Essays in Entrepreneurial Finance

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Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

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Statement of Cojoint Work

I certify that Chapter 3 was co-authored with Gianpaolo Caramellino, and that I contributed 50% of the work.

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Thesis Abstract

In Chapter 1, a theory of optimal fund size in venture capital is developed. Fund managers - the VCs - add value to the projects they finance, but their human capital is scarce. A matching model is proposed where VCs span their nurturing activity over more projects, and entrepreneurs, who own the projects, direct their search to VCs based on their projects' quality. The work provides necessary and sufficient conditions for positive and negative assortative matching over VC attention and project quality to emerge and shows when VCs fundraising decision is distorted by selection considerations. The chapter ends with an investigation of the effects of entry of less skilled intermediaries. By attracting the worse entrepreneurs, these new agents alleviate the adverse selection problem associated to managing a larger fund. This offers a new angle to think about policies encouraging entry in the venture capital industry.

In Chapter 2, the model developed in Chapter 1 is extended to a dynamic setting, where projects need time to develop and produce returns. VCs can choose to enter in a short-term contract with investors, giving them access to investors liquidity for a given period of time, and an open credit relationship that allows them to raise investors money at any point in time. The model illustrates a novel advantage of closed, finite-horizon funds, which emerge in equilibrium even when they are socially undesirable: they attract the best entrepreneurs, who value the most the exclusive relationship that only a closed-end fund can guarantee. The interpretation is that VCs benefit from committing to a size in the first place.

In Chapter 3, the focus moves to the study of the distortions in fund managers' behavior that may occur within a fund's life. A setting is introduced where information about a manager's ability is imperfect and managers are interested in their reputation. Given the application to investments in young firms, managers in the model are agents that create value because they can experiment and learn about a projects potential. Their incentive to take on risk is distorted by career concerns, and can result in under or over risk-taking. The result contrasts with Holmstrom (1999) where managers directly affect the project's success rate, and career concerns can only produce inefficiently low risk-taking. It is shown that the inefficiency is reduced when the market can also observe the outcome of projects with the same fundamental.

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Chapter 3

A Model of Risk Taking with Experimentation and Career Concerns

Chapter 1 - A Theory of Venture Capital Fund Size with Directed Search*

Abstract

I develop a theory of fund size and structure in venture capital where fund managers - the VCs - add value to the projects they finance, but their human capital is scarce. I propose a matching model where VCs span their nurturing activity over more projects, and entrepreneurs, who own the projects, direct their search to VCs based on their projects' quality. VCs differ in the ability to scale up their human capital. I derive necessary and sufficient conditions for positive and negative assortative matching over VC attention and project quality to emerge. Anticipating positive sorting, VCs shrink fund size below the efficient level. Entry of unskilled VCs feeds back into equilibrium sorting, increases returns at the top of the distribution - consistently with empirical evidence - and always results in a Pareto-improvement. This offers a new angle to think about policies encouraging entry in the venture capital industry.

1 Introduction

Venture capital has been undoubtedly a successful model of financing entrepreneurship. The common view among practitioners and academics is that venture capitalists (henceforth VCs) add value to the companies they finance, on top of the capital they provide them with.[1] There is evidence that VCs differ considerably in their ability to

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[1] For example they do so through a number of activities such as monitoring, selecting top management, and experimenting innovative business strategies.

generate returns and to help their companies get to the initial public offering stage.[2] The funds VCs raise often go oversubscribed, and recent evidence suggests they stay below the point where significant decreasing returns kick in.[3] In light of the role they play in boosting growth, it is important to understand how capital is allocated across, and used by, these scarce, and differently skilled VCs. Is there an efficient amount of capital put at the work in this industry?

This paper builds on the observation that in venture capital, those companies receiving financing are in turn interested in matching with the best VCs. I will argue that self-selection of different entrepreneurs seeking VC finance into different VC funds is responsible for an inefficient choice of fund size by VCs, and can explain some of the regularities we obseve in this industry.

The success of the venture capital model has motivated many governments to try and stimulate the provision of VC financing in various ways. This has generated a debate, and some scepticism among academics, on the role of the public sector in improving private VC activity. In a thorough analysis of the subject, Lerner (2009) argues that public measures encouraging VC investments may favour only the less efficient VCs, and even crowd out investments from the most knowledgeable ones.[4] But does allowing less sophisticated VCs in the economy necessarily result in bad outcomes?

I tackle these issues by developing a matching model of fund management in venture capital. There are two sets of agents in the economy: VCs, and entrepreneurs. To capture scarcity in the quality and quantity of a VC's human capital and expertise, I assume that VCs value added - or *attention* - to each investment dilutes as the number of projects they finance increases.[5] VCs differ in skill, which governs how efficient they are at increasing the size of their portfolio: the combination of VC skill and size ultimately determines the level of attention the VC can provide to each project under their management; more skilled VCs are those that can provide higher attention for any given portfolio size. For them, the diseconomies of scale are less severe. On the other side of the market, each entrepreneur owns one project. Projects are heterogeneous in quality. A projects needs the input of a VC to become profitable.

[2]The heterogeneity in skills among VCs has been documented for example by Sørensen (2007) and Korteweg and Sorensen (2017). For a survey of research in private equity, see Da Rin et al. (2011). Relevant empirical findings that motivate my modelling assumptions are in Kaplan and Schoar (2005), Harris et al. (2014) and Robinson and Sensoy (2016). In particular, it appears that 1) in the cross section, there is a positive size-(net-of-fees) returns relationship at the fund level and 2) accounting for fund managers fixed effects, average returns to investors are decreasing in fund size.

[3]See Rossi (2017).

[4]See in particular the discussion of the Canadian Labor Fund Program in Chapter 6.

[5]This is arguably one of the most significant drivers of the diseconomies of scale observed in the industry. For direct evidence of this, see for example Cumming and Dai (2011).

The return from a project is a deterministic function of its own quality and of the VC's attention.

In the model VCs move first and choose a fund size - or capacity - to which they commit. Entrepreneurs move after VCs. They first decide whether to enter the market and, if they do so, they observe their projects' quality; finally, they search for a suitable VC. Once a match is formed, returns are produced and shared exogenously between the VC and the entrepreneur. The focus on directed search is motivated by the application: one major distinction between the activity of VCs compared to that of other fund managers (e.g. buyouts, mutual funds) is that the former invest in targets that are in turn interested in their ability to add value; after all, entrepreneurs remain owners of a significant fraction of the firm they grow with the VC. The idea that entrepreneurs seeking venture capital money discriminate among VCs based on their reputation and perceived quality is supported by compelling evidence.[6]

Once entrepreneurs have directed their search, as many entrepreneurs as vacancies available are matched at random in a given VC skill-size combination, which defines a submarket. Since the measure of VCs in the economy, and the capacity they commit to, are limited, entrepreneurs in a given submarket may get rationed. Hence, when choosing which VC to search for, entrepreneurs trade off matching with VCs that can devote more attention to their projects, against the lower search frictions in markets where VCs attention is lower. Complementarity between the two inputs of the returns function mean that for the best entrepreneurs, the first force - the value attached to higher attention - is relatively more salient. This generates positive sorting between VCs' attention and entrepreneurial quality.

In turn, this has effects at the initial stage of the game, since VCs anticipate that managing a fund of larger size attracts low quality entrepreneurs. In equilibrium, some unskilled VCs shrink the size of their funds below what the welfare maximizing solution prescribes. The inefficiency arises because VCs don't internalize the effect their choice induces on the equilibrium assignment: what drives the separation among entrepreneurs is the increase in search frictions in markets where attention is higher compared to where it is lower. But if too many VCs offer high attention, this increase is too small, and entrepreneurs' separation is suboptimal. That is, some entrepreneurs whose quality is relatively low search for high-attention funds, lowering average quality in those submarkets. In addition, multiple equilibria may generally emerge, with Pareto-dominated equilibria being those characterized by smaller funds size.

In this environment, subsidizing entry of low skilled VCs that are inactive - for

[6] Hsu (2004) finds that entrepreneurs are willing to accept worse terms in order to affiliate with VCs that can provide greater value added. Recent empirical studies, starting from the seminal contribution by Sørensen (2007), show that there exist positive sorting in the industry between better VCs and start-up firms with greater potential.

example because their ability to generate returns is not sufficient to cover the fixed costs of starting operations - always results in net aggregate gains. The reason is that these agents will absorb low quality entrepreneurs; those efficient VCs who choose to provide higher attention will attract even better projects, because only the worse entrepreneurs they were originally matched to will find it worthwhile to switch in the now larger market associated to low attention. In some cases, the total measure of projects funded by incumbent VCs will also increase. This offers a new angle to think about public intervention in this market, and a more optimistic point of view on policies that encourage fundraising devoted to venture investments. Interestingly, Brander et al. (2014) find evidence that the presence of government-sponsored VCs does not crowd out, but rather increases investments from private VCs at the aggregate level.[7]

The model provides novel implications from entry of new VCs on the whole returns distribution. Specifically, when more unskilled VCs enter the market, while a larger share of funded projects end up in the lower side of the returns distribution, those at the top deliver higher returns. This is consistent with the findings in Nanda and Rhodes-Kropf (2013), who document that investment made "hot" periods are more likely to fail and give higher returns conditional on not failing, and in Kaplan and Schoar (2005), who find that, in times characterized by more intense activity in the industry, capital flows disproportionately to worse performing funds.

A benchmark model with random matching, or with homogeneous entrepreneurs would not produce the inefficiency in equilibrium fund size, nor the beneficial effect of entry of new VCs and its effect on the shape of the returns distribution described above.

Relation to the Literature. The paper directly contributes to the literature focusing on size determination in fund management, with particular application to the venture capital asset class. One natural reference is Berk and Green (2004), who derive several predictions concerning fund flows in the mutual fund industry; like in that paper, fund managers in my model possess scarce skills, and therefore receive all the rents from investors by choosing fund size and fees appropriately. However, while in Berk and Green (2004) this results in an efficient allocation of money across managers, adding entrepreneurs self selection in my model produces: 1) a generically inefficient outcome, 2) multiple equilibria that are not welfare equivalent and 3) a feedback effect of entry of unskilled managers on returns at the top of the distribution. Fulghieri and Sevilir (2009) model the optimal investment strategy of a VC who

[7]An empirical assesment of the effects of subsidized funds activity on the profitability of investments made by incumbent, non-subsidized VCs is still missing in the literature.

trades off the higher value added from a small portfolio, with the diversification gains from a large one. Inderst et al. (2006) hold portfolio size constant, and model the beneficial effect - through stronger competition among entrepreneurs - of having limited capital at the refinancing stage. I share with the first paper the view that VC's human capital dilutes with a larger portfolio, and with the second the idea that the amount of capital a VC raises affects the type of projects funded. But in my model the distribution of VCs size and structure affects the sorting; I study the equilibria that result from the interaction among VCs that anticipate this effect.

In terms of the entrepreneur-VC relationship, in my economy matches form between two parties whose payoffs are asymmetrically affected by the current match: while the entrepreneur is solely interested in the return from his project, the VC cares about the total fund's returns. The VC faces a typical quality-quantity of matches trade-off. This approach to modelling the venture capital environment, and the essential tension implicit to it, is shared with several recent works. In Michelacci and Suarez (2004), the focus is on identifying institutional market characteristics that increase total welfare by alleviating this trade-off and allowing VCs to free up their human capital quicker, without destroying too much of the monitored firm's value; Jovanovic and Szentes (2013) find conditions under which the optimal contractual arrangement in presence of moral hazard on the entrepreneurs' side takes the form of an equity contract. They also explain the returns premium to VC-backed firms; Silveira and Wright (2015) study project selection on the VC's side and optimal fund size when start-up costs are random but committing funds entails opportunity costs. Contrary to mine, none of the aforementioned models analyse sorting of different entrepreneurs with different VCs in presence of these forces. More importantly, while I also assume diseconomies of scale, I don't restrict intermediaries to run one project at a time. This more realistic assumption allows to study 1) the equilibrium choice of span of control and 2) the choice of how frequently go back to the market and actively search for new investments, possibly before the current one has produced returns. The VC-entrepreneur relationship has been also the subject of a large strand of literature focusing on the inherent agency problems associated to venture capital financing, and the contractual arrangements aimed at solving this problems: notable examples are Cornelli and Yosha (2003) and Repullo and Suarez (2004), both analysing optimal security design when new information is produced about the investment at an intermediate stage, which is an essential characteristic of this environment; in Schmidt (2003) the double moral hazard problem between the two parties justifies the use of convertible preferred equity, while Hellmann (2006) extends this analysis to allow for a distinction between exit via IPO and via private acquisition and finds that automatic conversion is only triggered under exit via IPO in the optimal contract; finally, Casamatta (2003) studies the endogenous emergence of external financing from venture capitalist who also provide human capital, and shows that the optimality of common stocks versus preferred equity depends on the relative amount invested by the venture capitalist. I abstract from these issues and take a reduced form approach to the determination of returns to a project, and assume an exogenous equity contract between the two parties. However, project's quality in my model could be interpreted as a (negative) measure of the severity of the moral hazard problem on the entrepreneur's side, naturally affecting total surplus from a match.

Like this paper, Marquez et al. (2014) builds upon the fundamental observation that investments in venture capital are special in that they are subject to a two-sided matching problem. Marquez et al. (2014) develop a signal-jamming model where VCs with differential ability to produce returns distort the fund size decision in order to affect entrepreneurs' learning; this, coupled with rigidity in fees adjustment ex-post, prevents them from extracting the full surplus from investors. In my model instead, the VCs ability is common knowledge. Moreover, while Marquez et al. (2014) take a reduced form approach to the determination of a fund's portfolio quality, I study and characterize sorting explicitly; since relative gains from committing higher attention are endogenous, I can derive conditions under which an equilibrium where every VC chooses a certain fund structure might unravel; plus, modelling sorting allows me to study efficiency of the funds allocation across VCs, and study the effects of entry of VCs on the entire allocation and returns distribution.

On a more abstract level, my paper provides conditions for sorting in a matching environment with non-transferable utilities and search frictions. Eeckhout and Kircher (2010) derive general results on the consequences of search frictions in an assignment problem where sellers commit on posted prices. Requirements on the match-value function for positive and negative sorting are found to depend on the elasticity of substitution in the matching technology. In my model, where utilities are non-transferable, the strongest form of supermodularity (and submodularity) is needed to guarantee sorting, under any specification of the matching function. More results related to my setting are in Eeckhout and Kircher (2016) who study the interaction between the choice of span of control and the sorting pattern in an assignment economy; they look at competitive equilibria where types are observable on both sides, and the allocation is not limited to one-to-one. In my model there will be no direct type complementarity, hence what will govern sorting is the interaction between the diseconomies of scale, the span-of-control complementarity and the managerial resource complementarity.

Roadmap: Section 2 introduces the setup, followed by the characterization of the equilibria; Equilibria are ranked in terms of welfare achieved and compared to a second best solution in Section 3; Section 4 explores the effects of entry of new VCs in the economy; Section 5 concludes; All proofs are relegated to the Appendix.

2 Model

Agents. The economy consists of heterogeneous venture capitalists (henceforth VCs), identical investors and ex-ante identical entrepreneurs. There is an arbitrarily large measure of investors. Each investor is endowed with money, which they can invest into funds, each managed by a single VC. VCs are exogenously endowed with ability, denoted x, according to the measure G, that admits a continuous density g with full support $[\underline{x}, \overline{x}] \subset \mathbb{R}_+$. The measure of VCs in the economy is fixed. Entrepreneurs are in large supply, and can enter the market upon paying startup cost c. If they do, they draw a type λ, the quality of the project they own, from a continuous distribution f strictly positive on the entire support $\left[\underline{\lambda}, \overline{\lambda}\right] \subset \mathbb{R}_+$. An higher λ is a better project in a way specified in the next paragraph. Entrepreneurs need money and the VCs' input to make their projects turn into profitable firms.[8]

Projects. All projects need only one unit of money to become a firm. Call m the measure of projects a given VC is matched to in equilibrium. Define a the attention the VC devotes to each project. Assume $a \in \{a_0, a_1, ..., a_N\}$, with $a_i > a_{i-1}$. VC's attention, or managerial input, is a function of his ability and the number of firms he is matched to, $a \coloneqq a(m, x)$. In particular $a(m, x)$ is the step function:

$$a(m,x) \ = \begin{cases} a_N & \forall m \in [0,\, m_N^x] \\ a_i & \forall m \in \left(m_{i+1}^x,\, m_i^x\right] \end{cases}$$

with $m_i^x - m_{i+1}^x = \Delta > 0$ for all x and i, and $\partial m_i^x / \partial x > 0$ for all i and all x. In words, the two conditions mean that 1) VCs' input gets diluted when working on more projects in parallel, 2) better managers can run more projects at a given level of attention. A manager with ability x can be matched to a maximum of m_0^x

[8] A natural interpretation - which fits the common view of the role of venture capitalists - is that young firms need to be constantly monitored, be it because entrepreneurs are unexperienced, or because the lack of collaterals makes it impossible to find alternative sources of financing.

projects.[9,10] Each project's return, R, is assumed to be a function of attention, a, and of the project's quality, λ. Call this function $R(a, \lambda)$.[11] It is natural to have $R_a(a, \lambda)$ and $R_\lambda(a, \lambda) > 0$. I further assume that $R(a, \lambda)$ is twice continuously differentiable in its arguments.

Matching and Information. While VCs' size and ability are common knowledge, the entrepreneur's type, λ, is his private information. Therefore, I study directed search from the long and informed side of the market, the entrepreneurs. Each VC's combination of size and ability, (w, x), will therefore form a submarket where entrepreneurs will select into, possibly depending on their type. Finally, assume that as many matches as possible are formed in each submarket; that is, the number of matches as a function of the measure of entrepreneurs searching, q_e, and the measure of money available (or "vacancies"), q_k, is given by $M(q_k, q_e) = \min\{q_k, q_e\}$.

Payoffs, Strategies and Timing. In the first stage of the game, each VCs offers investors a contract (w, p) , which specifies the size of the fund, w, and fixed fee p that the VC receives from the investors for every dollar invested.[12]As all projects require one unit of money, I will refer to fund size w as the fund's capacity, that is the maximum measure of entrepreneurs the VC can be matched to. Investors can accept the contract, and provide the VC with w dollars, or reject and invest in an alternative technology delivering constant returns R_0. When investing in a certain VC, they will get a fixed share $\alpha \in (0, 1)$ of the VC's average returns from the fund. In the second stage, entrepreneurs observe the joint distribution of (w, x) induced by the first stage, and choose whether or not to pay the startup cost. Those who do, can direct their search towards different VCs. Conditional on being matched, they receive the residual - $(1 - \alpha)$ - share of the returns from their projects. All agents are risk neutral and maximize expected returns.

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[9]The assumption that attention jumps discontinuously with m is of no consequence in terms of the qualitative results, but allows to guarantee existence of equilibria when size is the VC's choice.

[10]A more general setting could allow for Δ to be a function of x . In which case, to ensure separation of VCs in equilibrium, I would need to impose the single crossing condition $\partial\left(\frac{m_0^x}{m_0^x-\Delta(x)}\right)/\partial x < 0$, which is satisfied when Δ is constant across xs.

[11]The direct implication is that a is all that matters to a given type of entrepreneur. In other words, project's quality does not interact with VC's ability or fund size *per se.* This separability will greatly simplify the analysis.

[12]As it will be clear when studying size determination, the assumption that VCs receive no performance-based compensation is without loss of generality. This is due to: 1) the fact that there is no agency conflict between investors and VCs, nor uncertainty about the VC's ability, and 2) the presence of a large measure of investors, which implies that investors' participation constraint will bind in all equilibria.

2.1 The Entry and Sorting Subgames

Market Tightness. Let me first study the subgame where entrepreneurs make the entry decision and direct their search into different VCs. Assume that the allocation of investors' money generates fund size between $\underline{w}$ and $\overline{w}$ with $\overline{w} > \underline{w}$. Denote $H(w, x)$ the measure of venture capitalists with fund size below w and ability below x.[13] Upon entry, the search strategy for an entrepreneur is described by a distribution over $[\underline{w}, \overline{w}] \times [\underline{x}, \overline{x}]$. Formally, the entrepreneur strategy is a mapping

$$s : \left[\underline{\lambda}, \overline{\lambda}\right] \to \triangle\left([\underline{w}, \overline{w}] \times [\underline{x}, \overline{x}]\right).$$

The strategy generates for every λ a cumulative density function $S(w, x; \lambda)$. Calling E the measure of entrepreneurs who decide to enter. Define $\tilde{S}(w, x, E)$ the measure of entrepreneurs searching in market with size below w and ability below x, given E. This is given by summing the search strategy over all the entrepreneurs, so $\tilde{S}(w, x; E) = \int_{\lambda} ES(w, x; \lambda)\, dF(\lambda)$. On the other side of the market, as a VC managing fund of size w can follow up to w projects in parallel, the amount of vacancies in submarkets below (w, x) is given by $\int_{-\infty}^{x}\int_{-\infty}^{w} \hat{w} dH(\hat{w}, \hat{x})$. To define expected payoffs properly, let $\theta(w, x; E)$ be the expected ratio of vacancies to entrepreneurs in submarket (w, x), when a measure of E entrepreneurs has entered. I will refer to $\theta(w, x; E)$ as market tightness. The function will solve:

$$\int_{-\infty}^{x}\int_{-\infty}^{w} \hat{w} dH(\hat{w}, \hat{x}) = \int_{-\infty}^{x}\int_{-\infty}^{w} \theta(\hat{w}, \hat{x}; E)\, d\tilde{S}(\hat{w}, \hat{x}; E).$$

Finally, define $Q(w, x; E)$ the probability an entrepreneur finds a match when searching in market (w, x). Given that the matching function is Leontief, this is:[14]

$$Q(w, x; E) := \min\{\theta(w, x; E), 1\}.$$

I can now write type-λ entrepreneur's expected payoff from choosing to search in market (w, x) as:

$$(1-\alpha)\, Q(w, x; E)\, R(a(m(w, x; E), x), \lambda)$$

[13] This is endogenous, as it is determined by the investors and VCs equilibrium choice. Hence no assumption on H is made at this stage.

[14] The assumption that the matching function is Leontief does not affect the equilibrium characterization. However, it is relevant in the welfare analysis. By assuming that as many matches as possible are formed in every submarket, I can abstract from inefficiencies that might arise from matching frictions within the submarket, and focus on those coming from the directed search assumption alone.

where $m(w, x; E)$ is the measure of projects per VC in market (w, x). Note that $m(w, x; E) \leq w$, but the condition may, in principle, not bind. To save on notation, I will denote $\pi_\lambda(E, s^*)$ the *equilibrium* value of type-λ entrepreneur's expected payoff. I can now describe what is an equilibrium of this subgame.

Definition 1. ***(Equilibrium in the Subgame).*** *An equilibrium in the entry and sorting subgame is characterized by a vector (E, s^*) such that:*

$$(i) \qquad s^*(\lambda) = \arg\max_s \mathbb{E}_{w,x}\left[Q(w, x; E, s^*)(1-\alpha) R(a(m(w, x; E, s^*), x), \lambda)\right]$$

$$(ii) \qquad \int_\lambda \pi_\lambda(E, s^*)\, dF(\lambda) = c$$

Part (i) imposes optimality. Part (ii) from the unlimited number of entrepreneurs: it states that, ex-ante, entrepreneurs must be indifferent between entering the market and staying out.

An immediate observation to make is that in this model, not only the entrepreneur's search strategy imposes an externality to each other entrepreneur through its usual effect on search frictions, it also does by affecting VCs attention. In principle, this can generate a multitude of equilibria where the value of a VC is ultimately determined by the measure of entrepreneurs searching in a given submarket. However, one additional assumption can be shown to substantially simplify the sorting game. The assumption requires that lower VC's attention is not *too* detrimental to the average type, as formalized below.

Assumption A1. $(1-\alpha)\mathbb{E}_\lambda R(a_0, \lambda) > c \qquad \forall\lambda.$

A1 states that, ex-ante, an entrepreneur would strictly benefit from paying the startup cost and match to a VC in absence of search frictions, even when the VC's attention is fully diluted (at its lowest level it is given by a_0). When A1 holds, because entrepreneurs are in large supply, new ones will enter the market until search frictions kick in. This also implies that a situation where some VCs attract no entrepreneur can not be an equilibrium of the subgame, since those VCs would be able to provide the highest attention at no search friction, offering a strict incentive to deviate to entrepreneurs.

Lemma 1. *Under A1, in any equilibrium, in each submarket there are more entrepreneurs than vacancies. That is, $Q(w, x, E) < 1$ and $m(w, x, E) = w$, $\forall(w, x)$*

The implication of Lemma 1 is that all VCs operate at full capacity. The next result is a direct consequence of Lemma 1, and will help characterize the equilibrium strategies in the sorting subgame.

Lemma 2. *Given E, in any equilibrium, $Q(w, x; E)$ is a function of $a(w, x)$ only.*

Intuitively, because VCs must operate at full capacity in every equilibrium, attention in market (w, x) is given by $a(w, x)$. As returns are only a function of attention and project's quality, an entrepreneur must be indifferent between searching in two markets where attention is the same. This suggests that, in essence, the entrepreneur's strategy reduces to which attention levels a to seek matching with.

Lemma 3. *For a given E, any equilibrium of the sorting subgame is mirrored by one from a game where entrepreneurs can only direct their search to different attention levels, and are then matched with VCs that are at the chosen attention, in proportion to each VC's size.*

In words, because entrepreneurs must be indifferent between searching in any market where attention is the same, any equilibrium can equivalently be represented by one where their strategy is to simply choose to search over different levels of attention, which in this reduced model is a fixed, predetermined characteristic of the VC. The distribution of vacancies will reflect total size summed across all VCs at a given iso-attention locus in the original model.

Lemma 3 turns useful because it allows to focus on a particular type of sorting equilibrium, where the sole characteristic of a VC, hence what defines a sub-market to search in, is attention. The interest is then to study what requirements should the return function obey to, so that in a general setting, independently of the distribution of types, sorting would emerge. If such conditions are identified, one can conclude that the same sorting pattern would emerge in the original model, once mixed strategies are adjusted accordingly.

Let $\mathit{\Lambda}^s(a)$ be the set of entrepreneurs applying to market a under strategy s, $\mathit{\Lambda}^s(a) := \{\lambda \colon s(a; \lambda) > 0\}$.

Definition 2. *An equilibrium exhibits positive (negative) assortative matching if $\forall a, a'$* with $a > a'$

$$\lambda \in \mathit{\Lambda}^s(a) \cap \lambda' \in \mathit{\Lambda}^s\left(a'\right) \Rightarrow \lambda > (<)\, \lambda'.$$

Intuitively, under positive assortative matching (henceforth PAM), higher attention can not be associated with a worse entrepreneur; however, pooling of more entrepreneurs into a given attention level is allowed. I can now state the main result of this section, that estabilishes necessary and sufficient conditions for equilibria to exhibit PAM or NAM.

Proposition 1. ***(Sorting).*** *All equilibria exhibit PAM (NAM) if and only if $R(a,\lambda)$ is everywhere logsuper(sub)modular.*

Notice that logsuper(sub)modularity implies super(sub)modularity, while the opposite does not hold. To build intuition why a stronger form of supermodularity is necessary for PAM, notice that, as emphasized by Eeckhout and Kircher (2010), when allowing for search frictions in matching models, two forces drive the sorting pattern, in opposite directions: the "trading security motive", which motivates higher types to select into less crowded markets, and the "match value motive", which is related to the value of being matched to better types. In this setting, the latter motive corresponds to the value of the VC's attention, which is a bigger concern when λ is high[15]. This trade-off becomes evident if one looks at the difference in expected payoff from searching in any two markets, a and $a^{'}$, with $a > a^{'}$, and differentiates it with respect to λ. This difference is increasing in λ when:

$$\underbrace{\frac{-\left(Q(a)-Q\left(a^{'}\right)\right)}{Q(a^{'})}R_{\lambda}(a,\lambda)}_{\text{trading security motive}} < \underbrace{\frac{\partial\left(R(a,\lambda)-R\left(a^{'},\lambda\right)\right)}{\partial\lambda}}_{\text{match value motive}}.$$

In words, only when complementarities in the returns function between attention and quality are sufficiently strong does a higher-λ entrepreneur prefer to search for higher attention and face the larger search frictions in this, more crowded, market.
To understand why the logsupermodularity is sufficient, notice that a function $R(a,\lambda)$ is logsupermodular if and only if, for any $\left(a,a^{'}\right)$ with $a^{'} > a$, the ratio $R\left(a^{'},\lambda\right)/R(a,\lambda)$ is strictly increasing in λ. This means that, if for some type $\tilde{\lambda}$, $Q\left(a^{'}\right)R\left(a^{'},\tilde{\lambda}\right) > Q(a)R\left(a,\tilde{\lambda}\right)$, the same would be true for all $\lambda > \tilde{\lambda}$. This ensures separation.

The rest of the analysis focuses on the case when $R(a,\lambda)$ is logsupermodular. [16]

Assumption A2. *$R(a,\lambda)$ is everywhere logsupermodular.*

[15]It should be noted that the condition in Proposition 1 is particularly strong because utilities are non-transferable. In the framework proposed by Eeckhout and Kircher (2010), where sellers can commit on posted prices, it is shown that, although supermodularity per se is generally not sufficient, the requirements for PAM to emerge are milder. In particular, the degree of supermodularity depends on the elasticity of substitution in the matching function. Notably here, with directed search and non-transferable utilities, the result that R must be logsupermodular holds true under any specification of the matching function.

[16]Focusing on the case when R leads to assortative matching is motivated by the fact that, as it will be clear in the next section, this will guarantee all equilibria exhibit positive sorting between firms and managers, which is consistent with the evidence started by Sørensen (2007). Interestingly, the idea that better entrepreneurs are those that gain more by receiving VCs' advise appears to be at the core of the following quote by Fred Wilson, managing partner at Union Square Ventures: *"When it's clear the founder only wants your money and has no interest in your advice, it is hard to*

2.2 Choice of Fund Size and Equilibrium in the Supergame

In this section I study the VCs' choice at the initial stage, when contracting on size and fees with investors. Therefore, I endogenize the distribution $H(w,x)$, and hence will characterize equilibria of the entire game. I will restrict attention to equilibria where both VCs and entrepreneurs play symmetric, pure strategies.

As in Berk and Green (2004), VCs contract with competing investors over the fund's size and a per-dollar fee. Notice that, for every unit of money invested in the fund, investors' participation constraint gives:

$$\alpha\mathbb{E}\left[R\left(a\left(w,x\right),\lambda\right)|\lambda\in\Lambda^{s}\left(a\left(w,x\right)\right)\right]-p\geq R_{0}. \tag{1}$$

Since VCs have all bargaining power, it must be that the net return to investors equals their outside option, R_0. In other words (1) has to bind. It follows that VCs will choose w to maximize total excess returns, and then set p in such a way that investors' participation constraint binds, so to extract the full surplus and maximize total fees.

VC Strategy. The VC's decision can be further simplified by noting that, as entrepreneur's selection is affected by fund size only through its effect on attention, a VC will never set a size strictly in one region where the function $a(m,x)$ is constant. It follows that the relevant strategic choice from a VC is which attention a_i to offer. The VC will consequently propose investors the maximum size conditional on a_i, that is m_i^x. VC's strategy is therefore fully described by a mapping $\sigma:[\underline{x},\overline{x}]\rightarrow\{a_0,a_1,...,a_N\}$. I will sometimes refer to funds associated with higher attention as to more "focused" funds, although it should be emphasized that a more focused fund could well be of larger size than a less focused one, if it is managed by a more efficient VC. Define the set of VCs types choosing to offer a_i given σ, $X_i^\sigma:=\{x:\sigma(x)=a_i\}$. Finally, define the set of attention levels offered in equilibrium $I^*:=\{i:\ X_i^\sigma\neq\emptyset\}$. At any a_i with $i\in I^*$, and given s, E, and σ, one can then compute the probability for an entrepreneur to find a match, or, equivalently, market tightness as:

$$Q\left(a_i;\sigma,s,E\right)=\frac{\int_{x\in X_i^\sigma}m_i^x dG\left(x\right)}{E\int_{\lambda\in\Lambda^s(a)}dF\left(\lambda\right)}.$$

Before I define what is an equilibrium in the entire game, it is necessary to specify how VCs beliefs about the composition of entrepreneurs in a given market are formed. The notion of Weak Perfect Bayesian Equilbrium only disciplines beliefs on the equilibrium

get excited about the investment. When it seems that all the founder wants is your advice and isn't worried about getting money, it makes you want to work with that founder".

path, by restricting these to be computed via Bayes rule.[17] Formally, the belief β is a mapping:

$$\beta : \{a_0, .., a_N\} \to \triangle\left(\left[\underline{\lambda}, \overline{\lambda}\right]\right)$$

and, using Bayes rule, we have that, for $i \in I^*$,

$$\beta_\lambda(a_i) = \frac{f(\lambda)}{\int_{\lambda \in \Lambda^s(a)} dF(\lambda)}$$

where $\beta_\lambda(a_i)$ is the pdf $\beta(a_i)$ evaluated at λ. What about beliefs for markets where no VC is positioned, that is for any $j \notin I^*$? I am going to impose a restriction on these beliefs. The approach I follow is based on the same argument adopted by Guerrieri et al. (2010) in a similar setting. Let me first state the restriction, and then explain the intuition behind it.

Requirement 1. *Given a subgame equilibrium (s^*, E) and associated entrepreneurs expected payoff π^*_λ, the belief $\beta_\lambda(a_j)$ is strictly positive if and only if the set:*

$$Q(\lambda; a_j) := \{Q \in [0,1] \mid Q(1-\alpha) R(a_j, \lambda) \geq \pi_\lambda\}$$

is maximal.[18] *If $Q(\lambda)$ is empty for all λ, the VC expects no entrepreneur to search in market a_j.*

Essentially, for every λ, one can construct the set of Qs such that the entrepreneur would (weakly) benefit from deviating and search in market a_j. A VC that is contemplating to offer such level of attention must believe that this offer would attract the type(s) that are willing to face the highest search friction, that is, to deviate at the lowest level of Q.[19]

In comparing equilibria, I will sometime need to compute market tightness in empty markets. To do this, I will use the lowest Q such that the type(s) selected by Requirement 1 (weakly) benefits from the deviation. Armed with the definitions above, I can now formally state what is an equilibrium of the game.

[17] For a formal definition of Weak Perfect Bayesian Equilibrium see definition 9.C.3 in Mas-Colell et al. (1995)

[18] For a given collection of sets $Q(\lambda; a_j)$, $\lambda \in \left[\underline{\lambda}, \overline{\lambda}\right]$, $Q\left(\hat{\lambda}; a_j\right)$ is said to be maximal if it is not a subset of any other $Q(\lambda; a_j)$.

[19] Note that the value of $Q(\lambda; a_j)$ can come from the VCs off-equilibrium behavior, the vacancies posted at attention a_j. Requirement 1 can be then interpreted as follows: "the type that is expected to search in a_j is the one for which there is a larger set of VCs actions that would make this deviation profitable". In this sense, Requirement 1 is an adaptation of condition D1 introduced by Cho and Kreps (1987) for signaling games.

Definition 3. ***(Equilibrium).*** *An Equilibrium is a vector $(E, s^*, \sigma^*, \beta)$ constituting a Weak Perfect Bayesian Equilibrium, with the restriction that β satisfies Requirement 1 off the equilibrium path.*

In what follows, I characterize all equilibria of the game. The main message will be that better VCs will necessarily match with higher quality entrepreneurs. This comes directly from the result in the previous section, together with the properties of the function $a(m, x)$, ensuring that the best VCs have to give up fewer projects in order to provide higher levels of attention. To save on notation I denote Q_i^* the level of market tightness in market a_i in equilibrium.

Proposition 2. ***(Partitional Equilibria).*** *All equilibria are described by a partition of the set $[\underline{x}, \bar{x}]$ defined by cutoffs $\{\underline{x} = x_{-1}, ..x_i, .., x_N = \overline{x}\}$ and a partition of $\left[\underline{\lambda}, \overline{\lambda}\right]$ defined by cutoffs $\left\{\underline{\lambda} = \lambda_{-1}, ..\lambda_i, .., \lambda_N = \overline{\lambda}\right\}$ such that for any $i \in I^*$, $\Lambda_i^{s^*} = [\lambda_{i-1}, \lambda_i]$ and $X_i^{\sigma^*} = [x_{i-1}, x_i]$. If $i \notin I^*$,$\lambda_i = \lambda_{i-1}$ and $x_i = x_{i-1}$. For all adjacent $i, j \in I^*$ with $i > j$:*

$$(i) \quad m_j^{x_j}\Big(\alpha\mathbb{E}\left[R(a_j, \lambda) \mid \lambda \in \Lambda_j^{s^*}\right] - R_0\Big) = m_i^{x_j}\Big(\alpha\mathbb{E}\left[R(a_i, \lambda) \mid \lambda \in \Lambda_i^{s^*}\right] - R_0\Big).$$

$$(ii) \qquad Q_j^*(1-\alpha) R(a_j, \lambda_j) = Q_i^*(1-\alpha) R(a_i, \lambda_j).$$

$$(iii) \qquad \textit{For any } j \notin I^*,$$

$$m_i^{x_i}\left(\alpha\mathbb{E}\left[R(a_i, \lambda_i) | \lambda \in \Lambda_i^{s^*}\right] - R_0\right) \geq m_j^{x_i}(\alpha R(a_j, \lambda_j) - R_0) \qquad \forall i \in I^*.$$

In words the proposition states that all equilibria have the following form: entrepreneurs and VCs select into different attention levels according to their type, with successive subintervals of the equilibrium partitions $\Lambda_i^{s^*}$ and $X_i^{\sigma^*}$ corresponding to set of VCs and entrepreneurs selecting higher attention. Conditions (i) and (ii) impose that type at the limit of each subinterval are indifferent between the two adjacent attention levels where types right below and above are assigned to. Condition (iii) is where the requirement on off-equilibrium beliefs kicks in. Notice that, if $j \notin I^*$, $\lambda_j = \lambda_{j-1}$. Hence, condition (iii) is requiring that no VC finds it profitable to deviate to an off equilibrium a_j, given that this deviation would attract the highest entrepreneur in the set of those who select the closest lower a_i among those $i \in I^*$. Notice that this also implies that, when offering some out-of-equilibrium attention higher than in any non-empty market, a VC must expect to attract (if any) only type $\overline{\lambda}$, the type most willing to switch to that market. Similarly, offering attention lower than in any non-empty market can only attract the lowest type, $\underline{\lambda}$. Figure 1 provides a graphical representation of an equilibrium.

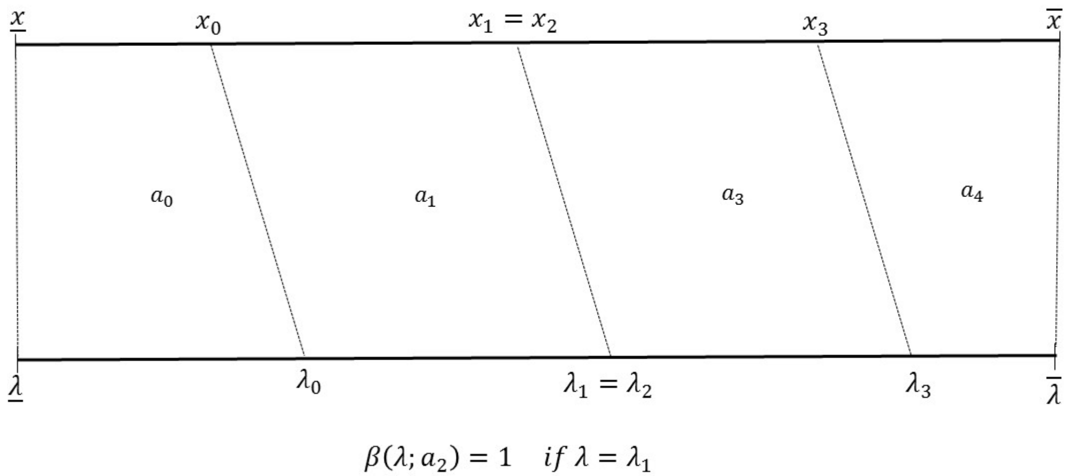

Figure 1: In this example, there are four non-empty submarkets in equilibrium. By offering attention a_2 - that no VC chooses in this example - a VC must believe to attract type λ_1, being the highest type searching in a_1 in equilibrium.

3 Efficiency

Ex-ante, total welfare in the economy amounts to the expected fees VCs receive from the investors. This is due to investors perfectly competing for VCs, and the entrepreneurs' free entry condition. In expectation, VCs are the only agents extracting rents. Denote W_i^* total vacancies in a given market i, in a given equilibrium. W_i^* depends on the particular equilibrium strategy profile that is examined. Since the fees VCs get equal the total excess returns to investors, for a given equilibrium, aggregate welfare is then given by:

$$V\left(E, s^*, \sigma^*\right) = \sum_{i \in \{1,..,N\}} W_i^* \left(\alpha \mathbb{E}\left[R\left(a_i, \lambda\right) | \lambda \in \Lambda_i^{\sigma^*}\right] - R_0\right).$$

Generally, equilibria need not be unique. A first question one can ask is whether some equilibria are more desirable then others, from an ex-ante point of view. The next proposition states that some type of equilibria can be unambiguously ranked. Interestingly, the undesirable equilibria are those where markets for higher level of attention are thicker, relatively to those for lower attention.

Proposition 3. ***(Ranking Equilibria).***

(i) An equilibrium of the game induces higher welfare than any another equilibrium where markets for higher attention are thicker, that is Q_i^*/Q_j^* is bigger for all (i, j) and $i > j$.

(ii) An equilibrium of the game induces higher welfare than any another equilibrium where the ratio W_i^*/W_j^* is bigger for all (i, j) and $i > j$.

The reason why equilibria where markets for higher level of attention are thicker are Pareto inferior is that, when increases in search frictions for any two adjacent markets are small, the resulting assignment is characterized by worse selection at the top, that is, each cutoff λ_i is lower, leading to lower average quality at each attention level. The second part of the Proposition is a consequence of this, and the fact that, whenever W_i/W_j is larger, entrepreneurs' search behavior adjusts so that the relative search friction between market i and j , Q_i/Q_j, is also larger. The emergence of Pareto dominated equilibria is due to a typical coordination failure on the VCs side: when many choose to raise more focused funds, it is relatively easy for entrepreneurs to find a match in the associated markets; as a result, only very low quality entrepreneurs are willing to give up the higher attention, and go for a less crowded market. In these equilibria, this exacerbates the adverse selection associated to setting a larger fund capacity, and the economy is stuck in a situation where (relatively) inefficient VCs choose to raise a focused fund.

I now study what would be the welfare maximizing allocation of VCs into fund sizes when the induced aggregate effect on sorting is taken into account. Below I define a Second Best Allocation as a solution to this problem. Because for a given profile of VCs strategies the sorting equilibrium need not be unique, call $\Lambda^{s^*}(\sigma)$ the collection of equilibrium partitions of the set $\left[\underline{\lambda}, \overline{\lambda}\right]$ associated to a strategy profile σ. Call $\Lambda^{s^*}(\sigma; n)$ one element of this set. By Proposition 2, $\Lambda^{s^*}(\sigma; n)$ is composed of successive intervals, each associated to a submarket a_i, and denoted $\Lambda_i^{s^*}(\sigma; n)$.

Definition 4. *A* ***Second Best Allocation*** *is a mapping $\tilde{\sigma} : [\underline{x}, \overline{x}] \to \{a_0, a_1, ..., a_N\}$ that solves:*

$$\tilde{\sigma} = \arg\max_{\sigma} \quad \textstyle\sum_{i \in \{1,..,N\}} \quad W_i^{\sigma} \left(\alpha \mathbb{E}\left[R\left(a_i, \lambda\right) | \lambda \in \Lambda_i\right] - R_0\right)$$

$$s.t. \qquad \forall i,\ \Lambda_i = \Lambda_i^{s^*}(\sigma; n) \qquad \qquad \textit{for some } n$$

It is easy to observe that a Second Best allocation must be characterized by a partition of $[\underline{x}, \overline{x}]$, with more skilled VCs being assigned to higher levels of attention. Call x_i^{sb} the limits of this partition. The next result compares the equilibrium with the second best, in an environment when attention can only be high are low.

Proposition 4. ***(Inefficiently small funds).*** *When $a \in \{a_0, a_1\}$, $x_0^{sb} > x_0$: in equilibrium, too many VCs choose high attention compared to the second-best solution.*

There is a simple intuition behind this result. A solution to the Second Best problem involves a tradeoff between allocating VCs to their optimal size, and the motive to increase relative search frictions so to induce a higher cutoff, and hence higher average quality in both markets; however, starting from any equilibrium - including the Pareto superior one - a marginal increase in x_0 come at a negligible (close to zero) cost in terms of the misallocation of VCs to a larger fund size, but has a strictly positive impact on the sorting outcome through the increase in λ_0.

4 Entry and Comparative Statics

In this section I conduct comparative statics around a refined set of equilibria. In particular, I will derive two sets of results: first, I will look at the effects of equilibrium sorting and welfare when an inflow of new VCs enter the market; second, I will compare economies that differ in how strong is entrepreneurs' self-selection, and derive results on average fund size.

Stable Equilibria. Let me restrict the analysis of this section to the case where attention can be either high or low, so that VCs and entrepreneurs can sort into two submarkets only. That is, $a \in \{a_0, a_1\}$. The main advantage is that, for a given equilibrium cutoff x_0, the induced equilibrium sorting is unique. This facilitates the comparative statics around a candidate equilibrium. First, it is convenient to define the function

$$\phi\left(a, a^{'}, \tilde{\lambda}\right) := \frac{\alpha \mathbb{E}\left[R\left(a, \lambda\right)|\lambda \geq \tilde{\lambda}\right] - R_0}{\alpha \mathbb{E}\left[R\left(a', \lambda\right)|\lambda \leq \tilde{\lambda}\right] - R_0}.$$

The function $\phi\left(a, a^{'}, \tilde{\lambda}\right)$ is the expected *per dollar* excess return from choosing attention, a, and attract entrepreneurs above some $\tilde{\lambda}$, *relative* to the excess return from choosing attention $a^{'}$ and attract entrepreneurs below the same threshold. The function $\phi\left(a, a^{'}, \tilde{\lambda}\right)$ need not be monotone in $\tilde{\lambda}$. Below is an example where it is always decreasing.

Example 1. Assume quality λ is uniformly distributed over the support $[0, 1]$. Returns are given by $R\left(a, \lambda\right) = a + \left(a - k\right)\rho\left(\lambda\right)$ with $a > k > 0$.[20] If $\rho\left(.\right)$ is any increasing linear function, it can be verified that the ratio $\phi\left(a, a^{'}, \tilde{\lambda}\right)$ is decreasing in $\tilde{\lambda}$ for any $a > a^{'}$ and any k, $R_0 > 0$.

[20]This function is logsupermodular whenever $\rho^{'} > 0$.

Unless the function is increasing everywhere, equilibria may not be unique. I introduce below one appealing property of a candidate equilibrium, that will help identify the comparative statics of this section. The property is based on a stability argument and will refine the set of equilibria. Notice that the equations identifying the equilibrium vector (x_0, λ_0) are

$$m_1^{x_0}(\alpha \mathbb{E}\left[R\left(a_1, \lambda\right) \mid \lambda \geq \lambda_0\right] - R_0) - m_0^{x_0}\left(\alpha \mathbb{E}\left[R\left(a_0, \lambda\right) \mid \lambda \leq \lambda_0\right] - R_0\right) = 0 \quad (2)$$

and

$$\frac{W_1\left(x_0\right)}{1 - F\left(\lambda_0\right)}\left(1-\alpha\right) R\left(a_1, \lambda_0\right) - \frac{W_0\left(x_0\right)}{F\left(\lambda_0\right)}\left(1-\alpha\right) R\left(a_0, \lambda_0\right) = 0. \quad (3)$$

Call $\eta\left(x_0, \lambda_0\right)$ the left hand side of (2) and $\mu\left(\lambda_0, x_0\right)$ the left hand side of (3).

Definition 5. ***(Stable Equilibria).*** *An Equilibrium* $\left(\tilde{x_0}, \tilde{\lambda_0}\right)$ *is stable if it is an attracting fixed point of the vector function:*

$$\Theta\left(x_0, \lambda_0\right) = \left[\begin{array}{c} \eta\left(x_0, \lambda_0\right) + x_0 \\ \mu\left(\lambda_0, x_0\right) + \lambda_0 \end{array}\right]$$

In words, a stable equilibrium is one that, after a small perturbation that forces some agents' strategies away from it, will eventually converge back to itself.[21]

[21] In the Appendix, it is shown that this is equivalent to requiring stability of the costant solution (x_0, λ_0) to a system of differential equations where x_0 is assumed to increase (decrease) proportially to the marginal benefit (loss) to type x_0 from chossing attention a_0 rather than a_1, given λ_0, and the same is assumed for the differential equation governing the changes of λ_0 for a given x_0.

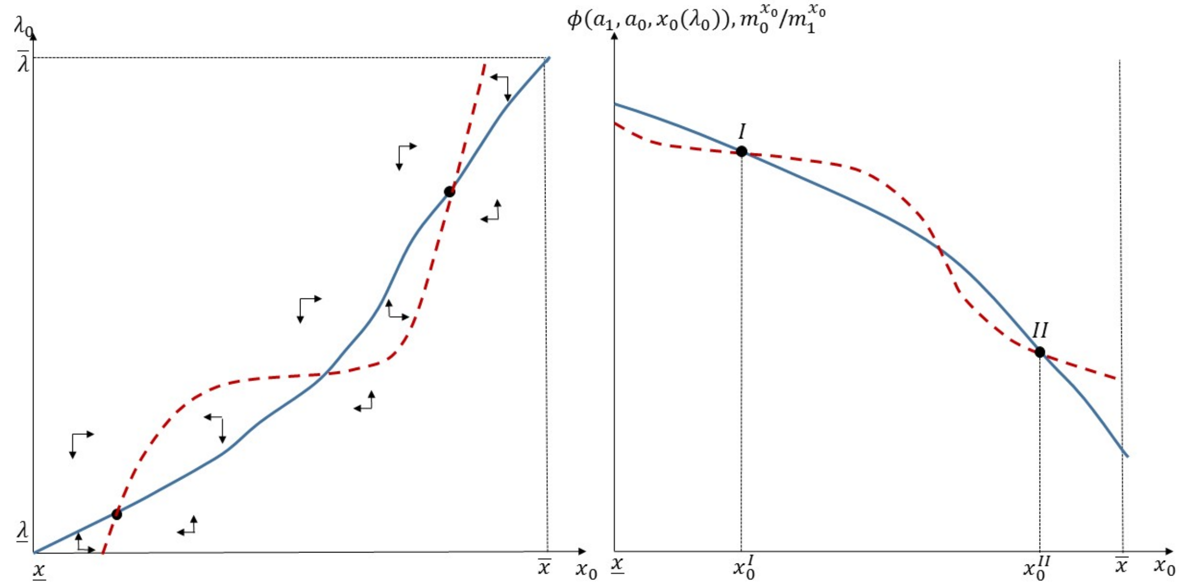

Figure 2: Left: The solid line is the solution to the entrepreneur's indifference condition for each level of x_0. Arrows above (below) this line point upwards (downwards) because if the population cutoff was type λ, he would strictly benefit (lose) from moving to market a_0. The dotted line connects all the indifferent VCs, for each λ_0. Arrows at the west (east) of the line point to the right (left) because if the population cutoff was type x, he would strictly benefit (lose) from moving to market a_0. The stable equilibria are the two intersection at the bottom-left and top-right of the picture. Right: The solid line is the function $m_0^{x_0}/m_1^{x_0}$, decreasing because the relative difference between m_0 and m_1 is smaller for better VCs. The dotted line is $\phi\left(a_1, a_0, \lambda_0\left(x_0\right)\right)$ which moves with x_0 through its effect on λ_0 and is decreasing because when x_0 increases, λ_0 increases, and $\phi_{\bar{\lambda}} < 0$ by assumption. An equilibrium is an intersection of this two curves, and stable equilibria (denoted I and II) are those where ϕ is flatter than $m_0^{x_0}/m_1^{x_0}$ at the intersection. In this example, there are three equilibria. I is the worse equilibrium, while II is the welfare maximizing equilibrium.

I will now conduct comparative statics around a stable equilibrium.

4.1 Inflow of Unskilled VCs

The analysis so far has focused on an economy where the measure and distribution of VCs is fixed. As introduced in Section 1, however, one object of interest of my analysis is to study the effect of entry of new VCs on the equilibrium allocation of investors money and projects to VCs. This is mainly motivated by the debate around the effectiveness of policies that encourage VC investments, and by the recent finding that government sponsored VC has not crowded out investments by private VCs at the aggregate level. Moreover, there exists evidence that money committed in the venture capital industry is highly volatile, that it is subject to booms and busts and that the number of funds dedicated to this asset class vary across time, sometimes in response to the business cycle. Determining the reason why these cycles occur is beyond the scope of this paper. However, the model can offer predictions on how the distribution of returns is affected by the inclusion of new VCs in the economy. One interesting exercise is to study what happens when new unskilled VCs enter the market. More precisely, imagine the distribution of skills g is defined on a support larger than $[\underline{x}, \overline{x}]$. Initially, only VCs in $[\underline{x}, \overline{x}]$ operate. What will happen if some of the worse VCs previously excluded decide to enter? In other words, what are the consequences of a decrease in $\underline{x}$? Notice that the exclusion of some VCs from the market could be resulting from the presence of barriers to entry. Since VCs expected payoff in equilibrium is strictly increasing in x, if being active in the market requires a fixed investment κ, the ex-ante payoff to the marginal VC - $\underline{x}$ - would be given, in an interior equilibrium, by:

$$m_0^{\underline{x}}\alpha\left(\mathbb{E}\left[R\left(a_0,\lambda\right) \mid \lambda \leq \lambda_0\right] - R_0\right) = \kappa.$$

In this environment, subsidizing the investment κ to the highest type outside the market would be equivalent to induce a marginal decrease in $\underline{x}$ in the venture capital market. The result below states what is the induced effect of this change on all stable equilibria.

Proposition 5. ***(Entry of unskilled VCs)***. *For every stable equilibrium* (x_0, λ_0)*:*

(i) $\frac{\partial \lambda_0}{\partial \underline{x}} < 0$.

(ii) As $\underline{x}$ *decreases, welfare increases.*

(iii) $\frac{\partial x_0}{\partial \underline{x}} > 0$ *if and only if* $\phi_{\tilde{\lambda}}\left(a_1, a_0, \lambda_0\right) > 0$.

In words, the inflow of unskilled VCs leads some entrepreneurs to switch to the low attention market: the indifferent entrepreneur's quality is higher; total welfare increases; when the function $\phi\left(a, a', \tilde{\lambda}\right)$ is decreasing in the cutoff $\tilde{\lambda}$ - hence the relative gain in attracting entrepreneurs above versus those below $\tilde{\lambda}$ is lower the higher is $\tilde{\lambda}$ - some VCs originally raising a small fund, opt for a large fund.

The intuition is simple. The relatively unskilled VCs who enter the market will select a_0. The larger number of vacancies in the market for low attention pushes the cutoff λ_0 up. This implies that those VCs who will keep raising relatively smaller funds will select better projects. Essentially, the now larger market for unfocused funds absorbs some of the low quality entrepreneurs from the economy. Because the increase in λ_0 increases average quality in both submarkets, total welfare increases. When the function $\phi\left(a, a', \tilde{\lambda}\right)$ is decreasing in $\tilde{\lambda}$, the adverse selection problem associated to managing a larger fund is less severe, inducing more VCs to raise one. In this circumstance, the inflow of unsophisticated venture capitalists increases average fund size and aggregate investments in the market. This is consistent with the findings in Brander et al. (2014).

Think now of the distribution of returns of funded projects in the industry. Returns will necessarily take values $R \in \left[R\left(a_0, \underline{\lambda}\right), R\left(a_1, \overline{\lambda}\right)\right]$. The shape of the returns distribution will depend on that of the distribution of projects quality - f - *and* on the equilibrium choices of VCs and entrepreneurs.

I show that the returns distribution is also affected by entry of new, unskilled VCs. In absence of sorting, the effect one should expect is mechanic: relatively more VC funds would now end up delivering low returns. When sorting is taken into account though, the positive externality induced to those incumbents VCs who keep raising focused fund results in higher returns at the top of the distribution. The corollary below formalizes this observation.

Corollary 1. ***(Entry and returns distribution)***. *For each equilibrium, as $\underline{x}$ decreases, there exists a point $\tilde{R}$ in the new distribution of returns, such that $\forall R > \tilde{R}$ returns are higher conditional on being above R and are more likely to be below $\tilde{R}$.*

The effect is consistent with the findings in Nanda and Rhodes-Kropf (2013), who document that investment made in "hot" periods are more likely to fail and give higher returns conditional on not failing, and in Kaplan and Schoar (2005), who find that, in times characterized by more intense activity in the industry, capital flows disproportionately to worse funds.

4.2 Intensity of Selection

Imagine the more realistic scenario where, conditional on setting a certain fund size, only a share $\rho \in [0, 1]$ of vacancies is filled via directed search. The residual share - $1 - \rho$ - is filled through random matching. The interpretation is that VCs partly select entrepreneurs from their own network. In essence, ρ measures how intense is entrepreneurs' self selection in the market, and what I have studied in the main model is the case where ρ equals one. Note that this specification has a convenient property: the entrepreneurs' search behavior is not affected by the parameter ρ. This is formally stated in the next claim.

Claim 1. *Given a strategy profile from VCs, and associated W_0^* and W_1^*, the indifferent entrepeneur λ_0 is independent on ρ.*

The next statement compares equilibrium funds size when the parameter ρ increases. Therefore it can be interpreted as a comparison between two economies that differ in how much of the fund manager's deal flow is endogenous and self-selected.

Proposition 6. *For every stable equilibrium* (x_0, λ_0), $\frac{\partial x_0}{\partial \rho} < 0$. *Consequently, average fund size is smaller the larger is* ρ .

This result is intuitive. Infact, it is immediate to observe that the ratio:

$$\frac{\rho\left(\alpha \mathbb{E}\left[R\left(a_1, \lambda\right) \mid \lambda \geq \lambda_0\right] - R_0\right) + (1-\rho)\left(\alpha \mathbb{E} R\left(a_1, \lambda\right) - R_0\right)}{\rho\left(\alpha \mathbb{E}\left[R\left(a_0, \lambda\right) \mid \lambda \leq \lambda_0\right] - R_0\right) + (1-\rho)\left(\alpha \mathbb{E} R\left(a_1, \lambda\right) - R_0\right)}$$

is *increasing* in the share of vacancies filled via directed search. Therefore, as the relative per dollar returns from offering a small fund compared to a large unfocused fund increase, more VCs will select the former option.

5 Conclusion

I have introduced a matching model of fund management where the two key ingredients are scarcity in fund managers human capital and directed search from entrepreneurs who are heterogeneous in their projects' quality. The two features are inspired by several stylized facts and empirical findings about the venture capital industry. In the model, entrepreneurs trade off matching with better VCs - those who opt for relatively smaller funds, and hence can devote more attention to their projects - against the lower search frictions associated to worse VCs. VCs are different in ability to scale up their human capital, and select fund size to maximise total returns. Anticipating that, due to complementarities in the returns function, higher quality entrepreneurs will sort into funds where attention is higher, VCs tend to shrink fund size excessively. In this environment, subsidizing entry of VCs that are inactive - for example because their ability to generate returns is not sufficient to cover the fixed costs of starting operations - always results in net aggregate gains. The reason is that these agents will absorb low quality entrepreneurs, and hence alleviate the adverse selection problem incumbent VCs are facing. This offers a more optimistic point of view on policies that encourage fundraising devoted to venture investments. Since the distribution of fund sizes feeds back into the equilibrium sorting, entry of VCs at the bottom of the skills distribution increases returns at the top. The effects of entry, as well as the inefficiency and the emergence of multiple, Pareto-dominated equilibria are a consequence of entrepreneurs self-selection and would not result from a model with random matching or homogenous entrepreneurs.

Appendix

Proof of Lemma 1. Consider first those (w,x) for which $\tilde{S}(w,x;E)>0$. Take one submarket $(\tilde{w},\tilde{x})$ where $Q(\tilde{w},\tilde{x})=1$. By entering and searching in it, an entrepreneur that is outside the market gets in expectation:

$$(1-\alpha)\mathbb{E}\left(a\left(m\left(\tilde{x},\tilde{w},E\right),\tilde{x}\right),\lambda\right)-c\geq(1-\alpha)\mathbb{E}R\left(a_0,\lambda\right)-c>0.$$

Hence, when $\tilde{S}(w,x;E)>0$, it must be that $Q(\tilde{w},\tilde{x})<1$. To show that $\tilde{S}(w,x;E)>0$ for all (w,x), assume not and denote $(\hat{w},\hat{x})$ the submarket where no entrepreneur searches. Then, any type λ who entered the market would like to deviate and search in $(\hat{w},\hat{x})$, as this would give:

$$(1-\alpha)R\left(a_N,\lambda\right)>Q\left(w,x;E\right)(1-\alpha)R\left(a\left(m\left(w,x\right),x\right),\lambda\right)\qquad\forall\left(w,x\right).$$

■

Proof of Lemma 2. By Lemma 1, returns to type λ in market (w,x) conditional on matching are given by $R\left(\left(a\left(w,x\right)\right),\lambda\right)$. Take two markets (w,x) and $\left(w^{'},x^{'}\right)$, with associated attention levels a and $a^{'}$, with $a=a^{'}$. Assume that $Q(w,x,E)>Q\left(w,x^{'},E\right)$. Then, any entrepreneur searching in $\left(w^{'},x^{'}\right)$ could deviate to (w,x) and get:

$$Q\left(w,x,E\right)R\left(a,\lambda\right)>Q\left(w^{'},x^{'},E\right)R\left(a^{'},\lambda\right).$$

■

Proof of Lemma 3. In the original model entrepreneurs maximize $Q(w,x,E)\,R\left(a\left(w,x\right),\lambda\right)$, and, by Lemma 2, $Q(w,x,E)=\theta(w,x,E)$. Because $R\left(a\left(w,x\right),\lambda\right)$ is costant across an iso-attention locus, and since Lemma 2 must apply to the transformed model, all that remains to show is that market tightness is the same in submarket a as it is at any point in the iso-attention locus in the original model. That is, formally, $\theta(a)=\theta(w,x,E)\ \forall(w,x):\ a(w,x)=a$. Call $\mathit{\Gamma}(a)$ the sum of vacancies across all VCs at a given iso-attention locus. From the definition of $\theta(a)$, one can write $\theta(a)=\frac{d\mathit{\Gamma}(a)}{dS(a)}$ with

$$S\left(a\right):=\int_{\tilde{a}\leq a}\int_{\{(w,x):a(w,x)=\tilde{a}\}}\int_{\lambda}EdS\left(w,x;\lambda\right)d\tilde{a}.$$

Recall that

$$\mathit{\Gamma}\left(a\right):=\int_{\tilde{a}\leq a}\int_{\{(w,x):a(w,x)=\tilde{a}\}}wdH\left(w,x\right)d\tilde{a}.$$

Therefore,

$$d\Gamma(a) = \int_{\{(w,x):a(w,x)=a\}} wdH(w,x)$$

and

$$dS(a) = \int_\lambda \int_{\{(w,x):a(w,x)=a\}} EdS(w,x;\lambda).$$

Notice that

$$d\tilde{S}(w,x,E)\,\theta(w,x,E) = wdH(w,x).$$

Integrating on both sides over a given iso-attention locus, and taking $\theta(w,x,E)$ outside of the integral by Lemma 2, it follows that

$$\theta(w,x,E)\int_{\{(w,x):a(w,x)=\tilde{a}\}} d\tilde{S}(w,x,E) = \int_{\{(w,x):a(w,x)=\tilde{a}\}} wdH(w,x).$$

Therefore,

$$\theta(w,x,E) = \left(\int_{\{(w,x):a(w,x)=a\}} wdH(w,x) \Big/ \int_{\{(w,x):a(w,x)=a\}} d\tilde{S}(w,x,E)\right) = \theta(a).$$

■

Proof of Proposition 1. (Sufficiency). Assume $R(a,\lambda)$ is logsupermodular everywhere. If there is an equilibrium that does not exhibit PAM everywhere, then there must exist at least two markets a_i, a_j with $a_i > a_j$, and two types λ', λ with $\lambda' > \lambda$ such that $\lambda \in \Lambda_i$ and $\lambda' \in \Lambda_j$. Optimality of the search strategy requires that type λ is at least as well off searching in a_i rather that in a_j and similarly λ' (weakly) prefers a_j to a_i, that is:

$$Q(a_i)R(a_i,\lambda) \geq Q(a_j)R(a_j,\lambda) \tag{4}$$

$$Q(a_j)R\left(a_j,\lambda'\right) \geq Q(a_i)R\left(a_i,\lambda'\right) \tag{5}$$

The two inequalities imply

$$\frac{R(a_i,\lambda)}{R(a_j,\lambda)} \geq \frac{R\left(a_i,\lambda'\right)}{R(a_j,\lambda')}$$

which contradicts the fact that $R(a,\lambda)$ is logsupermodular[22].

(Necessity). Assume $R(a,\lambda)$ is not logsupermodular at some point $\left(\hat{a},\hat{\lambda}\right)$. The continuity properties of $R(a,\lambda)$ (see Section 2) imply that there exists a number $\varepsilon > 0$,

[22]Logsupermodularity of $R(a,\lambda)$ implies that for any $\left(a,a'\right)$ with $a' > a$, the ratio $\frac{R\left(a',\lambda\right)}{R(a,\lambda)}$ is strictly increasing in λ.

s.t. the function is not logsupermodular anywhere in $[\hat{a}-\varepsilon,\hat{a}+\varepsilon]\times\left[\hat{\lambda}-\varepsilon,\hat{\lambda}+\varepsilon\right]$. I construct an economy where NAM could be supported, hence a contradiction arises. Let F be defined on $\left[\hat{\lambda}-\varepsilon,\hat{\lambda}+\varepsilon\right]$, and $a_i\in[\hat{a}-\varepsilon,\hat{a}+\varepsilon]$, for all i. By construction, all matches will be in $[\hat{a}-\varepsilon,\hat{a}+\varepsilon]\times\left[\hat{\lambda}-\varepsilon,\hat{\lambda}+\varepsilon\right]$. In order for only PAM sorting patterns to emerge, a necessary condition is that for at least two $\left(\lambda,\lambda^{'}\right)$ with $\lambda>\lambda^{'}$, and two $\left(a,a^{'}\right)$, with $a>a^{'}$,

$$Q(a)\,R(a,\lambda)\ \ \geq\ \ Q\left(a^{'}\right)R\left(a^{'},\lambda\right) \tag{6}$$

$$Q\left(a^{'}\right)R\left(a^{'},\lambda^{'}\right)\ \ \geq\ \ Q(a)\,R\left(a,\lambda^{'}\right) \tag{7}$$

and, crucially, at least one of the two inequalities is strict[23]. When either (7) or (8) or both are satisfied with strict inequality, it holds that

$$\frac{R(a,\lambda)}{R(a',\lambda)}>\frac{R\left(a,\lambda^{'}\right)}{R(a',\lambda')}$$

which means $R(a,\lambda)$ is logsupermodular somewhere in $[\hat{a}-\varepsilon,\hat{a}+\varepsilon]\times\left[\hat{\lambda}-\varepsilon,\hat{\lambda}+\varepsilon\right]$, a contradiction. ■

Proof Proposition 2. By Proposition 1, sorting in the subgame must exhibit PAM. Each point in the sequence that forms the equilibrium partition of the set $\left[\underline{\lambda},\bar{\lambda}\right]$ is a type that must be indifferent between searching in the two markets where types just above and just below are assigned.

Consider a VC with ability x that is choosing between a_i (and associated size, m_i^x) and a_j (with associated size m_j^x), with $i>j$ and $i,j\in I^*$. This VC will prefer the first option if and only if:

$$m_i^x\mathbb{E}\left[R(a_i,\lambda)\,|\lambda\in\mathit{\Lambda}_i^{s^*}\right]>m_j^x\mathbb{E}\left[R(a_j,\lambda)\,|\lambda\in\mathit{\Lambda}_j^{s^*}\right].$$

Which can be rewritten as:

$$\frac{m_i^x}{m_j^x}>\frac{\mathbb{E}\left[R(a_j,\lambda)\,|\lambda\in\mathit{\Lambda}_j^{s^*}\right]}{\mathbb{E}\left[R(a_i,\lambda)\,|\lambda\in\mathit{\Lambda}_i^{s^*}\right]}. \tag{8}$$

The right hand side of (9) is independent on x. The left hand side is continuous and *increasing* in x. Therefore, if (9) holds for some x, it will hold for any VC with ability above x. Moreover, if the same inequality is reverted for some $x^{'}<x$, then, by the

[23]Otherwise, it would be possible to support an equilibrium with NAM, and the contradiction would immediately arise.

Intermediate Value Theorem, there exist a level of ability $\tilde{x} \in \left[x^{'}, x\right]$ such that the payoff from a_i and a_j is the same.

Finally, consider a deviation to some a_j with $j \notin I^*$. Take the closest smaller market in I^* to a_j, call it a_i.

$$i := \arg \max_{h \in I^* \setminus \{h \geq j\}} h$$

It can be argued that the belief $\beta(a_j)$ has to place all support on type λ_i, which, since all markets in between a_i and a_j are empty, is also equal to λ_j. Assume not, and first, say that $\beta(a_j)$ is supported on another type $\lambda \in \Lambda_h$ with $h \in I^*$ and $h > j$. From the partitional equilibrium, $\lambda > \lambda_i$. If this type finds a deviation to a_j weakly beneficial for at least some Q, then, the set of $Q \in [0, 1]$ such that this is true is an interval $[Q_\lambda, 1]$, with Q_λ

$$Q(a_h) R(a_h, \lambda) = Q_\lambda R(a_j, \lambda)$$

However, the fact that $R(a_h, \lambda) / R(a_j, \lambda)$ is increasing in λ - by logsupermodularity - implies that, $\forall \lambda^{'} \in \Lambda_h$ and $\lambda^{'} < \lambda$,

$$Q(a_h) R\left(a_h, \lambda^{'}\right) < Q_\lambda R\left(a_j, \lambda^{'}\right).$$

That is, all types in Λ_h *lower* than λ would strictly benefit from deviating at Q_λ, and, similarly for some $Q > Q_\lambda$. Hence the set of Qs such that λ would deviate is a subset of the set of Qs at which these types would deviate. Because this set not maximal, β should place no density at λ. A contradiction.

Assume instead $\beta(a_j)$ is supported on some $\lambda \in \Lambda_h$ with $h \in I^*$ and $h < j$. A similar argument applies. In this case, $\forall \lambda^{'} \in \Lambda_h$ and $\lambda^{'} > \lambda$,

$$Q(a_h) R\left(a_h, \lambda^{'}\right) < Q_\lambda R\left(a_j, \lambda^{'}\right).$$

Given what the off-equilibrium deviation attracts, condition (iii) from the Proposition guarantees that no VC offers a_j. ■

Proof Proposition 3. *(i).* The proof of part (i) proceeds in two steps.

(Step 1). First, call I the equilibrium exhibiting lower ratio Q_i/Q_j for any two (i, j) with $i > j$, with II being the other equilibrium. Use the superscripts I and II to denote the limits of the equilibrium partitions X_i and Λ_i under equilibrium I and II. It can be shown that, for any i, $\lambda_i^I > \lambda_i^{II}$. This is easily seen by looking at the entrepreneur's λ_i indifference condition. Rewrite it as:

$$\frac{R\left(a_{i+1},\lambda_i\right)}{R\left(a_i,\lambda_i\right)}=\frac{Q_i}{Q_{i+1}} \tag{9}$$

Condition (10) has to hold under both equilibrium values λ_i^I and λ_i^{II}. The left hand side is increasing in λ_i by logsupermodularity. The right hand side is assumed to be larger under equilibrium I. Therefore, $\lambda_i^I > \lambda_i^{II}$ for all i.

(Step 2). Given $\lambda_i^I > \lambda_i^{II}$, it can be proven that welfare is higher under equilibrium I. Notice that, for any i, average quality is higher under I. That is:

$$\mathbb{E}\left[R\left(a_i,\lambda\right)|\lambda_{i-1}^I \leq \lambda \leq \lambda_{i+1}^I\right] > \mathbb{E}\left[R\left(a_i,\lambda\right)|\lambda_{i-1}^{II} \leq \lambda \leq \lambda_{i+1}^{II}\right].$$

Since equilibrium requires that VCs select a_i to maximise total returns, it must be that each one is strictly better off under I compared to II.

(ii). For part (ii), similarly call I the equilibrium exhibiting lower ratio W_i/W_j for any two (i,j) with $i>j$, with II being the other equilibrium. Use the superscripts I and II to denote the limits of the equilibrium partitions X_i and Λ_i under equilibrium I and II. It can be shown that, for any i, $\lambda_i^I > \lambda_i^{II}$. To prove this, assume this is not the case. That is, assume that, for at least some i , $\lambda_i^I \leq \lambda_i^{II}$. First focus on the case where the inequality is strict for some i. Take the *largest* i such that this holds. Rewrite the indifference condition for the indifferent type, λ_i , as:

$$\frac{R\left(a_{i+1},\lambda_i\right)}{R\left(a_i,\lambda_i\right)}=\frac{W_i}{W_{i+1}}\frac{F\left(\lambda_{i+1}\right)-F\left(\lambda_i\right)}{F\left(\lambda_i\right)-F\left(\lambda_{i-1}\right)}. \tag{10}$$

When λ_i is lower, the left hand side of (11) decreases (due to logsupermodularity). W_i/W_{i+1} is higher in equilibrium I by assumption. Hence, it must be that the ratio $\left(F\left(\lambda_{i+1}\right)-F\left(\lambda_i\right)\right)/\left(F\left(\lambda_i\right)-F\left(\lambda_{i-1}\right)\right)$ is lower under I. The numerator is higher under I , since i is the largest submarket for which $\lambda_i^I \leq \lambda_i^{II}$ (this is also true in case $i=N-1$ and hence $\lambda_{i+1}=\bar{\lambda}$). Therefore, it must be that $\lambda_{i-1}^I < \lambda_{i-1}^{II}$, giving a contradiction. It remains to show that it is impossible that $\lambda_i^I = \lambda_i^{II}$ for all i. Assume this is the case. This would imply that under the two equilibria, the left hand side of (16) stays constant, as well as the ratio $\left(F\left(\lambda_{i+1}\right)-F\left(\lambda_i\right)\right)/\left(F\left(\lambda_i\right)-F\left(\lambda_{i-1}\right)\right)$. Because W_i/W_{i+1} is not the same under the two equilibria, the desired contradiction arises.

Given $\lambda_i^I > \lambda_i^{II}$, welfare is higher under equilibrium I, as proven for part (i). This completes the proof. ■

Proof Proposition 4. First, observe that, for any allocation described by a cutoff x_0, so that VCs are assigned to the high attention market if and only if their ability is above x_0, the sorting outcome is described by a unique, increasing, and continuously differentiable cutoff $\lambda_0(x_0)$. To see why, rewrite the entrepreneur's indifference condition as:

$$\frac{R(a_1,\lambda_0)}{R(a_0,\lambda_0)} = \frac{W_0(x_0)}{W_1(x_0)}\frac{(1-F(\lambda_0))}{F(\lambda_0)}. \tag{11}$$

The left hand side of (12) is continuous and strictly increasing in λ_0 by assumption (as R is continuous in both arguments and assumed to be log-supermodular in this section). The right hand side is continuous and strictly decreasing in λ_0. Recall that $W_0 = \int_{\underline{x}}^{x_0} m_0^x dG(x)$, and $W_1 = \int_{x_0}^{\overline{x}} m_1^x dG(x)$. Therefore, the ratio $W_0(x_0)/W_1(x_0)$ is continuous and decreasing in x_0, as $\partial m_i^x/\partial x$ is positive and continuous, and the distribution g is continuous.

Denote x_0^* the largest equilibrium cutoff x_0, which is, by Proposition 2, the Pareto superior equilibrium. The proof proceeds in two steps.

(Step 1). First, I show that any allocation $\tilde{x}_0 < x_0^*$ delivers lower welfare than x_0^*. Notice that welfare induced by an allocation $\tilde{x}_0$ is bounded above by what total returns would be if, given the sorting subgame, VCs would optimally select fund size. Formally:

$$V(\tilde{x_0}) \leq \int_x \max\left\{m_0^x \mathbb{E}\left[R(a_0,\lambda)\,|\lambda \leq \lambda_0(\tilde{x_0})\right], m_1^x \mathbb{E}\left[R(a_1,\lambda)\,|\lambda \geq \lambda_0(\tilde{x_0})\right]\right\} dG(x).$$

Hence, for all $\tilde{x}_0 < x_0^*$:

$$V(x_0^*) = \int_x \max\left\{m_0^x \mathbb{E}\left[R(a_0,\lambda)\,|\lambda \leq \lambda_0(x_0^*)\right], m_1^x \mathbb{E}\left[R(a_1,\lambda)\,|\lambda \geq \lambda_0(x_0^*)\right]\right\} dG(x) > V(\tilde{x_0}).$$

(Step 2). Second, I show that the Second Best Problem can be improved by a marginal increase in x_0, starting from x_0^*. To see why, write the objective function:

$$V(x_0) = W_0(x_0)\,\mathbb{E}\left[R(a_0,\lambda)\,|\lambda \leq \lambda_0(x_0)\right] + W_1(x_0)\,\mathbb{E}\left[R(a_1,\lambda)\,|\lambda \geq \lambda_0(x_0)\right].$$

So,

$$\begin{aligned}\left.\frac{\partial V(x_0)}{\partial x_0}\right|_{x_0=x_0^*} &= \underbrace{\left(m_0^{x_0}\mathbb{E}\left[R(a_0,\lambda)\,|\lambda \leq \lambda_0(x_0)\right] - m_1^{x_0}\mathbb{E}\left[R(a_1,\lambda)\,|\lambda \geq \lambda_0(x_0)\right]\right) g(x_0)}_{=0} \\ &+ \left(W_0(x_0^*)\frac{\partial\left(\mathbb{E}\left[R(a_0,\lambda)|\lambda\leq\lambda_0\left(x_0^*\right)\right]\right)}{\partial\lambda_0} + W_1(x_0^*)\frac{\partial\left(\mathbb{E}\left[R(a_1,\lambda)|\lambda\geq\lambda_0\left(x_0^*\right)\right]\right)}{\partial\lambda_0}\right)\frac{\partial\lambda_0\left(x_0^*\right)}{\partial x_0} > 0,\end{aligned}$$

where the term in the first bracket is zero because x_0^* is indifferent in equilibrium. ■

Stable Equilibria. A fixed point of the vector function Θ - defined in Section 4.1 - is attracting if and only if all eigenvalues of the the Jacobian of Θ - denoted $J(\Theta)$ - are smaller than one in absolute value. Therefore, in this context a necessary condition for $\left(\tilde{x_0}, \tilde{\lambda}_0\right)$ to be an attracting fixed point is that the determinant of the 2x2 matrix $J(\Theta)$ is smaller than one in absolute value. Formally, the condition is:

$$\left| \det \begin{bmatrix} \eta_{x_0}\left(\tilde{x_0}, \tilde{\lambda}_0\right) + 1 & \eta_{\lambda_0}\left(\tilde{x_0}, \tilde{\lambda}_0\right) \\ \mu_{x_0}\left(\tilde{x_0}, \tilde{\lambda}_0\right) & \mu_{\lambda_0}\left(\tilde{x_0}, \tilde{\lambda}_0\right) + 1 \end{bmatrix} \right| < 1.$$

Since η_{x_0} and μ_{λ_0} are always positive, for all stable equilibria it has to be the case that:

$$\eta_{\lambda_0}\left(\tilde{x_0}, \tilde{\lambda}_0\right) \mu_{x_0}\left(\tilde{x_0}, \tilde{\lambda}_0\right) < \eta_{x_0}\left(\tilde{x_0}, \tilde{\lambda}_0\right) \mu_{\lambda_0}\left(\tilde{x_0}, \tilde{\lambda}_0\right). \tag{12}$$

An adjustment process. Consider the following dynamic adjustment process. Take an initial (x_0, λ_0). Impose that, starting from it, the cutoff x_0 increases (decreases) proportionally to the benefit (loss) from selecting a large fund size against a small size, given the rest of the agents are following the strategy described by the two cutoffs (x_0, λ_0). Similarly, impose that the cutoff λ_0 increases (decreases) proportionally to the benefit (loss) from searching in the low attention market versus searching for high attention. This process defines a system of autonomous differential equations as below:

$$\begin{cases} \dot{x}_0(t) = & -b\left[m_1^{x_0(t)}(\mathbb{E}\left[R(a_1, \lambda) \mid \lambda \geq \lambda_0(t)\right] - R_0) - m_0^{x_0(t)}\left(\mathbb{E}\left[R(a_0, \lambda) \mid \lambda \leq \lambda_0(t)\right] - R_0\right)\right] \\ \\ \dot{\lambda_0}(t) = & -b\left[\frac{W_1(x_0(t))}{1-F(\lambda_0(t))} R(a_1, \lambda_0(t)) - \frac{W_0(x_0(t))}{F(\lambda_0(t))} R(a_0, \lambda_0(t))\right] \end{cases} \tag{13}$$

for some $b > 0$. Notice the right hand side of the first part of (14) is $-b\eta(x_0, \lambda_0)$ and the right hand side of the second part is $-b\mu(x_0, \lambda_0)$. One interpretation is that at each point in time a fraction of the population readjusts their strategies, starting from a state where all agents are following cutoff strategies and taking those strategies as given. An equilibrium of the game - $\left(\tilde{x_0}, \tilde{\lambda}_0\right)$ - is clearly a constant solution to the system. One can then study local stability of such equilibria.

An equilibrium $\left(\tilde{x_0}, \tilde{\lambda}_0\right)$ is locally asymptotically stable if all eigenvalues of the Jacobian:

$$-b \begin{bmatrix} \eta_{x_0}\left(\tilde{x}_0, \tilde{\lambda}_0\right) & \eta_{\lambda_0}\left(\tilde{x}_0, \tilde{\lambda}_0\right) \\ \mu_{x_0}\left(\tilde{x}_0, \tilde{\lambda}_0\right) & \mu_{\lambda_0}\left(\tilde{x}_0, \tilde{\lambda}_0\right) \end{bmatrix}$$

have negative real parts.[24] Since the trace of this matrix is:

$$-b\left(\eta_{x_0}\left(\tilde{x}_0,\tilde{\lambda}_0\right)+\mu_{\lambda_0}\left(\tilde{x}_0,\tilde{\lambda}_0\right)\right)<0$$

local asymptotic stability of $\left(\tilde{x_0},\tilde{\lambda_0}\right)$ is implied by:

$$\eta_{\lambda_0}\left(\tilde{x_0},\tilde{\lambda_0}\right)\mu_{x_0}\left(\tilde{x_0},\tilde{\lambda_0}\right)<\eta_{x_0}\left(\tilde{x_0},\tilde{\lambda_0}\right)\mu_{\lambda_0}\left(\tilde{x_0},\tilde{\lambda_0}\right)$$

which is exactly equation (13).

Proof of Proposition 5. First, rewrite the system characterizing the vector of equilbrium cutoffs (x_0,λ_0) as*:*

$$m_1^{x_0}(\mathbb{E}\left[R\left(a_1,\lambda\right)\mid\lambda\geq\lambda_0\right]-R_0)-m_0^{x_0}\left(\mathbb{E}\left[R\left(a_0,\lambda\right)\mid\lambda\leq\lambda_0\right]-R_0\right)=0 \tag{14}$$

$$\frac{W_1\left(x_0\right)}{1-F\left(\lambda_0\right)}R\left(a_1,\lambda_0\right)-\frac{W_0\left(x_0,\underline{x}\right)}{F\left(\lambda_0\right)}R\left(a_0,\lambda_0\right)=0 \tag{15}$$

where I have made explicit in (15) the dependence of W_0 on $\underline{x}$. Infact, notice that $W_0=\int_{\underline{x}}^{x_0}m_0^x dG\left(x\right)$, so $\frac{\partial W_0}{\partial\underline{x}}=-m_0^{\underline{x}}g\left(\underline{x}\right)<0$. Call $\Phi\left(x_0,\lambda_0,\underline{x}\right)$ the left-hand side of (14) and $\Psi\left(x_0,\lambda_0,\underline{x}\right)$ the left-hand side of (15). Using the Implicit Function Theorem, one gets that:

$$\frac{\partial\lambda_0}{\partial\underline{x}}<0\iff\Phi_{\lambda_0}\left(x_0,\lambda_0,\underline{x}\right)\Psi_{x_0}\left(x_0,\lambda_0,\underline{x}\right)<\Phi_{x_0}\left(x_0,\lambda_0,\underline{x}\right)\Psi_{\lambda_0}\left(x_0,\lambda_0,\underline{x}\right)$$

which is implied by condition (13). Similarly, for x_0 one gets that:

$$\frac{\partial x_0}{\partial\underline{x}}>0\iff\frac{\Phi_{x_0}\left(x_0,\lambda_0,\underline{x}\right)}{\Phi_{\lambda_0}\left(x_0,\lambda_0,\underline{x}\right)}\Psi_{\lambda_0}\left(x_0,\lambda_0,\underline{x}\right)-\Psi_{\lambda_0}\left(x_0,\lambda_0,\underline{x}\right)>0$$

and, using condition (13) because we are looking at stable equilibria, it follows that the sign of $\partial x_0/\partial\underline{x}$ is the same as that $\Phi_{\lambda_0}\left(x_0,\lambda_0,\underline{x}\right)$, which is positive if and only if $\phi_{\tilde{\lambda}}\left(a_1,a_0,\lambda_0\right)$ is positive.

The effect on welfare directly follows from the fact that, as λ_0 increases, average quality in both submarkets increases. Since VCs choose fund size to maximize returns, it must be that all of them are better off after the change in λ_0. Hence, for all equilibria $V\left(E,s^*,\sigma^*\right)$ increases. ■

[24] For details see for example Theorem 2.5 from Acemoglu (2008).

Proof of Corollary 1. Consider the distribution of returns for a given equilbrium $(x_0(\underline{x}), \lambda_0(\underline{x}))$. The Cdf of the returns is a step function, denoted Y, taking the form:

$$Y(r;\underline{x}) = \begin{cases} \frac{W_0(x_0(\underline{x}),\underline{x})}{W_0(x_0(\underline{x}),\underline{x})+W_1(x_0(\underline{x}))} \frac{F\left(R^{-1}(r)\right)}{F(R^{-1}(R(a_1,\lambda_0(\underline{x}))))} & \text{if } r \leq R(a_1, \lambda_0(\underline{x})) \\ \\ \frac{W_0(x_0(\underline{x}))}{W_0(x_0(\underline{x}),\underline{x})+W_1(x_0(\underline{x}))} + \frac{W_1(x_0(\underline{x}))}{W_0(x_0(\underline{x}),\underline{x})+W_1(x_0(\underline{x}))} \frac{F\left(R^{-1}(r)\right)-F\left(R^{-1}(R(a_1,\lambda_0(\underline{x})))\right)}{F\left(R^{-1}\left(R\left(a_1,\bar{\lambda}\right)\right)\right)-F(R^{-1}(R(a_1,\lambda_0(\underline{x}))))} & o.w. \end{cases}$$

where $W_0(x_0(\underline{x}), \underline{x}) = \int_{\underline{x}}^{x_0(\underline{x})} m_0^x dG(x)$, and $W_1(x_0(\underline{x})) = \int_{x_0(\underline{x})}^{\bar{x}} m_1^x dG(x)$. The associated Pdf - y - is given by:

$$y(R;\underline{x}) = \begin{cases} \frac{W_0(x_0(\underline{x}),\underline{x})}{W_0(x_0(\underline{x}),\underline{x})+W_1(x_0(\underline{x}))} \frac{f\left(R^{-1}(\lambda)\right)}{F(R^{-1}(R(a_1,\lambda_0(\underline{x}))))} & \text{if } r \leq R(a_0, \lambda_0(\underline{x})) \\ \\ 0 \qquad \text{if } r \in (R(a_0, \lambda_0(\underline{x})), R(a_1, \lambda_0(\underline{x}))) \\ \\ \frac{W_1(x_0(\underline{x}))}{W_0(x_0(\underline{x}),\underline{x})+W_1(x_0(\underline{x}))} \frac{f\left(R^{-1}(\lambda)\right)}{F\left(R^{-1}\left(R\left(a_1,\bar{\lambda}\right)\right)\right)-F(R^{-1}(R(a_1,\lambda_0(\underline{x}))))} & o.w. \end{cases}$$

Since $\partial\lambda_0/\partial\underline{x} < 0$, it has to be the case that W_0/W_1 increases, so to satisfy the entrepreneur's indifference condition. Therefore, $W_0/(W_0 + W_1)$ is now higher. Hence, at $r = R(a_0, \lambda_0(\underline{x}))$, $Y(r;\underline{x})$ has increased.

Consider now the expectation of r conditional on being larger than $R(a_0, \lambda_0(\underline{x}))$. This is

$$\int_{R(a_1,\lambda_0(\underline{x}))}^{R\left(a_1,\bar{\lambda}\right)} r dY(r;\underline{x}) / (1 - Y(R(a_1, \lambda_0(\underline{x}));\underline{x})),$$

which is higher the higher is $\lambda_0(\underline{x})$. The same is true when conditioning on r being above any number in the set $(R(a_0, \lambda_0(\underline{x})), R(a_1, \lambda_0(\underline{x})))$. The conditional expectation of given that r is above some $\hat{r} > R(a_1, \lambda_0(\underline{x}))$ is instead given by

$$\int_{\hat{r}}^{R\left(a_1,\bar{\lambda}\right)} r dY(r;\underline{x}) / (1 - Y(\hat{r};\underline{x})),$$

which is unaffected by the change in $\underline{x}$ because $(1 - Y(\hat{r};\underline{x}))$ is equivalent to the function $\int_{\hat{r}}^{R\left(a_1,\bar{\lambda}\right)} dY(r;\underline{x})$, that is computed as:

$$\frac{W_1(x_0(\underline{x}))}{W_0(x_0(\underline{x}), \underline{x}) + W_1(x_0(\underline{x}))} \int_{\hat{r}}^{R\left(a_1,\bar{\lambda}\right)} \frac{f(R^{-1}(r))\, dr}{F\left(R^{-1}\left(R\left(a_1, \bar{\lambda}\right)\right)\right) - F(R^{-1}(R(a_1, \lambda_0(\underline{x}))))}$$

which therefore gives:

$$\frac{\int_{\hat{r}}^{R\left(a_1,\bar{\lambda}\right)} r dY\left(r;\underline{x}\right)}{\left(1-Y\left(\hat{r};\underline{x}\right)\right)}=\frac{\int_{\hat{r}}^{R\left(a_1,\bar{\lambda}\right)} r f\left(R^{-1}\left(r\right)\right) dr}{F\left(R^{-1}\left(R\left(a_1,\bar{\lambda}\right)\right)\right)-F\left(R^{-1}\left(\hat{r}\right)\right)}$$

■

Proof of Proposition 6. The system characterizing the two cutoffs (x_0, λ_0) is given by:

$$\begin{aligned} m_1^{x_0}\left[\rho\left(\mathbb{E}\left[R\left(a_1,\lambda\right)\mid\lambda\geq\lambda_0\right]-R_0\right)+\left(1-\rho\right)\left(\mathbb{E}R\left(a_1,\lambda\right)-R_0\right)\right] & - \\ -m_0^{x_0}\left[\rho\left(\mathbb{E}\left[R\left(a_0,\lambda\right)\mid\lambda\leq\lambda_0\right]-R_0\right)+\left(1-\rho\right)\left(\mathbb{E}R\left(a_1,\lambda\right)-R_0\right)\right] & =0 \end{aligned} \tag{16}$$

$$\frac{W_1\left(x_0\right)}{1-F\left(\lambda_0\right)}R\left(a_1,\lambda_0\right)-\frac{W_0\left(x_0\right)}{F\left(\lambda_0\right)}R\left(a_0,\lambda_0\right)=0 \tag{17}$$

Notice that the entrepreneur's indifference condition is unaffected by ρ. Call $\Phi\left(x_0,\lambda_0,\rho\right)$ the left-hand side of (16) and $\Psi\left(x_0,\lambda_0\right)$ the left-hand side of (17). Using the Implicit Function Theorem, one gets that:

$$\frac{\partial x_0}{\partial\rho}<0\iff\Phi_{\lambda_0}\left(x_0,\lambda_0,\underline{x}\right)\Psi_{x_0}\left(x_0,\lambda_0,\underline{x}\right)<\Phi_{x_0}\left(x_0,\lambda_0,\underline{x}\right)\Psi_{\lambda_0}\left(x_0,\lambda_0,\underline{x}\right)$$

which is implied by condition (13), and hence it holds for every stable equilibria. ■

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Chapter 2 - Closed-End Funds and Commitment in Venture Capital

Abstract

In this Chapter I extend the model developed in Chapter 1 to a dynamic setting, where projects need time to develop and produce returns. VCs can choose to enter in a short-term contract with investors, giving them access to investors liquidity for a given period of time, and an open credit relationship that allows them to raise money from investors at any point in time. The model illustrates a novel advantage of closed, finite-horizon funds, which emerge in equilibrium even when they are socially undesirable: they attract the best entrepreneurs, who value the most the exclusive relationship that only a closed-end fund can guarantee. The interpretation is that VCs benefit from committing to a size in the first place.

1 Introduction

Differently from, for example, mutual funds and hedge funds, private equity funds - like venture capital partnerships - are finitely lived: VCs' activity is restricted by a clear deadline when their investments must be exited, and capital does not flow in and out of a VC fund. Investors and VCs form a limited partnership. This arrangement can have the negative consequence of forcing VCs to give up investment opportunities that are discovered too late in the fund's life, and when much of the committed capital has been already invested. Kandel et al. (2011) find suggestive evidence that being closer to the end of the fund induces myopic behavior by VCs. Barrot (2016) finds that the length of the investment horizon is associated to selection of different startups, meaning that it has real effects on the VCs' investment strategy. The common understanding is that such fund configuration, despite introducing some potential distortions, helps mitigating agency problems between limited partners - the investors - and the VCs. But one could ask whether in absence of such problems, a different arrangement would emerge. In other words, is forming closed-ended fund also in the VC's best interest, or is it just an unavoidable cost?

To answer this question, in this Chapter I accommodate the model described in Chapter 1 to a dynamic setting where projects don't realize returns immediately, VCs can match to one entrepreneur every period, and follow the projects until they are ready to produce returns. I allow VCs to choose between a short-term contract and a long-lasting, open credit relationship with the investors. In the former case, VCs are forced to wait until the current project has realized its returns before they can get to manage a new fund, and go back to the market for entrepreneurs. In the latter, they have access to investors' money and can add a new project to the fund while the first investment is still ongoing. Projects under management of a VC that is in a short-term contract with investors won't overlap.[1] Thus, such contract allows the VC to commit its attention to the current project.

I show that there is no equilibrium where every VC is in a long-lasting credit relationship with investors, even when this is the most efficient arrangement. This happens because a deviating VC, by choosing the short-term contract - that is, by forming a closed-end fund - will be able to skim the market and attract the very best entrepreneurs, being them those who are willing to pay the highest search friction in order to match to a "committed" VC. This provides a new rationale for the prevalence of closed, finite-horizon funds in venture capital, as opposed to the open funds we observe in other contexts where fund managers invest in public securities and are not subject to a two-sided matching problem.

Relation to the Literature. The paper contributes to a literature focusing on the most observed features and contractual arrangements at the basis of investment funds: in Stein (2005) open-ended fund structure emerges because mutual fund managers compete for money flows and the best ones can credibly signal their ability by offering an open-end structure that can prevent them from fully exploiting arbitrage opportunities; Axelson et al. (2009) explain why buyout funds exhibit a mix of outside debt and equity financing in a setting where the key tension is between imposing discipline to privately informed managers while at the same time making efficient use of their superior screening ability.

Roadmap: Section 2 introduces the model; followed by the characterization of the equilibria; Equilibria are ranked in terms of welfare achieved and compared to a second best solution in Section 3; Section 4 explores the effects of entry of new VCs in the

[1] This is a stylized representation. Nonetheless, while it is true that a fund manager can open new funds in parallel, fundraising is typically time consuming, which strongly limits the extent to which VCs can put projects "on hold" until enough money is raised; the practice is also limited by contractual restrictions that are meant to protect current investors, so new funds can't be raised before the current has been substantially invested.

economy; Section 5 uses results in previous sections to analyse the choice between short and long-term investors-VC relationships, in an appropriately accommodated setup; Section 6 concludes; All proofs are relegated to the Appendix.

2 Model

In the next two subsections I will describe the physical environment, the timing and strategies, and comment on the interpretation of the two types of fund structures VCs will be assumed to choose among.

2.1 Setup

Agents. Time - denoted t - is countably infinite. The economy consists of long-lived homogeneous venture capitalists (henceforth VCs), identical investors and ex-ante identical entrepreneurs. There is an arbitrarily large measure of investors. Each investor is endowed with one unit of money in every period. The initial measure of VCs in the economy is fixed and normalized to one. Entrepreneurs are in large supply, and can enter the market upon paying startup cost c. If they do, they draw a type λ, the quality of the project they own, from a continuous distribution f strictly positive on the entire support $\left[\underline{\lambda}, \overline{\lambda}\right] \subset \mathbb{R}_+$.

Projects. All projects need only one unit of money to become a firm. It takes two periods for each project, independently on its quality, to develop and produce returns. Once a project has produced returns, the match expires. Each project's return, R, is assumed to be a function of attention, a, and of the project's quality, λ. Call this function $R(a, \lambda)$. As in Chapter 1, I assume $R_a(a, \lambda)$ and $R_\lambda(a, \lambda) > 0$. I further assume that $R(a, \lambda)$ is twice continuously differentiable in its arguments. Moreover, assume that $R(a, \lambda)$ is logsupermodular.

Diseconomies of Scale. To capture the same quality-quantity trade-off as in Chapter 1, I assume VCs attention into a particular project is determined by whether he has dealt with another one in any period of the project's life span. Denote the levels of a with and without overlapping projects a^h and a^l respectively, with $a^h > a^l$.

Fund Structure. At the beginning of each period, managers approaching investors can opt for either of two fund structures. In one case, they can choose an open credit line that allows them, at any time t to have the necessary cash to finance a new project. Alternatively, they can form a closed fund, with a finite, two periods long horizon. In the latter case, a fund consists essentially of a single investment, that matures returns two periods from the initial formation. Crucially, a VC that formed a closed fund at time t will not be able to raise any additional money before time $t+2$.

Matching and Information. Entrepreneurs are privately informed about their type, λ. Assume a VC can only match to one entrepreneur in every period. It follows that a VC searching for an entrepreneur can be in either of two states: he can be unmatched, or already be dealing with a project that is currently at its intermediate stage. Entrepreneurs observe what type of fund is every VC managing, and whether the VC is altready matched. VCs that are the same in these two, binary, dimensions form a submarket where entrepreneurs will select into, possibly depending on their type. Finally, assume that as many matches as possible are formed in each submarket; that is, the number of matches as a function of the measure of entrepreneurs searching, q_e, and the measure of money available (or "vacancies"), q_k, is given by $M(q_k, q_e) = \min\{q_k, q_e\}$.

Payoffs, Strategies and Timing. VCs offer investors a contract that specifies a fund structure and a management fee, p, that they receive from investos. Investors can accept the contract or reject and invest in an alternative technology delivering constant returns R_0. Normalize R_0 to zero. For every project invested with their money, investors will get a fixed share $\alpha \in (0,1)$ of the returns. Entrepreneurs observe VCs decision and choose whether or not to pay the startup cost. Those who do, can direct their search towards different VCs. Conditional on being matched, they receive the residual - $(1-\alpha)$ - share of the returns from their projects. All agents are risk neutral and maximize the sum of expected returns, with common discount factor $\delta \in (0,1)$.

Population. At every time t a measure N of new VCs enter the market; at random, an equal measure of VCs dies. Investors are infinitely lived. At each t a new generation of entrepreneurs is born. Those who choose not to enter the market, leave the economy forever. Those who enter and don't get matched die. Those who enter and get matched, receive their payoff after two periods and leave the economy forever.

2.2 Interpreting the Closed and Open Fund Structure

In case a VC opts for a closed fund strcture, a fund consists essentially of a single investment, that matures returns two periods from the initial formation. It is essential for the analysis that a VC that is in such contract with the investors can't raise additional money before the investment has produced returns. One can imagine that an investor writing this type contract will have its wealth at the intermediate period invested on the alternative asset. If approached by the VC in the intermediate period, the investor wouldn't have the liquidity to provide the VC with the money to start a new project. This is realistic: pension funds (representing a large share of investors in venture capital) usually meet capital calls by selling positions in liquid indexes.[2] Another interpretation is that in the typical private equity partnernship, many - dispersed - investors own a share of the fund. This makes it harder for VCs to agree with original investors on changes to the size of the fund. Whichever is the interpretation, all that matters is that a short-term contract creates an endogenous commitment not to start a new project before the original has produced its returns.

3 Analysis

Note that, as it occurs in the model presented in Chapter 1, VCs' choice maximizes the fund's total returns, because they can set fees so to hold investors to their participation constraint. Therefore a VC's objective is to maximize the discounted sum of expected excess returns. Observe also that the relevant dimension from the entrepreneurs' perspective on which VCs differ is how much attention will the VC eventually devote to their project. Each level of attention will hence define a submarket where entrepreneurs can select into.

Summary of the Game. Let me now summarize the timing of the game:

- At each time t newborn VCs and pennyless ones approach investors, choose a fund structure, and contract over the fee p that investors pay them upon realization of each project's returns. Managers opting for a closed fund can't approach investors before the project they are currently financing has produced returns.

- Entrepreneurs observe investors' strategy and make the entry choice. Those who enter the market privately observe their type λ and direct their search.

- At $t+1$ managers who chose a open credit line have the money to search for a new project. Every match formed in t generates returns $R\left(a,\lambda\right)$ at time $t+2$. By then, t-generation entrepreneurs who matched leave the market forever.

[2] See Robinson and Sensoy (2016).

The main difficulty is that a matched VC's behavior - whether to start a new project at the intermediate period - will in principle depend on the quality of projects he is expecting to be matched to.

Assumption A1. $(1+\delta)\, R\left(a^l, \lambda\right) > R\left(a^h, \lambda\right) \qquad \forall \lambda.$

The assumption above limits the extent to which a VC's human capital is destroyed when working on parallel projects. Under A1, *keeping quality fixed*, it is always optimal to start a new project every period. In turn this means that a VC that expects to attract the same type of projects is always going to search for new ones to finance, provided he has the necessary cash. Later on in the analysis I will restrict attention to a class of equilibria in which entrepreneurs sort into different VCs in a way that is constant over time. In such equilibria, a VC that follows the same strategy over time will always attract the same pool of entrepreneurs. Therefore, since the condition in A1 holds for every possible λ, VCs in the open credit relationship with investors will actively search in every period.

It is worth noting that, if instead the inequality in A1 was reversed, the choice by a VC of which contract to enter with the investors would be inconsequential: managers would never start a new project before the current has reached termination; the fact that they can't access liquidity at any point in time would not constrain their choices.

Under A1, in those equilibria I will study, entrepreneurs that are searching for a match will effectively face the choice between two distinct markets: one where "uncommitted" VCs will provide attention a^l, and the other where attention is at a^h, composed of fund managers that entered closed fund, who will not be able to search before the original investment has matured.

The Sorting Subgame. I first derive sorting behavior when a positive measure of vacancies is available in both markets. Normalize the mass of VCs to one and denote γ_t the share of managers running a finite-horizon fund at time t. It is immediate from results in previous sections to observe that equilibrium search behavior at time t is then characterized by a threshold λ^* such that entrepreneurs search in the high-attention market if and only if $\lambda \geq \lambda^*$. This is due to log-supermodularity of $R(a, \lambda)$. The threshold is implicitly defined by the equation:

$$\frac{\gamma_t}{1-\lambda^*} R\left(a^h, \lambda^*\right) = \frac{1-\gamma_t}{\lambda^*} R\left(a^l, \lambda^*\right). \tag{1}$$

Lemma 1. *The solution to* (1) *is unique: given a share of managers with finite-horizon funds* γ_t*, there is a unique equilibrium of the sorting subgame.*

In principle, VCs might choose different contracts at different times of their lives, and I allow for that. However, for the sake of simplicity, I restrict attention to equilibria where, whenever VCs are indifferent which fund structure to choose, the share of VCs going for either option stays constant. I show that such equilibria are always possible to construct, provided at a certain time t agents are indifferent about which contract to choose, thanks to the assumption that new VCs enter the market at each t.

Definition 1. *A stationary equilibrium is an equilibrium in which the shares of VCs choosing either fund structure is independent on t.*

Lemma 2. *If an equilibrium where $\gamma_t = z \in (0,1)$ for some t exists, then there exists a measure of newborns - N - such that an equilibrium where $\gamma_t = z$ for all t exists.*

Let me focus on the case when selecting the best entrepreneur is appealing to the VC. Formally, this means imposing the following restriction.

Assumption A2. $R\left(a^h, \bar{\lambda}\right) > (1+\delta)\,\mathbb{E}\left[R\left(a^l, \lambda\right)\right].$

Under A2 the returns from following the best entrepreneur exclusively are higher than those from financing two average projects in two subsequent periods. Finally, recall the definition from Chapter 1 of the function $\phi\left(a, a^{'}, \tilde{\lambda}\right)$. When a and $a^{'}$ take the values a^h and a^l respectively, this is given by:

$$\phi\left(a, a^{'}, \tilde{\lambda}\right) := \frac{\alpha\mathbb{E}\left[R\left(a^h, \lambda\right)|\lambda \geq \tilde{\lambda}\right]}{\alpha\mathbb{E}\left[R\left(a^l, \lambda\right)|\lambda \leq \tilde{\lambda}\right]}$$

that is the ratio between the returns per dollar invested into a fund that provides high attention and attracts entrepreneurs with quality above some $\tilde{\lambda}$, and the returns per dollar invested into one that provides low attention and attracts entrepreneurs below the same cutoff.

It is now possible to state the main result of this Chapter.

Proposition 1. ***(Equilibrium Fund Structure).***

(i) There is no stationary equilibrium where all VC choose the open credit line.

(ii) The equilibrium's measure of VCs with finite-horizon funds, γ, is the solution to the equation:

$$\mathbb{E}\left[R\left(a^h, \lambda\right)|\lambda \geq \lambda^*(\gamma)\right] = (1+\delta)\,\mathbb{E}\left[R\left(a^l, \lambda\right)|\lambda \leq \lambda^*(\gamma)\right]$$

whenever it exists.

(iii) When $\mathbb{E}\left[R\left(a^h, \lambda\right)\right] \geq (1+\delta)\,R\left(a^l, \underline{\lambda}\right)$, there is a stationary equilibrium where every VC chooses the finite-horizon fund.

Notice that, because of A2, if the function $\phi\left(a^h, a^l, \tilde{\lambda}\right)$ was decreasing in $\tilde{\lambda}$ - as it is in Example 1 - the condition in (ii) would have no solution, while condition (iii) would always hold. This means that a situation where every VC chooses the finite-horizon fund would be the unique equilibrium.

The picture below provides a description of the two types of equilibria as in Proposition 1.

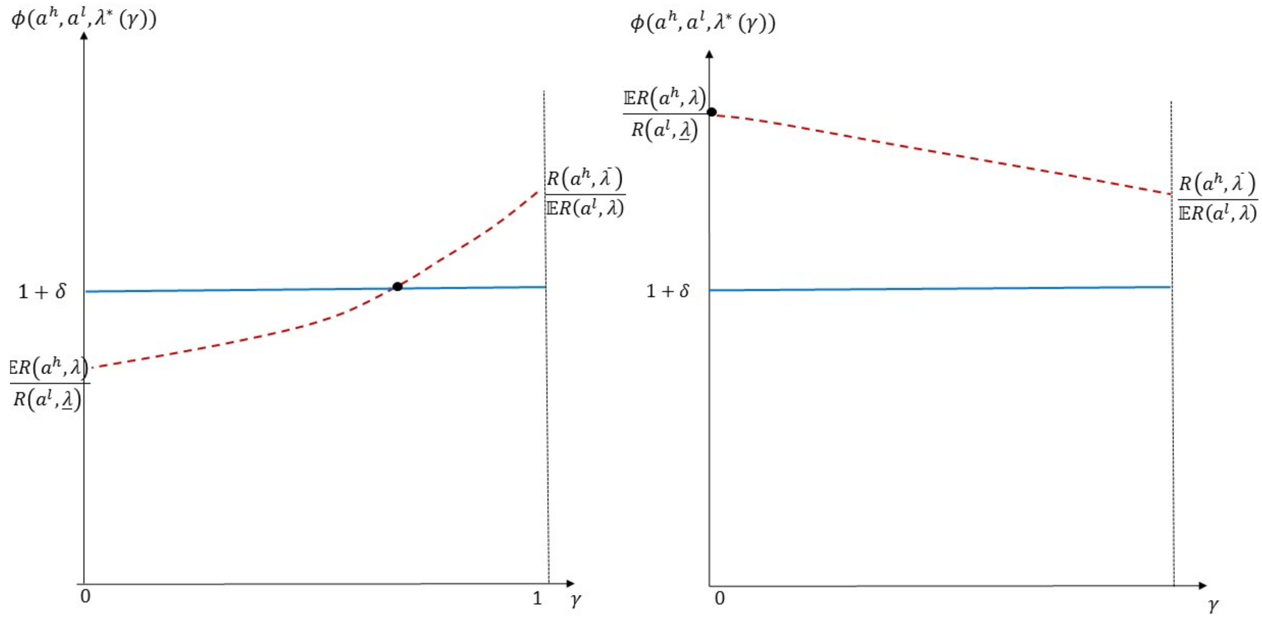

Figure 1: Left: On the horizontal axes, I represent the measure of VCs selecting the open fund structure. On the vertical axes, the function $\phi\left(a^h, a^l, \lambda^*(\gamma)\right)$ is represented, where $\lambda^*(\gamma)$ is the unique solution to the entrepreneur's indifference condition as in equation (1), as a function of γ. A situation where $\gamma = 1$ is not an equilibrium, as in that case the function would take value: $R\left(a^h, \bar{\lambda}\right)/\mathbb{E}\left[R\left(a^l, \lambda\right)\right]$ which is assumed to be larger than $1+\delta$. In this example, the function ϕ is monotonically increasing in $\tilde{\lambda}$, therefore it increases as γ increases, as this induces a larger cutoff in the entrepreneur's strategy. Hence, the unique equilibrium is an interior one. Right: In this example the function is monotonically decreasing in $\tilde{\lambda}$. Since $R\left(a^h, \bar{\lambda}\right)/\mathbb{E}\left[R\left(a^l, \lambda\right)\right]$ is larger then $1+\delta$, so will be the ratio $\mathbb{E}\left[R\left(a^h, \lambda\right)\right]/R\left(a^l, \underline{\lambda}\right)$. Since, when every VC is in the closed contract, deviating to the open fund structure would attract the worse type - $\underline{\lambda}$ - it follows that in this example the unique equilbrium is one where every VC opts for the closed contract, that is $\gamma = 0$.

Similarly to how established in the general static model, the motive to attract better entrepreneurs generates an inefficiency, as VCs don't internalize the aggregate effect - due to equilibrium sorting - of their choices. In particular, one natural and policy relevant question would be whether allowing these short-term contracts is desirable. It turns out that, under A1, banning the short-term contracts is always beneficial.

Proposition 2. ***(Banning finite-horizon funds).*** *Every equilibrium of the game delivers lower welfare than the case where every VC chooses the open credit line. That is, banning finite-horizon funds improves welfare.*

It is easy to see why the "corner" equilibrium where every VC chooses the short-term contract is welfare detrimental. Infact, notice that under A1, the choice of which contract to sign involves a simple trade-off: on the one hand, starting a new project every period allows the VC to make the best use of his human capital, as the dilution in attention is assumed to be small; on the other, committing to an exclusive relation helps the VC attract the best entrepreneurs. However, *in equilibrium* this commitment confers no benefit at all, since every VC will look alike. In all interior equilibria, VCs that opt for the infinite-horizon fund are attracting a negatively selected subset of entrepreneurs. Since expected returns are ultimately the same to all VCs - as they have to be indifferent which contract to choose - it follows that expected aggregate returns would be higher if all VCs would choose the long-term contract and get matched with the average entrepreneur.

4 Conclusion

In this Chapter, I have used the results from Chapter 1 to analyse the equilibrium choice between a closed, finite-horizon fund, versus an open, firm-like investors-VC relationship, in a simple dynamic version of the model. From the VC's point of view, finite-horizon funds come at the cost of giving up investment opportunities arriving when the current fund is still ongoing; entrepreneurs value this commitment, because it guarantees exclusive attention. A situation where all VCs opt for the open fund unravels, even when this would be the welfare maximising solution, due to the incentive to skim the market and attract the best entrepreneurs, who are the most willing to pay the highest search friction and get exclusive attention. Similarly, one where every VC has the closed fund is sustainable as an equilibrium outcome, because a deviation to the open-end fund would attract the worse entrepreneurs. This result suggests another reason why VCs raise funds with a finite horizon and with explicit limits on the investments that they can make while the current fund is still ongoing. They might benefit from committing to a size in the first place.

Appendix

Proof of Lemma 1. It is convenient to rewrite (1) as:

$$\frac{R\left(a^{h},\lambda^{*}\right)}{R\left(a^{l},\lambda^{*}\right)}=\frac{1-\gamma_{t}}{\gamma_{t}}\frac{\left(1-F\left(\lambda^{*}\right)\right)}{F\left(\lambda^{*}\right)} \tag{2}$$

The left-hand side of (2) is continuous and strictly increasing in λ^{*} by assumption (as R is continuous in both arguments and assumed to be log-supermodular in this section). The right hand side is continuous and strictly decreasing in λ^{*}. In particular, notice the left-hand side is positive and finite for all $\lambda^{*}\in\left[\underline{\lambda},\bar{\lambda}\right]$. The right hand side is zero as $\lambda^{*}\rightarrow\bar{\lambda}$ and tends to infinity as $\lambda^{*}\rightarrow\underline{\lambda}$ ■

Proof of Lemma 2. Take any period t where a share $z_{t}\in(0,1)$ of the VCs who have to choose fund structure opts for the open fund, and the residual selects the closed-end fund. Assume that - at time t - given the strategy profile in all periods other than t - there is a share of actively searching VCs - $y_{t}\in(0,1)$ - that will also search at $t+1$. This share will define how many VCs will be in the market where attention is a^{l}, relatively to how many are in the market where attention is a^{h}. By Lemma 1, this induces a unique cutoff in the entrepreneurs' strategy. Denote it $\lambda_{t}\left(y_{t}\right)$. Given that this is an equilibrium, it has to be the case that:

$$\mathbb{E}\left[R\left(a^{h},\lambda\right)|\lambda\geq\lambda_{t}\left(y_{t}\right)\right]=(1+\delta)\,\mathbb{E}\left[R\left(a^{l},\lambda\right)|\lambda\leq\lambda_{t}\left(y_{t}\right)\right] \tag{3}$$

To construct a stationary equilibrium, call L the measure of VCs that survive from time t to time $t+1$ and are actively searching. A measure N of newborns enters the economy. In a stationary equilibrium, one would have that $z_{t}=y_{t}=z$. Assign β newborns to the open fund structure and $(1-\beta)$ newborns to the closed-end fund. It must hold that:

$$\frac{zL+\beta N}{zL+N}=z. \tag{4}$$

In order for β to be strictly in $(0,1)$, a necessary and sufficient condition is that:

$$\frac{N}{L}>1-z$$

Notice that L is bounded above by $1-N$. It is easy to observe that - for any given z - an N such that $N/\left(1-N\right)$ exceeds $1-z$ always exists. Given that z stays constant, $\lambda_{t}\left(y_{t}\right)=\lambda_{t}\left(z\right)$ is also constant. Therefore condition (3) is satisfied at every t and VCs are indifferent in every period between choosing either fund structure. ■

Proof of Proposition 1. (i). I first show that there is no equilibrium where every manager has the open credit line. Take an equilibrium where $\gamma = 1$ and consider a manager who deviates to a finite-horizon structure. If there is a $Q\left(a^h\right)$ for which some λ benefits from deviating to a^h, all types above λ would strictly deviate. This means the manager must expect to attract type $\bar{\lambda}$. This is profitable as long as:

$$R\left(a^h, \bar{\lambda}\right) > (1+\delta)\,\mathbb{E}\left[R\left(a^l, \lambda\right)\right]$$

which is true by assumption.

(ii). The second part of the Proposition follows from the fact that all VCs are identical, therefore they must be indifferent in an interior equilibrium.

(iii). Finally, by an argument similar to that for part (i), when all managers are in a short-term contract with investors, a deviation to the open credit line must necesserily attract $\underline{\lambda}$. This is not profitable as long as:

$$\mathbb{E}\left[R\left(a^h, \lambda\right)\right] \geq (1+\delta)\,R\left(a^l, \underline{\lambda}\right)$$

■

Proof of Proposition 2. Recall that the mass of VCs is normalized to one in this section. Since all VCs are alike, ex-ante welfare, V, in the economy is the expected equilibrium payoff to the VC. Consider first an equilibrium where every VC opts for the finite-horizon fund. Welfare is:

$$V = \mathbb{E}\left[R\left(a^h, \lambda\right)\right] < (1+\delta)\,\mathbb{E}\left[R\left(a^l, \lambda\right)\right]$$

where the second inequality is a consequence of A3 once expectations are taken on both sides. Hence this equilibrium is dominated by a situation where every VC is forced to choose the open credit line. Second, consider welfare from any interior equilibrium. This is given by:

$$V = \mathbb{E}\left[R\left(a^h, \lambda\right)|\lambda \geq \lambda^*(\gamma)\right] = (1+\delta)\,\mathbb{E}\left[R\left(a^l, \lambda\right)|\lambda \leq \lambda^*(\gamma)\right] < (1+\delta)\,\mathbb{E}\left[R\left(a^l, \lambda\right)\right]$$

where the second inequality trivially holds since $\lambda^*(\gamma) < \bar{\lambda}$, for any $\gamma \in (0,1)$. ■

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Chapter 3 - A Model of Risk Taking with Experimentation and Career Concerns*

Abstract

We model an economy where managers create value through their ability to learn at an intermediate stage about the intrinsic profitability of a risky investment. Managers are heterogeneous in their ability to extract information from experiments, and care about their reputation. Their incentive to take on risk is distorted by career concerns, and can result in under or over risk-taking. This is determined by whether discarding a risky project following the experiment is more typical of better managers. Our result is in contrast with Holmstrom (1999) where managers' ability affects the project's success rate, and career concerns can only produce inefficiently low risk-taking. We show that the inefficiency is reduced in one extension of the model, where the market can also observe the outcome of similar projects. The novel implication is that the markets more plagued by career concerns distortions are those where managers engage in more idiosyncratic activities.

1 Introduction

Investing in young and innovative firms involves large uncertainty.[1] The Venture Capital financing model offers a solution to deal with the uncertainty inherent to the innovation process: venture capitalists (henceforth VCs) learn about firms over time and hence can condition their financing on the information they acquire. There is large evidence that they differ considerably in their ability to generate returns (see Korteweg and Sorensen (2017)), and that positive past performance by VCs increases their chances to raise a new fund (see the evidence in Kaplan and Schoar (2005)), and the fees they receive from assets under management. Thus, when making their choices they are arguably motivated by career concerns.

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[1]It has been calculated that around 50% of investments in venture capital exit with zero value, and only about 10% of total investments effectively make all the returns to venture capital vehicles (see Hall and Woodward (2010) and Nanda and Rhodes-Kropf (2013)).

Do career concerns prevent VCs from efficiently using their ability to learn about the projects they finance? And which markets are more prone to this problem? In this paper we show that career concerns generally lead to inefficient risk taking. In particular, our novel contribution is to find that the type of experiments that agents can undertake determines the direction of this inefficiency. Moreover, as the number of agents financing projects that are linked to the same state of the world increases, the inefficiency reduces. In the limit, the equilibrium risk taking approaches the first best.

In the pages that follow we develop a framework where managers can choose between a safe task and a risky one that can be abandoned after an experimentation phase. In our model, both the managers and the market do not know the state of the world - a binary variable - that determines the return on the risky project. Following the literature on career concerns, we also assume that the managers' ability to run the experiment is unknown to all agents. The manager, independently from his ability, privately receives an initial signal on the state of the world and, based on this, he chooses whether to select the safe or the risky project. If the risky task is chosen, both players observe the binary result of the experimental phase and they abandon the project in case the experiment conveys a bad signal. How informative the experiment is depends on the manager's ability. On average, a good manager extracts better information and produces higher returns from the risky task. However, in some states of the world the good managers might perform worse than a bad one. This happens when the high ability manager receives too often the good signal from the experiment, when it would be better to abandon the risky task, or when he receives too often the bad signal, when the state of the world is positive and the continuation of the risky project would yield high returns.

After characterizing the efficient risk taking rule, we turn the attention to the equilibrium characterization. We first show that every equilibrium features a cutoff strategy: the manager chooses to undertake the risky task if and only if the initial private signal implies that the likelihood of being in the good state of the world is high enough. Also the efficient risk taking rule prescribes a cutoff strategy. However, when we study the welfare properties of equilibria in our economy, we find that they are intrinsically plagued by inefficient risk taking. The marginal manager, rather than being purely motivated by financial returns, bases his choice also on his expected reputation. Because sometimes good managers are biased towards abandoning risky projects by mistake, the market could perceive the abandonment of the risky project as a good sign about the managerial ability. If so, in anticipation of the reputational gain that will come from abandonment, also managers that are not particularly optimistic about the state of the world might be induced to choose the risky task. In other circumstances, when abandoning risky projects is perceived by the market as a bad signal about manager's ability, they are inclined to a more prudent behaviour.

We show that our inefficiency result also holds when there are several managers that, upon choosing the same risky task, run independent experiments. We then show that the inefficiency is monotonically decreasing in the number of managers. The intuition of this results builds on the fact that, by observing the outcome of several experiments, the market figures out more often the true state of the world. It becomes, indeed, less and less likely that all managers, correctly in the bad state and by accident in the good one, abandon the risky project. When the market is expected to observe more often the true state of the world, in turn, the manager's expected reputation gets closer to the prior; in other words the manager can no longer use his private information to induce reputational gains. This is so because the initial signal is independent on the manager's ability and, thus, it offers no additional information about this ability once the state of the world is observed. Indeed, knowing the initial signal is useful to make better inference about the manager's talent only when the risky task is abandoned and the market doest not get to know whether it happened correctly or by mistake. We conclude by showing that in the limit when infinitely many managers operate, the inefficiency disappears.

Relation to the Literature. We contribute to three different strands of the literature. First, our paper is related to the recent theoretical literature on the effects of imperfect information about fund managers' abilities. Hochberg et al. (2013) model investors-managers bargaining in a sequential environment where incumbent investors are more informed than outside investors about managers' skills. Marquez et al. (2014), instead, develop a signal-jamming model where fund managers with differential ability to produce returns distorts the fund size decision in order to affect entrepreneurs' learning. Both papers can explain persistence in venture capital funds' returns. Similarly to these models, in our work there is uncertainty about managers skills. However, we focus on how this problem distorts managers' investment decisions once the fund has already been set. Second, we contribute to the discussion on experimentation in entrepreneurial finance. Recent works, such as and Kerr et al. (2014) and Ewens et al. (2017), emphasize the role of experimentation in nurturing the innovative activity of young firms. We provide a somewhat darker view on the amount of experimentation observed in the venture capital industry. In our model, there can be too much investments in experimental projects.

Third, on a more abstract level, our work is related to the literature on the effect of career concerns on managerial risk taking. In a seminal work, Holmström (1999) shows that when managerial ability directly affects the project success rate and managers care about their reputation, they underinvest in risky projects. A recent paper by Chen (2015) breaks this result by introducing managers' private information on their type and, hence, a signaling motive to take on risk. We maintain, instead, the assumption that managers do not know their ability, but we change the way in which managerial skills affect the returns from undertaking the risky activity. In our modified setting we characterize necessary and sufficient conditions for either type of inefficiency to emerge in equilibrium.

Finally, a setting where agents' learning ability differs in quality improvement - it is the same initially, but not in the intermediate stage - has been modeled by Li (2007). Unlike in our setting, agents are privately informed about their ability and - unlike in our setting - strategically change their actions as new information arrives. A signalling motive gives them an incentive to give inconsistent reports, similarly to what discarding the project would mean in our setting.

The rest of the paper is organized as follows. In section 2 we set up the model with one manager, we characterize the first best and the equilibria under career concerns, and we study their efficiency properties. In section 3 we extend the analysis to an economy with N managers. Section 4 concludes. Proofs that are not in the main text are relegated to the Appendix.

2 Model

2.1 Setup

Managers, Projects, and Experiments. An agent, called *manager*, can choose whether to undertake a safe project (S) or a risky project (R). The safe project costs 0, the risky one costs c. The safe one produces returns of v_s - with $v_s > c$ - in any state of the world, while the risky project pays returns v_r - with $v_r > v_s$ - in the good state and nothing in the bad state. Let x denote the state, and the state space be $\mathcal{X} = \{g,\ b\}$. The principal, often referred to as the *market* in this paper, assesses the capability of the manager to anticipate the state of the world.

When the manager chooses the risky task, he runs at no cost an informative experiment to gather additional information about the likelihood of success and, based on the information she gets from the experiment, can decide whether to pursue or to abandon the investment. Let i index the manager's type: a high type ($i = h$) is a manager that is able to extract *better information* from the experiment compared to a low type ($i = l$) in a sense that will become clear in the next lines. We assume that the experiment produces two signals only, denoted s, with $s \in \mathcal{S} = \{g, b\}$. An experiment is then fully described by the precision parameters defining the probability of receiving the *right* signal in each of the two states of the world, $\alpha^i = P(s = g \mid x = g)$ and $\beta^i = P(s = b \mid x = b)$. Notice that superscript i allows signals' precision to differ depending on the manager's type. We further assume that α^i, $\beta^i > \frac{1}{2}$ and that parameters are such that it is always optimal to follow the signal. Moreover, the cost of choosing the risky project, c, is only paid when the manager decides not to abandon the project, that is, after she observes a "good" signal ($s = g$), in which case returns realize.[2] Otherwise, in case of abandonment of the risky task because the signal from the experiment is "bad" $(s = b)$, the return is zero.

Information and Timing. Prior to choosing which project to select, the manager privately observes a signal $\omega \in \mathbb{R}_+$ (which we will refer to as the project's *intrinsic quality*), generated from a density f defined over the support $[\underline{\omega}, \overline{\omega}]$, which is independent of his type. The manager then updates his prior probability of success of the risky project to $p(\omega)$. Throughout the analysis we assume that $p'(\omega) > 0$. The signal ω is the only dimension where the manager's and the market's information don't coincide.

Players share a common prior belief, ρ, on the probability that the manager is high type. This probability, together with the precision parameters for the two types of agents, ultimately determines the average probabilities of receiving the right signal from the experiment in the two states of the world, that are defined as $\alpha \equiv \rho\alpha_H + (1-\rho)\alpha_L$ and $\beta \equiv \rho\beta_H + (1-\rho)\beta_L$. We denote instead γ the market posterior belief about the manager's type. If the safe project is chosen, returns v_s are realized and the market does not learn anything about the manager's ability. If the risky project is chosen, both the manager and the market observe the realization of the experiment, s. The result of the experiment depends both on the state of the world and on the manager's type; however, conditional on these two pieces of information, it is independent of the realization of the signal ω. The final realization of the risky project is also common knowledge and it is the outcome through which the market can update his prior on the manager's ability. We denote γ^l, γ^r the market's posterior when the risky project is pursued and the state of the world is revealed to be good and bad, respectively, and γ^0 the market's posterior if the manager abandons the risky task.

Payoffs. The manager's utility is increasing in profits from the project - which we will call π - and in the market's belief about the probability he is the high type. We will refer to the latter as the career concern motive. We assume the manager is risk-neutral, and that the career concern motive enters linearly in his utility. Specifically, we call the manager's utility $U(\pi, \gamma)$. We assume the following form:

$$U(\pi, \gamma) = (1-\lambda)\,\kappa\pi + \lambda\gamma$$

[2] The net returns are then $v_r - c$ in case of success or $-c$ in case of failure.

with $\lambda \in [0,1]$. Under this specification, the parameter λ measures the extent to which the manager is motivated by career concerns, as opposed to maximizing the payoff from the project. κ, the fraction of profits that the manager receives, is an exogenous parameter.

Parameter Assumptions. For the sake of simplicity, we assume that parameters are such that it is always optimal to follow the signal, that is, to continue the risky project if and only if the experiment delivers the good signal. Specifically, for this to be true we assume the following:[3]

$$\frac{\alpha^i p(\omega)}{\alpha^i p(\omega) + (1-\beta^i)(1-p(\omega))} v_r - c > 0 \quad \forall i, \omega$$

$$\frac{(1-\alpha^i) p(\omega)}{(1-\alpha^i) p(\omega) + \beta^i (1-p(\omega))} v_r - c < 0 \quad \forall i, \omega$$

It is evident that the higher α and β are, the more informative the experiment is. However, it might be the case that in a specific environment, detecting a succesfull project is relatively more beneficial than avoiding the loss associated to running a bad one, or viceversa. This will depend on the primitives of the model. We define the high type manager as the one that ensures higher expected profits. This leads to the following restriction.

Condition 1. *It must be the case that, for any ω:*

$$p(\omega)\alpha^h v_r - \left[\left(\alpha^h + \beta^h\right) p(\omega) - \beta^h\right] c \geq p(\omega)\alpha^l v_r - \left[\left(\alpha^l + \beta^l\right) p(\omega) + -\beta^l\right] c$$

This condition tells that, for a given ω, the expected return from the risky investment, obtained through the sum of the gain when the manager receives the right signal in the good state, $p(\omega)\alpha^i(v_r - c)$, and the loss in case the manager gets the wrong signal in the bad state, $-(1-\beta^i)(1-p(\omega))c$, is larger for the high type manager. Notice that the condition holds when $\alpha^h > \alpha^l$ and $\beta^h > \beta^l$, but could also be satisfied in some cases where the high type receives a more precise signal in one state, but a less precise one in the other state.

[3] Notice that, as the manager and market do not know the manager's type, this assumption is only a sufficient condition. That is, we might assume as well that for the average manager in the economy would be optimal to follow the result of the experiment, but not for one of the two types of agents.

2.2 Efficient Benchmark and Equilibrium

In order to make welfare considerations about the equilibrium outcome, we first characterize the efficient project choice, absent any career concerns motive. We then show that, under certain parameter restrictions, every equilibrium is characterized by a cutoff strategy: the manager chooses the risky project if and only if the first period signal, ω, is higher than come cutoff, denoted ω^*.

Through the analysis we call $\sigma : [\underline{\omega}\,;\,\overline{\omega}] \to [0\,;\,1]$ the manager's mixed strategy; $\sigma(\omega)$ denotes the probability that the manager chooses the risky project conditional on observing the signal ω.

Efficient benchmark

We define ω_{FB} the signal at which that expected payoffs from the risky and the safe project are equalized. That is, ω_{FB} solves

$$\underbrace{p(\omega_{FB})\alpha(v_r - c) - (1 - p(\omega_{FB}))(1-\beta)c}_{\pi(risk\,|\,\omega_{FB})} = \underbrace{v_s}_{\pi(safe\,|\,\omega_{FB})} \tag{1}$$

It is easy to see that, due to the assumption that $p'(\omega) > 0$, the expected returns of the risky project are monotonically increasing in ω. Therefore, efficient project choice prescribes to undertake the risky project if and only if $\omega \geq \omega_{FB}$. Rearranging equation 1, we can characterize the efficient project selection rule as follows.

Remark 2. *The efficient project selection rule is described by:*

$$\sigma(\omega) = \begin{cases} 1 & \text{if} \quad \omega \geq \omega_{FB} \equiv p^{-1}\left(\frac{v_s + (1-\beta)c}{\alpha(v_r - c) + (1-\beta)c}\right) \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Equilibrium

An equilibrium in this economy is a pair specifying the manager's strategy and the principal's posterior about managerial ability, $(\sigma(\omega),\ \gamma)$. Through the text, we consider (Weak) Perfect Bayesian Equilibria. As there might be cases where beliefs are not well defined, we further impose the restriction that players' beliefs are the limiting beliefs computed using totally mixed strategies.[4]

[4] The (Weak) Perfect Bayesian Equilibrium concept would not discipline beliefs in case the risky project is never chosen in equilibrium.

Let us first define and characterize the posteriors on manager's ability that a given strategy profile, $\sigma(\omega)$, would induce. The relevant events, as explained in the previous section, are that a risky project succeeds, fails or is abandoned following the experiment.

Call γ^r, γ^0 and γ^l the posteriors on manager's type when he chooses the risky project, conditional on the project being successful, discarded, or failing, respectively. These are derived in the Appendix and their expressions are given by:

$$\gamma^r \equiv \mathbb{P}(\theta = h \,|\, x = g,\, s = g) = \\ = \rho \frac{\int_\omega \mathbb{P}(s = g \,|\, x = g,\, \omega,\, \theta = h)\, p(\omega)\, \sigma(\omega)\, dF(\omega)}{\sum_{i \in \{l,\,h\}} \mathbb{P}(\theta = i) \int_\omega \mathbb{P}(s = g \,|\, x = g,\, \omega,\, \theta = i)\, p(\omega)\, \sigma(\omega)\, dF(\omega)}$$

$$\gamma^0 \equiv \mathbb{P}(\theta = h \,|\, s = b) = \\ = \rho \frac{\int_\omega \mathbb{P}(s = b \,|\, \omega,\, \theta = h)\, \sigma(\omega)\, dF(\omega)}{\sum_{i \in \{l,\,h\}} \mathbb{P}(\theta = i) \int_\omega \mathbb{P}(s = b \,|\, \omega,\, \theta = i)\, \sigma(\omega)\, dF(\omega)}$$

$$\gamma^l \equiv \mathbb{P}(\theta = h \,|\, x = b,\, s = g) = \\ = \rho \frac{\int_\omega \mathbb{P}(s = g \,|\, x = b,\, \omega,\, \theta = h)\, (1 - p(\omega))\, \sigma(\omega)\, dF(\omega)}{\sum_{i \in \{l,\,h\}} \mathbb{P}(\theta = i) \int_\omega \mathbb{P}(s = g \,|\, x = b,\, \omega,\, \theta = i)\, (1 - p(\omega))\, \sigma(\omega)\, dF(\omega)}$$

The posteriors computed above are clearly affected by the equilibrium strategy profile, since they depend on the set of individuals that choose to undertake the risky project. The analysis simplifies once we observe that, under some conditions, these sets take a simple interval representation. Notice infact, that since the expected payoff when choosing the risky project is an increasing function of the the realization of the signal ω, as long as career concern motives are not too strong, it is optimal from the manager's perspective to choose the risky project for high values of ω. The following Proposition characterizes the equilibrium strategy.

Proposition 1. *There exists $\tilde{\lambda} \in (0\,;\,1)$ such that, $\forall \lambda < \tilde{\lambda}$, the equilibrium $\sigma(\omega)$ takes the form :*

$$\sigma(\omega) = \begin{cases} 0 & if \quad \omega < \omega^* \\ 1 & if \quad \omega \geq \omega^* \end{cases} \tag{3}$$

for some $\omega^* \in [\underline{\omega};\, \overline{\omega}]$.

In words, Proposition 1 states that when career concerns are not too strong, *every* equilibrium will exhibit cutoff strategies. In this case, being optimistic enough about the probability of facing a good state of the world ($\omega \geq \omega^*$) is a necessary and sufficient condition for undertaking the risky project ($\sigma(\omega) = 1$). We will refer to the manager receiving the signal ω^* as the *marginal* manager.

Let us now restrict attention to cases where $\lambda < \tilde{\lambda}$. Let now $\tilde{p}$ be defined as the perceived probability of facing the good state of the world, according to the market, once the market observes that a risky project has been chosen. We rewrite the beliefs following the choice of a *risky* project using the cutoff strategy that agents follow in equilibrium. Moreover, in evaluating γ^0, we use the fact that $\mathbb{P}(s = b \,|\, \omega,\, \theta = i) = \mathbb{P}(s = b \,|\, x = g,\, \omega,\, \theta = i)\mathbb{P}(x = g \,|\, \omega,\, \theta = i) + \mathbb{P}(s = b \,|\, x = b,\, \omega,\, \theta = i)\mathbb{P}(x = b \,|\, \omega,\, \theta = i)$ and the definition $\int_{\omega^*}^{\overline{\omega}} \mathbb{P}(x = g \,|\, \omega) \frac{dF(\omega)}{1-F(\omega^*)} \equiv \tilde{p}$. We can then rewrite the posterior beliefs on managerial ability as follows:

$$\gamma^r \equiv \mathbb{P}(\theta = h \,|\, x = g,\, s = g) =$$
$$= \rho \frac{\int_{\omega^*}^{\overline{\omega}} \mathbb{P}(s = g \,|\, x = g,\, \omega,\, \theta = h)\, p(\omega)\; dF(\omega)}{\sum_{i \in \{l,\, h\}} \mathbb{P}(\theta = i)\; \int_{\omega^*}^{\overline{\omega}} \mathbb{P}(s = g \,|\, x = g,\, \omega,\, \theta = i)\, p(\omega)\; dF(\omega)} = \rho \frac{\alpha_h}{\alpha}$$

$$\gamma^0 \equiv \mathbb{P}(\theta = h \,|\, s = b) =$$
$$= \rho \frac{\int_{\omega^*}^{\overline{\omega}} \mathbb{P}(s = b \,|\, \omega,\, \theta = h)\; dF(\omega)}{\sum_{i \in \{l,\, h\}} \mathbb{P}(\theta = i)\; \int_{\omega^*}^{\overline{\omega}} \mathbb{P}(s = b \,|\, \omega,\, \theta = i)\; dF(\omega)} = \rho \frac{(1-\alpha_h)\tilde{p} + \beta_h(1-\tilde{p})}{(1-\alpha)\tilde{p} + \beta(1-\tilde{p})}$$

$$\gamma^l \equiv \mathbb{P}(\theta = h \,|\, x = b,\, s = g) =$$
$$= \rho \frac{\int_{\omega^*}^{\overline{\omega}} \mathbb{P}(s = g \,|\, x = b,\, \omega,\, \theta = h)\, (1 - p(\omega))\; dF(\omega)}{\sum_{i \in \{l,\, h\}} \mathbb{P}(\theta = i)\; \int_{\omega^*}^{\overline{\omega}} \mathbb{P}(s = g \,|\, x = b,\, \omega,\, \theta = i)\, (1 - p(\omega))\; dF(\omega)} = \rho \frac{1-\beta_h}{1-\beta}$$

In general, while every equilibrium features a cutoff strategy, the cutoff is not necessarily unique. This happens because, while the expected payoff from choosing the risky project is increasing with ω, the expected reputation might be a decreasing function of it. In some situations, summarized in the following result, however, we can find sufficient conditions that guarantee a unique equilibrium cutoff.

Corollary 1. *If* $\frac{1-\alpha_h}{\beta_h} > \frac{1-\alpha_l}{\beta_l}$ *and* $\forall \lambda < \tilde{\lambda}$, *the equilibrium cutoff is unique.*

2.3 Efficiency

In this section we show that our economy is intrinsically plagued by inefficient managerial investment decisions. To estabilish this, first recall the result in expression 2, stating that the first best project selection rule requires to undertake the risky project if only if $\omega \geq \omega_{FB}$. Therefore, since in the previous subsection we proved that equilibria are characterized by cutoffs, making welfare considerations about the level of risk taking in the economy boils down to comparing the initial signal of the marginal manager in equilibrium, ω^*, to the optimal ω_{FB}. The marginal manager takes on too much or not enough risk depending on the possibility of exploiting a reputational gain or avoiding a reputational loss, by undertaking the risky task. What matters is the wedge between what he and the market will think about his ability in running experiments. If his expected self-assessment is higher than the expectations of the market, the manager will be more cautious and choose the safe project. If, instead, he is less optimistic than the market about the probability of being perceived as high type, he chooses the risky task, even if efficiency requires the safe one. The second possibility arises when discarding risky projects is perceived by the market as a good signal about the managerial ability.

We start with a technical result, that helps to understand how different equilibria induce different market posteriors upon abandoning a risky project. It establishes conditions so that reputation following abandonment is higher, the higher is the market belief on the good state of the world.

Lemma 1. $\gamma^0(\omega^*)$ *is increasing (decreasing) in* ω^* *if and only if* $\frac{1-\alpha^H}{\beta^H} > (<) \frac{1-\alpha}{\beta}$

This result holds because while the threshold ω^* at which the VC is indifferent between the safe and risky project increases, the market becomes more and more optimistic about the state of the world. In this circumstance, a suspension of a risky task is increasingly associated to an error in the good state - which happens, on average, with probability $1-\alpha$ - rather than to a correct forecast when the state of the world is bad - which happens with probability β. The market is thus willing to believe that the manager is a high type when incorrectly discarding is a more salient behaviour of high types rather than low types managers, relative to how often the two correctly abandon the risky project.

As there is a one-to-one correspondence between ω and $p(\omega)$, in the previous Lemma we could have used the probability $p(\omega^*)$ rather than the initial signal on the state of the world, ω^*. Since the marginal manager (the one whose $\omega = \omega^*$) is less optimistic than the market about the state of the world (as $p(\omega^*) < \tilde{p}$) the previous Lemma also suggests that the self-assessment of the marginal manager, upon discarding the risky task, $\rho\frac{(1-\alpha^H)p(\omega^*)+\beta^H(1-p(\omega^*))}{(1-\alpha)p(\omega^*)+\beta(1-p(\omega^*))}$, is lower than the market posterior, $\rho\frac{(1-\alpha^H)\tilde{p}+\beta^H(1-\tilde{p})}{(1-\alpha)\tilde{p}+\beta(1-\tilde{p})}$, if and only if $\frac{1-\alpha^H}{\beta^H} > \frac{1-\alpha}{\beta}$. Viceversa the manager and the market would share the same posteriors in the events of success and failure of the risky project, as these beliefs are independent on the initial signal on the state of the world. The next result, thus, follows:

Lemma 2. *The marginal manager's expectation of his reputation induced by risk taking is higher than the prior if and only if* $\frac{1-\alpha^H}{\beta^H} > \frac{1-\alpha}{\beta}$.

The intuition for this result goes as follows. By the law of iterated expectations, conditional on his information, a manager's expected self-assesment is equal to the prior. However, in computing what his reputation will be, the manager takes into account that the posterior he and the market will form might not coincide when some additional asymmetry of information is still present. But when is this so? In those cases where the uncertainty about the state of the world is resolved by the manager's action, that is, when the manager gets a good signal in the experiment and pursues the risky project till the end, the manager and the market share the posterior beliefs on the manager's type. After all, conditional on the state of the world, the initial signal ω provides no extra help in figuring out the manager's type. This is because in the first stage all managers are equally good and receive the initial signal from the same distribution. In case the project succeeds the posterior is $\rho\frac{\alpha^H}{\alpha}$, while it is $\rho\frac{(1-\beta^H)}{(1-\beta)}$ if the project fails, independently on ω. Instead, when the manager discards the risky task after observing the outcome of the experiment, agents do not know if the project has been interrupted correctly or not. In this scenario, the manager and the market use their different beliefs on the state of the world, $p(\omega^*)$ and $\tilde{p}$ respectively, to draw a conclusion about the manager's type. The direction of disagreement in this event is disciplined by the condition provided in Lemma 1. When $\frac{1-\alpha^H}{\beta^H} > \frac{1-\alpha}{\beta}$ holds, γ^0 grows with $p(\omega)$, because getting the bad signal by mistake is more typical of an high type.

From this, it follows that the expected reputation induced by taking risk is higher, giving the marginal agent a strict additional gain from taking risk.

This also means that, in order for him to be indifferent, the project must be worse than the one equalizing monetary payoffs.

With the last Lemma at hand, we are now ready to state our main result providing conditions on the direction of the distortions associated to career concerns.

Proposition 2. *There is over(under) risk-taking in the economy if and only if* $\frac{1-\alpha^H}{\beta^H} > \frac{1-\alpha}{\beta}$ $\left(\frac{1-\alpha^H}{\beta^H} < \frac{1-\alpha}{\beta}\right)$.

To prove this result, we argue that the marginal individual chooses the risky project when the first best would require the safe one. Then we use the fact that any manager receiving the signal ω, where $\omega > \omega^*$, is also taking the risky project. By definition of *first best*, an manager should choose the risky project if and only if

$\omega > \omega^{FB}$. At the cutoff ω^{FB}, the expected profit from choosing the *safe* project is identical to the expected profit when choosing the *risky* one, that is

$$(1-\lambda)\kappa v_s = (1-\lambda)\kappa(p(\omega^{FB})\alpha(v_r - c) - (1 - p(\omega^{FB}))(1-\beta)c)$$

By definition, the *marginal* manager is indifferent between the *safe* and the *risky* projects when also the expected reputation is taken into account, that is:

$$(1-\lambda)\kappa v_s + \lambda\rho = (1-\lambda)\kappa(p(\omega^*)\alpha(v_r - c) - (1 - p(\omega^*))(1-\beta)c) + \lambda E(\gamma \mid \omega^*)$$

We put the two conditions together to get:

$$(1-\lambda)\kappa \overbrace{(p(\omega^{FB})\alpha(v_r - c) - (1 - p(\omega^{FB}))(1-\beta)c)}^{\pi(risk \mid \omega^{FB})} + \lambda\rho = (1-\lambda)\kappa v_s + \lambda\rho$$

and the right hand side is in turn equal to:

$$(1-\lambda)\kappa \underbrace{(p(\omega^*)\alpha(v_r - c) - (1 - p(\omega^*))(1-\beta)c)}_{\pi(risk \mid \omega^*)} + \lambda E(\gamma \mid \omega^*)$$

As $E(\gamma \mid \omega^*) > \rho$, it must be the case that $\pi(risk \mid \omega^{FB}) > \pi(risk \mid \omega^*)$. $\pi(\omega,\ risk)$ is an increasing function of ω because $p(\omega)$ is increasing in ω. This is equivalent to $\omega^{FB} > \omega^*$.

Our inefficiency result derives from the wedge between what the manager and the market think about the managerial capability in running experiments, that arises when the risky project is abandoned. Career concerns, in turn, have a bite in the managerial decision problem, because of the expected reputational gains or losses from choosing the risky task. Technically, this gains or losses emerge because the players cannot condition on the state of the world, $x = \{g, b\}$, in evaluating $\gamma^0 \equiv \mathbb{P}(\theta = h \,|\, s = b)$. In particular, the market conditions this inference on the equilibrium strategy profile, whereas the VC bases it on his observed signal ω. If the counterfactual state of the world in case of a bad draw in the experiment was observed, that is, if players observed what would have happened if the manager continued with the risky task, the expected reputation would coincide with the expected self-assessment, equal to the prior.[5] Similarly, if the VC did not have any private information on ω, no disagreement on γ^0 would result. In both these cases, no inefficiency would emerge.

[5] To see this, we just need to compute the posteriors in case players know that the manager was wrong in discarding the risky project, $\rho\frac{1-\alpha^H}{1-\alpha}$, and when the manager was right in doing that, $\rho\frac{\beta^H}{\beta}$. As none of the posterior would now depend on $p(\omega)$ and $\tilde{p}$, the expected reputation and the expected self-assessment would coincide and be equal to $p(\omega)\alpha\rho\frac{\alpha^H}{\alpha} + (1-p(\omega)(1-\beta)\rho\frac{(1-\beta^H)}{(1-\beta)} + p(\omega)(1-\alpha)\rho\frac{1-\alpha^H}{1-\alpha} + (1-p(\omega))\beta\rho\frac{\beta^H}{\beta} = \rho, \ \ \forall\omega$. This would also be true, in particular, for the marginal manager.

Notice that if the *good* manager is better in running experiments in both states of the world, that is $\alpha_h > \alpha_l$ and $\beta_h > \beta_l$, then $\frac{1-\alpha^H}{\beta^H} > \frac{1-\alpha}{\beta}$ never holds. In this scenario, as the following result states, the agents do not take enough risk.

Corollary 2. *If $\alpha_h > \alpha_l$ and $\beta_h > \beta_l$ there is underinvestment in the risky activity.*

2.4 How Robust is It to Signalling at the Experimentation Stage?

In the analysis so far, we have assumed that the outcome of the experiment is public information. This makes it compelling that a manager would follow the informative experiment and continue with the risky project if and only if the signal turns out to be good. However, one might argue that the action at the experimentation stage - whether or not to abandon the project - could be itself a signalling device, in case the experiment outcome is the manager's private information. In this paragraph we argue that, as long as the career concerns motive is not too strong, our main results are robust to adding this additional channel. That is, the unique equilibrium would be a separating equilibrium where managers follow the signal.

To see this, let us analyse the subgame where the manager has chosen the risky project, and he has (privately) observed the signal s. If we can show that the equilibrium in this signalling game is one where managers follow the signal, conditional on any ω, then we can conclude that our main results on risk taking behavior still hold. The reason is that in such equilibrium managers would play exactly as they are constrained to do by assumption in our original model.

Let us start focusing on the two possible pooling equilibria. First, consider the case where all managers - independently on s - abandon a project after the experiment. A manager that received $s = g$, would not deviate if and only if:

$$(1-\lambda) * 0 + \lambda\rho \geq (1-\lambda)\left[\frac{\alpha p(\omega)}{\alpha p(\omega) + (1-\beta)(1-p(\omega))} v_r - c\right] + \lambda\tilde{\gamma}$$

for some induced posterior - $\tilde{\gamma}$ - that is computed by specifying arbitrary off-equilibrium beliefs on s. By the parameters restrictions as in section 2.1, it is easy to observe that the condition can not be satisfied as long as λ is small enough, for any $\tilde{\gamma} \in [0, 1]$.

Similarly, consider the case where all managers - independently on s - continue with the project after the experiment. In this case a manager with $s = b$ would not deviate if and only if:

$$(1-\lambda)\left[\frac{(1-\alpha)\,p\,(\omega)}{(1-\alpha)\,p\,(\omega)+\beta\,(1-p\,(\omega))}v_r-c\right]+\lambda\rho\geq(1-\lambda)*0+\lambda\tilde{\gamma}$$

for some $\tilde{\gamma}\in[0,1]$. Again, this is impossible due to the same restrictions, when λ is small enough.

Finally, consider instead a candidate separating equilibrium where the manager continues with the risky project if and only if $s=g$. Here - due to our restrictions - a manager that is solely interested in the material payoffs would want to continue when $s=g$ and abandon when $s=b$. Therefore, since the expected reputation is bounded above by one and below by zero, such equilibrium must exist for some λ small enough.

3 An Extension: N Agents

In this section, we generalize the model by increasing the number of managers to $N>1$. The structure of the economy is the same of the one analyzed above, although we need some extra assumptions about the timing of the managerial investments, the initial signals ω and the experiments in case of risky project. As for the timing of the economy, we assume that each player observe the final outcome of each investment simultaneously.[6] We also assume that the managers share the same signal ω. With this assumption it follows that either all managers choose the safe project, or they all choose the risky one. As for the experiments, we assume that the realizations are *independent* across managers, conditionally on the state of the world and their types.

It is easy to observe that, also in the extended model, the efficient rule is the same of the simple model with one manager and also the cutoff strategy in equilibrium, when $\lambda<\tilde{\lambda}$, holds as before. Once again, we focus on this case.

Note that, with N managers, the following three facts become relevant. First, the market and a manager discarding the risky project after the experiment get to know that the state of the world is bad if at least an other manager pursues the investment and this fails. Similarly, they realize that the state of the world is good after suspending the project if there is at least a manager that continues and succeeds. In these two cases their assessment about the managerial ability coincide, as they now condition on the state of the world, rather than on their (different) probabilities of being in either state, pinned down by $p(\omega^*)$ and $\tilde{p}$. Third, even when all managers abandon the risky project after the experiment, and hence there is still uncertainty about the true state of the world, the posterior that market forms is a function of the number of managers in the economy. Indeed,

[6]The fact that other managers observe the various outcomes is irrelevant. What matters is that the market observe the realizations of the various projects.

the odds that *all* managers are right or wrong in receiving a bad signal from the experiment differ to the chance that only *one* of them receives the bad signal in either state of the world.

As the manager's type is independent on the signal ω, conditionally on the realization of the experiment and the state of the world, if a manager pursues the risky investment and succeeds the posterior of the market coincides with γ^r; if he fails the posterior equals γ^l. Through this section we rename the posteriors that coincide with the previous analysis as γ_N^r and γ_N^l, respectively. There are other three sets of events inducing different posteriors to the market on a manager's ability. One corresponds to the situation in which the manager abandoned the risky project, but at least another manager pursued it and failed. The second one happens when the manager abandoned the risky project, but at least another manager pursued it and succeeded. Finally, the third refers to the case in which all managers abandoned the risky project. We denote γ_N^{nl}, γ_N^{nr} and γ_N^0 the market posteriors associated to each of these three sets of events. The market's posteriors about the quality of the N^{th} manager, taking as given the performance of the other $N-1$ managers are, therefore, given by:

$$\gamma_N^{nl} \equiv \mathbb{P}(\theta_N = h \,|\, x = b,\, s_N = b) = \rho\,\frac{\beta_h}{\beta}$$

$$\gamma_N^{nr} \equiv \mathbb{P}(\theta_N = h \,|\, x = g,\, s_N = b) = \rho\,\frac{1-\alpha_h}{1-\alpha}$$

$$\gamma_N^{0} \equiv \mathbb{P}(\theta_N = h \,|\, s_N = b,\, s_{1,\dots,N-1} = b) = \rho\,\frac{(1-\alpha_h)(1-\alpha)^{N-1}\tilde{p} + \beta_h \beta^{N-1}(1-\tilde{p})}{(1-\alpha)^N \tilde{p} + \beta^N(1-\tilde{p})}$$

$$\gamma_N^{r} \equiv \mathbb{P}(\theta_N = h \,|\, x = g,\, s_N = g) = \rho\frac{\alpha_h}{\alpha}$$

$$\gamma_N^{l} \equiv \mathbb{P}(\theta_N = h \,|\, x = b,\, s_N = g) = \rho\frac{1-\beta_h}{1-\beta}$$

We first establish a result that mirrors the one in Proposition 2.

Proposition 3. *For any finite number of managers, N, the marginal manager takes too much risk if and only if* $\frac{1-\alpha^H}{\beta^H} > \frac{1-\alpha}{\beta}$.

Also in the new economy, there are circumstances in which the market cannot condition the analysis on whether the project was to deliver returns or not, upon observing that the manager got a bad signal from the experiment. Once again, the posterior on manager's ability in this event is the only one where two observers with different opinions on the state of the world would disagree on. As the market is more optimistic on the state of the world compared to the marginal manager, the latter enjoys a reputational benefit or suffers a cost when choosing the risky project, depending on whether the abandonment of it is perceived as a good signal for the market about his quality.

We are interested in comparing economies that differ in N - the number of managers running projects that are linked to the same state of the world. In particular, we want to assess which economies are more plagued by the inefficiency that inevitably results from the pressure of career concerns. To do this, one first observation to be made is that, provided the direction of the inefficiency is the same, it is possible to measure how "strong" is the inefficiency by only looking at how distant is the cutoff associated to the equilibrium marginal manager from the first best value - ω^{FB}. This is stated formally in the next Lemma and further explained in the following lines.

Lemma 3. *Take two economies - denoted 1 and 2 - and associated equilibrium cutoffs ω_1^* and ω_2^*. (i) If, $\frac{1-\alpha^H}{\beta^H} > \frac{1-\alpha}{\beta}$ the expected monetary payoff from economy 1 is higher than in economy 2 whenever $\omega_1^* > \omega_2^*$. (ii) If, $\frac{1-\alpha^H}{\beta^H} < \frac{1-\alpha}{\beta}$ the expected monetary payoff from economy 1 is higher than in economy 2 whenever $\omega_1^* < \omega_2^*$.*

To prove this result, consider the case when $\frac{1-\alpha^H}{\beta^H} > \frac{1-\alpha}{\beta}$. We know in this case equilibria will exhibit excessive risk taking, therefore the cutoffs ω_1^* and ω_2^* would be both lower than ω^{FB}. Assume now $\omega_1^* > \omega_2^*$. We can compare the equilibrium monetary payoffs for each realization of the initial signal ω . To do this, we identify three regions. When $\omega < \omega_2^*$, managers in both economies follow the efficient decision, that is, select the safe project. When $\omega > \omega_1^*$, managers in the two economies take the risky project. When $\omega_2^* \leq \omega \leq \omega_1^*$, managers in economy 2 are selecting the risky project, whereas those in economy 1 choose the safe alternative. Since the first best solution prescribes to select the safe project in these cases, it follows that returns are lower in economy 2 for any realization of ω within this region. Therefore, when taking expectations over all possible values of ω, the monetary payoff is higher in economy 1. The same logic applies to the case when $\frac{1-\alpha^H}{\beta^H} < \frac{1-\alpha}{\beta}$.

The result is useful because it provides us with a simple way to compare different economies in terms of how inefficient is project selection in equilibrium: it is sufficient to establish in which economy the marginal managers departs less from the indifferent one in the first best - absent the career concerns motive.

In the following, main result of this section, we show that the inefficiency decreases as the number of managers increases. Economies where N is larger induce equilibria in which expected returns are higher.

Proposition 4. *The inefficiency is monotonically decreasing in the number of managers, N.*

In the proof of this Proposition, we show that $E(\gamma_{N+1} \mid \omega^*) < E(\gamma_N \mid \omega^*)$ if $\frac{1-\alpha_h}{\beta_h} > \frac{1-\alpha_l}{\beta_l}$, while $E(\gamma_{N+1} \mid \omega^*) > E(\gamma_N \mid \omega^*)$ whenever $\frac{1-\alpha_h}{\beta_h} < \frac{1-\alpha_l}{\beta_l}$. That is, the marginal manager in the economy with $N+1$ managers finds less appealing to invest in the risky activity from a career perspective, compared to the same individual in the economy with N managers, exactly when there is a reputational gain by choosing it. Instead, the risky task is now becoming more appealing when it is associated to a reputational disadvantage. This clearly implies that the bite of career concerns is more loose and that the marginal individual is characterized by a signal ω^* closer to the first best cutoff, ω^{FB}.
The following Proposition states that in the limit the inefficiency disappears.

Proposition 5. *As the number of managers, N, goes to infinity, the inefficiency disappears. That is, ω^* approaches ω^{FB}.*

The proof of this result is very simple. Consider the expected reputation of the marginal manager, that we now denote $E(\gamma_N \mid \omega^*)$:

$$\begin{aligned} E(\gamma_N \mid \omega^*) \;\equiv\; & p(\omega^*)\alpha\rho\frac{\alpha^H}{\alpha} + ((1-p(\omega^*)(1-\beta))\rho\frac{(1-\beta^H)}{(1-\beta)} + \\ & + p(\omega^*)(1-\alpha-(1-\alpha)^N)\rho\frac{1-\alpha_h}{1-\alpha} + (1-p(\omega^*))(1-(1-\beta)-\beta^N)\rho\frac{\beta_h}{\beta} + \\ & + (p(\omega^*)(1-\alpha)^N + (1-p(\omega^*))\beta^N)\rho\,\frac{(1-\alpha_h)(1-\alpha)^{N-1}\tilde{p} + \beta_h\beta^{N-1}(1-\tilde{p})}{(1-\alpha)^N\tilde{p} + \beta^N(1-\tilde{p})} \end{aligned}$$

As $1-\alpha$ and β are numbers in the interval [0, 1], $(1-\alpha-(1-\alpha)^N)$ and $(1-(1-\beta)-\beta^N)$ tend to $(1-\alpha)$ and β, respectively. Furthermore, since the posterior belief associated to the event in which all managers suspend the investment in the risky project, $\frac{(1-\alpha_h)(1-\alpha)^{N-1}\tilde{p}+\beta_h\beta^{N-1}(1-\tilde{p})}{(1-\alpha)^N\tilde{p}+\beta^N(1-\tilde{p})}$, is also in the interval [0, 1], and the weight on this posterior approaches 0, we have that the following result:

$$\begin{aligned} \lim_{N\to\infty} E(\gamma_N \mid \omega^*) \;=\; & p(\omega^*)\alpha\rho\frac{\alpha^H}{\alpha} + ((1-p(\omega^*)(1-\beta))\rho\frac{(1-\beta^H)}{(1-\beta)} + \\ & + p(\omega^*)(1-\alpha)\rho\frac{1-\alpha_h}{1-\alpha} + (1-p(\omega^*))\beta\rho\frac{\beta_h}{\beta} \;=\; \rho \end{aligned}$$

This implies that the marginal manager has no reputational gain in expectations by choosing the risky task. Therefore ω^* and ω^{FB} must coincide and the inefficiency disappears.

The intuition behind this Proposition is straightforward. When the number of managers tends to infinity, the chance for the market not to observe the counterfactual state of the world, that occurs when all managers abandon the risky project, approaches zero. This is so because, when the state of the world is good, it is very unlikely that every manager has, incorrectly, a bad draw in the experiment. In contrast, when the state of the world is bad, it is very likely that at least one agent receives by mistake the signal to pursue the investment. In the limit, therefore, the wedge between the managers' self assessments and their expected reputations vanishes.

4 Conclusion

In this Chapter we proposed a setting where information about a manager's ability is imperfect and managers are interested in their reputation. Motivated by the application to investments in young firms, we modeled managers as agents that create value because they can experiment and learn about a projects potential. As it is greatly emphasized by, among others, Kerr et al. (2014), the ability to learn about a project's profitability at relatively early stages is a skill that venture capitalists must have in order to succeed in the industry. Infact, experimentation is desirable to the extent that it provides the incentive to finance innovative and young firms. However, it is reasonable to think that some venture capitalists are better at extracting information from early experiments than others. If this skill is so important, then naturally venture capitalists would benefit from making investors and entrepreneurs believe that they are good in this dimension. It would increase their bargaining power, and help them find better deals at the fundraising stage. In light of this observation, we studied a manager's incentive to take on risk when career concerns are at play, that is, outside observers are learning about his ability to experiment. Contrary to Holmström (1999), where managers add value because they directly increase a project's success rate and in equilibrium they become too risk-averse, managers in this model might take inefficiently high risk. The reason is that the abandonment of a promising project at an intermediate stage might be good news about the agent's ability. In particular, this is the case when a good manager is typically one whose experimenting technology is biased towards receiving negative outcomes. In this situation a manager would tend to opt excessively for risky, experimental business strategies, in anticipation of the reputational gain that comes from cutting off the investment at an intermediate stage. This result provides a somewhat darker point of view on the amount of experimentation and risk observed in the venture capital industry. We studied one solution to this problem in one extension of the model, where we show that when the observer gets information about the outcome of similar projects, the inefficiency is reduced. The reason is that what drives the inefficiency is the fact that the observer can't distinguish whether a project was abandoned because doomed to fail or due to a false negative in the experiment. The information from similar projects provides the observer with an imperfect signal about the counterfactual. It is usually argued that when agents interact less frequently, career concerns are more severe. Translated into the example of the venture capital industry, one would expect younger VC firms' decision to be particularly distorted. The novel empirical implication of our model is that the markets more plagued by career concerns distortions are those where agents engage in more unique and less correlated activities.

Appendix

Beliefs

$$\gamma^r \equiv \mathbb{P}(\theta = h \,|\, x = g,\, s = g) = \frac{\mathbb{P}(\theta = h,\, x = g,\, s = g)}{\mathbb{P}(x = g,\, s = g)} =$$

$$= \frac{\int \overbrace{\mathbb{P}(\theta = h \,|\, \omega)}^{=\mathbb{P}(\theta=h)=\rho} \mathbb{P}(x = g,\, s = g \,|\, \omega,\, \theta = h)\, \sigma(\omega)\, dF(\omega)}{\sum_{i\in\{l,h\}} \int \underbrace{\mathbb{P}(\theta = i \,|\, \omega)}_{=\mathbb{P}(\theta=i)} \mathbb{P}(x = g,\, s = g \,|\, \omega,\, \theta = i)\, \sigma(\omega)\, dF(\omega)} =$$

$$= \rho \frac{{}_{\Omega}\, \mathbb{P}(s = g \,|\, x = g,\, \omega,\, \theta = h)\, \overbrace{\mathbb{P}(x = g \,|\, \omega,\, \theta = h)}^{=\mathbb{P}(x=g\,|\,\omega)=p(\omega)}\, \sigma(\omega)\, dF(\omega)}{\sum_{i\in\{l,h\}} \mathbb{P}(\theta = i)\, \int \mathbb{P}(s = g \,|\, x = g,\, \omega,\, \theta = i)\, \underbrace{\mathbb{P}(x = g \,|\, \omega,\, \theta = i)}_{=\mathbb{P}(x=g\,|\,\omega)=p(\omega)}\, \sigma(\omega)\, dF(\omega)}$$

$$\gamma^0 \equiv \mathbb{P}(\theta = h \,|\, s = b) = \frac{\mathbb{P}(\theta = h,\, s = b)}{\mathbb{P}(s = b)}$$

$$= \frac{\int \overbrace{\mathbb{P}(\theta = h \,|\, \omega)}^{=\mathbb{P}(\theta=h)=\rho} \mathbb{P}(s = b \,|\, \omega,\, \theta = h)\, \sigma(\omega)\, dF(\omega)}{\sum_{i\in\{l,h\}} \int \underbrace{\mathbb{P}(\theta = i \,|\, \omega)}_{=\mathbb{P}(\theta=i)} \mathbb{P}(s = b \,|\, \omega,\, \theta = i)\, \sigma(\omega)\, dF(\omega)} =$$

$$= \rho \frac{\int \mathbb{P}(s = b \,|\, \omega,\, \theta = h)\, \sigma(\omega)\, dF(\omega)}{\sum_{i\in\{l,h\}} \mathbb{P}(\theta = i)\, \int \mathbb{P}(s = b \,|\, \omega,\, \theta = i)\, \sigma(\omega)\, dF(\omega)}$$

$$\gamma^l \equiv \mathbb{P}(\theta = h \,|\, x = b,\, s = g) = \frac{\mathbb{P}(\theta = h,\, x = b,\, s = g)}{\mathbb{P}(x = b,\, s = g)} =$$

$$= \frac{\int \overbrace{\mathbb{P}(\theta = h \,|\, \omega)}^{=\mathbb{P}(\theta=h)=\rho} \, \mathbb{P}(x = b,\, s = g \,|\, \omega,\, \theta = h)\, \sigma(\omega)\, dF(\omega)}{\sum_{i \in \{l,\,h\}} \int \underbrace{\mathbb{P}(\theta = i \,|\, \omega)}_{=\mathbb{P}(\theta=i)} \mathbb{P}(x = b,\, s = g \,|\, \omega,\, \theta = i)\, \sigma(\omega)\, dF(\omega)} =$$

$$= \rho\, \frac{{}_{\Omega}\, \mathbb{P}(x = b,\, s = g \,|\, \omega,\, \theta = h)\, \sigma(\omega)\, dF(\omega)}{\sum_{i \in \{l,\,h\}} \mathbb{P}(\theta = i) \int \mathbb{P}(x = b,\, s = g \,|\, \omega,\, \theta = i)\, \sigma(\omega)\, dF(\omega)} =$$

$$= \rho\, \frac{\int \mathbb{P}(s = g \,|\, x = b,\, \omega,\, \theta = h)\; \overbrace{\mathbb{P}(x = b \,|\, \omega,\, \theta = h)}^{\mathbb{P}(x=b \,|\, \omega)=1-p(\omega)}\; \sigma(\omega)\, dF(\omega)}{\sum_{i \in \{l,\,h\}} \mathbb{P}(\theta = i)\, \int \mathbb{P}(s = g \,|\, x = b,\, \omega,\, \theta = i)\; \underbrace{\mathbb{P}(x = b \,|\, \omega,\, \theta = i)}_{\mathbb{P}(x=b \,|\, \omega)=1-p(\omega)}\; \sigma(\omega)\, dF(\omega)}$$

Proof of Proposition 1. The expected utility of the agent with signal ω when choosing the *risky* project is

$$(1-\lambda)\kappa \underbrace{\left(p(\omega)\alpha(v_r - c) - (1 - p(\omega))(1-\beta)c\right)}_{\pi(risk \,\mid\, \omega)} + \lambda \mathbb{E}(\gamma \mid \omega)$$

We define ω^* as the signal that equalizes the managerial expected utilities when choosing the *safe* or the *risky* project

$$(1-\lambda)\kappa v_s + \lambda\rho = (1-\lambda)\kappa \underbrace{\left(p(\omega^*)\alpha(v_r - c) - (1 - p(\omega^*))(1-\beta)c\right)}_{\pi(risk \,\mid\, \omega^*)} + \lambda \mathbb{E}(\gamma \mid \omega^*)$$

It is sufficient to prove that the expected utility when choosing the *risky* project is an increasing function of ω. Notice that as $p(\omega)$ is increasing, the expected payoff $\pi(risk \mid \omega)$ is also growing with ω. For the whole utility to be increasing in ω, either $\mathbb{E}(\gamma \mid \omega)$ must be an increasing function of ω or the positive effect on $\pi(risk \mid \omega)$ must be larger than the negative one on $\mathbb{E}(\gamma \mid \omega)$. We study separately the sufficient conditions for these two cases.

Case 1: We can rewrite the expected reputation of a generic agent whose signal is ω, when choosing the risky project, as

$$\mathbb{E}(\gamma \mid \omega) \equiv p(\omega)\rho\alpha_H + (1 - p(\omega))\rho(1 - \beta_H) +$$

$$+ (p(\omega)(1-\alpha) + (1 - p(\omega))\beta) \underbrace{\frac{\int_{\sigma(l)=1} \rho((1-\alpha_h)p(l) + \beta_h(1 - p(l)))dF(l)}{\int_{\sigma(l)=1} (1-\alpha)p(l) + \beta(1 - p(l))dF(l)}}_{\equiv \gamma^0(risky)}$$

We can thus rewrite the expected reputation as the sum of two components: one independent on ω, the other one dependent on it

$$\overbrace{\rho(1-\beta_H)+\beta\gamma^0(risky)}^{\text{independent of }\omega}+p(\omega)(\rho\underbrace{(\alpha_H+\beta_H-1)}_{>0}+\gamma^0(risky)\underbrace{(1-\alpha-\beta)}_{<0})$$

As $p(\omega)$, is increasing, the expected reputation is increasing in ω if and only if

$$\rho(\alpha_H+\beta_H-1)+\gamma^0(w^*)(1-\alpha-\beta)>0$$

Using the definition of $\gamma^0(w^*)$, we can express this condition as:

$$\rho(\alpha_H+\beta_H-1)+\frac{\int_{\sigma(l)=1}\rho((1-\alpha_h)p(l)+\beta_h(1-p(l))dF(l)}{\int_{\sigma(l)=1}(1-\alpha)p(l)+\beta(1-p(l))dF(l)}(1-\alpha-\beta)>0$$

This is equivalent to

$$\underbrace{\frac{\rho}{\int_{\sigma(l)=1}(1-\alpha)p(l)+\beta(1-p(l))dF(l)}}_{>0}\times$$

$$\times\left(\int_{\sigma(l)=1}(\alpha_H+\beta_H-1)((1-\alpha)p(l)+\beta(1-p(l)))dF(l)+\right.$$

$$\left.+\int_{\sigma(l)=1}(1-\alpha-\beta)((1-\alpha_H)p(l)+\beta_H(1-p(l)))dF(l)\right)>0$$

This is true if and only if

$$\frac{1-\alpha_H}{\beta_H}<\frac{1-\alpha}{\beta}.$$

Case 2: When the expected reputation is decreasing in ω, we can nonetheless look for conditions that guarantee that the positive derivative of the expected payoff with respect to ω, when choosing the *risky* project, dominates.
In a similar way to what we did before, we express the portion of the managerial utility depending on his expected reputation as

$$\lambda\Big(\overbrace{(1-\beta_H)\rho+\beta\gamma^0(\omega^*)}^{\text{independent of }\omega}+p(\omega)(\rho\overbrace{(\alpha_H+\beta_H-1)}^{>0}+\gamma^0(\omega^*)\overbrace{(1-\alpha-\beta)}^{<0})\Big)$$

When $\rho(\alpha_H+\beta_H-1)+\gamma^0(w^*)(1-\alpha-\beta)<0$, the derivative of this expression with respect to ω cannot be lower than $\lambda p'(\omega)(\rho(\alpha_H+\beta_H-1)+(1-\alpha-\beta))$, as $\gamma^0(w^*)\in[\,0,1]$.
The derivative of the part of the utility function related to the return on the risky

project is, instead,

$$(1-\lambda)\kappa(p'(\omega)\alpha(v_r-c)+p'(\omega)(1-\beta)c)>0.$$

For the whole managerial utility to be an increasing function of ω it is then sufficient that

$$(1-\lambda)\kappa(p'(\omega)\alpha(v_r-c)+p'(\omega)(1-\beta)c>-\lambda p'(\omega)\quad\underbrace{(\rho(\alpha_H+\beta_H-1)+(1-\alpha-\beta))}_{\equiv\rho\alpha_H+\rho\beta_H-\rho+1-\rho\alpha_H-(1-\rho)\alpha_L-\rho\beta_H-(1-\rho)\beta_L}$$

that is, whenever:

$$\frac{\lambda}{1-\lambda}<\frac{\kappa(\alpha(v_r-c)+(1-\beta)c)}{(1-\rho)(\alpha_L+\beta_L-1)}.$$

To sum up, whenever $\frac{1-\alpha_H}{\beta_H}<\frac{1-\alpha}{\beta}$ the manager is better off choosing the *risky* project if and only if his signal ω is bigger than the equilibrium ω^*. When $\frac{1-\alpha_H}{\beta_H}>\frac{1-\alpha}{\beta}$, this is also true if career concerns are not too strong, that is λ is small enough. ■

Proof of Lemma 1. Consider two cutoff equilibria characterized by thresholds w_1^* and w_2^*, with $w_1^*>w_2^*$. Let us consider the conditions that guarantee that $\gamma^0(w_1^*)>\gamma^0(w_2^*)$. These are the beliefs in case a *risky* project is abandoned, under the two equilibria. We study in which circumstances the following holds:

$$\gamma^0(w_1^*)\equiv\frac{\int_{w_1^*}^{\overline{w}}\rho((1-\alpha_h)p(w)+\beta_h(1-p(w)))dF(w)}{\int_{w_1^*}^{\overline{w}}((1-\alpha)p(w)+\beta(1-p(w)))dF(w)}$$

$$>$$

$$\frac{\int_{w_2^*}^{\overline{w}}\rho((1-\alpha_h)p(w)+\beta_h(1-p(w)))dF(w)}{\int_{w_2^*}^{\overline{w}}((1-\alpha)p(w)+\beta(1-p(w)))dF(w)}\equiv\gamma^0(w_2^*)$$

As the two denominators are non negative, this is equivalent to:

$$(\int_{w_2^*}^{\overline{w}}((1-\alpha)p(w)+\beta(1-p(w)))dF(w))(\int_{w_1^*}^{\overline{w}}((1-\alpha_h)p(w)+\beta_h(1-p(w)))dF(w))$$

$$>$$

$$(\int_{w_1^*}^{\overline{w}}((1-\alpha)p(w)+\beta(1-p(w)))dF(w))(\int_{w_2^*}^{\overline{w}}((1-\alpha_h)p(w)+\beta_h(1-p(w)))dF(w))$$

Using the definitions of α and β, that is, $\alpha \equiv \rho\alpha_H + (1-\rho)\alpha_L$ and $\beta \equiv \rho\beta_H + (1-\rho)\beta_L$, we can rewrite this expression as:

$$((1-\alpha)\beta_h - \beta(1-\alpha_h))\int_{w_2^*}^{\overline{w}} p(w)dF(w)\int_{w_1^*}^{\overline{w}}(1-p(w))dF(w)$$

$$>$$

$$((1-\alpha)\beta_h - \beta(1-\alpha_h))\int_{w_1^*}^{\overline{w}} p(w)dF(w)\int_{w_2^*}^{\overline{w}}(1-p(w))dF(w)$$

Suppose now that $(1-\alpha)\beta_h \leq \beta(1-\alpha_h)$ - which is equivalent to $(1-\alpha_l)\beta_h \leq \beta_l(1-\alpha_h))$. Then the inequality holds if and only if:

$$\int_{w_2^*}^{\overline{w}} p(w)dF(w)\int_{w_1^*}^{\overline{w}}(1-p(w))dF(w) < \int_{w_1^*}^{\overline{w}} p(w)dF(w)\int_{w_2^*}^{\overline{w}}(1-p(w))dF(w)$$

that is, if and only if

$$\int_{w_2^*}^{\overline{w}} p(w)dF(w)\int_{w_1^*}^{\overline{w}} dF(w) - \int_{w_2^*}^{\overline{w}} p(w)dF(w)\int_{w_1^*}^{\overline{w}} p(w)dF(w)$$

$$<$$

$$\int_{w_1^*}^{\overline{w}} p(w)dF(w)\int_{w_2^*}^{\overline{w}} dF(w) - \int_{w_1^*}^{\overline{w}} p(w)dF(w)\int_{w_2^*}^{\overline{w}} p(w)dF(w)$$

As the second terms on each side of the inequality are the same, this simplifies to:

$$(1-F(w_1^*))\int_{w_2^*}^{\overline{w}} p(w)dF(w) < (1-F(w_2^*))\int_{w_1^*}^{\overline{w}} p(w)dF(w).$$

Because $p(w) < p(w_1^*)$ for any $w < w_1^*$, notice that the left hand side of this inequality is at most:

$$(1-F(w_1^*))(F(w_1^*)-F(w_2^*))p(w_1^*) + (1-F(w_1^*))\int_{w_1^*}^{\overline{w}} p(w)dF(w) - \epsilon_1$$

for some $\epsilon_1 > 0$. Therefore, the inequality necessarily holds if the following holds:

$$(1-F(w_1^*))(F(w_1^*)-F(w_2^*))p(w_1^*) - \epsilon_1 < (F(w_1^*)-F(w_2^*))\int_{w_1^*}^{\overline{w}} p(w)dF(w).$$

The right hand side of this equation cannot be lower than $(F(w_1^*) - F(w_2^*))(1 - F(w_1^*))p(w_1^*) + \epsilon_2$ for some $\epsilon_2 > 0$. Therefore this condition always holds and $\gamma^0(w_1^*) > \gamma^0(w_2^*)$ when $(1-\alpha_l)\beta_h \leq \beta_l(1-\alpha_h)$.
With a similar argument we could show that the opposite is true when $(1-\alpha_l)\beta_h \geq \beta_l(1-\alpha_h)$. ■

Proof of Corollary 1. Let ω_1^* and ω_2^* be two equilibrium cutoffs, such that $\omega_1^* > \omega_2^*$. By definition it must be the case that the expected utilities of the two *marginal* individuals choosing the *risky* project equals the utility when choosing the *safe* one

$$(1-\lambda)\kappa(v_s - c) + \lambda\rho$$

$$=$$

$$(1-\lambda)\kappa((p(\omega_1^*)\alpha(v_r - c) + (1-p(\omega_1))(1-\beta)(-c) + \lambda(p(\omega_1^*)\rho\alpha_h + (1-p(\omega_1^*))\rho(1-\beta_h)+$$

$$+(p(\omega_1^*)(1-\alpha) + (1-p(\omega_1^*))\beta)\frac{\int_{w_1^*}^{\overline{w}} \rho((1-\alpha_h)p(\omega) + \beta_h(1-p(\omega)))dF(\omega)}{\int_{w_1^*}^{\overline{w}}((1-\alpha)p(\omega) + \beta(1-p(\omega)))dF(\omega)})$$

$$=$$

$$(1-\lambda)\kappa((p(\omega_2^*)\alpha(v_r-c)+(1-p(\omega_2^*))(1-\beta)(-c)+\lambda(p(\omega_2^*)\rho\alpha_h+(1-p(\omega_2^*))\rho(1-\beta_h)+$$

$$+(p(\omega_2^*)(1-\alpha)+(1-p(\omega_2^*))\beta)\frac{\int_{w_2^*}^{\overline{w}}\rho((1-\alpha_h)p(\omega)+\beta_h(1-p(\omega)))dF(\omega)}{\int_{w_2^*}^{\overline{w}}((1-\alpha)p(\omega)+\beta(1-p(\omega)))dF(\omega)})$$

For this to hold, it must be the case that:

$$\frac{(1-\lambda)}{\lambda}\kappa((p(\omega_2^*)-p(\omega_1^*))\alpha(v_r-c)+(p(\omega_2^*)-p(\omega_1^*))(1-\beta)c)+$$

$$+(p(\omega_2^*)-p(\omega_1^*))\rho\alpha_h-(p(\omega_2^*)-p(\omega_1^*))\rho(1-\beta_h)$$

$$=$$

$$(p(\omega_1^*)(1-\alpha)+(1-p(\omega_1^*))\beta)\frac{\int_{w_1^*}^{\overline{w}}\rho((1-\alpha_h)p(\omega)+\beta_h(1-p(\omega)))dF(w)}{\int_{w_1^*}^{\overline{w}}((1-\alpha)p(\omega)+\beta(1-p(\omega)))dF(\omega)}+$$

$$-(p(\omega_2^*)(1-\alpha)+(1-p(\omega_2^*))\beta)\frac{\int_{w_2^*}^{\overline{w}}\rho((1-\alpha_h)p(\omega)+\beta_h(1-p(\omega)))dF(\omega)}{\int_{w_2^*}^{\overline{w}}((1-\alpha)p(\omega)+\beta(1-p(\omega)))dF(\omega)} \tag{4}$$

Notice that the left hand side of equation (4) is negative, as $p(\omega_2^*) < p(\omega_1^*)$ and $\alpha_h+\beta_h-1>0$.
As $p(\omega_1^*) \equiv p(\omega_2^*)+(p(\omega_1^*)-p(\omega_2^*)) > p(\omega_2^*)$, we can now rewrite the right hand side of (4) as:

$$(p(\omega_2^*)(1-\alpha)+(1-p(\omega_2^*))\beta+(p(\omega_1^*)-p(\omega_2^*))(1-\alpha-\beta)))\times$$

$$\times\frac{\int_{w_1^*}^{\overline{w}}\rho((1-\alpha_h)p(\omega)+\beta_h(1-p(\omega)))dF(\omega)}{\int_{w_1^*}^{\overline{w}}((1-\alpha)p(\omega)+\beta(1-p(\omega)))dF(\omega)}+$$

$$-(p(\omega_2^*)(1-\alpha)+(1-p(\omega_2^*))\beta)\frac{\int_{w_2^*}^{\overline{w}}\rho((1-\alpha_h)p(\omega)+\beta_h(1-p(\omega)))dF(\omega)}{\int_{w_2^*}^{\overline{w}}((1-\alpha)p(\omega)+\beta(1-p(\omega)))dF(\omega)}$$

and then as:

$$(p(\omega_1^*) - p(\omega_2^*))(1-\alpha-\beta))\frac{\int_{w_1^*}^{\overline{w}} \rho((1-\alpha_h)p(\omega) + \beta_h(1-p(\omega)))dF(\omega)}{\int_{w_1^*}^{\overline{w}}((1-\alpha)p(\omega) + \beta(1-p(\omega)))dF(\omega)} +$$

$$+(p(\omega_2^*)(1-\alpha) + (1-p(\omega_2^*))\beta)\times$$

$$\times(\frac{\int_{w_1^*}^{\overline{w}} \rho((1-\alpha_h)p(\omega) + \beta_h(1-p(\omega)))dF(\omega)}{\int_{w_1^*}^{\overline{w}}((1-\alpha)p(\omega) + \beta(1-p(\omega)))dF(\omega)} - \frac{\int_{w_2^*}^{\overline{w}} \rho((1-\alpha_h)p(\omega) + \beta_h(1-p(\omega)))dF(\omega)}{\int_{w_2^*}^{\overline{w}}((1-\alpha)p(\omega) + \beta(1-p(\omega)))dF(\omega)})$$

The first part of this term is negative as $\alpha + \beta > 1$ and it is bounded below by $(p(\omega_1^*) - p(\omega_2^*))(1-\alpha-\beta)$, as $\gamma_0(\omega_1^*) \in [0,1]$.
By the previous Lemma the second part is positive if and only if $(1-\alpha_l)\beta_h \leq \beta_l(1-\alpha_h)$. In this scenario, the right hand side of equation (4) has a negative component and a positive one. Thus, if $(1-\alpha_l)\beta_h \leq \beta_l(1-\alpha_h)$, (4) cannot hold whenever:

$$(1-\lambda)\kappa((p(\omega_2^*) - p(\omega_1^*))\alpha(v_r - c) + (p(\omega_2^*) - p(\omega_1^*)(1-\beta)c) +$$

$$+\lambda((p(\omega_2^*) - p(\omega_1^*))\rho\alpha_h - (p(\omega_2^*) - p(\omega_1^*))\rho(1-\beta_h))$$

$$<$$

$$\lambda(p(\omega_2^*) - p(\omega_1^*))(\alpha + \beta - 1).$$

This is equivalent to:

$$(1-\lambda)\kappa(\alpha(v_r - c) + (1-\beta)c) + \lambda\rho(\alpha_h + \beta_h - 1)) > \lambda(\alpha + \beta - 1)$$

and, after simplification, to:

$$\frac{\lambda}{1-\lambda} < \frac{\kappa(\alpha(v_r - c) + (1-\beta)c)}{(1-\rho)(\alpha_l + \beta_l - 1)}.$$

■

Proof of Lemma 2. The proof of this Lemma is straightforward. Assume

$\frac{1-\alpha^H}{\beta^H} > \frac{1-\alpha}{\beta}$. For the marginal manager, the expected reputation is given by:

$$E(\gamma \mid \omega^*) = p(\omega^*)\alpha\rho\frac{\alpha^H}{\alpha} + ((1-p(\omega^*)(1-\beta))\rho\frac{(1-\beta^H)}{(1-\beta)} +$$

$$+(p(\omega^*)(1-\alpha) + (1-p(\omega^*))\beta)\rho\frac{(1-\alpha^H)\tilde{p} + \beta^H(1-\tilde{p})}{(1-\alpha)\tilde{p} + \beta(1-\tilde{p})}$$

$$>$$

$$p(\omega^*)\alpha\rho\frac{\alpha^H}{\alpha} + ((1-p(\omega^*)(1-\beta))\rho\frac{(1-\beta^H)}{(1-\beta)} +$$

$$+(p(\omega^*)(1-\alpha) + (1-p(\omega^*))\beta)\rho\frac{(1-\alpha^H)p(\omega^*) + \beta^H(1-p(\omega^*))}{(1-\alpha)p(\omega^*) + \beta(1-p(\omega^*))} = \rho$$

On the left hand side of the inequality we have the expected reputation of the marginal individual, upon observing ω^* and choosing the risky project. On the right hand side we have his expected self-assessment. Notice that since the signal ω is independent of the manager's type, his expected self-assessment - at any ω - equals, of course, the prior, ρ. This is an immediate consequence of the law of iterated expectations. ■

Additional Beliefs With N Agents

$$\gamma_N^{nl} \equiv \mathbb{P}(\theta_N = h \,|\, x = b,\, s = b) = \frac{\int_{\omega^*}^{\overline{\omega}} \mathbb{P}(\theta_N = h,\, x = b,\, s = b\,|\omega)\sigma(\omega)dF(\omega)}{\int_{\omega^*}^{\overline{\omega}} \mathbb{P}(x = b,\, s = b\,|\omega)\sigma(\omega)dF(\omega)} =$$

$$= \frac{\int_{\omega^*}^{\overline{\omega}} \overbrace{\mathbb{P}(\theta_N = h \,|\, \omega)}^{=\mathbb{P}(\theta=h)=\rho}\, \mathbb{P}(x = b,\, s = b \,|\, \omega,\, \theta_N = h)\, \sigma(\omega)\, dF(\omega)}{\sum_{i\in\{l,\,h\}} \int_{\omega^*}^{\overline{\omega}} \underbrace{\mathbb{P}(\theta_N = i \,|\, \omega)}_{=\mathbb{P}(\theta_N=i)} \mathbb{P}(x = b,\, s = b \,|\, \omega,\, \theta_N = i)\, \sigma(\omega)\, dF(\omega)} =$$

$$= \rho\, \frac{\int_{\omega^*}^{\overline{\omega}} \mathbb{P}(x = b,\, s = b \,|\, \omega,\, \theta_N = h)\, \sigma(\omega)\, dF(\omega)}{\sum_{i\in\{l,\,h\}} \mathbb{P}(\theta_N = i) \int_{\omega^*}^{\overline{\omega}} \mathbb{P}(x = b,\, s = b \,|\, \omega,\, \theta_N = i)\, \sigma(\omega)\, dF(\omega)} =$$

$$= \rho \, \frac{\int_{\omega^*}^{\overline{\omega}} \mathbb{P}(s = b \,|\, x = b,\, \omega,\, \theta = h)\, \overbrace{\mathbb{P}(x = b \,|\, \omega,\, \theta_N = h)}^{=\mathbb{P}(x=b\,|\,\omega)=p(\omega)}\, \sigma(\omega)\, dF(\omega)}{\sum_{i\in\{l,\,h\}} \mathbb{P}(\theta_N = i) \int_{\omega^*}^{\overline{\omega}} (s = b \,|\, x = b,\, \omega,\, \theta_N = i)\, \underbrace{\mathbb{P}(x = b \,|\, \omega,\, \theta_N = i)}_{=\mathbb{P}(x=b\,|\,\omega)=p(\omega)}\, \sigma(\omega)\, dF(\omega)} = \rho \frac{\beta_h}{\beta}$$

$$\gamma_N^0 \equiv \mathbb{P}(\theta_N = h \,|\, s_N = b,\, s_1 = b, ..., s_{N-1} = b) = \frac{\mathbb{P}(\theta_N = h,\, s_1 = b, ..., s_N = b)}{\mathbb{P}(s_1 = b, ..., s_N = b)} =$$

$$= \frac{\int_{\omega^*}^{\overline{\omega}} \mathbb{P}(\theta_N = h, s_1 = b, ..., s_N = b \,|\omega)\sigma(\omega)dF(\omega)}{\int_{\omega^*}^{\overline{\omega}} \mathbb{P}(s_1 = b, ..., s_N = b \,|\omega)\sigma(\omega)dF(\omega)} =$$

$$= \frac{\int_{\omega^*}^{\overline{\omega}} \overbrace{\mathbb{P}(\theta_N = h \,|\, \omega)}^{=\mathbb{P}(\theta_N=h)=\rho}\, \mathbb{P}(s_1 = b, ..., s_N = b \,|\, \omega,\, \theta_N = h)\, \sigma(\omega)\, dF(\omega)}{\sum_{i\in\{l,\,h\}} \int_{\omega^*}^{\overline{\omega}} \underbrace{\mathbb{P}(\theta_N = i \,|\, \omega)}_{=\mathbb{P}(\theta_N=i)} \mathbb{P}(s_1 = b, ..., s_N = b \,|\, \omega,\, \theta_N = i)\, \sigma(\omega)\, dF(\omega)} =$$

Notice that $\mathbb{P}(s_1 = b, .., s_N = b \,|\, \omega, \theta = i)$ can be computed as:

$$\begin{aligned} &\mathbb{P}(s_1 = b, .., s_N = b \,|\, x = g, \omega,\, \theta = i)\mathbb{P}(x = g|\omega, \theta_N = i) \quad + \\ &+\mathbb{P}(s_1 = b, .., s_N = b|x = b, \omega, \theta = i)\mathbb{P}(x = b|\omega, \theta_N = i) \quad . \end{aligned} \tag{5}$$

Now, the first term of equation (5) - $\mathbb{P}(s_1 = b, ..., s_N = b \,|\, x = g,\, \omega,\, \theta_N = i)$ - is:

$$\mathbb{P}(s_N = b \,|s_1 = b, ..., s_{N-1} = b,\, x = g,\, \omega,\, \theta_N = i)\mathbb{P}(s_1 = b, ..., s_{N-1} = b \,|\, x = g,\, \omega,\, \theta_N = i)\times$$

$$\times\mathbb{P}(s_N = b \,|\, x = g,\, \theta_N = i)\mathbb{P}(s_{N-1} = b \,|\, x = g)...\mathbb{P}(s_1 = b \,|\, x = g) =$$

$$= (1 - \alpha_i)(1 - \alpha)^{N-1}.$$

Similarly, the second term of (5) can be computed as $\mathbb{P}(s_1 = b, ..., s_N = b \,|\, x = b,\, \omega,\, \theta_N = i) = \beta_i \beta^{N-1}$. Thus, we have:

$$\gamma_N^0 = \rho \frac{(1 - \alpha_h)(1 - \alpha)^{N-1} p(\omega) + \beta_h \beta^{N-1}(1 - p(\omega))}{(1 - \alpha)^N p(\omega) + \beta^N(1 - p(\omega))}$$

$$\gamma_N^{nr} \equiv \mathbb{P}(\theta_N = h \,|\, x = g,\, s = b) = \frac{\int_{\omega^*}^{\overline{\omega}} \mathbb{P}(\theta_N = h,\, x = g,\, s = b \,|\omega)\sigma(\omega) dF(\omega)}{\int_{\omega^*}^{\overline{\omega}} \mathbb{P}(x = g,\, s = b \,|\omega)\sigma(\omega) dF(\omega)} =$$

$$= \frac{\int_{\omega^*}^{\overline{\omega}} \overbrace{\mathbb{P}(\theta_N = h \,|\, \omega)}^{=\mathbb{P}(\theta_N = h)=\rho} \mathbb{P}(x = g,\, s = b \,|\, \omega,\, \theta_N = h)\, \sigma(\omega)\, dF(\omega)}{\sum_{i \in \{l,\, h\}} \int_{\omega^*}^{\overline{\omega}} \underbrace{\mathbb{P}(\theta_N = i \,|\, \omega)}_{=\mathbb{P}(\theta_N = i)} \mathbb{P}(x = g,\, s = b \,|\, \omega,\, \theta = i)\, \sigma(\omega)\, dF(\omega)} =$$

$$= \rho \, \frac{\int_{\omega^*}^{\overline{\omega}} \mathbb{P}(x = g,\, s = b \,|\, \omega,\, \theta_N = h)\, \sigma(\omega)\, dF(\omega)}{\sum_{i \in \{l,\, h\}} \mathbb{P}(\theta_N = i) \int_{\omega^*}^{\overline{\omega}} \mathbb{P}(x = g,\, s = b \,|\, \omega,\, \theta_N = i)\, \sigma(\omega)\, dF(\omega)} =$$

$$= \rho \, \frac{\int_{\omega^*}^{\overline{\omega}} \mathbb{P}(s = b \,|\, x = g,\, \omega,\, \theta = h)\, \overbrace{\mathbb{P}(x = g \,|\, \omega,\, \theta_N = h)}^{\mathbb{P}(x=g\,|\,\omega)=p(\omega)}\, \sigma(\omega)\, dF(\omega)}{\sum_{i \in \{l,\, h\}} \mathbb{P}(\theta_N = i)\, \int_{\omega^*}^{\overline{\omega}} \mathbb{P}(s = b \,|\, x = g,\, \omega,\, \theta_N = i)\, \underbrace{\mathbb{P}(x = g \,|\, \omega,\, \theta_N = i)}_{\mathbb{P}(x=g\,|\,\omega)=p(\omega)}\, \sigma(\omega)\, dF(\omega)} =$$

$$= \rho \frac{1 - \alpha_h}{1 - \alpha}.$$

Proof of Proposition 3. We start showing that Lemma 1 and Lemma 2 also hold in the generalized version of the model.

Consider, again, two cutoff equilibria characterized by thresholds w_1^* and w_2^*, with $w_1^* > w_2^*$. We want to show that $\gamma^0(w_1^*) > \gamma^0(w_2^*)$ if and only if $\frac{1-\alpha^H}{\beta^H} > \frac{1-\alpha}{\beta}$. In doing this, we use the new definition of belief in case of termination of the *risky* project. $\gamma^0(w_1^*) > \gamma^0(w_2^*)$ when:

$$\rho \frac{\int_{\omega_1^*}^{\overline{\omega}} (1 - \alpha_h)(1 - \alpha)^{N-1} p(w) + \beta_h \beta^{N-1}(1 - p(w)) dF(w)}{\int_{\omega_1^*}^{\overline{\omega}} ((1 - \alpha)^N p(w) + \beta^N (1 - p(w))) dF(w)}$$

$$>$$

$$\rho \frac{\int_{\omega_2^*}^{\overline{\omega}} (1 - \alpha_h)(1 - \alpha)^{N-1} p(w) + \beta_h \beta^{N-1}(1 - p(w)) dF(w)}{\int_{\omega_2^*}^{\overline{\omega}} (1 - \alpha)^N p(w) + \beta^N (1 - p(w)) dF(w)}$$

This is equivalent to:

$$
\begin{aligned}
&(1-\alpha_h)(1-\alpha)^{N-1}\beta^N \int_{\omega_1^*}^{\overline{\omega}} p(w)dF(w) \int_{\omega_2^*}^{\overline{\omega}} (1-p(w))dF(w) + \\
&\qquad\qquad + (1-\alpha)^N \beta_h \beta^{N-1} \int_{\omega_1^*}^{\overline{\omega}} (1-p(w))dF(w) \int_{\omega_2^*}^{\overline{\omega}} p(w)dF(w)
\end{aligned}
$$

$$>$$

$$
\begin{aligned}
&(1-\alpha_h)(1-\alpha)^{N-1}\beta^N \int_{\omega_1^*}^{\overline{\omega}} p(w)dF(w) \int_{\omega_2^*}^{\overline{\omega}} (1-p(w))dF(w) + \\
&\qquad\qquad + (1-\alpha)^N \beta_h \beta^{N-1} \int_{\omega_1^*}^{\overline{\omega}} (1-p(w))dF(w) \int_{\omega_2^*}^{\overline{\omega}} p(w)dF(w)
\end{aligned}
$$

We now divide everything by $(1-\alpha)^{N-1}\beta^{N-1}$ and rearrange, to get:

$$((1-\alpha)\beta_h - \beta(1-\alpha_h)) \int_{w_2^*}^{\overline{w}} p(w)dF(w) \int_{w_1^*}^{\overline{w}} (1-p(w))dF(w)$$

$$>$$

$$((1-\alpha)\beta_h - \beta(1-\alpha_h)) \int_{w_1^*}^{\overline{w}} p(w)dF(w) \int_{w_2^*}^{\overline{w}} (1-p(w))dF(w)$$

This is equivalent to what we had in the proof of Lemma 1. Therefore for w_1^* and w_2^*, with $w_1^* > w_2^*$, we have that $\gamma^0(w_1^*) > \gamma^0(w_2^*)$ if and only if $\frac{1-\alpha^H}{\beta^H} > \frac{1-\alpha}{\beta}$.

Now we need to show that, as in Lemma 2, upon observing ω^* the marginal agent is less optimistic than the principal in evaluating his own ability if and only if $\frac{1-\alpha^H}{\beta^H} > \frac{1-\alpha}{\beta}$. The proof of this claim is straightforward. Suppose $\frac{1-\alpha^H}{\beta^H} > \frac{1-\alpha}{\beta}$. Notice that, for the marginal agent:

$$E(\gamma \mid \omega^*) \;\; \equiv \;\; p(\omega^*)\alpha\rho\frac{\alpha^H}{\alpha} + ((1-p(\omega^*)(1-\beta))\rho\frac{(1-\beta^H)}{(1-\beta)} +$$

$$+p(\omega^*)(1-\alpha-(1-\alpha)^N)\rho\frac{1-\alpha_h}{1-\alpha}+(1-p(\omega^*))(1-(1-\beta)-\beta^N)\rho\frac{\beta_h}{\beta}+$$

$$+(p(\omega^*)(1-\alpha)^N+(1-p(\omega^*))\beta^N)\rho\,\frac{(1-\alpha_h)(1-\alpha)^{N-1}\tilde{p}+\beta_h\beta^{N-1}(1-\tilde{p})}{(1-\alpha)^N\tilde{p}+\beta^N(1-\tilde{p})}$$

$$>$$

$$p(\omega^*)\alpha\rho\frac{\alpha^H}{\alpha}+((1-p(\omega^*)(1-\beta))\rho\frac{(1-\beta^H)}{(1-\beta)}+$$

$$+p(\omega^*)(1-\alpha-(1-\alpha)^N)\rho\frac{1-\alpha_h}{1-\alpha}+(1-p(\omega^*))(1-(1-\beta)-\beta^N)\rho\frac{\beta_h}{\beta}+$$

$$+(p(\omega^*)(1-\alpha)^N+(1-p(\omega^*))\beta^N)\rho\,\frac{(1-\alpha_h)(1-\alpha)^{N-1}p(\omega^*)+\beta_h\beta^{N-1}(1-p(\omega^*))}{(1-\alpha)^Np(\omega^*)+\beta^N(1-p(\omega^*))} \quad = \quad \rho$$

Hence, the marginal agent takes too much risk if and only if $\frac{1-\alpha^H}{\beta^H} > \frac{1-\alpha}{\beta}$. ■

Proof of Proposition 4. We start from the definition of $E(\gamma_N \mid \omega^*)$ and $E(\gamma_{N+1} \mid \omega^*)$. These are, respectively:

$$E(\gamma_N \mid \omega^*) \equiv p(\omega^*)\alpha\rho\frac{\alpha^H}{\alpha}+((1-p(\omega^*)(1-\beta))\rho\frac{(1-\beta^H)}{(1-\beta)}+$$

$$+p(\omega^*)(1-\alpha-(1-\alpha)^N)\rho\frac{1-\alpha_h}{1-\alpha}+(1-p(\omega^*))(1-(1-\beta)-\beta^N)\rho\frac{\beta_h}{\beta}+$$

$$+(p(\omega^*)(1-\alpha)^N+(1-p(\omega^*))\beta^N)\rho\,\frac{(1-\alpha_h)(1-\alpha)^{N-1}\tilde{p}+\beta_h\beta^{N-1}(1-\tilde{p})}{(1-\alpha)^N\tilde{p}+\beta^N(1-\tilde{p})}$$

and:

$$E(\gamma_{N+1} \mid \omega^*) \equiv p(\omega^*)\alpha\rho\frac{\alpha^H}{\alpha}+((1-p(\omega^*)(1-\beta))\rho\frac{(1-\beta^H)}{(1-\beta)}+$$

$$+p(\omega^*)(1-\alpha-(1-\alpha)^{N+1})\rho\frac{1-\alpha_h}{1-\alpha}+(1-p(\omega^*))(1-(1-\beta)-\beta^{N+1})\rho\frac{\beta_h}{\beta}+$$

$$+(p(\omega^*)(1-\alpha)^{N+1}+(1-p(\omega^*))\beta^{N+1})\rho\,\frac{(1-\alpha_h)(1-\alpha)^N\tilde{p}+\beta_h\beta^N(1-\tilde{p})}{(1-\alpha)^{N+1}\tilde{p}+\beta^{N+1}(1-\tilde{p})}$$

We want to show that $E(\gamma_N \mid \omega^*) > E(\gamma_{N+1} \mid \omega^*)$ if and only if $\frac{1-\alpha_h}{\beta_h} > \frac{1-\alpha_l}{\beta_l}$. Noticing that the first two addends in the definitions of $E(\gamma_N \mid \omega^*)$ and $E(\gamma_{N+1} \mid \omega^*)$ coincide and dividing everything by ρ, $E(\gamma_N \mid \omega^*) > E(\gamma_{N+1} \mid \omega^*)$ if and only if

$$p(\omega^*)(1-\alpha-(1-\alpha)^{N})\frac{1-\alpha_h}{1-\alpha}+(1-p(\omega^*))(1-(1-\beta)-\beta^{N})\frac{\beta_h}{\beta}+$$

$$+(p(\omega^*)(1-\alpha)^{N}+(1-p(\omega^*))\beta^{N})\,\frac{(1-\alpha_h)(1-\alpha)^{N-1}\tilde{p}+\beta_h\beta^{N-1}(1-\tilde{p})}{(1-\alpha)^{N}\tilde{p}+\beta^{N}(1-\tilde{p})}$$

$$>$$

$$p(\omega^*)(1-\alpha-(1-\alpha)^{N+1})\frac{1-\alpha_h}{1-\alpha}+(1-p(\omega^*))(1-(1-\beta)-\beta^{N+1})\frac{\beta_h}{\beta}+$$

$$+(p(\omega^*)(1-\alpha)^{N+1}+(1-p(\omega^*))\beta^{N+1})\,\frac{(1-\alpha_h)(1-\alpha)^N\tilde{p}+\beta_h\beta^N(1-\tilde{p})}{(1-\alpha)^{N+1}\tilde{p}+\beta^{N+1}(1-\tilde{p})}$$

Now we multiply everything by $\Big((1-\alpha)^N\tilde{p}+\beta^N(1-\tilde{p})\Big)\Big((1-\alpha)^{N+1}\tilde{p}+\beta^{N+1}(1-\tilde{p})\Big)$ and adjust terms to get

$$(1-\alpha_h)\Bigg\{-p(\omega^*)(1-\alpha)^{N-1}\alpha\Big((1-\alpha)^N\tilde{p}+\beta^N(1-\tilde{p})\Big)\Big((1-\alpha)^{N+1}\tilde{p}+\beta^{N+1}(1-\tilde{p})\Big)+$$

$$+(1-\alpha)^{N-1}\tilde{p}\Big((1-\alpha)^Np(\omega^*)+\beta^N(1-p(\omega^*))\Big)\Big((1-\alpha)^{N+1}\tilde{p}+\beta^{N+1}(1-\tilde{p})\Big)+$$

$$-(1-\alpha)^N\tilde{p}\left((1-\alpha)^{N+1}p(\omega^*)+\beta^{N+1}(1-p(\omega^*))\right)\left((1-\alpha)^N\tilde{p}+\beta^N(1-\tilde{p})\right)\Bigg\}$$

$$>$$

$$\beta_h\Bigg\{(1-p(\omega^*))\beta^{N-1}(1-\beta)\left((1-\alpha)^N\tilde{p}+\beta^N(1-\tilde{p})\right)\left((1-\alpha)^{N+1}\tilde{p}+\beta^{N+1}(1-\tilde{p})\right)+$$

$$+\beta^N(1-\tilde{p})\left((1-\alpha)^{N+1}p(\omega^*)+\beta^{N+1}(1-p(\omega^*))\right)\left((1-\alpha)^N\tilde{p}+\beta^N(1-\tilde{p})\right)$$

$$-\beta^{N-1}(1-\tilde{p})\left((1-\alpha)^N p(\omega^*)+\beta^N(1-p(\omega^*))\right)\left((1-\alpha)^{N+1}\tilde{p}+\beta^{N+1}(1-\tilde{p})\right)\Bigg\}$$

We focus separately on the two sides of the inequality. From the left hand side we obtain:

$$(1-\alpha_h)\Bigg\{-p(\omega^*)(1-\alpha)^{N-1}\alpha\Big((1-\alpha)^{2N+1}\tilde{p}^2+(1-\alpha)^N\beta^{N+1}\tilde{p}(1-\tilde{p})+$$

$$+(1-\alpha)^{N+1}\beta^N\tilde{p}(1-\tilde{p})+\beta^{2N+1}(1-\tilde{p})^2\Big)+$$

$$+\tilde{p}(1-\alpha)^{N-1}\Big((1-\alpha)^{2N+1}p(\omega^*)\tilde{p}+(1-\alpha)^N\beta^{N+1}p(\omega^*)(1-\tilde{p})+$$

$$+(1-\alpha)^{N+1}\beta^N\tilde{p}(1-p(\omega^*))+\beta^{2N+1}(1-p(\omega^*))(1-\tilde{p})\Big)+$$

$$-\tilde{p}(1-\alpha)^N\Big((1-\alpha)^{2N+1}p(\omega^*)\tilde{p}+(1-\alpha)^{N+1}\beta^N p(\omega^*)(1-\tilde{p})+$$

$$+(1-\alpha)^N\beta^{N+1}\tilde{p}(1-p(\omega^*))+\beta^{2N+1}(1-p(\omega^*))(1-\tilde{p})\Big)\Bigg\}=$$

$$= (1-\alpha_h)\Bigg\{-\alpha(1-\alpha)^{2N-1}\beta^{N+1}p(\omega^*)\tilde{p}(1-\tilde{p})+$$

$$-\alpha(1-\alpha)^{2N}\beta^{N}p(\omega^*)\tilde{p}(1-\tilde{p})-\alpha(1-\alpha)^{N-1}\beta^{2N+1}p(\omega^*)(1-\tilde{p})^2+$$

$$(1-\alpha)^{2N-1}\beta^{N+1}\Big(p(\omega^*)\tilde{p}(1-\tilde{p})-(1-\alpha)(1-p(\omega^*))\tilde{p}^2\Big)+$$

$$+(1-\alpha)^{2N}\beta^{N}\Big(\tilde{p}^2(1-p(\omega^*))-(1-\alpha)p(\omega^*)\tilde{p}(1-\tilde{p})\Big)+$$

$$+(1-\alpha)^{N-1}\beta^{2N+1}\Big(\tilde{p}(1-p(\omega^*))(1-\tilde{p})-(1-\alpha)\tilde{p}(1-p(\omega^*))(1-\tilde{p})\Big)\Bigg\}=$$

$$= (1-\alpha_h)\Bigg\{(1-\alpha)^{2N-1}\beta^{N+1}\tilde{p}\Big(p(\omega^*)(1-\tilde{p})-(1-\alpha)(1-p(\omega^*))\tilde{p}-\alpha p(\omega^*)(1-\tilde{p})\Big)+$$

$$+(1-\alpha)^{2N}\beta^{N}\tilde{p}\Big(\tilde{p}(1-p(\omega^*)-(1-\alpha)p(\omega^*)(1-\tilde{p})-\alpha p(\omega^*)(1-\tilde{p})\Big)+$$

$$+(1-\alpha)^{N-1}\beta^{2N+1}\Big(\tilde{p}(1-p(\omega^*))-(1-\alpha)\tilde{p}(1-p(\omega^*))-\alpha p(\omega^*)(1-\tilde{p})\Big)\Bigg\}=$$

$$= (1-\alpha_h)\Bigg\{(1-\alpha)^{2N-1}\beta^{N+1}\tilde{p}\Big((1-\alpha)(p(\omega^*)-\tilde{p})\Big)+(1-\alpha)^{2N}\beta^{N}\tilde{p}\Big(\tilde{p}-p(\omega^*)\Big)+$$

$$+(1-\alpha)^{N-1}\beta^{2N+1}\Big(\alpha(\tilde{p}-p(\omega^*))\Big)\Bigg\}=$$

$$= (1-\alpha_h)\beta(\tilde{p}-p(\omega^*))\Bigg\{(1-\alpha)^{2N}\beta^{N-1}(1-\beta)\tilde{p}+(1-\alpha)^{N-1}\beta^{N}(1-\tilde{p})\alpha\Bigg\}.$$

From the hand side, instead, we obtain

$$\beta_h \Bigg\{ (1-p(\omega^*)(1-\beta)\beta^{N-1}\Big((1-\alpha)^{2N+1}\tilde{p}^2 + (1-\alpha)^N\beta^{N+1}\tilde{p}(1-\tilde{p})+$$

$$+(1-\alpha)^{N-1}\beta^N\tilde{p}(1-\tilde{p}) + \beta^{2N+1}(1-\tilde{p})^2\Big)+$$

$$+(1-\tilde{p})\beta^N\Big((1-\alpha)^{2N+1}p(\omega^*)\tilde{p} + (1-\alpha)^{N+1}\beta^N p(\omega^*)(1-\tilde{p})+$$

$$+(1-\alpha)^N\beta^{N+1}\tilde{p}(1-p(\omega^*)) + \beta^{2N+1}(1-p(\omega^*))(1-\tilde{p})\Big)+$$

$$-(1-\tilde{p})\beta^{N-1}\Big((1-\alpha)^{2N+1}p(\omega^*)\tilde{p} + (1-\alpha)^N\beta^{N+1}p(\omega^*)(1-\tilde{p})+$$

$$+(1-\alpha)^{N+1}\beta^N\tilde{p}(1-p(\omega^*)) + \beta^{2N+1}(1-p(\omega^*))(1-\tilde{p})\Big)\Bigg\}=$$

$$=\beta_h\Bigg\{(1-\alpha)^{2N+1}(1-\beta)\beta^{N-1}(1-p(\omega^*))\tilde{p}^2 + (1-\alpha)^N(1-\beta)\beta^{2N}(1-p(\omega^*))\tilde{p}(1-\tilde{p})+$$

$$+(1-\alpha)^{N+1}(1-\beta)\beta^{2N-1}(1-p(\omega^*))\tilde{p}(1-\tilde{p})+$$

$$-(1-\alpha)^{2N+1}(1-\beta)\beta^{N-1}p(\omega^*)\tilde{p}(1-\tilde{p})+$$

$$-(1-\alpha)^N\beta^{2N}(1-\tilde{p})\Big(p(\omega^*)(1-\tilde{p}) - \beta\tilde{p}(1-p(\omega^*))\Big)+$$

$$-(1-\alpha)^{N+1}\beta^{2N-1}(1-\tilde{p})\Big((1-p(\omega^*))\tilde{p} - \beta p(\omega^*)(1-\tilde{p})\Big)\Bigg\}=$$

$$=\beta_h(1-\alpha)\Bigg\{(1-\alpha)^{2N}(1-\beta)\beta^{N-1}\Big(\tilde{p}^2 - p(\omega^*)\tilde{p}^2 - p(\omega^*)\tilde{p} + p(\omega^*)\tilde{p}^2\Big)+$$

$$+(1-\alpha)^{N-1}\beta^{2N}(1-\tilde{p})\Big((1-\beta)(\tilde{p}-p(\omega^*)\tilde{p})-p(\omega^*)+p(\omega^*)\tilde{p}+\beta\tilde{p}-\beta p(\omega^*)\tilde{p}\Big)+$$

$$+(1-\alpha)^{N}\beta^{2N-1}(1-\tilde{p})\Big((1-\beta)(\tilde{p}-p(\omega^*)\tilde{p})-\tilde{p}+p(\omega^*)\tilde{p}+\beta p(\omega^*)-\beta p(\omega^*)\tilde{p}\Big)\Bigg\}=$$

$$=\beta_h(1-\alpha)\Bigg\{(1-\alpha)^{2N}(1-\beta)\beta^{N-1}\tilde{p}(\tilde{p}-p(\omega^*))+$$

$$+(1-\alpha)^{N-1}\beta^{2N}(1-\tilde{p})(\tilde{p}-p(\omega^*))+$$

$$+(1-\alpha)^{N}\beta^{2N}(1-\tilde{p})(p(\omega^*)-\tilde{p})\Bigg\}=$$

$$=\beta_h(1-\alpha)(\tilde{p}-p(\omega^*))\Bigg\{(1-\alpha)^{2N}\beta^{N-1}(1-\beta)\tilde{p}+(1-\alpha)^{N-1}\beta^{2N}(1-\tilde{p})\alpha\Bigg\}.$$

As $(\tilde{p}-p(\omega^*))$ and the term in braces are non negative and common between the left and the right hand side of the original inequality, $E(\gamma_N \mid \omega^*) > E(\gamma_{N+1} \mid \omega^*)$ if and only if $(1-\alpha_h)\beta_l > (1-\alpha_l)\beta_h$. In this case, $E(\gamma_N \mid \omega^*) > E(\gamma_{N+1} \mid \omega^*) > \rho$, meaning that the expected reputation of the marginal individual, higher than the prior, lowers as the number of agents, N, increases. The opposite holds whenever $(1-\alpha_h)\beta_l < (1-\alpha_l)\beta_h$. ■

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