i$ is assigned to mothers $i$ in municipality $m$ that gave birth in 1993, while this variable is set to zero for mothers in municipality $m$ who gave birth in 1992. $\mathbf{X_{im}}$ contains other time-invariant controls fixed at the mother level.[18]

### 3.5.3 Long Run Impacts on the Mother

We can now turn to the third pillar of the analysis. Namely, we want to shed light into whether the persistent part of the fertility shock had some long-lasting effects on the lives of the mother or the family environment in which the mothers live. In order to do this, we exploit the variation in night lights in the municipality a woman lives interacted with her feasibility of conceiving during the blackout as an instrument for the total number of children. This allows us to shed light on the impact of unplanned babies on the life-path of mothers and the family in which children are brought up. It is important to highlight that this design only captures the differential effect of increased total lifetime fertility due to the power outage and does not capture any effects that may be due to an unplanned child that did not result in an increase in lifetime fertility. Even though our treatment effect is on a very specific subset of women, we think this is an

[18]These include an indicator variable for the ethnicity status (mainly to control for indigenous populations) at the mother-level and some indicator of whether the location of the mother within the municipality is a population center, a head-town or dispersed population.

important contribution to the existing labor economics literature, which has highlighted the role of the family environment for long-term outcomes of household members.

As a preview of the forthcoming results, we first establish that the power outage had persistent effects on the total number of children born to a mother $i$. Next we use this variation to explore other margins through which a mother $i$ was affected through having a "power outage baby". We proceed with an IV estimation whereby we exploit the arguably exogenous variation in power-outage intensity that resulted in more babies being born. These additional children may affect women, as they may have to give up e.g. further education. They also may have had to enter the workforce at a younger age, which allowed them to build up assets earlier, even though their lifetime earnings prospects may be significantly lower.

The relevance of our instrument is ensured by the persistent effects of the power outage on long-run total number of children. The exclusion restriction requires that there is no other channel through which our instrument had an effect on some outcome $y_{mai}$ for a women $i$ of age cohort $a$ living in municipality $m$. We explore some falsification exercises that suggest that the excludability of the instrument is indeed satisfied.

The first stage for our Instrumental Variable specification is simply specification 3.2 from the previous section. We use the first-stage to generate fitted values for the total number of children and then estimate:

$$y_{mai} = b_{ma} + \theta t\hat{c}h_{mai} + \eta_1 M_i + \eta_2 O_m + \mathbf{X'_{im}}\delta + \nu_{ami} \tag{3.3}$$

The coefficient $\theta$ measures the rate at which a mothers' socio-economic outcomes change for an additional child that was born due to the power outage and who was not adjusted for in the long-run. Since we only capture variation in total fertility for women who did not dynamically adjust their fertility in the years after the power outage, this is a specific local average treatment effect.

The socio-economic outcomes we study at the mother level from 2005 are: ownership of accommodation, quality of accommodation, whether they are single mothers, whether they are self-employed and whether they are graduated from university.

We now present the key results from each of the tree steps of the analysis.

## 3.6 Results

### 3.6.1 Short Term Fertility Effect

The first set of results pertains to the short term fertility increases due to the power outages. These are presented in Table 3.1. The estimated coefficients on the interaction term between power outage intensity and treatment are positive and significantly different from zero. In column (2) we add time fixed effects, in column (3) we add municipality fixed effects and in column (4) we replace municipality fixed effects with mother fixed effects. At first, it may seem surprising that the coefficient remains very stable and does not change when adding the mother fixed effects. However, we may see this exercise as evidence that the treatment was quasi random, as adding the fixed effects does not change the estimated coefficient. In column (5) we add a control for economic development as measured by the log of lights in municipality $m$.[19] Finally we control for department trends in column (6). Even in the most demanding specification, our estimate remains stable and precisely measured. A 100% increase in power-outage intensity (i.e. complete blackout relative to previous year) increases the probability of having a birth by approximately 0.005 percentage points. A 0.005 percentage point increase is an increase of 5% in the probability of giving a birth in a given year when evaluated at the mean probability of giving birth. At the mean power-outage intensity of 32%, we estimate the additional number of children born due to the power-outage to be 9,994.[20] How does this compare to the total expected number of children being born in this period? Given a mean probability of giving birth of 9.9%, on average, 624,667 babies would have been born. Hence, we can estimate that 1.6% of the babies born were "power outage babies". Column (7) presents evidence on heterogeneous treatment effect. There is strong support for differential cohort effects. Most of the impact of the power-outage is being driven by the younger cohort - those who were aged between 15 and 30 in 1993. The effect on the older cohort is positive but marginally insignificant, possibly indicating that these women are less likely to be biologically responsive to the shock. The results indicate that the effect-sizes are relatively small, which may explain why demographers have failed to find evidence using aggregated data (see Udry, 1970).

We consider a few robustness checks to ensure the validity of our results. Column (1) of table 3.2 presents the preferred specification from table 3.1. Since the census sample was

[19]We use the mean lights to control for economic development as a time varying control. For years prior to 1992, we replace the light measure with the 1992 data as the satellite images are only available from 1992 onwards. Under the assumption that in the very short run, spatial development is time invariant, the 1992 light measure should be a good proxy for 1990 and 1991.

[20]We arrive at the figure by scaling up the point estimate by 10 and then multiplying it by the mean blackout intensity of 0.32 and by the number of women in the estimation sample. This implies, given our point estimate of 0.005, that there are $0.005 \times 0.32 \times 624,667 \times 10 = 9,994$. We multiply by 10, since the census micro-data pertains to only 10% of the population.

conducted in 2005, a major concern with our results is that mothers may have moved across municipalities since 1993, thus biasing our power-outage intensity assignment. If this is due to pure randomness, we would expect our measure to be noise and thus lead to attenuation bias. On the other hand if the re-location choice of the mother is correlated with some unobserved characteristic of the municipality in 1993, we would have biased estimates for the effect of power-outage on probability of birth. In order to address this, we restrict our sample to women who are born in the same municipality and have lived there all their lives. In column (2), we present the results of the sub sample to show that our point estimate remains stable. In column (3) we carry out a placebo test by re-assigning the treatment year to be 1992, reflecting children born who were conceived in 1991. Since 1991 was a normal year with respect to electricity provision, we would not expect any differential impact of power-outage intensity for children born in 1992. Indeed we find a smaller and statistically insignificant coefficient. In column (4) we use a different measure of the power-outage intensity and find that the point estimate for $\gamma$ remains robust to the alternative measure of power-outage intensity[21]. In column (5), we control for log luminosity to account for any economic development that maybe correlated with our measure of power outage.[22] Our measure of power outage still predicts fertility.

We now turn to the study of the persistent effects of the power outage on total fertility.

### 3.6.2 Incomplete Adjustment of Fertility Effect

The temporary fertility effect demonstrated in the previous section may be dynamically offset by having fewer children in the future. In this section we document that this is not the case. We study a cross section of total births for women in 2005. The results from this analysis are presented in table 3.3.

Column (1) is a simple difference in difference regression without any controls. The coefficient on the interaction term is positive and highly significant. This means that the mothers who gave birth due to the power outage were unable to fully compensate by having fewer future children relative to mothers who gave birth just a year prior to the power outage. In columns (2) and (3), we add municipality age fixed effects and mother level controls respectively. Column (3) suggests that if a municipality was exposed to 100% blackout in 1993, then approximately every 10th mother in the treatment

[21]The alternative measure is constructed by using the share of pixels that were lit in a given municipality rather than the mean value of lit pixels. This measure tries to control for measurement error in the average lights by just assigning a value equal to 1 if a pixel had any positive light and 0 otherwise. The construction of the outage intensity is done exactly in the same way by comparing total lit pixels in 1993 to total lit pixels in 1992.

[22]Log luminosity is measured as the log of total number of lit pixels in the municipality.

group within the municipality is likely to have one more child compared to the control group. Thus, out of the 9,994 power outages babies estimated in the previous section, approximately a 1000 babies were not fully adjusted for 12 years on. Evaluating at the mean power-outage intensity, we obtain an upper bound of 1,961 additional number of children 12 years later.[23] Column (4) and (5) report some robustness checks to ensure our results are indeed meaningful and do not simply capture differential trends between the cohorts. In column (4) we present a placebo check. We perform the same exercise comparing women who gave birth in 1992 to women who gave birth in 1991. Since there were no power outages in either of the two years, there is no reason to believe that the blackout should have any significant impact on the long run total number of children for these two groups of women. Indeed the difference in difference estimator is close to 0 and insignificant. In column (5) we look at the long run outcome by age groups. The heterogeneous treatment effect gives us a natural placebo, since younger cohorts have more time to adjust their fertility behavior, we would expect the impact to be smaller for them. Indeed, majority of the long run effect observed is being driven by the older cohort who potentially were not able to adjust their lifetime fertility post the black out. The coefficient for the younger cohort is smaller and positive, but marginally insignificant, indicating that there may still be younger women who have not been able to fully adjust to the unanticipated fertility shock.[24]

It becomes evident that the power outage had a significant effect on the number of children born for the treated cohort. This suggests that there is incomplete adjustment and an increase in total fertility. We can use these results one step further to answer the question: what are the impacts on the mother who had an unplanned child and was constrained in adjusting total fertility?

### 3.6.3 Long Term Effects on Mothers' Socioeconomic Outcomes

Bringing up a child is costly as it requires time spent away from working or obtaining a degree. In addition, women who had an unplanned child may find themselves in more unstable relationships. We study these questions using the increase in total fertility due to the power outage as a natural experiment for an instrumental variables design to estimate the local average effect of having an unplanned child. The results from this exercise are presented in table 3.4.

Column (1) presents the preferred first stage specification. In columns (2) and (3), we see that they were more likely to own the accommodation they live in, but this

[23] We arrive at this estimate by multiplying the estimated coefficient of 0.1 by the 61,282 mothers who had a child in 1993 by the mean outage intensity of 0.32

[24] Since total number of children is a count variable, we present the same specification using a Poisson model. The results remain robust to the Poisson model and are presented in Appendix 3.8.4

accommodation tends to be of lower quality. The quality of the accommodation is an index that takes a maximum value of four if the accommodation has dirt-floors, no solid walls, use of wood fuel and no access to running water.

In column (4) we find that women who were subject to the power-outage induced fertility shock were more likely to be single-mothers in 2005, possibly reflecting the social cost of having unwanted babies. We do not find any effect of an extra child on the likelihood of self-employment, see column (5). However, they are less likely to have graduated from university, see column (6). The last column serves as a placebo check. Here we see whether the total number of children had an effect on the mother's primary school educational attainment. As this was predetermined before the mother was in child-bearing age, it is reassuring that the total number of children appear not to have an effect on this outcome. It is important to highlight that the IV strategy provides some distinct results, compared to the simple OLS estimation of the above specifications.

Finally in table 3.5, we repeat the above exercise, but for the constrained sample of women who are not first time mothers. Column (1) presents the first stage results, which remain significant but become less precise. The IV estimates remain qualitatively similar to including all women in the sample. However, they lose precision due to the reduction in sample and a weaker first stage.

These results taken together paint a very interesting picture. It suggests that there were persistent effects on women, who had more children in total due to the power outage. These women are living in less stable family situations, as they are more likely to be single mothers. One potential explanation could be because the child was not planned. There are also repercussions on the educational attainment, with women not taking higher education. However: there are also some more positive results. Women may have had to enter the labor force at a younger age due to having a baby. This makes it more likely that these women can accumulate assets and own the house in which they live.

## 3.7 Conclusion

This paper set out to analyze the impact of vast power rationing in Colombia in the early 1990's on fertility behavior. This is the first paper to evaluate the impact of power rationing on population dynamics, going beyond the question whether power outages may cause "mini baby booms".

Such research was not possible, because we lacked reliable data on electricity consumption. However, we highlight that the satellite-based night lights measures may be used to identify places which were subject to power rationing and periods of blackout.

We use this measure to show that women who live in areas where the power rationing was more severe were more likely to give birth in the year following the rationing period. This suggests that there are indeed “mini baby booms”. We take these results further to answer the question whether fertility behavior dynamically adjusts over time. We find that there is persistence, as women do not fully adjust their overall fertility. Furthermore, we show that the power-outage induced baby boom had long-run consequences for the mothers. In particular, we found evidence that educational attainment, housing conditions and marital status are negatively affected by having a blackout baby. This suggests that there are significant “hidden costs” from variable or low quality power infrastructure, which have to be taken into account when estimating the returns of investment in public infrastructure.

Table 3.1: The Impact of Power Outage Intensity on Birth Probability

| | Different Fixed Effects | | | | | | Age Specific |
|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| Outage Intensity x Treated | 0.475*** | 0.475*** | 0.475*** | 0.475*** | 0.475*** | 0.457*** | |
| | (0.149) | (0.149) | (0.149) | (0.163) | (0.163) | (0.164) | |
| Outage Intensity x Treated x Younger than 30 | | | | | | | 0.650** |
| | | | | | | | (0.266) |
| Outage Intensity x Treated x Older than 30 | | | | | | | 0.290 |
| | | | | | | | (0.201) |
| Mother FE | No | No | No | Yes | Yes | Yes | Yes |
| Time FE | No | Yes | Yes | Yes | Yes | Yes | Yes |
| Municipality FE | No | No | Yes | . | . | . | . |
| Department Trends | No | No | No | No | No | Yes | No |
| Mean Birth Probability | .0985 | .0985 | .0985 | .0985 | .0985 | .0985 | .0985 |
| Observations | 3748002 | 3748002 | 3748002 | 3748002 | 3748002 | 3748002 | 3748002 |
| Number of Groups | 515 | 515 | 515 | 515 | 515 | 515 | 515 |

Notes: Significance levels are indicated as * 0.10 ** 0.05 *** 0.01. Standard errors in the parentheses are clustered at the municipality level. Outage x Intensity measures the proportional change in municipality-level luminosity between 1992 and 1993. The dependent variable is an indicator variable equal to one, in case the mother experiences a birth in a given year. Note that the municipality fixed effects are perfectly collinear with the mother fixed effects in specifications (4) - (6). The coefficients are multiplied by 100.

TABLE 3.2: Robustness of the Short-Run Fertility Effect of Power Outages

| | Robustness to Measures and Specification | | | | |
|---|---|---|---|---|---|
| | (1) Baseline | (2) Non-movers | (3) Placebo | (4) Lit Pixels | (5) Total Luminosity |
| Outage Intensity x Treated | 0.475*** | 0.437** | 0.257 | 0.302*** | 0.512*** |
| | (0.163) | (0.198) | (0.191) | (0.105) | (0.196) |
| log(Luminosity) | | | | | -0.001 |
| | | | | | (0.001) |
| Mother FE | Yes | Yes | Yes | Yes | Yes |
| Time FE | Yes | Yes | Yes | Yes | Yes |
| Mean Birth Probability | .0985 | .103 | .0985 | .0986 | .0977 |
| Observations | 3748002 | 2163630 | 3748002 | 3660234 | 3645339 |
| Number of Groups | 515 | 515 | 515 | 502 | 508 |

Notes: Significance levels are indicated as * 0.10 ** 0.05 *** 0.01. Standard errors in the parentheses are clustered at the municipality level. Outage x Intensity measures the proportional change in municipality-level luminosity between 1992 and 1993. Column (2) restricts the analysis to mothers whose location of birth is the same as the present location. Column (3) moves the treatment one year earlier. Column (4) uses a different measure of the outage as being simply the change in the share of lit pixels between 1992 and 1993. Column (5) controls for total luminosity in a municipality. The coefficients are multiplied by 100.

Table 3.3: The Persistent Effects of Power Outage Intensity on Total Number of Children

| | Different Controls | | | Placebo | |
|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) |
| Outage Intensity x Treated | 0.088*** | 0.093*** | 0.094*** | -0.043 | |
| | (0.033) | (0.031) | (0.030) | (0.036) | |
| Outage Intensity x Treated x Older than 30 | | | | | 0.109** |
| | | | | | (0.043) |
| Outage Intensity x Treated x Younger than 30 | | | | | 0.041 |
| | | | | | (0.026) |
| Outage Intensity | 0.464*** | | | | |
| | (0.085) | | | | |
| Muncipality x Age FE | No | Yes | Yes | Yes | Yes |
| Mother Controlls | No | No | Yes | No | Yes |
| Mean Number of Children | 3.55 | 3.55 | 3.54 | 3.55 | 3.54 |
| Observations | 103676 | 103676 | 101044 | 98499 | 103337 |

Notes: Significance levels are indicated as * 0.10 ** 0.05 *** 0.01. Standard errors in the parentheses are clustered at the municipality level. Outage x Intensity measures the proportional change in municipality-level luminosity between 1992 and 1993. Treated women are women who gave birth in 1993 while control group women are women who gave birth in 1992. The dependent variable is the total number of children born up to 2005. Column (4) is a placebo where we assign treatment to women who gave birth in 1992 and control group to women who gave birth in 1991. Column (5) shows that the persistence is mainly driven by women who cannot physically adjust their long term fertility anymore due to their age. Mother controls include an indicator variable for the ethnicity status (mainly to control for indigenous populations) and some indicator of whether the location of the mother within the municipality is in a population center, a head-town or considered to be dispersed population.

Table 3.4: The Persistent Effects of Power Outage Intensity on Socio-Economic Status of the Mother

| | (1)<br>First Stage | (2)<br>House | (3)<br>Low Quality House | (4)<br>Single Mom | (5)<br>Self-employed | (6)<br>University | (7)<br>Placebo |
|---|---|---|---|---|---|---|---|
| Outage Intensity x | 0.093*** | | | | | | |
| Treat | (0.031) | | | | | | |
| *Instrumental Variables* | | | | | | | |
| Total Children Born | | 0.280** | 0.479** | 0.163* | 0.496 | -0.119** | -0.125 |
| | | (0.128) | (0.216) | (0.093) | (0.472) | (0.059) | (0.112) |
| *Ordinary Least Squares*: | | | | | | | |
| Total Children Born | | -0.009*** | 0.098*** | -0.000 | 0.004** | -0.022*** | -0.079*** |
| | | (0.002) | (0.005) | (0.001) | (0.002) | (0.002) | (0.001) |
| *Reduced Form*: | | | | | | | |
| Outage Intensity x | | 0.027*** | 0.046** | 0.016** | 0.039** | -0.010** | -0.011 |
| Treat | | (0.010) | (0.018) | (0.007) | (0.018) | (0.004) | (0.010) |
| Municipality x Age Group FE | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Mean of Dependent Variable | 3.55 | .603 | .789 | .129 | .22 | .0514 | .638 |
| Observations | 103676 | 102721 | 102287 | 103358 | 30521 | 102800 | 102800 |

Notes: Significance levels are indicated as * 0.10 ** 0.05 *** 0.01. Standard errors in the parentheses are clustered at the municipality level. Treatment indicates mothers who gave birth in 1993, while control group constitutes of women who gave birth in 1992. Outage Intensity measures the proportional change in municipality-level luminosity between 1992 and 1993. The table presents the IV, OLS and the Reduced Form results in separate rows. The dependent variables, given in the column head, are various socio-economic variables of the mother measured in 2005. Column (1) is the first stage. Column (2) studies whether a mother owns the house in which she lives. Column (3) is an index for housing quality. Column (4) is an indicator whether the mother is a single mom. Column (5) studies an indicator whether the mother is self-employed, while column (6) studies whether the mother has some university education. Column (7) is a placebo test where the left hand side is a dummy indicating primary school completion, which is predetermined in the treatment year.

TABLE 3.5: The Persistent Effects of Power Outage Intensity on Socio-Economic Status of the Mother : Robustness to Not First Birth

| | (1) First Stage | (2) House | (3) Low Quality House | (4) Single Mom | (5) Self-employed | (6) University | (7) Placebo |
|---|---|---|---|---|---|---|---|
| Outage Intensity x Treat | 0.118*** | | | | | | |
| | (0.045) | | | | | | |
| *Instrumental Variables* | | | | | | | |
| Total Children Born | | 0.227 | 0.354 | 0.092 | 0.048 | -0.093* | -0.221 |
| | | (0.153) | (0.243) | (0.086) | (0.196) | (0.055) | (0.136) |
| *Ordinary Least Squares*: | | | | | | | |
| Total Children Born | | -0.009*** | 0.099*** | 0.001 | -0.001 | -0.017*** | -0.080*** |
| | | (0.002) | (0.005) | (0.001) | (0.003) | (0.002) | (0.002) |
| *Reduced Form*: | | | | | | | |
| Outage Intensity x Treat | | 0.025* | 0.039 | 0.011 | 0.008 | -0.010* | -0.023* |
| | | (0.015) | (0.025) | (0.009) | (0.032) | (0.005) | (0.014) |
| Municipality x Age Group FE | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Mean of Dependent Variable | 4.08 | .635 | .882 | .126 | .235 | .0364 | .58 |
| Observations | 57332 | 56811 | 56584 | 57020 | 15172 | 56637 | 56637 |

Notes: Significance levels are indicated as * 0.10 ** 0.05 *** 0.01. Standard errors in the parentheses are clustered at the municipality level. Treatment indicates mothers who gave birth in 1993 and had already given birth at least once before 1993, while control group constitutes of women who gave birth in 1992 and have given birth at least once before 1992. The sample is restricted to those mothers whose birth in 1993 was not their first birth. Intensity measures the proportional change in municipality-level luminosity between 1992 and 1993. The dependent variables, given in the column head, are various socio-economic variables of the mother measured in 2005. The left hand sides for column (2) - (6) are given in the column heads. Column (1) is the first stage, while column (7) is a placebo test where the left hand side is a dummy indicating primary school completion, which is predetermined in the treatment year.

## 3.8 Appendix

### 3.8.1 Theoretical model

#### 3.8.1.1 Setup

We propose a simple model to highlight the channels through which the blackout may have affected sexual behavior, pregnancy and fertility in the short run and accumulation of human capital in the long run. We simplify the analysis by assuming that a woman's partner chooses the sexual activity frequency, disregarding the consequences on her long-term outcomes. Women still have the last say on the number of children they bear and on the education level they want to achieve, but family planning is costly and education more difficult when resources have to be allocated to childcare.

The intuition behind the effect of the blackout is as follows. Sex is an activity that competes for time against work and leisure activities. The blackout increases sex frequency since scheduled blackouts imply shorter working hours. As sexual activity increases, so does pregnancy. Since family planning is costly, it is not optimal for woman to fully offset the increase in pregnancy. Childcare competes for time against study, therefore reducing a woman's human capital.

To simplify the analysis, we depart from the standard unitary household model. Instead, women and men assume differentiated roles and do not take into account the effect their decisions have on their partners. In particular, men choose sexual frequency disregarding the effect on their partners, while the costs of childcare and family planning are exclusively assumed by women. In contrast, a forward-looking household would take into account how sexual frequency affects human capital acquisition. Earlier versions of this paper included such a dynamic model, but the qualitative features of the model remained unchanged. The reader is asked to excuse us for presenting here a model which simplifies the analysis at the cost of acquiring sexist undertones.

The problem for a man is to choose the feasible amount of consumption and time allocated to sex and work that maximizes his utility. His utility depends on consumption ($c$) and on the fraction of time spent having sex ($s$). It is assumed that $u(c, s)$ is a quasi-concave function increasing in $c$ and $s$. Sex is costly in terms of time, hence only combinations of non-negative amounts of time $x$ and $s$ such that $x + s \leq 1$ are feasible. By working a fraction $x < \beta$ of time, he earns $vx$, where $v > 0$ represents the exogenous wage and $\beta > 0$ is an exogenous threshold. If working a fraction $x > \beta$ of the time, he earns $v\beta + \omega(x - \beta)$, where $\omega$ represents the marginal earnings working after a fraction $\beta$ of the time.

The problem for a woman's partner is:

$$\max_{s,x,c\geq 0} u(c,s)$$

subject to $x + s \leq 1$ and to

$$c = \begin{cases} \upsilon x & \text{if } x \leq \beta \\ \upsilon\beta + \omega(x-\beta) & \text{if } x > \beta \end{cases}$$

Denote by $x(\omega)$ the optimal labor supply for a man. We assume in all cases it holds that $x(\omega) \in [\beta, 1)$. In particular, if $x(\omega) \in (\beta, 1)$, the solution is characterized by equating the marginal utility of sex with its opportunity cost:

$$u_s(c(\omega), s(\omega)) = \omega u_c(c(\omega), s(\omega)) \tag{3.4}$$

where $s(\omega)$ and $c(\omega)$ represent the optimal choices of sex and consumption, respectively.

Women allocate their time between education ($e$) and childcare. If a woman has to take care of $n$ children, she has to spend a fraction $\gamma n$ of her time on childcare, where $\gamma > 0$ is a parameter that captures how costly is to raise children. The number of children $n$ is a function of the frequency of sex activity $s$, chosen by her partner, and the level of family planning $a$, chosen by her. In particular, the number of children is:

$$n = (1-a)\rho s \tag{3.5}$$

The monetary cost of family planning is given by $f(as)$, where $f(\cdot)$ is a real-valued, increasing, continuously differentiable and strictly convex function with $f'(0) = 0$ and $f'(\infty) = \infty$.

The utility a woman gets is a function of her net income, which is a function of the time she spends on education and the cost of family planning. The return to education is given by the parameter $\alpha > 0$. The problem for a woman is:

$$\max_{e\geq 0, a\in[0,1]} (\alpha e - f(as))$$

subject to $e + \gamma(1-a)s \leq 1$. Denote by $a(\omega)$ the optimal level of family planning for a woman when her partner chooses a frequency $s(\omega)$ of sex. If the optimal level of family planning is an interior solution, then the marginal cost of family planning equals its return in terms of income:

$$\alpha\gamma = f'(a(\omega)s(\omega)) \tag{3.6}$$

The fraction of time spend on education is obtained by noticing that $f'(\cdot)$ is an invertible function. Using the time allocation constraint, we obtain:

$$e(\omega) = 1 - \gamma s(\omega) + \gamma (f')^{-1}(\alpha\gamma) \tag{3.7}$$

Note that at interior solutions, women do not fully offset the fertility choices of their partners.

#### 3.8.1.2 Comparative statics

The blackout is modeled as a decrease in the marginal earnings from working at the scheduled blackout times, which is represented as a decrease in $\omega$. Note that if $\omega$ falls to zero, this is equivalent to restricting the number of hours that can be worked to $\beta$. Implicitly, it is assumed that labor and electricity are complementary inputs in the production process. As electricity starts to become rationed or prohibitively expensive, the demand for labor falls, decreasing $\omega$.

Let $\omega$ denote the marginal earnings in normal times and $\omega' < \omega$ the marginal earnings during the blackout. From a comparative statics exercise on equation (3.4) it can be seen that the sign of $s(\omega') - s(\omega)$ is ambiguous. This is not surprising: on one hand, the substitution effect pushes $s$ up as $\omega$ decreases. On the other, income and wealth effects pull in the opposite direction. Note, however, that if the fall on $\omega$ is big enough, the substitution effect dominates. In the extreme case where $\omega$ drops to zero, then $x(0) = \beta$, unambiguously increasing sex frequency.

Let $n(\omega)$ and $n(\omega')$ the equilibrium number of children in normal times and during the blackout, respectively. It follows from equations (3.5) and (3.6) that the effect on the fertility is given by:

$$n(\omega') - n(\omega) = s(\omega') - s(\omega) \tag{3.8}$$

where $s(\cdot)$ is characterized by equation (3.4). If the substitution effect dominates, the effect of the blackout on fertility is positive.

Testing whether there is an increase in fertility in response to the power outage is the first of two steps of the empirical analysis. We study the immediate short-term fertility effect, but also focus on the dynamic, long-run effect on total fertility twelve years after the power outage.[25]

[25] A more dynamic model would capture the long-run adjustment of fertility behavior.

The second major outcome which we address in our empirical strategy is human capital. Theoretically, the effect on human capital is given by:

$$e(\omega') - e(\omega) = -\gamma(s(\omega') - s(\omega)) \tag{3.9}$$

which is derived from equation (3.7). The sign of this effect is negative if the substitution effect dominates. We study this empirically in subsection 3.5.3, where we look at the effect that an unplanned child has on mother-level long-run socio-economic outcomes.

The blackout may affect fertility decisions through other channels. Other leisure activities (e.g. watching TV.) could be introduced into the woman's partner (or the household's) utility function. The blackout could be seen as an increase in the cost of carrying out these activities. In this case, the substitution effect of the blackout would be intensified, the increase in fertility would be higher and the increase in educational attainment would be lower.

### 3.8.2 Summary Statistics

TABLE 3.6: Summary Statistics for Short Run Analysis

| Variable | Mean | Std. Dev. | Min. | Max. | N |
|---|---|---|---|---|---|
| Birth (dummy) | 0.1 | 0.3 | 0 | 1 | 3748002 |
| Outage Intensity | 0.29 | 0.26 | -0.38 | 1 | 3748002 |
| Outage Intensity (lit pixel share) | 0.32 | 0.38 | -2.03 | 1 | 3690818 |
| TV ownership (dummy) | 0.72 | 0.45 | 0 | 1 | 3574890 |
| Electricity (dummy) | 0.92 | 0.27 | 0 | 1 | 3748002 |
| Refrigerator (dummy) | 0.63 | 0.48 | 0 | 1 | 3591732 |
| Total Number of Assets Owned | 2.02 | 1.62 | 0 | 6 | 3748128 |
| Aged between 18 and 29 in 1993 | 0.51 | 0.5 | 0 | 1 | 3748128 |

Source: 2005 micro sample census of Colombia.

TABLE 3.7: Summary Statistics for Long Run Analysis

| Variable | Mean | Std. Dev. | Min. | Max. | N |
|---|---|---|---|---|---|
| Outage Intensity | 0.32 | 0.27 | -0.38 | 1 | 162526 |
| Indigenous (dummy) | 0.17 | 0.37 | 0 | 1 | 161350 |
| Mother's age | 40.8 | 6.33 | 31 | 58 | 162526 |
| Age in 1992 | 27.8 | 6.33 | 18 | 45 | 162526 |
| Total Children Born | 4.08 | 2.38 | 1 | 24 | 159023 |
| House Ownership (dummy) | 0.62 | 0.49 | 0 | 1 | 160215 |
| Cheap Housing | 0.87 | 1.13 | 0 | 4 | 159458 |
| Single Mom (dummy) | 0.13 | 0.33 | 0 | 1 | 162024 |
| Self Employed (dummy) | 0.23 | 0.42 | 0 | 1 | 45801 |
| University Degree (dummy) | 0.05 | 0.21 | 0 | 1 | 160034 |
| Primary School Completion (dummy) | 0.59 | 0.49 | 0 | 1 | 160034 |

Source: 2005 micro sample census of Colombia.

Table 3.8: Summary Statistics: Comparison Between Control and Treatment Group for Long Run Analysis

| | Control | | Treatment | | P-val |
|---|---|---|---|---|---|
| Variables | N | Mean | N | Mean | |
| Outage Intensity | 56656 | 0.320 | 61282 | 0.324 | 0.015 |
| Indigenous (dummy) | 56265 | 0.173 | 60816 | 0.174 | 0.558 |
| Mother's Age | 56656 | 40.791 | 61282 | 39.997 | 0.000 |
| Mother's Age in 1993 | 56656 | 27.791 | 61282 | 26.997 | 0.000 |
| Total Children Born | 55436 | 4.140 | 59981 | 4.148 | 0.599 |
| House Ownership (dummy) | 55810 | 0.623 | 60618 | 0.610 | 0.000 |
| Cheap Housing (dummy) | 55533 | 0.896 | 60302 | 0.915 | 0.005 |
| Single Mom (dummy) | 56455 | 0.126 | 61116 | 0.125 | 0.641 |
| Self Employed (dummy) | 15822 | 0.225 | 16808 | 0.223 | 0.803 |
| University Degree (dummy) | 55777 | 0.046 | 60305 | 0.047 | 0.439 |
| Primary School Completion (dummy) | 55777 | 0.589 | 60305 | 0.590 | 0.682 |

Source: 2005 micro sample census of Colombia. P-value is the significance level of a t-test comparing the means of the respective variables in each row between the treatment and control groups.

### 3.8.3 Luminosity for 1994

As mentioned in the text, we lack luminosity data for the period before 1991, which would be the adequate control year for the construction of the power-outage intensity. However, the night light data is only available from 1992 onwards. That is why we had to compare the 1992 luminosity to the 1993 luminosity to construct the outage intensity variable. The following graphs depict the luminosity also for the year 1994 on the same scale. This highlights that the changes in luminosity from 1992 to 1993 is far from any "normal" year on year variation in luminosity, suggesting that we are really capturing the effect of the power outage through that variable correctly.

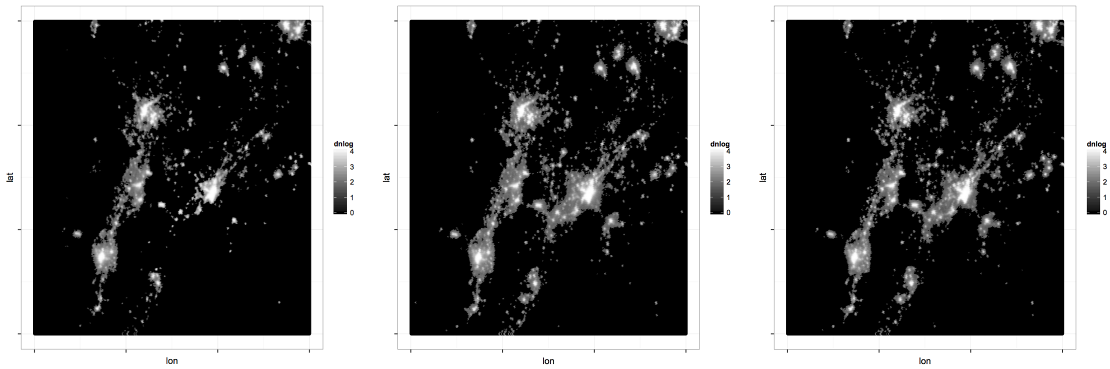


FIGURE 3.3: Light Intensity in Central Colombia, 1992 (left), 1993 (center) and 1994 (right)

### 3.8.4 Further Robustness Checks

Table 3.9: Robustness of Total Fertility Effect to using a Poisson Model

| | Different Controls | | | Placebo | |
|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) |
| Outage Intensity x Treated | 0.024*** | 0.024*** | 0.023*** | -0.011 | |
| | (0.009) | (0.008) | (0.008) | (0.010) | |
| Outage Intensity x Treated x Older than 30 | | | | | 0.027** |
| | | | | | (0.011) |
| Outage Intensity x Treated x Younger than 30 | | | | | 0.011 |
| | | | | | (0.007) |
| Outage Intensity | 0.129*** | | | | |
| | (0.024) | | | | |
| Muncipality x Age FE | No | Yes | Yes | Yes | Yes |
| Mother Controlls | No | No | Yes | No | Yes |
| Mean Number of Children | 3.55 | 3.55 | 3.54 | 3.55 | 3.54 |
| Observations | 103676 | 103643 | 101012 | 98478 | 103304 |

Notes: Significance levels are indicated as * 0.10 ** 0.05 *** 0.01. Regressions for a conditional fixed effect Poisson model. Standard errors in the parentheses are clustered at the municipality level. Outage x Intensity measures the proportional change in municipality-level luminosity between 1992 and 1993. The dependent variable is given in the column head.

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