The London School of Economics and Political Science

Essays in International Macroeconomics

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Declaration

I certify that the thesis I have presented for examination for the MPhil/PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

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I confirm that a small part of the work on which Chapter 3 is based was conducted during my studies for the MRes in Economics, in 2011. The paper I submitted towards that degree is based upon that work.

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Abstract

This thesis comprises three chapters. In the first chapter, I analyze three main facts from the recent experience of capital flows in the European monetary union. First, core and periphery countries ran widening current account surplus and deficit positions. Second, core countries intermediated gross capital flows from the rest of the world, which financed deficits in the periphery. Finally, a pervasive sovereign debt crisis took place. I argue that institutional features of the Economic and Monetary Union have contributed to these facts. First, I show in a theoretical model that subsidies on holdings of euro-denominated assets contribute to all three phenomena. Second, I build a dynamic model of an economic union. The model generates predictions for net and gross asset flows that quantitatively replicate the EMU experience. Finally, I propose a novel theoretical mechanism magnifying the severity of a debt crisis in an economic union. In the second chapter, I study the interaction between sovereign default risk, firm-level financial frictions, and fiscal policy. This research is motivated by the severe contraction observed in Italy during the euro area sovereign debt crisis. I show that a sovereign debt crisis causes a reduction of credit to firms, occurring through the channel of domestic fiscal policy. A fiscal tightening in the country in crisis causes a reduction of firms' profits and an increase in their default risk. Secondly, I show that firms are heterogeneous in the degree to which they are affected by a crisis: Firms in the non-tradable sector are more vulnerable, as demand for their output falls in a crisis. In the third chapter, I study the determinants of time-varying volatility in interest rates on emerging market economies' external debt. I show that a baseline model of endogenous sovereign default quantitatively replicates the pattern of time-varying volatility observed in the data. The model features a key non-linearity in the policy function for the interest rate on external debt. In the absence of shocks to the second moment of stochastic variables, the model generates a path of interest rates that is more volatile in bad times, when output is low and debt is high.

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Chapter 1

External Imbalances, Gross Capital Flows and Sovereign Debt Crises

1.1 Introduction

External imbalances, large gross capital flows and a pervasive sovereign debt crisis have characterized recent experience in the European monetary union. I introduce a framework to analyze jointly the determinants of these phenomena. This chapter shows that by focusing on the introduction of policy distortions in the institutional framework of the European Economic and Monetary Union (EMU), we can shed light on important aspects of this experience. A specific feature of the monetary union, the presence of subsidies on cross-border asset holdings, plays a crucial role in generating widening current account deficit and surplus positions, an expansion of gross capital flows with the rest of the world and pervasiveness and severity of a sovereign debt crisis. In addition, in an economic union, trade integration among member countries contributes to the transmission and amplification of a debt crisis.

Three major facts can be isolated as the most salient in the recent experience of euro area economies. First, current account deficits and surpluses widened for members of the monetary union, from its inception in the early 2000s until the global financial crisis in

2008-09. Second, gross financial positions expanded in magnitude, with surplus economies in the euro area issuing gross liabilities to the rest of the world and intermediating resources to deficit countries.[1] Third, a sovereign debt crisis took place in deficit countries. The crisis led to severe recessions in many countries, and to a collapse in net capital flows.

What role did institutional features of the EMU have in generating widening current account deficits, surpluses, and the expansion of gross asset and liability positions of euro area economies? How does trade integration among countries in an economic union affect severity and transmission of a debt crisis? This chapter addresses these questions by developing a theoretical framework of capital flows and policy distortions in a union of countries. In addition, I introduce an infinite-horizon, heterogeneous-countries model to quantitatively assess the importance of the channels highlighted by the theoretical framework, in relation to recent experience in the EMU.

The theoretical framework highlights the main channel through which policy distortions affect capital flows in a union of countries. First, intermediation of gross capital flows emerges as union residents leverage a subsidy they enjoy on assets issued within the union by expanding their gross liability position against the rest of the world. Second, countries in the union run wider deficit and surplus positions due to the subsidy. Debtor countries benefit from lower borrowing costs, while savers are induced by policy distortions to further accumulate net assets. Importantly, the result of a widening current account surplus in saving countries is not mechanically due to the larger deficit of debtors, as the union is fully integrated in international financial markets. Finally, this framework highlights a novel mechanism leading to amplification of a debt crisis in an economic union. Financial linkages in the form of cross-border exposures transmit the crisis within the union. The interaction of financial linkages with trade integration, in turn, exacerbates the recession in debtor countries. Amplification arises as export destinations of crisis countries coincide with the creditors on whom they default, hampering the ability of debtors to export during a crisis. The economic intuition behind this channel is related to the "secondary" burden of a transfer suggested by Keynes (1929), following from a feedback effect between

[1] This intermediation pattern has been documented by Waysand, Ross and de Guzman (2010) and Chen, Milesi-Ferretti and Tressel (2013). Hale and Obsfeldt (2014) document how borrowing from the rest of the world by banks in saving countries of the euro area helped fund their lending to deficit countries.

international transfers and changes in relative demand for goods.

I develop an infinite-horizon model of an economic union, represented by a continuum of countries trading in heterogeneous goods and assets, both internally and with the rest of the world. The model allows for a quantitative analysis of the effects of policy distortions in the euro area, and it is calibrated to replicate key features of countries in the monetary union. First, the model replicates the widening of the net foreign asset distribution observed after the inception of the EMU. Second, predictions of the model for net and gross capital flows replicate the experience of external imbalances and gross capital flow intermediation observed in the euro area. In the model, the introduction of a subsidy on cross-border asset holdings has non-linear effects on endogenous choices of the heterogeneous countries that constitute the union, depending on their position in the net foreign asset distribution. Finally, the model can be employed to analyze general equilibrium transmission and amplification of a debt crisis in an economic union, taking place via the external balance sheet of savers and through trade among union members.

The phenomena at the center of this analysis have been very significant in terms of their size, including the occurrence of a rare sovereign debt crisis in advanced economies. First, current account surplus and deficit positions in the core and periphery of the euro area both approached at their peak 3% of GDP of the monetary union.[2] Second, the expansion in gross flows brought the sum of gross asset and liabilities of the euro area against the rest of the world to 350% of GDP in 2007, a figure 50% higher than the one observed in the United States. Third, the output contraction associated with the debt crisis in peripheral countries amounted to 5% of GDP, between 2011 and 2013. The recovery in core countries slowed down significantly as well, with GDP growing by 1% only in the three years to 2013. In comparison, in the United States, GDP grew by 7% in the same period. Finally, I provide concrete examples of distortions subsidizing residents' purchases of assets issued by union-member economies in the EMU. These implicit subsidies take the form of financial regulation provisions on holdings of sovereign debt, the ability to use certain types of assets as collateral in transactions with the European Central Bank, as well as implicit expectations of bailouts on the part of asset holders.

[2]Throughout this chapter, the definition of core and periphery countries adopted will be the conventional one, with surplus countries Austria, Belgium, France, Germany and the Netherlands in the former group and deficit countries Greece, Ireland, Italy, Portugal and Spain in the latter.

An important contribution of this chapter lies in the analysis of trade in financial assets between a union of countries and the rest of the world. Capital flows between the euro area and the rest of the world have largely been overlooked by the literature on the Eurozone debt crisis. Current account surplus and deficit positions of core and peripheral countries were similar in magnitude in the period leading up to the crisis. Papers in this strand of the literature, then, have typically analyzed the crisis episode by modeling the monetary union as a set of open economies trading solely among each other.[3] The euro area, however, is closely integrated with international financial markets, displaying large gross asset and liability positions against the rest of the world. The analysis of trade in financial assets with the rest of the world allows for an investigation of two important issues. First, what factors induced core countries to save in the periphery, intermediating in addition resources borrowed from the rest of the world? Second, what are the implications of large gross external positions, as distinct from those of net ones? This latter issue appears to be especially important in the context of the euro area, where the external balance sheet of core countries featured asset and liability sides that were mismatched along important dimensions, notably in terms of nationality of counterparties.

The next subsection reviews the contribution of this chapter to the existing literature. Section 2 presents evidence on net and gross capital flows in the euro area, on the presence of distortions affecting trade in euro-denominated debt and on heterogeneous exposure to intra-union trade for European countries. In Section 3, I introduce the theoretical framework and I analyze the implications of a subsidy on asset holdings on current account imbalances, intermediation of gross capital flows and crisis. Section 4 presents the infinite-horizon model with heterogeneous countries, and quantitative results on the effects of policy distortions and of a debt crisis in an economic union. Section 5 concludes.

Literature review. A growing body of research has recently analyzed the European experience of current account imbalances, gross capital flows and sovereign debt crisis. This chapter contributes to that literature, focusing on the role of policy distortions in generating intermediation of gross capital flows by the core to the periphery and widening current account deficits and surpluses. In addition, I analyze how the interaction between

[3]Notable examples are Martin and Philippon (2015), Corsetti, Kuester, Meier, Müller (2014) and Guerrieri, Iacoviello and Minetti (2013).

financial linkages and trade integration in an economic union generates contagion and amplification of a debt crisis.

The emphasis in this chapter on the effects of policy distortions on capital flows in the euro area is motivated by the evidence on gross external positions presented by Waysand, Ross and de Guzman (2010) and Chen, Milesi-Ferretti and Tressel (2013). The latter suggest that core countries' intermediation behavior may have been induced by the presence of regulatory distortions in the EMU, leading them to issue gross liabilities to extra-euro area economies to finance lending to the periphery. Following that suggestion, I investigate in the context of a model the ability of subsidies on cross-border holdings of assets in an economic union to explain the observation of intermediation of gross capital flows and current account imbalances.

Hale and Obstfeld (2014) also provide evidence on the impact of the introduction of the EMU on international debt flows. In addition, they present a model where a reduction in financial transaction costs induces intermediation of capital flows by banks in the core. I describe here the implications of a similar distortion on intra-EMU trade in financial assets on net and gross financial positions. I analyze, in addition, crisis transmission and amplification effects of such a distortion, as well as providing an assessment of the optimality of its introduction.

The modelling approach adopted in my infinite horizon model is to describe the world economy as a continuum of countries subject to idiosyncratic income shocks, analyzing current account and external financial positions in this setting. I extend the traditional framework introduced by Clarida (1990) to allow a continuum of small open economies to trade risky assets with a large rest of the world. This extension, motivated by the observation that European economies engage in extensive financial trade with global financial markets, allows me to analyze motives inducing countries in the euro area core to hold positive financial positions against partners in the EMU, while being gross debtors with respect to extra-union economies. Fornaro (2014) employs a similar framework to analyze the effects of a deleveraging episode taking place in a monetary union. Here, the introduction of trade in goods and assets with the rest of the world allows me to analyze gross capital flows and the heterogeneous effects of a debt crisis on countries with varying exposure to trade with their union partners.

By analyzing transmission and amplification of a debt crisis in an economic union, this chapter is related to a rich literature on the Eurozone debt crisis.[4] Two key features define the economic union presented in this chapter. First, the presence of regulatory distortions affecting trade in assets issued within the union induces in equilibrium the creation of financial linkages among union members. A similar feature characterizes the economic union introduced by Broner, Erce, Martin and Ventura (2014). There, residents of a union share the preferential treatment given to domestic creditors by a sovereign debtor in default. Arellano and Bai (2014) also analyze contagion of a debt crisis when several countries borrow from a common lender. Secondly, countries in an economic union are tied in my model by close trade integration. In the model, asymmetric trade integration follows from lower transport costs that apply to trade in goods within the union. Anderson and van Wincoop (2004) document the presence of large variability in trade costs across goods and country pairs. Following Sachs (1982), I introduce heterogeneous goods, characterized by differences in transport costs. In my model, countries in the union differ in terms of specialization in output goods characterized by different degrees of tradability. Heterogeneous specialization determines different exposure of member economies to trade with union partners. In turn, asymmetric exposure to trade with union partners determines asymmetric amplification effects of a crisis episode. A similar feature is present in Benigno and Romei (2014), where fluctuations in the real exchange rate arising during a debt deleveraging episode have heterogeneous effects on countries specialized in production of different goods.

In my model, distortions on cross-border holdings of assets affect households' portfolio allocation problem. In particular, households that are offered a bailout promise fail to internalize the risk of losses associated with their purchases of gross assets. In Mendoza (2010), Bianchi (2011) and Benigno, Chen, Otrok, Rebucci and Young (2013) a similar externality is present. In those papers, however, a pecuniary externality arises as households do not take into account effects of their borrowing decision on a key relative price, determining in turn the severity of a borrowing constraint. Here the externality mainly affects countries with a positive net foreign asset position, where households' saving decision is critically affected by the presence of a bailout promise.

[4] A non-exhaustive list includes Aguiar, Amador, Farhi and Gopinath (2015), Guerrieri, Iacoviello and Minetti (2013), Martin and Philippon (2015), Corsetti, Kuester, Meier and Müller (2014) and Lane (2012).

1.2 Empirical evidence

I present in this section the main empirical evidence that motivates this chapter. First, I detail facts related to the widening of current account deficit and surplus positions observed in the euro area in the 2000s. Second, I describe evidence on the expansion of gross external asset and liability positions of euro area economies and on the pattern of intermediation of international capital flows. Third, I will present specific features of the regulatory and policy framework that have contributed to subsidize cross-border holdings of assets within the euro area. Finally, I present a measure of euro area economies' exposure to trade with European partners. This indicator will be crucial to determine the direction of the effect, in a crisis, of the interaction between financial linkages and trade integration.

Current account imbalances in the euro area. Current account dynamics of economies in the euro area displayed significant imbalances in the years preceding the global financial crisis, an observation that has been widely documented in the literature.[5] Figure 1.1 displays the sum of current account balances for countries in the two groups of core and periphery members of the monetary union, expressed as ratios to euro area GDP.[6] The gap in current account balances between countries in these two groups widened substantially in the period from the inception of the EMU to the global financial crisis. The current account surplus in core countries approached 3% of euro area GDP in 2007, from a balanced position in 2000. In the periphery, the current account deficit doubled in the same period, rising above 2% of euro area GDP in 2007.

Disaggregated data for individual countries are reported in Figure 1.2. In the periphery, the current account deficit rose in all countries but Portugal between 2000 and 2008. Prior to the global financial crisis, the current account deficit was above 10% of GDP in Portugal and Greece and above 5% in Ireland and Spain. Among core countries, the largest rise in current account surplus was observed in Austria, Germany and the Nether-

[5] For a detailed review of the pattern of capital flows in the EMU, see Lane (2013). The pattern of current account imbalances in the euro area has been recently analyzed in the context of a theoretical model by Siena (2015) and Jaccard and Smets (2015), among others.

[6] As already mentioned, the core includes Austria, Belgium, France, Germany and Netherlands. The periphery includes Greece, Ireland, Italy, Portugal and Spain.

lands. At its peak, in 2007, the German current account surplus amounted to 7% of GDP. A different pattern was observed in France and Belgium, where the surplus diminished in magnitude throughout this period.

Financial linkages and intermediation. Two key facts emerge from the analysis of bilateral international investment positions of euro area economies.[7] First, strong financial linkages tied together economies in the EMU, with large, and growing, cross-border positions among euro area economies. Second, euro area economies traded extensively in financial assets with the rest of the world, with an emerging pattern of intermediation by core countries of capital flows from outside the euro area.

Data on net and gross financial positions of euro area economies reveal the presence of close interdependence among these countries. First, the fraction of total gross external positions of economies in the EMU accounted for by euro area counterparties was equal to approximately half of their total gross positions. Second, economies in the euro area core represented major financial partners for peripheral countries. Gross liabilities of peripheral countries held by Germany, France and the Netherlands in 2008 ranged from 40% of GDP for Italy to 69% of GDP in Portugal.[8] Gross liabilities held by the three major core countries were even higher for Ireland, amounting to 268% of Irish GDP.[9] Finally, global international investment positions of peripheral countries could largely be explained by the bilateral positions of these countries against euro area partners. The fraction of the global international investment position of peripheral countries represented by their bilateral position against euro area partners ranged between 61% for Portugal to 80% for Greece, in 2008, highlighting how the majority of these countries' trade in

[7] All figures reported in this subsection are drawn from the dataset and paper presented by Waysand, Ross, de Guzman (2010).

[8] In addition, substantial amounts of liabilities corresponding to counterparties in the United Kingdom and Luxembourg are likely to partly reflect claimants ultimately based in the euro area core. Waysand, Ross, de Guzman (2010) discuss the merits of alternative residence reporting principles.

[9] The pattern observed in Ireland, however, is substantially different from the one of Italy, Spain, Portugal and Greece. Ireland, in fact, held a substantial amount of gross assets outside the euro area, amounting to approximately 400% of GDP just for the United Kingdom and the United States.

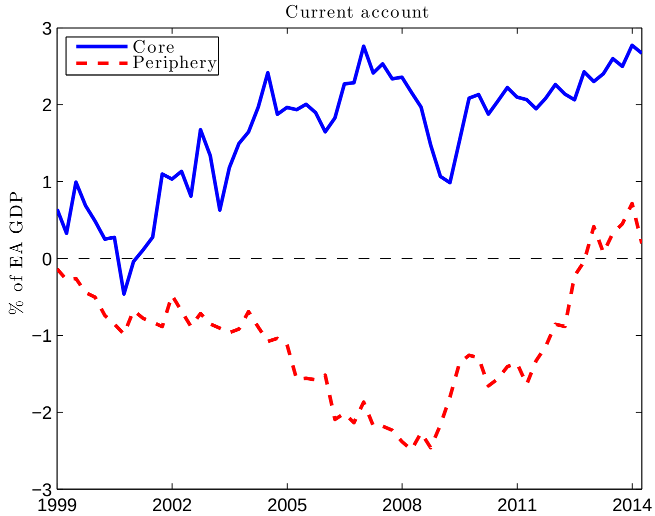

Figure 1.1: Current account balances of core and periphery countries in the euro area

Data are presented as fraction of total euro area GDP. Core includes Austria, Belgium, France, Germany and Netherlands. Periphery includes Greece, Ireland, Italy, Portugal and Spain.

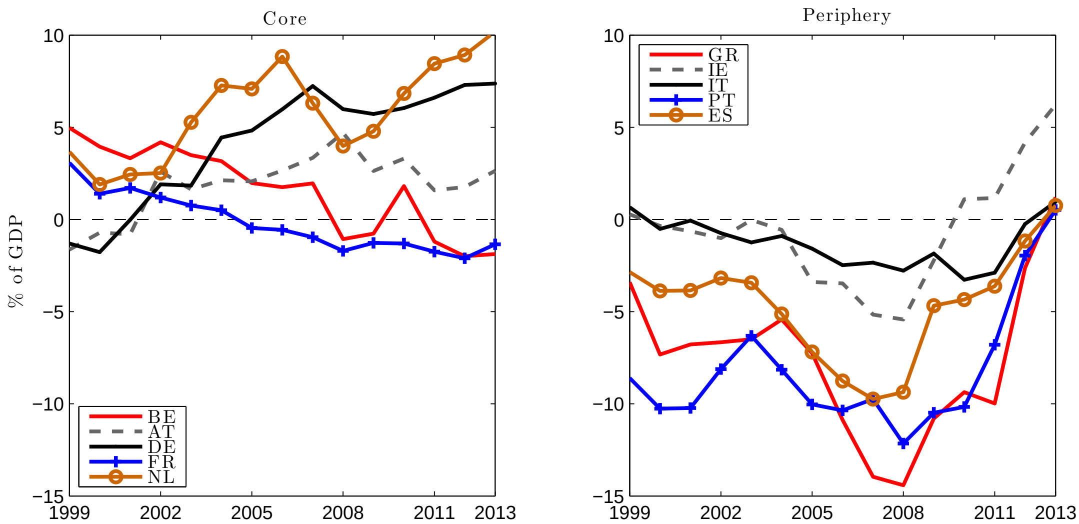

Figure 1.2: Current account balances of selected countries in the euro area

Data are presented as fraction of individual countries' GDP. The left-hand side panel represents so-called core countries, while the periphery is displayed in the right-hand side panel.

financial assets took place within the EMU.[10]

Trade in financial assets by economies in the euro area with the rest of the world was characterized by two main features. First, the evolution of gross financial positions of core euro area economies can be interpreted as describing intermediation of international capital flows by these countries. The bilateral international investment position of core economies with respect to the rest of the euro area rose between 2002 and 2007. This increase ranged between 30 percentage points of GDP for Belgium to 12 percentage points in France. At the same time, the global international investment position was deteriorating for some of these countries, signalling that they were funding gross saving in the euro area by borrowing from the rest of the world, rather than by increasing their net savings.[11] In Germany, the global international investment position was also growing at the time, but by less than the bilateral one with the euro area, so that the bilateral investment position with respect to the rest of the world was deteriorating.

Second, the magnitude of gross external positions of the euro area with the rest of the world was large by international standards, highlighting its financial openness and close integration with world financial markets. While the external net foreign asset position of the euro area was small when compared to that of the United States or Japan, the sum of its gross external asset and liabilities amounted to 350% of GDP in 2008, compared with less than 250% of GDP for the United States and 175% for Japan. Given the presence of large gross capital flows between the euro area and the rest of the world, it is important to consider the implications of financial trade by countries in the union with economies located outside of it, in order to conduct a comprehensive analysis of their pattern of external positions since the inception of the monetary union.

Distortions on asset holdings in the European Monetary Union. Several aspects of policy and regulation in the European Monetary Union are likely to have

[10]Again, the pattern is different in the case of Ireland. Given substantial holdings of gross assets outside the euro area, its global international investment position amounted to -56% of GDP, while its bilateral position against the rest of the euro area amounted to -123% of GDP.

[11]In France, Belgium and Austria, the global international investment position deteriorated by 14, 10 and 7 percentage points of GDP, respectively, between 2002 and 2008. In the Netherlands, on the other hand, a substantial improvement in the global international investment position was matched by a smaller one in the bilateral one with euro area partners.

contributed to the pattern of intermediation of capital flows observed in the euro area. I will briefly discuss here some aspects of banking regulation and collateral eligibility rules in transactions with the European Central Bank that subsidized holdings of euro-denominated assets, for institutions based within the monetary union. In addition, I will review ex-post policy actions in the context of the sovereign debt crisis that, similarly to a bailout, reduced losses suffered by holders of peripheral countries' debt.

First, financial regulation in place in the EMU has provided incentives for banks in the euro area to hold large amounts of euro-denominated sovereign debt. In the European Union, the Capital Requirements Directive allowed financial institutions to assign a "zero-risk weight" to government debt of member states denominated and funded in own currency.[12] According to Nouy (2012), the zero risk weight mandated by the Capital Requirements Directive, implied that local currency debt was largely regarded as risk-free by regulators. In addition, the author expressed a concern that banks were actively encouraged to accumulate sovereign debt by these aspects of regulation.[13]

Secondly, the ability to use sovereign debt issued by euro area governments as collateral in transactions with the European Central Bank has provided an additional incentive to financial institutions based in the monetary union in holding these assets. As suggested by Buiter and Sibert (2005), operational practices on collateral eligibility at the ECB have subsidized holdings by European banks of risky, short-maturity government debt. In particular, according to these authors, the eligibility as collateral of debt issued by all euro area governments has contributed to the compression of borrowing costs observed among euro area governments.

Third, actions taken by policy institutions in the euro area during the sovereign debt crisis, in the form of explicit bailouts or non-standard monetary policy, have contributed to reduce the adverse effects of the crisis on financial institutions with large exposures to peripheral governments. To the extent that banks in the euro area expected similar measures to be taken in the event of a crisis, such expectations induced them to exert less

[12] Regulation (EU) No 575/2013, Part Three, Title II, Chapter 2, Section 2, Article 114.4 contains the most recent version of relevant legislation.

[13] Ms. Nouy was then General Secretary of the Prudential Supervisory Authority at the Banque de France. She is currently Chair of the Supervisory Board of the Single Supervisory Mechanism at the European Central Bank.

caution regarding the occurrence of future crises, as they expected, to some degree, to be bailed out in such circumstances.[14] A concrete bailout example is reported by Acharya and Steffen (2013), who discuss the case of the Franco-Belgian banking group Dexia. The size of the sovereign bond portfolio of this institution amounted to "almost three times of its book equity", and it was largely composed of Italian and Greek government debt. In the second half of 2011, as sovereign bond prices fell and this bank found it harder to access sources of funding, Dexia was rescued by the governments of Belgium, France and Luxembourg. In addition, the Outright Monetary Transactions (OMT) programme launched by the ECB in 2012, with the aim of "safeguarding an appropriate monetary policy transmission and the singleness of the monetary policy", was successful in reducing yields on government debt of peripheral countries, thereby increasing the market value of such assets on financial institutions' balance sheets (Altavilla, Giannone and Lenza, 2014).

In the model presented in this chapter, I introduce a bailout promise to represent institutional distortions giving preferential treatment to residents of an economic union when purchasing risky assets issued by union partners. This modelling choice allows me to characterize in a simple way a wide range of subsidies and distortions leading residents of an economic union to perceive returns on union-issued assets to be higher, or safer, than what perceived by outside residents.[15]

Trade integration and specialization. Trade with partners in the European Union accounted in 2005 for approximately two thirds of total trade by EU economies.[16] Geographical proximity, similarity in tastes and explicit trade liberalization make it an unsurprising fact that European economies largely trade with each other, highlighting the high degree of goods market integration characterizing the Union.

Economies in the European Union, however, differ significantly in terms of relative specialization in those industries whose output is more extensively traded internationally.

[14] Acharya and Steffen (2013) document European banks' behavior of betting on the survival of the euro area, "choosing to hold peripheral sovereign bonds and financing their investments in short-term wholesale markets."

[15] Clearly, this assumption does not allow for a separate analysis of the individual channels suggested in the literature. Such analysis is left for future research.

[16] Source: Eurostat, International trade data.

Manufacturing accounted in 2005 for 23% of Gross Value Added (GVA) in Germany, while representing only 10% and 16% of GVA in Greece and Spain, respectively.[17] Peripheral economies, on the other hand, are relatively specialized in industries characterized by a lower degree of tradability of output. In the same year, construction accounted for only 4% of GVA in Germany, rising to 6% and 12% in Greece and Spain.[18]

In order to document the asymmetric pattern of specialization of European economies, accounting for heterogeneity in tradability of output produced by different industrial sectors, I construct a country-level measure of average tradability of output.[19] I define tradability of individual industrial sectors as the ratio, for each sector, of total exports and imports attributed to that sector in a sample of advanced economies, to total Gross Value Added (GVA) of that sector. Formally, tradability of sector s is defined as

$$\text{TDTY}_s = \frac{\sum_{i \in I} \text{EXP}_{s,i} + \text{IMP}_{s,i}}{\sum_{i \in I} \text{GVA}_{s,i}}$$

where $\text{EXP}_{s,i}$ and $\text{IMP}_{s,i}$ represent total exports and imports attributed to sector s in country i. I represents the set of all countries in my sample. Hence, $\sum_{i \in I} \text{GVA}_{s,i}$ represents total GVA of sector s in the entire sample.

At country level, average tradability of output is given by the average of tradability in all industrial sectors, weighted by the shares of GVA represented by each sector in the economy of each country.[20] Formally, this is given by

$$\overline{\text{TDTY}}_i = \sum_{s \in S} \text{TDTY}_s \frac{\text{GVA}_{s,i}}{\overline{\text{GVA}}_i}$$

where S represents the set of all sectors that compose the economy and $\overline{\text{GVA}}_i = \sum_{s \in S} \text{GVA}_{s,i}$ represents total GVA in economy i.

Figure 1.3 presents values of average tradability of output for EU economies. Consistently with the observation that tradable sectors such as manufacturing are under-represented in these economies, Greece, Spain and Portugal feature low average tradability of output. On the other hand, average tradability is high in Germany, especially in

[17] In the System of National Accounts, GVA "is defined as the value of output less the value of intermediate consumption and is a measure of the contribution to GDP made by an individual producer, industry or sector."

[18] Data are drawn from the OECD Structural Analysis (STAN) database.

[19] A complete description of the data used to construct this measure is presented in Appendix 1.A.

[20] This measure is analogous to that of tradedness in Betts and Kehoe (2001).

comparison to other very advanced economies. This variable, in fact, is closer in Germany to the level encountered in recent accession countries such as Hungary and Slovenia than in the United Kingdom or France. Low values for average tradability of output observed in Southern European economies are due to relative specialization in these countries towards those sectors characterized by lower sectoral tradability. Detailed evidence on EU economies' sectoral specialization is presented in Appendix 1.A.

In the context of the model presented in the following section, I will study how specialization in industries characterized by lower tradability determines an amplification of the adverse effects of a debt crisis, when debt is owed to partners in an economic union. The ability of a country to generate trade surpluses through exports, in fact, is hampered by weak economic conditions in union partner economies if industrial specialization determines a high degree of exposure to export demand from these countries.

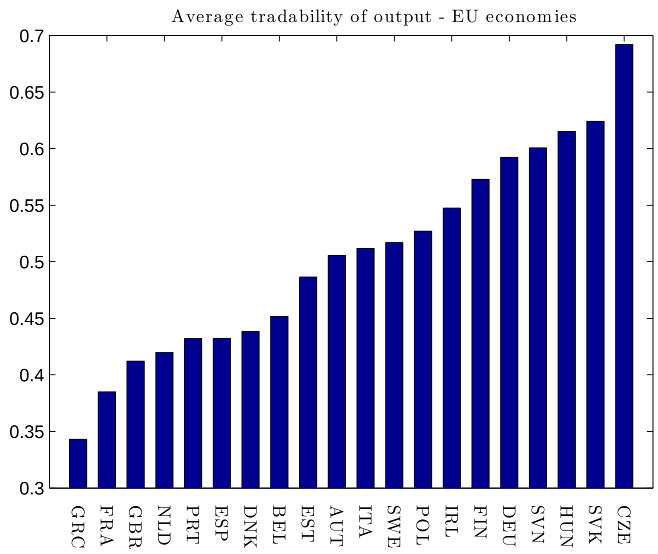

Figure 1.3: Average tradability of output in the European Union

This measure is computed as the weighted average of sector-level tradability of output, with weights given in each country by the shares of total GVA corresponding to each sector. Data source: OECD STAN and TiVA databases, year 2005.

1.3 A simple model of default, intermediation and trade

This section presents a formal analysis of net and gross capital flows between countries in an economic union. I analyze, in particular, the implications on current account balances and gross external positions of a subsidy on cross-border holdings of assets. In addition, I study under what conditions the interaction between trade integration and financial linkages introduced by the subsidy amplifies the severity of a debt crisis. Finally, I examine the rationale for the introduction of a subsidy on cross-border asset holdings, by assessing the implications of this distortion for welfare in the economic union. To this purpose, I introduce a simple model featuring trade in risky assets between two countries in an economic union and the rest of the world. To capture trade integration among members of an economic union, I introduce two types of consumption good, subject to heterogeneous transport costs across different country pairs.[21]

1.3.1 Model

Consider a two-period world economy. Time is discrete and indexed by $t \in \{1, 2\}$. The world economy is inhabited by two small open economies, H and F, and by the rest of the world, ROW. I will use the expression "economic union" when referring to the set of countries H and F. All countries receive endowments of two types of consumption good, A and B. Goods are tradable among all countries, subject to transport costs that are heterogeneous across country pairs and good types. All countries can issue one-period debt. H is subject to a limited commitment friction, and it cannot promise to always repay external debt issued in the initial period. F is not subject to limited commitment frictions.[22] Households in F enjoy a subsidy when purchasing H-issued assets, in the form of a bailout promise in the event of default by H. ROW is characterized as a large,

[21] Appendix 1.E presents a one-good version of this model. Main results on current account imbalances and intermediation of gross capital flows will be present in that version as well. In addition, the one-good framework serves as a benchmark case to evaluate the effect in a crisis of trade integration among members of a union of countries.

[22] This assumption, which is not essential, will be relaxed in the infinite-horizon model presented in Section 1.4.1, where all economies issue risky debt.

risk-neutral agent.

Country H. H is inhabited by a continuum of identical households and by a benevolent fiscal authority. Lifetime utility of the representative household is given by:

$$u(c_1) + \beta \mathbb{E}\left[u(c_2)\right], \tag{1.1}$$

where $0 < \beta < 1$ is the subjective discount factor and $\mathbb{E}[\cdot]$ is the mathematical expectation operator. Consumption in each period is given by c_t. The period utility function, $u(\cdot)$, is increasing and concave. I will impose that $u(\cdot)$ takes a logarithmic form: $u(c) = \log(c)$. In each period, consumption of the representative household, c_t, is given by an aggregator of two consumption goods, A and B:

$$c_t = \frac{(c_{A,t})^a (c_{B,t})^{1-a}}{a^a (1-a)^{1-a}}. \tag{1.2}$$

The parameter $a \in (0, 1)$ governs the utility weight of consumption good A, $c_{A,t}$. $c_{B,t}$ denotes consumption by the representative household in H of good B. Good A is the numéraire in the economy.

H receives a constant stream of endowments of good B: $y_{B,1} = y_{B,2} = y_B$. The stream of good A received by H is stochastic and its profile is skewed towards the terminal period:

$$y_{A,1} = y_A - \epsilon$$

where $y_{A,1}$ denotes initial-period endowment of good A and $\epsilon > 0$ represents the amount by which it falls short of a long-run value given by y_A. In the terminal period, H faces income risk: it only receives the amount y_A with probability π and, with probability $1-\pi$, a lower amount $y_{A,L} < y_A$:[23]

$$y_{A,2} = \begin{cases} y_A & \text{w.p. } \pi \\ y_{A,L} & \text{w.p. } 1-\pi. \end{cases} \tag{1.3}$$

Households in H do not have access to financial markets. Their problem is simple: As they cannot trade in financial assets, they act as hand-to-mouth agents, consuming in each period their endowment of both goods and net transfers they receive from the fiscal authority.

[23] I will limit attention to cases where the expected value of terminal-period resources is higher than the initial-period endowment: $y_{A,1} < \mathbb{E}[y_{A,2}]$.

The fiscal authority in H can issue debt on international financial markets in the initial period. Debt is denominated in units of numéraire, good A, and it is subject to a limited commitment friction. Bond holdings in H are denoted by b_H, which takes negative values when the fiscal authority issues debt. In the terminal period, the fiscal authority can choose to default on its debt, suffering a default cost ζ. The default cost takes the form of a reduction in the amount of good A endowment. I assume that default costs are increasing in the realization of the endowment process.[24] In particular, default costs are nil in the event of a low output realization and positive otherwise:

$$\zeta = \begin{cases} 0 & \text{if } y_{A,2} = y_{A,L} \\ \hat{\zeta} > 0 & \text{otherwise.} \end{cases} \tag{1.4}$$

The fiscal authority in H maximizes welfare of the representative household, (1.1), by issuing debt in the initial period and choosing between default and repayment in the terminal period. Resources raised on international financial markets are rebated in lump-sum fashion to the representative household.[25] Formally, the problem is given by:

$$\begin{aligned} V_H(\epsilon) = & \max_{\substack{b_H, c_1, c_2, \\ D \in \{0,1\}}} u(c_1) + \beta \mathbb{E}[u(c_2)] \\ & \text{s.t. } p_1 c_1 + q_H b_H = y_A - \epsilon + p_{B,1} y_B, \\ & \quad p_2 c_2 = y_{A,2} + p_{B,2} y_B - [D\zeta - (1-D) b_H], \end{aligned} \tag{1.5}$$

All quantities are denominated in units of the numéraire good A. The relative price of good B to good A is denoted by $p_{B,t}$. p_t is the price index of the consumption bundle c_t.[26] The fiscal authority takes good prices as given. The amount of debt issued by H in the initial period is given by $-b_H$, while q_H denotes the price of a unit of debt. D is an indicator taking value of unity in the event of default. The fiscal authority internalizes the effect of its choice for debt issued on the probability of terminal period default and on the debt price q_H.

[24]This is a common assumption in the endogenous sovereign default literature. Mendoza and Yue (2012) study how frictions in markets for imported inputs give rise to endogenous default costs that are increasing in TFP.

[25]The choice for initial-period debt determines, from the budget constraint, plans for initial and terminal-period consumption of the representative household.

[26]Given intratemporal preferences, the price index of the consumption bundle is given in each period by $p = (p_B)^{1-a}$.

Proceeding backwards from the terminal period, it is optimal for the fiscal authority to repay debt when $-b_H \leq \zeta$. When debt is positive, default occurs in the event of a low output realization, which is accompanied by the absence of default costs. Default would occur with certainty if debt were to be higher than the upper bound of default costs, $\hat{\zeta}$. The following proposition establishes optimality, from the point of view of the fiscal authority in H, of issuing debt carrying positive but not certain default probability:

Proposition 1. *H's optimal choice for b_H lies in the interval $\left[-\hat{\zeta}, 0\right)$ for a sufficiently low realization of the initial-period good A endowment, $y_A - \epsilon$, given values for $y_{A,L}$, π, and β.*

Proof. See appendix 1.B. □

Given this choice, as default in the terminal period only occurs in the event of a low output realization, the probability of terminal-period default is given by $1 - \pi$. The solution to the problem of the fiscal authority can be summarized by the optimality condition:

$$q_H \frac{u'(c_1)}{p_1} = \beta\pi \frac{u'(c_{2,R})}{p_{2,R}}. \tag{1.6}$$

where $p_{2,R}$ and $c_{2,R}$ denote the price index and consumption by H, respectively, in the terminal-period state of the world where repayment is optimal. The benefit of issuing a marginal unit of debt is equal to its marginal cost, at an optimum. The cost of repaying debt in the terminal period is only borne by the representative household in the state of the world where it is optimal not to default. Finally, demand for the two individual goods is given in each period by:

$$c_A = apc \text{ and } c_B = (1-a)\frac{p}{p_B}c. \tag{1.7}$$

Country F. As is country H, F is inhabited by a continuum of identical households. Preferences of households in F are identical to those of households in H, as defined in (1.2) and (1.1). Variables pertaining to country F are indexed by an asterisk, so that c_t^*, $c_{A,t}^*$ and $c_{B,t}^*$ indicate aggregate consumption and consumption of the two types of good by the representative household in country F in period t.

Households in F can trade assets on international financial markets in the initial period. In particular, households in F can issue risk-free debt b_F to *ROW* and purchase assets issued by H. Again, all assets are denominated in units of numéraire good A.

F-households are promised by their domestic fiscal authority that they will be bailed out of losses incurred on assets issued by H in the event of its default. The extent of partial bailout offered to households in F is captured by the parameter $\xi \in [0,1]$. Individual households have full information on the state variables determining the solution to the problem in H, (1.5), and they correctly understand what is the optimal default policy by H. Due to the presence of the bailout promise, the return on an asset issued by H in the event of default, as perceived by a resident of F, is equal to ξ. Absent a bailout promise, the return on debt issued by H in the event of default would be zero.[27] In the event of default by H, bailout transfers received by individual households in F are financed by a lump-sum tax T on all households in the country. Consumption of the representative household in the terminal period then fully bears the negative wealth shock implied by default of H. Each individual household, however, perceives that it will be bailed out of any losses on its bond holdings.

The problem solved by the representative household in F in the initial period is the following:

$$
\begin{aligned}
V_F(\epsilon) = & \max_{\substack{c_1^*, c_2^*, \\ b_F, b_{HF} \geq 0}} \; u(c_1^*) + \beta \mathbb{E}\left[u(c_2^*)\right] \\
& \text{s.t. } p_1 c_1^* + q_F b_F + q_H b_{HF} = y_A^* + p_{B,1} y_B^*, \\
& \quad p_2 c_2^* = y_A^* + p_{B,2} y_B^* + b_F + \left[D\left(\xi b_{HF} - T\right) + (1-D) b_{HF}\right],
\end{aligned}
\tag{1.8}
$$

The amount of H-issued assets purchased by the representative household in F is denoted by b_{HF}. Liabilities issued to ROW at price q_F are denoted by $-b_F$. The representative household in F acts as a price taker with respect to good prices. Debt prices q_H and q_F are also taken as given, as is the value of the lump-sum tax T. In equilibrium, tax receipts finance bailout payments in the default state of the world, $T = \xi b_{HF}$.

Restricting attention to cases where H issues risky debt and gross assets purchased by F are positive, $b_H \in \left(-\hat{\zeta}, 0\right)$ and $b_{HF} > 0$, the solution to the problem in F can be

[27] I restrict the bailout promise ξ to take a value of zero if $-b_H > \hat{\zeta}$. Absent this restriction, a positive price for b_H would be bid by households in F even when default would occur with certainty. The problem faced by the fiscal authority in H would not have a finite solution, as the choice of debt issued by H would be unbounded.

summarized by the following system of first-order conditions:

$$q_F \frac{u'(c_1^*)}{p_1} = \beta \mathbb{E}\left[\frac{u'(c_2^*)}{p_2}\right]$$
$$q_H = q_F - \beta(1-\pi)(1-\xi)\frac{u'\left(c_{2,D}^*\right)/p_{2,D}}{u'(c_1^*)/p_1}. \tag{1.9}$$

The subscript D denotes the terminal-period state of the world where default by H is optimal and, again, R denotes the one where repayment is preferred. The first equation is standard and it simply describes the optimal intertemporal allocation of consumption by F. The second equation characterizes the price q_H at which households in F are willing to purchase a positive amount of H-issued assets. F-households need to be compensated for default risk, given by the probability of default times the loss suffered on each unit of asset: $(1-\pi)(1-\xi)$. In addition, households need to be compensated for volatility in terminal-period consumption that default risk induces, as captured by the term $\frac{u'\left(c_{2,D}^*\right)/p_{2,D}}{u'\left(c_1^*\right)/p_1}$. Note that if a full bailout promise is offered, $\xi = 1$, the price that F-households are willng to bid for H-issued assets equals the one of risk-free liabilities, q_F. This is the same price they would bid on truly risk-free assets, if $\pi = 1$.

Finally, demand functions for individual goods are determined as in H:

$$c_A^* = apc^* \text{ and } c_B^* = (1-a)\frac{p}{p_B}c^*. \tag{1.10}$$

ROW. The representative household in *ROW* has risk-neutral preferences over consumption. Its intratemporal preferences are identical to those in H and F, as is its discount factor, β. The economy in *ROW* is large when compared to H and F. In particular, endowments of either good in H and F are of negligible size when compared to those in *ROW*. As in country F, the stream of endowments received by *ROW* is deterministic and constant: $y_{A,1}^{ROW} = y_{A,2}^{ROW} = y_A^{ROW}$, $y_{B,1}^{ROW} = y_{B,2}^{ROW} = y_B^{ROW}$. Bond holdings by *ROW* of risk-free assets are given by b_{ROW}. The price of a risk-free asset issued by *ROW* is given by q_{ROW}. Bond holdings by *ROW* of H-issued assets are given by $b_{H,ROW}$.[28]

Transport costs. Good A can be costlessly traded across all country pairs. International trade of good B is subject to an iceberg transport cost τ that only applies to trade with *ROW*: For each unit of good B that is shipped by an exporter, only $1-\tau$ units are

[28]The optimization problem solved by *ROW* is described in detail in Appendix 1.C.

received by the destination country. Absence of transport costs affecting trade between H and F captures strong goods market integration among members of an economic union. Heterogeneous transport costs affecting goods A and B serve instead as a proxy for heterogeneous tradability of output of different industrial sectors. In the model, countries that are relatively abundant in good B describe economies that are characterized by low average tradability of output. Relative abundance of the two types of good in H and F is captured by the ratios $\frac{y_A}{y_B}$ and $\frac{y_A^*}{y_B^*}$.

The relative price of good B to good A prevailing in H and F can be interpreted as the terms of trade of the country that is relatively abundant in good B. Due to the presence of transport costs, good prices in ROW may differ from those prevailing in the union formed by H and F. The relative price of good B to good A in ROW is denoted by p_B^{ROW}.

World prices. In equilibrium, world prices follow from the solution of ROW's problem, given the negligible size of H and F compared to the world economy. The world price of a risk-free bond is given by $q_{ROW} = \beta$, by risk-neutrality of ROW.[29] Bonds issued by F are risk-free, so that their price is also given by

$$q_F = q_{ROW} = \beta. \tag{1.11}$$

If ROW holds a positive amount of bond issued by H, the price of this asset must compensate ROW for default risk. Hence, $b_{H,ROW} > 0$ implies

$$q_H = \beta \mathbb{E}\left[1 - D\right].$$

In particular, if $b_H \in \left(-\hat{\zeta}, 0\right)$, the probability of repayment is given by π and $q_H = \beta\pi$. ROW is not willing to buy assets issued by H if $q_H > \beta \mathbb{E}\left[1 - D\right]$. In this event, F would be the marginal and sole buyer of assets issued by H: $b_{H,ROW} = 0$ and, by market clearing, $b_{HF} = -b_H$.

World good prices are also determined in ROW. When H or F export good B to ROW, arbitrage and transport costs imply $p_B = (1 - \tau)\, p_B^{ROW}$. Symmetrically, when

[29] Since assets are denominated in units of numeraire good A, the first order condition in ROW implies $q_{ROW} = \beta \frac{p_1^{ROW}}{p_2^{ROW}}$. Given the constant endowment streams, $p_1^{ROW} = p_2^{ROW}$, so that $q_{ROW} = \beta$ holds.

ROW exports good B to either H or F, $p_B = \frac{1}{(1-\tau)} p_B^{ROW}$. Finally, when $\tau > 0$, a no-trade equilibrium can arise, where price differentials are too small, compared to transport costs, for intratemporal trade in B to emerge. Indeed, when $p_B \in \left((1-\tau) p_B^{ROW}, \frac{1}{(1-\tau)} p_B^{ROW} \right)$, no trade in good B takes place between H or F and ROW. When trade in good B between H, F and ROW does not take place, the markets for good B in H and F and in ROW have to individually clear:

$$y_B + y_B^* = c_{B,s} + c_{B,s}^* \text{ and } y_B^{ROW} = c_{B,s}^{ROW}. \tag{1.12}$$

Demand for good B by H and F is then satisfied entirely by supply within these countries. Within this interval, the relative price of good B in H and F is determined by market clearing within the economic union. From intratemporal preferences, this relative price is given by:[30]

$$p_B = \frac{1-a}{a} \frac{c_A + c_A^*}{c_B + c_B^*} = \frac{1-a}{a} \frac{c_A + c_A^*}{y_B + y_B^*}. \tag{1.13}$$

Equilibrium. I will now proceed to define a competitive equilibrium in the world economy.

Definition 1. *An equilibrium is defined as a vector of quantities b_H, b_{HF}, b_F, b_{ROW}, $b_{H,ROW}$, c_s, $c_{A,s}$, $c_{B,s}$, c_s^*, $c_{A,s}^*$, $c_{B,s}^*$, c_s^{ROW}, $c_{A,s}^{ROW}$, $c_{B,s}^{ROW}$, prices q_H, q_F, q_{ROW}, $p_{B,s}$, p_s, $p_{B,s}^{ROW}$, p_s^{ROW} and default indicator D such that, given values for the bailout promise ξ, transport costs τ and output realization ϵ, in all time periods and states of the world, as indexed by $s \in \{1; 2, R; 2, D\}$:*

- *b_H and $\{c_s\}_s$ solve the intertemporal allocation problem in H, (1.5), given q_H and $\{p_s\}_s$.*
- *D solves optimal default policy by H, as defined by (1.5).*
- *b_F, b_{HF} and $\{c_s^*\}_s$ solve the intertemporal allocation problem in F, (1.8), given q_F, q_H and $\{p_s\}_s$.*
- *$\left\{c_{A,s}, c_{A,s}^*, c_{B,s}, c_{B,s}^*\right\}_s$ solve demand for individual goods by H and F, (1.7) and (1.10), given $\{c_s, c_s^*\}_s$ and prices $\{p_{B,s}, p_s\}_s$.*

[30]From (1.7) and (1.10), it can be seen that $c_A + c_A^* = \frac{a}{1-a} \frac{c_B + c_B^*}{p_B}$.

- b_{ROW}, $b_{H,ROW}$ *and* $\left\{c_s^{ROW}\right\}_s$ *solve the intertemporal allocation problem in ROW given* q_{ROW}, q_H *and* $\left\{p_s^{ROW}\right\}_s$. $\left\{c_{A,s}^{ROW}, c_{B,s}^{ROW}\right\}_s$ *satisfy ROW's demand for individual goods, given* $\left\{c_s^{ROW}\right\}_s$ *and prices* $\left\{p_{B,s}^{ROW}, p_s^{ROW}\right\}_s$.
- *All markets for assets clear:*

$$0 = b_H + b_{HF} + b_{H,ROW}$$

$$0 = b_F + b_{ROW}$$

- *The world market for good B clears, net of resources lost to transport costs between H,F and ROW:*

$$y_B + y_B^* + y_B^{ROW} = c_{B,s} + c_{B,s}^* + c_{B,s}^{ROW} + \tau \left| y_B^{ROW} - c_{B,s}^{ROW} \right|$$

- *The market for good A clears by Walras' law.*

$$c_{A,s} + c_{A,s}^* + c_{A,s}^{ROW} = y_{A,s} + y_A^* + y_A^{ROW}$$

1.3.2 Results

Intermediation and gross capital flows. What drives intermediation of international capital flows in this framework? When is it optimal for an economy like F to be the main holder of risky debt issued by its partner in the economic union? Risk aversion and the presence of a bailout promise play a crucial role in answering these questions.

In the absence of a bailout promise, risk-averse households in F are not willing to be the marginal buyers of debt issued by H. From F's Euler equation, (1.9), the price consistent with H-assets being priced by F-households is given by

$$q_{H,F} = \beta \left[1 - (1-\pi) \frac{u'\left(c_{2,D}^*\right)/p_{2,D}}{u'\left(c_1^*\right)/p_1} \right],$$

where $q_F = \beta$ follows from (1.11), as assets issued by F are risk-free. Due to risk-aversion in preferences of F, (1.1), this price is lower than the one consistent with pricing by ROW, $q_{H,ROW} = \beta\pi$.

$$\frac{u'\left(c_{2,D}^*\right)/p_{2,D}}{u'\left(c_1^*\right)/p_1} > 1.$$

F would suffer from low consumption in the state of the world where H defaults, if it were a holder of H-issued assets. F-households have to be compensated for this risk, whose price is captured by the ratio of marginal utilities across initial and terminal periods. Hence, given its risk-neutrality, ROW is the marginal buyer of assets issued by H, in the case where households in F are not offered a bailout promise on these assets. [31]

When a full bailout promise is offered, H-issued assets are perceived as risk-free by F-households. Accordingly, they are willing to purchase these assets at a price equal to that of risk-free liabilities:

$$q_{H,F} = q_F = \beta.$$

To the extent that these assets are not truly risk-free, $\pi < 1$, the price bid by H is not sufficiently high to compensate ROW for default risk, $\beta > \beta\pi$, and ROW will not buy any positive amount of H-issued debt. The entire debt issued by H is then held in equilibrium by F.

Gross asset purchases by F are partly financed with own resources, as well as by intermediating funds borrowed from ROW. The fraction of gross asset purchases that F finances by issuing gross liabilities lies in the interval:[32]

$$\frac{b_F}{b_H} \in \left(\frac{\beta}{1+\beta}, 1\right] \text{ for } \pi \in (0,1]. \tag{1.14}$$

Three main results emerge on intermediation of gross capital flows by F. First, if repayment by H is certain, F entirely finances its purchase of H-debt by borrowing from ROW, $b_F = b_H$. As debt issued by H is truly risk-free, the bailout promise has no effect on the equilibrium allocation: F fully intermediates gross asset purchases by issuing gross liabilities, and the allocation is equal to one where debt issued by H is held directly by ROW. Second, in the limit where default by H is certain, $\pi \to 0$, F is a net saver and its gross liabilities are smaller in magnitude than gross assets, $\lim_{\pi\to 0} b_F = \frac{\beta}{1+\beta} b_H$. Households in F face positive taxes with certainty in the terminal-period, to cover for certain losses on gross assets.[33] Hence, they act as net savers, in order to transfer resources

[31]In an equilibrium where $\frac{p_{2,D}}{p_1}$ is sufficiently high, it would be optimal for F to hold the entire amount of H-issued debt even in the absence of bailout promise, in order to transfer resources across states of the world and exploit differences in good prices. This equilibrium, however, would only arise in the presence of severe transport costs, and it is ignored here.

[32]This result follows from the solution to the system of optimality conditions in (1.9). I consider here for simplicity a setup where $\tau = 0$, so that good prices are constant across states of the world as determined in ROW, and normalized to unity. I define total endowment in F as $y^* = y_A^* + y_B^*$.

[33]Note that while individual households do not internalize the effects of their choices on taxation,

to the terminal period and mitigate the fall in consumption following default by H. Finally, when the terminal-period repayment is uncertain, $\pi \in (0, 1)$, households in F face uncertainty with regard to terminal-period taxation and consumption. The larger is the magnitude of consumption risk, the more F households will save to insure against it. Hence, the fraction of gross asset purchases that is financed with own resources is increasing in the relative size of gross assets to terminal-period endowment. The optimal amount of gross liabilities issued by F is presented graphically in Figure 1.4, for values of $\pi \in (0, 1]$ and for two different relative values of gross assets to total endowment.

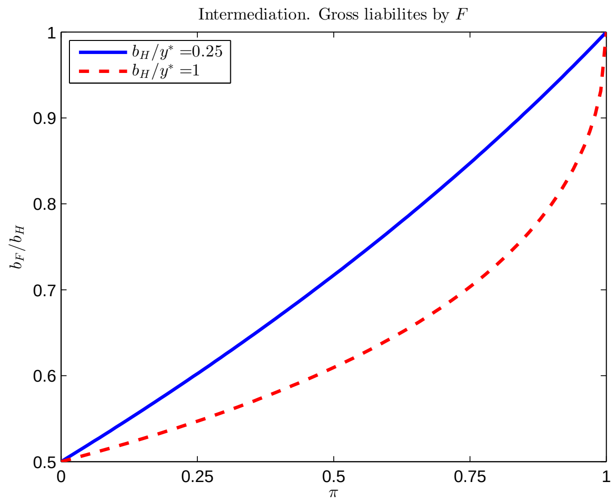

Figure 1.4: Issuance of gross liabilities by F

Gross liabilities issued by F are presented as a function of the probability of repayment by H and of the amount of gross assets. The blue, solid line represents the fraction of gross assets purchased by F that is financed by borrowing from ROW, for a low amount of gross assets relative to total income, $b_H/y^* = .25$. The red, dashed line represents the same variable for a higher ratio of gross assets to income, $b_H/y^* = 1$.

Debt crisis in an economic union. Motivated by recent experience in the euro area, we can employ the model here presented to study the transmission and amplification of a debt crisis taking place in an economic union. A crucial role is played in this setting by the interaction between trade integration and financial linkages characterizing countries in the economic union.

in equilibrium, they correctly form expectations on terminal-period taxation, $T = \xi b_{HF}$.

I analyze consumption by H and F in the terminal-period crisis state, where H faces a low output realization, $y_A = y_{A,L}$, and it does not repay its debt. By comparing the equilibrium allocation with bailout promise to the one with no bailout, we are able to analyze how policy distortions in the economic union lead to cross-border transmission and amplification of a debt crisis.[34]

The introduction of a bailout promise generates financial linkages between members of the economic union, through which a crisis is transmitted from debtors to creditors. This policy distortion induces F to take on exposure to default risk by H, issuing debt to finance asset purchases. In the crisis state, wealth in F is lower when this country intermediates resources towards H, as gross assets give no return and gross liabilities to ROW are repaid. Formally, aggregate consumption by F in the terminal-period crisis state is given by:

$$c^*_{2,D} = (y^*_A + p_B y^*_B + b_F)\,\frac{1}{p}$$

To the extent that F issues a positive amount of gross liabilities in order to purchase assets issued by H, policy distortions cause consumption in this country to be lower in the crisis state. The mismatch between risky gross assets and safe gross liabilities on F's external balance sheet leads to a transmission of the crisis to this country, with detrimental effects on consumption.

Amplification of the crisis in the debtor country arises when creditors on its external liabilities coincide with the destination of its exports. In the crisis state, H is reduced to financial autarky, but it is able to trade in goods A and B, according to relative abundance of the two in its endowment basket. Aggregate consumption by H in the crisis then crucially depends on the relative price of its exports. When transport costs imply that good B is not traded with ROW, its relative price is function of relative abundance within the union of H and F. In the crisis state, the relative price p_B is determined

[34] When discussing crisis transmission and amplification, it is worth noting that endowment realizations are not affected by features of the equilibrium of this economy, them being exogenous. However, endowment values in terms of consumption good are endogenous, due to the possibility of changes in goods relative prices, representing the terms of trade of countries in the economic union.

according to (1.13):

$$p_B = \frac{1-a}{a}\frac{c_{A,2,D} + c^*_{A,2,D}}{c_{B,2,D} + c^*_{B,2,D}} = \frac{1-a}{a}\frac{y_{A,L} + y^*_A + b_F}{y_B + y^*_B}$$

Intermediation of gross flows by F causes the relative price of good B to be lower in the crisis state. The bailout promise induces F to issue gross liabilities to ROW, $b_F < 0$. In the crisis state, the amount of good A available in the union is then lower than it would be had H borrowed directly from ROW, and defaulted on its liabilities. In turn, in the crisis state, relative scarcity of A causes the price p_B to be lower than it would be, had F not acted as an intermediary between H and ROW.

The effect of lower union wealth on the crisis country, H, depends on the status of this country as a net exporter or importer of the good subject to transport costs, B. Denote by $p_{B,I}$ the crisis-state relative price of good B in the equilibrium allocation where, due to the presence of a bailout promise, external borrowing by H is intermediated by F. Denote by $p_{B,NI}$ the relative price in the equilibrium where no bailout is promised, no intermediation occurs, and H borrows directly from ROW. As gross liabilities issued by F are higher in the equilibrium allocation with bailout, (1.13) implies $p_{B,I} < p_{B,NI}$.

Consider now two extreme cases of full specialization by H. First, when H is fully specialized in good B, $y_{A,L} = 0$, $y_B > 0$, it can be shown that its relative consumption across the two equilibrium allocations is proportional to the relative difference in p_B:

$$\frac{c_I}{c_{NI}} = \left(\frac{p_{B,I}}{p_{B,NI}}\right)^a. \tag{1.15}$$

Second, and conversely, if H is fully specialized in good A, its relative consumption is inversely related to the difference in p_B:

$$\frac{c_I}{c_{NI}} = \left(\frac{p_{B,I}}{p_{B,NI}}\right)^{a-1}, \tag{1.16}$$

as $a - 1 < 0$. Finally, when relative abundance of the two goods in B equals relative abundance in the union, the fall in p_B has no effect on consumption, as no intra-temporal trade occurs. Formally, $c_I = c_{NI}$ if

$$\frac{y_B}{y_B + y^*_B} = \frac{y_A}{y_{A,L} + y^*_A + b_{F,NI}} \tag{1.17}$$

with $c_I < c_{NI}$ for values of $\frac{y_B}{y_B+y^*_B}$ above this threshold and vice versa.[35] The higher the share of good B in the endowment basket of H, the more severe is the crisis in this

[35]The result in (1.17) holds locally, for $\frac{p_{B,I}}{p_{B,NI}} \approx 1$. A full derivation of results in this section is provided in Appendix 1.D.

country. Amplification occurs as demand in export destinations deteriorates in the event of the crisis, due to the presence of financial linkages between trade partners. If the crisis country were instead a net exporter of good A, it would benefit from cheaper imports of good B, while being able to export to ROW which is unaffected by the crisis.

Prior to the euro area sovereign debt crisis, Greece, Spain and Portugal were relatively specialized in sectors characterized by lower tradability of output, as detailed in Section 1.2. To the extent that lower output tradability implied stronger trade linkages with partners in the monetary union, this mechanism may have played a role in exacerbating the recession observed in the periphery of the euro area during the recent crisis. I present in Section 1.4 a quantitative evaluation of these channels, in the context of an infinite-horizon framework.

Optimal bailout promise. In order to analyze the rationale behind the introduction of distortions on cross-border holdings of assets, I discuss here conditions under which it is optimal, from the point of view of a union-wide benevolent planner, to introduce a bailout promise on risky assets. In an environment characterized by the presence of frictions that prevent the equilibrium allocation from being Pareto optimal, it need not be the case that the introduction of an additional distortion diminishes welfare.[36] In this framework, such frictions are represented by the inability of the fiscal authority in H to commit to repay debt in the terminal-period crisis state. There indeed exist equilibrium allocations in this model where aggregate welfare in the economic union with a full bailout promise is higher than that attained absent such a distortion. The introduction of a bailout promise distorts the allocation in F, inducing excessive risk-taking and leverage. In country H, subject to the limited commitment friction, the same distortion determines a higher price of debt issued, allowing for additional borrowing. When borrowing frictions are sufficiently severe, due for example to scarce initial-period resources in H, the benefits of redistribution would outweigh the costs of distortion, making a bailout promise desirable.

Consider welfare in the union as the sum of welfare in the two countries, as function of the initial period income shock in H, ϵ:

$$V(\epsilon) = V_H(\epsilon) + V_F(\epsilon)$$

[36] A canonical reference is Lipsey and Lancaster (1956).

We are interested in studying the change in aggregate welfare when comparing an allocation with no bailout promise, $\xi = 0$, to another one with bailout, e.g. $\xi = 1$. It is possible to analytically compare welfare in the union across these two different allocations by focusing on the limiting case where π, the probability of repayment by H, tends to zero.[37] Absent a bailout promise, neither H nor F would engage in financial trade, for opposite reasons. The flat endowment profile in F ensures that this country achieves in financial autarky the optimal allocation of consumption across periods and states, so that no borrowing or lending takes place:

$$c_1^* = c_2^* = y^*.$$

Scarcity of initial period resources, implies instead the presence of a borrowing motive in H. It cannot borrow if default is certain, however, as debt is issued at price $q_H = \beta\pi = 0$. The lower the initial period endowment, then, the higher is marginal utility of initial-period consumption in H and the more skewed will be the consumption profile in this country:

$$c_1 = y - \epsilon, \; c_{2,D} = y_L, \; c_{2,R} = y$$

When a bailout promise is offered, default by H still occurs with certainty, as $\pi \to 0$, but F is now willing to lend, financing asset purchases with own resources and by borrowing from ROW. Lending simply amounts to a free transfer of resources to H, so that wealth in F falls as consequence of the bailout distortion. H receives resources in the initial period, when they are the most needed, and its welfare rises in this allocation.[38] Importantly, the welfare gain in H is increasing in the initial period shock, ϵ, by concavity of the utility function. Hence, as welfare in F is unaffected by ϵ in either the full or no-bailout promise allocations, the following result can be established:

Proposition 2. *For a sufficiently high value $\bar{\epsilon}$ of the initial period shock ϵ, an allocation with bailout promise, $\xi = 1$ delivers aggregate welfare $V(\epsilon)$ that is equivalent to that*

[37] As in the previous discussion on intermediation, I am considering here an allocation with no transport costs and constant good prices determined by ROW. As prices are constant, I am also ignoring endowments of the two different goods and focusing simply on total endowments y and y^*.

[38] The amount of borrowing by H is bounded above by the amount of resources available in the repayment state, as except in the limit $\pi > 0$ holds, and marginal utility of consumption would otherwise be unbounded if H had to repay debt.

attained in the allocation where no bailout is promised, $\xi = 0$. The bailout promise allocation is preferable to the no-bailout one for values of ϵ above that threshold.

Proof. See appendix 1.F. □

Intuitively, the bailout distortion allows resources to be transferred across members of the union. This transfer, however, is only desirable if resources in the destination country are sufficiently scarce.

The above result on optimality of bailout promises can help to shed light on the rationale for the introduction of similar distortions in the euro area. At the cost of generating critical financial linkages, implicit subsidies on cross-border asset holdings may have contributed to a relaxation of borrowing limits faced by peripheral countries. The relaxation of such constraints could have been seen as desirable, in turn, if relative incomes in these countries were expected to converge.

1.4 An infinite-horizon model of gross capital flows and debt crises

I introduce in this section a quantitative model of countries in an economic union, trading among each other and with the rest of the world in goods and financial assets. The purpose of the model is to quantitatively assess the ability of policy distortions on financial trade in a union to explain the recent European experience in terms of capital flows.

In the model, the introduction of a subsidy on cross-border holdings of assets in an economic union generates predictions for current account and gross external positions that are in line with empirical evidence for the euro area. In addition, the model predicts that countries with substantial external liabilities are more severely affected by a debt crisis when their creditor countries coincide with the main destination of their exports.

The next subsection presents the infinite-horizon model. Two model economies are considered, characterized by different treatment of risky assets within the economic union. Thereafter, I present the calibration of the model to properties of euro area countries prior to the inception of the EMU and I describe the steady state allocation. Finally, I present results of the two quantitative experiments I consider. First, I consider the dynamic transition of the model economy after the introduction of subsidies on risky assets in the economic union. I analyze, in particular, the implications of this change on current account imbalances and on intermediation within the union of gross capital flows from the rest of the world. Second, I analyze a debt crisis, describing its transmission and amplification via financial and trade linkages among member countries of an economic union.

1.4.1 Infinite-horizon model

Environment. The world economy is inhabited by a continuum of unit measure of small open economies, I, and by the rest of the world, ROW. The small open economies in I will be referred to as countries and they are indexed by $i \in [0,1]$. Each country is inhabited by a continuum of identical households. Time is discrete and indexed by t. There are two types of goods, A and B. Good A is freely tradable across all country pairs, while trade in good B between ROW and countries in I is subject to an iceberg transport cost τ.[39] Each country in I receives a stream of endowments of either type of good. $F = [0, \kappa]$ denotes the subset of countries in I that receive endowments of good A, while countries in the complementary set, $H = (\kappa, 1]$, receive endowments of good B. Good A is the numeraire in the world economy. Each country receives a stream of endowments $\{y_{i,t}\}_{t=0}^{\infty}$ which is determined according to a stochastic Markov process. The process is i.i.d. across countries and there is no aggregate uncertainty in the world economy.

Households in each country i enjoy consumption of both types of good. Preferences of the representative household over consumption paths are indexed according to:

$$U_i = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{i,t}\right), \tag{1.18}$$

where $\mathbb{E}_0$ is the mathematical expectation operator conditional on information available at time zero, β is the subjective discount fator and $c_{i,t}$ denotes aggregate consumption in i. The period utility function takes the CRRA form $u\left(c\right) = \frac{c^{1-\gamma}}{1-\gamma}$, where the parameter γ captures the degree of relative risk aversion. In each period, aggregate consumption by

[39] This assumption on heterogeneous transport costs is the same one made in the model of Section 1.3.

the representative household is defined as the CES aggregator:

$$c_i = \left[a^{\frac{1}{\vartheta}} c_{A,i}^{\frac{\vartheta-1}{\vartheta}} + (1-a)^{\frac{1}{\vartheta}} c_{B,i}^{\frac{\vartheta-1}{\vartheta}} \right]^{\frac{\vartheta}{\vartheta-1}} \tag{1.19}$$

where ϑ represents the elasticity of substitution between the two types of consumption good and $a \in (0,1)$ is the utility weight of good of type A.

ROW is represented by a large, risk-neutral agent, enjoying consumption of both goods. *ROW* is relatively more patient than households in the economic union. Its stochastic discount factor, $\beta^{ROW} > \beta$ pins down the price of a risk-free asset in the world economy, $q^{ROW} = \beta^{ROW}$.[40] The relative price of good B in *ROW* is given by p_B^{ROW}.

Risky sovereign debt. All countries in I can buy risk-free assets issued by *ROW*. In addition, countries in the economic union issue risky debt on international financial markets. External debt is subject to a simple non-repayment friction: For a given amount of debt issued by i, lenders will fail to receive debt repayments with probability $1-\pi$. The exogenous repayment probability π is function of current-period output y_i and of the amount of debt issued. The function determining π is increasing in y_i and decreasing in issued debt.[41] I denote by b_i holdings by i of risk-free assets issued by *ROW*. Negative values of b_i indicate that i owes debt. Adopting recursive notation, b_i' denotes current-period purchases of assets or, for $b_i' < 0$, debt issuance. As assets issued by *ROW* are risk-free, $\pi = 1$ if $b' > 0$. All agents have full information on non-repayment probabilities. Lenders in *ROW* behave competitively, and they are willing to buy debt issued by i as long as the price of this asset compensates them for non-repayment risk. The price at which i can issue debt to *ROW* is then given by the function

$$q\left(y_i, b_i'\right) = q^{ROW} \pi\left(y_i, b_i'\right). \tag{1.20}$$

The debt price function inherits properties of π. In particular, the price of debt issued by i is decreasing in its quantity. Borrowers always repay debt in full, even though a fraction of debt repayments are not received by lenders.[42] This assumption captures in a simple

[40] As in other models of external debt this assumption ensures that $\beta/q^{ROW} < 1$, ruling out divergence of external positions in presence of a borrowing limit. Schmitt-Grohe and Uribe (2003) review properties of alternative stationarity-inducing methods in similar frameworks.

[41] In addition, I assume that the function is continuously differentiable in both arguments.

[42] I assume that resources paid by borrowers but not received by lenders are lost.

way sovereign risk faced by creditors on international financial markets. In addition, as the probability of non-repayment is priced by lenders, borrowers have to take into account how the quantity of debt they issue affects its price. These features will be crucial to our analysis on the direction of international capital flows and intermediation in an economic union.[43]

Equilibrium in no-bailout economy. Consider the case where, in all countries in the economic union I, no distortions are imposed on households' trade in financial assets. In particular, no bailout is offered in the event of losses suffered on risky assets. In this scenario, it is optimal for all countries in i to trade in financial assets solely with ROW.[44] The budget constraint of the representative household in i is given in each period by:

$$pc_i + q\left(y_i, b_i'\right) b_i' = p_i y_i + b_i. \tag{1.21}$$

The relative price of the endowment good received by i is given by p_i. This price takes value of unity if $i \in F$ and it receives an endowment of good A, and it is given by p_B otherwise. The relative price of the consumption basket c is given by $p = \left[a + (1-a)\, p_B^{1-\vartheta}\right]^{\frac{1}{1-\vartheta}}$. Again, bond holdings by i are given by b_i. Negative values of this variable indicate that the economy owes debt to ROW. Note that the amount of resources that i can raise on international financial markets is bounded, as the price at which debt is sold is decreasing in the amount issued.

The problem solved by the representative household in i is to maximize expected

[43]The simplifying assumption that debt is always repaid in full allows me to study the pattern of intermediation and current account imbalances while ignoring optimal default choice by borrowers. In particular, I disregard strategic considerations by borrowers that would arise when issuing defaultable debt whose price does not reflect the true default probability. While interesting, these issues fall beyond the scope of this chapter. The reader is referred to Conesa and Kehoe (2015) for an analysis of the strategic interaction between sovereign borrowers and lenders in a model of self-fulfilling debt crisis.

[44]Risky assets issued by other economies in I could be purchased by countries in the economic union. As these assets are priced by a risk-neutral agent, ROW, and they do not offer insurance against own income risk, it is not optimal for the risk-averse households in I to hold positive amounts of them.

lifetime utility (1.18) subject to the sequence of budget constraints (1.21):

$$\begin{aligned} V_i\left(y_i, b_i\right) &= \max_{b_i', c_i} \left\{u\left(c_i\right) + \beta \mathbb{E}\left[V_k\left(y_i', b_i'\right)\right]\right\} \\ \text{s.t. } & pc_i + q\left(y_i, b_i'\right) b_i' = p_i y_i + b_i, \end{aligned} \tag{1.22}$$

where V_i denotes the value function of a generic country i, and it is given by either V_H or V_F, depending on the type of good i is endowed with. The solution can be summarized by the following intertemporal optimality condition,

$$\frac{u'\left(c_i\right)}{p}\left(q\left(y_i, b_i'\right) + \frac{\partial q}{\partial b_i'} b_i'\right) = \beta \mathbb{E}\left[\frac{u'\left(c_i'\right)}{p'}\right], \tag{1.23}$$

equating the discounted marginal cost of an additional unit of debt with its marginal benefit, taking into account how a marginal increase in debt issued affects its price $q\left(y_i, b_i'\right)$.[45]

Households allocate consumption across the two goods according to standard CES demand:

$$c_{A,i} = a\left(\frac{1}{p}\right)^{-\vartheta} c_i \ , \ c_{B,i} = (1-a)\left(\frac{p_B}{p}\right)^{-\vartheta} c_i. \tag{1.24}$$

Finally, the aggregate amount of good B that countries in I are endowed with is given by

$$y_B = \int_{i \in H} y_i \, di.$$

Absent aggregate uncertainty, the aggregate endowment of goods in I is constant by the law of large numbers, $\int_{i \in I} y_i \, di = \mathbb{E}\left[y_i\right]$, where $\mathbb{E}\left[y_i\right]$ is the unconditional expected value of y_i. Given intra-temporal demand, (1.24), and defining aggregate consumption as $c = \int_{i \in I} c_i \, di$, aggregate consumption of good B in I is given by:

$$c_B = \int_{i \in I} c_{B,i} \, di = (1-a)\left(\frac{p_B}{p}\right)^{-\vartheta} c. \tag{1.25}$$

Due to the presence of transport costs, the relative price of B prevailing in I, p_B, may differ from the one in ROW, p_B^{ROW}. If gains from trade are not sufficiently large to outweigh transport costs, no trade in good B occurs between I and ROW.[46] When no

[45] Differently than in standard Euler equations in Eaton-Gersovitz frameworks, repayment occurs here in all states of the world. Hence, the expectation operator is conditional on information available in the current period only and not on a particular realization of the following-period exogenous state.

[46] Formally, no trade occurs in the range $p_B \in \left((1-\tau)\, p_B^{ROW}, \frac{1}{1-\tau} p_B^{ROW}\right)$. Trade in good B between I and ROW only emerges if total demand for good B, (1.25), would imply $p_B \notin \left((1-\tau)\, p_B^{ROW}, \frac{1}{1-\tau} p_B^{ROW}\right)$ when $c_B = y_B$.

trade with *ROW* occurs, the aggregate endowment in I of good B is entirely consumed within the union, and p_B is determined according to (1.25), with $c_B = y_B$.

I now define a competitive equilibrium in the world economy when no bailout is promised.

Definition 2. *A recursive competitive equilibrium in the no-bailout economy is defined as a set of functions p_B, c_i, $c_{A,i}$, $c_{B,i}$, b'_i for $i \in \{H, F\}$ that solve households' optimization problems and clear the market for good B, as well as by a transition function Γ for the joint distribution of (y, b):*

- *Consumption $c_i(y,b)$ and bond holdings $b'_i(y,b)$ solve households' maximization problem* (1.22) *for countries i in either H or F*

- *Consumption of individual goods $c_{A,i}(y,b)$, $c_{B,i}(y,b)$ solves the intra-temporal allocation* (1.24)

- *The relative price p_B ensures market clearing for good B, $y_B = \int_{i\in I}(1-a)\left(\frac{p_B}{p}\right)^{-\vartheta} c_i\, di$ if no trade in good B between I and ROW occurs, $y_B = c_B$, and it is determined consistently with transport costs and with the price prevailing in world markets otherwise:*

$$p_B = \begin{cases} (1-\tau)\, p_B^* & \text{if } y_B > c_B \\ \frac{1}{(1-\tau)} p_B^* & \text{if } y_B < c_B \end{cases} \tag{1.26}$$

- *The Markov transition probability of the exogenous shock process for y and the optimal choice of bond holdings determine the transition function Γ for the joint distribution of (y,b) in H and F.*

Equilibrium in the economy with full bailout promises. Consider now the case where, in all countries in I, the domestic government offers a bailout promise to all households holding risky assets issued by other members of the union. The government in each i offers to its residents a full bailout promise, if they were not to receive payments they are owed on holdings of risky sovereign debt. Due to the full bailout promise, households perceive assets issued by all other union members as being perfect substitutes for each other, even though they differ in terms of repayment probabilities. The bailout promise is only offered if the total amount of liabilities issued by each individual borrower

j falls below a certain threshold $\bar{b}$: $b_j \geq \bar{b}(y_j)$.[47] In every period, the government in each i finances bailout payments to asset holders in i by setting a lump-sum tax T_i that is paid by all households in the country. Define $\tilde{b}_{i,j}$ as holdings by the representative household in i of debt issued by j. The amount of resources that is lost by the representative household in i on risky assets issued by other union members is given by $\int_{j \in I} \left[1 - \pi\left(y_{j,-1}, \tilde{b}_j\right)\right] \tilde{b}_{ij} dj$. As the government fully compensates asset holders for losses incurred on risky debt, the amount of tax that is paid by the representative household in each period is exactly equal to the aggregate amount of resources that are lost by bond holders:[48]

$$T_i = \int_{j \in I} \left[1 - \pi\left(y_{j,-1}, \tilde{b}_j\right)\right] \tilde{b}_{ij} dj. \tag{1.27}$$

Countries in the union only differ ex-ante in terms of the type of good they are endowed with, which is given by A or B depending on whether i belongs to the set F or H. Depending on their wealth and realization of the exogenous shock, economies optimally choose to behave as gross savers, buying gross assets issued by union members and issuing gross liabilities to ROW, or whether to be borrowers instead, simply issuing debt to other countries in the union.

In gross saving countries, households can buy assets issued by other countries in the union and they can issue liabilities to ROW. I define $\tilde{b}_i = \int_{j \in I} \tilde{b}_{ij} dj$. as the aggregate amount of union-issued assets that is held by the representative household in country i. The price bid by union members for bonds issued by other countries in I is given by $\tilde{q}$. Due to perfect substitutability implied by the full bailout promise, the price $\tilde{q}$ is identical across all issuers in I. As in the no-bailout setting, b_i denotes holdings by i of risk-free bonds issued by ROW. This variable takes negative values when i owes gross liabilities to ROW. Beginning-of-period net wealth of the representative household in a gross saving country is defined as gross assets, net of gross liabilities owed to ROW and of taxes paid to the domestic government: $n_i = \tilde{b}_i + b_i - T_i$. Note that, even though it enjoys a full bailout promise, the representative household still incurs in losses on holdings of gross assets, as

[47] As in Section 1.3, this restriction on bailout promises is required to impose a borrowing limit and to prevent borrowers' liability positions from growing unbounded. In an endogenous sovereign default setting, a similar assumption is made by Ayres, Navarro, Nicolini and Teles (2015).

[48] By the law of large numbers, $\pi\left(y_j, b'_j\right)$ both indicates the probability of repayment by a given economy with $y = y_j$, $b' = b'_j$ as well as the total fraction of resources that are received by a lender on his holdings of assets issued by all economies with $y = y_j$, $b' = b'_j$.

bailout transfers are financed by taxation. The budget constraint of the representative household in a gross saving country is given by:

$$pc_i + \tilde{q}\tilde{b}_i' + q\left(y_i, b_i'\right) b_i' = p_i y_i + n_i \tag{1.28}$$

describing how purchases of consumption and gross assets are financed with own resources, n_i, and by issuing liabilities to ROW.

The problem faced by the representative household in each country i includes as a state variable aggregate wealth in the country itself, N_i. Aggregate wealth is a state variable as it allows individual households to forecast the next-period value of gross asset holdings in i, which determines in turn next-period taxation, according to (1.27).[49] I define $\Phi\left(y_i, N_i\right)$ as the forecast for the next-period value of aggregate wealth, N_i', given the current aggregate state in i, $\left(y_i, N_i\right)$. The other state variables for the household are given by its net wealth, n_i, and by the realization of the endowment shock y_i. The maximization problem solved by the representative household in a gross saving country is the following:

$$\begin{aligned} V_i^S\left(y_i, n_i, N_i\right) = & \max_{b_i',\, \tilde{b}_i' > 0,} \left\{u\left(c_i\right) + \beta \mathbb{E}\left[V_i\left(y_i', n_i', N_i'\right)\right]\right\} \\ & \text{s.t. } pc_i + q\left(y_i, b_i'\right) b_i' + \tilde{q}\tilde{b}_i' = p_i y_i + n_i \\ & n_i' = \tilde{b}_i' + b_i' - T_i' \\ & N_i' = \Phi\left(y_i, N_i\right) \end{aligned} \tag{1.29}$$

Country i is not restricted to be a gross saver in the following period, hence the continuation value of households is not given by $V_i^S(\cdot)$ but rather by $V_i(\cdot)$, to be defined shortly. I define ϖ as the subset of the state space where a solution to the problem in (1.29) exists with positive holdings of gross assets, $\tilde{b}_i' > 0$: $\varpi = \left\{(y, n, N) \mid \tilde{b}_i'(y, n, N) > 0\right\}$. The solution to the household problem implies a law of motion for net wealth of the representative household, $n'\left(y_i, n_i, N_i\right)$, which, in general, differs from the perceived aggregate one, $N_i' = \Phi\left(y_i, N_i\right)$. The two must coincide in equilibrium.

[49]The optimization problem in Bianchi (2011) exhibits similar features, in the presence of a pecuniary externality. Here, the household fails to internalize the effect on taxation of its choice for gross assets. There, household choices determine relative goods prices that affect in turn the severity of a borrowing constraint.

The problem can be summarized by the following system of two Euler equations, for gross liabilities and gross assets, respectively:

$$u'(c_i)\left(q(y_i, b_i') + \frac{\partial q}{\partial b_i'} b_i'\right) = \beta \mathbb{E}\left[u'(c_i')\right] \quad (1.30)$$

$$u'(c_i)\,\tilde{q} = \beta \mathbb{E}\left[u'(c_i')\right]. \quad (1.31)$$

In a gross saver country, the portfolio of gross asset and liability positions of the representative household is set by equating the price of gross assets $\tilde{q}$ with the amount of resources obtained by issuing a marginal unit of debt, $q(y_i, b_i') + \frac{\partial q}{\partial b_i'} b_i'$, as gross assets and gross liabilities are equally perceived as risk-free. In equilibrium, the amount of debt issued to *ROW* is pinned down by the arbitrage condition:

$$\tilde{q} = q(y_i, b_i') + \frac{\partial q}{\partial b_i'} b_i'. \quad (1.32)$$

In borrowing countries, the representative household issues debt to other union members at the constant price $\tilde{q}$. Country i is a borrower if, given the vector of state variables of the representative household (y_i, N_i, n_i), there exists no solution to the maximization problem in (1.29) with positive holdings of gross assets, $\tilde{b}_i' > 0$.[50] The budget constraint of the representative household in a borrowing country is given by:

$$pc_i + \tilde{q}\tilde{b}_i' = p_i y_i + n_i$$

The problem faced by the representative household in a borrowing country is the following:

$$\begin{aligned} V_i^B(y_i, n_i, N_i) = \max_{\tilde{b}_i' \geq 0} & \left\{u(c) + \beta \mathbb{E}\left[V_i\left(y_i', \tilde{b}_i', N_i'\right)\right]\right\} \\ \text{s.t. } & pc_i + \tilde{q}\tilde{b}_i' = p_i y_i + n_i \\ & \tilde{b}_i' \geq \bar{b}(y_i) \\ & n_i' = \tilde{b}_i' \\ & N_i' = \Phi(y_i, N_i). \end{aligned} \quad (1.33)$$

Households in borrowing countries do not purchase gross assets, hence no taxes are due in the following period, as there are no bailout transfers to be made to residents by the

[50] In principle, it should be possible for borrowing countries to be able to issue debt to *ROW*. This restriction can be relaxed only at substantial cost, from a computational point of view. In the numerical solution of this model, only a small subset of borrowing countries would issue debt at a higher price if trading with *ROW*.

domestic government. Next-period net wealth simply follows from the amount of debt issued in the current period, as borrowers always repay their debt in full. The borrowing constraint $\tilde{b}_i' \geq \bar{b}(y_i)$ stems from the limited bailout promise that is offered to gross savers in the union, as they are only willing to bid $\tilde{q}$ on debt issued by i if the amount issued is below $\bar{b}(y_i)$. The Euler equation describing the solution to borrowers' problem is the following:

$$u'(c_i)\,\tilde{q} = \beta\mathbb{E}\left[u'\left(c_i'\right)\right].$$

Borrowers do not take into account how marginal debt issues affect the repayment probability π. This occurs due to the presence of the bailout promise, as lenders do not require to be compensated for non-repayment risk associated with individual assets.

For gross savers and borrowers alike, the intra-temporal allocation of consumption across the two goods, A and B, is determined according to CES demand, (1.24), as in the no-bailout setting.

I can now define the value function of the representative household in a generic country i, $V_i(y_i, n_i, N_i)$, as the value function of a gross saving country, $V_k^S(y_i, n_i, N_i)$, in the subset of the state space ϖ where a solution to the problem in (1.29) with positive $\tilde{b}'$ exists. In the complementary subset of the state space, the country i is a borrower and its value function is given by $V_i^B(y_i, n_i, N_i)$:

$$V_i(y_i, N_i, N_i) = \begin{cases} V_i^S(y_i, N_i, N_i) & \text{if } (y, N) \in \varpi \\ V_i^B(y_i, N_i, N_i) & \text{otherwise.} \end{cases} \tag{1.34}$$

I now define an equilibrium in the economy with bailout promises.

Definition 3. *A recursive competitive equilibrium in the economy with bailout promises is defined as a set of functions* c_i, $c_{A,i}$, $c_{B,i}$, $\tilde{b}_i'$, b_i', θ_i *for* $i \in \{H, F\}$, p_B, $\tilde{q}$ *that solve households' optimization problems and clear the markets for assets traded in the union and for good* B*, as well as by a perceived law of motion for wealth* $\Phi(y_i, N_i)$ *and a transition function* Γ *for the joint distribution of* (y, N)*:*

- *In the state space subset* ϖ *where a solution to the problem in* (1.29) *with positive* $\tilde{b}$ *exists, consumption* $c_i(y, n, N)$*, bond holdings of union-issued assets* $\tilde{b}_i'(y, n, N)$ *and of ROW assets* $b_i'(y, n, N)$ *solve gross savers' maximization problem* (1.29) *for countries* i *in either* H *or* F

- *In the complement subset to* ϖ*, consumption* $c_i(y,n,N)$ *and bond holdings of union-issued assets* $\tilde{b}'_i(y,n,N)$ *solve borrowers' maximization problem* (1.33) *for countries* i *in either* H *or* F

- *In gross saving countries, taxes* T_i *are consistent with losses incurred on gross assets held in each country, according to* (1.27).

- *Consumption of individual goods* $c_{A,i}(y,b)$, $c_{B,i}(y,b)$ *solve intra-temporal allocation* (1.24) *for all* i

- *The price* $\tilde{q}$ *ensures market clearing for assets traded within the union:*

$$\int \tilde{b}_i di = 0$$

- *The relative price* p_B *ensures market clearing for good* B, $y_B = (1-a)\left(\frac{p_B}{p}\right)^{-\vartheta} \int_{i \in I} c_i \, di$ *if no trade in good* B *between* I *and* ROW *occurs,* $y_B = c_B$*, and it is determined consistently with transport costs and with the price prevailing in world markets otherwise:*

$$p_B = \begin{cases} (1-\tau)\, p_B^* & \text{if } y_B > c_B \\ \frac{1}{(1-\tau)} p_B^* & \text{if } y_B < c_B \end{cases} \tag{1.35}$$

- *The aggregate law of motion for wealth perceived by the representative household in each country,* $N' = \Phi(y,N)$*, coincides with the one implied by the optimization problems* (1.29) *and* (1.33).

- *The Markov transition probability for the exogenous shock and the optimal choice of bond holdings determine the the transition function* Γ *for the joint distribution of* (y,N) *in* H *and* F.

Calibration. The model cannot be solved analytically and I obtain its numerical solution via a global solution method. The algorithm is described in detail in Appendix A.6. Choices for parameter values closely follow the literature on open-economy macroeconomics. Parameter values and calibration targets are reported in Table 1.1. In order to compare simulated series from the model with empirical evidence on the current account, the model is calibrated at quarterly frequency.

The subjective discount factor in ROW, q^{ROW}, is set to match a world risk-free real interest rate of 2.5%, consistently with evidence reported in King and Low (2014) for the world real interest rate. Due to the presence of a downward secular trend in world interest rates, this value understates the risk-free rate prevailing in the pre-EMU period, while it can describe well the low interest rate environment observed in the world economy in the 2000s.[51]

The exogenous function $\pi(y, b')$, determining the probability that sovereign debt repayments are not received by lenders, takes value of unity for $b' \geq 0$ and it has the following functional form otherwise:

$$\pi\left(y, b'\right) = \frac{(\eta - 1)\,\bar{b}^{\eta}}{(\eta - 1)\,\bar{b}^{\eta} + (-b'/y)^{\eta}} \tag{1.36}$$

where $\bar{b}$ and η are parameters. The price of debt issued to ROW by economies in I is then given by $q(y, b') = q^{ROW}\pi(y, b')$ which is continuous, differentiable and positive for all real values of b'. The upper bound for resources raised on international financial markets when debt is priced by ROW is given by the solution to

$$\max_{b'} \left(-q\left(y, b'\right) b'\right).$$

The value for b' that maximizes resources raised by borrowing is given by

$$\arg\max_{b'} \left(-q\left(y, b'\right) b'\right)$$

which, from the function in (1.36), is given by $y\bar{b}$. I set the upper threshold for borrowers' debt above which bailout promises are not offered to this value: $\bar{b}(y) = y\bar{b}$.

The parameters $\bar{b}$, and η are set to match properties of the distribution of external positions and interest rates in euro area economies before the inception of the monetary union. I set the parameter $\bar{b}$ to calibrate the aggregate net foreign asset-to-GDP ratio for countries in I. Lane and Milesi-Ferretti (2007) report a value of -10.3% of this variable for the euro area in 1998. As data are not reported prior to this year, I set this variable to a slightly higher value, -7.5%, noting that the net foreign asset position of all major member economies was deteriorating in the years preceding 1998.

[51]Empirical evidence on the secular decline in interest rates is, again, presented by King and Low (2014). Thwaites (2015) presents an explanation based on the contextual observation of a secular decline in the price of investment goods.

By governing the elasticity of non-repayment probability with respect to liabilities, the parameter η can be set to target the dispersion in interest rates on debt issued by countries in I. The spread in the real interest rate paid by borrowers between the poorest and wealthiest decile of the stationary distribution in I is set to 50 basis points. While this value seems low when compared to the pre-EMU spread in benchmark nominal interest rates on European countries' government bonds, large inflation differentials that were present between these economies at the time need to be taken into account. For instance, the spread between benchmark nominal interest rates on Italian and German government debt equalled 320 basis points in 1996. This difference fell to 60 basis points when accounting for the inflation differential between the two countries.[52]

In order to rule out divergence of net foreign asset positions of individual countries in I, the subjective discount factor in I, β is set to be slightly lower that the one in ROW. By implication, the median real interest rate in I is slightly higher than the world risk-free rate, by 25 basis points.

Endowment streams are determined according to the autoregressive stochastic process:

$$\log(y_{i,t}) = (1-\rho_y)\,\mu_y + \rho_y \log(y_{i,t-1}) + \epsilon_{i,t}, \tag{1.37}$$

where $\epsilon_{i,t}$ is a normally distributed, zero-mean, i.i.d shock with standard deviation σ_ϵ. To normalize the unconditional mean of y_i to unity, μ_y, is set to equal $-\frac{1}{2}\frac{\sigma_\epsilon^2}{1-\rho_y^2}$. The standard deviation σ_ϵ and the autoregressive parameter ρ_y are set to match standard deviation and autocorrelation of 1.5% and 0.88, respectively, of HP-filtered GDP for euro-area economies. In the numerical solution, the process is approximated by a discretized process obtained via the Rouwenhorst method (Rouwenhorst, 1995, Kopecky and Suen, 2010).

Relative risk aversion of households in I, γ, is set to the standard value of 2.[53] The elasticity of intra-temporal substitution between A and B good, ϑ, is set to 0.85, following the value adopted in Corsetti, Dedola, Leduc (2008) for the elasticity between domestic and foreign goods. The relative mass of countries in the set H, specialized in output of high-trade cost good B is given by $1-\kappa$, which I set to 0.15 to match the relative size of Greece, Portugal and Spain to the euro area, consistently with evidence of low average

[52]Source: Eurostat, EMU convergence criterion bond yields and OECD, consumer price index.

[53]See, e.g. Backus, Kehoe, Kydland (1994), Corsetti, Dedola, Leduc (2008).

Table 1.1: Parameter values

Calibrated Parameter		Value	Target / Source
Discount factor in I	β	.9914	Average interest rate in I
Discount factor in ROW	β^{ROW}	.9938	World real interest rate
Relative risk aversion in I	γ	2	Standard
Bailout promise limit	$\bar{b}$	1.5	Average euro area NFA
Repayment probability, debt elasticity	η	4	Italy - Germany, pre-EMU spread
Relative size of H, F	κ	.85	Relative size of Greece, Portugal, Spain
Utility weight of good A	a	$= \kappa$	Normalize $p_B = 1$ in autarky
Intra-temporal elasticity of substitution	ϑ	.85	Corsetti, Dedola, Leduc (2008)
Output process, autocorrelation	ρ_y	.88	Euro area GDP
Output process, standard deviation	σ_ϵ	.015	Euro area GDP

tradability of output in these countries, as presented in Section 1.2. The utility weight a is set to equal relative abundance of the two goods, κ, in order to normalize to unity the relative price p_B, in autarky. Finally, the iceberg transport cost τ is set to the value of 3%. This value is extremely low in comparison with estimates for typical trade costs presented by Anderson and Van Wincoop (2004). Even for this value of τ, no trade in B with ROW emerges in the numerical solution of this model.[54]

1.4.2 Imbalances, intermediation and crisis

The objective of the model introduced in this section is to perform a quantitative analysis of the recent euro area experience, in terms of current account imbalances, expansion of gross financial positions and crisis. In particular, this model aims to evaluate the role of

[54]Relative demand and supply for the two goods are set to be equal in autarky. Trade in B with ROW would only emerge in the presence of extremely large changes in demand by countries in I, which are not considered here. Large shocks to supply are ruled out by stationariety of the process for y_i and by the law of large numbers.

subsidies on cross-border holdings of assets in contributing to these facts. To this purpose, I present here the main properties of the steady state equilibrium allocation in the two model economies I introduce, the one with no bailout and the one with a full bailout promise. I then conduct two main quantitative experiments, both describing the dynamic transition of the model economy after a shock. The shocks I consider allow me to analyze the introduction of a subsidy on cross-border asset holdings and the occurrence of a debt crisis in the economic union.

First, I define the steady state equilibrium allocation in the two economies of this model. Properties of the equilibrium differ substantially between the economies with and without bailout promise. I describe a key element of the steady state allocation, namely the policy function for the current account balance in individual countries. Properties of this policy function are crucial to determine the equilibrium distribution of wealth in the economic union I. I then proceed to analyze differences in the distribution of wealth that emerge in the two model economies considered, focusing in particular on differences in terms of dispersion of net foreign asset positions. I then compare features of the equilibrium distribution with empirical evidence for the euro area.

Second, I present results from the first of two quantitative experiments. I analyze the initial phase of the EMU, characterized by current account imbalances and intermediation of gross capital flows, as discussed in Section 1.2. In order to do so, I consider the effects of a shock consisting in the introduction of a subsidy on cross-border holdings of assets, represented by the full bailout promise. I perturb with this shock the steady state equilibrium of the model economy with no bailout promises, which is calibrated to replicate quantitative features of euro area countries prior to the introduction of the single currency. No additional shocks hit the model economy, which is left to transition towards its new steady state. To analyze current account imbalances, I describe in this context the simulated path of net foreign assets of individual countries in I, focusing in particular on choices by countries at the opposite ends of the wealth distribution. In addition, I compare model predictions in terms of gross external positions with the experience of the euro area core. To do so, I present the path of gross asset and liability positions of savers in the model, which describes a pattern of intermediation of international capital flows.

Finally, I conduct an additional experiment to study the later phase of the EMU, characterized by the occurrence of a sovereign debt crisis. In particular, the aim of this experiment is to investigate the role in a crisis of linkages that are distinctive of an economic union. In the steady state equilibrium of the economy with bailout promise, countries are linked by cross-border financial exposures, as well as by trade integration. To simulate a debt crisis, I perturb this equilibrium allocation with a shock amounting to a fall in the value of union-issued assets held by gross savers. This shock allows me to concentrate on general equilibrium effects of a debt crisis in an economic union, arising due to a deterioration in lenders' net wealth.[55] I analyze in this context the heterogeneous effects of the crisis on countries in I that differ in terms of wealth and exposure to trade with union partners, highlighting the interaction in a crisis between financial and trade linkages among countries in a union.

Steady states. A steady state is defined as an equilibrium allocation where all aggregate variables in I are constant, namely the wealth distribution, the relative price p_B and, in the economy with bailout promise, the price of union-issued assets $\tilde{q}$. Note that, even in a steady state allocation, variables characterizing individual countries are not necessarily constant. While aggregate uncertainty is not present, idiosyncratic endowment shocks induce individual countries to accumulate and run down assets. As their external assets fluctuate, countries move within the equilibrium wealth distribution, which is constant in steady state.

The policy function for current account balances of individual countries in I determines the evolution of their net foreign asset position. Differences in this policy function between the no-bailout and bailout economy stem from the key element characterizing the allocations, namely the determination of borrowing costs. In the no-bailout allocation, the price of debt is decreasing in the amount issued, via the function $q(y, b')$. This is not the case in the bailout allocation, where individual borrowers can issue debt at the constant price $\tilde{q}$. Figure 1.5 presents the current account balance policy function in the two model economies, for a country in F with realization of the endowment process equal to its stationary mean and for a range of values of net foreign assets.[56]

[55] A similar analysis is conducted by Arellano and Bai (2014), who focus on renegotiation after default. Direct output costs of default in crisis countries are ignored here, as I focus on feedback and amplification effects that are distinctive of an economic union. These have been treated extensively in the literature, see e.g. Mendoza and Yue (2012).

[56] In steady state, the policy function for a similar country in H would be practically identical

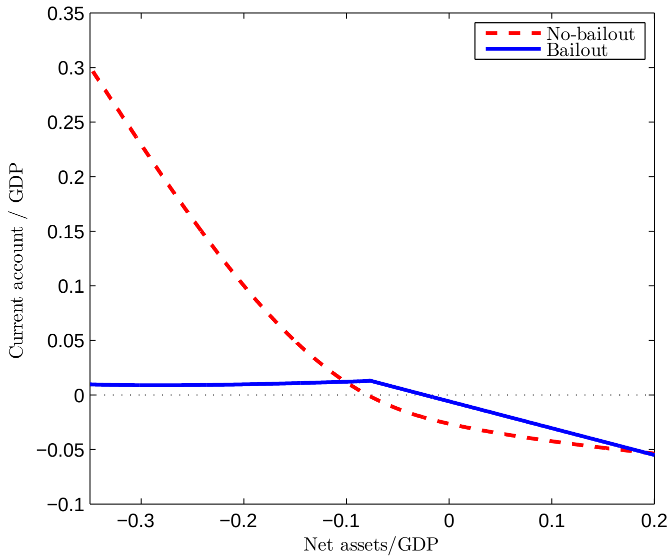

Figure 1.5: Steady state policy functions for current account, $N' - N$

The current account is presented as a function of net wealth, for a country in F with realization of the endowment process equal to its stationary mean. No-bailout (red, dashed line) and bailout (blue, solid line) allocations.

Two key differences emerge between the policy functions arising in equilibrium in the two model economies. First, a country with high debt chooses to optimally run a large current account surplus in the no-bailout economy, while this is not the case in the economy with bailout promises. This result is intuitive when we consider the difference in borrowing costs faced by debtors in the two models. Through the function $q(y, b')$, a highly indebted borrower pays a high interest rate in the no-bailout allocation, and it is induced to rapidly reduce its external liability position. Consider the case of a country with net liabilities equal to 35% of yearly output. In the no-bailout economy, this country would run an extremely large current account surplus, equal to 30% of quarterly output.[57]

and it is not shown here. This is due to the fact that the steady state relative price p_B is constant and close to unity.

[57]I express stock variables as ratios of yearly output, as is commonly done when discussing external debt ratios. As the model is calibrated at quarterly frequency, however, I express flow variables such as current account balances as ratios to quarterly output. Mean yearly output is simply given by mean quarterly output times four, so that debt equal to 35% of yearly output corresponds to 140% of quarterly output.

In the economy with bailout promise, the current account surplus of a country with the same level of debt would only equal 1.2% of quarterly output, as the constant price of debt $\tilde{q}$ induces more moderate savings.

Second, countries with low debt or positive net wealth are induced to save more in the economy with bailout promises. A country with net external liabilities equal to 8% of yearly output would run a balanced current account position in the no-bailout economy, while it would run a current account surplus in the economy with bailout promises, corresponding to 1.2% of quarterly output. In the bailout economy, current account surpluses by wealthy countries arise due to two channels. First, the bailout promise induces households to perceive union-issued assets as risk-free. The price of these assets is given by $\tilde{q}$, which is lower than that of a truly risk-free asset, q^{ROW}. The low price of assets perceived as risk-free constitutes an incentive to increase saving. Second, households in gross saving economies pay in each period a tax to their domestic government to finance bailout transfers on risky assets. The expectation of positive future taxes further induces saving, as households do not internalize the effect on taxation of their risky assets purchases.

Differences across the two model economies in terms of policy functions for current account balances are reflected in the distribution of net foreign assets arising in the two allocations. Figure 1.6 displays the steady state distribution of net foreign assets in the no-bailout economy and in the economy with full bailout promises.

The net foreign assets distribution is substantially wider in the economy with full bailout promises. The difference in net foreign assets between a country at the ninetieth percentile of the wealth distribution and one at the tenth percentile amounts to 25% of mean output in the steady state of the economy with full bailout promises. For comparison, the same difference amounts to just 3% of output in the economy with no bailout. As the stochastic process driving endowment realizations is the same in the two economies, the widening of the distribution is due to differences in agents' optimal choices across the two models.

The two main differences across policy functions in the two model economies are crucial to explain differences in the equilibrium distributions of net foreign assets. First, choices made by high-debt countries explain why we observe more mass to the left of the distribution in the economy with bailout promises. High saving by countries with high debt in the no-bailout economy implies that the mass of countries in this tail of the distribution falls steeply as net foreign assets decrease. In the economy with bailout promises, as high-debt countries run smaller current account surpluses, a larger mass of countries can be found in correspondence of high negative levels of net foreign assets. In addition, a positive mass of countries is against the borrowing constraint implied by $\bar{b}$. Second, in the economy with bailout promises, the stronger incentive to save implied by the high perceived return on union-issued assets leads wealthy countries to run smaller current account deficits. As economies with positive net foreign asset positions run down their wealth to a lesser degree when a bailout promise is offered, a larger mass of countries is found in the right tail of the wealth distribution.

The widening of net foreign assets distributions observed comparing steady state equilibria in the two model economies is comparable with the one observed in the euro area between the inception of the monetary union and the global financial crisis. Figure 1.7 shows net foreign asset positions of core and periphery countries in the euro area in two different years, in 2001, at the inception of the monetary union, and before the global financial crisis, in 2007, as ratio to euro area GDP.[58] The range of net foreign asset positions in these countries widened substantially in this period, rising from 6% of euro area GDP in 2001 to 16% in 2007, similarly to the the prediction of this model on the effects of of subsidies on cross-border holdings of assets.[59]

The inception of the EMU: Current account imbalances and intermediation. I present here results for the first of the two quantitative experiments I conduct by means of the infinite-horizon model. Aim of this experiment is to quantitatively assess the implications on current account balances and gross capital flows of the introduction of a full bailout promise on assets issued within the union. First, I will describe the type of experiment considered and the shock hitting the model economy. Second, I will present simulations for equilibrium net foreign asset positions in two countries located at opposite tails of the wealth distribution. Finally, I will analyze the path of gross capital flows in a country with relatively high initial net foreign assets.

[58] Data are drawn from Lane and Milesi-Ferretti (2007).

[59] The widening of euro area net foreign asset positions is not entirely surprising, given the observation of current account imbalances discussed in Section 1.2 and the close relationship between the two variables.

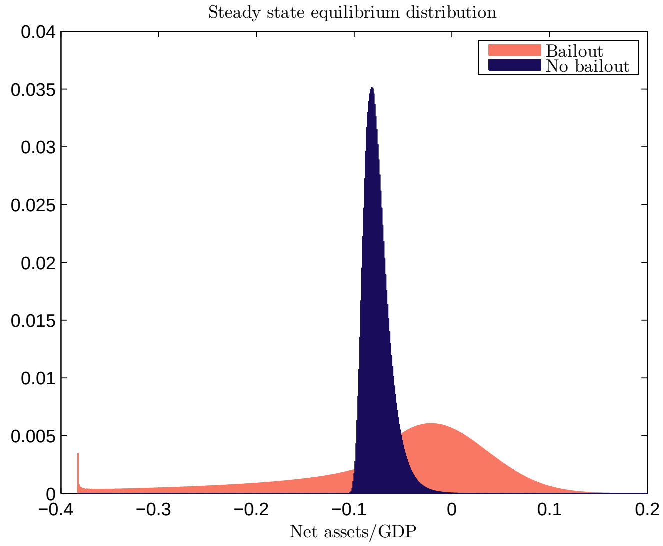

Figure 1.6: Steady state wealth distributions

The two distributions are the ones emerging in the equilibrium allocations of the no-bailout and bailout economies. Values on the y-axis represent masses of countries in intervals of the wealth distribution spanning 0.1% of mean yearly output.

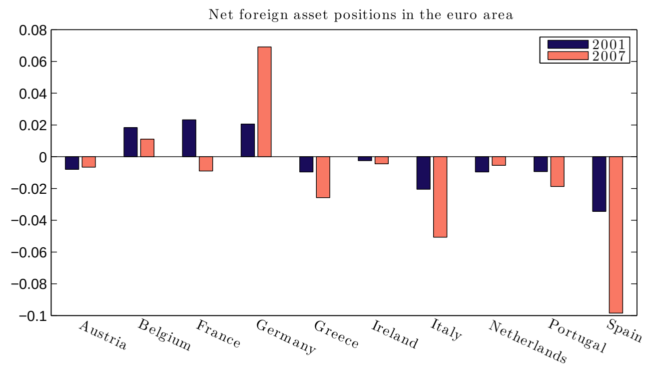

Figure 1.7: Net foreign asset positions of core and periphery euro area economies

Data are for 2001 and 2007, expressed as ratio of euro area GDP. Source: Lane and Milesi-Ferretti (2007) and own calculations.

The experiment I conduct is the following. Consider the steady state equilibrium allocation of the economy with no bailout promises. As discussed, parameter values are calibrated for quantitative properties of this allocation to replicate features of euro area economies, before the inception of the monetary union. I perturb this equilibrium allocation by introducing a full bailout promise on assets issued within the union. The shock is entirely unexpected by agents in the model. Thereafter, no other shocks hit the economy, which corresponds now to the economy with full bailout promise described in Section 1.4.1.

I consider the dynamic transition of variables chosen in equilibrium by individual countries during the transition towards the new steady state of the economy. The effects of the shock considered are heterogeneous across countries in the distribution.[60] In order to correctly isolate the effects of the shock on individual countries, I compare simulated series for choices made during the transition by a generic economy i, characterized by the pair (y_i, N_i), with choices the same country would make in a counterfactual simulation where the bailout promise is not introduced.[61]

To replicate the experience of peripheral countries in the euro area, characterized by low relative GDP and negative net foreign asset positions at the inception of the monetary union, I present in Figure 1.8 the path of net foreign assets chosen during the transition by a country with low endowment realization and high debt. This country is characterized by a realization of y_i two standard deviations below the stationary mean and net foreign assets corresponding to the tenth percentile of the wealth distribution.

Net foreign assets of the high debt country deteriorate substantially along the transition after the shock. In the first 24 quarters, net foreign liabilities approximately double, rising from 9% of yearly output to 17%. The magnitude of this increase in debt is comparable with the one observed in the periphery of the euro area. Between 2001 and 2007

[60] This is due to the fact that policy functions characterizing agents' optimal choice are not, in general, linear in the state variables.

[61] In general, choices made in the counterfactual simulation will not be constant, as the country is hit by idiosyncratic shocks and it moves within the stationary distribution. Hence, I need to ensure that simulated series do not reflect implications of a particular draw of idiosyncratic shocks during the transition. I address this concern by presenting median values of variables, given a large number of simulations for the sequence of idiosyncratic shocks hitting i. The same draws for the exogenous process are considered across transition and counterfactual simulations.

net foreign liabilities of peripheral countries increased by 160% as a ratio to euro area GDP, as shown already in Figure 1.7.

The deterioration of the net foreign asset position is due to the introduction of the bailout promise, and to the ensuing reduction in borrowing costs. Consider the path of net foreign assets chosen in the counterfactual simulation, where the bailout promise is not introduced. We would observe there a moderate increase in net foreign assets of this debtor country. Notwithstanding the borrowing motive implied by the low endowment realization, this country would face high borrowing costs due to high external liabilities, via the function $q(y, b')$. Hence, it would optimally choose to run current account surpluses, to reduce interest payments on external debt. This saving motive is not present when this country can issue debt to savers in the union at price $\tilde{q}$.

The experience of core countries in the euro area, characterized by higher than average income and wealth, can be analyzed in the model by focusing on the path of net foreign assets chosen by a country with high endowment realization and relatively high net foreign assets. Simulated series for a country at the ninetieth percentile of the wealth distribution and with a realization of the endowment shock two standard deviations above the long-run mean are presented in Figure 1.9.

The high wealth country optimally runs large current account surpluses in response to the introduction of the bailout promise. In the simulation, this country lets its net foreign asset position improve by 13 percentage points of yearly output in the first 24 quarters after the shock, rising to 8% of yearly output from a negative position of -5%. The rise in current account surplus observed in the model is especially large when compared to the one arising in the counterfactual simulation. There, this country would let its assets diminish to finance consumption, running current account deficits. As discussed, features of the allocation with bailout promise induce wealthy countries to increase their savings. Higher returns on union-issued assets and the expectation of future taxes, as needed to finance bailout transfers, induce households in wealthy countries to increase their net savings. This simulated pattern is similar to the one observed in some core euro area economies, where, for instance, a large increase in the net foreign asset position was observed in Germany between 2001 and 2007. Positive current account surpluses also characterized the experience of Austria, Belgium and Netherlands.

The evolution of gross external positions in the model displays a pattern of interme-

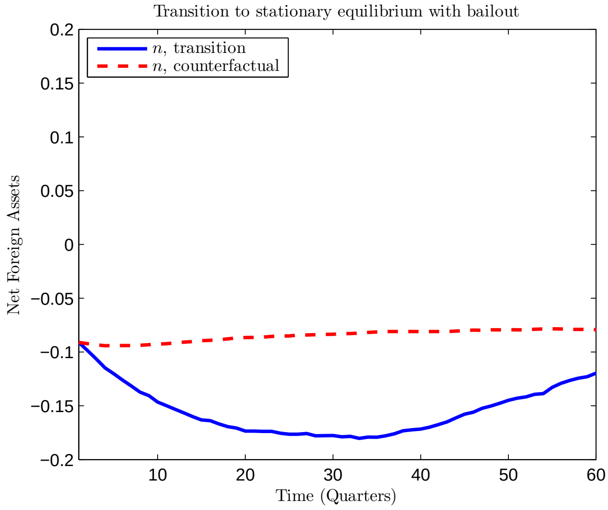

Figure 1.8: Transition after introduction of bailout promise. High-debt, low-income country

The blue, solid line represents the net foreign asset position chosen by a high-debt, low-income country after the introduction of the bailout promise. The red, dashed line displays the counterfactual simulation for the same country if no bailout promise is introduced.

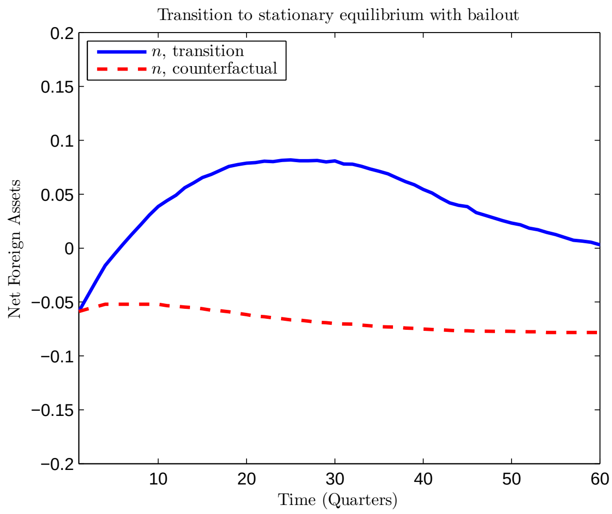

Figure 1.9: Transition after introduction of bailout promise. Low-debt, high-income country

The blue, solid line represents the net foreign asset position chosen by a low-debt, high-income country after the introduction of the bailout promise. The red, dashed line displays the counterfactual simulation for the same country if no bailout promise is introduced.

diation of international capital flows, not dissimilar to the one observed in core euro area economies. Figure 1.10 displays gross asset and liability positions after the introduction of the bailout promise, for the same high wealth country previously considered.[62]

Two main results emerge. First, we observe an expansion of gross external positions. In the first twenty quarters after the introduction of the bailout promise, gross asset and liability positions both widen in this country. By implication, the amount of gross assets held on the external balance sheet is larger than net foreign assets, as the country leverages by accumulating positive gross liabilities.

Second, gross asset and liability positions differ in terms of counterparty identity and risk. While the former are held by the wealthy country against partners in the union, the latter are issued to ROW. In addition, liabilities are always repaid in full, while losses are incurred on risky assets.

Predictions of the model for gross capital flows are quantitatively and qualitatively close to the observation of intermediation that characterized countries in the euro area. In particular, intermediation of gross capital flows was especially prominent in France. There, gross assets held against the rest of the euro area rose by 20 percentage points of GDP between 2001 and 2007, as shown in Figure 1.11. By comparison, gross assets rise by 15% of yearly output in the model experiment considered. External liabilities against the rest of the world also rose in France, by 30 percentage point of GDP. This amount is significantly larger than the one predicted by the model, corresponding to 8% of yearly output. Finally, the identity of counterparties of external positions is also similar to the one predicted in the model, with gross assets held against partners in the euro area and gross liabilities issued to the rest of the world.

Debt crisis in the economic union. I will now consider the quantitative effects of a debt crisis taking place in the equilibrium allocation of this model. The main purpose of the experiment conducted here is to analyze how heterogeneous countries in a union are differently affected by a crisis episode, depending on their wealth and on their exposure to trade with union partners. First, I will detail the nature of the experiment considered,

[62] I omit here results from the counterfactual simulation. The evolution of gross financial positions would be trivial, as all trade in financial assets occurs with ROW absent a bailout promise. I present again, however, the evolution of net foreign assets during the transition.

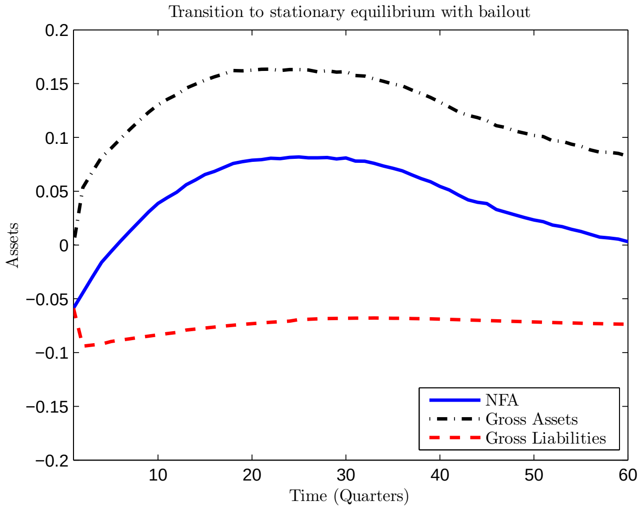

Figure 1.10: Transition after introduction of bailout promise. Gross positions, low debt, high income country

Gross foreign asset and liability positions of the low debt, high income country are represented by the black, dashed-dotted line and by the red, dotted line, respectively. The blue, solid line displays the transition for the net foreign asset position.

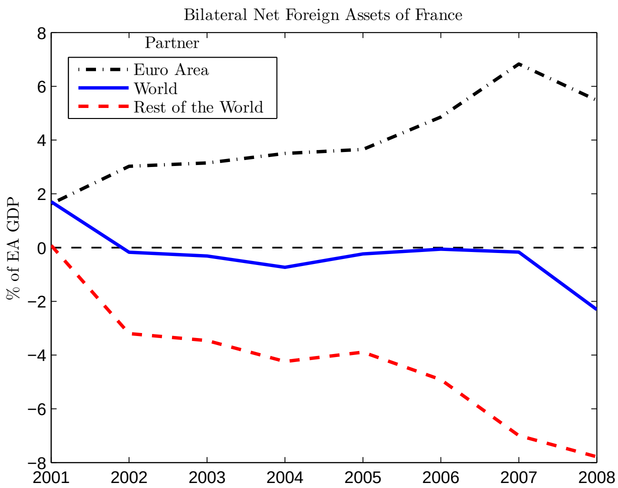

Figure 1.11: Bilateral international investment positions of France

Net positions against counterparties located in the euro area, in the rest of the world, and global position (World). Source: Waysand, Ross, de Guzman (2010).

and its relationship with the recent sovereign debt crisis. Second, I will present results on the implications of the crisis for aggregate consumption, interest rates and prices in the union. Finally, I will analyze heterogeneous consumption dynamics for large debtors with different export specialization.

To analyze a debt crisis in the economic union I conduct the following experiment. Consider the steady state allocation of the economy with full bailout promise. In this setting, countries in I are tied together by financial linkages in the form of cross-border exposures. The debt crisis is characterized as an unanticipated fall in the value of gross assets held by countries in I. The deterioration in asset values can be interpreted as the consequence of a shock inducing an exceptionally large subset of borrowing countries to default. After this shock, the economy is left to transition towards its steady state allocation. By modelling a debt crisis in this fashion, I deliberately abstract from direct effects of the crisis on the countries where it originates, focusing instead on transmission and amplification within the economic union. In the simulation presented, the debt crisis is described as a 16% fall in the aggregate value of gross assets held within the union.[63] The transition of aggregate gross assets after the shock is reported in Figure 1.12. As shown, the value of gross assets held in I reverts endogenously to its steady state value in approximately 50 quarters.

Aggregate consumption in the union falls by 1.6% in response to the debt crisis, as shown in Figure 1.13. The magnitude of this fall is comparable with the one observed in the euro area, where real consumption fell by 1.6% between 2011 and 2013.[64] Two main channels generate the fall in aggregate consumption. First, countries with positive gross assets are directly affected by the shock. As the value of gross assets deteriorates, their wealth falls and consumption falls in response. Second, in general equilibrium, debtor countries are also induced to consume less. As fewer resources are available to borrow from savers within the union, debtors are forced to reduce their liabilities, which they do by reducing consumption.

The interest rate charged on borrowers in the union rises by 280 basis points in the

[63] I assume that the fall in the value of gross assets simply amounts to a net loss in resources, as it is normally the case for non-repayments on risky assets in this economy.

[64] Clearly, the debt crisis in the euro area developed over a longer time horizon than the simple one-time shock considered in this experiment.

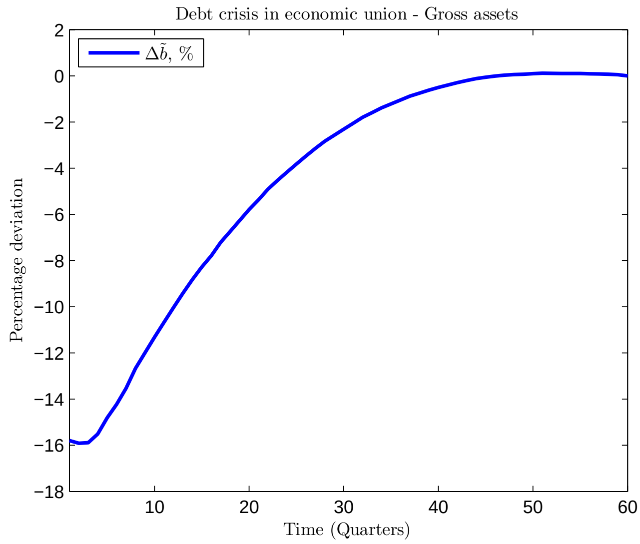

Figure 1.12: Transition after debt crisis, aggregate gross assets

Value of aggregate gross assets held in I, in percentage deviation from steady state.

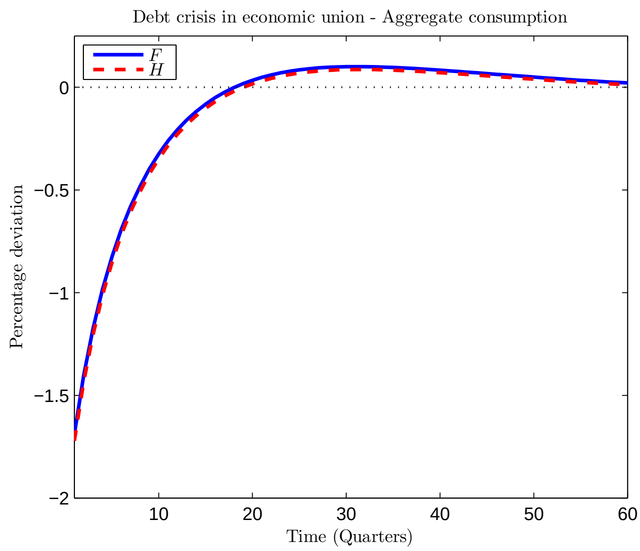

Figure 1.13: Transition after debt crisis, consumption

Aggregate consumption in H and F, in percentage deviation from steady state value.

event of the crisis, as shown in Figure 1.14.[65] By comparison, the spread between Italian and German benchmark bond yields rose by 320 basis points during 2011. The fall in wealth suffered by savers in the union implies that they are willing to purchase a lower amount of risky assets. The rise in the interest rate is the channel ensuring that the market for risky assets clears within I, by inducing debtor countries to deleverage and by making assets more attractive for savers.

The contraction in aggregate demand in the union determines a change in relative good prices, causing a fall in p_B. In response to lower demand in I, it is possible for countries in F to costlessly ship their output of good A to ROW, whose demand is unaffected by the crisis. This is not the case for economies in H producing good B: As transport costs hinder trade with ROW, when consumption demand is depressed within the union, the relative price of good B has to fall to clear the market within I.[66] As shown in Figure 1.14, the relative price of good B, falls in this experiment by 2.2% in response to the debt crisis shock.

The implications of a crisis episode are different across countries with high-debt in H and F. I focus here on countries in the poorest percentile of the wealth distribution, with income two standard deviations below the long-run mean. Figure 1.15 shows the deviation in these countries' consumption between the crisis episode and the counterfactual simulation that would be observed in steady state.[67] The high-debt country in H suffers to a greater degree from the crisis, due to its exposure to trade with partners in the union. Due to the crisis, consumption in this country is 2.5% lower than it would be in steady state. The fall in p_B implied by the demand deterioration in the union reduces the value of exports by this country. Being close to the borrowing constraint $\bar{b}$ due to its high debt, this country is unable to let changes in its foreign liabilities absorb the adverse effects of the shock, and the fall in the value of output is reflected in a large contraction

[65] The yearly interest rate presented here is defined as $\tilde{r} = \frac{1}{\bar{q}^4} - 1$.

[66] Note that the supply of goods A and B remains unchanged, as it is not affected by the debt crisis shock.

[67] Similarly to the first experiment considered in this section, the effects of the crisis are heterogeneous across countries in the wealth distribution. In order to compare the simulation arising from the crisis episode with the steady-state counterfactual, I present here the difference between the two series. As before, series displayed are the median from a large number of simulations, corresponding to different draws for sequences of the exogenous process.

in consumption. The high debt country in F, on the other hand, experiences a milder reduction in consumption, smaller than the average consumption fall for countries in I. As in H, consumption of this country is similar to that of a hand-to-mouth agent, due to the presence of the borrowing constraint. Here, however, the fall in p_B has a beneficial effect, determining a fall in the price of the aggregate consumption basket. In this country, the severity of a crisis episode is reduced by the availability of cheaper goods, caused by the collapse in aggregate demand in the union.

1.5 Concluding remarks

In the past fifteen years, capital flows in the euro area have been characterized by large current account imbalances and by a distinctive pattern of gross capital flows intermediation. In addition, a sovereign debt crisis led to severe recessions in debtor countries and to weak economic conditions throughout the union. The magnitude of the phenomena in question makes it important to gain a clear understanding of the causes of these facts, especially in light of their potential implications for policy.

This chapter shows that the recent experience of capital flows in the European monetary union can partly be explained by the introduction of policy distortions on trade in financial assets. The pattern of intermediation by core countries of gross capital flows and the widening of current account imbalances within the monetary union can be understood as the result of subsidies on cross-border holdings of assets, introduced in the euro area. The presence of such distortions in various forms has been widely documented in the literature. In addition, removal of currency risk and regulatory harmonization have contributed in inducing cross-border exposures for residents of the euro area.[68]

I contribute to the literature on capital flows in an economic union by developing a simple theoretical framework. This model allows me to study the implications for net and gross flows of policy distortions on financial assets trade. In addition, I introduce a rich heterogeneous-agents, infinite-horizon model to assess the quantitative importance of the theoretical channels identified by the simpler model.

Two important features of the euro area characterize the model introduced in this

[68]See, e.g. Buiter and Sibert (2005), Nouy (2012), Acharya and Steffen (2013), Coeurdacier and Martin (2009), Kalemli-Ozcan, Papaioannou and Peidro (2010), Hale and Obstfeld (2014).

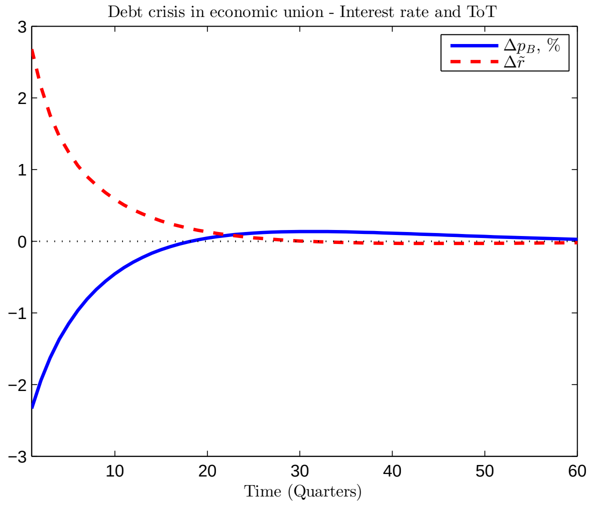

Figure 1.14: Transition after debt crisis, interest rate and terms of trade

Relative price of high-transport cost good, p_B, and interest rate on borrowers, $\tilde{r}$ in percentage deviation and difference, respectively, from steady state values.

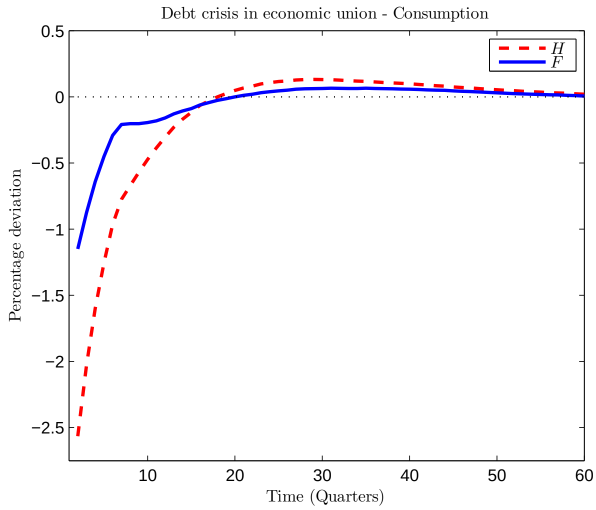

Figure 1.15: Transition after debt crisis. Consumption of high-debt, low-income economies

Consumption of the high-debt, low-income economy in F, specialized in good of type A, is given by the solid, blue line. Consumption of the corresponding economy in H, specialized in good of type B, is given by the dashed, red line. Figures are presented in percentage deviations from a counterfactual steady-state simulation.

chapter, the presence of extensive trade in financial assets with the rest of the world and close integration of goods markets within the union. First, by allowing for integration of the union with world financial markets, I am able to analyze the determinants of member countries' gross capital flows. In particular, I can study through the lens of this model the pattern of intermediation observed in the euro area. Second, I analyze the interaction between international capital flows and the pattern of imports and exports of individual countries, by allowing for trade in goods within the union and with the rest of the world. Specifically, I can consider in this framework how countries' specialization and their exposure to trade with union partners determine heterogeneous amplification of a detrimental debt crisis.

Intermediation of international capital flows emerges in my model as agents leverage the subsidy they enjoy on asset holdings. The presence of a subsidy leads residents of an economic union to issue gross liabilities to the rest of the world, in order to finance larger purchases of gross assets. The effects of policy distortions on gross capital flows predicted by the model closely resemble the pattern of intermediation observed in the euro area core, after the inception of the EMU. There, gross asset positions against partners in the periphery rose significantly, funded to a large extent by the creation of gross liability positions against the rest of the world. While the simple theoretical model here introduced clarifies how the introduction of subsidies constitutes a possible explanation for this observation, the infinite-horizon model presented highlights the quantitative relevance of this channel.

The empirical observation of rising current account imbalances is also consistent with predictions of the model for the effects of a subsidy on asset holdings. Importantly, the model generates a rise in the current account surplus of core countries, in response to the introduction of the policy distortion, even when these countries are fully integrated in international financial markets. This effect does not arise mechanically due to a larger deficit in peripheral countries, as it would if precluding trade in assets with the rest of the world. Rather, policy distortions actively induce core countries to increase their savings, by making assets issued by union partners more attractive.

A perverse feedback effect arises in the event of a sovereign debt crisis, when members of an economic union are tied together by financial as well as by trade linkages. When a debtor defaults on creditor countries that coincide with the main destination of its exports, a fall in the debtor's terms of trade ensues, amplifying the severity of the recession associated with the crisis. This mechanism is likely to have exacerbated the recent recession in peripheral countries of the euro area, as these were significantly exposed to trade with union partners that also constituted important creditors on external liabilities.

In future research, I plan to employ the flexible framework here introduced to consider two closely related issues, the introduction of common monetary policy in the euro area and the different treatment of sovereign and private-sector assets prescribed by financial regulation.

The introduction of common monetary policy has been a distinctive feature of the integration process represented by the EMU. In light of the diverse economic performance observed in countries of the euro area, and the of heterogeneous implications of policy distortions highlighted in this chapter, it is interesting to assess whether the presence of common monetary policy has magnified these effects. Fluctuations in demand in the event of a crisis are also heterogeneous in magnitude across union members, and this framework can contribute to our understanding of the stabilizing role of monetary policy in this setting. For instance, changes in the nominal exchange rate of the union against the rest of the world are likely to display a rich pattern of heterogeneous effects across union-member countries, as they differ in terms of gross external positions and trade specialization.

Prescriptions of financial regulation typically allow for different treatment of assets on banks' balance sheets, depending on the identity of their issuers. The prescription of a "zero-risk weight" on sovereign exposures in the Basel II regulatory framework is a notable example of this fact. Regulatory provisions of this type are likely to have had a significant impact on the nature of capital flows received by countries in the euro area periphery, by distorting the composition of flows towards specific counterparties. Recent research has highlighted how the misallocation of resources in the presence of large capital inflows is likely to have led to slow productivity growth in the euro area periphery.[69] By extending this framework to explicitly consider private sector agents and endogenous allocation across industrial sectors, it would be possible to further investigate the role of policy distortions and financial regulation in the euro area, analyzing their implications for productivity dynamics.

[69] See Aoki, Benigno, Kiyotaki (2009), Reis (2013), Gopinath et al. (2015).

Appendix

1.A Description of data and statistics in section 1.2

The data used to construct the measures of tradability of output presented in section 1.2 are drawn from the STructural ANalysis (STAN) and Trade in Value Added (TiVA) databases released by OECD and WTO (OECD 2011b and OECD/WTO, 2015).

I consider data for the year 2005, as that is the last year for which data from both datasets are available for a sufficiently wide sample of countries. The sample consists of all 34 OECD countries with the exclusion of Turkey and Chile, for which STAN data are not fully available. This sample does not include all EU economies: Bulgaria, Croatia, Cyprus, Latvia, Lithuania, Malta and Romania are EU-28 members but not OECD members, hence they are excluded from this sample.

Industrial sectors in my sample correspond to groupings of 2-digit industries from the ISIC Rev.3 Classification. Specifically, these are: Agriculture, hunting, forestry and fishing (01-05), Mining and quarrying (10 - 14), Food products, beverages and tobacco (15 - 16), Textiles, textile products, leather and footwear (17 - 19), Wood, paper, paper products, printing and publishing (20 - 22), Chemicals and non-metallic mineral products (23 - 26), Basic metals and fabricated metal products (27 - 28), Machinery and equipment, not elsewhere classified (29), Electrical and optical equipment (30 - 33), Transport equipment (34 - 35), Manufacturing not elsewhere classified; recycling (36 - 37), Electricity, gas and water supply (40 - 41), Construction (45), Wholesale and retail trade; Hotels and restaurants (50 - 55), Transport and storage, post and telecommunication (60 - 64), Financial intermediation (65 - 67), Business services (70 - 74), Other services (75 - 95).

In order to clarify the results described in section 1.2, I present in Figure 1.16 data for tradability of output disaggregated at the industrial sector level. Intuitively, services are characterized by low tradability of output, while subsectors of manufacturing, mining and agriculture are characterized by higher than average tradability.

Figure 1.17 decribes the industrial specialization of the EU economies in the sample, displaying the shares of GVA attributed to groups of industrial sectors ordered according to their tradability. Manufacturing is divided into two groups, with high tradability manufacturing including Textile, Transport equipment, Electrical equipment, Machinery and Chemical products and low tradability manufacturing including Metallic products, Manufacturing not elsewhere classifed, Food and Wood and paper products. Transport services are treated separately from other services due to their higher tradability. Consistently with the evidence presented on tradability of individual sectors, countries displaying higher average tradability are typically characterized by a larger share of the manufacturing sector.

1.B Proof of Proposition 1

Consider the case where a risk-neutral agent is the marginal buyer of debt issued by H. Given the terminal-period default policy by H, the price that the risk-neutral agent is willing to bid for b_H is given by

$$q_H = \begin{cases} \beta & \text{if } b_H \in [0, \infty) \\ \pi\beta & \text{if } b_H \in \left[-\hat{\zeta}, 0\right) \\ 0 & \text{if } b_H \in \left(-\infty, -\hat{\zeta}\right) \end{cases} \tag{1.38}$$

The interior solution for b_H in $\left[-\hat{\zeta}, 0\right)$ to the problem in (1.5) is given by the solution to the following first-order condition:

$$\pi\beta \frac{1}{p_1} u' \left(\frac{y_A - \epsilon + p_{B,1} y_B - \pi\beta b_H}{p_1} \right) = \pi\beta \frac{1}{p_{2,R}} u' \left(\frac{y_A + p_{B,2,R} y_B + b_H}{p_{2,R}} \right).$$

where $p_{B,2,R}$ and $p_{2,R}$ denote prices in the terminal-period state of the world with high output realization, where repayment by H is optimal.

Imposing logarithmic utility, the optimal amount of debt issued by H is given by

$$b_H = -\frac{\epsilon + y_B \left(p_{B,2} - p_{B,1} \right)}{1 + \pi\beta}.$$

I will restrict attention to cases where $\epsilon > y_B \left(p_{B,1} - p_{B,2,R} \right)$, where H optimally chooses to issue debt, $b_H < 0$. This interior solution for b_H is optimal if it is preferred by H to all values of b_H in $\left(-\infty, -\hat{\zeta}\right) \cup [0, \infty)$.

Consider now the interior solution with $b_H < 0$ and $q_H = \beta$. Such a solution is characterized by the following first-order condition:

$$\beta \frac{1}{p_1} u' \left(\frac{y_A - \epsilon + p_{B,1} y_B - \beta b_H}{p_1} \right) = \pi\beta \frac{1}{p_{2,R}} u' \left(\frac{y_A + p_{B,2,R} y_B + b_H}{p_{2,R}} \right) +$$
$$(1 - \pi)\beta \frac{1}{p_{2,D}} u' \left(\frac{y_{A,L} + p_{B,2,D} y_B + b_H}{p_{2,D}} \right).$$

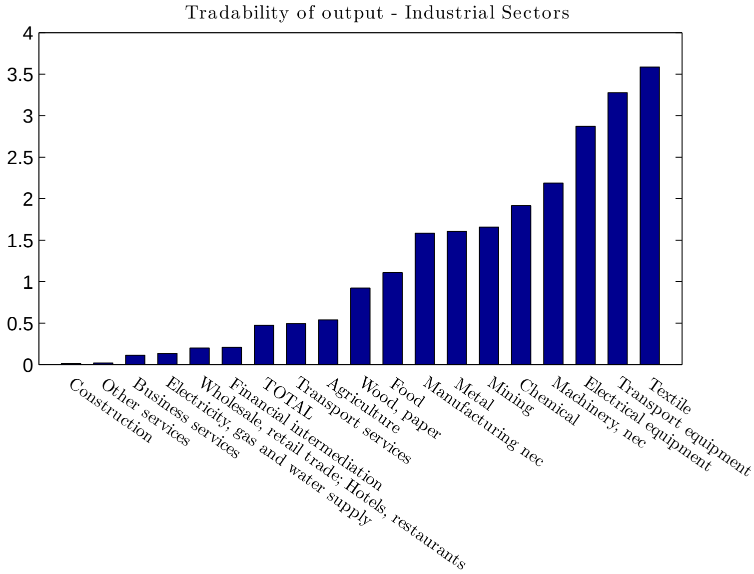

Figure 1.16: Tradability of output at the industrial sector level.

This measure of tradability is given by the ratio of total exports and imports attributed to a sector in all countries in the sample, to total GVA of that sector. Data source: OECD STAN and TiVA databases, year 2005.

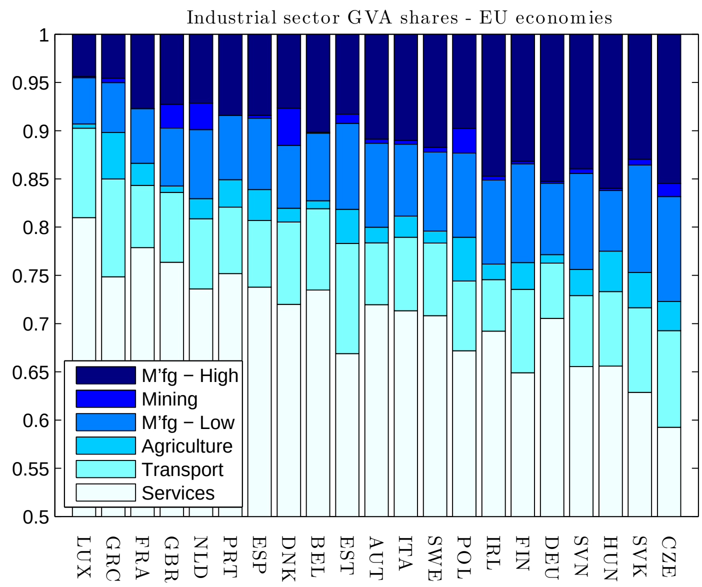

Figure 1.17: Shares of GVA attributed to broad industrial sectors

Sectors are ordered by tradability of output. EU economies. Data source: OECD STAN and TiVA databases, year 2005.

where $p_{B,2,D}$ and $p_{2,D}$ denote prices in the terminal-period state of the world with low output realization. Note that, for $b_H \geq 0$, default never occurs. Imposing logarithmic utility, the first-order condition becomes:

$$\frac{1}{y_A - \epsilon + p_{B,1} y_B - \beta b_H} = \pi \frac{1}{y_A + p_{B,2,R} y_B + b_H} + (1-\pi) \frac{1}{y_{A,L} + p_{B,2,D} y_B + b_H}.$$

Such a condition cannot be satisfied by a value of $b_H > 0$ whenever the initial-period output realization is sufficiently low:

$$\epsilon > y_A + p_{B,1} y_B - \left[\pi \frac{1}{y_A + p_{B,2,R} y_B} + (1-\pi) \frac{1}{y_{A,L} + p_{B,2,D} y_B} \right]^{-1}. \tag{1.39}$$

Let us now consider the corner solution $b_H = 0$. Welfare of the representative household is given in this case by :

$$\begin{aligned} V_{H,\text{corner}} = & \log\left(y_A - \epsilon + y_B p_{B,1}\right) - \log\left(p_1\right) \\ & + \pi\beta \left[\log\left(y_A + y_B p_{B,2,R}\right) - \log\left(p_{2,R}\right)\right] + (1-\pi)\,\beta \log\left(\frac{y_{A,L} + y_B p_{B,2,D}}{p_{2,D}}\right) \end{aligned}$$

Compare this with welfare under the interior solution for b_H in $\left[-\hat{\zeta}, 0\right)$, imposing log utility:

$$\begin{aligned} V_{H,\text{risky-debt}} = & -\log\left(p_1\right) - \pi\beta \log\left(p_{2,R}\right) \\ & + (1+\pi\beta) \left[\log\left(y_A - \frac{\epsilon}{1+\pi\beta} + \frac{1}{1+\pi\beta} y_B p_{B,1} + \frac{\pi\beta}{1+\pi\beta} y_B p_{B,2,R}\right)\right] \\ & + (1-\pi)\,\beta \log\left(\frac{y_{A,L} + y_B p_{B,2,D}}{p_{2,D}}\right). \end{aligned}$$

Removing identical terms, and ignoring differences in good prices across the two solutions, due to price-taking behaviour, the risky-debt choice is preferred to the corner solution if and only if

$$\begin{aligned} (1+\pi\beta) \log\left(y_A - \frac{\epsilon}{1+\pi\beta} + \frac{1}{1+\pi\beta} y_B p_{B,1} + \frac{\pi\beta}{1+\pi\beta} y_B p_{B,2,R}\right) > \\ \log\left(y_A - \epsilon + y_B p_{B,1}\right) + \pi\beta \log\left(y_A + y_B p_{B,2,R}\right) \end{aligned}$$

noting that

$$\begin{aligned} y_A - \frac{\epsilon}{1+\pi\beta} + \frac{1}{1+\pi\beta} y_B p_{B,1} + \frac{\pi\beta}{1+\pi\beta} y_B p_{B,2,R} = \\ \frac{1}{1+\pi\beta}\left(y_A - \epsilon + y_B p_{B,1}\right) + \frac{\pi\beta}{1+\pi\beta}\left(y_A + y_B p_{B,2,R}\right). \end{aligned}$$

this condition is satisfied due to concavity of the logarithm function and Jensen's inequality.

It is trivial to rule out choices for b_H in $\left(-\infty, -\hat{\zeta}\right)$. Given $q_H = 0$, such choices are dominated by setting $b_H = 0$. While the same amount of resources is obtained in the initial period, i.e. zero, by choosing $b_H < -\hat{\zeta}$ a positive output cost of default is suffered with positive probability in the terminal period.

Suppose an agent were willing to bid $q_H > \pi\beta$ for $b_H \in \left(-\hat{\zeta}, 0\right)$, $q_H = 0$ for $b_H < -\hat{\zeta}$ and $q_H = \beta$ for $b_H \geq 0$. A fortiori, it will still be optimal for H to issue b_H in the $\left(-\hat{\zeta}, 0\right)$ interval, as the value of this choice improves, while the value of the alternatives is unchanged.

The above proves that a risky borrowing choice is preferred to all other choices when

$$\epsilon > \min\left\{ y_{BH}\left(p_{B,1} - p_{B,2}\right), y_A + p_{B,1} y_B - \left[\pi \frac{1}{y_A + p_{B,2,R} y_B} + (1-\pi)\frac{1}{y_{A,L} + p_{B,2,D} y_B}\right]^{-1} \right\}.$$

This condition is function of good prices that are determined in general equilibrium and, in general, it must be verified on a case-by-case basis. In the special case where p_B is constant across periods and states of the world, for example being determined in *ROW* due to the absence of transport costs, $\tau = 0$, the condition simplifies to

$$\epsilon > (1-\pi)\left(y_A - y_{A,L}\right)\frac{y_A + p_B y_B}{\pi y_{A,L} + (1-\pi) y_A + p_B y_B}.$$

1.C Optimization problem in ROW

The *ROW* is inhabited by a risk-neutral representative household enjoying utility from consumption of goods of type A and B.

The problem faced by the representative household in *ROW* is the following:

$$\max_{\substack{c_1^{ROW}, c_{A,1}^{ROW}, c_{B,1}^{ROW},\\ c_2^{ROW}, c_{A,2}^{ROW}, c_{B,2}^{ROW},\\ b_{ROW}, b_{H,ROW} \geq 0}} c_1^{ROW} + \beta \mathbb{E} c_2^{ROW}$$

$$\text{s.t. } c_t^{ROW} = \left(c_{A,t}^{ROW}\right)^a \left(c_{B,t}^{ROW}\right)^{1-a}, \tag{1.40}$$

$$p_1^{ROW} c_1^{ROW} = y_A^{ROW} + p_{B,1}^{ROW} y_B^{ROW} + q_{ROW} b_{ROW} - q_H b_{H,ROW},$$

$$p_2^{ROW} c_2^{ROW} = y_A^{ROW} + p_{B,2}^{ROW} y_B^{ROW} - b_{ROW} + (1-D)\, b_{H,ROW},$$

where I allow relative prices and price indices in *ROW*, to differ from those prevailing in H and F. All other variables have their intuitive meaning, in particular, $b_{H,ROW}$ denotes the amount of assets issued by H and purchased by ROW and b_{ROW} denotes the amount of risk-free assets that are issued by ROW.

1.D Crisis amplification in H

Consider terminal-period consumption in H in the crisis state where it receives a low amount of good A endowment. This is given by

$$c = \frac{y_{A,L} + p_B y_B}{p}.$$

Denoting with subscripts I and NI allocations in the intermediation and non-intermediation scenarios, respectively, the crisis is amplified in H in presence of intermediation when

$$\frac{c_I}{c_{NI}} = \frac{p_{NI}}{p_I} \frac{y_{A,L} + p_{B,I} y_B}{y_{A,L} + p_{B,NI} y_B} < 1$$

It is easy to see that this condition holds when specialization in B good in H is extreme, namely when $y_{A,L} = 0$, $y_B > 0$. In this case,

$$\frac{c_I}{c_{NI}} = \frac{p_{NI}}{p_I} \frac{p_{B,I}}{p_{B,NI}} = \frac{p_{B,I}}{p_{B,NI}}^{a}.$$

Default by H liabilities owed to F rather than to ROW causes lower wealth in the union in the intermediation scenario. The ensuing lower price of the high-transport cost good B is unambiguously detrimental to consumption in H, when this economy displays extreme specialization in this type of good, as stated in (1.15). Consider now the opposite case where H is extremely specialized in good A, $y_{A,L} > 0$, $y_B = 0$. Now,

$$\frac{c_I}{c_{NI}} = \frac{p_{NI}}{p_I} = \frac{p_{B,I}}{p_{B,NI}}^{a-1}.$$

In this event, a fall in the relative price of good B implies an appreciation of the terms of trade and, thanks to cheaper imports of good B, intermediation dampens the severity of the crisis in H. Finally, consider the general case where both $y_{A,L} > 0$ and $y_B > 0$:

$$\frac{c_I}{c_{NI}} < 1 \leftrightarrow \frac{y_{A,L} + p_{B,I} y_B}{y_{A,L} + p_{B,NI} y_B} < \frac{p_I}{p_{NI}}$$

Noting that $\frac{p_I}{p_{NI}} = \left(\frac{p_{B,I}}{p_{B,NI}}\right)^{1-a}$, taking logs and making use of the fact that $\log(1+x) \approx x$, at first order, the condition above reduces to

$$\frac{(p_{B,I} - p_{B,NI})\, y_B}{y_{A,L} + p_{B,NI} y_B} < (1-a) \frac{p_{B,I} - p_{B,NI}}{p_{B,NI}}.$$

Imposing $p_{B,NI} = \frac{y_{A,L}+y_A^*+b_{F,NI}}{y_B+y_B^*}$, this reduces to the condition on relative abundance of good B stated in (1.17):

$$\frac{y_B}{y_B + y_B^*} > \frac{y_{A,L}}{y_B + y_B^*} y_{A,L} + y_A^* + b_{F,NI}.$$

1.E Model with homogeneous consumption good

This appendix presents a simplified version of the model in Section 1.3. In this version, all countries consume and are endowed with only one type of consumption good, freely tradable across all country pairs. This version of the model allows for a simpler analysis of the effects of policy distortions on intermediation of gross flows and current account imbalances. In addition, it can serve as a useful benchmark against which to compare the two-good model. This is especially true with regard to results on the amplification of a crisis due to the interaction between financial and trade linkages among members of a union of countries. The reader is referred to Section 1.3 for a full presentation of the model, as only major departures from the two-good version of the model will be described here.

Lifetime utility of households in H and F is given by (1.1), where c_t now simply denotes consumption of a homogeneous consumption good. The homogeneous consumption good is the numeraire of this simplified model economy.

H receives in the initial period an endowment of good given by

$$y_1 = y - \epsilon$$

where ϵ denotes the amount by which the initial-period endowment falls short of a long-run value, which is now given by y. In the terminal period, the endowment received by H is subject to uncertainty, as in the two-good case:

$$y_2 = \begin{cases} y & \text{w.p. } \pi \\ y_L & \text{w.p. } 1 - \pi. \end{cases}.$$

Again, the fiscal authority in H cannot commit to repay debt in the terminal period, and default costs are defined as in the two-good case, (1.4).

The problem faced by the fiscal authority in H is now the following:

$$\begin{aligned} V_H(\epsilon) = & \max_{\substack{b_H, c_1, c_2, \\ D \in \{0,1\}}} \; u(c_1) + \beta \mathbb{E}[u(c_2)] \\ & \text{s.t. } c_1 + q_H b_H = y - \epsilon, \\ & c_2 = y_2 + -[D\zeta - (1-D) b_H], \end{aligned} \tag{1.41}$$

The solution to this problem can be described by the optimality condition:[70]

$$q_H u'(c_1) = \beta \pi u'(c_{2,R}). \tag{1.42}$$

The problem solved in F is the following:

$$\begin{aligned} V_F(\epsilon) = & \max_{\substack{c_1^*, c_2^*, \\ b_F, b_{HF} \geq 0}} \; u(c_1^*) + \beta \mathbb{E}[u(c_2^*)] \\ & \text{s.t. } c_1^* + q_F b_F + q_H b_{HF} = y^*, \\ & c_2^* = y^* + b_F + [D(\xi b_{HF} - T) + (1-D) b_{HF}], \end{aligned} \tag{1.43}$$

and it can be summarized by the following system of first-order conditions[71]:

$$\begin{aligned} & q_F u'(c_1^*) = \beta \mathbb{E}\left[u'(c_2^*)\right] \\ & q_H = q_F - \beta(1-\pi)(1-\xi) \frac{u'\left(c_{2,D}^*\right)}{u'(c_1^*)}. \end{aligned} \tag{1.44}$$

The *ROW* is again a large, risk-neutral agent, with subjective discount factor β.

Intermediation of gross capital flows and current account imbalances. The one-good framework allows for a simple description of trade in financial assets in this model. Consider first the case where no bailout promise is offered to households in F, $\xi = 0$. F issues risk-free debt to *ROW* at price given by $q_F = \beta$. The price of H-issued assets consistent with pricing by households in F has to satisfy the system in (1.44). Imposing log-utility, the period budget constraints and $q_F = \beta$, this is given by:

$$\begin{aligned} \frac{1}{y - \beta b_F - q_H b_{HF}} &= \pi \frac{1}{y + b_F + b_{HF}} + (1-\pi) \frac{1}{y + b_F} \\ q_{H,F} &= \beta - \beta(1-\pi) \frac{y + \beta b_F + q_H b_{HF}}{y + b_F}. \end{aligned} \tag{1.45}$$

[70] As in the two-good case, this optimality condition only holds in the interior solution, when it is optimal for H to issue debt carrying positive but not certain default probability, see Proposition 1.

[71] Again, as in the two-good case, this is only true in cases where H issues risky debt and gross assets purchased by F are positive, $b_H \in \left(-\hat{\zeta}, 0\right)$ and $b_{HF} > 0$.

The price of H-assets consistent with the solution to the problem in F is higher than the price at which ROW is willing to buy these assets, $q_{H,F} > \beta\pi$, if and only if

$$\frac{y + \beta b_F + q_H b_{HF}}{y + b_F} < 1.$$

The solution to 1.45 implies that this inequality cannot be satisfied for $b_{HF} > 0$. Hence, F is not willing to purchase any positive amount of assets issued by H at price $q_{H,F} \geq \beta\pi$. From the above, it follows that the marginal buyer of debt issued by H is ROW if no bailout promise is offered to households in F, and that no amount of H-issued assets is purchased by F. In addition, no liabilities are issued by F, as $b_F = 0$, $b_{HF} = 0$ constitutes a solution to (1.44) when $\xi = 0$.

I have established that in the absence of a bailout promise, $\xi = 0$, the price of H-issued assets has to be consistent with pricing by ROW and it is therefore given by $q_H = \beta\pi$. From the optimality condition in H, it follows that assets issued satisfy

$$b_H = -\frac{\epsilon}{1 + \beta\pi}.$$

Intuitively, issuance of debt delivers insurance against the initial-period endowment shock, ϵ.

Consider now the case where a full bailout promise is offered to households in F, $\xi = 1$. It is trivial to show that the price of H-issued assets that is consistent with optimality in F, (1.44), is given by $q_H = \beta$, making F the marginal and sole buyer of these assets. Given this price, the amount of assets issued by H is given by

$$b_H = -\frac{\pi\epsilon + (1 - \pi)\, y}{1 + \beta\pi},$$

showing that H benefits from the higher price of its own debt by running a wider current account deficit.[72]

F finances purchases of H-issued assets with own resources and by borrowing from ROW. The amount of liabilities issued by F to ROW follows from the solution to (1.44), given $q_H = \beta$ and $b_H = -b_{HF}$. It can be shown that $\frac{b_F}{b_H}$ ranges between $\frac{-\beta}{1+\beta}$ for $\pi = 0$ and -1, for $\pi = 1$. Finally, to the extent that $\left|\frac{b_F}{b_H}\right| < 1$, F acts as a net saver, financing part of its gross asset purchases with own resources. The presence of a bailout promise, then, induces F to run a current account surplus, contributing to wider current account imbalances in the union of H and F.

[72] This is clearly true to the extent that $\epsilon < y$.

Crisis transmission and amplification. The analysis of a crisis event is extremely simple in this one-good framework. In the terminal-period crisis state, consumption in F is given by

$$c^*_{2,D} = y^* + b_F.$$

The analysis above showed that $b_F \neq 0$ if and only if $\xi > 0$: A bailout promise induces the creation of gross external positions in F. In the absence of the bailout promise, the crisis would not be transmitted to F, as $b_F = 0$ in that case. When F issues gross external liabilities to purchase risky gross assets issued by H, its consumption in the crisis state is adversely affected due to default by H.

In the absence of fluctuations in goods relative prices, financial linkages between H and F are not sufficient to deliver amplification of the crisis in H. In H, terminal-period consumption in the crisis state is simply given by

$$c_{2,D} = y_L.$$

In the absence of intra-temporal trade and unlike in the two-good framework, consumption in H in a crisis is independent of the identity of its external creditors.

1.F Proof of Proposition 2

Consider the allocation with $\xi = 0$, $\pi \to 0$. Consumption by H in financial autarky is given by $c_1 = y - \epsilon$, $c_{2,D} = y_L$, $c_{2,R} = y$, noting that the repayment state is realized with zero probability in the limit. In the limit, welfare in H is given by

$$V_H^{\xi=0}(\epsilon) = \log(y-\epsilon) + \beta \log(y_L).$$

Consumption by F is given by $c^*_s = y^*$ in all periods and states s. Welfare in F is

$$V_F^{\xi=0}(\epsilon) = (1+\beta)\log(y^*).$$

Under the bailout allocation, borrowing by H is bounded in the limit by the amount of resources available in the repayment state, $b_H = -y$. Consumption is given by $c_1 = 2y - \epsilon$, $c_{2,D} = y_L$ and $c_{2,R} \to 0$. Using the fact that $\lim_{x\to 0} x \log(x) = 0$, welfare in H tends to

$$V_H^{\xi=1}(\epsilon) = \log(2y-\epsilon) + \beta \log(y_L).$$

As established in (1.14) borrows $b_F = \frac{\beta}{1+\beta} b_H = -\frac{\beta}{1+\beta} y$ to finance asset purchases when $\pi = 0$. In the limit for $\pi = 0$, consumption is equalized across the initial period and terminal period default state: $c_1^* = c_{2,D}^* = y^* - \frac{\beta}{1+\beta} y$. Welfare in F in the bailout allocation is then given by

$$V_F^{\xi=1}(\epsilon) = (1+\beta)\log\left(y^* - \frac{\beta}{1+\beta} y\right).$$

Note that the change in F welfare induced by the bailout promise is independent of ϵ:

$$V_F^{\xi=1}(\epsilon) - V_F^{\xi=0}(\epsilon) = (1+\beta)\log\left(1 - \frac{\beta}{1+\beta}\frac{y}{y^*}\right).$$

The change in welfare in H is instead increasing in ϵ:

$$V_H^{\xi=1}(\epsilon) - V_H^{\xi=0}(\epsilon) = \log\left(\frac{2-\epsilon/y}{1-\epsilon/y}\right)$$

Hence, the change in union welfare induced by the bailout promise, $V^{\xi=1}(\epsilon) - V^{\xi=0}(\epsilon)$, is increasing in ϵ. Note that this change tends to infinity for $\epsilon \to y$. For $\epsilon = 0$, $V^{\xi=1}(\epsilon) - V^{\xi=0}(\epsilon) < 0$ under the following condition for y/y^* and β:

$$\left(1 - \frac{\beta}{1+\beta}\frac{y}{y^*}\right)^{1+\beta} < \frac{1}{2}.$$

By continuity of $V_H^{\xi=1}(\epsilon) - V_H^{\xi=0}(\epsilon)$ and the intermediate value theorem, it can be shown that there exist a value $\bar{\epsilon} \in (0, \infty)$ such that $V^{\xi=1}(\bar{\epsilon}) - V^{\xi=0}(\bar{\epsilon}) = 0$ and $V^{\xi=1}(\epsilon) - V^{\xi=0}(\epsilon) > 0$ for $\epsilon > \bar{\epsilon}$.

1.G Infinite-horizon model, solution method

The solution of this model is obtained by a numerical algorithm related to the one proposed by Guerrieri and Lorenzoni (2011), also employed by Fornaro (2014) in a model of external debt.

Solution for the equilibrium of the economy with no bailout promises is relatively simple. The risk-free real interest rate is exogenous in this model, as determined in ROW. I need to solve for the endogenous price of good B, p_B, which is determined in general equilibrium within the union I. To solve for the steady state of this model economy, I first need to obtain optimal policy functions of individual countries for net foreign assets, $b'(y, n)$. I discretize the state space on a grid for y and n, and I solve for the policy functions by iterating on the Euler equation that follows from the problem of individual countries, (1.33), given an initial guess for p_B, p. The price at which individual countries issue debt follows from q^{ROW} and from the function determining π, (1.36). Having obtained policy functions for net foreign asset accumulation, I update the distribution of net foreign assets in the economic union, until its convergence. Given distribution and policy functions, I solve for aggregate consumption in I and I check whether the market for good B clears within the union. If not, I update the for p_B and p I repeat this procedure until market clearing. The distribution obtained upon convergence of p_B is the steady state distribution of this model economy.

The algorithm I implement to solve for the steady state equilibrium of the economy with a full bailout promise is an extension of the one above presented. In addition to the distribution of net foreign asset and to the relative price p_B, I need to solve for the equilibrium price of union issued assets, $\tilde{q}$, and for the function determining the amount of tax paid by the representative household in gross saving countries. Again, I discretize the state space and I introduce an initial guess for aggregate variables, $\tilde{q}$, p_B and taxes. Given the initial guesses, I iterate on the Euler equation of gross savers and borrowers to determine their policy functions for external positions, $\tilde{b}'(y, n, N)$, $b'(y, n, N)$ and for the rule determining whether each country is a borrower or a gross saver, ϖ. Again, I use the policy functions to obtain a stationary distribution of net foreign assets. I now need to ensure that the market for good B and the market for assets issued within the union clear. In addition, the function determining taxes in gross saver economies has to be consistent with the amount of bailout transfers paid by in each period to holders of gross assets. To this purpose, I make use of the result that all assets issued within the union are perfect substitutes, thanks to the full bailout promise. I can impose that the portfolio of assets held by all gross saver countries is the market portfolio. By implication, losses on gross assets due to default are equal in each country to the aggregate fraction of non-repayments on liabilities, determined by the function π. Taxes paid in each period by the representative household in gross saver economies are then a constant fraction of holdings of gross assets. I iterate the above procedure until convergence of the net foreign asset distribution, p_B, $\tilde{q}$ and of the tax rate paid on gross assets.

Finally, I employ a standard algorithm to compute dynamic transitions after shocks in the two quantitative exercises. I introduce an initial guess for the path of aggregate variables for a large number of periods. Assuming that the economy will be in steady state in the final period, I solve the problem of individual countries backwards, given the guess for aggregate variables. In the initial period, the net foreign asset distribution of the economy is either the one of the steady state with no bailout promises, in the first exercise considered, or in the that arises after the shock to gross assets caused by the debt crisis. Starting from the initial period, I update the distribution forwards, given the policy functions obtained. In general, markets will not clear along the transition, and I update guesses for prices accordingly. Given the new guess, I repeat the procedure, until convergence of prices. Importantly, I need to check that in the final period of the transition the equilibrium of the economy is indeed given by the steady state one.

Chapter 2

Sovereign Debt Crises, Fiscal Austerity and Corporate Default

No Greece will not default. Please. In the euro area, the default does not exist.

Joaquín Almunia
European Commissioner for Economic and Financial Affairs
29 January 2010

2.1 Introduction

What is the impact of a sovereign debt crisis on credit to the private sector? What is the role of fiscal policy in transmitting the adverse effects of a crisis from the government to firms? The recent euro area sovereign debt crisis was accompanied by a severe recession. In Italy, the rise in government borrowing costs was followed by a sharp tightening in the stance of fiscal policy and by an increase in the cost of funding for firms. Moreover, during the crisis, the performance of Italian firms was impaired to a heterogeneous degree. In particular, the downturn had very severe effects for domestically-oriented firms who only exported a small fraction of their output.

I introduce in this chapter a model to analyze jointly sovereign default risk, fiscal policy and corporate default risk. I propose a transmission channel of a sovereign debt crisis to firms. Through the action of fiscal policy, a sovereign debt crisis causes a tightening in the severity of financial frictions faced by firms. An increase in sovereign default risk leads to a rise in the cost of government borrowing. In turn, higher borrowing costs induce the fiscal planner to reduce its exposure to international financial markets and to raise revenues domestically, instead. The contraction in fiscal policy leads to a reduction in output in the domestic economy. Firms' profits deteriorate due to the fiscal tightening and the risk of corporate default rises. Sovereign risk is thus transferred to firms. Firms' borrowing costs rise along with the increase in firms' default risk, leading to a contraction of firm-level financial frictions.

The model introduced here allows for a quantitative study of the three key elements of this analysis, sovereign default risk, fiscal policy, and firm-level financial frictions. The fundamental structure of this framework is that of an endogenous sovereign default model a la Eaton and Gersovitz (1981). Sovereign default emerges as the optimal choice of a benevolent planner weighting the benefits of integration in international financial markets against the cost of debt repayment. I augment the quantitative model of Arellano (2008) along three main directions. First, I introduce the presence of a meaningful corporate sector, composed of firms who hire labor supplied by households to produce output. Second, I calibrate parameter values of the model to replicate quantitative features of the Italian economy. In particular, I obtain empirically plausible levels of external debt, taxation and government expenditure. Finally, the model features the presence of distortionary taxation, so that government choices have implications for endogenous variables such as output.[1]

In the model economy, there is no direct linkage between the severity of financial frictions faced by firms and those faced by the government. Both firms and the government can borrow from risk-neutral lenders on international financial markets. Default by both types of agents can emerge in equilibrium, but default by the government does not necessarily trigger default by firms, nor it causes their exclusion from international financial markets. As firms can borrow from from foreign lenders, there is no meaningful role in this model for financial intermediation by domestic banks. In particular, firms in the model economy are insulated from the effects of a sovereign crisis on holders of

[1]Cuadra, Sanchez and Sapriza (2010) introduce a model of endogenous sovereign default where the government raises revenue domestically via distortionary taxation and it sets the level of government expenditure.

government debt. Credit supply conditions in the domestic markets are irrelevant for firms in this model, as they are able to raise funds from the rest of the world.

Two different types of firms are considered in the model. The two types of firms differ in their degree of exposure to the domestic economy. All firms equally rely on labor supplied by domestic households as their sole input in production. Firms producing a non-tradable type of good, however, are constrained by a lack of access to international goods markets. Firms in this sector are more reliant on the domestic economy, which is the sole destination market for their output. On the other hand, firms that produce tradable goods are less exposed to domestic considerations, as they can export their production to foreign residents.

Interest rates on government and firms' debt display positive comovement in the equilibrium of the model economy. The interest rate is high for both the government and firms in bad times, when aggregate productivity is low and government debt is high. First, in times of low productivity the government faces a strong incentive to default. Foreign lenders require compensation for the high default risk and charge a high interest rate on government debt. Low productivity is also detrimental to firms' profits. Low profits imply a high risk of default by firms, which leads in turn to a high interest rate on firms' debt. Second, a high level of government debt also strengthens the incentive for the government to default, thus implying a high interest rate on government debt. At the same time, high borrowing costs induce the government to run a surplus when government debt is high. The government thus sets higher taxes to raise revenue domestically. In turn, higher taxes lead to a contraction in firms' profits. As firms' profits fall, their default probability rises. Firms' borrowing costs rise along with their default probability, as lenders are compensated for higher default risk.

I consider in this framework the implications of a sovereign debt crisis that hits the small open economy of the model. A sovereign debt crisis is a shock leading to a sharp increase in the cost of borrowing for the government of the small open economy. The sovereign debt crisis causes a fiscal contraction, a tightening in financial frictions faced by firms, a reduction in output, and a more severe deterioration in the performance of non-exporting firms. The effects of the crisis in the model are similar to those observed in Italy during the recent recession. First, the higher cost of borrowing induces the government to reduce its stock of debt, leading to an increase in taxation. Second, higher taxes lead to a fall in output and to a reduction in firms' profits. Third, lower profits of firms lead to a rise in their default risk, and to a higher cost of firms' borrowing. Finally, the crisis leads to a contraction in domestic demand, making its effects particularly severe for firms in the non-tradable sector.

The rest of the chapter is organized as follows. Section 2.2 describes the contribution of this chapter to the literature on sovereign debt. Section 2.3 presents the key facts about the recent crisis in Italy that motivate this research. Section 2.4 introduces the model. Section 2.5 describes the calibration of the model economy and quantitative properties of the eqilibrium. Section 2.6 discusses a sovereign debt crisis episode. Section 2.7 concludes.

2.2 Related literature

This chapter is related to several strands of the literature on on sovereign debt. First, I contribute to the literature on endogenous sovereign default initiated by the seminal work of Eaton and Gersovitz (1981). The model in this chapter builds on the quantitative framework developed by Arellano (2008) and Aguiar and Gopinath (2006). I depart from the baseline model by introducing endogenous production of goods by firms facing idiosyncratic default risk. As in Cuadra, Sanchez and Sapriza (2010), I allow for the presence of distortionary taxation, government expenditure and endogenous labor supply by households. In addition, differently than in most papers in this literature, I calibrate the model economy to replicate quantitative features of an advanced economy like Italy, rather than those of emerging market economies.

Second, this chapter contributes to the literature on the interaction between sovereign debt and credit to the private sector. Mendoza and Yue (2012) study the endogenous determination of the costs of sovereign default. To this purpose, they assume the presence of a direct link between government and firms in their borrowing costs and in their access to international financial markets. I abstract here from assuming the presence of such direct linkages. I study instead how the correlation of borrowing costs and market access between the two agents emerges endogenously, as sovereign risk is transmitted to the private sector through fiscal policy.[2]

[2] In a New-Keynesian framework, Corsetti, Kuester, Meier and Müller (2014) also assume the presence of a direct linkage in borrowing costs of government and firms, to study the effects of

The endogenous determinants of linkages between sovereign debt markets and credit to the private sector have been analyzed extensively. Many authors have studied how a sovereign crisis is transmitted to firms because of its adverse effects on the balance sheet of domestic lenders. Gennaioli, Martin and Rossi (2014) is a notable example of a paper analyzing this transmission channel. Similarly, the adverse implications of sovereign debt for credit to the private sector stem from the exposure of domestic creditors to the government in Guerrieri, Iacoviello and Minetti (2013), Acharya, Drechsler and Schnabl (2014), Broner, Erce, Martin and Ventura (2014), Mallucci (2015), Perez (2015), Sosa-Padilla (2015) and Bocola (2016).[3] I contribute to this field of research by studying the transmission of shocks between sovereign and firms in a framework where debt of government and firms is held externally of the domestic economy. Here, fiscal policy set by the government affects fundamentals of the economy and the severity of firms' default risk, independently of the identity of holders of government debt. Similarly, Arellano and Kocherlakota (2014) introduce a model where a crisis is transmitted from the private sector to the government, as tax revenues fall in times of financial distress.

The correlation between credit conditions of government and firms has been empirically documented in the literature. I introduce here a theoretical framework that I employ to quantitatively analyze the importance of the fiscal channel of crisis transmission. Ağca and Igan (2015) show that policies aimed at reducing the stock of government debt have significant adverse affects on credit to the private sector. Effects are shown to be heterogeneous across firms, depending in particular on their degree of exposure to the domestic economy. Arteta and Hale (2008) document that sovereign default crises are associated with contractions in credit to the private sector in emerging market economies. Bofondi, Carpinelli and Sette (2013) and Baskaya and Kalemli-Ozcan (2016) empirically document the impact of sovereign risk on private sector credit, via the channel of domestic intermediaries.

Finally, this chapter is related to the growing area of research that has studied the recent euro area sovereign debt crisis. Lane (2012) and Shambaugh (2012) provide a

fiscal policy in a union of countries subject to sovereign debt crises.

[3]Chari, Dovis and Kehoe (2016) show that financial repression, in the form of incentives to domestic agents to hold domestic government debt can be welfare-improving, by improving the ability of the government to commit to repay debt.

detailed discussion of the economic and sovereign debt crises. Ardagna and Caselli (2014) discuss the conditions attached to the Greek government bailouts of 2010 and 2011. De Santis (2012) studies the spillover of the crisis from Greece to the rest of the euro area.

2.3 Empirical evidence

This section introduces three key sets of facts that motivate the analysis in this chapter. First, I present evidence on the adverse impact of the euro area sovereign debt crisis on borrowing costs of the Italian government and of Italian firms. Second, I show that fiscal policy in Italy reacted to the crisis with an increase in government saving, as the government primary balance rose along with the increase in government borrowing costs. Finally, I show that the performance of non-exporting firms was especially weak during the sovereign crisis and the subsequent contraction of fiscal policy.

Government and firms' interest rates Interest rates on liabilities of the Italian government and of Italian firms rose sharply during the euro area sovereign debt crisis of 2011-2013. A measure of borrowing costs for both is presented in Figure 2.1, displaying the spread of the interest rate paid by government and firms over a risk-free nominal benchmark, for the period from 1999 onwards.

The interest rate spread on Italian government debt rose to 485 basis points at its peak during the crisis in December 2011. I measure the interest rate spread as the difference between the interest rate on 12-month government debt and the interest rate on a nominal, risk-free security. I use the interest rate on main refinancing operations of the European Central Bank as a measure of the nominal, risk-free rate.[4] The increase in government borrowing costs was sharp, but relatively short-lived. While the spread on interest rates for government debt remained high and volatile until the second half of 2012, it quickly returned thereafter to a level close to its pre-crisis mean of just 20 basis points.[5]

[4]Both series are accessed via the Bank of Italy Statistical Database. For the government interest rate, I use the the interest rate on 12-month *Buoni ordinari del Tesoro.*

[5]In July 2012, Mario Draghi, President of the European Central Bank, expressed a commitment to do "whatever it takes to preserve the euro." This commitment was accompanied by the launch of non-standard monetary policy measures allowing the ECB to purchase government debt securities (Outright Monetary Transactions, OMT). The impact of non-standard monetary policy operations

The interest rate paid by Italian firms on their debt rose in coincidence with the rise in government borrowing costs.[6] The interest rate spread on firms' borrowing rose above 300 basis points, around the peak for the corresponding government variable. The interest rate on firms' debt persisted at a high level for longer than the government interest rate. Spreads remained high through 2012 and 2013, equal on average to 287 basis points. By comparison, the mean level of this spread equaled 130 basis points in the pre-crisis period between 2000 and 2007. Firms' borrowing costs started to fall considerably later than government borrowing costs, reaching a peak of 322 basis points in February 2014 and diminishing thereafter. As of December 2015, firms' interest rate spread was equal to 186 basis points, still above its pre-crisis peak.

Fiscal policy in the sovereign debt crisis A positive and rising government primary balance characterized the conduct of fiscal policy in Italy during the euro area sovereign debt crisis. Italy's primary balance rose in 2012 to 2.2% of GDP, in the context of a severe recession. During the same period, GDP fell by 5.2% between the second quarter of 2011 and the second quarter of 2013. Data for Italy's primary balance, government revenue and expenditure are presented in Figure 2.2.

A number of measures were enacted by the Italian government to achieve an increase in tax revenue. The ratio of government revenues to GDP rose by two percentage points between 2011 and 2012, to 47.8%. Implicit tax rates on consumption, labor employed and capital all rose in 2012.[7] Taxation on capital rose particularly substantially, with the implicit tax rate rising from 32.4% to 37%. Taxes on residential property also rose in this period, as did VAT and excise duties.[8] Finally, as government borrowing costs subsided

on government borrowing costs falls beyond the scope of this chapter. For an analysis of monetary policy in the euro area during the global financial crisis and during the euro area sovereign debt crisis, see Reichlin and Pill (2016).

[6]As a measure of the interest rate on firms' liabilities, I use the series for the harmonized interest rate on loans other than overdraft to non-financial corporations, average maturity, provided by the Bank of Italy. Again, I use the ECM main refinancing operations interest rate as a nominal risk-free benchmark.

[7]Data on implicit tax rates are compiled and published in Eurostat (2014). They are computed as the ratio of total tax revenues, by category of taxes, to an estimate of the tax base.

[8]Eurostat (2014) and Eurostat (2015) provide a detailed account of yearly changes in the tax policy of EU countries, including Italy. For an analysis of the impact on domestic demand of the

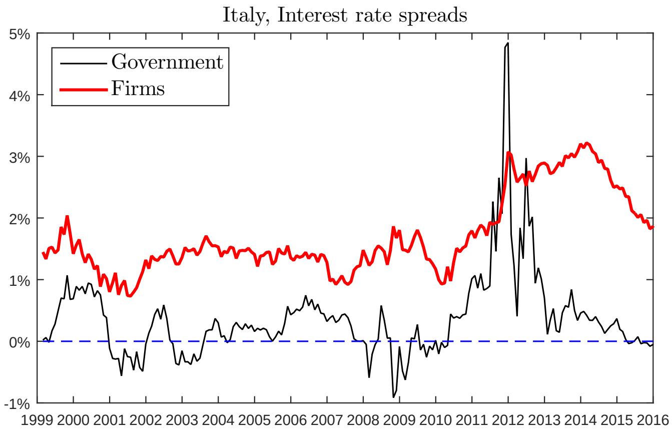

Figure 2.1: Interest rate spreads on liabilities of Italian government and firms

The series for the government is the difference between the interest rate over 12-month Italian government bonds (*Buoni ordinari del Tesoro*) and the risk-free rate. The series for firms is given by the spread over the risk-free rate of the interest rate for Italian non-financial corporations on non-overdraft loans, average across maturities. The nominal risk-free rate is given by the ECB main refinancing operations interest rate. All series are accessed via the Bank of Italy Statistical Database.

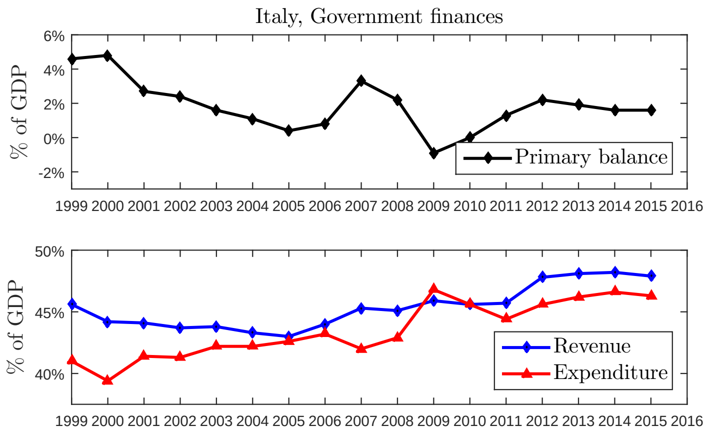

Figure 2.2: Italian government primary balance, revenue, and expenditure net of interest payments

All series are displayed as a share of GDP. The primary balance is simply defined as the difference between government revenue and government expenditure net of interest payments.

in 2013 and 2014, the primary balance fell as a share of GDP, reaching 1.6% in 2014 and 2015.

The increase in primary balance during the 2012 recession was in contrast with the fall in this variable observed during the global financial crisis, when the primary balance turned negative to -0.9% of GDP in 2009. In the period between the inception of the monetary union and the crisis, Italy had run a positive but diminishing primary balance, with this figure falling from 4.8% of GDP in 2000 to 0.4% in 2005. The increase in primary balance observed in 2007 and 2008 was abruptly interrupted by the emergence of the global financial crisis.

Heterogeneous impact of the crisis on firms The performance of non-exporting firms weakened substantially in Italy during the sovereign debt crisis. Firms for which a smaller share of sales is accounted for by exports observed the largest contraction, while the performance of all firms was, in general, adversely affected by the crisis. This pattern of heterogeneous performance is not common to all recessions and, for example, is not observed during the recession associated with the global financial crisis in 2009. I discuss here evidence on operating result, labor employed, sales and investment from the survey on industrial firms conducted by the Bank of Italy. Data are presented in Figure 2.3. This survey allows for an analysis of the heterogeneous effects of the crisis by presenting data on the performance of firms disaggregated by the share of their sales accounted for by exports.

First, the percentage of firms reporting an operating loss rose substantially for domestically-oriented firms in the year 2012, from 25% to 32%. This increase was the largest among firms exporting less than one third of their sales. As the number firms reporting a negative profit rose, it is likely that the risk of corporate default also rose substantially in this group. During 2012, the percentage of firms reporting an operating loss also rose in the group of firms exporting between one third and two thirds of their sales, while it remained constant for firms exporting more than two thirds of sales.

Second, hours employed fell the most for the group of firms exporting the smallest share of their output, between 2011 and 2013. Hours employed fell by 4% for firms exporting less than one third of output, and only by 1.5% for those exporting more than

increase in residential property tax rates, see Surico and Trezzi (2016).

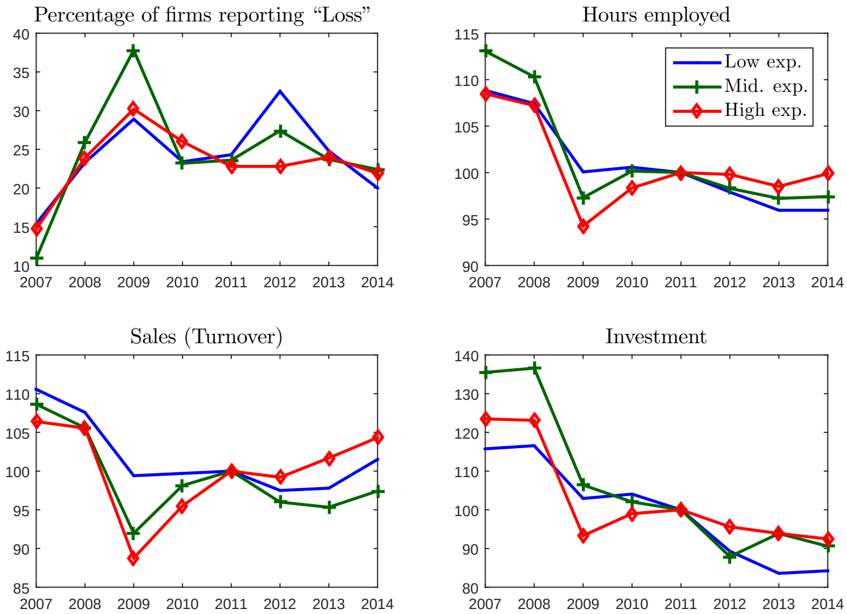

Figure 2.3: Heterogeneous impact of the crisis on firms

The four panels show data on the percentage of Italian firms reporting an operating result of "Loss", the amount of hours of labor employed, sales as measured by turnover, and investment. Figures are disaggregated by dividing firms according to the share of exports in their sales. Firms exporting less than one third of their output are classified as "Low exporters" (blue, solid line). Firms exporting between one third and two thirds of output are classified as "Middle exporters" (green line with cross markers) and the remainder of firms, exporting more than two thirds of output, are classified as high exporters (red line with diamond markers). All data, except those for operating result, are normalized to 100 for the year 2011. Data are drawn from Bank of Italy (2016).

two thirds. This pattern is in sharp contrast with the one observed for hours employed during the 2009 recession: Then, hours fell by 12% for high exporters, and by a more moderate 6.7% for low exporters.

Finally, data for sales and investment also corroborate the observation that the 2012 recession was characterized by a stronger contraction in the performance of domestically-oriented firms. While the turnover of high exporters fell by less than 1% in 2012 and rose in 2013, turnover fell by 2% and 4% for firms exporting less than one third, and between one third and two thirds of their output, respectively. Investment fell for all firms between 2011 and 2013, but while it fell by 6% for high exporters, the contraction amounted to 16% for low exporters. For both sales and investment, the 2009 recession was characterized instead by a sharper contraction in export-oriented firms.

2.4 Model

I consider an infinite-horizon small open economy trading in goods and assets with the rest of the world. Time is discrete and it is indexed by t. The small open economy is populated by three types of agents, a representative household, firms and a government. The household derives utility from leisure and from consumption of two different goods, a tradable and a non-tradable consumption good. The representative household owns the firms that operate in the economy. Firms hire labor to produce output of tradable or non-tradable good. The government purchases in each period an exogenous amount of goods. To fund this expenditure, the government can raise taxes on domestic households. In addition, the government is able to borrow from foreign lenders. The government borrows by issuing one-period, non-contingent debt. The government cares about welfare of the representative households and its objective is to maximize its utility.

2.4.1 Households

Expected lifetime utility of the representative household is given by

$$\mathbb{E}_0 \left[\sum_{t=1}^{\infty} \beta^t \frac{1}{1-\gamma} \left(c_t - \chi \frac{l_t^{1+\psi}}{1+\psi} \right)^{1-\gamma} \right] \tag{2.1}$$

where c_t is a composite consumption good and l_t denotes hours of labor supplied by the household. $\mathbb{E}_0$ is the expectation operator conditional on information available at time 0. The subjective discount factor is denoted by $\beta < 1$, the coefficient of relative risk aversion by $\gamma > 0$, $\psi > 0$ denotes the inverse of the elasticity of labor supply, and χ is a scaling parameter determining the relative marginal utility of consumption and leisure. Consumption c_t is defined as a CES aggregator of tradable and non-tradable consumption goods, $c_{T,t}$ and $c_{N,t}$:

$$c_t = \left[a^{\frac{1}{\theta}} c_{T,t}^{\frac{\theta-1}{\theta}} + (1-a)^{\frac{1}{\theta}} c_{N,t}^{\frac{\theta-1}{\theta}} \right] \tag{2.2}$$

where θ represents the elasticity of substitution across the two goods and a denotes the utility weight assigned in the aggregator to the tradable good.

In each period, the household faces a flow budget constraint given by

$$c_{T,t} + p_{N,t} c_{N,t} = w_t (1 - \tau_t) l_t + d_t - h_t \tag{2.3}$$

The budget constraint is expressed in units of tradable good, which is the numeraire in this economy. The relative price of the non-tradable good in units of numeraire is given by $p_{N,t}$. The left-hand side of this expression describes expenditure of the household for purchases of consumption goods. The right-hand side of the budget constraint describes income of the representative household. The wage rate per unit of labor supplied by the household is denoted by w_t. The household pays taxes on its labor income to the government, and τ_t is the proportional labor income tax rate. The household receives dividends from the firms it owns, and d_t is the aggregate amount of such dividends. h_t is a lump-sum social security contribution that is paid by the household.[9] Total consumption expenditure of the household can be expressed as $p_t c_t$, where p_t is the price index of the CES aggregator c_t:

$$p_t = \left[a + (1 - a) p_{N,t}^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{2.4}$$

The household cannot directly interact with foreign residents. However, the household indirectly borrows or lends from international financial markets through the actions of the domestic government.

Each period the household chooses $c_{T,t}$, $c_{N,t}$ and l_t to maximize its utility (2.1) subject to the flow budget constraint in period t, (2.3). The first-order conditions of this problem are given by

$$a^{\frac{1}{\theta}} \left(c_t - \chi \frac{l_t^{1+\psi}}{1+\psi}\right)^{-\gamma} \left(\frac{c_t}{c_{T,t}}\right)^{\frac{1}{\theta}} = \lambda_t \tag{2.5}$$

$$(1 - a)^{\frac{1}{\theta}} \left(c_t - \chi \frac{l_t^{1+\psi}}{1+\psi}\right)^{-\gamma} \left(\frac{c_t}{c_{N,t}}\right)^{\frac{1}{\theta}} = \lambda_t p_{N,t} \tag{2.6}$$

$$\chi l_t^{\psi} = \frac{w_t (1 - \tau_t)}{p_t}, \tag{2.7}$$

where λ_t is the marginal utility of wealth, or the Lagrange multiplier on the budget constraint. Neither this multiplier, nor consumption appear explicitly in the first-order

[9] In equilibrium, social security contributions serve to insure workers against losses due to default by firms on wage payments they are owed.

condition for hours, (2.7), due to the fact that the marginal utility of leisure is proportional to that of consumption, given the functional form for preferences here assumed.[10] Finally, (2.5) and (2.6) can be combined to obtain the intratemporal equilibrium condition linking the relative consumption of tradable and non-tradable goods to their relative price:

$$p_{N,t} = \left(\frac{1-a}{a}\right)^{\frac{1}{\theta}} \left(\frac{c_{T,t}}{c_{N,t}}\right)^{\frac{1}{\theta}}. \tag{2.8}$$

2.4.2 Firms

The small open economy is inhabited by a continuum of unit mass of firms. Firms are indexed by $i \in (0,1)$. A subset of firms with mass m_T has access to a technology to produce tradable good. The complementary set of firms, with mass $1 - m_T$, has access to a similar technology to produce non-tradable good. In both sectors, the only input required to produce output is labor supplied by the representative household. The amount of good produced by a generic firm i in each period t, $y_{i,t}$, is given by

$$y_{i,t} = z_t s_{i,t} l_{i,t}^{1-\alpha} - \varphi, \tag{2.9}$$

where $l_{i,t}$ is the amount of labor used in production by firm i, $s_{i,t}$ is an idiosyncratic productivity shock affecting output of firm i and z_t denotes an aggregate productivity shock. Output of all firms is net of a fixed cost, φ, which is denominated in units of the type of good produced by either firm. The elasticity of output, gross of fixed costs, to labor input is given by $1 - \alpha$.

Aggregate variables are known to all firms at the beginning of each period, before their production is realized. Such variables include aggregate productivity z_t and variables related to fiscal policy, the tax rate τ_t and assets held by the government $b_{G,t}$. Idiosyncratic productivity $s_{i,t}$ is not revealed to firms until after their choices are made and production is realized, at the end of each period. The stochastic process driving idiosyncratic productivity is i.i.d. across periods and firms.[11]

[10]Specifically, the results follows from $\lambda_t = \frac{1}{p_t}\left(c_t - \chi\frac{l_t^{1+\psi}}{1+\psi}\right)^{-\gamma}$. These preferences are also known as GHH, following Greenwood, Hercowitz and Huffman (1988). Note that in models with non-tradable goods, wealth can affect the choice of labor supply in general equilibrium, through its impact on the price level, p_t (Benigno, Chen, Otrok, Rebucci and Young, 2013).

[11]Aggregate and idiosyncratic productivity both take values in the support $[0, \infty)$.

Firms are subject to a working capital constraint. Due to the constraint, each firm must pay a fraction of its wage bill before production is realized. Firms cannot obtain equity from their shareholders, nor they can retain earnings to store resources internally. However, firms can borrow by issuing intraperiod, non-contingent and defaultable debt to foreign lenders, in order to obtain resources to pay workers. The working capital constraint is given by:

$$\zeta w_t l_{i,t} \leq q_{F,i,t} b_{F,i,t}, \tag{2.10}$$

where ζ denotes the fraction of the wage bill that the firm has to finance, $b_{F,i,t}$ is the amount of debt issued by the firm and $q_{F,i,t}$ is the unit price of such debt. The amount of resources that is raised by each firm on international financial markets at the beginning of each period is given by $q_{F,i,t} b_{F,i,t}$. The price of debt issued by each firm is affected by variables that are specific to the firm. In particular, it is a function of the amount of debt issued and of labor hired by the firm itself, as well as of the relative price of the good the firm produces. Such variables determine in equilibrium the expected return to lenders on firm-issued debt, hence they are necessary to determine the price at which such debt is traded.

At the end of each period, after idiosyncratic shocks are realized, all firms pay all resources left in the firm as dividends to the domestic household.[12] The dividend paid by each firm is given by

$$d_{i,t} = \max\left\{p_{i,t} y_{i,t} - (1-\zeta) w_t l_{i,t} - b_{F,i,t}, 0\right\}, \tag{2.11}$$

where $p_{i,t}$ is the relative price of output produced by each firm, given by $p_{N,t}$ if the firm produces non-tradable good, and by unity otherwise. The dividend that each firm pays amounts to the value of its output net of payments due to workers and lenders. In the event where the value of output is not sufficient to repay workers and lenders, $p_{i,t} y_{i,t} < (1-\zeta) w l_{i,t} + b_{F,i,t}$, the firm defaults on external lenders and it does not pay any dividend.[13] Upon default, a firm exits and it is replaced in the following period by a new and identical one.

[12] As firms do not have access to a storage technology, resources would be wasted if not transferred as dividends to the household.

[13] Workers' claims on firm resources are senior with respect to those of lenders. However, if output were insufficient to fully compensate workers, $p_{i,t} y_{i,t} < (1-\zeta) w l_{i,t}$, the firm would default on its obligations towards workers, too.

In each period, all firms hire labor and issue debt to maximize the expected value of dividend realized at the end of the period, subject to the working capital constraint (2.10) and taking into account how choices for labor and debt affect the price of debt. Dropping time subscripts, the problem of an individual firm is given by

$$\begin{aligned} \max_{l_i,b_i} \mathbb{E}_s\left(d_i\right) = \int_0^\infty \max\left\{p_i\left(zsl_i^{1-\alpha}-\varphi\right)-\left(1-\zeta\right)wl_i-b_i,0\right\} f_s\left(s\right)ds \\ \text{s.t. } \zeta wl_i \leq q_{F,i}b_{F,i}, \end{aligned} \tag{2.12}$$

where $\mathbb{E}_s$ denotes the mathematical expectation operator over the distribution of firm-level idiosyncratic productivity shocks, which is denoted by $f_s\left(s\right)$.[14] All firms have access to identical information sets and all enter each period with no resources inherited from the past. Hence, all firms in a given sector optimally make identical choices. In equilibrium, the working capital constraint is binding with equality. Each firm optimally equates the expected marginal product of an additional unit of labor with its marginal cost. Formally, the first-order condition associated with the firm's problem is given by

$$\left(1-\alpha\right)\mathbb{E}_s\left[s\middle|\, s\geq \hat{s}_i\right]p_i z l_i^{-\alpha} = w\left(1-\zeta+\frac{\zeta}{q_{F_b,i}b_i+q_{F,i}}\right)-\frac{q_{F_l,i}b_i}{q_{F_b,i}b_i+q_{F,i}} \tag{2.13}$$

where $q_{F_b,i}$ and $q_{F_l,i}$ denote the partial derivatives of the price of debt issued by the firm, $q_{F,i}$, with respect to debt issued and labor hired, respectively. The threshold for idiosyncratic productivity below which a firm defaults on external lenders is given by $\hat{s}_i$, which follows from the definition of firm dividends in (2.11). Given choices l_i and $b_{F,i}$ and aggregate variables z, w, and p_i, such threshold is given by

$$\hat{s}_i = \frac{\left(1-\zeta\right)wl_i + b_{F,i} + p_i\varphi}{p_i z l_i^{1-\alpha}}. \tag{2.14}$$

The policy function for default of individual firms is given by $D_{F,i}$, which is defined as the indicator variable for idiosyncratic productivity falling below the default threshold

$$D_{F,i} = \mathbb{1}\left(\hat{s}_i < s_i\right). \tag{2.15}$$

A firm defaults on wage payments due to workers if output is insufficient to fully cover for them, even after defaulting on external lenders. Default on workers occurs when idiosyncratic productivity falls below the threshold given by

$$\hat{\hat{s}}_i = \frac{\left(1-\zeta\right)wl_i + p_i\varphi}{p_i z l_i^{1-\alpha}}. \tag{2.16}$$

[14] Note that the choices of the firm only concern current-period variables, namely $l_{i,t}$, $b_{i,t}$, $d_{i,t}$, $q_{F,i,t}$. It is possible, then, to ignore time subscripts to simplify notation.

2.4.3 Government

The government is a benevolent agent who can trade bonds in international financial markets and sets taxes in the domestic economy, in order to maximize utility of the representative household. The only type of asset the government can trade in is a one-period, non-contingent and defaultable bond. The government can raise revenue domestically by charging a proportional labor income tax on the household. Revenues from taxation and external borrowing are used to finance a constant and exogenous stream of government expenditure. Government expenditure takes the form of a composite good, g, which is defined in analogous way to consumption

$$g = \left[a^{\frac{1}{\theta}} g_{T,t}^{\frac{\theta-1}{\theta}} + (1-a)^{\frac{1}{\theta}} g_{N,t}^{\frac{\theta-1}{\theta}} \right]. \tag{2.17}$$

Government purchases of tradable and non-tradable good are denoted by $g_{T,t}$ and $g_{N,t}$, respectively. The price index associated with g is the same one associated with private consumption, p_t.[15]

If the government has access to international financial markets, its budget constraint is given by

$$p_t g = \tau_t w_t l_t + b_{G,t} - q_G \left(z_t, b_{G,t+1} \right) b_{G,t+1}, \tag{2.18}$$

where $b_{G,t}$ denotes the amount of foreign assets held by the government and $b_{G,t+1}$ is the amount of bonds purchased by the government in the current period. Both are denominated in units of tradable good. A negative value for $b_{G,t+1}$ implies that the government is issuing debt in the current period, promising to repay one unit of consumption good in the following period for each unit of bond issued. The price at which bonds are issued or bought by the government is given by $q_G \left(z_t, b_{G,t+1} \right)$ which is a function of current aggregate productivity and of the amount of bond traded.

If the government does not have access to international financial markets, it cannot issue bonds and government expenditure is entirely financed by tax revenue. The government budget constraint is then given by

$$p_{G,t} g = \tau_t w_t l_t. \tag{2.19}$$

[15]Note that while aggregate government expenditure, g, is constant, the government can choose $g_{T,t}$ and $g_{N,t}$ in each period to minimize expenditure, $g_{T,t} + p_{N,t} g_{N,t}$.

Each period, if it has access to international financial markets, the government can choose whether to repay or to default on debt it owes to foreign lenders. The default choice is taken by the government comparing its benefits with the costs it would entail, in terms of utility of the domestic representative household. The benefit of default lies in the fact that debt is not repaid, so that a lower tax rate needs to be set to balance the government budget. On the other hand, default implies two different costs that are imposed onto the domestic economy. First, upon default, the government temporarily loses access to international financial markets, so that debt can no longer be used as a buffer against domestic shocks. Second, the representative household suffers from a direct welfare loss in the periods during which the government is excluded from international financial markets. Such a welfare loss is meant to capture disruptions that befall the economy in the aftermath of a default episode.[16]

The intertemporal choice problem of the government can be expressed recursively. When not excluded from international financial markets, the government chooses whether or not to default. Upon choosing repayment, it chooses how much debt to issue and it sets the labor income tax rate in order to satisfy the government budget constraint (2.18). The state variables for the government choice problem are the aggregate productivity shock, z, and the amount of foreign assets held by the government, b_G, where time subscripts have been removed making use of recursive notation.

The government value function, conditional on having access to international financial markets, is given by $V(z, b_G)$. The government chooses to repay or to default on debt by comparing the value function associated with default, $V_D(z)$, with the one associated with repaying debt and maintaining access to international financial markets, $V_R(z, b_G)$. The discrete choice default problem is given by

$$V(z, b_G) = \max_{D \in \{0,1\}} DV_D(z) + (1 - D) V_R(z, b_G), \tag{2.20}$$

where D denotes an indicator taking the value of unity in the event of default. The problem above defines the discrete policy function for default:

$$D(z, b_G) = \mathbb{1}(V_R(z, b_G) > V_D(z)). \tag{2.21}$$

[16]Cole and Kehoe (2000) and Arellano (2008) both introduce direct output costs of default in their models. A strand of the literature has more recently analyzed the endogenous determinants of sovereign default costs. Mendoza and Yue (2012) is a notable example.

If the government chooses to default, it sets the tax rate in order to satisfy the autarky budget constraint (2.19). While default causes the government to be excluded from financial markets, readmission occurs in the following period with exogenous probability λ. In the event of readmission, the value function of the government is given by $V(z,0)$, as the government regains access to markets with zero assets. If the government is not readmitted, it remains in financial autarky and it may again be readmitted in the following period with probability λ. During periods of exclusion from financial markets the welfare loss suffered by the representative household is increasing in aggregate productivity and it is given by $\delta(z)$. The value function in the event of exclusion is given by

$$\begin{aligned} V_D(z) = & \max_{\tau} \left\{ u(c^*, l^*) - \delta(z) + \beta \mathbb{E}\left[\lambda V(z', 0) + (1-\lambda) V_D(z') \middle| z \right] \right\} \\ & \text{s.t. } p^* g = \tau w^* l^* \end{aligned} \tag{2.22}$$

where $u(c,l)$ denotes the period utility function defined in (2.1) and c^* and l^* denote the equilibrium values of consumption and hours, respectively, that are consistent with the government choice for τ and with aggregate productivity z.[17] Similarly, w^* and p^* denote equilibrium values of the wage rate and of the price index. Prime superscripts denote next-period variable values. The expectation operator on next-period variables, conditional on information summarized by current-period productivity, is denoted by $\mathbb{E}[\cdot|\, z]$.

When the government has access to international financial markets, it can choose the amount of foreign assets to carry into the following period, b'_G, as well as the labor income tax rate that is charged to domestic households. When issuing debt to foreign lenders, the government internalizes how changes in the amount of debt affect its price, through the function $q_G(z, b'_G)$. The maximization problem is given by

$$\begin{aligned} V_R(z, b_G) = & \max_{\tau, b'_G} \left\{ u(c^*, l^*) + \beta \mathbb{E}\left[V(z', b'_G) \middle| z \right] \right\} \\ & \text{s.t. } p^* g = \tau^* w^* l^* + b_G - q_G(z, b'_G)\, b'_G. \end{aligned} \tag{2.23}$$

The problem above defines policy functions for the tax rate and for the amount of assets traded by the government, as a function of the aggregate state variables. Note that while star superscripts still denote equilibrium values of variables, such values differ, in general, from the ones in (2.22). This follows from the fact that the policy function for τ when the

[17] A complete description of the competitive equilibrium of this economy is presented in Section 2.4.6.

government has access to international financial markets differs from the one emerging under financial autarky.

2.4.4 Foreign lenders

The rest of the world is inhabited by a continuum of infinitely lived, identical and risk-neutral foreign lenders. Foreign lenders can trade one-period debt with the government of the small open economy and they can buy intraperiod bonds issued by firms. Foreign lenders are profit maximizers and they behave competitively.

The price of a unit of government debt that compensates lenders for sovereign default risk is given by

$$q_G\left(z, b'_G\right) = \beta_{ROW}\mathbb{E}\left[1 - D\left(z', b'_G\right)\middle| z\right] \tag{2.24}$$

where β_{ROW} denotes the subjective discount factor of foreign lenders. Lenders understand how the policy function for default depends on the state variables z and b_G and they correctly evaluate the risk of sovereign default by forming expectations over this policy function. Given the binary nature of the variable D, this expression can be rewritten as

$$q_G\left(z, b'_G\right) = \beta_{ROW}\left[1 - \Pr\left(D\left(z', b'_G\right) = 1\middle| z\right)\right],$$

showing how the price of sovereign debt is decreasing in the probability of next-period default.

The price at which firm-issued debt is traded must also compensate lenders for firms' default risk. However, when purchasing intra-period debt issued by firms, foreign lenders don't need to be compensated for their rate of time preference, as all transactions take place within each period. In the event of default, all output left in the firm, net of wage payments to workers and liquidation costs, is shared among lenders, proportionally to the amount of debt held by each. The amount of resources shared by lenders is given by

$$\max\left\{\xi\left(p_i y_i - (1-\zeta)\, w l_i\right), 0\right\} \tag{2.25}$$

where ξ is a parameter capturing the fraction of output left in the firm after losses due to liquidation costs. The price of debt issued by firm i that is consistent with optimization by foreign lenders is then given by

$$q_{F,i} = \mathbb{E}_s\left[(1 - D_{F,i}) + D_{F,i}\frac{\xi \max\left\{\left(p_i y_i - (1-\zeta)\, w l_i\right), 0\right\}}{b_{F,i}}\right]. \tag{2.26}$$

2.4.5 Market clearing

Market clearing for labor requires that the amount of labor demanded by firms equals labor supply by the representative household

$$l = \int_{i \in (0,1)} l_i di \tag{2.27}$$

For the non-tradable goods market to clear, the amount produced must equal the amount consumed by the representative household and by the government

$$c_N + g_N = y_N (1 - m_T) \tag{2.28}$$

where y_N is the amount of good produced by an average firm in the non-tradable sector and it is defined in Appendix 2.A.

By combining the household budget constraint (2.3), the budget constraint of the government (2.18), the definition of firm dividends (2.11), and the market clearing condition for non-tradable goods (2.28), we obtain the market clearing condition for tradable goods

$$c_T + g_T = m_T y_T + b_G - q_G b'_G. \tag{2.29}$$

Note that the condition above is satisfied by Walras' law.

Finally, social security contributions paid by the representative household must cover for losses on wage payments suffered by workers due to firms' default. This is established by (2.33) in Appendix 2.A.

2.4.6 Equilibrium

In equilibrium, firms choose the optimal amount of labor to hire and debt to issue. Households choose labor supply and consumption of tradable and non-tradable good. Both household and firms make their choices given government policy for taxation, debt issuance and default. In turn, the government sets policy taking into account optimal behavior of domestic households and firms and of foreign lenders. I define below the competitive equilibrium in the small open economy.

Definition 4. *A recursive equilibrium in the small open economy is characterized by*

- *A set of value functions for the government V, V_R, and V_D,*

- *Household policy functions for consumption c, c_T, and c_N, and labor supply l,*
- *Policy functions for firms in the tradable and non-tradable sector for labor, l_T, l_N and debt issued $b_{F,T}$, $b_{F,N}$,*
- *Government policy for taxation τ, asset holdings b'_G, default D and allocation of government expenditure g_N, g_T,*
- *Government debt price function q_G,*
- *Equilibrium good prices p_N, p, wage w, firm debt prices $q_{F,T}$, $q_{F,N}$, dividends d_T, d_N, default thresholds $\hat{s}_T$, $\hat{s}_N$, $\hat{\hat{s}}_T$, $\hat{\hat{s}}_N$, and social security contribution h,*

such that:

- *Given government policy, the policy functions of household and firms solve their maximization problems and they are consistent with the first-order conditions* (2.5) - (2.7) *and* (2.13)*,*
- *Given the debt price function q_G and the policy functions of households and firms, the government value and policy functions solve its maximization problem* (2.20), (2.22), (2.23)*,*
- *Government and firms' debt prices are consistent with optimization by foreign lenders,* (2.24) *and* (2.26)*,*
- *Markets for labor and non-tradable good clear domestically,* (2.27) *and* (2.28)*, and firm dividends and default thresholds are consistent with firm choices and given by* (2.11), (2.14) *and* (2.16).

2.5 Quantitative analysis

2.5.1 Calibration

The model is solved numerically. I analyze quantitative properties of the solution, focusing in particular on the nature of government policy and on its implications for firm-level variables. Empirical targets for the calibration will largely be drawn from data for Italy. The aim of this chapter is to study the experience in this country of a sovereign debt crisis followed by a fiscal policy tightening and by a contraction in credit to the private sector, motivated by the empirical evidence presented in Section 2.3. Table 2.1 shows calibrated parameter values and corresponding empirical targets. Parameters of the model economy are set to match means of variables from the ergodic distribution of the model-generated data with their empirical counterparts.

Parameters related to preferences of the domestic household are largely standard and drawn from the literature on international macroeconomics. One period corresponds to one quarter, for ease of comparison with data from national accounts. The subjective discount factor of the representative household, β, is set to 0.97, implying a standard deviation of the current account of 0.6%. The subjective discount factor of foreign lenders, β_{ROW}, is set to target an average world risk-free real interest rate of 2.5%. I set the intratemporal elasticity of substitution between tradable and non-tradable consumption θ and the relative share of tradable goods in consumption a to 0.5, following Stockman and Tesar (1995).

Parameters governing the technology of firms in the model are set to target key empirical moments of data on the Italian economy. The elasticity of gross output to labor input, $1-\alpha$, is consistent with the average share of labor income in GDP in this country, equal to 62.5%.[18] Bartelsman, Haltiwanger and Scarpetta (2013) show that the standard deviation of labor productivity is consistently higher than that of total factor productivity, across industries and countries. This result is delivered in the model by considering a positive value for the fixed cost of production, φ. I set φ to target a standard deviation of log labor productivity that is 50% higher than the one of log total factor productivity, as observed in the United States.[19] The recovery rate on payments received by lenders after default by firms, ξ, determines the price at which firms are able to issue debt, given parameters affecting the firms' default threshold and the standard deviation of idiosyncratic productivity. I set this parameter to be consistent with an average yearly interest rate of 1.3%, in line with evidence on the interest rate charged to firms by Italian banks.[20] The implied quarterly probability of default by a firm is equal to 0.5%. The parame-

[18] This figure is reported in European Commission (2007) for the period 1960-2006.

[19] Data are not available for Italy. The same ratio of standard deviations is also found for the United Kingdom, while the ratio is higher in France and the Netherlands.

[20] I make use of the same series for interest rates on loans to non-financial corporations used in Section 2.3, net of the nominal risk-free rate.

ter governing the tightness of the working capital constraint, ζ, is set to unity, implying that the entire wage bill has to be financed by firms. A similar assumption is made in Neumeyer and Perri (2005). This assumption is consistent with the observation that the stock of credit to non-financial corporations in the Italian economy is of the same order of magnitude as the share of GDP accruing to labor income.

The stochastic process driving aggregate productivity z is log-normal with autocorrelation of 0.98 and standard deviation of the innovations equal to 0.6%. These values are standard in the real business cycles literature. The mean of this process is set to normalize to unity the average output of firms. The process is discretized with the method of Tauchen (1986) on a very fine grid. On average, each grid point spans 0.036 unconditional standard deviations of the process. The very fine grid is needed to ensure that the function $q_G(z, b'_G)$ is sufficiently smooth. The presence of jumps in this function would cause the government to choose values of borrowing which correspond to the jumps, leading to an underestimation of the amount of sovereign debt issued and of the interest rate paid on such debt.

Idiosyncratic productivity of firms is driven by a log-normal process that is i.i.d. across periods and firms. The standard deviation of the shocks is set to 9%. This value is low in comparison to the ones reported by Bartelsman, Haltiwanger and Scarpetta (2013). In this model, however, transitory shocks account for the entirety of firm-level dispersion in productivity. As persistent shocks are likely to capture a sizable fraction of such dispersion, it appears conservative to use a lower value for such standard deviation. The mean of the log-normal process is set to normalize average idiosyncratic productivity to unity.

Government expenditure is equal to the average ratio of government consumption to GDP in Italy between 1999 to 2007, 18.5%.

The function that determines the welfare cost of default is akin to the one introduced by Arellano (2008). Differently than in that paper, the default cost is specified here as a lump-sum loss in welfare for the representative household. Thanks to this specification, the default cost has no implication for any of the variables determined in the competitive equilibrium. The default cost is defined as follows:

$$\delta(z) = \begin{cases} 0 & \text{if } z < \delta\mathbb{E}(z) \\ u_{aut}(z) - u_{aut}(\delta\mathbb{E}(z)) & \text{otherwise} \end{cases} \tag{2.30}$$

where $u_{aut}(z)$ denotes period utility of the representative household in autarky, when the tax rate is set so that tax revenue solely covers for government expenditure, for a given value of aggregate productivity. The above definition of welfare costs of defaults implies that, in case of exclusion from international financial markets, period utility of the representative household is increasing in z only for $z < \delta\mathbb{E}(z)$, and it is flat otherwise. This implies that the welfare cost of default is increasing in z. The parameter δ is set to match the average ratio of sovereign debt to GDP for Italy in 1999 - 2007. Given that sovereign debt is entirely held outside of the small open economy in this model, I correct the empirical target of 105% for the debt-to-GDP ratio by subtracting the fraction of sovereign debt that is held domestically, which equals to a half in the period preceding the global financial crisis, according to Brutti and Saurè (2014). The parameter λ, governing the probability of readmission into international financial markets is set to 0.1, a standard value in the endogenous sovereign default literature (e.g. Cuadra, Sanchez, Sapriza 2010).

2.5.2 Sovereign default, fiscal policy and firms in the model economy

I illustrate here key properties of fiscal policy and its implications for firms in the equilibrium of the calibrated model economy. First, I discuss the planner's optimal default decision, by presenting the equilibrium policy function for sovereign default and the implied function describing the price at which sovereign debt is issued. Second, I analyze optimal fiscal policy chosen by the planner, by presenting the policy functions for government net lending and for the tax rate on labor income. Finally, I discuss the implications of government policy for variables determined in the private sector. In particular, I show how a positive correlation emerges between interest rates on sovereign and corporate debt, and I discuss how fiscal policy differently affects the performance of firms in the tradable and in the non-tradable sector.

Table 2.1: Parameter values

Calibrated Parameter		Value	Target / Source
Discount factor	β	0.97	St. deviation of current account, 0.6%
Foreign lenders' discount factor	β_{ROW}	0.994	World risk-free rate, 2.5%
Relative risk aversion	γ	2	Standard
Frisch elasticity	$\frac{1}{\psi}$	2	Standard
Labor disutility	χ	2.49	Normalize $l = 1/3$
Share of tradable good	a	0.5	Stockman and Tesar (1995)
Intra-temporal elasticity of substitution	θ	0.5	Stockman and Tesar (1995)
Elasticity to labor input of gross output	α	0.46	Labor income share, 62.5%
Fixed cost in production	φ	0.5	Relative volatility of labor productivity
Recovery rate	ξ	0.24	Average real interest rate on firm debt
Working capital constraint	ζ	1	Ratio of total credit to labor income
Aggregate TFP standard deviation	σ_z	0.006	Standard
Aggregate TFP autocorrelation	ρ_z	0.98	Standard
Idiosyncratic TFP standard deviation	σ_s	0.09	Bartelsman et al. (2013)
Government expenditure	g	0.185	Government consumption to GDP ratio
Welfare cost of sovereign default	δ	0.9	Sovereign debt to GDP ratio
Readmission probability	λ	0.1	Average length of exclusion period

Sovereign default The policy function for sovereign default, as determined by the solution to (2.20), is presented in Figure 2.4. The pink, shaded area represents the subset of the state space where it is optimal to default, given aggregate productivity and sovereign debt. Assets held by the government are displayed on the horizontal axis, with negative values denoting sovereign debt.[21] Aggregate productivity is displayed on the vertical axis. The ratio of aggregate productivity to its ergodic mean is denoted by $\hat{z}$, so that e.g. $\hat{z} = 1.02$ implies that aggregate productivity is 2% above its unconditional mean.

Two main features of the default policy function are worth noting. First, the level of debt for which it is optimal to default is substantially higher than in most models of endogenous sovereign default. Default is optimal in this model for sovereign debt equal to 38.5% of yearly output, when aggregate productivity is at its ergodic mean. By way of comparison, mean debt is 6% of output in the seminal paper of Arellano (2008). As emphasized by Aguiar and Gopinath (2006), a high penalty for default is necessary to generate a high equilibrium level of debt. That is indeed the strategy adopted when setting $\delta(z)$ in calibrating this model.

Second, the highest sustainable level of debt is increasing in aggregate productivity. This result follows from the assumption that default costs are increasing in aggregate productivity, as in Arellano (2008). Here, default costs arise due to a lump-sum reduction in welfare, rather than from a reduction in productivity. This particular modeling choice, however, has no implications for this result on the slope of the default threshold.

Sovereign debt price function The sovereign debt price function $q_G(z, b'_G)$ describes the price at which the government can issue debt on international financial markets, given the amount issued b'_G and current productivity z, following (2.24). It is displayed in Figure 2.5 for three values of aggregate productivity, corresponding to its ergodic mean and to a high and a low value, 0.7 standard deviations above and below the mean, respectively.

First, changes in the value of current productivity cause the function to shift. For a

[21] Sovereign assets are a stock variable. I thus divide their value by four, for ease of comparison with empirical targets such as the debt-to-(yearly) GDP ratio. Note that quarterly output of an average firm is here normalized to unity.

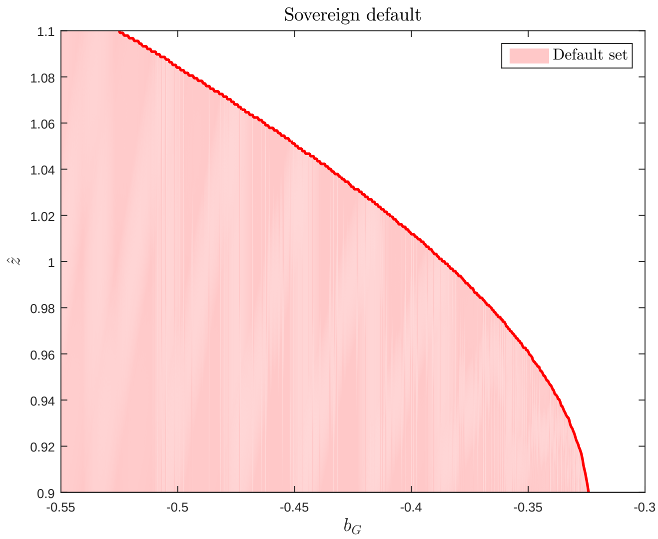

Figure 2.4: Policy function for sovereign default

The shaded area represents the region of the state space where it is optimal for the planner to default. $\hat{z}$ denotes aggregate productivity, expressed as the ratio to its ergodic mean, b_G denotes assets held by the government, with negative values denoting sovereign debt.

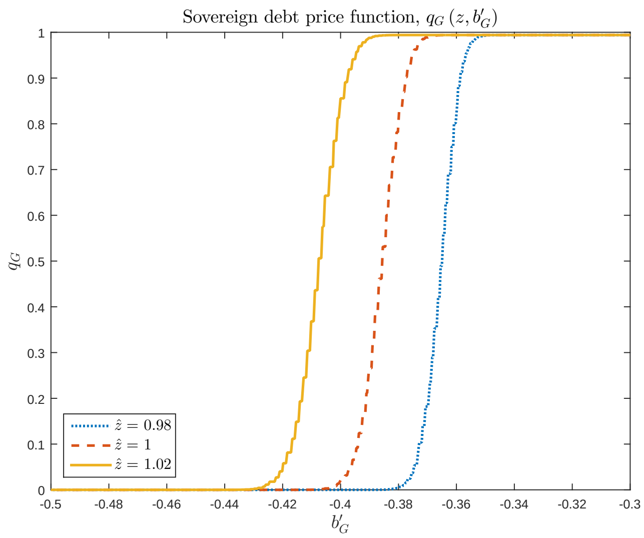

Figure 2.5: Sovereign debt price function

The function $q_G(z, b'_G)$ describes the price at which the government can issue debt on international financial markets, given the amount issued b'_G and current aggregate productivity $\hat{z}$, expressed as the ratio to its ergodic mean.

fall in productivity, the shift implies that a lower price of debt is compatible with any given choice of debt issued. Equivalently, a lower amount of debt must be issued by the government for the debt price to remain constant. The nature of the default policy function described is the cause of such shift, jointly with the presence of autocorrelation in the process for z. A low realization of current productivity implies that a similarly low realization is more likely in the following period. Lenders thus attach a high probability of default to low values of debt when current productivity is low, as default will be chosen for low values of debt if productivity will remain low in the following period. In turn, the increase in the default probability causes the price of sovereign debt to fall.

Second, the price at which debt is issued is decreasing in the amount of debt issued itself. This is a typical property of models of endogenous sovereign default, again following directly from the the properties of the default policy function. As the level of debt increases, default is chosen under a wider range of values for aggregate productivity. Hence, next-period default becomes more likely as debt rises and international lenders are willing to purchase sovereign debt only at a lower price.

Finally, the fact that the function $q_G(z, b'_G)$ is decreasing in b'_G implies that the planner faces an upper bound on the amount of resources that she can obtain from international financial markets. In addition, the distortionary nature of taxes implies the presence of an upper bound on the amount of tax revenue. Jointly, these two bounds place a limit on the feasible amount of external debt repayment. The presence of a bound on the amount of debt that can be feasibly repaid influences default incentives for the planner. Importantly, the presence of financial frictions in the firms' problem affects the amount of resources that can be raised by the planner via taxes. As the ability of the planner to raise tax revenues determines its incentive to default and the price of sovereign debt, the severity of financial frictions faced by firms affects in turn the severity of the constraint faced by the government.

Optimal fiscal policy The planner's choice of fiscal policy variables such as the tax rate and government borrowing is crucial in determining all quantities and prices in the equilibrium of this model. Policy functions for government net saving and for the labor income tax rate are presented in Figure 2.6, as resulting from the solution to (2.23).

Government surplus is defined as the policy function for the change in government-held assets. The government is the only agent in the economy who engages in intertemporal trade with foreign lenders, so net saving by the government corresponds with net saving of the economy as a whole. Government surplus is decreasing in government assets, or increasing in government debt, and it is decreasing in aggregate productivity, as shown in the left-hand side panel in Figure 2.6.[22] The high cost of borrowing that is associated with issuing large amounts of debt induces the government to run a positive surplus when owing a relatively large amount of debt. On the other hand, relative impatience of the domestic household compared to foreign lenders induces the government to run a deficit, allowing the household to front-load consumption when both government debt and government borrowing costs are low. A positive productivity shock causes a shift in the sovereign debt price function, relaxing the constraint on external borrowing faced by the government. Hence, by lowering borrowing costs, an improvement in aggregate productivity allows the government to choose a lower surplus.

Taxes on labor income allow the government to raise resources from the private sector, to be used to finance expenditure and payments to foreign lenders. The planner's optimal choice for the tax rate is displayed in the right-hand side panel in Figure 2.6, again as function of government held assets and for three different values of aggregate productivity. Three main facts can be noted regarding the policy function for the labor income tax rate. First, the tax rate chosen by the planner is relatively high as the government needs to finance high levels of government expenditure and debt. The tax rate set by the planner equals on average 28%, in the ergodic distribution of simulated series from the model solution. By comparison, the average implicit tax rate on labor income equalled 42% in Italy between 2001 and 2007.[23]

Second, the policy function for the tax rate largely inherits the properties of the tax policy for government surplus: the tax rate policy function is decreasing in government-

[22]Note that while assets are divided by four, to be expressed as the ratio of a stock variable to a yearly flow, this is not necessary for the government surplus, which is a flow variable already. This fact needs to be taken into account when comparing the policy function for government surplus with the level of government debt.

[23]Note that it would be possible to generate in the model a higher average tax rate by calibrating default costs for average sovereign debt to match the entire stock of general government debt owed by Italy, rather than the fraction of it held by foreign residents. Data source: Eurostat database, Implicit tax rate on labour, table `tec00119`.

held assets as well as in aggregate productivity. Intuitively, a higher tax rate needs to be set in order to generate high government savings, especially given the fact that the planner has no control over the level of expenditure.

Finally, the distortionary nature of taxation implies that the tax rate increases non-linearly with government surplus. Due to the presence of a concave "Laffer curve" in tax revenue, a higher increase in the tax rate is necessary to generate the same increase in tax revenue, when starting from a higher tax rate. In particular, the tax rate increases when productivity falls for two reasons. First, a higher government surplus is chosen given higher borrowing costs, as discussed above. Second, it is harder to generate tax revenues when productivity is lower. Indeed, a higher tax rate would have to be set, even when choosing to maintain a constant tax revenue.

Sovereign and firms' interest rates Borrowing costs of government and firms positively comove in this model economy. Several papers in this literature have studied the interaction between government default and the financial frictions faced by firms.[24] To my knowledge, however, this is the first paper where the positive comovement emerges due to positive correlation in the endogenous default risk of government and firms. The ability of the government to transfer its own default risk to the private sector via the channel of fiscal policy causes in the model the observed correlation between government and firms' default risk. This mechanism differs from the one of transmission via domestic financial intermediaries, which has extensively been studied in the literature. According to that channel, a fall in the value of sovereign debt held by domestic banks tightens their balance sheet, leading to a contraction in credit to firms. Here, the government directly achieves via taxation a transfer of resources from the private sector to the government itself. Such transfer causes an increase in the risk of default faced by firms, allowing for a relaxation in the budget constraint faced by the government. Policy functions for the interest rates on government and firms' debt are displayed in Figure 2.7.[25]

[24]See the literature review in Section 2.2 for a more extensive discussion of this strand of research.

[25]I define the interest rate as the inverse of the debt price, minus one. For ease of exposition, I consider annual interest rates, obtained by taking the fourth power of the debt price: $r_G = \frac{1}{q_G^4} - 1$. The firms' interest rate displayed is a weighted average of the interest rate paid by firms in tradable and non-tradable sectors.

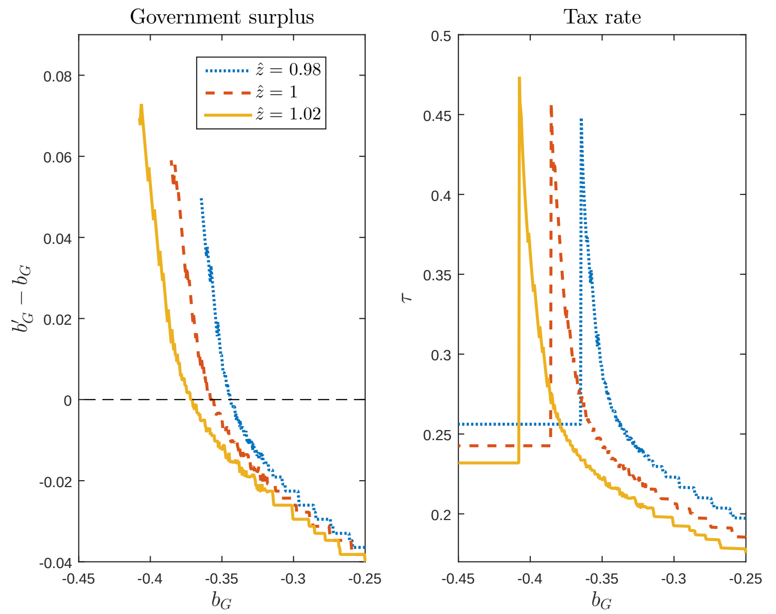

Figure 2.6: Policy functions for government surplus and labor income tax rate

The policy functions are displayed as function of the state variables of the model, government-held assets b_G and aggregate productivity, $\hat{z}$.

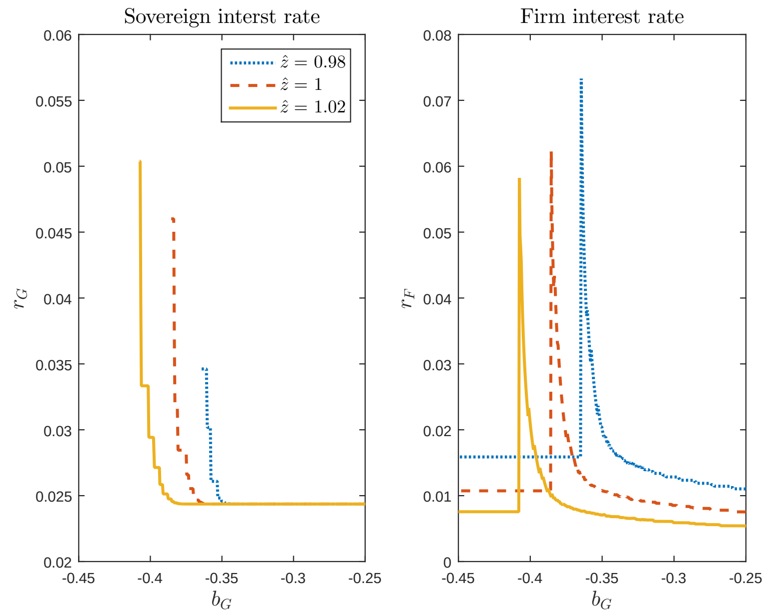

Figure 2.7: Policy functions for the interest rate paid by government and firms on their debt

The policy functions are displayed as a function of the state variables of the model, government-held assets b_G and aggregate productivity, $\hat{z}$.

Interest rates paid by both government and firms reflect, in the model, equilibrium default probabilities. Both are increasing in the amount of government debt $|b_G|$ and decreasing in aggregate productivity z. Different mechanisms, however, lie behind the behavior of the two variables. First, the increase in interest rates paid by firms as government debt rises is solely due to the increase in taxes through which sovereign risk is transmitted to the private sector. An increase in government debt from 34% to 38% of GDP causes an increase in firms' interest rate by 300 basis points, when aggregate productivity is equal to its mean. A higher level of debt causes the government to set a higher labor income tax rate, to generate higher government surplus. Firms' production costs rise along with the increase in the tax rate. In turn, their profits fall and the probability of default by firms rises, leading to a higher interest rate on their debt. For the government, a higher level of initial debt $|b_G|$ induces a corresponding increase in the level of debt issued in the current period $|b'_G|$. The choice for debt issuance thus moves along the sovereign debt price function of Figure 2.5 and debt is issued at a lower price, corresponding to a higher interest rate on government debt. The same increase in government debt from 34% to 38% of GDP causes the interest rate on government debt to rise by 35 basis points. The relatively moderate increase in government borrowing costs is partly due to the ability of this agent to shift the cost of repayment to firms, by increasing taxes to run a larger government surplus as debt rises.

Second, a fall in aggregate productivity also leads to a rise in the interest rate paid by firms. This result would occur already in the absence of fluctuations in fiscal variables, as lower productivity reduces the profits of firms and it leads to an increase in their default probability. The increase in tax rate that is associated with a decrease in productivity, however, magnifies the fall in profits, interacting with the direct effect of lower productivity and generating a rise in firms' interest rates. For the government, lower productivity leads to an increase in the probability of next-period defautl and to a shift in the sovereign default price function, causing the cost of sovereign borrowing to be higher for any given choice of debt issued. In turn, the shift in the curve leads the government to pay a higher interest rate on its debt in equilibrium given its choice for optimal fiscal policy.[26] A 2% fall in aggregate productivity from its unconditional mean value causes the interest rate

[26]This is a standard result in the endogenous sovereign default literature, as in e.g. Arellano (2008).

on firms' and government debt to rise by 250 and 10 basis points, respectively, when government debt equals 36% of GDP.

Finally, note that the interest rate on debt issued by firms is also defined in the region of the state space where sovereign default occurs. The interest rate on government debt is not defined in that region, as the government does not have access to international financial markets. The interest rate on firms' debt is determined there by the level of aggregate productivity and by the tax rate set in autarky to balance the government budget constraint, according to (2.22). In particular, the default cost has no bearing on firms' interest rate, as it only determines an exogenous reduction in household welfare. Default has no adverse impact per se on financial frictions faced by firms in this model. Rather, it is default risk and the incentive for the government to avoid default by increasing taxation that cause disruption to the performance of firms.[27]

Heterogeneous firm-level effects of sovereign debt Sovereign debt has heterogeneous effects on the two sectors of firms that inhabit the economy, producing tradable and non-tradable goods. High sovereign debt has more severe detrimental effects for firms in the non-tradable sector. These firms are exposed to fluctuations in domestic demand, as they cannot export and they are confined to selling their production locally. Domestic demand is lower when sovereign debt is higher, as the economy runs a current account surplus to eschew high costs of external borrowing.[28]

The heterogeneous effects of sovereign debt on the relative price of goods, on firms' interest rates, and on labor demand are presented graphically in Figure 2.8. First, a high amount of government debt causes a fall in the relative rice of the non-tradable good. Domestic demand for consumption goods is lower when government debt is high. Hence, a lower relative price of non-tradable is necessary to induce a rebalancing of consumption towards this good. As relative consumption of non-tradable to tradable rises, firms in the non-tradable sector are able to sell the entirety of their production domestically. An increase in government debt from 34% to 38% of GDP causes a 6% fall in the relative

[27]In light of this fact, joint consideration of the transmission channel through fiscal policy presented here and the one through domestic financial intermediaries appears as a promising avenue for future research.

[28]A current account surplus follows uniquely in this model from the presence of a government surplus, as the government is the only agent trading in intertemporal assets with foreign lenders.

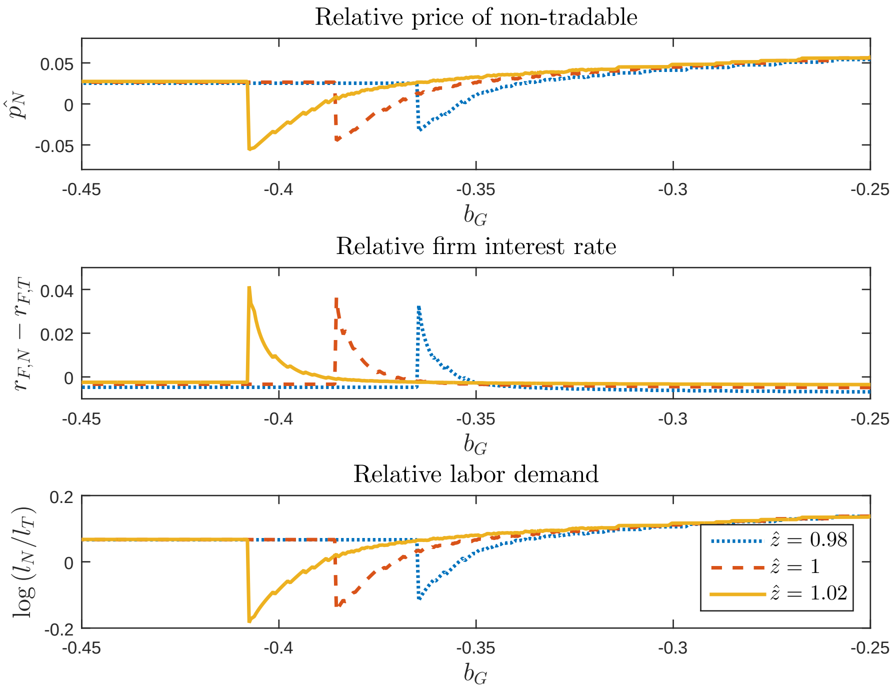

Figure 2.8: Heterogeneous firm-level effects of sovereign debt

The three panels display equilibrium values of variables as a function of the state variables of the model, government-held assets b_G and aggregate productivity, $\hat{z}$. The top panel displays the relative price of non-tradable good, expressed as deviation from its ergodic mean. The middle panel displays the difference between the interest rates paid on debt by firms in the non-tradable and in the tradable sector. The bottom panel displays the logarithm of relative labor input of firms in the two sectors.

price of non-tradable good, when aggregate productivity is equal to its unconditional mean value. The equilibrium relative price of non-tradable good is represented in the upper panel of Figure 2.8, as a function of the state variables, aggregate productivity and government-held assets.

Second, firms in the non-tradable sector pay a higher interest rate than their tradable-sector counterparts when sovereign debt is higher. The difference between the interest rates paid by firms in the two sectors is shown in the the middle panel of Figure 2.8. The low relative price of output hinders performance of firms in the non-tradable sector, while profits of all firms are depressed by the high taxes that are associated with high debt. Low relative demand interacts with high taxes, leading to a higher than average rise in the default probability of non-tradable firms and in their borrowing costs. Sovereign default risk is thus transmitted to firms to a heterogeneous degree, with firms that are more exposed to domestic demand suffering from a stronger contagion of sovereign risk. The difference between the interest rate paid by non-tradable and tradable sector firms rises by 140 basis points as debt rises from 34% to 38% of GDP and the relative price of non-tradable falls by 6%.

Finally, the low relative price of non-tradable induces a relative contraction in the production of this good, when government debt is high. Relative labor input of firms in the non-tradable sector is presented in the bottom panel of Figure 2.8. Firms in the non-tradable sector individually contract their production so that, on aggregate, a higher fraction of labor input is reallocated towards the tradable sector. Relative labor demand by non-tradable sector firms falls by sixteen percentage points, as debt rises from 34% to 38% of GDP, when aggregate productivity is equal to its unconditional mean value. Individual firms in the non-tradable sector would find it optimal to reallocate towards production of tradable goods, as domestic demand is low and expected dividends are higher in the tradable sector. Were such reallocation to be possible, the shift in relative supply would ensure the absence of fluctuations in the relative price of goods. The inability of firms in this model to choose the type of good they produce results in an incomplete reallocation of factors of production, in fluctuations in the relative price of non-tradable, and in equilibrium heterogeneity in firm size across sectors. In presence of decreasing returns to scale, as implied by $\alpha < 1$, the heterogeneity in firm size caused by the constraint on firms' reallocation is costly from an aggregate welfare perspective.

2.6 Sovereign debt crisis

I analyze in this section a sovereign debt crisis episode, studying its implications for the conduct of fiscal policy and for the performance of firms. First, I describe in detail the crisis episode considered, consisting of a shock to default incentives that reduces the amount of sustainable sovereign debt. Second, I analyze the implications of the crisis on the planner's choice for fiscal policy variables, namely for the amount of government borrowing and for the tax rate on labor income. Third, I show that the crisis is transmitted to the private sector, through its effect on fiscal policy. In particular, I show that the crisis leads to a rise in the cost of borrowing for firms, as well as for the government. Finally, I discuss how the effects of the crisis are heterogeneous for firms in the two sectors. The crisis causes a sharper contraction in credit and output for non-tradable firms, by inducing a fall in domestic demand.

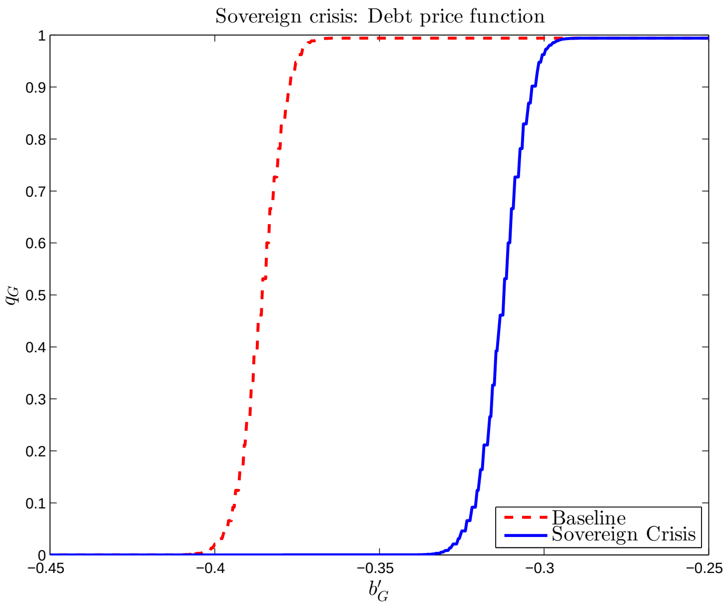

Figure 2.9: Sovereign debt crisis, shock

Shift in sovereign debt price function due to a shock to the welfare cost of default. The dashed, red line represents the sovereign debt price function under the baseline calibration of this model, for aggregate productivity equal to its ergodic mean. The solid, blue line represents the same sovereign debt price function after the sovereign crisis shock, as implied by the lower welfare cost of default.

Sovereign crisis shock I introduce a sovereign crisis as a shock giving rise to a contraction in lending to the government. In the model, the supply of funds to the government is expressed by the debt price function (2.24) which is determined, in turn, by the incentive for the government to default and by the optimality condition of lenders. A contraction in the supply of funds to the government follows in the model from a decrease in the welfare cost of sovereign default. A weaker incentive for the government to repay leads to a rise in the probability of default for any given level of debt. A higher probability of default reduces the price at which foreign lenders are willing to purchase sovereign debt, or to a contraction in the sovereign debt price function.

Analyzing the euro area sovereign debt crisis through the lens of this model, we can interpret events surrounding the Greek sovereign debt bailout in the summer of 2011 as an external shock causing a contraction in lending to other euro area economies with high debt, such as Italy.[29] In the initial phase of the crisis, EU officials had reassured financial markets stating that "In the Euro Area, the default does not exist."[30] Losses imposed on private sector lenders as part of the bailout package can be seen as disproving such statements, and showing that default in the Euro Area did, indeed, exist.

I model a sovereign debt crisis as an entirely unexpected shock amounting to a fall in the welfare cost of default. The steady state level of sovereign debt consistent with the new default cost is equal to 28% of yearly GDP, as opposed to 36% in the baseline calibration. The rise in borrowing costs associated with the sovereign debt crisis shock can be represented as a shift in the sovereign debt price function, as displayed in Figure 2.9 for aggregate productivity equal to its ergodic mean.

In the remainder of this section, I will analyze the implications of the sovereign crisis shock for fiscal policy and for variables of interest in the competitive equilibrium of the model. The effects of the crisis on equilibrium are different according to the state of the economy at the time of the shock, due to the presence of non-linearities in the policy functions that emerge from the solution of the model. I will consider here a crisis episode that hits the economy when aggregate productivity is at its ergodic mean and when sovereign debt equals 31% of yearly GDP. Finally, the dynamic implications of the crisis that emerge from a simulation of the model economy may differ depending on the particular path of exogenous shocks that is considered. This issue is due, again, to the non-linear nature of the solution of the model. To address this concern, I consider a large number of draws for the path of exogenous aggregate productivity shocks after the initial period. I compute the median path of each endogenous variable across the individual simulations that are consistent with each draw, and I show the difference between the median path after the sovereign crisis and the one in a baseline economy with no crisis. The baseline economy is not subject to the sovereign debt crisis shock, while it receives

[29] For a detailed analysis of the Greek bailout, see Ardagna and Caselli (2014).

[30] The quote was pronounced in January 2010 by Mr. Joaquín Almunia, then European Commissioner for Economic and Financial Affairs. See Reuters, http://uk.reuters.com/article/davos-almunia-idUSLAE00004520100129.

the same aggregate productivity shocks as the crisis economy and it starts from the same initial value of sovereign debt.

Fiscal policy in a sovereign crisis The optimal reaction of the fiscal planner to a sovereign crisis is to reduce the reliance of the domestic economy on international financial markets, increasing government surplus and taxes. The effects of the sovereign debt crisis on government surplus and on the labor income tax rate are displayed in Figure 2.10. I analyze here the impact of the crisis by studying the difference in the path of key fiscal policy variables between the crisis economy and the baseline no-crisis one. Government surplus rises on impact by 8% of quarterly GDP when the crisis hits. The higher cost of borrowing implied by the crisis makes it less desirable for the government to have a large and negative asset position. Hence, it is optimal for the planner to run a surplus, thus reducing the amount of external liabilities owed by the government. The increase in government surplus then gradually falls, as the stock of debt falls too.

The fiscal planner sets a higher tax rate on labor income to achieve the increase in government surplus. The median increase in the tax rate is very large, amounting to an increase of 23.5 basis points on impact. The increase in the tax rate is the largest in the first period of the crisis when the increase in government surplus is also the largest. The increase in the tax rate is proportionally larger than the increase in government surplus, however, due to the presence of a concave Laffer curve in tax revenue. After a sufficient number of periods following the crisis, the tax rate in the crisis economy is lower than in the baseline one. As the economy converges towards a new lower level of sovereign debt, a lower steady state tax rate is eventually required to cover for interest payments on debt and for government expenditure.

Finally, both the crisis economy and the baseline one converge towards a steady-state level of government debt. The two steady-state levels obviously differ, due to the difference in the incentive to default across the two economies. The median transitions of government assets in the crisis economy and in the baseline economy are displayed in Figure 2.11.

Government and firms' interest rates The cost of borrowing rises for both government and firms in the event of a sovereign debt crisis. The effects of the crisis on

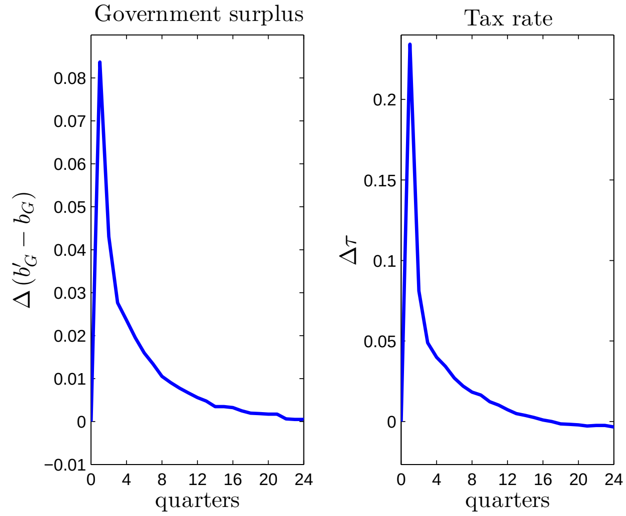

Figure 2.10: Sovereign debt crisis, fiscal policy

Government surplus and labor income tax rate after a crisis episode that hits the economy at $t = 1$. Series displayed are computed as the difference between the crisis and baseline economies for the median path of variables across simulations for exogenous aggregate productivity shocks.

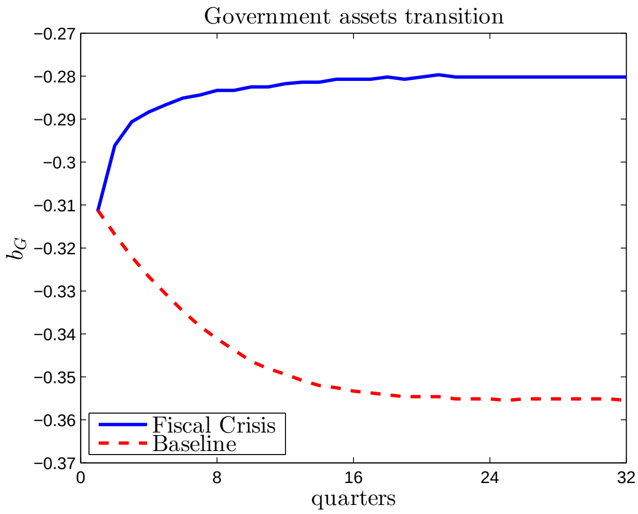

Figure 2.11: Sovereign debt crisis, transition

Transition of government-held assets in the crisis and in the baseline economy. Series displayed are the median across simulations for a large number of draws of exogenous aggregate productivity shocks.

interest rates paid by government and firms on their debt are presented in Figure 2.12.

Interest rates on government debt rise as a result of the interaction between fiscal policy choices and the crisis shock. On impact, the crisis causes a median increase in the interest rate on sovereign debt of 215 basis points.[31] A direct implication of the sovereign crisis is a fall in the price of sovereign debt for any given amount issued, i.e. a shift in the sovereign debt price function. The planner responds to this shock by letting the government run a larger surplus, hence reducing the amount of debt issued, as discussed. The reduction in the amount of debt issued is not large enough to completely undo the effects of the crisis, so that the interest rate on government debt rises in equilibrium. A larger increase in government surplus would be needed to completely offset the events of the shock on government debt prices. A large government surplus would be accompanied by a large rise in taxation, and by welfare losses that are associated with its distortionary effects. Such an increase would not be optimal from the point of view of the benevolent planner.

The sovereign crisis is transmitted to the private sector through the action of fiscal policy, inducing a rise in the interest rate paid by firms on their debt. The median increase in firms' interest rate due to the crisis equals 470 basis points.[32] The increase in taxes set by the planner in response to the crisis reduces the ability of firms to generate profits. Higher taxes result in a reduction in the probability of government default, by allowing for a higher government surplus. However, taxes cause resources to be transferred from the private sector to the government, increasing the probability of default by firms, as their profits fall. In turn, the rise in the probability of firms' default results in an increase in the interest rate they pay on their debt.

The transmission of the sovereign crisis to firms' borrowing costs is not due to changes in the net worth of lenders induced by the rise in sovereign default risk. In this model, firms are able to finance their working capital by borrowing from risk-neutral foreign residents. Hence, changes in the interest rate paid by firms are entirely due to changes in

[31] In the series for the government interest rate, the median is only computed in the subset of draws where the government has access to international financial markets, as the interest rate on government debt is not defined otherwise.

[32] The interest rate for firms is computed as the weighted average of the interest rate paid by firms in the two sectors, with weights given by the relative masses of firms.

their default probability, caused in turn by the evolution of fiscal policy.[33] If considering an economy characterized by the presence of a domestic banking sector holding claims on the domestic government, a sovereign crisis would potentially have important implication on the net worth of such domestic banks. In turn, the net worth of the domestic banking sector would affect the abundance of credit to firms, their borrowing costs, and, in general, their performance. This chapter abstracts from this particular transmission channel of a sovereign crisis to firms, which has previously been analyzed in the literature.[34] I focus instead on the effects of the crisis on domestic firms that arise due to the reaction of fiscal policy to such crisis. The two transmission channels are likely to be complementary to each other, for example if a fall in the net worth of lenders causes a rise in the required compensation for risk, at a time when the severity of firms' default risk rises due to contractionary fiscal policy.

Finally, the rise in interest rate on firms is more persistent than the one in government interest rates. The steepness of the sovereign debt price function makes it optimal for the planner to ensure that government debt quickly reverts to a level that is compatible with low borrowing costs, after the crisis. Even after government borrowing costs have fallen to a low level in the median simulation, however, the planner continues to let the government run a positive surplus, in order to further reduce the level of debt. As long as taxes remain higher than in the baseline simulation, firms' performance will be impaired, and their borrowing costs will remain high.

Heterogeneous firm-level implications of a debt crisis The sovereign crisis has heterogeneous implications for firms in the domestic economy, depending on the type of good they produce. I discuss here implications of the sovereign crisis shock on the relative price of non-tradable good, on the difference between the interest rates paid by firms in the two sectors, and on their relative demand for labor input. A graphical representation of these effects is presented in Figure 2.13.

First, the relative price of non-tradable goods falls in a crisis. Domestic absorption of goods contracts when the crisis hits due to the increase in government surplus. Demand

[33] I can ignore here shocks to aggregate productivity, as I analyze the median simulation from a large number of draws for the aggregate productivity process.

[34] Again, see Section 2.2 for a review of the literature in this field.

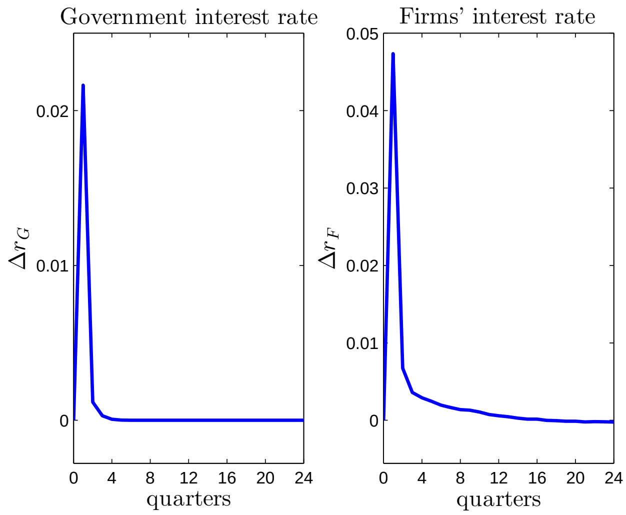

Figure 2.12: Sovereign debt crisis, interest rates

Government and firms' interest rates after a crisis episode that hits the economy at $t = 1$. Series displayed are computed as the difference between the median path of variables across simulations of the economy with and without crisis.

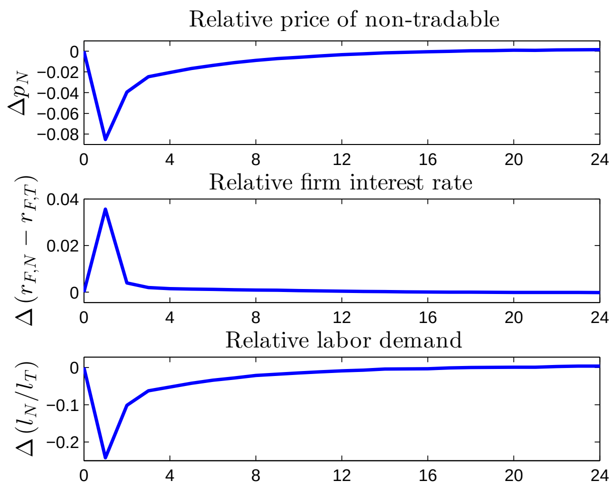

Figure 2.13: Sovereign debt crisis, heterogeneous firm-level effects.

Relative price of non-tradable good, difference in firms' interest rates across sectors and relative labor demand after a crisis episode that hits the economy at $t = 1$. Series displayed are computed as the difference between the median path of variables across the simulations of the economy with and without crisis. The relative price of non-tradable and the relative labor demand are presented in log-deviations from their ergodic mean.

for all goods falls, but as the non-tradable cannot be exported by domestic producers, its relative price must fall for this market to clear. The fall in this relative price also follows from the fact that the relative supply of non-tradable does not sufficiently contract during the crisis. In particular, the inability of firms to choose the type of good they produce causes goods' relative prices to fluctuate. In the presence of perfect mobility of firms across the two sectors, the relative price of non-tradable would be unitary independently of the state of the economy, by equalization of expected profits and wages across sectors. The relative price of non-tradable falls by 8.5% on impact due to the crisis in the median simulation.

Second, the interest rate on firms' debt rises by more for firms in the non-tradable sector than for those in the tradable sector. The median difference between the interest rates paid by firms in the two sectors is of 360 basis points when the crisis hits. The increase in nominal wage due to the rise in the labor income tax rate is the same across the two sectors. However, the fall in the relative price of non-tradable causes the value of output of firms producing this good to fall, at a time when their production costs are high. Their default probability thus rises by more than for tradable firms, as their average profits fall by more, too. The non-linear interaction between the tax rate increase and the fall in the non-tradable good price contributes to a sharp increase in the interest rate on debt for firms in this sector.

Finally, labor demand falls significantly for firms in the non-tradable sector as a consequence of the crisis. Relative median labor demand in the non-tradable sector falls by 24%. The larger fall in non-tradable sector labor demand is due to the combination of a fall in the relative price of output and a larger rise in production costs, due to the larger increase in interest rates. First, as the relative price of non-tradable falls, equalization of marginal products of labor across the two sectors implies that the amount of labor employed in the non-tradable sector is relatively lower. Second, the higher interest rate on debt paid by firms in the non-tradable sector implies that production costs are higher for these firms. The combination of higher production costs and lower marginal product of labor thus implies a strong contraction in labor demand by non-tradable firms. Finally, the reduction in labor demand implies a fall in the amount of non-tradable good that is produced domestically, as factors of production are partially reallocated towards the tradable sector. The fall in the amount produced of non-tradable good curbs the fall in its price.

2.7 Concluding remarks

This chapter shows that fiscal policy can act as a powerful transmission channel of a sovereign debt crisis to firms. I develop a framework where the actions of a government to avert sovereign default cause the profits of firms to deteriorate and default risk to be passed on to the private sector. In the model, no direct linkage is assumed between borrowing costs of the government and firms. However, a fundamental linkage is present between the two agents. Both the government and firms jointly rely on resources available in the domestic economy to satisfy their objectives. Domestic households represent for firms the key source of productive inputs. At the same time, the government relies on households' labor income as its tax base. Ultimately, the joint reliance of government and firms on the same pool of resources causes financial stress to be transmitted between the two.

Two main avenues for future research follow naturally from this chapter. First, the model can be extended to allow for an endogenous choice on the amount of government expenditure. Cuts in public expenditure were an important feature of fiscal austerity measures in crisis-stricken countries of the euro area. The analysis conducted here would be enriched by allowing for direct effects of government actions on domestic demand, especially when considering their heterogeneous implications for different types of firms. From a theoretical perspective, consideration of realistic forms of nominal rigidities would be of interest in this setting, in order to analyze aggregate demand externality effects of government expenditure.

Second, it would be promising to consider in the model the role of domestic financial intermediaries. This framework would allow for an analysis of the degree of complementarity between the crisis transmission channel through the balance sheet of domestic banks that has been studied in the literature and the one through fiscal policy here considered. In light of the recent crisis, several policy proposals have been made to address jointly the issue of high sovereign debt in euro area countries and the governance of the banking system.[35] The framework introduced here, if suitably extended to allow for domestic banks,

[35] Notable examples are Corsetti et al. (2015) and Brunnermeier et al. (2016).

would allow for example for an analysis of the costs and benefits of government-funded bank recapitalization. While such recapitalization could strengthen the domestic banking sector, its cost could weaken the fiscal position of the government. In turn, a higher level of government debt could be detrimental for the severity of financial frictions faced by firms.

Appendix

2.A Market clearing conditions

In each sector X, where X corresponds to either T or N, the average amount of output produced by a firm is given by

$$\begin{aligned} y_X = & \int_{\hat{s}_X}^{\infty} \left[zsl_X^{1-\alpha} - \varphi \right] f_s\left(s\right) ds + \\ & \int_{\hat{\hat{s}}_X}^{\hat{s}_X} \left[zsl_X^{1-\alpha} - \varphi - \left(1-\xi\right)\left(zsl_X^{1-\alpha} - \varphi - \left(1-\zeta\right) wl_X \right) \right] f_s\left(s\right) ds + \\ & \int_{\frac{\varphi}{zl_X^{1-\alpha}}}^{\hat{\hat{s}}_X} \left[zsl_X^{1-\alpha} - \varphi - \left(1-\xi\right)\left(zsl_X^{1-\alpha} - \varphi \right) \right] f_s\left(s\right) ds = \\ = & \int_{\hat{s}_X}^{\infty} \left[zsl_X^{1-\alpha} - \varphi \right] f_s\left(s\right) ds + \\ & \xi \int_{\frac{\varphi}{zl_X^{1-\alpha}}}^{\hat{s}_X} \left[zsl_X^{1-\alpha} - \varphi \right] f_s\left(s\right) ds + \left(1-\zeta\right) wl_X \Pr\left[s \in \left(\hat{\hat{s}}_X, \hat{s}_X \right) \right] \end{aligned} \tag{2.31}$$

where l_X is the amount of hours employed by a firm in sector X. Aggregate output equals aggregate output of non-defaulting firms, plus output net of liquidation costs of of defaulting firms.

Firms whose productivity falls below the threshold $\hat{\hat{s}}_X$, as defined in (2.16), are unable to fully compensate their workers as promised. The aggregate amount of resources promised to workers by these firms, in each sector, is given by

$$wl_X\left(1-\zeta\right)\Pr\left[s < \hat{\hat{s}}_X\right].$$

The amount of goods that these firms are able to pay workers, after liquidation, is equal to

$$\xi p_X \int_{\frac{\varphi}{zl_X^{1-\alpha}}}^{\hat{\hat{s}}_X} \left[zsl_X^{1-\alpha} - \varphi \right] f_s\left(s\right) ds.$$

The loss suffered by workers in defaulting firms in each sector is denoted by h_X and it is given by the difference between the two above defined terms:

$$\begin{aligned} h_X = & wl_X\left(1-\zeta\right)\Pr\left[s < \hat{\hat{s}}_X\right] + \xi p_X \varphi \Pr\left[s \in \left(\frac{\varphi}{zl_X^{1-\alpha}}, \hat{\hat{s}}_X \right) \right] \\ & - \xi z l_X^{1-\alpha} \Pr\left[s \in \left(\frac{\varphi}{zl_X^{1-\alpha}}, \hat{\hat{s}}_X \right) \right] \mathbb{E}\left[s \middle| s \in \left(\frac{\varphi}{zl_X^{1-\alpha}}, \hat{\hat{s}}_X \right) \right]. \end{aligned} \tag{2.32}$$

Finally, the total loss suffered by workers in all sectors is given by

$$h = m_T h_T + \left(1-m_T\right) h_N. \tag{2.33}$$

The aggregate amount of dividends paid by firms in sector X and is given by

$$\begin{aligned} d_X = & \int_{\hat{s}_X}^{\infty} \left[p_X \left(z s l_X^{1-\alpha} - \varphi \right) - (1-\zeta) w l_X - b_X \right] f_s(s)\, ds \\ = & \Pr(s \geq \hat{s}_X)\, p_X z l_X^{1-\alpha} \left[\mathbb{E}(s | s \geq \hat{s}_X) - \hat{s}_X \right] \end{aligned} \tag{2.34}$$

It can be shown that, in each sector, aggregate output equals the sum of wage and dividend payments made by firms, net of losses suffered by workers

$$y_X = w l_X + d_X - h_X. \tag{2.35}$$

This follows from:

$$d_X = \Pr(s \geq \hat{s}_X)\, p_X \left[z l_X^{1-\alpha} \mathbb{E}(s | s \geq \hat{s}_X) - \varphi \right] - \Pr(s \geq \hat{s}_X)\, w l_X \left(1 - \zeta + \frac{\zeta}{q_{F,X}} \right)$$

so that

$$\begin{aligned} d_X + w l_X - h_X = & \Pr(s \geq \hat{s}_X)\, p_X \left[z l_X^{1-\alpha} \mathbb{E}(s | s \geq \hat{s}_X) - \varphi \right] \\ & + w l_X \left[1 - (1-\zeta) \Pr(s \geq \hat{s}_X) - \zeta \frac{\Pr(s \geq \hat{s}_X)}{q_{F,X}} \right] - h_X \\ = & \Pr(s \geq \hat{s}_X)\, p_X \left[z l_X^{1-\alpha} \mathbb{E}(s | s \geq \hat{s}_X) - \varphi \right] \\ & + w l_X \left[(1-\zeta) \Pr(s \leq \hat{s}_X) \right] \\ & + w l_X \zeta \frac{q_{F,X} - \Pr(s \geq \hat{s}_X)}{q_{F,X}} - h_X. \end{aligned}$$

Noting that $b_X = \frac{\zeta w l_X}{q_{F,X}}$,

$$w l_X \zeta \frac{q_{F,X} - \Pr(s \geq \hat{s}_X)}{q_{F,X}} = \xi \int_{\hat{\hat{s}}_X}^{\hat{s}_X} \left[p_X \left(z s l_X^{1-\alpha} - \varphi \right) - (1-\zeta) w l_X \right]$$

and imposing the definition of h_X, we obtain the desired result.

Chapter 3

Time-Varying Volatility in Sovereign Interest Rates: The Role of Default Risk

3.1 Introduction

Interest rates on debt issued by emerging market economies display substantial fluctuations in volatility.[1] What is the driver of the observed fluctuations in the volatility of interest rates? Fluctuations in sovereign default risk can explain why emerging market interest rates display high volatility and a negative correlation with output.[2] Can fluctuations in endogenous default risk also explain the time-varying nature of volatility in interest rates on external debt of emerging market economies? What is the role played by shocks to exogenous financial variables for a country's incentive to default? Are fluctuations in the volatility of the world risk-free interest rate an important driver of uncertainty in emerging market economies?

This chapter introduces two main results. First, fluctuations in interest rate uncer-

[1] This observation is documented by Fernandez-Villaverde et al. (2011) for Argentina, Brazil, Ecuador and Venezuela.

[2] Arellano (2008) develops a quantitative, calibrated model where limited commitment to repay sovereign debt leads to countercyclicality in both the current account and borrowing costs for the government of an emerging market small open economy.

tainty emerge endogenously from a standard model of sovereign default a la Arellano (2008). This result occurs in the absence of shocks to the second moment of exogenous variables. Endogenous fluctuations in uncertainty emerge as the outcome of the same force driving fluctuations in the level of the interest rate, sovereign default risk. The intrinsic non-linearity of the sovereign default model considered implies that shocks leading to a high interest rate also cause such interest rate to be highly volatile. Second, fluctuations in the world risk-free real interest rate and in its volatility do not play a quantitatively important role in this framework. In particular, I show that for a standard calibration of the model, the role of output shocks is significantly larger than that of shocks to the world risk-free interest rate.

From a methodological point of view, this chapter builds on the framework developed by Arellano (2008). First, I define a model-based measure of interest rate uncertainty. Interest rate uncertainty is measured as the conditional variance of the error made by agents in the model when forecasting the next-period value of the interest rate on sovereign debt. I analyze the properties of this measure of uncertainty to study the ability of this model to replicate the observed pattern of interest rate volatility. Second, I extend the baseline model to consider shocks to the world risk-free interest rate. I simultaneously consider shocks to the level and to the volatility of this variable, as well as shocks to the level of output in the economy. Importantly, as the model is solved by means of a global solution method, no substantial modification of the solution algorithm is required when introducing shocks to the second moment of the risk-free interest rate.[3]

Quantitative properties of interest rate uncertainty in the model replicate the empirical features of time-varying volatility in emerging market economies' interest rates. In particular, the model-generated fluctuations in interest rate uncertainty are countercyclical and positively correlated with the level of the interest rate on external debt. A fundamental non-linearity in default risk lies behind this result. Default risk is bounded below by zero. In a boom, the cost of borrowing is close to the risk-free rate and it does not fluctuate substantially. In a recession, the economy is charged a premium for default

[3]The solution method adopted here is value function iteration with grid search over a finely discretized state space. Benigno, Benigno and Nisticò (2013) and Fernandez-Villaverde and Rubio-Ramirez (2013) discuss techniques to allow for second-moment shocks in larger models, for solution methods based on local approximation.

risk. Shocks cause the severity of default risk to fluctuate, leading to fluctuations in the cost of borrowing. The presence of fluctuations in the volatility of interest rates follows from this key mechanism, as does the fact that volatility correlates positively with the level of the interest rate and negatively with output.

Shocks to the level of the risk-free interest rate lead to fluctuations in the relative benefit from defaulting on external debt, as opposed to repaying. Intuitively, changes in interest rates have implications for the cost of refinancing debt, while leaving largely unaffected welfare under default and the subsequent exclusion from financial markets. Hence, a shock that causes the risk-free rate to rise strengthens the incentive to default. In turn, risk-free rate shocks affect interest rate spreads, the cost of borrowing and the choice over the amount of external debt to issue. The quantitative relevance of this qualitative mechanism is however minor in this framework. The level of debt owed by the small open economy to foreign lenders is low, on average, in this model. Hence, large shocks to the risk-free interest rate have minor implications for the amount of resources available for consumption. Shocks to the level of output have larger effects, both directly and through their effects on the severity of default costs. A fortiori, the implications of exogenous changes in the volatility of the risk-free interest rate process are negligible. In the absence of large and non-linear effects of shocks to the level of the risk-free interest rate, shocks to the volatility of this process do not significantly alter the equilibrium values of endogenous variables.

This chapter is related to three strands of the literature on business cycles in emerging market economies. First, I contribute to the literature investigating the role of interest rate uncertainty on emerging market economies' business cycles. Fernandez-Villaverde et al. (2011) document the presence of time-varying volatility in interest rates on sovereign debt of emerging market economies. In light of this observation, they calibrate a real business cycle model for a small open economy where the interest rate on external debt is driven by an exogenous process displaying time-varying volatility. I show that fluctuations in interest rate uncertainty can be explained without recourse to shocks to the second moment of exogenous variables. In the quantitative framework of Arellano (2008), I show that the non-linearities that characterize this workhorse model of sovereign default give rise to time-varying volatility in interest rates, when only a simple process for shocks to the level of output is considered.[4]

Second, this chapter is related to the literature on the effects of shocks to the world risk-free interest rate on emerging market economies' business cycles. Neumeyer and Perri (2005) find that while fluctuations in country spreads can explain a sizable fraction of volatility in emerging market business cycles, the contribution of risk-free interest rate shocks is more modest. Uribe and Yue (2006) argue that volatility in country spreads is itself explained to an important degree by shocks to the world risk-free interest rate. They find that US real interest rate shocks can significantly contribute to the volatility of business cycles in emerging markets, through their effect on these economies' interest rate spread. I contribute to this debate by analyzing the role of risk-free interest rate shocks in a model where the interest rate spread is determined endogenously. A similar analysis is conducted by Guimaraes (2011). He finds that risk-free rate shocks explain an important fraction of the volatility of external debt in small open emerging market economies, in a model of sovereign debt with costless renegotiation. I find that the quantitative effect of risk-free rate shocks is minor in the Arellano (2008) framework of endogenous sovereign default. Given the calibration of default costs in this model, output shocks have significantly larger effects than risk-free rate ones. Relatedly, Foley-Fisher and Guimaraes (2013) find that the overall correlation between emerging economies' default risk and the US real interest rate is negative.

Finally, I contribute to the literature on exogenous uncertainty shocks, by analyzing the role of shocks to the volatility of the risk-free real interest rate in an endogenous sovereign default model. Bloom (2009) is a seminal contribution on the aggregate implications of uncertainty shocks. I show that while a model of endogenous sovereign default can deliver endogenous fluctuations in interest rate uncertainty, the role of shocks to the risk-free rate volatility is extremely small. This result follows from the small role that shocks to the level of the risk-free rate have in this model. The role of exogenous uncertainty shocks has been analyzed in this strand of the literature by Seoane (2015), who

[4]This flexible framework is based on the seminal work by Eaton and Gersovitz (1981). It has been widely adopted in the literature on international capital flows and business cycles in emerging market economies. A non-exhaustive list of contributions in this strand of the literature includes Aguiar and Gopinath (2006), Mendoza and Yue (2012), Durdu, Nunes and Sapriza (2013), Hatchondo, Martinez and Sosa-Padilla (2016).

considers fluctuations in the volatility of the output process. In advanced economies, Benigno, Benigno and Nisticó (2012) show that fluctuations in monetary policy uncertainty have significant implications for fluctuations of the exchange rate.

The rest of this chapter is organized as follows. Section 3.2 presents the model. Section 3.3 analyzes the properties of interest rate uncertainty in the model economy. In section 3.4 I analyze the role of shocks to the world risk-free interest rate. Section 3.5 concludes.

3.2 Model

The model closely follows Arellano (2008). I extend the framework therein by introducing a stochastic process driving the world risk-free real interest rate and its volatility. I present here the model equations, highlighting the key differences from the original setup.

I consider an infinite-horizon small open economy trading in goods and assets with the rest of the world. Time is discrete and it is indexed by t. The small open economy is inhabited by a representative household and by a government. The household derives utility from consumption of a homogeneous, tradable and non-storable good. Welfare of the representative household is given by

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\frac{1}{1-\gamma}c_t^{1-\gamma}\right] \tag{3.1}$$

where c_t denotes consumption of the homogeneous good. As in previous chapters, $\mathbb{E}_0$ is the expectation operator conditional on information available at time 0. The subjective discount factor is denoted by $\beta < 1$ and the coefficient of relative risk aversion by $\gamma > 0$.

The government is a benevolent agent. Its objective is to maximize welfare of the representative household. The government can trade bonds in international financial markets, and it can transfer resources to the representative household. The government can also tax the household in order to repay liabilities issued to foreign residents. The government has access to lump-sum taxes and transfers on the representative household. Markets are incomplete and the government can only trade one-period, non-contingent bonds. The government can also choose whether to repay or to default on external debt, so that bonds are defaultable. By defaulting on external debt, the government loses access to international financial markets.

We can derive the aggregate resource constraint of the small open economy by consolidating the budget constraints of the government and of the representative household, due to the presence of lump-sum taxes and transfers. If the government has access to international financial markets, the resource constraint is given by

$$c = y + b - q\left(x, b'\right) b'. \tag{3.2}$$

The amount of foreign assets held by the government is given by b. The amount of bonds maturing in the next period that are purchased by the government in the current period is given by b', adopting recursive notation. c denotes current-period consumption. The household receives a stream of endowments of homogeneous good according to a stochastic process. y denotes the current-period realization of this process. The function $q\left(x, b'\right)$ describes the unit price at which the government trades bonds with the rest of the world. The price q is a function of the amount of bonds traded b' and of the current-period realization of the stochastic process x. x subsumes all the stochastic shocks that impinge on the small open economy. In particular, x includes the endowment process y, as well as a process for the risk-free real interest rate and its volatility. The presence of a stochastic process driving risk-free rates represents a departure from the model in Arellano (2008).

If the government does not have access to international financial markets, the small open economy is forced into financial autarky. Consumption of the representative household is given in autarky by

$$c = y - \delta\left(y\right) \tag{3.3}$$

where $\delta\left(y\right)$ represents an exogenous cost of default. In this model, the default cost is given by a reduction in the amount of endowment available for consumption by the household. The severity of the default cost is a function of the realization of the endowment process.

The choice problem of the government is the same as in Arellano (2008) and it is very similar to the one introduced in the previous chapter. When the government has access to international financial markets, it can choose to repay or to default on external debt. If choosing to default, the government loses access to markets. If choosing to repay, the government decides on the amount of assets to issue or purchase in the current period. The amount of assets traded with international financial markets determines the amount of taxes or transfers that are set on the representative household, given the financial position of the government as it enters the period, according to (3.2). The state variables for the government choice problem are given by the amount of foreign assets it holds b and by the realization of the exogenous process x, which determines the realization of the process for the endowment, for the risk-free rate and for the risk-free rate volatility.

When the government enjoys access to international financial markets, the value function for the representative household is given by $V(x,b)$. The default choice by the government is taken comparing the value function implied by default, $V_D(x)$, with the one implied by the choice to repay debt and to maintain access to international financial markets, $V_R(x,b)$. The discrete choice default problem is given by

$$V(x,b) = \max_{D\in\{0,1\}} DV_D(x) + (1-D)\,V_R(x,b)\,, \tag{3.4}$$

where D denotes an indicator for the choice of default. This discrete choice problem defines the policy function for default:

$$D(x,b) = \mathbb{1}\left(V_R(x,b) > V_D(x)\right). \tag{3.5}$$

The value function of the representative household under default and autarky is given by

$$V_D(x) = \left\{u\left(y-\delta(y)\right) + \beta\mathbb{E}\left[\lambda V\left(x',0\right) + (1-\lambda)\,V_D\left(x'\right)\middle|\, x\right]\right\}. \tag{3.6}$$

If the government chooses to default, no other meaningful choice is made by domestic agents, as the small open economy is in financial autarky. Consumption is given by the domestic endowment net of the default cost, according to (3.3). Default implies immediate exclusion from international financial markets, but the economy can be readmitted to markets in the following period with probability λ. If readmitted, the government regains access to markets with neither debt nor assets, so that the value function upon readmission is given by $V(x,0)$. With complementary probability $1-\lambda$ the government remains in autarky in the following period. The expectation operator on next-period variables, conditional on information summarized by the current-period realization of exogenous variables, is denoted by $\mathbb{E}[\cdot|\, x]$.

When it has access to international financial markets, the government decides over the amount of assets to trade with foreign residents. When issuing debt, the government takes the function $q(x,b')$ into account, understanding how the amount of debt it issues affects the price at which this is traded. The value function of the representative household is given by

$$V_R(x,b) = \max_{b'} \left\{ u\left(y + b - q\left(x,b'\right)b'\right) + \beta \mathbb{E}\left[V\left(x',b'\right) \middle| x \right] \right\}. \tag{3.7}$$

I impose the resource constraint (3.2) to determine household consumption. The continuation value $V(x',b')$ takes into account the option for the government to default in the next period. The solution to this maximization problem defines a policy function for the amount of assets traded as a function of the state variables, $b'_{pf}(x,b)$. In turn, the policy function for assets traded defines a policy function for the unit price of such assets:

$$q_{pf}(x,b) = q\left(x, b'_{pf}(x,b)\right). \tag{3.8}$$

The world economy is inhabited by a large number of foreign lenders, residing outside the small open economy. Foreign lenders are risk neutral, and they have access to a risk-free asset with unit price q_{RF}. When purchasing one unit of the risk-free asset, foreign lenders obtain with certainty one unit of consumption good in the following period. On the other hand, assets issued by the small open economy only pay off when the government does not default. For bonds issued by the government of the small open economy to be traded in equilibrium, their return has to be such that foreign lenders are indifferent between holding government bonds or risk-free assets. Hence, the price of bonds issued by the small open economy has to satisfy

$$q\left(x,b'\right) = q_{RF} \Pr\left[D\left(x',b'\right)|x\right]. \tag{3.9}$$

The interest rate on a risk-free asset is defined as

$$\tilde{r}_{RF} = \frac{1}{q_{RF}} - 1. \tag{3.10}$$

The model will be calibrated at quarterly frequency. We can then define the annualized risk-free interest rate as:[5]

$$r_{RF} = \left(\frac{1}{q_{RF}}\right)^4 - 1. \tag{3.11}$$

The risk-free real interest rate is determined according to an autoregressive stochastic process of order one

$$r_{RF} = (1-\rho_r)\mu_r + \rho_r r_{RF,-1} + \exp(\sigma_r)\epsilon_r \tag{3.12}$$

[5]Equivalently, we can define the risk-free asset price from the interest rate as $q_{RF} = (1+r_{RF})^{-\frac{1}{4}}$.

where μ_r denotes the unconditional mean of the process, ρ_r the autocorrelation coefficient, $r_{RF,-1}$ the previous period realization of the process and σ_r the logarithm of the standard deviation. ϵ_r is a normally distributed i.i.d. process with zero mean and unitary standard deviation. In turn, σ_r is also determined according to an autoregressive process, as in Fernandez-Villaverde et al. (2011):

$$\sigma_r = (1 - \rho_\sigma)\,\mu_\sigma + \rho_\sigma \sigma_{r,-1} + \eta_\sigma \epsilon_\sigma. \tag{3.13}$$

The parameters of the process for the logarithm of the standard deviation are defined as those of the process for the risk-free rate. μ_σ denotes the unconditional mean of the logarithm of the standard deviation, ρ_σ the autocorrelation coefficient, $\sigma_{r,-1}$ the previous period realization, and η_σ is the standard deviation of the process. Again, ϵ_σ is a normally distributed i.i.d. process with zero mean and unitary standard deviation. Finally, the endowment y is determined according to an autoregressive process in the logarithm

$$\log(y) = (1 - \rho_y)\,\mu_y + \rho_y \log(y_{-1}) + \sigma_y \epsilon_y \tag{3.14}$$

with the ususal definition of parameters. All stochastic processes impinging on the model economy are summarized by the stochastic vector x:

$$x \equiv \begin{bmatrix} r_{RF} \\ \sigma_r \\ y \end{bmatrix} \tag{3.15}$$

whose distribution follows from that of its individual components.

Equilibrium In the equilibrium of this model, the government sets the policy for default or repayment and for issuance or purchase of asset, in order to maximize welfare of the representative household, subject to the resource constraint of the small open economy and to the constraint implied by foreign lenders' pricing of debt. The equilibrium is formally defined below.

Definition 5. *A recursive equilibrium in the small open economy is characterized by*

- *A set of value functions for the representative household V, V_R, and V_D,*
- *Government policy for default D and asset holdings b'*

- *Government debt price function q*

such that:

- *The debt price function is consistent with optimization by foreign lenders,* (3.9)*,*

- *Given the debt price function q, the value functions of the household and the policy functions of the government solve the maximization problem* (3.4)*,* (3.6)*,* (3.7)*.*

- *The resource constraint of the small open economy,* (3.2) *or* (3.3)*, is satisfied*

Calibration The model is calibrated at quarterly frequency. Parameter values are standard and they are largely drawn from the literature on endogenous sovereign default. The main exception is given by parameters governing the exogenous process driving the risk-free real interest rate and its volatility. These variables are typically assumed to be constant in this literature. Parameter values are summarized in Table 3.1.

The parameter governing relative risk aversion, γ, is set to 2, as standard in the international real business cycles literature, and as in Arellano (2008). The subjective discount factor β is set to 0.97 as in Cuadra et al. (2010), who target through this parameter the volatility of consumption of Mexico. The probability of re-admission to financial markets, λ, is set to 0.1, again following Cuadra et al. (2010). This value implies an average period of exclusion from international financial markets of 10 quarters. Finally, default costs are asymmetric as in Arellano (2008). In models of endogenous sovereign default, asymmetric default costs deliver a relatively high level of sovereign debt to be sustainable in equilibrium. The small open economy does not suffer from an output cost of default if the realization of its endowment falls below a given threshold δ. For realizations of the endowment above the threshold, the amount of output available for consumption simply equals the threshold itself. This formulation implies that default costs are increasing in the endowment realization in a piecewise linear fashion. I set the threshold equal to 97% of the unconditional mean of the endowment process, as in Arellano (2008).

I feed the model with the same endowment process as Arellano (2008), who uses estimates from a process for Argentina's GDP. The autoregressive parameter ρ_y is set to 0.945 and the standard deviation σ_y is 0.0245. I normalize the unconditional mean of the endowment to unity by setting $\mu_y = .5\sigma_y^2/\left(1-\rho_y^2\right)$.

The process for the risk-free interest rate is calibrated by using parameter estimates from a quarterly process for the real interest rate on risk-free dollar-denominated debt. I construct a series for the risk-free rate as the quarterly average of the interest rate on the 3-month Treasury bill, net of expected inflation. I measure expected inflation as the growth of the United States GDP deflator over the past four quarters.[6] I estimate an AR(1) process over this measure of the risk-free real interest rate, including a linear trend in order to account for the observed secular decline in real interest rates.[7] I use the estimated autoregressive parameter and standard deviation to calibrate ρ_r and the mean standard deviation μ_σ. I calibrate μ_r as the unconditional mean of the real risk-free rate in the sample.

Fernandez-Villaverde et al. (2011) estimate a process for the time-varying volatility of risk-free real interest rates. I calibrate the autoregressive parameter ρ_σ using their estimate. I set the standard deviation η_σ to a higher value than the one estimated in that paper, 0.6 rather than 0.13. The effects of uncertainty shocks remain negligible in this model even under this exaggerated value of this process's standard deviation, as discussed in Section 3.4.

I study three different versions of the model. First, I ignore shocks to the risk-free rate, and I set this variable equal to its unconditional mean. The volatility of the risk-free rate obviously plays no role, either. For this version of the model, I discretize the process for the endowment on a vector of 551 points. The large number of points ensures that the discretized distribution approximates relatively well a continuous one. In particular, the function for q does not feature large discrete jumps as b' varies. Second, I consider shocks to the risk-free rate, as well as endowment shocks. In this version of the model, the standard deviation of the risk-free rate process is constant and equal to its unconditional mean. Finally, I consider all three shock processes simultaneously. The curse of dimensionality poses a limit on the fineness of the vectorized shock processes when multiple shocks are considered. In the versions of the model with two or three

[6] The same backward looking measure is adopted by Uribe and Yue (2006). The sample considered goes from 1987 to 2015, corresponding to the period from the beginning of the Greenspan era to the last available full year. All data are drawn from the Fred database of the Federal Reserve Bank of St. Louis.

[7] See King and Low (2014), Thwaites (2015).

Table 3.1: Parameter values in the sovereign default model

Parameter		Value
Discount factor	β	0.97
Relative risk aversion	γ	2
Output cost of sovereign default	δ	$0.97\ \mathbb{E}(y)$
Readmission probability	λ	0.1
Endowment, autoregressive	ρ_y	0.945
Endowment, standard deviation	σ_y	0.025
Endowment, mean	μ_y	$.5\ \sigma_y^2/\left(1-\rho_y^2\right)$
Risk-free rate, autoregressive	ρ_r	0.95
Risk-free rate, mean standard deviation	μ_σ	-5.5
Risk-free rate, mean	μ_r	0.01
Volatility, autoregressive	ρ_σ	0.94
Volatility, standard deviation	η_σ	0.6

shocks, I discretize the process for y on a vector of 111 points. I discretize the process for r on a vector of 15 points in the version of the model with constant volatility. When considering volatility shocks, too, I discretize the vector for σ_r on a vector of 3 points and, for each value of σ_r, I discretize the process for r on a vector of 5 points. All stochastic processes are discretized according to the procedure described in Tauchen (1986).

3.3 Endogenous fluctuations in interest rate uncertainty

The interest rate on external debt issued by the small open economy of this workhorse model of endogenous sovereign default displays time-varying volatility. Time-varying volatility is a well-documented empirical feature of emerging market economies.

This result emerges already when abstracting from exogenous volatility shocks. In order to inspect the mechanism through which time varying volatility arises endogenously, I focus in this section on the version of model with endowment shocks, only. First, I discuss some features of the model that are crucial for this result. In particular, I show that the policy function for the interest rate on external debt is non-linear. Second, I introduce a model-based measure of interest rate uncertainty. This measure is defined as the conditional variance of the one-period forecast error made by agents in the model when forming expectations over interest rates on external debt. Third, I show that this model generates substantial fluctuations in uncertainty. To this purpose, I analyze the dynamics of the above-defined measure of uncertainty, in a simulation of the model solution. Fourth, I show that interest rate uncertainty is positively correlated with the level of the interest rate in the model economy. Variation in interest rates explains a substantial fraction of the variation in uncertainty. This result is consistent with the observation that, in the model, the degree of non-linearity in the interest rate policy function increases along with the interest rate itself. Finally, I document how interest rate uncertainty is countercyclical. Interest rate uncertainty is negatively correlated with the endowment realization and positively correlated with the amount of debt of the economy.

Non-linearities in the interest rate policy function The interest rate on sovereign debt paid by the small open economy of this model is a non-linear function of the model state variables. The policy function describing how the interest rate depends on the level of assets held by the government and on the current-period realization of the endowment is defined as

$$r^y_{pf}\left(x, b\right) = \left(\frac{1}{q_{pf}\left(x, b\right)}\right)^4 - 1 \tag{3.16}$$

where r^y denotes the yearly interest rate and the policy function for the quarterly debt price is defined in (3.8). The policy function in (3.16) is presented graphically in Figure 3.1.

The policy function for the interest rate increases in the level of sovereign debt of the economy and it decreases in the level of the endowment. In addition, the first derivative of this function is also increasing in debt and decreasing in the endowment. The interest rate rises by 80 basis points when the endowment realization falls by 1% from its unconditional mean, when debt owed by the small open economy amounts to its mean level in a simulation of the model economy. The interest rate rises instead by 190 basis points for

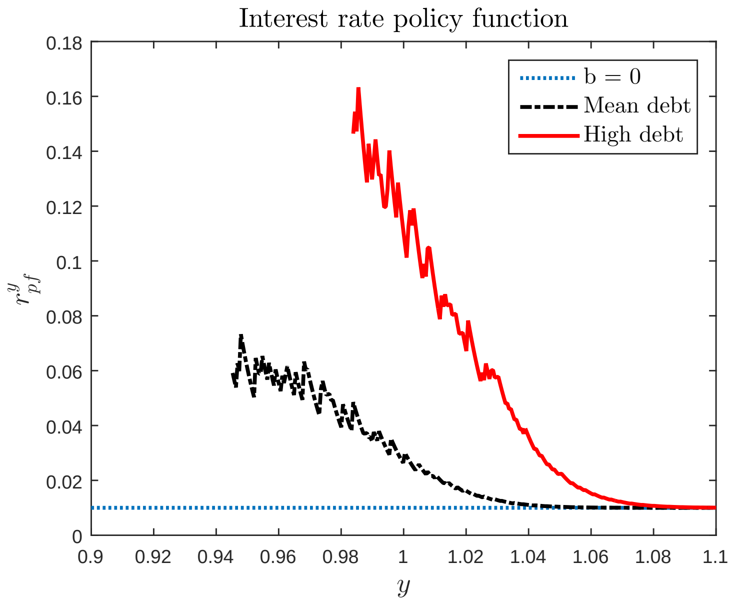

Figure 3.1: Policy function for the yearly interest rate on sovereign debt.

The policy function is presented as a function of the endowment realization y for three different levels of assets b. The three curves represent the policy function for a zero level of debt (dotted, blue line), for debt given by the ergodic mean of this variable in the simulation of the model (black, dashed line) and for debt given by the ninetieth percentile of this variable in the simulation (red, solid line). The model version considered here is the one with endowment shocks, only.

the same fall in the endowment when the economy owes a higher level of debt to foreign lenders, corresponding to the ninetieth percentile of the ergodic distribution of debt.[8] When the economy enters the period with no debt, it is optimal for the government to borrow at the risk-free rate independently of the endowment realization. In this region of the state space, the elasticity of the interest rate paid on external debt with respect to the endowment is thus equal to zero.[9]

[8]The mean level of debt corresponds to 11% of mean quarterly output. The high level of debt is equal to 22% of mean quarterly output.

[9]The effect on the interest rate of the same percentage fall in the endowment is lower when measured around a higher realization of the endowment. The elasticity of r_{pf}^{y} with respect to y is zero when the endowment is two standard deviations σ_y above its unconditional mean and debt is at its mean level. The interest rate rises instead by 165 basis points when the endowment falls by 1% from two standard deviations above the unconditional mean and debt is at the high level given by the ninetieth percentile of the ergodic distribution.

The non-linearity in the policy function for interest rates arises due to two facts, the presence of a lower bound on the interest rate that the small open economy pays on its debt, and the joint action of two forces leading the interest rate to rise when the endowment falls. First, the interest rate at which the small open economy trades assets with the rest of the world is bounded below by the risk-free interest rate. The presence of a perfectly elastic supply of risk-free assets ensures that when debt decreases the interest rate paid by the small open economy stops falling as debt once the probability of default reaches zero.

Second, two forces imply that the interest rate on sovereign debt rises as the level of the endowment falls. On the one hand, when the endowment falls the small open economy has a stronger desire to borrow, as the domestic supply of goods becomes scarcer. An increase in borrowing implies a higher probability of default and an increase in the interest rate paid on debt, given the sovereign debt price function (3.9). On the other hand, a fall in the endowment causes an increase in the cost of borrowing for any amount of debt issued. This is due to the fact that the sovereign debt price function is decreasing in y, as a low endowment implies a higher default probability.[10] The two effects jointly cause the interest rate at which the small open economy borrows to rise when the current endowment realization falls.

The presence of non-linearities in the policy function for the interest rate implies that shocks to the endowment cause fluctuations in the volatility of interest rates. As the degree of non-linearity of the function is increasing in the interest rate, the volatility of this variable is positively correlated with its level. To see this, first consider an economy receiving a sequence of positive endowment shocks. This economy pays a low and constant interest interest rate on its debt, moving towards the right-hand side of the graph in Figure 3.1. Second, consider an economy receiving instead a sequence of negative endowment shocks. This economy moves towards the left-hand side of the graph in Figure 3.1. It pays a high interest rate on debt and further shocks will cause such interest rate to fluctuate. Adverse endowment shocks thus imply a high and volatile interest rate. A discussion of the quantitative relevance of these effects is presented in the remainder of this section.

[10] Arellano (2008) analyzes these effects in detail. Chapter 2 of this dissertation also discusses in more detail the effects of exogenous productivity shocks on the tightness of the sovereign debt price function faced by a small open economy.

Model-based measure of interest rate uncertainty The model provides a measure of uncertainty over the interest rate on external debt as faced by agents in the model economy. This measure of uncertainty is given by the conditional variance of the error made by agents when forming expectations over the interest rate. Agents' conditional expectation over the next-period interest rate is defined as

$$\bar{r}^{y}_{pf}(y,b) = \mathbb{E}\left[r^{y}_{pf}\left(y', b'_{pf}(y,b)\right)\left|y, D\left(y', b'_{pf}(y,b)\right) = 0\right.\right] \tag{3.17}$$

where the expectation is formed conditioning on those next-period states of the world where the government does not default, as the policy function for the interest rate is not defined otherwise. This conditional expectation is a function of the state variables of the model, y and b.[11]

The conditional variance is analogously defined as

$$v(y,b) = \mathbb{E}\left[\left(r^{y}_{pf}\left(y', b'_{pf}(y,b)\right) - \bar{r}^{y}_{pf}(y,b)\right)^2\left|y, D\left(y', b'_{pf}(y,b)\right) = 0\right.\right]. \tag{3.18}$$

We can interpret the conditional variance as a measure of the precision of agents' forecast over interest rates. A high value of the variance implies that agents cannot form a precise forecast over the interest rate, or that a high degree of uncertainty is present. A graphical representation of the realized measure of uncertainty against the conditional mean of the interest rate is presented in Figure 3.2, displaying data from a long simulation of the model.

Time series estimates of interest rate uncertainty I analyze here the dynamic behavior of interest rate uncertainty. I estimate a time series process over realizations of this variable, from a long simulation of the model solution. Given a series of realizations for the conditional variance of interest rates, $\{v_t\}_{t=1}^{T}$, I estimate by ordinary least squares the following process in the logarithm of the standard deviation of interest rates in the model

$$\log\left(\sqrt{v_t}\right) = (1-\rho_v)\,\mu_v + \rho_v \log\left(\sqrt{v_{t-1}}\right) + \eta_v \epsilon_{v,t}. \tag{3.19}$$

The choice to estimate a process in this particular transformation of the variance makes it easy to compare parameter estimates with the ones obtained by Fernandez-Villaverde

[11] As risk-free rate and volatility shocks are not considered here, x and y describe the same information set.

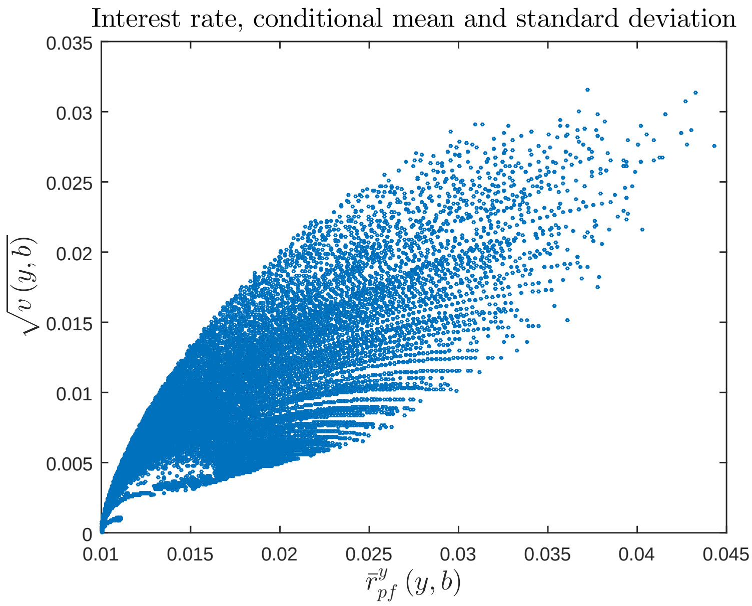

Figure 3.2: Conditional mean and standard deviation of the interest rate in the model simulation.

Each dot represents a time period in a simulation of the model for 25,000 periods. The horizontal axis represents the conditional expectation of the interest rate, defined in (3.17). The vertical axis represents the standard deviation, defined as the square root of the variance in (3.18). The model version considered here is the one with endowment shocks, only.

et al. (2011) for the interest rate on Argentine debt. A caveat is in order: while this model is calibrated at quarterly frequency, the process estimated in Fernandez-Villaverde et al. (2011) is monthly.

The model is simulated for 25,000 periods. To minimize the effects of the choice for the initial period state variables, I conduct the estimation starting from the first period in which the economy regains access to international financial markets, after the first episode of default. I thus ignore the initial T_1 time periods, where T_1 is defined as the first time period in which default by the small open economy occurs.

Parameter estimates from simulated data are extremely close to the ones estimated on Argentine debt. Both sets of parameter estimates are reported in Table 3.2. The volatility of the log-standard deviation, η_v is only slightly above the median posterior estimate for Argentina and within the 95% probability set of the posterior estimate. The autoregressive parameter estimated on the model-generated process is lower than the one

Parameter	Simulated data	Argentina
μ_v	-5.15	-5.71
	$[-5.35,\ -4.96]$	$[-6.39,\ -4.89]$
ρ_v	0.79	0.94
	[0.78, 0.80]	[0.83, 0.99]
η_v	0.52	0.46
	[0.515, 0.524]	[0.33, 0.63]

Table 3.2: Interest rate uncertainty, time series parameter estimates.

Number in square brackets report the 95% confidence interval for the OLS estimates on simulated data and the 95% probability set of the posterior estimate on Argentine data. Estimates on Argentine data are drawn from Fernandez-Villaverde et al. (2011). Note that the estimated process on Argentine data is monthly. The model is calibrated instead at quarterly frequency.

measured in the data. This estimate falls slightly below the 95 percent probability set of the posterior estimate reported by Fernandez-Villaverde et al. (2011). Finally, the unconditional mean of the log-standard deviation is very close to the one estimated on Argentine data. This result is not surprising, as the model analyzed here is known to be able to replicate the average volatility of interest rates on external debt. In light of these results, it is possible to argue that the baseline model of endogenous sovereign default is not only able to quantitatively replicate the average standard deviation of the interest rate on external debt, but also its dynamic behavior.

Cyclical behavior of interest rate uncertainty Fluctuations in interest rate uncertainty are systematically related to fluctuations in the other key variables of the model economy. I discuss here the correlation between interest rate uncertainty, the conditional expectation of the interest rate, and the state variables of the model. There are three key findings: interest rate uncertainty is positively correlated with the level of the interest rate, it is countercyclical, and it is related to the state variables of the model in a non-linear way.

First, the correlation between interest rate uncertainty and the interest rate paid by the small open economy on its debt is positive. The mechanism through which this result arises has been discussed above, and it hinges on the fact that the degree of non-linearity of the interest rate policy function increases as the interest rate itself increases. Figure 3.2 graphically illustrates the presence of positive correlation between the conditional standard deviation of the interest rate and the conditional expectation of the same variable. I formally test for the presence of such positive correlation by running a simple regression of the conditional standard deviation on the conditional expectation of the interest rate. Parameter estimates are reported in the first column of Table 3.3. The estimate for the regression coefficient of the conditional standard deviation on the conditional expectation is positive and significant. A 100 basis points increase in the interest rate results in an increase of the standard deviation by 78.5 basis points. In other words, the standard deviation roughly doubles when the interest rates rises from 1.5% to 2.5%. The fraction of the dispersion in interest rate uncertainty that is explained by the conditional expectation is high, as summarized by the regression R^2 of 0.64. Empirically, interest rate uncertainty is also found to be strongly positively correlated with the level of the interest rate by Fernandez-Villaverde et al. (2011).

Second, I analyze how interest rate uncertainty is related to the state variables of the model, namely to the realization of the endowment shock y and the asset position of the government b. To this purpose, I estimate a regression for the realized values of the interest rate standard deviation on the endowment shock and of the asset position. Results are presented in the second column of Table 3.3. Estimates from this regression show that uncertainty rises in bad times, as the regression coefficient is negative on both the endowment realization and on the government asset position. The R^2 of this regression is equal to 0.31, substantially lower than the one from the regression on the conditional mean, only. As the standard deviation of the interest rate is affected in a non-linear fashion by the state variables of the model, this simple linear regression is only able to explain a relatively small fraction of the dispersion in interest rate uncertainty.

Third, I present estimates from a regression of the standard deviation of the interest rate on its conditional mean and on the realizations of the endowment and of the government asset position. Results are reported in the third column of Table 3.3. All parameter estimates maintain their sign from the previous regression estimation, but the point estimate for the endowment process falls substantially in magnitude. When combined, the two state variables and the conditional expectation of the interest rate are able to explain

	(1)	(2)	(3)
Dependent variable	$\sqrt{v_{t+1}}$	$\sqrt{v_{t+1}}$	$\sqrt{v_{t+1}}$
$\bar{r}^y_{t+1}$	0.785		0.714
	(0.0027)		(0.0020)
y_t		-0.019	-0.001
		(0.0006)	(0.0003)
b_t		-0.063	-0.036
		(0.0006)	(0.0003)
R^2	0.639	0.307	0.805

Table 3.3: Cyclical behavior of model-based measure of interest rate uncertainty

Parameter estimates from a regression of simulated data for the conditional standard deviation of the interest rate on other key variables of the model economy. Variables analyzed are the conditional expectation of the interest rate and the state variables of the model, the endowment realization y and the asset position of the government b. All parameter estimates are significant at the 95% level. I do not report parameter estimates for the intercept of the regression equation.

80.5% of the dispersion in the standard deviation of interest rates. The inclusion of the conditional mean improves the fit of the regression as it is able to capture part of the non-linear relationship between interest rate uncertainty and the state variables.

Finally, I analyze the degree of non-linearity of the relationship between interest rate uncertainty and the state variables of the model. I estimate a regression of the standard deviation of the interest rate on a polynomial in the endowment realization and in the asset position of the government. The goodness of fit of the individual regressions is reported in Table 3.4 as a function of the order of the polynomial.[12] A second order polynomial is able to explain 87.6% of the disperison in interest rate uncertainty, amounting to 2.85 times the fraction explained by the simple linear regression on b and y. The fraction of the dispersion explained by the polynomial rises to 98.2% for a fourth order polynomial.

[12]Individual parameter estimates are omitted for brevity.

n	1	2	3	4	5
R^2	0.307	0.876	0.899	0.982	0.988

Table 3.4: Non-linearity of interest rate uncertainty.

R^2 from regressions of the conditional standard deviation of interest rates $\sqrt{v_{t+1}}$ on a polynomial of order n in the endowment realization y and in the asset position of the government b.

3.4 Interest rate shocks and sovereign default risk

Fluctuations in the risk-free real interest rate do not significantly affect the equilibrium of the model economy. I show in this section that the inclusion in the model of a stochastic process driving the risk-free real interest rate does not alter in a quantitatively significant way the behavior of the model economy. I discuss here the solution of the model economy when the risk-free rate follows a process that replicates the main empirical features of the risk-free real interest rate on US dollar-denominated debt, as discussed in Section 3.2. Unless specified otherwise, I consider here the solution of the model where the standard deviation of the risk-free interest rate is constant. First, I compare how the policy function for government default is affected by shocks of similar magnitude to the risk-free rate and to the endowment process. Second, I analyze the implications of the exogenous shocks for the interest rate at which the government can borrow on international financial markets. As changes in the risk-free interest rate and in the endowment level affect the probability of future default, shocks can affect the cost of borrowing both directly and through the spread over the risk-free rate that the government is charged. Third, I study how the policy functions for government borrowing and for the current account balance depend on the two stochastic processes that impinge on the small open economy in the model. Finally, I briefly analyze the implications of shocks to the volatility of the risk-free rate process.

Sovereign default incentives and the risk-free rate The incentive for the government to default is only marginally affected by the level of the risk-free real interest rate. Shocks to the endowment level, on the other hand hand, play an important role in affecting the decision for the government to default. The region of the endowment-assets

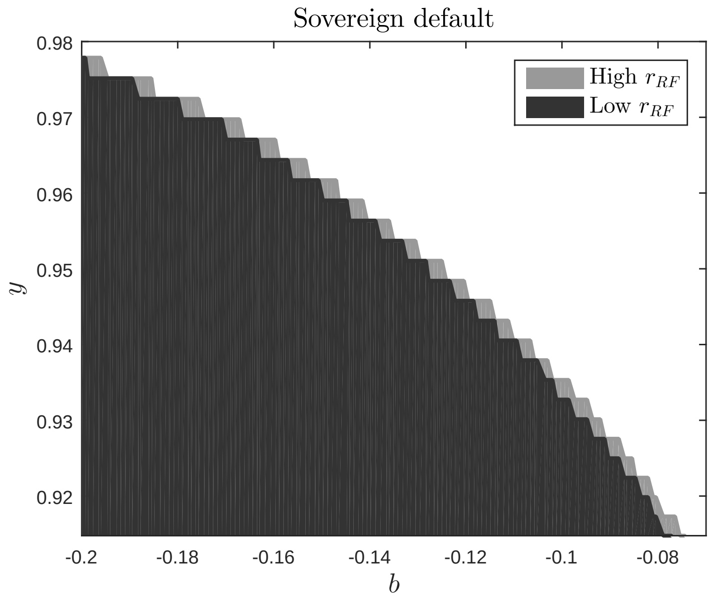

Figure 3.3: Policy function for sovereign default.

The light grey area labelled "High r_{RF}" represents the region of the endowment-assets state space where it is optimal for the government to default, when the risk-free interest rate is one standard deviation above the unconditional mean of the process. The dark-shaded area labelled "Low r_{RF}" represents the region where it is optimal to default when the risk-free interest rate is one standard deviation below the unconditional mean.

state space where it is optimal for the government to default is presented graphically in Figure 3.3. The two areas represented in the Figure correspond to the optimal default set for two different levels of the risk-free real interest rate. The light-shaded area represents the region where it is optimal to default when the risk-free interest rate is high, one standard deviation above the unconditional mean. The dark-shaded area represents instead the optimal default region when the risk-free interest rate is low, one standard deviation below the unconditional mean.[13]

The region of the state space where default is optimal is larger when the risk-free rate is high. Intuitively, a higher interest rate makes it more expensive to repay debt. In turn, as refinancing outstanding debt becomes more expensive, the value of accessing international financial markets diminishes. On the other hand, welfare under default is

[13]I refer here to the unconditional standard deviation of the process for the risk-free real interest rate, given by $\exp(\sigma_r)/\sqrt{1-\rho_r^2}$.

largely unaffected by the value of the risk-free rate. This occurs because the small open economy is excluded from international financial markets when defaulting on external debt.[14] Hence, an increase in the risk-free interest rate strengthens the incentive for the government to default, by reducing the incentive to maintain access to financial markets without affecting the payoff to default.

The quantitative magnitude of the effects of risk-free rate shocks on default incentives is small relative to that of shocks to the endowment process. An increase of the risk-free rate by two standard deviations causes the level of debt at which it is optimal to default to fall by 0.3% of mean quarterly output, when the risk-free rate is one standard deviation below its unconditional mean and output is equal to its mean. By comparison, a fall in the endowment realization of the same relative magnitude, namely of two standard deviations of the process, causes the default threshold for debt to fall by 64% of mean quarterly output, from 72% to 8%.

Two main facts explain the larger impact of endowment shocks on default incentives, relative to shocks to the risk-free rate. First, endowment shocks have a significantly larger impact on the amount of output available for consumption, relatively to shocks to the risk-free rate. This result is a direct implication of the low level of debt that is typically owed by the small open economy in models of endogenous sovereign default. When the small open economy owes a low amount of debt to foreign lenders, even relatively large shocks to the risk-free rate have small implications for consumption. In this model, the average amount of debt owed by the small open economy is equal to 11% of mean quarterly output. Second, endowment shocks directly affect the cost of default. The cost of default on external debt is increasing in the realization of the endowment process, given the functional form for the default cost $\delta(y)$ that is typically assumed in this literature. Shocks to the risk-free rate also generate a time-varying default cost, as default incentives are stronger when the risk-free interest rate is high. Given the present calibration of the model economy, however, these shocks are unable to affect the incentive for the government to default in a quantitatively substantial way.

[14] Since exclusion is only temporary, welfare is adversely affected by an increase in the risk-free rate even under exclusion. These effects are however negligible when compared to those of shocks to the endowment process.

Government borrowing costs Shocks to the risk-free real interest rate affect the interest rate at which the government can borrow, both directly and through their effects on the interest rate spread over the risk-free rate. As this spread is determined by the probability of next-period default, shocks to the risk-free real interest rate affect the spread through their impact on default incentives. The interest rate at which the government can borrow is presented graphically in Figure 3.4, as a function of the amount of debt issued. The Figure displays the interest rate spread over the risk-free rate, thus abstracting from the direct effect of the risk-free interest rate on the cost of borrowing.

The effects of shocks to the endowment process on the cost of government borrowing are, again, significantly larger than those of shocks to the risk-free rate. A fall in the risk-free rate reduces the interest rate spread on government debt, for all levels of debt issued by the government. For a one standard deviation fall in the risk-free rate, equivalent to a fall by 130 basis points, the shift in the function describing government borrowing costs implies a rise by 0.15% of mean quarterly output in the amount of debt at which the government can borrow for a spread of 50 basis points, when the endowment is equal to its mean. An increase in the endowment of comparable magnitude causes a significantly larger relaxation of borrowing costs for the government. A 7% rise in the endowment from its unconditional mean, equivalent to one standard deviation of this process, results in an increase by 22.5% of mean quarterly output in the amount of debt at which the government can borrow for the same 50 basis points spread, when the risk-free interest rate is equal to its mean.

The larger effect of endowment shocks on the interest rate spread follows directly from their larger effect on default incentives, relative to risk-free rate shocks. As processes for both the endowment and the risk-free rate display positive autocorrelation, a low current value of the risk-free rate is associated with a low expected value for the next-period realization of this variable. Hence, a low current risk-free rate helps predict a weak next-period incentive to default and it determines a low spread in the current cost of borrowing. The same mechanism applies if the endowment realization is relatively high in the current period. Given the significantly larger impact of endowment shocks on default incentives, however, the implications of endowment shocks for the cost of borrowing are consequentially larger.

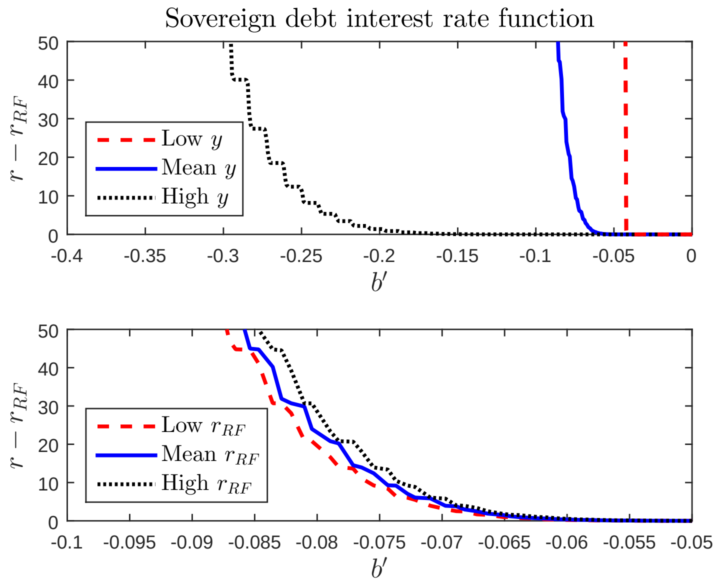

Figure 3.4: Interest rate spread on government debt over the risk-free interest rate, as a function of the amount of debt issued and of exogenous shocks.

The upper panel presents the function for three values of the endowment process, corresponding to the mean and to one standard deviation above and below it, for a risk-free real interest rate equal to its mean. The lower panel presents the same function for three values of the risk-free interest rate, again given by the mean and by realizations one standard deviation above and below it, for a mean value of the endowment. The horizontal axis represents the amount of government-purchased assets. Negative values denote debt issued. The interest rate spread is in basis points, where 100 basis points denote an interest rate spread of 1%. The two panels are presented in different scales.

Government borrowing, the current account and risk-free interest rate shocks Shocks to the risk-free real interest rate affect the government borrowing decision through their impact on the cost of government borrowing. A low risk-free interest rate causes the government to issue a higher level of debt, by lowering the cost of borrowing both directly and through a lower interest rate spread. A qualitatively similar effect is caused by a positive realization of the endowment process. A relatively high endowment causes the cost of borrowing to fall, by reducing the probability of next-period default associated with any level of debt issued. For a standard calibration in the endogenous sovereign default literature, the effects of endowment shocks on borrowing costs dominate the standard permanent-income mechanism that would cause borrowing to fall in times

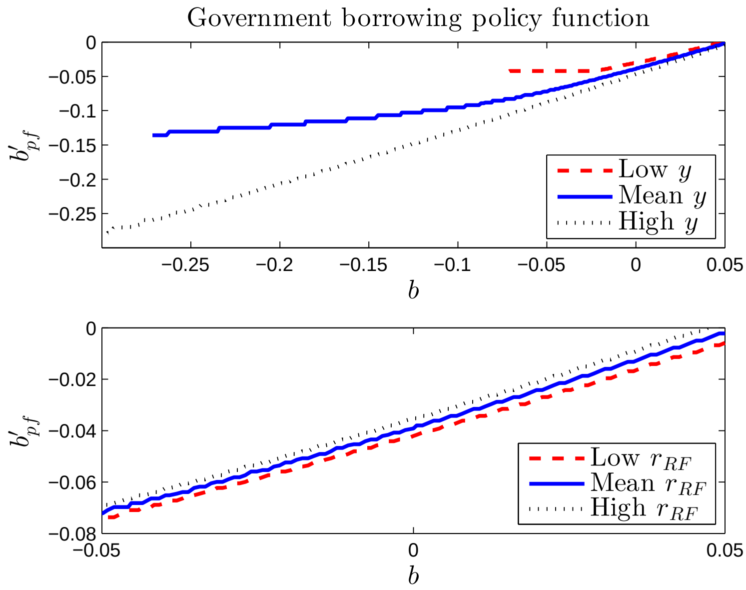

Figure 3.5: Policy function for government borrowing, as a function of the amount of assets held by the government and of exogenous shocks.

The upper panel presents the function for three different values of the endowment realization, equivalent to the unconditional mean of the process and to a level one standard deviation above and below the mean, for a risk-free real interest rate equal to its unconditional mean. The lower panel presents the same function for three different values of the risk-free interest rate, again given by the unconditional mean and by realizations one standard deviation above and below it, for a mean value of the endowment process. The horizontal axis represents the amount of asset held by the government, so that negative values denote debt. The two panels are presented in different scales, as the effects of the two shocks have substantially different magnitudes.

of high output.A common feature of models in this tradition is the prediction of a countercyclical current account balance, as commonly observed in emerging market economies (Aguiar and Gopinath, 2006, Arellano, 2008).

The impact of risk-free rate shocks on the government borrowing choice is relatively smaller than that of endowment shocks, given their relatively smaller impact on borrowing costs. The policy functions for assets purchased by the government is presented graphically in Figure 3.5. Figure 3.6 displays the policy functions for the current account, defined as the change in government assets. The amount of debt issued by the government equals 4.2% of mean quarterly output when the risk-free rate is one standard deviation below its mean and 3.5% when it is one standard deviation above it, when the small open

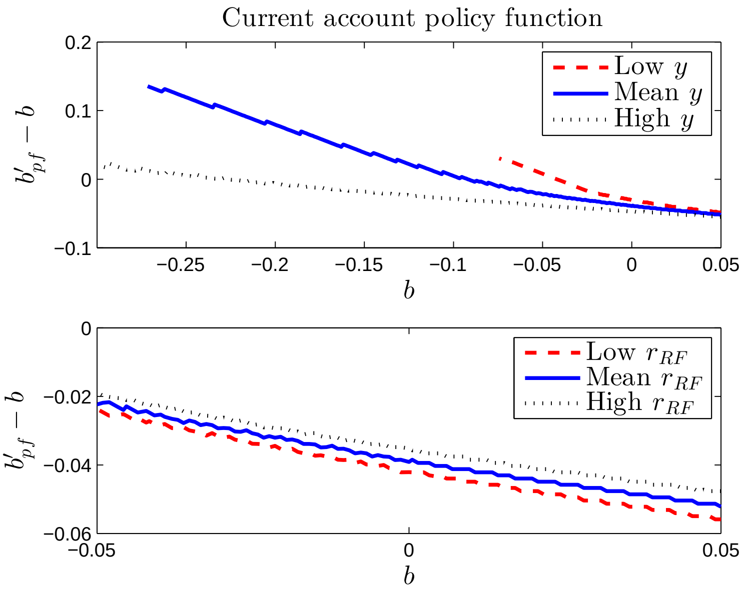

Figure 3.6: Policy function for the current account balance, as a function of the amount of assets held by the government and of exogenous shocks.

The current account balance is defined as the change in government assets over one period. The upper panel presents the function for three different values of the endowment realization, equivalent to the unconditional mean of the process and to a level one standard deviation above and below the mean, for a risk-free real interest rate equal to its unconditional mean. The lower panel presents the same function for three different values of the risk-free interest rate, again given by the unconditional mean and by realizations one standard deviation above and below it, for a mean value of the endowment process. The horizontal axis represents the amount of asset held by the government, so that negative values denote debt. The two panels are presented in different scales, as the effects of the two shocks have substantially different magnitudes.

economy does not owe any debt to the rest of the world nor does it have any assets and the endowment level equals the unconditional mean of the process. For the same level of initial assets, a high realization of the endowment process causes the government to issue debt amounting to 4.7% of mean output, while the amount issued falls to 3% if the endowment is low, when the risk-free interest rate is equal to its mean.[15]

The policy function for the current account that results from the government optimal choice is increasing in the level of debt of the economy, as the cost of refinancing such

[15]By "low" and "high" realization of the endowment process, I again refer here to realizations one standard deviation above and below the unconditional mean, respectively.

debt increases in the amount issued. When the economy enters the period with debt corresponding to 5% of output, the government runs a current account surplus of 2.4% of output if the risk-free rate is low, as opposed to 1.9% if the risk-free rate is high. For this level of assets, shocks to the endowment process have a substantially larger impact on the current account, as borrowing costs react sharply to these shocks. If the economy receives a low endowment shock, the government runs a current account surplus of 0.8% of mean quarterly output. On the other hand, if the endowment process realization is high, the government issues further debt by running a current account deficit amounting to 4% of mean output.

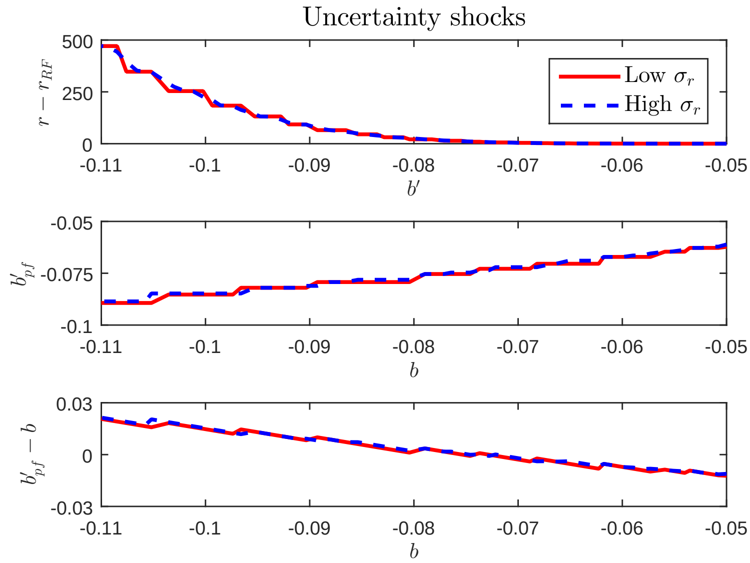

Figure 3.7: Effects of shocks to the volatility of the risk-free interest rate on key variables of the model.

The three panels describe the equilibrium of the model when the volatility of the risk-free interest rate is high or low, one standard deviation above or below the unconditional mean of the process, respectively. In all panels, the endowment and the risk-free interest rate are equal to their unconditional mean. The top panel describes the interest rate spread over the risk-free rate as a function of the amount of debt issued, for the two values of volatility. The middle panel describes the policy function for government borrowing, as a function of the amount of assets held by the government. The bottom panel describes the policy function for the current account balance.

Uncertainty shocks to the risk-free real interest rate The effects of uncertainty shocks on the model economy are minimal, at best. I discuss here the version of the model where all three exogenous shock processes are considered, focusing on the implications of changes in volatility for the interest rate at which the government borrows, for the government borrowing policy function, and for the current account. Results are presented graphically in Figure 3.7.

For all three variables considered, there are no visible implications of a shock to the volatility of the risk-free rate process. An increase in uncertainty does neither lead to a change in the interest rate at which the government borrows, nor to a change in the actual quantity of debt that the government chooses to issue.

The insignificance of the effects of a volatility shock is a direct implication of the small magnitude of the effects of shocks to the risk-free rate. As discussed above, even relatively extreme shocks to the level of the risk-free rate have only minor implications for the endogenous variables of the model. An increase in uncertainty amounts to an increase in the probability associated with the realization of tail events. In the absence of sizable and non-linear implications of such tail events, however, an increase in uncertainty is unable to generate substantial quantitative effects on the equilibrium of the model economy.

3.5 Concluding Remarks

This chapter highlights two important properties of a popular quantitative model of endogenous sovereign default. First, I document that in the absence of exogenous uncertainty shocks this model generates time-varying volatility in interest rates on external debt. The non-linearity of the policy functions that characterize the solution of this model lies at the heart of this result. Second, I show that in a standard calibration of the model, shocks to the level and to the volatility of the world risk-free interest rate have minor quantitative implications. Shocks to the level of output have significantly stronger effects on default incentives and on the choice for the external asset position of the economy.

In future research, I will investigate further the role of asymmetric default costs in relation to the two main results of this chapter. This assumption is likely to have important implications for both sets of findings. First, asymmetric default costs in output lead to stronger non-linarities in the policy function for the interest rate. Relaxation of this assumption is likely to lead to less sharp fluctuations in interest rate uncertainty. Second, the role of output shocks in driving default incentives is weaker in the absence of asymmetric default costs. The naturally asymmetric implications of risk-free rate shocks on default and repayment incentives would be likely to play a more important role in this setting.

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