REAL DISTURBANCES AND THE EXCHANGE RATE:
THEORETICAL AND EMPIRICAL ANALYSIS

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Thesis submitted in fulfilment of Ph.D. requirements of the University of London.

London School of Economics August 1984
This thesis analyses the effects of certain real shocks on the nominal and real exchange rates.

The first chapter analyses the impact and long run effects of an open market operation when domestic and foreign assets are imperfect substitutes. The nominal exchange rate overshoots its long run value. The real exchange rate depreciates on impact but appreciates in the long run.

The second chapter analyses the effects of monetary disinflation when the government budget constraint is introduced. It is shown that when a cut in the rate of monetary growth is accompanied by a cut in government expenditure we could have a jump depreciation of the real exchange rate. But the costs in terms of output foregone are identical to those in the literature.

The third chapter shows that under plausible assumptions an expansionary balanced budget fiscal policy leads to current account surpluses rather than deficits.

The fourth (and final) chapter examines empirically the effects of a foreign real interest shock on the real exchange rate for the UK.
ACKNOWLEDGEMENTS

Many people have helped shape the ideas which are to be found in this work. This thesis, surely, would not have been possible without the help and support of my three supervisors - Janet Yellen, Alasdair Smith and Willem Buiter.

David Demery, Pami Gugnani (now Dua) and Sushil Wadhwani helped with the econometric problems. Giancarlo Marini has been a constant source of encouragement. To all of them I am very grateful.
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INTRODUCTION AND OVERVIEW
I. ASSET MARKET APPROACH TO EXCHANGE RATE DETERMINATION

Floating exchange rates have been with us for the last 10 years. That
decade has seen major changes in the macroeconomic theory of the open
economy. New models have been advanced which seem to explain the reality
better than the traditional models.

The "consensus" macro model of the open economy in the 1960s was the
Mundell-Fleming model. Other models were usually just variants of it.
This model consisted of three key relations: (1) a goods market
equilibrium condition, (2) a money market equilibrium condition, and
(3) an assumption of perfect capital mobility. Often a fourth equation
explaining either the time rate of change or reserves (under fixed rates),
or a zero balance of payments condition determining the exchange rate for
the flexible rate case, was present. 1/

In the international macroeconomic literature of the last decade
the first two requirements have remained albeit with modifications. But
the third and fourth conditions mentioned above have been subjected to
much scrutiny and amendment.

Perfect capital mobility in the sense of the models of the 1960s
simply meant an equality of the domestic and foreign interest rates.
Expectations and forward rate did not figure in these models. The analysis
of the forward rate was done elsewhere but not integrated into these models.

The open economy models of the last ten years, on the other hand,
emphasize expectations and incorporate this explicitly into the interest
arbitrage condition. Perfect capital mobility is then taken to mean
perfect substitutability between assets and the absence of controls on free
movement of capital internationally.
Assets are perfect substitutes on a covered basis if the expected (nominal) return on domestic assets \((i)\) is equal to the return on foreign assets \((i^*)\) plus the forward discount on the home currency \((f)\).

\[ i = i^* + f \] (1)

This is the covered interest parity (CIP) condition. This requirement has been shown to be empirically robust. It ignores political risks (capital controls etc.). Almost all models assume this.\(^2\)

There is a much stronger assumption of uncovered interest parity (UIP) whereby risk-neutral speculators set the forward rate equal to the expected future spot rate.

\[ i = i^* + \lambda \] (2)

where \(\lambda\) is the expected depreciation of the home currency. UIP has met with less success empirically than CIP.

Thus, the forward market and the expectations of exchange rate changes have been introduced in asset market equilibrium conditions. This is why the new models are generally referred to as the "asset market" view.

The fourth building block of the traditional model has also received quite a battering in the last ten years. There the exchange rate was determined by the requirement that the balance of payments must equal zero since the central bank was not intervening under a floating exchange rate regime.\(^3\) The balance of payments was the sum of the balance on current account and the balance on capital account. This, in turn, implied that the capital account was thought of in flow terms. This accorded well
with the facts of the 1960s when even in the advanced capitalist countries capital controls were the rule rather than the exception.

The asset market view, on the other hand, assumes implicitly that the cost of transactions in the asset markets is negligible and preferred portfolios can be established instantaneously subject to the agents' opportunity sets. In order for the zero balance of payments condition to hold, the surplus on the current account is simply equal to the rate of growth of claims on the rest of the world.

II. REAL DISTURBANCES AND THE EXCHANGE RATE

The open-economy macroeconomics literature had been concerned with the effects of real disturbances even before the advent of the asset market models; fiscal policy and tariffs being two major examples. Since the emphasis was on the short run, in a model with unemployed resources even a monetary expansion was a real disturbance.

The 1970s witnessed, from the standpoint of the advanced capitalist countries, unprecedented real shocks combined with high rates of inflation. Many models with intermediate goods (oil) were built but it would be fair to say that the asset market literature has concerned itself primarily with the monetary side. This was probably due to the ascendancy of monetarist thinking and policy making at the beginning of that decade. In the exchange rate literature, it manifested itself in the purchasing power parity (PPP)
assumption. Increased government expenditure here could do nothing but crowd out net exports.

This is superficially similar to the Mundell-Fleming model. There under a regime of floating rates, expansionary fiscal policy was powerless to affect the level of output because it induced an equal decline in the trade balance by an appreciation of the home currency. But there the result was achieved due to a change in relative prices.

Later authors have re-examined these results for properly specified models which admit the possibility of fiscal policy giving rise to intermediate run dynamics (e.g. for the PPP case see Kouri [1976] and for the Mundell-Fleming case see Branson and Buit [1983]).

Although the literature continues to emphasize "monetary" disturbances, recently increasing attention has been paid to the effect of real disturbances on the exchange rate. To mention but two growing fields, (1) the effects of fiscal policy (see Chapter 3) and (2) commercial policy under various assumptions about wage setting (see e.g. Krugman [1981], Eichengreen [1981], Eichengreen [1983]. Also see Dornbusch and Fischer [1980]).

The approach to real disturbances has to be, by its very nature, piecemeal. Hence the bewildering variety of results and the lack of a consensus.

A real disturbance can be defined as a shock (a change in an exogenous variable) to the economic system which alters the values of real variables even when full equilibrium is re-established. This is in contrast to a nominal disturbance which may have transitory real effects but will leave all real variables unchanged in the new steady state. All nominal
endogenous variables, in this case, change in the same proportion as the exogenous nominal variable whose value has changed.

An example of the former in standard macroeconomics would be a change in government expenditure. Unless 100% direct crowding out occurs, the system will not go back to the old equilibrium. An example of a nominal shock is a step change in the money supply. If there are unemployed resources, this may indeed have real effects in the short run. But if the long run equilibrium is one of full employment then the values of all nominal variables would increase in the same proportion as the increase in money supply, leaving the values of all real variables unchanged. For this to happen, however, the money stock must be the only exogenous nominal variable. If there are two exogenous nominal variables and the quantity of one of them is changed (thus changing their relative magnitudes) then the new steady state would not be characterised by unchanged values of the endogenous real variables. This would be true even if the real variables are homogenous of degree zero in all nominal variables. Here we would have homogeneity but not neutrality.

III. OUTLINE OF THE THESIS

In the first three chapters we look at the effects of specific real disturbances on the nominal and real exchange rate in the context of three popular models. The fourth chapter derives a semi-reduced form for the real exchange rate and estimates it for the U.K.
In the first chapter we examine the effects of an open market operation in a model with three assets which are imperfect substitutes for one another. Two of these are exogenous and are denominated in nominal units. The nominal exchange rate overshoots its long run value. This is in contrast to the result obtained by Turnovsky [1981a]. The difference is that he imposed purchasing power parity and we do not. The authorities can get rid of this overshooting by announcing the open market operation sufficiently in advance. The real exchange rate, on the other hand, jump depreciates although the new long run equilibrium requires an appreciation. This "perverse" overshooting cannot be eliminated by the authorities no matter how long the period between announcement and implementation.

The second chapter considers the effect of a reduction in the rate of growth of the money supply on the real exchange rate and output. Two cases are examined: the first when the reduction in monetary growth is accompanied by an endogenous increase in taxes to keep the budget balanced. Here the real exchange jump appreciates and then depreciates along the adjustment path back to an unchanged long run value. Output falls on impact and then gradually goes back to its "natural" level. This is the case analysed by Buiter and Miller [1981], Turnovsky [1981] and Liviatan [1980]. The second case we consider is when government purchase of goods and services adjusts endogenously to maintain a balanced budget following the introduction of the policy of monetary disinflation. In the new long run equilibrium the real exchange rate is higher (a real depreciation). The impact effect is ambiguous and depends on parameter values. The transition path is one where output is below its natural rate. So here in addition to short run costs (loss of output) monetary disinflation has long run costs as well, viz. a worsening of the terms of trade.
In the third model we examine the effects of a balanced-budget increase in government expenditure directed towards home goods. Contrary to the popular Mundell-Fleming result, our model generates current account surpluses along the adjustment path.

In the final chapter we look at the effect of changes in the foreign real interest rate on the real exchange rate. First, a theoretical model is presented with wealth effects in the money demand and consumption functions and sticky prices. Wealth effects in the demand for money have been used to explain the twin mystery of "missing money" and "multiplying marks" (Frankel [1982]). The real exchange rate depends on the current values of the other state variables and on the (expected) foreign real interest rate from now to infinity, à la Blanchard and Kahn. We use Koyck transforms to generate two alternative but more tractable expressions. These equations are then estimated with (non-linear) restrictions imposed on the parameters. They track the actual performance of the real exchange rate quite well.

Each of the chapters is self-contained and discusses the assumptions made in detail. All the models below belong to the class which it has become fashionable to refer to as ad-hoc macro models, i.e. they do not start off from individuals' maximizing behaviour.

In all the models below five potentially interesting features are missing.
First, the supply side of the economy is not modelled at all. Output is either assumed to be at its (exogenous) full-employment value or is demand-determined. Some discussion of this issue is undertaken where relevant.

Second, the Marshall-Lerner condition is assumed to be satisfied always. A potentially rich exercise involving J-curves is thereby excluded. The Harberger-Laursen-Metzler effect is also assumed away. Recently, there has been growing interest in this topic. We just follow the traditional open economy macro literature and do not discuss this at all.

Third, real capital formation is ignored altogether. This is a feature of most recent models. By assuming a balanced budget and ignoring capital accumulation, the literature often identifies savings with a current account surplus. It is clear if domestic assets were growing at a non-zero rate then a current account surplus or deficit just redistributes wealth between countries.

Fourth, we ignore bond-financed government deficits. The reason for this is analytical tractability.

Finally, the home economy is assumed to be "small" in relation to the rest of the world in the sense that it takes all foreign variables as given. Foreign residents do not hold any domestic assets. The home country, however, is not small in the market for its good.
FOOTNOTES


2. See Eaton and Turnovsky [1983] and Girton and Henderson [1976].

3. The Mundell-Fleming model itself visualized the exchange rate being determined by the interest parity condition. In this respect it is akin to the new models rather than its contemporaries.


5. As a good example see the almost total exclusion of real factors from formal models in Shafer and Loopesko [1983].

6. See e.g. Obstfeld [1982] and Svensson and Razin [1983].
CHAPTER I

MONETARY POLICY IN A NON-NEUTRAL PERFECT FORESIGHT MODEL UNDER FLEXIBLE EXCHANGE RATES
1. **Introduction**

The recent literature on flexible exchange rate has been, to a large measure, concerned with the issue of "overshooting" of the exchange rate when agents possess rational expectations. The exchange rate is viewed as an asset price which responds to current and expected changes in exogenous variables. This "forward-looking" view of the exchange rate stands in sharp contrast to the view that the exchange rate moves to ensure a balance in the current account.

Overshooting occurs when the impact effect on the exchange rate of a change in an exogenous variable is greater than the long run effect. For example, overshooting occurs if a change in the level of the money stock depreciates the long run exchange rate by \( k \% \) but on impact the exchange rate depreciates by more than \( k \% \).

In perfect foresight models, if the exchange rate overshoots its long run value at all, it does so because there is some variable which is predetermined when the exogenous shock occurs. Thus the burden of adjustment in the short run falls on the non-predetermined variables, including the exchange rate. Over time, however, the predetermined variables are free to adjust and thus relieve the pressure on the exchange rate.

The predetermined variable which causes the exchange rate to overshoot when agents possess perfect foresight differs between models.
In one class of models, the price of domestic output adjusts sluggishly. So following an exogenous shock the exchange rate bears the major burden of adjustment in the short run. In the other class of models, the predetermined variable is the stock of foreign currency denominated asset. Foreign assets may only be accumulated by running current account surpluses because these assets are the only traded ones. If an exogenous change requires the real value of these foreign assets to rise, then the real exchange rate has to depreciate because in the short run the total quantity of foreign currency denominated assets is fixed.

The model presented below belongs to the latter class of models - the so-called portfolio balance models. However, it differs from other models in some crucial respects. First, there are three assets in the portfolios of domestic wealth holders which are viewed as imperfect substitutes for one another. Second, almost all the models which discuss the issue of overshooting are models in which long run neutrality prevails. A step change in the money supply would change all nominal variables in the same proportion in the long run and thus leave all real variables unchanged. Our model, however, has two exogenous assets denominated in nominal units and an operation that changes the proportions in which these assets are available to wealth holders has real effects even in the long run. It is important to note that this non-neutrality does not arise from "money illusion" since the real variables in the model are homogeneous of degree zero in all nominal variables. Actually, our model is very "classical" in all other respects except that there are two exogenous nominal assets.
In this setting, an expansionary open market operation causes the nominal exchange rate to overshoot its long run equilibrium value. Both the long run and the short run value of the nominal exchange rate increase if the interest service account is "small". The real exchange rate also overshoots its long run value, if the interest service account is small, but this overshooting is in the "wrong" direction. The long run value of the real exchange rate appreciates in response to the monetary expansion, while on impact its jump depreciates.

The implication of this is that when we look at real changes, it is no longer true that under perfect foresight, the impact effect of an exogenous shock is necessarily in the same direction as the long run effect. And changes in money supply (whether in its level or its rate of growth) in most cases are real shocks if the government budget constraint and the central bank balance-sheets are modelled carefully.

In the second section we set out the model and its long run properties. The next two examine the behaviour of the nominal exchange rate and the real exchange rate respectively, following an increase in the money supply through an expansionary open market operation. The fifth section briefly examines the case where these changes are preannounced. Finally, the sixth section analyses the conclusions that emerge.
2. The Model

The model used is an extension of Branson's model of the asset markets, see Branson (1977) or Branson, et al. (1977). It includes a goods market and it enables us to study the stock-flow interaction following an asset market intervention. This aspect was, at best, sketchy in the Branson papers.

We make the following five simplifying assumptions. First, domestic output is at its unique full employment level due to wage price flexibility. Second, all capital gains are saved. Third, the private sector's expenditure does not depend on the real (domestic) rate of interest. Fourth, the Government always balances its budget. Finally, the domestic economy is assumed to be in a net creditor position vis-a-vis the rest of the world.

The first four assumptions help us to reduce the dynamics of the model to two differential equations. Otherwise more differential equations in the price of domestic output, money stock or the stock of bonds would have to be added. The fifth assumption, while not crucial to the results obtained in this paper, helps us get around the ambiguities which would surround the signs of various wealth effects, otherwise.

The economy produces a good which is different from the one produced by the rest of the world. The home economy is large enough to determine the price of the domestic good and the interest rate on the
domestic bond. It takes the foreign interest rate and the foreign currency price of its importable as given. The foreign rate of inflation is assumed to be zero.

The model is set out in the following equations.

\[
\frac{M_d}{P} = m(i, i^* + x, qy/P, W) \tag{1}
\]

\[m_1 < 0 \quad m_2 < 0\]

\[0 < \frac{m_3}{m} \cdot \frac{qy}{P} < 1 \quad 0 < m_4 < 1\]

\[
\frac{B_d}{P} = b(i, i^* + x, qy/P, W) \tag{2}
\]

\[b_1 > 0 \quad b_2 < 0 \quad b_3 < 0 \quad 1 > b_4 > 0\]

\[
\frac{eF_d}{P} = f(i, i^* + x, qy/P, W) \tag{3}
\]

\[f_1 < 0 \quad f_2 > 0 \quad f_3 < 0 \quad 1 > f_4 > 0\]

\[P = P(q, eP^*) \tag{4}\]

\[W = \frac{M + B + eF}{P} \tag{5}\]
\[ x = \frac{De}{e} \]  

\[ m_1 + b_1 + f_1 = 0 \]  (7(a))

\[ m_2 + b_2 + f_2 = 0 \]  (7(b))

\[ m_3 + b_3 + f_3 = 0 \]  (7(c))

\[ m_4 + b_4 + f_4 = 1 \]  (7(d))

\[ B^c = M \]  (8)

\[ B^t - B^c = B \]  (9)

\[ M = P \cdot m( ) \]  (10)

\[ B = P \cdot b( ) \]  (11)

\[ t = \frac{IB}{P} + G \]  (12)

\[ \frac{qy}{P} = A(\frac{qy}{P} + \frac{IB}{P} - t + i*EF/P, \bar{W} - W) + G + T(A, eP*/q) \]

\[ 0 < A_1 < 1 \quad A_2 < 0 \quad -1 < T_1 < 0 \quad T_2 > 0 \]  (13)

\[ \bar{W} = \frac{kqy}{P} \]  (14)

\[ y = \bar{y} \]  (15)

\[ eDF/P = T(A, eP*/q) + i*EF/P \]  (16)
where

\[ M = \] nominal stock of domestic money

\[ B = \] domestic bonds denominated in domestic currency available to wealth holders

\[ P = \] the consumer price index

\[ i = \] the domestic nominal interest rate

\[ i^* = \] the foreign interest rate (nominal and real)

\[ e = \] the exchange rate, the domestic currency price of foreign exchange

\[ x = \] the expected rate of depreciation of the domestic currency

\[ \frac{De}{e} = \] the actual rate of depreciation of the domestic currency

\[ q = \] domestic currency price of domestic output

\[ y = \] the level of output

\[ \bar{y} = \] the full employment level of output

\[ W = \] real (financial) wealth of domestic wealth holders

\[ p^* = \] foreign currency price of the foreign output

\[ B^c = \] the central banks holdings of domestic securities

\[ b^t = \] total domestic bonds in existence

\[ G = \] government expenditure on goods and services, assumed to be denominated in units of the consumption basket

\[ t = \] lump-sum taxes denominated in units of consumption

\[ \bar{W} = \] target level of wealth.
For any variable $X$, $X_i$ denotes the partial derivative with respect to its $i$th argument.

Also $DX = \frac{dx}{dt}$, the derivative of $X$ with respect to time.

Equations (1), (2) and (3) are the demand functions for the three assets.

The first two arguments in the money demand function (equation (1)) relate to the opportunity costs of holding money rather than alternative assets. The nominal return on money is fixed at zero and on domestic bonds it is $i$. The expected return on the foreign bond is the foreign rate of interest $i^*$, plus the expected rate of depreciation of the domestic currency. The assumption of perfect foresight, equation (6), implies that except when new information is received the expected and actual rates of depreciation are equal. The third argument is intended to be a proxy for the transactions demand for money. While output ($y$) is always assumed to be at its full employment level ($\bar{y}$) (equation (15)), its real real value in terms of the consumption basket is free to vary. There is considerable disagreement on the appropriate choice of the transactions variable and our choice is no more arbitrary than most in the literature. We assume economies of scale in the use of transactions balances and, therefore, the elasticity of demand for such balances is less than unity. The demand for real balances is assumed to depend non-negatively on real wealth.
The demand for the other two assets (equations (2) and (3))
depend positively on own returns and negatively on gross returns (i.e.,
the assets are gross substitutes). When there is an increase in the
transactions demand for money, the demand for at least one type of bonds
must fall. The demand for both bonds depend positively on wealth.
Notice that all real variables are expressed in terms of the consumption
basket and deflated by the CPI defined in equation (4).

The price index is homogeneous of degree one in prices of the
domestic good and the domestic currency price of importables. We refrain
from writing the price index in Cobb-Douglas form because with real
disturbances, the shares of domestic and foreign outputs are not constant
even in the long run.

Equation (5) is the definition of real wealth of domestic
residents and equals the real value of their holdings of the three assets.

Equation (6) is the assumption of perfect foresight.

Equations (7a) to (7d) give the "adding-up" or "balance-sheet"
constraints that equations (1) to (3) must satisfy. Further the positive
partial derivative in equations (7a) to (7c) is assumed to be greater
than either of the other two, in absolute value. In view of equation (7)
only two of the asset demand functions are independent and one of them
can be dropped.
Equation (8) is the central bank's balance sheet. The supply of high powered money is equal to the central bank's holdings of private security. This equation assumes away a fractional reserve banking system and any holdings of any foreign securities by the central bank.

Equation (9) states that the total bonds available to domestic wealthholders is the total value of bonds in existence less the central bank's holdings of these.

Equations (10) and (11) are the asset market equilibrium conditions. When the nominal supplies of money and domestic bonds equal the nominal demand for them, all three asset markets clear. We need not, therefore, write down the equilibrium condition for the foreign asset market explicitly.

As mentioned above, we assume that the government continuously balances its budget. This is given by equation (12). Its expenditure consists of interest payment on the bonds held by the public and on currently produced domestic goods and services. For simplicity we assume that this demand is specified in units of the consumption basket. This simplifies the analysis but does not change any result. The government (including the central bank) holds no foreign bonds and thus receives no interest on them. The budget is balanced by changing the level of lump sum taxes. All taxes are lump-sum.

The market for domestic output clears when the demand for it equals...
its supply (equation (13)). Four points should be noted about this equation. First, although output is always at its full employment level (equation 15), the real value of output changes whenever the terms of trade change. Second all capital gains are assumed to be saved and thus do not figure in the equation. Capital gains arise either due to changes in the price level \( \frac{-\text{DPW}}{p} \) or due to exchange rate changes \((\text{De.F})\). Third absorption does not depend on interest rates. Finally, the wealth effect is expressed in terms of the gap between current wealth and the desired value (given by equation (14)). If the wealth effect term takes this form then in the long run equilibrium there are no wealth effects.

The absorption function in equation (13) is capable of generating a stock-adjustment type savings function, so popular in the literature, where the coefficient of adjustment depends on the disposable income but not on the real rate of interest.

The total expenditure by domestic residents is determined by their disposable income and the gap between target and actual wealth. Disposable real income is the real value of output plus the real value of interest receipts on the two types of bonds minus taxes. We have not shown capital gains because these are assumed to be saved.

The trade balance depends negatively on the total expenditure by the private sector (since the government does not demand any foreign output), and positively on the relative price of domestic output \((eP^*/q)\). The Marshall-Lerner condition is assumed to be satisfied at all times.
Equation (14) defines desired wealth, which is a multiple of real value of current labour income, in accordance with life cycle principles.

Equation (15) states that output is always at its full employment level.

Finally equation (16) gives the increase in the domestic residents claims on the rest of the world - the current account surplus. This is equal to the trade balance plus the interest service account (all in real terms).

For later derivations it is convenient to rewrite (1) and (2) using (10) and (11) as follows

\[ M = P \cdot m(i, i^* + De/e, qy/PW)W \] 
\[ B = P \cdot b(i, i^* + De/e, qy/PW)W \]  

This follows from the homogeneity of degree one of m and b in W.

We also substitute equations (5), (12), (14) and (15) into (13) to get

\[ \frac{qy}{P} = A(qy/P + i^*eF/P - G, kqy/P - \frac{M+B+eF}{P}) + G + T(A, eP^*) \frac{q}{q} \]
For a given state of expectations (17), (18) and (19) would determine the values of the endogenous variables i, q and e, given the exogenous variables and F. Over time the dynamics would be determined by the current account and any revisions in the expectations. But in our model agents are endowed with perfect foresight and therefore we cannot take the expectation of a change in the exchange rate as given. This depends on the entire future time path of the exogenous variables from now to infinity. We first turn to the long run solution of the model.

**Long Run Equilibrium**

Since there are two exogenous variables expressed in nominal units, namely M and B, the model does not exhibit neutrality in the long run. This in no way implies money illusion. A doubling of the money stock would raise prices by a proportion less than two because a doubling of the price level would lower real wealth. Non-neutrality implies not money illusion but simply that the value of the real variables is not independent of the composition of the nominal exogenous variables. Hence a nominal shock is a real shock as well.

In the long run equilibrium, which is a stationary state, we must have $De = DF = 0$.

The four endogenous variables are e, F, i and q, and four equilibrium
conditions can be derived from equations (16), (17), (18) and (19), using (14). Later we consider what happens to the real exchange rate

\[ M = m(\bar{I}, i^*, \frac{1}{k})kqy \]  

(20)

\[ B = b(\bar{I}, i^*, \frac{1}{k})kqy \]  

(21)

\[ kqy = M + B + \bar{e}F \]  

(22)

\[ T(A, \bar{e}P^*/q) + i^*\bar{e}F/P = 0 \]  

(23)

(A bar over a variable denotes its long run value).

Note that (22), (23), (12) and (13) imply that the entire disposable income is being consumed in the long run equilibrium.

**Long Run Comparative Statics**

We are primarily interested in the effects of an expansionary open-market operation by the central bank. Before we analyse the long run effects of such an intervention, it would be useful to look at some of the other long-run comparative-static results.

First, equations (20) and (21) determine the long run values of \( i \) and \( q \). This is due to the equality of desired and actual wealth in the long-run and the way we have specified target wealth. Given the value
of \( \dot{q} \) from these two equations we determine \( e \) and \( \dot{F} \) jointly from equations (22) and (23). Second, as in Branson's model, \( \frac{di}{di^*} \) cannot be signed - the sign depends on whether domestic money is a better substitute for foreign bonds than domestic bonds. If the two bonds are closer substitutes then \( \dot{I} \) rises. Finally, even though the exchange rate enters multiplicatively with the foreign price level or the stock of foreign assets everywhere, an increase in \( P^* \) is not offset by an appreciation of the exchange rate. If a complete offset was provided then the current account would go into deficit because the interest service account would now be smaller. To prevent this from happening the stock of foreign assets must rise. Thus, the model does not generate what could be considered the minimum insulation requirement of a system of flexible exchange rates - that it insulate the domestic economy from a once-and-for-all change in the foreign price level. This would be true even if the model was neutral as long as the foreign bond is not denominated in terms of the foreign output.\(^9\)

Before turning to the long-run consequences of an expansionary open market operation, it should be mentioned that the interest service account complicates the analysis considerably. In what follows, wherever there are ambiguities, we shall assume that this term is "small".\(^{10}\)

Following an open market operation \( dM = -dB \) or \( m^M = -b^B \) (where a (*) denotes percentage change), the interest rate falls and the price of
domestic output rises in the new long-run equilibrium. The interest
rate fall eliminates both the excess supply of money and the excess
demand for bonds. The rise in the price of domestic output creates
excess demand for both assets (via increased nominal wealth). These
effects are given by equations (24) and (25):

\[ \frac{d\bar{\delta}}{dM} = \frac{(b+m)}{PW(m_1 b-b_1 m)} \quad (24) \]

\[ \frac{d\bar{q}}{dM} = f_1/\frac{PW(m_1 b-b_1 m)}{\bar{q}} \quad (25) \]

The domestic currency value of foreign asset rises as is evident
from equation (22). On the left hand side \( \bar{q} \) rises, so on the right hand
side \( \bar{eF} \) must rise by the same proportion since the sum \( M + B \) is unchanged.
The stock of foreign bonds rises if the interest service account is
small (equation 26)

\[ \frac{\hat{F}}{dM} = \frac{1}{\Delta} (PW_f (M + B) (-T_1 qy e_2 /P + (1+T_1) i* eF e_1 /P + T_2 eP* /q)) \quad (26) \]

where \( \Delta = [- T_1 qy e_2 /P - (1+T_1) i* eF e_2 /P + T_2 eP* /q] \cdot eFPW^2 (m_1 b - b_1 m) \)

and \( \varepsilon_i \) is the elasticity of the price index with respect to its ith
argument (\( i = q, eP* \)).
The denominator in equation (26) is ambiguous in sign but would be negative for reasonable value of $i^*$ and $e_2$ (the share of the foreign good in the price index). If we assume this, then the stock of foreign bonds rises in the new long run equilibrium.

The interpretation of this result is as follows: a fall in the domestic interest rate and a rise in the price level raises the demand for (the domestic currency value of) foreign bonds. This could be achieved either by an exchange rate depreciation or an increase in $\bar{F}$.

The exchange rate and the stock of foreign bonds enter multiplicatively everywhere except in the price index and the relative price term. The term in the denominator, which is ambiguous in sign, is the difference between the current account effects of an increase in $e$ and of an increase in $\bar{F}$. The increase in $\bar{F}$ has no effect on the long run trade balance but has a larger effect than the exchange rate on the interest service account, because the exchange rate also enters the price index.

The effect of an open market operation on the nominal exchange rate is also ambiguous. This is given by equation (27). There is now an ambiguity in the numerator as well. For small $i^*$, we can still assume that the term inside the square brackets in the numerator is positive. So an expansionary open market operation depreciates the nominal exchange rate.

\[
\frac{\hat{e}}{dM} = \left[1 - (1+T_1)(M+B)i*/PD\right]f_1/W(mmbmbm) \tag{27}
\]

where $D = [-T_1qT^*_2/P - (l+T_1)i^*eF_2/P + T_2eF*/q]$. It is assumed positive above.
We finally wish to know the impact on the real exchange rate \( c \). We define it as \( e/P \). Others prefer to write the real exchange rate as \( e/q \). Our definition of \( c \) makes the algebra less messy and it is positively related to \( e/q \).

The real exchange rate definitely appreciates if \( \Delta \) is \( < 0 \) as is shown in equation (28)

\[
\frac{\hat{c}}{\hat{d}M} = \frac{- (1 + T_i) i \* \hat{c} \hat{F} \* (M + B) k y f \_}{\hat{A}}
\]

(28)

So if the interest service account is not very large, an expansionary open market operation raises \( \bar{q} \), \( \bar{e} \) and \( \bar{F} \) and lowers \( \bar{I} \) and \( c(=e/P) \).
3. Dynamics of the Nominal Exchange Rate

To analyse the motion of our economy over time it is convenient to reduce the system of equations (16) to (19) to two differential equations in terms of the state variables \( e \) and \( F \). Since both \( e \) and \( F \) are positive (the latter by assumption) we shall work with the logarithms of these variables.

We can solve equations (17), (18) and (19) for \( i, q \) and \( D\text{ln}e \) in terms of \( e \) and \( F \). In addition to the assumptions made so far we also assume that a depreciation creates an excess demand in the domestic goods market. A depreciation by raising the price level reduces the value of the output in terms of the consumption basket. It also diverts domestic and foreign demand away from the foreign good towards the domestic good, raises the interest income from foreign bonds and raises the domestic currency value of foreign bonds. All these tend to increase the demand for domestic output. But an increase in the price level also lowers the real value of \( M + B \). We have thus assumed that this effect is smaller than the excess demand creating effects. Note that this assumption is not required if \( f \geq \epsilon_2 \).

Given this additional assumption we have the first differential equation

\[
D\text{ln}e = \psi(\text{ln}e, \text{ln}F; \ M, B)
\]

\[
\psi_1 > 0 \quad \psi_2 > 0
\]
(The values of $\psi_1$ and $\psi_2$ and also $\pi_1$ and $\pi_2$ in equation (30) are given in the Appendix).

By substituting the solution of $q = q(e, F)$ into equation (16) we have the other differential equation

$$D\ln F = \pi(lne, lnF; M + B)$$

$$\pi_1 > 0 \quad \pi_2 = ?$$

Note that $M$ and $B$ feed through separately (through the asset market equilibrium conditions) into equation (29) but enter as a sum in equation (30) (through wealth effects only).

While $\pi_1$ is positive, it is not possible to sign $\pi_2$ unambiguously. Intuitively, an increase in $F$ tends to improve the current account via the interest service account, but worsen it due to increased expenditure, part of which falls on importables, resulting from increased wealth. The net effect is indeterminate.

Linearising the system of differential equations (29) and (30) around the long run equilibrium where $D\ln e = D\ln F = 0$, we have,

$$(D\ln e) = \begin{pmatrix} \psi_1 & \psi_2 \\ \pi_1 & \pi_2 \end{pmatrix} \begin{pmatrix} lne - lne^- \\ lnF - lnF^- \end{pmatrix}$$ (31)
Given the signs of the elements of the matrix of coefficients, the $D\ln e = 0$ curve must always be downward sloping in the $(\ln F, \ln e)$ space.

The $D\ln F = 0$ schedule could be either upward or downward sloping. If this schedule is upward sloping (figure 1) or downward sloping but flatter than the $D\ln e = 0$ schedule (figure 2), the long run equilibrium defined by the intersection of these two schedules is a saddle point. The arrows show the direction of motion of the variables and are obtained from equation (31). The stable arm in both the cases is downward sloping and flatter than $D\ln e = 0$. The unstable arm is positively sloped (and in figure 1, steeper than $D\ln F = 0$ locus).

Finally if the $D\ln F = 0$ locus is negatively sloped but steeper than the $D\ln e = 0$ curve, then the long run equilibrium is completely unstable. The economy cannot reach this equilibrium unless it happens to start there. We shall assume that the interest service account is small enough and rule out this last case (shown in figure 3).

Returning to figures 1 and 2, we see that only if the economy is on the saddle path will it reach the long run equilibrium. All other perfect foresight paths simply do not converge. We shall assume that the economy is always on the stable manifold. This is ensured by an appropriate jump in the exchange rate, whenever the system is subjected to a shock. The stock of foreign bonds is given at a point in time and
Figure 1

Figure 2
can be accumulated at a finite rate only by running current account surpluses. In assuming long run perfect foresight we invoke the transversality condition of optimising models without a rigorous justification (see Brock (1974), (1975)).

Now imagine a central bank intervention in the asset market involving a swap of domestic bonds for domestic money. Under the assumptions of the previous sections this leads to a higher steady state value of both \( lne \) and \( lnF \). In figure 4, therefore, we have the new long run equilibrium \( E_1 \) lying to the north-east of the old one \( E_0 \).

If the economy is to converge to the new long run equilibrium, then agents must choose that value of \( e \) which puts them at point \( E^R \), so that following the shock the economy is on the stable arm of the new equilibrium. The jump must occur at the moment the unanticipated purchase of bonds takes place since a jump in the exchange rate implies an infinitely large return on foreign bonds. If a jump was to take place at a future date then agents with perfect foresight would move now to eliminate these profitable opportunities in the future.

The new long run equilibrium is the intersection of the new \( DlnE = 0 \) schedule, which lies above the old schedule and the \( DlnF = 0 \) schedule which does not shift. This is because \( DlnF = 0 \) schedule unlike \( DlnE = 0 \) curve, depends not on the composition of \( M \) and \( B \) but just their sum.
So the nominal exchange rate overshoots its long run equilibrium value following the open market operation. The instantaneous effect on the other endogenous variables is as follows: \( q \) rises, \( \text{Dln}e \) is negative and the effect on \( i \) is uncertain (depends on, among other things, whether money or domestic bonds are better substitutes for foreign bonds).

As the economy moves from \( E_{01}^R \) to \( E_1 \), the exchange rate appreciates and the stock of foreign bonds rises, through current account surpluses. The adjustment is monotonic since the roots of the matrix of coefficients of the differential equation are real. If the equilibrium is a saddlepoint then the determinant of matrix \( A \) in equation (31) is negative.

Also note that the overshooting of the exchange rate is less with perfect foresight than if agents possessed static expectations. The adjustment path then would be along the \( \text{Dln}e' = 0 \) schedule and the momentary equilibrium value of the nominal exchange rate would be at \( E_{01}^S \) following the shock.
4. Dynamics of the Real Exchange Rate

In order to examine the dynamics of \( c(e/P) \) the real exchange rate, it is convenient to rewrite equations (16) to (19) with the wealth identity as

\[
M/P = m(i, i^* + D1ne, h(c)Y)W
\]  

(32)

\[
B/P = b(i, i^* + D1ne, h(c)Y)W
\]  

(33)

\[
W = \frac{M + B}{P} + cF
\]  

(34)

\[
h(c)Y = A(h(c)Y + i^*cF - G, kh(c)Y - W) + G + T(A, g(c)P^*)
\]  

(35)

\[
cDF = T(A, g(c)P^*) + i^*cF
\]  

(36)

where \( h(c) = q/P, \quad h' < 0 \)

and \( g(c) = e/q, \quad g' > 0 \).

We can solve equation (35) for \( W \) in terms of \( \ln c \) and \( \ln F \) alone given \( M + B \).

\[
W = j(\ln c, \ln F) \quad j_1 < 0, \quad j_2 < 0.
\]  

(37)
The signs of $j_1$ and $j_2$ follow since either an increase in $c$ or $F$ creates an excess demand in the domestic goods market and therefore requires a decline in wealth to eliminate it. Substituting equation (37) into (36) we get

$$D\ln F = \phi(\ln C, \ln F; M + B)$$

$$\phi_1 > 0 \quad \phi_2 > 0$$

Note that $\phi_2$ is unambiguously positive and equal to $i^*$ and

$$\phi_1 = \frac{T_1 y h^c + i^*cF(1+T_1) + T_2 cP^*g')}{cF(1+T_1)}$$

To get the other differential equation for $Dlnc$, we first solve for $i$ and $P$ from equations (32) and (34) and use (37). Substituting in equation (33) we get an equation for $Dlne$ as a function of $\ln c$ and $\ln F$

$$Dlne = \xi(\ln c, \ln F; M, B)$$

To get a value for $Dlnc$, we differentiate equation (34) with (37) substituted in to get,

$$D\ln P = \gamma(Dlnc, D\ln F; M + B)$$

Subtracting (40) from (39) we get an expression for $Dlnc$.

$$Dlnc = \delta(\ln c, \ln F; M, B)$$
Unfortunately, \( \delta_1 \) and \( \delta_2 \) are very messy and there is no hope of signing them.

Linearising equations (38) and (41) around the long run equilibrium where \( \text{Dln}c = \text{Dln}F = 0 \) we have

\[
\begin{pmatrix}
\text{Dln}c \\
\text{Dln}F
\end{pmatrix} = 
\begin{bmatrix}
\delta_1 & \delta_2 \\
\phi_1 & \phi_2
\end{bmatrix}
\begin{pmatrix}
\text{ln}c - \text{ln}\bar{c} \\
\text{ln}F - \text{ln}\bar{F}
\end{pmatrix}
\]

(42)

What can be said about the properties of the system represented by equation (42)? First, from equation (38), \( \text{Dln}F = 0 \) locus is downward sloping in the \((F, c)\) plane in figure 5. The slope of this curve gets flatter as the foreign interest rate gets smaller. In the limiting case where \( F \) is foreign currency, or the interest service account is ignored, the \( \text{Dln}F = 0 \) locus is horizontal.

Second, the \( \text{Dln}c = 0 \) locus must be downward sloping. This is because we know from the previous section that with static expectations the exchange rate depreciates on impact following an open market operation. The price of domestic output \((q)\) must also rise to maintain goods market equilibrium, with \( F \) fixed. But an equiproportionate increase in \( e \) and \( q \) creates excess supply in the goods market because \( e \) and \( q \) enter symmetrically everywhere except in the term \((M + B)/P\) where increases in either lower the real value domestic assets and thus create an excess supply of domestic goods. So on impact
\[ \hat{e} > \hat{q} \text{ or } (\hat{P}) \text{ and } \hat{c} > 0. \text{ So the new short run equilibrium lies vertically above the old one (on the new } Dln\text{c} = 0 \text{ locus).} \]

With perfect foresight, the initial jump in \( e \) is smaller, and therefore the jump in \( q \) is also smaller. Thus the impact effect on \( c \) is also smaller. So the stable arm must be flatter than the \( Dln\text{c} = 0 \) locus but steeper than the \( Dln\text{F} = 0 \) locus. The unstable arm is upward sloping.

Now consider the effects of an expansionary open market operation. We know that in the long run, given our assumptions about parameter values, the stock of foreign bonds increases and the real exchange rate appreciates because with a higher stock of foreign bonds and an unchanged real exchange rate the current account would move into surplus. The real exchange rate appreciation brings about current account balance. So it is the interest service account which causes the real exchange rate to appreciate. In currency substitution models, e.g. Calvo and Rodriguez (1977), the current account balance schedule is horizontal and the long run real exchange rate is determined exclusively by the trade balance.

In figure 6, the original long run equilibrium was at \( E_0 \) and the new long run equilibrium is at \( E_1 \). Both lie on the \( Dln\text{F} = 0 \) schedule. This schedule does not shift as it depends on \( M + B \) and not the composition of that sum. The \( Dln\text{c} = 0 \) line moves to the right to intersect the \( Dln\text{F} = 0 \) at \( E_1 \). The new stable arm is \( S'S' \). In order to get to the long run equilibrium \( E_1 \), rational agents must put the economy at point \( R_{01} \). The impact effect of the real exchange rate is a depreciation,
while the long run effect is an appreciation. So even with rational expectations, the long run and short run effects on an endogenous variable of a shock need not be in the same direction. Kouri's (1976) claim that it must be so is therefore not applicable to non-neutral disturbances. Thus the exchange rate overshoots but in the wrong direction. We dub this - "perverse overshooting".

For $E^R_{01}$, the real exchange rate appreciates monotonically to the new equilibrium as foreign assets are accumulated by the economy running current account surpluses.

It is worth mentioning that Obstfeld (1980) had got similar results for the fixed exchange rate case. The variable that jumped in the "wrong" direction there was the price of domestic output. The perverse overshooting in his model was also due to the interest service account.
5. Anticipated Future Shocks

Let us briefly consider the following question: can an announcement now (at time zero) that a certain increase in M through an open market operation is going to be implemented in the future (on date T) reduce the extent of overshooting? In particular, can we get rid of perverse overshooting. The answer to the first question, as we shall see, is in the affirmative, but it is not possible to get rid of perverse overshooting altogether.

Three points should be kept in mind. First, any jump in e or C must occur when the news of a policy change first arrive, ie, jumps are associated with unanticipated "news". Arbitrage rules out any future jumps. If jumps were allowed for t ≠ 0, this would imply opportunities of profits which rational agents in our model were ignoring. Second, between 0 and T, the dynamics of the system is defined with reference to the old long run equilibrium because the policy has not been implemented yet. And finally, from t = T onwards the dynamics is defined with reference to the new long run equilibrium and the system is on the stable manifold with reference to this equilibrium.

Let us first analyse the real exchange rate case, depicted in Figure 7. E₀ is the initial long run equilibrium. S₀S₀ is the stable arm of E₀ whereas U₀U₀ is the unstable arm. U₀U₀ is upward sloping. S₁S₁ is the stable arm of the new long run equilibrium, E₁. An
Figure 7.

Figure 8
unanticipated expansionary open market operation implemented immediately, causes a jump in the real exchange rate to a point such as J. An announcement that the same asset swap is going to be implemented $T_1$ periods from now leads to a jump to point A, as rational agents adjust their portfolios in anticipation of the policy change. From 0 to $T_1$ the path followed by the economy is given by AM. The point M is reached on date $T_1$ when the policy is put into effect. There are no jumps in C (ie $C(T^-_1) = C(T^+_1)$), other than the initial one from $E_0$ to A. If the policy was to be implemented on $T_2 > T_1$, then the initial jump would be smaller, from $E_0$ to L, and the path followed by the economy would be LN, reaching N on date $T_2$.

But it is impossible for the authorities to avoid perverse over­shooting altogether. The longer is the gap between announcement and implementation, the smaller the initial jump but an initial jump there must be for all finite T. Also for anticipated changes, the current account is in surplus but the real exchange rate is depreciating between A and M or L and N, whereas for an unanticipated increase in money supply implemented immediately, the popular notion of a covariation of an appreciating real exchange rate and a current account surplus was maintained (see Dornbusch and Fischer (1980)).

For the nominal exchange rate the story is quite different. The unstable arm of the initial long run equilibrium ($E_0$ in Figure 8) is, as before, upward sloping. But it must intersect the stable arm of the new long run equilibrium to the left of that long run equilibrium
position, $E_1$, since from the real exchange rate case, we know that the current account must be in surplus between $E_0$ and $E_1$.

Again, the longer the lag between announcement and implementation of the policy change, the less the jump in the nominal exchange rate. $J$ is, as before, the result of an unanticipated immediately implemented policy, $A$ when the policy is going to be implemented on $T_1$ and $L$ when the policy is going to be implemented on $T_2$. $J$, $A$, $L$, respectively are the response on date zero. $J$, $M$, and $N$ are, as before, the points on the stable arm of the new long run equilibrium when the policy is put into effect.

Four points should be noted for the nominal exchange rate case. First, the magnitude of the jump becomes less as the day the policy is to be implemented gets to be farther into the future. Second, for all policies to be implemented in the future, the pattern is a jump depreciation followed by further depreciation till the new stable arm is reached; from there to $E_1$ the exchange rate appreciates. Third, overshooting can be eliminated if the gap between announcement and implementation is sufficiently long (as for $T_2$). Finally, although overshooting can be eliminated, it is still true that the date on which the policy is implemented, the exchange rate is above the long run value. So, fluctuations in the nominal exchange rate in excess of that required to establish the new long run equilibrium position do still take place.
Conclusions

In this chapter we have analysed the impact and long run effects of an expansionary open market operation on the real and nominal exchange rate. The increase in money supply is a real shock in a model where neutrality does not obtain.

The main conclusions can be summarised as follows: (1) The nominal exchange rate overshoots its long run value following an unanticipated expansionary open market operation. (2) But the real exchange rate overshoots in the wrong direction. The long run effect on the real exchange rate is an appreciation, while in the short run it depreciates. This must happen to push the current account into surplus. The economy would end up with a higher stock of foreign bonds in the new long run equilibrium and current account surpluses are necessary in the adjustment process. (3) A preannounced increase in the money supply can prevent the overshooting of the nominal exchange rate, provided there is a sufficiently long gap between announcement and implementation. Even then, at some point the value of the nominal exchange rate would be in excess of the new long run value. (4) While the perverse overshooting in the real exchange rate can be reduced as the time that elapses between announcement and implementation gets longer, it cannot be eliminated altogether. (5) The fluctuations that do take place in the nominal and real exchange rates, when the policy is unanticipated and implemented immediately, are less with rational expectations than static expectations.
The main implication of the analysis is that for a non-neutral shock the impact and long run effects on endogenous variables could be in different directions. Where this happens, the presence of perfect foresight and a long lead time in implementation of the policy may reduce the fluctuations in the variables concerned, but cannot get rid of it altogether.

It is interesting to note in this context that with an identical model but where the domestic and foreign goods are perfect substitutes (i.e. purchasing power parity (PPP) holds), Turnovsky (1981a) shows that the nominal exchange rate overshooting need not occur. In Eaton and Turnovsky (1983a), this result is quoted as demonstrating that with flexible prices and imperfect asset substitutability nominal overshooting may not occur. Our analysis shows that these two assumptions are not responsible for that conclusion. It is the assumption of PPP which generates causes of undershooting.12.
In equation (31),

\[ \psi_1 = \frac{eF}{PD} ZPW (-m_1 b + b_1 m) + (Z - A_2 W)(m + b)qy(-m_2 b_3 + m_3 b_1), \]

where \( Z = qy\xi_2 (1-A_1 (1+T_1) - A_2 k(l+T_1)) + i\frac{eF}{P} A_1 (l+T_1) \)

\[ - A_2 W (l+T_1) + T_2 \frac{eP^*}{q}, \]

and \( D = eF Z P^2 W^2 (m_1 b_2 - m_2 b_1) > 0; \)

\[ \psi_2 = \frac{eF}{PD} [(A_2 (l+T_1) eF - A_1 (l+T_1) i\frac{eF}{P} PWqy(m_1 b_3 - m_3 b_1)] \]

\[ + Z PWEF(-m_1 b (1-\eta_{23}) + b_1 m (1-\eta_{13}))], \]

where \( \eta_{13} = \frac{m_3 qy}{m PW} \) and \( \eta_{23} = \frac{b_3 qy}{b PW} \),

\[ \pi_1 = \frac{P^2 W^2 (m_1 b_2 - m_2 b_1)}{D} \left[ T_1 qy\xi_2 / P - i\frac{eF}{P} (l+T_1) - T_2 \frac{eP^*}{q} \right] [A_2 M+B]. \]

and \( \pi_2 = \frac{P^2 W^2 (m_1 b_2 - m_2 b_1)}{D} \left[ Zi\frac{eF}{P} + (A_2 (l+T_1) eF - A_1 (l+T_1) i\frac{eF}{P} \right] \)

\[ (-\frac{qy}{P} \frac{\xi_2}{1+T_1} + i\frac{eF}{P} \frac{\xi_1}{1+T_1} + T_2 \frac{eP^*}{1+T_1}) \]

In equation (39),

\[ \ell_1 = \frac{1}{\Delta} \left[ -(m+b) (X - A_2 (l+T_1) W h/c) (m_1 b_2 \eta_{23} - b_1 m_1 \eta_{13}) \right. \]

\[ - (X + A_2 (l+T_1) W) (m_1 b - m b_1)] > 0 , \]
where \( \Delta = A_2(1+T_1)(m+b)(m_1b_2 - m_2b_1) < 0, \)

and \( X = (1-A_1(l+T_1) - A_2k(l+T_1)h'c'y - A_1(l+T_1)i*cF - T_2f*g'c < 0, \)

\[
\ell_2 = \frac{1}{\Delta} \left[ (m_1b_{23} - b_1m_{13})(m+b)A_1(l+T_1)i*cF + (m_1b - b_1m)f(-A_2(l+T_1) + A_1(l+T_1)i*cF) \right] > 0
\]

\[
\gamma_1 = \frac{1}{\Delta} (m_1b_2 - m_2b_1)(X + A_2(l+T_1)cF) > 0,
\]

\[
\gamma_2 = \frac{1}{\Delta} (m_1b_2 - m_2b_1)(A_2(l+T_1)cF - A_1(l+T_1)i*cF) > 0.
\]
FOOTNOTES

1. This literature originated with Dornbusch (1976). In these models changes in wealth through current account imbalances are ignored.

2. Representative papers here are Kouri (1976) and Calvo and Rodriguez (1977). Henderson (1980) attempts a synthesis of the two approaches when price and exchange rate expectations are regressive.


5. In our model the supply of labour is inelastic. If the supply curve was upward sloping and the supply price of labour is the nominal wage deflated by a consumer price index, which includes the price of imported goods, then full employment output would vary in response to a change in the real exchange rate.


7. See Dornbusch (1980) Chapter 4 Appendix

8. See Fischer (1981) for a discussion on the specification of wealth effects.

9. See e. g., Turnovsky (1979).

10. This has been recognised by many authors previously e. g., Rodriguez (1979), Branson and Buiter (1983), Dornbusch and Fischer (1980) and Allen and Kenen (1980).


12. There are other differences between Turnovsky's model and ours, e.g. he holds only one nominal asset exogenous, the other financing the budget deficit. Also, although he uses regressive expectations, they are incompatible with perfect foresight because the adjustment path is cyclical. This cannot be the case with one stable root.
CHAPTER II

FISCAL POLICY, MONETARY DISINFLATION AND

REAL EXCHANGE RATE
1. INTRODUCTION

Events in the UK and the US over the past few years seem to suggest that a reduction in the rate of inflation by reducing the rate of growth of the money stock may entail substantial costs. Theoretical models capable of explaining these costs, in an open economy context, have been proposed by Buiter and Miller (1981, 1982) and Turnovsky (1981), among others. An unanticipated, immediately implemented reduction in monetary growth causes an immediate appreciation of the real exchange rate. This worsens the trade balance and creates unemployment. The jump appreciation of the real exchange rate can be reduced, but not eliminated, by announcing the implementation of the reduction in monetary growth at a future date. Further, the loss of competitiveness would not occur if the reduction in the rate of growth of money was implemented in conjunction with a rise in the level of the money stock.

This conclusion seems very robust in the face of "sensitivity analysis", as long as it is assumed that there is sluggish adjustment in the goods market but speculators are endowed with forward looking expectations.

The models used to derive these results are variations of Dornbusch (1976). That in turn was a log-linear version of the familiar Mundell-Fleming model with the expected depreciation of the domestic currency included in the interest arbitrage condition. The drawbacks of the Mundell-Fleming
model, which the other models share, are well known. The most important
of these is that it ignores "intrinsic (asset) dynamics" altogether.
Thus, the long run equilibrium of the model does not necessarily imply
either a balanced government budget or a zero current account.

Within the confines of such a model we re-examine the results obtained
by Buiter and Miller and Turnovsky by introducing a government budget
constraint. A reduction in the rate of growth of money, in the absence of
residual bond financing, must be accompanied by either an increase in
taxes or a cut in government expenditure.

The first case, where a reduction in the rate of growth of money is
achieved by raising taxes, with government expenditure constant, generates
the results obtained by Buiter and Miller and Turnovsky, if capital gains
and losses on real balances are taken into account. With government
expenditure constant, a reduction in the inflation tax necessarily implies
an increase in explicit taxes. The long run level of the real exchange
rate is unaffected by the monetary slowdown. In the short run the real
exchange rate definitely overshoots its long run value.

In the second case, when monetary disinflation is accompanied by cuts
in government expenditure on domestic goods and services, the impact
effect on the real exchange rate depends on parameter values. Overshooting,
undershooting and no jumps are all possible candidates. If overshooting
occurs at all, it is in the "wrong" direction. The long run real
exchange rate depreciates while the impact effect of a monetary slowdown
is to generate an appreciation. In the long run, the level of domestic
output returns to its exogenous "natural" rate and the real exchange rate depreciates. The natural rate of output commands fewer units of the consumption bundle (which includes foreign goods whose relative price has risen as the terms of trade move against the home country). Notice that our model possibly understates the extent of the long run "welfare loss" because it takes the natural rate of output as exogenous. If it had been endogenous, and aggregate supply had been responsive to changes in the real exchange rate, the natural rate of unemployment would have risen. A given product wage rate commands fewer units of consumption and thus labour supply falls. This result is independent of the way expectations are generated.

Although in the second case (when government expenditure is cut) on impact the real exchange rate could go in any direction, the cumulative loss of output is identical to the Buiter-Miller case. The cumulative output loss depends on the difference between the value of the real exchange rate following the shock and its new long run value. This has obvious implications for those economists in the UK, who have tried to show that the pound did not "overshoot" in 1979 (see e.g., Annual Monetary Review, Centre for Banking and Finance (1980), p.12).

In Section 2 we set out the model. In Section 3 we examine what happens when a reduction in the rate of growth of money stock is accompanied by an increase in taxes. Section 4 examines the effects of cuts in government expenditure to match the monetary slowdown. Finally in Section 5 the conclusions are discussed.
2. THE MODEL

The model consists of the following equations.

\[ \frac{M}{P} = \ell(i, qy/P) \quad l_1 < 0, \quad l_2 > 0 \]  
(1)

\[ i = i^* + \lambda \]  
(2)

\[ \frac{qy}{P} = A(qy/P - \eta M/P - qt/P, i - \pi) + qG/P + T(A, eP^*/q) \]

\[ 0 < A_1 < 1, \quad A_2 < 0, \quad -1 < T_1 < 0, \quad T_2 > 0 \]  
(3)

\[ \frac{Dq}{q} = j(y-y^*) + \mu \quad j > 0 \]  
(4)

\[ \mu M/q = G - t \]  
(5)

\[ P = \phi(q, eP^*) \]  
(6)

\[ \lambda = De/e \quad ) \]

\[ \pi = DP/P \quad ) \]  
(7)

where \( M \) = the nominal money stock,
\( P \) = the consumer price index,
\( i \) = the domestic nominal rate of interest,
\( q \) = money price of domestic output,
\( y \) = domestic output assumed to be demand determined,
\( i^* \) = foreign nominal interest rate (= foreign real rate),
\( e \) = the exchange rate defined as the domestic currency price of foreign exchange,
\( \lambda \) = the expected rate of depreciation of the exchange rate,
\( t \) = lump sum taxes denominated in terms of domestic output,
\( G \) = government expenditure on domestic goods and services denominated in domestic output,
\( \pi \) = the expected rate of change in the price level,
\( A \) = the private sector's real absorption in terms of consumables,
\( T \) = the trade balance measured in terms of units of consumption,
\( P^* \) = the foreign currency price of the foreign good, given exogenously,
\( \text{D}q/q \) = the time derivative of \( \log q \),
\( \ddot{y} \) = exogenously given natural rate of output,
\( u \) = the rate of growth of the money stock,
\( \text{De}/e \) = the actual rate of depreciation of the domestic currency, the right hand time derivative of \( \log e \),
\( \text{DP}/P \) = the rate of change of the price level.

A prime denotes a derivative, a subscript \( i \), the \( i \)th partial derivative, and D the time derivative (\( \equiv d/dt \)).

Equation (1) is the money market equilibrium condition. It requires the real supply of money be equal to the real demand for it. The demand for money depends negatively on the nominal rate of interest, which is the opportunity cost of holding money, and positively on the real value of domestic output (\( qy/P \)). To simplify later derivations, let us assume, without loss of generality, that the elasticity of the transactions demand for money is unity.
Equation (2) is the interest arbitrage equation. Domestic and foreign bonds are perfect substitutes on a covered basis. Further, agents are assumed to be risk-neutral, so that the forward discount equals the expected depreciation of the home currency. The arbitrage condition always holds except when new information is received.

Equation (3) is the domestic goods market equilibrium condition. Domestic output is demand determined. Both demand and supply of output are expressed in units of the consumption basket. Demand for domestic output arises from three sources - the domestic private sector, the public sector and from abroad. Private sector demand depends on its real disposable income and the real interest rate. Real disposable income equals the real value of domestic output minus the sum of capital losses on real balances (the inflation tax) and lump-sum taxes. There are no wealth effects and capital gains or losses on other items of wealth are also ignored. This is done to keep the model analytically tractable by limiting the dynamics of the system to two differential equations.

Net foreign demand depends on domestic residents total expenditure (a part of which falls on the foreign good) and the relative price of the foreign good. The government demands only domestic goods.

Equation (4) is the expectations-augmented Phillips' curve. The rate of change of the price of domestic output depends on the gap between the
current level of output ($y$) and the exogenous natural level ($\bar{y}$), and on the expected rate of inflation. Expected inflation, as in Buiter and Miller (1981) is assumed to be its long run value, given by the rate of growth of the only exogenous nominal variable $M$. $\mu$ should be interpreted here as the right hand time derivative of $\log M$. The interpretation that we would be attaching to this specification in the analysis later is that the price of domestic output evolves continuously and does not jump in response to new information. We shall also discuss the case where expectations of inflation are fully rational (the case discussed by Buiter and Miller (1982), and Turnovsky (1981)).

The government budget constraint is given by equation (5). The actual growth of the nominal money stock must be equal to the nominal value of the government budget deficit ($q(G-t)$). We assume that government changes either taxes or its expenditure to ensure a given rate of growth of the money supply ($\mu$). Again we are ignoring the interest payment on outstanding government bonds.

Equation (6) is the definition of the price index. $P$ is homogeneous of degree one and increasing in its two arguments $q$ and $eP^*$. 

Equation (7) is the assumption of perfect foresight. The actual and expected rates of depreciation are equal. This holds everywhere except when new information is received. We shall permit the exchange rate to
make a jump at those points. De then is the right hand derivative of e.
The expected rate of inflation is equal to the actual rate of inflation.
The latter is simply a weighted average of the rate of increase of the price of domestic output and the exchange depreciation, since we assume that the foreign rate of inflation is zero.

As noted above, we have not included interest payments on domestic bonds and these do not figure in the definition of disposable income either. Nor does interest payments on bonds denominated in the foreign currency. Wealth is not an argument in any of the equations and it is not even required to be constant in the long run. These are well known drawbacks of the Mundell-Fleming model, of which ours is a variant. (1)
3. A TAX-"FINANCED" CUT IN $\mu$

Suppose the government wishes to reduce the rate of growth of money supply and thereby bring down steady-state inflation. This it does by increasing taxes i.e., it switches from an inflation tax to explicit taxation. The government budget constraint can be written as

$$t = G + \mu m.$$  \hfill (8)

$G$ and $\mu$ are both exogenous and as $m = M/q$ changes $t$ is altered to ensure a balanced budget.

The long run equilibrium conditions are given below. In the steady state equilibrium, all nominal variables grow at the rate of growth of money supply ($\mu$), and all real variables are constant.

Denoting the long run value of an endogenous variable by a bar over it, and given the assumption of unit elasticity of money demand with respect to the real value of output, we can write the long run equilibrium conditions as follows:

$$\bar{m} = L(i^* + \mu)\bar{y}$$  \hfill (9)

$$h(\bar{c})\bar{y} = A(h(\bar{c}) (\bar{y} - G), i^*) + h(\bar{c})G + T(A, \bar{c}P^*)$$  \hfill (10)

where $h(c) = q/P \quad h' < 0$
Note that in (10) because our definition of disposable income included capital losses on real balances due to inflation, we can write the disposable income in terms of $\bar{y}$ and $G$ only.

Equation (9) determines the steady-state value of $m$. A fall in $\mu$, reduces the opportunity cost of holding money and thus raises the steady-state value of $m$ ($\frac{d\bar{m}}{d\mu} = yL' < 0$). On the other hand a fall in $\mu$ does nothing to change competitiveness $c(\equiv e/q)$ ($\frac{dc}{d\mu} = 0$). This is because government expenditure on domestic goods and services is left unchanged and in the long run the sum of the inflation tax and explicit taxes must be equal the unchanged government expenditure. In other words, explicit taxes and inflation tax are identical in their primary burden, although in their secondary burden they may be different.

**DYNAMICS**

Because we assumed unit elasticity for the transaction balances, we can write equation (1) as

$$m = L(i)\bar{y} \quad (1')$$

We can then substitute for $i$, $P$, $\lambda$ and $\pi$ from equations (2), (6) and (7) respectively into (1'), (3) and (4). Then eliminating $t$, $DP/P$ and $\pi$ from the latter set of equations using (5) we can get

$$D\ln m = \psi(\lnm, \ln\mu) \quad (11)$$

$$D\ln c = \theta(\lnm, \ln\mu) \quad (12)$$
In (11) and (12) we have suppressed all other exogenous variables.

Linearising (11) and (12) around the long run equilibrium where \( \text{Dln}\_c = \text{Dlnm} = 0 \), we have

\[
\begin{bmatrix}
\text{Dlnm} \\
\text{Dln}\_c
\end{bmatrix} =
\begin{bmatrix}
A \\
A
\end{bmatrix}
\begin{bmatrix}
\text{lnm} - \ln\bar{m} \\
\text{ln}\_c - \ln\bar{c}
\end{bmatrix}
\]

(13)

where

\[
\begin{align*}
a_{11} &= \frac{m}{\Delta} [A_1 (1-a) (1+T_1) - A_2 a (1+T_1)] < 0 \\
a_{12} &= \frac{b}{\Delta} yL' < 0 \\
a_{21} &= \frac{m}{\Delta} [(1-A_1 (1+T_1) ) y / j' + A_1 (1+T_1) m] < 0 \\
a_{22} &= \frac{b}{\Delta} (L / j' + L') > 0
\end{align*}
\]

where \( \Delta = yL' [(1-A_1 (1+T_1)) h / j' + A_1 (1+T_1) hm] + (L / j' + L') (A_2 a (1+T_1) \\
- A_1 (1-a) (1+T_1)) \\
\]

\[
b = [(y-G) (1-A_1 (1+T_1)) h' c - T_2 cP^*] < 0
\]

where \( a = \text{dlnP/dlnq} \)
The signs of $a_{ij}$ above are given on the assumption that $\Delta$ is negative. A sufficient condition for this is that $L + L'j'$ be positive. This is the condition for the LM curve to be upward sloping in the real interest rate-output space. When output increases, the demand for money increases directly by $L$. But an increase in output raises the rate of inflation by $j'$ and so lowers the demand for money by $L'j'$. We assume the former effect dominates.

The determinant of $A$ in equation (13) is negative, which is a necessary and sufficient condition for the long run equilibrium to be a saddle-point locally.

In figure 1, the $\text{Dln} \equiv 0$ locus is upward sloping and the $\text{Dln}m \equiv 0$ locus is downward sloping. The saddle path SS is upward sloping and flatter than the $\text{Dln}c = 0$ locus and the unstable arm $WW$ is downward sloping and steeper than the $\text{Dln}m = 0$ locus.

In order for the economy to converge to the long run equilibrium, it must be on the stable manifold SS. All other perfect foresight paths diverge away from equilibrium approaching the unstable arm $WW$ asymptotically. We assume that the economy is always on the saddle path. This is achieved by a jump in $c$ and $e$. The price of domestic output $q$ is taken to be predetermined at a point in time. So $m$ is a predetermined variable except when the authorities increase the level of the money stock (M).
Figure 1

Figure 2
The monetary slowdown in this case increases the long run level of real balances and leaves the long run real exchange rate unchanged. It is easily verified that both $d\ln m = 0$ and $d\ln c = 0$ curve shift to the right by the same distance (-$\gamma L'\)$. The moment this unanticipated policy of disinflation is put into effect the real exchange rate appreciates to put the economy on the new stable arm at $E_{01}$ in figure 2. So the real exchange rate overshoots its (unchanged) long run value on impact. This is the case analysed by Buiter and Miller (1981) and Turnovsky (1981). Intuitively, real money balances have to be accumulated during the transition to the new long run equilibrium. So from equation (4) $y$ must fall below its natural level. This fall in $y$ is brought about by the jump appreciation of c.

From $E_{01}$ the economy moves steadily to $E_1$, accumulating real balances. At $E_1$ the old level of competitiveness is restored. A monetary slowdown accompanied by increased taxes therefore has real effects which are transitory. Money is superneutral in this case. Also notice, in the new long run equilibrium the trade balance surplus or deficit is the same (not necessarily zero) as in the old long run equilibrium.
4. REDUCTION IN $\mu$ WITH ENDOGENOUS $G$

LONG RUN EQUILIBRIUM

Since nothing crucial hinges on it and none of the results are modified, in this section we shall ignore capital gains on real balances.

The long run equilibrium condition can now be written as

$$\bar{m} = L(i^* + \mu)\bar{y}.$$  \hspace{1cm} (14)

$$h(c)\bar{y} = A(h(c)(\bar{y} - t),i^*) + h(c)\mu\bar{m} + T(A,cP^*)$$  \hspace{1cm} (15)

The long run equilibrium condition (14) is unchanged. A fall in the rate of growth of money supply would increase the long run level of real balances ($\frac{d\bar{m}}{d\mu} = yL' < 0$).

The effect of a change in $\mu$ on $c$ is given by the expression

$$\frac{dc}{d\mu} = h\bar{m}(1 - \mu yL'/\bar{m})/((\bar{y}(1-A_1(1+T_1)) - \mu \bar{m})h' - T_2P^*)$$  \hspace{1cm} (16)

Consider the numerator first. It is positive if the elasticity of money demand with respect to the interest rate is less than unity. We shall assume this to be the case. There is considerable empirical evidence to support this assumption e.g., Goldfeld (1973) found this elasticity to
be significantly less than unity. If, however, the elasticity were greater than unity then a reduction $\mu$ would raise government expenditure at an unchanged real exchange rate, and thereby create an excess demand for domestic goods.

The denominator in the equation above tells us the effect of a real exchange rate change on the demand for domestic goods. A real depreciation tends to create an excess demand by lowering the real value of a given level of output and switching domestic and foreign demand towards the domestic good. It also tends to create an excess supply by lowering the real value of a given level of government expenditure. We shall assume that the net effect is to create an excess demand in the domestic goods market.

A reduction in $\mu$ therefore requires a real depreciation. This is because a fall in $\mu$ raises real balances but the level of government expenditure falls ($G = \mu m$ and $\mu m$ falls). So a real depreciation is necessary to remove the excess supply.

If there are unemployed resources a real depreciation (with Marshall-Lerner conditions satisfied) reduces unemployment, but also reduces the value of domestic output in terms of the consumption basket. Usually (implicitly) it is assumed that the former effect outweighs the latter. But when the terms of trade worsen in long run equilibrium, there is an unambiguous welfare loss. With output at its (exogenous) full-employment
value, a real depreciation simply reduces the real purchasing power of that fixed output. If we had modelled the supply side of the economy, the natural rate of output would have been endogenous. Then the long run level of output would have been lowered. This happens if the labour supply curve is upward sloping and depends on the wage rate in terms of the consumption bundle, not the product wage rate. A worsening of the terms of trade would move the labour supply curve inwards at any given product wage rate and reduce employment.

So there are long run costs of monetary disinflation when this is achieved by reducing government expenditure. Real income is reduced unambiguously in the long run.

**DYNAMICS**

Proceeding as in the previous section we can eliminate $i$, $P$, $\lambda$, $\pi$, $G$, $DP/P$ and $y$ from equations (1) to (7). We are then left with two differential equations

\[ D\ln m = x(\ln m, \ln c; \mu) \]  \hspace{1cm} (17)  

\[ D\ln c = z(\ln m, \ln c; \mu) \]  \hspace{1cm} (18)  

where, as before, we have suppressed all exogenous variables except $\mu$.  

Linearising this pair of differential equations in the neighbourhood of the long run equilibrium where $D\ln m = D\ln c = 0$, we have

\[
\begin{bmatrix}
D\ln m \\
D\ln c
\end{bmatrix} =
\begin{bmatrix}
B \\
\lambda
\end{bmatrix}
\begin{bmatrix}
\ln m - \ln m_0 \\
\ln c - \ln c_0
\end{bmatrix}
\]

(19)

where

\[
b_{11} = \frac{m[yL'^n + A_2 a(1+T_1)]}{D} < 0
\]

\[
b_{12} = \frac{c[yL'^n]}{D} < 0
\]

\[
b_{21} = \frac{m[-(1-A_1 (1+T_1))h/j' + (yL' + L/j')yL]}{D} \geq 0
\]

\[
b_{22} = \frac{c [(L/j' + yL')n]}{D} > 0
\]

\[
D = -yL'(1 - A_1 (1+T_1))h/j' - A_2 a(1+T_1)(yL' + L/j)
\]

\[
n = [-I - A_1 (1+T_1)]yL' + umh' + T_2 \rho^*
\]

\[
a = d\ln P/d\ln q.
\]

As in the last section we assume $L > |yL'|$ so $D$ is positive. The expression $n$ is the effect of a depreciation on the commodity market. We assume a depreciation creates excess demand in that market.

To sign $b_{21}$ we notice that a rise in $m$ creates an excess supply in the money market directly and therefore $D\ln c$ tends to fall. But a rise in $m$
also raises government expenditure \((dG = \mu dm)\) and this raises output through a multiplier process \([\mu h/(1-A_1(1+T_1))]\). And this rise in income has the usual effects on the money market - a rise in transactions demand and a decline in the asset demand. The effect of \(m\) on \(Dln\) then depends on whether the direct excess supply creating effect is stronger than the indirect effects i.e., whether \(1 \geq (j'yl' + L)\mu h/(1-A_1(1+T_1))\). We assume the first inequality holds so \(b_{21} < 0\).

The long run equilibrium is again a saddle-point locally (see figure 1). The stable arm is upward sloping and flatter than the upward sloping \(Dln\) = 0 schedule and the unstable arm is negatively sloped and steeper than \(Dlnm = 0\) line, which is also downward sloping. We again assume that the economy is always on the saddle path by appropriate jumps in \(e\) and \(c\).

A decline in the rate of monetary growth raises the steady state values of both \(m\) and \(c\). The new long run equilibrium is therefore to the north-east of the old one. The \(Dlnm = 0\) schedule shifts to the right (by \((hm - A_2^a(l+T_1))/(\mu h + A_2^a(l+T_1)/yL'))\). The \(Dln = 0\) line may shift in either direction. The expression for its shift is

\[-yL' + nm(yL'j' + L)/(1 - A_1(l+T_1))/x\] where \(x = m - \mu hm(yL'j' + L)/(l - A_1(l+T_1))\) assumed positive above. Now the first term in the numerator is the excess demand in the money market if \(\mu\) is lowered. The second term is the change in money demand through the effect of \(\mu\) on \(y\). If the former effect dominates then \(Dln = 0\) shifts to the right.
If the $D\text{ln}c = 0$ schedule shifts to the left or does not shift at all when $y$ is reduced then on impact the real exchange rate undershoots its long run value.

If the $D\text{ln}c = 0$ schedule shifts to the right then three cases are possible. First, when the stable arm of the new long run equilibrium is flatter than the line joining the two long run equilibria. Second, when that line is flatter than the new saddle path. Finally, when the slopes are equal.

In the first case the real exchange rate undershoots its long run value and, therefore, this can be treated with $D\text{ln}c = 0$ line not shifting or shifting to the left. We examine this case and the other two in turn.

Undershooting of $c$

This happens as pointed above when $D\text{ln}c = 0$ stays put, shifts to the left or when it moves to the right but the stable arm of the new equilibrium is flatter than the line joining the old and the new long run equilibria. This is shown in figure 3. $E_0$ is the original long run equilibrium and $E_1$ is the new long run equilibrium. When the unanticipated and immediately implemented policy of reducing $y$ is put into effect, the real exchange rate depreciates (point $E_{01}$). This depreciation is short of the long run requirement. From $E_{01}$ the economy converges monotonically to $E_1$, with rising $c$ and $m$ (because the roots of the coefficient matrix are real).
If the policy had been announced (say at $T_0$) in advance of its implementation (at $T_1 > T_0$), then the economy would have jumped to position such as $E_{02}$ below $E_{01}$ in figure 4. From $E_{02}$, the system would be driven by a path (which approaches the unstable arm $U_0 U_0$ of $E_0$ asymptotically) that arrives on the convergent path $S_1 S_1$ of $E_1$ on date $T_1$ (point H). The only jump in c takes place when the policy change is announced (i.e., when it was unanticipated) and no jumps occur when it is implemented. Arbitrage would rule out such anticipated jumps since they would imply infinitely high real returns on bonds.

The delayed implementation means $m$ falls initially and starts rising only when the policy is put into effect. The real exchange rate depreciates steadily.

"Perverse Overshooting" of $c$

When the stable arm of the new long run equilibrium is steeper than the line joining the two long run equilibria then we have the situation portrayed in figure 5.

When the previously unanticipated policy of a reduction of monetary growth is put into effect, the real exchange rate appreciates to $E_{01}$ on impact. From $E_{01}$ the system converges monotonically to $E_1$ with a depreciating real exchange rate and rising real balances.
We refer to this case as "perverse overshooting" because the long run requirement is a depreciation of the real exchange rate and yet convergent (or long run) perfect foresight requires an appreciation on impact.

The authorities can reduce the initial overshooting by announcing the policy before (say at $T_0$) its implementation (at $T_1$). The economy jumps to a position such as $E_{O2}$ in figure 6. The longer the delay in implementation the smaller the initial jump. From $E_{O2}$, the real exchange rate appreciates and the real balances are built up till date $T_1$. From time $T_1$ onwards (when the policy is put into effect), the real exchange rate depreciates with real balances rising till the new long run equilibrium $E_1$ is reached.

No Jump in $c$

Here the economy at $E_0$ is on the stable manifold with reference to $E_1$ and hence no jump in $c$ is required (figure 7). From $E_0$ the economy converges monotonically to $E_1$.

A delayed implementation has no effect on $c$. The economy stays at $E_0$ till the policy is put into effect.

To summarise the three cases: when we recognise the interdependence of monetary and fiscal policy through the government budget constraint,
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the impact effect of c can be either the one associated with contractionary fiscal policy in the Mundell-Fleming model (a depreciation) or that associated with a contractionary monetary policy (an appreciation).

It is however important to recognise that whatever the impact effect on the real exchange rate, the output costs of disinflation are the same. In other words, whether the exchange rate jump appreciates or depreciates, unemployment in this model would rise above its natural rate when a contractionary monetary policy is put into effect.

This is important for the UK where some commentators have attributed the entire appreciation of the pound in the 1979-80 period to North Sea Oil. These authors were trying to shield the Conservative government from the charge of having triggered off a recession by causing an appreciation of the pound through its policy of bringing down inflation. Here we have shown that an appreciation of the pound is not a necessary consequence of such a policy. And now we turn to an examination of why the output costs are those implied by the Bui ter and Miller model even in such a case.

The expectations-augmented Phillips curve is reproduced for convenience

\[
\frac{Dq}{q} = j(y - \bar{y}) + \mu \quad j' > 0
\]  \tag{4}

or
\[
- \ln m = j(y - \bar{y})
\]  \tag{4'}

Integrating both sides of (4') we have
\[
\ln m(0) - \ln \bar{m} = \int_{0}^{\infty} j(y(t) - \bar{y}) \, dt
\]

or
\[
\ln c(0^+) - \ln \bar{c} = \int_{0}^{\infty} j(y(t) - \bar{y}) \, dt
\]

where \( X \) is the inverse of the slope of the stable manifold (and is positive).

Equation (20) tells us that the cumulative loss of output is proportional to the difference between the level of real balances when the policy is put into effect (at time \( 0 \)) and the long run level of real balances. Because the roots of the system of differential equations in equation (14) are real therefore \( y(t) \) is always less than \( \bar{y} \) in the transition to the new long run equilibrium.

Equivalently we could write the output costs in terms of the real exchange rate. We know that the slope of the stable arm is positive i.e.,

\[
(\ln c(t) - \ln \bar{c})/(\ln m(t) - \ln \bar{m}) = 1/X.
\]

So we can express (20), by using (22), equivalently as (21) where \( c(0^+) \) is the value of the real exchange after the jump.

The output costs of disinflation are very similar even if we had a fully rational Phillips' curve as in Turnovsky (1981). The Phillips curve can then be written as

\[
- D\ln c = j(y - \bar{y})
\]
and output costs can be represented by (20) or (21).

What happens when the policy is preannounced? In the undershooting case, this is accompanied by a decline in real balances, so we have output above the natural rate till the policy is implemented (at date $T_1$). From $T_1$ onwards until the new steady date is reached the level of output is below its natural rate. In the perverse overshooting case output starts falling from the time the announcement is made of the future cut in $\mu$. In the no change in $c$ case output stays at its capacity level till the policy is implemented and in the transition to the new steady state falls below the natural rate.
5. CONCLUSIONS

We have examined the relationship between the real exchange rate (and other endogenous variables) and a slowdown in monetary growth. Unlike the other papers where this relationship has been examined we have not ignored the government budget constraint. The long run effects and the time path of the endogenous variables depends on whether the reduction in $\mu$ is achieved by increased explicit taxes or a cut in government expenditure. When the former method is chosen, we get the usual result of an overshooting of the real exchange rate. The long run of the real exchange rate is not influenced by a fall in $\mu$.

On the other hand, if the cut in $\mu$ is accompanied by cuts in $G$ then we cannot say a priori which way the exchange rate moves on impact.

But there are both steady state and short run costs of such a policy. The short run costs occur because output falls below its capacity level when the policy of disinflation is put into effect. The reasoning is as in Buiter and Miller. The only way real balances can be accumulated (because higher real balances are desired in the new long run equilibrium), with a lower rate of growth of the nominal money stock is for the price of domestic output to lag behind this lower growth rate of the money stock. The only way the time path of prices can be lowered is by a fall in the level of output below its natural level.
But there are long run costs as well. The terms of trade move against the home country and thus reduces the real value of the given natural rate of domestic output. This happens because a fall in government demand creates an excess supply in the domestic goods market. The excess supply is eliminated by a change in the real exchange rate. In a closed economy, there would be a channel unavailable in our model due to the assumed perfect substitutability of assets, through which demand could be stimulated - the real rate of interest.

An announcement in advance of implementation may for some time increase the level of output above its natural level but when the policy is put into effect output must fall below its natural level.

The simple analytical solution to avoid these output costs is, as Buitier and Miller suggest, a step increase in the nominal money supply. But that poses serious problems of political credibility.
(1) If interest payment on government bonds was included, we would have to introduce some other endogenous variable in government budget constraints to ensure that a μ per cent growth rate of the nominal money stock was followed. This could be a G' policy of Tobin and Buitler (1976) or residual bond financing. In the latter case if government bonds are denominated in nominal units, then in steady state they have to grow at μ also.
CHAPTER III

A RE-EXAMINATION OF THE EFFECTS OF A TAX-FINANCED INCREASE

IN GOVERNMENT EXPENDITURE UNDER FLEXIBLE EXCHANGE RATES
In the last decade our understanding of how an economy operates under a system of flexible exchange rates has undergone substantial modifications. Few old models have emerged unscathed. One model which has proved quite durable is the Mundell-Fleming model. Not that it has not had its share of criticism, but it continues to be the consensus open-economy macro model.

It is well known that this model predicts the monetary policy is totally ineffective in affecting the level of output under a system of fixed rates. Similarly fiscal policy is impotent under a flexible rate regime. Both of these extreme results have received considerable attention. The fixed rate result led to the "sterilisation" and "offset coefficient" literature. Recently the fiscal policy ineffectiveness under flexible rates has come under close scrutiny and has been the subject of considerable criticism.

The Mundell-Fleming can be reduced to a system of three equations. Commodity market equilibrium requires domestic output to be equal to the demand for it from domestic and foreign sources. Money market is in equilibrium when a given nominal (and real, since prices are constant) quantity of money equals the demand for it. Finally domestic and foreign interest bearing assets are considered to be perfect substitutes, and with static expectations, domestic and foreign interest rates are equal. Now an increase in (tax-financed) government expenditure would tend to raise output, put upward pressure on interest rates by raising the transactions demand for money, thereby generating incipient capital inflows. The capital flows appreciate the domestic exchange rate leading to a substitution of foreign goods for domestic output at
Equilibrium is re-established when the trade balance is worsened by the amount by which government expenditure had increased. There is 100% crowding out. The model then predicts three consequences of an expansionary fiscal policy: (i) no change in the equilibrium level of output, (ii) an appreciation of the nominal (and real) exchange rate and (iii) a current account deficit (equal in magnitude to increased public expenditure).

The first prediction has been subjected to a detailed examination by Tobin and de Macedo (1980) and Branson and Buiter (1983). They find that this result is not robust to changes in model specification, e.g. introduction of wealth effects, deflating by a price index including the price of imports rather than the price of domestic goods alone, introducing an upward sloping aggregate supply curve so that the price of domestic output is related to the level of output, exchange rate expectations. On the other two predictions of the Mundell-Fleming model these two sets of authors seem to concur.

The second prediction has not been scrutinized by itself. In the literature one can find models where expansionary fiscal policy is accompanied by depreciation. This occurs in models which assume purchasing-power parity and thus are not strictly comparable to the other models.

The third prediction is the one that concerns us in this paper. Does increased government expenditure (financed by taxation) lead to an adjustment process with current account deficits?

It should be noted that the Mundell-Fleming model is a short-run model and hence the trade balance deficit is the impact effect of a
fiscal expansion. Branson and Buiter, while taking issue with the first prediction of the Mundell-Fleming model, show that the third prediction carries over to a model with rational expectations. An expansionary fiscal policy is now accompanied by current account deficits until the new long-run equilibrium is reached with a lower stock of foreign assets. Other models, e.g. Sachs (1980), Marion (1982), Rodriguez (1979), which look at the long-run equilibrium of a Mundell-Fleming model, get similar results.

In this chapter we show that the Branson and Buiter generalisation of the Mundell-Fleming model is not the only possible one. While on impact the nominal exchange rate does appreciate, during the transition to the new long-run equilibrium the expanding country runs current account surpluses.

The reason for this is quite simple. One of the important lessons of the flexible exchange rate literature of the last ten years is the effect of the exchange rate on the valuation of foreign assets. Now, if the domestic economy was a creditor to the rest of the world to begin with, then in the new long-run equilibrium, following the fiscal expansion, we would have a lower domestic currency value of foreign assets if the new long-run value of the exchange rate is lower. If in the adjustment process the economy was running deficits then this process is further exacerbated. Thus the Branson-Buiter results require a strong negative wealth effect and/or a shift away from foreign assets in portfolios of domestic asset holders. While this is quite possible, it is by no means inevitable.
The paper is organised as follows. Section 2 sets out the model. In Section 3, we look at the long-run solution of the model. Section 4 analyses the short-run behaviour of the model when agents possess static expectations, while Section 5 looks at the short run with rational expectations. Section 6 offers some concluding comments.

2. THE MODEL

The model is a full employment, currency substitution version of the standard IS-LM model of the open economy. There are two assets, domestic money and foreign money. There are also two goods, one in the production of which the home country specialises, and another which is produced abroad. Domestic currency is assumed to be non-traded, so that trade balance (equal to the current account) imbalances must be financed by changes in the stock of foreign currency. The home country is assumed to be in a net creditor position vis-à-vis the rest of the world.

The model can thus be represented by the following equations:

(i) \( \frac{M}{p} = \ell (x, qy/p, W) \quad l_1 < 0 \quad l_2 > 0 \quad 0 < l_3 < 1 \).

(ii) \( \frac{qy}{p} = A [q(y-S)/p, W-W] + qG/p + T(A, ep*/q) \)
    \( 0 < A_1 < 1 \quad A_2 > 0 \quad T_1 < 0 \quad T_2 > 0 \).

(iii) \( \frac{eF}{p} = T(A, ep*/q) \)

(iv) \( W \equiv (M + eF)/p \)

(v) \( \hat{W} \equiv kqy/p \)
(vi) \( S = G \)

(vii) \( y = \bar{y} \)

(viii) \( x = \begin{cases} \frac{\partial}{\epsilon} & \text{(a)} \\ \frac{\partial e}{\partial e} & \text{(b)} \end{cases} \)

(ix) \( p = p(q, \epsilon p^*) \)

where

\( M \) = stock of domestic (outside) money

\( p \) = domestic price level (consumer price index)

\( e \) = the exchange rate, defined as the domestic currency price of foreign exchange

\( x \) = the expected rate of change of \( e \)

\( q \) = the money price of domestic output

\( y \) = level of domestic output

\( W \) = real domestic wealth

\( \hat{W} \) = target level of wealth

\( S \) = lump sum taxes in units of domestic output

\( G \) = government expenditure in units of domestic output

\( p^* \) = foreign currency price of the foreign good (taken to be exogenous)

\( F \) = stock of foreign money held by domestic residents (assumed to be positive)
A dot over a variable denotes its derivative with respect to time, i.e. \( \dot{z} = \frac{dz}{dt} \).

A variable \( z \) with a subscript \( i \) denotes a partial derivative with respect to its \( ith \) argument.

Equation (i) is the money market equilibrium condition. The real supply of money must equal the demand for it. The demand for money depends on the opportunity cost of holding money rather than the alternative asset - foreign money, the transactions demand and on wealth. This opportunity cost is given by the expected depreciation of the domestic currency \( x \). We take the transactions proxy to be the real value of domestic output \( \frac{qy}{p} \). Some authors prefer to use \( y \), but nothing crucial hinges on this assumption. Note that \( y \) is fixed (at its full employment value) but \( \frac{qy}{p} \) is free to vary. The final argument in the demand for money function is real wealth. For later derivation it would be convenient to write (i) as:

\[
\text{(ia)} \quad \frac{M}{p} = L(x, \frac{qy}{p}W)W \quad 0 < \left( \frac{L_2}{L} \right) \left( \frac{qy}{p}W \right) < 1
\]

Equation (ii) is the goods market equilibrium condition. Equilibrium is attained when the real value of domestic output is equal to the demand for it. Demand arises from domestic consumers, the government and net exports abroad. Domestic consumption (on all goods) depends on disposable income and on the gap between actual and target wealth. This simplifies the long run solution of the model at no extra cost.\(^3\) Strictly speaking, the relative price of foreign to domestic output also belongs as a determinant of total consumption, i.e. the decision about the level of consumption and its allocation between the domestic and foreign goods is
simultaneous. Here we are following tradition in omitting the terms of trade from the $A$ function. It is assumed that all the expenditure on goods and services by the government is on the domestic good and is denominated in units of that good. The trade balance depends on the total consumption expenditure by domestic residents and the relative price of the foreign output to domestic output. The Marshall-Lerner condition is assumed to hold at every instant, so that a rise in the relative price of the foreign good, *ceteris paribus*, improves our trade balance. Further, all capital gains are assumed to be saved. Capital gains arise from changes in the value of assets through inflation and exchange rate changes. This helps us to keep the dynamics of the system to two differential equations.

Equation (iii) states that the net acquisition of foreign currency (measured in units of domestic consumption) is equal to the trade balance.

Equation (iv) is the wealth identity. Real wealth of domestic residents is equal to the real value of domestic and foreign currency.

Equation (v) is the definition of target wealth, which is assumed to be a constant multiple, $k$, of the real value of domestic output.

Equation (vi) states that the government budget is always in balance.

Output ($y$) is always at the full employment level ($\hat{y}$) according to equation (vii).

The expected rate of depreciation is either equal to the actual rate of depreciation when perfect foresight is assumed (equation viii(a)) (though this assumption does not hold when new information arrives) or equal to zero under static expectations (equation viii(b)).
Equation (ix) is the definition of the price index which is homogeneous of degree one in its two arguments.

We could reduce the dimensionality of the system by substituting (iv) to (ix) in (i) to (iii) to obtain the following:

\[ M = L(x, \frac{qy}{M+eF})(M+eF) \]  
(1)

\[ \frac{qy}{p(q, ep*)} = A \left( \frac{q(y-G)}{p(q, ep*)} \right) \frac{(M+eF)}{p(q, ep*)} - k\frac{qy}{p(q, ep*)} \]

\[ + \frac{qG}{p(q, ep*)} + T(a, ep*/q) \]  
(2)

\[ eF/p(q, ep*) = T(A, ep*/q) \]  
(3)

Before proceeding to analyse the behaviour of the model, two comments are in order. First, we have assumed that the alternative to holding domestic money is to hold foreign money. It is quite easy to introduce interest-bearing foreign asset. The major modification that this would require would be to introduce an interest service account. Also, we follow Branson and Buitier (1983) and Dornbusch and Fischer (1980) in ignoring the real interest rate as a determinant of expenditure. This is done to reduce the order of the dynamic system from three to two differential equations.

Second, and more important, is that in our specification it is the level of output and not disposable income which determines target wealth. If the latter variable were used we would still obtain similar results provided some further assumptions were made. In our defence we can say that an expansionary fiscal policy is accompanied by an appreciation of the real exchange rate. So even if we had used disposable income rather than \( y \), the effect on desired wealth \( q(y-G)/p \) of such a policy would still be ambiguous.
3. **LONG RUN SOLUTION**

Without a loss of generality, we can assume that the long run equilibrium of the model is a stationary state, i.e. the actual and expected rates of growth of all nominal and real variables is zero.

We thus have $\dot{e} = \dot{F} = 0$. Substituting these in equations (1), (2) and (3) we can write the long run solution as

$$M = L(1/k)\tilde{k}\tilde{q}\tilde{y}$$  \hspace{1cm} (4)

$$M + eF = k\tilde{q}\tilde{y}$$  \hspace{1cm} (5)

$$T[A(q(y-G)/p(q, e^*)), e^*/q] = 0$$  \hspace{1cm} (6)

(A tilde denotes the long run value of an endogenous variable.)

These three equations determine the values of $\tilde{q}$, $\tilde{e}$ and $\tilde{F}$. This system is recursive. For a given value of $M$ (exogenously fixed) we can determine the value of $\tilde{q}$ from (4). Substituting for $\tilde{q}$ in (6) we can find the value of $\tilde{e}$. And, finally, from equation (5) we can determine $\tilde{F}$ once we know $\tilde{q}$ and $\tilde{e}$.

A tax-financed increase in government expenditure on domestic goods and services has no effect on the value of $\tilde{q}$ because this fiscal expansion leaves the money market equilibrium condition unchanged in our model. So across steady states the value of $\tilde{q}$ is fixed. Now, from equation (6), it is obvious that an appreciation of the exchange rate is required for the trade balance to be zero. An increase in $G$, because it implies an increase in taxes, tends to improve the trade balance. Given the value of $\tilde{q}$, $\tilde{e}$ must fall. This appreciation
brings about a current account balance in two ways. First, it induces substitution away from home goods by both domestic and foreign residents. And second, by lowering the price level and thus increasing the real value of disposable income, it induces more imports.

Finally, from equation (5), it is clear that $eF$ must remain constant, so an appreciation of the currency would lead to an equal percentage increase in the stock of foreign bonds, i.e. $F = -e$ where $\Delta^{\circ}$ denotes percentage change. Thus across steady states, a fiscal expansion implies a higher stock of foreign bonds and an equal percentage appreciation of the exchange rate.

Two features of the long run equilibrium should be mentioned. First, equations (5) and (6) imply that all the disposable income of the domestic consumers is spent. Second, even if desired wealth was a multiple of disposable income, it is not necessary that $\Delta F$ would be negative. It would depend on, among other things, relative share in wealth of money and foreign assets, and on the extent of appreciation required to bring about balanced trade.

4. **Static Expectations**

If agents have static expectations then a momentary equilibrium is described by equations (1) and (2) with $x = 0$. These two equations (equations (7) and (8) below) determine the short-run values of $e$ and $q$, given the value of $F$ and the exogenous variables.
\[ M = L(\bar{q}y/(M+eF))(M+eF) \]  \hspace{1cm} (7)

\[ q(\bar{y}-G)/p = A \left( q(\bar{y}-G)/p, (M+eF)/p - k\bar{q}y/p \right) + T(A, e^*/q) \]  \hspace{1cm} (8)

An increase in \( G \) creates an excess demand for domestic output. A rise in the value of \( q \) eases the demand pressure in the goods market in three ways. First, it raises the real value of domestic output. Second, it creates an expenditure switch towards foreign goods by lowering \( e^*/q \). And, finally, it lowers the value of domestic wealth and raises the value of desired wealth and thus encourages savings.

An appreciation of the exchange rate also tends to lower the demand for domestic goods. It does this by lowering the price level and thus increasing the real value of domestic output, by switching demand towards foreign goods, by lowering the real value of foreign money and raising desired wealth. But a fall in the price level also raises real balances. We assume that the net effect of an appreciation is to create an excess supply of goods.

In the new short run equilibrium, following an increase in government expenditure, we have an appreciation of the home currency and an increase in the price level. This is apparent from the money market equilibrium condition. An increase in either \( q \) or \( e \) creates excess demand for money. Since the nominal quantity of money is exogenous and \( F \) is predetermined at any moment, we must have \( e \) and \( q \) moving in opposite directions. But a depreciation of \( e \) and a fall in \( q \) would add to the excess demand for output created by an increase in \( G \). So \( e \) must fall and \( q \) rise.
The dynamics of the system can be expressed in terms of equation (9) which is equation (3) rewritten emphasizing the dependence of $e$ and $q$ on $F$.

$$\frac{e(F)^*}{p(q(F), e(F)p^*)} = T \left[ A \left( \frac{q(F)(y-G)}{p(q(F), e(F)p^*)}, \frac{M+e(F)p^*}{p(q(F), e(F)p^*)} \right), \frac{e(F)p^*}{q} \right]$$ (9)

An increase in $F$ in (7) and (8) lowers both $e$ and $F$. This seems to suggest that $\frac{dF}{dF}$ is ambiguous in sign. However, it can be proved (see Appendix), after some tedious algebra, that $\frac{dF}{dF}$ is negative if an appreciation of the real exchange rate creates an excess supply in the market for the foreign currency. The valuation effect of a depreciation of the real exchange rate outweighs the increased demand through the wealth effect and lower transactions demand for money channels. This must be so because a real depreciation leads to an excess demand for money. So there must be an excess supply of the other asset. So the system is dynamically stable under static expectations.

5. RATIONAL EXPECTATIONS

When agents possess perfect foresight a momentary equilibrium depends on the entire (expected) time path of the economy from now to infinity. It is, therefore, no longer possible to analyse the short-run equilibrium and dynamics separately.

Going back to equations (1) to (3), we can write the money market equilibrium condition as
\[ \frac{\dot{e}}{e} = \psi(e, F, q) \quad \psi_1 > 0 \quad \psi_2 > 0 \quad \psi_3 > 0 . \quad (10) \]

An increase in any one of \( e \), \( F \) and \( q \) creates excess demand for money. This can be choked off by a rise in expected depreciation, given the stock of nominal balances.

The trade balance could be written as

\[ \dot{F} = \theta(e, F, q) \quad \theta_1 > 0 \quad \theta_2 < 0 \quad \theta_3 < 0 . \quad (11) \]

Finally, the goods market equilibrium condition defines a relationship between \( q \) on the one hand and \( e \) and \( F \) on the other. An increase in either \( e \) or \( F \) creates excess demand in the goods market under the assumptions made in the last section. Thus \( q \) has to increase to restore equilibrium. Therefore we have

\[ q = \Pi(e, F) \quad \Pi_1 > 0 \quad \Pi_2 > 0 . \quad (12) \]

Substituting this in equations (10) and (11) and linearising around the long run equilibrium, we obtain

\[
\begin{pmatrix}
\dot{e} \\
\dot{F}
\end{pmatrix} =
\begin{pmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{pmatrix}
\begin{pmatrix}
e - e \\
F - F
\end{pmatrix}
\begin{pmatrix}
a_{11} > 0 \\
a_{21} > 0
\end{pmatrix}
\begin{pmatrix}
a_{12} > 0 \\
a_{22} < 0
\end{pmatrix}
\]

\[ (13) \]

(A priori, the sign of \( a_{21} \) seems ambiguous, but in fact it is positive - see Appendix.)

The determinant of the matrix of coefficients in (13) is negative. So the long run equilibrium \((F, e)\) is a saddle-point. This is portrayed in Figure 1.
Unless the system happens to be on the stable arm, the economy never gets to the long run equilibrium. We make the common, but arbitrary, assumption that the system is always on the saddle path. This is ensured by jumps in the exchange rate, the stock of foreign assets evolving slowly over time. The implication of this assumption is that agents possess long-run perfect foresight and somehow rule out the non-convergent myopic perfect foresight paths as inoptimal. 4

The saddle path is flatter than the \( \dot{e} = 0 \) locus. It must be remembered that with static expectations the adjustment path of the economy is the \( \dot{e} = 0 \) schedule.

The effect of an expansionary fiscal policy is portrayed in Figure 2. Suppose the economy was in long-run equilibrium initially. Following a fiscal expansion the position of the new long-run equilibrium is \( E_1 \). The new \( \dot{e} = 0 \) locus is below and to the left of the old one. We know that this must be so from the fact that with static expectations the exchange rate appreciates in the short run. The impact effect of the tax financed fiscal expansion is to put the economy on the new stable manifold \( s's' \) at point \( E_{01}^R \). The nominal exchange rate thus undershoots its long run equilibrium value. Notice, however, that with rational expectations the jump appreciation is greater than with static expectations. As a general point, it is quite possible that with a real disturbance the jump in the exchange rate under rational expectations would be greater than with static expectations. Here, rational speculators take into account the long run appreciation of the exchange rate (see Dornbusch and Fischer (1980)).

Following the jump, the exchange rate appreciates steadily to its long run value. This appreciation is accompanied by trade balance surpluses.
Figure 1

Figure 2.
The price of domestic output \((q)\) rises when the expansionary fiscal policy is put into effect (as is shown in the Appendix). The increase under rational expectations is less than with static expectations. The moment the new policy is put into effect, the level of nominal wealth falls, due to the appreciation of the domestic currency, and the desired level of nominal wealth rises, as \(q\) rises. This gap must be big enough to reverse the initial excess demand caused by the increased government spending, and for a current account surplus to emerge.\(^5\)

This point needs to be emphasised. With rational expectations, a momentary equilibrium is that position which puts us on the unique convergent path to the new long-run equilibrium. If the new long-run equilibrium requires a higher stock of foreign currency, then \(e\) and \(q\) will take on values in the short run so that a current account surplus emerges.

6. **CONCLUSIONS**

In a full-employment portfolio balance macro model, we have shown that an expansion in government spending on domestic goods and services leads to an appreciation of the nominal exchange rate (as predicted by the Mundell-Fleming model) and a current account surplus (contrary to the prediction of that model and others). This latter result follows from the appreciation of the home currency which, in the absence of current account surpluses, would lead to a fall in the real value of foreign bonds (in domestic consumption units).
This result is possible in a model where the restrictive form of the target wealth function is absent. Whether a deficit or a surplus accompanies a fiscal expansion is, then, an empirical matter.

This result would carry over to a model with unemployment and sticky prices. Real wage rigidity could easily be introduced into the model by making $y = y(ep^*/q)$ with $y' < 0$. None of the conclusions are modified thereby. Imported intermediate goods would further strengthen the result.

Interest-bearing foreign assets would make no difference to the result. Introduction of domestic nominal bonds which are perfect substitute for the foreign bond also does not change any of the results. The assumption of net creditor position of the home country is crucial to the analysis.

It was also shown that it is not necessarily true that with static expectations, the jump in the exchange rate when a previous unanticipated policy is implemented is greater than when agents possess rational expectations.
Appendix

Static Expectations

To sign \( \frac{dF}{\partial q} \) write the endogenous variables as \( e/q \) and \( q \).

It can be verified that if a rise in \( e/q \) creates excess demand in the domestic money market (see p 14) then

\[
\frac{de}{dF} < 0 \quad \frac{dq}{dF} < 0
\]

Now the trade balance can be written as

\[
T(q, e/q, F) T_1 > 0 \quad T_2 > 0 \quad \text{and} \quad T_3 < 0
\]

from which \( \frac{dF}{\partial q} < 0 \).

Rational Expectations

The value of \( a_{21} \) is

\[
= \frac{1}{\Delta} \left[ WL_1 A_2 M/A \left( \frac{1}{p} \frac{(y-C)}{\varepsilon_1} \frac{(1+T_1)}{e_x} + \frac{T_2}{q_1} + \frac{ep^*}{q_2} \right) \right] > 0
\]

where

\[
\Delta = - eWL_1 \left[ \frac{(y-C)}{p} \varepsilon_2 (1-A_1 (1+T_1)) - k A_2 (1+T_1) y \varepsilon_2 + A_2 (1+T_1) \frac{w}{q_1} \varepsilon_1 + T_2 \frac{ep^*}{q_2} \right] > 0
\]

and \( \varepsilon_1 \) and \( \varepsilon_2 \) are the elasticities of the price index with respect to the price of domestic output and the price of importables respectively.
The Dynamics in Terms of $q$ and $F$

The goods market equilibrium condition (equation 2) gives us a relationship between $e$, $q$ and $F$.

$$e = e(q, F) \quad e_1 > 0 \quad e_2 < 0. \quad (A1)$$

Differentiating (A1) with respect to time and substituting the expression in the money market equilibrium (equation 1) we get

$$\dot{q} = Z(F, q, F) \quad Z_1 > 0 \quad Z_2 > 0 \quad Z_3 > 0. \quad (A2)$$

Substituting (A1) in the trade balance we get

$$\dot{F} = X(q, F) \quad X_1 > 0 \quad X_2 < 0. \quad (A3)$$

Substituting this expression in equation (A2) we get

$$\dot{q} = V(q, F) \quad V_1 > 0 \quad V_2 = 0. \quad (A4)$$

But we know from our discussion on the impact effect of fiscal policy under static expectations that the price of domestic output rises. Therefore the $\dot{q} = 0$ schedule must be downward sloping as in Figure 3.

$E_0$ is the initial long-run equilibrium. $E_1$ the new long-run equilibrium. The long-run value of $q$ does not change. The momentary equilibrium following the unanticipated fiscal expansion is at $E_{01}^S$ if agents have static expectations and at $E_{01}^R$ if they have perfect foresight.
Figure 3.
FOOTNOTES

1/ See, e.g., Dornbusch (1980), Chs. 10 and 11.

2/ E.g. Branson and Buiter (1983), Turnovsky (1981). Dornbusch and Fischer (1980) use national income. In our model the foreign asset is not interest-bearing and thus there is no interest income from abroad.

3/ On the issue of how wealth effects enter the expenditure equation, see Fischer (1981).

4/ In this appeal is usually made to the transversality conditions of optimising models, e.g. Brock (1974, 1975).

5/ A good example of this kind of result in a rational expectations model is the "undershooting" case in the Dornbusch (1976) model. There undershooting occurs when the rise in income caused by the depreciation, following an increase in money supply, raises the demand for money so much that the nominal interest rate has to rise to clear the money market.
CHAPTER IV

THE FOREIGN REAL INTEREST RATE AND THE REAL EXCHANGE RATE:

AN EMPIRICAL ANALYSIS
1. **Introduction**

In this chapter we look at the relationship between the foreign real rate of interest and the real exchange rate. We first derive a semi-reduced form expression for the real exchange rate from an extended Mundell-Fleming model when agents have rational expectations. The exchange rate equation is then estimated.

The Mundell-Fleming model has been extended to take account of secular inflation by a number of authors (see e.g., Buiter and Miller (1981), (1982), (1983) and Turnovsky (1981)). We extend this model further to take account of wealth effects in the demand for money and the consumption function. Through this channel the net claims on the rest of the world (and therefore the current account) influences the exchange rate. This study is an attempt to integrate two sources of (intrinsic) exchange rate dynamics - the first arising from sticky prices and the second due to asset accumulation. ¹

The model can be represented by three state variables - the real exchange rate, real money balances and the stock of foreign bonds. Given the sufficient conditions (which are well known in the literature), there are two stable roots and one unstable root. We can derive a semi-reduced form expression for the real exchange rate following the methods of Blanchard and Kahn (1980) and Buiter (1984) (see also Mussa (1982)). The real exchange rate, a non-predetermined or jump variable, depends on the current values of the two predetermined variables and the expected future time paths of all exogenous variables.

In this study, we limit ourselves to the study of one exogenous variable only - the foreign real interest rate. The analysis could be extended to the study of more exogenous variables if more monthly data was available.

The expression for the real exchange rate involves an infinite "lead" in the future path of the exogenous variable. We simplify
this expression by taking a Koyck transform. Two alternatives are investigated. The first, where the transform enables us to express the current real exchange rate in terms of current predetermined variables and lagged variables. The second method expresses the current value of the real exchange rate in terms of the current predetermined and exogenous variables and values of the state variables in the next period.

The model is then estimated, in turn, by ordinary least squares (OLS), non-linear least squares (NLS) and non-linear instrumental variables (NLIV). OLS ignores the non-linear restrictions on the parameters. NLS gives consistent estimates only when certain special conditions obtain.

The coefficients in the equations estimated turned out to be of the expected sign\(^2\) and "significant".

The results are encouraging especially since some previous efforts to implement empirical models of the exchange rates for the U.K. have yielded very poor results (see, e.g., Hacche and Townsend (1981)).

In section 2 we outline the model and then we solve for the real exchange rate. In section 3 we estimate the real exchange rate equation. Finally, section 4 offers some comments on the results and the drawbacks of the model.
2. THE MODEL

The model consists of the following equations:

\[ \frac{M}{P} = L(i^* + E_t(De/e), y, W) \]  \hspace{1cm} (1)

\[ L_1 \leq 0, \quad L_2 \geq 0, \quad 0 \leq L_3 \leq 1 \]

\[ \frac{qy}{P} = A(qy/P + i^*eF/P, i^* + E_t(De/e) - E_t(DP/P), W) + T(A, y^*, eP*/q). \]  \hspace{1cm} (2)

\[ 0 \leq A_1 \leq 1, \quad A_2 \leq 0, \quad A_3 \geq 0, \quad 0 \leq T_1 \leq 1, \quad T_2 \geq 0, \quad T_3 > 0 \]

\[ \frac{Dq}{q} = j(y - \bar{y}) + E_t(De/e) + \frac{DP^*/P^*}{1} \] \hspace{1cm} (3)

\[ j' > 0 \]

\[ \frac{eDF}{P} = T(A, y^*, eP*/q) + i^*eF/P \] \hspace{1cm} (4)

\[ W = \frac{(M + eF)}{P} \] \hspace{1cm} (5)

\[ P = P(q, eP^*) \] \hspace{1cm} (6)

\[ i^* = r^* + \frac{DP^*/P^*}{1} \] \hspace{1cm} (7)

where M is the nominal money supply, P the price index, i* the foreign nominal rate of interest, e the exchange rate (domestic currency price of foreign exchange), y (y*) the level of domestic (foreign) output, W, the real financial wealth, q, the domestic currency price of domestic output, F, the stock of foreign bonds, P*, the price of foreign output in foreign currency, \( \bar{y} \), the "natural" level of output, and r*, the real foreign interest rate. D denotes the time derivative (\( \Xi \frac{d}{dt} \)), and E is the expectations operator.

Equation (1) is the money market equilibrium condition. Real demand for money depends on the domestic nominal interest rate, domestic output and the level of real wealth. The domestic interest
rate is equal to foreign nominal interest rate plus the expected rate of depreciation of the domestic currency i.e., uncovered interest parity holds. Since we impose rational expectations, expected and actual rates of change of a variable are always equal except when new information arrives.

Equation (2) is the goods market equilibrium condition. Demand for domestic output from domestic consumers depends on real national income, the expected domestic real rate of interest, real wealth and the terms of trade. Exports depend on foreign output \( (y^*) \) and the terms of trade. Note, that all real variables are expressed in terms of the consumption basket i.e., deflated by the CPI.

Equation (3) is the expectations-augmented-Phillips' curve. The value of \( q \), which is predetermined at any instant, changes over time if the level of output is different from its natural rate and/or if the cost of the imported good is changing. This form can be derived from a mark-up pricing rate where money wage changes depend on the output gap and the change in the cost of living.\(^4\)

All bonds in the model are foreign currency, short bonds and domestic money is non-traded. Therefore foreign asset accumulation is equal to the current account surplus (equation (4)).

Real wealth is defined in equation (5) and is the real value of domestic money and the stock of foreign bonds. The domestic economy is assumed to be a net creditor country and so \( F \) is positive.\(^5\)

Equation (6) is the definition of the price index. \( P \) is homogeneous of degree one and increasing in its arguments.

Finally equation (7) defines the foreign nominal interest rate as the sum of the foreign real rate and the foreign rate of inflation. The latter is assumed to be contemporaneously observed.

Equations (1) to (7) represent an open economy IS-LM model with wealth effects and rational expectations. It can be reduced to a
system of three differential equations. Defining the state variables as \( c = e/q \), \( l = M/q \), and \( F \), and linearising the system we get

\[
\begin{bmatrix}
DC \\
Dl \\
DF \\
\end{bmatrix}
= \begin{bmatrix}
A \\
\end{bmatrix} \begin{bmatrix}
C \\
l \\
F \\
\end{bmatrix} + BZ
\]  

(8)

where \( Z \) represents a vector of exogenous variables.

If the sufficient conditions in Appendix A are met, then the matrix \( A \) has two stable roots and one unstable root. We can "associate" the real exchange rate with the unstable root and derive a solution for it in terms of the other two state variables and the expected future time path of the exogenous variables following the method outlined by Blanchard and Kahn (1980) and Buiter (1984). This is done in equation (9). The other two state variables are associated with stable roots and are backward looking.

\[
C(t) = -\frac{x_{31}}{x_{33}} l_t - \frac{x_{32}}{x_{33}} F_t - \frac{1}{x_{33}} \int_t^\infty \exp \lambda_3 (t-\tau) SE_t Z(\tau) d\tau.
\]  

(9)

where \( X = \begin{bmatrix}
x_{11} & x_{12} & x_{13} \\
x_{21} & x_{22} & x_{23} \\
x_{31} & x_{32} & x_{33} \\
\end{bmatrix} \) and \( E \) is the expectation operation, \( t \) being the date on which expectations are formed. The columns of \( X \) are the (left) eigen-vectors associated with the three roots \( \lambda_1 \), \( \lambda_2 \) and \( \lambda_3 \), \( \lambda_3 \) being the unstable root.

The vector \( S \) is defined in equation (10)

\[
S \equiv \left( \sum_{i=1}^3 x_{31}^i b_{11}, \sum_{i=1}^3 x_{31}^i b_{12}, \ldots, \sum_{i=1}^3 x_{31}^i b_{1k} \right)
\]  

(10)

where \( b_{ij} \) is the \((i,j)\)-th element of \( B \).
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balances and reduces absorption. \( \delta \) is the "discount factor" and must be less than unity because in deriving equation (11) (or (9)) we have eliminated the unstable roots.

Equation (11) is a forward-looking expression - an infinite "lead". There are three methods used to estimate such an equation. The first method uses lagged values of the exogenous variable to predict future values of \( r^* \). The second method involves using actual values of the future variables and instruments. The third method reduces the infinite forward looking expression to a more manageable one by using a Koych transform. If there are \( n \) exogenous variables then we have \( n \)-period ahead variables (rather than expressions from \( t \) to \( \infty \)). Alternatively, we could transform equation (11) into one in which lagged variables rather than future variables appear (see Abel (1981), Flavin (1981) and Hayashi (1982)).

In this study we have followed the third method. Since we have one exogenous variable \( (r^*) \) we can have either a one period lead (Method A) or a one period lag (Method B).

For the estimation, monthly data from December 1971 to January 1980 was used (see Appendix B for details). Two dummies were used - one for monetary targets (MMD) which was zero up to August 1976 and the other for exchange controls (ECD) which was zero up to August 1979.

**METHOD A**

Shift equation (11) forward one period to get:

\[
C_{t+1} = a_0 + a_1 F_{t+1} + a_2 L_{t+1} + a_3 \sum_{i=0}^{\infty} \delta^i E\left( r^*_{t+1+i} / I_{t+1} \right) \quad (12)
\]

Taking the expectation of equation (12) conditional on the information available at \( t \):
TABLE 1

A. \[ C_t = 0.0467 + 0.979C_{t+1} - 0.833F_t - 0.832F_{t+1} + 0.388\ell_t \]
(0.21) (70.13) (-14.45) (14.58) (4.22)
- \[ -0.396\ell_{t+1} + 0.00247r_t + 0.00043ECD - 0.1512MMD \]
(-3.99) (1.86) (0.04) (-2.86)

\[ \text{RSS} = 0.0127 \quad \text{SE of Regression} = 0.0120 \]
\[ R^2 = 0.997 \quad \tilde{R}^2 = 0.996 \quad F(8,87) = 3813 \]
\[ \text{Log likelihood} = 292.3 \quad DW = 2.10 \]

B. Parameter | Estimate NLS | Estimate NLIV |
--- | --- | --- |
constant | 0.0167 (1.240) | 0.0103 (0.48) |
\( \delta \) | 0.991 (198.97) (a) | 0.993 (144.42) (a) |
\( \alpha_1 \) | -0.837 (-14.69) (a) | -0.761 (-3.70) (a) |
\( \alpha_2 \) | 0.391 (4.29) (a) | 0.366 (2.39) (a) |
\( \alpha_3 \) | 0.0024 (3.53) (a) | 0.00225 (3.19) (a) |
ECD | 0.00546 (0.78) | 0.00631 (0.85) |
MMD | -0.133 (-2.75) (a) | -0.01 (1.16) |

\[ \text{RSS} = 0.0128 \quad \text{SE of Regression} = 0.0131 \]
\[ \text{Log Likelihood} = 291.7 \quad \text{Log Likelihood} = 25.87 \]

LM TEST

| 1st Order TR^2 | 2.35 | 6.07 |
| 12th Order TR^2 | 20.46 | 25.87 |

In all cases figures in parentheses are t-statistics. The number of observations was 96.

\[ \chi^2_1(.10) = 6.63 \quad \chi^2_{12}(.10) = 26.22 \]

Additional Instruments for NLIV, \( F_{t-1}, \ell_{t-1}, r^*_t, C_{t-1}, y^*_t, P^*_t, r^*_{t-1}, p^*_{t-1} \).

All variables other than \( r^* \), ECD and MMD are in logs.

(a) Significant at 95%
(b) Significant at 90%
The signs of all the parameters of interest are correct and all (except the constant and ECD) are significant (r* at 10%).

OLS estimates are inefficient because they ignore the non-linear restrictions on the parameters. To remedy the inefficiency, we re-estimated equation (14) by NLS using estimates derived from OLS as starting values of parameters. The results are reported in Table 1B.

All the parameter estimates are of the correct sign and are significant (at 5%). The value of \( \delta \) is high though less than unity.

A Lagrange Multiplier test for first and twelfth order serial correlation (or moving average) found these to be absent (the test procedure is set out in Appendix C).

OLS and NLS give biased and inconsistent estimates of the parameters because (i) \( C_{t+1} \) appears on the right hand side and (ii) the predetermined variables may not be free from realization errors. We therefore re-estimated equation (14) by NLIV, to get around this errors in variables problem using instruments for \( C_{t+1} \), \( F_{t+1} \), \( I_{t+1} \). The results are reported in Table 1B.

The estimates are of the correct sign and significant, though the value of \( \delta \) is again very high.

We again did LM tests for first and twelfth order moving average or serial correlation. These tests are fairly complicated to implement (see Appendix C for details) but no evidence of serial correlation was found (at 90%).

METHOD B

Equation (11) lagged one period is

\[
C_{t-1} = a_0 + a_1 F_{t-1} + a_2 I_{t-1} + a_3 \sum_{i=0}^{\infty} \delta^i E(r_{t+i-1}^*/I_{t-1})
\]  

(15)
Multiplying equation (15) by \(1/\delta\) and subtracting from equation (11) we get:

\[
C_t = \alpha_0 \delta^{-1} + \delta^{-1}C_{t-1} + \alpha_1F_t - \alpha_1\delta^{-1}F_{t-1} + \alpha_2\ell_t
\]

\[- \alpha_2\delta^{-1}\ell_{t-1} - \alpha_3\delta^{-1}r^*_{t-1}
\]

\[+ \alpha_3 \sum_{i=0}^{\infty} \delta^i[E(r^*_{t+i}/I_t) - E(r^*_{t+i}/I_{t-1})] + U_t \tag{16}
\]

The term in the square brackets in equation (16) represents revisions in expectations about the future time path of \(r^*\). This term is uncorrelated with all other terms on the right hand side of equation (16) (and can be treated as part of \(U_t\)).

The OLS estimates are presented in Table 2A. The estimate of \(\delta\) from the coefficient of \(C_{t-1}\) is greater than unity but if we use the coefficients of \(\ell_t\) and \(\ell_{t-1}\) then we can recover a value for \(\delta\) less than unity. All the coefficients other than on the dummies and the constant are significant.

Next we imposed the non-linear restrictions and re-estimated equation (16) by NLS. All the coefficients of interest are of the correct sign and significant. The value of \(\delta\) is also less than unity. No serial correlation of either first or twelfth order was detected.

NLS estimates are consistent if (i) \(F_t\) and \(\ell_t\) are contemporaneously observable and (ii) there was no error term in equation (11).

If there was an error term in (11) then we would get a first order moving average error process \(U_t = \varepsilon_t - \delta^{-1}\varepsilon_{t-1}\). This would be correlated with \(C_{t-1}, \ell_t\) and \(F_t\). We therefore re-estimated equation (16) using NLIV. Instruments were used for \(C_{t-1}, \ell_t\) and \(F_t\) and the results are reported in Table 2B.
TABLE 2

A. \[ C_t = -0.161 + 0.998c_{t-1} - 0.825r_{t-1} + 0.826f_{t-1} + 0.450e_{t} \]
\[ (-0.70) \quad (69.3) \quad (-14.04) \quad (13.84) \quad (4.55) \]
\[-0.421r_{t-1} - 0.031r^{*}_{t-1} - 0.009oe_{C} + 0.00966m_{MMD} \]
\[ (-4.47) \quad (-2.27) \quad (-0.10) \quad (1.77) \]

\[ \text{RSS} = 0.0134 \quad \text{SE of Regression} = 0.0124 \]
\[ R^2 = 0.997 \quad R^2 = 0.966 \quad F(8,87) = 3818.4 \]
\[ \text{Log likelihood} = 289.7 \quad \text{DW} = 2.09 \]

B. Parameter Estimate NLS Estimate NLIV
constant -0.0061 (-0.51) \(-0.0124 (-0.56) \)
\[ \frac{1}{\delta} \] 1.004 (186.78) \(a\) 1.005 (159.43) \(a\)
\[ \alpha_1 \] -0.824 (-14.22) \(a\) -0.907 (-3.80) \(a\)
\[ \alpha_2 \] 0.429 (4.63) \(a\) 0.415 (2.32) \(a\)
\[ \alpha_3 \] 0.00232 (3.55) \(a\) 0.00236 (3.27) \(a\)
ECD -0.00296 (-0.45) \(-0.00294 (-0.43) \)
MMD 0.00889 (1.75) \(b\) 0.0119 (1.28)

\[ \text{RSS} = 0.0135 \quad \text{SE of Regression} = 0.0123 \quad 0.0138 \]
\[ \text{Log Likelihood} = 289.47 \quad 0.0124 \quad - \]

LM TEST

1st Order TR$^2$ 1.94 2.02
12th Order TR$^2$ 17.51 13.87

In all cases figures in parentheses are t-statistics. The number of observations for OLS and NLS estimation was 96 and for NLIV it was 95.

\[ \chi^2_{1(0.05)} = 3.84 \quad \chi^2_{12(0.05)} = 21.03 \]

Additional Instruments for NLIV, $c_{t-2}$, $c_{t-3}$, $f_{t-2}$, $f_{t-2}$.

All variables other than \( r^* \), ECD and MMD are in logs.

(a) Significant at 95%
(b) Significant at 90%
NLIV results are not very different from the NLS results. All coefficients of interest are of the correct sign and significant. The value of δ is less than unity though it is high.

We tested for the presence of moving average or serial correlation of the first and twelfth order by an LM test. These were found to be absent (at 95%).

To sum up the econometric results, the estimates were found to be similar across all estimation techniques. The values of $\alpha_1$, $\alpha_2$ and $\alpha_3$ are all of the correct sign and significant. The value of δ was found to be less than unity though very high.
4. CONCLUSIONS

We first set up a theoretical model and obtained a semi-reduced form expression for the real exchange rate in terms of the other state variables and the expected time path of the foreign real interest rate from now to infinity, conditional on currently available information.

The empirical estimates were found to be of the right sign and significant. The results suggest significant forward-looking behaviour on the part of the agents.

The drawbacks of the analysis are fairly obvious. We have looked at only one exogenous variable, though this was, in part, due to data considerations.

For the empirical analysis, we obtained the value of £ by dividing nominal balance (EM3) by the price index (CPI).\textsuperscript{8} So strictly speaking it was not a predetermined variable.

In spite of these shortcomings the results are on the whole encouraging.
The determinant of matrix $A$ in equation (8) is

$$\Delta = \frac{1}{\Delta T_1}[ (y^h T_1 + T_2 P^*) ( - T_1 A_3 h g - (1 + T_1 A_1) i^* g h (1 - L_3)) + (1 - L_3) h (1 + T_1) i^* g F]$$  \hfill (A.1)

where $\Delta = g L_1 [(1 - A_1 (1 + T_1)) h/j' + A_2 (1 + T_1) a]$ \hfill (A.2)

and $g \equiv e/P, \ h \equiv q/P$ and $\alpha$ is the share of the domestic good in the price index (i.e. $\alpha \equiv P_1 q/P$).

The trace of $A$ is equal to

$$\frac{1}{\Delta} [ g L_1 (y (1 - A_1 (1 + T_1)) h' - A_1 (1 + T_1) i^* F g' - A_3 (1 + T_1) m h' - A_3 (1 + T_1) F g' - T_2 P^*)$$

$$+ g ((L_1 - L_2/j') (- A_3 (1 + T_1) h) - (1 - L_3) h ((1 - A_1 (1 + T_1)) h/j' + A_2 a (1 + T_1))$$

$$+ L_1 h/j' (y (1 - A_1) i^* g + T_1 A_3 g + A_2 a (1 + T_1) i^* g))]$$  \hfill (A.3)

For $A$ to have two stable roots (negative real parts) and one unstable root the following two conditions taken together are sufficient:

(i) Determinant of $A$ be positive

(ii) Trace of $A$ be negative.

The following three sufficient conditions ensure that these requirements are met:

(a) $(1 - A_1 (1 + T_1)) h/j' + A_2 (1 + T_1) a < 0$

(b) $|T_1 A_3 g F| > (1 - A_1) i^* g F$

(c) $L_3 \neq 1$. 
Condition (a) states that an increase in output \( y \) should create excess supply in the goods market. Supply increases by \( h y \) in real terms and demand by \( A_1(l + T_1)hy + A_2(l + T_1)\alpha j'y \). \( A_1(l + T_1)hy \) is the increase (in real terms) in demand for domestic output out of increased output. But an increase in output also increases the rate of inflation (for a given \( De/e \)) and thereby lowers the real rate of interest by \( \alpha j'y \). This, in turn, increases expenditure on domestic goods and services by \( A_2(l + T_1)\alpha j'y \).

Condition (b) requires the interest service account to be "small". It is somewhat stronger than the condition usually encountered in the literature, namely that an increase in foreign assets should worsen the current account.

Finally, condition (c) requires wealth effects in the demand for money to be high. A higher value of \( m \) requires, *ceteris paribus*, a high \( Dm \) to clear the money market. If \( L_3 \) is high then the extent of excess supply due to increased real balances is correspondingly lower and the tendency towards instability is also lower. In the fixed exchange rate literature, however, a high value of \( L_3 \) was seen to be "destabilising" (see e.g. Kingston and Turnovsky [1978]).
1. The nominal exchange rate data is the trade-weighted effective exchange rate (EER) published in the Bank of England Quarterly Bulletin (BEQB). It is defined as the foreign currency price of domestic currency. In our analysis, the exchange rate is defined as the domestic currency price of foreign exchange.

2. The foreign nominal interest rate is a weighted average of short term interest rates of the UK's major trading partners. The weights are MERM weights. The data is a monthly average and is taken from BEQB.

3. The nominal money stock figures refer to EM3. These were taken from International Financial Statistics (IFS) published by the IMF.

4. The foreign real income and price level figures are also from the IFS. MERM weights were used to construct the series.

5. The domestic price level (retail price index with Jan 1974 = 100) series was taken from the Monthly Digest of Statistics (MDS) published by the CSO.

6. The net claims on the rest of the world series was constructed by adding (subtracting) current account surpluses (deficits) from the net foreign asset position of the UK economy. The current account figures are from MDS. The net foreign asset position is published in the June issue of the BEQB (and is subject to major revisions).
APPENDIX C

Lagrange Multiplier (LM) Test for Non-Linear Least Squares (NLS)

We set out the procedure for Model B, the test for Model A is analogous.

The estimated equation was:

\[ C_t = \beta + \gamma C_{t-1} + \alpha_1 F_t + \alpha_2 L_t + \alpha_3 \gamma L^*_{t-1} + \alpha_4 ECD + \alpha_5 \text{MMD} + U_t \]

where \( \gamma = 1/\delta \)

or more compactly

\[ (1 - \gamma L)C_t - \alpha_1 (1 - \gamma L)F_t - \alpha_2 (1 - \gamma L)L_t + \alpha_3 \gamma L^*_{t-1} - \beta - \alpha_4 ECD - \alpha_5 \text{MMD} = U_t \]

where \( L X_t = X_{t-1} \).

To test for serial correlation (or moving average - the LM test does not distinguish between them) of order \( n \), write

\[ (1 - \gamma L)(1 - \sum_{i=1}^{n} \rho_i L^i)C_t - \alpha_1 (1 - \gamma L)(1 - \sum_{i=1}^{n} \rho_i L^i)F_t - \alpha_2 (1 - \gamma L)(1 - \sum_{i=1}^{n} \rho_i L^i)L_t \]

\[ + \alpha_3 \gamma L(1 - \sum_{i=1}^{n} \rho_i L^i) - \alpha_4 \gamma L(1 - \sum_{i=1}^{n} \rho_i L^i)ECD - \alpha_5 \gamma L(1 - \sum_{i=1}^{n} \rho_i L^i)\text{MMD} - \beta(1 - \sum_{i=1}^{n} \rho_i) = \epsilon_t \]

where \( \epsilon_t \) is white noise.
Now take the derivatives of $\epsilon_t$ with respect to all the parameters $(\beta, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \gamma$ and $\rho_1$) successfully and evaluate at the estimated values $\hat{\beta}$ etc and $\rho_1 = 0 \forall i$). This gives us the set of regressors

$$(1 - \hat{\gamma}L)F_t, (1 - \hat{\gamma}L)\ell_t, -\hat{\gamma}Lr_t^*, L(C_t - \hat{\alpha}_1 F_t - \hat{\alpha}_2 \ell_t - \hat{\alpha}_3 r_t^*),$$

ECD, MMD and $\hat{\Upsilon}_{t-1}$ $i = 1, \ldots, n$. \quad (C.2)

Finally, we regress $\hat{\Upsilon}_t$ on the set of regressors above. Then

\[
\text{LM Test for Non-Linear Instrumental Variables (NLIV) estimation}
\]

For the reasons indicated in the text, $C_{t-1}, F_t$ and $\ell_t$ would be correlated with the error term and thus give rise to inconsistent estimates. So we instrumented these variables ($C_{t-2}, C_{t-3}, F_{t-2}, \ell_{t-2}$ were the additional instruments used).

To perform the LM test, first, we need to regress all the instrumented variables (i.e. $C_{t-1}, F_t$ and $\ell_t$) on all the instruments and exogenous variables ($F_{t-1}, \ell_{t-1}, r_t^*, ECD, MMD, F_{t-2}, \ell_{t-2}, C_{t-2}$ and $C_{t-3}$). Call the fitted values $\hat{C}_{t-1}, \hat{F}_t$ and $\hat{\ell}_t$.

Next, regress the residuals from NLIV $\hat{U}_{t-1}$ $i = 1, \ldots, n$ on all the exogenous variables and the additional instruments. Call the fitted
values $\hat{U}_{t-1} \ i = 1, \ldots, n$.

Finally, replace the values of $C_{t-1} , F_t , \hat{U}_{t-1} \ i = 1, \ldots, n$ in (C.2) by the fitted values and regress $\hat{U}_t$ on the set of regressors thus generated.

Again $T_n^2 \ Asy \ \chi^2(n)$. 
1. Since the Mundell-Fleming model incorporates uncovered interest parity, analysing wealth accumulation (ignoring real capital) would involve looking at the current account deficit in the balance of payments and the budget deficit of the government. Since there is no reliable monthly data on government expenditure, taxes, etc., we have ignored the government budget constraint and, indeed, government bonds in what follows.

2. The expected sign of a parameter refers, in this context, to the sign associated with the dynamic adjustment paths (the stable arms) of sticky price and asset accumulation models respectively.

3. Their study, which is the most comprehensive with UK data, comes to the following conclusion:

"... We have not succeeded in finding any empirical regularities in the data to help explain in any satisfactory way the fundamental determinants of sterling's effective exchange rate during the floating period." (p.243)

The performance of exchange rate models for other countries in predicting out-of-sample is very poor (see Meese and Rogoff (1983)). Frankel (1983) contains an excellent survey of the empirical implementation of exchange rate models. Hooper and Morton (1982) discuss some of the issues related to exchange market intervention in the context of a two-country model similar to ours. Their reduced form expression is very different from ours.

4. Write the wage-Phillips curve as:

\[ \frac{DV}{V} = k(y-y) + E_t DP/P \]
Now if the price of domestic output is a constant mark-up over wage costs then (a) becomes:

\[ \frac{Dq}{q} = k(y - \bar{y}) + E_tDP/P. \]  

Equation (3) follows from (b) if we substitute the definition of \( P \) (equation (6)), where \( j = k/(i-\alpha) \) where \( \alpha = \frac{dp}{dq} \cdot \frac{q}{P}. \)

Note that in equation (3), \( q \) is a predetermined variable, even though \( e \) is a jump variable.

5. As mentioned in footnote (1) we have ignored domestic currency government bonds. This model would be consistent with the view that domestic bonds do not constitute net wealth. If government expenditure is constant and a constant growth rate of money is maintained with residual bond financing, we would get a model identical to the one above.

6. Abel and Mishkin (1983) contains a good discussion of these models.


8. \( \text{£M3} \) is an inappropriate monetary aggregate from a theoretical viewpoint for this study, where the own return on money is zero. We tried the same regressions with M1 and the results were not dissimilar to the ones reported.


_________ "Real Exchange Rate Overshooting and the Output Cost of Bringing Down Inflation", European Economic Review, 18 (1982), 85-123.


Centre for Banking and Finance, The City University, Annual Monetary Review, 2 (1980), 11-12.


Open Economy Macroeconomics, Basic Books (New York, 1980).


