MACROECONOMIC MODELS FOR INFLATION TARGETING IN ECONOMIES WITH FINANCIAL DOLLARISATION

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ABSTRACT

After an introductory chapter, the thesis is divided in three parts. In the first part, chapter 2 includes domestic financial dollarisation into an otherwise standard DSGE model of a small open economy. Domestic financial dollarisation implies that some of the assets of households and some liabilities of financial intermediaries are denominated in a foreign currency. The main implication is that exchange rate swings affect the financial wealth of households and disrupt production. The chapter also derives a New-Keynesian Phillips curve augmented with agency costs. Chapter 3, sets up a framework whereby demand substitution occurs when cheaper imported goods appear and trigger a propagation mechanism in non-tradeable prices. As in the previous chapter, Chapter 3 disentangles the dynamics of inflation exploring yet another effect that explains how the fall in world inflation might drag down non-tradeable inflation in a small open economy.

The second part of the thesis deals with operational issues; notably the inflation forecast and instrument setting. Chapter 4 proposes a Bayesian method to combine model-based density forecasts with policy makers' subjective priors. Next, Chapter 5 estimates forward-looking interest rate rules by quantile regressions. The advantage of quantile regressions is that we can learn about the likely feedback from forecasts to instruments, not only on the mean value but on different quantiles of the inflation forecast distribution. Thus, we can gain some added information about monetary authorities' risk balance or the nature of their loss function.

In the last part of the thesis, Chapter 6 provides an econometric evaluation of the effects of inflation targeting adoption on the dynamics of inflation. This evaluation covers developed and emerging-market inflation targeters alike.

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CHAPTER 1

INTRODUCTION

Ever since monetary policy authorities in New Zealand embraced Inflation Targeting (IT from now on) back in 1990, many countries have followed suit. Currently, twenty-one central banks in the world conduct monetary policy under the guidelines set by this framework (ITers from now on). Remarkably, more than a half of all ITers are catalogued as emerging market economies.

This thesis presents research on monetary policy under the IT framework in small open economies with special emphasis on emerging-market and Latin American countries. Models of varying degree of complexity are put forward both to tackle key elements of small, emerging-market economies relevant for policy making and to better understand the features of IT in this environment.

The research has benefited from the interplay between theory and practice provided by my years at the London School of Economics and my fieldwork at the Central Bank of Peru. Although the primary concern of the thesis is to draw practical monetary policy implications for Peru, the issues studied are broad and cover aspects concerning IT in general and monetary policy in small open economies in particular.

The thesis contains three parts. The first part introduces dynamic models aimed at understanding two key issues that have shaped the monetary policy debate in recent times. The first is the role of exchange rates in the transmission mechanism of monetary policy and the second is the role of increasing competition in goods markets because of expanding trade globalisation. The thesis approaches these two topics in chapters 2 and 3 respectively.

The second part concentrates on operational issues of IT. Chapter 4 considers an approach towards applying an inflation density forecast and then chapter 5 estimates forward-looking instrument rules for Latin American ITers. In the third and last part, the thesis provides a novel empirical evaluation of whether IT affects inflation dynamics.

In standard small-open economy models, the monetary policy transmission mechanism considers the exchange rate channel. The dynamics of the exchange rate[1] affects prices and inflation by both; the pass-through and the aggregate demand. The extent of the pass-through depends on the exchange-rate regime and the relative size of the tradable and non-tradeable production, while the aggregate demand impact originates from the expenditure switching effect owing to real exchange rate swings. Studies like Sutherland (120), Svensson (124) and more recently Devereux and Engel (38) and Gali and Monacelli (52) distinguish between total CPI inflation and non-tradable inflation[2] as targets for monetary policy, as well as the degree of pass-through. The relevant trade-off faced by policymakers, given the degree of pass-through, is to induce lower exchange rate fluctuations (associated with lower CPI inflation variability) at the expense of higher "non-tradeable" inflation volatility.

The policy conclusions that arise from this literature implicitly assume that central banks do know the tradable and non-tradable price components[3]. Nevertheless, for reasons of transparency, accountability, and opportunity, central banks base their targets mostly on observable measures like the Consumer-Price-Index

[1]Throughout the thesis, the nominal exchange rate, unless stated otherwise, will be understood to be the effective exchange rate, i.e. the domestic price of a weighted basket of foreign currencies.

[2]Also known as "domestic inflation".

[3]In practice this exercise is not easy, the definition of what is tradeable or not, in a statistical sense, is not standard. Measures of tradeable and non-tradeable inflation share the same feature as definitions of underlying and non-underlying inflation. They are unobservable and indirectly estimated with errors.

(CPI) inflation[4]. Therefore, exchange rate swings concerns IT practice in small open economies insofar as the degree of exchange rate pass-through is high. The documented decline of the degree of pass-through in developed and developing economies[5] have eased IT practice in this particular issue.

However, against the backdrop of emerging-markets, there is another fundamental concern; the tradeoff between exchange rate flexibility and financial stability. This is especially the case in financially dollarised economies where exchange rate risk is not properly hedged.

In extreme cases[6] of financial fragility[7], sizeable unexpected exchange rate depreciations against the dollar[8] increase the burden of dollar-denominated debts, weakening balance sheets, and increasing the risks of financial distress. Policy makers living in this dangerous environment cannot afford to neglect exchange rates. The practice of IT in emerging markets has therefore been shaped by the dilemma imposed by financial fragility. This is for example outlined in Amato and Gerlach (1) which points out that on the path towards fully-fledged IT, many countries kept exchange rate targets and only slowly relinquished them. In fact, abandonment of exchange rate targets has usually not been undertaken until measures to mitigate financial vulnerability have been put in place.

The purpose of chapter 2 is therefore to include domestic financial dollarisation into an otherwise standard DSGE model of a small open economy. Domestic financial dollarisation implies that some of the assets of households and some liabilities of financial intermediaries are denominated in a foreign currency[9]. The main

[4]Sometimes they might also target wholesale or retail price index inflation. Though they might operationally observe a number of underlying and non-tradeable inflation measures.

[5]See Goldfjan and Werlang (55) and Frankel et al. (49).

[6]See Mishkin (90) and Calvo and Mishkin (24).

[7]A high degree of financial dollarisation and currency mismatches in the denomination of assets and liabilities of agents enhances such fragility.

[8]A depreciation of the currency means an increase in the domestic price of dollar.

[9]The dollar.

implication of this is that exchange rate fluctuations affect the financial wealth of households and disrupt production. The impact of financial dollarisation on monetary policy has also been studied in a number of papers, most notably in Céspedes et.al (26), Cook (33) and Devereux et.al (39) where explicit balance-sheet channels are built in. Chapter 2 shares important features of these papers; however, the scope is different. My aim is to introduce financial dollarisation frictions into the dynamics of inflation in a structural form. Therefore, the sources of inflation dynamics can be disentangled into their various components. One of them is the agency costs relevant to a dollarised financial system. The resulting friction-augmented Phillips curve is relevant to the assessment of monetary policy and inflation, the key elements in any IT regime[10].

Another important development treated in the thesis, is the low inflation scenario that has characterised monetary policy-making through the 90's and the current decade. As suggested by Andersen and Wascher (2), Bowman (19), Rogoff (107), and Chen et.al (28), several explanations have been proposed: for instance; institutional factors such as increasing central bank independence, strong commitments to anti-inflationary policies, and the *increased competition* hypothesis in price setting behaviour. According to this hypothesis, both the rising trade globalisation and deregulation witnessed worldwide in the 90s have contributed to the fall in the market power of price setting firms. As a result, inflation rates have reached historically low levels both in developed and developing countries[11].

In order to undertake an investigation of the *increasing competition* hypothesis, Chapter 3 sets up a framework whereby substitution on the demand side occurs when cheaper imported goods appear and trigger a propagation mechanism

[10]It is worth noticing that the theoretical models in Céspedes et.al (26), Cook (33) and Devereux et.al (39), and the model developed in Chapter 2, are models for tranquil times, not for crisis episodes akin to a structural regime shift. An important research avenue followed for example in Caballero and Krishnamurthy (22) does treat financial fragility within such a crisis context.

[11]Country specific examples can be found in Rogoff (107).

in non-tradeable prices. This mechanism is conveyed in the claim made in Rogoff (107, p. 18): *"(...) sharp reductions in [tradable goods] prices are bound to create spillover effects on other sectors. Many traded goods are intermediate goods or, to some degree, substitutes for non-traded goods"*.

As in the previous chapter, Chapter 3 disentangles the dynamics of inflation exploring yet another type of effect. The resulting inflation equation allows us to explain how the fall in world inflation might drag down non-tradeable inflation in a small open economy. This is done by deriving a New-Keynesian Phillips curve using the assumption of translog preferences[12] that allows the price elasticity of domestically produced goods to depend on foreign price movements. As a result, the coefficients of the Phillips curve turn out to depend on the real exchange rate. This chapter is based on Vega and Winkelried (133)[13] where translog preferences are introduced in the same vein.

The second part of the thesis deals with operational issues; notably the inflation forecast and instrument setting.

The aim for price stability has led many central banks to be keen inflation forecasters. This has been even more noticeable with the advent of IT. Inflation forecasts are important in this regime because they are intermediate targets at the operational level, as proposed in Svensson (121).

Also, inflation forecasts made by central banks, and the formal explanations of the reasons behind those forecasts, serve as a signalling device for central banks to communicate how appropriately their actions have been taken. However, forecasts are in practice subject to a myriad of asymmetric risks that unavoidably affect the asymmetry of the inflation forecast itself. This has prompted central banks to turn

[12] As in Bergin and Feenstra (11, 12).

[13] This paper won the 2004 Rodrigo Gomez Award at the Centre of Latin American Monetary Studies, a research centre sponsored by Latin American central banks.

attention to density, instead of point, forecasts[14].

IT practitioners rely on model-based forecasts, but also understand the future is subject to risks that even highly sophisticated models cannot foresee. In fact, most of the balance of risks, even though they might be rationalised by models and statistical toolkits on a first pass, are fed by judgements and priors of decision makers. The purpose of Chapter 4 is precisely to explore this topic. There, a method, based on Bayesian techniques, is developed to combine model-based density forecasts with policy makers' priors.

Density forecast combination is an important area of current research and of main interest for IT practice. Recently a series of papers by Stephen Hall and James Mitchell[15] propose a powerful method for forecast combination. The method outlined in Chapter 4 differs from those papers in the definition of what is "optimal". In the cited papers, optimality is rightly related to the forecast evaluation view of forecast error minimisation. In my approach, optimality is taken from a policy-maker's perspective: those who are to decide based on a model forecast know the model is just one input in the decision process. At the time the decision is made, they might hold priors about the risks likely to unfold in the forecast horizon, irrespective of the ex-post forecast performance of the model. The final density forecast then is related to a maximisation of policy-maker's utility that depends on a trade-off between his own priors and the model-based density forecast.

Next, in Chapter 5, and following the lead of Chapter 4, I turn to the interest rate decision, based on the future outlook for inflation[16]. During policy deliberations, policy makers consider the latent risks, the low-probability, high-impact events and the nature of the shocks[17] that shape the probabilistic distribution of

[14] See Goodhart (56).
[15] See for example Hall and Mitchell (61), Hall and Mitchell (62) and Hall and Mitchell (63).
[16] This chapter is based on a version of Vega (131).
[17] The persistent or transitory nature of the shocks, and the assessment of the shock as supply or demand driven.

forecasts. Therefore, I estimate forward-looking interest rate rules first in the same vein as Clarida et.al (30), Orphanides (98) and Goodhart (58) but next I perform quantile regression estimations. The advantage of quantile regressions in this context is that we can learn about the likely feedback from forecasts to instruments, not on the mean value as standard estimations suggest, but on different quantiles of the distributions[18]. Thus, I can obtain some added information about the evaluation of the risks implied in every decision.

Part 3 of the thesis provides an econometric evaluation of the effects of IT adoption on the dynamics of inflation. The ultimate benchmark of the success of IT for a country is the delivery of superior outcomes relative to all other possible monetary policy regimes that might have been adopted instead of IT. The exercise is complicated because it needs comparison of outcomes with unobservable counterfactuals.

In the IT evaluation literature, papers like Ball and Sheridan (6), Neumann and Von Hagen (97), and Levin et.al (83) have performed this evaluation. However, such exercises are hindered by various reasons: they are mostly concerned with the evaluation of IT in advanced economies, their choice of counterfactuals tend to be limited, and they miss robustness checks on different possible IT adoption dates.

In Chapter 6, this exercise is carried out using a technique borrowed from the programme evaluation literature. First, IT adoption is defined as a *treatment*, the ITers are the *treated* group and all the non-ITers are the *control* set. Then the choice of counterfactuals is entirely data-driven from the distribution of countries which are summarised in a metric called "propensity score". The comparison of outcomes of ITers against their counterfactuals is governed by the propensity scores, that is, ITers are compared to control countries according to how similar the countries were before IT adoption. The result of the evaluation confirms the

[18]Quantile regressions were introduced in Koenker and Bassett (77).

overwhelming benefit IT has had over the mean and variance of inflation not only in advanced economies but most significantly in emerging-market countries[19].

To sum up, the thesis provides a rigorous treatment of key issues about IT practice in small open economies. I have introduced models to understand phenomena such as financial dollarisation and the increasing competition hypothesis in relation to the dynamics of inflation. I have then introduced original modelling techniques in the monetary policy literature about the implementation of IT considering the risk embedded not only in the inflation forecast but also in the instrument decision itself. Finally the thesis provides a preliminary answer of whether IT can deliver superior outcomes.

As a member of a central bank in an emerging-market country such as Peru, I am a direct witness of the value of rigorous and model-based thinking as well as the sheer amount of out-of model analysis of risks in doing policy. It is the aim of this thesis to contribute to this process bridging the gap between theory and practice in this type of economy.

[19] The paper version of this chapter was recently published in the first volume of the International Journal of Central Banking as Vega and Winkelried (132).

CHAPTER 2

THE ROLE OF EXCHANGE RATES IN A DSGE MODEL OF A FINANCIALLY DOLLARISED ECONOMY

This chapter presents a DSGE model with financial dollarisation features. The role of financial dollarisation in this type of models is tantamount to the existence of a non-trivial role for financial intermediation (through the presence of agency costs) and therefore to the presence of a general credit channel of monetary policy. The specific form of this credit channel in the context of New-Keynesian Phillips curves has not been directly treated in the current literature. One contribution of this chapter is to provide an inflation equation that takes into account the presence of agency costs and financial dollarisation.

A second purpose of the chapter is to study the link between agency costs, financial dollarisation and the restrictions they impose to monetary policy. In particular, the question the chapter intends to address is to what extent different types of inflation targets affect the evolution of the economy under the presence of agency costs.

In the chapter, financial dollarisation is explicit as both the assets of households and the liabilities of firms that produce and generate non-tradeable income are dollarised. It is assumed that there are two productive sectors in the home country; the sector that produces non-tradable goods $Y_{h,t}$ and a sector that produces an exogenous amount of a "traditional" tradable good $Y_{f,t}$. The sector that produces non-tradable goods is composed of heterogeneous wholesalers who face a credit-in-advanced constraint as in Cooley and Nam (34) or Carslstrom and Fuerst

(25). The heterogeneity of wholesalers (borrowers) stems from idiosyncratic productivity shocks affecting these firms. The resulting structure allows for the existence of standard debt contracts between banks and each wholesaler. A particular feature of this contract is the existence of a mark-up margin in wholesale prices that results in order to cover the deadweight losses imposed by the existence of agency costs.

In order to model a non-trivial role for monetary policy, sticky-prices are introduced by assuming monopolistic retailers as in Bernanke et.al (13). As known, retailer prices will also sell at a mark-up over marginal cost due to the market power structure assumed. The overall result is a dynamics of prices and inflation influenced by these two distortions: agency costs and monopolistic competition. In fact, a key contribution of the chapter is the derivation of the Phillips curve that bears the same New-Keynesian features as observed in Clarida et al. (31) or Woodford (137) but incorporates a term that depends on the degree of agency cost distortions.

The chapter is organised as follows: Section 2.1 provides the general modelling framework, section 2.2 sets up the canonical log-linearised system and section 2.3 performs the assessment of three different types of inflation targeting regimes under a series of shocks and section 2.4 concludes. Appendix A provides technical derivations.

2.1 Framework

This chapter presents a small open economy model where imports are traded using the dollar as a medium of exchange within the boundaries of the domestic country. In order to have a role for monetary policy the nominal rigidity introduced is a staggered price setting structure on the part of firms. The broad view is that there are two productive sectors in the home country. The country produces non-tradable goods $Y_{h,t}$ and an exogenous amount of a "traditional" commodity tradable good $Y_{f,t}$

whose price is determined exogenously in the world market[1]. Non-tradable goods production is made by monopolistic competitive firms that set prices. However, the setting of prices is made in a staggered way due to the fact that pricing decisions can not be made continuously. In my framework, this results in a Phillips kind of curve for the supply of non-tradables with both a backward and a forward looking component in inflation[2].

The next subsections analyse the behaviour of households, firms, foreigners and the monetary authority. Before doing so, it is convenient to summarise the model environment:

- A small open economy is analysed. However, domestic consumers do not have access to internationally traded assets. The country is not financially sophisticated. In this sense the financial market is fairly incomplete.
- However there is foreign trade in goods. Consumers are offered foreign goods, firms depend on foreign inputs and there are export-only firms that produce primary commodities.
- Within the borders of the economy, consumers do have access to assets denominated in both, pesos and dollars. These are offered by domestic financial intermediaries. This feature captures dollarisation of assets on the portfolio of domestic consumers.
- Domestic financial intermediaries do have access to foreign borrowing/lending.

[1]One feature of emerging market economies is precisely the fact that their exports heavily depend on commodities.

[2]These hybrid Phillips curves have been analysed in Gali and Gertler (51). A negative assessment is found in Ball et.al (5).

2.1.1 Households

A typical household maximises the expected present value of utility[3] over future consumption levels and labour.

$$\sum_{s=t}^{\infty} E_t \left[\beta^{s-t} \left(\frac{C_s^{1-\delta} - 1}{1-\delta} - \frac{N_s^{1+\nu} - 1}{1+\nu} \right) \right] \tag{2-1}$$

subject to the following resource constraint

$$D_{s+1} + \mathcal{E}_s B_{s+1} = \mathcal{I}_{s-1} D_s + \mathcal{E}_s \mathcal{I}_{s-1}^f B_s + (\mathcal{E}_s - E_{s-1}\mathcal{E}_s) B_s + W_s N_s - P_s C_s + \Omega_s \tag{2-2}$$

For every period $s = t, t+1, ...$ and where D_s and B_s represent peso and dollar denominated assets purchased at the beginning of time $s-1$ and held up to the beginning of time s when a new decision about assets holdings is made, $\mathcal{I}_{s-1} = (1 + i_{s-1})$ is the gross interest rate paid by the peso assets bought at the beginning of time $s-1$, likewise $\mathcal{I}_{s-1}^f = (1 + i_{s-1}^f)$ is the corresponding gross interest rate paid by the dollar asset. $\mathcal{E}_s$ is the nominal exchange rate defined as the peso price of one dollar. Both types of assets (D_s and B_s) have only a one-period maturity and can be thought of as deposits in a domestic financial intermediary. Households in this economy do not trade assets directly with the foreign sector, they are net savers [4]. The term $(\mathcal{E}_s - E_{s-1}\mathcal{E}_s)B_{s-1}$ captures the accounting adjustment needed to explain capital gains or losses. This means that if there is an unexpected depreciation of the currency, then there is a positive peso valued capital gain from holding dollar-denominated assets.

[3] Given that monetary policy uses the nominal interest rate rule as instrument, money holdings from the utility function are left out.

[4] To ensure that households are net savers in the steady-state, certain conditions on the parameters are needed.

There are two arguments in the above utility function[5]; an overall consumption index C_t and a measure of labour supply N_t.

The variable C_t is an aggregate Constant Elasticity of Substitution (CES) consumption index

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{h,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{f,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{2-3}$$

Where $\eta > 1$ is the elasticity of substitution between home and foreign goods. A large value of η indicates high substitution while a value of $\eta \to 1$ imposes almost no possibility of substitution.

In this world, home goods (non tradables) are consumed in a variety of ways which are aggregated in the index $C_{h,t}$ which is defined as

$$C_{h,t} = \left[\int_0^1 C_{h,t}(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \tag{2-4}$$

Here the parameter $\theta > 1$ measures the degree of substitutability among the different home goods. High substitutability implies lower market power to the producers of the different types. Let's define two important relative prices

- The real domestic price ratio is the price of non-tradable prices $P_{h,t}$ relative to the consumer based price index P_t (to be defined later)

$$S_t = \frac{P_{h,t}}{P_t} \tag{2-5}$$

- The real exchange rate is defined as the ratio of the peso price of imports $P_{f,t}$ to the consumer based price index[6]

[5] In this equation the parameters $1/\nu$, and $1/\delta$ measure constant intertemporal elasticities of substitution.

[6] It is perhaps important to define a more accurate measure of real exchange rate; the price of tradables in terms of non-tradables (sometimes also refereed as terms of trade): $\mathcal{T}_t = \frac{P_{f,t}}{P_{h,t}} = \frac{Q_t}{S_t}$

$$Q_t = \frac{P_{f,t}}{P_t} = \frac{\mathcal{E}_t P_t^*}{P_t} \tag{2-6}$$

Note that from the perspective of the home country, the dollar price of the imported good abroad P_t^* is given[7], which means that the domestic price of that good evolves according to: $P_{f,t} = \mathcal{E}_t P_t^*$. The domestic price of the imported good moves one-to-one with the nominal exchange rate which implies a pass-through equal to one; however, the pass-through to the consumer price index P_t depends also on the equilibrium effect of the exchange rate on domestic producer prices set by firms that sell final goods.

Intratemporal consumption decisions:

Given an optimal choice of C_t in a specific period, the intratemporal consumption decision hinges on the choices of home and foreign consumption that minimise the expenditure for given prices $P_t, P_{h,t}$ and $P_{f,t}$. The solution is given by the following decision rules

$$C_{h,t} = (1-\alpha)\, S_t^{-\eta} C_t \tag{2-7}$$

$$C_{f,t} = \alpha Q_t^{-\eta} C_t \tag{2-8}$$

It is clear from these equations that the home and foreign good consumption levels depend negatively on the real domestic price ratio and on the real exchange rate respectively. For a constant overall consumption C_t, an exchange rate spot depreciation reduces S_t and raises Q_t, thereby there is a substitution in consumption from foreign goods to home goods.

The consumption based price index summarises the relationship between $P_{h,t}$ and $P_{f,t}$ and it is given by[8]

[7]As usual, starred variables designate variables in the foreign country.

[8]Note that from the definition of the overall consumer price index:

$$P_t = \left[(1-\alpha)P_{h,t}^{1-\eta} + \alpha P_{f,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{2-9}$$

Using the same previous procedure, the demand for the different varieties of goods produced domestically is given by

$$C_{h,t}(j) = \left(\frac{P_{h,t}}{P_{h,t}(j)}\right)^{\theta} C_{h,t} \tag{2-10}$$

These consumption rules are defined given an overall home price index $P_{h,t}$, a price for the specific variety of good (set by the retailer) $P_{h,t}(j)$ and by the level of overall home consumption $C_{h,t}$. Likewise, the aggregate home price index is defined by

$$P_{h,t} = \left[\int_0^1 P_{h,t}(j)^{1-\theta} dj\right]^{\frac{1}{1-\theta}} \tag{2-11}$$

Knowledge of these equations is important insofar as they depict the evolution of prices, given the retailer's price setting behaviour to be described in Subsection 2.1.4.

Intertemporal consumption decision:

The first order condition for the optimal intertemporal consumption decision that solves [2-1] subject to [2-2] is

$$\frac{C_t^{-\delta}}{P_t} = \beta E_t \left[\frac{C_{t+1}^{-\delta}}{P_{t+1}} \mathcal{I}_t\right] \tag{2-12}$$

This equation has the standard meaning; the left hand side is the utility loss of forgoing consumption of $\frac{1}{P_t}$ units of the composite consumption basket while the right hand side is the gain from the extra utility generated by the additional next period consumption made possible by higher current savings.

$(1-\alpha)S_t^{1-\eta} + \alpha Q_t^{1-\eta} = 1$

Intratemporal portfolio decisions:

In order for both types of assets to be valued positively in consumer's preferences and hence to avoid corner solutions, it must be true that the uncovered interest parity holds between peso dollar asset returns (see Appendix A1)

$$\mathcal{I}_t = \frac{E_t\left[\mathcal{E}_{t+1}\right]}{\mathcal{E}_t}\mathcal{I}_t^f \tag{2-13}$$

Intratemporal labour supply decision:

The labour supply decision is made according to a standard condition that equates the real wage and the marginal disutility of labour

$$N_t^{\nu} C_t^{\delta} = \frac{W_t}{P_t} \tag{2-14}$$

As with the previous household choice rules, the supply of labour depends on the aggregate consumption index. The dynamic properties of labour supply depend upon the dynamics of the aggregate consumption index C_t through the Euler condition.

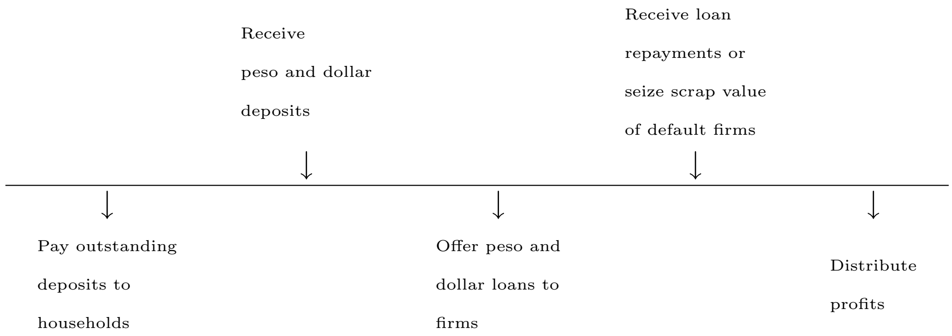
2.1.2 Financial intermediaries

They receive deposits from households and foreigners and lend to domestic firms. The timing of the actions is as follows

- At the beginning of time t they pay the outstanding deposit debt plus the interest rate accrued to households and foreigners for funds offered the previous period.

$$\mathcal{I}_{t-1}D_t + \mathcal{E}_t\mathcal{I}_{t-1}^f B_t + \mathcal{E}_t\mathcal{I}_{t-1}^f B_t^* + \left(\mathcal{E}_t - E_{t-1}\mathcal{E}_t\right)\left(B_t + B_t^*\right) \tag{2-15}$$

Figure 2.1: Timeline of financial intermediary's actions within any period.



Where: $\mathcal{I}_t^f = \mathcal{I}_t^* V_t$. The domestic dollar interest rate incorporates the foreign benchmark interest rate $\mathcal{I}_t^*$ and a factor $V_t = (1 + \nu_t)$ that accounts for country risk[9].

- Immediately afterwards, financial intermediaries offer households new stocks of both types of deposits: D_{t+1} and B_{t+1}. At the same time, an amount of deposits is offered to foreigners at the return[10] $\mathcal{I}_t^f$.

- Next, financial intermediaries offer loans to wholesale firms. These firms need to borrow in advance to be able to buy production inputs. The amounts lent by financial intermediaries in pesos and dollars are $L_{h,t}$ and $L_{f,t}$ respectively. The sources of fund available to financial intermediaries are twofold; the pesos and dollars deposited by domestic consumers plus any amount of pesos borrowed from the central bank and dollars borrowed abroad. Financial intermediaries have to hold compulsory reserves calculated as a fraction of deposits made last period.

$$L_{h,t}^s \equiv D_{t+1} + \Delta M_{b,t} - \zeta_D D_t \tag{2-16}$$

[9]This variable can be endogenised like in Céspedes et.al (26) or Mendoza (94). However, this is not done here because the purpose of the chapter is different.

[10]Due to the country-risk parameter, foreigners need to be paid more than the riskless benchmark foreign rate $\mathcal{I}_t^f > \mathcal{I}_t^*$.

$$L^s_{f,t} \equiv B_{t+1} + B^*_{t+1} - \zeta_B \left(B_t + B^*_t\right) \tag{2-17}$$

Here $\Delta M_{b,t}$ is the net position of financial intermediaries assets at the central bank and B^*_{t+1} is the net position of financial intermediaries dollar assets with the foreign sector[11]. If $\Delta M_{b,t}$ is positive then financial intermediaries take a short-term loan (to be re-paid in the same period), otherwise they make deposits at the central bank.

- The loan repayment is subject to agency costs because there is asymmetric information regarding the productivity of firms. Firms learn about their idiosyncratic shock to productivity before due repayment of their debts. Unproductive firms are insolvent and cannot pay their debt. Hence, financial intermediaries sign the same debt contract with all firms so that they can raise "enough" expected funds from intermediation.

2.1.3 Wholesale firms

Every period a continuum of firms in the unit interval is born. They all produce a homogeneous good. They face a credit-in-advance constraint in their purchases of production inputs. As in Cooley and Nam (34), this means that before production takes place, they have to borrow an amount equal to their entire input bill.

They borrow pesos and dollars before the idiosyncratic productivity shock realises and they repay or default after production and sale but before the next period starts. At the end of each period all firms die; either after setting their transfers to households or after default.[12]

[11]The presence of $\Delta M_{b,t}$ mimics the typical standing facility offered by the central bank at date t (a marginal lending facility or a deposit facility). In fact, this is the rationale whereby the central bank can control the short term interest rate of the economy. Though, the specific process of nominal interest rate setting is not modelled here. Here $\Delta M_{b,t}$ only works as an extra variable left to clear the market.

[12]This crucial assumption precludes accumulation of net worth by firms.

The technology they use to produce these goods is given by

$$Y_{h,t}(i) = \varpi_{it} A_t N_{it}^a J_{it}^{1-a} \tag{2-18}$$

Here, ϖ_{it} is an idiosyncratic productivity shock assumed to be $i.i.d$ across time and firms with density function $\phi(\varpi)$, c.d.f $\Phi(\varpi)$, unconditional expectation $E[\varpi_{it}] = 1$ and support on the bounded interval $[\varpi_l, \varpi_u]$. A_t is an aggregate productivity shock. N_{it} is the labour input and J_{it} is the imported intermediate input.

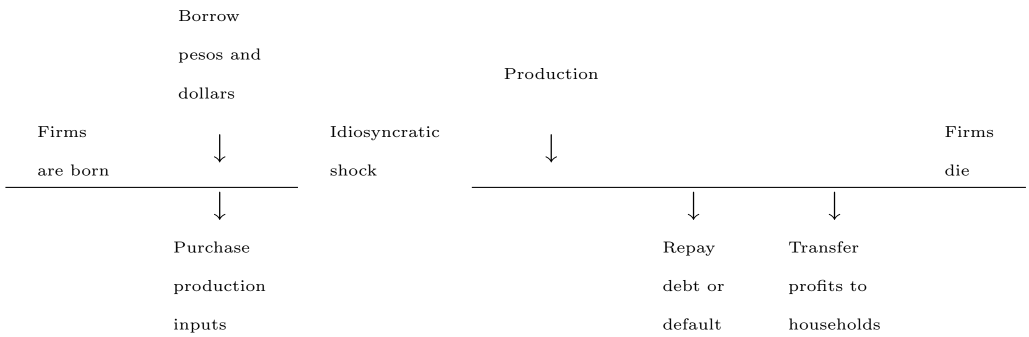
The credit-in-advance constraints for any firm i in pesos and dollars are given respectively by

$$L_{h,i,t} \equiv W_t N_{it} \tag{2-19}$$

$$L_{f,i,t} \equiv P_t^* J_{it} \tag{2-20}$$

Where W_t and P_t^* are the peso price of labour and the dollar price of the imported input respectively.

Figure 2.2: Timeline of firms actions within any period.



The nominal value of wholesale production considers the fact that non-tradeable production is sold at the wholesale home price $P^w_{h,t}$. Conveniently replacing [2-19] and [2-20] into [2-18] yields

$$P^w_{h,t}Y_{h,t}(i) = G_t \varpi_{it} L^a_{h,i,t} L^{1-a}_{f,i,t} \tag{2-21}$$

Where $G_t = A_t S^w_t \left(\frac{P_t}{W_t}\right)^a \left(\frac{P_t}{P^*_t}\right)^{1-a}$ groups the aggregate determinants of firm i production and $S^w_t = \frac{P^w_{h,t}}{P_t}$ represents the relative price of wholesale goods.

The design of the financial contract

A key assumption to endogenise financial intermediation is that after loans are taken and inputs enter into production, each firm i privately observes its idiosyncratic shock ϖ_{it}. If any other agent wants to learn about firm i's shock, that agent has to incur in auditing or monitoring costs. The existence of asymmetric information between firms and the rest of the agents and the introduction of a costly hidden-state verification induces the existence of financial intermediation as shown in Diamond (41).

The optimal contract that emerges from this type of setup has been solved in Gale and Hellwig (50)[13]. For risk neutral firms and financial intermediaries, the optimal, incentive compatible contract is a risky-debt contract.

The contract[14] at each time t and for every firm i hinges on finding the optimal loan demand levels of $L_{h,i,t}$, $L_{f,i,t}$, the return to the financial intermediary $\widetilde{\mathcal{I}}_t$ and a cutoff level of idiosyncratic productivity shock $\varpi_{o,i,t}$ that breaks even performing and non-performing loans. These optimal values are such that a) they maximise the expected return of the firm (Equation [2-22])and b) they allow the

[13]And applied in Bernanke et.al (13) and Carslstrom and Fuerst (25) among others.

[14]The contract in this setup has an intra-periodic nature. Long-term contracting is not possible given my assumption about the type of borrowers (short-lived and atomistic). Inter-periodic contracting made by long-lived agents would induce less severe agency costs.

financial intermediary to get expected returns from intermediation at least as high as its cost of funds (its participation constraint - Equation [2-23]). Formally,

$$\underset{\substack{L_{h,i,t},\, L_{f,i,t} \\ \widetilde{\mathcal{I}}_t,\, \varpi_{o,i,t}}}{Max} \int_{\varpi_{o,t}}^{\varpi_u} \left[G_t \varpi L_{h,i,t}^{a} L_{f,i,t}^{1-a} - \widetilde{\mathcal{I}}_t \left(L_{h,i,t} + \mathcal{E}_t L_{f,i,t} \right) \right] \phi(\varpi) d\varpi \tag{2-22}$$

s.a:

$$\begin{gathered} \int_{\varpi_{o,t}}^{\varpi_u} \widetilde{\mathcal{I}}_t \left[L_{h,i,t} + \mathcal{E}_t L_{f,i,t} \right] \phi(\varpi) d\varpi + \cdots \\ \int_{\varpi_l}^{\varpi_{o,t}} \left[G_t \varpi L_{h,i,t}^{a} L_{f,i,,t}^{1-a} - \lambda G_t \varpi L_{h,i,t}^{a} L_{f,i,,t}^{1-a} \right] \phi(\varpi) d\varpi + Z_t \geq X_t \end{gathered} \tag{2-23}$$

$$G_t \varpi_{o,t} L_{h,i,t}^{a} L_{f,i,t}^{1-a} = \widetilde{\mathcal{I}}_t \left(L_{h,i,t} + \mathcal{E}_t L_{f,i,t} \right) \tag{2-24}$$

Where

$$X_t = \mathcal{I}_t D_{t+1} + \mathcal{I}_t \Delta M_{b,t} + \mathcal{E}_t \mathcal{I}_t^{f} \left(B_{t+1} + B_{t+1}^{*} \right) + \left(\mathcal{E}_t - E_{t-1} \mathcal{E}_t \right) \left(B_t + B_t^{*} \right)$$

$$Z_t = \zeta_D D_t + \zeta_B \mathcal{E}_t \left(B_t + B_t^{*} \right)$$

The expected return of the firm is given by the expected production value minus the loan repayment. Loan repayment is only possible if the firms does not default. If the firm defaults, it obtains nothing.

On the other hand, the expected return of lending considers the expected repayment received from firms and the expected residual claims of the financial intermediary over the firms production in case of default. Monitoring costs are a proportion of the size of the production value. The constraint [2-23] means that the expected return of the financial intermediary plus the zero gross return from holding "required reserves" have to be at least equal to the funds the financial intermediaries promised to depositors (X_t) which also includes the funds to make up for the expected capital losses or gains[15]. On the other hand, Z_t is an exogenous amount of cash that financial intermediaries have to hold (obligatory reserve requirements as is standard in some emerging market economies). This amount of reserves is determined as a fraction ζ of the value of deposits made in the previous period.

Appendix A1 follows Gertler et.al (53) to show that this problem can be written in the following compact form

$$\underset{\substack{L_{h,i,t},\, L_{f,i,t} \\ \varpi_{o,i,t}}}{Max} \quad [1 - \Gamma(\varpi_{o,i,t})]\, G_t L^{a}_{h,i,t} L^{1-a}_{f,i,t} \tag{2-25}$$

s.a.

$$[\Gamma(\varpi_{o,i,t}) - \lambda\Upsilon(\varpi_{o,i,t})]\, G_t L^{a}_{h,i,t} L^{1-a}_{f,i,t} + \zeta_D D_t + \zeta_B \mathcal{E}_t \left(B_t + B^*_t\right) \geq X_t \tag{2-26}$$

The functions $\Gamma(.)$ and $\lambda\Upsilon(.)$ represent the expected share of output that goes to the financial intermediary and the expected monitoring costs[16] respectively. The cutoff point $\varpi_{o,i,t}$ is positive and finite and does not depend on idiosyncratic factors (hence $\varpi_{o,i,t} = \varpi^{e}_{o,t}$). A variable that rises as an important determinant on the solutions is the ratio S^w_t/mc_t which represents how much higher the real price of wholesale goods (S^w_t) has to be in excess of the marginal financial cost mc_t that arises in the absence of agency costs.

The optimal equilibrium loan levels are give by

[15] The funds to be obtained by financial intermediation treat realised capital gains and losses alike. *Ceteris-paribus*, more funds are needed to make up for capital losses and less funds for the case of capital gains. This does not need be so.

[16] The properties of $\Gamma(.)$ and $\Upsilon(.)$ are outlined in Appendix A1 along the lines of Bernanke et.al (13).

$$L_{h,t} = \frac{a}{\mathcal{I}_t} \frac{R_{r,t}}{f_{m,t}} \tag{2-27}$$

$$L_{f,t} = \frac{(1-a)}{\mathcal{E}_t \mathcal{I}_t^f} \frac{R_{r,t}}{f_{m,t}} \tag{2-28}$$

Where $R_{r,t}$ represent the provisions to deal with the opportunity cost of holding non-interest bearing reserves and capital gains or losses. It is defined by

$$R_{r,t} = \zeta_D(\mathcal{I}_t - 1)D_t + \zeta_B \mathcal{E}_t(\mathcal{I}_t^f - 1)\left(B_t + B_t^*\right) + \left(\mathcal{E}_t - E_{t-1}\mathcal{E}_t\right)\left(B_t + B_t^*\right)$$

And $f_{m,t}$ is the financial margin defined as the return of the lending activity in excess of the payment of interests to depositors

$$f_{m,t} = \left[\Gamma(\varpi_{o,t}^e) - \lambda\Upsilon(\varpi_{o,t}^e)\right]\left(\frac{S_t^w}{mc_t}\right) - 1$$

Both equilibrium peso and dollar loan levels depend positively on the respective share in the Cobb-Douglas production function and on the provision $R_{r,t}$, whereas they depend negatively on the financial margin $f_{m,t}$. The sign of the dependence of the interest rate is not conclusive because rising interest rates mean also that the provisions must also rise.

Lastly, the lending interest rate determined by the financial contract is proportional to both the cutoff productivity point and the ratio S_t^w/mc_t. Namely, the size of the lending rate is directly given by the extent of agency costs.

$$\widetilde{\mathcal{I}}_t = \varpi_{o,t}\left(\frac{S_t^w}{mc_t}\right) \tag{2-29}$$

2.1.4 Retailers and price setting

Following Bernanke et.al (13) and Gertler et.al (53), the model assumes that there is a continuum of monopolistically competitive retailers on the unit range. Retailers buy the amount $\widetilde{Y}_{h,t}$ of wholesale goods from firms and financial intermediaries[17] at the price $P^{w}_{h,t}$ and then costlessly differentiate the product. As a result the cost function results in:

$$Cost\left(P^{w}_{h,t}\right) = P^{w}_{h,t}\widetilde{Y}_{h,t}\left(P^{w}_{h,t}\right) \tag{2-30}$$

Importantly, prices are set in a staggered way. So, following Calvo (23) and Yun (138) the chapter derives a Phillips curve relationship between home inflation and "marginal costs" incurred in the acquisition of non-tradables from wholesalers.

It is assumed that, at any time, state of the world and regardless of history, any firm j has a probability γ to face institutional restrictions that make it impossible to set current prices in an optimal way[18]. With probability $1-\gamma$ instead, any firm has the opportunity to choose a new optimal price $P^{op}_{h,t}(j)$ that maximises the discounted sum of expected future profits. Because each home producer that chooses its new price in period t faces exactly the same problem, the optimal price $P^{op}_{h,t}(j)$ is the same for each of them. Hence, in equilibrium, all optimally chosen prices are equal to $P^{op}_{h,t}$.

Woodford (137) shows that in order to account for reasonable impulse response functions (hump-shaped response of inflation) after a monetary policy shock, the inflation rate must have some backward looking component. This is achieved through non-optimal indexation of prices through past inflation. Which implies

[17] Given that a fraction of firms default, financial intermediaries get the scrap value of production after the monitoring cost is incurred. Afterwards, they sell the seized product to retailers. Basically $\widetilde{Y}_{h,t} < Y_{h,t}$.

[18] So γ is a measure of price stickiness. A high value of this parameter on the unit range means that the degree of price stickiness is high.

that the home price index evolves according to

$$P_{h,t}^{1-\theta} = (1-\gamma)\left[P_{h,t}^{op}\right]^{1-\theta} + \gamma\left[\Pi_{h,t-1}P_{h,t-1}\right]^{1-\theta} \tag{2-31}$$

The dynamics of this price index, is determined recursively by knowing its initial value and the single new price $P_{h,t}^{op}$ that is chosen each period. The determination of $P_{h,t}^{op}$, in turn, depends upon current and expected future demand conditions for the individual home good. The choice of $P_{h,t}^{op}$ is such that it maximises the present value of the expected future profit conditional on the price being indexed through past accumulated inflation whenever it can not be adjusted optimally.

$$\underset{P_{h,t}^{opt}}{Max} E_t\left[\sum_{k=0}^{\infty}\gamma^k\beta_{t,t+k}^f\left\{\left[\frac{P_{h,t-1+k}}{P_{h,t-1}}\right]P_{h,t}^{op} - P_{h,t+k}^w\right\}\widetilde{Y}_{h,t+k}\right] \tag{2-32}$$

Subject to a sequence of demand constraints

$$\widetilde{Y}_{h,t+k}(j) = \left[\frac{P_{h,t+k}}{\left(\frac{P_{h,t-1+k}}{P_{h,t-1}}\right)P_{h,t}^{op}(j)}\right]^{\theta} C_{h,t+k} \tag{2-33}$$

Where $\beta_{t,t+k}^f$ is the discount factor of the $t+k$ monetary flows back to period t. Given that households are the ultimate owners of all type of firms, this monetary discount factor takes into account the discount factor implicit in the consumption Euler equation. Namely

$\beta_{t,t+k}^f = \beta^k\frac{U_c(C_{t+k})}{U_c(C_t)}\frac{P_t}{P_{t+k}}$. Maximisation of the above problem yields

$$E_t\left[\sum_{k=0}^{\infty}\gamma^k\beta_{t,t+k}^{firm}\widetilde{Y}_{h,t+k}\left\{\left[\frac{P_{h,t-1+k}}{P_{h,t-1}}\right]P_{h,t}^{op} - \mu P_{t+k}.S_{t+k}^w\right\}\right] = 0 \tag{2-34}$$

This condition states that the best retailers can do, given that they cannot set prices flexibly every period is to set the price such that it incorporates all the chances that they will keep the chosen price in the future. Instead of setting prices

$P^{op}_{h,t}$ equal to a mark-up over marginal cost (as a flexible price-setter would do), these constrained price setters set $P^{op}_{h,t}$ roughly equal to a weighted average of future expected marginal costs that will prevail given that $P^{op}_{h,t}$ remains unchanged.

2.1.5 Foreigners

The resource constraint in the foreign sector imposes the following equality valued in dollars.

$$P_{f,t}(Y_{f,t} - C_{f,t} - J_t) + \mathcal{E}_t B^*_{t+1} - \mathcal{I}^f_{t-1}\mathcal{E}_t B^*_t - (\mathcal{E}_t - E_{t-1}\mathcal{E}_t)\, B^*_t = 0 \qquad (2\text{-}35)$$

2.1.6 Monetary policy authority

Monetary policy is conducted by means of an ad-hoc rule. The instrument is the gross domestic interest rate $\mathcal{I}_t$ which is assumed to behave according to a rule that reacts systematically to inflation and output.

$$\mathcal{I}_t = (\mathcal{I}_{t-1})^{\rho}\left[\left(\frac{\Pi_{h,t+1}}{\widetilde{\Pi}}\right)^{\chi_{\pi h}}\left(\frac{Q_t}{Q_{t-1}}\right)^{\frac{\alpha\chi_\pi}{1-\alpha}}\left(\frac{\widetilde{Y}_{h,t}}{\widetilde{\widetilde{Y}}_h}\right)^{\chi_y}\mathcal{I}^f\right]^{(1-\rho)}\exp(\xi^m_t) \qquad (2\text{-}36)$$

Where $\mathcal{I}^f$ is the steady-state domestic dollar interest rate and ξ^m_t represents monetary policy shocks. The parameter ρ captures monetary policy inertia. Within the systematic component of the rule $\chi_{\pi h}$ and χ_π measure the sensitivity of the instrument to inflation deviations and χ_y measures the policy makers concern about economic activity.

The systematic behaviour defines three possible types of central banker. If the inflation targeting regime is in place, the values of the coefficients $\chi_{\pi h}$, χ_π and χ_y characterise possible types of inflation targeting.

The *strict home-inflation* targeting regime reacts only to deviations of home inflation from target $\Pi_{h,t+1}$, ($\chi_{\pi h} > 0$, $\chi_{\pi} = \chi_y = 0$). Real exchange rate movements are only of concern insofar as they affect the marginal cost of firms and hence home-price setting behaviour.

The *strict CPI inflation* targeting regime is defined as interest rates reacting to total CPI inflation only ($\chi_{\pi h} = \chi_{\pi} > 0$ and $\chi_y = 0$). This implies a concern for imported goods prices as well and therefore for a stronger concern about real exchange rate movements than that of the *strict-home inflation* targeting regime.

The third regime to be considered is a *flexible inflation targeting* regime where $\chi_{\pi h} = \chi_{\pi} > 0$ with $\chi_y > 0$. In this case the monetary authority also tries to smooth fluctuations in non-tradeable output. In this regime, therefore, the monetary authority is even more concerned about real exchange rate movements.

2.2 The solution to the log-linear approximation

2.2.1 The steady-state

The deterministic steady-state[19] is characterised by values of exogenous variables equal to their unconditional means: $Y_{f,t} = Y_f$, $I_t^* = I^*$, $I_t^f = I^* V$, $\Pi_t^* = \Pi^* = \beta^* I^*$, $A_t = A$ and a long-run monetary policy stance that sets the domestic interest rate such that: $I = I^f$. Also, in the long run, the real exchange rate Q_t clears the market for both the imported and exported goods. Given an infinitely elastic world net demand, it is assumed that the real exchange rate at which world net demand is infinitely elastic is $Q = 1$. This assumption is helpful insofar as it allows the real retail price $S = 1$ and $p_h^{op} = 1$. The direct implication is that aggregate consumption of non-tradeables and imported goods are $C_h = (1 - \alpha)\, C$ and $C_f = \alpha C$. Inasmuch as the monetary authority sets the domestic nominal interest rate in such away that

[19]The steady-state value of any variable x_t will be denoted by x.

it will not depart from the foreign monetary policy, then the nominal exchange rate evolution, as defined by the UIP condition (equation [2-13]), will result in a constant path ($\mathcal{E}_{t+1} = \mathcal{E}_t = \mathcal{E}$). Namely, the long-run trajectory of the nominal exchange rate is basically a function of the long-run monetary policy stance.

From the Euler equation the real interest rate $\mathcal{R}$ consistent with consumption decisions is assumed to be equal to the long-run US real interest $\mathcal{R}^* = \frac{1}{\beta^*}$ rate adjusted by country risk V. With the real interest rate already pinned down by preference parameters, the resulting steady-state inflation is conditioned by the long-run monetary policy stance using $\mathcal{I}/\Pi = 1/\beta$. Since monetary policy sets the interest rate $\mathcal{I}$ equal to $\mathcal{I}^f = \mathcal{I}^* V$ then the inflation rate achieved in the steady-state is exactly the same as the steady-state world inflation: $\Pi = \Pi^*$

The households budget constraint in real terms can be determined denoting $d_{t+1} = \frac{D_{t+1}}{P_t}$, $b_{t+1} = \frac{B_{t+1}}{P_t^*}$ and $b^*_{t+1} = \frac{B^*_{t+1}}{P_t^*}$. After some manipulation of the households budget constraint (equation [2-2])

$$d + b = \left(\frac{\beta}{1-\beta}\right)(C - wN - \omega) \tag{2-37}$$

Here, $wN + \omega$ denotes the total real wage income and the real value of transfers households receive from all firms and financial intermediaries. A positive amount of steady-state real deposits is only possible if $C > wN + \omega$. This is tantamount to households being able to afford high real consumption given the steady stream of interest rate gain on deposits.

Tradeable production

Since tradeable production is obtained from a costless and labourless random effort, its net production value is transferred to their ultimate owners, the households, then from equation [a12] in the appendix

$$\omega^f = Y_f \tag{2-38}$$

Non-tradeable wholesale production

The marginal cost[20] of the wholesaler if there were no agency costs is denoted by mc

$$mc = \frac{\Lambda}{A}\mathcal{I}^f w^a \tag{2-39}$$

The real wholesale price S^w has been defined as the ratio of the wholesale price to the CPI price level . The presence of frictions in the financial system implies that S^w needs to be larger than the real marginal cost mc. Wholesale goods are sold at a premium due to the deadweight losses imposed by the presence of agency costs. The ratio $\frac{S^w}{mc}$ is defined by

$$\frac{S^w}{mc} = \frac{A}{\mu\Lambda\mathcal{I}^f w^a} \tag{2-40}$$

The amount of real profits that non-tradeable wholesale firms have to transfer to households (their ultimate owners) is determined by the expected value of production kept by firms (see Appendix A1, equation [a11])

$$\omega^h = \frac{[1 - \Gamma(\varpi_o)]}{\mu} Y_h \tag{2-41}$$

Retailers

The pricing equation [2-34], together with the fact that $p_h^{op} = \left(P_{h,t}^{op}/P_t\right) = 1$ imposes the standard result whereby the marginal cost to the retailer S^w has to equal the inverse of the markup $\frac{1}{\mu}$. On the other hand, the equilibrium aggregate supply of retailer firms has to equal non-tradeable consumption

$$\widetilde{Y}_h = C_h = (1 - \alpha)C \tag{2-42}$$

Finally, retailers transfer monopolistic profits due to the mark-up of retailer prices over wholesale prices.

$$\omega^r = \left(\frac{\mu - 1}{\mu}\right)\widetilde{Y}_h = \left(\frac{\mu - 1}{\mu}\right)[1 - \lambda\Upsilon(\varpi_o)]\, Y_h \tag{2-43}$$

[20] See the Definition A1.2 in Appendix A1.

Financial intermediaries

From equation [a10] in Appendix A1, the transfers from financial intermediaries to households amounts to

$$\omega^b = \left(\mathcal{I}^f - \frac{\mathcal{I}^f}{\Pi^*}\right)(d + b + b^*) \tag{2-44}$$

Total transfers

Summing up all the transfers in [2-38], [2-41], [2-43] and [2-44] allows us to obtain the total transfers going to households

$$\omega = Y_f + \left(\frac{\Pi^*}{\beta} - \frac{1}{\beta}\right)(d + b + b^*) + \left(\frac{1 - \Gamma(\varpi_o)}{1 - \lambda\Upsilon(\varpi_o)} + \mu - 1\right)\frac{(1-\alpha)}{\mu}C \tag{2-45}$$

Replacing [2-45] in [2-37]

$$\begin{aligned} &(\Pi^* - \beta)(d + b) + (\Pi^* - 1)\, b^* = \\ &\left[1 - \left(\tfrac{1-\Gamma(\varpi_o)}{1-\lambda\Upsilon(\varpi_o)} + \mu - 1\right)\tfrac{(1-\alpha)}{\mu}\right]\beta C - \beta w N - \beta Y_f \end{aligned} \tag{2-46}$$

Labour market

The supply of labour is given by $N = w^{\frac{1}{\nu}} C^{\frac{-\delta}{\nu}}$ while the demand is $N = \frac{l_h}{w}$. The demand for labour depends on the real peso loan quantity[21] l_h which is given by[22]

$$l_h = \frac{a}{\mathcal{I}^f}\frac{\left(\mathcal{I}^f - 1\right)\frac{\zeta}{\Pi^*}(d + b + b^*)}{\left(\frac{\Gamma(\varpi_o) - \lambda\Upsilon(\varpi_o)}{\mu}\right)\frac{1}{mc} - 1} \tag{2-47}$$

Importantly, this real peso loan quantity is equal to the real peso deposits

$$l_h = wN = d\left(1 - \frac{\zeta}{\Pi^*}\right) \tag{2-48}$$

Market for imported input

[21] Derived from the equilibrium loan equation [2-27].

[22] For ease of solution, a convenient assumption is $\zeta_D = \zeta_B = \zeta$

In steady-state equilibrium the quantity of imported input is determined by the real dollar loan quantity which in turn is equal to the real dollar deposits in the domestic financial system

$$J = l_f = (b + b^*)\left(1 - \frac{\zeta}{\Pi^*}\right) \tag{2-49}$$

Given this condition, the imported input is determined by

$$J = \frac{(1-a)}{\mathcal{I}^f}\frac{\left(\mathcal{I}^f - 1\right)\frac{\zeta}{\Pi^*}(d + b + b^*)}{\left(\frac{\Gamma(\varpi_o) - \lambda\Upsilon(\varpi_o)}{\mu}\right)\frac{1}{mc} - 1}$$

Asset and Liability dollarisation in the steady state

From the previous equations, the asset and liability dollarisation ratios are the same and equal to the share of imported inputs in the production of non-tradable goods

$$l_{dr} = \frac{l_f}{l_f + l_h} = 1 - a \quad \text{and} \quad a_{dr} = \frac{b + b^*}{b + b^* + d} = 1 - a$$

In steady-state, non-tradable production can be defined in terms of the loan capacity of the financial system (long run liquidity) $(d + b + b^*)$ net of compulsory reserves, the nominal cost of funds $\mathcal{I}^f$ and the benchmark financial marginal cost mc. From solving the first order conditions in Appendix A1 and using equations [2-48] and [2-49]

$$Y_h = \frac{\mathcal{I}^f}{mc}\left(1 - \frac{\zeta}{\Pi^*}\right)(d + b + b^*) \tag{2-50}$$

External sector

From equation [2-35] Equilibrium vis-a-vis the rest of the world implies

$$Y_f = \alpha C + J + \left(\frac{1-\beta}{\beta}\right) b^* \tag{2-51}$$

Solution procedure

The solution hinges in replacing $wN = d\left(1 - \frac{\zeta}{\Pi^*}\right)$ and $Y_f = \alpha C + (b+b^*)\left(1 - \frac{\zeta}{\Pi^*}\right) + \left(\frac{1-\beta}{\beta}\right) b^*$ within [2-46] to get

$$\left(\Pi^* - \frac{\beta\zeta}{\Pi^*}\right)(d+b+b^*) = \left(\frac{\Gamma(\varpi_o) - \lambda\Upsilon(\varpi_o)}{1-\lambda\Upsilon(\varpi_o)}\right)\frac{(1-\alpha)}{\mu}\beta C \tag{2-52}$$

Taking the market clearing condition for retail goods $\widetilde{Y}_h = (1-\alpha)C$ and knowing that the amount of retail goods is related to the amount of wholesale goods via $\widetilde{Y}_h = [1-\lambda\Upsilon(\varpi_o)]\, Y_h$

$$Y_h = \frac{(1-\alpha)}{1-\lambda\Upsilon(\varpi_o)} C \tag{2-53}$$

This allows to write [2-50] as

$$(d+b+b^*) = \frac{mc}{\mathcal{I}^f\left(1-\frac{\zeta}{\Pi^*}\right)}\frac{(1-\alpha)}{1-\lambda\Upsilon(\varpi_o)} C \tag{2-54}$$

And combining the expressions for $(d+b+b^*)$ in [2-52] results in an expression that relates $\frac{S^w}{mc}$ to the equilibrium cutoff level ϖ_o

$$SW1: \quad ... \quad \frac{S^w}{mc} = \frac{1-\frac{\beta\zeta}{(\Pi^*)^2}}{\left(1-\frac{\zeta}{\Pi^*}\right)(\Gamma(\varpi_o)-\lambda\Upsilon(\varpi_o))} \tag{2-55}$$

Equation [2-55] together with the solution for ϖ_o in terms of $\frac{S^w}{mc}$ characterised in the intra-period equilibrium analysed in [a6] and [a7]

$$SW2: \quad ... \quad \varpi_o = \varpi_o(\frac{S^w}{mc}) \tag{2-56}$$

determine the equilibrium values for ϖ_o and $\frac{S^w}{mc}$

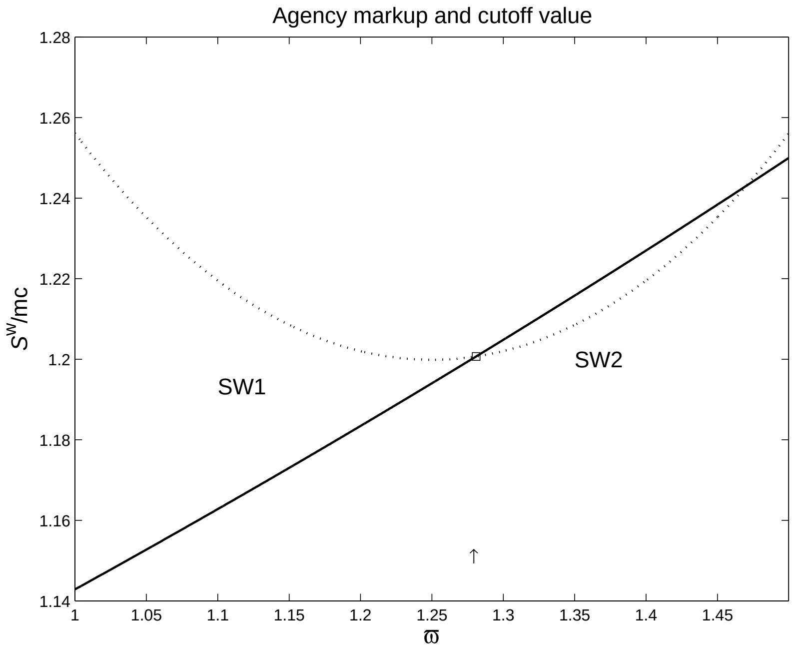


Figure 2.3: Equilibrium values of $\frac{S^w}{mc}$ and ϖ_o.

Once these values are pinned down, it is straightforward to disentangle the other variables.The equilibrium real wage rate is determined using the definition of mc

$$w = \left(\frac{Amc}{\Lambda \mathcal{I}^f}\right)^{\frac{1}{a}} \tag{2-57}$$

In order to determine the steady-state consumption level, the equilibrium labour has to be solved first. On the labour supply schedule

$$N = (w)^{\frac{1}{\nu}} (C)^{\frac{-\delta}{\nu}} = A_1 (C)^{\frac{-\delta}{\nu}} \tag{2-58}$$

So $A_1 = (w)^{\frac{1}{\nu}}$

On the other hand, the labour demand schedule

$$N = \frac{a}{w\left(\mathcal{I}^f\right)^2} \frac{\zeta\left(1-\alpha\right) mc}{\Pi^*\left(1-\frac{\zeta}{\Pi^*}\right)\left(1-\lambda\Upsilon\left(\varpi_o\right)\right)} \frac{1}{\left[\frac{\Gamma(\varpi_o)-\lambda\Upsilon(\varpi_o)}{\mu}\frac{1}{mc}-1\right]} C = A_2 C \tag{2-59}$$

Where $A_2 = \frac{a}{w\left(\mathcal{I}^f\right)^2} \frac{\zeta(1-\alpha)mc}{\Pi^*\left(1-\frac{\zeta}{\Pi^*}\right)(1-\lambda\Upsilon(\varpi_o))} \frac{1}{\left[\frac{\Gamma(\varpi_o)-\lambda\Upsilon(\varpi_o)}{\mu}\frac{1}{mc}-1\right]}$

Therefore

$$C = \left(\frac{A_1}{A_2}\right)^{\frac{1}{1+\frac{\delta}{\nu}}} \tag{2-60}$$

Once consumption is determined all the rest of the variables are uniquely pinned down

2.2.2 The log-linear approximation

In the approximation the variables are expressed in the form $\widehat{x}_t = (x_t - x)/x$, where x is the steady-state value of the variable x_t. The model outlined here can be approximated by 10 structural equations[23]

1. The equation for home prices is a typical hybrid Neo-Keynesian Phillips curve with past and expected next-period inflation. It also depends positively on the real exchange rate and the wholesale real price (See Section A2.1 in Appendix A2)

$$\widehat{\Pi}_{h,t} = \left(1 - B_1\right)\widehat{\Pi}_{h,t-1} + B_1 E_t\left[\widehat{\Pi}_{h,t+1}\right] + B_2\widehat{S}_t^w + B_3\widehat{Q}_t \tag{2-61}$$

Where:

$$\begin{aligned} B_1 &= \tfrac{\beta}{1+\beta} > 0 \\ B_2 &= \tfrac{1}{(1+\beta)}\tfrac{1-\gamma}{\gamma}\left(1-\gamma\beta\right) > 0 \\ B_3 &= \tfrac{\alpha}{1-\alpha}B_2 > 0 \end{aligned}$$

[23] See Appendix A2 for the derivation of the structural equations.

The wholesale real price $\widehat{S}_t^w$ represents the marginal cost the retailer has to face. This wholesale real price is affected by agency costs as seen later in [2-67]. The extent of how $\widehat{S}_t^w$ affects home inflation is determined by the parameter B_2. When the degree of price stickiness γ is small (more firms can adjust their prices in every period) then B_2 tends to be large and therefore home inflation is more responsive to changes in $\widehat{S}_t^w$

The real exchange $\widehat{Q}_t$ appears in the equation because it affects the pricing decisions of those retailers that can optimally choose new prices in period t. An increase in $\widehat{Q}_t$ prompts a consumption substitution towards home goods and therefore affects the demand conditions home-good producers face. The parameter B_3 can be interpreted as a partial pass-through coefficient. Note that the pass-through coefficient is positively related to the degree of openness α but it is negatively related to the degree of price stickiness γ

2. The aggregate consumption equation is the standard log-linearised form of the consumption Euler equation [2-12]. Movements in the nominal policy rate $\widehat{\mathcal{I}}_t$, insofar as they produce similar movements in the real interest rate[24], affect consumption directly via the intertemporal elasticity of consumption substitution δ^{-1}. A higher value of δ^{-1} makes aggregate consumption more reactive to changes in nominal interest rates

$$\widehat{C}_t = E_t\left[\widehat{C}_{t+1}\right] - \frac{1}{\delta}\left(\widehat{\mathcal{I}}_t - E_t\left[\widehat{\Pi}_{t+1}\right]\right) \tag{2-62}$$

[24]Note that [2-62] can be solved forward:

$$\widehat{C}_t = \lim_{s\to\infty} E_t\left[\widehat{C}_{t+s}\right] - \frac{1}{\delta}E_t\left[\sum_{s=0}^{\infty}\left(\widehat{\mathcal{I}}_{t+s} - \widehat{\Pi}_{t+s+1}\right)\right]$$

From here, the long-run real interest is $\mathcal{R}_t^{lr} = E_t\left[\sum_{s=0}^{\infty}\left(\widehat{\mathcal{I}}_{t+s} - \widehat{\Pi}_{t+s+1}\right)\right]$. Then $\widehat{C}_t = -\frac{1}{\delta}\mathcal{R}_t^{lr}$ i.e consumption is affected only to the extent that $\mathcal{R}_t^{lr}$ is affected.

3. The policy rate set by the monetary authority has a simple log-linear form (See Appendix A2). It is a weighted average of persistent and systematic behaviour. The systematic behaviour implies interest rates reacting to three possible components. The way these components are weighted characterise the types of policy regime under analysis. For example, a *strict home-inflation* targeting regime is defined by in $\chi_{\pi h} > 0$, $\chi_{\pi} = \chi_y = 0$. A *strict CPI inflation* targeter is obtained by setting $\chi_{\pi h} = \chi_{\pi} > 0$ and $\chi_y = 0$ and a *flexible inflation* targeter is obtained by setting $\chi_{\pi h} = \chi_{\pi} > 0$ with $\chi_y > 0$.

$$\widehat{\mathcal{I}}_t = \rho \widehat{\mathcal{I}}_{t-1} + (1-\rho)\left[\chi_{\pi h} E_t\left[\widehat{\Pi}_{h,t+1}\right] + \left(\tfrac{\alpha}{1-\alpha}\right)\chi_{\pi}\left(\widehat{Q}_t - \widehat{Q}_{t-1}\right) + \chi_y \widehat{C}_{h,t}\right] + \xi_t^m \tag{2-63}$$

Direct isolation of the policy stance from real exchange rate fluctuations is only possible under the *strict home inflation* targeting regime.

4. From the non-arbitrage condition between peso and dollar interest rates

$$\widehat{\mathcal{I}}_t = E_t\left[\widehat{\mathcal{E}}_{t+1}\right] - \widehat{\mathcal{E}}_t + \widehat{\mathcal{I}}_t^f \tag{2-64}$$

This is the standard uncovered interest parity condition. This equation governs the nominal exchange rate dynamics[25]

5. From the definition of the real exchange rate

$$\widehat{Q}_t - \widehat{Q}_{t-1} = \widehat{\mathcal{E}}_t - \widehat{\mathcal{E}}_{t-1} + \left(\widehat{\Pi}_t^* - \widehat{\Pi}_t\right) \tag{2-65}$$

6. The overall CPI inflation rate is defined in terms of the home inflation and the real exchange rate change

[25] Note that [2-64] can be solved forward to get
$\widehat{\mathcal{E}}_t = \lim_{i\to\infty} E_t\left[\widehat{\mathcal{E}}_{t+s}\right] - E_t\left[\sum_{s=0}^{\infty}\left(\widehat{\mathcal{I}}_{t+s} - \widehat{\mathcal{I}}_{t+s}^f\right)\right]$

$$\widehat{\Pi}_t = \widehat{\Pi}_{h,t} + \tfrac{\alpha}{1-\alpha}\left(\widehat{Q}_t - \widehat{Q}_{t-1}\right) \tag{2-66}$$

7. The wholesale real price $\widehat{S}_t^w$ depends on two broad terms, the first term in braces in 2-67 represents the real marginal costs wholesale producer would face in the absence of agency costs. The second term in braces describes the additional amount the wholesale producer would have to charge in order to recoup the deadweight losses imposed by the presence of agency costs

   The real marginal cost in turn has two parts. The first terms represents the "peso" financial cost of hiring labour. The second term is the "dollar" financial cost. Monetary policy has direct and indirect effects on the real wholesale price: the direct effect stems from the fact that a rise in $\widehat{\mathcal{I}}_t$ affects marginal costs and hence inflation positively through the parameter a which measures the weight of domestic factors in production, the indirect effects are manifold. Monetary policy affect $\widehat{S}_t^w$ through its effect on real wages ($\widehat{w}_t$), the real exchange rate ($\widehat{Q}_t$) and the benchmark idiosyncratic productivity level ($\widehat{\varpi}_{o,t}$)

$$\widehat{S}_t^w = \left\{a\left(\widehat{\mathcal{I}}_t + \widehat{w}_t\right) + (1-a)\left(\widehat{Q}_t + \widehat{\mathcal{I}}_t^f\right) - \widehat{A}_t\right\} + \left\{\frac{H_2}{H_1}\varpi_o\widehat{\varpi}_{o,t}\right\} \tag{2-67}$$

   Here the two parameters H_1 and H_2 depend on steady-state levels of ϖ_o and mc

$$H_1 = \tfrac{1}{[\Gamma(\varpi_o)-\lambda\Upsilon(\varpi_o)]\left(\frac{S^w}{mc}\right)-1} > 0$$

$$H_2 = \left[\tfrac{\lambda\Upsilon"(\varpi_o)-\Gamma"(\varpi_o)}{\lambda\Upsilon'(\varpi_o)-\Gamma'(\varpi_o)} - \tfrac{\Gamma"(\varpi_o)}{\Gamma'(\varpi_o)} - \tfrac{\Gamma'(\varpi_o)}{1-\Gamma(\varpi_o)} - \tfrac{[\Gamma'(\varpi_o)-\lambda\Upsilon'(\varpi_o)]\left(\frac{S^w}{mc}\right)}{[\Gamma(\varpi_o)-\lambda\Upsilon(\varpi_o)]\left(\frac{S^w}{mc}\right)-1}\right]$$

   The effect of variations in the cutoff level $\widehat{\varpi}_{o,t}$ upon the real price $\widehat{S}_t^w$ depends on the magnitude of H_1 and H_2 which in turn depends on the specific parameterisation of the probabilistic process for idiosyncratic productivity ϖ. In the solution, the special case of a uniform distribution for ϖ is considered.

8. The real wage depends on a direct income effect represented by a term in consumption and on the level of peso loans.

$$\widehat{w}_t = \frac{\nu}{1+\nu}\widehat{l}_{h,t} + \frac{\delta}{1+\nu}\widehat{C}_t \tag{2-68}$$

If ν is large (i.e. the elasticity of intertemporal elasticity of substitution small), then labour supply is inelastic. In such a case, real wage changes are driven by labour demand movements derived from the dynamics of real peso loans.

On the other hand, the elasticity of consumption substitution has to be very low in order for consumption to have a strong effect on wage dynamics.

9. The loanable funds equilibrium dynamics is governed by equation [2-19] in log-linearised form. Peso loans are increasing in the amount of reserves that banks need to hold. The overall effect of the interest rate is negative and the effect of the cutoff value $\widehat{\varpi}_{o,t}$ is determined by the sign of H_3.

$$\begin{aligned}\widehat{l}_{h,t} = {} & \left(\tfrac{\mathcal{I}^f}{\mathcal{I}^f-1}\right)\left[a_{dr}\widehat{\mathcal{I}}_t^f + (1-a_{dr})\widehat{\mathcal{I}}_t\right] + (1-a_{dr})\,\widehat{d}_t + a_{dr}\left(\widehat{Q}_t + \widehat{b}_t + \tfrac{1}{b}b_t^*\right) + ... \\ & + \tfrac{a_{dr}}{\zeta\left(\mathcal{I}^f-1\right)}\left(\widehat{\mathcal{E}}_t - E_{t-1}\widehat{\mathcal{E}}_t\right) - a_{dr}\widehat{\Pi}_t^* - (1-a_{dr})\,\widehat{\Pi}_t - \widehat{\mathcal{I}}_t - H_3\widehat{\varpi}_{o,t}\end{aligned} \tag{2-69}$$

Where

$$a_{dr} = \tfrac{b}{d+b}$$

$$H_3 = \left(\tfrac{[\Gamma'(\varpi)-\lambda\Upsilon'(\varpi)]\left(\frac{S^w}{mc}\right)}{G_1/mc-1} + \tfrac{G_1/mc}{G_1/mc-1}\tfrac{H_2}{H_1}\right)\varpi_o$$

In turn, equilibrium loans denominated in dollars is given by

$$\widehat{l}_{f,t} = \widehat{l}_{h,t} + \widehat{\mathcal{I}}_t - \widehat{Q}_t - \widehat{\mathcal{I}}_t^f \tag{2-70}$$

This equation results from the Cobb-Douglas specification of the production function. Additionally, the supply of both peso and dollar-denominated loans is linked to the evolution of both denomination of deposits

$$\widehat{l}_{h,t} = \left(\tfrac{1}{1-\zeta/\Pi^*}\right)\widehat{d}_{t+1} + \left(\tfrac{1}{1-\zeta/\Pi^*}\right)\tfrac{\Delta m_{b,t}}{d} - \left(\tfrac{\zeta/\Pi^*}{1-\zeta/\Pi^*}\right)\left(\widehat{d}_t - \widehat{\Pi}_t\right) \tag{2-71}$$

$$\widehat{l}_{f,t} = \left(\tfrac{1}{1-\zeta/\Pi^*}\right)\widehat{b}_{t+1} + \left(\tfrac{1}{1-\zeta/\Pi^*}\right)\tfrac{b^*_{t+1}}{b} - \left(\tfrac{\zeta/\Pi^*}{1-\zeta/\Pi^*}\right)\left(\widehat{b}_t + \tfrac{b^*_t}{b} - \widehat{\Pi}^*_t\right) \tag{2-72}$$

The policy rate has two type of effects: It will tend to reduce peso loans as the cost of peso funds increases. However, the increase in the peso cost of funds means that the relative dollar cost of funds falls. This substitution effect is partially offset by the production scale effect: As production grows, the economy does not want to depart from the optimal combination of peso and dollar loan levels. The extent of the effect is given by the weight of dollar loans (the parameter $a_{dr} < 1$)

10. Foreign sector equilibrium

$$J\left(\widehat{l}_{h,t} + \widehat{\mathcal{I}}_t - \widehat{Q}_t - \widehat{\mathcal{I}}^f_t - \tfrac{1}{b}b^*_{t+1}\right) = \eta C_f \widehat{Q}_t - C_f \widehat{C}_t - \tfrac{1}{\beta}b^*_t + Y_f \widehat{Y}_{f,t} \tag{2-73}$$

Solution procedure

The system of linear expectational difference equations [2-61] to [2-73] summarises the dynamics of the model which can be solved numerically for given values of the deep parameters. In order to perform the solution exercise a standard solution algorithm[26] is used.

$$\mathcal{Y}_t = \left[\begin{array}{cccccccccccccc} \widehat{C}_t & \widehat{\Pi}_t & \widehat{\Pi}_{h,t} & \widehat{\mathcal{I}}_t & \widehat{\mathcal{E}}_t & \widehat{Q}_t & \widehat{l}_{h,t} & \widehat{l}_{f,t} & \widehat{S}^w_t & \widehat{\varpi}_{o,t} & \widehat{w}_t & \widehat{d}_{t+1} & \widehat{b}_{t+1} & \widehat{i}^*_t & \widehat{b}^*_{t+1} \end{array}\right]^t$$

[26]I use the algorithm described in Klein (76). First I define a set of endogenous state variables grouped in the vector $\mathcal{Y}_t$.

The solutions will depend on a vector of predetermined state variables called $\mathcal{X}_t$ and a vector of exogenous variables $\mathcal{Z}_t$ which are defined respectively as

$$\mathcal{X}_t = \left[\begin{array}{ccccccccc} \widehat{Q}_{t-1} & \widehat{\Pi}_{h,t-1} & \widehat{\mathcal{E}}_{t-1} & \widehat{\mathcal{I}}_{t-1} & E_t(\widehat{\mathcal{E}}_{t-1}) & \widehat{i}^*_{t-1} & \widehat{b}^*_t & \widehat{b}_t & \widehat{d}_t \end{array} \right]^t$$

$$\mathcal{Z}_t = \left[\begin{array}{ccccccc} \widehat{A}_t & \xi^i_t & \widehat{\Pi}^*_t & \widehat{Y}_{f,t} & \xi^{i^*}_t & \xi^{b^*}_t & \xi^{\Delta mb}_t \end{array} \right]^t$$

The system can be written in compact form as:

$$AE_t \begin{bmatrix} \Upsilon_{t+1} \\ K_{t+1} \end{bmatrix} = B \begin{pmatrix} \Upsilon_t \\ K_t \end{pmatrix} + C\mathcal{Z}_t \tag{2-74}$$

$$\mathcal{Z}_{t+1} = \Theta \mathcal{Z}_t + U_{t+1} \tag{2-75}$$

The solution is given in a state-space representation where the predetermined state variables are updated according to

$$\begin{pmatrix} K_{t+1} \\ \mathcal{Z}_{t+1} \end{pmatrix} = \begin{pmatrix} P & Q \\ 0 & \Phi \end{pmatrix} \begin{pmatrix} K_t \\ \mathcal{Z}_t \end{pmatrix} + \begin{pmatrix} 0 \\ U_{t+1} \end{pmatrix} \tag{2-76}$$

And the endogenous state is observed according to

$$\Upsilon_t = \begin{pmatrix} M & N \end{pmatrix} \begin{pmatrix} K_t \\ \mathcal{Z}_t \end{pmatrix} \tag{2-77}$$

2.2.3 Calibration of model parameters

To calibrate de model, Peruvian data is used whenever it is possible. The Peruvian economy is a typical emerging market country with financial dollarisation features, just what the present model tries to portrait.

Parameters describing household preferences

- The subjective discount factor β is calibrated such that it implies a steady-state domestic real interest rate equal to 6% per year, considering that the US steady-state real rate is considered to be 4% per year. This implies $\beta = 0.9852$, $\beta^* = 0.9901$ and the risk premium factor $V = 1.005$

- The elasticity of intertemporal consumption substitution measures the degree of reactiveness of aggregate consumption to real interest rate movements. This value is set to $1/\delta = 1/5$ which is relatively low and suggests that this channel might be weak in emerging market economies.

- The elasticity of intertemporal labour substitution $1/\nu$ is set to 2.2, this value is however relatively high and reflects the idea that labour demand might be more responsive to wages in this type of economies.

- For the elasticity of intratemporal substitution between consumption of foreign goods and home goods a value $\eta = 2$ is chosen suggesting an environment where people find difficult to substitute consumption of foreign goods by that of home goods.

- The elasticity of substitution across the different varieties of home goods is set to be $\theta = 11$. This value is consistent with a steady-state mark-up of 10%[27]

- The proportion of foreign consumption out of total consumption in steady state is given by the parameter α. This parameter is set to $\alpha = 0.25$ as Céspedes et.al (26)

Parameters describing the production technology

- Production scale parameter $A = 1$

[27] Recall that the mark-up is expressed in terms of that elasticity $\mu = \frac{\theta}{\theta - 1}$

- The Cobb-Douglas coefficient a is econometrically estimated to be between 0.6 and 0.8, the mean value of 0.68 is used, which means that the liability dollarisation ration is about 32%. Official estimates of dollarisation ratios in Peru are as high as 60%. The value assumed here is a lower bound.

  The idiosyncratic productivity shock is assumed to follow a uniform distribution with unconditional mean equal to one. Specifically the p.d.f is $\phi(\varpi) = \frac{1}{2\Delta}$ and the c.d.f is $\Phi(\varpi) = \frac{1}{2\Delta}(\varpi - 1 + \Delta)$, with $\Delta = 0.5$.

Parameter describing the institutional restriction on price setting

- The probability that an individual firm does not change its price at any date is γ and the average duration of this price quotation is $1/(1-\gamma)$ quarters. The standard value for a developed, stable economy is $\gamma = 0.75$. Instead, a value $\gamma = 0.5$ is assumed, which means that price quotations last two quarters only, namely, prices are more flexible than the standard case.

Parameters describing monetary policy

- The interest rate smoothing coefficient is set to $\rho = 0.7$

- The parameterisation of the three regimes is as follows:

  Strict home-inflation: $\chi_{\pi h} = 1.5$, $\chi_{\pi} = \chi_y = 0$.

  Strict CPI inflation:$\chi_{\pi h} = \chi_{\pi} = 1.5$ and $\chi_y = 0$.

  Flexible inflation targeting:$\chi_{\pi h} = \chi_{\pi} = 1.5$ with $\chi_y = 0.5$

Parameters describing the foreign nominal variables

- The US steady-state inflation rate is set to be 2% per year, which means that $\Pi^* = 1 + 0.02/4$

- The mean US nominal interest rate is considered to be 6% per year (given a real rate of 4% and an inflation rate of 2%). Hence $\mathcal{I}^* = 1 + 0.06/4$

Parameters describing financial conditions

Financial conditions depend heavily on two parameters; monitoring costs as a proportion λ of the size of borrowers production and the reserve requirement ration ζ. The value of these two parameters are likely to be high in emerging market economies and they should be such that the steady-state lending interest rate results in reasonable values. Hence, these values are set to $\lambda = 0.2$ and $\zeta = 0.2$ such that the lending interest rate is $\widetilde{\mathcal{I}} = 17\%$.

Parameters describing the data generating process of exogenous variables

- The exogenous variables of the model contained in the vector $\mathcal{Z}_t$ are assumed to follow an AR(1) representation. The respective parameters (AR(1) coefficients and standard deviations) are grossly estimated from data.

A note about the steady-state solution

The calibrated parameters define a steady-state solution shown in Table [2.1]. The probability of default in steady-state is as high as 78 percent. This number is not realistic.

2.3 The agency-cost channel and the Phillips curve

The chapter analyses the responses of the model economy to three types of shocks relevant to an emerging market economy; an aggregate productivity shock, a dollar

REAL QUANTITIES		
Aggregate consumption	C	0.745
Home consumption	C_h	0.559
Foreign consumption	C_f	0.186
Labour	N	1.276
Imported output	J	0.154
Household's peso deposits	d	0.409
Household's dollar deposits	b	0.192
Peso credit	l_h	0.328
Dollar credit	l_f	0.154
Wholesale production	Y_h	0.649
Retailer production	Y_{hr}	0.559
TRANSFERS		
From financial intermediaries	ω_b	0.003
From wholesale producers	ω_{wh}	0.014
From retailers	ω_r	0.051
From tradeable production	ω_f	0.340
PRICES AND INTEREST RATES		
Nominal gross interest rate	I	1.020
Real wholesale price	S^w	0.909
Real domestic price	S	1.000
Real exchange rate	Q	1.000
Real wage	w	0.257
MARKUPS		
Domestic prices over wholesale prices	S/S^w	1.100
Wholesale prices over marginal cost	S^w/mc	1.201
FINANCIAL FRICTIONS		
Idiosyncratic productivity cutoff value	ϖ_0	1.281
Lending rate	$\widetilde{I}$	1.165
Probability of default	PD	0.781
Failure rate	h	1.141

Table 2.1: Steady-state values.

interest rate shock and a commodity production shock. Then these shocks are compared under three possible types of monetary policy regimes; *strict home-inflation* (HIT), *strict CPI-inflation* (CIT) and *flexible inflation targeting* regimes (FIT).

A key feature that emerges from this set up is the positive correlation between unexpected depreciations and the probability that borrowers default on their loans. Higher default probabilities constitute a heavy burden on wholesale price setting which is then transmitted to final goods.

Financial intermediaries have liabilities denominated in both pesos and dollars. When an unexpected depreciation occurs, they suffer capital losses against households. The good news is that financial intermediaries also hold assets denominated in both currencies and that they have agreed on loan contracts stipulating that loan quantities are adjusted in the same direction as movements in their liabilities[28]. However, the amount of loans offered cannot quickly jump to recoup capital losses, the variable that does adjust quickly is the cutoff productivity value[29] that determines the shares of production that goes to both borrowers and financial intermediaries. An increase in the cutoff value due to a an unexpected depreciation is built in the structure of the contract as an equilibrium outcome; firms that did not default are better off even though they have a small proportion of the cake because they were able to produce more and financial intermediaries are not worse off because they can compensate their capital losses by increasing the share they can grab from the production process.

The hidden cost of the above mechanism however is the increasing amount of business defaults that emerge in equilibrium due to an unexpected depreciation of the exchange rate.

2.3.1 A positive aggregate productivity shock

When a positive aggregate productivity shock hits the economy (See figures [2.4], [2.5] and [2.6]) the standard result is that the marginal cost of producers firms, producer prices and final goods inflation all tend to fall, whereas consumption and output tend to increase.

In this setup, the presence of agency costs can offset or magnify those standard effects. For example, when the monetary authority is characterised by the HIT regime, then mechanism that serves to stabilise home-inflation hinges on inducing a negative correlation between the real exchange rate and the wholesale real domestic price in equation [2-61][30]. A positive aggregate productivity shock

[28]See equation [2-69].

[29]The cutoff productivity value moves in the same direction of the lending rate and the probability of default.

[30]This mechanism might sometimes imply that real exchange rates are not smoothed at all but are used as a device to offset domestic home inflationary factors.

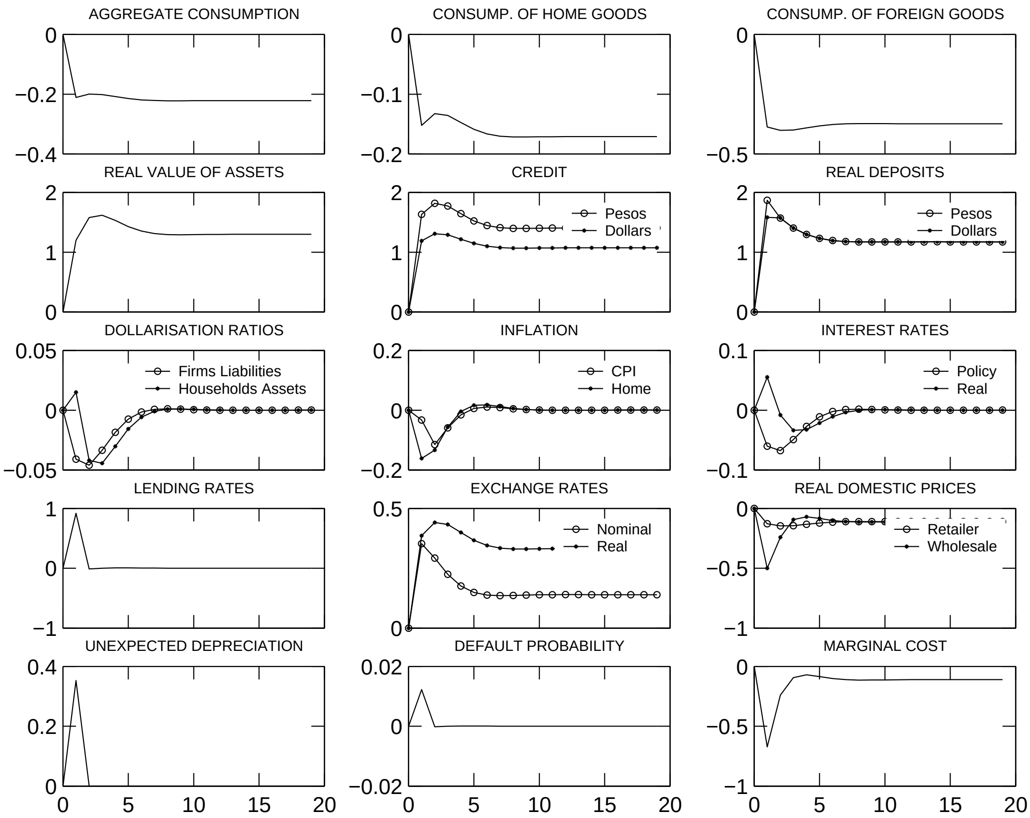


Figure 2.4: Strict home inflation target: Responses to a one-standard-deviation positive productivity shock: (Responses are measured as percentage deviations from the respective steady-state values)

tends to reduce producer marginal costs which the HIT mechanism tries to undo by inducing an real exchange rate depreciation and as a result an unexpected nominal exchange rate depreciation. This last effect is the link between the productivity shock and the financial conditions in the economy described above. Both households and financial intermediaries increase their holdings of assets. As households save more, they reduce overall consumption. The transmission of the productivity shock to home inflation is hampered by the fact that the agency markup (the difference between the real wholesale price S_t^w and the wholesaler marginal cost mc_t) increases due to the more stringent agency conditions. This means that reduction of marginal costs imply a less than proportional reduction in wholesale prices.

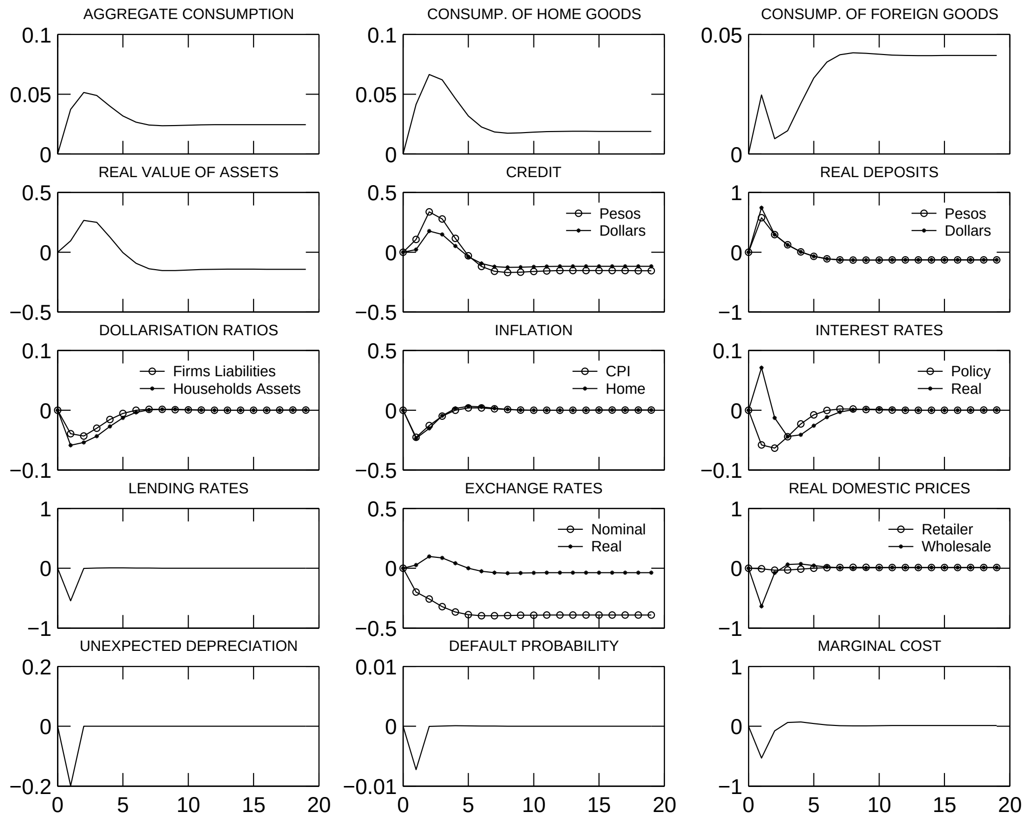


Figure 2.5: CPI-home inflation target: Responses to a one-standard-deviation positive productivity shock:(Responses are measured as percentage deviations from the respective steady-state values)

Under both the CIT and FIT regimes there is a concern for smoothing real exchange rate deviations per se and not to use it as an offsetting device. This implies that disinflationary pressures brought about by a positive productivity shock are absorbed by a nominal exchange rate appreciation [see equation 2-65]. The unexpected appreciation in turn, triggers the opposite effects on the financial side of the economy to the ones under the HIT regime; credit, deposits after some quarters, the default probability, the lending rate and the real value of households assets fall whereas consumption increases.

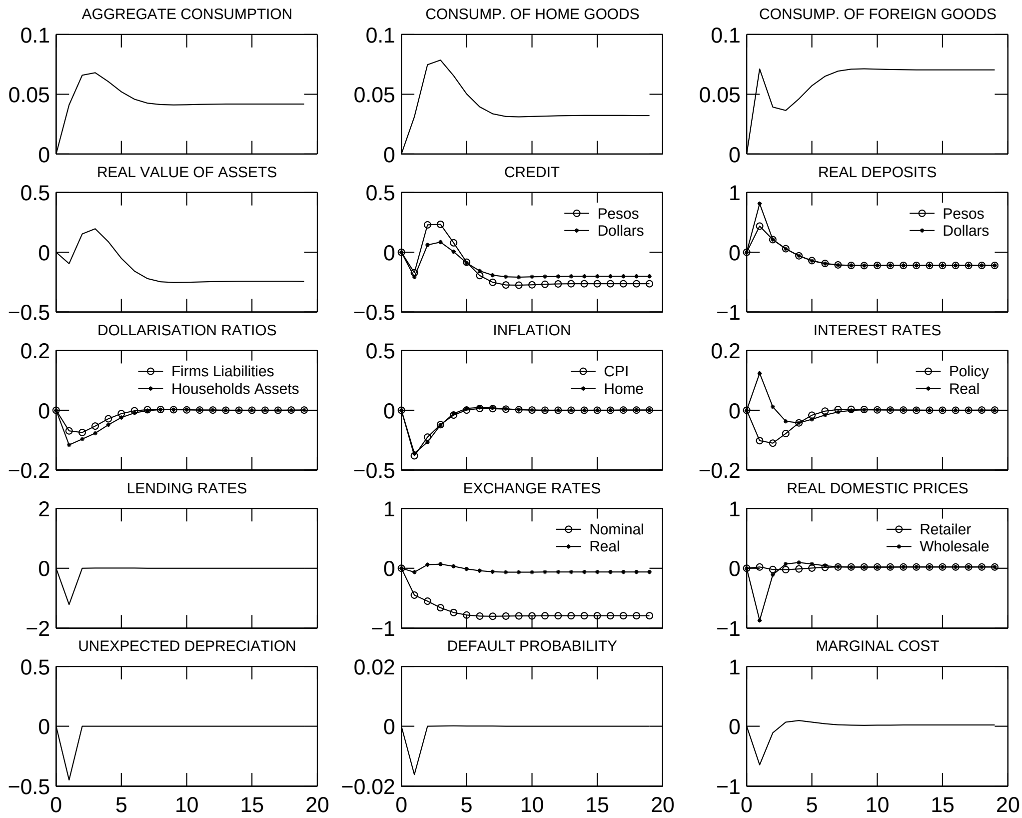


Figure 2.6: Flexible inflation target: Responses to a one-standard-deviation positive productivity shock:(Responses are measured as percentage deviations from the respective steady-state values)

2.3.2 A Dollar interest rate shock

An increase in the dollar interest rate has a standard effect of causing a spot depreciation of the nominal exchange rate. Though it is not the case under the HIT regime due to the fact that marginal cost of wholesalers tends to increase due to higher interest rates and as a result the home inflation stabilisation mechanism calls for a real exchange rate reduction which triggers a nominal exchange rate appreciation. The results are depicted on figures [2.7], [2.8] and [2.9]. Upon inspection of the diverse responses to this shock, it turns out that the HIT regime fares better to smooth inflation and even exchange rates but not home and aggregate consumption.

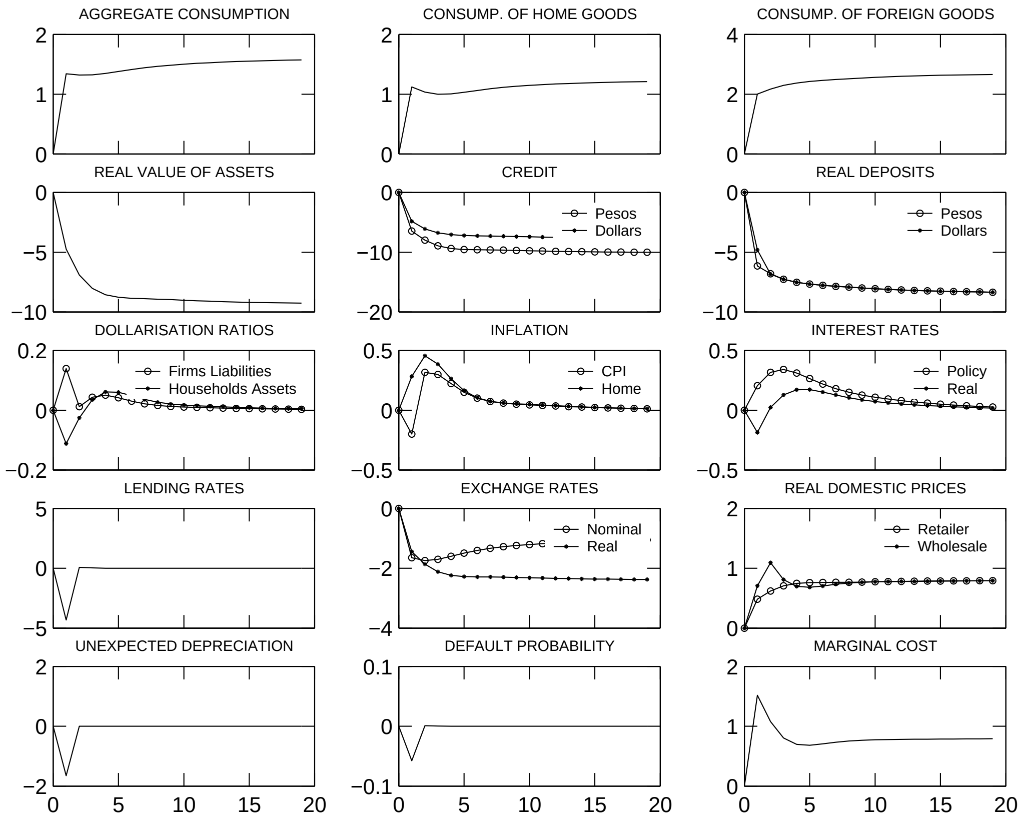


Figure 2.7: Strict home inflation target: Responses to a one-standard-deviation dollar interest rate shock(Responses are measured as percentage deviations from the respective steady-state values)

2.3.3 A commodity production shock

A positive shock to commodity production coupled with the fact that the net asset position with foreigners is bound to remain fixed implies that imports should adjust in the same direction on impact, in particular imports of production inputs. This also implies an increase in the demand for labour due to the complementary of the Cobb-Douglas production function.

In order to be able to hire more labour, the wage rate must adjust upwards, forcing the marginal cost of wholesale producers to increase. The increase in the marginal cost represents an inflationary pressure. Again, under the HIT regime this

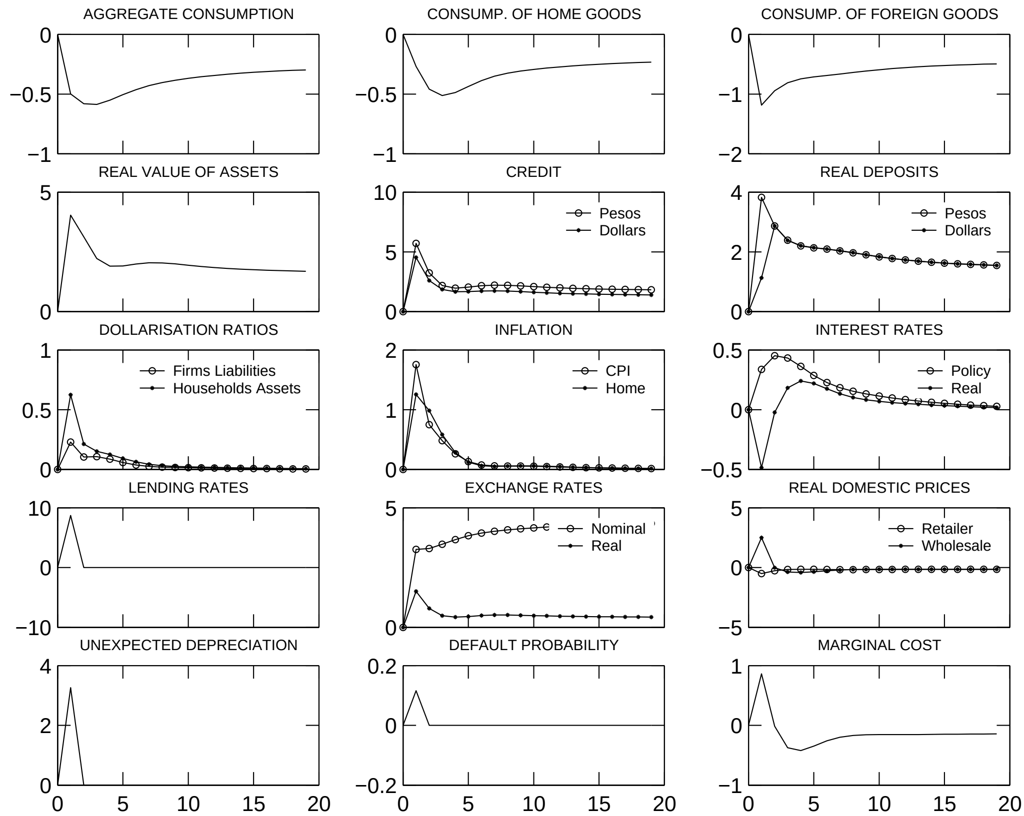


Figure 2.8: CPI-home inflation target: Responses to a one-standard-deviation dollar interest rate shock (Responses are measured as percentage deviations from the respective steady-state values)

upward movement in the marginal cost is partially offset in equilibrium by a real exchange rate appreciation that triggers an unexpected nominal appreciation and all the consequences that follow through.

On the CIT and FIT regimes, the above mechanism is not present. Real exchange rates hardly change and as a result nominal exchange rates move to compensate higher CPI prices. In this case, the unexpected depreciation triggers the adverse effects of agency costs on consumption.

These results are depicted in [2.10], [2.11] and [2.12]. In this case, the particular specification of the model economy also favours the HIT regime to stabilise

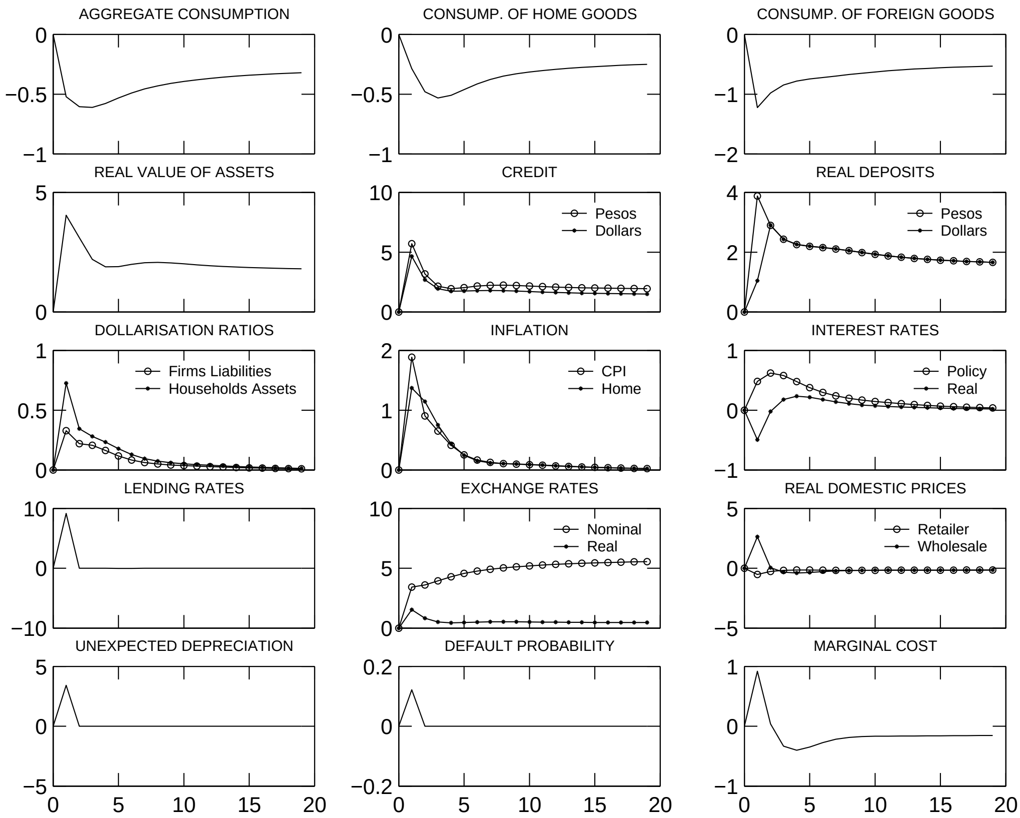


Figure 2.9: Flexible inflation target: Responses to a one-standard-deviation dollar interest rate shock (Responses are measured as percentage deviations from the respective steady-state values)

inflation but at the cost of higher real exchange rate and consumption fluctuations. On the other hand, the CIT and FIT regimes render lower variability of real exchange rate and consumption but at the cost of generating more inflation variability.

2.4 Conclusion

The model presented in this chapter tries to capture one element often disregarded in the analysis of dollarisation in emerging market economies; the fact that both assets and liabilities are dollarised and that increasing dollarisation might not be

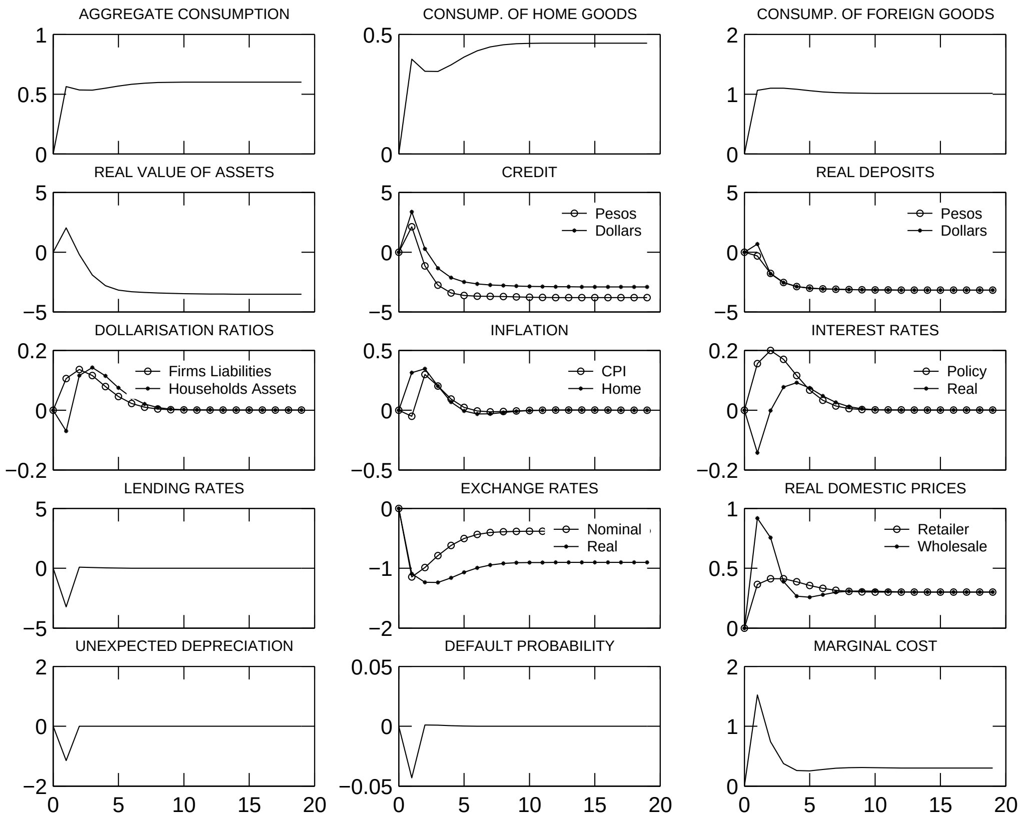


Figure 2.10: Strict home inflation target: Responses to a one-standard-deviation positive commodity production shock (Responses are measured as percentage deviations from the respective steady-state values)

necessarily bad for certain types of agents and certain types of shocks, in fact they result from optimising behaviour of agents.

The key mechanism captured in the model is that unexpected nominal exchange rate depreciations are closely linked with the probability of default by borrower firms. Any unexpected movement of the exchange rate turns out to be a powerful mechanism to move the real value of households' assets (savings) and therefore to move aggregate consumption. On the other hand, the default probability is a manifestation of whether agency costs become higher or not. When agency costs increase (increasing probability of default) the markup of real wholesale prices

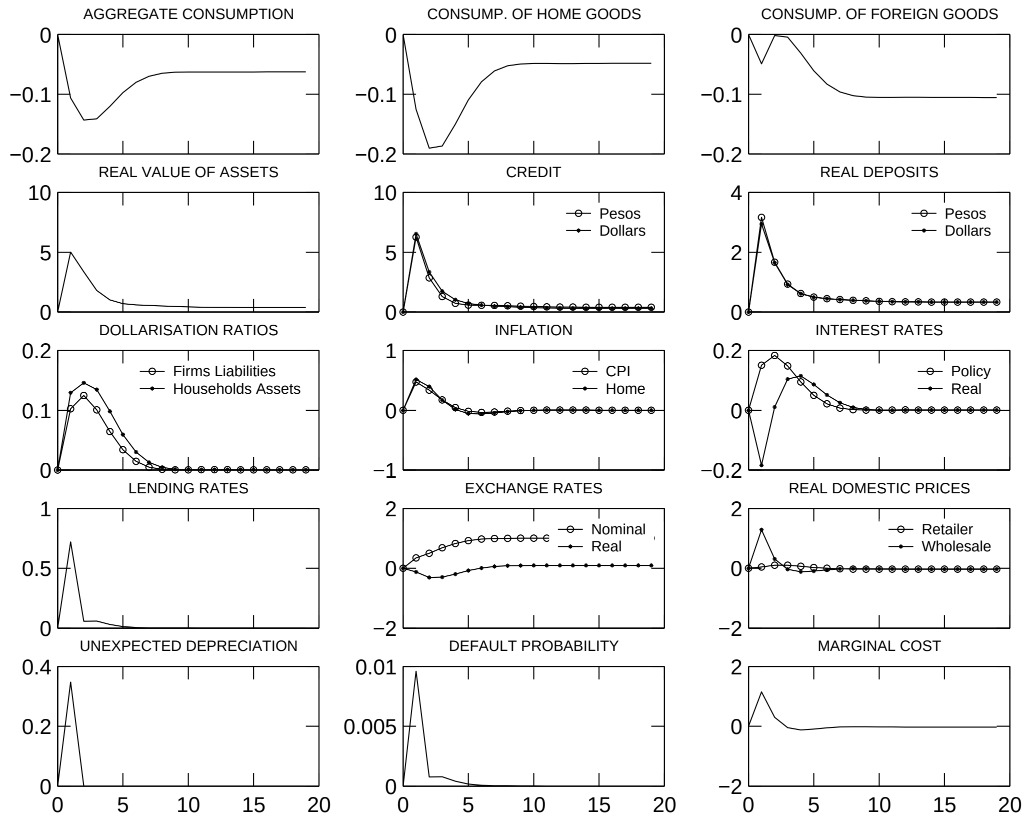


Figure 2.11: CPI-home inflation target: Responses to a one-standard-deviation positive commodity production shock: (Responses are measured as percentage deviations from the respective steady-state values)

over wholesale marginal costs increases which in turn shapes the dynamics of home inflation.

Within this environment, three possible inflation targeting regimes are evaluated; a strict *home-inflation targeting* (HIT), a strict *CPI-inflation targeting* (CIT) and a *flexible inflation targeting* (FIT). The core mechanism in the HIT regime is the use of the real exchange rate as a marginal cost stabilising devise in order to smooth home inflation deviations. The CIT and FIT regimes are defined such that the concern about real exchange rate fluctuations are built within the structure of the equilibrium. In order to assess these three regimes three types of shocks

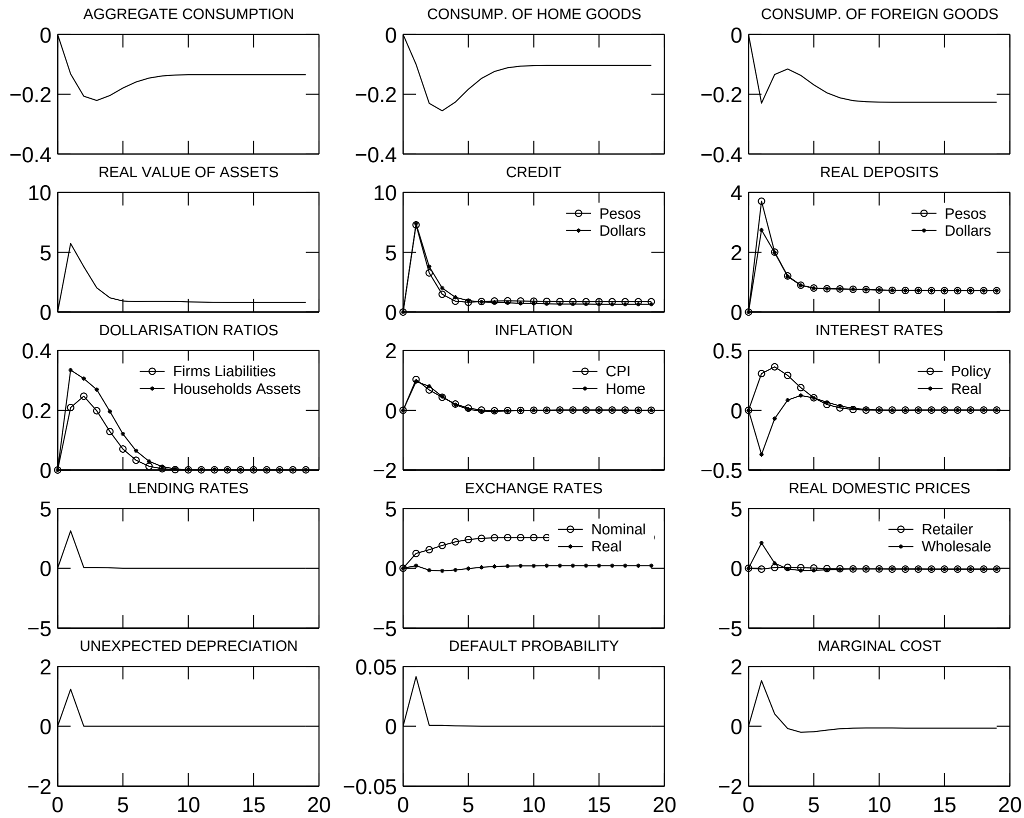


Figure 2.12: Flexible inflation target: Responses to a one-standard-deviation positive commodity production shock: (Responses are measured as percentage deviations from the respective steady-state values)

dominant in emerging market economies are analysed; an aggregate non-tradeable productivity shock, a shock to the dollar interest rate and a tradeable commodity production shock. As is standard in these evaluations, the HIT regime renders in small inflation fluctuations at the cost of higher real exchange rate and consumption fluctuations whereas the CIT and FIT regimes produce the converse results. In all the cases, the sign of the unexpected depreciation is positively correlated to the real value of assets and negatively correlated to aggregate consumption.

In this chapter, monetary policy is conducted without absolute concern about the financial health of firms; namely, firms defaults produce no further costs to society other than the liquidation costs that financial firms have to incur. In reality, defaults or a potential systemic failure are seen as a fundamental threat to central bankers. Further research is necessary to seek for monetary policy regimes that take into account a loss function for the monetary authority that considers for example financial stability aspects in addition to the usual inflation and real activity concerns. In line with this research agenda, a recent contribution of the author in Bigio and Vega (16) suggest that dirty-floating regimes are optimal even if the monetary authority is uncertain about the strength of the pervasive effect of financial dollarisation.

A1 Appendix: Optimal decisions

A1.1 Households

Given the reward function and the budget constraint outlined in the main text, the households problem can be expressed as

$$V(D_t, B_t) = \underset{\{C_t, N_t, B_{t+1}\}}{Max} \left\{ \frac{C_t^{1-\delta} - 1}{1-\delta} - \frac{N_t^{1+\nu} - 1}{1+\nu} + \beta E_t \left[V\left(D_{t+1}, B_{t+1}\right) \right] \right\}$$

Where

$$D_{t+1} = -\mathcal{E}_t B_{t+1} + \mathcal{I}_{t-1} D_t + \mathcal{E}_t \mathcal{I}_{t-1}^f B_t + \left(\mathcal{E}_t - E_{t-1}\mathcal{E}_t\right) B_t + W_t N_t - P_t C_t + \Omega_t$$

The standard optimality conditions are:

Consumption	:	$C_t^{-\delta} = \beta E_t \left[V_{D_{t+1}} P_t \right]$
Labour supply	:	$N_t^{\nu} = \beta E_t \left[V_{D_{t+1}} W_t \right]$
Nominal dollar deposits	:	$E_t V_{B_{t+1}} = \mathcal{E}_t E_t V_{D_{t+1}}$
Envelope Theorems:	:	$V_{D_t} = \beta E_t \left[V_{D_{t+1}} \mathcal{I}_{t-1} \right]$
		$V_{B_t} = \beta E_t \left[V_{D_{t+1}} \left\{ \mathcal{I}_{t-1}^f \mathcal{E}_t + \left(\mathcal{E}_t - E_{t-1}\mathcal{E}_t\right) \right\} \right]$

Combining the equation for nominal dollar deposits and envelope theorems:

$$E_t \left[V_{D_{t+2}} \left(\mathcal{I}_t - \left\{ \mathcal{I}_t^f \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} + \frac{\left(\mathcal{E}_{t+1} - E_t \mathcal{E}_{t+1}\right)}{\mathcal{E}_t} \right\} \right) \right] = 0$$

Knowing that $\mathcal{I}_t$ and $\mathcal{I}_t^f$ are known as of time t, then after some algebraic manipulation:

$$\frac{1}{\beta^2}\frac{C_t^{-\delta}}{P_t}E_t\left[\mathcal{I}_t - \left\{\mathcal{I}_t^f\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} + \frac{(\mathcal{E}_{t+1} - E_t\mathcal{E}_{t+1})}{\mathcal{E}_t}\right\}\right] = 0$$

Hence the standard UIP condition is obtained

$$\mathcal{I}_t = \mathcal{I}_t^f\frac{E_t\left[\mathcal{E}_{t+1}\right]}{\mathcal{E}_t}$$

Likewise, equations [2-12] and [2-14] appearing in the main text can be derived

A1.2 Financial intermediaries, firms and financial contracting

Given that $\phi(\varpi)$ is the density function and $\Phi(\varpi)$ is the cumulative distribution function then, the expected return level to the financial intermediaries and firms can be defined (ignoring time and firm subscripts)

Expected return to the financial intermediary

$$ER_{fint} = \int\limits_{\varpi_o}^{\varpi_u} \widetilde{\mathcal{I}}\left[L_h + \mathcal{E}L_f\right]\phi(\varpi)d\varpi + ...$$
$$\int\limits_{\varpi_l}^{\varpi_o} G\varpi L_h^a L_f^{1-a}\phi(\varpi)d\varpi - \lambda\int\limits_{\varpi_l}^{\varpi_o}\varpi G L_h^a L_f^{1-a}\phi(\varpi)d\varpi \geq X$$

In the problem outlined in the text

$$G\varpi_o L_h^a L_f^{1-a} = \widetilde{\mathcal{I}}\left(L_h + \mathcal{E}L_f\right)$$

Then

$$ER_{fint} = GL_h^a L_f^{1-a}\left[\int_{\varpi_o}^{\varpi_u}\varpi_o\phi(\varpi)d\varpi + \int_{\varpi_l}^{\varpi_o}\varpi\phi(\varpi)d\varpi - \lambda\int_{\varpi_l}^{\varpi_o}\varpi\phi(\varpi)d\varpi\right]$$

Here, the following definitions are helpful

Definition 1

$\Gamma(\varpi_o)$ is the gross share of output that goes to the financial intermediary $\Gamma(\varpi_o) = \varpi_o\int_{\varpi_o}^{\varpi_u}\phi(\varpi)d\varpi + \int_{\varpi_l}^{\varpi_o}\varpi\phi(\varpi)d\varpi$. This share $\Gamma(\varpi_o)$ has the following features:

It is increasing in $\varpi_o : \Gamma'(\varpi_{o,t}) = 1 - \Phi(\varpi_{o,t}) > 0$

$\Phi(\varpi_{o,t})$ represents the default probability

Definition 2

$\lambda\Upsilon(\varpi_o)$ is the expected monitoring cost. $\lambda\Upsilon(\varpi_o) = \lambda\int_{\varpi_l}^{\varpi_o}\varpi\phi(\varpi)d\varpi$

It is increasing in $\varpi_o : \lambda\Upsilon'(\varpi_{o,t}) = \lambda\varpi_0\phi(\varpi_{o,t}) > 0$

And by definition: $0 < \Gamma(\varpi_o) - \lambda\Upsilon(\varpi_o) < 1 - \lambda$

Definition 3

$h(\varpi_o)$ is the firm's failure (or hazard) rate defined as $h(\varpi) = \frac{\phi(\varpi)}{1-\Phi(\varpi)}$

Using these definitions, the expected return to the financial intermediary as

$$ER_{fint} = [\Gamma(\varpi_o) - \lambda\Upsilon(\varpi_o)]\, GL_h^a L_f^{1-a}$$

Expected returns to the firm

$$ER_{firm} = \int_{\varpi_o}^{\varpi_u} G\varpi L_h^a L_f^{1-a}\phi(\varpi)d\varpi - \int_{\varpi_o}^{\varpi_u}\widetilde{\mathcal{I}}\left(L_h + \mathcal{E}L_f\right)\phi(\varpi)d\varpi$$

Applying the definition of $\widetilde{\mathcal{I}}\left(L_h + \mathcal{E}L_f\right)$

$$ER_{firm} = \left[\int_{\varpi_o}^{\varpi_u}\varpi\phi(\varpi)d\varpi - \varpi_o\int_{\varpi_o}^{\varpi_u}\phi(\varpi)d\varpi\right] GL_h^a L_f^{1-a}$$

Using the same notation as above

$$ER_{firm} = [1 - \Gamma(\varpi_{o,t})]\, G_t L_{h,t}^a L_{f,t}^{1-a}$$

These expressions for expected returns allow us to formulate the problem in compact form in the main text

The solution in the general case

The Lagrangian function for problem [2-22] in the main text, with associated multiplier ψ is

$$\begin{aligned}\mathcal{L}\left(L_h, L_f, \varpi_o, \psi\right) &= [1 - \Gamma(\varpi_o)]\, GL_h^a L_f^{1-a} + \\ &\quad \psi\left[X - [\Gamma(\varpi_o) - \lambda\Upsilon(\varpi_o)]\, GL_h^a L_f^{1-a} - \zeta\left[D + \mathcal{E}\left(B + B^*\right)\right]\right]\end{aligned} \tag{a1}$$

The f.o.c's are

$$\{1 - \Gamma(\varpi_o) - \psi\left[\Gamma(\varpi_o) - \lambda\Upsilon(\varpi_o)\right]\}\, Ga\frac{L_h^a L_f^{1-a}}{L_h} + \psi\mathcal{I} = 0 \tag{a2}$$

$$\{[1-\Gamma(\varpi_o)] - \psi\,[\Gamma(\varpi_o) - \lambda\Upsilon(\varpi_o)]\}\,(1-a)\,G\frac{L_h^a L_f^{1-a}}{L_f} + \psi\mathcal{E}\mathcal{I}^f = 0 \tag{a3}$$

$$\left[-\Gamma'(\varpi_o) - \psi\left(\Gamma'(\varpi_o) - \lambda\Upsilon'(\varpi_o)\right)\right] G L_h^a L_f^{1-a} = 0 \tag{a4}$$

$$\begin{aligned} &- [\Gamma(\varpi_o) - \lambda\Upsilon(\varpi_o)]\, G L_h^a L_f^{1-a} + \mathcal{I}L_h + \mathcal{E}\mathcal{I}^f L_f + ... \\ \zeta &\left[(I-1)D + \mathcal{E}(I^f-1)\,(B+B^*)\right] + (\mathcal{E} - E_{-1}\mathcal{E})\,(B+B^*) = 0 \end{aligned} \tag{a5}$$

From [a4] the equilibrium value of ψ in terms of the cutoff value ϖ_o is

$$\psi^e = \frac{\Gamma'(\varpi_o)}{\lambda\Upsilon'(\varpi_o) - \Gamma'(\varpi_o)} \tag{a6}$$

Provided $\psi > 0$, dividing [a3] from [a2]:

$$\frac{a}{(1-a)}\frac{L_f}{L_h} = \frac{\mathcal{I}}{\mathcal{E}\mathcal{I}^f}$$

This allows to express both the unconditionally expected product and loan repayment in terms of L_h only

$$G L_h^a L_f^{1-a} = G\left(\frac{1-a}{a}\right)^{1-a}\left(\frac{\mathcal{I}}{\mathcal{E}\mathcal{I}^f}\right)^{1-a} L_h$$

$$\left(\mathcal{I}L_h + \mathcal{E}\mathcal{I}^f L_f\right) = \frac{1}{a}\mathcal{I}L_h$$

Also, making the following definition

Definition 4

The marginal cost of the wholesale firm for producing one unit of its good in the absence of agency costs is defined as:

$$mc = \tfrac{\Lambda}{A}\,(\mathcal{I}w)^a\left(Q\mathcal{I}^f\right)^{1-a} \text{ with } \Lambda = \left(\tfrac{1}{a}\right)^a\left(\tfrac{1}{1-a}\right)^{1-a}$$

Replacing these expressions in [a2] or [a3] (actually one of them is redundant) to get

$$\psi^e = \frac{[1-\Gamma(\varpi_o)]\left(\frac{S^w}{mc}\right)}{[\Gamma(\varpi_o) - \lambda\Upsilon(\varpi_o)]\left(\frac{S^w}{mc}\right) - 1} \tag{a7}$$

In order to characterise the solution, the following assumptions are needed:

- Assumption 1: $1 < \left(\frac{S^w}{mc}\right) < \frac{1}{1-\lambda}$
- Assumption 2: $\varpi h(\varpi)$ is increasing in ϖ.

Solution in the case ϖ follows a uniform distribution

In this case: $\phi(\varpi) = \frac{1}{2\Delta}$ and $\Phi(\varpi) = \frac{1}{2\Delta}(\varpi - 1 + \Delta)$ with $\varpi \in [1-\Delta, 1+\Delta]$. The expressions for $\Gamma(\varpi_o)$ and $\lambda\Upsilon(\varpi_o)$ are given respectively by $\Gamma(\varpi_o) = \frac{1}{2\Delta}\varpi_o(1+\Delta) - \frac{\varpi_o^2}{4\Delta} - \frac{(1-\Delta)^2}{4\Delta}$ and $\lambda\Upsilon(\varpi_o) = \lambda\frac{1}{4\Delta}\left(\varpi_o^2 - (1-\Delta)^2\right)$

The derivatives of the above two functions are given by

$$\Gamma'(\varpi_o) = \frac{1}{2\Delta}\left[1 + \Delta - \varpi_o\right] \text{ and } \lambda\Upsilon'(\varpi_o) = \lambda\frac{1}{2\Delta}\varpi_o$$

Hence, $\Gamma'(\varpi_o) - \lambda\Upsilon'(\varpi_o) = \frac{1}{2\Delta}\left[1 + \Delta - (1+\lambda)\varpi_o\right]$

Using these definitions, the corresponding expressions for the Lagrangian multiplier as outlined in [a6] and [a7] are

$$\psi^e = \frac{1 + \Delta - \varpi_o}{\varpi_o(1+\lambda) - 1 - \Delta} \tag{a8}$$

$$\psi^e = \frac{\left[1 - \Gamma(\varpi_o)\right]\left(\frac{S^w}{mc}\right)}{\left[\Gamma(\varpi_o) - \lambda\Upsilon(\varpi_o)\right]\left(\frac{S^w}{mc}\right) - 1} \tag{a9}$$

From the budget constraint of financial intermediaries Transfers are given by

$$\omega_t^b = \mathcal{I}_t d_{t+1} + Q_t \mathcal{I}_t^f\left(b_{t+1} + b_{t+1}^*\right) - \frac{\mathcal{I}_{t-1}}{\Pi_t} d_t - Q_t \frac{\mathcal{I}_{t-1}^f}{\Pi_t^*}\left(b_t + b_t^*\right) \tag{a10}$$

Budget constraint of wholesale producers

$$\omega_t^h = \left[1 - \Gamma\left(\varpi_{o,t}\right)\right] S_t^w Y_{h,t} \tag{a11}$$

A1.3 Tradeable production and retailers

Export producer firms: They produce the exogenous exportable good $Y_{f,t}$ at zero cost, hence the profits generated are given by: $\Omega_t^f = P_{f,t}Y_{f,t}$. These profits are transferred to households. In real terms

$$\omega_t^f = Q_t Y_{f,t} \tag{a12}$$

It is assumed that this exportable output follows is i.i.d: $Y_{f,t} \sim N(Y_f, \sigma_{yf}^2)$

Retailers: Production of retailers is lower than the expected production of wholesalers due to agency costs

$$\widetilde{Y}_{h,t} = [1 - \lambda \Upsilon (\varpi_{o,t})] Y_{h,t} \tag{a13}$$

Transfers to households

$$\omega_t^r = (S_t - S_t^w) [1 - \lambda \Upsilon (\varpi_{o,t})] Y_{h,t} \tag{a14}$$

A2 Appendix: The log-linearised approximation

A2.1 The Phillips curve

There are two logical steps in the log-linearisation. First, using the definition of the home price index, the derivation of a relationship between home price inflation and the optimal home price ratio, second using the optimality condition [2-34] the determination of an equation for the optimal price ratio

First Step

From the definition of the home price index $P_{h,t}$ in equation [2-11] under the assumed indexation scheme:

$$\widehat{\Pi}_{h,t} = \widehat{\Pi}_{h,t-1} + \frac{1-\gamma}{\gamma} \left[\widehat{\rho}_{h,t}^{op} + \frac{\alpha}{1-\alpha} \widehat{Q}_t \right] \tag{a15}$$

There is a positive relationship between deviations of the optimal price ratio and deviations of current home inflation. A rise in $\widehat{\rho}_{h,t}^{op}$ produces a similar reaction in the domestic price index (and hence it affects home price inflation in the same way). Also, as the domestic price index increases, so does the total consumer price index and hence, the real exchange rate falls for given nominal exchange rates and foreign prices. The increase in both $\widehat{\rho}_{h,t}^{op}$ and $\widehat{\Pi}_{h,t}$, together with the fall in $\widehat{Q}_t$ are governed by equation [a15] just derived. If the probability γ is on the vicinity of 1, then the desired optimal price has a small effect on both domestic inflation and real exchange rates. On the contrary, when γ is close to zero, optimal price changes are strongly transmitted to domestic prices and to the consumer price index.

The sensitivity to the real exchange rate strongly depends on the degree of economic openness (α); when α is low the economy puts little weight on foreign goods consumption and therefore purely domestic price changes have a strong impact over total CPI which at the same time implies larger changes in the real exchange rate

Thus, it seems that low backward-lookingness (high forward-lookingness, i.e.γ low) of price setters and an economy relatively closed (α) is associated with strong real exchange rate movements in response of the set of factors that affect optimal price setting decisions.

Second Step

Taking the optimisation condition of firms:

$$
\begin{aligned}
& E_t\left[\sum_{k=0}^{\infty}(\gamma\beta)^k\,\frac{U_c(C_{t+k})}{P_{t+k}}\left(\frac{P_{h,t+k}}{\left[\frac{P_{h,t-1+k}}{P_{h,t-1}}\right]P^{op}_{h,t}}\right)^{\theta}\left[\frac{P_{h,t-1+k}}{P_{h,t-1}}\right]P^{op}_{h,t}C_{h,t+k}\right] \\
= \quad & E_t\left[\mu\sum_{k=0}^{\infty}(\gamma\beta)^k\,U_c(C_{t+k})\left(\frac{P_{h,t+k}}{\left[\frac{P_{h,t-1+k}}{P_{h,t-1}}\right]P^{op}_{h,t}}\right)^{\theta}C_{h,t+k}S^w_{t+k}\right]
\end{aligned}
$$

The following definitions[31] are used:

Definition 5

$$
\frac{P^{op}_{h,t}}{P_{t+k}}=\frac{P_t}{P_{t+1}}\frac{P_{t+1}}{P_{t+2}}...\frac{P_{t+k-1}}{P_{t+k}}\frac{P^{op}_{h,t}}{P_t}=\frac{1}{\Pi_{t+1}}\frac{1}{\Pi_{t+2}}...\frac{1}{\Pi_{t+k}}\rho^{op}_{h,t}=\frac{\nu^{op}_{h,t}}{\Pi_{t+1.t+k}}
$$

Definition 6

$$
\frac{P_{h,t+k}}{P^{op}_{h,t}}=\frac{P_{h,t+k}}{P_{h,t+k-1}}\frac{P_{h,t+k-1}}{P_{h,t+k-2}}...\frac{P_{h,t+1}}{P_{h,t}}\frac{P_{h,t}/P_t}{P^{op}_{h,t}/P_t}=\Pi_{h,t+k}\Pi_{h,t+k-1}...\Pi_{h,t+1}\frac{S_t}{\rho^{op}_{h,t}}=
\Pi_{h,t+1.t+k}\frac{S_t}{\rho^{op}_{h,t}}
$$

Definition 7

$$
\frac{P_{h,t+k-1}}{P_{h,t-1}}=\frac{P_{h,t}}{P_{h,t-1}}\frac{P_{h,t+1}}{P_{h,t}}...\frac{P_{h,t+k-1}}{P_{h,t+k-2}}=\Pi_{h,t}\Pi_{h,t+1}...\Pi_{h,t+k-1}=\Pi_{h,t.t+k-1}
$$

Then the above optimality condition can be written as:

$$
\begin{aligned}
& E_t\left[\sum_{k=0}^{\infty}(\gamma\beta)^k\,U_c(C_{t+k})\left(\frac{\Pi_{h,t+1.t+k}}{\Pi_{h,t.t+k-1}}\frac{S_t}{\rho^{op}_{h,t}}\right)^{\theta}\frac{[\Pi_{h,t.t+k-1}]}{\Pi_{t+1.t+k}}\rho^{op}_{h,t}C_{h,t+k}\right] \\
= \quad & E_t\left[\mu\sum_{k=0}^{\infty}(\gamma\beta)^k\,U_c(C_{t+k})\left(\frac{\Pi_{h,t+1.t+k}}{\Pi_{h,t.t+k-1}}\frac{S_t}{\rho^{op}_{h,t}}\right)^{\theta}C_{h,t+k}S^w_{t+k}\right]
\end{aligned}
$$

[31] Note that $\Pi_{t.t+k}$ represents the cumulative inflation rate from period t to $t+k$.

Working with the term inside the expectation operator in the left hand side of the above equation and calling it LHS_t.

$$LHS_t = \sum_{k=0}^{\infty} (\gamma\beta)^k U_c(C_{t+k}) \frac{[\Pi_{h,t.t+k-1}]}{\Pi_{t+1.t+k}} \left(\frac{\Pi_{h,t+1.t+k}}{\Pi_{h,t.t+k-1}} \frac{S_t}{\rho_{h,t}^{op}} \right)^{\theta} \rho_{h,t}^{op} C_{h,t+k}$$

The value of this expression in the deterministic steady state is:

$$LHS = \sum_{k=0}^{\infty} (\gamma\beta)^k U_c(C)C_h = \frac{U_c(C)C_h}{1-\gamma\beta}$$

A similar kind of argument can be applied to expression on the expectation operator in the right hand side:

$$RHS_t = \mu \sum_{k=0}^{\infty} (\gamma\beta)^k U_c(C_{t+k}) \left(\frac{\Pi_{h,t+1.t+k}}{\Pi_{h,t.t+k-1}} \frac{S_t}{\rho_{h,t}^{op}} \right)^{\theta} C_{h,t+k} S_{t+k}^{w}$$

In steady state:

$$RHS = \mu \sum_{k=0}^{\infty} (\gamma\beta)^k U_c(C)C_h S^w = \frac{\mu U_c(C)C_h S^w}{1-\gamma\beta}$$

And hence, a standard result emerges:

$$1 = \mu S^w$$

In steady-state monopolistic pricing is embedded in the total domestic price because all firms have monopolistic power. Hence the ratio of optimal domestic prices to overall prices is equal to 1 (the left hand side of the above equation). At the same time, this optimal price ratio has to be equal to a mark-up over marginal cost (the right hand side)

$$\widehat{LHS}_t = (1-\gamma\beta) \left[\begin{array}{c} \sum\limits_{k=0}^{\infty} (\gamma\beta)^k \left(\widehat{C}_{h,t+k} + \widehat{U}_{c,t+k} \right) + \frac{\left\{ \theta\left(\widehat{S}_t - \widehat{\rho}_{h,t}^{op} \right) + \widehat{\rho}_{h,t}^{op} \right\}}{1-\gamma\beta} \\ + \sum\limits_{k=1}^{\infty} (\gamma\beta)^k \sum\limits_{j=1}^{k} \left\{ \widehat{\Pi}_{h,t+j-1} - \widehat{\Pi}_{t+j} + \theta \left(\widehat{\Pi}_{h,t+j} - \widehat{\Pi}_{h,t+j-1} \right) \right\} \end{array} \right]$$

$$\widehat{RHS}_t = (1-\gamma\beta)\left[\begin{array}{c} \sum\limits_{k=0}^{\infty}(\gamma\beta)^k\left(\widehat{U}_{c,t+k}+\widehat{C}_{h,t+k}+\widehat{S^w_{t+k}}\right)+\frac{\theta\left(\widehat{S}_t-\widehat{\rho}^{op}_{h,t}\right)}{1-\gamma\beta}+ \\ \sum\limits_{k=1}^{\infty}\left\{(\gamma\beta)^k\sum\limits_{j=1}^{k}\theta\left(\widehat{\Pi}_{h,t+j}-\widehat{\Pi}_{h,t+j-1}\right)\right\} \end{array}\right]$$

Taking expectations conditional on information at time t both terms and disregarding Jensen's inequality:

$$\frac{\widehat{\rho}^{op}_{h,t}}{1-\gamma\beta} = E_t\left[\sum_{k=0}^{\infty}(\gamma\beta)^k_{t+k}\,S^w_{t+k}\right] - E_t\left[\sum_{k=1}^{\infty}(\gamma\beta)^k\sum_{j=1}^{k}\left(\Pi_{h,t+j-1}-\widehat{\Pi}_{t+j}\right)\right]$$

This is the link between deviations of the optimal price relative to the overall price index and the expected future values of the real marginal cost and future overall inflation rate differentials

$$\frac{\gamma\beta}{(1-\gamma\beta)}E_t\left[\widehat{\rho}^{op}_{h,t+1}\right] = E_t\left[\sum_{k=0}^{\infty}(\gamma\beta)^{k+1}\,\widehat{S}^w_{t+k+1}\right]+E_t\left[\sum_{k=1}^{\infty}(\gamma\beta)^{k+1}\sum_{j=1}^{k}\left(\Pi_{h,t+j}-\widehat{\Pi}_{t+1+j}\right)\right]$$

On the original expression:

$$\frac{1}{(1-\gamma\beta)}\widehat{\rho}^{op}_{h,t} = \widehat{S}^w_t + E_t\left[\sum_{k=1}^{\infty}(\gamma\beta)^k\,\widehat{S}^w_{t+k}\right] + E_t\left[\sum_{k=1}^{\infty}(\gamma\beta)^k\sum_{j=1}^{k}\left(\Pi_{h,t+j-1}-\widehat{\Pi}_{t+j}\right)\right]$$

Then, summing both last expressions adequately:

$$\widehat{\rho}^{op}_{h,t} = (\gamma\beta)\,E_t\left[\widehat{\rho}^{op}_{h,t+1}\right] + (1-\gamma\beta)\,\widehat{S}^w_t + (\gamma\beta)\,E_t\left[\widehat{\Pi}_{t+1}-\widehat{\Pi}_{h,t}\right] \qquad \text{(a16)}$$

Plugging the definition of $\widehat{\rho}^{op}_{h,t}$ found in [a15] and the definition of the overall price index:

$$\widehat{\Pi}_{h,t}-\widehat{\Pi}_{h,t-1} = \beta E_t\left[\widehat{\Pi}_{h,t+1}-\widehat{\Pi}_{h,t}\right] + \frac{1-\gamma}{\gamma}\frac{\alpha\,(1-\gamma\beta)}{1-\alpha}\widehat{Q}_t + \frac{1-\gamma}{\gamma}\,(1-\gamma\beta)\,\widehat{S}^w_t$$

From here:

$$\widehat{\Pi}_{h,t} = \left(\frac{1}{1+\beta}\right)\widehat{\Pi}_{h,t-1} + \left(\frac{\beta}{1+\beta}\right)E_t\left[\widehat{\Pi}_{h,t+1}\right] + \frac{\alpha}{1-\alpha}\frac{1}{(1+\beta)}\frac{1-\gamma}{\gamma}(1-\gamma\beta)\widehat{Q}_t + ...$$
$$\frac{1}{(1+\beta)}\frac{1-\gamma}{\gamma}(1-\gamma\beta)\widehat{S}^w_t$$

Equation [2-58] in the main text is obtained.

A2.2 Consumption dynamics

Log-linearisation of the Euler equation implies

$$\widehat{C}_t = E_t\left[\widehat{C}_{t+1}\right] - \frac{1}{\delta}\left(\widehat{\mathcal{I}}_t - E_t\left[\widehat{\Pi}_{t+1}\right]\right) \tag{a17}$$

It is straightforward to derive the dynamics of consumption of home and foreign goods

$$\widehat{C}_{h,t} = -\eta\widehat{S}_t + \widehat{C}_t$$

$$\widehat{C}_{f,t} = -\eta\widehat{Q}_t + \widehat{C}_t$$

The real prices $\widehat{S}_t$ and $\widehat{Q}_t$ are related through

$$\widehat{S}_t = -\frac{\alpha}{1-\alpha}\widehat{Q}_t \tag{a18}$$

A2.3 Monetary policy

The rule is described as

$$\widehat{\mathcal{I}}_t = \rho\widehat{\mathcal{I}}_{t-1} + (1-\rho)\left[\chi_{\pi h}E_t\left[\widehat{\Pi}_{h,t+1}\right] + \left(\frac{\alpha}{1-\alpha}\right)\chi_\pi\left(\widehat{Q}_t - \widehat{Q}_{t-1}\right) + \chi_y\widehat{\widetilde{Y}}_{h,t}\right] + \xi^m_t$$

Replacing the equilibrium condition for home goods

$$\widehat{\widetilde{Y}}_{h,t} \equiv \widehat{C}_{h,t} = \frac{\alpha\eta}{1-\alpha}\widehat{Q}_t + \widehat{C}_t$$

Allows us to obtain

$$\widehat{\mathcal{I}}_t = \rho\widehat{\mathcal{I}}_{t-1} + R_\pi E_t\left[\widehat{\Pi}_{h,t+1}\right] + R_q\widehat{Q}_t + R_{q1}\widehat{Q}_{t-1} + R_c\widehat{C}_t + \xi_t^m \tag{a19}$$

Where

$$\begin{aligned}
R_\pi &= (1-\rho)\,\chi_{\pi h} \\
R_q &= (1-\rho)\left(\chi_y\tfrac{\alpha\eta}{1-\alpha} + \chi_\pi\tfrac{\alpha}{1-\alpha}\right) \\
R_{q1} &= -(1-\rho)\,\chi_\pi\tfrac{\alpha}{1-\alpha} \\
R_c &= (1-\rho)\,\chi_y
\end{aligned}$$

A2.4 The wholesale real price

In order to derive the dynamics of the wholesale real price, The derivation of the equation for the frictionless marginal cost has to be done first(from A1.2 in Appendix A1)

$$\widehat{mc}_t = a\left(\widehat{\mathcal{I}}_t + \widehat{w}_t\right) + (1-a)\left(\widehat{Q}_t + \widehat{\mathcal{I}}_t^f\right) - \widehat{A}_t \tag{a20}$$

On the other hand, the relationship between the agency cost mark up S_t^w/mc_t and the cutoff level is given by equations [a6] and [a7] in Appendix A

$$\frac{[1-\Gamma(\varpi_{o,t})]\left(\frac{S_t^w}{mc_t}\right)}{[\Gamma(\varpi_{o,t}) - \lambda\Upsilon(\varpi_{o,t})]\left(\frac{S_t^w}{mc_t}\right) - 1} = \frac{\Gamma'(\varpi_o)}{\lambda\Upsilon'(\varpi_o) - \Gamma'(\varpi_o)}$$

The log-linearisation of the above expression takes the form

$$\widehat{S}_t^w - \widehat{mc}_t = \frac{H_2}{H_1}\varpi_o\widehat{\varpi}_{o,t} \tag{a21}$$

Then:

$$\widehat{S}_t^w = a\left(\widehat{\mathcal{I}}_t + \widehat{w}_t\right) + (1-a)\left(\widehat{Q}_t + \widehat{\mathcal{I}}_t^f\right) - \widehat{A}_t + \frac{H_2}{H_1}\varpi_o\widehat{\varpi}_{o,t} \tag{a22}$$

Where

$$H_1 = \frac{1}{[\Gamma(\varpi_o) - \lambda\Upsilon(\varpi_o)]\left(\frac{S^w}{mc}\right) - 1}$$

$$H_2 = \left[\frac{\lambda\Upsilon''(\varpi_o) - \Gamma''(\varpi_o)}{\lambda\Upsilon'(\varpi_o) - \Gamma'(\varpi_o)} - \frac{\Gamma''(\varpi_o)}{\Gamma'(\varpi_o)} - \frac{\Gamma'(\varpi_o)}{1-\Gamma(\varpi_o)} - \frac{[\Gamma'(\varpi_o) - \lambda\Upsilon'(\varpi_o)]\left(\frac{S^w}{mc}\right)}{[\Gamma(\varpi_o) - \lambda\Upsilon(\varpi_o)]\left(\frac{S^w}{mc}\right) - 1}\right]$$

In the special case of a uniform distribution for the idiosyncratic shock

$$\begin{array}{ll} \Gamma(\varpi_o) = \frac{1}{2\Delta}\varpi_o(1+\Delta) - \frac{1}{2}\frac{\varpi_o^2}{2\Delta} - \frac{(1-\Delta)^2}{4\Delta} & \lambda\Upsilon(\varpi_o) = \lambda\frac{1}{4\Delta}\left(\varpi_o^2 - (1-\Delta)^2\right) \\ \Gamma'(\varpi_o) = \frac{1}{2\Delta}\left[1+\Delta-\varpi_o\right] & \lambda\Upsilon'(\varpi_o) = \lambda\frac{1}{2\Delta}\varpi_o \\ \Gamma"(\varpi_o) = \frac{-1}{2\Delta} & \lambda\Upsilon"(\varpi_o) = \frac{\lambda}{2\Delta} \end{array}$$

Defining the following auxiliary variables

$$\begin{array}{l} G_1 = \left[\Gamma(\varpi_o) - \lambda\Upsilon(\varpi_o)\right] S^w = \frac{[\Gamma(\varpi_o)-\lambda\Upsilon(\varpi_o)]}{\mu} \\ G_2 = 1 - \lambda\Upsilon(\varpi_o) \end{array}$$

A2.5 Labour market equilibrium and the real wage rate

The interaction between the labour demand and labour supply gives a market equilibrium representation for wage rates

The supply of labour is

$$\nu\widehat{N}_t + \delta\widehat{C}_t = \widehat{w}_t$$

Labour demand given by

$$\widehat{N}_t = \widehat{l}_{h,t} - \widehat{w}_t$$

Hence

$$\widehat{w}_t = \frac{\nu}{1+\nu}\widehat{l}_{h,t} + \frac{\delta}{1+\nu}\widehat{C}_t$$

A2.6 Loans

From the solution for peso loans - equation [2-27] in the main text

$$l_{h,t} = \frac{ar_{r,t}}{\mathcal{I}_t f_{m,t}}$$

Where, assuming $\zeta_D = \zeta_B = \zeta$

$$r_{r,t} = \zeta\left((\mathcal{I}_t - 1)\frac{d_t}{\Pi_t} + \frac{Q_t}{\Pi_t^*}\left(\mathcal{I}_t^f - 1\right)(b_t + b_t^*)\right) - \left(1 - \frac{E_{t-1}\mathcal{E}_t}{\mathcal{E}_t}\right)\frac{Q_t}{\Pi_t^*}(b_t + b_t^*)$$

And

$$f_{m,t} = \left[\Gamma(\varpi_{o,t}^e) - \lambda\Upsilon(\varpi_{o,t}^e)\right]\left(\frac{S_t^w}{mc_t}\right) - 1$$

Log-linearisation of the above expressions yields

$$\begin{aligned}\widehat{l}_{h,t} &= \left(\tfrac{\mathcal{I}^f}{\mathcal{I}^f-1}\right)\left[a_{dr}\widehat{\mathcal{I}}_t^f + (1-a_{dr})\widehat{\mathcal{I}}_t\right] + (1-a_{dr})\,\widehat{d}_t + a_{dr}\left(\widehat{Q}_t + \widehat{b}_t + \tfrac{b_t^*}{b}\right) + ... \\ &+ \tfrac{a_{dr}}{\zeta(\mathcal{I}^f-1)}\left(\widehat{\mathcal{E}}_t - E_{t-1}\widehat{\mathcal{E}}_t\right) - a_{dr}\widehat{\Pi}_t^* - (1-a_{dr})\,\widehat{\Pi}_t - \widehat{\mathcal{I}}_t - \widehat{f}_{m,t}\end{aligned} \quad \text{(a23)}$$

Where:a_{dr} stands for the asset dollarisation ratio

$a_{dr} = \frac{b}{d+b}$ (with $b^* = 0$ in steady state)

The log-linearised form $\widehat{f}_{m,t}$, considering equation [a21] is given by

$$\widehat{f}_{m,t} = \left[\frac{[\Gamma'(\varpi) - \lambda\Upsilon'(\varpi)]\left(\frac{S^w}{mc}\right)}{G_1/mc - 1} + \frac{G_1/mc}{G_1/mc-1}\frac{H_2}{H_1}\right]\varpi_o\widehat{\varpi}_{o,t} \quad \text{(a24)}$$

Plugging equation [a24] into [a23]

$$\begin{aligned}\widehat{l}_{h,t} &= \left(\tfrac{\mathcal{I}^f}{\mathcal{I}^f-1}\right)\left[a_{dr}\widehat{\mathcal{I}}_t^f + (1-a_{dr})\widehat{\mathcal{I}}_t\right] + (1-a_{dr})\,\widehat{d}_t + a_{dr}\left(\widehat{Q}_t + \widehat{b}_t + \tfrac{b_t^*}{b}\right) + ... \\ &+ \tfrac{a_{dr}}{\zeta(\mathcal{I}^f-1)}\left(\widehat{\mathcal{E}}_t - E_{t-1}\widehat{\mathcal{E}}_t\right) - a_{dr}\widehat{\Pi}_t^* - (1-a_{dr})\,\widehat{\Pi}_t - \widehat{\mathcal{I}}_t - H_3\widehat{\varpi}_{o,t} +\end{aligned} \quad \text{(a25)}$$

Where: $H_3 = \left(\frac{[\Gamma'(\varpi)-\lambda\Upsilon'(\varpi)]\left(\frac{S^w}{mc}\right)}{G_1/mc-1} + \frac{G_1/mc}{G_1/mc-1}\frac{H_2}{H_1}\right)\varpi_o$

Suitable expressions for the real asset values in terms of the loan quantities are needed. Consider the log-liberalisations of equations [2-16] and [2-17]

$$\widehat{l}_{h,t} = \left(\frac{1}{1-\zeta/\Pi^*}\right)\widehat{d}_{t+1} + \left(\frac{1}{1-\zeta/\Pi^*}\right)\frac{\Delta m_{b,t}}{d} - \left(\frac{\zeta/\Pi^*}{1-\zeta/\Pi^*}\right)\left(\widehat{d}_t - \widehat{\Pi}_t\right) \quad \text{(a26)}$$

$$\widehat{l}_{f,t} = \left(\frac{1}{1-\zeta/\Pi^*}\right)\widehat{b}_{t+1} + \left(\frac{1}{1-\zeta/\Pi^*}\right)\frac{b_{t+1}^*}{b} - \left(\frac{\zeta/\Pi^*}{1-\zeta/\Pi^*}\right)\left(\widehat{b}_t + \frac{b_t^*}{b} - \widehat{\Pi}_t^*\right) \quad \text{(a27)}$$

From appendix A, the relationship between peso and dollar loan dynamics is given by

$$\widehat{l}_{f,t} = \widehat{l}_{h,t} + \widehat{\mathcal{I}}_t - \widehat{Q}_t - \widehat{\mathcal{I}}_t^f \quad \text{(a28)}$$

Equation [a25] to [a28] characterise the equilibrium dynamics in the market for loanable funds

A2.7 The foreign sector resource constraint

Log-linearising equation [2-35] and after replacing the expressions for $\widehat{J}_t$ and $\widehat{C}_{f,t}$:

$$0 = \eta C_f \widehat{Q}_t - C_f \widehat{C}_t - J \widehat{b}_{t+1} - \frac{1}{\beta} b_t^* + Y_f \widehat{Y}_{f,t}$$

Replacing the expression for $\widehat{b}_{t+1}$ results in equation [2-73] in the main text of this chapter.

A2.8 Additional equations

The production of wholesale goods is loglinearised as

$$\widehat{Y}_{h,t}^{whole} = \widehat{A}_t - a\widehat{w}_t - (1-a)\widehat{Q}_t + (1-a)\left(\widehat{I}_t - \widehat{I}_{f,t}\right) + \widehat{l}_{h,t}$$

Non-tradable consumption in equilibrium is equal to the net production of goods, this comes from equation [a13] in Appendix A1

$$\ln \widetilde{Y}_{h,t} = \ln \left[1 - \lambda \Upsilon \left(\varpi_{o,t}\right)\right] + \ln Y_{h,t}$$

$$\widehat{\widetilde{Y}}_{h,t}^{whole} = \widehat{Y}_{h,t} - \left[\frac{\lambda \Upsilon' \left(\varpi_o\right) \varpi_o}{1 - \lambda \Upsilon \left(\varpi_o\right)}\right] \widehat{\varpi}_{o,t}$$

Asset dollarisation ratio:

This ratio is defined as:

$$a_{dr,t} = \frac{\left(B_{t+1} + B_{t+1}^*\right) \mathcal{E}_t}{\left(B_{t+1} + B_{t+1}^*\right) \mathcal{E}_t + D_t}$$

which, upon linearisation becomes

$$a_{dr,t} = \frac{d}{d + b + b^*} \left(\widehat{Q}_t - \widehat{d}_{t+1}\right) + \frac{db}{\left(b + b^*\right)\left(d + b + b^*\right)} \left(\widehat{b}_{t+1} + \frac{\widehat{b}_{t+1}^*}{b}\right)$$

Households dollarisation ratio:

$$H_{dr,t} = \frac{B_{t+1} \mathcal{E}_t}{B_{t+1} \mathcal{E}_t + D_t}$$

$$H_{dr,t} = \frac{d}{d+b}\left(\widehat{Q}_t + \widehat{b}_{t+1} - \widehat{d}_{t+1}\right)$$

Real value of assets to households:

$$rva_t = d_{t+1} + b_{t+1}$$

$$\widehat{rva}_t = \frac{d}{d+b}\widehat{d}_{t+1} + \frac{b}{d+b}\widehat{b}_{t+1}$$

Liability dollarisation ratio:

$$L_{dr,t} = \frac{(L_{f,t})\,\mathcal{E}_t}{(L_{f,t})\,\mathcal{E}_t + L_{h,t}}$$

$$\widehat{L_{drt}} = L_{dr}\left(\widehat{Q}_t + \widehat{l}_{f,t} - \widehat{l}_{h,t}\right)$$

CHAPTER 3

INCREASING COMPETITION AND INFLATION NON-LINEARITIES IN SMALL OPEN ECONOMIES

This chapter presents a theoretical framework to incorporate increasing competition effects to the inflation process in a small open economy. This topic is of relevance in terms of policy because the monetary policy transmission mechanism to drive inflation changes in ways which can affect its strength.

Increasing competition in traded goods is part of a recent research interest on globalisation and inflation. The relevance of globalisation stems from the fact that, by increasing openness and competition, it delivers a rising number of cheaper goods to consumers which eventually affects the pricing decision of goods within domestic economies.

Recent literature provides evidence on the importance of increasing openness on inflation. Chen et.al (28) present a theoretical and empirical setup to examine whether more trade has effects over mark-ups and productivity in the Euro area, and find evidence supporting the idea that increased openness has significantly lowered inflation. On the other hand, Dexter et.al (40) show that increasing international trade is important to identify the forces behind Phillips curve equations, and in particular they find that a higher availability of imported consumption goods tends to lower inflation. Kamin et.al (73) study the effect of cheap Chinese exports on inflation in the US and find a modest but significant impact on US import prices. Chapter 3 of the World Economic Outlook [IMF (68)] also finds significant effects of trade openness in the reduction of inflation in a panel of industrialised countries.

Borio and Filardo (18) as well as Mumtaz and Surico (96) find evidence that global factors play a stronger role in explaining the decline of the level of inflation than domestic factors[1].

Therefore, the existing evidence points that increasing globalisation has some bearing on prices[2], suggesting that external pressures have gained more importance in determining inflation whereas domestic factors might have become less important. In virtue of this, a lucid article in The Economist (October 2005) reads,

> *Increased global competition has thus limited the room for firms to pass on higher costs. This makes a nonsense of [...] models of inflation, which virtually ignore globalisation and assume that companies set prices by adding a mark-up over unit costs [...] In reality, when setting prices firms are increasingly likely to be constrained by global competition. Given the price the market will bear, they design and make their products as profitably as they can. As a result, domestic cost pressures [...] no longer lead automatically to higher inflation.*

The aim of this chapter is to provide a formal treatment of this statement and to study its implications for monetary policy. It is worth emphasizing that the analysis here focuses on the effect of globalisation on the markets of final goods; I take the factor markets as given.

The higher degree of competition implied by globalisation affects the balance of domestic and external factors of inflation. From a macroeconomic perspective, this suggests a form of non-linearity or state-dependency of the Phillips curve. The slope of the Phillips curve, i.e. the coefficient associated with real marginal

[1] In Borio and Filardo (18) a concept of global output gap versus individual domestic output gaps is used while in Mumtaz and Surico (96) a factor-augmented VAR is used.

[2] The influential study by Romer (108) shows that more open economies have lower prices. According to time consistency theories of inflation, monetary authorities in open economies have less incentives to inflate. However, Temple (127) finds no strong enough evidence about steeper Phillips curves in more open economies implied but not tested in Romer (108). This chapter does not imply a flatter or steeper slope but a changing one, depending on world inflation swings.

costs affecting inflation (associated in turn to the domestic output gap), can be interpreted as a measure of domestic factors importance in price setting. On the other hand, the partial pass-through coefficient, i.e. that associated with a measure of foreign inflation in the Phillips curve, can be understood as a measure of external influences.

The chapter provides a simple theoretical explanation for these changing weights due to external factors. This is done by modelling demand substitutability between foreign and home goods using translog preferences instead of relying on the widespread constant elasticity of substitution (CES) assumption. CES preferences would be at odds against the backdrop of increasing global competitiveness.[3] The advantage of the translog specification is that it allows the price elasticity of domestically produced goods to depend on foreign price movements. An approach that incorporates competition effects on the demand side is that of Chen et.al (28) where the price elasticity of substitution depends on the number of firms in the supply side.

Within the context of a model for a small open economy, the Phillips curve resulting from the translog assumption implies strong strategic complementarities that render variations in both non-tradable and tradable prices following a world inflation shock. Importantly, the inflation effect is such that the traditional demand channel of monetary policy weakens in favour of the external inflation components. This poses a key challenge for policymakers.

The rest of the chapter is organized as follows. Section 3.1 derives Phillips curves assuming both CES and translog preferences, section 3.2 performs world disinflation experiments with a stylised general equilibrium model to study the effects on the ability of monetary policy to affect inflation and section 3.3 contains the conclusions. Then, Appendix B is introduced to outline the details of the analytical derivations.

[3] Translog preferences on monetary models are introduced in Bergin and Feenstra (11, 12).

3.1 A simple model

Two types of goods – a home, non-tradable good and a world, tradable good – which enter into households consumption basket according to either a CES or translog aggregator. In what follows, lower cases refer to the natural logarithms of the respective upper cases. Also, the h and w superscripts refer to home and world variables, respectively. Variables with no superscript are aggregate figures.

The price of the world good is determined by the law of one price. That is, if P_t^* denotes the international price of the world good and S_t is the nominal exchange rate, then the domestic currency price of this good is $P_t^w = S_t P_t^*$ and its inflation is $\pi_t^w = \Delta s_t + \pi_t^*$.

On the other side, to model stickiness in home prices, I adopt the cost-of-changing-prices setup of Rotemberg (112). This approach consists first in finding desired prices, as if having firms operating in a flexible price environment and then introducing costs of adjustment to move observed prices towards the optimal ones.

Two simplifying assumptions are made for analytical convenience. First, linearity in the home good production function is assumed to shut off the direct demand effect on marginal costs and hence on prices. Since this effect is almost the same under both aggregators, the gains from working with the standard concave production function are negligible to my purposes. Also, provided both preference assumptions do not qualitatively make difference in the sensitive parts of marginal costs, labour demand is assumed as give. Next, real domestic wages are defined in terms of the home price rather than the consumption price. This allows us to draw inflation equations that are easy to handle and interpret, without altering the main conclusions of the model.

3.1.1 Inflation dynamics with a CES aggregator

Under the CES aggregator, the consumption of the home good C_t^h depends negatively on its relative price P_t^h/P_t and positively on aggregate consumption C_t. In logs,

$$c_t^h = \ln(1-\alpha) - \eta(p_t^h - p_t) + c_t \tag{3-1}$$

In this equation, $\eta > 1$ measures the degree of substitutability between the two goods and $\alpha \in (0,1)$ is usually interpreted as the degree of openness.

It is easy to show that if the steady-state relative price P^h/P^w is equal to one, the consumer-based price inflation can be approximated by

$$\pi_t = (1-\alpha)\pi_t^h + \alpha\pi_t^w \tag{3-2}$$

Overall inflation does depend on α but not on η. Thus, under CES preferences, the degree of goods substitutability plays no fundamental role on aggregate dynamics.

Home firms and flexible price setting

The domestic good is produced and sold by a large number of identical monopolistically competitive firms. The focus is on a representative firm. Production Y_t^h is made with a technology that exhibits constant returns on labour. So, for given nominal wages W_t, the total nominal costs are defined by $\text{Costs}(Y_t^h) = W_t Y_t^h$.

Every period, each producer chooses the price P_t^h to maximize profits, subject to the equilibrium condition $Y_t^h = C_t^h$. The optimal price decision reduces to the standard markup pricing over marginal cost. Taking logs to the markup pricing, the working expression appears $p_t^{\text{ces}} = \ln(\mu) + w_t$, where μ is the flexible-price markup $\mu = \frac{\eta}{\eta-1}$. As the chapter notes later, the differentiated expression for p_t^{ces} is a key variable that feeds into the inflation processes and is simply defined as

$$\Delta p_t^{\text{ces}} = \Delta w_t \tag{3-3}$$

Introducing price rigidity

Now suppose that firms cannot set their desired optimal price due to the existence of adjustment costs, so firms maximize profits net of the loss incurred by inducing variability in the price path. After approximating the profit function around the flexible price equilibrium (the optimal price level in the absence of adjustment costs, p_t^{ces}) and introducing adjustment costs, the firms' problem can be reformulated as the following cost minimization program

$$\min_{p_s^h} E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \left\{ \left(p_s^h - p_s^{\text{ces}}\right)^2 + \frac{1}{2c} \left(p_s^h - p_{s-1}^h\right)^2 \right\} \right] \tag{3-4}$$

where $\beta \in (0,1)$ is the firms' discount factor, $c > 0$ is a measure of the speed of adjustment and E_t is the expectation operator.

The optimal price plan obtained by solving (3-4) implies the following inflation process

$$\pi_t^h = \left(\frac{\beta}{1+\beta}\right) E_t \left[\pi_{t+1}^h\right] + \left(\frac{1}{1+\beta}\right) \pi_{t-1}^h + \left(\frac{2c}{1+\beta}\right) \Delta\varpi_t + \xi_t \tag{3-5}$$

where $\Delta\varpi_t$ is the growth of real wages defined as $\varpi_t = w_t - p_t^h$. The term ξ_t is a combination of *iid* forecast errors and is treated as a shock. Crucially, the importance of this derivation is that the shock *per se* does not affect home prices.[4]

Aggregate inflation

It is straightforward to plug (3-5) into the aggregator (3-2) to obtain

$$\begin{aligned} \pi_t \quad = \quad & \left(\tfrac{\beta}{1+\beta}\right) E_t[\pi_{t+1}] + \left(\tfrac{1}{1+\beta}\right) \pi_{t-1} + (1-\alpha) \left(\tfrac{2c}{1+\beta}\right) \Delta\varpi_t + ... \\ & ... + \alpha \left(\pi_t^w - \left(\tfrac{\beta}{1+\beta}\right) E_t[\pi_{t+1}^w] - \left(\tfrac{1}{1+\beta}\right) \pi_{t-1}^w\right) + (1-\alpha)\xi_t \end{aligned} \tag{3-6}$$

The result is a standard hybrid Phillips curve with the following features: (i) it has a dynamic linear homogeneity property implying nominal neutrality in the long run;

[4]In a general equilibrium setting, domestic inflation would respond to changes in $\Delta\varpi_t$ generated, for instance, by a policy reaction to the external shock. This is analyzed in Section 3.2.

(ii) it depends on the real marginal cost defined by $\Delta\varpi_t$ and on the expectation shock ξ_t; and (iii) it depends on the world price inflation. Here, world inflation affects the aggregate inflation just by a direct pass-through effect on import prices.

3.1.2 Inflation dynamics with a translog aggregator

With two consumption goods, the aggregate log price p_t is defined as[5].

$$p_t = (1-\alpha)\,p_t^h + \alpha p_t^w - \frac{\gamma}{2}\left(p_t^w - p_t^h\right)^2 \tag{3-7}$$

In this aggregator, the parameters $\alpha \in (0,1)$ and $\gamma > 0$ are such that both goods enter symmetrically in consumption preferences. Also, homogeneity in the demand functions is imposed. Since the translog can be understood as an augmented CES aggregator (if $\gamma = 0$), the parameter α is the same as in (3-1).

The log of the compensated demand for the domestic good is then

$$c_t^h = \ln(1-\alpha+\gamma q_t) - (p_t^h - p_t) + c_t \tag{3-8}$$

which differs from the demand under the CES specification in an important way: it depends on the relative price of the world good to the home good, $q_t = p_t^w - p_t^h$.

Differencing equation (3-7) leads to aggregate inflation

$$\pi_t = (1-\alpha_t)\pi_t^h + \alpha_t \pi_t^w \tag{3-9}$$

This expression resembles equation (3-2) for the CES case. However, the weights are state-dependent now. In this case $\alpha_t = \alpha - 0.5\gamma\,(q_t + q_{t-1})$, so the inflation process is a changing weighted average of domestic and foreign inflation.[6] As the relative price of the world good falls, q_t turns negative and therefore, world inflation gradually becomes more important to determine overall inflation.

[5]For the general form and properties of the translog aggregator see Bergin and Feenstra (11, 12).

[6]For the shares of either home or world good expenditure to be bounded between zero and one, both γ and q_t should not to be too large. Empirically and for practical purposes, these conditions always hold.

Home firms and flexible price setting

Under translog aggregation, the home firms take into account the fact that the demand for their good depends on the world good price. Then, the expression for the change in prices under a flexible-price scenario becomes

$$\Delta p_t^{\text{trans}} = 0.5(\pi_t^w + \Delta w_t) \tag{3-10}$$

Namely, the optimal price change $\Delta p_t^{\text{trans}}$ is an average of world inflation and marginal costs growth. To prevent consumers from substituting away the consumption of home goods, the home producers will find optimal to follow up the world trend, so a falling world inflation will drag home inflation. In the opposite case, when the world price increases, it is on the interest of the profit-maximizing firms to raise its price against the backdrop of a higher demand for the non-tradable good.

Introducing price rigidity

In the presence of adjustment costs, the domestic inflation process is

$$\begin{aligned} \pi_t^h \;=\; & \left(\tfrac{\beta}{1+\beta+c}\right) E_t[\pi_{t+1}^h] + \left(\tfrac{1}{1+\beta+c}\right) \pi_{t-1}^h + \dots \\ & \dots + \left(\tfrac{c}{1+\beta+c}\right) \pi_t^w + \left(\tfrac{c}{1+\beta+c}\right) \Delta\varpi_t + \zeta_t \end{aligned} \tag{3-11}$$

where ζ_t is an *iid* shock.

This equation is quite different from that in the CES case in (3-5). Particularly, home inflation now depends positively on world inflation.[7]

[7] The degree of dependence is captured by the adjustment cost parameter c. When adjustments costs are high (c is small), the degree of dependence weakens and the situation is close to the CES case.

Aggregate inflation

To aggregate the inflation dynamics (3-11) is plugged into (3-9) to get

$$\begin{aligned}\pi_t &= \left(\tfrac{\beta}{1+\beta+c}\right) E\left[\pi_{t+1}\right] + \left(\tfrac{1}{1+\beta+c}\right)\pi_{t-1} + \left(\tfrac{c}{1+\beta+c}\right)\pi_t^w + ... \\ &\quad ... + (1-\alpha_t)\left(\tfrac{c}{1+\beta+c}\right)\Delta\varpi_t + ... \\ &\quad ... + \alpha_t\left(\pi_t^w - \left(\tfrac{\beta}{1+\beta+c}\right)E_t\left[\pi_{t+1}^w\right] - \left(\tfrac{1+c}{1+\beta+c}\right)\pi_{t-1}^w\right) + (1-\alpha_t)\zeta_t\end{aligned} \tag{3-12}$$

The above Phillips curve not only has the basic properties of (3-6) but also exerts more interesting dynamics. The slope (the coefficient multiplying $\Delta\varpi_t$) depends negatively on α_t, the share of the imported good in the consumption basket, whereas the pass-through coefficient is directly related to α_t. Since α_t increases as the relative price q_t decreases, a drop of external prices (relative to home prices) causes the slope of the Phillips curve to fall and the pass-through coefficient to increase.

This result has an intuitive interpretation. Recall that in an open economy Phillips curve, the slope parameter could be roughly interpreted as a measure of the importance of domestic factors in the formation of prices. A fall in the price of tradables or a rise in the price of non-tradables leads to demand substitution, implying a higher share of tradable goods in domestic expenditure. Under such circumstances, foreign shocks disturbing tradable prices would become more important in equilibrium determination. As a result, the Phillips curve becomes flatter. This is also consistent with the negative correlation between q_t and the pass-through.[8]

Besides and perhaps more importantly, an external shock directly affects home price-setting, magnifying the response of aggregate inflation. Hence, in this case the pass-through effect of world price fluctuations is reinforced by the existence of a further dragging effect.

[8] This result is in line with empirical findings in Goldfjan and Werlang (55).

3.2 Implications for monetary policy

In an environment of global disinflation due to globalisation, monetary policy may lose effectiveness because the fall in the slope of the Phillips curve ends up weakening a channel whereby domestic shocks affect inflation. Regardless of the expectation or exchange rate transmission channels, monetary policy also affects inflation through marginal costs, so the lower the slope is, the weaker this channel becomes. In other words, the effectiveness of monetary policy to affect inflation is inversely related to the dragging effect of world inflation.[9]

The chapter analyses this conjecture formally by including the two inflation equations derived in Section 3.1 into a stylised model with general equilibrium features. Then, the system is shocked to study policy implications.

3.2.1 A stylized general equilibrium for a small open economy

The model is quarterly and consists of six equations. The first is the law of motion of world inflation, which is exogenous and follows a simple AR(1) process,

$$\pi_t^* = (1-\rho)\bar{\pi} + \rho\pi_{t-1}^* + \epsilon_t \text{ with } \epsilon_t \sim iid(0, \sigma_\epsilon^2) \tag{3-13}$$

where $|\rho| < 1$ and $\bar{\pi}$ is the steady-state world inflation rate. The second equation is a Phillips curve derived either for the CES, (3-6), or the translog, (3-12), preferences.

The third equation, (3-14) below, establishes the link between the monetary policy interest rate instrument i_t and the growth of real wages

$$\Delta\varpi_t = E_t\left[\Delta\varpi_{t+1}\right] - b_r(i_t - E_t\left[\pi_{t+1}\right] - r) + \epsilon_{\varpi,t} \tag{3-14}$$

where r is the equilibrium real interest rate (assumed fixed) and $b_r > 0$. Typically this equation is specified in terms of the output gap and is interpreted as an IS

[9] To be more precise, in an open economy the degree of price stickiness is lowered by the presence of imported goods and nominal exchange rate fluctuations. Since real effects of monetary policy shocks occur mainly because of nominal rigidities, the decline of monetary policy effectiveness is a consequence of the decrease of overall price stickiness implied by the model.

curve.[10] However, in the absence of demand effects due to the assumed linearity of the production function, marginal costs solely depend on the real wage rate. The important feature of (3-14) is the negative relation between the real interest rate (gap) and the indicator of marginal cost used in the present setup.

Equation (3-15) describes a monetary policy rule that incorporates a concern about deviations of future expected inflation rates from the target $\bar{\pi}$ and the measure $\Delta\varpi_t$

$$i = (r + \bar{\pi}) + f_p \left(E_t \left[\pi_{t+1} \right] - \bar{\pi} \right) + f_{\varpi} \Delta\varpi_t + \epsilon_{i,t} \tag{3-15}$$

where $f_p > 1$ and $f_{\varpi} > 0$.

Equation (3-16) is the definition of the relative price process

$$q_t = q_{t-1} + 0.25 \left(\pi_{w,t} - \pi_{h,t} \right) \tag{3-16}$$

Finally, exchange rate dynamics is embedded into the model in two alternative forms,

$$\begin{aligned} s_t &= s_{t-1} - \chi q_{t-1} && \text{PPP Model} \\ s_t &= E_t \left[s_{t+1} \right] - 0.25 \left(i_t - \left\{ r + f_p^* \pi_t^* + (1 - f_p^*) \bar{\pi} \right\} \right) && \text{UIP Model} \end{aligned} \tag{3-17}$$

These alternatives are chosen given the fact that there is no consensus about the correct nominal exchange rate model. However, despite our ignorance about how exchange rate dynamics actually evolves, this section shows that the dragging effect is present under both exchange rate specifications.

The two model representations in (3-17) depict two extreme cases regarding the way the exchange rate adjusts to shocks. In the PPP model, the exchange rate moves only insofar as the real exchange rate is misaligned (i.e. whenever there are deviations from purchasing parity or disequilibria in the goods market). The parameter χ measures the speed of nominal exchange rate adjustments to real exchange rate deviations from its steady-state value ($q = 0$). Under this setting,

[10]See Clarida et al. (31) and Smets and Wouters (118).

the exchange rate shows smooth and persistent dynamics. Also there will be no response to shocks on impact, since s_t depends on lagged values of q_t.

In contrast, in the UIP case the spot exchange rate reacts to current and future expected values of the interest rate differential, so that the non-arbitrage condition holds. To prevent from undue jumps in the spot exchange rate, I allow the world nominal interest rate to move in response to world inflation shocks. Insofar as domestic and world interest rates will tend to move in the same direction, the spot exchange rate jump will not be magnified. This means that a falling world inflation will decrease the world interest rate.[11] The UIP model renders a more volatile exchange rate than the PPP model, with a non-zero response on impact.

3.2.2 Calibration

The steady-state real interest rate r is set to 3 percent, which implies a value $\beta = 0.993$. The annual steady-state inflation rate $\bar{\pi}$ equals 2.5 percent which is about the actual inflation target for various countries. For the world inflation process, the autoregressive parameter is assumed to be $\rho = 0.5$ i.e. the effect of a shock dies away in about a year, which roughly corresponds to international empirical estimates.

Regarding the aggregators, for both the CES and translog cases the parameter that measures the degree of openness α is set to 0.35, Cook (33) uses a value of 0.3 while Gali and Monacelli (52) work with 0.4. For the translog case, $\gamma = 1$[12] and the parameter c is set such that the slopes of both Phillips curves are equal in steady state[13].

[11] In fact, the term in braces in equation (3-17) states that the world interest rate is set by the policy rule $i_t^* = (r + \bar{\pi}) + f_p \left(E_t \left[\pi_{t+1}^* \right] - \bar{\pi} \right) = (r + \bar{\pi}) + f_p \rho \left(\pi_t^* - \bar{\pi} \right)$ so $f_p^* = f_p \rho$. With this, I am assuming that both home and foreign policy makers have the same response to inflation deviations.

[12] Qualitatively similar results were obtained for $\gamma = 0.5$ and $\gamma = 2$.

[13] This means that if I set c^{trans} in the translog case, then $2(1 + \beta + c^{\text{trans}})c^{\text{ces}} = c^{\text{trans}}(1 + \beta)$.

On the other hand, in equation (3-14), $b_r = 0.2$, which is about the inverse of the intertemporal elasticity of substitution estimates reported in Smets and Wouters (118). In the policy rule (3-15), the standard values $f_p = 1.5$ and $f_\varpi = 0.5$ are chosen. For the exchange rate PPP equation (3-17) the value $\chi = 0.2$ is used, which implies a half-life of a misalignment of about a year, consistent with the mean group estimates reported in Imbs et al. (67). Finally, for the UIP model for exchange rate the values are $f_p^* = f_p\rho = 0.75$.[11]

3.2.3 The exercise

This part performs two experiments regarding the way a world disinflation may hit an economy initially resting on its steady state. First, a one-period-only disinflation shock ϵ_0 that brings world inflation from $\bar{\pi} = 2.5$ to 1 percent on impact is evaluated. This shock will illustrate the dynamics of the model. Second, world inflation is hit such that the level of world inflation remains at 1 percent for a year (4 quarters).[14] Through this type of persistent shock, the exercise tries to replicate the global disinflation phenomenon. I then compare the responses of the model variables under the two Phillips curve specifications for the PPP model and then repeat the procedure with the UIP model.[15]

The PPP case

The results for inflation are displayed in Figure 1(a) where the first row depicts the responses under the one-quarter shock and the second, under the persistent one-year shock.

[14] To do this, I simulate the model subject to the following history of world inflation shocks: $\epsilon_0 = 1 - \bar{\pi}$, $\epsilon_1 = \epsilon_2 = \epsilon_3 = (1-\rho)\epsilon_0$ and $\epsilon_k = 0$ for $k > 3$.

[15] Additionally, the model was shocked considering different sizes and signs for the shocks in order to exploit the non-linearities in (3-12). Although some differences were found in the responses of the endogenous variables, none of them were sizeable enough to be reported.

The CES specification produces a moderate fall while the translog case generates a deeper drop in aggregate inflation. The home inflation behaviour provides a better insight. It remains basically unperturbed in the CES case while the translog home inflation reacts in the same direction as the world inflation shock. In this case, the falling world inflation drags the home inflation down, a fact that becomes even more apparent under the persistent shock.

Figure 3.1(b) shows the effect on other three key variables for monetary policy: the real wage growth rate, the nominal interest rate and the nominal depreciation. Under both types of shocks, the monetary policy rule calls for a stronger, expansionary response of the policy instrument in the more disinflationary environment, i.e the translog case. The stronger response of interest rates in turn implies a stronger effect upon the real wage growth. It is remarkable that although monetary policy performs in an unduly expansionary way, the effect on inflation is flimsy.

These results are compatible with the two key features observed in the empirical part. Namely, the existence of a positive correlation between the slope of the Phillips curve and the real exchange rate on the one hand but a negative correlation between the pass-through and the real exchange rate on the other hand.

The UIP case

Figure 3.2 displays the responses of the different variables under the UIP model. It is important to recall that the main difference relative to the previous results originates from the response of the nominal exchange rate. The shock causes a strong depreciation on impact because the interest rate cut is anticipated. The nominal exchange rate depreciation more than offsets the shock so, world inflation in domestic currency rises. Under translog preferences, this leads to an increase in the domestic inflation that eventually turns into a higher aggregate inflation.

Nonetheless, after the shock, the translog effect operates and the results are qualitatively the same as the ones obtained in the PPP model. Note, however, that the depreciation on impact under the persistent shock calls for a subsequent real appreciation that magnifies the dragging effect of the disinflation shock.

3.3 Conclusions

This chapter provides a possible theoretical explanation of how the world disinflation might drag down domestic inflation in small open economies. It also argues that globalisation and the increasing availability of cheaper foreign goods make world prices ever more important to the price setting of domestic non-tradable goods.

A simple Phillips curve based on translog preferences (with state-dependent elasticity of substitution) arises within the increasingly competitive environment induced by globalisation. This is due to the fact that the best response from home price setters to avoid losing market share is to follow up the world inflation trend. The usual CES preferences cannot generate this strategic complementarity in price setting. In the disinflation experiments the CES specifications is outperformed by the translog assumption in explaining the importance of competition effects.

The existence of this dragging effect of world inflation has important consequences for monetary policy in small open economies. The domestic interest rate channel of monetary policy loses strength to affect inflation, as the domestic spending in tradable goods increases relative to that of non-tradables. Therefore, if globalisation drives an economy to a low-inflation trap, policymakers may find that inducing a currency depreciation may be the only way out this trap.

A possible extension to this research is to move the model economy towards a more detailed general equilibrium framework. For instance, it is necessary to complement the results in this chapter with the study of the labour market and its relation to marginal costs. In this case, a shock that pushes down the relative price of tradables to non-tradables might expand the demand in the tradable sector and reduce that of the non-tradable sector. This could lower non-tradable sector relative real wages and therefore further reduce home good prices.

B1 Appendix: Flexible price setting

B1.1 The CES case

The consumption basket is given by

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{h,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{w,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{b1}$$

where $C_{h,t}$ and $C_{w,t}$ denote the quantity of domestic and imported goods respectively. Standard intratemporal choice condition for the home good implies

$$C_{h,t} = (1-\alpha)\left(\frac{P_{h,t}}{P_t}\right)^{-\eta} C_t \tag{b2}$$

which is the version in levels of (3-1) in the text. After imposing the conditions $Y_t^h = C_t^h$, $\text{Costs}(Y_t^h) = W_t Y_t^h$ and (b2) we obtain the profit function

$$B(P_t^h) = (1-\alpha)\left(P_t^h - W_t\right)\left(\frac{P_t^h}{P_t}\right)^{-\eta} C_t \tag{b3}$$

which is maximized by $P_t^{\text{ces}} = \left(\frac{\eta}{\eta-1}\right) W_t$, its percent change being equation (3-3).

B1.2 The translog case

Define the log expenditure function as a sum of log aggregate consumption and log consumption-based price index, $g_t = p_t + c_t$, where p_t is defined in (3-7). The demand for the domestic good can be determined using Shephard's Lemma (note that $G_t = P_t C_t$)

$$C_{h,t} = \frac{\partial G_t}{\partial P_{h,t}} = \frac{\partial g_t}{\partial p_{h,t}}\left(\frac{G_t}{P_{h,t}}\right) = (1-\alpha+\gamma q_t)\left(\frac{P_{h,t}}{P_t}\right)^{-1} C_t \tag{b4}$$

which is the version in levels of (3-8). In this case, the profit function is

$$\mathcal{B}(P_{h,t}) = (1-\alpha+\gamma q_t)\left(P_{h,t} - W_t\right)\left(\frac{P_{h,t}}{P_t}\right)^{-1} C_t \tag{b5}$$

The optimal price level is $P_{h,t}^{trans} = \left(1 + \frac{1-\alpha+\gamma q_t}{\gamma}\right) W_t$ which cannot be solved explicitly since q_t depends on $p_{h,t}^{trans} = \ln(P_{h,t}^{trans})$. However we can approximate the optimal price by taking logs and using the fact that $\ln(1+x) \simeq x$ for a small number x. Then,

$$p_{h,t}^{trans} = \frac{1-\alpha}{2\gamma} + \frac{p_{w,t}}{2} + \frac{w_t}{2} \tag{b6}$$

After differentiation of (b6) we get equation (3-10) in the text.

B2 Appendix: Price setting with adjustment costs

The quadratic approximation of the profit function around its desired price level $P^*_{h,t}$ is

$$\mathcal{B}\left(P_{h,t}\right) \simeq \mathcal{B}\left(P^*_{h,t}\right) + \mathcal{B}'\left(P^*_{h,t}\right)\left(P_{h,t} - P^*_{h,t}\right) - c_a\left(p_{h,t} - p^*_{h,t}\right)^2 \tag{b7}$$

where $c_a = -\frac{1}{2}\mathcal{B}''\left(P^*_{h,t}\right)\left(P^*_{h,t}\right)^{-2} > 0$. The linear term disappears due to the optimality of $P^*_{h,t}$ while the constant term is irrelevant to the firms' decision-making.

The adjustment costs for price changes are given by $c_b\left(p_{h,t} - p_{h,t-1}\right)^2$. Therefore the firm pricing problem can be reformulated as an overall minimization problem (Assuming $c \neq 0$)

$$\min_{\{p_{h,s}\}_{s=t}^{\infty}} E_t\left[\sum_{s=t}^{\infty}\beta^{s-t}\left\{\left(p_{h,s} - p^*_{h,s}\right)^2 + \frac{1}{2c}\left(p_{h,s} - p_{h,s-1}\right)^2\right\}\right] \tag{b8}$$

subject to the transversality condition

$\lim_{s\to\infty}\beta^s E_t\left[\left(p_{h,s} - p^*_{h,s}\right) + \frac{1}{2c}\left(p_{h,s} - p_{h,s-1}\right)\right] = 0,$

where $\frac{1}{2c} = \frac{c_b}{c_a} > 0$. The Euler equation in period t is

$$2c\left(E_t p_{h,t} - E_t p^*_{h,t}\right) + \left(E_t p_{h,t} - E_t p_{h,t-1}\right) - \beta\left(E_t p_{h,t+1} - E_t p_{h,t}\right) = 0 \tag{b9}$$

where E_t is the expectation operator conditional on the information up to and including period t (when the pricing decision is made), hence $E_t p_{h,s} = p_{h,s}$ for $s \leq t$. Due to rational expectations, the price forecasting error based on this period information set is an *iid* sequence of random variable, $E_t p_{h,t+1} - p_{h,t+1} = \frac{2c}{\beta}\xi_{t+1}$. Replacing and reordering yields

$$\left[1 - \frac{(2c+1+\beta)}{\beta}L + \frac{1}{\beta}L^2\right]p_{h,t+1} = -\left(\frac{2c}{\beta}\right)\left(p^*_{h,t} + \xi_{t+1}\right) \tag{b10}$$

where L is the lag operator. The lag-polynomial in brackets can be factorized as $(1 - \mu_1 L)(1 - \mu_2 L)$, with $\mu_1 + \mu_2 = (2c + 1 + \beta)\beta^{-1}$ and $\mu_1\mu_2 = \beta^{-1}$. The roots are such that $0 < \mu_1 < 1$ and $\mu_2 > \beta^{-1}$, with $\mu \equiv \mu_1$ being the sable root. It can be verified that $\beta\mu^2 + 1 - 2c\mu = (1+\beta)\mu$.

Replacing the factorized polynomial and multiplying by $(1 - \mu_2 L)^{-1}$ allows us to get

$$(1 - \mu L)\,p_{h,t+1} = -\left(1 - \mu_2 L\right)^{-1}\left(\frac{2c}{\beta}\right)\left(p^*_{h,t} + \xi_{t+1}\right) \tag{b11}$$

After expanding $(1 - \mu_2 L)^{-1}$ the expression becomes

$$p_{h,t} = \mu p_{h,t-1} + \frac{2c}{\beta}E_t\left[\sum_{j=t}^{\infty}(\beta\mu)^{j-t+1}p^*_{h,j}\right] \tag{b12}$$

This is the key solution to the problem. To derive an inflation process, we forward (b12) one period, take time t expectations and multiply by $\beta\mu$,

$$\beta\mu E_t\left[p_{h,t+1}\right] = \beta\mu^2 p_{h,t} + \frac{2c}{\beta}E_t\left[\sum_{j=t+1}^{\infty}(\beta\mu)^{j-t+1}p^*_{h,j}\right] \tag{b13}$$

Then, taking (b12) out of (b13) and rearranging leads

$$\left(1+\beta\mu^2\right)\pi_{h,t} = \beta\mu_1 E_t\pi_{h,t+1} + \mu\pi_{h,t-1} + 2c\mu\Delta p^*_{h,t} + iid \tag{b14}$$

where the optimal price $p^*_{h,t}$ depends on the consumption aggregator used.

B2.1 The CES case

According to equation (3-3), $\Delta p^*_{h,t} = \Delta p^{ces}_{h,t} = \Delta w_t = \Delta\varpi_t + \pi_{h,t}$, so (b14) becomes

$$(1+\beta\mu^2 - 2c\mu)\pi_{h,t} = \beta\mu E_t\pi_{h,t+1} + \mu\pi_{h,t-1} + 2c\mu\Delta\varpi_t + \beta\mu\varepsilon_t \tag{b15}$$

Considering that $\beta\mu^2 + 1 - 2c\mu = (1+\beta)\mu$, allows us to obtain equation (3-5) in the main text that does not depend on μ as the production function is assumed to be linear.

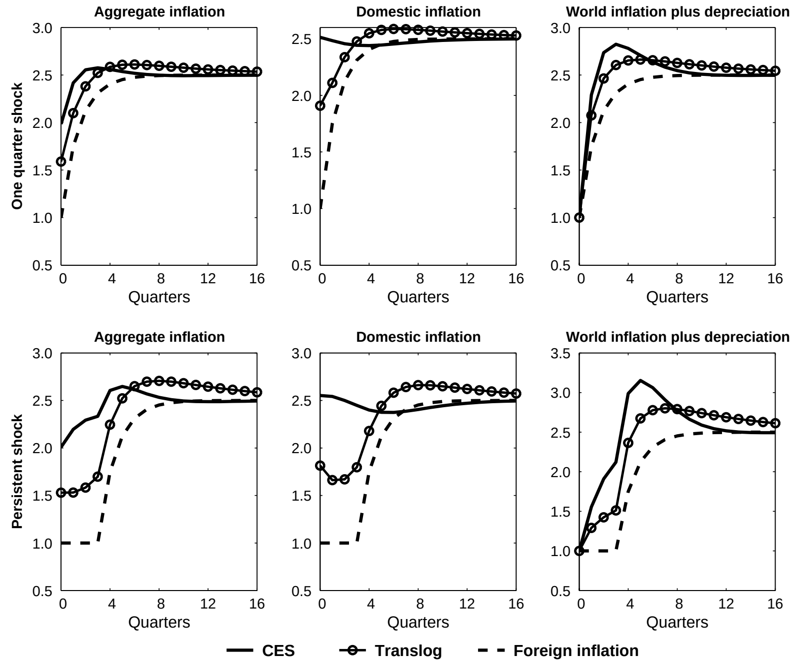
B2.2 The translog case

Now we replace $\Delta p^*_{h,t} = \Delta p^{trans}_{h,t} = \frac{1}{2}\pi_{w,t} + \frac{1}{2}\Delta w_t = \frac{1}{2}\pi_{w,t} + \frac{1}{2}\Delta\varpi_t + \frac{1}{2}\pi_{h,t}$ into (b14) and get

$$(1+\beta\mu^2 - c\mu)\pi_{h,t} = \beta\mu E_t\pi_{h,t+1} + \mu\pi_{h,t-1} + c\mu\pi_{w,t} + c\mu\Delta\varpi_t + \beta\mu\varepsilon_t \tag{b16}$$

Again, the equality $\beta\mu^2 + 1 - 2c\mu = (1+\beta)\mu$ allows to simplify equation (b16) into (3-11). Then, after aggregating with (3-9) we get the time-varying Phillips curve (3-12).

(a) Inflation



(b) Other variables

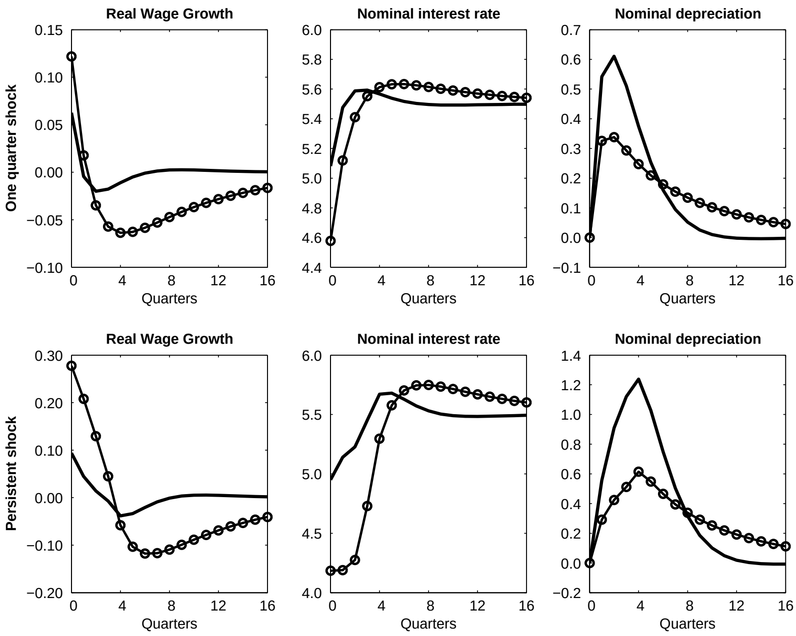
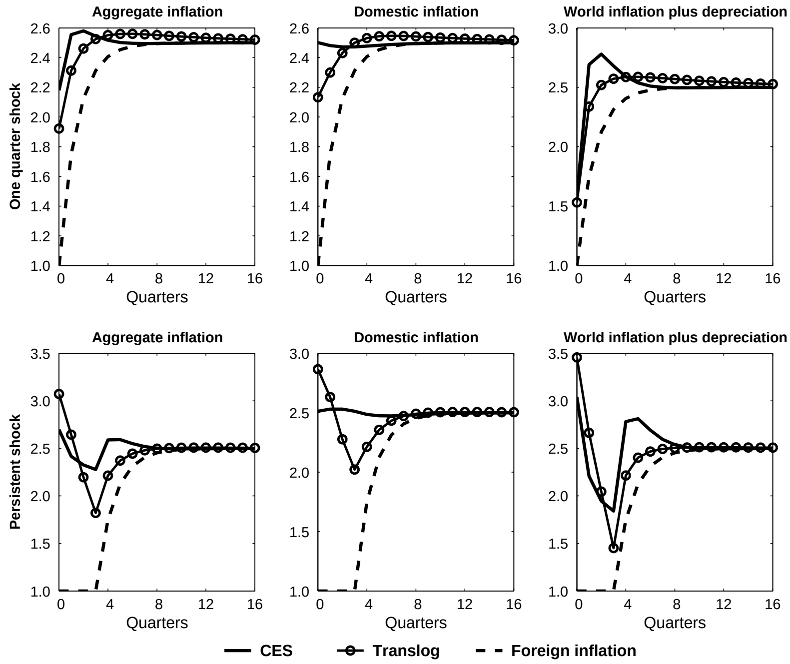


Figure 3.1: Responses to world inflation shocks, PPP case.

(a) Inflation



(b) Other variables

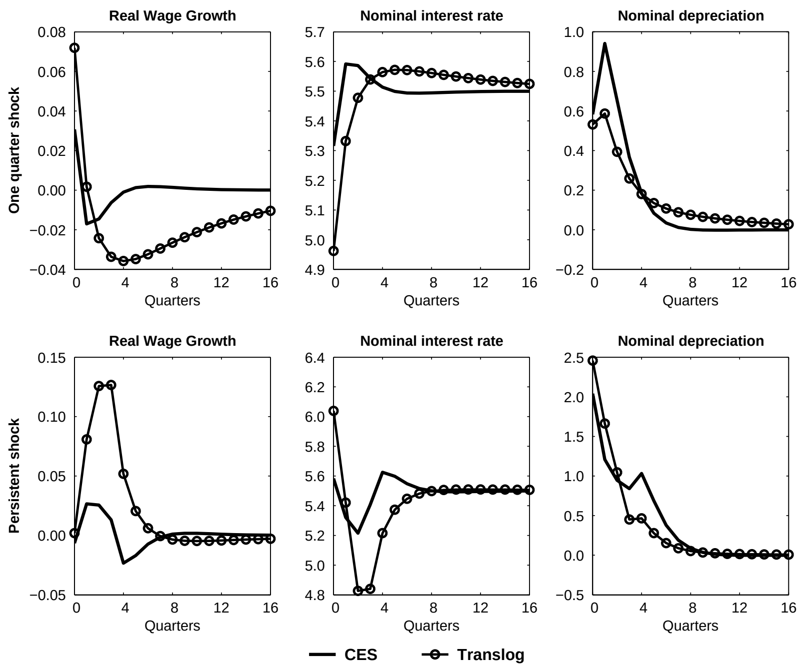


Figure 3.2: Responses to world inflation shocks, UIP case.

CHAPTER 4

THE INFLATION FORECAST AND POLICY MAKERS JUDGEMENTS

The purpose of this chapter is to build a methodology to obtain marginal inflation density forecasts. The approach lies in estimating a parametric inflation density forecast where uncertainty, asymmetry and central tendency profiles are brought about mainly from the exogenous variables through the use of a forecasting model. The estimated parameters are combined with policy maker's prior views through an explicit Bayesian approach. The prior views encompass all other factors of risk and uncertainty that may strike at the inflation forecast. The formulation postulates that policy makers weigh their confidence in both; their prior beliefs and their model via a utility function of the sorts used in information-theoretic design as proposed by Lindley (85).

This is a more realistic way of combining prior beliefs with model-based density forecasts. The approach is particularly important, in environments where macroeconometric formulation of models is hindered by measurement errors and poor data availability[1]. Nevertheless, even in stable and developed countries with quality data rich environments, prior inputs are essential.

The chapter proceeds as follows, section 4.1 outlines the density forecast framework, section 4.2 illustrates the methodology with a simple example for forecasting Peruvian inflation. Finally, section 4.3 draws the conclusions. Appendix C contains technical derivations.

[1] Which is the case in most emerging-market economies.

4.1 Density forecast framework

The forecasting literature has recently turned attention from point forecasts towards density forecasts[2]. The reasons to provide complete representations of probability distribution lie on the failure of the certainty equivalence principle in a world overwhelmingly characterised by asymmetric risks. This failure is particularly relevant in the fields of financial risk management and modern monetary policy where decision theory plays a substantial role.

Some central banks like the Federal Reserve in the USA or the Bank of England have a long tradition in macroeconomic point forecasts. Only recently, the Bank of England has pioneered the presentation of density forecast by means of fan charts. Since then, a number of ITers publish a density forecast with varying degrees of detail. About twelve out of twenty-one ITers regularly publish a fan chart[3].

Leading density forecast central banks[4] have favoured the use of specific parametric methods to construct their density forecasts. The parameters governing the forecast densities directly control for uncertainty and the asymmetry of the distribution. This is the approach taken in the next subsection.

The role of models in the forecasting process has been recognised by academics and practitioners alike. In a recent survey of central banks practising IT (Schmidt-Hebbel and Tapia (115)), basically all 20 surveyed banks refer the use of some kind of model. The key evidence is that most central banks, specially ITers endorse the use of one core forecasting model that helps centre policy discussions within the bank.

[2]See Diebold et.al (42) and Tay and Wallis (125).

[3]In alphabetical order: Brazil, Chile, Colombia, Hungary, Iceland, Israel, Norway, Peru, South Africa, South Korea, Sweden, Thailand, and United Kingdom. In Fracasso et.al (47), Israel appears as not publishing a Fan chat because the inflation report under assessment exceptionally did not have one. Colombia is not considered in their sample due to "limited information".

[4]For the Bank of England the references are Briton et.al (20) and Wallis (135). For the Riksbank the reference is Blix and Sellin (17).

But the use of models in forecasting does not mean that subjective views are filtered out in the forecasting process. In fact, a factor also mentioned in the Schmidt-Hebbel and Tapia (115) survey is that in most central banks; the published forecasts are a *"balanced combination"* of technical forecasts and decision makers' views. The practice of including subjective approaches to macroeconomic forecasting within central banks is also recognised in Sims (117) and Goodhart (56).

Papers like Hall and Mitchell (61, 62, 63) propose a powerful method for forecast combination that allows the incorporation of subjective forecasts. The combination procedure in these works hinges on forecast error minimisation. Instead, this chapter proposes a methodology based on the interaction between the policy decision maker and the producers of forecasts. The central bank staff implements simulations using a forecasting model and policy makers input priors about parameters that reflect uncertainty, risk balance and baseline forecast values.

4.1.1 The parametric density forecast

The economists at a central bank own a forecasting process at time t about future realizations of an inflation sequence up to horizon H. This sequence is generated by a forecasting model and is denoted by[5] $\{\widehat{\pi}_s\}_{s=t+1}^{H}$

$$\pi_s = M_s(Y_t, X_t; \theta, I_t) \text{ for } s = t+1, t+2...H \tag{4-1}$$

In equation [4-1], Y_t denotes the known history of endogenous macroeconomic variables y_t in the model (including inflation π_t) Formally

$$Y_t = \{y_t, ..., y_{t-n}\}$$

This model-based forecast is conditional upon various factors that can be controlled in the process. These factors are X_t, θ, and I_t. The first one denotes the history and likely future realizations of the exogenous variables: $X_t =$

[5] Hatted variables are forecasts of either exogenous or endogenous variables. In the case of the instrument setting, it refers to the stance assumed by the policy maker.

$\{x_{t-n}, .., x_t, \widehat{x}_{t+1} ... \widehat{x}_{t+H} ...\}$, θ denotes the set of parameters that describes the particular economic model in use. This set of parameters is included in the broader set of parameters Θ that defines model uncertainty. The last factor, I_t denotes the history as well as the particular stance of the central bank instrument assumed at time t: $I_t = \left\{ i_{t-n,} ..., i_t, \widehat{i}_{t+1}, ... \right\}$.

Model M is general enough and need not be explicit as it may correspond to a rational expectations equilibrium solution. I make the following definition:

Definition 1 A central forecast[6] is an inflation sequence $\{\widehat{\pi}_{c,s}\}_{s=t+1}^{H}$ obtained by conditioning the model to: (a) the most likely sequence of exogenous variables within the forecast horizon $\{\widehat{x}_{c,s}\}_{s=t+1}^{H}$, (b) parameter values θ_c and (c) the monetary policy instrument setting $I_{c,t}$

Also, the economists at this central bank have to provide a technical assessment of risk and uncertainty about the inflation forecast. This relies on random realizations of exogenous variables from suitably calibrated probability distribution functions. The random draws take into account a chosen parameterised standard deviation, skewness and the "most-likely" sequence of exogenous variables. The parameters of these probability density functions reflect the technical staff historical estimates as well as subjective and the informed view of sectorial experts.

Among the distinct probability density functions that are suitable to perform random draws are the Beta and the Split Normal. The latter is used intensively in Blix and Sellin (17), Briton et.al (20) and Vega (130). These two types of distributions are useful because their parameters illustrate the distributional characteristics that matter most in a density forecast; a central point; a measure of dispersion and skewness.

Performing simulated histories of exogenous variables within the forecast horizon allows to determine alternative trajectories of inflation. Evaluated at each

[6] In this definition, the subscript c denotes both central forecasts and assumed central values.

point in time within the forecast horizon, the distinct inflation points originated in the simulations can be hypothesised as coming from a generic probability function. The determination of the explicit form of this inflation forecast probability distribution function (pdf) resulting from this exercise is hindered by two facts (a) the mapping from the exogenous variables to inflation imply a solution like [4-1] which can be highly non-linear and (b) even if we manage to find the exact form of the distribution; its communication to the policy makers would not be easy. A way to circumvent the problem is to assume a parametric form for the distribution function that can serve two purposes; be a good approximation to the true pdf and allow a communication strategy that can easily be grasped by the policy maker. A good candidate for the assumed pdf is the Split Normal, given that its parameters can be easily communicated in terms of straightforward balance of risks.

Definition 2 A model-based parametric inflation density forecast is a sequence of parameters $\left\{\widehat{\Lambda}_{c,s}\right\}_{s=t+1}^{H}$ describing a probability density function of the inflation forecast at every point in time s.

The parameters involved in the above definition can be obtained by a likelihood estimation procedure assuming the Split Normal distribution and using the simulated data.

Henceforth, I am going to concentrate on a relevant horizon H and drop time subscripts. After S number of stochastic simulations on the exogenous variables are performed, I obtain a mapping from data conditional on the model parameters and the instrument setting to object ω

$$\left(\{X_t\}_{j=1}^{S}, Y_t; \Theta, I_t\right) \rightarrow \omega \tag{4-2}$$

The variable ω contains the elements upon which both, the econometrist and the policy maker care about[7]. Namely, the inflation forecast at horizon H, and the three parameters that underlie policy discussions. I group these three parameters

[7]Observe that the parameter Θ as well as the instrument may remain constant or vary exogenously along the simulations.

in the vector $\Lambda = (m, \sigma^2, \gamma)$, with m being the modal point[8], σ^2 the uncertainty measure and γ the skewness of the distribution of the inflation forecast. These three parameters precisely define the Split Normal $SN\left(m, \sigma^2, \gamma\right)$. This distribution collapses into a Normal $N\left(m, \sigma^2\right)$ whenever the skewness parameter γ equals zero. The γ parameter varies on the range $\langle -1, 1\rangle$ and is closely linked to the balance of risks made at central banks (see Appendix B). Specifying ω in a compact way

$$\omega = \left(\left\{\pi\right\}_{j=1}^{S}, \Lambda\right) \tag{4-3}$$

I treat ω parameters in a Bayesian context[9] and characterise its posterior probability density conditional on all the information acquired after performing S simulations of the model conditional on all the given factors Ω (observe that S itself is a conditioning factor)

$$p(\omega|\ \Omega) = p(\Lambda|\ \Omega)p(\left\{\pi\right\}_{j=1}^{S} \mid \Lambda, \Omega) \tag{4-4}$$

where

Ω is the given information set: $\Omega = \left\{\left\{X_t\right\}_{j=1}^{S}, Y_t; \Theta, I_t\right\}$

$p(\Lambda|\ \Omega)$ is the prior density elicited by the policy maker, and

$p\left(\left\{\pi\right\}_{j=1}^{S} \mid \Lambda, \Omega\right)$ is the probability of the simulated inflation forecast data given the information Ω and the parameters of interest. The likelihood principle implies that this probability is equivalent to the likelihood of the parameters given the simulated data and the information set: $L(\Lambda|\ \left\{\pi\right\}_{j=1}^{S}, \Omega)$.

My interest is to draw probabilistic judgments of the inflation forecast distribution, thus I need to find the posterior conditional distribution of the parameters. This is achieved by making use of Bayes theorem

$$p(\Lambda|\ \left\{\pi\right\}_{j=1}^{S}, \Omega) = \frac{p(\Lambda|\ \Omega)L(\Lambda \mid\ \left\{\pi\right\}_{j=1}^{S}, \Omega)}{p(\left\{\pi\right\}_{j=1}^{S} \mid \Omega)} \tag{4-5}$$

[8] When risks are asymmetric, there are three measures of tendency that central banks can look at. In practice, central banks tend to pay more attention to modal points (See Goodhart (56) and Vega (130))

[9] Namely, it is itself a random variable.

Given that both, the prior distribution and the likelihood are known parameterised functions, the posterior distribution can be explicitly determined. Furthermore, by holding constant a pair of parameters, I can determine the conditional distribution of the remaining parameter.

4.1.2 Elicitation of the priors as the outcome of policy makers views

Upon learning the outcome of the model-based density forecast, policy-makers views are formed. These views take into account other forms of uncertainties not included in the forecast; model-uncertainty, measurement errors or any other type. It remains an internal operational task the way to optimally extract these views and to translate them into tractable distribution functions.

For my purpose, I assume that the first subjective view is that the three parameters are independent random variables, so that the joint prior is

$$p(\Lambda|\ \Omega) = p(\sigma^2|\ \Omega)p(\gamma|\ \Omega)p(m|\ \Omega) \tag{4-6}$$

Prior for uncertainty parameter σ^2

Following the literature (Bauwens et.al (8)), I assume that σ^2 is driven by the Inverted Gamma-2 distribution $iG_2(b,a)$. The parameters (a,b) are chosen by the policy maker. This distribution has support $\langle 0,\infty\rangle$ and its parameters can be specified using the two moments and the mode of the distribution as guidelines

$$E(\sigma^2|.) \equiv \frac{b}{a-2} \text{ for } a>2$$

and

$$V(\sigma^2|.) \equiv \frac{2}{a-4}\left(\frac{b}{a-2}\right)^2 \text{ for } a>4$$

while the mode is

$$mode(\sigma^2|.) \equiv \frac{b}{a+2}$$

It can be observed that the mean is always higher than the mode, by taking the estimated $\widehat{\sigma}_c^2$ in Definition 2 as a reference point, possible values of b and a can be evaluated by weighing the resulting mode, mean and variance.

Prior for skewness parameter γ

For the skewness parameter I need a distribution with bounded support. I assume a slight transformation of a Beta distribution and name it as $\widetilde{B}(c, d)$. This allows γ to vary in the interval $\langle -1, 1\rangle$. To do this, I make a transformation of a random variable z lying on the interval $\langle 0, 1\rangle$ with a Beta distribution $B(c, d)$ (the transformation applied is $\gamma = 2z - 1$). The first two moments are defined as

$$E(\gamma|\Omega) \equiv \frac{c-d}{c+d}$$

and

$$V(\gamma|\Omega) \equiv \frac{4cd}{(c+d+1)(c+d)^2}$$

with mode

$$mode(\gamma|\Omega) \equiv \frac{c-d}{c+d-2}$$

Prior for mode parameter m

I impose a non-informative uniform distribution for the mode m

$$p(m|a_m, b_m) \propto \text{constant} \tag{4-7}$$

4.1.3 The posterior distribution

Given the Split Normal likelihood assumption[10], the kernel of the joint posterior distribution of the three parameters of interest is

$$p\left(\Lambda \middle|\ \pi_{t+H}, \Omega\right) \propto \left(\tfrac{\gamma+1}{2}\right)^{c-1}\left(\tfrac{1-\gamma}{2}\right)^{d-1}\left(\sigma^2\right)^{\frac{-(a+2)}{2}} e^{\left(\frac{-b}{2\sigma^2}\right)} \left(\frac{\left(\sigma^2\right)^{-\frac{1}{2}}}{\sqrt{1-\gamma}+\sqrt{1+\gamma}}\right)^N e^{\left(\frac{-1}{2}\left\{\sum_{i=1}^{S_1}\left(\frac{\pi_{t+H}-m}{\sigma\sqrt{1-\gamma}}\right)^2+\sum_{i=S_1+1}^{S}\left(\frac{\pi_{t+H}-m}{\sigma\sqrt{1+\gamma}}\right)^2\right\}\right)} \quad (4\text{-}8)$$

From this joint pdf, I obtain the posterior conditional distribution of σ^2. As expected, this distribution is also an Inverted Gamma-2

$$p\left(\sigma^2 \middle|\ \gamma, m, \pi_{t+H}, \Omega\right) \propto \left(\sigma^2\right)^{\frac{-(a+N+2)}{2}} e^{\left(\frac{-(\vartheta(m,\gamma)+b)}{2\sigma^2}\right)} \quad (4\text{-}9)$$

where $\vartheta(m,\gamma) = \left\{\sum_{i=1}^{S_1}\left(\frac{(\pi_{t+H}-m)^2}{1-\gamma}\right) + \sum_{i=S_1+1}^{S}\left(\frac{(\pi_{t+H}-m)^2}{1+\gamma}\right)\right\}$

The other two relevant conditional distributions are given by

$$p\left(m \middle|\ \gamma, \sigma^2, \pi_{t+H}, \Omega\right) \propto e^{\left(\frac{-1}{2\sigma^2}\left\{\sum_{i=1}^{S_1}\left(\frac{(\pi_{t+H}-m)^2}{(1-\gamma)}\right)+\sum_{i=S_1+1}^{S}\left(\frac{(\pi_{t+H}-m)^2}{(1+\gamma)}\right)\right\}\right)} \quad (4\text{-}10)$$

and

$$p\left(\gamma \middle| m\ , \sigma^2, \pi_{t+H}, \Omega\right) \propto \left(\tfrac{\gamma+1}{2}\right)^{c-1}\left(\tfrac{1-\gamma}{2}\right)^{d-1}\left(\frac{2}{\sqrt{1-\gamma}+\sqrt{1+\gamma}}\right)^S e^{\left(\frac{-1}{2\sigma^2}\left\{\sum_{i=1}^{S_1}\left(\frac{(\pi_{t+H}-m)^2}{1-\gamma}\right)+\sum_{i=S_1+1}^{S}\left(\frac{(\pi_{t+H}-m)^2}{1+\gamma}\right)\right\}\right)} \quad (4\text{-}11)$$

The conjugacy of the prior distribution of σ^2 allows to express the conditional moments from the posterior from an inverted gamma distribution $iG_2(\frac{a+S}{2}, \frac{2}{\vartheta(m,\gamma)+b})$. The moments are

$$E(\sigma^2|.) \equiv \frac{\frac{a+S}{2}}{\frac{2}{\vartheta(m,\gamma)+b}-2} \text{ for } \frac{2}{\vartheta(m,\gamma)+b} > 2$$

[10]See Appendix [B] for details about this distribution.

and

$$V(\sigma^2|.) \equiv \frac{2}{\frac{2}{\vartheta(m,\gamma)+b} - 4} \left(\frac{\frac{a+S}{2}}{\frac{2}{\vartheta(m,\gamma)+b} - 2} \right)^2 \text{ for } \frac{2}{\vartheta(m,\gamma)+b} > 4$$

while the mode is

$$mode(\sigma^2|.) \equiv \frac{\frac{a+S}{2}}{\frac{2}{\vartheta(m,\gamma)+b} + 2}$$

From this explicit representation, we observe that as the sample size increases, the posterior mean and mode would collapse to the model-based estimates. In that case, the prior view has a small effect on the posterior outcome. In an econometric estimation environment, a larger sample size is always good because it improves the model-based information. The context here is rather different. It is based on the willingness of a Bayesian policy maker to learn about the properties of the inflation forecast from a general perspective instead of a non-Bayesian econometrist who wants to learn the properties of its model-based forecast.

4.1.4 The choice of sample size as an information theoretic design problem

In the proposed methology, the sample size S is a choice variable as well. If a high enough sample size is considered, the prior view of the policy makers becomes useless. On the other hand, if the sample size is small, then the model-based estimation turns less accurate so that the simulation experiment becomes informationally poor.

Policy makers need to weigh the information provided by the model and the prior beliefs they may hold. In practice, this process appears complex as it is bound to the subjective beliefs of the policy makers coupled with out-of-model information they might have.

Under this circumstance, the information-theoretic approach [11] common in

[11] This view was proposed by Lindley (85). Applications of Lindley's approach are found for

the field of "experimental design" seems plausible. What is the experiment the policy maker performs? In my view, the experiment consists in updating the policy makers prior beliefs about the inflation forecast modal point, uncertainty and risks by means of a forecasting model provided by econometricians. The outcome of this updating process depends crucially on the simulation sample size under evaluation. The choice of sample size S is made so that policy makers maximise their expected utility resulting from the experiment. In other words

$$S^* = \arg\max_{S} \{KL(S) - \lambda S\} \tag{4-12}$$

This expected utility of experimentation with sample size S depends on two factors; a) the Kullback-Leibler (KL hereon) divergence between the posterior and prior distribution of the parameters $KL(S)$ and b) the linear loss function λS. The KL number provides the value of the information provided by the forecasting model under use[12]. The loss term is rationalised by the unwillingness to disregard their own priors[13]. So, as the sample size increases, the prior of the policy maker is downweighted and thus reduces the utility of a policy maker who considers her priors are indeed somewhat important. In this case, the utility parameter λ is the degree of importance of the prior in the overall utility function [14].

The KL divergence number is defined as

$$KL(S) = \int_{\Lambda} \int_{\Pi} \log \left[\frac{p(\Lambda|\Pi, S)}{p(\Lambda)} \right] p(\Pi, \Lambda|S) d\Pi d\Lambda \tag{4-13}$$

Where $\Pi = \{\pi\}_{j=1}^{S}$ is the simulated inflation data of size S, $p(\Lambda)$ is the prior distribution of the parameters and $p(\Lambda|\Pi, S)$ is the posterior distribution.

example in Ryan (114), Clyde (32), Parmigiani and Berry (100), Chaloner and Verdinelli (27) and Muller and Parmigiani (95). Most of these applications belong to the design of clinical experiments.

[12] KL(S) is increasing in S and concave. See Lindley (85).

[13] These priors might indeed not be correct ex post and as studied by Bigio and Vega (16), they are shaped by their fears and uncertainties about the driving forces in the economy.

[14] λ can also be interpreted as the inverse of policy makers credibility on the model.

4.2 An example

In order to provide an example, I use a simple ad-hoc univariate model[15] for quarterly inflation estimated using ordinary least squares [16]. I run the inflation rate at quarter t against the following regressors: the exchange rate depreciation at lag 3 (Δe_{t-3}), GDP growth at lag 2 (g_{t-2}), the mean interbank interest rate at lag 1 (i_{t-1}), the mean three months Libor rate at lag 3 (i^*_{t-3}) and the trade growth at lag 4 (Δtot_{t-4}).

$$\begin{array}{lllllllll} \pi_t = & 0.69\pi_{t-1} & +0.24\Delta e_{t-1} & +0.23 g_{t-2} & -0.30 i_{t-1} & +0.55 i^*_{t-3} & +0.06\Delta tot_{t-4} & +\varepsilon_t \\ & (9.23) & (3.58) & (3.06) & (-1.95) & (1.72) & (1.70) & \end{array} \quad (4\text{-}14)$$

The estimation[17] is carried out using data from the first quarter of 1994 to the second quarter of 2003. Except for lagged inflation, all the variables on the right-hand side are considered as exogenous. Hence, to start the density forecast I need to construct a baseline scenario and uncertainty and risk profiles for the set of exogenous variables: $(g_t, i_t, \Delta e_t, i^*_t, \Delta tot_t)$. In particular, I assume the following distributions

Exogenous variable	**Balance of risk**	**Distribution**	**Mode**	σ^2
Libor rate	upside 70%	Split normal	3.57	1.2
Nominal exchange rate depreciation	upside 55%	Split normal	0.00	10.6
GDP growth	upside 60%	Split normal	3.90	8.3
Terms of trade growth	neutral	Normal	0.5	4.9

Table 4.1: Distributional assumptions for exogenous variables at the end of the forecast horizon.

In Figure [4.1] I show the historical, central scenario and the 90 per cent

[15]The univariate model is used only to ease the exposition. In practice, structural models as the ones developed in Luque and Vega (88) and Llosa et.al (86) for Peru should be used.

[16]I use data from Peru. The Central Bank of Peru has recently adopted the Inflation Targeting framework (January 2002).

[17]In equation [4-14] the lag structure minimises the sum of squared residuals. As usual, the t-values are in parenthesis.

central prediction interval for the exogenous variables along the forecast periods. The asymmetry as well as the uncertainty increases linearly until it reaches the values specified in Table [4.1]. In each forecast period, I also consider random realizations of the unforecastable shock ε_t, drawn from a normal distribution $N(0, 0.3)$. This last feature is important for two reasons; first it makes the first-period-ahead inflation forecast random given that all the exogenous determinants are predetermined for this horizon. Second, it allows the inflation uncertainty to increase even in the absence of uncertainty in the exogenous variables.

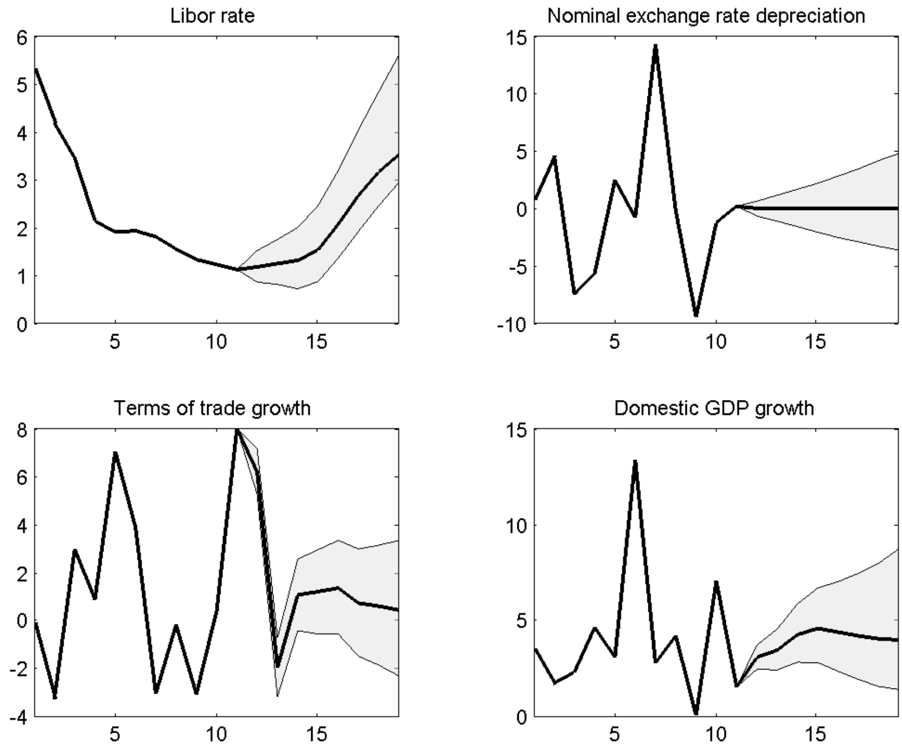


Figure 4.1: forecast interval and modal forecast.

To complete the conditioning factors, I also need to assume a particular monetary policy setting within the forecast horizon. In this case, I consider a constant-interest-rate forecast with the rate kept at 2.75 per cent during the forecast period.

The inflation density forecast is then achieved by estimating the parameters of an assumed split normal distribution $SN(m, \sigma^2, \gamma)$ for the simulated sample of size[18] S_T for each forecast period.

An important conclusion emerges from this exercise: Notwithstanding that the exchange rate depreciation, GDP growth and the Libor rate all show considerable asymmetry[19] (especially at the end of the forecast horizon). There is no build up of asymmetry in both inflation measures; the quarterly and the year-on-year rate. In Figure [4.2] I show the estimated densities at each of the eight forecast periods along with the estimated parameters; mode m, σ^2 and γ. The gamma parameter is close to zero in all periods.

The reasons why the increasingly asymmetric nature of exogenous variables does not pass on to inflation are twofold; the lag structure and the interplay between the variability versus asymmetric forces. Regarding the lag structure, as the asymmetric exogenous variables affect quarterly inflation with some lags, then full asymmetry is not transferred to inflation at the end of the forecast horizon. As of the relation variability/asymmetry, it is know that when the variability of inflation increases the asymmetric forces that affect inflation are dampened (see for example Blix and Sellin 2000). Inflation variability does grow because the exogenous variability increases linearly and because the persistent nature of inflation (as it depends strongly on its own lags) exacerbates all the sources of uncertainty in inflation, even the one that corresponds to the inflation shock itself.

The estimated mode from the simulations are quite different from the one computed using only the central scenario values of exogenous variables (the modes). There is an upward bias (See Figure [4.3]) in both the quarterly inflation and the year-on-year inflation. The reason is that at the end of the forecast horizon, the

[18]In this step, the sample size S_T can be as large as possible. The objective here is to get the most accurate distributional representation originated from the forecasting model alone.

[19]In Figure [4.3] in the appendix the estimated means differs from the modes of the asymmetric exogenous variables. In Figure [4.4] the asymmetry parameter γ for the exogenous variables becomes larger towards the end of the forecast horizon.

simulated distribution is quite symmetric around the mean. The mean is the central tendency that is preserved in both the point and the density forecast.

Once the results of the simulation are known, I proceed to introduce the information provided by the policy maker. To do this, I concentrate in forecast horizon $H = 8$. I need to assume a prior distribution for the set of parameters $\Lambda = (m, \sigma^2, \gamma)$. I take the distributional assumptions outlined in Section 4.1.2. Namely, the mode follows a uniform distribution; $m \sim U(m_{low}, m_{high})$ with parameters $m_{low} = -0.22$ and $m_{high} = 5.78$ such that the distribution is centred in an year-on-year inflation rate of 2.78 percent.

The uncertainty parameter follows an inverted gamma-2 distribution; $\sigma^2 \sim iG_2(b, a)$. In order to find the parameters, I can consider that the estimated $\widehat{\sigma}^2$ from the simulation step is too low. Policy makers may consider that there are other factors that necessarily drive forecast uncertainty to a higher level. For example they can assume that $E_{prior}(\sigma^2) = 1.95$ and the $mode_{prior}(\sigma^2) = 1.8$. This implies the corresponding parameters $(a, b) = (38, 72)$

The asymmetry parameter follows a beta type of distribution considered in the previous section; $\gamma \sim \widetilde{B}(c, d)$. In this case, policy makers believe that the inflation forecast at horizon H will have an upside risk, as opposed to the model-based case which considers a slight downside risk. Let's suppose that the mean prior gamma is $E_{prior}(\gamma) = 0.3$ (which is close to a 60 percent upside risk) and that they believe about this asymmetry quite strongly $V_{prior}(\gamma) \approx 0.006$. This implies parameter values $(c, d) = (92.857, 50)$.

Before combining the prior information given by the policy maker, it is necessary to establish the sample size to use in the Bayesian procedure. This sample size is obtained from solving the problem in equation [4-12)]. The calculation of the utility measure requires to get the KL divergence number via some numerical integration procedure. In Appendix D, I follow Ryan (114) by using a MCMC estimation. The optimal value S^* depends on the parameter λ. A small λ about

0.007 is related to a large sample size (about 164), a "large" λ, around 0.017, generates a sample size of about 33. Hence, I interpret the sample size as the weight of confidence in the prior. In this example, I assume $\lambda = 0.01$. Therefore the optimal sample size is $S^* = 120$ (see Figure [4.7])

Next, I sample from the Bayesian conditional posterior distributions. The corresponding mean values are shown in Table (4.2) and a graphical representation of conditional posterior against prior distributions is shown in Figure [4.8]).

		Prior Mean	**Model-based Estimation**	**Posterior Mean**
Mode	m	2.78	3.03	2.75
Uncertainty	σ^2	1.95	0.83	0.78
Risk	γ	0.30	-0.05	0.34

Table 4.2: Mean values of the parameters under the prior distribution, the ML estimation and the posterior distributions.

The distributional means of the prior and posterior turn out to be very close to each other except for the uncertainty parameter σ^2. The model-based estimate of uncertainty is low while the prior belief about this parameter is too high relative to the model. Also, the model-based estimate of the asymmetry is slightly negative (-0.05) as opposed to the prior belief which posits a strong upside risk ($\gamma = 0.3$). It seems that the model strongly rejects the combination of high levels of uncertainty and sizeable upside risks as defined by the prior. Thus, in terms of the posterior, the prior view of the policy makers is taken into account for the modal and the risk forecasts, yet it is not the case for the uncertainty parameter estimation. In fact, the posterior calculation hints that a lower uncertainty seems necessary in order to "make room" for a high value of asymmetry provided in the likelihood[20].

[20] This particular result does not always hold. It depends on the relative prior variances of the parameters. If policy makers are highly confident about their prior view of uncertainty, then the distributional variance is in fact very low. Therefore, the resulting posterior might be closer to this posterior.

4.3 Conclusion

This chapter contributes to the understanding of how central banks do forecasts in the context of monetary policy making. It posits attention to Bayesian policy makers who hold or develop prior views on key features of the inflation density forecast. The decision makers interact with the technical staff in charged of running the macroeconomic model-based density forecast.

In reality, neither the prior views nor the model-based forecast are per se true. Prior views are subject to human imperfection while models are always false. However, policy makers in fact use both types of inputs to make quantitative inference about their forecasts.

In the present approach, policy makers weigh both the prior view and the information provided by the model via a utility function advocated in Information Theory. The utility function considers the trade-off between the importance of policy makers priors and the "faith" on the core forecasting model. If the model is given full "faith" then priors are irrelevant and viceversa.

A further application of the approach developed in this chapter would be to reverse engineer this density forecasting process to extract λ and thus to find a metric on the amount of the importance of judgement relative to pure objective model-based forecasts.

C1 Appendix: Inflation forecast : Prior distributions

C1.1 Prior for σ^2

In the main text I assume that σ^2 follows an Inverted Gamma 2 distribution with parameters (b, a)

$$p(\sigma^2|.) = \left(\Gamma(\tfrac{a}{2})\left(\tfrac{2}{b}\right)^{\frac{a}{2}}\right)^{-1} \left(\sigma^2\right)^{\frac{-(a+2)}{2}} e^{\left(-\frac{b}{2\sigma^2}\right)} \tag{c1}$$

where

$$E(\sigma^2|.) \equiv \frac{b}{a-2} \text{ for } a > 2$$

and

$$V(\sigma^2|.) \equiv \frac{2}{a-4}\left(\frac{b}{a-2}\right)^2 \text{ for } a > 4$$

while the mode is

$$mode(\sigma^2|.) \equiv \frac{b}{a+2}$$

C1.2 Prior for γ

I start assuming that a random variable z follows a Beta distribution with parameters (c, d)

$$g(z|c,d) = \tfrac{\Gamma(c+d)}{\Gamma(c)\Gamma(d)} z^{c-1} (1-z)^{d-1} \text{ for } 0 < z < 1$$

with

$$E(z|\Omega) \equiv \frac{c}{c+d}$$

and

$$V(z|\Omega) \equiv \frac{cd}{(c+d+1)(c+d)^2}$$

with mode

$$mode(z|\Omega) \equiv \frac{c-1}{c+d-2}$$

Then I define γ in terms of the following transformation

$$\gamma = 2z - 1$$

Hence, the prior distribution of γ can be expressed as

$$p(\gamma|.) = g(z(\gamma)|c,d)\left|\frac{d}{d\gamma}z\right|$$

As a result, the prior distribution for γ is

$$p(\gamma|.) = \frac{\Gamma(c+d)}{\Gamma(c)\Gamma(d)}\left[\frac{1+\gamma}{2}\right]^{c-1}\left[\frac{1-\gamma}{2}\right]^{d-1} \text{ for } -1 < \gamma < 1 \tag{c2}$$

C1.3 Prior for m

As for m, I assume a uniform, non-informative prior. The exact determination for this prior is inconsequential for the Bayesian posterior sampling. However, it is used in the sample size determination given that I require sampling from the priors. Hence, I assume $m \sim Uniform(m_{low}, m_{high})$

$$p(m|.) = \frac{1}{m_{high} - m_{low}} \text{ for } m_{low} < m < m_{high} \tag{c3}$$

C2 Appendix: Model-based density simulation and estimation

C2.1 Fitting the simulated data

I define a Split Normal pdf for the data with parameters (m, σ^2, γ) in the following way

$$f(x; m, \sigma^2, \gamma) = \begin{cases} \frac{2}{\sqrt{\sigma^2}\left(\sqrt{1-\gamma}+\sqrt{1+\gamma}\right)}\phi\left(\frac{x-m}{\sqrt{\sigma^2(1-\gamma)}}\right) & \text{if } x < m \\ \frac{2}{\sqrt{\sigma^2}\left(\sqrt{1-\gamma}+\sqrt{1+\gamma}\right)}\phi\left(\frac{x-m}{\sqrt{\sigma^2(1+\gamma)}}\right) & \text{otherwise} \end{cases}$$

Where $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2}$

Given a simulated sample $\{x\}_{s=1}^{S_T}$; I can sort the data in ascending order and split the ordered data $\{\widetilde{x}\}_{s=1}^{S_T}$ in two sub-samples

$$\mathcal{S}_1 = \{\widetilde{x}_i \mid \widetilde{x}_i < m\}$$

$$\mathcal{S}_2 = \{\widetilde{x}_i \mid \widetilde{x}_i \geq m\}$$

Let S_1 and $S_T - S_1$ be the number of elements of $\mathcal{S}_1$ and $\mathcal{S}_2$ respectively. Then the likelihood of the sample is given by

$$L(x; m, \sigma^2, \gamma) = \left(\frac{2/\sqrt{2\pi\sigma^2}}{\sqrt{1-\gamma}+\sqrt{1+\gamma}}\right)^{S_T} e^{\left(\frac{-1}{2}\left\{\sum_{i=1}^{S_1}\left(\frac{x-m}{\sqrt{\sigma^2(1-\gamma)}}\right)^2 + \sum_{i=S_1+1}^{S_T}\left(\frac{x-m}{\sqrt{\sigma^2(1+\gamma)}}\right)^2\right\}\right)} \tag{c4}$$

while the log-likelihood is

$$\mathcal{L}(x;m,\sigma^2,\gamma) = S_T \log\left(\frac{2/\left(2\pi\sigma^2\right)^{\frac{1}{2}}}{\sqrt{1-\gamma}+\sqrt{1+\gamma}}\right) - \frac{1}{2}\sum_{i=1}^{S_1}\left(\frac{x-m}{\sqrt{\sigma^2(1-\gamma)}}\right)^2 - \ldots$$
$$\frac{1}{2}\sum_{i=S_1+1}^{S_T}\left(\frac{x-m}{\sqrt{\sigma^2(1+\gamma)}}\right)^2$$

and further expressed as

$$\begin{aligned}\mathcal{L}(x;m,\sigma^2,\gamma) &= S_T \log\left(2/\sqrt{2\pi}\right) - \frac{S_T}{2}\log\left(\sigma^2\right) - S_T\log\left(\sqrt{1-\gamma}+\sqrt{1+\gamma}\right) \\ &\quad -\frac{1}{2\sigma^2}\sum_{i=1}^{S_1}\left(\frac{x-m}{\sqrt{1-\gamma}}\right)^2 - \frac{1}{2\sigma^2}\sum_{i=S_1+1}^{S_T}\left(\frac{x-m}{\sqrt{1+\gamma}}\right)^2\end{aligned}$$

Estimation of the parameters requires the computation of the firs order conditions of the likelihood problem:

For the uncertainty parameter I have

$$\frac{\partial}{\partial\sigma^2}\mathcal{L}(x;\sigma^2,\gamma,m) = -\frac{S_T}{2\sigma^2} + \frac{1}{2\left(\sigma^2\right)^2}\sum_{i=1}^{S_1}\left(\frac{x-m}{\sqrt{1-\gamma}}\right)^2 + \frac{1}{2\left(\sigma^2\right)^2}\sum_{i=S_1+1}^{S_T}\left(\frac{x-m}{\sqrt{1+\gamma}}\right)^2 = 0$$

$$\widehat{\sigma}^2 = \frac{1}{S_T\left(1-\widehat{\gamma}\right)}\sum_{i=1}^{S_1}\left(x-\widehat{m}\right)^2 + \frac{1}{S_T\left(1+\widehat{\gamma}\right)}\sum_{i=S_1+1}^{S_T}\left(x-\widehat{m}\right)^2 \tag{c5}$$

For the risk parameter I find

$$\begin{aligned}\frac{\partial}{\partial\gamma}\mathcal{L}(x;\sigma^2,\gamma,m) &= -\frac{S_T/2}{\sqrt{1-\gamma}+\sqrt{1+\gamma}}\left(\frac{\sqrt{1-\gamma}-\sqrt{1+\gamma}}{\sqrt{1+\gamma}\sqrt{1-\gamma}}\right) \\ &\quad -\frac{1}{2\sigma^2\left(1-\gamma\right)^2}\sum_{i=1}^{S_1}\left(x-m\right)^2 + \frac{1}{2\sigma^2\left(1+\gamma\right)^2}\sum_{i=S_1+1}^{S_T}\left(x-m\right)^2\end{aligned}$$

which collapses to the following equation in the estimators

$$\frac{\sum_{i=S_1+1}^{S_T}\left(x-\widehat{m}\right)^2}{\left(1+\widehat{\gamma}\right)^2} - \frac{\sum_{i=1}^{S_1}\left(x-\widehat{m}\right)^2}{\left(1-\widehat{\gamma}\right)^2} = \frac{\widehat{\sigma}^2 S_T\sqrt{1+\widehat{\gamma}}\sqrt{1-\widehat{\gamma}}}{1}\left(\frac{\sqrt{1-\widehat{\gamma}}-\sqrt{1+\widehat{\gamma}}}{\sqrt{1-\widehat{\gamma}}+\sqrt{1+\widehat{\gamma}}}\right) \tag{c6}$$

For the mode parameter I have the expression

$$\frac{\partial}{\partial m}\mathcal{L}(x;\sigma^2,\gamma,m)=\frac{\sum\limits_{i=1}^{S_1}(x-m)}{\sigma^2\left(1-\gamma\right)^2}+\frac{\sum\limits_{i=S_1+1}^{S_T}(x-m)}{\sigma^2\left(1+\gamma\right)^2}=0$$

$$\frac{\sum\limits_{i=1}^{S_1}x-\sum\limits_{i=1}^{S_1}m}{\left(1-\gamma\right)^2}+\frac{\sum\limits_{i=S_1+1}^{S_T}x-\sum\limits_{i=S_1+1}^{S_T}m}{\left(1+\gamma\right)^2}=0$$

which is simplified as

$$\frac{\sum\limits_{i=1}^{S_1}x}{\left(1-\widehat{\gamma}\right)^2}+\frac{\sum\limits_{i=S_1+1}^{S_T}x}{\left(1+\widehat{\gamma}\right)^2}=\left[\frac{S_1}{\left(1-\widehat{\gamma}\right)^2}+\frac{S_T-S_1}{\left(1+\widehat{\gamma}\right)^2}\right]\widehat{m} \tag{c7}$$

Equations [c5], [c6] and [c7] are solved to find the triple of MLE parameters $\widehat{\Lambda}=\left(\widehat{m},\widehat{\sigma}^2,\widehat{\gamma}\right)$.

C3 Appendix: The posterior distribution

C3.1 The joint posterior

The joint posterior distribution is given by

$$\begin{aligned} p\left(\Lambda|\ \{\pi\},\Omega\right) \ \propto \ & \left(\tfrac{\gamma+1}{2}\right)^{c-1}\left(\tfrac{1-\gamma}{2}\right)^{d-1}\left(\sigma^2\right)^{\frac{-(a+2)}{2}}e^{\left(\frac{-b}{2\sigma^2}\right)} \\ & \left(\frac{\left(\sigma^2\right)^{-\frac{1}{2}}}{\sqrt{1-\gamma}+\sqrt{1+\gamma}}\right)^{S_T}e^{\left(\frac{-1}{2}\left\{\sum\limits_{i=1}^{S_1}\left(\frac{\pi_i-m}{\sqrt{\sigma^2(1-\gamma)}}\right)^2+\sum\limits_{i=S_1+1}^{S^*}\left(\frac{\pi_i-m}{\sqrt{\sigma^2(1+\gamma)}}\right)^2\right\}\right)} \end{aligned}$$

In the main text I have determined the conditional posterior distribution kernel of σ^2 by fixing the other two parameters

$$p\left(\sigma^2|\ \gamma,m,\{\pi_H\},\Omega\right)\propto\left(\sigma^2\right)^{\frac{-\left(a+S^*+2\right)}{2}}e^{\left(-\frac{\vartheta(m,\gamma;S^*)+b}{2\sigma^2}\right)} \tag{c8}$$

where $\vartheta(m,\gamma;S^*)=\left\{\sum\limits_{i=1}^{S_1}\left(\frac{(\pi_i-m)^2}{1-\gamma}\right)+\sum\limits_{i=1+S_1}^{S^*}\left(\frac{(\pi_i-m)^2}{1+\gamma}\right)\right\}$

The implied posterior distribution of σ^2 is also a iG2 distribution with parameters: $(\vartheta(m,\gamma;S^*)+b, a+S^*)$. From here, it is straightforward to determine the mean of σ^2 under the conditional posterior

$$E\left(\sigma^2|\ .\right)_{post} = \frac{\vartheta(m,\gamma;S^*)+b}{a+S^*-2}$$

On the other hand, the prior mean was given by

$$E\left(\sigma^2|\ .\right)_{prior} = \frac{b}{a-2}$$

While the fitted estimation with simulated data according to equation [c5] gives

$$\widehat{\sigma}^2|_{\ fit} = \frac{\vartheta(m,\gamma;S^*)}{S^*}$$

$$\text{If } E\left(\sigma^2|\ .\right)_{prior} > \widehat{\sigma}^2|_{\ fit}\ \text{, then } E\left(\sigma^2|\ .\right)_{prior} > E\left(\sigma^2|\ .\right)_{post} > \widehat{\sigma}^2|_{\ fit}$$

Starting with the conditional: $\frac{b}{a-2} > \frac{\vartheta(m,\gamma;S)}{S}$:

(a) I post multiply and add the term $b(a-2)$ in both sides:

$$bS + b(a-2) > (a-2)\,\vartheta(m,\gamma;S) + b(a-2)$$

$$b\,(a+S-2) > (a-2)\,(\vartheta(m,\gamma;S)+b)$$

$$\frac{b}{a-2} > \frac{\vartheta(m,\gamma;S)+b}{a+S-2}$$

(b) I post multiply and add the term $\vartheta(m,\gamma;S)S$ in both sides:

$$bS + \vartheta(m,\gamma;S)S > (a-2)\,\vartheta(m,\gamma;S) + \vartheta(m,\gamma;S)S$$

$$S\,(b+\vartheta(m,\gamma;S)) > \vartheta(m,\gamma;S)\,(a-2+S)$$

$$\frac{b+\vartheta(m,\gamma;S)}{a-2+S} > \frac{\vartheta(m,\gamma;S)}{S}$$

The basic result when $E\left(\sigma^2|\ .\right)_{prior} > \widehat{\sigma}^2|_{\ fit}$ is:

$$\frac{b}{a-2} > \frac{b+\vartheta(m,\gamma;S^*)}{a-2+S^*} > \frac{\vartheta(m,\gamma;S^*)}{S^*}$$

As the simulated sample becomes large, the procedure implemented here downweights the prior; and thus the simulated variance does not differ from the posterior.

The other two relevant conditional distributions are given by

$$p\left(m|\ \gamma,\sigma^2,\pi_{t+H},\Omega\right) \propto e^{\left(\frac{-1}{2\sigma^2}\left\{\sum_{i=1}^{S_1}\left(\frac{(\pi_{t+H}-m)^2}{(1-\gamma)}\right)+\sum_{i=S_1+1}^{S}\left(\frac{(\pi_{t+H}-m)^2}{(1+\gamma)}\right)\right\}\right)} \qquad \text{(c9)}$$

and

$$p\left(\gamma|m\ ,\sigma^2,\pi_{t+H},\Omega\right) \ \propto \ \left(\frac{\gamma+1}{2}\right)^{c-1}\left(\frac{1-\gamma}{2}\right)^{d-1}\left(\frac{2}{\sqrt{1-\gamma}+\sqrt{1+\gamma}}\right)^{S} \quad \text{(c10)}$$
$$e^{\left(\frac{-1}{2\sigma^2}\left\{\sum_{i=1}^{S_1}\left(\frac{(\pi_{t+H}-m)^2}{1-\gamma}\right)+\sum_{i=S_1+1}^{S}\left(\frac{(\pi_{t+H}-m)^2}{1+\gamma}\right)\right\}\right)}$$

C3.2 Sampling from the posterior

In order to make inferences about the posterior distribution of the parameters, it is necessary to obtain samples from the three posterior distributions. The posterior distribution of σ^2 is an inverted gamma-2 (equation [c8]) and thus, poses no problem. However, the other two kernels (equations [c9] and [c10)] are of unknown form. This calls for a sampling procedure commonly known as Metropolis-Hastings within Gibbs sampling. The sampling algorithm takes the following steps:

1. Initialize the parameters at an arbitrary value $\left(m_0, \sigma_0^2, \gamma_0\right)$.
2. Generate a k_{th}-draw $\sigma_k^2 \sim p\left(\sigma_{k-1}^2 |\ \gamma_k, m_k, .\right)$
3. Metropolis step to get m update:
   Consider the function from equation [c9]:

$$c_m(m;\sigma^2,\gamma) = e^{\left(\frac{-1}{2\sigma^2}\left\{\sum_{i=1}^{S_1}\left(\frac{(\pi_{t+H}-m)^2}{(1-\gamma)}\right)+\sum_{i=S_1+1}^{S}\left(\frac{(\pi_{t+H}-m)^2}{(1+\gamma)}\right)\right\}\right)}$$

   (a) Calculate a function value: $M_{k-1} = c_m(m_{k-1};\sigma_k^2,\gamma_{k-1})$
   (b) Generate a candidate draw from: $m_k^* \sim m_{k-1} + cN(0,1)$; where c is an appropriate constant.
   (c) Calculate the corresponding function value: $M_k = c_m(m_k^*;\sigma_k^2,\gamma_{k-1})$
   (d) Calculate the ratio: $\rho = \min(\frac{M_k}{M_{k-1}}, 1)$
   (e) Draw a uniform random variable between zero and one $\rho_u = Uniform(0,1)$
   (f) if $\rho_u < \rho$, make the candidate m_k^* draw be the selected draw m_k. Otherwise go back to [a.] and repeat the procedure.
4. Metropolis step to get γ update: Considering the function from equation [c9]

$$c_\gamma(\gamma;\sigma^2,m) \ \propto \ \left(\tfrac{\gamma+1}{2}\right)^{c-1}\left(\tfrac{1-\gamma}{2}\right)^{d-1}\left(\frac{2}{\sqrt{1-\gamma}+\sqrt{1+\gamma}}\right)^{S}$$
$$e^{\left(\frac{-1}{2\sigma^2}\left\{\sum_{i=1}^{S_1}\left(\frac{(\pi_{t+H}-m)^2}{1-\gamma}\right)+\sum_{i=S_1+1}^{S}\left(\frac{(\pi_{t+H}-m)^2}{1+\gamma}\right)\right\}\right)}$$

And repeat [a.] to [f.] as in Step 3.

After a number of draws, the sampling scheme is equivalent to sampling from the true posterior distributions outlined above. In the example developed in the paper, the number of total draws amounts to 50,000 from which, the first 5,000 were excluded.

C4 Appendix: The optimal design of the sample size

As stated in the main text, the optimal sample size design maximises the expected utility

$$S^* = \arg\max_{S \in \mathcal{D}} \{KL(S) - \lambda S\} \tag{c11}$$

Where the KL divergence number is defined as

$$KL(S) = \int_\Lambda \int_\Pi \log\left[\frac{p(\Lambda|\Pi, S)}{p(\Lambda)}\right] p(\Pi, \Lambda|S) d\Pi d\Lambda$$

Where $\Pi = \{\pi\}_{j=1}^{S}$ is the simulated inflation data of size S, $p(\Lambda)$ is the prior distribution of the parameters and $p(\Lambda|\Pi, S)$ is the posterior distribution.

Following Ryan (114), it is straightforward to show that the KL information number is

$$KL(S) = \int\int \log\left[p(\Pi|\Lambda, S)\right] p(\Pi, \Lambda|S) d\Pi d\Lambda - \int \log\left[p(\Pi|S)\right] p(\Pi|S) d\Pi$$

Hence, this number can be estimated by a MCMC procedure that does not rely in sampling from the posterior distribution of the parameters. The estimator is

$$\widehat{KL}(S) = \frac{1}{N}\sum_{i=1}^{N} \{\log[p(\Pi_i|\Lambda_i, S)] - \log\left[\widehat{p}(\Pi_i|S)\right]\} \tag{c12}$$

Where (Π_i, Λ_i) for $i = 1, ..., N$ is a sample from $p(\Pi, \Lambda|S)$ and $\widehat{p}(\Pi_i|S)$ is an estimator of the marginal density of the data $p(\Pi_i|S)$. The dependent pair $(\Pi_i,\ \Lambda_i)$ drawn from $p(\Pi, \Lambda|S) = p(\Pi|\Lambda, S)p(\Lambda)$, is obtained by first drawing Λ_i from the prior distribution $p(\Lambda)$ and then Π_i from the conditional distribution $p(\Pi|\Lambda_i, S)$.

The estimation of the marginal density of the data is obtained by an importance sampling based estimator as in Ryan (114)

$$\widehat{p}(\Pi_i|S) = \frac{1}{M}\sum_{j=1}^{M} p(\Pi_i|\Lambda_{ij}^*, S) \tag{c13}$$

Where $\{\Lambda^*_{ij}\}$ for $i = 1, ..., N$ and $j = 1, ..., M$ are N samples of size M drawn from the prior $p(\Lambda)$ obtained independently of the N pairs (Π_i, Λ_i) drawn before.

The sampling algorithm to get the estimator [c12] follows exactly that of Ryan (114)

1. Generate a large sample of size N_Λ from $p(\Lambda)$, $\{\Lambda, ..., \Lambda_{N_\Lambda}\}$.

2. Generate an index set for MCMC estimator [c12] as a size $N \leq N_\Lambda$ random sample without repetition of the integers 1 to N_Λ. Call this sample $\{out_i\}_{i=1}^N$

3. Generate index sets for importance sampling estimator [c13] as N independent size $N \leq N_\Lambda$ random samples without repetition of the integers 1 to N_Λ. Call these samples $\{in_{ij}\}_{j=1}^M$ for $i = 1, ..., N$.

4. For $k = 1, ..., n_d$, let S_k represent n_d designs to be compared. Generate one dataset Π_{ki} from $p(\Pi|\Lambda_{out_i}, S_k)$ for each $k = 1, ..., n_d$ and each $i = 1, ..., N$.

5. For $k = 1, ..., nd$, compute

$$\widehat{KL}^M(S_k) = \frac{1}{N}\sum_{j=1}^{N}\widehat{KL}_i^M(S_k) \tag{c14}$$

where

$$\widehat{KL}_i^M(S_k) = \log[p(\Pi_i|\Lambda_{out_i}, S_k)] - \log\left[\frac{1}{M}\sum_{j=1}^{M} p(\Pi_i|\Lambda^*_{ij}, S)\right]$$

To implement the estimation, I considered the following values: $N_\Lambda = 5000$, $N = 1000$, $M = 100$, and $n_d = 200$. Also, I consider a sample size higher than 30 via: $S_k = (k-1) + 30$.

In figure [4.6], I depict the MCMC draws of KL together with a smoothed version of it. The smoothed version is combined with the loss term in [c11] to get the utility function shown in figure [4.7].

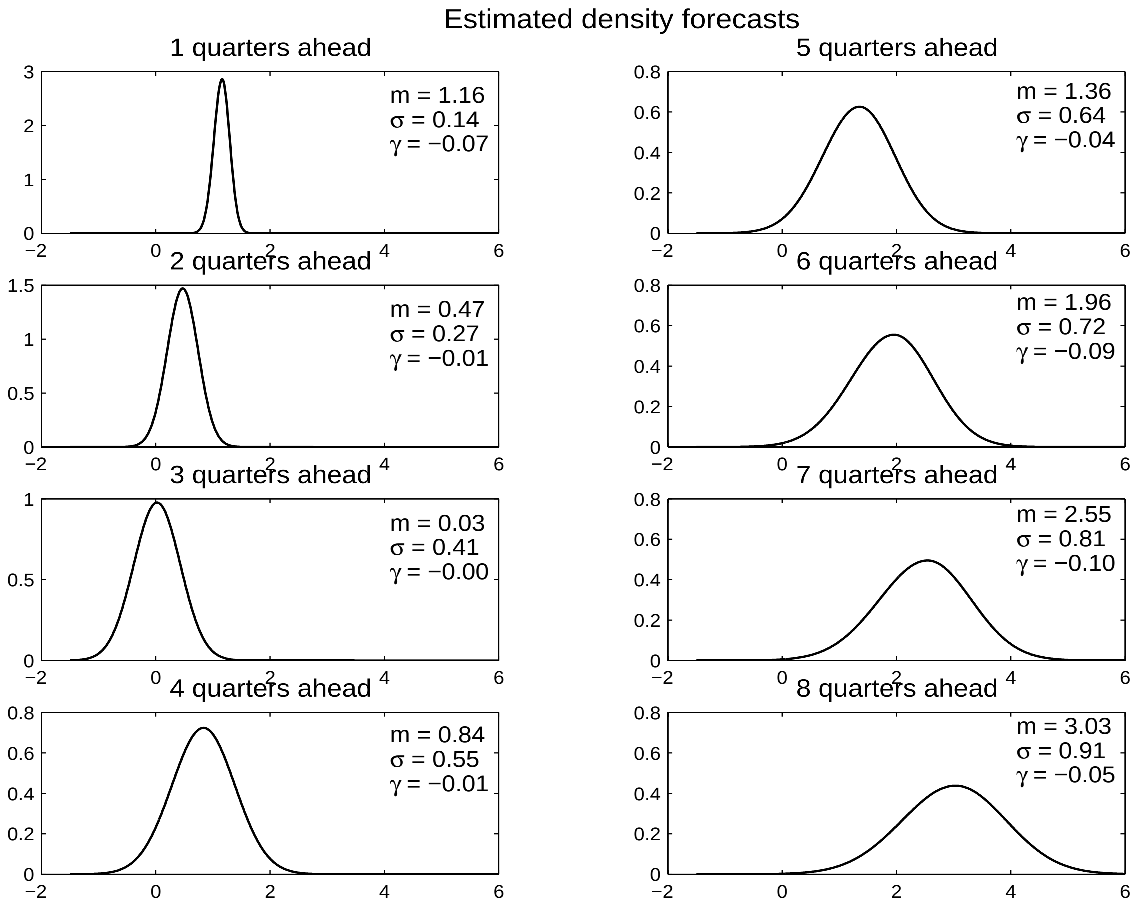


Figure 4.2: Estimated SN pdf's for the year-on-year inflation forecast.

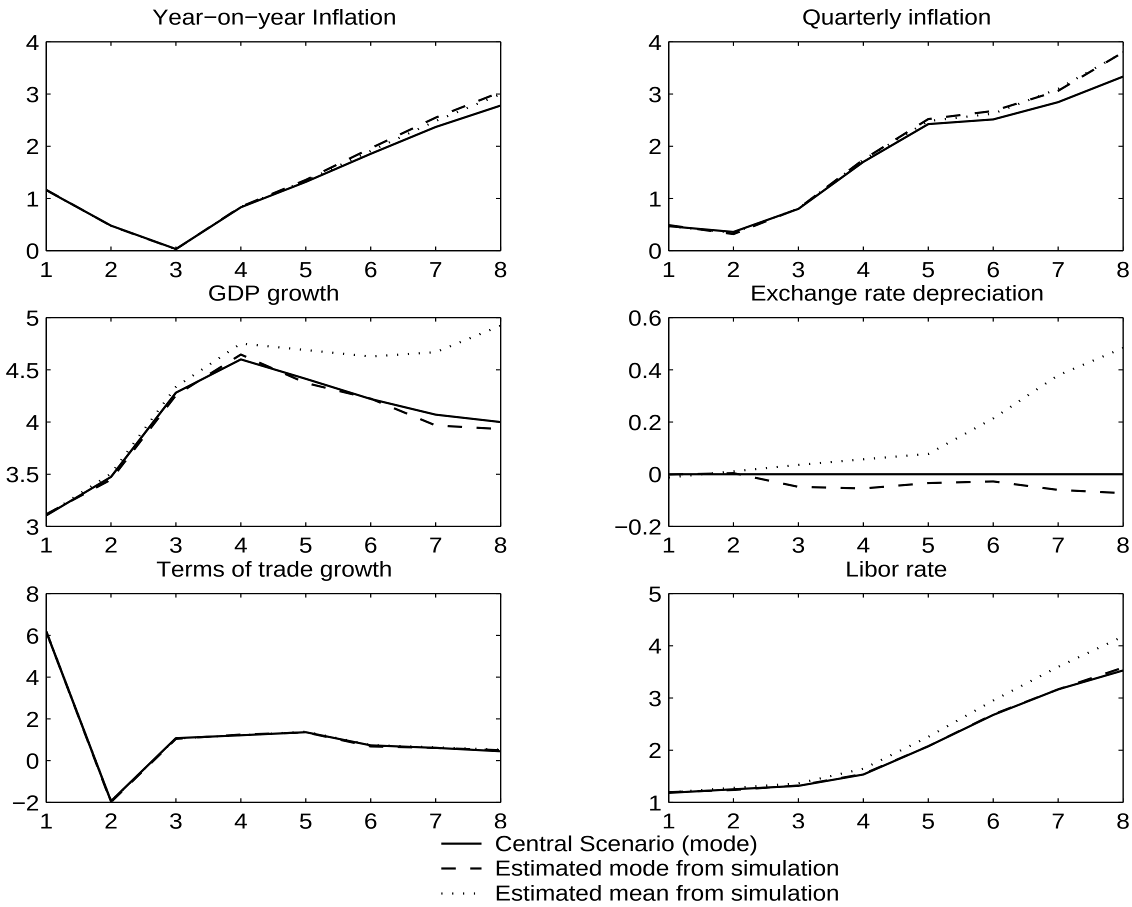


Figure 4.3: Central measures of tendency.

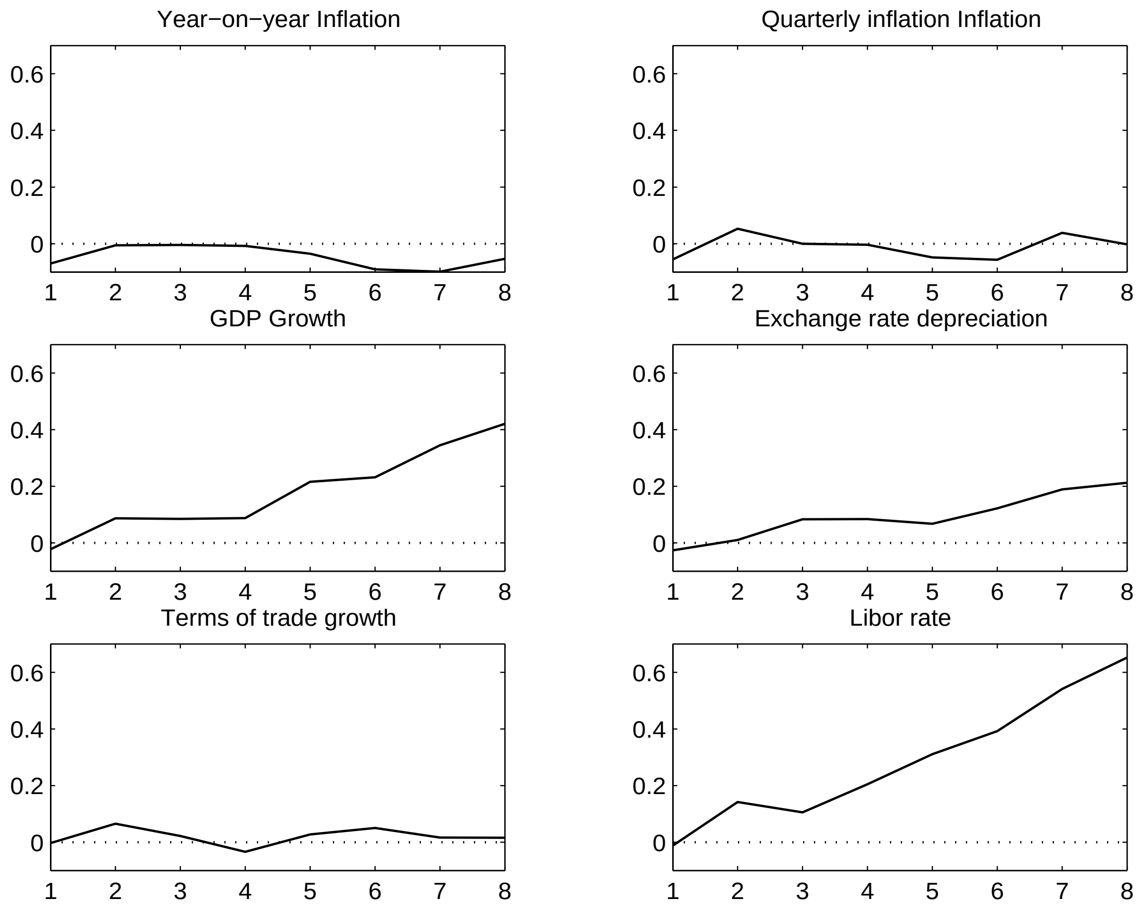


Figure 4.4: Evolution of the gamma parameter.

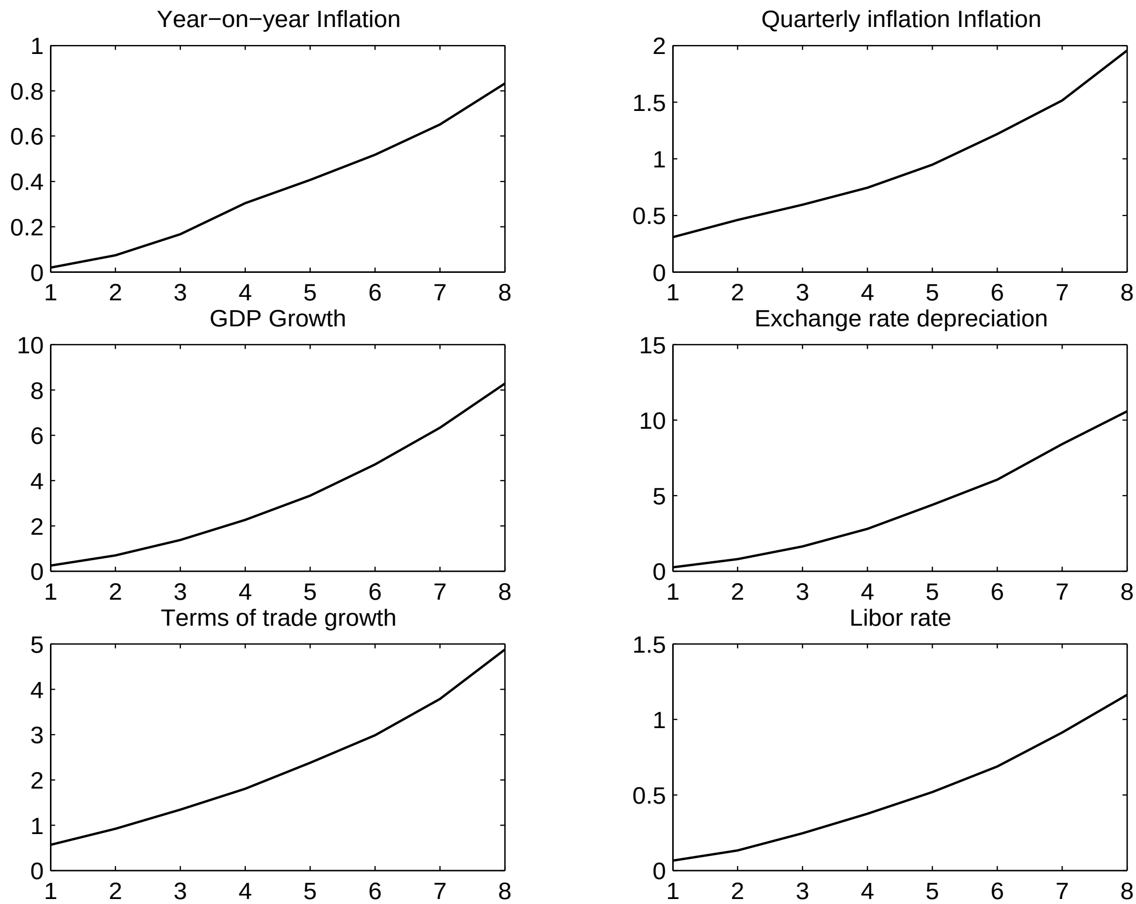


Figure 4.5: Evolution of the uncertainty parameter.

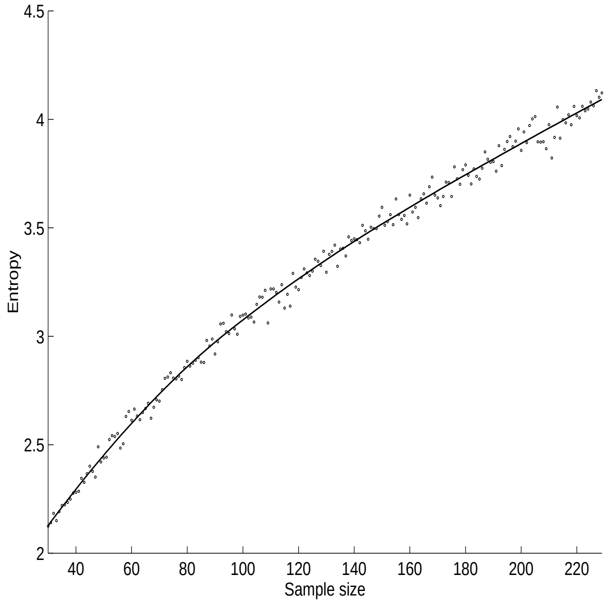


Figure 4.6: The KL divergence number (a.k.a entropy). The scatter plot is the estimation with monte carlo variation, the line is the smoothed version.

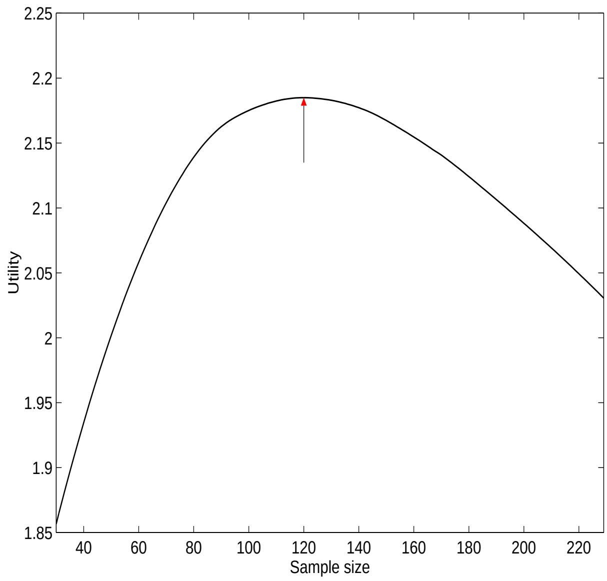


Figure 4.7: The utility function of the policymaker as a function of the simulation sample size.

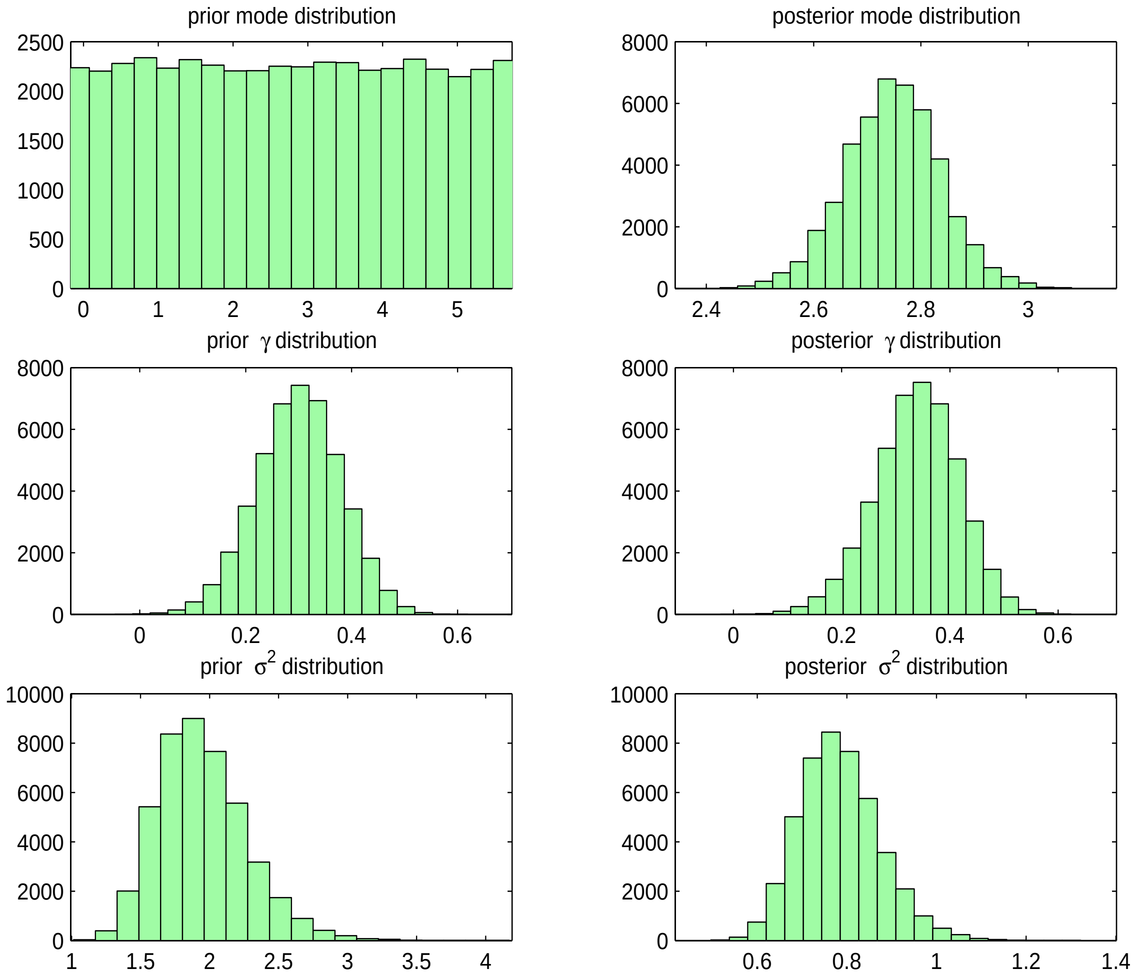


Figure 4.8: The utility function of the policymaker as a function of the simulation sample size.

CHAPTER 5

SKEWED FORWARD-LOOKING MONETARY POLICY BEHAVIOUR

The purpose of this chapter is to empirically estimate forward-looking monetary policy behaviour in the five countries in Latin America that have adopted the inflation targeting regime so far (IT henceforth): Brazil, Chile, Colombia, Mexico and Peru[1].

In recent times, monetary policy in Latin America has been characterised by the evolving pattern in the use of intermediate targets and policy instruments, as a result, central banks and specially ITers[2] have tended to use a controllable short term nominal interest rate as their preferred policy instrument[3]. This has been very important because it has allowed to have a better measure of monetary policy stance and has opened the possibility to perform formal econometric analysis.

Regarding the management of the policy instrument, most central bankers in the world either in developed or emerging-market countries, either ITers or non-ITers; justify forward-looking monetary policy making.[4]. At the theoretical level, inflation forecasts can be considered as intermediate targets in the implementation

[1]Monetary policy options in Latin America have in general converged to the three strategies outlined in Mishkin and Savastano (92); full-dollarisation, monetary targeting and inflation targeting (IT).

[2]See for example Armas and Grippa (4), Minella et.al (89), Landerretche et.al (81), Restrepo (105) and Truman (128) for country-specific cases.

[3]Adoption of IT by developing countries is not the only reason. The changing structure of their economies together with developments in interbank markets and financial institutions have facilitated central banks to endorse interest rates instead of other instruments.

[4]A perusal of Inflation Reports and formal communication from web pages of various, heterogeneous central banks easily confirms that assertion.

of forward-looking policy[5]. On the empirical side, Clarida et.al (30) and Orphanides (98) initiated a research agenda devoted to the estimation of forward-looking interest rate feedback rules.

However, there is one dimension of analysis that has had scant attention in the empirical estimation of monetary policy reaction functions. As suggested by Goodhart (57) and recently by Greenspan (60) and King (75), when policy makers take decisions, they pay considerable attention to the risks in the foreseeable future. It is not only the most likely or baseline forecasts that is important. The low-probability, high-impact events and the nature of the shocks[6] that shape the probabilistic distribution of forecasts are also key.

In the discussion to FED Chairman Alan Greenspan's "Risk and Uncertainty in Monetary Policy", during the 2004 Annual Meeting of the American Economic Association, Mervin King, governor of the Bank of England, reflects on the *risk management approach* to central banking

> *Greenspan defines the [risk management] approach by saying that policy makers should look at a range of "risks" to output and inflation; and give due consideration to those risks when setting policy. He argues that policy makers cannot just rely on the forecasts from a structural model of the economy when even deep parameters are drifting. They should also use their judgement; compare current experiences with previous, similar episodes; and continually test and update a range of reduced-form models, which should help give some insight into how the economy is evolving.*
>
> *This is the approach taken at the Bank of England, where the Monetary Policy Committee takes into account the entire distribution of future*

[5] See Svensson (121).

[6] Their persistent or transitory features and their qualification as supply or demand driven shocks.

outcomes for inflation and output when setting interest rates. A "fan-chart" for its forecasts of both inflation and output is published in the quarterly Inflation Report.

This is also the case within Latin American ITers. The systematic inclusion of balance of risks discussions within their Inflation Reports suggests that their views and decisions are somehow shaped by the outlook of risks surrounding the inflation forecast.

In light of these considerations, the aim of this chapter is to estimate forward-looking behaviour encompassed in the dynamics of interest rates in relation to measures of inflation forecasts. To this end, I define the lagged interest rate and a predetermined inflation forecast as the conditioning variables that affect the interest rate setting at any given time.

First, I am interested in the mean interest rate effect. In order to do so, simple linear forward-looking interest rate rules are estimated by standard ordinary least squares techniques at different possible forecast horizons.

Second, in order to have a broader information than that provided by the mean OLS estimates, I perform the estimation of quantile effects; namely, the response of the interest rate at the different quantiles of its conditioning distribution. This is done by estimating linear quantile regression models as documented in Koenker (79). The quantile estimates provide a broader picture of interest rate behaviour and can potentially shed light on the probabilistic nature of interest rate responses against the backdrop of the myriad of risks Latin American ITers face.

Therefore, the technique applied in the chapter provides one way to extract information from the data to characterise forward-looking behaviour under both the spectrum of risks and the attitudes towards those risks policy makers have. This is particularly important in Latin America, given the many risk factors affecting baseline inflation forecasts.

The chapter proceeds as follows, section 5.1 sets up the linear forward-looking response regression, section 5.2 does so for the quantile regression model, section 5.3 describes the data used in the estimations and section 5.5 concludes.

5.1 Mean forward-looking responses

The empirical literature on forward-looking interest rate rules have focused primarily on developed countries; Clarida et.al (30) and Orphanides (98)[7] showed for the first time the relevance of policy driven by future expected outcomes. In the specific context of Latin America, several country specific studies like Restrepo (105), Minella et.al (89), Truman (128) and Ramos and Torres (103) deal with the estimation of forward-looking policy rules for Brazil, Chile, Colombia, and Mexico.

The econometric approach for the estimation of this type of rules, follows two directions. First the GMM methodology advocated in Clarida et.al (30) which is followed by Restrepo (105) and Ramos and Torres (103). The second approach - pioneered by Orphanides (98) - consists in using real-time forecasts available at the time of every interest rate decision and it is used for example in Jansson and Vredin (69), Kuttner (80) and Goodhart (58). For the Latin American case, Ramos and Torres (103) use forecasts from surveys instead of own-central bank forecasts while Minella et.al (89) construct estimates with central bank forecasts.

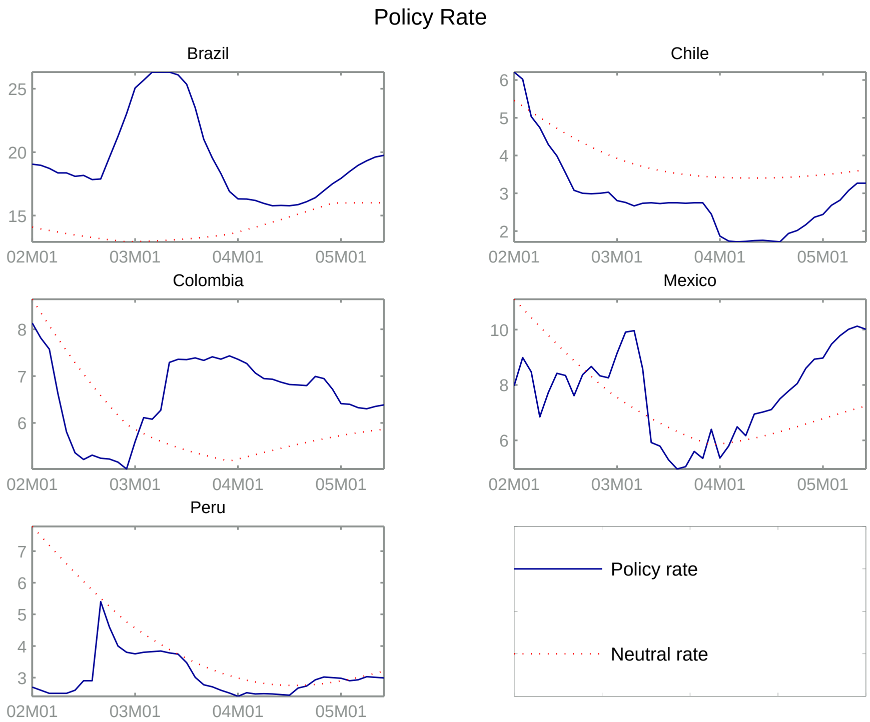
In this chapter I follow more closely this latter approach of treating forecasts directly as explanatory variables. As it will be explained in section 5.3, I use monthly series. Also, given that it is practically impossible to obtain central banks' own forecasts for the period under study, I rely instead on consensus forecasts of private agents gathered by *Consensus Economics*. These forecasts, in the form of monthly vintages, mimic the real-time data sets used for example in Orphanides

[7]The working paper versions appeared both in 1997.

(98). However, it is important to reckon that these forecasts might not be appropriate because they might indeed differ from central banks' own forecasts[8]. For the time being, I need to assume that the data set at hand captures the fundamental dynamics of central banks' own forecasts.

In all the countries under study I use a relevant interbank rate as the monetary policy operational target(See figure 5.1)[9]. This is not exactly true for Mexico where the policy instrument is defined as the cumulative balance of commercial banks' current accounts at the Central Bank[10]. Nevertheless, according to Truman (128), during the period under study the interbank rate is already a good indicator of Banco de Mexico monetary policy stance.

Figure 5.1: Policy rates and estimated neutral interest rates.



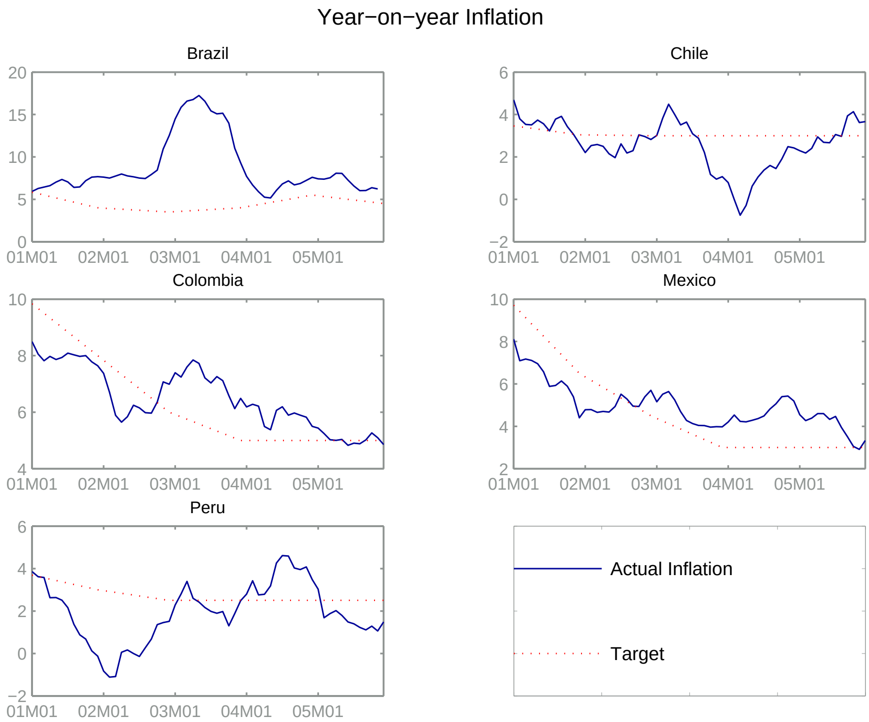
[8]It is reasonable to think that central banks react basically to their internal forecasts.

[9]In this chapter, the operational target is also the policy instrument as operational issues are totally abstracted.

[10]Known as the 'corto'.

As apposed to IT practice in advanced economies, Latin American IT still displays different degrees of convergence. Some countries are still on the way or have just converged to a stationary inflation target (See Figure 5.2), in such cases, the policy horizon is not clearly discernible. Others, like Chile have explicitly announced a fixed policy horizon of more than a year. Unfortunately, the data at hand allow us to have complete times series only up to 13-months-ahead inflation forecasts[11]. This will limit the results along the horizon dimension as responses to horizons more than 13 months ahead can not be calculated. Yet, the data can already show some important effects at available longer horizons.

Figure 5.2: Inflation rates and ex-ante targets.



In this chapter I assume that the monthly interest rate behaves according to the following equation

[11] Including the month when the decision is taken.

$$i_t = \rho i_{t-1} + (1-\rho)\left[i_t^n + a_\pi\left(\pi^f_{t-1,t+h} - \pi^o_{t,t+h}\right)\right] + \varepsilon_t \tag{5-1}$$

Where i_t is the policy rate, $\pi^f_{t-1,t+h}$ is the year-on-year, h-months-ahead inflation forecast made in the month prior the policy decision is taken, $\pi^o_{t,t+h}$ is the numerical, ex-ante inflation target known at time t and to be achieved at time $t+h$, i_t^n is the neutral short-term interest rate, and ε_t represents all other possible sources of interest rate change[12].

To be able to diminish the bias arising from simultaneous dependence, interest rate decisions at time t depend on forecasts made before the decision (time $t-1$). However, those forecasts made at time $t-1$ implicitly assume an expected path of interest rates and a particular value of interest rates for period t that is highly correlated with period $t-1$ interest rates[13]. Therefore I postulate a relatively strong assumption of exogeneity of last-period forecasts to the current and future interest rate decisions.

According to equation [5-1] I can calculate the mean interest rate decision conditional on information available at each decision step

$$E[i_t \mid \Omega_t] = \rho i_{t-1} + (1-\rho)\left[i_t^n + a_\pi\left(\pi^f_{t-1,t+h} - \pi^o_{t,t+h}\right)\right] \tag{5-2}$$

Where Ω_t is the information set policy makers have before any time-t interest rate decision. This set is comprised by the lagged interest rate, the neutral interest rate and the deviations of predetermined, last-period inflation forecast from the planned target[14]. I assume that $E[\varepsilon_t \mid \Omega_t] = 0$.

[12] These sources of interest rate variations can be serially correlated.

[13] See Kim and Nelson (74). There, it is argued that to for the exercise to be clean, the forecasts must assume a constant interest rate, to avoid simultaneous equation bias.

[14] I use de term "planned" target because in some circumstances such as Brazil, targets have been adjusted ex-post. See Minella et.al (89).

5.2 Quantile forward-looking responses

The key element in standard rule estimations of [5-2] is the use of linear regressions and the least squares method to estimate what I call the mean response of the instrument. If the estimated errors are normal, the mean response is a good descriptor and not much else can be said. However, if the errors are not gaussian, Koenker and Bassett (77) show that some features can be extracted from applying quantile regressions.

In order to setup the quantile regression framework, the model in [5-1] can be transformed in:

$$\widetilde{i}_t = \rho\widetilde{i}_{t-1} + \alpha\widetilde{\pi}^f_{t-1,t+h} + \varepsilon_t \tag{5-3}$$

Where I have transformed the variables in $\widetilde{i}_t = i_t - i^n_t$ and $\widetilde{i}_{t-1} = i_{t-1} - i^n_t$ as interest rate deviations from their neutral values, and $\alpha = (1-\rho)\, a_\pi$ together with $\widetilde{\pi}^f_{t-1,t+h} = \pi^f_{t-1,t+h} - \pi^o_{t,t+h}$ denoting the sensitivity of interest rates and the inflation deviations from target respectively.

The quantile regression model considers:

$$\widetilde{i}_t = \rho\left(\gamma\right)\widetilde{i}_{t-1} + \alpha\left(\gamma\right)\widetilde{\pi}^f_{t-1,t+h} + \varepsilon_{\gamma t} \tag{5-4}$$

Where $\gamma \in [0,1]$ represents the orders upon quantiles are calculated (for example, when $\gamma = 0.5$ I calculate median effects). The distribution of $\varepsilon_{\gamma t}$ is not know, it is only assumed that the conditional quantile of the error term is $Q_\gamma\left(\varepsilon_{\gamma t} \mid \Omega_t\right) = 0$. Then, the conditional γ-quantile response is

$$Q_\gamma\left(\widetilde{i}_t \mid \widetilde{i}_{t-1}, \widetilde{\pi}^f_{t-1,t+h}\right) = \rho\left(\gamma\right)\widetilde{i}_{t-1} + \alpha\left(\gamma\right)\widetilde{\pi}^f_{t-1,t+h} \tag{5-5}$$

Koenker (79) show that the parameters of the regression model for any

$\gamma \in [0,1]$ can be estimated by minimising the sum of sample quantile regression functions[15]

$$\min_{\rho(\gamma),\alpha(\gamma)} \left\{ \frac{1}{T} \sum_{t=0}^{T} q_{\gamma}(\varepsilon_{\gamma t}) \varepsilon_{\gamma t} \right\} \tag{5-6}$$

Where $q_{\gamma}(\varepsilon_{\gamma t})$ is the quantile regression weight function given by $q_{\gamma}(\varepsilon_{\gamma t}) = \gamma - I_{(\varepsilon_{\gamma t}<0)}$ (note that $I_{(\varepsilon_{\gamma t}<0)}$ is the standard indicator function). For example, in the median case $\gamma = \frac{1}{2}$ then $q_{0.5}(\varepsilon_{\gamma t})$ is either $\frac{1}{2}$ or $-\frac{1}{2}$ depending on the sign of $\varepsilon_{\gamma t}$. In that case, deviation above or below $\varepsilon_{\gamma t}$ are weighted similarly. In all other cases within the space $[0,1]$, deviations are weighted asymmetrically.

The minimisation and hence the estimation of the parameters of interest relies on linear programming methods outlined first in Koenker and Bassett (77)[16]. In order to get confidence intervals, the standard errors can be obtained by bootstrap methods.

The quantile regression approach outlined here is potentially useful for assessing monetary policy behaviour. It can shed light on the response of interest rates at the lower and upper ends of the distribution of the inflation forecast.

For example, during the period of analysis I might find that for a particular ITer, interest rates might react strongly at the upper end of the distribution (at the higher quantiles) but less strongly at the lower end of the distribution (at the lower quantiles). If the distribution of inflation forecasts have been such that the upper end of the distribution have been outside permissible ranges but the lower end have been mostly closed to the target then the above finding is compatible with a central bank trying to curve upside risks. This is the asymmetric-risks interpretation related to the risk management approach quoted in the introductory section.

[15] As explained in Koenker and Bassett (77); Koenker (79), this is a parallel to the ordinary least squares minimisation where the aim is to minimise the sum of squared functions.

[16] See Koenker (79) for details and more references of time series applications and quantile autoregressions.

Another possible interpretation is that the above behaviour might have been the result of an asymmetric loss function of a central bank that, given overall balanced risks, have reacted more to upper end parts of the forecast distribution than to the lower parts. Hence central bank behaviour can be driven by asymmetric risks, asymmetric losses or a combination of both. Unfortunately, given the available data I can not identify the sources of such a behaviour, only that the particular behaviour has been present throughout the historical sample.

5.3 The data

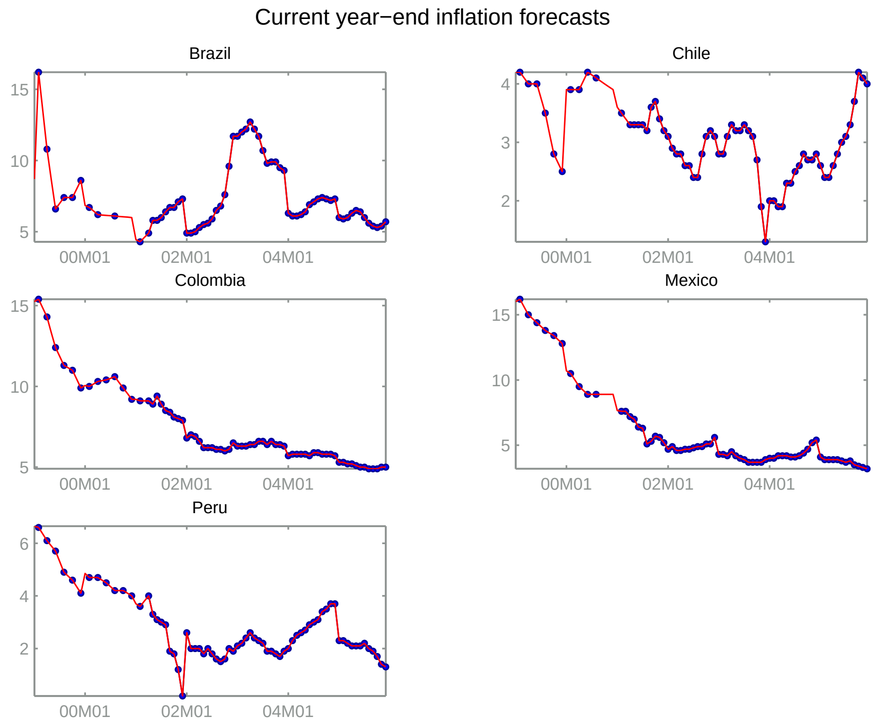
Using the nominal interest rate series, I construct ex-post real interest rate series which are then decomposed in trend and cycle. The trend is used as a proxy for time-varying neutral real interest rates which are then summed to corresponding inflation targets to obtain neutral nominal interest rate series to be used in the regressions.

Regarding the consensus forecast, the surveys only contain forecast for the current and next year-end inflation rates[17]. The survey reports are released on the second half of every month and therefore the current month is always part of the forecast. Given observed inflation rates within the year, the current end-year inflation forecast imply a residual inflation for the rest of the current year. Additionally using next year-end forecasts, it is possible to construct h-month implied forecasts. Given the pattern of the surveys, it is only possible to obtain complete times series of 13-months ahead implied inflation forecast.

The data set covers the period until November 2005. For the regressions, I consider periods starting in 2000 for Brazil, mid-2001 for Chile and Colombia and 2002 in Mexico and Peru.

[17]See figures 5.3 and 5.4 where these series are plotted. Minor interpolation is done there to complete missing data.

Figure 5.3: Current year-end consensus forecasts: Dotted lines are the actual data and continuous lines are interpolated data for missing observations.



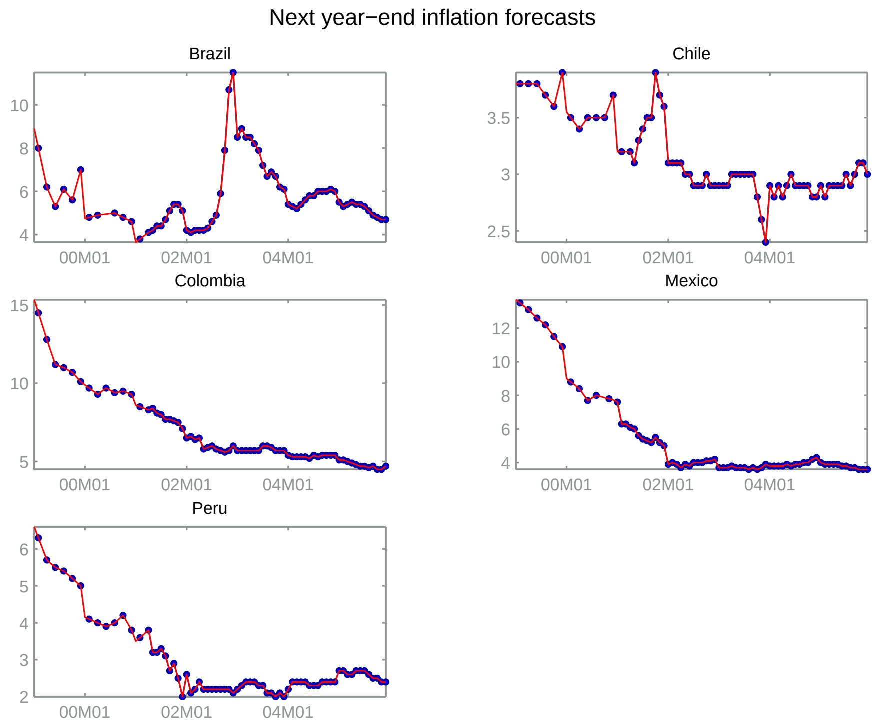
5.4 Results

5.4.1 Mean responses

On figure 5.5 I observe the different responses of the systematic part of interest rates to deviations of inflation forecasts for horizons 0 to 12 months ahead together with their one-standard deviation confidence interval. If the mean estimate statistically exceeds unity then I have some evidence that the stabilising Taylor principle applies.

I observe that the responses increase as the forecast horizons rise in the case of Brazil, Chile and Mexico, reaching values of near or more than one for the 12-month ahead forecasts. These results at the end-horizons are in line with those reported in Minella et.al (89) for Brazil, and Restrepo (105) for Chile and Ramos

Figure 5.4: Next year-end consensus forecasts: Dotted lines are the actual data and continuous lines are interpolated data for missing observations.

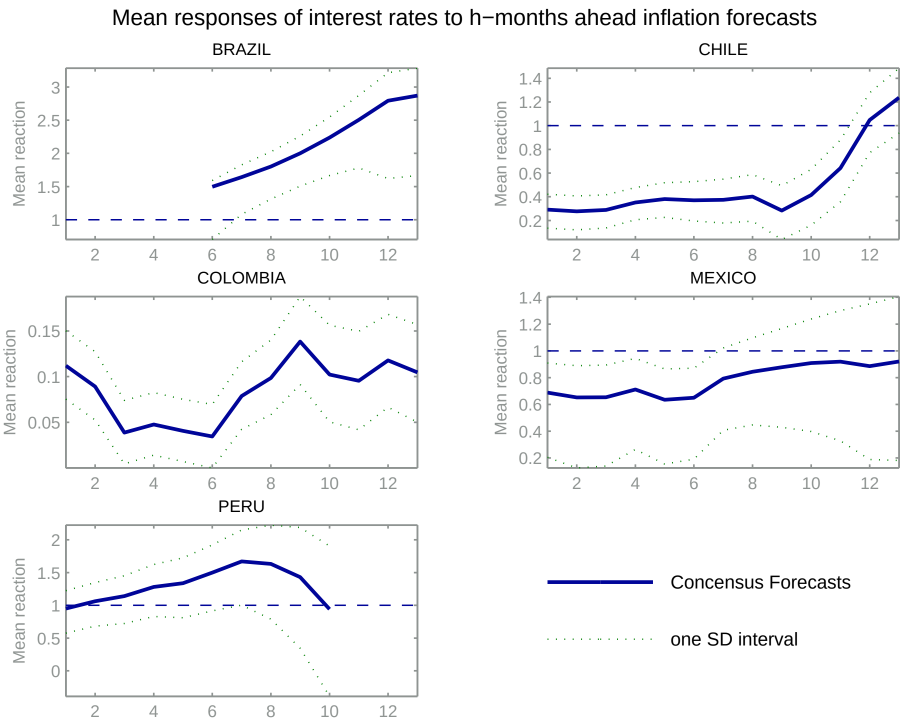


and Torres (103) for Mexico.

In the case of Colombia the results show a very mild and statistically lower-than-one response of interest rate at the higher-end horizons. Taken at face value, this would indicate that monetary policy in Colombia might not have been responding enough to stabilise inflation. However, I should warn that these results might reflect the fact that the consensus forecast data for Colombia might be ill-suited for the case at hand. Also, it might reflect the failure to adequately capture monetary policy stance throughout the whole sample.

In the case of Peru, the responses to consensus forecasts are statistically significant and close to unity up to about 7 months ahead inflation forecasts. For longer horizons the statistical significance vanishes. In this case, the results suggest that the monetary policy horizon in Peru has been lower than a year. This result

Figure 5.5: Mean responses to h-period ahead inflation forecasts.



might reflect the fact that - during the period of analysis - the policy target in Peru was set on a calendar year-end basis and not o a fixed horizon of a year or more which is the approximate monetary control lag in Peru [18].

As in the Colombian case, however I warn that the result might be just the mirror of an inadequate forecast series and that the use of the own-inflation forecast might change the results in a significant way.

What are the lessons to be learned from these pieces of evidence? First the chapter tends to confirm previous findings of forward-looking behaviour for Brazil, Chile and Mexico. Second, it opens the question of the proper characterisation of monetary policy in Colombia and Peru within the sample; robustness, additional explanatory variables, etc.

[18] See Luque and Vega (88) and Llosa et.al (86) for details about Peruvian data and monetary policy.

5.4.2 Quantile responses

Figure 5.6 depicts 5 panels showing the quantile responses of interest rates to one-year-ahead inflation forecasts together with the mean responses and their respective 95 percent confidence intervals. For the case of Peru I have considered 7 months ahead inflation forecasts because this is the relevant horizon reflected in the data.

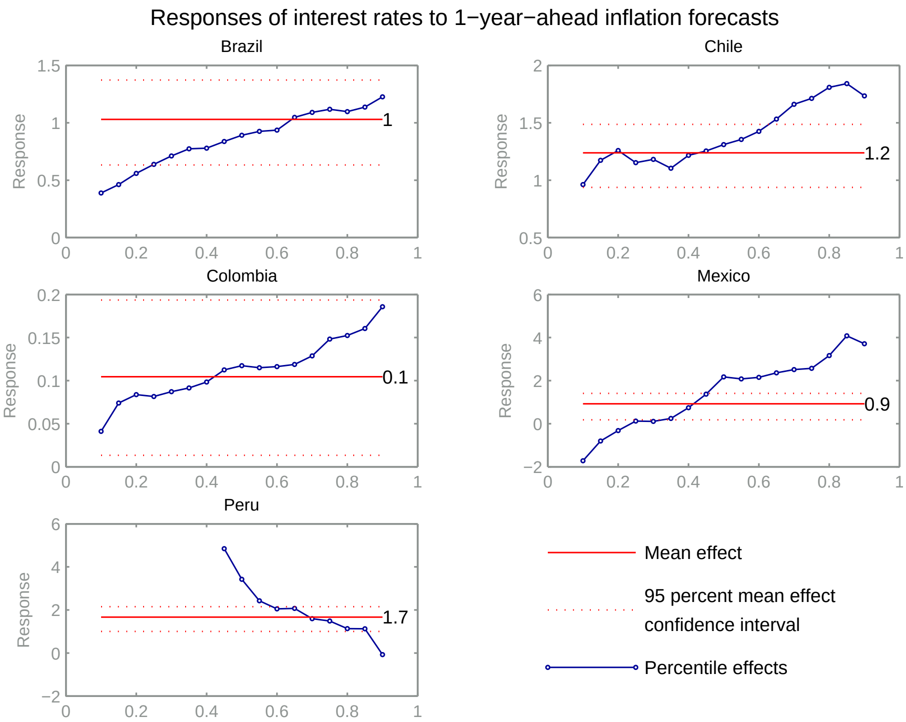
For example, a 0.9 percentile effect (the responses on the right hand side of the panels) shows how the interest rate responds to inflation forecast deviations that are higher than the 90 percent of all forecast deviations, namely the response of the interest rates at the upper tail of the inflation forecast deviation distribution. Conversely, the 0.1 percentile effect shows the responses at the lower tail. In other words, the effects at the edges of the panels show how interest rates would respond under extreme expected inflation deviations. If the forecast distributions are skewed to the right on average then a central bank might react statistically more, equal or less than the mean response.

In a completely symmetric world, I would expect the responses at all points of the distribution to be very close to the mean responses and statistically the same.

When a response is low at the lower tail and high at the upper tail such as the case of Brazil, Chile and Mexico I can interpret that - provided that the monetary policy loss functions are symmetric - the inflation risks during the sample might have been to the upside and that monetary policy have in fact reacted aggressively against those risks, even more than the median effect would suggest.

For the case of Peru, policy responses at the upper tails of the inflation forecast distribution have been lower than the mean responses. This is an indication that the Central Bank of Peru have tended not to strongly respond to upside risks to their inflation forecasts. With such a low policy horizon (7 months), upside risk balances reflect inflationary factors to which it is not desirable to respond aggressively.

Figure 5.6: Quantile responses to relevant forecast horizons.



All in all, these results point to the fact that symmetry is not a feature of policy behaviour within Latin American ITers. Rather, skewed risks and particular responses to them tend to deny the standard quadratic loss functions used in the literature about optimal policy rules.

5.5 Conclusion

I have performed mean and quantile response estimations of forward-looking monetary policy behaviour for the five ITers in Latin America.

Using the mean response estimation I have found that monetary policy behaviour in these countries is forward-looking. Moreover, the use of a control lag of more than a year suggested in the results for Brazil, Chile and Mexico is akin to the practice of central banks in developed countries. Possible data problems or possible shorter control lags characterise the Colombian and Peruvian case.

The quantile regression estimates give us some key directions of the risks surrounding monetary policy decisions in these countries. I have interpreted that Brazil, Chile and Mexico have faced upside risks to inflation during their recent monetary policy history and that these upside risks have somewhat prompted stronger interest rate responses[19]. I find some weak evidence that Peru is likely to have faced upside risk to which the authorities did not reacted in the expected fashion, possibly due to the short policy horizon in place.

Further research is necessary in order to relate the above findings to institutional features of each ITer. For example, the way the central bank policy mandate is defined, the type of IT design or the macroeconomic structure of the country might all shape the specific way monetary policy is conducted.

The above econometric assessment of forward-looking behaviour is positive. An avenue of future research is to analyse the interplay between optimal policy under skewed risks conditional on a typical economic structure of Latin American inflation targeters.

[19]The fact that the skewness of the inflation forecast distribution might affect the interest rate setting in a forward-looking central bank is explained for example in Goodhart (57).

CHAPTER 6

THE EFFECTS OF INFLATION TARGETING ON INFLATION

The goal of this chapter is to evaluate the behaviour of inflation dynamics brought about by the adoption of IT. I do so by studying three measures that distinguish inflation dynamics: mean, variance and persistence. Key interesting questions emerge from the study of these measures.

First, IT has been adopted by countries either to credibly disinflate (or converge) or, as asserted by some authors, to lock-in the gains obtained from episodes of disinflation. Would countries have done better or worse had they adopted any other regime?

Second, it is generally stated that inflation uncertainty results from factors exogenous to the scope of the transmission mechanism of monetary policy (terms of trade or supply shocks, for instance) as well as from monetary policy shocks. In this sense, inflation can be made less uncertain up to the limits set out by the amount of exogenous uncertainty. Modern monetary policy practice, whether IT or not, hinges precisely on making monetary policy more predictable and hence less uncertain. Once again, a fair question for a country that adopted IT is whether inflation uncertainty has fallen more or less in comparison to the counterfactual situation of not having adopted IT.

Last, the theory of IT emphasises that the overall features of the framework are built upon the pillar of credibility. Credibility is understood as the ability the central bank has to anchor medium to long run expectations, to avoid expectation traps that may render persistently high or low inflation rates. On the other hand, “flexible” IT implies that shocks that drive inflation away from the target should revert at a pace that does not harm real activity. Hence, the speed of adjustment seems to depend on the degree of flexibility[1]. Too fast an adjustment is equivalent to a strict IT, likely in situations whereby the central bank needs to gain or strengthen credibility. When the adjustment is slow, a more flexible IT is in place. In the fast-adjustment case, undue real volatility might emerge whereas in the slow-adjustment case either credibility is strong enough that the central bank can reap some benefits of flexibility, or the nominal anchor is lost and the inflation falls to the expectation trap.

Thus, the effects of IT adoption on persistence are ambiguous. More persistence can result from successful flexible ITers or unsuccessful ITers not gaining credibility. Once more, what does an empirical evaluation of IT over persistence tell about the adopting ITers?

In recent years, a growing body of literature has provided insights on the empirical assessment of IT. Corbo et al. (35), for instance, compare policies and outcomes in fully-fledged IT countries to two groups, potential ITers and non-ITers. They find that sacrifice ratios were lower in ITers, that IT countries have reduced inflation forecast errors and that inflation persistence has declined strongly among ITers.

Johnson (70), by comparing five ITers to six non-ITers, all of them in industrialised economies, finds that the period after the announcement of IT is associated with a statistically significant reduction in the level of expected inflation. Also, he finds that IT has not reduced absolute average forecast errors in targeting countries relative to those in non-targeting countries. However, ITers did avoid even larger forecast errors than those that would have occurred in the absence of IT.

On the other hand, Neumann and Von Hagen (97) consider a group of six industrial IT countries and three non-IT countries and perform an event study to

[1]See Svensson (124).

quantify the response of inflation and long-run as well as short-run interest rates to a supply shocks (increases in the world oil price in 1978 - 1979 and in 1998 - 1999[2]). They find that the effect of IT is not significantly different from zero for average inflation, but it is for interest rates, meaning a gain in credibility among ITers .

Pétursson (102) analyses a bigger sample (twenty-one ITers) that includes developing economies. He evaluates the performance of a set of macroeconomic outcomes using a dummy variable for pre and post IT periods on a country basis and finds that IT has been beneficial to reduce the level, persistence and variability of inflation[3]. However, the technique offered by this study, does not tackle the fundamental question of relative performance. Its contribution hinges in giving a clear and robust account for the evidence of the absolute benefits of IT and corroborates previous findings on this line.

Levin et.al (83) study inflation persistence using five industrial ITers which are compared to seven industrial non-ITers. The study performs univariate regressions on inflation for each country and finds that inflation persistence is estimated to be quite low within ITers whereas the unit root hypothesis cannot be rejected for non-ITers. Levin and Piger (84), on the other hand, in a similar empirical framework with twelve industrial countries allow for structural breaks and finds that inflation in general exhibits low persistence[4]. They also suggest that IT does not seem to have had a large impact on long-term expected inflation for a group of eleven emerging market economies.

Finally, Ball and Sheridan (6) provides evidence on the irrelevance of IT. They look at seven OECD countries that adopted IT in the early 90's and thirteen countries that did not. They claim that ITers that reduced higher-than-average

[2]This type of shock creates a dilemma because it implies more inflation coupled with a downturn of economic activity.

[3]There are other studies that provide mixed evidence about inflation persistence. Benati (10) and Levin et.al (83) find that inflation has become less persistent within the OECD and specially IT countries.

[4]These results confirm those of Benati (10) that studies inflation dynamics in twenty OECD countries.

inflation rates towards equilibrium levels were merely reflecting *regression to the mean* and not a proper effect of IT. Once they control for regression to the mean, they conclude that IT did not improve macroeconomic performance. In their words, *"Just as short people on average have children who are taller than they are, countries with unusually high and unstable inflation tend to see these problems diminish, regardless of whether they adopt inflation targeting"*.

In my view, rather than challenging the previous evidence and beliefs about IT effects, the crucial point of the claim made in Ball and Sheridan (6) is methodological. If there is an ITer with poor performance before IT, then it should be compared with a non-ITer with equally poor initial performance. Otherwise, the targeting effect would be overstated. The methodology in this chapter hinges precisely on this matter of comparability.

Following Johnson (70) and Ball and Sheridan (6) the chapter uses a difference-in-difference estimator approach to evaluate the effects on key measures of inflation dynamics resulting from IT adoption. As I argue later, the previous studies on this issue may suffer from sample selection bias (a few industrialised countries, for instance) and, importantly, select counterfactuals for the ITers in an arbitrary fashion. The contribution is twofold: first, I use all the twenty-three IT experiences so far, the *widest possible control group* of non-ITers (86 countries) and different possible dates of IT adoption. With this, I understand IT as an alternative monetary policy framework worldwide, for both industrialised and developing economies. Second, I interpret the IT adoption as a "natural experiment", so I seek to reestablish the conditions of a randomised experiment where the IT adoption mimics a *treatment*. This naturally leads us to perform *propensity score matching* as an alternative to the widely used regression approach. In a nutshell, I seek to overcome the aforementioned methodological limitations by letting the data select the controls for ITers.

The rest of the chapter is organised as follows. Section 6.1 briefly describes the propensity score and matching techniques for evaluation, section 6.2 discusses some empirical issues regarding the robustness of the results and presents the inflation outcomes to be evaluated, section 6.3 shows the main findings while section 6.4 concludes. Appendix D details the empirical estimation.

6.1 Methodology

As mentioned, the chapter uses microeconometric techniques usually applied in non-experimental contexts, borrowed from the programme evaluation literature. To be consistent with this literature, in this section I refer to the adoption of IT as *treatment*, to the ITers as the *treated* group and to the non-ITers as the *control* group.

6.1.1 The fundamental problem

Let D be a binary indicator that equals one if a country has adopted IT and zero otherwise. Also, let Y_t^1 denote the value of a certain outcome in period t if the country has adopted the IT regime and Y_t^0 if not. Given a set of observable country attributes X, the average effect of being an ITer on Y_t is[5]

$$\xi = E\left[(Y_t^1 - Y_t^0) \mid X, D = 1\right] = E\left[Y_t^1 \mid X, D = 1\right] - E\left[Y_t^0 \mid X, D = 1\right] \qquad (6\text{-}1)$$

It is clear from 6-1 that I face an identification problem since $E[Y_t^0 \mid X, D = 1]$ is not observable. It is convenient to rewrite 6-1 in a slightly different way, closer to what I actually use in the empirical work. Suppose that IT was adopted in period k. Then, for $t > k > t'$, 6-1 is equivalent to

$$\xi = E\left[(Y_t^1 - Y_{t'}^0) \mid X, D = 1\right] - E\left[(Y_t^0 - Y_{t'}^0) \mid X, D = 1\right] \qquad (6\text{-}2)$$

[5]The quantity ξ refers to what is defined in the literature as the *average treatment effect on the treated*, i.e the average effect of IT only across those countries who adopted the regime.

This way of representing ξ allows us to exploit the panel data nature of the sample, and hence to control for fixed factors that could be correlated with the outcomes (i.e. most developed countries having less volatile inflation rates).

A common approach to estimate the expectation $E[(Y_t^0 - Y_{t'}^0) \mid X, D = 1]$ is to replace it with the observable average outcome in the untreated state $E[(Y_t^0 - Y_{t'}^0) \mid X, D = 0]$[6]. However, this could result in biased estimates of ξ from two sources[7]. The first arises from the presence of ITers in the sample that are *not comparable* with non-ITers and vice versa. The second is due to *different distributions* of X between the treated and the control groups, which is usual in non-randomised samples (like a dataset of countries). Fortunately, matching methods deal with these shortcomings.

6.1.2 Matching methods

The idea behind matching techniques is to eliminate the aforementioned biases by pairing ITers with non-ITers that have similar observed characteristics. The goal is to estimate a suitable *counterfactual* for each ITer, to reestablish the conditions of a randomised experiment (that is, random assignment of X) when no such data are available. Under these circumstances, the difference between the outcome of the treated and that of a matched counterfactual can be attributed to the treatment.

The propensity score

Usually, determining along which dimension to match the countries or what type of weighting scheme to use is a difficult task. Rosenbaum and Rubin (110) reduce the dimensionality of this problem by suggesting that the match can be performed

[6]See, for instance, Johnson (70) and Ball and Sheridan (6).
[7]See Heckman et al. (65).

on the basis of a single index that summarises all the information from the observable covariates. This index, the *propensity score*, is the probability of treatment conditional on observable characteristics,

$$p(X) = E\left[D \mid X\right] = \Pr\left(D = 1 \mid X\right) \tag{6-3}$$

and should satisfy the *balancing hypothesis*, which states that observations with the same propensity score must have the same distribution of X independently of the treatment status[8]. Hence, equation 6-1 can be rewritten as

$$\xi = E\left[(Y_t^1 - Y_{t'}^0) \mid p(X), D = 1\right] - E\left[(Y_t^0 - Y_{t'}^0) \mid p(X), D = 1\right] \tag{6-4}$$

The non-comparability bias can be eliminated by only considering countries within the *common support*, the intersection on the real line of the supports of the distributions $\{p(X) \mid D = 1\}$ and $\{p(X) \mid D = 0\}$. The bias from different distributions of X is eliminated by reweighing the non-ITer observations.

Estimating the propensity score is straightforward, as any probabilistic model suits 6-3. For instance, I can adopt the parametric form $\Pr\left(D_i = 1 \mid X_i\right) = F(h(X_i))$ where $F(.)$ is the logistic cumulative distribution (a logit). However, two points are to be handle with care. First, the estimation requires choosing a set of conditioning variables X that are not influenced by the adoption of the IT regime. Otherwise, the matching estimator will not correctly measure the treatment effect, because it will capture the (endogenous) changes in the distribution of X induced by the IT adoption. For this reason, the X variables should measure country attributes *before* the treatment[9]. Second, the model selection, i.e. the form of $h(X_i)$, can be used to test the balancing hypothesis. Dehejia and Wahba (37) suggest using a polynomial according to the following steps:

[8] Rosenbaum and Rubin (110) show that the conditions $D \perp \{Y^1, Y^0\} \mid X$ and $0 < p(X) < 1$ together (*strong ignorability of the treatment*) are sufficient to identify the treatment effect. In practice, I require a weaker and *testable* condition of ignorability for identification: *conditional mean independence*, $E[Y^0 \mid X, D] = E[Y^0 \mid X]$ and $E[Y^1 \mid X, D] = E[Y^1 \mid X]$.

[9] However, even these variables could be influenced by the programme through the effects of expectations.

- Start with a parsimonious logit specification (i.e. $h(X_i)$ linear)
- Stratify all observations on the common support such that estimated propensity scores within a stratum for treated and control countries are close. For example, start by dividing observations into strata of equal score range $(0-0.2, \ldots, 0.8-1)$.
- For each interval, test whether the averages of X of treated and control units do not differ. If covariates are balanced between these groups for all strata, the specification satisfies the balancing hypothesis[10]. If the test fails in one interval, divide it into smaller strata and reevaluate.
- If a covariate is not balanced for many strata, a less parsimonious specification of $h(X_i)$ is needed. This can be achieved by adding interaction and/or higher-order terms of the covariate.

It is important to emphasise that the role of the propensity score is to reduce the dimensionality of the matching, it does not necessarily convey a behavioural interpretation. Indeed, the logit regressions do not seek to find the determinants that made a central bank adopt an IT regime, but to characterise and summarise the economic state in which the ITers began to implement the regime. The difference is subtle but allows us to control for variables that although are useful to define the profile of a particular economy (importantly, relative to others), are not theoretically included in the central bank's decision to change the monetary policy regime[11].

The matched estimator

Given the propensity score, there are various methods available for finding a counterfactual for ITer i[12]. Following Heckman et al. (64) and Heckman et al. (65), I can compute a consistent estimator of the counterfactual by means of a kernel weighted average of outcomes. This approach not only has good statistical properties but is also a convenient way to work with a sample of countries, as it could be difficult to find an actual non-ITer for each ITer. Let $\mathcal{C}$ denote the set of non-ITer

[10] Actually, the weaker version of mean conditional independence. See footnote 8.

[11] See Mishkin and Schmidt-Hebbel (93) for an attempt to interpret a cross sectional logit of the IT adoption in behavioural terms.

[12] See Smith and Todd (119) for a review and examples.

countries whose propensity scores are over the region of the common support. The counterfactual of the outcome $Y_{i,t}^0$ is

$$\tilde{Y}_{i,t}^0 = \frac{\sum_{j\in\mathcal{C}} K_b(p_j - p_i) Y_{j,t}^0}{\sum_{j\in\mathcal{C}} K_b(p_j - p_i)} \tag{6-5}$$

where $K_b(z) = K\left(\frac{z}{b}\right)$ is a kernel function (with bandwidth parameter b) that weights the outcome of country i inversely proportionally to the distance between its propensity score value (p_i) and the one of the non-ITer j (p_j).

Having found the matched pairs of ITers and non-ITers, the treatment effect estimator for country i in period t can be written as

$$\hat{\xi}_{i,t} = \left(Y_{i,t}^1 - \frac{1}{k-1}\sum_{\tau=1}^{k-1} Y_{i,\tau}^0\right) - \left(\tilde{Y}_{i,t}^0 - \frac{1}{k-1}\sum_{\tau=1}^{k-1} \tilde{Y}_{i,\tau}^0\right) \tag{6-6}$$

where the pre-treatment outcome $Y_{t'}^0$ has been replaced by the time averages of $Y_{i,\tau}^0$ and $\tilde{Y}_{i,\tau}^0$ before the treatment[13]. The estimator 6-6 has no analytical variance, so standard errors are to be computed by bootstrapping (i.e. resampling the observations of the control group). Finally, the average of all possible $\hat{\xi}_{i,t}$ constitutes an unbiased estimator of 6-2,

$$\hat{\xi} = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{1}{T_i}\sum_{t=1}^{T_i}\hat{\xi}_{i,t}\right) \tag{6-7}$$

where N is the number of ITers in the sample an T_i is the number of years ITer i has been conducting its monetary policy under an IT regime.

6.2 Empirical issues

Before presenting the propensity score estimations and the "inflation outcomes" to be used in the evaluation, it is convenient to briefly discuss some issues regarding the dates the various central banks adopted their IT regime, i.e the period when treatment occurred.

[13]Heckman et al. (65) and Smith and Todd (119) suggest using a weighted average of the pre-treatment observations instead of a sole observation to control for possible outliers or trend effects. In 6-6 I have used a simple average (equal weights).

6.2.1 Adoption dates

In a number of cases the exact IT adoption timing is unclear: authors and central banks use different criteria. To address this ambiguity and for the sake of robustness, I use two possible adoption dates for each country. First, I consider dates when countries started some form of IT (*soft* IT), typically by simply announcing numerical targets for inflation or by stating that they were switching to IT. On the other hand, I use dates of *fully-fledged* IT adoption, namely, an explicit IT adoption as publicised by central banks and implying numerical targets for inflation together with the absence of nominal anchors other than the inflation target[14].

This approach contrasts with previous studies as it considers that many developing-country ITers used a soft version of IT as a strategy to reduce inflation from two-digit to international levels[15]; once inflation reached a stable low level, their central banks would reinforce the regime, by abandoning other nominal anchors and committing exclusively to target inflation. For example, Chile may appear as an early IT adopter (1991) in other studies but it ran exchange rate regimes not compatible with fully-fledged IT until 1999. For Peru, authors such as Corbo et al. (35) use a soft IT adoption date (1994), when the central bank announced an inflation target consistent with a money growth operational target, while Levin et.al (83) use its fully-fledged date (2002).

The year of IT adoption for developed economies is less controversial. In New Zealand for instance, the beginning of IT can be dated as far as 1988 when a numerical target for inflation was announced in the Government budget statement. Or, following Mishkin and Schmidt-Hebbel (93), 1990 when the first Policy Targets Agreement between the Minister of Finance and the Governor of the Reserve Bank of New Zealand was published, specifying numerical targets for inflation and the dates by which they were to be achieved. In 1991, a target range of 0 to 2 percent

[14]This information is available from the various central bank's web sites.

[15]See Fraga et al. (48) for a comprehensive survey of IT in developing countries.

for 1993 was announced[16].

In the case of Sweden, I follow Ball and Sheridan (6) for my fully-fledged classification given that the first announced inflation target was 2 for 1995 even though the Riksbank announced its shift to IT during 1993. For Canada, the first target range was announced in 1991. In 1993, a range of 1 to 3 percent was established for 1994 onwards.

In Table 6.1, I compare adoption dates among five different studies and provide my two possible adoption dates. Column “Class. 1” refers to the soft IT adoption dates while “Class. 2” accounts for fully-fledged IT adoption. In 6 cases I have more than a three-year difference between both dates: Chile (8 years), Colombia (4 years), Israel (5 years), Mexico (4 years), Peru (8 years) and Philippines (7 years). In others, such as Australia and the UK, both classifications coincide.

6.2.2 Propensity score estimations

In order to estimate 6-3, I built a yearly dataset for 109 countries containing a set of variables that broadly define an economy (X). The sources were the Penn World Table (PWT version 6.0) for GDP per capita and national accounts data, the IFS for international reserves, money and credit markets data and Romer and Romer (109) for exchange rate regimes[17].

The variables entered in the regression are the averages of the five years prior to the IT adoption for ITers. To check for robustness, for non-ITers I use either the average since 1990 up to 2004 or the 5 years previous to 1996 (for Classification 1) or 1998 (for Classification 2)[18]. As described earlier, I tested for the balancing hypothesis and selected the most parsimonious specification.

[16]The upper bound of this range was changed to 3 percent in the 1996 Policy Target Agreement.

[17]I also considered social indicators from the World Bank and other sources for central bank staff and geographical controls. These variables were not significative in the regressions.

[18]These are the average adoption dates in each classification.

In Table 6.3, I show the variables whose coefficients were statistically significant in the four estimated models: from the PWT, Investment to GDP, exports plus imports to GDP (namely, openness ratio) and the share of world GDP (GDP for a particular country to the sum of GDPs of the 109 countries in the database); from the IFS, the fiscal balance to GDP, inflation and its coefficient of variation (inflation volatility) and the money to GDP ratio; finally, the average number of years that a country was classified as *freely floating* by Romer and Romer (109).

Figure 6.1 displays the density of the propensity score for ITers and non-ITers derived for each of the estimated models. It can be seen that the densities for model (1) are close to those of model (3); similarly, model (4) resembles (2). For this reason, I will work with the first two specifications, where the differences between the propensities scores are driven by the alternative IT adoptions dates, and not by variations in the control group.

6.2.3 Inflation outcomes

A shortcoming of working with a wide control group is the low availability of data. Even though the Consumer Price Index (CPI) time series are readily available for most of the countries, this is not true with some interesting variables. Such is the case for inflation expectations (from surveys) or forecasts errors (from polls) that are directly influenced by IT adoption[19] or cross-sectional higher moments (skewness and kurtosis) of the CPI distribution.

Hence, the outcomes I use are quantities that can be extracted from conventional CPI data that broadly characterise inflation dynamics: level, variation and persistence. I built a yearly dataset from quarterly CPI information from the IMF's database (IFS), computed the counterfactuals and estimated the ITs effects

[19] See Johnson (70) for an application to a sample of selected countries.

as in 6-6 and 6-7[20,21]. For each year t the *level* of inflation is defined as the mean of the annualised quarterly inflation rates of years t and $t-1$. The same logic applies to the *standard deviation* of inflation.

The interesting debate on measuring inflation *persistence*[22] can be summarised in the equation

$$\pi_t - \mu_t = \rho(\pi_{t-1} - \mu_{t-1}) + \sum_{\tau=1}^{p} \beta_\tau \Delta(\pi_{t-\tau} - \mu_{t-\tau}) + \epsilon_t \tag{6-8}$$

that is a reparameterisation of a simple AR(p) process for $(\pi_t - \mu_t)$, the deviation of inflation (π_t) from its mean (μ_t). A common practice is to set $\mu_t = \mu$ and estimate the parameter ρ, which equals the sum of all the autoregressive coefficients in the original AR(p) representation[23]. The closer ρ is to one, the more persistent the inflation.

However, Robalo-Marques (106) has pointed out that if the true process in 6-8 has a time-varying mean, imposing $\mu_t = \mu$ leads to misleading conclusions. Particularly, a series that quickly reverts to a time-varying mean may be estimated as highly persistent (ρ close to one) if it is assumed to revert to an imposed constant level. To control for this undesirable effect, he suggests estimating μ_t as a smooth trend of π_t. Considering this, I use two measures of inflation persistence: the estimated ρ with $\mu_t = \mu$ and with μ_t approximated by the HP filter[24]. To compute these quantities I use rolling windows with between 10 and 15 years of quarterly data[25].

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[20] As a baseline I consider the pre-treatment period to be the average of the five years before the IT adoption (k in equation 6-6), as I did in the propensity score estimations. I also tried different definitions, though the results were not sensitive to this assumption.

[21] It is important to note that the number of years after IT (T_i in equation 6-7) varies as IT adoption dates do. For Classification 1 [2] there are $\sum_{i=1}^{N} T_i = 175\,[132]$ post-IT observations.

[22] See Robalo-Marques (106) for a survey. This author also shows that the approach followed here to measure persistence, even tough having some limitations, seems to the most reliable among simple alternatives.

[23] It is well known that the OLS estimator of ρ is biased when $\rho \simeq 1$. An alternative (and popular) estimator, that is adopted here, is proposed in Andrews and Chen (3).

[24] I use a smoothing parameter of $\lambda = 1600$. Different choices of λ do not qualitatively change the results.

[25] The lag length in 6-8, p, was selected to minimise the Schwarz criterion.

6.3 The effects of inflation targeting

In Table 6.2, I present the estimated average effects of IT for all ITers, for the group of industrialised countries as well as developing ones. I report effects on inflation dynamics according to the two alternative classifications of IT adoption. In the spirit of the mean-regression hypothesis of Ball and Sheridan (6), I also include the results obtained by controlling for initial (pre-treatment) conditions[26].

The first key result is that IT has significantly reduced mean inflation in all the cases. In general, I find that the benefits of *soft* IT adoption are stronger than those of *fully-fledged* IT adoption. This was expected due to high-inflation countries adopting IT to stabilise (the dates in Classification 1). Also, the benefits on developing countries have been significantly stronger than those on industrialised ones, which confirms previous findings in Bernanke et al. (14), Corbo et al. (35), Neumann and Von Hagen (97) and Pétursson (102). The results also suggest that regression to the mean is indeed an important phenomenon, since the effects of IT tend to be smaller once I control for initial conditions. However, by considering a substantially wider treatment and control groups than the ones in Ball and Sheridan (6), I find that there is no sufficient evidence to discard the benefits of IT: IT matters for mean inflation in both industrial and developing countries alike.

As mentioned in Faust and Henderson (46), "*Common wisdom and conventional models suggest that best-practice policy can be summarised in terms of two goals: first, get mean inflation right; second, get the variance of inflation right*". The finding regarding mean inflation supports the idea that IT in fact helps achieving the first goal. What about the second goal? During the period of analysis, inflation has been falling worldwide, and together, the variance of inflation has been decreasing everywhere as well[27]. The second finding precisely indicates that the observed

[26] To control for initial conditions, as in Ball and Sheridan (6), I perform estimations of average treatment effects on the treated by performing the regression $Y_{i,t} - Y_{i,t'} = \alpha + \beta Y_{i,t'} + e_{i,t}$.

[27] See Pétursson (102).

fall in the variance of inflation has been particularly strong within ITers, such that the treatment effect has been that of a marked reduction in inflation volatility. The pattern of this effect across country groups and IT classifications is similar to the one found for the level of inflation. Neumann and Von Hagen (97) and Corbo et al. (35) also provide evidence suggesting that IT has contributed to the fall in inflation volatility[28].

What can we say about IT effects on inflation persistence? As mentioned, there is no a straightforward theoretical prediction of the effects of IT on persistence. Adoption of IT can be linked to either lower or higher inflation persistence, it all hinges on two opposing effects: how fast central banks allow inflation to revert back to its mean after a shock and how price formation changes if expectations become more anchored. Studies like Levin et.al (83) show that persistence is lower in ITers than that in non-ITers whereas Ball and Sheridan (6) show there is no evidence that ITers achieve lower inflation persistence[29].

I find that the results depend on the measure of persistence (ρ) used. If I consider a constant unconditional mean in the inflation process ($\mu_t = \mu$) I find that IT increases persistence, though the estimates are not statistically significant and different from zero. Contrarily, if I allow for a time varying mean inflation ($\mu_t = \text{HP}$) I find that IT does reduce the persistence parameter. Interestingly, some sort of mean-regression is present under Classification 1 (soft IT): once I control for the initial persistence, the fall in ρ disappears. However, under Classification 2 (fully-fledged IT) the fall in ρ is significant even after controlling for mean-regression (which seem to exist in industrialised economies).

This last effect, although different from zero, is at most modest. The half

[28] Johnson (70) and Ball and Sheridan (6) suggest that IT increases inflation uncertainty. The finding in Johnson (70) in fact refers to volatility of expected inflation from surveys, a variable related to observed inflation volatility but with a dynamics of its own.

[29] Time series studies on persistence for industrial countries like Benati (10), Levin and Piger (84) or Robalo-Marques (106) point to the conclusion that high inflation persistence is not a robust feature of inflation processes in the euro-area.

life of a shock to inflation is, roughly speaking, $\tau \approx -\ln(2)/\ln(\rho)$[30]. The change in ρ implied by the results varies around -0.04; hence, considering an initial $\rho = 0.85$[31] the change in τ is just one quarter. All in all, the evidence on the effect of IT on inflation persistence, if any, is not as categorical as the one associated with the reduction in mean and volatility.

6.4 Conclusion

The increasing popularity of IT as a framework for conducting monetary policy calls for the evaluation of its benefits in comparison to alternative schemes. In this chapter I have combined data of IT adoption and inflation dynamics with programme evaluation techniques to assess the dimensions in which IT is a beneficial regime. The central findings support the idea that the adoption of IT, either in its soft or explicit form, delivers the theoretically promised outcomes: low mean inflation (around a fixed target or within a target range) and low inflation volatility.

I also find that IT has reduced the persistence of inflation in developing countries. Given that IT is understood to be flexible, the reduction in persistence is likely to be the effect of the anchoring of expectations to a defined nominal level. Nevertheless, the small magnitude of the reduction is such that it prevents us from categorically concluding in favour of IT in this particular dimension of the inflation dynamics. In the future, it would be useful to contrast these results with alternative measures of persistence. Also, a promising area for further research is to formalise the theoretical link between IT, inflation persistence and long-run expectations (credibility), which can guide subsequent empirical efforts.

The interpretation I gave to IT adoption, that of a "natural experiment", allowed us to use powerful evaluation tools normally applied in microeconometrics,

[30] This formula is exact if the estimated model is an AR(1).

[31] This is a generous value. The sample mean of the computed ρ after de-trending is just below 0.40.

where the odds to identify policy effects are by far higher than in macroeconomics. I also reckon that the study of the response of other macroeconomic variables (for instance, the business cycles and interest rates) to IT is essential in order to having a complete appraisal of the effects of the IT regime. Hence future research can explore further, within the IT adoption evaluation, the advantages of these techniques on a wider variety of macro indicators.

Table 6.1: Inflation targeters and dates of adoption †

	Corbo et al. (2002)	Fracasso et al. (2003)	Fraga et al. (2003)	Levin et al. (2004)	Pétursson (2004)	Ball & Sheridan (2004)	Class. 1	Class. 2
Australia	1994	1994	1993		1993	1994	1994	1994
Brazil	1999	1999	1999	1999	1999		1999	1999
Canada	1991	1991	1991		1991	1992(94)	1991	1994
Chile	1991	1991	1991	1991	1990		1991	1999
Colombia	1999	1999	1999	1999	1999		1995	1999
Czech Republic	1998	1998	1998	1998	1998		1998	1998
Finland ‡	1993					1994	1993	1993
Hungary		2001	2001	2001	2001		2001	2001
Iceland	2001	2001	2001		2001		2001	2001
Israel	1992	1992	1992	1992	1992		1992	1997
Mexico	1999	1999	1999	1999	1999		1995	1999
New Zealand	1990	1988	1990		1990	1990(93)	1990	1991
Norway	2001	2001	2001	2001	2001		2001	2001
Peru	1994	2002	1994	2002	2002		1994	2002
Philippines		2002			2002		1995	2002
Poland	1998	1998	1998	1998	1998		1998	1998
South Africa	2000	2000	2000	2000	2000		2000	2000
South Korea	1998	1998	1998	1998	1998		1998	1998
Spain ‡	1995					1994(95)	1994	1995
Sweden	1993	1993	1993		1993	1995	1993	1995
Switzerland	2000	2000	2000	2000	2000		2000	2000
Thailand	2000	2000	2000	2000	2000		2000	2000
United Kingdom	1992	1992	1992		1992	1993	1992	1992

† Blank cells mean the authors did not provide a clear reference of the date of IT adoption.
‡ Finland and Spain abandoned inflation targeting and adopted the Euro in 1999.

Table 6.2: Average treatment effect of Inflation Targeting †

	All ITers		Industrialized countries		Developing countries	
Classification 1			*Difference in means*			
Level	–4.802	(0.440)	–3.335	(0.627)	–6.320	(0.631)
Standard Deviation	–2.099	(0.323)	–1.546	(0.468)	–2.671	(0.452)
Persistence ($\mu_t = \mu$)	0.027	(0.042)	0.031	(0.068)	0.024	(0.050)
Persistence ($\mu_t =$ HP)	–0.028	(0.026)	–0.092	(0.023)	–0.039	(0.011)
Classification 2			*Difference in means*			
Level	–2.863	(0.235)	–1.327	(0.334)	–5.382	(0.297)
Standard Deviation	–1.551	(0.318)	–1.103	(0.386)	–2.286	(0.557)
Persistence ($\mu_t = \mu$)	0.027	(0.032)	0.003	(0.047)	0.066	(0.036)
Persistence ($\mu_t =$ HP)	–0.016	(0.024)	–0.061	(0.018)	–0.058	(0.012)
Classification 1			*Regression, controls for initial conditions*			
Level	–3.874	(0.745)	–2.804	(0.868)	–4.907	(1.269)
Standard Deviation	–1.863	(0.413)	–0.988	(0.568)	–2.708	(0.657)
Persistence ($\mu_t = \mu$)	0.030	(0.039)	0.012	(0.057)	0.049	(0.058)
Persistence ($\mu_t =$ HP)	–0.015	(0.031)	–0.006	(0.022)	–0.023	(0.024)
Classification 2			*Regression, controls for initial conditions*			
Level	–2.621	(0.312)	–1.603	(0.421)	–3.242	(0.337)
Standard Deviation	–1.798	(0.308)	–1.284	(0.383)	–2.112	(0.478)
Persistence ($\mu_t = \mu$)	0.043	(0.023)	0.012	(0.035)	0.094	(0.035)
Persistence ($\mu_t =$ HP)	–0.047	(0.021)	–0.033	(0.016)	–0.055	(0.016)

† Figures in parenthesis are bootstrapped standard errors (5000 replications).

Table 6.3: Propensity score estimation, logit regressions †

	(1)		(2)		(3)		(4)	
Classification for ITers	Class. 1		Class. 2		Class. 1		Class. 2	
Classification for non-ITers	> 1990		> 1990		Class. 1		Class. 2	
Investment to GDP	0.337	(0.099)	0.250	(0.073)	0.402	(0.111)	0.282	(0.076)
Openness ratio	–0.057	(0.012)	–0.042	(0.013)	–0.010	(0.027)	–0.065	(0.019)
Share of world GDP	–0.591	(0.199)	–0.342	(0.161)	–0.712	(0.313)	–0.437	(0.244)
Fiscal balance to GDP	0.291	(0.166)	0.147	(0.103)	0.325	(0.150)	0.159	(0.120)
CPI Inflation	0.428	(0.133)	0.254	(0.099)	0.351	(0.126)	0.242	(0.097)
Inflation volatility	–5.206	(1.926)	–3.599	(1.543)	–4.523	(1.957)	–2.929	(1.752)
Money to GDP	0.033	(0.015)	0.027	(0.013)	0.051	(0.021)	0.028	(0.015)
Exchange rate regime	–0.232	(0.079)	–0.154	(0.061)	–0.207	(0.079)	–0.141	(0.055)
Observations	100		100		100		100	
Pseudo R^2	0.6114		0.4704		0.6066		0.4940	
LR stat, $\chi^2(8)$	65.95		50.74		65.43		53.28	
Common support region	[0.036, 0.998]		[0.037, 0.994]		[0.030, 0.993]		[0.015, 0.995]	
non-ITers in common support	28		31		30		43	

† Figures in parenthesis are robust standard errors.

D Appendix: Propensity score estimations

I present some details on the propensity score estimations under various definitions of IT adoption dates. It is important to recall that the role of the propensity score is to reduce the dimensionality of the matching, it does not necessarily convey a behavioral interpretation. Indeed, the logit regressions below do not seek to find the determinants that made a central bank adopt an IT regime, but to characterize and summarize the economic state in which the ITers began to implement the regime. The difference is subtle but allows us to control for variables that although are useful to define the profile of a particular economy (importantly, relatively to others), are not theoretically included in the central bank's decision to change the monetary policy regime[32].

I built a yearly dataset for 109 countries containing a set of variables that broadly define an economy. The sources were the Penn World Table (PWT version 6.0) for GDP per capita and national accounts data, the IFS for international reserves, money and credit markets data, Romer and Romer (109) for exchange rate regime, the World Bank for social indicators and other sources for central bank staff and geographical controls.

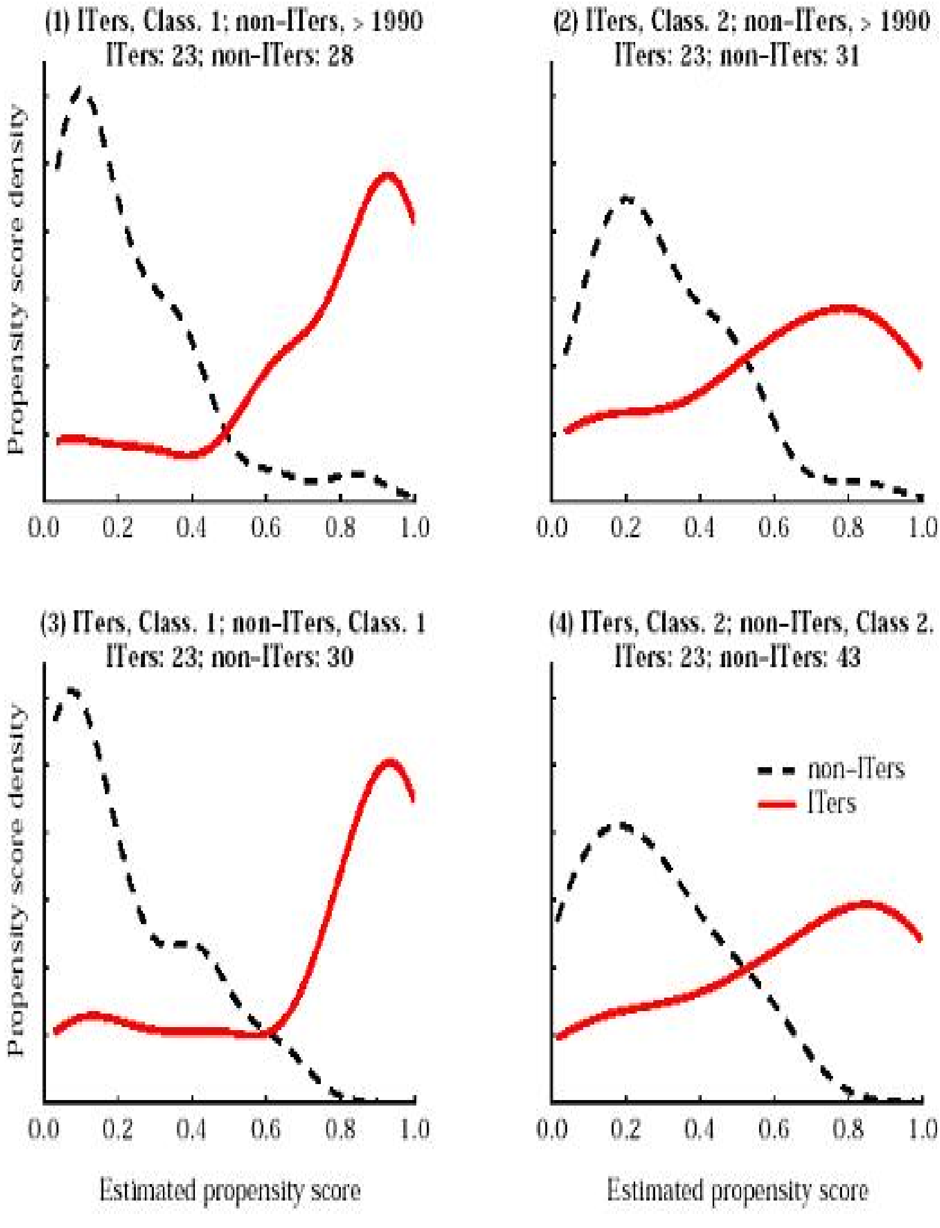
The variables entered in the regression are the averages of the five years previous to the IT adoption for ITers. To check for robustness, for non-ITers I use either the average since 1990 up to 2004 or the 5 years previous to 1996 (for Classification 1) or 1998 (for Classification 2)[33]. As described in the main text of the chapter, I tested for the balancing hypothesis and selected the most parsimonious specification.

In Table 6.3, above we show the variables whose coefficients were statistically significant in the four estimated models: from the PWT, Investment to GDP,

[32]See Mishkin and Schmidt-Hebbel (93) for an attempt to interpret a cross sectional logit of the IT adoption in behavioral terms.

[33]These are the average adoption dates in each classification.

Figure 6.1: Propensity score densities by IT adoption date.



exports plus imports to GDP (namely, openness ratio) and the share of world GDP (GDP for a particular country to the sum of GDPs of the 109 countries in the database); from the IFS, the fiscal balance to GDP, inflation and its coefficient of variation (inflation volatility) and the money to GDP ratio; finally, the average number of years that a country was classified as *freely floating* by Romer and Romer (109).

Figure 6.1 displays the density of the propensity score for ITers and non-ITers derived for each of the estimated models. It can be seen that the densities for model (1) are close to those of model (3); similarly, model (4) resembles (2). For this reason, we work with the first two specifications in the text, where the differences between the propensities scores are driven by the alternative IT adoptions dates, and not by variations in the control group.

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