The London School of Economics and Political Science

Three Essays on Macro Labour Economics

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Declaration

I hereby declare that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is my own work. Chapter 3 was undertaken as joint work with Xijie Gao.

I declare that my thesis consists of 31,117 words.

Jiajia Gu
Abstract

The first chapter investigates the role of financial intermediation in explaining the occupation choices. A large fraction of the labour force in developing countries is own-account workers who work for themselves and have no paid employees. This paper argues that imperfect financial intermediation drives a wedge between the return on saving and the cost of borrowing. A larger wedge generates a lower return on saving and a higher borrowing cost. The lower return induces individuals with some wealth but low entrepreneurial ability to manage their own wealth. Together with a wage fall when financial intermediation worsens, the model predicts higher share of own-account workers and lower share of wage workers.

The second chapter explores the impact of One-Child Policy on human capital and aggregate income. A quantity-quality trade-off predicts an increase in human capital when fertility falls. The higher human capital level contributes to aggregate output but the lower fertility reduces the size of future labour force, hence reduces aggregate output. In a quantitative OLG model, I show that the human capital level of children born under the strict One-child Policy increases, but the policy’s effect on aggregate income turned negative in around 2000 due to smaller size of labour force.

The third chapter examines the effects of a decline in transaction cost of information good. We classify industries into information sector and non-information sector, and we classify labour into information labour and non-information labour. We make two observations from the data. The first is the increase in the share of information intermediate input in total intermediate input. The second is the increase in return to information labour relative to non-information labour. In a two sector model, We find that under reasonable parameter assumptions, a decline in transaction cost of information good cannot explain both facts.
To my friend, Rui Zhang.
Acknowledgements

I am extremely lucky to have both Rachel Ngai and Alwyn Young as my supervisors in the last two years of my PhD studies. They did not give up on me, they were patient with me, and they were always there when I needed help. I learned from them not just how to do research, but also how to be a better person.

I am also very grateful to Keyu Jin. When I had no clue what research is like, she offered help and I started to work on the One-Child Policy project under her supervision.

I am very grateful to Wouter Den Haan for the helpful discussions on solving numerical models and all the help he gave me during the job market.

A special thank you goes to Shengxing Zhang and the reading group he organized. I learned how to read paper and how to explain intuition.

I also would like to thank all participants at LSE Money/Macro Student WiP seminars.

And finally, last but by no means least, I thank my parents, who have always supported me, emotionally and financially, no matter what I do.
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Chapter 1

Financial Intermediation and
Occupation Choice

1.1 Introduction

A large fraction of the labour force in developing countries is own-account workers. According to International Labour Organizations (ILO), in 2013, own-account workers accounted for about 50% of the total employment in low income and lower-middle income countries, about 25% in upper-middle income countries, and about 9.3% in high income countries. Own-account workers are self-employed without employees. They are different from wage workers in that they work for themselves and manage their own wealth. They are different from employers in that they typically have no paid employees and do not work with much capital.

Distinguishing between own-account workers and employers is important as own-account workers withdraw their labour input from the market while employers actively create jobs for others. Often, studies focusing on developed economies treat all self-employed (those with or without employees) as en-
In grouping both own-account workers and employers as business owners, cross-country data implies a negative relationship between share of business owners and per capita income (Figure 1.3), which may lead to a rather misleading suggestion that developing countries have more entrepreneurial activities. A closer look at the data reveals that the negative relationship only holds for own-account workers. The share of employers is actually positively correlated with income, suggesting the entrepreneurship rate is lower in developing countries (Figure 1.4b).

The main claim of this paper is that the higher share of own-account workers in developing countries is due to imperfect financial intermediation, which drives a wedge between return on savings and the cost of borrowing. A higher wedge generates lower return on savings and a higher borrowing cost. The former induces individuals with some wealth but low entrepreneurial ability to manage their own wealth; while the later acts as a barrier to higher ability but not so wealthy individuals to become employers. As financial intermediation deteriorates, the share of own-account workers increases, and the share of wage workers falls. As more individuals choose to manage wealth by themselves, the share of capital intermediated through the market falls.

The set-up of the model in this paper differs from the standard occupation choice models in two ways. First, I explicitly model own-account workers. Agents with heterogeneous wealth and ability choose among becoming wage workers, own-account workers or employers. The difference between an own-account worker and an employer is that an own-account worker hires no paid employees, and the difference between a wage worker and an own-account worker is that wage workers hand all wealth to intermediaries, but an own ac-

---

1 For example, see Evans and Jovanovic (1989), Evans and Leighton (1989) and Hamilton (2000).

2 Poschko (2015) argues that a skilled biased technology change can explain why entrepreneurship rates fall when productivity increases, and generates cross-country variations in firm size distribution. His entrepreneurship rate includes both business owners with and without employees.
count worker can manage some wealth through investing in their own-account business. The model also implies that the amount of capital an agent can manage as an own-account worker is small relative to what this agent could manage as an employer. Even the most capable agent in this model manages relatively small amount of capital. This prediction captures the idea that own account workers usually operate on a small scale.

The second difference is that I model financial inefficiency as the cost incurred when intermediating capital. When the cost rises, it leads to a higher cost of borrowing and a lower return on savings. The higher cost of borrowing is similar to any borrowing constraint that prevents people from borrowing. The lower return on savings, however, is a new mechanism emerging from this model.

In the perfect financial intermediation case, the return on savings is the same as the cost of borrowing, so wealth is irrelevant for occupation choices. Two ability cut-offs exist. Low ability agents, those with abilities below the lower cut-off, become wage workers. Medium ability agents, those with abilities in between the two cut-offs, become own-account workers and high ability agents, those with abilities higher than the higher cut-off, become employers.

When financial intermediation falls, the higher cost of borrowing and lower return on savings distort the occupation choices. There are two channels that increase the share of own-account workers. The first is that the lower return on savings encourages the medium-low ability agents (at low cut-off) who have some wealth to become own-account workers, managing small businesses to avoid the low return. The second channel is that the higher borrowing cost makes it expensive for medium-high ability agents (at high cut-off) to borrow to become employers, and they become own-account workers instead. There are also two channels that decrease the share of own-account workers. The first is that the medium-high ability agents who have relative high wealth to abilities
are more likely to switch from being own-account workers to entrepreneurs, increasing the amount of capital they manage. The second channel is that due to the higher cost of borrowing, the medium-low ability agents who have very low wealth are less likely to borrow to become own-account workers, and switch to becoming wage workers.

The predictions of this model differ from a standard binary occupation choice model in two ways. First, this model is able to quantitatively generate a large share of self-employment, while it is difficult for models with binary occupation choices to do so. In a binary model with wage workers and entrepreneurs, frictions only affect one margin: the marginal entrepreneurs. A higher borrowing interest rate makes borrowing expensive, so the low wealth marginal agents become wage workers. A low return on savings induces high wealth marginal agents to become entrepreneurs to manage wealth. As entrepreneurial ability is relatively rare, the number of marginal agents who respond to the frictions are quantitatively small. Hence the predicted change in share of self-employment, entrepreneurs, is small. However, when own-account workers are explicitly modelled, frictions can affect two margins, the medium-low ability agents and the medium-high ability agents, as explained in the previous paragraph. The ability and wealth needed to be own-account workers are not high, and there is a relative abundance of medium-low ability agents. Frictions have the potential to generate large quantitative effects on the share of self-employment.

The second important difference is that in this paper, the wealth holding of medium-low ability agents is very important, while it is less so in binary occupation choice models. In a binary model with wage workers and entrepreneurs, the medium-low ability agents become wage workers no matter what. Their wealth holding has no direct impact on occupation choices other than through affecting the general equilibrium wage rate and interest rate(s). Modelling own-account workers as a separate group, however, makes the wealth of medium-low ability agents very important. These agents have the potential to become own-
account workers, but whether they choose to do so will depend on how badly their wealth income is hurt by friction and, in this case, it is the fall in return on savings. The wealth income of an agent with zero-wealth is not affected at all by changes in return on savings. For a given fall in return on savings, the larger the wealth, the larger the fall in wealth income. Agents of a given ability are more likely to move from being wage workers to own-account workers if they have higher wealth. As the amount of capital required to become an own-account worker is small, relatively small changes in wealth holdings can affect the occupation decision of such agents.

The strength of the four channels mentioned above depends on the wealth holding of medium-low ability agents. Thus, the effect of financial intermediation inefficiency on the share of own-account workers depends on the joint distribution of wealth and ability, which is not directly observable. Thus, a dynamic model with endogenous saving decision is necessary. I present such a model with a steady state joint distribution of wealth and ability. I then calibrate the model’s perfect intermediation case to match relevant moments in the US, and vary the financial intermediation level to assess how well the model matches the data. The quantitative results show that as financial intermediation falls, borrowing interest rates increases, and saving interest rates falls. Together with the fall of wage rate, they lead to a larger share of own-account workers and smaller share of wage workers and employers. The calibrated model accounts for more than 70% of the variation in the share of own-account workers in the data for 2013. Furthermore, the model’s main occupation change comes from the responses of the medium-low ability agents. The model also predicts that as financial intermediation efficiency falls, the share of own-account workers who manage their own wealth increases, leading to a lower fraction of total capital intermediated through the market.

To summarize, this paper makes two contributions; first, I emphasize the new channel of return on savings, which is key in generating the a large share
of own-account workers; second, I emphasize the importance of the wealth holdings of individuals with medium-low abilities.

The large fraction of self-employment in developing countries is well known. Gindling and Newhouse (2014) provide a descriptive analysis. As a country develops, employment moves from the agricultural sector to the non-agricultural own-account sector and then to wage workers. De Mel et al. (2010), using survey data on Sri Lanka, finds that 70% of own account workers share the characteristics of wage workers, instead of employers. Gollin (2008) argues that difference in TFP generates the cross country pattern in the share of self-employment.

Recent papers by Cuberes and Teignier (2015) and Cuberes and Teignier (2017) model own account workers in a similar way as I do here; however, they do not study the role of financial frictions. One caveat is that these models, including the one I present here, does not capture those own account workers who do not work with capital.

This paper is also related to the literature on financial development and occupation choice (See a survey by Buera et al. (2015)). This literature features a binary occupation choice, where agents choose between wage workers and entrepreneurs. The entrepreneurs have production function as in Lucas (1978). Banerjee and Newman (1993) is different in that they include an occupation category similar to own-account workers, but they do not model heterogeneous abilities, and their focus is on borrowing constraint, the typical way of modelling financial development in the literature. The borrowing constraint can be motivated from a moral hazard problem. In my model, the increase in cost of borrowing is similar to the borrowing constraint, but this paper differs by having the extra channel through lower return on savings. Antunes et al. (2008) model both the borrowing constraint and a wedge between cost of borrowing.

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Gollin (2008) is essentially a capital accumulation story. In his model, holding capital stock constant, change in TFP does not change occupation shares. However, holding TFP constant, change in capital stock does affect occupation choice. A higher TFP is associated with a higher steady state capital stock, and with an elasticity of substitution between labour and capital less than one, higher capital stock generates lower share of entrepreneurs.
and return on savings, but as in most other papers, they focus on the binary choice between wage workers and entrepreneurs.

This paper is organized as follows: Section 1.2 presents the observed relationship between occupation shares and net interest margin, an empirical measure of cost of intermediation. Section 1.3 presents a static model with three occupations, and Section 1.4 illustrates this model’s mechanisms and emphasizes the importance of the joint distribution of wealth and ability. Section 1.5 presents a dynamic model with endogenous saving decision, calibrate the model, and then compares the model’s prediction with the data. Section 1.6 concludes.

1.2 Motivating Facts

How does the entrepreneurship rate change with development level? Figure 1.3 shows the negative relationship between the share of entrepreneurs (business owners) and per capita income, using data from the Global Economic Monitor, Adult Population Survey 2013 (GEM APS). The left panel excludes business owners in the agricultural sector while the right panel includes them.

Very often all business owners are regarded as entrepreneurs, regardless of whether they have no paid employees. Using this definition of entrepreneurs, it has a puzzling implication that entrepreneurship rate is higher in developing countries (see for example (Poschke 2015)). However, the small business owners without employees are very different from those who have employees. The former withdraw labour services from the market while the latter actively create jobs for others. For this reason it is reasonable to use the finer classification of employment provided by the ILO. Here, I focus on three employment groups: employees, own-account workers and employers. Employees (wage
workers) are those who get a basic remuneration not directly dependent on the revenue of the employer. Employers are those who hold self-employment jobs and engage one or more person to work for them as employees on a continuous basis. Own-account workers are those who hold self-employment jobs and do not engage employees on a continuous basis. Figure 1.4a shows the correlation between the share of own-account workers and GDP per capita, and Figure 1.4b shows the correlation between the share of employers and GDP per capita.

The correlation is negative for own-account workers and positive for employers. The different correlations suggest that own-account workers and employers need to be treated separately. Furthermore, if we think of employer as a proxy for entrepreneur, this suggests that the entrepreneurship rate will be lower in developing countries.

This paper focuses on the role of financial intermediation efficiency in affecting the occupation shares. A widely used measure of financial intermediation efficiency is the net interest margin. It is the accounting value of bank’s net interest revenue as a share of its average interest-bearing assets. A higher net interest rate margin indicates a large difference between borrowing and saving interest rate, and hence a lower efficiency of financial intermediation.

ILO also reports three other categories: Members of producers’ cooperatives, who hold self-employment jobs in a cooperative producing goods and services, where the members take part on an equal footing in making major decisions concerning the cooperative; Contributing family workers, who hold self-employment jobs in an establishment operated by a related person, with a too limited degree of involvement in its operation to be considered a partner; Workers not classifiable by status. The shares of the members of producers’ cooperatives and the unclassified are negligible, but in some countries, the share of contributing family workers is not negligible in size. However, this paper chooses to focus on own-account worker, wage workers and employers. One reason is that ILO notes for some countries the data on contributing family workers are unreliable.

Demirguc-Kunt et al. (2004), Beck (2007), Antunes et al. (2008), and Antunes et al. (2013) all use net interest margin as a measure of financial efficiency. The measure that directly corresponds to my model would be the interest rate spread, the difference between lending rate and deposit rate. However interest rate spread has limited cross country comparability, so I follow the literature by using the net interest margin. Beck (2007) notes that the main difference between interest rate spreads and net interest margins are lost interest revenue on non-performing loans.
Table 1.1: Conditional correlation

<table>
<thead>
<tr>
<th></th>
<th>2013 share of own-account workers</th>
</tr>
</thead>
<tbody>
<tr>
<td>private credit GDP ratio</td>
<td>-0.0164 (0.0230)</td>
</tr>
<tr>
<td>bank net interest margin</td>
<td>3.266*** (0.426)</td>
</tr>
<tr>
<td>Constant</td>
<td>7.736** (3.901)</td>
</tr>
<tr>
<td>Observations</td>
<td>86</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.539</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Data from ILO aggregate series and WB.

Figure 1.5 plots the observed net interest margin and occupation shares for 2006 and 2013, before and after the peak of the financial crisis. In places of a high interest rate margin, the share of own-account worker is higher and there is almost no correlation between the share of employers and the net interest margin.

Table 1.1 shows that the correlation between the share of own-account workers and bank net interest margins still exists conditional on another widely used measure of financial development, the credit GDP ratio. A similar pattern holds true for other years as well.

Some may argue that one reason there are so many own-account workers in developing countries is that labour forces in those countries have lower human capital level. Here I show some evidence suggesting that, at least, the lower human capital story alone cannot explain all. I use the GEM APS 2013 data to calculate the share of own-account workers within each of three education categories: some secondary education (no degree), secondary degree,

---

For other years with data available, correlation with credit GDP ratio is sometimes significant. Correlation with net interest margin is always significant. Some may argue that people become own account workers because they do not have bank accounts. I have not looked at all countries yet, but one example is Brazil. The share of population with an account in a financial institution increased from 61.1 in 2011 to 71.1 in 2014 for men, and from 51 to 64.8 for women. Yet the share of own account workers actually increased from 23.7 in 2009 to 25.3 in 2014.
and post secondary education. The GEM survey asked business owners how many employees they had. If the answer was zero, I classify this person as an own-account worker. Figure 1.6 plot the share of own-account workers in one education category against the share in another. It shows that in places where a higher share of the less educated become own account workers, a higher share of the better educated also become own account workers. This suggest that in some countries, people of all education categories are more likely to become own-account workers. In the Appendix 1.7.2 I show the education distributions of occupations for a set of countries. Own account workers come from all education categories, despite they have lower education level on average.

Table 1.2 uses the IPUMS data to calculate the share of own-account workers among working age males and regress it on net interest margin and other control variables. It suggests that controlling for TFP and industry share, the correlation between share of own account workers and net interest margin survives.

### 1.3 A Static Model

This section presents a static occupation choice model, taking into account the inefficiency of financial intermediation. The inefficiency is modelled as the fraction of resources lost during the intermediation process. This can be thought of as the cost of intermediation, a deadweight loss. For each unit of capital intermediated, a fraction $1 - \lambda$ is lost, and the fraction $\lambda$ arrives with the borrowers. Let $R_s$ denote the gross return on savings and $R_b$ the cost of borrowing, thus:

$$R_s = \lambda R_b \tag{1.1}$$

where $0 < \lambda < 1$. This creates a wedge between the cost of borrowing and return on savings. While $\lambda$ is treated as an exogenous parameter, both interest
Table 1.2: Share of own account worker regression, IPUMS data

<table>
<thead>
<tr>
<th></th>
<th>share of oaw</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank net interest margin</td>
<td>0.0126**</td>
</tr>
<tr>
<td></td>
<td>(0.00609)</td>
</tr>
<tr>
<td>TFP</td>
<td>-1.41e-05**</td>
</tr>
<tr>
<td></td>
<td>(6.37e-06)</td>
</tr>
<tr>
<td>Share with less than primary education</td>
<td>-0.0682</td>
</tr>
<tr>
<td></td>
<td>(0.453)</td>
</tr>
<tr>
<td>Share with primary education completed</td>
<td>-0.119</td>
</tr>
<tr>
<td></td>
<td>(0.441)</td>
</tr>
<tr>
<td>Share with secondary education completed</td>
<td>-0.329</td>
</tr>
<tr>
<td></td>
<td>(0.513)</td>
</tr>
<tr>
<td>Share of individuals working in wholesale and retail</td>
<td>-0.756*</td>
</tr>
<tr>
<td></td>
<td>(0.407)</td>
</tr>
<tr>
<td>Share of individuals working in manufacturing</td>
<td>-0.594</td>
</tr>
<tr>
<td></td>
<td>(0.414)</td>
</tr>
<tr>
<td>Capital output ratio</td>
<td>0.00697</td>
</tr>
<tr>
<td></td>
<td>(0.0167)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.624</td>
</tr>
<tr>
<td></td>
<td>(0.445)</td>
</tr>
<tr>
<td>Observations</td>
<td>63</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.593</td>
</tr>
</tbody>
</table>

rates $R_b$ and $R_s$ are determined in equilibrium. \[8\]

There is a continuum of measure one agents, indexed by $i$. Agents potentially differ in wealth $a_i$, and ability $\gamma_i$. Ability is drawn from a random distribution $f(\gamma)$, and it affects an agent’s income in every occupation. Based on wealth and ability, each agent chooses the occupation to maximizes total income.

If an agent decides to be a worker, she receives total wage income $\gamma_i w$, where $w$ is the wage rate and $\gamma_i$ can be thought of as the efficiency labour units this agent provides to the labour market. As a wage worker, she supplies all her wealth to financial intermediaries and receives $a_i R_s$ in return. The total income is

$$I_w(a_i) = \gamma_i w + a_i R_s$$  \((1.2)\)

For the agent who becomes an own account worker, the production technology is

$$A_o \gamma_i k_{o,i}$$  \((1.3)\)

Productivity has two components. The aggregate productivity $A_o$ applies to all own account workers. The idiosyncratic productivity is the ability $\gamma_i$. The only labour input is the own account worker’s own time. Given ability $\gamma_i$ and wealth $a_i$, an own account worker chooses capital input $k_{o,i}$ to maximize his income:

$$\max_{k_{o,i}} A_o \gamma_i k_{o,i}^\alpha + \begin{cases} (1 - \delta)k_{o,i} - R_b (k_{o,i} - a_i); & \text{borrow} \\ (1 - \delta)a_i; & \text{own wealth} \\ (1 - \delta)k_{o,i} + R_s (a - k_{o,i}) + R_s a_i; & \text{save} \end{cases}$$  \((1.4)\)

where $\delta$ is the depreciation rate. She can either borrow, use her own wealth or save part of her wealth with intermediaries.

---

\[8\] This paper focuses on how interest rate spread affects occupation choices rather than explaining why it varies across countries.
The maximized total income, depending on wealth $a_i$ and ability $\gamma_i$, is the following:

$$I_o(a_i, \gamma_i) = \begin{cases} (1 - \alpha) (\gamma_i A_o)^{\frac{1}{1-\alpha}} \left( \frac{\alpha}{R_b + \delta - 1} \right)^{\frac{\alpha}{1-\alpha}} + R_b a_i; & a_i < k_o(\gamma_i, A_o) \\ \gamma_i A_o \alpha^\alpha + (1 - \delta) a_i; & k_o(\gamma_i, A_o) < a_i < k_o(\gamma_i, A_o) \\ (1 - \alpha) (\gamma_i A_o)^{\frac{1}{1-\alpha}} \left( \frac{\alpha}{R_s + \delta - 1} \right)^{\frac{\alpha}{1-\alpha}} + R_s a_i; & a_i > k_o(\gamma_i, A_o) \end{cases}$$

with the corresponding capital demand:

$$k_o(a_i, \gamma_i) = \begin{cases} a(\gamma_i, A_o); & a_i < k_o(\gamma_i, A_o) \\ a_i; & k_o(\gamma_i, A_o) < a_i < k_o(\gamma_i, A_o) \\ a(\gamma_i, A_o); & a_i > k_o(\gamma_i, A_o) \end{cases}$$

where $k_o(\gamma_i, A_o) = \left( \frac{\alpha \gamma_i A_o}{R_b + \delta - 1} \right)^{\frac{1}{1-\alpha}}$ and $\overline{k_o}(\gamma_i, A_o) = \left( \frac{\alpha \gamma_i A_o}{R_s + \delta - 1} \right)^{\frac{1}{1-\alpha}}$.

Figure 1.7 illustrates the total income of an own-account worker for a given borrowing interest rate $R_b$. When financial intermediation is perfect ($\lambda = 1$), the total income $I_o(a_i, \gamma_i)$ is a straight line and $k_o(\gamma_i, A_o)$ is equal to $\overline{k_o}(\gamma_i, A_o)$. Agents with wealth below $k_o(\gamma_i, A_o)$ borrow and any agents with wealth above $k_o(\gamma_i, A_o)$ save. When financial efficiency deteriorates ($\lambda < 1$), return on savings $R_s$ falls for any given level of borrowing interest rate $R_b$\footnote{To make the difference between perfect intermediation case and imperfect intermediation case more visible and for illustration purposes, I use the $\lambda$ value of 0.5}. The total income of net borrowers, those with wealth level below $k_o(\gamma_i, A_o)$, is not affected. They still borrow up to $k_o(\gamma_i, A_o)$, and pay interest rate $R_b$ on the borrowed capital $k_o(\gamma_i, A_o) - a_i$. For agents with wealth in between $k_o(\gamma_i, A_o)$ and $\overline{k_o}(\gamma_i, A_o)$, investing all wealth in the own-account business generates a marginal return to capital in between $R_b$ and $R_s$, so they invest all own wealth and do not participate in the capital market. For agents with wealth above $\overline{k_o}(\gamma_i, A_o)$, investing more than $\overline{k_o}(\gamma_i, A_o)$ in the own-account business would
generate a marginal return lower than the market return on savings \( R_s \), so they invest \( k_o(\gamma_i, A_o) \) in their own-account business and save the rest \( a_i - k_o(\gamma_i, A_o) \) with financial intermediaries to earn return \( R_s \). These agents will be net savers.

For an agent who becomes an employer, the production function is:

\[
A_e \gamma_i k_{e,i}^\alpha l_{e,i}^\beta
\]

(1.7)

with \( 0 < \alpha + \beta < 1 \). \( A_e \) is the aggregate productivity of all employers. The employers choose capital \( k_{e,i} \) and labour \( l_{e,i} \), depending on ability and wealth. Similar to the own-account workers, an employer can borrow, save or use own capital. Given ability \( \gamma_i \) and wealth \( a_i \), an employer chooses \( k_{e,i} \) and \( l_{e,i} \) to maximize the income:

\[
\max_{(k_{e,i}, l_{e,i})} A_e \gamma_i k_{e,i}^\alpha l_{e,i}^\beta + \begin{cases} 
(1 - \delta)k_{e,i} - wl_{e,i} - R_b (k_{e,i} - a_i); & \text{borrow} \\
(1 - \delta)a_i - wl_{e,i}; & \text{own wealth} \\
(1 - \delta)k_{e,i} - wl_{e,i} + R_s (a - k_{o,i}); & \text{save}
\end{cases}
\]

(1.8)

Similar to own-account workers, employers can either borrow, use their own wealth or save.

The maximized total income of an employer with ability \( \gamma_i \) and wealth \( a_i \) is:

\[
I_e(a_i, \gamma_i) = \begin{cases} 
(1 - \alpha - \beta) \left[ \gamma_i A_e \left( \frac{a}{R_b + \delta - 1} \right)^\alpha \left( \frac{\beta}{w} \right)^\beta \right]^{\frac{1}{1 - \alpha - \beta}} + R_b a_i; & a_i < k_e(\gamma_i, A_o) \\
(1 - \beta) \left[ \gamma_i A_e \left( \frac{\beta}{w} \right)^\beta a_i^\alpha \right]^{\frac{1}{1 - \beta}} + (1 - \delta)a_i; & k_e(\gamma_i, A_o) < a_i < k_e(\gamma_i, A_o) \\
(1 - \alpha - \beta) \left[ \gamma_i A_e \left( \frac{\beta}{w} \right)^\beta a_i^\alpha \right]^{\frac{1}{1 - \alpha - \beta}} + R_s a_i; & a_i > k_e(\gamma_i, A_o)
\end{cases}
\]

(1.9)
with the labour demand:

\[ l(a_i, \gamma_i) = \begin{cases} 
\gamma_i A e \left( \frac{\alpha}{R_b + \delta - 1} \right)^{\alpha} \left( \frac{\beta}{w} \right)^{1-\alpha} ; & a_i < k_e(\gamma_i, A_o) \\
\gamma_i A e \left( \frac{\alpha}{R_b + \delta - 1} \right)^{\alpha} \left( \frac{\beta}{w} \right)^{1-\alpha} ; & k_e(\gamma_i, A_o) < a_i < \overline{k}_e(\gamma_i, A_o) \\
\gamma_i A e \left( \frac{\alpha}{R_s + \delta - 1} \right)^{\alpha} \left( \frac{\beta}{w} \right)^{1-\alpha} ; & a_i > \overline{k}_e(\gamma_i, A_o) 
\end{cases} \]

and the capital demand:

\[ k_e(a_i, \gamma_{e,i}) = \begin{cases} 
k_e(\gamma_{o,i}, A_o) ; & a_i < k_e(\gamma_i, A_o) \\
a_i ; & k_e(\gamma_i, A_o) < a_i < \overline{k}_e(\gamma_i, A_o) \\
\overline{a}(\gamma_{e,i}, A_e) ; & a_i > \overline{k}_e(\gamma_i, A_o) 
\end{cases} \]

where

\[ k_e(\gamma_i, A_o) = \left[ \gamma_i A e \left( \frac{\alpha}{R_b + \delta} \right)^{1-\beta} \left( \frac{\beta}{w} \right)^{\beta} \right]^{\frac{1}{1-\alpha-\beta}} \]

and

\[ \overline{k}_e(\gamma_i, A_o) = \left[ \gamma_i A e \left( \frac{\alpha}{R_s + \delta} \right)^{1-\beta} \left( \frac{\beta}{w} \right)^{\beta} \right]^{\frac{1}{1-\alpha-\beta}} . \]

When the employer is using her own wealth only, the labour demand does not depend on the market interest rates. Figure 1.8 shows the income of an employer, for the case where financial intermediation is perfect \((\lambda = 1)\) and and the case where it is imperfect \((\lambda = 0.5)\), for given borrowing interest rates \(R_b\) and wage rate \(w\). In the perfect case \((\lambda = 1)\), income is a straight line. In the imperfect case \((\lambda < 1)\), those with wealth below \(k_e(\gamma_i, A_o)\) borrow and become net borrowers, but those with wealth in between \(k_e(\gamma_i, A_o)\) and \(\overline{k}_e(\gamma_i, A_o)\) use own wealth and do not participate in capital market. Those with wealth above \(\overline{k}_e(\gamma_i, A_o)\) use exactly \(\overline{k}_e(\gamma_i, A_o)\) in their business and save \(a_i - \overline{k}_e(\gamma_i, A_o)\) in the capital market.

One thing to note is that here I do not model the fixed cost of operating businesses. The fixed cost component is more important if the goal is to
match establishment size or firm size distribution, but that is not the object here. Moreover, in this paper, financial development is modelled as the cost of intermediation instead of a borrowing constraint. Agents can borrow as much as they wish at a given borrowing interest rate; adding an extra fixed cost would simply mean they need to borrow more.

1.3.1 Perfect Financial Intermediation

In this case, financial intermediation incurs no cost at all, and the borrowing interest rate $R_b$ is the same as return on savings $R_s$. Given wealth $a_i$, interest income $R_s a_i$ is the same for all occupations. Thus when deciding on occupations, agents only consider the occupation income, the difference between total income and interest income $R_s a_i$. In this case it can be explicitly written out. The occupation income of a wage worker is $\gamma_i w_i$, the occupation income of an own-account worker is $(1 - \alpha) \left( \gamma_{o,i} A_o \right)^{\frac{1}{1-\alpha}} \left( \frac{\alpha}{R_s + \delta - 1} \right)^{\frac{\alpha}{1-\alpha}}$, and the occupation income of an employer is $(1 - \alpha - \beta) \left( \gamma_{e,i} A_e \left( \frac{\alpha}{R_s + \delta - 1} \right)^{\alpha} \left( \frac{\beta}{w} \right)^{\beta} \right)^{\frac{1}{1-\alpha-\beta}}$. An own-account worker claims a share $(1 - \alpha)$ of total output, higher than the share an employer claims, $(1 - \alpha - \beta)$. However, as ability increases the output of an employer increases by much more than the output of an own-account worker. This is because the ability is raised to power $\frac{1}{1-\alpha}$ for own-account workers, and it is raised to $\frac{1}{1-\alpha-\beta}$ for employer. As a result, as ability increases, income as an employer exceeds that of an own-account worker. This is illustrated in Figure 1.9, in which there are two ability cut-offs shown. The agents with ability higher than $\gamma_h$ will be employers, those with ability lower than $\gamma_l$ will be wage workers and those in between will choose to be own-account workers.

Figure 1.10 plots the capital demand of own-account workers and employers for agents of different abilities. As shown, when ability is extremely low, the capital demand of an own-account worker is actually higher than that of the employer. As ability increases, the amount of capital an employer can manage
increases at a much faster rate than that of the own-account worker. Even for the very capable agents, the amount of capital they manage as an own-account workers is small. This captures the idea that own-account workers work with small amount of capital.

The labour market clearing condition is:

\[
\int_{\gamma_l}^{\gamma_l^\text{max}} \gamma f(\gamma) d\gamma = \int_{\gamma_h}^{\gamma_h^\text{max}} \left[ \gamma_i A_e \left( \frac{\alpha}{R_s + \delta - 1} \right)^{\alpha} \left( \frac{\beta}{w} \right)^{1-\alpha} \right]^{\frac{1}{\alpha}} f(\gamma) d\gamma
\]

(1.10)

where the left hand side is the total efficiency labour units supplied to the market by agents choosing to be wage workers, and the right hand side is the total labour demand from employers.

The capital market clearing condition is:

\[
K = \int_{\gamma_l}^{\gamma_h} \left[ \frac{\alpha \gamma_i A_o}{R_s + \delta - 1} \right]^{\frac{1}{1-\alpha}} f(\gamma) d\gamma + \int_{\gamma_h}^{\gamma_h^\text{max}} \left[ \gamma_i A_e \left( \frac{\alpha}{R_s + \delta - 1} \right)^{1-\beta} \left( \frac{\beta}{w} \right)^{\beta} \right]^{\frac{1}{1-\alpha-\beta}} f(\gamma) d\gamma
\]

(1.11)

where the left hand side is the total wealth in the economy, the sum of individual wealth \(a_i\). Under perfect financial intermediation, no resource is lost in intermediation, and all wealth become capital used in production, as shown on the right hand side. The first term on right hand side is the total capital used by all own-account workers, and the second term is the capital used by all employers.

The lower ability cut-off \(\gamma_l\) is implicitly determined by

\[
\gamma_l w = (1 - \alpha)(\gamma_l A_o)^{\frac{1}{\alpha}} \left( \frac{\alpha}{R_s + \delta - 1} \right)^{\frac{\alpha}{\alpha}}
\]

(1.12)

which states that an agent with ability \(\gamma_l\) must be indifferent towards becoming a wage worker or an own-account worker. The higher ability \(\gamma_h\) is implicitly
determined by

\[(1-\alpha)(\gamma_h A_o)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{R_s + \delta - 1}\right)^{\frac{1}{1-\alpha}} = (1-\alpha-\beta) \left[ \gamma_h A_e \left(\frac{\alpha}{R_s + \delta - 1}\right)^{\alpha} \left(\frac{\beta}{w}\right)^{\beta} \right]^{\frac{1}{1-\alpha-\beta}} \]

which states that an agent with ability $\gamma_h$ is indifferent towards becoming an
own-account worker or an employer.

To summarize, for a given joint distribution of ability and wealth, the equilib-
rium is composed of the market interest rates $R_b = R_s$, a wage rate $w$, and
the two ability cut-offs $\gamma_l$ and $\gamma_h$ such that equations (1.10) - (1.13) are all
satisfied. The total output $Y$ in this economy is:

\[ Y = Y_o + Y_e \]
\[ = \int_{\gamma_l}^{\gamma_h} \left[ \gamma A_o \left(\frac{\alpha}{R_s + \delta - 1}\right)^{\alpha} \right] f(\gamma)d\gamma + \int_{\gamma_h}^{\gamma_{max}} \left[ \gamma A_e \left(\frac{\alpha}{R_s + \delta - 1}\right)^{\alpha} \left(\frac{\beta}{w}\right)^{\beta} \right]^{\frac{1}{1-\alpha-\beta}} f(\gamma)d\gamma \]
\[ (1.14) \]

where $Y_o$ is the total output from all own-account workers and $Y_e$ is the total
output from all employers.

**Proposition 1.** In the case of perfect financial intermediation, the level and
distribution of wealth do not matter for occupation choice.

**Proof.** Rearrange equation (1.12) as:

\[ \gamma_l = \left\{ \frac{1}{A_o} \left[ w^{1-\alpha} (R_s + \delta - 1)^{\alpha} \right]^{\frac{1}{1-\alpha}} \left[ \frac{1}{\alpha^{1-\alpha}} \right] \right\}^{\frac{1}{1-\alpha}} \]
\[ (1.15) \]

and equation (1.13) as:

\[ \gamma_h = \left\{ \frac{A_o (1-\alpha)^{1-\alpha} \alpha^\alpha}{A_e (1-\alpha-\beta)^{1-\alpha-\beta} \alpha^\alpha \beta^\beta} \right\}^{\frac{1-\alpha-\beta}{1-\alpha}} \left[ w^{1-\alpha} (R_s + \delta - 1)^{\alpha} \right] \]
\[ (1.16) \]

Let $x = w^{1-\alpha} (R_s + \delta - 1)^{\alpha}, \gamma_l = \gamma_l(x, A_o)$ and $\gamma_h = \gamma_h(x, A_e, A_o)$ and substitute
into equation (1.10):

\[
\gamma_l(x,A_o) \hat{1} \gamma_i = \int_1^{\gamma_{\max}} f(\gamma) \frac{\gamma_i A_e \alpha^{\beta - 1 - \alpha}}{x} \gamma_1 \gamma_\beta \hat{1} \gamma \hat{1} d\gamma \tag{1.17}
\]

Given the parameter values, equation (1.17) solves \(x\), the only unknown. This value of \(x\) does not depend on wealth distribution or quantity of wealth. Hence \(\gamma_l\) and \(\gamma_h\) do not depend on wealth distribution or quantity of wealth. \(\square\)

Even though wealth level does not affect the occupation outcome under perfect intermediation, it does affect the equilibrium wages rate and interest rates. An increase in aggregate wealth \(K\) leads to higher wage rate \(w\) and lower interest rates \(R_b\) and \(R_s\). \(^{10}\)

**Proposition 2.** In the case of perfect financial intermediation, if \(A_o\) and \(A_e\) simultaneously increase by the same percentage, \(\gamma_l\) and \(\gamma_h\) remain unchanged and output increases by the same percentage. \(^{11}\)

**Proof.** Equation (1.17) implies that when \(A_o\) and \(A_e\) increases by the same percentage, \(x\) increase by the same percentage. Then, equations (1.15) and (1.16) imply that \(\gamma_l\) and \(\gamma_h\) remains unchanged. \(\square\)

A sector neutral productivity increase does not affect occupation choice. \(^{12}\)

However when \(A_e\) increases relative to \(A_o\), \(\gamma_l\) will increase and \(\gamma_h\) will fall, resulting in higher shares of wage workers and employers, and a lower share of own-account workers. If \(A_e\) continues to increase relative to \(A_o\), then eventually there will be no own-account workers in the economy, but wage workers and

\(^{10}\)The result that quantity of wealth does not matter for occupation choice under perfect financial intermediation is due to the Cobb-Douglas production function. In this case, the elasticity of substitution between capital and labour is equal to one. Lucas (1978) shows that capital accumulation lead to a declining share of entrepreneurs if and only if the elasticity of substitution was less than one. Gollin’s(2008) quantitative result is based on capital accumulation and a CES production function with elasticity less than one.

\(^{11}\)This is also true for the case when financial intermediation is not perfect.

\(^{12}\)This is also true even when financial intermediation is imperfect.
employers will always exist. On the other extreme if \( A_o \) increases indefinitely relative to \( A_e \), then, eventually, to be an own-account worker will be the only occupation agents choose.

### 1.3.2 Imperfect Financial Intermediation

In the case of imperfect intermediation, \( \lambda \) is smaller than one and some resource is lost in transit. Both wealth and ability matter for occupation decisions. Figure 1.11 illustrates this. The dotted lines represent the ability cut-offs under perfect financial intermediation, and they are independent of wealth. When \( \lambda \) is less than one, a wedge exist between cost of borrowing \( R_b \) and return on savings \( R_s \). The ability cut-offs \( \gamma_l(a) \) and \( \gamma_h(a) \) now depend on the wealth level \( a_i \). As wealth \( a_i \) decreases, both \( \gamma_l(a) \) and \( \gamma_h(a) \) increase, meaning that the ability required to become own-account workers or employers will be higher if agents have less wealth. For agents with medium-low abilities close to \( \gamma_l(a) \), as wealth increases, the decrease in income due to a lower return on savings becomes larger, so they are more likely to become own-account workers. For agents with medium high abilities close to \( \gamma_h(a) \), if they choose to be own account workers, the amount of wealth handed to intermediaries is increasing in total wealth, because the amount of capital an own-account workers can manage is limited. The low return on savings \( R_s \) reduces what they receive from intermediaries, and to avoid this, they become employers, which allows them to manage larger amount of capital and hire labour to make their capital more productive.

For given wage rate \( w \) and interest rates \( R_b \) and \( R_s \), agents decide on their occupations. Depending on agents’ saving and borrowing decisions, agents can be classified into seven finer categories: \( o(a_i, \gamma_i) = w \) for those who choose to be wage workers, \( o(a_i, \gamma_i) = o_b \) for own-account workers who borrow, \( o(a_i, \gamma_i) = o_a \) for own-account workers who use their own wealth, \( o(a_i, \gamma_i) = o_s \) for own-
account workers who save, \( o(a_i, \gamma_i) = e_b \) for borrowing employers, \( o(a_i, \gamma_i) = e_a \) for employers who use their own wealth and \( o(a_i, \gamma_i) = e_s \) for employers who save.

For a given joint distribution of wealth and ability \( G(a, \gamma) \), the equilibrium wage rate \( w \) and interest rates \( R_b \) and \( R_s \) satisfy the interest wedge \( R_s = \lambda R_b \), the labour market clearing condition and capital market clearing condition. The labour market clear is given by

\[
\int \int \gamma G(da, d\gamma) = \int \int l(\gamma_i, a_i) G(da, d\gamma)
\]

where the left hand side is the supply of wage workers and the right hand side is the demand from the employers. The total capital supplied to the market is:

\[
K_s = \int \int a_i G(da, d\gamma) + \int \int (a - k_o(a_i, \gamma_i)) G(da, d\gamma)
\]

\[
+ \int \int (a_i - k_e(a_i, \gamma_i)) G(da, d\gamma)
\]

It includes the capital supplied to the market by wage workers, own-account workers who save and employers who save. The total demand for capital from the market is

\[
K_d = \int \int (k_o(a_i, \gamma_i) - a_i) G(da, d\gamma) + \int \int (k_e(a_i, \gamma_i) - a_i) G(da, d\gamma)
\]

For business owners who want to use more capital than they have, they borrow from the market. The first term is the amount own-account workers want to borrow from the market and the second term is the amount employers wants.
to borrow. Then the capital market clearing condition is:

$$\lambda K_s = K_d$$  \hspace{1cm} (1.19)$$

which states that the total amount of capital supplied to the market multiplied by the fraction successfully intermediated is equal to the amount agents want to borrow.

The output from the employer sector is

$$Y_e = \int \int \left[ \gamma_i A_e \left( \frac{\alpha}{R_s + \delta - 1} \right)^{\alpha} \left( \frac{\beta}{w} \right)^{\beta} \right] \frac{1}{1 - \alpha - \beta} G(da, d\gamma)$$

$$+ \int \int \left[ \gamma_i A_e \left( \frac{\beta}{w} \right)^{\beta} a_i^o \right] \frac{1}{1 - \beta} G(da, d\gamma)$$

$$+ \int \int \left[ \gamma_i A_e \left( \frac{\alpha}{R_b + \delta - 1} \right)^{\alpha} \left( \frac{\beta}{w} \right)^{\beta} \right] \frac{1}{1 - \alpha - \beta} G(da, d\gamma)$$

which is the sum of output from employers who save, employers who use their own wealth and employers who borrow. The output from the own-account sector is:

$$Y_o = \int \int \left[ \gamma_i A_o \left( \frac{\alpha}{R_s + \delta - 1} \right)^{\alpha} \right] \frac{1}{1 - \alpha} G(da, d\gamma) + \int \int A_o \gamma_i a_i^o G(da, d\gamma)$$

$$+ \int \int \left[ \gamma_i A_o \left( \frac{\alpha}{R_b + \delta - 1} \right)^{\alpha} \right] \frac{1}{1 - \alpha} G(da, d\gamma)$$

which is the sum of output from own-account workers who save, who use their own wealth, and who borrow. Then the total output in this economy is $Y = Y_e + Y_o$.

Whenever financial intermediation is imperfect, the marginal product of capital across business is not equalized and ranges between $R_b$ and $R_s$.  

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1.4 Role of Financial Intermediation in the Static Model

In this section, I present some comparative statics of the static model to illustrate the model’s mechanisms. The parameters used are from Table 1.3 and they will be explained in Section 1.5.

1.4.1 Role of Financial Intermediation

As $\lambda$ falls, financial intermediation efficiency deteriorates. For given borrowing interest rate $R_b$ and wage rate $w$, the decrease in $\lambda$ causes the return on savings $R_s$ to decline, so anyone with positive saving sees decline in total income.

A wage worker with sufficient high wealth is more likely to become an own-account workers to manage his or her own wealth. This reduces capital and labour supply to the market.

An own-account worker who saves with intermediaries is more likely to expand the size of their own account business or switch to become an employer. Figure 1.12 illustrates this by showing the income of an agent who is indifferent between becoming an own-account worker and an employer under perfect financial intermediation ($\lambda = 1$). The left panel of Figure 1.12 shows that the income from becoming an own-account worker and an employer exactly coincide. The right panel illustrates when $\lambda$ falls to 0.5, keeping borrowing interest rate $R_b$ and wage rate $w$ constant. This leads to a fall in return on savings $R_s$. If the agent has little wealth and borrows to become an own-account worker or employer, then this agent is not affected by the fall of $\lambda$. However if his wealth is high enough such that he saves some wealth with intermediaries as an own-account workers, then he would switch to be an employer when $\lambda$ falls. The income for both own-account workers and employers decreases but
the income for employers declines by less because employers can manage more wealth. This increases capital and labour demand.

As the return on savings falls, employers who save will expand their businesses and save less with intermediaries. This also increases the capital and labour demand.

To summarize, for a given interest rate $R_b$ and wage rate $w$, a fall in intermediation efficiency leads to excess demand for capital and labour. Interest rates $R_b$ and $R_s$ and wage rate $w$ will adjust to clear the market. An increase in wage rate, given the borrowing interest rate, reduces the gap in both the labour and capital markets. As the wage rate increases, becoming a wage worker becomes more attractive. When own-account workers switch to be wage workers, they supply their wealth to capital market, increasing both the labour and capital supply. The higher wage rate also reduces capital and labour demand from employers. Hence, it helps to clear the labour and capital market.

An increase in the borrowing interest rate does the same. An increase in borrowing interest rate, given financial intermediation efficiency $\lambda$, also increases the return on savings. Hence, own-account workers and employers borrow less and reduce the amount of own wealth invested in businesses, increasing the supply to capital market. The labour demand also falls with higher borrowing cost. Some marginal own-account workers switch to be wage workers. Thus, the higher interest rate also reduces both the gap in the labour and capital market. As a result, what happens to the equilibrium borrowing interest rate $R_b$ and wage rate $w$ will depend on parameters and the joint distribution of wealth and ability, but to clear both capital and labour markets, either interest rate $R_b$ or wage rate $w$ needs to increase.

For illustration Figure 1.13 shows the effects of financial intermediation, for a given joint distribution between wealth and ability. The distribution used here is the stationary steady state distribution obtained from the perfect in-
termediation case from the dynamic model in Section 1.5. For now, I just take this distribution as given. As financial intermediation deteriorates, borrowing interest rate increases and saving interest rate decreases. The share of own-account workers increases and the share of wage workers decreases. With this joint distribution, the channels that increase the share of own-account workers dominate. The fall in return on savings leads medium-low ability agents to become own-account workers and manage some own wealth.

1.4.2 Role of Wealth Distribution

To illustrate the importance of wealth distribution, I change the distribution used in Section 1.4.1 but keep the aggregate wealth the same, and then I study how the model responds to changes in financial intermediation. For agents with abilities lower than the 80 percentile, I change their wealth to zero. I then redistribute this wealth equally to agents with abilities in the top 20% percentile. Under perfect financial intermediation, this change of distribution does not affect occupation choices. Figure 1.14 presents the results responding to changes in financial intermediation.

Figure 1.14 and Figure ?? have different occupation shares, interest rates and wage rate when $\lambda < 1$. In Figure 1.14 the share of own-account workers and employers sum up to 20% at most.

The reason is that the bottom 80% agents have zero wealth. A fall in return on savings does not affect their interest income because they have zero wealth anyway. Their decision is between becoming a wage worker or a borrowing own-account worker. However the cost of borrowing increases with a fall in financial intermediation, so these agents remain to be wage workers. At the same time, some agents with abilities in the top 20% switch from own-account workers to employers, because this allow them to manage more wealth (the re-distribution increases their wealth).
As stated, the only difference between Figure 3 and Figure 1.14 is the wealth distribution used. In the latter the vast majority of the agents, the medium-low ability agents, have zero wealth. This points to the importance of the wealth holdings of medium-low ability agents in determining occupations. This is one of the key differences between this model and a model without own account workers. In the standard model with wage workers and entrepreneurs only, the wealth of the medium low ability agents are not important. Their abilities are too low to become entrepreneurs, so they always become wage workers. Their wealth does not directly affect their occupation choices except through general equilibrium effects on the wage rate and interest rate(s). Hence, in the binary choice model, frictions affect only the marginal entrepreneurs. Given that entrepreneurial ability is rare, the agents on this margin constitute a small share of the population. The large share of medium-low ability agents do not respond to frictions. This limits the magnitude of occupation changes induced by frictions. However, in this paper, becoming an own-account workers is an option. This introduces an extra margin. Frictions affect (i) agents who are on the margin between own account workers and employers, the medium-high ability agents, and (ii) those who on the margin between wage workers and own-account workers, the medium-low ability agents. Given the relatively large share of medium-low ability agents, this model has the potential to generate large occupation changes responding to frictions.

As illustrated, the joint distribution of wealth and ability is the key for understanding occupation shares under imperfect financial intermediation, but it cannot be directly observed. I now turn to a dynamic model where agents endogenously choose saving and wealth.
1.5 A Dynamic Model

This section presents a dynamic model where wealth decision is endogenous. At the beginning of each period, all agents receive a shock to ability. With probability $\tau$ agents keep their abilities, and with probability $1 - \tau$ they receive an independent new draw of ability.

After the ability shock, agents learn if they die at the end of this period. All agents have equal probability of death $d$. The value function of an agent who will die at the end of the period is given by:

$$V_d(\gamma, a) = \frac{c^{1-\sigma}}{1-\sigma} + \frac{a'^{1-\sigma}}{1-\sigma}$$

where $\phi$ captures the bequest motive. For agents who survive to the next period, the value function is given by:

$$V_s(\gamma, a) = \frac{c^{1-\sigma}}{1-\sigma} + \rho \{ \tau (1 - d) V_s(\gamma, a') + \tau d V_d(\gamma, a') \}$$

$$+ (1 - \tau) (1 - d) E_{\gamma'} [V_s((\gamma', a')] + (1 - \tau) d E_{\gamma'} [V_d(\gamma', a')]$$

where $\rho$ is the discount factor. The four terms inside the large bracket correspond to the future value if this agent, at the beginning of next period, receives no ability shock or death shock, receives no ability shock but receives a death shock, receives an ability shock but no death shock, and receives both shocks.

\[13\]If agent are infinity lived, and ability remains forever, the equilibrium features financial autarky. If agents are infinitely lived, but face shocks to ability, in equilibrium, low ability agents saves nothing. This is because the expected future ability will be higher than today, and if anything, they want to borrow against future income. In a case when financial intermediation deteriorates, borrowing becomes more expensive, but agents can still borrow as much as they wish.
The budget constraints are the same for all agents irrespective of death shock:

\[ c + a' = I(a, \gamma) \]
\[ I(a, \gamma) = \max\{I^w(a, \gamma), I^o(a, \gamma), I^e(a, \gamma)\} \]

where \( I^w(a, \gamma), I^o(a, \gamma), I^e(a, \gamma) \) are as described before. If an agent does not die at the end of the period, this agent carries wealth \( a' \) to the next period. If an agent dies, \( a' \) is the bequest left to offspring. The offspring inherit the wealth and ability, and when they join the labour market at the beginning of next period, they receive an ability shock like all other agents.

In this setting the death shock is irrelevant for occupation choices. When agents choose occupations, their state variables include wealth \( a \) and ability \( \gamma \). A stationary equilibrium is composed of: a borrowing interest rate \( R_b \), a saving interest rate \( R_s \), a wage rate \( w \), an invariant joint distribution of wealth and abilities \( G(a, \gamma) \), occupation policy function \( o(a, \gamma) \), consumption policy function \( c_t(a, \gamma) \) and saving policy function \( a_{t+1}(a, \gamma) \) such that:

(i) \( R_s = \lambda R_b \);

(ii) agents optimally make occupation, consumption and saving decisions.

(iii) the labour market clears as described by equation (1.18).

(iv) the capital market clears. This requires the total capital demand from those who borrow to be equal to the total capital supply net of the intermediation cost, as described in equation (1.19).

In this model, saving behaviour will depend on ability. For the highest ability agents, they save because of the ability shock and the death shock. When they receive a shock to ability, they are likely to draw an ability lower than their current ability, and their income would be smaller. They save for this uncertainty in income. They also save because they care about bequest. For the
lowest ability agents, the death shock and ability shock work in the opposite direction. If they receive a shock to ability, the chances are that the new ability will be higher than the current one, generating a higher income. They have the incentive to borrow against that future income. However, similar to high ability agents, they also have the incentive to save for bequest.

The model is calibrated such that under perfect financial intermediation, it matches moments of the US economy. Then I vary the financial intermediation parameter to country specific level. This allows me to study the role of financial intermediation alone, holding all other parameters constant.

1.5.1 Calibration

I calibrate the model such that under perfect intermediation, the model matches occupation shares and certain moments observed in the US. The model is calibrated to match annual data. Following Buera et al. (2011) I assume $\gamma_i$ has Pareto distribution. With an upper bound $\gamma_{\text{max}}$, the probability distribution is given by:

$$f(\gamma) = \frac{\eta \gamma^{-(1+\eta)}}{1 - \gamma_{\text{max}}^{-(1+\eta)}}$$

(1.21)

The parameters of the model include $\alpha$, $\beta$, $\delta$, $A_o$, $A_e$, $\gamma_{\text{max}}$, $\eta$, $\rho$, $d$ and $\sigma$. Table 1.3 summarises the baseline parameter values. Three parameters are exogenously chosen. This includes $\sigma$, $\delta$ and $\rho$. The relative risk aversion parameter $\sigma$ is 1.5, a standard value in the literature. The annual capital depreciation rate $\delta$ is set to be 0.55, and the annual utility discount $\rho$ is set to 0.98.

Two parameters are calibrated outside the model. The probability of death $d$

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14 De Nardi (2004) uses the same value in a framework with bequest.
is 0.22, implying an average working life of about 45 years. The probability of keeping the existing ability \( \tau \) is chosen such that a wage worker remains a wage worker again with probability 0.997. It is set to match the occupation transition in \cite{Beckhusen2014}.\footnote{Beckhusen (2014) presents monthly transitions. I use this to calculate the probability that this agent will be in the same occupation after 12 months. The probability that the agent will remain self-employed after one month is the same as the probability that the agent will remain self-employed after 12 months.}

The other parameters are model calibrated. The capital income share is calibrated to match an aggregate capital output ratio of 3.1. Under perfect financial intermediation, capital output ratio is:

\[
\frac{K}{Y} = \frac{\alpha}{R_b + \delta - 1}
\]

Targeting an interest rate of \( R_b - 1 = R_s - 1 = 0.04 \), together with the annual capital depreciation rate, the implied value of \( \alpha \) is 0.299, close to the standard capital share.

For the two aggregate productivity, I normalize \( A_o \) to be one. As shown in Proposition 2, the levels of \( A_o \) and \( A_e \) do not matter. \( A_e \) and \( \gamma_{\text{max}} \) are chosen such that, given the other parameters, the occupation shares match the observed ones in the US. The target values are those reported from the BLS report. In 2015, US has 89.9% wage workers, 7.6% own-account workers and 2.5% employers. \( \beta \) and \( \eta \) are set jointly to match two targets. The first target is the total compensation of employees as a percent of GDP, which was between 53\% to 54\% from 2010 to 2015\footnote{Calculated from BEA Table 1.10. Gross Domestic Income by Type of Income.}. Here I target a value of 53\%.

The second target is the labour income of the top 1\%. Piketty et al. (2016) report that the top 1\% income share in the US is around 21\% but around 9\% is labour related income. In the model’s perfect financial intermediation case, occupation income and capital income can be separated. I match the model’s top 1\% occupation income to the labour income in the data.
Table 1.3: Parameters

<table>
<thead>
<tr>
<th>Exogenously set</th>
<th>Calibrated</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>1.5</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.055</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.299</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.98</td>
</tr>
<tr>
<td>$d$</td>
<td>0.022</td>
</tr>
<tr>
<td>$A_o$</td>
<td>1</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.97</td>
</tr>
<tr>
<td>$A_e$</td>
<td>1.0115</td>
</tr>
<tr>
<td>$\gamma_{max}$</td>
<td>2.7884</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.579</td>
</tr>
<tr>
<td>$\eta$</td>
<td>7.9</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Finally, the incentive of bequest parameter $\phi$ is chosen such that, given all the other parameters, it generates an annual interest rate of 4% in the perfect financial intermediation case ($R_b = R_s = 1.04$).

1.5.2 Model Prediction and Data

Figure 1.15 summarizes the model’s responses to changes in the financial intermediation efficiency $\lambda$ while keeping all other parameters the same. I focus on $\lambda$ values ranging between 0.9 and 1, as this generates the relevant net interest margin observed in the data. When comparing the model to the data, I match the model’s prediction of the interest rate spread with the net interest margin in the data. In the model, interest rate spread is $R_b - R_s = (1 - \lambda)R_b$, so $\lambda$ does not directly correspond to $R_b - R_s$. For each country, I find the $\lambda$ value such that the model generated interest rate spread $(1 - \lambda)R_b$ is equal to the net interest rate margin observed in the data. Each country is assigned a $\lambda$ value.

As $\lambda$ falls to 0.90, the share of wage workers decreases from about 90% to 64.8%, the share of own-account workers increases from 7% to 33.4%, and
the share of employers falls from 2.5% to about 1.8%. With the fall of $\lambda$, the output in the own-account worker sector increases and the share of the output in the employer sector falls. Overall output falls by about 17.72%. Relative to the occupation share changes, the effect on output is modest. One reason is that most of the occupation change comes from the medium-low ability agents. When they change occupations, the impact on output is, thus, relatively modest.

The magnitude of share of occupation change is roughly consistent with the data. To compare the model with the data, Figure 1.16 and Figure 1.17 plot the model’s prediction of the share of own-account workers and wage workers against the observed data, focusing on own-account workers and wage workers. For own-account workers, the correlation between the model prediction and data is about 0.75 and for wage workers it is about 0.7.

1.5.3 Decomposing Channels

As explained, there are four types of individuals affected by changes in financial intermediation level. Figure 1.20 labels the four categories. When financial intermediation falls, Type I switch from wage workers to own account workers, Type II switch from own account workers to wage workers, Type III switch from own account workers to employers, and Type IV switch from employers to own account workers. When $\lambda$ falls from 1 to 0.9, share of own account workers increase by 26.6 percentage point. Table 1.4 summarizes the four channels’ contribution to this increase. Obviously, type I agents, medium low ability agents with some wealth, is the driving force of the pattern we observe in the data.
Table 1.4: Decomposing Channels

<table>
<thead>
<tr>
<th>Total Change</th>
<th>Type I</th>
<th>Type II</th>
<th>Type III</th>
<th>Type IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>+26.6</td>
<td>+26.8</td>
<td>−0.8</td>
<td>−0.1</td>
<td>+0.8</td>
</tr>
</tbody>
</table>

Note: This is the change in own account worker share when $\lambda$ falls from 1 to 0.9.

1.6 Conclusion

In this paper, I have argued that own-account workers and employers are different and need to be treated separately. Having a large share of self-employment and small business owners does not mean a country is more entrepreneurial. I have argued that financial intermediation can help explaining the cross country differences in occupation shares. A lower financial intermediation efficiency leads to a higher cost of borrowing and lower return on savings. Agents who save with financial intermediaries are more likely to seek alternative occupations to manage more wealth. Wage workers are more likely to become an own-account workers, operating small businesses to manage their wealth. Agents who need to borrow to become employers choose to be own-account worker instead. The result is less capital and labour intermediated through the market. The quantitative results presented here has shown that, by varying financial efficiency, the model can account for over 70% of the cross country variation in the share of own-account workers.

This paper takes an agnostic view on whether becoming an own-account workers is a good thing. There are discussions on uncertainties associated with own-account workers, and becoming an own-account workers may affect the human capital accumulated from taking on a wage job. If for some reason we wished to reduce the share of own-account workers, this paper suggests that improving the efficiency of financial intermediation and reducing the gap between the borrowing interest rate and deposit interest rate would help in achieving this goal.
1.7 Appendix

1.7.1 Data

List of countries in the GEM data 2013: United States, Russia, South Africa, Greece, Netherland, Belgium, France, Spain, Hungary, Romania, Switzerland, United Kingdom, Sweden, Norway, Poland, Germany, Peru, Mexico, Argentina, Brazil, Chile, Colombia, Malaysia, Indonesia, Thailand, Japan, South Korea, Vietnam, China, Turkey.

List of countries used in constructing Figure 1.5 Australia, Austria, Burundi, Burkina Faso, Bulgaria, Bosnia and Herzegovina, Bolivia, Canada, Switzerland, Chile, Colombia, Costa Rica, Cuba, Cayman Islands, Czech Republic, Germany, Denmark, Dominican Republic, Ecuador, Egypt, Spain, Estonia, Finland, United Kingdom, Georgia, Greece, Hong Kong, Honduras, Hungary, Indonesia, Ireland, Iceland, Israel, Italy, Jamaica, Japan, Kazakhstan, Kyrgyzstan, Republic of Korea, Sri Lanka, Luxembourg, Latvia, Macau, Morocco, Republic of Moldova, Malta, Mauritius, Malaysia, Nicaragua, Netherlands, Norway, New Zealand, Pakistan, Panama, Peru, Philippines, Palestine, Romania, Russian Federation, Serbia, Sweden, Thailand, Tunisia, Ukraine, Uruguay, United States, and South Africa.

1.7.2 Education distribution for individual countries

In developed economies, own-account workers come from all education groups. Figure 1.21 and Figure 1.22 plot the education distribution for each occupation type for US and Canada. The education distributions of own-account workers are very similar to those of the whole sample and other occupations as well. For developing countries, Figures 1.23 to 1.27 illustrate the occupation distri-
butions for Brazil, China, Mexico, Panama, and Venezuela. While it is true from the figures that for some countries the distribution of own-account workers is slightly left skewed towards the lower education level when compared with the whole sample, it is also obvious that they come from all education categories.
1.7.3 Solving the Dynamic Model

Here I explain the algorithm used to solve the dynamic model presented in Section 1.5. I consider 11 values of $\lambda$ ranging from 1 to 0.9. Instead of iterating on the joint distribution of ability and wealth as most papers do, I iterate on the wage rate and interest rate. I set the maximum of wealth agents can have to be 2000, and the smallest wealth to be zero. I draw 150 $\gamma$ values from the distribution and assign the weight on the grid. Let $P$ be the joint distribution matrix at the beginning of the period. For each given value of $\lambda$, i do the following:

Step 1: Start with guesses of wage rate $w$ and borrowing interest rate $R_b$. Let return on savings $R_s$ be $\lambda R_b$.

Step 2: For the given wage rate $w$ and interest rates $R_b$ and $R_s$, I find the optimal occupations of each ability-wealth pair $(a_i, \gamma_i)$ and calculate the resulting total income.

Step 3: Based on the income matrix obtained, I do value function iterations to find agents’ optimal saving matrix $a'_i(a_i, \gamma_i)$. From this, I can obtain a transition matrix $\Pi$ that connects wealth distribution at the beginning of the period to that at the end of the period.

Step 4: Find the stationary joint distribution between ability and wealth associated with this transition matrix. This can be done through iterations.

$$P' = f(\tau, d, \Pi, P) \quad (1.22)$$

With a given $P$, a transition matrix $\pi$ and a probability $\tau$ of keeping the same ability in the next period, and the probability of death $d$, I can work out the beginning of period distribution of the next period, $P'$. I keep iterating until the difference between $P$ and $P'$ is small enough. For this process, I can
start with any arbitrary joint distribution and it will converge to the same stationary distribution.

Step 5: Given this stationary distribution just obtained, find the wage rate $w'$, borrowing interest rate $R_b'$ and depositing interest rate $R_b = \lambda R_s$ that clear both labour and capital markets. This is done through a static iteration.

Step 6: If $w'$ and $R_b'$ are close enough to $w$ and $R_b$, stop. If not, I update the wage rate and interest rates, go back to Step 1 and repeat the process.


URL: https://EconPapers.repec.org/RePEc:ags:aaea14:170114


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Another prediction of the model is that as financial inefficiency decreases, the share of own-account workers who use their own wealth will increase. They are merely managing their own wealth without participating in the market.
Figure 1.3: Share of business owners

Note: Weighted share calculated from GEM APS 2013. Left panel excludes business owners in the agricultural sector and right panel includes them.

Figure 1.19 plots the fraction of own-account workers who uses own wealth among all own account workers against the financial intermediation efficiency parameter $\lambda$. As $\lambda$ falls, the fraction of own-account workers who do not participate in the market increases. This reduces the capital supplied to the market. This is related to the most widely used measure of financial development, the private credit to GDP ratio. As a comparison, Figure 1.18 plots the model predictions and data observed. With this set of parameters, the model generated credit to GDP ratio, a measure of financial development, is around 2.4 when financial intermediation is perfect. The total credit to the private sector as a share of GDP is around 2 in the US in year 2013. The model generated credit to GDP ratio is closely related to the observed in the data. Given that the intermediation cost is the only friction in the model and no other constraint on borrowing is imposed, the model does a good job in delivering a correlation coefficient of about 0.75.
Figure 1.4: Occupation Shares

(a) Share of own-account workers

(b) Share of employers

Note: data from PWT and ILO 2013.
Figure 1.5: Net interest margin and occupation choices: 2006 and 2013

Data: WB and ILO.
Figure 1.6: Share of own-account workers in each education category

Data: GEM APS 2013. GEM has four education categories: some secondary education (no degree obtained), secondary degree, post secondary education, and graduate experience. Here I focus on the first three categories and I calculate the share of own account workers within each education category.

Figure 1.7: Total income of an own-account worker \( I_o(a_i, \gamma_i) \)

Notes: Parameters: \( \gamma_i = 3, \alpha = 0.35, \beta = 0.5, R_b = 1.05, R_s = \lambda R_b, \delta = 1. \)
Figure 1.8: Total income of an employer

Parameters: $\gamma_i = 3$, $\alpha = 0.35$, $\beta = 0.5$, $R_b = 1.05$, $R_s = \lambda R_b$, $w = 1.5$, $\delta = 1$.

Figure 1.9: Occupation income and ability

Note: Occupation income is total income minus interest payment.
Figure 1.10: Capital Demand

Note: The parameter values, wage rate and interest rates used to create this plot are from Section 1.5. The only difference is $\delta = 1$. 

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Note: The parameter values, wage rate and interest rates used to create this plot are from Section 1.5. The only difference is $\delta = 1$. 

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Figure 1.11: Occupation choices

Wealth

Ability $\gamma_i$

$\gamma_l(a)$

$\gamma_h(a)$

own account workers

employers

wage workers

Figure 1.12: Income when $\lambda$ falls

$\lambda = 1$

$\lambda = 0.5$

Note: On the left panel, the two lines exactly coincide.
Figure 1.13: Static Model Response to Different $\lambda$

Note: Parameter values are from Table 1.3. Distribution used is stationary distribution under perfect financial intermediation from the dynamic model in Section 1.5.
Figure 1.14: Static Model Response to Different \( \lambda \): a different wealth distribution

Note: Parameters are the same as in Figure ???. However, the wealth of the agents with abilities at the bottom 80% are removed and redistributed to the top 20%.
Figure 1.15: Dynamic Model Response to Different $\lambda$

Figure 1.16: Own-account Workers

Figure 1.17: Wage Workers
Figure 1.18: Credit over GDP: model and data

![Credit over GDP: model and data](image)

Correlation: 0.7

Figure 1.19: Model: share of non-borrowing own-account workers

![Model: share of non-borrowing own-account workers](image)
Figure 1.20: Occupation choices
Figure 1.21: Occupations and Education: US

Data: 2015 CPS March. Education Attainment left to right: less than 1st grade; 2nd, 3rd or 4th grade; 5th or 6th grade; 7th or 8th grade; 9th grade; 10th grade; 11th grade; 12th grade no diploma; high school grad-diploma or equiv. (GED); some college but no degree; associate degree-occupational/vocational; associate degree-academic program; Bachelor’s degree (ex: BA, AB, BS); Master’s degree (ex: MA, MS, MEng, MEd, MSW); Professional school deg (ex: MD, DDS, DVM); Doctorate Degree (EX: PhD, EdD)
Figure 1.22: Occupations and Education: Canada

Data: IPUMS. Education categories left to right: below grade 5; grades 5-8; grades 9-13; high school graduation certificate; trades certificate or diploma; non-university without trades or college certificate or diploma; non-university with trades certificate or diploma; non-university with college certificate or diploma; university, no certificate, diploma or degree; university or college certificate or diploma; bachelor or first professional degree; certificate or diploma above bachelor level; Master’s degree; Doctoral degree.
Figure 1.23: Occupations and Education: Brazil

Data: IPUMS. Years of schooling starts from 0.
Figure 1.24: Occupations and Education: China

Data: 2013CHFS. Education categories from left to right: no school, primary school, junior high school, senior high school, vocational high school, vocational college, college degree, master, phd.
Figure 1.25: Occupations and Education: Mexico

Data: IPUMS. Years of schooling starts from 0.

Figure 1.26: Occupations and Education: Panama

Data: IPUMS. Years of schooling starts from 0.
Figure 1.27: Occupations and Education: Venezuela

Data: IPUMS. Years of schooling starts from 0.
Chapter 2

Fertility, Human Capital and Aggregate Income: The One-Child Policy

2.1 Introduction

After 35 years of mandatory fertility restriction, China ended its controversial One-child Policy at the end of year 2015. The repercussions of the One-child Policy have not fully unfolded yet since the oldest born under this policy are still young adults. As a policy to curb population growth and promote modernization, it certainly achieved its goal of reducing fertility, but its effects on macro economic outcome such as aggregate income has not been carefully studied. This paper tries to fill this gap.

The strict One-child policy was implemented in China in 1979. Since then urban families were allowed only One-child. However, China’s family planning policy started long before that, mostly through propaganda campaign. This led to voluntary fertility decline already before the One-child Policy. In 1971,
propaganda slogan “One-child isn’t too few, two are just fine, and three are too many” appeared, and in 1973, China started to encourage couples to get marriage later, increase the time gap between the having the first and the second child and have fewer number of children in total (Zhang, 2017). Fertility rate fell sharply between 1971 and 1978, shown in Figure 2.1. Urban fertility fell to about 1.5 children per family prior 1979. It continued to fall to close to 1 after the mandatory implementation of One-child Policy.

This paper examines the effects of this fertility policy on individual human capital and aggregate output. Data shows that as fertility declines, the share of household expenditure spent on the education of a single child increases significantly. In 1992, the share of household expenditure spent on the education of a child around age 20 is less than 5%. In 2002, this share has reached to about 20%, and remains at that level since then. This increase in education spending, if translated into the human capital of the affected children, could potentially increase their individual income and contribute positively to aggregate income. However, the fertility decline itself reduces the size of future labour force, and hence negatively affects aggregate income. To understand the net effects, a quantitative model is necessary.

I first present a three period OLG model with a quantity-quality trade off in a framework with intergenerational transfer. Parents give birth to children not just because they love children. They also receive old age support from children in the form of transfers, and the transfers receive are increasing in both the number and human capital level of children. Parents, therefore, optimally choose the number and the human capital level of children. When an exogenous binding constraint on fertility rate is imposed, parents choose the maximum number of children allowed and increase the human capital spending on each

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1 Zhang (2017) provides a detailed summary of the fertility policy going back to 1950s and discusses how it is affected by changes of political leaders.

2 This is the first year that I have data available.

3 The structure of transfer is from Choukhmane et al. (2014). They also provide evidence on the importance of the intergenerational transfer channel in China.
child. This gives rise to the quantity-quality trade-off. The binding fertility restriction results in a smaller size of the future labour force but a higher level of individual human capital. However, the effect on aggregate human capital, which depends on both labour force size and individual human capital level, is unclear, and needs to be assessed quantitatively. Hence, I extend the model to a 16 period OLG model, which allows me to carefully account for the timing of human capital accumulation. I then calibrate the quantitative model and compare the case with fertility restriction to the counter-factual case had there been no restrictions on fertility.

The effects of a fertility restriction policy crucially depend on the difference between the fertility rate under policy intervention and the counter-factual fertility rate. As shown in Figure 2.1, the fertility started to decline prior to the implementation of the One-child policy due to the propaganda campaign. To match this, the fertility policy I feed into the model is a 2.5 children policy in 1970 to 1974, a 1.5 children policy in 1975 to 1979, and a strict One-child policy since 1980. Getting the right counter-factual fertility is not so straightforward, since the counter-factual fertility itself is not observable, and it is extremely unlikely that it has remained at the same level before the fertility policies. To address this issue, I feed in the model a time-varying cost of raising and educating children estimated from the data, and this cost applies in both the case with and without fertility restrictions. In the case with no fertility intervention, the model generates a time-varying fertility rate, and I use this as the counter factual fertility rate. The model’s calibration suggests that counter-factual fertility falls from 3 children per family in 1960s to 1.6 children per family in 2010s. This suggests that the current Two-children Policy is non-binding on average for urban households.

The main results of this paper depend on the comparison between the model’s prediction under fertility intervention with the counter-factual case. Calibration results suggest that generations born under the One-child policy see their
income increase by 33%. However, policy intervention’s effect on aggregate income has turned negative in around 2000, due to a smaller labour force. It also implies that even if the quantity-quality trade-off exists on individual level, it does not exist on aggregate level.

By addressing the counter-factual fertility issue, this paper contributes to the debates on by how much the One-child Policy contribute to the observed fertility decline. Lively and Freedman (1990), Yang and Chen (2004) and Li et al. (2015), for example, argue that family planning policy has an important role, but some others disagree. Whyte et al. (2015) argues that the fertility has already started to decline even before the One-child Policy is implemented, and Schultz and Zeng (1995) points to the concurrent voluntary fertility decline in other east and south-east Asian countries. This paper’s result suggests that the One-child Policy itself is still binding on average, even though the counter-factual fertility has fallen due to the increase in the cost of children.

One caveat is that I only focus only on the urban households, so the aggregate output in this paper is not directly comparable to China’s aggregate GDP. One reason to focus on the urban household is that the policy is only strictly implemented in urban area. Its enforcement in rural areas varies over time and across provinces. Baochang et al. (2007) has a detailed summary of this. Another reason is that the quality-quantity trade off could potentially be different in urban and rural area. In rural areas, sometimes children are expected to help with the farm work. In this case, having more children could mean that each child does less farm work and they may have a higher chance of receiving more education. For these reasons, I focus on the urban household. Another caveat is that the measure of human capital is based on the education investment children receive instead of outcome variable. However, as Choukhmane et al. (2014) shows in their sample, the difference between outcomes of children in families with and without twins is significantly different.
This paper is related to the literature on the trade off between number of children and quality of children, a mechanism that has been formally theorized ever since Becker and Lewis (1973) and Becker and Tomes (1976) and empirically examined using data of different countries (Rosenzweig and Wolpin (1980)). Evidence is not always consistent. In the case of China, Li et al. (2008) and Rosenzweig and Zhang (2009) find evidence supporting quantity quality trade-off looking at education outcomes, using twins as the exogenous variation. Qian (2009) exploits the the variation of the One-child Policy in rural area and finds that having an additional child actually increase the enrolment probability of the first child. These papers use the education outcomes as the variable of interests. Choukhmane et al. (2014) focuses on education input and points to a sizeable difference in education expenditure on each child between single child families and twin families.

This paper is also related to the studies on the interactions between fertility and aggregate growth (See Barro and Becker (1989) and Ehrlich and Lui (1991)). Empirical work on this is relatively scant. Li and Zhang (2007) examine China’s family planning policy and conclude negative causal effect of population on economic growth, supporting the Malthusian claim. However, the results presented in this paper disagree.

Finally this paper is also related to the literature that explores the unintended consequences of the One-child Policy. Banerjee et al. (2014) and Choukhmane et al. (2014) investigate its role in explaining China’s high household saving rate. Ebenstein (2008) and Li et al. (2011) examine One-child policy and the distorted high male-female gender ratio in China.

In Section 2.2, I present a model of fertility and human capital choice to illustrate the main mechanisms. In Section 2.3 I analyze the model’s implication

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4Some of outcomes in Rosenzweig and Zhang (2009) are based on expectations as they surveyed ageing 7 to 18 year old in 2012/2013 and their education is not completed yet. Qian (2009) does not address the issue of average quality of children.
when a binding fertility restriction is imposed. I then extend the model into a 16-period quantitative OLG model in Section 2.4, calibrate it and discuss the results. Section 2.5 concludes.

2.2 The Model

2.2.1 Agent Problem

This section presents a three-period OLG model with inter-generational transfer, a channel that has been emphasized by Ehrlich and Lui (1991). I take the transfer function used by Choukhmane et al. (2014). An agent lives for three periods. An agent born at time $t-1$ is a child in that period, a young agent at time $t$ and an old agent at time $t+1$.

A child receives human capital investment in the form of education goods paid for by parents. The child does not make any decisions. A young agent supplies labour and earns wage income. The young agent also decides on the number of children, $n_t$, and the units of human capital goods $E_t$ to give to each child. The human capital formation takes this form:

$$h_t = A_h E_t^\gamma$$

(2.1)

with $0 < \gamma < 1$. $E_t$ is the unit of human capital goods a child born at time $t$ receives. $A_h$ reflects the efficiency of the human capital formation. For simplicity I do not model a separate sector that produces human capital goods. Instead I assume human capital goods can be converted from consumption goods at price $p_{E,t}$, so the total spending on human capital goods is $n_t p_{E,t} E_t$.

---

5 This formation of human capital abstracts from the years of schooling, which is usually a factor considered. For example, the Ben-Porath (1967) human capital production function is a Cobb-Douglas function of time spent, existing human capital and resource input. Manuelli and Seshadi (2014) also assume it to be a function of both time input and resource input.
The young agent at time $t$, together with the agent’s siblings, makes transfers to support their parents, who are old aged at time $t$. Each young agent gives a fraction $\psi n_{t-1}^\omega$ of wage income to parents. The total transfer received by parents, the old aged at time $t$, is $\psi n_{t-1}^\omega w_{yt}$. $\psi$ captures children’s generosity to parents, and $\omega < 1$ captures the free-riding between siblings. When $n_{t-1}$ increase, $n_{t-1}^\omega$ falls. Each agent transfers a smaller share of their income to parents.

Similarly, when the time $t$ young agent becomes old at time $t+1$, the total transfer this agent receives from children will be $\psi n_{t+1}^\omega w_{yt+1}$.

Assuming agents have log utility, the agent born at time $t-1$ becomes an active agent at time $t$ and maximizes the following utility:

$$\max_{\{n_t, E_t, c_{yt}, a_{yt}, c_{ot+1}\}} U_t = \ln(c_{yt}) + v \ln(n_t) + \beta \ln(c_{ot+1})$$

subject to:

$$c_{yt} + a_{yt} = \left(1 - n_t \phi_f - \psi n_{t-1}^\omega \right) w_{yt} - n_t P E_tE_t \quad (2.2)$$

$$c_{ot+1} = R_{t+1} a_{yt} + \psi n_{t}^\omega w_{yt+1} \quad (2.3)$$

where $v$ in the utility function represents the love of children. This agent chooses fertility rate $n_t$, units of human capital goods $E_t$, young-age consumption $c_{yt}$, saving at the end of the young period $a_{yt}$, and old age consumption $c_{ot+1}$. Equation 2.2 is the budget constraint when this agent is young. $n_t \phi_f$ is the fraction income used to pay for the fixed cost of raising children, and $\psi n_{t-1}^\omega$ is fraction transfered to parents. Equation 2.3 is the budget constraint when this agent becomes old.
2.2.2 Production

I assume the production function takes the standard Cobb-Douglas form with labour augmenting productivity:

\[ Y_t = K_t^{1-\alpha}(A_t L_t)^\alpha \]  \hspace{1cm} (2.4)

where

\[ L_t = N_y h_{t-1} \]  \hspace{1cm} (2.5)

is the efficient labour units. The agents who are in the labour force at time are \( t \) are young at time \( t \), and the size of this cohort is \( N_{y,t} \). They were born at time \( t-1 \) and received human capital goods \( E_{t-1} \), with the human capital level \( h_{t-1} \).

2.2.3 Optimality Conditions and Equilibrium

Define \( k_t = \frac{K_t}{A_t L_t} \) to be the efficient capital labour ratio. Assuming competitive labour and capital market, wage rate and interest rate in this economy are:

\[ w_t = \alpha A_t k_t^{1-\alpha} h_{t-1} \]  \hspace{1cm} (2.6)

\[ R_t = (1 - \alpha) k_t^{-\alpha} + 1 - \delta \]  \hspace{1cm} (2.7)

where \( R_t \) is the gross return to capital and \( \delta \) is the depreciation rate.

The equilibrium of this model is composed of the series of factor prices \( \{w_t, R_t\}_{t=0}^\infty \) and the series of agents choice variables \( \{n_t, E_t, c_{y,t}, a_{y,t}, c_{o,t+1}\}_{t=0}^\infty \) that solve the individual maximization problem and satisfy the the aggregate capital condition:

\[ K_t = N_{y,t-1} a_{y,t-1} \]
The consumption and saving decisions are standard and are shown in Appendix 2.6.1. Here I focus on the fertility and human capital conditions. The first order condition for human capital spending $E_t$ is

$$n_t p_{E,t} = \frac{1}{R_{t+1}} \frac{\psi n_t^\omega}{\omega} \frac{\partial w_{y,t+1}}{\partial h_t} \frac{\partial h_t}{\partial E_t}$$

Rearrange it to be:

$$n_t p_{E,t} = \frac{1}{R_{t+1}} \frac{\psi n_t^\omega}{\omega} \frac{w_{y,t+1}}{E_t}$$ (2.8)

where the left hand side is the marginal cost of buying one extra unit of human capital goods for each child, and the right hand side is the discounted marginal increase in transfer received next period. Holding interest rate and price of human capital goods constant, equation (2.8) shows a negative relationship between fertility and human capital goods. When there are more children, the marginal cost of buying one unit of human capital goods increases linearly with the number of children, but marginal benefit is diminishing. This with a higher fertility rate, parents buy less human capital goods for each child.

This equation generates the key quantity quality trade-off in this model. It also indicates that when interest rate $R$ is higher, parents invest less in human capital, because the alternative investment channel through saving becomes more profitable.

The first order condition for fertility $n_t$ is given by:

$$\frac{\nu}{n_t} = \frac{1}{c_{y,t}} \left( \phi_f w_{y,t} + p_{E,t} E_t - \frac{1}{R_{t+1}} \psi n_t^{\omega-1} w_{y,t+1} \right)$$ (2.9)

The left hand side is the direct gain in utility, while the right hand side is the discounted net marginal cost of a child. $\phi_f$ represent the fixed cost, and $p_{E,t} E_t$ is the spending on human capital goods. $\psi n_t^{\omega-1} w_{y,t+1}$ is the marginal transfer received next period.

The fertility first order condition (equation (2.9), combined with consumption
decision (equation (2.26)) and human capital first order condition (equation (2.8)) gives:

\[
\frac{v}{n_t} = \frac{(1 + \beta) \left( \phi_f w_{y,t} + \left( 1 - \frac{\omega}{\gamma} \right) p_{E,t} E_t \right)}{\left( 1 - \psi \frac{n_{t-1}}{\omega} - n_t \phi_f \right) w_{y,t} - \left( 1 - \frac{1}{\gamma} \right) n_t p_{E,t} E_t}
\]  \hspace{1cm} (2.10)

and it can be further rearranged as:

\[
(1 + \beta + v) (1 - \lambda) \frac{p_{E,t} E_t}{w_{y,t}} = -(1 + \beta + v) \phi_f + \frac{v \left[ \left( 1 - \psi \frac{n_{t-1}}{\omega} \right) \right]}{n_t}
\]  \hspace{1cm} (2.11)

where \( \lambda = \frac{(1 + \beta) \omega + v}{\gamma (1 + \beta) + \gamma v} \).

**Assumption 2.1.** Assume \( \omega > \gamma \). This is a sufficient condition for \( \lambda > 1 \).

When Assumption 2.1 is satisfied, equation (2.11) implies a positive relationship between the fertility rate \( n_t \) and quantity of human capital goods \( E_t \). When children are of higher human capital, parents would like to have more children. An increase in human capital level of the children increases both the marginal cost of and the marginal benefit from children, but when \( \omega > \gamma \), the marginal return from having more children increases more than the cost, so the parents are willing to have more children. Intuitively, a larger \( \omega \) means less free-riding between siblings, hence making investing in children more profitable.

The aggregate capital stock used in production at time \( t+1 \) comes from the savings of time \( t \) young agent, with \( K_{t+1} = N_{y,t} a_{y,t} \). Substitute in the human capital condition equation (2.8) and saving decision equation (2.28), it becomes

\[
K_{t+1} = N_{y,t} \left\{ \frac{\beta}{1 + \beta} \left[ \left( 1 - \psi \frac{n_{t-1}}{\omega} - n_t \phi_f \right) w_{y,t} - n_t p_{E,t} E_t \right] - \frac{1}{1 + \beta} \frac{n_t p_{E,t} E_t}{\gamma} \right\}
\]
divide both sides by $N_{y,t}w_{y,t}$ gives:

$$
(1 + \beta) \frac{k_{t+1}}{\alpha k_t^{1-\alpha}} g_A \frac{E_t^\gamma}{E_{t-1}^\gamma} n_t = \beta \left[ 1 - \psi \frac{n_{t-1}^{\omega-1}}{\omega} - n_t \phi_f \right] - n_t E_t \frac{p_{E,t}}{w_{y,t}} \frac{p_{E,t}}{\gamma} w_{y,t} 
$$

(2.12)

Given exogenous human capital goods price series $p_{E,t}$, the human capital condition equation (2.8), the fertility condition equation (2.11) and physical capital condition equation (2.12), together with the wage rate equation (2.6), interest rate equation (2.7) and human capital formation equation (2.1) characterize the equilibrium of the model in \{n_t, E_t, h_t, w_{y,t}, R_t, k_t\}_{t=0}^\infty.

### 2.2.4 Steady State

The model admits a steady state equilibrium when price of human capital goods grows at the same rate as wage, such that $\frac{p_{E,t}}{w_{y,t}} = \frac{p_{E}}{w_y}$. For now I assume it is the case, and I will present more data on this later. I also assume TPF grows exogenously at rate $\frac{A_{t+1}}{A_t} = g_A$. With these assumptions, we can further simplify the equilibrium conditions. Substitute in these assumptions, wage rate equation (2.6), interest rate equation (2.7) and human capital formation equation (2.1) into the the human capital condition equation (2.8), the fertility condition equation (2.11) and physical capital condition equation (2.12), we get:

Equation (2.11) now becomes:

$$(1 + \beta + \nu) (1 - \lambda) \frac{p_{E}}{w_y} E = - (1 + \beta + \nu) \phi_f + \frac{\nu \left[ 1 - \psi n^{\omega-1} \right]}{n}$$

(2.13)

Equation (2.8) now becomes:

$$
\frac{p_{E}}{w_y} E = \frac{g_A \gamma \psi n^{\omega-1}}{R} \frac{1}{\omega}
$$

(2.14)
Equation (2.12) now becomes:

\[(1 + \beta) \frac{1 - \alpha \eta_n}{\alpha} \frac{g_n}{R} = \beta \left(1 - \psi \frac{n^{\omega - 1}}{\omega} - n\phi_f\right) - \left(\beta + \frac{1}{\gamma}\right) \frac{p_e}{w_y} n E\]  

Equation (2.13), (2.14) and (2.15) characterize the steady state equilibrium in consumption of human capital goods \(E_{ss}\), fertility rate \(n_{ss}\) and interest rate \(R_{ss}\).

To better understand the mechanisms of the model, I provide some graphical illustration. For a given fertility rate \(n\), Figure 2.2 plots the human capital condition equation (2.14) and physical capital condition equation (2.15) in units of human capital goods \(E\) and interest rate \(R\). The human capital condition is downward sloping, because when interest rate is high, saving becomes more attractive than investing in children, so parents purchase less human capital goods for children. Physical capital condition is upward sloping, because when parents spend more on human capital goods, they save less in physical capital, so return from saving is higher because of the diminishing marginal productivity of capital.

Figure 2.3 then illustrates the movement of human capital and physical capital conditions to a decline in fertility. Human capital condition shifts upward, because for any given interest rate, when there is less children, marginal return from an extra child is higher, so parents increase the spending on human capital goods for each child. Physical capital condition shifts leftwards, because for any given human capital level, a fall in fertility decrease the total spending and increases saving, which reduces interest rate. The movements of the human capital and physical capital conditions together imply a higher human capital spending per child when fertility declines. This negative relationship between fertility and human capital goods is summarized by the downward sloping curve in Figure 2.4 labeled as capital condition. Mathematical description of this curve can be obtained by combining equation (2.14) and equation (2.15).
to substitute out $R$ and get an expression in terms of $n$ and $E$. It inherits the quality-quantity trade-off from equation (2.14).

The other upward sloping curve in Figure 2.4 is the fertility condition, equation (2.13). The intersection gives the steady state fertility $n_{ss}$ and human capital goods consumption $E_{ss}$ in this economy. In this steady state, interest rate $R_{ss}$ and capital labour ratio $k_{ss}$ will also be constant. Wage rate grows at the rate of TFP growth. All other aggregate variables grow at the rate of TFP growth multiplied by the fertility rate.

\[
\frac{w_{y,t+1}}{w_{y,t}} = g_A \quad (2.16) \\
\frac{L_{t+1}}{L_t} = \frac{K_{t+1}}{K_t} = \frac{Y_{t+1}}{Y_t} = g_A n_{ss} \quad (2.17)
\]

### 2.2.5 Comparative Statics

In this section, I show some of the model’s response to changes in parameters.

**Proposition 3.** As the fixed cost of raising a child $\phi$ increases, the number of children will decrease. \( \frac{\partial n}{\partial \phi} < 0 \).

This is intuitive. However, its effects on human capital spending per child, \( \frac{\partial E}{\partial \phi} \), is ambiguous. Two forces operates at opposite direction. As parents reduce the number of children, quantity quality trade-off means that they tend to invest more in each child. However, the higher fixed cost also implies parents have a smaller budge set, and they buy less of everything, including human capital goods. Hence the effect on human capital goods $E$ is ambiguous.

**Proposition 4.** When the price of human capital good increases relative to income, it reduces the human capital of each child. \( \frac{\partial E}{\partial p_{E}} < 0 \).

This is also intuitive, but its effects on fertility \( \frac{\partial n}{\partial p_{E}} \) is ambiguous. As education price increases, investing in children for old-age support becomes less appealing...
than saving. This reduces both the quality and quantity of children. However, as parents invest less in the education of each child, the total cost of children is lower, and because parents also enjoy having children, fertility tend to increase. The net effect on fertility rate is ambiguous and will depend on parameters.

Proposition 5. A higher TFP growth increase both fertility rate and human capital good consumption. \( \frac{\partial n}{\partial g} > 0; \frac{\partial E}{\partial g} > 0 \)

A higher TFP growth indicates a larger productivity difference between now and future. Children who join labour force in the future will have higher wage income, and this benefit parents through higher transfers. As a result, parents will increase both quantity and quality of children.

2.3 The Model with Fertility Restriction

In this section, I present the model with a fertility restriction. All variables with subscript \( ss \) denotes the unconstrained steady state, while variable with an upper bar denotes the constrained steady state. A binding exogenous fertility restriction is imposed such that \( \bar{n} < n_{ss} \).

2.3.1 Equilibrium Characterization

In this case agents cannot have a fertility rate higher than \( \bar{n} \), so they just choose \( \bar{n} \). The fertility optimality condition, equation (2.13), becomes irrelevant and is replaced by \( n = \bar{n} \). The equilibrium is then characterized by the constrained version of the human capital decision and physical market clearing condition. The human capital condition (2.14) now becomes:

\[
\frac{p_E}{w_y} E = \frac{g_A \gamma \bar{n}^{\omega - 1}}{R \omega} \tag{2.18}
\]
and the physical capital market condition (2.15) now becomes:

\[
(1 + \beta)^{1 - \alpha} \frac{g \bar{n}}{R} = \beta \left( 1 - \frac{\bar{n}^{\omega-1}}{\omega} - \bar{n} \phi_f \right) - \left( \beta + \frac{1}{\gamma} \right) \frac{pE}{w_y} \bar{E} \tag{2.19}
\]

The above two equations solve the constrained steady state \( \bar{E} \) and \( \bar{R} \) under a binding fertility restriction \( \bar{n} \).

### 2.3.2 Comparative Statics

**Proposition 6.** As long as \( \bar{n} < n_{ss} \), \( \frac{\partial E}{\partial \bar{n}} < 0 \).

When fertility choice is restricted by \( \bar{n} < n_{ss} \), the upward sloping fertility condition in Figure 2.4 is irrelevant. The downward sloping capital condition in Figure 2.4 alone summarizes the equilibrium. When parents are allowed less children, they invest more in each children’s human capital, but the effect on total human capital spending, \( \bar{n} \bar{E} \), is ambiguous. The reason is that when interest rate is held constant, a decrease in fertility will decrease the total spending on education, and this tend to increase saving. As saving increases, interest rate will fall. This then makes investing in children’s human capital more attractive than saving, and tend to increase human capital spending. Hence, the net result on total human capital spending, is ambiguous.

Since the spending on children’s human capital, together with the number of children, determine the efficient labour units, the ambiguity in total human capital spending lead to ambiguity in the change of the total efficient labour units, the key determinant of aggregate income. To answer the question of how a decline in fertility affects aggregate income, a quantitative exercise will be necessary.\(^\text{6}\)

\(^{6}\)In a model where children human capital directly depends on parental human capital, similar results hold.
2.4 A Quantitative OLG Model

This section presents an extended quantitative OLG model and uses it to assess the effect of fertility restriction on output. This extended model allows me to match the observed timing of education expenditure. One period in the model corresponds to 5 years in real life. I now describe the timing of the lifetime events.

**Human Capital Investment:** In the first five periods of agents’ life, they make no active decisions. They receive human capital goods paid for by parents. I assume that in the first three periods they receive compulsory education investment. Parents must pay for this. This corresponds to the nine-year compulsory education in China, which children usually finish at around age 15. The investments in the latter two periods, period four and five, corresponds to high school and college education, and they are optional. Parents choose the amount of education goods to be invested on their children in these two periods.

**Working and Saving:** Agents start working in the fifth period, the same period when their human capital is finalized. For simplicity, I assume that in period five, the agent consumes the wage income and do not borrow against future income. Starting from period six, agents optimally choose consumption and saving.

**Child birth:** Agents decide on the number of children in the beginning of the 6th period in life. In the 6th, 7th and 8th period, agents pay the fixed cost of children, and in the 9th and 10th period, agents choose the education goods to be invested in children’s human capital.

**Transfer:** Agents transfer to their parents from the 8th to the 11th period of their lifetime, which corresponds to the last four periods of their parents
Table 2.1: The Timing of child-birth and transfer of an agent born at time $t - 4$

<table>
<thead>
<tr>
<th>Age</th>
<th>1-5</th>
<th>Human Capital Investment</th>
<th>Transfers</th>
</tr>
</thead>
<tbody>
<tr>
<td>t-4</td>
<td>1</td>
<td>born</td>
<td></td>
</tr>
<tr>
<td>t-3</td>
<td>2</td>
<td>6-10</td>
<td></td>
</tr>
<tr>
<td>t-2</td>
<td>3</td>
<td>11-15</td>
<td></td>
</tr>
<tr>
<td>t-1</td>
<td>4</td>
<td>16-20</td>
<td></td>
</tr>
<tr>
<td>t</td>
<td>5</td>
<td>21-25 receive $h_t$</td>
<td></td>
</tr>
<tr>
<td>t+1</td>
<td>6</td>
<td>26-30 give birth $n_{t+1}$ compulsory</td>
<td></td>
</tr>
<tr>
<td>t+2</td>
<td>7</td>
<td>31-35 compulsory</td>
<td></td>
</tr>
<tr>
<td>t+3</td>
<td>8</td>
<td>36-40 compulsory</td>
<td>to parents</td>
</tr>
<tr>
<td>t+4</td>
<td>9</td>
<td>41-45 discretionary</td>
<td>to parents</td>
</tr>
<tr>
<td>t+5</td>
<td>10</td>
<td>46-50 discretionary</td>
<td>to parents</td>
</tr>
<tr>
<td>t+6</td>
<td>11</td>
<td>51-55 to parents</td>
<td></td>
</tr>
<tr>
<td>t+7</td>
<td>12</td>
<td>56-60 from children</td>
<td></td>
</tr>
<tr>
<td>t+8</td>
<td>13</td>
<td>61-65 from children</td>
<td></td>
</tr>
<tr>
<td>t+9</td>
<td>14</td>
<td>66-70 from children</td>
<td></td>
</tr>
<tr>
<td>t+10</td>
<td>15</td>
<td>71-75 from children</td>
<td></td>
</tr>
<tr>
<td>t+11</td>
<td>16</td>
<td>76-80 die</td>
<td>from children</td>
</tr>
</tbody>
</table>

Note: The human capital level $h_t$ of the agent born in time $t - 4$ depends on human capital goods received from time $t - 4$ to $t$. Each agent’s human capital level are determined in the fifth period in life.

Similarly, when they are in the last four periods of their own lifetime, they receive transfers from children. Table 2.1 summarizes the timing of these events.

Follow the previous log utility assumption, the utility function of an agent born at time $t - 4$ and enters the labour market at time $t$ is:

$$U_t = \sum_{s=5}^{16} \beta^{s-t} \log(c_{t+s-5}(s)) + v \log(n_{t+1})$$

where the subscript $t$ denotes time period, and the subscript $s$ denotes the age. Taking into the timing of the education expenditure and transfers, the agent
faces the following constraints:

\[ c_t(5) = w_t(5) \]

\[ c_{t+1}(6) + a_{t+1}(6) = (1 - n_{t+1}\phi_6)w_{t+1}(6) \]

\[ c_{t+2}(7) + a_{t+2}(7) = (1 - n_{t+1}\phi_7)w_{t+2}(7) + R_{t+2}a_{t+1}(6) \]

\[ c_{t+3}(8) + a_{t+3}(8) = \left(1 - n_{t+1}\phi_8 - \frac{\psi n_{t+1}^{\omega-1}}{\omega}\right)w_{t+3}(8) + R_{t+3}a_{t+2}(7) \]

\[ c_{t+4}(9) + a_{t+4}(9) = \left(1 - \frac{\psi n_{t+1}^{\omega-1}}{\omega}\right)w_{t+4}(9) + R_{t+4}a_{t+3}(8) - n_{t+1}p_{E_{t+4},t+4}E_{t+4}(4) \]

\[ c_{t+5}(10) + a_{t+5}(10) = \left(1 - \frac{\psi n_{t+1}^{\omega-1}}{\omega}\right)w_{t+5}(10) + R_{t+5}a_{t+4}(9) - n_{t+1}p_{E_{t+5},t+5}E_{t+5}(5) \]

\[ c_{t+6}(11) + a_{t+6}(11) = \left(1 - \frac{\psi n_{t+1}^{\omega-1}}{\omega}\right)w_{t+6}(11) + R_{t+6}a_{t+5}(10) \]

\[ c_{t+7}(12) + a_{t+7}(12) = w_{t+7}(12) + R_{t+7}a_{t+6}(11) \]

\[ c_{t+8}(13) + a_{t+8}(13) = \frac{\psi n_{t+1}^{\omega-1}}{\omega}w_{t+8}(8) + R_{t+8}a_{t+7}(12) \]

\[ c_{t+9}(14) + a_{t+9}(14) = \frac{\psi n_{t+1}^{\omega-1}}{\omega}w_{t+9}(9) + R_{t+9}a_{t+8}(13) \]

\[ c_{t+10}(15) + a_{t+10}(15) = \frac{\psi n_{t+1}^{\omega-1}}{\omega}w_{t+10}(10) + R_{t+10}a_{t+9}(14) \]

\[ c_{t+11}(16) = \frac{\psi n_{t+1}^{\omega-1}}{\omega}w_{t+11}(11) + R_{t+11}a_{t+10}(15) \]

Here, \( \phi_6, \phi_7 \) and \( \phi_8 \) represent the compulsory education costs per child. \( p_{E_{t+4},t+4} \) (\( p_{E_{t+5},t+5} \)) is the time \( t + 4 \) (\( t + 5 \)) price of the human capital goods that a child receives in the child’s fourth period in life. \( E_{t}(s) \) is the amount of human capital goods an age-\( s \) agent receives at time \( t \). Parameters \( \psi \) and \( \omega \) are the same as before.

The human capital accumulation now has the following form:

\[ h_{t+5} = A_h \left[ E_{t+4}(4)^{\tau} E_{t+5}(5)^{1-\tau} \right]^\gamma \quad (2.20) \]

where \( 0 < \tau < 1 \) and \( \gamma < 1 \). It depends on human capital investment received in the 4th and 5th period in life.\(^\text{[9]}\) I assume it only depends on the voluntary human capital investment across multiple

\(^{[9]}\) Cunha and Heckman (2007) discusses the how human capital investment across multiple
investment for two reasons. The first reason is that the education investments in the first three periods are compulsory, and the compulsory education is very well implemented in urban China. Moreover, Choukhmane et al. (2014) shows that the difference in education spending received by a single child and a twin-child are not obvious before they reach 15 and becomes pronounced after entering age 15.

The production function is the same as before, \( Y_t = K_t^{1-\alpha} (A_t L_t)^\alpha \) and \( k \equiv \frac{K_t}{A_t L_t} \). \( L_t \) includes the whole labour force at time \( t \).

\[
L_t = [e_5 N_t(5)h_t + e_6 N_t(6)h_{t-1} + e_7 N_t(7)h_{t-2} + e_8 N_t(8)h_{t-3} + e_9 N_t(9)h_{t-4} + e_{10} N_t(10)h_{t-5} + e_{11} N_t(11)h_{t-6} + e_{12} N_t(12)h_{t-7}] \tag{2.21}
\]

It includes all people who are in the 5th period to the 12th period of their life, \( e_s \) is the efficiency of age \( s \) agents. \( N_t(s) \) refers to the size of the time-\( t \) age-\( s \) cohort. Similarly, capital at time \( K_t \) would include all the saving stock from all agents last period.

\[
K_t = N_{t-1}(6)a_{t-1}(6) + N_{t-1}(7)a_{t-1}(7) + N_{t-1}(8)a_{t-1}(8) + N_{t-1}(9)a_{t-1}(9) + N_{t-1}(10)a_{t-1}(10) + N_{t-1}(11)a_{t-1}(11) + N_{t-1}(12)a_{t-1}(12) + N_{t-1}(13)a_{t-1}(13) + N_{t-1}(14)a_{t-1}(14) + N_{t-1}(15)a_{t-1}(15) \tag{2.22}
\]

The wage income and interest rate in this economy are:

\[
w_t(s) = \alpha A_t k_t^{1-\alpha} e_s h_{t+5-s} \tag{2.23}
\]

\[
R_t = (1 - \alpha) k_t^{-\alpha} \tag{2.24}
\]

where \( w_t(s) \) is the wage rate of an \( s \)-year old agent at time \( t \). In this model, the choices for this agent born at time \( t - 4 \), whose human capital is determined periods should be aggregated and emphasizes human capital investment in different time periods are not perfect substitutes, so they are all important for the formation of human capital.
at time \( t \) and joins labour market at time \( t + 1 \), includes fertility decision \( n_{t+1} \), human capital spending \( E_{t+4}(4) \), \( E_{t+5}(5) \), consumption \( c_t(s) \) series and saving \( a_t(s) \) series.

### 2.4.1 Calibration of the quantitative OLG model

This section explains how I calibrate the model. The general approach is: first, I calibrate the model to match data observed in 2000s. Assuming fertility intervention is binding in these time period, this allow me to ignore the fertility optimality condition. I use the cost of raising children observed in 2000s to find the appropriate parameter values such that the model’s prediction under a constrained fertility rate match certain features of the data observed in 2000s. Then I keep the values of the matched parameters, but change the cost of raising children observed in the early 1990s, and then I find the love of children parameter \( \nu \) such that the optimal unconstrained fertility is about 3 children per family, the pre-1970 fertility rate before policy intervention. Table 2.2 summarizes the values of the parameters.

**Matching Data Observed in the 2000s**

The labour share \( \alpha \) is taken from the average share of labour income from year 1978 to 2003 reported by [Bai et al. (2006)](Baietal2006). \( \beta \) is assumed to be 0.99 per annum. \( g_A \) represents the TFP growth in this period. I choose it to be 5% growth. Ideally I would want to match the growth rate produced by the model to the observed growth rate of China, but given the limited scope of this paper, it does not make sense. One reason is I am only looking at urban households. Also during this period, China went through major changes that affected the growth. The privatization of state owned enterprises, the flow of migrant workers from rural to urban area, and the huge inflow of foreign capital

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Table 2.2: Baseline Calibration

<table>
<thead>
<tr>
<th>Parameters</th>
<th>value</th>
<th>target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.52</td>
<td>labour share in Bai et al. (2006)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>per annum</td>
</tr>
<tr>
<td>$e_6$</td>
<td>0.733</td>
<td>average of UHS 1992 to 1995</td>
</tr>
<tr>
<td>$e_7$</td>
<td>0.854</td>
<td>average of UHS 1992 to 1995</td>
</tr>
<tr>
<td>$e_8$</td>
<td>0.951</td>
<td>average of UHS 1992 to 1995</td>
</tr>
<tr>
<td>$e_9$</td>
<td>1.05</td>
<td>average of UHS 1992 to 1995</td>
</tr>
<tr>
<td>$e_{10}$</td>
<td>1</td>
<td>average of UHS 1992 to 1995</td>
</tr>
<tr>
<td>$e_{11}$</td>
<td>0.882</td>
<td>average of UHS 1992 to 1995</td>
</tr>
<tr>
<td>$e_{12}$</td>
<td>0.655</td>
<td>average of UHS 1992 to 1995</td>
</tr>
<tr>
<td>$\phi_6$, $\phi_7$, $\phi_8$</td>
<td>time varying</td>
<td>UHS 1992 to 2009</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.45</td>
<td>absolute spending share</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.6</td>
<td>relative spending share</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.026</td>
<td>3 children per family pre-1970</td>
</tr>
<tr>
<td>$p_E$</td>
<td>timing varying</td>
<td>changes in fees</td>
</tr>
<tr>
<td>$g_A$</td>
<td>1.05</td>
<td>annual growth of 5%</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.1</td>
<td>Choukhmane et al. (2014)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.65</td>
<td>Choukhmane et al. (2014)</td>
</tr>
</tbody>
</table>

None of these is incorporated here, so it would be a stretch too far to match the model’s growth to the observed aggregate growth in China. Instead I use an exogenous productivity growth of 5% per year. This is in the range of numbers provided by studies examining the productivity growth in China in the relevant periods (for example see Zhu (2012) and Young (2003)).

$\psi$ and $\omega$ are parameters related to the transfer from middle-aged child to old-aged parents. For now I use the estimates from Choukhmane et al. (2014), because I also use their my transfer function.

$e_s$ represent agent’s lifetime income profile. I regress log of wage on education dummy and age category dummy for each year from 1992 to 1995 and average them across years to get the values.

$\phi_6$, $\phi_7$, $\phi_8$: these three parameters capture the fixed cost of raising children in the first, second and third period of a child’s life. $\phi_6$, $\phi_7$ and $\phi_8$ correspondingly match the fraction of wage income spent on the education of children between 1 to 5 year old, 6 to 10 year old and 11 to 15 years old. Figure 2.5 shows the
education spending as a fraction of household income for single child family from year 1992 to year 2009. This fraction increases in the 1990s but has more or less stabilized since 2000. I match the \( \phi_6 \), \( \phi_7 \) and \( \phi_8 \) to the average of the spending share on children less than 15 years old observed in 2000s. The values are 0.033, 0.067, and 0.068 correspondingly.

\( \gamma \) and \( \tau \): These two parameters are related to parents’ voluntary spending on children’s education, the human capital goods. \( \tau \) is used to match the relative spending. In any steady state,

\[
\frac{p_{E_4}E_4}{p_{E_5}E_5} = \frac{\tau}{1 - \tau}
\]  

(2.25)

Since the left had side of Equation 2.25 is directly observable in the data, this pins down the value of \( \tau \). \( \gamma \) is the return to human capital parameter in this model, and is used to match the absolute share of spending observed in the data in the 2000s. In this exercise I assume a fixed \( \gamma \) through out.

One concern might be that the return to human capital in China has been increasing. Table 2.4 summarizes the yearly mincer return from year 1992 to 2009. It has increased significantly since the privatization in 1990s and then it has remained relatively stable in 2000s. For children born under the One-child Policy, when they enter labour market, the return has stabilized, so perhaps a constant \( \gamma \) may not be that a bad idea. I use \( \gamma \) to target the absolute expenditure share on education. Figure 2.6 summarizes the data and the model target on education spending.

Matching Pre-policy Fertility

So far I have not used the fertility optimality condition. The only parameter left undetermined is \( \nu \), which captures the love of children. I proceed by keeping the most of parameter values above constant. Two changes are made.
Table 2.3: Education Fees per person

<table>
<thead>
<tr>
<th>Year</th>
<th>College fee per person level</th>
<th>College fee per person growth rate</th>
<th>high school fee per person level</th>
<th>high school fee per person growth rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1996</td>
<td>1477.12</td>
<td>307.49</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1997</td>
<td>1823.75</td>
<td>0.23</td>
<td>382.63</td>
<td>0.24</td>
</tr>
<tr>
<td>1998</td>
<td>2144.72</td>
<td>0.18</td>
<td>419.11</td>
<td>0.10</td>
</tr>
<tr>
<td>1999</td>
<td>2921.71</td>
<td>0.36</td>
<td>484.34</td>
<td>0.16</td>
</tr>
<tr>
<td>2000</td>
<td>3463.60</td>
<td>0.19</td>
<td>576.41</td>
<td>0.19</td>
</tr>
<tr>
<td>2001</td>
<td>3927.71</td>
<td>0.13</td>
<td>673.50</td>
<td>0.17</td>
</tr>
<tr>
<td>2002</td>
<td>4324.25</td>
<td>0.10</td>
<td>737.93</td>
<td>0.10</td>
</tr>
<tr>
<td>2003</td>
<td>4561.89</td>
<td>0.05</td>
<td>810.66</td>
<td>0.10</td>
</tr>
<tr>
<td>2004</td>
<td>4857.09</td>
<td>0.06</td>
<td>898.23</td>
<td>0.11</td>
</tr>
<tr>
<td>2005</td>
<td>5070.67</td>
<td>0.04</td>
<td>994.13</td>
<td>0.11</td>
</tr>
<tr>
<td>2006</td>
<td>4931.45</td>
<td>-0.03</td>
<td>1008.33</td>
<td>0.01</td>
</tr>
<tr>
<td>2007</td>
<td>6489.44</td>
<td>0.32</td>
<td>1470.71</td>
<td>0.46</td>
</tr>
<tr>
<td>2008</td>
<td>7016.87</td>
<td>0.08</td>
<td>1585.81</td>
<td>0.08</td>
</tr>
<tr>
<td>2009</td>
<td>7182.25</td>
<td>0.02</td>
<td>1672.75</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Average: 0.135 0.144

Data source: Educational Statistics Yearbook of China.

One is changing $\phi_6$, $\phi_7$ and $\phi_8$ to the values observed in 1992, and another change is to assume the pre-1970 TFP growth to be zero. Given these, I find the value of $\nu$ such that the unconstrained steady state fertility would be three children per family, which is the observed fertility in the 1960s.\(^8\)

2.4.2 Dynamic Transition

Over this time period, nominal education price has increased significantly. As a proxy for the change in education price, I calculated the changes in high school and college fees. Data is transcribed from Educational Statistics Yearbook of China. I use the total fees divided by enrolment in the corresponding education level to get the fee person. Table 2.3 summaries the change over this period. On average, college fee increases by 13.5% annually and high school fee increases by 14.4%.

\(^8\)Ideally I want to match $\phi_6$, $\phi_7$ and $\phi_8$ to the values before 1970, but there is no data available for that time.
The question is whether this change is matched by nominal increase in wage rate. I focus on the average wage increase of those between 40 to 50 years old. Using data from UHS, I calculate that average wage increase by 14.8% annually between year 1996 and 2001. Then from year 2002 to year 2009 average wage increase by 10%. This suggests that the growth of nominal wage income roughly matches the education price increase before year 2000 and then fall below that after 2000. I use the growth of fees relative to the growth rate of wage to construct the relative price $\frac{P_{E_{t+4}}}{W_{t+4}}$ and $\frac{P_{E_{t+5}}}{W_{t+5}}$. Since one period in the model is equal to five years in real life, I use the five year average.

To solve the model, I also need initial distributions of population age, their corresponding human capital, and wealth distribution. I use the urban population distribution in year 1970 as the initial population distribution. I assume all agents born before 1970 have equilibrium human capital level that is predicted by the unconstrained pre-1970 steady state of the model, and they also hold the capital as in the pre-1970 steady state.

### 2.4.3 Results

This section presents the model’s results with and without fertility intervention. The fertility intervention policy I feed into the model is a 2.5-children policy from 1970 to 1974 and a 1.5-children policy from 1975 to 1979 and a One-child policy since year 1980. In the case with no fertility intervention, agents optimally choose fertility rate, and I call this the natural transitional fertility.

The response of the model’s fertility rate and individual human capital is summarized in Figure 2.7. The fall of fertility under the natural transition is

---

9First I calculate the wage increase of the 40 to 50-year-old for each education category and then take the average of the wage increase. The reason I calculate for these two sub-periods is because there is a change of classification of education category. I also calculated the weighted mean of the growth of wage income. The results are very similar to the numbers reported here.
due to the higher cost of raising children. However, it rises to above 1.5 (three children per family) before it starts to fall. This is due to the construction of the model. Agents have perfect foresight, so when parents decide on the number of children to give birth to in 1970s, they foresee the higher productivity growth after 1980, hence they will want to invest more in children, both in quantity and quality. Under natural transition fertility stabilizes at around 1.6 children per family (0.8 in the figure).

For individual human capital, the model predicts that the human capital level of agents born under the One-child policy (after 1980) is about around 30% higher than that of the agents born under the natural transition.

Figure 2.8 shows the results for aggregate capital stock, aggregate labour units, and output. Despite not obvious on the graph, the aggregate capital stock, as in equation (2.22), is slightly higher under the One-Child policy case before 1990. This is due to that agents whose fertility choices are restricted spent less on children and save more. Later the natural transition path takes over because of the larger population. The second panel in 2.8 shows the paths for efficient labour units, calculated as in equation (2.21). After year 1995, the efficient labour units under One-child policy is lower than that under natural transition. In other words, even though the fertility policy increases human capital per child, the aggregate human capital in the economy is lower. Hence the output will eventually fall below the natural transition case, as shown in the third panel. The model’s prediction suggests the negative effect of fertility policy started to show in around year 2000.

These results suggest that despite there is a quality quantity trade-off on the individual level, it does not exist on aggregate level. The increase in individual human capital is unable to overturn the effect of a fertility decline.

This model’s prediction of fertility rate at 1.6 under natural transition suggests that the current Two-children policy is not binding on average, if the
cost of raising and educating children remains at its current level. The above calibration takes education cost variations as exogenous. It is possible to theorize a situation where the increase in education price is actually the result of a binding fertility policy. When parents are only allowed to have less than their ideal number of children, they may be willing to pay a higher price for a given amount of human capital goods. Similarly, when they give birth to more children, parents are willing to pay less for the same human capital level, because parents are paying for more children. If education price respond endogenously respond to fertility, it is possible that the model under predicts the counter-factual fertility rate.

2.5 Conclusion

In this paper, I present an OLG model of fertility and human capital choice in a framework with inter-generational transfer. I illustrate that when an exogenously binding fertility is imposed, agents choose the maximum fertility allowed, and this leads to an increase in the human capital level and wage income of the generations affected. To assess the effect on total income, I extend the model to 16-period and calibrate the quantitative model. The calibration exercise shows that the One-child policy improves the human capital of the generations born under the policy by around 30%. However, even though it improves the individual human capital and income, its effect on total income has turned negative. The increase in individual income cannot offset the decline in fertility. This suggests that even though there is quantity quality trade-off on the individual level, there is no quantity-quality trade-off on the aggregate level. Based on this calibration exercise, the unconstrained fertility is around 1.6 children per family given the current cost of raising and educating children. The relaxation of the One-child policy at the end of 2015 to a Two-children policy is not binding.
2.6 Appendix

2.6.1 Consumption and Saving in the Three Period Model

The consumption decisions of a young agent at time $t$:

\[ c_{y,t} = \frac{1}{1 + \beta} \left[ \left( 1 - \psi \frac{n_{t-1}^{w-1}}{\omega} - n_{t}\phi_{f} \right) w_{y,t} - n_{t}p_{E,t}E_{t} + \frac{\psi n_{t}^{w} w_{y,t+1}}{\omega R_{t+1}} \right] \quad (2.26) \]

\[ c_{m,t+1} = \frac{\beta R_{t+1}}{1 + \beta} \left[ \left( 1 - \psi \frac{n_{t-1}^{w-1}}{\omega} - n_{t}\phi_{f} \right) w_{y,t} - n_{t}p_{E,t}E_{t} + \frac{\psi n_{t}^{w} w_{y,t+1}}{\omega R_{t+1}} \right] \quad (2.27) \]

and the corresponding saving decisions is given by:

\[ a_{y,t} = \frac{\beta}{1 + \beta} \left[ \left( 1 - \psi \frac{n_{t-1}^{w-1}}{\omega} - n_{t}\phi_{f} \right) w_{y,t} - n_{t}p_{E,t}E_{t} \right] - \frac{1}{1 + \beta} \frac{\psi n_{t}^{w} w_{y,t+1}}{\omega R_{t+1}} \quad (2.28) \]

2.6.2 Optimality Conditions of the Quantitative Model

The consumption decision of this agent is standard.

\[ c_{t}(5) = w_{t}(5) \]

\[ c_{t+1}(6) = \frac{1}{10} W_{t+1}(6) \]

\[ c_{t+i}(5 + i) = \beta R_{t+i} c_{t+i-i}(4 + i), \forall i = 2, ..., 11 \]
where $W_{t+1}(6)$ is given by:

$$
W_{t+1}(6) = (1 - n_{t+1} \phi_6) w_{t+1}(6) + \frac{(1 - n_{t+1} \phi_7) w_{t+2}(7)}{R_{t+2}} + \frac{(1 - n_{t+1} \phi_8 - \frac{\psi n_{t+4}^{c-1}}{\omega}) w_{t+3}(8)}{R_{t+2} R_{t+3}} + \frac{(1 - n_{t+1} \phi_9 - \frac{\psi n_{t+4}^{c-1}}{\omega}) w_{t+4}(9) - n_{t+1} p_{t+4} E_{t+4}(4)}{R_{t+2} R_{t+3} R_{t+4} R_{t+5}} + \frac{(1 - \psi n_{t+4}^{c-1}) w_{t+5}(10)}{R_{t+2} R_{t+3} R_{t+4} R_{t+5}} - n_{t+1} p_{t+5} E_{t+5}(5)
$$

The optimal choice of fertility:

$$
\frac{v}{n_{t+1}} + \frac{\beta^7}{c_{t+8}(13)} \psi n_{t+1}^{c-1} w_{t+8} + \frac{\beta^8}{c_{t+9}(14)} \psi n_{t+1}^{c-1} w_{t+9} + \frac{\beta^9}{c_{t+10}(15)} \psi n_{t+1}^{c-1} w_{t+10} + \frac{\beta^{10}}{c_{t+11}(16)} \psi n_{t+1}^{c-1} w_{t+11} = \frac{1}{c_{t+1}(6)} \left( \phi_6 w_{t+1}(6) + \frac{\phi_7 w_{t+2}(7)}{R_{t+2}} + \frac{\phi_8 w_{t+3}(8)}{R_{t+2} R_{t+3}} + \frac{p_{E_{t+4} E_{t+4}}(4)}{R_{t+2} R_{t+3} R_{t+4} R_{t+5}} + \frac{p_{E_{t+5} E_{t+5}}(5)}{R_{t+2} R_{t+3} R_{t+4} R_{t+5}} \right)
$$

where the left hand side is marginal gain from an extra child and right hand side is the marginal cost of raising another child.

The optimal choice of human capital spending in the fourth and fifth period of children’s life is given by:

$$
n_{t+1} p_{E_{t+4} E_{t+4}}(4) = \frac{\psi n_{t+1}^{c-1}}{\omega} \frac{\partial h_{t+5}}{\partial E_{t+4}(4)} \left( \frac{1}{R_{t+8} \ldots R_{t+5}} \frac{\partial w_{t+8}(8)}{\partial h_{t+5}} + \frac{1}{R_{t+9} \ldots R_{t+5}} \frac{\partial w_{t+9}(9)}{\partial h_{t+5}} + \frac{1}{R_{t+10} \ldots R_{t+5}} \frac{\partial w_{t+10}(10)}{\partial h_{t+5}} + \frac{1}{R_{t+11} \ldots R_{t+5}} \frac{\partial w_{t+11}(11)}{\partial h_{t+5}} \right)
$$

$$
n_{t+1} p_{E_{t+5} E_{t+5}}(5) = \frac{\psi n_{t+1}^{c-1}}{\omega} \frac{\partial h_{t+5}}{\partial E_{t+5}(5)} \left( \frac{1}{R_{t+8} \ldots R_{t+6}} \frac{\partial w_{t+8}(8)}{\partial h_{t+5}} + \frac{1}{R_{t+9} \ldots R_{t+6}} \frac{\partial w_{t+9}(9)}{\partial h_{t+5}} + \frac{1}{R_{t+10} \ldots R_{t+6}} \frac{\partial w_{t+10}(10)}{\partial h_{t+5}} + \frac{1}{R_{t+11} \ldots R_{t+6}} \frac{\partial w_{t+11}(11)}{\partial h_{t+5}} \right)
$$
### 2.6.3 Mincer Return

Table 2.4 shows the mincer regression from 1992 to 2009 using Urban Household Survey Data.

#### Table 2.4: Mincer Return: 1992 to 2009

<table>
<thead>
<tr>
<th>Year</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>yos</td>
<td>exp</td>
<td>exp2</td>
<td>Constant</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2009</td>
<td>0.121***</td>
<td>0.0991***</td>
<td>-0.00169***</td>
<td>7.018***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00241)</td>
<td>(0.00217)</td>
<td>(0.00154***</td>
<td>(0.0484)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td>0.123***</td>
<td>0.0916***</td>
<td>-0.00154***</td>
<td>6.895***</td>
<td>0.217</td>
<td>0.189</td>
</tr>
<tr>
<td></td>
<td>(0.00250)</td>
<td>(0.00237)</td>
<td>(0.00162***</td>
<td>(0.0531)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2007</td>
<td>0.132***</td>
<td>-0.000915</td>
<td>-1.23e-07</td>
<td>7.917***</td>
<td>0.126</td>
<td>0.126</td>
</tr>
<tr>
<td></td>
<td>(0.00249)</td>
<td>(0.000652)</td>
<td>(4.14e-07)</td>
<td>(0.0452)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2006</td>
<td>0.129***</td>
<td>0.0817***</td>
<td>-0.00130***</td>
<td>6.665***</td>
<td>0.165</td>
<td>0.165</td>
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<tr>
<td></td>
<td>(0.00247)</td>
<td>(0.00223)</td>
<td>(3.54e-05)</td>
<td>(0.0492)</td>
<td></td>
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</tr>
<tr>
<td>2005</td>
<td>0.132***</td>
<td>0.0820***</td>
<td>-0.00129***</td>
<td>6.325***</td>
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<td>0.167</td>
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<td></td>
<td>(0.00244)</td>
<td>(0.00225)</td>
<td>(3.59e-05)</td>
<td>(0.0488)</td>
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</tr>
<tr>
<td>2004</td>
<td>0.125***</td>
<td>0.0870***</td>
<td>-0.00138***</td>
<td>6.437***</td>
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<tr>
<td></td>
<td>(0.00260)</td>
<td>(0.00236)</td>
<td>(3.77e-05)</td>
<td>(0.0512)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Observations | 24,940 | 25,323 | 26,765 | 26,341 | 25,958 | 25,168 |
| R-squared     | 0.217  | 0.189  | 0.126  | 0.165  | 0.167  | 0.152  |

<table>
<thead>
<tr>
<th>Year</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
<th>(11)</th>
<th>(12)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>yos</td>
<td>exp</td>
<td>exp2</td>
<td>Constant</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2003</td>
<td>0.127***</td>
<td>0.0909***</td>
<td>-0.00139***</td>
<td>6.169***</td>
<td>0.154</td>
<td>0.179</td>
</tr>
<tr>
<td></td>
<td>(0.00265)</td>
<td>(0.00238)</td>
<td>(0.00162***</td>
<td>(0.0509)</td>
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<tr>
<td>2002</td>
<td>0.127***</td>
<td>0.102***</td>
<td>-0.00219***</td>
<td>6.031***</td>
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<tr>
<td></td>
<td>(0.00276)</td>
<td>(0.00249)</td>
<td>(0.00401)</td>
<td>(0.0526)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2001</td>
<td>0.111***</td>
<td>0.124***</td>
<td>-0.00187***</td>
<td>6.043***</td>
<td>0.180</td>
<td>0.180</td>
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<tr>
<td></td>
<td>(0.00459)</td>
<td>(0.00401)</td>
<td>(0.00350)</td>
<td>(0.0854)</td>
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<tr>
<td>2000</td>
<td>0.123***</td>
<td>0.113***</td>
<td>-0.00197***</td>
<td>5.895***</td>
<td>0.193</td>
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<tr>
<td></td>
<td>(0.00413)</td>
<td>(0.00350)</td>
<td>(0.00327)</td>
<td>(0.0752)</td>
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<tr>
<td>1999</td>
<td>0.0947***</td>
<td>0.115***</td>
<td>-0.00198***</td>
<td>6.194***</td>
<td>0.186</td>
<td>0.186</td>
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<tr>
<td></td>
<td>(0.00390)</td>
<td>(0.117***</td>
<td>(0.00302)</td>
<td>(0.0713)</td>
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<tr>
<td>1998</td>
<td>0.0904***</td>
<td>0.117***</td>
<td>-0.00198***</td>
<td>6.142***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00367)</td>
<td>(0.117***</td>
<td>(0.00302)</td>
<td>(0.0665)</td>
<td></td>
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</tr>
</tbody>
</table>

| Observations | 23,256 | 21,332 | 9,093 | 9,177 | 9,710 | 9,996 |
|             | 0.154  | 0.179  | 0.180 | 0.193 | 0.186 | 0.200 |

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<tr>
<th>Year</th>
<th>(13)</th>
<th>(14)</th>
<th>(15)</th>
<th>(16)</th>
<th>(17)</th>
<th>(18)</th>
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<td>exp2</td>
<td>Constant</td>
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<tr>
<td>1997</td>
<td>0.0845***</td>
<td>0.119***</td>
<td>-0.00290***</td>
<td>6.137***</td>
<td>10.246</td>
<td>10.148</td>
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<tr>
<td></td>
<td>(0.00358)</td>
<td>(0.00304)</td>
<td>(0.00207***</td>
<td>(5.06e-05)</td>
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<td>1996</td>
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<td>-0.00207***</td>
<td>6.212***</td>
<td>10.278</td>
<td>10.278</td>
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<td></td>
<td>(0.00328)</td>
<td>(0.00288)</td>
<td>(0.00285)</td>
<td>(4.79e-05)</td>
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<td>1995</td>
<td>0.0659***</td>
<td>0.120***</td>
<td>-0.00208***</td>
<td>6.202***</td>
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<td></td>
<td>(0.00319)</td>
<td>(0.00285)</td>
<td>(0.00278)</td>
<td>(4.77e-05)</td>
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<td>1994</td>
<td>0.0803***</td>
<td>0.119***</td>
<td>-0.00201***</td>
<td>5.804***</td>
<td>10.289</td>
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<td></td>
<td>(0.00321)</td>
<td>(0.119***</td>
<td>(0.00240)</td>
<td>(4.63e-05)</td>
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<td></td>
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<tr>
<td>1993</td>
<td>0.0584***</td>
<td>0.116***</td>
<td>-0.00196***</td>
<td>5.832***</td>
<td>10.798</td>
<td>10.798</td>
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<tr>
<td></td>
<td>(0.00272)</td>
<td>(0.113***</td>
<td>(0.00225)</td>
<td>(4.03e-05)</td>
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<tr>
<td>1992</td>
<td>0.0524***</td>
<td>0.113***</td>
<td>-0.00187***</td>
<td>5.704***</td>
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<td></td>
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<td></td>
<td>(0.00242)</td>
<td>(0.113***</td>
<td>(0.00225)</td>
<td>(3.88e-05)</td>
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</table>

| Observations | 10,246 | 10,148 | 10,278 | 10,243 | 10,289 | 10,798 |
| R-squared    | 0.187  | 0.200  | 0.199  | 0.209  | 0.221  | 0.214  |

Note: Standard errors in parentheses. ***p<0.01, ** p<0.05, * p<0.1.
References


Figure 2.1: Fertility rate: rural and total


Figure 2.2: Human capital and Physical capital conditions, given fertility

Note: $\beta = 0.6; \nu = 0.08; \alpha = 0.52; g_A = 1.3; \phi_f = 0; A_h = 0.5; \gamma = 0.4; \psi = 0.2; \omega = 0.65; \frac{p_h}{w_h} = 0.01$
Figure 2.3: Human capital and physical capital conditions, changing fertility

Note: $\beta = 0.6; \nu = 0.08; \alpha = 0.52; g_A = 1.3; \phi_f = 0; A_h = 0.5; \gamma = 0.4; \psi = 0.2; \omega = 0.65; \frac{pE}{w_0} = 0.01$
Figure 2.4: Steady state fertility and human capital goods

Figure 2.5: Education Spending on Child: 1992 to 2009

Note: This is the average expenditure on education in Single-child household, divided by household income. Data: Calculated from UHS.
Figure 2.6: Education spending on One-child

Note: Data from UHS 2002 to 2009 average

Figure 2.7: Fertility and human capital
Figure 2.8: Aggregate changes
Chapter 3

Information Technology, Input Structure, and Inequality

3.1 Introduction

With the recent development in information technology, the transmission of information has been made so much easier. In this paper, we analyze its implications on the structure of intermediate input and wage inequality.

We obtain an information score for each occupation, based on the the level of the skill of analyzing data or information needed in this occupation. We calculate the weighted national average info score in 1980, and we group industries into information (info) sector and non-information (non-info) sector based on industry average information score. Industries with information score higher than national average are classified into info sector and the rest are in the non-info sector. Similarly, workers are grouped into info workers and non-info workers. Those in occupations with information score higher than 1980 national average are info workers, and the rest are classified into non-info workers.

Based on this classification, we make two observations. The first is that over
time, info goods, output of the info sector, accounts for a larger share in total intermediate input, while non-info goods, output of the non-info sector, accounts for a smaller share in total intermediate input. The second observation is that wage rate of info labour over non-info labour is increasing over time, controlling for other observables.

We interpret the development of information technology as the decline in the transaction cost of info sector output. In a two sector model with an info sector and a non-info sector, we assess if this decline can explain the observations on the intermediate input structure and wage inequality. The model has two goods, info good and non-info good. There are also two types of labour, info labour and non-info labour. Production in both sectors require an aggregate info input and an aggregate non-info input. The aggregate info input in each sector is provided by info labour and market purchase of intermediate info goods. Similarly, the aggregate non-info input is provided by non-info labour and intermediate non-info goods.

In this framework, we show that with reasonable assumptions on the elasticities, a decline in transaction cost of info good cannot simultaneously explain the rise of info good share in intermediate input and the rise of relative wage rate of info worker. Two elasticities are crucial in this framework. One is the elasticity of substitution between the aggregate info input and the aggregate non-info input. The other one is the elasticity of substitution between labour and intermediate good for a given aggregate input.

Vaguely speaking, when transaction cost of info good falls, to generate an increase in the share of info intermediate input, we need the elasticity of substitution between aggregate info input and aggregate non-info input to be larger than one. In other words, they need to be good substitutes. In this setting, it is natural to assume that substituting between labour and intermediate goods used to produce a given aggregate input is easier than substituting between the
two aggregate inputs. If so, a decline in transaction cost of info good cannot increase the relative wage rate of info worker. When transaction cost of info good declines, both labour types benefit because it represents a technological improvement. However, non-info labour benefits more, because compared with info labour, non-info labour is less substitutable with info good. In a summary, with these assumptions on elasticities, we can explain the increase of info intermediate input share but not the increase of the relative wage rate of info worker.

Our industry classification follows the BEA 15 sector classification, and the only difference is that we take out the manufacturing of Computers and Electronic Services from the manufacturing sector to be an individual industry. Based on this industry classification, those that are classified as info sector includes: Wholesale; Utilities; Finance, Insurance and Real Estate; Professional Business Service; Educational Services, Health Care and Social Assistance; and Manufacturing of Computer and Electronic Products. This is similar to the sector classification by Jorgenson et al. (2005) in that our info sector largely corresponds to what he calls IT-producing and IT-using sector, and our non-info sector is similar to what he calls non-IT using industries. Berlingieri (2013) also notes the increasing importance of finance and professional business service sector as intermediate input.

This paper proceeds as follows: Section 3.2 presents the empirical motivations. Section 3.3 presents a two sector model where sectors differ in labour intensity. Section 3.4 calibrate the model and analyze the quantitative effects. Sections 3.5 concludes.
3.2 Empirical Motivation

Data used in this section is from IPUMS ASEC CPS, BEA Input-output table and O*Net database.

3.2.1 Information Score and Sector Classification

We make use of the question "Analyzing data or information" in the O*Net data. It is one of the 41 questions on work activities asked for each occupation in the O*Net database V15.1. For each question, an importance score and a level score are reported. We use the level score which is reported on a scale of 0 to 7. A higher score indicates that a higher degree of a particular descriptor is required or needed to perform the occupation. We take the reported score to be our occupation information score, and we calculate the sector level information score as the weighted average of occupation information score. The weights are occupation shares in each sector calculated from the IPUMS CPS ASEC. We calculate an information score for each industry based on a 16-industry classification. This classification is the same as the BEA 15 aggregate sector classification, except for one difference; we take out the manufacturing of Computer and Electronic Products (NAICS 334) and re-
We then calculate the unweighted average of industry information score, and we classify industries with information score above this average as information sector, and those below as non-information sector. Based on this classification, information sector includes Wholesale; Utilities; Finance, Insurance and Real Estate; Professional Business Service; Educational Services, Health Care and Social Assistance; and Manufacturing: Computer and Electronic Products. The rest is classified into non-information sector. The information score for each industry in 1980 is reported in Table 3.6 in the Appendix 3.6.2

### 3.2.2 Intermediate Input Share

Based on the sector classification, we present the changes in intermediate input structure. In both info and non-info sector, the share of info goods in total intermediate increases, and the share of non-info goods declines, shown in Figure 3.1 and Figure 3.2 respectively. These figures also indicate that the share of total intermediate input in each sector is relatively stable.

Figure 3.3 plots the ratio of info intermediate input over non-info intermediate input in each sector. This ratio increases more in info sector.

### 3.2.3 Returns to Info Labour

First we show that returns to occupations with higher information score has been increasing overtime. For each occupation, we calculate its mean real

---

4The industries are: 1 Agriculture; 2 Mining; 3 Utilities; 4 Construction; 5 Manufacturing, excluding Computer and Electronic Products; 6 Wholesale trade; 7 Retail trade; 8 Transportation and Warehousing; 9 Information; 10 FIRE; 11 PBS; 12 Educational Services, Health Care and Social Assistance; 13 Arts, Entertainment, Recreation, Accommodation and Food Services; 14 Other services, except Government; 15 Manufacturing: Computer and Electronic Products, 16 Government. We ignore the government sector.

5Based on this classification, industry 9, the Information industry is classified into non-info sector.
Table 3.1: Regression on Information Score

<table>
<thead>
<tr>
<th>log hourly wage</th>
<th>1980</th>
<th>1990</th>
<th>2000</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>info score</td>
<td>0.0935***</td>
<td>0.121***</td>
<td>0.141***</td>
<td>0.166***</td>
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<tr>
<td></td>
<td>(0.00322)</td>
<td>(0.00289)</td>
<td>(0.00322)</td>
<td>(0.00272)</td>
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<td></td>
<td>(0.0643)</td>
<td>(0.0542)</td>
<td>(0.0747)</td>
<td>(0.0707)</td>
</tr>
<tr>
<td>Race Dummy</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Educ Dummy</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Sex Dummy</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>years of experience</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>experience square</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
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<td>42,146</td>
<td>41,454</td>
<td>36,344</td>
<td>50,383</td>
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<tr>
<td>R-squared</td>
<td>0.209</td>
<td>0.263</td>
<td>0.283</td>
<td>0.319</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

hourly wage by year. Figure 3.4 shows the scatter plot for 1980, 1990, 2000 and 2010. We also fit a linear line for each year, and for ease of comparison, Figure 3.5 summarizes all four lines on the same graph. Clearly the lines are become steeper over time, indicating higher returns to occupations with higher information score.

To show this more formally, we regress the log real hourly income on information score and other control variables. Table 3.1 summarizes the results. The coefficient on information score is increasing over time.

In section 3.6.3 in the Appendix, we show a comparison of the measure used here and the ones proposed by Acemoglu and Autor (2011).

To better match the model in Section 3.3, we can also group workers into info and non-info worker and calculate the relative wage rate. We use the same threshold as the one we used for sector classification. All workers in occupations with information score higher than national average are classified as info workers, and all workers in occupations with score lower than that are classified as non-info workers. Figure 3.6 shows the results. It is increasing
over time until around 2008.

3.3 Model

In this section, we present a two-sector model. Assume there are info sector $I$ and non-info sector $N$. In each sector, production requires both info input and non-info input. Info input can be produced using info labour and info sector output. Similarly, non-info input can be produced using non-info labour and non-info sector output. Production function has the following form:

$$ Q_j = A_j \left[ \left( A_{jj} L_{jj} \right)^{\frac{\sigma_1}{\sigma_2}} + \left( \frac{M_{jj}}{\lambda_{jj}} \right)^{\frac{\sigma_1}{\sigma_2}} \right]^{\frac{\sigma_2}{\sigma_1}} + \left( A_{j,-j} L_{j,-j} \right)^{\frac{\sigma_1}{\sigma_2}} + \left( \frac{M_{j,-j}}{\lambda_{j,-j}} \right)^{\frac{\sigma_1}{\sigma_2}} \right]^{\frac{\sigma_2}{\sigma_1}} \right] (3.1) $$

where $Q_j$ and $A_j$ are the output and aggregate productivity of sector $j$, $j \in \{I, N\}$. When $j = N$, then $-j = I$, and vice versa. $A_{jj}$ and $A_{j,-j}$ are the labour productivity of type $j$ and type $-j$ labour working in sector $j$. $L_{jj}$ and $L_{j,-j}$ are type $j$ and type $-j$ labour employed in sector $j$. $M_{jj}$ and $M_{j,-j}$ are the intermediate inputs that sector $j$ purchases from sector $j$ and sector $-j$, and $\lambda_{jj}$ and $\lambda_{j,-j}$ represent the transaction costs incurred when sector $j$ uses intermediate inputs from sector $j$ and sector $-j$. $\sigma_2$ is the elasticity of substitution between the aggregate info input and aggregate non-info input, and $\sigma_1$ is the elasticity of substitution between labour and intermediate input.

When $\sigma_2 > 1$, aggregate info input and aggregate non-info input are good substitutes.

There are two types of homogeneous labour: a measure $S_I$ of info worker and a
measure $S_N$ of non-info worker. Workers receive wage payment and consume. They have the following preference over info and non-info goods:

$$C = \left[ \zeta C_N^{\frac{\epsilon-1}{\epsilon}} + (1 - \zeta) C_I^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{1}{\epsilon-1}} \tag{3.2}$$

and the corresponding price index is given by:

$$P = \left[ \zeta P_N^{1-\epsilon} + (1 - \zeta) P_I^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \tag{3.3}$$

### 3.3.1 Optimality Conditions

Each sector maximizes the following problem:

$$\max_{L_{j,j}, L_{j,-j}, M_{j}} P_j Q_j - W_j L_{j,j} - W_{-j} L_{j,-j} - P_j M_{j,j} - P_{-j} M_{j,-j}$$

where $P_j$ refers to the price of sector $j$ output. $W_j$ and $W_{-j}$ are the wage rates of type $j$ and type $-j$ labour. The first order conditions for sector’s optimal input is summarized in Appendix 3.6.4.

The consumers maximizes the following aggregate consumption, subject to their total wage income. The problem is written as:

$$\max_{C_N, C_I} C = \left[ \zeta C_N^{\frac{\epsilon-1}{\epsilon}} + (1 - \zeta) C_I^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{1}{\epsilon-1}} \tag{3.4}$$

subject to $$P_N C_N + P_I C_I = S_N W_N + S_I W_I$$

The optimal consumption decision satisfies:

$$\frac{P_I C_I}{P_N C_N} = \left( \frac{1 - \zeta}{\zeta} \right)^\epsilon \left( \frac{P_I}{P_N} \right)^{1-\epsilon} \tag{3.5}$$
3.3.2 Equilibrium

In equilibrium, goods market must clear, and hence:

\[ Q_j = C_j + M_{j,j} + M_{-j,j}; j \in I, N \]  

(3.6)

Labour market satisfies:

\[ S_N = L_{IN} + L_{NN}; \quad S_I = L_{NI} + L_{II}; \]  

(3.7)

In a summary, the equilibrium satisfies the first order conditions, equation (3.10) to equation (3.17), production functions as in equation (3.1), the optimal consumption decision and budget constraint, equation (3.5) and (3.4), the labour market and goods market clearing conditions, equation (3.6) and (3.7) and the aggregate the price index equation (3.3) with \( P = 1 \).

3.3.3 Effects of a Fall in Transaction Cost of Info Good

We interpret a fall in transaction cost of info goods as a simultaneous fall in both \( \lambda_{II} \) and \( \lambda_{NI} \) by the same percentage. This is similar to a decrease in iceberg cost. We can write the marginal product of info labour over the marginal product of on-info labour in info sector as:

\[
\frac{MP_{II}}{MP_{IN}} = \left( \frac{A_{II} L_{II}}{A_{IN} L_{IN}} \right)^{\frac{\sigma_2 - 1}{\sigma_2}} + \left( \frac{M_{II}}{M_{NI}} \right)^{\frac{\sigma_2 - 1}{\sigma_2}} \left( \frac{M_{II}}{M_{NI}} \right)^{\frac{1}{\sigma_2}} \left( \frac{L_{II}}{L_{IN}} \right)^{\frac{1}{\sigma_2}}
\]  

(3.8)

The direct effect of a fall in \( \lambda_{II} \) on relative marginal productivity, holding labour allocation and intermediate input constant, depends on the sign of \( \sigma_2 - \sigma_1 \). If \( 0 < \sigma_1 < \sigma_2 \), then a fall in \( \lambda_{II} \) increase the relative productivity of info worker. If \( \sigma_1 > \sigma_2 > 0 \), then it decreases the relative productivity of
info worker. This is intuitive, when $\sigma_1 > \sigma_2 > 0$, the substitution between labour and intermediate input in the production of a given aggregate input is easier than the substitution between two types of aggregate input. This means that info intermediate input is more substitutable with info labour and non-info labour, and hence, the type of labour that is less substitutable with intermediate info input benefits more. I used the info sector as an illustration, but it is the same with the non-info sector.

What happens to the overall substitution between intermediate info and intermediate non-info will depend on $\sigma_2$, which determines the substitutability between aggregate info input and aggregate non-info input. Manipulating the first order conditions, we can write the nominal relative intermediate input in info sector as:

$$\frac{P_I M_{II}}{P_N M_{IN}} = \left( \frac{P_I \lambda_{II}}{P_N \lambda_{IN}} \right)^{1-\sigma_2} \left( \frac{(\frac{P_I \lambda_{II}}{W_I} A_{II})^{\sigma_1-1} + 1}{(\frac{P_N \lambda_{IN}}{W_N} A_{IN})^{\sigma_1-1} + 1} \right)^{\frac{\sigma_2 - \sigma_1}{\sigma_1 - 1}} \tag{3.9}$$

Without the second term on the right hand side, this equation is the same as the expenditure share derived from a standard CES function. In that case, with $\sigma_2 > 1$, holding prices constant, a fall in $\lambda_{II}$ lead to a higher info intermediate input share. With a reasonable assumption of $\sigma_2 < \sigma_1$, the second term predicts the same qualitative results as the first term.

In a summary, to generate the rising of info intermediate input share when transaction costs fall, we need $\sigma_2$ to be greater than one, and together with a reasonable assumption of $\sigma_2 < \sigma_1$, this model will generate an increase in the info intermediate share, but will lead to a decline of the relative wage rate of info worker.

Of course the above discussion above is based on partial equilibrium. In Section 3.4, we numerically solve the model and discuss the results under general
Before moving to the quantitative result, there is another point worth mentioning. Despite that our focus is on the decline of transaction cost of info good, a simultaneous fall in $\lambda_{II}$ and $\lambda_{NI}$, an increase of $A_I$ has similar effects.

Intuitively, $A_I$ is making info sector more productive, in both the production of consumption info good and intermediate info good, while a decline in transaction cost of info good represents an increase in productivity of info sector in producing intermediate input, not consumption good. This is because we assume the transaction costs appears when a good is used as intermediate input, not when it is consumed. If we were to assume that the same transaction cost also applies when a good is consumed, then an increase info sector’s productivity $A_I$ will have the same effects as the fall of transaction cost $\lambda_{NI}$ and $\lambda_{II}$.

### 3.4 Quantitative Results

#### 3.4.1 Parameter Calibration

In this section, I first discuss how other parameters are calibrated for a given pair of elasticities $\sigma_1$ and $\sigma_2$. We normalize both $A_I$ and $A_N$ to be 1.

We calibrate the parameters in consumption CES in equation (3.2) using the relative price $\frac{P_I}{P_N}$ calculated from US KLEMS data and the consumption share $\frac{P_{IC_I}}{P_{NC_N}}$ calculated from BEA data.

The other parameters are calibrated to match certain moments in the 1980 data. We treat all workers in occupations with information score lower than

---

6We show in Appendix 3.6.5 that $A_I$ actually increase less than $A_N$.
7We regress the log first difference version of equation 3.5 and get a $\epsilon$ value of about 0.9. Then we back out the $\zeta$ using yearly data. Each year gives a different $\zeta$, and it suggest that $\zeta$ is declining over time. In other words, consumers increasingly prefer info goods over time. However, in the baseline, we set $\zeta = 0.6$.  

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the national average in 1980 as non-info worker, and workers in occupations with information score higher than national average as info workers. The 1980 CPS data implies that total hours supplied by info worker relative to that of non-info worker is 0.8196. We normalize the supply of non-info labour to be one, $S_N = 1$, hence the supply of of info labour is $S_I = 0.8196$. Also, about 50.91% of the total info hours is supplied to non-info sector and the rest to info sector. Together with the homogenous labour assumptions, we get $L_{NI} = 0.5091S_I$, and $L_{II} = 0.4909S_I$. For non-info hours, 74.49% is supplied to non-info sector. Thus $L_{NN} = 0.7449S_N$, and $L_{IN} = 0.2551S_N$.

The average hourly wage rate of an info worker relative to that of an non-info worker calculated from 1980 data is $\frac{W_I}{W_N} = 1.4893$.8

The intermediate input presented in Figure 3.1 and Figure 3.2 is divided by gross GDP, and it includes capital. Our model does not have capital, We also need to to exclude the share of capital. Using KLEMS data to calculate capital share, we can calculate the intermediate shares $\frac{P_I M_{NI}}{P_N Q_N}$, $\frac{P_N M_{NN}}{P_N Q_N}$, $\frac{P_N M_{IN}}{P_I Q_I}$, and $\frac{P_I M_{II}}{P_I Q_I}$.

Since we also observe the labour allocation and relative wage rate, we can also work out $\frac{W_I L_{II}}{P_I Q_I}$, $\frac{W_N L_{IN}}{P_N Q_N}$, $\frac{W_I L_{NI}}{P_N Q_N}$, and $\frac{W_N L_{NN}}{P_N Q_N}$.

With the labour inputs and intermediate share given, we can calculate the implied nominal consumption share $\frac{P_I C_I}{P_N C_N}$. Then by equation (3.5), we find the relative price $\frac{P_I}{P_N}$ that is consistent with this relative nominal consumption. Together with the aggregate price index equation (3.3), it solves $P_I$ and $P_N$.

Finally the last set of parameters to be identified are the productivities $A_I$ and $A_N$, the transaction cost $\lambda_{II}$, $\lambda_{IN}$, $\lambda_{NI}$ and $\lambda_{NN}$. These can be backed out from the first order conditions, equation (3.10) to equation (3.17), given that we now have all the input allocations.

---

8The sector level relative wage rate differ in two sectors. Given our homogenous labour assumption, we cannot, at the same time, match the relative wage rate of both sectors.
### Table 3.2: Parameters: $\sigma_1 > \sigma_2 > 1$

<table>
<thead>
<tr>
<th>Exogenously Set Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_1$</td>
<td>1.5</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>1.2</td>
</tr>
<tr>
<td>normalization</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Normalization</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_I$</td>
<td>1  normalization</td>
</tr>
<tr>
<td>$A_N$</td>
<td>1  normalization</td>
</tr>
<tr>
<td>$A_{II}$</td>
<td>1  normalization</td>
</tr>
<tr>
<td>$s_N$</td>
<td>1  normalization</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Calibrated Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon$</td>
<td>0.9 consumption data</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.6 consumption data</td>
</tr>
<tr>
<td>$s_I$</td>
<td>0.8196 hours worked by info and non-info workers</td>
</tr>
<tr>
<td>$A_{IN}$</td>
<td>0.3673 labour allocation and intermediate input share</td>
</tr>
<tr>
<td>$\lambda_{II}$</td>
<td>4.4483 labour allocation and intermediate input share</td>
</tr>
<tr>
<td>$\lambda_{IN}$</td>
<td>5.7742 labour allocation and intermediate input share</td>
</tr>
<tr>
<td>$A_{NI}$</td>
<td>0.4007 labour allocation and intermediate input share</td>
</tr>
<tr>
<td>$A_{NN}$</td>
<td>0.8476 labour allocation and intermediate input share</td>
</tr>
<tr>
<td>$\lambda_{NI}$</td>
<td>11.2401 labour allocation and intermediate input share</td>
</tr>
<tr>
<td>$\lambda_{NN}$</td>
<td>2.4587 labour allocation and intermediate input share</td>
</tr>
</tbody>
</table>

### 3.4.2 Case 1: $\sigma_1 > \sigma_2 > 1$

In this case, we look at the case where the aggregate info input and aggregate non-info input are good substitutes $\sigma_2 > 1$, and that substitution between labour and intermediate input is easier than substitution between two aggregate input $\sigma_1 > \sigma_2$. Table 3.2 summarizes the parameter values. All other parameters are calibrated following the steps described in Section 3.4.1.

We look at both the increase of info sector productivity $A_I$ and fall in transaction costs $\lambda_{II}$ and $\lambda_{NI}$. The results are presented in Model 2 and Model 3 in Table 3.3. In both Model 2 and Model 3, the relative price of info good declines, the relative wage rate of info worker declines, and both sectors use more info input than non-info input. Model 3 makes it clear that with reasonable assumptions on the elasticities, the decline in transaction cost of info goods can explain the increase in the usage of info intermediate input, but not
Table 3.3: Model results: $\sigma_1 > \sigma_2 > 1$

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Data</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_1 = 1.5$, $\sigma_2 = 1.2$</td>
<td>1980</td>
<td>Baseline</td>
<td>$A_I \uparrow$</td>
<td>$\hat{\lambda}<em>{II} \downarrow$, $\hat{\lambda}</em>{NI} \downarrow$</td>
</tr>
<tr>
<td>$P_I/P_N$</td>
<td>1.2096</td>
<td>0.9704</td>
<td>1.1768</td>
<td></td>
</tr>
<tr>
<td>$W_I/W_N$</td>
<td>1.4893</td>
<td>1.4893</td>
<td>1.4633</td>
<td>1.4704</td>
</tr>
<tr>
<td>$(P_I M_{II})/(P_N M_{IN})$</td>
<td>0.8580</td>
<td>0.8580</td>
<td>0.9429</td>
<td>0.9400</td>
</tr>
<tr>
<td>$(P_I M_{NI})/(P_N M_{NN})$</td>
<td>0.2914</td>
<td>0.2914</td>
<td>0.3206</td>
<td>0.3196</td>
</tr>
</tbody>
</table>

Note: These data moments are are direct targets, so naturally the baseline case matches them exactly. Model 2 represent a 20% increase in $A_I$ relative to the baseline case, meaning $A_{I_{\text{new}}} = 1.2 A_{I_{\text{old}}}$. Model 3 represents a fall in $\lambda_{II}$ and $\lambda_{NI}$. $\lambda_{II}^{\text{new}} = \left(\frac{1}{1+\frac{1}{\sigma_2}}\right)^{\frac{\sigma_1-1}{\sigma_2}} \lambda_{II}^{\text{old}}$ and $\lambda_{NI}^{\text{new}} = \left(\frac{1}{1+\frac{1}{\sigma_2}}\right)^{\frac{\sigma_1-1}{\sigma_2}} \lambda_{NI}^{\text{old}}$.

the relative wage rate change.

3.4.3 Case 2: $\sigma_2 < 1$ and $\sigma_1 > \sigma_2$

In the above case, we assume that aggregate info input and non-info aggregate input are good substitutes $\sigma_1 > 1$, I now show examples where aggregate info input and non-info input are not good substitutes, $\sigma_1 < 1$. We follow the same steps described in Section 3.4.1 in calibrating the other parameters. For this reason, in Table 3.4, both Case 2A and Case 2B have the same baseline results as Case 1. Case 2A is the case where labour and intermediate input is bad substitute with $\sigma_1 < 1$, and Case 2B is the case where they are good substitutes with $\sigma_1 > 1$.

In both case 2A and 2B, a fall in the transaction cost leads to a lower relative wage rate and a lower ratio of info intermediate over non-info intermediate. Similar as before, the lower relative wage rate is because of the assumption $\sigma_1 > \sigma_2$, and the lower ratio of info intermediate over non-info intermediate is due to the low elasticity of substitution between aggregate info and aggregate non-info input.

As these cases demonstrate, the elasticity parameters $\sigma_1$ and $\sigma_2$ are crucial in

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Table 3.4: Model results: $\sigma_2 < 1$ and $\sigma_1 > \sigma_2$

<table>
<thead>
<tr>
<th>Case 2A</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_1 = 0.9$, $\sigma_2 = 0.8$</td>
<td>Baseline</td>
<td>$A_I \uparrow$</td>
</tr>
<tr>
<td>$P_I/P_N$</td>
<td>1.2096</td>
<td>0.9757</td>
</tr>
<tr>
<td>$W_I/W_N$</td>
<td>1.4893</td>
<td>1.4693</td>
</tr>
<tr>
<td>$(P_IM_{II})/(P_NM_{IN})$</td>
<td>0.8580</td>
<td>0.8393</td>
</tr>
<tr>
<td>$(P_IM_{NI})/(P_NM_{NN})$</td>
<td>0.2914</td>
<td>0.2850</td>
</tr>
</tbody>
</table>

Case 2B

<table>
<thead>
<tr>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_1 = 1.2$, $\sigma_2 = 0.8$</td>
<td>Baseline</td>
</tr>
<tr>
<td>$P_I/P_N$</td>
<td>1.2096</td>
</tr>
<tr>
<td>$W_I/W_N$</td>
<td>1.4893</td>
</tr>
<tr>
<td>$(P_IM_{II})/(P_NM_{IN})$</td>
<td>0.8580</td>
</tr>
<tr>
<td>$(P_IM_{NI})/(P_NM_{NN})$</td>
<td>0.2914</td>
</tr>
</tbody>
</table>

Note: Model 2 in both cases represents a 20% increase in $A_I$ relative to the baseline case, meaning $A_I^{new} = 1.2A_I^{old}$. Model 3 in both cases represents a fall in $\lambda_{II}$ and $\lambda_{NI}$. $\lambda_{II}^{new} = (1/\sigma_1)^{\sigma_1 - 1} \lambda_{II}^{old}$ and $\lambda_{NI}^{new} = (1/\sigma_1)^{\sigma_1 - 1} \lambda_{NI}^{old}$.

As illustrated in table determining whether we can explain the two facts with a decline in transaction cost of info good. A $\sigma_1$ greater than one is required to generate the substitution between info and non-info intermediate input. However, a natural assumption that $\sigma_2$ is less than $\sigma_1$ imply that the relative wage rate of info worker falls when with a decline in transaction cost.

3.5 Conclusion

In this paper, we first present two facts from the data. One is the increase in share of info goods in total intermediate input, and the second one is the rise in return to info worker. We ask if it these two facts can be explained by a decline in transaction costs of info good. We build a two sector model, and we show that with reasonable assumptions on the elasticities, a decline in cost of transaction of the info good cannot explain these two facts at the same time.
3.6 Appendix

3.6.1 14 Questions Related to Information

Table 3.5 lists the 14 questions that have the word information in either the question itself or question explanatory notes.

3.6.2 Information Score by Industry

The information score presented in Table 3.6 is based on the level score to the question "Analyzing data or information".

3.6.3 Comparisons with Existing Measures

Our measure of occupation information score is very similar to Acemoglu and Autor (2011)'s measure of non-routine cognitive analytical ability. They create five measures, non-routine cognitive analytical, non-routine interpersonal, routine cognitive, routine manual and non-routine manual. Table 3.7 summarizes them. Top panel shows they have two extra questions in their non-routine cognitive analytical skill measure, apart from analyzing data and information.

To see which of the five measures better explain wage variations, we calculate the partial $R^2$ values (net of experience, education, gender and race) using each of the 5 task measures and using all five measures in combination. Left panel of Figure 3.7 reports $R^2$ value from the residual wage dispersion regressions. The explanatory power of 4 out of 5 task measures rise from late 1980s, and it increases most for non-routine analytical task increases. In fact the increase in the $R^2$ of the regression using all 5 task measures together is driven by the non-routine analytical measure alone. Starting from late 1980s, the explanatory
<table>
<thead>
<tr>
<th>Number</th>
<th>Work activity</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Getting Information</td>
<td>Observing, receiving, and otherwise obtaining information from all relevant sources.</td>
</tr>
<tr>
<td>2</td>
<td>Identifying Objects, Actions, and Events</td>
<td>Identifying information by categorizing, estimating, recognizing differences or similarities, and detecting changes in circumstances or events.</td>
</tr>
<tr>
<td>3</td>
<td>Monitoring Processes, Materials, or Surroundings</td>
<td>Monitoring and reviewing information from materials, events, or the environment to detect or assess problems.</td>
</tr>
<tr>
<td>4</td>
<td>Estimating the Quantifiable Characteristics of Products, Events, or Information</td>
<td>Estimating sizes, distances, and quantities; or determining time, costs, resources, or materials needed to perform a work activity.</td>
</tr>
<tr>
<td>5</td>
<td>Evaluating Information to Determine Compliance with Standards</td>
<td>Using relevant information and individual judgment to determine whether events or processes comply with laws, regulations, or standards.</td>
</tr>
<tr>
<td>6</td>
<td>Processing Information</td>
<td>Compiling, coding, categorizing, calculating, tabulating, auditing, or verifying information or data.</td>
</tr>
<tr>
<td>7</td>
<td>Analyzing Data or Information</td>
<td>Identifying the underlying principles, reasons, or facts of information by breaking down information or data into separate parts</td>
</tr>
<tr>
<td>8</td>
<td>Making Decisions and Solving Problems</td>
<td>Analyzing information and evaluating results to choose the best solution and solve problems.</td>
</tr>
<tr>
<td>9</td>
<td>Working with Computers</td>
<td>Using computers and computer systems (including hardware and software) to program, write software, set up functions, enter data, or process information.</td>
</tr>
<tr>
<td>10</td>
<td>Documenting/Recording Information</td>
<td>Entering, transcribing, recording, storing, or maintaining information in written or electronic/magnetic form.</td>
</tr>
<tr>
<td>11</td>
<td>Interpreting the Meaning of Information for Others</td>
<td>Translating or explaining what information means and how it can be used.</td>
</tr>
<tr>
<td>12</td>
<td>Communicating with Supervisors, Peers, or Subordinates</td>
<td>Providing information to supervisors, coworkers, and subordinates by telephone, in written form, e-mail, or in person.</td>
</tr>
<tr>
<td>13</td>
<td>Communicating with People Outside the Organization</td>
<td>Communicating with people outside the organization, representing the organization to customers, the public, government, and other external sources. This information can be exchanged in person, in writing, or by telephone or e-mail.</td>
</tr>
<tr>
<td>14</td>
<td>Performing Administrative Activities</td>
<td>Performing day-to-day administrative tasks such as maintaining information files and processing paperwork.</td>
</tr>
</tbody>
</table>

Note: This table shows the 14 questions on work activities that have the word "information" in either the question or question explanation.
Table 3.6: Sector Information Score: 1980

<table>
<thead>
<tr>
<th>Industry</th>
<th>Score</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Agriculture</td>
<td>2.41</td>
<td>3.23</td>
</tr>
<tr>
<td>2. Mining</td>
<td>3.07</td>
<td>3.23</td>
</tr>
<tr>
<td>3. Utilities</td>
<td>3.59</td>
<td>3.23</td>
</tr>
<tr>
<td>4. Construction</td>
<td>3.03</td>
<td>3.23</td>
</tr>
<tr>
<td>5. Manufacture, excluding Computer and Electronic Products</td>
<td>3.06</td>
<td>3.23</td>
</tr>
<tr>
<td>6. Wholesale Trade</td>
<td>3.47</td>
<td>3.23</td>
</tr>
<tr>
<td>7. Retail Trade</td>
<td>3.16</td>
<td>3.23</td>
</tr>
<tr>
<td>8. Transportation and Warehousing;</td>
<td>3.10</td>
<td>3.23</td>
</tr>
<tr>
<td>9. Information</td>
<td>3.20</td>
<td>3.23</td>
</tr>
<tr>
<td>10. Finance, Insurance, Real Estate;</td>
<td>3.66</td>
<td>3.23</td>
</tr>
<tr>
<td>11. Professional Business Service</td>
<td>3.79</td>
<td>3.23</td>
</tr>
<tr>
<td>12. Educational Services, Health Care and Social Assistance</td>
<td>3.51</td>
<td>3.23</td>
</tr>
<tr>
<td>13. Arts, Entertainment, Recreation, Accommodation and Food Services</td>
<td>2.60</td>
<td>3.23</td>
</tr>
<tr>
<td>14. Other Services, except Government</td>
<td>3.01</td>
<td>3.23</td>
</tr>
<tr>
<td>15. Manufacture: Computer and Electronic Products</td>
<td>3.60</td>
<td>3.23</td>
</tr>
</tbody>
</table>

Note: Numbers are rounded to two digits. We ignore the government sector. Both industry scores and average score are weighted by the corresponding occupation share, with weights calculated from IPUMS CPS ASEC. I use the weighted associated with individual multiplied by hours worked. We can also do this by pooling all years together and results are similar. One exception is Sector 2: mining. In 1980, its info score is lower than average, but in 2010, its info score has exceeded the average of that year.

On the right panel we compare the residual $R^2$ of the non-routine cognitive analytic measure with the information intensity we use, which is the question on "analyzing information and data". They tack each other closely, confirming that the analyzing data or information is the most relevant task measure in explaining wage dispersions, at least among the measures looked in this section.
<table>
<thead>
<tr>
<th>Task Measures</th>
<th>Acemoglu and Autor (2011)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-routine analytical</td>
<td>Analyze data/information</td>
</tr>
<tr>
<td></td>
<td>Think creatively</td>
</tr>
<tr>
<td></td>
<td>Interpret information for others</td>
</tr>
<tr>
<td>Non-routine interpersonal</td>
<td>Establish/maintain personal relationships</td>
</tr>
<tr>
<td></td>
<td>Guide/direct/motivate subordinates</td>
</tr>
<tr>
<td></td>
<td>Coach/develop others</td>
</tr>
<tr>
<td>Routine cognitive</td>
<td>Importance of repeating the same tasks</td>
</tr>
<tr>
<td></td>
<td>Importance of being exact or accurate</td>
</tr>
<tr>
<td></td>
<td>Structured v. Unstructured work (reverse)</td>
</tr>
<tr>
<td>Routine manual</td>
<td>Control machines and processes</td>
</tr>
<tr>
<td></td>
<td>Spend time making repetitive motions</td>
</tr>
<tr>
<td>Non-Routine manual</td>
<td>Operate vehicles/mechanized devices</td>
</tr>
<tr>
<td></td>
<td>Use hand to handle objects/tools</td>
</tr>
<tr>
<td></td>
<td>Manual dexterity</td>
</tr>
<tr>
<td></td>
<td>Spatial orientation</td>
</tr>
</tbody>
</table>
3.6.4 Optimal Input Conditions

The optimality conditions of the sector optimization problem pretend in Section 3.3.1 are:

$$P_I A_I Q_I^T \left( (A_{II}L_{II}) \frac{\sigma_{1-1}}{\sigma_1} + \left( \frac{M_{II}}{\lambda_{II}} \right) \frac{\sigma_{2-1}}{(\sigma_1-1)\sigma_2} \right) A_{II}^\dagger L_{II}^\dagger = W_I$$

(3.10)

$$P_I A_I Q_I^T \left( (A_{II}L_{II}) \frac{\sigma_{1-1}}{\sigma_1} + \left( \frac{M_{II}}{\lambda_{II}} \right) \frac{\sigma_{2-1}}{(\sigma_1-1)\sigma_2} \right) \frac{1}{\lambda_{II}^\dagger} M_{II}^\dagger = P_I$$

(3.11)

$$P_I A_I Q_I^T \left( (A_{IN}L_{IN}) \frac{\sigma_{1-1}}{\sigma_1} + \left( \frac{M_{IN}}{\lambda_{IN}} \right) \frac{\sigma_{2-1}}{(\sigma_1-1)\sigma_2} \right) A_{IN}^\dagger L_{IN}^\dagger = W_N$$

(3.12)

$$P_I A_I Q_I^T \left( (A_{IN}L_{IN}) \frac{\sigma_{1-1}}{\sigma_1} + \left( \frac{M_{IN}}{\lambda_{IN}} \right) \frac{\sigma_{2-1}}{(\sigma_1-1)\sigma_2} \right) \frac{1}{\lambda_{IN}^\dagger} M_{IN}^\dagger = P_N$$

(3.13)

$$P_N A_N Q_N^T \left( (A_{NN}L_{NN}) \frac{\sigma_{1-1}}{\sigma_1} + \left( \frac{M_{NN}}{\lambda_{NN}} \right) \frac{\sigma_{2-1}}{(\sigma_1-1)\sigma_2} \right) A_{NN}^\dagger L_{NN}^\dagger = W_N$$

(3.14)

$$P_N A_N Q_N^T \left( (A_{NN}L_{NN}) \frac{\sigma_{1-1}}{\sigma_1} + \left( \frac{M_{NN}}{\lambda_{NN}} \right) \frac{\sigma_{2-1}}{(\sigma_1-1)\sigma_2} \right) \frac{1}{\lambda_{NN}^\dagger} M_{NN}^\dagger = P_N$$

(3.15)

$$P_N A_N Q_N^T \left( (A_{NI}L_{NI}) \frac{\sigma_{1-1}}{\sigma_1} + \left( \frac{M_{NI}}{\lambda_{NI}} \right) \frac{\sigma_{2-1}}{(\sigma_1-1)\sigma_2} \right) A_{NI}^\dagger L_{NI}^\dagger = W_I$$

(3.16)

$$P_N A_N Q_N^T \left( (A_{NI}L_{NI}) \frac{\sigma_{1-1}}{\sigma_1} + \left( \frac{M_{NI}}{\lambda_{NI}} \right) \frac{\sigma_{2-1}}{(\sigma_1-1)\sigma_2} \right) \frac{1}{\lambda_{NI}^\dagger} M_{NI}^\dagger = P_I$$

(3.17)
3.6.5 Sector Productivity Change

Figure 3.8 plot the productivity of info and non-info sector calculated from the data. Data show that over time $A_I$ increase less than $A_N$. 
References


Figure 3.1: Intermediate input used by info sector

![Graph showing intermediate input used by info sector.]

Data: BEA Input-Output table. Intermediate inputs are divided by sector nominal GDP.

Figure 3.2: Intermediate input used by non-info sector

![Graph showing intermediate input used by non-info sector.]

Data: BEA Input-Output table. Intermediate inputs are divided by sector nominal GDP.
Data: BEA Input-Output table. This figure plots the intermediate info input over intermediate non-info input used by each sector.

Data: CPS ASEC from IPUMS. Real hourly wage is the occupation mean.
Figure 3.5: real wage and information score

Data: CPS ASEC from IPUMS

Figure 3.6: real wage and information score

Data: CPS ASEC from IPUMS. Real hourly wage is the occupation mean.
Figure 3.7: Partial $R^2$ net of experience, education, race and gender

Data: IPUMS CPS ASEC and O*Net. The partial $R^2$ values presented above are calculated as follows: Regress Log hourly wages on a quartic in experience, and dummies of race, gender and education. The residual log hourly wages are regressed separately on the variable groups of interest. $R^2$ from each regression is plotted above for each year. Both panels plot the results for the all task measures used in ?.

Figure 3.8: $A_I$ and $A_N$ over time

Data: We compute $A_I$ and $A_N$ using March 2017 release world KLEMS available at [http://www.worldklems.net/data.htm](http://www.worldklems.net/data.htm). For each industry, we calculate productivity growth as $\ln A_{st} - \ln A_{st-1} = \ln Q_{st} - \ln Q_{st-1} - 0.5(\Theta_K_{st} + \Theta_K_{st-1})(\ln K_{st} - \ln K_{st-1}) - 0.5(\Theta_L_{st} + \Theta_L_{st-1})(\ln L_{st} - \ln L_{st-1}) - 0.5(\Theta_M_{st} + \Theta_M_{st-1})(\ln M_{st} - \ln M_{st-1})$ where $\Theta_{i,st}$ is the factor share of input $i = K, L, M$ for industry $s$ at year $t$. We normalize productivities in all industries to be unity in 1980, and then we aggregate the industry level productivities into two aggregate sector productivities, using the gross output share of each individual industry such that $\ln A_{jt} = \sum_{s \in j} \frac{\text{Gross Output}_{st}}{\text{Gross Output}_{jt}} \ln A_{st}$, $j \in I, N$. 