Essays on Financial Macroeconomics

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A thesis submitted to the Department of Economics of the London School of Economics for the degree of Doctor of Philosophy.

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Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

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Statement of conjoint work

I confirm that the third and final chapter of the thesis was jointly co-authored with Michel Azulai and I contributed to 50% of this work.
Abstract

In this thesis, I study various aspects of the financial system particularly relevant to macroeconomics, focusing on securitization and financial product complexity.

The first chapter is devoted to developing a model dealing with the interaction between securitization and recourse (limited liability) laws, and its effect on the housing market. The model finds that securitization of mortgage loans allows originators to pass on risk. As a consequence, investor borrowers start receiving loans, and when these loans are non-recourse, there is a put option that pushes up house prices during a demand boom. I thus have the novel prediction that the interaction between securitization and non-recourse status should lead to higher house prices.

The second chapter proceeds to test this prediction, making use of heterogeneity in recourse laws in US states. I find that non-recourse status roughly doubles the size of the positive relationship between securitization and house prices in a state, and can explain 75% of the difference in prices between recourse and non-recourse states. To address potential endogeneity concerns, I propose a new instrument for securitization, the distance of a housing market to the headquarters of ‘originate and securitize’ institutions, and find further empirical support for the predictions of the model.

In the last chapter (joint work with Michel Azulai), we turn our attention away from the behaviour of banks to asking why regulators have difficulties in regulating them. We develop a framework focusing on financial product complexity and how it can make it costly for regulators to screen them. Bad financial products created by banks can lead to moral hazard issues, as banks are bailed out in case of adverse shocks. Thus regulators must incentivise banks so that they do not ‘abuse’ complexity by making bad products complex. We show what the optimal contract is like for when regulators can commit, and discuss how the contract would be with limited regulator commitment.
Dedication and Acknowledgments

Dedication

I dedicate this thesis firstly to my parents, who jointly inspired me to study Economics when I was a teenager, through countless discussions and debates, and who have always supported me throughout my many years of study.

I also dedicate it to my loving wife, whose care and support these last three years have been a cornerstone of my life.

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Chapter 1

Introduction

Economics, like any area of research, requires a never ending effort to keep improving itself, as what we do not know dwarfs, and seems to always dwarf, what we do. This is true particularly in macroeconomics, I believe, where areas of research previously considered of more secondary importance, are becoming ever more prominent, areas such as housing, finance and bounded rationality, and this thesis is written very much in the spirit of exploring these new areas.

One of the most important reasons for this shift in academic importance lies with the Great Recession. Research has turned towards the understanding and incorporation of financial topics into macro models, as we, macroeconomists, severely underestimated the importance of this topic previously, as is shown by the way that both the causes and the size of the Great Recession are strongly linked with the financial crisis.

Similarly, the Great Recession also showed us how much more important housing, in particular house prices, seems to be for the economy as whole, and how more research is needed in this area as well. Fortunately, housing, as an area of study, has greatly benefited from the ‘empirical revolution’ economics as a whole is experiencing, with the advent of new data sources and techniques which has expanded the breadth and quality of empirical research.

This thesis seeks to explore topics that approach and combine both these areas of study, finance and housing, precisely because of their growing importance to macroeconomics. I hope that by doing so, it might contribute to our better understanding of the causes behind the Great Recession, and might help better prevent such events from happening again.

It does so by tackling two separate, but somewhat intermingled questions surrounding finance. The first two chapters of the thesis focus on the housing boom and bust that took place in the US during the 2000s, and both offers and tests the hypothesis that securitization, when interacted with non-recourse (limited liability) mortgage loans, helped push house prices upwards during the boom. The final chapter deals with the regulation of financial products, focusing particularly on how complexity might be used to dodge regulatory screening of banks. The common thread between the two topics is securitization, being as it is, a complex financial product.

The first chapter presents the theoretical side of this investigation into housing. I model the housing market by dividing it into two separate markets, an origination/borrowing sector, and a securitization/secondary market sector. I solve my model in a period of increased demand of uncertain duration, with two types of borrowers, safe ‘owner-occupiers’, and risky
investors’. We find two equilibrium results, in the first, loan originators, who are risk-averse, opt to screen out borrowers and only extend loans to the owner-occupiers. House prices then simply reflect the fundamentals of the demand boom, whether loans are recourse or non-recourse.

In the second, loans are sold to the securitization market. The risk-shifting that happens when loans are sold combined with the lack of self selection by borrowers results in loan originators stop screening, and speculators receive loans. And as the mortgages are non-recourse, these speculators will have an option value on defaulting that, given the chance for them to become marginal sellers, pushes up house prices.

I.e., my model predicts that the more securitization happens, the greater the likelihood that speculators will receive loans and have a chance of becoming marginal sellers, which can then push up house prices, this is the key prediction of the model. To a lesser extent, the model also predicts that the interaction between securitization and non-recourse loans leads to house prices falling further in a bust (i.e., if the demand boom stops ‘early’) and that defaults will then be higher.

This mechanism may seem like a purely theoretical one, but it has potentially important implications. The increase in house prices this mechanism creates will deviate from what some argue is the fundamental value of housing. This can, for example, then result in a misleading signal to the construction sector, or other potential home buyers. To the extent that this might then reinforce the trends that are driving the demand boom, something I do not formally explore in this thesis, this mechanism might lead to an overconstruction of housing for the former or create a self-reinforcing mechanism for increase in house prices.

To test whether this mechanism is empirically relevant, the second chapter of the thesis makes uses of heterogeneity in recourse laws between US states. Virtually all US state laws concerning recourse have been stable since shortly after the Great Depression. I make use of this ‘exogenous’ heterogeneity and regress house prices, on a state and city level, on a measure of securitization of new loans, the recourse status of a state/city and the interaction between the two, for 2004 to 2006.

What I find is that this interaction is associated with a twice as large, positive effect from securitization on house prices. More specifically, a 1% increase in securitization in recourse states is associated with a 1% increase in house prices on average, whereas the same increase leads to a approximately 2% increase in prices in non-recourse states. I also find that this mechanism can potentially explain around 75% or around 3.5 p.p. of the difference in increase in house prices between recourse and non-recourse states. Given that average house prices in the US in 2004 had already increased around 40% in the previous 4 years, this further increase of 3.5 p.p. comes from a high plateau and indicates how potent this mechanism can be.

As there is a potential issue of endogeneity between house prices and securitization, I use a novel instrument, the distance from a given housing market to the headquarters of ‘originate and securitize’ loan originators. Although the instrument currently has limitations, it provides further evidence for the model mechanism. Similarly, I find some favourable evidence that house prices did indeed fall further due to this mechanism, although there is no significant evidence of increased defaults during the bust.

As such, this chapter seems to validate concerns about how securitization and other financial developments that might allow for shifting of risk from originators, and how recourse laws interact with each other. Given the evidence I find, jurisdictions that have non-recourse
laws, such as some US states and countries like Brazil, should be aware of these possibilities and may wish to mitigate these effects actively.

The final chapter follows straight into the last point, by trying to find the optimal regulation that authorities should use when screening banks and dealing with complex financial products. Moral hazard concerns surrounding bank bailouts have existed since the 19th century at least, and the increased pace of development of financial innovation in the last 40 years raises questions of whether banks deliberately engineer products so as to make it harder for regulators to realize that they are de facto gambling, knowing that authorities will bailout them out should the gamble fail.

To address this question, Michel Azulai and I formally model what is the optimal regulatory regime that regulators should aim to use. In our framework, banks randomly receive new products, that are either good and complex, or bad and simple, and must essentially decide whether to turn these bad products into complex (and still bad) ones; bad products, when they fail, result in large costs to regulators. The incentive of banks to make bad products complex is that complex products are costly to screen by regulators, and this discourages them from doing so.

Furthermore, as we argue is the case in real life, we assume that there can be no direct transfers between regulators and banks. Thus the regulator’s problem lies mainly with finding the optimal schedule of screening, conditional on public information available (summarized into the expected payoff for banks in each period), conditional on a promise keeping constraint and incentive compatibility constraints.

What we find is that, when regulators can make binding commitments, the optimal schedule is for regulators to begin by giving out low payoffs and do no screening. Despite the latter, banks will not choose to make their bad products complicated, as they are otherwise punished, and they are promised a reward phase should they keep to the schedule. This reward phase, which is eventually reached after a number of periods, consists of banks then making full use of complexity, and regulators neither screening, nor punishing the banks, and is continuous henceforth once reached.

We argue that this result, whilst intriguing, is unlikely to hold, however, if regulators cannot make such binding commitments and, as we discuss, there are likely several reasons why regulators are unable to make binding commitments in practice. In the absence of binding commitments, we conjecture that the reward phase is then unsustainable, as regulators would have incentives to deviate at that point and start screening banks instead, unraveling the equilibrium.

Our results thus make it clear how difficult it is for financial regulators to push banks towards good behaviour, even with binding commitments, crises are not stopped, but postponed; regulators face numerous constraints in what they can currently do. This suggests that, given the current institutional set-up, bailouts and financial crises may be to some extent a inevitable part of the financial system.

As such, to improve regulatory regimes and further minimize future crisis, changing these constraints would be a necessary step. Some possibilities may be along the lines of increasing the scope for punishments of banks which are not just direct fines, such as making a portion of remuneration be deferred and conditional on good future results for individual bankers\(^1\), or perhaps encouraging more whistleblowers even when clear evidence of criminal behaviour is

\(^1\)Something which some regulators are currently exploring.
not available.

This result ties together the themes of this thesis, which explores the institutional framework that surrounds financial systems and the consequences this has on housing and finance itself, both areas of studies that macroeconomics cares about greatly. If the insights that this thesis has found can help assist the economics profession in better understanding these areas, then my ambitions coming into the PhD have been fulfilled.
Chapter 2

Modelling Securitization and Non-Recourse Loans in the Housing Market

Abstract

We study the effects of securitization and recourse (limited liability) laws on housing markets. Securitization allows originators to pass on the risk of loans they originate. As a consequence, originators stop screening due to the absence of credible signalling to securitizers. This allows speculator borrowers, who are interested in buying for investment purposes, to start receiving loans. When these loans are non-recourse, there is a put option that pushes up house prices during a demand boom. We thus predict that the interaction between securitization and non-recourse status should lead to higher house prices. We also make predictions concerning the housing market in a bust period.

JEL Classification Numbers: E00, E44, G20, R31.

Keywords: House prices, Securitization, Screening, Non-recourse loans.

2.1 Introduction

The national rise and fall of house prices experienced in the US during the 2000s was unparalleled in the last 70 years, and the literature is still grappling with trying to explain the cause of this boom and bust. Given that many prominent economists, including Bernanke (2010) and Mian and Sufi (2014), have argued that the financial crisis and Great Recession that followed were a direct consequence of what happened in the US housing market in that period, the importance of trying to understand this phenomenon cannot be understated.

Many explanations have been put forward to try to explain the pattern in house prices. Amongst others, it has been proposed that moral hazard in mortgage originations caused an increase in supply of loans (Mian and Sufi, 2009); that a decline in lending standards by originators led to an increase in demand for housing (Duca, Muellbauer and Murphy, 2011, and Dell’Ariccia, Igan and Laeven, 2012); that there was a large degree of misrepresentation of the quality of mortgages (Piskorski, Seru and Witkin, 2013); and that house buyers experienced

All these papers, with the exception of Case, et al., emphasize the importance that private securitization, such as CDOs and MBOs, had in affecting prices, which is not surprising, as private securitization also reached unprecedented levels in the 2000s. We seek to add to this literature by proposing a mechanism by which private securitization, when combined with ‘recourse’ laws pertaining to foreclose, can affect house prices.

Our model follows the approach pioneered in Allen and Gorton (1993), where asymmetries of information and agency problems result in a mechanism which affects asset prices\(^1\), resulting in prices being higher than they would otherwise be\(^2\). Their results have been extended to many different areas, such as between different sectors of the economy in Allen and Gale (2000) and Barlevy (2011), and there is experimental evidence that this mechanism can affect asset prices (Holmen, Kirchler and Kleinlercher, 2014). In particular, Barlevy and Fisher (2010), hereafter B&F, extend this mechanism to the housing sector; we use their framework to build our model.

In B&F’s model, there exist two types of borrowers, those that value owning a house (high types) who can be interpreted as ‘traditional’ owner-occupiers, and those who do not (low types) who can be thought as speculators\(^3\), with lenders unable to tell them apart. They find that under certain conditions, house prices can be higher than their fundamental value\(^4\), arising during a housing boom triggered by an increase in housing demand.

This appreciation in prices is mainly due to these loans being non-recourse, that is, of limited liability, where in the case of a default, lenders can only recover the asset securing the loan\(^5\). This creates a put option value for speculators, as they can default cheaply/costlessly should prices fall. When speculators subsequently become marginal sellers, this pushes house prices up. The model predicts that either demand keeps increasing for long enough such that a new, permanent high level of house prices becomes the equilibrium price, or, if housing demand stops rising before that, that prices immediately drop and defaults happen.

We use B&F’s framework, but introduce two novel elements to this literature: a screening technology that allows originators to costly screen borrowers, and a securitization market for loans. We choose to add these elements for several reasons, most saliently because there is empirical evidence that securitization interacted in important ways with screening by lenders during the 2000s boom in the US; Mian and Sufi (2009), Keys, Mukherjee, Seru and Vig (2010) and Elul (2011) all find that more securitization of loans led to less screening by lenders. Both Elul and Keys et al. find that this caused an increase in default rates in subprime mortgages, whilst the former also finds an increase in privately securitized prime loans, suggesting that this decrease in screening happened in all types of mortgages. Our addition of screening and securitization may also help explain why this mechanism may have not played a significant

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\(^1\)This literature denotes the effects of such mechanisms as rational bubbles.

\(^2\)This increase happens within the context of an agent-principal problem, where there are asymmetric payoffs for a risky investment, such that the upside rewards the agent, but the downside is born mainly or completely by the principal; this incentivizes the agent to over-invest in the risky asset

\(^3\)As we discuss in the Appendix, there is substantial evidence that speculators were a significant part of house buyers during the 2000s housing boom in the US.

\(^4\)As per Allen, Morris, and Postlewaite (1993): ‘Value of an asset in normal use as opposed to (...) as speculative instrument.’

\(^5\)Few countries outside of the US offer non-recourse mortgages, but Brazil became an important exception in 1997, when ‘alienacao fiduciaria’ loans (article 27 of law 9.514) were established, which are not only non-recourse, but in the case of defaults, if the market value of the repossessed asset is greater than the contractual value, borrowers are entitled to the value in excess of the contract, after costs; such loans may be sold via a ‘cessao de credito’ operation.
2.2. PARTIAL EQUILIBRIUM WITH EXOGENOUS PRICES

role prior to the 2000s, as we discuss in the next chapter.

With these two new elements, we find that, under some parameter restriction, in housing markets where loans are non-recourse there are two possible equilibria. In one, borrowers are screened and speculators are denied loans, however, counter-factually, no loans are securitized. In the other, no screening happens and loans are securitized. This is due to loan originators being unable to credibly signal to the securitization market whether a loan has been made to a speculator type or not, combined with the non-recourse nature of loans.

As a consequence, in the absence of securitization, house prices follow fundamentals during a housing boom, but when securitization occurs, speculators’ access to loans pushes up house prices as in B&F. Furthermore, if the boom stops, house prices fall further and defaults can take place when loans are being securitized. If loans are recourse, however, there is never an option value for borrowers, and prices always follow the fundamentals, independently of whether there is securitization or not.

We thus predict that the combination of both factors, securitization and the presence of non-recourse laws, should have a positive effect on house prices in US states, compared to states where either or both factors are missing. Depending on the size of originators and how much they can affect house prices, this equilibrium can exist when we introduce down-payments.

This chapter and its results are related to the mainly empirical and growing literature on recourse law in the United States, in particular how recourse laws affected the recent housing boom and bust, and whose inception happened largely due to the influential paper, Ghent and Kudlyak (2011). To the best of our knowledge, only one other paper, Nam and Oh (2014), explores a similar hypothesis to what we do in this chapter, that is, looking into the relationship between non-recourse status on house prices on the basis of risk-shifting mechanism. Their paper only proposes the possibility of this mechanism, without an explicit housing model, and seeks only to investigate this question empirically.

This chapter is organized as follows. The next section presents a two period version of the model with static prices to illustrate our basic model mechanism of how securitization and screening interact in our general equilibrium model. We then present in Section 2.3 our general equilibrium model with endogenous prices, and discuss its implications in detail. Section 2.4 concludes this chapter.

2.2 Partial equilibrium with exogenous prices

There are two periods in this version of the model. In the first period, transactions between borrowers (B) and originators (O) happen. Borrowers consist of two types, owner-occupiers/high types (denoted by H) and speculators/low types (denoted by L). This is followed by a securitization period, where originators can sell mortgages to securitizers (S). In the second period, an exogenous house price increase/decrease happens and borrowers must decide whether to default or repay.

Houses initially cost 1 in period 1, and in period 2 will be $1 + \Pi$. $\Pi$ is a random variable that equals $\pi$ with probability $q$ and $-\pi$ with probability $(1 - q)$. All loans are of size 1 with total repayment in period 2 equal to $1 + r$, and we restrict interest rates to be positive for all cases. As we only have one repayment period, any default is for 100% of the loan. If a loan
is in default, the house is immediately taken as collateral and sold for the prevailing market price.

We discuss the assumptions we make for each agent, including borrowers, and each market of our model setup in Appendix A.

2.2.1 Borrowers

Borrowers derive a stock utility from owning a house. They are required to take on a loan to purchase a house and can only acquire one house. If they receive a loan in the first period, in period two they can either repay the loan from their income or from selling their house, or the can default on the loan. Borrowers can choose which originator to approach for a loan in period one.

Borrowers consist of two types, \( \zeta \in \{ H, L \} \), with \( \gamma \) low/speculators and \((1 - \gamma) \) high types/owner-occupiers; we use these terms interchangeably. Both types have an income of \( y \), realized in period two, and where \( y \) is large enough to fully cover any level of mortgage repayments.

Borrowers’ utility function is linear and separable between consumption goods and house ownership, such that for a borrower \( i \):

\[
U^B_i = c_i + \kappa_i B_i (1 - D_i) (1 - S_i)
\]

where \( c_i \) is the consumption in period 2; \( \kappa_i \) is the stock utility from owning a house at the end of period 2, with \( \kappa_i = 1 + \kappa \) for \( \zeta_i = H \) and \( \kappa_i = 0 \) for \( \zeta_i = L \); and \( B_i, D_i \) and \( S_i \) are indicator functions, where a 1 indicates whether a borrower has bought a house, defaulted on a loan and sold a house, respectively.

The budget constraint of a borrower \( i \) is:

\[
c_i + B_i (1 - D_i) (1 + r_{ij}) = y + B_i (1 - D_i) S_i (1 + \Pi)
\]

where \( r_{ij} \) is the interest rate on a loan from originator \( j \) and \( y \) is the income of borrowers, high enough such that \( 1 + r_{ij} < y \) for any \( r_{ij} \). Although borrowers are risk neutral, we can make borrowers of both types be risk averse and find that our equilibrium results can hold, depending on the level of risk aversion; Appendix A provides a further discussion.

Borrowers have a set of 3 actions, \( S_{\zeta, B} \). In the first period, they decide which originator they approach for a loan, choosing \( j_i^B \) in \( J \) (where \( J \) is the set of originators). They do so taking into account that each originator posts a set of information \( \Lambda_j \). In the second period, borrowers decide whether to default on a loan (\( D_i \)), and whether to sell a house (\( S_i \)).

The information set \( \Lambda_j = (SC(j), BO(j), \{ r(\cdot) \}) \) consists of the following. \( SC(j) \) is an indicator function which takes value 1 if \( j \) screens borrowers. \( BO(j) \) is an indicator function which takes value 1 if loans will be given to both types (as opposed to only high types). \( \{ r(\cdot) \} \) is the set of interest rates on the loans being offered, which we divide into different cases, depending on \( SC(j) \) and \( BO(j) \). If \( SC(j) = 0 \), this is a non-screening Originator, so there will be a single interest rate \( r^{P,j} \) (where \( P \) stands for pooled). If \( SC(j) = BO = 1 \), this is a screening Originator who gives loans to both types, so the set of rates will be \( r^{H,j} \) and \( r^{L,j} \). If \( SC(j) = 1 \) and \( BO = 1 \),

\[6\]Although assuming that speculators derive zero utility from owning a house may seem extreme, we could renormalize this to some positive number without loss of generality.
the set will consist of \( r_{H,i} \). Note that we use the notation \( r_{i,j} \) to denote posted interest rates, as opposed to \( r_{i,j} \) to denote interest rates in originated loans.

### 2.2.1.1 Borrowers’ optimal behaviour

We assume that \( \pi \leq \kappa \), as the equivalent condition always holds when we endogenize prices, that is, house prices never exceed their value by owner-occupiers.

Borrowers’ optimal behaviour can be determined through a standard dominated action analysis. The resulting set of optimal strategies is very similar to the set that buyers have when playing a Bertrand competition, so we deem this as ‘Bertrand-like’ competition.

As borrowers have the option to costlessly default in the second period, both types are always at least weakly better off borrowing and buying a house; buying a house is a weakly-dominating strategy. Note that as low types have zero utility from owning a house, they only benefit from buying if they can sell the house at a profit.

To decide which \( j^β \), high types will choose the Originator with lowest posted interest rate, \( j^H = \arg\min_j r_{i,j}, j \in J \). Low types will first find the subset \( J' \in J \), such that \( \Lambda_{j'} \) indicates they will receive a loan, and then choose \( j^L = \arg\min_j r_{i,j}, j \in J' \). As \( \Lambda_j \) is common knowledge for borrowers, in equilibrium we find that all borrowers of a given type must have the same interest rate on their mortgages. A proof for this is shown in the Appendix.

Owner-occupiers never wish to sell the house as \( \pi \leq \kappa \). As long as \( r_{i,j} \leq \kappa \), they never wish to default in the second period and, as \( \pi \leq \kappa < r \), if \( r > \kappa \), they default on the loans in period 2, irrespective of what happens to house prices.

For speculators, if house prices decrease, their best action is to default as house prices are now worth less than loans \( \pi < 0 \). If house prices increase, then if \( \pi \geq r \), they can make a profit by not defaulting and then selling the house; otherwise the cost of repaying is greater and they then default.

As we show ahead, in equilibrium originators will set interest rates \( \pi \geq r_{i,j} \), such that the set of optimal actions for borrowers, \( S^*_B \) will consist of the following, which resembles the set of actions they will take in the general equilibrium model:

**Conclusion 1** Owner-occupiers choose an originator \( j^H \), which is offering the lowest interest rate from set \( I \) of all originators. They never default or sell in the second period. Speculators choose originator \( j^L \), with the lowest interest rate from set \( J' \) of originators offering loans to speculators. In the second period, they default when prices fall, otherwise they do not default and sell the house.

### 2.2.2 Originators

Originators should be understood as the banks and other financial agents that create mortgages. As such, they have two separate, but intertwined roles in our model. They decide whether to extend loans to borrowers and at what interest rates, and they choose whether to sell loans to securitizers.

We assume that originators are risk averse, with the following utility function:

\[
U^O_j = E(W^O_j) - aV(W^O_j) - n_j \ast C
\]
where $W^j_2$ is the wealth they hold at the end of period two, $E$ is the expectation operator, $a$ is a parameter determining risk aversion, $V$ is the variance operator, $n_j$ is the number of borrowers screened and $C$ is the cost of screening per borrower screened.

The assumption that originators are risk averse and securitizers are risk neutral is an integral part of our model; as we discuss in the next section, these two assumptions are required to model the securitization process\textsuperscript{7}.

Although we use Bertrand-like competition and generic set $J$ of originators for our model results, as an alternative, we can model our originators as having deep pockets and there being free-entry into the origination market and focus on a representative originator.

Originators will take 2 sets of actions in the model. They begin by posting the set of information $\Lambda_i$, that is, whether they screen borrowers or not; if they screen, whether they grant loans to both types or just owner-occupiers; and choosing the interest rates on loans. After a borrower approaches an originator, the originator acts according to their $\Lambda_i$ and make to type $q_i$. For low types, when prices are high, they repay, so $X(i) = X(i)$, with probability $q_i$; otherwise, $X(i) = -\pi$, with probability $(1-q_i)$, so $E(X(i)) = qr_i - (1-q_i)\pi$.

Now we define $Y(Q^j_1, SC, BO, I(j))$ as the returns obtained for all 3 possible courses of action that an originator can take concerning loan origination, that is, not screening, screening and lending to both types, and screening and lending to owner-occupiers, in addition to their choices of selling/keeping a loan. We thus have that:

\[
Y(Q^j_1, 0, I(j)) = \sum_{i \in I(j)} q^j_i (P^0(r_i) - 1) + (1 - q^j_i)X_H(r_i)
\]

\[
Y(Q^j_1, 1, I(j)) = \left\{ \sum_{i \in I(j)} q^j_i (P^0(r_i) - 1) + (1 - q^j_i)X_H(r_i) \right\}
\]

\[
+ \left\{ \sum_{i \in I(j)} q^j_i (P^0(r_i) - 1) + (1 - q^j_i)E(X_L(r_i)) \right\}
\]

\[
Y(Q^j_0, 0, \emptyset, I(j)) = \left\{ \sum_{i \in I(j)} q^j_i (P^0(r_i) - 1) + (1 - q^j_i)X_H(r_i) \right\}
\]

\[
+ \left\{ \sum_{i \in I(j)} q^j_i (P^0(r_i) - 1) + (1 - q^j_i)E(X_L(r_i)) \right\}
\]

The wealth of an originator $j$ at the end of period 2 will thus be:

\[
W^j_2(I(j)) = SC_1(1 - BO)(Y(Q^j_1, 0, I(j)) + SC_1BOY(Q^j_1, 1, I(j))
\]

\[
+ (1 - SC_1)Y(Q^j_0, 0, \emptyset, I(j))
\]

\textsuperscript{7}If originators are risk neutral, our results are very similar those of B&F and securitization plays no significant role in determining house prices.

\textsuperscript{8}A fall leads to speculators defaulting; the house is then repossessed and immediately sold at the market price.
Finally, note that the interest rates on loans can be used as signal of loan quality by originators to securitizers. Although this is the only signal we allow between originators and securitizers, in practice other characteristics of a loan, such as differentiated loan-to-value ratios/down-payments, might also be used as such; a further discussion of the results of our model with down-payments can be found in Appendix A. So the strategy set of an originator $j$, $S_j$, consists of choosing the set of $\Lambda_j$ and of choosing whether to sell each loan, $Q_j$.

### 2.2.3 Securitizers

In this chapter and the next, the securitization market in our model refers to the private sector securitization exclusively, and we opt not to include securitization done by government sponsored enterprises (GSEs); a further discussion of GSE securitization may be found in Appendix A. Securitizers in our model consist of a single risk-neutral agent who buys loans from originators and holds on to them, and only cares about their expected wealth at the end of period 2, such that their utility function is:

$$U^S = E(W^S)$$

where $W^S$ is the wealth they hold at the end of period two. Note that securitizers in our model play both the role of the financial intermediates who securitize the loan and the financial agents who buy the loan. We opt to keep both roles in one agent to simplify our model. We use risk neutrality as a reduced form for the securitization process, in particular, as the reduction of uncertainty that stems from securitization. Appendix A provides a further discussion for both issues.

We assume there is free-entry into the securitization market, so we can model our equilibrium results through a representative agent. Much like with originators, this would be equivalent to using a set of securitizers and ‘Bertrand-like’ competition. The securitizer’s only action will be to post the price $P_r$ for which they will be willing to buy a loan of interest rate $r$. Securitizers cannot condition their purchase of loans to specific originators.

As we discuss in the Appendix A, due to the complexities of the securitization process, securitizers cannot distinguish between high and low-type loans. Due to asymmetry of information, the price securitizers are willing to pay depends on their beliefs, denoted by $\Omega(r)$ which is the probability that a loan of a given interest rate is of a low type.

The wealth of a securitizer at the end of period 2 will be:

$$W^S(Q_j) = \sum_{j \in J} \left\{ \left[ \sum_{i \in I(j,H)} q_{ij}^O (X_H(r_i) - P^*(r_i)) \right] + \left[ \sum_{i \in I(j,L)} q_{ij}^O (E(X_L(r_i)) - P^*(r_i)) \right] \right\}$$

So the strategy set $S^S$ of securitizer consists of a set of $P_r$.

### 2.2.4 Timeline and definition of the equilibrium

To summarize the possible actions taken in our model, we now present the timeline of actions taken in Figure 2.1.

We now define the equilibrium of our model. As this is a signalling game, we focus our attention on a Perfect Bayesian Equilibrium (PBE), under which beliefs are consistent with Bayesian updating. This also means that we solve parts of the game via backwards induction.
A PBE in our model consists of a strategy profile \( (S^*_B, S^*_O, S^*_S) \) and a set of beliefs \( \Omega_S \) for all agents, that is, for all \( \forall i \) and \( \forall j \in J \), we have that:

**Borrowers:**

\[
S^*_B \in \arg \max c_i + \kappa B_i (1 - D_i)(1 - S_i)
\]

s.t.

\[
c_i + B_i (1 - D_i)(1 + r_i - (1 + \Pi)S_i) = y
\]

**Originators:**

\[
S^*_O \in \arg \max E(W^O_j(I(j)^*)) - aV(W^O_j(I(j)^*)) - n_j(I(j)^*) * C
\]

where their wealth \( W^O_j \) is defined above, \( I(j)^* \in S^*_{i,B} \).

**Securitizers:**

\[
S^*_S \in \arg \max E[(W^S(Q^*_j))/\Omega)]
\]

where their wealth is defined above and \( \{Q^*_j\} \in S^*_{j,O} \) and securitizer’s beliefs \( \Omega(r) \), must satisfy Bayes’ law.

In other words, our model consists of a signalling game played between originators and securitizers, where the interest rate for a loan put on sale is the signal, and where originators are constrained in their actions by the actions taken by borrowers, \( S^*_j \).

### 2.2.5 Securitizers’ optimal behaviour

The price paid by securitizers for a loan will depend on the interest rate and the beliefs that securitizers have about the composition of that loan, i.e., \( P_r = f(\Omega(r), r) \). Securitizers buy and then hold on to the loans until they pay off in the next period. The equilibrium price, conditional on beliefs, will be such that expected utility of securitizers will be equal to zero due to free entry; a proof of this can be found in Appendix B.

We now establish what is the expected utility of securitizers given their beliefs and establish necessary conditions on the prices. Let the belief structure of securitizers be such that any
given loan of interest rate $r_\Omega$ has probability $\Omega$ of being of a low type, noting again that we restrict ourselves to $r \leq \pi$:

$$EU^S_\Omega(X_H, X_L) = (1 - \Omega)(1 + r_\Omega) - \Omega[(1 + qr_\Omega - (1 - q)(1 - \pi)] - P_\Omega.$$  

With free entry, $EU^S_\Omega = 0$, so $P_\Omega = (1 - \Omega)(1 + r_\Omega) - \Omega[(1 + qr_\Omega) - (1 - q)(1 - \pi)]$. In particular, if $\Omega = 0$, a belief that a loan is to a of high type, we have that with free entry:

$$P^*_H = 1 + r_H$$

where we abuse notation. If $\Omega = 1$, a belief that loans consists only of low types, then with free entry:

$$P^*_L = 1 + qr_L - (1 - q)\pi$$

with similar abuse of notation. With this, we have established the full set of optimal actions of securitizers with free entry, $S^*_S$, conditional on their beliefs.

Note that, as expected, $P^*_H \geq P^*_L$ for two loans with the same interest rate but different beliefs about their types and that the price paid is monotonically decreasing in $\Omega$, and that all $P_\cdot$ are monotonically increasing in $r$.

**2.2.5.1 Preview of results and strategy**

To help the exposition that follows, we first show the equilibrium when originators are restricted from selling loans. We then proceed to find the equilibrium under two different set of actions for originators, whether they screen borrowers or not. Also note that we make several assumptions about our model parameters to find our results, so we discuss what happens otherwise for each key assumption.

In a screening equilibrium with ‘low probabilities’, because the price paid for low type loans is less than the cost of lending, if any selling of loans to securitizers were to happen, the only loans that could be sold to securitizers would be those consisting of owner-occupiers. However, originators are capable of masquerading speculators as owner-occupiers by offering them the same equilibrium interest rates, which would be a profitable deviation. Securitizers are thus unwilling to pay a high enough price for any loan put on sale, so none are sold. Originators will screen loans and only lend to owner-occupiers. With ‘high probabilities’, a screening equilibrium where speculator loans are sold and owner-occupier loans are held by originators can be sustained.

In a no-screening equilibrium, which can only be sustained when costs are high enough to stop originators from ‘skimming the cream’, loans are sold to securitizers and both types receive loans. Thus, if loans are securitized, speculators will receive loans.

**2.2.5.2 Restricted selling equilibrium**

As high types never default there is no uncertainty from them, thus when originators are restricted from selling, utility is additive.

As we show in Appendix B, if originators are sufficiently risk averse, satisfying $a \geq \max\{\bar{q}, \pi\}$, will always suffer negative utility by lending to speculators; thus, if possible, they will screen
and only give loans to owner-occupiers. This assumption is required for tractability, as if \( a < \max\{\bar{a}, \pi\} \), originator’s set of optimal actions is very large and becomes conditional on our other parameters.

Furthermore, as originators are competing among each other via Bertrand-like pricing, we have to have that \( EU^{O,H} = 0 \). In equilibrium, interest rates must be such that their utility is zero and the equilibrium interest rate will be \( r_H = \frac{C}{(1-\gamma)} \). Note that this requires that \( -\frac{C}{(1-\gamma)} \leq \pi \), which implies that \( \bar{a} < 0 \), making \( \bar{a} \) redundant. If \( \frac{C}{(1-\gamma)} > \pi \), loans are too costly and no lending takes place.

Finally, borrowers will have utility \( EU_H = \kappa + y - \frac{C}{(1-\gamma)} > y = EU_L \).

**Conclusion 2** If originators are restricted from selling loans to securitizers and we have \( a \geq \bar{a} \) and \( \frac{C}{(1-\gamma)} \leq \pi \), a unique screening equilibrium exists where only owner-occupiers receive loans, \( r_H = \frac{C}{(1-\gamma)} \).

### 2.2.6 Screening equilibrium

We begin by setting \( \frac{a}{1-\eta} < 1 \), as we are interested in cases with asymmetry in house price movements analogous to the general equilibrium version of our model. From \( W^O_j \), the profit from selling a loan is \( P_i(r_i) - 1 \), so originators will want to sell only if, for any given loan with interest rate \( r_i \), \( P_i(r_i) \geq 1 \). As \( \frac{a}{1-\eta} < 1 \), \( P_L(r_L) < 1 \)\(^9\), speculator loans are not profitable and not granting loans to speculators dominates.

A screening equilibrium where loans are extended only to owner-occupiers and are then sold to securitizers, and speculators are denied loans cannot be thus sustained. In such a case, first assume that the equilibrium posted interest rates \((r^L, r_H)\) are different. A originator \( j' \) could then profitably deviate by posting \( \Lambda_{j'} \) where they offer to grant loans to speculators and set \( r^{L,j'} = r_H \), masking speculators as owner-occupiers. Speculators would then choose \( j = j' \) and this originator would have higher payoff, as \( P_H \geq 1 \).

Originators will not wish to hold on to loans made to speculators with \( a \geq \bar{a} \) as we discussed in the previous subsection. So in a screening equilibrium, if \( \frac{a}{1-\eta} < 1 \), no equilibrium can exist where screening takes place and low types receive loans. We can sustain this equilibrium by setting the off the equilibrium path beliefs of securitizers such that any loan put on sale is a low type loan (\( \Omega = 1 \)) for any interest rate, in which case no originator would want to deviate and sell a loan, making these beliefs consistent. If, alternatively, \( r^L = r_H \), then the equilibrium would not be sustained as not screening would strictly dominate screening for originators due to the cost of screening.

If \( \frac{a}{1-\eta} > 1 \), then it is possible to sustain an equilibrium where speculator loans are sold and owner-occupiers receive loans which are not sold, which we discuss further in the Appendix.

**Conclusion 3** In a screening equilibrium, if speculators are sufficiently risk, only owner-occupiers receive loans and originators do not sell loans to securitizers. If speculators are not too risky, then their loans are sold and owner-occupiers loans are held on to by originators.

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\(^9\)If the equilibrium interest rate \( r' \) was such that \( EU' > 0 \) for a originator making loans, a different originator could offer \( 0 < r'' < r' \) attracting those borrowers and increase their profits.

\(^{10}\)The highest possible interest rate such low types do not default is \( r_L = \pi \), and for that interest rate, \( P_L = 1 + \pi(2\eta - 1) < 1 \) for \( \frac{a}{1-\eta} < 1 \).
2.3. GENERAL EQUILIBRIUM

2.2.7 No screening equilibrium

From our results when originators are restricted from selling loans, if the cost of screening is not incurred, then originators would never want to extend loans to simply hold-on to them. As such, if there is no screening taking place, an equilibrium can only exist if originators sell loans to securitizers.

Conclusion 4 In a no screening equilibrium, originators offer interest rates of \( r = \frac{\gamma(1-q)}{(1-\gamma)+q} \), for any borrowers. Securitizers will set \( P^* = \frac{1}{\pi} \) for any loans with an interest rate of \( r \) (which implies \( \Omega(r) = \gamma \)) and they have off-the-equilibrium path beliefs that \( \Omega(r \neq \frac{1}{\pi}) = 1 \).

We show that this is an equilibrium in Appendix B, under two additional parameter restrictions, that \( \gamma \leq \frac{1}{2(1-q)} \), so that interest rates are not too high, and that \( \frac{(1-\gamma)\gamma(1-q)}{(1-\gamma)+q} \leq C \), which guarantees that originators will not wish to ‘skim the cream’. Otherwise, a no-screening equilibrium cannot be sustained.

2.2.8 Summary and discussion

Under the conditions that \( a > \overline{a} \) (sufficient risk aversion), \( \frac{q}{1-q} < 1 \) (low-types present a bad risk), \( \frac{C}{1-\gamma} \leq \pi \) (sufficiently low screening costs), \( \gamma \leq \frac{1}{\pi(1-q)} \) (sufficiently low number of low types) and \( \frac{(1-\gamma)\gamma(1-q)}{(1-\gamma)+q} \leq C \) (sufficiently high costs)\(^{11} \) we find there are two equilibria. The first is such originators screen and only lend to owner-occupiers and no loans are sold to securitizers. In the second, originators do not screen, thus allowing both types to have access to loans, and originators sell both loans to the securitization market. Alternatively, if \( \frac{q}{1-q} > 1 \), then both types receive loans, both are screened and only speculator loans are sold.

This result illustrates the basic mechanism that will drive our results in the general equilibrium set-up. The non-recourse nature of loans and their 100% LTV ratio means that borrowers will not self-select, putting the onus on originators to screen out low and high types. In the absence of a credible signalling, originators could mask speculators as owner-occupiers when selling them to securitizers, which impedes any equilibrium wherein owner-occupiers loans are sold.

As we discuss further below, we believe that the equilibrium we find in our model when there is no securitization taking place may describe the state of the world before the securitization boom of the 2000s, whereas the equilibrium where it does may delineate how the market started operating once securitization increased.

2.3 General equilibrium

2.3.1 Setup

The general equilibrium model differs from the partial equilibrium one as we now endogenize the prices of houses, by including house sellers in addition to house buyers/borrowers. The model is of finite duration and finishes at period \( N \).\(^{12} \)

\(^{11}\)Note that \( \gamma \leq \frac{1}{\pi(1-q)} \) guarantees that both our high cost and low cost conditions will hold simultaneously.

\(^{12}\)The model generalizes to a infinite horizon model, as there is a simple mapping from flow utility of owning houses and receiving a stock utility at the end of time in a finite period model.
2.3. GENERAL EQUILIBRIUM

We assume the same settings for this model as in our partial equilibrium model, unless noted, and a discussion of our modeling choices may be found in Appendix A, so all variables and agents are defined analogously to the partial equilibrium model.

House owners, prospective borrowers or otherwise, remain divided into two types, with analogous utility functions to their partial equilibrium model, such that for borrower \( i \) of type \( \zeta \) arriving at \( \rho \) utility is:

\[
U_{i}^{\rho} = \sum_{t=\rho+1}^{N} c_t + \kappa_i B_{\rho} ND_{\rho+1} ND_{\rho+2} \prod_{t=\rho+1}^{N} (1 - S_t)
\]

who faces a budget constraint such that aggregate expenditure is:

\[
\sum_{t=\rho}^{N} c_t + B_{\rho} ND_{\rho+1} (A_{\rho} + r_{\rho, j} + ND_{\rho+2} A_{\rho} + r_{\rho, j})
\]

and aggregate income is:

\[
\sum_{t=\rho}^{N} y + B_{\rho} ND_{\rho+1} ND_{\rho+2} \sum_{t=\rho+1}^{N} S_{t} A_{t,j}
\]

where \( c_t \) is consumption, \( y \) income, \( A_t \) house prices, \( r_{t,j} \) interest rate from a loan by originator \( j \), \( B_t \), \( ND_t \) and \( S_t \) indicator functions for buying a house, not defaulting and selling a house. In addition, period by period budget constraints exist, as house buyers cannot save or borrow except via their (potential) single house purchase.

Originators will now seek to maximize

\[
U_{j}^{O} = \sum_{t=1}^{N} E(W_{j,t}^{O}) - aV(W_{j,t}^{O}) - n_{j,t} * C
\]

where \( W \) is their wealth/profits in period \( t \), \( a \) is the coefficient of risk aversion, \( n_{j,t} \) total borrowers screened and \( C \) is the cost of screening per borrower. We further define wealth analogously to the partial equilibrium model, as

\[
W_{j,t}^{O}(I(j, t)) = SC_{j,t}(1 - BO_{j,t}) Y(Q^{0}, 1, 0, I(j, t), t)
+ SC_{j,t} BO_{j,t} Y(Q^{0}, 1, 1, I(j, t), t)
+ (1 - SC_{j,t}) Y(Q^{0}, 0, \emptyset, I(j, t), t)
\]

where again \( SC_{j,t}, BO_{j,t} \) are indicator functions for screening and type lending, \( Y(Q^{0}, SC, BO, I(j, t), t) \) is expected profit earned on conditional loans originated \( (I(j, t)) \) and on loans sold \( (Q^{0}) \) at every period \( t \). More precise definitions of \( Y(.) \) can be found in the Appendix A, and they are defined analogously to the partial equilibrium model.

Finally, the representative securitizer seeks to maximize

\[
U^{S} = \sum_{t=1}^{N} E(W_{j,t}^{S}(Q_{j}))
\]

i.e., the sum of their expected utility, where their wealth/profit per period is
We assume that there exists a fixed\textsuperscript{13} housing stock at $t = 1$ such that $\Psi$ of houses are owned by low types and that all current high types own houses. To simplify our analysis, we assume there does not exist a renters market for this housing market\textsuperscript{14} and we exclude the possibility of borrowers owning multiple houses.

At each time period, starting at 1, with probability $q$ a cohort of size 1 of new borrowers will enter this housing market and may buy houses, with $(1 - \gamma)$ borrowers being owner-occupiers. This is conditional on a cohort having arrived in the last period, so if a cohort does not arrive in period $M$, no cohorts arrive in $M + 1, M + 2...$ Arriving speculators, as in the partial equilibrium model, will want to buy houses with the intent of reselling them, and will optimally default if a cohort fails to arrive at any period.

We have two necessary conditions on the size of the housing stock, such that $2(1 - \gamma) < \Psi \leq 2 - \gamma$\textsuperscript{15}, and for analytical convenience, we assume that $\Psi = 2 - \gamma$. We discuss how our results would change if we altered the size of our cohorts and/or housing stock in Appendix A.

The loan structure is such that loan repayments occur over two periods of time, so for a loan originated in $t$, half of the total loan payment of $A_t(1 + r_t)$ is paid in $t + 1$ and the other half at $t + 2$. Loans remain non-recourse and if defaults happen, whoever owns the loan contract at the moment of default proceeds to repossess the house and sell it in the market for the prevailing price.

Originators can costlessly distinguish between new arrivals and buyers from previous periods and will only extend loans to buyers of a new cohort. Buyers are required to acquire a loan to buy a house\textsuperscript{16}. Borrowers’ income is such that they can always cover their loan payments in every period and/or make early repayment of loans, for which there is no penalty.

The timing within each period is now as follows: at the start of each period, a new cohort does or does not arrive and, after this, buyers with outstanding loans decide whether to default or not. New house buyers proceed to establish conditional prices\textsuperscript{17} for houses via a Walrasian auctioneer. New buyers can then approach originators for loans and if they succeed, proceed to buy houses, with new owner-occupiers moving first in acquiring houses from existing owners (as arriving owner-occupiers always value houses more than speculators, they could always bid some positive $\epsilon$ to guarantee this). Finally, originators can sell loans to securitizers. This timeline is summarized in Figure 2.2.

\textsuperscript{13}We can relax this restriction as long as the amount of housing being added every period is smaller than the size of new cohorts of borrowers. A further discussion may be found in B&F and Glaeser, Gyourko, and Saiz (2008).

\textsuperscript{14}One could be incorporated without loss of generality, as in B&F.

\textsuperscript{15}The first guarantees that the housing stock is greater than the number of new high types until at least period 3; the second guarantees that, in period 2, if houses are sold, then at least 1 was bought by a low type who arrived in the cohort of period 1. With more time periods and longer loans, we would have less strict conditions.

\textsuperscript{16}This would not be necessary in a model with a more lengthy loan payment schedule where the price of houses can always be above the income of buyers, making it necessary to acquire loans to buy a house.

\textsuperscript{17}If a new cohort fails to arrive, we have no way of establishing the price of houses, as no transactions take place, so in such cases, we simply establish the price that would prevail if a single new high type arrived and sought to buy a house, i.e., the value of a marginal seller.
2.3. GENERAL EQUILIBRIUM

2.3.1 Prices and Fundamental Value

The key uncertainty in our model is whether at the end of time, the number of new high types exceeds the housing supply or not.

In periods 4 and beyond, if cohorts have arrived in all periods, the number of owner-occupiers exceeds the stock of houses immutably, so the equilibrium price must be equal to the valuation of the marginal buyer, owner-occupiers borrowers, which is $\kappa$.\(^{18}\)

If not, housing supply exceeds the number of owner-occupiers forever, so the equilibrium price for houses will be equal to the value of the marginal seller, 0.\(^{19}\)

So if a cohort fails to arrive prior to period 4, the price is equal to 0 from that period onwards. If cohorts arrive in the first 3 periods, then the price will be equal to $\kappa$ for all periods onwards. In particular, as there is never any uncertainty for periods 4 and beyond, the price must either be $\kappa$ or 0, as either enough cohorts have arrived or not.

To compare our house prices with some notion of fundamental value, we define fundamental value of housing. Following Allen et al. (1993) and related literature, this is the ‘Value of an asset in normal use as opposed to (...) as speculative instrument’. That is, the fundamental value is the value/price an asset would have if house buyers did not have loan contracts that skewers their incentives by ‘safeguarding them from a negative shock’.

For periods 4 and beyond, as there is no ‘speculative’ element, the prices we have established are equal to the fundamental value. For periods 1 to 3, the fundamental value is by definition equal to the expected value for what the price will be in period 4, as this is the value of a house if buyers could buy houses outright, without loans. Appendix B discusses and proves this claim. As such, the fundamental value after the arrival of new cohort in period 3 is equal to $\kappa$, in 2 the value is $q\kappa$ and in 1, it is $q^2\kappa$. As we will demonstrate below, this will be equal to the price that prevails when no securitization takes place.

2.3.1.2 Borrower’s optimal actions

We briefly outline the optimal actions of borrowers, which is identical to the partial equilibrium case. Owner-occupiers arriving in $\rho$ gain $\kappa$ from owning a house, as long as the overall cost of a mortgage is lower than their utility value, they will always be willing to take on a loan and repay the loan fully, that is

\[^{18}\]If the price was lower, then any owner-occupier who currently does not own a house would be willing to bid for a house at a higher price $A' = A + \epsilon \leq \kappa$. And as no high type is willing to sell for a price less than $\kappa$, the only equilibrium price is $\kappa$.

\[^{19}\]As proof, first note that there is no chance of being able to re-sell the house in the future for a greater price, as no new cohorts can arrive, so all speculators/low types value the house at zero. If the equilibrium price was some $A' > 0$, then any low type seller who is not selling could post a price $A' - \epsilon \geq 0$ instead and make a profit, so only $A = 0$ can be an equilibrium price.
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\[ \kappa \geq A_p (1 + r_{p,j}) \]

As \( A_p \) and \( (1 + r_{p,j}) \) are fully determined when they arrive and in equilibrium will always hold for all time periods, owner-occupiers will always seek loans and will always repay them.

Speculators will wish to take a loan and buy a house as long as there is some potential appreciation value, stemming from the increased probability that prices will reach \( \kappa \), as they can always default costlessly in the next period. Furthermore, as we know that in equilibrium, speculators who arrive in \( t \) will be marginal sellers in \( t + 1 \), we can focus on the potential appreciation in between those periods, so speculators arriving in \( \rho \) buy if:

\[ q(A_{\rho+1} - A_{\rho}(1 + r_{p,j})) \geq 0 \]

As this will always hold in equilibrium, speculators will always seek to buy, at any price and interest rate. This means there is no self-selection by speculators. Furthermore, they will not default in \( \rho + 1 \) when a new cohort arrives only if \( A_{\rho+1}/A_{\rho} \geq (1 + r_{p,j}) \), which constrains the interest rate.

A speculator who bought in \( \rho \) will have 3 actions in \( \rho + 1 \), defaulting, selling and waiting. If no cohort arrives, they will optimally default, as they have no further chance of selling a house to future cohorts and houses will now be worth zero. If a cohort arrives, they will sell if the returns from selling this period exceed the expected returns from waiting. The latter consists of the expected gains of the appreciation of the house next period minus the cost of the mortgage installment today:

\[ q[A_{\rho+2} - \frac{A_{\rho}(1 + r_{\rho})}{2}] + (1 - q) \times 0 - \frac{A_{\rho}(1 + r_{\rho})}{2} \]

The return from selling the house this period is equal to \( A_{\rho+1} - A_{\rho}(1 + r_{\rho}) \). So speculator sell when:

\[ A_{\rho+1} - A_{\rho}(1 + r_{\rho}) \geq q[A_{\rho+2} - \frac{A_{\rho}(1 + r_{\rho})}{2}] + (1 - q) \times 0 - \frac{A_{\rho}(1 + r_{\rho})}{2} \]

On the other hand, a no-loan low type, who initially owns the housing stock, has no put option, so they sell in any period \( t \) if the value of selling is greater than the expected appreciation, \( A_t \geq qA_{t+1} \).

Finally, the optimal choice of originators remains exactly the same as in the partial equilibrium case. This means that when contemplating a deviation from equilibrium, originators will not be able to offer a higher interest rate from the equilibrium, as no borrower would take that interest rate.

2.3.1.3 Restricted Selling Equilibrium

Like in the partial equilibrium case, we first find the equilibrium when we restrict originators from selling loans to securitizers. We solve the model via a PBE, much like in the partial equilibrium model, solving backwards and finding the equilibrium actions that prevail in periods 3, 2 and 1 assuming that cohorts have arrived in every such period; for all other cases, we know what the equilibrium actions and prices are. We use analogous results from our partial equilibrium model where applicable, and our equilibrium result is similar: as long as risk
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aversion is high enough and speculators are a minority, speculators will not receive loans.

**Period 3** We begin by assuming that in periods 1 and 2, only arriving owner-occupiers, not speculators, have bought houses, which we show will be an equilibrium action. If a new cohort fails to arrive, then owner-occupiers who bought houses in previous periods will not default as long as the total cost of their loan is less than their value of the house, which we will show will hold if costs are not too high. So, equilibrium prices will be 0 and no defaults happen.

If a new cohort has arrived, as owner-occupiers move first when buying houses, all houses will be purchased by owner-occupiers. This is because there will be \( 3(1 - \gamma) \) new owner-occupiers, and the housing stock, \( \Psi = 2 - \gamma \), is smaller for \( \gamma < \frac{1}{2} \).

The new owner-occupiers thus exhaust the supply of housing, meaning that even if a speculator were to receive a loan, they would never be able to purchase a house. As there is no risk, there is no need to screen borrowers by originators. As a consequence, originators will post a single interest rate, will not screen borrowers and interest rates will be, due to the Bertrand-like competition, \( r_{P,3} = 0 \). This means that all high types receive loans, so the equilibrium price of houses must be equal to the value of the marginal buyer:

\[ A_3 = \kappa \]

**Period 2** We show in Appendix B that if originators have risk aversion such that

\[ a \geq a'' = \sqrt{\gamma^2 + \frac{(1 - \gamma(1 - q))^2}{q(1 - q)} - \gamma} \frac{2q^2\kappa}{2q^2\kappa} \]

then originator’s utility from not-screening and lending to both types is always less than or equal to zero. This means that the unique equilibrium action will be for originators to screen borrowers and only lend to owner-occupiers. We assume \( a \geq a'' \) mainly for tractability, as otherwise originators’ actions encompass a wide range of possibilities, depending on the values of our parameters and how risky speculators are, given \( q \) and \( \gamma \).

Thus, in period 2, the number of buyers is smaller than the number of sellers, so price will be equal to the value of the marginal seller. As in equilibrium no speculators receive loans, the equilibrium price is the value of no-loan low types, so the price is:

\[ A_2 = q\kappa \]

Under Bertrand competition/free entry, expected utility of originators remains zero, so the equilibrium interest rate will be \( r_{H,2} = \frac{C}{(1 - \gamma)q\kappa} \), for which we need that \( C \leq q\kappa(1 - q)(1 - \gamma) \) for high types to accept loans; otherwise the cost of screening is too high and originators do not lend. So under \( a \geq a'' \), originators screen and only lend to owner-occupiers.

**Period 1** Under our assumptions \( a > a'' \) and \( \gamma < \frac{1}{2} \), as we show in Appendix B, speculators would not receive loans in either a non-screening or a screening equilibrium. As the number of owner-occupiers is smaller than the housing stock, equilibrium house prices are determined by the expected value of the marginal sellers, the no-loan low type house owners. In this case, \( A_1 = qA_2 = q^2\kappa \) and the equilibrium interest rate will be the same as in period 2, \( r_{H,1} = \frac{C}{(1 - \gamma)q\kappa} \).
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To summarize, we find our results by assuming that originators are sufficiently risk averse, \( a > a'' \), that speculators are a minority, \( \gamma < \frac{1}{2} \), and that costs are not too high, \( C < q\kappa(1 - q)(1 - \gamma) \). Under such conditions, we find a unique equilibrium when originators are restricted from selling loans. As long as a new cohort of borrowers arrives every period, house prices experience a boom, progressing from \( q^2\kappa \) to \( q\kappa \) to \( \kappa \), loans are only ever extended to high-types, with interest rates that eventually fall to zero at the end of the boom, and no defaults ever happen. If a new cohort fails to arrive at any point, then house prices immediately collapse to 0 and remain there; no new loans are extended, but no defaults happen as only high types have received loans.

2.3.2 Screening and non-screening equilibrium

To distinguish our variables from the previous case, we denote the variables in this equilibrium with a tilde. We begin by providing some intuition and a discussion of the results we find.

We have 4 possible equilibrium results, but focus our attention on just two equilibria, analogous to the partial model results, with either screening or no-screening taking place in both periods 1 and 2. The other two possible equilibria outcomes consists of no-screening taking place in period 1 and screening taking place in period 2 or vice-versa.

We can trim this set of 4 equilibria by assuming that securitizers will not switch beliefs about the quality of loans between periods, a refinement that we believe seems reasonable in this context\(^{20}\). We prove in Appendix B that the other two equilibria produce outcomes in house prices identical to the screening equilibrium, i.e., there is no securitization, and also discuss how relaxing this refinement of no belief switching would affect our results in a more general model.

In the no-screening equilibrium, we have that both types receive loans in periods 1 and 2. This means that in period 2, the marginal seller of houses will be a speculator. This seller will have a put option value, so house prices in period 2 are now higher. As a consequence, the price in period 1 is also greater than the fundamental value, due to rational expectations. If a cohort fails to arrive, this also implies that the fall in house prices will be much greater than would happen in the non-securitized market. We find that speculators who receive loans will default, as they lack further opportunities to sell.

We now proceed to prove and discuss, period-by-period, our two main equilibria. For both cases, in periods 4 and beyond, prices are equal to the fundamental value, as we discuss above.

2.3.2.1 Period 3

To establish the price securitizers are willing to pay, we establish the beliefs of securitizers and then make sure that these beliefs are consistent, in a Bayesian sense, with what actually happens in equilibrium.

In period 3, if a cohort arrives, as the housing supply is exhausted, only owner-occupiers buy houses and take on loans. As a consequence, this is the belief that securitizers will have of the composition of borrowers behind a loan, so that they can expect returns of:

\(^{20}\)As is discussed in Lewis (2011), there is anecdotal evidence that the beliefs about loans of securitizers / buyers of securitized assets were largely unchanging throughout the 2000s up until the house prices themselves stopped increasing around 2007. As in our model the only big ‘shock’ that can take place is a cohort failing to arrive, we believe it is reasonable to assume that securitizers will maintain their belief structure as long as cohorts are arriving.
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\[ \bar{U}_{H,3} = \bar{A}_3(1 + \bar{r}_{H,3}) - P_{H,3} \]

With free entry, we have that \( P_{H,3} = \bar{A}_3(1 + \bar{r}_{H,3}) \). If originators choose to sell their loans, they will have a payoff of:

\[ \bar{U}^O_{H,3} = P_{H,3} - \bar{A}_3 = \bar{A}_3 \bar{r}_{H,3} \]

As housing supply is exhausted and only high-types receive loans / buy houses, prices must be, in equilibrium, \( \bar{A}_3 = \kappa \). Due to Bertrand-like competition, in equilibrium \( \bar{r}_{H,3} = 0 \).

This means that \( P_{H,3} = \bar{A}_3 \), so originators are indifferent between selling and not selling loans, and either equilibria can be sustained.

If a cohort fails to arrive, prices collapse and are equal to 0.

2.3.2.2 Period 2

If a speculator receives a loan in period 2, they will default if a cohort does not arrive in 3. As we show in the Appendix, the equilibrium cost of the loan granted in period 2 is always smaller than the price in 3, so if a new cohort arrives in 3, all speculators from period 2 sell their houses to the new arrivals and repay the loan completely.

Thus, the prices that securitizers will be willing to pay will depend on their belief about the loan composition. As we show in Appendix B, if securitizers believe a loan to consist exclusively of high types or low types, the price is respectively

\[ P_{H,2} = \bar{A}_2(1 + \bar{r}_{H,2}) \]
\[ P_{L,2} = \bar{A}_2q \bar{r}_{L,2} \]

If \( P_{L,2} \leq \bar{A}_2 \), that is, if the cost of loan \( \bar{A}_2 \) is higher than the amount they receive for the loan, \( P_{L,2} \), then originators will not sell speculator loans. The price is lower than the cost of the loan if \( (1 + \bar{r}_{L,2}) \leq \frac{1}{q} \); in Appendix B, we show that this is true for all values of \( \bar{r}_{L,2} \). If \( 1 + \bar{r}_{L,2} > \frac{1}{q} \), then speculator always default in period 3 and the price of the loan is zero. As such, we have an analogous situation to that of the exogenous price case, as loans believed to consist only of low types will never be sold in equilibrium.

As we now show, this means only two possible equilibria can exist. Either originators screen and do not sell their loans, or originators do not screen and sell loans to securitizers.

**Screening equilibrium** Similar to the partial equilibrium case, as the price that a loan believed to consist of speculators is too low to compensate originators, we cannot have an equilibrium outcome where low type loans are sold to securitizers.

We can equally rule out a screening equilibrium where loans are extended only to owner-occupiers and are then sold to securitizers, with speculators denied loans, as originators would deviate by not screening and masking speculators as owner-occupiers by setting interest rates to be the equilibrium rate. This would be a profitable deviation for both originators, as \( P_{H,2} \geq 0 \), and for speculators, who would gain access to loans.

Consequently, if originators choose to screen, then the only possible equilibrium outcome is for them to hold-on to loans. We can sustain this with off-the-equilibrium-path beliefs by securitizers that any loans sold are speculator loans. The equilibrium outcome is thus for
originators to screen, deny loans to speculators and set \( \tilde{r}_{H,2} = \frac{C}{(1-\gamma)\kappa} \) as the interest rate. Owner-occupiers receive loans and no loans are put on sale on the securitization market.

As we are assuming there is no belief switching, we must have that this is the action that happened in period 1. So the outcome of the housing market is identical to that which we find with restricted selling. That is, only high types received loans in period 1 and only they receive loans in period 2. For this, we need the same set of assumptions, \( a > a'' \) and \( C < q\kappa(1-q)(1-\gamma) \). House prices will then be equal to \( \tilde{A}_{SC,2} = q^2\kappa \), where \( SC \) denotes a screening equilibrium.

**No-screening equilibrium** The other equilibrium is where originators choose to not screen and sell the loans in the securitization market. Securitizers believe that any loan sold by the equilibrium interest rate \( \hat{r}_{NSC,2} \) (where \( NSC \) denotes the no-screening equilibrium) has \( \Omega = \gamma \) low types and any loan sold off the equilibrium path has \( \Omega = 1 \). From our assumption that there is no belief switching, in period 1 both types received loans. As \( \Psi = 2 - \gamma \), in period 2, at least one speculator who bought a house in period 1 will sell in period 2, and so becomes the marginal seller.

The price of loans is then:

\[
P_{NSC,2} = \tilde{A}_2(1 - \gamma(1 - q))(1 + \hat{r}_{NSC,2})
\]

The expected utility of originators is \( EUO_{NSC,2} = p_{NSC,2} - \tilde{A}_2 \), such that due to Bertrand-like competition, this will be equal to zero and the equilibrium interest rate is \( 1 + \hat{r}_{NSC,2} = \frac{1}{1-\gamma(1-q)} \).

As we discuss in the partial equilibrium case, there is only one possible profitable deviation for originators, which would be to ‘skim the cream’\(^{21}\), wherein originators screen, sell speculator loans and hold on to owner-occupier loans. In the Appendix, we show that originators will not deviate if \( C > \frac{\tilde{A}_2(1-\gamma)(1-q)}{1-\gamma(1-q)} \), which holds simultaneously with \( C < q\kappa(1-q)(1-\gamma) \). If costs are too low, then we will not be able to sustain this equilibrium and only a screening, no selling equilibrium remains.

The marginal sellers are now speculators who bought in period 1. As per the discussion of their optimal actions, speculators sell, in equilibrium, only if

\[
\tilde{A}_2 - \tilde{A}_1(1 + r_{NSC,1}) \geq q\kappa - (1 + q)\tilde{A}_1(1 + \hat{r}_{NSC,1})
\]

which implies that \( \tilde{A}_2 \geq q\kappa + \frac{1-d}{2} \tilde{A}_1(1 + r_{NSC,1}) \), a deviation from the fundamental value of houses.

As in period 2 we have more sellers than buyers, the equilibrium price will be exactly equal to that of the marginal seller, and \( \tilde{A}_2 = q\kappa + \frac{1-d}{2} \tilde{A}_1(1 + r_{NSC,1}) \). Combined with our results below from period 1, this implies that equilibrium prices will be:

\[
\tilde{A}_2 = q\kappa \frac{2(1 - \gamma(1 - q))}{2(1 - \gamma(1 - q)) - q(1-q)} > q\kappa\]

\(^{21}\)That is, any deviation with higher interest rates would not attract borrowers, and any deviation to a lower interest rate would, given securitizers’ beliefs and the lack of self-selection by speculators, result in lower prices for loans.

\(^{22}\)Note that if \( \tilde{A}_2(1 + r_{NSC,2}) \geq \kappa \), the price would simply become that the value the high type borrower has of the house, as otherwise high types would default. As we show in the Appendix, this never holds in equilibrium and \( \tilde{A}_2 < \kappa \).
So in a no-screening equilibrium, originators set interest rates $\tilde{r}_{NSC,2} = \frac{1}{1-\gamma(1-q)} - 1$, and sell these loans to securitizers; securitizers believe that any loan with a different interest rate consists of a low type. Both types of borrowers buy houses and, as the marginal seller will be a speculator who bought a house in period 1 and has the put option value, house prices are higher than the screening equilibrium.

### 2.3.2.3 Period 1

Speculators who buy in period 1 will have identical actions to speculators who buy in period 2, that is to say, if no cohorts arrive, they default on their loans. If a cohort arrives, all arriving speculators are capable of selling their houses to the new buyers, and no defaults happen. As consequence, the beliefs of securitizers map onto prices in the same way as before, so we have the same equilibrium price function as in period 1.

**Screening equilibrium**  Analogously to our discussion of period 2, a screening equilibrium can then be sustained if the off-the-equilibrium path beliefs are set such that any loan sold consists of a low type. Thus, originators post a single interest rate $\tilde{r}_{S,1} = \frac{C}{1-\gamma q}$, and choose to screen and deny loans to speculators. As such, only owner-occupiers receive loans and no loans are put on sale in the securitization market, so prices are equal to the fundamental value:

$$\tilde{A}_{S,1} = q^2 \kappa$$

**No-screening equilibrium**  A no-screening equilibrium, can also be sustained by setting identical conditions to the no-screening equilibrium of period 2, which, as we show in the Appendix, will stop originators from ‘skimming the cream’. Thus originators extend loans to both types, and then sell these loans to securitizers.

As the marginal seller, a no-loan low type, has no put option value, we have to have the result that in equilibrium:

$$\tilde{A}_1 = q \tilde{A}_2$$

As the equilibrium interest rates are the same as that in period 2, we can combine this with our previous result that $\tilde{A}_2 = q\kappa + \frac{1}{q^2} \tilde{A}_1 (1 + r_{NSC,1})$, to find that

$$\tilde{A}_1 = q^2 \kappa \frac{2(1-\gamma(1-q))}{2(1-\gamma(1-q)) - q(1-q)}$$

### 2.3.3 Summary and discussion

Our results are found under the assumptions that $a > a''$ (sufficiently high risk aversion), $\frac{2(1-\gamma(1-q))}{2(1-\gamma(1-q)) - q(1-q)} > 1$, and $\frac{2(1-\gamma(1-q))}{2(1-\gamma(1-q)) - q(1-q)} > 0$; this shows that house prices deviate from fundamentals.

We find two mains results under no belief switching. In the screening equilibrium, originators do not sell loans to securitizers, aka, our prediction when there is no securitization. Assuming cohorts arrive every period, as high types are the only borrowers to receive loans,
house prices follow $A_1 = q^2 \kappa$, $A_2 = q \kappa$, $A_3 = \kappa = A_4 = \ldots = A_N$, and if a cohort fails to arrive before period 4, house prices collapse immediately to 0, and no defaults happen.

In the no-screening equilibrium, both borrower types receive loans, which are sold to the securitization market every period. As a consequence of the put option value of non-recourse loans, house prices deviate from fundamentals. Assuming cohorts arrive every period, starting from period 1, we have that $\tilde{A}_1 = q^2 \kappa \frac{2(1-\gamma(1-q))}{(1-\gamma(1-q)) - q(1-q)}$, $\tilde{A}_2 = q \kappa \frac{2(1-\gamma(1-q))}{(1-\gamma(1-q)) - q(1-q)}$, $\tilde{A}_3 = \kappa = \tilde{A}_4 = \ldots = \tilde{A}_N$. Finally, if a cohort fails to arrive before period 4, house prices collapse immediately to 0, and defaults happen from speculators who received loans.

House prices thus experience a ‘boom-like’ behaviour for both cases, but without screening/when securitization is taking place, the put option value of speculators pushes house prices above the other. Furthermore, as we show in the Appendix, when loans are recourse and lacking the put option value, prices follow fundamentals.

Thus the combination of securitization and non-recourse laws has a positive effect on house prices. We illustrate this graphically in Figure 2.3 by comparing house prices when there are both non-recourse laws and securitization, and a housing market where at least one is not present, during a boom.

![Figure 2.3: House prices in a sustained boom.](image)

Note that cohorts arrive until the housing stock is exhausted in both cases, thus house prices eventually become equal at a new high point $\kappa$, and the recourse/no-securitization market experiences a much larger increase in prices at the end.

Should a cohort fail to arrive at, for example, period 3, then prices in both markets would immediately fall to zero. In such a case, a market with non-recourse and securitization would fall from a higher price level and, subsequently, experience a greater boom and a greater bust. This is illustrated in Figure 2.4.

This is the core prediction of our model. We would also expect to find that, in case of a housing bust, defaults are non-zero in the non-recourse and securitization market.

Concerning welfare, as we discuss in Appendix A, the no-screening/securitization equilibrium is ex-ante, welfare increasing, as it leads to a reduction in the screening costs. Note
that this result stems from our assumption that securitizers are risk-neutral, which itself stems from our use of risk-neutrality as a proxy for the securitization process.

Our equilibrium results are partly due to the 100% loan-to-value ratios, so we explore what happens when we introduce them to our model. Although we do not fully characterize all the possible set of equilibria, in the Appendix, we discuss and show we can sustain our no-screening equilibrium with the addition of down-payments, as long as originators are sufficiently small\textsuperscript{25}.

Taken literally, our model predicts that if a securitization markets exists, then 100% of loans would be sold off by originators and securitized. As we have omitted many important characteristics that matter to participants of these markets, we would not expect to, and do not find, such levels of securitization. Instead, the key prediction in our model comes from the extent that securitization allows for originators to extend loans to speculators.

That is to say, in US states where mortgage loans are non-recourse, we would expect that with higher levels of securitization, the probability that speculators have managed to buy houses and become the marginal seller in subsequent periods is increased.

Thus, our model predicts that there is a positive effect on house prices coming from the interaction between percentages of loans (privately) securitized in US states and whether that state has non-recourse laws on mortgage loans; this goes beyond the positive effect that securitization has been found to have empirically by the literature (Keys et. al. (2010), among others). This is the primary prediction of our paper that we wish to test in the next chapter.

In addition, it is possible to test if this interaction effect subsequently lead during the bust period to greater drops in house prices and to higher levels of defaults in non-recourse states. As our model is static after prices collapses, which takes place immediately after the drop in demand, we do not believe that our model is as well suited for dealing with the bust period. This is particularly true as defaults, foreclosures and bankruptcies in practice can be lengthy

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\textsuperscript{25}And incomes are restricted, as with high enough incomes, we find a trivial equilibrium where borrowers buy houses with a 100% down-payment.
processes, and a richer model in those characteristics would be better suited to testing this mechanism in the bust period.

Finally, concerning other mechanisms that the literature has suggested as causes for the boom and bust, from moral hazard issues to overoptimism, from increase in loan supply to mispriced loans, we surmise that these most likely would interact with our own model in such a way as to enhance each other.

For example, consider moral hazard issues such as outright fraud and, more generally, anything that makes it such that securitizers are not fully aware/misled about the composition of mortgage loans and thus mispricing of loans. We expect that this would make our conditions for an equilibrium less stringent (particularly the cost restriction for no ‘skimming the cream’), whilst simultaneously providing an additional reason for why house prices might have increased, and also making it easier for an equilibrium such as ours to come about, even with large originators and down-payments.

We believe that by setting up the model as we do, we are likely finding a lower bound for under what conditions our mechanism might contribute to increased house prices.

2.4 Conclusion

The literature on the housing market has sought out many explanations for what led to the unprecedented boom and bust in the US during the 2000s. We seek to add to this literature by proposing that the non-recourse status of mortgage loans in some states in the US, when combined with the increase in private securitization experienced at that time, pushed up the prices of houses in those states beyond their fundamental value during the boom.

Our model generates an increase in house prices using the ‘risk shifting’ mechanism of Allen and Gorton (1993), where the asymmetry of payoffs between loan originators and borrowers creates a put option value for the latter. We base our model on the work of Barlevy and Fisher (2010), but introduce two important, novel elements: screening of borrowers by originators and a securitization market.

We find that during a demand boom, when securitization takes place, originators shift the risks associated with the loans they extend by selling them to the securitization market. Furthermore, when these loans are non-recourse, originators cannot credibly signal whether a loan has been extended to a risky speculator or a owner-occupier. As result of this, a non-screening equilibrium is possible, where originators extend loans to both types and sell both to securitizers. Due to the put option in non-recourse loans, speculator’s value of housing increases, such that house prices increase when they can become marginal sellers.

We show that in the absence of securitization taking place and/or with recourse loans, house prices remain at the fundamental level. We thus predict that the interaction of private securitization and non-recourse laws should lead to increased house price growth during a boom. Under some restrictions, we show that our results can exist even when borrowers are risk-averse or there are down-payments on loans.

Our model also makes predictions for the bust period, although, due to the static nature of the bust period, is less suited for doing so. It predicts that the interaction effect between securitization and non-recourse status from the boom period should lead to greater falls in house prices and more defaults during a subsequent bust.
2.4. CONCLUSION

We thus have established how, from a theoretical perspective, securitization and recourse laws might have significant effects on house prices. We now proceed, in the next chapter, to test these predictions for the case of the states in the US.
Appendices to the Chapter

2.A Appendix A - Model and empirics discussion

2.A.1 Borrowers

We can think of low type borrowers as being speculators as they only wish to buy a house to take advantage of (potential) capital gains by selling it in a later period. There is substantial evidence that house buyers seeking to make capital gains were an important part of the housing market in the US during the recent boom, as can be seen in Haughwout, Lee, Tracy and van der Klaauw (2011), Bhutta (2015) and Bayer, Mangum and Roberts (2016). Haughwout et al. (2011) for example, finds that around half of mortgage originations for house purchases in states with greatest price appreciation at the top of the housing boom were done by such borrowers. In our model, we need not assume such a large number of speculators: a small number of low types is sufficient to generate increased house prices, as long as they have the opportunity to become the marginal sellers.

For the purposes of our model, we have that speculators and owner-occupiers are a separate set of agents, but in practice there is likely a ‘spectrum’ of intentions from borrowers concerning what they wish to do with houses. That is, borrowers utility from owning a house likely span a wide range of values, which would turn owner-occupiers into speculators if prices have risen sufficiently. We surmise that making such an addition to our model would generate analogous results, and opt for two separate sets of agents for tractability.

2.A.1.1 Risk Averse Borrowers

If we made borrowers risk averse with equivalent utility to that of the originators, then there is no change to the optimal behaviour of owner-occupiers, as their optimal decision is still to never default, so there is no uncertainty/variance.

With non-recourse loans, speculators remain protected from a fall in house prices. However, the uncertainty stemming from the change in house prices means that they only take a loan if the negative utility from this variance is smaller than the expected benefit, i.e., if $aq(\pi - r_P) \geq q(1 - q)(\pi - r_P)^2$, where $r_P$ is the equilibrium, no-screening interest rate.

This is true if $a \leq \frac{1}{(1-q)(\pi - r_P)}$, which we can show hold simultaneously with $a \geq a' = \frac{1}{2(1-q)\pi}$, thus any $a$ that satisfies both restrictions allows for both risk averse borrowers and originators and finds the same result in a no-screening equilibrium.

If, however, risk aversion is sufficiently high, then speculators never wish to take on loans in either equilibrium. In such a case, our model predicts that only owner-occupiers receive loans.

We show in Appendix B, that the analogous result holds in general equilibrium for a large enough probability, $q > 0.25$.

26Unlike our model, Haughwout et al. (2011) find evidence of speculator-like buyers by looking at second home purchases, but as they conclude that this was done by speculators “apparently misreporting their intentions to occupy the property”, as “(...) many of the borrowers who claimed on the mortgage application that they planned to live in the property they were purchasing had multiple first-lien mortgages when the transaction was complete”. This can be thought as a part of the information that good screening discovers.
2.A.2 Originators

The interaction within the group of originators and between originators and borrowers is similar to that of a Bertrand competition, as borrowers can see the interest rate schedule and the loan decision before deciding which originator to approach. Because of this Bertrand-like marketplace, in the absence of scale effects, and with linear costs, we need not specify the number of originators that exist; the model would work equally well with just two originators as with a continuum of them.

Furthermore, this means that with deep pockets and free entry, we would find the same results, which allows us to use a representative originator to find some results.

Originators’ capacity to distinguish between the two types of borrowers comes at a cost of $C$ per each individual they screen. We can think of this being the cost needed to obtain extra information necessary to tell apart two borrowers whose observable characteristics are identical (that is, any and all information used to price loans/mortgage securities). This could, for example, be thought of as information that only becomes available from when experienced bank managers carefully investigate and vet potential borrowers.

A 2013 article by The Economist illustrates how this cost of screening can be significant: “Marquette’s [a bank] (...) approach was to have a lending officer accompanied by one of the bank’s trustees (board members, in effect) visit every mortgage applicant on the Saturday after each application was filed.” Originators may have problems in being able to signal the quality of loans, affecting the incentives to screen if originators can sell: “Its overseers wanted it to sell its mortgages to protect itself from swings in property prices. (...) The subprime crisis revealed so much slapdash issuance that buyers of mortgages consider valuations provided by the originators worthless. So Marquette can no longer conduct its own appraisals. Saturday visits have ended.27”

As Keys, et al argue, only contractual terms (such as LTV ratios and interest rates) and FICO scores were used by investors to evaluate the quality of a securitized pool. As such, although we have restricted the signal of loan quality between originators and securitizers to be interest rates, other ‘hard’ characteristic of loans might have been used instead, in particular, the loan-to-value ratio, which we discuss below.

2.A.3 Securitization and Securitizers

We focus on the private securitization market in our modelling, as opposed to the GSE market. We do this for several reasons, firstly because the incentives of GSEs are difficult to ascertain and model, given their dual role as for-profit companies which were also tasked with helping finance low and middle income loans.28

Secondly, the literature has evidence that lending standards of loans associated with GSEs changed little during the 2000s. GSE securitization has a long history in the US, and their overall participation rate in creating loans changed little in the decade preceding the crisis. As Adelino, Schoar and Severino (2016) summarize “Loans that were sold to (and then guaranteed by) the GSEs had to conform to higher origination standards than those sold to other entities”. This can be seen in the default rates of GSE loans in 2008 and 2009 which, although

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28 As per the Housing and Community Development Act of 1992, they ‘have an affirmative obligation to facilitate the financing of affordable housing for low- and moderate-income families in a manner consistent with their overall public purposes, while maintaining a strong financial condition and a reasonable economic return’
high by historical standards, were not even half as large as those at the lower end of the housing market sold to the private sector (Angelides and Thomas, 2011).

Note as we do find evidence that house prices were affected by GSE loans in our regressions, albeit negatively, whether our mechanism may have had some effect in the 2000s remains an open question, and we cannot exclude such a possibility.

Private securitization expanded to unprecedented levels in the late 90s and the early 00s. This resulted in new financial agents buying these products and becoming exposed to the US housing market for the first time, including agents outside of the US. Many of these agents, as discussed by Lewis (2011), had very little understanding of the US housing market, much less of the constituent mortgage loans they held via the CDOs and MBS they owned. As we discuss in the previous section, products were typically evaluated over aggregated/averaged information over the thousands of loans backing a single product, such as the average FICO score. In addition, Elul (2011) finds evidence that originators of loans may have had access to private information beyond that typically used by buyers of securitized products, resulting in adverse selection. For these reasons, we model securitizers as not being able to distinguish between the different types of borrowers behind a loan.

Buyers of securitized products would use such averaged information in part due to the complexity of the securitization process, which may have been deliberately made more complex due to moral hazard issues (Hofmann, 2008).

This makes securitization quite complex to model, as it involves numerous agents and steps\textsuperscript{29}. We can summarize it as consisting of aggregating loans from a number of different markets, on the assumption that they exhibit some statistical independence from each other, and then slicing the returns from these loans into different tranches, so that the senior tranches receive priority in payments, and losses are absorbed by the lower tranches first. The whole process involves a number of intermediaries and steps, most saliently a loan originator and a securitization arranger, normally an investment bank.

The aggregation should result, if loans are statistically independent at least to some extent, in both a reduction in uncertainty about the outcomes and a reduction in the variance. Combined with the creation of tranches, this would theoretically allow for high tranches to be relatively safe products; the arranger banks would normally hold on to the lowest tranches, the so called ‘residual’ or ‘junior’ tranches.

Given these complexities, we focus our attention on just one consequence of the securitization process, the reduction in uncertainty. We do this by having securitizers be risk neutral as opposed to risk averse, allowing us to proxy this aspect of securitization in a tractable way. That is, as they do not have any loss of utility from risk aversion, securitizers will be willing to pay a higher price for loans compared to originators, much like owners of securitized products should do if securitization reduces uncertainty (especially for owners of high tranches).

We can rationalize this approach as being directly equivalent to having a continuum of identical and independent housing markets, and allowing risk-averse securitizers to purchase a diversified portfolio of each, and having all markets have the same equilibrium outcome of screening or no-screening. In such a case, there is no uncertainty about outcomes for securitizers, so no risk-aversion, and our ex-ante results become ex-post too.

This way of modeling securitization, although tractable, does have limitations. Although

\textsuperscript{29}See Ashcraft and Schuermann (2008) for a discussion of the stages and the problems that might arise in each of these.
we capture to some degree the benefits of securitization due to a reduction in risk and uncertainty, the absence of tranches is a significant omission. It may be possible, through having multiple housing markets, to create tranches in our model, albeit a previous version with just two markets had unreasonable had impractically difficult dynamics. We surmise that including tranching would not rule out our equilibrium results, even if originators held on to junior tranches; we conjuncture this would result in higher interest rates/prices to compensate originators, whilst still being low enough to be accepted by speculators.

Note finally that in our model we do not split up securitizers into the intermediaries that securitize the products and the ultimate holders of the securitized loans. This should be thought of as a reduced form of the process and simplifies the analysis by not adding an additional agent.

2.A.4 General equilibrium

We define the profits for Originators conditional on what actions they take, \( Y(\cdot, \cdot) \), as follows.

If they screen and only lend to owner-occupiers:

\[
Y(Q^0, 1, 0, I(j)) = \sum_{i \in I(j)} q^0_{ij} \left( P^*(r_i) - 1 \right) + (1 - q^0_{ij}) X_H(r_i)
\]

If they screen and lend to both types:

\[
Y(Q^0, 1, 1, I(j)) = \left\{ \sum_{i \in I(j), H} q^0_{ij} \left( P^*(r_i) - 1 \right) + (1 - q^0_{ij}) X_H(r_i) \right\} + \left\{ \sum_{i \in I(j), L} q^0_{ij} \left( P^*(r_i) - 1 \right) + (1 - q^0_{ij}) E(X_L(r_i)) \right\}
\]

Finally, if they don’t screen:

\[
Y(Q^0, 0, \emptyset, I(j)) = \left\{ \sum_{i \in I(j), H} q^0_{ij} \left( P^*(r_i) - 1 \right) + (1 - q^0_{ij}) X_H(r_i) \right\} + \left\{ \sum_{i \in I(j), L} q^0_{ij} \left( P^*(r_i) - 1 \right) + (1 - q^0_{ij}) E(X_L(r_i)) \right\}
\]

The choice of \( N \) periods of time may seem arbitrary and it is, to some extent. Our set-up is isomorphic to having 3 periods of time by extending the set of actions taken in period 3 and allowing borrowers to choose to fully repay or default on their loans. Consequently, for the purposes of our model, our interest lies, in particular, with periods 1, 2 and 3, where a securitized and non-securitized market might differ; periods 4 and beyond are necessary only because we have loans that are paid out in 2 periods of time and that loans granted in period 3 need the subsequent periods to be repaid\(^{30}\).

The entry of new borrowers, which increases the stock of high types, can be interpreted as an increase of demand for housing, and might be endogenized via the mechanisms of Duca, et al. (2011) or Case et al. (2012), among others, for the recent boom. Consequently, our model cannot explain why fundamentals are changing, but may explain why our mechanism can have potent effects on house prices during a boom.

The assumption that mortgage loans are repaid in two, equal sums is not innocuous, as the higher the first repayment is in the first period, the less house prices will deviate from

\(^{30}\)B&F have a similar modeling technique, as even though they have an infinite number of periods, their model experiences no further changes once a sufficient number of periods have passed and either cohorts stop arriving, or the number of high types exceeds the housing stock.
their fundamental value, as the option value of waiting is decreased in proportion to that. Similarly, having repayments happen over only two periods of time and a LTV ratio of 100% both increase how much prices deviate. A richer model would most likely have house price deviate from fundamentals over a longer period of time and by smaller amounts\(^{31}\).

2.A.4.1 Housing stock and cohort size

We make specific assumptions about our housing stock and cohort size, unlike B&F. We do so primarily so we can focus our attention on a model where the fundamental uncertainty, whether high types will exceed the housing supply, is resolved by period 3.

This is the smallest number of periods where it is possible to illustrate our model mechanism, as we require at least one period where low types can buy, and at least one period where they can become marginal sellers before high types exceed the housing supply.

As we discuss in Section 2.B.2 of Appendix B, we surmise that our model results would generalize to changes in housing supply and cohort size such that the fundamental uncertainty is resolved for some period greater than 3. The only substantial difference would be that price deviation from fundamentals would not necessarily take place only if securitization happens every period, but more generally, when there is ‘sufficient securitization’ such that speculators have had access to loans.

2.A.4.2 Recourse loans

When loans are recourse, we can show that speculator’s marginal value of housing is always equal to the fundamental value. A speculator obtaining a loan in period 1 can choose, in period 2, to either sell and repay today, \(A_2 - A_1(1 + r)\). The can also pay the installment \(-A(1 + r)/2\) and wait, and with probability \(q\) a cohort arrives, so they sell and repay \(A_3 - A_1(1 + r)/2\), but with probability \((1 - q)\) a cohort fails to arrive, in which case they still have to repay their loan installment, \(A_1(1 + r)/2\). Ergo, as speculators are the marginal sellers in period 2, we have that their value determines house prices, so \(A_2 - A_1(1 + r) = -A(1 + r)/2 + q(A_3 - A_1(1 + r)/2) + (1 - q)(-A_1(1 + r)/2) = qA_3 - A_1(1 + r)\), so \(A_2 = qA_3\).

As can be seen in the math, this holds more generally for any period \(t\).

2.A.4.3 Loan-to-value ratio

Adding loan-to-value to our base model, as a choice variable for originators, results in a trivial equilibrium wherein down-payments are set to 100% and borrowers buy houses outright. The reason for this is that with risk neutrality and no discounting, the payment of interest rates is always negative to utility of both types, so simply letting borrowers buy houses outright would be the trivial equilibrium result.

Instead, if we wish to have a meaningful result with down-payments, we have to restrict incomes of borrowers to \(y_x\), such that there is a maximum down-payment \(d\), \(y_x = dA_{y}\) when they arrive.

As we show in the Appendix B, a no-screening equilibrium identical to our main result can now be sustained, but only if loan originators are small and take the prices of houses as given;

\(^{31}\)We also surmise that in a more general model, having teaser rates (smaller fixed rates that only last for the beginning of the loan), would lead to greater deviations of prices, as both of which make it cheaper for a borrower to wait.
otherwise we will have a new equilibrium wherein down-payments are set to be as high as possible, \( \bar{d} \), and interest rates are set high enough so as to discourage speculators, creating self-selection without the need for screening.

The main reason for this result is the asymmetric effects of (restricted) down-payments for speculators and owner-occupiers. Any positive \( d < 1 \) means that there is now a range of interest rates where speculators, but not owner-occupiers, do not wish to take-on loans, as defaults are no longer costless. Owner-occupiers will be sufficiently compensated with a high enough \( d \) if house prices ‘switch’ to the fundamental price level when originators deviate, ergo, originators are large enough; if house prices remain at the no-screening level when owner-occupiers contemplate the deviation, then no deviation exists, and we maintain our no-securitization equilibrium with zero down-payments\(^{32}\).

As such, the extent to which our model results may be true depends on the LTV ratios that prevail. Concerning the housing boom and bust of the 2000s, the literature has found that these were very close to 100%. For example, Mayer, Pence, and Sherlund, (2009) find that for securitized assets, “median combined loan-to-value ratio for subprime purchase loans rose from 90 percent in 2003 to 100 percent in 2005”, meaning that over 50% of these loans had 100% LTV ratios, and they find a similar result for Alt-A mortgages, with LTV ratios increasing from 90% to 95%. Given in the no-speculator equilibrium we described above our model predicts down-payments that are as high as possible, versus the zero down-payments in the no-screening result, this suggests that most likely we were in an equilibrium where speculators were receiving loans.

Finally, although our model predicts that speculators will not accept loans with positive down-payments, if this were to happen then house price growth would depend on the down-payment, as the higher the down-payment, the smaller the put-option value, so the less house prices deviate from fundamentals.

2.A.4.4 Welfare

Note that in all our equilibria, originators never hold speculator loans, thus their risk-aversion portion of utility is never a part of welfare considerations. This means that the difference between the screening and a no-screening equilibria will consist of transfers of wealth between ‘de facto’ risk-neutral agents. This would suggest that welfare is the same for both situations; however, in the screening equilibria, there are real costs of screening, which increase the interest rates, lowering overall welfare.

Note that originators and securitizers, due to Bertrand-like competition/free entry always have ex-ante zero utility in both cases, although if price collapses, whoever holds loans will have losses in a no-screening equilibrium. Instead, ex-ante, welfare changes happen between borrowers and no-loan low types. In particular, we have that interest rates on loans are smaller without screening, \( C/(1 - \gamma) > 1 - \gamma(1 - q) \) due to our no-skimming condition. However, as house prices are also higher, the overall effect is ambiguous for owner-occupiers. On the other hand, speculators, who proceed to gain access to loans and can potentially gain from

\(^{32}\)In addition to these results, there are several ways we can maintain a no-screening equilibrium with large originators by departing from our model assumptions. The simplest is if neither interest rates nor the LTV ratio are used as signaling devices to securitizers. In that case, securitizers assume that loans are given out to both speculators and owner-occupiers, i.e., \( P_{NSC} = P_L = P_H \). This means that originators will wish to sell as many loans as possible, as they do not benefit from signalling, so they optimize by setting \( d = 0 \). Thus, this result is equivalent to a mispricing of securitized assets by the securitization market, which Nam and Oh (2014) among others have found evidence for.
speculation, and no-loan low types, who receive higher prices for their houses, are unambiguously better off in a no-screening equilibrium.

2.B Appendix B - Proofs

2.B.1 Partial Equilibrium Proofs

In equilibrium, securitizers’ expected utility will be equal to zero due to free entry. Much like with Bertrand competition and the previous proof, if the equilibrium $P'$ were such that $E(U'/\Omega, r) > 0$ for a securitizer, a different securitizer could enter the market offering $P'' > P'$, buy the same loans and increase their payoff. In equilibrium, only a price such that $E(U'/\Omega, r) = 0$ is sustainable.

If originators are restricted from selling their loans and are sufficiently risk averse, they screen and only lend to owner-occupiers.

In a screening equilibrium, if a loan is composed of high types the expected utility of holding the loan is $EU_{O,H} = E(X_H) = r_H$ and for low types it is $EU_{O,L} = E(X_L) - aV(X_L) = -aq \times (r_L)^2 (1-q) + q[1 - 2a(1-q)\pi] \times r_L - \pi(1-q)[1 + aq\pi]$.

A sufficient condition for loans never be extended to low types is $EU_{O,L} \leq 0$ for all $r_L$, and a sufficient condition for this to hold is if $q[1 - 2a(1-q)\pi] < 0$, which is true if:

$$a > \frac{1}{2(1-q)\pi} = \pi$$

So for $a \geq \pi$, originators only lend to high types in a screening equilibrium (SC) and have expected utility of $EU_{O,SC} = (1 - \gamma)r_H - C$.

If originators don’t screen (NSC), their utility is:

$$EU_{O,NSC} = \gamma EU_{O,H} + (1 - \gamma)EU_{O,L}$$

This implies that their utility in a no-screening equilibrium will be smaller or equal to that in a screening equilibrium, $EU_{O,NSC} \leq EU_{O,SC}$, if and only if:

$$(1 - \gamma)EU_{O,H} + \gamma EU_{O,L} \leq (1 - \gamma)EU_{O,H} - C \iff U_L^O \leq -\frac{C}{\gamma}$$

This will hold if $\pi > \frac{C}{q(1-q)\gamma a^2}$.

If $(1 - q)/q > 1$, then an equilibrium where both types receive loans and only speculator loans are sold can be sustained.

With $(1 - q)/q > 1$, then $P_L \geq 1$, so speculators will receive loans which are sold. In that case, owner-occupier loans cannot be sold, as either the equilibrium interest rates are the same, in which case not-screening dominates, or they are different, in which case not-screening and setting interest rates equal to the lowest equilibrium interest rate dominate, in both cases due to the real cost of screening.

33This guarantees that the term independent of $r$ in $U_{O,L}$ is less than or equal to zero.
Thus, the only possible equilibrium is to hold on to originator loans and sell speculators’. This can be sustained if speculator loans are the lowest possible \((r_L = (1 - q)\pi/q)\), there exists a \(r_H\) such that \(EU^O = (1 - \gamma)r_H + \gamma(qr_L - (1 - q)\pi) - C \geq 0\), which is \(r_H = C/(1 - \gamma) < \pi < \kappa\), so owner-occupiers accept. Note that as \(r_H = C/(1 - \gamma) < (1 - q)\pi/q = r_L\), there is no way for a originator to deviate by not screening and setting \(r_H = r_L\), as owner-occupiers would not accept a higher interest rate.

In partial equilibrium when there is no screening, the equilibrium is for loans to be sold by originators.

Recall our posited equilibrium actions: originators do not screen, and offer interest rates of \(r_P = \frac{\gamma(1 - q)\pi}{1 - \gamma + q}\) to any borrower who approaches them, borrowers approach any originator posting that information set, and securitizers pay \(P^*_P = 1\) for any loan with interest rate \(r_P\) and have beliefs that any loan with a different interest rate is composed of low types. First note that our equilibrium price and interest rate make the expected utility of originators and securitizers equal to zero, as is required by the way we model our markets. Also note that from our previous restriction on the value of rates, we must have that \(r_P \leq \pi\), which is true if \(\gamma \leq 1/2(1 - q)\).

What are the possible actions that an originator could contemplate that would deviate from this equilibrium\(^{34}\)? They can choose to not screen and offer an interest rate different from the equilibrium interest rate (and choose to hold or sell their loans)(\(PD_1\)). They could choose to screen and: not grant loans to high types (\(PD_2\)); not grant loans to low types (\(PD_3\)); grant loans to both and offer a different interest rate to high types (and choose to sell or keep these loans)(\(PD_4\)); grant loans to both and offer a different interest rate to low types (and choose to sell or keep these loans)(\(PD_5\)); grant loans to both and offer a different interest rates to both types (and choose to sell or keep these loans)(\(PD_6\)); grant loans to both at the equilibrium interest rate and choose to not sell either one or both of the loans(\(PD_7\)).

If they choose to not screen and offer a different interest rate than the equilibrium interest rate, first note from our previous results, due to risk aversion, they would never wish to hold-on to loans. So if they wished to sell loans, the deviation interest rate they would contemplate could only be lower than the equilibrium interest rate, as otherwise they will not attract any borrowers\(^{35}\), who are better off at the equilibrium interest rate. As prices are monotonically decreasing in \(\Omega\) and increasing in \(r_\zeta\), any deviation would result in a loan with a lower interest rate and a higher \(\Omega\) on the part of securitizers, so the price would be smaller than the one they receive by staying in the equilibrium, ruling out \(PD_1\).

We now show that it is never optimal for originators to screen and not grant loans to high types. In all versions of our model, high types will never default\(^{36}\) and originators can always hold-on to any loan granted. As such, originators can always be made better, vis-a-vis choosing to screen and deny loans to high types, by granting a loan to a high type and holding on

\(^{34}\)Note again that we have established the optimal actions of borrowers and securitizers, conditional on the actions of originators and the beliefs of securitizers.

\(^{35}\)Originators are thus constrained by the actions of the borrowers, as any successful deviation from a equilibrium interest rate must be such that it not only increases the payoff of the originator, but must also increase (at least weakly) the payoff of the borrowers too. This is a somewhat surprising result that comes from the peculiarities of the Bertrand-like competition between originators, and we surmise that it would still hold even if other forms of competition were used instead.

\(^{36}\)This will be shown to be true in the general equilibrium model.
to these loans, as they will have a payoff of at least 0 for each high type. So we need not contemplate this deviation action further and rule out PD2. Similarly, their payoff would be smaller by screening and denying loans to low types, as they would have the cost of screening and the same revenue, so PD3 is ruled out.

For all our other possible deviations, note that from our discussion of PD1, the only possible deviation interest rate that originators could offer would necessarily be lower, and as we have demonstrated, that would result in a lower payoff by selling these loans, which means that we can rule out all other possible deviations except PD7. For PD7, originators might be better off by screening, offering loans to both types at the equilibrium interest rate, holding on to loans made to high types and only selling loans composed of low types, ‘skimming the cream’.

However, the resulting expected utility is lower or equal to the expected utility of taking the equilibrium action if:
\[\gamma(P_p^* - 1) + (1 - \gamma)P_p - C \leq P_p^* - 1,\]
which will hold as long as \(\frac{(1-\gamma)(1-q)}{(1-\gamma) + q^\gamma} \leq C\).

So, as long as \(\gamma \leq \frac{1}{2(1-q)}\) and \(\frac{(1-\gamma)(1-q)}{(1-\gamma) + q^\gamma} \leq C\) holds, we have a unique equilibrium.

2.2.2 General Equilibrium Proofs

The fundamental value of houses is the expected value of houses in period 4.

From our definition of fundamental value, we note that if loans are needed to buy a house, then there is no ‘speculative’ element, as the consequences of buying a house are fully born by any house buyer, both positive and negative.

In such a case, arriving new low types buyers will have an identical valuation to existing low type house sellers, such that if the price that prevails is that of no-loans low type sellers, new low types will be indifferent between buying and selling a house, so, for simplicity, we assume they do not buy with the intention of selling.

As they both value the house at 0, this consists only the value that they might gain from waiting and selling the house in a future period, which is the expected value of house prices in period 4. Finally, for periods 1 and 2, housing supply will exceed the number of new high type buyers arriving, the price that prevails will be that of the marginal seller; in period 3, either a cohort arrives such that high types exceed the number of low sellers and the price is equal to the marginal buyer’s value, \(\kappa\), or it does not, so the price will be 0.

Under restricted selling, there exists a unique equilibrium such that originators screen and only lend to high types in period 2.

Under Bertrand-like competition, we know that equilibrium interest rates will be such that expected utility of originators is equal to zero. As in the no-screening equilibria there is no cost of screening and there is the additional revenue from high types, the utility that originators have from lending with the highest possible interest rate is always greater than or equal to the

\[\text{If a loan granted to a high type in a given period is } A > 0, \text{ then high types will repay } A(1 + r_H) \text{ in total, so originators will have a payoff of } Ar_H \text{ by holding on to the loan. As } r_H \geq 0, Ar_H \geq 0.\]

\[\text{Since the equilibrium price is 1, for every given loan, the revenue originators achieve by selling the loans is simply 0.}\]
utility they receive if they were to screen and lend to both types. So any condition that satisfies
the former, will guarantee the latter.

Noting again that defaults happen with probability $q$ if lending happens to low types\(^{39}\) and
utility is separable between types as there is no risk associated with high types, the expected
utility of lending in a no screening equilibrium for originators is

$$EU_{O,2}^P = (1 - \gamma)EU_{O,2}^H + \gamma EU_{O,2}^L = -aq(A_2)^2rP_2 + (1 - \gamma(1 - q))A_2rP_2 - (1 - q)[\gamma + aA_2]$$

This is a quadratic function of $rP_2$, with a positive coefficient only in the first order term, so the function only has non-negative values between its roots, if it has any, and it is mono-
tonically decreasing in $a$. The weakest condition that guarantees that no lending will hap-
pen is if $a$ is large enough so that there are no real roots in this equation, the condition that
$$Aa^2 + a\gamma - \frac{(1 - \gamma(1 - q))^2}{4q(1 - q)A} \geq 0,$$
which itself is satisfied by setting $a$ larger than the largest positive root, $a \geq a'.\]

Note here that $a$ is inversely proportional to the value of house prices in this period, this
means that this risk aversion condition (that makes originators wish to screen and deny loans
to low types) is decreasing in the price of houses. As giving loans to low types can only in-
crease house prices, a sufficient condition comes from setting prices to be the smallest possible
value.

So setting $A$ to be the smallest possible value, $A = q^2\kappa$, we guarantee that originators
will always be better off by screening and only lending to high types as long as they have
$$a \geq a'' = \sqrt{\frac{\gamma^2 + (1 - \gamma(1 - q))^2}{4q(1 - q)\kappa}} = a'.\]

Under restricted selling, originators will wish to screen and only lend to high types if $a \geq a''$
in period 1.

From our previous proof, we need only show that in a no-screening equilibrium originators
have utility less than or equal to zero to guarantee a unique equilibrium where screening takes
place and only high types receive loans. Due to the assumption of $\Psi = 2 - \gamma$, in period 2, if
low types received loans in period 1 and bought houses, in period 2 high types will end up
buying at least some houses from these new low types. That is, even if high types first buy
houses from old low types, they will, at least, have to buy a house from one new low type who
bought a house in period 1.

If we assume that $\gamma < \frac{1}{2}$, and if high types arriving in period 2 buy first from new low types
who bought in period 1, they could buy all the houses these low types own, in which case new
low types would not default and would proceed to repay their loans. As this happens with
probability $q$, and if a cohort fails to arrive low types would default, this is identical to what
happens in period 2, so the condition $a \geq a''$ is a sufficient condition.

If high types do not first acquire their houses from low types who bought in period 1, then
a portion of these low types would proceed to only partially repay their loans and may or
may not repay them in full in period 3. In such a case, this portion of low types are riskier
than the low types who repay in full in period 2, so the risk of lending to low types would be
even higher, so the utility that originators would have is necessarily less than or equal to the
previous case, such that $a \geq a''$ is a sufficient condition for originators to wish to screen and
only lend to low types.

\(^{39}\)That is, they default if no new cohort arrives, and sell houses and repay their loans early if a cohort arrives.
The two equilibria when we relax our refinement of no belief switching by securitizers is identical to the screening equilibrium.

Assume that securitizers’ beliefs changed between periods 1 and 2, such that now, it is possible for a screening equilibrium to happen in period 1 (AE1), followed by a no-screening equilibrium in period 2, or vice-versa (AE2).

In AE1, as screening took place in period 1, no low types acquired loans in that period, which means there cannot be any low types with mortgages in period 2 selling houses, ergo, prices cannot deviate from fundamentals in period 2 and, by extension, in period 1. We could make this equilibrium hold by setting beliefs of securitizers analogously to their counterparts in the screening equilibrium in period 1 and the no-screening equilibrium in period 2.

In AE2, under our assumption that $\Psi = 2 - \gamma$, in the first period, the housing supply falls by 1, as both high and low types buy houses, leaving the supply of houses equal to $1 - \gamma$. In period 2, arriving high types are of size $1 - \gamma$, so they are exactly equal to the housing stock still owned by old low types. As such, they buy only from old low types and prices cannot deviate from fundamentals. We could sustain this equilibrium in an analogous manner to the way we discuss AE1.

Note that in both the above cases, as the risks of low types is lower than in the no-screening equilibrium throughout, low types will not receive loans in all periods. This implies that our conditions for the no-screening equilibrium will be sufficient for these to hold too.

In a more general model, where the initial housing stock is different from our assumption that $\Psi = 2 - \gamma$, we would find different results. For example, if $(1 - \gamma) < \Psi' \leq 2 - \gamma$, then AE1 would have an identical outcome, but in AE2, high types will buy from low types who bought in period 1 and we would have a deviation in house prices.

Similarly, if the size of the housing stock was larger and/or the size of cohorts smaller, such that the point at which high types exceed the housing supply is later than 3, we might also have equilibrium outcomes where securitization takes place in some, but not all periods. Nevertheless arriving low types become marginal sellers before high types exceed the housing stock, again resulting in a deviation from fundamentals.

However, in such scenarios, it would still be necessary for ‘sufficient’ amounts of securitization to take place for there to be deviations from fundamental price. The higher the number of periods where securitization takes place, the more likely it is for prices to deviate and, potentially, the larger the deviations\textsuperscript{40}.

\textbf{The equilibrium cost of the loan in period 2 is always smaller than the price in 3.}

When prices deviate from fundamentals, we have that $\tilde{A}_2 = q\kappa + q(1 - q)(1 + r)/2$, so $\tilde{A}_2(1 - q + q(1 + r)/2) = \kappa$. So for a speculator to accept a loan in period 2, we must have that $\kappa - \tilde{A}_2(1 + r_{NSC}) \geq 0$. Given the equilibrium no-screening interest rates $r_{NSC,2} = 1/(1 - \gamma(1 - q))$, we can rewrite the above to be $2 \geq 2q + q(1 - q) + 2\gamma(1 - q)$, so at most, when we have that $\gamma = 0.5$, this will be equal to $1 > 2q - q^2$, which holds for all values of $q < 1$. If house prices are lower, the cost of the loan is smaller, so this must also hold.

\textsuperscript{40}If arriving low types become marginal sellers in more than one period, say two periods, than their put option value will increase their value of waiting in both the last two periods before high types are equal or higher than the housing supply, pushing prices higher in the penultimate period than the pure RE effect.
Securitizers pay $P_{H,2} = \hat{A}_2(1 + \hat{r}_{H,2})$ for loans they believe to consist of high types and $P_{L,2} = \hat{A}_2q(1 + \hat{r}_{L,2})$ for low types.

The expected utility of securitizers for an $\omega$-type loan with belief that it has $\Omega$ of low types will be $\hat{U}^2_{\Omega,2} = \hat{A}_2(1 - \Omega)(1 + \hat{r}_{\Omega,2}) + \hat{A}_2\Omega[q(1 + \hat{r}_{\Omega,2}) + (1 - q)\hat{A}_{3,2}] - P_{\Omega,2}$, where $\hat{A}_{3,2}$ is the price that prevails if low types default, so that with free entry, $P_{\Omega,2} = \hat{A}_2(1 - \Omega)(1 + \hat{r}_{\Omega,2}) + \hat{A}_2\Omega[q(1 + \hat{r}_{\Omega,2}) + (1 - q)\hat{A}_3]$. As low types default in period 3 only if a cohort fails to arrive, we will have that $\hat{A}_3 = 0$, so $P_{\Omega,2} = \hat{A}_2(1 - \Omega)(1 - q)(1 + \hat{r}_{\Omega,2})$.

\[ \text{We must have that } (1 + \hat{r}_{L,2}) \leq \frac{1}{q} \text{ holds for all values of } \hat{r}_{L,2}. \]

Note that there is a lower bound on the value of house prices whenever a cohort arrives, which is equal to the value houses take when there is no securitization market $A_2 = q\kappa$. House prices cannot be valued by less if cohorts are arriving every period, this is the value old low types have for houses, and they value houses in such a way that is always less than or equal to the value of high types, $\kappa$, and new old types, which may be higher due to the default option value. Low types won’t default in the next period, assuming a new cohort arrives, if and only if $\hat{A}_3 = A_3 - \hat{A}_2(1 + \hat{r}_{L,2}) \geq 0$. We have that $A_3 = \kappa$, so $A_3 = \hat{A}_2(1 - \Omega)(1 - q)(1 + \hat{r}_{\Omega,2})$, which implies that the largest possible interest rate that can be charged is when $A_2$ is at its lower bound, $q\kappa$, so $A_3 = (1 + \hat{r}_{L,2}) \leq \frac{1}{q}$.

\[ \text{Originators will not wish to ‘skim the cream’.} \]

The expected utility of securitizing the cream is less than zero if and only if the cost of screening is higher than the benefits of ‘skimming, which comes from selling low types and holding on to high types, which is equal to $C > (1 - \gamma)A_2\hat{r}_{P,2} + \gamma A_2(1 - \gamma)(1 - q)(1 + \hat{r}_{P,2}) - 1$. For the equilibrium interest rate $\hat{r}_{P,2}$, this is equal to $C > A_2\frac{(1 - \gamma)(1 - q)}{1 - \gamma(1 - q)}$. This will hold as long as there exists values of $C$ such that it satisfies this equation and that $C < q\kappa(1 - q)(1 - \gamma)$ at the same time, which now proceed to show.

Note that as $A_2$ is increasing in $\gamma$ and for $\gamma < \frac{1}{2}$, so is $\frac{(1 - \gamma)(1 - q)}{1 - \gamma(1 - q)}$, it is sufficient to prove there can exist values of $C$ for the limit value of $\gamma = \frac{1}{2}$. In such case, we must have that $q\kappa(1 - q)\frac{1}{2} > A_2\frac{(1 - q)}{1 - \frac{1}{2}(1 - q)}$, which equal to $2q\kappa(1 - \frac{1}{2}(1 - q)) > A_2$. For $\gamma = \frac{1}{2}$, $A_2 = \kappa\frac{q^2 + 1}{1 + q^2}$, so $2q\kappa(1 - \frac{1}{2}(1 - q)) > \kappa\frac{q^2 + 1}{1 + q^2}$, which simplifies to $1 > \frac{1}{1 + q^2}$, which holds for all real $q$. Note as $C > A_2\frac{(1 - \gamma)(1 - q)}{1 - \gamma(1 - q)}$ is increasing in $A_2$, as long as $A_1 < A_2$, this condition will hold in period 1 as well.

\[ \text{Prices in period 1 and 2 are less than } \kappa. \]

For our equilibrium values, $A_{P,2} \leq \kappa$ is equal to $\kappa \geq (q\kappa + \frac{q^2}{2\gamma}k\frac{1}{2\gamma(1 - q)} - \frac{1}{2\gamma(1 - q)})q^2$, which can be rewritten and simplified into $1 - \gamma \geq \frac{q^2}{2\gamma(1 - q)} - q(1 - q) - \frac{q^2}{2\gamma}$ and further simplified into $\gamma^2(1 - q) - \gamma(1 + (1 - q)^2) + 1 - q(1 - q) - \frac{q^2}{2\gamma} \geq 0$. First note that the zero order term, $1 - q(1 - q) - \frac{q^2}{2\gamma}$ is always greater than zero for $q \in [0, 1]$. Then note that $\gamma^2(1 - q) - \gamma(1 - q) + 1^2$.

\[ As we have discussed previously, this is a heavily stylized assumption, in so much that house prices never decline to 0 in real life. We could renormalize this value upwards as in B&F; but choose not to, as, essentially, what we must have is that the risk for buyers of loans to end up with houses post-defaults, which will happen when house prices fall, is larger than the gains from lending to them if they don’t. Any changes would still have to satisfy this in our model. \]

\[ This holds strictly if we opt to have low types default when $A_3 = A_3(1 + \hat{r}_{L,2}) = 0$. \]
2.B. APPENDIX B - PROOFS

(1 − q)^2\) has two roots, \(\gamma = 0\) and \(\gamma = \frac{1}{1-q} + 1 - q > 1\), such that for \(\gamma \in [0, 1]\), \(\gamma^2(1 - q) - \gamma(1 + (1 - q)^2) \leq 0\), which means that our condition always holds.

Note that as long as \(\hat{A}_1 \leq \hat{A}_2\), this condition holds for period 1 as well.

### 2.B.3 Extensions Proofs

We can have risk averse borrowers in general equilibrium.

Like in partial equilibrium, we need to have that the benefits of screening outweigh the costs stemming from risk aversion. This means that we have to have:

\[
q(A_{t+1} - A_t(1 + r)) \geq aq(1 - q)(A_{t+1} - A_t(1 + r))^2
\]

We focus purely on speculators in period 1. If our results do not hold for speculators in period 2, we could change our housing stock size to be equal to 2, such that in the second period the housing supply is 2(1 − γ), which would guarantee that arriving owner-occupiers in period 2 will buy from at least 1 speculator, and at the same time, they will not exhaust the housing supply, thus leading to the put option value changing house prices. Thus, the benefits outweigh the costs if

\[
q(a_{t+1} - a_t(1 + r)) \geq a(1 - q)(a_{t+1} - a_t(1 + r))^2
\]

We test this numerically for our equilibrium values, using double decimal precision for \(q\) and \(\gamma\), and find that this holds for all \(q\) greater than \(q_{min} \in (0.27, 0.41)\), depending on the value of gamma.

With restricted incomes and down-payments, a no-screening equilibrium can exist with small enough loan originators, otherwise only owner-occupiers receive loans.

As per our discussion in Appendix A, we have a trivial result when incomes are unrestricted, wherein the equilibrium is for borrowers to purchase homes with 100% down-payment. We thus focus on the case where there is a restricted income \(y_p = \hat{d}A_\rho\), and thus a maximum down-payment, \(\hat{d}\).

We begin by discussing what happens when originators wish to hold on to loans. For any initial down-payment \(d\), the addition of down-payments mean that there is now reduced uncertainty, so a pooling equilibrium is now:

\[
EU_{p_{r}} = -aq((1 - d)A)^2r_{p_{r}} + (1 - \gamma(1 - q)(1 - d)A) r_{p_{r}} - (1 - q)(1 - d)[\gamma + aA]
\]

In the limit of \(d = 1\), there is no uncertainty, so our previous results do not hold. For any \(\hat{d}\), we have that they will hold if \((1 - \hat{d})Aa^2 + a\gamma - \frac{(1-\gamma)(1-d)^2}{4q(1-q)(1-d)A} \geq 0\), which itself is satisfied by setting \(a\) larger than the largest positive root of this equation, \(a''''\). We use this new restriction, \(a > a''''\) for the results below.

We now show that speculators will now not take on loans if the interest rate is too high. Using the result from our previous extension proof, we need only show the conditions under
which a speculator from period 1 will not wish to take a loan. Given \( d \), they now take on a
loan if \( q(A_2 - (1 - d)A_1(1 + r_1)) - dA_1 \geq 0 \).

Given that \( A_1 = qA_2 \), this will be strictly less than zero if and only if \( 1 + r > \frac{1}{q(1 - d)} \), thus an originator may be able to deviate from equilibrium by selling loans with a positive \( d \) and
\[ 1 + r = \frac{1}{q(1 - d)} \], knowing that this is a credible deviation and that speculators will take on such
loans. In such a case, the price of these loans, where \( D \) stands for deviation, will be strictly
positive and originators will be able to make positive profits. Originators will compete to
attract borrowers by setting the lowest possible interest rate and the highest possible down-
payment, i.e., \( \bar{d} \) and \( 1 + r = \frac{1}{q(1 - d)} \).

From the above results, originators will not wish to hold on to speculators, and that if
\( 1 + r < \frac{1}{q(1 - d)} \), speculators will not self-select. Thus, the only possible deviation is the afore-
mentioned one, with \( \bar{d} \) and \( 1 + r = \frac{1}{q(1 - d)} \). The only question that remains is whether owner-
occupiers will wish to accept these new loans. They will not accept the new loans if the total
cost of the new loan is larger, that is, if at the equilibrium value for interest rates

\[
(\bar{d} + 1/q)A_D > (\bar{d} + (1 - \bar{d})/(1 - \gamma(1 - q)))A_{NSC}
\]

If loan originators are small, such that the contemplated deviation from a no-screening
equilibrium remains with house prices at the no-screening equilibrium, we have that \( A_D = A_{NSC} \), such that they will not deviate if \( 1/q > (1 - \bar{d})/(1 - \gamma(1 - q)) \). Given that \( 1/q > 1/(1 - \gamma(1 - q)) \), there can be no \( \bar{d} \in (0, 1) \) that makes this true. Thus, the no-screening
equilibrium is sustainable.

However, if loan originators are large enough to ‘influence’ house prices in equilibrium,
then using our equilibrium results, they will deviate if

\[
(\bar{d} + 1/q)(1 - (1 - \bar{d})(1 - q)q(1 + r_{NSC})) < (\bar{d} + (1 - \bar{d}(1 + r_{NSC}))
\]

Which we can rewrite as

\[
2 + (1 - q) - 2/(q(1 + r_{NSC})) < \bar{d}(2 - q) + \bar{d}^2(1 - q)q
\]

We verify this numerically and find that it holds for all values of \( q \) and \( \gamma \). Thus this result
is a possible deviation from the no-screening equilibrium, in which case there can be no equi-
librium where speculators receive loans, and the equilibrium will have owner-occupier loans
sold to securitizers, with the highest possible \( d \) and ‘high’ interest rates.
Chapter 3

House Prices, Securitization and Non-Recourse Loans in the US during the 2000s

Abstract

We conduct an empirical study of the effects that private securitization and recourse (limited liability) laws had on house prices. From our previous chapter, we have the prediction that the interaction between private securitization and non-recourse status should lead to higher house prices during a boom. We use heterogeneity across recourse laws in US states to test this. As predicted, non-recourse status roughly doubles the size of the positive relationship between securitization and house prices and can explain around 75% of the difference in prices between recourse and non-recourse states. To address potential endogeneity concerns, we propose a new instrument for securitization, the distance of a housing market to the headquarters of ‘originate and securitize’ institutions, and find further empirical support for our prediction. We conduct further tests concerning predictions for the interaction and the housing market and find further, if more mixed, evidence for our model predictions.

JEL Classification Numbers: E00, E44, G20, R31.

Keywords: House prices, Securitization, Screening, Non-recourse loans.

3.1 Introduction

The literature in macroeconomics is becoming ever more concerned with the importance of the housing market. Iacoviello and Neri (2010) and Mian and Sufi (2014), among many recent papers, form part of the growing literature which finds evidence that the housing market, particularly how house prices change, forms an important part of both short and long term dynamics in macroeconomics. As such, a better understanding of which factors lead to changes in house prices, and a more general understanding of the housing market as a whole, now forms a significant part of the research agenda in macroeconomics.

This chapter seeks to add this literature, by testing the predictions from the previous chapter. There, we build a model which deals with the interaction between private securitization
3.1. INTRODUCTION

and recourse, that is, limited liability laws and the effect of this interaction on house prices. When securitization takes place, it allows risk-averse mortgage loan originators to sell loans to the securitization market. Non-recourse loans means that borrowers do not self-select when taking mortgages so there is no credible way to signal loan quality to the securitization market.

Combined, these factors make originators stop screening borrowers, allowing for speculator-type borrowers to start receiving loans. As these mortgages are non-recourse, speculators have a put option that pushes up their value for house prices, and as they become marginal sellers, house prices increase more during a boom; this mechanism should lead to a larger fall in house prices during a bust. We thus need both causes, securitization and non-recourse loans, to have significant effects on house prices, and we predict that US states that have non-recourse laws for mortgages and experienced higher levels of securitization should have higher house prices during the boom period prior to 2007.

Some tentative evidence for this mechanism can be seen in Figure 3.1, which plots house prices in recourse and non-recourse states from 1991; the latter experiences higher house price growth starting at a similar period to when the private securitization market ‘took-off’, around 2003. In the Appendix C, an identical plot starting from 1975 but with lower quality data, shows a similar pattern.  

We test our model predictions using US state and MSA (Metropolitan Statistical Area, that is, cities) level data from 2004-2006, making use of heterogeneity in US states’ recourse laws. We do this by regressing house prices on a measure of the percentage of securitized new loans and interact this with non-recourse status of a state/MSA (and controls). We find evidence that securitization is positively correlated with house prices, wherein for every extra 1 p.p. of new mortgages that were securitized, house prices increased by around 1% in the period. Moreover, this effect was roughly doubled when laws are non-recourse. This result survives a number of robustness checks. We conclude that the interaction between securitization and non-recourse status can potentially explain around 75% of the difference in house prices in the average recourse and non-recourse states from 2004-2006.

\footnote{However it also shows two other detachments, one from around 1985 to 1992, wherein recourse states experienced higher house growth compared to non-recourse states, and one since around 2012 with non-recourse states growing faster; we discuss this issue more in Section 3.3}
Due to possible endogeneity/reverse-causality issues between house prices and securitization, we use novel instrument for securitization, the weighted distance of a given MSA to the two closest headquarters of ‘originate and securitize’ mortgage institutions. We find similar results to our main regressions, but we note that there may be first stage problems with our instrument; in particular our first stage coefficients are inconsistent in recourse states. We take this result as more evidence, but not conclusive evidence, that our mechanism played a role in the US housing boom.

We also test our model predictions for the bust period, that house prices should fall more and there should be more defaults due to the interaction effect stemming from the boom period. We use a similar empirical strategy to the boom period, but replace our contemporaneous measure of securitization with a measure for the average amount of securitization of the boom-period instead. We find weak evidence for our house price prediction, and no evidence of increased defaults due to the interaction effect. We also find some evidence that securitization as a whole played an role in determining house prices in both boom and bust, and for defaults in the bust.

This chapter and its results are closely related to the growing literature on the effects of recourse laws in the United States, and in particular exploits the heterogeneity of states’ laws going back to the Great Depression. Ghent and Kudlyak (2011)’s paper, by providing a benchmark classification of recourse status and showing its effects on defaults, has proven very influential, with Dobbie and Goldsmith-Pinkham (2014), Westrupp (2015) and Chan, Haughwout, Hayashi, and Van der Klaauw (2016) being recent papers that explore the effects of recourse status on defaults.

As we discuss in the previous chapter, our paper is closely related to Nam and Oh (2014), which proposes a similar hypothesis to the one our paper explores, that is, that non-recourse status may have affected prices in a risk-shifting mechanism. They investigate this possibility empirically by looking at the effects of recourse on house prices during the boom period by using state border discontinuities. They find that non-recourse states experienced greater house price increases during the boom, and that borrowers were actively taking advantage of non-recourse status by taking on greater leverage and investing more in housing, and that lenders were aware of the added risks; unlike our paper, they do not control for levels of securitization. Our paper chooses to not follow their empirical strategy, however, following some of the concerns raised by Westrupp (2015).

This chapter is organized as follows. We briefly summarize the predictions of the last chapter and discuss our data in Section 3.2. We present our empirical strategy and results in Section 3.3. Section 3.4 concludes this chapter.

### 3.2 Model Predictions and Data

We begin by first briefly summarizing the predictions obtained in the previous chapter of this thesis, and how they pertain to the empirical analysis. We then proceed to describe our dataset and sources, and how we define what constitutes a non-recourse state. This is followed by a discussion of issues with our data and how we address them, and by a discussion concerning the literature on the non-recourse status of mortgage laws in the US.
3.2. MODEL PREDICTIONS AND DATA

3.2.1 Model Predictions

Our model makes predictions about how house prices should evolve over time during a period of increased demand, depending on whether loans are recourse or not, and whether private securitization is taking place. In all scenarios, house prices experience ‘boom-like’ behaviour, reflecting the changes in fundamentals driving the increased demand. In markets where loans are non-recourse, the put option value of defaulting associated with limited liability can always potentially push up house prices, but in our model this will only happen when there is securitization, as otherwise loan originators screen out the risky speculators who must receive loans for the option value to matter.

Our model has binary predictions for what happens to loans with the private securitization market, as either all loans are securitized, or none are. As 100% securitization does not happen in practice, we need to interpret our model results in that light. We do so by seeing the model as suggesting that the higher the levels of securitization, the greater the probability that speculators have managed to buy houses and, in subsequent periods, become the marginal seller\(^2\).

So, our primary prediction is that there is a positive effect on house prices due to the interaction between the percentage of loans securitized in a US state and whether that state has non-recourse law on mortgage loans. This prediction is distinct from the more established prediction that there is a positive effect on house prices coming from private securitization, which has been found previously by the literature.

We also have the prediction that this interaction effect, during a boom, subsequently leads to greater drops in house prices during the bust period, and to higher levels of defaults, in non-recourse states. The former is due to house prices converging to the same level in all scenarios during a bust, thus falling more in non-recourse states that experienced higher levels of securitization, and the latter from defaults of speculators who no longer profit from the demand boom. As our model less well suited to dealing with the bust period, we treat these predictions as being of a secondary nature.

3.2.2 Securitization

To test the predictions of our model, we use the LAR datasets of the HMDA. The HMDA act was originally passed to collect data to check for discrimination in the US housing market. It requires most loan originators to report certain types of information on any loan request, successful or not. The act now covers around 80% of the mortgage loans according to Fishbein and Essene (2010), and is available in the aggregated LAR datasets, on an annual basis.

Amongst other things, originators must report to whom they sell a loan if the loan is sold within the same calendar year of being originated. The possible categories to whom a loan is sold were changed in 2004, and have remained the same since. This change included the addition of the category ‘Private Securitization’, which consists of any sale to a non-GSE entity where the originator believes the loan will be securitized in the private market\(^3\).

Our main measure of private securitization, ‘Sec’ is thus the percentage of loans which are

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\(^2\)This also corresponds to what we conclude in our discussion in the Appendix of the previous chapter concerning ‘sufficient levels of securitization’ required for the interaction effect to take place.

\(^3\)If an institution selling a loan knows or reasonably believes that the loan will be securitized by the institution purchasing the loan, then the seller should use code ‘5’ for “private securitization” regardless of the type or affiliation of the purchasing institution.”, according to http://www.ffiec.gov/hmda/faqreg.htm#purchaser.
believed by originators will be privately securitized, out of all successfully originated loans intended for houses purchases. Using the same category, we also find which loans were originated and sold to/securitized by GSE entities\(^4\), and from that we obtain our variable ‘GSE’, the percentage of loans sold to GSEs of all successfully originated loans intended for houses purchases.

### 3.2.3 Recourse in the US

The question of whether mortgages are non-recourse in practice in the US\(^5\) is more complex than seems at first. State laws dictate the procedures taken after mortgage repayments stop, and there is a great deal of heterogeneity in how each state deals with this, as with the possible ways borrowers and lenders can proceed once a default happens\(^6\). For the purposes of this paper, what matters is if the state permits deficiency judgments to be made on defaulted mortgages during a foreclosure procedure.

A deficiency judgment consists of a judicial ruling, during a foreclosure process, that the proceeds from the sale of an asset was insufficient to fully cover the loan that was secured by that asset. As such, these permit lenders to recover the difference between the contracted value of a house and the value obtained through selling the house, either by using the borrower’s income, or other assets they possess.

A borrower can avoid this by declaring bankruptcy, but only if they file for chapter 7, which, according to Ghent and Kudlyak (2011), is not always possible in every American state; a chapter 13 filing does not eliminate the possibility of a deficiency judgment. Furthermore, the possibility of filing for chapter 7 bankruptcies was restricted in 2005, via the BAPCPA law, particularly for borrowers with higher income. From the perspective of our model, what matters crucially is the relative ease/how cheaply a borrower can walk away from his mortgage obligations, and as we discuss in introduction, the most recent evidence seems to suggest that non-recourse status matters significantly in this respect.

Regarding how to classify the recourse status of a state, Ghent et al. (2011) and Mitman (2015), amongst others, use a very similar list that has a high degree of concurrence, with Arizona, California, Iowa, Minnesota, Montana, North Dakota, Oregon and Washington being considered non-recourse; they differ only in that Alaska, North Carolina and Wisconsin are also considered non-recourse by Ghent et al.

This is a fairly typical result in the literature, as there is a degree of subjectivity in classifying which states are recourse or not. However, most papers’ classifications have a large degree of overlap in states classified as non-recourse, particularly the West Coast and Northern states; they normally only differ in their classification on a small number of states. Consequently, and following most recent papers, we opt to use the Ghent et al. (2011) classification as the benchmark for our regressions. We opt to use Mitman (2015) as a robustness check, and find that our results are unaffected by this.

Finally, as Ghent et al. (2011) and Ghent (2012) document, recourse law for the vast majority of states has largely stayed the same since the Great Depression until 2008, with, in some cases,\(^7\)

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5 As mentioned in the previous chapter, few other countries have non-recourse mortgage loans, with Brazil being a notable exception.

6 According to Ghent and Kudlyak (2011), borrowers may give the house deed to the lender in exchange for no further actions, they may find a purchaser for the house or they may enter a foreclosure procedure, which may or may not be contested, during the former of which deficiency judgments may happen.
the laws remaining unchanged since the 19th century. Subsequently and in accordance with the literature, I treat recourse status of states as exogenous for the purposes of the empirics.

### 3.2.4 Other data

We obtain house price levels from the FHFA’s HPI, which uses data from Freddie Mac and Fannie Mae. For state-wide prices, this index uses the standard weighted repeat-sales methodology. For MSA-level data, we use the all-transactions index, due to greater coverage, despite its methodological limitations. We believe that the limitations of using data stemming only from GSEs transactions is not a great one because there should be no significant segmentation between housing markets when it comes to the growth in house prices. Nevertheless, we do perform a robustness check using the Case-Shiller price index for 20 MSAs.

We define our controls and the other variables that we use, and the sources they come from in Table 3.1. On an MSA-level, we have less data available and our data is of poorer quality, so we use our state-level regressions as a baseline whenever possible. As we use fixed-effects and time dummies in most of our regressions, thus capturing nominal effects, we normalize house prices, population, income/income growth, unemployment and interest rate measures to be 100 in 2004, to facilitate the interpretation of the results.

<table>
<thead>
<tr>
<th>Variable</th>
<th>State-level</th>
<th>MSA-level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income</td>
<td>‘Per Capita Personal Income’, U.S. Department of Commerce</td>
<td>Mean reported income of all loan transactions, HMDA*</td>
</tr>
<tr>
<td>Unemployment</td>
<td>‘Unemployment rate’, US. Bureau of Labor Statistics</td>
<td>N/A</td>
</tr>
<tr>
<td>Subprime loans</td>
<td>% of purchase, originated loans with rates above 3%, HMDA**</td>
<td>% of purchase, originated loans with rates above 3%, HMDA**</td>
</tr>
<tr>
<td>Unemployment</td>
<td>‘Unemployment rate’, US. Bureau of Labor Statistics</td>
<td>N/A</td>
</tr>
<tr>
<td>LTI</td>
<td>Loan Amount divided by Income of the applicant, HMDA***</td>
<td>N/A</td>
</tr>
<tr>
<td>Interest rates</td>
<td>Interest rate on ‘conventional loans’, FHA</td>
<td>N/A</td>
</tr>
<tr>
<td>Defaults</td>
<td>% of mortgage debt 90+ days delinquent, FRBNY</td>
<td>N/A</td>
</tr>
</tbody>
</table>

*Including non-originated loans, around 10-15% of observations missing per annum.
**In ‘RateSpread’, following Mayer and Pence (2008), see also the Appendix A.
***Only for originated loans, around 10-15% of observations missing per annum.

Table 3.1: Data sources for controls and other variables

A summary of the descriptive statistics of our main variables for the boom period (2004-2006) and the overall period of our analysis (2004-2012) can be found in Table 3.2. The average values of our main variables are very similar for both types of states; non-recourse states do experience slightly less defaults than recourse states, but this difference is not statistically significant.

### 3.2.5 Discussion of Data and Recourse

#### 3.2.5.1 Securitization

There are two important limitations for our securitization data. Firstly, as the category ‘Private Securitization’ only began to be used from 2004 onwards, we have a limited amount of

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*For data starting at 1991, we use the all-transactions index.*
3.2. MODEL PREDICTIONS AND DATA

Table 3.2: Descriptive statistics for Boom and full sample periods

<table>
<thead>
<tr>
<th></th>
<th>Non-Recourse States</th>
<th>Recourse States</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Securitization (%)</strong></td>
<td>3.66 (2.47)</td>
<td>3.94 (2.47)</td>
</tr>
<tr>
<td><strong>Income</strong></td>
<td>34523 (3360)</td>
<td>35481 (6451)</td>
</tr>
<tr>
<td><strong>Income Growth (%)</strong></td>
<td>4.91 (1.65)</td>
<td>5.29 (1.88)</td>
</tr>
<tr>
<td><strong>Population</strong></td>
<td>6922 (9464)</td>
<td>5485 (5296)</td>
</tr>
<tr>
<td><strong>Unemployment (%)</strong></td>
<td>4.98 (1.09)</td>
<td>4.83 (1.05)</td>
</tr>
<tr>
<td><strong>Mortgage Defaults (%)</strong></td>
<td>0.85 (0.32)</td>
<td>1.21 (0.52)</td>
</tr>
<tr>
<td><strong>Subprime (%)</strong></td>
<td>19.23 (7.97)</td>
<td>21.84 (7.13)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Non-Recourse States</th>
<th>Recourse States</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Securitization (%)</strong></td>
<td>1.86 (2.07)</td>
<td>1.75 (2.19)</td>
</tr>
<tr>
<td><strong>Income</strong></td>
<td>38350 (4957)</td>
<td>39021 (7882)</td>
</tr>
<tr>
<td><strong>Income Growth (%)</strong></td>
<td>3.77 (3.48)</td>
<td>3.47 (3.19)</td>
</tr>
<tr>
<td><strong>Population</strong></td>
<td>7147 (9704)</td>
<td>5627 (5469)</td>
</tr>
<tr>
<td><strong>Unemployment (%)</strong></td>
<td>6.37 (2.28)</td>
<td>6.34 (2.25)</td>
</tr>
<tr>
<td><strong>Mortgage Defaults (%)</strong></td>
<td>2.93 (2.60)</td>
<td>3.47 (3.00)</td>
</tr>
<tr>
<td><strong>Subprime (%)</strong></td>
<td>10.42 (8.37)</td>
<td>12.54 (8.86)</td>
</tr>
</tbody>
</table>

Standard deviation in parenthesis.

Data points available. Our main regressions will thus cover the period from 2004-2006, giving us only 3 years’ worth of data. The second concern is that the category ‘Private Securitization’ only reports originators’ beliefs on how sold loans will be put to use, not on whether securitization actually took place, and only for loans sold within the same calendar year of origination.

This means that there could be misreporting of the actual securitization levels of loans within states/MSAs due to at least 3 effects. Originators might report loans as having been securitized when they were not, it is possible that loans reported as sold to other types of purchasers were, subsequently, securitized and not reported as such, and loans may have been sold to be securitized in a subsequent calendar year. We strongly suspect that the second and/or third of these effect is dominant, as the LPS data used in Krainer and Laderman (2014), using somewhat different criteria for what loans to include, report that around 38% of loans in California were privately securitized in 2006, whereas we find it to be around 10%.

If our measure is under-reporting the amount of securitization in each state by the same fixed amount (for example, by 5 p.p. in each state), then this error would be captured by our state fixed-effects. However, if this error is proportional to the level of securitization taking place in each state, then our estimates for the coefficient of securitization will be biased upwards. Finally, there are likely to be the classic measurement error bias with our measure that should not affect our results significantly.

Fortunately for the purposes of this paper, we are mainly concerned with relative, cross-state measures of securitization, not absolute measures. Unless there are systematic differences in the way originators in each state/MSA reported this category, then it should provide an accurate measure of relative securitization.

As such, our estimates for the coefficient of securitization may be biased. However, the relative effects of securitization, when compared to other sources of variation on house prices, should be captured more accurately as we are using fixed-effects and time dummies.

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8Of particular concern are the categories “Life insurance company, credit union, mortgage bank, or finance company” and “Other type of purchaser”.

3.3. EMPIRICAL STRATEGY AND RESULTS

3.2.5.2 Non-recourse literature

The evidence concerning recourse and how it affects the housing market has changed over time, and earlier evidence for the importance of recourse is more ambiguous: Pence (2003) summarizes the literature up to that point, noting that the effects of recourse/deficiency judgments on mortgages was ambiguous. They state that “lenders rarely pursue deficiency judgments” and they find weak empirical evidence for its importance. Despite this, they suggest that recourse matters for people purchasing houses to speculate and when borrowers are in a ‘non-hardship’ situation, both being cases where deficiency judgments are more likely to be pursued.

Other papers, particularly more recent papers, finds otherwise, however. The seminal paper, Ghent et al. (2011), finds that non-recourse states have higher default rates and that non-recourse alters the way borrowers default, which they take as evidence of strategic defaults on the part of borrowers. Pennington-Cross (2003) finds significant evidence that loans being recourse increases the amounts recovered by lenders in case of a default. Dobbie and Goldsmith-Pinkham (2014) find evidence in the recent bust that home-owners in non-recourse states experienced greater declines in debt, which they attribute to the protections afforded by these laws, but also saw greater falls in house prices (due to increased foreclosures), leading to a greater fall in consumption and income when compared to recourse states. Chan et al. (2016) find similarly that non-recourse status increases defaults on all types of housing debt. Westrupp (2015) also finds evidence of the importance of recourse status on the volume and discount levels of foreclosure sales.

Thus, the most recent evidence for whether a state’s recourse law affects how borrowers and lenders behave seems to indicate that it has significant effects. Given the results of Pence (2003), this suggests that recourse may have become more important during the boom and bust of the 2000s.

3.3 Empirical strategy and results

3.3.1 Boom period

We first test our model predictions for the boom period. To do this, we regress house prices on the interaction effect between securitization and the non-recourse status of a US state, at both a state and a MSA level:

\[
H_{Price,i,t} = \beta_1 \text{Sec}_{i,t} + \beta_2 \text{NonRec}_i + \beta_3 \text{Sec} \times \text{NonRec}_{i,t} + \gamma_{i,t} + D_{20XX} + \epsilon_{i,t}
\]

\( H_{Price,i,t} \) are house prices in state/MSA \( i \) at time \( t \), \( \text{Sec}_{i,t} \) is the percentage of mortgages that are privately securitized of all house-purchase loans originated in \( i, t \), \( \text{NonRec}_i \) is a dummy for whether state \( i \) (or the state in which a MSA is located) is non-recourse, \( \gamma_{i,t} \) are the controls\(^9\), consisting of income (Inc), population\(^10\) (Pop), income growth (IncG) and unemployment (Unemp) for a state \( i \), and income (Inc) and population (Pop) for a MSA \( i \), and \( D_{20XX} \) are year dummies.

\(^9\)One potentially important control that we do not include is the loan-to-value ratio, due to data availability issues for those years, and this may result in a omitted variable bias in our results; see also Appendix A.

\(^10\)As most regressions will have state or MSA fixed-effects and land can be supposed to be largely fixed for those 3 years, this means that we are controlling for population density.
3.3. EMPIRICAL STRATEGY AND RESULTS

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1) HPrice</th>
<th>(2) HPrice</th>
<th>(3) HPrice</th>
<th>(4) HPrice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Securitization</td>
<td>1.031***</td>
<td>1.177**</td>
<td>0.726***</td>
<td>0.784***</td>
</tr>
<tr>
<td></td>
<td>(0.388)</td>
<td>(0.484)</td>
<td>(0.192)</td>
<td>(0.225)</td>
</tr>
<tr>
<td>NonRecourse</td>
<td>-1.015</td>
<td>0.582</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.953)</td>
<td>(0.586)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Securitization × NonRecourse</td>
<td>1.018**</td>
<td>1.262**</td>
<td>0.215</td>
<td>0.526**</td>
</tr>
<tr>
<td></td>
<td>(0.473)</td>
<td>(0.542)</td>
<td>(0.232)</td>
<td>(0.253)</td>
</tr>
<tr>
<td>Observations</td>
<td>153</td>
<td>153</td>
<td>1,076</td>
<td>1,076</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.880</td>
<td>0.821</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of State/MSA</td>
<td>51</td>
<td>51</td>
<td>359</td>
<td>359</td>
</tr>
<tr>
<td>Dataset</td>
<td>State</td>
<td>State</td>
<td>MSA</td>
<td>MSA</td>
</tr>
<tr>
<td>Method</td>
<td>RE</td>
<td>FE</td>
<td>RE</td>
<td>FE</td>
</tr>
</tbody>
</table>

Robust, clustered standard errors in parentheses, *** p < 0.01, ** p < 0.05, * p < 0.1.
Regressions include controls and year dummies. Annual data from 2004 to 2006.

Table 3.3: Boom period, main regressions

The regressions are run for 2004-2006 period using either fixed-effects (FE) or random-effects (RE), clustering the standard errors at either state or MSA level; when using FE, non-recourse is automatically absorbed and is thus omitted. We treat our control variables as exogenous, that is, we assume that they are not affected by changes in house prices in the 3 year period of our regressions. As we regress house prices using year dummies, from a normalized index, these regressions deal only with house price growth.

As the prediction of our model is that the interaction effect between securitization and non-recourse status should have positive effects, we focus our attention on the interaction effect in the subsequent discussion. Wherever possible, we focus on the results of the FE regressions, which can control for omitted variables bias, and on the state level results, as discussed in the previous section. The results of our regressions can be seen in Table 3.3, with the results for our controls in Table 3.8 in Appendix B.

The interaction effect is positive and significant in all but one specification. As securitization has a coefficient of around 1\(^{11}\), this means that a 1 p.p. increase in securitization is associated with a 1% increase in house prices in recourse states. Moreover, when a state is non-recourse, a 1 p.p. increase in securitization is associated with a 2% increase in prices in non-recourse.

We are not too concerned that the coefficient in the MSA-RE specification is not significant at 10%, as MSAs should have higher levels of heterogeneity than states and there should be higher uncertainty when using a RE specification. In all other cases, the value of the interaction coefficient is at a similar level when compared to the securitization coefficient.

These results suggest that the association between securitization and house prices is around double in size in non-recourse states when compared to recourse states, and this is consistent with our model predictions.

\(^{11}\) However, as discussed in Section 3.2, we are likely underestimating the amount of securitization that took place, this means that the estimated effects is likely larger than the actual effect and should be treated as an upper bound.
3.3. EMPIRICAL STRATEGY AND RESULTS

3.3.1 Robustness checks

We perform 22 baseline robustness checks (including variations on state/MSA), and each test is described and discussed in detail in Appendix A. We opt not to include certain controls in our baseline regressions, such as our measure of subprime mortgages and our measure of GSE securitized loans, due to concerns about endogeneity.

Among our robustness checks, we reinterpret our model predictions, by taking it more literally and assuming that there is a cut-off point at which the effects of high securitization are felt and speculators start receiving loans. We propose and test this interpretation in two different ways. We first restrict our sample to those MSAs with high level of securitization and run the same regressions as the baseline. Alternatively, we create a dummy for MSAs above the average value of securitization (‘Top Securitization MSA’) and use it instead our main measure of securitization, using the same specifications as our baseline regressions.

Some of these robustness checks are shown in Table 3.4, specifically, the results when we include a measure of subprime mortgages, when we exclude California, when restrict our sample to non-Western states/non-Coastal states, and when we use the ‘Top Securitization MSA’ dummy. Our other robustness test results are reported in Appendix B, in Tables 3.9 and 3.10\textsuperscript{12}.

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1) HPPrice</th>
<th>(2) HPPrice</th>
<th>(3) HPPrice</th>
<th>(4) HPPrice</th>
<th>(5) HPPrice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Securitization</td>
<td>0.409 (0.600)</td>
<td>0.456 (0.598)</td>
<td>1.189 (0.726)</td>
<td>0.160 (0.335)</td>
<td></td>
</tr>
<tr>
<td>Securitization × NonRecourse</td>
<td>0.969* (0.518)</td>
<td>1.264* (0.667)</td>
<td>-0.862** (0.403)</td>
<td>1.487 (0.954)</td>
<td></td>
</tr>
<tr>
<td>Top Securitization MSA</td>
<td>1.655*** (0.552)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top Securitization MSA × NonRecourse</td>
<td>2.938** (1.172)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Observations | 153 | 150 | 114 | 81 | 1,074 |
R-squared | 0.888 | 0.886 | 0.844 | 0.887 | |
Number of State/MSA | 51 | 50 | 38 | 358 | |
Dataset | State | State | State | State | MSA |
Method | FE | FE | FE | RE | |
Change | Subprime | No Cali | Non-Western | Non-Coastal | Top 50% |

\textsuperscript{12}Results for our controls are omitted for brevity’s sake.

We interpret the results of our numerous robustness checks as showing that our baseline results largely hold; the notable exception consists of when we regress only in non-Western states, which we discuss below. The interaction effect otherwise is positive and mostly significant, with a coefficient ranging from 0.36 to 3.7. This is a fairly high range, as is expected given that we change the range, measurement and add variables to our regressions in these tests. The coefficient is not significant when we extent our range to 2007, when we use the Case-Shiller index, when we do Coastal and non-Coastal level regressions and when our measure of subprime loans (on a MSA, but not state level) is added. However, the coefficients we find in these tests are compatible with our baseline regressions and it is only in these 5 of our 21 robustness checks (excluding the non-Western result) that the interaction term is non-significant.

The coefficient for securitization is significant in 9 of the 19 robustness checks that include it, although always being positive and (excluding Subprime MSA-level and Non-Coastal regressions) with a coefficient ranging from 0.3 to 1.4, all of which suggest that this is a less
robust result. We also note that, aside from concerns about endogeneity, the coefficient for subprime is positive and statistically significant at 1% in all regressions, indicating that subprime mortgages were tightly linked with house prices at that time period\footnote{Similarly, the coefficient for GSE is negative and mostly significant in our specifications, whereas its interaction is positive and significant, suggesting that GSE securitization had some effects on house prices at that time, albeit differently in recourse and non-recourse states; this result is somewhat surprising and may warrant further research.}.

### 3.3.1.2 Non-Western states

Concerning the non-Western state result, which seems to go against our model predictions, we find that the average securitization level of MSAs in non-Western, non-recourse states was 2.2%, compared to 5.9% in Western, non-recourse states. Similarly, whereas in Western, non-recourse states 85% of MSAs are among the top 50% of our measure, in non-Western, non-recourse states, only 19% of MSAs (24 in total) could be classified as such. Finally, the highest level percentage of securitization experienced by any MSA in the latter was only 5.9%, compared to 15.7% in the former.

Given these statistics, we conclude that non-Western, non-recourse states experienced relatively low levels of securitization. This suggests that, unlike most other states, non-Western, non-recourse states may have been closer to the no-securitization prediction of our model. Assuming this is the case, the most reliable regressions for these states should be the ones where we use our dummy for top 50% highest securitization, as we can then focus on the few MSAs that did have ‘sufficiently high’ levels of securitization.

We thus run the same regressions for these states, using the TopSecuritization dummy instead of our measure of securitization $Sec$, and either using it directly (with RE) or interacted with year dummies (so we can use FE). The results can be found in Table 3.12 in Appendix B and we find that they are closer to what our model predicts. Although the coefficients on the interaction effect are not significant and, for FE regression, smaller in size when compared to our baseline results, they are nevertheless positive, in the direction that our model predicts.

We thus conclude that there very likely were not enough MSAs in non-Western, non-recourse states that had high enough levels of securitization for us to test our predictions on a state level. When we test these states using our securitization dummy, the few MSAs in non-recourse, non-Western states that did have higher levels of securitization did experience higher house prices as our model predicts, albeit with coefficients that are non-significant.

### 3.3.1.3 Discussion of results

From these regression results, we conclude that we have fairly robust evidence that the interaction between securitization and non-recourse is associated with higher growth in house prices in that period. This conforms to our model prediction and we take this as evidence for our model mechanism. On that basis, it is possible to try to quantify how much this mechanism is associated with the difference in house price growth in the average recourse and average non-recourse state in the period.

To do so, we use the results from our main regressions using our state-FE results as a benchmark. We take the average changes in each of our explanatory variables and, using the coefficients from the state-FE, report the % that each variable explains of the total fitted change of house prices in Table 3.5. By doing this, we conclude that securitization is associated with
around 31% of the increase of house prices in non-recourse states, compared to 18% in recourse states.

Moreover, by the same method we can estimate how much the difference in growth of house prices between states in that period was related to the interaction effect. Non-recourse states experienced an increase in prices of around 21%, compared to around 16.5% for recourse states in the period, of which around 75% is associated with our mechanism (around 3.4 p.p. increase from 2004). The rest is mainly associated with differences in the effects of population growth (25%), as securitization and income growth were largely similar for both types of states, and the effects of unemployment and income roughly cancel each other out.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Non-Recourse</th>
<th>Recourse</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sec</td>
<td>15%</td>
<td>18%</td>
</tr>
<tr>
<td>SecNonRec</td>
<td>16%</td>
<td>N/A</td>
</tr>
<tr>
<td>Inc</td>
<td>41%</td>
<td>55%</td>
</tr>
<tr>
<td>Pop</td>
<td>18%</td>
<td>16%</td>
</tr>
<tr>
<td>IncG</td>
<td>-1%</td>
<td>-1%</td>
</tr>
<tr>
<td>Unemp</td>
<td>11%</td>
<td>11%</td>
</tr>
</tbody>
</table>

At average values, shares each covariate explains fitted, average house price growth in recourse and non-recourse states from 2004-2006.

Table 3.5: Share of covariates in explaining average, fitted house price growth

3.3.1.4 House prices

One concern about our main empirical strategy is the possibility that non-recourse states may have systematically experienced bigger housing booms in the past, compared to recourse states. Our data for house prices ranges from 1975 until 2016, and during that time, according to Agnello and Schuknecht (2011), the US experienced a single housing boom and a single housing bust, which makes it difficult to identify whether this is the case.

Observing the previously discussed Figure 3.1 finds no observable detachment prior to 00s. Figure 3.2, although consisting of lower quality data, show house price growth in recourse and non-recourse states tends to differ little except for around 3 periods, from around 1985 to 1992, from around 2003 to 2011 and from around 2013 to 2016. We interpret that both the joint-growth long-term trend and the first detachment, being the inverse of what a put-option would predict, as suggesting that risk-shifting mechanisms were likely not boosting house prices prior to the 2000s, which is in accordance with our model predictions.

However, the subsequent detachment in house prices after 2012 is more concerning, and raises the question whether our interaction term is perhaps acting as a proxy for a more general detachment in house prices in non-recourse states starting in the 2000s. Given the observable convergence during the bust period from 2008-2011, this is not a simple question to answer and we believe that further investigation may be necessary for periods subsequent to the previous bust.
3.3. EMPIRICAL STRATEGY AND RESULTS

3.3.2 Endogeneity and IV strategy

3.3.2.1 Discussion of endogeneity

There is a critical issue of endogeneity/reverse causality for our interaction effect. For example, non-recourse states should be inherently more risky for lenders and owners of mortgage loans (i.e., securitizers), when compared to recourse states, due to the lack of deficiency judgments. In addition, lenders and securitizers may believe that when house prices are growing faster, loans are safer, as prices would have to fall more before the value of a house goes below the loan value. I.e., higher price growth may provide a buffer margin against defaults. If both these hypothesis hold, then higher levels of house prices would be required in non-recourse states to achieve similar levels of securitization, when compared to recourse states.

As this is what we find in the data and is an equivalent prediction when compared to our model, we cannot rule out the possibility that house price growth is reverse-causing securitization. If this is the case, then the standard problems with endogeneity apply and our baseline regression results may not be consistent.

In addition, the hypothesis that higher house price growth increases the buffer margin against defaults has other implications for endogeneity. In particular, this would lead to reverse causality between securitization and house prices, which would also bias our results.

We opt for an instrumental variable approach to deal with these issues. As omitting any constituent component of an interaction term biases the estimates of the interaction itself, we choose to focus our attention on finding an instrument for securitization. A valid instrument for securitization would also result in a valid instrument for our interaction term, by interacting the instrument with non-recourse, as non-recourse can be considered exogenous. This would solve both issues with endogeneity simultaneously; having fixed-effects means we need not worry about endogeneity from the non-recourse term directly.

3.3.2.2 Instrumental Variable strategy and results

To address the issue of endogeneity, we use what we believe is a new instrumental variable for securitization. There is a long tradition of using geographic distance as an instrumental variable, one widely cited example being Hall and Jones (1999)\textsuperscript{14}. Inspired by this and similar approaches, we use distance from an MSA as a instrument, specifically the minimal distance to the headquarters of the largest ‘originate and securitize’ mortgage originators in the period.

A common element in the explanation for the housing boom and bust, such as seen in Lewis (2011) among others, is that certain loan originators were ‘originate and sell’ institutions. These are institutions that specialized in creating mortgages and selling them to other entities so that these loans could be privately securitized, particularly subprime loans. We seek to identify an analogous subset of such originators in our HMDA data. To do so, for 2004, 2005 and 2006, we select the top 15 originators who originated the most loans destined for home purchases and that were sold to be privately securitized. We then verify that at least 30% of their total loans originated were sold in such a fashion, and we select originators who satisfy both criteria. This leaves us with a total of 18 institutions, covering 33%, 78% and 87% of loans originated for private securitization in 2004, 2005 and 2006 respectively. We proceed to identify 17 of these institutions via their ‘RespondentID’ codes\textsuperscript{15} and investigate where their

\textsuperscript{14}They use distance from the equator as an instrumental variable for social infrastructure of a country.

\textsuperscript{15}One, covering 1% of loans in 2005, was impossible to identify.
3.3. EMPIRICAL STRATEGY AND RESULTS

headquarters are located and the year they were founded.

Our IV strategy assumes that if an MSA is closer to where the headquarters of these originators is located, than it is easier for the originators to participate in the housing market of that MSA. We propose that closer MSAs make it easier for headquarters to monitor branches, that headquarters will more likely have greater knowledge, including legal knowledge, of closer housing markets, headquarters will have incurred the fixed cost of complying with state regulations, etc. At the same time, the validity of our instrument requires that these headquarters are not based in or near locations where house prices were expected to grow more in our period of 2004 to 2006. For this reason, we exclude 3 originators who were founded post-1996, leaving us with the originators found in Table 3.16 in Appendix B, all of which were founded at least 8 years before our 2004, which we conjecture is enough time to satisfy the exclusion restriction.

In addition, as some of these institutions are very large, and some are quite small, we wish to give appropriate weights reflecting their size. For this, we use the amount of loans originated\textsuperscript{16} in 2003 as weights. Finally, we opt to select the two closest originators, as a proxy for the level of competition that a MSA encountered between originators. We give more weight to the distance of the larger one of the two and this results in our instrument, DistW.

We acknowledge the limitations of this instrument. Of particular concern is that our model results imply that no-screening takes place with securitization, which would suggest that a deeper knowledge of local markets would not matter in equilibrium, weakening our instrument’s power. In addition, it is possible that as loan originators have less knowledge of more distant MSAs, this could lead to moral hazard problems and more loans being securitized the further away the MSA is from the headquarters, as opposed to being held. Thus, if the heterogeneity of securitization from MSA to MSA has a larger effect than the effect of expansion of branches, then our instrument may have had the opposite effect on securitization. A further discussion of these issues may be found in Appendix A.

To implement our IV, we use both DistW and its interaction, DistW × NonRec, as instruments. The former is used to instrument Sec and the latter to instrument Sec × NonRec. We then proceed to estimate a similar same equation to our baseline results:

\[ HPrice_{i,t} = \beta_1 Sec_{i,t} + \beta_2 NonRec_i + \beta_3 Sec_{i,t} \times NonRec_{i,t} + D20XX_t + \gamma_i + \epsilon_{i,t} \]

We use fixed effects, using a clustered (at a MSA level), robust standard errors; as both our instruments are time invariant, to be able to use fixed effects on these regressions, we interact both our instruments with year dummies\textsuperscript{17}.

Before discussing our IV regression results, we first focus on the first stage results seen in Table 3.6, as they are not as expected. When instrumenting securitization, the coefficients for distance (interacted with our year dummies) are positive, the opposite of what we would expect; the F-statistic on excluded instruments is 8.28, which may be a sign of a weak instrument. When instrumenting for the interaction between securitization and non-recourse, however, the coefficients on both distance and its interaction are negative as expected, with an F-statistic of 15.71.

These results suggest that our instrument is not working as intended for all MSAs or, possi-

\textsuperscript{16}Which resulted in excluding one more originator, LOAN CENTER OF CALIFORNIA, as there is no available data for loan origination prior to 2004; we believe it was previously exempt from reporting to the HMDA.

\textsuperscript{17}Which assumes a time-differential effect of the instrument, another limitation of our approach.
3.3. EMPIRICAL STRATEGY AND RESULTS

bly due to some of problems we discuss above, has differential effects depending on the status of recourse law of the state; we do not currently have a working proposition for the latter. To deal with this issue, we split our sample into recourse and non-recourse states and run them separately. We acknowledge, however, that an instrument that only works in part of our sample is concerning and raises the question of whether it is truly satisfying the first stage restrictions.

\[
\begin{array}{cccc}
\text{VARIABLES} & \text{Securitization} & \text{Securitization} & \text{Securitization} \\
& \times \text{NonRecourse} & \times \text{NonRecourse} & \\
D2005 \times \text{Distance} & 0.00097^{***} & -0.00009 & 0.00098 \\
& (0.0002) & (0.0001) & (0.00025) \\
D2005 \times \text{Distance} & 0.00073^{***} & -0.00021 & 0.00076 \\
& (0.0002) & (0.0002) & (0.00026) \\
D2005 \times \text{Distance} \times \text{NonRecourse} & -0.0027^{***} & -0.0018^{***} & \\
& (0.0005) & (0.0003) & \\
D2006 \times \text{Distance} \times \text{NonRecourse} & -0.0022^{***} & -0.0018^{***} & \\
& (0.0005) & (0.0003) & \\
\end{array}
\]

| Observations | 1,076 | 1,076 | 792 | 284 |
| Number of MSA | 359 | 359 | 264 | 95 |
| Dataset | MSA | MSA | MSA | MSA |
| Method | FE | FE | FE | FE |
| States | All | All | Recourse | Non-Recourse |
| F-Stat on excluded instruments | 8.28 | 15.71 | 7.73 | 7.69 |

Table 3.6: First stage regressions for distance as an instrument

The first stage coefficients in recourse states remain positive and significant and have a low F-statistic, at 7.7, meaning that the possibility of a weak instrument problem remains, and the positive coefficient has the same implications as before. However, for non-recourse states, our first stage has instruments with the correct signs, in addition to being significant. The F-statistic is still below 10, also at 7.7, which means that our results may be biased.

In addition, in Table 3.13 in Appendix B, we also present the results of the reduced form estimates for our instruments. We find similar results as the first stage regressions, with evidence that our instrument is weak, and is inconsistent and/or has differentiated effects for recourse states, but works as expected for non-recourse states.

We present the results of our instrumented regressions for all three cases, with all states and with just recourse or non-recourse states, in Table 3.7.

Our results for the whole sample are consistent with our previous results, as the interaction effect is positive and significant. The results for when we split our sample are also consistent, as securitization in recourse states has no significant effect on house prices (although the negative coefficient is unexpected), whereas the effect is positive and significant in non-recourse states. Furthermore, the coefficient is larger than in the estimates from our baseline regressions, which we take as suggestive evidence that our mechanism is playing a role in increasing house prices in these states.

We also regress for our whole sample including our subprime measure as a control. We do this without subprime being instrumented, and find that although not significant, our coefficient for the interaction effect remains positive and in line with our previous result. Overall, given the first stage issues we report, we take these results as further, but not conclusive evi-

\footnote{18}{If there is a differential effect of our instrument on securitization due to recourse status, this would allow it to work as intended}

\footnote{19}{Given the potential endogeneity between house prices and subprime mortgages, however, this estimation may not be consistent.}
3.3. EMPIRICAL STRATEGY AND RESULTS

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1) HPrice</th>
<th>(2) HPrice</th>
<th>(3) HPrice</th>
<th>(4) HPrice</th>
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<tr>
<td>Securitization</td>
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<td>-1.201</td>
<td>4.072***</td>
<td>-2.161</td>
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<tr>
<td></td>
<td>(1.683)</td>
<td>(1.713)</td>
<td>(1.093)</td>
<td>(3.403)</td>
</tr>
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<td>Securitization × NonRecourse</td>
<td>3.527**</td>
<td>3.394</td>
<td></td>
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<tr>
<td></td>
<td>(1.725)</td>
<td>(2.070)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
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<td>792</td>
<td>284</td>
<td>1,076</td>
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<tr>
<td>Number of MSA</td>
<td>359</td>
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<td>95</td>
<td>359</td>
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<td>Dataset</td>
<td>MSA</td>
<td>MSA</td>
<td>MSA</td>
<td>MSA</td>
</tr>
<tr>
<td>Method</td>
<td>FE</td>
<td>FE</td>
<td>FE</td>
<td>FE</td>
</tr>
<tr>
<td>States</td>
<td>All</td>
<td>Recourse</td>
<td>Non-Recourse</td>
<td>All+Subprime</td>
</tr>
</tbody>
</table>

Robust, clustered standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1. Regressions include controls and year dummies. Annual data from 2004 to 2006.

Table 3.7: Instrumental variable regressions

3.3.3 Bust period

Our model also predicts that the cumulative effect of securitization in a non-recourse state should lead to greater falls in house prices during the bust period.

To explore this prediction, we create a new variable, PastSec, which is the average securitization done from 2004 to 2006 in each state. From this variable, we can derive the interaction effect between this measure and the non-recourse status, PastSec × NonRec. With this, we can test our prediction by regressing house prices on these variables:

\[
HPrice_{it} = \beta_1 D20XX_i \ast PastSec_i + \beta_2 D20XX \ast NonRec_i \\
+ \beta_3 D20XX \ast PastSec \times NonRec_i + \gamma_i + D20XX_i + \nu_i
\]

where \(\gamma_i\) is the same set of controls as in the boom period and \(D20XX_i\) are year dummies. That is, to be able to include state fixed effects, we interact our static variables with year dummies; this assumes a time-differentiated effect of these variables during the bust.

As it is less clear when the bust period ended, we vary the end date of our regressions, using as end 2009/2010, with the first period being 2007. We also include in some of our main regressions PastSubprime, which is the average percentage of subprime mortgages in new originations during 2004-2006. When included, PastSubprime is also time interacted. Our results can be seen in table 3.14 in Appendix B.

Our results provide evidence for our mechanism, but with important caveats. In the absence of subprime mortgages, the coefficient for our interaction effect is negative and significant for two years, 2008 and 2009, suggesting that our mechanism is explaining some of the drop in house prices, more than doubling the effects of securitization stemming from the boom period. Securitization itself seems to have also caused significant drops in house prices, especially in 2009 and 2010, conforming to the results found in the literature.

However, when subprime mortgages are included in these regressions, we find that our interaction coefficients, whilst still negative and of a similar order of magnitude, are no longer
3.3. EMPIRICAL STRATEGY AND RESULTS

statistically significant. Given the importance the literature has found for subprime mortgages in explaining the housing market in the period, we feel that the results from the specification that includes our measure of subprime mortgages should receive significant weight.

Securitization still has significant and negative coefficients, as expected, and we find that subprime mortgages, although not reported here, also adversely affected house prices in the period, with negative and statistically significant coefficients 20.

Unlike our boom period regressions, as both PastSec and PastSubprime are measured prior to when we run our regressions, it less likely that there will be reverse causality with respect to house prices. Endogeneity issues may still exist if securitization was higher in states where house prices were expected to grow the most during 2004-2006 and these states experienced the biggest falls in prices afterwards. As a simple test of this, we calculate the correlation between house price growth in states from 2004-2006 with 2007-2009. We find a correlation of -0.56 which we interpret as being sufficiently high for this to remain a concern, meaning that our bust period results should be treated with a degree of caution.

As such, we find further, albeit weaker evidence for our mechanism’s effects on house prices during the bust period, and evidence that securitization and subprime mortgages created in the boom period played an important role in determining house prices during the bust.

3.3.3.1 Defaults

Our model also predicts that defaults are increased in a bust when there are higher levels of securitization in non-recourse states during a boom.

For defaults, we use MDefault, the percent of mortgage debt balance that is 90+ days delinquent in each state, for all mortgages. This may be a too broad measure of defaults, however, as our model focuses on defaults of home-purchase mortgages created during the boom. As before, we use the start date of 2007 and use as end dates of 2009/2010. We then regress defaults on past securitization and its interaction effect:

\[
M_{\text{Default}}_{it} = \beta_1 D_{20XX} \times \text{PastSec}_i + \beta_2 D_{20XX} \times \text{Past} \times \text{SecNonRec}_i + \gamma_i + D_{20XXt} + \pi_{it}
\]

As before, we interact our static variables with year dummies to allow for fixed effects. Our results are found in table 3.15 in Appendix B.

In none of our regressions are the coefficients of our interaction significant, nor are they consistently positive, from which we conclude that there is little evidence that our mechanism led to increased defaults during the bust, albeit given the limitation of using a broad measure of defaults. We do find evidence that securitization from the boom period increased defaults in the bust period, a result that is robust, as the literature has previously found. Similarly, subprime mortgages from the boom period, the results of which we do not report here, also led to increased defaults during the subsequent bust.

\[\text{For every extra 1% of (average) mortgages that were subprime in a given state during the boom period, there was a corresponding fall in house prices of around 0.5% for every year from 2008-2010, in 2004 prices.}\]
3.4 Conclusion

This chapter seeks to add to the empirical literature exploring the causes behind the boom and bust in house prices observed in the US during the 2000s. In the previous chapter, we build a model that explores the effects on the housing market, particularly on house prices, that securitization and non-recourse loans might have. Our model predicts that the interaction between these two factors should lead to increased house prices during a demand boom, compared to markets absent at least one of the two factors.

We find some empirical evidence for this prediction. We regress house prices, on an MSA and state level from 2004 to 2006, on securitization and its interaction with non-recourse status of states. We find that the positive effect of securitization on house prices in the US was nearly double in non-recourse states, compared to recourse states. This effect is such that for every 1 p.p. of securitized mortgage loans is associated with an increase in house prices by 1% in recourse states, and 2% in non-recourse states. The mechanism can potentially explain around 75% of the differences in growth of house prices between recourse and non-recourse states from 2004-2006.

To control for potential endogeneity issues between house prices and securitization, we use as an IV the weighted distance between an MSA and the two closest headquarters of ‘originate and securitize’ mortgage institutions. Our results largely hold when doing so, particularly for non-recourse states, but note that the first stage F-statistic low and our first stage coefficients may be inconsistent or possibly have differentiated effects for recourse states. As such, we take this as more evidence, but not conclusive evidence, that this mechanism influenced house prices at the time.

Our model also makes predictions for the bust period, although, due to the static nature of the bust period, is less suited for doing so. It predicts that the interaction effect between securitization and non-recourse status from the boom period should lead to greater falls in house prices and more defaults during a subsequent bust. We find weak evidence for our house price predictions during the bust period, and no evidence that our mechanism affected defaults during the bust.

Finally, we also find evidence that higher levels of securitization during the boom led to increased house prices during the boom itself, and lead to greater falls in house prices and increased defaults during the subsequent bust, as is consistent with what the literature has previously found.

We conclude by noting that there is currently an ongoing debate about whether mortgage originators should be forced to have a ‘skin in the game’, that is, to hold on to at least some percentage of any loan they originate. Given the results discussed above, we believe that it would be wise to make these rules binding, particularly in jurisdictions where mortgage loans are non-recourse, or, as an alternative, proportional to the loan-to-value ratio of a mortgage.

Although securitization levels have currently fallen, it or similar financial innovations which allow mortgage originators to sell their loans could easily reappear in the near future. In that sense, the relatively recent changes\(^\text{21}\) by US regulators that relaxed the Dodd-Frank laws may have been counterproductive. The law originally required originators to have a substantial ‘skin in the game’ and these changes have instead exempted the vast majority of mortgages in the US from such requirements.

\(^{21}\)As reported in “Banks Again Avoid Having Any Skin in the Game”, New York Times, 23rd of October, 2014.
Taking note that our model results and, to a lesser extent, our empirical results are independent of the existence of subprime mortgages, these changes might be re-laying the foundations for future problems, even if subprime mortgages are more tightly controlled and regulated, or even non-existent\textsuperscript{22}.

\textsuperscript{22}Similar concerns may also lie with Brazil, due to the relatively new innovation of similarly non-recourse ‘alienacao fiduciaria’ mortgages.
Appendices to the Chapter

3.A Appendix A - Empirics discussion

3.A.1 Loan-to-value ratio

As we discuss in the Appendix to the previous chapter, house price growth may depend on the down payment set on mortgages; higher down-payments would lead to lower house prices, assuming speculators take on loans, which our model would never predict happens. Thus, as a precaution, we would wish to include the LTV as an explanatory variable in our regressions as a robustness check, although there would also be concerns about reverse-causality and endogeneity if we had such a measure (for much the same reasons as there are for securitization and subprime loans). At this point, we do not have data that would allow us to include a measure of the average LTV for states or MSAs in the time period in which we do our regressions. As a consequence, we acknowledge that our regressions may suffer from omitted variable bias.

3.A.2 Subprime variable

To obtain our measure of subprime mortgages, we follow the methodology of Mayer and Pence (2008). They find that classifying loans with interest rate spread above 3% in the HMDA dataset provides a measure of subprime that is largely similar to alternative measures, such as using the HUD classification. Our main measure of subprime thus consists of the percentage of all loans destined for house purchases successfully originated which have that interest rate spread, to differentiate it from those originated from subprime specialists, which may otherwise lead to issues with our IV strategy.

3.A.3 Robustness checks

All robustness checks use the same basic regression structure as our main regressions, but vary in the following ways: the start (2005) and end (2007) dates of our regressions are changed (state only); we use an alternate measure (Case-Shiller) of house prices (MSA only); we use an alternate definition (NonRecKM) of recourse (state only); we include a measure of the percentage of subprime loans (Subprime) originated for house purchases (both state and MSA); we include a measure of state (Int) interest rates on mortgage loans (state only); we omit California from our sample (both state and MSA); we run our regressions only in coastal or non-coastal states (state only); we run our regressions only in Western or non-Western states (state only); we extend the range of our regressions to start from 1991 until 2004, for which we assume, incorrectly, that securitization is zero prior 2004 (state only); we restrict our sample to the MSAs that were in the top 50% of MSAs with highest percentage of securitization (MSA only); instead of Sec and Sec × NonRec, we use a dummy for the MSAs with the top 50% highest percentages of securitization, also including the related interaction effect with non-recourse (TopSec and TopSec × NonRec), both interacted with year dummies (MSA only); we include our measure of securitization done by government sponsored agencies GSE and run with and without its interaction with non-recourse GSENonRec (state only); we include
a measure of the loan-to-income ratio $LTI$ (state only); and we use all our additional controls, GSE and interaction, subprime, interest rates and loan-to-income $All$ (state only).

We include these for a number of reasons. State interest rates are used as a proxy for financing conditions, but omitted in the main regressions due to concerns about endogeneity. Similarly, including subprime loans is due to concerns that our measure of securitization may be just capturing the effects of subprime loans instead of securitization, with similar concerns about endogeneity. As California is a non-recourse state that experienced particularly high increases in house prices and high levels of securitization, omitting it serves to verify that we are not just capturing the effects of that state. Similarly, by using just coastal or Western states, we try to guarantee that we are not just capturing the effects of securitization in coastal vs inland or Western vs non-Western states. As GSE were responsible for a large amount of securitization at the period, we aim to make sure that we are not capturing their effects with private securitization with and without the interaction with non-recourse status.

### 3.A.3.1 IV discussion

One way to test the validity of our instrument is to compare the percentage of securitization done in each city by each originator. If we have correctly identified our set of originators as ‘originate and securitize’ types, then the variation of securitization done in each city should change little. To test for this, for every year and for every originator, we calculate the average securitization level done for each city. We then calculate the standard deviation of each subset for each originator in each year, finding that the average standard deviations for each year are as follows: in 2004, 15.8%; in 2005, 15.5% and in 2006, 21.3%, relatively high numbers.

Furthermore, the evidence for each originator is mixed, and it seems that some, but not all of our originators are satisfying our criteria. Both ‘LOAN CENTER OF CALIFORNIA’ and ‘CHAPEL MORTGAGE’ originators have low standard deviation of these frequencies, at less than 1%, and ‘COUNTRYWIDE HOME LOANS’, ‘AEGIS MORTGAGE’ and ‘FREMONT INVESTMENT & LOAN’ have persistently low standard deviations, at less than 15%. On the other hand, ‘EAGLE HOME MORTGAGE’, ‘DELTA FUNDING’ and ‘FIRST RESIDENTIAL MORTGAGE’ all have standard deviations of above 30% in at least one year, suggesting that they may not be satisfying our criteria. These results suggest that further refinement of originators being used as instruments may be required, and it not yet clear whether the issues we raise in the main text are being dealt with appropriately.
### 3.B. APPENDIX B - TABLES

#### 3.B Appendix B - Tables

#### 3.B.1 Additional tables for boom period

<table>
<thead>
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<th>(2)</th>
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<td>1.025***</td>
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<td>(0.0619)</td>
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<td>0.524**</td>
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<td>Method</td>
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Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Regressions include controls and year dummies. Annual data from 2004 to 2006.

Table 3.8: Boom period, main regression control results

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<td>HPrice</td>
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<td>HPrice</td>
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<td>0.110</td>
<td>0.851***</td>
<td>1.117**</td>
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<td></td>
<td>(0.752)</td>
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<td>(0.491)</td>
<td>(0.243)</td>
<td>(0.223)</td>
<td>(0.492)</td>
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Robust, clustered standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1 Regressions include controls and year dummies. Annual data from 2004 to 2006. NonRecourseKM uses an alternate recourse classification.

Table 3.9: Boom period, additional robustness checks - 1
## 3.B. APPENDIX B - TABLES

### Table 3.10: Boom period, additional robustness checks - 2

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Robust clustered standard errors in parentheses, *** p < 0.01, ** p < 0.05, * p < 0.1, Regressions include controls and year dummies. Annual data from 2004 to 2006. 1991-2006 assumes securitization is zero prior to 2004.

### Table 3.11: Boom period, additional robustness checks - 3

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<td>Securitization × NonRecourse</td>
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<td>2.429***</td>
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<td>(0.423)</td>
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<td>D2005 × TopSecuritization</td>
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Robust clustered standard errors in parentheses, *** p < 0.01, ** p < 0.05, * p < 0.1, Regressions include controls and year dummies. Annual data from 2004 to 2006.
### 3.B. APPENDIX B - TABLES

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<td>D2006 × TopSecuritization × NonRecourse</td>
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Robust, clustered standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1. Regressions include controls and year dummies. Annual data from 2004-2006.

Table 3.12: Non-Western states, robustness checks

#### 3.B.2 Additional tables for instrumental variable regressions

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<td>(0.0012)</td>
<td>(0.0011)</td>
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<td>-0.0076**</td>
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<td></td>
<td>(0.0024)</td>
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Robust, clustered standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1. Regressions include controls and year dummies as instruments. Annual data from 2004 to 2006.

Table 3.13: Reduced form regressions for instrument
### 3.B.3 Bust period regressions

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<td>D2009 × PastSecuritization</td>
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<td>-1.654**</td>
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<td>D2010 × PastSecuritization</td>
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Robust, clustered standard errors in parentheses, *** p < 0.01, ** p < 0.05, * p < 0.1. Regressions include controls and year dummies. Annual data from 2007 to 2009/2010.

**Table 3.14:** Bust period, house price regressions

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Robust, clustered standard errors in parentheses, *** p < 0.01, ** p < 0.05, * p < 0.1. Regressions include controls and year dummies. Annual data from 2007 to 2009/2010.

**Table 3.15:** Bust period, default regressions
3.C. APPENDIX C - OTHER FIGURES

3.B.4 Other tables

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<td>NOVASTAR MORTGAGE</td>
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<tr>
<td>FIRST HORIZON HOME LOAN</td>
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<td>FIRST RESIDENTIAL MORTGAGE</td>
<td>1995</td>
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<tr>
<td>GATEWAY FUNDING DIVERSIFIED</td>
<td>1994</td>
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<td>AEGIS MORTGAGE</td>
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<td>Bellevue, WA</td>
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Table 3.16: ‘Originate and securitize’ institutions

3.C Appendix C - Other figures

Figure 3.2: House Prices in recourse and non-recourse states, from 1975
Chapter 4

Dynamics of Regulation of Strategically Complex Financial Products *

Abstract

Banks can use complexity to make new financial products costly to screen by regulators. Bad financial products created by banks can lead to moral hazard issues, as banks are bailed out in case of negative shocks. Thus, regulators should incentivise banks so that they do not ‘abuse’ complexity by making new bad products complex. We find the optimal regulation regime in a framework without transfers and where regulators can make binding commitments, with low discount factors. Initially, regulators do not screen and banks do not make use of complexity. Once a bank has received enough positive shocks, in the form of good financial products, regulators then stop screening and allow banks to make use of bad, complex products as a reward for their previous actions. We discuss why this mechanism may not hold when regulators cannot commit, and discuss the implications of a lack of commitment.

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4.1 Introduction

As was seen in the recent financial crisis and in prior crises, banks and other financial institutions are likely to be bailed out in situations where depositors could lose their savings, when intra-banking capital flows are at risk of stopping or when ‘systemic’ institutions may collapse. This creates a moral hazard problem between bankers/banks and regulators/governments, as banks and bankers have incentives to take risky bets that result in a large payoff to themselves should they work, but upon failure, impose large costs on regulators due to bailouts, with little cost to the banks.

To avoid such costly consequences, financial regulators should want to prevent banks from taking such risks, in particular, from issuing, holding and/or buying risky financial products.

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4.1. INTRODUCTION

As these regulations stop banks from enjoying the moral hazard rents arising from limited liability and bailouts, banks might want to avoid the oversight of financial regulatory agencies. Given these incentives, how should regulatory agencies design their monitoring procedures over time, taking into account that banks are trying to dodge oversight?

In this paper, we consider the problem of optimal dynamic regulation of a bank, large enough to affect risk at a systemic level, which might try to avoid financial regulation by making its financial products complex to avoid regulatory oversight. In our model, each period of time starts with banks devising an idea for a new financial product. This new idea could be thought of as a new form of aggregating and distributing risk, such as a new securitization method, or a new way of making payoffs contingent on certain outcomes, such as a new derivative. In the model and from the perspective of regulators, a new product idea will either be a good idea involving no risk, or a bad idea, akin to a risky bet with the potential to result in a bailout and/or large social welfare losses. For simplicity, we assume that banks will hold on to new products they create, thus exposing themselves directly to the product risk; we discuss the implications of having more banks below.

Once banks have developed their new financial product, they know what quality and complexity the product has; for simplicity, we assume all good products are inherently complex and all bad products are inherently simple. In practice, this inherent complexity might be a consequence of the number of states/outcomes on which a risky derivative’s outcome is conditional, or, as another example, as the consequence of having a securitization process be applied multiple times to a group of assets\(^1\), such as with a CDO squared. Crucially, added complexity brings about costs to regulatory oversight, and can be manipulated by banks, which can turn simple assets into complex assets - for instance, by securitizing a group of highly correlated assets, increasing complexity without significantly reducing risk.

These assumptions seem to be realistic features of the problem that financial regulators face. Firstly, regulatory authorities make frequent mention in official documents of how resources must be used wisely and how costly it is to monitor banks. The Office of the Comptroller of the Currency (OCC), concerning the frequency of on site examinations of banks\(^2\), states that ‘Before increasing the frequency of examinations, supervisory offices should consider how OCC resources can be used most efficiently(...)\(^3\)’. Eisenbach, et al. (2015), a report by the New York Federal Reserve describing the regulatory activity of the Federal Reserve, stresses that, in the context of supervisory activities, ‘Resources are often constrained in the sense that there is often more work that could be done than there is time or staff to do it.’ Finally, there is an empirical literature dealing the costs regulators face when regulating banks, such as Hirtle and Lopez (1999) or Eisenbach, Lucca and Townsend (2016), which also takes the costs regulators face as substantial.

Secondly, banks seem to be able to manipulate the complexity of their financial products and at least anecdotal evidence of such behaviour exists. For example, a former trader at Salomon Brothers, when discussing the bank in the 80s, claimed that ‘Many “new products”

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\(^1\)It seem plausible that many good assets are inherently complex, both in principle and in practice. For example, securitizing a sufficiently large number of uncorrelated mortgages can create high quality tranches that are of very low risk, but understanding how uncorrelated these mortgages are requires a great deal of time and effort, particularly when the loans backing them may in the thousands.

\(^2\)Bank Supervision Process - Comptrollers Handbook

\(^3\)Similarly, the Prudential Regulatory Authority (PRA), concerning the amount of information that banks should send when requested, states that ‘It is essential, however, that supervisors are not overwhelmed by the amount of information that they have to analyze.’, in Section 188, The Prudential Regulation Authority’s approach to banking supervision March 2016.
invented by Salomon Brothers were outside the rules of the regulatory game (...). To attract new investors and to dodge new regulations, the market became ever more arcane and complex.4 There is also the evidence from the related literature which deals with complexity in retail financial markets, wherein complexity may be used to mislead or defraud consumers, by making it hard for them to fully comprehend the nature (i.e., risks and potential payoffs) of the product they might be purchasing, resulting in product mispricing; see for example Agnew and Szykman (2005), Carlin (2009) or Celerier and Vallee (2013). Thus, evidence that banks might deliberately structure products to be complex, with the intent of misleading other agents and/or bending/breaking the rules, exists.

We study this set-up when the bank and the regulator interact over time repeatedly, and the regulator can design dynamic regulatory procedures to provide incentives for banks to not hide bad assets by making them complex. In doing so, however, the regulator faces limitations in designing its optimal regulation. The key assumption we make is that it cannot design mechanisms with fines and bonuses for banks as rewards for keeping their bad financial products simple. The reason for this assumption is that such a mechanism would require punishments, such as a fine, to take place when banks have been caught out creating bad and complex financial products.

Banks are likely to be caught out, however, when there is a crisis happening, meaning that, instead of fines, regulators will be forced into bailouts. Given the need for these bailouts and their size, regulators would then need to provide large bonuses/transfers to banks during good times to create an effective incentive system, which would be politically dubious given the high payments that would be required5.

Our paper proceeds to firstly find the optimal mechanism for regulators when they can make binding commitments. We find that in such cases, the resulting mechanism will be such that banks either choose full complexity or no complexity at any moment of time. Regulators begin by promising and delivering relatively low payoff values to banks, these values being the sole state variable in our model, close to the Nash Equilibrium values. During this initial period, banks will choose not to use complexity and regulators will not screen products, despite the latter creating incentives for banks to deviate.

This outcome is made possible by the fact that regulators promise to punish the banks if they opt to make use of complexity, and by not deviating, banks will receive positive shocks, mainly in the form of bad products, that lead to higher promised continuation values. Eventually, both bad products and good products deliver the same, positive continuation values, and these higher continuation values are delivered to the banks via a ‘reward’ phase for banks, wherein regulators stop screening altogether and banks are free to use bad, complex products with no punishment. As such, this results in a path for bank payoffs resembling a random walk with a positive drift, followed by a set increase every period until the ‘reward’ phase is met and payoffs become completely static.

However, it is likely that regulators cannot make such binding commitments. In particular, we conjecture and discuss that the regulator cannot commit to not screening when it is in the ‘reward’ phase of this mechanism, as once we relax the commitment constraint, we surmise that a completely lax screening regime and absence of punishment for using bad, complex products would be infeasible, as regulators would want to deviate towards screening at this
point. So, in the absence of commitment, this would require regulators to have more a moder-
ate ‘reward’ phase instead. Thus, we conjecture, but do not prove, that the optimal dynamic
regulatory mechanism without commitment is similar to the above, but with more moderate
‘punishment’ and ‘reward’ phases, with neither regulators, nor banks choosing extremum val-
ues for screening and complexity. We also argue that similar results happen when regulators
are sufficiently more patient than banks. Our results thus highlight the constraints placed on
financial regulators, and how difficult it is to eliminate financial crises.

Our paper is related to the literature on optimal banking regulation in the context of moral
hazard, such as in Prescott (2004) and Blum (2008), although these and other papers mainly
focus on questions related to bank capital, and also to the larger, mainly empirical literature on
banking regulations, such as Barth, Caprio and Levine (2004). It is also related to the literature
on optimal bank opacity, such as Dang, Gorton, Holmstrom, and Ordonez (2014), which posits
that banks use opacity in their balance sheet loans to avoid issues from information external-
ities from investment projects and liquidity concerns. Our approach in finding the optimal
regulatory mechanism stems from the literature on dynamic games with one-sided private
information, such as in Abreu, Pearce and Stacchetti (1990), and Li, Matouschek and Powell
(2015). To the best of our knowledge, this is the first time this approach is taken in the bank
regulation literature.

This paper is organized as follows. In Section 4.2 we state the model set-up and equilibrium
concepts, as well as finding a benchmark, static result. In Section 4.3 we set up the regulator’s
optimal contract problem and discuss and delineate some useful results. In Section 4.4 we
execute the optimization program via a Lagrangian, find the optimal regulation contract for
when the regulator can make binding commitments and then discuss the optimal solution we
should find when the regulator cannot make such commitments. In Section 4.5 we discuss
some possible model extensions and in Section 4.6 we conclude.

4.2 Model

4.2.1 Basic set-up

In the beginning of each period \( t = 1, 2, \ldots \), a bank creates a single, new financial product.
This product can be either one of two types: good products (G) or bad products (B). Banks
will hold-on to the products they create, and good products will pay \( p_t = M \) with probability
1 to banks that create them, with a guaranteed payoff \( r_t \) normalized to zero to regulators. Bad
financial products, on the other hand, suffer idiosyncratic shocks at the end of a period. With
probability \( \theta \), they produce an identical positive outcome of \( p_t = M \) and \( r_t = 0 \) to banks and
regulators. But with probability \( 1 - \theta \), the product suffers a negative shock, and banks, due
to their systemic nature, are bailed out by regulators. Consequently, banks receive \( p_t = 0 \) and
regulators have a payoff of \( r_t = -R \). Regulators will thus wish to impede banks from using B.

How are financial products created? As with the literature in innovation and technologi-
growth, here innovation is modeled in a reduced form and we abstract from using more
detailed mechanisms. In the beginning of a period, a bank will receive a financial product
exogenously, that will be a good product with probability \( \mu \) and be a bad one with probability

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6We surmise that this form of provision of incentives is likely to create cycles of complex bad financial products,
and analogous cycles of strong/weak regulatory oversight of these products.
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1 - \( \mu \). For tractability, we assume the draw of quality of the financial product to be iid over time and that all products can only be used once by banks\(^7\).

Once these financial products are created, banks may decide to make bad products ‘complex’ or ‘simple’. We assume that all new good products are complex\(^8\) for tractability, and that all new bad products are initially simple, or can be made complex at a cost \( D \). Simple products are easily inspected and understood by regulators, and they can distinguish a simple bad product from a good one costlessly. Complex products are costly for regulators to inspect and we model this by assuming regulators must pay a cost \( C \) if they wish to tell whether a new complex product is bad or good.

In the context of mortgage backed securities, the process of securitization can used to bundle together a small set of mortgages with similar characteristics and clear diversification, thus creating a simple product, or, by using various different methods\(^9\), bundle them together in ways that the regulator might only understand after considerable effort analyzing the product (a complex product). In practice, the timing of when this analysis/screening happens will depend on how large a product is, relative to a bank’s balance sheet, and how quickly it is growing. In our model and for simplicity, we assume that a new complex product can be screened within the same period of time as it is created. We can justify this by assuming that the product at this point will have grown enough to be a worry, but is still small enough that if prohibited, it will not result in significant payoffs to the bank or the regulator.

Note that we allow regulators to veto products from being used only if they have been screened. This can be rationalized as a requirement that regulators must clearly justify their veto to a bank, otherwise such vetoes might be contested in the courts of law. It may also serve as a reduced form for the fact that good products can create positive externalities for the society as a whole, and regulators do not wish to impede good products from being created indiscriminately\(^10\). That is, when complex good products are relatively frequent enough relative to complex bad products, and the social benefits are high enough, regulators would not wish to indiscriminately veto complex products.

4.2.2 Timing

Each period \( t \) has the following timing:

1. A public random variable \( x \in [0, 1] \) is realized.

2. Nature determines whether a new product is good (with probability \( \mu \)) or bad (with probability \( 1 - \mu \)), and the bank receive this new product.

3. A bank with a good product (\( G \)) has no actions they can take. A bank with a bad product (\( B \)) chooses the probability \( \alpha \) at which it will make B complex at a cost \( D \), such that at probability \( (1 - \alpha) \) the product is kept simple at no cost.

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\(^7\)An example of the latter could be a bank finding a pool of mortgages that is not diversified, and developing a complex securitization product from that pool.

\(^8\)This is a reduced form for having some new good products be inherently simple and some be inherently and irreducibly complex.

\(^9\)Such as using very large and opaque sets of mortgages or applying the securitization process on top of already securitized products, such as ‘CDO squared’.

\(^10\)Although we do not explicitly incorporate the latter into the regulators’ payoffs and this may be an area for future research.
4. Any new complex product falls under the supervision of the regulator before payoffs are realized, and the regulator chooses whether to screen the product at cost $C$ with probability $\gamma$ to not screen it with probability $1 - \gamma$.

5. The regulator then forbids/prohibits the bank from using any simple bad product or any screened, complex bad product, and both the regulator and the bank receive payoff 0, in addition to any costs they may have incurred. If the product is not screened or is screened and is determined to be good, then the bank is allowed to use it with the associated payoffs described above.

Assuming the existence of the public randomization device $x$ considerably simplifies the analysis of such games, and is frequently used in the theoretical and applied literature on games with imperfect public monitoring. In particular, it allows us to assume that the set of possible equilibrium payoffs is convex\(^{11}\).

We also assume that $\gamma$, the probability of screening, is directly publicly observable by the bank and the regulator. The main motivation for this is to maintain simplicity in our model and results; in particular, it makes dealing with some of the constraints pertaining to the regulator in the optimization problem easier, and it should be possible to relax this assumption. We also argue that this is likely not an unrealistic assumption to make: most big banks in the US have existed for many decades, and thus have ample experience interacting with regulators, whose institutional setting are likely to change slowly. Thus banks can infer, based on their experience and public information, what level of scrutiny regulators currently have.

Per the literature, we assume that both the bank and the regulator discount payoffs at rate $\delta \in (0, 1)$. We normalize present discounted payoffs by multiplying them by $1 - \delta$. With this, the present discounted value payoff of the regulator is

$$
\phi = (1 - \delta) E[\sum_{t=0}^{\infty} \delta^t (r_t - C \gamma_t)]
$$

while the present discounted value payoff of the bank is

$$
v = (1 - \delta) E[\sum_{t=0}^{\infty} \delta^t (p_t - D \alpha_t)]
$$

Finally, we assume that the parameters of the model satisfy the following:

**Assumption 1**

(i) $(1 - \mu)(1 - \theta) R > C$

(ii) $\theta M > D$

We use these two assumptions so that our analysis focuses on the more economically interesting cases. The first assumption requires that the costs of screening a complex product is not higher than the expected loss to regulators, if banks always opted to make their products complicated. Otherwise, the regulator would simply choose to never screen a product. The second assumption makes the expected profits in making a bad financial product complex higher than the cost of making it complex. Much like our previous assumption, we would otherwise have that banks would never choose to make their products complex, and the regulatory problem in question would not exist.

\(^{11}\)See also Abreu, Pearce and Stacchetti (1990) and p. 41 in Mailath and Samuelson (2006)
4.2.3 Equilibrium concept

To solve this game, we consider a Perfect Public Bayesian Equilibrium, which is a subset of Perfect Bayesian Equilibria where strategies are conditional only on public information. We will refer to these as PPEs, or sometimes, when it is not ambiguous to do so, as equilibria. First let $c_t \in \{0, 1\}$ be a dummy indicating complexity choice of a bank in period $t$, and $s_t \in \{0, G, B\}$ be the outcome of the screening process by the regulator (where $s_t = 0$ denotes no screening). A public history of this game can be summarized as $h_t = (c_1, \gamma_1, s_1, p_1, \ldots, c_{t-1}, \gamma_{t-1}, s_{t-1}, p_{t-1})$, where $p_t \in \{\emptyset, \{0, M\}\}$ denotes the performance of the product, with returns of $\{0, M\}$ if the product is allowed, and $p_t = \emptyset$ if the product is forbidden by the regulator. Let the set of such histories be $H_t$.

Then, we consider as a Perfect Public Bayesian Equilibrium a set of strategies $\alpha_t : H_t \rightarrow \Delta\{0, 1\}$ (that is, conditional on public history, a mixed decision of complexity) and $\gamma_t : H_t \times \{0, 1\} \rightarrow \Delta\{0, 1\}$ (that is, conditional on public history and a realization of complexity, a mixed decision of screening the product), such that all players best respond to each other. We then look at the best such equilibrium for the regulator. We need not consider private strategies as part of the set of strategies as all sequential equilibrium outcomes will be a PPE outcome too.

4.2.4 Benchmark

Before solving for the full dynamic game, it is useful to consider the benchmark case of what happens in a Perfect Bayesian Equilibrium of the one-shot stage game. As per our setup, all simple products are bad products and are identified as such costlessly. Thus, regulators will forbid the product as it is bad, and payoffs are equal to zero.

On the other hand, a regulator that has observed a complex product solves the following maximization problem:

$$\max_{\gamma} -\gamma C - (1 - \gamma)(1 - \hat{\mu})(1 - \theta)R$$

where $\hat{\mu} = \frac{\mu}{\mu + \alpha(1 - \mu)}$ is the expected probability that any complex product is a good one. Clearly, the regulator will choose to regulate with positive probability if and only if $C \leq (1 - \hat{\mu})(1 - \theta)R$ (and in particular, he is willing to mix strategies if this expression holds with equality).

A bank with a good product has no actions and thus need not be considered in this analysis. A bank with a bad financial product, however, faces the following problem:

$$\max_{\alpha} -D + (1 - \gamma)\theta M$$

As with the regulator, the bank decides to make its bad financial product complex as long as $(1 - \gamma)\theta M \geq D$ (and in particular, he’s willing to mix strategies if this condition holds with equality).

With all this in mind, it can be easily seen that this stage game has one unique Perfect Bayesian equilibrium under assumption 1. In particular, the bank chooses to make the bad product complex so that $\hat{\mu} = 1 - \frac{C}{(1 - \theta)R}$ (which implies $\alpha = \frac{\mu}{1 - \mu(1 - \theta)R - C} \in (0, 1)$ by assump-

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12Note that $p_t$, the return to banks, is informationally equivalent to $r_t$.

13For more details, see p.330 in Mailath and Samuelson (2006)
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4.3. Setting up the problem

We begin by characterizing the constraints that must be satisfied by the payoffs of the regulator and banks such that they belong to the set of values that can be attained in equilibrium. To do so, we follow Abreu, Pearce and Stacchetti (1990), turning our problem of finding the optimal equilibrium into a recursive problem. More precisely, instead of designing the optimal set of strategies to be followed by players subject to strategies being an equilibrium, we focus on the problem of choosing current actions and continuation values for both the bank and the regulator.

In doing so, we face the constraints that given their current actions and continuation values, the bank and the regulator must want to follow the strategies we assign to them (IC constraints), and secondly, that the continuation values we are assigning for the bank and the regulator must be values that they can in fact attain in equilibrium (which we designate as E constraints).

We then proceed to find the optimal actions and new continuation values for the players, subject to the IC and E constraints and a further promise keeping constraint (PKC). The PKC is used to guarantee that, in recursive problem, the continuation value promised to the banks in the previous period is delivered in the current period. We then proceed analogously for every stage of the game.

Note then that in any given period $t$, the IC constraints will only depend on the incentives the player has from $t$ onwards, and that conditional on the continuation values the player is receiving, the IC constraint is not affected by the history of play prior to $t$. The same applies for the E constraints as the repeated game from $t+1$ onwards is the same as the repeated game from $t$ onwards, so that the set of feasible continuation payoffs that can be given to players in equilibrium is the same over time, and independent of history.

Finally, the promise keeping constraint only depends on the continuation value promised to players in period $t-1$, and on the discounted present values from $t$ onwards. As a consequence, this problem of designing actions and continuation payoffs in stage $t$ only depends on the history up until stage $t$ through the last promised continuation values, which we treat as a state variable. This is what allows this methodology to turn the problem of finding the optimal equilibrium into a recursive problem. Similar approaches are taken in the literature pioneered by Spear and Srivastava (1987) and Phelan and Townsend (1991).

With this in mind, the continuation values in a PPE in stage $t$ will depend on the past promised value (which we omit, since for most of the analysis, this can be taken to be a generic
\( v, \phi \), and on the set of public outcomes/information in \( t \). In our problem, this will consist of the set \( \{c, \gamma, s, p\} \); that is, respectively, the complexity choice \( c \), the choice of intensity of regulation \( \gamma \), the outcome of the screening process \( s \) and the outcome (or no outcome, if the product is forbidden) of the product in said period \( p \). In particular, as we are focusing on solving the optimal contract problem for the regulator, we will characterize the ‘promised’ continuation values that the regulator proposes to the banks and to itself, to induce both agents to choose a specific set of actions.

Let \( v \) be the value or payoff that the bank receives in any given period and the value or payoff the bank was promised last period. Moreover, let \( w(c, \gamma, s, p) \) be the promised continuation value the bank gets after public information is set in that period. I.e., \( w(c, \gamma, s, M) \) will be the continuation value of the bank for a given \( (c, \gamma, s) \) and a positive return on the product, \( M \), and \( w(c, \gamma, s, 0) \) will be the continuation value the bank gets after a failure 0 of the product.

Our first constraint will deal with the requirement that regulators are consistent with their promises over time. This requires that:

\[
v = (1 - \delta) \{ \mu M + (1 - \mu) \alpha [-D + (1 - \gamma) \theta M] \} \\
+ \delta \{ \mu E_{c,\gamma,p}[w(c, \gamma, s, p)]|G] + (1 - \mu) E_{c,\gamma,p}[w(c, \gamma, s, p)]|B] \}
\]

That is, if the regulator promised the bank a continuation value \( w(c, \gamma, s, M) \) after output of \( M \) and \( w(c, \gamma, s, 0) \) after output of 0, having promised in the last period a continuation value of \( v \), then any optimal contract must have the equation above satisfied so that the regulator’s last promise is kept. Hence, as per the literature, we will also refer to this as the Promise Keeping Constraint (PKC).

Secondly, notice that the regulator will never want to set \( \gamma > 0 \) and to set \( (1 - \delta)[-D + (1 - \gamma) \theta M] + \delta \times \text{cont. value} < 0 + \delta \times \text{cont. value} \) as a way of inducing no complexity for bad products, that is, inducing the bank to strictly prefer no complexity with \( \gamma > 0 \). To see why, consider the counterfactual: in such a case, the regulator could provide incentives for the same choice of no complexity by reducing \( \gamma \) (and the associated cost of screening complex products) while still keeping the inequality weakly satisfied. This implies that in equilibrium, it must be that if the regulator wants to provide incentives for the bank to choose \( \alpha \in [0, 1) \), then it must be that:

\[
(1 - \delta)[-D + (1 - \gamma) \theta M] + \delta E_{c,\gamma,p}[w(1, \gamma, s, p)]|B] \leq \delta E_{c,\gamma,p}[w(0, \gamma, s, p)]|B]
\]

where the left hand side is the expected payoff of the bank with a bad product who chooses to make it complex, and the right hand side is the payoff if it chooses to keep the product simple. The above holds with equality whenever \( \gamma > 0 \). Similarly, if the regulator wants to provide incentives for \( \alpha = 1 \), it needs to be the case that:

\[
(1 - \delta)[-D + (1 - \gamma) \theta M] + \delta E_{c,\gamma,p}[w(1, \gamma, s, p)]|B] \geq \delta E_{c,\gamma,p}[w(0, \gamma, s, p)]|B]
\]

As per norm, we refer to these two constraints as forming the IC\( _B \) set, that is, the Incentive Compatibility constraint set for complexity choice by banks.

Thirdly, we need to guarantee that the regulator has incentives to comply with our de-
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signed regulatory mechanism. That is, we have to design a regulation that provides incentives for the regulator to not deviate, by having some punishment in case of deviation, as is standard in the literature. Denote by $\phi(c, \gamma, s, p)$ the continuation value of the regulator after the set of outcomes $c, \gamma, s, p$. To make sure that the regulator has incentives to implement the amount of screening $\gamma$, it needs to be weakly better off doing so rather than pursuing an alternative level of screening $\gamma \neq \gamma$. Since $\gamma$ is observable by the bank and the regulator, we assume that after $\gamma \neq \gamma$ is chosen, the bank and the regulator move, in the next stage, to the worse equilibrium for the regulator. Denote the payoff of this worse equilibrium for the regulator as $\phi$. Then, the regulator prefers to choose $\gamma$ rather than $\gamma$ as long as:

$$(1 - \gamma)(1 - \delta)\{1 - \delta\delta \phi(1, \gamma, 0, 0) + (1 - \theta)\phi(1, \gamma, 0, 0)\}
+ (1 - \gamma)\phi(1, \gamma, 0, M) + \gamma \{1 - \delta\}C + \delta\phi(1, \gamma, C, M) + \delta(1 - \delta)\phi(1, \gamma, B, \emptyset)
\geq (1 - \delta)\max\{(1 - \delta)(-R), -C\} + \delta\phi$$

To elaborate, notice that the first line is the present value payoff the regulator gets if he does not screen and the product turns out to be bad. The second line contains, firstly, the payoff the regulator gets if he does not screen and the product turns out to be good, and secondly, the payoff the regulator gets if he screens. This payoff in the first two lines needs to be higher than the payoff from deviating towards $\gamma$, which is the payoff of either screening fully or of not screening at all, plus the continuation value upon $\gamma$, which is $\phi$. We will refer to this set of constraints as the set of Incentive Compatibility constraints for the regulator, $IC_R$.

Finally, it must be that for any given feasible $v$ promised in the last period by the regulator, the set of chosen promised continuation values, $w(c, \gamma, s, M), w(c, \gamma, s, 0)$, and $\phi(c, \gamma, c, p)$ are also feasible in equilibrium. That is, the regulator cannot promise continuation values that are unfeasible to attain in equilibrium. To satisfy this restriction, let the minimum and maximum attainable values in equilibrium be $[\underline{w}, \overline{w}]$, we must then have that $w(c, \gamma, s, M), w(c, \gamma, s, 0) \in [\underline{w}, \overline{w}]$ for all $c, s$.

Denote the present value payoffs for the regulator and the bank in a given period to be $\phi, v$, and let $\mathcal{E}$ be the set of $(\phi, v)$ that can happen in equilibrium. Consider now the problem $\phi(v) = \sup\{(\phi, v) \in \mathcal{E}\}$ - that is, the problem of maximizing the regulator’s payoff subject to delivering $v$ to the bank, and subject to $(\phi, v)$ being attainable in equilibrium. The following lemma is then useful in solving this problem:

**Lemma 5** The set $\mathcal{E}$ is compact and convex. The function $\phi(v)$ is concave. Finally, any solution to the problem $\phi(v)$ with $a^*, \gamma^*$ as the current period actions and with $w(c, \gamma^*, s, p)$ as continuation values for the bank, must have $\phi(c, \gamma^*, s, p) = \phi(w(c, \gamma^*, s, p))$ as the continuation value to the regulator on the equilibrium path.

A proof of this lemma may be found in the Appendix. The lemma serves several different purposes: firstly, it shows that the problem $\phi(v)$ has a solution, since the set $\mathcal{E}$ is compact. As mentioned before, it also implies that we can take $w(c, \gamma, s, p) \in [\underline{w}, \overline{w}]$ as an appropriate description of the set of continuation values the bank can obtain in equilibrium. Thirdly, it guarantees that whenever the regulator promises a continuation value $w$ for the bank, it promises himself a continuation value $\phi(w)$ that is efficient - that is, the regulator does not punish himself by promising himself a continuation value $\phi(c, \gamma, s, p) < \phi(w(c, \gamma, s, p))$. The intuition for this is clear: if the regulator was choosing $\phi(c, \gamma, s, p) < \phi(w(c, \gamma, s, p))$ on the equilibrium
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path, it could deviate towards offering the same \( w(c, \gamma, s, p) \) to the bank (hence, keeping PKC and \( IC_R \) valid) but offering himself \( \phi(w(c, \gamma, s, p)) \) on the equilibrium path (relaxing \( IC_R \) and increasing his own payoff).

We can now state the schedule to find the regulator’s optimal contract. Firstly, we maximize the recursive problem of the regulator, by choosing \( \gamma \) and the set of promised continuation values to the banks, subject to the four previous sets of constraints:

\[
\phi(v) = \mu \{ (1 - \delta)(-\gamma C) + \delta E_\gamma[\phi(w(1, \gamma, s, M))|G] \} \\
\max_{\gamma, \alpha} \{ v(\gamma, s, p) | \gamma, s, p \} \\
\text{s.t. } IC_B, IC_R, PKC, w(c, \gamma, s, p) \in [\bar{w}, \underline{w}]
\]

Once the we have solved this recursive problem, we need to further find the optimal \( v \) for the first stage. This will come from solving for \( v^* = \arg \max_{v \in [\bar{w}, \underline{w}]} \phi(v) \), which is equivalent to solving to the first period problem.

Once we have the set of values for \( \gamma, \{w(c, \gamma, s, p)\} \) and \( v \), we can then characterize the optimal regulation contract. Start with the optimal \( v^* \), which maximizes the problem of the regulator. Then select \( \gamma, \alpha \) and continuation values to maximize the problem \( \phi(v^*) \) of the regulator conditional on assigning value \( v^* \) to the bank. Once complexity, screening and performance is realized, then, the design problem determines a new value \( w(c, \gamma, s, p) \) to be assigned to the bank. This new value then determines next period choices of \( \gamma, \alpha \) (which are made conditional on the history of the game, since \( w \) depends on the first period history of the game). By proceeding in this way progressively, we can characterize the optimal regulation.

### 4.3.2 Useful results

Before we perform the aforementioned schedule, we further restrict the optimization problem by proving some useful Lemmas. We begin with the following lemma on \( IC_B \):

**Lemma 6** Without loss of generality, it is optimal to either set:

\[
\delta w(0, \gamma, B, \emptyset) \geq (1 - \delta)[-D + (1 - \gamma)\theta M] + \delta E_{\gamma,p}[w(1, \gamma, s, p)|B]
\]

or to set \( w(0, \gamma, B, \emptyset) = \bar{w} \). That is, it is either weakly optimal to give weakly more incentives for \( c = 0 \) or to set \( w(0, \gamma, B, \emptyset) = \bar{w} \).

This has a simple intuition: if we were giving strictly more incentives for \( c = 1 \), we could increase \( w(0, \gamma, B, \emptyset) \) up until \( IC_B \) is binding without affecting payoffs - since payoffs for no complexity would be out-of-equilibrium payoffs when providing strict incentives for \( c = 1 \). This will be feasible as long as we can do so while still setting \( w(0, \gamma, B, \emptyset) \leq \bar{w} \).

We can also provide some helpful bounds on \( \bar{w} \) and \( \underline{w} \):

\[\text{As we explain above, finding the optimal contract is equivalent to finding the optimal equilibrium actions, and need not be interpreted literally; the natural question of whether regulators make explicit promises to banks in practice is an open question.}\]
Lemma 7 In equilibrium, \( w = \mu M \). Moreover, \( \phi \in \left[-C, -\frac{\mu(1-\theta)R_C}{(1-\theta)R - C}\right] \). \( w \leq \mu M + (1-\mu)(-D + \theta M) \), holding with equality if the regulator had commitment (or IC\( _R \) never binds), and with strict inequality if the regulator has no commitment (IC\( _R \) binds for some \( v \)).

This is a straightforward lemma to prove: first, note that the bank might get payoff \( \mu M \) if we play the repeated static PBE, and that the bank could never get a payoff smaller than \( \mu M \) because the bank could always deviate and choose \( c = 0 \) in all periods that his product happens to be bad.

Similarly, the bank cannot get a payoff above \( M \) when his product is good, and above \( -D + \theta M \) when his product is bad, due to assumption 1. As a consequence, his discounted present value payoff cannot be above \( \mu M + (1-\mu)(-D + \theta M) \). In the Appendix, we show that this limit can be attained with commitment, but cannot be attained without commitment.

Finally, note that the regulator could always be awarded his static PBE payoff in every period, which guarantees that \( \phi \leq -\frac{\mu(1-\theta)R_C}{(1-\theta)R - C} \). Also, the regulator cannot be awarded less than \( -C \), since if he got less than that, he could always deviate towards monitoring whenever the product is complex and get a payoff of \(-C[\mu + (1-\mu)\alpha]\), which is at least \(-C\).\(^{16}\)

With this shown, we can prove the next lemma, which turns out to be crucial for the analysis:

Lemma 8 \( \phi(v) \) is weakly decreasing in \( v \) for \( v \geq v^* \), with \( v^* \in \left[w, \mu M + (1-\delta)(1-\mu)(-D + \theta M)\right] \).

While the proof is somewhat long, the intuition for this result is fairly simple. Consider a problem where we first fix \( \gamma \) and choose \( a \) and continuation values subject to all constraints, with value \( \phi(v,\gamma) \). Clearly, if \( \gamma^*(v) \) is the optimal solution to \( \phi(v) \), then \( \phi(v) = \phi(v,\gamma^*(v)) \).

Now, consider relaxing the PKC constraint in the problem \( \phi(v,\gamma) \), by letting the bank receive weakly more than \( v \) and by letting the bank receive continuation values consistent with this relaxed problem (as opposed to continuation values in the original problem). Clearly, the value of this new problem would be decreasing in \( v \).\(^{17}\) We then show that, in this problem, either PKC is binding, or \( \gamma \neq \gamma^*(v) \): more precisely, we show that for all \( \gamma \) values that can be candidate solutions for \( \gamma^*(v) \), if PKC was not binding, we could decrease \( w(c,\gamma,s,p) \) by \( \epsilon \) for some state \( c,\gamma,s,p \) while satisfying all constraints,\(^{18}\) and improving continuation values in the relaxed problem (after all, the continuation values in the relaxed problem are decreasing in \( v \)). In that case, if PKC is binding in this relaxed problem, this relaxed problem yields a feasible and optimal solution for \( \phi(v,\gamma) \) whenever \( \gamma^* \) is a plausible candidate for the optimal \( \gamma^*(v) \).

Although we have that \( \phi(v,\gamma) \) must be weakly decreasing in \( v \), we have yet to show that we are able to satisfy all of our constraints. To tackle this issue, we first show that the regulator is constrained to pick:

\[
\gamma \geq \max \left\{ \gamma^{PBE} - \frac{v - \mu M}{(1-\delta)(1-\mu)\theta M}, 0 \right\}
\]

after all, the maximum punishment we can offer with continuation values is \( w = \mu M \). If monitoring is low enough, we will end up being forced to deliver more than \( v \) to the bank.

\(^{16}\)Note also that \(-C\) is the regulator’s minmax payoff.

\(^{17}\)As for \( v < v^* \), any solution feasible for the problem \( v^* \) is also feasible for problem \( v \).

\(^{18}\)More precisely, we show that whenever such a deviation does not satisfy one of the constraints, then \( \gamma^* \) cannot be optimal, and is not a candidate for \( \gamma^*(v) \).
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Then, for \( v < \mu M + (1 - \delta)(1 - \mu)(-D + \theta M) \), the lower is \( v \), the more this constraint limits the choice of \( \gamma \), and this constrains the value \( \phi(v) \) from increasing in \( v \) as we reduce \( v \) below \( v^* \leq \mu M + (1 - \delta)(1 - \mu)(-D + \theta M) \).

This however shows implicitly that as \( \delta \to 1 \), \( \phi(v) \) becomes decreasing for almost all \( v \). This result will be important in the numerical computations of the model.

In the next lemma, we show that the choice of optimal \( \alpha \) by the banks is restricted when \( \gamma > \gamma^{PBE} \). For this, let \( \alpha^{MIN} \) be the minimum value of \( \alpha \) consistent with IC\(_R\) for given levels of \( \gamma \) and \( w(1, \gamma, \cdot) \) (note that continuation values \( w(0, \gamma, B, \emptyset) \) do not affect IC\(_R\), since incentives for monitoring are only an issue after the bank shows up with a complex asset). Similarly, let \( \alpha^{MAX} \) be the maximum value of \( \alpha \) consistent with IC\(_R\) for a given \( \gamma \) and \( w(1, \gamma, \cdot) \). Then, we have the following lemma.

**Lemma 9** If the optimal dynamic regulatory mechanism implies \( \gamma > \gamma^{PBE} \) in a given period, then the optimal choice of \( \alpha \) by the banks will be \( \hat{\alpha} = \alpha^{MIN} \). \( \alpha^{MIN} \) is weakly increasing in \( \gamma \).

A sketch of proof follows, with further details provided in the Appendix. Begin by assuming otherwise, such that \( \alpha > \alpha^{MIN} \). We show that there exists a possible deviation, wherein a regulatory mechanism with equilibrium \( \hat{\alpha} = \alpha^{MIN} \) can be used instead, by changing the promised continuation values. We then show that this deviation satisfies the constraints of the regulator’s problem, and that the bank is indifferent between the two mechanisms. We then show that this results in a weakly higher payoff for the regulator. Consequently, the deviation is feasible and has a weakly higher payoff for the regulator, so that there is a solution to the problem where, whenever \( \gamma > \gamma^{PBE} \), \( \alpha = \alpha^{MIN} \).

From this result, the following corollary immediately follows:

**Corollary 10** It cannot be optimal to set \( \gamma > \gamma^{PBE} \) in the dynamic regulatory mechanism.

The proof follows the fact that the higher the \( \gamma \), the higher the expected cost of screening and the higher is \( \alpha^{MIN} \). Moreover, given that we are requiring the bank to choose \( \alpha^{MIN} \geq 0 \), we have to reward the bank with higher continuation values for choosing complexity when \( \gamma > \gamma^{PBE} \) (since the bank’s static incentives with \( \gamma > \gamma^{PBE} \) are to choose \( \alpha = 0 \)). Thus, from our Lemma 9, if we start at \( \gamma > \gamma^{PBE} \), we could deviate and obtain a lower level of complex bad products (lower \( \alpha^{MIN} \)), with a lower cost of monitoring (lower \( \gamma^{PBE} \) and lower \( C\gamma^{PBE} \)), and with lower continuation value rewards for complexity, up until we reach \( \gamma = \gamma^{PBE} \).

The next few lemmas further simplify the solution to the optimal dynamic regulation problem.

**Lemma 11** Whenever it is optimal to set \( \gamma > 0 \) and \( \alpha = 0 \), then it is optimal for IC\(_B\) to hold with equality.

If it is optimal to set \( \gamma = 0 \) and \( \alpha = 0 \), then if \( v < \mu M + \frac{1-\delta}{\theta}[-D + \theta M] \), it is optimal to set IC\(_B\) to hold with equality.

If it is optimal to set \( \gamma = 0 \) and \( \alpha = 0 \), then if \( v \geq \mu M + \frac{1-\delta}{\theta}[-D + \theta M] \), it must be optimal to set \( w(1, \gamma, 0, M) = w(0, \gamma, B, \emptyset) = \frac{v-(1-\delta)\mu M}{\theta} \), and IC\(_B\) becomes weakly non-binding.

The intuition for this lemma is fairly simple: we could try to give strict incentives for \( \alpha = 0 \) by setting IC\(_B\) not to be binding. However, if we are also setting \( \gamma > 0 \), this cannot be optimal, since we could reduce \( \gamma \), keep providing weak incentives for \( \alpha = 0 \), we would not change
the bank’s payoffs (since he chooses \( \alpha = 0 \) either way), and the fall in monitoring costs when \( \alpha = 0 \) would be profitable for the regulator.

If we want to provide incentives for \( \alpha = 0 \) with \( \gamma = 0 \), on the other hand, we need to be more careful. We can show that when \( \alpha = \gamma = 0 \), IC\(_R\) always holds. Moreover, if we started providing strict incentives for \( \alpha = 0 \), we could try deviating in the direction of \( w(1, \gamma, 0, M) = w(0, \gamma, B, \emptyset) \) while keeping the average of these two continuation values constant. If we do so, we will still satisfy PKC, we will still satisfy IC\(_B\) for a small deviation, and we will still have continuation values in \([w, \bar{w}]\). However, this will improve the regulator’s payoff, given that \( \phi(w) \) is concave and the regulator dislikes risk in continuation values. This implies that having \( \alpha = 0, \gamma = 0, w(1, \gamma, 0, M) \neq w(0, \gamma, B, \emptyset) \) and IC\(_B\) non-binding cannot be optimal.

For \( v < \mu M + \frac{1-\delta}{\delta}[-D + \theta M] \), we will not be able to fully move towards \( w(1, \gamma, 0, M) = w(0, \gamma, B, \emptyset) \) while still having IC\(_B\) non-binding. The reason for that is that for \( v \) small enough, having \( w(1, \gamma, 0, M) = w(0, \gamma, B, \emptyset) \) would require a very strong punishment in the form of \( w(1, \gamma, 0, 0) \) in order to keep IC\(_B\) satisfied. However, if \( v \) is small enough, \( w(1, \gamma, 0, M) = w(0, \gamma, B, \emptyset) \) would be also small (due to PKC), and this would in turn imply that the punishment \( w(1, \gamma, 0, 0) \) would be smaller than \( w \). Hence, for \( v < \mu M + \frac{1-\delta}{\delta}[-D + \theta M] \), we will have that if \( \alpha = \gamma = 0 \), IC\(_B\) needs to be binding on the optimal regulatory mechanism, and that \( w(1, \gamma, 0, M) \neq w(0, \gamma, B, \emptyset) \).

If, on the other hand, \( v \geq \mu M + \frac{1-\delta}{\delta}[-D + \theta M] \), then we can show that if it is optimal to set \( \alpha = \gamma = 0 \), it is optimal to set \( w(1, \gamma, 0, M) = w(0, \gamma, B, \emptyset) = \frac{v-(1-\delta)\mu M}{\delta} \), and to have IC\(_B\) non-binding. The reason for that is, as discussed two paragraphs above, this will allow the regulator to have lower continuation value risk.

Note also that lemma 6 states that if we were to give more incentives for the bank to set \( c = 1 \), we could always increase \( w(0, \gamma, s, p) \) up until IC\(_B\) is binding or up until it hits \( \bar{w} \). As a consequence, the only circumstances in which we might not want to have IC\(_B\) binding are if either \( \alpha = 0 \) and \( \gamma = 0 \), or if \( \alpha = 1 \) and \( w(0, \gamma, B, \emptyset) = \bar{w} \).

The next lemma further simplifies the problem by showing that whenever the regulator detects that the product is bad and complex (either by engaging in costly monitoring or by observing a failure), the regulator should give the same continuation value to the bank.

**Lemma 12** It is optimal to set \( w(1, \gamma, B, \emptyset) = w(1, \gamma, 0, 0) \)

The proof for this is immediate: if this was not the case, the regulator could deviate towards a mechanism that delivers \( w(1, \gamma, B, \emptyset) = w(1, \gamma, 0, 0) \) whilst maintaining the same expected values for complex bad products, which reduces the risk in continuation values that the regulator faces (which is good for the regulator given that \( \phi(v) \) is concave), and, furthermore, this would increase the value of the objective function and relax IC\(_R\). Moreover, by deviating in a way that keeps constant the expected continuation value for the bank conditional on detected complex bad products, we will be trivially satisfying IC\(_B\) and keeping PKC.

The next step also considerably simplifies the analysis by reducing the optimal space of \( \alpha \) under consideration. We begin by defining some special values for \( \alpha \). Let \( \alpha^{\text{MIN}} \in [0, 1] \) be the minimum \( \alpha \) consistent with IC\(_R\) and let \( \alpha^{\text{MAX}} \in [0, 1] \) be the maximum \( \alpha \) consistent with IC\(_R\) holding (for given levels of \( \gamma \) and continuation values). Then define \( \alpha^T \) to be:

\[
\alpha^T = \frac{\mu}{1 - \mu (1 - \delta)(1 - \theta)R - C} \left[ (1 - \delta)C + \delta[\phi(w(1, \gamma, 0, M)) - \phi(w(1, \gamma, G, M))] \right] 
\]
As can be seen from the payoff function of the regulator, $\alpha^T$ is the value of $\alpha$ that makes the regulator indifferent between increasing or decreasing $\gamma$. We then prove that:

**Lemma 13** Whenever $\gamma > 0$, or $\gamma = 0$ and the IC$_B$ holds with equality, we have that:

(i) If $\alpha^T \geq 1$, the optimal solution for the regulator’s problem is to set $\alpha \in \{\alpha^{\text{MIN}}, 1\}$, that is, either $\alpha^{\text{MIN}}$ or 1. Additionally, the regulator’s objective function is decreasing in $\gamma$.

(ii) If $\alpha^T < 1$, then $\alpha^T \in [\alpha^{\text{MIN}}, \alpha^{\text{MAX}}]$. Moreover, for every solution to the optimal regulation problem, there is an alternative solution where $\alpha \in \{\alpha^{\text{MIN}}, \alpha^{\text{MAX}}\}$. If $\alpha = \alpha^{\text{MIN}}$, then the regulator’s objective function is weakly decreasing in $\gamma$. If $\alpha = \alpha^{\text{MAX}}$, then the regulator’s objective function is weakly increasing in $\gamma$ (and strictly increasing if $\alpha^{\text{MIN}} \neq \alpha^{\text{MAX}}$).

The intuition for this lemma is also relatively simple: whenever we can actually choose any value of $\alpha$ (that is, whenever IC$_B$ holds with equality, implying that the bank is indifferent between payoffs), then given that the payoff function from the regulator is linear in $\alpha$, the regulator can either prefer to set the minimum feasible $\alpha$, the maximum feasible $\alpha$, or be indifferent. Hence, it is possible, without loss of generality, to restrict the set of optimal values to the extreme $\alpha$ choices. Moreover, the lemma shows that given a level of $\gamma$ and continuation values that make IC$_B$ hold with equality, we can see what are the maximum and minimum levels of $\alpha$ simply by looking at IC$_B$: after all, with $\gamma$ and continuation values that make IC$_B$ hold with equality, the bank will be indifferent between different levels of $\alpha$, and thus changing the values of $\alpha$ will not affect the PKC.

Another property of the optimal solution which also helps to simplify the analysis follows:

**Lemma 14** If either (a) $\alpha^T \geq 1$, (b) it is optimal to set $\alpha = \alpha^{\text{MIN}}$ or (c) it is optimal to set $\alpha = \alpha^{\text{MAX}} = \alpha^T$, then it is optimal to set $\phi'(w(1, \gamma, B, \emptyset)) = \phi'(w(1, \gamma, 0, 0)) \geq 0$.

Moreover, if either (i) given $\gamma$, $w(1, \gamma, 0, M)$ and $w(1, \gamma, G, M)$, we have $\alpha^{\text{MIN}} = 0$, or (ii) $\gamma = 0$, then whenever it is optimal to set $\alpha = \alpha^{\text{MIN}} = 0$, it is optimal to set $w(1, \gamma, B, \emptyset) = w(1, \gamma, 0, 0) = w$.

The intuition for this lemma is that, if we want to minimize the incentives the bank has in choosing complexity and implement the minimum feasible $\alpha$, then the choice of $w(1, \gamma, B, \emptyset)$ such that $\phi'(w(1, \gamma, B, \emptyset)) < 0$ will either be irrelevant (if the minimum feasible $\alpha$ is 0) in which case the above setting is trivially possible; or will require us to compensate for choosing a lower punishment of $w(1, \gamma, B, \emptyset)$ by increasing $\gamma$. In this case, we could reduce monitoring (which is desirable under $\alpha^{\text{MIN}} \leq \alpha^T$ or when $\alpha^T \geq 1$) and reduce $w(1, \gamma, B, \emptyset)$ in a way that keeps the payoff from complexity constant (and hence, keeps the overall payoffs from the bank constant), while reducing the costs of monitoring and the costs of providing a higher continuation value for the bank in the future (which is costly under $\phi'(w(1, \gamma, B, \emptyset)) < 0$). Since, from lemma 12, $w(1, \gamma, 0, 0) = w(1, \gamma, B, \emptyset)$ in the optimal solution, the same logic applies to $w(1, \gamma, 0, 0)$.

On the other hand, whenever $\alpha^{\text{MIN}} = 0$, then if it is optimal to set $\alpha = \alpha^{\text{MIN}} = 0$, then it is optimal to engage in maximum punishments $w(1, \gamma, B, \emptyset) = w(1, \gamma, 0, 0) = w$. The reason for that is that under $\alpha = 0$, states $(1, \gamma, B, \emptyset)$ and $(1, \gamma, 0, 0)$ never happen on equilibrium, so that increasing this punishment comes at no direct cost, and it is without loss of generality to assume so. Moreover, if $\gamma > 0$, deviating towards such maximum punishments further allow us to reduce $\gamma$, which is good for the regulator when $\alpha = 0$.

Finally, note also from lemma 8 that this result suggests that if $\delta \to 1$, then $w(1, \gamma, B, \emptyset) = w(1, \gamma, 0, 0) \to w$, even if $\alpha^{\text{MIN}} > 0$. 
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The final lemma does the analogous of the previous lemma for \( \alpha = \alpha^{\text{MAX}} \), at least when \( \alpha^{\text{MAX}} = 1 \).

**Lemma 15** If \( \alpha^T < 1 \) and if it is optimal to set \( \alpha = 1 \), then it must be optimal to set \( w(1, \gamma, s, p) = w^* \) for all \( s, p \).

The intuition for this lemma is simple: first, when \( \alpha = \alpha^{\text{MAX}} = 1 > \alpha^T \), the regulator’s objective function is increasing in \( \gamma \). Hence, if we were setting \( w(1, \gamma, s, p) \) non constant, we could deviate in the direction of a constant continuation value \( w(1, \gamma, s, p) \) - and increase the regulator’s equilibrium value because of concavity of \( \phi(w(\cdot)) \) - and simultaneously increase \( w(0, \gamma, B, \emptyset) \) to keep \( IC_R \) binding - which makes no difference to the regulator, since \( \alpha = 1 \).

From this, we proceed to describing the optimal contract.

## 4.4 The Optimal Dynamic Contract

### 4.4.1 Regulatory mechanisms with commitment

With the previous lemmas, we can proceed to characterize the optimal solution to the regulatory problem. We do this by solving the problem when the regulator can commit, that is, when \( IC_R \) is not present/not binding, so that \( \alpha^{\text{MIN}} = 0 \) and \( \alpha^{\text{MAX}} = 1 \).

In this case, if we want to implement \( \alpha = \alpha^{\text{MAX}} = 1 \), lemma 15 indicates that continuation values after complexity are constant at \( w^* \). As a consequence, given \( PKC \) will be holding with \( \alpha = 1 \), we have

\[
\delta w^* = v - (1 - \delta) \{ \mu M + (1 - \mu)(-D + (1 - \gamma)\theta M) \}
\]

From \( IC_B \), then, all we need to do is to adjust \( w(0, \gamma, B, \emptyset) \) so that it holds with equality.

Once this is arranged, the regulator’s payoff will be:

\[
-(1 - \delta)[\gamma C + (1 - \mu)(1 - \gamma)(1 - \theta)R] + \delta \phi(w^*)
\]

which then allow us to take first order conditions with respect to \( \gamma \), replacing in \( w^* \) by its value implied from \( PKC \). Taking derivatives with respect to \( \gamma \) then yields the following:

\[
-\phi'(w^*) = \begin{cases} 
-\phi'(W_s) & \text{if } -\phi'(W_s) \geq \frac{(1 - \mu)(1 - \theta)R - C}{(1 - p)M} \\
\frac{(1 - \mu)(1 - \theta)R - C}{(1 - p)M} & \text{if } -\phi'(W^*) > \frac{(1 - \mu)(1 - \theta)R - C}{(1 - p)M} > -\phi'(W_s) \\
-\phi'(W^*) & \text{if } -\phi'(W^*) \leq \frac{(1 - \mu)(1 - \theta)R - C}{(1 - p)M}
\end{cases}
\]

where \( W_s = \max \left\{ \frac{v - (1 - \delta)(\mu M + (1 - \mu)(-D + \theta M))}{\delta} \right\} \), and \( W^* = \min \left\{ v, \frac{v - (1 - \delta)\mu M}{\delta} \right\} \). Note that the second derivative of the objective function with respect to \( \gamma \) is given by:

\[
\phi''(w^*) \left[ (1 - \delta)(1 - \mu)\theta M \right]^2 \leq 0
\]

With this in mind, we can take the interior solution; and if the derivative with respect to \( \gamma \) is always positive, we just cap \( w^* \) by either \( \bar{w} \) or by the value of \( w^* \) when \( \gamma = \gamma^{\text{PBE}} \); if the derivative with respect to \( \gamma \) is always negative, we just cap \( w^* \) to be either \( \bar{w} \) or by the value of \( w^* \) when \( \gamma = 0 \).
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Crucially, once we decide to set \( \alpha = \alpha^{MAX} \), if \( \gamma \in (0, \gamma^{PRE}) \) and \( w^* \in (w, \bar{w}) \), then we will decide to award a continuation value \( w^* \) to the bank that is independent of \( \gamma, \alpha \) and \( v \). In particular, this means that once we set \( \alpha = \alpha^{MAX} = 1 \), we will set future actions and future continuation values in a way that is independent from the level of \( v \) that led to \( \alpha = \alpha^{MAX} = 1 \). In this sense, once we set \( \alpha = \alpha^{MAX} = 1 \), we are doing the equivalent of erasing history for the next period onwards. Moreover, \( \gamma \) changes in a way that allows the bank to get \( v \).

If we set \( \gamma \in \{0, \gamma^{PRE}\} \), we will award a continuation value that depends on \( v \), and hence, the future of play will depend on the history that led to a promise of \( v \).

If, on the other hand, we want to implement \( \alpha = \alpha^{MIN} = 0 \), then lemma 14 indicates that without loss of generality, we will have \( w(1, \gamma, B, \emptyset) = w(1, \gamma, 0, 0) = \bar{w} \). Moreover, Section 4.A.1 of the Appendix shows that, from the first order conditions to the Lagrangian of this problem, we get \( w(1, \gamma, G, M) = v \). We need to further determine \( w(1, \gamma, 0, M) \) and \( w(0, \gamma, B, \emptyset) \).

However, we know that if \( \gamma > 0 \), then lemma 11 tells us that \( IC_B \) is binding, so that the bank is indifferent between complexity and no complexity. Consequently, from \( PKC \), we get that:

\[
v = (1 - \delta)\mu M + \delta \{ \mu [\gamma v + (1 - \gamma)w(1, \gamma, 0, M)] + (1 - \mu)w(0, \gamma, B, \emptyset) \}
\]

(once again, using the fact that we can write the bank’s payoff as the payoff from no complexity after a bad product). Also, at \( \gamma > 0 \), we know that due to \( IC_B \) binding:

\[
(1 - \delta)[-D + (1 - \gamma)\theta M] + \delta[\gamma + (1 - \gamma)(1 - \theta)]w + (1 - \gamma)\theta w(1, \gamma, 0, M)] = \delta w(0, \gamma, B, \emptyset)
\]

These two equations give us, implicitly, for each value of \( \gamma \), a value for \( w(0, \gamma, B, \emptyset) \) and for \( w(1, \gamma, 0, M) \). Conditionally on \( \alpha = 0 \), then, we get that the regulator’s objective function is given by:

\[
\mu \{ \delta[\gamma \phi(v) + (1 - \gamma)\phi(w(1, \gamma, 0, M))] - (1 - \delta)\gamma C \} + (1 - \mu)\delta \phi(w(0, \gamma, B, \emptyset))
\]

From these last three equations, we can easily verify that conditionally on \( \alpha = \alpha^{MIN} = 0 \), it is optimal to set \( \gamma \) to make the derivative of the objective function above with respect to \( \gamma \) equal to zero. This means that:

\[
\frac{(1 - \mu)}{\mu} \phi'(w(0, \gamma, B, \emptyset)) \frac{\partial w(0, \gamma, B, \emptyset)}{\partial \gamma} = \frac{1 - \delta}{\delta} C + \phi(w(1, \gamma, 0, M)) - \phi(v) - (1 - \gamma)\phi'(w(1, \gamma, 0, M)) \frac{\partial w(1, \gamma, 0, M)}{\partial \gamma}
\]

This expression has the following intuition: on the first line, we have the marginal benefits of monitoring, which allows for the regulator to reduce the prize (in terms of continuation values) given to banks that do not make their bad assets complex. Naturally, the higher the monitoring, the lower this prize needs to be.

The second line, on the other hand, shows the marginal costs of monitoring. First of all, we have the direct costs of monitoring \( \frac{1 - \delta}{\delta} C \). Secondly, we have an indirect cost of monitoring: when the bank increases \( \gamma \), it increases the number of times it finds out the product is good, leading to more times giving continuation values \( v \) rather than continuation values
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\( w(1, \gamma, 0, M) \) (note that \( w(1, \gamma, 0, M) \) is set to be a punishment relative to \( v \), since the state \( 1, \gamma, 0, M \) might happen when the bank has a bad product). The final term is a second indirect cost of monitoring: increasing monitoring reduces the profits from bad products (by increasing the number of times these products are forbidden), so that to keep delivering the promise of \( v \) to the bank (needed to satisfy \( PKC \)), this requires the regulator to increase \( w(1, \gamma, 0, M) \) with \( \gamma \).

Once we get these two solutions - one with \( \alpha = 1 \), the other with \( \alpha = 0 \), all there is left to do is to compare the value of these two solutions when they're both feasible. In particular, note that while the solution with \( \alpha = 1 \) is always feasible (that is, we can always adjust \( \gamma \) and \( w^* \) so that we deliver value \( v \) to the bank and have \( \alpha = 1 \)), if \( v > (1 - \delta)\mu M + \delta[\mu M + (1 - \mu)(-D + \theta M)] = \mu M + \delta(1 - \mu)(-D + \theta M) \), then it is impossible to attain value \( v \) while having \( \alpha = 0 \) and some continuation value \( w(\cdot) \leq \bar{w} \).

4.4.1.1 Numerical results for the optimal regulatory schedule with commitment when delta is close to 1

We first present the numerical results for when delta is close to one, as a benchmark for our analysis. As can be seen in Step 3.1 of Lemma 8, as delta gets closer to 1, the portion of the domain of \( \phi \) that is weakly decreasing increases and tends towards the whole span of the domain. Thus, we set relatively high value of \( \delta \) to guarantee that the resulting function will be weakly decreasing through its span.

Concerning which parameter values to use, we currently have no clear way of calibrating them to match moments from the data, or a way of micro-founding them, and this remains an area for future research. As such, we choose our parameters to satisfy the constraints coming from assumptions (i) and (ii), and opt to have the bank reward and bailout cost be of similar size (equal, in this case), the costs of screening and making financial products complicated be of similar size and significantly smaller than the aforementioned parameters, and choosing probabilities for \( \mu \) and \( \theta \) such that even when no screening takes place and bad products are always make complex, bailouts are relatively rare (around 10% of the time). The values used can be seen in Table 4.1:

<table>
<thead>
<tr>
<th></th>
<th>( \mu )</th>
<th>( \theta )</th>
<th>( M )</th>
<th>( D )</th>
<th>( C )</th>
<th>( K )</th>
<th>( \delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.7</td>
<td>0.7</td>
<td>1</td>
<td>0.05</td>
<td>0.05</td>
<td>1</td>
<td>0.995</td>
</tr>
</tbody>
</table>

**Table 4.1:** Parameters used when delta is close to one

We plot the resulting \( \phi \) function, and optimized values of \( \alpha \) and \( \gamma \) for the span of possible values for \( v \), that is to say, between \( \bar{w} \) and \( w \), in Figures 4.1 and 4.2.

---

19Aside from a small approximation error for the very first step.
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Figure 4.1: Optimal $\phi$ for $\delta = 0.995$

Figure 4.2: Optimal $\alpha$ and $\gamma$ for $\delta = 0.995$
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As can be seen in the above figures, \( \phi \) is essentially static until \( v \) become \( \bar{w} \), at which point it declines significantly. The reason for this can be seen in the behaviour of \( \alpha \) and \( \gamma \). When \( v \) is equal to \( \bar{w} \), we are in the ‘punishment’ phase, so \( \alpha = 0 \) and \( \gamma = 1 \), the PBE result that constitutes the maximum punishment possible by the regulators. We then have that both \( \alpha \) and \( \gamma \) are zero for all values of \( v \) except for when \( v \) is equal to \( \bar{w} \); as expected, regulators can incentivise banks to not make use of complexity even when \( \gamma \) is zero. And this is because there is a reward phase, happening when \( v \) is exactly equal to \( \bar{w} \), where \( \alpha \) is equal to 1 and banks are not punished.

Note that in these graphs and the graphs ahead, there are ‘sharp’ drops in the values we find, most importantly when \( v \) reaches it’s highest value. This is to be expected, as when we hit the extremum in the value of \( v \), we reach the ‘reward’ phase for the banks, which is an absorbing state, such that nothing changes from that point onwards: bad products are always complex and banks are never punished, so expected values are constant.

This can also be seen in how the promised continuation values change over time, normalized here by their deviations from the current \( v \) and seen in Figure 4.3 and, in more detail, in Figure 4.4.

![Figure 4.3: Optimal promised continuation values for \( \delta = 0.995 \)](image)

At the extremum, banks are promised continuation values equal to their current outcomes no matter what they do, as these are absorbing states\(^{20}\). For values in between, should banks deviate, by being discovered using a bad complex product (either \( w(1, \gamma, B, \emptyset) \) or \( w(1, \gamma, 0, 0) \)), they receive the maximum punishment available, as can be seen by the increasing fall of \( w(1, \gamma, B, \emptyset) \). \( w(1, \gamma, G, M) \) is very simple remains at \( v \) throughout these values of \( v \).

Similarly, they are initially punished when a complex, unscreened product that succeeds, \( w(1, \gamma, 0, M) \), which is by far the most likely outcome in the dynamics given that \( \mu = 0.7 \) and that \( \gamma \) is zero at this point. This is small punishment, significantly smaller than the grid size

\(^{20}\)As \( \delta \) is not exactly equal to 1, the function is increasing in the very first step in the grid of \( v \), as we move from \( v \)'s lowest value and, as such, that second-to-lowest value should be the chosen one, as it is the highest \( \phi \).
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Figure 4.4: Optimal promised continuation values for $\delta = 0.995$ in more detail

used for $v^{21}$, quickly increases to a positive promised continuation value and converges, after reaching the threshold specified by Lemma 11 to the value of $w(0, \gamma, B, \emptyset)$. $w(0, \gamma, B, \emptyset)$ itself starts positive and remains positive throughout, as we would expect, since the regulator is particularly concerned with what banks do when they have a bad product.

With this, the dynamics of the optimal regulation is as follows. The regulator chooses to deliver the closest value $v$ to $w$ that it can, promising the above schedule for $\alpha$, $\gamma$ and the continuation values for the banks. Banks then experience an erratic upwards increase $v$ and for values very close to $w$, bad, uncomplex products increase $v$, and good products decrease $v$.

For the majority of values of $v$, however, receiving either type of product simply means an identical increase in $v$ such that, on average, their promised continuation values increases over time and neither the bank, nor the regulator incurs any flow cost. The values of $w(0, \gamma, B, \emptyset)$ and $w(1, \gamma, 0, M)$ we find are such that, when considering their expected values, the average expected increase in $v$ is only slightly above the current value of $v$. This can be seen in Figure 4.5, the expected increase in $v$, as a percentage of the current value of $v$.

\[\text{\footnotesize 21Such that, due to the numerical approximation methods used, if this outcome were to occur at a value of } v \text{ that is close to } w, \text{ in the next period, the promised continuation value } v \text{ that our algorithm approximates to, would still be above } w.\]

\[\]
4.4. THE OPTIMAL DYNAMIC CONTRACT

This no screening and no complex, bad products regime lasts until it hits the upper boundary of $v$, $\bar{w}$, at which point both the bank and the regulator switch to the ‘reward’ phase, where regulators no longer screen and banks always make products complex, and the continuation values are exactly equal to $\bar{w}$. Thus, by promising future rewards, regulators can sustain for a significant amount of time a no-screening regime where no bailouts will happen, although this does lead, inevitably, to a future, permanent ‘reward’ phase where bailouts happen at their highest probability.

4.4.1.2 More results for the optimal regulatory schedule with commitment

We now execute the optimization schedule for two values of $\delta$, above and below the threshold specified by Lemma 11, 0.95 and 0.90, whilst maintaining the same parameter values as in the above case.

We begin by first finding the solution for 0.95, above the threshold, and like above, we plot the resulting $\phi$ function, and optimized values of $\alpha$ and $\gamma$ for the span of possible values for $v$ in Figures 4.6 and 4.7, and the promised continuation values, in Figures 4.8 and, again in more detail, in 4.9.
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Figure 4.6: Optimal φ for δ = 0.95

Figure 4.7: Optimal α and γ for δ = 0.95
4.4. THE OPTIMAL DYNAMIC CONTRACT

Figure 4.8: Optimal promised continuation values for $\delta = 0.95$

Figure 4.9: Optimal promised continuation values for $\delta = 0.95$ in more detail
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The resulting optimal schedule is very similar to our results when \( \delta \) is close to 1. We find that \( \phi \) has a small portion in which it is increasing in \( v \), as expected and corresponding to the region of \( v < v^* \) in Lemma 8; however, the function is not convex as expected, which we surmise is due to numerical errors in our algorithm. In this region, \( \alpha \) remains zero, as expected, and \( \gamma \) starts at the maximum punishment of \( \gamma^{PBE} \), before slowly decreasing as we move away from \( \bar{w} \), in such a way that allows the regulator to satisfy the ICB with these progressively lower values of \( \gamma \).

We then have a wide range of values of \( v \) where our \( \phi, \gamma \) and \( \alpha \) behave much like in the ‘no complexity, no screening’ region we find when \( \delta \) is close to 1\(^{22}\). Finally, when we get close to the upper boundary of \( v, \bar{w} \), we enter a ‘reward’ phase wherein \( \gamma \) remains at zero, and \( \alpha \) begins to increase up until \( \alpha = 1 \) at \( v = \bar{w} \), such that \( \phi \) decreases more smoothly until its lowest point, the reason for which we discuss below.

The promised continuation values, in deviation from \( v \), all behave identically to the case when \( \delta \) is close to 1, with a similar convergence between \( w(0, \gamma, B) \) and \( w(1, \gamma, 0, M) \) once the threshold of Lemma 11 is reached.

The biggest difference between the two cases, aside from the behaviour of our optimized values when \( v < v^* \), is from the expected increase in \( v \) as a percentage of the current \( v \), as can be seen Figure 4.10. Its average increase is significantly higher for this value of \( \delta \), around \( 10 \times \) as much.

\[ \text{Figure 4.10: Expected increase in } v \text{ as a percentage of current } v \text{ for } \delta = 0.95 \]

As such, the dynamics that take place for these parameters are very similar to the previous case. The key differences are that, firstly, instead of selecting as the initial value of \( v \) the closest possible value of \( v > \bar{w} \), instead, the regulator opts to choose the value of \( v \) at which \( \phi \) no longer increases, i.e., \( v^* \). At this point, again, depending on the type of product the bank receives and the region of \( v \) they are in, the promised continuation value fluctuates up and down, but with a positive and increasing expected value.

\(^{22}\)Although, in this case, the value of \( \phi \) is further away from zero as our ‘reward’ phase for banks begins earlier.
This process, which moves on average around $10 \times$ faster than when $\delta$ is close to 1, eventually reaches a point where all promised continuation values are positive, and then converges to a stable phase where increases no longer depend on the product received (i.e., it is no longer erratic). Finally, it converges to the ‘reward’ phase, where the second significant difference is apparent, as this phase now starts at an earlier value of $v < \bar{v}$, with a gradually increasing $\alpha$ all the way up to 1.

Both these differences, much like the faster convergence to the ‘reward’ phase, are expected for cases when $\delta$ is not close to 1. By selecting a smaller $\delta$, we allow for a ‘non-zero’ region where $\phi$ is increasing, and as the optimal choice for regulators must be to choose to highest point of the function, that choice changes. Similarly, as banks are more impatient now, they must be more highly rewarded for not using bad, complex products, and that is achieved by having the convergence take place faster and by having the ‘reward’ phase begins at an earlier value of $v$. However, as mentioned above, the overall dynamics are not greatly changed.

We now consider the case where $\delta$ is 0.9, which greatly resembles the cases above. We plot the first four optimal results in Figure 4.11 and the expected increase in $v$ in Figure 4.12. The only significant changes relate to the speed of convergence, which is even faster as expected, and to how long the ‘erratically’ increasing portion of the dynamics lasts, which is for almost all values of $v$. 

![Figure 4.11: Optimal results for $\delta = 0.9$](image)
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4.4.2 Regulatory mechanisms without commitment and dynamic cycles

The previous section’s results depend crucially on regulators being able to make binding commitments; however, we believe that it is likely that regulators cannot make such commitments. For one thing, regulators’ beliefs that they are in a commitment regime might lead to complacency: “I made a mistake in presuming that the self-interests of organizations, specifically banks and others, were such that they were best capable of protecting their own shareholders and their equity in the firms...”\(^{23}\). Furthermore, bank regulators are likely to be unable to make renegotiation proof agreements, as they have broad powers and can impose, albeit in a lobbying-limited extent, rules on banks. Finally, even if regulators were not complacent and had the will to implement their commitments perfectly, the mechanism we find previously, although optimal from a societal perspective, results in a ‘reward’ phase where regulators need to be ‘purposefully negligent’, which we deem to be politically unfeasible, and thus unenforceable in practice.

Consequently, what would happen if we relax the commitment constraint? We proceed now to conjecture, but not prove, what we expect to find. Without commitment, regulators are unable to commit to the promise of a high return to banks in the future if they do not use complexity now, making the ‘reward’ phase for banks unfeasible. We believe this is the case as in the ‘reward’ phase, regulators must commit to never screening again, but knowing that banks are creating bad, complex products, they are tempted to deviate towards screening and, without commitment, would do so.

As such, to prevent this sort of deviation, we believe that regulators must opt to create a mechanism wherein they do not reduce screening to zero in the ‘reward’ phase, and instead can only reduce it by some degree, and that the regulators can subsequently punish the banks when they are caught out using bad products, by increasing screening again. This would allow

\(^{23}\)Alan Greenspan to the House oversight committee, 2008
for a credible mechanism, as regulators do not wish to deviate as much, due to their low-but-not-zero levels of screening would lead to a lower use of complexity by banks, and will lead to a ‘reward’ phase for the regulators themselves when screening increases again.

We thus conjecture that this mechanism would consist of having the regulator start screening financial products carefully, in a high $\gamma$ phase, with the bank making low use of complexity with bad products. As more time passes without the detection of complex, bad financial products, the regulator slowly reduces the amount of screening it undertakes, while the bank slowly increases the rate at which it makes use of complexity, which corresponds to the ‘reward’ phase of the optimal mechanism with commitment. This laxer regime, however, will at some point lead to a bad and complex product being detected, either through a direct screening or through a product failure. Once either case happens, it subsequently triggers a ‘punishment’ phase, and regulators return to high levels of screening and banks reduce the amount of bad complex products they create.

These dynamics would be optimal as, much like in the commitment mechanism, they creates incentives for banks to not make bad financial assets complex via its ‘punishment’ and ‘reward’ phases. Banks will wish to create few complex bad products, as this allows them to enter a period of temporarily lax screening, and banks know that they will be punished by more screening from regulators if they choose to deviate. If correct, this means that the dynamics in this sort of mechanism would be cyclical, with periods of strict screening and few bad products that are unlikely to lead to bailouts, and periods of lax screening where bad products are unlikely to be detected and, as a consequence, the probability of a bailout/crisis increases. This seems to correspond to how financial regulation and financial crises happen in reality.

Note also that if this is the optimal mechanism, although it would have a smaller net-present value than the mechanism with commitment for regulators, it would still be an improvement over just having the mixed-strategy Nash Equilibrium PBE. This would be because it would both reduce the total cost of screening of regulators, since they would need to screen less frequently\(^{24}\), and it would reduce the probability, and thus the total cost, of bailouts.

4.5 Discussion

As is the case with many theoretical papers, we have made many simplifying and stylized assumptions that we might wish to explore further in the context of the model. In this section, we discuss these assumptions and how our results would change if we altered them: the assumption that regulators are unable to use fines/transfers to punish/reward banks, the assumption that regulators and banks have equal discount factors and the assumption that there is a single bank for regulators to screen.

4.5.1 Transfers and Fines

As we discuss in the introduction, our model does not include the possibility of direct punishments or rewards to banks, such as via fines. In the context of dynamic games with one-sided private information, when it possible to make use of such transfers, the optimal

\(^{24}\)Indeed, we surmise that the same restriction on $\gamma$, that it must be smaller or equal to $\gamma^{PBE}$, will hold in this mechanism.
mechanism normally becomes a static one, as can be seen in the more general setting of Levin (2003). This is a consequence of being able to make these transfers to and from the regulator and banks conditional on specific outcomes, and an optimal mechanism would then incentivise banks to adopt no screening at any point by providing sufficient rewards for not deviating and/or punishment for deviating.

However, as we discuss in the introduction, we do not believe that is a realistic assumption in the case of our model. Begin with the issue of whether bankers face ex-post punishment for creating deliberately complex, bad products. In the aftermath of the financial crisis of 2008, very few bankers were directly punished, via the criminal justice system. For example, the New York Times reported in 2014 (Eisinger, 2014), that only 1 Wall Street executive had been convicted due to their actions prior to and during the financial crisis\textsuperscript{25}, and a similar pattern seems holds in most major economies, such as the UK. Furthermore, in the immediate aftermath of a bank bailout, as was seen in the case of AIG, it can be necessary to maintain the individuals directly responsible for creating the bad, complex products at the head of the bailed out firms. Indeed, it can even be necessary to give them high bonuses at such times, as their knowledge of these products can subsequently help minimize the costs of the bailout itself (Sorkin, 2010).

Therefore, incentives for individual bankers to engage in moral hazard behaviour has been a problem and is likely to still exist. Aside from the criminal justice system, bankers have limited liability in case things go wrong and the maximum punishment they then face is being fired. Indeed, regulators, recognizing this problem, have started to try to change this; to take one example, the PRA has created rules wherein senior bankers now face deferred variable remuneration, such that the deferred payments these bankers are due to receive may be less if banks suffer losses in subsequent periods, even if the banker has left his post. The effectiveness of these rules has yet to be tested, however.

So, if individual bankers are unlikely to face punishment post-fact, the question then is whether this is true for banks themselves, where the answer is a bit more ambiguous. In a strict sense of timing, such punishments are not possible, as it is precisely during a bailout/crisis that money has to be put into the banks, making punishments such as fines impossible. Once past the crisis, however, it might be possible to then levy fines, but the evidence that this happens and has happened in the past is mixed at best.

Concerning the most recent crisis, banks have paid fines due to their misconduct in mortgage products, such that total fines paid by US banks to financial regulators due to mortgages and mortgage derivatives in the 8 years since the crisis has totaled around 80 billion dollars\textsuperscript{26}. However, these fines are perhaps not as large as they seem, if compared to total profits of the banking sector in the US of around 660 billion dollar earned in 4 and half years from 2010\textsuperscript{27} or the around 450 billion dollars used in the Troubled Asset Relief Program (TARP), which itself was used mainly to bailout banks.

Equally important, it should be noted that these (relatively) large fines are somewhat unprecedented if compared to the Savings and Loan Crisis, the most recent, previous ‘financial

\textsuperscript{25}Although the same article does stress that in at least one previous financial crisis, the Savings and Loans crisis, more people working in the financial sector were jailed, although these were ‘low ranking’ individuals and not high ranking Wall Street bankers.

\textsuperscript{26}Authors’ estimates using data from the Financial Times, http://blogs.ft.com/ftdata/2015/07/22/bank-fines-data/

bailout’ crisis in the US. In that crisis, fines were little applied, even though the total cost to the US government was over 120 billion dollars (Curry and Shibut, 2000). As such, the evidence that regulators can use fines as an effective punishment mechanism post-bailouts is mixed at best, as although fines have been imposed since 2008, these fines are a relatively new development, and their size may not be large enough to discourage future negative behaviour.

Finally, if bankers and banks cannot be punished directly post-fact, can they instead be rewarded directly for good behaviour? We believe that this cannot be the case, as in models that have optimal mechanisms using positive transfers, regulators have to reward agents they regulate with positive transfers that are multiples of the size of the the payoff in case of an negative shock. As the TARP involved 450 billion dollars and the Savings and Loan Crisis cost 120 billion dollars, this would involve sums of money that are, for both practical and political reasons, simply unfeasible.28

4.5.2 Different discount factors by banks and regulators

In our model, we assume that both regulators and banks have the same discount factor. However, in practice, given the incentives that banks face for short term results, there is a possibility that governments and regulators might be more patient than the banks they regulate, which although an unorthodox assumption, can produce interesting results. To explore this possibility, we focus our attention on the FOC with respect to $\gamma$ when $\alpha$ is equal to 1, in Section 4.4.

Note that the term of interest is $\frac{(1-\mu)(1-\theta)R-C}{(1-\mu)BM}$, and that the $\delta$ term has been canceled out from the numerator and denominator. Yet this cancelling has only happened because regulators and banks are assumed to have the same discount factor, such that if the regulators and banks have different $\delta$s, then the this FOC becomes $\frac{\delta_B \delta_R}{\delta_R} \frac{(1-\mu)(1-\theta)R-C}{(1-\mu)BM}$, where $\delta_R$ and $\delta_B$ are the regulator’s and bank’s discount factor respectively.

Note that this means that this FOC term will be, if the regulator is more patient than the bank, now smaller than previously, and also note that our results for when $\alpha = 0$ remain unchanged. As a consequence, if this $\frac{\delta_B \delta_R}{\delta_R}$ is small enough, then the value of $-\phi'(w_*)$ falls such that the promised continuation value upon $\alpha = 1$ falls into the region of $\alpha = 0$. Consequently, we create a cycle of complexity and no complexity by the part of the banks even if the regulator has full commitment.

4.5.3 More than one bank

Finally, we assume that only one bank is being regulated in our model, and we do not discuss how a bailout to one individual bank can be analogous to a financial crisis. To address both issues, we must first discuss how exactly the way we model our financial products translates into how they work in practice, as we have, deliberately, kept this aspect abstract so far.

Our model is kept simple by assuming that banks are creating and holding on to products. However, it is possible to interpret the model differently and assume that it means that banks are not just creating products and then holding on to them in their balance sheets, but that

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28One possibility, however, which lies outside the framework in this paper, is for regulators to make better use of whistleblowers.

29See Terry (2015), for example.
these products can and do spread out into the more general financial system. For one thing, new products, if perceived to be successful, can rarely be kept a secret in financial markets for very long. So it can be possible to interpret our model in such a way that these products can and are likely to be held on to the creating bank’s balance sheet, but that they are also sold to clients, some of which are banks themselves, such that if a bank is holding on to a particular asset, than other banks might choose to replicate this strategy. That is to say, once the ‘genie is out of the bottle’, and a bad product is left unscreened by regulators, that product will subsequently spread out, and banks have incentives to maintain regulators ignorant of its true nature. In this set-up, with multiple banks, this translates into correlated product quality shocks $\mu$.

With this in mind, an immediate extension of this model to a set-up with multiple banks is to assume that both product quality shocks $\mu$ and payoff shocks $\theta$ are perfectly correlated across banks. In such a case, then our model results are unchanged and there is no difference between the one bank and multiple banks case. Then the risk that individual banks face is effectively an aggregate risk for the economy, and a bailout of a individual bank in our model is akin to a more general bailout of the financial sector in a financial crisis. We believe that this a likely scenario for many new types of financial products, especially when institutions that did not create the product and have not fully screened it/understood its risks, choose to make use of it nevertheless, due to the moral hazard incentives they face, which justifies a high product quality correlation. Additionally, as was largely the case of MBS and CDS products in the subprime mortgage crisis, in many cases these products returns face aggregate risk which explains the correlations in payoffs.

It is less immediate, however, if $\mu$ is still very highly correlated but the correlation of the payoff $\theta$ of new products is lower or even non-existent, which might be the case for many other classes of financial products in practice. In this case, an individual failure of one bank will indicate which other banks also have a bad complex product. This means that the problem regulators face changes, as regulators now receive new information about the quality of a product which multiple banks hold after bailing out only one bank. We conjecture that, as in Levin (2002), the optimal mechanism in this case would result in a collective punishment/heightened scrutiny towards all banks holding that failed product, and that regulatory cycles akin to the ones we have described previously might thus happen, even without an aggregate financial crisis.

Thus, our assumption of having a single bank, which helps keep the model tractable, can likely be generalized into results that closely resemble our own.

### 4.6 Conclusion

Bank bailouts and limited liability create incentives for banks to evade oversight of regulators so as to use risky financial products which can payout substantially if the bet on them works, but leaves regulators with large costs if the gamble fails. One way banks might evade regulations is by deliberately making such risky products complex, making it difficult to fully grasp the possible range of outcomes a product has, while knowing the costs that regulators

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30Even if the expertise to fully understand these products might be.

31In particular, the multi-bank public history of the game is the same as the individual banks’ histories.
4.6. CONCLUSION

face when trying to determine the nature of financial products and the limited resources they have.

Consequently, regulators are faced with the challenge of designing an optimal regulatory regime that minimizes the possibility of bailouts without incurring substantial screening costs, especially as two possible methods to incentivise banks, the punishment of banks when things go wrong, and positive transfers when things go right, are difficult to use in practice. The former would involve fines of banks at the same time as bailouts are happening, thus making them impossible, whereas the latter would involve transfers many times the size of bank bailouts and, thus, probably unfeasible in practice.

With these limitations in mind, we find that when regulators can make binding commitments, the optimal regulatory regime results in, initially, banks not making bad financial products complex and regulators not having to screen, and this no screening and no bad, complex products regime lasts for a considerable amount of time. Banks choose not to deviate towards complexity because regulators can commit towards a ‘punishment’ phase and a ‘reward’ phase should banks choose to deviate or not deviate. The ‘reward’ phase, in particular, consists of allowing banks to make bad products complex without regulators ever screening, which increases the payoff of banks, and once it is triggered, lasts until the end of time. However, this ‘reward’ phase also results in the highest probability that banks will have to be bailed out, henceforth, and the ‘reward’ phase starts at the end of the no screening and no bad, complex products regime.

We discuss why such binding commitments are likely not possible for regulators in practice, among other reasons because regulators would have incentives to deviate in the ‘reward’ phase. Instead, we conjecture, but do not prove, that the optimal regulatory regime when there is no commitment by regulators should involve cycles of strong/weak regulatory oversight and cycles of banks using and refraining from using complexity on bad products, which, if correct, would still be better than the PBE of Nash Equilibrium. We also argue that such cycles may prevail if regulators are sufficiently more patient than banks.

Our results thus highlight how constrained financial regulators are with their current tools to incentivise banks towards good behaviour, such that, given the current institutional set-up, bailouts and financial crises may be to some extent an inevitable part of the financial system. This suggests that regulators may have to make even more significant changes to the financial system, if they wish to minimize further the chances of crises. Along with some of the possibilities we discuss above, such as whistle-blowers and deferred remuneration, regulators may wish to increase the scope for punishments of banks which are not just direct fines.
Appendices to the Chapter

4.A Proofs

Proof of Lemma 5.
Compactness of $\mathcal{E}$ is a direct consequence of theorem 4 in Abreu, Pearce and Stacchetti (1990). The convexity of $\mathcal{E}$ is a consequence of the existence of a public randomization device $x$, and the fact that the set of payoffs feasible in equilibria under the presence of such public randomization devices is convex (as is usual under correlated equilibria). From the compactness and convexity of the equilibrium payoff set, we can assume that if $(\phi, v) \in \mathcal{E}$ (that is, $\phi, v$ are equilibrium present value payoffs for the regulator and the bank, respectively), then $v \in [w, \bar{w}]$ for some $w, \bar{w}$.

To prove that $\phi(v)$ is concave, assume otherwise, that $\phi(v)$ is strictly convex in an interval $[w, w^*] \in [w, \bar{w}]$, with $v \in (w, w^*)$. Let $b \in (0, 1)$ be such that $bw_s + (1 - b)w^* = v$. Given that $\phi(v)$ is not concave, deviate from equilibrium towards delivering the promised value of $v$ by delivering $w_s$ with probability $b$ and $w^*$ with probability $1 - b$ (which is feasible because of the existence of the public randomization device $x$). Given that $w_s, w^* \in [w, \bar{w}]$, then it is clear that it is feasible to deliver these as equilibrium payoffs to the bank. Moreover, note that $\phi(w_s), \phi(w^*)$ are equilibrium payoffs for the regulator, since the definition of the function $\phi(v)$ requires that $\phi(v) \in \mathcal{E}$.

As a consequence, this deviation satisfies all equilibrium conditions. However, this delivers an expected payoff for the regulator of $b\phi(w_s) + (1 - b)\phi(w^*) > \phi(bw_s + (1 - b)w^*) = \phi(v)$, contradicting the definition of $\phi(v)$ as $\phi(v) = \sup \{ \phi : (\phi, v) \in \mathcal{E} \}$, as there exists an alternative $(\phi, v) \in \mathcal{E}$ with $\phi > \phi(v)$. Thus $\phi(v)$ is (at least) weakly concave.

Finally, we now prove that given the equilibrium values $\alpha^*, \gamma^*$ and $w(c, \gamma^*, s, p), \phi(c, \gamma^*, s, p) = \phi(w(c, \gamma^*, s, p))$. Assume otherwise, that the solution to the problem $\phi(v)$ specifies actions $\alpha^*, \gamma^*$ in the current period, continuation payoffs $w(c, \gamma, s, p)$ for all $\gamma$ to the bank, and delivers continuation payoffs $\phi(c, \gamma, s, p)$ to the regulator, with $\phi(c, \gamma^*, s, p) < \phi(w(c, \gamma^*, s, p))$, that is, the continuation values for the regulator on the equilibrium path are not the maximum attainable continuation values for the regulator (in particular, for $\gamma^*$).

Then, deviate by keeping $w(c, \gamma^*, s, p), \gamma^*, \alpha^*$ constant, but changing the continuation payoffs for the regulator from $\phi$ towards $\phi^*(c, \gamma, s, p) = \phi(c, \gamma, s, p)$ for $\gamma \neq \gamma^*$ and $\phi^*(c, \gamma^*, s, p) = \phi(w(c, \gamma^*, s, p))$. As the $w$ have not changed, $IC_B$, $PKC$ and $w \in [w, \bar{w}]$ must still hold. Moreover, since we increased continuation payoffs for the regulator after $\gamma^*$, without changing the payoffs after $\gamma \neq \gamma^*$, it must be that the regulator has weakly more incentives to choose $\gamma^*$ due to the weakly increased payoff, and hence $IC_R$ still holds. So in this deviation, all constraints still hold. However, as the optimal choice of the regulator is still $\gamma^*$ (because $IC_R$ still holds) and the optimal choice of the banks is still $\alpha^*$, then the flow payoff to the regulator has not changed, but the continuation payoffs for the regulator has increased after the proposed deviation. This then contradicts the optimality of $\phi(v)$, that was delivered with less than optimal continuation payoffs for the regulator on the equilibrium path, thus $\phi(c, \gamma^*, s, p) = \phi(w(c, \gamma^*, s, p))$.

Proof of Lemma 6.
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Assume that it is instead optimal to set

$$(1 - \delta)[-D + (1 - \gamma)\theta M] + \delta E_{\gamma,p}[w(1, \gamma, s, p)\mid B] > \delta E_{\gamma,p}[w(0, \gamma, s, p)\mid B]$$

that is, to provide strict incentives for $\alpha = 1$. Consider then, firstly, the case where there exists two states $(c = 1, \gamma, s, p)$ and $(c = 1, \gamma, s', p') \neq (c = 1, \gamma, s, p)$ such that $w(1, \gamma, s, p) \neq w(1, \gamma, s', p')$. Let $w^{AVG}$ be the expected continuation value obtained by the bank, unconditionally on the quality of the product but conditional on $c = 1$ (notice that with $\alpha = 1$, implied by the assumption on $IC_B$ made in the proof, $w^{AVG} = \mu E_{\gamma}[w(1, \gamma, s, M)\mid G] + (1 - \mu) E_{\gamma,p}[w(1, \gamma, s, p)\mid B]$). Moreover, take a number $\kappa \in (0, 1)$, for $\epsilon$ small. Then, deviate by implementing $w(1, \gamma, s, p)$ with probability $1 - \kappa$, and by implementing $w(1, \gamma, s, p) = w^{AVG}$ with probability $\kappa$. Keep $w(0, \gamma, B, \emptyset)$ (the continuation payoff after no complexity) unchanged.

Given that we have assumed that the bank has strict incentives to set $\alpha = 1$, then for $\kappa$ small enough, we will have that:

$$(1 - \delta)[-D + (1 - \gamma)\theta M] + \delta(1 - \kappa)E_{\gamma,p}[w(1, \gamma, s, p)\mid B] + \kappa w^{AVG} \geq \delta E_{\gamma,p}[w(0, \gamma, s, p)\mid B]$$

which implies $IC_B$ still holds. Moreover, since we give the bank the same expected continuation value as before with probability $1 - \kappa$, and give him the average continuation value with probability $\kappa$ (notice that since $\alpha = 1$, we never give the bank the continuation value after no complexity), this must mean that the average continuation value did not change, so that the bank must be receiving the same payoffs and PKC must still hold. Moreover, by concavity of $\phi(\sigma)$, notice that with probability $\kappa$, we are giving the bank the same expected continuation value but with no variance, so with probability $\kappa$, the regulator’s continuation value after complexity must weakly improve. Moreover, with probability $1 - \kappa$, the regulator’s continuation value after complexity does not change, since we keep giving the bank the same continuation values as before in these events. As a consequence, the continuation value for the regulator weakly improves, relaxing $IC_B$ and increasing the value of the objective function, without changing $\alpha, \gamma$. Finally, this deviation must obviously be feasible, since it must be that if $w(1, \gamma, s, p) \in [w, \bar{w}]$, then $w^{AVG} \in [w, \bar{w}]$. As a consequence, it is clear that this deviation is feasible, and weakly more profitable than the conjectured original regulatory mechanism, contradicting the optimality of the conjectured mechanism, and showing that either (i) the bank must have weakly more incentives for $c = 0$ as opposed to $c = 1$ or (ii) given $c = 1$, we must have $w(1, \gamma, s, p)$ constant.

Assume then that it is optimal to set strict incentives for $c = 1$ and $w(1, \gamma, s, p) = w^*$ is constant across states $s, p$ (or, to set $\delta w(0, \gamma, B, \emptyset) < (1 - \delta)[-D + (1 - \gamma)\theta M] + \delta w^*$). Deviate towards setting $\delta \bar{w}(0, \gamma, B, \emptyset) = (1 - \delta)[-D + (1 - \gamma)\theta M] + \delta w^*$, while keeping $\alpha, \gamma, w^*$ unchanged. Notice that $w(0, \gamma, B, \emptyset)$ does not affect the regulator’s payoff nor the bank’s payoff (since state $c, \gamma, s, p = 0, \gamma, B, \emptyset$ happens with zero probability), so that changing $w(0, \gamma, B, \emptyset)$ leaves the regulator indifferent and so that PKC must still hold. Since $IC_R$ only matters after complexity, $w(0, \gamma, B, \emptyset)$ does not affect $IC_R$. Moreover, $\bar{w}(0, \gamma, B, \emptyset) > w(0, \gamma, B, \emptyset) \geq \bar{w}$, so that the lower bound on continuation values is satisfied. As long as $\bar{w}(0, \gamma, B, \emptyset) = \bar{w}$, this deviation is thus feasible and does not affect payoffs, so without loss of generality, we can take $IC_B$ to give weak incentives for $\alpha = 0$, or to set $w(0, \gamma, B, \emptyset) = \bar{w}$. ■

Proof of Lemma 7.
As mentioned in the text, \( w \leq \mu M \) because playing the static PBE in all periods yields discounted present payoff to the bank of \( \mu M \). Similarly, \( w \geq \mu M \) because if the bank got a lower payoff than this, the bank could always deviate towards playing \( a = 0 \) whenever his product is bad, and this alternative strategy would yield a discounted present value payoff of \( \mu M \). Hence, \( w = \mu M \).

As for \( \phi \), notice that \( \phi \leq -\frac{\mu(1-\theta)RC}{(1-\theta)R-C} \), since it is feasible to get \(-[\mu+(1-\mu)a^{PBE}]\gamma^{PBE}C-(1-\mu)a^{PBE}(1-\gamma^{PBE})(1-\theta)R = -\frac{\mu(1-\theta)RC}{(1-\theta)R-C} \) as a payoff to the regulator by playing the static PBE in all periods. \( \phi \geq -C \) because if the regulator was getting something worse than this, he could always deviate towards always monitoring complex products (or \( \gamma = 1 \)), which yields discounted present value payoff \(-C[\mu+(1-\mu)a] \) (note that when the regulator always monitors, he never allows bad products to fail). The lowest value \(-C[\mu+(1-\mu)a] \) can attain is \(-C \) (also note that \(-C = \min_a \max_\gamma -\gamma C[\mu+(1-\mu)a]-(1-\gamma)(1-\alpha)(1-\theta)R, \) so that \(-C \) is the minmax payoff for the regulator).

On the other hand, \( \bar{w} \leq \mu M+(1-\mu)\{\{D+\theta M\} \) after all, the bank will always get a payoff of \( M \) after good products, and cannot get a payoff higher than \( -D+\theta M \) after bad products. If there is commitment, sustaining \( \bar{w} = \mu M+(1-\mu)\{\{D+\theta M\} \) as a promise is feasible: after all, prescribe \( \alpha = 1, \gamma = 0, \) and \( w(c,\gamma,s,p) = \bar{w} \). Then, IC\(_B\) is trivially satisfied, since \((1-\delta)(-D+\gamma\theta M) + \delta \bar{w} = (1-\delta)(-D+\theta M) + \delta \bar{w} > \delta \bar{w}. \) PKC is also trivially satisfied after a promise of \( \bar{w}, \) since \( \bar{w} = \bar{v} = (1-\delta)\{M+(1-\mu)\{\{D+\gamma\theta M\} + \delta \bar{w} = (1-\delta)\{M+(1-\mu)\{\{D+\theta M\} + \delta \bar{w} \), which implies \( \bar{w} = M+(1-\mu)\{\{D+\theta M\} as conjectured. All continuation values are trivially \( w(c,\gamma,s,p) = \bar{w} = (\bar{w},\bar{w}). \) With commitment, IC\(_R\) is not present, so that this then satisfies all constraints.

Without commitment, on the other hand, it is not feasible to deliver \( \bar{w} \) because IC\(_R\) is violated. After all, the only way to implement a discounted present value payoff to the bank of \( \mu M+(1-\mu)\{\{D+\theta M\} is to set, on the equilibrium path, \( \alpha = 1 \) and \( \gamma = 0 \) for all periods. However, under this scenario, the regulator is getting a discounted present value payoff of \(-\mu M\{\{D+\theta M\} < -C \) from assumption 1. Hence, if the regulator unilaterally deviates towards monitoring in all periods the product comes complex, he gets a discounted present value payoff of \(-C \), and this is a profitable deviation. This in turn shows that without commitment, it is not feasible to deliver payoff \( \mu M+(1-\mu)\{\{D+\theta M\} to the bank. \)

Proof of Lemma 8.

**Step 1: A relaxed problem** - Consider the following two maximization problems: the first one is simply:

\[
\phi(v, \gamma) = \mu \{(1-\delta)(-\gamma C) + \delta E_\gamma[\phi(w(1,\gamma,s,M), \gamma)|G]\}
\]

\[
\max_a \{w(c,\gamma,s,p)|w(c,\gamma,s,p)\} \{w(c,\gamma,s,p,|\gamma)|\gamma)\} + (1-\mu)\{\{1-\alpha\}((1-\delta)0 + \delta \phi(w(0,\gamma,1,\Omega), \gamma)\}
\]

s.t. PKC, IC\(_B\), IC\(_R\), \( w(c,\gamma,s,p) \in [\bar{w}, \bar{w}], \phi(c,\gamma,s,p) = \phi(w(c,\gamma,s,p, \gamma)\}

This problem is of interest because \( \phi(v) = \arg \max_\gamma \{\phi(v, \gamma) : \exists eq. with value v for bank, level \gamma of monitoring\} = \arg \max_\gamma \{\phi(v, \gamma) : solution to \phi(v, \gamma) exists\}. \) With this in mind,
we are only interested in problem $\phi(v, \gamma)$ when there is an equilibrium with value $v$ for the bank when the level of monitoring is $\gamma$. Throughout, we will typically denote the set of optimal $\gamma$ in the problem $\phi(v)$ as $\gamma^*(v)$.

The second problem we consider relaxes the promise keeping constraint in the problem above:

$$\hat{\phi}(v, \gamma) = \max_{\{w(c,\gamma,s,p)\}_{\gamma(s)} \land (\gamma,s,p)} \left\{ \begin{array}{l}
\mu \{ (1 - \delta)(-\gamma C + \delta E_\gamma [\phi(w(1, \gamma, s, M), \gamma) | G]) \\
+ (1 - \mu) \{(1 - a)(1 - \delta)0 + \delta \phi(w(0, \gamma, 1, \mathcal{Q}, \gamma)) \\
\} + a[(1 - \delta)(-\gamma C - (1 - \gamma)(1 - \theta)R) + \delta E_\gamma [\phi(w(1, \gamma, s, p), \gamma) | B]) \\
\} + \delta \{ \mu E_{a,\gamma,p}[w(c, \gamma, s, p) | G] + (1 - \mu) E_{a,\gamma,p}[w(c, \gamma, s, p) | B] \}
\end{array} \right.
\] s.t. $v \leq (1 - \delta) \{ \mu M + (1 - \mu) a[-D + (1 - \gamma)\theta M] \}
+ \delta \{ \mu E_{a,\gamma,p}[w(c, \gamma, s, p) | G] + (1 - \mu) E_{a,\gamma,p}[w(c, \gamma, s, p) | B] \}$,

That is, we are taking problem $\phi(v, \gamma)$ and firstly, relaxing PKC, and secondly, letting the regulator have continuation payoffs $\hat{\phi}(v, \gamma)$ as opposed to $\phi(v, \gamma)$.

Let a feasible solution to the relaxed problem above be given by a function $\hat{\phi}(v, \gamma)$, strategy $a$ and continuation values $\{w(c, \gamma, s, p)\}_{\gamma(s)} \land (\gamma,s,p)$ such that all constraints hold. That is, a feasible solution is one where all constraints hold and the recursive equation for the regulator’s payoff is satisfied.

**Step 2: Properties of the relaxed problem** - First, notice that the solution to our original problem specified in the main text is a feasible solution to the relaxed problem shown in this proof, and as a consequence, it must be that $\hat{\phi}(v, \gamma) \geq \phi(v, \gamma)$. Secondly, notice that $\hat{\phi}(v, \gamma)$ must be weakly decreasing in $v$: after all, for $v < v'$, notice that any solution feasible for the problem $v'$ is also feasible for problem $v$.

Thirdly, if we find that there exists a solution to problem $\hat{\phi}(v, \gamma)$ with PKC binding, then this solution is feasible for problem $\phi(v, \gamma)$. Hence, if a given solution to $\hat{\phi}(v, \gamma)$ has binding PKC, then there exists a set of continuation values and a choice of $a$ that yields value $v$ to the bank (since PKC is binding) when the level of monitoring is $\gamma$, so that there is an equilibrium with value $v$ for the bank and $\gamma$ monitoring.

In this case, if the solution to $\hat{\phi}(v, \gamma)$ has binding PKC for all $\gamma$ that allow for equilibrium with value $v$ for the bank, then the optimal solution to problem $\hat{\phi}(v, \gamma^*(v))$ specifies the optimal $a, w(\cdot)$ for problem $\phi(v)$.

**Step 3.1:** If, for a given $v$, we have weakly more incentives for $c = 1$, continuation values $w(1, \gamma, s, p) = w$ and PKC is still non-binding, then there is no equilibrium with value $v$ for the bank and level $\gamma$ of monitoring - To see this, note that if we have weakly more incentives
for \( c = 1 \) and \( w(1, \gamma, s, p) = w \), we must have that:

\[
(1 - \delta)[-D + (1 - \gamma)\theta M] + \delta w \geq \delta w(0, \gamma, B, \emptyset)
\]

Moreover, due to this indifference, a non-binding PKC can be re-written as:

\[
v < (1 - \delta)\{\mu M + (1 - \mu)[-D + (1 - \gamma)\theta M]\} + \delta w
\]

Note, however, that any equilibrium where \( \gamma \) is played yields payoff to the bank:

\[
(1 - \delta)\{\mu M + (1 - \mu)\alpha[-D + (1 - \gamma)\theta M]\} + \delta E[w(c, \gamma, s, p)] \geq
\]

\[
(1 - \delta)\{\mu M + (1 - \mu)[-D + (1 - \gamma)\theta M]\} + \delta w
\]

where the inequality comes from (i) \( IC_B \) giving weakly more incentives for complexity and (ii) from \( w(\cdot) \geq w \). But then, this implies that if we find that whenever \( w(1, \gamma, s, p) = w \) and \( IC_B \) binding, PKC is non-binding, then there is no equilibrium with level of monitoring \( \gamma \) yielding value \( v \) to the bank.

**Step 3.1:** For a given level of \( v \), if there is an equilibrium with weakly more incentives for \( c = 1 \) and value \( v \) for the bank, then \( \gamma \geq \max \{ \gamma^{PBE} - \frac{v - \mu M}{(1 - \delta)(1 - \mu)\theta M}, 0 \} \) (or, if there are weakly more incentives for \( c = 1 \), then \( \gamma \geq \max \{ \gamma^{PBE} - \frac{v - \mu M}{(1 - \delta)(1 - \mu)\theta M}, 0 \} \) is a necessary condition for there to exist an equilibrium with \( \gamma \) monitoring and value \( v \) for the bank) - Note that with weakly more incentives for \( c = 1 \), we have:

\[
(1 - \delta)[-D + (1 - \gamma)\theta M] + \delta E[w(1, \gamma, s, p)|B] \geq \delta w(0, \gamma, B, \emptyset)
\]

Moreover, in this case, we can write PKC as:

\[
\gamma = \frac{\mu}{(1 - \mu)\theta} + 1 - \frac{D}{\theta M} + \frac{\delta E_{1,p}[w(1, \gamma, s, p)] - v}{(1 - \delta)(1 - \mu)\theta M}
\]

\[
\geq \max \left\{ \frac{(1 - \delta)\mu M}{(1 - \delta)(1 - \mu)\theta M} + 1 - \frac{D}{\theta M} + \frac{\delta w - v}{(1 - \delta)(1 - \mu)\theta M}, 0 \right\}
\]

\[
= \max \left\{ \gamma^{PBE} - \frac{v - \mu M}{(1 - \delta)(1 - \mu)\theta M}, 0 \right\}
\]

For \( v = \mu M = w \), this states that \( \gamma \geq \gamma^{PBE} \), while for \( v \rightarrow \mu M + (1 - \mu)[-D + \theta M] \) (an upper bound on \( \bar{w} \)), this suggests \( \gamma \geq -\frac{\delta - (1 - \delta)}{(1 - \delta)} \gamma^{PBE} \). Finally, this lower limit on \( \gamma \) is 0 when \( \mu M + \gamma^{PBE}(1 - \delta)(1 - \mu)\theta M = \mu M + (1 - \delta)(1 - \mu)[-D + \theta M] = v \)

**Step 4:** For all \( \gamma \leq \gamma^{PBE} \) and \( \gamma \geq \max \{ \gamma^{PBE} - \frac{v - \mu M}{(1 - \delta)(1 - \mu)\theta M}, 0 \} \), if there are weak incentives for \( \alpha = 0 \), then it is optimal in problem \( \hat{\phi}(v, \gamma) \) to set the promise keeping constraint to be binding - If there are weak incentives for \( \alpha = 0 \), we must have:

\[
(1 - \delta)[-D + (1 - \gamma)\theta M] + \delta E_{1,p}[w(1, \gamma, s, p)|B] \leq \delta w(0, \gamma, B, \emptyset)
\]

With \( \gamma \leq \gamma^{PBE} \), \(-D + (1 - \gamma)\theta M \geq 0\), so that we must have \( E_{1,p}[w(1, \gamma, s, p)] \leq \delta w(0, \gamma, B, \emptyset) \). Moreover, we have \( E_{1,p}[w(1, \gamma, s, p)] < \delta w(0, \gamma, B, \emptyset) \) if either \( \gamma < \gamma^{PBE} \) or \( IC_B \) is non-binding. But then, it must be that at least \( \delta w(0, \gamma, B, \emptyset) > w \).

With this in mind, assume for a contradiction that it is optimal to set \( \gamma \leq \gamma^{PBE} \) but to set
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PKC not to be binding. Then, first note that if \( w(1, \gamma, G, M) > \bar{w} \), we can reduce \( w(1, \gamma, G, M) \) by \( \epsilon \), which will still satisfy the relaxed PKC for \( \epsilon \) small enough; this will not change \( IC_B \) (since \( (c, \gamma, s, p) = (1, \gamma, G, M) \) never happens when the product is bad, so it does not affect the incentives for banks with bad products); it will relax the modified \( IC_R \) in the relaxed problem by weakly increasing \( \hat{\phi}(w(1, \gamma, G, M), \gamma) \) (and hence, it will increase the payoff the regulator gets by choosing \( \gamma \) as opposed to \( \hat{\gamma} \neq \gamma \), and will still satisfy \( w(\cdot) \in [\bar{w}, \bar{w}] \) for \( \epsilon \) small enough. Yet, this will weakly increase the objective function by weakly increasing \( \hat{\phi}(w(1, \gamma, G, M), \gamma) \), while not changing anything else.

If \( w(1, \gamma, G, M) = \bar{w} \) but there is some other state \((c, \gamma, s, p) = (1, \gamma, s^*, p^*) \) such that \( w(1, \gamma, s^*, p^*) > \bar{w} \), then we can deviate by reducing \( w(0, \gamma, s, p) \) by \( \delta \) and reducing \( w(1, \gamma, s^*, p^*) \) by \( \epsilon \), in a way that keeps \( E, \bar{\gamma}, \bar{p} \) \((w(1, \gamma, s, p), B) - w(0, \gamma, B, \emptyset) \) constant. By construction, this will keep \( IC_B \) holding. For \( \epsilon \) small enough, \( \delta \) will also be small enough and relaxed PKC will still be satisfied; and we will also have \( w(\cdot) \in [\bar{w}, \bar{w}] \). Finally, this will relax \( IC_R \) by weakly increasing \( \hat{\phi}(w(\cdot), \gamma) \). Hence, this deviation is feasible, while weakly increasing the objective function by increasing \( \hat{\phi}(\cdot) \).

Finally, if \( w(1, \gamma, s, p) = \bar{w} \) for all \( s, p \), then if \( IC_B \) is non-binding (that is, \( \delta w(0, \gamma, B, \emptyset) > (1 - \delta) [ -D + (1 - \gamma) \theta M] + \delta \bar{w} \), we can deviate by reducing \( w(0, \gamma, B, \emptyset) \) by \( \epsilon \) while still satisfying \( IC_B \) and relaxed PKC (as long as \( \epsilon \) is small enough); we will still satisfy \( w(0, \gamma, B, \emptyset) \geq \bar{w} \) after the deviation for \( \epsilon \) small enough (since \( \gamma \leq \gamma^{PBE} \) and non-binding \( IC_B \) implies \( w(0, \gamma, B, \emptyset) > \bar{w} \)), this will not affect \( IC_R \) since \( IC_R \) only matters after complex products show up, and this will weakly increase the objective function for the regulator by weakly increasing \( \hat{\phi}(w(0, \gamma, B, \emptyset), \gamma) \) and changing nothing else.

If \( w(1, \gamma, s, p) = \bar{w} \) for all \( s, p \), \( IC_B \) is binding, then \( \gamma \geq \max \left\{ \gamma^{PBE} - \frac{\bar{v} - \mu M}{(1 - \delta)(1 - \mu) \theta M}, 0 \right\} \) guarantees either PKC is binding or relaxed PKC is violated, since
\[
\gamma \geq \frac{\max \left\{ \gamma^{PBE} - \frac{\bar{v} - \mu M}{(1 - \delta)(1 - \mu) \theta M}, 0 \right\}}{\gamma^{PBE} - \frac{\bar{v} - \mu M}{(1 - \delta)(1 - \mu) \theta M}} \geq \gamma^{PBE} - \frac{\bar{v} - \mu M}{(1 - \delta)(1 - \mu) \theta M}
\]
which imply that (using the formula for \( \gamma^{PBE} \)):
\[
\bar{v} \geq (1 - \delta) \{ (1 - \mu) [-D + (1 - \gamma) \theta M] + \mu M \} + \delta \mu M
\]
\[
= (1 - \delta) \{ \mu M + (1 - \mu) [-D + (1 - \gamma) \theta M] \} + \delta \bar{w} \]
\[
= (1 - \delta) \{ \mu M + (1 - \mu) [-D + (1 - \gamma) \theta M] \} + \delta E_{\gamma, p} [w(1, \gamma, s, p)]
\]
which, given \( IC_B \) is binding, is the payoff the bank obtains in case \( w(1, \gamma, s, p) = \bar{w} \). This concludes step 4.

Step 5: If there exists \( \gamma \in \gamma^*(v) \) with \( \gamma > \gamma^{PBE} \) providing weak incentives for \( \alpha = 0 \) and non-binding PKC, then there exists \( \gamma' \in [\gamma^{PBE}, \gamma] \) with \( \gamma' \in \gamma^*(v) \) such that the solution to \( \hat{\phi}(v, \gamma') \) has binding PKC constraint - Whenever \( E_{\gamma, p} [w(1, \gamma, s, p)]B \leq w(0, \gamma, B, \emptyset) \) or whenever \( w(1, \gamma, G, M) > \bar{w} \), the proof is exactly the same as that in step 4 (moreover, note that \( \gamma > \gamma^{PBE} \) guarantees \( \gamma \geq \max \left\{ \gamma^{PBE} - \frac{\bar{v} - \mu M}{(1 - \delta)(1 - \mu) \theta M}, 0 \right\} \) for \( v \geq \bar{w} = \mu M \)). Hence, we focus from now on on the case with \( E_{\gamma, p} [w(1, \gamma, s, p)]B > w(0, \gamma, B, \emptyset) \) and with \( w(1, \gamma, G, M) = \bar{w} \). This immediately implies that there exists a state \((c, \gamma, s, p) = (1, \gamma, s^*, p^*) \neq (1, \gamma, G, M) \) with \( w(1, \gamma, s^*, p^*) > \bar{w} \).

Hence, assume for a contradiction PKC is not binding. Then, if \( w(0, \gamma, B, \emptyset) > \bar{w} \), then once again, deviate by reducing \( w(1, \gamma, s^*, p^*) \) by \( \epsilon \) and reducing \( w(0, \gamma, B, \emptyset) \) by \( \delta \) in a way


that keeps \( E_{x,p}[w(1, \gamma, s, p)|B] - w(0, \gamma, B, \varnothing) \) constant. This will keep satisfying \( IC_B \); for \( \varepsilon \) small enough, \( \delta \) will be small enough and PKC will still be satisfied; this will relax \( IC_R \) by weakly increasing \( \hat{\phi}(w(1, \gamma, 0, M)) \) and will still satisfy \( w(\cdot) \geq w \) for \( \varepsilon \) small enough. Moreover, this will also increase the regulator’s objective function by increasing \( \hat{\phi}(w(\cdot), \gamma) \).

If, on the other hand, \( w(0, \gamma, B, \varnothing) = w \), then deviate towards \( \hat{\alpha} = \min\{a^T, \alpha\} \), where \( a^T \) is defined to be:

\[
a^T = \frac{\mu}{1 - \mu} \frac{(1 - \delta)C + \delta \phi(w(1, \gamma, 0, M)) - \phi(w(1, \gamma, G, M))}{{(1 - \delta)C + \delta \phi(w(1, \gamma, B, \varnothing)) - \phi(w(1, \gamma, 0, M))}}
\]

(that is, \( a^T \) is the level of \( \alpha \) below which the regulator finds it profitable to reduce \( \gamma \), and above which the regulator finds it profitable to increase \( \gamma \)). Moreover, reduce \( \gamma \) by \( \delta \), set \( \tilde{w}(1, \gamma - \delta, s^*, p^*) = w(1, \gamma, s^*, p^*) - \varepsilon \) and, for \( c, s, p \neq 1, s^*, p^* \), set \( \tilde{w}(c, \gamma - \delta, s, p) = w(c, \gamma, s, p) \).

Set \( \delta \) and \( \varepsilon \) in a way that keeps \( (1 - \delta)[D + (1 - \gamma + \delta)\theta M + \delta E_{x,p}[w(1, \gamma - \delta, s, p)|B]] = (1 - \delta)[D + (1 - \gamma)\theta M + \delta E_{x,p}[w(1, \gamma, s, p)|B]] \) constant (note that reducing \( \gamma \) increases this payoff, so that we need to reduce \( w(1, \gamma, s^*, p^*) \) to keep this constant).

Note that if the original solution was feasible for the problem \( \hat{\phi}(w, \gamma, \alpha) \), then this postulated solution must be feasible for problem \( \hat{\phi}(w, \gamma, \alpha - \delta) \): after all, \( IC_B \) must still hold since we have kept the payoff from complex bad products unchanged and because we have kept the payoff from non-complex products unchanged. PKC must still hold for \( \varepsilon \) small enough, since originally it was not binding. If \( \alpha \leq a^T \), then \( IC_R \) must have been weakly relaxed, since the only changes were (i) to weakly increase the regulators continuation value by reducing \( w(1, \gamma, s^*, p^*) \) and (ii) to reduce \( \gamma \) when the regulator finds it profitable to do so. If \( a > a^T \), on the other hand, we have deviated towards \( a^T \), and at \( \hat{\alpha} = a^T \), \( IC_R \) always holds (to see this, note that if \( - (1 - \hat{\mu})(1 - \theta)R \geq - C \), we have that

\[
(1 - \gamma)(1 - \hat{\mu})\{(1 - \delta)(1 - \theta)(-R) + \delta[\theta \phi(1, \gamma, 0, M) + (1 - \theta)\phi(1, \gamma, 0, 0)]\}
+ (1 - \gamma)\hat{\mu}\{\delta \phi(1, \gamma, 0, M)\} + \gamma \{-(1 - \delta)C + \delta \hat{\mu}\phi(1, \gamma, G, M) + \delta(1 - \hat{\mu})\phi(1, \gamma, B, \varnothing)\}
= (1 - \delta)(1 - \mu)\{(1 - \delta)(1 - \theta)(-R) + \delta[\theta \phi(1, \gamma, 0, M) + (1 - \theta)\phi(1, \gamma, 0, 0)]\}
+ \gamma \{-(1 - \delta)C + \delta \hat{\mu}\phi(1, \gamma, G, M) + \delta(1 - \hat{\mu})\phi(1, \gamma, B, \varnothing)\}
\geq (1 - \delta)(1 - \mu)\{(1 - \delta)(1 - \theta)R + \delta \hat{\mu} = (1 - \delta)\max\{-(1 - \hat{\mu})(1 - \theta)R, -C\} + \delta \hat{\mu}
\]

(where the first equality comes from indiffference between some monitoring and no monitoring, and the second line is an implication of \( \hat{\phi} \leq \phi(1, \gamma, s, p) \)), which implies that when \( -(1 - \hat{\mu})(1 - \theta)R > -C \), the \( IC_R \) constraint holds. On the other hand, if \( -C > -(1 - \hat{\mu})(1 - \theta)R \), note that:

\[
(1 - \gamma)(1 - \hat{\mu})\{(1 - \delta)(1 - \theta)(-R) + \delta[\theta \phi(1, \gamma, 0, M) + (1 - \theta)\phi(1, \gamma, 0, 0)]\}
+ (1 - \gamma)\hat{\mu}\{\delta \phi(1, \gamma, 0, M)\} + \gamma \{-(1 - \delta)C + \delta \hat{\mu}\phi(1, \gamma, G, M) + \delta(1 - \hat{\mu})\phi(1, \gamma, B, \varnothing)\}
= \{-(1 - \delta)C + \delta \hat{\mu}\phi(1, \gamma, G, M) + \delta(1 - \hat{\mu})\phi(1, \gamma, B, \varnothing)\} \geq -(1 - \delta)C + \delta \hat{\mu}
\]

(where the first equality comes from indiffference between some monitoring and full monitoring, and the second line is also an implication of \( \hat{\phi} \leq \phi(1, \gamma, s, p) \)), once again, implying that \( IC_R \) holds after the first deviation.)

Finally, this deviation is profitable for the regulator: if \( \alpha \leq a^T \), we are reducing \( \gamma \) when
it’s profitable to do so, and weakly increasing the regulator’s continuation value by reducing the bank’s continuation value in state $c, s, p = 1, s^*, p^*$. If $\alpha > \alpha_T$, the regulator first gains by reducing complexity towards $a^T$ when the product is bad, and then the move in $\gamma$ does not change his payoffs, while his continuation payoff increases from the reduction in the bank’s continuation payoff in state $c, s, p = 1, s^*, p^*$.

This then says that if PKC is not binding, $w(1, \gamma, G, M) = w(0, \gamma, B, \emptyset) = \bar{w}$, $\exists s^*, p^*$ with $w(1, \gamma, s^*, p^*) > \bar{w}$ and $\gamma > \gamma_{PBE}$, then there exists $\gamma' < \gamma$ such that $\hat{\phi}(v, \gamma) \leq \phi(v, \gamma')$, with $\bar{w}(1, \gamma', s^*, p^*) < w(1, \gamma, s^*, p^*)$, $w(1, \gamma, s, p) = \bar{w}(1, \gamma', s, p)$ for $s, p \neq s^*, p^*$ and weakly lower payoffs for the bank. Then, we will still have this deviation available up until either (i) $w(1, \gamma, s, p) = \bar{w}$ for all $s, p$, (ii) PKC becomes binding or (iii) $\gamma$ becomes equal than $\gamma_{PBE}$. In any of these cases, we have either found a solution with PKC binding or we have moved to the cases covered by step 4, where we do not need to deviate with $\gamma$ anymore, guaranteeing the statement of the step 5.

**Step 6:** If there are strict incentives for $\alpha = 1$ and $\gamma \geq \max\{\gamma_{PBE} - \frac{v - \mu M}{(1 - \delta)(1 - \mu)\theta M}, 0\}$, then PKC is binding - According to Lemma 6, this case is only relevant if $w(0, \gamma, B, \emptyset) = \bar{w}$. So assume $\alpha = 1$, PKC is not binding, $IC_B$ provides strict incentives for $\alpha = 1$ and $w(0, \gamma, B, \emptyset) = \bar{w}$. Then, if there exists some state $(c, s, p) = (1, \gamma, s, p)$ with $w(1, \gamma, s, p) > \bar{w}$, we can trivially improve the payoff for the regulator by reducing $w(1, \gamma, s, p)$ (and weakly increasing, as a consequence, $\hat{\phi}(w(1, \gamma, s, p), \gamma)$), while still satisfying PKC for a deviation small enough (since it was not binding), still satisfying $IC_B$ (because of strict incentives for $\alpha = 1$), still satisfying $w(1, \gamma, s, p) \in [\bar{w}, \bar{w}]$ for a small enough deviation, and relaxing $IC_R$ (by increasing $\phi(w(1, \gamma, s, p), \gamma)$).

If, on the other hand, $w(1, \gamma, s, p) = \bar{w}$ for all $s, p$, while PKC is not binding, then the fact that we have strictly more incentives for $c = 1$ implies that $\gamma \leq \max\{\gamma_{PBE} - \frac{v - \mu M}{(1 - \delta)(1 - \mu)\theta M}, 0\}$: after all, if there are strictly more incentives for $c = 1$, then we can write the bank’s payoff as:

$$(1 - \delta)\{\mu M + (1 - \mu)[-D + (1 - \gamma)\theta M]\} + \delta \bar{w} > v$$

from the non-binding PKC constraint, which in turn implies:

$$\gamma < \gamma_{PBE} - \frac{v - \mu M}{(1 - \delta)(1 - \mu)\theta M} \leq \max\{\gamma_{PBE} - \frac{v - \mu M}{(1 - \delta)(1 - \mu)\theta M}, 0\}$$

**Step 7:** $\phi(v) = \arg \max_\gamma \hat{\phi}(v, \gamma)$ s.t. $\gamma \geq \max\{\gamma_{PBE} - \frac{v - \mu M}{(1 - \delta)(1 - \mu)\theta M}, 0\}$ - If there is $\gamma \in \gamma^*(v)$ with $\gamma > \gamma_{PBE}$, then steps 5 and 6 can be used show that there exists $\gamma' \in [\gamma_{PBE}, \gamma]$ (so that $\gamma' \geq \max\{\gamma_{PBE} - \frac{v - \mu M}{(1 - \delta)(1 - \mu)\theta M}, 0\}$), such that $\hat{\phi}(v, \gamma') \geq \phi(v, \gamma)$ and such that $\hat{\phi}(v, \gamma')$ has PKC binding. But then, it must mean that the solution to problem $\hat{\phi}(v, \gamma')$ is feasible to problem $\phi(v, \gamma')$, and that $\hat{\phi}(v, \gamma') \geq \phi(v, \gamma')$. Superimposing this with $\phi(v, \gamma') \leq \phi(v, \gamma')$ from step 2 then yields existence of a solution to problem $\phi(v, \gamma')$ and that $\hat{\phi}(v, \gamma') = \phi(v, \gamma')$. Moreover, this logic suggests $\phi(v, \gamma') = \hat{\phi}(v, \gamma') \geq \phi(v, \gamma) \geq \phi(v, \gamma')$. However, $\gamma \in \gamma^*(v)$ suggests $\phi(v, \gamma) \geq \phi(v, \gamma')$, so that we must have $\phi(v, \gamma) = \hat{\phi}(v, \gamma) = \phi(v, \gamma') = \phi(v, \gamma')$ whenever $\gamma \in \gamma^*(v)$, with $\gamma > \gamma_{PBE}$. But then, this implies that if there is $\gamma \in \gamma^*(v)$ with $\gamma > \gamma_{PBE}$, there exists a solution to the problem $\phi(v, \gamma)$ and that $\hat{\phi}(v) = \arg \max_\gamma \{\hat{\phi}(v, \gamma) : \exists \text{ solution to } \phi(v, \gamma)\} = \arg \max_\gamma \{\hat{\phi}(v, \gamma) : \gamma \geq \max\{\gamma_{PBE} - \frac{v - \mu M}{(1 - \delta)(1 - \mu)\theta M}, 0\}\}$. If there exists $\gamma \in \gamma^*(v)$ satisfying $\gamma \leq \gamma_{PBE}$, then by step 3.1', it must be that $\gamma \geq$
max \{ γ^{PBE} - \frac{v - µM}{(1 - δ)(1 - µ)PM}, 0 \}. Moreover, whenever this was the case, steps 4 and 6 shown that there exists a solution to problem φ(v, γ) with PKC is binding. But then, this solution is feasible for problem φ(v, γ), so that φ(v, γ) ≤ φ(v, γ) whenever γ ≤ γ^{PBE} and γ ≥ max \{ γ^{PBE} - \frac{v - µM}{(1 - δ)(1 - µ)PM}, 0 \}. This together with φ(v, γ) ≥ φ(v, γ) from step 2 guarantees then that \hat{φ}(v, γ) = φ(v, γ) for γ in the range analyzed here. But then, this guarantees existence of a solution to problem φ(v, γ) whenever γ ≤ γ^{PBE} and γ ≥ max \{ γ^{PBE} - \frac{v - µM}{(1 - δ)(1 - µ)PM}, 0 \}.

Given that we know there exists no solution to problem φ(v, γ) when γ < max \{ γ^{PBE} - \frac{v - µM}{(1 - δ)(1 - µ)PM}, 0 \} (from step 3.1'), this means that conditionally on the existence of γ ∈ γ*(v) with γ ≤ γ^{PBE}, we can write

φ(v) = \arg \max_γ \{ φ(v, γ) : γ ≥ \max \{ γ^{PBE} - \frac{v - µM}{(1 - δ)(1 - µ)PM}, 0 \} \} = \arg \max_γ \{ \hat{φ}(v, γ) : γ ≥ \max \{ γ^{PBE} - \frac{v - µM}{(1 - δ)(1 - µ)PM}, 0 \} \}.

Putting the two paragraphs together, this logic guarantees the statement in step 7 is true.

Step 8: There exists v* ∈ [\bar{w}, µM(1 - δ)(1 - µ)(-D + θM)] such that φ(v) is decreasing in v for v > v*. This is an immediate consequence of the envelope theorem applied to the problem in step 7, of the fact that \hat{φ}(v, γ) is decreasing in v from step 2, and of the fact that for v ≥ µM + (1 - δ)(1 - µ)[-D + θM], the constraint γ ≥ \max \{ γ^{PBE} - \frac{v - µM}{(1 - δ)(1 - µ)PM}, 0 \} becomes simply γ ≥ 0.

Proof of Lemma 9.

(Note: check step 7 from the proof of lemma 8). Assume otherwise, that we are in a contract with equilibrium values of α > α^{MIN} (the minimum α consistent with ICα) and γ > γ^{PBE}. We now propose the following deviation: first, change α towards \tilde{α} = α^{MIN}. Additionally, change the continuation payoff for the bank after a choice of no complexity to be:

\[ E_{\tilde{α}, γ, P}[\tilde{w}(0, γ, B, \emptyset)] = \tilde{w}(0, γ, B, \emptyset) \]
\[ = E_{\tilde{α}, γ, P}[w(1, γ, s, p)|B] - D\frac{(1 - δ)}{δ} + (1 - γ)θM\frac{(1 - δ)}{δ} \]

Leave γ and all other continuation payoffs unchanged. Note that \(-D\frac{(1 - δ)}{δ} + (1 - γ)θM\frac{(1 - δ)}{δ}\) must be less than zero, as γ > γ^{PBE}.

With this deviation, the new payoff for banks is33:

\[ \tilde{v} = µ(1 - δ)M + δE_{\tilde{α}, γ, P}[w(1, γ, s, p)|G] + (1 - µ)(1 - δ)0 + δ\tilde{w}(0, γ, B, \emptyset) \]
\[ = µ(1 - δ)M + δE_{\tilde{α}, γ, P}[w(1, γ, s, p)|G] + (1 - µ)(1 - δ)[-D + (1 - γ)θM] \]
\[ + δE_{\tilde{α}, γ, P}[w(1, γ, s, r)|B] \]

where the RHS denotes the payoff for the bank in the initial equilibrium, v, which was specified in the promise keeping constraint. Note that when α ∈ (0, 1), banks must be indifferent between choosing α = 0 and α = 1 (otherwise, the bank would not be willing to choose c = 0 and c = 1 both with positive probability), so we can substitute out the payoff from c = 0 by the payoff from c = 1 and the resulting payoff from α ∈ (0, 1) must be the payoff from c = 1. Similarly, if the bank is choosing α = 1, then it must be that it’s getting the payoff from c = 1. Either way, since we had α > 0 originally, it must be that the bank is getting the payoff from c = 1.

33 We can omit α from the expectation term when the product is good, since no decision is available to banks.
As can be seen above, \( v = \tilde{v} \), such that the PKC is still satisfied after the deviation (and relaxed PKC is satisfied with no changes). Furthermore, as the continuation value after choosing complexity has not changed and the payoff of choosing no complexity is equal to the payoff of choosing complexity, it must be that the choice of \( \tilde{\alpha} = \alpha^{MIN} \) is optimal and the IC\(_R\) is still satisfied after the deviation.

Note that continuation values after complexity have not changed, and \( \gamma \) has not changed. As a consequence, it must be that the minimum \( \alpha \) consistent with IC\(_R\) has not changed, so that \( \alpha^{MIN} \) remained unaltered. Then, by definition of \( \alpha^{MIN} \), it must be that IC\(_R\) is satisfied.

Thirdly, notice that for \( \alpha > \alpha^{MIN} \geq 0 \)

\[
\delta [E_{\alpha, \gamma, p}[w(1, \gamma, s, p)|B] - w(0, \gamma, B, \emptyset)] \geq [D - (1 - \gamma)\theta M](1 - \delta) > 0
\]

(since the bank must be either indifferent or have weakly more incentives for \( \alpha = 1 \) if it is choosing \( \alpha \geq 0 \), and since \( \gamma > \gamma^{PBE} \) implies \( -D + (1 - \gamma)\theta M < 0 \) and that

\[
\delta [E_{\alpha, \gamma, p}[w(1, \gamma, s, p)|B] - \hat{w}(0, \gamma, B, \emptyset)] = [D - (1 - \gamma)\theta M](1 - \delta) > 0
\]

This implies that \( \hat{w}(0, \gamma, B, \emptyset) \geq w(0, \gamma, B, \emptyset) \). From the definition of \( \hat{w}(0, \gamma, B, \emptyset) \), we have that \( \hat{w}(0, \gamma, B, \emptyset) < E_{\alpha, \gamma, p}[w(1, \gamma, s, p)|B] \) (after all \( \gamma > \gamma^{PBE} \) implies \( -D + (1 - \gamma)\theta M < 0 \). Since both \( w(1, \gamma, s, p) \) and \( w(0, \gamma, B, \emptyset) \) are promised values in the original optimal regulation, they must have been feasible, which imply \( E_{\gamma, p}[w(1, \gamma, s, p)|B], w(0, \gamma, B, \emptyset) \in [\hat{w}, \tilde{w}] \). But then, as a consequence, it must be that \( \hat{w}(0, \gamma, B, \emptyset) \in [\hat{w}, \tilde{w}] \).

Finally, we need now show that the deviation improves the payoff of the regulator. Now, in terms of the flow payoff in the current period, since \( \alpha \) reduces, the regulator’s expected cost of screening when the product is bad falls, keeping \( \gamma \) unchanged, and the number of bad products approved reduces, so the cost of negative shocks to bad products is reduced. Consequently, the flow utility of the regulator is improved.

For the continuation values, first note that if we originally specified \( \alpha = 1 \), the continuation value for the banks was originally specified to be \( w(1, \gamma, s, p) \), compared to the deviation value of \( \tilde{w}(0, \gamma, B, \emptyset) < E[w(1, \gamma, s, p)] \) with some probability and unchanged continuation values with the complementary probability. Now, we know that for fixed \( \gamma \), \( \phi(v, \gamma) \) is weakly decreasing. But then \( E[\phi(w(1, \gamma, s, p), \gamma)] \leq \phi(E[w(1, \gamma, s, p)], \gamma) \leq \phi(\hat{w}(0, \gamma, B, \emptyset), \gamma) \), the former inequality by Jensen’s inequality and concavity of \( \phi(v, \gamma) \) (shown in the Appendix as analogous to the proof in lemma 1), the latter by \( \phi \) being decreasing in \( v \); thus, the continuation values are higher in the deviation. Hence, if \( \alpha = 1 \), it is clear that both flow payoffs and continuation payoffs for the regulator increase after the deviation, contradicting the optimality of the original dynamic regulatory mechanism.

If we had in the original mechanism \( \alpha \in (0, 1) \), then \( \tilde{w}(0, \gamma, B, \emptyset) = w(0, \gamma, B, \emptyset) < E_{\gamma, p}[w(1, \gamma, s, p)|B] \), so that, analogously to the previous result, \( \phi(\tilde{w}(0, \gamma, B, \emptyset), \gamma) = \phi(w(0, \gamma, B, \emptyset), \gamma) \geq E[\phi(w(1, \gamma, s, p), \gamma)] \); thus, by decreasing \( \alpha \), we’re increasing the frequency of continuation values \( \phi(w(0, \gamma, B, \emptyset), \gamma) \) and reducing the frequency of continuation values \( \phi(\tilde{w}(1, \gamma, s, p), \gamma) \) with \( E_{\gamma, p}[\phi(w(1, \gamma, s, p), \gamma)|B] < \phi(w(0, \gamma, B, \emptyset), \gamma) \). Hence, the continuation values are also higher in the deviation. Once again, for \( \alpha \in (0, 1) \), it is clear that both flow payoffs and continuation payoffs for the regulator increase after the deviation, contradicting the optimality of the original dynamic regulatory mechanism.
This concludes the proof that, whenever $\gamma > \gamma_{PBE}$, then if $\alpha > \alpha_{MIN}$, we can find a deviation that reduces $\alpha$ towards $\alpha_{MIN}$, satisfies all constraints, and that makes the regulator better off in problem $\hat{\phi}(v, \gamma)$.

Finally, it is easy to see that $\alpha_{MIN}$ is weakly increasing in $\gamma$: to see this, take $\alpha^T$ as defined in the proof of Lemma 8 (which can be easily seen to be above zero under assumption 1). We have seen there that $IC_R$ holds at $\alpha^T$ for all $\gamma, w(1, \gamma, \cdot)$, so that it must be that $\alpha_{MIN} \leq \alpha^T$.

Moreover, below $\alpha^T$, the regulator loses from increasing monitoring. As a consequence, at any given level of $\alpha \leq \alpha^T$, the higher is $\gamma$, the lower is the regulator’s payoff after complexity.

Secondly, note that at $\alpha_{MIN}$, the regulator’s on-equilibrium path payoff from choosing $\gamma$ after complexity must be weakly increasing in $\alpha$: if it was not, then we could decrease $\alpha$ by $\epsilon$, increase the regulator’s payoff from $\gamma$ after complexity and keep satisfying $IC_R$, which would contradict the assumption that the minimum $\alpha$ consistent with $IC_R$ is $\alpha_{MIN}$.

With these two arguments, it can then be easily seen that as $\gamma$ increases, it must be that $\alpha_{MIN}$ increases by the implicit function theorem.

**Proof of Lemma 11.**
Assume now $\gamma > 0$ but
\[
(1 - \delta)[-D + (1 - \gamma)\theta M] + \delta E_{\gamma,p}[w(1, \gamma, s, p)|B] < \delta E_{\gamma,p}[w(0, \gamma, s, p)|B]
\]
If this is the case, then $\alpha = 0$. Then, deviate by reducing $\gamma$. If the reduction in $\gamma$ is small enough, $IC_B$ will still hold. Given that $\alpha = 0$, $\gamma$ will not affect the bank’s payoffs, so that PKC still holds. Also, given that $\alpha = 0$, we can see that the regulator’s payoff after complexity is decreasing in $\gamma$, so that reducing $\gamma$ will flexibilize $IC_R$. Finally, given $\alpha = 0$, the regulator’s payoff is decreasing in $\gamma$, so that decreasing $\gamma$ weakly increases the regulator’s payoff. Hence, the original mechanism can’t be optimal, suggesting that if it is optimal to set $\gamma > 0$, then it must be that $IC_B$ is binding.

Assume now $\gamma = 0$ but it is optimal to set
\[
(1 - \delta)[-D + (1 - \gamma)\theta M] + \delta E_{\gamma,p}[w(1, \gamma, s, p)|B] < \delta E_{\gamma,p}[w(0, \gamma, s, p)|B]
\]
so that $\alpha = 0$ on the optimal mechanism. Assume initially that $w(1, \gamma, 0, M) \neq w(0, \gamma, B, \emptyset)$ (we will later analyze the case when they’re equal). Note first of all that for all continuation values, when $\alpha = \gamma = 0$, $IC_R$ becomes:
\[
\delta \phi(1, \gamma, 0, M) \geq \delta \phi
\]
which is trivially true given the assumption that $\phi$ is the worse continuation value for the regulator that is consistent with equilibrium. Hence, all continuation values we can possibly set will still satisfy $IC_R$. Also, note that when $\alpha = \gamma = 0$, PKC is given by:
\[
v = (1 - \delta)\mu M + \delta \{\mu w(1, \gamma, 0, M) + (1 - \mu)w(0, \gamma, B, \emptyset)\}
\]
and that $IC_B$ is given by:
\[
(1 - \delta)[-D + \theta M] + \delta [\theta w(1, \gamma, 0, M) + (1 - \theta)w(1, \gamma, 0, 0)] < \delta w(0, \gamma, B, \emptyset)
\]
With this in mind, then, let $w^{AVG} = \mu w(1, \gamma, 0, M) + (1 - \mu)w(0, \gamma, B, \emptyset)$. Deviate, with probability $1 - \kappa$, towards setting $\bar{w}(1, \gamma, 0, M) = w(1, \gamma, 0, M)$ and $\bar{w}(0, \gamma, B, \emptyset) = w(0, \gamma, B, \emptyset)$, while with probability $\kappa$, set $\bar{w}(1, \gamma, 0, M) = w^{AVG}$ and $\bar{w}(0, \gamma, B, \emptyset) = w^{AVG}$. Keep all other continuation values unchanged, and keep $\gamma$ unchanged. Clearly, this deviation satisfies $\bar{w}(c, \gamma, s, p) \in [w, \bar{w}]$ because the deviation has kept continuation values either unchanged, or moved them towards an average of continuation values $w^{AVG}$. Moreover, this deviation must, by construction, keep PKC satisfied. Finally, for a small enough deviation, it must be that $IC_B$ is satisfied. As shown previously, in this case, $IC_R$ is always satisfied. As a consequence, this deviation is feasible. Moreover, it improves the regulator’s payoff due to concavity of $\phi(v)$. Hence, we cannot have that it is optimal to have $a^{MIN}$ implemented with $IC_B$ non-binding, with $\gamma = 0$, and with $w(1, \gamma, 0, M) \neq w(0, \gamma, B, \emptyset)$.

Next, assume $w(1, \gamma, 0, M) = w(0, \gamma, B, \emptyset) = w^{AVG}$, that $\gamma = 0$, $\alpha = 0$ and that $IC_B$ is not binding. For this to be the case, then, we must have that, from PKC

$$v = (1 - \delta)\mu M + \delta w^{AVG} \rightarrow w^{AVG} = \frac{v - (1 - \delta)\mu M}{\delta}$$

while, from $IC_B$, we must have:

$$(1 - \delta)[-D + \theta M] + \delta(1 - \theta)w(1, \gamma, 0, 0) < \delta(1 - \theta)\frac{v - (1 - \delta)\mu M}{\delta}$$

Re-writing the expression above, we obtain that:

$$\delta(1 - \theta)w(1, \gamma, 0, 0) < (1 - \theta)[v - \mu M] - (1 - \delta)[-D + \theta M] + \delta(1 - \theta)\mu M$$

Now, this is feasible as long as the right hand side above is above $\delta(1 - \theta)\bar{w} = \delta(1 - \theta)\mu M$ (after all, in such a case, we are just requiring that $\delta(1 - \theta)w(1, \gamma, 0, 0)$ is lower than something above $\delta(1 - \theta)\bar{w}$). This is the case as long as:

$$(1 - \theta)[v - \mu M] \geq (1 - \delta)[-D + \theta M]$$

Noting that $v \leq \mu M + (1 - \mu)[-D + \theta M]$ from lemma 7, we have that $(1 - \theta)[v - \mu M] \leq (1 - \theta)(1 - \mu)[-D + \theta M]$, so that a necessary condition for the above to hold for some $v < \bar{w}$ (with commitment) and for some $v \leq \bar{w}$ (without commitment) is that $1 - \delta \leq (1 - \theta)(1 - \mu)$. In this sense, if $\delta < \theta(1 - \mu) + \mu$, it is impossible to have $w(1, \gamma, 0, M) = w(0, \gamma, B, \emptyset)$ while still satisfying PKC and $IC_B$ providing strict incentives for $\alpha = 0$. For $\delta \geq \theta(1 - \mu) + \mu$, on the other hand, we will need to have $IC_B$ holding with equality for all:

$$v < \mu M + \frac{1 - \delta}{1 - \theta}[-D + \theta M]$$

Finally, assume $v \geq \mu M + \frac{1 - \delta}{1 - \theta}[-D + \theta M]$, that it is optimal to set $\alpha = 0$, $\gamma = 0$, and that $w(1, \gamma, 0, M) \neq w(0, \gamma, B, \emptyset)$. Note that in such a case where $\alpha = \gamma = 0$, the regulator’s payoff is:

$$\mu \delta \phi(w(1, \gamma, 0, M)) + (1 - \mu)\delta \phi(w(0, \gamma, B, \emptyset))$$
Given that PKC must hold and $\alpha = 0$, note that:

$$v = (1 - \delta)\mu M + \delta \{ \mu w(1, \gamma, 0, M) + (1 - \mu)w(0, \gamma, B, \emptyset) \}$$

$$\rightarrow \mu w(1, \gamma, 0, M) + (1 - \mu)w(0, \gamma, B, \emptyset) = \frac{v - (1 - \delta)\mu M}{\delta}$$

so that the expected continuation value is given as above. Finally, as before, when $\alpha = \gamma = 0$, $IC_R$ always holds for all continuation values we might set.

Now, with these considerations in mind, deviate from the proposed mechanism above towards setting $\tilde{w}(1, \gamma, 0, M) = \tilde{w}(0, \gamma, B, \emptyset) = \frac{v-(1-\delta)\mu M}{\delta}$ and $\tilde{w}(1, \gamma, 0, 0) = w = \mu M$. Keep $\alpha = \gamma = 0$, and all other continuation values unchanged.

Clearly, this deviation must still satisfy PKC. Moreover, under the deviation, note that $IC_B$ becomes:

$$(1 - \delta)[-D + \theta M] + \delta \left\{ \theta \frac{v - (1 - \delta)\mu M}{\delta} + (1 - \theta)\mu M \right\} \leq \frac{\delta}{\delta} - (1 - \delta)\mu M$$

(where the inequality holds because $v \geq \mu M + \frac{1-\delta}{\delta}[-D + \theta M]$). This is clearly consistent with $\alpha = \gamma = 0$. $IC_B$, as noted before, always holds. Finally, note that if the previous mechanism was feasible, it delivered an expected continuation value of $\frac{v-(1-\delta)\mu M}{\delta}$, so that it must still be feasible to move towards these deviation values. As a consequence, if it was optimal to set $\gamma = \alpha = 0$, it is still feasible to do so after our proposed deviation. However, this improves the regulator’s payoff because $\phi(v)$ is concave, and we’re keeping the same average continuation value while reducing risk.

$\blacksquare$

**Proof of Lemma 12.**

Assume $w(1, \gamma, B, \emptyset) \neq w(1, \gamma, 0, 0)$. Deviate towards $\tilde{w}(1, \gamma, B, \emptyset) = \tilde{w}(1, \gamma, 0, 0) = w^* = \frac{\gamma w(1, \gamma, B, \emptyset) + (1-\gamma)(1-\theta)w(1, \gamma, 0, 0)}{\gamma + (1-\gamma)(1-\theta)}$. This deviation still satisfies $IC_B$ because it keeps the payoffs from complex bad products constant. Given that it does not change the payoff from non-complex bad products, nor the payoffs from good products, it follows immediately that the overall payoff from the banks does not change, so the PKC still holds. Thirdly, given that $w(1, \gamma, B, \emptyset), w(1, \gamma, 0, 0) \in [\underline{w}, \overline{w}]$, then $w^* \in [\underline{w}, \overline{w}]$ because it is a weighted average of $w(1, \gamma, B, \emptyset)$ and $w(1, \gamma, 0, 0)$. Finally, the continuation payoffs from the regulator after detected complex bad products must have weakly increased, since we kept the average continuation value of the bank in this state constant, and since $\phi(v)$ is a concave function and we have reduced the risk the regulator faces. This implies $IC_R$ is relaxed and still holds, and that this is a weakly profitable deviation.

$\blacksquare$

**Proof of Lemma 13.**

As we discuss in the main text, $a^T$ delineates the point of indifference for the regulator between screening and non-screening. This implies that when $\alpha > a^T$, the objective function of the regulator will be increasing in $\gamma$, when $\alpha < a^T$, the objective function of the regulator is decreasing in $\gamma$, and with $\alpha = a^T$, the regulator is indifferent between different values of $\gamma$.

Secondly, note that at $\alpha = a^T$, the $IC_R$ holds. To see this, first note that at $\alpha = a^T$, the regulator is made indifferent between different levels of $\gamma$, so the regulator’s payoff after complexity is equal to the payoff of no monitoring and to the payoff from monitoring. This in turn implies that $IC_R$ always holds under this deviation: after all, if $-(1 - \hat{\mu})(1 - \theta)R \geq -C$, we
have that

\[(1 - \gamma)(1 - \hat{\mu})\{(1 - \delta)(1 - \theta)(-R) + \delta[\theta \phi(1, \gamma, 0, M) + (1 - \theta)\phi(1, \gamma, 0, 0)]\}
+ (1 - \gamma)\hat{\mu}\{(1 - \delta)(1 - \theta)(-R) + \delta[\theta \phi(1, \gamma, 0, M) + (1 - \theta)\phi(1, \gamma, 0, 0)]\}
+ \gamma\{(1 - \delta)\hat{\mu}\phi(1, \gamma, G, M) + \delta(1 - \hat{\mu})\phi(1, \gamma, B, \emptyset)\}
= -(1 - \delta)(1 - \theta)(-R) + \delta\phi = (1 - \delta) \max\{-C, -(1 - \hat{\mu})(1 - \theta)R\} + \delta\phi
\]

(where the first equality comes from indiffERENCE between some monitoring and no monitoring, and the second line is an implication of \(\phi \leq \phi(1, \gamma, s, p)\)), which implies that when \(-(1 - \hat{\mu})(1 - \theta)R > -C\), the \(IC_R\) constraint holds. On the other hand, if \(-C > -(1 - \hat{\mu})(1 - \theta)R\), note that:

\[(1 - \gamma)(1 - \hat{\mu})\{(1 - \delta)(1 - \theta)(-R) + \delta[\theta \phi(1, \gamma, 0, M) + (1 - \theta)\phi(1, \gamma, 0, 0)]\}
+ (1 - \gamma)\hat{\mu}\{(1 - \delta)(1 - \theta)(-R) + \delta[\theta \phi(1, \gamma, 0, M) + (1 - \theta)\phi(1, \gamma, 0, 0)]\}
+ \gamma\{(1 - \delta)\hat{\mu}\phi(1, \gamma, G, M) + \delta(1 - \hat{\mu})\phi(1, \gamma, B, \emptyset)\}
= -(1 - \delta)(1 - \theta)(-R) + \delta\phi = (1 - \delta) \max\{-C, -(1 - \hat{\mu})(1 - \theta)R\} + \delta\phi
\]

(where the first equality comes from indiffERENCE between some monitoring and full monitoring, and the second line is also an implication of \(\phi \leq \phi(1, \gamma, s, p)\)), once again, implying that \(IC_B\) holds after the first deviation. Given that \(a^T\) allows \(IC_R\) to hold, then it is clear that \(a^T \in [a^{MIN}, a^{MAX}]\). Moreover, this implies that if we choose \(a = a^{MIN}\), the regulator’s payoff will be weakly decreasing in \(\gamma\) and that at \(a = a^{MAX}\), the regulator’s payoff will be weakly increasing in \(\gamma\).

Now, assume there is an optimal regulatory mechanism with \(a \in (a^{MIN}, a^{MAX})\) (either \(a\) is in this interval or \(a \in \{a^{MIN}, a^{MAX}\}\) not to violate \(IC_B\)). Consider either one of the following two deviations: either deviate to \(a' = a^{MIN}\), or towards \(a'' = a^{MAX}\). Both deviations still satisfy \(IC_B\) because we have assumed that \(IC_B\) is binding (after all, we have assumed \(a \in (a^{MIN}, a^{MAX}) \in (0, 1)\)). They also satisfy PKC because the bank is indifferent between different levels of \(a\) due to \(IC_B\) binding. By construction, both deviations satisfy \(IC_R\). Since continuation values did not change, it is clear then that all constraints are satisfied with this deviation.

Yet, notice that by increasing \(a\), the regulator’s objective function increases by:

\[-(1 - \delta)[\gamma C + (1 - \gamma)(1 - \theta)R]
+ \delta\{(\gamma + (1 - \gamma)(1 - \theta))[\phi(\gamma, B, \emptyset) - \phi(\gamma, B, \emptyset)]
+ (1 - \gamma)\theta[\phi(\gamma, 0, M) - \phi(\gamma, B, \emptyset)]\}\]

While the first line is clearly negative, the second and third lines are positive. Moreover, \(a\) does not show up in the expression above, so that the regulator’s objective function is linear in \(a\). If the total effect is weakly negative, this shows that by increasing \(a\), the regulator’s objective function weakly decreases, so that decreasing \(a\) towards \(a' = a^{MIN}\) is a weakly profitable and feasible deviation. Alternatively, if the total effect is weakly positive, this shows that by increasing \(a\), the regulator’s objective function weakly increases, so that increasing \(a\) towards \(a^{MAX}\) is a weakly profitable and feasible deviation. \(\blacksquare\)
4.A. PROOFS

Proof of Lemma 14.

Case 1: $\gamma > 0$ - Assume instead that it is optimal to set $\alpha = \alpha^{MIN}$ or that $\alpha^T \geq 1$, $\gamma > 0$, but to have $\phi'(w(1, \gamma, B, \emptyset)) < 0$ Deviate then by reducing $\gamma$ by $\epsilon$ (which increases payoffs from complex bad products by $\epsilon \theta M(1 - \delta)$) and reducing $w(1, \gamma, B, \emptyset)$ by $\omega = \epsilon \frac{\delta M}{1 - \delta} \frac{1 - \delta}{\delta}$ (which reduces payoffs from complex bad products by $\omega \delta (\gamma - \epsilon) = \epsilon \theta M(1 - \delta)$, after the reduction in $\gamma$). By construction, $IC_B$ will still hold after the deviation, and since there was no change in (i) the payoff from complex bad products, (ii) the payoff from simple bad products and (iii) the payoff from good products, this implies that $PKC$ still holds. Given that $w(1, \gamma, B, \emptyset)$ fell marginally, $\phi(w(1, \gamma, B, \emptyset))$ rises, so that $IC_R$ becomes more relaxed. Moreover, when $\gamma$ rises, the difference in on-equilibrium path payoffs for the regulator vs. off equilibrium path deviations changes by something proportional to:

$$(1 - \mu)\alpha^{MIN}\{[(1 - \mu)(1 - \theta)R - C] + \delta \theta [\phi(w(1, \gamma, 0, 0)) - \phi(w(1, \gamma, 0, M))]\}$$  

$$-\mu(1 - \delta)C + \delta \mu [\phi(w(1, \gamma, G, M)) - \phi(w(1, \gamma, 0, M))]$$

Note that the expression above is rising in $\alpha^{MIN}$ and would be equal to zero if $\alpha^{MIN} = \alpha^T$. Given that $\alpha^{MIN} \leq \alpha^T$, this implies the expression above is negative, and when $\gamma$ rises with $\alpha = \alpha^{MIN}$, $IC_R$ becomes “more binding”. Conversely, when $\gamma$ falls, $IC_R$ is relaxed, and this suggests that our proposed deviation relaxes $IC_R$.

Finally, note that at $\alpha = \alpha^{MIN} \leq \alpha^T$, the fall in $\gamma$ weakly increases the regulator’s payoff, and the fall in $w(1, \gamma, B, \emptyset)$ further increases the regulator’s payoff given the assumption that $\phi'(w(1, \gamma, B, \emptyset)) < 0$. Hence, the proposed deviation is feasible and weakly profitable, suggesting that without loss of generality, when $\alpha = \alpha^{MIN}$ and $\gamma > 0$, it is optimal to set $\phi'(w(1, \gamma, B, \emptyset)) \geq 0$. The logic is analogous for $w(1, \gamma, 0, 0)$, and finishes the proof.

Moreover, if for a given $\gamma > 0$, $w(1, \gamma, 0, M)$ and $w(1, \gamma, G, M)$, we still have $\alpha^{MIN} = 0$, start by assuming for a contradiction that $w(1, \gamma, B, \emptyset) > \bar{w}$. Then, given $\alpha^{MIN} = 0$, the payoff of the regulator is not affected by $w(1, \gamma, B, \emptyset)$, and it is weakly increasing in $\gamma$. Then, deviate by reducing $w(1, \gamma, B, \emptyset)$ by $\epsilon$, and reducing $\gamma$ by $\omega$, in a way that keeps $(1 - \delta)(-D + (1 - \gamma)\theta M) + E_{x,y}[w(1, \gamma, s, p)]B$ constant. Clearly, this deviation will still satisfy $IC_B$. Given that we haven’t changed the payoff for the bank from good products, and that the payoff for the bank from bad products is $\delta w(0, \gamma, B, \emptyset)$ when $\alpha = 0$, we haven’t changed the bank’s ex-ante payoffs, and $PKC$ is still satisfied. Trivially, for $\epsilon$ small enough, we must still satisfy $w(1, \gamma, B, \emptyset) - \epsilon \in [\bar{w}, \bar{w}]$. Finally, since we have reduced $\gamma$ while keeping $\alpha = 0$, and $w(1, \gamma, 0, M)$ and $w(1, \gamma, G, M)$ constant, it must have been that we made $IC_R$ more flexible and $\alpha^{MIN}$ remained equal to zero. Hence, this deviation is feasible, and weakly profitable due to the fall in $\gamma$ when $\alpha = \alpha^{MIN} = 0 < \alpha^T$.

Case 2: $\gamma = 0$ - Assume instead that it is optimal to set $\alpha = \alpha^{MIN}$ or that $\alpha^T \geq 1$, $\gamma = 0$, but to have $w(1, \gamma, B, \emptyset) > \bar{w}$. In this case, first note that when $\gamma = 0$, $\alpha^{MIN} = 0$, after all, at $\gamma = \alpha = 0$, $IC_R$ reduces to:

$$\delta \phi(w(1, \gamma, 0, M)) \geq \delta \phi \bar{w}$$

which is trivially true (so that the minimum $\alpha$ consistent with $IC_R$ given $\gamma = 0$ is $\alpha^{MIN} = 0$). Then, deviate by reducing $w(1, \gamma, B, \emptyset)$. $IC_B$ will still hold, since the bank will find it strictly optimal to set $\alpha = 0$ after the deviation. The $PKC$ constraint will still hold because the bank
will keep receiving utility \( \delta w(0, \gamma, B, \emptyset) \) after bad products, and because the utility from good products did not change. As shown above, \( IC_R \) holds. Finally, given that \( \alpha = 0, w(1, \gamma, B, \emptyset) \) happened with zero probability, so that it does not affect the regulator’s payoff. Hence, without loss of generality, the optimal solution to the regulator’s problem has \( w(1, \gamma, B, \emptyset) = w \). The same logic applies to \( w(1, \gamma, 0, 0) \).

Proof of Lemma 15.
Assume that it is optimal to set \( \alpha = a^{MAX} \) and that \( a^T < 1 \). In this case, lemma 13 guarantees that the regulator’s objective function is increasing in \( \gamma \), and that the higher is \( \gamma \), the higher is the regulator’s on-equilibrium payoff after complexity (so that \( IC_R \) gets relaxed when \( \gamma \) increases). Moreover, lemma 11 guarantees that under \( \alpha = a^{MAX} \), either \( IC_B \) is binding (after all, \( a^{MAX} \geq a^T > 0 \)) or \( w(0, \gamma, B, \emptyset) = w \). Finally, remember that corollary 1 suggests that \( \gamma \leq \gamma^{PBE} < 1 \) on the optimal solution, which guarantees that either \( IC_B \) is binding and \( w(0, \gamma, B, \emptyset) \geq E(w(1, \gamma, s, p) | B) \), or that \( w(0, \gamma, B, \emptyset) = \bar{w} \). Finally, let

\[
\begin{align*}
    w^* &= \frac{\mu}{\mu + (1 - \mu)a^{MAX}} [\gamma w(1, \gamma, G, M) + (1 - \gamma)w(1, \gamma, 0, M)] \\
    &\quad + \frac{(1 - \mu)a^{MAX}}{\mu + (1 - \mu)a^{MAX}} [\gamma (1 - \gamma)(1 - \theta)w(1, \gamma, B, \emptyset) + (1 - \gamma)\theta w(1, \gamma, 0, M)]
\end{align*}
\]

(that is, \( w^* \) is the expected continuation payoff for the bank conditional on the product being complex but unconditional on the quality of the product).

Case 1: \( a^{MAX} = 1 \) - In this case, deviate then towards \( w(1, \gamma, s, p) = w^r \) for all \( s, p \), and towards \( \bar{w}(0, \gamma, B, \emptyset) \) set so that \( IC_B \) holds as below:

\[
\min\{(1 - \delta)[-D + (1 - \gamma)\theta M] + \delta w^* , \bar{w} \} = \delta \bar{w}(0, \gamma, B, \emptyset)
\]

Keep \( \alpha \) and \( \gamma \) unchanged.

By construction, it must be that \( IC_B \) is either still binding and consistent with the choice of \( \alpha \), or that \( w(0, \gamma, B, \emptyset) = \bar{w} \leq (1 - \delta)[-D + (1 - \gamma)\theta M] + \delta w^* \), once again, consistent with the choice of \( \alpha = 1 \). Also by construction, \( w^* \) is a weighted average of the original continuation values, which guarantee that \( w^* \in [w, \bar{w}] \) as long as the original continuation values are in \([w, \bar{w}]\). Moreover, given that we’re setting \( \alpha = a^{MAX} = 1 \), we can write the bank’s payoff after deviation as:

\[
(1 - \delta)\{\mu M + (1 - \mu)[-D + (1 - \gamma)\theta M]\} + \delta w^*
\]

\[
= (1 - \delta)\{\mu M + (1 - \mu)[-D + (1 - \gamma)\theta M]\} + \delta \mu [\gamma w(1, \gamma, G, M) + (1 - \gamma)w(1, \gamma, 0, M)]
\]

\[
+ (1 - \mu)\{\gamma (1 - \gamma)(1 - \theta)w(1, \gamma, B, \emptyset) + (1 - \gamma)\theta w(1, \gamma, 0, M)\} = v
\]

where the last equality comes from the original promise keeping constraint, and the first equality is from the definition of \( w^* \). This ensures the deviation still satisfies PKC.

It is clear that, by construction, the new expected continuation value did not change (using the assumption that \( \alpha = a^{MAX} = 1 \), this is solely due to the continuation values after complexity not depending on \( w(0, \gamma, B, \emptyset) \)). However, we have reduced risk in the bank’s continuation value, and due to concavity in \( \phi \), this increases the regulator’s continuation value (which does not depend on \( w(0, \gamma, B, \emptyset) \) because \( \alpha = a^{MAX} = 1 \)). This same argument suggests that
the regulator’s expected continuation payoff after complexity must weakly increase, which weakly relaxes $IC_R$.

Hence, this deviation is feasible. Moreover, this deviation increases the regulator’s ex-ante payoff by reducing risk in continuation values. This shows the original mechanism containing non-constant $w(1, \gamma, s, p)$ cannot be optimal when $\alpha = \alpha^{MAX} = 1$.

4.A. PROOFS

To derive the next results, we discuss here the first order conditions for the Lagrangian. For this, we will use the envelope theorem showing that $\phi'(v) = \lambda_{PKC}$ (where $\lambda_{PKC}$ is the Lagrange multiplier on the promise keeping constraint). Then, we can write the first order condition with respect to $w(1, \gamma, 0, M)$ as:

$$[FOC_{w(1,\gamma,0,M)}] - \phi'(w(1, \gamma, 0, M)) = -\frac{\phi'(v) + \lambda_B \frac{\theta}{\mu + (1-\mu)\theta}}{1 + \frac{\lambda_R}{\mu + (1-\mu)\theta}} \tag{4.1}$$

where $\lambda_B$ is the Lagrange multiplier on $IC_B$ and $\lambda_R$ is the Lagrange multiplier on $IC_R$. That is: if the $IC_B$ is binding, in the sense that we need to provide incentives for the bank to make its asset complex, then we need to reduce $\phi'$ relative to $\phi'(v)$ and increase $w(1, \gamma, 0, M)$ relative to $v$. Similarly, remembering that $\phi' < 0$ and $\phi'' < 0$ (so that $-\phi'' > 0$) notice that if $IC_R$ is binding, then $\lambda_R > 0$, and we’re dividing the right hand side by something higher than 1, reducing $-\phi'$ and reducing $w(1, \gamma, 0, M)$.

The first order condition with respect to $w(1, \gamma, G, M)$ is given by:

$$[FOC_{w(1,\gamma,G,M)}] - \phi'(w(1, \gamma, G, M)) = -\frac{\phi'(v)}{1 + \frac{\lambda_R}{\mu + (1-\mu)\theta}} \tag{4.2}$$

The intuition for this is also clear: since $w(1, \gamma, G, M)$ does not show up in the $IC_B$, then as long as $IC_R$ is not binding, the easiest way to meet the promise keeping constraint is to keep $w(1, \gamma, 0, M) = v$. If $IC_R$ is binding, though, we need to increase the continuation value of the regulator, which we do by reducing $-\phi'$ (achieved by reducing $w(1, \gamma, G, M)$).

The first order condition with respect to $w(0, \gamma, B, \emptyset)$ is given by:

$$[FOC_{w(0,\gamma,B,\emptyset)}] - \phi'(w(0, \gamma, B, \emptyset)) = -\phi'(v) - \frac{\lambda_B}{(1-\mu)(1-\alpha)} \tag{4.3}$$

The first order condition with respect to $w(1, \gamma, B, \emptyset)$ is given by:

$$[FOC_{w(1,\gamma,B,\emptyset)}] - \phi'(w(1, \gamma, B, \emptyset)) = -\frac{\phi'(v) + \lambda_B \frac{1}{(1-\mu)\alpha}}{1 + \frac{\lambda_R}{\mu + (1-\mu)\theta}} \tag{4.4}$$

The first order condition with respect to $w(1, \gamma, 0, 0)$ is given by:

$$[FOC_{w(1,\gamma,0,0)}] - \phi'(w(1, \gamma, 0, 0)) = -\frac{\phi'(v) + \lambda_B \frac{1}{(1-\mu)\alpha}}{1 + \frac{\lambda_R}{\mu + (1-\mu)\theta}} \tag{4.5}$$
The first order condition with respect to $\gamma$ is given by:

$$
\left[1 + \frac{\lambda_R}{\mu + (1 - \mu)\alpha}\right] \left\{ - \left(1 - \delta\right) \left[\gamma C - (1 - \gamma)(1 - \theta)R\right] + \gamma \delta \phi(w(1, \gamma, 0, M)) \right\}
$$

$$+ [\lambda_B + (1 - \mu)\alpha \lambda_{PKC}] \left\{ \left[\gamma C - (1 - \gamma)(1 - \theta)R\right] + \gamma \delta \phi(w(1, \gamma, B, \varnothing)) \right\} + \lambda_{PKC} \delta \mu[w(1, \gamma, 0, M) - w(1, \gamma, G, M)] = 0
$$

In compact notation, let $D_{\gamma}(R|c = 1)$ be the derivative of the regulator’s on-equilibrium path payoff when $c = 1$ but letting $\beta = \mu$. Similarly, let $D_{\gamma}(Bank|B)$ be the derivative of the bank’s on-equilibrium path payoff when he has a bad asset. Then, we can re-write the above as:

$$
[(1 - \mu)\alpha \lambda_{PKC} - \lambda_B] D_{\gamma}(Bank|B) + \lambda_{PKC} \delta \mu[w(1, \gamma, 0, M) - w(1, \gamma, G, M)] = \left[1 + \frac{\lambda_R}{\mu + (1 - \mu)\alpha}\right] D_{\gamma}(R|c = 1)
$$

(4.6)

The first order condition with respect to $a$ is given by:

$$
- \delta \phi(w(0, \gamma, B, \varnothing)) + (1 - \delta)[-\gamma C - (1 - \gamma)(1 - \theta)R] + \gamma \delta \phi(w(1, \gamma, 0, M)) + (1 - \gamma)(1 - \theta)\delta \phi(w(1, \gamma, 0, 0))
$$

$$+ \frac{\lambda_R}{\mu + (1 - \mu)\alpha} \left\{ \left[\gamma C - (1 - \gamma)(1 - \theta)R\right] + \gamma \delta \phi(w(1, \gamma, 0, M)) + (1 - \gamma)(1 - \theta)\delta \phi(w(1, \gamma, 0, 0)) \right\} = 0
$$

where $L = -R$ if $(1 - \mu)\alpha R \leq C[\mu + (1 - \mu)\alpha]$ and $L = -C$ otherwise. Note that the PKC terms do not show up because we can only change $a$ on the margin when the bank is indifferent between $c = 1$ and $c = 0$, and as a consequence, in these situations, $a$ will not affect the bank’s payoff nor the promise keeping constraint. Similarly, note that $IC_B$ does not show up because it clearly does not depend on $a$. Note finally that several terms inside $IC_R$ cancel out by using the complementary slackness condition. With this in mind, the FOC condition above simplifies to:

$$
\left[1 + \frac{\lambda_R}{\mu + (1 - \mu)\alpha}\right] \left\{ (1 - \delta)[-\gamma C - (1 - \gamma)(1 - \theta)R] + \delta \phi(w(1, \gamma, B, \varnothing)) \right\}
$$

$$+ \frac{\lambda_R}{\mu + (1 - \mu)\alpha} \left[\gamma C - (1 - \gamma)(1 - \theta)R\right] + \gamma \delta \phi(w(1, \gamma, 0, M)) + (1 - \gamma)(1 - \theta)\delta \phi(w(1, \gamma, 0, 0)) = \delta \phi(w(0, \gamma, B, \varnothing)) + \frac{\lambda_R}{\mu + (1 - \mu)\alpha} \left[(1 - \delta)L + \delta \phi(v)\right]
$$

(4.7)

4.4.2 Regulator’s $IC_R$ not binding

If this is the case, $\lambda_R = 0$. From (4.4) and (4.5), we know that $w(1, \gamma, B, \varnothing) = w(1, \gamma, 0, 0)$.

If $\lambda_B > 0$, then (4.3) and (4.4) show that $w(0, \gamma, B, \varnothing) < w(1, \gamma, B, \varnothing)$. Also, from (4.1) and from (4.3), we know that $w(1, \gamma, 0, M) > w(0, \gamma, B, \varnothing)$. With this in hands, we know that $E_{\gamma,p}[w(1, \gamma, s, p)|B] > E_{\gamma,p}[w(0, \gamma, s, p)|B]$. From corollary 1, we also know that $\gamma \leq \gamma_{PBE}$, which shows that $-D + (1 - \gamma)\theta M \geq 0$. But then, the discounted payoffs from $c = 1$ are higher than the discounted payoffs from $c = 0$, which show that $IC_B$ cannot be binding, and $\lambda_B = 0$, contradicting $\lambda_B > 0$.

What happens when $\lambda_B = 0$? Note then from (4.1)-(4.5) that $w(c, \gamma, s, p) = v$ for all $c, \gamma, s, p$. But then, from (4.7), we get that:

$$(1 - \delta)[-\gamma C - (1 - \gamma)(1 - \theta)R] + \delta \phi(v) < \delta \phi(v)$$
(since $-\gamma C - (1 - \gamma)(1 - \theta)R < 0$). But then, (4.7) implies that $\alpha = 0$, that is to say, that we have a corner solution. But then, notice that the flow payoff from $c = 1$ must be 0, such that $IC_B$ holds strictly, which then implies $\gamma = \gamma_{PBE}$. Replacing this on 4.6, we get that:

$$\phi(v) = \frac{1 - \delta}{\delta} \frac{\mu}{1 - \mu} C$$

At the same time, $\phi(v)$ must satisfy the recursive equation for the regulator, that is, inputting the optimized values into the maximization problem, which here implies that:

$$\phi(v) = \mu \{- (1 - \delta) \gamma_{PBE} C + \delta \phi(v)\} + (1 - \mu) \delta \phi(v)$$

$$\rightarrow \phi(v) = - \mu \gamma_{PBE} C$$

which yields a contradiction. As a consequence, we must have in the solution that $\lambda_B < 0$.

From Lemma 13 we know that we can split our analysis into two cases, $\alpha = 0$ or $\alpha = 1$. We solve and find the optimal values for the continuation values as functions of $v$ and $\gamma$ in each case, which will allow us to find the optimal solution numerically.

**Subcase 1: $\alpha = 0$**

From Lemma 14, when $\alpha = 0$ it must be that $w(1, \gamma, B, \varnothing) = w(1, \gamma, 0, 0) = w$, and from 4.2 we have that $v = w(1, \gamma, G, M)$. This means we need only find the values of $w(0, \gamma, B, \varnothing)$ and $w(1, \gamma, 0, M)$ and to do so, we focus our attention now on the PKC and the $IC_B$.

With $\lambda_R = 0$ and $\lambda_B < 0$, the $IC_B$ is binding and is:

$$\delta w(0, \gamma, B, \varnothing) = (1 - \delta)[-D + (1 - \gamma)\theta M]$$

$$+ \delta \{\gamma w(1, \gamma, B, \varnothing) + (1 - \gamma)[\theta w(1, \gamma, 0, M) + (1 - \theta) w(1, \gamma, 0, 0)]\}$$

which we can rewrite, using the above results as:

$$(1 - \mu) \delta w(0, \gamma, B, \varnothing) = (1 - \mu)(1 - \delta)[-D + (1 - \gamma)\theta M]$$

$$+ (1 - \mu) \delta \gamma + (1 - \gamma)(1 - \theta) w + (1 - \mu) \delta (1 - \gamma) \theta w(1, \gamma, 0, M)$$

As the $IC_B$ is binding, banks are indifferent between the total payoff they receive from choosing $\alpha = 0$ or $\alpha = 1$, so we choose to set $\alpha = 0$ and have that the PKC is:

$$v = (1 - \delta) \mu M + \delta \mu [\gamma w(1, \gamma, G, M) + (1 - \gamma) w(1, \gamma, 0, M)] + (1 - \mu) w(0, \gamma, B, \varnothing)$$

Now, note that $v = w(1, \gamma, G, M)$, so we can rewrite the PKC as:

$$(1 - \mu) \delta w(0, \gamma, B, \varnothing) = (1 - \mu) \gamma \delta v - \mu (1 - \delta) M - \mu (1 - \gamma) \delta w(1, \gamma, 0, M)$$

So by subtracting $IC_B$ from PKC, we get that:

$$(1 - \gamma) \delta w(1, \gamma, 0, M) = [\mu + (1 - \mu) \theta]^{-1} \times$$

$$\times \{(1 - \mu) \gamma \delta v - \mu (1 - \delta) M - (1 - \mu)(1 - \delta)(-D + (1 - \gamma) \theta M)$$

$$- (1 - \mu)(\gamma + (1 - \gamma)(1 - \theta)) \delta w\}$$

note: if $v = w$ then $(1 - \gamma) \delta w(1, \gamma, 0, M) = \delta (1 - \gamma) w - (1 - \delta) M + \frac{(1 - \delta)((1 - \mu)(D + \gamma \theta M) + w)}{\mu + (1 - \mu) \theta} \geq$
\[(1 - \gamma)\delta w \iff w \geq \mu M + (1 - \mu)(-D + (1 - \gamma)\theta M),\] which is true at \(\gamma = \gamma^{PBE}\) (after all, the bank could always choose to not make its assets complex in all periods and get \(\mu M\) guaranteed), but not necessarily true at \(\gamma < \gamma^{PBE}\). In particular, if \(\gamma \to 0\), this says that \(\delta w(1, \gamma, 0, M) \geq \delta w\) iff \(w \geq \mu M + (1 - \mu)(-D + \theta M)\), which is clearly not true (after all, \(-D + \theta M > 0\) is the maximum payoff the bank can obtain after drawing a bad product, and this requires that \(w\) is higher than getting the payoff of good products with probability \(\mu\) and getting the maximum payoff of bad products with probability \((1 - \mu)\) in all periods). 

Similarly, for \(v > w\), we have \((1 - \gamma)\delta w(1, \gamma, 0, M) = \frac{(1 - \mu)\delta w}{\mu + (1 - \mu)\theta}(v - w) + \delta (1 - \gamma) w - (1 - \delta) M + \frac{1 - \gamma - v}{\mu + (1 - \mu)\theta}(D + (1 - \gamma)\theta M)\) \((v - w) + w \geq (1 - \delta) M w + (1 - \delta)(1 - \mu)(-D + (1 - \gamma)\theta M)\). Noting that \((1 - \mu - \gamma) \leq v - w\) then yields that a necessary condition for \((1 - \gamma)\delta w(1, \gamma, 0, M) \geq (1 - \gamma)\delta w\) is that \(v \geq (1 - \delta) M w + (1 - \delta)(1 - \mu)(-D + (1 - \gamma)\theta M)\), which is, again, impossible for \(\gamma \to 0\).

This says that with \(v \to w\), it is impossible to set \(v = 0\), \(\gamma\) low enough, PKC binding and \(IC_B\) binding. But then, with \(v \to w\), it is impossible to set \(v = 0\), \(\gamma\) close enough to zero, PKC binding and strict incentives for \(\alpha = 0\) (since this would require \(w(1, \gamma, 0, M)\) even lower than the one above, to create larger punishments for \(\alpha = 0\)). To see this more clearly, note that \(IC_B\) provides strict incentives for \(\alpha = 0\) iff:

\[(1 - \mu)\delta w(0, \gamma, B, \emptyset) > (1 - \mu)(1 - \delta)(-D + (1 - \gamma)\theta M)\]

\[+ (1 - \mu)\delta [\gamma + (1 - \gamma)(1 - \theta)] w + (1 - \mu)\delta (1 - \gamma)\theta w(1, \gamma, 0, M)(1 - \mu)\delta\]

and using PKC to replace for \((1 - \mu)\delta w(0, \gamma, B, \emptyset)\) yields

\[(1 - \gamma)\delta w(1, \gamma, 0, M) < [\mu + (1 - \mu)\theta]^{-1} \times\]

\[\times \{(1 - \mu - \gamma)v - \mu(1 - \delta)M - (1 - \mu)(1 - \delta)(-D + (1 - \gamma)\theta M)\]

\[- (1 - \mu)(\gamma + (1 - \gamma)(1 - \theta))\delta w\]

and we have just shown that this upper limit to \((1 - \gamma)\delta w(1, \gamma, 0, M)\) is below \(w\) when \(\alpha = 0\), PKC is binding and \(v \to w\).

This is relatively intuitive: when \(v\) is low enough, this means that \(w(0, \gamma, B, \emptyset)\) is relatively low. But then, to provide incentives for \(\alpha = 0\), relying on punishments based only on continuation values and not on monitoring requires very large punishments \(w(1, \gamma, 0, M)\), beyond \(w\). Hence, when \(v\) is low enough, it is not feasible to enforce \(\alpha = 0\) with \(\gamma = 0\) while having optimal continuation values.

Similarly, note that when \(v = w\), then we have that \((1 - \gamma)\delta w(1, \gamma, 0, M) = (1 - \mu)(1 - \gamma)\theta w - (1 - \delta) M + \frac{(1 - \gamma - v)}{\mu + (1 - \mu)\theta}(D + (1 - \gamma)\theta M)\). Now, this is lower than \((1 - \gamma)\delta w\) as long as \(\mu(1 - \mu)\theta w - (1 - \delta) M + \frac{(1 - \gamma - v)\mu + (1 - \mu)\theta w - (1 - \delta) M}{\mu + (1 - \mu)\theta}\). Note then that \(\omega \leq \mu M + (1 - \mu)(-D + \theta M)\), while \(w \geq \mu M\), so that \(\omega - w \leq (1 - \mu)(-D + \theta M)\).

Now, we turn to \(w(0, \gamma, B, \emptyset)\). To do so, we can replace \(w(1, \gamma, 0, M)\) on PKC, to find that:

\[\delta w(0, \gamma, B, \emptyset) = [\mu + (1 - \mu)\theta]^{-1}\times\]

\[\times \{-\theta\mu(1 - \delta)M + \mu(1 - \delta)(-D + (1 - \gamma)\theta M) + \mu(1 - \gamma)(1 - \theta))\delta w\]

\[+ \theta(1 - \mu - \gamma\theta)\} w\]
We thus have all the continuation values as functions of \( v \) and \( \gamma \).

**Subcase 2: \( \alpha = 1 \)**

From Lemma 15 we know that when \( \alpha = 1 \), \( w(1, \gamma, s, p) = w^* \) for all values of \( s \) and \( p \). This means that can trivially show by the PKC that \( \delta w(1, \gamma, s, p) = \delta w^* = \delta v - (1 - \delta)(\mu M + (1 - \mu)[-D + (1 - \gamma)\theta M]) \). With this result, we have all the continuation values as functions of \( v \) and \( \gamma \).

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34Note that \( w(0, \gamma, s, p) \) need not be defined in this case, as it happens with zero probability.
Bibliography


