THESIS: WAGES AND CAPITALIST PRODUCTION.

By Victor Edelberg

Inscribed to

Professor Irving Fisher.

MOTTO: To maintain and augment...consumption is the sole end and purpose of both fixed and circulating capitals.

SMITH.

MOTTO: Machinery and labour are in constant competition with each other and the former can frequently not be employed until labour rises.

RICARDO.

MOTTO: We can have no reference to the use of capital, without introducing time as the essence of the matter.

JEVONS.

MOTTO: It appears to me of the highest possible importance that we should be quite clear in our science about the causal interrelation of goods.
.........the idea of causal connection is inseparable from the idea of time........

MENGER.

MOTTO: Time is the formal condition *a priori* of all phenomena whatsoever.

KANT.

MOTTO: "The passage of time" - there is no such thing.

WITTGENSTEIN.

MOTTO: IN EVERY ECONOMIC PLAN, FUTURE TIME IS THE UNIVERSAL PARAMETER

The Scope and the Abstract of the thesis:

"WAGES AND CAPITALIST PRODUCTION"

My purpose is to build an explanation of production that takes time into account. Adam Smith and the English Classical economists approached the problem by studying the relation between profits and wages or wages and capital. They emphasised the time aspect of production by saying that capitalists "advance" wages to labour long before its final consumption product is ready. On the back ground of that conception they painted a rough picture of production through time.

I follow the classical tradition both in the title of my work and in parts 2. and 3. of my work where I approached the problem of production by studying equilibrium between the rates of interest and wages.

Some light is thrown on the nature of this equilibrium by the statistics of distribution of the national income.

My main results are in part 4. which contains a fairly general theory of production that takes time into account. I found the use of mathematics indispensable.

The best known attempts to formulate such a theory are Böhm-Bawerk's, Professor Fisher's and J.B. Clark's (which is examplified by Dr. Hicks' Theory of Wages). As I show, Böhm-Bawerk's famous doctrine of the "average period of production" is based on a mathematical mistake. Professor Fisher's analysis of the "output streams" while very instructive, contains a greater number of unknowns than equations to determine them. Dr. Hicks assumes that capital is a homogeneous factor of production assisting the production of consumption goods which emerge either instantaneously or after time lags. If there are time - lags, Dr. Hicks assumes it makes no difference what these lags are. This is an attempt to deal with the time aspect of production by means of abstracting from it.

In recent articles (Economica and the Economic Journal) a very distinguished economist, Professor F. Knight, appears to think that the problem of capital is incomprehensible and that it is no use trying to understand it.

All these difficulties of theory mean that the pronouncements of economists on every concrete question of production - such as wage policies - are necessarily surrounded by a penumbra of considerable doubt. And no substantial improvement can take place until matters of general principles are cleared up.

I try to clear them up. The essence of my analysis in part 4 is this. Assuming for simplicity that each competing firm controls a vertically completely integrated process of production of a consumption good, and writing $t=0$ as "the present", an entrepreneur finds himself in possession of a set of goods c at $t=0$ and is aware of a large number of alternative production plans. Each plan provides for future investment at a rate $f(t)$ in value terms and for output of the consumption good at a rate $u(t)$. For simplicity we can take the good as the "numeraire". As between all the alternative plans the functions u(t), f(t) are different. I.e. they have certain fields of variation. The central idea of the analysis is this: for each particular input function f(t) the output function u(t) can be varied within a certain field by varying the methods of production. This is the "missing link" which the earlier theories failed to bring out.

Given the interest rate ρ, the condition which determines which plan is chosen is that the present value of the collection of goods c

$$x = \int_0^\infty e^{-\rho t}[u(t)-f(t)]\,dt \qquad (1)$$

(is finite since $\rho > 0$)

is maximised. I.e. that the present value of future profits is maximised (and is zero under perfect competition).

That means that for each possible input function f(t) such an output function u(t) has to be considered as maximises x. I.e. the first partial variation

$$\delta_u x = 0 \qquad (2)$$

As f(t) is varied, different output functions u(t) become possible, and for each f(t) an output function u(t) is segregated which maximise x i.e. satisfies (2). Thence, in (1) f(t) becomes the only independent variable, and the equilibrium condition is that x is maximised with respect to the input function f(t), i.e.

$$\delta x = 0 \qquad (3)$$

This condition determines which plan is chosen i.e. determines the unknown functions f(t), u(t).

From (1) (2) (3)

$$\delta \int_0^\infty e^{-\rho t} u(t)\, dt = \delta \int_0^\infty e^{-\rho t} f(t)\, dt$$

i.e. the present value of the future marginal product equals the present value of the future marginal cost.

In this way a marginal productivity theory is built up which takes time into account.

Then the theory is extended: account is taken of uncertainty, of "vertical disintegration" etc., etc., until a fairly comprehensive picture is reached of the course of production through time.

INTRODUCTION

I wish to explain the course of production through time.

To explain how planners (entrepreneurs) adopt production-plans and "revise" them as they learn new facts.

In spite of appearance to the contrary, that explanation was attempted by the so called "theory of productivity of capital". The history of the theory is given in part 1.

Part 2 is a comment on the state of the theory to date, and part 3 deals with some statistics.

Part 4 is completely self contained and provides a better (more general) "picture" of the course of production through time.

Note: Peano's system is used to number the parts of this work with integral figures, and the sections with decimals. Equations have numbers in brackets. Parts, sections and equations are referred to by their numbers. Thus 2.1 means 'part two, section one', and (5) means 'equation number five'.

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I am indebted to books. Especially to Smith, Ricardo, to Kant and to Volterra's book on functionals.

TABLEAU ÉCONOMIQUE.

Objets à considérer, 1°. Trois sortes de dépenses ; 2°. leur source ; 3°. leurs avances ; 4°. leur distribution ; 5°. leurs effets ; 6°. leur reproduction ; 7°. leurs rapports entr'elles ; 8°. leurs rapports avec la population ; 9° avec l'Agriculture ; 10°. avec l'industrie ; 11°. avec le commerce ; 12°. avec la masse des richesses d'une Nation.

DÉPENSES PRODUCTIVES relatives à l'Agriculture, &c.	DÉPENSES DU REVENU, l'Impôt prélevé, se partagent aux Dépenses productives et aux Dépenses stériles.	DÉPENSES STÉRILES relatives à l'industrie, &c.
Avances annuelles pour produire un revenu de 600.tt sont 600.tt	Revenu annuel de	Avances annuelles pour les Ouvrages des Dépenses stériles, sont
600.tt produisent net	600.tt	300.tt
Productions (moitié passe icy)	(moitié passe icy)	Ouvrages, &c.
300.tt reproduisent net	300.tt	300.tt
(moitié passe icy)		(moitié passe icy)
150. reproduisent net	150.	150.
(moitié &c.)		(moitié &c.)
75. reproduisent net	75.	75.
37..10.s reproduisent net	37..10.	37..10
18..15. reproduisent net	18..15.	18..15
9...7...6.d reproduisent net	9...7...6.d	9...7...6.d
4..13...9. reproduisent net	4..13...9.	4..13...9
2..6..10. reproduisent net	2..6..10.	2..6..10
1..3..5. reproduisent net	1..3..5.	1..3..5
0..11...8. reproduisent net	0..11...8.	0..11....8
0...5..10 reproduisent net	0..5..10.	0..5...10
0...2..11 reproduisent net	0...2..11.	0...2..11
0....1...5 reproduisent net	0....1...5	0....1....5

&c.

REPRODUIT TOTAL .... 600.tt de revenu ; de plus, les frais annuels de 600.tt et les interets des avances primitives du Laboureur, de 300.tt que la terre restitue. Ainsi la reproduction est de 1500.tt compris le revenu de 600.tt qui est la base du calcul, abstraction faite de l'impôt prélevé, et des avances qu'exige sa reproduction annuelle, &c. Voyez l'Explication à la page suivante.

1. Historical Introduction

1.1 François Quesnay, the founder of the French school of Physiocrats or Les Economistes, gives the earliest analysis of the process of production as a whole, Tableau Oeconomique 1759, cf bibliography. The table describes behaviour of production in a country during a short interval of time. It shows production in a stationary equilibrium, p i. A given money investment in agriculture - and in extractive industries in general - produces a return sooner or later, greater than the investment by a surplus which gives a Revenue to the "proprietor" who is assumed to have made the investment called an "advance". The return is double the advance, and produces a surplus of 100% . Agriculture is "productive" in the sense of being productive of a surplus over investment.

Certain advances of food and materials produced by agriculture are made to certain producers - as town artisans - who transform these advances into manufactured goods - "clothes, utensils, instruments", pii. They merely transform food and materials into manufactures and they add nothing to their value. They produce no surplus over the advances. Hence manufacturing industry is "Sterile" - sterile of a surplus.

Economic criteria are not used and the distinction between productive and unproductive investments is due to Physiocratic - view of production as a mere physical process.

The germ of truth is that land rent is, in the Ricardian sense, a surplus. Further, the distinction is artificial: if we consider advances in agriculture to comprise advances to proprietors as well as to farmers and labourers, the return is equal to the advances, in equilibrium, and agriculture becomes as "unproductive" as manufacturing.

The table shows transactions between three classes: proprietors, agriculturalists and manufacturers. The annual income consisting of consumption goods, materials and instruments exchanged is 1500liv. 600 is the "Revenue" of landowners 600 of agriculturalists and 300 of manufacturers. The 300 of manufacturers is not an addition to material income but is a result of double counting. The 300 is the value of food and materials advanced to them by landlords and agriculturalists, which are merely transformed into manufactures. Hence only 1200 liv. worth is produced during the year. It is the value of the output of food and materials by the extractive industries. The annual advance of 600 to agriculture produces - sooner or later - the 1200 liv. of output which is distributed in the manner described. The annual advance of 600 worth of food and materials and instruments maintains an output of

1200 liv ,piii, every year and the whole process of production is in equilibrium, pi.

The zigzag of the table indicates that the transactions between the classes of society do not take place in lump sums once a year, but continuously during the year.

If the 'productive' advance is diminished from 600 to 500 agricultural capital is not maintained and the output falls to 1000 liv, pii. If productive investment is increased to 700 the annual output rises to 1400, pii.

Quesnay analyses the maintenance, decumulation and accumulation of capital and vaguely suggests that the rate of output of consumption goods varies in the same direction as the value of capital.

1.2 Physiocratic theory reaches in many ways its fullest expression in Turgot's Reflexions sur la Formation et la Distribution des Richesses, written in 1766. But Turgot's weakness is that he gives no view of production in an economy as a whole. In § Lii the time factors necessitating "advances" are brought out. Production takes time. "Even if one should till the land with one's hands, it would be necessary to sow before reaping: it would be necessary to live until after the harvest." A workman must have an advance of tools and materials before he can set to work, and food to enable him to "subsist while waiting for the sale of his finished goods." § Liii describes the primitive process of production through time of a hunter who turns cultivator.

§ Lx describes several time-stages of leather making from construction of "canals and different buildings" and training of apprentices to the application of labour of workmen "during several months until the leather is sold". Certain productive acts are roughly repeated, wear and tear of tools and cattle etc. must be annually replaced, §Lxii, and "materials must be replaced as they are being consumed," §Lxvi.

Advances in production yield a surplus over investment. Hence business men are prepared to pay interest on the capital they borrow, §Lxx. The normal rate of interest is determined by conditions of supply of, and demand for, loans.

"A loan..... is the use of a certain quantity of values during a certain time,"

The rate of interest tends to fall as capital is accumulated, § Lxxxii, xc.

1.3

Adam Smith describes the behaviour of production both over a short and a long period of time.

The behaviour of national production over a long period is described in the sections on division of labour, Wealth of Nations 1776, Introduction, Bk. 1. chs. i, ii., and the chapter on the component parts of price, Bk. 1. Ch. vi. It is shown that production of consumption goods is a process which spreads over a long period of time, involves 'vertical' division of labour, and the function of intermediate products - fixed and circulating capital goods - is explained. Ch.vi explains that the amount of money, for which a unit quantity of a consumption good is bought, is equal to the wages and rents of all the labour and land which have contributed to its production over a long period of time, plus interest on these "advances" earned before the commodity is "brought to market"

It is seen that Prof. Taussig's statement on p. 150 of his "Wages and Capital" that Smith did not describe the process of production through time as a whole is quite incorrect.

Moreover, the process through time is implicit in any analysis of using up and of production of capital goods, and is implicit in Smith's analysis in Bk. 2, Chs. 1, 2, 3 of the behaviour of the process of production during a short interval of time, one year.

In ch i, on the Division of Stock, the using up and production of intermediate and final products is described in terms of a distinction between "fixed" and"circulating" capital.

Smith means by fixed capital goods those which are sold slowly, and by circulating those sold quickly in the form of products of a business. The first include durable equipment of the entrepreneur. The second also includes durable goods, but those only which are held for sale by their makers or middlemen. "Circulating capital" goods include various material and consumption goods in shops.

In the beginning of ch ii distinction is made between"gross" and"net" annual revenue. "The gross revenue of all the inhabitants of a great country comprehends the whole annual produce ~~(sic)~~ of their land and labour; the net revenue, what remains free to them after deducting the expenses of maintaining, first their fixed, and secondly their circulating capital, or what, without enroaching upon their capital, they can ....spend upon their

subsistence, conveniences and amusements."
i.e. Gross revenue is the value of all the intermediate and final products produced during a year. To illustrate the concept: in a stationary equilibrium gross revenue is equal to the value of national capital multiplied by one year. Total depreciation during the year is the value of capital multiplied by one year and, in this case, the value of capital is maintained constant. The "net revenue" is the difference between "gross revenue" and total depreciation, during a year, and is the value of the annual income of consumption goods (+ net appreciation or - net depreciation of the value of national capital). Smith does not deal with changes in the value of capital which are due, as one could put it, to changes in stock-exchange valuations. He thinks only of genuine investment (positive or negative) of money which would otherwise be spent on consumption. Hence his net revenue always equals the value of the

+ annual output of consumption

national income of consumption goods. All these concepts are fundamental and in their terms Smith outlines the process of using up of capital goods and production of capital and consumption goods, in a country during a year.

In Ch. iii On the Accumulation of Stock there is a change of definitions. While in Ch. ii "annual revenue" is "gross revenue", in Ch. iii "annual revenue" signifies the income of consumption goods, and "Neat" revenue is annual profits plus rents, and is not to be confused with "net" revenue of Ch. ii, which means the income of consumption goods. Prof. Cannan does not see this change in definitions and his attack on pp75-7 of his "<u>Production and Distribution</u>" 3rd ed. against Smith is based on a misunderstanding. Smith also draws a curious distinction between "productive" and "unproductive" labour. Labour producing intermediate "products" is called "productive", that yielding consumable services - as servants - is "unproductive". In the case of the first interest accumulates on the wage advance before the final product is sold, in the case of the second the product is immediate and there is no time in which interest can accumulate. Hence "productive" and "unproductive" labour means "productive" or "unproductive" of a <u>surplus</u> over the wage advances of interest. In this distinction Smith follows the Physiocrats in calling something "productive" if it yield a surplus over the advances. Only the Physiocrats thought that natural resources alone were productive of such a surplus. Smith's analysis of land in this connection is sketchy and as such is criticised by Ricardo, "Principles" xxi p174 of "Works" ed. McCulloch, and Mill, "Principles" Bk iv. Ch. iv § i.

In terms of all these distinctions in Ch. iii Smith again shows how the value of the national capital may be maintained, increased or diminished. His analysis of gross and net saving is remarkable and contains most of the best ideas of Mr. Keynes' "Treatise on Money". E.g. of net saving Smith writes "What is annually saved is as regularly consumed, and nearly in the same time too; but by a different set of people."

Smith's theory of the productivity of capital is set out in his chapters on division of labour, and Bk ii: introduction and Chs iii, v.

Smith's view is well summarised in ~~a quotation from~~ Bk. ii Ch. iii: "The productive power of the same (a given) number of labourers cannot be increased, but in consequence either of some addition and improvements to those machines and instruments which facilitate and abridge labour; or of a more proper (vertical) division and distribution of employments. In either case additional capital is almost always required."

Smith well observed the positive correlation, in a country in the value of capital per head and the annual income of consumption goods. He explained this fact by superior productiveness, in the long run, of the more capitalistic methods of production.

As to the details of the theory of capital, he observed that increase in the value of capital per head tended to increase real wages and diminished the market rate of interest, and conversely , Bk i Ch. ix , Bk ii Ch v, c.f. Bibliography.

Adam Smith provides an outline of all the essentials of the theory of capital.

1.4

These essentials were brought out more clearly by Ricardo. An account of Ricardo's theory of capital, with exact page references to his writings, is available in the article "The Ricardian Theory of Profits" , Economica Feb. 1933. According to Ricardo time is the essence of capital. He writes about "the ... time that must elapse before the results of ...labour can be brought to market," and adds, "All questions of fixed capital come under the ... rule, which I will endeavour to explain to you if you should wish it." (Ricardo, Letters to MacCulloch, p.65.) Elsewhere he writes that circulating capital goods involve time in the same way. Ricardo calls intermediate products fixed and circulating according as they are durable or not. The distinction, he writes, is one of degree only. The difference between Smith's and Ricardo's definitions of the terms 'fixed' and 'circulating' capital should be noted.

As to the productivity of capital, Ricardo shows that the annual output of consumption goods and the rate of real wages are increasing functions of "the proportion capital bears to the population." and the rate of interest is a diminishing function of that proportion. The concept of the value of capital per head, as Böhm shows in an appendix to the later editions of his Positive Theory, is equivalent to the average period of production. It should be noted that Ricardo and his school used the word "capital" in several senses, and in two senses when writing of the proportion of capital to labour. In the first sense "capital", as above means the value of all the intermediate products. In the second, it means consumption goods advanced to labour as wages. Using the second sense, "the proportion of capital to labour" equals the average real wage, which is the famous identity of the theory of the wages-fund.

In Ch. i Sec. iii of his "Principles" Ricardo gives an example where a given output is produced by 100 men working during a year or by 85 making a machine in one year, the machine yielding the following year the same output as that of 100 men. The first is direct process, the second, roughly, is of one year average duration (waiting). The average period in the first process is 0 in the second 1. The example shows that the process of longer average period is more productive <u>per capita</u>. Ricardo implies that in general the longer the average period, the greater the product p.c.: "...in real value (labour cost) commodities made by machines fall, and fall in <u>proportion to their durability</u>."

Ricardo adds that the lower is the market rate of interest, the longer is the average period of the processes employed and the higher the <u>real</u> rate of wages tends to be. It should be noted that in ch i and sometimes elsewhere, the term "wages" is used in a curious sense of, roughly, the increase of the rate of interest, and the phrase, "rise of wages" means a fall in the rate of interest. This point which is essential for the understanding of Ricardo is explained at length in the article "The Ricardian Theory of Profits". Böhm (Capital and Interest) and particularly Wicksell (Uber Wert Kapital Und Rente p10, Vorlesungen p 233, Untersuchungen pp. 26,7) realise that this example of Ricardo illustrates the fundamental proposition that the total product depends on the way that it accrues through time. Ricardo shows that, in the language of mathematics, the total product is a functional of time.

1.5.

John Rae in his New Principles of Political Economy 1834, writes that capital goods produce future consumption goods. With given labour force and state of knowledge different sets of capital goods can be produced and arranged in order according as they yield consumption goods more and more distantly on the whole, in the future. Sets giving more future consumption output yield a greater total output, but at a diminishing rate.
A horizon of foresight is

implied. The increase of durability of fixed capital goods -houses- is shown to be one of the principal ways of projecting final output further into the future and increasing its total amount.

Rae anticipates the concept of (62) for a fixed set $f_1 \ldots f_n$, 4. The set of "instruments" adopted depends on "the effective desire of accumulation." Invention is analysed.

1.6

Stanley Jevons in <u>Theory of Political Economy</u> 1871 illustrates production through time by diagrams similar to fig. 2 , 2.3. He calls the flow of consumption goods advanced as wages 'capital investment' and the value of capital in terms of wage-goods 'capital invested', his formula for the latter is restated by (6).

He formulates final product as a function of time. He assumes a case described in 2.2, 2.6 where a given labour force is applied at a moment say t=o and its whole final product f(t) emerges at <u>t</u>. If <u>t</u> is lengthened and f(t) postponed by <u>δt</u> the product increases by $f(t+\delta t) - f(t)$. $f(t+\delta t) - f(t)$ is the <u>amount</u> of interest on the on the principal f(t) earned during δt . . . . The rate of interest per unit-time is in the limit $\frac{f'(t)}{f(t)}$ as $\delta t \to 0$, p 346 4th.ed. Its dimension is $f(t).t^{-1}.f(t)^{-1} = t^{-1}$, p249.

If f(t) is written for anything which grows with time, $\frac{f'(t)}{f(t)}$ becomes the general expression for the rate of interest per moment.

The rate of interest can only be >0 if f(t) being >0, $f'(t)>0$, eg. $f(t)= a.t$, $f'(t)=a$, where a is a positive constant, p 247; $\frac{d}{dt}\left(\frac{f'}{f}\right)<0$ pp254-5, and 246-7.

Jevons hints that in f(t), t can also be interpreted as an average period of production, pp228-9, 246.

Jevons exposition is explicit. He begins by saying p223 that he agrees with Ricardo that the essence of capital is time. "The views which I shall endeavour to establish on this subject are in fundamental agreement with those adopted by Ricardo".

1.7.

Carl Menger is the founder of the Austrian theories of wants and production. In his Grundsätze der Volkswirtschaftslehre 1871 he writes that man has a system of wants. Certain goods - bread, dwelling houses- directly cause satisfaction of wants. They are consumption goods: "goods of the first order." Goods of the first order are caused by goods of the "higher order" - instruments, materials, natural resources, labour pp 7-10. Goods of higher order produce either consumption goods directly or other goods of less high order and through a more or less long series of good of diminishing order they indirectly produce consumption goods. This causal system is called production. All causal connections imply time, hence production takes time, I § 4. The value of all goods depends on the satisfaction they are expected to cause, p133. This value is found on the basis of the principle of marginal productivity, pp 138-. The behaviour of production as a whole over a long period of time is described, pp 129 - 30.

The greater, on the whole, is the period between the application of goods of the highest order and the emergence of the imputed consumption goods, the greater is the total final output. Or else combinations of different final products can be produced by longer methods which are preferable to any combinations of goods producible by the shorter methods. All this is due to the utilisation of different laws of nature pp 131-2, 136 footnote.

Menger's article, cf. bibliography, adds that in a changing economy the value of capital goods extant at a moment of time is not equal to their cost of production incurred in the past. Their value depends on the future satisfactions they are expected to cause. In that sense in economics the past is irrelevant, cf. 4.2.

1.8

Böhm Bawerk in his Positive Theorie des Kapitales, 1888, elaborates Menger's theory about time in production (c.f. bibliography). The central concept is the average period of production. At any given technical knowledge, enterpreneurs can choose among certain alternative methods of production involving different average periods. With a given labour-force and with a given average period, the method selected is the one giving the largest output.

The market rate of interest cannot be > 0 unless product p.c. is an increasing function of the average period of production. It is an empirical law that, within limits, the product of a given labour force is an increasing function of the average period, and increasing at a diminishing rate, cf 2.7 and 2.13,14.

(46) gives the definition of Böhm's average period. In Part ii Book iv section ii c on the determination of equilibrium in the labour: capital market the following is implied. Of the rates of interest and wages, one being given, such an average period is adopted by the entrepreneur, that the other is maximised. The more coherent part of Böhm's theory is mathematically restated in 2.3, 2.7, 2.11; 2.7 shows that it is necessary for Böhm to assume the rates of wages and interest constant through time at undetermined levels. Otherwise equilibrium is undetermined. His analysis is limited to stationary equilibrium and non-durable, circulating, capital goods.

1.9.

In Über Wert Kapital und Rente 1893, pp 95-105, Wicksell mathematically restates Böhm's theoryand on pp121-3 adds that land is on the same footing as labour and writes $p=F(t,h)$, where p is product per man, h land per man, and t the duration of the process. He really means $F(t,h,i)$, i standing for unit-labour. Application of the unit-labour and h is evenly spread during t and $\frac{t}{2}$ is the average period, cf 1.11. There are **three** equilibrium equations $p = (l + h.r)(1 + \frac{z.t}{2})$

$p = F(t, h, i)$?

$$\frac{\partial p}{\partial t} = (l + h.r) \cdot \frac{z}{2}$$

$$\frac{\partial p}{\partial h} = r\left(1 + \frac{t.z}{2}\right)$$

where l = wages per man, r = rent per acre and z is simple rate of interest. Given l and r, z is maximised and the four unknowns h, p, t, z are determined by the four equations.

Similarly equilibrium is determined if l, z or r, z are supposed given.

Wicksell determines l, r by equations

$$h = \frac{B}{A}\ ; \quad K = \frac{t}{2} \times (A.l + B.r) \qquad (7)$$

where B is acreage, A the number of workers and K the capital in a country. A, B, K are supposed given. Given A, B, for each given value of K the six equations determine a specific equilibrium.

Wicksell shows in a simple example how the theory of production-through time applies to foreign trade, pp128-9, and to internal trade in two commodities, pp130-5. The analysis is limited to stationary equilibrium, constancy through time of the levels of the rates of wages, rent and interest, and to circulating capital. Cf 2.3,7 and 2.2,6.

In an article, cf bibliography, Wicksell gives an analysis of two cases of durable goods. The analysis is reformulated in 2.4, 2.8.

1.10

Irving Fisher's Capital and Income 1906 distinguishes between various meanings of "capital" and "income"

It contains two false propositions.

First, that "services" of goods (events caused by goods, p145) are not goods but are "abstract" (?), p105 etc.

Second, that the total of "services" consumed by consumers is identical with "net income", p158 etc.

These two false propositions are supressed in the following summary.

"Psychic income---------is one's whole conscious life", p176.

It is a succession of "satisfactions" (atomic mental states).

To a man, possible psychic incomes are ends.

Goods are "material objects owned by human beings", p2.

Goods are part-causes of satisfactions. They are means to psychic incomes (ends).

The ends are chosen by choosing the means. To a man: "The desirability or utility of goods (of a totality of goods) is the present esteem in which the future satisfactions from these goods are held", pp43-4.

Comment: I think the proposition: "There are scales of value for goods (material objects)" has no sense. Because material objects are not concrete experiences but are formal space-—time aspects of experiences. I.e. because of "complementarity" between goods and satisfactions.

Only concrete experiences exist. Only alternative totalities of them (psychic incomes) can be objects of valuation--can be arranged in order of preference. C.f. 4.18 —

Indifference curves etc. for goods are meaningless. cf. 4

Continuing the summary, "capital" means the collection of goods "existing" during an "instant" (a very short interval) at a time t, p 51.

Symbolizing, capital $= \overset{\omega}{\underset{\alpha}{S}}\, g(t)$

meaning: "sum" from good α to good ω, of the quantity of any good G(t) extant at t. Money value of capital $= \overset{\omega}{\underset{\alpha}{S}}\, p(t) \cdot g(t)$

where p(t) symbolizes price of any good, at t.

"Gross income" during an interval (a, b)

$$= \underset{\alpha}{\overset{\omega}{S}} \int_{t=a}^{b} dg_p(t)$$, where $dg_p(t)$ is

quantity of any good produced at t, pp119--.

Logical genesis of the integral $\int_a^b dg_p(t)$ is complicated, 2.1 .

"Outgo" $= \underset{\alpha}{\overset{\omega}{S}} \int_a^b dg_c(t)$ where $dg_c(t)$

is quantity of any good consumed at <u>t</u>.

"Net income" $= \underset{\alpha}{\overset{\omega}{S}} \int_a^b dg_p(t) - dg_c(t)$

Its money value $= \underset{\alpha}{\overset{\omega}{S}} \int_a^b p(t) \left[dg_p(t) - dg_c(t)\right]$.

N.B. Net income consists of 0 , >0 , <0 quantities of every kind of good: foods, machines, men etc. etc. , and not only of "consumption" goods.

These concepts (Capital, gross and net income, and their money values) are originally due to the Physiocrats, 1.1 , also 1.2,3 .

The dimension of capital and gross and net incomes are complexes of the dimensions of the good of which they are composed.

Thus, if B, β, C signify the dimensions of bread beer and cheese, respectively, a net income consisting of bread, beer, cheese.....etc is of dimensions B, β, C.......etc.

Rates of income per interval of time have a dimension T^{-1}, say where T is the time-dimension.

Thus average net money income during (a,b) is

$$\frac{\sum_{\alpha}^{\omega} \int_a^b p(t)\left[dg_p(t) - dg_c(t)\right]}{b - a}$$

and is of dimension $£T^{-1}$, where £ means the dimension of money.

Suppose a business own a collection of goods in the "present", at t = o. Let f(t). dt be the money value of its product sold at t. The total future net income from the collection is

$$\int_0^{\infty} f(t)\, dt.$$

In equilibrium, the present value of the collection is

$$\int_0^{\infty} f(t)\, e^{-\rho t}\, dt,$$ where

ρ is the market rate of interest, p384.

a time dt and income which is not. The dimension of the value of capital is M.t and that of money income is the timeless dimension M.

In the Rate of Interest 1907 Fisher analyses the technical opportunities to produce in their time aspect as a determinant of the rate of interest. The 1930 version, The Theory of Interest, is quoted.

Stated rigorously his theory is as follows: A given collection of labour and other capital goods existing in the present moment $t=o$, produces a future income stream $f(t)$ over a period of time. By varying the methods of production the "time shape" of $f(t)$ can be varied within limits set by the given technical knowledge. Mathematically we have a functional $f(t) = F\left[f(t)\right]$. The field $\left[f(t)\right]$ denotes all streams which are technically possible. Fisher calls $[f]$ "the investment opportunity... the opportunity to shift from one optional stream to another" p151. Given the rate of interest such a stream $f(t)$ is selected which has the maximum present value, viz. maximises the integral $\int_0^\infty e^{-\rho t} f(t)dt$ which is the present value of the given collection of capital goods, p175. Different shapes of income streams are illustrated on pp 169-70.

The analysis is restated in detail in 2.9 where $\Theta(\tau) = \int_0^\tau f(t)dt$, and for simplicity only labour is assumed in place of Fisher's heterogeneous collection of capital goods.

Unlike the theory of Böhm and Wicksell, Fisher's analysis is explicitly future-looking and is not limited to stationary equilibrium. Capital and Income deals with fluctuating interest, p390, and risk, Ch xvi and pp403-11.

Fisher's analysis has two limitations. He does not show how parts of the stream f(t) are ditributed between seperate capital goods and labour in the given collection, and how f(t) itself is segregated from the total stream of a society. Secondly he assumes f(t) to be independent of - for example- the quantities of labour with which the given collection may have to co-operate at t=o and in the future at successive moments t (Present factors co-operate with future labour via intermediate products). He assumes that the "investment opportunity" [f(t)] is given independently of future labour etc. and that the form of the stream f(t) can be varied independently of the future labour etc.

which is never really the case.

These problems are solved in 4.

1.11.

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"To the memory of John Rae and Eugene von Böhm-Bawerk who laid the foundations on which I have endeavoured to build."

On history of the theory:

Böhm Bawerk <u>Kapital und Kapitalzins</u> 1884

<u>Capital and Interest</u> 1890

translated by Smart;

Böhm's interpretations are more accurate than his evaluations. E. g. he outlines correctly the theories of Ricardo and Jevons, and then dismisses Ricardo as "colourless" and Jevons as an "eclectic....a genius - perhaps - but still an eclectic."

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The positive part describes the time-structure of production. The period of production is projected into the past pp 10-13 and so is made irrelevant to questions of productivity since nothing which has happened in the past can be influenced by anything in the present, 4.2. Hence the productivity aspect is neglected. Erroneously change in technical knowledge, invention, is said to be a necessary condition of superior productiveness of more roundabout methods, pp9-10. It is not stated that the superior productiveness of more roundabout methods is a pre-condition of a positive market rate of interest.

The historical part accurately brings out what the main theories studied have to say on the time-structure of production and omits what they have to say on the productivity aspects, except pp 311, 313

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2 One Original Factor Production

A Description of processes in equilibrium.

Note:

If some propositions of the table 2.1 are not clear on a first reading, they can be understood by studying particular cases 2.2 - 2.5.

2.1 Table of general definitions and expressions.

The concrete historic process of production of consumption goods is causally organic. All goods produced in an interval of time have some common causal antecedents, e.g. invention of the steam engine.

But for certain theoretical purposes independent processes having simultaneous parts and yet having no common causal antecedents or consequents within the field of description, can be imagined. Postulating for simplicity homogeneous labour and homogeneous consumption goods, we do not inquire into the causes of labour and the consequences of consumption goods, but suppose labour to be the ultimate causal antencedent of consumption goods within the field of description delimited by labour and consumption goods.

This implies that production must start with an application of labour without co-operation with any factor. The first application causes, say, some intermediate product with which another application of labour will co-operate and so the process is started. The process is completed, if ever, when no more labour is applied and no more consumption goods are produced which are direct or indirect (via the intermediate products) effects of the previous applications of labour.

Theoretically, several such processes can be started simultaneously and in succession. The condition of independence of one such process from another is this: no labour application in one process is a part cause of any consumption good in another process. This is a formal or tautological definition of independent processes.

t is a point on the continuous time-scale.

$t=0$ is the present.

The time-scale and the time-dimension of goods are analysed in 4.1-8.

A unit of a concrete quantity has a duration δt. E.g. a unit of labour "labour x hour" has a duration δt = one hour.

Writing X for a quantum of labour, $\dot{X} = \frac{X}{\delta t}$ is the rate of flow of labour per unit time. Economising symbols and using same symbols for the dimensions of quantities as for the quantities themselves, dimension $\dot{X} = Xt^{-1}$; and the dot is used to indicate clearly that in constructing $\dot{X} = \frac{X}{\delta t}$ the time-dimension of the concrete quantity X is abstracted (cancels out) and that $\dot{X}$ is a timeless quantity. In what follows I use the dot notation in the same way. A dotted symbol represents a rate of a concrete quantity per unit time i.e. represents a timeless quantity.

I assume that in a firm independent processes are started one at a time continuously i.e. at successive moments <u>dt</u>. I study a process which starts at $t=0$ and ends at $t=s$. The meaning of starting and ending a process has been explained above.

$d\dot{X}_t$ is the rate of labour input into the process at <u>t</u>.

$d\dot{Y}_t$ is the rate of output of the consumption good from the process at $\underline{t}$.

$\dot{Y}_\tau = \int_0^\tau d\dot{Y}_t$ is the total rate of output up to $t = \tau$, i.e. $\dot{Y}_t dt$ is the total output in the process up to t.

A unit of good Y in its timeless dimension $\dot{Y}$ is taken as a unit of account. All prices are expressed in terms of it.

$w = \frac{\delta \dot{Y}}{\delta \dot{X}}$ is the rate of wages, a rate at which a rate $\delta \dot{Y}$ of the good is exchanged against a labour rate $\delta \dot{X}$ in the market.

$d\dot{B}_t = w d\dot{X}_t$ is the rate of wage-bill input at $\underline{t}$.

In cases of "point input" 2.2,4,5,6,8

$\dot{x}$ is written for $d\dot{X}_0$

x is written for $\dot{x}dt$

$\dot{b}$ is written for $d\dot{B}_0$

b is written for $\dot{b}dt$

In cases of "point output" 2.2,3,6,7

$\dot{y}$ is written for $d\dot{Y}_3$

y is written for $\dot{y}dt$

The $\underline{\text{amounts}}$ x,b,y depend on dt and are therefore arbitrary. They are introduced in order to be able to speak simply of inputs and outputs etc., instead of the input and output rates.

This especially simplifies exposition in cases 2.3-5 involving second derivatives of concrete quantities with respect to time e.g. in 2.3 $\dot{X}_s = \int_0^s \frac{d\dot{X}}{dt} dt$.

ρ is the compound rate of interest. Interest is assumed to accumulate continuously.

e is the base of natural logarithms.

$$\dot{Z}_\tau = \int_{t=0}^{\tau} e^{\rho(\tau-t)} (d\dot{B}_t - d\dot{Y}_t) \quad \ldots\ldots\ldots\ldots(1)$$

is the sum of net increments $d\dot{B}-d\dot{Y}$ invested plus interest upon them, up to $t = \tau$.

= the cost of the rate of flow of the intermediate products in the process at τ.

Under perfect competition, when the process finishes the entrepreneur makes neither profit on it or loss i.e.

$$0 = Z_s = \int_0^s e^{-\rho t} d\dot{B}_t - \int_0^s e^{-\rho t} d\dot{Y}_t \quad \ldots\ldots(2)$$

$$\dot{W}_\tau = \int_0^\tau d\dot{B}_t - d\dot{Y}_t \quad (3)$$

is the net value invested less interest accumulated up to τ, net wages-bill rate, invested up to τ or the wage compound of $\dot{Z}_\tau$.

$$\dot{H}_\tau = \int_0^\tau d\dot{X}_t - \frac{1}{W} d\dot{Y}_t \quad (4)$$

is W_τ in terms of labour rate, a concept of classical economists.

$$K = \int_0^\sigma \dot{Z}_\tau \, d\tau \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

using (1), integrating by parts and using (2),

$$= \left[-\frac{1}{\rho} \int_s e^{\rho\tau} e^{-\rho\tau} (d\dot{B}_\tau - d\dot{Y}_\tau) \right]_{\tau=0}^{s}$$

and writing $\dot{Y}_s = \int_0^s d\dot{Y}_t$, $B_s = \int_0^s dB_t$;

$$\frac{\dot{Y}_s - \dot{B}_s}{\rho} \qquad \ldots\ldots\ldots\ldots\ldots(5')$$

is of dimension $Y = \dot{Y}.t$ since the dimension of ρ is t^{-1},

is (a) value of "capital disposal" or "waiting" necessary to finance the whole process from $t = 0$ to $t = s$ (is analogous to Prof. Cassel's concept in his Nature and Necessity of Interest and Theory of Social Economy)

is (b) the sum of values of intermediate products extant at $t = s$ of a continuum s/dt of coexisting identical processes started successively at intervals dt from $t = 0$ to $t = s$.

is (c) the value at any moment of the stock of intermediate products owned by a firm in a state of statuary equilibrium and amploying labour at a constant total rate $\dot{X}_s = \int_0^s d\dot{X}_t$, identical processes being started successively and continuously.

(5') proves the familiar proposition that the value of capital equals the total income from it per annum divided by the rate of interest.

$$F = \int_0^s \dot{W}_\tau \, d\tau \qquad (6)$$

is the famous Wages-Fund (the source of wages) of the calssical economists and Böhm-Bawerk, extant in stationary equilibrium; to be distinguished from $\dot{B}_s$ which the classics also called the wages-fund (source) conceived as a rate of flow.

$$A = \int_0^s H_t \, dt \qquad (7)$$

is the calssical concept of capital as accumulated labour extant in stationayy equilibrium.

Point input, Point output.

This is the simplest (Wicksell-Lectures) case to consider. Here a labour force x is applied at a point dt of time and no labour is applied later in the process - point input. After an interval of time a consumption product y emerges - also at a point of time - point output. An example is a labour force preparing raw wine - in a negligible time - which matures, some years later, into a consumable wine which is then sold and consumed at once, Wicksell. The input separated from the output by s, the duration of the process.

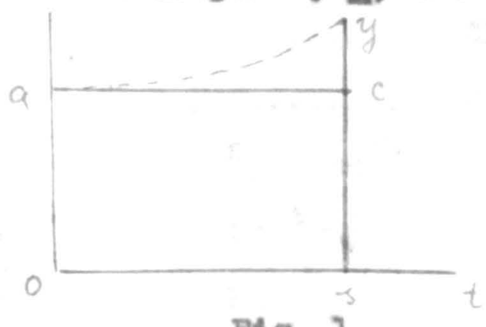


Fig. 1.

Fig. 1. depicts the process. At $t = 0$, a labour force x is applied i.e. the wages-bill $w.\ x$ measured by oa is invested and earns compound interest, the value of the intermediate product at each moment being measured by the ordinate of the dotted curve which is exponantial. At $t = s$ the output y emerges at once and is indicated by the line sy. The area under the curve ay measures K, the capital disposal.

The area also measures the sum of the values of intermediate products, existing at $\underline{t}=\underline{s}$ if the like processes were continuously started from $t=0$ to $t=s$, i.e. measures the value of capital in stationary equilibruim. The area of the rectangel $\underline{o}\ \underline{a}\ \underline{c}\ \underline{s}$ is capital as the 'wages-fund' and is equal to the area $\underline{o}\ \underline{a}\ \underline{y}\ \underline{s}$, K, less area $\underline{a}\ \underline{y}\ \underline{c}$, representing interest accumulated. If we take $\underline{w}=1$ then area of the rectangle $\underline{o}\ \underline{a}\ \underline{c}\ \underline{s}$ measures also capital as 'accumulated labour' $\acute{A}$.

Analytically, in the point - input - point - output case:

$d\dot{X}_0=\dot{x}$ and all other dX's are o

$d\dot{B}_0=\dot{b}$ .... $d\dot{B}$'s .....

$d\dot{Y}_s=\dot{y}$ .... $d\dot{Y}$'s .....

$\therefore$ from (1) $\qquad \dot{z}_t=\dot{b}e^{\rho t}$

from (2) $\qquad \dot{b}e^{\rho s}=\dot{y} \qquad$ (8)

from (5) $\qquad K=\dfrac{\dot{y}-\dot{b}}{\rho} \qquad$ .... (9)

from (6) $\qquad F=\dot{b}s \qquad$ .... (10)

and from (7) $\qquad A=\dot{x}s \qquad$ .... (11)

2.3 <u>Continuous input and point output.</u>

In this case labour is applied gradually through time (gradual input) from $t=o$ when a process is started to $t=s$, when the final product point output emerges all at once.

Fig. 2 depicts the special Böhm - Bawerk - Wicksell U. W. K. R. case in which continuous input takes place at a constant rate.

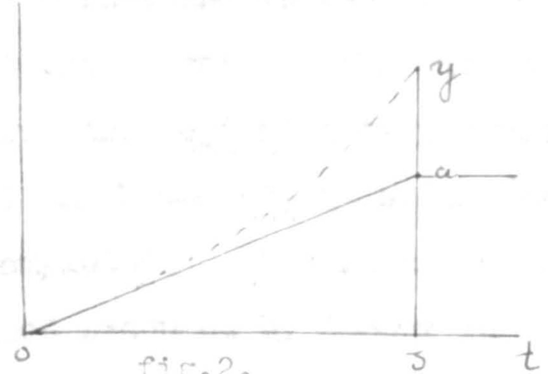


fig.2.

Taking w equal to unity, the same line o a is the curve of labour and wages-input up to t; o a is a straight line the slope of which indicates the constant rate of input; o y is the curve of the value of the intermediate product at t; s y shows the output y, s a the total wages bill and a y the interest component of the output. The value of capital K is shown by the area o s y, the wages fund F and capital as 'accumulated labour' A by the area o s a.

Analytically:

$\frac{d\dot{X}_t}{dt}$ is constant between t = o and s.

$\frac{dB_t}{dt}$ is constant = k say, between t = 0 and s (taking w constant)

$d\dot{Y}_s = \dot{y}$, and all other $d\dot{Y}$'s are 0.

$\therefore$ from (2) $\frac{k(e^{\rho s}-1)}{\rho} = \dot{y}$ (12)

from (5) $K = \frac{\frac{k}{\rho}(e^{\rho s}-1) - ks}{\rho} = \frac{\dot{y} - \dot{B}_s}{\rho}$ (13)

from (6) $F = \frac{ks^2}{2} = \frac{\dot{B}_s \cdot s}{2}$ (14)

from (7) $A = \frac{\dot{X}_s \cdot s}{2}$ (15)

which is the well known formula of Böhm Bawerk.

2.4 <u>Point input - continuous output.</u>

In this process all the labour force is applied at one moment and the output accrues continuously from the beginning of the process until it stops at which moment, $t = s$, the process is said to be completed. This is a case of a durable consumption good produced by labour in a negligible time dt. Åkerman and Wicksell assume the rate of output $\frac{d}{dt}\dot{Y}_t\,dt$ to be constant, (for each value of <u>dt</u>) say kdt between $t = 0$ and $t = s$.

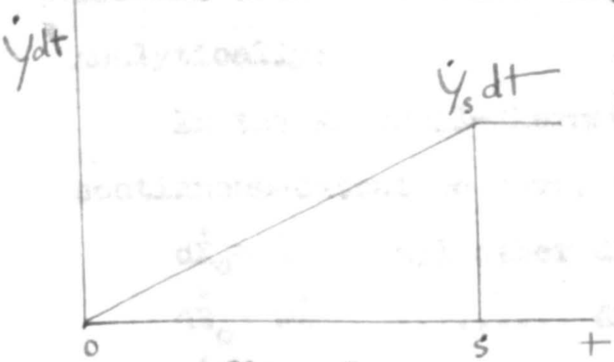


fig. 3.

Fig. 3 represents this process. The straight line <u>o</u> $\dot{Y}_s$ of which the slope measures the constant rate of output, is a portion of the graph of output $\dot{Y}_t\,dt$ accrued up to t. Beyond <u>s</u> the graph becomes parallel to the <u>t</u> axis, indicating that no further output accrues. The general relations (1) - (7) of the table 2.1 can be illustrated in the point-input-continuous-output case on a three dimensional diagram:

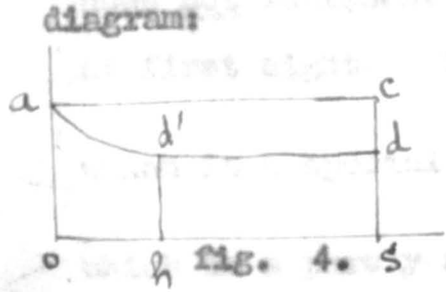


fig. 4.

In fig. 4 <u>ac</u> is the graph of $\frac{d\dot{Y}_t}{dt} = k$, <u>ad</u> of $e^{-\rho t}k$. The area <u>o a d s</u> represents the wages bill rate b, or the

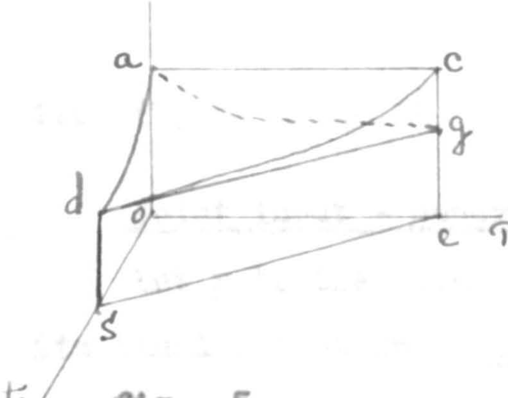


fig. 5.

value of the intermediate product rate at $t = o$. The value of the intermediate product rate at any moment $t = \tau$ is measured by area o a h d', where $h = s - \tau$ The integral of these areas x dt between $\tau = o$ and $\tau = s$ is the volume o a e c s d of fig. 5, and represents the value of capital K. If $\frac{d}{dt}\dot{Y}_t$ is not taken constant, geometirc representation becomes even more cumbersome.

Analytically:

In the Wicksell-Akerman case of point-input-continuous-output we have:

$d\dot{X}_o = \dot{x}$ all other dX's $= o$

$d\dot{B}_o = w\dot{x}$ .... $d\dot{B}$'s $= o$

$d\dot{Y}_t$ = constant k between $t = o$ and $t = s$.

$\therefore$ from (2) $\dot{b} = \frac{k(1 - e^{-\rho s})}{\rho}$ (16)

" (5') $K = \frac{ks - b}{\rho}$ (17)

(6) $F = \dot{b}s - \frac{ks^2}{2}$ (18)

Note: The volume o a e g s d (where ag is exponential) does not represent the wages fund, as may be expected at first sight. It represents $\int_o^s \int_\tau^s k e^{-\rho t} dt\, d\tau$ which is a special case of $\int_o^s \int_o^\tau (d\dot{B}_t - e^{-\rho t} d\dot{Y}_t)\, d\tau$

which is a purely artificial expression.

from (7) $$A = \dot{x}s - \frac{Ks^2}{2w} \qquad (19)$$

2.5 Point input - general output.

Let x be the point input and $\dot{Y}_t dt = \Theta(t)$ (20)

its final output up to t. In the most general case Θ can be of any form either continuous or discontinuous or both. 2.1, 2.2 and 2.3 are special cases of Θ

from (2) $$\dot{b} = \int_{t=0}^{s} e^{-\rho t}\, d\dot{y}_t \qquad (21)$$

" (3) $$K = \frac{\dot{y}_s - b}{\rho} \qquad (22)$$

etc.

The output Θ (t) can be interpreted as either independently given, or, more generally, as imputed to x according to its total marginal productivity, 4.

2.6 The second interpretation of Θ (t) can be illustrated as follows.

A house may be imagined constructed by a continuous input of labour and when ready, giving continuous output of consumable house service, in co-operation with the labour of a housemaid.

For simplicity we take the rate of continuous input in house-building, the rate of service from the house, and the rate of servant's labour all constant, we assume that the rate of interest ρ is zero, and for further simplicity, contemplate a state of stationary equilibruim to describe which in terms of a $\theta(t)$ function is the purpose of the present illustration.

Let x be the total labour force employed, consisting of two components x_1, builders and x_2 servants, let u be the time it takes to build a house, and n the life of a house. Let any moment when x begins to be applied be taken as $t=0$.

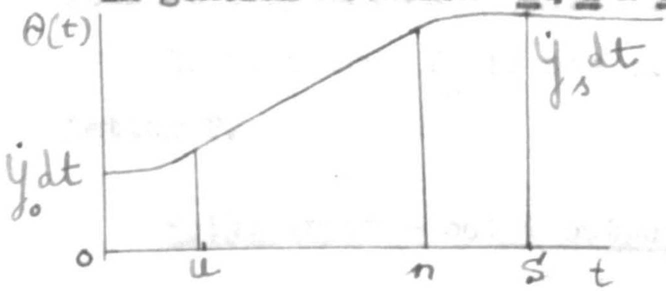
The output imputable to the servants x_2 accrues at once and is $Y_0^{\#}$ The house builders x_1, are spread evenly between all the houses under construction in a stationary equilibruim at $t=0$.

It is easy to see that on the above assumptions the quantity of finished houses imputable to x_1, will accrue at a constant rate k from $t=0$ when the house which was at the point of completion just before $t=0$, is ready, to $t=u$, when the house just started at $t=0$ is completed.

From $t=u$ to $t=n$ the quantity of houses standing, imputable to x_1, is constant (since n is the life of a house) and diminishes at a constant rate k to zero between $t=n$ and $t=n+u$.

The rate of service from houses is proportionate to the number of houses. Therefore the rate of service imputable to x_1, increases from o at a constant rate k from $t=0$ to $t=u$, is constant between $t=u$ and $t=N$, and diminishes to zero, at a constant rate k, between $t = n$ and $t = \underline{n} + \underline{u}$.

Hence the graph of output $\theta(t)$ imputable to $\underline{x}$ is as shown in fig. 6. It starts at Y_0 at $t = 0$, is a quadratic between $t = 0$ and $t = u$, linear between $t = \underline{u}$ and $t = \underline{n}$, and quadratic again between $t = \underline{n}$ and $t = \underline{n} + \underline{u}$, and is parallel to the $\underline{t}$ axis beyond. In general notation $\underline{n} + \underline{u} = \underline{s}$.



fig, 6.

The general method of deriving $\theta(t)$ functions applicable to nonstationary cases, is given in 4.

B. Product functionals and functions.

First are examined product functions on the basis of the special kinds of processes studied by the earlier writers. Then those founded on $\theta(t)$ output-functions. I write $\dot{Y}_t dt = Y_t$ and $\dot{B}_t dt = B_t$.

The form of the productivity functions will be determined by the empirical condition that the relative shares of capitalists and labour in the final product are constant i.e. that $Y_s : B_s$ is constant, based on the statistical evidence of 3.

To avoid irrelevant complication, the labour force $\dot{X}_s$ employed by an entrepreneur is assumed constant and "economies of scale" are abstracted from. All productivity functions are assumed homogeneous and linear with respect to $\dot{X}_s$. Perfect competition is postulated.

Total output Y_s is now expressed by the letter P.

2.7 Point input - point output.

It is supposed that the known technical methods of production consist of a domain of point input-point-output processes of various durations s.

i.e. P is a functional of the output function Y_t

i.e. $$P = F\left[Y_t\right]$$

Given the rate of interest, the entrepreneur will select such a method (such a Y_t) that the rate of wages he pays is maximised,

i.e. using (8) $$w.x = b = P.e^{-\rho s} \qquad (23)$$

is maximised.

Whence, regarding ρ as an independent variable, we can write the output P as a function of s.

i.e. $$P = f(s) \quad \ldots\ldots\ldots\ldots\ldots (24)$$

which is Wicksell's period productivity function, Lectures.

From the above maximal condition it follows that f(s) means that for a given duration s, only the method giving the greatest output P is used, in a case, if there is one, where there is more than one method of the same duration.

Since ρ is given and b is at a maximum, using (23)

$$\rho = \frac{dP}{ds \cdot P} \quad \ldots\ldots\ldots\ldots\ldots (25)$$

That b is maximum means

$$\frac{d(Pe^{-\rho s})}{ds} = 0$$

and $\frac{d^2(Pe^{-\rho s})}{ds^2} < 0$; whence using (25)

$$\frac{d^2P}{ds^2} < \left(\frac{dP}{ds}\right)^2 \frac{1}{P}$$

which is the condition of the possibility of maximising b, i.e. of the possibility of competitive equilibrium at given rates of interest. Therefore $\frac{d^2P}{d\gamma^2}$ can be positive in some cases e.g. when $f(s) = s^2$

From (25) if $\rho > 0$, $\frac{dP}{ds}$ must be positive, since P cannot be negative i.e. the technical condition for the existence of a positive rate of interest is the condition that the methods of longer duration are more productive.

The empirical condition 3. is

$$\frac{P}{b} = e^{\rho s} = \text{constant}$$

$$\therefore \quad \rho s = \text{constant.}$$

$$= \alpha \text{ say,}$$

α is > 0 since in general $P > b$, or the output is greater than the wage-input

using (25)

$$\frac{f'(s)}{f(s)} = \frac{\alpha}{s}$$

$$\therefore \quad f(s) = Cs^{\alpha} \qquad \text{..........(26)}$$

(where C is an integration constant) which is the empirical form of the function.

also $\frac{d\rho}{ds} = -\alpha s^{-2} < 0$, since $\alpha > 0$.

All the above conclusions are similarly obtained when w is regarded as given and ρ as maximised.

2.8 In Böhm's case P is a functional $F\left[X_t\right]$ of the function $X_t = \dot{X}_t dt = dt \int_0^T d\dot{X}_s dt$ which is linear in the interval (o,s) and constant for $t > s$. As $X_s = \dot{X}_s dt$ is constant (for each value of dt) $\frac{X_s}{s}$ is determined by s.

Of the rates of interest ρ and wages w one being given, such an X_t is selected that the other is maximised in the equation

$$P = \frac{X_s}{s} \frac{w(e^{\rho s} - 1)}{\rho} \quad (12)$$

Made subject to this maximal condition the functional $F\left[X_t\right]$ becomes a function $P = f(s)$, say.

Böhm and Wicksell U. W. K. R. make w determinate by assuming it to be constant during a process of production. This assumption limits analysis to stationary equilibria. This is an independent assumption as there is no reason why in a world of Böhm's processes with rigid linear inputs w should be so constant and determinate.

But granted their assumption, we have $X_s w = B_s$, the empirical condition is $P : B_s$ is constant whence using (12) s is constant as between different stationary equilibruim positions. Using (12) and taking say given and B_s maximised we have

$$0 = \frac{dB_s}{ds} = (e^{\rho s} - 1)\left(s\frac{dP}{ds} + P\right) - \rho s P e^{\rho s}$$

$$\therefore \frac{dP}{ds.P} = \left(\frac{\rho s e^{\rho s}}{e^{\rho s} - 1} - 1\right)\frac{1}{s} = \frac{\text{constant}}{s} = \frac{\alpha}{s} \text{ say}$$

$$\therefore \quad P = C s^{\alpha} \quad \ldots\ldots\ldots\ldots\ldots\ldots(27)$$

2.9 <u>Point input - linear output.</u>

We now suppose that all the known methods are of that type, studied in 2.4, and are of various durations <u>s</u>.

As before P is a <u>functional</u> of the output function Y_t

$$P = F\left[Y_t\right]$$

Given the rate of interest, the entrepreneur uses such a Y_t as maximises the present value

$$\frac{P}{s}\,\frac{(1 - e^{-\rho s})}{\rho} = b \quad \ldots\ldots\ldots\ldots\ldots\ldots(16)$$

of the output.

Therefore P can be regarded as a function of <u>s</u> $\qquad P = f(s).$

From the above maximal condition, using (16)

and differentiating logarithmically:

$$0 = \frac{db}{ds.b} = \frac{dP}{ds.P} + \frac{\rho e^{-\rho s}}{1-e^{-\rho s}} - \frac{1}{s} .$$

If relative shares are constant

P : b and $\therefore \rho s$ are constant (using (16))

$\therefore \quad \frac{dP}{ds.P} = \frac{\alpha}{s}$ where α is a constant

$\therefore \quad P = Cs^{\alpha}$ (28)

Footnote.

This is the form which Wicksell selected at random and deduced from it the constancy of the relative shares, his article, cf 1,11, instead of vice versa. Also, following Rae and Åkerman, he formulates it in a roundabout way, which leads to circumlocution in his mathematics. He writes $a = kn^{v}$; where a is the amount of labour invested in each fixed capital good, k and v are constants, and n = s is the life time of the capital good, the goods, of whatever life-time being supposed to yeild services at a constant rate as n varies.

In his article Wicksell also studies the following case: A fixed capital good (durable consumption good) is constructed by a point input x_1 while another labour force x_2 co-operates with extant capital goods and the output P_2 imputed to x_2 is <u>direct</u>, i.e. emerges all at once. The output P_1 imputable to x_1, emerges continuously at a constant rate $\frac{P_1}{s}$. The life <u>s</u> of the fixed capital good is assumed constant, though in other respects its quality can be altered, but total output $P = P_1 + P_2$ can be varied by altering the ratio $\frac{x_1}{x_2} = r$, keeping $x_1 + x_2 = x = \text{constant}$, i.e. $r = \frac{x_1}{x - x_1}$ i.e. by using a smaller or greater proportion of the labour force x <u>indirectly</u>, through the intermediary of the capital goods, i.e. $P = f(r)$

Assuming the rate of wages <u>w</u> given and constant through time and using (16) we have

$$P = P_1 + P_2 = wx_2 \frac{\rho s w x_1}{1 - e^{-\rho s}} = w(x - x_1) + \frac{\rho s w x_1}{1 - e^{-\rho s}},$$

since ρ is then maximised

$$\frac{dP}{dr} = -w\frac{dx_1}{dr} + \frac{w \rho s}{1 - e^{-\rho s}} \cdot \frac{dx_1}{dr} = w\frac{dx_1}{dr}\left(\frac{\rho s}{1 - e^{-\rho s}} - 1\right);$$

$w, \rho, \frac{dx_1}{dr}$ are positive, hence $\frac{dP}{dr}$ is positive if $\rho s > (1 - e^{-\rho s})$, which is so, since $\rho s > 0$.

This analysis is limited to a comparison of different states of stationary equilibrium by:

(a) The assumption that the rate of wages paid to the makers of capital goods is the same as that paid later to labour co-operating with these capital goods, and (b) the assumption that the total output P is imputed to the use of the labour force x for a small interval dt, which presupposes that for each value of r a full stock of capital goods has been accumulated (i.e. that at any moment the number of capital goods made by x_1 , is equal to the number, of the same kind, that become worn out and fall out of use).

This part of Wicksell's work, as he himself realises, does not state the conditions or the possible causes of stationary equilibrium. It does not explain the transition from one stationary equilibrium to another, or what can prevent such a transition, it does not study the process of displacement of some kinds of capital goods by others, but merely compares the different states of stationary equilibrium: i.e. shows how P is correlated with r in different stationary equilibria. The "neighbouring point" of a stationary equilibrium is not another stationary equilibrium.

But his analyses of point input - point output, and point-input-continuous output do state the conditions of stationary equilibrium (at a uniform rate of interest) because neither of the above assumptions (a) (b) has to be made in these cases. In these cases only one rate of wages - that at the moment of input is to be determined and the product of the labour input is independent of the co-operation of the extant capital goods, because the point input is assumed to take place without co-operation with extant capital goods. Hence given either ρ or W the euqilibrium is determined in these tow cases. But all his analysis is limited by the postulate that ρ is constant through time at an undetermined level.

2.10 <u>Point input - general output</u>.

On the assumption that θ (t) is capable of independent variation e.g. is independent of the future labour with which x may co-operate we have the following analysis.

A given state of technical knowledge implies a domain of $\theta(t)$ functions viz. output up to t is a functional of $\theta(t)$ i.e. $Y_t = F\left[\theta(t)\right]$.

Under competition, given the rate of interest ρ, the entrepreneur adopts θ_ρ such that the rate of wages w and the wages bill b are greater than if any other $\theta(t)$ had been adopted

viz $$b = \int_0^\infty \theta'_\rho e^{-\rho t}\, dt > \int_0^\infty \theta' e^{-\rho t}\, dt \qquad (29)$$

where θ signifies any alternative output function other than the selected one θ_ρ. For simplicity in notation θ's are assumed to be continuous. The analysis in case of discontinuous θ's is the same in principle. The upper limits of the integrals are written as ∞ instead of s to avoid distinguishing between different values of s when different θ's are used. This does not affect the values of the inegrals because after $t = s$ $dY_t = 0$.

(29) means also that given b, θ_ρ gives a greater rate of interest ρ than any other θ. Hence whether ρ or b of the inequality (29) are regarded as given, the method adopted by the entrepreneur is the same θ_ρ.

Inequality (29) isthe condition of equilibrium at a given rate of interest or wages. I.e. (29) explains the euqilibrium.

2.11 Parametric method will be used to yield details.

Let ξ be the hypothetical interest rate so that supposing the market rate were ξ , the entrepreneur would regard ξ as given and would adopt such a θ as would maximise the present value

$$\int_0^\infty \theta' e^{-\xi t} dt \qquad (29)$$

By imagining different values of ξ to be successively given to the entrepreneur, from $\xi = -\infty$ to $\xi = \infty$ contimuously, there will be a particular θ_ξ adopted at each supposed value of ξ .

The totality of these θ_ξ forms a surface

$$Y_t = \theta(t, \xi) \qquad (30)$$

where Y_t is the hypothetical equilibrium output up to $\underline{t}$ at the market rate ξ , and the index ξ of θ_ξ is now put inside the brackets and is a parameter (contimuous index).

One purpose of introducing the parameter ξ is that each potential equilibrium output function θ_ξ should have a name - a particular, corresponding, value of ξ - attaded to it.

Assuming the field of variation $\left[\theta(t)\right]$ to be continuous, the surface (30) is also continuous.

As before, ρ is the actual market rate of interest. Given ρ the entrepreneur maximises the present value of output and the equilibrium condition (29) can now be stated as follows.

Writing present value

$$b(\xi, \rho) = \int_0^\infty \frac{\partial}{\partial t}\theta(t, \xi)\, e^{-\rho t} dt$$

the maximal condition is

$$\frac{\partial b(\xi,\rho)}{\partial \xi} = 0 > \frac{\partial^2 b(\xi,\rho)}{\partial \xi^2} \qquad (31)$$

and the solution which satisfies (31) is $b(\xi,\rho)$ in which $\xi = \rho$

i.e. is

$$b(\rho,\rho) = \int_0^\infty \frac{\partial}{\partial t}\theta(t,\rho)\, e^{-\rho t} dt .$$

If there are several maxima at different values of ξ, the entrepreneur selects the greatest maximum viz. $b(\rho,\rho)$.

(31) states that at the market rate ρ the output function with an index $\xi=\rho$, $\theta(t,\rho)$ gives greater present value, by definitions of (30), than any function with a neighbouring index $\xi=\rho+\delta\xi$, $\theta(t, \rho+\delta\xi)$.

If the market rate ρ changes, the total derivative of the equilibrium wages bill, keeping $\xi=\rho$, is

$$\frac{d\, b(\rho\rho)}{d\rho} = \frac{\partial}{\partial \xi} b(\xi,\rho) + \frac{\partial}{\partial \rho} b(\xi,\rho) \qquad (32)$$

using (31)

$$= -\int_0^\infty t \cdot \frac{\partial}{\partial t}\theta(t,\rho)\, e^{-\rho t} dt \qquad (33)$$

Excluding negative goods, the rate of output $\frac{\partial}{\partial t}\theta$ cannot be negative and is positive at some values of t; also $t \geq 0$, $e^{-\rho t} > 0$

$$\therefore \frac{d\, b(\rho,\rho)}{d\rho} < 0 \qquad (34)$$

Viz. if the market rate of interest rises, the equilibrium wages bill and the wages rate $w = \frac{b(\rho,\rho)}{x}$ diminish. If the market interest rate falls the wages bill and the wage rate increase.

This proposition is stated by Smith, Ricardo, Böhm, Wicksell, 1.3, 1.4, bibliography, and readily follows in the special cases 2.7,8,9.

The total potential equilibrium output

$$P(\xi) = \int_0^\infty \frac{\partial}{\partial t}\theta(t,\xi)\,dt$$

unlike $b(\xi,\rho)$ is a function of the parameter ξ only. The equilibrium total output is

$$P(\rho) = \int_0^\infty \frac{\partial}{\partial t}\theta(t,\rho)\,dt$$

and the maximal condition (31) tells us nothing about the sign of $\frac{dP(\xi)}{d\xi}$

and $$\therefore \frac{dP(\rho)}{d\rho} \gtreqless 0 \qquad (35)$$

Viz. The total equilibrium output can increase, diminish and remain constant, or oscillate, as the market rate ρ rises or as it falls.

Fig. 7 shows a case in which total output $P(\rho)$ diminishes as the rate of interest rises. At the rate $\rho=\beta$, the output stream θ'_β gives the greatest present value $\int_0^\infty \theta'_\beta e^{-\beta t} dt > \int_0^\infty \theta'_\gamma e^{-\beta t} dt$ and is adopted. If the rate rises to $\rho = \gamma > \beta$, another stream θ'_γ is adopted, which yeilds output at a greater rate at first and at a smaller rate later. This can be the case because a rise in discount rate ρ diminishes the present values $\theta' e^{-\rho t} dt$ of earlier items of output $\theta' dt$ in a smaller degree than the present values of the later items. As they are drawn, the stream θ'_β can only be preferred to θ'_γ when $\rho=\beta$, if the excess of output given by θ'_β after t = a is greater than the excess of output given by θ'_γ before t = a.

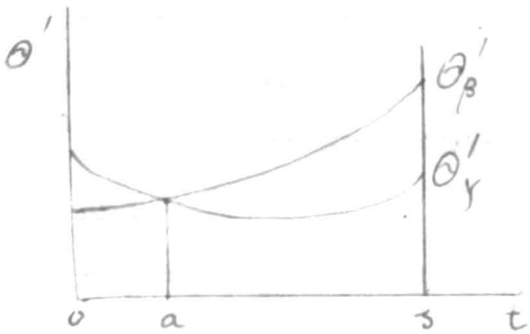


fig. 7

i.e. if $$\int_a^\tau (\theta'_\beta - \theta'_\gamma)\, dt > \int_0^a (\theta'_\gamma - \theta'_\beta)\, dt ,$$
i.e. if θ'_β yeilds greater total output.
As the rate of interest rises from β to γ, total output diminishes from that indicated by the area under θ'_β to that under the curve θ'_γ.

Fig. 8 shows a case in which total output increases as the rate of interest rises.

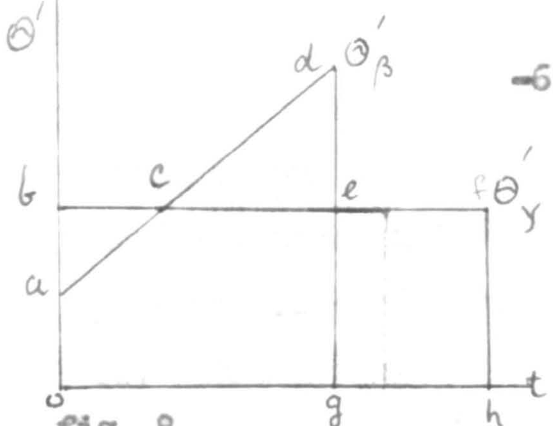


fig. 8

The area under θ'_γ is greater than the area under θ'_β. Area o a c e g is common to the two areas and need not be considered. θ'_β is adopted when $\rho=\beta$, and θ'_γ is adopted when $\rho=\gamma>\beta$, and the following possible condition holds:

at $\rho=\beta$ present value of area abc + present value of area gefh is $<$ present value of area cde;

and

at $\rho=\gamma>\beta$ the inequality sign $<$ is reversed.

In words: when the rate rises to γ, the effect of the excesses of output by θ'_γ in the immediate future outweighs the effect of the excess of θ'_β given somewhat later, on the present value $\int_0^\infty \frac{\partial}{\partial t}\theta(t,\xi)e^{-\gamma t}dt$

Böhm and his followers (in connection with his famous theory of 'Mehrergiebigkeit' of more 'roundabout' methods) believed that as the market rate of interest falls, the total equilibrium output <u>per capita</u> necessarily increases.

Their belief was due to mistakes about the "average period of production."

2.12 The average period of production $m(\xi,\rho)$ is defined as that single period for which the total output $P(\xi)$ has to be discounted at the market rate of interest ρ in order to obtain its present value $b(\xi, \rho)$. Böhm, Wicksell article, cf. bibliography.

I.e. $P(\xi) e^{-\rho \cdot m(\xi,\rho)} = b(\xi, \rho)$

$$\therefore m(\xi,\rho) = \frac{1}{\rho}\left[\log P(\xi) - \log b(\xi,\rho)\right] \qquad (36)$$

$$= \frac{1}{\rho}\left[\log \int_0^\infty \frac{\partial}{\partial t}\Theta(t,\xi)\,dt - \log \int_0^\infty \frac{\partial}{\partial t}\Theta(t,\xi) e^{-\rho t}\,dt\right]$$

$$\therefore \rho \frac{\partial m(\xi,\rho)}{\partial \xi} = \frac{1}{P(\xi)}\frac{\partial P(\xi)}{\partial \xi} - \frac{1}{b(\xi,\rho)} \cdot \frac{\partial b(\xi,\rho)}{\partial \xi} \qquad (37)$$

At $\xi = \rho \quad \frac{\partial b(\xi,\rho)}{\partial \xi} = 0 \quad (31)$

$$\therefore \rho \frac{\partial}{\partial \xi} m(\rho,\rho) = \frac{1}{P(\rho)} \cdot \frac{\partial P(\rho)}{\partial \xi} \qquad (38)$$

Since P, ρ are positive,

$$\frac{\partial m(\rho,\rho)}{\partial \xi} \gtreqless 0 \text{ according as } \frac{\partial P(\rho)}{\partial \xi} \gtreqless 0 \qquad (39)$$

The equilibrium period is

$$m(\rho,\rho) = \frac{1}{\rho}\left[\log P(\rho) - \log b(\rho,\rho)\right]$$

$$\therefore \rho \frac{dm(\rho,\rho)}{d\rho} + m(\rho,\rho) = \frac{1}{P(\rho)}\frac{dP(\rho)}{d\rho} - \frac{1}{b(\rho,\rho)}\frac{db(\rho,\rho)}{d\rho} \qquad (40)$$

$\frac{db(\rho,\rho)}{d\rho} < 0$ (34), and P,b,m are positive

$\therefore$ for $\frac{dP(\rho)}{d\rho} \geq 0$

$$\frac{dm(\rho,\rho)}{d\rho} \gtreqless 0 \text{ according as } \frac{1}{P(\rho)} \cdot \frac{dP(\rho)}{d\rho} - \frac{1}{b(\rho,\rho)} \frac{db(\rho,\rho)}{d\rho} \gtreqless m(\rho,\rho) \quad (41)$$

and for $\frac{dP(\rho)}{d\rho} < 0$ $\frac{dm(\rho,\rho)}{d\rho} \gtreqless 0$ according as

$$-\frac{1}{b(\rho,\rho)} \cdot \frac{db(\rho,\rho)}{d\rho} \gtreqless \left[m(\rho,\rho) - \frac{1}{P(\rho)} \cdot \frac{dP(\rho,\rho)}{d\rho} \right] .. \quad (42)$$

I.e. The equilibrium average period of production can shorten, remain constant or increase as the market rate of interest rises, and the period can increase, remain constant or diminish as the rate falls.

13 Böhm says that certain propositions must hold in all cases of competitive equilibrium and that unless they hold, competitive equilibrium is impossible.

I mark his generalisations with letters, and my comments with the same letters dashed.

Such a generalisation must be true in all cases of competitive equilibrium. If it is not true in some cases I say that that generalisation is false.

A. The product <u>per capita</u> depends on the average period of production.

A'. There are tow meanings to the average period: the non-equilibrium concept $m(\xi,\rho)$ and the equilibrium concept $m(\rho,\rho)$.

From (37), $P(\xi)$ can vary (except at $\xi = \rho$)

as $m(\rho, \rho)$ remains constant, and from (41), (42) the equilibrium output $P(\rho)$ can vary while the equilibrium average period $m(\rho, \rho)$ remains constant.

A is false.

B Given the market rate, the entrepreneur so adjusts the average period that the rate of wages he pays is maximised (competition).

B' B means $$\frac{\partial b(\xi,\rho)}{\partial \xi} \Big/ \frac{\partial m(\xi,\rho)}{\partial \xi} = 0$$

at $\xi = \rho$. From (37) this is true if

$\frac{\partial m(\xi,\rho)}{\partial \xi} \neq 0$. If $\frac{\partial m(\xi,\rho)}{\partial \xi} = 0$

$\frac{\partial b(\xi,\rho)}{\partial \xi} \Big/ \frac{\partial m(\xi,\rho)}{\partial \xi}$ is indeterminate since $\frac{\partial b(\xi,\rho)}{\partial \xi} = 0$

B also says

$$0 > \frac{\partial}{\partial m(\xi,\rho)}\left[\frac{\partial b}{\partial \xi} \Big/ \frac{\partial m}{\partial \xi}\right] = \frac{\partial}{\partial \xi}\left[\qquad\right]\frac{\partial \xi}{\partial m(\xi,\rho)}$$

From (38) $\frac{\partial \xi}{\partial m(\xi,\rho)} \gtreqless 0$

B is false.

C In equilibrium, the marginal product of the average period is proportional to the market rate of interest.

C' Böhm deduces C from A and B, and as A, B are false, C is false.

NOTE: Böhm's school confuses (40) with (37). They think that since in (37)

$$\rho = \frac{1}{P(\rho)} \frac{\frac{\partial P(\rho)}{\partial \xi}}{\frac{\partial m(\rho,\rho)}{\partial \xi}}$$

then in (40) $\rho = \frac{1}{P(\rho)} \frac{dP(\rho)}{dm(\rho,\rho)}$

which is false. In (40);

$$\rho = \frac{d\rho}{dm(\rho,\rho)}\left[\frac{1}{P(\rho)}\cdot\frac{dP(\rho)}{d\rho} - \frac{1}{f(\rho,\rho)}\cdot\frac{df(\rho,\rho)}{d\rho} - m(\rho,\rho)\right].$$

D The market rate of interest cannot be positive, unless more 'roundabout' methods are more productive.

D' By more 'roundabout' methods Böhm means methods of production invo~~k~~lving longer average periods. D means either

$$\frac{\partial P(\xi)}{\partial \xi} \Big/ \frac{\partial m(\xi,\rho)}{\partial \xi} > 0 \quad \text{or} \quad \frac{dP(\rho)}{dm(\rho,\rho)} > 0,$$

is deduced from A, B, C and is false. Böhm also thought $\frac{d^2P(\rho)}{d\rho^2} < 0$. For details see (37), (41) and (42).

E The euqilibrium average period lengthens as the market rate of interest rises, and shortens if the rate falls.

E' From (41), (41) $\frac{d\,m(\rho,\rho)}{d\rho}$ can be $\gtreqless 0$

∴ The E is false.

F The equilibrium output diminishes if the market rate rises, and increases if the rate falls.

F' F is deduced from D and E.

∴ F is false. Is false from (35).

G The equilibrium wage rate increases as the equilibrium average period lengthens, etc.

G' G is deduced from B and is false. Is false from (34), (41), (42).

H The equilibrium wage rate diminishes as the market rate of interest rises, and falls as the latter rises.

H' H means $\frac{d\,b(\rho,\rho)}{d\rho} < 0$ and is true from (34).
Böhm could hardly fail to see proposition H.
H i.e. (34) easily follows from the principle that given the rate of interest, the present value of output is maximised, (29), (31).

2.14 The analysis of 2.11, 12, 13 applies to the particular case studied by Böhm as follows:

The less restricted case of Böhm assumes point-output accrues in the present $t = 0$ and is produced by a continuous labour input commenced in the indefinite past $t = -\infty$.

The total labour applied is constant, x say, but it can spead over the past differently, and gives rise to different point-outputs P, say. As in 2.1 let $X_t = \dot{X}_t dt$ be labour input up to t. Simple rate of interest is used. The rate of wages w, say, is assumed uniform from $t = -\infty$ to $t = 0$.

Suppose the simple market rate is ξ.
Then w is defined by the equation: $w \int_{t=-\infty}^{0} d\, X_t \cdot (1 + \xi t) = P$
Supposing ξ given, such an input-function X_t is used as maximises w. We can write the maximal function $X_t = X(t, \xi)$ and the maximised $w = w(\xi)$, and corresponding output $P(\xi)$, say. By assigning a continuous range of values to ξ, $X(t\ \xi)$ becomes a surface of potential equilibrium input-functions. $X(t, \xi)$ is similar to (30) and ξ as before is a parameter.

Let the actual simple market be i.

The wage rate to be maximised is

$$w(\xi, i) = \frac{P(\xi)}{\int_{t=-\infty}^{0} d\, X(t, \xi)(1 + it)} \qquad (43)$$

and the maximal condition is $\frac{\partial w(\xi,i)}{\partial \xi} = 0 > \frac{\partial^2 w(\xi,i)}{\partial \xi^2}$ (44)

and the solutions are w(i,i), X(t,i), and P(i).

The average period R(ξ) is the single period for which the output P(ξ) has to be discounted at the market rate i to obtain the wages bill x.w. (ξ ,i).

$$xw.(\xi,i).(1+R(\xi).i) = P(\xi)$$

$$\therefore \quad R(\xi) = \frac{1}{xi}\left[\frac{P(\xi)}{w(\xi,i)} - 1\right] \qquad (45)$$

using (43) and since $\int_{-\infty}^{0} dX_t = x$

$$= \frac{\int_{t=-\infty}^{0} dX(t,\xi)\cdot t}{\int_{-\infty}^{0} dX(t,\xi)} \qquad (46)$$

is a weighted average, is independent of i, and is a function of the ~~parameter~~ ξ only.

Differentiating (45) and using (44)

$$\text{at } \xi = i \qquad i\frac{\partial R(i)}{\partial \xi} = \frac{\frac{\partial P(i)}{\partial \xi}}{x.w(i,i)} \qquad (47)$$

$$\text{and} \quad i\frac{dR(i)}{di} + R(i) = \frac{w(i,i)\frac{dP(i)}{di} - P(i)\frac{dw(i,i)}{di}}{x\left[w(i,i)\right]^2} \qquad (48)$$

and similarly to 2.11, 12, 13

$$\frac{dw(i,i)}{di} < 0 ;$$

$$\frac{\partial P(\xi)}{\partial \xi}, \frac{dP(i)}{di} \gtreqless 0 ; \quad \frac{\partial R(i)}{\partial \xi} \gtreqless 0$$

according as $\frac{\partial P(i,i)}{\partial \xi} \gtreqless 0$; $\frac{dR(i,i)}{di} \gtreqless 0$;

and Böhm's proposiions A to G are false and H right.

The possibility of the equilibrium average period lengthening as the market rate rises $\left[\frac{dR(i,i)}{di} > 0\right]$ is suggested by the possibility that the increase in R(i) and in the discount rate from i_1 to i_2, say, is offset by an increase in output from $P(i_1)$ to $P(i_2)$, so that $w(i_2, i_2) > w(i_1, i_2)$ I.e. a larger output is discounted at a higher rate over a longer average period.

In Böhm's more restricted case X_t is linear $= \frac{x}{2}$ 2.3,8. Substituting in (46) $R(i) = \frac{s}{2}$ which is Böhm's familiar formula. In the linear input case, all Böhm's conclusions A to H are true, see 2.8 But in the analysis of the linear input case the i independent variable is the total period of production, the concept of the average period is superfluous and therefore senseless. In fact Wicksell, see 1.9, does not regard $\frac{s}{2}$ as independent and differentiates with respect to s, in restating the case.

Böhm's mistake was to think that all that is true in his 'linear' case is true in all cases.

15. 2.15 I revert to the point input-continuous output case.

What can be called the equilibrium average <u>investment</u> period is m(b) $= \dfrac{\int_0^\infty t \frac{\partial}{\partial t}\theta(t,\rho)e^{-\rho t}dt}{\int_0^\infty \frac{\partial}{\partial t}\theta(t,\rho)\,dt}$

using (33) $= -\dfrac{db(\rho,\rho)}{d\rho} \Big/ b(\rho,\rho)$ ......(49)

$$\frac{dm(b)}{d\rho} = -\frac{b(\rho,\rho)\frac{d^2b}{di^2} - \left(\frac{db(\rho,\rho)}{di}\right)^2}{\left[b(\rho,\rho)\right]^2}$$

and is $\gtreqless 0$.

The average investment period m(b) is less affected by the future items $\frac{\partial}{\partial t}\theta\,dt$ of output than the average production period m(ρ,ρ), (36).

Because m(b) is not defined in terms of undiscounted output, but discounted output. Nevertheless, the average investment period m(b) can lengthen as the market ~~of~~ rate of interest rises.

It is impossible to assign empirical probability of truth of pwwpositions <u>a priori</u> whehter the average period — on any definition— is most likely to diminish, remain constant, or increase in practice, if market rate of interest either rises or falls.

But using the statistical datum of 3 that relative shares are constant,

$$\rho \cdot m(\rho, \rho) = \text{constant} = \alpha \text{ say,} \quad (36)$$

and $\frac{dm(\rho,\rho)}{d\rho} = \frac{\alpha}{\rho^2} < 0$ since $\alpha > 0$.

In the "statistical" case the average period shortens if the market interest rate rises.

The statistics place little other restrictions on the conclusions of 2.11, 12, 13.

In particular, we still don't know enough about $\frac{dP(\rho)}{dm(\rho,\rho)}$ in (40) to find the empirical form of the product as a function of the average period $m(\rho,\rho)$, if such a function exists. The empirical forms of similar period-productivity-functions have been found in the simple cases (26), (27), (28). Nothing of the sort can be done in the case of point input and continuous output. Because the output function $\theta(t)$ has a high degree of freedom.

2.16 2.11............2.15 clear up Böhm's errors.

Böhm tried to solve a mathematical problem of variations without using mathematics. He thought (continuous input - point output case) that output and wages depned on the weighted average value of <u>t</u> in the input function dX_t and not at all on the form of dX_t.

2.14 show sthat wages $w(\xi, i)$ and output $P(\xi)$ depend on the form of dX_t and need not be correlated with the average period $R(\xi)$.

Böhm thought that his "triangle" ("linear case" 2.3.fig 2, 2.8) is a good model of the real world.

2.11............ 2.15 prove that the world generally is not like Böhm's "triangle".

<u>Note.</u> Many authors fall into the same errors as Böhm-Bawerk. E.g. Mr. C. Gifford (<u>Economic Journal</u> Dec. 1933, <u>Econometrica</u> April 1935), assumes the case of continuous input-point output. He does not restrict the input X_t to linear functions. Yet he says that the cost per unit output is minimised and the wage rate <u>w</u> maximised with respect to the average period R. This is false. As I showed in general <u>w</u> cannot be regarded as function of R.

We have an unknown input function X_t

on which w depends i.e. of which w is a functional. The equilibrium condition is that $\underline{w}$ is maximised with respect to the variable function X_t i.e. the first variation $\delta w = 0$ and $\delta^2 w < 0$.

That does not imply that $\underline{w}$ is maximised with respect to R i.e. that $\frac{dw}{dR} = 0 > \frac{d^2w}{dR^2}$. In saying that $\delta w = 0$ implies $\frac{dw}{dR} = 0$, Bohm-Bawerk and Mr. Gifford make a purely mathematical mistake. Mr. Gifford's fundamental equation is illegitimate. The mistake arises from trying to solve a problem of variations without using the calculus of variations. This shows how important it is to use this caluclus if we are to understand the "problem of capital".

While with Wicksell 1,7 the assumption of simple interest is made only as a simplification for purposes of teaching, with Mr. Gifford it plays a more important role: all his conclusions are based on it and for his purposes, as he says the assumption of compound interest is "unmanageable" and destroys all his conclusions. This is so because in (45) the assumption of simple interest renders the concept of the average period R independent of the market rate of interest, which is not the case when the compound rate ρ is assumed.

It is therefore worth pointing out that the assumption of simple interest is quite unreasonable. For it means that £100 invested for two years at 5% p.a. simple interest is worth £105 at the end of the first year and that this £105 earns the same £5 during the second year as £100 invested at the end of the first year for one year. I.e. the assumption of simple interest means that during the same year the same interest of £5 is earned by £100 as by £105. Which cannot be the case in the real world.

P.16 2.17 In the point-input - continuous output case, using (5')

$$\rho \frac{dK}{d\rho} + K = \frac{d\dot{P}(\rho)}{d\rho} - \frac{d\dot{B}(\rho,\rho)}{d\rho}$$

K, ρ are positive

$$\therefore \frac{dK}{d\rho} \gtreqless 0 \text{ according as } \left(\frac{d\dot{P}(\rho)}{d\rho} - \frac{d\dot{B}(\rho,\rho)}{d\rho} \right) \gtreqless K \quad (50)$$

N.B. (33).

I.e. the equilibrium capital disposal K can increase, remain constant, or diminish if the market rate of interest rises, and can increase, remain constant or diminish if the rate of interest falls.

E.g. A rising demand curve for capital (disposal) is possible under conditions of competition, fig. 9. That curve is similar to the rising demand curve of Pareto, Manuel and of Dr. Hicks, Economica March, 1934.

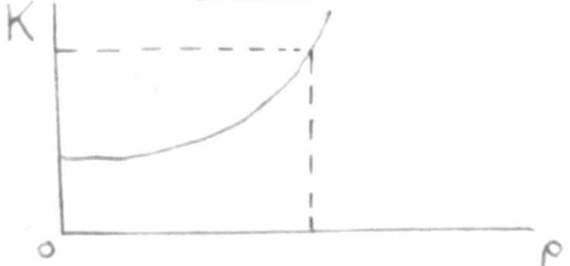


fig. 9.

In fig. 9 at a given value of an entrepreneur uses the amount of capital indicated. He will not use more, since if he did, he would fail to maximise the present value.

$b(\xi, \rho)$ of his output.

2.1? **Productivity of capital.**

There is a theory that in equilibrium the marginal productivity of capital equals the market rate of interest; that $\frac{d\dot{P}(\rho)}{dK} = \rho$.

That theory is false.

For, using (5′)

$$\frac{d\dot{P}(\rho)}{dK} = \rho + K\frac{d\rho}{dK} + \frac{d\dot{b}(\rho,\rho)}{dK}$$

$$\neq \rho$$

Except when both $dK \neq 0$ and $K\frac{d\rho}{dK} = -\frac{d\dot{b}(\rho,\rho)}{dK}$.

2.1? **Demand curve for labour.**

In a theoretical economy in which all firms are identical, let U signify the total capital disposal, and $\dot{D}$ the total labour-force employed. U is assumed given and constant.

We have $\dot{D} = \dot{x}\frac{U}{K}$, $w(\rho) = \frac{\dot{b}(\rho,\rho)}{\dot{x}}$

and $\dot{D}(w)$ is the labour-force employed as a function of the market wage rate. The graph of $\dot{D}(w)$ is the total demand curve for labour-force.

$$\frac{d\dot{D}}{dw(\rho)} = -\dot{x}^2 U K^{-2}\frac{dK}{d\rho}\frac{d\rho}{d\dot{b}(\rho,\rho)},$$

$X, U, K > 0$ and (33) $\frac{d\dot{b}(\rho,\rho)}{d\rho} < 0$

$$\therefore \frac{d\dot{D}}{dw(\rho)} \gtreqless 0 \quad \text{as} \quad \frac{dK}{d\rho} \gtreqless 0 \quad (51)$$

and falling, horizontal and rising demand graphs $\dot{D}(w)$ are possible. N.B. the demand curves $\dot{D}(w)$ and $K(\rho)$ have the same sense.

Let D_e = the elasticity of $\dot{D}(w)$.

$$D_e = \frac{d\dot{D}(w)}{dw(\rho)} \frac{w(\rho)}{\dot{D}(w)} = -K^{-1}\dot{b}(\rho\rho)\frac{dK}{d\rho}\frac{d\rho}{d\dot{b}(\rho,\rho)}$$

and is $\gtreqless 0$ as $\frac{dK}{d\rho} \gtreqless 0$. (52)

I.e. under competition the (partial) elasticity of demand for labour can be negative, zero, or positive.

According to statistics of 3. $\dot{P}(\rho):\dot{b}(\rho,\rho) =$ constant $= c$, say.

Then $K = \frac{(c-1)}{\rho d\rho}\dot{b}(\rho,\rho)$

and $D_e = -1 + \frac{\rho d\rho}{d\dot{b}(\rho,\rho)}\frac{\dot{b}(\rho,\rho)}{\rho}$ (53)

and is < -1 using (33).

From (52) the conditions of competition are so unrestrictive that they tell us nothing about the degree of (partial) elasticity of the total demand for labour-force.

From (53) the statistics suggest that in the real world the demand for labour is "elastic". If so, and if the national supply of capital disposal does not increase, the effect of a rise in wages, due to Trade Union and State intervention, will be to diminish the national real wages bill.

2.20. <u>Fluctuating ρ.</u>

If at $t = o$ £1 say is invested and becomes $f(t)$ at <u>t</u>, the rate of interest at t is $\frac{f'(t)}{f(t)}$ 1.5 and ρ is defined by the equation $f(o) = f(t)\,e^{-\rho t}$

whence $\rho = \frac{\log f(t) - \log f(o)}{t} = \rho(t)$ say.

So far $\rho(t)$ has been assumed constant, in which case $f(t)$ is an exponential, and $\frac{f'}{f} = \rho$.

But in the more general case $\rho(t)$ can vary with <u>t</u>.

When $\rho(t)$ varies with t, the potential-equilibrium output surface (30), and the average periods (36) (45) (49) cannot even be constructed; (5') breaks down but (5) stands.

The equilibrium condition (29) is still valid.

2.21 The analysis of point input- general output 2.10, 11, 12, 13, 15, 16, 17, 18, 19 is a useful propaedeutic. Because every application of a factor at a moment of time constitutes a point input.

It does not assume the rate of wages constant through time. Hence it is not limited to stationary equilibrium as are 2.8 2.9 (second case).

True, ρ is taken as constant at an undetermined level. But this is a fair approximation to the real world.

4. examines the restrictions imposed on 2.10.....13, 15........ 19 by the assumption that all the alternative output functions $\theta(t)$ are independent of the factors applied after t = o.

3. ## Statistics of Distribution of the National Income.

3.1 By the "national income" I mean the sum of money payments to all the inhabitants of a country during a year, for the use of their labour and property in production.

3.2 In the case 2.10 $\Theta(t)$ is so unrestricted that statistics of national income can help us to draw only a few conclusions 2. The more general theory of production in 4. is still less restricted and the statistical datum that relative shares are constant is insufficient to lead to any particular conclusions. It seems possible so to restrict the models of production in 4. as to enable the statistics to be used analogously as in 2. But at the moment I have not developed my theory far enough to be able to use the statistics very much. Therefore I shall content myself with only a short note on the income statistics for the United Kingdom from 1843 to 1924.

3.3 R. Giffen gives a table showing the distribution of the national income in 1843 with a reservation that his "figures.... make no pretence to exactness". (Essays in Finance second series p.467).

Using his figures we have the following: table: distribution of the national income in 1843 in £ millions.

Income from property	190
Income of all classes from work	325
Total	515
The proportion of the total going to work	63%

3.4 Professor Bowley (Changes in the National Income 1880-1913, p. 24-6) gives the proportion of the national income "earned" as 62½% in 1880, rising to an annual average of about 65% in the next twenty years, and falling again to 62½% in 1913. Professor Bowley estimates (Economic Journal 1904 p. 459) that in 1860 the proportion was nearly the same as in 1913.

The word 'earned' is used in the special sense employed in assessing income tax and includes not only income from work but also income derived by persons from employment of their own capital in private firms.

Professor Bowley estimates that subtracting from the "earned" category the income due to capital in private firms owned by their members, the proportion of income from work in 1913 is 61%. This is to be compared to Giffen's estimate for 1843 of 63%. Professor Bowley says (p.24) that farmers' incomes which are mainly due to work areunderestimated in Schedule B, and therefore the proportion due to work should be taken greater than 61%. Therefore it does not seem necessary to distinguish income from work and the earned income which is 62½% of the national income in 1913. Giffen's estimate of 63% can be taken as relating to earned income.

The approximate constancy of the earned proportion in the period from 1843 to 1913 is remarkable. Comparing 1880 with 1913 Professor Bowley writes "though it is not proper to lay stress on the exact equality of the proportions at the two dates, yet the evidence is sufficient that any change there may have been is inconsiderable. The constancy of so many proportions found in the investigation seems to point to a fixed system of causation and has an appearance of inevitableness".

A part of the income from property is interest on capital invested abroad. Interest from abroad in 1880 was about £50 mns. and in 1913 about £200 mns.

The "home produced national income" is the national income less net interest, from abroad. Using Professor Bowley's figures the earned income as a proportion of the home-produced income in 1880 is 65½% and in 1913 is 69%.

The difference 3½% between the two proportions is not great.

3.5 Professor Bowley and Dr. Stamp exclude Southern Ireland and estimate the earned proportion for 1911 as 75½% and for 1924 as 78% (National Income in 1924, p. 50, using Professor Bowley's estimates for 1911 published in his Division of the Product of Industry).

The difference between 75½ for 1911 and 69% for 1913 is due to a difference in the basis of the two estimates. In estimating for 1913 Professor Bowley appears to take one quarter of the gross assessment under Sch.D as earned (N.I. 1880-1913, p.24) and a greater proportion for 1911 (N.I. 1924 p.51). Southern Ireland accounts for some 4% of the national income and its inclusion or omission affects the proportion negligibly.

Using the figures of N.I. 1924, the earned proportion of the national income in 1911 (including £194 mns interest from abroad) is 70%. This is to be compared to 62½% for 1913 and the difference must be due to the change of "basis".

Conclusion: whether we take the annual national income or its home-produced component, the earned proportion is roughly constant between 1843 and 1913 on the old (Giffen) basis and as between 1911 and 1924 on the new Bowley-Stamp basis. The estimate of the income from work should be nearly the same as for the earned income. Hence, the proportion due to work was roughly constant.

Mr. Clark carries analysis of the income statistics up to the recent years (National income 1924-1931, and Economic Journal 1933,4). But they are not comparable to the Bowley-Stamp estimates. Thus instead of the Bowley-Stamp estimate of the earned proportion 75½% for 1911 Mr. Clark gives only 54½% in table XXV of his book. I do not go into Mr. Clark's figures because it seems difficult to recast them for purposes of comparison with the earlier estimates, especially as his methods present certain difficulties which have been pointed out in a review of his book (Statistical Journal 1933, p. 110).

3.6 There are three main difficulties about the income statistics.

a) The distinction between the value of a firm's capital and its profits (on ordinary shares) is somewhat arbitrary. The firm's gross annual profits are the difference between what it pays out and what is paid to it during a year. Its net profits are gross profits less "normal capital depreciation" and net borrowing during the year. "Normal depreciation" is an arbitrary thing, it includes neither capital written off nor "capital appreciation" such as may be due to a stock exchange boom or to monetary inflation. Profits are arbitrary because normal depreciation is so. Profits are what the business world thinks they ought to be.

But in the mass and in the long run profits are not arbitrary. It is found that the average rate of profit on the nominal value of ordinary shares over ten or twenty years approximates to the average rate earned, by debentures. The rough rule which directors follow in declaring profits is this: they declare such a rate as is compatible with maintaining profits at the above average rate in the long run.

b) The estimate of profits is complicated by the fact that Schedule D of income tax returns does not separate income from capital in private businesses from income due to work of the owners of these businesses. Dr. Stamp (British Incomes and Property) goes into this, as also R. Giffen and Professor Bowley (loc cit). A conventional basis is adopted for dividing income under Sch. D between work and property. Till the recent growth of the number of the small retailing businesses, the proportion of the national property controlled by limited companies has been constantly growing. "There has been a transference of income from the earned to the unearned category without any real change in its source", (N.J. 1924). Hence the use of a fixed basis for dividing the gross assessment under Sch. D into the earned and unearned components over long periods is misleading. Unfortunately statistitians have not been able to separate income from work and income from property in private firms. That would have solved the above difficulty. In theory the two components are separable according to the principle of marginal productivity. With reference to the purpose to which I put the statistics, there is a third difficulty.

c) A country is not a closed economic system. Production in United Kingdom is only a part of the world process of production. This country imports materials and foodstuffs and exports manufactures, etc. I want information on income distribution in the whole process but only a part of it is covered by the statistics I use. I cannot use the statistics for Germany, The United States etc., etc., as it is a greater task than a single investigator can safely undertake. Foreign statistics of distribution present difficulties of interpretation and comparison.

There is no reason why the relative shares of labour and property in the national incomes of different countries should be the same. Yet among industrialised countries they do not seem to differ very much. Thus for the United States in 1910 King estimates the proportion of the national income due to work as 74½% (The Wealth and Income of the United States, table XXXI). This is very close to Professor Bowley's estimate of 75½% as the earned share of the home produced income of United Kingdom in 1911.

As a rough approximation I could perhaps assume for my purposes that the relative shares for the whole world are the same as in the home-produced income of the United Kingdom. But there is no need to assume that.

Instead, I regard this country as a quasi closed economy. This is done by regarding imported goods as produced in this country. They are produced by producing the export-goods which are exchanged for them. In that sense we have a closed structure of production. The total income yielded by this structure is the home-produced national income.

From 3.3-5 the relative shares of property and labour in this were roughly constant. Assuming the relative share of land as an original factor was also constant (as in U.S.A., in any case it is small) we can say roughly that the relative share of labour and land together has been constant. Meanwhile, the rate of interest and national capital per head varied (there was great capital consumption during the war). So we can say that while s varied the relative shares $(\frac{P}{B})$ remained constant (though b,P are not simultaneous as are the income and its original factor component). Hence we get the period productivity functions (26),(27),(28) where original factors are represented by homogeneous labour. These functions are simple models of the laws of distribution of the home- produced income between capital on the one hand and labour and land on the other.

3.7 In connection with the statistics I shall discuss the point input-point output case in some detail and try to estimate the numerical value of α in (26). To estimate α I use the following statistics for 1911, arranged for convenience in the form of a table showing amounts in £ millions, from table, II N.J. 1924

	1911
The home produced income less pensions and national debt interest	1868
Earned	1409
From table II, "Division of the Product of Industry"	
Ownership of lands	34
" " buildings	144

"Buildings" include land on which they stand, and the share of the land in the income from building has been variously estimated to be from $\frac{1}{5}$ to $\frac{1}{2}$ of the total (British Incomes and Property, pp. 344-5), and I take its share as, say, £50 mns. Treating earned income as from work, the total income of land

and labour is £1493 mns.

Using the point-input point output model 2. and assuming all firms in the economy are homogeneous (though they need not be of same size) and expressing

$$\frac{\dot{P}}{\dot{b}} = \frac{1868}{1493}$$

as the ratio of the national income to its original factor component, and using 2.7

$$\log\left(\frac{\dot{P}}{\dot{b}}\right) = .224 = \frac{9}{40} \quad \text{approximately.}$$

A slight inaccuracy is introduced by the fact that in (26) $\dot{P}$ is a rate of output of consumption goods whereas the home produced income in 1911 consisted of about 90% of the value of consumer's goods and of 10% of capital saved.

The input $\dot{b}$ occurs s years (taking 1 year as unit) earlier than the output rate $\dot{P}$. Therefore above I should not have used the 1911 estimate of £1493 mns. as the correlative of $\dot{b}$ but an estimate relating to a date s years before 1911. Roughly s equals the ratio between the value of the national capital and the national income p.a. For pre-war years the ratio is around 6. (checked from s=7) Therefore to calculate α I would have to use the share of the "original factors" about 6 years before 1911 say the share in 1905 instead of using the 1911 estimate of £1493 mns. Also, to compare incomes on different dates I should express them in money of the same purchasing power in terms of consumption goods. This to eliminate effects of changes in the consumption price level.

The national income in 1905 was between 1700 and 1800£mns. (c.f. British Incomes and Property p.427). I take it as £1800 mns. which is an overestimate, to compensate for the fall in purchasing power of money between 1905 and 1911.

The share of the original factors in 1905 is roughly

$$\frac{1493\times 1800}{2012}\text{£mns.}$$

Hence $\log\left(\frac{\dot{P}}{\dot{b}}\right) = .336$

and $\alpha = \frac{1}{3}$ approximately.

Using a higher value for s of 8 years (as the equation $s = \frac{\alpha}{\rho}$ suggests) does not alter the estimate $\alpha = \frac{1}{3}$ appreciably. Because the national income in 1903 is over 1700£mns.

The real income of the original factors is greater in 1911 than in 1905 because of invention and the growth of the employed population. The effect of invention is to increase the coefficient C in (26).

The first estimate $\alpha = \frac{9}{40}$ is obtained by regarding the 1911 position as if it were a position of "stationary equilibrium" and the amount of the national income and the absolute shares were the same year after year. This is a first approximation. Then an allowance is made for "progress" and an estimate $\alpha = \frac{1}{3}$ is obtained. The two estimates are not very different. Only a very rough estimate is needed for the conclusionswhich follow. In what follows I use $\alpha = \frac{1}{3}$ as a rough estimate of the value of α which would be obtained (using the point input - point output model) if exact measurements were available of the components of the national income. (26) becomes

$$P = Cs^{\frac{1}{3}} \qquad (54)$$

and is plotted in fig. 10.

From (54) $\frac{d\dot{P}}{ds} = \frac{1}{3} Cs^{-\frac{2}{3}}$ and for $s > 1$ year $\dot{P}$ and the productivity per capita $\frac{\dot{P}}{\dot{x}}$ vary little with s.

From (9) and using the result obtained in 2.7 that $\rho s = \alpha$

$$K = s\dot{P}\,\frac{(1-e^{-\alpha})}{\dot{p}\quad\alpha}$$

and we see that for $K > 1$ increases little with K. This suggests that in the real world neither capital consumption nor accumulation influences productivity $(\frac{\dot{P}}{\dot{x}})$ per capita very much. I.e. a change in the planned capital-disposal does not lead to expect any great cahnge in the consumption product per head in the (finite) future.

So far the labour force $\dot{x}$ employed in the firm was assumed constant. Now I allow $\dot{x}$ to vary and assume that the rate of output $\dot{P}$ is proportional to $\dot{x}$

i.e. $$\dot{P} = C\dot{x}s^{\alpha} \qquad (26')$$

The rate of wages paid is

$$w = \frac{\dot{b}}{\dot{x}}$$

Using (26') and since $\dot{b} = e^{-\alpha}\dot{P}$,

$$w = e^{-\alpha}.\,Cs^{\alpha}$$

and $$s = (e^{-\alpha}C)^{-\frac{1}{\alpha}}.\,w^{\frac{1}{\alpha}}$$

Writing (9) as

$$K = \frac{(e^{\alpha}-1)}{\alpha}\,s\dot{b}$$

$$\dot{x} = \frac{\dot{b}}{w} = \frac{\alpha(e^{-\alpha}C)^{\frac{1}{\alpha}}}{e^{\alpha}-1}\,Kw^{-(1+\frac{1}{\alpha})}$$

Putting the constants $\frac{\alpha(e^{-\alpha}C)^{\frac{1}{\alpha}}}{e^{\alpha}-1} = C_1$ and $1 + \frac{1}{\alpha} = \beta$

$$\dot{x} = C_1 Kw^{-\beta}$$

which is an equation of demand for labour force $\dot{x}$ in terms of capital and the rate of wages. The formula has the advantage of showing succinctly how a demand for labour depends not only on the rate of wages (as is usually emphasised) but also on the amount of capital available to the entrepreneur to finance the process started by investing $\dot{x}$.

Assuming an economy consisting of identical firms (except they need not be of the same size) and writing $\underline{\dot{L}}$ for the total employed labour-force and $\underline{M}$ the capital disposal available to finance to their completion the processes started by investing $\dot{L}$ (in the whole economy),

$$\dot{L} = \frac{M}{X}\dot{x} = C_1 Mw^{-\beta}$$

is the labour force demanded in the economy as a whole as a function of the wage rate $\underline{w}$ and the capital disposal $\underline{M}$. In this formula $\underline{M}$ signifies the amount of capital-disposal which the entrepreneurs expect at the moment of investing $\underline{\dot{L}}$ to be forthcoming in the future to finance the processes so started till their completion. This concept of M applies to dynamic problems where we assume uncertainty.

Putting $\alpha = \frac{1}{3}$

$$\dot{L} = C\ Mw^{-4} \qquad (55)$$

which is a model of demand conditions for labour in this country.

For each value of $\underline{M}$, regarding $\underline{w}$ as an independent variable we have an ordinary demand curve for labour. It is plotted in fig. 11, and logarithmically in fig. 12.

In (55) the elasticity of the demand with respect to $\dot{w}$ is
$$\frac{\partial L w}{\partial w L} = -\beta = -4$$
and is very high. This suggests that the elasticity of the total demand for labour in this country is high, and that employment could be increased by 4% if the average real wage rate were lowered by about 1%. Stress should not be laid on the exact number - 4 as an estimate of the partial labour demand elasticity with respect to wages in this country. Nevertheless the result suggests that the real partial elasticity must be of that order and I should expect it to lie between - 2 and - 6.

It seems very unlikely that it is numerically lower than 2 because $\beta = 1 + \frac{1}{\alpha}$ and even if α were unity (and there were no diminishing returns to the production period s, i.e. in (26) $\frac{d\dot{P}}{ds} = 0$) the elasticity would be - 2, and higher than - 1 for any finite positive value of α.

The partial elasticity in (55) with respect to the capital disposal M is positive and equal to unity.

The conclusion from theoretical considerations that the elasticity of demand with respect to the real wage rate w (it will be remembered that I was taking the timeless dimensions $\dot{Y}$ of the consumption good as numeraire) is borne out by observation. It has been observed that trade unions were powerless to raise the rates of wages appreciably above what they probably would have been in the absence of their pressure.

It seems probable that Trade Union pressure and the state regulation of minimum wage rates were partly responsible for the post war unemployment of about 1 mn. without influencing real wage rates appreciably. The high elasticity of the demand for labour with respect to the average wage rate is a disadvantage in that wages can not be raised institutionally without causing great unemployment. It is an advantage in that a small reduction in wages will expand employment considerably.

The analysis suggests that the real incomes of the wage-earners as a class cannot be increased by raising wages institionally. It can be diminished if wages are so raised, by the increase in unemployment. The only effective way of increasing the real incomes of the wage earning class is through taxing the incomes of the rich.

That latter policy has its limits. Taxation especially the death duties tend to diminish the supply of capital disposal. Using the simple model (55) they tend to diminish M, and since the partial elasticity $\frac{\partial L . M}{\partial M . L} = 1$, they tend to diminish employment.

The total "elasticity differential" (55) is

$$\frac{\partial L . w}{\partial w . L} dw + \frac{\partial L . M}{\partial M . L} dM = dM - 4 dw$$

and taking dM, dw as unities is - 3 when w and M are increased simultaneaously, and is - 5 when w is increased and M diminished. In the second case it is particularly high. This suggests that in the real world a continual institutional upward pressure on wages and a downward pressure on saving will in combination easily lead to severe and prolonged unemployment. On the other hand it appears that if determination of wages is left to competition while a policy favourable to saving is persued, the problem of "permanent unemployment" would disappear because its main causes will have been eliminated.

The apparent high demand elasticity for labour has long been noted. But the explanations of it that were fothcoming were of a purely hypothetical nature (e.g. Professor Pigou's <u>Unemployment</u>; Dr. Hicks' <u>Theory of Wages</u>). They say the elasticity is high if the "productivity" functions have certain properties. The prupose of 3. is to discover those properties ($\dot{P} = Cs^{\alpha}$ etc.,) from statistics (<u>a posteriori</u>).

The point input- point output case is very simple. So, it is not a good model of production. But it enables us to use interesting statistics. And it is noteworthy that taking account of the facts of distribution of the national income leads to the same general results as have been reached by other methods. Only the analysis 3. yeilds the results in a more concrete form of empirical functions and numerical solutions.

Tables of numerical values.

Fig. 10, graph of $\dot{P} = Cs^{\frac{1}{3}}$ (54), taking C = 1.

s	o	.33	½	1	2	3	4	5	6
$\dot{P}$	o	.69	.79	1	1.26	1.44	1.59	1.71	1.8

Fig 11, graph of $\dot{L} = C_1Mw^{-4}$, taking $C_1M = 1$.

w	o	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
$\dot{L}$	∞	1	.682	.482	.351	.263	.191	.153	.120	.095	.077	.062

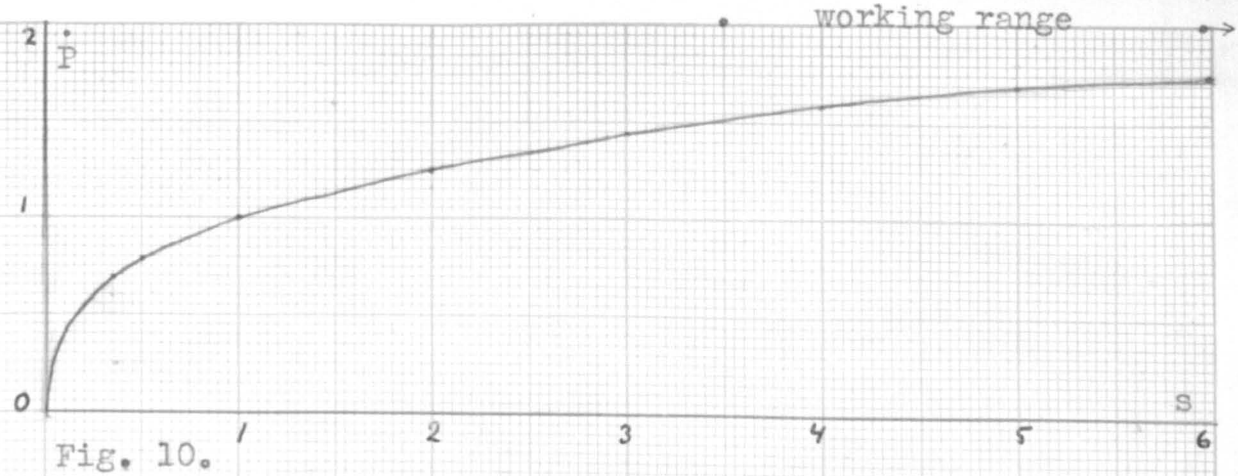


Fig. 10.

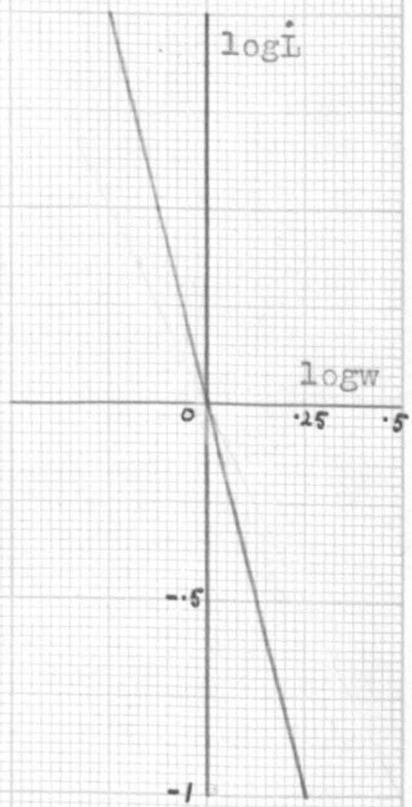


Fig 12.

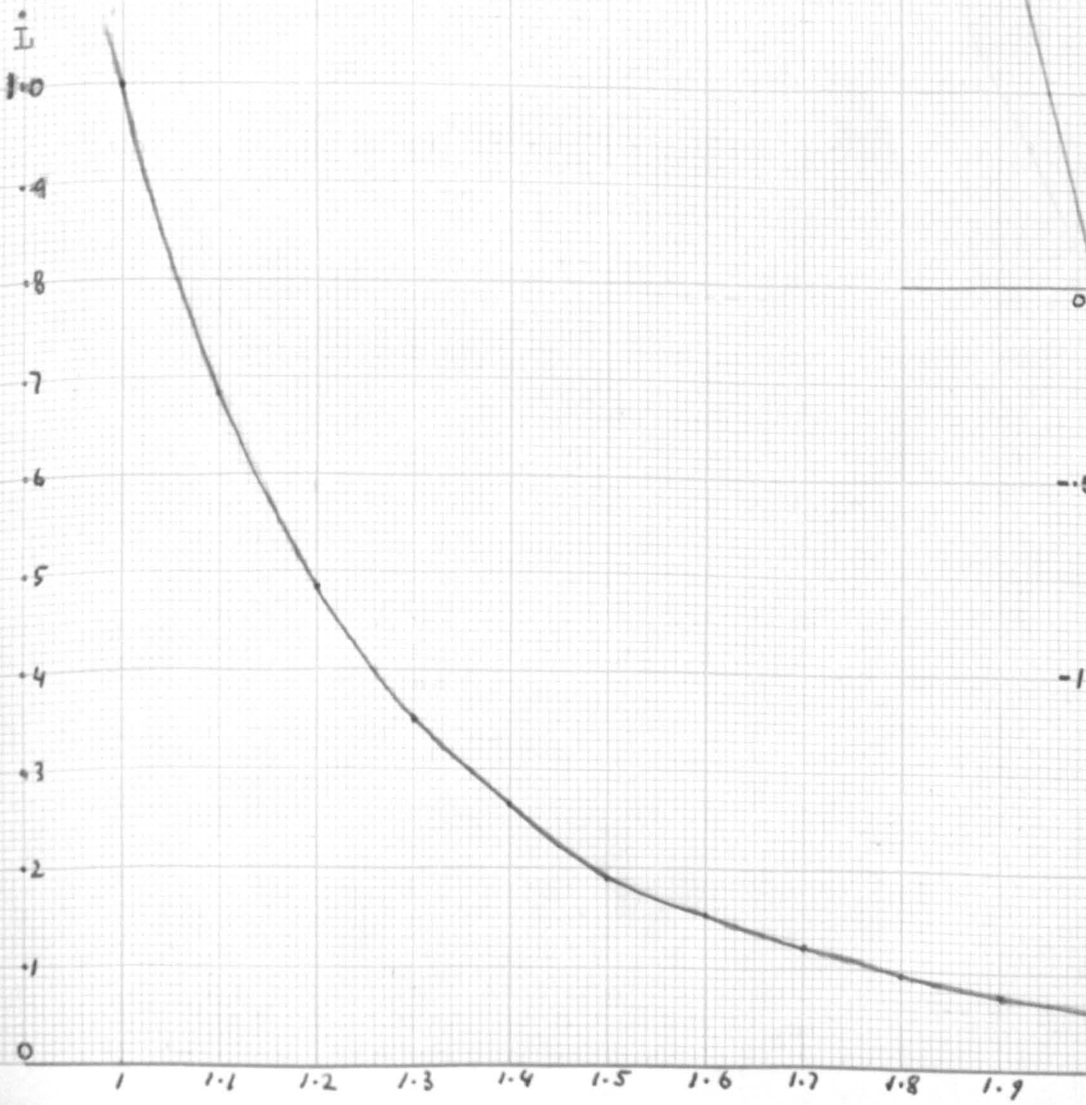


Fig. 11. A model demand curve for labour force in U.K.

4. COURSE OF PRODUCTION.

In 4. I try to explain the course of production through time. It will be useful to have a precise notion of time.

4.1 Deduction of Time.

Man has "impressions" (Hume), or observes phenomena (Kant), events, or facts.

A phenomenon happens before another, is copresent with it, or succeeds it.

A "succession" is a set of phenomena a, b, c, d, ... no one of which is coperesent with any other. E.g. b occures after a, c comes after b, d after c, etc. An example of successions is the succession of days and nights.

If we take a succession a, b, c........., and abstract from the particular phenomena a, b, c......., we are left with the mere order of their succession. Thus, if we take a succession of days and nights and abstract from the "light" of the days and the"dark "of the nights (for ease assuming "light" and "dark" are the only qualities of days and nights), we are left with the mere order of the succession of days and nights. That order is called time. Time is the form common to all successions. Time is entirely a priori (Kant). I.e. time cannot be an object of experience. Time is not a fact. It is the form of all successions of facts. "Time is the universal form of perception of all phenomena whatever" (Kant). Because every fact is a member of a succession.

Space (form) is deduced similarly.

Logical forms are a bit difficult to think about. Time is simply a rule that we do distinguish whether an event happens before or after another, or is copresent with it.

4.2 Time scales.

For convenience a succession is often used as a standard to compare other successions. E.g. they are compared by means of the succession of angular displacements (Hours, minutes, seconds) of a hand of a clock. The order of events of a succession (e.g. trains leaving Waterloo Station), is expressed ny noting the angular displacements copresent with the events (trains leaving) of that succession.

A railway time-table is an example of a succession expressed in that way.

A clock "succession" is a concrete time-scale. Any "succession" actually observed has a finite number of terms (is discontinuous).

A continuous "succession" such that between any two events in it we can imagine to occur as many other successive events as we please. The imaginary events are, say, imaginary angular displacements which are as small as we please. By marking off the imaginary events copresent with the events of some real succession (say clock-revolutions), we get a continuous time-scale of which the "marked" events are the degrees (hours). The time-scale is a succession (imaginary) and therefore has time form. Above the concrete successions and the continuous time-scale are ordinal. Suitably restricting the continuous ordinal time-scale, we construct a continuous cardinal time-scale. The intervals of the scale are continuously divisible. The cardinal continuous time scale is universally applicable (it is "frequent" enough to express even the most frequent imaginary successions of physics). It lends itself to mathematical handling. It is also used in every day life.

The word "time" means either the time form, or the time scale. The time form is the universal form of all successions according to which all phenomena are observed, and is fundamental. The continuous cardinal time scale is arbitrary and is merely useful.

4.3

It follows that the answer whether goods (material objects) and "satisfactions" have a time dimension (duration) or not, is arbitrary.

"Duration" and "time dimension" are alternative names for the time form of successions.

Only successions have time form. A term of a succession is not itself a succession and therefore has no time form.

But a single term (a phenomenon) can have time form at second hand, so to speak. I.e. a phenomenon can be copresent with a number of events not copresent with each other, i.e. can be copresent with a succession. Thus Waterloo Station is copresent with the succession of departing trains. A phenomenon copresent with a succession borrows the time form of that succession.

The only way in which we can conceive a phenomenon to be copresent with a succession is to regard it as itself being a succession (repetition) of terms which are all identical with each other. These identical terms are copresent with some of the terms of the given successions.

The phenomenon copresent with a succession is itself a succession (repetion) and therefore has time form.

There are cases of an event (lightning) not copresent with any concrete succession. Such events have no time form.

Therefore some phenomena have time form and some have not.

But if we agree to use the continuous time scale, then every phenomenon has time form. This is shown as follows. Every concrete event is copresent with a continuous succession of identical terms. Every event is a succession and therefore has time form, or duration, or a time dimension.

Everybody assumes the arbitarary continuous time scale. Therefore from everyone's point of view, all goods and satisfactions have duration or a time dimension.

4.4.

Writing t for a value on the continuous time scale a phenomenon has a duration δt. This δt has a minimum and a maximum.

For example in the case of a house δt is a maximum if we regard δt as the interval during which the house does not visibly alter; and δt is minimum if we define it as the smallest time in which a house can be observed.

It would be convenient to define all phenomena as having the same standard duration δt. But the maximum duration of one kind of phenomena (a house) can be greater than the maximum of another kind (lightning).

For this reason (as for reasons of incommensurability) we cannot reduce all phenomena to a standard empirical (finite) duration. It is obviously inconvenient to specify the duration of each "good" seperately. This inconvenience is removed by the following artifice.

4.5

Let G be a good having empirical duration δt. Then $\frac{G}{\delta t}$ is the (average) rate of flow of the good per unit time at every instant during the δt. It does not matter if we take the maximum duration δt, or the minimum, or any intermediate value. The rate $\frac{G}{\delta t}$ is independent of the possible values of δt. We can specify the rate of flow at a moment of time, without troubling ourselves to specify the duration δt of the good. Adopting a"dot" notation, we can write the rate of flow $\frac{G}{\delta t} = \dot{G}$. An "instant" is dt, and $\dot{G}$<u>dt</u> is the quantity of the good at an instant. The rate $\dot{G}$ is independent of the size of the instant dt. The quantity of the good $\dot{G}$<u>dt</u> at an instant depends on the size of the instant <u>dt</u>, and to make $\dot{G}$<u>dt</u> determinate we have to specify dt. Hence, in describing a good at an instant it is easier to speak in terms of the rate $\dot{G}$ than in terms of the amount $\dot{G}dt$. For in this way we avoid having to fix the size of the instant. In the same way we can describe all kinds of goods in terms of rates at an instant. The advantage of this procedure is that we avoid having to specify particular empirical durations of "goods" out of all possible durations, that we don't have to fix the size of the instant, and that infinitesimal calculus becomes applicable.

If a rate of flow of a good $\dot{G}$ is regarded as a function of time t, the function is often assumed to be continuous. The assumption rules out the concept of the concrete good G. Because G has a finite duration δt and $\dot{G} = \frac{G}{\delta t}$ cannot vary in the interval δt. But once having constructed the concept of $\dot{G}dt$ we leave out of consideration the real good G altogether. We treat henceforth $\dot{G}$ dt as an independent concept; as an imaginary event of infinitesimal duration dt. Having done that, $\dot{G}$.dt can be regarded as varying continuously at successive moments dt. Similarly we can regard $\dot{G}$ as a continuous function of t (if we want to for mathematics). $\dot{G}$ has the advantage over $\dot{G}dt$ of eliminating the arbitrary element dt. $\dot{G}$ itself is not of arbitrary size as it is obtained by construction

$\dot{G} = \frac{G}{\delta t}$ where G is a quantum of a real good, and G, δt are given empirically.

A slightly different manner of constructing $\dot{G}$ is this: we take a quantum G of a good and abstract from the interval of time δt it takes to observe it (whatever the length of δt is). I.e. we consider the quantum G in all its aspects except its duration. We get a time-less entity $\dot{G}$ which is in all respects indentical with G except that it has no "time dimension". This is done when we measure a quantum of a good in units of space (yards etc.) or weight(pounds etc.) or a number of elements (number of loaves etc), and leave out of account (abstract) the time taken to make these measurements and the time goods take to observe. And we do not abstract other attributes besides duration. Thus "a pound of apples" does not mean "a mere pound weight".

Having abstracted the duration we can assume for mathematical convenience that a measurement of the good (weight etc.) is a continuous function of time. But empirically the measurement can vary only discontinuously. It varies by jerks from one interval δt to another.

4.6

Logically, the way of abstracting duration I have just described is the same as the way when $\dot{G}$ is defined as $\frac{G}{\delta t}$. Therefore $\dot{G}$ stands for the size of the good in all its dimensionseexcept duration. The two ways are different ways of expressing the same thing. But the two ways of expression lay emphasis on different aspects. The definition $\dot{G} = \frac{G}{\delta t}$ stresses that $\dot{G}$ is a rate of flow of a concrete good per unit time. But saying that δt is abstracted emphasises that $\dot{G}$ is some quantum which is timesless, and does not make clear that $\dot{G}$ is a rate of flow of a concrete good per time. Hence this way of abstracting duration often leads to the confusion of the rate of flow of the good $\dot{G}$ with the concrete good G itslef. In written and spoken discussion economists often speak of a concrete good when they mean its rate of flow. But often the meaning is indeterminate. Thus Prof. Fisher says " A stock of wealth (goods) existing at an instant of time is called capital"(Nature of Capital and Income p52) By "Wealth" he means "material object owned by human beings" p3. But he does not say whether by"a material object at an instant" he means a concrete good G with a duration δt containing that "instant" of the rate of flow of the good G with a duration δt containing that "instant" or the rate of the good).

flow of the good G at that instant, or the good at an instant $\dot{G}$ dt. Quite rightly he finds that "It is no exaggeration to say that at present the state of economic opinion on this important subject is deplorably confused and conflicting", p101.

In 1.10 I have restated his theory on the assumption that by a concrete good he means a good with an empirical duration δt. When assuming continuity he must mean either $\dot{G}$ or $\dot{G}dt$. The whole of his theory can be most conveniently formulated on the assumption that by a "concrete" good he means the rate $\dot{G}$. But the word "concrete" frequently used by him in this connection in inappropriate. For "concrete" means "observable" and neither the good at an instant $\dot{G}.dt$ nor the rate $\dot{G}$ is observable.

It may be thought that my distinction between G and $\dot{G}$ is the familiar one between "stock" and "rate of flow" That is not the case. My distinction is a special case of the popular distinction. According to the popular distinction (Fisher and others) a "stock" can be G, or $\dot{G}$ or $\dot{G}dt$, and a rate of flow can be $\frac{\Delta G}{\Delta t}$, or $\frac{d\dot{G}}{dt}$ or $\frac{d\dot{G}dt}{dt}$ respectively. According to my definition G or $\dot{G}dt$ is a "stock" and $\dot{G} = \frac{G}{\delta t}$ is a rate of flow (where δt is the empirical duration of the good).

Again, Jevons and other say that "static economics" is timeless because it deals with rates of flow. They say that the time dimension disappears by dividing, say, the output of a good produced during a unit-time by the unit of time.

The proposition is true if by the amount of good we mean G. The proposition is false if by the good is meant $\dot{G}$ (as is often done) for $\dot{G}$ has no time dimension and in this case there is no time dimension to disappear when dividing by the unit of time. Professor Cassel's proposition that the dimension of capital disposal K is money x time (5) is true if by "money" he means $\dot{G}$ and not G. Usually the term "good" or "object" etc., is applied to certain phenomena and is used in the sense of G. To use the same term "good" for the "good with duration abstracted" ($\dot{G}$) is very confusing.

A case of this confusion is the popular definition of "labour-really-labour x time". A recent writer says "<u>A Unit of Labour</u> is defined as equal to a unit of population (i.e. one man) multiplied by a unit of time". Here the "unit of labour" and "one man" have different dimensions and "one man" is timeless ($\dot{G}$) due to an implicit abstraction of the man's empirical duration δt.

4.7

N.B. A remark is required on the measurement of $\dot{G}$. $\dot{G}$ is time-less and its size can vary in some dimensions and not in others. A variation of a good can be measured only in a dimension in which it can vary. E.g. an amount of milk produced can vary in volume (space dimensions) but not in colour (its colour dimension). When by an amount of a good we mean the number of elements (loaves) in a collection of goods (loaves of bread) all the elementary dimensions of $\dot{G}$ are fixed and only the "pure number" of elements can vary. The collection as a quantum can be written as C.G, where C is a variable integer.

4.8 SUMMARY

The traditional obscurity of economic discussion of the time-dimension of goods is due to two difficulties. First, the proposition that every good has a duration δt (has a time dimension) is true only on the arbitrary assumption of the continuous time-scale. Secondly, the concepts of the good G and its rate of flow $\dot{G}$ are easily confounded with each other.

4.9

Causality is the form of successions whose term are causally connected. If we have a succession of two phenomema A, B and if it is believed (a priori) that if A occurs then B necessarily occurs also, then A and B are said to be causally connected and A is called the "cause" and B the "effect" (Hume, Kant). We can write this causal succession as A→B, and the other causal successions similarly.

Causality (causal necessity) is the form (→) of causal successions of events (A→B) and is not itself an event.

This clear notion of causality will be useful because (a) economic analysis deals with causal connections, and (b) the economic subject (consumer, producer) also deal with them.

ECONOMIC CONDUCT.

Man has wants.
(Menger).

Man has Libido.
(Freud).

Know thyself.
(Apollo's Temple Delos).

4.10. There are two theories of value.

One is psychological (Menger, Freud etc.).

The other is apsychological (Pareto etc.).

The "apsychological theory of value" sounds a contradiction in terms. It is better named "the theory of exchange" - of the externally observable acts of buying and selling etc.

4.11. It (Pareto) says: suppose an individual has a "scale of preference" for all material goods (about details see the Hicks-Allen "Reconsideration of the Theory of Value", Economica 1934).

The scale consists of "sets" of all possible "combinations" of goods. A "set" consists of "combinations" between which the individual is "indifferent". The "sets" are arranged in order of "preference". The whole scale is constructed by an imaginary experiment of giving the individual all possible different money-incomes and charging him all possible different prices.

The scale is depicted by a mathematical "map". The "map" consists of a continuum of hypersurfaces with n goods as the coordinates. Each hypersurface depicts a set of "indifferent" combinations of quantities of the n goods. Each hypersurface is an "indifference - locus."

If market prices are to be determined, the preference "map" must have two formal properties. The indifference hypersurfaces must be concave from the origin of the coordinates, and the individual must "prefer" more of a good to less (ceteris paribus) i.e. must "prefer" the surfaces further from the origin to those nearer it. This second attribute is a note written on the "map" (so to speak).

Given such a "map", given the individual's "income", and given market prices, the amounts of goods he buys are determined by the condition that he "moves" on to the furthest possible hypersurface from the origin. I.e. where the "price hypersurface-plane" is tangential to a hypersurface. The price hyper - plane is the locus of all "combinations" of goods which the given income can buy at given prices. Similarly prices etc., etc., are determined when a large number of individuals compete in a market.

The preference "map" describes the individual's total potential "external" or "objective" market behaviour (buying so much of this and so much of that etc.,).

The concept of the "map" can be extended to the whole potential behaviour whith respect to all external objects (e.g. Robinson Cruso). The map is based on imaginary experiments and is not based on any inquiry into the "internal motives" of of "external" behaviour. A "preference" map like that can describe the behaviour of a platform ticket machine which "exchanges" tickets for pennies. Such maps can be made to describe the potential behaviour of all dead and liv ng matter. Paretian Theory makes no psychological assumptions. It does not assume that man feels wants etc. It frees economics from dependendence on the existence of psychology (Robbins, <u>Nature and Significance of Economic Science</u> pp. 83 and 86.)

4.12. There is an objection to this apsychological theory. The imaginary experiments are not real experiments. Till real experiments have been made, we have a mere possibility that a Paretian map can be obtained experimentally. I.e. the "map" is an <u>a priori</u> assumption, its two properties 4.11. are <u>a priori</u>.

That being the case, the "map" says merely that any observed external behaviour can be "explained" by imagining a preference "map" of an appropriate form to determine it. The "map" says merely that causal necessity, which rules throughout nature, applies also to an individual's external behaviour. But (in the absence of real experiments) it says nothing in particular about the causal laws of the external behaviour. For this reason Paretian economics is often referred to as "purely formal" (Robbins, idem p. 86, p. 98.)

My criticism can be restated quite clearly in the following way:-

Consider a consumer x in a market at a time t to $(t + \Delta t)$. He has a sum of money M at t and the market price structure during Δt is p (p is sufficient to describe the nature of the market.) Let c be "the collection of commodities he buys during Δt (i.e. his external behaviour).

Consider c as an effect. Its cause is a set of conditions (x M p). The task of a scientific explanation of c is to find its cause (x M p) and to describe it. One of the conditions to be described is x (consumer). The immediately relevant attributes of x are the elements (desires etc) of his psychic behaviour during Δt. The apsychological theory leaves that out of account. I.e. it leaves out some of the causal antecedents of the external behaviour c. Hence the apsychological "explanation" is indeterminate. It is as if we attempted to fix a value of a mathematical function f(u, v, z ...) without assigning a particular value to everyone of the independent variables u, v, z......... Therefore the apsychological theory of an individual's external behaviour is not a causal explanation at all.

For simplicity I was assuming that x is a part cause of the effect c. In reality, during Δt, the earlier parts of c may be partcauses of the later parts of x. I.e. "internal" and "external" behaviour interact causally. But psychic elements still have causal significance and because Pareto leaves them out, his "explanation" is indeterminate.

In itself Paretian apsychological theory is useless. But its mathematical apparatus can be used in the causal theory of "Economic Conduct".

THEORY OF ECONOMIC CONDUCT.

(The psychological theory of value).

4.13. The theory of economic conduct is an elaboration of the proposition that a man does something because he wants to, and that his thinking affects his actions, In contradistinction to the apsychological theory, it takes the psychic causal antecedents of individual behaviour into account.

4.14. Our total experience (consciousness) is a manifold of elements (phenomena). The manifold is a complex "succession" all its elements are arranged in time. Some are also ordered in space. These comprise the physical world -the world of material objects or "goods". The remaining other elements are not ordered in space. They are feelings and thoughts, and comprise the "inner world". The manifold of experience, therefore can be divided into two worlds - the physical and the "inner". The first has space-form, the second has not. The criterion of distinction between the two is space-form. Therefore in contradistinction to the "inner" or "internal" world the physical world is called "external". Here "external" does not mean "outsiede one's body". Because the body is a part of the external world. "External" means "in space". Sometimes the external world is càlààd "objective" and the internal "subjective" Thus Menger's theory is called the "subjective theory of value" because it pays attention to internal factors - feelings and thoughts. But the terms "objective" and subjective" have a lot of other meanings and their use is likely to lead to confusions. Also, the term "psyché" is often used in the sense of the inner world. In 4.12. I was using "psychic" in that narrow sense. In the wider sense "psyche" includes the whole conscious or all perceptions (and the unconscious), i.e. includes an individual's outer, physical world, as well as his inner one.

4.15. Part of the inner world consists of thoughts - of theore-

tical

knowledge about various parts of the "manifold". Various elements of the manifold are thought of as causally connnected with each other, 4.9. Physical etc., knowledge deals with causal connections in the external world. Psychological knowledge deals with causal connections in the inner world as well as causal connections between inner events and certain external events (e.g. exchange.).

Some thoughts in the manifold are about the "future". They are representations of some possible future events. Future events are thought of as subject to the same causal laws as those governing similar events which happened in the past. The future is "remembered" like the past.

As a scientist I assume that mental events do not happen at random but according to causla laws. I think "free will" is "Causality viewed from within" (Schopenhauer).

4.16. Economic conduct ("[illegible]"), is a series of mental and outer events.

As a purely preliminary definition economic conduct consists of "choosing". But is seems difficult to define "economic conduct to mean anything less than the whole consciousness. One is tempted to exclude things like dreams or solging a mathematical problem. But people choose a lot of thingsin their dreams

and mathematical problems are not

worked out somehow in the abstract, but in the concrete - on paper or in the head and mathematicians always plan to produce maximum "elegance" in their proofs and to economise time and paper. It seems quite impossible to say that one series of mental events is "Choosing" and another is not. This will appear more clearly if we try and see what we mean by "choosing". We all have a rough idea of the meaning.

4.17. All past and present events and some future events (the weather etc.) are not in our control. But some future potential events are. Our wills cause some of these potential events to take place. Which of these potential events do take place is determined by "choosing".

"Choosing" consists of certain feelings and thoughts causing other events either in the inner or the outer world or in both. Certain representations (thoughts) of alternative courses of possible events coupled with a certain feeling of "effort" or "tension" or "desire" cause one of the "courses" (or something like it) to happen, and this course of events is said to have been "chosen" by the individual.

An act of choosing may range form a swift decision "I want that cup of tea" to a prolonged stream of representations of alternative "courses" accompanied by continuous tension (continuous weighing of alternatives) which do not cause any of the alternatives to take place (but N.B. 4.48.). The latter case is roughly described as "inner conflict which results in no action" and is called neurosis, (Hamlet etc). Normal acts of choosing fall between the above two extreme cases.

Choosing is neither a necessary nor a sufficient condition of "action". It is not sufficient to bring action about in the case of neurosis. It is not necessary in the case of "impulsive" actions. But in the majority of "actions", "choosing" is necessary. It is unlikely that a mathematician can hit upon the right solution of a difficult problem quite accidentally. He has to think. He has to "choose", the correct solution he wishes to obtain. Neither would solutions be found for our private and social "problems" without some "choosing".

4.18. It may be noted that the objects of choosing (the representation of the alternative courses of possible events) are not confined to the "outer" physical objects, (goods). We do not only control our outer environment. We control ourselves. We control our feelings, thoughts and the whole mental attitude. Even in writing this sentence I was choosing each particular word thought.

Usually the two types of control were studied separately. *Ethics* studied "controlling ourselves" and improving our "dispositions", and *economics* was concerned with choice about external objects or goods (Menger). For the sake of supposed advantages of division of labour ethics and economics dealt with these two things separately. That presupposes that the future part of the inner and outer worlds are causally independent of each other. I.e. that the two "future worlds" are independent causal systems. That is obviously untrue. Every perception of an external object (or good) is associated with some feeling - tone. E.g. watching a ballet performance. Eating a fine dinner causes one to feel happy, and the dinner was served because you had thought to order the waiter to serve it. Similarly, one cannot be virtuous if one has either too little or too much to eat. The inner and the outer worlds are causally interconnected and the attempt of ethics and economics to separate them has been a violent and unsuccessful *tour de force*. Of course, such a separation cannot be carried out with complete rigour. Ethics does preach moderation in consumption, and some economists do talk of "philosophic" ends. But it remains true that Kant (*Lectures on Ethics* and etc), does not discuss "price-determination", and Menger (not to mention Pareto) does not include the moral perfection of mankind "as the supreme end" in the "system of wants" of his "economic subject".

In the above paragraph I was speaking of the positive and applied aspects of ethics, and not of the metaphysical aspects (God etc) as the latter do not belong to a causal-scientific analysis of "choosing".

The future inner and outer worlds are causally interconnected and are therefore - planned for simultaneously and together. Only concrete experiences exist. We choose concrete experiences and these have both spatial (outer) and "inner" aspects. I.e. we have representations of potential future "manifolds of experience". Which comprise both the inner and the outer future worlds. In a word, we choose a "life - plan".

4.19. I objected to the concept of choice between goods alone, by saying that goods alone do not exist and "chooing" goods alone does not exist. This is a question of fact. I may be told that when economists say "the individual thinks of two collections of goods x, y" they assume that the future feelings which are independent of whether x or y is possessed, are given and constant. Just like a "map" of indefference curves between two goods A.,B. is constructed by assuming the quantities of other goods C.,D.......Z given and constant. And I reply first, why don't the economists say so? And, secondly, what happens ifthe individual thinks of x in association with feelings α and of y in association with feelings β ? This is the more general and usual case and cannot be described in terms of the collections of goods x, y alone. It could be described only if x could not be associated with any other feelings than α so that to affirm x is to imply α, and if y were similarly inseparable from β. But such is not the case. We often choose "feelings" and thoughts which are independent of given collection of goods x. E.g. I can choose between x (including some glasses of whiskey this evening) plus "feeling very gay", and the same x (including the same glasses of whiskey) plus a feeling of aristocratic boredom etc., which I can affect. Similarly I can choose to feel gay or bored with another combination y (including cocktails instead of whiskey).

4.20. Menger's theory is not only confined to choice of "goods" but is further limited to a narrow class of goods only, to "consumer's" goods. "Consumer's" goods are things like food stuffs, clothes, houses etc. They alone are "valued" as "ends", and "producer's goods" are values only as "means" to these ends. Choosing "consumer's goods" has "logical priority" over choosing "producer's goods".

All this part of Menger's theory is a mistake. The distinction between "consumer's" and "producer's" goods is useless for the theory of choosing. "Consumer's goods" are not thought of as "ends" only but also as means to other things - health, capacity for heavy work etc. "Producer's goods" are not "means" only but also "ends" in themselves. E.g. producer's goods called "conditions of work", whole towns and countries to live in etc., etc., are as much objects of choice as are bread and butter This truth is not new. Adam Smith hinted it when he said that a wage-earner seeks to maximise "net advantages" - taking working conditions etc., into account.

4.21. The dichotomy of "ends" and "means" is a formal fiction. Everything is both a means and and end.

"Scarcity economics" says "conduct consist of allocating given scarce means among given competing ends". The proposition has sense only if "given means" signifies events which are outside the individual's control (the "whole present", the weather etc). But it expresses it badly. I think its sense is more clearly expressed thus. Simply we are presented with a small number of alternatives at a time. We have representations of bits of different potential "manifolds". These representations (thought events) cause, under certain conditions of tension etc., one of those manifolds actually to happen. Strictly the manifold that actually happens is never quite like any of the representations. In every experience there is an element of surprise.

4.22. Goods and feelings etc., are chosen "together" because the outer world and the inner world are causally interconnected.

I am afraid I agree a little with Professor Spann. He says (<u>Types of Economic Theory</u>), that Menger's value-scale and Pareto's "map" show goods only, that that is not enough, and that they ought to take "everything" into account.

4.23. Next I wish to see if there is any rule according to which we choose one alternative future "manifold" and not another, out of the many of which we are aware.

The rough rule is: "a man chooses what he wants most". A man "wants" something if he has a representation of it associated with a special feeling of tension - "desire" or "libido" (Freud), and if that tension is diminished when he gets that something. As a clear instance, a lover away from his love - object thinks of her and experiences a feeling of severe tension which is relieved when he comes near her.

Libido has causal significance. E.g. it causes the lover to approach his love - object.

The feeling of tension has several degrees. The rough rule is that we choose that alternative experience which has the smallest degree of tension in it. I.e. we minimise libido or economise it. So, choosing is aptly called "economic conduct" (Freud). Economic conduct consists of economising libido not only in this way but also according to the "aesthetic" principle explained in 4.25.

4.24 This "discharge from libido" theory is true is true about some isolated acts. E.g. when a man eats to satisfy hunger. But it is not true of many acts. E.g. actions from principle and against inclinations: "curbing oneself" as when forcing oneself to work hard.

The truth seems that the "discharge" theory applies only to a part - perhaps a considerable part of our actions - to living from "hand to mouth" without bothering of the consequences. S. Freud calls the general rule obeyed by these actions "the pleasure principle" (the "discharge" hypothesis). With this he contracts the "reality principle" (Beyond the Pleasure Principle). The principle explains those of our actions which are preceded by "planning" which takes account of probable future consequences of present actions. The principle includes the "pleasure principle" as a limiting case. The reality principle has been worked out also by economists. Particularly by Karl Menger. But he does not go far beyond the pleasure principle.

Next I deal with the reality principle i.e. with planning.

4.25 "The individual adopts that life-plan which gives him less tension to contemplate than any alternative plan". This may be called the "aesthetic" theory of planning. The individual works out and thinks of as many different plans as he can and "adopts" that one which causes least tension to look at. He is like an artist painting a picture. The artist adds paint and experiments with details till any further strokes of the brush do not produce any further diminution of "tension" of looking at the picture. When he has reached that point of minimum tension the picture is complete, i.e. he cannot "improve" it any further. And so in all works of art. In the case of works of genius such as the pianoforte sonatas of Mozart - the tension drops to zero.

4.26. The "aesthetic" theory can be stated in psychoanalytic terms Planning is seeking the return to the peace of the pre-natal state. It is modelled on this experience. The plan that gives least tension or greatest peace of mind to contemplate, resembles most closely the pre-natal peace and we adopt it in order to simulate the "return".

Psychoanalytical hypotheses often seem disturbing. But fear should not deter us from using them whenever they seem to explain things better than do alternative assumptions.

4.27. The artist sometimes completes a picture, but no plan is laid down once and for all. And the reason is "uncertainty".

Strictly speaking the artist also never completes the picture. So called pictures are only discarded experiments in a search for perfection which ends only with the artist's life.

All future events are uncertain (both those which are not in our control and those that are). I.e. a plan provides for a number of alternative probable events. E.g. "it will rain on Sunday" and "it will not rain on Sunday" (case of uncontrolable events) and "if it rains I shall take my umbrella and it will protect me" and "the wind will turn my umbrella inside-out and it won't protect me", (case of controlable events). To all probable alternatives we attach various degrees of "belief".

When "uncertainty" is assumed in the analysis, the selected plan no longer provides for a single "manifold" but for a number of probable "manifolds."

When one of the probable alternative events actually occurs, our "expectation structure" is altered. E.g. if it does rain on Sunday my belief in having a cold on Monday increases. Or if a draught actually occurs, the speculator's belief that wheat prices will rise increases, and that prices will fall - diminishes, etc., etc.

Another reason why "expectation structure" changes is that some quite unexpected events take place. We make complete mistakes of anticipation. And in general, as I said, every experience contains an element of surprise. E.g. scientific disconeries". We live and learn ...... and forget .

Realisation of a probable event or occurence of an unexpected event is a "change of data". Economists were trying to say that.

Every potential event is either probable or unexpected. Therefore every actual event is a "change of data". Therefore change of data is incessant. It causes the adopted plan to be revised continiously. The change of date is due to "uncertainty". If we abstract uncertainty, there is no change of data.

In the plan only actions intended for the immediate future are 'represented' in detail. More future peobable events are represented in outline. Very distant events are represented by some such thin reservation as "something will happen" or "I shall be alive" or "not alive".

4.28. The plan is revised chiefly by filling - in the details as "the immediate future" becomes part of "the past". "Outlines" and concrete decisions are also revised. E.g. an appointment to Canada causes me to substitute the outline "I will live in England" by the outline "I will live in Canada". And an attack of nervousness will cause a bridegroom to change at the very last moment a detailed decision "I will marry" into "I will not". This is due to a "change of data" - the fit of fear.

As data change (as events occur) at each instant a new plan yields minimum tension and is adopted. Economic conduct consists of a continious revision of the minimum-tension (adopted) life-plan in face of incessant changes of data (events). This view of conduct is "dynamic" - like the facts.

4.29. Strictly, no causal process can be continious. Economic conduct is not continious, it proceeds in stages. Alternative plans are successively made. Their tensions are compared. The least tension one of them is adopted. But the causal stages succeed each other so rapidly that the process appears continious. The orator select each next word from a number of alternatives and plans his "effects". Yet the flow of eloquence he produces appears to be continuous.

4.30. To say that the tensions of plans are compared is to imply that somehow the plans are held in mind simultaneously. This is effected by a sort of mental shorthand of thinking of a lot of things in a "single mental effort". I.e. at any moment we have a somplex of representation and feeling tone. The representations include perhaps a word or two but not more, as words can be thought only successively and to think a succession of them takes time.

4.31. How do I know which plan I adopt? How does the "adopted" plan differ from a rejected plan?

If we abstract from uncertainty, the answer is simple. The adopted plan consists of representations of events (manifold) which I am certain will take place. A rejected plan consists of representations of events which I am certain will not take place.

If we admit uncertainty, my adopted plan consists of "chance - distributions". It consists of representations a number of manifolds (which are alternative by definition of a "manifold") to which I attach specific degrees of belief. A rejected plan consists of representations of manifolds with degrees of belief which I can imagine myself attaching to them, but which are different from the degrees which I do actually attach. I.e. it consists of imaginary, not actual, chance-distributions. Thus in the case of certainty, a rejected plan attaches certainty to events which I am certain will not happen. This analysis has the air of a paradox. But I am afraid that that is what we really mean to say when we try to distinguish between an adopted and a rejected plan. If any one doubts just let him try and think of some other ground of distinction.

The distinction can be put in different words. A plan is a "possible structure of belief". The adopted plan is "the actual structure of belief". The rejected plans are all the remaining "possible structures of belief".

Choosing a life plan consists of making a possible structure of belief actual. "To choose what to do" means "to choose what to believe will happen". And "to choose" means "to follow the path of least tension". "Revision means succession of one belief - sturcture by another, according to the principle of least tension, as "data" change.

4.32. Let us consider the connection between the belief - structure at one "instant" and the events at the next instant. "I believe I shall walk into this shop and buy a box of ten Player's cigarettes for sixpence.". The next moment I do so. Has that belief been the cause of my action? Perhaps the answer will be clearer if I say thatbelief is the same thing as "Will". E.g. "I believe in socialism" means "I will socialism". The answer is that belief or will has causal signigicence. This is expressed in the saying "faith moves mountains."

The will is only one of the causal conditions and by itself does not determine the nature of the "effect". E.g. the will "I shall buy ten Player's" is not enough to make me do so. Several other conditions have to be fulfilled. For instance, there must be these Cigarettes in the shop. If they are not, I ask for Will's and if they have not got them, for State - Express. And if there are no cigarettes I like (as in France) I simply walk out of the shop.

To determine "uniquely" the action in theory it is not enough to know the individual's adopted plan. It would be enough if there were no uncertainty: then actions would be absolutely according to plan. But there is uncertainty. The plan provides for a number of probable alternatives. The plan does not determine which of the expected "manifolds" will occur (if any). Each experience is a change of data, 4.28. To get "unique solutions" we have got to account for the change of data (i.e. the actual events).

4.33. To do that the economist has to pretend that he knows more than the individual whose behaviour he wishes to explain. The economist has to pretend that he knows all the necessary causal conditions of the individual's behaviour. E.g. he has to pretend he knows if there are Player's cigarettes in the shop or not etc., etc. He says: "given X's plan to buy a sixpenny Player's etc., etc. and given they are in the shop etc., etc. his action is unequally determined: he buys the sixpenny - Player's etc., etc., etc."

4.34. The general conclusion is this. The plan at one moment is a causal condition of the actions (events) at the following moments. Abstracting uncertainty, the causal connection is very simple: all events occur according to plan and the plan is the whole cause. Admitting uncertainty, the plan (representations <u>plus</u> belief) is only one of the many causal conditions of behaviour (actions) and the causal connection between the two is complecated. The plan does not say which of the probable events will happen. Only parts of the experiences have been forseen by the plan. Unexpected events occur, and behaviour is not according to plan. Still, the plan has something to do with it.

Some events in the immediate future are almost certain. They happen almost according to plan.

Some events in the immediate future are almost certain. They happen almost according to plan. The causal connection between the plan and these events is probabilistic (incomplete) but approximates to a complete causal connection. E.g. I know the next word I am going to say and say it almost in every case. The proposition "I say the next word <u>because</u> I consciously will it" is almost true.

Each concrete action is immediately preceded by a plan. Assuming it was forseen almost with certainty, it can be explained with fair accuracy by explaining how the plan was determined.

4.35. To take a simple case. I consider only two alternative manifolds; to buy cigarettes for a sixpence (and the rest of the manifold I) or to pass the shop retaining the sixpence (plus the rest of the manifold II). Say, I gives less tension to think of I than II and I adopt I in accordance with the esthetic rule. In this case the esthetic rule determines my action, exchange of a sixpence for cigarettes, with almost complete accuracy. This can be put into mathematics. Let l(I) be the tension given by looking at plan I and l(II) - that of looking on II, and given that $l(I) < l(II)$ the solution is I.

4.36. The matter is put more like the usual mathematical economics as follows.

Let there be in my pocket quantity <u>C</u> of cigarettes and <u>M</u> of money.

Let <u>m</u>, <u>c</u> be the quantities of money and cigarettes planned to be in my right trouser pocket. Let l(m, c) be the tension function of looking at different plans (differing in respect of m, c). Let p be the price of cigarettes i.e. the range of alternatives is described by the equation:

$$m = \left(M + \frac{C}{p}\right) - pc \qquad \text{i.e. } dm = -p.dc \quad .$$

The minimum tension condition is:

$$0 = dl = \frac{\partial l}{\partial m} dm + \frac{\partial l}{\partial c} dc = -\frac{\partial l}{\partial m} p.dc + \frac{\partial l}{\partial c} dc$$

$$\therefore \qquad \frac{\partial l}{\partial c} : \frac{\partial l}{\partial m} = p \qquad \text{.......... (56)}$$

which is something like the celebrated Jevons-Walras "equation of exchange".

Given l(m,c), M, C, p (56) determines the adopted plan. I.e. it "almost" determines the amount of cigarettes I buy. (56) means that if I am to decide to buy a determinate amount of cigarettes at the price p, my function l( ) must admit of a minimum. I.e. My "indifference curves" must be concave upwards.

An individual makes exchanges successively in time. Applying an equation of exchange such as (56) we can explain each exchange. At each instant the form of the tension - function e.g. l(m, c) will be different. Because of changes of data.

4.37. The ordinary mathematical method has advantages and disadvantages. It is insufficient: it cannot describe vague ideas ("outlines") nor "changes of data" nor "revision". And in the last resort it fails on its own ground: it cannot symbolize a collection of goods. For each unit of a good has a different space & time index (position). These indicies cannot be expressed in a mathematical function such as $\ell(m, c)$. I got round the difficulty by saying that both money and cigarettes are in my right trouser pocket. That is not generally the case. They can be in different pockets. I have to decide which. And suggesting "pocket-indicies" won't help really.

All that curtails the usefulness of 'calculus' much more than my friends, the mathematical economists would like to admit. If I may repeat, in economics mathematics speak clearly but say little. The logical form of mathematics is not general enough to express the form of economic conduct as a whole.

4.38. Theoretical determination of the adopted life-plan (intentions) and theoretical determination of behaviour (actions) are two different things. If uncertainty is abstracted, they are the same. Jevons, Walras, Pareto etc., abstracted uncertainty. Therefore they did not distinguish between the two things. Menger did distinguish: he said something about learning (the unexpected) from experience. Pareto also abstracted from inner phenomena - from the plan. So he explains neither the intentions nor the actions. His is an empty theory of <u>a priori</u> "tendencies to action", c.f. 4.11.

The theory of economic conduct deals with the determination of both the intentions and the actions, and with more. It deals with "changes of data" and "revision".

4.39. The aesthetic theory of economic conduct can be summarised as follows.

Economic conduct is a process in time. It consits of cycles each having two phases;adopting a plan and action. We have a stream of repesentations of possible future events , coupled with a varying feeling of tension. That stream contains a number of alternative plans.

By another set of representations the tensions attached to each plan are compared, and the plan giving least tension to think of asadopted, is adopted.

"A life-plan is adopted" means "the least tension imagined structure of belief becomes actual". "The plan" is determined. Next "action" occurs. It contains a number of events forseen by the plan as probable and also some unexpected events which are coupled with an "element ofsurprise". Both the becoming of particular probable events certain when they happen, and the surprising events cause the plan to be revised. The beginning of revision is the beginning of a new cycle. New representations of a number of possible "structures of belief" about potential "manifolds" take place, and the new cycle proceeds according to the same general rule, and is in turn followed by another cycle, and so on.

Economic conduct is complicated. But its general form is simple. Here is its formula: Economic conduct is a succession of cycles, each of two phases. The first phase is selection of the plan according to the aesthetic principle of least tension. The second phase is "action". The bi-phasal cycle is a bit "Hegelian".

4.39. There are two criticisms about the aesthetic theory. First, that "tension" does not exist. Second, that if it exists, it has nothing to do with determining "the plan" (the psychophysical view).

Both criticisms deny that "a plan is adopted in accordance with the aesthetic least-tension".

Whether the tension exists or not is question of fact. I think it exists. Degrees of "Peace of mind" certainly exist. And the degrees of Peace of mind ~~of mind~~ are simply degrees of tension read upside down. "tension" and "Peace of mind" are different symbols for the same thing, for the same feeling.

Our peace of mind depends on what we suppose we decide to do. If a lover asks himself "suppose I decide to leave her etc", his peace of mind becomes distrubed. If he then asks "suppose I decide to stay with her etc.," the unpleasant tension is at once relieved. The second plan is aesthetically more agreable to him i.e. gives less tension. From observation we find that he adopts the second plan (in absence of ther alternatives.) If a healthy person reads news about a suicide and has a thought "suppose I decide to die", the mere thought fills him with some apprehension. He next thinks "suppose I decide to live" and experiences a sense of relief. Again it is found that he adopts the second (the least-tension) plan. With a deep melancholic it is the other way about. The first plan gives least tension and the melancholic adopts it.

Many examples can be quote . But these are sufficient. The rule "the least-tension" plan is adopted is a generalisation from experience.

Freud calls the tension "libido". The rule becomes: "economic conduct follows the path of minimum libido".

The rule seems imbedded in the nature of man.

4.40. If l(t) is the cardinal index of libido at time t, the rule is that l(t) is at a minimum, which implies that $\int_a^b l(t)\,dt$ is at a minimum.

In the language of the mathematical theory of variations (slightly generalised) we have this.

Let P(t) be a possible plan the individual can adopt at t. It is a function of t.

Let $\left[P(t)\right]_a^b$ be the field of variation of the function P(t) in the interval t=a, t=b. The field consists of all the alternative plans the individual is aware of at each instant in the interval a, b.

The index l(t) of libido at t depends on the plan P(t) and on t. The same possible plan imagined at different times gives different tension. We have the function $l(t) = l\{P(t), t\}$ in the interval a, b.

The integral $\int_a^b l(t)\,dt$ is a <u>functional</u> of the function l(t), i.e. of P(t).

I.e.
$$\int_a^b l(t)\,dt = F\left[P(\overset{b}{\underset{a}{t}})\right]$$

Assuming continuity and letting P(t) vary in its field, the aesthetic minimum libido condition is: the first variation

$$\delta F\left[P(\overset{b}{\underset{a}{t}})\right] = 0 \quad \text{and} \quad \delta^2 > 0. \qquad \ldots\ldots (57)$$

Given the plan-field $\left[P(\overset{b}{\underset{a}{t}})\right]$ and the libido function $l\{P(t), t\}$ this condition determines the function P(t) in the interval a, b. I.e. it determines every adopted plan.

I.e. it determines <u>the path of intentions through time.</u>

But the theory of economic conduct is not content with assuming the field of possible plans $\left[P(\overset{b}{\underset{a}{t}})\right]$ and the libido function $l\{P(t), t\}$ as given. It also explains actions and studies how both the field and the function are determined by "changes of data" (by experiences). Only then is the explanation of economic conduct logically complete. It is logically complete because there is nothing left to explain - except "details" 4.47.

In economics ordinary maths are not much use, 4.37. But the generalised calculus of variations is. It expresses the principles of economics (e.g. the aesthetic) with absolute rigour.

4.41 The discharge from tension theory of 4.23 says that a man does something because he wants it and in so doing satisfies his want or discharges tension. But actions from the pleasure principle apart, tension is often not discharged and is sometimes deliberately accumulated as during watching some moving tragedy on the theatre. True, the less immediate tendency of seeing tragic plays isto diminish tension as Aristotle pointed out. These plays do this by removing our unconscious fears by making them conscious. The case of the plays suggest that we often incur immediate tension if we expect to be recompensed later by a discharge of tension.

The pleasure principle is a limiting case of the aesthetic. It is a case where the individual forsees alternative events only in the immediate future. The alternative plans are confined to the immediate future. And that one usually has least tension attached to it and is chosen which provides for a discharge of tension at the next moment.

The pleasure principle being a special case it is liable to be confused with the general aesthetic principle. Thus, aesthetic theory says that a plan is chosen which (uncertainty apart) comprises the "next" action. From outside it may seem that the individual chose this action by itself without thinking of subsequent actions, and because the "next" action seemed to him a means of discharging tension. That is how the pleasure principle would explain his action. But the easthetic theory says that he chose this action <u>and</u> the rest of the chosen plan which may provide for a considerable future period. The fact that he also chose "the rest" is liable to be overlooked. If it is overlooked the difference between the general aesthetic and the particular "discharge" theories becomes blurred. The two theories become confused as was the case when each kind of goods was supposed to have a separate "utility" (Jevons, Walras, Menger).

4.42

The aesthetic principle of minimum libido is a unifying idea. It coordinates under a rule an economist's ideas out of the first phase of a conduct-cycle, 4.39. About the immediate inner causal antecedents (representations and feelings) of an "action".

To help describing the less-immediate inner and outer causal antecedents, another unifying idea is required. The <u>unconcsious</u>. It is a convenient fiction. The unconscious is the world of forgotten experiences.

The experiences of the first few years have a strong influence on the rest of our lives. Most of our education stops at the age of five (Freud). These early experiences are forgotten in later life. To explain their influence on the subsequent life clearer, we imagine them to be hidden in the unconscious. They are stored up inside the unconscious and from there exercise continuous influence on the conscious life. The ways they do it are called Freudian Mechanisms. Freud psychoanalised his patients i.e. he made them recall forgotten experiences. This proves that forgotten events are remembered by the unconscious.

Similarly, all experiences up to time t influence life after t. At t most of these experiences are pushed into the unconscious. At any moment we remember consciously very little about the past. The content of the unconscious at t is a quasi part-cause of the next action after t. The real immediate causes are the content of the conscious and such external factors as "there are Player's cigarettes in the shop", 4.32. But the content of the conscious, the plan, is itself an effect of certain causes in the past. I.e. of past experiences most ofwhich are repressed into the unconscious. The conscious is a part-result of the unconscious. This is the Freudian way of searching deeply into the past for the roots of "actions."

That method explains how an individual builds up possible life-plans, compares them and adopts one of them. We can have only a few clear ideas at a time. A possible plan "suppose I go to Paris for holidays" takes a long time to work out in detail. The working out consists of a succession of representations some of which are clear ideas. Viz. "I shall stay at Monmartre, I shall eat at the Dominique, see my friends X. Y. Z. and go on the top floor of the Louvre to see "Cézanne" and a lot of other details. At an instant when I have worked out the plan I find that I have forgotten that million details. I cannot think so many clear ideas simultaneously. Instead I have a vague representation of the "Paris plan" as a whole. Rather, I have a vague idea of a hundred alternative "Paris," "London","Bierritz" and "Margate" plans for the next holidays. It seems incomprehensible how I can choose where to go if I have only such a vague idea. The hypothesis of the unconscious explains the puzzle quite elegantly. It says that in fact I remember all the details of the hundred plans. Only in my unconscious. The hundred plans are simultaneously compared in my unconscious and one of them is adopted by the unconscious. It is useful to personify the unconscious a little. When all this is done for me by the unconscious, the unconscious informs my conscious (me) of its decision. It informs me by means of the conscious vague idea of the hundred plans by the feeling that I attach least libido to the plan it has chosen, and by making me feel belief into the plan or to will this plan . I.e. the unconscious makes up my conscious mind for me.

In Freudian literature the conscious and the unconscious are usually said to disagree (in cases of illness). But they do not seem to disagree about my holidays. The reason is that the unconscious is not "consistent". That part of it which is my forgotten reflection about the holidays agrees with me, but certain other "repressed desires" may be hostile to my adopted holiday-plan.

Anyhow, the unconscious is a metaphor. When I say it makes up my mind for me, I mean that each "present" decision I make (according to the aesthetic principle) is partly the result of reflection and feelings extending deep into the past. And I mean also that practically all the past thoughts and feelings are forgotten.

Freud explains how repressed sexual desires cause neurosis and hysteria. But little detailed work was done about the connection between repressed experiences and "normal" behaviour as whole.

An important forgotten experience is the unconscious memory of the prenatal state. This "memory" seems to cause our conduct to follow the path of minimum libido. Just as it appears to be the cause of religious wishes for paradise and Nirvana.

4.43. Another unifying idea is "psychic energy."

As physical energy this energy is a pure quantity. It is entirely a priori. Nobody ever saw it or knows what it looks like.

The amount of energy in the conscious is constant. All psychic processes are transformations of energy. The causal laws of psychic processes can be expressed as rules of transformation of energy.

Freudian Mechanisms are transfers of energy. E.g. the OEdipus complex. Parental education represses incestuous desires and the young man (Hamlet) develops "symptoms". Energy is transferred from the desire to the symptoms. And so with all other complexes and sublimation. The patient is cured by transfering energy from his symptoms "on the analyst".

4.44. Freud talks of "transference of libido" when he means "transference of energy." Libido (a feeling) exists. Energy does not exist. Freud's verbal confusion does not lead to any mistakes of diagnosis etc. Because Freud implicitly expresses a quantum of energy by an "equivalent" degree of libido. Just as in physics, an energy quantum has a "heat-equivalent" or a "work - equivalent" etc.

Freud's terminology has some advantages. "It fits the facts". All the complexes etc., have to do with repressed libido which is as it were transformed into symptoms. Explicitly: energy is transformed from libido into symptoms. There is a rough quantitative connection. The intenser the libido which is repressed, the more violent is the tension -

component (libido) ofthe resulting symptoms.

Jung (Contributions to Psychoanalysis, essay on energy) makes a worse confusion. He calls evergy "libido". He says "libido" is a priori and does not exist. At the same time he says "libido" is a feeling which exists.

COnfusion between libido the phenomenon, and energy the unifying idea a priori , seems to run through the whole of the psychoanalytic leterature.

4.45. The concept of energy helps to connect up ideas about the conduct process. For example, we cannot think more than a few clear ideas at a time. If we think some, others are excluded from the conscious. The total of energy is constant. If some takes the form of certain ideas and feelings there is less energy left to take other forms at the time. This has applications in many typical economic problems. E.g. advertising. When an advertisement is seen it introduces new content into the conscious, and represses some of the ideas etc., previously held in the conscious. We explain this by saying that energy is constant. This transformation of energy affects subsequent conduct. E.g. I pass a tabacconist's and his window show causes me to go in and buy some tobacco instead of proceeding uninterruptedly on my way. The window causes energy to be transformed along that particular "path". Economists describe such effects of advertisements by saying "advertising raises the demand-curve for the good advertised" etc., etc.

More precisely, advertising influences the individual's field of possible plans $\left[P(\overset{b}{\underset{a}{t}})\right]$ and the libido-function $l\left\{P(t), t\right\}$. i.e. (in older terminology) the individual's "knowledge" and the "scale of value" are altered.

How the field and the function are altered can be described in some detail. For example: during an interval a, b advertising causes us to keep in mind the goods advertised and to forget the goods not advertised to us. An elegant and appealing advertisment intensifies the libido. A successful advertisement so alters the function $l\{\ \}$ that it is minimised only when the individual plans to purchase more of the good then he planned before he saw its advertisement.

The plan field $\left[P(\overset{b}{\underset{a}{t}})\right]$ is determined by energy flowing from some thoughts etc., to thoughts about the good. The function $l\left\{\ , \overset{b}{\underset{a}{t}}\right\}$ is determined by some energy taking the form of the intensified libido and by different libidoes becoming attached to the same possible plans (if any) as compared with a case when the advertisement is not seen but something else occurs instead. To express clearly these complex relations it seems necessary to use the concepts of the <u>variable</u> field $\left[P(\overset{b}{\underset{a}{t}})\right]$ and the varieble function $l\left\{P(t), \overset{b}{\underset{a}{t}}\right\}$.

In a similar way all other external stimuli affect the field and the function.

Similarly they are influenced by all inner events. The field and the function are <u>determined</u> by the totality of the inner and of the relevant outer events.

4.46. The determination can be explained by means of the principle of conservation of energy.

We have these generalities. The life of an individual is determined by his environment.

Symbolically:-

L(t) = the individual's life-function, the flow of his experience.

V(t) = the environment-function, the stream of events which affect his life but are unaffected by it. E.g. the weather, public-school education and so on.

The life L(t) consists of the inner aspects, inner life I(t) and external aspects X(t) his perceptions of the physical world.

$$L(t) = I(t) + X(t) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (58).$$

His inner life consists of representations of possible plans P(t) and of libido-distribution $\ell\{P(t), t\}$ on them, and of the rest R(t).

I.e. we have:

$$I(t) = [P(t)] + \ell\{P(t), t\} + R(t) \quad \ldots \quad (59)$$

The rest R(t) is made up of memories, pleasures, pains, sensations of colour, sound and so on.

The life is transformation of energy. The principle "energy at each instant is constant" means "given the environment, the life is determined by causal laws". I adopt the convention that if life deviates from the causal path, energy is not constant but decreases (is lost). "Life follows the causal path" becomes "energy is conserved". Now the rule "energy is constant" means "energy is maximised (conserved)".

I can now proceed symbolically. Let E(t) be energy at t, and $\left[L(t)\right]$ the field of the function L(t).

Then given the environment function V(t), the life function L(t) is determined by the condition

$$\delta \int_b^d E(t)\,dt = \delta F\left[L(t)\right] = 0 \qquad (60)$$

where the limits b, d signify the dates of birth and death of the individual's conscious.

4.47. The formula is the *parent* - formula of all economic formulae.
These formulae are a family.

For example (56), (57) can be stated in terms of energy by saying that energy is maximised. I am not quite sure that the aesthetic minimum-libido principle is true. It seems to be. But if it is not, mathematical formulae of conduct are always possible in terms of energy.

The formula (60) is a clearer way of saying "life follows the causal path". It distinguishes sharply between the environment V(t) and the life L(t).

The formula isthe most general proposition in psychology. Therefore it says little. It asserts psychic determinism. I hope it is lucid.

It says "given the environment V(t), life L(t) is completely determined by causal laws". It is interesting to know what those laws are.

4.48. Freud describes several. He explains the more general aspects of the life L(t) by tracing them to the environmental factors in early childhood etc. Details of the life L(t) are explained by the aesthetic theory of conduct and by the energy theory. They trace them to the more immediate inner and environmental factors. E.g. "actions" are traced to the actual belief structure (the adopted plan) and to environmental facts such as "there are cigarettes in the shop". The adopted plan is the result of a reflective process of plan building, of plan comparison and plan selection according to the aesthetic principle of minimum libido. Conduct consists of bi-phasal cycles:"will-action". The process of plan building goes far into the past. But conduct cycles are rapid: we have to decide what word to think next in the very process of working out plans. Little details e.g. "how I think of the next word" etc. are explained by the laws of association etc. E.g. law of opposites "I say 'black' and think of 'white' etc." All these laws can be brought under the concept of energy.

Ultimately $\left[P(t)\right]$ and $L\left\{P(t), t\right\}$ are determined by V(t)

(59) (60).

But concrete explanations of them can be given only in terms of the more immediate causal antecedents.

Each experience determines the plan field [P(t)] the individual is aware of at the next moment. And the libido distribution among those plans, l { }. E.g. seeing an advertisement. These changes are also partly due to inner factors. E.g. thinking out a practical problem on can have a bright idea - an unexpected plan to solve the problem.

4.49 Menger apart, traditional economics assumes the plan field $\left[P(t)\right]$ and the libido (utility) function $l\{P(t),t\}$ as its logical data and deduces from those logical implications (Robbins idem p.p.115-6). But that cannot be done. Because the plan field $\left[P(t)\right]$ and the libido-function $l\{P(t),t\}$ are parts of conduct and have themselves to be explained. The plan field and libido are functions with respect to time, and have been called "changes in given data". They are part of conduct and do not come from "outside". They are not the logical data of a conduct explanation at all. Following MengerI explain them as caused by the individual's past experiences e.g. by an advertisement. The so-called "changes in given data" are incessant because the flow of experience is continuous.

The true logical data are: (a) introspective evidence.
(b) observations of others' behaviour.
(c) hypotheses.

4.40 shows clearly how taking the plan field (P t) and the function l ,t as given, fails to explain conduct. In 4.40 I say "supposing economic conduct is determined as in (60). Then the field and the function are determined. There is a relation between these two and the path of intentions $P(t)_a^b$ according to the aesthetic principle. This relation is express ed by (57)". That is all one can deduce from the "changes in given data" [P(t)], 1{,t}. The plan field and the libido function are not determined independently of the path of intensions. All these are determined "simultaneously" as parts of conduct, (59), (60). The "changes in given data" explain neither conduct nor that part of it which I called the path of intensions.

A like objection applies if "changes in given data" are interpreted in the apsychological sense as a human body's field of potential behaviour (tendencies to action), and as its environment V(t)

4.50 Mind and matter interact. The interaction is called "conduct".

Pareto tries to explain it. He abstracts from mind. So, he explains nothing.

I take mind into account. I find that a hard task. I am tempted to give it up and to go back to Pareto's behaviourism. But that can't be done.

My account of conduct is not very good. It is a sketch. May others do better.

NOTE.

I have asked some economists to criticise my account of conduct. On the whole they objected to the "aesthetic" principle while broadly admitting that the rest of my account - the account of plan-building, of the adopted plan as the "actual structure of belief", of uncertainty, of "revision" - and of accumulation of experience in general, does seem to touch on the real problems of pure economics.

I wish to consider some objections to the aesthetic principle. Some objections were not of a scientific character. For instance it has been suggested that psychoanalysis ought not to have been brought into the discussion because it is very painful to many people and so, antagonises them to the argument and to its author. As if to illustrate that point, unwholesome motives have been imputed to me for making use of psychoanalysis. It was suggested that I used it solely because I read a few books on it and was impatient to make that fact known to other people; and that my appeal to psychoanalysis would not have been genuine because my knwoledge of it is inexpert.

I do not believe, however, that it is necessary (though desireable) for an economist to have a degree in psychology before he is competent, to use psychologists' results. If an economist understands his own work he should be able to understand the work of the psychologist because they study the same thing - conduct. Sometimes the aspects of conduct they study are different. Sometimes theyare the same - as the process of building the life-plan. Psychoanalysis and economics, as they now are, are specialisms within the same science - psychology in its widest sense as a study of interaction of mind and matter (Geistwissenschaften). A unity underlies them both. Psychoanalysis is mainly concerned with the more remote antecedents (childhood experiences etc.,) of conduct, while economics deals with the more immediate antecedents with the process of reflection etc., preceding an act of choice.

I make use of Freud's results because:

(a) his picture of conduct is more dynamic, than the picture which economists have so far succeeded in working out, therefore it is useful to study his picture;

(b) because it is useful to realise the unity underlying his and the economist's work; and

(c) because psychoanalysis appears to give theoretical support of the "return" hypothesis to the empirical evidence in favour of the aesthetic minimum-tension principle of choice.

The scientific objections concern the existence of tension and the validity of the aesthetic principle.

It has been said that introspection does not clearly show whether tension exists, and that probably it does not exist.

In an average individual (a statictical mode) tension is most easily seen by him a s a component of physical pain for

for example when crushing a finger. It again occurs in mental pain as when receiving bad news. It is isòlated as an element which physical and mental pain have in common. That feeling-element, tension, is f und to admit of different degrees of intensity. Both physical and mental degrees of pain can be arranged in order of intensity. The intensities of physical and mental pains can be compared. Manya mental pain is said to be "greater" than asphysical pain. The mere comparison proves that the two kinds of pain have something in common. That something is tension. Strictly, physical pain is a special case of mental pain in general. A degree of tension is sometimes called a degree of peace of mind, and "great peace of mind" means "small tension" and "small mental pe ce" means "great tension". These remarks show that tension exists and c n be observed as an element in the feeling of pain for instance when crushing one's finger or learning bad news.

The difficulty of detecting tension is due to the absence of a strict correlation between tension and the correlated phenomena, for example crushing a finger. Thus, while to the average (mode) person crushing the finger causes suffering, to a masochist it causes enjoyment. I.e. in the mode it is accompanied by high tension and in the masochist by a low degree of tension. We quite often seek peace of mind in apparently violent experiences such as Tchaikovski's 6th Pathetic Symphony or Shakespear's "King Lear". An unsuccessful suitot goes biggame hunting, and a gambler satisfies his passion by staking his fortune on the Stock Exchange. As a poet put it many are "Seeking peace in storms." There is no strict correlation between tension and the apparently violent experiences. Failure to realise this lack of correlation underlies the objections that have been made against the aesthetic principle of choice.

It has been objected, for instance, that the principle is false because a masochist adopts a plan to mutilate himself etc., rather than to develop an athlete's body etc. So, the objection runs, he adopts the greater tension-plan instead of the lesser tension-plan and the principle that he adopts the least tension-plan is in this case clearly false.

The objection fails to realise that the masochist _enjoys_ self-mutilation, and perjaps would _suffer_ if he underwent the training of an athlete.

To the masochist the plan to hurt himself holds forth a prospect of pleasure (small tension) and may give less tension to think of aas adopted, than any other plan. There is no difficulty in showing how the aesthetic principle applies to masochistic conduct: the plan to hurt himself is the least tension one, and the masochist adopts it. This disposes of the objection to the aesthetic principle.

It may be noted that the tension of contemplating a plan does not depend in any simple way on the prospective tensions on successive dates, provided for in the plan. On the whole the smaller the latter the smaller the tension of contemplating the plan. But theoretically one can imagine a case where the plan providing for the greatest tensions, gives least tension to think of and is adopted. It is something in favour of the aesthetic theory that it appears in one guise or in other in most psychologies, and in the value-theories of economists. These latter can be viewed as early attempts to formulate the aesthetic principle. The principle appears in the form of Jevonian hedonism, of Edgworth's conception of man as a pleasure-machine; it takes the form of Walras' principle of maximum desiredness, of the widespread doctrine of "maximum satisfaction" and even of Pareto's principle of maximising "ophelimité". All these state the aesthetic principle in a more or less vague way.

The reason why formulations of something like the aesthetic principle have always been made is this. How can an individual know which alternative he likes better unless there is some mark or feeling to indicate it to him? There must be some feeling and some rule how it causes the individual to choose a particular alternative. An "indeifference curve" (in a psychological sense as referring to intentions) says an individual <u>feels</u> indifferent about certain alternatives. It assumes that "feeling indifferent" exists. Hence it assumes tensions exist and are equal. For introspection shows that "feeling indifferent" is awareness of equal tensions being attached to the alternatives. The arbitrariness of the <u>index 1</u> of the degree of tension has nothing to do with the question whether tension exists or not.

"Utility", "desiredness", "ophelimité", etc., all have been more or less undefined terms for tension read upside down. Jevonian hedonism is an embrionic and unsatisfactory form of the aesthetic principle.

Desire for a particular thing or a particular event is a species of mental pain and contains the element of tension or libido. Desire in general is the same thing as libido or tension.

Tension in the sphere of conduct appears to play a "biological" or an "evolutionary" role like that of physical pain in animal behaviour. The animal flies from pain, man seeks an escape from tension. The aesthetic principle appears well founded. But I am perfectly ready to renounce it if good evidence is brought against it. Supposing that principle disproved, the greater part of my account of conduct (of uncertainty and of learning from experience etc.) still may be true. For it does not depend for its truth on the truth of the aesthetic principle of choice.

It is seen that even Pareto mentions a psychic entity "ophelimité" and conceives of conduct as an equilibrium between "tastes" and "obstacles". My objection 4.12 is directed only against Pareto the behaviourist (of the mathematical appendix) who seems to have had the most influence. There is a behaviourist Pareto in every economist. We are all a bit afraid of psychology.

The case against behaviourism is this: introspection is useful and there is no reason why we should give it up as behaviourists would have us to do. By imagining ourselves in the position of an "economic subject" we are able in a large measure to understand his actions. By assuming that he is conscious we can account for his behaviour on analogy with our own conduct. This assumption is made in every-day life and is equally useful in economics and psychology in general. If a shipping firm insures its ships, we explain its action by assuming that its directors feel uncertain and have a variable structure of anticipations (belief). We can study how this structure varies with external stimuli - e.g. the submarine menace during the war, and how the firm's policy is influenced by this. And in general, imputing consciousness and learning from experience to an individual we can discover laws saying how is it that he behaves similarly under the same circumstances, and still more usefully, how is it that he behaves differently under apparently the same circumstances. The introspective method helps to analyse and to predict behaviour of other people. We are able to discover uniformities in the incessant changes of the irreversible conduct - process.

I have gone into the general problem of economic conduct in order to provide a suitable background for my analysis of the business plan which follows. The general analysis solves the problem of plan-adoption in a general way and the results are easily applicable to business planning. Further, the general analysis will bring out clearly the artificiality of the usual assumptions about a firm and will help us to see how the abstract and restricted analysis of production which follows, may be brought closer to the facts.

PRODUCTION.

4.51. Production is the same thing as conduct (interaction of mind and matter.).

At first sight it does not seem so. It seems as if production is a purely material process. Actually it is also psychological.

E.g. it seems that production of rails is a purely chemical process of smelting the ore and a physical process of rolling the steel etc. Actually, the process of production of rails partly also consists of thoughts and feelings of the workers, the engineers, the scientists, and the management. These inner events are part-causes of the rails.

The element of human will etc., is the essence of the concept of production. E.g. if the rails were brought about without the assistance of human will, they would be regarded as a gift of nature and not as a result of production. Production is conduct.

4.52. The world economy through time is a system of individual "lives" L(t) and material goods. Some lives affect others. E.g. Watt's invention affected the succeeding generations. Some lives interact, as in families, schools, a market, a business. The whole process is irreversible. And is complicated. E.g. the world economy to-day is a system of, among other things, some 2,000 million "lives".

To help to think of the world-economy evolution as a deterministic structure we can symbolise:

$$n\left(\overset{0}{\underset{-\infty}{t}}\right) = F\left[v\left(\overset{0}{\underset{-\infty}{t}}\right)\right] \qquad \text{.............. (61)}$$

where n(t) is the state of the economy at t, v(t) is its environment at t, t = 0 is the present, t = -∞ is the infinite past, and F is the rule how the environment determines the economy.

4.53. It is impossible to describe the causal structure of the whole evolution of the world economy (the whole historical process of production). One can only state the method of description. The method is this. We take a date in the past as far as we like. Note the state of the economy then. Consider the environment of the economy N(t) say, up to the present. The environment N(t) is a stream of events acting on the economy and is itself outside human influence ("the weather".). Having the initial state of the economy and the environment, and assuming we know all causal laws, we trace out the resulting causal process, viz., the evolution of the economy. A simpler method is to take the state of the economy to-day and follow up its causal antecedents till we reach events N(t) which are outside human influence. These are the true "original factors". The second method would miss out those features of the economy in the past which have not influenced the state of the economy in the present.

4.54. To illustrate the first method, I shall give a simplified version of a bit of a process of production of some commodity, bread for example.

I shall start the description at some arbitrary point. On some spring day (March) a field is ploughed and sown with wheat. The details of the process are familiar. In the next few weeks intervals of rain and sunshine occur and the average daily temperature rises. The seeds in the ground germinate and soon the field becomes green with young shoots. After further intervals of rain, sun and rising temperature, the shoots grown and ripen into golden corn. Sometimes in August the corn is reaped, threshed out and the grain is sent, say, to the local town where it is milled some day. The flour is sent to the baker's where dough is made from it (applying the known properties of yeast). The dough is cut into leaves and these are baked in the oven at 450° F. When ready these are sent to retail-shops and I can leave off describing the bread-process at that date (September). The human element came in at innumerable stages of the process. The farmer gives instructions to the labourers and they sometimes pay attention to them. The labourers do not let the plough-horses wander at random, but guide them and endeavour to get the furrows reasonably straight. They use some intelligence in the control of animals and machines. Similarly the element of human will comes at various other points of the process, in milling, baking and the sale of bread.

There are other and less evident parts of the bread process, occuring in the same time-interval (March - September).

For example the coal with which the oven is fired to bake the bread, was mined, say, when the field was being ploughed, and went through various stages of storage and transportation before reaching the oven. Similarly the oven may be a new one, and may have been made while the corn was rîpening, and the clay for the bricks may have been mined while the ploughing went on. The coal and the bricks were brought by a railway company and everything which is necessary to keep the railway in sufficient order to effect this transportation, has to be included in the process of production of bread. There are many other parts of the process and it is evident that the total process of production of the loaves between March and September, is highly complicated.

The causal structure of the process is more sharply expressed by a causal "map".

Consider a small patch of the field. We have this table events:-

a	=	the farmer says certain instructions just before ploughing begins (a psychic event),
b	=	the labourer hears and memorizes them.
A	=	the labourer guides the plough over the patch
B	=	a segment of furrow appears
C	=	the sowing mahhine passes over the segment

D = the segment is seen sowed and closed

R = rain

E = young shoots of corn

S = sunshine

F = ripe corn

H = a harvester passes over the patch

J = straw

K = grain

L = chaff

M = milling

N = flour.

etc.

So, with respect to a patch of the field we have the following causal "map".

$$\begin{array}{ccccccccccccc} & & & & & & R\searrow & & S\searrow & & & J & \\ a \to & b \to & A \to & B \to & D & \to & E & \to & F & \longrightarrow & \Big\{ & K & \longrightarrow N \text{ etc.} \\ & & & C\nearrow & & & & & & H\nearrow & & L & M\nearrow \end{array}$$

We have a like map for every patch.

In theory a similar map can be constructed for the whole bread process. Also for the whole evolution of the world economy.

R, S are placed above the other letters to indicate that environmental events - rain and sunshine as also coal-deposits and clay for the bricks - come from outside the production process. C, D, M, are placed below to indicate that they come from within the process. For simplicity their antecedents are omitted.

4.55 A production map is built up of elements. An element is an elementary causal connection such as A → B. Given the environmental events R, S etc., what causal connections occur in the map depends on what inner events occur. Thus A → B would not occur if a did not occur. E.g. if the farmer did not will to grow wheat, some other crops would grow in the field, or it might run fallow.

This brings us more specifically to the question of the conscious control of production. That question is solved in a perfectrly general way by the theory of economic conduct, 4.13. etc. I now feel entitled to proceed on a number of restrictive assumptions which will yield various details.

4.56 NOTE ON FUNCTIONALS.

This note is based on Volterra's Theory of Functionals, English translation, 1930.

Definition of a functional.

".....a quantity z is a functional of the function x(t) in the interval a, b when it depends on all the values taken by x(t) when t varies in the interval a, b; or, alternatively when a law is given by which to every function x(t) defined within a, b (the independent variable within a certain functional field) there can be made to correspond one and only one quantity z, perfectly determined, and we shall write..... .... $z = F\left[x\overset{b}{\underset{a}{(t)}}\right]$ or also $F\left[x(t)\right]$".

In words of mathematical logic a functional is a "propositional function", of which the "argument" is a mathematical function.

Similarly, we can have a functional depending on several functions:

$$z = F\left[x(t),\ y(u),\ n(v)\ \ldots\ldots\ldots\ldots\right]$$

The simplest instance of a functional is a definite integral $\int_a^b x(t)\,dt$. The integral depends on the function x(t) and will vary if we vary the function, i.e. if we use different particular functions.

Functional first derivative and variation δ .

Consider a functional $F\left[y\left(\overset{b}{\underset{a}{t}}\right)\right]$. "We give y(t) an increment $\delta y(t) = \vartheta(t)$ which does not change in sign, and such that $|\vartheta(t)| < \epsilon$, $\vartheta(t) = 0$ outside an interval m, n of a, b of amplitude h, containing ξ in its interior.

We suppose:

(i) that the ratio $\frac{\Delta F}{\epsilon h}$ is always less than a finite number M;

(ii) that, putting $\sigma = \int_m^n \vartheta(t)\,dt$ there exists a determinate and finite limit of $\frac{\Delta F}{\sigma}$ when ϵ and h tend simultaneously to 0, subject to the condition that the interval m, n always contains ξ in its interior;

(iii) that the ratio $\frac{\Delta F}{\sigma}$ tends to its limit uniformily with respect to all possible functions y(t) and to all points ξ .

The limit of this ratio will then in general depend on the function y(t); it will therefore be a functional of y(t), and will also depend on the parameter ξ and will be a function of ξ. It will be denoted by the symbol $F'[y(t), \xi]$ and will be called the first derivative of the functional F with respect to the function y(t) at the point ξ."

Analogously to the total differential $df = \sum_i \frac{\partial f}{\partial y_i} dy_i$ of a function $f(y_1, y_2, \ldots y_i \ldots \ldots y_n)$ where the discontinuous index i of the independent variables y_i corresponds to the continuous parameter t, we have:

$$\delta F = \int_a^b F'[y(t), \xi] \, \delta y(\xi) \, d\xi$$

which is called the differential or the first variation of the functional F.

The first variation δF is itself a functional of y(t) and of $\delta y(t)$. The first variation of F with respect to y(t) is called the second variation of F and is written $\delta^2 F$.

For details see Volterra, op cit p.p. 22 - 8.

DETERMINATION OF THE BUSINESS - PLAN.

4.57.4.57 Table of variables provided for in the business - plan.

t	=	time - scale, the universal parameter,
t	= 0	is "the present"
c	=	the collection of goods owned by the business at t < 0, is given and constant,
u(t)	=	the output function, the rate of output of the consumption good at t

$p_u(t)$ = the good-price function,
the price of the consumption good at *t*

$f_1(t), f_2(t).....f_n(t)$ are input functions,
the rates of input of different "original factors" 1, 2 ...n at *t*

$p_1(t), p_2(t).....p_n(t)$ are factor-price functions,
prices of factors 1, 2 ....n at *t*

$\rho(t)$ the discount function,
the rate of interest for period *t* used to calculate present values, 2.19.

4.58 Assumptions.

(a) Only one consumption good, say bread, is produced in the economy.

(b) All firms are completely integrated vertically.

I.e. a firm buys only original factors and sells only bread. The original factors are different kinds of labour and natural resources for sale in the market and outside the firm's control (in that sense "original"). Once a unit of labour or land etc., is purchased by the firm it passes under its control. Its properties can be altered. Labour can be trained. Its quality is influenced by conditions of work. Similarly land is "improved" etc. An original factor becomes an "intermediate product" as soon as it is purchased.

(c) Competition is perfect.

Bread and the factors are exchanged for money in the market. The prices $p_u(t), p_1(t).........p_n(t)$ and the discount rate $\rho(t)$

are outside the influence of an individual firm.

(d) Foresight is certain.

(e) The firm is controlled by an entrepreneur. His field of possible business plans $P(0)$ at $t = 0$ is given. It provides for the price and discount functions $p_u(t)$, $p_1(t)$.......$p_n(t)$, $\rho(t)$ and for alternative methods of production, 4.59.

(f) The entrepreneur's tension function $\ell\{P(0), 0\}$ is simple: the libido is minimised if the capitalised value of the business (the present value of c) is maximised. He is not interested in anything except his business. I.e. he seeks to maximise the present value of future profits. From (c) realised profits are zero.

4.59 Details of assumption (e).

The plan-field $[P(0)]$ provides for alternative total processes of production of the firm's output form $t = 0$ to $t = \infty$. Each process is a causal structure and can be described by a causal map M. See the map-fragment 4.54. All the alternative structures are described by a field of maps [M]. A map M is a picture of phenomena as shown in 4.54, and not of price and discount functions. These latter are not phenomena but concepts. Thus a price is not a thing but a ratio of quantities of two things. A plan P(0) contains both a map M and the entrepreneur's knowledge of market price and discount functions. Since certainty is assumed, market functions and the entrepreneur's ideas of them can be represented by the same symbols

$p_u(t)$, $p_1(t)$.......,$p(t)$.

The map M has a range of values in its field [M]. The field is limited by the enterpreneur's knowledge of causal-connections by the number of structures he can imagine. Another limitation is that the collection of goods c (equipment, materi ls etc.) inherited by the firm at t = 0 is given and constant. It is beyond control because its causes lie in the past and so are beyond control of the entrepreneur at t = 0. The field [M] is *not* restricted by fa tor-inputs. The input functions $f_1(t)$.......$f_n(t)$ have perfect freedom to vary (except that they can't have negative values). Causal processes consist of jerks, and the input functions a etc are assumed continuous only for convenience.

Consider a set of particular input functions $f_1(t)$, $f_2(t)$..$f_n(t)$, There is a class of different maps M having that set. They differ in the ways the inputs are used. Consequently they have different ouput-functions u(t). By "using the inputs in different ways" I mean that any given element of a factor f(t)dt is a member (part-cause) of a different "elementary causal connection", 4.54, as between different maps of the class. I.e. at t = 0 original factors can be "combined" with each other and the goods c in may different ways. Each combination causes a different set of goods at the next moment. This set can be "combined" in different ways with the original factors employed at the next moment, and so on from t = 0 to t = ∞ .

So, each structure contains a different set of capital goods or intermediate products. Each one of the maps having the same set of given input functions, has an output stream of a particular "time-shape", or has a different ouput function u(t).

EXAMPLE.

By way of an easy artificial example we can imagine the entrepreneur as employing a constant labour force say f(t) constant on a given estate. Consider two plans P(O). One provides for ploughing the fields next year etc. The other provides for letting the land rest and lie fallow next year, and for employing labour so released in effecting various "improvements" on the estate. As a result, the second plan in comparison with the first gives no harvest next year, but bigger harvests afterwards (because of the improvements).

In consequence, we may suppose, the second plan provides for a smaller output of bread next year and for more afterwards, than the first plan does.

The difference in the time-shape of the bread output-streams given by the two plans may be large. Slight variations of the output function u(t) can be had by baking a little more or a little less bread (from the stock of wheat) on successive days. Also by transferring small amounts of labour (instead of en bloc) from field work to improvements, etc., etc.

A change in the earlier stages of production (e.g. labour transfer from field-work to improvements) does not determine the output function u(t). It enfluences the field of possible variation of u(t) on future dates.

A simple case of a small variation of u(t) is shown in fig. 13. In the case of II, say, there has been a slight transfer of labour from field-work to making improvements. Of course a "transfer" of labour is associated with a transfer of other things - tools, horses etc., etc.

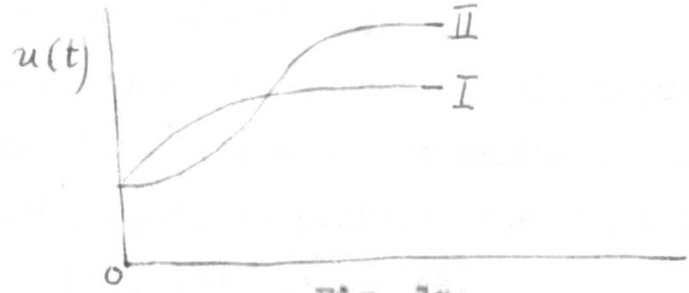


Fig. 13

The general principles which are exhibited in the variation of the output function u(t), apply also to all outputs in the world economy.

I have shown that u(t) has a field of variation $[u(t)]$ as between different maps M all having the same set of input functions $f_1(t), f_2(t) \ldots\ldots\ldots f_n(t)$.

In general for different sets $f_1(t), f_2(t) \ldots\ldots\ldots f_n(t)$ the field $[u(t)]$ is different. I.e. the field is a functional of the input functions:

$$[u(t)] = G\left[f(t),\ f(t) \ldots\ldots\ldots f_n(t)\right] \quad . \quad . \qquad (62)$$

For example, if more men are employed, greater output streams become possible.

4.60 We write:

$$v = \int_0^\infty p_u(t).u(t)\, e^{-\rho t}\, dt$$

= the present value of future output

$$z = \int_0^\infty \{p_1(t).f_1(t) + p_2(t).f_2(t) + \cdots p_n(t).f_n(t)\} e^{-\rho t}\, dt$$

= the present value of future input

x = v - z

the capitalised value of the business, the presnet value of the collection c.

I assume: the output and the input and the price functions are all finite (do not have infinite values), and the rate of discount $\rho = \rho(t)$ is positive for all values of t. On assumption of continuity this is enough.

It follows that very future values have negligible present values on account of discounting them at compound positive rates of interest, and that the integrals v, z, x, are all finite limits.

The collection c is given. The price functions $p_u(t)$, $p_1(t)$, $p_2(t)$.........$p_n(t)$ and the discount function $\rho(t)$ are given by the market. The field $[u(t)]$ is a dependent variable, (62).

The only independent variables are the input functions, $f_1(t)$.......$f_n(t)$ and the output function u(t) within its dependent field $[u(t)]$.

Now, v, z, are functionals of the independent functions on which they depend

$$v = v[u(t)], \quad z = z[f_1(t), f_2(t) \ldots f_n(t)]$$

According to assumption (f) the entrepreneur maximises the capitalised value of the business $x = v - z$.

Both v and z are obviously positive. Hence for each given value of z i.e. for each given set of functions $f_1(t) \dots f_n(t)$ the entrepreneur maximises $\underline{v}$ i.e.:

$$\delta_u v = 0 > \delta_u^2 v \qquad (63)$$

where $\delta_u v$, $\delta_u^2 v$ are variations of $\underline{v}$ with respect to u(t) in its field $\left[u(t)\right]$. (63) holds for all possible sets $f_1(t) \dots f_n(t)$. As the input functions are varied, the field $\left[u(t)\right]$ changes as per (60), and (63) <u>segregates out</u> the maximal values of $\underline{v}$ and the respective output function yeilding the maximum, corresponding to each possible set $f_1(t), f_2(t) \dots\dots f_n(t)$ of the input functions.

Writing V for the maximal values of v, we have: V depends on the input functions alone:

$$V = V\left[f_1(t), f_2(t) \dots\dots f_n(t)\right].$$

Now $x = V - z$ becomes a functional of the input functions alone:

$$x = F\left[f_1(t), f_2(t) \dots\dots f_n(t)\right]. \qquad (64)$$

That $\underline{x}$ is maximised means that all the first partial variations of x are indentically zero

$$0 = \delta_1 F = \delta_2 F = \dots = \delta_n F \qquad (65)$$

and that the second partial variations $\delta_1^2 F, \delta_2^2 F, \dots \delta_n^2 F$ are all negative; where $\delta_1 F$ is the first variation of x with respect to $f_1(t)$, $\delta_2 F$ with respect to f(t) and so on.

In (65) the numbers of functional equations equals the number of the unknown input functions. We have also three unknowns x, u(t), and u(t) and three equations (64), (63), (62). The total number of equations equals the total number of unknowns. The unknowns are determined. Strictly, the functions u(t), $f_1(t)$, $f_2(t)$.......$f_n(t)$ are determined for all finite values of $t>0$. For $t=\infty$ they are indeterminate as it makes no difference to the capitalised value x what they are at $t=\infty$.

All the variables provided for in a business plan are determined. The adopted business plan P(0) is determined. On assumption (d) of certainty, production is according to plan. Hence the firm's actual course of production is determined.

The whole causal map M of the process is determined on the assumption that for a given set of output and input functions there corresponds one and only one map M.

4.61 Using (65) we get following details.

We have $0 = \delta_1 x = \int_0^\infty F'_{1\xi}\, \delta f_1(\xi)\, d\xi$

where $F'_{1\xi}$ is the first partial functional derivative with respect to function $f_1(t)$ at $t=\xi$, $F'_{1\xi} = \lim \dfrac{\Delta_1 F}{\int_m^n \delta f_1(t)\,dt}$ as $h=n-m$ and $\epsilon > |\delta f_1(t)|$ simultaneously tend to 0. See 4.56.

When we have a function $f(y_1, y_2 \ldots\ldots y_n)$ of $y_1, y_2 \ldots\ldots y_n$ and the total differential $df = \sum_i \frac{\partial f}{\partial y_i} dy_i$ is zero, and the variables $y_1, y_2 \ldots\ldots y_n$ are independent of one another, then each partial derivative $\frac{\partial f}{\partial y_i}$ is identically zero because each differential dy_i : $dy_1, dy_2 \ldots\ldots dy_n$ is entirely arbitrary.

Similarly, the values of the function $f_1(t)$ at different values of t are mutually independent, so that the function $\delta f_1(\xi)$ which corresponds to dy_i, is entirely arbitrary.

Hence $$F'_{1\xi} = 0$$

for all ξ from 0 to ∞.

See Volterra op cit p.p. 27 - 8.

Since $x = V - z$

$$0 = F'_{1\xi} = V'_{1\xi} - z'_{1\xi} \qquad (65')$$

where $V'_{1\xi}$, $z'_{1\xi}$ are first partial functional derivatives of V, z with respect to $f_1(t)$ at ξ.

Now $z'_{1\xi} = e^{-\rho\xi} \cdot p_1(\xi)$, see definition of z.

$$\therefore \quad V'_{1\xi} = e^{-\rho\xi} \cdot p_1(\xi) \qquad (66).$$

at all values of ξ from 0 to ∞.

I.e. the "present - value" marginal productivity (per unit time) of the factor input rate $f_1(t)$ at ξ, is equal to the present value of the factor's price at ξ.

This is analogous to the well known proposition of static economics: "in competitive equilibrium, the marginal value-productivity of a factor is equal to the facotr's price".

And analogously:

$$\delta_1 V = \delta_1 z \qquad (67).$$

or the present-value of the "marginal product" of a variation of the whole input stream $f_1(t)$ between $t = 0$ and $t = \infty$ equals the present cost of the marginal variation of the factor throughout the future.

The same argument applies to all other factor streams

$f_2(t)$, $f_3(t)$.......$f_n(t)$.

4.62 Any input function say $f_1(t)$ has perfect freedom of variation subject to the condition $f_1(t) \geq 0$. Not so the output function u(t). For a given set of input functions, its freedom is restricted in certain ways, by the field $\left[u(t)\right]$. Thus as shown by fig. 13 greater rates of output in the more distant future may involve smaller tates of output in the nearer future. In every day language: "saving is a sacrifice". A positive increment $\delta u(t)$ may involve negative increments $\delta u(t)$ at other values of t. A function u(t) may be given and increment at one value of t without deminishing its value on any other (finite) date. But such a function can obviously maximise v, x in no circumstances. It represents waste. For by giving it the above increment, v, x can be increased. We leave out of consideration all such output functions. We have only to deal with those functions which cannot be given an increment at one value of t without causing a decrement at some other finite value of t. The relevant field of output functions $\left[u(t)\right]$ consits only of functions of that latter sort. That field can be written analogously to an implicit function as $F\left[u(t)\right] = 0$. Just like a stone thrown in the centre of a pond sends out rings throughout the surface of the pond. The field of variation of the water's surface is limited by the laws of mechanics. The field of variation $\left[u(t)\right]$ is restricted in an analogous way by the laws of nature and the entrepeneur's limited knowledge of them.

This can be put by saying that taking a particular function u(t) and writing $\varphi(t) = \delta u(t)$ as a distance from it, the function $\varphi(t)$ has a restricted field.

Hence it follows returning to (62) that in

$$0 = \delta_u v = \int_0^\infty v'_{u\xi}\, \varphi(\xi)\, d\xi$$

the functional derivative $v'_{u\xi}$ is not identically zero for all possible ξ, but can be $> 0, = 0, < 0$. Because $\varphi(\xi)$ is not entirely arbitrary but is restricted (compare $F'_{1\xi} = 0$ in 4.61). The way it is restricted is analogous to the restirctive condition dm = pdc of 4.36.

Unless $\varphi(t)$ were suitably restricted, $v = \int_0^\infty e^{-\rho t} u(t)\,dt$ could not be maximised as it is an increasing linear functional of u(t). (63) pressupposes that $[\varphi(t)]$ is suitably restricted.

.4.63

THE MARGINAL PRODUCT.

From (62) and (63) the output function u(t) which maximises v is a functional of the output functions

$$u(t) = u\left[f_1(t), f_2(t) \ldots\ldots\ldots\ldots f_n(t)\right].$$

Similarly:

$$\int_0^\infty u(t)dt = U\left[f_1(t), \ldots\ldots\ldots f_n(t)\right]$$

and varying an input stream f_i, the "marginal product" of the factor is the partial variation

$$\delta_i U$$

and both the marginal products and the factor are spread through time. Similarly the functional derivative

$$U'_{i\xi}$$

with respect to the f_i at ξ, is the marginal productivity per unit-time of the rate of input at ξ $f_i(\xi)$. The marginal product of the factor i employed at ξ is $U'_{i\xi}\,\delta f_i(\xi)\,d\xi$ and

$$\int_0^\infty U'_{i\xi}\,\delta f_i(\xi)\,d\xi = \delta_i U \qquad (68)$$

In general, as when a workman assists in building a durable good - a house, the marginal product $U'_{i\xi}\,\delta f_i(\xi)\,d\xi$ is itself spread over time. It can be shown as an integral of a function over time.

Consider u(τ) at $t = \tau$

and $\{u(\tau)\}'_{i\xi}$ its functional derivative with respect to function $f_i(t)$ at ξ . Then

$$\int_0^\infty \{u(\tau)\}'_{i\xi}\,d\tau = U'_{i\xi} \qquad (69)$$

or

$$U'_{i\xi}\,\delta f_i(\xi)\,d\xi = \delta f_i(\xi)\,d\xi \int_0^\infty \{u(\tau)\}'_{i\xi}\,d\tau \qquad (69')$$

and the integral part clearly expresses how the marginal product of a factor employed at a point of time ξ may be spread over time. The last expression is analogous to a differential obtained in differential calculus by "differentiating a function under an integral" viz., $dx \int_a^b \frac{\partial f}{\partial x}\,dy$

Finally,

$$\delta_i U = \int_0^\infty U'_{i\xi}\,\delta f_i(\xi)\,d\xi = \int_0^\infty\int_0^\infty \{u(\tau)\}'_{i\xi}\,\delta f_i(\xi)\,d\tau\,d\xi \qquad (70)$$

64.4.64 If a workman assists in making a durable good it appears impossible to assign any marginal product to him if the good (a machine) is indivisible. Actually it is generally possible. E.g. a workman assists, on any day, in making a plough. The plough is indivisible. But the plough will influence future harvests and so the bread output stream u(t). Because of the additional workman the plough is, say, stronger and will last longer, and the stream u(t) will experience an increment $\delta' u(t)$. The integral of those increments $\int_0^\infty \delta u(t)\,dt$ is the workman's product i.e. is the marginal product of labour. Thus the principle of "variation of methods " - having a stronger instead of a less strong plough, rescues the marginal productivity theory from the seeming impasse created by indivisibility of certain "intermediate products".

There is a paradoxical point that the marginal product a factor application $f_i(\xi)\,dt$ at $t=\xi$ may begin to accrue before ξ. This is so because an increment $\delta f_i(\xi)dt$ may make it profitable to change the production structure before ξ in a manner to change u(t) before ξ. The marginal product before ξ can comprise either positive or negative elements $\delta_i u(t)dt$. Example: the farmer-entrepreneur expects to increase his labour force next year and with its help to carry out a programme of improvements. So he decides to employ his present labour force in field-work instead of on improvements. As a result his harvest this year is greater than if he did not decide to employ more labour next year. I.e. the bread-stream u(t) in the near future is bigger. Conversely, he may decide to transfer some labour from field work this year to making dwellings for the new labour next year, and u(t) is diminished before the new labour has arrived. In this case the marginal product of new labour before its arrival is negative. Something like that happened in new countries in periods of rapid immigration. The expected marginal product can accrue before, as well as after the expected application of a factor.

1.5 .4.65 The preceding analysis of determination of the business plan can be restated in terms of differential calculus. But functional calculus is better suited to the problem.

The period $0, \infty$ is divided into equal atomic moments dt which we fix once and for all at as small a size as we please. The amount of output at t is u(t).dt and of input is f(t).dt. I deal only with one kind of "original factor", Several can be taken into account similarly. The amounts of input and output at t are measured by u(t), f(t) since dt is fixed.

The amounts of output at all moments depend on the amounts of input at all moments, and on each other. We have the implicit function determined by the entrepreneur's "knowledge".

$$\psi\left\{u(t),\ f(t)\right\} = 0 \qquad (71)$$

where u(t) symbolises the rates output and f(t) the rates of input at all moments from $t = 0$ to $t = \infty$.

We have: $$v = \int_0^\infty u(t)\, e^{-\rho t}\, dt$$

As before v has to be maximised for any given set of inputs f(t) i.e.

$$0 = dv = \frac{\partial v}{\partial f(1)}\, df(1) + \frac{\partial v}{\partial f(2)}\, df(2) + \dots + \frac{\partial v}{\partial f(\infty)}\, df(\infty) \qquad (72)$$

where $\frac{\partial v}{\partial f(1)}$ is the partial derivative of v with respect to the factor input during the first moment dt, $\frac{\partial v}{\partial f(2)}$ during the second moment dt,...... and $\frac{\partial v}{\partial f(\infty)}$ is the partial derivative with respect to the moment dt at $t = \infty$. The differentials $df(1)$, $df(2)$.......$df(\infty)$ are determined by the function ψ.

Writing V for the maximal values of v, and letting the inputs at successive moments vary, we have a function

$$V = V\left\{f(t)\right\}.$$

Writing

$$Z = \int_0^\infty p(t).\, f(t)\, e^{-\rho t}$$

where p(t) is the factor-price at t, $x = V - z$ has to be maximised i.e.

$$0 = \frac{\partial x}{\partial f(1)} = \frac{\partial x}{\partial f(2)} = \ldots\ldots = \frac{\partial x}{\partial f(\infty)} \quad (73)$$

and the second partial derivatives $\frac{\partial^2 x}{\partial f(t)^2}$ are all negative.

It is seen that we have a determinate system of an infinite number (an infinity of second order) of simultaneous equations. Therefore the adopted plan is determined.

As before, the rates $u(\infty)$ and $f(\infty)$ are indeterminate. But $f(\infty)$ has no influence on V, because of discounting at the positive rate $\rho(t)$ and because the input $f(\infty)dt$ is assumed to have no elements of marginal product $\frac{\partial u(t)}{\partial f(\infty)}dt$ accuring within finite future before its application.

From (73)

$$\frac{\partial V}{\partial f(\xi)} = p(\xi)\, e^{-\rho\xi} \quad (74)$$

for all ξ, where $f(\xi)$ is the rate of input at ξ. The marginal product of the factor-input is

$$df(\xi) \int_0^\infty \frac{\partial u(t)}{\partial f(\xi)}\, dt \quad (75)$$

which I am perfectly entitled to express thus, because I take the atomic interval dt as small as I please. I use limits $0, \infty$ for the sake of generality. I do not imply that all factor-inputs $f(\xi)dt$ have their marginal products spread from $t = 0$ to ∞.

Equations (72) (73) (74) (75) respectively correspond to (63) (65') (66) (69').

It is seen that the "differential" and the "functional" methods both lead to the same results. As they should.

§4.66 COMMENTS.

The central idea of the theory of the business plan 4.57-60 is the variable output function u(t). It extends Fisher's concept of the output stream 1.10 and reconciles it with the well known timeless theory of marginal productivity. A theory of marginal productivity is built up that takes time into account. It completes what Fisher left undone and generalises Wicksell's attempt in his special case 1,9. This is the main result of these researches.

§4.67 On my definitions, Fisher's stream is that imputed to the collection c of goods extant at t = 0. Fisher does not show how this is done. It is done as follows:-

The capitalised value of this stream x is given by (64) and x can be expressed as the "residual share" after the shares in the present value V of the variable factor inputs:

$$\int_0^\infty f_1(t)\,dt, \quad \int_0^\infty f_2(t)\,dt \;\ldots\ldots\; \int_0^\infty f_n(t)\,dt$$

have been imputed, viz.

$$x = V - z = V - \sum_{i=1}^{i=n} \int_0^\infty V'_{i\xi} f_i(\xi)\,d\xi \qquad \ldots\ldots\ldots(76)$$

Analogously, writing $\psi(t)$ for the rate of output imputable to c the rate is given by the equation

$$\psi(t) = u(t) - \sum_{i=1}^{i=n} \int_0^\infty \{u(t)\}'_{i\xi}\, f_i(\xi)\,d\xi \qquad (77)$$

and the total stream of output imputed to c is

$$\int_0^\infty \psi(t)\, dt$$

Fisher does not give (77) or its genesis. In so far his theory is incomplete - is indeterminate. But even with those omissions he is right in saying that the field of variation $[\psi(t)]$ is restricted and that such a $\psi(t)$ is adopted that x is maximised. For $\delta_u x = 0$, (63), hence per (77) x is maximised with respect to $\psi(t)$ i.e.

$$\delta_\psi x = 0 > \delta_\psi^2 x$$

But he is wrong in suggesting that the discout function $\rho(t)$, the field $[\psi(t)]$ and the condition that x is maximised, are sufficient to determine x and $\psi(t)$. They are not sufficient. Tracing back the genesis of (77) and interpreting $[\psi(t)]$ as a field for all possible sets of original factor input functions $f_1(t)$..........$f_n(t)$, it is seen that the price functions $p_u(t)$, $p_1(t)$..........$p_n(t)$ also have to be known if x, $\psi(t)$ are to be determined.

Assuming the price functions given, Fisher's assumptions become sufficient in the sense that they complete the necessary list of assumptions. Then for each $\rho(t)$ there will be a corresponding plan P(0) as I explained and hence a corresponding $\psi(t)$. On special assumptions, see 4.69 the price functions are determined by the discout-function as in the simple cases 2.29.

On these assumptions the discout-function determines every thing

But Fisher does not state these assumptions. Hence his analysis is indeterminate: the data $\rho, [\psi(t)], \delta_{\psi} x = 0$ are insufficient to determine x, $\psi(t)$. Had he said that he was concerned with a theoretical case where $\psi(t)$ is independent of future factor inputs, his three conditions would be sufficient and his analysis determinate. But it is clear he was not thinking of such a restricted case.

Fisher's excellent work has been much misunderstood and underrated precisely because his analysis is not determinate and so leaves the reader somewhat in the air. Almost all the analyses of production - through time I have seen (except Ricardo's and Wicksell's in special cases 2.6,8) are logically indeterminate. They have more unknowns than determining conditions. This explains why controversies about capital have usually been vague.

4.68 The analysis of point input - continious output 2.10 - 13, 15 - 18 has similar limitations. There the labour-point input roughly corresponds to the collection of goods c. The analysis 2.10 - 13, 15 - 18 assumes labour to be the only scarce original factor. Writing $f_1(t)$ for the rate of labour input stream, $f_1(o)dt$ is the point input, and its imputed rate of output at t is:-

$$\theta'(t) = \{u(t)\}'_{1,\xi=0} \cdot f_1(o) \qquad (78)$$

and therefore is a <u>functional</u> of $f_1(t)$ and u(t), as well as an ordinary function of t; i.e.

$$\theta' = F[f_1(t), u(t), t]$$

where u(t) is restircted by (62).

I.e. θ' partly depends on all the values of $f_1(t)$ for $t > 0$. Hence the assumption that ρ and the field $[\theta']$ are given and the rate of wages $w = V'_{1,\xi=0}$ maximesed are clearly insufficient. The price functions $p_1(t)$.......$p_n(t)$ for $t > 0$ are also required (and $p_u = 1$ in2. as product is taken there as the "numeraire"). Supposing factor prices given, the rate of wages need not be maximised with respect to the point input or with respect to θ'. For there is no reason why the second derivative $V''_{1,\xi=0} = w'$ should be 0 (it must not be > 0), and from condition

$$\delta_u v = 0 \qquad (63)$$

it neither follows that $\delta_u w = 0$ nor that $\delta_\theta w = 0$. Hence "wage rate is maximised" in general is not an equilibrium condition at all. It becomes such only if θ' is independent of the future input rates $f_1(t)$ by Assumption. This assumption limits analysis to artificial cases like durable consumption goods (dwellings) constructed by labour in negligible time. But the artifice is instructive. It helps to dispose easily of the Böhm-Bawerkian fallacies, 2.13. And it yields results which are conveniently generalised as follows,

.69 4.69 Assuming a bread-economy containing a large and constant number n of identical firms such as the firm in 4.58 and given the total streams of factors outside a firm's control available in the economy in the future i.e. given the population-growth and the using up of the natural resources (mines etc.,) etc., the factor input rates $f_1(t)\ldots\ldots\ldots f_n(t)$ employed by an individual firm are determined. Taking a unit of bread as a unit of account, $p_u = 1$, and given $\rho(t)$, the factor prices are determined (65'),

$$p_1(\xi) = V'_{1\xi}\, e^{\rho\xi}, \quad p_2(\xi) = V'_{2\xi}\, e^{\rho\xi}, \ldots\ldots p_n(\xi) = V'_{n\xi}\, e^{\rho\xi}$$

and the plan P(o) is determined, 4. 60.

As a simple preliminary assuming ρ constant with respect to <u>t</u>; writing $u(t,\rho)$ for the adopted function $u(t)$, and parameter α for a possible value of ρ, and $u(t,\alpha)$ for the locus of the output function $u(t)$, that <u>would</u> be adopted <u>if</u> the rate of discount were α, I write

$$\int_0^\infty u(t,\alpha)\, e^{-\rho t}\, dt = V(\alpha,\rho)$$

so that

$$V(\rho,\rho) = V$$

and using (63) at $\alpha = \rho$

$$\frac{\partial V(\alpha,\rho)}{\partial \alpha} = 0$$

and at $\alpha = \rho$

$$\frac{dV(\alpha,\rho)}{d\rho} = \frac{\partial V(\alpha,\rho)}{\partial \alpha} + \frac{\partial V(\alpha,\rho)}{\partial \rho}$$

$$= \frac{\partial V(\alpha,\rho)}{\partial \rho}$$

$$= -\int_0^\infty t.\, u(t,\alpha)\, e^{-\rho t}\, dt$$

$$\therefore \frac{dV}{d\rho} < 0 \qquad (79)$$

since u,t are positive, c.f. (34).

Since $x = V - z$

$$\frac{dx}{d\rho} = \frac{dV}{d\rho} - \frac{dz}{d\rho}$$

and is negative if $\frac{dz}{d\rho} \gtreqless 0$, and if $\frac{dz}{d\rho} < 0$

$$\frac{dx}{d\rho} \gtreqless 0 \quad \text{according as} \quad \left|\frac{dz}{d\rho}\right| \gtreqless \left|\frac{dV}{d\rho}\right| \qquad (80)$$

That result is disappointing as it is impossible to say whether the capitalised value of the business (of c) is an increasing, or a constant, or a diminishing function of the rate of discount ρ. It merely disproves the common belief that the value of a firm's (and of "natural") stock of capital goods (relatively to prices of consumption goods) necessarily increases if the discount rate falls, and necessarily diminishes if the discount rate rises.

I now drop the assumption that ρ, α are constant in respect to t. Corresponding to the functions $u(t, \alpha)$, $V(\alpha, \rho)$ we now have the functionals

$$u[\alpha(t), t], \quad V[\alpha(t), \rho(t)].$$

Analogously to (79), varying $\rho(t)$, the total variation

$$\delta V = \delta V[\alpha(t), \rho(t)] = -\int_0^\infty t\, u[\alpha(t), t]\, e^{-\rho(t).t}\, \delta\rho(t)\, dt$$

for $\alpha(t) = \rho(t)$ at all t, and is negative if we take $\delta\rho(t) > 0$ and positive if we take $\delta\rho(t) < 0$ for all t.

I.e. the present value of future output, $V = V[\rho(t), \rho(t)]$ diminishes if the discount rate $\rho(t)$ increases for all periods t, and increases if the discount rate falls "throughout".

This conclusion is by no means self-evident as the output function u(t) varies with the discount function ρ (t).

Analogously to (80), making $\delta\rho(t)$ constant and positive,

$$\delta x = \delta V - \delta z$$

and can be $\gtrless 0$. Again, the result is important only in so far far as it prevents certain errors. The "national capital" in the economy is simply n.x, and the result disproves the error that under competition etc., the "national capital" (in terms of consumption goods) is necessarily a diminishing functional of the discount function.

4.70 This shows, as pointed out in 2.11, how little restrictive is the assumption of competition and positive interest-rate. The output function u(t) has a degree of freedom even greater than that of $\theta'(t)$ explained in 2.11 fig. 8. etc. The relevant field of output functions $[u(t)]$ 4.62 can be restricted further by expirical data over and above the datum that there is (roughly) competition and positive interest-rate. E.g. observation suggests that in 4.68 a diminution $-\delta u(t)$ in the rate of output in the nearer future allows an increase $+\delta u(t)$ in the more distant future so that the total output $\int_0^m u(t)\,dt$ up to a distant finite date m is increased. Conversely for an increase $+\delta u(t)$ in the near future. This assumption - that there is a premium on postponement - is constantly made in studies of saving and production. It is a separate empirical assumption. It is not pressupposed in the assumption of a positive rate of discount and competition, as thought Böhm-Bawerk.

To say it more generally, observation suggests that given the state of the world economy today, a general diminution in the alternative combinations of consumption goods that can be produced in the near future will allow increased alternative combinations of consumption goods to be produced in the more distant future, etc. Concisely: narrowing the field of the alternatives in the present extends it in the future, etc.

71 4.71 The assumption (d) of certainty implies that the actual business-production process is according to plan and we know it if we know the plan (the entrepreneur's mental state) at t=0. But this mental state is not a sufficient cause of the production process from t=0 onwards. This mental factor and the collection of goods c and the original factor market at t=0 are only the initial causes of the process. Many other causal factors - materials, labour, resources, etc., figure after t=0). And to make the theory more realistic we can regard the mental factor of entrepreneurial control to be a continually recurring one. I.e. though the plan is adopted at t=0 once and for all, we can imagine that the entrepreneur has continually to keep it in mind if production is to run according to plan. Thus we have a psychic causal factor at every stage of the production process. But if we are to be still more realistic it is necessary to assume that labour employed by the firm is not purely "mechanical" and mind-less, but that labourers take part in the production process partly as a result of their

choice. Moreover, many operations of labour call for forethought and judgement on their part. To be realistic it is not enough to know how the entrepreneur's plan is determined we must also know how the plan of every individual concerned with the firm is determined at every stage of the process. We have to explain the path of intensions and actions of every individual in the firm. This is an exceedingly intricate problem. But it is useful even to mention it. It is clear for example that the entrepreneur's plan and labour's plans are of a different character: the entrepreneur has to think of the business process as a whole, labour has not. Therefore in the explanation of the production - process it is useful to take entrepreneurial planning as the starting point. But in the last analysis one has to treat the business process and (because of market exchanges etc.,) the whole world economy process - as a complicated system of individual conducts. The great merit of Pareto was to have shown how a theoretical economy is a system of innumerable individual "equilibria".

In my analysis the problem is much simplified by assuming that the conduct of the owners of the original factors is governed by a simple principle. I assume that factor owners are interested in one thing only: their earnings and that they seek to get the highest factor prices possible. Under assumption (c) of competition their conduct-principle is sufficiently described by saying that factors are available to a firm at a given time at a constant supply-price. The simplification that factor owners seek maximum prices is similar to the assumption that the entrepreneur seeks to maximise the capitalised value of his business. These simplifications do not take us too far from reality, and they are useful since even on these simplifications the analysis of the business-plan 4.57-70 is complicated enough. It may be noted that in reality the entrepreneur's plans are based largely on their knowledge of practical psychology and are chiefly concerned with controlling the subordinates.

The assumption of certainty precludes inventions. For if inventions are forseen they must be already known. But that does not mean that changes in the methods of production are precluded. Supplies of natural resources and labour may change. National capital $\underline{x}.\underline{n}$ may vary i.e. the discount function ρ (t) is a function of time and is associated with changes in methods of production. E.g. if the trend of the discount rate is falling as time goes on more durable goods are constructed. In general the bread-economy is conditioned at each moment by everything that happened before and therefore is different from what it was at any moment before. The economy evolves in "open cycles". The "stationary state" is a special case where the cycles are closed.

4.72 It is interesting to note how the positive rate of discount ρ (t) "cuts off" the distant future and so renders the business plan determinate 4.60. It is not a sufficient condition of determinacy. Nor is it a necessary condition. For the distant future is similarly cut off by uncertainty. Uncertainty can cut off the remote future even supposing the entrepreneur can think in full details about all the possible future events.

Without going into complicated details, this is shown as follows. Even the near consequences of present actions are uncertain. E.g. sowing the field may lead to harvests of different sizes on account of different probable climatic conditions etc. And very distant consequences of given actions in the nearer future are exceedingly uncertain. The number of the probable distant alternative consequences tends to infinity and the degree of belief the entrepreneur attaches to each, tends to zero, as the "distance" tends to infinity. This is so because these future events depend on less future events which are not certain, and these in turn on still nearer events which are also not certain, and so on till we reach the present actions. Even these, actions in the immediate future, are not quite certain. There is an element of surprise in every experience.

Observation shows that in planning we totally neglect the probable events in which our belief is negligible. Therefore the entrepreneur leaves out of account all the alternative "distant consequences" since he attaches negligible degrees of belief to them. I.e. uncertainty "cuts off" the distant future. Uncertainty prevents infinite future from being a source of indeterminateness of the business plan.

The assumption that the entrepreneur has clear ideas about all the possible events is a convenient fiction. It helps to isolate the element of belief as one of the determinants of actions.

In reality the problem of plan indeterminancy on account of infinite future does not arise at all. Because, no entrepreneur has any idea of it. For him it does not exist. Even astronomers' period of foresight is finite. The entrepreneur's ideas of the future are vague in proportion to the "distance."

4.73 The analysis of plan - determination 4.57-65 can be made to take account of uncertainty in many ways. Leaving out for simplicity the problem of vague ideas, the easiest preliminary way is to use the concept of mathematical probability. The entrepreneur's possible plan can be expressed as a probability function. As explained in 4.34,9 under uncertainty a possible plan provides for a number of probable alternatives. Writing <u>s</u> for a probable alternative a constellation of probable price functions $p_u(t), p_1(t)$..... and the output functions $u(t)$ etc., etc., and p for the degree of probability of <u>s</u> occuring (the index of the degree of belief attached to <u>s</u>) we have a probability functional given by the entrepreneur's expectation-structure at t 0,

$$p = F\left[\, p_u(t), p_1(t) \ldots\ldots\ldots\ldots u(t) \text{ etc.} \right] \qquad (81)$$

which can be expressed as a shortened function

$$p = f(s) \quad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (82)$$

and reduced to a simple probability function of one variable

$$p = \varphi(x) \quad \ldots\ldots\ldots\ldots\ldots \qquad (83)$$

for the entrepreneur is interested only in the present value of his business $\underline{x}$. A possible plan is judged by the entrepreneur by the corresponding probability function $\varphi(x)$. And to the field of possible plans $\left[P(o)\right]$ there corresponds a field of probability functions $\left[\varphi(x)\right]$. Thus choosing between different plans comes down to choosing between alternative "distributions" of degrees of probability of different present values x.

The alternative distributions are illustrated by fig. 14

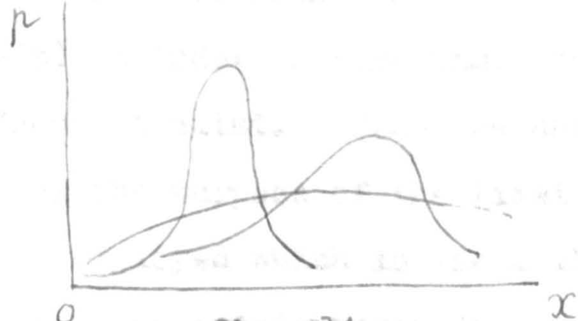


fig. 14.

This analysis is similar to Dr. Hicks' (Economica May 1931)

Next it is necessary to postulate some principle of choosing between the alternative distributions. The simplest principle is to assume a distribution is selected which gives the greatest "first moment" or greatest average probable value of x, $\int_a^b p.x\,dx$ where the limits a, b contain all possible values of x. Generalising we can write the entrepreneur's tension function $1\left\{P(o), \emptyset\right\}$ as a functional of the probability function:

$$F\left[\varphi(x)\right] \qquad (84)$$

and give the functional operator F any desired property to indicate the entrepreneur's valuation of different kinds of risk (different distributions).

Then the "equilibrium" condition is

$$\delta_{\varphi} \ell = 0 > \delta^2 \qquad (85)$$

and the chosen φ and the corresponding plan P(o) are determined.

Above I split the finding of <u>x</u> into two operations. The first operation is discounting all future probable values at ρ(t) and so finding the field of schemes $[\varphi(x)]$. The second operation is seeing which φ(x) is chosen (85) i.e. has the greatest risk discounted value <u>x</u>.

Now ρ(t) as index of pure time preference assumes certainty which does not exist. Hence we have to adopt the convention that for the purpose of the first operation the discount rate p(t) is employed which is the market rate used to discount the least uncertain money incomes e.g. annuities on gilt-edged stock.

On assumption of competition the selected scheme (x) is that which has the greatest market value at t = o. We have to assume a sort of market for uncertainties i.e. simply a market for goods <u>c</u>. Assumption of competition implies a large number of enterpreneurs (owners of businesses) with risk preferences F (x) which need not be identical but such that they all would give the same maximum price for the business i.e. for <u>c</u>. And the present value <u>x</u> of the business is simply the market value of <u>c</u>.

Similarly redefining V,z as risk interest discounted values, the minimum tension condition (f) implies that

$x = V - z$ is maximised, viz. the total variation

$$\delta x = \delta V - \delta z = 0 \qquad (86)$$

Making the business a model joint stock company with n shares outstanding, the equilibrium condition (85) becomes: "the entrepreneur adopts such a plan P(o) that the market value x of the n shares is maximised".

The entrepreneur becomes a hired directorband influences the price of shares by making his policy public. The "path" of the business becomes the path of maximum price per share. The number of shares may vary with time.

Writing $f_1(o)$ for the present rate of input of labour and $p_1(o)$ its (almost) certain price and using (86)

$$V'_{1,o} = p_1(0) \qquad (87)$$

or the present wage rate equals the risk: interest discounted marginal value - productivity. The more future wage rates and rates of input etc., are uncertain and we have no such equation as (87) for them. With uncertainty, marginal productivity theory applies only to immediate factor inputs.

4.74 At each successive instant the entrepreneur learns new facts (prices, inventions etc., etc.,) and his expectation - structure varies and he continually revises his plan. The actual "path" of the business - the realised production-process through time is a bead-string of the realised initial "t=0" stages of the plans successively adopted. Even that is not quite accurate. For even the most immediate actions are not absolutely to expectation and there are always "surprises". But it is a good approximation to say that (as seen by the entrepreneur) the path of the business is a bead-string of realised initial bits of the successively adopted plans. To determine the business path completely, the economist has to pretend that he knows the entrepreneur's environment function V(t).

That is how production looks like in the light of the general theory of planning 4.13-49.

4.75 The reason why it is realistic to assume that giving a negative increment $-\delta u(t)$ in the nearer future will lead later to a positive increment $+\delta u(t)$ of a greater absolute average size, 4.62, may be due to uncertainty. For, from the general analysis of planning, the entrepreneur's ideas about the uncertain future are vague. And the only way of taking advantage of a lower rate $\rho(t)$ he can see is to produce less in the near future and much more later. He cannot see other more complicated ways which may exist, because they are too complicated and his vague ideas do not apprehend them. Vague ideas allow him to see only simple possibilities. I.e. the rough rule "more roundabout methods are more productive" may be founded on the vagueness of planning, and to that extent, on uncertainty.

4.76 The assumption (b) of vertical integration is useful: it gives a view of the production process as a whole. The view is similar to the view of production as a whole given by the Physiocrats and Adam Smith 1.1-3. The bread-economy is shown as a sum of parallel independent firm-processes.

On assumption of certainty, each process from $t = o$ is according to plan determined at $t = o$. The question arises how the process is determined when the assumption of vertical integration is dropped, and the whole process is not controlled by a single firm and when there is no single production plan for the process from $t = o$.

Let an individual process be divided up between a number of firms in such a way that some firms produce "intermediate products" which they sell to other firms. Intermediate products are minerals, meterials, equipment. A firm that buys intermediate products (e.g. materials) may use these to produce other intermediate products (e.g. tools) which it sells in turn. It may even sell to the firm from which it buys. I.e. I assume an individual bread-process to be divided between firms in the sort of way in which we see the world-economy is divided into firms etc. So that no two firms do precisely the same thing.

I retain assumptions (a)(c)(d)(e) 4.58. The collection of goods $\underline{c}$ is divided between all the firms. As before we can take as the equilibrium condition the condition that the value $\underline{x}$ of $\underline{c}$ is maximised. We have again the equations (62)....(70). In addition we require the condition that $\underline{x}$ is maximised with respect to the prospective pattern of firms into which the process is divided i.e. with respect to the pattern function $r(t)$ say. In shorthand form:

$$\delta_r x = o \qquad (88)$$

i.e. a small variation in the future from the equilibrium number and kind of firms is unprofitable to any individual firm as it diminishes the aggregate present value of goods c distributed between different firms at $t = o$. This distribution is a datum. Given also the market price functions of intermediate products $p_a(t), p_b(t) \ldots\ldots\ldots p_z(t)$, it is seen that all the unknowns are determined. The plan of each firm is determined. The pattern function $r(t)$, the output function $u(t)$ etc., are all determined. From assumption (d) of certainty, the realised whole individual process is in accordance with the plans made at $t = o$. As if there was a single plan $P(o)$ for the whole process.

I assumed for simplicity that though there are competitive markets for intermediate products, every firm producing intermediate products sells them only to firms in the same individual bread-process. I did that because equations (62)-(70) apply to an individual bread-process and because it is easy to see that they apply whether the process is integrated or divided up into firms exchanging intermediate products at given market prices.

I now drop the assumption of parallel individual bread-processes. Instead I assume a bread-economy and that the whole bread-economy is a single "organic" process, like the real world production-process (contrast 4.69). All firms in the economy are more or less distant relations. Two firms are related if they use equipment, say, made by another firm with the assistance of the same plant etc. In this sort of way (by common causal antecedents) all firms in the economy are causally related. As before I assume that there is no vertical integration: every firm either buys or sells intermediate products, or does both.

Now writing c for all the goods existing in the bread economy at t = o and given the way r(o) these are distributed among all the fimrs at t = o, we can determine the realised total process of production in much the same way as above and in 4.60. Given total supplies of original factors from t = o, and the discount function ρ (t), the equilibrium condition is that x the value of c (N.B. the extended interpretation of (x) is maximised. Equations (62)-(70) again apply and together with the condition

$$\delta_t x = o \qquad (88')$$

determine all the separate firm-plans at t= o, i.e. determine all the unknowns: prices, the pattern function r(t) and the output function u(t) for the whole bread-economy, etc., etc. On assumption of certainty the realised path of the economy is in accordance with the plans adopted at t= o.
For each discount function ρ (t) there will correspond a particular economy-path. I.e. the path is conditioned by saving.

As before, uncertainty can be brought in and the analysis can also be extended to all goods produced in the real world.

All theses successive approximations show that the abstract analysés of production and business planning 4.51-65 is a serviceable theoretical model of the production-process in the real world.

4.77 In the last fifty years controversy about the time-aspect of production (capital) was unsatisfactory. Vague ideas such as "the average production period in the real world", the "marginal productivity of capital" in an undefined sense, etc., have been used. Concepts of secondary analytical significance such as "capital disposal" and "real capital" were suggested as the basis of theory. Sometimes even time was abstracted from in an attempt to deal with the time-aspect of production, and a "homogeneous factor capitalC" was supposed to have an instantaneous marginal consumption product $\frac{\partial P}{\partial C}$ dC, etc., etc. Most of the analysès put forward had more unknowns than determining conditions and were indeterminate. Even mistakes of reasoning were sometimes comitted. The controversy was bitter, obscure and inconclusive. That was due to vague ideas.

Therefore my aim is rigour and clarity. I study the time-dimension of goods, individual conduct, the causal structure of production. I construct theoretical models using functional calculus which suits my problem best. Only abstract models can give clarity.

My results are: a "time-conscious" theory of marginal productivity (that completes Professor Fisher's work), and a clear concept of production as an evolutionary process.

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