Three Essays on Money and Banking

Effects of Monetary Policy on Liquidity Risk

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A thesis presented for the degree of

Doctor of Philosophy

Department of Economics

London School of Economics and Political Science

July 2018

Declaration

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Abstract

This thesis studies the effects of monetary policy on liquidity risk. I extend the model of financial intermediation developed by Diamond and Dybvig (1983) to include a monetary authority. Through the lens of different versions of this model, I study the effects of negative interest on reserves, of payment of positive interest on reserves and of a large central-bank balance sheet.

In the first essay, I study optimal monetary policy in the model's liquidity trap. I find that a negative interest on reserves is effectively a tax on the banking system. As such, it leads to less effective financial intermediation and therefore increases liquidity risk. On the other hand, it also acts as a tax on saving and therefore has the effect of boosting aggregate demand. I find that in the liquidity trap, when aggregate demand is insufficient to absorb the economy's full productive capacity, it is optimal for the central bank to set a strictly negative interest on bank reserves.

The second essay adds financial markets to the model. Banks faced with competition for savings from financial markets are unable to fully insure depositors' liquidity risk. In this setting, I ask whether appropriate monetary policy can improve the economy's equilibrium outcome. I find that paying a positive interest on bank reserves is welfare improving. There exists a strictly positive level of interest on reserves that implements the economy's efficient allocation.

In the last essay, I make the model's term structure of interest rates endogenous. I find that the central bank can control the term premium by varying the size of its balance sheet. In particular, issuing bank reserves lowers the return on long-term assets. I show that in this setting optimal monetary policy requires a large central-bank balance sheet.

Contents

Dedication

I dedicate this thesis to my family.

My mother Paola and my father Franco supported me with incredible generosity both morally and materially during the many years that it took to come to this point. My grandparents, Alberto, Assunta, Dorino and Lidia, have always had the right words to comfort me during setbacks and to share the joy of some successes. To my brother Michael I owe an intellectual debt, too. He went through all of my work and gave me many precious comments and suggestions.

In these important years, I have also found the love of my life. I am extremely lucky that Florentina is family now. This thesis is dedicated to her, too.

Many friends have contributed to the creation of this thesis in a myriad of ways. Freddy and Claudi are not only wonderfully fun-loving friends. As flatmates, I could always count on their patience and kindness when I needed it. Angelo, Lorenzo, Matteo and Roberto are simply great friends. No distance can change this. I would like to thank Federico, Francesco, Gianpaolo, Kilian, Maria and Matteo, especially for the lunches and chats in the kitchen.

Acknowledgments

First, I would like to express my deep gratitude to my supervisor, Dr Gianluca Benigno, for his support during my PhD studies and for his continuous guidance in my research projects.

I also sincerely thank Prof. Wouter Den Haan, Prof. Ricardo Reis and Dr Kevin Sheedy for their generous and invaluable advice. My thinking and my work has benefited immensely from it.

The participants of the Money-Macro seminars of the Centre for Macroeconomics have killed off my worst ideas and questioned my better ones. I learnt that this is how progress takes place in research and thus I thank them for spending their energies this way.

Finally, I would also like to thank the administrative staff of the LSE's Department of Economics and Centre for Macroeconomics for their precious work.

1 | Optimal Negative Interest on Reserves

A negative interest rate on reserves is effectively a tax on the banking system. The rationale for adopting such policy in the context of a classic liquidity trap is its expansionary effect, via the reduction in the incentive to save. However, critics argue that the resulting distortion in the banking system may have adverse effects which outweigh the welfare gains from economic stimulus. This paper contributes to the debate by modelling the banking sector in accordance with the maturity-transformation framework of Diamond and Dybvig (1983). I show that, on the one hand, negative interest on reserves leads to financial disintermediation, which reduces the extent of maturity transformation below the first-best level; while, on the other hand, it is expansionary, as consumers save less overall. The paper's main finding is that, while in normal times taxing banks is inefficient, optimal monetary policy in the liquidity trap prescribes a strictly negative interest on reserves.

1.1 Introduction

In the aftermath of the Great Recession, the interest rate on bank reserves has become the active monetary policy tool in advanced economies. Central banks increased the supply of reserves massively. This turned the monetary system into a floor system, in which the stance of monetary policy is determined by the interest rate on reserves rather than by the scarcity of bank reserves (i.e., open market operations).[1] Also, the implementation of negative interest rate policies,

[1] For example, the Federal Reserve in the last quarter of 2008 alone issued $800bn in bank reserves, amounting to a nine-fold increase in the supply of reserves. It did not have the authority to pay interest on bank reserves. So, the Emergency Economic Stabilization Act of 2008 granted it such authority on 1 October 2008. This allowed the central bank to continue controlling the fed funds rate, as banks became satiated in reserves.

performed by several advanced-economy central banks, required the active use of the interest on reserves.[2]

In light of this development in the practice of monetary policy, in this paper I study the role of the interest on reserves in pursuing the central bank's policy objectives. A literature has recently developed on the role of reserves and of the interest on reserves in monetary policy (Kashyap and Stein, 2012; Ennis, 2014; Reis, 2016). However, such literature abstracts from negative interest rates and the challenges to monetary policy posed by the lower bound on nominal interest rates. A separate literature studies the effectiveness of negative interest rates in pursuing macroeconomic objectives when an economy is in the liquidity trap (Rognlie, 2016; Brunnermeier and Koby, 2017). However, as most of the literature on negative interest rates, their analysis concentrates on only one interest rate, the interbank rate. This simplification is more restrictive than usual when negative interest rates are the subject, because it does not allow the analysis to focus on the key issue of the transmission of monetary policy to the money market. I provide a theoretical contribution to the literature on negative interest rates, which I expand by including insights from the literature on reserves.

In the model, I explicitly model monetary policy as a choice of interest on reserves and open market operations, which determines the interbank rate. This a more realistic description of the economy and of monetary policy. Central banks influence multiple interest rates. This explicit modelling of monetary policy transmission allows me to study whether the lower bound on nominal interest rates constrains different interest rates in different ways, and if this has implications for monetary policy in the liquidity trap. The contribution of my paper moves the current debate on negative interest rates closer to the original literature on the liquidity trap, as developed from the seminal paper by Krugman (1998), which is founded on the idea that the interbank rate cannot fall below a lower bound.

The lower bound on the interbank rate represents the idea that a low enough interest on reserves will fail to ease monetary policy via the conventional channel,

[2]Please refer to figure 1.A.1 in appendix 1.A for data on the implementation of negative interest on reserves.

by lowering the interbank rate and thus, for example, reducing the rewards from saving. In policy circles the question is not whether there is a lower bound, but how to estimate it (Alsterlind et al., 2015; Bech and Malkhozov, 2016; Dell'Ariccia et al., 2017). A recently disclosed Federal Reserve internal memo (Burke et al., 2010) estimates that the federal funds rate can at most be lowered to -35 basis points.[3] Considering that the current interbank rate in the Euro Area is -0.35%, hitting the lower bound is a real possibility in case further monetary easing were desirable.[4] Therefore, a full evaluation of the desirability of negative interest rate policies must take this eventuality into account.[5]

The interest on reserves is not constrained by a lower bound in the same way as the interbank rate is. A first-pass explanation for this is that the central bank can decide by decree to pay whatever interest on bank reserves, while the interbank rate is the equilibrium price that clears the interbank market. The central bank controls the interbank rate only indirectly by carrying out open market operations and by setting the interest on reserves, and I show that the lower bound emerges as an equilibrium requirement due to the existence of zero-interest-paying physical currency.[6]

Nonetheless, this does not imply that the central bank can ease monetary policy just by lowering the interest on reserves, once the interbank rate is stuck at the the lower bound. Consider the canonical New Keynesian model, where the banking sector is just a veil behind which consumers lend to firms. In such model, lowering the interest on reserves per se would not be expansionary, because, even if banks were forced to hold bank reserves, consumers would move all of their savings out of deposits into the money market if the cut in the interest on

[3]Burke et al. (2010) is the only publicly accessible paper that reports the methodology with which the lower bound on the interbank rate is estimated. Viñals et al. (2016) reports estimates by IMF staff ranging from -75 basis points to -200 basis points, but without disclosing the analysis.

[4]The Eonia rate is the overnight interbank rate in the Euro Area. It is -0.35% as of 6 November 2017. Concurrently, the interest on excess reserves in the Euro Area is -0.4%.

[5]Bernanke (2016) makes this point to argue that negative interest rate policies are unlikely to have large stimulative effects.

[6]Hicks (1937) was the first to describe the lower bound on the money-market interest rate in these terms.

reserves led to a reduction in the deposit rate. In other words, in such framework deposits and bonds are perfect substitutes from the consumers' perspective and therefore the interest on reserves has a macroeconomic role only insofar as it steers the interest rate in the bond market. In order to meaningfully study the macroeconomic effects of the interest on reserves, we need a theory of banking which generates a demand for deposits.

The role of banks in the model that I develop is based on the classical framework by Diamond and Dybvig (1983) on maturity transformation. Banks are depository institutions. Consumers are subject to idiosyncratic liquidity shocks, which may force them to wind up their investments early and thus forego the liquidity premium on their assets. The combination of liquidity shocks and liquidity premium gives rise to liquidity risk for consumers. Banks emerge in the model to provide insurance against liquidity risk. They do so by issuing redeemable deposits and holding long-term assets. In other words, banks perform maturity transformation, which means that even short-term depositors enjoy to some extent the higher yields on the long-term assets that are held by their bank. The key role of banks in this model is to facilitate saving behaviour, as they provide consumers with assets that are more liquid than the assets available in direct asset markets. This also implies that deposits and direct holdings of assets are not perfect substitutes from the perspective of consumers.

An influential strand of the literature on negative interest rates also has its focus on the banking system, but uses a different theory of banking. It studies the interaction of a negative interbank rate with the banking system, by emphasising the asset side of banks: the key role of the banking system in their model is to invest, because banks are defined by their high productivity at making investments. They show that, if deposit rates are downwardly sticky at zero, there is a a level of the interbank rate beyond which cuts are contractionary, because they harm bank profitability and thus investment (Brunnermeier and Koby, 2017; Eggertsson et al., 2017). The framework that I propose is not in contradiction with this strand of the literature. In fact, it complements it by focusing on another one of the many different roles that banks play in the economy: maturity transformation.

Since in my model deposits and direct holdings of assets are not perfect substitutes, I find that a reduction in the interest on reserves does not only lead to disintermediation. Interestingly, it also leads to a reduction in savings and thus to an increase in aggregate demand. The mechanism whereby the interest on reserves affects consumers' propensity to save runs through the deposit rate. Banks hold reserves, either because they are required to do so or because reserves are useful in the payment system among banks. Thus, a reduction in the interest on reserves translates into a lower deposit rate. The reduction in the deposit rate makes the assets available to consumers less attractive overall, because deposits and other assets are imperfectly substitutable. Hence, consumers respond both by moving their wealth out of deposits into direct holdings of assets and by saving less in total.

A benevolent central bank finds that boosting aggregate demand by reducing the interest on reserves, while holding the interbank rate constant, is beneficial, if aggregate demand is insufficient because the interbank rate is at its lower bound. However, this policy has a drawback: it reduces the level of bank activity. In this economy banks provide liquidity-risk insurance, a desirable service. Therefore, an interesting policy tradeoff emerges between stimulating the economy and preserving the banking system.

The main finding of the paper is that, when the lower bound on the interbank rate is binding and the economy needs further easing, it is optimal to lower the interest on reserves strictly below the interbank rate. This boosts employment but also reduces the effectiveness of the banking system. The key is that the point at which the negative effect of the latter outweighs the benefits of the former is strictly below the lower bound. When the economy is in the liquidity trap, the central bank should exploit this channel of monetary policy in order to stimulate the economy by boosting demand. The result is important for monetary policy in the liquidity trap, because it identifies an unconventional policy instrument that can improve the performance of the economy when further reductions in the interbank rate cannot be implemented.

Rogoff (2017) identifies the lower bound on the conventional monetary policy instrument (i.e., the interbank rate) as the key constraint currently facing central bankers. Moreover, there is a growing literature (Karabarbounis and Neiman, 2013; Summers, 2014; Carvalho et al., 2016) arguing that structural factors are pushing down the real interest rate, resulting in a higher probability of hitting the lower bound in the future. This suggests that the literature on unconventional monetary policy, in which this paper fits, is relevant and will remain relevant in the future.

1.1.1 Related literature

Unconventional monetary policies, defined as policies within the remit of the central bank other than targeting the spot short-term interbank interest rate, have attracted a large academic literature.

This paper is closely related to a small number of papers that study negative interest rate policies. Rognlie (2016) postulates a demand for currency with a bliss point within a New Keynesian set-up and finds that transmitting a negative interest rate to the bonds market is feasible. Moreover, he finds that it is desirable when the economy experiences a liquidity trap. Brunnermeier and Koby (2017) and Eggertsson et al. (2017) study the interaction between negative rates and banks. They assume a lower bound on deposit rates and find that negative interest rate policies reduces banks' interest margin and profits. Since banks are subject to a capital constraint and are the only lenders in the economy, negative interest rate policies, which lower bank profits, are contractionary. The evidence for a lower bound on deposit rates is mixed. Bech and Malkhozov (2016) and Heider et al. (2017) find evidence for it. The latter show that retail banks in the Eurozone more reliant on deposits suffered more from the European Central Bank's negative interest rate policy. On the other side of the argument, Basten and Mariathasan (2017) show that Swiss retail banks funded with more deposits, which should have been hit harder by negative interest rate policies in the presence of a binding zero lower bound on deposit rates, increased fees on depositors more and hence did not suffer in terms of profits. Effectively, this means that negative interest rate policies were passed on to depositors. In my paper, I model the banking sector in a novel way in the context of this literature, as a maturity transformer, and I focus on the lower bound on the interbank rate, which has a long tradition in the New

Keynesian liquidity trap and can be derived from the presence of paper money.

A theory of banking based on the notion of maturity transformation was first formalised by Diamond and Dybvig (1983). A subsequent theoretical literature worked on expanding the framework with general-equilibrium elements. Jacklin (1987) and Allen and Gale (2004) add financial markets and discuss the implications for the incentive compatibility of the demand deposit contract.[7] Hellwig (1994) and Farhi et al. (2009) introduce competitive forces in the banking sector.

The existence of a lower bound on the nominal interest rate prevailing in the bonds market, which acts as a constraint on monetary policy, was first hypothesised by Hicks (1937). The seminal paper for the modern literature is Krugman (1998), and a very large literature followed. An integration of the lower-bound friction in the canonical New Keynesian framework, as laid out by Woodford (2003) and Galí (2008), is reached in Eggertsson and Woodford (2003).[8] Demand shocks are commonly modelled as simple time-preference shocks within the literature. More structure on the large adverse demand shock that leads into the liquidity trap is provided by Guerrieri and Lorenzoni (2011) and Eggertsson and Krugman (2012), who model the adverse demand shock as a consequence of a credit crisis.

This paper finds that the interest rate on reserves has macroeconomic effects per se, beyond the mere transmission of the monetary policy stance to the interbank rate. Bank reserves and the interest rate on reserves recently received a great deal of attention. Reis (2016) discusses monetary policy by central banks with large balance sheets. He finds that maintaining banks satiated in their reserve holdings is desirable, because it makes the choice of interest rate independent of the balance-sheet size. This frees quantitative easing for use as an independent policy instrument that can achieve independent policy goals. Cúrdia and Woodford (2011) also advocate a floor system of monetary policy on the ground that a system which sets the interest rate on reserves below the interbank rate is effectively taxing the banking sector. Eliminating such tax implements the Friedman rule and is therefore optimal. Moreover, they also find that the floor system has the

[7] See also Diamond (1997) and von Thadden (1998) on this subject. A discussion of the incentive compatibility of maturity transformation with multiple assets is available in von Thadden (1997).

[8] In Werning (2011) a similar model is presented in continuous time.

benefit of making the central bank's balance-sheet size independent of the target interest rate. Thus, it can be used to react to financial-intermediation disturbances. Kashyap and Stein (2012) contrast these views by pointing out that, if short-term debt issued by the banking sector poses a systemic risk, it is optimal to set a non-zero wedge between the interbank rate and the interest rate on reserves in order to pursue macroprudential objectives. The interest-rate wedge serves as a tax, disincentivising excessive issuance of short-term debt by the banking sector. This paper's finding that optimal monetary policy in the liquidity trap requires a positive wedge between the interbank rate and the interest on reserves is similar: the interest rate wedge acts as a tax on bank deposits, discouraging excessive saving by consumers.

1.2 Technology and Preferences

This section lays out the assumptions of the model on technology and preferences. They are mostly simllar to the assumptions made in Diamond and Dybvig (1983), the seminal paper in the literature on maturity transformation, with differences that I will highlight.

There is an investment technology, which transforms consumption goods into capital goods, according to function

$$K = f(I), \tag{1.1}$$

with f twice-continuously differentiable, $f' > 0$ and $f'' < 0$. This assumption is different from Diamond and Dybvig (1983), where $f' = 1$ for all I. I need this new assumption to have an endogenous real interest rate in the model.

Each unit of the capital good returns one unit of the consumption good if liquidated after one period, and $R > 1$ units of the consumption good if liquidated after two periods. Hence, there is a reward from holding the asset longer. I call this the liquidity premium.

The economy is inhabited by a unit mass of ex-ante identical consumers indexed by j. Consumers live for three periods. At time zero, they receive an endowment of consumption goods and make a saving decision. In the subsequent time periods, consumers consume the proceeds of their time-0 savings.

Consumers' preferences are represented by utility function

$$U(C_{j,0}, C_{j,1}, C_{j,2}, \theta_j) = u(C_{j,0}) + \hat{\beta} \cdot [(1-\theta_j) \cdot u(C_{j,1}) + \beta \cdot \theta_j \cdot u(C_{j,2})], \tag{1.2}$$

with felicity function u satisfying Inada conditions. $\beta \in [R^{-1}, 1]$ is the discount factor between time 1 and time 2. $\hat{\beta}$ is the discount factor between time 0 and the future. Shocks to $\hat{\beta}$ represent demand shocks at time 0. A necessary assumption is that the coefficient of relative risk aversion is greater than 1:

$$\frac{-C \cdot u''(C)}{u'(C)} \equiv \gamma(C) \geq 1 \quad \forall C > 0. \tag{1.3}$$

Risk aversion needs to be high enough to generate the demand for liquidity-risk insurance that is at the heart of this paper. The utility function is different than in Diamond and Dybvig (1983) in that consumers enjoy consumption also at time 0. To analyse interest-rate setting, the model needs a meaningful saving decision.

Random variable θ_j represents a liquidity shock and, as such, it is only known at time 1. It takes on values 0 or 1. At time 0, agents know the objective probability of the liquidity shock's realisations:

$$Pr(\theta_j) = \begin{cases} \phi & \text{if } \theta_j = 0, \\ 1-\phi & \text{if } \theta_j = 1. \end{cases} \tag{1.4}$$

A consumer whose realisation for θ_j is 0 is hit by the liquidity shock. I refer to these consumers throughout the paper as early types and to the other consumers, who have not been hit by the liquidity shock, as late types. There is no uncertainty at time 0 about the share of consumers who will be hit by the liquidity shock. Hence, there is no aggregate risk.

1.3 Social Planner

The allocation of the social planner is the benchmark for efficiency in the following analysis of the decentralised economy.

The social planner maximises the expected value of aggregate welfare subject to the economy's resource constraints and non-negativity of consumption. Informational frictions, which are a key characteristic of the decentralised economy, do not constrain the social planner. It follows that the social planner represents a high standard of efficiency.

Since the social planner finds it optimal to let consumers of the same type θ_j consume equally (i.e., $C_{j,t}(\theta_j) = C_{h,t}(\theta_j) = C_t(\theta_j)$), expected aggregate welfare can be written as

$$u(C_0) + \hat{\beta} \cdot \{\phi \cdot u[C_1(0)] + (1-\phi) \cdot \beta \cdot u[C_2(1)]\}. \tag{1.5}$$

The economy's resource contraints are given by:

$$C_0 + I \leq Y, \tag{1.6}$$

$$\phi \cdot C_1(0) + (1-\phi) \cdot C_1(1) \leq L, \tag{1.7}$$

$$\phi \cdot C_2(0) + (1-\phi) \cdot C_2(1) \leq R \cdot [f(I) - L], \tag{1.8}$$

where L stands for the quantity of capital goods liquidated at time 1.

Consumption must be non-negative at all points in time and for all consumers, as according to

$$C_t(\theta_j) \geq 0 \quad \forall\, t, \theta_j. \tag{1.9}$$

The allocation that solves the social planner's problem is by definition first-best efficient. The system of equations that pins down the efficient allocation of consumption across types and time $\{C_0, C_1(\theta_j), C_2(\theta_j)\}_{\theta_j \in \{0,1\}}$ is given by:

$$u'(C_0) = f'(Y - C_0) \cdot \hat{\beta} \cdot u'[C_1(0)], \tag{1.10}$$

$$\frac{u'[C_1(0)]}{\beta \cdot u'[C_2(1)]} = R, \tag{1.11}$$

$$(1-\phi) \cdot C_2(1) = R \cdot [f(Y - C_0) - \phi \cdot C_1(0)], \tag{1.12}$$

$$C_1(1) = C_2(0) = 0. \tag{1.13}$$

Definition 1.
An allocation $\{C_0, C_1(\theta_j), C_2(\theta_j)\}_{\theta_j \in \{0,1\}}$ is first-best efficient if it satisfies equations (1.10), (1.11), (1.12) and (1.13).

Equation (1.10) is an Euler equation. The optimal saving decision at time zero sets the marginal rate of substitution between time 0 and time 1 equal to the corresponding marginal rate of transformation.

Equation (1.11) is familiar from the optimality conditions in the standard Diamond-Dybvig model. It implies that it is optimal to insure liquidity risk in the economy. If consumers do not insure each other, then early types, who are forced to liquidate their asset early, consume too little. Ex ante, in a fully efficient economy the provision of liquidity-risk insurance implements equation (1.11).

1.4 Decentralised Economy

In this section, I describe the characteristics of the four types of agents who inhabit the decentralised economy: capital-producing firms, consumers, banks and the government. I

1.4.1 Capital-Producing Firms

Capital-producing firms purchase consumption goods and transform them into capital goods, which they sell. They are active at time 0 within a perfectly competitive market for capital goods. A representative firm statically maximises its profits Π_F, given by

$$\Pi_F = Q \cdot K - I, \tag{1.14}$$

where Q is the price of capital in terms of consumption goods, I is the quantity of consumption goods purchased by the firm and invested to produce K units of capital goods. They produce capital goods according to technology

$$K = f(I), \tag{1.1}$$

with f twice-continuously differentiable, $f' > 0$ and $f'' < 0$.

As described above, capital production implies an upward-sloping supply of capital goods. The Diamond-Dybvig model is a limiting case, where capital supply is perfectly elastic at $Q = 1$. The underlying technological assumption of Diamond and Dybvig (1983) is that $f'(I) = 1$ and $f''(I) = 0$ for all I. Decreasing returns to investment, as assumed in this paper, are necessary to make the interest rate endogenous.

1.4.2 Consumers

Consumers make saving and portfolio allocation decisions under idiosyncratic liquidity risk. There is a unit measure of consumers, who are identical as of time 0. Consumer j's utility function is given by

$$U(C_{j,0}, C_{j,1}, C_{j,2}, \theta_j) = u(C_{j,0}) + \hat{\beta} \cdot [(1-\theta_j) \cdot u(C_{j,1}) + \beta \cdot \theta_j \cdot u(C_{j,2})]. \tag{1.2}$$

All consumers enjoy consumption at time 0. Then, at time 1 they are subject to a privately-observed liquidity shock θ_j: with probability $\phi \in (0,1)$ they become early types, with $\theta_j = 0$, and enjoy consumption only at time 1; with probability $1-\phi$ they become late types, with $\theta_j = 1$, and enjoy consumption only at time 2. The consumer's expected utility function is therefore

$$U[C_{j,0}, C_{j,1}(0), C_{j,2}(1)] = u(C_{j,0}) + \hat{\beta} \cdot \{\phi \cdot u[C_{j,1}(0)] + (1-\phi) \cdot \beta \cdot u[C_{j,2}(1)]\}. \tag{1.15}$$

Consumers are subject to budget constraints. At time 0, they receive an endowment Y, transfers from the government T and profits from banks Π_B and firms Π_F. They use this income to consume and save. Savings are held in bank deposits $\int_0^1 D_{j,k}\,\mathrm{d}k$, where k is an index that represents a bank, or invested directly in capital goods. So, the time-0 budget constraint is given by

$$C_{j,0} + Q_0 \cdot K_j + \int_0^1 D_{j,k}\,\mathrm{d}k = Y + T + \Pi_B + \Pi_F. \tag{1.16}$$

The ability of the consumer to invest directly in capital goods at time 0 is key in the model. It makes the demand for financial intermediation endogenous in the model. If deposits become less attractive, consumers have the possibility to change the portfolio allocation of their savings tilting it more towards direct finance.

At time 1, the idiosyncratic liquidity shock is realised. Thus, consumer's decisions at time 1 are contingent on their type realisation. To finance consumption $C_{j,1}(\theta_j)$, they can liquidate $L_j(\theta_j)$ units of the physical capital or withdraw $\int_0^1 W_{j,k}\,\mathrm{d}k$ from their bank deposit. Consumer j's budget constraint at time 1 is then given by

$$C_{j,1}(\theta_j) = L_j(\theta_j) + \int_0^1 W_{j,k}(\theta_j)\,\mathrm{d}k. \tag{1.17}$$

I do not allow consumers to sell their capital goods on a secondary market at time 1 or invest in capital goods at time 1 at all. Since it removes any incentive for late-type consumers to withdraw their deposits early, this assumption eliminates the incentive-compatibility constraint from the bank's problem and greatly simplifies the model. A restrictive assumption on the time-1 market for used capital is necessary to escape the Jacklin critique (Jacklin, 1987) and retain maturity-transforming banks. According to the Jacklin critique, if consumers are allowed to frictionlessly borrow and lend at time 1 (or buy and sell used capital), then the banking mechanism is unable to provide any liquidity-risk insurance. For example, Diamond and Dybvig (1983) has maturity transformation in equilibrium, because consumers are implicitly not allowed to lend and borrow among themselves at time 1. They can only reinvest in new capital goods at time 1. In this paper, I make a more extreme assumption. However, I think this is justified on the grounds that my focus is not the fragility of the banking contract, for which late-type consumers' desire to withdraw early is key, but the demand for deposits due to banks' maturity-transformation activity.

At time 2, consumers use their remaining assets to purchase consumption goods $C_{j,2}(\theta_j)$ according to

$$C_{j,2}(\theta_j) = R \cdot \left[K_j - L_j(\theta_j)\right] + \int_0^1 (1 + d_{k,1}) \cdot \left[(1 + d_{k,0}) \cdot D_{j,k} - W_{j,k}(\theta_j)\right] \mathrm{d}k. \tag{1.18}$$

$d_{k,0}$ and $d_{k,1}$ are the real interest rates, respectively from time 0 to time 1 and from time 1 to time 2, offered in the deposit contract of bank k.

Before I present the bank's problem, it is useful to work out consumers' optimal decisions with regard to withdrawing and depositing. The consumer's withdrawing behaviour is simple, thanks to the assumption that there is no market for loans and capital goods at time 1. Early types withdraw early and late types withdraw late, as formalised by

$$W_{j,k}(\theta_j) = \begin{cases} (1 + d_{k,0}) \cdot D_{j,k} & \text{if } \theta_j = 0, \\ 0 & \text{if } \theta_j = 1. \end{cases} \tag{1.19}$$

Consumer j's demand for bank k's deposits depends on the return and liquidity-insurance characteristics that the bank offers relative to other deposit contracts on offer. It is a Bertrand-like demand function, where the consumer deposits everything with the bank that offers the best contract from the consumer's viewpoint. From the consumer's problem, I derive a valuation function for deposit contracts according to which the consumer chooses her bank. It determines the value of a unit of bank k's deposits in utility terms for consumer j:

$$V_j(d_{k,0}, d_{k,1}) \equiv (1 + d_{k,0}) \cdot \hat{\beta} \cdot \{\phi \cdot u'[C_{j,1}(0)] + (1 - \phi) \cdot (1 + d_{k,1}) \cdot \beta \cdot u'[C_{j,2}(1)]. \quad (1.20)$$

Demand for bank k's deposits is given by

$$D_{j,k} = \begin{cases} D_j & \text{if } V_j(d_{k,0}, d_{k,1}) > \max_{z \neq k}\{V_j(d_{z,0}, d_{z,1})\} \\ [0, D_j] & \text{if } V_j(d_{k,0}, d_{k,1}) = \max_{z \neq k}\{V_j(d_{z,0}, d_{z,1})\} \\ 0 & \text{otherwise.} \end{cases} \quad (1.21)$$

$D_j \equiv \int_0^1 D_{j,k}\, \mathrm{d}k$ is defined as the total amount of deposits held by consumer j. It is determined in equilibrium. The consumer decides to hold positive amount of deposits if the value of the best deposit contract, $\max_k\{V_j(d_{k,0}, d_{k,1})\}$, is at least as good as the value of investing directly in capital goods, given by

$$V_{j,K} \equiv Q \cdot \hat{\beta} \cdot \{\phi \cdot u'[C_{j,1}(0)] + (1 - \phi) \cdot R \cdot \beta \cdot u'[C_{j,2}(1)]\}. \quad (1.22)$$

1.4.3 Banks

There is a unit mass of banks in a perfectly competitive market. They are profit-maximising agents that supply deposits D_k, characterised by interest rates $(d_{k,0}, d_{k,1})$, and use the resources they collect to invest in capital assets K_k, bank reserves B_k, currency Cu_k and interbank loans F_k. The latter three assets are nominal. Bank k's profits are defined as

$$\Pi_k + Q \cdot K_k + \frac{B_k + Cu_k + F_k}{P_0} = D_k. \quad (1.23)$$

The bank's portfolio allocation decision is constrained by a reserve requirement. Banks must hold at least a proportion $\rho \in [0, 1]$ of their deposits in reserves, according to

$$\frac{B_k}{P_0} \geq \rho \cdot D_k. \quad (1.24)$$

From equation (1.19), banks know how many of their deposits will be withdrawn at time 1. They use their short-term asset holdings, reserves, currency and interbank loans, and liquidate capital goods L_k to satisfy early withdrawal requests, as according to

$$\phi \cdot (1 + d_{k,0}) \cdot D_k = L_k + \frac{(1 + i^B) \cdot B_k + Cu_k + (1 + i) \cdot F_k}{P_1}. \tag{1.25}$$

The nominal interest rates on reserves and interbank loans are respectively given by i^B and i. Of course, currency pays a zero nominal interest rate. This generates a zero lower bound on the interest rate that can be paid on interbank loans. The absence of storage costs is an assumption made for notational convenience. Storage costs that are proportional to currency holdings would lower the effective lower bound to negative territory but would not change the paper's results once such effective lower bound becomes binding. Late deposit withdrawals are met with the bank's remaining assets, according to

$$(1 - \phi) \cdot (1 + d_{k,1}) \cdot (1 + d_{k,0}) \cdot D_k = R \cdot (K_k - L_k). \tag{1.26}$$

Banks know the consumers' demand for their their deposits $\int_0^1 D_{j,k}\, \mathrm{d}k$, given by equation 1.21. Accordingly,

$$D_k = \int_0^1 D_{j,k}\, \mathrm{d}j. \tag{1.27}$$

1.4.4 Government

The monetary base, M, is supplied by the government at time 0. On the asset side, the government engages in open-market operations, whereby it purchases capital goods K_g. Accordingly, the government's time-0 budget constraint is

$$Q \cdot K_g + T = \frac{M}{P_0}. \tag{1.28}$$

T are lump-sum transfers to consumers whereby the government rebates its seigniorage revenue.

The split of the monetary base in reserves and currency is not directly determined by the government. It is determined in equilibrium by relative demand for bank reserves and currency. Given that there is no special role for currency in the model, except as an asset that pays a zero percent nominal return, currency is only held in non-zero amount if the interest on reserves i^B is smaller than zero. The government controls directly the rate of interest on reserves i^B. At time 1, the government uses its capital holdings to pay off holders of the monetary base, according to

$$K_g = \left(1 + i^B\right) \cdot \rho \cdot \int_0^1 D_k \, \mathrm{d}k + \left(1 + \max\left\{i^B, 0\right\}\right) \cdot \left(\frac{M}{P_1} - \rho \cdot \int_0^1 D_k \, \mathrm{d}k\right). \tag{1.29}$$

At time 2, the government plays no role in the economy.

Note that throughout the paper I also refer to the government as central bank and monetary authority, because I focus on the monetary prerogatives of government.

1.5 Equilibrium

In this section of the paper, I define two equilibrium concepts. First, I define the competitive equilibrium with flexible prices. This allows me to find the real interest rate that must prevail for markets to clear: the natural real rate of interest. Second, I define an equilibrium concept with sticky prices. In this equilibrium, I can meaningfully study monetary policy, which has real effects. Demand and supply are not automatically equated by changes in the price level. Hence, it is the central bank's task to manage demand in order to ensure the full employment of the endowment. I will assume that the central bank is benevolent and knows exactly the interest rate that ensures market clearing. However, if monetary policy is constrained by the lower bound, then the equilibrium features rationing in the time-0 goods market.

1.5.1 Flexible-Price Equilibrium and the Natural Real Interest Rate

The flexible-price equilibrium is a useful benchmark to understand the workings of the model. Moreover, it pins down the natural real rate of interest, the real rate of interest at which the goods market clears.

In the flexible-price equilibrium, firms, banks and consumers solve their optimisation problems and prices adjust to ensure market clearing. The government controls the interest on reserves, i^B, and the interest on the interbank market, i. I formalise the equilibrium concept as follows.

Definition 2. *Given policy $\{i^B, i\}$ with $i^B \leq i$, the flexible-price equilibrium consists of $\{C_{j,0}, K_j, D_{j,k}, C_{j,1}(\theta_j), C_{j,2}(\theta_j), L_j(\theta_j), W_{j,k}(\theta_j), \Pi_f, K, I, \Pi_k, B_k, Cu_k, F_k, K_k, D_k, L_k, M, K_g, T\}_{j,k,\theta_j}$ and $\{P_0, P_1, Q, d_{k,0}, d_{k,1}\}_{k\in[0,1]}$, respectively a vector of quantities and a vector of prices, such that*

1. *The representative capital-producing firm chooses $\{\Pi_f, K, I\}$ to maximise its profits, Π_f, defined by equation (1.14) subject to (1.1).*

2. *Consumer j chooses $\{C_{j,0}, K_j, D_{j,k}, C_{j,1}(\theta_j), C_{j,2}(\theta_j), L_j(\theta_j), W_{j,k}(\theta_j)\}$ to maximise (1.15) subject to budget constraints (1.16), (1.17), (1.18) and non-negativity constraints*
$$C_{j,t}(\theta_j) \geq 0 \quad \forall t, \theta_j. \tag{1.30}$$

3. *Bank k chooses $\{\Pi_k, B_k, Cu_k, F_k, K_k, D_k, L_k, d_{k,0}, d_{k,1}\}$ to maximise its profits, Π_k, subject to budget constraints (1.23), (1.25), (1.26), the reserve requirement (1.24), demand for deposits given by equations (1.21) and (1.27), and non-negativity constraints*
$$\left(B_k, Cu_k, K_k\right) \geq 0. \tag{1.31}$$

4. *$\{M, K_g, T\}$ are such that the government's budget constraints (1.28) and (1.29) hold.*

5. *The goods market clears with*
$$\int_0^1 C_{j,0}\,\mathrm{d}j + I = Y. \tag{1.32}$$

6. *The market for reserves clears with*
$$\int_0^1 B_k + Cu_k\,\mathrm{d}k = M. \tag{1.33}$$

7. *The market for interbank loans clears with*
$$\int_0^1 F_k\,\mathrm{d}k = 0. \tag{1.34}$$

8. The market for capital goods clears with

$$\int_0^1 K_j \,\mathrm{d}j + \int_0^1 K_k \,\mathrm{d}k + K_g = K. \tag{1.35}$$

From here on, I drop the k index for banks, since banks behave symmetrically. Moreover, I drop the index j for consumers realising that heterogeneity between consumers is uniquely due to different realisations of a consumer's liquidity shock, θ.

First of all, I find the zero lower bound as an equilibrium condition. A policy that sets the interest rate in the interbank market strictly below zero is not compatible with equilibrium, because all banks would have an incentive to borrow in order to hold money in the form of currency. The zero lower bound is given by

$$i \geq 0. \tag{1.36}$$

A storage cost for currency would create an effective lower bound at a negative level. Although the empirical evidence is that the lower bound is effectively at a negative level, in this paper I adopt a zero lower bound for simplicity. The same mechanism as studied in the paper would operate with a negative effective lower bound.

The interbank rate of interest, i, is important in the economy because, by exploiting arbitrage opportunities, banks make sure that it is equated to the short-term return on capital assets, according to

$$(1+i) \cdot \frac{P_0}{P_1} = Q^{-1}. \tag{1.37}$$

In fact, if the real interest rate on interbank loans was lower than the short-term return on capital assets, then every bank would want to borrow in this market and this would violate the market clearing condition. Conversely, if the real interbank interest rate was higher, all banks would want to lend in this market.

It is interesting to study whether the short-term real interest rate can be influenced by monetary policy. As is usual in set-ups without nominal rigidities, I find that the short-term real interest rate is decoupled from monetary policy. It is pinned down by the supply schedule for capital goods and by the market-clearing condition of the goods market (1.48), as

$$Q^{-1} = f'(Y - C_0). \tag{1.38}$$

In equilibrium, banks offer a deposit contract that only pays off for early types. Late-type consumers do not get any of their deposits back. This is a direct consequence of the absence of an investment technology for consumers at time 1 and thus of the absence of an incentive-compatibility constraint on the bank. Moreover, given that banks Bertrand-compete to supply deposits, banks make zero profits in equilibrium.

Lemma 1. *Define* $\tau \equiv \rho \cdot \frac{i-i^B}{1+i}$. *In equilibrium, the prevailing deposit contract* $\{d_0, d_1\}$ *is given by:*

$$1 + d_0 = (1+i) \cdot \frac{P_0}{P_1} \cdot \frac{1-\tau}{\phi}, \tag{1.39}$$

$$1 + d_1 = 0. \tag{1.40}$$

Proof. Please refer to appendix 1.B. □

The absence of incentive-compatibility considerations has the drawback of giving an unrealistic deposit contract, where late-type consumers ex-post receive nothing from their time-0 deposits. However, the upside of this specification is that the role of banks as providers of liquidity-risk insurance becomes very clear. It is worth noting that from an ex-ante perspective, all consumers are willing to deposit part of their wealth in bank deposits because of this insurance role.

Using the consumer's first-order condition with regard to capital goods combined with arbitrage condition (1.37) and the short-term real interest rate (1.38), I find the Euler equation of the economy in the flexible-price equilibrium, given by

$$u'(C_0) = f'(Y - C_0) \cdot \hat{\beta} \cdot u'[C_1(0)] \cdot \left\{ \phi + (1-\phi) \cdot R \cdot \frac{\beta \cdot u'[C_2(1)]}{u'[C_1(0)]} \right\}. \tag{1.41}$$

Define the level of implicit taxation on the banking system due to the reserve requirement as

$$\tau \equiv \rho \cdot \frac{i - i^B}{1+i}. \tag{1.42}$$

Consumer's first-order condition with respect to capital goods and deposits combined determine the consumer's investment allocation. She holds a portfolio such that:

$$\frac{u'[C_1(0)]}{\beta \cdot u'[C_2(1)]} = \min\left\{\frac{1-\phi}{1-\phi-\tau}, R^{\gamma[C_2(1)]-1}\right\} \cdot R. \tag{1.43}$$

A higher level of taxation on banks, which is passed on to depositors as lower interest on deposits, incentivises consumers to hold fewer deposits in their portfolio, although they are imperfectly insured against liquidity shocks. In the absence of taxation, consumers decide to fully insure themselves, in the same way as the social planner would decide. Notice that the level of liquidity risk to which consumers are exposed cannot be worse than the case without deposits, where $C_2(1) = R \cdot C_1(0)$.

Market-clearing conditions combined with all the agents' budget constraints imply that the resource constraint holds as follows:

$$(1-\phi) \cdot C_2(1) = R \cdot [f(Y - C_0) - \phi \cdot C_1(0)]. \tag{1.12}$$

Consumers have no incentive to consume when they do not enjoy consumption. Hence, we have that in equilibrium

$$C_1(1) = C_2(0) = 0. \tag{1.13}$$

In terms of optimal monetary policy, the first-best efficient allocation, as per definition 1, can be implemented by setting the interest on reserves equal to the interest rate on interbank loans. This is a version of the Friedman rule. Paying the market interest rate on bank reserves implements the efficient allocation by eliminating the distortionary taxation implied by the reserve requirement.

In the flexible-price equilibrium, the level of the nominal interest rate prevailing in the interbank market does not matter at all for the allocation. Monetary policy conducted by changing the interbank rate has no real effects, because prices change to ensure market clearing for any level of the interbank rate. The short-term real interest rate in the flexible-price equilibrium, Q^{-1}, is determined by the supply curve for capital goods and market clearing condition, according to equation (1.38). I define this short-term real interest rate, which is consistent with market clearing, the natural short-term real rate of interest.

Definition 3 (Natural short-term real rate of interest).
Consider flexible-price equilibrium outcomes $\{Q, C_0, C_1(\theta), C_2(\theta), \tau\}_\theta$*, given by equations (1.41), (1.42), (1.43), (1.12) and (1.13).*

Define the natural short-term real rate of interest r^n *as equal to the short-term real return on capital in the flexible-price equilibrium,*

$$1 + r^n \equiv Q^{-1} = 1 + r^n(\tau, \hat{\beta}). \tag{1.44}$$

The natural short-term interest rate is a function of the extent of financial repression operated by the government through the reserve requirement, τ, and of the consumer's time-0 discount factor. Financial repression worsens the risk-reward profile of saving and therefore leads to an increase in the natural short-term real interest rate. An increase in the discount factor means that consumers are more patient. Hence, for markets to clear the short-term real interest rate must fall.

In the sticky-price equilibrium, the natural short-term real rate of interest becomes an important concept. If prices are sticky, they cannot change to make markets clear and guarantee full employment. Since the natural short-term real rate of interest is what the economy needs for markets to clear, it is the short-term real interest rate that a benevolent monetary authority must attempt to implement.

1.5.2 Sticky-Price Equilibrium

I define an equilibrium with sticky prices in order to study optimal monetary policy. Nominal rigidities are necessary for interest-rate setting by monetary authorities to have real effects.

I assume a simplified notion of nominal rigidity. In the short run, prices are fixed with $P_1 = P_0 = \overline{P}$. Since prices do not adjust to ensure market clearing, demand is not generally equal to supply. It is up to monetary policy to manage demand so that there are no spare resources.

The sticky-price equilibrium is formalised as follows:

Definition 4. *Given policy* $\{i^B, i\}$ *with* $i^B \leq i$*, the sticky-price equilibrium consists of* $\{C_{j,0}, K_j, D_{j,k}, C_{j,1}(\theta_j), C_{j,2}(\theta_j), L_j(\theta_j), W_{j,k}(\theta_j), \Pi_f, K, I, \Pi_k, B_k, Cu_k, F_k, K_k, D_k, L_k, M,$

$K_g, T\}_{j,k,\theta_j}$ *and* $\{P_0, P_1, Q, d_{k,0}, d_{k,1}\}_{k\in[0,1]}$*, respectively a vector of quantities and a vector of prices, such that*

1. *The representative capital-producing firm chooses* $\{\Pi_f, K, I\}$ *to maximise its profits,* Π_f*, defined by equation (1.14) subject to (1.1).*

2. *Consumer j chooses* $\{C_{j,0}, K_j, D_{j,k}, C_{j,1}(\theta_j), C_{j,2}(\theta_j), L_j(\theta_j), W_{j,k}(\theta_j)\}$ *to maximise (1.15) subject to budget constraints (1.16), (1.17), (1.18) and non-negativity constraints*
$$C_{j,t}(\theta_j) \geq 0 \quad \forall t, \theta_j. \tag{1.45}$$

3. *Bank k chooses* $\{\Pi_k, B_k, Cu_k, F_k, K_k, D_k, L_k, d_{k,0}, d_{k,1}\}$ *to maximise its profits,* Π_k*, subject to budget constraints (1.23), (1.25), (1.26), the reserve requirement (1.24), demand for deposits given by equations (1.21) and (1.27), and non-negativity constraints*
$$\left(B_k, Cu_k, K_k\right) \geq 0. \tag{1.46}$$

4. $\{M, K_g, T\}$ *are such that the government's budget constraints (1.28) and (1.29) hold.*

5. *Prices are sticky with*
$$P_0 = P_1 = \overline{P}. \tag{1.47}$$

6. *The goods market has*
$$\int_0^1 C_{j,0}\, \mathrm{d}j + I = Y. \tag{1.48}$$

7. *The market for reserves clears with*
$$\int_0^1 B_k + Cu_k\, \mathrm{d}k = M. \tag{1.49}$$

8. *The market for interbank loans clears with*
$$\int_0^1 F_k\, \mathrm{d}k = 0. \tag{1.50}$$

9. *The market for capital goods clears with*
$$\int_0^1 K_j\, \mathrm{d}j + \int_0^1 K_k\, \mathrm{d}k + K_g = K. \tag{1.51}$$

Rationing takes place when at the going price there is not enough demand to absorb all the goods supplied, because the return on saving is too high.[9] The role of monetary policy is to adjust its interest rates in order to discourage consumers from saving excessively, because this leads to rationing. However, the presence of a lower bound on the interbank rate may make it impossible for the central bank to offset large adverse demand shocks. In these cases, which I call liquidity traps, rationing takes place in equilibrium.

From here on, I drop the k index for banks, since banks behave symmetrically. Moreover, I drop the index j for consumers realising that heterogeneity between consumers is uniquely due to different realisations of a consumer's liquidity shock, θ.

There is a zero lower bound on the interbank rate, because of the option of exchanging bank reserves one-for-one with currency, given by

$$i \geq 0. \tag{1.52}$$

If the interbank rate was set below zero, then all banks would want to borrow and hold the borrowings as currency. This is inconsistent with market clearing in the interbank market. In the equilibrium with sticky prices, the lower bound restricts the allocations that monetary policy can attain.

The consumer's first-order condition with respect to direct holdings of capital goods and the bank's arbitrage condition (1.37) give us the economy's Euler equation

$$u'(C_0) = (1+i) \cdot \hat{\beta} \cdot u'[C_1(0)] \cdot \left\{ \phi + (1-\phi) \cdot R \cdot \frac{\beta \cdot u'[C_2(1)]}{u'[C_1(0)]} \right\}. \tag{1.53}$$

The equilibrium deposit contract is the same as in the flexible prices equilibrium, described in lemma 1. Substituting this in the consumer's first-order condition with respect to bank deposits and using the first-order condition with respect to capital goods, we find the consumer's time-0 portfolio allocation. It is such that

$$\frac{u'[C_1(0)]}{\beta \cdot u'[C_2(1)]} = \min \left\{ \frac{1-\phi}{1-\phi-\tau}, R^{\gamma[C_2(1)]-1} \right\} \cdot R. \tag{1.43}$$

[9] Buyers are rationed. Sellers do not consume the part of their endowment that they are unable to sell, because by assumption consumers do not like their own endowment and thus trade is necessary.

Consumers hold enough deposits to fully insure themselves against liquidity risk. τ, defined in equation (1.42), is the effective tax on bank deposits caused by the reserve requirement. A burdensome reserve requirement, which implies lower interest rates on deposits, makes consumers hold fewer deposits in their asset portfolio and thus exposes them to more liquidity risk.

Market-clearing conditions imply that at time 1 and at time 2 consumption satisfies the intertemporal budget constraint

$$(1-\phi)\cdot C_2(1) = R\cdot[f(I)-\phi\cdot C_1(0)]. \tag{1.54}$$

And consumers do not consume in time periods when they do not enjoy consumption, so that

$$C_1(1) = C_2(0) = 0. \tag{1.13}$$

The level of investment in the economy, I, is determined by the interbank rate via the supply curve for capital goods. This is true, unless an excessively low interbank rate implies that demand for consumption goods at time 0 is greater than supply. In this case, investment is limited by the availability of consumption goods to invest.

$$f'(I) = \max\{1+i, f'(Y-C_0)\}. \tag{1.55}$$

Lemma 2. *In the sticky-price equilibrium, the allocation* $\{I, C_0, C_1(\theta), C_2(\theta), \tau\}_\theta$ *is given by equations (1.53), (1.42), (1.43), (1.54), (1.13) and (1.55). Policy instruments* $\{i^B, i\}$ *are subject to the following restrictions:*

$$i \geq 0, \tag{1.56}$$

$$i \geq i^B. \tag{1.57}$$

1.6 Optimal Monetary Policy

Policymakers choose policy instruments $\{i^B, i\}$ in order to maximise aggregate welfare in the sticky-price equilibrium.

It is useful to define liquidity traps in this setting, because optimal monetary policy differs depending on whether the economy has fallen in the liquidity trap or not.

Definition 5. *If $r^n(0, \hat{\beta}) < 0$, then the economy is in the liquidity trap.*

It can be shown that there are realisation of $\hat{\beta}$ such that the economy is in the liquidity trap. A high realisation of $\hat{\beta}$ implies that consumers are patient in their consumption decisions. Thus, a low real interest rate is required for demand to fully absorb the supply of goods at time 0. If a negative interest rate is required, then we say that the economy is in the liquidity trap.

Lemma 3. *Consider the function $r^n(\tau, \hat{\beta})$ in definition 3. There exists a threshold $\overline{\beta}$ such that, if $\hat{\beta} > \overline{\beta}$, then $r^n(0, \hat{\beta}) < 0$.*

The presence of the lower bound on the interbank rate makes the liquidity trap a special economic contingency for policymakers.

In the first subsection, I discuss optimal monetary policy when the economy is not in the liquidity trap. Then, I move on to the case of an economy with very high propensity to save and study the implications for optimal monetary policy.

1.6.1 Out of the Liquidity Trap

When a positive level of the interest rate makes the market for consumption goods clear, then the monetary authority should set the interbank rate at that level. Simultaneously, by setting the interest on bank reserves at the same level, the central bank can implement full liquidity-risk insurance in the economy.

Proposition 1. *If the economy is not in the liquidity trap with $r^n(0, \hat{\beta}) \geq 0$, then the monetary authority can implement the first-best efficient allocation with*

$$i^B = i = r^n(0, \hat{\beta}). \tag{1.58}$$

Proof. Pleaser refer to appendix 1.B. □

There is no tension between between making sure none of the endowment is wasted and not interfering with the provision of liquidity-risk insurance by banks. The central bank should follow the Friedman rule and not tax holders of liquid assets. Demand management can be accomplished exclusively by setting the interest rate in the interbank market. In summary, outside of the liquidity trap it is fine to study optimal interest-rate setting in the absence of maturity transformation.

1.6.2 Liquidity Trap

In this paragraph, I study the optimal setting of the interest on reserves in the liquidity trap. As a consequence of large adverse demand shock hitting the economy, a negative short-term real interest rate is necessary to ensure enough demand to fully absorb supply. In this case, the lower bound on the interbank rate represents a constraint on the central bank's ability to ensure full utilisation of the economy's resources. I show that in the liquidity trap the central bank faces a trade-off between liquidity-risk insurance and ensuring full employment. This is because banks that perform maturity transformation encourage consumers to save. And in the liquidity trap insufficient demand is caused by an excessive propensity to save from consumers.

First of all, the lower bound on the rate of interest prevailing in the interbank market is binding in the economy's liquidity trap. That is, it is optimal for the central bank to set the interbank rate to zero, whenever the economy is in the liquidity trap.

Lemma 4. *For any level of* $\tau \equiv \rho \cdot \frac{i-i^B}{1+i}$*, if the economy is in the liquidity trap as per definition 5, then it is optimal to set* $i = 0$.

Proof. Please refer to appendix 1.B. □

It is hardly surprising that in the liquidity trap it is optimal for the central bank to hold the interbank rate at the lower bound.

Consider $\tau = 0$ so that maturity transformation leads to full liquidity-risk insurance, as according to equation (1.43). In this case, since the interbank rate is larger than the natural short-term real interest rate, there is rationing in the goods market. An alternative policy is to set $\tau > 0$. This reduces liquidity-risk insurance to a suboptimal level. On the other hand, less liquidity-risk insurance reduces consumers' incentive to save, as according to the Euler equation (1.53), which can reduce wasted resources. We learn two things about an economy in the liquidity trap from this reasoning: 1) the central bank cannot attain the first-best efficient allocation and 2) there is an interaction between aggregate demand and maturity transformation that the central bank may want to exploit.

The interaction between demand management and maturity transformation is represented by the *IMP* schedule in the cartesian plane of figure 1.A.2. Point A is the combination of liquidity-risk insurance and time-0 consumption that prevails if the central bank sets $i^R = 0$ and thus implements perfect liquidity-risk insurance. The economy moves along the *IMP* constraint to the left as the interest on reserves is cut. Lower interest on reserves reduces the attractiveness of deposits, as banks are effectively taxed. Hence, consumers substitute their wealth away from deposits into direct capital holdings. However, deposits and capital are not perfect substitutes, in that the former provides liquidity-risk insurance. It follows that the consumer partially substitutes away from saving overall and increases her current consumption. The quasi concavity of the *IMP* curve is given by an income effect, which pulls in the other direction. Consumers are made poorer by the reduction in the attractiveness of investment opportunities. This increases the value of saving. At the point where the implementability curve peaks, the income effect and the substitution effect perfectly cancel each other out. To the left of the peak, the income effect dominates. However, the range of liquidity-risk insurance where the income effect dominates is not important for the optimal monetary policy exercise, because it is always suboptimal for monetary policy to move the economy to it.

The central bank's role is to maximise aggregate welfare with the policy instruments at its disposal. Aggregate welfare is represented on the Cartesian plane by a family of indifference curves, labeled *IND*. Aggregate welfare is increasing with current consumption and with liquidity-risk insurance . If $u'[C_1(0)] = R \cdot \beta \cdot u'[C_2(1)]$, consumers are satiated in liquidity-risk insurance. This means that, if they have perfect liquidity-risk insurance, consumers are willing to reduce it marginally in exchange for any strictly positive increase in current consumption, however small. This is captured in the figure by the flatness of the indifference curves at that point.

Optimality requires that the rate at which consumers would trade off liquidity-risk insurance for more consumption remaining equally well off be equal to the rate at which the central bank can increase consumption by decreasing liquidity-risk insurance. This is given by the tangency point of curve *IND* and *IMP*, point B in figure 1.A.2.

Proposition 2 (**Optimal Monetary Policy in Liquidity Trap**).
If the economy is in the liquidity trap, optimal policy prescribes $i = 0$ *and* $i^B < 0$. *It implements a second-best allocation.*

Proof. Please refer to appendix 1.B. □

Point B in figure 1.A.2, which represents the second-best equilibrium allocation, always features partial liquidity-risk insurance. So, setting the interest on reserves strictly below the lower bound on the interbank rate is optimal. Notice that this monetary policy does not transmit to the economy through the conventional intertemporal substitution channel via a reduction in the real interest rate. It solely transmits by making deposits less attractive for consumers, relative to direct holdings of capital.

1.7 Conclusion

In this paper, I develop a monetary model with two important characteristics: multiple interest rates, which monetary policy controls, and a meaningful banking sector. With reference to the former, I explicitly model the interbank rate, which is the conventional instrument of monetary policy, and the interest on reserves separately. And as regards the latter, banks are modelled as maturity transformers, in accordance with the framework of Diamond and Dybvig (1983).

I have four main findings in the paper. First, I show that the lower bound does not apply to the interest on reserves. A first-pass explanation for this is that the central bank can decide by decree to pay whatever interest on bank reserves, while the interbank rate is the equilibrium price that clears the interbank market. The lower bound emerges as equilibrium requirement on the latter because of the presence of currency. Nonetheless, if reserves and other assets are perfect substitutes as in the canonical New Keynesian model, changing the interest on reserves per se has no macroeconomic effect.

Second, I show that reserves and other assets become imperfect substitutes if banks perform maturity transformation à la Diamond and Dybvig (1983). From the consumer's perspective, deposits and other assets are not perfect substitutes because of the benefits of maturity transformation. Since reserves are necessary to supply redeemable deposits, bank reserves also become imperfect substitutes of other assets. Thus, a reduction in the interest on reserves, which leaves the interbank rate unchanged, increases aggregate demand.

Third, I find that setting a negative interest on reserves in order to boost aggregate demand in the liquidity trap involves an interesting trade-off with the preservation of a fully functional banking system. A lower interest on reserves transmits to the deposit rate. Thus, consumers respond by moving their wealth out of deposits into direct asset holdings. Such disintermediation is detrimental to welfare in this setting, because deposits provide valuable liquidity-risk insurance. In summary, stimulus can be provided to the economy by means of negative interest on reserves only against the backdrop of a weakening banking system.

The last and most important finding of the paper is that, when the interbank rate is constrained by the zero lower bound, optimal monetary policy prescribes a strictly negative interest on reserves. In other words, I find that, when demand is insufficient, there is always some space to stimulate the economy by cutting the interest on reserves below the interest rate prevailing in the money market without damaging the banking sector excessively.

In reality, central banks implement negative interest rate policies by cutting the interest on reserves without knowing where the lower bound on the interbank rate is. They do not know when the cuts to the interest on reserves will stop transmitting to the money market. A discussion has developed over the extent to which central banks should therefore be prudent in lowering their interest on reserves (Dell'Ariccia et al., 2017). The result of my paper can be read as policy recommendation not to be prudent, because lowering the interest on reserves below the lower bound on the interbank rate is the optimal monetary policy.

Moreover, my theory contributes a relevant and measurable indicator for the health of the banking sector, which could guide monetary authorities in the implementation of negative interest rates. The degree of liquidity-risk insurance, which represents the effectiveness of the banking sector at providing liquidity, is expressed in terms of observables and therefore can in principle be measured.

The theory predicts that negative interest rates lead to a decline in liquidity-risk insurance, and makes clear that, while a moderate reduction is necessary in order to stimulate the economy, an excessive reduction is undesirable. Therefore, monitoring this measure would allow to adjust interest-rate setting in accordance with the level of stress that it imposes on the banking system.

This paper is the first to study the role of the interest rate on bank reserves in the liquidity trap of a model with maturity-transforming banks. Maturity transformation is an important aspect of banking. However, it is by no means the only one. Future research should enrich the modelling of the banking sector, on the asset side (Brunnermeier and Koby, 2017; von Thadden, 1997) and on the liability side with capital constraints. This will let us establish quantitatively the extent to which bank characteristics matter for the effectiveness of negative interest rates.

Appendix

1.A Figures

Figure 1.A.1: Interest rate on excess reserves, 2012-2017

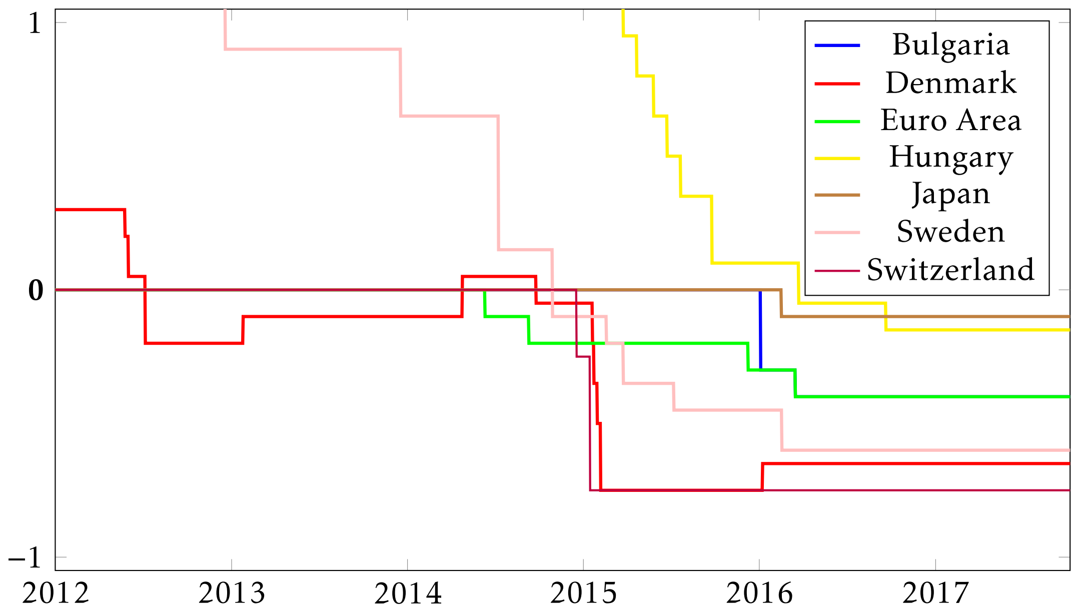
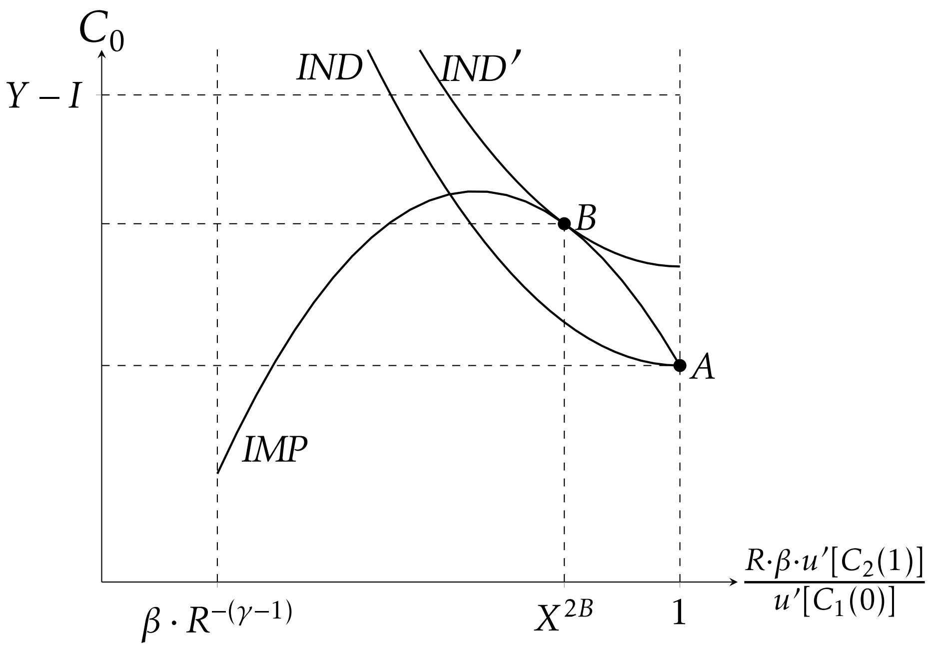


Figure 1.A.2: The second-best problem



1.B Proofs

Lemma 1. *Define* $\tau \equiv \rho \cdot \frac{i-i^B}{1+i}$. *In equilibrium, the prevailing deposit contract* $\{d_0, d_1\}$ *is given by:*

$$1 + d_0 = (1+i) \cdot \frac{P_0}{P_1} \cdot \frac{1-\tau}{\phi}, \tag{1.39}$$

$$1 + d_1 = 0. \tag{1.40}$$

Proof. Consider the flexible-price equilibrium allocation $\{C_0, C_1(\theta), C_2(\theta), \tau\}_\theta$ governed by equations (1.41), (1.42), (1.43), (1.12) and (1.13).

By Bertrand competition, banks make zero profits. It follows from banks' budget constraints, (1.23), (1.25), (1.26), the reserve requirement, (1.24, and non-negativity of capital holdings, $L \leq K$, that feasible deposit contracts are restricted to

$$1 + d_0 = (1+i) \cdot \frac{P_0}{P_1} \cdot (1-\tau) \cdot \frac{R}{\phi \cdot R + (1-\phi) \cdot (1+d_1)}, \tag{1.59}$$

with

$$1 + d_1 \geq 0. \tag{1.60}$$

I show that there is no deposit contract that is feasible and that consumers prefer in equilibrium.

Take the deposit valuation equation (1.20) and substitute in the equilibrium allocation and feasibility conditions for deposit contract. We have that

$$V = (1+i) \cdot \frac{P_0}{P_1} \cdot (1-\tau) \cdot \frac{R}{\phi \cdot R + (1-\phi) \cdot (1+d_1)} \cdot \hat{\beta} \cdot [\phi \cdot Z + (1-\phi) \cdot (1+d_1) \cdot \beta] \cdot u'[C_2(1)], \tag{1.61}$$

with

$$Z = \min\left\{\frac{1-\phi}{1-\phi-\tau}, R^{\gamma[C_2(1)]-1}\right\} \cdot \beta \cdot R. \tag{1.62}$$

The value of the deposit contract is continuously non-increasing in d_1. Hence, there is no feasible deposit contract that consumers strictly prefer to the deposit contract with $1 + d_1 = 0$.

□

Lemma 4. *For any level of* $\tau \equiv \rho \cdot \frac{i-i^B}{1+i}$, *if the economy is in the liquidity trap as per definition 5, then it is optimal to set* $i = 0$.

Proof. Consider the sticky-price equilibrium illustrated in lemma 2 with τ given. From equation (1.55), a reduction in i increases investment I. Through equation (1.54), the increase in investment increases $C_1(0)$ and $C_2(1)$.
From equation (1.53), a reduction in i leads to an increase in C_0.
Holding τ constant, an increase in i weakly increases consumption in all time periods. Hence, it is unambiguously (weakly) welfare improving.
It follows that in the liquidity trap, for any level of τ, it is optimal to hold the interbank rate against the lower bound.

□

Proposition 2 (**Optimal Monetary Policy in Liquidity Trap**).
If the economy is in the liquidity trap, optimal policy prescribes $i = 0$ *and* $i^B < 0$. *It implements a second-best allocation.*

Proof. Consider the graphical representation of the second-best problem given in figure 1.A.2.
The implementability constraint, labelled by *IMP*, is given by equations (1.53), (1.54) and (1.55) with $i = 0$ because by lemma 4 this is optimal. The first-order derivative of time-0 consumption C_0 with respect to the level of liquidity-risk insurance $\frac{\beta \cdot R \cdot u'[C_2(1)]}{u'[C_1(0)]}$ is given by

$$\frac{dC_0}{d\frac{\beta \cdot R \cdot u'[C_2(1)]}{u'[C_1(0)]}} = \frac{\phi \cdot (1-\phi) \cdot \hat{\beta} \cdot u'[C_1(0)]}{u''[C_0]} \cdot \left(\frac{R \cdot \frac{\gamma[C_2(1)]}{\gamma[C_1(0)]} \cdot \frac{C_1(0)}{C_2(1)} \cdot \frac{\beta \cdot R \cdot u'[C_2(1)]}{u'[C_1(0)]} - 1}{\frac{\beta \cdot R \cdot u'[C_2(1)]}{u'[C_1(0)]} \cdot \left\{ \phi \cdot R \cdot \frac{\gamma[C_2(1)]}{\gamma[C_1(0)]} \cdot \frac{C_1(0)}{C_2(1)} + 1 - \phi \right\}} \right). \tag{1.63}$$

Notice that at $\frac{\beta \cdot R \cdot u'[C_2(1)]}{u'[C_1(0)]} = 1$, with full liquidity-risk insurance, the derivative is negative and that as liquidity-risk insurance decreases the first-order derivative of the *IMP* schedule increases. Thus, the graphical representation is accurate.
The indifference curve, labelled *IND*, gives bundles of liquidity-risk insurance and time-0 consumption such that expected utility is constant. The first-order derivative of time-0 consumption C_0 with respect to level of liquidity-risk insurance $\frac{\beta \cdot R \cdot u'[C_2(1)]}{u'[C_1(0)]}$ on the schedule is given by

$$\frac{dC_0}{d\frac{\beta \cdot R \cdot u'[C_2(1)]}{u'[C_1(0)]}} = \frac{\phi \cdot (1-\phi) \cdot \hat{\beta} \cdot \{u'[C_1(0)]\}^2}{u''[C_1(0)] \cdot u'(C_0)} \cdot \frac{1}{\phi \cdot R \cdot \frac{\gamma[C_2(1)]}{\gamma[C_1(0)]} \cdot \frac{C_1(0)}{C_2(1)} + 1 - \phi} \cdot \frac{1 - \frac{\beta \cdot R \cdot u'[C_2(1)]}{u'[C_1(0)]}}{\frac{\beta \cdot R \cdot u'[C_2(1)]}{u'[C_1(0)]}}. \tag{1.64}$$

Notice that the derivative of the indifference curves is equal to zero if $\frac{\beta \cdot R \cdot u'[C_2(1)]}{u'[C_1(0)]} = 1$. This means that at perfect liquidity-risk insurance the consumer is satiated in liquidity.

It follows that for the central bank setting $\frac{\beta \cdot R \cdot u'[C_2(1)]}{u'[C_1(0)]} = 1$ is inefficient. While a marginal increase in liquidity risk does not reduce expected utility, the resulting increase in current consumption does.

□

2 | A Theory of Liquidity and Interest on Reserves

A well-known result of the Diamond-Dybvig model is that the existence of financial markets curtails provision of liquidity-risk insurance by the banking system. The model's equilibrium allocation has inefficiently low investment in short-term assets. Farhi et al. (2009) propose imposing a reserve requirement on banks in order to increase investment in short-term assets. In a model where consumers are restricted to investing their wealth in bank deposits, they show that a reserve requirement can implement the efficient allocation. First, I show that the fully efficient allocation can equivalently be implemented by payment of positive interest on reserves. Then, I extend the model to allow consumers to invest their wealth directly in illiquid capital assets as well as in bank deposits. Such generalisation breaks down the equivalence between reserve requirement and interest on reserves. A reserve requirement does not lead to more aggregate investment in short-term assets, because it is effectively a tax on bank deposits and consumers respond to it by reducing their holdings of deposits. The paper's main result is that, in the generalised setting, the efficient level of liquidity-risk insurance can only be implemented by a policy of paying strictly positive interest on bank reserves. Interest on reserves provides an incentive for banks to hold more short-term assets and does not lead to disintermediation.

2.1 Introduction

Consumers hold assets for the purpose of smoothing their consumption over time and over states of the world. An asset that is hard to sell or that goes for a low price if sold before its maturity date is an illiquid asset. This is a poor asset to hold for consumption-smoothing purposes.[1]

Assets are illiquid for a variety of reasons. For example, it takes time to build a plant and, until it is completed, the plant has no value except the resale value of the individual parts. Because of asset illiquidity, financial intermediaries play an important role in the economy. They buy illiquid assets and fund such purchases by selling more liquid assets, in the form of bank deposits, to consumers. This business plan is viable because banks can predict deposit withdrawals. A large literature formalises this notion of banks providing liquidity. The seminal paper is Diamond and Dybvig (1983).

There is a limitation to the profitable provision of liquid assets by the banking system. If we accept that assets are illiquid so that agents end up selling their assets at a low price. Then, a direct consequence is that there are great buying opportunities in the economy. In his critique of the Diamond-Dybvig model, Jacklin (1987) points out that in these circumstances no consumer would keep her wealth in a bank deposit. They would all withdraw their deposits in order to buy the cheap assets on sale. It turns out that, if the market for used capital works frictionlessly, then private provision of liquid assets is impossible.

A strand of the literature has found reasons why the Jacklin critique may not hold fully, so that private provision of liquidity is possible. Diamond (1997) adds limited participation in financial markets, Antinolfi and Prasad (2008) assume that consumers who interact in financial markets are subject to borrowing constraints and Hasman et al. (2014) assume transaction costs in the market for used capital. However, the existence of the frictions that these papers advocate only implies that private liquidity is possible to a degree. Even if this is the case, the Jacklin critique still implies that private provision of liquidity alone is insufficient for full efficiency in the economy. Thus, it is important to study whether policy can improve the economy's liquidity outcomes.

Farhi et al. (2009) take the Jacklin critique seriously and study the effectiveness of monetary policy in improving the equilibrium allocation in the Diamond-Dybvig model. They find that a reserve requirement is effective at making assets

[1]For a discussion of the several definitions of liquidity that recur in different literatures, see von Thadden (1999).

more liquid in the economy. As banks hold more short-term assets, the deposits held by consumers become more liquid. In fact, a reserve requirement can implement the first-best efficient allocation. Panetti (2017) includes systemic liquidity risk in a similar model and also concludes that imposing a reserve requirement is optimal. A key assumption of these papers is that consumers do not have a choice of how much of their wealth to hold directly in illiquid assets and how many in bank deposits.

In this paper, first I show that in the same setting as in Farhi et al. (2009) the payment of interest on reserves can equivalently implement the first-best efficient allocation. I stress the effect of interest on reserves on incentives: of course, it increases the returns from holding short-term assets and therefore the incentive to hold them.

Next, I generalise the model. I allow consumers to invest their wealth directly in illiquid assets at time 0, so that the degree of financial intermediation is endogenously determined. I find that this matters for the effects of monetary-policy instruments. In fact, this generalisation breaks the equivalence between reserve requirement and payment of interest on reserves. In the new setting, the central bank can only implement the first-best allocation by paying a positive interest on reserves. In contrast, a reserve requirement is ineffective.

A reserve requirement is effectively a tax on banks. It is a regulation that mandates banks to hold a portfolio of assets that is suboptimal from the banks' viewpoint and thus reduces the return on banks' asset portfolios. The tax is passed on to depositors in the form of lower returns on deposits. Thus, if consumers can also invest their wealth directly in financial markets, they will respond to the reserve requirement by reducing their investment in bank deposits and holding more illiquid assets instead. I find that such financial disintermediation offsets the improvements in liquidity due to the introduction of a reserve requirement. In other words, the effects of a reserve requirement are that (1) banks hold more short-term assets as a share of their asset portfolio, which implies more liquid deposits, but (2) consumers decide to hold fewer deposits. The net effect is that the average liquidity of consumers' assets does not change.

The key result of this paper is that a central bank should pay a strictly positive rate of interest on bank reserves. Interest on reserves provides an incentive for banks to hold more short-term assets and therefore promotes the creation of more liquid deposits. At the same time, unlike liquidity regulation, it does not represent a tax on the banking system. Therefore, it does not lead to financial disintermediation by consumers. It follows that with interest on reserves the central bank can implement the first-best efficient allocation in the economy.

Especially since the Federal Reserve acquired the authority to pay interest on reserves on 1 October 2008, a large literature has focused on the costs and benefits of such policy. An important result, first developed in Sargent and Wallace (1985), is that payment of the market rate of interest on bank reserves decouples the quantity of money from the determination of inflation. Several papers have built on this to argue that paying interest on reserves frees the central bank to use the size of its balance sheet as an additional policy instrument. Thus, it improves the institution's ability to stabilise the economy (Cúrdia and Woodford, 2011; Ennis, 2014; Reis, 2016). This paper is more closely related to another set of papers in this literature, which points out that payment of interest on reserves eliminates the implicit taxation on monetary instruments. Taxing reserves makes the banking system reduce their holdings of reserves to inefficiently low levels. If the tax is eliminated, banks choose to be satiated in reserves. The Friedman rule prescribes such policy as socially optimal (Ennis and Weinberg, 2007; Cochrane, 2014; Canzoneri et al., 2017). In this paper, the inefficiently low level of short-term assets held by financial institutions is structural, rather than the result of implicit distortionary taxation, and payment of positive interest on reserves is better understood as a subsidy on short-term assets. While the literature that looks back to the Friedman rule concludes that the interest on reserves should be set equal to to the market short-term rate of interest, this paper's conclusion is that the government should push up the market short-term interest rate in order to incentivise banks to hold more short-term assets.

2.1.1 Related Literature

The Diamond-Dybvig model is mostly used to study the fragility of the banking system. A large literature studies whether and how policy can improve the financial system's resilience (Allen and Gale, 2004; Allen et al., 2014). I abstract from bank runs altogether. The model's bank-run equilibrium can be eliminated by assuming that deposits carry a guarantee by the government or that banks have the right to suspend convertibility. Building on the literature on global games started by Carlsson and van Damme (1993), Rochet and Vives (2004) and Goldstein and Pauzner (2005) obtain a unique equilibrium in which bank runs take place with positive probability. This is an important step forward for the study of the effects of policies in the context of the Diamond-Dybvig model. Integrating global games in this paper's analysis of the effects of interest on reserves would allow a discussion of the policy's impact on the likelihood of crises. I leave this to future work.

2.2 Technology and Preferences

In this section, I lay out the assumptions on technology and preferences of a standard model of financial intermediation, similar to the model in Diamond and Dybvig (1983) and in Allen and Gale (2004).

The economy is inhabited by a unit mass of ex-ante identical consumers, each of which is indicated by j. Consumers live for three periods. In period 0, each consumer receives an endowment E and decides how to invest it. In period 1 and 2, she consumes.

There is one good in the economy, which can be consumed or invested. There are two investment technologies. The short-term investment technology gives one unit of the good after one period for each unit of goods invested. The long-term investment technology gives $R > 1$ goods after two periods for each unit of goods invested.

Consumers are endowed with preferences represented by utility function

$$U(C_{j,1}, C_{j,2}, \theta_j) = (1 - \theta_j) \cdot u(C_{j,1}) + \beta \cdot \theta_j \cdot u(C_{j,2}), \tag{2.1}$$

where the felicity function u complies with Inada conditions. The discount factor is restricted to $\beta > R^{-1}$. An additional necessary assumption is that the coefficient of relative risk aversion is everywhere weakly greater than 1:

$$\frac{-C \cdot u''(C)}{u'(C)} = \gamma(C) \geq 1 \quad \forall C > 0. \tag{2.2}$$

Risk aversion needs to be sufficiently high to generate a demand for liquidity-risk insurance. Random variable θ_j represents a liquidity shock and, as such, it is only known at time 1. For simplicity, it only takes on values 0 or 1. At time 0, consumers know the objective probability of the liquidity shock's realisations:

$$Pr(\theta_j) = \begin{cases} \phi & \text{if } \theta_j = 0, \\ 1 - \phi & \text{if } \theta_j = 1. \end{cases} \tag{2.3}$$

Consumers with $\theta_j = 0$ are hit by the liquidity shock. Throughout the paper, I refer to them as early types and to other consumers as late types. The liquidity shocks $\{\theta_j\}_{j \in [0,1]}$ are idiosyncratic. The share of consumers who will be hit by the liquidity shock is known at time 0. So, there is no aggregate risk.

2.3 Social Planner

The social planner observes the realisations of individual liquidity shocks. Hence, in her maximisation of welfare, the social planner is exclusively constrained by technology; not by incentive-compatibility considerations. It is useful to study the social planner's problem to derive a benchmark against which we can evaluate the efficiency of the equilibrium allocation in the decentralised economy. The social planner's allocation corresponds to the first-best efficient allocation.

The social planner faces a choice of non-negative type-contingent consumption paths $\{C_1(\theta_j), C_2(\theta_j)\}_{\theta_j \in [0,1]}$, investment in the short-term technology K_S and investment in the long-term technology K_L. She maximises aggregate welfare, given by

$$\int_0^1 \mathbb{E}_{\theta_j}\Big\{U\Big[C_{j,1}(\theta_j), C_{j,2}(\theta_j), \theta_j\Big]\Big\} \mathrm{d}j = \phi \cdot u[C_1(0)] + (1 - \phi) \cdot \beta \cdot u[C_2(1)]. \tag{2.4}$$

The maximisation is subject to resource constraints. In period 0, the social planner uses the aggregate endowment to invest in the short-term technology and in the long-term technology, according to

$$K_S + K_L = E. \tag{2.5}$$

In period 1, the output of short-term investments finance early consumption, as given by

$$\phi \cdot C_1(0) + (1-\phi) \cdot C_1(1) = K_S. \tag{2.6}$$

In the last time period, the social planner uses the proceeds of her long-term investments to finance time-2 consumption, according to

$$\phi \cdot C_2(0) + (1-\phi) \cdot C_2(1) = R \cdot K_L. \tag{2.7}$$

The social planner's allocation $\{C_1(\theta_j), C_2(\theta_j)\}_{\theta_j \in [0,1]}$ satisfies the following equations:

$$\phi \cdot C_1(0) + (1-\phi) \cdot \frac{C_2(1)}{R} = E, \tag{2.8}$$

$$\frac{u'[C_1(0)]}{u'[C_2(1)]} = \beta \cdot R, \tag{2.9}$$

$$C_1(1) = C_2(0) = 0. \tag{2.10}$$

These equations define the first-best allocation of the model.

Definition 6 (First-best efficient allocation)**.** *An allocation is first-best efficient if it satisfies equations (2.8), (2.9) and (2.10).*

Notice that definition 6 represents a high standard of efficiency. Incentive compatibility, an important restriction to risk-sharing in the decentralised economy, does not constrain the social planner.

2.4 Diamond-Dybvig Model with Hidden Trades

The model described in this section is known as the hidden-trade Diamond-Dybvig model. Its key feature is that consumers can trade one-period bonds at time 1 in an anonymous market. Since this bond market is anonymous, deposit contracts cannot restrict consumers' ability to trade in it; hence the name hidden trades. This model is used as a benchmark to study the possibility of coexistence of maturity-transforming banks with financial markets. The key result of the literature is that the banking system's provision of liquidity-risk insurance is curtailed by the existence of financial markets and therefore the equilibrium allocation of an economy where maturity-transforming banks coexist with financial markets is inefficient.

In the first part of this section, I describe the hidden-trade Diamond-Dybvig model without government intervention. I show that the model's competitive equilibrium allocation is inefficient. There is insufficient investment in short-term assets, which implies insufficient liquidity-risk insurance.

In the second part, I study government policies that can improve on the model's laissez-faire equilibrium allocation. I show that a reserve requirement on the banking system can implement first-best efficiency. This is the central result of Farhi et al. (2009). Moreover, I show that first-best efficiency can equivalently be implemented with the payment of positive interest on reserves.

2.4.1 No Government Intervention

Two unit masses of agents inhabit the economy: a unit mass of consumers indicated by j and a unit mass of banks indicated by k.

The only source of risk in the economy is idiosyncratic liquidity risk. However, though idiosyncratic, liquidity risk cannot be insured away with contingent bonds because of a friction: private observability of liquidity shocks. Consumer j's type, θ_j, is her own private information. Thus, other agents must rely on what consumer j reveals about the realisation of θ_j. Contingent bonds fail because, regardless of the true realisation, each consumer always has an incentive to claim she was hit by the liquidity shock in order to pocket the bond's payout. Banks emerge as a mechanism to insure liquidity risk.

At time 0, consumers choose how much to deposit in each bank k, $\{D_{j,k}\}_{k\in[0,1]}$. Obviously, the consumer will choose to deposit all her endowment in the bank that offers the best deposit rates $(d_{k,0}, d_{k,1})$. Also, at time 1 the consumer decides how much to withdraw from each deposit $\{W_{j,k}(\theta_i)\}_{k\in[0,1]}$ and how much to lend on the bond market $S_j(\theta_j)$ at price Q. These choices determine a type-contingent path for consumption $\{C_{j,1}(\theta_j), C_{j,2}(\theta_j)\}_{\theta_j \in \{0,1\}}$. The consumer maximises

$$\mathbb{E}[U(C_{j,1}, C_{j,2}, \theta_j)] = \phi \cdot u[C_{j,1}(0)] + (1-\phi) \cdot \beta \cdot u[C_{j,2}(1)], \tag{2.11}$$

subject to constraints

$$\int_0^1 D_{j,k} \, \mathrm{d}k = E, \tag{2.12}$$

$$C_{j,1}(\theta_j) + Q \cdot S_j(\theta_j) = \int_0^1 W_{j,k}(\theta_j) \, \mathrm{d}k, \tag{2.13}$$

$$C_{j,2}(\theta_j) = \int_0^1 (1 + d_{k,1}) \cdot [(1 + d_{k,0}) \cdot D_{j,k} - W_{j,k}(\theta_j)] \, \mathrm{d}k + S_j(\theta_j), \tag{2.14}$$

$$C_{j,t}(\theta_j) \geq 0 \quad \forall t, \theta_j. \tag{2.15}$$

Banks are profit-maximising firms that compete to attract deposits by offering a deposit contract, which specifies deposit rates (d_0, d_1). Banks know the demand for deposits and can anticipate consumers' withdrawing behaviour. So, before I lay out the banks' maximisation problem, I shall work out from the consumer's maximisation problem the demand for deposits and the consumers' withdrawing behaviour.

Regarding the consumers' withdrawing behaviour, early-type consumers withdraw everything early as long as they are not better off holding on to their deposits and borrowing in the bond market to finance their time-1 consumption. On the other hand, late types withdraw everything at time 2 as long as they are not better off withdrawing at time 1 and buying bonds. This can be formalised as

$$W_{j,k}(\theta_j) = \begin{cases} (1 + d_{k,0}) \cdot D_{j,k} & \text{if } 1 + d_{k,1} < Q^{-1}, \\ [0, (1 + d_{k,0}) \cdot D_{j,k}] & \text{if } 1 + d_{k,1} = Q^{-1}, \\ 0 & \text{if } 1 + d_{k,1} > Q^{-1}. \end{cases} \tag{2.16}$$

Notice that there can only be a separating equilibrium, in which consumers of different types withdraw in different time periods, in the indifferent case with $1 + d_{k,1} = Q^{-1}$.

Consumers deposit their endowment with the bank that offers the best deposit contract, $(d_{k,0}, d_{k,1})$. To formalise consumers' demand for deposits, I need to derive the consumers' valuation function for deposit contracts from the consumer's maximisation problem. This is given by

$$V_j(d_{k,0}, d_{k,1}) \equiv \phi \cdot u'[C_{j,1}(0)] \cdot (1 + d_{k,0}) \cdot \max\{1, (1 + d_{k,1}) \cdot Q\} + \\ + (1 - \phi) \cdot \beta \cdot u'[C_{j,2}(1)] \cdot (1 + d_{k,0}) \cdot \max\{1 + d_{k,1}, Q^{-1}\}. \tag{2.17}$$

Using this equation, it is easy to write the consumer's demand for deposits as

$$D_{j,k}(d_{k,0}, d_{k,1}) = \begin{cases} E & \text{if } V_j(d_{k,0}, d_{k,1}) > \max_{z \neq k}\{V_j(d_{z,0}, d_{z,1})\}, \\ [0, E] & \text{if } V_j(d_{k,0}, d_{k,1}) = \max_{z \neq k}\{V_j(d_{z,0}, d_{z,1})\}, \\ 0 & \text{if } V_j(d_{k,0}, d_{k,1}) < \max_{z \neq k}\{V_j(d_{z,0}, d_{z,1})\}. \end{cases} \tag{2.18}$$

This is a Bertrand demand correspondence, where demand is fully absorbed by the firm that offers the best conditions to consumers.

Taking as given the price of bonds Q and the deposit contracts offered by other banks $\{d_{z,0}, d_{z,1}\}_{z \neq k}$, bank k offers a deposit contract $(d_{k,0}, d_{k,1})$, which implies a quantity of deposits and of withdrawals from each consumer $\{D_{j,k},\, W_{j,k}(\theta_j)\}_{j \in [0,1]}$, and makes a portfolio decision $(K_{k,S}, K_{k,L})$ in order to maximise profits Π_k.[2] The constraints faced by banks are:

$$K_{k,S} + K_{k,L} + \Pi_k = \int_0^1 D_{j,k}\,\mathrm{d}j, \tag{2.19}$$

$$\int_0^1 W_{j,k}(\theta_j)\,\mathrm{d}j = K_{k,S}, \tag{2.20}$$

$$(1 + d_{k,1}) \cdot \left[(1 + d_{k,0}) \cdot \int_0^1 D_{j,k}\,\mathrm{d}j - \int_0^1 W_{j,k}(\theta_j)\,\mathrm{d}j\right] = R \cdot K_{k,L}, \tag{2.21}$$

$$(K_{k,S}, K_{k,L}) \geq 0. \tag{2.22}$$

In equilibrium, consumers maximise their expected utility subject to budget constraints, banks offer deposit contracts to maximise their profits and markets clear. I formalise the equilibrium in the Diamond-Dybvig economy with hidden trade as follows.

Definition 7. *The equilibrium consists of a vector of quantities and a vector of prices, respectively given by* $\left\{D_{j,k}, S_j(\theta_j), C_{j,1}(\theta_j), C_{j,2}(\theta_j), W_{j,k}(\theta_j), K_{k,S}, K_{k,L}\right\}_{(j,k,\theta_j)}$ *and* $\left(Q, \{d_{k,0}, d_{k,1}\}_{k \in [0,1]}\right)$*, such that:*

[2] In principle, profits are distributed to consumers, who own the banks. I do not specify this because bank profits turn out to be 0 in equilibrium.

1. *Consumer j chooses quantities* $\left\{D_{j,k}, S_j(\theta_j), C_{j,1}(\theta_j), C_{j,2}(\theta_j), W_{j,k}(\theta_j)\right\}_{(k,\theta_j)}$ *to maximise (2.11) subject to constraints (2.12), (2.13), (2.14) and (2.15).*

2. *Bank k chooses quantities* $\left(K_{k,S}, K_{k,L}, \{D_{j,k}, W_{j,k}(\theta_j)\}_{(j,\theta_j)}\right)$ *and a deposit contract* $(d_{k,0}, d_{k,1})$ *to maximise profits* Π_k *subject to constraints (2.16), (2.18), (2.19), (2.20), (2.21) and (2.22).*

3. *The bond market clears:* $\int_{j=0}^{1} S_j(\theta_j)\,\mathrm{d}j = 0$.

Key to the provision of liquidity-risk insurance by the banking system is the equilibrium price of bonds in the hidden market, Q. A high rate of return on bonds increases the incentive of those consumers who are not hit by the liquidity shock to withdraw early. This makes it harder for banks to separate early-type consumers from late-type ones and therefore reduces the scope for providing liquidity-risk insurance.

Following a large literature (Jacklin, 1987; Allen and Gale, 2004; Farhi et al., 2009), I find that without government intervention the equilibrium return on bonds is equal to the marginal rate of transformation of the economy between time 1 and time 2: R.

Lemma 5. *In the equilibrium of the Diamond-Dybvig economy with hidden trades, the price of bonds traded at time 1 in the anonymous market is given by*

$$Q = \frac{1}{R} \tag{2.23}$$

and the prevailing deposit contract is given by

$$(d_0, d_1) = (0, R-1). \tag{2.24}$$

Proof. Please refer to appendix 2.A. □

The logic is that, if the return in the bond market is greater than the return that banks can make on their assets between time 1 and time 2, then the best deposit contract from the consumer's viewpoint pays off at time 1, regardless of the consumer's type realisation, and thus allows the consumer to exploit the high return in the bond market. At price $Q < \frac{1}{R}$ all consumers want to lend in the bonds market and none wants to borrow. This is impossible in equilibrium.

Similarly, if the interest rate on the bond market is lower than R. The best deposit contract will be such that all consumers hold off withdrawing until time 2. Those consumers who want to consume at time 1 should just take advantage of the low interest rate by borrowing in the bond market. However, more borrowing than lending in the bond market is of course not possible in equilibrium.

Strikingly, it turns out that without government intervention the equilibrium price of bonds is so low that banks are unable to provide any liquidity-risk insurance at all. The allocation in equilibrium corresponds to what the literature on maturity transformation calls autarky: the allocation that would be achieved if consumers did not trade at all.

Proposition 3. *The equilibrium allocation of the Diamond-Dybvig model with hidden trades satisfies equations (2.8), (2.10) and*

$$\frac{C_2(1)}{C_1(0)} = R. \tag{2.25}$$

It is not first best.

Proof. Please refer to appendix 2.A. □

Consumers who are hit by the liquidity shock end up consuming too little relative to late-type consumers. Ex-ante, all consumers would have gained from sharing this risk. The provision of liquidity-risk insurance by banks, which implies investing in short-term assets and paying an above-market return to those depositors who withdraw early, is not profitable because it gives an incentive to late-type consumers not to hold on to their deposits at time 1 but to withdraw early too.

2.4.2 Interest on Reserves and Reserve Requirement

In this section, I study the effects of government interventions on the model's equilibrium allocation. In particular, I analyse whether appropriate monetary policy can implement the first-best allocation.

I introduce bank reserves that pay interest and a reserve requirement. The former means that the government issues reserves, B, a short-term asset that pays an interest rate $i \geq 0$. The latter policy measure is a requirement imposed by the government on banks to hold a minimum share of their asset portfolios in reserves.

The problem that the government is trying to solve is that banks do not have an incentive to invest a sufficient share of their portfolio in short-term assets and thus to provide the socially optimal amount of liquidity-risk insurance. In equilibrium, too little investment is directed to short-term assets. A reserve requirement compels banks to hold more short-term assets and interest on reserves gives banks incentive to hold more short-term assets.

Importantly, neither of the two proposed policy interventions require an informational advantage for the government in order to be implemented. In particular, the government does not need knowledge of the distribution of types.

The government supplies reserves B and promises to pay on them interest rate i. I assume that it invests in short-term capital, $K_{G,S}$, and uses lumpsum taxes, T, on consumers to balance its budget constraints:

$$K_{g,S} = B_g + T, \tag{2.26}$$

$$(1+i) \cdot B_g = K_{g,S}. \tag{2.27}$$

I formalise a reserve requirement adding a constraint to the bank's profit maximisation problem:

$$B_k \geq \rho \cdot \int_0^1 D_{j,k} \, \mathrm{d}j. \tag{2.28}$$

Since $i \geq 0$, banks weakly prefer holding bank reserves to short-term capital. Thus, the results would be exactly identical if I allowed banks to satisfy the requirement with other short-term assets. Banks would still choose to hold reserves.

The equilibrium with government intervention is different from the equilibrium defined in the previous subsection, because (1) there is a government that supplies reserves by buying other short-term assets, (2) consumers are subject to lumpsum taxation, (3) banks hold reserves and are subject to a reserve requirement. A formalisation of the equilibrium is given by

Definition 8. *Given policy instruments $i \geq 0$ and $\rho \in [0,1]$, the equilibrium consists of* $\left\{D_{j,k}, S_j(\theta_j), C_{j,1}(\theta_j), C_{j,2}(\theta_j), W_{j,k}(\theta_j), B_k, K_{k,S}, K_{k,L}, K_{g,S}, T, B_g\right\}_{(j,k,\theta_j)}$, *a vector of* *quantities, and* $\left(Q, \{d_{k,0}, d_{k,1}\}_{k\in[0,1]}\right)$*, a vector of prices, such that:*

1. *Consumer* j *chooses quantities* $\left\{D_{j,k}, S_j(\theta_j), C_{j,1}(\theta_j), C_{j,2}(\theta_j), W_{j,k}(\theta_j)\right\}_{(k,\theta_j)}$ *to maximise (2.11) subject to constraints*

$$\int_{k=0}^{1} D_{j,k}\,\mathrm{d}k = E - T, \tag{2.29}$$

   (2.13), (2.14) and (2.15).

2. *Bank* k *chooses quantities* $\left(B_k, K_{k,S}, K_{k,L}, \{D_{j,k}, W_{j,k}(\theta_j)\}_{(j,\theta_j)}\right)$ *and a deposit contract* $(d_{k,0}, d_{k,1})$ *to maximise profits* Π_k *subject to constraints (2.16), (2.18), (2.28),*

$$B_k + K_{k,S} + K_{k,L} + \Pi_k = \int_{j=0}^{1} D_{j,k}\,\mathrm{d}j, \tag{2.30}$$

$$\int_{j=0}^{1} W_{j,k}(\theta_j)\,\mathrm{d}j = K_{k,S} + (1+i)\cdot B_k, \tag{2.31}$$

   *(**??**) and*

$$(B_k, K_{k,S}, K_{k,L}) \geq 0. \tag{2.32}$$

3. $(K_{g,S}, T, B_g)$ *are such that the government's budget constraints (2.26) and (2.27) hold.*

4. *The bond market clears:*

$$\int_{j=0}^{1} S_j(\theta_j)\,\mathrm{d}j = 0. \tag{2.33}$$

5. *The market for reserves clears:* $\int_{k=0}^{1} B_k\,\mathrm{d}k = B_g$*.*

The impact of the policy instruments on the interest rate prevailing in the bonds market at time 1 is key. Remember that a high interest rate in the bonds market makes banks ineffective at providing liquidity-risk insurance, since it gives an incentive to all depositors, regardless of their type, to withdraw their deposits early. Interest on reserves and the reserve requirement succeed in reducing returns in the bond market by increasing investment in the short-term asset and therefore the quantity of resources available at time 1.

Lemma 6. *In the equilibrium of the hidden-trade Diamond-Dybvig economy with reserve requirement and interest on reserves, the price of bonds traded at time 1 in the anonymous market is given by*

$$Q = \frac{(1+i)\cdot \max\left\{1, \frac{(1-\phi)\cdot\rho}{\phi\cdot(1-\rho)}\right\}}{R}. \tag{2.34}$$

and the prevailing deposit contract is given by:

$$1 + d_0 = \max\left\{1, \frac{\rho}{\phi}\right\} \cdot (1+i), \tag{2.35}$$

$$1 + d_1 = \frac{R}{(1+i)\cdot \max\left\{1, \frac{(1-\phi)\cdot\rho}{\phi\cdot(1-\rho)}\right\}}. \tag{2.36}$$

Proof. Please refer to appendix 2.A. □

If $i = 0$ and $\rho \leq \phi$, then the price of bonds, Q, is the same as in the economy without government intervention. This is because reserves with $i = 0$ are perfect substitutes of short-term capital and the reserve requirement is not binding as long as $\rho \leq \phi$. Increasing the interest on reserves to above 0 or tightening the reserve requirement, setting $\rho > \phi$, have the effect of increasing the price of bonds above the laissez-faire level. As a consequence, these policies support the provision of liquidity-risk insurance.

Lemma 7. *The equilibrium allocation of the hidden-trade Diamond-Dybvig economy with a reserve requirement and interest on reserves satisfies equations (2.8), (2.10) and*

$$\frac{C_2(1)}{C_1(0)} = \frac{R}{(1+i)\cdot \max\left\{1, \frac{(1-\phi)\cdot\rho}{\phi\cdot(1-\rho)}\right\}}. \tag{2.37}$$

Proof. Please refer to appendix 2.A. □

Comparing the system of equations in lemma 7 with the first-best efficient allocation in definition 6, it is obvious that the government can make the economy fully efficient using its policy instruments. It is interesting to look at the combinations of interest on reserves and reserve requirement that achieve this, as this gives an insight in the relationship between the two policy instruments.

Proposition 4. *Define $h(i,\rho) = 0$ as the locus of policies with $i \geq 0$ and $\rho \in [0,1]$ that implement the first-best efficient allocation in the hidden-trade Diamond-Dybvig economy. We have that:*

1. *There exist policies (i,ρ) with $i \geq 0$ and $\rho \in [0,1]$ such that $h(i,\rho) = 0$.*

2. *There exists $\hat{\rho} \in [0,1]$ such that $h(0,\hat{\rho}) = 0$. $\hat{\rho} > \phi$.*

3. *There exists $\hat{i} \geq 0$ such that $h(\hat{i},0) = 0$. $\hat{i} > 0$.*

4. *On the domain $i \geq 0$ and $\rho \in [0,1]$, $\left.\frac{\mathrm{d}\rho}{\mathrm{d}i}\right|_{h(i,\rho)=0} < 0$.*

From the perspective of the government, interest on reserves and minimum reserve ratios are perfectly substitutable policies. If the government pays zero interest on reserves, there is a level of the reserve requirement that implements full efficiency. Conversely, in the absence of a reserve requirement, the government can implement optimality by paying a sufficiently high interest rate. Even only focusing on policies that use both instruments, if the reserve requirement is reduced, it is always possible to implement the optimal allocation by increasing the interest on reserves, and viceversa. In other words, it is unnecessary for the government to have both policy instruments at its disposal. It could forgo the use of either interest on reserves or the minimum reserve ratio and achieve the same level of welfare in the economy.

In this section, I replicated Farhi et al. (2009)'s result, according to which a minimum reserve ratio obtains the first-best allocation in the hidden-trade Diamond-Dybvig model. In addition, I showed that a policy of paying interest on reserves can equally implement full efficiency. In the next section, I extend the model making the level of intermediation of savings endogenous. I show that, if consumers can decide how much of their savings to deposit and how much to invest directly, the two policy instruments have different effects. It turns out that a reserve requirement is ineffective at promoting liquidity-risk insurance, because it leads to financial disintermediation. On the other hand, with interest on reserves the government can still implement the first best.

2.5 Endogenous Intermediation

As shown in the previous section, a reserve requirement or the payment of positive interest on bank reserves can equally implement the first-best allocation in the hidden-trade Diamond-Dybvig model. The fundamental problem that these policy measures solve is insufficient investment in short-term assets. The two policies fix this problem in different ways. A reserve requirement is a mandate for banks to hold a minimum amount of short-term assets and interest on reserves provides an incentive to hold more short-term assets. In effect, the two policies are equivalent: they make banks hold more short-term assets. Indeed, from the policymaker's prospective the policies are perfectly substitutable.

In this section, I show that the equivalence of interest on reserves and reserve requirement depends on an assumption. In the hidden-trade Diamond-Dybvig model, consumers must invest their entire endowment in bank deposits. They cannot invest directly in capital assets at time 0. In other words, full intermediation of consumers' savings through the banking system is imposed exogenously. I relax this assumption allowing consumers to choose the level of intermediation of their savings.

Compared to the hidden-trade Diamond-Dybvig model, the consumer's time-0 portfolio decision becomes more complicated. In addition to deposits $D_{j,k}$, she can choose to save directly in long-term capital $K_{j,L}$, as shown in the time-0 budget constraint:

$$\int_0^1 D_{j,k}\,\mathrm{d}k + K_{j,L} = E - T. \tag{2.38}$$

T are lumpsum taxes levied by the government. I restrict the consumer's portfolio allocation to long-term capital and bank deposits for simplicity. The same results would go through if I allowed the consumer to also hold short-term capital and reserves directly. This is because bank deposits are always a good short-term investment relative to alternatives.

The consumer's other two flow budget constraints are given by equation (2.13) and

$$C_{j,2}(\theta_j) = \int_0^1 (1 + d_{k,1}) \cdot [(1 + d_{k,0}) \cdot D_{j,k} - W_{j,k}(\theta_j)] \, \mathrm{d}k + S_j(\theta_j) + K_{j,L}. \tag{2.39}$$

I derive the consumer's demand for deposits from the optimisation problem. The key novelty in this version of the model is that consumers have an outside option: they also can hold long-term capital. So, the consumer will only hold deposits if she does not strictly prefer capital assets. Hence, in order to write the demand for deposits, I need to define a valuation function for long-term capital, which allows me to compare the benefits of investing in a bank deposit, as given by equation (2.17), with the benefits of direct capital investment. The value expressed in expected utility terms of a unit of the long-term asset at time 0 for a consumer j is given by:

$$V_{j,L} \equiv \phi \cdot u'[C_{j,1}(0)] \cdot R \cdot Q + (1 - \phi) \cdot u'[C_{j,2}(1)] \cdot R. \tag{2.40}$$

Using equations (2.17) and (2.40), I can write the demand correspondence for deposits as

$$D_{j,k}(d_{k,0}, d_{k,1}) = \begin{cases} E - T & \text{if } V_j(d_{k,0}, d_{k,1}) > \max_{z \neq k} \left\{ V_j(d_{z,0}, d_{z,1}), V_{j,L} \right\}, \\ [0, E - T] & \text{if } V_j(d_{k,0}, d_{k,1}) = \max_{z \neq k} \left\{ V_j(d_{z,0}, d_{z,1}), V_{j,L} \right\}, \\ 0 & \text{if } V_j(d_{k,0}, d_{k,1}) < \max_{z \neq k} \left\{ V_j(d_{z,0}, d_{z,1}), V_{j,L} \right\}. \end{cases} \tag{2.41}$$

A consumer decides to hold all of her wealth in a deposit account if that is strictly the best investment for her. If another investment strictly dominates the deposit account, then the consumer will choose not to hold any of her wealth in the bank deposit. The consumer holds a mixed portfolio if she is indifferent between assets.

The definition of equilibrium in the hidden-trade Diamond-Dybvig model with endogenous intermediation is as follows.

Definition 9. *Given policy instruments $i \geq 0$ and $\rho \in [0,1]$, the equilibrium consists of* $\left\{ D_{j,k}, K_{j,L}, S_j(\theta_j), C_{j,1}(\theta_j), C_{j,2}(\theta_j), W_{j,k}(\theta_j), B_k, K_{k,S}, K_{k,L}, K_{g,S}, T, B_g \right\}_{(j,k,\theta_j)}$*, a vector of quantities, and* $\left(Q, \{d_{k,0}, d_{k,1}\}_{k \in [0,1]} \right)$*, a vector of prices, such that:*

1. *Consumer i chooses quantities* $\left\{ D_{j,k}, K_{j,L}, S_j(\theta_j), C_{j,1}(\theta_j), C_{j,2}(\theta_j), W_{j,k}(\theta_j) \right\}_{(k,\theta_j)}$ *to maximise (2.11) subject to constraints (2.38), (2.13), (2.39),*

$$(D_{j,k}, K_{j,L}) \geq 0 \quad \forall k \tag{2.42}$$

and (2.15).

2. *Bank k chooses quantities* $\left(B_k, K_{k,S}, K_{k,L}, \{D_{j,k}, W_{j,k}(\theta_j)\}_{(j,\theta_j)}\right)$ *and a deposit contract given by* $(d_{k,0}, d_{k,1})$ *to maximise profits* Π_k *subject to constraints (2.16), (2.41), (2.28), (2.30), (2.31), (2.21) and (2.32).*

3. $(K_{g,S}, T, B_g)$ *are such that the government's budget constraints (2.26) and (2.27) hold.*

4. *The bond market clears:* $\int_0^1 S_j(\theta_j)\,\mathrm{d}j = 0.$

5. *The market for reserves clears:* $\int_0^1 B_k\,\mathrm{d}k = B_g.$

Policies are effective at promoting liquidity-risk insurance insofar as they push up the price of bonds at time 1, Q. A low level of Q represents an incentive for late-type consumers to withdraw deposits early and reinvest. This incentive curtails banks' ability to provide liquidity-risk insurance, because it makes it impossible to separate early types, who have liquidity needs, from late types.

When consumers are allowed to disintermediate, I find that a reserve requirement, regardless of the level at which it is set, has no effect at all on price Q. Only a policy of paying interest on reserves can put upward pressure on Q and therefore reduce the return from investing in the bonds market at time 1.

Lemma 8. *In the equilibrium of the hidden-trade Diamond-Dybvig economy with endogenous intermediation, a reserve requirement and interest on reserves, the price of bonds traded at time 1 in the anonymous market is given by*

$$Q = \frac{1+i}{R}. \tag{2.43}$$

and the prevailing deposit contract is given by:

$$d_0 = i, \tag{2.44}$$

$$1 + d_1 = \frac{R}{1+i}. \tag{2.45}$$

Proof. Please refer to appendix 2.A. □

In the economy without government intervention, there is too little investment in short-term assets. Reserve requirements and interest on reserves attempt to fix this inefficiency in different ways, the former forces banks to invest more in short-term assets and the latter provides an incentive to invest more in short-term assets. However, in a model where intermediation is endogenous and therefore consumers react to an effective taxation of bank deposits by depositing less, a reserve requirement only has the effect of prodding consumers to disintermediate and invest directly in long-term assets. As a consequence, the reserve requirement does not change the economy's portfolio of short-term and long-term investment. On the other hand, interest on reserves provides an incentive to invest in short-term assets that succeeds at shifting the economy's investment portfolio towards short-term assets. In conclusion, interest on reserves promotes liquidity-risk insurance, while a reserve requirement does not.

Lemma 9. *The equilibrium allocation of the hidden-trade Diamond-Dybvig economy with endogenous intermediation, reserve requirements and interest on reserves satisfies equations (2.8), (2.10) and*

$$\frac{C_2(1)}{C_1(0)} = \frac{R}{1+i}. \tag{2.46}$$

Proof. Please refer to appendix 2.A. □

Clearly, by using the interest on reserves the government can implement the efficient allocation.

Proposition 5. *For any $\rho \in [0,1]$, there exists a unique optimal interest on reserves $i = i^* > 0$ that implements the first-best allocation in the equilibrium of the hidden-trade Diamond-Dybvig model with endogenous intermediation.*

While the reserve requirement does not have an impact on risk sharing in equilibrium, interest on reserves promotes liquidity-risk insurance in the economy. The level of interest on reserves that implements full efficiency in the economy is always strictly positive.

To study in greater detail how the optimal interest on reserves, i^*, varies with the model's parameters, I choose to restrict the utility function to feature constant relative risk aversion.

Proposition 6. *Consider utility function* $u(C) = \frac{C^{1-\gamma}-1}{1-\gamma}$*, with* $\gamma \geq 1$ *coefficient of relative risk aversion. The resulting formula for the optimal interest on reserves is given by*

$$\ln(1+i^*) = \ln(R) - \frac{1}{\gamma} \cdot \ln(\beta \cdot R). \tag{2.47}$$

Properties of the optimal interest on reserves are that:

1. *The optimal interest on reserves is non-decreasing in the return on long-term investments R, with* $\frac{\partial \ln(1+i^*)}{\partial \ln(R)} = \frac{\gamma-1}{\gamma} \geq 0$.
2. *The optimal interest on reserves is strictly decreasing in the consumers' discount factor* β*, with* $\frac{\partial \ln(1+i^*)}{\partial \ln(\beta)} = -\frac{1}{\gamma} < 0$.
3. *The optimal interest on reserves is increasing in the consumers' coefficient of relative risk aversion at a decreasing rate* γ*, with* $\frac{\partial \ln(1+i^*)}{\partial \gamma} = \frac{\ln(\beta \cdot R)}{\gamma^2} > 0$ *and* $\frac{\partial^2 \ln(1+i^*)}{\partial \gamma^2} = -2 \cdot \frac{\ln(\beta \cdot R)}{\gamma^3} < 0$.

Since the rationale for paying positive interest on reserves is to improve liquidity-risk sharing, it stands to reason that parametrisations that increase demand for such insurance, i.e. a low discount factor β and a high coefficient of relative risk aversion γ, call for a higher interest on reserves. A higher return on long-term assets R implies a higher optimal interest on reserves, too. Nonetheless, an optimally set interest rate on reserves is never greater than the long-term rate of return in the economy: $1+i^*$ tends to R as relative risk aversion tends to infinity.

A reserve requirement, the policy measure proposed by Farhi et al. (2009) to deal with insufficient liquidity-risk insurance, is ineffective in a model where consumers can choose to invest directly in capital in addition to depositing their wealth in banks. It causes consumers to disintermediate and invest directly in long-term assets, since banks are effectively taxed by the obligation to hold a suboptimal asset portfolio and this lowers the returns of bank deposits.

Interest on reserves promotes liquidity-risk insurance in a Diamond-Dybig model in which financial markets coexist alongside the banking sector and the level of financial intermediation is endogenous. By means of an appropriately set interest on reserves, the government can implement the first-best allocation in equilibrium.

2.6 Fiscal Consequences

Offering a positive interest on reserves requires the government to levy taxes, because a positive interest on reserves is a subsidy on short-term assets in this model. Using the government's budget constraints (2.26) and (2.27), we can see that

$$T = i \cdot B_g. \tag{2.48}$$

Notice that, since i is weakly positive, I do not consider negative taxes, i.e. transfers, in this section.[3]

In the previous sections, I assumed that taxes do not distort any of the decisions made in the economy. A more serious analysis of the fiscal consequences of policies should allow for taxes to reduce the incentives to produce and thus generate a deadweight loss. Accordingly, I postulate that the endowment depends inversely on the level of taxation, defining a function as follows.

Definition 10. *Define a twice-continously differentiable function $E : \mathbb{R} \to \mathbb{R}_+$ that maps taxes, T, into endowment, E. The function is characterised by:*

1. $\frac{\partial E(T)}{\partial T} \leq 0 \quad \forall\, T,$
2. $\frac{\partial^2 E(T)}{\partial T^2} \leq 0 \quad \forall\, T.$

The definition of the endowment function nests the case of non-distortionary taxation, which is the assumption maintained so far in the paper. Non-distortionary taxation corresponds to a function with first-order derivative equal to zero at all levels of taxation.

In the rest of this section, I study optimal monetary policy in a model with deadweight loss due to taxation. I organise the section in two subsections, the first dedicated to the hidden-trade Diamond-Dybvig model with exogenous intermediation and the second to the model with endogenous intermediation.

[3] At negative values of i, there would be no demand for reserves. Hence, taxes would be zero.

2.6.1 Exogenous Intermediation

In the hidden-trade Diamond-Dybvig model with exogenous intermediation, consumers can only save through the banking system at time 0. The model is described in section 2.4.2 of this paper. In this analysis, I let the endowment depend on the level of taxes according to the function in definition 10.

As according to equation (2.48), the level of taxation that the government needs depends on the demand for reserves and on the interest paid on them. In turn, the demand for reserves is increasing in the endowment, in the probability ϕ of being hit by the liquidity shock and in the reserve requirement ρ. In equilibrium, taxes are given by:

$$T = \frac{\max\{\phi, \rho\} \cdot i}{1 + \max\{\phi, \rho\} \cdot i} \cdot E(T) \tag{2.49}$$

While a strictly positive interest rate per se implies a strictly positive amount of taxation, any level of the reserve requirement is compatible with zero taxes as long as the interest on reserves is zero.

Proposition 7. *Consider $E(T)$ with $\frac{\partial E(T)}{\partial T} < 0$ for all T. There exists a unique policy (i, ρ) with $i = 0$ and $\rho = \rho^* \in (\phi, 1)$ that implements the first-best efficient allocation in the equilibrium of the hidden-trade Diamond-Dybvig model with exogenous intermediation and distortionary taxation.*

In the case, illustrated in section 2.4.2, of non-distortionary taxation, I found that interest on reserves and a reserve requirement are perfectly substitutable policy instruments from the prospective of welfare-maximising policy makers.

Distortionary taxation breaks the equivalence of the policy instruments in this model. Welfare-maximising policy makers should avoid policies that require taxes to be sustained. Since interest on reserves makes taxation necessary whereas a reserve requirement does not, a reserve requirement is a superior instrument to facilitate liquidity-risk insurance and implement the first-best efficient allocation.

2.6.2 Endogenous Intermediation

In the hidden-trade Diamond-Dybvig model with endogenous intermediation, consumers can save at time 0 through financial intermediaries or directly in financial markets. The model is described in section 2.5 of this paper. In this subsection, taxes generate a deadweight loss by reducing the endowment, as described in definition 10.

Unlike in the model with exogenous intermediation, demand for bank reserves does not depend on the reserve requirement when consumers can decide how much of their savings to deposit. An increase in the reserve requirement makes banks hold more of their assets as reserves but also makes consumers hold less of the their wealth in bank deposits. It turns out that the two effects cancel out. Thus, the level of taxation is given by

$$T = \frac{\phi \cdot i}{1 + \phi \cdot i} \cdot E(T). \tag{2.50}$$

A reserve requirement does not require taxation to be imposed. However, in this model a reserve requirement also has no effect on the overall level of investment in short-term assets. Therefore, it is useless in terms of improving liquidity-risk insurance in the economy. To this aim, the government can use interest on reserves effectively. It faces a trade-off in doing so though, because interest on reserves requires taxation.

Since policy makers cannot in general obtain the first-best efficient allocation, I define the optimal monetary policy problem.

Definition 11. *Optimal policy (i, ρ) in the hidden-trade Diamond-Dybvig model with endogenous intermediation and distortionary taxation maximises aggregate welfare, given by (2.11), subject to variables being determined in equilibrium, as described in definition 9, with additional endogenous variable E determined by*

$$E = E(T). \tag{2.51}$$

A welfare-maximising policy maker would choose (i, ρ) to solve the optimal policy problem, as defined above. The result of such optimisation follows.

Lemma 10. *Optimal policy (i, ρ) in the hidden-trade Diamond-Dybvig model with endogenous intermediation and distortionary taxation implements the allocation given by*

$$\frac{u'[C_1(0)]}{u'[C_2(1)]} = \beta \cdot R \cdot \min\left\{ R^{\gamma[C_2(1)]-1}, 1 - \frac{\left\{\phi \cdot \left(\frac{R}{1+i}\right)^{\gamma[C_2(1)]} + 1 - \phi\right\} \cdot E'(T)}{(1-\phi) \cdot \{1 + [1 - E'(T)] \cdot \phi \cdot i\}} \right\}, \tag{2.52}$$

$$\phi \cdot C_1(0) + (1-\phi) \cdot \frac{C_2(1)}{R} = E(T), \tag{2.53}$$

and equation (2.46).

Proof. Please refer to appendix 2.A. □

From equation (2.52), we can see that, if taxation is distortionary, a welfare-maximising government reduces its use of interest on reserves to support liquidity-risk insurance.

Proposition 8. *If $E'(T) < 0$ for all T, then the optimal allocation in the hidden-trade Diamond-Dybvig model with endogenous intermediation is not first-best efficient.*

In the model with endogenous intermediation, the only way for the government to promote liquidity-risk insurance, which otherwise is inefficiently low, is to pay interest on reserves. However, interest on reserves is a policy that needs to be financed with taxes. Hence, if taxation creates distortions, the government should implement a less than first-best level of liquidity-risk insurance in order to reduce the inefficiencies created by taxes.

2.7 Conclusion

The existence of financial markets in the Diamond-Dybvig model reduces the scope for banks to provide liquidity-risk insurance, as shown by Jacklin (1987). The equilibrium allocation features inefficiently low aggregate investment in short-term assets.

Farhi et al. (2009) conclude that the inefficiency can be fixed by imposing a reserve requirement on the banking system. This forces banks to invest more in short-term assets and therefore increases the economy-wide level of investment in short-term assets. My first result is that within the Diamond-Dybvig framework adopted by Farhi et al. (2009) a policy of paying positive interest on reserves can achieve the same allocation as a reserve requirement. What a reserve requirement achieves with regulation, interest on reserves achieves by providing an incentive to invest more in short-term assets.

The Diamond-Dybvig model restricts consumers to investing their wealth solely in bank deposits. In the central section of this paper, I relax this assumption and allow consumers to also invest directly in financial markets. In this more general set-up, I find that a reserve requirement is unable to increase aggregate investment in short-term assets. While it does increase the share of the banks' asset portfolio invested in short-term assets, it also makes consumers decide to hold fewer deposits, since it acts as a tax on bank deposits. It turns out that the two effects, the increase in the share of bank investment in short-term assets and consumer disintermediation, cancel out. Hence, a reserve requirement has no effect on the aggregate level of short-term assets. On the other hand, the incentive to invest more in short-term assets provided by the payment of interest on bank reserves effectively increases aggregate investment in short-term assets, as it does not discourage financial intermediation.

The paper's key conclusion is that interest on reserves is effective at promoting liquidity-risk insurance. It is a superior policy to the setting of a reserve requirement, because it does not lead to financial disintermediation.

In the last section of the paper, I study the fiscal consequences of interest on reserves. Paying interest on reserves is costly for the government and must therefore be financed with taxation. I postulate an inverse relation between the endowment and taxes, justified by the presence of a deadweight loss associated with distortionary taxes. If consumers cannot disintermediate their wealth, then the government finds it optimal to impose a reserve requirement and not to pay interest on reserves, because a reserve requirement does not require taxes. On the other hand, if intermediation is endogenous, as discussed above, a reserve requirement is ineffective at promoting liquidity-risk insurance. Thus, it is optimal for the government to pay interest on reserves. However, payment of interest on reserves involves a policy trade-off for the government: a higher interest on reserves improves liquidity-risk insurance but reduces output via the distorting effects of taxes. Ultimately, I find that the more distorting taxes are, the lower the interest on reserves should be.

Studying optimal interest-rate setting and its relationship with the payment of interest on reserves and liquidity would be interesting. However, this paper's model is too simple to study interest-rate setting. Crucially, there is no meaningful saving decision taken by consumers at time 0. The necessary elements to study interest-rate setting in a Diamond-Dybvig model can be found in Porcellacchia (2018). I leave this extension to future work.

Since in reality banks do perform maturity transformation and financial markets exist, the merit of interest on reserves should be studied in a setting where banks have scope to provide liquidity-risk insurance despite their coexistence with financial markets. Diamond (1997) is an important example of a model with such characteristics.

The fragility of deposit contracts is a key result of the Diamond-Dybvig model, which this paper treats as orthogonal to the monetary policies analysed. This is because a financial crisis is the model's bad equilibrium, which simple deposit insurance can prevent from ever taking place. Once deposits are insured, there is no relationship between monetary policy and bank runs. Global games, developed in Carlsson and van Damme (1993), deliver a unique equilibrium in which financial crises take place with non-zero probability. Hence, global games hold promise to shed light on the relationship between monetary-policy instruments, such as interest on reserves, and the probability of financial crises.

Appendix

2.A Proofs

Lemma 5. *In the equilibrium of the Diamond-Dybvig economy with hidden trades, the price of bonds traded at time 1 in the anonymous market is given by*

$$Q = \frac{1}{R} \tag{2.23}$$

and the prevailing deposit contract is given by

$$(d_0, d_1) = (0, R - 1). \tag{2.24}$$

Proof. Define $\zeta = \frac{\phi \cdot W(0) + (1-\phi) \cdot W(1)}{(1+d_0) \cdot E}$, the share of deposits withdrawn at time 1. The market clearing condition in the hidden bond market

$$\phi \cdot S(0) + (1 - \phi) \cdot S(1) = 0, \tag{2.54}$$

the consumer's budget constraints (2.12), (2.13) and (2.14), and the fact that $C_2(0) = C_1(1) = 0$ imply that in equilibrium

$$\zeta = \phi. \tag{2.55}$$

By the withdrawing behaviour equation (2.16), then it must be that in equilibrium $1 + d_1 = Q^{-1}$. Otherwise, all consumers would either not withdraw at all at time 1, so that $\zeta = 0$, or withdraw all their deposits, so that $\zeta = 1$.

Banks offer competitively deposit contracts (d_0, d_1), which imply a withdrawing behaviour ζ. Banks make zero profits because of Bertrand-style competition. By the bank's budget constraints (2.19), (2.20) and (2.21), this implies that

$$1 + d_0 = \frac{R \cdot Q}{\zeta \cdot R \cdot Q + 1 - \zeta}. \tag{2.56}$$

A bank can deviate and earn strictly positive profits unless in equilibrium the deposit contract on offer solves

$$\max_{\zeta \in [0,1]} \frac{1}{Q} \cdot \Big\{\phi \cdot u'[C_1(0)] + (1 - \phi) \cdot \beta u'[C_2(1)]\Big\} \cdot \frac{R \cdot Q}{\zeta \cdot R \cdot Q + 1 - \zeta}. \tag{2.57}$$

Note that banks can choose the timing of depositor withdrawal because $1+d_1 = Q^{-1}$ makes consumers indifferent.

The solution of the above is given by the deposit contract which gives

$$\zeta = \begin{cases} 0 & \text{if } R \cdot Q > 1, \\ [0,1] & \text{if } R \cdot Q = 1, \\ 1 & \text{if } R \cdot Q < 1. \end{cases} \tag{2.58}$$

Since, as shown above, $\zeta = \phi$ in equilibrium, we have that

$$Q = \frac{1}{R}. \tag{2.59}$$

□

Proposition 3. *The equilibrium allocation of the Diamond-Dybvig model with hidden trades satisfies equations (2.8), (2.10) and*

$$\frac{C_2(1)}{C_1(0)} = R. \tag{2.25}$$

It is not first best.

Proof. With the budget constraints (2.12), (2.13), (2.14) and the equilibrium prices given in lemma 5, we find that the equilibrium allocation satisfies equations (2.8), (2.10) and (2.25).

Now, given a coefficient of relative risk aversion larger than 1 as implied by restriction (2.2) on the felicity function, I show that the equilibrium allocation is not first best.

By Euler's homogeneous function theorem, function $u'(C)$ is homogenous of degree $\frac{u''(C)\cdot C}{u'(C)} < -1$ for all C. This implies that

$$\frac{u'[C_1(0)]}{u'[C_2(1)]} = R^{-\frac{u''(R\cdot E)\cdot R\cdot E}{u'(R\cdot E)}} > R. \tag{2.60}$$

Thus, the allocation is not first-best efficient. □

Lemma 6. *In the equilibrium of the hidden-trade Diamond-Dybvig economy with reserve requirement and interest on reserves, the price of bonds traded at time 1 in the anonymous market is given by*

$$Q = \frac{(1+i)\cdot \max\left\{1, \frac{(1-\phi)\cdot\rho}{\phi\cdot(1-\rho)}\right\}}{R}. \tag{2.34}$$

and the prevailing deposit contract is given by:

$$1 + d_0 = \max\left\{1, \frac{\rho}{\phi}\right\} \cdot (1 + i), \tag{2.35}$$

$$1 + d_1 = \frac{R}{(1+i)\cdot\max\left\{1, \frac{(1-\phi)\cdot\rho}{\phi\cdot(1-\rho)}\right\}}. \tag{2.36}$$

Proof. Define $\zeta = \frac{\phi\cdot W(0)+(1-\phi)\cdot W(1)}{(1+d_0)\cdot E}$, the share of deposits withdrawn at time 1. The market clearing condition in the hidden bond market

$$\phi \cdot S(0) + (1-\phi)\cdot S(1) = 0, \tag{2.61}$$

the consumer's budget constraints (2.29), (2.13) and (2.14), and the fact that $C_2(0) = C_1(1) = 0$ imply that in equilibrium

$$\zeta = \phi. \tag{2.62}$$

By the withdrawing behaviour equation (2.16), then it must be that in equilibrium $1 + d_1 = Q^{-1}$. Otherwise, all consumers would either not withdraw at all at time 1, so that $\zeta = 0$, or withdraw all their deposits, so that $\zeta = 1$.

Banks offer competitively deposit contracts (d_0, d_1), which imply a withdrawing behaviour ζ. Banks make zero profits because of Bertrand-style competition. By the budget constraints (2.30), (2.31) and (2.21) and by the reserve requirement (2.28), this implies that

$$1 + d_0 = \min\left\{\frac{R\cdot Q\cdot(1+i)}{\zeta\cdot R\cdot Q + (1-\zeta)\cdot(1+i)}, \frac{1-\rho}{1-\zeta}\cdot R\cdot Q\right\}. \tag{2.63}$$

A bank can deviate and earn strictly positive profits unless in equilibrium the deposit contract on offer solves

$$\max_{\zeta\in[0,1]} \frac{1}{Q}\cdot\left\{\phi\cdot u'[C_1(0)] + (1-\phi)\cdot\beta u'[C_2(1)]\right\}\cdot\min\left\{\frac{R\cdot Q\cdot(1+i)}{\zeta\cdot R\cdot Q + (1-\zeta)\cdot(1+i)}, \frac{1-\rho}{1-\zeta}\cdot R\cdot Q\right\}. \tag{2.64}$$

Note that banks can choose the timing of depositor withdrawal because $1+d_1 = Q^{-1}$ makes consumers indifferent.

The solution of the above is given by the deposit contract which gives

$$\zeta = \begin{cases} \frac{\rho\cdot(1+i)}{\rho\cdot(1+i)+(1-\rho)\cdot R\cdot Q} & \text{if } R\cdot Q > 1+i, \\ [\rho, 1] & \text{if } R\cdot Q = 1+i, \\ 1 & \text{if } R\cdot Q < 1+i. \end{cases} \tag{2.65}$$

Since, as shown above, $\zeta = \phi$ in equilibrium, we have that

$$Q = \frac{(1+i)\cdot\max\left\{1, \frac{(1-\phi)\cdot\rho}{\phi\cdot(1-\rho)}\right\}}{R}. \tag{2.66}$$

□

Lemma 7. *The equilibrium allocation of the hidden-trade Diamond-Dybvig economy with a reserve requirement and interest on reserves satisfies equations (2.8), (2.10) and*

$$\frac{C_2(1)}{C_1(0)} = \frac{R}{(1+i)\cdot\max\left\{1, \frac{(1-\phi)\cdot\rho}{\phi\cdot(1-\rho)}\right\}}. \tag{2.37}$$

Proof. With the budget constraints (2.29), (2.13), (2.14) and the equilibrium prices given in lemma 6, we find that the equilibrium allocation satisfies equations (2.8), (2.10) and (2.37).

□

Lemma 8. *In the equilibrium of the hidden-trade Diamond-Dybvig economy with endogenous intermediation, a reserve requirement and interest on reserves, the price of bonds traded at time 1 in the anonymous market is given by*

$$Q = \frac{1+i}{R}. \tag{2.43}$$

and the prevailing deposit contract is given by:

$$d_0 = i, \tag{2.44}$$

$$1 + d_1 = \frac{R}{1+i}. \tag{2.45}$$

Proof. Define $\zeta = \frac{\phi\cdot W(0)+(1-\phi)\cdot W(1)}{(1+d_0)\cdot D}$, the share of deposits withdrawn at time 1. The market clearing condition in the hidden bond market

$$\phi \cdot S(0) + (1-\phi)\cdot S(1) = 0, \tag{2.67}$$

the consumer's budget constraints (2.29), (2.13) and (2.14), and the fact that $C_2(0) = C_1(1) = 0$ imply that in equilibrium

$$\phi \cdot \int_{j=0}^{1} K_{j,L}\,\mathrm{d}j = (1+d_0)\cdot(1+d_1)\cdot D\cdot(\zeta-\phi). \tag{2.68}$$

By non-negativity of deposits and capital holdings, from this equation we can conclude that in equilibrium $\zeta \geq \phi$. Moreover, if in equilibrium $\zeta > \phi$, then $D > 0$ and $\int_{j=0}^{1} K_{j,L}\, \mathrm{d}j > 0$. And, if $\zeta = \phi$, then $D > 0$ and $\int_{j=0}^{1} K_{j,L}\, \mathrm{d}j = 0$.
Focus on the case $\zeta > \phi$. This requires $V(d_0, d_1) = V_K$, so that consumers are indifferent between deposits and direct capital holdings and hold both in equilibrium. We find that

$$V(d_0, d_1) - V_K = \frac{1}{Q} \cdot \Big\{\phi \cdot u'[C_1(0)] + (1-\phi) \cdot \beta u'[C_2(1)]\Big\} \cdot \frac{\zeta}{\zeta \cdot R \cdot Q + (1-\zeta) \cdot (1+i)} \cdot \Bigg(1 + i - R \cdot Q\Bigg). \tag{2.69}$$

Hence, in equilibrium if $\zeta > \phi$, then $Q = \frac{1+i}{R}$, $d_0 = i$ and $1 + d_1 = \frac{R}{1+i}$.
Now consider $\zeta = \phi$ and $\int_{j=0}^{1} K_{j,L}\, \mathrm{d}j = 0$. The equilibrium is identical to the one described by lemma **??**. In such equilibrium,

$$Q = \frac{(1+i) \cdot \max\Big\{1, \frac{(1-\phi)\cdot\rho}{\phi\cdot(1-\rho)}\Big\}}{R} \tag{2.70}$$

and

$$(1 + d_0, 1 + d_1) = \Bigg(\max\Big\{1, \frac{\rho}{\phi}\Big\} \cdot (1+i), \frac{R}{(1+i) \cdot \max\Big\{1, \frac{(1-\phi)\cdot\rho}{\phi\cdot(1-\rho)}\Big\}}\Bigg). \tag{2.71}$$

If we verify whether consumers have an incentive to invest one unit of their endowment in directly in capital rather than in their deposit, we find that

$$V(d_0, d_1) - V_K = \frac{1}{Q} \cdot \Big\{\phi \cdot u'[C_1(0)] + (1-\phi) \cdot \beta u'[C_2(1)]\Big\} \cdot \frac{\phi \cdot (1+i)}{\phi \cdot R \cdot Q + (1-\phi) \cdot (1+i)} \cdot \Bigg(1 - \max\Big\{1, \frac{(1-\phi)\cdot\rho}{\phi \cdot (1-\rho)}\Big\}\Bigg) \leq 0. \tag{2.72}$$

Hence, whenever $\rho > \phi$ consumers have a strict incentive to hold capital directly rather than deposits. The equilibrium features $\zeta = \phi$ if and only if $\rho \leq \phi$. Also in this case we have that $Q = \frac{1+i}{R}$.
For any policy (ρ, i), in equilibrium $Q = \frac{1+i}{R}$. □

Lemma 9. *The equilibrium allocation of the hidden-trade Diamond-Dybvig economy with endogenous intermediation, reserve requirements and interest on reserves satisfies equations (2.8), (2.10) and*

$$\frac{C_2(1)}{C_1(0)} = \frac{R}{1+i}. \tag{2.46}$$

Proof. With the budget constraints (2.29), (2.13), (2.14) and the equilibrium prices given in lemma 8, we find that the equilibrium allocation satisfies equations (2.8), (2.10) and (2.46). □

Lemma 10. *Optimal policy* (i, ρ) *in the hidden-trade Diamond-Dybvig model with endogenous intermediation and distortionary taxation implements the allocation given by*

$$\frac{u'[C_1(0)]}{u'[C_2(1)]} = \beta \cdot R \cdot \min\left\{R^{\gamma[C_2(1)]-1}, 1 - \frac{\left\{\phi \cdot \left(\frac{R}{1+i}\right)^{\gamma[C_2(1)]} + 1 - \phi\right\} \cdot E'(T)}{(1-\phi)\cdot\{1+[1-E'(T)]\cdot\phi\cdot i\}}\right\}, \tag{2.52}$$

$$\phi \cdot C_1(0) + (1-\phi)\cdot\frac{C_2(1)}{R} = E(T), \tag{2.53}$$

and equation (2.46).

Proof. Equations (2.52), (2.53) and (2.46) are the first-order conditions of the maximisation of objective function (2.11) with respect to variables $(C_1(0), C_2(1), i, T)$ subject to equations that define the variables in equilibrium:

$$C_1(0) = \frac{1+i}{1+\phi\cdot i}\cdot E(T), \tag{2.73}$$

$$C_2(0) = \frac{R}{1+\phi\cdot i}\cdot E(T), \tag{2.74}$$

$$T = \frac{\phi\cdot i}{1+\phi\cdot i}\cdot E(T), \tag{2.75}$$

$$i \geq 0. \tag{2.76}$$

□

3 | A Theory of the Optimal Quantity of Reserves

I incorporate a reserve-issuing central bank in the financial-intermediation framework of Diamond and Dybvig (1983) and let the term premium be determined endogenously. First, I find that the issuance of reserves, by increasing the relative supply of short-term assets, can affect the yield curve reducing the term premium. In contrast, interest on reserves, the other policy instrument, shifts the entire yield curve and therefore has no impact on the term premium. Second, I find that it is optimal for the central bank to have a large balance sheet, because the resulting reduction in the term premium reduces the economy's inefficiently high liquidity risk. The theory determines an optimal size of the central bank's balance sheet, which implements the first-best efficient allocation in the economy.

3.1 Introduction

What is the optimal size of a central bank's balance sheet? As monetary authorities in advanced economies normalise monetary policy, this has become a momentous question. In response to the Great Financial Crisis, the Federal Reserve nearly sextupled the size of its balance sheet and the European Central Bank tripled it. Now that the crisis is over, what size should they go back to?

I use a model of financial intermediation à la Diamond and Dybvig (1983). Assets have a positive term premium and consumers are subject to idiosyncratic liquidity shocks. In combination, these two elements generate liquidity risk for consumers: if the consumer is hit by a liquidity shock and has to sell her assets early, then she loses out on the term premium. I extend the model to include a central bank, which controls the quantity and price of reserves, and I let the model's term premium be determined endogenously. First, I find that by increasing the supply of reserves the central bank can increase the price of long-term assets and hence reduce the term premium. Other papers have worked out models that have this feature (Vayanos and Vila, 2009). Second, I find that it is optimal for the central bank to issue a large quantity of reserves in order to flatten the yield curve. In fact, the theory delivers an optimal slope of the yield curve, which implies an optimal size of the central bank's balance sheet. The optimal slope of the yield curve reduces liquidity risk to zero.

A famous irrelevance result holds for the central bank's balance sheet in the standard New-Keynesian model. This result has been found by Eggertsson and Woodford (2003) and the idea dates back to Wallace (1981). They find that the size and composition of the central bank's balance sheet has no effect on the model's outcomes, because, if the central bank decides to hold more assets, then consumers react by holding fewer and thus perfectly offset the central bank's policy.

An important literature has departed from the irrelevance result by assuming that a demand exists for assets that only the central bank can create: money. The justification for this assumption is that money helps agents to carry out economic transactions. The seminal paper of this literature is Friedman (1969). The main result is that, since the central bank can create money costlessly, the central bank should create money to the point that agents are satiated in it. This is referred to as the Friedman Rule. Within a modern monetary system with banks and bank reserves, recent papers make the same point: a central bank should have a sufficiently large balance sheet (Ennis and Weinberg, 2007; Cochrane, 2014; Canzoneri et al., 2017). This paper similarly concludes that the central bank's balance sheet should be large. A key difference of the policy implication is that, while the Friedman Rule is a lower bound for the central bank's balance-sheet size, my paper indicates an exact quantity of reserves that is optimal. An excessively large quantity of reserves is also undesirable.

Another related literature focuses on the use of the central bank's balance sheet as a policy instrument during financial crises. A financial crisis is defined as a state of the world in which private financial intermediaries are less effective at channeling consumers' savings to capital markets. They find that in such contingency the central bank should directly purchase assets in capital markets that, because of the crisis, are malfunctioning (Gertler and Karadi, 2011; Cúrdia and Woodford, 2011). In other words, the central bank's balance sheet is an important monetary-policy instrument in a crisis. It remains broadly irrelevant in normal times.

An implication of my paper's results is that, given a standard model of financial intermediation, government intervention in the form of a large supply of bank reserves is necessary to achieve an efficient allocation. This contrasts with the original Diamond and Dybvig (1983) model, where private financial intermediation leads to the efficient allocation. Jacklin (1987) showed that the result in the Diamond-Dybvig model crucially depends on restrictions to trading. Following Farhi et al. (2009), I relax these restrictions and analyse an economy in which private financial intermediation alone is unable to provide the efficient level of liquidity-risk insurance.

3.2 Technology, Preferences and Social Planner's Problem

The characteristics of the supply schedule for capital goods are the key difference of this model's set-up, compared to the seminal Diamond-Dybvig model. The consumers' endowment is received not in consumption goods but in capital goods. Thanks to this assumption, the model features an inelastic supply of capital goods, which allows the price of capital goods to be determined by demand. The Diamond-Dybvig model features perfectly elastic supply of capital goods.

The other assumptions on technology and preferences are directly taken from the model in Diamond and Dybvig (1983).

The economy is inhabited by a unit mass of consumers, each indexed by c. Consumers live for three periods and have preferences over consumption patterns given by

$$U(C_{c,1}, C_{c,2}, \theta_c) = (1-\theta_c) \cdot u(C_{c,1}) + \theta_c \cdot \beta \cdot u(C_{c,2}), \tag{3.1}$$

where the felicity function u complies with Inada conditions. I impose a restriction on the functional form of the felicity function:

$$\frac{-C \cdot u''(C)}{u'(C)} \equiv \gamma(C) \geq 1 \quad \forall C > 0. \tag{3.2}$$

It is necessary to have a sufficiently high coefficient of relative risk aversion in order to obtain positive demand for liquidity-risk insurance in the model.

θ_c determines consumer c's preference for consumption over time. For simplicity, it can only take on value 0 or 1 with probability

$$Pr(\theta_c) = \begin{cases} \phi & \text{if } \theta_c = 0, \\ 1 - \phi & \text{if } \theta_c = 1. \end{cases} \tag{3.3}$$

Notice that there is no aggregate risk. I interpret θ_c as a liquidity shock. Agents with realisation $\theta_c = 0$ are hit by the liquidity shock, in that they only enjoy consuming early. Throughout the paper, I will refer to consumers with $\theta_c = 0$ as early types and to those with $\theta_c = 1$ as late types. The realisation of the shock is only known at time 1 and importantly it is privately known. The fact that liquidity shocks are private knowledge is the key friction of the model, because it makes liquidity shocks not contractible and therefore not directly insurable.

A capital good takes two periods to mature and gives a gross return $R > \beta^{-1}$. It can be liquidated already after one period, in which case it gives one unit of the consumption good.

Given the structure of the economy, I study the social planner's problem. The outcome is the first-best efficient allocation, which I will use as the benchmark for efficiency when analysing the decentralised economy.

The social planner maximises the expected welfare of consumers, given by

$$\phi \cdot u[C_1(0)] + (1 - \phi) \cdot \beta \cdot u[C_2(1)]. \tag{3.4}$$

As discussed, at time 0 each consumer receives an endowment of E units of the capital good. Of the endowment, the social planner decides how much to liquidate, L, at time 1 to finance time-1 consumption according to

$$\phi \cdot C_1(0) + (1 - \phi) \cdot C_1(1) = L, \tag{3.5}$$

and how much to hold until time 2 to finance consumption according to

$$\phi \cdot C_2(0) + (1 - \phi) \cdot C_2(1) = R \cdot (E - L). \tag{3.6}$$

The social planner is also constrained by non-negativity of consumption:

$$C_t(\theta_c) \geq 0 \quad \forall t, \theta_c. \tag{3.7}$$

It is easy to show that the social planner's allocation $\{C_1(\theta_c), C_2(\theta_c)\}_{\theta_c \in \{0,1\}}$ is given by:

$$\frac{u'[C_1(0)]}{\beta \cdot u'[C_2(1)]} = R, \tag{3.8}$$

$$(1-\phi) \cdot C_2(1) = R \cdot [E - \phi \cdot C_1(0)], \tag{3.9}$$

$$C_1(1) = C_2(0) = 0. \tag{3.10}$$

Equation (3.8) is familiar from the optimality conditions in the standard Diamond-Dybvig model. It implies that it is optimal to fully insure liquidity risk in the economy. If consumers do not insure each other, then early types, who are forced to liquidate their assets early, consume too little. Ex ante, in a fully efficient economy the provision of liquidity-risk insurance implements equation (3.8).

Equation (3.9) is the economy's resource constraint and, according to equation (3.10), the social planner does not give consumption to consumers at points in time when they do not enjoy consumption.

Definition 12.
An allocation $\{C_1(\theta), C_2(\theta)\}_\theta$ is first-best efficient if it satisfies equations (3.8), (3.9) and (3.10).

3.3 Decentralised Economy

In models of financial intermediation à la Diamond and Dybvig (1983), the key friction that prevents the decentralised economy from achieving efficiency through the trading of Arrow-Debreu securities is that liquidity shocks are privately observed. If consumers were to write an insurance contract contingent on liquidity shocks, all the parties would always have the incentive to falsely reveal that they were hit by the liquidity shock in order to pocket the pay out. It follows that liquidity shocks are not contractible.

In this paper, I crucially allow consumers to frictionlessly sell and buy capital assets at time 1. In the literature on maturity transformation, this is known as hidden trading. As a consequence of consumers trading after liquidity risk is resolved, banks are unable to provide liquidity-risk insurance, as originally shown by Jacklin (1987). The result is that the model features inefficiently low liquidity-risk insurance in equilibrium. An interesting question emerges then: can monetary policy improve the liquidity outcome?

Since banks are unable to provide liquidity-risk insurance and thus are a mere veil, for simplicity I do not include them in the model. In subsection 3.4.1, I show that banks can be safely abstracted from.

In this section, I describe the characteristics of the two types of agents who inhabit the decentralised economy: consumers and the government.

3.3.1 Consumers

Consumers make a portfolio-allocation decision under idiosyncratic liquidity risk.

There is a unit measure of ex-ante identical consumers. Since consumers are ex-ante identical, from here on I drop the index c. Consumers only differ because of different realisations of the idiosyncratic liquidity shock θ. Given the utility function defined in (3.1) and the probability distribution of the liquidity shock in (3.3), a consumer's expected utility function is given by

$$\phi \cdot u[C_1(0)] + (1-\phi) \cdot \beta \cdot u[C_2(1)]. \tag{3.4}$$

At time 0, each consumer receives an endowment of capital goods E, which she can either sell, S_0, on the time-0 market for capital goods or hold, K. This is represented by a budget constraint for capital goods, given by

$$K + S_0 = E. \tag{3.11}$$

Consumers cannot hold a negative quantity of capital goods. This is formalised with the following non-negativity constraint:

$$K \geq 0. \tag{3.12}$$

The time-0 price of capital goods in terms of consumption goods is Q_0. Reserves B, the price of reserves Q^B and taxes T are also set in terms of consumption goods. This gives a constraint of the form

$$Q^B \cdot B = Q_0 \cdot S_0 - T. \tag{3.13}$$

Combining equations (3.11) and (3.13), we obtain a time-0 budget constraint defined as

$$Q_0 \cdot K + Q^B \cdot B = Q_0 \cdot E - T. \tag{3.14}$$

At time 1, consumers already know their type θ. Hence, they can decide on the basis of their liquidity realisation how many of their capital goods to liquidate, L, or sell, S. The time-1 budget constraint is given by

$$C_1(\theta) = B + L(\theta) + Q_1 \cdot S(\theta), \tag{3.15}$$

where Q_1 is the time-1 price of capital goods.

At time 2, the consumers use their remaining assets to consume, according to

$$C_2(\theta) = R \cdot [K - L(\theta) - S_1(\theta)]. \tag{3.16}$$

First, it is useful to characterise the supply and demand of capital goods at time 1. Consumers who turn out to be early types sell their capital assets, in order to consume today. This is true unless the price is so low that they are better off liquidating capital goods. Supply of used capital goods is therefore given by

$$S(0) = \begin{cases} K & \text{if } Q_1 > 1, \\ [0, K] & \text{if } Q_1 = 1, \\ 0 & \text{if } Q_1 < 1. \end{cases} \tag{3.17}$$

Late-type consumers buy used capital assets using the proceeds of their short-term investments. So, late types' demand for capital goods at time 1 is given by

$$S(1) = \begin{cases} -\frac{B}{Q_1} & \text{if } Q_1 > 1, \\ \left[-\frac{B+K}{Q_1}, -\frac{B}{Q_1}\right] & \text{if } Q_1 = 1, \\ -\frac{B+K}{Q_1} & \text{if } Q_1 < 1. \end{cases} \tag{3.18}$$

The consumer makes her time-0 portfolio-allocation decision on the basis of the relative value of holding capital goods and reserves. The value of investing in reserves is given by the following first-order condition:

$$Q^B \cdot \lambda = \phi \cdot u'[C_1(0)] + (1-\phi) \cdot \frac{R}{Q_1} \cdot \beta \cdot u'[C_2(1)] \tag{3.19}$$

If you turn out to be an early type, you can use your reserves to buy consumption goods and consume at time 1. If you are a late type, you have to buy used capital in the market at price Q_1 and you get a return R on each unit of capital.

The first-order condition with respect to capital holdings is given by

$$Q \cdot \lambda = \phi \cdot \max\left\{1, Q_1\right\} \cdot u'[C_1(0)] + (1-\phi) \cdot R \cdot \max\left\{1, \frac{1}{Q_1}\right\} \cdot \beta \cdot u'[C_2(1)] + \mu, \tag{3.20}$$

where μ is the Kuhn-Tucker multiplier associated with the non-negativity constraint on capital (3.12). At time 1, early-type consumers can either sell their capital goods at the price Q_1 or liquidate it. If the consumer is a late type, she gets a return R, unless the price of used capital goods is extremely low. In this case, the consumer is better off liquidating her capital at time 1 and using the proceeds to buy used capital.

3.3.2 Government

The government chooses as policy instruments the size of the monetary base, $B^S \geq 0$, and its price Q^B. Of course, choosing the price of reserves corresponds to choosing the interest on reserves.

On the asset side, the government engages in open-market operations, whereby it purchases capital goods K_g in order to issue reserves. If necessary, the government levies lump-sum taxes T to back up its monetary policy. Therefore, the government's time-0 budget constraint is

$$Q_0 \cdot K_g = Q^B \cdot B^S + T. \tag{3.21}$$

At time 1, the government uses its capital holdings to pay off holders of the monetary base, according to

$$K_g = B^S. \tag{3.22}$$

At time 2, the government plays no role in the economy.

Note that throughout the paper I also refer to the government as central bank and monetary authority, because I focus on the monetary prerogatives of the government.

3.4 Equilibrium

In this section of the paper, I illustrate the definition of equilibrium and I describe the equilibrium allocation given the central bank's policy. This allows me to discuss the general-equilibrium effects of changes in the interest on reserves and in the quantity of reserves. I conclude showing that the presence of an explicitly modelled banking system does not change the equilibrium allocation.

In equilibrium, consumers solve their optimisation problems, the government's budget constraints hold and prices adjust to ensure market clearing. The government controls the price and quantity of reserves. I formalise the equilibrium concept as follows.

Definition 13. *Given policy $\{Q^B, B^S\}$, the equilibrium consists of quantities and prices, respectively $\{K, B, C_1(\theta), C_2(\theta), L_\theta, S(\theta), K_g, T\}_\theta$ and $\{Q_0, Q_1\}$, such that*

1. *Each consumer chooses $\{K, B, C_1(\theta), C_2(\theta), L_\theta, S(\theta)\}_\theta$ to maximise (3.4) subject to budget constraints (3.14), (3.15), (3.16) and non-negativity constraint (3.12).*

2. *$\{K_g, T\}$ are such that the government's budget constraints (3.21) and (3.22) hold.*

3. *The market for new capital goods clears with*

$$K + K_g = E \tag{3.23}$$

4. *The market for used capital goods clears with*

$$\phi \cdot S(0) + (1 - \phi) \cdot S(1) = 0. \tag{3.24}$$

5. *The market for reserves clears with*

$$B = B^S. \tag{3.25}$$

First, I characterise the equilibrium price of used capital goods, Q_1. From a quick look at supply and demand, respectively equations (3.17) and (3.18), it is clear that the price cannot be lower than 1 in equilibrium, because early types would be better off liquidating their capital assets rather than selling them.

The equilibrium price of used capital depends on the quantity of short-term assets in the economy. If there are few reserves in the economy, the demand for used capital by late type consumers is small. Early types can satisfy this demand by selling some of their capital and liquidating the capital that they cannot find buyers for. As the monetary base expands, the demand for used capital increases. Once demand outstrips the capital that early types have for sale, the equilibrium price must rise for the market to clear. In equilibrium, the price of a bond at time 1 is given by

$$Q_1 = \begin{cases} \frac{1-\phi}{\phi} \cdot \frac{B}{K} & \text{if } \frac{B}{K} > \frac{\phi}{1-\phi}, \\ 1 & \text{otherwise.} \end{cases} \tag{3.26}$$

The price of used capital is important because it determines the ratio of late-type consumption and early-type consumption. In other words, it determines the term premium that accrues to investors who hold assets for a longer time. In fact, combining the consumer's budget constraint (3.14) and (3.15) with the optimal time-1 demand and supply for used capital, equations (3.17) and (3.18), we have that

$$\frac{C_2(1)}{C_1(0)} = \frac{R}{Q_1}. \tag{3.27}$$

An increase in the price of used capital implies a transfer of resources from late types to early types.

Equation (3.26) determines the equilibrium price of used capital, taking as given the consumer's portfolio allocation. I need to find out whether the portfolio allocation that the central bank attempts to implement by issuing reserves can in fact hold in equilibrium and what price changes are required for this to happen. In this regard, I find that the price of new capital, Q_0, adjusts to ensure that consumers are willing to hold the quantity of reserves that the central bank determines. If reserves pay a high return relative to capital, consumers sell their capital goods to hold more reserves. Since in aggregate the quantity of reserves is fixed, the result of this behaviour is a reduction in the price of capital assets. At the end of this process, we have that

$$\frac{Q_1}{Q_0} = \frac{1}{Q^B}, \tag{3.28}$$

and the consumer is indifferent between holding reserves and holding capital assets.

In equilibrium, the central bank can control the term premium, which is given by

$$\frac{C_2(1)}{C_1(0)} = \begin{cases} R \cdot \frac{\phi}{1-\phi} \cdot \frac{E-B^S}{B^S} & \text{if } B^S > \phi \cdot E, \\ R & \text{otherwise.} \end{cases} \tag{3.29}$$

At low levels, an increase in the quantity of reserves has no effect on the term premium. There is a threshold above which more reserves succeed in reducing the term premium. In particular, the central bank must issue reserves that more than cover for the short-term consumption demands of consumers.

The resource constraint holds in equilibrium and is given by

$$(1-\phi) \cdot C_2(1) = R \cdot [E - \phi \cdot C_1(0)]. \tag{3.9}$$

Moreover, consumers do not consume in time periods in which they do not enjoy consumption:

$$C_1(1) = C_2(0) = 0. \tag{3.10}$$

Definition 14. *Given policy $\{Q^B, B^S\}$, the equilibrium allocation $\{C_1(\theta), C_2(\theta)\}_\theta$ is determined by equations (3.29), (3.9) and (3.10).*

It is notable that the level of real interest on reserves does not affect the equilibrium allocation. Only the quantity of reserves does.

Firstly, this is true because the model does not feature a meaningful saving decision. Hence, interest on reserves does not affect savings. Moreover, the supply of capital goods is inelastic. Thus, interest on reserves does not affect the consumers' portfolio allocation in aggregate.

A change in the quantity of reserves changes the relative supply of short-term and long-term assets. More reserves imply that in the next period there will be an increase in liquid assets chasing a reduced quantity of long-term capital assets.

The consequence is that the price of long-term investments rises. Hence, the policy benefits those agents that sell their used capital: the early types.

3.4.1 Banks

It is customary to include a banking sector in models of maturity transformation. In this subsection, I show that, because of the frictionless trading of used capital, banks are unable to provide liquidity-risk insurance. They are but a veil in the model. Hence, I can avoid modelling them explicitly.

Consider a representative bank offering a deposit contract that allows consumers to withdraw their deposits at any point in time. It offers a net interest rate d_0 on funds withdrawn at time 1 and a net interest rate d_1 on funds kept in the account between time 1 and time 2.

Suppose that early-type consumers withdraw at time 1 and late types withdraw at time 2. Then, the value to the consumer of such contract in utility terms is given by

$$\lambda^D = \phi \cdot (1 + d_0) \cdot u'[C_1(0)] + (1 - \phi) \cdot (1 + d_0) \cdot (1 + d_1) \cdot \beta \cdot u'[C_2(1)]. \qquad (3.30)$$

However, at time 1 late-type consumers would only hold on to their deposits rather than withdraw and purchase used capital so long as

$$1 + d_1 \geq \frac{R}{Q_1}. \qquad (3.31)$$

This is the incentive-compatibility constraint on banks.

At time 0, the bank uses the deposits it collects, D, to purchase capital goods K_b and reserves B_b, according to

$$Q_0 \cdot K_b + Q^B \cdot B_b = D. \qquad (3.32)$$

At time 1, the bank liquidates part of its capital goods, L_b, and uses its reserves to pay the early-type consumers, who withdraw. The bank's budget constraint at time 1 is given by

$$\phi \cdot (1 + d_0) \cdot D = L_b + B_b. \qquad (3.33)$$

At time 2, the bank pays off the late types with its remaining assets, according to

$$(1-\phi)\cdot(1+d_0)\cdot(1+d_1)\cdot D = R\cdot(K_b - L_b). \tag{3.34}$$

The bank maximises the value of deposits (3.30) subject to the incentive-compatibility constraint (3.31) and budget constraints (3.32), (3.33) and (3.34).

Such bank offers deposits with the following interest rates:

$$1+d_0 = \frac{Q_1}{Q_0}, \tag{3.35}$$

$$1+d_1 = \frac{1}{Q_1}. \tag{3.36}$$

Setting $1+d_1 < \frac{1}{Q_1}$, which would imply a higher pay off for early types and thus liquidity-risk insurance, violates the incentive-compatibility constraint. Late-type consumers would have an incentive to withdraw their deposits early, too. The converse, $1+d_1 > \frac{1}{Q_1}$, is possible but not desirable. Consumers would not hold such deposits because they actually increase liquidity risk.

The conclusion is that banks do not play any role in this economy, because of the frictionless trade in used capital. Hence, the banking system can be safely abstracted from. A friction in the market for used capital, for example a transaction cost, would allow banks to provide liquidity-risk insurance. If the market for used capital was not perfectly frictionless, it would be necessary to explicitly model financial intermediaries.

However, a friction in the market for used capital, which allows banks to provide liquidity-risk insurance, would not eliminate the need for government intervention altogether. For private intermediation to be able to supply the efficient level of liquidity-risk insurance, the friction needs to be sufficiently large. Therefore, effective private provision of liquidity-risk insurance does not rule out a role for the government to improve liquidity risk in the economy.

3.5 Optimal Monetary Policy

Policymakers choose the price and quantity of reserves $\{Q^B, B^S\}$ in order to maximise aggregate welfare in the equilibrium.

The first-best efficient allocation, described in definition 12, can be implemented in equilibrium with a policy of keeping the central bank's balance sheet sufficiently large. There is a threshold above which an increase in bank reserves reduces the term premium and therefore mitigates liquidity risk. Above such threshold, there is a quantity of reserves that is optimal. The resulting term premium is such that liquidity risk is eliminated. It is noticeable that, unlike the quantity of reserves, the interest on reserves has no effect on the economy's equilibrium allocation. In fact, the interest on reserves shifts the entire yield curve. It does not affect its shape.

Proposition 9. *Implementing the first-best efficient allocation requires a large quantity of reserves, with*

$$B^S > \phi \cdot E. \tag{3.37}$$

The price of reserves, Q^B, has no impact on the equilibrium allocation.

By restricting the functional form of the felicity function to constant relative risk aversion, I find a closed-form solution for the optimal quantity of reserves.

Proposition 10. *Consider felicity function $u(C) = \frac{C^{1-\gamma}-1}{1-\gamma}$, with $\gamma \geq 1$ coefficient of relative risk aversion. The resulting formula for the optimal quantity of reserves is given by*

$$B^S = \frac{R^{\frac{\gamma-1}{\gamma}}}{\phi \cdot R^{\frac{\gamma-1}{\gamma}} + (1-\phi) \cdot \beta} \cdot \phi \cdot E. \tag{3.38}$$

The optimal quantity of reserves is a share of the total amount of capital assets in the economy. This share is increasing in the coefficient of relative risk aversion and in the return of long-term capital assets. It is decreasing in the discount factor. In other words, more demand for liquidity-risk insurance calls for a greater supply of reserves.

It is important to note that this theory calls for a specific quantity of reserves as optimal. Too many reserves lower the term premium excessively. This stands in contrast with the prescriptions of theories that call for sufficiently large quantities of reserves in order to implement the Friedman rule. According to the Friedman rule, a large central-bank balance sheet is desirable to have banks satiated in reserves. Once banks are satiated, printing more reserves has no effect. This paper proposes a different justification for a policy of expanding the central bank's balance sheet: a large quantity of reserves flattens the yield curve. Since a large term premium implies high liquidity risk, doing so is optimal.

3.6 Fiscal Implications

In this model, in order to have a large balance sheet the central bank needs fiscal backing. Given monetary policy, taxes required in equilibrium are given by

$$T = \begin{cases} \frac{(B^S - \phi \cdot E) \cdot B^S}{\phi \cdot (E - B^S)} & \text{if } B^S > \phi \cdot E, \\ 0 & \text{otherwise.} \end{cases} \tag{3.39}$$

Importantly, optimal monetary policy, according to which $B^S > \phi \cdot E$, implies negative seigniorage revenue.

Hall and Reis (2015) analyse the impact on central-bank solvency of a large balance sheet. They conclude that no fiscal backing is necessary for a central bank that issues reserves and holds short-term assets, regardless of the size of its balance sheet. They argue that arbitrage makes the return on the central bank's assets and liabilities equal. In this paper's model their assertion is not true. In fact, when the central bank's balance sheet is large, private agents, who are awash in reserves, start to hold them as an alternative to longer-term assets. Hence, the private sector arbitrages between reserves and the longer-term assets that it would otherwise hold. In conclusion, the short-term assets that the central bank holds on its asset side have a lower return in equilibrium than reserves. Otherwise, the private sector would not be willing to hold all of the reserves supplied.

3.7 Conclusion

In this paper, I study a model of maturity transformation with inefficiently low liquidity-risk insurance in equilibrium. Liquidity risk is due to the presence of a term premium. If consumers are hit by a liquidity shock, they have to unwind their assets early and thus miss out on the term premium. Private financial intermediaries are unable to provide liquidity-risk insurance, because of the existence of financial markets where consumers can trade after uncertainty about their liquidity needs is resolved.

I find that a policy of keeping a large central-bank balance sheet can reduce reduce the term premium. It does so by increasing the relative supply of short-term assets in the economy. As long-term assets become scarcer, their price increases, which translates into a reduction in the term premium.

The paper's main finding is that it is optimal for the central bank to flatten the yield curve by reducing the term premium. A lower term premium mitigates liquidity risk. A policy that keeps the central bank's balance sheet large at the optimal level can implement the efficient allocation.

In contrast to the the Friedman rule, this paper delivers a unique optimal quantity of reserves. According to the Friedman rule, the central bank must have a large balance sheet in order to make the banking system satiated in reserves. Once banks are satiated, additional reserves do not affect the equilibrium allocation, as banks regard them as perfectly substitutable with the short-term assets that the central bank purchases in open-market operations. In this paper, the rationale for a large balance sheet is to flatten the yield curve until liquidity risk is zero. A yield curve with an excessively small, or even negative, slope is undesirable.

Interestingly, the optimal monetary policy requires fiscal backing to be implemented even if the central bank holds assets of equal duration to its liabilities. This is because, if the central bank's balance sheet is large, the private sector has more than enough short-term assets. It wants to hold long-term assets. Hence, it will accept to hold reserves only if the interest on reserves is larger than the interest rate on other short-term assets in the economy.

Bibliography

Allen, Franklin, Elena Carletti, and Douglas Gale. 2014. "Money, Financial Stability and Efficiency." *Journal of Economic Theory*, 149(C): 100–127.

Allen, Franklin, and Douglas Gale. 2004. "Financial Intermediaries and Markets." *Econometrica*, 72(4): 1023–1061.

Alsterlind, Jan, Hanna Armelius, David Forsman, Björn Jönsson, and Anna-Lena Wretman. 2015. "How Far Can the Repo Rate Be Cut?." Sveriges Riksbank, URL: `http://www.riksbank.se/Documents/Rapporter/Ekonomiska_kommentarer/2015/rap_ek_kom_nr11_150929_eng.pdf`.

Antinolfi, Gaetano, and Suraj Prasad. 2008. "Commitment, Banks and Markets." *Journal of Monetary Economics*, 55(2): 265–277.

Basten, Christoph, and Mike Mariathasan. 2017. "Bank Profitability And Risk-Taking When Interest Rates Are Negative."

Bech, Morten Linnemann, and Aytek Malkhozov. 2016. "How Have Central Banks Implemented Negative Policy Rates?." *BIS Quarterly Review*.

Bernanke, Ben. 2016. "What Tools Does the Fed Have Left? Part 1." Brookings Institute, URL: `https://www.brookings.edu/blog/ben-bernanke/2016/03/18/what-tools-does-the-fed-have-left-part-1-negative-interest-rates/`.

Brunnermeier, Markus, and Yann Koby. 2017. "The Reversal Interest Rate: The Effective Lower Bound of Monetary Policy." Unpublished manuscript.

Burke, Chris, Spence Hilton, Ruth Judson, Kurt Lewis, and David Skeie. 2010. "Reducing the IOER Rate: an Analysis of Options." Federal Reserve, URL: `https://www.federalreserve.gov/monetarypolicy/files/FOMC20100805memo05.pdf`.

Canzoneri, Matthew, Robert Cumby, and Behzad Diba. 2017. "Should the Federal Reserve Pay Competitive Interest on Reserves?." *Journal of Money, Credit and Banking*, 49(4): 663–693.

Carlsson, Hans, and Eric van Damme. 1993. "Global Games and Equilibrium Selection." *Econometrica*, 61(5): 989–1018.

Carvalho, Carlos, Andrea Ferrero, and Fernanda Nechio. 2016. "Demographics and Real Interest Rates: Inspecting the Mechanism." Working Paper Series 2016-5, Federal Reserve Bank of San Francisco.

Cochrane, John H. 2014. "Monetary Policy with Interest on Reserves." *Journal of Economic Dynamics and Control*, 49(C): 74–108.

Cúrdia, Vasco, and Michael Woodford. 2011. "The Central-Bank Balance Sheet as an Instrument of Monetary Policy." *Journal of Monetary Economics*, 58(1): 54–79.

Dell'Ariccia, Giovanni, Vikram Haksar, and Tommaso Mancini-Griffoli. 2017. "Negative Interest Rate Policies – Initial Experiences and Assessments." International Monetary Fund.

Diamond, Douglas W. 1997. "Liquidity, Banks, and Markets." *Journal of Political Economy*, 105(5): 928–956.

Diamond, Douglas W, and Philip H Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy*, 91(3): 401–419.

Eggertsson, Gauti B., Ragnar E. Juelsrud, and Ella Getz Wold. 2017. "Are Negative Nominal Interest Rates Expansionary?" Working Paper 24039, National Bureau of Economic Research.

Eggertsson, Gauti B., and Paul Krugman. 2012. "Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach." *The Quarterly Journal of Economics*, 127(3): 1469–1513.

Eggertsson, Gauti B., and Michael Woodford. 2003. "The Zero Bound on Interest Rates and Optimal Monetary Policy." *Brookings Papers on Economic Activity*, 34(1): 139–235.

Ennis, Huberto M. 2014. "A Simple General Equilibrium Model of Large Excess Reserves." Working Paper 14-14, Federal Reserve Bank of Richmond.

Ennis, Huberto M, and John A Weinberg. 2007. "Interest on Reserves and Daylight Credit." *Economic Quarterly*, 93(2): 111–142.

Farhi, Emmanuel, Mikhail Golosov, and Aleh Tsyvinski. 2009. "A Theory of Liquidity and Regulation of Financial Intermediation." *Review of Economic Studies*, 76(3): 973–992.

Friedman, Milton. 1969. "The Optimum Quantity of Money." In *The Optimum Quantity of Money and Other Essays*. Chicago: Aldine Publishing Company.

Galí, Jordi. 2008. "Introduction to Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework." In *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*.: Princeton University Press.

Gertler, Mark, and Peter Karadi. 2011. "A Model of Unconventional Monetary Policy." *Journal of Monetary Economics*, 58(1): 17–34.

Goldstein, Itay, and Ady Pauzner. 2005. "Demand-Deposit Contracts and the Probability of Bank Runs." *Journal of Finance*, 60(3): 1293–1327.

Guerrieri, Veronica, and Guido Lorenzoni. 2011. "Credit Crises, Precautionary Savings, and the Liquidity Trap."Technical report, National Bureau of Economic Research, Inc.

Hall, Robert E., and Ricardo Reis. 2015. "Maintaining Central-Bank Financial Stability under New-Style Central Banking." NBER Working Papers 21173, National Bureau of Economic Research, Inc.

Hasman, Augusto, Margarita Samartín, and Jos van Bommel. 2014. "Financial Intermediation in an Overlapping Generations Model with Transaction Costs." *Journal of Economic Dynamics and Control*, 45(C): 111–125.

Heider, Florian, Farzad Saidi, and Glenn Schepens. 2017. "Life Below Zero: Bank Lending Under Negative Policy Rates." September.

Hellwig, Martin. 1994. "Liquidity Provision, Banking, and the Allocation of Interest Rate Risk." *European Economic Review*, 38(7): 1363–1389.

Hicks, John R. 1937. "Mr. Keynes and the "Classics"; a Suggested Interpretation." *Econometrica*, 5(2): 147–159.

Jacklin, Charles J. 1987. "Demand Deposits, Trading Restrictions, and Risk Sharing." In *Contractual Arrangements for Intertemporal Trade*. eds. by Edward C. Prescott, and Neil Wallace: University of Minnesota Press, 26–47.

Karabarbounis, Loukas, and Brent Neiman. 2013. "The Global Decline of the Labor Share." NBER Working Papers 19136, National Bureau of Economic Research, Inc.

Kashyap, Anil K., and Jeremy C. Stein. 2012. "The Optimal Conduct of Monetary Policy with Interest on Reserves." *American Economic Journal: Macroeconomics*, 4(1): 266–282.

Krugman, Paul R. 1998. "It's Baaack: Japan's Slump and the Return of the Liquidity Trap." *Brookings Papers on Economic Activity*, 29(2): 137–206.

Panetti, Ettore. 2017. "A Theory of Bank Illiquidity and Default with Hidden Trades." *Review of Finance*, 21(3): 1123–1157.

Porcellacchia, Davide. 2018. "Optimal Negative Interest on Reserves: the Maturity-Transformation Channel of Monetary Policy Transmission."

Reis, Ricardo. 2016. "Funding Quantitative Easing to Target Inflation." Discussion Papers 1626, Centre for Macroeconomics (CFM).

Rochet, Jean-Charles, and Xavier Vives. 2004. "Coordination Failures and the Lender of Last Resort: Was Bagehot Right After All?." *Journal of the European Economic Association*, 2(6): 1116–1147.

Rognlie, Matthew. 2016. "What Lower Bound? Monetary Policy with Negative Interest Rates." Unpublished manuscript.

Rogoff, Kenneth. 2017. "Dealing with Monetary Paralysis at the Zero Bound." *Journal of Economic Perspectives*, 31(3): 47–66.

Sargent, Thomas, and Neil Wallace. 1985. "Interest on Reserves." *Journal of Monetary Economics*, 15(3): 279–290.

Summers, Lawrence H. 2014. "U.S. Economic Prospects: Secular Stagnation, Hysteresis, and the Zero Lower Bound." *Business Economics*, 49(2): 65–73.

von Thadden, Ernst-Ludwig. 1997. "The Term-Structure of Investment and the Banks' Insurance Function." *European Economic Review*, 41(7): 1355–1374.

von Thadden, Ernst-Ludwig. 1998. "Intermediated versus Direct Investment: Optimal Liquidity Provision and Dynamic Incentive Compatibility." *Journal of Financial Intermediation*, 7(2): 177–197.

von Thadden, Ernst-Ludwig. 1999. "Liquidity Creation Through Banks and Markets: Multiple Insurance and Limited Market Access." *European Economic Review*, 43(4-6): 991–1006.

Vayanos, Dimitri, and Jean-Luc Vila. 2009. "A Preferred-Habitat Model of the Term Structure of Interest Rates." CEPR Discussion Papers 7547, CEPR.

Viñals, Jose, Simon Gray, and Kelly Eckhold. 2016. "The Broader View: the Positive Effects of Negative Nominal Interest Rates." International Monetary Fund, URL: `https://blogs.imf.org/2016/04/10/the-broader-view-the-positive-effects-of-negative-nominal-interest-rates.`

Wallace, Neil. 1981. "A Modigliani-Miller Theorem for Open-Market Operations." *American Economic Review*, 71(3): 267–274.

Werning, Ivan. 2011. “Managing a Liquidity Trap: Monetary and Fiscal Policy.” nber working papers, National Bureau of Economic Research, Inc.

Woodford, Michael. 2003. *Interest and prices*. Princeton, NJ: Princeton Univ. Press.