The London School of Economics and Political Science

ESSAYS IN FINANCIAL ECONOMICS

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A thesis submitted to the Department of Finance of the London School of Economics for the degree in Doctor of Philosophy
London, October 2016

Declaration

I certify that the thesis I have presented for examination for the MPhil/PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

I warrant that this authorisation does not, to the best of my belief, infringe the rights of any third party.

I confirm that Chapter 1 was jointly co-authored with Dr. Zijun Liu, Dr. Christian Julliard, and Dr. Kathy Yuan.

Abstract

This thesis explores topics in financial intermediation and macro-finance. In the first chapter, my co-authors and I analyse the structure of the UK repo market using a novel dataset. We estimate the extent of collateral rehypothecation, and address the question of which variables determine haircuts using transaction-level data. We find that collateral rating and transaction maturity have first order of importance in setting haircuts. Banks charge higher haircuts when they transact with non-bank institutions even after controlling for measures of counterparty risk. We examine the structure of the repo market network and we find out that banks with higher centrality measures ask for more haircuts on reverse repos and pay lower haircuts on repos.

In the second chapter, I study the real effects of benchmarking in fund management industry in a general equilibrium model where stocks of productive sectors are traded in a competitive equity market. Investors delegate their portfolio decisions to managers whose performance is benchmarked against an index. Managers hedge themselves by tilting their portfolio toward the index which increases the demand and price of the stocks that feature prominently in the index. In equilibrium there is an inefficient shift towards extreme states in which big sectors dominate the economy. I show that in presence of benchmarking, index inclusion is preceded by a rise in firm's investment rate relative to its capital stock.

In the third chapter, I examine balance sheet recessions in a general equilibrium model where agents have heterogeneous beliefs about future technology growth. I show a channel which describes the risk concentration through belief dispersion rather than ad-hoc constraints on aggregate risk sharing. Endogenous stochastic consumption volatility arises from constant fundamental volatility. I study the role of static vis-à-vis dynamic disagreement and examine the effect of financing constraints on the equilibrium when there is belief dispersion among agents.

Acknowledgements

This thesis would have not been possible without the tremendous help from my advisor Christian Julliard, who has guided and supported me throughout my PhD study. I would like to thank him dearly for all the nurturing he provided me with throughout these years.

I would like to thank Georgy Chabakauri, my second advisor, and Dimitri Vayanos for numerous helpful comments and discussions. My MSc advisor Michela Verardo was an exceptional source of advice and encouragement during my MSc and PhD. I would also like to thank Kathy Yuan and Zijun Liu for their contribution to the first chapter of this thesis. I am grateful to Rohit Rahi and Daniel Ferreira, who provided me with the opportunity to study at this distinguished institution.

I wish to express my gratitude to all the staff at the Department of Finance, especially Mary Comben who is an invaluable asset to all PhD students. I would like to thank Osmana Raie who trusted me with the teaching fellowship opportunity.

My fellow PhD students Michael Punz, Una Savic, Olga Obizhaeva, and Jesus Gorrin were great friends and companions in times of joy and distress during the student life. I wish them best of luck in their life.

I would like to gratefully acknowledge the support of the Economic and Social Research Council (ESRC) in funding the Systemic Risk Centre who provided me with financial support.

I am indebted to my father for his moral support during the years away from the family, and to my late mother for the high example she set for my life. My utmost gratitude goes to my wife Arefeh, who supported me with love and compassion, and has been with me in all the ups and downs. Last but certainly not least, I am grateful to my daughter, Maryam, who reminded me to focus on important things in life, and to my newborn son, Masih, who made the last days of my student life so memorable and blissful.

Contents

Chapter 1

The Repo Market and the Determinants of Haircuts

Abstract

We analyse the structure of the UK repo market using a regulatory dataset that covers about 70% of this market. We examine the maturity structure, collateral types and different counterparty types that engage in this market and estimate the extent of collateral rehypothecation by the banks. We address the question of what variables determine haircuts using transaction-level data. We find that collateral rating and transaction maturity have first order importance in setting haircuts. Banks charge higher haircuts when they transact with non-bank institutions. In particular, hedge funds as borrowers receive a significantly higher haircut even after controlling for measures of counterparty risk. We find that larger borrowers with higher ratings receive lower haircuts, but this effect can be overshadowed by collateral quality, because weaker borrowers try to use higher quality collateral to receive a lower haircut. Finally, we examine the structure and attributes of the repo market network and assess if the network structure has an influence over haircuts. We observe that the banks with higher centrality measures ask for more haircuts on reverse repos and pay lower haircuts on repos.

1.1 Introduction

The repurchase agreement (repo) market is a major tool for short-term funding of financial institutions. Although there is no definitive data about the size of this market, the International Capital Market Association suggests that the value of the global commercial market can be up to €15 trillion (International Capital Market Association, 2013). During the recent financial crisis repo markets experienced various disruptions and potentially contributed to the severity of the crisis. For example Copeland et al. (2010) provide evidence that shows that during the days prior to bankruptcy, the amount of collateral Lehman Brothers financed in tri-party repo fell drastically. Gorton and Metrick (2012) argue that the repo market experienced a run during the crisis, manifested in rise of haircuts, which exacerbated the crisis.

Although using collateral has stabilising effects for individual lenders, it may aggravate systemic risk by intensifying the correlation between asset prices and funding costs. This is highlighted by Valderrama (2010), who shows, that when a lender deleverages its reverse repo contracts in face of a liquidity shock, it can cause a wave of contagion along a chain of market players. Arrival of new information about the quality of assets used as collateral can change the asset prices dramatically and cause a systemic event in the repo market. Another type of disruption happens when lenders respond to the perceived creditworthiness of a counterparty (Adrian et al., 2013).

Given the importance of the repo market and its contribution to the systemic risk of the financial system—especially in the wake of the recent crisis—there is ample interest in better understanding and monitoring it from academics, policy makers and members of the public. However, as Adrian et al. (2013) indicate in their analysis of the data requirements for screening repo and securities markets, existing data sources are incomplete and there is a need for a more inclusive data collection from both academic and policy-making perspectives. In particular, they enumerate six important characteristics of repo trades that need to be gathered at the firm level: principal amount, interest rate, collateral, haircut, tenor, and counterparty.

Public sources providing information on the aforementioned data items at disaggregated level are very limited. Adrian et al. (2013) provide an overview of the sources that provide information for the US repo market and conclude that, though some sources provide data on interest rates and values used in repo trades, very little is known about haircuts, tenor and counterparties.

Systemic importance of the repo market on one hand and shortage of micro-data on the other hand prompted the UK regulator to require banks to disclose transaction-level data on their repo books. We were given the opportunity to work with this regulatory dataset to analyse the structure of the UK repo market. Among the six important trade characteristics required to generate an inclusive picture of the repo market discussed above, we have access to all at trade level, but repo rates. We examine the maturity structure, collateral types and different counterparty types that engage in this market and estimate the extent of collateral rehypothecation by the reporting banks. Due to the importance of haircuts and the fact that they control the amount of inside liquidity that shadow banking system can generate, we try to answer the question of what variables determine haircuts using transaction-level data. We also examine the structure and attributes of the repo market network and assess if the network structure has an influence on haircuts.

1.1.1 Background Information on Repurchase Agreements

A repurchase agreement is the simultaneous sale and forward agreement to repurchase of securities at a specific price, at a future date (Duffie, 1996). In effect a repo is a collateralised loan, where the underlying security serves the collateral role. The party who borrows cash and delivers collateral is said to be doing a repo, and the party who lends cash and receives collateral is doing a reverse repo. The difference between the original loan value and the repayment specifies the repo rate. The haircut or margin on the other hand is determined by the difference between the loan and collateral value. Usually the borrower has to post collateral in excess of the notional amount, and the haircut is defined as $h = 1 - F/C$ with collateral value C and notional amount F (Krishnamurthy et al., 2014). For example, if a borrower receives \$98 against \$100 value of collateral, the haircut is 2%.

In Europe, the legal title to the collateral is transferred to the cash lender by an outright sale. In the US, this is not the case, but the repo collateral is not subject to an automatic stay and can be sold by the lender should the borrower defaults (International Capital Market Association, 2013).

Repurchase agreements are broadly classified in two categories. Tri-party repo is a transaction for which post-trade services like collateral management (e.g. selection, valuation, and verifying eligibility criteria), payment, margining, etc. are outsourced to a third-party agent which is a custodian bank.[1] A tri-party agent settles the repos on its book, but in a bilateral repo settlement usually occurs on a delivery versus payment basis, and the cash lender must have back-office capabilities to receive and manage the collateral (Adrian et al., 2013).

A growing number of repos are cleared via central (clearing) counterparties (CCPs). CCPs place themselves between the two sides of a trade, leading to a less complex web of exposures (Rehlon and Nixon, 2013). They provide benefits such as multilateral netting and facilities to manage member defaults in an orderly manner, but can also pose systemic risks to the financial system. CCPs always receive a haircut, whether in a reverse repo or repo. So banks doing a reverse repo with a CCP will need to give a haircut, which amounts to a negative value for haircut.

1.1.2 Literature on Repurchase Agreements

The financial crisis rekindled interest in the theoretical and empirical study of the short-term funding market. Brunnermeier and Pedersen (2009) model the link be-

[1] There are two tri-party agents in the US, Bank of New York Mellon and JP Morgan. In Europe, the main tri-party agents are Clearstream, Euroclear, Bank of New York Mellon, JP Morgan, and SegaInter-Settle.

tween traders' funding liquidity—i.e. their capital and margin requirements—and the assets' market liquidity, and show how rising haircuts can have destabilising effects on financial markets and cause a margin spiral. Jurek and Stafford (2011) develop a static model for the characterisation of margins and rates in collateralised lending markets. In their framework, haircut protects lenders from changes in the liquidation value of collateral. Spreads and margins for assets are expected to move with each other and proxy for the aggregate risk. Securities that have quickly declining recovery values will be financed at higher rates and haircuts, and will respond more strongly in volatile market conditions. Dang et al. (2011) explain the existence of haircuts by sequential transactions and the information asymmetry between agents in trading chains. Haircuts exist to protect a lender who may need to sell the collateral in case of borrower default, facing the possibility of adverse selection. Based on their theory, the haircut size should depend on the default probability of the borrower, the liquidity needs of the lender, the default probability of the lender in a subsequent repo transaction and the nature of the collateral. In this model a run on repo occurs when information-insensitive collateral asset becomes information sensitive. Rytchkov (2009) studies time-varying margin requirements in a dynamic setup. Acharya et al. (2011) model freezes in the market for short-term financing in form of sudden collapse in debt capacity of collateral in an information-theoretic framework.

The empirical studies of repurchase agreements have been mostly focused on the US repo market. Several papers have studied developments in this market during the financial crisis. Broadly speaking two distinct phenomena can be identified in the US bilateral and tri-party repo markets. In the bilateral market, as argued by Gorton and Metrick (2012), a run occurred in form of rapid increases in haircut levels. This is further supported by multiple hedge funds failing due to margin calls (Adrian et al., 2013). In contrast, in the tri-party market haircuts moved very little and the amount of funding remained fairly stable, but instead, lenders refused to extend financing altogether to the most troubled institutions—namely Bear Stearns and Lehman Brothers (Copeland et al., 2010). Krishnamurthy et al. (2014) argue that there indeed was a run on tri-party market but only for non-agency MBS/ABS, which constituted a relatively small and insignificant part of the short-term debt market. So in the tri-party market, tension seemed to affect specific institutions rather than the broad collateral classes, except maybe the private-label securitised assets (Adrian et al., 2013). Martin et al. (2014) develop a dynamic equilibrium model and relate the differences between the behaviour of these two markets to their microstructure. In the tri-party market, haircuts are fixed in custodial agreements that are revised infrequently, but this is not the case in the bilateral market.

Adrian and Shin (2010) empirically show that repo transactions have contributed the most to the procyclical adjustments of the leverage of banks. From this perspective, rapid increase of haircuts in bilateral repos during the crisis, can also be viewed as (forced) deleveraging of broker-dealers (Adrian et al., 2013).

1.2 Overview of the Data

The regulatory dataset is a snapshot of the repo books of 10 banks that are major players in the UK repo market. The total size of the repo books—the sum of repos and reverse repos—of the 10 banks is around £1.5 trillion as at end-2012.[2] According to Financial Stability Board (2013) the UK-resident deposit-taker banks hold around £2.1 trillion in gross repo activity on their balance sheets, so our data accounts for around 70% of the total repo activity in this market. The majority of this activity is with non-UK resident banks, including the activity between UK and foreign branches of the same consolidated group, and is highly concentrated (Financial Stability Board, 2013).

Each of the 10 banks report their outstanding repo transactions as at end-2012, including the gross notional, maturity, currency, and counterparty. Out of the 10 banks, only 6 banks reported data on haircuts and collaterals. The 6 banks have a total repo book of £511 billion and 27,886 transactions.

We have supplemented this dataset with additional data about securities, counterparties, and the reporting banks from Datastream and Bloomberg.

In what follows we report information and results for reverse repos (REVR) and repos (REPO) separately. This classification is from the point of view of reporting banks, so in a reverse repo the reporting bank is lending to a counterparty, and in a repo the reporting bank is borrowing money from a counterparty.

Tables 1.1 and 1.2 presents an overview of our dataset in terms of key variables. It shows the breakdown of the data along four categories: maturity, currency, counterparty type, and collateral type (Panels A, B, C, and D respectively). For each category we report the sum of the notional amounts of deals for each subcategory in Table 1.1 and the weighted average of haircuts for each subcategory in Table 1.2. Table 1.1 also shows the percentage of each category in terms of the notional values. The average haircuts in Table 1.2 are weighted by the gross notional of transactions. Both values and haircuts are reported for reverse repos and repos separately. Since repo shows bank borrowing, we denote the repo values with negative numbers. In Panels A, B, and C in Table 1.1 the total values are based on the data from the ten reporting banks. The values in Panel D in Table 1.1, as well as the all the haircuts in Table 1.2 are based on the data from the six banks that report haircut and collateral information.

Comparing the total value of reverse repos and repos shows that banks are net borrowers in the repo market. They borrow more heavily in transactions with shorter maturity. Panel A in Table 1.1 shows that most of the transactions have maturity less

[2] The actual reporting periods differ slightly across the banks but all are towards the end of 2012.

than three month. It is also evident from Panel A in Table 1.2 that haircuts monotonically increase with maturity both for repos and reverse repos. We can see that except for very long maturities, the reporting banks are able to borrow at slightly lower haircuts than they lend. This observation means that they can use the collateral they receive in a reverse repo to obtain more funding than originally lent. A similar pattern exists for different currencies, except for EUR. A potential explanation is that a large proportion of Euro repos is against securitisation collateral, which is subject to a higher haircut. The same does not hold for the other currencies, because most USD securitisation business is done in the US (hence not included in the dataset) and no securitisation market exists for other currencies.

It is worth noting that the GBP haircuts seem relatively high. This phenomenon is driven by relatively high haircuts against UK government bonds (see Panel D in Table 1.2). To investigate this point, we cast a closer look at contracts with sovereign bond collateral in Table 1.5. The data suggests that higher haircuts on gilt collateral are driven by transactions with other asset managers in reverse repos, and transactions with government entities in repos. However, to obtain a comprehensive view of the determinants of haircuts we will use regression analysis as described in Section 1.3.

While borrowing exceeds lending for maturities less than three month, lending is more for transactions with three month to one year maturity. This observation suggest that the banks conduct maturity transformation to some extent, however, for maturities longer than one year they are still net borrowers.

Panel C in Tables 1.1 and 1.2 exhibits the breakdown based on counterparty type. Data in this panel makes it clear that the above-mentioned haircut advantage arises from trades with hedge funds, insurance and pension funds and other asset managers. In the transactions with these counterparties, the banks can receive funding at significantly lower margins than they lend. First row in Panel C describes the values and haircuts when counterparty is another member of the reporting banks. The reporting banks report on a UK consolidated basis, but counterparties are reported on a global basis. Therefore there may be discrepancies between the reverse repos and repos with the reporting banks.

Finally, Panel D in Tables 1.1 and 1.2 shows the breakdown based on collateral type. It displays how margins depend on the quality of collateral. For example both repos and reverse repos for German government bonds have a low average haircut, while haircuts for troubled European countries and securitisation are very high.

Tables 1.3 and 1.4 present a finer breakdown of the reverse repos and repos respectively. Each column shows the percentage of deals (in terms of the notional value) against different counterparty types. This information is further divided by maturity, currency and collateral type. For example the entry in first column and first row denotes the percentage of the notional value in contracts with overnight maturity where the counterparty is another reporting bank.

1.2.1 Collateral Rehypothecation

An important aspect of the repo market is rehypothecation—reusing pledged collateral. Large banks and dealers often reuse collateral pledged by hedge funds, pension funds, and other institutions, and this practice means that collateral has a velocity. Collateral rehypothecation creates liquidity and lubricates the financial system, but can have adverse consequences for financial stability. Singh (2011) argues that during the financial crisis debt capacity declined through two channels: (1) lower availability of collateral due to fall in asset prices and increase in haircuts and (2) lower leverage multiplier due to shortening of intermediation chains.

We measure to what extent collateral is reused in our dataset. Table 1.6 shows what percentage of the collateral received by the banks in reverse repo transactions is reused in repos to borrow against. The results in Panel A are weighted by the value of transactions, and in Panel B they are equally weighted. This information provides an indication of the the extent of collateral reuse collateral received in reverse repo transactions by banks. In Table 1.6, high and low quality refer to investment-grade and non-investment-grade securities respectively.

Overall, the table suggests that government securities have the highest reuse rate, which is as expected given the liquidity of government bond markets. Collateral with longer residual maturity tends to have a higher reuse rate, except for securitisation with 10Y+ residual maturity. Collateral quality seems to have mixed effects on the reuse rate. Investment grade government bonds tend to have higher reuse rate than non-investment grade ones; however the opposite is true for corporate bonds. Comparing the value-weighted and non-value-weighted results in Table 1.6 shows that rehypothecation is concentrated on particular securities, especially high quality government bonds.

1.3 The Determinants of Haircuts

We now analyse what explanatory variables govern haircuts and in what ways these variables affect them. For this purpose we run multiple regressions on reverse repo and repo data separately, with different specifications as described below.

For the most part of the regression analysis, we focus on the sample excluding the trades with CCPs. In practice, CCPs often calculate haircuts (or initial margin requirements) on a portfolio basis. That is, the over-collateralisation of repo positions is calculated at the portfolio or netting set level, without applying haircuts on individual transactions. In our dataset, firms still report a transaction-level haircut, but this is often zero given that the 'true' haircut is applied at the portfolio level. In such cases, it is not meaningful to look at haircuts on individual transactions that are centrally cleared. In addition, there is basically only one CCP in our sample, which uses a fixed schedule of haircuts available online. Therefore, we focus on the sample that excludes CCP transactions.

There are a lot of zero haircuts in the data as illustrated by the histogram of haircuts in Figure 1.1. Some of these zero haircuts are due to the way haircuts are reported in CCP trades as explained above, but even excluding CCP trades, zero-haircut trades are still prevalent. A summary of these trades is presented in Table 1.10. This finding is not surprising and has been confirmed by other data collections done at the global level. In order to make sure that the multitude of zero haircuts do not distort our results, in addition to the ordinary least square regressions, we perform two sets of additional regressions. We use Tobit model with truncation at zero as our baseline model. In addition, we use logit transform to generate more variation in haircuts and to run logistic regression.

Table 1.7 explains all the explanatory variables used in different regressions. We have dummy variables for currencies as well as collateral and counterparty types. Other than dummy variables we use trade-specific variables, collateral rating and maturity, and counterparty characteristics. We also have two measures for counterparty and collateral concentration. Counterparty concentration measures the share of transactions with a specific counterparty in total, evaluated using the notional amount of transactions. It represent how systemically important that counterparty is to the bank. Similarly, collateral concentration is measured by the share of transactions against a specific collateral in total, evaluated using the notional amount of transactions. We also include an interaction term between collateral rating and counterparty rating. The logic behind the this term is to find whether counterparty and collateral quality can compensate for each other.

Table 1.8 shows summary statistics for haircuts and non-dummy explanatory variables for the sample used in the baseline regressions which excludes the deals with CCPs. Except collateral and counterparty ratings which are categorical, other variables in this table are continuous. The summary statistics are represented separately for reverse repos and repos in Panels A and B respectively, given that haircut practices can potentially differ significantly between the two instruments. The sample only includes the six banks that provided data on haircuts and collateral ID. Variables have been winsorised at 0.5% level.

Even though haircuts can have as high value as 46%, the weighted average of haircuts is about 6% both for reverse repos and repos. Notional values are log-transformed. Maturity values, both for transactions and collateral, are in year. The weighted average of maturity for the transactions is about 50 to 70 days, while the mean is less at around 20 days. Average collateral maturity used is between 2 and 3 years. Collateral and counterparty ratings are modified into numeric scale from 1 to 20, with 20 being the highest rating. The average collateral quality in this scale is about 17, while the average counterparty rating is between 14 and 15.

The summary statistics for counterparty return on assets (RoA), leverage, CDS spread, and cash ratio are also presented in Table 1.8, and the respective definitions are in Table 1.7. The logic for including RoA is to see how profitability of the counterparty can affect haircuts, and cash ratio is intended to proxy for liquidity needs. Overall the summary statistics for reverse repos and repos are not significantly different.

Table 1.9 exhibits summary of the dummy variables for the sample used in the baseline regressions which excludes the deals with CCPs. The sample includes the six banks that provide data on haircuts and collateral ID, and the percentages represent frequency of transactions.

Over 50% of transactions are conducted with other banks and broker-dealers, and hedge funds and other asset managers are next prominent counterparties. Government bonds account for 50% of collateral in reverse repos but only 36% in repos. In repos, corporate bonds constitute 54%, considerably higher than government bonds. This evidence suggests that the reporting banks borrow more than they lend against corporate bonds and securitised products, while the opposite is true about government bonds. This observation is confirmed by inspecting the total notional values reported in Table 1.1 Panel D.

In Tables 1.11–1.16 we present the main results of the paper. These tables show regression results to understand what factors determine haircuts. The dependent variable is haircut in all tables and explanatory variables are listed in second column. We have classified explanatory variables into several categories. These categories are shown in the first column. The columns that are labeled with numbers display regression coefficients for different explanatory variables. Standard errors, which are not reported, are clustered at reporting bank level. One and two stars denote 10% and 5% significance levels respectively. The tables exhibit the results for Tobit, OLS, and Logistic regressions for reverse repos and repos. The next two sections elaborate on the main results presented in Tables 1.11–1.16.

1.3.1 Reverse Repo Results

The results in Tables 1.11–1.13 are for reverse repo transactions. In these transactions the reporting bank lends cash and receives collateral, and the counterparty borrows money and delivers collateral to the bank. Hence, counterparty characteristics correspond to borrower characteristics in these transactions.

Table 1.11 presents the outcome of the baseline Tobit regression, and Tables 1.12 and 1.13 show the OLS and Logistic regressions respectively. Overall the Tobit model seem to describe the data better and results have more significance using this model, however, the main results that we will emphasise below are robust to the choice of model.

Column (1) in these tables, reports the result when smallest set of explanatory variables is used. In this column, we include currency dummies, notional and maturity of transaction, collateral characteristics (rating and maturity) and collateral type dummies, and dummies for counterparty type, but we leave out counterparty characteristics. In column (2) we add counterparty characteristics and concentration measures for counterparties and collateral. Column (3) uses the largest set of regressors by adding the reporting bank characteristics to the ones used in column (2). In column (4) we add reporting bank characteristics to the initial set of regressors used in (1), excluding the counterparty characteristics. Finally columns (5) and (6) are similar to column (4), but they include network centrality measures described in Section 1.4.

In columns (1) and (2) we do not include the reporting bank characteristics, instead we look for haircut determinants by assessing the effects of explanatory variables within transactions conducted by each reporting bank. This is achieved by including reporting bank fixed effects in the regressions. We also report regressions including the reporting bank characteristics (own variables) in columns (3) and (4). These variables are specially relevant for repos where the reporting banks are borrowers and we can expect that their attributes should matter in specifying haircut level.

Notional value of transaction does not influence haircuts. On the other hand maturity has a significant positive and robust effect on haircuts across all specifications, so the higher the maturity of transaction the higher would be the haircut. As column (1) in Table 1.11 shows, 1% increase in maturity increases haircuts by 11.9 basis points.[3] Another variable that has a significant and robust effect is collateral rating. As expected higher quality collateral receives lower haircut. One notch increase in collateral rating, reduces haircuts by 1.3 percentage points. Controlling for collateral rating, the maturity and type of collateral (government, corporate, or securitisation) does not significantly affect haircuts.

Among the counterparty types, hedge funds receive significantly higher haircuts in all specifications. Relative to the baseline haircut received by banks, hedge funds face haircuts that are on average 13.4 percentage points higher, as shown in column (1) in Table 1.11. Some, but not all of this effect can be described by counterparty characteristics. Broker-dealers receive lower haircut in most specifications, but the effect becomes weak once we account for other counterparty characteristics such as rating and size.

Results in column (2) of Tables 1.11–1.13 show that larger counterparties and borrowers with higher ratings receive lower haircut. A one-unit increase in log of counterparty size or one notch increase in its rating decrease haircuts by 3.1 and 1.8 percentage points respectively. Higher counterparty leverage increases the haircut, although this effect is insignificant except in Tobit regressions. An important question about haircuts is that how collateral risk and counterparty risk interact. There

[3]Deal maturity is log transformed.

is a significant and positive coefficient on the interaction term between counterparty and collateral rating. Excluding this interaction term from the regression weakens the magnitude and significance of the effect of counterparty characteristics. This observation means that collateral quality can overshadow counterparty characteristics. It seems that borrowers with lower ratings try to use higher quality collateral to receive a lower haircut, and as a consequence the influences of counterparty attributes are concealed. After accounting for this interaction we can observe that larger counterparties and borrowers with higher ratings receive lower haircut.

Haircuts increase with the counterparty concentration, therefore the banks charge significantly higher haircut when dealing with institutions that borrow relatively large sums from them and hence are systematically important to them. On the other hand, collateral concentration plays no significant role.

We should interpret the results related to the reporting bank characteristics (own variables) in columns (3)–(4) with a grain of salt due to the small size of the sample of reporting banks. Nevertheless, they show that profitable lenders with more cash available to them charge lower haircuts on average. We defer the discussion of the results in columns (5)–(6) to Section 1.4 on network analysis.

1.3.2 Repo Results

We present analogous results for repos in Tables 1.14–1.16. In these transactions the reporting bank borrows cash and delivers collateral, and the counterparty lends money and receives collateral. Hence, counterparty characteristics correspond to lender characteristics in these transactions. Columns (1)–(6) are equivalent to the corresponding columns in Tables 1.11–1.13.

Overall some of the results for repos are less stable across specifications and more difficult to interpret. Similar to the reverse repos, higher maturity increases haircut on a transaction. As column (1) in Table 1.14 shows, 1% increase in maturity increases haircuts by 5.7 basis points. Notional value in Tobit regressions has a negative and significant effect, but the sign alters in the other two specifications (but with no or marginal significance). The effect of collateral rating is the most robust result in all specifications and also across repos and reverse repos. Similar to reverse repos, higher quality collateral always receives lower haircut, with one notch increase in collateral rating reducing haircuts by 0.4 percentage points. As can be seen in Table 1.1 Panel D, most of the securitisation collateral is used in repos, and although there was no visible effect for reverse repos, haircut is higher for it relative to government bond after controlling for other variables in repos. This result is weak nonetheless. The effect of collateral maturity is insignificant as before.

Insurance and pension funds charge higher haircuts as lender relative to other banks. There seems to be a similar effect for hedge funds, but it is not robust and the coefficient changes sign in Tobit regressions by adding counterparty characteristics. Similar unstable results exist for broker-dealers and central banks and governments.

The counterparty—i.e. lender in a repo—characteristics appear to matter for haircuts one way or another, although the results are less stable and difficult to interpret. The result that is most stable is that counterparties with higher CDS spread charge a lower haircut. Lender size, RoA, leverage, and cash ratio also have unstable effects across some of the specifications. Both collateral and counterparty concentration have positive coefficients although they are not significant in all of the specifications.

In theory borrower characteristics should be more relevant in determining haircuts and therefore ought to be included in the explanatory regressions for repos. We represent the results with the reporting bank characteristics (own variables) in columns (3)–(4). As expected, reporting bank characteristics, have significant impact on haircuts that are stable unlike the lender attributes. Larger, more profitable, less leveraged banks who have a low CDS spread and high cash ratio receive smaller haircuts. Significance levels change, but the signs are stable and the attributes have the expected impact on the dependent variable. Nevertheless the concern about this regression is the small size of the sample of reporting banks.

1.3.3 Bank versus Non-Bank Counterparties

In order to see how haircuts applied between two banks differ from the haircuts between a bank and a non-bank entity, we run a different set of regressions, with results reported in Tables 1.17–1.22. These regressions are analogous to the ones in Tables 1.11–1.16, except that there is only one dummy variable for counterparty type which takes value of 1 if the counterparty is a bank or broker-dealer, and 0 otherwise.

The coefficient on bank/dealer dummy is negative both for repos and reverse repos in all specifications. It is significant in all repo specifications, and Tobit reverse repo regressions. This result shows that when banks trade with institutions similar to themselves they charge lower haircuts, controlling for all observables like counterparty or collateral rating, maturity, etc. This observation can be related to the fact that the two parties in a repo contract may disagree on the collateral value and charging haircut is a tool to mitigate the disagreement. Similar institutions use comparable models and therefore it is more likely that two banks have less disagreement than two completely different entities, say a bank and a hedge fund, hence the lower haircuts for bank/dealer counterparty in our sample.

1.4 Network Analysis

The financial crisis has shown the importance of the interconnectedness of the banking system and the need to analyse risk not by looking at individual institutions in isolation, but by assessing network structure and interplay between institutions. As a result various studies have used network analysis tools to study the interbank and inter-dealer markets (e.g. Denbee et al. (2014) and Li and Schürhoff (2012)).

In this part we try to examine the network structure of the UK repo market using our dataset. We use network centrality measures borrowed from the literature on network analysis and employed by Li and Schürhoff (2012). Table 1.23 provides summary statistics of these measures (for definitions see Li and Schürhoff (2012)).

Figure 1.2 displays the repo market network plot. The network plot shows the reporting banks in yellow and size of the nodes is proportional to total degree measure. Panel A is based on all transactions of the entire sample (ten reporting banks) while Panel B is based on the sample of the six banks that report haircut and collateral information and are used in the regression analysis.

In order to see if network structure affects haircuts in the repo market, we use principal component of the unweighted and weighted centrality measures in the explanatory regressions. The results are presented in columns (5)–(6) of Tables 1.11–1.16 for reverse repos and repos. These columns are similar to column (4) in their corresponding tables, but they include the principal component of either unweighted (pcu) or weighted (pcw) centrality measures in column (5) and (6), respectively. We see that the banks with higher centrality measures ask for more haircuts on reverse repos and pay lower haircuts on repos, which is consistent with the story that these banks have higher market power. The results using weighted or unweighted measures are virtually the same.

In unreported regressions we use the entire sample including the CCP deals. None of the results mentioned above changes significantly, with two notable exceptions. Firstly, with CCP transactions, the two network measures are not significant in any case, so we do not observe any meaningful network effect when CCP transactions are included. Furthermore, including CCP transactions attenuates the impact of counterparty concentration on increasing the haircuts. Overall, given the issues described in Section 1.3, it seems that including CCP transactions introduces some noise in the way that the architecture of the market affects haircuts and it is to be expected that the results related to the network measures and counterparty concentration become less significant.

1.5 Conclusion

In this study we analyse the structure of the UK repo market using a regulatory dataset collated by the UK regulator. We examine the maturity structure, collateral types and different counterparty types that engage in this market and try to estimate the extent of collateral rehypothecation by the reporting banks. We try to answer the question of what variables determine haircuts using transaction-level data. We find that collateral rating and transaction maturity have first order importance in setting haircuts. Banks charge higher haircuts when they transact with non-bank institutions. In particular, hedge funds as borrowers receive a significantly higher haircut even after controlling for measures of counterparty risk. Larger borrowers with higher ratings receive lower haircuts, but this effect can be overshadowed by collateral quality, because weaker borrowers try to use higher quality collateral to receive a lower haircut. Banks charge higher haircut when trading with institutions that are systematically important to them. Finally, we examine the structure and attributes of the repo market network to assess if the network structure has an influence over haircuts. We observe that the banks with higher centrality measures ask for more haircuts on reverse repos and pay lower haircuts on repos.

1.6 Appendix

1.6.1 Figures

Figure 1.1: Histogram of haircuts

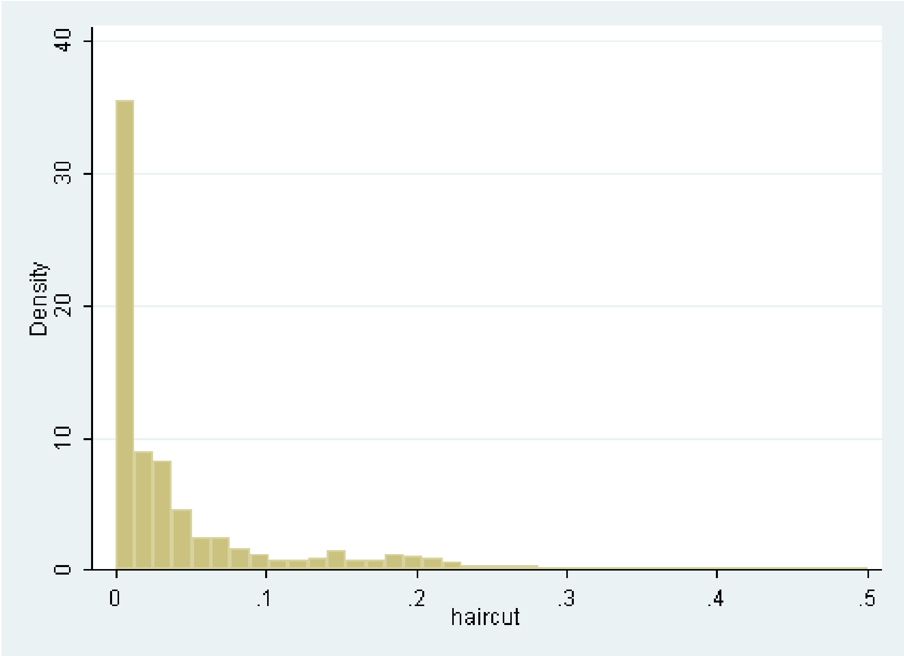
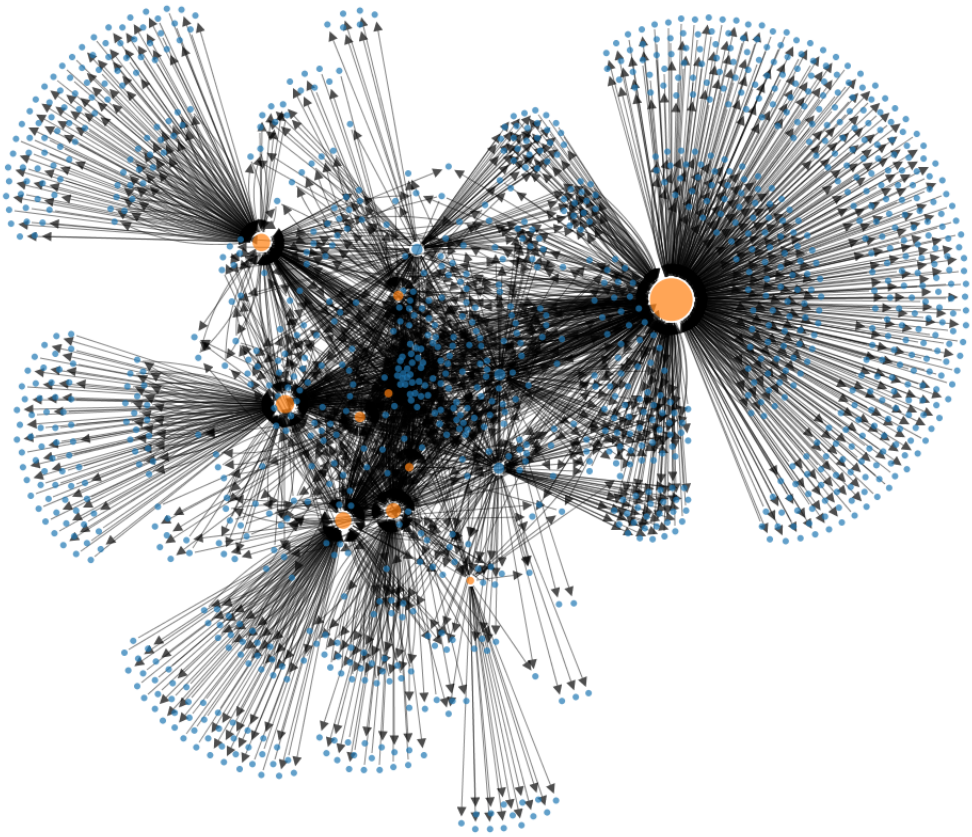
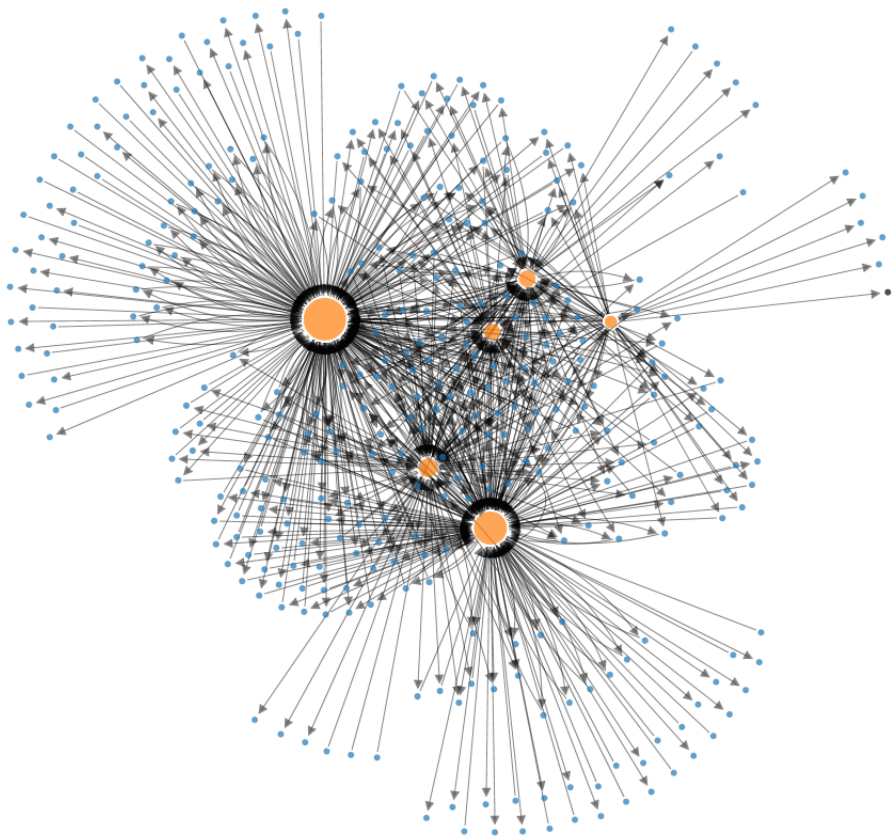


Figure 1.2: Network flows plot

A. Whole sample (10 banks)



B. Sample of banks that report haircut and collateral info (6 banks)



1.6.2 Tables

Table 1.1: The breakdown of value of contracts (in £bn) by maturity, currency, counterparty type, and collateral type

	REVR		REPO	
	Value	Percent	Value	Percent
A. Maturity				
Overnight	41.4	6.0%	-62.2	8.1%
<3m	432.8	62.3%	-469.4	61.3%
3m-1y	173.6	25.0%	-166.6	21.8%
1y-5y	28.6	4.1%	-41.0	5.3%
5y+	18.5	2.7%	-26.4	3.5%
Total	694.9	100.0%	-765.6	100.0%
B. Currency				
GBP	240.3	34.6%	-297.8	38.9%
EUR	247.7	35.6%	-211.7	27.7%
USD	168.2	24.2%	-235.8	30.8%
JPY	21.9	3.1%	-10.0	1.3%
Other	16.8	2.4%	-10.2	1.3%
Total	694.9	100.0%	-765.6	100.0%
C. Counterparty type				
Another reporting bank [a]	49.4	7.1%	-54.5	7.1%
Other banks	138.4	19.9%	-128.9	16.8%
Broker-dealer [b]	78.0	11.2%	-66.6	8.7%
Hedge fund	87.2	12.6%	-63.1	8.2%
MMF [c]	0.7	0.1%	-9.4	1.2%
Other asset managers [d]	53.9	7.8%	-106.2	13.9%
CCP [e]	227.6	32.8%	-206.2	26.9%
Insurance and pension	32.8	4.7%	-24.9	3.2%
Central bank & govt	15.8	2.3%	-90.3	11.8%
Other [f]	11.0	1.6%	-15.7	2.0%
Total	694.9	100.0%	-765.6	100.0%
D. Collateral type				
US govt	10.9	6.0%	-5.4	2.9%
UK govt	83.1	45.7%	-111.7	59.1%
Germany govt	25.5	14.0%	-19.1	10.1%
France govt	16.9	9.3%	-7.2	3.8%
Supranational	3.7	2.0%	-4.6	2.4%
GIIPS [g]	0.5	0.3%	0.0	0.0%
Other sovereign	31.4	17.3%	-15.7	8.3%
Corporate	7.5	4.1%	-11.7	6.2%
Securitisation	2.0	1.1%	-13.5	7.1%
Other	0.0	0.0%	0.0	0.0%
Total	181.6	100.0%	-188.9	100.0%

The table presents the breakdown of the deals by maturity, currency, counterparty type, and collateral type (Panels A, B, C, and D respectively). For each category, it shows the value of the trades in billion Pounds and the percentage of total trades for the reverse repos and repos respectively. In Panels A, B, and C, the total values are based on the data from the ten reporting banks. The total values in Panel D are based on the data from the six banks that report haircut and collateral information.
[a] The reporting banks report on a UK-consolidated basis, but counterparties are reported on a global basis. Therefore there may be discrepancies between the reverse repos and repos with the reporting banks.
[b] Broker-dealers are mostly securities firms that are subsidiaries of large banks. [c] Money market fund. [d] Non-leveraged non-MMF mutual funds—asset managers that are not hedge fund or MMF. [e] Central (clearing) counterparty. [f] Includes corporations, schools, hospitals and other non-profit organisations. [g] Greece, Italy, Ireland, Portugal, and Spain government bonds.

Table 1.2: The breakdown of average haircuts by maturity, currency, counterparty type, and collateral type

	REVR	REPO
A. Maturity		
Overnight	1.8%	0.5%
<3m	5.2%	4.8%
3m-1y	8.8%	8.5%
1y-5y	13.7%	14.8%
5y+	N/A	15.9%
B. Currency		
GBP	8.1%	7.6%
EUR	3.7%	7.1%
USD	5.8%	3.2%
JPY	4.3%	0.6%
Other	6.8%	2.1%
C. Counterparty type		
Another reporting bank [a]	4.0%	5.6%
Other banks	6.1%	6.3%
Broker-dealer [b]	3.1%	6.1%
Hedge fund	18.7%	5.0%
MMF [c]	N/A	1.5%
Other asset managers [d]	10.7%	1.7%
CCP [e]	-5.3% [f]	5.7%
Insurance and pension	10.1%	8.8%
Central bank & govt	2.7%	5.0%
Other [g]	0.1%	3.7%
D. Collateral type		
US govt	2.6%	1.4%
UK govt	7.7%	6.2%
Germany govt	0.9%	2.1%
France govt	5.9%	2.3%
Supranational	0.4%	1.4%
GIIPS [h]	18.4%	14.0%
Other sovereign	4.3%	3.7%
Corporate	5.4%	5.9%
Securitisation	17.7%	16.0%
Other	15.8%	N/A
Overall average [i]	5.9%	6.2%

The table presents the breakdown of the deals by maturity, currency, counterparty type, and collateral type (Panels A, B, C, and D respectively). For each category, it shows the average haircut for the reverse repos and repos respectively. The averages are weighted by the gross notional of the transactions. The haircuts are based on the data from the six banks that report haircut and collateral information.

[a] The reporting banks report on a UK-consolidated basis, but counterparties are reported on a global basis. Therefore there may be discrepancies between the reverse repos and repos with the reporting banks.

[b] Broker-dealers are mostly securities firms that are subsidiaries of large banks. [c] Money market fund. [d] Non-leveraged non-MMF mutual funds—asset managers that are not hedge fund or MMF. [e] Central (clearing) counterparty. [f] CCPs always receive a haircut, whether in a reverse repo or repo. So banks doing a reverse repo with a CCP will need to give a haircut, which amounts to a negative value for average haircut. [g] Includes corporations, schools, hospitals and other non-profit organisations. [h] Greece, Italy, Ireland, Portugal, and Spain government bonds. [i] Excludes the deals with CCPs.

Table 1.3: The breakdown of reverse repos

	Counterparty type										
	1	2	3	4	5	6	7	8	9	10	Total
A. Maturity											
Overnight	0.6	0.9	1.9	0.4	0.0	0.1	1.9	0.1	0.0	0.0	5.9
<3m	3.9	11.8	6.9	10.5	0.1	5.0	19.4	2.6	1.1	1.0	62.3
3m-1y	1.9	5.8	1.8	1.0	0.0	2.0	9.6	1.6	0.9	0.4	25.0
1y-5y	0.3	0.6	0.4	0.3	0.0	0.7	1.3	0.2	0.3	0.1	4.2
5y+	0.4	0.7	0.2	0.4	0.0	0.0	0.7	0.2	0.0	0.0	2.6
Total	7.1	19.8	11.2	12.6	0.1	7.8	32.9	4.7	2.3	1.5	100.0
B. Currency											
GBP	2.5	4.8	2.2	2.2	0.1	4.7	15.7	1.4	0.7	0.3	34.6
EUR	2.1	7.3	3.7	2.7	0.0	1.3	15.7	2.0	0.7	0.2	35.7
USD	2.3	6.0	3.8	6.3	0.1	1.6	1.4	1.1	0.7	1.0	24.3
JPY	0.0	0.9	0.8	1.0	0.0	0.2	0.0	0.0	0.1	0.2	3.2
Other	0.1	0.9	0.7	0.3	0.0	0.0	0.0	0.1	0.1	0.0	2.2
Total	7.1	19.8	11.2	12.6	0.1	7.8	32.9	4.7	2.3	1.5	100.0
C. Collateral type											
US govt	1.6	0.8	1.5	0.6	0.0	0.7	0.0	0.0	0.8	0.0	6.0
UK govt	0.5	0.3	0.2	0.3	0.0	3.7	37.7	1.5	0.2	1.2	45.6
Germany govt	0.2	0.3	0.2	0.5	0.0	0.5	10.7	0.5	1.0	0.1	14.0
France govt	0.0	1.1	0.2	0.3	0.0	0.1	6.4	0.8	0.1	0.3	9.3
Supranational	0.1	1.1	0.2	0.0	0.0	0.0	0.5	0.1	0.1	0.0	2.1
GIIPS	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.3
Other sovereign	1.1	5.2	1.7	0.8	0.0	0.4	7.0	0.1	0.9	0.3	17.4
Corporate	0.5	0.6	1.0	0.9	0.0	0.2	0.2	0.4	0.0	0.4	4.2
Securitisation	0.1	0.3	0.2	0.2	0.0	0.1	0.1	0.1	0.0	0.1	1.2
Other	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total	4.1	9.7	5.2	3.9	0.0	5.7	62.5	3.5	3.1	2.4	100.0

This table exhibits a finer breakdown of the reverse repo contracts. The numbers are in percentage points and indicate the percentage of notional value in each category. The data is double sorted by counterparty type (columns) and maturity, currency and collateral type in Panels A, B, and C respectively. The table is based on the data from the the six banks that report haircut and collateral information. Columns 1–10 refer to the following counterparty types:
1. Another reporting bank
2. Other banks
3. Broker-dealer
4. Hedge fund
5. MMF
6. Other asset managers
7. CCP
8. Insurance and pension
9. Central bank & govt
10. Other

Table 1.4: The breakdown of repos

	Counterparty type										
	1	2	3	4	5	6	7	8	9	10	Total
A. Maturity											
Overnight	0.8	3.1	1.3	0.6	0.0	1.3	0.7	0.1	0.2	0.0	8.1
<3m	3.7	7.7	4.5	5.8	1.2	11.5	15.1	2.5	7.7	1.7	61.4
3m-1y	1.6	3.9	1.4	0.9	0.0	0.8	9.5	0.6	2.7	0.3	21.7
1y-5y	0.6	1.0	1.2	0.0	0.0	0.0	1.6	0.0	0.9	0.0	5.3
5y+	0.4	1.2	0.3	1.0	0.0	0.2	0.0	0.1	0.3	0.0	3.5
Total	7.1	16.9	8.7	8.3	1.2	13.8	26.9	3.3	11.8	2.0	100.0
B. Currency											
GBP	2.5	4.2	2.0	1.1	0.7	4.5	18.3	1.2	4.1	0.2	38.7
EUR	1.7	5.8	2.3	2.8	0.2	2.8	8.0	0.2	3.7	0.2	27.6
USD	2.8	5.8	3.9	3.9	0.3	6.4	0.6	1.8	3.8	1.6	30.8
JPY	0.0	0.4	0.1	0.4	0.1	0.2	0.0	0.1	0.1	0.0	1.4
Other	0.1	0.6	0.5	0.1	0.0	0.0	0.0	0.0	0.1	0.0	1.4
Total	7.1	16.9	8.7	8.3	1.2	13.8	26.9	3.3	11.8	2.0	100.0
C. Collateral type											
US govt	0.3	1.7	0.4	0.0	0.0	0.1	0.0	0.0	0.4	0.0	2.9
UK govt	0.2	0.6	0.3	0.6	1.0	1.5	49.9	0.5	4.1	0.4	59.2
Germany govt	0.0	2.5	0.2	1.6	0.0	0.4	3.1	0.0	2.2	0.2	10.2
France govt	0.0	1.2	0.0	0.6	0.0	0.4	1.2	0.0	0.4	0.0	3.8
Supranational	0.0	0.8	0.3	0.1	0.0	0.1	0.7	0.1	0.2	0.1	2.4
GIIPS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Other sovereign	0.2	2.0	0.8	1.0	0.0	0.1	2.5	0.1	1.4	0.1	8.2
Corporate	0.2	2.1	0.8	0.4	0.0	0.6	0.0	1.9	0.2	0.0	6.2
Securitisation	1.7	2.6	2.4	0.0	0.0	0.2	0.0	0.0	0.2	0.0	7.1
Other	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total	2.6	13.5	5.2	4.3	1.0	3.4	57.5	2.6	9.1	0.8	100.0

This table exhibits a finer breakdown of the repo contracts. The numbers are in percentage points and indicate the percentage of notional value in each category. The data is double sorted by counterparty type (columns) and maturity, currency and collateral type in Panels A, B, and C respectively. The table is based on the data from the the six banks that report haircut and collateral information. Columns 1–10 refer to the following counterparty types:

1. Another reporting bank
2. Other banks
3. Broker-dealer
4. Hedge fund
5. MMF
6. Other asset managers
7. CCP
8. Insurance and pension
9. Central bank & govt
10. Other

Table 1.5: Haircuts on sovereign bond collateral

	A. REVR							
	UK		Germany		France		US	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
A1. Collateral maturity								
<10Y	31.91%	3.8%	76.53%	0.6%	77.49%	3.4%	84.61%	2.2%
>10Y	68.09%	9.5%	23.47%	3.1%	22.51%	4.8%	15.39%	4.6%
A2. Counterparty type								
Oth managers [a]	35.17%	12.6%	16.20%	0.0%	4.31%	5.6%	13.19%	1.2%
Other	64.83%	5.1%	83.80%	1.4%	95.69%	3.6%	86.81%	2.9%
	B. REPO							
	UK		Germany		France		US	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
B1. Collateral maturity								
<10Y	71.68%	4.8%	64.11%	2.6%	61.60%	2.0%	96.88%	1.3%
>10Y	28.32%	9.4%	35.89%	0.0%	38.40%	1.3%	3.12%	9.1%
B2. Counterparty type								
Government	61.7%	8.2%	48.8%	2.0%	0.8%	0.1%	2.4%	0.2%
Other	38.3%	4.5%	51.2%	1.4%	99.2%	2.2%	97.6%	1.8%

This table shows the breakdown of reverse repo (Panel A) and repo (Panel B) contracts on government bond collateral. The information is broken down by 1. collateral maturity, and 2. counterparty type. For each sovereign bond (e.g. UK government bond) column 1 shows the percentage of the notional amount against that collateral for the given category, and column 2 shows the corresponding average haircut for that category (the averages are weighted by the gross notional of the transactions). The sample only includes the six banks that provided data on haircuts and collateral.
[a] Other asset managers

Table 1.6: The extent of collateral reuse

		Collateral maturity				
Collateral type	Quality	<1Y	1-5Y	5-10Y	10Y+	Mean
		A. Value-weighted				
Government	High	50%	41%	57%	62%	52%
	Low	18%	10%	18%	28%	18%
Corporate	High	15%	18%	23%	11%	17%
	Low	29%	36%	27%	23%	28%
Securitisation	High	14%	16%	51%	7%	21%
	Low	26%	37%	21%	0%	24%
Mean		47%	39%	53%	58%	49%
		B. Non-value-weighted				
Government	High	19%	26%	35%	13%	22%
	Low	36%	20%	33%	51%	35%
Corporate	High	16%	17%	22%	15%	17%
	Low	27%	28%	31%	33%	29%
Securitisation	High	9%	14%	25%	5%	13%
	Low	15%	20%	19%	0%	17%
Mean		18%	21%	29%	15%	21%

This table shows what percentage of the collateral received by the banks in reverse repo transactions is reused in repos to borrow against. This information provides an indication of the the extent of collateral reuse. The results in Panel A are weighted by the value of transactions, and in Panel B there are equally weighted. The sample only includes the six banks that provided data on haircuts and collateral. High and low quality refer to investment-grade and non-investment-grade securities respectively.

Table 1.7: Description of the explanatory variables

Variable	Description
gbp	Dummy variable = 1 if transaction is in GBP.
eur	Dummy variable = 1 if transaction is in EUR.
jpy	Dummy variable = 1 if transaction is in JPY.
othercurrency	Dummy variable = 1 if transaction is not GBP, EUR or JPY.
notional	Notional of the transaction in million Pounds (log transformed).
maturity	Maturity of the transaction in years (log transformed).
collrating	Rating of the collateral: 20 is highest and 1 is lowest.
collmaturity	Maturity of the collateral in years (log transformed).
corpdebt	Dummy variable = 1 if collateral is corporate bond.
securitisation	Dummy variable = 1 if collateral is securitisation.
brokerdealers	Dummy variable = 1 if counterparty is broker-dealers.
hedgefund	Dummy variable = 1 if counterparty is hedge fund.
othermanager	Dummy variable = 1 if counterparty is other asset managers.
ccp	Dummy variable = 1 if counterparty is CCP.
insur&pension	Dummy variable = 1 if counterparty is insurance company or pension fund.
cb&govt	Dummy variable = 1 if counterparty is central bank or government.
other	Dummy variable = 1 if counterparty is other type.
cptysize	Size of the counterparty in million Pounds (log transformed).
cptyroa	RoA of the counterparty (log transformed).
cptyrating	Rating of the counterparty: 20 is highest and 1 is lowest.
cptyleverage	Leverage ratio of the counterparty (RWA over equity) (log transformed).
cptycds	CDS spread of the counterparty (log transformed).
cptycashratio	Cash ratio of the counterparty (cash over short-term debt) (log transformed).
nocptydata	Dummy variable = 1 there is no counterparty data.
cptycon	Concentration of the counterparty measured by the share of transactions with that counterparty in total: higher number indicates more concentration.
collcon	Concentration of the collateral measured by the share of transactions against that collateral in total: higher number indicates more concentration.
cpty&collrating	Interaction term between counterparty rating and collateral rating
ownsizelog	Size of the reporting bank in million Pounds (log transformed).
ownroa	RoA of the reporting bank (log transformed).
ownleverage	Leverage ratio of the reporting bank (RWA over equity) (log transformed).
owncds	CDS spread of the reporting bank (log transformed).
owncashratio	Cash ratio of the reporting bank (cash over short-term debt) (log transformed).
pcu	Principal component of the network centrality measures for unweighted network.
pcw	Principal component of the network centrality measures for weighted network.
bank/dealer	Dummy variable = 1 if counterparty is a bank or broker-dealer.

Table 1.8: Summary statistics for the sample excluding deals with CCPs

Variable	Obs	Mean	Std dev	Min	Max	Average [a]
			A. REVR			
Haircut	7397	5.94%	10.17%	0.00%	46.11%	5.89%
Notional	10435	6.25	0.86	3.44	8.32	7.55
Maturity	10435	0.06	0.11	0.00	1.39	0.14
Collateral maturity	7085	2.36	0.75	0.80	3.81	2.25
Collateral rating	5729	14.5	4.8	1.0	20.0	17.2
Ctpy size	6512	5.17	0.70	3.57	6.25	5.41
Ctpy RoA	6506	0.00	0.00	-0.01	0.02	0.00
Ctpy leverage	6469	1.69	0.23	1.09	2.40	1.78
Ctpy CDS	5593	0.01	0.01	0.01	0.04	0.02
Ctpy cash ratio	6428	-1.68	1.07	-3.40	1.47	-1.46
Ctpy rating	6495	14.6	1.4	3.0	20.0	14.4
			B. REPO			
Haircut	7386	2.37%	5.82%	0.00%	46.11%	6.24%
Notional	11896	6.18	0.79	3.44	8.32	7.70
Maturity	11905	0.05	0.18	0.00	1.39	0.20
Collateral maturity	8993	2.03	0.63	0.80	3.81	2.09
Collateral rating	8629	14.3	5.0	1.0	20.0	18.3
Ctpy size	8380	5.37	0.62	3.57	6.25	5.43
Ctpy RoA	8367	0.00	0.00	-0.01	0.02	0.00
Ctpy leverage	7300	1.74	0.25	1.09	2.40	1.79
Ctpy CDS	5908	0.02	0.01	0.01	0.04	0.02
Ctpy cash ratio	8092	-1.31	1.14	-3.40	1.47	-1.51
Ctpy rating	8445	15.2	2.0	3.0	20.0	14.8

The table shows the summary statistics of variables used in the regressions excluding the deals with CCPs, for repo and reverse repo transactions. The sample only includes the six banks that provided data on haircuts and collateral. Variables have been winsorised at 0.5% level. Rating scale is 1–20, with 20 being the highest rating.
[a] Average is weighted by the gross notional of transactions.

Table 1.9: Summary of the regression sample excluding deals with CCPs

Category	Subcategory	REVR	REPO
Currency	GBP	17.3%	7.9%
	USD	44.2%	42.3%
	EUR	34.1%	46.5%
	JPY	1.6%	0.8%
	Other	2.8%	2.5%
Counterparty type	Another reporting bank	2.1%	4.2%
	Other banks	40.9%	43.6%
	Broker-dealer	23.0%	23.0%
	Hedge fund	12.8%	10.6%
	Other asset managers	8.0%	9.4%
	Insurance and pension	6.1%	5.1%
	Central bank and govt	1.5%	3.2%
	Other	5.6%	0.7%
Collateral type	Sovereign	49.9%	36.1%
	Corporate debt	42.1%	54.0%
	Securitisation	7.6%	9.9%
	Other	0.5%	0.0%

The table presents breakdown of deals by currency, counterparty and collateral type, and collateral maturity, for the sample used in the regressions excluding the deals with CCPs. The sample only includes the six banks that provided data on haircuts and collateral. The percentages represent frequency of deals.

Table 1.10: Summary of the zero-haircut sample excluding deals with CCPs

Category	Subcategory	REVR	REPO
Currency	GBP	33.6%	6.3%
	USD	22.1%	40.0%
	EUR	40.5%	51.0%
	JPY	1.6%	0.9%
	Other	2.2%	1.9%
Counterparty type	Another reporting bank	4.3%	2.2%
	Other banks	53.4%	68.7%
	Broker-dealer	6.1%	9.5%
	Hedge fund	0.9%	0.0%
	Other asset managers	6.4%	16.0%
	Insurance and pension	11.6%	1.4%
	Central bank and govt	2.2%	1.6%
	Other	15.2%	0.5%
Collateral type	Sovereign	36.7%	44.2%
	Corporate debt	63.0%	43.9%
	Securitisation	0.3%	11.9%
	Other	0.0%	0.0%

The table presents breakdown of deals by currency, counterparty and collateral type, and collateral maturity, for the sample of deals with zero haircut, excluding the deals with CCPs. The sample only includes the six banks that provided data on haircuts and collateral. The percentages represent frequency of deals.

Table 1.11: Reverse repo Tobit regressions excluding CCPs

Category	Variable	(1)	(2)	(3)	(4)	(5)	(6)
Deal var	notional	0.005	0.003	0.006	0.010	0.007	0.007
	maturity	0.119*	0.125**	0.111*	0.101	0.114*	0.114*
Collateral var	collrating	-0.013**	-0.016**	-0.016**	-0.012**	-0.012**	-0.012**
	collmaturity	-0.003	0.001	0.001	-0.003	-0.002	-0.002
	corpdebt	-0.008	-0.011	-0.012	-0.009	-0.008	-0.008
	securitisation	0.040	0.022	0.022	0.039	0.040	0.040
Cpty type	brokerdealers	-0.028**	-0.015**	-0.014	-0.027**	-0.028**	-0.028**
	hedgefund	0.134**	0.093**	0.089**	0.133**	0.133**	0.133**
	othermanager	0.080**	0.048	0.042	0.076**	0.078**	0.078**
	insur&pension	0.038	0.016	0.012	0.035	0.037	0.037
	cb&govt	-0.010	-0.008	-0.008	-0.010	-0.011	-0.011
	other	0.043	0.012	0.008	0.041	0.043	0.043
Cpty var	cptysize		-0.031**	-0.030**			
	cptyroa		-3.331*	-3.249			
	cptyrating		-0.018**	-0.018**			
	cptyleverage		0.093**	0.089**			
	cptycds		0.503	0.217			
	cptycashratio		-0.002	-0.001			
	nocptydata		-0.058	-0.057			
Misc	cptycon		0.225**	0.224**			
	collcon		0.139	0.145			
	cpty&collrating		0.001**	0.001**			
Own var	ownsizelog			-0.104	-0.149**	-0.143*	-0.150*
	ownroa			-25.998**	-30.534**	-20.146*	-20.146*
	ownleverage			0.456	0.844	1.219**	1.063*
	owncds			-6.444	-13.869*	-11.510	-17.639**
	owncashratio			-0.142**	-0.191**	-0.215**	-0.263**
Network var	pcu					0.044**	
	pcw						0.045**
	Bank FE	Yes	Yes	No	No	No	No
	Currency FE	Yes	Yes	Yes	Yes	Yes	Yes
	Obs	4414	4414	4414	4414	4414	4414
	Pseudo R^2	2.89	2.97	2.97	2.89	2.90	2.90

The table shows Tobit regression results for reverse repos excluding deals with CCPs, where the Tobit model with truncation at zero is used. The dependent variable is haircut and explanatory variables are listed in the second column. The first column shows the category of explanatory variable. The columns that are labeled with numbers display regression coefficients for different explanatory variables. Standard errors (not reported) are clustered at reporting bank level. One and two stars denote 10% and 5% significance levels respectively.

Table 1.12: Reverse repo OLS regressions excluding CCPs

Category	Variable	(1)	(2)	(3)	(4)	(5)	(6)
Deal var	notional	-0.001	0.001	0.002	0.001	0.000	0.000
	maturity	0.077**	0.093**	0.086**	0.069*	0.071**	0.071**
Collateral var	collrating	-0.009**	-0.014**	-0.014**	-0.009**	-0.009**	-0.009**
	collmaturity	-0.002	0.002	0.003	-0.002	-0.002	-0.002
	corpdebt	-0.005	-0.006	-0.006	-0.005	-0.005	-0.005
	securitisation	0.040	0.028	0.027	0.039	0.040	0.040
Cpty type	brokerdealers	-0.019**	-0.013	-0.012	-0.019**	-0.019**	-0.019**
	hedgefund	0.135**	0.103**	0.100**	0.135**	0.134**	0.134**
	othermanager	0.063	0.049	0.045	0.061	0.061	0.061
	insur&pension	0.018	0.012	0.010	0.018	0.018	0.018
	cb&govt	0.020	0.028	0.026	0.019	0.019	0.019
	other	0.017	0.004	0.004	0.018	0.018	0.018
Cpty var	cptysize		-0.028**	-0.027**			
	cptyroa		-3.410	-3.135			
	cptyrating		-0.019**	-0.020**			
	cptyleverage		0.036	0.034			
	cptycds		-0.720	-0.963			
	cptycashratio		0.003	0.003			
	nocptydata		-0.187**	-0.194**			
Misc	cptycon		0.153**	0.157**			
	collcon		0.039	0.032			
	cpty&collrating		0.001**	0.001**			
Own var	ownsizelog			-0.085	-0.094	-0.093	-0.094
	ownroa			-22.611*	-22.570*	-20.952**	-20.952**
	ownleverage			0.657	0.790	0.855	0.830
	owncds			-3.993	-7.119	-6.775	-7.786
	owncashratio			-0.024*	-0.046**	-0.049**	-0.057
Network var	pcu					0.007	
	pcw						0.007
	Bank FE	Yes	Yes	No	No	No	No
	Currency FE	Yes	Yes	Yes	Yes	Yes	Yes
	Obs	4414	4414	4414	4414	4414	4414
	R^2	0.541	0.588	0.592	0.545	0.545	0.545

The table shows OLS regression results for reverse repos, excluding deals with CCPs. The dependent variable is haircut and explanatory variables are listed in the second column. The first column shows the category of explanatory variable. The columns that are labeled with numbers display regression coefficients for different explanatory variables. Standard errors (not reported) are clustered at reporting bank level. One and two stars denote 10% and 5% significance levels respectively.

Table 1.13: Reverse repo Logistic regressions excluding CCPs

Category	Variable	(1)	(2)	(3)	(4)	(5)	(6)
Deal var	notional	-0.016	0.000	0.017	0.003	0.005	0.005
	maturity	0.939**	1.086**	1.032**	0.874**	0.862**	0.862**
Collateral var	collrating	-0.103**	-0.151**	-0.148**	-0.101**	-0.101**	-0.101**
	collmaturity	0.001	0.043	0.049	0.007	0.007	0.007
	corpdebt	-0.021	-0.039	-0.036	-0.019	-0.020	-0.020
	securitisation	0.439	0.307	0.303	0.434	0.433	0.433
Cpty type	brokerdealers	-0.298**	-0.284*	-0.284	-0.297*	-0.297*	-0.297*
	hedgefund	1.431**	1.094**	1.066**	1.420**	1.422**	1.422**
	othermanager	0.668	0.501	0.465	0.646	0.645	0.645
	insur&pension	0.202	0.107	0.091	0.198	0.198	0.198
	cb&govt	0.215	0.256	0.236	0.200	0.199	0.199
	other	0.131	-0.030	-0.021	0.149	0.148	0.148
Cpty var	cptysize		-0.359**	-0.343**			
	cptyroa		-26.874	-23.195			
	cptyrating		-0.205**	-0.214**			
	cptyleverage		0.437	0.393			
	cptycds		1.183	-2.074			
	cptycashratio		0.029	0.029			
	nocptydata		-2.234**	-2.377**			
Misc	cptycon		1.748**	1.870**			
	collcon		0.200	0.062			
	cpty&collrating		0.008**	0.008**			
Own var	ownsizelog			-1.433	-1.556	-1.561	-1.554
	ownroa			-282.835	-286.001	-295.187*	-295.187*
	ownleverage			9.285	10.881	10.511	10.657
	owncds			-58.087	-93.594	-95.545	-89.807
	owncashratio			-0.775**	-1.024**	-1.003**	-0.957**
Network var	pcu					-0.041	
	pcw						-0.042
	Bank FE	Yes	Yes	No	No	No	No
	Currency FE	Yes	Yes	Yes	Yes	Yes	Yes
	Obs	4414	4414	4414	4414	4414	4414
	R^2	0.709	0.739	0.744	0.714	0.714	0.714

The table shows Logistic regression results for reverse repos, excluding deals with CCPs. The dependent variable is logit-transformed haircut and explanatory variables are listed in the second column. The first column shows the category of explanatory variable. The columns that are labeled with numbers display regression coefficients for different explanatory variables. Standard errors (not reported) are clustered at reporting bank level. One and two stars denote 10% and 5% significance levels respectively.

Table 1.14: Repo Tobit regressions excluding CCPs

Category	Variable	(1)	(2)	(3)	(4)	(5)	(6)
Deal var	notional	-0.004**	-0.004**	-0.004**	-0.003**	-0.003**	-0.003**
	maturity	0.057**	0.034**	0.037**	0.060**	0.059**	0.059**
Collateral var	collrating	-0.004**	-0.004**	-0.004**	-0.004**	-0.004**	-0.004**
	collmaturity	0.002	0.003	0.003	0.002	0.002	0.002
	corpdebt	0.001	0.000	-0.001	0.000	0.001	0.001
	securitisation	0.004	0.001	0.001	0.003	0.003	0.003
Cpty type	brokerdealers	0.017**	0.016**	0.014**	0.015**	0.015**	0.015**
	hedgefund	0.009**	-0.022**	-0.018**	0.011**	0.010**	0.010**
	othermanager	-0.028	-0.027	-0.028	-0.029	-0.028	-0.028
	insur&pension	0.110**	0.093**	0.094**	0.111**	0.112**	0.112**
	cb&govt	0.037**	0.025**	0.023**	0.034**	0.035**	0.035**
	other	-0.011	-0.015**	-0.015**	-0.012	-0.012	-0.012
Cpty var	cptysize		0.014	0.015			
	cptyroa		2.420	2.513			
	cptyrating		-0.005**	-0.006**			
	cptyleverage		0.076**	0.074**			
	cptycds		-2.609**	-2.599**			
	cptycashratio		0.007**	0.007**			
	nocptydata		0.130	0.125			
Misc	cptycon		0.328	0.372			
	collcon		0.196**	0.208**			
Own var	ownsizelog			-0.002	0.011	0.018	0.020
	ownroa			7.587**	9.811**	8.815*	8.815*
	ownleverage			-0.009	-0.074	-0.241*	-0.198
	owncds			9.398**	9.245**	9.529**	11.239**
	owncashratio			0.007	0.001	0.008	0.022**
Network var	pcu					-0.012**	
	pcw						-0.012**
	Bank FE	Yes	Yes	No	No	No	No
	Currency FE	Yes	Yes	Yes	Yes	Yes	Yes
	Obs	5289	5289	5289	5289	5289	5289
	Pseudo R^2	-1.00	-0.91	-0.91	-1.00	-1.01	-1.01

The table shows Tobit regression results for reverse repos excluding deals with CCPs, where the Tobit model with truncation at zero is used. The dependent variable is haircut and explanatory variables are listed in the second column. The first column shows the category of explanatory variable. The columns that are labeled with numbers display regression coefficients for different explanatory variables. Standard errors (not reported) are clustered at reporting bank level. One and two stars denote 10% and 5% significance levels respectively.

Table 1.15: Repo OLS regressions excluding CCPs

Category	Variable	(1)	(2)	(3)	(4)	(5)	(6)
Deal var	notional	0.005*	0.004*	0.004*	0.004*	0.004*	0.004*
	maturity	0.049**	0.031**	0.032**	0.049**	0.048**	0.048**
Collateral var	collrating	-0.002**	-0.002**	-0.002**	-0.002**	-0.002**	-0.002**
	collmaturity	0.002	0.003	0.003	0.003	0.003	0.003
	corpdebt	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003
	securitisation	-0.001	-0.001	0.001	0.001	0.001	0.001
Cpty type	brokerdealers	-0.001	-0.001	0.000	0.001	0.001	0.001
	hedgefund	-0.001	-0.002	-0.003	-0.001	-0.002	-0.002
	othermanager	-0.001	-0.002	-0.001	0.000	0.000	0.000
	insur&pension	0.077**	0.072**	0.071**	0.075**	0.076**	0.076**
	cb&govt	0.000	0.000	0.002	0.002	0.003	0.003
	other	0.006	0.003	0.003	0.007	0.007	0.007
Cpty var	cptysize		0.011**	0.012**			
	cptyroa		0.568	0.683			
	cptyrating		0.000	0.000			
	cptyleverage		-0.010	-0.009			
	cptycds		-0.274	-0.441			
	cptycashratio		0.001	0.001			
	nocptydata		0.051*	0.052*			
Misc	cptycon		0.159	0.169			
	collcon		0.056	0.061			
Own var	ownsizelog			-0.033**	-0.042**	-0.036**	-0.034**
	ownroa			-6.719**	-7.846**	-8.278**	-8.278**
	ownleverage			0.287**	0.337**	0.213**	0.243**
	owncds			0.284	-1.041	-0.710	0.461
	owncashratio			-0.024**	-0.030**	-0.025**	-0.016**
Network var	pcu					-0.008**	
	pcw						-0.009**
	Bank FE	Yes	Yes	No	No	No	No
	Currency FE	Yes	Yes	Yes	Yes	Yes	Yes
	Obs	5289	5289	5289	5289	5289	5289
	R^2	0.372	0.344	0.347	0.377	0.377	0.377

The table shows OLS regression results for repos, excluding deals with CCPs. The dependent variable is haircut and explanatory variables are listed in the second column. The first column shows the category of explanatory variable. The columns that are labeled with numbers display regression coefficients for different explanatory variables. Standard errors (not reported) are clustered at reporting bank level. One and two stars denote 10% and 5% significance levels respectively.

Table 1.16: Repo Logistic regressions excluding CCPs

Category	Variable	(1)	(2)	(3)	(4)	(5)	(6)
Deal var	notional	0.050	0.037	0.034	0.047	0.047	0.047
	maturity	0.845**	0.575**	0.606**	0.864**	0.831**	0.831**
Collateral var	collrating	-0.037**	-0.039**	-0.040**	-0.038**	-0.037**	-0.037**
	collmaturity	0.064	0.073*	0.075*	0.067	0.067	0.067
	corpdebt	0.018	0.020	0.019	0.015	0.019	0.019
	securitisation	0.082**	0.087*	0.101*	0.096**	0.097**	0.097**
Cpty type	brokerdealers	0.072	0.052	0.060*	0.088*	0.090*	0.090*
	hedgefund	0.164**	0.202**	0.222**	0.170**	0.157**	0.157**
	othermanager	-0.082	-0.103	-0.095	-0.072	-0.069	-0.069
	insur&pension	1.294**	1.223**	1.206**	1.272**	1.283**	1.283**
	cb&govt	0.067	0.043	0.046	0.073	0.088	0.088
	other	0.036	-0.016	-0.006	0.045	0.043	0.043
Cpty var	cptysize		0.115**	0.138**			
	cptyroa		10.034	11.950			
	cptyrating		-0.011	-0.021			
	cptyleverage		0.113	0.118			
	cptycds		-6.750*	-8.536**			
	cptycashratio		0.010	0.009			
	nocptydata		0.708**	0.671**			
Misc	cptycon		3.851**	4.302**			
	collcon		0.377	0.418			
Own var	ownsizelog			-0.662**	-0.828**	-0.593**	-0.537**
	ownroa			-55.784**	-66.551**	-82.777**	-82.777**
	ownleverage			4.490**	5.072**	0.409	1.528**
	owncds			27.146	11.941	24.392**	68.411**
	owncashratio			-0.549**	-0.627**	-0.440**	-0.090**
Network var	pcu					-0.317**	
	pcw						-0.322**
	Bank FE	Yes	Yes	No	No	No	No
	Currency FE	Yes	Yes	Yes	Yes	Yes	Yes
	Obs	5289	5289	5289	5289	5289	5289
	R^2	0.545	0.534	0.534	0.545	0.547	0.547

The table shows Logistic regression results for repos, excluding deals with CCPs. The dependent variable is logit-transformed haircut and explanatory variables are listed in the second column. The first column shows the category of explanatory variable. The columns that are labeled with numbers display regression coefficients for different explanatory variables. Standard errors (not reported) are clustered at reporting bank level. One and two stars denote 10% and 5% significance levels respectively.

Table 1.17: Reverse repo Tobit regressions excluding CCPs, with bank/dealer dummy

Category	Variable	(1)	(2)	(3)	(4)	(5)	(6)
Deal var	notional	0.002	0.001	0.005	0.007	0.004	0.004
	maturity	0.129*	0.135**	0.117*	0.108	0.124	0.124
Collateral var	collrating	-0.015**	-0.018**	-0.018**	-0.015**	-0.015**	-0.015**
	collmaturity	-0.002	0.002	0.002	-0.003	-0.002	-0.002
	corpdebt	-0.015	-0.017	-0.018	-0.016	-0.014	-0.014
	securitisation	0.022	0.005	0.004	0.020	0.022	0.022
Cpty type	bank/dealer	-0.084**	-0.047	-0.042	-0.082**	-0.083**	-0.083**
Cpty var	cptysize		-0.024**	-0.024**			
	cptyroa		-3.482	-3.391			
	cptyrating		-0.022**	-0.021**			
	cptyleverage		0.084**	0.081**			
	cptycds		0.303	-0.002			
	cptycashratio		-0.003	-0.002			
	nocptydata		-0.056	-0.055			
Misc	cptycon		0.260**	0.255**			
	collcon		0.101	0.110			
	cpty&collrating		0.001**	0.001**			
Own var	ownsizelog			-0.110	-0.158*	-0.149*	-0.158*
	ownroa			-28.536**	-33.163**	-20.494*	-20.494*
	ownleverage			0.466	0.878	1.330**	1.141**
	owncds			-7.862	-16.135**	-13.179*	-20.594**
	owncashratio			-0.156**	-0.216**	-0.243**	-0.302**
Network var	pcu					0.053**	
	pcw						0.054**
	Bank FE	Yes	Yes	No	No	No	No
	Currency FE	Yes	Yes	Yes	Yes	Yes	Yes
	Obs	4414	4414	4414	4414	4414	4414
	Pseudo R^2	2.89	2.97	2.97	2.89	2.90	2.90

The table shows Tobit regression results for reverse repos excluding deals with CCPs, where there is only one dummy variable for counterparty type which takes value of 1 if the counterparty is a bank or broker-dealer, and 0 otherwise. The Tobit model with truncation at zero is used. The dependent variable is haircut and explanatory variables are listed in the second column. The first column shows the category of explanatory variable. The columns that are labeled with numbers display regression coefficients for different explanatory variables. Standard errors (not reported) are clustered at reporting bank level. One and two stars denote 10% and 5% significance levels respectively.

Table 1.18: Reverse repo OLS regressions excluding CCPs, with bank/dealer dummy

Category	Variable	(1)	(2)	(3)	(4)	(5)	(6)
Deal var	notional	-0.005	0.000	0.002	-0.003	-0.003	-0.003
	maturity	0.087**	0.105**	0.094**	0.075*	0.080*	0.080*
Collateral var	collrating	-0.011**	-0.016**	-0.016**	-0.011**	-0.011**	-0.011**
	collmaturity	-0.001	0.004	0.004	-0.001	-0.001	-0.001
	corpdebt	-0.014	-0.014	-0.014	-0.014	-0.014	-0.014
	securitisation	0.016	0.008	0.007	0.016	0.016	0.016
Cpty type	bank/dealer	-0.050	-0.034	-0.032	-0.049	-0.049	-0.049
Cpty var	cptysize		-0.016	-0.015			
	cptyroa		-3.727*	-3.408			
	cptyrating		-0.022**	-0.022**			
	cptyleverage		0.013	0.011			
	cptycds		-1.222**	-1.495**			
	cptycashratio		0.003**	0.003**			
	nocptydata		-0.167**	-0.174**			
Misc	cptycon		0.187**	0.188**			
	collcon		-0.057	-0.060			
	cpty&collrating		0.001**	0.001**			
Own var	ownsizelog			-0.092	-0.094	-0.093	-0.095
	ownroa			-27.666**	-27.251**	-23.857**	-23.857**
	ownleverage			0.721	0.835	0.972	0.918
	owncds			-6.639	-10.239*	-9.500*	-11.616
	owncashratio			-0.037**	-0.065**	-0.073**	-0.090*
Network var	pcu					0.015	
	pcw						0.015
	Bank FE	Yes	Yes	No	No	No	No
	Currency FE	Yes	Yes	Yes	Yes	Yes	Yes
	Obs	4414	4414	4414	4414	4414	4414
	R^2	0.541	0.588	0.592	0.545	0.545	0.545

The table shows OLS regression results for reverse repos excluding deals with CCPs, where there is only one dummy variable for counterparty type which takes value of 1 if the counterparty is a bank or broker-dealer, and 0 otherwise. The dependent variable is haircut and explanatory variables are listed in the second column. The first column shows the category of explanatory variable. The columns that are labeled with numbers display regression coefficients for different explanatory variables. Standard errors (not reported) are clustered at reporting bank level. One and two stars denote 10% and 5% significance levels respectively.

Table 1.19: Reverse repo Logistic regressions excluding CCPs, with bank/dealer dummy

Category	Variable	(1)	(2)	(3)	(4)	(5)	(6)
Deal var	notional	-0.059	-0.011	0.012	-0.032	-0.035	-0.035
	maturity	1.074**	1.251**	1.154**	0.965**	0.977**	0.977**
Collateral var	collrating	-0.125**	-0.179**	-0.176**	-0.123**	-0.123**	-0.123**
	collmaturity	0.015	0.063	0.068	0.020	0.021	0.021
	corpdebt	-0.125	-0.129	-0.124	-0.123	-0.122	-0.122
	securitisation	0.186	0.083	0.085	0.183	0.185	0.185
Cpty type	bank/dealer	-0.523	-0.323	-0.310	-0.520	-0.520	-0.520
Cpty var	cptysize		-0.236	-0.221			
	cptyroa		-35.787	-31.754			
	cptyrating		-0.213**	-0.222**			
	cptyleverage		0.214	0.179			
	cptycds		-5.629	-9.232			
	cptycashratio		0.012	0.012			
	nocptydata		-1.669**	-1.817**			
Misc	cptycon		2.324**	2.406**			
	collcon		-0.894	-0.994			
	cpty&collrating		0.010**	0.010**			
Own var	ownsizelog			-1.492	-1.573	-1.568	-1.575
	ownroa			-339.800*	-338.241*	-329.157*	-329.157*
	ownleverage			9.886	11.486	11.851	11.707
	owncds			-84.317	-128.207	-126.229	-131.892
	owncashratio			-0.892**	-1.234**	-1.255**	-1.300**
Network var	pcu					0.041	
	pcw						0.041
	Bank FE	Yes	Yes	No	No	No	No
	Currency FE	Yes	Yes	Yes	Yes	Yes	Yes
	Obs	4414	4414	4414	4414	4414	4414
	R^2	0.666	0.712	0.718	0.672	0.672	0.672

The table shows Logistic regression results for reverse repos excluding deals with CCPs, where there is only one dummy variable for counterparty type which takes value of 1 if the counterparty is a bank or broker-dealer, and 0 otherwise. The dependent variable is logit-transformed haircut and explanatory variables are listed in the second column. The first column shows the category of explanatory variable. The columns that are labeled with numbers display regression coefficients for different explanatory variables. Standard errors (not reported) are clustered at reporting bank level. One and two stars denote 10% and 5% significance levels respectively.

Table 1.20: Repo Tobit regressions excluding CCPs, with bank/dealer dummy

Category	Variable	(1)	(2)	(3)	(4)	(5)	(6)
Deal var	notional	-0.005**	-0.005**	-0.005**	-0.005**	-0.005**	-0.005**
	maturity	0.107**	0.067**	0.069**	0.109**	0.109**	0.109**
Collateral var	collrating	-0.006**	-0.006**	-0.006**	-0.007**	-0.007**	-0.007**
	collmaturity	-0.006	-0.003	-0.003	-0.007	-0.007	-0.007
	corpdebt	-0.001	-0.001	-0.002	-0.002	-0.002	-0.002
	securitisation	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
Cpty type	bank/dealer	-0.030**	-0.022**	-0.022**	-0.030**	-0.030**	-0.030**
Cpty var	cptysize		0.005	0.005			
	cptyroa		1.980	2.030			
	cptyrating		-0.015**	-0.016**			
	cptyleverage		0.094**	0.092**			
	cptycds		-3.704**	-3.701**			
	cptycashratio		0.007	0.008			
	nocptydata		-0.045	-0.047			
Misc	cptycon		0.411*	0.434*			
	collcon		0.221**	0.230**			
Own var	ownsizelog			-0.015	-0.001	-0.001	-0.001
	ownroa			3.712*	5.419	5.444	5.444
	ownleverage			0.060	-0.006	-0.002	-0.003
	owncds			7.043**	6.485**	6.477**	6.431**
	owncashratio			-0.012*	-0.020**	-0.020**	-0.020**
Network var	pcu					0.000	
	pcw						0.000
	Bank FE	Yes	Yes	No	No	No	No
	Currency FE	Yes	Yes	Yes	Yes	Yes	Yes
	Obs	5289	5289	5289	5289	5289	5289
	Pseudo R^2	-1.00	-0.91	-0.91	-1.00	-1.01	-1.01

The table shows Tobit regression results for repos excluding deals with CCPs, where there is only one dummy variable for counterparty type which takes value of 1 if the counterparty is a bank or broker-dealer, and 0 otherwise. The Tobit model with truncation at zero is used. The dependent variable is haircut and explanatory variables are listed in the second column. The first column shows the category of explanatory variable. The columns that are labeled with numbers display regression coefficients for different explanatory variables. Standard errors (not reported) are clustered at reporting bank level. One and two stars denote 10% and 5% significance levels respectively.

Table 1.21: Repo OLS regressions excluding CCPs, with bank/dealer dummy

Category	Variable	(1)	(2)	(3)	(4)	(5)	(6)
Deal var	notional	0.004*	0.004*	0.003*	0.004*	0.004*	0.004*
	maturity	0.083**	0.055**	0.054**	0.080**	0.080**	0.080**
Collateral var	collrating	-0.004**	-0.004**	-0.004**	-0.004**	-0.004**	-0.004**
	collmaturity	-0.001	0.001	0.001	0.000	0.000	0.000
	corpdebt	-0.005	-0.004	-0.004	-0.005	-0.005	-0.005
	securitisation	0.001	0.002	0.004	0.003	0.003	0.003
Cpty type	bank/dealer	-0.016**	-0.014**	-0.014**	-0.016**	-0.016**	-0.016**
Cpty var	cptysize		0.009**	0.011**			
	cptyroa		0.577	0.740			
	cptyrating		-0.003	-0.004			
	cptyleverage		-0.012**	-0.010*			
	cptycds		-0.652**	-0.870*			
	cptycashratio		0.002	0.001			
	nocptydata		-0.019	-0.015			
Misc	cptycon		0.160	0.172			
	collcon		0.077**	0.078**			
Own var	ownsizelog			-0.040**	-0.048**	-0.046**	-0.045**
	ownroa			-9.656**	-11.187**	-11.339**	-11.339**
	ownleverage			0.349**	0.406**	0.362**	0.372**
	owncds			-1.539	-3.188**	-3.068**	-2.649**
	owncashratio			-0.036**	-0.042**	-0.040**	-0.037**
Network var	pcu					-0.003	
	pcw						-0.003
	Bank FE	Yes	Yes	No	No	No	No
	Currency FE	Yes	Yes	Yes	Yes	Yes	Yes
	Obs	5289	5289	5289	5289	5289	5289
	R^2	0.372	0.344	0.347	0.377	0.377	0.377

The table shows OLS regression results for repos excluding deals with CCPs, where there is only one dummy variable for counterparty type which takes value of 1 if the counterparty is a bank or broker-dealer, and 0 otherwise. The dependent variable is haircut and explanatory variables are listed in the second column. The first column shows the category of explanatory variable. The columns that are labeled with numbers display regression coefficients for different explanatory variables. Standard errors (not reported) are clustered at reporting bank level. One and two stars denote 10% and 5% significance levels respectively.

Table 1.22: Repo Logistic regressions excluding CCPs, with bank/dealer dummy

Category	Variable	(1)	(2)	(3)	(4)	(5)	(6)
Deal var	notional	0.040	0.035	0.029	0.036	0.036	0.036
	maturity	1.424**	0.975**	0.985**	1.408**	1.389**	1.389**
Collateral var	collrating	-0.067**	-0.063**	-0.063**	-0.066**	-0.066**	-0.066**
	collmaturity	0.001	0.024	0.027	0.006	0.005	0.005
	corpdebt	-0.010	0.003	0.005	-0.010	-0.007	-0.007
	securitisation	0.094*	0.121**	0.139**	0.112*	0.111*	0.111*
Cpty type	bank/dealer	-0.247**	-0.223**	-0.225**	-0.248**	-0.251**	-0.251**
Cpty var	cptysize		0.071*	0.102**			
	cptyroa		10.765	13.152			
	cptyrating		-0.082**	-0.093**			
	cptyleverage		0.084*	0.102*			
	cptycds		-14.651**	-17.250**			
	cptycashratio		0.016	0.008			
	nocptydata		-0.678	-0.661			
Misc	cptycon		3.950**	4.323**			
	collcon		1.522**	1.623**			
Own var	ownsizelog			-0.778**	-0.922**	-0.762**	-0.724**
	ownroa			-103.133**	-117.016**	-127.741**	-127.741**
	ownleverage			5.385**	5.918**	2.763**	3.517**
	owncds			-0.823	-19.907	-11.416	18.257
	owncashratio			-0.734**	-0.814**	-0.688**	-0.452**
Network var	pcu					-0.214**	
	pcw						-0.217**
	Bank FE	Yes	Yes	No	No	No	No
	Currency FE	Yes	Yes	Yes	Yes	Yes	Yes
	Obs	5289	5289	5289	5289	5289	5289
	R^2	0.462	0.458	0.461	0.466	0.467	0.467

The table shows Logistic regression results for repos excluding deals with CCPs, where there is only one dummy variable for counterparty type which takes value of 1 if the counterparty is a bank or broker-dealer, and 0 otherwise. The dependent variable is logit-transformed haircut and explanatory variables are listed in the second column. The first column shows the category of explanatory variable. The columns that are labeled with numbers display regression coefficients for different explanatory variables. Standard errors (not reported) are clustered at reporting bank level. One and two stars denote 10% and 5% significance levels respectively.

Table 1.23: Centrality measures summary

Network type	Measure	Mean
Unweighted	in degree	6.60E+01
	out degree	6.70E+01
	eigenvector centrality	-2.23E-01
	betweenness	1.57E+04
	closeness out	1.87E-01
	closeness in	4.81E-02
	kcore in	3.67E+00
	kcore out	4.17E+00
	clustering coefficient	4.12E-02
Weighted	in degree (trade number)	1.51E+02
	out degree (trade number)	1.93E+03
	in degree (value)	4.09E+09
	out degree (value)	3.86E+10
	eigenvector centrality (trade number)	-2.68E-01
	eigenvector centrality (value)	-2.40E-01

Chapter 2

Fund Managers and Misallocation of Capital

Abstract

I study the real effects of benchmarking in the professional fund management industry. Stocks of the productive sector are traded in a competitive equity market. Investors delegate their portfolio decisions to managers whose performance is benchmarked against an aggregate stock market index. Benchmarking affects market prices and returns, which consequently changes firms' investment behaviour. Managers hedge themselves by tilting their portfolio toward the index which increases the demand and price of the stocks that feature prominently in the index. Higher prices lead to even more investment by larger firms, generating a positive feedback loop. In equilibrium there is an inefficient shift towards extreme states in which big sectors dominate the economy. In presence of benchmarking, index inclusion is preceded by rise in firm's investment rate relative to its capital stock, which drops by a smaller degree after the inclusion.

2.1 Introduction

An essential service that the financial sector provides to foster economic growth is to reallocate capital to the highest value use (Rajan and Zingales, 1998). Absent any frictions, the market prices should signal the most efficient use of resources, which leads to highest growth. This paper presents a model in which distortions in the capital market hinder this crucial role of the financial sector and lead to misallocation of resources. The delegation of portfolio allocation to professional fund managers and their incentives play a key role in the misallocation.

An overwhelming and increasing share of the money invested in financial markets is managed professionally. Individuals held 47.9% of US equities in 1980 and only 21.5% in 2007 (French, 2008). Not only asset managers matter in financial markets, they are in fact the marginal investors (He et al., 2016).

Asset managers' performance is measured relative to benchmarks, which affects their incentives in investment and ultimately affects market prices. In this paper I study the interaction between fund managers' behaviour in equity markets and firms' investment strategy. Unlike prior literature that only looks at the effect of delegated portfolio management on prices, I focus on the implications for the real economy. In my model firms' investment decisions that lead to their growth is explicitly modelled, and there is a feedback loop between the financial market and the real economy. Distortions introduced by benchmarking in the equity market can have real effects.

The model, presented in Section 2.2, is as follows. There is a continuous-time infinite-horizon economy with multiple sectors represented by firms. These firms have risky productive capital and produce intermediate goods which are then aggregated to consumption good. The firms are equity financed, they invest part of their earnings to grow and the rest to pay dividends to their shareholders. Shares of the firms are traded in a competitive stock market. Households who are ultimate claimholders cannot invest directly in financial assets and need to entrust their money with fund managers to invest on their behalf. The behaviour of fund managers is modelled in a reduced-form way to capture the documented phenomena in asset management industry. Fund managers care about their reputation which is driven by how their investments perform relative to a benchmark index.

Absent benchmarking, there is decreasing returns to scale to each individual sector in the economy. As a firm grows bigger compared to other firms, its returns diminish relative to its size. Benchmarking introduces a positive feedback between firms' size and their investment levels. Since managers care about their performance relative to a benchmark index, they hedge themselves by tilting their portfolio toward the index. That increases the demand for stocks of the relatively large firms that feature prominently in the index, resulting in elevated price levels for these stocks. Firms' investment strategy is driven by the neoclassical investment theory, so higher valuations in the stock markets leads to increased levels of investment which reinforces the firms growth, making it feature even more prominently in the benchmark, which leads to higher prices and further investment and so on. Moreover, the composition of the index may differ from the actual market portfolio. To the extent that the benchmark and market portfolios are different, individual stocks are disproportionately affected by benchmarking.

Introducing benchmarking has two effects on the probability distribution of the states of the world. Firstly, the distribution of the relative size of the firms has fatter tails with benchmarking, so the economy is more likely to end up in extreme cases where one sector dominates. Secondly, different firms can be disproportionately affected by benchmarking, for example if a firm is over-represented in the benchmark, the distribution shifts towards the states in which it dominates the economy.

Finally I show that benchmarking generates an index inclusion effect on market valuation and investment strategy of firms. In the run-up to index inclusion, market valuation relative to capital stock rises substantially. Investment rate also exhibits a similar pattern that will revert back to downward trend after inclusion. Absent benchmarking this phenomenon is not observed. I provide suggestive empirical evidence of the effect of index inclusion on investment rates.

My paper is related to the literature in asset pricing theory that studies the effects of delegated portfolio management in financial markets. Most of this literature focuses on the the impact of asset managers on equilibrium prices. A group of papers study managers contractual incentives in detail. Buffa et al. (2015) examine the joint determination of fund managers' contracts and equilibrium asset prices, and show how benchmarking can be part of optimal contracts in presence of agency frictions. In their setup a negative relationship between risk and return arises and benchmarking biases the aggregate market upwards. Basak and Pavlova (2013) include benchmark directly into institutional investors preferences, and show that demand by these investors raises prices of the assets in the benchmark and makes them more volatile and correlated with each other. Also Cuoco and Kaniel (2011) analyse the asset pricing implications of contracts linking the compensation of fund managers to the performance relative to a benchmark portfolio. They show that symmetric fees have positive effect on the prices of assets included in the benchmark, while asymmetric fees have more complex effects that fluctuate stochastically. Other papers in this literature focus on managers' reputation concerns (e.g. Dasgupta et al. (2011) and Guerrieri and Kondor (2012)) and on fund flows (e.g. Vayanos and Woolley (2013) and He and Krishnamurthy (2012)).

My work is also inspired by the literature in economic growth that examines the effects of capital misallocation. For example Banerjee and Duflo (2005), Jeong and Townsend (2007) and Hsieh and Klenow (2009), among others, argue that the misallocation of resources in poor countries explain a large part of the total factor productivity gap between rich and poor countries. This argument is substantiated by evidence documenting significant dispersion in the marginal products of capital in developing countries. This literature focuses mainly on the role of frictions in credit market in misallocation of capital. Firms face credit constraints, which are heterogeneous across firms. They cannot borrow as much as they like to rent capital. The credit frictions result in misallocation of capital across firms and sectors and create inefficiencies. My analysis on the other hand, explores the equity market and how frictions in this market can distort allocation of resources in the economy.

My paper is also related to the theoretical literature that studies the real effects of financial markets that stem from the informational role of market prices. The key insight of this literature goes back to Hayek (1945), who asserts that market prices provide an important source of information for decision makers. Market prices aggregate dispersed information and therefore real decision makers learn from this information and use it to guide their decision, in turn affecting prices. A recent example in this literature is Goldstein et al. (2013) who study a model in which investors learn from the price of a firm's security and use it to determine how much capital to provide for new investment, and show that this feedback effect gives rise to trading frenzies. See Bond et al. (2012) for a comprehensive review of this literature. Unlike the models in this literature, in my paper prices do not play informational role as there is no information asymmetry. However prices affect and guide the investment decisions through the neoclassical q-theory channel.

The rest of the paper is organised as follows. In Section 2.2, I describe the model. Section 2.3 describes the solution methodology. Section 2.4 illustrates the equilibrium results and Section 2.5 concludes. Additional results, details of the derivations, complete solution and numerical method are provided in the Appendices.

2.2 The Model

The model consists of firms, households and fund managers. Time is continuous and runs from zero to infinity. There are N intermediate goods and a perishable consumption good from which agents obtain utility and is taken as a numeraire. The components of the models are described with more detail in the subsequent sections.

2.2.1 Firms

There are N firms in the economy, indexed by $j = 1, \dots, N$, that produce N differentiated intermediate goods. We can think of these firms as entities representing various sectors in the economy producing different goods. I use the words "firm" and "sector" interchangeably in this article to emphasise the point that these firms represent different industries in the economy. Since the nature and output of these industries are different, we can expect their means of production to be different as well. The productive capital used in one industry cannot be directly utilised in another industry. Hence, I assume that capital is specific to sectors. The firm-specificity of capital is introduced by inserting technological illiquidity to the model. Firms can adjust their capital holdings by investment or disinvestment, but adjustment is costly.

Each firm owns its productive capital, which is firm-specific, i.e. the capital stock of firm j, at time $t \in [0, \infty)$, denoted by k_t^j, can only be used to produce intermediate good j, at rate

$$y_t^j = a^j k_t^j,$$

per unit of time, where a^j is the productivity of firm j.

The intermediate-good producers sell their products to a final-good producer. The final-good producer transforms the intermediate goods into consumption good (or simply the output), and sells its products to consumers. I assume that the consumption good is produced by a representative firm who behaves competitively. It has a production function which exhibits constant elasticity of substitution (CES), so consumption good is produced at rate

$$y_t = \left(\sum_{j=1}^{N} \left(y_t^j \right)^{\frac{s-1}{s}} \right)^{\frac{s}{s-1}}, \tag{2.1}$$

per unit of time, where s is the elasticity of substitution.

Parameter s in (2.1) captures the degree of substitutability of intermediate goods. For $s = \infty$ they are perfect substitutes, and for $s = 0$ there is no substitutability. I always assume that s has an intermediate value, so that although there is some degree of substitutability, goods are not perfect substitute. If the productivity of different sectors are equal, the upshot of imperfect substitutability is that for a given total of productive capital, the most productive allocation is the one with capital equally allocated to each sector.

I take the consumption good as numeraire and set its price to one per unit. All other prices and values are henceforth expressed in terms of the consumption good. Since the consumption good market is competitive, the final-good producer should earn zero profit in equilibrium. As a result, we can derive the prices of intermediate goods in terms of consumption good as

$$p_t^j = \left(\frac{y_t}{y_t^j} \right)^{\frac{1}{s}}.$$

The focus of this paper is on the intermediate-good producers rather than the final-good producer. They own their means of production, so they can use their revenue to finance investment and also to pay dividend to their owners. On the other hand the final-good producer earns zero profit and can conceptually be thought of as a machine that converts inputs to outputs. Therefore, I refer to the intermediate-good producers simply as firms.

Firms can invest to accumulate capital, with negative levels of investment tantamount to disinvestment. I assume that capital accumulation follows the Ito process

$$\frac{dk_t^j}{k_t^j} = \mu_t^j dt + \sigma^j dZ_t^j, \tag{2.2}$$

where dZ_t^j is an exogenous firm-specific Brownian shock and σ^j is the exposure to this shock, which is constant by assumption. The expected growth rate of capital μ_t^j depends on the investment decision of firm j. If firm j invests at rate i_t^j per unit of capital (i.e. $i_t^j k_t^j$ is the total investment rate in terms of the final good), its capital stock grows at the expected rate

$$\mu_t^j \equiv i_t^j - \Phi\left(i_t^j\right) - \delta^j, \tag{2.3}$$

where δ^j is the depreciation rate and $\Phi(.)$ is an investment cost function. The function $\Phi(.)$ exhibits convex adjustment costs, and satisfies $\Phi(0) = 0$, $\Phi' > 0$, and $\Phi'' > 0$. The existence of convex adjustment costs reflects technological illiquidity, i.e. the cost due to converting the output to new capital and vice versa.

The technological illiquidity is related to the firm-specificity of capital. If investment technology was fully liquid (i.e. $\Phi(i) = 0, \forall i$), firms could adjust their level of capital without any cost, which could result in infinite investment or disinvestment rates. Adjustment costs ensure that investment rates are always finite, and therefore each firm's capital stock only changes smoothly across time.

The Brownian shocks dZ_t^j, $j = 1, \ldots, N$, are independent across firms. If we think of k_t^j as efficiency units rather than physical units of capital, we can interpret dZ_t^j as shocks to how effective the capital is. Alternatively we could model the shocks directly as factor productivity shocks. The alternative formulation generates isomorphic results, but for analytical simplicity the former formulation is chosen.[1]

2.2.2 Capital Market

I assume that the firms described in Section 2.2.1 are equity financed only. Investors can trade their shares in a competitive stock market. A shareholder can profit from holding a stock in two ways, changes in the value of the firm (capital gains) and dividends. Consider firm $j = 1, \ldots, N$ which produces intermediate output y_t^j. The gross revenue generated by this firm from selling its product to the final-good producer is

$$y_t^j p_t^j = a^j k_t^j p_t^j,$$

[1] The alternative formulation is to assume that $y_t^j = a_t^j x_t^j$, where a_t^j is factor productivity which follows $da_t^j = a_t^j \sigma^j dZ_t^j$, and x is now physical capital rather than efficiency units and evolves deterministically. For details of this formulation see Seyedan (2015).

per unit of time. Therefore, if it invests at rate i_t^j per unit of capital, the net profit that is available to be distributed as dividend to shareholders is

$$a^j k_t^j p_t^j - i_t^j k_t^j,$$

per unit of time.[2] Let V_t^j denote the total market value of firm j. Hence, the instantaneous rate of return from investing in its shares can be written as

$$dr_{k,t}^j = \frac{dV_t^j}{V_t^j} + \frac{a^j k_t^j p_t^j - i_t^j k_t^j}{V_t^j} dt. \tag{2.4}$$

The first term in (2.4) is the shareholders' capital gains rate due to changes in the market value of the firm, and the second term is the dividend yield distributed by firm j.

At each point in time firm j holds k_t^j in physical assets, and its market value is V_t^j. It is useful to define q_t^j as

$$q_t^j \equiv \frac{V_t^j}{k_t^j}, \tag{2.5}$$

which is essentially the average Tobin's q of firm j. Since capital is encapsulated within firms, we can interpret q_t^j as the market price of type j capital. Trading activity in the stock market determines V_t^j and in turn q_t^j. Let's postulate the following endogenous process for q_t^j

$$\frac{dq_t^j}{q_t^j} = \mu_{q,t}^j dt + \left(\sigma_{q,t}^j\right)' dZ_t, \tag{2.6}$$

where $dZ_t = \left[dZ_t^1 \cdots dZ_t^N\right]'$ is the $N \times 1$ vector of independent Brownian shocks, and

$$\sigma_{q,t}^j = \left[\sigma_{q,t}^{j1} \cdots \sigma_{q,t}^{jj} \cdots \sigma_{q,t}^{jN}\right]' \tag{2.7}$$

is the vector of the exposures of price q_t^j to the shocks. The drift term $\mu_{q,t}^j$ and the volatility vector $\sigma_{q,t}^j$ are endogenous objects and are determined in equilibrium. As it is evident in formulation (2.7), the price of the firm j capital, q_t^j, can be exposed to shocks to other firms as well.

The definition of q_t^j in (2.5) means that the market value of firm j can be written as $V_t^j = q_t^j k_t^j$, so the capital gains term dV_t^j / V_t^j in (2.4) can be derived by Ito's Lemma using the postulated q_t^j process (2.6) and the law of motion of k_t^j in (2.2). Doing so,

[2]In theory there is no restriction on dividends to be positive. Negative values can be interpreted as capital injections by shareholders to finance investment.

we can rewrite the returns from holding stock j in (2.4) as

$$dr^j_{k,t} = \mu^j_{k,t} dt + \left(\sigma^j_{k,t}\right)' dZ_t, \tag{2.8}$$

where

$$\mu^j_{k,t} = \mu^j_t + \mu^j_{q,t} + \sigma^j \sigma^{jj}_{q,t} + \frac{a^j p^j_t - i^j_t}{q^j_t}, \tag{2.9}$$

and

$$\sigma^j_{k,t} = \begin{bmatrix} \sigma^{j1}_{q,t} \\ \vdots \\ \sigma^{jj}_{q,t} + \sigma^j \\ \vdots \\ \sigma^{jN}_{q,t} \end{bmatrix}. \tag{2.10}$$

The term $\mu^j_{k,t}$ in (2.8) is the instantaneous expected return from holding shares in firm j. Its components are shown in (2.9). The last term is the dividend yield and is the same as in (2.4), now expressed in terms of q^j_t. The terms μ^j_t and $\mu^j_{q,t}$ capture the expected growth rates in stock and value of capital respectively. Finally, the Ito-term $\sigma^j \sigma^{jj}_{q,t}$ reflects the covariance between the exogenous volatility of the capital stock of firm j and its endogenous price risk.

The volatility vector in (2.8) represents the total risk from holding shares in firm j. By assumption, shocks to the stock of capital are independent across firms, hence the capital stock of firm j is only affected by dZ^j_t, with σ^j capturing the size of this exposure. Hence σ^j is the fundamental volatility, which is constant by assumption. However, firms are exposed to shocks to other firms through the market prices. These exposures are summarised in $\sigma^j_{q,t}$. Therefore, as (2.10) shows the total risk is the sum of the fundamental volatility and the endogenous, market-induced volatility.

To succinctly represent the investment opportunity set in the stock market, we can collect all the individual stock returns $dr^j_{k,t}$, $j = 1, \ldots, N$, in a vector of returns denoted by $dr_{k,t}$, such that

$$dr_{k,t} = \mu_{k,t} dt + \sigma_{k,t}' dZ_t, \tag{2.11}$$

where

$$\mu_{k,t} = \left[\mu^1_{k,t} \cdots \mu^j_{k,t} \cdots \mu^N_{k,t}\right]'$$

is the vector of expected returns and

$$\sigma_{k,t} = \left[\sigma_{k,t}^{1} \cdots \sigma_{k,t}^{j} \cdots \sigma_{k,t}^{N} \right]$$

is the volatility matrix. Each element in the $N \times N$ matrix $\sigma_{k,t}$ is a column vector as demonstrated in (2.10).

In addition to the risky assets described above, there is a risk-free asset in the economy. We can make either of the two following assumptions about the supply of the risk-free asset. We can presume that the risk-free asset is in zero net supply and the risk-free rate is determined in equilibrium. Alternatively we can assume that the supply is elastic at an exogenously fixed rate. This can be the case if the economy in question is a small open economy.

The main results of this paper are not sensitive to the assumption about the risk-free rate. In the Appendices, I analyse the implication of each assumption on the solution and results.

2.2.3 Investment

The only decision taken within the firms in this model is investment. The firms' decision makers (managers) are not modelled explicitly because it is assumed that there is no conflict of interest between shareholders and managers, and hence unmodelled managers behave in the best interest of their shareholders. For example we can assume that they own a fraction of the firm they are running and hence their incentives are aligned with rest of the shareholders.

Therefore the investment decision at each point in time boils down to maximising the rate of return from holding shares expressed in (2.8). The investment rate affects the expected return and is chosen to solve

$$\max_{i^j} \mu_{k,t}^{j}.$$

This objective pins down the investment choice at each moment in the following way

$$\Phi'\left(i^j\right) = 1 - \frac{1}{q_t^j}. \tag{2.12}$$

This condition shows that investment solely depends on the current price of capital,

$$i_t^j = i\left(q_t^j\right). \tag{2.13}$$

In addition, higher prices lead to more investment as

$$\begin{aligned} i'(q) &= \frac{1}{\Phi''(i)}\frac{1}{q^2} \\ &= \frac{\Phi'(i)^2}{\Phi''(i)} > 0. \end{aligned} \tag{2.14}$$

The optimal investment rule (2.13) is incorporated in the expressions for the return on capital, $dr^j_{k,t}$, hereafter.

We defined q^j_t in (2.5) as Tobin's q of firm j (average q). Hayashi (1982) laid out conditions under which marginal q and average q are equal, which hold in this model.[3] Hence, as it is evident in (2.12), investment rate of firm j is pinned down solely by q^j_t.

In this study I use a quadratic adjustment cost function, i.e.

$$\Phi(i) = \frac{\varkappa}{2}i^2,$$

where $\varkappa$ controls the degree of technological illiquidity. Using this cost function in (2.12), we can solve for the optimal investment function as

$$i(q^j) = \frac{1}{\varkappa}\left(1 - \frac{1}{q^j}\right).$$

2.2.4 Agents

There are two groups of agents in the economy, a continuum one of households and fund managers. Each group can be viewed as a representative agent. Households derive utility from consuming the consumption good. Their objective is to maximise the expected utility of their life-time consumption, expressed by the utility function

$$\mathbb{E}\left[\int_0^\infty e^{-\beta t}\log(c_t)\,dt\right]. \tag{2.15}$$

Households are the ultimate claim-holders to all the firms in the economy. They invest their wealth in the capital market as explained below.

There is a friction in the capital market though, which gives rise to the second group of agents, fund managers. Households cannot invest in the capital market directly, but they delegate their investment decision to fund managers. Fund managers raise money from households and invest on their behalf in a portfolio of risky and risk-free assets.

The delegation of portfolio allocation decision can take two forms depending

[3] Marginal q is the marginal value of a unit of capital, i.e. $\frac{\partial V}{\partial k}$. The conditions concern homogeneity of production function and adjustment cost function, both of which are satisfied in this model.

on our assumption. In the full-delegation case, households entrust all their financial wealth to fund managers and only make consumption decision. Fund managers then allocate their endowment to a portfolio of risky assets and the risk-free asset. In the partial-delegation case, in addition to consumption choice, households decide on the fraction of wealth allocated to the risk-free asset and funds. Fund managers then allocate the amount raised as before. For clarity and brevity, in the main text I adhere to the simplest case where all the financial decisions are delegated. In this case, if a consumer's wealth is denoted by w_t and the fund she has invested her wealth in returns dR_t over period dt, her wealth evolves according to

$$dw_t = w_t dR_t - c_t dt. \tag{2.16}$$

I explore the partial delegation case in Appendix 2.6.3. The results are very similar nonetheless.

Although fund managers are instrumental in choosing the investment portfolio as described further below, it is assumed that their share of aggregate consumption is negligible relative to the household sector. In other words we can ignore their consumption decision and assume that they do not consume goods. This formulation which is convenient when clearing the goods market is borrowed from He and Krishnamurthy (2014). Hence, we do not have to keep track of the fund managers' consumption decisions and the distribution of wealth between households and managers, which makes the model more tractable and enables us to focus on the essence of this analysis, which is the capital allocation across sectors.

A fund manager chooses portfolio $z_t \equiv \left[z_t^1 \cdots z_t^N\right]'$ of stocks at each time t, where z_t^j denotes the fraction of fund's resources invested in stock j. The remaining resources are invested in the risk-free asset. Therefore the return on the fund is

$$dR_t = z_t{}' dr_{k,t} + \left(1 - z_t{}'1\right) r_t dt. \tag{2.17}$$

The behaviour of fund managers is modelled in a reduced form to capture the benchmarking phenomenon, which is common practice to evaluate performance in the financial industry. The intention is to capture two features of fund management practice: fund flows depend on past performance and hence managers care about their performance, and performance is evaluated relative to a benchmark. As described above, we ignore their consumption, and assume that fund managers are motivated by their "reputation", which is affected by their performance relative to a benchmark portfolio. The formulation using reputation is adopted from He and Krishnamurthy (2014). Formally, managers choose their portfolio z_t to maximise their future reputation ϵ_t. A given fund manager may also "die" at Poisson intensity η. Therefore, the objective of a manager is to maximise

$$\mathbb{E}\left[\int_0^\infty e^{-\eta t}\log(\epsilon_t)dt\right]. \tag{2.18}$$

The evolution of a manager's reputation depends on the performance of the manager relative to the benchmark portfolio $\omega_{b,t}$. The law of motion of ϵ_t is

$$\frac{d\epsilon_t}{\epsilon_t} = dR_t - \tau\,\omega_{b,t}{}'dr_{k,t} - \eta dt, \tag{2.19}$$

where τ is the sensitivity of the reputation to the return on the benchmark portfolio, and dR_t is the rate of return delivered by the manager expressed in (2.17). The first two terms in (2.19) capture the return of the fund relative to the benchmark portfolio return and the last term captures the exit rate of managers (or alternatively interpreted, the rate at which reputation decays). The benchmark portfolio $\omega_{b,t} \equiv \left[\omega^1_{b,t}\ \cdots\ \omega^N_{b,t}\right]'$ is an exogenously given portfolio. In Section 2.4.1 I discuss examples of different benchmark portfolios.

2.2.5 Equilibrium

Since there is a continuum of households with unit mass, individual wealth w_t also denotes aggregate wealth. Moreover, it is shown later that all households consume the same fraction of their wealth, therefore individual consumption c_t denotes aggregate consumption as well.[4] Similarly for fund managers, z_t represents aggregate portfolio chosen by the fund industry.

We can assume a representative household who has wealth w_t and chooses consumption c_t to maximise utility (2.15) subject to dynamic budget constraint (2.16). The representative household entrusts its wealth to a representative fund manager who invests in a portfolio z_t of risky assets and the risk-free asset on behalf of the households, which delivers return dR_t in (2.17). The representative manager's objective is to maximise her expected future reputation expressed in (2.18), subject to the law of motion of reputation (2.19).

For any given initial condition on the firms' capital stock $\left\{k^j_0 > 0 : j = 1,\ldots,N\right\}$, an equilibrium is described by a set of stochastic processes: the market prices of capital $\left\{q^j > 0 : j = 1,\ldots,N\right\}$, interest rate r, wealth process $w \geq 0$, investment decisions $\left\{i^j > 0 : j = 1,\ldots,N\right\}$, portfolio choice z by fund managers, and consumption choice c by households, such that

1. each agent maximises her objective function, taking prices as given, and
2. markets for consumption good and capital clear, i.e.

[4] The CRRA utility function ensures this property.

$$c_t = \sum_j \left(a^j p_t^j - i_t^j\right) k_t^j, \tag{2.20}$$

$$z_t^j = \frac{q_t^j k_t^j}{w_t}, \quad j = 1, \ldots, N. \tag{2.21}$$

Market clearing for the risk-free asset follows from Walras' Law.

As illustrated in Section 2.2.2, each firm $j = 1, \ldots, N$ distributes $\left(a^j p_t^j - i_t^j\right) k_t^j$ in terms of consumption good as dividends to shareholders . Good market clearing condition (2.20) indicates that aggregate household consumption should equate the sum of dividends received by households.

The total wealth entrusted to fund managers is w_t and they invest a fraction z_t^j of it in shares of firm j. The capital market clearing condition (2.21) says that $z_t^j w_t$ should be equal to the total market value of firm j, i.e. $V_t^j = q_t^j k_t^j$.

2.3 Solution Methodology

All agents in the economy behave competitively, they take prices as given and act accordingly. Knowing prices fully characterises the behaviour of agents. In this section first I define a set of state variables. Next, I solve for all the equilibrium outcome variables in terms of the prices and the state variables. I then solve for a Markov equilibrium where prices are functions of the state variables.

To describe the state of the economy, it is enough to inspect how capital is allocated across firms. In addition, since the technology is linear in all firms, it is enough to inspect the ratio of capital across firms. Therefore, I normalise the capital stock of all firms by the level of capital stock in one of the firms, firm N, to form a state variable. Hence, to describe the state of the economy, we need $N-1$ state variables. I conjecture and later verify that the equilibrium can be derived in terms of

$$v_t^i = \log\left(\frac{k_t^i}{k_t^N}\right) \qquad i = 1, \ldots, N-1. \tag{2.22}$$

I denote the collection of state variables by v_t. For notational convenience, it is useful to define $v_t^N \equiv 0$ for all t.

We can easily derive the law of motion of the state variables by applying Ito's Lemma to the dynamics of firm capital described in (2.2) as

$$dv_t^i = \mu_{v,t}^i dt + \sigma^i dZ_t^i - \sigma^N dZ_t^N, \qquad i = 1, \ldots, N-1, \tag{2.23}$$

where $\mu_{v,t}^i \equiv \mu_t^i - \mu_t^N - \frac{1}{2}\left({\sigma^i}^2 - {\sigma^N}^2\right)$ by Ito's Lemma.

To characterise the equilibrium, first I solve for all the equilibrium outcome variables in terms of the state variables and $q^j, j = 1, \ldots, N$, which in turn are functions of the state variables themselves, i.e.

$$q_t^j = q^j(v_t), \qquad j = 1, \ldots, N,$$

and are conjectured to be twice continuously differentiable. The dynamics of these prices are linked to the law of motion of v_t in (2.23) by Ito's Lemma as described in Appendix 2.6.2. Finally, I derive a system of partial differential equations for q^js by imposing the equilibrium conditions. This system is solved numerically to find q^js which are used to compute all other desired equilibrium quantities.

The details of the solution are presented in Appendix 2.6.2. The next section will review the key results and implications. In particular I focus on two examples with 2 and 3 sectors in the economy.

2.4 Equilibrium Result

Let's denote the variance-covariance matrix of returns with $\Sigma_t = \sigma_{k,t}{}'\sigma_{k,t}$. If there is no benchmarking, i.e. $\tau = 0$, the optimal portfolio chosen by managers is

$$z_t = \Sigma_t{}^{-1}\left(\mu_{k,t} - r_t 1\right), \tag{2.24}$$

which is a simple mean-variance portfolio. Given the logarithmic utility of households, this is the same portfolio they would have chosen if they take the allocation decision themselves. In this case, fund managers do not introduce any distortion in the capital allocation.

With benchmarking the optimal portfolio chosen by fund manager is

$$z_t = \Sigma_t{}^{-1}\left(\mu_{k,t} - r_t 1\right) + \tau\omega_{b,t}. \tag{2.25}$$

Managers want to tilt their portfolio toward the benchmark and away from the optimal portfolio in terms of the risk-return trade-off. The second term of the portfolio is a hedging component against the risk of underperforming the benchmark.

Ultimately fund managers should hold the entire supply of risky assets in the economy and clear the capital market. For simplicity let's denote the market clearing portfolio of risky assets with $\omega_{m,t}$. Hence the capital market clearing condition becomes

$$z_t = \omega_{m,t}. \tag{2.26}$$

To make the analysis more interesting, I assume that ω_m and ω_b are different. This can be because ω_m is the market portfolio and ω_b is an index that differs from it, for example it does not include some stocks, or has different weights on stocks relative to the market (similar to Vayanos and Woolley (2013)). Alternatively, suppose that there are unmodelled investors (e.g. firm managers) that behave as buy-and-hold investors for reasons such as corporate-control, hedging, or other considerations. In this case, we can interpret ω_m as the free-float portfolio, that is the residual supply left from buy-and-hold investors, and ω_b is the aggregate market index against which the managers' performance is benchmarked (similar to Buffa et al. (2015)). In either case the results are similar, and for clarity I take the first assumption, i.e. ω_m is the market portfolio and ω_b is an arbitrary index against which the managers' performance is benchmarked.

To the extent that ω_b and ω_m are different, individual stocks are disproportionately affected by benchmarking. Managers want to tilt their portfolio toward the benchmark. Prices, expected returns and covariances should alter to clear the capital market. These changes affect firms investment policies and their relative size, and changes the allocation of resources in the economy. In the next two sections I look at two specific examples to illustrate how these changes affect the economy. Section 2.4.1 examines an economy with two sectors in which ω_b and ω_m are different and the composition of the index is constant in terms of the share of each sector. Section 2.4.2 explores an economy with three sectors in which the index is composed of the two largest firms by market capitalisation at each point in time.

2.4.1 Two-Sector Economy

In this section I consider an economy with two firms, indexed by $j = 1, 2$. To focus on the role of benchmarking, I assume all parameters are symmetric, i.e. both firms have the same productivity parameter, fundamental volatility, etc. The only difference is that the stock of firm 1 is under-represented in the benchmark portfolio and the stock of firm 2 is over-represented. To be specific, the benchmark index is composed of 50% of the value of firm 1, and 100% of firm 2. Although the composition of the index is constant across time in terms of the share of each sector, the weights change continuously due to fluctuations in the market values. If either of the firms is big enough it can dominate the index.

Figure 2.1 shows the probability density function (pdf) of the state variable $v^1 = \log(K^1/K^2)$, which captures the relative size of the sectors. I have used the Kolmogorov Forward Equation to derive the stationary distribution as described with more detail in Appendix 2.6.2. Since the parameters are symmetric and both sectors are equally productive, the maximum output is produced when $K^1 = K^2$, i.e. $v^1 = 0$. When there is no benchmarking (i.e. $\tau = 0$), the distribution is bell-shaped and peaks at $v^1 = 0$. Introducing benchmarking (i.e. $\tau > 0$) has two effects on the probability distribution. First, the distribution of the relative size of the firms has fatter tails with benchmarking, so the economy is more likely to end up in extreme cases where one sector dominates. Second, due to the fact that different firms are disproportionately affected by benchmarking and one is over-represented in the benchmark, the distribution shifts towards the state in which it dominates the economy (firm 2 in this example). In other words, it is more probable that firm 2 predominate in this economy which corresponds to low values of the state variable (low values of v^1 occur when K^2 is much higher than K^1).

Relative to the case with no benchmarking, both tails are fatter (first effect, due to benchmarking per se), and the left tail is fatter because firm 2 is over-represented in the benchmark so it is more likely to be in states in which this sector dominates the economy (second effect, due to the composition of the benchmark).

The reason behind the change in distribution is clear when we inspect relative prices and investment in different sectors in Figure 2.2. Benchmarking results in over-pricing and over-investment in the sector that constitutes big part of the market, and that increases the probability of extreme outcomes. Bigger sectors attract more investment and grow even bigger consequently. The magnitude of welfare loss due to misallocation in my simulations is in the order of 1% depending on the parameter values.

The left panel of Figure 2.2 shows the ratio of the prices of capital. Expectedly, as one sector grows bigger, the relative price of capital in that sector falls. However, benchmarking tilts the relative prices such that the firm with higher contribution to the index has higher relative price compared to the equilibrium without benchmarking. This would happen more often for the stock with over-representation in the index. The right panel in Figure 2.2 shows the corresponding picture of firms' internal investment. Since investment is strictly increasing in q, we see the same picture in relative investment of the firms. A firm with high representation in the index accumulates capital at a rate higher than what it would have achieved in a regime with no benchmarking.

2.4.2 Three-Sector Economy

In this section I consider an economy with three firms, indexed by $j = 1, 2, 3$. As in the previous section all parameters are symmetric, i.e. firms have the same productivity parameter, fundamental volatility, etc. The performance of the managers is benchmarked against an index which is constructed as follows: at each point in time the two largest firms by market capitalisation are included in the index and the smallest firm is excluded from the index. The primary purpose of this exercise is to see how inclusion in the index affects the firm characteristics and behaviour.

Figure 2.3 demonstrates the investment rate of a firm around the index inclusion. The horizontal axis shows the ratio of capital stock to the starting amount of capital stock (hence the initial value being 1 on the horizontal axis). The vertical axis shows

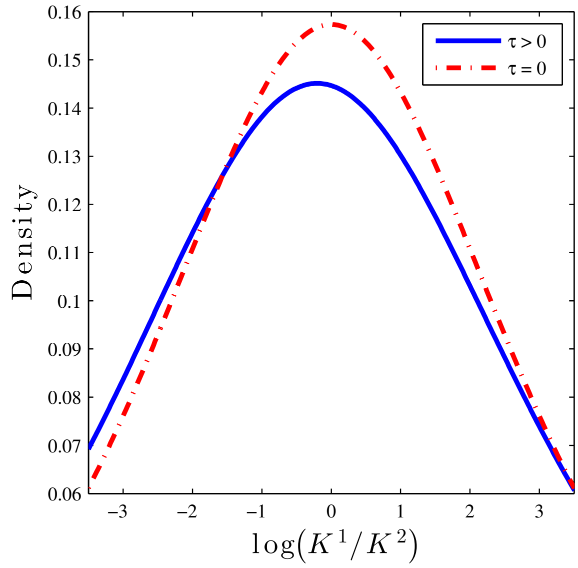


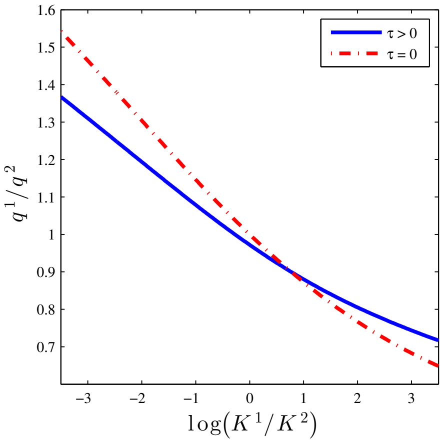
Figure 2.1: Stationary probability distribution of the state variable $v^1 = \log\left(\frac{K^1}{K^2}\right)$ with benchmarking in solid blue and without in dashed-dotted red. All parameters are symmetric, but the benchmark index is composed of 50% of the value of firm 1, and 100% of firm 2.

investment rate (i.e. the total investment normalised by capital stock at each point). The vertical dashed-dotted line marks the index inclusion event. To the left of this line stock 1 is not in the index and to the right it is, since it becomes large enough in terms of the market capitalisation.[5]

Top panel in Figure 2.3, depicts an economy in which there is benchmarking ($\tau > 0$). As it is evident in this panel, benchmarking introduces an index inclusion effect on the investment behaviour of the firm. Prior to inclusion, firm increases its investment rate dramatically. Once the firm is included in the index, the investment rate gradually drops as now the firm is a bigger firm and fundamental forces drive the investment rate down. The bottom panel shows the corresponding graph for an economy where there is no benchmarking, i.e. $\tau = 0$. In this economy the investment rate declines monotonically with increase in the size of the firm.

Figure 2.4 demonstrates the changes in investment rates of firms around inclusion in S&P 500 index. The details of the data used to draw these figures are described in Appendix 2.6.4. In the top panel, quarter 0 corresponds to the quarter in which the stock is included in the index, and I inspect the investment rate in a window of 10 quarters before and after the inclusion. The bottom panel shows investment rate versus the ratio of the capital stock to the initial level of capital stock. We can see in

[5]In Figure 2.3 the ratio of the capital stock of the other two firms is held constant at 1, i.e. the second state variable $v^2 = \log(v^2/v^3)$ is held constant in this graph at 0 throughout.



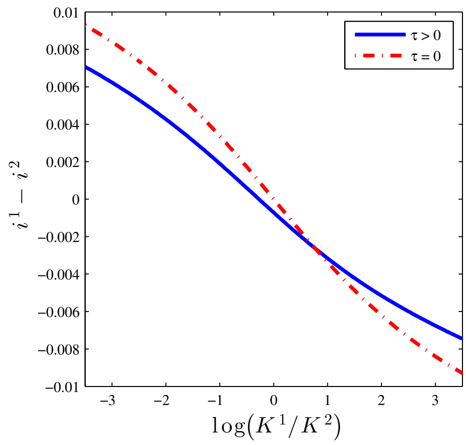


Figure 2.2: The left panel plots the ratio of the price of firm 1 capital to firm 2 capital, and the right panel the difference between the investment rates of the two firms, with benchmarking in solid blue, and without in dashed-dotted red. All parameters are symmetric, but the benchmark index is composed of 50% of the value of firm 1, and 100% of firm 2.

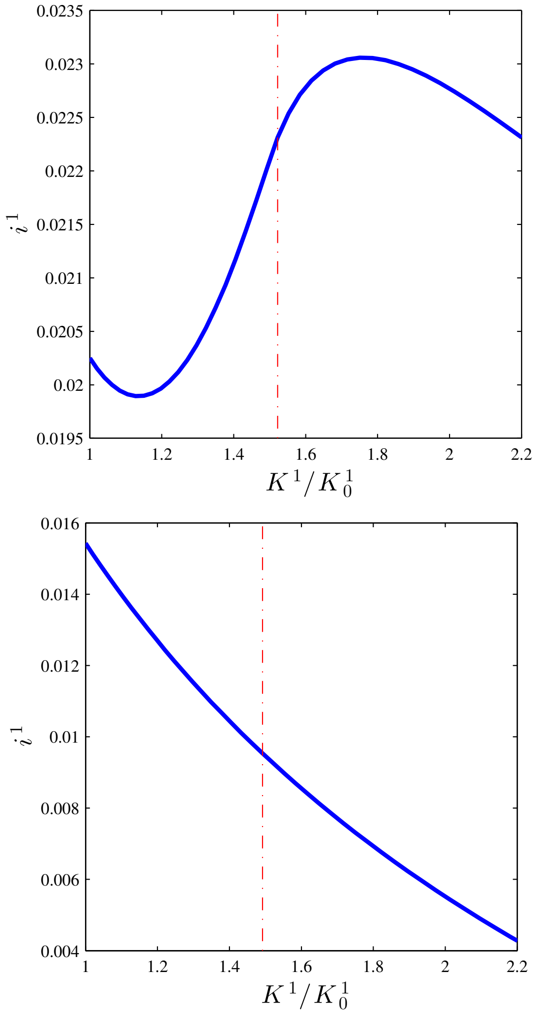


Figure 2.3: Investment rate around index inclusion with benchmarking in the top panel, and without in the bottom panel. The horizontal axis shows the capital stock normalised by the initial amount (hence the initial value of 1). The vertical dashed-dotted line marks the index inclusion event. There are three firms in the economy with symmetric parameters, and the index is composed of the two largest firms in terms of market capitalisation. The ratio of the capital stock of the other two firms is held constant at 1, i.e. $v^2 = \log(v^2/v^3) = 0$.

these graphs that investment rate around the index inclusion demonstrates an increase followed by a smaller decrease, which is similar to the behaviour shown in the model with benchmarking in Figure 2.3. The mechanism through which index inclusion affects the investment policy in the model is the following: market prices are affected by fund managers who care about the index because their performance is benchmarked against it, and these market prices in turn guide corporate investment policy. This figure provides suggestive evidence that being in an index can affect firms' investment behaviour.

2.5 Conclusion

I present a model of delegated asset management in a fully stochastic multi-sector production economy, and I show that benchmarking against an index in financial market has real effects in terms of the capital allocation to different sectors in the economy. I study the implications of the model for prices and capital allocation in general and also I examine examples of the effects of index composition in economies with two and three sectors in particular. Benchmarking generates a positive feedback loop from the market prices to firm's size and investment strategy, and creates an inefficient shift towards extreme states in which big sectors dominate the economy. In addition I show that benchmarking generates an index inclusion effect. Index inclusion event is preceded by rise in firm's valuation and investment rate relative to its capital stock, which drops by a smaller degree after the inclusion. I provide suggestive empirical evidence of the effect of index inclusion on investment rates.

2.6 Appendix

2.6.1 Additional Results

In all of the figures shown in this section, the left panel refers to an economy with endogenous interest rate and zero net supply of the risk-free asset and the right panel refers to an economy with an exogenously fixed interest rate with the risk-free asset in elastic supply.

Figure 2.5 demonstrates prices and interest rate in the economy described in Section 2.4.1 under two assumptions: endogenous interest rate (left) and exogenous interest rate (right), where interest rate is fixed at $r = 3.3\%$. Panel A shows changes in the interest rate versus the state variable $v^1 = \log\big(K^1/K^2\big)$. The risk-free asset is in zero net supply in the left Panel, therefore the interest rate should adjust to preclude aggregate borrowing and lending. With intermediate levels of v^1, return to capital is high (as demonstrated in Figure 2.7), so agents are willing to borrow to invest in the stock market. This fact raises interest rate for intermediate levels v^1. Moreover, in

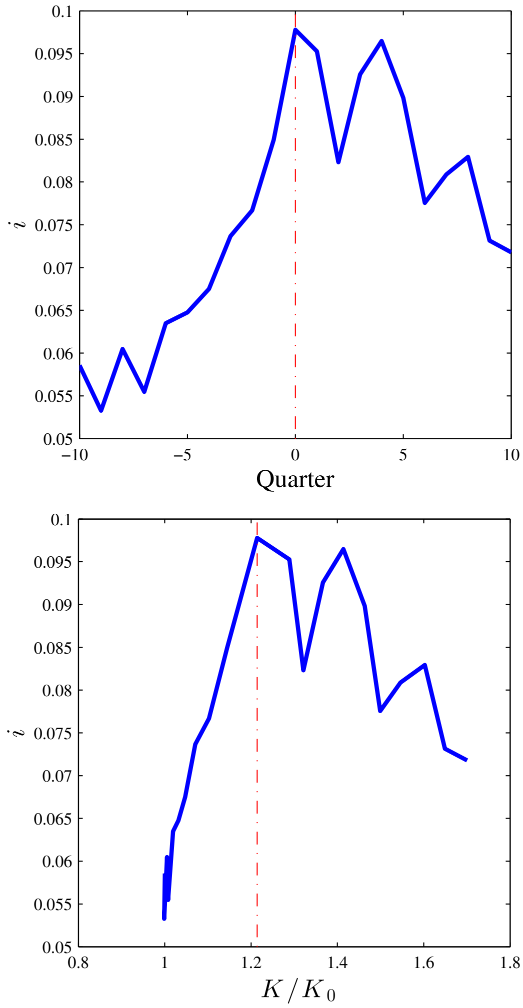


Figure 2.4: Investment rate around inclusion in S&P 500 index, from a sample of 103 index inclusions described in Appendix 2.6.4. The top panel plots the investment rate against time in a window of 10 quarters before and after inclusion, where quarter 0 corresponds to the quarter in which a firm is included in the index. The bottom panel plots investment rate versus the capital stock normalised by the initial amount (hence the initial value of 1). The vertical dashed-dotted line marks the index inclusion event.

the economy with benchmarking interest rate takes higher values for all levels of v^1. In the right panel the interest rate is exogenously fixed at $r = 3.3\%$ and the risk-free asset has elastic supply at this rate.

Panel B in Figure 2.5 shows changes in prices of capital in the two firms. With endogenous risk-free rate, benchmarking results in over-valuation for a firm that dominates the index and under-valuation for a firm that is dominated. As discussed in Section 2.4.1 these over-valuation and under-valuation feed into the investment strategies of firms and make big firms grow bigger and small firms become smaller.

With exogenous interest rate, benchmarking always increases the prices of the risky assets due to shift from risk-free asset to the risky assets which is a result of the fact that the index is composed of the risky assets only. However increase in the price of an asset is the highest when the asset dominates the index.

Figure 2.6 illustrates the drift on the capital stock of each firm μ^j. The overall shape of the drift on capital is dictated by the fundamental decreasing returns to scale to each sector in the economy. As the capital stock of a firm grows bigger compared to other firms, its returns diminish relative to its size, which reduces the average q and thereby reduces the investment and drift on capital. On the other hand, if the capital stock becomes small relative to other sectors, the opposite occurs. As Figure 2.6 shows, benchmarking acts as a counterbalance to this fundamental force and introduces a positive feedback between firms' size and their investment levels. Therefore, with endogenous risk-free rate benchmarking results in over-valuation for a firm that dominates the index and under-valuation for a firm that is dominated which then feed into the investment strategies and ultimately affect the growth rate of capital in firms.

With exogenous interest rate, benchmarking always increases the prices of the risky assets due to shift from the risk-free asset to the risky assets as described before, which leads to increase in the growth rate of capital stock due to higher investment levels.

Figure 2.7 illustrates the return on capital for the two firms. The effect of benchmarking on the return on capital is somewhat the mirror image of the effect on the prices shown in Figure 2.5. That's because the higher return in capital is associated with lower price of capital and vice versa.

Panel A in Figure 2.8 illustrates the volatility of return on capital for both firms i.e. $|\sigma_k^j|$. As evident in the left panel with endogenous interest rate, benchmarking reduces return volatility once the firm is dominant in the benchmark and increases it otherwise. Similar to the effect on prices, in presence of exogenous interest rate benchmarking always increases the volatilities of risky assets due to shift from risk-free asset to the risky assets as described above.

Panel B in Figure 2.8 shows the correlation between the return on capital of the two firms. With both endogenous and exogenous rates, benchmarking reduces the correlation between the returns.

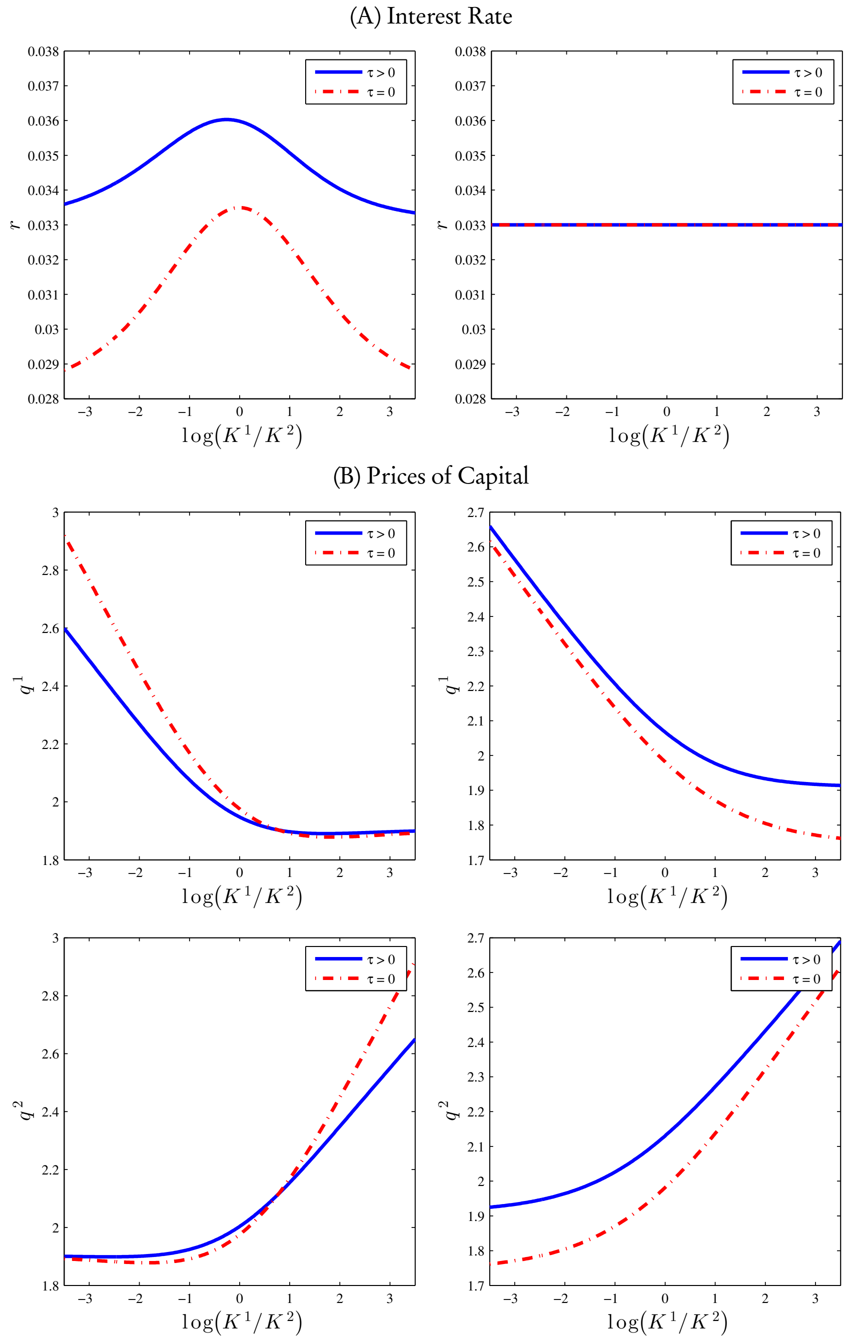


Figure 2.5: Prices of capital and interest rate in a two-sector economy with benchmarking in solid blue and without in dashed-dotted red, under two assumptions: endogenous interest rate on the left and exogenous interest rate on the right, where interest rate is fixed at $r = 3.3\%$. Panel A plots the interest rate and panel B plots the price of capital of the two firms. All parameters are symmetric, but the benchmark index is composed of 50% of the value of firm 1, and 100% of firm 2.

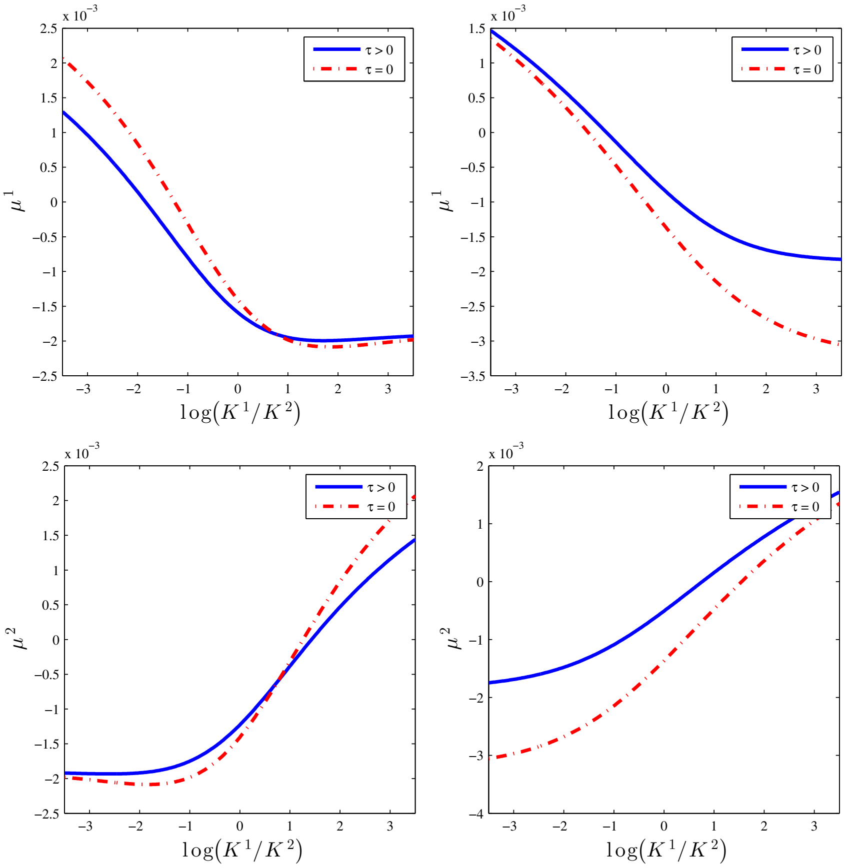


Figure 2.6: Growth rate of capital in a two-sector economy with benchmarking in solid blue and without in dashed-dotted red, under two assumptions: endogenous interest rate on the left and exogenous interest rate on the right. All parameters are symmetric, but the benchmark index is composed of 50% of the value of firm 1, and 100% of firm 2.

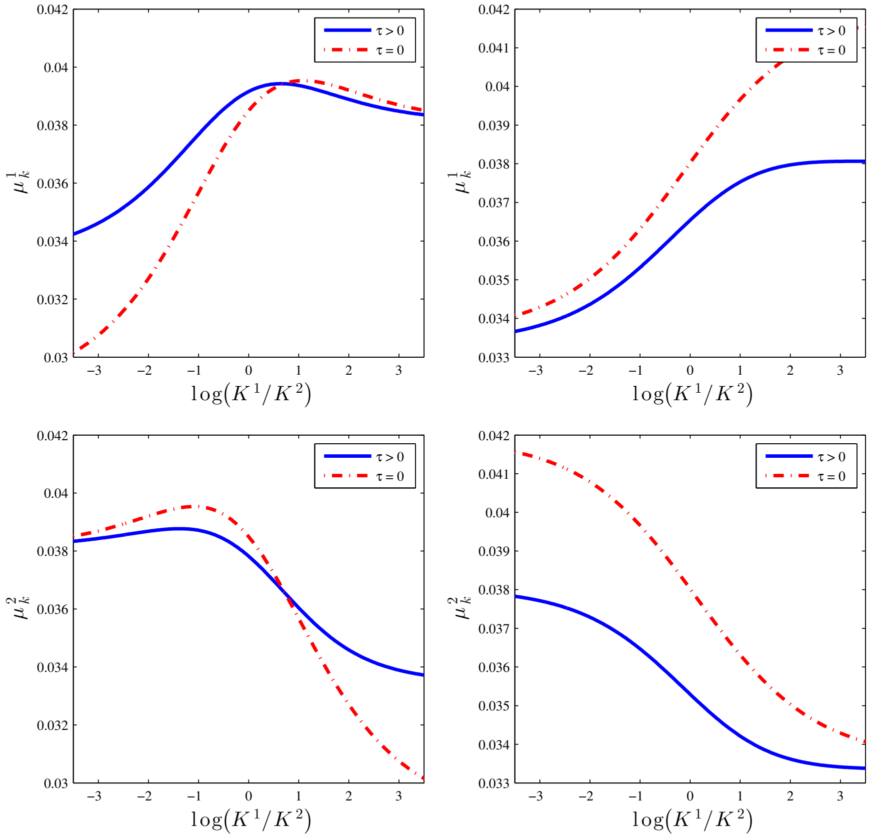


Figure 2.7: Return on capital in a two-sector economy with benchmarking in solid blue and without in dashed-dotted red, under two assumptions: endogenous interest rate on the left and exogenous interest rate on the right. All parameters are symmetric, but the benchmark index is composed of 50% of the value of firm 1, and 100% of firm 2.

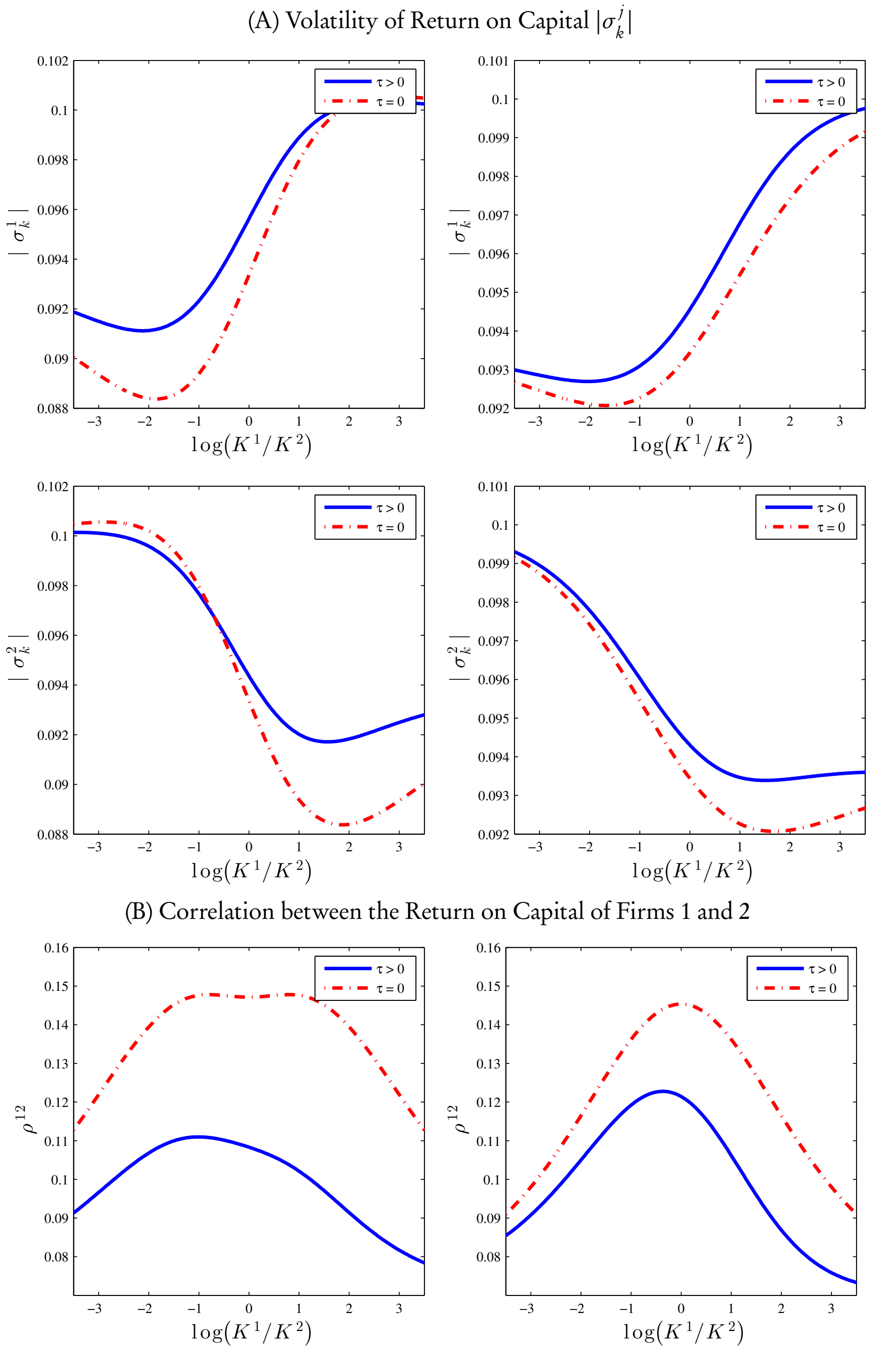


Figure 2.8: Volatility of return on capital and correlation between the returns in a two-sector economy with benchmarking in solid blue and without in dashed-dotted red, under two assumptions: endogenous interest rate on the left and exogenous interest rate on the right. Panel A plots the volatility of return on capital for each firm, and panel B plots the correlation between the two returns. All parameters are symmetric, but the benchmark index is composed of 50% of the value of firm 1, and 100% of firm 2.

2.6.2 Solution

Although state variable is v_t, for notational convenience it is useful to define

$$\psi_t^j = e^{v_t^j} = \frac{k_t^j}{k_t^N}, \qquad j = 1, \ldots, N. \tag{2.27}$$

which has a one-to-one relationship with v_t. I will express equilibrium conditions in terms of ψ_t, which is equivalent to expressing them in terms of v_t.

I will drop the subscript t from the variables in most of what follows for brevity.

Agents' Optimisation

When households delegate all their financial decisions to managers, they only choose their consumption level. Given the logarithmic utility, the consumption choice is very simple, and the optimal consumption is

$$c_t = \beta w_t. \tag{2.28}$$

Fund managers choose z_t to maximise (2.18) subject to (2.19). Because of logarithmic utility, the value function of the representative fund manager is of the form

$$V^m = \underbrace{\frac{1}{\eta} \log(\epsilon)}_{J^m(\epsilon)} + H^m,$$

where H^m depends only on aggregate state. HJB equation is

$$\max_z (\mathscr{D} V^m + \log(\epsilon)) - \eta V^m = 0,$$

where $\mathscr{D}$ is the Dynkin operator.[6] The optimal portfolio z is given by

$$\arg\max_z \mathscr{D} V^m = \arg\max_z \mathscr{D} J^m.$$

Since $J^m(\epsilon) = \frac{1}{\eta}\log(\epsilon)$, we can use Ito's Lemma and the law of motion of ϵ in (2.19), substituting dR_t from (2.17) to obtain

$$\mathscr{D} J^m = \frac{1}{\eta}\Big(z'(\mu_k - r1) + r - \tau\omega_b'\mu_k - \eta - \frac{1}{2}(z - \tau\omega_b)'\sigma_k'\sigma_k(z - \tau\omega_b)\Big).$$

Taking first-order condition of $\mathscr{D} J^m$ with respect to z yields (2.25).

[6]The Dynkin operator is the drift term of a process S, when applying Ito's Lemma to derive dS.

Market Clearing Conditions

In this section I rewrite the market clearing conditions for consumption and capital in terms of ψ and combine them with optimality conditions. Since ψ is directly linked to the state variables via (2.27), this is equivalent to writing the conditions in terms of the state variables.

First, divide both sides of (2.20) by k_t^N to derive

$$D(\psi) = \frac{c_t}{k_t^N}, \tag{2.29}$$

where $D(\psi) \equiv \sum_j \left(a^j p_t^j - i_t^j\right)\psi_t^j$. Note that D is a function of ψ only. Firstly, investment is only function of q as shown in (2.13) which in our Markov equilibrium is function of ψ (or v) itself. Secondly, intermediate good prices $p_t^j, j = 1, \ldots, N$ are also pinned down by ψ according to

$$p_t^j = \left(\frac{f(\psi_t)}{\alpha^j \psi_t^j}\right)^{\frac{1}{s}}$$

where $\alpha^j \equiv \frac{a^j}{a^N}$, and

$$f(\psi) \equiv \left[\sum_j \left(\alpha^j \psi_t^j\right)^{\frac{s-1}{s}}\right]^{\frac{s}{s-1}},$$

which is obtained by dividing both sides of (2.1) by $y_t^N = a^N k_t^N$.

Next, we can combine the consumption market clearing (2.29) and the optimal consumption rule (2.28) to derive

$$\frac{w_t}{k_t^N} = \frac{D(\psi_t)}{\beta}. \tag{2.30}$$

The left-hand side of (2.30) is the aggregate financial wealth in the economy normalised by the level of capital in firm N. We will use this equation later in the capital market clearing condition.

Next, let's turn our attention to the capital market clearing condition in (2.21). I have expressed the capital market clearing condition in terms of the market portfolio $\omega_{m,t}$. The share of wealth invested in firm j is $q_t^j k_t^j / w_t$, therefore in the equilibrium market portfolio we should have

$$\omega_{m,t}^j = \frac{q_t^j k_t^j}{w_t}, \qquad j = 1, \ldots, N. \tag{2.31}$$

Normalising the numerator and denominator in the right-hand side of (2.31), we obtain

$$\omega_{m,t}^{j} = \frac{q_t^j \psi_t^j}{w_t / k_t^N},$$

which together with (2.30) characterises the market portfolio in terms of the state variables as

$$\omega_m(\psi) = \frac{\beta}{D(\psi_t)} \begin{bmatrix} q_t^1 \psi_t^1 \\ \vdots \\ q_t^N \psi_t^N \end{bmatrix}. \tag{2.32}$$

PDEs

In this section I combine the equilibrium conditions to derive a system of partial differential equations that determines prices $q^j, j = 1, \ldots, N$ in terms of the state variables $v^i, i = 1, \ldots, N-1$. First let's rewrite the optimal portfolio allocation (2.25) as

$$\mu_k = \Sigma(\omega_m - \tau\omega_b) + r1, \tag{2.33}$$

where I have imposed the capital market clearing condition (2.26). Equation (2.33) characterises a system of N PDEs. To see this, note that μ_k and $\Sigma = \sigma_k' \sigma_k$ are linked to μ_q and σ_q which are given below in terms of the partial derivatives of $q^j, j = 1, \ldots, N$ with respect to the state variables:

$$\mu_q^j = \frac{1}{q_j} \left[\sum_i \frac{\partial q^j}{\partial v^i} \mu_v^i + \frac{1}{2} \sum_{i \neq k} \frac{\partial^2 q^j}{\partial v^i \partial v^k} \sigma^{N2} + \frac{1}{2} \sum_i \frac{\partial^2 q^j}{\partial v^{i2}} \left(\sigma^{i2} + \sigma^{N2} \right) \right], \tag{2.34}$$

and

$$\begin{aligned} \sigma_q^{ji} &= \frac{1}{q^j} \frac{\partial q^j}{\partial v^i} \sigma^i, \qquad i = 1, \ldots, N-1, \\ \sigma_q^{jN} &= -\frac{1}{q^j} \sum_{i=1}^{N-1} \frac{\partial q^j}{\partial v^i} \sigma^N, \end{aligned} \tag{2.35}$$

where the summations are from 1 to $N-1$, since we have only $N-1$ state variables ($v_t^N \equiv 0, \forall t$). In the Markov equilibrium that we are looking for, prices $q^j, j = 1, \ldots, N$ are functions of the state variables $v^i, i = 1, \ldots, N-1$. Therefore, we can apply Ito's Lemma using the law of motion of the state variables described in (2.23) to obtain (2.34)–(2.35). Combining (2.34)–(2.35) with (2.9)–(2.10) yields expressions for μ_k and σ_k in terms of the partial derivatives of $q^j, j = 1,\ldots,N$ with respect to the state variables, that can be used in (2.33) to form a system of PDEs.

In addition, the market portfolio ω_m is given in terms of the state variables and prices in (2.32), and the composition of the benchmark portfolio ω_b is determined exogenously (it depends on prices as well).

If interest rate is given exogenously, the system of N PDEs in (2.33) can be solved numerically as described in Appendix 2.6.2 to find N unknown functions $q^j, j = 1,\ldots,N$ in terms of the state variables, which then can be used to compute all other desired equilibrium quantities.

If the risk-free rate is endogenous and risk-free asset is in zero net supply, the sum of the market value of all firms should be equal to the aggregate wealth in the economy, i.e. $w_t = \sum_j q_t^j k_t^j$. Dividing this equation by k_t^N and using (2.30) we obtain

$$\frac{D(\psi_t)}{\beta} = \sum_j q_t^j \psi_t^j. \tag{2.36}$$

We can solve the system of N PDEs in (2.33) together with Equation (2.36) to find N functions $q^j, j = 1,\ldots,N$ and interest rate r in terms of the state variables. The details of the numerical solution are described in the next section.

Numerical Algorithm

To solve for the infinite-horizon equilibrium numerically, I use the explicit 4-step Runge-Kutta method expounded below. The goal is to find functions $q^j\left(v^1,\ldots,v^{N-1}\right)$, $j = 1,\ldots,N$, where v^i, $i = 1,\ldots,N-1$ are state variables as defined in (2.22). All other desired values can be recovered from these functions. The problem is solved numerically by adding time dimension t.

The state variables and t are discretised with step sizes equal to $dv^i = 0.02$, and $dt = 0.01$. Since $v^i \in (-\infty, +\infty), i = 1,\ldots,N-1$, I impose upper and lower limits on the values of v^is. Adding the time dimension, Ito's Lemma implies that the q-drifts change to

$$\tilde{\mu}_q^j = \mu_q^j + u^j, \qquad j = 1,\ldots,N,$$

where

$$u^j = \frac{\dot{q}^j}{q^j}, \qquad j = 1,\ldots,N, \tag{2.37}$$

μ_q^j is the drift from the original problem given by (2.34) without the time derivative, and $\dot{q}^j$ is the derivative of q^j with respect to time. Incorporating this change in (2.9) means that the drift of capital μ_k^j should also alter to

$$\tilde{\mu}_k^j = \mu_k^j + u^j, \qquad j = 1, \ldots, N,$$

where μ_k^j is the drift from the original problem given by (2.9) without the time derivative. In vector notation we can write

$$\tilde{\mu}_k = \mu_k + u,$$

where u is the vector of scaled time derivatives with elements specified in (2.37) and μ_k is the vector of drifts from the original problem without the time derivatives.

Let's call the function of interest $f(v,t)$ where f can be $q^j, j = 1, \ldots, N$. We begin with some terminal values, that is $f(v,T)$ for all values of v in the grid (more details on the choice of terminal values below) and move backwards in time as described further below by computing $f(v, T-dt)$, $f(v, T-2dt)$, and so on until the time derivatives vanish, i.e. until u^j converges to zero for all $j = 1, \ldots, N$.

At each point in time, having $q^j, j = 1, \ldots, N$, I use the finite difference method to approximate first- and second-order derivatives of f with respect to v, i.e. $f_i(v,t)$, $f_{ii}(v,t)$, and $f_{ik}(v,t)$ for $i, k = 1, \ldots, N-1$. Having the prices and their derivatives, we can compute μ_k, σ_k, and ω_m for all values of the state variables following the procedure described in the previous section.

Since the modified problem contains the time dimension, μ_k should be replaced with $\tilde{\mu}_k$ in the system of PDEs (2.33), so it becomes

$$\tilde{\mu}_k = \Sigma(\omega_m - \tau\omega_b) + r1. \tag{2.38}$$

If interest rate r is exogenously given and is known, the next step which is computing the time derivatives is easily done by inverting (2.38) to find u according to

$$u = \Sigma(\omega_m - \tau\omega_b) + r1 - \mu_k. \tag{2.39}$$

If interest rate is endogenous, I differentiate both sides of the condition (2.36) with respect to t to get

$$\sum_j B^j u^j = 0. \tag{2.40}$$

where $B^j, j = 1, \ldots, N$, are defined as

$$B_j = q^j \psi^j \left(\beta + \frac{di}{dq^j}\right).$$

Combining (2.39) and (2.40), we can find r as

$$r = -\frac{\sum_j B^j F^j}{\sum_j B^j}, \tag{2.41}$$

where F^j is j-th element of vector $\Sigma(\omega_m - \tau\omega_b) - \mu_k$. Substituting (2.41) in (2.39) yields the vector of time derivatives u.

Having computed the time derivatives for all values of v in the grid, we can take a step back in time and compute new values for $q^j, j = 1, \ldots, N$. Formally, let the time derivative computed in (2.39) be

$$\dot{f} = g(v, t, f),$$

where f can be $q^j, j = 1, \ldots, N$, and function $g(.)$ is the corresponding time derivative from (2.39). Then we take a step of size dt back according to

$$f(v, t - dt) = f(v, t) - g(v, t, f)dt, \qquad \forall v. \tag{2.42}$$

Equation (2.42) corresponds to the Euler method for solving differential equations. For better convergence properties, instead of (2.42), I use a 4-step Runge-Kutta method (RK4). Intuitively, instead of taking a step back of size dt, I only step back a fraction of that, recompute the time derivatives at that step and use it to take another fractional step, and so on through the 4 steps of RK4. Formally, we move back in time according to

$$f(v, t - dt) = f(v, t) - \frac{dt}{6}(k_1 + 2k_2 + 2k_3 + k_4) \qquad \forall v,$$

where

$$\begin{aligned}
k_1 &= g(v, t, f), \\
k_2 &= g(v, t - \frac{dt}{2}, f - \frac{dt}{2}k_1), \\
k_3 &= g(v, t - \frac{dt}{2}, f - \frac{dt}{2}k_2), \\
k_4 &= g(v, t - dt, f - (dt)k_3).
\end{aligned}$$

We repeat this process until time derivatives converge to zero, which means that the solution found satisfies the infinite-horizon problem. The solution will converge to the stationary solution for a wide range of terminal values at $t = T$, as long as these values satisfy the market clearing conditions.

Stationary Distribution

Let's restate the dynamics of the state variable given by (2.23) as

$$dv_t = \mu_v dt + \sigma_v' dZ_t.$$

If a stationary distribution, $\lim_{t\to\infty} f(v,t) = f(v)$ exists, it satisfies the following differential equation

$$0 = -\frac{d}{dv}\left[\mu_v f(v)\right] + \frac{1}{2}\frac{d^2}{dv^2}\left[\sigma_v^2 f(v)\right], \tag{2.43}$$

which is known as Kolmogorov Forward Equation. In case of multivariate state variable the multidimensional analogue of (2.43) should be used.

To derive the stationary distribution, differential equation (2.43) is solved numerically by adding a time dimension. If the distribution is a function of time, the Kolmogorov Forward Equation becomes

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial v}\left[\mu_v f(v,t)\right] + \frac{1}{2}\frac{\partial^2}{\partial v^2}\left[\sigma_v^2 f(v,t)\right]. \tag{2.44}$$

We start from an initial guess distribution and solve forward until the time derivative converges to zero, which means that the solution satisfies (2.43) and therefore is a stationary distribution.

2.6.3 Partial Delegation

Households decide how much to invest in the fund and how much to consume

$$\max_{c_t, x_t} \mathbb{E}\left[\int_0^\infty e^{-\beta t} \log(c_t) dt\right],$$

s.t.

$$\frac{dw_t}{w_t} = x_t dR_t + (1 - x_t) r_t dt - \frac{c_t}{w_t} dt. \tag{2.45}$$

Because of logarithmic utility, the value function of the representative household is of the form

$$V^h = \underbrace{\frac{1}{\beta}\log(w)}_{J^h(w)} + H^h,$$

where H^h depends only on aggregate state. HJB equation is

$$\max_{c,x}\left(\mathscr{D}V^h + \log(c)\right) - \beta V^h = 0,$$

where $\mathscr{D}$ is the Dynkin operator.[7] The optimal fraction of wealth invested in the fund the and optimal consumption are given by

$$\arg\max_{c,x}\ \mathscr{D}V^h + \log(c) = \arg\max_{c,x}\ \mathscr{D}J^h + \log(c).$$

Since $J^h(w) = \frac{1}{\beta}\log(w)$, we can use Itos Lemma and the law of motion of w in (2.45), substituting dR_t from (2.17) to obtain

$$\mathscr{D}J^h = \frac{1}{\beta}\left(xz'(\mu_k - r1) + r - \frac{c}{w} - \frac{1}{2}x^2 z'\sigma_k'\sigma_k z\right).$$

Taking first-order condition of $\mathscr{D}J^h$ with respect to x yields the following optimal portfolio

$$x_t = \frac{z_t{}'\left(\mu_{k,t} - r_t 1\right)}{z_t{}'\Sigma_t z_t}, \tag{2.46}$$

where $\Sigma_t = \sigma_{k,t}{}'\sigma_{k,t}$. Optimal consumption is given by $c = \beta w$ as before. It can be easily shown that if there is no benchmarking, i.e. $\tau = 0$, then $x_t = 1$, meaning that households invest all their wealth in the fund. That's because absent benchmarking, fund managers invest exactly the same way as households would have done themselves.

Although households' decision affects the total amount of money invested in the stock market, it does not affect the portfolio allocation within the risky assets disproportionately, because the decision on how to allocate funds to different stocks is taken by fund managers who behave as before.

2.6.4 Index Inclusion Data

The index inclusion events data is taken from Jeffrey Wurgler website. the data is used and described in Wurgler and Zhuravskaya (2002). The data includes both additions and deletions but I only consider additions because most deletions are related to corporate events such as takeover. I have excluded inclusions around which a corporate event such as merger or takeover, spin-offs or divestiture had taken place (these events are recorded in Wurgler's data).

This data is merged with the Compustat data to track investment and capital levels around the inclusion event. Quarterly investment amount ($I = iK$) is computed

[7] The Dynkin operator is the drift term of a process S, when applying Ito's Lemma to derive dS.

from the cumulative capital expenditure (CAPXY). Replacement value of capital (K) is computed using the perpetual inventory method as described in Whited (1992) from the gross capital stock (PPEGTQ). The investment rate i is the total quarterly investment I divided by the replacement value of capital K. I have only kept events for which the data is fully available around a window of 10 quarters before and after the event with no missing values in investment and gross capital levels. As a result there are 103 unique events in the data during years 1986–2000.

Chapter 3

A Production Economy with Heterogeneous Beliefs

Abstract

This article examines balance sheet recessions in a general equilibrium model when agents have heterogeneous beliefs about future technology growth. I show a channel which describes the risk concentration through belief dispersion rather than ad-hoc constraints on aggregate risk sharing, which is the case in most macro models with financial frictions. Endogenous stochastic consumption volatility arises from constant fundamental volatility, and prices and investment are stochastic even with static disagreement. Recursive preferences allow me to disentangle the role of risk aversion and elasticity of intertemporal substitution in the model. I study the role of static vis-à-vis dynamic disagreement and examine the effect of financing constraints on the equilibrium when there is belief dispersion among agents. In my model financing constraints have an ambiguous effect on the equilibrium risk sharing. In some regions they mitigate the endogenous risk and diminish the balance sheet channel by limiting the concentration of the aggregate risk while in others they have the opposite effect.

3.1 Introduction

The seminal works of Kiyotaki and Moore (1997) and Bernanke et al. (1999) and more recent papers by Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2012), among others, have highlighted the role of the financial sector in amplifying and propagating shocks to the economy and generating balance sheet recessions. Some agents in the economy lever up and hold a higher fraction of aggregate risk.

Shocks to the economy erode their net worth and depress asset prices, reducing the funding capacity and aggravating financial frictions, which in turn deepen and prolong the initial slump. While the role of balance sheets in an economy with financial frictions is well studied, the reason why aggregate risk is so concentrated in the first place is less clear. Di Tella (2012) shows that if contracts on aggregate state of the economy are introduced in standard models of balance sheet recessions, the balance sheet channel disappears. Such contracts sever the link between leverage and risk sharing, and are easy to implement using standard financial instruments. In this paper I show a channel which generates risk concentration via heterogeneous beliefs rather than ad-hoc constraints on aggregate risk sharing, and I demonstrate that a nonlinear relationship between financial variables and real investment need not be the result of financial frictions. I study the role of static vis-à-vis dynamic disagreement and also examine the effect of financing constraints on the equilibrium when there is belief dispersion among agents.

There are two groups of agents in my model that have heterogeneous beliefs about the future path of technology growth. I abstract from any other heterogeneity to focus on belief dispersion, hence agents are otherwise identical. In particular, unlike Kiyotaki and Moore (1997), Bernanke et al. (1999), Brunnermeier and Sannikov (2014), He and Krishnamurthy (2012), and many other models that study balance sheet recessions, the two groups have the same productive technology. At each point in time agents in the two groups have the same productivity level, but they project different paths for future levels. In the baseline model, the disagreement is modelled as dogmatic, time-invariant subjective beliefs. A static disagreement is enough to induce stochastic dynamics in investment opportunities, prices, and investment, because of the dynamics of wealth distribution among agents which is irrelevant in a homogeneous-belief economy. The model is later extended to cases with time-varying disagreement which adds further dynamics to the economic activity. The extension with time-varying disagreement can accommodate two channels for belief dispersion: (i) different subjective priors as in Basak (2000) or Buraschi and Jiltsov (2006); and (ii) different subjective models as in David (2008), Dumas et al. (2009), and Whelan (2014). In all cases agents have the same information set, but agree to disagree how to process information, which is represented by different filtrations.

Belief dispersion motivates trade in the model. Agents who are optimistic about future technology growth are the natural buyers of capital; they apply leverage to hold more capital relative to their wealth and have large exposure to the aggregate risk. This capital allocation profile generates endogenous volatility in the economy which affects the real sector via volatile investment, prices and interest rate. Although the fundamental volatility is constant, aggregate consumption exhibits stochastic volatility which is induced by belief dispersion.

When natural buyers are well capitalised, they are able to absorb most of the risk in the economy and this regime would comprise normal times in the model. Negative shocks however can erode their wealth and weaken their balance sheets so that they are not able to hold as much capital as they would in normal times. As a result, asset prices drop and risk premia rise to induce the pessimist agents to hold the assets. Lower prices in turn reduce investment and deepen the recession.

This paper predominantly is related to two strands of literature. First and foremost, it relates to the literature on balance sheet channel for business cycles and builds on the idea that adverse price movements affect the borrowers net worth. Many papers in this tradition focus on the role of financial constraints and build on the seminal works of Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997), and Bernanke et al. (1999) who studied financial frictions in infinite-horizon macro settings. These classic examples study how financial frictions amplify (unanticipated) shocks near the steady state using log-linear approximation. Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2012) study the full dynamics in continuous time with anticipated shocks. The intuition behind my research is in spirit similar to the one in Geanakoplos (2010) who examines the role of heterogeneous beliefs in creating leverage cycles. In a two-period exchange economy, he illustrates how the identity of the marginal buyer affects prices in the economy. Instead I study a fully dynamic model in a continuous-time setup with production.

In recent macro-finance papers, like Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2012), the driving force is a constraint on raising equity for experts. Experts have to hold (at least) a pre-specified share of the risk on their balance sheet. A feature of these models is that, as long as the equity constraint is not binding, there is no role for the debt market. Debt financing only arises in the constrained region, when firms cannot raise equity any more. To address that, He and Krishnamurthy (2013) impose an ad-hoc assumption that a fraction of households can only invest in debt market. In Brunnermeier and Sannikov (2014), the economy is always constrained (no equity issuance allowed at all). This counterfactual feature does not arise in my setting. In the baseline model there are no constraints on financing and firms can raise debt or equity. However, since agents disagree about the prospects of the economy, they would not invest in the equity of agents in the other belief group and the introduction of constraint on raising equity would not change the equilibrium. Introducing other financing restrictions into the model such as short-selling ban and leverage cap can indeed curb the level of endogenous risk in this model, therefore although the channel in which balance sheet plays a role is different from models with financial frictions, introducing frictions can interact with this channel.

My paper is also motivated by the consumption-based asset pricing literature that studies the role of heterogeneous agents in general, and heterogeneous beliefs in particular in generating trade in financial markets and aims to expand this framework to link it with real economic activity and show how it can explain economic fluctuations. While in the consumption-based asset pricing literature the level of aggregate consumption is a given exogenous process, in this research aggregate production, investment and consumption are all endogenous equilibrium outcomes. I borrow most of the modeling apparatus from this literature. A few examples of related papers in this literature are Dumas (1989), Bhamra and Uppal (2009), Basak and Pavlova (2013), Chabakauri (2013). In these papers, agents have different risk aversions, and Sharpe ratios and volatilities change with changes in wealth distribution. Basak (2005) and Chabakauri (2014) investigate the asset-pricing implications of models where agents have heterogeneous beliefs. David (2008) shows that disagreement can describe the equity premium with low risk aversion. Bhamra and Uppal (2014) solve for asset prices with heterogeneous preferences and beliefs. Borovička (2013) studies survival and long-run dynamics when agents have heterogeneous beliefs and recursive preferences. Similarly, Kogan et al. (2006) study the survival of agents when traders have erroneous beliefs.

All of these papers are set up in a Lucas economy with a given aggregate consumption stream with constant volatility. I use the insight developed in this literature and implement it in a production economy, hence I am able to discuss features like investment and endogenous production and consumption growth. As a result, in my model aggregate consumption, growth rates, and investment exhibit stochastic volatility endogenously. As illustrated by the long-run risk literature following Bansal and Yaron (2004), consumption stochastic volatility can explain a high and time-varying equity premium. In my model an endogenous stochastic volatility arises from constant fundamentals.

The closest paper in spirit to my research is Baker et al. (2014), who also study heterogeneous beliefs in a production economy. My paper is different to theirs in four different ways. First, I model belief dispersion in a different and more empirically-relevant way.[1] Second, they use CRRA preferences in their model and therefore they cannot differentiate risk aversion from elasticity of intertemporal substitution (EIS). As a result, to avoid counterfactual results for prices and investment, they focus on risk-aversion coefficient less than one. On the contrary, I use Epstein-Zin preferences to separate the effects of risk aversion and EIS. While they only study dogmatic beliefs, I study the model when disagreement is dynamic in the extension. Finally, I also study the model when there are financing constraints, namely short-selling and leverage constraints.

In terms of the modeling and solution methodology my paper is similar to Di Tella (2012) who studies the effect of uncertainty shocks. In a setting akin to Brunnermeier and Sannikov (2014) he shows that when agents are allowed to write contracts on the

[1]They model heterogeneous beliefs as disagreement about the depreciation rate of capital. However, at least in empirical asset pricing literature calibrations, there is little disagreement about proper values for the depreciation rate. In my model belief dispersion is about how technology evolves over time.

aggregate state of the economy, the balance sheet channel vanishes, but if uncertainty shocks hit the economy, they can drive this channel, therefore the type of aggregate shock can explain the concentration of aggregate risk.

The rest of the paper is organised as follows. In Section 3.2, I describe the baseline model. Section 3.3 solves for the equilibrium in the baseline case and explores its properties. Section 3.4 extends the model to include financial frictions. Dynamic disagreement is studied in Section 3.5, and Section 3.6 concludes. Proofs, details of derivations, complete solutions and numerical method are provided in the Appendices.

3.2 The Baseline Model

There are two groups of agents in the economy, a continuum one of each. Agents within each group are identical and the wealth distribution among them is irrelevant, therefore I represent each group with a representative agent indexed with $j = A, B$. They are identical except that they have different beliefs about the uncertainty in the economy. There is a perishable consumption good from which agents obtain utility. The consumption good is taken as numeraire. There are two assets, a productive asset called capital (risky) and a risk-free asset. The capital stock is endogenous with dynamics described below, and the risk-free asset is in zero net supply. Capital can be used to produce consumption good, and consumption good can be invested to accumulate capital. Both groups can hold capital and produce output with the same productivity.

The model closely follows Di Tella (2012) and Brunnermeier and Sannikov (2014). Unlike these two papers, agents in my model have the same productivity level, and in the baseline case they face no financial constraints. Instead agents have heterogeneous beliefs about the future productivity growth.

3.2.1 Technology and Belief Dispersion

Consider representative agent $j = A, B$ with capital holding k_t^j, where $t \in [0, \infty)$ is time. The agent can use capital to produce flow of output at rate

$$y_t^j = a_t k_t^j$$

per unit of time, where a_t is the current level of productivity in the economy. I model the belief dispersion as different beliefs about the evolution of the technology. Agents $j = A, B$ have their own probability spaces $\left(\Omega, \left\{\mathscr{F}_t^j\right\}, \mathbb{P}^j\right)$, and believe that technology will evolve according to this law of motion

$$\frac{da_t}{a_t} = \mu^j_{a,t} dt + \sigma_a dZ^j_t, \qquad j = A, B, \tag{3.1}$$

where Z^j is a standard Brownian motion under agent j's subjective probability measure $\mathbb{P}^j$, $\mu^j_{a,t}$ is the agent j's subjective belief about technology growth rate, and σ_a is the constant volatility of productivity growth process, on which agents concur. The true dynamics of the productivity growth under the objective probability measure $\mathbb{P}$ is

$$\frac{da_t}{a_t} = \mu_{a,t} dt + \sigma_a dZ_t, \tag{3.2}$$

where Z is a Brownian motion under the objective probability space denoted by $(\Omega, \{\mathscr{F}_t\}, \mathbb{P})$.

Since agents should agree on observables in general and on the productivity growth da_t/a_t in particular, the innovation processes $dZ^j_t, j = A, B$ are linked via

$$dZ^B_t = dZ^A_t + \phi_t dt, \tag{3.3}$$

where

$$\phi_t = \left(\mu^A_{a,t} - \mu^B_{a,t}\right)/\sigma_a \tag{3.4}$$

is dubbed the disagreement process. In the baseline model I assume that $\mu_{a,t} = \mu_a$ is a constant and agents A and B have dogmatic beliefs μ^A_a and μ^B_a about this parameter, which they do not update. This simplification is not crucial and is later relaxed to accommodate more general cases. The underlying intuition and mechanism carries over to the general case. Furthermore, I assume that μ_a lies between both investors' beliefs in this fashion

$$\mu^B_a < \mu_a < \mu^A_a,$$

therefore agent A is more optimistic about the future than agent B, and the disagreement process is $\phi_t = \phi > 0$.

Agents can invest the consumption good to accumulate capital. If representative agent $j = A, B$ invests at i^j_t rate per efficiency unit of capital (i.e. $i^j_t a_t$ per unit of capital), the capital stock held by her evolves according to

$$\frac{dk_t^j}{k_t^j} = \left(\Phi\left(i_t^j\right) - \delta\right)dt, \tag{3.5}$$

where δ is the depreciation rate and $\Phi(.)$ reflects the cost of investment. $\Phi(.)$ is an investment function with adjustment costs, such that $\Phi(0) = 0$, $\Phi^{'} > 0$, and $\Phi^{''} < 0$, and i_t^j is the investment rate per efficiency unit of capital. The concavity of Φ reflects decreasing returns to scale, and for negative values of investment, corresponds to technological illiquidity. An agent needs to invest at the rate of $i_t^j a_t k_t^j$ to accumulate capital at rate $\left(\Phi\left(i_t^j\right) - \delta\right)k_t^j$. This formulation which is similar to Adrian and Boyarchenko (2012) implies that the costs of adjusting capital are higher when technology is more advanced, reflecting the intuition that more developed economies are more specialised and higher technology levels require more investment to acquire and to maintain.

3.2.2 Asset Markets

Agents can trade in two assets, the risk-free asset in zero net supply with instantaneous rate of return of r_t, and capital which is risky and has endogenous supply.

Agents trade capital in the competitive market and the price of capital per efficiency unit is denoted by p_t. Therefore agents can trade capital at rate $p_t a_t$. Both prices p and r are determined in equilibrium. I postulate the following endogenous Ito process for p_t

$$\frac{dp_t}{p_t} = \mu_{p,t}^j dt + \sigma_{p,t} dZ_t^j, \qquad j = A, B. \tag{3.6}$$

Since agents must agree on the prices, the difference in price drifts is given by

$$\mu_{p,t}^A - \mu_{p,t}^B = \sigma_{p,t}\phi_t. \tag{3.7}$$

Because the price of capital is volatile, holding capital is risky. If an agent holds k_t^j units of capital and invests i_t^j per efficiency unit of capital, the value of her capital holding is $p_t a_t k_t^j$, which using Ito's Lemma evolves according to

$$\frac{d\left(p_t a_t k_t^j\right)}{p_t a_t k_t^j} = \left(\Phi\left(i_t^j\right) - \delta + \mu_{p,t}^j + \mu_{a,t}^j + \sigma_a \sigma_{p,t}\right)dt + \left(\sigma_a + \sigma_{p,t}\right)dZ_t^j, \qquad j = A, B. \tag{3.8}$$

Since capital is a productive asset, her capital holding generates a dividend yield as well. Given the capital and investment levels, her total production rate is $a_t k_t^j$, and production rate net of investment is $a_t k_t^j - i_t^j a_t k_t^j$. Therefore the dividend yield from holding capital is

$$\frac{a_t k_t^j - i_t^j a_t k_t^j}{p_t a_t k_t^j} = \left(1 - i_t^j\right) / p_t. \tag{3.9}$$

Combining the returns from the capital gains and the dividend yield, the total returns from holding capital can be summarised as

$$d r_{k,t} = \mu_{k,t}^j dt + \sigma_t dZ_t^j, \qquad j = A, B, \tag{3.10}$$

where $\mu_{k,t}^j$ is the instantaneous expected return on capital under the subjective measure of agents

$$\begin{aligned} \mu_{k,t}^j &\equiv \mathbb{E}_t^j \left[d r_{k,t} / dt \right] \qquad (3.11) \\ &= \frac{\left(1 - i_t^j\right)}{p_t} + \Phi\left(i_t^j\right) - \delta + \mu_{p,t}^j + \mu_{a,t}^j + \sigma_a \sigma_{p,t}, \qquad j = A, B, \end{aligned}$$

and σ_t is the instantaneous volatility of the return on capital

$$\begin{aligned} \sigma_t &\equiv \mathbb{E}_t^j \left[d r_{k,t}^2 / dt \right]^{1/2} \\ &= \sigma_a + \sigma_{p,t}. \qquad (3.12) \end{aligned}$$

The difference in the drifts is derived from (3.4) and (3.7) to be

$$\mu_{k,t}^A - \mu_{k,t}^B = \sigma_t \phi_t. \tag{3.13}$$

The volatility term in (3.10) represents the total risk from holding the risky asset. It has two components, σ_a the fundamental risk, and $\sigma_{p,t}$ the endogenous risk. The fundamental component is assumed to be constant, but the endogenous component is time-varying. As the distribution of wealth across agents changes through time, the endogenous risk varies.

Although agents have different probability measures and process information differently, they must agree on observables and conditions (3.3), (3.4), (3.7), and (3.13) ensure that their beliefs are consistent. We can summarise these *consistency conditions* in the following relationship

$$\frac{\mu_{k,t}^A - \mu_{k,t}^B}{\sigma_t} = \frac{\mu_{p,t}^A - \mu_{p,t}^B}{\sigma_{p,t}} = \frac{\mu_{a,t}^A - \mu_{a,t}^B}{\sigma_a} \equiv \phi_t. \tag{3.14}$$

3.2.3 Agents' Problem

If representative agent $j = A, B$ with wealth w_t^j invests a fraction x_t^j of her wealth in capital, and $1 - x_t^j$ in the risk-free asset, and consumes at rate c_t^j, her wealth evolves according to

$$\begin{aligned} \frac{dw_t^j}{w_t^j} &= x_t^j dr_{k,t} + \left(1 - x_t^j\right) r_t dt - \frac{c_t^j}{w_t^j} dt \\ &= \left(r_t + x_t^j \left(\mu_{k,t}^j - r_t\right) - \frac{c_t^j}{w_t^j}\right) dt + x_t^j \sigma_t dZ_t^j, \end{aligned} \tag{3.15}$$

where the second line follows from (3.10). The portfolio choice of agents can be viewed as a choice about how much risk they want to take. Investing a fraction x_t^j of wealth in capital results in wealth volatility equal to $x_t^j \sigma_t$. Hence, we can rewrite the wealth dynamics in terms of

$$\sigma_{w,t}^j = x_t^j \sigma_t, \tag{3.16}$$

i.e. the exposure of agent j to aggregate risk. Using the new choice variable $\sigma_{w,t}^j$ and defining the price of risk under agents' subjective measures as

$$\pi_t^j = \frac{\mu_{k,t}^j - r_t}{\sigma_t}, \tag{3.17}$$

we can rewrite the wealth dynamics (3.15) as

$$\frac{dw_t^j}{w_t^j} = \left(r_t + \sigma_{w,t}^j \pi_t^j - \hat{c}_t^j\right) dt + \sigma_{w,t}^j dZ_t^j, \tag{3.18}$$

where $\hat{c}_t^j = c_t^j / w_t^j$ is the consumption-wealth ratio of agent j. An agent earns risk-free rate r_t plus a premium π_t^j for each unit of exposure to the aggregate risk that she chooses to take on. Nevertheless, agents disagree about the price of risk in the economy and this disagreement leads to disproportionate risk sharing. Consistency condition (3.14) implies that the disagreement about the risk premium is

$$\pi_t^A - \pi_t^B = \phi_t. \tag{3.19}$$

All agents have Epstein-Zin preferences with the same discount rate ρ, risk aversion γ, and elasticity of intertemporal substitution (EIS) ψ^{-1}.[2] Agent $j = A, B$ faces

[2] For more information see Duffie and Epstein (1992).

the following problem

$$\max_{\hat{c}_t^j, \sigma_{w,t}^j, i_t^j} U^j,$$

subject to the solvency constraint $w_t^j \geq 0$, and the dynamic budget constraint (3.18). Utility U^j is defined recursively as

$$U_t^j = \mathbb{E}_t^j \left[\int_t^\infty f(c_u, U_u) \, du \right],$$

where

$$f(c, U) = \frac{1}{1-\psi} \left\{ \frac{\rho c^{1-\psi}}{((1-\gamma) U)^{\frac{\gamma-\psi}{1-\gamma}}} - \rho (1-\gamma) U \right\}.$$

3.2.4 Equilibrium

Given that the wealth distribution within each group is irrelevant and agents within each group behave similarly, I present the equilibrium conditions in terms of variables for representative agents A and B.

For any given initial condition on the aggregate capital stock K_0 and its distribution among agents $\left\{k_0^j > 0 : j = A, B\right\}$ such that $\sum_{j=A,B} k_0^j = K_0$, an equilibrium is described by a set of stochastic processes: the price of capital per efficiency unit p, interest rate r, wealth processes $\left\{w^j \geq 0\right\}$, capital holdings $\left\{k^j\right\}$, investment decisions $\left\{i^j\right\}$, and consumption choices $\left\{c^j\right\}$ of agents $j = A, B$ such that

1. initial wealth satisfy $w_0^j = p_0 a_0 k_0^j$,
2. each agent solves her problem, taking prices as given, and
3. markets for consumption good and capital clear, i.e.,

$$\sum_{j=A,B} c_t^j = \sum_{j=A,B} a_t k_t^j \left(1 - i_t^j\right), \tag{3.20}$$

$$\sum_{j=A,B} k_t^j = K_t, \tag{3.21}$$

where

$$dK_t = \left(\sum_{j=A,B} \left(\Phi\left(i_t^j\right) - \delta\right) k_t^j \right) dt.$$

Market clearing for the risk-free asset follows from Walras' Law.

3.3 Solving the Baseline Model

The solution methodology in this section closely follows Di Tella (2012). In the baseline model, the disagreement is static and there are no funding constraints. Even a static disagreement is enough to induce stochastic dynamics in investment opportunities, prices, and investment and portfolio decisions by agents. These dynamics are described by how wealth is distributed among the two groups, and a single state variable defined as

$$s_t = \frac{w_t^A}{w_t^A + w_t^B},$$

which is the share of wealth of representative agent A, is enough to portray the state of the economy. In Section 3.3.4, it is shown that the equilibrium conditions above can be restated in terms of s_t, and hence it is enough to describe the system in terms of it. Since the technology is linear in capital, the economy is scale-invariant and we can abstract from the level of capital stock. I solve for the equilibrium by solving the agents' optimisation problem and imposing market-clearing conditions and I look for a Markov equilibrium with state variable s_t. When we allow for time-varying disagreement, a second state variable is required.

The state variable s_t can be interpreted in terms of the strength of agent A balance sheet relative to agent B. In an economy with homogeneous beliefs, s_t will be constant and there is no balance sheet channel. With belief dispersion risk will be concentrated on the balance sheet of optimists and negative shocks reduces their share of wealth, generating volatility in prices and investment.

For consistency, I write down the equilibrium dynamics mostly under the subjective measure of representative agent A, which is called measure A. It is easy to change the measure to the objective measure or the subjective measure of agent B (measure B) using the relevant consistency condition.

I employ the dynamic programming approach to solve the agents' problem. I conjecture that the value functions have the following form

$$V_t^A = \frac{\left(\xi_t w_t^A\right)^{1-\gamma}}{1-\gamma},$$

for agent A, and

$$V_t^B = \frac{\left(\zeta_t w_t^B\right)^{1-\gamma}}{1-\gamma},$$

for agent B. The processes ξ_t and ζ_t are wealth multipliers for agents A and B and capture the forward looking stochastic investment opportunities that agents face. For a given level of wealth, agents can attain higher utility with when ξ_t or ζ_t are larger, therefore these quantities capture how attractive investment opportunities are. I assume the following dynamics for these processes that need to be derived endogenously in equilibrium

$$\frac{d\xi_t}{\xi_t} = \mu_{\xi,t} dt + \sigma_{\xi,t} dZ_t^A,$$

$$\frac{d\zeta_t}{\zeta_t} = \mu_{\zeta,t} dt + \sigma_{\zeta,t} dZ_t^B.$$

Notice that the dynamics for the wealth multipliers are written under each agent's respective probability measure.

To characterise the equilibrium, first I solve for all the equilibrium outcome variables in terms of p, ξ, and ζ (and their derivatives with respect to s_t), which in turn are functions of the state variable themselves, i.e.

$$p_t = p(s_t), \quad \xi_t = \xi(s_t), \quad \zeta_t = \zeta(s_t),$$

and are conjectured to be twice continuously differentiable. Because p, ξ, and ζ are functions of s_t, their dynamics are linked to the law of motion of s_t by Ito's Lemma as described in Appendix 3.7.5. Once these functions and the law of motion of s_t are determined, $\sigma_p, \sigma_\xi, \sigma_\zeta, \mu_p, \mu_\xi$, and μ_ζ are all known functions of s_t as well. Finally, I derive a system of differential equations for p, ξ, and ζ by imposing the equilibrium conditions. This system is solved numerically to find p, ξ, and ζ, which are used to compute all other desired equilibrium quantities.

The rest of this section is laid out as follows. First I describe the agents' problem in terms of the Hamilton-Jacobi-Bellman equations. These equations are then used to pin down agents' optimal choices of investment, consumption and portfolio in terms of p, ξ, and ζ. Next the law of motion of s_t is derived and the equilibrium is restated in terms of a Markov equilibrium, and the conditions are rewritten in terms of s_t. Combining these conditions with agents' optimal choices, I discuss the properties of the equilibrium, in particular capital holdings and risk sharing, and compare it with a homogeneous-belief benchmark.

3.3.1 Agents' HJB Equations

We can use the conjectured value function to derive the Hamilton-Jacobi-Bellman equation for each agent. Agents A and B solve the following HJB equations respectively

$$\frac{\rho}{1-\psi} = \max_{\hat{c}^A, \sigma_w^A, i^A} \left\{ \frac{\left(\hat{c}^A\right)^{1-\psi}}{1-\psi} \rho \xi^{\psi-1} + r + \sigma_w^A \pi^A - \hat{c}^A + \mu_\xi - \frac{\gamma}{2} \left(\left(\sigma_w^A\right)^2 + \sigma_\xi^2 - 2\frac{1-\gamma}{\gamma} \sigma_w^A \sigma_\xi \right) \right\}, \quad (3.22)$$

$$\frac{\rho}{1-\psi} = \max_{\hat{c}^B, \sigma_w^B, i^B} \left\{ \frac{\left(\hat{c}^B\right)^{1-\psi}}{1-\psi} \rho \zeta^{\psi-1} + r + \sigma_w^B \pi^B - \hat{c}^B + \mu_\zeta - \frac{\gamma}{2} \left(\left(\sigma_w^B\right)^2 + \sigma_\zeta^2 - 2\frac{1-\gamma}{\gamma} \sigma_w^B \sigma_\zeta \right) \right\}. \quad (3.23)$$

3.3.2 Investment Choice

Investment shows up in agents' HJB equation only through the instantaneous expected return on capital (3.11) (through the risk premiums $\pi^j, j = A, B$). Every agent who holds capital chooses investment to maximise the instantaneous expected return on capital. Therefore, optimal investment per efficiency unit of capital solves

$$\max_i \frac{(1-i)}{p_t} + \Phi(i).$$

This objective pins down the investment choice at each moment in the following way

$$\Phi'(i) = \frac{1}{p_t}.$$

This condition shows that investment solely depends on the current price of capital per efficiency unit, and agents show the same investment behaviour, i.e.,

$$i_t^A = i_t^B = i(p_t). \quad (3.24)$$

In addition, higher prices lead to more investment because

$$\begin{aligned} i'(p) &= -\frac{1}{\Phi''(i(p))} \frac{1}{p^2} \\ &= -\frac{\Phi'(i)^2}{\Phi''(i)} > 0. \end{aligned} \quad (3.25)$$

The optimal investment rule (3.24) is incorporated in the expressions for return on capital $dr_{k,t}$ hereafter.

3.3.3 Consumption Choice

The first-order conditions of the HJB equations (3.22) and (3.23) pin down the agents' consumption choices

$$\hat{c}_t^A = \rho^{\frac{1}{\psi}} \xi_t^{-\frac{1-\psi}{\psi}}, \tag{3.26}$$

$$\hat{c}_t^B = \rho^{\frac{1}{\psi}} \zeta_t^{-\frac{1-\psi}{\psi}}. \tag{3.27}$$

If the elasticity of intertemporal substitution $\psi^{-1} > 1$, agents increase their consumption relative to their wealth when investment opportunities are low and vice versa (the substitution effect dominates). On the other hand, EIS < 1 means that the consumption-wealth ratio is increasing in the investment opportunities (the income effect dominates).

Before proceeding to the portfolio choice problem of agents, I review the equilibrium concept in the next section.

3.3.4 Equilibrium Revisited

In this section the equilibrium is restated in terms of a Markov equilibrium, and the conditions are rewritten in terms of s_t. First I characterise the law of motion of $s_t = w_t^A / \left(w_t^A + w_t^B\right)$ in the following Lemma using the agents dynamic budget constraints.

Lemma 1. *The equilibrium law of motion of s_t under probability measure A is*

$$\frac{ds_t}{s_t} = \mu_{s,t}^A dt + \sigma_{s,t} dZ_t^A, \tag{3.28}$$

where

$$\mu_{s,t}^A = \sigma_{w,t}^A \pi_t^A - \sigma_t \left(\pi_t^A + \sigma_{w,t}^A\right) + \sigma_t^2 + (1 - s_t)\left(\hat{c}_t^B - \hat{c}_t^A\right),$$

and

$$\begin{aligned} \sigma_{s,t} &= \sigma_{w,t}^A - \sigma_t \\ &= \sigma_{w,t}^A - \left(\sigma_a + \sigma_{p,t}\right). \end{aligned}$$

Proof: See Appendix 3.7.2.

Next, I redefine the equilibrium conditions in terms of a Markov equilibrium. For brevity, subscript t is dropped from most of the variables hereafter. I have already imbedded the optimal investment choice of agents in the definition below and I have used the fact that investment is only a function of the price of capital. We are looking for a Markov equilibrium which is a set of functions p, ξ, and ζ in s and policy functions $\hat{c}^j, \sigma_w^j, i^j, j = A, B$, such that

1. the policy functions are optimal given the HJB equations (3.22) and (3.23), and ξ and ζ satisfy the HJB equations,

2. the consumption good market clears according to

$$s\hat{c}^A + (1-s)\hat{c}^B = \frac{1-i(p)}{p}, \tag{3.29}$$

3. the capital market clears according to

$$s\sigma_w^A + (1-s)\sigma_w^B = \sigma_a + \sigma_p, \tag{3.30}$$

4. the law of motion of s satisfies (3.28).

Technology and capital levels can be recovered from (3.1) and (3.5). Interest rate r and market prices of risk $\pi^j, j = A, B$ are functions of p, ξ, and ζ. The market clearing conditions (3.29) and (3.30) are derived in Appendix 3.7.3.

3.3.5 Capital Holdings and Risk Sharing

The portfolio choice of agents is pinned down by the first-order condition of their HJB equations. The exposure of representative agent A is given by

$$\sigma_w^A = \frac{\pi^A}{\gamma} - \frac{\gamma - 1}{\gamma}\sigma_\xi. \tag{3.31}$$

The first term in (3.31) represents the myopic demand and the second term is driven by the hedging motive due to the stochastic investment opportunities. A risk-averse agent prefers to have more wealth when ξ_t is low to smooth her utility, while a risk-neutral agent prefers higher wealth when ξ_t is high to achieve a higher level of expected utility. Agent B follows a similar rule to choose her exposure

$$\sigma_w^B = \frac{\pi^B}{\gamma} - \frac{\gamma - 1}{\gamma}\sigma_\zeta. \tag{3.32}$$

Disproportionate risk sharing between the two groups generates the balance sheet channel. Using (3.31), (3.32), and (3.19) we obtain an expression for the difference between the risk exposures of agents A and B

$$\sigma_w^A - \sigma_w^B = \frac{\phi}{\gamma} + \Delta, \tag{3.33}$$

where Δ is the relative hedging motive of agents defined as

$$\Delta \equiv -\frac{\gamma-1}{\gamma}\left(\sigma_{\xi}-\sigma_{\zeta}\right). \tag{3.34}$$

The difference in risk exposures has two components. The first component, ϕ/γ, is a positive constant in the baseline model. According to our assumption, agent A is more optimistic about the risk and she attaches a higher risk-premia to the risky asset. Therefore her myopic demand is always larger than the myopic demand of agent B. The second component is the relative hedging motive of agents, Δ, which is itself induced by the disagreement and risk aversion and is therefore dynamic.

We can link the strength of the agent A balance sheet to the difference in the risk exposures (3.33). From Lemma 1, we know that $\sigma_{s,t} = \sigma^A_{w,t} - \sigma_t$. We can use the capital market clearing condition (3.30) and the definition of σ_t in (3.12) to rewrite this as

$$\begin{aligned}\sigma_s &= \sigma^A_w - \left(s\sigma^A_w + (1-s)\sigma^B_w\right)\\ &= (1-s)\left(\sigma^A_w - \sigma^B_w\right).\end{aligned}$$

Using (3.33) we obtain

$$\sigma_s = (1-s)\left[\frac{\phi}{\gamma}+\Delta\right]. \tag{3.35}$$

In equilibrium, ξ and ζ are functions of s, therefore the relative hedging motive is itself a function of σ_s by Ito's Lemma (see Appendix 3.7.5 for the details). Ultimately we obtain

$$\sigma_s = \frac{(1-s)\frac{\phi}{\gamma}}{1+s(1-s)\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{\xi_s}{\xi}-\frac{\zeta_s}{\zeta}\right)}. \tag{3.36}$$

Since s is the state variable, its dynamics drives all other stochastic variables. In Section 3.2.2 total risk in the economy σ_t is decomposed into two components of fundamental and endogenous risks. The fundamental component σ_a is constant. Using (3.36) and Ito's Lemma, the endogenous component is

$$\begin{aligned}\sigma_p &= \frac{p_s}{p}s\sigma_s\\ &= \frac{s(1-s)\frac{p_s}{p}\frac{\phi}{\gamma}}{1+s(1-s)\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{\xi_s}{\xi}-\frac{\zeta_s}{\zeta}\right)}.\end{aligned} \tag{3.37}$$

The foremost contributor to price volatility is the disagreement. Under the conditions discussed below, it is going to generate a positive price volatility that amplifies the fundamental risk. The risk-aversion parameter γ on the other hand has a dampening effect on the balance sheet channel. If the agents are very risk averse, they do not over-expose themselves to the risky asset as much, even if they are very optimistic about it. Finally the sign of price derivative with respect to s determines the sign of σ_p. It turns out that the sign of p_s depends on EIS parameter ψ^{-1}. If EIS > 1, p_s is positive and price is increasing in group A share of wealth s, because the natural buyers cut investment and consume more with weak balance sheets and high risk premium. With EIS < 1, the p_s would be negative and price volatility would diminish the fundamental risk. If CRRA preferences were used instead of EZ, risk aversion and EIS were tangled and could not be separately studied. This is the reason why Baker et al. (2014) focus only on the risk-aversion parameter less than one, otherwise balance sheet channel has a dampening rather than amplifying effect. I can allow for $\gamma > 1$ case, which is the empirically relevant case, without hampering the balance sheet channel due to using EZ preferences.

Figure 3.1 illustrates how risk aversion γ and EIS ψ^{-1} affect the equilibrium risk sharing. In the baseline case (blue solid line), both risk aversion and EIS are bigger than one. The red dash-dotted line depicts the equilibrium quantities when $\gamma < 1$ and EIS > 1, the green dashed line when $\gamma > 1$ and EIS < 1, and the black dotted line when both $\gamma < 1$ and EIS < 1. In all cases $\sigma_s > 0$, so negative shocks to technology growth decreases the share of wealth of optimists (natural buyers). In other words, the inherent interpretation of the model is that lower values of s (when natural buyers are poorly capitalised) correspond to slumps while higher values comprise normal times and booms. Negative technology shocks not only weaken optimists' balance sheet and reduce their share of wealth, but also increase its volatility.

As Figure 3.1 shows, to rule out unrealistic price changes during downturns, EIS needs to be greater than 1. If EIS > 1, intertemporal substitution effect dominates and agents cut back on investment in the asset market and increase their consumption when investment opportunities are low, therefore price of capital and investment fall down in a slump, as expected. But when EIS < 1, the price of capital and investment in capital both increase in a downturn. In other words in this case a negative shock to technology, increases the price of the productive asset, dampening the effect of the original shock ($\sigma_p < 0$) instead of amplifying it ($\sigma_p > 0$).

Figure 3.1 also depicts the relative hedging motive of agents, Δ, under different scenarios. From (3.33) we know that Δ is one of the components of the difference in the risk exposures. The other component, ϕ/γ, is decreasing in risk aversion and is higher when agents are less risk averse. On the other hand, γ has a different effect on the relative hedging motive. When $\gamma < 1$, the relative hedging motive is negative and reduces the difference in the risk exposures. When $\gamma > 1$, agents are more risk averse, and the relative hedging motive is positive and increases the difference. As evident from the σ_s plot, overall the direct effect of γ is greater than the indirect effect through the hedging motive.

Figure 3.2 shows how equilibrium outcomes change with the disagreement ϕ. The curves with red dash-dotted line have higher disagreement (by reducing the fundamental volatility) relative to the ones with solid blue line. As a result, the relative hedging motive, price volatility, state variable volatility, optimists fraction of wealth invested in capital (or leverage, x^A), the interest rate, the price of capital and investment increase and risk premium decreases.

3.3.6 Benchmark with Homogeneous Beliefs

If agents have homogeneous beliefs ($\phi = 0$), the balance sheet channel vanishes. In Equation (3.35) both the direct effect via ϕ itself, and the indirect channel via the hedging motives disappear. The price is constant, and there is a balanced growth path in which the economy grows at a constant rate. Price is a fixed point in the following relationship

$$p = \frac{1 - i(p)}{\rho - (1-\psi)\left(\Phi(i(p)) - \delta + \mu_a - \frac{\gamma}{2}\sigma_a^2\right)}, \tag{3.38}$$

where μ_a is the consensus technology growth rate. For the derivation and the regularity conditions under which this solution is admissible, see Appendix 3.7.4.

3.4 Financing Constraints

In this section I examine the model when financing constraints are introduced. In the baseline model there are no constraints on financing and firms can raise debt or equity. However, since agents disagree about the prospects of the economy, they would not invest in the equity of agents in the other group. Therefore introducing a constraint on raising equity would not change the equilibrium studied in the previous section and I focus on two other restrictions.

The first constraint is the short-selling restriction. If agents are pessimist enough about the risky asset, they may want to short sell it in the baseline model and depending on the parameter values, short-selling can occur in equilibrium. Since this paper deals with physical capital first and foremost, it is natural to think that short selling may be infeasible. While I study cases both with and without short selling, similar papers like Brunnermeier and Sannikov (2014) and Di Tella (2012) rule out short-selling altogether, and Baker et al. (2014) do not consider the model with constraints at all.

The short-selling constraint only affects the pessimists' problem (group B). There cannot be an equilibrium where the optimists also want to sell the capital short, since capital is in positive supply. Group B agents solve their HJB equation (3.23) subject to the non-negativity constraint $\sigma^B_{w,t} \geq 0$. As a result, optimal exposure to risk changes from (3.32) to

$$\sigma_w^B = \frac{\pi^B}{\gamma} - \frac{\gamma - 1}{\gamma}\sigma_\zeta + \frac{\lambda^B}{\gamma}, \tag{3.39}$$

where $\lambda^B \geq 0$ is the Lagrange multiplier of the non-negativity constraint. When the constraint is not binding, λ^B is zero and the portfolio choice is unrestricted. If the sum of the myopic and hedging demands is negative, the agent stays out of the market for the risky asset such that $\sigma_w^B = 0$ and $\lambda^B > 0$. In this case the entire capital stock is held by optimists. This change in the portfolio choice affects the aggregate risk sharing and as a result the volatility of the state variable changes to

$$\sigma_s = (1-s)\left[\frac{\phi - \lambda^B}{\gamma} + \Delta\right], \tag{3.40}$$

where Δ is the relative hedging motive defined in (3.34). The short-selling restriction has two effects on σ_s. Since $\lambda^B \geq 0$, comparing (3.40) with (3.35) shows the fact that short-selling constraint limits the concentration of the risk on optimists' balance sheet and dampens the balance sheet channel. This is the direct channel. On the other hand, there is an indirect channel through the relative hedging term Δ. Importantly, the direct channel is active only when the constraint binds, but the indirect effect also exists in the range where the constraint does not bind. The overall effect is ambiguous as I illustrate later.

The second type of constraint that I consider is the leverage constraint. This restriction may be imposed by regulators or can arise endogenously if agents are subject to idiosyncratic discontinuous shocks. Agents' leverage (the ratio of their asset holding to their wealth) cannot exceed a certain threshold. This restriction only affects the optimists' problem (group A). They want to borrow from the other group to hold more assets than their wealth, and this constraint limits their ability to do so. On the other hand, pessimists never hold more assets than their wealth and are net lenders, so they are unaffected by the leverage restriction. Group A agents solve their HJB equation (3.22) subject to the leverage constraint $\sigma_{w,t}^A \leq \sigma_t/\theta$, where $\theta \in (0,1)$ is a constant. Due to (3.16), this restriction is equivalent to $x_t^A \leq 1/\theta$, i.e. a constraint on the fraction of wealth invested in the risky asset (e.g. $\theta = 1/3$ means that agents cannot invest more than three times their wealth in the risky asset). As a result, optimal exposure to risk changes from (3.31) to

$$\sigma_w^A = \frac{\pi^A}{\gamma} - \frac{\gamma - 1}{\gamma}\sigma_\xi - \frac{\lambda^A}{\gamma}. \tag{3.41}$$

where $\lambda^A \geq 0$ is the Lagrange multiplier of the leverage constraint. When the constraint is not binding, λ^A is zero and the portfolio choice is unrestricted. If the sum of the myopic and hedging demands exceeds the leverage threshold, then λ^A is positive and leverage will be bound at $\sigma_w^A = \sigma_t/\theta$. In this case agent A holds less capital than she wishes to do and part of the capital stock is offloaded to agent B. This change in the portfolio choice affects the aggregate risk sharing and as a result the volatility of the state variable changes to

$$\sigma_s = (1-s)\left[\frac{\phi - \lambda^A}{\gamma} + \Delta\right]. \tag{3.42}$$

Similar to the previous case, there can be two effects. Since $\lambda^A \geq 0$, comparing (3.42) with (3.35) shows that the leverage constraint also limits the concentration of the risk on optimists' balance sheet and dampens the balance sheet channel. This is the direct channel. On the other hand, there is an indirect channel through the relative hedging term Δ. The direct channel is active only when the constraint binds, but the indirect effect also exists in the range where the constraint does not bind.

Figure 3.3 compares the solution in the baseline case to a model where both restrictions are active. For a range of the domain of the state variable s, these restrictions have anticipated effects from their direct channel, i.e. endogenous volatility is decreased and the balance sheet channel dampened. However there is another range of intermediate values, where in fact the indirect channel through the hedging motives dominates. In this region risk is more concentrated on optimists' balance sheet than the benchmark case, and endogenous volatility and the volatility of s are higher.

To summarise, unlike the macro models with financial constrains like Kiyotaki and Moore (1997), Bernanke et al. (1999), Brunnermeier and Sannikov (2014), and He and Krishnamurthy (2012), in this model financing restrictions can have ambiguous effects. In some regions they mitigate the endogenous risk and diminish the balance sheet channel by limiting the concentration of the aggregate risk while in others they have the opposite effect. For a full characterisation of the solution with financing constraints, see Appendix 3.7.6.

3.5 Dynamic Disagreement

In this section the baseline model is extended to the cases with time-varying disagreement. The extension with time-varying disagreement can accommodate two channels for belief dispersion: (i) different subjective priors with updating as in Basak (2000) or Buraschi and Jiltsov (2006); and (ii) different subjective models as in David (2008), Dumas et al. (2009), and Whelan (2014). In all cases agents have the same information set, but agree to disagree about how to process information, which is represented by different filtrations.

In the first case, agents start from different subjective priors about μ_a which they update through time in a Bayesian fashion, and therefore their disagreement changes with time. In the second case agents can have disagreement about the correct model of the technology growth dynamics, and they can learn about it from the realised technology process.

In either case, the disagreement process is stochastic. Let's assume it has the following law of motion under measure A

$$d\phi_t = \mu_{\phi,t} dt + \sigma_{\phi,t} dZ_t^A. \tag{3.43}$$

The terms $\mu_{\phi,t}$ and $\sigma_{\phi,t}$ depend on the particular channel for belief dispersion and the specifics of the information environment.[3] Here I abstract from those specifics and assume that the disagreement dynamics are exogenously given by (3.43).

When the disagreement is time-varying, we need another state variables to characterise the Markov equilibrium, the disagreement ϕ_t. Therefore we need to find functions p, ξ, and ζ such that

$$p_t = p(\phi_t, s_t), \quad \xi_t = \xi(\phi_t, s_t), \quad \zeta_t = \zeta(\phi_t, s_t),$$

which are conjectured to be twice-continuously differentiable. The solution approach is very similar to the baseline model, save for the fact that p, ξ, and ζ are now functions of two variables. Agents' HJB equations remain the same as before, and optimal investment and consumption choices do not change. Capital and consumption good market clearing conditions stay unchanged as well. Agents optimal risk exposures will still be (3.31) and (3.32), and hence aggregate risk sharing rule is given by (3.35), i.e.

$$\sigma_s = (1-s)\left[\frac{\phi}{\gamma} + \Delta\right],$$

where $\Delta = -\frac{\gamma-1}{\gamma}\left(\sigma_\xi - \sigma_\zeta\right)$. Because ξ and ζ are now multivariate functions, the relative hedging motive term now has two components, a part driven by the wealth distribution dynamics (as before), and a part driven by changes in the disagreement (new). Formally, I derive expressions for σ_ξ and σ_ζ using Ito's Lemma (provided in Appendix 3.7.7), from which I obtain the following expression for the relative hedging

[3] In one such specification for example is similar to Whelan (2014), where agents agree on the long run technology growth rate, but have subjective beliefs on the persistence of the growth rate shocks.

motive term

$$\Delta = -\frac{\gamma - 1}{\gamma}\left[\underbrace{\left(\frac{\xi_s}{\xi} - \frac{\zeta_s}{\zeta}\right)s\sigma_s}_{\text{endogenous}} + \underbrace{\left(\frac{\xi_\phi}{\xi} - \frac{\zeta_\phi}{\zeta}\right)\sigma_\phi}_{\text{exogenous}}\right]. \tag{3.44}$$

The first term is endogenous with respect to the wealth distribution, and the second term is exogenous. Because of the endogenous term, there is a feedback between the relative hedging motive term and the aggregate risk-sharing rule. Plugging (3.44) into the expression for σ_s and solving for it explicitly, we obtain the following expression for the volatility of the agent A wealth ratio which is analogous to (3.36) in the baseline model

$$\sigma_s = \frac{(1-s)\left[\overbrace{\frac{\phi}{\gamma}}^{\text{static}} \overbrace{-\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{\xi_\phi}{\xi} - \frac{\zeta_\phi}{\zeta}\right)\sigma_\phi}^{\text{dynamic}}\right]}{1 + s(1-s)\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{\xi_s}{\xi} - \frac{\zeta_s}{\zeta}\right)}. \tag{3.45}$$

The disagreement now contributes to the disproportionate risk sharing in two channels: the static effect as the baseline model and the dynamic effect due to time-variation in the disagreement which affects the relative investment opportunity sets and hedging motives. The risk-sharing rule in the baseline model (3.36) only contains the static effect ϕ/γ.

Empirical evidence and economic intuition tells us that uncertainty and belief dispersion increase during economic downturns. Baker et al. (2013), among others, provide evidence for this claim. In the model, this phenomenon corresponds to $\sigma_\phi \leq 0$. A negative shock to technology growth decreases optimists' share of wealth and increases the disagreement ϕ_t, which increases σ_s and endogenous volatility σ_p, amplifies the effects of the original shock and worsens risk-sharing. Therefore, the dynamic effect of belief dispersion provides further amplification mechanism for the balance sheet channel in addition to the baseline static effect.

With the new expression for σ_s in (3.45), the rest of the solution is similar to the baseline model, aside from the drift terms μ_p, μ_ξ, and μ_ζ that should change to reflect that p, ξ, and ζ are functions of ϕ as well. Expressions for the new drifts plus the modifications to the numerical solution are provided in Appendix 3.7.7.

3.6 Conclusion

In this paper I examine balance sheet recessions in a general equilibrium model when agents have heterogeneous beliefs. Agents have different subjective beliefs about the future technology growth rate. I show a channel which describes the risk concentration via heterogeneous beliefs rather than ad-hoc constraints on aggregate risk sharing, and I demonstrate that the nonlinear relationship between financial variables and real investment need not be the result of financial frictions. In my model aggregate consumption, growth rates, and investment exhibit stochastic volatility endogenously, and endogenous stochastic consumption volatility arises from constant fundamental volatility. Even with static disagreement, prices and investment opportunities are stochastic.

In addition, I examine the effect of financing constraints on the equilibrium when there is belief dispersion among agents. In my model financing constraints have an ambiguous effect on the equilibrium risk sharing. In some regions they mitigate the endogenous risk and diminish the balance sheet channel by limiting the concentration of the aggregate risk while in others they have the opposite effect. Finally I extend the model to accommodate dynamic belief dispersion in a general way to accommodate different cases.

3.7 Appendix

3.7.1 Figures

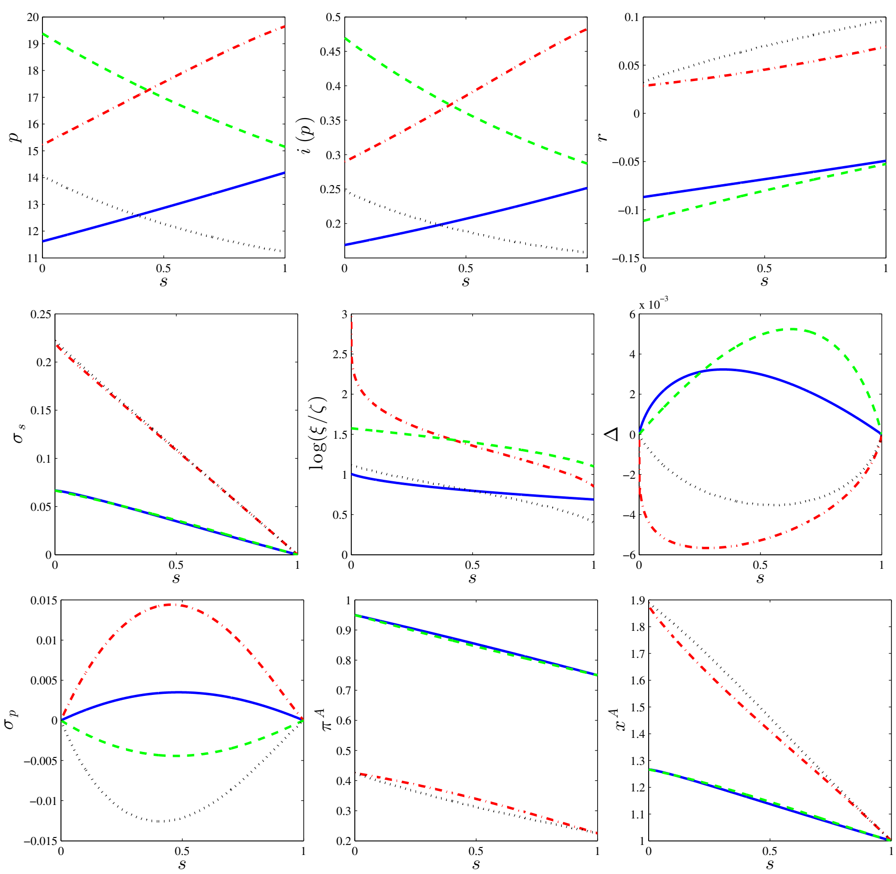


Figure 3.1: The price of capital p, investment i, interest rate r, volatility of s, σ_s, log of the investment opportunities ratio $\log(\xi/\zeta)$, relative hedging motive Δ, price volatility σ_p, market price of risk (as perceived by agent A) π^A, and agent A's fraction of wealth invested in capital x^A, for various combinations of γ and ψ^{-1}. $\gamma = 3$ and $\psi^{-1} = 1.5$: solid blue, $\gamma = 0.9$ and $\psi^{-1} = 1.5$: dash-dotted red, $\gamma = 3$ and $\psi^{-1} = 2/3$: dashed green, $\gamma = 0.9$ and $\psi^{-1} = 2/3$: dotted black.

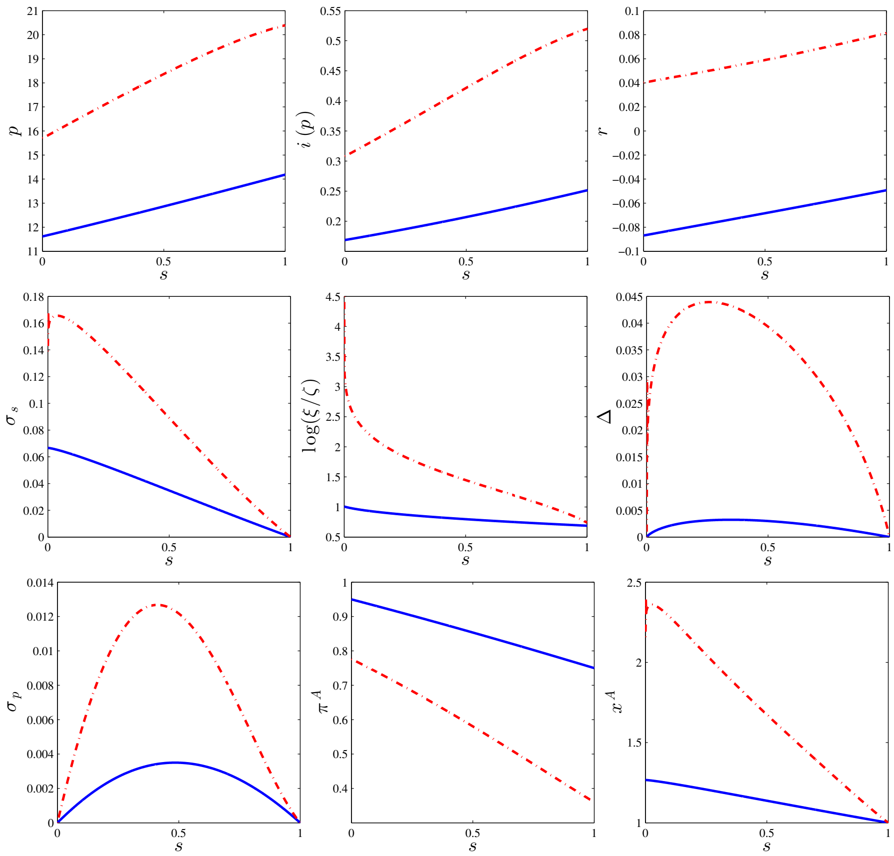


Figure 3.2: The price of capital p, investment i, interest rate r, volatility of s, σ_s, log of the investment opportunities ratio $\log(\xi/\zeta)$, relative hedging motive Δ, price volatility σ_p, market price of risk (as perceived by agent A) π^A, and agent A's fraction of wealth invested in capital x^A, for $\phi = 0.2$: solid blue, and $\phi = 0.42$: dash-dotted red.

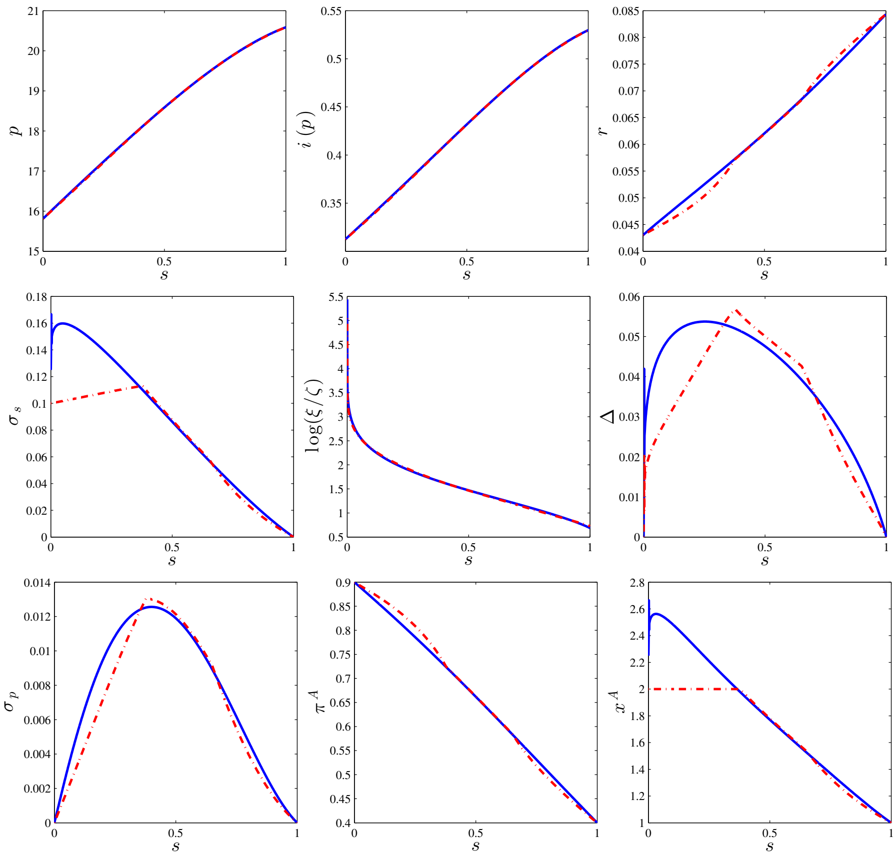


Figure 3.3: The price of capital p, investment i, interest rate r, volatility of s, σ_s, log of the investment opportunities ratio $\log(\xi/\zeta)$, relative hedging motive Δ, price volatility σ_p, market price of risk (as perceived by agent A) π^A, and agent A's fraction of wealth invested in capital x^A, for unrestricted model: solid blue, and restricted model (both short-selling and leverage constraints with $\theta = 1/2$): dash-dotted red.

3.7.2 Proof of Lemma 1

We can use (3.3) to rewrite the wealth dynamics of agent B in (3.18) under measure A

$$\begin{aligned}\frac{dw_t^B}{w_t^B} &= \left(r_t + \sigma_{w,t}^B \pi_t^B - \hat{c}_t^B + \sigma_{w,t}^B \phi_t\right) dt + \sigma_{w,t}^B dZ_t^A, \\ &= \left(r_t + \sigma_{w,t}^B \pi_t^A - \hat{c}_t^B\right) dt + \sigma_{w,t}^B dZ_t^A,\end{aligned}$$

where the second line follows from (3.19). Combining this law of motion with the wealth dynamics of agent A in (3.18) using Ito's Lemma, the law of motion of the total wealth in the economy is

$$\begin{aligned}\frac{d\left(w_t^A + w_t^B\right)}{w_t^A + w_t^B} &= \left(r_t + \pi_t^A\left(s_t \sigma_{w,t}^A + (1-s_t)\sigma_{w,t}^B\right) - \left(s_t \hat{c}_t^A + (1-s_t)\hat{c}_t^B\right)\right) dt + \sigma_t dZ_t^A, \\ &= \left(r_t + \sigma_t \pi_t^A - \frac{1-i(p_t)}{p_t}\right) dt + \sigma_t dZ_t^A,\end{aligned}$$

where the second line follows from the goods and capital market clearing conditions (3.29) and (3.30). Since $s_t = w_t^A / \left(w_t^A + w_t^B\right)$, and we have the law of motion of w_t^A and $w_t^A + w_t^B$ separately, we can employ Ito's Lemma again to get the desired result stated in Lemma 1.

3.7.3 Derivation of the Market Clearing Conditions (3.29) and (3.30)

According to (3.24), agents invest at the same rate, which only depends on prices, therefore the right-hand side of equation (3.20) becomes

$$a_t(1 - i(p_t)) \sum_{j=A,B} k_t^j = a_t(1 - i(p_t)) K_t.$$

In the left-hand side, I substitute c_t^j with $w_t^j \hat{c}_t^j$, and then divide both sides by the total wealth in the economy. Since the risk-free asset is in zero net supply, the aggregate wealth in the economy is equal to the aggregate value of capital, i.e.

$$w_t^A + w_t^B = p_t a_t K_t. \tag{3.46}$$

Hence, dividing both sides of by $p_t a_t K_t$, leads to (3.29).

For the capital market clearing, first from Section 3.2.3, remember that x_t^j is the fraction of wealth of agent j invested in capital. Hence the money invested by agent j in capital is $x_t^j w_t^j$ and the capital holding is

$$k_t^j = \frac{x_t^j w_t^j}{p_t a_t}.$$

Therefore, we can rewrite the capital market clearing condition in (3.21) as

$$\frac{x_t^A w_t^A}{p_t a_t} + \frac{x_t^B w_t^B}{p_t a_t} = K_t$$

$$\Rightarrow s_t x_t^A + (1 - s_t) x_t^B = 1, \tag{3.47}$$

where I have used (3.46), and the definition of s_t. The last step is to multiply both sides of (3.47) by $\sigma_t = \sigma_a + \sigma_{p,t}$, which results in equation (3.30).

3.7.4 Derivation of the Homogeneous-Belief Price

Equation (3.36) hints that in the absence of disagreement, $\sigma_{s,t} = 0, \forall t$, and the share of wealth of agents remains constant through time at initial values. Consequently, p, ξ, and ζ are constants with no dynamics (zero drifts and volatilities). Investment per efficiency unit of capital is constant and identical for agents and is equal to $i(p)$ according to (3.24). Moreover, agents face the same investment opportunity set, therefore $\xi = \zeta$, which together with (3.26), (3.27), and (3.29) means that consumption wealth ratio for agents is

$$\hat{c}^A = \hat{c}^B = \rho^{\frac{1}{\psi}} \xi^{-\frac{1-\psi}{\psi}} = \frac{1 - i(p)}{p}.$$

Agents choose their portfolios similarly, and since there are no hedging motives, exposure to risk is

$$\sigma_w^A = \sigma_w^B = \frac{\pi}{\gamma} = \sigma_a,$$

where the last equality follows from the capital market clearing condition (3.30). As a result, the market price of risk is $\pi = \gamma\sigma_a$, which along with (3.17) and (3.11) yields this expression for the interest rate

$$r = \frac{1 - i(p)}{p} + \Phi(i(p)) - \delta + \mu_a - \gamma\sigma_a^2.$$

Finally we can substitute the above quantities in the agents' HJB equations (3.22) and (3.23) (which are now identical) to arrive at the equation (3.38) which implicitly defines the price.

3.7.5 Solving for the Equilibrium

In this section, I derive a system of differential equations for p, ξ, and ζ by imposing the equilibrium conditions. This system is solved numerically to find these functions, which are used to compute all other desired equilibrium quantities.

I start from Equation (3.36) which provides a solution for σ_s. Because p, ξ, and ζ are functions of s_t, their volatilities are linked to σ_s by Ito's Lemma according to

$$\begin{aligned}\sigma_p &= \frac{p_s}{p} s \sigma_s, \\ \sigma_\xi &= \frac{\xi_s}{\xi} s \sigma_s, \\ \sigma_\zeta &= \frac{\zeta_s}{\zeta} s \sigma_s. \end{aligned} \tag{3.48}$$

Lemma 1 implies agent A's exposure to risk is $\sigma_w^A = \sigma_s + \sigma_a + \sigma_p$, which can be used to determine her perceived market price of risk

$$\pi^A = \gamma \sigma_w^A + (\gamma - 1)\sigma_\xi,$$

using (3.31). Agent B's perceived risk premium is linked to π^A by the consistency condition $\pi^B = \pi^A - \phi$, which I use to derive σ_w^B from (3.32)

$$\sigma_w^B = \frac{\pi^B}{\gamma} - \frac{\gamma - 1}{\gamma}\sigma_\zeta.$$

Next, I determine μ_s from Lemma 1

$$\mu_s = \sigma_w^A \pi^A - \left(\sigma_a + \sigma_p\right)\left(\pi^A + \sigma_w^A\right) + \left(\sigma_a + \sigma_p\right)^2 + (1 - s)\left(\hat{c}^B - \hat{c}^A\right).$$

The drift terms μ_p, μ_ξ, and μ_ζ are given by Ito's Lemma

$$\begin{aligned}\mu_p &= \frac{p_s}{p} s \mu_s + \frac{1}{2}\frac{p_{ss}}{p} s^2 \sigma_s^2, \\ \mu_\xi &= \frac{\xi_s}{\xi} s \mu_s + \frac{1}{2}\frac{\xi_{ss}}{\xi} s^2 \sigma_s^2, \\ \mu_\zeta &= \frac{\zeta_s}{\zeta} s \left(\mu_s - \sigma_s \phi\right) + \frac{1}{2}\frac{\zeta_{ss}}{\zeta} s^2 \sigma_s^2. \end{aligned} \tag{3.49}$$

Note that I have dropped the superscript from the drifts, but it should be remembered that all drifts are written under measure A, except μ_ζ which is under measure B (this point is valid throughout this section).

The expression for the interest rate is obtained from agent B's HJB (3.23), where

$\hat{c}^B$ is replaced from (3.27)

$$r = \frac{\rho}{1-\psi} - \frac{\psi}{1-\psi}\rho^{\frac{1}{\psi}}\zeta^{-\frac{1-\psi}{\psi}} - \sigma_w^B \pi^B - \mu_\zeta + \frac{\gamma}{2}\left(\left(\sigma_w^B\right)^2 + \sigma_\zeta^2 - 2\frac{1-\gamma}{\gamma}\sigma_w^B \sigma_\zeta\right). \quad (3.50)$$

We obtain two second-order differential equations in p, ξ, and ζ by plugging the terms provided above in

$$\frac{(1-i(p))}{p} + \Phi(i(p)) - \delta + \mu_p + \mu_a^A + \sigma_a \sigma_p - r = \left(\sigma_a + \sigma_p\right)\pi^A, \quad (3.51)$$

and

$$\frac{\rho}{1-\psi} = \frac{\psi}{1-\psi}\rho^{\frac{1}{\psi}}\xi^{-\frac{1-\psi}{\psi}} + r + \sigma_w^A \pi^A + \mu_\xi - \frac{\gamma}{2}\left(\left(\sigma_w^A\right)^2 + \sigma_\xi^2 - 2\frac{1-\gamma}{\gamma}\sigma_w^A \sigma_\xi\right), \quad (3.52)$$

where the first expression follows from the definition of the market price of risk in (3.17) together with (3.11) and (3.12), and the second one follows from (3.22) replacing $\hat{c}^A$ from (3.26). Finally, replacing $\hat{c}^A$ and $\hat{c}^B$ from (3.26) and (3.27) in the goods market clearing condition (3.29), we get

$$\rho^{\frac{1}{\psi}}\left(\xi^{-\frac{1-\psi}{\psi}} s + \zeta^{-\frac{1-\psi}{\psi}}(1-s)\right) = \frac{1-i(p)}{p}, \quad (3.53)$$

which along with the two differential equations characterise the equilibrium. This system is solved numerically with details described below.

Numerical Algorithm

To solve for the infinite-horizon equilibrium numerically, I use the explicit 4-step Runge-Kutta method expounded below. The ultimate goal is to find three functions functions $p(s)$, $\xi(s)$, and $\zeta(s)$, where $s \in (0,1)$ is the state variable.[4] All other desired values can be recovered from these functions. The problem is solved numerically by adding time dimension t. Both s and t are discretised with step sizes equal to $ds = 0.001$ and $dt = 0.001$.[5] Adding the time dimension, Ito's Lemma implies that the drifts of these three variables change to

$$\tilde{\mu}_p = \mu_p + \frac{\dot{p}}{p}, \quad \tilde{\mu}_\xi = \mu_\xi + \frac{\dot{\xi}}{\xi}, \quad \tilde{\mu}_\zeta = \mu_\zeta + \frac{\dot{\zeta}}{\zeta},$$

[4]The solution for $s = 0$ and $s = 1$ is provided in the Appendix for Boundary Conditions.

[5]For some specifications the size of the time step is changed to $dt = 0.0005$ for the algorithm to converge.

where the terms in red are derivatives with respect to time and μ_p, μ_ξ and μ_ζ are given by (3.49).

Let's call the function of interest $f(s,t)$ where f can be p, ξ, or ζ. We begin with some terminal values, that is $f(s,T)$ for all values of s in the grid (more details on the choice of terminal values below) and move backwards in time as described further below by computing $f(s,T-dt)$, $f(s,T-2dt)$, and so on until the time derivatives vanish. I use the finite difference method to approximate first- and second-order derivatives of f with respect to s, i.e. $f_s(s,t)$ and $f_{ss}(s,t)$, $\forall s$.[6] Having p, ξ, and ζ and their s-derivatives, consumption-wealth ratios $\hat{c}^A$ and $\hat{c}^B$ are given by (3.26) and (3.27), and σ_s is computed from (3.36), using which we are able to compute σ_p, σ_ξ, and σ_ζ from (3.48). Next, values of σ_w^A, π^A, π^B, σ_w^B, μ_s, μ_p, μ_ξ, and μ_ζ are computed following the procedure described in the previous section.

Since the modified problem contains the time dimension, μ_ξ and μ_ζ should be replaced with $\tilde{\mu}_\xi$ and $\tilde{\mu}_\zeta$ in the agents' HJB equations (3.22) and (3.23). Consequently, we obtain the following expression from agent B's HJB (3.23) for the new interest rate

$$\tilde{r} = r - \frac{\dot{\zeta}}{\zeta},$$

where r is the interest rate from the original problem without the time derivative given by (3.50). Plugging $\tilde{r}$ and $\tilde{\mu}_\xi$ into agent A's HJB (3.22), we obtain

$$\begin{aligned} \frac{\dot{\zeta}}{\zeta} - \frac{\dot{\xi}}{\xi} &= -\frac{\rho}{1-\psi} + \frac{\psi}{1-\psi}\rho^{\frac{1}{\psi}}\xi^{-\frac{1-\psi}{\psi}} + r + \sigma_w^A\pi^A + \mu_\xi \\ &\quad - \frac{\gamma}{2}\left(\left(\sigma_w^A\right)^2 + \sigma_\xi^2 - 2\frac{1-\gamma}{\gamma}\sigma_w^A\sigma_\xi\right) \\ &\equiv F_1. \end{aligned} \tag{3.54}$$

Similarly, we should modify (3.51) by replacing μ_p and r with $\tilde{\mu}_p$ and $\tilde{r}$, which yields

$$\begin{aligned} \frac{\dot{p}}{p} + \frac{\dot{\zeta}}{\zeta} &= -\left[\frac{(1-i(p))}{p} + \Phi(i(p)) - \delta + \mu_p + \mu_a^A + \sigma_a\sigma_p - r - \left(\sigma_a + \sigma_p\right)\pi^A\right] \\ &\equiv F_2. \end{aligned} \tag{3.55}$$

Finally, I differentiate both sides of the goods market clearing condition (3.53) with

[6] The derivatives are approximated by

$$f_s(s,t) \approx \frac{f(s+ds,t) - f(s-ds,t)}{2ds}, \qquad f_{ss}(s,t) \approx \frac{f(s+ds,t) - 2f(s,t) + f(s-ds,t)}{ds^2}.$$

respect to t to get

$$B_1 \frac{\dot{p}}{p} = B_2 \frac{\dot{\xi}}{\xi} + B_3 \frac{\dot{\zeta}}{\zeta}, \tag{3.56}$$

where B_1, B_2, and B_3 are defined as

$$\begin{aligned}
B_1 &\equiv \frac{i(p) - i'(p)p - 1}{p}, \\
B_2 &\equiv \left(1 - \frac{1}{\psi}\right) s\hat{c}^A, \\
B_3 &\equiv \left(1 - \frac{1}{\psi}\right)(1-s)\hat{c}^B.
\end{aligned}$$

For each value of s in the discretised grid, Equations (3.54), (3.55), and (3.56) provide three equations to find the three time derivatives. The solution to this system is

$$\begin{aligned}
\frac{\dot{\zeta}}{\zeta} &= \frac{B_1 F_2 + B_2 F_1}{B_1 + B_2 + B_3} \\
\frac{\dot{\xi}}{\xi} &= \frac{\dot{\zeta}}{\zeta} - F_1, \\
\frac{\dot{p}}{p} &= F_2 - \frac{\dot{\zeta}}{\zeta}.
\end{aligned} \tag{3.57}$$

Having computed the time derivatives for all values of s in the grid, we can take a step back in time and compute new values for p, ξ, and ζ. Formally, let the time derivative computed in (3.57) be

$$\dot{f} = g(s, t, f),$$

where f can be p, ξ, or ζ, and function $g(.)$ is the corresponding time derivative from (3.57). Then we take a step of size dt back according to

$$f(s, t - dt) = f(s, t) - g(s, t, f)dt \qquad \forall s. \tag{3.58}$$

Equation (3.58) corresponds to the Euler method for solving differential equations. For better convergence properties, instead of (3.58), I use a 4-step Runge–Kutta method (RK4). Intuitively, instead of taking a step back of size dt, I only step back a fraction of that, recompute the time derivatives at that step and use it to take another fractional step, and so on through the 4 steps of RK4. Formally, we move back in time according to

$$f(s,t-dt)=f(s,t)-\frac{dt}{6}(k_1+2k_2+2k_3+k_4) \qquad \forall s,$$

where

$$\begin{aligned}
k_1 &= g(s,t,f),\\
k_2 &= g(s,t-\frac{dt}{2},f-\frac{dt}{2}k_1),\\
k_3 &= g(s,t-\frac{dt}{2},f-\frac{dt}{2}k_2),\\
k_4 &= g(s,t-dt,f-(dt)k_3).
\end{aligned}$$

We repeat this process until time derivatives converge to zero, which means that the solution found satisfies the infinite-horizon problem. The solution will converge to the stationary solution for a wide range of terminal values at $t=T$, as long as these values satisfy the goods market clearing condition. I set the terminal conditions at values such that the consumption-wealth ratios for agents are the same and equal to some arbitrary number and the corresponding market-clearing price.

Derivation of the Boundary Conditions

This section addresses the equilibrium solution when the share of wealth of either group goes to zero, i.e. at the boundary values $s \in \{0,1\}$. The solution is very similar to the benchmark case with homogeneous beliefs, because at the boundaries the entire wealth is held by agents with the same beliefs, and the other group has no wealth.

When $s \to 1$, the aggregate wealth is held by group A agents. As argued in Appendix 3.7.4, $\sigma_s = 0$ when $s=1$, and as a result

$$\begin{aligned}
\sigma_p &= \sigma_\xi = \sigma_\zeta = 0,\\
\mu_p &= \mu_\xi = \mu_\zeta = 0.
\end{aligned}$$

From (3.26) and (3.29) we obtain the consumption-wealth ratio for agent A

$$\hat{c}^A = \rho^{\frac{1}{\psi}}\xi^{-\frac{1-\psi}{\psi}} = \frac{1-i(p)}{p},$$

which also provides an expression for ξ. Capital market clearing condition (3.30) shows that the total risk in the economy is absorbed by agent A, i.e.

$$\sigma_w^A = \frac{\pi^A}{\gamma} = \sigma_a,$$

which means that $\pi^A = \gamma\sigma_a$. Reasoning similar to Appendix 3.7.4 provides the expression for the interest rate using (3.23)

$$r = \frac{1 - i(p)}{p} + \Phi(i(p)) - \delta + \mu_a^A - \gamma\sigma_a^2.$$

Finally we can substitute the above quantities in HJB equation (3.22) to derive

$$p = \frac{1 - i(p)}{\rho - (1-\psi)\left(\Phi(i(p)) - \delta + \mu_a^A - \frac{\gamma}{2}\sigma_a^2\right)},$$

which implicitly defines the price. This expression is identical to the homogeneous belief price (3.38), except that μ_a^A has replaced μ_a.

Agent B has zero wealth when $s = 1$, nonetheless we can compute her perceived price of risk and desired exposure from the equilibrium conditions

$$\pi^B = \pi^A - \phi = \gamma\sigma_a - \phi,$$
$$\sigma_w^B = \frac{\pi^B}{\gamma} = \sigma_a - \frac{\phi}{\gamma},$$

which I substitute in her HJB equation to find ζ from

$$\frac{\rho}{1-\psi} = \frac{\psi}{1-\psi}\rho^{\frac{1}{\psi}}\zeta^{-\frac{1-\psi}{\psi}} + r + \frac{1}{2\gamma}(\gamma\sigma_a - \phi)^2.$$

The solution at $s = 0$ is very similar. Again, $\sigma_s = 0$ and therefore p, ξ, and ζ have zero drifts and volatilities. Consumption-wealth ratio for agent B is

$$\hat{c}^B = \rho^{\frac{1}{\psi}}\zeta^{-\frac{1-\psi}{\psi}} = \frac{1 - i(p)}{p},$$

which also provides an expression for ζ. She also holds the aggregate capital stock, therefore

$$\sigma_w^B = \frac{\pi^B}{\gamma} = \sigma_a,$$

which means that $\pi^B = \gamma\sigma_a$. The interest rate is

$$r = \frac{1 - i(p)}{p} + \Phi(i(p)) - \delta + \mu_a^B - \gamma\sigma_a^2,$$

and the price of capital is implicitly defined by

$$p = \frac{1 - i(p)}{\rho - (1-\psi)\left(\Phi(i(p)) - \delta + \mu_a^B - \frac{\gamma}{2}\sigma_a^2\right)}.$$

Agent A has zero wealth when $s = 0$, nevertheless we can compute her perceived price of risk and desired exposure from the equilibrium conditions

$$\pi^A = \pi^B + \phi = \gamma\sigma_a + \phi,$$
$$\sigma_w^A = \frac{\pi^A}{\gamma} = \sigma_a + \frac{\phi}{\gamma},$$

which I substitute in her HJB equation to find ξ from

$$\frac{\rho}{1-\psi} = \frac{\psi}{1-\psi}\rho^{\frac{1}{\psi}}\xi^{-\frac{1-\psi}{\psi}} + r + \frac{1}{2\gamma}(\gamma\sigma_a + \phi)^2.$$

3.7.6 Solving the Model with Financing Constraints

The Lagrangian for the problem with the short-selling constraint is (optimal investment already imbedded)

$$\max_{\hat{c}^B, \sigma_w^B} \frac{\left(\hat{c}^B\right)^{1-\psi}}{1-\psi}\rho\zeta^{\psi-1} + r + \sigma_w^B\pi^B - \hat{c}^B + \mu_\zeta - \frac{\gamma}{2}\left(\left(\sigma_w^B\right)^2 + \sigma_\zeta^2 - 2\frac{1-\gamma}{\gamma}\sigma_w^B\sigma_\zeta\right) + \lambda^B\sigma_w^B,$$

subject to

$$\begin{aligned} \sigma_w^B &\geq 0, \\ \lambda^B &\geq 0, \\ \lambda^B\sigma_w^B &= 0. \end{aligned}$$

From the first-order conditions, optimal consumption-wealth ratio is identical to (3.27), and optimal exposure to risk is (3.39). Similar to the derivation in Section 3.3.5, we can obtain (3.40) and consequently

$$\sigma_s = \frac{(1-s)\frac{\phi-\lambda^B}{\gamma}}{1 + s(1-s)\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{\xi_s}{\xi} - \frac{\zeta_s}{\zeta}\right)},$$

using the new value for σ_w^B. When the short-selling constraint binds, $\sigma_w^B = 0$, and the aggregate risk is concentrated on the balance sheet of agent A, hence

$$s\sigma_w^A = \sigma_a + \sigma_p,$$

which is concluded from (3.30). This condition along with $\sigma_s = \sigma_w^A - \left(\sigma_a + \sigma_p\right)$ from Lemma 1 and $\sigma_p = \frac{p_s}{p} s\sigma_s$ from Ito's Lemma, leads to the following relations for σ_s and λ^B when we are in the constrained region

$$\sigma_s = \frac{(1-s)\sigma_a}{s\left(1-(1-s)\frac{p_s}{p}\right)},$$

$$\lambda^B = (\gamma-1)\sigma_\zeta - \pi^B.$$

In the numerical solution, these conditions are used in the constrained region and the rest of the solution is similar to the baseline case.

The Lagrangian for the problem with the leverage constraint is (optimal investment already imbedded)

$$\max_{\hat{c}^A,\sigma_w^A} \frac{\left(\hat{c}^A\right)^{1-\psi}}{1-\psi}\rho\xi^{\psi-1} + r + \sigma_w^A\pi^A - \hat{c}^A + \mu_\xi - \frac{\gamma}{2}\left(\left(\sigma_w^A\right)^2 + \sigma_\xi^2 - 2\frac{1-\gamma}{\gamma}\sigma_w^A\sigma_\xi\right)$$
$$-\lambda^A\left(\sigma_w^A - \sigma_t/\theta\right),$$

subject to

$$\begin{aligned}
\sigma_w^A &\leq \sigma_t/\theta, \\
\lambda^A &\geq 0, \\
\lambda^A\left(\sigma_w^A - \sigma_t/\theta\right) &= 0.
\end{aligned}$$

From the first-order conditions, optimal consumption-wealth ratio is identical to (3.26), and optimal exposure to risk is (3.41). Similar to the derivation in Section 3.3.5, we can obtain (3.42) and consequently

$$\sigma_s = \frac{(1-s)\frac{\phi-\lambda^A}{\gamma}}{1+s(1-s)\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{\xi_s}{\xi}-\frac{\zeta_s}{\zeta}\right)}.$$

using the new value for σ_w^A. When the leverage constraint binds, $\sigma_w^A = \left(\sigma_a + \sigma_p\right)/\theta$, which together with $\sigma_s = \sigma_w^A - \left(\sigma_a + \sigma_p\right)$ from Lemma 1 and $\sigma_p = \frac{p_s}{p}s\sigma_s$ from Ito's Lemma, results in the following relation for σ_s

$$\sigma_s = \frac{\left(\frac{1}{\theta}-1\right)\sigma_a}{1-s\left(\frac{1}{\theta}-1\right)\frac{p_s}{p}}.$$

Since in the constrained region $\sigma_w^A = \left(\sigma_a + \sigma_p\right)/\theta$, the capital market clearing condition (3.30) implies that

$$\sigma_w^B = \frac{\sigma_a + \sigma_p}{1-s}\left(1 - \frac{s}{\theta}\right),$$

in this region. Market risk premia for the representative agents are

$$\begin{aligned}
\pi^B &= \gamma \sigma_w^B + (\gamma - 1)\sigma_\zeta \\
\pi^A &= \pi^B + \phi.
\end{aligned}$$

Finally, the Lagrange multiplier λ^A is

$$\lambda^A = \pi^A - (\gamma - 1)\sigma_\xi - \gamma \sigma_w^A.$$

In the numerical solution, these conditions are used in the constrained region and the rest of the solution is similar to the baseline case.

3.7.7 Solving the Model with Dynamic Disagreement

When the disagreement process is time varying and evolves according to (3.43), p, ξ, and ζ are multivariate functions, i.e.,

$$p_t = p(s_t, \phi_t), \quad \xi_t = \xi(s_t, \phi_t), \quad \zeta_t = \zeta(s_t, \phi_t),$$

thus, Ito's Lemma indicates that the volatilities and drifts of these functions to follow

$$\begin{aligned}
\sigma_p &= \frac{p_s}{p} s\sigma_s + \frac{p_\phi}{p}\sigma_\phi, \\
\sigma_\xi &= \frac{\xi_s}{\xi} s\sigma_s + \frac{\xi_\phi}{\xi}\sigma_\phi, \\
\sigma_\zeta &= \frac{\zeta_s}{\zeta} s\sigma_s + \frac{\zeta_\phi}{\zeta}\sigma_\phi,
\end{aligned}$$

and

$$\begin{aligned}
\mu_p &= \frac{p_s}{p} s\mu_s + \frac{p_\phi}{p}\mu_\phi + \frac{1}{2}\left(\frac{p_{ss}}{p}s^2\sigma_s^2 + 2\frac{p_{s\phi}}{p}s\sigma_s\sigma_\phi + \frac{p_{\phi\phi}}{p}\sigma_\phi^2\right), \\
\mu_\xi &= \frac{\xi_s}{\xi} s\mu_s + \frac{\xi_\phi}{\xi}\mu_\phi + \frac{1}{2}\left(\frac{\xi_{ss}}{\xi}s^2\sigma_s^2 + 2\frac{\xi_{s\phi}}{\xi}s\sigma_s\sigma_\phi + \frac{\xi_{\phi\phi}}{\xi}\sigma_\phi^2\right), \\
\mu_\zeta &= \frac{\zeta_s}{\zeta} s(\mu_s - \sigma_s\phi) + \frac{\zeta_\phi}{\zeta}\left(\mu_\phi - \sigma_\phi\phi\right) + \frac{1}{2}\left(\frac{\zeta_{ss}}{\zeta}s^2\sigma_s^2 + 2\frac{\zeta_{s\phi}}{\zeta}s\sigma_s\sigma_\phi + \frac{\zeta_{\phi\phi}}{\zeta}\sigma_\phi^2\right).
\end{aligned}$$

Using the new volatilities and drifts and expression (3.45) for σ_s in the procedure described in Appendix 3.7.5 we derive second-order partial differential equations in p, ξ, and ζ, that can be solved numerically in conjunction with the market-clearing condition (3.53). The disagreement process moves exogenously according to (3.43).

In the numerical solution the range of possible values for ϕ_t needs to be discretised as well, so we will have a two-dimensional grid of values in (s, ϕ). We next follow the same algorithm described in Appendix 3.7.5 for each point in this grid: add time dimension to the problem, guess some terminal values, evaluate time derivatives and move backward in time until the time derivatives converge to zero.

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