The London School of Economics and Political Science

Essays on Delegated Portfolio Management

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Declaration

I certify that the thesis I have presented for examination for the MPhil/PhD degree of the London School of Economics and Political Science is solely my own work.

 I declare that my thesis consists of 26,590 words.

Abstract

This thesis studies how financial market outcomes are affected by the reputational concerns of fund managers. The first chapter presents a model in which a fund manager trades in an environment with uncertain market liquidity. The fund manager trades off expected profits in the initial period and learning relating to the investment strategy in the successive period. Surprisingly, the indirect incentives do not cause the manager to focus on short-term returns to impress investors but result in a behaviour that may be described as inefficient "long termism". The model may help explain empirical puzzles such as the limits of arbitrage, the convex flow-performance relationship and the excessive trading of fund managers.

The second chapter focuses on the asset pricing implications of fund flows motivated by past performance. By investing in an out-performing asset, fund managers can improve their reputations and therefore experience inflows of money into their funds. In my model, the value of a fund manager's reputation is state dependent. In the case of an inefficient asset management market, I show that asset prices are increasing in their beta. Furthermore, the asset price depends on asset supply in my model.

The third chapter analyses the size of the active management sector in a model where fund managers have reputational concerns. I show that the size of the active management sector depends on the skill of the fund managers in the sector in a non-monotone manner. The asset choices of fund managers are

influenced by reputational concerns, and the information revelation of the skill of the individual fund managers depends on market outcomes. The model predicts that the amount of money invested in the active management sector may shrink sharply following rare events.

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Chapter 1

Delegated Portfolio Management and Uncertain Liquidity

1.1 Introduction

Hedge funds and mutual funds are an increasingly dominant force in today's financial markets. They spend vast resources finding and exploiting trading opportunities. However, their success in trading based on their information has been questioned by an empirical literature pointing towards an inability to outperform various benchmarks. Since these institutions are typically large, the price impact of their trades is often a major concern and a key determinant of the scalability of their investments. The ability to manage trading costs is also of great importance in the case of "smart-beta" strategies that follow mechanical rules that have derived from asset market anomalies.[1] For these types of strategies, the core competencies that an investor should evaluate include not only the stock-picking skill of a fund, but also its ability to implement the strat-

[1] For example, money management companies such as AQR and Dimensional Fund Advisors offer funds that invest based on momentum, value, size and quality factors. Ratcliffe, Miranda and Ang (2016) discuss the capacity of various smart beta strategies.

egy with low trading costs. In this paper, I study the interplay between a fund manager's incentives that stem from investor flows and the need to manage price impact efficiently. My model indicates that these incentives can cause inefficient management of the price impact of trades. Surprisingly, the short-term flow-performance relationship caused by return-chasing investors, induces inefficient "long-termism" on the part of the fund manager. The fund is incentivised to give up too much of its profits early on in order to learn how to implement the investment strategy in the future.

The main reason an investment strategy has limited capacity, is that typically the more you trade on an idea, the more you move prices, decreasing the strategy's profitability. For many strategies, there is a significant amount of uncertainty regarding their scalability. Uncertainty about scalability is closely related to uncertainty about the price impact costs of the trades.[2] Given this uncertainty, large financial institutions spend a considerable amount of resources trying to estimate price impact costs. They purchase information technology systems and hire consulting firms in order to overcome this problem.[3] Uncertainty about scalability is an important issue even for well-known market anomalies, as confirmed by a current debate among practitioners and academics on the scalability of such strategies. A recent paper by Frazzini, Israel and Moskowitz (2012) received great attention among practitioners as it hints at the much greater scalability of some well-known anomalies (e.g. momentum) than estimated by previous academic research, but it found that others are limited in their scalability (e.g.

[2] In general, other explicit trading costs such as commissions and bid-ask spreads are small and easy to measure for large institutions. For example, Stoll and Whaley (1983) examine the effects of commissions and spreads on size portfolios.

[3] There is a firm called "Investment technology group" which provides these services. One of their core services is trading analytics. Their consultants are experts in market microstructure and financial engineering and they help to forecast price impact costs. (http://www.itg.com/product/trading-analytics/)

short-term reversal).[4] Since the most reliable way to learn about price impact costs is to actually trade in the market, realised returns of funds may help in learning about the scalability of a strategy. For example, when asked whether investors can profit from momentum, Fama and French state, "Many academics claim that trading costs will wipe out any benefits of trying to trade actively on momentum. This will now be tested by live funds. The results will be interesting."(Fama and French Forum 2010). Fama and French suggest that they will update their opinion about momentum after seeing the first returns of money management companies trading on the anomaly.

Motivated by these stylised facts, I consider a model of strategic trading where an insider trades in the presence of uncertain liquidity. The investment strategy of the insider is based on perfect information about the asset payoff. In a model inspired by Kyle (85), uncertain liquidity means that the price response to large market orders is uncertain. This is equivalent to uncertainty with regards to scalability of the investment strategy. I compare the behaviour of two types of insiders: a fund manager and a profit-maximising trader, who implements the first best trading strategy. The goal is to investigate how indirect incentives influence the fund manager's behaviour. The fund manager is compensated based on the amount of money he manages and the returns he is expected to generate. Competitive outside investors supply funds to the manager until they earn a zero net (after fees) expected return (similar to Berk and Green (2004)). In this way, the fund manager extracts the entire surplus he generates in any given period. The size of this surplus and consequently, the size of the fund depends on two factors. The first is the expected level of liquidity, a higher liquidity means less price impact and more scalability. The second is the level of information about liquidity. Better information results in better management of the price impact.

[4] For example, in a blog article by alpha architect the issue is discussed http://blog.alphaarchitect.com/2016/08/17/surprise-the-size-value-and-momentum-anomalies-survive-after-trading-costs

In a two-period model, the realised price impact in the first period is a noisy signal of liquidity and, thus, of the scalability of the strategy. A price that is not affected by the trade of the fund signals a liquid market. The informativeness of the signal that the price sends, is influenced by the aggressiveness of the trade in the first period. Both the fund manager and the trader face a trade off between an optimal execution, which maximises profits in the initial period, and the optimal level of learning about the trading environment for the next period. A higher trade risks price impact today, but also provides more informative signals. The trader optimally trades off these two effects. The fund manager shares the costs of the excessive trading with current investors. In the following period the fund manager can capture the full benefit of learning through investor inflows. Thus, I find that the fund manager overweighs learning for the long run and, in turn, gives up too much of its expected profits today. This happens despite the fact that investors react positively to higher returns with inflows. The outside investors always break even, so the cost of inefficient learning is borne by the fund manager. The initial inflows reflect the surplus the fund manager is expected to create in the present period only. The initial investors do not benefit from the learning of the fund manager, since the fund will open up again in the interim period and new inflows result in zero after-fee returns going forward. The fund manager is unable to commit to not trading excessively, so the initial size of the fund will be small, resulting in a low fee income for the fund.

This paper sheds light on the existing debate on the scalability of investment strategies among academics and practitioners. Based on my model, a strategy could well be scalable, but the fund manager has incentives to implement the strategy with excessively aggressive trades. Then outside observers, such as Fama and French, rationally conclude that there is a high probability that the strategy is not scalable. Thus, trading strategies that would appear to be implementable in studies with estimated price impact, may often deliver disappointing returns when implemented by fund managers. The problem of inefficient management of price impact identified in my model, can also help explain why some scalable investment strategies are not implemented by funds to such a scale so that the trading opportunities disappear.

My model is also related to a number of other stylised empirical facts. It points toward a new explanation of the asymmetric and convex flow-performance of mutual funds and hedge funds as documented in empirical papers (e.g. Chevalier and Ellison (1997)). The flow-performance relationship is positive since higher performance increases the estimate of the scalability of the fund's strategy. Additionally, a very high performance allows the fund manager to be fairly certain about the state of liquidity. This certainty increases his expected profits. The two effects together yield convexity. Furthermore, the model may help explain excessive trading by fund managers as documented in empirical studies such as Edelen, Evans and Kadlec (2007).[5] I find that in the initial period the fund manager trades excessively as he tries to learn more about market liquidity. In my model, the fund manager would like to commit to closing in the interim period. If I allow for this possibility, my model generates return persistence. A high return is more likely to come from a scalable strategy, resulting in higher expected returns going forward. This implication of my model may help us understand the empirical evidence of hedge fund returns persistence (see Jagannatha, Malakhov and Novikov (2010); Fung, Hsieh, Naik and Ramadorai (2008)). In particular, Aggarwal and Jorion (2010) document that return persistence is stronger for younger funds. For younger funds the problem of uncertain scalability may be more relevant.

This paper is broadly related to a stream of research that studies how reputational concerns influence the trading decisions of fund managers and the functioning of financial markets (e.g. Dow and Gorton (1997), Dasgupta and Prat (2006,2008), Dasgupta, Prat and Verardo (2011), Cuoco and Kaniel (2011), Guerrieri and

[5]Other explanations based on reputational concerns of the same phenomena are given in Dow and Gorton (1997) and Dasgupta and Prat (2006)

Kondor (2012)). There is also a connection to Makarov and Plantin (2015), who study risk shifting of fund managers. Furthermore, my model is related to papers that study how investor flows may affect limits of arbitrage (Shleifer and Vishny (1997)). The main difference from previous papers on the agency problems of fund managers is that the mechanism in my model is based on excessive learning *by* the fund manager and not learning *about* the fund's type by investors. In these papers, the distortion comes from the fund's attempts to impress investors; e.g. in Dasgupta and Prat (2006), the fund manager trades without information for a chance to appear informed. In my model the fund manager also trades excessively, but here, the fund manager does so in order to enable more learning about the strategies' scalability. The model also contributes to a vast literature on strategic trading and price impact (Kyle (85), Easley and O'Hara (1987), Glosten and Harris (1988), Huberman and Stanzl (2000)). Hong and Rady (2002) present a model with uncertain liquidity in which each trader only trades once. Thus, they cannot analyse how optimal learning distorts trading decisions. To the best of my knowledge, this is the first paper in which trades are partly "experiments" to learn about liquidity. Furthermore, papers that study the aggressiveness of trading based on private information do not take into account agency problems that these large traders might face. My paper fills this gap, since this is particularly relevant for firms such as hedge funds and mutual funds, for which indirect incentives are a large part of the total compensation of the decision maker. Finally, this paper is also related to a vast theoretical literature on bandit problems and experimentation dating to Robbins (1952). Strategic experimentation is also analysed in theoretical economics (e.g. Aghion, Bolton, Harris and Jullien (1991), Bolton and Harris (1999), Manso (2011)). Recent applications of the paradigm in corporate finance include Bergemann and Hege (1998,2005). The paper is also related to papers that study managerial "short-termism". A bias for short-term projects may be due to career concerns (Narayanan (1985)), concerns about stock prices (Stein

(1989)) or herding behaviour (Zwiebel (1995)). In contrast to this literature, in my model there is inefficient "long-termism".

The remainder of the paper is organised as follows. In Section 2, I illustrate the main mechanism of the paper using an example model in the spirit of Berk and Green (2004). In Section 3, I present the main model and solve for the equilibrium. Then, in section 4, I relate my model and its results to the empirical evidence and conclude. All proofs are included in the Appendix.

1.2 Long Termism of Fund Managers

1.2.1 An Example

I introduce a very stylised model in the spirit of Berk and Green (2004) in order to illustrate the main mechanism of my paper. There are two time periods $t \in \{1, 2\}$. For the rest of the paper, everyone is risk neutral, and there is no discounting between the two time periods. There is a fund manager with access to a technology that produces excess returns. If the fund manager[6] puts $x > 0$ dollars into the technology, then the x dollars become

$$xR - C_t(x) \tag{1.1}$$

where $R > 1$, and C_t denotes the trading costs. For simplicity, I assume that R is non-stochastic. I assume, as in Berk and Green (2004), that C_t has the properties $C_t(x) > 0, C_t'(x) \geq 0$ and $C_t''(x) > 0$, with $C_t(0) = 0$ and $\lim_{x\to\infty} C_t'(x) = \infty$. In addition to this investment technology, the fund manager can invest in a benchmark technology with a risk free return $R^{BM} = 1$. There are outside investors who can flow into the fund or into the benchmark technology. In each period, there is a morning and an afternoon. The inflows happen in the morning and the investment decision x happens in the afternoon. The total investment

[6] Sometimes simply referred to as "fund".

in the fund is given by $\tilde{f}_t$. The fund is compensated by the following exogenous contract at the end of a time period $t \in \{1, 2\}$

$$\gamma_M \tilde{f}_t + \gamma_P \tilde{f}_t(\tilde{R} - 1) \tag{1.2}$$

where $\tilde{R}$ is the realised gross return of the fund. I assume that $\gamma_M \in (0, 1)$ and $\gamma_P \in [0, 1)$.[7]

This contract includes a few special cases, such as $\gamma_P = 0$, the contract that Berk and Green (2004) focuses on. This case is mainly observed in the mutual fund space.[8] Hedge funds, however, typically charge a substantial incentive fee.[9] I summarises the outcome in the Berk and Green (2004) model when $C_1(x) = C_2(x) = C(x)$. The fund's returns is

$$\tilde{R} = \frac{xR - C(x) + \tilde{f} - x}{\tilde{f}}. \tag{1.3}$$

Once the fund manager received the inflows $\tilde{f}$, he invests $\hat{x}$ to maximise his compensation

$$\max_{x \in [0,\infty]} \gamma_M \tilde{f} + \gamma_P (xR - C(x) + (\tilde{f} - x) - \tilde{f}). \tag{1.4}$$

Let me write $\hat{x}$ as the solution to this problem. The inflow $\tilde{f}$ in each period will be such that outside investors are indifferent between investing with the fund or on their own. Thus, the after fee excess return of the fund needs to be equal to the excess return of the benchmark technology, i.e.

$$(1 - \gamma_P)(\tilde{R} - 1) - \gamma_M = 0. \tag{1.5}$$

[7]Since this is a deterministic framework, where the fund does not produce any losses, I can write the contract in this simple way. Everything would be exactly the same if I would write the incentive fee part as $\gamma_P \max(\tilde{f}_t(\tilde{R} - 1), 0)$

[8]ICA of 1940 prohibits mutual funds from charging asymmetric incentive fees

[9]For hedge funds the typical contract we see in the real world is "a 2-20" contract. This would correspond to $\gamma_M = 0.02$ and $\gamma_P = 0.2$.

From $\hat{x}$, (1.3) and (1.5) the inflow $\tilde{f}$ can be found. This also yields the compensation of the fund.

Lemma 1. *[Berk and Green (2004)] For $t \in \{1, 2\}$, the amount of investment in each period is*

$$\hat{x} := C'^{-1}(R - 1). \tag{1.6}$$

The fund manager is paid the full expected profits (NPV) that he makes in any given period. These profits are given by

$$\Pi = (R - 1)\hat{x} - C(\hat{x}). \tag{1.7}$$

The amount invested with the fund each period is given by

$$\tilde{f} = \Pi \frac{1 - \gamma_P}{\gamma_M}. \tag{1.8}$$

Any contract of the form $\gamma_M, \gamma_P \in (0, 1)$ such that $\tilde{f} > \hat{x}$ achieves the efficient outcome.

An important implication of this model is that the need for outside investors to break even results in a fund manager who always collects the full profits he makes in any given period. These are given by Π in both periods in the lemma. Thus, in total, the fund makes $2\,\Pi$.

I introduce one new assumption to this framework. There is the possibility of *learning by trading*. Specifically, I assume that when the fund manager operates the strategy at a sufficiently large scale, he is able to learn from the experience. I assume

$$C_2(x) = \begin{cases} \delta C(x) \iff x_1 \geq \bar{x} \\ C(x) \iff x_1 < \bar{x} \end{cases} \tag{1.9}$$

where $0 < \delta < 1$. This means that if the strategy was implemented with more than $\bar{x}$ dollars invested, the fund can operate more efficiently in the next period.

This assumption seems natural, as a larger investment today lets the fund learn more and reduces the cost of the same strategy tomorrow. I will provide a microfoundation for this assumption in the main model in the next section based on an uncertain price impact. I assume that $\hat{x} < \bar{x}$, so there is a trade-off in the first period. A key friction in my model is that the fund manager *cannot commit* to a certain size of investment ex-ante.

At time $t = 2$, the total compensation of the fund will be the expected profits in that period. In the case of learning, the profit in the second period is $\Pi_H := (R-1)\hat{x}_H - \delta C(\hat{x}_H)$ where $\hat{x}_H := C'^{-1}(\frac{R-1}{\delta})$. It is obvious that $\Pi_H > \Pi$. The expected compensation of the fund at $t = 2$ will thus be either Π_H or Π depending on the choice of x in the first period.

In the initial period, the fund essentially chooses between two investment levels x. One possibility would be to invest $\hat{x}$, which delivers profits Π. Since $\hat{x} < \bar{x}$, this is not enough to reduce costs in the next period. Another possibility would be to select an investment of $\bar{x}$. This investment is inefficiently high today, but is the best way to capture increased efficiency in the next period. Let $\Pi_L := (R-1)\bar{x} - C(\bar{x})$, we clearly have $\Pi_L < \Pi$, since $\hat{x} < \bar{x}$.

In order to obtain the best overall profits (to implement the first best strategy), it would be optimal to invest $\hat{x}$ in the case

$$\Pi_L + \Pi_H < 2\Pi \iff \Pi_H - \Pi < \Pi - \Pi_L \tag{1.10}$$

and $\bar{x}$ otherwise. When the loss in the first period $\Pi - \Pi_L$ is higher than the gain from the increased efficiency in the second period $\Pi_H - \Pi$, then it is not worthwhile to pick $\bar{x}$ in order to learn. Suppose that δ is large enough that (1.10) is satisfied. What will the fund do?[10]

Proposition 2. *The fund manager invests excessively relative to the first best strategy.*

[10] I assume here that γ_M, γ_P are small enough so that $\frac{1-\gamma_P}{\gamma_M}\Pi_L > \bar{x}$. Alternatively, I could allow borrowing at the risk-free benchmark rate.

Suppose that the fund receives an initial inflow $\tilde{f}_1$. In the initial period, the fund maximises

$$\max_{x\in[0,\infty]} \gamma_M \tilde{f}_1 + \gamma_P(xR - C(x) + (\tilde{f}_1 - x) - \tilde{f}_1) + \Pi + \mathbf{1}_{x\geq\bar{x}}(\Pi_H - \Pi). \qquad (1.11)$$

Once the fund has collected $\tilde{f}_1$, he takes this inflow as given, and the optimal $x \in [0,\infty]$ could be either $\hat{x}$ or $\bar{x}$. The last two terms represent the compensation of next period. It is clear that no $x \in [0,\bar{x})$ can be a better choice than $\hat{x}$. Similarly, a $x > \bar{x}$ is always worse than $x = \bar{x}$, by the convexity assumption on $C(x)$. The fund will choose $x = \bar{x}$ if and only if

$$\gamma_P \Pi_L + \Pi_H > \gamma_P \Pi + \Pi \iff \Pi_H - \Pi > \gamma_P(\Pi - \Pi_L). \qquad (1.12)$$

Comparing (1.10) and (1.12) makes the "long-termism" problem clear. The benefit of learning in the *long run* $\Pi_H - \Pi$ is overweighted relative to the *short-term* cost of learning $\Pi - \Pi_L$.

Since $\gamma_P \in [0,1)$, we might well have that both (1.10) and (1.12) are satisfied. The fund puts full weight on the benefit of learning, but only γ_P weight on the cost of learning. This results in an inefficiently large amount of investment. In particular, when there is no incentive fee, the fund always chooses to invest heavily in order to learn, even if this is potentially very inefficient. The investors of the morning of the initial period anticipate the excessive investment. Since investors need to break even in the first period, the compensation of the fund in that period totals Π_L. The fund manager's total compensation then is

$$\Pi_L + \Pi_H < 2\,\Pi. \qquad (1.13)$$

Therefore, the fund manager suffers from the excessive investment through his first period inflows. He is unable to commit to a level of investment and once the investment is determined he has incentive to over-invest as he takes the current assets under management as given. Thus, in the end, the fund manager is worse off when he has the opportunity to learn than when there is no learning potential, as in lemma 1.

Discussion. The short-term inflows, which the fund collects at the interim date, lead the fund to care more about the long run and so he over-invests in the strategy to improve for the next period. The first important assumption is that the fund manager can open the fund to new investors after the first period and thus capture the full benefits of learning in the first period. The second important assumption is that the fund manager cannot commit to not engage in excessive investment. In this framework, where there is no uncertainty, the after-fee returns of the funds are always equal to outside option 1; the fund grows over time in this example: $\frac{\tilde{f}_2}{\tilde{f}_1} = \frac{\Pi_H}{\Pi_L}$. Growth happens even though the returns of the fund are constant and the inflow in the fund is not related to investors updating their opinions about the fund (as in Berk and Green (2004)). Here, inflows are due to learning by the fund about the investment strategy and not by investors learning about the fund. In the full model that I develop in the rest of the paper, there is a positive flow-performance relationship. I will consider an environment where there is uncertainty about the price impact of trades and, thus, room for learning. In my model, more aggressive trades result in better learning and higher average profits in the next period, which is consistent with (1.9). Furthermore, in this model, high returns signal a scalable strategy and, therefore, results in high inflows. One might expect that in light of this flow-performance relationship, the fund increasingly cares about the short term and would try to impress investors. However, I find that the fund chooses to trade too aggressively and that the "long termism" problem remains.

1.3 The Model

1.3.1 Setup

In this paper, I contrast the behaviour of two types of insiders I^{11}. Our insider I could be a profit-maximising trader or a fund manager, i.e., $I \in \{trader, fund\}$. I use the trader as the agent that implements the first best and compare her outcome to that of the fund manager who invests on behalf of investors. The insider I has private information about the dividends of a traded asset. The insider then seeks to optimally take advantage of this information. There is a risky asset that pays a dividend v, where $v \in \{0, 1\}$, at the end of the period. I assume that

$$\mathbb{P}(v = 1) = 1/2. \tag{1.14}$$

The insider has perfect information about v and can submit a market order of $\tilde{x}$ shares of the asset. I will first consider a one-period model and then a two-period model. I assume the following for the price of the risky asset

$$P \in \{0, 1/2, 1\}. \tag{1.15}$$

The insider does not know how deep the market in which she trades is. A deep market means a substantial amount of trading is possible with little price impact.[12] Market depth L has the property $L \in \{\Delta_L, \Delta_H\}$. A high value $L = \Delta_H > \Delta_L > 0$ means that the market is deep. I assume that

$$\frac{\Delta_H}{\Delta_L} < 2. \tag{1.16}$$

[11] I use the term "insider" as in Kyle (85), but the situation I have in mind is not necessarily insider trading. It simple means that this market trading participant has an informational advantage.

[12] In my paper, "high liquidity" and "deep market" are different terms for the same concept and are used interchangeably. A deep market also means that the fund strategy is more scalable.

The insider I only knows that

$$\mathbb{P}(L = \Delta_H) = q. \tag{1.17}$$

The realised price then has the following property

$$\mathbb{P}(P = 1/2) = \begin{cases} 1 - \frac{\tilde{x}}{L} & \forall \tilde{x} \in [0, L] \\ 1 + \frac{\tilde{x}}{L} & \forall \tilde{x} \in [-L, 0). \end{cases} \tag{1.18}$$

$$\mathbb{P}(P = 1) = \frac{\tilde{x}}{L} \qquad \forall \tilde{x} \in [0, L] \qquad \mathbb{P}(P = 0) = \frac{-\tilde{x}}{L} \qquad \forall \tilde{x} \in [-L, 0). \tag{1.19}$$

For general $\tilde{x}$ we have

$$\begin{aligned} \mathbb{P}(P = 1/2) &= \max\left(1 - \frac{|\tilde{x}|}{L}, 0\right) \\ \mathbb{P}(P = 1) &= \min\left(\max(\frac{\tilde{x}}{L}, 0), 1\right) \\ \mathbb{P}(P = 0) &= \min\left(\max(\frac{-\tilde{x}}{L}, 0), 1\right). \end{aligned}$$

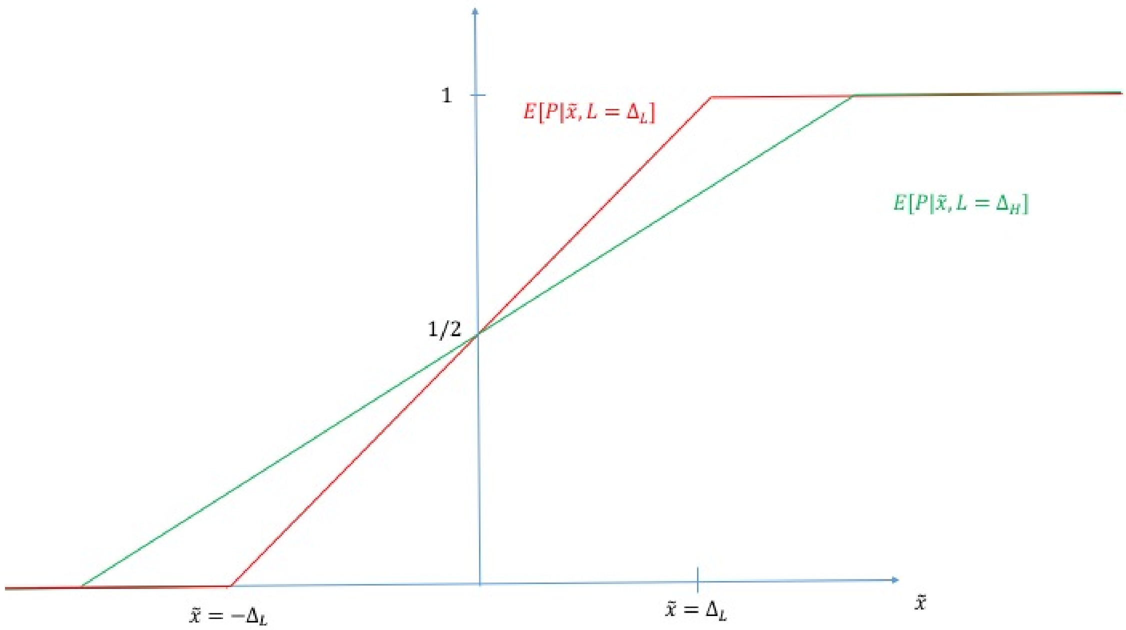

Figure 1.1: The expected price as a function of the market order $\tilde{x}$

Microfoundation. In the Appendix, I provide a simple micro-foundation for this type of price function. The main idea behind the price function is as follows: the uncertain liquidity corresponds to an uncertain variance of noise trading. Noise traders submit market orders that are uniform on an interval of length L. A market maker knows L, but assumes the probability of the presence of an insider to be negligible. The market maker observes the sum of the market orders of noise traders and the insider. Whenever the total amount of trade is in the interval, the price is the expectation. A price outside the interval reveals the presence and the direction of the informed trade. In this case, the price is v. The probability of the total market orders slipping out of the interval and information being revealed is then given as in the assumption above.

Comments on the model. The model is motivated by the model of strategic trading in Kyle (85). The more shares the insider buys, the more likely it becomes that $P = 1$ is realised. Similarly, large sales mean that a low price becomes more likely, i.e., that $P = 0$. The prices in this model switch between different extremes for tractability. This model could represent a variety of real world situations. It is quite easy to backtest an investment strategy and find out if the strategy would have delivered high abnormal return "on paper", i.e., returns without incorporating trading costs. However, it is often unclear to the investment manager to what extent the strategy survives trading costs. The importance of uncertainty in scalability and price impact is also confirmed by a current debate among practitioners and academics about the scale at which some investment strategies could be deployed. A recent paper by Frazzini, Israel and Moskowitz (2012) hints at the much higher scalability of some well-known anomalies than had been estimated by previous academic research. This is also in contrast with previous studies that have found the opposite results. Chen, Stanzl, and Watanabe (2002) conclude that only small fund sizes are possible before costs eliminate any profits on value, momentum and size portfolios. Furthermore, Lesmond, Schill and Zhou (2003) find that trading costs make the profits from momentum strategies vanish. However, Asness, Frazzini and Moskowitz (2014) argue that one of the great myths of momentum is that it does not survive trading costs. My model aims to capture a situation where the insider has found a way to predict v (for simplicity, she always perfectly predicts v) and starts trading in the presence of uncertain liquidity. The insider does not know the effect of trades on the asset price. In order to mitigate the problems associated with uncertain price impact, large financial institutions rely on information technology to estimate price impact. Furthermore, some consulting firms help traders estimate price impact. Some fund management companies spend significant resources developing proprietary price-impact estimation techniques. The problem of uncertain price impact should be even more relevant for investment strategies where the trader obtains private information that is related to a specific time window around an event or that is short lived. Furthermore, trading could be required in situations where the price impact is different from the usual, such as shortly before corporate events. For example, there is evidence that there is a greater price impact prior to earnings announcements as shown in, e.g., Kim and Verrecchia (1994). In this case, price-impact costs may be even harder to estimate and predict.

The price impact in this model is linear, the expected price is given by

$$\mathbb{E}[P|\tilde{x}, L] = 1/2 + 1/2\frac{\tilde{x}}{L} \tag{1.20}$$

for some $\tilde{x}$ such that $|\tilde{x}| < L$.

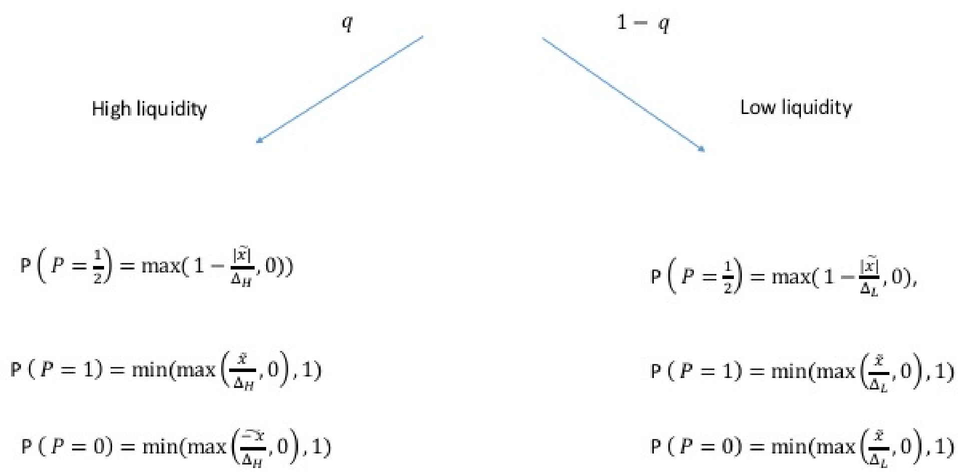

Figure 1.2: Illustration of the model structure

Trader. An insider of type *trader* chooses a market order $\tilde{x}$ in each period to maximise her expected profits. Thus, the trader will serve as the frictionless benchmark. The profit maximisation for the trader means that she will chooses a market orders $\tilde{x}$ in order to maximise her expected profits over the whole game. Each period the profits are given by

$$\mathbb{E}[\tilde{x}(v-P)|v]. \tag{1.21}$$

Fund manager. The fund manager invests on behalf of outside investors that put their money into his fund. To model the behaviour of the investors in our fund manager, I utilise the approach of Berk and Green (2004) as in section 2. Our period has a morning and an afternoon. There is a large mass of outside investors endowed with one dollar each in the morning of the period. They are risk neutral and want to maximise consumption in the afternoon. Their outside option is to invest in a vehicle with a normalised expected return $\mathbb{E}[R^{BM}] = 1$. I fix a "2-20" contract between the fund manager and his investors. As in section 2, the fund manager cannot commit to a trading strategy ex-ante. The compensation contract is as in Section 2. Given some inflows $\tilde{f}$ and some realised return $\tilde{R}$, the fund manager obtains a share $\gamma_M \in (0, 1)$ of the inflow as a management fee and a share $\gamma_P \in [0, 1)$ as a incentive fee. This means that the compensation of the fund received in the afternoon of each period is given by

$$\gamma_M \tilde{f} + \gamma_P \tilde{f}(\tilde{R} - 1) \tag{1.22}$$

where $\tilde{R}$ is the realised return of the fund in that period. The fund manager chooses trades in order to maximise his fee income over the whole game.

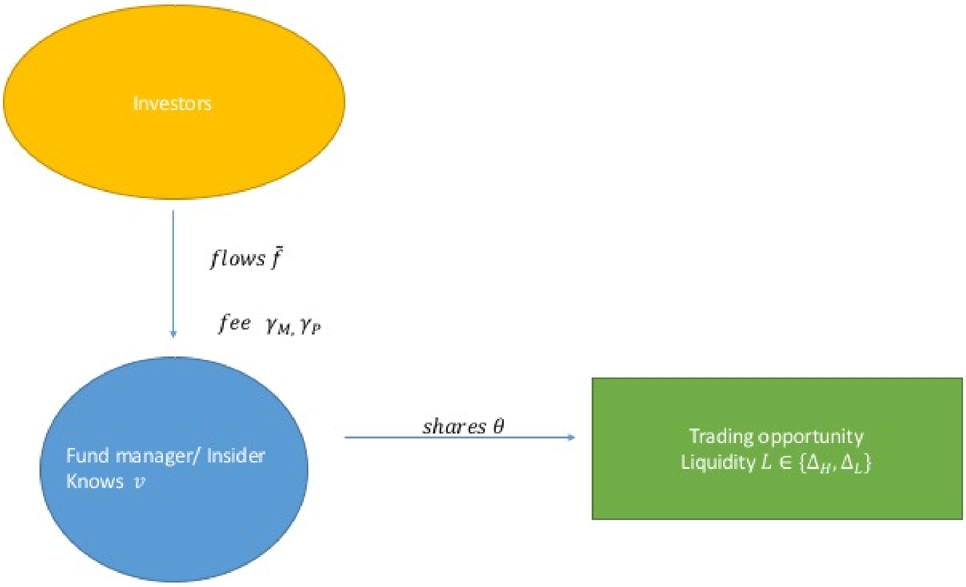

Figure 1.3: The structure of the model

1.3.2 One-period model

For simplification and clarification, I first solve a one-period model in which the insider only faces the opportunity one time. $\Pi(q, \theta) := \mathbb{E}[\tilde{x}(v - P)|v, \tilde{x} = -1^{v+1}\theta]$ is the expected profit of the informed trader as a function of the probability of a deep market when submitting a symmetric market order $\tilde{x} = \theta$ when $v = 1$ and $\tilde{x} = -\theta$ when $v = 0$. It is clear by the symmetry of the model that the insider always buys/sells the same amount of shares given some private information. The parameter θ_I can be interpreted as the *aggressivness* of the insider of type I. This aggressiveness is one of the main objects in which I am interested.

The following lemma summarises the equilibrium of the one-period model.

Lemma 3. *Insiders of both types will trade the same way. Insiders of both types I submit a market order* $\tilde{x}_I = (-1)^{v+1}\theta_{BM}$ *where*

$$\theta_{BM} = 1/2\,\frac{\Delta_H\,\Delta_L}{\Delta_L q + (1-q)\Delta_H}. \tag{1.23}$$

The expected profits of the trader and the expected compensation of the fund manager is given by

$$\Pi(q) := \Pi(q, \theta_{BM}) = 1/8\;\frac{\Delta_H\,\Delta_L}{\Delta_L q + (1-q)\Delta_H}. \tag{1.24}$$

The probability that the information will be revealed is given by

$$\mathbb{P}(P \in \{1, 0\}) = 1/2 = \mathbb{P}(P = 1/2). \tag{1.25}$$

The inflows in the fund are

$$\tilde{f}(q) = \frac{1-\gamma_P}{\gamma_M}\Pi(q). \tag{1.26}$$

The fund is indifferent between any contract γ_M, γ_P*.*

Proof. See the Appendix. □

1.3.3 Discussion

I find that some implications of the Kyle (85) model survive in this framework. One-half of the time, the private information of the insider is fully revealed in the price. This is similar to Kyle (85), where 1/2 of the insiders' private information is incorporated into prices. The amount of information conveyed through prices is not dependent on the variance of noise trading in Kyle (85) and it does not depend on the expected market depth in my model.

Suppose that $q \in \{0, 1\}$. Then, the insiders profits are simply $1/8L$ and, thus, linear in L. This is similar to Kyle (85), where the profits of the insider are linearly increasing in the volatility of noise trading. The assumption that prices switch between different extremes makes the analysis more tractable compared to what I would find with a classical Kyle (85) model when I consider the main results of this paper. In terms of the inflows, the mechanism here is as in the Berk and Green (2004) model. The funds flow in until they obtain the same return as their outside option. The fund manager collects all the rents and trades in the same way and obtains the same expected profit as the trader in the one-period case. Since in the one-period case the two types of insiders behave in the same way, there is no need to distinguish between the two for the remainder of this section.

Corollary 4. *Holding expected liquidity constant, the expected profits of an insider I are lower when there is uncertainty about L compared to when there is not.*
Holding expected liquidity constant, the insider trades less aggressively when there is uncertainty about L compared to when there is not.

Proof. See the Appendix. □

It makes sense that the profits are lower with uncertainty, since uncertainty affects the choice of θ. With uncertainty, the market order is not ideal in both cases, that is, when $L = \Delta_H$ or when $L = \Delta_L$.[13]

The following lemma will be important for the rest of our analysis.

Lemma 5. *The expected profits of an insider given by $\Pi(q)$ are convex in the probability of high liquidity q.*

Proof. See the Appendix. □

[13] The results are similar in Hong and Rady (2002), suggesting that the main mechanism in this paper is similar to what we would find had we assumed a standard Kyle (85) model.

What is the intuition behind that result? A higher probability of the high-liquidity state clearly increases the profits of the insider, since she can trade more without revealing information. However, starting from a situation where $q = 0$, an increase in q also makes the trader more uncertain. This increase in uncertainty causes a small increase in expected utility. As the initial q becomes larger, the situation involves greater uncertainty, so the effect is weaker and the increase is larger. At some point, an increase in q not only increases expected liquidity but also makes the insider more certain of the true state of high liquidity. Thus, the increase in expected utility is large.

1.3.4 Two-period model

In this section, I assume that the insider obtains perfect information about v for two periods in a row. The insider can trade based on her information two times, but L stays the same over the two periods. I still assume that in the beginning of the model everyone knows that $\mathbb{P}(L = \Delta_H) = q$.

In this two-period model, the insider gains experience from trading based on the information in the first period. Her price impact tells her about the depth of the market. In the following, we will see that a price that does not reveal information, i.e., $P = 1/2$ in the first period increases the insider's confidence in high liquidity. A large price impact, i.e., $P \in \{0, 1\}$ will signal low liquidity.

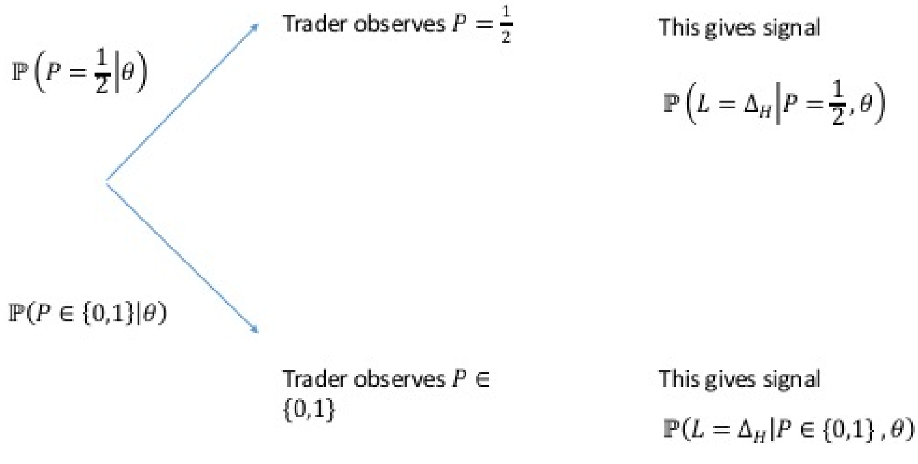

Figure 1.4: Signal structure

The market order θ influences the precision of the signal about L provided by the realised price at time $t = 1$. The price is an asymmetric binary signal. Suppose that $\theta < \Delta_L$. Then, the likelihood ratio is strictly increasing in θ for $P = 1/2$

$$\frac{\mathbb{P}(L = \Delta_H|P = 1/2)}{\mathbb{P}(L = \Delta_L|P = 1/2)} = \frac{1 - \theta/\Delta_H}{1 - \theta/\Delta_L} \frac{q}{1 - q} \tag{1.27}$$

but stays constant for $P \in \{0, 1\}$

$$\frac{\mathbb{P}(L = \Delta_H|P \in \{0, 1\})}{\mathbb{P}(L = \Delta_L|P \in \{0, 1\})} = \frac{\Delta_L}{\Delta_H} \frac{q}{1 - q}. \tag{1.28}$$

Thus, it is clear that the expected informativeness of the price signal is increasing in θ for $\theta < \Delta_L$. Similarly, I can show that expected informativeness is decreasing

in θ for $\theta > \Delta_L$. Let me denote the posterior probabilities $q_2 = \mathbb{P}(L = \Delta_H | P, \theta)$. This is the probability of high liquidity given the first period's realised price P and aggressiveness θ. Updating with Bayes' rule results in
$\mathbb{P}(L = \Delta_H | P_1 = 1/2, \theta) > q$ and $\mathbb{P}(L = \Delta_H | P_1 \in \{0, 1\}, \theta) < q$. On the other hand, if $\theta > \Delta_L$ we obtain $\mathbb{P}(L = \Delta_H | P_1 = 1/2, \theta) = 1$ and $\mathbb{P}(L = \Delta_H | P_1 \in \{0, 1\}, \theta) < q$.

This shows that a high price impact is a sign of low liquidity.

Lemma 6. *(Learning by trading) The expected next-period profits* $\mathbb{E}_1[\Pi(q_2)|\theta]$ *are increasing in aggressiveness* θ *at* $t = 1$ *for* $\theta < \Delta_L$. *The expected profits are maximised with aggressiveness* $\theta = \Delta_L$.

The expected profits at $t = 2$ are increasing in the informativeness of the price signal the insider receives about the state of liquidity L, and this signal depends on aggressiveness θ. A more aggressive trade up to Δ_L increases the informativeness of the price, which can be seen from figure 1.3.1. The difference between the expected prices in the two states of liquidity is highest for $\theta = \Delta_L$. A more precise signal then allows the trader to submit a market order that is more appropriate given the perceived liquidity of the market. This lemma can be seen as a micro-foundation of assumption (1.9) from section 2.

The trader. The trader's trade-off involves managing the price impact today and adjusting her trades in order to learn optimally for tomorrow. She chooses her aggressiveness

$$\theta_{trader} = \underset{\theta}{\operatorname{argmax}}\ \Pi(q, \theta) + \mathbb{E}_1[\Pi(q_2)|\theta]. \tag{1.29}$$

In the following, I consider θ_{BM}, the optimal aggressiveness we would have in the one-period model according to lemma 3 for both types of insiders $I \in \{trader, fund\}$. If an insider would only care about this periods' profits, the optimal order would be $(-1)^{v+1}\theta_{BM}$, so the difference between the optimal trade in the first period of the two-period model and θ_{BM} is how learning influences the trade decision.

Proposition 7 (Learning trader)**.** *The optimal demand of the trader in the first period is given by*
$\theta_{trader} = \frac{\left(3\Delta_H(1-q)+3\,q\Delta_L-\sqrt{5\,(\Delta_H-\Delta_L)^2q^2-6\,(\Delta_H-2/3\,\Delta_L)(\Delta_H-\Delta_L)q+\Delta_H{}^2}\right)\Delta_L\,\Delta_H}{4\Delta_H{}^2(1-q)+4\Delta_L{}^2q}.$
The learning trader trades more aggressive at $t = 1$ compared to the one-period benchmark with

$$\theta_{trader} > \theta_{BM}. \tag{1.30}$$

Proof. See the Appendix. □

I show that the incentive to learn about liquidity leads the trader to "experiment" in the first period. Starting from the optimal trade θ_{BM} that maximises the profits in this period, the trader can perform better by slightly increasing the size of the order in this period. By doing so, she does not hurt this periods' profits much, but she is able to increase the precision of the signal she receives from the price. An unrevealing price then provides a stronger signal of a deep market. This confidence allows her to expect to better exploit the trading opportunity in next period.

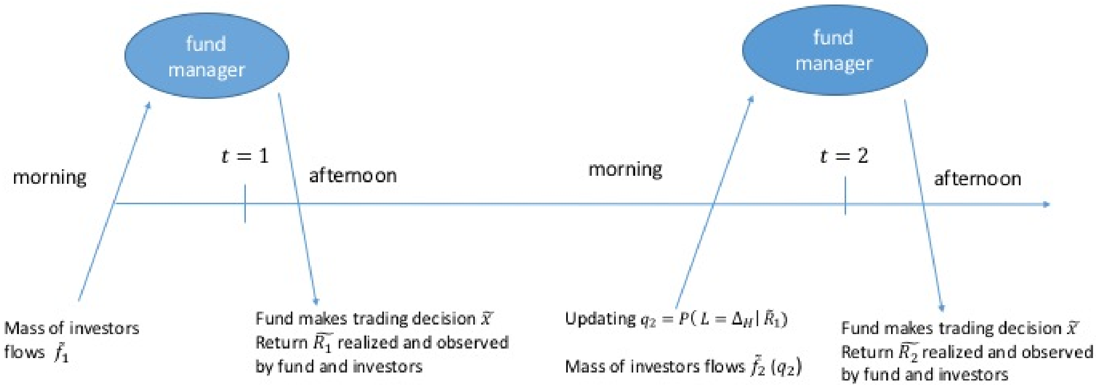

Figure 1.5: Timeline of the model

The fund manager. The fund manager would like to trade in order for him to maximise his overall compensation. At time $t = 1$, he has both direct and indirect incentives. His direct incentives stem from the share γ_P that he obtains of the fund profits at the end of the period, so he would like to produce high returns that result in a high payment. He also has to consider the indirect incentives of future inflows. In the morning of the second period, the forward-looking investors rationally flow in (out) after good (bad) performance, depending on the outcome of the previous period. The flows are not related to skill of the manager in terms of his knowledge of the payoff v. Everyone knows that the manager has perfect information about v. Furthermore, there is also no asymmetric information between the fund manager and the investors. The investors observe the return that the fund realises $\tilde{R}_1$ and can thus deduce the price impact and market order $\tilde{x}$. Then, the inflows at time $t = 2$ given by $\tilde{f}_2(q_2)$ will depend on the signal that the returns send about liquidity. By the results of the previous section, q_2 is increasing in $\tilde{R}_1$; thus, higher returns result in higher inflows. This is consistent with a large empirical literature documenting a positive short-term flow-performance relationship. In my model, high returns signal the scalability of the investment strategy at hand. The next proposition indicates how the fund manager behaves.

Proposition 8 (Learning fund manager)**.** *The fund manager is excessively concerned with the long run. The fund is more aggressive than the trader is in the first period*

$$\theta_{fund} > \theta_{trader}. \tag{1.31}$$

The total expected compensation of the fund manager is less than the expected profits of the trader.

In case $\gamma_P = 0$, we get $\theta_{fund} = \Delta_L$.

Proof. See Appendix. □

1.3.5 Discussion

This result shows that in the presence of an uncertain price impact, the fund manager is not able to optimally take advantage of the investment opportunity at hand. This may shed light on why practitioners are worried about price impact and sometimes fail to implement investment strategies optimally. In my model, it could very well be that a strategy is very scalable ($L = \Delta_H$) but would appear not to be most of the time because funds trade too aggressively. For example, despite evidence that momentum is very scalable from Frazzini, Israel and Moskowitz (2012), Carhart (1997) concludes that transactions costs consume the gains from a momentum strategy implemented by mutual funds. My model suggests that these mutual funds were trading too aggressively.

The fund manager in the first period collects the inflow $\tilde{f}_1$ and then decides on his trade. As for the trader, there are two factors that play into the determination of the optimal aggressiveness θ. The choice of θ determines the first period's profits as well as the informativeness of the price signal. On the one hand, the fund manager obtains a share γ_P of the profits in this period. On the other hand, the full value of the increase in information that can be obtained from the price signal accrues to the fund manager. The fund sets

$$\theta_{fund} = \underset{\theta}{\operatorname{argmax}}\ \gamma_P \Pi(q, \theta) + \mathbb{E}_1[\Pi(q_2)|\theta]. \tag{1.32}$$

As we see from lemma 6, the next period's profits are increasing in aggressiveness. Holding some level of expected liquidity constant, the fund manager is able to deliver a higher return in the second period with a more precise signal. This knowledge increases his compensation through two channels: high inflows from investors who know that the fund gained experience in the morning of period 2 and high expected profits, of which the fund receives a share γ_P. These channels together let the fund capture the full expected profits at $t = 2$, as can be seen by lemma 3. Thus, as in section 2, the fund manager is over-incentivised to learn. The fund manager gives up too much profit in the first period to let his investors and himself learn more.

However, a high amount of experimentation and therefore a high expected price impact in the first period is anticipated by investors. The first period inflows $\tilde{f}_1$ depend only on the expected profit in the first period. Since the expected profit suffers from excessive trading, the initial inflows are small. The competitive outside investors always need to break even. In the end, the fund manager bears the costs of excessive experimentation, and his overall expected compensation suffers. The fund obtains, in expectation, $\Pi(q, \theta_{fund}) + \mathbb{E}_1[\Pi(q_2)|\theta_{fund}] < \Pi(q, \theta_{trader}) + \mathbb{E}_1[\Pi(q_2)|\theta_{trader}]$.

The result that funds are excessively concerned with learning is quite surprising. Consistent with reality, in my model, a high return is followed by high inflows into the fund. Thus, at first glance, it might seem that a fund manager who cares how investors view him would try to obtain the highest possible expected return today to gather more flows tomorrow. This might lead to insufficient learning, and the trades of the fund would be close to θ_{BM} to impress investors and capture inflows. However, trades θ at $t = 1$ cannot change the expected estimation of liquidity of investors and funds at $t = 2$. The only thing it can influence is the precision of the signal about L. Thus, the fund chooses a strategy with low *average* returns that does not deliver high returns very often. However, if returns are high, they are very high, and it can be learned with high certainty that the market is deep. The convexity result from lemma 5 comes into play here. The high aggressiveness of the fund results in low average returns but high average inflows. There is empirical evidence that indirect incentives coming from future flows are of fundamental importance for both mutual funds and hedge funds. For example, this is documented in Brown, Harlow, and Starks (1996) and Chevalier and Ellison (1997) for mutual funds. For hedge funds, Lim, Sensoy and Weisbach (2015) show that future compensation from future flows matters roughly four times as much as the direct compensation.

1.4 Implications

1.4.1 Limits of Arbitrage

In reality, we find some investment strategies where trading based on observable signals generates abnormal returns. These asset market anomalies do not seem to have fully disappeared after articles about them are published and market participants should have become aware of them (McLean and Pontiff (2015)). For many of the anomalies, it is quite difficult to find risk-based explanations. This raises the question of why these anomalies persist. The limits of arbitrage have been suggested as a potential reason (Shleifer and Vishny (1997)). My model suggests that uncertain scalability could also be a reason why there is insufficient trade on these anomalies to make them disappear. A leading example may be momentum, the scalability of which practitioners and academics debate. For example, Dimensional Fund Advisors refuses to implement a large-scale momentum strategy because the fund claims that the robustness of the strategy to trading costs in light of the higher turn-over involved is questionable.[14] The fund clearly acknowledges the existence of momentum before trading costs. As noted by Assness et al. (2014), the low scalability of momentum seems to be a myth. My model indicates that uncertainty about scalability alone could lead funds to shy away from exploiting opportunities even in a situation where the expected scalability is quite high. I assume that the trading opportunity with uncertain $L \in \{\Delta_L, \Delta_H\}$ is one strategy in which the insider could engage. Furthermore, I assume that the insider can, as an alternative, choose and commit to an "old" strategy with known liquidity L_{old} and obtain private information about a different asset with dividend v_{old} for two periods. The only difference between the old and the new strategy is that the new strategy has uncertain liquidity.

I look at how the strategy choice differs between a trader and a fund manager.

[14]This is the answer they give investors when asked why they do not offer a momentum fund (www.ifa.com/articles/momentum-factor-empirical-update).

In the case $L_{old} = \mathbb{E}[L] = (1-q)\Delta_L + q\Delta_H$, both the fund and the trader would obviously choose the old strategy, since they like the certainty of the liquidity regime. Let

$$\hat{L}^{I}_{old} \qquad\qquad I \in \{trader, fund\} \tag{1.33}$$

denote the cutoff liquidity of the old strategy where an insider of type I would choose and commit to a new strategy. Clearly, a low $\hat{L}^{I}_{old}$ indicates a preference for certainty.

Corollary 9. *The fund will be more adverse to uncertainty about liquidity*

$$\hat{L}^{trader}_{old} > \hat{L}^{fund}_{old}. \tag{1.34}$$

We always have $\hat{L}^{trader}_{old} > \Delta_L$, but we might have $\hat{L}^{fund}_{old} < \Delta_L$.

Proof. See the Appendix. □

The corollary shows that even though it is clear that the new strategy is more scalable than the old one ($\hat{L}^{fund}_{old} < \Delta_L$), the fund manager may still prefer to deploy the old strategy. The low compensation that the fund receives in the initial period because of inefficient learning may outweigh the benefits of higher scalability. The problem of the fund is the inability to commit to not engage in inefficiently aggressive trades.

1.4.2 Asymmetric Flow-Performance Relationship

The profit that the manager generates before fees $\tilde{R}$ could be either zero (in the case of a price impact) or positive. In the following, I find that learning results in a convex flow-performance relationship. The return that the fund can generate is dependent on θ. The fund can generate profits $\frac{1}{2}\theta$ with probability $\mathbb{P}(P = 1/2|\theta)$ or 0 with probability $\mathbb{P}(P \in \{0, 1\}|\theta)$.

Corollary 10. *Suppose $\theta < \Delta_L$. The inflows $\tilde{f}_2 - \tilde{f}_1$ are convex in the profits that the fund makes in the first period.*

Even though there are strong inflows into a fund after high performance, there might not be any outflows following poor performance. This is because the initial inflows were small based on anticipated experimentation and the fund's lack of knowledge about L. Over time, the flows are related to learning *about* the fund (which type of strategy L) and also by learning *by* the fund. Given some expected level of L, the second period's inflows will be higher, since the fund has learned and no longer has an incentive to experiment. There is a lot of empirical evidence documenting that the flow-performance relationship is asymmetric and convex for mutual funds (see Chevalier and Ellison (1997) and Sirri and Tuffano (1998)). In particular, Chevalier and Ellison (1997) show that the relationship is more convex for young funds. This suggests that the learning explanation given in my model could indeed be correct. Furthermore, evidence of the same phenomenon for hedge funds is documented by Baquero and Verbeek (2013).

1.4.3 Excessive trading

Edelen, Evans and Kadlec (2007) show that for mutual funds, the scale effects are broadly consistent with Berk and Green (2004). However, they show that funds seem to trade well past the point at which the marginal cost of a trade is equal to the marginal profit increase. This means that they find that mutual funds seem to trade excessively. My model may help explain this seemingly excessive trading.

1.4.4 Return persistence and Hedge fund closures

So far, I have maintained the assumption that the fund opens up in the interim period. The problem of inefficient "long termism" can be overcome if the fund can commit to remaining closed to new investment in the interim period. In principle, hedge funds can choose to remain close to new investment. Indeed, there is evidence that many hedge funds are closed, as shown in Yin (2015). Why a hedge fund would remain closed may be puzzling, since in principle, a hedge fund could always increase its compensation by accepting more money, and the hedge fund would not necessarily have to invest all the available funds. Furthermore, there is empirical evidence that hedge fund managers' total compensation grows with fund size (see Yin (2015)). If I assume that the fund could commit to closing, the fund would always choose to do so in my model. In this case, my model may speak to the empirical evidence that hedge funds exhibit return persistence. A series of papers note that hedge funds exhibit persistent abnormal performance (see Jagannatha, Malakhov and Novikov (2010) and Fung, Hsieh, Naik and Ramadorai (2008)). The problem described in my paper should be more relevant to new hedge funds. It may explain the finding in Aggarwal and Jorion (2010) that hedge fund persistence is significantly stronger for new hedge funds.[15]

Corollary 11. *Suppose that the fund can commit to closing on the morning of $t = 2$. The fund is able to implement the first-best strategy, i.e., $\theta_{fund} = \theta_{trader}$ and is paid the expected profits of the trader. There is return persistence, i.e.,*

$$\mathbb{E}[\tilde{R}_2 | \tilde{R}_1] \tag{1.35}$$

is increasing in $\tilde{R}_1$.

If the fund manager can commit to closing in the intermediate period, he would very much like to do so. This increases the inflows that he can gather in the first period. The initial investors are now willing to suffer subpar returns in the first period, since they know that they will also benefit from the fund's learning in the next period. Since the fund manager shares both the profits and benefits of learning in the same way, he now experiments optimally and is paid the full expected profits.

[15] As one commentator from the financial industry with regards to young hedge funds that close to new investment said, "What you see with small or newer managers is they are engaging in strategies that are different and new and haven't been seen before" (NYT September 7,2011).

1.4.5 Conclusion

This paper presents a new type of agency problem that prevails between fund managers and investors given the way fund managers are rewarded in the real world. I show that the short-term incentives of fund managers may result in inefficient "long termism". In the context of an uncertain price impact, the fund manager faces incentives to learn excessively about his trading opportunity and the market in which he is trading. In my model, this means that, on average, he trades too aggressively and has too great an impact on the price. I show that the model may explain some empirical facts about the investment management industry, such as the limits of arbitrage, excessive trading, convexity of the flow-performance relationship and hedge fund return persistence.

It may be interesting to consider a dynamic model with more periods. Furthermore, it would be interesting to consider a more standard Kyle (85) framework, as in Collin-Dufresne and Fos (2016), with uncertain stochastic noise trade volatility and to analyse how the insider experiments over time. One could also focus on the case of a profit-maximising trader and leave out the agency problem.

Chapter 2

Reputation, Fund Flows and Asset Prices

2.1 Introduction

Over the last few decades, we have experienced strong growth in the holding of financial assets by money managers. Typical compensation schemes in the industry indicate that money managers would like to impress investors and thereby increase the assets under their management. In reality, there are frictions in the market for asset management, as it is costly for investors to search for a new manager (Sirri and Tufano (1998)). How convincing the results of an asset management company are to the wider universe of investors affects how many more investors will invest with the company. This flow-performance relationship for mutual funds is well documented in the empirical literature (Chevalier and Ellison (1997)). In this paper, I develop a model of an asset market and a market for portfolio management. Endogenous flows arise from changes in a fund manager's reputation. I show that when the asset management market is inefficient, fund managers' incentives may result in the over-pricing of high-beta assets, which is consistent with the empirical evidence (Black, Jensen, and Scholes (1972)).

My model features a large set of investors that receives endowments and invests with fund managers. Some fund managers have better private information about the payoff of an asset than others. The type of a fund manager is private information. Investors use realised returns to update their opinions about their managers. Fund managers that achieve high returns experience inflows, while under-performing fund managers experience outflows. The more investors switch to informed funds, the smaller the benefits of switching. The size of the inflows is related to the benefits of investing with an informed fund manager. In a model similar to that of Grossman and Stiglitz (1980), the benefit of information is decreasing in the amount of money managed by informed fund managers. Furthermore, I assume that it is costly for investors to search for a better fund manager. An investor who is richer cares more about having an informed fund and cares less about the constant cost of switching. Thus, the value of reputation is state dependent. In the initial period, fund managers take into account the future inflows they could receive. The state-dependent value of a reputation can produce a high-beta asset that trades at a premium. Furthermore, assets differ in the change in reputation they provide. In my model, an asset in short supply will be held by few funds, which potentially provides large reputation improvements. However, holding an asset in short supply is very costly in the case of under-performance. In this case, investors in an under-performing fund would be very inclined to switch funds. I show that in case of inefficient asset management markets, the former effect dominates, and the smaller the asset supply, the higher the asset price.

There is a growing literature describing the pricing of assets in the presence of career concerns. These papers typically do not consider endogenous fund size. Dasgupta and Prat (2006) provide a microfoundation for career concerns and show that fund managers may have incentives to make trades that are not based on information. In Dasgupta and Prat (2008), information aggregation in prices with career-concerned managers is analysed. Finally, Dasgupta, Prat and Verardo (2011) show that assets may trade at a reputational premium (discount) in a sequential trading model. The paper that is most closely related to my paper is that of Guerrieri and Kondor (2012). In this paper, the simplistic structure of the asset market is similar. However, Guerrieri and Kondor (2012) make an extreme assumption of decreasing returns to scale, as each fund manager can only invest 1$. One of the central results of their paper reflects the impact of career concerns on asset prices – risky assets will trade at a reputational premium (discount). When the probability of a high payoff is high, a career-concerned fund manager wants to hold the asset not only for the chance of a high payoff but also for the relatively high chance of improving his or her reputation. These preferences drive up the prices of the risky asset. In this paper, I go a step further and explicitly model the rewarding of a good reputation with inflows. A recent paper that shares some predictions with this paper is that of Garlenau and Pedersen (2015). Both papers consider the market for asset managers with search costs. In their paper, there are no reputational concerns, so the two papers consider quite different issues. Their paper focuses on the relations between price efficiency and asset management market efficiency, fee determination and related questions. They obtain some predictions that are similar to those in this paper in a more standard asset pricing framework. However, they abstract from agency problems that can distort asset prices. In contrast, this paper focuses on the asset pricing implications of the reputational concerns of fund managers. My model is related to the literature on reputation-based herding, which can be traced back to Scharfstein and Stein (1990). In my paper, the fund managers do not make sequential choices. They make their decisions in isolation. The rewards stemming from flows depends on reputations in an endogenous form. Under certain conditions, the asset that most funds hold will trade at a premium, whereas the opposite pattern may hold if the asset markets are inefficient. In Vayanos and Woolley (2008), fund flows generate momentum, reversal, amplification, co-movement and lead-lag effects. Their multiple-period setting allows them to concentrate on different issues. In their model, flows into a single active fund are generated by exogenous changes the active fund's efficiency parameter. In my model, there is a set of active funds, and fund flows are motivated by differences in skill within a mass of fund managers. Kaniel and Kondor (2008) introduce an exogenous convex flow performance relationship in a standard Lucas economy. There is a large empirical literature testing the CAPM. Black, Jensen, and Scholes (1972) show that the security market line is too flat relative to the CAPM. There is also some research that tries to explain the beta anomaly. A related paper is that of Karceski (2002). In his model, funds care more about out-performing the benchmark in good times because there are more inflows, which is similar to this paper. However, in Karcesky (2002), the flows are completely exogenous, whereas in this paper, the flows are endogenous and create the anomaly. This provides new predictions about the beta anomaly; for example, it is stronger when asset management markets are less efficient. Baker, Bradley, and Wurgler (2011) posit benchmarking as a possible explanation for the phenomenon. Here, irrational investors demand high-beta assets, and fund managers and benchmarking create the limits of arbitrage, resulting in the anomaly. In a related paper, Buffa, Vayanos, and Woolley (2013) develop a theoretical framework where benchmarking amplifies the high-beta/low-return anomaly. In their model, managers wanting to reduce deviation from a benchmark have incentives to buy more volatile (high-beta) stocks because these stocks explain a large share of overall market volatility. In contrast to these paper, in my model, there is no benchmarking. Frazzini and Peddersen (2013) provide an alternative explanation for the beta anomaly. In their paper, leverage-constrained investors hold high-beta stocks, since they would like high expected returns and cannot move on the capital market line because they are constrained. The result is that high-beta stocks have low alphas. However, there is some empirical evidence that the relationship between beta and expected returns is almost flat (Baker, Bradley, and Wurgler (2011)). The mechanism I discuss in the present paper may help explain this anomaly. In the empirical part of their paper, Frazzini and Peddersen confirm that the results of Black, Jensen, and Scholes (1972) hold 40 years later not only for stocks but also for other asset classes. Furthermore, Hong and Sraer (2014) generate the anomaly through disagreement about market fundamentals. The disagreement is higher for high-beta stocks, since those are more sensitive to market movements. In combination with short-sale constraints, this leads to high prices for these stocks driven by the demand of optimists.

The remainder of this paper is organised as follows. In Section 2, I illustrate the main mechanism of the paper using an example. In Section 3, I present the main model. In Section 4, I solve for the equilibrium. In Section 5, I relate my model and results to the empirical evidence and conclude. All proofs are included in the appendix.

2.2 The main mechanism in a nutshell

This section illustrates the main mechanism of this paper and shows how it relates to the previous literature on asset prices and career concerns. Suppose there is a large mass of risk-neutral funds that can invest in a risk-less asset with exogenous return R or in a risky asset that gives a dividend $v = 1$ with probability $(1 - q)$ and $v = 0$ otherwise. Let us suppose that for the market to clear, these managers have to be indifferent between the two assets. Let me introduce a reward W, which funds can obtain if they pick the right asset. The reward is related to the funds' reputation they get from picking the right asset. Let γ denote the share of profits funds receive. *Suppose that W is a constant*; then, we need the price P to clear the market:

$$\gamma \frac{1-q}{P} + (1-q)W = \gamma R + qW. \tag{2.1}$$

The left hand side of (2.1) is the expected payoff of a mananger who invests in the risky asset. The right hand side is the expected payoff of a manager who invests in the risk-less asset. These two expected payoffs should be equal to make uninformed fund managers indifferent.

In Dasgupta, Prat and Verardo (2011) and Guerrieri and Kondor (2012), the *reputational premium* (discount) derives from the fact that when $q < (>)1/2$, $P > (<)\frac{1-q}{R}$. It is clear that when $q = 1/2$, there is no reputational premium. However, in reality, it is unlikely that W is a constant. It may depend on the state of the world. A fund manager's compensation depends on the assets under management. Following good performance, they manage to improve their reputations, which might in turn generate inflows. In my model, I focus on the case when $q = 1/2$ to shut down the familiar reputational premium effect, but the price distortion stems from the fact that the reward W may be *state dependent.*[1] The price equation is then

$$\gamma\frac{1}{2}\frac{1}{P} + \frac{1}{2}\mathbb{E}[W|v = 1] = \gamma R + \frac{1}{2}\mathbb{E}[W|v = 0]. \tag{2.2}$$

The risky asset may then trade at a premium (discount) when

$$\mathbb{E}[W|v = 1] > (<)\mathbb{E}[W|v = 0]. \tag{2.3}$$

I call this the *flow premium.* First, the same reputation may lead to different inflows in different states of the world. For example, it is possible that a reputation is more valuable in good times. In these times, investors have more money to invest, and they will invest with reputed funds. Second, how many other funds invest in the risky asset may matter. If the asset is in low supply in equilibrium, fewer funds will do so. Thus, out-performing by buying the asset induces a large improvement in reputation and high inflows. These flows come from the large set of all other funds, so the outflows will be distributed among a large set of funds and might be small.

To analyse this mechanism, I develop a model where investors face a decision to

[1] This reward should denote the difference in the expected utility of a fund manager who was right compared to that of a fund manager who was wrong.

either invest in a new fund or stay with their current fund. Investors will invest in funds that have out-performed until the expected utility of doing so is equal to that of staying with the current fund. The endogenous asset prices will make the decreasing returns to scale endogenous, so the flows into active funds will be endogenous. Unlike the seminal paper by Berk and Green (2004), decreasing returns to scale do not come from capacity constraints at the fund level but from capacity constraints at the industry level. In my model, what matters is the aggregate amount of money managed by informed active funds. This paper combines rational flows and an asset market model with career-concerned fund managers.

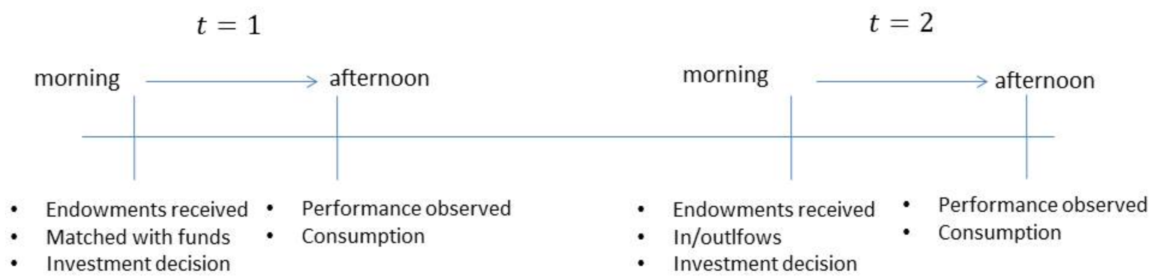

Figure 2.1: Timeline of the model

2.3 The Model

I consider a two-period economy with time $t \in \{1, 2\}$, where there is a round of trading in each period t. There is a risky asset and a risk-free asset. Each period can be divided into morning and afternoon. Everyone is risk-neutral.

There is a unit mass of *investors*. In each period, each investor obtains an endowment in the morning. Furthermore, before each trading round, investors invest with fund managers. The investors can only consume in the afternoon. The only way for investors to transfer their endowment to the afternoon is by investing with a fund manager. The endowment process represents a state of the economy in which a large endowment represents good times when the overall economy is doing well. In the initial period, each investor receives one dollar

$$e_1 = 1 \tag{2.4}$$

and in the second period they each receive the same e_2, where

$$\mathbb{P}(e_2 = 1 + \delta) = 1/2 \qquad \mathbb{P}(e_2 = 1 - \delta) = 1/2. \tag{2.5}$$

I assume that $\delta \in (0, 1)$.

There is a unit mass of *fund managers*[2]. A small mass θ of these fund managers is informed, and the type of the fund is private information. Fund managers are paid through an exogenous contract; they simply receive a share of the fund $\gamma > 0$. If a fund manager manages α dollars at the beginning of the period and the return realised is $\tilde{R}$, his compensation in the afternoon of a period is[3]:

$$\alpha\gamma\tilde{R}. \tag{2.6}$$

The goal of fund managers is to maximise the fee they earn from investors.[4] The

[2]Sometimes referred to simply as "funds".

[3]By returns, I mean gross returns.

[4]The assumption on the contract perfectly aligns the incentives of fund managers and investors in the absence of fund flows and reputation concerns.

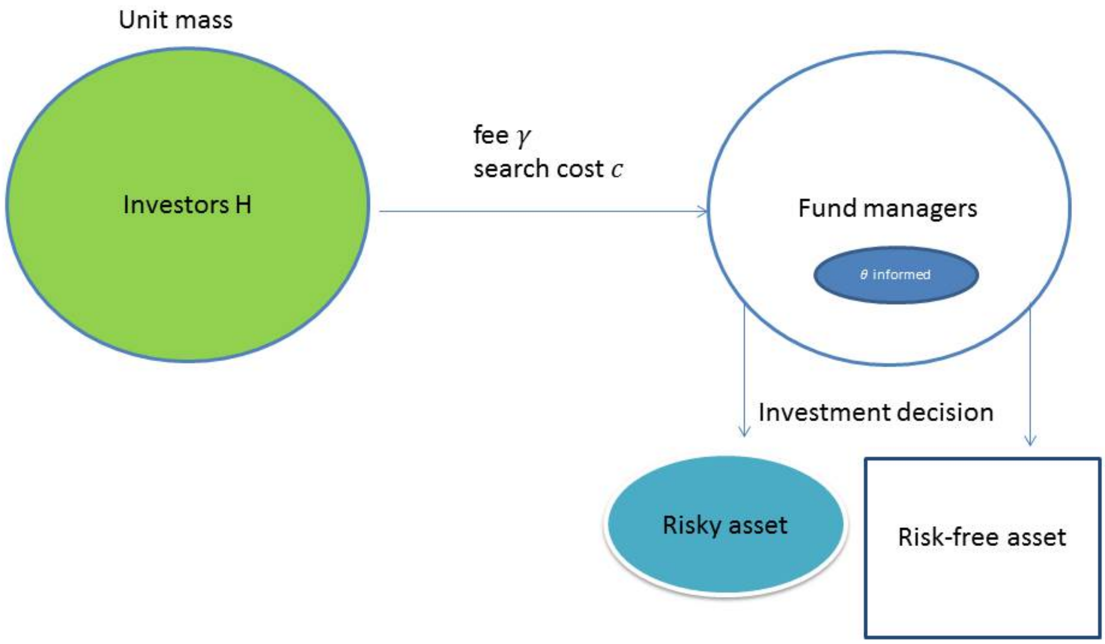

Figure 2.2: Summary of flows in the model

investors and the fund managers discount future payoffs at a rate $\omega > 0$.[5]

Whenever investors want to find a new fund manager, they incur a fixed search cost $c > 0$. The cost c of switching funds can be interpreted in a number of ways. It could represent the effort cost of becoming informed about the fund's performance relative to other funds, which seems reasonable for retail investors. It could also represent due diligence for new funds. In reality, due diligence is a costly and sometimes lengthy process, consisting of evaluating various aspects of the asset management firm. Garlenau and Pedersen (2015) make a similar

[5]The discounting is between the time periods t. There is no discounting from the afternoon to the morning of a period.

assumption in their paper.[6] We could also think of c as simply a measure of investor "responsiveness" to fund performance.

The fund managers can invest in an asset that pays aa dividend of $v_t \in \{1-d, 1+d\}$ in the afternoon of each period t. I assume that $d \in (0,1)$. Furthermore, there is a risk-less asset with a perfectly elastic supply, which gives an exogenous risk-free return R. I assume

$$\mathbb{P}(v_t = 1+d) = 1/2. \tag{2.7}$$

I assume

$$\mathbb{P}(e_2 = 1+\delta | v_1 = 1+d) = \beta \qquad \mathbb{P}(e_2 = 1+\delta | v_1 = 1-d) = 1-\beta \tag{2.8}$$

where $\beta \in (0,1)$. The risky asset has a nominal supply $b > 0$.[7] The informed fund managers have perfect private information about v_t in the morning of each time period t.
An asset with a high β in my model has a payoff that is positively correlated with good times in the overall economy. That is, if the asset has a high payoff, most of the time, investors receive a large endowment ($e_2 = 1+\delta$) in the next period. The interpretation is that the investors also have other non-modelled investments that resemble a "market portfolio". Following up high returns of this portfolio, they have a lot of money to invest with fund managers next period (e_2 high).

There is a mass of *noise traders* who have a total dollar amount $\Delta > 0$ to invest.

[6] In my model, paying the cost c allows investors to match with a new fund, but in contrast to Garlenau and Pedersen (2015), it does not inform investors about the type of fund (informed or uninformed). The only information the investors have to update their opinions about the funds is past performance. Furthermore, in contrast to their model, I have no need for "noise allocators". All investors are the same, have the same search cost c, and are rational.

[7] As in Guerrieri and Kondor (2012), the nominal supply can be interpreted as a mass of b one period-lived borrowers that supply inelastically assets to finance one unit of consumption. These borrowers repay a low amount in the afternoon with probability 1/2.

I assume that $\tilde{\Delta}_t$ dollars are invested in the risky asset. I assume

$$\tilde{\Delta}_t \in U[0, \Delta]. \tag{2.9}$$

Here U denotes the uniform distribution. The realisation of $\tilde{\Delta}_t$ is independent of everything else and is not observed by investors or fund managers. The noise traders become uninformed funds in the next period.[8]

The asset markets are similar to those in the model of Guerrieri and Kondor (2012).[9] In the morning of each period, all funds submit demand schedules to an auctioneer. I restrict the fund managers to three choices: they can invest all funds in the risky asset $d = 1$, they can fully invest in the risk-free asset $d = 0$, or they can be indifferent between the two options $d = \{0, 1\}$. A demand schedule is a map $d : \mathbb{R}_+ \to \{0, 1, \{0, 1\}\}$, for each price $p \geq 0$ it contains a demand. The auctioneer collects the demand schedules and selects the price. The managers receive either the risky asset only or the risk-free asset only.[10]

2.3.1 Optimisation problems

I denote by $a_t := (\tilde{\Delta}_t, v_t, e_t)$ the realisation of the shocks to the model at time t. I define $P_t(a_t)$ as the equilibrium price function at time t. In our rational expectations equilibrium, both uninformed and informed managers will maximise their expected utility conditional on the prices they observe. Thus, at each time t, uninformed managers choose the demand schedules that maximise their expected utility:

[8]The noise traders could represent emerging funds and have Δ investors that may move funds in the next period. However, the Δ investors may mistake their noise trader for an informed fund.

[9]The way I define the portfolio choice problem makes simplifying assumptions. There are no short sales, and there is no possibility to take on leverage. Both of these assumptions seem consistent with reality when we think about mutual funds.

[10]This assumption is less restrictive then it may seem. Under reasonable assumptions regarding out-of-equilibrium beliefs, risk-neutral fund managers would never want to diversify.

$$V_t^U(\alpha_t) = \max_{d_t \in \{0,1,\{0,1\}\}} \mathbb{E}_t[\gamma \alpha_t \tilde{R}_t + \omega V_{t+1}^U(\alpha_{t+1})|P_t] \tag{2.10}$$

where the informed managers face the same problem, except that they can condition on v_t. Here, V_t^U is the expected utility of an uninformed fund, and we have $V_t^U = 0 \forall t \geq 3$ (since the model has only two periods).

I will focus on a symmetric equilibrium where all informed fund managers and all uninformed fund managers submit the same demand schedules. Thus, there are two distinct demand schedules. There are the demand schedules for informed funds $d_t^I(P_t, v_t)$ and the schedules for uninformed funds $d_t^U(P_t)$. The auctioneer then sets an equilibrium price P_t to satisfy the market-clearing condition. Let $X_t(d_t, a_t)$ denote the equilibrium probability of obtaining the risky asset given the realisation of a_t and demand schedule d_t. Let $\tilde{\theta}_t$ denote the total money managed by informed funds at time t. Then, market clearing requires

$$(e_t - \tilde{\theta}_t)X_t(d_t^U(P_t), a_t) + \tilde{\theta}_t X_t(d_t^I(P_t, v_t), a_t) + \tilde{\Delta}_t = b. \tag{2.11}$$

Since all the funds in the model are infinitesimal, by the law of large numbers, $(e_t - \tilde{\theta}_t)X_t(d_t^U(P_t), a_t)$ is the total dollar amount of the risky asset held by uninformed funds, and $\tilde{\theta}_t X_t(d_t^I(P_t, v_t), a_t)$ is the amount held by the informed funds. An allocation $X(d_t, a_t)$ for a given demand schedule d_t is *consistent with a manager's demand* if and only if $\forall a_t \; X(1, a_t) = 1, X(0, a_t) = 0, X(\{0,1\}, a_t) \in [0,1]$.

Since the investors all have to match with a fund in the first period and the type of a fund is unkown to investors, the investor optimisation problem before $t = 1$ is trivial. Each investor matches with a fund randomly. An investor observes the performance of his fund after $t = 1$ and can choose to pay the matching cost c to invest in a new fund. Each investor i has two options. The investor can search for a new fund and invest with the fund that has the best reputation. In this case, the investor incurs the cost c.[11]

[11] Given risk neutrality and infinitesimal investors, it is clear that an investor will always invest all

The investor can also choose to stay with his current fund throughout the period and the game. In this case, the investor does not incur search costs. In an equilibrium, no investor can gain by changing their switching decision.

2.3.2 Equilibrium concept

I summarise the equilibrium concept in the following definition.

Definition 12. *A rational expectations equilibrium constitutes demand schedules $d_t^I(P_t, v_t)$, $d_t^U(P_t)$ a price function $P_t(a_t) \in [\frac{1-d}{R}, \frac{1+d}{R}]$, an allocation function $X_t(d_t, a_t)$ and investors investment decisions such that for each $t \in \{1, 2\}$:*
1.) For each realisation a_t, a price P_t such that the asset market clears, i.e.,

$$(e_t - \tilde{\theta}_t) X_t(d_t^U(P_t), a_t) + \tilde{\theta}_t X_t(d_t^I(P_t, v_t), a_t) + \tilde{\Delta}_t = b. \tag{2.12}$$

2.) The demand schedules solve the informed and uninformed managers' optimisation problem (2.10)*;*
3.) The asset allocation is consistent with the demand schedules;
4.) The investors beliefs are consistent with Bayes' Rule on the equilibrium path;
5.) The investors fund-switching decisions are optimal.

2.3.3 Assumptions

The following two assumptions make sure that there are no corner solutions. I assume that there exists a real number $\kappa \in (1/2, 1)$ such that

$$b/(1+\Delta) \in (1/2, \kappa) \qquad \frac{\Delta}{\theta} \in \left(1+\delta, \frac{1-\delta}{\kappa}\right). \tag{2.13}$$

I assume that δ and κ are small enough that this interval is non-empty. I assume for c

$$c \in \left(0, (1 - \frac{(1+\delta)\theta}{\Delta}) \frac{(1-\delta)(1-\gamma)\theta d R}{2b}\right). \tag{2.14}$$

of his or her money in one of the funds. Hence, it is no restriction to assume this.

Furthermore, I assume

$$2\Delta < \min\left(b, 1-\delta-b\right) \tag{2.15}$$

and for the discount factor ω

$$\omega < \frac{\theta}{\Delta}\frac{d}{d+1}. \tag{2.16}$$

I assume that all these assumptions hold throughout the paper in all sections. I say that a fund *makes the right decision* when the fund fully invests in asset that makes the higher return ex post. Thereby, a fund maximises the returns possible given the investment options.

2.4 Equilibrium

2.4.1 Price function

Let $\tilde{\theta}_t$ denote the mass of of money managed by informed funds at time t.

The next definition introduces the asset market equilibrium I will focus on in my model. Let the random variable z_t denote the total dollar demand for the risky asset of noise traders and informed managers

$$z_t = \tilde{\Delta}_t + \tilde{\theta}_t \mathbf{1}_{v_t=1+d}. \tag{2.17}$$

.

Definition 13. *A simple equilibrium is a rational expectations equilibrium in which at time $t \in \{1,2\}$ there exist the following revealing equilibrium regimes:*
If $z_t \in [0, \tilde{\theta}_t)$, then $P_t = \frac{1-d}{R}$.
If $z_t \in (\Delta, \tilde{\theta}_t + \Delta]$, then $P_t = \frac{1+d}{R}$.
In the revealing, regimes fund managers submit $d_I(P_t, v_t) = d_U(P_t) = \{0,1\}$.
There exists an unrevealing equilibrium regime:
If $z_t \in [\tilde{\theta}_t, \Delta]$, then $P_t = \hat{P}_t$,

where $\hat{P}_t$ is the price that makes it optimal for uninformed fund managers to submit $d_U(\hat{P}_t) = \{0,1\}$, and informed fund managers submit $d_I(\hat{P}_t, v_t) = \mathbf{1}_{v_t=1+d}$.

The main goal is to find the price $\hat{P}_1$ such that a simple equilibrium exists. It is clear that $\hat{P}_2 = \mathbb{E}[v]/R = 1/R$, since there are no more reputational concerns at $t = 2$ and no information about v_2 is transmitted in an unrevealing regime[12]. The above definition implies that an uninformed manager employed at $t = 2$ always has expected returns of R and that uninformed fund managers are the marginal investors. Hence, I simply have:

$$\mathbb{E}[V_2^U(\alpha_2)] = \alpha_2 \gamma R \tag{2.18}$$

where α_2 denotes the fund's assets under management at time $t = 2$ (at $t = 1$, each fund has 1 dollar). Hence, the expected utility of uninformed fund managers depends on the assets under management in a simple linear way. It is important to note that informed fund managers always make the right decision in a simple equilibrium. It is clear that $\tilde{\theta}_1 = e_1\theta = \theta$, since $e_1 = 1$.

Lemma 14. *In a simple equilibrium, suppose we had an unrevealing regime at $t = 1$. Let π denote the reputation of the out-performing funds of the first period. In the second period, the expected per-dollar benefit of investing with one of those funds is given by:*

$$\Pi(\tilde{\theta}_2) = (1-\gamma)\pi\frac{1}{2}dR\frac{\Delta - \tilde{\theta}_2}{\Delta}. \tag{2.19}$$

Then, a manager who does not make the right decision is uninformed with probability one. A manager who makes the right decision is informed with probability

$$\pi = \frac{\theta}{b} \text{ if } v_1 = 1 + d \qquad \pi = \frac{\theta}{1 - b + \Delta} \text{ if } v_1 = 1 - d. \tag{2.20}$$

After a revealing regime, no investor will switch.

[12]That this is the case will be shown in the proof of proposition 16

2.4.2 Investor flows

Investors compare the expected benefit of changing their fund to staying after the first period. It is obvious that no investor will shift into an underperforming active fund, since such a fund is certainly uninformed. The first term in (2.19) reflects the fees. The probability to match with an informed fund is given by π. In this case, the expected benefit is $\frac{1}{2}dR$ but only the in case of an unrevealing regime, which will happen with probability $\frac{\Delta-\tilde{\theta}_2}{\Delta}$. The probability that informed managers can exploit their informational advantage is decreasing in the total amount of money they manage.

Let me analyse the possible flows after $t = 1$. Inflows will depend on the wealth of investors after $t = 1$. In an interior equilibrium, the marginal investor is indifferent between switching fund and not switching. This means

$$e_2\Pi(\tilde{\theta}_2) = c. \tag{2.21}$$

The equation results in the marginal utility of switching being equal to the marginal cost of switching. The assumptions make sure that we always have an interior equilibrium. The next lemma looks at the assets under management α_t at the fund-manager level. It is clear that $\alpha_1 = 1$ for all funds. α_2 depends on the performance of a fund manager and on the outcomes of the endowment process and the asset markets.

In a simple equilibrium, suppose we had an unrevealing regime at $t = 1$ and let π denote the reputation of the out-performing funds in the first period. At time $t = 2$, we have the following:

Lemma 15. *In a simple equilibrium, suppose we had an unrevealing regime at $t = 1$ and let π denote the reputation of the out-performing funds in the first period. At time $t = 2$, we have the following: The assets under management α_2 of out-performing funds are given by*

$$\frac{\Delta}{\theta}(1-2\frac{c}{(1-\gamma)e_2\pi dR}). \tag{2.22}$$

In the case in which the risky asset was the right choice, the expected assets under management α_2 of underperforming funds are given by $e_2 - \frac{b}{1-b+\Delta}(\frac{\Delta}{\theta}(1-2\frac{cb}{(1-\gamma)e_2\theta dR}) - e_2)$. In the case in which the risk-free asset was the right choice, the expected assets under management α_2 of underperforming funds are given by $e_2 - \frac{1-b+\Delta}{b}(\frac{\Delta}{\theta}(1-2\frac{c(1-b+\Delta)}{(1-\gamma)e_2\theta dR}) - e_2)$.

This lemma shows that the per-fund assets under the management of funds that out-perform are increasing in their reputation π. The level of reputation matters only in cases with search frictions c, as can be seen from (2.22). Furthermore, the assets under management of an out-performing fund are increasing in e_2, as can be seen from (2.22).

2.4.3 Flow premium

Suppose that we have an unrevealing equilibrium at $t = 1$. In this case, uninformed fund managers should be indifferent between the two assets. Thus, we need the expected utility of both investments to be equal. They obtain their share of the returns in this period and take into account the expected assets under management in the next period in various cases, since by (2.18), their expected utility depends on them. I denote by $\psi \in \{V, S\}$ the realised investment of the fund manager.[13] We have $\psi = V$ in the case in which the manager is allocated the risky asset. In order for the fund manager to be indifferent between the two assets, we need:[14]

[13] Suppose that the allocation is consistent. If a fund submits $d \in \{0,1\}$, the realised investment is V or S for sure. If a fund submits $d = \{0,1\}$, ψ is random and equal to V with probability $X(d,a)$.

[14] This equation resembles (2.2) from the introduction. The microfoundation of W would be $\mathbb{E}[W|v = 1] = \mathbb{E}[\alpha_2|\psi = V, v_1 = 1]R - \mathbb{E}[\alpha_2|\psi = S, v_1 = 1]R$

$$\gamma \mathbb{E}[v]/\hat{P}_1 + \omega\gamma\mathbb{E}[\alpha_2|\psi = V]R = \gamma R + \omega\gamma\mathbb{E}[\alpha_2|\psi = S]R. \tag{2.23}$$

Thus, I obtain (noting that $\mathbb{E}[v] = 1$)

$$\hat{P}_1 = \frac{1}{R\left(1 + \omega(\mathbb{E}[\alpha_2|\psi = S] - \mathbb{E}[\alpha_2|\psi = V])\right)}. \tag{2.24}$$

The uninformed funds care about the rewards in terms of assets under management in the next period given the possible outcomes. We see the expected assets under management in lemma 14. In this model, it can be seen that the risky asset trades at a premium (discount) when the expected assets under management in the next period are higher (lower) for the risky asset compared to the risk-free asset. If

$$\mathbb{E}[\alpha_2|\psi = S] - \mathbb{E}[\alpha_2|\psi = V] < 0, \tag{2.25}$$

then the risky asset trades at a premium. This is what I call the *flow premium*. It is important to distinguish this *flow premium* from the *reputational premium* identified in previous studies, such as those of Guerrieri and Kondor (2012) and Dasgupta, Prat and Verardo (2011). In these papers, the effect of price changes on the probability of a high payoff was the focus. The reputational premium in these papers leads the risky asset to trade at a discount (premium) relative to the risk-neutral benchmark for a high (low) probability of $v = 1 - d$, since the high probability of a reputational loss makes uninformed managers unwilling to invest in an asset that has a high chance of underperforming. I shut down this effect by setting the probability to 1/2. In my paper, the premium stems from inflows that differ based on the state of the world, although the states are equally likely. The flow premium stems from two parts. The difference in assets under management after having improved reputation $\mathbb{E}[\alpha_2|\psi = S, v_1 = 1-d] - \mathbb{E}[\alpha_2|\psi = V, v_1 = 1+d]$ and the difference after having lost reputation $\mathbb{E}[\alpha_2|\psi = S, v_1 = 1+d] - \mathbb{E}[\alpha_2|\psi = V, v_1 = 1-d]$. The sum of the two determines the sign of the premium.

Proposition 16. *There exists a simple equilibrium, and* $\hat{P}_1$ *is given by* (2.24) *and* $\hat{P}_2 = 1/R$.

2.5 Implications

2.5.1 The Beta Anomaly

The price is given by (2.24). When the risky asset has β close to one, this means that almost surely a fund investing in that asset would have a good reputation in the high-endowment state and a poor reputation in the low-endowment state. How will the price of the risky asset depend on β? The search frictions in the market for asset management are the key variable in this analysis.

Proposition 17. *There exists a cutoff search cost* $\hat{c}$ *such that if search frictions are high* $c > \hat{c}$ *then*

$$\frac{\partial \mathbb{E}_1[R_1]}{\partial \beta} < 0. \tag{2.26}$$

If $c < \hat{c}$ *then*

$$\frac{\partial \mathbb{E}_1[R_1]}{\partial \beta} > 0. \tag{2.27}$$

Equation (2.26) presents the main result of this paper. The higher the correlation of the asset payoff with the realisation of the high-endowment state, the lower the expected return of the risky asset. Contrary to most models in finance, the fund managers' reputational concerns result in paying a higher price for an asset that does well in good states of the world. The β in my model effects the asset price through two channels. First, an investor with a larger endowment cares more about securing a good fund manager and less about the cost c. Thus, more investors will switch, and out-performing in such a state leads to more inflows through this effect. Second, the total amount of flows is bounded because of liquidity constraints. The more money managed by informed funds, the smaller the benefit of switching to them. The larger the endowment, the more money is already managed by informed funds before flows, and thus, there is less room for additional inflows. In cases where c is sufficiently high, the former channel is more important.

This result could help explain the negative relationship found in the data between alpha and expected return. Furthermore, as documented in Frazzini and Pedersen (2014), the relationship between beta and expected returns is almost flat during 1916-2012. Baker, Bradley and Wurgler (2011) find that the relationship becomes negative during 1968-2008. They also document that during this time institutional ownership of equities increased substantially. The leverage constraints explanation of the beta anomaly is not sufficient to explain the anomaly between beta and expected returns. A higher expected return is needed for a leverage constrained investor to pick high-beta assets.

My model predicts that the beta anomaly should be stronger when the market for asset management is less efficient. Search costs c may be higher for hedge funds and funds that hold more opaque assets, since in these cases, understanding fund performance is more difficult. This is consistent with the work of Coval, Jurek and Stafford (2009). They show that there is evidence of mis-pricing systemic risk in senior trenches of CDOs.

2.5.2 Asset Supply

Proposition 18. *There exists a cutoff search cost $\hat{\hat{c}}$ such that if search frictions are low*

$c < \hat{\hat{c}}$

then

$$\frac{\partial \mathbb{E}_1[R_1]}{\partial b} < 0 \tag{2.28}$$

if $c > \hat{\hat{c}}$

$$\frac{\partial \mathbb{E}_1[R_1]}{\partial b} > 0. \tag{2.29}$$

This proposition shows that when search frictions are low, the asset's return is decreasing in the nominal supply b. A higher nominal supply lets a fund "share the blame" with other funds if the investment turns out to be a wrong decision. In the case of low search frictions, this is very valuable. A smaller supply of the asset allows a fund to potentially stand out from the masses. This is favourable when the investment was correct. A higher c makes the search frictions more important relative to the liquidity constraints. In the case where search frictions are important, the level of reputation matters more, and it is more valuable to stand out from the masses then to share the blame.

2.5.3 Asset market efficiency

The following highlights some results regarding funds performance. By price efficiency at time t, I mean the probability that the asset price reveals the informed fund managers $\mathbb{P}(P_t \in \{\frac{1-d}{R}, \frac{1+d}{R}\}) = \frac{\tilde{\theta}_t}{\Delta}$.

Proposition 19. *At* $t = 2$*, the following results hold:*
i) informed fund managers out-perform uninformed fund managers in expectation
ii) the higher the search cost c*, the higher the expected out-performance of informed fund managers*
iii) investors whose fund underperformed and thus shift funds out expect over-performance that just offsets their search cost
iv) the price efficiency is decreasing in c
v)all else equal the price efficiency is higher in good times ($e_2 = 1 + \delta$*) than in bad times (*$e_2 = 1 - \delta$*)*

This proposition links the efficiency of the asset market to the efficiency of the asset management market. In my model, there is room for fund manager out-performance net of fees and return persistence as long as $c > 0$. For example, Kosowski, Timmermann and White (2006) find significant net-of-fee performance differences in mutual fund returns, and Kosowski, Naik and Teo (2007) find a similar result for hedge funds. It seems reasonable that c would be higher in the hedge fund industry compared to the mutual fund industry. Thus, this model predicts that performance persistence should be higher for hedge funds. This is consistent with the empirical literature that generally finds greater performance persistence for hedge funds.[15]

2.6 Conclusion

This paper presents a model of career concerns with endogenous flows. I show that the value of a manager's reputation can be state dependent when there are frictions in the asset management market. This gives rise to a potential explanation for the beta anomaly and shows why the asset supply may affect asset pricing. An important next step in this line of research would be to consider a model with multiple risky assets.

[15] These results are similar to those of Garlenau and Pedersen (2015)

Chapter 3

Rare Events and Active Management

3.1 Introduction

This paper seeks to understand how the amount of funds under active management evolves over time and interacts with outcomes in financial markets. The size of the asset management industry is changing over time but has remained large. In recent years, there has been a significant shift from active management to passive management. This trend accelerated after the financial crisis. I provide a model that studies how flows both within and out from the sector are influenced by events in financial markets. One implication of my model is that following rare events in asset markets, the amount of funds under active management may shrink dramatically. I show that this also results in reduced issuance of risky assets. Furthermore, I show that the active management sector may be large, despite a low amount of skill in the sector.

I consider a two-period model in which investors have the opportunity to invest with fund managers or by themselves. There is a small fraction of informed fund managers that have superior information about the payoff of an asset. The type of fund manager – informed or uninformed – is private information, and investors learn about the type from returns. Thus, fund managers consider their reputational concerns when investing in risky assets. In equilibrium, the expected rewards in terms of reputation of risky and safe assets must be the same. This requirement leads to a high demand for assets that are more likely to enable managers to maintain their reputations, and few funds bet on rare events that would provide a chance to really stand out. If a rare event – such as the default of a highly rated bond – occurs, the amount of money under active management may shrink. Such events result in a few fund managers obtaining 'star' status and many fund managers suffering damage to their reputations. Reduction in uncertainty in terms of who is informed surprisingly results in shrinkage of the active management sector. In equilibrium, households have to be indifferent between investing in active funds or by themselves. In the case in which the money mostly goes to skilled funds, it does not take that much money until the decreasing returns to scale at the sector level make opportunities go away.

The paper is related to a stream of research focusing on reputational concerns and asset prices (Dasgupta and Prat (2006,2008), Dasgupta, Prat and Verardo (2011), Guerrieri and Kondor (2012)). The paper differs because of its focus on the amount of funds subject to active management, which the aforementioned papers could not analyse because the investors do not not have a choice between investing in actively managed funds or not doing so. The paper is related to Malliaris and Yan (2015). They also get outflows out of the fund management sector after a rare event. In their paper the reason is an aggregate loss of reputation of fund managers employing a certain strategy. The mechanism differs in my model. After a rare event, the aggregate reputation of the sector stays constant, many fund managers lose their reputation, but some fund manager improve their reputation substantially. The paper is also related to Berk and Green (2004). Unlike Berk and Green (2004), in my model, there are endogenous decreasing returns to scale on the industry level of the active management sector. Increasingly, the more funds trade on information, the more often they reveal information and consequently reduce trading profits. Garcia and Vanden (2009) study the size and asset pricing implications of the mutual funds industry in a more standard competitive noisy rational expectations framework, but their paper does not consider the reputational concerns of fund managers. More related is the study of Pastor and Stambaugh (2012), who, in their paper, consider the amount of funds subject to active management in a setting with exogenous decreasing and uncertain returns to scale at the industry level. Paster and Stambaugh study how the size of the industry evolves over time in the context of learning about the aggregate skill of the industry over time. However, since they do not explicitly model the asset markets, they are not able to link the size of the sector to outcomes in the asset markets. Furthermore, they do not consider the reputational concerns of fund managers. My model also further relates to Gennaioli, Shleifer, and Vishny (2012), who consider a behavioural explanation for the demand for safe assets and the decreased issuance of such assets as a result of low returns due to neglected risks. My model makes similar predictions in a rational framework.

The remainder of the paper is organised as follows. In Section 2, I present the model. In Section 3, I solve for the equilibrium. Then, in section 4, I analyse the results of my model. All proofs are included in the Appendix.

3.2 The Model

Consider a two-period economy with $t \in \{1, 2\}$, where there is a round of trading in each period t. There is no discounting between the periods. Each period has a morning and an afternoon. Everyone is risk-neutral.

There is a large measure H of *households*. In each period, each of the H households is endowed \$ 1 to invest in the morning, and their aim is to maximise their consumption in the afternoon of each period. They have no private information and can invest either by themselves or with a fund manager.[1] The matching is random. I assume that the endowment must be fully invested in the morning of a period and fully consumed in the afternoon of a period. There is no saving decision. The households observe the returns of the funds after each period. There is a unit mass of *fund managers*. Of those fund managers, a mass θ is informed. The type of a fund manager is private information. I call the unit mass the *active-management sector*. Fund managers get paid through an exogenous contract; they simply get a share of the fund $\gamma > 0$. If a fund manager manages α dollars at the beginning of the period and the return realised is $\tilde{R}$, his compensation is

$$\alpha\gamma\tilde{R}. \tag{3.1}$$

The compensation is received and consumed in the afternoon of a period. There is no discounting between the two periods. The goal of fund managers is to maximise the fee that they earn from investors. In the above, 'returns' refers to gross returns.[2]

For households, the benefit of investing with a fund manager is the potential to be matched with an informed one. The drawback is the fee γ that households must pay.

The money can be invested in two assets, a risk-free asset and a risky asset. The risk-free asset has exogenous risk-free return R. There is a competitive financial intermediary who has a technology to produce a risky asset that pays $v_t \in \{0, 1\}$ with

$$\mathbb{P}(v_t = 0) = q. \tag{3.2}$$

[1]Investing by themselves can be interpreted as a passive strategy. Investing with a fund manager represents investing via active management.

[2]The above assumption perfectly aligns the incentives of fund managers and investors in the absence of fund flows and reputational concerns.

The intermediary can choose to supply a number of risky assets b_t and has no information about v_t. The supply b_t is chosen *before* each trading round in the morning of a period t. The price at time t of the asset P_t will be determined in equilibrium and may depend on the supply choice b_t. The informed fund managers receive perfect private information about v_t in the morning of a period. Households, the intermediary and uninformed fund managers only know q. *I assume that households by themselves cannot hold risky assets.*[3] The only method for households to invest in a risky asset is through fund managers.[4]

There is a mass of *noise traders* who have a total dollar amount $\Delta > 0$ to invest and are of the same size as the funds. I assume that in each period, $\tilde{\Delta}_t$ dollars are to be invested in the risky asset.[5] I assume

$$\tilde{\Delta}_t \in U[0, \Delta]. \tag{3.3}$$

The noise traders become uninformed funds with investors in the next period.[6] The realisation of $\tilde{\Delta}_t$ is independent of everything else and is not observed by households and fund managers.

The asset markets are similar to the model in Guerrieri and Kondor (2012). There is an *auctioneer* that collects the demand schedules, selects the equilibrium price and allocates assets to clear the market. In the morning of each period, all funds submit demand schedules to the auctioneer. I restrict the fund managers to three choices: they can invest completely in the risky asset ($d = 1$), they can fully invest in the risk-free asset ($d = 0$), or they can be indifferent to the two

[3]This assumption is not essential for the results of the paper. For structured products such as collateralised debt obligations (CDOs), the assumption seems realistic.

[4]Since everyone is risk-neutral, the crucial difference between investing via a fund and by themselves is not that the households invest risk-free by themselves; rather, it is that fund managers may be informed, but they also charge fees. Thus, investing via a fund and by themselves can be interpreted as active and passive investment, respectively.

[5]If the nominal supply of the risky asset is zero, I assume that these dollars go to the risk-free asset.

[6]The noise traders could represent emerging funds. This assumption is made for technical reasons.

options ($d = \{0, 1\}$). A demand schedule is a map from $\mathbb{R}_+ \to \{0, 1, \{0, 1\}\}$; for each price, it specifies a demand. The auctioneer collects the demand schedules and selects the price. The managers receive either the risky asset only or the risk-free asset only.

3.2.1 Equilibrium definition

I assume that

$$\theta < 1 \qquad\qquad \gamma < \frac{\theta q(1-\theta)}{1+q\theta-q\theta^2} \tag{3.4}$$

and

$$\frac{R}{1-q}\frac{\Delta}{\theta} < H. \tag{3.5}$$

The first assumption makes sure that enough money goes to the active management sector. The second assumption makes sure that some money is also invested by households by themselves.

At each time t, each of the households in H wants to maximise their expected utility. Household i can choose from two options, which we denote by $\tilde{x}_t^i \in \{0, 1\}$. Household i can either invest with a fund (set $\tilde{x}_t^i = 1$)[7] or invest on its own ($\tilde{x}_t^i = 0$). The total inflow into the active management sector at time t is then given by $\int_H \tilde{x}_t^i = \tilde{x}_t$. The households can base their decision on the observed returns after $t = 1$.

Let us denote by $a_t := (\tilde{\Delta}_t, v_t)$ the realisation of the shocks to the model at time t. Let us denote by b_t the choice of supply of the financial intermediary. Let us define as $P_t(a_t, b_t)$ the equilibrium price function at time t. In our rational expectations equilibrium, both uninformed and informed managers will maximise their expected utility conditional on the prices they observe. At each time

[7]By investing with a fund, I mean that the household invests with a fund with the highest reputation possible. In equilibrium, no household would want to invest with a fund with a lower reputation.

t, uninformed managers choose demand schedules to maximise their expected utility:

$$V_U^t(\alpha_t) = \max_{d_t \in \{0,1,\{0,1\}\}} \mathbb{E}_t[\gamma \alpha_t \tilde{R}^t + V_U^{t+1}(\alpha_{t+1}) | P_t] \tag{3.6}$$

where α_t is the dollar amount the fund manages. The informed fund managers face the same problem except that they can also condition on v_t.

I focus on a symmetric equilibrium in which all informed fund managers submit the same demand schedules and all uninformed fund managers also submit the same demand schedules. I denote the schedules from informed funds by $d_t^I(P_t, v_t)$ and those from uninformed funds by $d_t^U(P_t)$. Furthermore, let me denote schedules of households who invest by themselves by $d_t^H(P_t)$.[8] The auctioneer then sets an equilibrium price P_t to satisfy the market clearing condition. Let $X^t(d_t, a_t)$ denote the equilibrium probability of receiving the risky asset given the realisation of a_t and demand schedule d_t. Let $\tilde{\theta}_t$ denote the total money managed by informed funds at time t. Then, the following is required for the asset market to clear:

$$(\tilde{x}_t - \tilde{\theta}_t) X^t(d_t^U(P_t), a_t) + \tilde{\theta}_t X^t(d_t^I(P_t, v_t), a_t) + (H - \tilde{x}_t) X^t(d_t^H(P_t), a_t) + \tilde{\Delta}_t = b_t P_t. \tag{3.7}$$

The first term is the amount of money managed by uninformed funds multiplied by the probability of receiving the risky asset. Thus, this is the amount of dollars in the risky asset held by uninformed funds. Then, we have the amount held by informed funds, households and noise traders. This must be equal to the nominal supply of the risky asset in equilibrium.

We say an allocation $X^t(d_t, a_t)$ for a given demand schedule d is *consistent with a managers demand* if and only if $\forall a_t \; X^t(1, a_t) = 1, X^t(0, a_t) = 0, X^t(\{0,1\}, a_t) \in$

[8] Households are not allowed to invest in a risky asset, so $d_t^H = 0 \iff \mathbb{P}(v_t/P_t < R) > 0$.

$[0, 1]$.

The intermediary needs to break even; this means that we need to have b_t such that in equilibrium at each time t,

$$\mathbb{E}[P_t] = \frac{1-q}{R}. \tag{3.8}$$

Then, the intermediary makes zero profit.

A rational expectation equilibrium is defined as follows.

Definition 20. *A rational expectation equilibrium constitutes demand schedules, a price function $P_t(a_t, b_t) \in [0, 1/R]$, a risky asset supply b_t and an allocation function $X^t(d_t, a_t)$ such that at each time t, the following hold:*
1.) for each realisation of shocks a_t, there is a price $P_t(a_t, b_t)$ such that the asset market clears;
2.) the demand schedules solve the optimisation problems;
3.) the asset allocation is consistent with the demand schedules;
4.) the households update their opinion about their funds using Bayes' rule;
5.) the household flows $\tilde{x}_t$ into the active-management sector are optimal;
6.) the risky asset supply b_t is such that the intermediary breaks even.

3.3 Analysis

In my model, I focus on an equilibrium in which the price function takes the following simple form. Let us define the random variable $z_t = \tilde{\theta}_t v_t + \tilde{\Delta}_t$.

Definition 21. *I call a simple equilibrium a rational expectation equilibrium as in definition 20, for which the price function at time t takes the form:*
If $z_t \in (\Delta, \tilde{\theta}_t + \Delta]$, then $P_t = \frac{1}{R}$.
If $z_t \in [\tilde{\theta}_t, \Delta]$, then $P_t = (1-q)/R$.
If $z_t \in [0, \tilde{\theta}_t)$, then $P_t = 0$.
Furthermore, in the unrevealing regime, where the price is $P_t = (1-q)/R$, in-

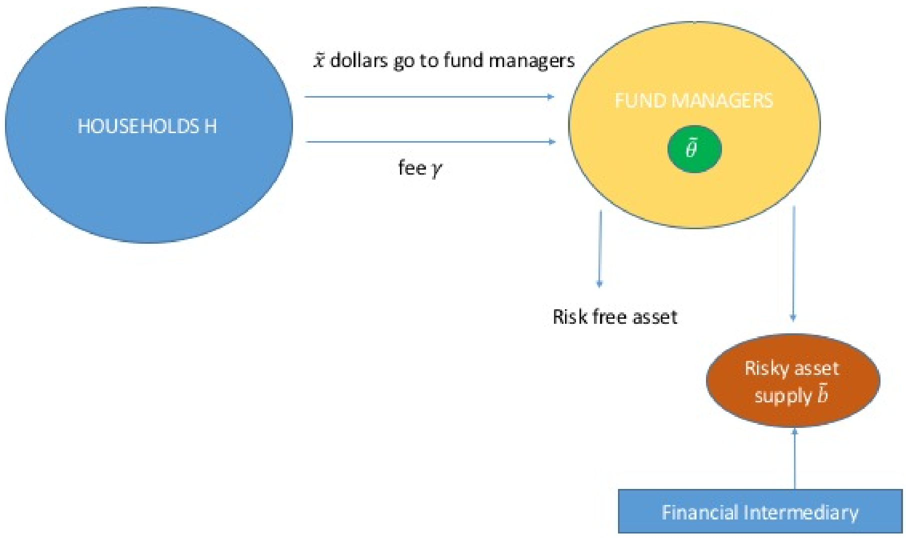

Figure 3.1: Summary of the structure of the model

formed fund managers submit $d_t^I = v_t$*, and uninformed fund managers submit indifference,* $d_t^U = \{0, 1\}$.

Let us denote by $\tilde{b}_t = b_t \frac{1-q}{R}$ the nominal supply of the risky asset in the unrevealing regime. For such an equilibrium to exist, we need

$$\tilde{\theta}_t < \Delta \tag{3.9}$$

and[9]

$$\Delta < \tilde{b}_t \qquad \tilde{b}_t < \tilde{x}_t. \tag{3.10}$$

In a simple equilibrium, an uninformed manager employed at t always makes an expected return of R (this follows since it is required that they find it optimal to be indifferent in that regime). Furthermore, we obtain that the probability of being in an unrevealing regime at time t is given by

$$\mathbb{P}(P_t = \frac{1-q}{R}) = q\mathbb{P}(\tilde{\Delta}_t \in [\tilde{\theta}_t, \Delta]) + (1-q)\mathbb{P}(\tilde{\Delta}_t \in [0, \Delta - \tilde{\theta}_t]) = \frac{\Delta - \tilde{\theta}_t}{\Delta}. \tag{3.11}$$

In a simple asset market equilibrium, uninformed funds are indifferent in the unrevealing regime, and their expected return is R. The expected benefit of having an informed manager in that regime is thus

$$(1-q)R/(1-q) + qR - R = qR. \tag{3.12}$$

This equation represents the expected return of an informed fund manager minus the expected return of an uninformed fund manager in the unrevealing regime. Let us compare the expected utility benefit of a household that invests one dollar in a fund who was correct compared with a household investing on its own. Let

[9] The first condition means that there is always sufficient nominal supply to satisfy the demand of noise traders and informed fund managers in the unrevealing regime. The second ensures that there is sufficient money to clear the markets coming from fund managers. Recall that households are not allowed to invest in risky assets.

$\tilde{\theta}_t$ denote the money managed by informed funds at time t. Let us denote by π_t the highest reputation of funds in the market.[10] The expected per dollar benefit of investing with an active fund is

$$(1-\gamma)qR\pi_t\frac{\Delta-\tilde{\theta}_t}{\Delta}-\gamma R. \tag{3.13}$$

In order for the households to be indifferent, in equilibrium, the above expression must equal zero. The flows affect the equation by affecting the mass of money managed by informed funds, $\tilde{\theta}_t$. The first term of the equation denotes the benefit of investing with a fund manager, which is the after-fee return benefit of having an informed manager in the unrevealing regime $(1-\gamma)qR$ multiplied by the probability of being matched with an informed manager and the probability of being in the unrevealing regime.

3.3.1 One-period benchmark

In order to illustrate that the results in this paper are driven solely by the reputational concerns of the fund managers, I consider first a one-period model.[11] Then, we can construct the following equilibrium:

Lemma 22. *There exists a simple equilibrium. The size of the active management sector is given by*

$$\tilde{x}=\frac{\Delta}{\theta}\left(1-\frac{\gamma}{\theta q(1-\gamma)}\right). \tag{3.14}$$

The mass of money managed by informed managers is $\tilde{\theta}=\theta\tilde{x}$. The supply choice of the intermediary b is undetermined.[12]

Proof. See Appendix. □

[10]Only these funds will receive inflows, and π_t is the probability of matching with an informed fund

[11]I drop time subscripts because there is only one period

[12]The financial intermediary can choose any b such that $\frac{1-q}{R}b<\tilde{x}$ and $\frac{1-q}{R}b>\Delta$

The amount of funds invested in the active management sector depends on liquidity, fees and the measure of informed fund managers.

Corollary 23.

$$\frac{\partial \tilde{x}}{\partial \gamma} < 0 \qquad\qquad \frac{\partial \tilde{x}}{\partial \Delta} > 0 \tag{3.15}$$

and

$$\frac{\partial \tilde{x}}{\partial \theta} > 0(< 0) \iff \theta < (>) 2 \frac{\gamma}{q\,(1-\gamma)}. \tag{3.16}$$

The choice b of risky assets produced is undetermined.

Proof. The proof is obvious. □

This benchmark has an interesting implication. It may be the case that greater fractions of informed fund managers (higher θ) correspond to smaller active management sectors, $\tilde{x}$. Suppose that the fee γ is low relative to θ; this means that in equilibrium, the before-fee advantage of investing with a fund manager must be small to make investors indifferent. To make the before-fee advantage small, prices have to often be revealing. This can only happen when a significant fraction of funds invested are managed by informed fund managers. If θ is low, only a small proportion of the total dollars $\tilde{x}$ that go to funds go to informed funds $\theta\tilde{x}$. Thus, a very high value of $\tilde{x}$ is necessary to make $\theta\tilde{x}$ sufficiently large to achieve the in-equilibrium required probability of price revelation. This explains (3.16).

This result may be interesting when it comes to the debate why active-management remains large in spite of the significant evidence of only few funds that are able to beat the benchmark.

3.3.2 Two-period model

Next, I consider a two-period model. In this model, the first period is interesting and different from the one-period model, since in that period, fund managers have reputational concerns. To obtain a simple equilibrium at $t = 1$, fund managers have to be indifferent between the risky asset and the risk-free asset in an unrevealing regime. Furthermore, the price of the risky asset needs to be $P_1 = \frac{1-q}{R}$. This is only possible if the expected reward in terms of future inflows is the same for both assets.

Since everyone dies after $t = 2$, prices are not affected by career concerns in the second period, and our analysis is straightforward; we thus set $V_U^3 = V_I^3 = 0$.

Since an uninformed fund manager makes an expected return of R in a simple equilibrium, we obtain for a fund that has α_2 dollars at time $t = 2$

$$\mathbb{E}[V_U^2(\alpha_2)] = \alpha_2 \gamma R. \tag{3.17}$$

3.3.3 Investor flows

Suppose that we have a simple equilibrium; what can the households learn from the funds' actions at $t = 1$? Let us denote the nominal supply of the risky asset in the unrevealing regime in the initial period by $\tilde{b}_1$ $(= b_1 \frac{1-q}{R})$ and the initial inflow of households (or equivalently dollars) as $\tilde{x}_1$.

Lemma 24. *Suppose that an unrevealing regime occurred in a simple equilibrium at $t = 1$.*
A manager who does not make the right decision is uninformed with probability one. A manager who makes the right decision when the risky asset (risk-free asset) was the right choice ex post is informed with a probability of

$$\pi_2 = \frac{\tilde{\theta}_1}{\tilde{b}_1} \tag{3.18}$$

$$(\pi_2 = \frac{\tilde{\theta}_1}{\tilde{x}_1 + \Delta - \tilde{b}_1}). \tag{3.19}$$

At $t = 2$, all households either invest on their own or flow into one of the funds *who made the right choice.*

Proof. See Appendix. □

The lemma demonstrates an intuitive result. If the nominal supply $\tilde{b}_1$ is low, in equilibrium, few fund managers can invest in that asset. Since all informed fund managers choose the risky asset when it performs well, the increase in reputation of funds buying the risky asset is then high. The next lemma considers inflows into the active management sector in a simple equilibrium.

Lemma 25. *At $t = 1$, the initial inflow in a simple equilibrium is given by*

$$\tilde{x}_1 = \frac{\Delta}{\theta}(1 - \frac{\gamma}{(1-\gamma)q\theta}). \tag{3.20}$$

If at $t = 1$ there is a revealing equilibrium, then no household will flow in or out of the sector. Suppose that there is an unrevealing equilibrium at $t = 1$.

Then, the total investment in the active management sector is given by

$$\tilde{x}_2 = \frac{\Delta}{\pi_2}(1 - \frac{\gamma}{(1-\gamma)q\pi_2}), \tag{3.21}$$

and π_2 can take values as in lemma 24, depending on v_1.

Proof. See Appendix. □

The fund management sector in my model has endogenous decreasing returns to scale at the industry level. Households will choose to flow into funds who could increase their reputation. As more households flow in due to the endogenous prices, I obtain decreasing returns to scale. As the skilled funds grow, it becomes less likely that they can provide a superior return compared with households investing on their own. The households will flow into the funds until they are indifferent between investing on their own or choosing to invest with a fund.

The total dollar amount managed per fund can be obtained from the fact that there is a mass θ of informed funds (that will manage the same amount as uninformed funds) and $\tilde{\theta}_2 = \pi_2 \tilde{x}_1$. Consequently, each fund with a positive reputation manages $\frac{\pi_2}{\theta}\tilde{x}_1$ dollars.[13] Although an increase in the reputation of funds (increase in π_2) does not generally lead to an increase in money invested with the sector, an increase in reputation always increases the money invested with an individual fund.

3.3.4 Fund manager incentives and asset prices

Suppose in the following that at $t = 1$ in the unrevealing regime, we have a supply b_1 of the risky asset. In a simple equilibrium we need

$$\tilde{x}_1 > b_1 \frac{1-q}{R} \qquad b_1 \frac{1-q}{R} > \Delta \qquad \tilde{x}_2 > \Delta. \tag{3.22}$$

Here $\tilde{x}_2$ depends on b_1 as outlined in lemma 25. For an equilibrium to exist, uninformed fund managers must be indifferent between investing in the two assets in the unrevealing regime. Thus,

$$\begin{aligned} &\tilde{x}_1 \gamma \frac{1-q}{P_1} + (1-q)\gamma R \frac{\Delta}{\theta}(1 - \frac{P_1 b_1 \gamma}{(1-\gamma) q \tilde{\theta}_1}) \\ &= \tilde{x}_1 \gamma R + q \gamma R \frac{\Delta}{\theta}(1 - \frac{(\tilde{x}_1 - P_1 b_1 + \Delta)\gamma}{(1-\gamma) q \tilde{\theta}_1}). \end{aligned} \tag{3.23}$$

The left-hand side of the above equation denotes the expected utility of an uninformed fund who invests in the risky asset. The right-hand side corresponds to the expected utility of an uninformed fund who fully invests in the risk-free asset. The fund manager needs to be indifferent, and the financial intermediary needs to break even; consequently, we need $P_1 = \frac{1-q}{R}$. In equilibrium, it is required that the expected reward for the two assets in terms of inflows in the future is the same. If an asset has a low probability of being the "right" choice, then the reputational reward must be high in case it is the right choice ex post. To achieve

[13] Let α_2 denote the money managed per fund; then, we have $\tilde{\theta}_2 = \theta\alpha_2$ and $\tilde{\theta}_2 = \pi_2 \tilde{x}_2$, so $\alpha_2 = \frac{\pi_2}{\theta}\tilde{x}_1$.

indifference, we can solve (3.23) for b_1 and find

$$b_1 = \frac{R}{1-q}\frac{2\left(\left(1-\gamma\right)q\left(\gamma q - q + 1/2\right)\theta^2 + 3/2\,\left(1-\gamma\right)\left(q-1/3\right)\gamma\theta - 1/2\,\gamma^2\right)\Delta}{\gamma\theta^2\left(1-\gamma\right)}. \tag{3.24}$$

Theorem 26. *Suppose that b_1 given by* (3.24) *satisfies conditions* (3.22)*. There exists a simple equilibrium in which the production of risky assets in the first period is given by b_1. The production of the risky asset in period 2 is undetermined.*

Proof. See Appendix. □

I take the limit to make the analysis more tractable and to focus on the important aspects of the model. I define the parameters $\kappa_L, \kappa_F > 0$ such that

$$\Delta = \kappa_L \theta \tag{3.25}$$

and

$$\gamma = \kappa_F \theta. \tag{3.26}$$

I have, by my assumptions, $\kappa_L < H$.

In the following, I keep my parameters κ_L, κ_F constant and I let the measure of informed fund managers go to zero

$$\theta \to 0. \tag{3.27}$$

In my model all of θ, γ and Δ are naturally interpreted to be small.

We want to see the conditions under which a simple equilibrium as described above exists. In a simple equilibrium, we need

$$\tilde{x}_t > \tilde{b}_t \qquad\qquad \tilde{b}_t > 0. \tag{3.28}$$

Furthermore, it must be optimal for uninformed fund managers to be indifferent between the risky asset and the risk-free asset. Thus, (3.23) must be satisfied.

Theorem 27. *For $q \leq 1/2$, there exists a simple equilibrium if and only if*

$$\kappa_F \in \left(\frac{q - 2q^2}{1 - q}, q \right). \tag{3.29}$$

For $q > 1/2$, there exists a simple equilibrium if and only if

$$\kappa_F \in (2q - 1, q). \tag{3.30}$$

The nominal supply of the risky asset in period 1 is given by

$$\tilde{b}_1 = \kappa_L(3q - 1) - \kappa_L \kappa_F + \frac{\kappa_L}{\kappa_F}(q - 2q^2). \tag{3.31}$$

The nominal supply of the risky asset in period 2 is undetermined.

Proof. See Appendix. □

In the limit case the conditions on κ_F are easy to find such that a simple equilibrium exists.[14]

3.4 Implications

We will distinguish two kinds of assets, safe assets,

$$q < 1/2, \tag{3.32}$$

and lottery tickets,

$$q > 1/2. \tag{3.33}$$

3.4.1 Equilibrium level of risky asset holding

The fraction of funds holding the risky asset is given by $\frac{\tilde{b}_1}{\tilde{x}_1}$.

[14]The limit case also makes clear that there are parameters with θ, Δ, γ close to zero so that conditions (3.22) are satisfied and a simple equilibrium exists away from the limit.

Proposition 28. *The holding of the risky asset is given by*

$$\frac{\tilde{b}_1}{\tilde{x}_1} = \frac{q(\kappa_F + 1 - 2q)}{\kappa_F}. \tag{3.34}$$

For $q < 1/2$, we have

$$\frac{\tilde{b}_1}{\tilde{x}_1} \in (1 - q, 1), \tag{3.35}$$

and this is decreasing in κ_F. We have for $q = 1/2$ that

$$\frac{\tilde{b}_1}{\tilde{x}_1} = 1/2. \tag{3.36}$$

For $q > 1/2$, we have

$$\frac{\tilde{b}_1}{\tilde{x}_1} \in (0, 1 - q), \tag{3.37}$$

and this is increasing in κ_F.

Proof. See Appendix. □

Suppose that the we have a lottery ticket ($q > 1/2$). In this case the fear of losing reputation is high when investing in the risky asset. Thus, the risky asset must provide a high reward in terms of reputation in case it turns out against the odds have a high return. In this case relatively few uninformed funds will hold the risky asset. Suppose that the risky asset is safe ($q < 1/2$). Now most uninformed funds will hold the risky asset and a few uninformed funds will bet on the "rare disaster". If the latter turn out to be right, they will have a very high reputation and be rewarded with big inflows.

An example of such an episode could be the subprime market collapse in 2006. A large majority of fund managers lost a lot money as a result of that event. However, there was a small set of fund managers, including John Paulson and Greg Lippman, that were betting on the possibility of a housing market crash. Such a crash was considered an unlikely event at the time. Consequently, these

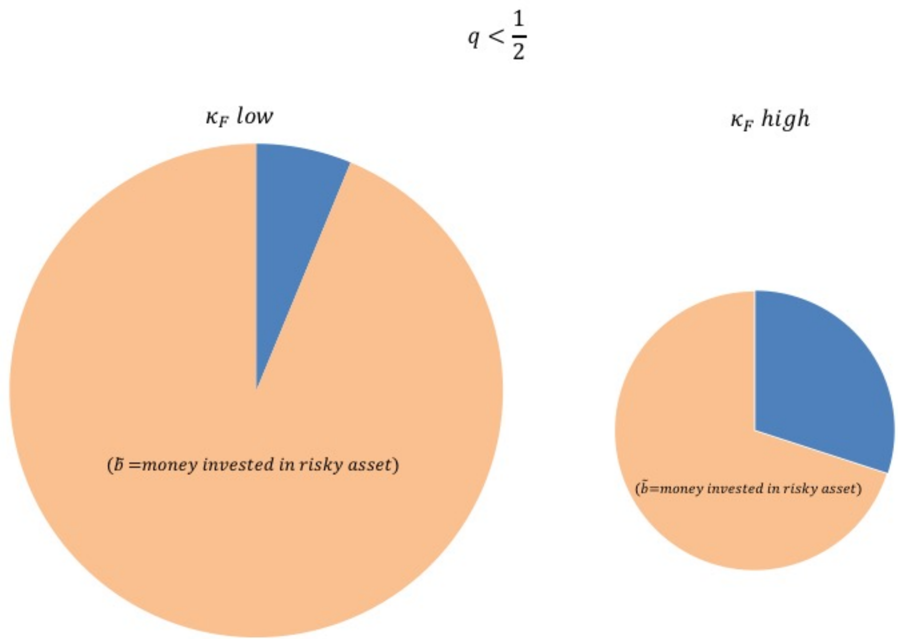

Figure 3.2: Asset holdings

fund managers made huge profits and gained widespread recognition. As an additional reward, these funds received large inflows by new investors into their funds.

As illustrated in Figure 3.2, the asset management sector is more concentrated in one asset the lower the relative fee level κ_F. The funds that lose their reputation by investing in the wrong asset get zero assets under management in the next period. Suppose for concreteness that $q < 1/2$. The inflows of the funds that maintain a positive reputation depends on the level of their reputation. If κ_F is low, the inflows are not very sensitive to reputation, thus the reputation increase in the event when the risky asset defaults (which happens with $q < 1/2$) must be very high to induce funds to invest in the risk-free asset. The result is that for low κ_F only very few funds bet on default and invest in the risk-free asset.

3.4.2 The evolution of the amount of funds under active management

The growth of the active management sector is given by

$$\frac{\tilde{x}_2}{\tilde{x}_1}. \tag{3.38}$$

In the next proposition, I analyse how the size of the sector grows depending on market outcomes.

Proposition 29. *Suppose that we had an unrevealing regime period 1. If there is no default in the initial period ($v_1 = 1$), we get*

$$\frac{\tilde{x}_2}{\tilde{x}_1} = \frac{(\kappa_F - 2\,q)\,q^2\,(\kappa_F - 2\,q + 1)}{\kappa_F\,(\kappa_F - q)}. \tag{3.39}$$

If there is a default in the initial period ($v_1 = 0$), we obtain

$$\frac{\tilde{x}_2}{\tilde{x}_1} = \frac{(1-q)\,(\kappa_F\,q - 2\,q^2 - \kappa_F + q)\,(\kappa_F - 2\,q)}{\kappa_F\,\,(q - \kappa_F)}. \tag{3.40}$$

Suppose that we have a safe asset $q < 1/2$. Then we get for $v_1 = 1$

$$\lim_{\kappa_F \to \frac{q-2q^2}{1-q}} \frac{\tilde{x}_2}{\tilde{x}_1} = 1 \tag{3.41}$$

and for $v_1 = 0$, we have

$$\lim_{\kappa_F \to \frac{q-2q^2}{1-q}} \frac{\tilde{x}_2}{\tilde{x}_1} = 0. \tag{3.42}$$

Proof. See Appendix. □

Suppose that the asset is quite safe ($q < 1/2$). This proposition shows that for an active management sector of significant size ($\kappa_F \approx \frac{q-2q^2}{1-q}$), there will be a major outflow of the active management sector following a disappointing return ($v_1 = 0$). This happens although everyone understands that going forward, another comparably disastrous event is still very unlikely. The reason is that in the sector will be quite concentrated in the risky asset in this case (as illustrated in Figure 3..2). Of the few funds that survive a high fraction is informed. Thus, with the same logic as in the one-period model (3.16), the size of the sector is going to be very small.

As we see from (3.42), the largest active management sectors vanish in the most extreme form following a rare disaster. Furthermore, in this case, there is a sharp decline in the issuance of risky assets. Since $\tilde{b}_1 \approx \tilde{x}_1$ for low q, by proposition 28 and $\tilde{b}_2 < \tilde{x}_2$. This result could be related to the idea that the growth of asset backed commercial paper was fuelled by the demand of money market funds which were reaching for yield. After the financial crisis demand by money market funds dried up and the commercial paper market collapsed (Kacperczyk and Schnabl, 2010).

What if we have a lottery ticket?

Proposition 30. *Suppose* $q > 1/2$ *and* $v_1 = 1$

$$\lim_{\kappa_F \to 2q-1} \frac{\tilde{x}_2}{\tilde{x}_1} = 0, \tag{3.43}$$

and if $v = 0$,

$$\lim_{\kappa_F \to 2q-1} \frac{\tilde{x}_2}{\tilde{x}_1} = 1. \tag{3.44}$$

Proof. See Appendix. □

The results for the lottery ticket are symmetric to the case with a safe asset.

3.5 Conclusion

In this paper, I develop a model to analyse the time evolution in the amount of funds under active management. The size of the sector depends on the reputation of fund managers. A rare-event such as a surprisingly low return for a risky asset reveals the low skill of many fund mangers and results in shrinkage of the sector.

Future research could seek to develop a more dynamic model with an infinite number of periods.

Chapter 4

APPENDIX

4.1 Proofs of Chapter 1

4.1.1 Proof of proposition 2 (see page 22)

Proof. This follows from the argument in the main text. □

4.1.2 Proof of lemma 3 (see page 32)

Proof. Let me first solve the model when I have a trader, i.e., $I = trader$.
The trader chooses her optimal demand, and I assume $v = 1$ in the proof (the other case is symmetric). It is clear that a market order $\tilde{x} < 0$ cannot be optimal. Furthermore, the trader always makes zero profit with a market order $\tilde{x} > \Delta_H$. Let us write $\tilde{x} = \theta$. I will distinguish between two possible ranges of θ. There are two possibilities: $\theta < \Delta_L$ and $\theta > \Delta_L$. For the high values of θ, the trader will always reveal her information in case of $L = \Delta_L$. Thus, the traders' problem is

$$\max_{\theta \in [0, \Delta_H]} \Pi(q, \theta) = \max_{\theta \in [0, \Delta_H]} \mathbb{E}[\theta(v - P) | \tilde{x} = \theta, v = 1] \tag{4.1}$$

$$= q\theta\mathbb{E}[(1 - P)|\tilde{x} = \theta, L = \Delta_H] + (1 - q)\theta\mathbb{E}[(1 - P)|\tilde{x} = \theta, L = \Delta_L].$$

This can be written as

$$\max_{\theta\in[0,\Delta_H]} \Pi(q,\theta) = \max_{\theta\in[0,\Delta_H]} \mathbf{1}_{\theta\leq\Delta_L} f(\theta) + \mathbf{1}_{\theta>\Delta_L} g(\theta). \tag{4.2}$$

I thus have

$$f(\theta) = q\frac{\Delta_H-\theta}{\Delta_H}\theta(1-1/2) + (1-q)\frac{\Delta_L-\theta}{\Delta_L}\theta(1-1/2) \tag{4.3}$$

and

$$g(\theta) = q\frac{\Delta_H-\theta}{\Delta_H}\theta(1-1/2). \tag{4.4}$$

The function $\mathbf{1}_{\theta\leq\Delta_L} f(\theta) + \mathbf{1}_{\theta>\Delta_L} g(\theta)$ is continuous.
The function f has a global maximum at $\theta_{BM} = 1/2\frac{\Delta_H\,\Delta_L}{\Delta_L q+(1-q)\Delta_H}$. Furthermore, g is strictly decreasing in θ for $\theta > \Delta_H/2$, since $g'(\theta) = 1/2\,\frac{q(-2\,\theta+\Delta_H)}{\Delta_H}$. Since $\Delta_L > \Delta_H/2$, g is strictly decreasing in θ in the relevant range for $\theta \in (\Delta_L, \Delta_H]$. Since, by continuity, $f(\theta_{BM}) > f(\Delta_L) = g(\Delta_L) > g(\theta)\forall\theta \in [\Delta_L, \Delta_H]$ and $f(\theta_{BM}) > f(\theta)\forall\theta$, I find that θ_{BM} is the optimal order. All the other results follow from plugging in the optimal θ and I obtain expected profits of $\Pi(q) = \frac{\Delta_H\,\Delta_L}{(8-8q)\Delta_H+8\,\Delta_L\,q}$.
The case $v = 0$ is symmetric, with $\tilde{x} = -\theta_{BM}$, so the symmetric conjecture is correct and $\tilde{x} = (-1)^{v+1}\theta_{BM}$.
Let us now assume that $I = fund$
I solve the game starting in the afternoon. Given some inflows $\tilde{f}$ and some contract γ_M, γ_P, the fund chooses θ to maximise his expected compensation, which is given by

$$\gamma_M\tilde{f} + \gamma_P\tilde{f}(\mathbb{E}[\tilde{R}|\theta] - 1). \tag{4.5}$$

The only term that depends on θ is given by $\mathbb{E}[\tilde{R}|\theta]$. It is clear that the fund receives the highest possible expected return with the same market order as the trader, so $\theta_{fund} = \theta_{BM}$. The expected profits that the fund makes are then given by $\Pi(q)$.
In the morning, I have the following: Given our contract with γ_M and γ_P, households will flow into the fund until the expected return is equal to the outside

option 1, so the following equation needs to be satisfied

$$\frac{\tilde{f} - \gamma_M \tilde{f} + (1-\gamma_P)\Pi(q)}{\tilde{f}} = 1. \tag{4.6}$$

This yields $\tilde{f} = 1/8 \, \frac{1-\gamma_P}{\gamma_M} \Pi(q)$. The fund is paid $\gamma_M \tilde{f} + \gamma_P \tilde{f}(\mathbb{E}[\tilde{R}|\theta_{fund}] - 1) = \Pi(q)$.

$\square$

4.1.3 Proof of corollary 4 (see page 34)

Proof. Let me first show that the trader is less aggressive when uncertain about L. If the trader knew that $L = q\Delta_H + (1-q)\Delta_L$ by lemma 3, we would an optimal $\theta = 1/2 \, \Delta_H \, q + 1/2 \, (1-q) \, \Delta_L$. I see that

$$1/2 \, \frac{\Delta_H \, \Delta_L}{q\Delta_L + (1-q) \, \Delta_H} < 1/2 \, \Delta_H \, q + 1/2 \, (1-q) \, \Delta_L \tag{4.7}$$

$$\iff \frac{q \left(\Delta_H - \Delta_L\right)^2 (1-q)}{(2 \, \Delta_H - 2 \, \Delta_L) \, q - 2 \, \Delta_H} < 0. \tag{4.8}$$

The inequality follows from rearranging, and the expression is obviously negative. From here, it is clear that $\Pi(q) = 1/4 \; \theta_{BM} < 1/4(1/2 \, \Delta_H \, q + 1/2 \, (1-q) \, \Delta_L)$, so the profits are smaller when the insider is uncertain.

$\square$

4.1.4 Proof of lemma 5 (see page 34)

Proof. The expected profit is given by $\frac{\Delta_H \, \Delta_L}{(8-8q)\Delta_H + 8 \, \Delta_L \, q}$ by lemma 3. The second derivative of the expected profit with respect to q is given by

$$\Pi(q)'' = 2 \, \frac{\Delta_H \, \Delta_L \, (8 \, \Delta_H - 8 \, \Delta_L)^2}{((8-8q) \, \Delta_H + 8 \, q\Delta_L)^3} > 0. \tag{4.9}$$

Thus, we see that the profits are convex.

$\square$

4.1.5 Proof of lemma 6 (see page 37)

Proof. Suppose that the market order in the first period is $\tilde{x} = (-1)^{v+1}\theta$ and $\theta < \Delta_L$. In this case, the signals have the following form

$$\mathbb{P}(L = \Delta_H | P_1 = 1/2, \theta) = \frac{\mathbb{P}(L = \Delta_H|\theta)\mathbb{P}(P_1 = 1/2|L = \Delta_H, \theta)}{\mathbb{P}(P_1 = 1/2|\theta)}$$
$$= \frac{q\frac{\Delta_H - \theta}{\Delta_H}}{\left(q\frac{\Delta_H-\theta}{\Delta_H} + (1-q)\frac{\Delta_L - \theta}{\Delta_L}\right)} > q \tag{4.10}$$

and

$$\mathbb{P}(L = \Delta_H | P_1 \in \{0,1\}, \theta) = \frac{q\frac{\theta}{\Delta_H}}{\left(q\frac{\theta}{\Delta_H} + (1-q)\frac{\theta}{\Delta_L}\right)} = \frac{q\Delta_L}{\Delta_H - q(\Delta_H - \Delta_L)} < q.$$

On the other hand, if $\theta \in (\Delta_L, \Delta_H]$ I obtain

$$\mathbb{P}(L = \Delta_H | P_1 = 1/2, \theta) = 1 \tag{4.11}$$

and

$$\mathbb{P}(L = \Delta_H | P_1 \in \{0,1\}, \theta) = \frac{q\frac{\theta}{\Delta_H}}{q\frac{\theta}{\Delta_H} + 1 - q} \leq q. \tag{4.12}$$

I compute that

$$\mathbb{E}_1[\Pi(q_2)|\theta] = \mathbb{P}(P_1 = 1/2|\theta)\Pi(\mathbb{P}(L = \Delta_H|P_1 = 1/2, \theta)) \tag{4.13}$$
$$+\mathbb{P}(P_1 \in \{0,1\}|\theta)\Pi(\mathbb{P}(L = \Delta_H|P_1 \in \{0,1\}, \theta)). \tag{4.14}$$

The function $\mathbb{E}_1[\Pi(q_2)|\theta]$ is continuous on $[0, \Delta_H]$ and differentiable everywhere but at $\theta = \Delta_L$.

I compute for $\theta < \Delta_L$ that

$$\frac{\partial}{\partial\theta}\mathbb{E}_1[\Pi(q_2)|\theta]$$
$$\tag{4.15}$$
$$= 1/8\,\frac{{\Delta_H}^2{\Delta_L}^2q^2\left(\Delta_H - \Delta_L\right)^4\left(q-1\right)^2}{\left(\left(1-q\right){\Delta_H}^2 + q{\Delta_L}^2\right)\left(\left(-\theta + \Delta_L\right)\left(q-1\right){\Delta_H}^2 - q\Delta_H\,{\Delta_L}^2 + \theta\,q{\Delta_L}^2\right)^2} > 0$$

for all $\theta \in [0, \Delta_L)$.

I compute that for $\theta > \Delta_L$:

$$\frac{\partial}{\partial\theta}\mathbb{E}_1[\Pi(q_2)|\theta] = -1/8\,\frac{q{\Delta_H}^2\left(\Delta_H - \Delta_L\right)^2(q-1)^2}{\left((q-1)\,{\Delta_H}^2 - \theta\,\Delta_L\,q\right)^2} < 0 \quad \forall\theta \in (\Delta_L, \Delta_H]. \quad (4.16)$$

Since for $\theta > \Delta_H$, we have $\mathbb{E}_1[\Pi(q_2)|\theta = \Delta_H] = \mathbb{E}_1[\Pi(q_2)|\theta]$, a $\theta > \Delta_H$ cannot be optimal. Thus, $\theta = \Delta_L$ maximises $\mathbb{E}_1[\Pi(q_2)|\theta]$. □

4.1.6 Proof of proposition 7 (see page 38)

Proof. The trader will choose $\tilde{x}$ in the first period to achieve two goals, she wants to have high profits in the first period and have high expected profits next period. So the trader will choose $\tilde{x} = (-1)^{v+1}\theta$ at time $t = 1$ to maximise

$$\mathbb{E}_1[\tilde{x}(v - P_1) + \Pi(q_2)|v, \tilde{x}]. \quad (4.17)$$

Let us assume that $v = 1$, so $\tilde{x} = \theta$ (the case $v = 0$ is completely symmetric, with $\tilde{x} = -\theta$). It is clear that $\tilde{x} < 0$ and $\tilde{x} > \Delta_H$ cannot be optimal.

Then, problem (4.17) can be written as

$$\max_{\theta\in[0,\Delta_H]} \mathbf{1}_{\theta<\Delta_L} f(\theta) + \mathbf{1}_{\theta\geq\Delta_L} g(\theta). \quad (4.18)$$

The function f is given by

$$f(\theta) = q\frac{\Delta_H - \theta}{\Delta_H}\theta(1 - 1/2) + (1-q)\frac{\Delta_L - \theta}{\Delta_L}\theta(1 - 1/2) + \mathbb{E}_1[\Pi(q_2)|\theta], \quad (4.19)$$

and the function g is given by

$$g(\theta) = q\frac{\Delta_H - \theta}{\Delta_H}\theta(1 - 1/2) + \mathbb{E}_1[\Pi(q_2)|\theta]. \quad (4.20)$$

I maximise over a continuous piecewise function. The first two terms of f and the first term of g are the profits for this period. The first candidate for a maximum is the maximum of $f(\theta)$ for $\theta \in [0, \Delta_L]$. In the case $\mathbb{P}(L = \Delta_H) = q$ at $t = 2$, the expected profit in period 2 is obtained by lemma 3:

$$\Pi(q) = 1/8 \frac{\Delta_H \, \Delta_L}{(1-q)\, \Delta_H + \Delta_L \, q}. \tag{4.21}$$

I see that

$$\mathbb{E}_1[\Pi(q_2)|\theta] = \mathbb{P}(P_1 = 1/2|\theta)\Pi(\mathbb{P}(L = \Delta_H|P_1 = 1/2, \theta) \tag{4.22}$$

$$+\mathbb{P}(P_1 \in \{0,1\}|\theta)\Pi(\mathbb{P}(L = \Delta_H|P_1 \in \{0,1\}, \theta). \tag{4.23}$$

I then compute

$$f(\theta) = 1/2\, \frac{q\theta\, (\Delta_H - \theta)}{\Delta_H} + 1/2\, \frac{(1-q)\,(\Delta_L - \theta)\, \theta}{\Delta_L} \tag{4.24}$$

$$-\frac{((\Delta_L + (q-1)\,\theta)\, \Delta_H - q\Delta_L\, \theta)^2}{8\, (\Delta_L - \theta)\, (q-1)\, {\Delta_H}^2 - 8\, q\Delta_H\, {\Delta_L}^2 + 8\, q{\Delta_L}^2\theta} \tag{4.25}$$

$$-\frac{((\Delta_H - \Delta_L)\, q - \Delta_H)^2\, \theta}{\left(8\, {\Delta_H}^2 - 8\, {\Delta_L}^2\right) q - 8\, {\Delta_H}^2}.$$

This function is continuous and has no singularity for $\theta \in [0, \Delta_L]$. I also find that $f'(\theta)$ is continuous and has no singularity for $\theta \in [0, \Delta_L]$.

I find that $f'(\theta) = 0$ has 3 solutions in the following set
$\{1/4\, \frac{\left(3\Delta_H(1-q)+3\,q\Delta_L - \sqrt{5\,(\Delta_H-\Delta_L)^2 q^2 - 6\,(\Delta_H - 2/3\,\Delta_L)(\Delta_H-\Delta_L)q + {\Delta_H}^2}\right)\Delta_L\,\Delta_H}{{\Delta_H}^2(1-q)+{\Delta_L}^2 q}$,
$1/4\, \frac{\left(3\Delta_H(1-q)+3\,q\Delta_L + \sqrt{5\,(\Delta_H-\Delta_L)^2 q^2 - 6\,(\Delta_H - 2/3\,\Delta_L)(\Delta_H-\Delta_L)q + {\Delta_H}^2}\right)\Delta_L\,\Delta_H}{{\Delta_H}^2(1-q)+{\Delta_L}^2 q}$,
$1/2\, \frac{\left(\left(q^2-3\,q+2\right){\Delta_H}^2 - 2\,q\Delta_L\,(q-1)\Delta_H + q{\Delta_L}^2(q+1)\right)\Delta_H\,\Delta_L}{\left((q-1){\Delta_H}^2 - {\Delta_L}^2 q\right)((q-1)\Delta_H - q\Delta_L)}\}$. The solutions are real, since the term in the square root is positive. This can be written as

$$5\,(\Delta_H - \Delta_L)^2\, q^2 - 6\,(\Delta_H - 2/3\,\Delta_L)\,(\Delta_H - \Delta_L)\, q + {\Delta_H}^2 \tag{4.26}$$

$$> 5\,(\Delta_H - \Delta_L)^2\, q^2 - 4\,(\Delta_H)\,(\Delta_H - \Delta_L)\, q + {\Delta_H}^2 \tag{4.27}$$

$$> 4\,(\Delta_H - \Delta_L)^2\, q^2 - 4\,(\Delta_H)\,(\Delta_H - \Delta_L)\, q + {\Delta_H}^2 = (\Delta_H - 2(\Delta_H - \Delta_L)q)^2 > 0. \tag{4.28}$$

The first inequality follows from $\Delta_L > \Delta_H/2$.

Let me denote the first element in the set as θ_{trader}. Let me show that $\theta_{trader} > 0$. Clearly, the denominator is greater than zero. For the numerator, I have $(\Delta_H\Delta_L)(3\Delta_H - 3(\Delta_H - \Delta_L)q - \sqrt{5\,(\Delta_H - \Delta_L)^2 q^2 - 6\,(\Delta_H - 2/3\,\Delta_L)(\Delta_H - \Delta_L)\,q + {\Delta_H}^2})$
$> (\Delta_H\Delta_L)(3\Delta_H - 3(\Delta_H - \Delta_L)q - \sqrt{\Delta_H^2 - 2\Delta_H(\Delta_H - \Delta_L) + (\Delta_H - \Delta_L)^2 q^2})$
$= (\Delta_H\Delta_L)(3\Delta_H - 3(\Delta_H - \Delta_L)q - (\Delta_H - (\Delta_H - \Delta_L)q))$
$= 2\Delta_H\Delta_L(\Delta_H - (\Delta_H - \Delta_L)q) > 0$.

For the last element in the set, I see $1/2\,\frac{\left(\left(q^2-3\,q+2\right){\Delta_H}^2 - 2\,q\Delta_L\,(q-1)\Delta_H + q{\Delta_L}^2(q+1)\right)\Delta_H\,\Delta_L}{\left((q-1){\Delta_H}^2 - {\Delta_L}^2 q\right)((q-1)\Delta_H - q\Delta_L)} > \Delta_L$ whenever $\Delta_L < \Delta_H$, so it is not a candidate for an optimum. I have

$$f'(0) = -1/8\,\frac{\left((\Delta_H - \Delta_L)^2 q^2 + \left(-3\,{\Delta_H}^2 + 2\,\Delta_H\,\Delta_L + {\Delta_L}^2\right)q + 2\,{\Delta_H}^2\right)^2}{\left(\left({\Delta_H}^2 - {\Delta_L}^2\right)q - {\Delta_H}^2\right)\left((\Delta_H - \Delta_L)\,q - \Delta_H\right)^2} > 0 \tag{4.29}$$

and

$$f'(\Delta_L) = 1/16\,\frac{\Delta_H^2(\Delta_H - 2\Delta_L) + \Delta_L^2(\Delta_H^3 - 2\Delta_L^2) - \Delta_H^2\Delta_L^3}{\Delta_H\,{\Delta_L}^2\left({\Delta_H}^2 + {\Delta_L}^2\right)} < 0. \tag{4.30}$$

I see that the numerator is negative, by $\Delta_H < 2\Delta_L$. By continuity of $f'(\theta)$, there is an odd number of roots in $[0, \Delta_L]$. Since there are three roots in total and the last root of the set is not in $[0, \Delta_L]$, we need to have exactly one root in the interval. It has to be θ_{trader}, since $\theta_{trader} > 0$, and the second root in the set is greater than θ_{trader}. Thus, we have $\forall\theta \in [0, \Delta_L] f'(\theta) > 0 \iff \theta < \theta_{trader}$, and θ_{trader} is a candidate for our maximum.

Next, let me consider the possibility of an optimal $\theta \in (\Delta_L, \Delta_H]$.

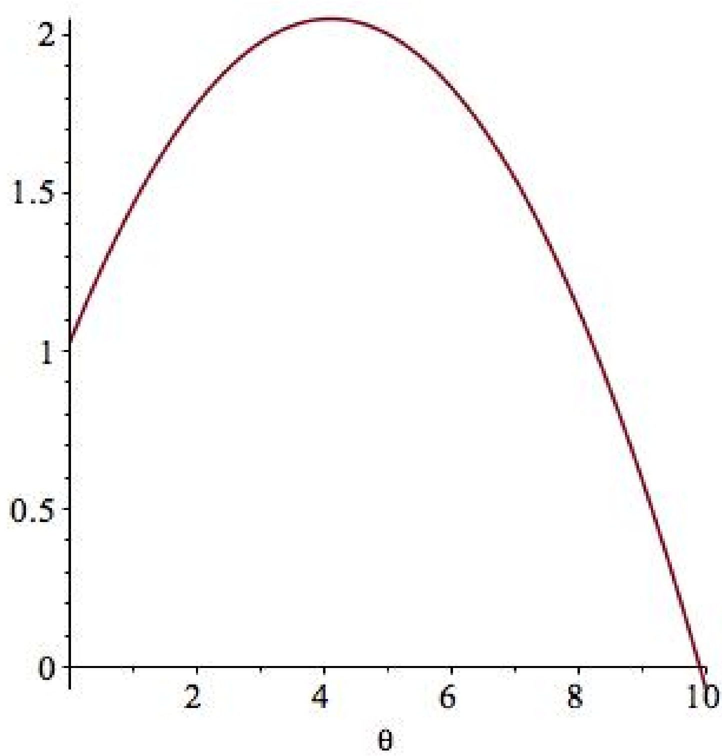

Figure 4.1: The function $\mathbf{1}_{\theta<\Delta_L} f(\theta) + \mathbf{1}_{\theta\geq\Delta_L} g(\theta), with \Delta_H = 10, \Delta_L = 7, q = 0.5$

I compute $\frac{\partial g(\theta)}{\partial\theta} = 1/2\,\frac{q(-2\,\theta+\Delta_H)}{\Delta_H} + \frac{\partial}{\partial\theta}\mathbb{E}_1[\Pi(q_2)|\theta]$. I note that the second term is negative as shown in lemma 6 and I obtain

$$\frac{\partial g(\theta)}{\partial\theta} < 0 \qquad\qquad \forall\theta \geq \Delta_L. \tag{4.31}$$

Thus, I obtain $f(\theta_{trader}) > f(\Delta_L) = g(\Delta_L) > g(\theta)\forall\theta \in [\Delta_L, \Delta_H]$, and the optimal order is θ_{trader}.

Finally, I need to show that $\theta_{BM} < \theta_{trader}$.

Let $f^{BM} = q\frac{\Delta_H-\theta}{\Delta_H}\theta(1-1/2) + (1-q)\frac{\Delta_L-\theta}{\Delta_L}\theta(1-1/2)$.

Thus, I have by lemma 3 and lemma that 6

$$\frac{\partial}{\partial\theta}f(\theta_{BM}) = \frac{\partial}{\partial\theta}f^{BM}_{\theta=\theta_{BM}} + \frac{\partial}{\partial\theta}\mathbb{E}_1[\Pi(q_2)|\theta]_{\theta=\theta_{BM}} > 0. \tag{4.32}$$

Here, I use $\frac{\partial}{\partial\theta}f^{BM}_{\theta=\theta_{BM}} = 0$ and the fact that the second term is positive by lemma 6. Furthermore, it is clear that $\theta_{BM} \in [0, \Delta_L]$. Since for $\theta \in [0, \Delta_L]$, we have $f'(\theta) > 0 \iff \theta < \theta_{trader}$, we obtain $\theta_{BM} < \theta_{trader}$.

□

4.1.7 Proof of proposition 8 (see page 40)

Proof. I solve the game backwards.

Given some probability $\mathbb{P}(L = \Delta_H) = q$, I know the outcome of the game at $t = 2$ from lemma 3. In the afternoon of $t = 1$, the fund manager at $t = 1$ chooses an optimal trade to maximise the expected compensation over the whole game.

Here, as usual, I assume that $v = 1$ in the first period, and the other case is symmetric. Then, the fund maximises given any inflow $\tilde{f}_1$ trade θ at $t = 1$. It is clear that the optimal $\theta \in [0, \Delta_H]$.

In $t = 2$, the situation is as in lemma 3. Thus, given that $\mathbb{P}(L = \Delta_H) = q$ at $t = 2$, the fund receives an inflow $\tilde{f}_2(q) = \frac{1-\gamma_P}{\gamma_M}\Pi(q)$. The expected compensation of a trader will be as in the case of an informed trader $\Pi(q)$ by lemma 3. Thus, the problem in the afternoon of $t = 1$ reduces to

$$\max_{\theta\in[0,\Delta_H]} \mathbb{E}_1[\gamma_M\tilde{f}_1 + \gamma_P(1-P)\theta + \Pi(q_2)|\theta]. \tag{4.33}$$

I see that the second period does not depend on γ_P, and the profits of the second period are thus weighted more. Recall the functions

$$f(\theta) = q\frac{\Delta_H - \theta}{\Delta_H}\theta(1-1/2) + (1-q)\frac{\Delta_L - \theta}{\Delta_L}\theta(1-1/2) + \mathbb{E}_1[\Pi(q_2)|\theta] \tag{4.34}$$

and

$$g(\theta) = q\frac{\Delta_H - \theta}{\Delta_H}\theta(1-1/2) + \mathbb{E}_1[\Pi(q_2)|\theta] \tag{4.35}$$

from proposition 7.

Let $f(\theta) = f_{BM}(\theta) + f_{SEC}(\theta)$ and $g(\theta) = g_{BM}(\theta) + g_{SEC}(\theta)$, with $f_{BM}(\theta) = q\frac{\Delta_H-\theta}{\Delta_H}\theta(1-1/2)) + (1-q)\frac{\Delta_L-\theta}{\Delta_L}\theta(1-1/2)$ and $g_{BM}(\theta) = q\frac{\Delta_H-\theta}{\Delta_H}\theta(1-1/2)$denoting the first period's profits. Problem (4.33) results in the same function as proposition 7, except that the fund manager weights the first period's profits only by γ_P. Then the problem (4.33) can be written as

$$\max_{\theta\in[0,\Delta_H]} \mathbf{1}_{\theta<\Delta_L}(\gamma_P f_{BM}(\theta) + f_{SEC}(\theta)) + \mathbf{1}_{\theta\geq\Delta_L}(\gamma_P g_{BM}(\theta) + g_{SEC}(\theta)). \tag{4.36}$$

I maximise over a continuous function that has a kink at $\theta = \Delta_L$. What is the optimal θ? I see that with θ_{trader} from proposition 7, I obtain

$$0 = f'_{BM}(\theta_{trader}) + f'_{SEC}(\theta_{trader}) < \gamma_P f'_{BM}(\theta_{trader}) + f'_{SEC}(\theta_{trader}). \tag{4.37}$$

Since $f'_{BM}(\theta_{trader}) < 0$ and $f'_{SEC}(\theta_{trader}) > 0$, $\gamma_P \in [0,1)$. Note that f_{BM} and f_{SEC} are continuously differentiable on $[0,\Delta_L]$. Thus, an increase in θ increases the expected utility of the fund. It is clear that no θ below θ_{trader} can be optimal (the function f is strictly increasing in θ in that region). By the results of proposition 7, a $\theta_{fund} > \Delta_L$ is not possible (the function g from that proof is still strictly decreasing when the first period's profits are weighted by γ_P and $\theta > \Delta_H/2$). Thus, I have $\theta_{fund} \in (\theta_{trader}, \Delta_L]$. Suppose that $\gamma_P = 0$, then the fund manager chooses θ to maximise the second period profits by theorem 8. By lemma 6, in this case $\theta_{fund} = \Delta_L$.[1]

The initial inflow will be such that the after-fee expected return of the fund is equal to the outside option. I obtain $\tilde{f}_1 = \frac{(1-\gamma_P)\Pi(q,\theta_{fund})}{\gamma_M}$, and the first period's expected compensation of the fund is given by $\gamma_M\tilde{f}_1 + \gamma_P\tilde{f}_1(\tilde{R}_1-1) = \Pi(q,\theta_{fund})$.

$$\begin{aligned} &\Pi(q,\theta_{fund}) + \mathbb{E}_1[\Pi(q_2)|\theta = \theta_{fund}] < \\ \max_\theta \Pi(q,\theta) + \mathbb{E}_1[\Pi(q_2)|\theta] = {}&\Pi(q,\theta_{trader}) + \mathbb{E}_1[\Pi(q_2)|\theta = \theta_{trader}]. \end{aligned} \tag{4.38}$$

[1] As shown in the figures for γ_P high enough $\theta_{fund} < \Delta_L$ and for low γ_P $\theta_{fund} = \Delta_L$.

The last inequality follows by the definition of θ_{trader}, solving the maximisation problem in proposition 7. This inequality shows that the expected compensation of the fund is lower than the expected profits of the trader.

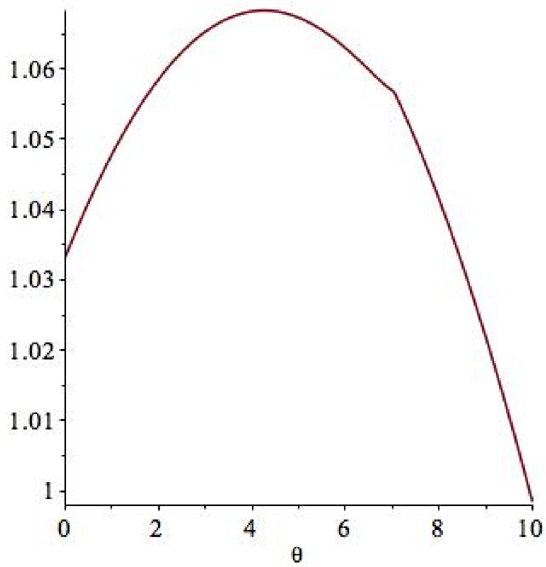

Figure 4.2: The function $\mathbf{1}_{\theta<\Delta_L} f(\theta) + \mathbf{1}_{\theta\geq\Delta_L} g(\theta)$, *with* $\Delta_H = 10, \Delta_L = 7, q = 0.5, \gamma_P = 0.05$

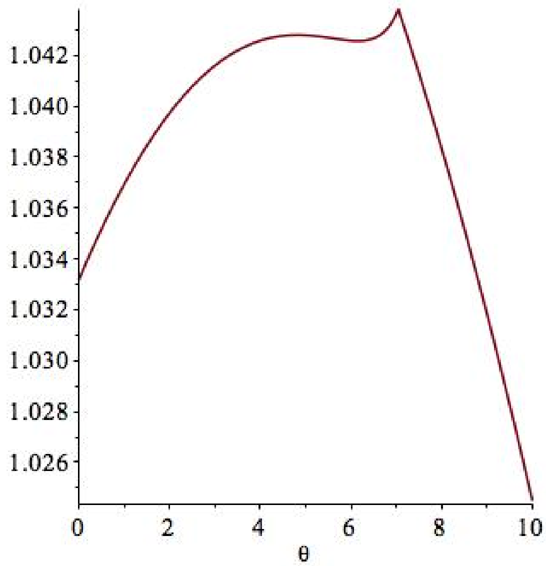

Figure 4.3: The function $\mathbf{1}_{\theta<\Delta_L} f(\theta) + \mathbf{1}_{\theta\geq\Delta_L} g(\theta)$, *with* $\Delta_H = 10, \Delta_L = 7, q = 0.5, \gamma_P = 0.01$

$\square$

4.1.8 Proof of corollary 9 (see page 44)

Let $\hat{L}_{old}^{trader}$ denote the cutoff point of the trader. This means that for this level of liquidity, the trader is indifferent. The trader's expected compensation with uncertain liquidity is always higher than that of the fund manager by proposition 8. Furthermore, without uncertainty about liquidity, the expected compensation of the fund is the same as that of the trader. The result follows. If $\gamma_P = 0$, we have by proposition 8 $\theta_{fund} = \Delta_L$. If the dispersion between Δ_H, Δ_L is relatively low, the expected compensation of the fund might be lower with uncertain liquidity compared to the case when $L = \Delta_L$.

4.1.9 Proof of corollary 10 (see page 44)

The inflow is given by

$$\frac{1-\gamma_P}{\gamma_M}\Pi(q_2) - \tilde{f}_1. \tag{4.39}$$

The inflow is convex if the profits in the second period $\Pi(q_2)$ are convex in the first period profits. Given a market order $\tilde{x} = \theta$, the profits if non zero are given by $1/2\theta$. Thus, we have to show that $\Pi(q_2)$ is convex in θ. The second derivative is given by

$$\frac{\partial^2}{\partial^2\theta^2}\Pi(q_2) = 1/4\,\frac{{\Delta_L}^2\left(\Delta_H - \Delta_L\right)^2(1-q)\left((q-1)\,{\Delta_H}^2 - q{\Delta_L}^2\right)q{\Delta_H}^2}{\left((\Delta_L-\theta)\,(q-1)\,{\Delta_H}^2 - q\Delta_H\,{\Delta_L}^2 + \theta\,q{\Delta_L}^2\right)^3} > 0. \tag{4.40}$$

It is easy to see that the expression satisfies the inequality when $\theta < \Delta_L$.

4.1.10 Proof of corollary 11 (see page 46)

Let the first period inflow denote $\tilde{f}_1$. The fund commits to closing and thus finds θ_{fund} to maximise

$$2\gamma_M\tilde{f}_1 + \gamma_P(\Pi(q,\theta) + \mathbb{E}_1[\Pi(q_2)|\theta]). \tag{4.41}$$

The fund solves the same problem as the trader. Thus, the fund chooses $\theta_{fund} = \theta_{trader}$. $\tilde{f}_1$ is such that investors break even over the two periods and this gives the result that the fund gets the full NPV (the expected profits of the trader). A high return increases q_2 and $\Pi(q_2)$ is increasing in q_2. Thus, we get the return persistence.

4.1.11 Micro-foundation of the trading opportunity

There is a large continuum of assets with $v_i \in \{0, 1\}$, where $i \in A$, and A is the universe of assets. The trader is informed about exactly one asset j and can trade in only that asset. The asset for which he receives information is random and private information. For each asset, there are noise traders who submit market orders and a market maker. Suppose that the noise traders are uniform on the open interval $(-L/2, L/2)$. They demand

$$\tilde{y}_i \sim U(-L/2, L/2) \tag{4.42}$$

shares of each assets. The noise trade is independent across assets. I assume that $L \in \{\Delta_H, \Delta_L\}$ and that the market maker knows L, which is the same for each asset. The informed trader submits a market order of $\tilde{x}_i$ shares (which can be different from zero only for $i = j$). The market maker observes the total order flow

$$\tilde{z}_i = \tilde{y}_i + \tilde{x}_i. \tag{4.43}$$

However, the market maker cannot observe the individual orders $\tilde{x}_i$ or $\tilde{y}_i$ separately. I assume each competitive market maker sets the following price for each asset

$$P_i = \mathbb{E}[v_i|\tilde{z}_i] \qquad \forall i \in A. \tag{4.44}$$

Lemma 31. *Suppose that the informed trader buys when $v_j = 1$ ($\tilde{x}_j > 0$) and sells otherwise. The price of asset j set by the market maker has the property*

$$P_j \in \{0, 1/2, 1\} \tag{4.45}$$

and

$$\mathbb{P}(P_j = 1/2) = \max(1 - \frac{|\tilde{x}_j|}{L}, 0) \quad \mathbb{P}(P_j = 1) = \min(\max(\frac{\tilde{x}_j}{L}, 0), 1) \tag{4.46}$$

$$\mathbb{P}(P_j = 0) = \min(\max(\frac{-\tilde{x}_j}{L}, 0), 1). \tag{4.47}$$

Proof. Let the market maker set the following price function for each asset i:
If $\tilde{z}_i \in (-\infty, -L/2]$ then he sets $P_i = 0$.
If $\tilde{z}_i \in (-L/2, L/2)$ then he sets $P_i = 1/2$.
If $\tilde{z}_i \in [L/2, \infty)$ then he sets $P_i = 1$.
Let $j \in A$ denote the asset for which there is insider trade. It is clear that $\mathbb{E}[v_i|\tilde{z} \in (-\infty, -L/2]] = \mathbb{E}[v_i|\tilde{z}_i \in (-\infty, -L/2], j = i, \tilde{x}_j < 0] = 0$, since it is only possible to obtain this region when the insider places a negative order. Similarly, I can see $\mathbb{E}[v_i|\tilde{z}_i \in [L/2, \infty)] = \mathbb{E}[v_i|\tilde{z}_i \in [L/2, \infty), j = i, \tilde{x}_j > 0] = 1$. I obtain that
$\mathbb{E}[v_i|\tilde{z}_i \in (-L/2, L/2)] = \mathbb{P}(i = j|\tilde{z}_i \in (-L/2, L/2))\mathbb{E}[v_i|\tilde{z}_i \in (-L/2, L/2), i = j]$
$+\mathbb{P}(i \neq j|\tilde{z}_i \in (-L/2, L/2))\mathbb{E}[v_i|\tilde{z}_i \in (-L/2, L/2), i \neq j] = \mathbb{E}[v_i|\tilde{z}_i \in (-L/2, L/2), i \neq j] = 1/2$.

This follows since I have $\mathbb{P}(i = j|\tilde{z}_i \in (-L/2, L/2)) = 0$, and
$\mathbb{E}[v_i|\tilde{z}_i \in (-L/2, L/2), i = j] \in [0, 1]$. Suppose that $\tilde{x} \in [0, L]$ is the market order for asset j:
I find that $\mathbb{P}(P_j = 1/2|\tilde{x}) = \mathbb{P}(\tilde{z}_j \in (-L/2, L/2)|\tilde{x}) = \mathbb{P}(\tilde{y}_j \in (-L/2, L/2-\tilde{x})) = 1 - \tilde{x}/L$. Furthermore, I see that $\mathbb{P}(P_j = 1|\tilde{x}) = \mathbb{P}(\tilde{z}_j \in [L/2, \infty)|\tilde{x}) = \mathbb{P}(\tilde{y}_j \in [L/2 - \tilde{x}, L/2)) = \tilde{x}/L$ and $\mathbb{P}(P_j = 0|\tilde{x}) = \mathbb{P}(\tilde{z}_j \in (-\infty, -L/2]|\tilde{x}) = 0$.
If $\tilde{x} > L$, we obtain that $\mathbb{P}(P_j = 1|\tilde{x}) = 1$ and $\mathbb{P}(P_j = 1/2|\tilde{x}) = \mathbb{P}(P_j = 0|\tilde{x}) = 0$.
Similarly, I can show symmetric results for $\tilde{x} < 0$.

□

4.2 Proofs of Chapter 2

4.2.1 Proof of lemma 14 (see page 62)

Proof. In an unrevealing regime in a simple equilibrium at $t = 2$, the uninformed funds make R, as noted in the main text. The informed funds make $1/2\frac{1+d}{P} + 1/2R = R + 1/2dR$, since the conjectured price is $\hat{P}_2 = 1/R$. The probability of an unrevealing regime is given by $\frac{\Delta-\tilde{\theta}_2}{\Delta}$, and the probability of matching with an informed fund is π. Thus, the per-dollar benefit is given as in the lemma.

That funds that make the wrong decision are uninformed is trivial (informed funds always make the right decision in a simple equilibrium). A manager with a high return could be informed or lucky and uninformed. If lucky, this could be a previous noise trader or an uninformed fund. By market clearing,[2] the probability of obtaining the risky asset for the uninformed fund manager who is indifferent is given by $\frac{(b-z_1)}{(1-\theta)}$ at $t = 1$. Let us denote by $\tilde{R}$ the realised return of the fund. We can calculate the reputation of a fund that out-performed with the risky asset $\mathbb{P}(informed|v_1 = 1{+}d, \tilde{R} = (1{+}d)\frac{1}{\hat{P}_1}) = \frac{\mathbb{P}(informed|v_1=1+d)\mathbb{P}(\tilde{R}=(1+d)\frac{1}{\hat{P}_1}|informed,v_1=1+d)}{\mathbb{P}(\tilde{R}=(1+d)\frac{1}{\hat{P}_1}|v_1=1+d)} = \frac{\frac{\theta}{1+\Delta}}{\frac{\theta}{1+\Delta}+\frac{(\tilde{\Delta}/\Delta)\Delta}{1+\Delta}+\frac{(1-\theta)(b-\tilde{\Delta}-\theta)}{(1+\Delta)(1-\theta)}} = \frac{\theta}{b}$
and for a fund that out-performed with the risk-free asset:

$$\mathbb{P}(informed|v_1 = 1-d, \tilde{R} = R) = \frac{\mathbb{P}(informed|v_1=1-d)\mathbb{P}(\tilde{R}=R|informed,v_1=1-d)}{\mathbb{P}(\tilde{R}=R|v_1=1-d)}$$
$$= \frac{\frac{\theta}{1+\Delta}}{\frac{\theta}{1+\Delta}+\frac{((\Delta-\tilde{\Delta})/\Delta)\Delta}{1+\Delta}+\frac{(1-\theta)}{1+\Delta}(1-\frac{(b-\tilde{\Delta})}{1-\theta})} = \frac{\theta}{1+\Delta-b}.$$

It is clear that an investor does not learn anything from returns following a revealing regime. Thus, no investor will pay the switching cost c.

$\square$

[2] As will be confirmed in proposition 16

4.2.2 Proof of lemma 15 (see page 63)

Proof. I have to show that the assets under management given in the lemma are the result of optimal flows. For the flows to be optimal, no investor should want to switch their decision given the decisions of all other investors. It is clear that all investors will stay with a fund that has outperformed.

Suppose $v_1 = 1 + d$, so the risky asset was the right choice in the first period. Thus, the reputation of the outperforming fund is given by $\pi = \frac{\theta}{b}$, as learned from lemma 14. In an interior equilibrium with flows, I have $e_2\Pi(\tilde{\theta}_2) = c$, so investors with underperforming funds are indifferent. For an interior equilibrium to exist, we need $\alpha_2 > e_2$, so there are some flows and some money invested with uninformed funds. Since all the informed funds are among the outperforming funds, I obtain $\theta\alpha_2 = \tilde{\theta}_2$. Solving for α_2, I find $\alpha_2 = \frac{\Delta}{\theta}(1 - 2\frac{c/e_2 b}{(1-\gamma)\theta dR})$ by assumption (2.14) $\alpha_2 > e_2$, so I have an equilibrium with flows. The inflow comes from a mass of funds $1 - b + \Delta$. This inflow goes to a mass of b funds. Thus, the total expected outflow per fund is $\frac{b}{1-b+\Delta}(\frac{\Delta}{\theta}(1 - 2\frac{c/e_2 b}{(1-\gamma)\theta dR}) - e_2)$. I need to show that $e_2 - \frac{b}{1-b+\Delta}(\frac{\Delta}{\theta}(1 - 2\frac{c/e_2 b}{(1-\gamma)\theta dR}) - e_2) > 0$ so that there are enough funds for these flows to be feasible. I obtain $e_2 - \frac{b}{1-b+\Delta}(\frac{\Delta}{\theta}(1 - 2\frac{c/e_2 b}{(1-\gamma)\theta dR}) - e_2) > 1 - \delta - \frac{\kappa}{1-\kappa}(\frac{\Delta}{\theta} - (1 - \delta)) > 0$. The last inequality follows follows by assumption (2.13), i.e, $\frac{\Delta}{\theta} < \frac{1-\delta}{\kappa}$.

Suppose now that $v_1 = 1 - d$. In an interior equilibrium similar to the above, we obtain $\alpha_2 = \frac{\Delta}{\theta}(1 - 2\frac{c/e_2(1+\Delta-b)}{(1-\gamma)\theta dR})$. First, we need to show that $\alpha_2 > e_2$ in this case. This follows by assumptions (2.13) and (2.14). The inflow goes to a mass of $1+\Delta-b$ funds, and those fund managers have chosen the risk-free asset. The inflow comes from a mass of b funds, that is, those that chose the risky asset. I need to show that $e_2 - \frac{1-b+\Delta}{b}(\frac{\Delta}{\theta}(1 - 2\frac{c/e_2(1+\Delta-b)}{(1-\gamma)\theta dR}) - e_2) > 0$. I obtain that $e_2 - \frac{1-b+\Delta}{b}(\frac{\Delta}{\theta}(1 - 2\frac{c/e_2(1+\Delta-b)}{(1-\gamma)\theta dR}) - e_2) > (1-\delta) - (\frac{\Delta}{\theta} - (1-\delta)) > 0$ by assumptions (2.13).

$\square$

4.2.3 Proof of proposition 16 (see page 66)

Proof. The proof is quite similar to the one in Guerrieri and Kondor (2012). The price schedule is as follows.

There exist the following revealing equilibrium regimes:

If $z_t \in [0, \tilde{\theta}_t)$, then $P_t = \frac{1-d}{R}$.

If $z_t \in (\Delta, \tilde{\theta}_t + \Delta]$, then $P_t = \frac{1+d}{R}$.

There exists a non-revealing equilibrium regime:

If $z_t \in [\tilde{\theta}_t, \Delta]$, then $P_t = \hat{P}_t$.

Here, $\hat{P}_t$ are as in the proposition.

I take the investors fund switching decisions from the lemma 15 as given and then:

1) I will construct the demand schedules of the fund managers.

2) I will construct asset allocations consistent with these demand schedules.

3) I will show that in the unrevealing regime, no information is transmitted through the price.

4) I will verify that the demand schedules are optimal given these prices.

5) I will show that the markets clear.

1)

The informed funds submit the following demand schedule[3]

$$d_I(P) = \{0, 1\} \qquad\qquad P_t \in \{\frac{1+d}{R}, \frac{1-d}{R}\} \tag{4.48}$$

$$d_I(P) = \mathbf{1}_{v=1+d}. \qquad\qquad P_t = \hat{P}_t. \tag{4.49}$$

The uninformed funds submit:

$$d_U(P) = \{0, 1\} \qquad\qquad \forall P_t. \tag{4.50}$$

[3] $\mathbf{1}$ is the indicator function

2)

The allocation of the auctioneer is:[4]

$$X_t(d) = d \qquad d \in \{0,1\} \tag{4.51}$$

$$X_t(\{0,1\}) = \frac{b - z_t}{e_t - \tilde{\theta}_t} \qquad z_t \in [\tilde{\theta}_t, \Delta] \tag{4.52}$$

$$X_t(\{0,1\}) = \frac{b - \tilde{\Delta}_t}{e_t} \qquad z_t \notin [\tilde{\theta}_t, \Delta] \tag{4.53}$$

3)

The updated probability is calculated using Bayes' rule.

$$\mathbb{P}(v_t = 1 + d | P_t = \hat{P}_t) = \frac{\mathbb{P}(v_t = 1 + d)\mathbb{P}(P_t = \hat{P}_t)|v_t = 1 + d)}{\mathbb{P}(P_t = \hat{P}_t)} \tag{4.54}$$
$$= \frac{1/2\mathbb{P}(\tilde{\Delta}_t \in [0, \Delta - \tilde{\theta}_t])}{1/2\mathbb{P}(\tilde{\Delta}_t \in [0, \Delta - \tilde{\theta}_t]) + 1/2\mathbb{P}(\tilde{\Delta}_t \in [\tilde{\theta}_t, \Delta])} = 1/2.$$

Thus, I have shown that there is no information revealed in this case. It is clear that the other prices fully reveal v_t, by the demand of informed fund managers.

4)

At $t = 1$, the informed and the uninformed fund manager anticipate the optimal investor flows and the resulting assets under management as summarised in lemma 15. It is clear that the informed managers strategy is optimal. In a revealing regime, the strategy of uninformed managers is optimal since they perfectly mimic informed managers. For uninformed managers to submit the above demand schedule in the unrevealing regime, they have to be indifferent between the two assets. Given risk neutrality and the price that makes their expected utility the same (guaranteed by (2.23)), this is the case when the following condition is satisfied:

$$\mathbb{E}[X_t(\{0,1\}))|v_t = 1 + d, P_t = \hat{P}_t] = \tag{4.55}$$

[4]Since by assumption (2.15) $1 - \delta - b > 2\Delta > 0$, we have that all $X(d) \in [0,1]$

$$\mathbb{E}[X_t(\{0,1\}))|v_t = 1-d, P_t = \hat{P}_t].$$

This means that the probability of getting the risky asset does not depend on v_t, so that $d = \{0,1\}$ is optimal. A price $P_t = \hat{P}_t$ and $v_t = 1+d$, means $\tilde{\Delta} \in [0, \Delta - \tilde{\theta}_t]$. A price $P_t = \hat{P}_t$ and $v_t = 1-d$, means $\tilde{\Delta} \in [\tilde{\theta}_t, \Delta]$. We get

$$\mathbb{E}[X_t(\{0,1\})|v_t = 1-d, P_t = \hat{P}_t] = \int_{\tilde{\theta}_t}^{\Delta} \frac{b - \tilde{\Delta}}{(e_t - \tilde{\theta}_t)} \frac{1}{(\Delta - \tilde{\theta}_t)} d\tilde{\Delta} \tag{4.56}$$

$$= \frac{1}{2(e_t - \tilde{\theta}_t)}(2b - \tilde{\theta}_t - \Delta) \tag{4.57}$$

and

$$\mathbb{E}[X_t(\{0,1\})|v_t = 1+d, P_t = \hat{P}_t] = \int_{0}^{\Delta - \tilde{\theta}_t} \frac{b - \tilde{\theta}_t - \tilde{\Delta}}{(e_t - \tilde{\theta}_t)} \frac{1}{(\Delta - \tilde{\theta}_t)} d\tilde{\Delta} \tag{4.58}$$

$$= \frac{1}{2(e_t - \tilde{\theta}_t)}(2b - \tilde{\theta}_t - \Delta). \tag{4.59}$$

I see that both expressions are the same and so the demand schedule is optimal.

The assumption (2.16) makes sure that $\hat{P}_1 \in [\frac{1-d}{R}, \frac{1+d}{R}]$. It is obvious that $\mathbb{E}[\alpha_2|\psi = S] - \mathbb{E}[\alpha_2|\psi = V] > -\frac{\Delta}{\theta}$. Thus, I get $\hat{P}_1 = \frac{1}{R(1+\omega(\mathbb{E}[\alpha_2|\psi=S]-\mathbb{E}[\alpha_2|\psi=V]))} < \frac{1}{R(1-\omega\frac{\Delta}{\theta}))} < \frac{1}{R(1-\frac{d}{d+1})} = \frac{1+d}{R}$. It is also clear that $\mathbb{E}[\alpha_2|\psi = S] - \mathbb{E}[\alpha_2|\psi = V] < \frac{\Delta}{\theta}$, and so I get that $\hat{P}_1 > \frac{1-d}{R}$.

5)

Suppose $v_t = 1-d$ and $P_t = \hat{P}_t$, then I have $(e_t - \tilde{\theta}_t)\frac{b - \tilde{\Delta}_t}{e_t - \tilde{\theta}_t} + \tilde{\Delta}_t = b$. Suppose $v = 1+d$ and $P = \hat{P}_t$, then I obtain $(e_t - \tilde{\theta}_t)\frac{b - \tilde{\Delta}_t - \tilde{\theta}_t}{e_t - \tilde{\theta}_t} + \tilde{\Delta}_t + \tilde{\theta}_t = b$. Suppose that $v = 1+d$ and $P = \frac{1+d}{R}$. I obtain: $e_t\frac{b - \tilde{\Delta}_t}{e_t} + \tilde{\Delta}_t = b$.

The other cases are similar.

It is clear that given this price function the switching decisions of investors as in lemma 25 are optimal.

□

4.2.4 Proof of proposition 17 (see page 66)

Proof. I differentiate the expression for the expected return $\mathbb{E}_1[R_1] = \frac{1}{\hat{P}_1}$ where $\hat{P}_1$ is given by (2.24) and obtain
$\frac{\partial \mathbb{E}_1[R_1]}{\partial \beta} = 2\,\frac{\delta\left(-b^3 c\Delta+\left(-c\Delta^2-c\Delta+Rd\theta^2\left(\delta^2-1\right)\right)b^2-(1+\Delta)\left(-2\,c\Delta^2-2\,c\Delta+Rd\theta^2\left(\delta^2-1\right)\right)b-c\Delta\,(1+\Delta)^3\right)\omega}{\theta^2 db(\delta^2-1)(b-1-\Delta)}$,
a continuous function given our assumptions. This expression has a single simple root in c, which is given by

$$\hat{c} = \frac{Rbd\theta^2(1+\Delta-b)(1-\delta^2)}{\Delta\,(\Delta^3-2\,b\Delta^2+b^2\Delta+b^3+3\,\Delta^2-4\,b\Delta+b^2+3\,\Delta-2\,b+1)}. \tag{4.60}$$

I evaluate the function at $c = 0$ and find
$\frac{\partial \mathbb{E}_1[R_1]}{\partial \beta}(0) = 2\delta R\delta\omega > 0.$

Since the function $\frac{\partial \mathbb{E}_1[R_1]}{\partial \beta}$ is a linear and decreasing function in c, I obtain $\frac{\partial \mathbb{E}_1[R_1]}{\partial \beta} > 0 \iff c < \hat{c}$ and $\hat{c} > 0$.

□

4.2.5 Proof of proposition 18 (see page 67)

Proof. I take the derivative of the expression for the expected return with respect to b and get $\frac{\partial \mathbb{E}_1[R_1]}{\partial b} = \omega\Delta(\kappa_1 + \kappa_2 c)$, where $\kappa_1 := -\frac{R(\Delta+1)\left(1/2\Delta^2-b\Delta+b^2+\Delta-b+1/2\right)}{\theta\,b^2(b-1-\Delta)^2}$ and

$\kappa_2 :=$
$\frac{\left(b^2(b-1-\Delta)^2\delta^2+\left(b^4+(-2\,\Delta-2)b^3+(\Delta+1)^2b^2-2\,(\Delta+1)^3b+(\Delta+1)^4\right)(1-2\beta)\delta-2\left(b^2+(-1-\Delta)b+1/2\,(\Delta+1)^2\right)(\Delta+1)^2\right)}{\theta^2 db^2(\delta^2-1)(b-1-\Delta)^2}$.

$\frac{\partial \mathbb{E}[R_1]}{\partial b}$ is a continuous function which is linear in c. I set the equation to zero and solve for c. I see that the function has a single root in c given by the threshold level
$\hat{\hat{c}} =$
$\frac{(\delta^2-1)(\Delta+1)\left(\Delta^2-2\,\Delta\,b+2\,b^2+2\,\Delta-2\,b+1\right)Rd\theta}{2\,b^2(b-\Delta-1)^2\delta^2-4\,(-1/2+\beta)\left(b^4+(-2\,\Delta-2)b^3+(\Delta+1)^2b^2-2\,(\Delta+1)^3b+(\Delta+1)^4\right)\delta-4\,\left(b^2+(-\Delta-1)b+1/2\,(\Delta+1)^2\right)(\Delta+1)^2}$.
The function $\frac{\partial \mathbb{E}_1[R_1]}{\partial b}$ is linear and increasing[5] in c. Therefore, I obtain $\frac{\partial \mathbb{E}_1[R]_1}{\partial b} <$

[5] It is easy to check that given our assumptions $\kappa_2 > 0$

$0 \iff c < \hat{\hat{c}}$. I obtain
$\frac{\partial \mathbb{E}_1[R_1]}{\partial \beta}(0) = -\frac{\Delta\, R(\Delta+1)\left(1/2\,\Delta^2+(-b+1)\Delta+b^2-b+1/2\right)\omega}{\theta\, b^2(b-\Delta-1)^2} < 0$. Thus, I obtain $\hat{\hat{c}} > 0$.

□

4.2.6 Proof of proposition 19 (see page 68)

Proof. i) This is clear.

ii) This is also clear since the expected utility benefit of switching fund is equal to c.

iii) This follows from the fact that in equilibrium $e_2\Pi(\tilde{\theta}_2) = c$

iv) The price efficiency is given by $\frac{\tilde{\theta}_2}{\Delta}$ and we have $e_2\Pi(\tilde{\theta}_2) = c$. Thus, the higher c the lower $\tilde{\theta}_2$ the lower the price efficiency.

v) Suppose that the reputation of out-performing funds is π. The price efficiency is given by

$$\frac{\tilde{\theta}_2}{\Delta} = 1 - 2\frac{c}{e_2 dR(1-\gamma)\pi} \tag{4.61}$$

and clearly increasing in e_2.

□

4.3 Proofs of Chapter 3

4.3.1 Proof of lemma 22 (see page 80)

Proof. The proof will be trivial given the results of the two-period model from theorem 26. Note that by assumption (3.4) $\Delta < \tilde{x}$ and $\tilde{x} < H$. Thus, we can find b such that $\tilde{b} < \tilde{x}$ and $\tilde{b} > \Delta$. $\square$

4.3.2 Proof of lemma 24 (see page 82)

Proof. Let us calculate the probability of being informed given that $v_1 = 1$ using Bayes' rule. Let $\tilde{R}$ denote the realised return of a fund. By market clearing, the probability of an uninformed fund manager obtaining the risky asset when $v_1 = 1$ in the unrevealing regime is given by $\frac{(\tilde{b}_1 - \tilde{\Delta}_1 - \tilde{\theta}_1)}{\tilde{x}_1 - \tilde{\theta}_1}$. Thus, we obtain $\mathbb{P}(\tilde{R} = \frac{R}{1-q} | v_1 = 1) = \frac{\theta}{1+\Delta/\tilde{x}_1} + \frac{(\tilde{\Delta}_1/\Delta)\Delta/\tilde{x}_1}{1+\Delta/\tilde{x}_1} + \frac{(1-\theta)(\tilde{b}_1 - \tilde{\Delta}_1 - \tilde{\theta}_1)}{(1+\Delta/\tilde{x}_1)(\tilde{x}_1 - \tilde{\theta}_1)}$. The first term is the probability of getting an informed fund who invested in the risky asset[6], the second is the probability of a noise trader investing in the risky asset, and the last term is the probability of an uninformed fund manager investing in the risky asset. I also note that $\tilde{\theta}_1 = \theta \tilde{x}_1$.

$\mathbb{P}(informed | v_1 = 1, \tilde{R} = \frac{R}{1-q}) = \frac{\mathbb{P}(informed|v_1=1)\mathbb{P}(\tilde{R}=\frac{R}{1-q}|informed, v_1=1)}{\mathbb{P}(\tilde{R}=\frac{R}{1-q}|v_1=1)}$

$$= \frac{\frac{\theta}{1+\Delta/\tilde{x}_1}}{\frac{\theta}{1+\Delta/\tilde{x}_1} + \frac{(\tilde{\Delta}_1/\Delta)\Delta/\tilde{x}_1}{1+\Delta/\tilde{x}_1} + \frac{(1-\theta)(\tilde{b}_1 - \tilde{\Delta}_1 - \tilde{\theta}_1)}{(1+\Delta/\tilde{x}_1)(\tilde{x}_1 - \tilde{\theta}_1)}} = \frac{\tilde{\theta}_1}{\tilde{b}}.$$

In the case of $v_1 = 0$, I get

$\mathbb{P}(informed | v_1 = 0, \tilde{R} = R) = \frac{\mathbb{P}(informed|v_1=0)\mathbb{P}(\tilde{R}=R|informed, v_1=0)}{\mathbb{P}(\tilde{R}=R|v_1=0)}$

$$= \frac{\frac{\theta}{1+\Delta/\tilde{x}_1}}{\frac{\theta}{1+\Delta/\tilde{x}_1} + \frac{((\Delta - \tilde{\Delta}_1)/\Delta)\Delta/\tilde{x}_1}{1+\Delta/\tilde{x}_1} + \frac{(1-\theta)}{(1+\Delta/\tilde{x}_1)}(1 - \frac{\tilde{b} - \tilde{\Delta}_1}{\tilde{x}_1 - \tilde{\theta}_1})} = \frac{\tilde{\theta}_1}{\tilde{x}_1 + \Delta - \tilde{b}}.$$

$\square$

[6] By definition, in a simple equilibrium, informed fund managers always make the right choice

4.3.3 Proof of lemma 25 (see page 83)

Proof. Suppose the probability to match with an informed manager is π. In equilibrium, we need that given the set of all households decisions $\tilde{x}$, no household i wants to change its decision. This happens when $(1-\gamma)\pi q R(\frac{\Delta-\tilde{\theta}_t}{\Delta}) - \gamma R = 0$. If this equation is satisfied, there is no utility gained by any household changing its decision. An informed fund manager is never fired at $t = 1$; therefore, we have $\tilde{\theta}_2 = \pi\tilde{x}_2$. Then, we can solve for $\tilde{x}_2$. The proof is similar for $\tilde{x}_1$.

□

4.3.4 Proof of theorem 26 (see page 85)

Proof. I need to show that at each time there is a simple equilibrium. Suppose that b_t is given as in the theorem. I take the flows as in lemma 25 at each time as given and I consider the following pricing function:
If $z_t \in (\Delta, \tilde{\theta}_t + \Delta]$ then $P_t = \frac{1}{R}$.
If $z_t \in [\tilde{\theta}_t, \Delta]$ then $P_t = (1-q)/R$.
If $z_t \in [0, \tilde{\theta}_t)$ then $P_t = 0$.
I proceed with the following steps:
1) I construct demand schedules of the fund managers.
2) I construct asset allocations consistent with these demand schedules.
3) I show that in the unrevealing regime, no information is transmitted through the price.
4) I verify that the demand schedules are optimal given these prices.
5) I show that markets clear
6) I show that the intermediary breaks even.
7) I show that the household flows are optimal
Demand schedules

The informed submit the following demand schedule

$$d_t^I(1/R) = \{0,1\} \tag{4.62}$$

$$d_t^I(\frac{1-q}{R}) = v_t \tag{4.63}$$

$$d_t^I(0) = 0. \tag{4.64}$$

The uninformed submit

$$d_t^U(P_t) = \{0,1\} \qquad \forall P_t \neq 0 \tag{4.65}$$

$$d_t^U(0) = 0. \tag{4.66}$$

The households submit[7]

$$d_t^H(1/R) = \{0,1\} \tag{4.67}$$

$$d_t^H(P_t) = 0 \qquad \forall P_t \neq 1/R. \tag{4.68}$$

Asset allocations

The allocation of the auctioneer is

$$X^t(d) = d \qquad d \in \{0,1\} \tag{4.69}$$

$$X^t(\{0,1\}) = \frac{b_t\frac{1-q}{R} - z_t}{\tilde{x}_t - \tilde{\theta}_t} \qquad z_t \in [\tilde{\theta}_t, \Delta] \tag{4.70}$$

$$X^t(\{0,1\}) = \frac{b_t\frac{1}{R} - \tilde{\Delta}_t}{H} \qquad z_t \in (\Delta, \tilde{\theta}_t + \Delta] \tag{4.71}$$

$$X^t(\{0,1\}) = 0 \qquad z_t \in [0, \tilde{\theta}_t) \tag{4.72}$$

3)

Using Bayes' rule, I calculate

$$\mathbb{P}(v_t = 1 | P_t = \frac{1-q}{R}) = \frac{\mathbb{P}(v_t = 1)\mathbb{P}(P_t = \frac{1-q}{R})|v_t = 1)}{\mathbb{P}(P_t = \frac{1-q}{R})} \tag{4.73}$$

[7]The households asset allocation did not come up in the main text since they are restricted to invest in risk-free assets.

$$= \frac{(1-q)\mathbb{P}(\tilde{\Delta}_t \in [0, \Delta - \tilde{\theta}_t])}{(1-q)\mathbb{P}(\tilde{\Delta}_t \in [0, \Delta - \tilde{\theta}_t]) + q\mathbb{P}(\tilde{\Delta}_t \in [\tilde{\theta}_t, \Delta])} = 1 - q.$$

The result follows, since we have for the uniform distribution $\mathbb{P}(\tilde{\Delta}_t \in [0, \Delta - \tilde{\theta}_t]) = \mathbb{P}(\tilde{\Delta}_t \in [\tilde{\theta}_t, \Delta])$. It is clear that the other prices fully reveal v_t, by the demand of informed fund managers.

Optimal demands

It is clear that the informed fund managers demand schedules are optimal given their information and the that inflow they can obtain. Furthermore, when prices are fully revealing, it is clear that the demand schedules of uninformed managers are optimal because they mimic the informed fund managers. In order for uninformed funds to submit indifference in the unrevealing regime, the expected utility of submitting $d = 1$ and $d = 0$ must be the same. This is the case, since b_1 is such that $\tilde{b}_1$ solves (3.23) at $t = 1$. Furthermore, we need to check that the probability of getting the risky asset does not depend on v_t, so that $d = \{0, 1\}$ is optimal. A price $P_t = \frac{1-q}{R}$ and $v_t = 1$, means $\tilde{\Delta} \in [0, \Delta - \tilde{\theta}_t]$. A price $P_t = \frac{1-q}{R}$ and $v_t = 0$, means $\tilde{\Delta} \in [\tilde{\theta}_t, \Delta]$. We get

$$\mathbb{E}[X^t(\{0,1\})|v_t = 0, P_t = \frac{1-q}{R}] = \int_{\tilde{\theta}_t}^{\Delta} \frac{b_t \frac{1-q}{R} - \tilde{\Delta}}{(\tilde{x}_t - \tilde{\theta}_t)} \frac{1}{(\Delta - \tilde{\theta}_t)} d\tilde{\Delta} \tag{4.74}$$

$$= \frac{1}{2(\tilde{x}_t - \tilde{\theta}_t)} (2b_t \frac{1-q}{R} - \tilde{\theta}_t - \Delta) \tag{4.75}$$

and

$$\mathbb{E}[X^t(\{0,1\})|v_t = 1, P_t = \frac{1-q}{R}] = \int_0^{\Delta - \tilde{\theta}_t} \frac{b_t \frac{1-q}{R} - \tilde{\theta}_t - \tilde{\Delta}}{(\tilde{x}_t - \tilde{\theta}_t)} \frac{1}{(\Delta - \tilde{\theta}_t)} d\tilde{\Delta} \tag{4.76}$$

$$= \frac{1}{2(\tilde{x}_t - \tilde{\theta}_t)} (2b_t \frac{1-q}{R} - \tilde{\theta}_t - \Delta). \tag{4.77}$$

It is easy to see that the expressions are the same. Then we see that submitting $\{0, 1\}$ is optimal.

Allocation functions

It is clear that the allocations are consistent with the demand schedules.[8]

[8]The allocation probabilities $X^t(d) \in [0, 1]$,since H is large and the assumptions on b

Market clearing

Suppose that $z_t \in (\Delta, \tilde{\theta}_t + \Delta]$. Then, the price is $P = 1/R$. Everyone is indifferent, and since $b\frac{1}{R} = \tilde{\Delta}_t + \frac{b/R - \tilde{\Delta}_t}{H} H$, markets clear.[9] Suppose that $z_t \in [\tilde{\theta}_t, \Delta]$; then, we have $b\frac{1-q}{R} = z_t + \frac{b\frac{1-q}{R} - z_t}{\tilde{x}_t - \tilde{\theta}_t}(\tilde{x}_t - \tilde{\theta}_t)$, and thus markets clear. If $z_t \in [0, \tilde{\theta}_t)$, then markets clear since there is no demand for the risky asset and no nominal supply.

Intermediary

In the simple equilibrium, we get that $\mathbb{E}[P_t] = q(\frac{\Delta - \tilde{\theta}_t}{\Delta}\frac{1-q}{R} + \frac{\tilde{\theta}_t}{\Delta} 0) + (1-q)(\frac{\Delta - \tilde{\theta}_t}{\Delta}\frac{1-q}{R} + \frac{\tilde{\theta}_t}{\Delta} 1)$

$= \frac{1-q}{R}$. Thus, the intermediary breaks even when producing b_t as specified in the theorem. This means that b_1 is given as specified in the theorem and b_2 is can be chosen such that $\tilde{b}_2 > \Delta$ and $\tilde{b}_2 < \tilde{x}_2$.

Household flows

It is clear that given this price function the flows as in lemma 25 are optimal.

□

4.3.5 Proof of theorem 27 (see page 86)

Proof. The limit equilibrium is the obvious limit case of proposition 26.

We have $\mathbb{P}(P_t = \frac{1-q}{R}) = 1 - \frac{1}{\kappa_L}\tilde{x}_t$. As we approach the limit, $\tilde{b}_1$ converges to $\kappa_L(3q-1) - \kappa_L\kappa_F + \frac{\kappa_L}{\kappa_F}(q - 2q^2)$. An equilibrium exists when $\tilde{b}_1 > 0$ and $\tilde{x}_1 > \tilde{b}$. We have that $\tilde{x}_1 = \kappa_L(1 - \frac{\kappa_F}{q})$. Let $q < 1/2$; we find that $\tilde{b}_1 > 0$ and $\tilde{x}_1 > 0$ if and only if $\kappa_F < q$. For $\kappa_F < q$, we have $\tilde{b}_1 < \tilde{x}_1$ as long as $\frac{q-2q^2}{1-q} < \kappa_F$.

Suppose that $q > 1/2$. We need $\tilde{x}_1 > 0$ so $\kappa_F < q$; as long as $\kappa_F > 2q - 1$ we have $\tilde{x}_1 > \tilde{b}_1 > 0$.

[9] By the assumption (3.5) and the assumptions on b_t, we always have that the probability term is in $[0, 1]$.

Suppose that the conditions are satisfied. We have $\tilde{x}_2 = \frac{\tilde{b}_1 \kappa_L}{\tilde{x}_1}\left(1 - \frac{\tilde{b}_1 \kappa_F}{\tilde{x}_1\, q}\right)$ for the case $v_1 = 1$ and $\tilde{x}_2 = \frac{(\tilde{x}_1 - \tilde{b}_1)\kappa_L}{\tilde{x}_1}\left(1 - \frac{(\tilde{x}_1 - \tilde{b}_1)\kappa_F}{\tilde{x}_1\, q}\right)$ for the case $v_1 = 0$. We have $\frac{\tilde{b}_1}{\tilde{x}_1} < 1$ and $\frac{(\tilde{x}_1 - \tilde{b}_1)}{\tilde{x}_1} < 1$ and so it is clear that $\tilde{x}_2 > 0$. Thus, b_2 can be chosen so that $\tilde{x}_2 > \tilde{b}_2 > 0$.

□

4.3.6 Proof of proposition 28 (see page 87)

Proof. We have that $\tilde{x}_1 = \kappa_L(1 - \frac{\kappa_F}{q})$ and $\tilde{b}_1$ is as in theorem 27. We take the ratio and plug in $\tilde{b}_1$ and find (3.34). Suppose $q < 1/2$; then, we find the interval from plugging in the possible values of κ_F. Similarly, for the case $q > 1/2$. We find that $\frac{\partial}{\partial \kappa_F} \frac{\tilde{b}_1}{\tilde{x}_1} = \frac{q}{\kappa_F} - \frac{q(-2\,q + \kappa_F + 1)}{\kappa_F{}^2} > 0 \iff q > 1/2$.

□

4.3.7 Proof of proposition 29 (see page 89)

We get the expression from plugging in $\tilde{x}_1 = \kappa_L(1 - \kappa_F/q)$, $\tilde{b}_1 = \kappa_L(3q-1) - \kappa_L\kappa_F + \frac{\kappa_L}{\kappa_F}(q - 2q^2)$ and $\tilde{x}_2 = \frac{\tilde{b}_1 \kappa_L}{\tilde{x}_1}\left(1 - \frac{\tilde{b}_1 \kappa_F}{\tilde{x}_1\, q}\right)$ for the case $v_1 = 1$. If $v_1 = 0$, we have $\tilde{x}_2 = \frac{(\tilde{x}_1 - \tilde{b}_1)\kappa_L}{\tilde{x}_1}\left(1 - \frac{(\tilde{x}_1 - \tilde{b}_1)\kappa_F}{\tilde{x}_1\, q}\right)$. The results for the limits are straightforward.

4.3.8 Proof of proposition 30 (see page 90)

The limits are straightforward from proposition 29.

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